Theoretical analysis and mitigation of inter-carrier interference in orthogonal frequency division multiplexing system

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Theoretical Analysis and Mitigation of Inter-Carrier Interference in Orthogonal Frequency Division Multiplexing System

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctorate of Philosophy

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Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research done by me and has not been submitted for a higher degree to any other University or Institute.

9/1/2006
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Date

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Ng Wee Teck

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Abstract

One of the main problems of high data rate transmission over wireless channel is the delay spread. Orthogonal Frequency Division Multiplexing (OFDM) system, which is robust against multipath propagation, is often employed for such hostile wireless environments. However, due to fast time varying (Doppler spread) channels, the performance of OFDM system degrades considerably. It is found that by using directional antenna or beamforming, the amount of Doppler spread can be reduced. To investigate the effect, Doppler Power Spectral Density (PSD) equations are derived where directional antennas are employed at the mobile or/and the base station. The Doppler PSDs are derived for two different Doppler spread channel models, the two dimensional (2-D) and three dimensional (3-D) models. The 2-D model considers only the azimuthal plane, whereas the 3-D model takes into account the effect of both azimuthal and elevation planes. The 2-D model is derived from the popular classical Clarke/Jakes' model, whereas the 3-D model is extended from the uniform Doppler spread model.

The Doppler PSDs for the 2-D model with directional antennas employed at the receivers are derived and generalized into two cases. The first case happens when the mobile is moving within the range specified by the beamwidth of the directional antenna mounted at the mobile. The second case occurs when the mobile is moving further away, out of the range specified by the first case. The Doppler PSDs are also derived for systems employing directional antennas at both the transmitter and receiver. For

the case of the 3-D model, the Doppler PSD is obtained when a directional antenna is employed at the receiver.

Next, the amount of Inter-carrier Interference (ICI) in an OFDM system due to Doppler spread is characterized. The ICI is analyzed when the Angle-of-Arrival (AOA) of the scattering components are not of conventional form. The general form for the ICI power is derived by taking into consideration the Probability Density Function (PDF) of the AOA, including both azimuthal and/or elevation planes, depending on the channel model used. From the general equation obtained for the 2-D channel model, it is found that the ICI power at the receiver will change marginally when a directional antenna is employed at the base station. However, the ICI power can be reduced substantially when directional antenna is employed at the mobile, together with frequency compensation. The ICI power expression is also derived for the 3-D model when the directional antenna is employed at the receiver. Furthermore, it is found from the 3-D model that the elevation angle can reduce the ICI power irrespective of the beam pattern in the azimuthal plane.

The generalized ICI power of OFDM system is extended to include multiple Rician channel. It is shown that in a multiple Rician channel, the reduction in ICI power is not as much as in the case of Rayleigh channel. However, in the presence of only one Rician path, the reduction in ICI power is marginally better than the Rayleigh channel. Furthermore, multiple beamforming using Uniform Linear Array (ULA) is used for OFDM systems to reduce the Doppler spread and exploit the advantage of diversity. The simulation results show that angular diversity in an OFDM system can provide significant improvement as compared to using only single beamforming. It shows that with just two antenna elements, the Bit Error Rate (BER) can be improved significantly.

The use of directional antennas cannot remove the ICI in some situations. Coding is used to further reduce the ICI power. The general expressions for the ICI power, for the various ICI reduction codes available in literature, are shown for general channels. The current ICI reduction coding schemes have either limited ICI reduction capability or require bandwidth expansion (non-full rate codes). A novel coding technique, which can mitigate some of these disadvantages, is also proposed in the thesis, with its ICI power expression characterized. The upper and lower bounds for the proposed code and some of the other ICI reduction codes (found in literature) are also derived. Using the bounds, the absolute reduction in the amount of ICI power, for each of the various coding schemes, are obtained.

Another technique of mitigating ICI is to employ a filter/equalizer at the receiver. Nevertheless, the knowledge of the fast time varying channel response is required in designing filters/equalizers. A simple channel estimation algorithm for fast time varying channel is proposed. The algorithm is based on the polyphase sequence and is employed in the frequency domain. It can be applied to Multiple Input, Multiple Output (MIMO) OFDM systems as well. The proposed algorithm is compared with other channel estimation algorithms and is found to have a better performance for high Doppler spread channels.

List of Abbreviations and Symbols

Abbreviations

2-D	two dimension
3-D	three dimension
A/D	analog-to-digital converter
ADSL	asymmetric digital subscriber line
AOA	angle-of-arrival
AP	azimuthal plane
AWGN	additive white Gaussian noise
BER	bit error rate
BPSK	binary phase shift keying
CDMA	code division multiplex access
CFO	carrier frequency offset
CP	cyclic prefix
D/A	digital-to-analog converter
DAB	digital audio broadcasting
DFT	discrete Fourier transform
DSP	digital signal processing
DVB-T	digital video broadcasting-terrestrial

EGC	equal gain combining
EP	elevation plane
FDMA	frequency division multiplex access
FFT	fast Fourier transform
ICI	inter-carrier interference
IDFT	inverses discrete Fourier transform
IFFT	inverses fast Fourier transform
ISI	inter-symbol interference
KD	Kronecker-delta
LOS	line-of-sight
LS	least square
MBOA	multiband OFDM alliance
MC-CDMA	multicarrier code division multiplexing access
MCM	multicarrier modulation
MIMO	multiple input, multiple output
MMSE	minimum mean square error
MRC	maximum ratio combining
MSE	mean square error
NMSE	normalized mean square error
OFDM	orthogonal frequency division multiplexing
PAPR	peak-to-average power ratio
PCC	polynomial cancellation codes
PDF	probability density function
PIC	parallel interference cancellation
PSD	power spectral density
PSK	phase shift keying
QAM	quadrature amplitude modulation

QPSK	quadrature phase shift keying
SFN	single frequency network
SIC	successive interference cancellation
SIR	signal to interference ratio
SINR	signal to interference plus noise ratio
SNR	signal to noise ratio
TDMA	time division multiplex access
ULA	uniform linear array
WLAN	wireless local area network
WMAN	wireless metropolitan area network
WPAN	wireless personal area network
WSS	wide sense stationary
ZP	zero padding

Symbols

$A(\theta)$	array pattern
B_{coh}	coherence bandwidth
BW	bandwidth of the system
$\mathbf{C}$	set of time domain index for the cyclic prefix
$d_{n,m}$	information symbol
$E[\cdot]$	expectation operator
f_0	carrier frequency offset
f_d	maximum Doppler frequency
f_k	frequency of the k th subcarrier
${}_2F_1(\cdot, \cdot; \cdot, \cdot)$	Hypergeometric function
$\mathbf{G}$	frequency domain channel matrix

$G(\theta, \beta)$	beam pattern of a directional antenna
$g_{k,m}$	frequency domain channel impulse responses
$\mathbf{h}$	time domain channel impulse response in vector
$h(t, \tau)$	time domain channel impulse responses
$J_0(\cdot)$	zeroth-order Bessel function of the first kind
$J_{CE}(b, c)$	cost function of the channel estimator
$J_{vc}(d, f_0 T_s)$	cost function for virtual carrier frequency and timing estimator
K	Rician K factor
L	number of paths
$M(s)$	cost function for Cox's timing synchronization algorithm
N_c	number of subcarriers
N_r	number of Rician LOS path
N_{trx}	number of transmit antennas
$\mathcal{O}(\cdot)$	complexity level
P_{ICI}	ICI power
$\mathbf{P}_i$	set of index for subcarriers with pilot signal
$p(\cdot)$	PDF for AOA
R	baud rate, in term of symbols per second
$r(f_d T_s x)$	time domain autocorrelation function of the channel
s	estimated timing for the beginning of the OFDM symbol
$S(\cdot)$	PSD
$s(t)$	transmitted signal
T_{coh}	coherence time
T_{CP}	duration of the CP
T_s	symbol duration
$tr(\cdot)$	trace operator
$\mathbf{V}$	set of index for virtual carrier

$w_{b,c}$	filter weight
$x(t)$	received signal
$y_{k,m}$	demodulated OFDM symbol
z	amount of CFO in multiple of intercarrier spacing
β	elevation angle
ε	vehicle travelling direction
ζ	frequency compensation parameter
ϕ_n	modulating waveform for the n -th subcarrier
$\Phi(\cdot)$	random variable caused by phase noise
$\omega(t)$	AWGN in time domain
$\omega_{k,m}$	AWGN in frequency domain
η	fine CFO within the length of the intercarrier spacing
σ_s^2	transmitted information symbol energy
σ_n^2	noise variance
θ	azimuthal angle

Contents

Acknowledgement	i
Abstract	ii
List of Abbreviations and Symbols	v
List of Figures	xvi
List of Tables	xx
1 Introduction	1
1.1 Motivation	3
1.2 Organization of the Thesis	4
1.3 Main Contributions	6
2 Basics of Orthogonal Frequency Division Multiplexing	8
2.1 MCM and OFDM Basics	10

2.1.1	Cyclic Extension	12
2.1.2	Zero Padding to Replace CP	14
2.2	Synchronization in OFDM	16
2.2.1	Synchronization based on training symbols	16
2.2.2	Synchronization based on Virtual Carriers	18
2.2.3	Synchronization Based on Cyclic Extension	19
2.3	Channel Estimation	20
2.3.1	Two Dimensional Channel Estimation	21
2.3.2	One Dimensional Channel Estimation	23
2.4	High Peak-to-Average Power Ratio	24
2.5	Summary	24
3	OFDM in Time Varying Channel	25
3.1	Various Time Varying Channel Models	27
3.1.1	Uniform Doppler Spread Model	28
3.1.2	Doppler Shift Model	28
3.1.3	2-rays Doppler Spread Model	29
3.1.4	Other Channels	29
3.2	System Impairments Which Results in ICI	30

3.2.1	Oscillator Phase Noise as a Channel	30
3.3	Quantifying ICI in OFDM	32
3.4	Channel Estimation for Time Varying Channel	35
3.4.1	Kronecker-Delta Scheme	36
3.4.2	Stamoulis' Method	37
3.5	Equalization of OFDM Signal in Time-Varying Channel	39
3.5.1	Conventional Methods	40
3.5.2	Frequency Domain Sub-Optimal method	41
3.5.3	Frequency Domain Successive Interference Cancellation	43
3.5.4	Time Domain Linear Filter	44
3.6	Summary	46
4	Doppler Power Spectral Density Using Directional Antennas	48
4.1	Doppler Power Spectral Density When Using Directional Antennas for 2-D Model	49
4.1.1	Doppler PSD Function for Directional Antenna at Mobile	49
4.1.2	Doppler PSDs When Both Directional Antennas are Employed	53
4.2	Doppler Power Spectral Density When Using Directional Antennas for 3-D Model	60

4.2.1	Doppler PSD Function for Directional Antenna at Mobile for 3-D Model	61
4.3	Summary	63
5	ICI Reduction for OFDM System using Directional Antennas	66
5.1	Computing ICI Power in terms of PDF of AOA	67
5.1.1	In Rayleigh Environment	67
5.1.2	In Rician Environment	69
5.1.3	3-D Channel Environment	72
5.2	Numerical results of the ICI Power When Using Directional Antenna in the 2-D Rayleigh Environment	73
5.2.1	ICI Power When Using Directional Antenna at the Base Station in Rayleigh Environment	73
5.2.2	ICI Power When Using Directional Antennas at Both Base and Mobile Stations in Rayleigh Environment	74
5.3	Numerical Results of the ICI Power When Using Directional Antennas at Mobile Station in 2-D Rician Environment	78
5.4	Numerical results of the ICI Power When Using Directional Antenna in the 3-D Rayleigh Environment	81
5.5	Application of Angular Diversity using Linear Uniform Array in Rayleigh Environment	84

5.5.1	Diversity Combining	85
5.5.2	Equal Doppler Spread and Frequency Compensating Parameters Computation	87
5.5.3	Computation of Steering Angle and Antenna Elements Needed .	89
5.5.4	Simulation Results	91
5.6	Summary	92
6	Analysis of ICI Reduction Coding Techniques in OFDM Systems for Time-Varying Channel	95
6.1	OFDM with ICI with Correlation Among Subcarriers	97
6.1.1	Background	97
6.1.2	Coding for ICI Reduction	98
6.2	Performance Analysis and Comparison	100
6.2.1	Exact Calculation	100
6.2.2	Performance Bounds	108
6.3	Comparison of Results	112
6.4	Summary	118
7	Polyphase Sequence Based Pilot Channel Estimation for OFDM in Doubly Fading Channel	120

7.1	Channel Estimation Technique and Sequence	121
7.1.1	Channel Estimation Technique	121
7.2	Extension to MIMO OFDM	124
7.2.1	Sequence Properties and Construction	126
7.3	Computation of NMSE	128
7.4	Simulation Results	130
7.5	Summary	134
8	Conclusions and Future Works	135
8.1	Conclusions	135
8.2	Suggestions on Future Research	139
	Author's Publications	143
	Bibliography	145

List of Figures

2.1	Illustration of Cyclic Prefix	13
2.2	OFDM System Model	15
2.3	An example of pilot placement DVB-T system	21
3.1	PSD of the Oscillator Phase Noise	31
3.2	Numerical and Calculated ICI Comparison	34
3.3	Kronecker-Delta scheme’s placement of training sequences before and after an OFDM symbol	36
3.4	Example of the placement of frequency domain pilot sequences in an OFDM symbol	39
3.5	Extraction of Sub-Matrix from the General Frequency Domain Matrix .	42
4.1	Illustration of geometry of mobile unit	51
4.2	Doppler PSD with Directional Antenna at Mobile	54

4.3	Illustration of the AOA at the mobile when a directional antenna is implemented at Base Station	55
4.4	PDF of AOA and Doppler PSD with directional antenna at base station	56
4.5	Doppler PSD with Directional Antenna at Both Base and Mobile Stations	59
4.6	Illustration of the 3-D geometry of the AOA of the receiving signal . .	61
4.7	Doppler PSD with Directional Antenna for 3-D model	64
5.1	P^{ICI} when Directional Antenna is employed at the Base Station. Beamwidth of Directional Antenna, $2\beta_b = \pi/18$ and $R=1\text{km}$, $D=3\text{km}$	74
5.2	P^{ICI} when Directional Antenna of Beamwidth $2\theta_m = \pi/6$ is deployed at the Mobile	76
5.3	P^{ICI} when Directional Antennas are implemented at the Both Base and Mobile Stations, with $\varepsilon = 0$, Beamwidth of Directional Antennas, $2\beta_b = \pi/18$ and $2\theta_m = \pi/6$ and $R = 1\text{km}$, $D = 3\text{km}$	77
5.4	P^{ICI} of Scenario 1	79
5.5	P^{ICI} of Scenario 2	80
5.6	P^{ICI} of Scenario 3	80
5.7	P^{ICI} of of the 3-D model with vehicle moving direction, $\epsilon = 0$	82
5.8	P^{ICI} of of the 3-D model with vehicle moving direction, $\varepsilon = \pi/2$, with 2 different directional antennas	83
5.9	Setup of the Angular Diversity OFDM system	85

5.10 Geometry showing the relationship between θ , ψ and the antenna separation distance when ULA based smart antenna is used 86

5.11 Array Pattern with respect to the beam interval where Doppler Spreads are equal for all diversity beams, with different curve representing different beams 91

5.12 Comparing BERs With Different Number of Antenna Elements 93

6.1 Results of Various Coding in CFO 113

6.2 Results of Various Coding in Clarke/Jakes' Doppler Spread Channel Model 114

6.3 Results of Various Coding in Uniform Doppler Spread Channel Model . 115

6.4 Results of Various Coding in 2-Rays Channel Model 116

6.5 Simulation BER of Various Coding Scheme with maximum Doppler frequency of $f_d T_s = 0.1$ for Clarke/Jakes' Doppler Spread Model 116

6.6 Simulation BER of Various Coding Scheme with maximum Doppler frequency of $f_d T_s = 0.1$ for Uniform Doppler Spread Model 117

6.7 Simulation BER of Various Coding Scheme with maximum CFO frequency of $f_d T_s = 0.1$ for CFO Model 117

7.1 Frequency Domain Channel Impulse Response for different normalized Doppler spread, $f_d T_s$, using Clarke/Jakes' Model 123

7.2 Frequency Domain Channel Impulse Response for different g for $f_d T_s = 0.5$, using Clarke/Jakes' Model 126

7.3 Numerical results showing the NMSE (with $\sigma_n^2 = 0$, $\lambda^2 = 1$ and $N_{trx} = 1$)
for different $f_d T_s$ using Clarke/Jakes' Model 130

7.4 Comparison of Different Channel Estimation Algorithms with SNR=30dB132

7.5 Comparing Theoretical and Simulated MSE for 4 transmit antennas . . 133

List of Tables

5.1	Scenario 1: Equal power (K-factor) for Different Rician Paths	78
5.2	Scenario 2: Different power (K-factor) for Different Rician Paths	79
5.3	Beam Intervals Position for Equal Doppler Spread (in rad)	88
5.4	Frequency Compensation Parameters, ζ_n , for Different Number of Beams Required	89
5.5	Steering Angles of the ULA (in rad)	90
6.1	γ_1 and γ_2 for different time varying channels	109
6.2	Comparing Different ICI Reduction Coding Schemes	118

Chapter 1

Introduction

The introduction of high-bandwidth applications, such as streaming of video and high speed internet connection, has created huge demands for high data transmission to individual user. On top of these requirements, the demand for high mobility, often in a very large area, has steered research effort into high data rate wireless communications.

The use of high data rate wireless technologies is not limited to outdoor environments. In indoor offices, using wireless technologies to reconfigure network connection has been shown to be simpler in implementation and relatively cheaper to install, as compared to using wired connection. This is particularly true for older buildings which are not designed to have network connection.

However, the wireless propagation environment is not as friendly as the wired channel. Signal can experience rapid fluctuations, in both time and frequency, caused by both RF hardware impairments and mobile environments. Furthermore, fast time varying multipath environment can distort the transmitted signals, making the demodulation and decoding computationally intensive and complicated. Therefore, a wireless communication system has to be designed (with limited computational cost) to be ro-

CHAPTER 1. INTRODUCTION

bust against the effects of RF hardware impairments, high path losses, fast time varying environments and large multipath delay spread (which can cause Intersymbol Interference (ISI)). Even for a small amount of delay spread of $5.54 \mu\text{s}$ (as in COST 207 [1]), the high speed transmission will cause the number of ISI symbols to be as high as a hundred, when the bit rate goes up to a few hundreds Mbps [2]. Thus, there is an urgent need to compensate for the ISI in a high speed wireless system. For conventional single carrier systems, an adaptive filter of at least a few hundred taps (implying high computational complexity) is required to provide satisfactory performance.

Multicarrier modulation (MCM) has provided a simpler way to avoid the enormous amount of computational load using the procedure of divide and conquer. It divides the high data rate system into a few similar low data rate streams and transmit them in parallel. It separates the parallel low data rate systems by using different orthogonal frequencies. Thus, at the receiver, data at different frequencies can be differentiated. There is an obvious advantage of low data rate system. As mentioned earlier, the effect of multipath delay is comparatively less for a low data rate wireless system. Orthogonal Frequency Division Multiplexing (OFDM) has been cited as a highly efficient way of implementing MCM. It has much better spectral efficiency as compared to traditional MCM methods, where there is a guard band between adjacent parallel low data rate systems.

However, OFDM suffers more from the effect of fast fluctuations in the channel. It results in Inter-carrier Interference (ICI) between these parallel channels [3],[4],[5],[6],[7], which is the main issue that this thesis is tackling. This fast fluctuation is normally caused by the mobile environment when either the mobile station or its environment is in motion. The fast time varying effect is normally modelled as the Doppler spread channel. Through the technique of using directional beam pattern (or limiting the Angle-of-Arrival (AOA)), the speed of fluctuation can be reduced. Thus, the amount

CHAPTER 1. INTRODUCTION

of Doppler spread is reduced. The reduced ICI power of the OFDM system can be found by analyzing the amount of Doppler spread reduction. Nevertheless, directive beam pattern fails to reduce ICI in some situations. Therefore, ICI reduction coding is introduced to eliminate this limitation. ICI reduction codes have been shown, in this thesis, to mitigate ICI irrespective of the channel conditions. Other means to mitigate ICI include filtering or equalizing, but provided that the fast time varying channel response is known. In the thesis, a simple channel estimation algorithm to obtain the instantaneous fast time varying channel impulse response is proposed.

1.1 Motivation

OFDM system is still operationally limited to static or slow varying environments, mainly due to the ICI effects. For example, OFDM is implemented in several standards, including Wireless Metropolitan/Local/Personal Area Network (IEEE 802.11a/g [8],[9],[10], possible IEEE 802.15.3a [11], IEEE 802.16 [12]) and Digital Video and Audio Broadcasting (DVB and DAB) [13],[14]. It can be seen that all these applications are targeted at slow time varying channel (IEEE Wireless MAN/LAN/PAN) or static channel like DVB and DAB. On the other hand, for systems like cellular or low orbit satellite, OFDM is not adopted, mainly because of ICI. Nevertheless, if ICI effects in OFDM system can be mitigated successfully, systems deployed in fast time varying environment can employ OFDM to communicate at a higher data rate. In addition, as mentioned before, OFDM systems can be implemented with reduced complexity, as compared to single carrier systems, in channels with large delay spread.

1.2 Organization of the Thesis

In Chapter 2, a brief introduction to OFDM, its characteristics and applications is given. The history of the OFDM technique and its motivation are described in detail. The choice for the number of subcarriers is examined, as well as considerations for the length of the guard interval. The various strategies of implementing the symbol to symbol guard interval and its usages are also covered in the chapter. Some of the common techniques for channel estimation and timing/frequency synchronization are also discussed.

In Chapter 3, the effects on OFDM systems from the loss of orthogonality are presented. The ICI power is used as a measure of the amount of interference. The various fast time varying channel models, like the classic Clarke/Jakes' Doppler spread model, uniform Doppler spread model, Doppler shift model and 2-rays Doppler spread model are also described. As mentioned earlier, system impairments will also cause interference. Thus, an example of system impairment, phase noise, which can be modelled as a time-varying channel is also shown. The popular instantaneous channel impulse response estimators for fast time varying channels are further described. Finally, the various channel equalizers for fast time varying channel are examined.

In Chapter 4, we focus on the calculation of the Doppler Power Spectral Density (PSD) when using directional antennas, at the base and/or mobile stations. The PSDs for the two dimensional (2-D) channel when directional antennas are employed at the transmitter or at the receiver, or at both transmitter and receiver are shown. Furthermore, a three-dimensional (3-D) Doppler spread model, which takes into account both azimuthal and elevation angles, is shown. The PSD for the 3-D channel model is displayed when directional antenna is employed at the receiver.

CHAPTER 1. INTRODUCTION

The equations for ICI power of an OFDM system that operates in the fast Rayleigh or Rician environment are described in Chapter 5. The ICI power equations are derived from the Probability Density Function (PDF) of Angle-of-Arrival (AOA) of the multipath components, which can include biased or restricted AOA's PDF. The ICI power is obtained for both the Rayleigh and Rician channel models, based on both 2-D and 3-D AOA channel models. Examples of using directional antennas at the mobile and/or base station are also described in the chapter. The ICI power equation is applied to angular diversity (using antenna array).

Coding techniques, including correlative coding and Polynomial Cancellation Code (PCC), have been proposed to reduce the total ICI power. In Chapter 6, the ideas behind using coding to reduce ICI are introduced. The general expressions for quantifying the ICI power of correlative coding and PCC are shown. The general equations are derived in terms of the time domain correlation function of the channel conditions. In view of the limitations that occur for the above mentioned coding methods, a novel coding technique is proposed, and its general expression derived. The example of Carrier Frequency Offset (CFO), 2-rays, Clarke/Jakes' and uniformly Doppler spread channel model have been applied to examine the validity of the general expressions. The tight upper and lower bounds on the above mentioned ICI power reduction codes have been derived using the general expressions.

Other than coding to reduce ICI, equalization/filter techniques have also been proposed for ICI mitigation. However, such equalizer or filter techniques require immediate channel response information for them to be successfully implemented. In Chapter 7, a frequency domain channel estimation algorithm is proposed. The algorithm is based on optimizing the pilots, which have been developed using the polyphase sequence. The algorithm is simulated and compared with the conventional time domain methods, like the Kronecker-Delta (KD) approach and Stamoulis' algorithm.

CHAPTER 1. INTRODUCTION

Finally, in Chapter 8, the conclusion and summary of this report are presented. Further research areas are also included in the chapter.

1.3 Main Contributions

1. Doppler PSDs for various channels are derived when directional antennas are mounted at the transmitter, and/or at the receiver. The Doppler PSDs are derived by extending and modifying the PDFs of AOA (when directional antennas are employed) that are found in the literature. The Doppler PSDs are derived based on the AOA information, and are applied to both 2-D and 3-D channel models.
2. The reduction of ICI power in OFDM system is investigated by characterizing the reduction in the Doppler spread when directional antennas are employed. The ICI power equations are derived for the case when directional antennas are utilized at the transmitter and/or receivers. Using the AOA information investigated previously, the AOA based ICI power equations are derived. The AOA based ICI equation is extended to both 2-D and 3-D channel models, where both azimuthal and elevational planes are involved. Furthermore, an ICI reduction technique, based on limiting the AOA of the received signal, using antenna array, is proposed. An angular diversity technique is also proposed to harness and combine the received signals from different AOAs.
3. Nevertheless, it has been found that directional antennas will not be able to reduce ICI in all scenarios. Another approach of reducing ICI is through the use of special coding. There are some common ICI reduction codes found in literature, but their analysis are limited to the CFO channel. In the thesis, the general expressions of the ICI power, for an OFDM system, are derived for some common ICI reduction codes. The derived general ICI power equations for the

CHAPTER 1. INTRODUCTION

various coding schemes can be applied to all channel models. The upper and lower bounds of the various coding techniques are derived from the general ICI power equations. The performance gain for the various coding schemes is shown from the bounds, irrespective of the channel. A novel full rate coding scheme has also been proposed and compared with the existing coding techniques. The general expression, bounds and performance gain are also derived for the proposed scheme.

4. Filters or equalizers can also be deployed to mitigate ICI. However, instantaneous time varying channel impulse response is required to implement these filters/equalizers. In view of this, a pilot based channel estimation algorithm for OFDM system in a multipath and fast time varying channel is proposed. The pilots are designed using polyphase sequences. The algorithm is to be deployed in the frequency domain, which is in contrast to the published techniques that work in time domain. It is further shown that the proposed channel estimation technique can also be utilized in multiple input, multiple output (MIMO) OFDM systems. The normalized mean square error of the estimate is also derived.

Chapter 2

Basics of Orthogonal Frequency Division Multiplexing

In conventional single carrier modulation, data is transmitted over the channel serially. The speed of transmission is at a baud rate of R symbols per second, and time duration per symbol is $1/R$. The relative amount of delay spread will be significant if the time duration per symbol is short. Thus, in a broadband transmission, the delay spread can be as large as hundred symbols duration. This results in high ISI, which requires a few hundred to thousand taps equalizer to compensate the channel distortion. The need for broadband signalling is inevitable, as high speed data rate is a necessity for future wireless communications. The way to mitigate the problem is to divide and conquer. By dividing the broadband into multiple narrow band signals, the relative delay spread for each of the narrow band signals is much lower and thus ISI is reduced. This is the basic idea of MCM [15],[16].

