Investigation of micro-optofluidic components and their applications

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**INVESTIGATION OF MICRO-OPTOFLUIDIC
COMPONENTS AND THEIR APPLICATIONS**

SONG CHAOLONG

**SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING**

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INVESTIGATION OF MICRO-OPTOFLUIDIC COMPONENTS AND THEIR APPLICATIONS

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Abstract

Investigation of Micro-optofluidic Components and Their Applications

by

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The field of optofluidics is a combination of optics and microfluidics to generate new functionalities and to improve the integration level of optical components. Liquid replacing, liquid refractive index control and interface control are the three main methods to make reconfigurable optofluidic components. This thesis focuses on the manipulation of the interface between different flows through hydrodynamic spreading to develop tunable in-plane optofluidic components. A circular chamber is used to hydrodynamically develop the optofluidic lenses with liquid core liquid cladding (L^2) structure. The two-dimensional dipole flow theory and the hydrodynamic spreading theory are applied to describe single-fluid and two-fluid models respectively. The model described here provides a method to configure optofluidic lenses with a mathematically pre-defined curvature, whose interface is atomically smooth. Furthermore, the curvature of the interface as well as the focal length can be tuned by adjusting the flow rate ratio.

An optofluidic bi-concave lens, which can both focus and diverge the light, is first proposed and demonstrated. In the focusing mode of the optofluidic concave lens, a liquid with low refractive index is used as the core stream and a liquid with higher refractive index serves as the cladding stream. An auxiliary cladding liquid with a very low flow rate and a refractive index matching that of polydimethylsiloxane (PDMS) is introduced to prevent the light scattering. The light emitted from an optical fibre can be focused and the focal length can be tuned by adjusting the flow rate ratio between the core and the cladding streams. If the core inlet is blocked, the liquid from cladding inlets will converge into a new core stream and the liquid from the auxiliary cladding inlet will serve as cladding streams. Therefore, the device can seamlessly switch from the focusing mode to diverging mode. In the diverging mode, a liquid with high refractive index works as the core stream, and a liquid with refractive index matching PDMS is used as cladding streams. Owing to the higher refractive index of the core stream and the tuneable lens interface, the divergence of the light beam can be expanded and adjusted. Combining the performance of focusing and diverging, the device can greatly enhance the tunability of the focal length of an optofluidic lens.

Based on the L^2 structure, an optofluidic prism is developed to manipulate the in-plane light beam. This tunable micro optofluidic prism is hydrodynamically formed by one core and two cladding streams inside a sector-shape chamber. Liquid with a higher refractive index is employed as the core stream, which forms the geometry of a triangular prism. Liquid with a lower refractive index serves as cladding streams. Due to the higher refractive index of the core stream, this configuration acts as a prism. The apex angle of this optofluidic prism can be tuned by adjusting the flow rate ratio between

core and cladding streams, and therefore the deviation angle of an incident light beam can be changed accordingly. Since the propagation direction of refracted light beam can be accurately controlled by choosing a proper flow rate ratio, this tunable prism can continuously scan the light beam, and therefore can be engaged in the alignment of optical path, or in the development of optical switches.

Aperture stop or entrance pupil is an important optical component in imaging and detection systems. The component determines the amount of light passing through the optical system as well as the angular aperture on the object side. A tunable optofluidic aperture stop, which can be dynamically re-configured according to the flow condition, is proposed and demonstrated. A liquid core liquid cladding structure was used to form this aperture. The core liquid is optically transparent allowing light rays to pass through, while the ink with negligible optical transmittance ($\approx 0\%$) works as the cladding liquid blocking the propagation of light. When the aperture is tuned with a given flow rate ratio, the amount of light which reaches the image space as well as the angular aperture on the image side can be adjusted accordingly.

Due to the solid-based lens interfaces, previous optofluidic flow cytometer designs suffer from light scattering, which degrades the quality of the focused light beam. Divergence or scattering of the light beam results in a larger beam width. One obvious disadvantage resulting from the large beam width is that the cytometer would miscount if multiple particles enter the large illumination area because the multiple particles would give only one burst in the collected signal. Normally this burst owing to multiple particles has a larger peak value than that of one particle. Thus the other consequent problem is that this burst of the signal would be misleading as it would be wrongly interpreted as

a larger particle. A new design of a microfluidic flow cytometer with an optofluidic lens formed in a circular chamber is proposed and tested. The optofluidic lens has a mathematically predictable focal length, and is immune to light scattering. A well-focused light beam is achieved with a much smaller beam width of $23\text{ }\mu\text{m}$ compared to all previous flow cytometers. This flow cytometer presents a good performance on particle counting, and the signal intensity shows small aberrations and strong dependence on the particle size.

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List of Symbols

Romans

A Interface position

a Coordinate of source and sink position

B Interface position

C Streamline constant

d The shortest normal distance from the interface

$\vec{F}$ Force exerted on object

f Focal length of a lens

H Heaviside function

h Depth of channel

L Characteristic length

l Distance

m Mass of object

n	Refractive index of liquid
P	Press inside liquid
Q	Flow rate of fluid
R	Radius of a circular chamber
r	Radius of interface
S	Source and sink
t	Time
U	Characteristic velocity
u	Velocity along x -axis
u^*	Dimensionless velocity along x -axis
v	Velocity along y -axis
$\vec{v}$	Velocity vector
v^*	Dimensionless velocity along y -axis
W	Complex potential function for the flow field
w	Velocity along z -axis
w^*	Dimensionless velocity along z -axis
x	Dimension in x -axis
y	Dimension in y -axis

z Dimension in z -axis

Greeks

α Angle

β Viscosity ratio

Γ Interface between fluids

θ Apex angle of a prism

λ Wavelength of light

μ Viscosity of liquid

ρ Density of liquid

Φ Distance function

ϕ Flow rate ratio

φ Velocity potential function

ψ Stream function for the flow field

Ω Domain of numerical computation

Acronyms

AOD Acousto-optical deflector

CCD Charge-coupled device

CVD Chemical vapor deposition

DI	Deionized
DOE	Diffractive optical element
FWHM	Full width at half maximum
HOT	Holographic optical tweezers
IPA	Isopropyl alcohol
LOC	Lab-on-chip
LSM	Level-set method
MEMS	Micro-electro-mechanical system
MMF	Multi-mode fiber
NA	Numerical aperture
PDMS	Polydimethylsiloxane
PIV	Particle image velocimetry
PVD	Physical vapor deposition
RI	Refractive index
SMF	Single-mode fiber
TAS	Total analysis system
UV	Ultraviolet

Chapter 1

Introduction

1.1 Background of optofluidics

Optofluidics is referred to the science and technology that combines optics with microfluidics to create a functional system. The system can use optics to manipulate fluidics or utilize fluidics to realize optical functions. This field has summoned growing interests as a result of unique optical properties of fluids, fluid interfaces, technological advance in device fabrication, and the demand for highly integrated chips which have a variety of applications in biotechnology, chemical synthesis and analytical chemistry.

For centuries, the majority of optical devices are fabricated based on solids, such as glass lenses, mirrors, prisms, or semiconductor laser diode and etc. These solid optical devices have limited tunability and the optical interface greatly depends on the fabrication process. Until recently, insights into the unique properties of fluids or liquids have diverted the attention of research community from solid-based to liquid-based optics [1]. Replacing a liquid with another liquid inside an optical component can easily tune

its optical performance or function. Gradients in optical properties can be created by the control of diffusion of two miscible fluids. A smooth optical interface can be naturally formed between two immiscible liquids. These advantages of using liquids give the opportunity to optics to achieve a high level of tunability and performance.

The fabrication technologies in Micro-Electro-Mechanical System (MEMS) can, in principle, be adapted to fabricate optofluidic devices. Although most of the implementations until now have been with soft lithography, the expectation to integrate all optical components into one chip calls for the use of more complex technologies to realize complex three-dimensional structures. A good example is the work reported by Armani and Vahala [2], who employed wet etching to fabricate an ultra-high-Q microcavity to detect biomolecules. Considering the integration of a light source or lenses into a chip, it should be required to utilize different deposition methods such as Physical Vapor Deposition (PVD) or Chemical Vapor Deposition (CVD). Poly (dimethylsiloxane) (PDMS) has been preferred as the main material for channel fabrication. Many other materials, such as photocurable hydrogels, thermoset plastics and elastomers, are also suitable. However, highly integrated chips of the future will contain not only channel structures but also other optical components. Therefore the anticipated material should involve silicon, oxide, metals or even gas-phase substances.

Microfluidics has been an increasingly growing field with important applications in areas such as biotechnology, chemical synthesis and analytical chemistry as a result of the novel effects in microscale. On the one hand, behavior of fluids at the microscale can differ from macrofluidic behavior, because factors such as surface tension, energy dissipation, and fluidic resistance start to dominate the system. The physical properties

of fluids in microchannels that are available only on small spatial scales can be exploited for a variety of device functionalities, many of which can also be used to control optical properties. On the other hand, optics provides mechanically non-contact means to transfer energy or momentum. The modulated light can serve as a signal. Highly focused light can be used as scalpel or the well-known optical tweezers. Imaging small features or sensing variables in microscale can be achieved utilizing certain parameters of light, such as intensity, phase, or spectrum response. Highlighted by the unique effects through the interaction between optics and fluidics, their integration gives a promising future for the functionality of a micro-device.

From a functional point of view, methods to develop optofluidic devices can be categorized into two types: employing fluids to enable and tune the optical properties and using optics to manipulate and detect the fluids. The subsequent literature review will follow this structure.

1.2 Literature review

1.2.1 Configuration and manipulation of optics by tuning fluidic properties

Optical components, such as waveguide and lens, play indispensable roles in biological and chemical analysis. These components are needed for both manipulation and sensing such as optical trapping of micro-particle or bio-cells [3, 4, 5] and single molecule detection [6, 7]. Previously, optical systems for these functionalities were built with bulky external devices consisting of solid optical components. The fabrication process inherently limit the tunability and quality of these systems. To date, the miniaturization

and integration of optical components into a lab-on-a-chip (LOC) system is one of the big challenges and summon great interests from research community. A lot of efforts have been devoted to exploit novel materials for optical components and reducing their size. Using micromachining technologies and soft-lithography, liquid-filled lens arrays or polydimethylsiloxane (PDMS) microlens [8, 9, 10, 11, 12] and liquid-core waveguide [13, 14, 15] were realized with immobile optical properties. Wenger *et al.* [7] managed to enhance the numerical aperture (NA) of a lens system using a latex micro sphere. However, these optical components with fixed properties are short of flexibility and tunability. Moreover, the optical interface is formed by a solid material, whose smoothness is limited and greatly dependent on the fabrication process.

The concept of optofluidics aims to improve the quality and thus enhance the performance of the optical components. One of the motivations to investigate micro-optofluidic components is to downsize the whole system and make it cost-efficient. Instead of the bulky external optical system, the optofluidic systems can be compact and portable for field testing. Secondly, the liquids/fluids boast the instinct of deformability under shear stress to make the optofluidic components dynamically tunable. Besides the tunability, recent research works have shown that the optical properties of optofluidic components can be mathematically pre-defined. The combination of optics and microfluidics leads to a cost-efficiently miniaturized, flexibly tunable and analytically predictable platform for chemical and biological analysis. Therefore, building an optical system fully configured by liquids/fluids with multiple functionalities can be highly expected.

Liquid replacing

The function and performance of the traditional optical components are permanently fixed after the fabrication due to the immobility of the solid-based design. Recently, the infiltration or injection of liquid in some hollow structures has brought unique light-liquid interactions and excellent tunability of the optical components. Examples are the infiltrated photonic crystal devices, such as photonic crystal fibers or planar photonic crystals (Fig. 1.1). Thermal actuation [16], manual use of micropipette [17], soft lithography nanofluidic technique [18] have been utilized to fill liquid into photonic crystal structures. In porous photonic crystal devices, the selectively introduced liquids can significantly tune the transmission behavior of light, such as emission wavelength [19], propagation direction [20] and speed [21] or spectrum response [22, 23]. The enhanced interaction between liquid and light in the photonic crystal devices has brought many applications, such as chemical sensor [21, 24, 25], dye laser [19], and optical switch [16, 23].

Another example taking advantage of replacing liquid inside an optical component is the optical switch based on the phenomenon of total internal reflection [26]. A mirror channel was filled with water (low refractive index) or salt solution (high refractive index) to determine whether total internal reflection happens or not, and therefore whether to allow the light beam to transmit through the mirror channel. Optical path of interference can also be tuned by changing the refractive index (RI) of the medium where interference happens. Groisman *et al.* reported another optical switch using blazed diffractive grating [27]. In Figure 1.2, the fluidic mediums above the grating structure can be replaced by liquids with different RI, and therefore the interference optical path can

Figure 1.1: Schematics of optical switch using optofluidic photonic crystal fiber [16].

be changed accordingly to diffract the transmitted light beam with different propagation direction.

