Performance analysis of digital communication systems in two-wave with diffuse power fading

Oh, Soon Huat

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Performance Analysis of Digital Communication Systems
in
Two-Wave with Diffuse Power Fading

Oh Soon Huat

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
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Summary

The received signal strength at a mobile wireless receiver as it moves over a small, local area is subjected to severe and rapid fluctuation. This rapid fluctuation in signal strength arises mainly due to multipath propagation in the immediate vicinity of the receiver.

A proper characterization of this small-scale or fast fading is therefore necessary when investigating the performance of a receiver in the presence of noise and interference. The fading probability density function (pdf), which represents the probability density of the amplitude of the received signal, should be carefully chosen so that the system analysis gives accurate prediction of the bit-error rate (BER) or symbol-error rate (SER) performance.

Rayleigh and Rician pdfs are commonly used to characterize small-scale fading, but experimental measurements of local fading signal amplitudes have sometimes shown that either one of these two pdfs is not adequate in this respect. The two-wave with diffuse power (TWDP) fading pdf, recently introduced with the aim to complement the above classical pdfs, characterizes the physical situation where there are two specular (dominant) waves in the presence of other diffuse components. The use of the TWDP pdf to characterize small-scale fading has the advantage that it has, as its special cases, the Rayleigh and Rician pdfs. Hence, BER and SER expressions derived for TWDP fading will encompass the corresponding expressions for Rayleigh and Rician fading. An initial study of the properties of the TWDP pdf suggests that it can produce a link poorer than that of Rayleigh fading. This is a significant claim, which needs to be investigated further, as Rayleigh fading is often used as the “worst-case” situation in the design of communication systems.

This thesis details further investigations of TWDP fading, which include analyses of BER performance results of binary PSK, DPSK and FSK systems in TWDP fading. The

analyses are also extended to multichannel, pre-detection maximal ratio combining (MRC), binary PSK and DPSK as well as M -ary PSK and DPSK systems, as the proven benefits of diversity combining had led to many modern communication systems incorporating it in their design.

The performance analyses in TWDP fading are performed using an approximate TWDP pdf as there is no exact closed-form TWDP pdf expression. The approximate expression is given by the weighted sum of Rician pdfs, the number of which depends on the order of approximation used. This complex expression makes analysis much more difficult compared with analysis using the Rician pdf. Nevertheless, the BER and SER expressions derived here for TWDP fading are presented in a compact and closed-form, allowing for efficient calculation of BER and SER values. As the Rayleigh and Rician pdfs are special cases of the TWDP pdf, the BER and SER expressions derived for diversity combining systems also provide a convenient platform for investigating mixed-mode fading.

The results of analysis show that under certain conditions (to be quantified in this study), TWDP fading does produce a link poorer than that of Rayleigh fading due to the two specular waves canceling each other when they are equal in amplitude but opposite in phase.

The BER and SER expressions derived are to the author's knowledge the first available closed-form analytical expressions for TWDP fading channels and they are able to provide more detailed information and greater insight into TWDP fading than the previous studies through the shape of its pdf. The BER and SER expressions in TWDP fading for MRC diversity systems show that they are given by the weighted sum of the BERs of a number of effective Rician diversity systems that are generated when the physical TWDP channels of the MRC diversity systems are combined.

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Chapter 1

Introduction

1.1 Motivation

When transmitted, wireless communication signals are subjected to degradation due to the addition of noise and interference and the effects of multipath fading. Fading arises because the signal traverses multiple paths on its way to the receiver. These multipath components combine to give a resultant signal whose amplitude varies, depending on the phases and amplitudes of the individual multipath components. As a mobile wireless receiver moves over a relatively small local area, multipath fading causes rapid and severe fluctuations in the received signal strength [1, 2]. This short-term fluctuation in the received signal strength is also referred to as small-scale or fast fading to distinguish it from the much longer-term variation in the mean signal level (slow fading) [2].

To investigate the performance of the wireless receiver in the presence of noise and interference, a proper characterization of the signal fading is necessary. Hence an appropriate probability density function (pdf), which accurately represents the probability density of the received signal strength, has to be carefully chosen. This will facilitate the proper design and use of equalization methods, diversity schemes and error correcting codes for a communication link [3, 4].

Rayleigh and Rician pdfs till now, are the most commonly used to characterize the small-scale local fading experienced by wireless receivers. However, these pdfs are sometimes found to be lacking, unable to characterize many narrow-band fading measurements, like those for mobile radio, observed by Suzuki [5] and Turin[6]. Durgin *et al*, in their paper [4], formulated a number of new pdfs for the received signal magnitude, which help towards overcoming the above-mentioned shortcoming of the Rayleigh and

Rician pdfs. These new pdfs, together with the Rayleigh and Rician pdfs, provide a “nearly complete set” of analytical models for the study of local area fading.

Of all the new pdfs presented, the two-wave with diffuse power (TWDP) fading envelope pdf is most notable. This pdf characterizes the physical situation where there are two dominant (specular) multipath components in the presence of other diffusedly propagating waves. One important consideration with this fading channel is when the two specular waves are approximately equal in magnitude but opposite in phase. Their resulting cancellation leaves a remaining, weak diffused signal component. It will be shown that in such situations, TWDP fading can give rise to a weaker link than that of Rayleigh fading. This is significant, as Rayleigh fading had often been considered the “worst-case” fading situation [7,8], commonly used as the basis in designing communication links.

In [4], the TWDP fading channel was investigated through analyses of the shapes of the pdf of the TWDP fading envelope. Useful conclusions were drawn from them about receiver performance in this fading channel, relative to the Rayleigh and Rician fading channels.

However, there are limitations with this graphical method of analysis. It could qualitatively give relative measures of performance in TWDP fading against the other fading distributions, but is unable to quantify it with bit-error rate (BER) figures. While it reveals that TWDP fading can give rise to poorer performance than Rayleigh fading when K and Δ (the two parameters of TWDP fading) approaches large enough values, it fails to be more specific or precise about the values of K and Δ for poor performance to occur. There is hence the motivation to explore more fully TWDP fading in terms of BER performance analysis of digital modulation systems in this fading environment.

1.2 Objectives and Contributions

The performance analysis starts with deriving closed-form BER expressions for binary PSK, DPSK and FSK systems in TWDP fading. In the analysis, an approximate expression of the TWDP pdf suggested by Durgin *et al* [4] is used. It is in the form of a weighted sum of a number (depending on the order, L , of the approximation used) of Rician pdfs. The Rician K parameter of each of the Rician pdfs is different and is evaluated using the TWDP K and Δ parameters. It is well known that analysis with the Rician pdf presents a greater challenge compared with analysis using the Rayleigh pdf because of the presence of the modified Bessel function in the Rician pdf. With the more complex TWDP pdf expression, the analysis is made even more difficult.

The proven effectiveness of diversity combining [3, 9] techniques (whether using polarisation, frequency, time, space or space-time diversity [10]) in combating signal fading had resulted in many modern communication systems, e.g., 3G wideband CDMA systems [11, 12], incorporating diversity combining in their design. The BER analysis is hence also carried out for multichannel, maximal ratio combining (MRC), binary PSK and DPSK as well as M -ary PSK and DPSK systems in TWDP fading. The author feels that the contribution from this portion of the thesis is significant as it reveals that the BER of a diversity combining system in TWDP fading is given by the weighted sum of the BERs of $(2L)^N$ effective Rician diversity systems generated when the N physical TWDP fading paths are combined.

The derivation of BER expressions for diversity combining systems in TWDP fading provides a convenient platform for investigating mixed-mode fading. This is a consequence of the Rayleigh and Rician pdfs being the limiting cases of the TWDP fading pdf [4].

To the author's knowledge, all the above closed-form expressions derived are the first ones available for the TWDP fading channel. The validity of the mathematical method used in deriving the analytical expressions is verified through simulation of the physical systems in TWDP fading. One significant result from this study of TWDP fading is the specification of the conditions under which TWDP fading produces a link poorer than that of Rayleigh fading. In wireless system design, when Rayleigh fading is assumed to be the worst-case fading, we foresee problem of 'under provision' in locations deemed to be having Rician fading and having large Rician K values. In these locations, there is a high possibility that instead of just only one dominant path (with diffuse signals) there could be two or more such paths leading to the particular TWDP fading situation (which is worse than Rayleigh) where the strong dominant (large K) signals cancels, leaving only the (weak) diffuse signal components. A system design based on the assumption that Rayleigh fading is the worst case fading, can hence under-provide in locations having good line-of-sight between transmitter and receiver (and deemed to be Rician fading locations). The situation could be exacerbated if the wireless system uses wide-band signals or directional antennas, as the likelihood for TWDP fading would also increase (see Section 2.7).

The analytical BER expressions for TWDP fading obtained are more specific and precise in quantifying TWDP channel performance compared with the earlier observation [4] where only the shape of the pdf is used. It also provides more information and insight into TWDP fading, like for example how quick the system performance changes with variation in the parameters' values.

The Rician pdf is a special case of the TWDP pdf. The BER analytical expression for BPSK and coherent FSK systems in *Rician* fading, derived in this way from the TWDP BER expression, is in an efficient form of a rapidly converging infinite series. This is an

improvement over presently available ones which is either much more complex and difficult to use [13] or remained in an integral form [14].

1.3 Organization of thesis.

The next chapter looks at various small-scale fading pdfs, and in particular the TWDP pdf. The TWDP parameters, the approximate TWDP pdf expression and its important characteristics are detailed. Chapters 3 and 4 present the analyses of BPSK, DPSK and FSK systems in TWDP fading. Important information and conclusions on TWDP fading were drawn from the plots of BER versus signal-to-noise ratio (SNR). In Chapters 5 and 6, the TWDP analysis is extended to multichannel, pre-detection MRC binary PSK and DPSK systems and then further to multichannel, MRC M -ary PSK and DPSK systems. The analytical results from the foregoing chapters provided a convenient platform for the study of mixed-mode fading which is investigated and discussed in Chapter 7. Chapter 8 discusses and summarizes the findings of this work. There are also recommendations for further work.

Chapter 2

Fading Envelope Probability Density Functions

2.1 Introduction

In mobile wireless communications, small-scale fading of the transmitted signal is commonly characterized by the classical fading pdfs such as the Rayleigh and Rician distributions. Recently, a newer distribution has been proposed [4]. Known as two-wave with diffuse power (TWDP) fading, it characterizes the physical situation where the received signal is composed of two specular (direct) waves in the presence of other diffusedly propagating waves. A significant claim for TWDP fading is that it can give rise to a communication link worst than that of Rayleigh fading [4, 15]. In the following, information on the TWDP pdf, like the parameters used to describe the TWDP pdf, the family of pdfs that approximate the TWDP pdf and the properties of the TWDP pdf are presented. But first, a brief summary of other pdfs used to characterize small-scale fading is given.

2.2 Rayleigh fading pdf

This characterizes the probability density of the received signal strength in a physical situation where all multipath powers are diffused (i.e. there is no dominant or specular signal components). The envelope pdf expression is given as [4]

$$p(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) \quad r \geq 0 \quad (2.1)$$

where r is the amplitude of the received signal and $2\sigma^2$ represents the mean-squared value (or average power) of the diffuse signal.

For mobile wireless communications in an urban built up environment, it is likely that almost all of signal power received is diffused and of approximately equal values. Hence, the Rayleigh pdf has been used extensively to describe narrow-band local area fading for mobile radios. A plot of the normalized (note that the horizontal scale is r/σ) Rayleigh distribution is given in Figure 2.1. As can be observed, it shows the signal strengths spending a great deal of time at low levels, hence explaining why Rayleigh fading is traditionally taken as the worst case of fading.

Figure 2.1 Rayleigh Fading pdf

2.3 Rician fading pdf

The Rician pdf describes the fading of diffused power in the presence of a dominant, specular multipath component [16, 17]. The pdf expression is given by [4]

$$p(r) = \frac{r}{\sigma^2} \exp\left(\frac{-r^2 - V_1^2}{2\sigma^2}\right) I_0\left(\frac{rV_1}{\sigma^2}\right), \quad r \geq 0 \quad (2.2)$$

where V_1 is the amplitude of the specular component and $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind.

Situations where Rician fading are likely to occur include wireless indoor transmissions and transmissions in the small cells of a cellular radio system, where the close proximity between transmitter and receiver improves the probability of having a line-of-sight path or a dominant incoming wave.

Figure 2.2 Rician pdfs for varying K values

The Rician K factor is the ratio of the power of the specular component to the power of the diffused components and is hence given by $K = V_1^2 / 2\sigma^2$. Note that when

$K = 0$ ($-\infty$ dB), the Rician pdf reduces to the Rayleigh pdf. Figure 2.2 shows the Rician pdf for various values of K .

2.4 Other small-scale fading pdfs

These include the one-wave, two-wave and three-wave pdfs. For the two-wave pdf, the envelope fading is caused by the interference of only two constant amplitude waves in a local area. The envelope pdf expression is [4]

$$p(r) = \frac{2r}{\pi \sqrt{4V_1^2 V_2^2 - (V_1^2 + V_2^2 - r^2)^2}}, \quad |V_1 - V_2| \leq r \leq V_1 + V_2 \quad (2.3)$$

where V_1 and V_2 are the amplitudes of the two waves. To relate the relative magnitudes of V_1 and V_2 to one another, a parameter Δ is employed. The parameter Δ is defined as

$$\Delta = \frac{\text{Peak Specular Power}}{\text{Average Specular Power}} - 1 = \frac{2V_1 V_2}{V_1^2 + V_2^2}$$

Note that $\Delta=1$ when $V_1 = V_2$, and $\Delta=0$ when either V_1 or V_2 equals 0.

The expressions for one-wave and three-wave envelope pdfs are available in [4]. The analysis in [4] does not go beyond the three-wave pdf as it is argued that “even if it were possible to analytically calculate a four-wave pdf, its usefulness would be limited since the most general cases will appear to be even more Rayleigh distributed than the three-wave pdf”. The explanation is that due to the central limit theorem, the addition of more constant amplitude waves makes the total multipath power more diffused. This trend was observed in the examination of the three-wave cumulative density function (cdf) in [4], where it was observed that the different plots of the three-wave cdf (obtained using different combinations of the three waves’ amplitudes) are much more similar to each other (compared to the two-wave case).

2.5 TWDP Fading Pdf

This pdf characterizes fading situations where there are two dominant specular components in the presence of diffusedly propagating waves.

2.5.1 Parameters of TWDP pdf

The two parameters used [4] to classify the shape of the TWDP pdf are K and Δ . K is the ratio of the total specular power waves to the power of the diffused waves, i.e.,

$$K = \frac{\text{Total Specular Power}}{\text{Diffused Power}} = \frac{V_1^2 + V_2^2}{2\sigma^2} \quad (2.4)$$

where V_1 and V_2 are amplitudes of the two specular waves and $2\sigma^2$ is the average power of the diffused waves. Δ , which relates the relative strength of the two specular components, similar to the two-wave situation, is therefore given by

$$\Delta = \frac{2V_1V_2}{V_1^2 + V_2^2} \quad (2.5)$$

Note that similar to the Rician pdf situation, when $K = 0$, the TWDP pdf degenerates to the Rayleigh pdf for which only diffused waves exist. When $K \neq 0$ and $\Delta = 0$, the TWDP pdf reduces to a Rician pdf, as in this case, only one specular wave remains. A summary TWDP fading for limiting parameter cases is shown in Table 2.1.

Table 2.1 Limiting behaviour of approximate TWDP pdf

Parameter Value		Fading Type
$K=0$	---	Rayleigh
$K>0$	$\Delta=0$	Rician
$K \rightarrow \infty$	$\Delta=0$	One-Wave
$K \rightarrow \infty$	$\Delta>0$	Two-Wave

When $\Delta = 1$, we have $V_1 = V_2$ and we have a TWDP channel which can fade more severely than a Rayleigh channel for large enough K values. This is elaborated on later in this chapter.

2.5.2 The approximate TWDP pdf

The TWDP pdf is given by [4]

$$p(r) = \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2} - K\right) \frac{1}{\pi} \int_0^\pi \exp(\Delta K \cos \theta) I_0\left(\frac{r}{\sigma} \sqrt{2K(1 - \Delta \cos \theta)}\right) d\theta \quad (2.6)$$

This is not in a ‘closed-form’ and thus, does not lend itself to efficient numerical calculations necessary for intensive simulation of fading communication links. It will be of limited use for derivation of BER performance of communication systems. However, using the above, a family of pdfs can be derived that approximates the TWDP pdf. This approximate TWDP pdf is given as [4]

$$p(r) = \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2} - K\right) \sum_{i=1}^L a_i D\left(\frac{r}{\sigma}; K, \alpha_i\right) \quad (2.7)$$

where $D(z; K, \alpha_i) = \frac{1}{2} \exp(\alpha_i K) I_0\{z \sqrt{2K(1 - \alpha_i)}\} + \frac{1}{2} \exp(-\alpha_i K) I_0\{z \sqrt{2K(1 + \alpha_i)}\}$,

$\alpha_i = \Delta \cos\left(\frac{\pi(i-1)}{2L-1}\right)$, $I_0(\cdot)$ is the modified Bessel function of zero order, and L is the order

of the approximate TWDP pdf. It is recommended in [4] that $L \geq K\Delta/2$ should be used so that (2.7) does not deviate significantly from the exact pdf. The values of a_i for $L = 1$ to 5 are given in Table 2.2 [4]:

Table 2.2 Values of a_i for $L=1$ to 5

Order (L)	a_1	a_2	a_3	a_4	a_5
1	1				
2	$\frac{1}{4}$	$\frac{3}{4}$			
3	$\frac{19}{144}$	$\frac{25}{48}$	$\frac{25}{72}$		
4	$\frac{751}{8640}$	$\frac{3577}{8640}$	$\frac{49}{320}$	$\frac{2989}{8640}$	
5	$\frac{2857}{44800}$	$\frac{15741}{44800}$	$\frac{27}{1120}$	$\frac{1209}{2800}$	$\frac{2889}{22400}$

2.5.3 Characteristics of Approximate pdf

Besides allowing for efficient numerical calculations, this family of pdfs possesses some important characteristics that are independent of L , K and Δ [4]. They are mathematically exact pdfs as they integrate to 1 over the range $0 \leq r < \infty$. They are accurate over the upper and lower tails regions, which is important when modeling noise-limited mobile communication systems. Over the most useful range of K and Δ , they give an accurate representation of the true TWDP pdf. (The approximate pdf will only deviate from the exact pdf if the specular power is much larger than the diffused power (i.e., K is very large) and if the two specular components are relatively equal in magnitude (i.e., Δ approaches 1). This approximate pdf expression is hence used in the performance analysis of the various communications systems described in the following chapters.

2.6 Analysis of TWDP fading through its pdf

The fading pdf represents the probability density of the received signal strength. Hence, the shape of a pdf determines the performance of a wireless receiver in the presence of

noise and interference [2, 18]. Durgin *et al* [4] analyzed the plots of the TWDP pdf, comparing them against the corresponding plots of Rayleigh and Rician pdfs, and drew conclusions about TWDP fading performance.

Figures 2.3—2.6 show the TWDP pdf for various values of K from 0 dB to 10 dB. From Figures 2.3 and 2.4, it can be seen that for $K < 3$ dB, the TWDP pdf is very close to the Rician pdf. However, the difference between the two gradually becomes more pronounced as K increases, and when the two specular components are closer in strength (i.e., Δ approaches 1). Another noteworthy observation is that for $K = 10$ dB and $\Delta = 1$ (see Figure 2.6), the TWDP pdf curve shows the signal strength to be at low levels for significantly much longer than when it is Rician fading with $K = 10$ dB. In fact, it has been found [4] that some of the TWDP pdfs with $K = 10$ dB shows signal strength reaching low

Figure 2.3 TWDP pdf with $K = 0$ dB

Figure 2.4 TWDP pdf with $K=3$ dB

Figure 2.5 TWDP pdf with $K=6$ dB

Figure 2.6 TWDP pdf with $K=10$ dB

levels more often than Rayleigh fading distributions of equal average power [4]. This is an important conclusion (which is investigated further in this work, in the later chapters) as Rayleigh fading has always been assumed to be the worst case fading [7, 8], resulting in it being often used when designing communication systems.

Table 2.3 Conclusions on TWDP Fading Channel (PDF Analysis)

Parameter(s) Value	Conclusion on TWDP Fading
$K \leq 2$	$\cong$ Rician Fading
$K\Delta < 2$	$\cong$ Rician Fading
$K < \min\{2/\Delta, 1/\sqrt{1-\Delta^2}\}$	$\cong$ Rayleigh Fading
As $K \rightarrow 10$, $\Delta \rightarrow 1$	Deviate from Rician
Large K , $\Delta \approx 1$	BER poorer than Rayleigh

A summary of Durgin's graphical analysis of the TWDP fading pdf is given in Table 2.3.