MCM uses the principle of transmitting high data rate by dividing it into several parallel streams (known commonly as subcarriers, which N_c is denoted as the total num-

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

ber of subcarriers from here onwards), where each of them has a much lower baud rate, R/N_c . These subcarriers are then modulated by different carrier frequencies. MCM was first deployed in the military HF radio links in the late 1950's and early 1960's. Orthogonal Frequency Division Multiplexing (OFDM), proposed in 1960's and patented in 1970 by Chang [17], is a special form of MCM with densely spaced subcarriers and overlapping spectra. It abandoned the use of steep bandpass filter to completely separate the spectrum of subcarriers which is common to older Frequency Division Multiplex Access (FDMA) systems.

Traditional OFDM system [18],[19],[20] uses Discrete Fourier Transform (DFT) technique, at both transmitter and receiver [21],[22], to ensure the mutual orthogonality among subcarriers even while the spectra overlap. Although OFDM is shown to be a bandwidth efficient technique, its applications were limited for a relatively long time mainly because of the complexity in implementing DFT in a real-time system. But with the discovery of Fast Fourier Transform (FFT) by Cooley and Tukey and the breakthrough of Digital Signal Processing (DSP) technologies, OFDM has become feasible. The need for higher data rate has placed OFDM back into the spotlight. With intensive research since the 1980's, MCM or OFDM techniques appear to be on the verge of a breakthrough. Many of the implementation problems seem solvable and MCM or OFDM has already become part of several standards, for example, Wireless Metropolitan/Local/Personal Area Network (which include IEEE 802.11a/g [8],[9],[10], possible IEEE 802.15.3a [11], IEEE 802.16 [12]) and Digital Video and Audio Broadcasting [13],[14]. Moreover, the basic OFDM system had been proposed to combine with other modulation or access techniques to enhance the overall system performance or to provide multiple accessibility. A good example is the Multi-Carrier Code Division Multiplex Access (MC-CDMA) [15], which combines CDMA with OFDM.

In this chapter, the basic principles of the OFDM are introduced. Various popular

synchronization algorithms for OFDM systems are also shown, which can provide a platform for the frequency compensation technique discussed in Chapter 5. The various channel estimation algorithms and equalization techniques, for non-time varying channel, are also discussed in this chapter.

2.1 MCM and OFDM Basics

The transmitted MCM signal can be represented as

$$s(t) = \sum_{m=-\infty}^{+\infty} \sum_{n=0}^{N_c-1} d_{n,m} \psi_n(t - mT_s) \quad (2.1)$$

where $d_{n,m}$ is the information symbol on the n -th subcarrier in the interval of $(m-1)T_s$ to mT_s , $T_s = N_c/R$ is the time taken to transmit one set of information symbols on N_c subcarriers. ψ_n is the modulating waveform for the n -th subcarrier, assuming that the waveform is similar for each of the time interval. As seen from the previous section, the timing interval of T_s must be sufficiently long with N_c , as compared to the maximum delay spread of the channel. At the same time, the bandwidth of the subband must be smaller than the coherence bandwidth of the channel. With both of the conditions fulfilled, each subcarrier will experience flat fading and a much simpler equalizer is sufficient to compensate the channel. Conventionally, the channel is assumed stationary during the period of $(m-1)T_s$ to mT_s , for whichever m , but may be varying for different value of m ¹.

As described previously, OFDM ensures the mutual orthogonality among subcarriers even while the spectra overlap. Thus, the waveform ψ in (2.1) must ensure the

¹Please refers to the next chapter for the case when the channel is not stationary during one time interval.

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

orthogonality among different transmitted information symbols, that is

$$\int_0^{T_s} \psi_k(t) \psi_l^*(t) dt = \delta(k - l) \quad (2.2)$$

where $(\cdot)^*$ denotes complex conjugate and $\delta(\cdot)$ denotes Kronecker delta function. Alternatively, the orthogonality between subcarriers in the discrete implementation is to be fulfilled, seen as

$$\sum_{n=0}^{N_c-1} \psi_k\left(\frac{nT_s}{N_c}\right) \psi_l^*\left(\frac{nT_s}{N_c}\right) = \delta(k - l) \quad (2.3)$$

where $t = \frac{nT_s}{N_c}$ in the discrete case. Thus, the orthogonal sets of waveform in both continuous and discrete time domains can be easily achieved by

$$\psi_k(t) = \begin{cases} \frac{1}{\sqrt{T_s}} \exp(j2\pi f_k t) & t \in [0, T_s) \\ 0 & \text{otherwise} \end{cases} \quad (2.4)$$

where f_k is the subcarrier frequency for $k = [0, 1, \dots, N_c - 1]$, and $f_k = f_0 + k\Delta f$. The frequency f_0 is the lowest subcarrier frequency, and Δf is the spacing between adjacent subcarrier. Moreover, the orthogonal waveform, $\psi_k(t)$ is restricted in the time window of $[0, T_s)$, the subcarrier spacing must also satisfy $\Delta f = \frac{1}{T_s} = \frac{R}{N_c}$.

At the receiver, the received signal can be seen as

$$x(t) = h(t, \tau) \otimes s(t) + \omega(t) \quad (2.5)$$

where $h(t, \tau)$ is the time domain channel response, $\omega(t)$ is the additive white Gaussian noise (AWGN) and $\otimes$ denotes convolution. With the orthogonality condition satisfied (see (2.2)) and $h(t, \tau) = h(mT_s, \tau)$ for the duration between $(m - 1)T_s$ to mT_s (which indicate that channel does not change with time), the demodulation of the OFDM

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

symbol can be based on a bank of matched filters which implements

$$y_{k,m} = \int_{(m-1)T_s}^{mT_s} x(t) \psi_k^*(t - mT_s) dt \quad (2.6)$$

The discrete equivalent of (2.6) can be seen as (using (2.3))

$$y_{k,m} = \sum_{n=0}^{N_c-1} x \left(mT_s + \frac{nT_s}{N_c} \right) \psi_k^* \left(\frac{nT_s}{N_c} \right) \quad (2.7)$$

Since the orthogonality condition (see (2.2)) is satisfied, the received signals can be simplified to

$$y_{k,m} = g_{k,m} d_{k,m} + \omega_{k,m} \quad (2.8)$$

where $g_{k,m}$ and $\omega_{k,m}$ are the frequency domain channel impulse response and the AWGN after FFT, at subcarrier k in the m -th OFDM symbol respectively. However, it should be noted that the basic assumption is that each subcarrier experiences flat fading, thus the coherence bandwidth of the channel must be greater than the subcarrier bandwidth, or $B_{coh} \gg BW/N_c$. It is also assumed that the channel is time invariant during each OFDM symbol, thus, the coherence time must be longer than the OFDM symbol duration, $T_{coh} \gg T_s = N_c/R$. So, N_c will have to be carefully chosen in such a way as to achieve a delicate balance such that $RT_{coh} \gg N_c \gg BW/B_{coh}$.

2.1.1 Cyclic Extension

When the signal passes through a delay spread channel, the received signal experiences ISI. There are two methods to mitigate the effect of ISI, the first is to use complex equalizer with a lot of taps, or to introduce guard interval between symbols. By implementing equalizers, we will increase the hardware complexity at the receivers. But by employing guard interval, we will reduce the spectral efficiency. In practical OFDM sys-

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

tems, both methods are applied concurrently. But in this section, only the motivations of implementing the guard interval are discussed.

In a delay spread channel, the maximum amount of delay spread is assumed to be known at the transmitter. Therefore, in order to conserve spectrum efficiency, the length of guard interval is set to be equal or slightly larger than the maximum amount of delay spread. As mentioned in Section 1.1, the ratio of a OFDM symbol time to guard interval is short as compared to conventional single carrier system. This is the major advantage of OFDM.

The main issue of this section is to consider on the construction of the guard interval. The most obvious way would be to pad the guard interval with zeros, as similar to the Time Division Multiplex Access (TDMA) case. This eliminates the ISI. But in the OFDM system, the orthogonal condition cannot be satisfied, resulting in Intercarrier Interference (ICI). ICI is the crosstalk between different subcarriers when the orthogonality is lost. To mitigate the problem, Peled and Ruiz [23] introduced the cyclic prefix (CP) in 1980. A CP is a copy of the last portion of a OFDM symbol that is pre-pended to the transmitted OFDM symbol, as shown in Fig. 2.1. It is removed at the receiver before demodulation. Similar idea can be applied by appending the front part of the OFDM symbol to the end. It is known as cyclic postfix. Both these techniques are known collectively as cyclic extension, but it is commonly denoted as CP because of the term Cyclic Prefix/Postfix.

Figure 2.1: Illustration of Cyclic Prefix

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

As mentioned earlier, CP is implemented to mitigate ISI and its most important role is to act as a cyclic convolution in the time domain and to maintain orthogonality in frequency domain by translating it as a scalar multiplication. However, with CP, there is an overhead, which can be characterized as

$$\eta = \frac{N_c}{N_c + L} \quad (2.9)$$

And additional energy (of the CP) is required to be transmitted, thus the signal to noise ratio (SNR) loss is given by

$$SNR_{loss} = -10 \log_{10} \left(\frac{T_s}{T_{CP} + T_s} \right) \quad (2.10)$$

where T_{CP} is the duration of the CP. The spectrum efficiency can be seen as $R \left(\frac{T_s}{T_{CP} + T_s} \right)$. Due to the loss in spectrum efficiency and SNR, the length of CP should be kept as short as possible. If T_{CP} is chosen to be equal to the length of the delay spread of the channel, the ratio of $\left(\frac{T_s}{T_{CP} + T_s} \right)$ is still relatively insignificant. Therefore, the use of CP only results in small amount of SNR loss, but it ensures ISI and ICI-free transmission.

However, there are a lot of advantages if the length of the CP is made slightly longer. As there is a strict requirement for OFDM symbol to be symbol-wise synchronized, the use of CP can help to relax the requirement. Moreover, CP can be employed for estimating the timing and frequency offset, as will be described in the coming section.

2.1.2 Zero Padding to Replace CP

OFDM does not experience ISI and ICI in a multipath channel when CP is used. From another point of view, pre-processing has been done at the transmitter (in term of CP) to

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

Figure 2.2: OFDM System Model

remove the ISI and ICI. Nevertheless, zero padding (ZP) [24], for guard interval, can be employed to remove the ISI. However, it does not ensure the orthogonality of subcarriers at frequency domain, which will result in ICI. Thus, instead of the pre-processing (in term of CP) done at the transmitter, the process to ensure the orthogonality between subcarriers can be shifted to the receiver.

There are a few advantages of ZP OFDM system. The most obvious advantage is that there will not be any SNR loss due to the CP, as no signal will be transmitted during that period. Furthermore, it is mentioned in [24] that the channel with ZP implemented is much more well-conditioned, as it is always invertible, as compared to the channel with CP. The implementation of blind channel estimators and equalizers will also be much easier with ZP-based OFDM systems. The low-complexity equalizers deployed in CP-based OFDM system can also be applied to ZP OFDM, after some simple processing.

2.2 Synchronization in OFDM

OFDM system requires synchronization in both frequency and symbol timing. In this section, the various synchronization algorithms that are based on training symbols, CP or unused subcarriers (known as virtual carriers) are discussed.

2.2.1 Synchronization based on training symbols

Frequency Synchronization

One of the most important work in synchronization is due to Moose [25], which is based on OFDM training symbols. This synchronization technique is suggested for the Wireless Local Area Network (WLAN) standard IEEE 802.11a/g [8],[9]. This algorithm estimates the carrier frequency offset by comparing the phase difference between two adjacent training symbols. Using those two symbols, under the influence of frequency offset, the two results can be calculated as

$$\begin{aligned} y_{k,1} &= d_{k,1} + \omega_{k,1} \\ y_{k,2} &= d_{k,2} \exp(j2\pi f_0 T_s) + \omega_{k,2} \end{aligned} \quad , \text{ for } k = 0, \dots, N_c - 1 \quad (2.11)$$

where f_0 is the Carrier Frequency Offset (CFO), and $f_0 T_s$ is normalized with respect to the intercarrier frequency spacing. If $d_{k,1} = d_{k,2}$, it is shown that the maximum likelihood estimate of the offset to be [25]

$$\widehat{f_0 T_s} = \frac{1}{2\pi} \arctan \left(\frac{\sum_{k=0}^{N_c-1} \text{Imag}(y_{k,2} y_{k,1}^*)}{\sum_{k=0}^{N_c-1} \text{Real}(y_{k,2} y_{k,1}^*)} \right) \quad (2.12)$$

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

It can be seen from [25] that the Moose's frequency offset estimation algorithm achieves a mean square error (MSE) that is inversely proportional to the SNR of the received signals. However, for this algorithm, $\widehat{f_0 T_s}$ is limited to values between -0.5 and $+0.5$. For example, the Moose's algorithm may mistakenly estimates the actual $f_0 T_s = 2.3$ as 0.3 . A similar technique proposed by Schmidl and Cox [26] removes this limitation. In their context, the CFO should be defined as

$$f_0 T_s = \eta + z \quad (2.13)$$

where z is an integer that corresponds to the amount of CFO in multiple of intercarrier spacing and η is the fine CFO within the length of the intercarrier spacing. Schmidl & Cox optimized the training symbols to achieve the results. The method encodes the even subcarriers of the OFDM training symbols with a PN sequence and the odd subcarriers with zeros. To differentiate between the two OFDM training symbols, they are differentially encoded, that means that $\frac{d_{k,1}}{d_{k,2}} = \pm 1, \pm j$. At the receiver, the fine (fractional) carrier frequency offset is estimated using the Moose's algorithm, while the integer frequency offset, z , is estimated through

$$\hat{z} = \max_z \left| \sum_{k=0}^{N_c-1} y_{k+2z,1}^* \frac{d_{k,1}}{d_{k,2}} y_{k+2z,2}^* \right|^2 \quad (2.14)$$

As both Schmidl & Cox and Moose algorithms shared the same idea, the MSE of the CFO estimation is inversely proportional to the SNR in both cases.

Timing Synchronization

Using the same set of training symbols as frequency synchronization mentioned above, Schmidl & Cox proposed an algorithm that is able to estimate the symbol timing. By

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

correlating the input signals in time domain (before the FFT), the corresponding timing equation can be derived.

$$M(s) = \frac{\left| \sum_{n=0}^{N_c-1} x(s+n)x^*(s+n+N_c) \right|^2}{\left(\sum_{n=0}^{N_c-1} |x(s+n+N_c)|^2 \right)^2} \quad (2.15)$$

The OFDM symbol starting position is when $M(s)$ is the largest. The MSE of the estimation, given in [26], is inversely proportional to the $(N_c \times SNR)$ of the received signal. Thus, the longer the training symbol length, the better the estimate. This technique can be applied using one OFDM symbol, when the first half is the same as the second half of the OFDM symbol. This can be achieved only when the even number subcarriers are inserted with pilot signals and the odd subcarriers are inserted with zeros, which is a requirement of the Schmidl & Cox algorithm as mentioned in the previous case for frequency offset estimation.

2.2.2 Synchronization based on Virtual Carriers

Synchronization by using virtual carriers was first proposed by Liu *et al* in 1999 [27],[28]. Virtual carriers are unused subcarriers that are not utilized for data or pilots. Normally, these subcarriers are placed at both ends of the spectrum, so as to reduce the out-of-band spectrum. Thus, these unused subcarriers are loaded with zeros. To estimate the start of the OFDM symbol block, the algorithm focuses on minimizing the output of a correlator that demodulates at the frequency of those virtual carriers. Effectively, the motivation is to reduce the following cost function [28].

$$J_{vc}(s, f_0 T_s) = \left| \sum_{u \in \mathbf{V}} \sum_{k=0}^{N_c-1} \sum_{l=0}^{N_c-1} x(s) \exp(j2\pi l f_0 T_s) \exp(j2\pi u k) \right|^2 \quad (2.16)$$

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

where $\mathbf{V}$ is the set of index for virtual carriers (unused subcarriers) in the OFDM system. If $J_{vc}(s, f_0 T_s)$ is 0, then it is known that $x(s)$ is an interference free N_c time domain sampled window of the OFDM symbol. The algorithm is based on a sliding N_c -window on the received signal, as suggested by (2.16).

The algorithm can be implemented in two steps, the first is to assume that time synchronization is achieved, and the cost function $J_{vc}(s, f_0 T_s)$ is calculated for the estimated range of values for $f_0 T_s$. If it is below a certain threshold for a certain value of $f_0 T_s$, the value for s and $f_0 T_s$ is estimated. If not, the second step is to shift s and go back to the first step of searching through all the values of $f_0 T_s$.

2.2.3 Synchronization Based on Cyclic Extension

Cyclic extension has been employed in timing synchronization and frequency offset estimation of OFDM system. It is based on the idea of pairwise correlation between the last part of the OFDM symbol with the CP, and the uncorrelated properties of the rest of the symbol, which can be seen as

$$E[x(s)x^*(s+n)] = \begin{cases} \sigma_s^2 + \sigma_n^2 & \text{for } n = 0 \\ \sigma_s^2 \exp(-j2\pi f_0 T_s) & \text{for } n = N_c \\ 0 & \text{otherwise} \end{cases} \quad \forall k \in \mathbf{C} \quad (2.17)$$

where $\mathbf{C}$ is the set of time domain index for the cyclic prefix and σ_s^2 is the transmitted symbol energy. Thus, it has been shown in [29],[30] that the joint maximum likelihood estimation of the timing and frequency offset can be seen as

$$(\hat{s})_{ML} = \arg \max_s [|\gamma(s) - \rho\Phi(s)|] \quad (2.18)$$

$$(\widehat{f_0 T_s})_{ML} = -\frac{1}{2\pi} \text{angle} \{ \gamma[(\widehat{s})_{ML}] \} \quad (2.19)$$

where

$$\begin{aligned} \gamma(n) &= \sum_{k=n}^{n+L-1} x(k)x^*(k+N) \\ \Phi(n) &= \frac{1}{2} \sum_{k=n}^{n+L-1} |x(k)|^2 + |x(k+N)|^2 \\ \rho &= \frac{SNR}{SNR + 1} \end{aligned}$$

The algorithm described above must be implemented over a satisfactory number of OFDM symbols, depending on the length of the CP. This is because the estimated values for one symbol is often corrupted by noise and smoothing or averaging is required.

2.3 Channel Estimation

Channel estimation can be implemented in various ways, with or without the help of channel statistics, using blind or training based algorithm. Normally, channel statistics, together with a parametric channel model [31], can help to estimate the required parameters of the channel. Usually, these parametric channel models require fewer parameters to estimate and perform better. However, there is a drawback. This approach may degrade the performance considerably if the parametric channel model employed differs from the actual channel.

Channel statistics, in terms of the time and frequency correlation properties, can assist in the estimation of channel, even when no specific channel model is defined. However, there is still the basic problem of estimating the unknown channel statistics

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

at the receiver. Even then, if the channel statistics estimated at the receiver are different from the actual values, the system will experience some loss in the performance.

2.3.1 Two Dimensional Channel Estimation

Typical channel estimation algorithms for OFDM systems are implemented over two dimensions (2-D), namely time and frequency domains. The pilot symbols are placed on several subcarriers in both frequency and time directions. An example of an practical application can be found in the Digital Video Broadcasting-Terrestrial (DVB-T) [14].

Figure 2.3: An example of pilot placement DVB-T system

By choosing the separation of pilots with respect to the channel coherence bandwidth and coherence time, estimation of the channel impulse response can be obtained by interpolation and filtering. The basic 2-D channel estimation can be divided into two steps. In the first step, the algorithm estimates the channel impulse response at positions where the pilots are located. It is obtained using the least square approach, seen as

$$\begin{aligned}\hat{g}_{b,c} &= \frac{y_{b,c}}{d_{b,c}} \\ &= g_{b,c} + \frac{\omega_{b,c}}{d_{b,c}} \quad \text{for } (b, c) \in \mathbf{P}_i\end{aligned}\tag{2.20}$$

where $\mathbf{P}_i$ is the indexes for the piloted subcarriers. In the second step, the final estimates

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

of the complete channel transfer function are obtained from the initial estimates, $\hat{g}_{b,c}$, by 2 dimension interpolation or filtering, which is given by

$$\tilde{g}_{b_d, c_d} = \sum_{(b,c) \in \mathbf{P}_i} w_{(b_d, c_d), (b, c)} \hat{g}_{b, c} \quad (2.21)$$

where the subscript (b_d, c_d) denotes the information at subcarrier b_d in the c_d OFDM symbol and the d denotes that the channel impulse responses are for the data subcarriers. In the same equation, $w_{(b_d, c_d), (b, c)}$ is the weight of the two dimensional interpolator. The number of weights will be equal to the number of pilot subcarriers. There are a few methods to calculate the values for the weights, and those include the method of linear or polynomial interpolation and the optimal Wiener solution.

The criteria for the Wiener solution is that the mean square error (MSE) of the estimator has to be minimum, which can be seen as

$$\begin{aligned} J_{CE}(b, c) &= E[|e_{b,c}|^2] \\ &= E[|g_{b,c} - \tilde{g}_{b,c}|^2] \end{aligned} \quad (2.22)$$

From [32],[33], it is known that the MSE is minimum when the orthogonal principle is fulfilled. And the orthogonal principle states that

$$E[|e_{b_1, c_1} \hat{g}_{b_1, c_1}^*|^2] = 0 \quad \text{for } (b_1, c_1) \in \mathbf{P}_i,$$

which will lead to

$$\begin{aligned} E[g_{b_1, c_1} \hat{g}_{b_2, c_2}^*] &= E[g_{b_1, c_1} g_{b_2, c_2}^*] \\ &= \sum_{(b,c) \in \mathbf{P}} w_{(b,c), (b_1, c_1)} E[\hat{g}_{b,c} \hat{g}_{b_2, c_2}^*] \end{aligned} \quad (2.23)$$

where $E[g_{b_1, c_1} g_{b_2, c_2}^*]$ is the discrete time-frequency correlation function of the channel

CHAPTER 2. BASICS OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

and the autocorrelation function of the estimated channel can be given by

$$\begin{aligned} E[\hat{g}_{b_1,c_2} \hat{g}_{b_2,c_2}^*] &= E[g_{b_1,c_1} g_{b_2,c_2}^*] + E[\omega_{b_1,c_1} \omega_{b_2,c_2}^*] \\ &= E[g_{b_1,c_1} g_{b_2,c_2}^*] + \sigma_n^2 \delta_{(b_1,c_1),(b_2,c_2)} \end{aligned} \quad (2.24)$$

Therefore, from [32],[33], the optimum weights, $w_{(b,c),(b_1,c_1)}$, that minimize the MSE can be found using (2.23).

2.3.2 One Dimensional Channel Estimation

Nevertheless, two dimensional interpolation and filtering results in large computational complexity. In order to reduce complexity without much trade off in performance, two 1-dimensional filters [34],[35],[36] are proposed. The method divides the former second step into another 2 separate steps. In the step 1 of the modified second step, filtering/interpolation is applied in the frequency domain, as depicted by

$$\hat{g}_{b,c_1} = \sum_{(b_1,c_1) \in P_i} w_{(b,c_1),(b_1,c_1)}^f \hat{g}_{b_1,c_1} \quad (2.25)$$

After that, interpolation/filtering is accomplished between OFDM symbols, among frequencies, like

$$\hat{g}_{b,c} = \sum_{(b,c) \in P_i} w_{(b,c),(b,c_1)}^t \hat{g}_{b,c_1} \quad (2.26)$$

In this way, the filter weights, $w_{(b,c_1),(b_1,c_1)}^f$ and $w_{(b,c),(b,c_1)}^t$ are dependent only on either frequency or time channel index. For the two 1-dimensional Wiener solution, the weights are only dependent on the time-domain channel correlation for $w_{(b,c),(b,c_1)}^t$ and frequency domain channel correlation for $w_{(b,c_1),(b_1,c_1)}^f$.

2.4 High Peak-to-Average Power Ratio

From (2.1), it can be seen that the OFDM signal is made up of N_c modulating waveforms. Thus, it can lead to large Peak-to-Average Power Ratio (PAPR) [15] when all the waves are added up coherently. In this case, the peak power is N_c times the average power. A large PAPR will bring many disadvantages. It will increase the complexity of the Analog-to-Digital Converter (A/D) and Digital-to-Analog Converter (D/A) and reduce the efficiency of the RF power amplifier [37]. However, there is still no optimal solution to it yet. Several methods have been suggested to reduce the PAPR effect. The first method uses the idea of signal distortion technique, which reduces the peak amplitudes by nonlinearly distorting the OFDM signal at or around the predefined limiting value. This method includes clipping, peak windowing and peak cancellation [15]. The second category uses scrambling and/or coding to exclude OFDM symbols from a large PAPR. Example of PAPR reduction codes used are Golay complementary code [38] and polyphase sequence [39]. Scrambling techniques include selected mapping and partial transmit sequences [15].

2.5 Summary

In this chapter, the basic principles of OFDM are given, focusing on the attributes that makes it so popular. Moreover, several issues on OFDM implementation are highlighted. These include the use of CP to remove ISI, the various synchronization algorithms and channel estimation techniques. Nevertheless, the channel gain is assumed to be constant within one OFDM symbol. But this may not be the case always. In the next chapter, various aspects of fast time varying channel on the OFDM system are discussed.

Chapter 3

OFDM in Time Varying Channel

As mentioned in the previous chapter, it is assumed that the channel is time invariant for a whole OFDM symbol, satisfying the condition $RT_{coh} \gg N_c \gg BW/B_{coh}$. However, this assumption may not be valid in practical situations. Once this condition cannot be realized, the channel will become time variant within an OFDM symbol. In this scenario, there will be an erosion of orthogonality between subcarriers, resulting in ICI or "FFT leakage". Therefore, there is a need to model and evaluate the performance of the OFDM systems under such a condition and find means to mitigate it.

In this chapter, the various aspects of OFDM in time-varying channel will be reviewed. Various time-varying channels will be introduced, together with their impact on the OFDM systems. Furthermore, channel estimation and equalization techniques for the OFDM systems in time-varying channels, that are found in literatures, are also being introduced.