Waveguide is one of the key elements in optical systems, equivalent to the channel or tube on the fluidic side [28]. The integration and functionality of waveguide in microfluidic systems are of great interest in the research community. A tunable and reconfigurable waveguide is critical in chemical analysis and bio-sensing assays [29]. A simple method to realize a tunable waveguide is to make a hollow channel filled with higher RI liquid embraced by lower-RI solid material [13, 14]. The optical characteristics can be easily adjusted by replacing the liquid inside the channel.

Llobera *et al.* reported a hollow prism built in a microfluidic system for optical sensing [30, 31]. The sensing performance and efficiency were investigated using solutions of different concentrations of fluorescein. The devices were found to have good sensitivity

Figure 1.2: An optofluidic switch based on blazed diffraction grating [27].

in the absorption and shift of refractive index of the solutions.

Liquid refractive index control

Instead of replacing the liquid inside an optical component, tuning the refractive index of liquid by subjecting it to a flow field, temperature field, electrical field, acoustic field or mechanical strain [32], are alternative methods to manipulate the optical properties of light propagating in the optical material such as the optical path, phase change and polarization [33]. The generation of refractive index gradient is the most common method to manipulate light properties. The concept of refractive index gradient was already widely used in traditional fiber optics [34, 35, 36, 37]. Light rays propagate to follow sinusoidal paths in the gradient-index fiber, which has an advantage of decreasing the modal dispersion over the multi-mode step-index fiber. Gradient-index also lends its

Figure 1.3: Comparison between step-index and gradient-index fibers in transmission of signals.

strength to the lens optics [38, 39, 40, 41]. A proper distribution of index gradient along the perpendicular direction to the optical axis in the lens can help to eliminate the aberrations caused by traditional spherical lenses without requiring variation to the shape of the lens. The problems of fabrication of the previous solid-based gradient-index optics is the limited tunability. The distribution of the gradient-index is fixed after fabrication. Moreover, the fabrication process to establish such an index gradient is quite complex, requiring field-assisted ion-exchange, micro-controlled dip-coating or vapor deposition [42].

The recent optofluidic technologies exploit the unique microfluidic properties for applications in optics [1, 43]. The distribution of index gradient can be controlled through the variation in concentration of a solution, which could be well defined by diffusion of microfluidic laminar flows. Based on this motivation, two-dimensional (in-plane) [42]

Figure 1.4: In-plane optofluidic gradient refractive index lenses [42].

and three-dimensional (out-of-plane) [44] optofluidic gradient refractive index lenses were proposed and demonstrated. The laminar flow in microscale gives the relative stability of optical performance to these devices, and the most prominent strength of using fluids to configure the index-gradient lens is the flexibility of tuning the optical properties. For example, the focal length can be controlled by the diffusion process, which can be well defined by setting a proper flow rate. More applications of optical devices using diffusion are the optical attenuator [45] and switch or splitter [46].

Another mechanism to manipulate the refractive index gradient is placing the liquid into a temperature gradient field, which generates refractive index contrast along the cross-section perpendicular to the optical axis [47]. In this work, liquid core liquid cladding configuration is employed to build an optofluidic waveguide. Two cladding streams at higher temperature sandwich a core stream at lower temperature, which creates a temperature gradient field. The refractive index and many other physical prop-

erties are temperature dependent. Therefore the temperature gradient field generates a distribution of refractive index with higher value at the center of the channel. This refractive index contrast enable a waveguiding function of liquid core liquid cladding flow. However, the thermo-optical coefficient dn/dT of most liquids is relatively small. So a large temperature contrast is required to enable an effective waveguiding function. Another drawback is the decay of temperature contrast along the flow direction due to dissipation of heat, which leads to un-evenly distributed refractive index along the optical axis. This phenomenon causes a lot of optical energy loss during the propagation of light.

Sandwiching liquid crystal between two transparent electrodes and applying electric field is another method to generate refractive index gradient. A popular implementation to build a spherical lens is the combination a passive solid-state lens and a liquid crystal modulator [48, 49]. In spite of the merits of this method, such as simple fabrication and single electrode, one of the problems is the roughness of the inner surface, which leads to severe light scattering. And the other problem is the limited tunable range of focal length [40]. To improve the light beam quality and tuning range, a spherical glass shell mounted on the liquid crystal cell was used [40, 50] (Figure 1.5). A centro-symmetric gradient of refractive index can be generated within the liquid crystal cell upon the applied voltage, and the filling material inside the glass shell can greatly help to enhance the tunable range of the focal length.

Liquid interface control

Recently, a lot of optofluidic components based on liquid interface have been proposed and demonstrated, and most of the applications based on liquid interface control

Figure 1.5: Configuration of a tunable spherical lens combining glass shell and liquid crystal cell.

are the developments of optical lenses. These liquid lenses possess atomically smooth interfaces and make use of interfacial tension to maintain the curvature [51, 52, 53]. The characteristics of laminar flow in microscale was used to dynamically develop the shape of the fluidic lens [54, 55]. According to the light propagation direction relative to the plane of the substrate, which normally carries the test sample, micro lenses can be divided into two categories: in-plane and out-of-plane types. In-plane lenses modify light properties in the plane of the substrate. Out-of-plane lenses manipulate the incident light perpendicular to the plane of the substrate [54].

One advantage of the out-of-plane lens is the compatibility to conventional solid-state optical system and thus the feasibility to replace the lens in applications such as digital camera or cell phones. A popular method to develop an out-of-plane optofluidic lens is to fill working fluid into a circular chamber sealed with a deformable membrane. One of the actuation method to deform the membrane is using controllable pressure., which is referred to as pneumatic tuning [33]. Inspired by a human eye's crystalline lens, Ahn *et al.* proposed to use a deformable glass diaphragm with a thickness of $40\ \mu\text{m}$ to enable a variable focal length of a single piece of lens [56]. Silicone oil with a refractive index of 1.65 was employed as the working fluid to match that of the glass diaphragm. An external pump was used to actuate the deformation of the glass diaphragm, with

Figure 1.6: Pneumatically tunable out-of-plane fluidic micro-lenses.

which the curvature of the interface can be adjusted to achieve a tunable focal length. But the problem of this configuration is the high Young's modulus of the glass diaphragm which limits the tuning range. Only up to $50\text{ }\mu\text{m}$ of deformation for a 10 mm lens can be achieved. To solve this problem, Zhang *et al.* proposed and demonstrated using PDMS membrane instead of glass to develop the lens interface (Figure 1.6 (a)) [57, 58]. Up to $6\text{ }\mu\text{m}$ deformation for $200\text{ }\mu\text{m}$ lens can be achieved in the work of Chronis *et al.*. Werber *et al.* also reported a microfluidic lens developed with dry-etched silicon substrate bonding to a piece of PDMS membrane [59]. The spin-coated PDMS can have a smoothness up to 8 nm . Chen *et al.* propose to use a micro-lens film to serve as the membrane to obtain a shorter focal length [60]. These designs were based on a plano-convex structure, and the focal length is still quite large. Jeong *et al.* improved the design by combining a negative solid elastomer interface with a membrane to build a micro-doublet lens [8]. The doublet lens can either be tuned as bi-convex or meniscus (Figure 1.6 (b)). Zhang *et al.* further replaced the solid elastomer interface with another deformable membrane so as to tune the two lens interface independently (Figure 1.6 (c)) [61].

Instead of using pneumatic tuning method to control the membrane, other concepts, such as piezoelectric, magnetic and electrochemical actuation can be used to modulate the pressure in the liquid lens chamber. Embedded microcoil [62] or electroactive polymer (EAP) [63] can be attached to the PDMS membrane to control its deformation so as to increase or decrease the pressure in the chamber (Figure 1.7 (a)). The pressure change will drive the lens membrane to deflect so as to vary the focal length. However the problem of using PDMS membrane is non-uniformity of the membrane thickness, which depends on the fabrication process. A number of configurations have been pro-

posed to take advantage of the liquid/air or liquid/liquid interface maintained by surface tension to develop lenses with variable focal length. Hirs'a's group demonstrated the use of internal electrodes [64] or external acoustic force [52] to tune the liquid/air interface of a droplet (Figure 1.7 (b)). Compared to using membrane, the advantages of utilizing liquid/air interface are the optically smooth surface and no need of matching the refractive index between lens liquid and the membrane. Recently a ferrofluid droplet has been used as a piston to create pressure difference between the top and the bottom chamber to modulate the interface of a surrounding transparent droplet (Figure 1.7 (c)) [65]. This system is free of moving solid parts, and suitable for long-term operation without wear. Other tuning methods to manipulate the liquid/air interface include using hydrogels to create pressure difference [66], electrowetting to generate electrostatic force [67] as well as electric gradient field to obtain a dielectrophoretic force [68].

The advantage of in-plane lenses over out-of-plane lenses is the possibility of integrating them into microfluidic networks which perform the sample analysis [69]. A high level of integration furthermore helps to avoid the manual alignment of optical components as well as to reduce the cost of external bulky components. Different configurations of in-plane liquid/liquid or liquid/air interface optical components have been reported recently. They can be categorized into two groups: (a) interfacial tension based tuning and (b) hydrodynamic tuning. The balance between the applied pressure and the interfacial tension can be used to tune the curvature of the liquid lens interface. By filling liquid PDMS, Lien *et al.* developed spherical micro-lens with pre-defined focal length inside microfluidic networks [70]. The interface curvature was controlled by surface interaction between the liquid and the channel. Silane was used to treat the hydrophobic PDMS

Figure 1.7: Electro-magnetic actuation of liquid lens: (a) external electro-magnetic force actuating membrane; (b) internal electro-chemical force deforming droplet; (c) external electro-magnetic force actuating droplet.

channel and make it more hydrophilic for forming the lens. The different durations of silane treatment can lead to different curvatures of the PDMS lens interface to obtain a programmable focal length. The problems of this fabrication method are the difficulty to pump PDMS, a highly viscous liquid, into a micro-channel within a short period of time. To solve this problem, several configuration have been proposed. Dong and Jiang manually pumped a liquid droplet into a microchannel and utilized the pressure difference at the two interfaces to manipulate the interface curvature (Figure 1.8 (a)) [71]. The

drawback of this work is the rough air-solid interface scatters the light beam, leading to a poor beam quality. Shi *et al.* generated a smooth lens interface between a flow and air by capillary force (Figure 1.8 (b)) [69]. Manipulating the flow rate can obtain different curvatures of the interface. A similar design was proposed to use a flared microfluidic structure to manipulate the liquid meniscus [72]. CaCl_2 solution was injected into the microfluidic channel as working fluid. And when fluid advances through the flange, the contact angle θ remains constant while the curvature of the interface decreases (Figure 1.8 (c)).

Hydrodynamic tuning is another method to manipulate the liquid/liquid interface. One excellent example is the liquid core liquid cladding (L^2) waveguide developed based on hydrodynamic spreading (Figure 1.9) [73]. By adjusting the flow rate ratio, the optical performance of this waveguide can be dynamically tuned. Mao *et al.* first proposed a hydrodynamically tunable optofluidic lens fabricated with standard soft lithography (Figure 1.10) [74]. This lens consists of two working fluids: water and CaCl_2 solution. The two-fluid flow was driven through a 90-degree curved micro-channel, and the CaCl_2 solution bows outward into water owing to the centrifugal force induced in the curve. By adjusting the flow rate ratio between these two fluids, different curvatures of the interface as well as different focal lengths can be obtained. Tang *et al.* developed an optofluidic lens with liquid core liquid cladding structure in a rectangular expansion chamber (Figure 1.11) [54]. A fluid with higher refractive index works as the core stream and one with lower refractive index serves as cladding stream. Manipulating the flow rate ratio between core and cladding streams can tune the curvature of the interface formed in the expansion chamber accordingly. A similar design of the same concept was

Figure 1.8: (a) Liquid lens tuned by controlling pressure different; (b) Liquid lens tuned by controlling flow rate; (c) tunable micro-lens based on liquid meniscus.

Figure 1.9: Liquid-core liquid cladding waveguide configuration [73].

proposed later by Seow *et al.* [75]. Combining the 90-degree curved micro-channel and a L^2 -based structure, Rosenauer *et al.* [55] constructed an optofluidic lens which can perform three-dimensional (3-D) focusing of light. Thanks to the simple tuning mechanism, adjusting the flow rate ratio between fluids, and optically smooth interface, the method of hydrodynamical tuning has generated a large number of functions of in-plane optofluidic components, such as optical attenuator [45], optical switch [76], flow cytometer [77], evanescent field based sensor [78] and etc. during the past decade.

Figure 1.10: Hydrodynamically tunable optofluidic cylindrical lens [74].

Figure 1.11: Liquid-core liquid cladding bi-convex lens configuration [54].

1.2.2 Manipulation and detection in microfluidics using optical methods

Manipulation and sorting of micro-particles

Microscale particles and droplets are the main objects of interest in microfluidics; thus their manipulation is of great interests. The manipulation involves trapping, sorting, separation and etc., which extend their applications in the fields of chemistry, biology and colloidal science. Thanks to the high resolution, stable control, notable flexibility and the non-invasive manner, optical tweezers have attracted worldwide attention in the past 20 years since it was first proposed by Ashkin and his colleagues [79, 80]. In short, the forces at work may be broadly described as the "radiation pressure" forces driving particles in the direction of light propagation. Another component of force is similar to that exploited by dielectrophoresis, driving particles of higher refractive index than the surrounding medium to the region of maximum optical intensity.