2.7 Likely scenarios for TWDP fading

For typical narrow-band receiver operation, the scenario of TWDP fading, where two dominant specular multipath waves are received in the presence of other weaker diffuse waves, is always a possibility. However, the likelihood for TWDP fading should increase with the use of

- (i) directional antennas and
- (ii) wideband signals

The use of directional antennas or arrays at the receiver amplifies several of the strongest multipath waves that arise in a particular direction while attenuating the remaining waves [4, 19]. This will increase the ratio of the specular to diffuse received power, turning a Rayleigh or Rician fading channel into a TWDP fading channel. Similarly, wide-band signal fading also exhibits TWDP fading characteristics because a wide-band receiver has the ability to reject multipath components that arrive with largely different propagation time delays [4, 20]. This action will separate specular multipath components from other diffused multipath waves causing the ratio of specular to diffused received power to increase for a given propagation delay, hence leading to a TWDP fading scenario.

Transmissions in large indoor facilities like a factory floor, an office or shopping complex is also more likely to give rise to TWDP fading because the closer proximity between transmitter and receiver and the presence of large reflective surfaces (e.g., doors, walls and partitions) will enhance chances of there being more than one dominant wave in the presence of diffused components.

The assumption that there is only *one* specular wave with diffused components (i.e., Rician fading) is hence very restrictive, especially when directional antennas and wideband signals are used. The assumption of TWDP fading would be more representative and definitely less restrictive as the analytical expression for TWDP fading given in (2.7) provides a near-complete description of the possible envelope fading pdfs in a local area [4]. As Table 2.1 shows, it includes the one-wave, two-wave, Rayleigh and Rician pdfs as limiting cases. Also, employing the same argument that was used for not going beyond three-wave fading, i.e., to four-wave, five-wave, etc (see Section 2.4), Durgin *et al* explained why it is also not necessary to go beyond three-wave *with diffuse fading* [4].

There are evidences of TWDP fading applications provided by the close fit of TWDP cumulative distribution functions (cdfs) to published cdfs of measured signals. Figure 2.7 shows the cdf of temporal signal levels measured by Bultitude [21, Figure 8] inside a building with an ‘open-office’ environment of small offices and laboratories. It is shown in the figure that it fits closely with the cdf of a TWDP fading with $K = 11$ dB, $\Delta = 0.2$. TWDP fading with $K = 11$ dB, $\Delta = 0.2$ represents an environment where there are two direct waves (with $\Delta = 0.2$ indicating one of them being significantly larger than the other) in the presence of diffused signals 11 dB weaker than the direct signals combined. This is a likely fading situation of the laboratory/office environment described in [21] (with obstruction of the transmitted signal caused by movement of personnel and machinery). Note that in Figure 2.7, the cdf for Rician fading with $K = 11$ dB is also shown. This was Bultitude’s choice of the best fit but as can be observed, the TWDP fading cdf provides a better fit. This observation is reaffirmed with a calculation of the ‘difference’ between the measured-signal cdf and the cdfs of TWDP and Rician fading. The sum of the square

Figure 2.7 Cdf from Bultitudes's measurement with cdFs of TWDP fading, $K= 11$ dB, $\Delta = 0.2$ and Rician fading, $K = 11$ dB

Figure 2.8 Cdf from Rappaport's measurement with cdFs of TWDP fading, $K = 3$ dB, $\Delta = 1$ and Rayleigh fading

of difference in ordinate values for the 19 points used to plot the cdfs of TWDP fading and ‘measured signal’ is $1.6575\text{e-}3$ while the corresponding figure for ‘difference’ between the Rician cdf and the ‘measured signal’ cdf is $3.3826\text{e-}3$. Figure 2.8 shows the close fit of the cdf for TWDP fading ($K = 3$ dB, $\Delta = 1$) and the cdf of measured-signal levels in a factory site *with line of sight* between the fixed transmitter and a moving receiver, in the presence of surrounding heavy clutter. The latter cdf is extracted from [22, Figure 6]. In [22], this measured cdf was fitted against the Rayleigh cdf. We note that the cdf of TWDP with $K = 3$ dB, $\Delta = 1$ fits just as well, as Figure 2.8 shows. In fact, we feel that TWDP with $K = 3$ dB and $\Delta = 1$, which describes a fading situation of two dominant waves of equal strength in the presence of diffused signals two times weaker than the direct waves is a more appropriate fading model for this factory site (with surrounding heavy clutter, and *line-of-sight* between the transmitter and the receiver), than Rayleigh fading (which is the model for fading scenarios with only diffused signals and no direct waves).

2.8 BER Performance Analysis in TWDP Fading

The above conclusions (in Section 2.6) on the TWDP fading channel were derived from the analysis of the shape of the TWDP pdf. The following chapters in this thesis report on further analysis of TWDP fading through the investigation of the BER performance of various communication systems. It includes derivation of the closed-form BER expressions for binary PSK, DPSK, and FSK in TWDP fading. The analysis is also extended to MRC, binary PSK and DPSK as well as M -ary PSK and DPSK systems. Simulations will be performed to verify the validity of the method used in deriving the analytical expressions. In fact, the analytical expressions derived are more precise and specific than the analysis, in [4], in qualifying TWDP performance. They also provide

additional information and insight not revealed directly through Durgin's 'graphical' analysis of the pdf shape. To the author's knowledge, the analytical and simulation BER results obtained in this thesis are the first available quantitative performance studies on TWDP channels.

Chapter 3

Performance Analysis of BPSK systems in TWDP fading

3.1 Introduction

The derivation of BER expressions for an uncoded BPSK communication system in TWDP fading is presented in this chapter. The validity of the expressions derived will be verified through simulation. The plots of BER against SNR obtained using the analysis and simulation results allow useful conclusions to be drawn about TWDP fading.

3.2 Bit-Error Rate (BER) Performance Analysis of BPSK Systems

The BER expression for a communication system allows the performance of the system in the presence of noise and fading to be evaluated. This is expressed as probability of error when a data bit is transmitted.

The conditional probability of bit error for a communication system, $P(E|r)$, denotes the probability of error at the output of the receiver when a signal propagated through a channel with gain magnitude of r is received. The signal amplitude varies due to channel fading and if this fading is characterized by fading envelope pdf, $p(r)$, the average BER performance of the system, P_E , is derived using [3]

$$P_E = \int_0^{\infty} P(E | r) p(r) dr \quad (3.1)$$

i.e., through averaging the conditional probability of error over the pdf of r .

For an uncoded BPSK system, with a transmit bit energy of E_b , the conditional probability of bit error is given by [3]

$$P(E | r) = Q\left(\sqrt{\frac{2r^2 E_b}{N_0}}\right) \quad (3.2)$$

where $r^2 E_b$ represents the instantaneous energy received per bit and $N_0/2$ is the double-sided power spectral density of additive white Gaussian noise (AWGN). Therefore, the BER expression for an uncoded BPSK system in a fading channel with envelope pdf $p(r)$ is given by

$$P_E = \int_0^{\infty} Q\left(\sqrt{\frac{2r^2 E_b}{N_0}}\right) p(r) dr \quad (3.3)$$

In the following, the results of studies into BER performance of a BPSK system in Rayleigh and Rician fading are first discussed. These results are important because analytical expressions of BER performance for BPSK systems in Rayleigh and Rician fading are limiting cases of that in TWDP fading derived in this chapter and, hence, can be used for the purpose of verification.

3.2.1 BPSK Performance Analysis for Rayleigh Fading

For a Rayleigh fading channel, the fading pdf is given in (2.1) as

$$p(r) = \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right), \quad r \geq 0.$$

Substituting the above into (3.3) gives

$$P_E = \int_0^{\infty} Q\left(\sqrt{\frac{2r^2 E_b}{N_0}}\right) \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right) dr.$$

The result of the integration has been shown in [3] as

$$P_E = \frac{1}{2} \left(1 - \sqrt{\frac{2\sigma^2 \frac{E_b}{N_0}}{1 + 2\sigma^2 \frac{E_b}{N_0}}} \right) \quad (3.4)$$

It will be shown in Section 3.2.3 that this expression can also be obtained as a limiting case ($K=0$) of the closed-form BER expression (3.12) derived there for a BPSK system in TWDP fading.

3.2.2 BPSK Performance Analysis for Rician fading

The Rician fading pdf given in equation (2.2) is more difficult to handle than the Rayleigh pdf in the BER analysis. There are fewer reported results on Rician fading channel BER analyses, among which are those by Lindsay [13], Sun and Reed [14, 23] and Roberts [25].

In Lindsey's classic paper [13], the performance characteristics of coherent (binary) and non-coherent (M -ary) multireceivers in Rician fading channels were derived. However, his results were iterative in form (for multichannel systems) and computation can be formidable when considering a large number of channels. For single channel systems, the expression involves calculations using the Marcum Q -function [24] and the modified Bessel function. Sun and Reed derived BER expressions in Rician fading for non-diversity [14] and diversity combining [23] M -ary modulation systems. Their results for MPSK, however, remained as integrals which require numerical integration. In [25], the BER expressions in Rician fading are derived for binary DPSK and non-coherent FSK systems only.

The above results are closely related and relevant to the thesis' investigations into the closed-form BER expressions for the above systems in TWDP fading. This is because of the close relationship (discussed later in this Section) between the Rician and TWDP pdf. Also, Rician and Rayleigh fading are special cases of TWDP fading and therefore results obtained for them serves as a mean of corroborating the results for TWDP fading obtained in the thesis.

A related work of interest is [26], which presents in a single common framework, the characteristic function method of computing error rate performance of different modulation formats, with or without diversity in a myriad of fading environments.

Other related works in this area include those where ‘closed-form’ performance expressions in Rician fading were derived for various QAM systems, DS-CDMA systems and multicarrier (MC) systems. The QAM systems investigated include maximal ratio combining (MRC) MQAM [27], [28] and 16 STAR-QAM [29]. The DS-CDMA systems considered in [30—35] employed BPSK or/and DPSK modulation schemes. MC systems in which BER expressions in Rician fading were derived include those where the schemes of modulation are MDPSK [36], MFSK [37] and DPSK [38].

Closed-form BER expressions were also derived for coded speech modulated systems [39] and Block coded modulation and Trellis coded modulation systems [40] in Rician fading channels.

An examination of the approximate TWDP pdf, given by (2.7), reveals that it is made up of the weighted sum of $2L$ Rician pdfs. The order to be used for the approximate TWDP pdf, L , depends on the TWDP K and Δ parameter values. (Note that $L \geq K\Delta/2$ according to [4], but as will be discussed later in Chapter 5, it is found that this order recommendation is not sufficiently accurate for the purpose of verifying the analytical results of BER performance against simulation results when considering diversity combining, where the BER reaches very low values, in the order of 1 in 1 million).

Other than the graphical analysis by [4], there is no known performance analysis in TWDP fading. Analysis of TWDP fading is much more involved compared with Rician fading analysis, not just because of the greater number of Rician terms. The effective K factors of the $2L$ Rician components in TWDP fading also have to be individually calculated using the TWDP K and Δ parameters.

It will also be shown in the next section that the average BER expressions for Rician fading derived in this thesis (as a limiting case of the BER expression for TWDP fading) is different from and is an improvement on the ‘numeral integration’ form derived in [14, 23]. Our expressions are given as an infinite series, which converges quickly and hence can be evaluated efficiently.

3.2.3 BPSK Performance Analysis for TWDP Fading

The BER expression for an uncoded BPSK system with an optimum detector [pp. 255, 3], [pp. 707, 48] over a fading channel is given as (3.3),

$$P_E = \int_0^\infty Q\left(\sqrt{\frac{2r^2 E_b}{N_0}}\right) p(r) dr$$

where $p(r)$ is the pdf of the fading channel, $r^2 E_b$ is the instantaneous energy received per bit and $N_0/2$ is the double-sided power spectral density of additive white Gaussian noise (AWGN). In order to obtain an infinite series representation of the BER expression, we rewrite the Q -function as [41]

$$Q(\sqrt{x}) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(\frac{-x}{2\sin^2 \theta}\right) d\theta, \quad x \geq 0 \quad (3.5)$$

Substituting (3.5) and the expression of TWDP pdf given in (2.7) into (3.3), we have

$$P_E = \frac{1}{2\pi} \sum_{i=1}^L a_i \int_0^\infty \int_0^{\pi/2} \exp\left(\frac{-r^2 \left(\frac{E_b}{N_0}\right)}{\sin^2 \theta}\right) d\theta \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2} - K\right) \left[\exp(\alpha_i K) I_0\left(\frac{r}{\sigma} \sqrt{2K(1-\alpha_i)}\right) + \exp(-\alpha_i K) I_0\left(\frac{r}{\sigma} \sqrt{2K(1+\alpha_i)}\right) \right] dr \quad (3.6)$$

Note that P_E is the sum of $2L$ terms, each of them involving an integral of the same form.

In (3.6), exchanging the order of integration between θ and r for each of the $2L$ terms and re-arranging gives

$$P_E = \frac{1}{2\pi\sigma^2} \sum_{i=1}^L a_i \int_0^{\frac{\pi}{2}} \int_0^{\infty} r \exp \left(-r^2 \left(\frac{\left(\frac{N_0}{E_b} \right) \sin^2 \theta + 2\sigma^2}{2\sigma^2 \left(\frac{N_0}{E_b} \right) \sin^2 \theta} \right) \right) \cdot \left[\exp(-K(1-\alpha_i)) I_0 \left(\frac{1}{\sigma} \sqrt{2K(1-\alpha_i)} r \right) + \exp(-K(1+\alpha_i)) I_0 \left(\frac{1}{\sigma} \sqrt{2K(1+\alpha_i)} r \right) \right] dr d\theta \quad (3.7)$$

From [pp. 324, 42],

$$\int_0^{\infty} x \exp(-a^2 x^2) I_0(\beta x) dx = \frac{1}{2a^2} \exp \left(\frac{\beta^2}{4a^2} \right). \quad (3.8)$$

Therefore, (3.7) simplifies to

$$P_E = \frac{1}{2\pi} \sum_{i=1}^L a_i \left\{ \exp[-K(1-\alpha_i)] \int_0^{\frac{\pi}{2}} \frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} \exp \left(K(1-\alpha_i) \frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} \right) d\theta \right. \\ \left. + \exp[-K(1+\alpha_i)] \int_0^{\frac{\pi}{2}} \frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} \exp \left(K(1+\alpha_i) \frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} \right) d\theta \right\} \quad (3.9)$$

In (3.9), expanding the exponential terms of the integrands as power series yields

$$P_E = \frac{1}{2} \sum_{i=1}^L a_i \left[\exp\{-K(1-\alpha_i)\} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left\{ \frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} + K(1-\alpha_i) \left(\frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} \right)^2 + \frac{[K(1-\alpha_i)]^2}{2!} \left(\frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} \right)^3 + \dots d\theta \right\} \right. \\ \left. + \exp\{-K(1+\alpha_i)\} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left\{ \frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} + K(1+\alpha_i) \left(\frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} \right)^2 + \frac{[K(1+\alpha_i)]^2}{2!} \left(\frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} \right)^3 + \dots d\theta \right\} \right] \quad (3.10)$$

Since the integrands in (3.10) are uniformly convergent series, the integrals can therefore be evaluated through integration term-by-term. Using [(73) and (74) in 43], simple calculations (refer to Appendix A.1) show that

$$I_{n+1}\left(\frac{\pi}{2}; c\right) = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{\sin^2 \theta}{\sin^2 \theta + c} \right)^{n+1} d\theta = \left[\frac{1}{2} - \frac{1}{2} \sqrt{\frac{c}{1+c}} \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} \right] \quad (3.11)$$

for $n = 0, 1, 2, 3, \dots$. Using (3.11) in (3.10) gives the infinite series representation of P_E as

$$P_E = \frac{1}{2} \sum_{i=1}^L a_i \left\{ \exp[-K(1-\alpha_i)] \sum_{n=0}^{\infty} H_{n+1}\left(K(1-\alpha_i); 2\sigma^2 \frac{E_b}{N_0}\right) + \exp[-K(1+\alpha_i)] \sum_{n=0}^{\infty} H_{n+1}\left(K(1+\alpha_i); 2\sigma^2 \frac{E_b}{N_0}\right) \right\} \quad (3.12)$$

where

$$H_{n+1}(b; c) = \frac{b^n}{n!} \left[\frac{1}{2} - \frac{1}{2} \sqrt{\frac{c}{1+c}} \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} \right] \quad (3.13)$$

Note that the sequence $\{I_1, I_2, I_3, \dots\}$ defined by (3.11) can easily be generated by a recursive formula,

$$I_{n+1}\left(\frac{\pi}{2}; c\right) = I_n\left(\frac{\pi}{2}; c\right) - \frac{1}{2} \binom{2n}{n} \left[\frac{1}{[4(1+c)]^n} \sqrt{\frac{c}{1+c}} \right] \quad (3.14)$$

with $I_0 = 1/2$. (Details are shown in Appendix A.2). Since $\{I_1, I_2, I_3, \dots\}$ is a decreasing sequence with positive terms less than 1, the series

$$\sum_{n=0}^{\infty} H_{n+1}(b; c) = \sum_{n=0}^{\infty} \frac{b^n}{n!} I_{n+1}\left(\frac{\pi}{2}; c\right)$$

will converge faster than the exponential series expansion $\sum_{n=0}^{\infty} b^n / n!$. Hence, the infinite series representation of P_E as given in (3.12) can be evaluated efficiently. Also note that over the most useful range of K and Δ parameters, the values of L used in (3.12) recommended by Durgin *et al* [4] are 1 through 5. As explained earlier, the value of L should be chosen to satisfy $L \geq K\Delta/2$.

The TWDP pdf reduces to the Rayleigh pdf when $K = 0$. Putting $K = 0$ in (3.12), we obtain the BER expression in (3.4) for a BPSK system in Rayleigh fading. The TWDP pdf

also reduces to the Rician pdf for $\Delta = 0$. With Δ set to 0 in (3.12), all α_i 's, given by (2.7),

i.e., $\alpha_i = \Delta \cos\left(\frac{\pi(i-1)}{2L-1}\right)$, will be zero, and because the sum of the α_i 's for any order L ,

equals 1 (see Table 2.2), (3.12) reduces to

$$P_E = \exp(-K) \sum_{n=0}^{\infty} H_{n+1} \left(K; 2\sigma^2 \frac{E_b}{N_0} \right) \quad (3.15)$$

where

$$H_{n+1}(b; c) = \frac{b^n}{n!} \left[\frac{1}{2} - \frac{1}{2} \sqrt{\frac{c}{1+c}} \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} \right].$$

Equation (3.15) is therefore the BER expression for BPSK system in *Rician* fading. As mentioned earlier, this expression, in the form of an infinite series which converges rapidly, is an improvement to those offered in [13] and [14, 23]. Note that Equation (3.15) derived using Equation (3.12) is exact, even though the latter itself is approximate, being derived using Equation (2.7), which is an approximation of the TWDP pdf. With Δ set to 0 in Equation (3.12) and due to fact that the α_i 's sum to 1, Equation (3.15) is derived to be independent of the order of the approximation of the TWDP pdf.

A closer examination of Equation (3.12) and Equation (3.15) also reveals an important relationship—Equation (3.12) is a weighted sum of a number of BER expressions in Rician fading of the form shown in Equation (3.15). Hence we have in Equation (3.15), a BER expression for Rician fading in a form that allows us to easily use it to obtain the BER expression in TWDP fading. The other forms of Rician BER expressions derived in [13, 14, and 23] do not afford this advantage.

3.3 Simulation of BPSK Systems in TWDP Fading Channel

To verify the correctness and validity of the method used to derive the analytical expressions, simulation of an uncoded BPSK system in TWDP fading was performed

using Matlab's Simulink [44]. Simulation was also performed for a BPSK system with a Rake receiver [3]. Note that the Rake receiver presented here employs maximal ratio combining (MRC) [9] of four incoming signals—two specular components plus two Rayleigh components. This is equivalent to a TWDP fading channel and another Rayleigh channel, which is a special case of multi-TWDP fading channels. The analysis of MRC systems with multi-TWDP fading channels will be presented later in Chapters 5 and 6.

In the next section, the physical model of the TWDP channel used in the simulations of the two wireless communications systems is briefly described.

3.3.1 TWDP Channel—Physical Model

In the model of the TWDP channel, the two specular paths can be written as $\mathbf{V}_1 = V_1$ and $\mathbf{V}_2 = V_2 \exp(j\theta)$, where θ is a random variable uniformly distributed over the interval $[0, 2\pi]$. The scalar constants V_1 and V_2 represent the amplitude gains of the two specular paths. The diffused component, R , is Rayleigh distributed with a pdf given in (2.1),

$$p(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

where $2\sigma^2$ is the variance. Figure 3.1 shows this model of the TWDP fading channel.

The Matlab Simulink models of the TWDP fading channels for the uncoded BPSK systems (with and without MRC) operating in the presence of additive white Gaussian noise (AWGN), are shown in Appendices A.3 and A.4, respectively.

Figure 3.1 TWDP Fading Model

3.4 Discussion of Results

The results of analysis and simulation for the two systems are presented next. In the discussion, important information, insights and conclusions on TWDP fading, not revealed by the earlier ‘graphical’ analysis [4] will be highlighted.