Nevertheless, the effects on the loss of orthogonality with OFDM subcarriers will

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

be first investigated, by re-writing (2.5), where the convolution can be seen as

$$x(t) = \sum_{l=0}^{L-1} h(t, \tau_l) s(t - \tau_l) + \omega(t) \quad (3.1)$$

where τ_l is the time delay of the l -th path. As the channel is not fixed within every OFDM symbol, the above equation have to be re-written with ICI as

$$\begin{aligned} y_k &= \frac{1}{T_s} \int_0^{T_s} x(t) \psi_k^*(t) dt \\ &= \frac{1}{T_s} \int_0^{T_s} \left[\sum_{l=0}^{L-1} h(t, \tau_l) s(t - \tau_l) + \omega(t) \right] \psi_k^*(t) dt \\ &= g_k d_k + \frac{1}{T_s} \int_0^{T_s} \left(\sum_{l=0}^{L-1} h(t, \tau_l) \sum_{n=0, n \neq k}^{N_c-1} d_n \psi_n(t - \tau_l) \right) \psi_k^*(t) + w_k \end{aligned} \quad (3.2)$$

for OFDM symbol m

The subscript m has been removed from y_k , g_k and d_k , but it is indicated in the equation that the transmitted and received symbols are for the m -th OFDM symbol. It can be seen from the equation that there is an additional interference term (the second term). It is caused by the loss of orthogonality due to the time varying channel. The amount of interference is very much dependent on the channel. There are various ways of modelling the time varying channel, and a few of the most popular ones are the Clarke/Jakes' classical Doppler spread model, the Uniform Doppler spread model, the single ray and two rays Doppler shift models. However, there can be various modifications to these models which may include the other effects of "FFT leakage", such as phase noise, non-linearity of the amplifier, frequency offset, ISI, etc. However, it has been shown that these effects can be separately investigated [3],[4].

3.1 Various Time Varying Channel Models

Various time varying channel models have been proposed in the literatures. We classify them based on their time domain autocorrelation function, as defined by

$$\begin{aligned} r(t_1, t_2) &= E[h(t_1)h'(t_2)] \\ &= r(t_1 - t_2) \\ &= r(T) \end{aligned} \tag{3.3}$$

where $h(t)$ is the time domain channel impulse response at time t and T is the timing interval between sampling timings t_1 and t_2 . The above assumption holds when the channel is wide sense stationary (WSS), which will be assumed throughout the thesis.

Classic Clarke/Jakes' Doppler Spread Model

The Clarke/Jakes' classical Doppler spread model is a stochastic channel model, which is based on the long term averaging of the channel. In the model, the AOA of the incoming signals is assumed to be uniformly distributed among the azimuthal plane. And the autocorrelation is given as [40],[41]

$$r_J(f_d T_s x) = J_0(2\pi f_d T_s x), \tag{3.4}$$

where $J_0(x)$ is the zeroth-order Bessel function of the first kind and f_d is the maximum Doppler frequency. The Power Spectral Density (PSD) is seen as [40],[41]

$$S_J(f) = \begin{cases} \frac{1}{\pi f_d \sqrt{1 - \left(\frac{f}{f_d}\right)^2}} & \text{for } |f| < f_d \\ 0, & \text{otherwise} \end{cases} \tag{3.5}$$

3.1.1 Uniform Doppler Spread Model

This is also a stochastic channel model where PSD is uniformly distributed from $-f_d < f < +f_d$. It is found that this model matches scattering environment in three dimensions. And the autocorrelation can be written as [42]

$$r_U(f_d T_s x) = \text{sinc}(2f_d T_s x), \quad (3.6)$$

where $\text{sinc}(x)$ is $\frac{\sin(\pi x)}{\pi x}$. The PSD is given by [42]

$$S_U(f) = \begin{cases} \frac{1}{2f_d} & \text{for } |f| < f_d \\ 0, & \text{otherwise} \end{cases} \quad (3.7)$$

3.1.2 Doppler Shift Model

The Doppler shift model is useful for Line-of-sight (LOS) propagation where the signal is only received from one direction. This model is also useful for modelling a communication system with CFO. The correlation of the model can be seen as [42]

$$r_1(f_0 T_s x) = \exp(-j2\pi f_0 T_s x), \quad (3.8)$$

where the PSD is [42]

$$S_1(f) = \delta(f - f_0) \quad (3.9)$$

where f_0 is the Doppler shifted frequency.

3.1.3 2-rays Doppler Spread Model

The 2-rays Doppler spread model is useful for channel with a LOS and a far-field dominating reflection of the signal. In this case, the signal is received from only two specific angle-of-arrivals. The receiver thus experiences two different Doppler shifts. As it is assumed that the two rays are uncorrelated, the autocorrelation can be defined as

$$r_2(f_d T_s x) = p_1 \exp(-j2\pi f_1 T_s x) + p_2 \exp(-j2\pi f_2 T_s x), \quad (3.10)$$

where p_1 and p_2 are the received powers from the first and second ray respectively, which $p_1 + p_2 = 1$. The parameters f_1 and f_2 are their respective Doppler shifts. With the autocorrelation defined, the PSD can be seen as

$$S_2(f) = p_1 \delta(f - f_1) + p_2 \delta(f - f_2) \quad (3.11)$$

3.1.4 Other Channels

Most of the channels can be considered as a combination of the above mentioned channels. For example, in the Rician case, there is a LOS component with some Rayleigh scatterings. It can be modelled as combination of the Doppler shift channel and the classic Clarke/Jakes' Doppler spread channel. In this case, there will be a K factor for the Rician channel determining the distribution of power between the LOS channel and the Doppler spread channel. However, the total received power has to be normalized to one.

3.2 System Impairments Which Results in ICI

The motivation of this section is to provide a unified analysis of ICI, for time-varying channels as well as practical system impairments, such as oscillator phase noise, CFO. Like the ICI caused by the fast time varying channel, these impairments contribute to the reduction of reliability of the OFDM system. These impairments can be modelled as a channel response with an autocorrelation function, as seen in the previous section.

3.2.1 Oscillator Phase Noise as a Channel

Oscillator phase noise can be treated in a similar manner as the Doppler spread caused by flat fading. The distortion, or 'time varying channel gain' can be modelled as a phase component, denoted by $\exp[j\Phi(n)]$. The phase noise process is assumed to be zero-mean and WSS with a narrow band PSD described by a polynomial expression with finite power, σ_Φ^2 [43]. However, simplification of the phase noise can be achieved if $\Phi(n)$ is assumed to take on a small value, which will justify the approximation $\exp[j\Phi(n)] \approx 1 + j\Phi(n)$, for $\Phi(n) \ll 1$ [44]. In [44], an approximation of the PSD of the phase process has been proposed as

$$S_\Phi(f) = 10^{-c} + \begin{cases} 10^{-a} & |f| < f_l \\ 10^{-(f-f_l)\left(\frac{b}{f_h-f_l}\right)-a} & f > f_l \\ 10^{(f+f_l)\left(\frac{b}{f_h-f_l}\right)-a} & f < -f_l \end{cases} \quad (3.12)$$

where c is the white phase noise floor, a gives the phase noise level near the centre frequency up to $\pm f_l$ and b determines the gradient of noise reduction with increasing frequency distance up to $\pm f_h$. The PSD can be seen in Fig. 3.1. It is generated with $a = 8$, $b = 2$, $c = 12$, and $f_l = 10$ kHz and $f_h = 100$ kHz which is normalized to a

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

symbol period of $T_s = 3.2 \text{ us}$.

Figure 3.1: PSD of the Oscillator Phase Noise

The time domain autocorrelation of the oscillator phase noise modelled above is seen as

$$r_\Phi(f_l T_s x) \approx 1 + \xi_\Phi(f_l T_s x) \quad (3.13)$$

where

$$\xi_\Phi(f_l T_s x) = 10^{-c} \delta(x) + 2 \times 10^{-a} \left[\text{sinc}(\pi f_l T_s x) + \frac{p \cos(2\pi f_l T_s x) - 2\pi x \sin(2\pi f_l T_s x)}{p^2 + (2\pi x)^2} \right] \quad (3.14)$$

and

$$p = \frac{b \ln 10}{(f_h - f_l) T_s}$$

It is found in [45] that the amount of ICI may seriously limit the performance of OFDM system, especially for higher order modulation schemes.

3.3 Quantifying ICI in OFDM

ICI can be quantified using its power by applying (3.2). With the FFT defined as

$$\psi_k^*(t) = \frac{1}{\sqrt{T_s}} \exp(j2\pi f_k t),$$

the ICI power is defined as covariance of the second term in (3.2), and it can be written as

$$\begin{aligned} P^{ICI} &= \frac{1}{T_s} E \left[\left| \int_0^{T_s} \left(\sum_{l=0}^{L-1} h(t, \tau_l) \sum_{n=0, n \neq k}^{N_c-1} d_n \psi_n(t - \tau_l) \right) \psi_k^*(t) dt \right|^2 \right] \\ &= \frac{1}{T_s} E \left[\left| \int_0^{T_s} \sum_{n=0, n \neq k}^{N_c-1} \sum_{l=0}^{L-1} h(t, \tau_l) d_n \exp \left(j \frac{2\pi n \tau_l}{T_s} \right) \exp \{j2\pi(f_n - f_k)t\} dt \right|^2 \right] \end{aligned} \quad (3.15)$$

In order to simplify the above equation, it is assumed that there is no multipath in the channel, i.e., $L = 1$. The assumption is valid [42] as the total energy of the received signal is the same, which is usually normalized to unity. Thus,

$$\begin{aligned} P^{ICI} &= \frac{1}{T_s} E \left[\left| \int_0^{T_s} \sum_{n=0, n \neq k}^{N_c-1} h(t) d_n \exp \{j2\pi(f_n - f_k)t\} dt \right|^2 \right] \\ &= E \left[\left| \sum_{n=0, n \neq k}^{N_c-1} d_n g_{n-k} \right|^2 \right] \end{aligned} \quad (3.16)$$

where

$$g_{n-k} = \frac{1}{T_s} \int_0^{T_s} h(t) \exp \{j2\pi(f_n - f_k)t\} dt. \quad (3.17)$$

Therefore, by assuming that the data symbols, d_k , are independent, which is

$$E(d_k d_l^*) = \delta(k - l),$$

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

the ICI power can be seen as

$$\begin{aligned}
 P^{ICI} &= E \left[\left| \sum_{n=0, n \neq k}^{N_c-1} d_n g_{n-k} \right|^2 \right] \\
 &= \sum_{n=0, n \neq k}^{N_c-1} \sum_{n'=0, n' \neq k}^{N_c-1} E[d_n d_{n'}^*] E[g_{n-k} g_{n'-k}^*] \\
 &= \sum_{n=0, n \neq k}^{N_c-1} E[g_{n-k} g_{n-k}^*]
 \end{aligned} \tag{3.18}$$

With g_{n-k} defined in (3.17), the correlation can be seen as

$$\begin{aligned}
 P^{ICI} &= \sum_{n=0, n \neq k}^{N_c-1} E[g_{n-k} g_{n-k}^*] \\
 &= \frac{1}{T_s^2} \int_0^{T_s} \int_0^{T_s} E[h(t_1) h^*(t_2)] \exp(j2\pi(f_n - f_k)(t_1 - t_2)) dt_1 dt_2
 \end{aligned} \tag{3.19}$$

With the results from [46] and the time domain channel correlation expression from Section 3.1, the ICI power can finally be derived as

$$P^{ICI} = \sum_{n=0, n \neq k}^{N_c-1} \int_{-1}^1 (1 - |x|) r(f_d T_s x) \exp\left(\frac{j2\pi(n - k)T_s x}{N_c}\right) dx \tag{3.20}$$

The above equation is the final ICI power expression. The ICI power expression can be simplified when infinite number of subcarriers are assumed [42][47], as follows

$$P^{ICI} = 1 - \int_{-1}^1 (1 - |x|) r(f_d T_s x) dx \tag{3.21}$$

The theoretical outcomes calculated by (3.21) are similar to those simulated ICI power results and can be seen from Fig. 3.2. The simulation results are obtained by the Clarke/Jakes' Doppler spread channel model with 100000 Monte Carlos trials. From

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

Fig. 3.2, it can be seen that with $N_c \geq 4$, the differences between the numerical and calculated ICI power using (3.21) are less than 10%. When $N_c \geq 16$, the differences between the numerical and calculated ICI power decrease till below 1%. Similar results can be obtained from other channel models.

Figure 3.2: Numerical and Calculated ICI Comparison

Moreover, it has been shown in literatures [42],[6] (using simulations and theoretical analysis) that the assumption of infinite number of subcarriers in ICI power calculation shows no significant difference as N_c is usually quite large. Thus, this assumption will be used throughout the thesis. From (3.21), it can be seen the useful energy for an OFDM subcarrier is

$$\begin{aligned}
 P^{useful} &= 1 - P^{ICI} \\
 &= \int_{-1}^1 (1 - |x|) r(f_d T_s x) dx
 \end{aligned} \tag{3.22}$$

Using the time-domain channel autocorrelation function defined earlier, the ICI power for each of the different channels can be found [6]. For the classic Clarke/Jakes' Doppler

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

spread channel, it can be seen as

$$P^{ICI} = 1 - {}_1F_2\left(\frac{1}{2}; \frac{3}{2}, 2; -(f_d T_s \pi)^2\right) \quad (3.23)$$

where ${}_1F_2(\cdot, \cdot; \cdot, \cdot)$ is the generalized hypergeometric function [48]. For the uniform Doppler spread channel, it can be found as

$$P^{ICI} = 1 + \frac{1 - \cos(2\pi f_d T_s) - 2\pi f_d T_s \text{Si}(2\pi f_d T_s)}{2(\pi f_d T_s)^2} \quad (3.24)$$

where $\text{Si}[\cdot]$ is the sine integral function [48]. For the Doppler shift model, P^{ICI} can be shown as

$$P^{ICI} = \text{sinc}^2(\pi f_d T_s). \quad (3.25)$$

And for two rays model, the ICI power is written as

$$P^{ICI} = p_1 \text{sinc}^2(\pi f_1 T_s) + p_2 \text{sinc}^2(\pi f_2 T_s) \quad (3.26)$$

There is no closed form solution for the oscillator phase noise model, but the ICI power can be numerically calculated using (3.21) and (3.13).

3.4 Channel Estimation for Time Varying Channel

A novel method to estimate the time varying channel will be introduced in Chapter 7, for the case when channel statistics are not available. But in this section, other alternative algorithms are shown. These algorithms will be compared in detail with the proposed algorithm in Chapter 7.

3.4.1 Kronecker-Delta Scheme

Perhaps the simplest and most well-known scheme of estimating time-varying multipath channel impulse response (for OFDM system) is the Kronecker-Delta (KD) scheme [49], which places the pilot sequence at the start and the end of the OFDM symbol, as shown in Fig. 3.3. It estimates a particular OFDM symbol's instantaneous time domain channel impulse response by linear interpolation between channel estimates from these two sets of pilot sequence.

Figure 3.3: Kronecker-Delta scheme's placement of training sequences before and after an OFDM symbol

The scheme operates in two steps. The first is to estimate the channel response of the pilot sequences at both ends of the OFDM symbol. The length of the pilot sequence at each end is set to be greater or equal to the length of the multipath, L . The second step is performed once the instantaneous channel impulse responses at the two ends are known. Using the estimated values from the first step, the instantaneous channel response of that OFDM symbol is estimated through linear interpolation. The interpolation is performed in sets, where the first path channel responses are estimated using the first training sample at both ends, and the channel responses for the second path is estimated by interpolating the second training sample from both ends, with the effects of the first path removed. This will carry on till the channel responses of all the paths are estimated.

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

This method can also be extended to the case with known channel statistics. In this case, the channel interpolation used is no longer linear, but depends on the channel statistics.

3.4.2 Stamoulis' Method

This algorithm is quite similar to the KD method described previously. It also estimates the instantaneous time domain channel using interpolation among known points in time. But in this case, the known points in time are within an OFDM symbol. The algorithm uses a few groups of subcarriers to estimate the known points. By using these known points, we are able to employ interpolation to obtain the full time varying channel responses within an OFDM symbol.

We begin by characterizing the discrete time demodulation of subcarrier q , which can be seen as

$$\begin{aligned} y_q &= \frac{1}{N_c} \sum_{n=0}^{N_c-1} \sum_{l=0}^{L-1} h(n, l) s(n - \tau_l) \exp\left(\frac{-j2\pi qn}{N_c}\right) \\ &= \frac{1}{N_c} \sum_{n=0}^{N_c-1} \sum_{k=0}^{N_c-1} \sum_{l=0}^{L-1} h(n, l) d_k \exp\left(\frac{j2\pi(k-q)n}{N_c}\right) \exp\left(\frac{j2\pi k\tau_l}{N_c}\right) \end{aligned} \quad (3.27)$$

where $h(n, l)$ is the discrete representation of $h(t, \tau_l)$. As similar to the channel estimation algorithm described for a non-time varying channel, weighting for interpolation can be employed, which is written as

$$\tilde{h}(n, l) = \sum_{m=0}^{M-1} w_m(n) h(m, l)$$

where $h(m, l)$ is assumed to be the known points in time domain, $w_m(n)$ is the weight for the filter and M is a constant $\gg 1$. By placing the weights and the known channel

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

response back in the original equation, it can be seen as

$$\begin{aligned} y_q &= \frac{1}{N_c} \sum_{n=0}^{N_c-1} \sum_{k=0}^{N_c-1} \sum_{l=0}^{L-1} \sum_{m=0}^{M-1} w_m(n) h(m, l) d_k \exp\left(\frac{j2\pi(k-q)n}{N_c}\right) \exp\left(\frac{j2\pi k \tau_l}{N_c}\right) \\ &= \sum_{l=0}^{L-1} \sum_{m=0}^{M-1} h(m, l) b_l^q(m) \end{aligned} \quad (3.28)$$

where

$$b_l^q(m) = \frac{1}{N_c} \sum_{n=0}^{N_c-1} \sum_{k=0}^{N_c-1} w_m(n) d_k \exp\left(\frac{j2\pi(k-q)n}{N_c}\right) \exp\left(\frac{j2\pi k \tau_l}{N_c}\right) \quad (3.29)$$

Re-writing the above equation into matrix form,

$$y_q = \mathbf{h}^T \mathbf{b}^q \quad (3.30)$$

where

$$\begin{aligned} \mathbf{h} &= [h(0, 0) \cdots h(m-1, 0), h(0, 1) \cdots h(m-1, L-1)]^T \\ \mathbf{b}^q &= \text{vec}(\mathbf{B}^q) \end{aligned}$$

where $\mathbf{B}_{m,n}^q = b_m^q(n)$. The operator $\text{vec}(\mathbf{X})$ is the vector obtained by stacking the column of $\mathbf{X}$ after each other. Let $\mathbf{y} = [y_p \cdots y_q]$ (where $(p, \dots, q) \in \mathbf{P}$) and $\mathbf{P}$ denotes the index for piloted subcarriers, the final equation becomes

$$\begin{aligned} \mathbf{y} &= [\mathbf{b}^0 \cdots \mathbf{b}^{p-1}]^T \mathbf{h} \\ &= \mathbf{B} \mathbf{h} \end{aligned} \quad (3.31)$$

Finally, the estimated time-varying channel responses in time domain can be found by

$$\hat{\mathbf{h}} = \mathbf{B}^\dagger \mathbf{y} \quad (3.32)$$

where $(\cdot)^\dagger$ denotes pseudo-inverse. It must be noted that the pilots' placement will be

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

different from the time-invariant channel case. It is found that the pilots have to be placed in a cluster-comb way [50], as seen in Fig. 3.4. Moreover, the number of clusters has to exceed $K \times m$, as the main objective of each cluster is to provide smoothing effect for the noise and interference so that it can estimate the various instantaneous time domain channel gains.

Figure 3.4: Example of the placement of frequency domain pilot sequences in an OFDM symbol

Similarly, the Stamoulis' channel estimation algorithm can be extended to the case for channel with known fading statistics. It can be seen from (3.29) that the time domain channel can be interpolated using several points, by the term $a_m(n)$. In the case when channel fading statistics are not known, it will become a linear interpolater and interpolate the unestimated points linearly between two known time domain samples. However, for channel with known fading statistics, the weight of $a_m(n)$ can be constructed more optimally, for example, using the Wiener equations.

3.5 Equalization of OFDM Signal in Time-Varying Channel

With the instantaneous channel impulse responses known, the next logical step before demodulation of the received symbol will be to equalize it. In this section, some of the

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

most popular equalization methods (for OFDM systems in time varying channel) are briefly explained. The received signal can be written in matrix form as

$$\mathbf{y} = \mathbf{G}\mathbf{d} + \boldsymbol{\omega} \quad (3.33)$$

where the complex vector $\mathbf{d} = \{d_n\}$ is the transmitted vector of data symbols, $\mathbf{y} = \{y_n\}$ is the received signal at the subcarriers, $\boldsymbol{\omega} = \{\omega_n\}$ is the AWGN noise vector, for $n = 0, \dots, N_c - 1$. In time varying case, OFDM subcarriers will lose their orthogonality. This is reflected in the frequency domain channel matrix, $\mathbf{G}$, where the (m, n) element of $\mathbf{G}$ is written as

$$\mathbf{G}_{m,n} = \{g_{m,n}\}$$

3.5.1 Conventional Methods

These techniques have been applied extensively to both single carrier and multicarrier systems. In this section, their applications on OFDM systems with ICI are discussed.

Matched Filter

The expression for matched filter can be written as [51]

$$\hat{\mathbf{d}} = \mathbf{G}^H \mathbf{y} = (\mathbf{G}^H \mathbf{G})\mathbf{x} + \mathbf{G}^H \boldsymbol{\omega} \quad (3.34)$$

In the time-invariant channel, $\mathbf{G}^H \mathbf{G}$ will become a diagonal matrix, as $\mathbf{G}$ is diagonal. But this assumption is no longer valid for the time-varying channel. Thus, matched filter cannot remove ICI, unless $\mathbf{G}^H \mathbf{G}$ is orthogonal.

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

Least Square (LS)

The classic method of LS [51],[52] can be written as

$$\hat{\mathbf{d}} = \mathbf{G}^\dagger \mathbf{y} \quad (3.35)$$

where $\mathbf{G}^\dagger = (\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}^H$ is a $N_c \times N_c$ matrix that is the pseudo-inverse of $\mathbf{G}$. This will lead to the removal of ICI.

Minimum Mean Square Error (MMSE)

The main idea behind MMSE equalizer [51] is to minimize the function $E[|\hat{\mathbf{d}} - \mathbf{d}|^2]$. Therefore, it is shown that the MMSE equalizer can be implemented as

$$\hat{\mathbf{d}} = (\mathbf{G}^H \mathbf{G} + \sigma_n^2 \mathbf{I}_{N_c})^{-1} \mathbf{G}^H \mathbf{y} \quad (3.36)$$

where $\mathbf{I}_{N_c}$ is the $N_c \times N_c$ identity matrix, and σ_n is the noise variance. As compared to the LS method, the inclusion of σ_n^2 is to reduce the effect of noise enhancement.

3.5.2 Frequency Domain Sub-Optimal method

The full matrix inverse required, as seen in the conventional techniques, $N_c \times N_c$ multiplications. This can be a problem in real time applications, especially if N_c is large. In the frequency domain sub-optimal technique, it tries to reduce the complexity by constructing smaller sub-matrices within $\mathbf{G}$. So instead of inverting the full matrix size $\mathbf{G}$, it inverts the smaller sub-matrix inside. In this way, the computational load will be reduced as the size of the sub-matrix is much smaller than N_c [53],[54],[55],[56],[57].

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

It begins by assuming that some of the ICI terms in the instantaneous frequency domain channel response can be neglected. Thus, by assuming that those non-dominant terms that are far away from the diagonal elements to be zeros, the channel matrix can be seen as

$$\hat{\mathbf{G}} = \begin{bmatrix} g_{0,0} & g_{0,1} & \dots & g_{0,q} & 0 & \dots & \dots & 0 \\ g_{1,0} & g_{1,1} & \dots & \dots & g_{1,q+1} & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & & & & & \\ g_{q,0} & & \ddots & & \ddots & & & \vdots \\ 0 & & \ddots & & \ddots & & & 0 \\ \vdots & & & & \ddots & & g_{N_c-1-q,N_c-1} & \\ & & & & \ddots & & \vdots & \\ 0 & \dots & 0 & g_{N_c-1,N_c-1-q} & \dots & g_{N_c-1,N_c-1} & & \end{bmatrix} \quad (3.37)$$

where $g_{m,n}$ denotes the frequency domain interference of the data at the n subcarrier to the m subcarrier. With this assumption, the sub-matrix can be extracted, as shown, in Fig. 3.5.

Figure 3.5: Extraction of Sub-Matrix from the General Frequency Domain Matrix

The square sub-matrix $\mathbf{G}_n^{sub}$ of size $2q+1$, is assumed to be extracted from $\mathbf{G}$, where

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

n indicates the frequency domain channel the sub-matrix represents. In this case, n will span from q to $N_c - q$. Thus, the received signals can be equalized using LS, shown here as

$$\hat{d}_n = (\mathbf{G}_n^{sub})^\dagger \bar{\mathbf{y}}_n \quad (3.38)$$

where $\bar{\mathbf{y}}_n = [y_{n-q}, \dots, y_{n+q}]^T$. Or as similar to the previous section, the sub-optimal MMSE equalizer [53] can be implemented using

$$\hat{d}_n = [(\mathbf{G}_n^{sub})^H (\mathbf{G}_n^{sub}) + \sigma_n^2 \mathbf{I}_{N_c}]^{-1} (\mathbf{G}_n^{sub})^H \bar{\mathbf{y}}_n \quad (3.39)$$

The equalized signal $\hat{d}_n$ is a scalar now, denoting the equalized signal of subcarrier index n . The number of channel impulse parameters where the equalizer can be applied is from n equals q to $N_c - q$. It indicates that there will be some subcarriers (from 0 to $q - 1$ and $N_c - q + 1$ to $N_c - 1$) that this algorithm fails to apply. The solutions provided by the authors in [54] are that frequency domain channel sub-matrix for those subcarriers can be interpolated using known channel sub-matrices for other subcarriers. So similar equalizer can still be employed using the interpolated channel sub-matrices for those 0 to $q - 1$ and $N_c - q + 1$ to $N_c - 1$ subcarriers. So far, the techniques described in this section is based on serial equalization, and other similar (parallel) techniques of implementing the sub-optimum matrix inversion can be found in [55],[56].

3.5.3 Frequency Domain Successive Interference Cancellation

The basic idea of interference cancellation detectors is to remove the interference from the desired signals. There are two basic schemes, and they are known as Parallel Interference Cancellation (PIC) and Successive Interference Cancellation (SIC). In SIC scheme, each stage of the detector will demodulate the received signal of a certain

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

subcarrier, regenerate it and remove its effects on the rest of the subcarriers. In PIC scheme, all the interference is subtracted in parallel. Nevertheless, it has been shown that the SIC outperforms the PIC [58].

The detection algorithm can be described below.

1. At the step i , detect the remaining undetected subcarriers with the conventional methods, for example, the LS or MMSE.
2. Define the set U_i which contains the index of subcarriers not yet detected till the i -th stage. Determine the subcarrier which has the highest SNR, which can be seen as

$$\arg \max_k (SNR_k) = \arg \max_k (g_{k,k})$$

3. Make a hard decision on that subcarrier and its neighbouring $2q$ subcarriers.
4. Regenerate the signals for the group of subcarriers using the frequency domain channel impulse response.
5. Subtract the regenerated signals from the received signal, $\mathbf{y}_i$, yielding $\mathbf{y}_{i+1}$. Increase the index of the group of subcarriers from U_i to U_{i+1} .
6. Repeat steps 1 to 5 until all the subcarriers have been demodulated or the pre-defined number of iteration is reached.