The technique of optical tweezers has been improved and developed in the past 20 years. Of great interest is the development of multiple tweezers when one wants to control and manipulate multiple particles simultaneously. Several methods have been developed to achieve this goal. A simple way is to guide several beams on the objective lens at different angles, which can lead to multiple traps after the beams are tightly focused [81]. However, this method becomes cumbersome when there are more than a few traps or when one is interested in arbitrary trap pattern. An improved method for manipulating multiple particles uses holographic optical tweezers (HOT) [82, 83, 84]. In this method, a single beam is passing through a diffractive optical element (DOE), which can modulate the phase and the magnitude of the optical wave. The diffractive waves

will interfere and form trap arrays in the sample. Multiple tweezers can also be achieved by creating interference between several coherent laser beams [81, 85].

An alternative method to achieve multiple tweezers is to use mirrors to rapidly scan the laser beam among a number of positions to create a time-averaged extended trapping pattern. A unique advantage of this method is a trapping pattern that can be changed with a high frequency in a programmable way. Misawa *et al.* used computer-controlled galvano mirrors to scan the laser beam at 12.5 Hz [86]. Mio *et al.* have boosted the frequency to a higher level using piezoelectric mirrors. The frequency was as high as 106 Hz [87]. Another scanning method is using acousto-optic deflector (AOD) to guide the laser beam. Due to the non-mechanical characteristic and programmable scanning mode, the deflected laser beam can access arbitrary locations in a pre-defined way within scanning scale in the sample at a high frequency only limited electronically [88]. Single beam trapping can be used in microfluidic systems for a variety of functions, such as cell sorting, separation and interrogation. Terray *et al.* exploited optical propulsion in a weakly focused beam propagating against the flow direction to achieve separation [89], even in a three-dimensional microfluidic device [90].

Combining laminar flow characteristics in a microfluidic system with linear trapping, sorting of particle or cells can also be achieved. The absence of turbulence in a microchannel gives the particle or cells a constant drag force along flow direction. The linear trap will exert a gradient force along a defined direction, which combined with drag force results in a net force that can orientate particles from the main channel into branches as illustrated in Fig 1.12 [84]. Another configuration that can generate linear trap was described by Applegate *et al.* A diode laser bar was focused in a microchannel

Figure 1.12: Schematic concept of particle sorting using linear optical trapping [84].

to achieve particle sorting [91]. With this configuration, Bovine red blood cells can also be captured by the linear trap, which was placed at an angle to the flow direction, and the cells were conveyed along it before being released at its end, thereby sorting cells according to their sizes.

The evanescent field trapping is another type of optical methods to manipulate particles yet still based on a gradient force. Kawata *et al.* demonstrated the propulsion of particles in an evanescent field, which was generated by the light incidents on the base of the prism beyond the critical angle [92]. Schmidt *et al.* investigated the forces exerted by the evanescent field. The investigation shows that there are two kinds of forces: (a) attraction of particles to the waveguide surface; (b) propulsion of particles in the direction of optical propagation, Figure 1.13 [93].

Figure 1.13: Optical trapping and propulsion of particles in a microfluidic system [93].

Investigation of microscale flow field by micro-particle image velocimetry

Micro-particle image velocimetry (μ PIV) has become a widely used tool in the investigation of microfluidic flow field due to its intuitive and quantitative flow visualization [94]. This technique has been adaptively used in many research areas such as lab-on-a-chip system, analytical chemistry and biomedical engineering. The measurement method is similar to Particle Image Velocimetry for large scale applications. The regions of measurement depend on the magnifications of the microscope objectives ranging from a few hundreds of nanometer to several millimeters. The employed particle sizes depend on the dimensions of the flow channels and the premise that the seeding of particles has negligible impact on the flow field. Particle with a diameter smaller than $1 \mu\text{m}$ are used in some applications, where the Brownian motion introduces a significant error in the evaluation of the velocity vectors [95]. However, the impact of Brownian motion can be eliminated by the correlation averaging method during the data evaluation

Figure 1.14: A typical setup for μ PIV measurement in the investigation of a microfluidic flow field [63].

process [96, 97, 98]. Light source for PIV measurement can be incoherent or coherent and continuous or pulsed laser. The choice of illumination source depends on the flow velocity and the magnification. A typical μ PIV setup employs a pulsed light source [94]. Coherent laser source is used in fluorescence imaging for the improvement of the quality of a μ PIV recording [99]. Figure 1.14 shows a typical setup for μ PIV measurement in the investigation of microfluidic flow field.

μ PIV technique was robustly adapted to the measurements of flow fields in different microfluidic environments. Several groups used the μ PIV to characterize the quantities related to the fluid-solid interaction in the near-wall flow measurements, such as determination of channel wall shapes [100] and shear stress distribution of an endothelial cell layer [101]. Micro-mixer is a critical component in many microfluidic systems

working for μ TAS, biological/biochemical assays, drug delivery process [102, 103]. μ PIV has been widely used as an important tool to study the flow characteristics in micro-mixing devices. The microscale segmented gas-liquid flow was investigated using μ PIV to visualize the velocity field inside the liquid plug. Flow recirculations were observed for both straight and meandering channel networks [104]. The velocity field of the two-phase flow in hydrodynamic focusing was characterized by Wu *et al.* using μ PIV. Diffusive mixing effect was studied between the sheath and the sample flow [105]. A stereo-microscope was combined with PIV technique to investigate the three-dimensional velocity field at a T-junction micromixer [106]. Bottausci *et al.* quantified the mixing performance using μ PIV at the intersection of primary and secondary channels where mixing was enhanced [107]. Micro-PIV was used to measure the velocity and vorticity field by Sheen *et al.* to study the device that possesses both pumping and mixing capabilities. In this study, the visualization of flow by PIV technique indicated the flow circulation at triangular structures placed along the channel, which strengthened the mixing effect [108]. Electrokinetic flow applications are commonly used in microfluidic systems due to the electro-osmosis effect to drive flow and the feasibility to manipulate and sort micro-particles. Although the particle movement due to the electrophoretic and dielectrophoresis effects is a critical issue, particle image velocimetry still can be used to visualize the flow field if an earlier calibration is done before the measurement. Devasenathipathy *et al.* derived the true absolute velocity field through the measurement of zeta potential [109]. The other way is to use a two-color PIV approach [110, 111]. Beyond the above mentioned the different microfluidic environments, the μ PIV has been applied to investigate the biological flow [112, 113], two-phase flow [104, 114], and turbulent flow [115].

Generating RI gradient field through diffusion and manipulating interface between fluids are the basic methods to develop tunable optofluidic components. Both methods are related to the velocity field of the flow. Therefore the diagnosis of the velocity field is critical to the investigation of optofluidic systems. Micro-PIV provides such a powerful tool for this diagnosis. Prior to the present work, micro-PIV had not been applied to study these optofluidic components. But the future work should involve micro-PIV to measure the velocity field and visualize the flow field for better characterization of the optofluidic system.

Flow cytometry in a microfluidic system

Flow cytometry is the measurement technique to quantify the physical and chemical properties of suspended bio-cells or particles, such as size, shape, cell surface makers, DNA content and sample concentration [116]. Niehren *et al.* proposed and demonstrated the concept of using solid-state optical system in flow cytometry, although the system and device were not micro-fabricated [117]. The identification and enumeration of virus was demonstrated in a capillary tube using laser excitation and fluorescence signal collection by Ferris et al [118]. Although these conventional flow cytometers possess the ability to measure multiple parameters of micro-particles or bio-samples for bio-chemical studies, they are still bulky, expensive and not portable. The emergence of micro-fabrication technologies provide the feasibility to integrate flow cytometers into microfluidic systems. Other functions, such as sorting or separation, can also be flexibly added into the microfluidic network. Flow cytometers with high performance need a sheath flow surrounding the sample flow to align the samples along the center of the

Figure 1.15: Schematic setup of solid-state flow cytometer for counting fluorescent micro-particles. The emitted fluorescent light is collected by two aspherical lenses and detected by a single photon avalanche diode. The excitation plane is orthogonal to the fluorescent light detection plane [117].

stream. The structure becomes mechanically complex for the conventional flow cytometers based on glass capillary tubing. This problem can be easily solved using microfluidics to realize the cytometer function. The micro-fabrication technology possess the ability to pattern desired structures rapidly and cost-effectively, which is not achievable with conventional flow cytometers using glass machining techniques.

Microfluidics provides flow cytometry a robust platform for fast and accurate analysis of the particles in the streaming fluids. One of the most important advantage to employ microfluidics is the hydrodynamic focusing of cells or particles in a single line along the flow direction, which is critical for the analysis of the detected signals [119]. Three-dimensional hydrodynamic focusing was applied for microfluidic flow cytometer to precisely and continuously deliver the samples to the detection area [120]. To further

Figure 1.16: Hydrodynamic focusing of sample for micro-fabricated flow cytometer [119].

improve the level of integration of the whole system, optical fiber, which can be inserted into the microfluidic networks, was used to perform both excitation and collection of light instead of using external bulky objectives [121, 122]. The problem of merely using optical fiber for excitation include the low intensity in illumination and inaccuracy in detection owing to the divergent light beam emitted from the fiber. Therefore, some on-chip lenses have been proposed to focus the light beam from the fiber. Wang *et al.* designed a flow cytometer with a PDMS-based lens [123]. Godin *et al.* demonstrated a fluidic lens for flow cytometry [124]. Barat *et al.* built both waveguide and lens based on polymer substrate for a flow cytometer [125]. The combination of optics and microfluidics to develop flow cytometry can make the whole system more compact, low cost, and portable for field use [121].

1.3 Objective and scope

Liquid replacing, diffusion control and liquid-liquid interface control are the three main methods to tune the optical functions by adjusting liquid/fluidic properties. The objectives of the works in this thesis are to realize micro optofluidic components for manipulating a light beam and to use conditioned light beam to manipulate particles and fluidic properties. The present thesis will focus on the development of models for liquid-liquid interface and their experimental verification. Firstly, analytical models are developed to mathematically describe the fluidic characteristics of the optofluidic components. The theory of two-dimensional dipole-flow is used to calculate the lens shape that is hydrodynamically formed in a circular chamber with single fluid. The flow fields in symmetric and asymmetric geometries, also with multiple inlets/outlets, can be modelled. The theory of two-dimensional dipole flow presents a convenient and accurate way to analyze the fluid dynamics in a circular domain. Furthermore, the theory of hydrodynamic spreading is used to address the two-fluid flow in the circular chamber as well as a sector-shape chamber. To analyze the fluid dynamics and capture the interface between fluids in a rectangular chamber, a numerical model is developed using level set method. Experiments are carried out to verify the modelling. Ray-tracing method are also applied to investigate the optical performance of the optofluidic components numerically and experimentally.

The effective integration of optofluidic components into a microfluidic network help to miniaturize the whole system and make it cost-effective. The tunability of the fluidic properties enable optofluidic components to be more flexible and adaptive. The mathematical predictability of the optical performance make the optical alignment inside

an on-chip system more precise and convenient. The optofluidic lens takes advantages of optically smooth liquid-liquid interface to focus the light beam without scattering. The good quality of the light beam and the enhancement of intensity make an optofluidic lens adaptive and feasible for detection. This thesis will present a microfluidic flow cytometer system using an optofluidic lens. Due to the well-focused and non-scattered light beam from the optofluidic lens, the flow cytometer shows a more efficient and accurate performance on particle counting and sizing as compared to previous on-chip flow cytometers [123].

1.4 Structure of the thesis

In this thesis, Chapter 1 gives a introduction to the background of optofluidics. A review on the previous works helps to identify the objectives and scopes of the works in this thesis. Chapter 2 provides some theoretical background from which the analytical and numerical models are developed. Chapter 3 presents the materials and experimental methods which are employed during this study. Chapter 4 presents the analytical models and experimental validation of an micro optofluidic lens developed in a circular chamber. Chapter 5 describes a bi-concave lens which can perform both light focusing and diverging. Numerical simulation is carried out to further investigate the characteristics of the system. Chapter 6 presents the development of a liquid core liquid cladding optofluidic prism formed in a sector-shape chamber. A simple analytical model is given to describe the position of the interface. Chapter 7 demonstrates a micro optofluidic aperture stop which can control the radiant flux through the optical system as well as the angular aperture on both object and image spaces. Chapter 8 shows a highly efficient flow cytometer

in particle counting and sizing using optofluidic lens. At last, chapter 9 concludes the works of this thesis and presents a perspective for future works.