3.4.1 BER Performance of uncoded BPSK Receiver

Figures 3.2—3.4 show some results of the simulation for the uncoded BPSK receiver without diversity. In the figures, system BER is plotted against signal-to-noise ratio (SNR), defined as $2\sigma^2(1+K)E_b/N_0$. This SNR is the ratio of the average energy received per bit to the noise power density [13].

In Figure 3.2, the performance curves for various K values with $\Delta = 1$ show that for K larger than 6 dB, TWDP fading results in poorer receiver performance than with Rayleigh fading for $\text{SNR} < 35$ dB. This results from cancellation of the two specular waves when they are anti-phase and when K is large enough, such that the remaining diffuse components have low power. Note that analysis of the TWDP pdfs in [4] predicted this

happening for large K values when $\Delta = 1$. However, *our simulation results provide a specific value of K beyond which this will happen.*

Figure 3.2 BPSK Receiver Performance–TWDP Fading, $\Delta = 1$, Varying K

Figures 3.3 and 3.4 show the performance curves of BPSK receivers in TWDP fading channels where $K = 2$ (or 3 dB) and 10 (or 10 dB), respectively, with Δ varying between 0 and 1. They show the deterioration in performance when Δ approaches 1. It is important to note that for $K \leq 10$ dB, as long as Δ is not larger than 0.8, performance results of BPSK receivers in TWDP channels are still significantly better than in a corresponding Rayleigh channel. That is, *performance degrades to below that for Rayleigh fading only when the two specular waves cancel almost completely.*

Figure 3.3 BPSK Receiver Performance–TWDP Fading, $K = 3$ dB, Varying Δ

Figure 3.4 BPSK Receiver Performance–TWDP Fading, $K = 10$ dB, Varying Δ

The analysis in [4] of the pdfs indicated that for $K\Delta < 2$, TWDP fading is very similar to Rician fading. Figure 3.3 corroborates this, but we can (by looking at Figures 3.3 and 3.4) also add that for $K \leq 10$ dB, the performances of BPSK receivers without MRC move from one in a Rician channel towards one in a Rayleigh channel as Δ varies from 0 to 1. (Nevertheless, Figure 3.3 does show that the performance curves are not well spaced for $K = 3$ dB. That is why when K is small, receiver performance in a TWDP channel is still close to that in a Rician channel, even at $\Delta = 1$).

The validity of the physical model and simulation method was verified by the closeness of the BER curves obtained through simulation and the corresponding curves obtained through analysis. This is illustrated in Figure 3.3 for the case of $K = 3$ dB and $\Delta = 0.8$ and in Figure 3.4 for $K = 10$ dB, $\Delta = 1$ and 0.8.

3.4.2 BER Performance of BPSK Rake Receiver

To study the performance in TWDP fading when there are multiple paths, simulation results for an uncoded BPSK receiver are also obtained for the situation where the channel consists of two Rayleigh paths and two specular waves. (Note that detailed analysis of pre-detection MRC PSK and DPSK systems in *multi-TWDP* fading channels will be presented in Chapters 5 and 6. The results presented here are just preliminary simulation studies of diversity systems).

The results are broadly similar to that of the BPSK receiver without diversity. For example, when $\Delta = 1$, the BER performance in a TWDP channel becomes poorer than in a Rayleigh channel when the value of K is 6 dB or higher (see Figure 3.5). However, as anticipated, the BER curves for the Rake receiver are exponential in shape (and hence more spread out as compared to the curves in Figure 3.2) as the benefits of coherent combining become significant at higher SNR values. As a result, for $K = 6$ dB and $K = 10$ dB, when $\Delta = 1$, though cancellation of specular waves causes the Rake receiver

performance at low SNR to be poorer than in a Rayleigh fading channel—this is no longer true when SNR reaches 19 dB and 27 dB, respectively. This shows that at large enough SNR, the Rake receiver is more effective in a TWDP channel than in a Rayleigh channel.

Figure 3.5 BPSK Rake Receiver—TWDP Fading

Also in Figure 3.5, the curves for $\Delta = 0$ and $\Delta = 1$ when $K = 3$ dB are seen to be well spaced out—no longer like those in the case of the BPSK receiver without MRC. Thus for $K\Delta < 2$, the performance of Rake receivers in a TWDP channel is not close to that in a Rician fading channel. *So for $K\Delta < 2$, the conclusion in [4] that TWDP fading approaches Rician fading (due to the close resemblance in their pdfs' shapes) is not universally true (except for $K\Delta \approx 0$) as illustrated by the case of the Rake receiver.*

It can be observed from Figure 3.3 that though TWDP fading for $K\Delta < 2$ approaches Rician fading, there is a noticeable difference. In the Rake receiver, the improvement

afforded by diversity combining expanded the difference significantly, so much so that we no longer can say that TWDP fading approaches Rician fading for $K\Delta < 2$.

However, it remains true that for $K \leq 6$ dB, as Δ varies from 0 to 1, the performance of Rake receivers in a TWDP channel varies from one that is close to Rician towards one that is close to Rayleigh.

A summary of TWDP fading channel behavior, derived from the above BER curves, is given in Table 3.1.

Table 3.1 Conclusion on TWDP Channel Behaviour (from BER plots)

	Parameter(s) Value	Conclusion on TWDP Fading
Uncoded BPSK	$K \leq 6$ dB as Δ vary $0 \rightarrow 1$	Performance worsens, from Rician $\rightarrow$ Rayleigh
	$K > 6$ dB; $\Delta = 1$	Poorer than Rayleigh
	$\Delta \leq 0.8$ as K vary $0 \rightarrow 10$ dB	Performance improves, from Rayleigh $\rightarrow$ Rician
	$K\Delta < 2$	$\cong$ Rician
MRC Receiver	$K \leq 6$ dB as Δ vary $0 \rightarrow 1$	Performance worsens, from Rician $\rightarrow$ Rayleigh
	$K > 6$ dB; $\Delta = 1$	Poorer than Rayleigh for lower SNRs; as SNR increases, performance better than Rayleigh due to benefit of coherent combining, e.g. $K = 6$ dB, better than Rayleigh above 19 dB SNR.
	$K\Delta < 2$	No longer $\cong$ Rician, except when $K\Delta \approx 0$; otherwise, poorer than Rician.

3.5 Conclusion

BER performance of an uncoded BPSK system in TWDP fading was investigated through analysis and simulation. For a BPSK system with a Rake receiver, simulation results of BER performance were obtained. The results for the two communication

systems studied confirm the findings in [4] that performance in a TWDP channel can be poorer than a Rayleigh channel when there is a cancellation of the two specular waves in TWDP. This happens for values of $K > 6$ dB when $\Delta = 1$. However, for a Rake receiver system, as the SNR reaches higher values, the advantage of coherent combining comes in and pushes TWDP channel performance above that of Rayleigh. For values of $K \leq 6$ dB, as Δ varies from 0 to 1, the performance curves of both receiver types move from one in a Rician channel towards one in a Rayleigh channel. For the uncoded BPSK system without diversity, when $K\Delta < 2$, the performance is close to Rician and this is as predicted in [4] from their analysis of the TWDP pdf.

Chapter 4

Performance Analysis of DPSK, FSK Systems in TWDP Fading

4.1 Introduction

In this chapter, the closed-form BER expressions for binary DPSK and FSK (coherent and non-coherent) systems in TWDP fading are derived. The derivation follows closely that for BPSK in the previous chapter. Like in the case for BPSK system, these results are the first available closed-form TWDP fading BER expressions for DPSK and FSK systems. Again, it should be noted that the BER expressions for these systems in Rayleigh and Rician fading can be derived as limiting cases of the TWDP expressions. It is worth noting that the *Rician* BER expression for coherent FSK derived as such, is in the form of a rapidly converging infinite series, and hence is an improvement over currently available expressions which are less efficient or inconvenient to use (also see the comments in Section 3.2.2—‘BPSK Performance Analysis for Rician fading’). From the plots of BER versus SNR, the performance of these systems under TWDP fading is evaluated and discussed.

4.2 DPSK System Performance in TWDP fading

For a DPSK system, the conditional probability of error is given by [3],

$$P(E | r) = \frac{1}{2} \exp\left(-\frac{r^2 E_b}{N_0}\right) \quad (4.1)$$

where $r^2 E_b$ is the instantaneous energy received per bit and $N_0/2$ is the double-sided power spectral density of additive white Gaussian noise (AWGN). Substituting (4.1) and the approximate TWDP pdf expression given by (2.7) into (3.1), the error probability becomes

$$P_{E_{DPSK}}(TWDP) = \frac{1}{2\sigma^2} \int_0^\infty r \exp\left(-r^2 \left(1 + 2\sigma^2 \frac{E_b}{N_0}\right)\right) \cdot \sum_{i=1}^L a_i \left[\frac{1}{2} \exp(-K(1-\alpha_i)) I_0\left(\frac{r}{\sigma} \sqrt{2K(1-\alpha_i)}\right) + \frac{1}{2} \exp(-K(1+\alpha_i)) I_0\left(\frac{r}{\sigma} \sqrt{2K(1+\alpha_i)}\right) \right] dr \quad (4.2)$$

Using the equation

$$\int_0^\infty r \exp(-a^2 r^2) I_0(\beta r) dr = \frac{1}{2a^2} \exp\left(\frac{\beta^2}{4a^2}\right)$$

from [pp.324, 42] in (4.2) gives

$$P_{E_{DPSK}}(TWDP) = \frac{1}{4 \left(1 + 2\sigma^2 \frac{E_b}{N_0}\right)} \cdot \sum_{i=1}^L a_i \left\{ \exp(-K(1-\alpha_i)) \exp\left[\frac{K(1-\alpha_i)}{1 + 2\sigma^2 \frac{E_b}{N_0}}\right] + \exp(-K(1+\alpha_i)) \exp\left[\frac{K(1+\alpha_i)}{1 + 2\sigma^2 \frac{E_b}{N_0}}\right] \right\}$$

Simplification yields

$$P_{E_{DPSK}}(TWDP) = \frac{1}{4 \left(1 + 2\sigma^2 \frac{E_b}{N_0}\right)} \exp\left[\frac{-2K\sigma^2 \frac{E_b}{N_0}}{1 + 2\sigma^2 \frac{E_b}{N_0}}\right] \cdot \sum_{i=1}^L a_i \left\{ \exp\left[\frac{2K\sigma^2 \frac{E_b}{N_0} \alpha_i}{1 + 2\sigma^2 \frac{E_b}{N_0}}\right] + \exp\left[\frac{-2K\sigma^2 \frac{E_b}{N_0} \alpha_i}{1 + 2\sigma^2 \frac{E_b}{N_0}}\right] \right\}$$

or

$$P_{E_{DPSK}}(TWDP) = \frac{1}{2 \left[1 + 2\sigma^2 \frac{E_b}{N_0}\right]} \exp\left[\frac{-2K\sigma^2 \frac{E_b}{N_0}}{1 + 2\sigma^2 \frac{E_b}{N_0}}\right] \sum_{i=1}^L a_i \left\{ \cosh\left(\frac{2\alpha_i K \sigma^2 \frac{E_b}{N_0}}{1 + 2\sigma^2 \frac{E_b}{N_0}}\right) \right\} \quad (4.3)$$

For $K=0$, (4.3) reduces to the Rayleigh BER expression for DPSK

$$P_{E_{DPSK}}(Rayleigh) = \frac{1}{2 \left[1 + 2\sigma^2 \frac{E_b}{N_0} \right]} \quad (4.4)$$

which is similar to that given in [3]. For $\Delta=0$ (where TWDP pdf becomes Rician pdf), all α_i 's in (4.3) equals zero and noting that the a_i 's sum to 1 (for any order L), the Rician BER expression for DPSK is obtained as

$$P_{E_{DPSK}}(Rician) = \frac{1}{2 \left[1 + 2\sigma^2 \frac{E_b}{N_0} \right]} \exp \left[\frac{-2K\sigma^2 \frac{E_b}{N_0}}{1 + 2\sigma^2 \frac{E_b}{N_0}} \right] \quad (4.5)$$

similar to that derived in [25].

4.3 FSK System Performance in TWDP Fading

We look at both coherent orthogonal FSK and non-coherent orthogonal FSK [3]. In coherent FSK it is assumed that the reference of the carrier frequencies transmitted is generated and used at the receiver for the detection process, while in the non-coherent case, a bank of two filters tuned to the carrier frequencies, followed by envelope detectors, are used in the detection of the transmitted signal.

4.3.1 Coherent FSK

For coherent FSK, the conditional probability of bit error is given by [3]

$$P(E | r) = Q \left(\sqrt{r^2 \frac{E_b}{N_0}} \right) \quad (4.6)$$

It can be seen that this is very close to the case of BPSK where $P(E | r) = Q \left(\sqrt{2r^2 \frac{E_b}{N_0}} \right)$.

The method to derive the closed-form BER expression is hence similar to that used in Chapter 3 to analyze BPSK performance. The result is

$$P_{E_{cohFSK}}(TWDP) = \frac{1}{2} \sum_{i=1}^L a_i \left\{ \exp[-K(1-\alpha_i)] \sum_{n=0}^{\infty} H_{n+1} \left(K(1-\alpha_i); \sigma^2 \frac{E_b}{N_0} \right) \right. \\ \left. + \exp[-K(1+\alpha_i)] \sum_{n=0}^{\infty} H_{n+1} \left(K(1+\alpha_i); \sigma^2 \frac{E_b}{N_0} \right) \right\} \quad (4.7)$$

where $H_{n+1}(b; c)$ has been defined in (3.13). The difference between (3.12) and (4.7) is that instead of $c = 2\sigma^2 E_b / N_0$ in (3.12), we now have $c = \sigma^2 E_b / N_0$ in (4.7).

The limiting cases of (4.7) for $K=0$ (Rayleigh) and $\Delta=0$ (Rician) gives

$$P_{E_{cohFSK}}(Rayleigh) = \frac{1}{2} \left(1 - \sqrt{\frac{\sigma^2 \frac{E_b}{N_0}}{1 + \sigma^2 \frac{E_b}{N_0}}} \right) \quad (4.8)$$

as given in [3] and

$$P_{E_{cohFSK}}(Rician) = \exp(-K) \sum_{n=0}^{\infty} H_{n+1} \left(K; \sigma^2 \frac{E_b}{N_0} \right). \quad (4.9)$$

As discussed in Chapter 3, this is a rapidly converging series and hence (4.9) is a BER expression for coherent FSK with Rician fading that can be used for efficient computation.

4.3.2 Non-coherent FSK

For non-coherent orthogonal FSK, the conditional probability of error is given by [3]

$$P(E | r) = \frac{1}{2} \exp \left(-r^2 \frac{E_b}{2N_0} \right) \quad (4.10)$$

Here we see the similarity between (4.10) and that of the DPSK system in (4.1). The analysis is hence similar to that in Section 4.2 and the result is

$$P_{E_{noncohFSK}}(TWDP) = \frac{1}{2 \left[1 + \sigma^2 \frac{E_b}{N_0} \right]} \exp \left[\frac{-K \sigma^2 \frac{E_b}{N_0}}{1 + \sigma^2 \frac{E_b}{N_0}} \right] \sum_{i=1}^L a_i \left\{ \cosh \left(\frac{\alpha_i K \sigma^2 \frac{E_b}{N_0}}{1 + \sigma^2 \frac{E_b}{N_0}} \right) \right\} \quad (4.11)$$

The limiting cases for $K=0$ and $\Delta=0$ gives, respectively,

$$P_{E_{\text{noncohFSK}}}(\text{Rayleigh}) = \frac{1}{2 \left[1 + \sigma^2 \frac{E_b}{N_0} \right]} \quad (4.12)$$

as in [3] and

$$P_{E_{\text{noncohFSK}}}(\text{Rician}) = \frac{1}{2 \left[1 + \sigma^2 \frac{E_b}{N_0} \right]} \exp \left[\frac{-K \sigma^2 \frac{E_b}{N_0}}{1 + \sigma^2 \frac{E_b}{N_0}} \right] \quad (4.13)$$

as in [25].

4.4 Results and Discussion

The plots of BER versus SNR for the DPSK and non-coherent FSK systems in TWDP fading are shown in Figures 4.1–4.5. The SNR used in the plots is the average bit SNR, $2\sigma^2(1+K)E_b/N_0$, where K is the TWDP K parameter value.

It is of interest again to investigate the performance of the DPSK, coherent and non-coherent FSK systems when $\Delta = 1$. We recall that the result in Chapter 3 confirmed poorer performance of the BPSK system in TWDP fading than in Rayleigh fading when K goes beyond 6 dB—the explanation being that when $\Delta = 1$, the two specular components are equal in amplitudes and will cancel if they are anti-phase. For large enough K values, the remaining diffuse component will hence be weaker in signal power than a system in Rayleigh fading with transmitted power equal to the one in TWDP fading.

Figure 4.1 Performance of DPSK System in TWDP Fading, $\Delta=1$, K varying

In Figure 4.1, the plots for $K = 3$ dB, 6 dB and 10 dB when $\Delta = 1$ are shown for DPSK systems. It shows that when $K > 6$ dB, the DPSK system in TWDP fading can perform poorer than that in Rayleigh fading. The curve for $K = 3$ dB and $\Delta = 1$ illustrates that when the K value is lower, cancellation of specular components in TWDP still left a large enough diffuse component for performance to be better than in Rayleigh fading, though only marginally. In the same figure, the plot for $K = 10$ dB and $\Delta = 0.8$ is also shown to illustrate how fast the performance improves when Δ decreases from the value of 1. It shows that once Δ is not really close to 1, the performance improves significantly. The simulation results for $K = 10$ dB and $\Delta = 0.8$, also plotted in the figure, verify the validity of the analytical methodology and results for TWDP fading.

Figure 4.2 Performance of DPSK system in TWDP Fading, $K=3$ dB and 10 dB

In Figure 4.2, BER curves for the DPSK system are plotted for $K=3$ dB and 10 dB, with Δ varying between 0 and 1. It shows that for $K=3$ dB, varying Δ does not affect performance significantly and it remains close to that for Rician fading. This corroborates results of the analysis in [4] of the TWDP pdf envelope which concludes that for $K \leq 3$ dB, TWDP fading is close to Rician fading. His graphical analysis of TWDP pdf could not quantify the degree of closeness, unlike the BER plots we show in Figure 4.2.

Figures 4.3 and 4.4 are the corresponding plots for non-coherent FSK systems in TWDP fading. The plots are similar to those for the DPSK system, except for a 3 dB deterioration in performance—i.e., for the same BER performance, the non-coherent FSK system requires a 3 dB increase in SNR. It can be seen from Figure 4.3 that, when $\Delta = 1$, $K = 6$ dB or higher in TWDP fading will result in poorer performance than in Rayleigh

Figure 4.3 Performance of non-coherent FSK System in TWDP Fading, $\Delta=1$, K varying

Figure 4.4 Performance of non-coherent FSK System in TWDP Fading, $K=3$ and 10 dB

fading, just as in DPSK. The variation of performance with change in value of Δ for various K values is also similar as Figure 4.4 shows. In Figure 4.3, a plot for *coherent* FSK for $K = 10$ dB, $\Delta = 0.8$ is shown alongside the corresponding plot for non-coherent FSK. It shows the 3 dB improvement of coherent FSK over non-coherent FSK.

Figure 4.5 BER Performances of DPSK, Coherent and Non-coherent FSK in TWDP Fading, $K=3$ dB, $\Delta=1$

Figure 4.5 compares the performance results of the three systems in TWDP fading when $K=3$ dB and $\Delta=1$. As in Rayleigh fading, DPSK and coherent FSK system performances are equally good while non-coherent FSK is 3 dB poorer compared with them.

4.5 Conclusion

The closed-form BER expressions for DPSK, coherent and non-coherent FSK systems in TWDP fading were derived. The validity of the method used was verified through

simulation. The resulting plots of BER against SNR show that TWDP fading does give rise to poorer performance than in Rayleigh fading when the direct signal components in TWDP fading cancel with each other (because they are equal in amplitude but opposite in phase) and only a weak diffuse component remains. This happens when the value of the TWDP K parameter exceeds 6 dB. However, performance improves quickly when Δ reduces from 1 to 0, and with increasing values of K .

Chapter 5

Performance of pre-detection MRC Binary PSK and DPSK Systems in TWDP Fading Channels

5.1 Introduction

Various transmit/receive diversity techniques, such as time, frequency, space, polarization and space-time coding [10] have been proposed to combat degradation due to fading. The proven effectiveness of multichannel techniques has led to their widespread adoption in modern communication systems [11, 12]. This chapter analyzes the performance of BPSK and DPSK systems in N TWDP channels when the pre-detection maximal ratio combining (MRC) technique is employed. In Appendix D, the BER analysis of a non-coherent FSK system, which employs switched diversity combining in TWDP channels is included (This is an addendum to Chapter 5 (see Section 5.5), arising from the recommendation of Examiner 1 to include analysis of other diversity combining techniques)

Linear diversity analyses presented thus far for phase modulation schemes, e.g., in [13, 23], had assumed Rayleigh and Rician fading, where the complexity of the analysis is much lower compared to analysis when TWDP fading is involved. This increased complexity arises because the TWDP pdf used in the analysis comprises of the weighted sum of Rician pdfs, the number of which increases with the K and Δ parameter values of the TWDP pdf. The Rician pdfs have Rician K values that are dependent on the K and Δ values of the TWDP pdf.