This scheme, proposed in [58], has been shown to outperform those conventional techniques. Furthermore, they have also shown that the MMSE based SIC performs much better than the LS based SIC.

3.5.4 Time Domain Linear Filter

Most of the detection/equalization algorithms describe in the previous section are to be employed in frequency domain. However, there are time domain filtering schemes in

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

literature and one of the most outstanding proposals is the linear time domain filter can be found in [50]. This time domain linear filter is based on the solution for standard generalized eigenvalue problem. The basic idea of using this solution is to maximize the Signal to Interference plus Noise Ratio (SINR). The filter structure can be seen as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{G}}\mathbf{d} + \tilde{\mathbf{w}} \quad (3.40)$$

where $\tilde{\mathbf{G}}$ is the frequency domain channel responses after filtering and can be written by $\tilde{\mathbf{G}} = \mathbf{Q}\mathbf{W}\mathbf{H}\mathbf{Q}^H$, where $\mathbf{W}$ is the weighting (filtering) matrix applied to the OFDM system in time domain, $\mathbf{Q}$ is the FFT matrix and $\mathbf{H}$ is the channel matrix in time domain. In (3.40), $\tilde{\mathbf{w}}$ is the colored noise, which has the variance as $\sigma_n^2 \mathbf{W}\mathbf{W}^H$. The main idea of this algorithm is to maximize the SINR using the filtering weight, $\mathbf{W}$. The SINR is defined as [50]

$$\text{SINR}_m = \frac{E[|\tilde{g}_{m,m}|^2]}{\frac{\sigma_n^2}{N_c} \text{tr}(\mathbf{W}\mathbf{W}^H) + \sum_{n \neq m} E[|\tilde{g}_{m,n}|^2]} \quad (3.41)$$

where $E[|\tilde{g}_{m,m}|^2]$ is the energy of the desired signal after filtering, $\sum_{n \neq m} E[|\tilde{g}_{m,n}|^2]$ is the interference from other subcarriers after filtering and $\text{tr}(\cdot)$ is the trace operator. The filtered noise power is represented by $\frac{\sigma_n^2}{N_c} \text{tr}(\mathbf{W}\mathbf{W}^H)$. After some manipulations, the SINR can be written as

$$\text{SINR}_m = \frac{\mathbf{w}_m^H \hat{\mathbf{g}}_m \hat{\mathbf{g}}_m^H \mathbf{w}_m}{\mathbf{w}_m^H (\sigma_n^2 \mathbf{I}_{N_c} + \mathbf{R}_m) \mathbf{w}_m} \quad (3.42)$$

where $\hat{\mathbf{g}}_m = [g_{m,0}^*, \dots, g_{m,N_c}^*]^T$ and $\mathbf{w}_m = \mathbf{W}^H \mathbf{Q}^H \mathbf{e}_m$, which $\mathbf{e}_m$ is defined as the m -th unit vector. The matrix $\mathbf{R}_m$ is defined as $\mathbf{H}\mathbf{H}^H - \mathbf{g}_m \mathbf{g}_m^H$. Moreover, it is assumed that $\mathbf{w}_m^H \mathbf{w}_m = 1$, thus, $\text{tr}(\mathbf{W}\mathbf{W}^H) = N_c$.

Therefore, it becomes a optimization problem, as seen as

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

$$\begin{aligned} & \max_{\mathbf{w}_m} \mathbf{w}_m^H \mathbf{h}_m \mathbf{h}_m^H \mathbf{w}_m \\ & \text{subject to } \mathbf{w}_m^H (\sigma_n^2 \mathbf{I} + \mathbf{R}_m) \mathbf{w}_m = 1 \text{ and } \mathbf{w}_m^H \mathbf{w}_m = 1 \end{aligned}$$

This is a generalized eigenvalue problem, whose solution can be determined by

1. Define the Cholesky factorization [59] of $(\sigma_n^2 \mathbf{I}_{N_c} + \mathbf{R}_m) = \mathbf{U} \mathbf{U}^H$
2. Compute the dominant eigenvalue of the matrix $\mathbf{U}^{-1} \mathbf{h}_m \mathbf{h}_m^H \mathbf{U}^{-H}$, and obtain the corresponding eigenvector, $\mathbf{v}_{dom}$.
3. Compute optimal weight $\mathbf{w}_{m,opt} = \xi \mathbf{U}^{-H} \mathbf{v}_{dom}$, where ξ is chosen to make $\mathbf{w}_{m,opt}$ a unit-norm vector.
4. The optimal SINR for subcarrier m can be written as

$$\text{SINR}_{m,opt} = \mathbf{h}_m^H (\sigma_n^2 \mathbf{I}_{N_c} + \mathbf{R}_m)^{-1} \mathbf{h}_m \quad (3.43)$$

It is mentioned in [50] that the performance of the filter is comparable (in term of MSE) with the case when there is no ICI in OFDM.

3.6 Summary

In this chapter, the effects of OFDM in time varying channels are discussed. The amount of interference, in terms of ICI power, is quantified for an OFDM system. Moreover, various time varying channel models are also discussed and their effect on OFDM system is clearly quantified. The results will be used in the subsequent chapters to quantify the level of ICI for the various ICI reduction techniques.

Various ICI mitigating filter/equalizer algorithms found in literatures are also been discussed in this chapter. These algorithms require the knowledge of the instantaneous channel impulse response. Moreover, various channel estimation algorithms for fast time

CHAPTER 3. OFDM IN TIME VARYING CHANNEL

varying channel are also examined in the chapter. These channel estimation algorithms will be compared in detail with the proposed channel estimation algorithm in Chapter 7.

Other ICI mitigation techniques can be found in Chapter 5 and 6. In Chapter 5, the idea of using directional antennae for ICI reduction is proposed and the theoretical analysis of the reduced ICI power are shown. In Chapter 6, the means of coding to reduce ICI is illustrated. Detailed analysis of various coding techniques are also shown.

Chapter 4

Doppler Power Spectral Density Using Directional Antennas

In Chapter 3, the ICI power was derived as a function of the time-domain channel autocorrelation function, $r(f_d T_s)$. Therefore, it can be seen that by modifying autocorrelation function (by limiting the bandwidth of the PSDs) ICI can be reduced.

In this chapter, channel models when directional antennas are employed at either the base station and/or the mobile station are proposed to modify and reduce the bandwidth of the Doppler PSDs. Moreover, a 3-D channel model with directional antenna at the mobile will also be shown.

Nevertheless, we will investigate the modified autocorrelation function for all the channel models when directional antennas are deployed. The modified Doppler PSDs, when directional antennas are employed, will be characterized to obtain an expression for ICI power in Chapter 5. It can be easily shown that the Doppler PSDs are much narrower for the case when directional antennas are used, as compared to the case when omni-directional antennas are employed.

4.1 Doppler Power Spectral Density When Using Directional Antennas for 2-D Model

Signals received from wireless channel normally include multipath components and Doppler effect which shifts the transmitted carrier frequency when the receiver is moving. Thus, signals through wireless channel are distorted and fading occurs both in time and frequency domains. However, the Doppler spread tends to reduce when directional antennas are employed at either the mobile or the base station or both. Smaller value of Doppler spread helps in transforming the fast fading of the channel into a slow fading channel, as described in [60]. The two dimensional (2-D) channel model of the classic Clarke/Jakes' Doppler spread channel using directional antennas will be investigated in this section.

4.1.1 Doppler PSD Function for Directional Antenna at Mobile

Usually, the receiving signals arriving at different Angle-of-Arrival (AOA) at the same frequency. The received signal experiences Doppler shift when the receiver is moving. Furthermore, the different Doppler shifts, with different AOA, cause the fluctuations in the received signals.

Before the fast fluctuation of the channel is quantified, the Doppler frequency shift of an individual signal have to be described, which can be seen by

$$f = f_d \cos(\theta - \varepsilon) \quad (4.1)$$

where f_d is the maximum Doppler shift, and it is given by $f_d = v/\lambda$; v is the velocity

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

of the mobile unit and λ is the carrier wavelength. As seen in Fig. 4.1, the notation θ represents the AOA with respect to the base line, from base station to mobile station and ε denotes the angle between the travelling direction and that base line. With some mathematical manipulations, it can be shown as

$$d\theta = - \left[f_d \sqrt{1 - \left(\frac{f}{f_d} \right)^2} \right]^{-1} df \quad (4.2)$$

Using (4.2) and total power received $P_r = \int G(\theta) p(\theta) d\theta$, the Doppler PSD of the received signal is [41],[40]

$$S(f) = \frac{G(\theta - \varepsilon) p(\theta) + G(-\theta - \varepsilon) p(-\theta)}{f_d \sqrt{1 - \left(\frac{f}{f_d} \right)^2}} \quad (4.3)$$

where $G(\cdot)$ is the gain of the antenna, $p(\cdot)$ is the PDF of AOA. The beam pattern of the directional antenna is defined as

$$G(\theta) = \begin{cases} G_o & \text{if } |\theta| \leq \theta_m \\ 0 & \text{otherwise} \end{cases} \quad (4.4)$$

where G_o is the constant gain for an omni-directional antenna and the beamwidth of the directional antenna is $2\theta_m$. Fig. 4.1 shows the relationship of the directional antenna mounted on the mobile station (which is always pointing to the base station) and the received multipath components with respect to the mobile travelling direction. From (4.4) and Fig. 4.1, it can be seen that the directional antenna only receives the signals from certain angles. For an omni-directional antenna, it is commonly assumed that the AOA is uniformly distributed over $(0, 2\pi)$ [41],[40] at the mobile, and thus $p(\theta) = 1/2\pi$.

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

And with a directional antenna at the receiver, the PDF of AOA can be modified into

$$p(\theta) = \begin{cases} \frac{1}{2\theta_m} & \text{for } |\theta| \leq \theta_m \\ 0 & \text{otherwise} \end{cases} \quad (4.5)$$

where $2\theta_m$ is the beamwidth of the directional antenna.

Figure 4.1: Illustration of geometry of mobile unit

By following the derivation described in [61], it can be generalized into two separate cases. The first case happens when the moving direction ε is within the angle specified by half of the beamwidth, that is $0 \sim \theta_m$ or $(\pi - \theta_m) \sim \pi$. In this case, there will be ‘folding back’ for some range of the PSD. This case will include the situation when the mobile is moving towards the transmitter or away from it. Therefore, the Doppler PSD

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

for the first condition, $S_1(f)$, is seen as

$$S_1(f) = \begin{cases} \frac{G_o}{2\pi f_d \sqrt{1 - \left(\frac{f}{f_d}\right)^2}} & \begin{array}{l} \text{either if } f_d \cos(\theta_m + \varepsilon) < f < f_d \cos(\theta_m - \varepsilon) \\ \text{or if } -f_d \cos(\theta_m - \varepsilon) < f < -f_d \cos(\theta_m + \varepsilon) \end{array} \\ \frac{G_o}{\pi f_d \sqrt{1 - \left(\frac{f}{f_d}\right)^2}} & \begin{array}{l} \text{either if } f_d \cos(\theta_m - \varepsilon) \leq f < f_d \\ \text{or if } -f_d < f \leq -f_d \cos(\theta_m - \varepsilon) \end{array} \\ 0 & \text{otherwise} \end{cases} \quad (4.6)$$

When mobile is travelling in $0 \leq \varepsilon \leq \theta_m$, the range for (4.6) will be

$$f_d \cos(\theta_m + \varepsilon) < f < f_d \cos(\theta_m - \varepsilon)$$

and

$$f_d \cos(\theta_m - \varepsilon) \leq f < f_d$$

For mobile travelling direction $(\pi - \theta_m) \leq \varepsilon \leq \pi$, the range for (4.6) will be

$$-f_d \cos(\theta_m - \varepsilon) < f < -f_d \cos(\theta_m + \varepsilon)$$

and

$$-f_d < f \leq -f_d \cos(\theta_m - \varepsilon)$$

The second condition happens when the vehicle is moving at an angle further away, which is more than θ_m from 0 or π (i.e. $\theta_m < \theta < (\pi - \theta_m)$), where no ‘folding back’ occurs. The Doppler PSD for the second condition, $S_2(f)$, is shown as

$$S_2(f) = \begin{cases} \frac{G_o}{2\pi f_d \sqrt{1 - \left(\frac{f}{f_d}\right)^2}} & \text{if } f_d \cos(\theta_m + \varepsilon) < f < f_d \cos(\theta_m - \varepsilon) \\ 0 & \text{otherwise} \end{cases} \quad (4.7)$$

Fig. 4.2 shows the Doppler PSDs in the two cases, described by (4.6) and (4.7). The

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

beamwidth of the mobile directional antenna, $2\theta_m$ is set to $\pi/3$, which is a practical value. It is assumed that the directional antenna is always pointing directly towards the base station. The f_d is assumed to be 100 Hz. For Fig. 4.2a, the plot of (4.6), ε has the values of $\pi/18$ and $17\pi/18$. Whereas, ε is set to $21\pi/36$ for the plot of (4.7) in Fig. 4.2b. All plots are normalized so $\int_{-\infty}^{\infty} S(f)df = 1$.

When the mobile travelling direction, ε , is near or at the angle 0 or π rad, the resultant Doppler PSDs are either positive or negative. And as observed from Fig. 4.2a, the Doppler PSDs are not continuous functions. In the plot of (4.7), it shows that the Doppler PSDs have a restricted asymmetrical shape as compared to Clarke/Jakes' Doppler PSD.

4.1.2 Doppler PSDs When Both Directional Antennas are Employed

The Doppler PSD when directional antenna is employed at the base station is presented first, before deriving the Doppler PSDs when directional antennas are employed at both mobile and base stations. It is shown in [60] that the AOA is not uniformly distributed, when directional antenna is implemented at the base station. By using the model developed in [60],[62], the PDF of AOA is defined by

$$p(\theta) = \begin{cases} \frac{(D^2 \tan(\phi_b))^2}{I(\sin(\theta) + \cos(\theta) \tan(\phi_b))^2} & \text{if } \theta_1 < |\theta| \leq \theta_2 \\ \frac{R^2}{I} & \text{otherwise} \end{cases} \quad (4.8)$$

where

$$I = 2R^2(\pi + \theta_1 - \theta_2) + 4D \sin(\phi_b) \sqrt{R^2 - D^2 \sin^2(\phi_b)} \quad (4.9)$$

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

(a)

(b)

Figure 4.2: Doppler PSD with Directional Antenna at Mobile

Figure 4.3: Illustration of the AOA at the mobile when a directional antenna is implemented at Base Station

In (4.8) and (4.9), D is the distance between mobile and the base station, and it is assumed that the scatterers around the mobile are located within a circle of radius R . The beamwidth of the antenna at the base station is $2\phi_b$. From (4.3), (4.8) and (4.9), the following equation is derived[60],[62].

$$S_{base}(f) = \begin{cases} \frac{G_o}{f_d \sqrt{1 - \left(\frac{f}{f_d}\right)^2}} \left[p\left(\varepsilon + \left|\arccos\left(\frac{f}{f_d}\right)\right|\right) + p\left(\varepsilon - \left|\arccos\left(\frac{f}{f_d}\right)\right|\right) \right] & \text{if } |f| < f_d \\ 0 & \text{otherwise} \end{cases} \quad (4.10)$$

where $p(\cdot)$ can be found in (4.9). For Fig. 4.4, an antenna of beamwidth $\pi/18$ at the base station is applied to plot the PDF of AOA. The value of D is equal to 3 km whereas the radius, R , of 1 km is adopted. The travelling direction, ε , of $\pi/2$, is adopted for the plot of Doppler PSD in Fig. 4.4. In the figure, the directional antenna is having beamwidth of $2\phi_b > 2\arctan(R/D)$. From the PDF of AOA, it reveals that the AOA

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

is not uniformly distributed.

(a) PDF of AOA with Directional Antenna at Base Station

(b) Doppler PSD with Directional Antenna at Base Station

Figure 4.4: PDF of AOA and Doppler PSD with directional antenna at base station

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

With (4.6), (4.7) and (4.8), the Doppler PSDs when directional antennas are employed at both base and mobile stations are introduced in (4.11) and (4.12). The two cases are the same as that described in Section 4.1.1. The first case will be represented by (4.11), where $0 < \varepsilon < \theta_m$ and $(\pi - \theta_m) < \varepsilon < \pi$. For the second situation, as depicted by (4.12), happens when $\theta_m \leq \varepsilon < (\pi - \theta_m)$.

$$S_1(f) = \left\{ \begin{array}{ll} \frac{G_o}{f_d \sqrt{1 - \left(\frac{f}{f_d}\right)^2}} p\left(\varepsilon - \left|\arccos\left(\frac{f}{f_d}\right)\right|\right) & \begin{array}{l} \text{either if } f_d \cos(\theta_m + \varepsilon) < f < f_d \cos(\theta_m - \varepsilon) \\ \text{or if } -f_d \cos(\theta_m - \varepsilon) < f < -f_d \cos(\theta_m + \varepsilon) \end{array} \\ \frac{G_o}{f_d \sqrt{1 - \left(\frac{f}{f_d}\right)^2}} \left[p\left(\varepsilon + \left|\arccos\left(\frac{f}{f_d}\right)\right|\right) + p\left(\varepsilon - \left|\arccos\left(\frac{f}{f_d}\right)\right|\right) \right] & \begin{array}{l} \text{either if } f_d \cos(\theta_m - \varepsilon) \leq f < f_d \\ \text{or if } -f_d < f \leq -f_d \cos(\theta_m - \varepsilon) \end{array} \\ 0 & \text{otherwise} \end{array} \right. \quad (4.11)$$

When mobile is travelling in $0 \leq \varepsilon \leq \theta_m$, the range for (4.11) will be

$$f_d \cos(\theta_m + \varepsilon) < f < f_d \cos(\theta_m - \varepsilon)$$

and

$$f_d \cos(\theta_m - \theta) \leq f < f_d$$

For mobile travelling direction $(\pi - \theta_m) \leq \varepsilon \leq \pi$, the range for (4.11) will be

$$-f_d \cos(\theta_m - \varepsilon) < f < -f_d \cos(\theta_m + \varepsilon)$$

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

$$\begin{aligned}
 & \text{and} \\
 & -f_d < f \leq -f_d \cos(\theta_m - \varepsilon) \\
 S_2(f) = & \begin{cases} \frac{G_o}{f_d \sqrt{1 - \left(\frac{f}{f_d}\right)^2}} p\left(\varepsilon - \left|\arccos\left(\frac{f}{f_d}\right)\right|\right) & \text{if } f_d \cos(\theta_m + \varepsilon) < f < f_d \cos(\theta_m - \varepsilon) \\ 0 & \text{otherwise} \end{cases} \quad (4.12)
 \end{aligned}$$

Fig. 4.5 is plotted with $f_d = 100$ Hz, $D = 3$ km, radius of scatter, $R = 1$ km and $G_o = 1$. The parameters, ε in (4.11), is set to $\pi/18$ and $17\pi/18$, which are used in plotting of Fig. 4.5a. In the second situation, the parameters, ε , of the value $15\pi/36$ is assigned in Fig. 4.5b. The PDF of AOA, $p(\theta)$, is the same as the one described by (4.8). The beamwidth of the directional antenna at base station, $2\phi_b$, of $\pi/18$ is chosen. It is also a practical value as the base station can afford to have a large area for placement of antenna array. At the mobile station, beamwidth of $2\theta_m$ with $\pi/3$ is applied. It is also assumed that the base station and mobile directional antennas are always pointing directly to each other.

In Fig. 4.5, similar results are seen as compared with Fig. 4.2. However, the AOA for Fig. 4.5 is not longer uniformly distributed, and the distribution is described by (4.8). However, the Doppler PSD does not follow the outline of the classic Clarke/Jakes' Doppler PSD now.

Lastly, although the directional antenna at the mobile is assumed to be steered directly pointing to the base stations in the analysis, simple modifications (by change the span of $\theta_m + \varepsilon$ and $\theta_m - \varepsilon$) can be easily applied to the equations in Section 4.1.2 for the case where the directional antenna at the mobile is steered towards other direction.

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

(a)

(b)

Figure 4.5: Doppler PSD with Directional Antenna at Both Base and Mobile Stations

4.2 Doppler Power Spectral Density When Using Directional Antennas for 3-D Model

As seen from the previous section, that for 2-D model, the PSD changes with respect to the directional antenna. However, the effect can be modified for three dimensional (3-D) model [63],[64],[65]. And as mentioned from the previous chapter, the uniform Doppler spread model is derived from the situations where scatterers are in 3-D. These effect can be modelled as having AOA in both azimuthal and elevation planes.

For the uniform Doppler spread model, it is assumed that the PDF of the AOA for the azimuth plane (AP) is uniform, that is

$$p_{AP}(\theta) = \begin{cases} \frac{1}{2\pi} & \text{for } |\theta| \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad (4.13)$$

As there is a consideration for the elevation plane (EP), the PDF of the EP has to be specified, which can be seen as

$$p_{EP}(\beta) = \begin{cases} \frac{\cos(\beta)}{2} & \text{for } |\beta| \leq \pi/2 \\ 0 & \text{otherwise} \end{cases} \quad (4.14)$$

The notations, β and θ , are illustrated in Fig. 4.6, with the X and Y axis representing the azimuthal plane and the Z axis representing the elevation plane. As the travelling direction is assumed to be only on the azimuthal plane, it is only represented with the angle ε in the azimuthal plane.

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

Figure 4.6: Illustration of the 3-D geometry of the AOA of the receiving signal

With $\beta_{\max} = \pi/2$, the time domain autocorrelation function can be seen as

$$\begin{aligned} r_U(f_d T_s x) &= \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \cos[2\pi f_d T_s x \cos(\theta) \cos(\beta)] p_{AP}(\theta) p_{EP}(\beta) d\beta d\theta \\ &= \text{sinc}(2f_d T_s x) \end{aligned} \quad (4.15)$$

which can be seen from the uniform Doppler spread model in the previous chapter.

4.2.1 Doppler PSD Function for Directional Antenna at Mobile for 3-D Model

Using the same idea from the previous section, the Doppler spread can be restricted using directional antenna. In this case, the PDF of the AOA is no longer the expression

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

shown in (4.13) and (4.14). It can be seen as

$$p_{AP}(\theta) = \begin{cases} \frac{1}{2\theta_m} & \text{for } |\theta| \leq \theta_m \\ 0 & \text{otherwise} \end{cases} \quad (4.16)$$

and

$$p_{EP}(\beta) = \begin{cases} \frac{\cos(\beta)}{\mathcal{N}} & \text{for } \beta_{\min} \leq \beta \leq \beta_{\max} \\ 0 & \text{otherwise} \end{cases} \quad (4.17)$$

where $\mathcal{N}$ is the normalizing factor which fulfilled

$$\mathcal{N} = \int_{\beta_{\min}}^{\beta_{\max}} \cos(\beta) d\beta$$

and $2\theta_m$ is the azimuthal beamwidth of the directional antenna and $\beta_{\max} \sim \beta_{\min}$ is the range of angle in elevation plane that the directional antenna can receive the signal. Thus, the general equation for the time domain autocorrelation function can be seen as

$$r(f_d T_s x) = \int_{-\pi/2}^{\pi/2} \int_{-\pi}^{\pi} G(\theta, \beta) \cos[2\pi f_d T_s x \cos(\theta) \cos(\beta)] p_{AP}(\theta) p_{EP}(\beta) d\theta d\beta \quad (4.18)$$

where $G(\theta, \beta)$ is the directional antenna gain. As assumed previously, the beam pattern of the directional antenna is assumed to be ideal, which can be seen as

$$G(\theta, \beta) = \begin{cases} G_0 & \text{for } |\theta| \leq \theta_m \text{ and } \beta_{\min} \leq \beta \leq \beta_{\max} \\ 0 & \text{otherwise} \end{cases} \quad (4.19)$$

Eq. (4.19), however, cannot be extended to the case where there are directional antennas mounted at the base station, due to the complexity in modelling these channels. Thus, in this thesis, we shall limit the 3-D modelling to the case where directional antenna is only deployed at the mobile station. The PSD of the 3-D Doppler spectrum [65] can

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

be seen as

$$S^{3D}(f) = G_0 \int_{-\theta_m}^{\theta_m} \int_{\beta_{\min}}^{\beta_{\max}} p_{AP}(\theta) p_{EP}(\beta) \delta[f \pm f_d \cos(\beta) \cos(\theta)] d\theta d\beta \quad (4.20)$$

Bringing in the movement of the vehicle into the equation (as seen in Fig. 4.6, assuming that it moves only in the azimuthal plane), it can be seen as

$$S^{3D}(f) = G_0 \int_{-\theta_m}^{\theta_m} \int_{\beta_{\min}}^{\beta_{\max}} p_{AP}(\theta) p_{EP}(\beta) \delta[f \pm f_d \cos(\beta) \cos(\varepsilon - \theta)] d\theta d\beta \quad (4.21)$$

An example of (4.21) is shown in Fig. 4.7. In Fig. 4.7a, the direction on the vehicle movement, $\varepsilon = \pi/2$, with $\theta_m = \pi/6$ for both directional antennas. The maximum Doppler frequency, f_d is set to 100Hz. The elevation angle is different between the two directional antennas, with antenna 1 having $\beta_{\min} = \pi/4$ and $\beta_{\max} = \pi/2$, while antenna 2 has $\beta_{\min} = 0$ and $\beta_{\max} = \pi/4$. For Fig. 4.7b, the beamwidth of the directional antennas in the azimuthal plane is set to $\theta_m = \pi/6$ with the beamwidth in the elevation plane fixed at $\beta_{\min} = \pi/4$ and $\beta_{\max} = \pi/2$. The first directional antenna is operating in the case where the vehicle is moving in $\varepsilon = 0$, while the second is operating in the situation where $\varepsilon = \pi$.

For the case of 3-D channel modelling with directional antenna at base station, it is complicated and is beyond the scope of this thesis. Nevertheless, it can be a future research topic.

4.3 Summary

In this chapter, 2-D and 3-D channel models are shown when directional antennas are employed. The 2-D channel models with directional antennas are intended for channel

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

(a)

(b)

Figure 4.7: Doppler PSD with Directional Antenna for 3-D model

CHAPTER 4. DOPPLER POWER SPECTRAL DENSITY USING DIRECTIONAL ANTENNAS

surrounded by relative low-rise building. For the case of the 3-D model, it will be more suited for environments with relative high-rise building.

And it has been shown in this chapter, the directional antennas are able to reduce the Doppler PSDs for both the 2-D and 3-D models. Therefore, it is possible to slow down the fast time varying effect of the channel using directional antennas. Nevertheless, the channel is still fast fading for some values of the vehicle travelling direction because the Doppler PSDs are frequency shifted. Thus, frequency offset corrections have to be performed for these cases so that they truly become slow time varying channels.