Chapter 2

Theoretical background

2.1 Potential function in shallow microfluidic structures with low Reynolds number flow

This section is aimed to provide the theoretical background on how to simplify the Navier-Stokes equation for the application of low Reynolds number flow in a thin microfluidic structures. The basic equations to describe fluid dynamics start from the Newton's second law:

$$\vec{F} = m \frac{d\vec{v}}{dt} \quad (2.1)$$

Considering a specific fluid particle, equation (2.1) gives the relationship between the acceleration of the fluid particle and the exerted force. In the absence of viscosity, equation (2.1) for an incompressible fluid of density ρ becomes

$$\rho(\vec{v} \cdot \nabla)\vec{v} + \rho \frac{\partial \vec{v}}{\partial t} = \rho \vec{F} - \nabla P \quad (2.2)$$

where P is the local pressure and $\vec{F}$ is the applied external force, such as gravity. If the fluid is considered as viscous, equation (2.2) transforms into the well-known Navier-Stokes

equation:

$$\rho(\vec{v} \cdot \nabla)\vec{v} + \rho \frac{\partial \vec{v}}{\partial t} = \rho \vec{F} - \nabla P + \mu \nabla^2 \vec{v} \quad (2.3)$$

where μ is the viscosity of the fluid. It is difficult to solve equation (2.3) and obtain an analytical solution due to the non-linear item $(\vec{v} \cdot \nabla)\vec{v}$. However the simplification of equation (2.3) still can be carried out if some assumptions are made preliminarily. Since the purpose is to investigate the steady-state flow field where 2-dimensional optofluidic components are developed, the structure of the fluidic networks has a much smaller depth in z -direction than the dimensions in xy -plane. In such shallow structures, the viscous force dominates and is much larger than the inertia force resulting in a low Reynolds number and a negligible external force. Under these assumptions, equation (2.3) can be greatly simplified.

Nondimensionalization is a method to remove units from an equation by proper substitution of variables and simplification of the equations. Before nondimensionalization, equation (2.3) can be written in its scalar form with ignoring the external force $\vec{F}$.

The flow field is time-independent:

$$\rho(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}) = -\frac{\partial P}{\partial x} + \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) \quad (2.4)$$

$$\rho(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}) = -\frac{\partial P}{\partial y} + \mu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}) \quad (2.5)$$

$$\rho(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}) = -\frac{\partial P}{\partial z} + \mu(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}) \quad (2.6)$$

where u , v and w are the velocities along x , y and z directions respectively. For the non-dimensionalization, suppose that

$$u = U \cdot u^*, v = U \cdot v^*, w = U \cdot w^* \quad (2.7)$$

where U is the characteristic velocity, and

$$x = L \cdot x^*, y = L \cdot y^*, z = L \cdot z^* \quad (2.8)$$

where L is the characteristic length. Substituting equation (2.7) and (2.8) into equation (2.4) to (2.6) results in the dimensionless equations:

$$\frac{\rho LU}{\mu} (u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*}) = -\frac{\partial P^*}{\partial x^*} + (\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}) \quad (2.9)$$

$$\frac{\rho LU}{\mu} (u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*}) = -\frac{\partial P^*}{\partial y^*} + (\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}) \quad (2.10)$$

$$\frac{\rho LU}{\mu} (u^* \frac{\partial w^*}{\partial x^*} + v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*}) = -\frac{\partial P^*}{\partial z^*} + (\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}) \quad (2.11)$$

where the item $\rho LU/\mu$ is actually the Reynolds number, which is assumed to have a very small value $Re \ll 1$. Therefore for the steady-state case, the Navier-Stokes equation can be simplified after non-dimensionalization into:

$$\nabla P - \mu \nabla^2 \vec{v} \sim 0 \quad (2.12)$$

In spite of the simplification of the Navier-Stokes equation, it is still not easy to solve this equation for an arbitrary boundary conditions. However most of the optofluidic components are developed in a shallow structures, such as circular [126] and sector-shape [127] chamber. In this case, the velocity function $\vec{v}(x, y, z)$ can be separated into functions $\vec{V}(x, y)$ and $f(z)$ [128]:

$$\vec{v}(x, y, z) = f(z) \vec{V}(x, y) \quad (2.13)$$

Firstly for the very shallow structures (the z depth $h \ll$ all x, y dimensions), the solution of the 1-D Navier-Stokes equation must be parabolic:

$$f(z) \sim 1 - (\frac{2z}{h})^2, -\frac{h}{2} \leq z \leq \frac{h}{2} \quad (2.14)$$

Substituting equation (2.13) and (2.14) into (2.12) results in:

$$\frac{1}{\mu} \nabla P = \nabla^2 [f(z) \vec{V}(x, y)] \quad (2.15)$$

The scalar form of equation (2.15) can be written as:

$$\frac{1}{\mu} \frac{\partial P}{\partial x} = f(z) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + u \frac{\partial^2 f(z)}{\partial z^2} \quad (2.16)$$

$$\frac{1}{\mu} \frac{\partial P}{\partial y} = f(z) \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + v \frac{\partial^2 f(z)}{\partial z^2} \quad (2.17)$$

The method of non-dimensionalization can be used to simplify equation (2.16) and (2.17).

Suppose that:

$$u = U_0 \cdot u^*, v = U_0 \cdot v^* \quad (2.18)$$

where U_0 is the characteristic velocity, and

$$x = L \cdot x^*, y = L \cdot y^*, z = h \cdot z^* \quad (2.19)$$

Substituting equation (2.18) and (2.19) into equation (2.16) and (2.17) leads to:

$$\frac{P_0}{\mu L} \frac{\partial P^*}{\partial x^*} = \frac{U_0}{L^2} f(z^*) \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right) + \frac{U_0}{h^2} u^* \frac{\partial^2 f(z^*)}{\partial z^{*2}} \quad (2.20)$$

$$\frac{P_0}{\mu L} \frac{\partial P^*}{\partial y^*} = \frac{U_0}{L^2} f(z^*) \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right) + \frac{U_0}{h^2} v^* \frac{\partial^2 f(z^*)}{\partial z^{*2}} \quad (2.21)$$

Noted that $h \ll L$ due to the assumption of shallow structure, the magnitudes of the first terms on the right hand side of equation (2.20) and (2.21) are much smaller than those of the second terms. Therefore the governing equation can be further simplified and written in its vector form:

$$\vec{V}(x, y) = -\frac{h^2}{8\mu} \nabla P \quad (2.22)$$

Applying the ∇ operator to equation (2.22) results in:

$$\nabla \times \vec{V}(x, y) = -\frac{h^2}{8\mu} \nabla \times (\nabla P) \quad (2.23)$$

Noted that the curl of the pressure gradient equals to zero and $\nabla \times \vec{V}(x, y) = \nabla \times (\nabla \times \vec{\psi})$.

And $\vec{\psi} = (0, 0, \psi)$ is the vector form of stream function ψ . So the governing equation can be presented using the stream function:

$$\nabla \times (\nabla \times \vec{\psi}) = \nabla(\nabla \cdot \vec{\psi}) - \nabla^2 \vec{\psi} = 0 \quad (2.24)$$

Since the divergence of $\vec{\psi} = (0, 0, \psi)$ is equal to zero, eventually the stream function turns out to satisfy the Laplace's equation:

$$\nabla^2 \psi(x, y) = 0 \quad (2.25)$$

According to the definition of the stream function:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (2.26)$$

Substituting equation (2.26) into (2.25) results in:

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \quad (2.27)$$

Therefore, there must be a potential function $\varphi(x, y)$ which satisfies:

$$u = \frac{\partial \varphi}{\partial x}, v = \frac{\partial \varphi}{\partial y} \quad (2.28)$$

Combining equation (2.28) and continuity equation leads to the Laplacian partial differential equation for the velocity potential function $\varphi(x, y)$:

$$\nabla^2 \varphi(x, y) = 0 \quad (2.29)$$

Since $\psi(x, y)$ and $\varphi(x, y)$ satisfy the Cauchy-Riemann equation:

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y}, \frac{\partial \varphi}{\partial y} = -\frac{\partial \psi}{\partial x} \quad (2.30)$$

a complex analytic function $W(z)$, called the complex potential, can be built to address the flow field in shallow structures:

$$W(z) = \varphi + i\psi \quad (2.31)$$

The velocity field can be simply obtained by the differential of W with respect to z :

$$\frac{dW}{dz} = u - iv \quad (2.32)$$

2.2 The theory of hydrodynamic focusing/spreading

The phenomenon of hydrodynamic focusing and spreading has been widely used for lab-on-chip applications. One of the examples is flow cytometry, which utilizes two sheath flows to sandwich a sample flow. The micro-particles, such as cells, are carried and transported by the sample flow to the detection area with optical diagnosis [129]. Hydrodynamic focusing is also a powerful method to improve the mixing behaviors in microfluidic channels. The core stream can be squeezed by two sheath flows, which results in a small stream width and consequently a short mixing path [130]. Recently a number of optofluidic components have been built based on the phenomenon of hydrodynamic focusing/spreading, such as waveguides [73, 47] in a straight channel, lenses [54] in a expansion chamber and prism [127] in a sector-shape chamber.

An analytical model of hydrodynamic focusing/spreading was reported by Wu *et al.* [105]. Figure 2.1 illustrates the model of this phenomenon: one core stream sandwiched by two cladding streams inside a microfluidic channel with a rectangular cross-section. $2W$ and H are the width and height of the channel respectively. The interface between core and cladding streams is located at $y = rW$. Owing to the low

Figure 2.1: Rectangular cross-section of microfluidic channel: (a) actual geometry; (b) the dimensionless model.[105]

Reynolds number in this case, the non-linear item $(\vec{v} \cdot \nabla) \vec{v}$ can be neglected in the Navier-Stokes equation. Therefore the velocity distributions of the core and cladding streams can be obtained by solving the following simplified N-S equations:

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} = \frac{1}{\mu_1} \frac{\partial p}{\partial x} \quad (2.33)$$

$$\frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_2}{\partial z^2} = \frac{1}{\mu_2} \frac{\partial p}{\partial x} \quad (2.34)$$

where indices 1 and 2 denote core and cladding streams respectively, and μ represents the viscosity. Equation (2.33) and (2.34) can be non-dimensionalized by the method mentioned in the above section, which results in a dimensionless model:

$$\frac{\partial^2 u_1^*}{\partial y^{*2}} + \frac{\partial^2 u_1^*}{\partial z^{*2}} = \frac{W}{\mu_1 u_0} \frac{\partial p}{\partial x^*} \quad (2.35)$$

$$\frac{\partial^2 u_2^*}{\partial y^{*2}} + \frac{\partial^2 u_2^*}{\partial z^{*2}} = \frac{W}{\mu_2 u_0} \frac{\partial p}{\partial x^*} \quad (2.36)$$

with boundary conditions of:

$$u_2^*(1, z^*) = 0 \quad (2.37)$$

$$\frac{\partial u_1^*}{\partial y^*} \big|_{y^*=0} = 0 \quad (2.38)$$

The velocity and viscous rate are continuous at the interface between core and cladding streams:

$$u_2^*(r, z^*) = u_1^*(r, z^*) \quad (2.39)$$

$$\frac{\partial u_1^*}{\partial y^*} \big|_{y^*=r} = \beta \frac{\partial u_2^*}{\partial y^*} \big|_{y^*=r} \quad (2.40)$$

where β is the viscosity ratio μ_2/μ_1 . The position of the interface can be estimated as:

$$r = \frac{1}{1 + 2\beta\phi} \quad (2.41)$$

where ϕ is the flow rate ratio Q_2/Q_1 . The solution of equations (2.35) and (2.36) was given by Wu *et al.* [105] as:

$$u_1^*(y^*, z^*) = P_0 \left[\frac{z^{*2} - h^2/4}{2} + \sum_{n=1}^{\infty} \cos\theta z^* (A_{n1} \cosh\theta y^* + B_{n1} \sinh\theta y^*) \right] \quad (2.42)$$

$$u_2^*(y^*, z^*) = \frac{P_0}{\beta} \left[\frac{z^{*2} - h^2/4}{2} + \sum_{n=1}^{\infty} \cos\theta z^* (A_{n2} \cosh\theta y^* + B_{n2} \sinh\theta y^*) \right] \quad (2.43)$$

with $P_0 = \frac{W}{\mu_1 u_0} \frac{\partial p}{\partial x^*}$ and $\theta = (2n-1)\pi/h$. The coefficients in the solution can be found in the paper of Wu *et al.* [105].

2.3 Numerical simulation: level-set method

Incident light on a interface will lead to reflection and refraction as long as there is a difference of refractive index across this interface. The curvature of the interface can bend the incident light accordingly to Snell's law. Therefore, manipulating the liquid/air

or liquid/liquid interface is an efficient way to tune optical properties of optofluidic components as well as the light beam. How to mathematically describe the geometry of the interface has become a critical issue in the investigation of an optofluidic components. Although the theories of potential flow and hydrodynamic spreading can give analytical solutions only if simple boundary conditions are involved. Complex boundary geometries will demand other methods to work out the mathematical prediction of the interface position.