The methodology used in arriving at the analytic expressions was verified through simulation for the BPSK MRC system. In TWDP fading, MRC diversity systems enjoy

significant improvement in performance over non-diversity systems, just like in other fading environments. These are shown in the plots of BER against SNR for both BPSK and DPSK systems.

The derivation of the BER expression for an MRC diversity system in TWDP fading in the next section starts with deriving the BER expression for an MRC diversity system in *Rician* fading. The reason is that, in TWDP fading, the BER for an MRC diversity system will be shown to be given by the weighted sum of the BERs of a number of effective Rician MRC systems generated when combining the physical TWDP channels. The Rician BER expressions derived here is in a form that is much simpler (compared with [13], where the results obtained are iterative and can involve formidable calculations when the number of channels is large) and in the form of a rapidly converging infinite series, unlike that in [23] which remains as an integral. As the Rician and Rayleigh fading pdfs are the limiting cases of the TWDP pdf, the Rician and Rayleigh BER expressions can also be obtained from the TWDP BER expressions derived in this chapter.

5.2 Performance of Pre-Detection MRC Systems in TWDP Fading

The analyses for binary PSK and DPSK diversity systems in TWDP fading are presented in the following sections after the communication model used in the analyses is defined. The results of the analyses are general in the sense that they are valid for any of the MRC diversity schemes mentioned above.

5.2.1 The Communication Model

The system communicates using binary signaling over an N -channel link. In each of the N channels, the signal is subjected to TWDP fading and there is also corruption due to additive white Gaussian noise (AWGN) with a two-sided power spectral density (PSD) of

$N_0/2$. It is assumed that (a) fading is slow over the bit duration and flat over the signal bandwidth, (b) fading on the different channels are mutually independent, and (c) the AWGN in the different channels are statistically independent and all have the same PSD value of $N_0/2$.

The above assumptions simplify analyses but are close approximations to practical situations. At the receiver, a linear combining technique, in this case pre-detection MRC, is adopted so that the contribution to the resultant signal by each signal is proportional to the rms value of the signal and inversely proportional to the mean square noise in the channel [9].

5.2.2 Performance of an N -path MRC BPSK System in TWDP fading

The BER expression of an N -path MRC diversity system in TWDP fading will be shown to be the weighted sum of BERs of $(2L)^N$ N -path MRC diversity systems in *Rician* fading, where L is the order of approximation of the approximate TWDP pdf expression. Hence, the BER of an N -Rician-path diversity system is first presented.

A. BER of an N -path MRC diversity BPSK system in Rician Fading

The BER of a communication system in a fading channel can be derived by averaging the conditional probability of error over the fading pdf, $p(r)$, i.e.,

$$P_E = \int_0^\infty P(E | r) p(r) dr .$$

For uncoded BPSK with an optimum detector [8][9],

$$P(E | r) = Q\left(\sqrt{2r^2 \frac{E_b}{N_0}}\right) = \frac{1}{\pi} \int_0^\pi \exp\left(\frac{-r^2 \frac{E_b}{N_0}}{\sin^2 \theta}\right) d\theta \quad (5.1)$$

where $r^2 E_b$ is the instantaneous energy received per bit.

Consider an N -path MRC diversity system where X is the resultant signal power fading variable. Then

$$X = r_1^2 + r_2^2 + \dots + r_N^2 \quad (5.2)$$

where $r_1, r_2, \dots, r_N$ are the fading amplitudes of the N paths, respectively. Using (5.2) in

$P_E = \int_0^\infty P(E | r) p(r) dr$, and well-known method in probability theory [45], the

BER expression for a BPSK diversity system is shown (see Appendix B.1) to be

$$P_{E_{BPSK}}(N) = \int_0^\infty \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{X E_b/N_0}{\sin^2 \theta}\right) d\theta p(X) dX \quad (5.3)$$

where $p(X)$ is the pdf of X . From (11) of [13], for N -Rician paths MRC diversity,

$$p(X) = \frac{1}{2\sigma^2} \left(\frac{X}{P}\right)^{\frac{N-1}{2}} \exp\left[-\frac{(X+P)}{2\sigma^2}\right] I_{N-1}\left(\frac{\sqrt{PX}}{\sigma^2}\right) \quad (5.4)$$

where $P = \sum_{m=1}^N V_m^2$. $V_1^2, V_2^2, \dots, V_N^2$ are respectively the average powers of the direct wave

signals in the N Rician paths (assuming a unit bit energy signal was transmitted). If

$K_1, K_2, \dots, K_N$ are the K parameters of the N Rician paths, then $P = 2\sigma^2 K_X$ where

$K_X = K_1 + K_2 + \dots + K_N$. Therefore, using (5.4) in (5.3),

$$P_{E_{BPSK}}(N - \text{Rician}) = \int_0^\infty \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{X E_b/N_0}{\sin^2 \theta}\right) d\theta \cdot \frac{1}{2\sigma^2} \left(\frac{X}{P}\right)^{\frac{N-1}{2}} \exp\left[-\frac{(X+P)}{2\sigma^2}\right] I_{N-1}\left(\frac{\sqrt{PX}}{\sigma^2}\right) dX \quad (5.5)$$

Using the relation derived from a result in [46], we have

$$\int_0^\infty x^\nu e^{-\alpha x} I_{2\nu}(2\beta\sqrt{x}) dx = \alpha^{-(2\nu+1)} \beta^{2\nu} \exp\left(\frac{\beta^2}{\alpha}\right) \quad (5.6)$$

Substituting (5.6) in (5.5) and simplifying give

$$P_{E_{BPSK}}(N - \text{Rician}) = \frac{\exp(-K_X)}{\pi} \int_0^{\pi/2} \left(\frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 E_b/N_0} \right)^N \exp\left(\frac{K_X \sin^2 \theta}{\sin^2 \theta + 2\sigma^2 E_b/N_0} \right) d\theta \quad (5.7)$$

Expanding the exponential term as a power series,

$$P_{E_{BPSK}}(N - \text{Rician}) = \frac{\exp(-K_X)}{\pi} \int_0^{\pi/2} \left\{ \left(\frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} \right)^N + \frac{K_X}{1!} \left(\frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} \right)^{N+1} + \frac{K_X^2}{2!} \left(\frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2 \left(\frac{E_b}{N_0} \right)} \right)^{N+2} + \dots \right\} d\theta$$

and using the result (3.11), derived from [43] that

$$\frac{1}{\pi} \int_0^{\pi/2} \left(\frac{\sin^2 \theta}{\sin^2 \theta + c} \right)^{n+1} d\theta = \left[\frac{1}{2} - \frac{1}{2} \sqrt{\frac{c}{1+c}} \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} \right],$$

a closed-form expression can be derived as

$$P_{E_{BPSK}}(N - \text{Rician}) = \exp(-K_X) \sum_{n=N-1}^{\infty} G_{n+1} \left(K_X; 2\sigma^2 \frac{E_b}{N_0} \right) \quad (5.8)$$

where

$$G_{n+1}(b; c) = \frac{b^{n-(N-1)}}{[n - (N-1)]!} \left[\frac{1}{2} - \frac{1}{2} \sqrt{\frac{c}{1+c}} \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} \right]. \quad (5.9)$$

B. BER of N -path MRC Diversity BPSK System in TWDP Fading

As the approximate TWDP pdf given by (2.7) is not a simple expression, the BER performance of a diversity system for TWDP fading is first derived here for the case of two-path MRC diversity rather than for the general case of N -path MRC diversity. This approach of presentation will enhance clarity of the derivation process of the BER

expression and, along the way, it will be apparent how its extension to an N -path diversity system can be carried out.

By re-expressing (2.7) in detail, it can be seen that the approximate TWDP pdf is the weighted sum of $2L$ Rician pdfs as follows:

$$\begin{aligned}
 p(r) = & \frac{a_1}{2} \frac{r}{\sigma^2} \exp\left(-\frac{(r^2 + 2\sigma^2 K(1 - \alpha_1))}{2\sigma^2}\right) I_0\left(\frac{r}{\sigma} \sqrt{2K(1 - \alpha_1)}\right) \\
 & + \frac{a_1}{2} \frac{r}{\sigma^2} \exp\left(-\frac{(r^2 + 2\sigma^2 K(1 + \alpha_1))}{2\sigma^2}\right) I_0\left(\frac{r}{\sigma} \sqrt{2K(1 + \alpha_1)}\right) \\
 & + \frac{a_2}{2} \frac{r}{\sigma^2} \exp\left(-\frac{(r^2 + 2\sigma^2 K(1 - \alpha_2))}{2\sigma^2}\right) I_0\left(\frac{r}{\sigma} \sqrt{2K(1 - \alpha_2)}\right) \\
 & + \frac{a_2}{2} \frac{r}{\sigma^2} \exp\left(-\frac{(r^2 + 2\sigma^2 K(1 + \alpha_2))}{2\sigma^2}\right) I_0\left(\frac{r}{\sigma} \sqrt{2K(1 + \alpha_2)}\right) \\
 & + \dots + \frac{a_L}{2} \frac{r}{\sigma^2} \exp\left(-\frac{(r^2 + 2\sigma^2 K(1 - \alpha_L))}{2\sigma^2}\right) I_0\left(\frac{r}{\sigma} \sqrt{2K(1 - \alpha_L)}\right) \\
 & + \frac{a_L}{2} \frac{r}{\sigma^2} \exp\left(-\frac{(r^2 + 2\sigma^2 K(1 + \alpha_L))}{2\sigma^2}\right) I_0\left(\frac{r}{\sigma} \sqrt{2K(1 + \alpha_L)}\right)
 \end{aligned} \tag{5.10}$$

i.e., $p(r)$ is the weighted sum of $2L$ Rician amplitude pdfs with weightings of

$\frac{a_1}{2}, \frac{a_1}{2}, \frac{a_2}{2}, \frac{a_2}{2}, \dots, \frac{a_L}{2}$ and these terms have ‘effective’ Rician K values

of $K_{11} = K(1 - \alpha_1)$, $K_{12} = K(1 + \alpha_1)$, $K_{21} = K(1 - \alpha_2)$, $K_{22} = K(1 + \alpha_2)$, ..., $K_{L2} = K(1 + \alpha_L)$.

Note that K and α ’s are the parameters of the approximate TWDP pdf.

At this point, the method of characteristic function will be used to derive the pdf of the decision variable in a two-path MRC diversity system in TWDP fading. We shall start by finding the characteristic function in the single TWDP fading channel situation.

If we let $Y = r^2$ be the square of a TWDP channel's amplitude variable, then the characteristic function of Y , $\Phi_Y(\omega)$ can be shown to be given by [pp. 119, 45]

$$\Phi_Y(\omega) = \int_0^\infty e^{j\omega Y} p(Y) dY \quad (5.11)$$

Using (5.10) and (5.11), it can be derived that

$$\begin{aligned} p(Y) = & \frac{a_1}{2} \left[\frac{1}{2\sigma^2} \exp\left(-\frac{Y + V_{Y_{11}}^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{YV_{Y_{11}}^2}}{\sigma^2}\right) + \frac{1}{2\sigma^2} \exp\left(-\frac{Y + V_{Y_{12}}^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{YV_{Y_{12}}^2}}{\sigma^2}\right) \right] \\ & + \frac{a_2}{2} \left[\frac{1}{2\sigma^2} \exp\left(-\frac{Y + V_{Y_{21}}^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{YV_{Y_{21}}^2}}{\sigma^2}\right) + \frac{1}{2\sigma^2} \exp\left(-\frac{Y + V_{Y_{22}}^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{YV_{Y_{22}}^2}}{\sigma^2}\right) \right] \\ & + \dots \\ & + \frac{a_L}{2} \left[\frac{1}{2\sigma^2} \exp\left(-\frac{Y + V_{Y_{L1}}^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{YV_{Y_{L1}}^2}}{\sigma^2}\right) + \frac{1}{2\sigma^2} \exp\left(-\frac{Y + V_{Y_{L2}}^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{YV_{Y_{L2}}^2}}{\sigma^2}\right) \right] \end{aligned} \quad (5.12)$$

where

$$V_{Y_{11}}^2 = 2\sigma^2 K_Y (1 - \alpha_{Y_1}) = 2\sigma^2 K_{Y_{11}},$$

$$V_{Y_{12}}^2 = 2\sigma^2 K_Y (1 + \alpha_{Y_1}) = 2\sigma^2 K_{Y_{12}},$$

$$V_{Y_{21}}^2 = 2\sigma^2 K_Y (1 - \alpha_{Y_2}) = 2\sigma^2 K_{Y_{21}},$$

.....

and

$$V_{Y_{L2}}^2 = 2\sigma^2 K_Y (1 + \alpha_{Y_L}) = 2\sigma^2 K_{Y_{L2}}.$$

K_Y and α_{Y_i} , $i = 1, 2, \dots, L$ are the parameters associated with the approximate TWDP pdf of this path. We can rewrite (5.12) as

$$p(Y) = \frac{a_1}{2} [p_{11}(Y) + p_{12}(Y)] + \frac{a_2}{2} [p_{21}(Y) + p_{22}(Y)] + \dots + \frac{a_L}{2} [p_{L1}(Y) + p_{L2}(Y)] \quad (5.13)$$

where

$$p_{11}(Y) = \frac{1}{2\sigma^2} \exp\left(-\frac{Y + V_{Y_{11}}^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{YV_{Y_{11}}^2}}{\sigma^2}\right),$$

$$p_{12}(Y) = \frac{1}{2\sigma^2} \exp\left(-\frac{Y + V_{Y_{12}}^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{YV_{Y_{12}}^2}}{\sigma^2}\right),$$

.....

and

$$p_{L2}(Y) = \frac{1}{2\sigma^2} \exp\left(-\frac{Y + V_{Y_{L2}}^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{YV_{Y_{L2}}^2}}{\sigma^2}\right).$$

Now consider the two-path MRC diversity system where the two TWDP paths have amplitude pdfs of $p(r_1)$ and $p(r_2)$. If we define $W = r_1^2$ and $Z = r_2^2$. Then we have

$$p(W) = \frac{a_1}{2} [p_{11}(W) + p_{12}(W)] + \frac{a_2}{2} [p_{21}(W) + p_{22}(W)] + \dots + \frac{a_L}{2} [p_{L1}(W) + p_{L2}(W)]$$

and

$$p(Z) = \frac{a_1}{2} [p_{11}(Z) + p_{12}(Z)] + \frac{a_2}{2} [p_{21}(Z) + p_{22}(Z)] + \dots + \frac{a_L}{2} [p_{L1}(Z) + p_{L2}(Z)].$$

The characteristic functions of W and Z are, respectively, given by

$$\Phi_W(\omega) = \frac{a_1}{2} [\Phi_{W_{11}}(\omega) + \Phi_{W_{12}}(\omega)] + \frac{a_2}{2} [\Phi_{W_{21}}(\omega) + \Phi_{W_{22}}(\omega)] + \dots + \frac{a_L}{2} [\Phi_{W_{L1}}(\omega) + \Phi_{W_{L2}}(\omega)]$$

and

$$\Phi_Z(\omega) = \frac{a_1}{2} [\Phi_{Z_{11}}(\omega) + \Phi_{Z_{12}}(\omega)] + \frac{a_2}{2} [\Phi_{Z_{21}}(\omega) + \Phi_{Z_{22}}(\omega)] + \dots + \frac{a_L}{2} [\Phi_{Z_{L1}}(\omega) + \Phi_{Z_{L2}}(\omega)].$$

If we represent the amplitude square of the two-path MRC decision variable by U , then

$U=W+Z$, and its characteristic function is [45]

$$\begin{aligned}\Phi_U(\omega) &= \Phi_W(\omega)\Phi_Z(\omega) \\ &= \frac{a_1^2}{4}\Phi_{W_{11}}(\omega)\Phi_{Z_{11}}(\omega) + \frac{a_1^2}{4}\Phi_{W_{11}}(\omega)\Phi_{Z_{12}}(\omega) \\ &\quad + \frac{a_1a_2}{4}\Phi_{W_{11}}(\omega)\Phi_{Z_{21}}(\omega) + \dots + \frac{a_L^2}{4}\Phi_{W_{L2}}(\omega)\Phi_{Z_{L2}}(\omega)\end{aligned}\tag{5.14}$$

The pdf of U will therefore be

$$\begin{aligned}p(U) &= \mathcal{F}^{-1}[\Phi_U(\omega)] \\ &= \frac{a_1^2}{4}\mathcal{F}^{-1}[\Phi_{W_{11}}(\omega)\Phi_{Z_{11}}(\omega)] + \frac{a_1^2}{4}\mathcal{F}^{-1}[\Phi_{W_{11}}(\omega)\Phi_{Z_{12}}(\omega)] \\ &\quad + \frac{a_1a_2}{4}\mathcal{F}^{-1}[\Phi_{W_{11}}(\omega)\Phi_{Z_{21}}(\omega)] + \dots + \frac{a_L^2}{4}\mathcal{F}^{-1}[\Phi_{W_{L2}}(\omega)\Phi_{Z_{L2}}(\omega)] \\ &= \frac{a_1^2}{4}p_1(U) + \frac{a_1^2}{4}p_2(U) + \frac{a_1a_2}{4}p_3(U) + \dots + \frac{a_L^2}{4}p_{4L^2}(U)\end{aligned}\tag{5.15}$$

where

$$p_1(U) = \mathcal{F}^{-1}[\Phi_{W_{11}}(\omega)\Phi_{Z_{11}}(\omega)],$$

$$p_2(U) = \mathcal{F}^{-1}[\Phi_{W_{11}}(\omega)\Phi_{Z_{12}}(\omega)],$$

.....

and

$$p_{4L^2}(U) = \mathcal{F}^{-1}[\Phi_{W_{L2}}(\omega)\Phi_{Z_{L2}}(\omega)].$$

We now observe that each of the above $(2L)^2$ terms of the form $\mathcal{F}^{-1}[\Phi_{W_{ij}}(\omega)\Phi_{Z_{kl}}(\omega)]$ is exactly the pdf of the amplitude square of a two-path MRC diversity system in Rician fading with the N in (5.4) set to 2. Applying this step $(2L)^2$ times, each time with the

appropriate P value substituted into (5.4), we can obtain $p_1(U), p_2(U), \dots, p_{4L^2}(U)$. For example,

$$p_4(U) = \mathcal{F}^{-1}[\Phi_{W_{11}}(\omega)\Phi_{Z_{22}}(\omega)] = \frac{1}{2\sigma^2} \left(\frac{U}{P_4} \right)^{\frac{1}{2}} \exp\left[-\frac{(U+P_4)}{2\sigma^2} \right] I_1\left(\frac{\sqrt{P_4 U}}{\sigma^2} \right)$$

where $P_4 = V_{W_{11}}^2 + V_{Z_{22}}^2 = 2\sigma^2 K_4$ with $K_4 = K_{W_{11}} + K_{Z_{22}}$. $K_{W_{11}}$ and $K_{Z_{22}}$ are the K parameters of the effective Rician pdfs, p_{11} and p_{12} , associated with the random variables W and Z , and they are given by $K_{W_{11}} = K_W(1 - \alpha_{W_1})$ and $K_{Z_{22}} = K_Z(1 + \alpha_{Z_2})$. (Note that if there are N diversity paths then each of the $p_j(U)$, $j = 1, 2, \dots, (2L)^N$ is the inverse Fourier transform of the product of N appropriate characteristic functions. Therefore, extension to N path diversity can be carried out easily.

Upon substituting (5.15) into

$$P_{E_{BPSK}}(2 - TWDP) = \int_0^\infty \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(\frac{-U \frac{E_b}{N_0}}{\sin^2 \theta} \right) d\theta p(U) dU,$$

we obtain the BER expression for a two-path diversity BPSK system in TWDP fading. The BER expression becomes the corresponding weighted sum of $(2L)^2$ terms of (5.8) with $N=2$, and each of the terms has its respective K_j , $j = 1, 2, \dots, 4L^2$. (Note that (5.8) derived earlier, is the BER expression of an N -path MRC BPSK system in Rician fading). Hence,

$$\begin{aligned} P_{E_{BPSK}}(2 - TWDP) = & \frac{a_1^2}{4} \exp(-K_1) \sum_{n=1}^{\infty} G_{n+1} \left(K_1; 2\sigma^2 \frac{E_b}{N_0} \right) \\ & + \frac{a_1^2}{4} \exp(-K_2) \sum_{n=1}^{\infty} G_{n+1} \left(K_2; 2\sigma^2 \frac{E_b}{N_0} \right) \\ & + \frac{a_1 a_2}{4} \exp(-K_3) \sum_{n=1}^{\infty} G_{n+1} \left(K_3; 2\sigma^2 \frac{E_b}{N_0} \right) \\ & + \dots + \frac{a_L^2}{4} \exp(-K_{4L^2}) \sum_{n=1}^{\infty} G_{n+1} \left(K_{4L^2}; 2\sigma^2 \frac{E_b}{N_0} \right) \end{aligned} \quad (5.16)$$

where

$$G_{n+1}(b; c) = \frac{b^{n-(N-1)}}{[n-(N-1)]!} \left[\frac{1}{2} - \frac{1}{2} \sqrt{\frac{c}{1+c}} \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} \right].$$

For N -path MRC diversity, it can be deduced easily from the above that

$$P_{E_{BPSK}}(N - TWDP) = \frac{1}{2^N} \sum \beta_{i_1 i_2 \dots i_N} P_{i_1 i_2 \dots i_N}(N) \quad (5.17)$$

where the summation is to be taken over all combinations of the N number of subscripts, each of which is an integer chosen from $1, 2, \dots, 2L$. There will hence be $(2L)^N$ terms in the summation (see Appendix B.2). The actual values taken by the subscripts of a particular $\beta_{i_1 i_2 \dots i_N}$ define how it (and the corresponding $P_{i_1 i_2 \dots i_N}(N)$) should be computed.