The idea of slowing down the time varying channel can be extended to the OFDM systems. The evaluation of ICI reduction in an OFDM system, using directional antennas, will be investigated in the next chapter.

Chapter 5

ICI Reduction for OFDM System using Directional Antennas

As seen in the previous chapters, OFDM is very effective in mitigating the effects of delay spread due to the channel. However, the performance is compromised when the channel exhibits time varying distortions, which result in the loss of orthogonality among sub-carriers in OFDM systems. This problem has been investigated by many researchers [7],[42],[6] and various techniques have been suggested to circumvent this problem [66],[67],[68],[54],[69],[70]. However, previous investigations on the ICI power caused by Doppler spreading are confined to omni-directional antennas or uniform distribution of AOA. It has been shown in Chapter 4 that directional antennas (or smart antennas) can reduce the amount of Doppler spread in single-carrier systems. In this chapter, the amount of reduction in ICI power of an OFDM system with non-uniform distribution of AOA is investigated.

5.1 Computing ICI Power in terms of PDF of AOA

For convenience, (3.21) in Chapter 3 which defines the ICI power at the k -th subcarrier [7],[42] is repeated here, and can be written as

$$\begin{aligned} P^{ICI} &\triangleq E \left| \sum_{n=0, n \neq k}^{N_c-1} g_{k-n} d_n \right|^2 \\ &= 1 - \int_{-1}^{+1} (1 - |x|) r(f_d T_s x) dx \end{aligned} \quad (5.1)$$

By using the above equation, it will be shown in this sub-section that the ICI power can be represented in terms of AOA.

5.1.1 In Rayleigh Environment

With the total received power normalized to unity, the correlation function of the Jakes/Clarke 2-D model [40],[41] with unknown PDF of AOA can be shown as

$$r(f_d T_s x) = \int_{-\pi}^{+\pi} p(\theta) \exp\{j2\pi f_d T_s x \cos(\theta)\} d\theta \quad (5.2)$$

Substituting (5.2) into (5.1), the ICI power is shown to be

$$\begin{aligned} P^{ICI} &= 1 - \int_{-1}^{+1} \int_{-\pi}^{+\pi} (1 - |x|) p(\theta) \exp\{j2\pi f_d T_s x \cos(\theta)\} d\theta dx \\ &= 1 - \int_{-\pi}^{+\pi} p(\theta) \text{sinc}^2[f_d T_s \cos(\theta)] d\theta \end{aligned} \quad (5.3)$$

where f_d is the maximum Doppler shift and $p(\theta)$ is the PDF of AOA of the multipath components, which is normalized to $\int_{-\pi}^{+\pi} p(\theta) d\theta = 1$.

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

It is seen from Fig. 4.1 the relationship between the mobile travelling direction, ε , and AOA of the multipath components, θ , with respect to the base line between base and mobile stations. However, when frequency compensation is employed, there should be a parameter that changes the carrier frequency at the receiver. So bringing in these parameters into the equation, (5.3) can be re-written as

$$P^{ICI} = 1 - \int_{-\pi}^{+\pi} p(\theta) \text{sinc}^2 \{f_d T_s [\cos(\varepsilon - \theta) - \zeta_{ray}]\} d\theta \quad (5.4)$$

where ζ_{ray} is the correction factor for frequency compensation algorithm that minimizes the P^{ICI} due to Doppler spread. The PDF of AOA can be applied from (4.5), (4.8) or both after normalized. However, for simple beam pattern of $p(\theta)$ as in (4.5), ζ_{ray} can be easily estimated by (5.5). The parameters f_0 and T_s are assumed to be the same in both transmitter and receiver. However, by observing the resulted Doppler PSD (from Chapter 4, Fig. 4.2) for situations where directional antenna is mounted at the mobile, it is found that ζ_{ray} can be estimated through

$$\zeta_{ray} = \cos \left(\frac{\theta_1 + \theta_2}{2} \right) \quad (5.5)$$

where θ_1 and θ_2 are the maximum and minimum angle that the directional antenna is able to receive signal from, which has the range of $[0, \pi]$. If the angles are over the range, one of the θ will be either 0 or π . For example, if the ideal beam pattern is $\varepsilon - \theta_m = -2\pi/18, \varepsilon + \theta_m = 2\pi/18$, the angles will be $\theta_1 = 0$ and $\theta_2 = 2\pi/18$. Through the results obtained from numerical method, it is found that the estimation using (5.5) is quite accurate. Thus, (5.5) can be employed in practical implementation. The numerical results of (5.4) will be presented in Section 5.2.

5.1.2 In Rician Environment

In this section, the correlation equations for both single and multiple Rician channels will be derived. For a mixture of multiple Rician and Rayleigh channels, some of the Rician K -factors are set to be zero.

Single Dominating Path

In a single discrete Rician channel, its time domain impulse response [71] will be

$$\begin{aligned}
 h(T_s x) = & \frac{1}{\sqrt{K+1}} \lim_{B \rightarrow \infty} \left\{ \frac{1}{\sqrt{B}} \sum_{b=1}^B a_b \exp\{j[2\pi f_d T_s x \cos(\theta_b) + \phi_b]\} \right\} \\
 & + \sqrt{\frac{K}{K+1}} \exp\{j[2\pi f_d T_s x \cos(\theta^0) + \phi^0]\}
 \end{aligned} \tag{5.6}$$

where the Rician K -factor [72],[64],[73] is defined as

$$K = \frac{\text{fixed component power}}{\text{diffused component power}} = \frac{P}{\sigma^2} \tag{5.7}$$

and θ_b is the angle between the incoming wave and the mobile travelling direction, which has a distribution defined by $p(\theta)$, where $p(\theta)$ is the PDF of AOA of the diffused components. The phase component of the channel ϕ_b is a random variable that is uniformly distributed on $(-\pi, \pi]$ and the gain a_b is normally assumed to be equal for all b [40]. θ^0 is the AOA of the fixed Rician component and ϕ^0 is the phase of the signal associated with it.

It is assumed that the diffused component and the fixed Rician component are uncorrelated. Thus, the correlation of the Rician channel can be derived from [71], and

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

it is seen as

$$r_{rice}(f_d T_s x) = \frac{1}{K+1} \int_{-\pi}^{+\pi} p(\theta) \exp\{j2\pi f_d T_s x \cos \theta\} d\theta + \frac{K}{K+1} \exp\{j2\pi f_d T_s x \cos \theta^0\} \quad (5.8)$$

However, it is noted that gain of the antenna will affect the received power, thus the fixed component power as in (5.7) will be $G(\theta^0)P$, where $G(\theta^0)$ is the normalized gain of the antenna in the θ^0 direction. The diffused component power will be treated as in P^{ICI} , and the total power received will be unity. Thus, it is seen here as

$$K = \frac{G(\theta^0)P}{\sigma^2} \quad (5.9)$$

Multiple Dominating Rician Paths

However, if there are N_r fixed Rician paths, the channel impulse response [72] will be

$$h(T_s x) = \frac{1}{N_r} \sum_{n=1}^{N_r} h_n(T_s x - \tau_n) \quad (5.10)$$

where

$$\begin{aligned} h_n(T_s x - \tau_n) = & \frac{1}{\sqrt{K_n + 1}} \lim_{B \rightarrow \infty} \left\{ \frac{1}{\sqrt{B}} \sum_{b=1}^B a_b \exp\{j[2\pi f_d T_s x \cos(\theta_b) + \phi_b]\} \right\} \\ & + \sqrt{\frac{K_n}{K_n + 1}} \exp\{j[2\pi f_d (T_s x - \tau_n) \cos(\theta_n^0) + \phi_n^0]\} \end{aligned} \quad (5.11)$$

It is known from [64], that each of the fixed Rician components will be Doppler shifted by $f_d \cos(\theta_n^0)$, which is different from the case in [72]. Thus, it can be assumed that they are just slightly correlated, which can be neglected. This assumption is rea-

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

sonably true when each OFDM sub-carrier is considered as an identical, independently distributed narrow-band channel, with large value of $f_d T_s$ and large separation distance between the transmitter and receiver [74] (typical in marco-cell situation). Therefore, the generalized correlation equation of the channel response can be written as

$$r_{rice}(f_d T_s x) = \frac{1}{N_r} \sum_{n=1}^{N_r} \left\{ \frac{1}{K_n + 1} \int_{-\pi}^{+\pi} p(\theta) \exp[j2\pi f_d T_s x \cos(\theta)] d\theta \right. \\ \left. + \frac{K_n}{K_n + 1} \exp[j2\pi f_d T_s x \cos(\theta_n^0)] \right\} \quad (5.12)$$

Defining the K -factor as

$$K_n = \frac{G(\theta_n^0) P_n}{\sigma_n^2} \quad (5.13)$$

As similar to the single Rician path case, each of the received fixed Rician components will be affected by the beam pattern of the antenna. Therefore, it is possible to filter out those paths that cannot be easily frequency compensated. Using (5.12) into (5.1) with the frequency compensation parameter, ζ_{rice} , and mobile moving direction, ε , the ICI power becomes

$$P^{ICI} = 1 - \frac{1}{N_r} \sum_{n=1}^{N_r} \left\{ \frac{1}{K_n + 1} \int_{-\pi}^{+\pi} p(\theta) \text{sinc}^2 \{f_d T_s [\cos(\varepsilon - \theta) - \zeta_{rice}]\} d\theta \right. \\ \left. + \frac{K_n}{K_n + 1} \text{sinc}^2 \{f_d T_s [\cos(\varepsilon - \theta_n^0) - \zeta_{rice}]\} \right\} \quad (5.14)$$

It is shown in the previous section that the frequency compensating parameter of a Rayleigh channel can be estimated by (5.5). However, in this case, it has to be modified for the mixed Rician and Rayleigh channel, which can be easily approximated to

$$\zeta_{rice} = \frac{1}{N_r} \sum_{n=1}^{N_r} \left\{ \frac{1}{K_n + 1} \cos\left(\frac{\theta_1 + \theta_2}{2}\right) + \frac{K_n}{K_n + 1} \cos(\varepsilon - \theta_n^0) \right\} \quad (5.15)$$

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

The numerical results of (5.14), using frequency compensated using ζ_{rice} from (5.15), will be shown in Section 5.3.

5.1.3 3-D Channel Environment

The time domain autocorrelation function for a 3-D model [63],[65] can be written as

$$r_U(f_d T_s x) = \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \cos[2\pi f_d T_s x \cos(\varepsilon - \theta) \cos(\beta)] p_{AP}(\theta) p_{EP}(\beta) d\beta d\theta \quad (5.16)$$

where $p_{AP}(\theta)$ and $p_{EP}(\beta)$ can be found in (4.16) and (4.17) respectively. Substituting (5.16) into (5.1), the ICI power is given by

$$\begin{aligned} P^{ICI} &= 1 - \int_{-1}^{+1} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \cos[2\pi f_d T_s x \cos(\varepsilon - \theta) \cos(\beta)] p_{AP}(\theta) p_{EP}(\beta) d\beta d\theta \\ &= 1 - \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} p_{AP}(\theta) p_{EP}(\beta) \text{sinc}^2 [f_d T_s \cos(\varepsilon - \theta) \cos(\beta)] d\beta d\theta \end{aligned} \quad (5.17)$$

With ζ_{3D} , which is the frequency compensation parameter that minimizes the P^{ICI} due to Doppler spread, (5.17) is rewritten as

$$P^{ICI} = 1 - \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} p_{AP}(\theta) p_{EP}(\beta) \text{sinc}^2 \{f_d T_s [\cos(\varepsilon - \theta) \cos(\beta) - \zeta_{3D}]\} d\beta d\theta \quad (5.18)$$

As in the case of the 2-D model, the frequency correction factor, ζ_{3D} , that minimized the ICI power can be written as

$$\zeta_{3D} = \cos\left(\frac{\theta_1 + \theta_2}{2}\right) \cos\left(\frac{\beta_1 + \beta_2}{2}\right) \quad (5.19)$$

where θ_1 , θ_2 and β_1 , β_2 is the minimum, maximum azimuthal and elevation angle that the directional antenna are receiving the signal from, respectively. The numerical results will be presented in Section 5.4.

5.2 Numerical results of the ICI Power When Using Directional Antenna in the 2-D Rayleigh Environment

5.2.1 ICI Power When Using Directional Antenna at the Base Station in Rayleigh Environment

The PDF of AOA when ideal directional antenna is mounted at the base station is given in Chapter 4. Fig. 5.1 is obtained using (4.9) and (5.4). In this situation, frequency compensation, similar to (5.5), will not be able to work, just by observing the resulted Doppler PSD (Fig. 4.4).

It is examined from the plot that P^{ICI} may not decrease when a directional antenna is deployed at the base station. P^{ICI} even increases for $\varepsilon = 0$ when frequency compensation is applied. From the PDF of AOA plot in Fig. 4.4a, it can be seen that there are two peaks that occur around 0 and $\pm\pi$. It means that the majority of the energy will be concentrated at both ends of the Doppler PSD. In this case, it is impossible to compensate the Doppler spread by just applying frequency compensation. To remedy this, other methods [66],[67],(or results presented in Chapter 6) or equalization [54],[69],[50], (or the algorithm presented in Chapter 7) must be implemented.

The ICI power, P^{ICI} , reduces by a few dB for $\varepsilon = \pi/2$, even without frequency

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

Figure 5.1: P^{ICI} when Directional Antenna is employed at the Base Station. Beamwidth of Directional Antenna, $2\beta_b = \pi/18$ and $R=1\text{km}$, $D=3\text{km}$

compensation. In this situation, as well as all situations of Rayleigh channel in this chapter where $\varepsilon = \pi/2$, there is no need for frequency compensation as the original carrier frequency is at an optimum value. The Doppler spread will be concentrated around the original carrier frequency, thus causing the P^{ICI} to decrease.

5.2.2 ICI Power When Using Directional Antennas at Both Base and Mobile Stations in Rayleigh Environment

The PDF of AOA, $p(\theta)$, can be assumed to be directly related to the beam pattern of the directional antenna, without any loss of generality. Thus, when ideal directional

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

antenna is mounted at the mobile [41],[40], it can be shown as

$$p(\theta) = \begin{cases} \frac{1}{2\theta_m} & \text{if } |\theta| < \theta_m \\ 0 & \text{otherwise} \end{cases} \quad (5.20)$$

where $2\theta_m$ is the beamwidth of the directional antenna at the mobile. Similarly, the gain of the directional antenna at the mobile is assumed to be constant within the beamwidth $2\theta_m$ and zero outside.

Fig. 5.2 is plotted using (5.4) and (5.20). It can be noticed from Fig. 5.2a, that the P^{ICI} increases when no frequency compensation algorithm is applied. It is because the Doppler spreading occurs around $+f_d$, as seen in Fig. 4.2. However, P^{ICI} drops substantially when frequency compensation is implemented, thus bringing the carrier frequency towards $+f_d$. This clearly brings out the importance of frequency compensation when using directional antenna at the mobile.

In Chapter 4, it has been shown that the amount of Doppler spread is not always constant at different ε . The spreading is maximum when mobile is travelling perpendicular to the direction of the antenna, $\varepsilon \approx \pi/2$ and is minimum when the vehicle is moving in the direction towards the base station or away from it, i.e. $\varepsilon = 0$ or π . In Fig. 5.2b, the bounds for P^{ICI} when a directional antenna is mounted at the mobile with frequency compensation are presented. The results show that the maximum amount of P^{ICI} occurs when $\varepsilon \approx \pi/2$ and the minimum P^{ICI} occurs at $\varepsilon = 0$ or π . It is good to note that the optimum ζ_{ray} equals to 0.991 for $\varepsilon = 0$, 0 for $\varepsilon = \pi/2$ and -0.991 for $\varepsilon = \pi$. Thus, the amount of P^{ICI} can vary between 13.5dB to as much as 36.6dB (Fig. 5.2b).

When using directional antennas at both mobile and base stations, the PDF of AOA, $p(\theta)$, is derived by multiplying (4.8) with (5.20). Thus, by applying (4.8), (5.4)

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

(a) P^{ICI} when $\varepsilon = 0$

(b) Comparing P^{ICI} with Different ε

Figure 5.2: P^{ICI} when Directional Antenna of Beamwidth $2\theta_m = \pi/6$ is deployed at the Mobile

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

and (5.20), Fig. 5.3 is plotted.

Figure 5.3: P^{ICI} when Directional Antennas are implemented at the Both Base and Mobile Stations, with $\varepsilon = 0$, Beamwidth of Directional Antennas, $2\beta_b = \pi/18$ and $2\theta_m = \pi/6$ and $R = 1\text{km}$, $D = 3\text{km}$

It can be seen from Fig. 5.3 that the P^{ICI} curves are similar to Fig. 5.2a, because the PDF of AOA $p(\theta)$, in this case, is approximately equal to (5.20). Therefore, by employing directional antenna at the mobile, along with frequency compensation, will greatly improve the performance of an OFDM system, while there is limited advantage in using a directional antenna at the base station. Using (5.5) to estimate the frequency compensation parameter, ζ , in this section, it is found to be approximately equal to 0.991, as seen in Fig. 5.2.

5.3 Numerical Results of the ICI Power When Using Directional Antennas at Mobile Station in 2-D Rician Environment

In this section, an ideal beam pattern at mobile station is assumed, similar to the assumption made in Section 5.2. In this case, the beamwidth is $2\pi/9$. Thus, $G(\theta)$ is also constant within the beamwidth and zero outside. Based on the equations derived in Section 5.1.2, the P^{ICI} is plotted according three scenarios, namely, multiple Rician channel with 5 equal power paths, multiple Rician channel with 5 unequal power paths, 1 dominate path Rician channel. For the first scenario, as seen in Table 5.1, the 5 Rician paths are of equal power. For the omni-directional antenna case, path 5 is not removed. But when directional antenna is implemented, the signal coming from path 5 is being attenuated as it falls outside the beamwidth, thus $K'_5 = 0$. Path 6 represents the pure Rayleigh channel. The numerical results of ζ for Situation 1 is shown in the plot, whereas the estimated ζ using (5.15) for omni-directional antenna and beamforming are 0.510 and 0.971 respectively. The plot is presented in Fig. 5.4.

	Path1	Path2	Path3	Path4	Path5	Path6
K_n	10	10	10	10	10	0
K'_n	10	10	10	10	0	0
θ_n^0	0.000	0.3491	-0.3316	0.2618	-1.3962	–

Table 5.1: Scenario 1: Equal power (K-factor) for Different Rician Paths

For the second scenario, as seen in Table 5.2, the 5 Rician paths are of different power and the signal coming from path 5 is also being attenuated when directional antenna is employed, thus $K'_5 = 0$. Path 6 is also representing the Rayleigh channel. The estimated frequency compensation parameter, ζ , using (5.15) are 0.624 and 0.984

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

Figure 5.4: P^{ICI} of Scenario 1

for employing omni-directional and directional antenna respectively. The plot is shown in Fig. 5.5.

	Path1	Path2	Path3	Path4	Path5	Path6
K_n	10	5	4	1	3	0
K'_n	10	5	4	1	0	0
θ_n^0	0.000	0.3491	-0.3316	0.2618	-1.3962	—

Table 5.2: Scenario 2: Different power (K-factor) for Different Rician Paths

In the third scenario, there is only one Rician path, which has a K-factor of 10, arriving at $\pi/36$ rad. The estimated ζ are 0.906 and 0.995 for omni-directional and directional antenna respectively. The plot can be seen in Fig. 5.6.

Comparing the plots with that of a pure Rayleigh channel, as seen in Fig. 5.2, it is found that the improvement is less for scenario 1 (Fig. 5.4) and scenario 2 (Fig. 5.5). This is because some of the Rician components cannot be compensated fully and will

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

Figure 5.5: P^{ICI} of Scenario 2

Figure 5.6: P^{ICI} of Scenario 3

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

contribute more to the P^{ICI} . However, for the third scenario (Fig. 5.6), the improvement is greater than the pure Rayleigh channel. In this situation, there is only one Rician path, which can be compensated easily. Lastly, from the plots of all the 3 situations, it is found that the ICI power can be reduced significantly by employing directional antenna, when compared to those cases where only omni-directional antenna is employed.

For the case where directional antenna is employed at the base station, no readily channel model for Rician is available. Thus, to really derive the results for Rician channel with directional antenna either at base station, or at both base and mobile stations required much investigation into the Rician channel model, especially the channel with directional antenna at the base station. However, it is beyond the scope of the thesis and will be considered as a future extension to the work of this thesis.

5.4 Numerical results of the ICI Power When Using Directional Antenna in the 3-D Rayleigh Environment

With the directional antenna mounted at the mobile, the ICI power of the OFDM system is plotted in Fig. 5.7 and Fig. 5.8, with the consideration of the AOA in a 3-D channel model. Fig. 5.7 is plotted with directional antenna in the azimuthal plane of $\theta_m = \pi/6$ and in the elevation plane of $\beta_{\min} = \pi/4$, $\beta_{\max} = \pi/2$. The vehicle moving direction is set to $\varepsilon = 0$.

It can be seen from Fig. 5.7 that the ICI power with the directional directional antenna is lower than that of the omni-directional antenna. This is different from the case of the 2-D model, where the ICI power for the directional antenna with frequency

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

Figure 5.7: P^{ICI} of of the 3-D model with vehicle moving direction, $\epsilon = 0$

compensation, $\zeta_{3D} = 0$ is larger than that of the omni-directional antenna. This is because in the 3-D model, the elevation pointing angle of the directional antenna does have some effects. In this situation, the elevation angle helps to reduce the Doppler spread of the channel, which can be seen from the Doppler PSD obtained in the previous chapter. But it can be seen from the same figure that the effect of the frequency compensation, ζ_{3D} , is crucial, in reducing the ICI power, when directional antenna is employed.

In Fig. 5.8, there are two different directional antennas employed. Both the directional antennas are pointing in the azimuth plane of $\theta_{\min} = -\pi/6$, $\theta_{\max} = \pi/6$. The first directional antenna is pointing in the direction of $\beta_{\min} = 0$, $\beta_{\max} = \pi/4$ in the elevation plane, whereas the second directional antenna is pointing at the direction of $\beta_{\min} = \pi/4$, $\beta_{\max} = \pi/2$ in the elevation plane. The vehicle moving direction is set to $\epsilon = \pi/2$. For the case of both antennas in Fig. 5.8, $\zeta_{3D} = 0$ is the optimum value.

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

Figure 5.8: P^{ICI} of the 3-D model with vehicle moving direction, $\varepsilon = \pi/2$, with 2 different directional antennas

It can be seen from Fig. 5.8 that the effect of elevation pointing angle of the directional antenna is critical in the ICI power reduction. As seen in Fig. 4.7, the amount of Doppler spread is smaller when the elevation pointing angle is close to $\pi/2$, as seen in the case of the second directional antenna. Thus, the ICI power for the second directional antenna is lower than the first. From the case of both 2-D and 3-D models, the ICI power can be reduced significantly when directional antennas (or directive beam/smart antenna) are employed at the mobile.

5.5 Application of Angular Diversity using Linear Uniform Array in Rayleigh Environment

Antenna arrays have been shown to increase the capacity of wireless communication systems. By applying the PDF of AOA, $p(\theta)$, as the equivalent of the array pattern, it has been shown in the previous section that beamforming can reduce the P^{ICI} at the mobile. However, with one narrow beam, the improvement [75],[76] will be limited. Furthermore, it has been shown practically, by Perini [77],[78] and others, that the effects of angular diversity are similar to space diversity, especially in the dense urban area.

In this section, Doppler compensation using antenna array for OFDM system [79] is shown. Furthermore, angular diversity is applied to OFDM system using multiple beamforming, as seen in the system setup in Fig. 5.9. Various beams will be pointing towards different directions, and the signal from each beam will be combined using Equal Gain Combining (EGC). While achieving the advantage of diversity in the system, SNR will be maximized. It will be shown that the combined SNR will be maximum when each of the beams is designed to have equal Doppler spread. Thus, the amount of interference (or Signal to Interference Ratio, SIR) between each beam will be similar. Even by using Maximum Ratio Combining (MRC), because of the similar SIR among each branch, the results will be the same as EGC.

A method to calculate the beam interval (or beamwidth at different steering angle), to achieve equal Doppler spread, will also be shown in this chapter. The problem of estimating the frequency compensating parameters for different beamwidths and steering angles will also be addressed.

Figure 5.9: Setup of the Angular Diversity OFDM system

5.5.1 Diversity Combining

In this section, the concept of diversity using multi-beam will be introduced. The SIR when diversity combining [61],[40] is utilized (by assuming the interference is white), can be written as

$$SIR = \frac{\gamma^2}{2W} \quad (5.21)$$

where $\gamma = \sum_{n=1}^{N_d} \mu_n \gamma_n$ and $W = \sum_{n=1}^{N_d} \mu_n^2 W_n$. N_d is the total number of diversity (or beams), μ_k is dependent on the diversity combining scheme, γ_n is the received signal level and W_n is the interference and noise power of one beam. In the analysis, it is assumed that the AWGN is negligible. By applying (5.4) as W_n , W is defined as

$$W = \sum_{n=1}^{N_d} \mu_n^2 P_{ICI}^n \quad (5.22)$$

However, in this section P_{ICI} in (5.4) is redefined. Without any loss in generality,

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

the 0 rad is placed as the mobile travelling direction, thus, it is re-written here as

$$P^{ICI} = 1 - \int_{-\pi}^{+\pi} p(\theta) \text{sinc}^2 \{f_d T_s [\cos(\theta) - \zeta]\} d\theta \quad (5.23)$$

and W becomes

$$W = \sum_{n=1}^{N_d} \mu_n^2 \left[1 - \int_{\theta_{n-1}}^{\theta_n} p(\theta) \text{sinc}^2 \{f_d T_s [\cos(\theta) - \zeta_n]\} d\theta \right] \quad (5.24)$$

and θ_{n-1} and θ_n are shown in Fig. 5.10, which correspond to the minimum and maximum angle within the beamwidth of the ideal array pattern. It also depends on the number of antenna elements and the geometry of the array.

Figure 5.10: Geometry showing the relationship between θ , ψ and the antenna separation distance when ULA based smart antenna is used

In order to simplify analysis, EGC will be implemented, i.e. $\mu_n=1$. Using ideal array pattern, $p(\theta) = \frac{1}{\theta_n - \theta_{n-1}}$, (5.24) becomes

$$W = \sum_{n=1}^{N_d} \left[1 - \frac{1}{\theta_n - \theta_{n-1}} \int_{\theta_{n-1}}^{\theta_n} \text{sinc}^2 \{f_d T_s [\cos(\theta) - \zeta_n]\} d\theta \right] \quad (5.25)$$

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

For EGC to be the optimum combining technique [61],[40], the amount of received signal power (r_n^2) to interference power (W_n), SIR of r_n^2/W_n , (or $r_n^2/(P^{ICI})_n$) from each branch should be equal. When there are equal Doppler spread in each beam, the amount of W_n in each branch is approximately the same. As the total received power is assumed to be unity when deriving (5.23), the received wanted signal power will be similar when all the interference powers are equal, thus, all r_n equal to r_o . The overall SIR can be expressed as

$$\gamma = \frac{K r_o^2}{2W_o} \quad (5.26)$$

where W_o is the ICI power from any one branch or beam and r_o^2 denotes the received signal power from any one branch. The notation K indicates the total number of branches.