The level-set method (LSM) is one of the numerical tools for tracking interfaces and shapes, which is proposed and demonstrated by Osher and Sethian [131]. One of the advantages of LSM is the computation of interfaces or shapes without parameterizing these objects on a fixed Cartesian grid. Up to now, level-set method has been performed successfully for tracking the moving interface and free boundary problems in microfluidics [132]. In this study of optofluidic components, level-set method is used for tracking the liquid/liquid interface between different fluids in microfluidic networks. Since most of the situations for developing optofluidic components are thin structures (when dimensions in $x - y$ plane $\gg h$ depth in z direction, the flow field can be described by equations (2.13)-(2.25), a two-dimensional numerical model is built based on the specific geometry of the device. The level-set method is applied to treat the interfaces between different fluids [131]. The level-set function is defined as

$$\Phi = \begin{cases} -d, & \vec{x} \in \Omega_{\text{cladding}} \\ 0, & \vec{x} \in \Gamma \\ +d, & \vec{x} \in \Omega_{\text{core}} \end{cases} \quad (2.44)$$

where x is the position vector in the computational domain Ω , and d is the shortest normal distance from the interface Γ . The domain of interest consists of a single special

fluid of variable properties. The density ρ and the dynamic viscosity μ of the special fluid are expressed as

$$\rho = H\rho_{\text{core}} + (1 - H)\rho_{\text{cladding}} \quad (2.45)$$

$$\mu = H\mu_{\text{core}} + (1 - H)\mu_{\text{cladding}} \quad (2.46)$$

where the smeared Heaviside function H is given by

$$H(\Phi) = \begin{cases} 0, & \Phi < -\varepsilon \\ \frac{\Phi + \varepsilon}{2\varepsilon} + \frac{1}{2\pi} \sin\left(\frac{\pi\Phi}{\varepsilon}\right), & |\Phi| \leq \varepsilon \\ 1, & \Phi > +\varepsilon \end{cases} \quad (2.47)$$

The motion of this special fluid is governed by the Navier-Stokes equations, i.e.

$$\nabla \cdot \vec{u} = 0 \quad (2.48)$$

$$\nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \nabla \cdot [\mu (\nabla \vec{u} + \nabla \vec{u}^T)] \quad (2.49)$$

The motion of the interface, i.e. the level-set function Φ , is given by

$$\vec{u} \cdot \nabla \Phi = 0 \quad (2.50)$$

To maintain Φ as a distance function, Φ is set to the steady-state solution of the following equation:

$$\frac{\partial \Phi'}{\partial \bar{t}} = \overline{\text{sgn}}(\Phi)(1 - |\nabla \Phi'|) \quad (2.51)$$

where $\bar{t}$ is a pseudo-time that is employed for redistancing purpose only, and sgn is given by

$$\overline{\text{sgn}}(\Phi) = \frac{\Phi}{\sqrt{\Phi^2 + |\nabla \Phi|^2 (\Delta x)^2}} \quad (2.52)$$

For boundary conditions, the average velocity at the inlet is set as: $u_c = q_c/(wh)$, where q_c is the volumetric flow rate set by the syringe pump in the corresponding experiment and w and h are the channel width and height respectively. Outflow boundary conditions are used for the outlet. The initial level-set function is set to an assumed shape and evolved under Eq. 2.50 to its steady state shape.

The finite-volume method is employed to address the equations (2.48) and (2.49) to solve the velocity-pressure coupling. A staggered grid is used in this computation. The scalar variables are stored at the centers of the control volume, while the velocities are located at the control volume faces. The level-set and redistancing functions (2.50) and (2.51) are implemented within a narrow-band region near the interface instead of the whole computational domain to reduce computational effort. The level-set equation (2.45) is solved with the high order fast sweeping method [133, 134]. The computed positions of the interface can be transferred into a custom Matlab program for later implementing the numerical ray-tracing based on Snell's law.

Chapter 3

Materials and methods

3.1 Fabrication technology

Most optofluidic components are built on the same platform as other microfluidic system for biological or biochemistry analysis. A variety of methods for fabrication of microfluidic networks have been proposed and demonstrated. Micro-machining techniques derived from microelectronic industry are widely used to fabricate microchannels. Bulk [135] and surface micro-machining [136] are the two main methods. Bulk micro-machining normally employs wet etching to pattern on the silicon substrate. Whereas surface micro-machining is a process to build patterns on a layer deposited on the substrate. The deposition layer can be selectively etched by wet etching or dry etching methods. The advantages of surface micro-machining over bulk micro-machining are the feasibility to produce smaller features and low cost which is due to the possible recycling of the expensive silicon wafer. However, the procedures of surface micro-machining are relatively complex, which include deposition of polysilicon and silicon dioxide layers,

photolithography, wet etching or ionized gas etching. Soft lithography technique has become the most popular method to fabricate microfluidic systems as well as optofluidic devices since it was proposed in 1998 [137]. The advantages of soft lithography are the convenience and the rapid preparation of high-resolution laser printed mask in stead of a chrome mask [138], easy and fast deposition of a SU-8 layer by spin coating, and the durable master mold. This chapter focuses on the process of soft lithography to fabricate the optofluidic devices.

Figure 3.1 shows the process-flow diagram of the device fabrication [139]. Before the deposition of SU-8, the silicone wafer should be dry and clean to achieve maximum process reliability. Dilute acid and DI water are used to rinse the wafer followed by etching and cleaning using piranha (H_2SO_4 and H_2O_2). Baking at 120°C for 10 minutes on a hot plate is necessary to dehydrate the wafer surface. Table 3.1 shows the spin coating speed for a required thickness. SU-8 products with different viscosities can satisfy a variety of requirements of deposition thickness. The fabrication of the devices reported in this thesis employs SU-8 2050, which can be deposited with a $150\ \mu\text{m}$ thickness when applying a spin speed of 1000 rpm. The thickness of our device depends on the outer diameter of the optical fiber. Soft bake is necessary to evaporate the solvent in the SU-8 and solidify the deposited film. The time of soft bake depends on the thickness. It takes 30 minutes to soft bake our wafer at 95°C after a short pre-bake at 65°C .

After the soft bake step, the SU-8 wafer is ready to be exposed with a transparency mask that has a resolution of 8000 dpi. The optimized bandwidth for SU-8 exposure is near Ultraviolet wavelength (350-400nm). A long-pass filter is used to eliminate UV light below 350nm to achieve a relatively vertical side-wall. Table 3.2 shows

Figure 3.1: The flow-process diagram of the fabrication of optofluidic devices using soft lithography: (a) Silicon wafer after pre-treatment; (b) Deposition of SU-8 layer by spin coating; (c) mold development by UV light exposure; (d) PDMS casting; (e) Bonding PDMS after surface treatment by oxygen plasma; (f) Introducing optical fiber into the optofluidic chip.

Table 3.1: Thickness as function of spin coating speed for a selected SU-8 photoresist [139]

Product Name	Thickness (μm)	Spin Coating Speed (rpm)
SU-8 2035	35	3000
	55	2000
	110	1000
SU-8 2050	50	3000
	75	2000
	150	1000
SU-8 2075	75	3000
	110	2000
	225	1000
SU-8 2100	100	3000
	140	2000
	260	1000

Table 3.2: Exposure dose as a function of film thickness [139]

Thickness (μm)	Exposure Energy (mJ/cm^2)
25-40	150-160
45-80	150-215
85-110	215-240
115- 150	240- 260
160-225	260-350

the exposure energy for a corresponding thickness. Over-exposure will cause the pattern to lose small features because of the diffraction. Whereas inadequate exposure will bring about slanting the side-wall or even the lift-off of SU-8 during the developing process.

Before the development, a post exposure bake should be carried out on a hot plate to selectively cross-link the exposed portions of the film. For our devices, post-exposure bake takes 15 minutes at 95°C after a short pre-bake at 65°C . After the post exposure bake and cooling down the wafer to room temperature, development can be carried out. Table 3.3 shows the development time dependent on the thickness for a corresponding SU-8 product. Agitation or ultrasonics can be used to enhance the process after the immersion of the wafer in the developer for several minutes. When development is completed, the SU-8 wafer can be cleaned with isopropyl alcohol (IPA) as well as acetone. After drying the wafer with a nitrogen gun, polydimethylsiloxane (PDMS)

Table 3.3: Development time as a function of thickness for a selected SU-8 photoresist [139]

Product Name	Thickness (μm)	Development time (minutes)
SU-8 2035	35	5
	55	6
	110	10
SU-8 2050	50	6
	75	7
	150	12
SU-8 2075	75	7
	110	10
	225	12
SU-8 2100	100	10
	140	15
	260	20

casting can be performed.

PDMS was mixed in a weight ratio of 10:1 and poured onto the SU-8 master. Thereafter, the PDMS was vacuumed for two hours to remove gas bubbles. After the degassing process, the PDMS was heated at 80°C for two hours. The PDMS piece was then peeled off from the master mold, and 0.75 mm diameter access holes were punched using the Harris uni-core puncher. The PDMS with the optofluidic designs was subsequently bonded to another flat PDMS part after treating both surfaces with oxygen plasma. The bonded PDMS was then placed in an oven at 150°C for twenty minutes to ensure good bonding between both surfaces. Needles with an inner diameter of 0.33 mm and an outer diameter of 0.64 mm were press fit into the access holes and worked as fluidic interconnects. With this soft lithography technique, our optofluidic device can be fabricated with a 150 μm channel height.

3.2 Experimental setup

Figure 3.2 illustrates the schematic of the setup for the experiments reported in this thesis. The liquids are kept in a 5 mL syringe (Hamilton glass), which are driven by syringe pumps (KDS230, KD Scientific Inc, USA) to deliver any flow rate needed. A multi-mode optical fiber (AFS105/125Y, THORLABS Inc.) was inserted into a pre-fabricated microchannel with a width of $130\ \mu\text{m}$ to work approximately as a point light source. The light is coupled into the fiber from a Nd:Yag laser using a $10\times$ objective lens. The testing results are imaged using a microscopic objective (Nikon EclipseTE 2000-S, Japan) and a sensitive CCD camera (HiSense MKII). The color bright field images were recorded with a digital camera (DCRDVD803E, SONY). By switching the filter, separated fluorescent bands can be detected and recorded. In the experiment, fluorescence dye (fluorescein disodium salt $\text{C}_{20}\text{H}_{10}\text{Na}_2\text{O}_5$, Acid Yellow 73 or C.I. 45350) and $3\ \mu\text{m}$ red fluorescent particles (Duke Scientific Co.) were used to visualize the flow. Fluorescence dye Rhodamine B (Sigma Aldrich, excitation wavelength of 540 nm, emission wave length of 625 nm) diluted in a mixture of glycerol (60% by weight) and water (40% by weight) which has a matching refractive index of $n = 1.412$ was injected into the ray-tracing chamber to visualize the light rays emitted from a Nd:YAG laser with a wavelength 532 nm.

Figure 3.2: Schematic of experimental setup.

Figure 3.3: Spectrum of PDMS with the optical loss in dB/cm [140].

3.3 Materials

3.3.1 Polydimethylsiloxane (PDMS)

Generally, optofluidic devices are optical systems based on microfluidic networks. The materials for the fabrication of a microfluidic system can be silicon or glass. However, fabrication using polymers is easier, less expensive and more flexible than using glass. Polydimethylsiloxane (PDMS) is a polymeric organosilicon compound that has become one of the most popular materials for microfluidics for its good properties, such as optical transparency, mechanical elasticity, chemical inertia and non-toxicity. The transmittance spectrum of PDMS shows that it is transparent to all the visible light wavelengths (Figure 3.3) [140]. PDMS has a refractive index of 1.412 [141].

PDMS is mechanically elastic and possesses a tunable Young's modulus, typi-

cally around 750kPa [141] when it is solid; liquid PDMS is viscoelastic [142, 143]. After casting liquid PDMS on a mold surface and leaving it naturally flowing over the surface, PDMS will mold to any small or even the undercut features on the surface. Therefore, microfluidic channels as well as other small patterns can be replicated without any deformation. Replicas with small features below 2 nm have been demonstrated [144].

After polymerization and cross-link, the solid-phase PDMS possesses a hydrophobic surface [145] because of the repeating units $-\text{OSi}(\text{CH}_3)_2-$ groups. After oxygen plasma treatment, the surface of PDMS can become hydrophilic due to the introduction of silanol (SiOH) groups. Oxidized surface can maintain its hydrophilic property for about 30 minutes, after which the hydrophobic recovery of the surface is inevitable independently of the surrounding medium [146]. Aqueous solvents are not allowed to penetrate or swell the material of solid-phase PDMS. Thus water as well as other alcohol solvents can be used with PDMS without deformation or corrosion of the material. However, most organic solvents can diffuse into PDMS and make it swell [145].