Specifically,

$$\beta_{i_1 i_2 \dots i_N} = b_{i_1} b_{i_2} \dots b_{i_N}$$

with $b_{2k-1} = b_{2k} = a_k$ for $k = 1, 2, \dots, L$ and

$$P_{i_1 i_2 \dots i_N}(N) = \exp(-K_{i_1 i_2 \dots i_N}) \sum_{n=N-1}^{\infty} G_{n+1} \left[K_{i_1 i_2 \dots i_N}; 2\sigma^2 \frac{E_b}{N_0} \right]$$

where

$$G_{n+1}(b; c) = \frac{b^{n-(N-1)}}{[n-(N-1)]!} \left[\frac{1}{2} - \frac{1}{2} \sqrt{\frac{c}{1+c}} \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} \right],$$

and

$$K_{i_1 i_2 \dots i_N} = K_{i_1 1} + K_{i_2 2} + \dots + K_{i_N N}.$$

Using the subscripts l to indicate which one of the N channels, and k to indicate which one of the L number of α parameters in the channel is to be used, $K_{i_1 1}, K_{i_2 2}, \dots, K_{i_N N}$ are

computed with

$$K_{2k-1 l} = K_l (1 - \alpha_{kl})$$

$$K_{2k l} = K_l (1 + \alpha_{kl})$$

and

$$\alpha_{kl} = \Delta_l \cos\left(\frac{\pi(k-1)}{2L-1}\right), \quad k = 1, 2, \dots, L, \quad l = 1, 2, \dots, N,$$

where K_l and Δ_l are respectively the TWDP K factor and Δ parameter of the l^{th} channel. It can be verified easily (see Appendix B.3) that for $N=1$, (5.17) reduces to (3.12), the BER performance of a non-diversity BPSK system in TWDP fading, derived in Chapter 3.

5.2.3 Performance of an N -path MRC DPSK System in TWDP Fading

For a DPSK system, the conditional probability of error is given as [3]

$$P_{DPSK}(E | r) = \frac{1}{2} \exp\left(-r^2 \frac{E_b}{N_0}\right) \quad (5.18)$$

The BER expression for an N -path MRC diversity, DPSK system in *Rician* fading is first obtained by substituting (5.18) and (5.4) into $P_E = \int_0^\infty P(E | r) p(r) dr$. As we have done for a BPSK system, simplifying with the help of (5.6) (see Appendix B.4) gives

$$P_{E_{DPSK}}(N - \text{Rician}) = \frac{1}{2[1 + 2\sigma^2 E_b/N_0]^N} \exp\left[\frac{-2K_X \sigma^2 E_b/N_0}{1 + 2\sigma^2 E_b/N_0}\right] \quad (5.19)$$

where $K_X = K_1 + K_2 + \dots + K_N$ is the sum of the K parameters of the N Rician pdfs.

Like in the case of BPSK, the BER expression for an N -path MRC diversity DPSK system in *TWDP* fading is then derived to be the weighted sum of $(2L)^N$ terms, with each term in the form of (5.19), L being the order of approximation of the TWDP pdf used. Therefore,

$$P_{E_{DPSK}}(N - \text{TWDP}) = \frac{1}{2^N} \sum \beta_{i_1 i_2 \dots i_N} P_{i_1 i_2 \dots i_N}(N) \quad (5.20)$$

where the summation is to be taken over all combinations of the N number of subscripts, each of which is an integer chosen from $1, 2, \dots, 2L$. The coefficient

$$\beta_{i_1 i_2 \dots i_N} = b_{i_1} b_{i_2} \dots b_{i_N}$$

with $b_{2k-1} = b_{2k} = a_k$ for $k = 1, 2, \dots, L$, and

$$P_{i_1 i_2 \dots i_N}(N) = \frac{1}{2[1 + 2\sigma^2 E_b/N_0]^N} \exp\left[\frac{-2K_{i_1 i_2 \dots i_N} \sigma^2 E_b/N_0}{1 + 2\sigma^2 E_b/N_0}\right]$$

with $K_{i_1 i_2 \dots i_N} = K_{i_1 1} + K_{i_2 2} + \dots + K_{i_N N}$, where $K_{i_1 1}, K_{i_2 2}, \dots, K_{i_N N}$ are computed with

$$K_{2k-1l} = K_l(1 - \alpha_{kl})$$

$$K_{2kl} = K_l(1 + \alpha_{kl})$$

and

$$\alpha_{kl} = \Delta_l \cos\left(\frac{\pi(k-1)}{2L-1}\right), \quad k = 1, 2, \dots, L, \quad l = 1, 2, \dots, N.$$

Note that K_l and Δ_l are respectively the TWDP K factor and Δ parameter of the l^{th} channel. For $N=1$, (5.20) will reduce to (4.3), the expression for the bit-error rate performance for a non-diversity DPSK system in TWDP fading.

5.3 Results and Discussion

Using (5.17), the plots of BER against SNR are drawn in Figures 5.1 and 5.2 to display the performance of the BPSK system with pre-detection MRC. The SNR used in the plots is the average total bit SNR, i.e., $2\sigma^2(1 + K_{AV})E_b/N_0$, where K_{AV} is the average TWDP K parameter value for the N paths in the MRC system. To simplify calculations, all the paths are simply chosen to have the same TWDP fading parameters.

Figure 5.1 shows the performance of the BPSK system in two different TWDP fading conditions, one where $K=3$ dB and $\Delta=1$ and the other where $K=3$ dB and $\Delta=0$ (equivalent to Rician fading). As the diversity order N is varied from 1 to 4, the significant improvement in performance in both fading situations is clearly seen. For example, to achieve a BER of 10^{-3} , a system with $N=2$ (two-path diversity) requires an SNR which is more than 20 dB lower, compared with a system with $N=1$. The plots in Figure 5.1 also show that with the application of pre-detection MRC, the change in performance

behaviour of the system are similar in the two fading conditions. For example, in both situations, it can be observed that at higher SNR values, the performance improvement is greater, compared to the case when the SNR is lower. The rate of improvement with an increase in N , is also similar for the two fading conditions. The BER *simulation* results for the two fading conditions when $N=2$ are also plotted in Figure 5.1. The close concurrence of the analytical and simulation results verifies the validity of the methodology used in derivation of the closed-form BER expressions.

In Figure 5.2, the plots for a BPSK system in TWDP fading where $\Delta \neq 1$ ($K=10$ dB, $\Delta=0.8$) are shown against those for $K = 3$ dB and $\Delta = 1$. It shows diversity combining accentuating more (compared with Figure 5.1), the performance difference between the two fading situations as N increases from 1 to 4. For example, it shows that the performance of the system with $K=10$ dB and $\Delta=0.8$ when $N = 3$ almost similar to the other system with order of diversity $N = 4$.

Figure 5.3 shows the improvement in BER performance of the DPSK system for $K=3$ dB, $\Delta=1$ when N is varied from 1 to 4. It is observed that under diversity combining, there is no difference in behaviour between the BPSK and DPSK systems in TWDP fading, except for the inherent superiority of BPSK systems over DPSK.

Figure 5.1 Performance of MRC BPSK Systems in TWDP Fading ($K=3$ dB, $\Delta=0$ or $\Delta=1$)

Figure 5.2 Performance of MRC BPSK Systems in TWDP Fading ($K=3$ dB, $\Delta=1$ and $K=10$ dB, $\Delta=0.8$)

Figure 5.3 Performance of MRC DPSK System in TWDP Fading ($K=3$ dB, $\Delta=1$)

In the analytical plot for $K = 3$ dB, $\Delta = 1$ in Figure 5.1, the order L of the approximate TWDP pdf was initially chosen as equal to 2. This was following the recommendation [4] that $L \geq K\Delta/2$ in order that the approximate TWDP does not deviate too much from the exact TWDP pdf. But it was then found that the corresponding simulation plot deviates very slightly, though noticeably, from the analytical plot. However, when L was increased to 5, the discrepancy disappears. So the guide that $L \geq K\Delta/2$ should be used carefully. In fairness, it was mentioned in [4] that as $\Delta \rightarrow 1$, the approximate TWDP pdf will be less accurate. Therefore, a larger L value should be chosen when Δ is close to 1.

5.4 Conclusion

In TWDP fading, the BER performance of a multi-channel MRC diversity system was shown to be given by the weighted sum of BERs of the $(2L)^N$ effective Rician MRC

diversity systems when combining the N physical TWDP channels, where L is the order of the approximate TWDP pdf used. Hence, the BER performance of a multi-channel MRC system in Rician fading was first derived and then used to obtain the closed-form BER expressions for the BPSK and DPSK systems employing pre-detection MRC in TWDP fading. The validity of the methodology of derivation was verified through simulation. As with Rayleigh and Rician fading, the results show that there is significant performance improvement arising from employing diversity, especially when SNR is large.

5.5 Addendum: Non-coherent FSK System with Switched Diversity

Combining

In Appendix D, the BER performance of a non-coherent FSK (NCFSK) system with switched diversity combining is analyzed. The design and implementation of this switched diversity system is much simpler (in terms of modulation and diversity combining technique) compared with the MRC, BPSK system described in the above Sections. Hence, as expected, its performance is poorer. For example, for a BPSK system in TWDP fading ($K = 10$ dB, $\Delta = 0.8$), the BER performance at SNR = 20 dB for a two-channel MRC system is 10^{-6} , versus 10^{-4} for the corresponding BER figure of a two-channel switched combining NCFSK system (See Figures 5.2 and D.5). We would expect the performance of systems employing other diversity combining methods, like equal gain combining and selection combining, to be somewhere between those of MRC and switched combining—as equal gain combining and selection combining have efficiencies between these two methods [9].

Chapter 6

Performance of Pre-detection MRC M -ary PSK and DPSK Systems in TWDP Fading

6.1 Introduction

In this thesis, the BER performance analysis in TWDP fading started with the investigation of single-channel, binary phase/frequency modulation systems in Chapters 3 and 4. This was then extended in Chapter 5 to multichannel binary PSK and DPSK systems employing pre-detection MRC. In this chapter we look at *multi-level*, multichannel phase modulation systems [3, 49] and derive symbol-error rate (SER) expressions for pre-detection MRC, M -ary PSK and DPSK systems in TWDP fading.

As the TWDP pdf was only recently proposed and analyzed [4, 15], these are, to the author's knowledge, the first available SER performance results for MPSK and MDPSK systems in TWDP fading. (There are however, performance analyses of MPSK and MDPSK systems with diversity combining investigated over Rayleigh and Rician fading [13, 23]. These are limiting cases of TWDP fading).

The validity of the method employed in the analysis is verified through simulation.

6.2 Performance Analysis Procedure

6.2.1 The Communication Model

The system communicates using M -ary signaling over an N -channel link. In each of the N channels, the signal is subjected to TWDP fading and there is also corruption due to AWGN with a two-sided power spectral density of $N_0/2$. It is assumed that (a) fading is slow over the symbol duration and flat over the signal bandwidth, (b) fading on the

different channels are mutually independent and (c) the AWGN in the different channels are statistically independent and all have the same power spectral density value of $N_0/2$. The above assumptions simplify analysis but are close approximations to practical situations. At the receiver, a linear combining technique, in this case MRC, is adopted so that the contribution to the resultant signal by each signal is proportional to the rms value of the signal and inversely proportional to the mean square noise in the channel [9].

6.2.2 Analysis for Rician Fading

The BER expression for an N -path, pre-detection MRC system in TWDP fading has been shown in the previous chapter to be the weighted sum of BERs of $(2L)^N$ effective N -Rician-path MRC diversity systems, where L is order of the approximate TWDP pdf expression. Hence, in the same manner, the SER expression of an N -Rician-path, MRC diversity system is first presented before it is employed in the SER expression of an N -TWDP-path, MRC diversity system.

6.3 MPSK System Performance in TWDP fading

6.3.1 SER of an N -path MRC MPSK System in Rician Fading

The SER of an M -ary communication system is derived by averaging the conditional probability of symbol error over the envelope pdf of the fading channel, i.e.,

$$P_E = \int_0^\infty P(E | r) p(r) dr \quad (6.1)$$

For an MPSK system the conditional probability of symbol error is [47]

$$P_{MPSK}(E | r) = \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \exp\left(-\frac{r^2(E_s/N_0)\sin^2(\pi/M)}{\sin^2\theta}\right) d\theta \quad (6.2)$$

where $r^2 E_s$ is the instantaneous energy received per symbol and $N_0/2$ is the double-sided power spectral density of additive white Gaussian noise (AWGN). Consider an N -path MRC diversity system where

$$X = r_1^2 + r_2^2 + \cdots + r_N^2 \quad (6.3)$$

and $r_1, r_2, \dots, r_N$ are the fading amplitudes of the N paths, respectively. Using (6.1)—(6.3), and the well-known method in probability theory [45], the SER expression for an MPSK diversity system can be written as

$$P_{E_{MPSK}}(N) = \int_0^\infty \frac{1}{\pi} \int_0^{\pi-\frac{\pi}{M}} \exp\left(\frac{-X(E_s/N_0)\sin^2(\pi/M)}{\sin^2 \theta}\right) d\theta p(X) dX \quad (6.4)$$

where $p(X)$ is the pdf of X (see Appendix B.1). From (11) of [13], for N -Rician paths MRC diversity,

$$p(X) = \frac{1}{2\sigma^2} \left(\frac{X}{P}\right)^{\frac{N-1}{2}} \exp\left[-\frac{(X+P)}{2\sigma^2}\right] I_{N-1}\left(\frac{\sqrt{PX}}{\sigma^2}\right) \quad (6.5)$$

where $P = \sum_{m=1}^N V_m^2$. Note that $V_1^2, V_2^2, \dots, V_N^2$ are, respectively, the power levels of the direct wave signals in the N Rician paths (assuming a unit energy symbol was transmitted). If $K_1, K_2, \dots, K_N$ are the K parameters of the N Rician paths, then $P = 2\sigma^2 K_X$ where $K_X = K_1 + K_2 + \cdots + K_N$. Therefore, using (6.5) in (6.4), we get

$$P_{E_{MPSK}}(N - \text{Rician}) = \int_0^\infty \frac{1}{\pi} \int_0^{\pi-\frac{\pi}{M}} \exp\left(\frac{-X(E_s/N_0)\sin^2(\pi/M)}{\sin^2 \theta}\right) d\theta \cdot \frac{1}{2\sigma^2} \left(\frac{X}{P}\right)^{\frac{N-1}{2}} \exp\left[-\frac{(X+P)}{2\sigma^2}\right] I_{N-1}\left(\frac{\sqrt{PX}}{\sigma^2}\right) dX. \quad (6.6)$$

Using the relation

$$\int_0^\infty x^\nu e^{-\alpha x} I_{2\nu}(2\beta\sqrt{x}) dx = \alpha^{-(2\nu+1)} \beta^{2\nu} \exp\left(\frac{\beta^2}{\alpha}\right), \quad (6.7)$$

which can be derived from a result in [46], in (6.6) and simplifying yields

$$P_{E_{MPSK}}(N - \text{Rician}) = \frac{\exp(-K_X)}{\pi} \int_0^{\pi - \frac{\pi}{M}} \left(\frac{\sin^2 \theta}{\sin^2 \theta + 2\sigma^2(E_s/N_0)\sin^2(\pi/M)} \right)^N \exp\left(\frac{K_X \sin^2 \theta}{\sin^2 \theta + 2\sigma^2(E_s/N_0)\sin^2(\pi/M)} \right) d\theta \quad (6.8)$$

Expanding the exponential term in (6.8) as a power series, followed by re-writing the results in term-by-term integration, we get

$$P_{E_{MPSK}}(N - \text{Rician}) = \exp(-K_X) \sum_{n=N-1}^{\infty} G_{n+1} \left(K_X; 2\sigma^2 \frac{E_s}{N_0} \sin^2 \frac{\pi}{M} \right) \quad (6.9)$$

where

$$G_{n+1}(b; c) = \frac{b^{n-(N-1)}}{(n-(N-1))!} \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \left(\frac{\sin^2 \theta}{\sin^2 \theta + c} \right)^{n+1} d\theta \quad (6.10)$$

Using [(73) and (74) in 43], (6.10) can be written as

$$G_{n+1}(b; c) = \frac{b^{n-(N-1)}}{(n-(N-1))!} I_{n+1} \left(\pi - \frac{\pi}{M}; c \right) \quad (6.11)$$

where

$$I_{n+1}(\phi; c) = \frac{1}{\pi} \int_0^{\phi} \left(\frac{\sin^2 \theta}{\sin^2 \theta + c} \right)^{n+1} d\theta$$

$$= \frac{\phi}{\pi} - \frac{\beta}{\pi} \left[\left(\frac{\pi}{2} + \tan^{-1}(\delta) \right) \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} + \sin\{\tan^{-1}(\delta)\} \sum_{k=1}^n \sum_{i=1}^k \left(\frac{T_{ik}}{(1+c)^k} [\cos\{\tan^{-1}(\delta)\}]^{2(k-i)+1} \right) \right] \quad (6.12)$$

in which $-\pi \leq \phi \leq \pi$, $\beta = \sqrt{c/1+c} \operatorname{sgn} \phi$, $\delta = -\beta \cot \phi$, and

$$T_{ik} = \frac{\binom{2k}{k}}{\binom{2(k-i)}{k-i} 4^i (2(k-i)+1)}.$$

6.3.2 SER of N -path MRC MPSK System in TWDP Fading

The BER expression for pre-detection MRC systems in TWDP fading has been shown in Chapter 5 to be the weighted sum of BERs of $(2L)^N$ effective N -Rician-path MRC diversity systems, where L is order of the approximate TWDP pdf expression. Apparently the SER expression for a pre-detection MRC MPSK system can therefore be expressed in a form similar to the BER expression for a pre-detection MRC BPSK system (see (5.17), Chapter 5), i.e.,

$$P_{E_{MPSK}}(N-TWDP) = \frac{1}{2^N} \sum \beta_{i_1 i_2 \dots i_N} P_{i_1 i_2 \dots i_N}(N) \quad (6.13)$$

where the summation is to be taken over all combinations of the N subscripts, each of which is an integer chosen from the set $\{1, 2, \dots, 2L\}$. (There will hence be $(2L)^N$ terms in the summation). The actual values taken by the subscripts of a particular $\beta_{i_1 i_2 \dots i_N}$ define how it (and the corresponding $P_{i_1 i_2 \dots i_N}(N)$) should be computed. Specifically,

$$\beta_{i_1 i_2 \dots i_N} = b_{i_1} b_{i_2} \dots b_{i_N}$$

with $b_{2k-1} = b_{2k} = a_k$ for $k = 1, 2, \dots, L$ and

$$P_{i_1 i_2 \dots i_N}(N) = \exp(-K_{i_1 i_2 \dots i_N}) \sum_{n=N-1}^{\infty} G_{n+1} \left[K_{i_1 i_2 \dots i_N}; 2\sigma^2 \frac{E_s}{N_0} \sin^2 \frac{\pi}{M} \right]$$

where $G_{n+1}(b; c) = \frac{b^{n-(N-1)}}{(n-(N-1))!} I_{n+1} \left(\pi - \frac{\pi}{M}; c \right)$, and $K_{i_1 i_2 \dots i_N} = K_{i_1} + K_{i_2} + \dots + K_{i_N}$.

$I_{n+1}\left(\pi - \frac{\pi}{M}; c\right)$ is computed as given in (6.12). Using the subscripts l to indicate which channel, and k to indicate which one of the L number of α parameters in the channel is to be used, $K_{i_1 1}, K_{i_2 2}, \dots, K_{i_N N}$ are computed with

$$\begin{aligned} K_{2k-1l} &= K_l(1 - \alpha_{kl}) \\ K_{2kl} &= K_l(1 + \alpha_{kl}) \end{aligned}$$

and

$$\alpha_{kl} = \Delta_l \cos\left(\frac{\pi(k-1)}{2L-1}\right), \quad k = 1, 2, \dots, L, \quad l = 1, 2, \dots, N,$$

where K_l and Δ_l are respectively the TWDP K factor and Δ parameter of the l^{th} channel.