5.5.2 Equal Doppler Spread and Frequency Compensating Parameters Computation

The amount of Doppler spreading depends on the beamwidth, which are denoted as θ_n and θ_{n-1} in Fig. 5.10. Thus, the amount of Doppler spread can be expressed as

$$f_d \cos(\theta_n) < f < f_d \cos(\theta_{n-1}) \quad (5.27)$$

Hence, to achieve equal Doppler spread criterion, (5.28) has to be satisfied.

$$\begin{aligned} \cos(\theta_{N_d}) - \cos(\theta_{N_d-1}) &= \cos(\theta_{N_d-1}) - \cos(\theta_{N_d-2}) \\ &= \cos(\theta_{N_d-2}) - \cos(\theta_{N_d-3}) \\ &= \vdots \\ &= \cos(\theta_1) - \cos(\theta_0) \end{aligned} \quad (5.28)$$

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

The equation shows that the beamwidth will be wider around 0 rad and π rad, which can be easily implemented by uniform linear array (ULA). However, ULA suffers from the effect of angular ambiguity [80]. In this case, the W_n contribution in $0 \sim \pi$ is similar to that where θ ranges from $0 \sim -\pi$. Therefore, the analysis will focus on the region of $[0, \pi)$ and set $\theta_{N_d} = \pi$ and $\theta_0 = 0$.

N_d	2	3	4	5	6	7
θ_0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
θ_1	1.5708	1.2310	1.0472	0.9273	0.8411	0.7752
θ_2	3.1416	1.9106	1.5708	1.3694	1.2310	1.1279
θ_3		3.1416	2.0944	1.7722	1.5708	1.4274
θ_4			3.1416	2.2143	1.9106	1.7141
θ_5				3.1416	2.3005	2.0137
θ_6					3.1416	2.3664
θ_7						3.1416

Table 5.3: Beam Intervals Position for Equal Doppler Spread (in rad)

Therefore, the value of $[\cos(\theta_{N_d}) - \cos(\theta_0)]$ is equal to -2 . From (5.28), it can be seen that the beam interval is divided evenly into K intervals. So, for every interval, $[\cos(\theta_n) - \cos(\theta_{n-1})] = -2/N_d$. As $\cos(\theta_0) = 1$, $\cos(\theta_1) = \frac{N_d - 2}{N_d}$ and so on. Therefore, the beam interval, θ_k , can be easily obtained by solving (5.29) and the solutions are shown in Table 5.3.

$$\theta_n = \arccos\left(\frac{N_d - 2n}{N_d}\right), \quad n = 0, 1, \dots, N_d \quad (5.29)$$

With the beam interval known, the frequency compensating parameters can be determined. From (5.4), it can be seen that in order to minimize W in (5.25), to the optimum value, for each branch, it has to satisfy

$$\max_{-1 \leq \zeta \leq 1} \left\{ \int_{\theta_{n-1}}^{\theta_n} \text{sinc}^2 \{f_d T_s [\cos(\theta) - \zeta_n]\} d\theta \right\} \quad (5.30)$$

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

Using numerical methods and θ_n from Table 5.3, Table 5.4 can be obtained. Comparing Table 5.4 and (5.5) in the previous section, it is found that the difference between the approximate ζ and the numerical solution is less than 0.06. Hence, (5.5) can be easily employed for practical implementation.

N_d	2	3	4	5	6	7
ζ_1	0.641	0.770	0.830	0.865	0.890	0.905
ζ_2	-0.641	0.000	0.260	0.410	0.510	0.580
ζ_3		-0.765	-0.255	0.000	0.170	0.290
ζ_4			-0.825	-0.405	-0.168	0.000
ζ_5				-0.860	-0.505	-0.288
ζ_6					-0.885	-0.575
ζ_7						-0.900

Table 5.4: Frequency Compensation Parameters, ζ_n , for Different Number of Beams Required

5.5.3 Computation of Steering Angle and Antenna Elements Needed

With the beam interval known, the number of antenna elements needed has to be calculated. When distance between adjacent antenna elements is equal to half wavelength and for a given beamwidth and steering angle, it can be seen from [80] that the number of antenna elements needed in the ULA is

$$\text{No. of Antennas} = N_{ant} = \sqrt{\left[\frac{4\sqrt{3}}{\pi \sin(\psi) \text{BeamWidth}} \right]^2} + 1 \quad (5.31)$$

where $\text{BeamWidth} = (\theta_n - \theta_{n-1})$ is the half power beamwidth of the array pattern, and ψ is the steering angle of the ULA, which is between θ_n and θ_{n-1} . When the beamwidth specified by Table 5.3 and ψ as the middle of θ_n and θ_{n-1} , i.e. $\frac{1}{2}(\theta_n + \theta_{n-1})$, it can be easily shown from (5.31) that the number of antenna elements needed, N_{ant} ,

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

is approximately equal to the number of beams, N_d , required, i.e. $N_d = N_{ant}$.

Now the steering angle, ψ , has to be further refined. The array patterns, with beamwidth as shown in Table 5.3, can be achieved by the simple Delay-and-Sum beamforming [80]. The array pattern can be seen as

$$A(\theta) = \sum_{n=1}^{N_{ant}} \exp \left\{ j\pi \left(n - \frac{N_{ant} - 1}{2} \right) [\cos(\theta) - \cos(\psi)] \right\} \quad (5.32)$$

Thus, with $N_d = N_{ant}$ and using trial and error, ψ can also be approximated using (5.32), provided that the beamwidth is within the angle specified by Table 5.3. The results are shown in Table 5.5.

N_d	2	3	4	5	6	7
ψ_1	1.0297	0.8168	0.7156	0.6109	0.5533	0.5105
ψ_2	2.1118	1.5708	1.3089	1.1483	1.0316	0.9200
ψ_3		2.3248	1.8326	1.5708	1.3982	1.2422
ψ_4			2.4260	1.9932	1.7369	1.5708
ψ_5				2.5307	2.0977	1.8363
ψ_6					2.5883	2.1675
ψ_7						2.6311

Table 5.5: Steering Angles of the ULA (in rad)

With the steering angle known, the array patterns with respect to the ideal beam interval (Table 5.3) and steering angle (Table 5.5) are plotted in Fig. 5.11.

Fig. 5.11a and Fig. 5.11b are plotted with simple delay and sum beamforming algorithm. The different beams formed by the beamforming algorithm are represented by the different curves in the figures. It can be seen from the figures that each of the beams formed are having similar beam interval calculated in Table 5.3. Thus, it highlights that the system is constructible in practice.

Although ζ_n is known from (5.5) and Table 5.4, there is still a need to know the

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

Figure 5.11: Array Pattern with respect to the beam interval where Doppler Spreads are equal for all diversity beams, with different curve representing different beams

maximum Doppler frequency, f_d , to correctly compensate the frequency shift. This can be easily estimated just by obtaining the velocity of the vehicle that the antenna is mounted on, as $f_d = v/\lambda$, where v is the velocity and λ is the wavelength of the transmitted carrier frequency. The velocity of the vehicle can be obtained by the sensor [81], or even by the deployed of ad-hoc network communications between vehicles [82].

5.5.4 Simulation Results

Using ideal array patterns, Fig. 5.12 is plotted through simulations, with the number of antenna elements equal to 2 and 3 respectively. The other parameters are specified in Tables 5.3 ~ 5.5. The OFDM system is simulated with 64 subcarriers of BPSK, and the AWGN is assumed to be negligible. The restricted Doppler spectrum simulator is adopted from [83], with the maximum Doppler frequency varying. The simulation results are obtained from 100000 Monte-Carlos trials.

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

From Fig. 5.12a, it can be seen that with just two beams, the bit error rate (BER) reduces by more than ten times as compared to using only one beam. The BER reduction is not significant when comparing one beamforming with omni-directional antenna. From Fig. 5.12b, it can be seen that the BER further reduces when angular diversity is applied. It shows that the reduction in BER is of the order of 10^{-3} as compared to using just one beamforming or omni-directional antenna. It can also be seen that the improvement when single beamforming is applied is not very significant. This is because the beamwidth of the array pattern is still quite wide, thus the reduction in Doppler spreading is not very significant. The second reason is that when $f_d T_s$ is too high ($f_d T_s > 0.04$ for Fig. 5.12a and $f_d T_s > 0.12$ for Fig. 5.12b), it will reach a upper ceiling which even the directional antenna (of that given beamwidth) can't provide significant improvement.

The results clearly show that with array antennas, it will be more advantageous to employ angular diversity rather than just using single beamforming. Although the simulation results are presented only for 2 and 3 antenna elements, preliminary investigation of employing more antenna elements are showing promising results.

5.6 Summary

In this chapter, the ICI power of an OFDM system is quantified for cases where directional antennas are employed. It is shown that the amount of ICI reduction is marginal when directional antenna is employed at the base station. Whereas for the context when directional antenna is employed at the receiver, the amount of ICI reduction can be quite significant, provided that frequency compensation is implemented. The ICI power is derived for both 2-D and 3-D channel models, in both Rayleigh and Rician channels.

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

(a) Comparing BERs for 2 Antennas Array

(b) Comparing BERs for 3 Antennas Array

Figure 5.12: Comparing BERs With Different Number of Antenna Elements

CHAPTER 5. ICI REDUCTION FOR OFDM SYSTEM USING DIRECTIONAL ANTENNAS

By applying the same idea of ICI reduction using directional antenna, angular diversity is applied to an OFDM system to reduce ICI and achieve diversity gain. It has been shown through simulation that the BER performance of the OFDM system improves significantly when angular diversity is employed.

Chapter 6

Analysis of ICI Reduction Coding Techniques in OFDM Systems for Time-Varying Channel

As mentioned in the previous chapters, the OFDM system has many desirable properties such as robustness to frequency selectivity and inter-symbol interference. However, OFDM is very sensitive to time varying channels [5],[7],[42],[6] which generates ICI. Various methods, which include windowing [84], equalization [54],[50] or even using directional antenna at the receiver [85] as mentioned in the previous chapter, have been proposed to reduce the ICI power. Coding techniques to reduce ICI, first proposed in [70],[66], have been shown to reduce ICI significantly in carrier frequency offset (CFO) channel.

One of the most popular coding schemes, known as the polynomial cancellation coded OFDM (PCC-OFDM) [70],[86],[67] has also been found (using simulation) to reduce ICI in OFDM over classic Clarke/Jakes' Doppler spread channel model [67].

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

However, a theoretical equation to quantify the amount of ICI power of the coding scheme for general channel condition is not available. Furthermore, PCC-OFDM is a half rate code in its most efficient form. Full rate codes have also been proposed in [66] (correlative coding), but the main pitfall of the codes is they can only obtain limited ICI reduction performance. The theoretical equation for the ICI power of correlative coding, for channels other than CFO, is also not available. In this chapter, the general theoretical expressions of the ICI power for the two above mentioned coding schemes, in terms of the correlation function of the channel, are derived. It is found that by maintaining appropriate correlation between different subcarriers, the ICI power caused by these subcarriers can be significantly reduced. To alleviate some of the drawbacks in either PCC or correlative coding, an alternative ICI reduction code is introduced. It is a deterministic coding technique and is similar to precoding method for interference reduction.

The general expressions for all the above mentioned coding schemes are verified using examples of the CFO model, the classic Clarke/Jakes' Doppler spread model, the uniform Doppler spread model, and the 2-rays model. Simulation results for these channel conditions agree very well with the results calculated numerically using the theoretical equations. In order to get a quantitative value for the amount of ICI power reduction of the three coding schemes, tight upper and lower bounds are also developed.

6.1 OFDM with ICI with Correlation Among Subcarriers

6.1.1 Background

From Chapter 3, the demodulated signal at the receivers when the channel gain is fast time varying is repeated here and can be written as

$$y_k = g_{0,k}d_k + \sum_{n=0, n \neq k}^{N_c-1} g_{n-k,k}d_n + w_k \quad (6.1)$$

where the frequency domain channel coefficient is redefined here in discrete frequency as

$$g_{n-k,k} = \int_0^1 \sum_{l=0}^{L-1} [h(T_s x) \exp(j2\pi n \tau_l / N_c)] \exp\{-j2\pi(n-k)x\} dx, \quad (6.2)$$

where L is the number of paths and τ_l is the discrete normalized path delay of the l -th path with respect to T_s/N_c , which T_s is the OFDM symbol time. $h(t, \tau) = h(T_s x) \exp(j2\pi n \tau_l / N_c)$ is the time-varying channel gain of the l -th path at $T_s x$ time instant¹. By assuming that there is no multipath ($L = 1$), and using [42],[46], its covariance can be written as

$$\begin{aligned} E[g_{b-q,q} g_{c-q,q}^*] &= E[g_b g_c^*] \\ &= \begin{cases} \int_{-1}^1 \frac{-\sin[\pi(b-c)|x|]}{\pi(b-c)} r(f_d T_s x) \exp\{-j\pi(b+c)x\} dx, & \text{for } b \neq c \\ \int_{-1}^1 (1-|x|) r(f_d T_s x) \exp\{-j2\pi b x\} dx, & \text{for } b = c \end{cases}, \end{aligned} \quad (6.3)$$

¹Instead of the conventional use of $h(t, \tau)$ as the time-varying channel gain with respect to continuous time, t , $T_s x$ is used instead where x is the continuous variable

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

This is different from the previous chapter which assumed that $E[d_k, d_l] = \delta_{k,l}$. Here, that assumption is no longer valid, as coding is involved and there is correlation between the coded d_k .

6.1.2 Coding for ICI Reduction

In this section, the possibility of reducing ICI by utilizing coding is investigated. Let us define the total ICI power, at the q -th subcarrier, induced by an set of arbitrary adjacent subcarriers denoted by $\mathbf{R}$ (where $q \notin \mathbf{R}$), in a frequency flat fading channel, as

$$P_{\mathbf{R}}^{ICI} = E \left| \sum_{n \in \mathbf{R}} g_n d_n \right|^2, \quad (6.4)$$

where d_n ($E|d_n|^2 = 1, \forall n$) is assumed to be correlated to each other in $\mathbf{R}$, with N_P elements. It also means that the coding causes the signal to be correlated among these N_P subcarriers. Let us assume that $\mathbf{R} = \{1, \dots, N_P\}$ and $q = 0$, the total ICI power[42] at the zero-th subcarrier generated by the $\mathbf{R}$ subcarriers is

$$P_{\mathbf{R}}^{ICI} = \int_{-1}^1 (1 - |x|) r(f_d T_s x) \sum_{n=1}^{N_P} \{\exp\{-j2\pi n x\}\} dx - \int_{-1}^1 r(f_d T_s x) I_{code} dx \quad (6.5)$$

where

$$I_{code} = \begin{bmatrix} \sum_{n=1}^{N_P-1} E\{d_n d_{n+1}^*\} \exp\{-j2\pi n x\} & \sum_{n=1}^{N_P-2} E\{d_n d_{n+2}^*\} \exp\{-j2\pi n x\} & \dots \end{bmatrix} \begin{bmatrix} \frac{\sin[2\pi|x|]}{\pi} \\ \vdots \\ \frac{\sin[2(N_P-1)\pi|x|]}{(N_P-1)\pi} \end{bmatrix}.$$

The motivation here is to reduce the ICI power as much as possible. From (6.5), the $P_{\mathbf{R}}^{ICI}$ equation can be divided into two terms, where the first term is the ICI power when no coding is utilized and the second term is due to correlation between d_n and

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

d_{n+k} ($k \in \{1, \dots, N_P\}$), which is due to coding. In order to reduce the ICI power, the second term has to be made as large as possible. However, the result of the integration from the second term will be negative if I_{code} is positive. Thus, in order to reduce the ICI power, I_{code} must be as large a negative value as possible. It is seen that the scalar I_{code} can be represented by the multiplication of two vectors, where the first vector in I_{code} indicates the code that is implemented to reduce the ICI power. Furthermore, the portion that contributes most to ICI reduction comes from the second term of the first vector of I_{code} , $\sum_{n=1}^{N_P-1} \text{real}(E\{d_n d_{n+1}^*\})$, as the impact of other elements is reduced by a factor of $2, \dots, N_P - 1$. In the case for the coded signal correlation with group size of $N_P = 2$, it can be seen that as long as $E[d_n d_{n+1}^*]$ is negative, the ICI power will be reduced. This is the main idea behind correlative code [66], which ensures that the adjacent subcarriers are correlated by using the correlative polynomial $F(D) = (1 - D)$.

For the case of correlative code, its ICI reduction capability is limited, and is found to be slightly over 4dB (seen later in the chapter). In the case of PCC-OFDM [70],[86], it is achieved by coding an input data symbol to a group of adjacent subcarriers. The coding ensures that in $\mathbf{R}$, the adjacent subcarriers have opposite phases. However, PCC-OFDM greatly reduces the efficiency of the system, as at least two subcarriers are needed to represent a data symbol. Therefore, a new ICI reduction scheme is proposed in this chapter to improve the ICI reduction capability of correlative coding and maintain its full rate advantage.

6.2 Performance Analysis and Comparison

6.2.1 Exact Calculation

In this section, the ICI power of the various codes with respect to the correlation function of the channel is analyzed. The total ICI power, at subcarrier q is defined as

$$\begin{aligned}
 P^{ICI} &= E \left| \sum_{\substack{n=-N_c/2 \\ n \neq q}}^{N_c/2-1} g_n d_n \right|^2 \\
 &= \sum_{\substack{n=-N_c/2 \\ n \neq q}}^{N_c/2-1} \sum_{\substack{n'=-N_c/2 \\ n' \neq q}}^{N_c/2-1} E[g_n g_{n'}^*] E[d_n d_{n'}^*]
 \end{aligned} \tag{6.6}$$

For uncoded OFDM, there is no correlation between subcarriers. As the ICI power for each of the subcarriers is the same [42], the equations can be simplified by setting $q = 0$, which means that it only needs to consider the total ICI power at the zero-th subcarrier. Thus, from Chapter 3, the total ICI power at the zero-th subcarrier of an uncoded OFDM system is repeated here for sake of convenience, and can be written as

$$\begin{aligned}
 P_{unc}^{ICI} &= \sum_{\substack{n=-N_c/2 \\ n \neq 0}}^{N_c/2-1} E|g_n|^2 \\
 &\approx \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} E|g_n|^2 \\
 &= 1 - \int_{-1}^1 (1 - |x|) r(f_d T_s x) dx.
 \end{aligned} \tag{6.7}$$

For the derivation of (6.7), infinite number of subcarriers is assumed. This assumption does not alter the ICI power significantly, and is found to be quite accurate [42],[6], as seen from the simulation results in Chapter 3. However, under this assumption, the ICI

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

power can be expressed in a more convenient way. Therefore, throughout this chapter, it is assumed that $N_c \rightarrow \infty$.

Correlative Coding

Correlative coding, proposed in [66], makes use of the high correlation property of the adjacent subcarriers to achieve ICI mitigation. It modulates the data bits by using the difference between them. For example, subcarrier '0' is used as a reference signal, with the information symbol starting at subcarrier '1'. The output coded symbol at subcarrier '1' is being modulated using the differences between the current symbol and the one before it, the information symbol at subcarrier '0'. The coded information symbol at subcarrier '1' can be seen in the relationship, $\hat{d}_1 = d_1 - d_0$, where $\hat{d}_n$ is the correlative coded information symbol for the n -th subcarrier. These operations will be applied to the rest of the subcarriers. Hence, for the case of correlative coding, the signal correlation can be written as [66]

$$E[\hat{d}_n \hat{d}_{n'}^*] = \begin{cases} 1 & n = n' \\ -0.5 & n = n' \pm 1 \\ 0 & \text{otherwise} \end{cases} \quad (6.8)$$

Therefore, applying (6.6) and (6.8), the ICI power, of the zero-th subcarrier, can be expressed as

$$P_{cc}^{ICI} = \sum_{\substack{n \neq 0 \\ n = -N_c/2}}^{N_c/2-1} E|g_n|^2 - \sum_{\substack{n \neq 0,1 \\ n = -N_c/2}}^{N_c/2-1} E[g_n g_{n-1}^*] \quad (6.9)$$

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

For the first term, it is exactly the same as an uncoded system, which can be seen in (6.7). For the second term, by applying (6.3), it can be seen as

$$\begin{aligned}
 \sum_{\substack{n \neq 0,1 \\ n=-\infty}}^{\infty} E[g_n g_{n+1}] &= \int_{-1}^1 \frac{\sin(\pi|x|)}{\pi} [r(f_d T_s x)(1 + \exp\{-j2\pi x\}) - 1] \exp\{j\pi x\} dx \\
 &= \int_{-1}^1 \frac{\sin(\pi|x|)}{\pi} [r(f_d T_s x)(1 + \exp\{-j2\pi x\})] \exp\{j\pi x\} dx \\
 &= \int_{-1}^1 \frac{\sin(2\pi|x|)}{\pi} r(f_d T_s x) dx
 \end{aligned} \tag{6.10}$$

Finally, the ICI power equation for the correlative code can be written as

$$P_{cc}^{ICI} = \int_{-1}^1 (1 - |x|) \{1 - r(f_d T_s x)\} dx - \int_{-1}^1 \frac{\sin(2\pi|x|)}{\pi} r(f_d T_s x) dx. \tag{6.11}$$

Comparing the general expression for correlative code with that of OFDM system without coding, as given in (6.7), there is an additional term, $\int_{-1}^1 \frac{\sin(2\pi|x|)}{\pi} r(f_d T_s x) dx$. By comparing (6.11) with (6.5), it can be seen that it has a group size for correlated coding of $N_P = 2$, which also means that the coded signals are correlated between adjacent subcarriers.

1/2 Rate Polynomial Cancellation Coded OFDM

For PCC-OFDM [70],[86], the relative weightings of the subcarriers in a group that will modulate the same information symbol are given by the coefficients of the polynomial expansion of $(1 - x)^{c-1}$, where $1/c$ represent the coding rate. For example, if d_0 is to be transmitted using $c = 2$ subcarriers, the first subcarrier will be loaded with d_0 , while the second subcarrier loaded with $-d_0$. Thus, it reduces the efficiency of the system by two, as the coding rate is now $1/2$.

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

For the specific case of $c = 2$ and with the total transmit power of a data symbol equal to one, the ICI power at subcarrier q (after decoding) can be written as [70],[86]

$$P_{PCC,q}^{ICI} = \frac{1}{4} \sum_{\substack{n+q=\text{even} \\ n+q \neq 0 \\ n+q=-N_c/2}}^{N_c/2-1} E| -g_{n+q-1,q} + 2g_{n+q,q} - g_{n+q+1,q} |^2, \quad (6.12)$$

which can be expressed in a simpler form as

$$P_{pcc}^{ICI} = \frac{1}{4} \left[\begin{aligned} & \sum_{\substack{n \neq 0 \\ n=-N_c/2}}^{N_c/2-1} E|g_n|^2 + \sum_{\substack{|n| \neq 0,1 \\ n=-N_c/2}}^{N_c/2-1} E|g_n|^2 + 2 \sum_{\substack{n=\text{even} \\ n \neq 0 \\ n=-N_c/2}}^{N_c/2-1} E|g_n|^2 \\ & - 4 \sum_{\substack{|n| \neq 0,1 \\ n=-N_c/2}}^{N_c/2-1} E[g_n g_{n-1}^*] + 2 \sum_{\substack{n=\text{odd}, \\ |n| \neq 1 \\ n=-N_c/2}}^{N_c/2-1} E[g_n g_{n-2}^*] \end{aligned} \right]. \quad (6.13)$$

For the first term, it is exactly the same as an uncoded system, which can be seen in (6.7). Again using (6.3), the second term of (6.13) can be seen as

$$\sum_{\substack{|n| \neq 0,1 \\ n=-\infty}}^{\infty} E|g_n|^2 = \int_{-1}^1 (1 - |x|) [1 - r(f_d T_s x) \{1 + 2 \cos(2\pi x)\}] dx. \quad (6.14)$$

For the third term, the summation is over even numbers, and the summation of the exponential terms with channel correlation can be expressed as

$$r(f_d T_s x) \sum_{\substack{n=\text{even} \\ n \neq 0 \\ n=-\infty}}^{\infty} \exp(-j2\pi n x) = 1 - r\left(\frac{f_d T_s}{2} x\right).$$

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

Thus, the third term of (6.13) can be expressed as

$$\sum_{\substack{n=\text{even} \\ n \neq 0 \\ n=-\infty}}^{\infty} E|g_n|^2 = \int_{-1}^1 (1 - |x|) \left[1 - r\left(\frac{f_d T_s}{2} x\right) \right] dx. \quad (6.15)$$

Using (6.3), the fourth term in (6.13) can be shown to be

$$\sum_{\substack{|n| \neq 0,1 \\ n=-\infty}}^{\infty} E[g_n g_{n-1}] = \int_{-1}^1 \frac{\sin(\pi|x|)}{\pi} r(f_d T_s x) [1 + 2 \cos(2\pi x)] \exp(j\pi x) dx. \quad (6.16)$$

For the fifth term in (6.13), summation of the exponential term with $r(f_d T_s x)$ is over odd numbers. For this case, the summation can be written as

$$\begin{aligned} r(f_d T_s x) \sum_{\substack{n=\text{odd} \\ |n| \neq 1 \\ n=-\infty}}^{\infty} \exp(-j2\pi n x) &= \exp(-j2\pi x) \left\{ r(f_d T_s x) \sum_{\substack{n=\text{even} \\ n \neq 0, -2 \\ n=-\infty}}^{\infty} \exp(-j2\pi n x) \right\} \\ &= \exp(-j2\pi x) \left\{ 1 - [1 + \exp(j2\pi x)] r\left(\frac{f_d T_s}{2} x\right) \right\}. \end{aligned}$$

Using the above mentioned result, the fifth term in (6.13) can be written as

$$\sum_{\substack{n=\text{odd} \\ |n| \neq 1 \\ n=-\infty}}^{\infty} E[g_n g_{n-2}^*] = \int_{-1}^1 \frac{\sin(2\pi|x|)}{2\pi} [1 + \exp(j2\pi x)] r\left(\frac{f_d T_s}{2} x\right) dx. \quad (6.17)$$

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

Therefore, the expression for P_{pcc}^{ICI} in (6.13), after simplification, can be shown as

$$P_{pcc}^{ICI} = \frac{1}{2} - \frac{1}{2} \int_{-1}^1 (1 - |x|) r(f_d T_s x) dx \quad (i)$$

$$- \frac{1}{2} \int_{-1}^1 (1 - |x|) \cos(2\pi x) r(f_d T_s x) dx \quad (ii)$$

$$- \int_{-1}^1 \frac{\sin(\pi|x|)}{\pi} r(f_d T_s x) [1 + 2 \cos(2\pi x)] \exp(j\pi x) dx \quad (iii) \quad (6.18)$$

$$+ \frac{1}{2} \left\{ 1 - \int_{-1}^1 (1 - |x|) r\left(\frac{f_d T_s}{2} x\right) dx \right\} \quad (iv)$$

$$+ \frac{1}{2} \int_{-1}^1 \frac{\sin(2\pi|x|)}{2\pi} (1 + \exp(j2\pi x)) r\left(\frac{f_d T_s}{2} x\right) dx. \quad (v)$$

From (6.18), (6.18-i) is the same as that of the uncoded OFDM system, as in [42], with only the power reduced by half to ensure the total transmitted power of each symbols to one. For the 1/2 rate PCC-OFDM, a data symbol is coded to a pair of subcarriers, where the first subcarrier will be loaded with the original data symbol, and the second subcarrier loaded with the same data symbol, but is phase shifted by π . Thus, for the whole OFDM symbol, the even subcarriers are loaded with the original data symbols, while the odd subcarriers are loaded with the phase-shifted data symbols. The decoding of the data symbols is achieved by subtracting the odd subcarriers from the even. Thus, the common ICI between the coded pairs of subcarriers can be effectively removed. This effect can be seen in in both (6.18-ii) & (6.18-iii). Whereas for (6.18-iv) & (6.18-v), they add to the overall ICI power. This is because the pair of PCC coded subcarriers, though they have almost similar ICI, is not exactly the same. The ICI at those pair of subcarriers have a small amplitude difference. Therefore, these differences result in (6.18-iv) & (6.18-v). Although only 1/2 rate PCC-OFDM system is analyzed here, the

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

extension to PCC with $d > 2$ can be easily achieved [86].