3.3.2 Working liquids

Optofluidic devices employ liquids to develop optical components [73, 54], to transport samples for optical detection [147] or manipulation [5]. The properties of liquids are critical to the functions [148] and efficiencies [149] of optofluidic components. Refractive index (RI) of a liquid is one of the most important properties. The mismatch of refractive indices of two media can introduce refraction or reflection of the light that is incident upon the interface between the two mediums. A well controlled distribution of RI, either by interface-control or diffusive control, can enable some specific optical func-

Table 3.4: Properties of working liquids for micro optofluidics

Liquids	CAS code	Refractive index	Viscosity (mPa·s)	Density (kg/m ³)
DI-water	7732-18-5	1.33	0.89	997
Ethanol	64-17-5	1.36	1.04	789
Ethylene glycol	107-21-1	1.43	16.1	960
Benzyl alcohol	100-51-6	1.54	8.0	1044
Benzothiazole	95-16-9	1.64		1272
Cinnamaldehyde	104-55-2	1.62	5.70	1050
Glycerol	56-81-5	1.47	1200	1261
Methanol	67-561	1.33	0.59	787
Trifluoroethanol	75-89-8	1.291	1.24	1382
Isopropanol	67-63-5	1.33	1.96	786
Silicone oil	63148-62-9	1.375-1.405	0.494-976	760-976
73.5% ethylene glycol 26.5% ethanol		1.412	9.80	915

Table 3.5: Properties of glycerol water mixtures

Glycerol % by weight	Refractive index	Viscosity (mPa·s)	Density (kg/m ³)
100	1.47	1200	1261
90	1.45	207.6	1230
80	1.44	55.3	1200
70	1.43	21.5	1180
60	1.41	8.99	1150
50	1.40	5.90	1120
40	1.38	3.79	1090
30	1.37	2.58	1070
20	1.36	1.85	1040
10	1.34	1.36	1020
0	1.33	1.33	997

tions, such as waveguide [73] or lens-effect [54, 42]. Interface-control can be achieved by hydrodynamic spreading [105] or electro-osmotic effect [150] in a straight micro-channel [73] or chambers with different geometries [54, 75, 126].

Table 3.4 shows the properties of some commercial liquids. Since most optofluidic devices are fabricated in PDMS, the refractive index 1.412 of PDMS is considered as the reference for choosing the working liquids [33]. The liquid with RI matching that of PDMS has an important role in the optofluidic systems, especially in liquid-core liquid-cladding configurations. The PDMS side-wall of micro-channels or other struc-

tures fabricated by soft lithography has an optically rough surface, which will scatter the incident light upon itself [149]. A fluid with a matching RI attached to the side-wall can prevent light-scattering and improve the optical performance of the optofluidic components [148]. Table 3.5 shows the properties of the mixture between glycerol and DI-water. The RI of the mixture can be fine tuned by different proportions of the blend. The mixture of glycerol 60% and water 40% by weight has a RI matching that of PDMS. The optical performance greatly depends on the difference of refractive indices between fluids in the optofluidic system. Considering the RI of PDMS as the reference, benzothiazole is a liquid with a high refractive index ($n = 1.64$), but it is irritant and toxic to human. Benzyl alcohol is a relatively safe liquid and it is often used as preservative due to its bacteriostatic and antipruritic properties. But its refractive index ($n = 1.54$) is not as high as benzothiazole. Cinnamaldehyde is a relatively safe liquid with high refractive index ($n = 1.62$). For example, cinnamaldehyde is a good choice to serve as core liquid for the development of a L^2 convex lens to focus light. Furthermore, cinnamaldehyde is slightly soluble in ethanol and water, which makes it easy to form a stable flow with an almost immiscible interfaces [33].

Viscosity of the liquid is the parameter that greatly affects the fluid dynamics in microscale. The position of the interface between fluids depends on the viscosity ratio of the fluids [76]. A rheometer (Contraves Low Shear 40) is employed to measure the viscosities of the liquids that are used in the following discussed experiments. Figure 3.4 shows the variation of shear stresses with different shear rates of these liquids. The measurement result presents a good linear relationship between shear stress and shear rate. And the estimated shear rate in the following discussed experiments ranges approx-

Figure 3.4: The measurement of shear stresses with different shear rates. The discrete points denote the experimental measurements, and the solid lines represents the fitting whose slopes are the viscosity values.

imately from 10 s^{-1} to 50 s^{-1} . So these liquids are considered to be Newtonian fluids in the development of theoretical analysis as well as numerical simulation.

Chapter 4

Tunable optofluidic lenses

developed in a circular chamber ¹

4.1 Motivation and device concept

Recently, micro optofluidics has been emerging as an active research field, where liquid flows in microchannels are used to realize optical components. The small size of microchannels leads to a small Reynolds number and a stable laminar flow. The smooth and relatively stable interfaces between different liquids are ideal for designing different optical components such as waveguide, lenses and mirrors [1]. The simplest application of laminar liquid streams with mismatched refractive indices is the liquid-core/liquid-cladding waveguide [46]. Hydrodynamic control of the liquid streams can be used to define the light propagation direction. The curved interface between two immiscible liquids can be achieved with the contact angle at the liquid/liquid/solid interface [151, 68] or with the Dean flow in a curved channel [74]. In the later case, a small radius of curvature

Figure 4.1: The concept of the micro optofluidic lens with a circular lens chamber. The shapes and dimensions of the lens are tuneable by the flow rates of core and cladding streams: (a) double-convex lens; (b) plano-convex lens; (c) convergent-meniscus lens.

and a short focal length can be achieved. Recently, Tang *et al.* reported a dynamically reconfigurable liquid-core liquid-cladding lens using the curved interfaces in a rectangular chamber [54]. This paper reported experimental results of an in-plane lens formed by hydrodynamic spreading of a core stream, which has a higher refractive index than that of the side stream. However, little theoretical work has been done to characterize the fluid dynamics in the chamber where optofluidic lens are developed with liquid-core liquid-cladding configuration. Good analytical models to describe the interfaces between fluids are critical to realize optofluidic applications with high precision.

In this chapter, we propose a new design with a circular chamber to realize a perfect arc-shape lens curvature. This chapter presents the development of mathematical model and the validation of the model by experimental results. The two-dimensional dipole flow theory and the hydrodynamic spreading theory are applied to describe single-

fluid and two-fluid models respectively. Traditional precision engineering can fabricate optical lens with a pre-defined curvature of interface by using grinding and polishing, which is a contact method. Thus the optical smoothness of the interface is limited. The model described here provides a method to configure optofluidic lenses with a mathematically pre-defined curvature, whose interface is atomically smooth. Figure 4.1 depicts the basic concept of a liquid-core liquid-cladding optofluidic lens in a circular chamber. The design based on liquid flow at low Reynolds numbers, and thus promises to form laminar streams in the lens chamber. The core stream sandwiched by two cladding streams enters the circular chamber and then expands to develop the lens-shape. Manipulating the flow rates of the three streams allows tuning the curvature of the interface of the lens, and consequently changes the focal length. In the design described here, the width of the channel is much smaller than the other dimensions of the chamber. So the entrance and exit of the circular chamber act approximately as a source and sink, respectively. Therefore, the flow field in the circular chamber can be approximately described by a source-sink pair model bounded in a circular domain. Based on this model, the curvature of the interface between core and cladding streams, and consequently the focal length of the fluidic lens can be calculated.

4.2 Analytical modelling

4.2.1 Single-fluid model using dipole flow theory

Symmetric geometry

Koplik *et al.* [152] discussed the trajectories of tracer particles in a two-dimensional dipole flow field in a circularly bounded domain. In this source-sink pair model, a source of strength Q is placed at $(-a, 0)$ in the Cartesian coordinates, and a corresponding sink of strength $-Q$ located at $(a, 0)$. The flow field is determined by two parameters: the distance $2a$ between the source and sink, and their strength Q . This field can be described by the complex potential:

$$w(z) = \varphi + i\psi = \frac{Q}{2\pi}(\lg(z + a) - \lg(z - a) + \lg(R^2 + az) - \lg(R^2 - az)) \quad (4.1)$$

where x and y are the coordinates in the two dimensional Cartesian system, R is the radius of the circular domain and $z = x + iy$, with i as the imaginary unit. The real part ϕ and imaginary part ψ of the complex potential are the velocity potential and stream function respectively. The streamlines are equipotential lines of the function ψ .

In the case of the lenses depicted in Fig. 4.1, the source and sink can be assumed to be located at the circumference of the circular domain ($a = R$). The complex potential can be written as:

$$w(z) = \varphi + i\psi = \frac{Q}{2\pi}(\lg(z + R) - \lg(z - R) + \lg(R^2 + Rz) - \lg(R^2 - Rz)) \quad (4.2)$$

The corresponding streamlines are shown in Fig. 4.2. For mathematical simplicity, only the positive half of the y -axis ($y > 0$) is taken into consideration. In this

Figure 4.2: Streamlines of source-sink pair in a circularly bounded domain with source and sink located at the circumference.

region, the stream function can be simplified into:

$$\psi = \frac{Q}{2\pi} \left(2 \arctan \frac{y}{R+x} + 2 \arctan \frac{y}{R-x} - \pi \right) \quad (4.3)$$

The streamlines are a cluster of curves $\psi = C_i (i = 1, 2, 3, \dots)$. Thus the coordinates (x, y) at each streamline should satisfy the equation:

$$\arctan \frac{y}{R+x} + \arctan \frac{y}{R-x} = (C_i + \pi)/2 \quad (4.4)$$

For each specific C_i , equation (4.4) defines a curve representing a streamline. The two terms at the left hand side of equation (4.4) are the angles α and β depicted in Fig. 4.3 (a). Since the sum of the angles α and β is a constant for each specific streamline, thus the angle $\angle MPN = \pi - (\alpha + \beta)$ is also a constant. According to the law of sines, the point P is located at a circle with a radius of $R/\sin(\alpha + \beta)$. Therefore, the track of point P satisfying equation (4.4) should be an arc of radius r . Thus, each

Figure 4.3: (a) The coordinate of point P satisfies the equation (4) and the trajectory of point P is an arc with a radius ; (b) Interface positions of a lens.

streamline in Fig. 4.2 is an arc with a radius r :

$$r = R / \sin(\alpha + \beta) = R / \left(\frac{y}{\sqrt{(R+x)^2 + y^2}} \frac{R-x}{\sqrt{(R-x)^2 + y^2}} + \frac{R+x}{\sqrt{(R+x)^2 + y^2}} \frac{y}{\sqrt{(R-x)^2 + y^2}} \right) \quad (4.5)$$

Because of the symmetry, the streamlines in the other semi-plane ($y < 0$) should also comply with the conclusion above. With a given interface position, the radius of curvature can be determined using equation (4.5). A liquid interface with well-defined

radius of curvature and a mismatch in refractive indices can serve as an optofluidic lens. According to the above model, the curvature of the interface is tunable by controlling the flow rate ratio of the streams. Following, the relationship between the flow rate ratio and the curvature of the interface are derived.

According to the theory of potential flow [153], the first derivative of the complex potential with respect to z represents the velocity field $\vec{V} = (u, v)$. The real and imaginary parts of this first derivative represent the velocity components in x -axis and y -axis respectively. Differentiating W in equation (4.2) with respect to z results in the velocity components:

$$\begin{cases} u = \frac{2(x+R)}{(x+R)^2+y^2} - \frac{2(x-R)}{(x-R)^2+y^2} \\ v = \frac{2y}{(x+R)^2+y^2} - \frac{2y}{(x-R)^2+y^2} \end{cases} \quad (4.6)$$

At the center cross-section of the chamber ($x = 0$) the velocity components are:

$$\begin{cases} u = \frac{4R}{R^2+y^2} \\ v = 0 \end{cases} \quad (4.7)$$

The curvature of the interfaces determines the positions of A_2 and B_2 (Fig. 4.3 (b)), and therefore decides the flow rates for the core and cladding flows:

$$\begin{cases} Q_{\text{cladding A}} = \int_{A_2}^{A_1} u \cdot dy = \int_{A_2}^{A_1} \frac{4R}{R^2+y^2} \cdot dy = 4[\arctan(y_{A_1}/R) - \arctan(y_{A_2}/R)] \\ Q_{\text{core}} = \int_{B_2}^{A_2} u \cdot dy = \int_{B_2}^{A_2} \frac{4R}{R^2+y^2} \cdot dy = 4[\arctan(y_{A_2}/R) - \arctan(y_{B_2}/R)] \\ Q_{\text{cladding B}} = \int_{B_1}^{B_2} u \cdot dy = \int_{B_1}^{B_2} \frac{4R}{R^2+y^2} \cdot dy = 4[\arctan(y_{B_2}/R) - \arctan(y_{B_1}/R)] \end{cases} \quad (4.8)$$

Simply by substituting $x = 0$ into equation (4.5), the relationship between the radius of interface and the position of the intersection between the interface and the y -axis is retrieved by solving the equation:

$$y^2 - 2ry + R^2 = 0 \quad (4.9)$$

Figure 4.4: Radius of interface versus the position of intersection between the corresponding interface and y-axis (both radius and position are normalized by the radius R of the circular chamber).

or

$$|y| = r - \sqrt{r^2 - R^2} \quad (4.10)$$

This relationship of the radius of interface versus position of intersection is illustrated in Fig. 4.4. As the position of the interface approaches the wall of the chamber, the radius of interface approaches the radius of the chamber.