6.4 MDPSK System Performance in TWDP Fading

As in the above method for MPSK, the symbol error rate of an N Rician paths diversity MDPSK system is first presented.

6.4.1 SER of N-path MRC MDPSK System in Rician Fading

For an MDPSK system, the conditional probability of symbol error [47] is given by

$$P_{MDPSK}(E | r) = \frac{1}{\pi} \int_0^{\pi - \frac{\pi}{M}} \exp\left(-\frac{r^2 \frac{E_s}{N_0} \sin^2\left(\frac{\pi}{M}\right)}{1 + \cos\left(\frac{\pi}{M}\right) \cos \theta}\right) d\theta \quad (6.14)$$

where $r^2 E_s$ is the instantaneous energy received per symbol and $N_0/2$ is the double-sided power spectral density of AWGN. Consider an N -path MRC diversity system where $X = r_1^2 + r_2^2 + \dots + r_N^2$ as in (6.3), $r_1, r_2, \dots, r_N$ being the fading amplitudes of the N

paths, respectively. Using (6.1), (6.3) and (6.14), and the well-known method in probability theory [45], the SER expression for a MPSK diversity system can be written as

$$P_{E_{MDPSK}}(N) = \int_0^\infty \frac{1}{\pi} \int_0^{\pi-\frac{\pi}{M}} \exp\left(-\frac{X \frac{E_s}{N_0} \sin^2\left(\frac{\pi}{M}\right)}{1 + \cos\left(\frac{\pi}{M}\right) \cos \theta}\right) d\theta p(X) dX \quad (6.15)$$

where $p(X)$ is the pdf of X (see Appendix B.1). Using (6.5),

$$p(X) = \frac{1}{2\sigma^2} \left(\frac{X}{P}\right)^{\frac{N-1}{2}} \exp\left[-\frac{(X+P)}{2\sigma^2}\right] I_{N-1}\left(\frac{\sqrt{PX}}{\sigma^2}\right),$$

where $P = \sum_{m=1}^N V_m^2$. Note that $V_1^2, V_2^2, \dots, V_N^2$ are, respectively, the power levels of the direct wave signals in the N Rician paths (assuming a unit energy symbol was transmitted). If $K_1, K_2, \dots, K_N$ are the K parameters of the N Rician paths, then

$P = 2\sigma^2 K_X$ where $K_X = K_1 + K_2 + \dots + K_N$. Therefore, using (6.5) in (6.15), we get

$$P_{E_{MDPSK}}(N) = \int_0^\infty \frac{1}{\pi} \int_0^{\pi-\frac{\pi}{M}} \exp\left(-\frac{X \frac{E_s}{N_0} \sin^2\left(\frac{\pi}{M}\right)}{1 + \cos\left(\frac{\pi}{M}\right) \cos \theta}\right) d\theta \cdot \frac{1}{2\sigma^2} \left(\frac{X}{P}\right)^{\frac{N-1}{2}} \exp\left[-\frac{(X+P)}{2\sigma^2}\right] I_{N-1}\left(\frac{\sqrt{PX}}{\sigma^2}\right) dX \quad (6.16)$$

Using (6.7) in (6.16) and simplifying, we yield

$$P_{E_{MDPSK}}(N - \text{Rician}) = \frac{\exp(-K_X)}{\pi} \int_0^{\pi-\frac{\pi}{M}} \left(\frac{1 + \cos\left(\frac{\pi}{M}\right) \cos \theta}{2\sigma^2 \frac{E_s}{N_0} \sin^2 \frac{\pi}{M} + 1 + \cos\left(\frac{\pi}{M}\right) \cos \theta} \right)^N \exp\left(\frac{K_X \left(1 + \cos\left(\frac{\pi}{M}\right) \cos \theta\right)}{\frac{E_s}{N_0} \sin^2 \frac{\pi}{M} + 1 + \cos\left(\frac{\pi}{M}\right) \cos \theta} \right) d\theta$$

Letting $a = \sec \frac{\pi}{M}$ and $b = \sec \frac{\pi}{M} + 2\sigma^2 \frac{E_s}{N_0} \tan \frac{\pi}{M} \sin \frac{\pi}{M}$

in the above equation gives

$$P_{E_{MDPSK}}(N - Rician) = \frac{\exp(-K_X)}{\pi} \int_0^{\pi-\frac{\pi}{M}} \left(\frac{a + \cos \theta}{b + \cos \theta} \right)^N \exp \left[K_X \left(\frac{a + \cos \theta}{b + \cos \theta} \right) \right] d\theta$$

and expanding the exponential term in the integrand gives

$$P_{E_{MDPSK}}(N - Rician) = \frac{\exp(-K_X)}{\pi} \int_0^{\pi-\frac{\pi}{M}} \left[\left(\frac{a + \cos \theta}{b + \cos \theta} \right)^N + \frac{K_X}{1!} \left(\frac{a + \cos \theta}{b + \cos \theta} \right)^{N+1} + \frac{K_X^2}{2!} \left(\frac{a + \cos \theta}{b + \cos \theta} \right)^{N+2} + \dots \right] d\theta$$

or

$$P_{E_{MDPSK}}(N - Rician) = \frac{\exp(-K_X)}{\pi} \sum_{n=N-1}^{\infty} E_{n+1}(K_X; a, b) \quad (6.17)$$

where

$$E_{n+1}(K_X; a, b) = \frac{K_X^{n-(N-1)}}{[n-(N-1)]!} \int_0^{\pi-\frac{\pi}{M}} \left(\frac{a + \cos \theta}{b + \cos \theta} \right)^{n+1} d\theta.$$

The integral $\int_0^{\pi-\frac{\pi}{M}} \left(\frac{a + \cos \theta}{b + \cos \theta} \right)^{n+1} d\theta$ can be evaluated either through numerical integration

or after expanding the integrand as a series, we can use the results for integrals shown in [pg.180, 46]. The details on the latter evaluation method are given in Appendix C.

6.4.2 SER of N -path MRC MDPSK System in TWDP Fading

The SER expression for an MDPSK system in TWDP fading, obtained in a manner similar to deriving SER of an MPSK system, is

$$P_{E_{MDPSK}}(N - TWDP) = \frac{1}{2^N} \sum \beta_{i_1 i_2 \dots i_N} P_{i_1 i_2 \dots i_N}(N) \quad (6.18)$$

where the summation is to be taken over all combinations of the N number of subscripts, each of which is an integer chosen from $1, 2, \dots, 2L$. The actual values taken by the

subscripts of a particular $\beta_{i_1 i_2 \dots i_N}$ define how it (and the corresponding $P_{i_1 i_2 \dots i_N}(N)$) should be computed. Specifically,

$$\beta_{i_1 i_2 \dots i_N} = b_{i_1} b_{i_2} \dots b_{i_N}$$

with $b_{2k-1} = b_{2k} = a_k$ for $k = 1, 2, \dots, L$ and

$$P_{i_1 i_2 \dots i_N}(N) = \frac{\exp(-K_{i_1 i_2 \dots i_N})}{\pi} \sum_{n=N-1}^{\infty} E_{n+1}(K_{i_1 i_2 \dots i_N}; a, b)$$

$$\text{where } E_{n+1}(c; a, b) = \frac{c^{n-(N-1)}}{[n-(N-1)]!} \int_0^{\pi - \frac{\pi}{M}} \left(\frac{a + \cos \theta}{b + \cos \theta} \right)^{n+1} d\theta, \quad a = \sec \frac{\pi}{M},$$

$$b = \sec \frac{\pi}{M} + 2\sigma^2 \frac{E_s}{N_0} \tan \frac{\pi}{M} \sin \frac{\pi}{M} \quad \text{and} \quad K_{i_1 i_2 \dots i_N} = K_{i_1 1} + K_{i_2 2} + \dots + K_{i_N N}.$$

Using the subscripts l to indicate which channel, and k to indicate which one of the L number of α parameters in the channel is to be used, $K_{i_1 1}, K_{i_2 2}, \dots, K_{i_N N}$ are computed with

$$K_{2k-1l} = K_l(1 - \alpha_{kl})$$

$$K_{2kl} = K_l(1 + \alpha_{kl})$$

and

$$\alpha_{kl} = \Delta_l \cos\left(\frac{\pi(k-1)}{2L-1}\right), \quad k = 1, 2, \dots, L, \quad l = 1, 2, \dots, N,$$

where K_l and Δ_l are, respectively, the TWDP K factor and Δ parameter of the l^{th} channel.

6.5 Results and Discussion

Using (6.13) and (6.18), the graphs of SER versus SNR for the MPSK and MDPSK systems with two-channel pre-detection MRC are plotted. The SNR used in the plots is the average total SNR, i.e., $2\sigma^2(1 + K_{AV})E_s/N_0$ where K_{AV} is the average TWDP K parameter

value for the two paths and E_s is the energy of symbol transmitted. In the plots, the two channels are chosen to have the same TWDP fading parameters for calculation convenience. The K and Δ parameter values of TWDP fading are chosen to be 3 dB and 1, respectively.

In Figure 6.1, the plots for the MPSK systems are shown for $M = 2, 4$ and 8. It shows the improvement in performance afforded by diversity combining. (The SER curve for a single channel 8PSK system is also displayed in the same figure for comparison). The curves also show the deterioration in performance with the increase in M , which is to be expected.

Figure 6.1 Performance of MPSK, two-Channel MRC System in TWDP Fading

The plot of the simulation curve, in Figure 6.1, for a two-channel 4-PSK system shows close concurrence with the corresponding analytical curve. This verifies the validity of the method employed in deriving the analytical expressions.

In Figure 6.2, the corresponding plots for the MDPSK system are displayed. They show similar characteristics to the MPSK system—i.e., performance improvement with diversity and deterioration in performance with an increase in M values.

The plots for the 2-PSK and 2-DPSK systems in Figures 6.1 and 6.2 respectively, were also checked to be consistent with the corresponding plots (see Figures 5.1 and 5.3) in Chapter 5 for binary PSK and DPSK systems. This corroborates the results obtained earlier.

Figure 6.2 Performance of MDPSK, two-Channel MRC System in TWDP Fading

6.6 Conclusion

The SER expressions of MPSK and MDPSK systems with pre-detection MRC in TWDP fading were derived. The validity of the derivation method was verified through simulation. Though the approximate TWDP pdf expression is much more complex than the Rayleigh or Rician pdf, the analytical SER expressions derived are expressed in a compact and convenient form which facilitates efficient calculations of SER values for various N (diversity order), M and TWDP parameters values.

Chapter 7

Mixed-Mode Fading in Pre-detection MRC Systems

7.1 Introduction

For a multi-channel communication system with diversity combining, there is a possibility of mixed-mode fading where some or all of the channels may not be subjected to the same fading characteristics. In this chapter we discuss how the results in this thesis provide a convenient platform to investigate mixed-mode fading. We also present some results obtained using this platform.

7.2 Application of Analytical Results to Mixed-Mode Fading Studies

In the previous chapters, analytical BER and SER expressions were derived for commonly-used digital modulation systems in TWDP fading. As the Rayleigh and Rician pdfs are limiting cases of the TWDP pdf, the TWDP BER and SER expressions easily reduce to the BER and SER expressions in Rayleigh fading (when K is set to 0) and the corresponding expressions in Rician fading (when Δ is set to 0, $K \neq 0$), where K and Δ are the two parameters describing the TWDP pdf. Moreover, the BER and SER expressions derived in Chapters 5 & 6 were for multichannel diversity systems with pre-detection MRC.

Hence at this juncture, there is flexibility in configuring the system, with regards to the order of diversity, the fading types, and the number of channels for investigating system BER or SER performance. We could have a non-diversity system ($N = 1$) in either Rayleigh, Rician or TWDP fading or we could set up a diversity system with two or more channels and choose the fading type in each of these channels by specifying an

appropriate set of TWDP parameter values. A convenient platform is hence made available for the study of mixed-mode fading. We could investigate, for example, a multichannel system consisting of a mixture of Rayleigh and TWDP fadings, or a mixture of Rayleigh and Rician fadings or any other combinations of two or more of the different fading types. In a practical diversity combining system, there is always a possibility of mixed-mode fading. For example, in a space diversity system where the two transmitters are placed at different locations, the two channels to the receiver would experience possibly different types of fading.

In [13, 23], mixed-mode fading for multichannel reception was investigated. However, their results are special cases of those presented here. In their studies, TWDP fading channels were not considered. It should also be noted that in [13], the BER/SER expressions derived are in a more complex form and does not lend themselves to efficient calculations, especially when the number of channels is large. It is also important to note that the BER/SER expressions given in [23] remained in integral forms.

Some useful results arising from investigation of mixed-mode fading could include for example, the number and types of channels required to achieve a desired BER or SER performance, the variation in system performance with different combinations of fading modes and the effect on BER or SER performance when the K and Δ parameter values of the fading pdfs are varied. The results can impact the design of a communication system as the type of fading experienced by a transmitted signal can, to a certain extent, be decided by the physical design of system—e.g. with space-diversity, choice of location of the transmitters and height of antenna would determine the type of fading or the fading parameter values.

7.3 Results and Discussions

Some results of the investigation into mixed-mode fading for a BPSK system is shown in Figures 7.1 to 7.3. In the figures, BER is plotted against the total average bit SNR, i.e., $2\sigma^2(1 + K_{AV})E_b/N_0$ where K_{AV} is the average TWDP K parameter value of all the channels in the diversity system. The BER values were calculated using (5.17) in Chapter 5:

$$P_{E_{BPSK}}(N - TWDP) = \frac{1}{2^N} \sum \beta_{i_1 i_2 \dots i_N} P_{i_1 i_2 \dots i_N}(N)$$

Detailed procedures for calculating the BER using the above expression are given in Chapter 5.

Figure 7.1 Performance of Two-Channel Diversity System with Mixed-mode Fading (Rayleigh and Rician Fading)

Figure 7.1 shows the BER performance for a two-channel diversity system with different combinations of Rayleigh and Rician fading channels. Also included is a

combination of two Rician fading channels with different K parameter values. Quantitatively, one would expect the Rayleigh—Rician ($K=6$ dB) BER performance curve to fall somewhere between the Rayleigh—Rayleigh and the Rician—Rician (both $K = 6$ dB) BER performance curves. This is made precise by our analytical results, which showed exactly where it lies relative to the two non mixed-mode BER performance curves. The Rician ($K=6$ dB)—Rician ($K=3$ dB) BER performance curve could represent a situation where the first Rician fading channel has a (stronger) line-of-sight path while the other Rician fading channel has a (weaker) reflected, specular path.

Figure 7.2 Performance of Two-Channel Diversity System with Mixed-mode Fading (Rician and TWDP Fading)

Figure 7.2 shows a combination of TWDP and Rician fading channels for a two-channel diversity system. There is also a TWDP-TWDP combination with the same K value but different Δ values.

Figure 7.3 illustrates a three-channel diversity system with various combinations of Rayleigh, Rician and TWDP fading channels. The study could be extended to more channels and other combinations of fading modes if necessary.

Figure 7.3 Performance of three-Channel Diversity System with Mixed-mode Fading (Rayleigh, Rician and TWDP Fading)

The above figures illustrate the flexibility and convenience afforded by the use of the analytical BER and SER expressions (derived for TWDP fading in a multi-channel diversity system) in the investigation of mixed-mode fading. These advantages arise because the TWDP pdf expression used is able to provide a near-complete description of the possible fading envelopes in small-scale fading. Particularly, it has included in its limiting cases, the one-wave, two-wave, Rayleigh and Rician pdfs (see Table 2.1).

Chapter 8

Conclusions and Future Work

8.1 Conclusions

In this thesis, the BER and SER performances of various digital modulation systems were analyzed in two-wave with diffusion power (TWDP) fading. The TWDP fading pdf had been introduced recently to complement the Rayleigh and Rician pdfs which are traditionally the pdfs used to characterize the fading channels when studying small-scale local fading experienced by mobile wireless receivers.

In TWDP fading, there are two dominant (specular) signal components in the presence of other diffuse components. There is hence the possibility, when the two specular components are approximately equal in amplitude (parameter $\Delta=1$) and opposite in phase, that cancellation occurs, leaving weak remaining diffuse signal components. A study of the TWDP pdf envelope [4] reveals that, when K (the ratio of specular to diffuse powers) is large, it does show in some instances the signal spending long periods of time at low levels. This tends to support the contention that when the combined powers of the two specular components is high (large K) and they cancel each other (when $\Delta=1$), what remains is a weak diffuse component. Hence, TWDP fading can give rise to a communication link worse than that of Rayleigh fading. This is significant as Rayleigh fading is often taken as the “worst-case” fading situation [7, 8], and used in the design of communication links. There is hence the motivation to look beyond the conclusions derived from studying the pdf envelope shape [4], a method that has its limitations, especially it being unable to quantify BER and SER performances.

In this work, the author analyzes and investigates the BER and SER performances of different modulation systems in TWDP fading as the TWDP parameters, K and Δ , are

varied. In these analyses, an approximate TWDP pdf which parameterizes a family of fading envelopes is used as there is no exact closed-form TWDP pdf expression. Though approximate, the pdf is mathematically exact (e.g. the area under the pdf curve is one) and is hence suitable for use in the derivation of analytical expressions for BER and SER. Also, for the useful range of parameter values, the approximate pdf is a good approximation of the exact pdf. However, it is much more complex (and hence more difficult to handle in analysis) than the Rayleigh and Rician pdf—comprising of the weighted sum of a number of Rician pdfs, each having a different Rician K value evaluated from the TWDP parameter values. It should be noted that the Rayleigh and Rician pdfs are limiting cases of the TWDP pdf.

The BER performance studies starts with an uncoded BPSK system where a close-form BER expression in the form of infinite series was derived. This series is rapidly converging and it therefore permits efficient calculation of BER when the TWDP K and Δ parameters values are varied. The resulting plots of BER versus SNR confirm that when the two specular components are equal in strength ($\Delta = 1$) and K is large enough, TWDP fading does give rise to a link poorer than that of Rayleigh fading. The threshold K value, beyond which this happens, is found to be 6 dB. However, it is noted that increasing K to 10 dB from 6 dB does not cause too significant a deterioration. For example, to maintain a BER of 10^{-3} , the increase in SNR required is less than 3 dB. For $K \leq 3$ dB, varying Δ from 0 (Rician fading) to 1 does not cause performance to deteriorate too much from that of Rician fading. This had been surmised earlier when a study of the TWDP pdf envelope [4] reveals that for $K \leq 3$ dB, there is close concurrence between the Rician and TWDP pdf envelopes. The BER investigation goes further as it reveals quantitatively, how close are the effects of the two pdfs on BER performances, as Δ of the TWDP pdf is varied from 0 to 1 for $K = 3$ dB. When K values are large (e.g., 10 dB), the BER curves show a great

variation on BER performance as Δ is varied from 0 to 1. Hence, compared with the pdf envelope analysis, the BER performance analysis provided more precise, quantitative information and better insight into TWDP fading.

In Chapter 4, the BER analysis was extended to DPSK and FSK (coherent and non-coherent) systems. The BER performance curves plotted with the derived expressions show characteristics similar to those of the BPSK system. The threshold TWDP K value (when $\Delta = 1$) beyond which the systems perform poorer than in Rayleigh fading is also 6 dB. The variation in performance as K and Δ are varied is also similar in trend to those for the BPSK system.

The proven benefits of diversity combining [3, 9] had resulted in many modern communication systems incorporating it in their design. In Chapter 5, the BER analysis was extended to pre-detection MRC, binary PSK and DPSK systems. The method used for analyzing diversity systems operating in a TWDP channel involves deriving first the BER expression of the systems in Rician fading. This is because it was proven in the analysis that the BER expression of the system in TWDP fading is given by the weighted sum of the BER expressions of $(2L)^N$ effective Rician diversity systems resulting from combining N physical TWDP channels. (L is the order of the approximate TWDP pdf used to characterize TWDP fading). The BER expressions derived are in a compact and closed-form which permits efficient evaluation of BER performance as N is varied for each set of K and Δ parameter values in TWDP fading. For $N = 1$, these expressions reduce to those derived earlier in Chapters 3 and 4 for corresponding single channel binary PSK and DPSK systems. Performance behaviour of the systems with diversity combining, in TWDP fading, is not different from that in Rayleigh or Rician fading in the sense that diversity combining systems offers significant improvement over single channel systems. For example, to achieve a BER of 10^{-3} , a BPSK system with $N = 2$ (two-channel diversity)

requires a significantly lower SNR, (more than 10 dB lower in this case) than a system with $N = 1$. As in the case with Rayleigh and Rician fading [13], improvement in BER performance with MRC diversity is greater in the higher SNR regime.

In Chapter 6, the performance analysis of digital communication systems was extended to multi-level phase modulation systems with pre-detection MRC. Again, the symbol error rate (SER) expressions derived for MPSK & MDPSK are shown to be in a compact form allowing efficient calculations of SER values. Curves of SER versus SNR plotted with the help of the derived SER expressions have exposed the performance deterioration as the number of levels, M , varies from 2 to 4 in a two-channel diversity system. The analytical results for 2-PSK and 2-DPSK also agree with those obtained earlier for binary PSK and DPSK.