Proposed Deterministic Scheme

The mentioned ICI reduction codes can be presented as a precoding to the OFDM symbol in terms of a matrix, as seen in

$$\hat{\mathbf{d}} = \mathcal{C}\mathbf{d} \quad (6.19)$$

where $\mathcal{C}$ is the precoding matrix to reduce ICI. As seen from the PCC code, the $\mathcal{C}$ can be seen as

$$\mathcal{C} = \begin{bmatrix} 1 & 0 & \cdots \\ -1 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ 0 & -1 & 0 & \cdots \\ \vdots & & \ddots \end{bmatrix} \quad (6.20)$$

which has the size of $N_c/2 \times N_c$. And the data vector $\mathbf{d}$ has only length of $N_c/2$. The general approach can be extended to correlative coding, with $\mathcal{C}$ defined as

$$\mathcal{C} = \begin{bmatrix} \sqrt{0.5} & -\sqrt{0.5} & 0 & \cdots \\ 0 & \sqrt{0.5} & -\sqrt{0.5} & 0 & \cdots \\ \vdots & & \ddots \end{bmatrix} \quad (6.21)$$

From the above equation, it can be seen that the correlation of the code can be defined as the vector multiplication between any two of the columns. And the transmitted power is not modified as $\mathcal{C}$ is unitary. This idea can be further extended to other

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

coding schemes. Assuming full rate coding, we define $\mathcal{C}$ as

$$\mathcal{C} = \begin{bmatrix} c_0 & c_1 & \cdots & c_{N_c} \\ c_{-1} & c_0 & c_1 & \cdots \\ \vdots & & \ddots & \\ c_{-N_c+1} & \cdots & c_0 \end{bmatrix} \quad (6.22)$$

And as describe earlier, the autocorrelation values of a code can be easily determined by

$$E[\hat{d}_n \hat{d}_{n'}^*] = \mathcal{R}_{n-n'} = \mathbf{c}_n^T \mathbf{c}_{n'} \quad (6.23)$$

where $\mathbf{c}_n$ denotes the n -th column of the matrix $\mathcal{C}$. Using the values found from (6.23), the ICI power can be derived as

$$P_{det}^{ICI} = \sum_{\substack{n \neq 0 \\ n=-N_c/2}}^{N_c/2-1} E|g_n|^2 + 2 \sum_{k=1}^{N_p} \mathcal{R}_k \sum_{\substack{n \neq 0, \dots, k \\ n=-N_c/2}}^{N_c/2-1} E[g_n g_{n-k}^*] \quad (6.24)$$

And by assuming that there are infinite number of subcarriers, the ICI power can be simplified as

$$\begin{aligned} P_{det}^{ICI} &= \int_{-1}^1 (1 - |x|) \{1 - r(f_d T_s x)\} dx \\ &+ \int_{-1}^1 \sum_{k=1}^{N_p} \frac{2\mathcal{R}_k \sin(2k\pi|x|)}{k\pi} r(f_d T_s x) dx. \end{aligned} \quad (6.25)$$

From (6.25), it can be seen that the first term is the same as that of the uncoded OFDM system, as seen in (6.7). Like correlative codes, there exist additional terms similar to (6.5). However, in this proposed coding scheme, there are more terms which can be applied to optimize the ICI reduction. Before that, we have to define the bandwidth of a matrix [87]. It refers to the width of the diagonal elements, which is denoted as $2N_p$ here. For example, if there is 1 non-zero element immediately adjacent to both the right and left of the diagonal elements, the matrix will have a matrix bandwidth of

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

$$2N_p = 2.$$

Coming back to the proposed coding scheme, the main objective is to optimize the ICI reduction given the matrix bandwidth of $2N_p$. However, the exact equation is complicated and cannot be easily applied for the optimization. Therefore, approximation will have to be made using bounds. This will be considered in the next section, when the bounds are derived.

6.2.2 Performance Bounds

In the previous subsection, exact expressions for the ICI power are derived. However, the equations are complicated and cannot be obtained in closed-form, and thus may not be able to provide much insight into the improvement of each of the ICI reduction coding schemes. In this section, tight bounds on the ICI power of the various codes are obtained. Upper bounds for quantifying the improvement in ICI power reduction, independent of channel conditions, are also derived for each coding schemes.

For the ICI power bound to be calculated, the bounds for the time domain fading correlation function, $r(f_d T_s x)$, have to be known. From [42], $r(f_d T_s x)$ can be expressed as follows for the various channels,

$$1 - \frac{\gamma_1}{2}(2\pi f_d T_s)^2 x^2 + \frac{\gamma_2}{24}(2\pi f_d T_s)^4 x^4 \geq r(f_d T_s x) \geq 1 - \frac{\gamma_1}{2}(2\pi f_d T_s)^2 x^2. \quad (6.26)$$

The value of γ_1 and γ_2 in (6.26) are given in Table 6.1 for various channels. As the list is not exhaustive, γ_1 and γ_2 can be calculated [42] for other channel models using

$$\gamma_k = \frac{1}{f_d^{2k}} \int_{-f_d}^{f_d} f^{2k} S(f) df, \quad (6.27)$$

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

Model	γ_1	γ_2
Classical	1/2	3/8
Uniform	1/3	1/5
2-Rays	1	1

Table 6.1: γ_1 and γ_2 for different time varying channels

where $S(f)$ is the PSD of the channel model. For example, in the case of CFO, where $r(f_0 T_s x) = \exp(-j2\pi f_0 T_s x)$, [42] does not provide the bound. Nevertheless, it is easily obtained from (6.27), as $S(f) = \delta(f - f_d)$. The bound can be written as

$$1 - \frac{1}{2}(2\pi f_d T_s)^2 x^2 + \frac{1}{24}(2\pi f_d T_s)^4 x^4 \geq r(f_d T_s x) \geq 1 - \frac{1}{2}(2\pi f_d T_s)^2 x^2. \quad (6.28)$$

By comparing Table 6.1, (6.26) with (6.28), it can be seen that the results (for CFO) are the same for the 2-rays model. This is not surprising as the real value of the $r(f_d T_s x)$ of CFO is the same as the 2-rays model found in [42]. For the case of the 2-rays model in Table 6.1, it is assumed that each ray arrives with equal power but exactly opposite magnitude of the Doppler shift. Normally, the two rays are assumed to be uncorrelated and therefore, the bounds can be calculated independently for each of the rays and they are summed according to their power ratio. Using the bounds for the correlation function, it can be easily shown that the upper and lower bounds for uncoded OFDM can be written as [42]

$$P_{unc}^{ICI} \leq \frac{\gamma_1}{12}(2\pi f_d T_s)^2, \quad (6.29)$$

$$P_{unc}^{ICI} \geq \frac{\gamma_1}{12}(2\pi f_d T_s)^2 - \frac{\gamma_2}{360}(2\pi f_d T_s)^4. \quad (6.30)$$

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

Correlative Coding

By employing (6.11) and (6.26), the upper and lower bounds of the correlative code can be easily derived as

$$P_{cc}^{ICI} \leq \frac{\gamma_1}{12}(2\pi f_d T_s)^2 \left[1 - \frac{6}{\pi^2}\right] - \frac{\gamma_2}{360}(2\pi f_d T_s)^4 \left[1 + \frac{45 - 15\pi^2}{\pi^4}\right], \quad (6.31)$$

$$P_{cc}^{ICI} \geq \frac{\gamma_1}{12}(2\pi f_d T_s)^2 \left[1 - \frac{6}{\pi^2}\right]. \quad (6.32)$$

From the bounds in (6.31) and (6.32), it is found that the reduction of ICI power for the correlative coding is upper bounded by $\frac{\pi^2 - 6}{\pi^2} = 4.1\text{dB}$.

1/2 Rate Polynomial Cancellation Coded OFDM

The upper and lower bounds of the 1/2 rate PCC-OFDM, by applying (6.13) and (6.26), can be written as

$$P_{pcc}^{ICI} \leq \frac{\gamma_1}{12}(2\pi f_d T_s)^2 \left[\frac{20\pi^2 - 177}{32\pi^2}\right] - \frac{\gamma_2}{360\pi^4}(2\pi f_d T_s)^4 \left[\frac{544\pi^4 - 15060\pi^2 + 59715}{1024}\right], \quad (6.33)$$

$$P_{pcc}^{ICI} \geq \frac{\gamma_1}{12}(2\pi f_d T_s)^2 \left[\frac{20\pi^2 - 177}{32\pi^2}\right]. \quad (6.34)$$

By using the bounds, it can be seen that the reduction of ICI power for the 1/2 rate PCC-OFDM is less than $\frac{20\pi^2 - 177}{32\pi^2} = 11.9\text{dB}$. PCC-OFDM achieves such a substantial performance gain by sacrificing bandwidth efficiency. The extension of the bounds for other coding rate of PCC-OFDM can be easily derived using a similar approach. It is shown by the analysis of 1/3 rate PCC-OFDM in CFO [86], that the ICI reduction is much better than 1/2 rate PCC-OFDM. Therefore, the improvement of PCC-OFDM at other lower coding rates will be much greater than 11.9dB.

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

Proposed Deterministic Scheme

Using (6.25) and (6.26), the upper bound of the proposed deterministic coding scheme is seen as

$$P_{det}^{ICI} \leq \frac{\gamma_1}{12} (2\pi f_d T_s)^2 \left[1 + \sum_{k=1}^{N_p} \frac{\mathcal{R}_k}{(k\pi)^2} \right], \quad (6.35)$$

Thus, to minimize the ICI, $1 + \sum_{k=1}^{N_p} \frac{\mathcal{R}_k}{(k)^2}$ should be minimized. It is achieved by maximizing $\sum_{k=1}^{N_p} \frac{\mathcal{R}_k}{k^2}$, with the constraint that $\mathcal{R}_k = 1$. But remember, $\mathcal{R}_k$ is derived (6.23). Thus, expressed in matrix, it can be seen as

$$\begin{aligned} -\sum_{k=1}^{N_p} \frac{\mathcal{R}_k}{k^2} &= -\mathbf{c}_0^T [\mathbf{c}_0, \mathbf{c}_1, \dots, \mathbf{c}_{N_c}] \mathbf{M}_{0,N_c} \\ &= -\mathbf{c}_1^T [\mathbf{c}_0, \mathbf{c}_1, \dots, \mathbf{c}_{N_c}] \mathbf{M}_{1,N_c} \end{aligned} \quad (6.36)$$

where $\mathbf{M}_{k,N_c}$ is defined as

$$\mathbf{M}_{k,N_c} = \left[0, \dots, \frac{1}{N_p^2}, \dots, 1, 0, 1, \dots, \frac{1}{N_p^2}, \dots, 0 \right]^T \quad (6.37)$$

and the zero-th value is at the k -th position of the vector. By optimizing (6.36), the optimal value for matrix bandwidth of $2N_p = 2$ can be found as

$$\begin{aligned} c_0 &= 1/\sqrt{2} \\ c_{\pm 1} &= -1/2 \end{aligned} \quad (6.38)$$

and the correlation is

$$\mathcal{R}_k = \begin{cases} 1 & \text{for } k = 0 \\ -\frac{1}{\sqrt{2}} & \text{for } |k| = 1 \\ 1/4 & \text{for } |k| = 2 \\ 0 & \text{otherwise} \end{cases} \quad (6.39)$$

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

Again by assuming that there are infinite number of subcarriers, the ICI power can be simplified as

$$P_{det}^{ICI} = \int_{-1}^1 (1 - |x|) \{1 - r(f_d T_s x)\} dx - \int_{-1}^1 \frac{\sqrt{32} \sin(2\pi|x|) - \sin(4\pi|x|)}{4\pi} r(f_d T_s x) dx. \quad (6.40)$$

Employing (6.40) and (6.26), the upper and lower bounds of the proposed deterministic coding scheme are seen as

$$P_{det}^{ICI} \leq \frac{\gamma_1}{12} (2\pi f_d T_s)^2 \left[1 - \frac{24\sqrt{2} - 3}{4\pi^2} \right], \quad (6.41)$$

$$P_{det}^{ICI} \geq \frac{\gamma_1}{12} (2\pi f_d T_s)^2 \left[1 - \frac{24\sqrt{2} - 3}{4\pi^2} \right] - \frac{\gamma_2}{360} (2\pi f_d T_s)^4 \left[1 + \frac{(63.64 - 19.34\pi^2)}{\pi^4} \right]. \quad (6.42)$$

From the bounds, it can be shown that the reduction of ICI power for the proposed deterministic scheme has an approximate gain of 6.65dB. The proposed technique provides better performance as compared to correlative code. It compares well with PCC-OFDM scheme and it is full rate.

6.3 Comparison of Results

The purpose of this section is to validate by simulations the theoretical equations derived for various codes. The simulated OFDM system employs 64 subcarriers, where the random input data are modulated with BPSK. As only the effect of ICI is considered, the noise is assumed to be negligible. It is also assumed that there is no multipath, with perfect timing synchronization.

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

The theoretical and simulation results of the ICI power in CFO, with different coding schemes employed, are presented in Fig. 6.1. Both the upper and lower bounds are also plotted on the same figure. Comparing theoretical results with the equation in [88] (for PCC-OFDM) (derived for CFO only), the differences are marginal. Therefore, our theoretical expressions are compatible with published results. As can be seen from Fig. 6.1, the simulated and theoretical results for the proposed algorithm are close, verifying the analytical result. Fig. 6.2 shows the theoretical and simulation results of the ICI power in classics Clarke/Jakes' Doppler spread channel, with both the upper and lower bounds, for coded and uncoded OFDM systems.

Figure 6.1: Results of Various Coding in CFO

The results for uniform and 2-rays channel model are shown in Figs. 6.3 and 6.4 respectively. For the 2-rays model, a power ratio of $p_1 = 0.6$ and $p_2 = 0.4$ is assigned, where the Doppler shift is $f_d T_s = f_1 T_s = 2 f_2 T_s$.

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

Figure 6.2: Results of Various Coding in Clarke/Jakes' Doppler Spread Channel Model

All the plots show that 1/2 rate PCC-OFDM has the lowest ICI power. However, as it is half rate, its efficiency is half compared to the uncoded OFDM. The next lowest ICI power is the proposed deterministic coding. The scheme is full rate and has lower ICI power, as compared to uncoded OFDM. Lastly, the correlative coding has the worst performance, as compared to the rest of the codes. From the plots, it is seen that the bounds are quite tight, except for the lower bound of PCC-OFDM in the range of $f_d T_s > 0.3$.

The BER of the various coding scheme has also been simulated. The numerical results with $f_d T_s = 0.1$ are shown in Figs. 6.5, 6.6 and. 6.7 for the Clarke/Jakes' Doppler spread channel model, uniform Doppler spread channel model and the CFO model respectively. The results are obtained by 100000 Monte Carlos trials. Hard decision decoding is implemented for all the codes.

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

Figure 6.3: Results of Various Coding in Uniform Doppler Spread Channel Model

From the figures, it is shown that the 1/2 rate PCC scheme has no error floor for the BER performance in a time-varying channel. Nevertheless, the efficiency is half that of the other coding schemes. The proposed deterministic scheme has a poorer BER performance than PCC, but performs better than the correlative code, as more ICI has been eliminated.

In our discussion, a flat fading channel is assumed. For OFDM with multipath, the cyclic extension can be applied to remove ISI. Under this assumption, it is seen in [42] that ICI power is similar for both flat and dispersive fading channels, as long as the total received power is normalized to 1. Moreover, the analysis is also valid when the length of the cyclic extension is not enough to remove the ISI [3],[4], by analyzing the amount of interference power caused by ISI and the time-varying channel independently.

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

Figure 6.4: Results of Various Coding in 2-Rays Channel Model

Figure 6.5: Simulation BER of Various Coding Scheme with maximum Doppler frequency of $f_d T_s = 0.1$ for Clarke/Jakes' Doppler Spread Model

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

Figure 6.6: Simulation BER of Various Coding Scheme with maximum Doppler frequency of $f_d T_s = 0.1$ for Uniform Doppler Spread Model

Figure 6.7: Simulation BER of Various Coding Scheme with maximum CFO frequency of $f_d T_s = 0.1$ for CFO Model

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

6.4 Summary

In this chapter, the general expressions of the ICI power for two of most popular ICI reduction codes, correlative coding and PCC have been derived. The equations are applicable to general channel models, and are expressed in terms of the time domain correlation function. A novel ICI reduction coding technique, deterministic coding, is proposed, with its general expression derived. The general expressions of all the codes have been verified with simulation of popular channel models, including CFO, classic Clarke/Jakes' Doppler spread model, uniform Doppler spread model and 2-rays model. The ICI power calculated using the general expressions have also been verified by the simulation. The tight upper and lower bounds for those ICI reduction codes have also been developed.

	PCC	Correlative Code	Proposed Deteministic Code
ICI Reduction Capability	11.9dB	4.1dB	6.65dB
BER Performance	1st	3rd	2nd
Full Rate?	No	Yes	Yes
Complexity	Low But Require special decoder	High May need soft demodulation to improve BER	High May need soft demodulation to improve BER

Table 6.2: Comparing Different ICI Reduction Coding Schemes

A brief summary about the various ICI reduction codes is presented in Table 6.2. It can be seen that there is no clear winner, as each coding scheme has its own pros and cons. But generally, the proposed deterministic code performs better than the correlative code under all conditions. Lastly, although the performance of PCC OFDM

CHAPTER 6. ANALYSIS OF ICI REDUCTION CODING TECHNIQUES IN OFDM SYSTEMS FOR TIME-VARYING CHANNEL

is the best, in terms of ICI reduction capability and BER performance, it is not full rate.

Chapter 7

Polyphase Sequence Based Pilot Channel Estimation for OFDM in Doubly Fading Channel

The application of OFDM to highly-mobile (doubly fading) channels has been quite limited [42],[7]. Although there are various ways to reduce ICI, like utilizing of directional antennas, or the use of ICI reduction codes, there is still a requirement to know the channel in order to correctly decode the received signal. Furthermore, equalizers/filters can also mitigate the channel impairments and assist in the correct detection of the received signal. It is known from [54],[51],[50],[53] that the impulse response for the doubly fading channel is required in order to equalize the received signal and to combat ICI. Most of the available OFDM channel estimation algorithms assume knowledge of exact or close to exact channel statistical parameters [51]. However, estimating the channel statistical parameters is another significant problem in itself.

In this chapter, we address the problem of channel estimation under the assumption

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

that channel statistical parameters are not available. In particular, polyphase sequences are proposed for constructing a simple cluster-comb pilot based channel estimation technique using LS algorithm. The estimated channel response is in frequency domain and can be directly applied in the design of frequency domain equalizers [54],[51],[53].

7.1 Channel Estimation Technique and Sequence

The general representation of the received OFDM signal in matrix form can be written as (from Chapter 3)

$$\mathbf{y} = \mathbf{G}\mathbf{d} + \boldsymbol{\omega} \quad (7.1)$$

where the complex vector $\mathbf{d} = \{d_n\}$, $\mathbf{y} = \{y_n\}$, $\boldsymbol{\omega} = \{\omega_n\}$ for $n = 0, \dots, N_c - 1$ and $\mathbf{G}$ is the channel matrix where the element at c row and $(b + c)$ column is denoted as $g_{b,c}$.

7.1.1 Channel Estimation Technique

Conventional OFDM system channel estimation algorithms estimate the time domain $N_c \times L$ channel impulse responses using one or multiple OFDM symbols [54],[51],[50],[53]. Moreover, as noted in [51],[50],[53], for the frequency domain channel estimation algorithm, N_c^2 values are required to be estimated, more than the $N_c \times L$ in time domain. However, the number of values to be estimated can be reduced by only estimating those significant channel responses [54]. In this case, as seen later, the number of values to be estimated in frequency domain is similar to that in time domain. Furthermore, the channel responses in time domain usually need frequency transformation before it can be applied to the equalizers [54],[51],[53], thus increasing the overall complexity.

Here, a simple linear channel estimator in frequency domain is proposed which

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

estimates the significant channel impulse responses. We begin by assuming a frequency flat fading, time dispersive channel, where all the diagonal elements of $\mathbf{G}$ (which is $g_{b,c}$, given a constant b and c) are the same. By assuming that only 2α number of adjacent subcarriers [54] are dominant, there is only a need to estimate $2\alpha + 1$ values. This is especially true when the Doppler spread of the channel is high.

An example of the spreading effect in frequency domain is shown in Fig. 7.1, which is simulated with Clarke/Jakes' Doppler spread channel model [40],[41], averaged over 1000 simulation results. In this figure, the desired subcarrier index is 17, where all the power at other subcarrier indexes are the ICI resulted from subcarrier 17. As seen in the case when $f_d T_s = 0.1$, the relative amount of ICI power to adjacent subcarriers is around 10 times less, as compared with the power at subcarrier 17. However, as $f_d T_s$ gets higher, the amount of ICI generated by subcarrier 17 to adjacent subcarrier becomes more significant. For the case of $f_d T_s = 0.3$, the relative amount of ICI power is around 1/3 of the power at the subcarrier 17. In the case of $f_d T_s = 0.5$, the amount of ICI power at adjacent subcarriers around 17 is even higher than the desired signal. It is in this situation that the adjacent channels around the desired subcarrier are required to be estimated, so as to remove the dominant ICI.

Therefore, by concentrating on these dominant ICI and neglecting the others, the channel matrix is approximated with a matrix of bandwidth α [87] (see Chapter 6). In order to ensure that non-pilot subcarriers do not interfere, it requires a buffer of α pilots on each side of the $2\alpha + 1$ subcarriers. This can be easily achieved by placing a cluster of pilots of length $4\alpha + 1$. Thus, with χ as the subcarrier number in the middle (the $(2\alpha + 1)$ -th) position of a pilot cluster, the received signal at subcarriers $\chi + (-2\alpha, \dots, 2\alpha)$ can be approximated as

$$\mathbf{y}_\chi \approx \overline{\mathbf{G}}_\chi \mathbf{p}_\chi + \mathbf{z}_\chi + \boldsymbol{\omega}_\chi \quad (7.2)$$

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

Figure 7.1: Frequency Domain Channel Impulse Response for different normalized Doppler spread, $f_d T_s$, using Clarke/Jakes' Model

The vector $\mathbf{p}_\chi = \{p_{\chi+n}\} := \{p_k\}$, $n = -2\alpha, \dots, 2\alpha$, $k = 0, \dots, 4\alpha$, is a cluster of $4\alpha + 1$ number of pilots and $\mathbf{y}_\chi = \{y_{\chi+n}\}$, $n = -2\alpha, \dots, 2\alpha$ is the corresponding receiver output of the cluster of pilots. The $(4\alpha + 1) \times (4\alpha + 1)$ matrix $\overline{\mathbf{G}}_\chi$ is the channel response matrix for a specific cluster of pilots and $\mathbf{z}_\chi$ is the amount of interference caused to the specific cluster of pilots from other subcarriers. As the channel is assumed to be flat fading, the diagonal elements in the channel matrix, $\overline{\mathbf{H}}_\chi$ are highly correlated within the $4\alpha + 1$ subcarriers in the cluster of pilots and $\overline{\mathbf{H}}_\chi$ is seen as having a Toeplitz structure with $(2\alpha + 1)$ -diagonal non-zero elements (which are required to be estimated). So the matrix is rearranged by extracting the portion where the element of $\mathbf{z}_\chi$ can be approximated as zero (by neglecting the non-significant ICI), it becomes

$$\tilde{\mathbf{y}}_\chi = \tilde{\mathbf{P}}_\chi \tilde{\mathbf{g}}_\chi + \tilde{\mathbf{w}}_\chi \quad (7.3)$$

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

where the vector $\tilde{\mathbf{y}}_\chi = \{y_{\chi+n}\}$, $\tilde{\mathbf{g}}_\chi = \{g_{n,\chi}\}$ and $\tilde{\boldsymbol{\omega}}_\chi = \{\omega_{\chi+n}\}$ for $n = -\alpha, \dots, \alpha$. $\tilde{\mathbf{P}}_\chi$ is a square $(2\alpha+1) \times (2\alpha+1)$ Hankel matrix, with the first and the last row equal to $\{p_k\}$, $k = 0, \dots, 2\alpha+1$ and $\{p_k\}$, $k = 2\alpha+1, \dots, 4\alpha$ respectively. Using the LS approach, the channel can be estimated with

$$\hat{\tilde{\mathbf{g}}}_\chi = \left(\tilde{\mathbf{P}}_\chi^H \tilde{\mathbf{P}}_\chi \right)^{-1} \tilde{\mathbf{P}}_\chi^H \tilde{\mathbf{y}}_\chi \quad (7.4)$$

7.2 Extension to MIMO OFDM

The expression of (7.1) can be extended to the case of multiple transmit and received antennas, known commonly as multiple input, multiple output (MIMO) OFDM system. It is written as

$$\mathbf{y}^n = \sum_{m=0}^{N_{trx}-1} \mathbf{G}^{m,n} \mathbf{d}^m + \boldsymbol{\omega}_n \quad (7.5)$$

where N_{trx} denotes the number of transmit antennas and the complex vector $\mathbf{d}^m$ is the transmitted data vector of the m -th transmitted antenna, and $\mathbf{y}^n$ is the corresponding received vector. The channel matrix, $\mathbf{G}^{m,n}$ corresponds to the channel from the m -th transmit antenna to the n -th receive antenna. Similarly, (7.5), with respect to (7.3), can be approximated by neglecting non-significant ICI around the χ subcarriers, as

$$\tilde{\mathbf{y}}_\chi^n = \sum_{m=0}^{N_{trx}-1} \mathbf{P}_\chi^m \tilde{\mathbf{g}}_\chi^{m,n} + \tilde{\boldsymbol{\omega}}_\chi^n \quad (7.6)$$

where $\tilde{\mathbf{y}}_\chi^n$ is the equivalent of $\tilde{\mathbf{y}}_\chi$ in (7.3) for the n -th received antenna and $\tilde{\mathbf{g}}_\chi^{m,n}$ is the channel vector at center position χ for the m -th transmit antenna to the n -th received antenna. The pilot matrix, $\mathbf{P}_\chi^m$, is the pilot sequence from the m -th transmit antenna. The size of $\tilde{\mathbf{y}}_\chi^n$, $\mathbf{P}_\chi^m$ and $\tilde{\boldsymbol{\omega}}_\chi^n$ will be different from (7.3). This is because for MIMO system, the length of the pilot sequence has to be longer in order to distinguish

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

between antennas. Thus, it has been found that the vector length for $\tilde{\mathbf{y}}_x^n$ and $\tilde{\mathbf{w}}_x^n$ will be $2\alpha(N_{trx} + 1) + N_{trx}$ and the size of the Hankel pilot matrix will be $(2\alpha + 1) \times (2\alpha(N_{trx} + 1) + N_{trx})$.