Suppose the radii for $S_- \widehat{A_2} S_+$ and $S_- \widehat{B_2} S_+$ are r_1 and r_2 respectively, according to equation (4.10), the position of intersections between these arcs and the y -axis are calculated as:

$$\begin{cases} y_{A_2} = r_1 - \sqrt{r_1^2 - R^2} \\ y_{B_2} = \sqrt{r_2^2 - R^2} - r_2 \end{cases} \quad (4.11)$$

Substituting equation (4.11) into (4.8) results in the fluxes of core and cladding

Figure 4.5: Hydrodynamic formation of optofluidic lens in a circular chamber with the lens-forming channel shifted off-axis. Optical fibers are aligned along the optical axis of the lens for testing its focusing ability

flows, which correspond to the stream flow rates:

$$\left\{ \begin{array}{l} Q_{\text{cladding A}} = \int_{A_2}^{A_1} u \cdot dy = 4[\pi/4 - \arctan \frac{r_1 - \sqrt{r_1^2 - R^2}}{R}] \\ Q_{\text{core}} = \int_{B_2}^{A_2} u \cdot dy = 4[\arctan \frac{r_1 - \sqrt{r_1^2 - R^2}}{R} - \arctan \frac{\sqrt{r_2^2 - R^2} - r_2}{R}] \\ Q_{\text{cladding B}} = \int_{B_1}^{B_2} u \cdot dy = 4[\arctan \frac{\sqrt{r_2^2 - R^2} - r_2}{R} + \pi/4] \end{array} \right. \quad (4.12)$$

Asymmetric geometry

This section discusses the case of a channel placed with an offset to the axis of a circular chamber as depicted in Fig. 4.5. Here, a mathematic model is established to describe the relationship between flow rate ratio and the shape of lens. In the design, the width of the channel is much smaller than the dimension of the chamber. So the entrance and exit of the circular chamber act approximately as a source and sink, respectively. Assuming that the sink is placed at (x_0, y_0) , and the source is symmetrically located at $(-x_0, y_0)$ illustrated in Fig. 4.6. The complex potential of this flow field can be written

Figure 4.6: Streamlines of source-sink pair in a circularly bounded domain with source and sink located at the boundary.

as:

$$W = \varphi + i\psi = \ln \frac{(z + x_0 - i \cdot y_0)(z + \frac{R^2}{x_0 + i \cdot y_0})}{(z - x_0 - i \cdot y_0)(z + \frac{R^2}{-x_0 + i \cdot y_0})} \quad (4.13)$$

The streamlines showed in Fig. 4.6 is the equipotential lines of function ψ , which is the imaginary part of the complex potential W . According to the theory of potential flow, the first derivative of the complex potential with respect to z represents the velocity field $\vec{V} = (u, v)$. The real and imaginary parts of this first derivative represent the velocity components in x -axis and y -axis respectively. Differentiating W in equation (4.13) with respect to z results in the velocity components, and the velocity distribution along y -axis is as follows:

$$\begin{cases} u = \frac{4x_0}{R^2 + y^2 - 2y_0 y} \\ v = 0 \end{cases} \quad (4.14)$$

The position of A_2 and B_2 (Fig. 4.7), as well as the flow rates for core and

Figure 4.7: Interface positions of the optofluidic lens.

cladding streams can be determined by the curvature of the two interfaces:

$$\left\{ \begin{array}{l} Q_{\text{cladding B}} = \int_{B_2}^{B_1} u \cdot dy = \frac{4x_0}{\sqrt{R^2 - y_0^2}} [\arctan((R - y_0)/\sqrt{R^2 - y_0^2}) - \arctan((y_{B_2} - y_0)/\sqrt{R^2 - y_0^2})] \\ Q_{\text{core}} = \int_{A_2}^{B_2} u \cdot dy = \frac{4x_0}{\sqrt{R^2 - y_0^2}} [\arctan((y_{B_2} - y_0)/\sqrt{R^2 - y_0^2}) - \arctan((y_{A_2} - y_0)/\sqrt{R^2 - y_0^2})] \\ Q_{\text{cladding A}} = \int_{A_1}^{A_2} u \cdot dy = \frac{4x_0}{\sqrt{R^2 - y_0^2}} [\arctan((y_{A_2} - y_0)/\sqrt{R^2 - y_0^2}) - \arctan((-R - y_0)/\sqrt{R^2 - y_0^2})] \end{array} \right. \quad (4.15)$$

Two pairs of source-sink bounded in a circular domain

The theory of dipole flow now can be further applied to address the case of multiple source-sink pairs bounded in circular domain. Figure 4.8 illustrates the design of an optofluidic concave lens developed in a circular chamber by placing two pairs of inlets/outlets that can be considered as two source-sink pairs bounded in a circular domain. Assuming that the two sources are placed at $(-x_0, y_0)$ and $(-x_0, -y_0)$, and the

Figure 4.8: Schematic of optofluidic concave lens developed in a circular chamber

two sinks are positioned at (x_0, y_0) and $(x_0, -y_0)$ respectively as illustrated in Figure 4.9.

The complex potential of this flow field can be written as:

$$W = \varphi + i\psi = \ln \left[\frac{(z + x_0 - i \cdot y_0)(z + \frac{R^2}{x_0 + i \cdot y_0})}{(z - x_0 - i \cdot y_0)(z + \frac{R^2}{-x_0 + i \cdot y_0})} \frac{(z + x_0 + i \cdot y_0)(z + \frac{R^2}{x_0 - i \cdot y_0})}{(z - x_0 + i \cdot y_0)(z + \frac{R^2}{-x_0 - i \cdot y_0})} \right] \quad (4.16)$$

The streamlines shown in Figure 4.9 is the equipotential lines of the function ψ , which is the imaginary part of the complex potential.

4.2.2 Two-fluid model-the theory of hydrodynamic spreading

The theory of dipole flow allows the analyze of the flow field in a circular domain accurately and conveniently. The streamlines and velocity field can be easily formulated. Subsequently the relationship between the interface curvature of the optofluidic lens and the flow rate ratio can be described analytically. Even a relatively complicated situation of a concave lens as discussed above, can be addressed by this model using multiple pairs of source and sink. However, the assumed condition to apply the dipole flow theory is uniformity of viscosity of the liquid across the whole flow field. Normally building the

Figure 4.9: Streamlines of two source-sink pairs in a circularly bounded domain.

optofluidic lenses requires that the refractive index of the core liquid is different from that of the cladding liquid. Therefore, different liquids are used as the core and cladding streams in the actual optofluidic lens. So the viscosities of the different streams are not unanimous consequently. As a result, the dipole flow theory is not applicable to address most of cases of the liquid-core liquid-cladding configuration possessing different refractive indices, unless preliminary adjustment of the viscosity are carried out by mixing of different liquids.

This section introduces an analytical model based on hydrodynamic spreading theory [105, 76] to address the problem of mismatched viscosities of the core liquid and the cladding liquid in a symmetric geometry. Figure 4.10 (a) shows the the model of a L^2 system for optical waveguide based on hydrodynamic spreading theory [105, 76]. Assuming that the three streams comprise the liquids with different viscosities and flow

Figure 4.10: Schematic of hydrodynamic spreading in microfluidic channels of different geometries:

(a) straight-line channel; (b) circularly bounded chamber.

rates, and the viscosity ratios and flow rate ratios are

$$\beta_1 = \mu_1/\mu_0, \beta_2 = \mu_2/\mu_0 \quad (4.17)$$

and

$$\phi_1 = Q_1/Q_0, \phi_2 = Q_2/Q_0 \quad (4.18)$$

respectively. In the above equations, μ and Q represent the viscosity and the flow rate respectively. The core stream and the cladding streams are denoted with the subscripts 0, 1, and 2, respectively. Based on the model of hydrodynamic focusing reported in the literature [105, 76], the width ratios can be estimated as

$$W_0/W_1 = 1/(1 + 2\beta_1\phi_1), W_0/W_2 = 1/(1 + 2\beta_2\phi_2). \quad (4.19)$$

Considering a symmetric system (the two cladding streams of the same liquid have the same flow rate), the relationship between the flow rate ratio ($Q_{\text{core}}/Q_{\text{cladding}}$) and the widths of the core and cladding streams can be derived from equation (4.19):

$$\beta \cdot \frac{Q_{\text{core}}}{Q_{\text{cladding}}} = \frac{2W_0}{W_1 - W_0} \quad (4.20)$$

where $\beta = \mu_{\text{core}}/\mu_{\text{cladding}}$ is the viscosity ratio between cladding and core liquid. In this configuration as shown in Fig. 4.10 (a), all the streamlines are parallel to the side wall of the channel because of the characteristic laminar flow in micro-scale.

Fig. 4.10 (b) shows the situation of hydrodynamic spreading in a circular chamber. The streamlines spread from a source point A and flow back to a sink point B in a circularly bounded domain. All streamlines inside the circular domain have an arc-shape. At the beginning of the streamlines (source point A), the flow rate ratio as well as the viscosity ratio conjunctly decides the opening angles of the core stream and the

cladding stream. Consequently, the curvature of the interface between the core stream and the cladding stream can be determined, because each streamline path has its unique arc-shape in a circular chamber. Equation (4.20) can be written in the angular form as:

$$\beta \cdot \frac{Q_{\text{core}}}{Q_{\text{cladding}}} = \frac{2\theta}{\pi/2 - \theta} \quad (4.21)$$

Where 2θ and the $\pi/2 - \theta$ are the opening angles of the core stream and the cladding stream at the source point A, respectively. Equation (4.21) also assumes that the two cladding streams are symmetric. Since $\overline{AM}$ is the tangent of the arc $\widehat{APB}$, the angle $\angle PAO$ is derived from geometric rules as:

$$\angle PAO = \frac{\theta}{2} = \arctan(y/R) \quad (4.22)$$

Where O is the center of the circular domain, P is the middle point of $\widehat{APB}$, y is the length of $\overline{PO}$, and R is the radius of the circular domain. The length y can be represented by a function of the radius r of the curvature $\widehat{APB}$ as:

$$y = r - \sqrt{r^2 - R^2} \quad (4.23)$$

By substituting equation(4.22) and (4.23) into (4.21), the relationship between flow rate ratio and the radius of the interface can be analytically described as:

$$\frac{Q_{\text{core}}}{Q_{\text{cladding}}} = \frac{1}{\beta} \cdot \frac{2 \arctan(r/R - \sqrt{r^2 - R^2}/R)}{\pi/4 - \arctan(r/R - \sqrt{r^2 - R^2}/R)} \quad (4.24)$$

If the core liquid and cladding liquid have the same viscosity, equation (4.24) reduces to the same form previously derived by the model based on the theory of dipole flow bounded in a circular domain discussed in section 4.2.1.

Considering the asymmetric case where the two cladding streams of the same liquid have different flow rates, and the flow rate of cladding 1 (Figure 4.10 (c)) is equal

to that of the core stream. By varying the flow rate ratio, asymmetric lenses can be formed, and the radii of the upper and lower interface can be derived in the same way as shown in equation (4.17-4.19):

$$\begin{aligned} \frac{Q_{\text{cladding2}}}{Q_{\text{core}}} = \frac{Q_{\text{cladding2}}}{Q_{\text{cladding1}}} &= \frac{(1+\beta)(2 \arctan(r_1/R - \sqrt{r_1^2 - R^2}/R) + \pi/2) - \pi}{\beta(\pi/2 - 2 \arctan(r_1/R - \sqrt{r_1^2 - R^2}/R))} \\ &= \frac{(1+\beta)(\arctan(r_2/R - \sqrt{r_2^2 - R^2}/R) + \pi/4)}{\beta(\pi/4 - \arctan(r_2/R - \sqrt{r_2^2 - R^2}/R))} \end{aligned} \quad (4.25)$$

where r_1 and r_2 are the radii of interfaces $\widehat{APB}$ and $\widehat{AQB}$ respectively (Figure 4.10 (c)).

When the core and cladding streams are of same liquid ($\beta=1$), equation (4.25) can be simplified into the same form as discussed in the above section.

4.2.3 Gaussian optics model

In the case of the liquid-core liquid-cladding optofluidic lens, optically smooth interfaces are formed by three streams of flow, and each interface has a constant curvature of radius. This structure can be simply and sufficiently described by a combination of two surfaces which are rotationally symmetric about the optical axis as illustrated in Figure 4.11. In this situation, the principal planes located across the surface vertices and their positions l_H and l'_H are given as:

$$\begin{cases} l_H = -f' \frac{n-1}{n} \frac{d}{r_2} \\ l'_H = -f' \frac{n-1}{n} \frac{d}{r_1} \end{cases} \quad (4.26)$$

where n is the refractive index of the lens, d is the distance between two surface vertices, r_1 and r_2 are the radii of the two interface, and f' is the focal length of the lens constructed by a combination of these two surfaces:

$$f' = \frac{nr_1r_2}{(n-1)[n(r_2-r_1) + (n-1)d]} \quad (4.27)$$

Figure 4.11: A thick lens is constructed by a combination of two surfaces with given radii of curvature that are rotationally symmetrical about the optical axis.

Combining equation (4.26) and (4.27), the positions of focal points with respect to front and rear surface vertices are respectively:

$$\begin{cases} l_F = -f' \left(1 + \frac{n-1}{n} \frac{d}{r_2} \right) \\ l'_F = -f' \left(1 - \frac{n-1}{n} \frac{d}{r_1} \right) \end{cases} \quad (4.28)$$

Taking the cladding as the reference, the relative refractive indices of core and cladding streams are n and 1, respectively.