There will be an increased possibility of TWDP fading where directional antennas and wideband signals are used [4]. Transmission in a large indoor environment (e.g., inside a factory or office building) where there are large reflective surfaces (e.g., walls or doors) are also likely to increase the occurrence of TWDP fading, with the direct signal and a strong reflected component(s) forming the two-wave signal, together with other diffuse components. The author is of the opinion that the assumption of Rician fading, where there is only one specular wave in the presence of diffuse components, is too restrictive and hence less likely to occur. This will be especially true in systems where directional antennas and wideband signals are used. The TWDP fading pdf provides a near-complete description of possible envelope fading pdfs in small-scale fading, including as its limiting cases, the one-wave, two-wave, Rayleigh and Rician pdfs. The case for using the TWDP fading pdf is further enhanced with the argument that three (or more) waves with diffuse power can, at most times, be reduced to two-wave with diffuse power [4].

As mentioned earlier, the Rayleigh and Rician pdfs are limiting cases of the TWDP pdf. More specifically, when the TWDP K parameter is 0, the TWDP pdf reduces to the Rayleigh pdf, and when $\Delta=0$ (with $K \neq 0$), it gives the Rician pdf. Therefore, in all the above BER and SER expressions derived, setting $K = 0$ gives BER and SER performance expressions in Rayleigh fading, while setting $\Delta=0$ (when $K \neq 0$) gives the BER and SER performance expressions in Rician fading. For BPSK and coherent FSK systems, the *Rician* BER expressions derived as such are in the form of a rapidly converging infinite series. This form of the expressions is new and an improvement over the forms of earlier derived Rician BER expressions that are either more complex [13] or remain in the form of integrals [14, 23].

Due to the fact that the one-wave, two-wave, Rayleigh and Rician BER and SER expressions are special cases of the TWDP BER and SER expressions, and because the TWDP BER and SER expressions were also derived for multichannel diversity systems, a convenient platform is hence available to investigate mixed-mode fading. In Chapter 7, this likely scenario of having different fading types in the channels of a diversity system was discussed and investigated. The results presented show the flexibility afforded by this platform in the investigation of mixed-mode fading as different combinations of fading types and different number of channels could be selected as desired.

The validity of the methodology used in the derivation of the analytical BER and SER expressions in Chapters 3 to 6 was verified through simulation, using Matlab's Simulink.

8.2 Recommendations for Further Work

8.2.1 Performance Evaluation with Correlated TWDP fading channels

In this thesis, it was assumed, when evaluating the performance of the pre-detection MRC systems, that the channels in the diversity branches were mutually independent (see

Sections 5.2.1 and 6.2.1). The work can be extended to encompass performance studies where there are correlations between the TWDP fading channels. This is important, as the effectiveness of diversity combining in combating fading will be expected to vary with changes in the degree of correlation amongst the channels [3, 9].

8.2.2 Performance study of MQAM system in TWDP fading

In this thesis, we have studied systems with constant energy per bit or constant energy per symbol. Nevertheless, multi-level quadrature amplitude modulation (MQAM) systems are becoming popular in high rate data transmission over wireless links. In the performance analysis of the various modulation systems (with or without predetection MRC) in this thesis (e.g., BPSK or MPSK), the study involves averaging the Q -function $Q(\sqrt{\gamma})$ over the fading distribution, where γ is the SNR involved. However, MQAM systems [48, 49] requires in addition to $Q(\sqrt{\gamma})$, averaging of $Q^2(\sqrt{\gamma})$ [50]. The various techniques for evaluating $Q^2(\sqrt{\gamma})$ in fading channels (other than TWDP fading channels) had been reported in [50–55]. These could be employed to facilitate the derivation of SER expressions for MQAM systems in TWDP fading.

8.2.3 Experimental Measurements

It will be interesting to conduct experimental measurements in an environment where TWDP fading is likely to occur. This could be performed inside a factory or a large office where there are large reflective surfaces, as mentioned earlier in this chapter. These can then be compared against predicted results from analysis or simulations, assuming TWDP fading. The experiments could confirm if there are fading channels that will actually give rise to a link poorer than in Rayleigh fading. The experiments could also be used to investigate if the performance does vary as predicted when the parameter values of K and

Δ are varied and how these parameters depend on the physical environment of the experimental set-up. Note that while the measurements of received signal amplitudes taken in such environments [21, 22, 56–58] had been compared against simulation results for Rayleigh, Rician, Nakagami- m or log-normal fading, they were performed before TWDP fading was proposed.

The question of “which of the non-Rayleigh and non-Rician fading pdf can better describe a particular physical transmission environment” is still an interesting open problem.

8.2.4 Comparative study of Nakagami- m and TWDP pdfs

The Nakagami- m pdf [59–61] is a versatile fading model as varying the m parameter gives special cases of fading. For example, $m=1$ gives Rayleigh fading while $m=0.5$ reduces it to the one-sided Gaussian distribution [2]. Also, for $m=0.75$, the Nakagami- m pdf gives system performance poorer than that given by the Rayleigh fading pdf [62]. The Nakagami- m model has also been used to approximate a Rician distribution [63–65]. It seems that there are many similarities between Nakagami- m and TWDP pdfs—in that both can be readily used to study Rayleigh and Rician fading as well as fading which is poorer than Rayleigh. A comparative study of the two would be interesting and useful in furthering the understanding of fading channels characterization.

8.2.5 Study of MIMO systems in TWDP Fading

Multiple-Input-Multiple-Output (MIMO) systems is presently an active area of study as it is an effective strategy for improved performance in wireless systems. However investigation of MIMO system performance so far is conducted mainly in Rayleigh [66–

69] and Rician [70-74] fading environments. Including the study of MIMO systems in TWDP fading would result in a more comprehensive report on MIMO systems.

Author's Publications

- [1] S. H. Oh and K. H. Li, "BER performance of BPSK receivers over two-wave with diffuse power fading channels," *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, pp. 1448-1454, Jul. 2005.
- [2] S. H. Oh, K. H. Li and W. S. Lee, "Performance of pre-detection MRC systems over two-wave with diffuse power fading channels," revised and re-submitted to *IEEE Trans. Wireless Commun.* for possible publication.
- [3] S. H. Oh and K. H. Li, "Performance Analyses of DPSK and FSK systems in two-wave with diffuse power fading channels," to be submitted to *EURASIP Journal on Wireless Communications and Networking* for possible publication.
- [4] S. H. Oh and K. H. Li, "Performance of pre-detection MRC M -ary PSK and DPSK systems in two-wave with diffuse power fading channels," to be submitted for publication.

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Appendices

A.1 Proof of Equation (3.11)

$$\text{Showing } I_{n+1}\left(\frac{\pi}{2}; c\right) = \left[\frac{1}{2} - \frac{1}{2} \sqrt{\frac{c}{1+c}} \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} \right]$$

From [43],

$$I_{n+1}(\phi; c) = \frac{\phi}{\pi} - \frac{\beta}{\pi} \left[\left(\frac{\pi}{2} + \tan^{-1}(\delta) \right) \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} + \sin\{\tan^{-1}(\delta)\} \sum_{k=1}^n \sum_{i=1}^k \left(\frac{T_{ik}}{(1+c)^k} [\cos\{\tan^{-1}(\delta)\}]^{2(k-i)+1} \right) \right]$$

where $-\pi \leq \phi \leq \pi$, $\beta = \sqrt{c/1+c} \operatorname{sgn} \phi$, $\delta = -\beta \cot \phi$, and

$$T_{ik} = \frac{\binom{2k}{k}}{\binom{2(k-i)}{k-i} 4^i (2(k-i)+1)}.$$

If $\phi = \pi/2$, $\beta = \sqrt{c/1+c} \operatorname{sgn}(\pi/2) = \sqrt{c/1+c}$, $\delta = -\beta \cot(\pi/2) = 0$, and therefore

$$\begin{aligned} I_{n+1}\left(\frac{\pi}{2}; c\right) &= \left[\frac{\pi}{2} \cdot \frac{1}{\pi} - \frac{1}{\pi} \sqrt{\frac{c}{1+c}} \frac{\pi}{2} \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} \right] \\ &= \left[\frac{1}{2} - \frac{1}{2} \sqrt{\frac{c}{1+c}} \sum_{k=0}^n \binom{2k}{k} \frac{1}{[4(1+c)]^k} \right] \end{aligned}$$

A.2 Proof of Equation (3.14)

Showing $I_{n+1}\left(\frac{\pi}{2}; c\right) = I_n\left(\frac{\pi}{2}; c\right) - \frac{1}{2}\binom{2n}{n}\left[\frac{1}{[4(1+c)]^n}\sqrt{\frac{c}{1+c}}\right]$ with $I_0 = 1/2$

For $n=0$,

$$I_1 = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{c}{1+c}}\sum_{k=0}^0\binom{0}{0}1 = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{c}{1+c}}.$$

For $n=1$,

$$I_2 = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{c}{1+c}}\sum_{k=0}^1\binom{2k}{k}\frac{1}{4[(1+c)]^k} = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{c}{1+c}}\left[1 + \frac{1}{2(1+c)}\right] = I_1 - \frac{1}{4(1+c)}\sqrt{\frac{c}{1+c}}.$$

For $n=2$,

$$I_3 = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{c}{1+c}}\left[1 + \frac{1}{2(1+c)} + \frac{6}{16(1+c)^2}\right] = I_2 - \frac{3}{16(1+c)^2}\sqrt{\frac{c}{1+c}}.$$

For $n=3$,

$$I_4 = I_3 - \frac{5}{32(1+c)^3}\sqrt{\frac{c}{1+c}}.$$

For $n=4$,

$$I_5 = I_4 - \frac{35}{256(1+c)^4}\sqrt{\frac{c}{1+c}}$$

and so on. Therefore,

$$I_{n+1}\left(\frac{\pi}{2}; c\right) = I_n\left(\frac{\pi}{2}; c\right) - \frac{1}{2}\binom{2n}{n}\left[\frac{1}{[4(1+c)]^n}\sqrt{\frac{c}{1+c}}\right]$$

with $I_0 = 1/2$.

A.3 TWDP Channel for Simple BPSK Receiver Simulation

Figure A.1 TWDP Channel for Simple BPSK Receiver Simulation

A.4 TWDP Channel for BPSK Rake Receiver Simulation

Figure A.2 TWDP Channel for BPSK Rake Receiver Simulation

B.1 Proof of Equation (5.3)

$$P_{E_{BPSK}}(N) = \int_0^\infty \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(\frac{-X \frac{E_b}{N_0}}{\sin^2 \theta}\right) d\theta p(X) dX$$

where $X = r_1^2 + r_2^2 + \dots + r_N^2$.

If $X = r^2$ (i.e., X is square of fading amplitude),

$$dX = 2rdr \text{ or } dr = \frac{1}{2\sqrt{X}} dX \quad (a)$$

From [(5-6), 45],

$$p_X(X) = \frac{p_r(r)}{(dX/dr)} = \frac{1}{2\sqrt{X}} p_r(r)$$

or

$$p_r(r) = 2\sqrt{X} p_X(X) \quad (b)$$

Substituting (a) and (b) into $P_E = \int_0^\infty P(E | r) p(r) dr$ gives

$$P_E = \int_0^\infty P(E | X) p(X) dX$$

for a single channel system.

For a multi-channel system where MRC is applied at the receiver, the effective SNR ,

$X \frac{E_b}{N_0}$ is given by [9],

$$X \frac{E_b}{N_0} = r_1^2 \frac{E_b}{N_0} + r_2^2 \frac{E_b}{N_0} + \dots + r_N^2 \frac{E_b}{N_0}$$

assuming equal transmitting power per bit of E_b and assumption of equal noise power N_0 .

This is equivalent to a single channel where $X = r_1^2 + r_2^2 + \dots + r_N^2$ and therefore using

(5.1) in

$$P_E = \int_0^\infty P(E | r) p(r) dr$$

gives

$$P_{E_{BPSK}}(N) = \int_0^\infty \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(\frac{-X \frac{E_b}{N_0}}{\sin^2 \theta}\right) d\theta p(X) dX .$$

B.2 Number of terms in Equation (5.17)

$$\ln \sum \beta_{i_1 i_2 \dots i_N} P_{i_1 i_2 \dots i_N}(N)$$

“...the summation is taken over all combinations of the N number of subscripts, each of which is an integer chosen from $1, 2, \dots, 2L$ ”.

For example if N (order of diversity) = 2 and L (order of approximate TWDP pdf) = 2,

Then,

$$\sum \beta_{i_1 i_2 \dots i_N} P_{i_1 i_2 \dots i_N}(N) = \sum \beta_{i_1 i_2} P_{i_1 i_2} \quad (2)$$

i.e. the number of subscripts = 2 for β (and P) and each subscript “*is an integer chosen from 1,2,3...2L*”, i.e. each subscript is an integer from 1,2,3,4. Then “*all combinations of the N number of subscripts*” in this case would be 11, 12, 13, 14, 21, 22, 23, 24, 31,...,44.

Hence

$$\begin{aligned} \sum \beta_{i_1 i_2} P_{i_1 i_2}(2) = & \beta_{11} P_{11} + \beta_{12} P_{12} + \beta_{13} P_{13} + \beta_{14} P_{14} \\ & + \beta_{21} P_{21} + \beta_{22} P_{22} + \beta_{23} P_{23} + \beta_{24} P_{24} \\ & + \\ & + \beta_{44} P_{44} \end{aligned}$$

which is a total of $(2L)^N = 16$ terms.

B.3 Equation (5.7) when $N = 1$

Showing $P_{E_{BPSK}}(N - TWDP) = \frac{1}{2^N} \sum \beta_{i_1 i_2 \dots i_N} P_{i_1 i_2 \dots i_N}(N)$ **reduces to**

$$P_E = \frac{1}{2} \sum_{i=1}^L a_i \left\{ \exp[-K(1 - \alpha_i)] \sum_{n=0}^{\infty} H_{n+1} \left(K(1 - \alpha_i); 2\sigma^2 \frac{E_b}{N_0} \right) + \exp[-K(1 + \alpha_i)] \sum_{n=0}^{\infty} H_{n+1} \left(K(1 + \alpha_i); 2\sigma^2 \frac{E_b}{N_0} \right) \right\} \quad (3.12)$$

for $N = 1$.

For $N = 1$,

$$P_{E_{BPSK}}(N - TWDP) = \frac{1}{2^N} \sum \beta_{i_1 i_2 \dots i_N} P_{i_1 i_2 \dots i_N}(N)$$

gives

$$P_{E_{BPSK}}(1 - TWDP) = \frac{1}{2} \sum \beta_{i_1} P_{i_1}(1)$$

with i_1 chosen from 1,2,3,4.

Therefore,

$$P_{E_{BPSK}}(1 - TWDP) = \frac{1}{2} [\beta_1 P_1 + \beta_2 P_2 + \beta_3 P_3 + \beta_4 P_4]$$

The **β terms** are calculated using

“ $\beta_{i_1 i_2 \dots i_N} = b_{i_1} b_{i_2} \dots b_{i_N}$ with $b_{2k-1} = b_{2k} = a_k$ for $k = 1, 2, \dots, L$ ”.

Hence,

$$\beta_1 = a_1, \beta_2 = a_1, \beta_3 = a_2 \text{ and } \beta_4 = a_2$$

since $b_1 = b_2 = a_1$ for $k=1$, and $b_3 = b_4 = a_2$ for $k=2$.

The **P terms** are evaluated using

$$P_{i_1 i_2 \dots i_N}(N) = \exp(-K_{i_1 i_2 \dots i_N}) \sum_{n=N-1}^{\infty} G_{n+1} \left[K_{i_1 i_2 \dots i_N}; 2\sigma^2 \frac{E_b}{N_0} \right]$$

where $K_{i_1 i_2 \dots i_N} = K_{i_1 1} + K_{i_2 2} + \dots + K_{i_N N}$.

$K_{i_1 1}, K_{i_2 2}, \dots, K_{i_N N}$ are computed with

$$K_{2k-1l} = K_l(1 - \alpha_{kl})$$

$$K_{2kl} = K_l(1 + \alpha_{kl})$$

and

$$\alpha_{kl} = \Delta_l \cos\left(\frac{\pi(k-1)}{2L-1}\right), \quad k=1,2,\dots,L, \quad l=1,2,\dots,N.$$

Therefore,

$$\begin{aligned} P_1(1) &= \exp(-K_1) \sum_{n=1}^{\infty} G_{n+1} \left[K_1; 2\sigma^2 \frac{E_b}{N_0} \right] \\ &= \exp(-K_{11}) \sum_{n=0}^{\infty} G_{n+1} \left[K_{11}; 2\sigma^2 \frac{E_b}{N_0} \right] \\ &= \exp[-K_1(1 - \alpha_{11})] \sum_{n=0}^{\infty} G_{n+1} \left[K_1(1 - \alpha_{11}); 2\sigma^2 \frac{E_b}{N_0} \right] \\ &= \exp[-K(1 - \alpha_1)] \sum_{n=0}^{\infty} G_{n+1} \left[K(1 - \alpha_1); 2\sigma^2 \frac{E_b}{N_0} \right] \end{aligned}$$

since for $N=1$, we used $\alpha_{11} = \alpha_1$ and $K_1 = K$ in (3.12).

In the same manner we get,

$$\begin{aligned} P_2(1) &= \exp[K(1 + \alpha_1)] \sum_{n=0}^{\infty} G_{n+1} \left[K(1 + \alpha_1); 2\sigma^2 \frac{E_b}{N_0} \right] \\ P_3(1) &= \exp[-K(1 - \alpha_2)] \sum_{n=0}^{\infty} G_{n+1} \left[K(1 - \alpha_2); 2\sigma^2 \frac{E_b}{N_0} \right] \\ P_4(1) &= \exp[K(1 + \alpha_2)] \sum_{n=0}^{\infty} G_{n+1} \left[K(1 + \alpha_2); 2\sigma^2 \frac{E_b}{N_0} \right]. \end{aligned}$$

Therefore,

$$\begin{aligned}
 P_{E_{BPSK}}(1 - TWDP) &= \frac{1}{2} [\beta_1 P_1 + \beta_2 P_2 + \beta_3 P_3 + \beta_4 P_4] \\
 &= \frac{1}{2} \sum_{i=1}^L a_i \left\{ \exp[-K(1 - \alpha_i)] \sum_{n=0}^{\infty} H_{n+1} \left(K(1 - \alpha_i); 2\sigma^2 \frac{E_b}{N_0} \right) \right. \\
 &\quad \left. + \exp[-K(1 + \alpha_i)] \sum_{n=0}^{\infty} H_{n+1} \left(K(1 + \alpha_i); 2\sigma^2 \frac{E_b}{N_0} \right) \right\}
 \end{aligned}$$

which is (3.12).

B.4 Derivation of Equation (5.19)

Showing $P_{E_{DPSK}}(N - \text{Rician}) = \frac{1}{2 \left[1 + 2\sigma^2 \frac{E_b}{N_0} \right]^N} \exp \left[\frac{-2K_X \sigma^2 \frac{E_b}{N_0}}{1 + 2\sigma^2 \frac{E_b}{N_0}} \right]$

$$P_{E_{DPSK}}(N) = \int_0^\infty \exp\left(-X \frac{E_b}{N_0}\right) p(X) dX$$

Since

$$p(X) = \frac{1}{2\sigma^2} \left(\frac{X}{P} \right)^{\frac{N-1}{2}} \exp\left[-\frac{(X+P)}{2\sigma^2}\right] I_{N-1}\left(\frac{\sqrt{PX}}{\sigma^2}\right),$$

$$\begin{aligned} P_{E_{DPSK}}(N) &= \int_0^\infty \exp\left(-X \frac{E_b}{N_0}\right) \frac{1}{2\sigma^2} \left(\frac{X}{P} \right)^{\frac{N-1}{2}} \exp\left[-\frac{(X+P)}{2\sigma^2}\right] I_{N-1}\left(\frac{\sqrt{PX}}{\sigma^2}\right) p(X) dX \\ &= \frac{1}{2(2\sigma^2) P^{\frac{N-1}{2}}} \int_0^\infty X^{\frac{N-1}{2}} \exp\left[-\left(\frac{E_b}{N_0} + \frac{1}{2\sigma^2}\right)X\right] \exp\left[-\frac{P}{2\sigma^2}\right] I_{N-1}\left(\frac{\sqrt{PX}}{\sigma^2}\right) dX \end{aligned}$$

Using $\int_0^\infty x^\nu e^{-\alpha x} I_{2\nu}(2\beta\sqrt{x}) dx = \alpha^{-(2\nu+1)} \beta^{2\nu} \exp\left(\frac{\beta^2}{\alpha}\right)$ in the above equation,

$$\begin{aligned} P_{E_{DPSK}}(N) &= \frac{1}{2(2\sigma^2) P^{\frac{N-1}{2}}} \exp\left[-\frac{P}{2\sigma^2}\right] \\ &\quad \cdot \left(\frac{E_b}{N_0} + \frac{1}{2\sigma^2}\right)^{-N} \left(\frac{\sqrt{P}}{2\sigma^2}\right)^{N-1} \exp\left[\left(\frac{P}{2\sigma^2}\right) \left(\frac{1}{2\sigma^2 \frac{E_b}{N_0} + 1}\right)\right] \end{aligned}$$

Simplifying and noting that $K_X = K_1 + K_2 + \dots + K_N = \frac{P}{2\sigma^2}$, we get

$$P_{E_{DPSK}}(N - \text{Rician}) = \frac{1}{2 \left[1 + 2\sigma^2 \frac{E_b}{N_0} \right]^N} \exp \left[\frac{-2K_X \sigma^2 \frac{E_b}{N_0}}{1 + 2\sigma^2 \frac{E_b}{N_0}} \right]$$

C. Evaluating expression in equation (6.17)

$$\text{In } \int_0^{\pi-\frac{\pi}{M}} \left(\frac{a + \cos \theta}{b + \cos \theta} \right)^{n+1} d\theta,$$

$$\begin{aligned} \left[\frac{a + \cos \theta}{b + \cos \theta} \right]^{n+1} &= \left[1 + \frac{a-b}{b + \cos \theta} \right]^{n+1} \\ &= 1 + \binom{n+1}{1} (a-b) \left(\frac{1}{b + \cos \theta} \right) \\ &\quad + \binom{n+1}{2} (a-b)^2 \left(\frac{1}{b + \cos \theta} \right)^2 + \cdots + (a-b)^{n+1} \left(\frac{1}{b + \cos \theta} \right)^{n+1} \end{aligned}$$

Formula 2.554 [pg.180, 46] can then be used to evaluate $\int_0^{\pi-\frac{\pi}{M}} \left(\frac{1}{b + \cos \theta} \right)^{n+1} d\theta$ for

different values of n .