Again, using the LS approach, the channel can be estimated as

$$\widehat{\tilde{\mathbf{g}}^{m,n}} = ([\mathbf{P}^m]^H [\mathbf{P}^m])^{-1} [\mathbf{P}^m]^H \tilde{\mathbf{y}}^n \quad (7.7)$$

As the size of $\tilde{\mathbf{P}}_x$ is $(2\alpha + 1) \times (2\alpha(N_{trx} + 1) + N_{trx})$, the computational complexity will not be very large as it is $\mathcal{O}[(2\alpha + 1)^2]$, and α is normally less than 3. It will be shown in next section by calculating the normalized mean square error (NMSE) that the improvement is marginal for $\alpha \geq 3$.

In the case of multipath channel, the effects of frequency selectivity have to be considered. The proposed method is still applicable in frequency selective channel, since the channel responses from adjacent subcarriers are still highly correlated. An example for multipath fast time-varying channel is simulated in Fig. 7.2, with the Clarke/Jakes [40],[41] Doppler spread model of $f_d T_s = 0.5$. The results for the immediate channel impulse response of all the subcarriers and their dominant ICI ($g_{b,c}$ for $b = [-2, -1, 0, 1, 2]$, $c = [0, \dots, N_c]$) are plotted. As seen from the figure, the channel impulse response of all the subcarriers and their dominant ICI, $g_{b,c}$ for $b = [-2, -1, 0, 1, 2]$, are quite close, which coincide with the results found in Fig. 7.1. And as seen from Fig. 7.2, $g_{b,c}$ can be linearly interpolated between two values of c for a given value of b . Thus, in the first step, the channel and their ICI, for each of the clusters of pilot, have to be estimated. The second step uses those estimated values and linearly interpolates them to estimate the values of the channel response and their ICI for those data subcarriers. This approach is similar to the one dimensional channel estimation algorithms [34],[35],[36] for those non-time varying channel, as described in Chapter 2. In this case, the channel

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

matrix, for a given matrix bandwidth [87], can be estimated. Nevertheless, the proposed algorithm may suffer small amount of degradation when α is large, as shown in the later sections.

Figure 7.2: Frequency Domain Channel Impulse Response for different g for $f_d T_s = 0.5$, using Clarke/Jakes' Model

7.2.1 Sequence Properties and Construction

For optimum pilot sequence of length $\{2\alpha(N_{trx} + 1) + N_{trx}\}$, any two rows of the Hankel pilot matrix, $\mathbf{P}^m$, have to be orthogonal [89],[90],[52]. When the transmitted power of each chip of the pilot sequence is equal, the correlation of the pilot sequence can be written as

$$[\mathbf{P}^m]^H [\mathbf{P}^m] = N_{trx}(2\alpha + 1)\lambda_m^2 \mathbf{I} \quad (7.8)$$

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

where $|p_q^m|^2 = \lambda_{q,m}^2 = \lambda_m^2$. Nevertheless, in order to differentiate the signals from different transmit antennas, it has to have zero cross-correlation between different sets of pilot for different antennas. So it must fulfill

$$[\mathbf{P}^{m_1}]^H [\mathbf{P}^{m_2}] = \mathbf{0}, \quad \text{for } m_1 \neq m_2 \quad (7.9)$$

The Zadoff-Chu's [91],[92],[93] sequence is adopted for constructing the pilot sequence, which can be written as

$$p_q^m = \exp\left(\frac{j2\pi r \kappa q^2 + bq}{c}\right),$$

$$\text{for } q = (m-1)(2\alpha+1) + (1, \dots, \{2\alpha(N_{trx}+1) + N_{trx}\}) \quad (7.10)$$

$$\text{mod } [2\alpha(N_{trx}+1) + N_{trx}]$$

where r is any integer relatively prime to c , b is any arbitrary integer and κ equals to $\frac{1}{2}$ when N_{trx} is odd and equals to 1 when N_{trx} is even.

From [36], Boer *et al* proposed a technique (known as diagonally loaded preamble) for the proposed MIMO-OFDM IEEE 802.11n Wireless LAN standard. By extending it to a time varying case and using polyphase pilot sequence for just one antenna, the number of pilots needed for every antenna element will be $\{2\alpha(N_{trx}+1) + N_{trx}\}$. Comparing the proposed method with Boer *et al* diagonally loaded preamble with polyphase pilot sequence case, the proposed method can significantly reduce the number of pilots with increasing number of antennas, N_{trx} or number of dominant neighboring interfering subcarriers, α .

7.3 Computation of NMSE

Taking into account the amount of interference caused by the non-pilot subcarriers, the actual received signal can be written as

$$\tilde{\mathbf{y}}^n = \sum_{m=0}^{M-1} (\mathbf{P}^m \tilde{\mathbf{g}}^{m,n} + \tilde{\mathbf{o}}^{m,n}) + \tilde{\mathbf{w}}^n$$

where the vector $\tilde{\mathbf{o}}_\chi$ is the sum of non-dominant interference caused by the non-pilot subcarriers. From [7], it is known that $\tilde{\mathbf{o}}_\chi$ can be modelled as white noise, which is uncorrelated with AWGN, $\tilde{\mathbf{w}}_\chi$, where $E(|\tilde{\mathbf{w}}_\chi|^2) = \sigma_n^2 \mathbf{I}$. By assuming that the transmitted power of each non-pilot random signal is the same as each chip of the pilot sequence ($E\{|s_n|^2\} = \lambda^2$), the $\tilde{\mathbf{o}}_\chi$ covariance can be expressed as [42]

$$\begin{aligned} & \sum_{m=0}^{N_{trx}-1} E(|\tilde{\mathbf{o}}^{m,n}|^2) \\ &= \sum_{m=0}^{N_{trx}-1} \left\{ \lambda_m^2 \mu^{m,n} \sum_{\substack{n=0 \\ n \notin \mathbf{m}_\chi}}^{N_c-1} \left[\int_{-1}^1 r^{m,n}(f_d T_s x) (1 - |x|) \exp\{-j2\pi(n - \chi)x\} dx \right] \right\} \quad (7.11) \\ &= \sum_{m=0}^{N_{trx}-1} \lambda_m^2 \mu^{m,n} \sigma_{ICI,\{m,n\}}^2 \mathbf{I} \end{aligned}$$

where $r^{m,n}(f_d T_s x)$ is the time domain autocorrelation function of the channel from the m -th transmit antenna to the n -th received antenna. The set $\mathbf{m}_\chi$ is the set of subcarrier index of that pilot sequence, where $\mu^{m,n}$ is the average power of the $\{m, n\}$ channel. Thus, from the LS approach, the estimated channel response for a cluster of

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

pilot sequence is given by

$$\begin{aligned}\widehat{\mathbf{g}}^{m,n} &= \left([\tilde{\mathbf{P}}^m]^H [\tilde{\mathbf{P}}^m] \right)^{-1} [\tilde{\mathbf{P}}^m]^H \tilde{\mathbf{r}}^n \\ &= \tilde{\mathbf{g}}^{m,n} + \left([\tilde{\mathbf{P}}^m]^H [\tilde{\mathbf{P}}^m] \right)^{-1} [\tilde{\mathbf{P}}^m]^H \left(\sum_{g=0}^{N_{trx}-1} \tilde{\mathbf{o}}^{g,n} + \tilde{\mathbf{w}}^n \right)\end{aligned}\quad (7.12)$$

And finally, the NMSE of the estimate can be written as

$$\begin{aligned}E \left(\left\| \widehat{\mathbf{g}}^{m,n} - \tilde{\mathbf{g}}^{m,n} \right\|^2 \right) &= \left\{ (2\alpha + 1)\sigma_n^2 + \sum_{g=0}^{N_{trx}-1} \lambda_g^2 \mu^{g,n} \sigma_{ICI,\{g,n\}}^2 \right\} \text{tr} \left([\tilde{\mathbf{P}}^m]^H [\tilde{\mathbf{P}}^m] \right)^{-1} \\ &= \frac{1}{N_{trx} \lambda_m^2} \left(N_{trx} \sigma_n^2 + \sum_{g=0}^{N_{trx}-1} \lambda_g^2 \mu^{g,n} \sigma_{ICI,\{g,n\}}^2 \right)\end{aligned}\quad (7.13)$$

It is understood from (7.13) that if different power is assigned to transmit antennas (similar to waterfilling concept), the NMSE will be different. The higher the transmit power of a certain antenna, the lower will be the NMSE. However, if the transmit power of each antenna is equal, then the NMSE will be the same for each channel, independent of M . It is because $M\lambda_m^2 = 1$.

Numerical examples of the NMSE using Clarke/Jakes' [41],[40] model for $N_{trx} = 1$, are shown in Fig. 7.3. It can be seen with just $\alpha = 1$, the reduction in the estimation error is about 8 dB compared to $\alpha = 0$. The amount of estimation error can be reduced as α increases. However, for $\alpha > 3$, the reduction in NMSE is only around 1 dB for every increase of α by 1.

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

Figure 7.3: Numerical results showing the NMSE (with $\sigma_n^2 = 0$, $\lambda^2 = 1$ and $N_{trx} = 1$) for different $f_d T_s$ using Clarke/Jakes' Model

7.4 Simulation Results

In this section, the simulated NMSE is shown for two different systems, the single transmit antenna (SISO) system and the MIMO system. The number of subcarriers for both the SISO and MIMO systems is $N_c = 1024$ for different amount of Doppler spread, depending on the $f_d T_s$. The Clarke/Jakes' [41],[40] Doppler spread channel model is simulated with SNR=30dB. For the SISO system, the transmitted power is set to $\lambda^2 = 1$. It is simulated in two different wireless scenarios, frequency flat fading (in Fig. 7.4a) and frequency selective fading (in Fig. 7.4b). In the case of frequency selective fading, the total number of paths is set to 3 and the delay of each path is arbitrary and uniformly distributed between 0 and 32 sampling time. The power of the 3 paths are (0, -10, -20) dB and are normalized to unity total power. The proposed

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

algorithm, with $\alpha = 2$, is compared with the Kronecker-Delta (KD) [54] method and the algorithm proposed by Stamoulis *et al*[50]. (Both are mentioned in Chapter 3.) The total number of pilots deployed is the same for both Stamoulis' algorithm and the proposed algorithm. The KD method and Stamoulis' algorithm are chosen because they can be implemented without the need for the channel statistics. Thus, the proposed algorithm can be compared in a fairer manner.

From Fig. 7.4, it can be seen that the proposed algorithm outperforms the Stamoulis' method in the frequency flat channel. Whereas, for the frequency selective channel, the proposed algorithm outperforms other algorithms for $f_d T_s > 0.25$. Comparing with the KD algorithm, the proposed technique also performs better for higher value of $f_d T_s > 0.4$ in both cases. This is because KD methods assumed that the channel changes linearly within one OFDM symbols. For the proposed scheme, it does not make this assumption. Thus, the proposed algorithm is found to be suitable for situations where $f_d T_s$ is high. The theoretical NMSE of the proposed method, for the SISO case, matches very well with the results for the frequency flat fading, but increased by 2dB for frequency selective channels. It is because fading characteristics within α is no longer a constant in frequency selective channels, but with small amount of variations. Therefore, there will be small discrepancies between the assumptions made for the proposed algorithm and the practical channel conditions. In this case, the NMSE can serve as a lower achievable bound for the proposed algorithm.

The MIMO system simulation results are shown in Fig. 7.5, where $N_{tx} = 4$. Single path and multipath Clarke/Jakes' Doppler spread channel models are employed for simulations. As is the case of SISO, the power of the 3 paths are set to (0, -10, -20) dB respectively and are normalized to unity total power. The delay of each path is arbitrary and uniformly distributed between 0 and 32 sampling time. The transmitted power of each of the antenna is assumed the same, $\lambda_m^2 = 1/4$, without channel adaptation and

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

(a) Frequency Flat Fading Channel

(b) 3-path Frequency Selective Fading Channel

Figure 7.4: Comparison of Different Channel Estimation Algorithms with SNR=30dB

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

the channel gain of each channel $\mu^{m,n}$ is set to 1. Each of the channels between the m -th transmit antenna and the n -th receive antenna is assumed to be independent, but have the same statistics, and the same amount of Doppler spread, $f_d T_s$. However, these are not the requirements of the proposed algorithm, as each channel is estimated independently.

Figure 7.5: Comparing Theoretical and Simulated MSE for 4 transmit antennas

It is seen from Fig. 7.5 that the amount of degradation due to the multipath channel is less significant when the $f_d T_s$ is large. Nevertheless, the degradation may be significant for low $f_d T_s$, which can be compensated through a filter if the multipath channel characteristics are known. This knowledge of multipath response is slowly changing and can be easily estimated using conventional channel estimator seen in Chapter 2, after some smoothing to remove the ICI. The degradation of the multipath channel is slightly higher than the SISO case, due to the fact that there are interferences received from other transmit antennas.

CHAPTER 7. POLYPHASE SEQUENCE BASED PILOT CHANNEL ESTIMATION FOR OFDM IN DOUBLY FADING CHANNEL

7.5 Summary

In this chapter, a polyphase based sequence is proposed as the pilot cluster to estimate the channel in OFDM systems. It is able to estimate the channel and the dominant neighboring interference subcarriers due to fast time-varying channel. The NMSE for the estimate has been derived and verified using simulations. The algorithm has been compared with other algorithms mentioned in Chapter 3. It has been found that the proposed algorithm outperforms other algorithms when the amount of Doppler spread is high. The proposed algorithm has been shown to be easily extendable to include MIMO OFDM systems.

Chapter 8

Conclusions and Future Works

8.1 Conclusions

OFDM is robust against multipath but suffers degradation in fast time varying channel. This phenomenon is worrying to many researchers, as OFDM is often termed as the basis for next generation wireless communication system [2]. In fact, many researchers are currently working on resolving the effects of fast time varying channels which cause ICI in OFDM system. From literature, it is found that there are channel dependent and channel independent methods to reduce the ICI. The channel independent methods include ICI reduction codes and pulse shaping, whereas the channel dependent methods include the use of directional antennas and filters/equalizers. Both channel dependent and channel independent methods are investigated in the thesis.

The ICI power for the directional antenna channel is first investigated. The PDFs of the AOA and PSDs of the Doppler spread channel are examined before calculating the ICI power. The PDFs of the 2-D channel model are derived when directional antennas are employed. The Doppler PSDs are derived based on the PDFs of AOA. It

CHAPTER 8. CONCLUSIONS AND FUTURE WORKS

is found that the amount of Doppler spread can be significantly reduced when directional antennas are used. This insight leads to the extension of reduction of Doppler spread in the 3-D channel model. The amount of Doppler spread is shown to reduce if the beam pattern is restricted in both azimuthal and elevation planes for the 3-D model. It is found that with careful planing of the steering angle of the directional antenna (at the mobile receiver), the amount of Doppler spread can be decreased significantly.

Once the PDFs of the AOA and the Doppler PSDs are characterized, the amount of ICI power in an OFDM system using directive antennas is investigated. The general expressions for calculating the ICI power of the OFDM system, in terms of the PDF of AOA, have been derived. The expressions are derived for both 2-D and 3-D channel models. From the equations, it has been found that the ICI power may not necessarily reduce when a directional antenna is deployed at the base station. The amount of ICI can be lessened only in the case when the mobile is moving in the perpendicular direction or around it. However, the ICI power is shown to decrease significantly in all possible scenarios when a directional antenna is employed at the mobile, with frequency compensation appropriately employed. The equation for estimating the ICI power in a Rician environment has also been derived. In a multiple Rician environment, it is shown that the ICI power may not reduce as much as in a pure Rayleigh channel.

The use of angular diversity in OFDM system is proposed using the general ICI power expression. It reduces the fading rate and also provides the advantage of diversity. Nevertheless, each branch of the diversity system should be optimized such that their average ICI power will be minimized. In order to have the minimum average ICI power when using ULA, the amount of Doppler spread from each beam has to be approximated to be equal. A method has been proposed to calculate the beam width for equal Doppler spread. The use of a frequency compensating factor is demonstrated for the case when the beamforming has different beam intervals and steering angles. It is shown that the

CHAPTER 8. CONCLUSIONS AND FUTURE WORKS

use of multiple beamforming will not only reduce the Doppler spreading, but will also enhance the system performance by exploiting diversity. It is found through simulations that the improvement can be substantial when only just two antenna elements are used. The improvement is even more significant when three antenna elements are employed.

Next, a ICI reduction technique which does not depend on the channel model is considered. The amount of ICI power of the coding approach has been calculated for different channel models. The general equations of the ICI power have been derived for two of the most popular ICI reduction codes, namely, correlative coding and PCC. The equations are expressed in terms of the time domain correlation function of the channel models. A novel coding method has also been proposed. The proposed coding scheme is introduced to mitigate some of the disadvantages of the existing coding schemes. The general expressions of all the codes have been verified by using popular channel models, including CFO, classic Clarke/Jakes' Doppler spread model, uniform Doppler spread model and 2-rays model. From the simulation results, calculations of the ICI power using the general expressions are verified.

The tight upper and lower bounds for the ICI reduction codes are also derived as the general ICI power expressions for the various codes are complicated. Using the bounds, it is found that the upper bounds for ICI power reduction are 6.65dB, 4.1dB and 11.9dB for the proposed deterministic coding scheme, the correlative codes and the 1/2 rate PCC-OFDM respectively. A table comparing the advantages and disadvantages of various coding schemes is given. It can be seen that there are no clear winner, as each code excel in either one or more of the categories.

Another technique of mitigating ICI is to employ an equalizer at the receiver. Nevertheless, equalization requires the knowledge of immediate channel impulse response. In this thesis, a MIMO-OFDM frequency domain channel estimation algorithm is pro-

CHAPTER 8. CONCLUSIONS AND FUTURE WORKS

posed. The main difference between the proposed algorithm and the conventional channel estimation for time varying channel is that it is in frequency domain. The proposed algorithm is a pilot cluster based channel estimation algorithm, where the pilot sequence is constructed using a polyphase sequence. It is able to estimate the channel and the dominant neighboring interfering subcarriers due to fast time-varying channel. The NMSE is also derived for the algorithm and verified using simulation. By comparing the proposed algorithm with the other conventional algorithms, it has been found that the proposed channel estimation algorithm outperforms other algorithms, especially in the case of high Doppler spread.

In this thesis, a total of 3 ICI reduction techniques are proposed, with their respective merits and limitations. In the case of employing directional antennas, additional space resources are required to accommodate the (larger) antenna(s). The advantage is that this methodology will be relatively straight forward to implement, without any modification to the original OFDM system. The amount of ICI power reduction is also quite significant.

For the ICI reduction codes, several modifications are required to be made to the OFDM transmitter and receiver, in terms of the modulation and demodulation schemes employed for each subcarrier. However, most of them do not require additional (space, frequency or time) resources to reduce the ICI power and can achieve reasonable performance improvements. Lastly, the proposed channel estimator with equalizer does not require modification to the modulation schemes, but it needs special arrangement for the pilots and requires special design for the pilot sequences. Nevertheless, the performance is acceptable, as compared to the other ICI reduction techniques.

8.2 Suggestions on Future Research

There are tremendous ongoing research efforts in resolving the ICI effects in OFDM systems. Novel solutions, including ICI reduction codes and directional antennas, have been proposed and thoroughly investigated in the thesis. Nevertheless, there are alternative methods of reducing ICI. Moreover, the techniques mentioned in the thesis can be applied to other areas of research.

In the case of ICI reduction codes, they belong to the idea generally known as precoding. As seen in Chapter 6, the BER performances of the precoding techniques are not that desirable when only simple decodings are applied. The post processing technique requirement may be relevant. In this case, the main motivation of ICI reduction will not only be on the precoding, but on both the precoding and the post processing. For this approach, the optimum solution has been shown to deduce to a generalized eigenvector problem, where the received subcarriers' channel gains are enhanced while the ICI level is reduced. However, the transmitter and receiver must know the instantaneous channel impulse response. Nevertheless, it has been shown that by exploiting channel statistics to replace the true channel response, the amount of ICI can be lessened. In this case, the ICI power reduction will not be as effective as the case when the instantaneous channel impulse response is used. A further extension to this will be to come up with another approach to maximize the received subcarriers' channel gain. It can utilize the technique of decision feedback where the estimated channel using the proposed algorithm is fed back and applied to the post processing by posing it as the general eigenvalue problem, where both channel statistics and estimated channel impulse response can be applied. In this case, there will not be much differences between the channel parameters employed at the receiver and the actual channel parameters.

The precoding and post-processing can also be generalized to include more extensive

CHAPTER 8. CONCLUSIONS AND FUTURE WORKS

methods. This generalization can be accomplished by employing the concept of Frame [94], which is commonly applied in area of image signal processing. For example, in the case of using antenna array to compensate Doppler [95],[79], the signal processing applied can be generalized to the Frame structure. The idea of Frame can be extended to the precoding and post-processing techniques as well, as the precoding and post-processing matrix. Moreover, it can be seen that the extension of the Frame structure to include waveform design for ICI reduction.

Again as shown in Chapter 6, certain coding techniques can effectively reduce ICI power. Nevertheless, these coding techniques do not provide error control and correction properties. Although conventional forward error correction codes are found to be able to reduce ICI [96], the results are only based on simulation. Moreover, the amount of ICI reduction are really dependent on which specific code word applied. Therefore, it is not very effective and lacks theoretical understanding and insight. Hence, a way to approach this problem is through the basic of coding, which can be defined in Galois field. As conventional block codes can be represented with a generator polynomial [97], it can be related to the PCC.

Another insight, from the investigation of ICI power reduction in Chapters 3 and 6, is on the theoretical analysis of the BER performance of OFDM system in time-varying channels. Although ICI power is commonly assumed as Guassian distributed in order to calculate the BER of an uncoded OFDM systems [98], the validity of this assumption is still very much debated. Moreover, regarding the validity of this assumption in OFDM system with ICI reduction codes, there isn't much discussion yet, except in [99]. Even in [99], only the loose BER bounds are shown, based on the Central Limit Theorem assumption. Therefore, theoretical derivation of the BER equations of OFDM system in time-varying channels (for both coded and uncoded cases), can be an interesting extension to the work of this thesis.

CHAPTER 8. CONCLUSIONS AND FUTURE WORKS

In OFDM, PAPR is another huge issue that has not been really solved yet. Researchers have been going round the problem by either using phase rotation or mapping. These approaches require the receiver to have prior knowledge of the mapping or the amount of phase rotation, which might change from symbol to symbol. Other than the methods mentioned, brute force methods (clipping and/or filtering) introduce interference due to loss of orthogonality, which is also one of the causes of ICI. However, as in the case of time-varying channels, the ICI can be mitigated using other methods, like the PCC or correlative code. Thus, ICI reduction for both PCC and correlative code in clipping situation can be investigated. Furthermore, it has been found that certain ICI reduction technique can reduce the PAPR. Therefore, joined ICI and PAPR reduction techniques can also be investigated. Moreover, this area of research can include the case where multiple antennas are being applied to OFDM systems.

As mentioned in Chapter 7, the time-varying MIMO-OFDM channel has to be estimated in order for the receiver to equalize the MIMO signal and demodulate it. But instead of using equalizers to mitigate the ICI problem, the use of space time pre-compensation can be considered. Various space-time codes (full diversity full rate complex space time code [100] or space-time-Doppler block code [101]) for fast time varying channel have been developed in literature, but they involve high computational complexity to mitigate both the multipath and time varying channels. Furthermore, these fast time varying channel space-time codes do not perform as well as those conventional space-time codes [102], in term of diversity gain. As in conventional MIMO-OFDM systems, OFDM techniques can be applied for mitigating of the multipath channel, while the underlying MIMO/space-time codes can be designed for enhancing capacity/diversity while mitigating the fast time varying channels.

Lastly, ICI reduction and mitigation concepts and analysis can be further extended to other MCM systems, such as the multi-carrier CDMA (MC-CDMA) system. As

CHAPTER 8. CONCLUSIONS AND FUTURE WORKS

most MCM systems are based on FFT, MC-CDMA and other similar systems also suffer degradations from the effects of time-varying channels. However, the extension of the ICI reduction and analysis methodologies into other MCM systems have not yet been investigated extensively in literature. Thus, this direction of research can be one of the most interesting and meaningful future research topics, and may find its practical usage for 4G systems.

Author's Publications

Journal Paper

1. W.T. Ng and V.K. Dubey, "Comments on "On the Doppler Spectrum at the Mobile Unit Employing a Directional Antenna"," *IEEE Commun. Lett.*, vol. 6, no. 11, pp. 472–474, Nov. 2002.
2. W.T. Ng and V.K. Dubey, "Effect of Employing Directional Antennas on OFDM Systems with Time-Varying Channel," *IEEE Commun. Lett.*, vol. 7, no. 4, pp. 165–167, Apr. 2003.
3. W.T. Ng and V.K. Dubey, "Analysis of PCC in OFDM Systems for Time-Varying Channel," to be published in *IEEE Commun. Lett.*, 2005.
4. W.T. Ng and V.K. Dubey, "Analysis of ICI Reduction Coding Techniques in OFDM Systems for Time-Varying Channel," submitted to *IEEE Trans. Vehicular Technology*, 2004.

CHAPTER 8. CONCLUSIONS AND FUTURE WORKS

Conference Paper

1. W.T. Ng and V.K. Dubey, "Effect of Employing Beamforming on OFDM Systems in Rician Channel," in *Proc. of 8th International Conference on Communication System*, 2002, Singapore.
2. W.T. Ng and V.K. Dubey, "Application of Angular Diversity in OFDM Systems," in *Proc. of IEEE International Conference on Communication*, 2003, Anchorage, Alaska, USA.
3. W.T. Ng and V.K. Dubey, "On Coded Pilot Based Channel Estimation for OFDM in Very Fast Multipath Fading Channel," in *Proc. of 4th International Conference on Information, Communications & Signal Processing*, 2003, Singapore.

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