D. Dual switch-and-stay diversity systems in TWDP channels

1. Introduction

In this appendix, we shall consider the bit-error rate (BER) performance of switch-and-stay diversity systems in two-wave with diffuse power (TWDP) fading channels. Switch-and-stay diversity is one of the simplest techniques for overcoming low BER performance of communication systems operating in fading channels. It is simpler to implement than equal gain combining and maximal ratio combining (both require cophased branch signals), and selection combining (which requires simultaneous monitoring of instantaneous signal-to-noise ratios of two or more channels). The receiver is especially low cost when noncoherent frequency shift keying (NCFSK) is chosen as the modulation scheme. Performances for this type of receiver in Rician channels and Nakagami- m channels were reported by [1] and [2], respectively. Unified analysis of switched-and-stay diversity systems by employing the moment generating function approach was reported in [3]. However, to our best knowledge, the performance of switch-and-stay diversity NCFSK systems in TWDP channels has not been reported by any researchers.

For simplicity, we shall consider a dual diversity system in which two antennas are employed. At any time, only one of the antennas is connected to the input of a NCFSK receiver, where the instantaneous power of the received signal is monitored. At a sampling instant, if the instantaneous received power of the currently connected antenna (i.e. the first branch) falls below a predetermined threshold, the input of the receiver will be switched to the other antenna (i.e. the second branch) via an antenna switch. If the instantaneous received power of the second branch is above the threshold, switching of antennas ceases. However, if the instantaneous received power of the second branch is below the threshold, the connection is immediately reverted to the first branch and if

necessary, the switching is continued rapidly between the antennas until one of them has received power above the threshold. After the input of the receiver has established its connection to an antenna whose received power is above the threshold, the connection remains in this branch at least until the next sampling instant regardless of its instantaneous received power is above or below the threshold. Note that in this switch-and-stay strategy, only a single receiver front-end and intermediate-frequency electronics are used, thus avoiding the duplication of expensive high-frequency analog circuits required in other diversity systems.

In the following analysis, for brevity of presentation, we shall assume that the fading channels are independent and have identical fading parameters. In Section 2, we derive the pdf of decision variable in switch-and-stay diversity NCFSK systems and employ it to obtain a general expression for the BER in terms of the switching threshold. In Section 3, expressions for optional switching threshold and minimum probability of error are derived specifically for identical and independent TWDP channels. In Section 4, the expressions obtained in Section 3 are employed for plotting graphical results, which allow comparison to be made with NCFSK systems with no diversity.

2. The pdf of decision variable in switch-and-stay diversity NCFSK systems

For switch-and-stay diversity systems, the cumulative distribution function of the instantaneous power at the output of the combiner can be written as (see Equation (4) of [2])

$$\begin{aligned} F_{Z_n}(u) &= P_r\{Z_n \leq u\} \\ &= P_r\{Z_n = X_n \text{ and } X_n \leq u\} + P_r\{Z_n = Y_n \text{ and } Y_n \leq u\} \end{aligned} \quad (1)$$

where X_n and Y_n denote, respectively, the instantaneous power of the signal received by antenna 1 and antenna 2 at $t = nT$, and Z_n is the instantaneous signal power at the output

of the switch-and-stay combiner at $t = nT$. Let the sample period T be sufficiently large such that the fading process introduces small or no correlation in the samples in each branch, then X_{n-1} and X_n are independent and Y_{n-1} and Y_n are independent. Under this condition, we can write, with ξ denoting the switching threshold (for which the optimal value is to be derived in Section 3), the probabilities on the right hand side of Equation (1) as

$$P_r\{Z_n = X_n \text{ and } X_n \leq u\} = P_r\{\xi \leq X_n \leq u\}P_r\{Z_{n-1} = X_{n-1}\} \\ + P_r\{Y_n < \xi \text{ and } X_n \leq u\}P_r\{Z_{n-1} = Y_{n-1}\}$$

and

$$P_r\{Z_n = Y_n \text{ and } Y_n \leq u\} = P_r\{\xi \leq Y_n \leq u\}P_r\{Z_{n-1} = Y_{n-1}\} \\ + P_r\{X_n < \xi \text{ and } Y_n \leq u\}P_r\{Z_{n-1} = X_{n-1}\}$$

Therefore Equation (1) can be rewritten as

$$F_{Z_n}(u) = [P_r\{\xi \leq X_n \leq u\} + P_r\{X_n < \xi \text{ and } Y_n \leq u\}]P_r\{Z_{n-1} = X_{n-1}\} \\ + [P_r\{\xi \leq Y_n \leq u\} + P_r\{Y_n < \xi \text{ and } X_n \leq u\}]P_r\{Z_{n-1} = Y_{n-1}\} \quad (2)$$

To avoid cluttering of notations, we shall omit the time index in the following. Since

$$P_r\{\xi \leq X \leq u\} = \int_0^u f_X(x) dx - \int_0^\xi f_X(x) dx \quad \text{for } u \geq \xi \\ = 0 \quad \text{for } u < \xi$$

$$P_r\{X < \xi \text{ and } Y \leq u\} = \int_0^\xi \int_0^u f_{X,Y}(x, y) dy dx$$

and

$$P_r\{\xi \leq Y \leq u\} = \int_0^u f_Y(y) dy - \int_0^\xi f_Y(y) dy \quad \text{for } u \geq \xi \\ = 0 \quad \text{for } u < \xi$$

$$P_r\{Y < \xi \text{ and } X \leq u\} = \int_0^\xi \int_0^u f_{X,Y}(x, y) dx dy$$

where $f_X(\cdot)$, $f_Y(\cdot)$ and $f_{X,Y}(\cdot, \cdot)$ denote, respectively, the marginal probability densities and the joint probability density of the instantaneous signal powers received by the antennas. After substituting these expressions into Equation (2) and differentiating with respect to u , we can express the probability density function of the instantaneous signal power at the output of the combiner as

$$f_Z(u) = \left[f_X(u) V(u - \xi) + \int_0^\xi f_{X,Y}(x, u) dx \right] P_r\{Z = X\} + \left[f_Y(u) V(u - \xi) + \int_0^\xi f_{X,Y}(u, y) dy \right] P_r\{Z = Y\} \quad (3)$$

where $V(\cdot)$ is the unit step function

$$V(\omega) = 1 \quad \text{for } \omega \geq 0 \\ = 0 \quad \text{for } \omega < 0$$

Recall that we have assumed that the channels have identical fading parameters (and hence identical fading statistics). By symmetry, we can conclude that

$$P_r\{Z = X\} = P_r\{Z = Y\} = \frac{1}{2}$$

Therefore, Equation (3) becomes

$$f_Z(u) = \frac{1}{2} \left[f_X(u) V(u - \xi) + \int_0^\xi f_{X,Y}(x, u) dx \right] + \frac{1}{2} \left[f_Y(u) V(u - \xi) + \int_0^\xi f_{X,Y}(u, y) dy \right]. \quad (4)$$

Now invoking the fact that the two channels are independent, that is,

$$f_{X,Y}(x, u) = f_X(x) f_Y(u)$$

and

$$f_{X,Y}(u, y) = f_X(u) f_Y(y)$$

so that we can write

$$\int_0^\xi f_{X,Y}(x, u) dx = F_X(\xi) f_Y(y)$$

and

$$\int_0^\xi f_{X,Y}(u, y) dy = f_X(u)F_Y(\xi)$$

$$\text{where } F_X(\xi) = \int_0^\xi f_X(x) dx \quad \text{and} \quad F_Y(\xi) = \int_0^\xi f_Y(y) dy$$

Since the channels are identical, $f_X(u) = f_Y(u)$ and $F_X(\xi) = F_Y(\xi)$, therefore Equation (4) becomes

$$f_Z(u) = f_X(u)[V(u - \xi) + F_X(\xi)] \quad (5)$$

It is well known that the probability of error for noncoherent detection of binary frequency shift keying signals in additive white Gaussian noise (see [4]) is

$$P_e(\gamma) = \frac{1}{2} \exp\left(-\frac{\gamma}{2N_0}\right)$$

where γ is the signal power and N_0 is the noise power spectral density at the sampling instant for detection. For the case of switch-and-stay diversity NCFSK systems, $\gamma = Z$ and the average BER subjected to fading conditions is

$$P_e = \frac{1}{2} \int_0^\infty \exp\left(-\frac{u}{2}\right) f_Z(u) du \quad (6)$$

where, without loss of generality, $N_0 = 1$ is assumed. After substituting Equation (5) into Equation (6), we can write

$$P_e = \frac{1}{2} \left[\int_\xi^\infty \exp\left(-\frac{u}{2}\right) f_X(u) du + F_X(\xi) \int_0^\infty \exp\left(-\frac{u}{2}\right) f_X(u) du \right] \quad (7)$$

Note that P_e is a function of ξ .

In the next section, we shall obtain an expression of P_e by evaluating the three integrals

$$F_X(\xi) = \int_0^\infty f_X(u) du \quad (8)$$

$$\int_0^\infty \exp\left(-\frac{u}{2}\right) f_X(u) du \quad (9)$$

and

$$\int_{\xi}^{\infty} \exp\left(-\frac{u}{2}\right) f_X(u) du \quad (10)$$

with $f_X(\cdot)$ specified as the TWDP pdf.

3. BER performance in TWDP channels

The TWDP pdf is given by [5] as

$$f_R(r) = \frac{r}{\sigma^2} \exp(-K) \exp\left(-\frac{r^2}{2\sigma^2}\right) \frac{1}{\pi} \int_0^\pi \exp(\Delta K \cos \theta) \cdot I_0\left[\frac{r}{\sigma} \sqrt{2K(1 - \Delta \cos \theta)}\right] d\theta$$

which is not in a “closed form” and thus does not lend itself to efficient numerical evaluations. However, a family of pdfs can be derived to approximate the TWDP pdf. The general approximate TWDP pdf is also given in [5]. It is

$$f_R(r) = \frac{r}{\sigma^2} \exp(-K) \exp\left(-\frac{r^2}{2\sigma^2}\right) \sum_{i=1}^L a_i D\left(\frac{r}{\sigma}; K, \alpha_i\right) \quad (11)$$

where

$$D\left(\frac{r}{\sigma}; K, \alpha_i\right) = \frac{1}{2} \left\{ \exp(\alpha_i K) I_0\left[\frac{r}{\sigma} \sqrt{2K(1 - \alpha_i)}\right] + \exp(-\alpha_i K) I_0\left[\frac{r}{\sigma} \sqrt{2K(1 + \alpha_i)}\right] \right\},$$

$$\alpha_i = \Delta \cos\left[\frac{(i-1)\pi}{2L-1}\right],$$

$$K = \frac{V_1^2 + V_2^2}{2\sigma^2}, \quad (K \text{ is known as the TWDP } K \text{ factor})$$

$$\Delta = \frac{2V_1 V_2}{V_1^2 + V_2^2}.$$

V_1 and V_2 are the amplitudes of the two specular components and $2\sigma^2$ is the variance of the diffuse component in the TWDP channel, $I_0(\cdot)$ is the zeroth-order modified Bessel

function of the first kind, and L is the order of approximation. The values of the a_i coefficients for $L = 1$ to 5 are given in Table 2 of [5].

By expanding Equation (11) out in full and after grouping related terms, we have

$$f_R(r) = \frac{1}{2} \sum_{i=1}^L a_i \left\{ \frac{r}{\sigma^2} \exp(-K_{i1}) \exp\left(-\frac{r^2}{2\sigma^2}\right) I_0\left(\frac{r}{\sigma} \sqrt{2K_{i1}}\right) + \frac{r}{\sigma^2} \exp(-K_{i2}) \exp\left(-\frac{r^2}{2\sigma^2}\right) I_0\left(\frac{r}{\sigma} \sqrt{2K_{i2}}\right) \right\} \quad (12)$$

That is, the approximate TWDP pdf is the weighted sum of $2L$ “effective” Rician pdfs with the $2L$ “effective” Rician K factors defined by

$$K_{i1} = K(1 - \alpha_i)$$

and

$$K_{i2} = K(1 + \alpha_i)$$

for $i = 1, 2, \dots, L$. In [5], $L \geq \frac{1}{2} K\Delta$ was recommended as the order to be used for achieving good approximation.

Now we can substitute the expression of $f_R(\cdot)$ in Equation (12) for $f_X(\cdot)$ in Equation (8) to evaluate $F_X(\xi)$. By using the following properties of the Marcum Q-function (see [4]),

$$Q(\sqrt{2a}, \sqrt{2b}) = \int_b^\infty \exp[-(y+a)] I_0(2\sqrt{ay}) dy$$

and

$$Q(\sqrt{2a}, 0) = 1$$

and after some tedious and lengthy calculations, we obtain

$$F_X(\xi) = \frac{1}{2} \sum_{i=1}^L a_i \left[2 - Q\left(\sqrt{2K_{i1}}, \sqrt{\frac{\xi}{\sigma^2}}\right) - Q\left(\sqrt{2K_{i2}}, \sqrt{\frac{\xi}{\sigma^2}}\right) \right] \quad (13)$$

By using similar substitution and techniques, we can also obtain

$$\begin{aligned}
 & \int_{\xi}^{\infty} \exp\left(-\frac{u}{2}\right) f_X(u) du \\
 &= \frac{1}{2} \sum_{i=1}^L a_i \left(\frac{1}{1+\sigma^2} \right) \left[\exp\left(-\frac{\sigma^2 K_{i1}}{1+\sigma^2}\right) \mathcal{Q}\left(\sqrt{\frac{2K_{i1}}{1+\sigma^2}}, \sqrt{\frac{(1+\sigma^2)\xi}{\sigma^2}}\right) \right. \\
 & \quad \left. + \exp\left(-\frac{\sigma^2 K_{i2}}{1+\sigma^2}\right) \mathcal{Q}\left(\sqrt{\frac{2K_{i2}}{1+\sigma^2}}, \sqrt{\frac{(1+\sigma^2)\xi}{\sigma^2}}\right) \right]
 \end{aligned} \tag{14}$$

and

$$\begin{aligned}
 & \int_0^{\infty} \exp\left(-\frac{u}{2}\right) f_X(u) du \\
 &= \frac{1}{2} \sum_{i=1}^L a_i \left(\frac{1}{1+\sigma^2} \right) \left[\exp\left(-\frac{\sigma^2 K_{i1}}{1+\sigma^2}\right) + \exp\left(-\frac{\sigma^2 K_{i2}}{1+\sigma^2}\right) \right]
 \end{aligned} \tag{15}$$

To find the optimal switching threshold ξ^* such that P_e achieves a minimum, we substitute Equations (13), (14) and (15) into Equation (7). After differentiating the resulting expression of P_e with respect to ξ and setting

$$\frac{dP_e}{d\xi} = 0$$

we found the optimal switching threshold

$$\xi^* = -2 \log_e \left\{ \frac{1}{2(1+\sigma^2)} \sum_{i=1}^L a_i \left[\exp\left(-\frac{\sigma^2 K_{i1}}{1+\sigma^2}\right) + \exp\left(-\frac{\sigma^2 K_{i2}}{1+\sigma^2}\right) \right] \right\}$$

and the minimum BER

$$\begin{aligned}
 P_e^* = & \frac{1}{4} \left(\frac{1}{1+\sigma^2} \right) \left\{ \sum_{i=1}^L a_i \left[\exp \left(-\frac{\sigma^2 K_{i1}}{1+\sigma^2} \right) Q \left(\sqrt{\frac{2K_{i1}}{1+\sigma^2}}, \sqrt{\frac{(1+\sigma^2)\xi^*}{\sigma^2}} \right) \right. \right. \\
 & \left. \left. + \exp \left(-\frac{\sigma^2 K_{i2}}{1+\sigma^2} \right) Q \left(\sqrt{\frac{2K_{i2}}{1+\sigma^2}}, \sqrt{\frac{(1+\sigma^2)\xi^*}{\sigma^2}} \right) \right] \right\} \\
 & + \frac{1}{8} \left(\frac{1}{1+\sigma^2} \right) \left\{ \sum_{i=1}^L a_i \left[\exp \left(-\frac{\sigma^2 K_{i1}}{1+\sigma^2} \right) + \exp \left(-\frac{\sigma^2 K_{i2}}{1+\sigma^2} \right) \right] \right\} \\
 & \cdot \left\{ \sum_{i=1}^L a_i \left[2 - Q \left(\sqrt{2K_{i1}}, \sqrt{\frac{\xi^*}{\sigma^2}} \right) - Q \left(\sqrt{2K_{i2}}, \sqrt{\frac{\xi^*}{\sigma^2}} \right) \right] \right\}.
 \end{aligned}$$

In the next section, we will plot and discuss the optimal BER performance and optimal switching threshold against the average SNR, which is defined as $SNR = 10 \log_{10} [2\sigma^2(1+K)]$ dB.

4. Results and discussions

By using the expressions of the optimal switching threshold ξ^* and the optimal BER P_e^* obtained in Section 3, we can plot and study the performance characteristics of switch-and-stay diversity NCFSK systems operating in TWDP fading channels.

We shall first make use of the fact that when $\Delta = 0$, TWDP fading becomes Rician fading. This allows us to plot the curves for this special case and compare them with known results to verify the correctness of our equations. In Figure 1, the curves of P_e^* against average SNR for $K = 1$ (or 0 dB) and $K = 10$ (or 10 dB) when $\Delta = 0$ are plotted. The corresponding curves of ξ^* against average SNR are plotted in Figure 2. Direct comparison show that our results are identical to those given in Figure 1 and Figure 2 of [1]. Note that in Figure 1 and Figure 2, the curves for Rayleigh fading channels are also plotted. This is obtained by setting $K = 0$ in TWDP fading.

Next we shall plot the curves of P_e^* and ξ^* for the limiting case where $\Delta = 1$. This corresponds to the physical situation where the specular components in the TWDP channels can cancel one another such that approximately, only the diffuse power component remained (see [5] for details). The curves shown in Figure 3 and Figure 4 clearly illustrated this fact. However, it is important to note from Figure 3 that even in this extreme situation, dual switch-and-stay diversity NCFSK systems are able to perform much better than NCFSK systems with no diversity when the average SNR is sufficiently large.

For completeness, we also plotted the curves of P_e^* and ξ^* for the situation where $\Delta = 0.8$. It can be seen from Figure 5 that dual switch-and-stay diversity NCFSK systems significantly out-performed NCFSK systems with no diversity over a wide range of average SNR for both $K = 1$ and $K = 10$, although in Figure 6, their optimal switching threshold curves are somewhat closer, as opposed to the situation where $\Delta = 0$ in which the ξ^* curves for $K = 1$ and $K = 10$ are well separated.

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Figure D.1 Optimum BER Performance of Switch Diversity NCFSK System (Rician Fading)

Figure D.2 Optimum Switching Threshold of Switch Diversity NCFSK System (Rician Fading)

Figure D.3 Optimum BER Performance Of Switch Diversity NCFSK System (TWDP Fading, $\Delta = 1$, $K = 1$ or 10)

Figure D.4 Optimum Switching Threshold of Switch Diversity NCFSK System (TWDP Fading, $\Delta = 1$, $K = 1$ or 10)

Figure D.5 Optimum BER Performance Of Switch Diversity NCFSK System (TWDP Fading, $\Delta = 0.8$, $K = 1$ or 10)

Figure D.6 Optimum Switching Threshold of Switch Diversity NCFSK System (TWDP Fading, $\Delta = 0.8$, $K = 1$ or 10)