Focal length is an important parameter to characterize a lens. At given refractive indices, the focal length is a function of the radii of the two interfaces. For mathematical simplicity, we confine the degrees of freedom to only one by assuming a case of symmetrical double-convex lens (Fig. 4.1 (a)) or fix one radius of interface at an infinitely large value, which is actually the case of plano-convex lens, Fig. 4.1 (b). The viscosities of core and cladding liquids are assumed to be the same. Then we discuss the flow rate ratios between core and cladding streams with a given focal length.

In the case of symmetrical bi-convex lens (Fig. 4.1 (a)), the moduli of radii of two interfaces are both equal to r . The focal length is formulated as:

$$f = \frac{nr^2}{(n-1)[(n-1)d - 2nr]} \quad (4.29)$$

where r is the radius of the interface of the lens, n is the relative index of the core stream, and d is the distance between the vertices of two interfaces. In the case of $r \gg d$, the focal length in equation (4.29) has an approximate expression of:

$$f = \frac{r}{2(n-1)} \quad (4.30)$$

Under the assumption that the moduli of radii of two interfaces are equal, the flow rate ratio between cladding and core streams is according to equation (4.12):

$$\frac{Q_{\text{cladding}}}{Q_{\text{core}}} = \frac{\pi}{8 \arctan\left(\frac{r - \sqrt{r^2 - R^2}}{R}\right)} - \frac{1}{2} \quad (4.31)$$

Substituting Equation (4.30) into (4.31), the relationship between the flow rate ratio and the focal length of the symmetrical bi-convex optofluidic lens is:

$$\frac{Q_{\text{cladding}}}{Q_{\text{core}}} = \frac{\pi}{8 \arctan\left(\frac{2(n-1)f - \sqrt{4(n-1)^2 f^2 - R^2}}{R}\right)} - \frac{1}{2} \quad (4.32)$$

Figure 4.12 (a) depicts the relationship between the focal length and the flow rate ratios as well as the refractive index. The focal length is almost linearly proportional to the flow rate ratio. At the same refractive index, the higher the flow rate ratio between the cladding and the core streams, the longer is the focal length. At the same flow rate ratio, the higher the refractive index the shorter is the focal length.

In the case of a plano convex lens as depicted in Fig. 4.1 (b), one radius of an interface is infinitely large $r_2 = \infty$; while the other has a finite dimension $r_1 = r$. Then

the focal length can simplified into:

$$\lim_{r_2 \rightarrow \infty} f = \frac{r}{n-1} \quad (4.33)$$

Under this assumption, the flow rate ratio between cladding A and core stream is calculated as follows:

$$\frac{Q_{\text{cladding A}}}{Q_{\text{core}}} = \frac{\pi - 4 \arctan\left(\frac{(n-1)f - \sqrt{(n-1)^2 f^2 - R^2}}{R}\right)}{4 \arctan\left(\frac{(n-1)f - \sqrt{(n-1)^2 f^2 - R^2}}{R}\right)} \quad (4.34)$$

It is obvious that the cladding flow from inlet B takes up a half space of the circular chamber, which means the flow rate of cladding B is simply the sum of the flow rates of core stream and cladding stream A. Figure 4.12 (b) shows the relationship between the focal length and flow rate ratio between the cladding stream A and the core stream.

In the case of plano-convex lens achieved with the asymmetric geometry (Figure 4.7), one radius of an interface is infinitely large $r_2 = \infty$; while the other has a finite dimension $r_1 = r$. Then the flow rate ratio as a function of focal length can be written as:

$$\frac{Q_{\text{cladding B}}}{Q_{\text{core}}} = \frac{\pi/3 - \arctan\left(\frac{y_0 + (n-1)f - \sqrt{(n-1)^2 f^2 - x_0^2 + 0.5R}}{\sqrt{3R/2}}\right)}{\arctan\left(\frac{y_0 + (n-1)f - \sqrt{(n-1)^2 f^2 - x_0^2 + 0.5R}}{\sqrt{3R/2}}\right)} \quad (4.35)$$

and the above relationship is illustrated in Figure 4.13.

In the case of a two-fluid flow in a symmetric circular chamber, the focal length as a function of flow rate ratio between core and cladding streams can be formulated by combining Eqn. (4.24) and (4.30):

$$\frac{Q_{\text{core}}}{Q_{\text{cladding}}} = \frac{1}{\beta} \cdot \frac{2 \arctan\left(\frac{2(n-1)f - \sqrt{4(n-1)^2 f^2 - R^2}}{R}\right)}{\pi/4 - \arctan\left(\frac{2(n-1)f - \sqrt{4(n-1)^2 f^2 - R^2}}{R}\right)} \quad (4.36)$$

Figure 4.12: Relationships between flow rate ratio and focal length (the dimension of focal length is normalized by the radius of the circular chamber): (a) symmetrical bi-convex lens; (b) plano-convex lens.

Figure 4.13: Relationships between flow rate ratio and focal length (the dimension of focal length is normalized by the radius of the circular chamber). A plano-convex lens is developed in the asymmetric geometry. To fabricate a lens with a required focal length, the corresponding flow rate ratio is illustrated.

4.3 Experimental results and discussions

4.3.1 Single-fluid model in symmetric geometry

The testing devices were fabricated with soft lithography technique, which is elaborated in Chapter 2. In the experiment, cladding liquid is de-ionized (DI) water mixed with fluorescence dye (fluorescein disodium salt $C_{20}H_{10}Na_2O_5$, Acid Yellow 73 or C.I. 45350) and $3\text{-}\mu\text{m}$ red fluorescent particles (Duke Scientific Co.) were used to visualize the cladding liquid and its stream lines. The separated fluorescent bands of the fluorescent particles (540/610 nm) and of the dye (490/520 nm) allows easy imaging by switching the epi-fluorescent attachments on the microscope (Nikon EclipseTE 2000-S,

Figure 4.14: The streamlines of the flow in the circular chamber are extracted by tracing of fluorescent particles. The asterisks in the figure represent the experimental results, and the lines are the fittings arcs (all dimensions are normalized by the radius of the lens chamber).

Japan). DI water works as the core liquid. The liquids were kept in 5-ml glass syringes, which are driven by two syringe pumps (KDS230, KD Scientific Inc, USA). Any flow rate ratio can be adjusted. A sensitive CCD camera (HiSense MKII) attached to the microscope was used to capture the fluorescent images.

First, flow visualization with fluorescent particles verified that the streamlines in the circular chamber have a perfect arc-shape, which is of great importance for designing the optofluidic lens. As mentioned above, red fluorescent particles are diluted in the two cladding flows that maintain the same flow rates as that of core flow. A sufficiently long exposure time of 85 ms was chosen, within which a particle can travel from the inlet to the outlet of the circular chamber. In this way, the particles will leave a streak line on the image as illustrated in the inset of Fig. 4.14. Since the flow is laminar in our case, the streak lines of the particles correspond to the streamlines of the flow in the chamber.

We extract a group of positions for each of three paths showed in Fig. 4.14, and use an arc to fit each of them. The good agreement between the extracted positions and the fitting arcs confirm that the streamlines in the circular chamber have a perfect arc-shape, which can not be achieved with a rectangular chamber [54].

Figure 4.15: Symmetrical bi-convex lens. The flow rates of two cladding streams are kept the same. The flow rate of core stream varies to achieve symmetrical biconvex lenses with different interface curvatures. The insets illustrate the fluorescence images of the lenses. The dimensions are normalized by the radius of circular chamber. The flow rate of core stream was kept at 0.6mL/h, and the flow rate of cladding stream varies from 0.1mL/h to 1.2mL/h. The corresponding Reynolds number ranges from 0.44 to 1.67, and Peclet number ranges from 444 to 1667.

For the case of a symmetric lens as illustrated in Fig. 4.15, the flow rate of core

stream was fixed at 0.6mL/h, while the flow rates of two cladding streams vary equally from 0.1 mL/h to 1.2 mL/h. The corresponding Reynolds number ranges from 0.44 to 1.67, and Peclet number estimated for the fluorescent dye ranges from 444 to 1667. By this way, a symmetrical biconvex lens was achieved, and the curvature of interface is tuned by varying the flow rates of cladding streams. When the flow rates of cladding streams decrease, the interface of this fluidic lens approaches the wall of the chamber but still remains as an arc-shape. In the case of the lens reported by Whitesides' group [54], the paraxial region of the lens was not bent with an perfect arc-shape, but flat especially when the interface approaches the wall of their rectangular chamber. Therefore our circular design to form the fluidic lens is demonstrated here to have inherently robust wide-range tunability. This advantage allows the realization a smaller radius of interface, and thus a shorter focal length, which shows a potential to increase the level of integration of a lab-on-a-chip optical component.

The relationship between the flow rate ratio and the curvature of the interface can be analytically described by deriving Eqn. (4.37):

$$\frac{Q_{\text{core}}}{Q_{\text{cladding}}} = \frac{2 \arctan\left(\frac{r - \sqrt{r^2 - R^2}}{R}\right)}{\pi/4 - \arctan\left(\frac{r - \sqrt{r^2 - R^2}}{R}\right)} \quad (4.37)$$

where R is the radius of the circular chamber, and r denotes the radius of curvature of interface. Figure 4.15 shows the normalized curvature of the interface (R/r) versus the flow rate ratio between the core and the cladding stream. The results show that experimental data agrees well with the theoretical prediction. Thus this analytical model of the optofluidic lens can serve as a basic mathematic tool to design a fluidic lens if specific radii of interfaces are required.

To test the fluidic lens in the case of an asymmetric lens, the flow rates of

Figure 4.16: Asymmetrical lens. The flow rates of upper cladding and core streams are kept the same, and the flow rate of lower cladding stream varies to achieve biconvex, plano-convex, and meniscus lenses. The insets illustrate the fluorescence images of the lenses. The dimensions are normalized by the radius of the circular chamber. The flow rates of upper cladding and core streams are fixed equally at 0.6 mL/h, and the flow rate of lower cladding varies from 0.15 mL/h to 2.7 mL/h. The corresponding Reynolds number ranges from 0.75 to 2.17, and Peclet number ranges from 750 to 2167.

the core and the upper cladding streams are fixed equally at 0.6mL/h (insets in Fig. 4.16), and varied the flow rate of the lower cladding stream in a range from 0.15mL/h to 2.7mL/h. The corresponding Reynolds number ranges from 0.75 to 2.17, and the Peclet number ranges from 750 to 2167. A plano-convex lens is achieved when the flow rate of the lower cladding stream is equal to the sum of the flow rates of the core and the upper cladding stream or a flow rate ratio of 2. When the flow rate ratio decreases, a biconvex lens is formed. If the flow rate ratio is larger than 2, a meniscus lens can be constructed. With this test, different kinds of lenses were demonstrated in our circular chamber design.

The curvatures of lower and upper interfaces can also be mathematically defined by flow rate ratio derived from Eqn. (4.12) assuming equal flow rates of the core stream and the upper stream:

$$\frac{Q_{\text{cladding B}}}{Q_{\text{core}}} = \frac{2 \arctan\left(\frac{r_1 - \sqrt{r_1^2 - R^2}}{R}\right)}{\pi/4 - \arctan\left(\frac{r_1 - \sqrt{r_1^2 - R^2}}{R}\right)} = \frac{\pi/4 + \arctan\left(\frac{r_2 \pm \sqrt{r_2^2 - R^2}}{R}\right)}{\pi/8 - \arctan\left(\frac{r_2 \pm \sqrt{r_2^2 - R^2}}{R}\right)/2} \quad (4.38)$$

where r_1 and r_2 are the radii of upper and lower interfaces, respectively. The solid lines in Fig. 4.16 depict the predicted curvatures of the two cladding streams versus the flow rate ratio between lower cladding and core. We define the sign of curvature to be positive if the center of curvature lies at the lower side of channel. The experimental results agree well with the theoretical analysis.

The device was also tested at higher flow rates, whose Reynolds numbers range from 2 to 9. When the Reynolds number is relatively small, the flow can be fully developed in the circular chamber and follows stream lines predicted with the dipole theory. However with an increasing Reynolds number, the inertia causes flow separation at the entrance of the chamber. The flow passes through the chamber without following the

Figure 4.17: Effect of flow separation on lens shape: (a) low flow rate 3.6 mL/h ($Re = 2$) (b) high flow rate 16.2 mL/h ($Re = 9$).

streamlines as in low flow rate case. This phenomenon causes the lens to be shifted along the flow direction and the shape of the lens is distorted.

4.3.2 Single-fluid model in asymmetric geometry

To test the design, in which the lens-injecting channel is placed with an offset to the axis of circular chamber, a plano-convex lens was fabricated to validate its mathematically predictable shape. To achieve a plano-convex lens, the y -axis position of A_2 should be as the same as the source or sink, which is equal to y_0 . Substituting $y_{A_2} = y_0$ into Eqn. (4.15), the relationship between the position of upper interface $S_+\widehat{B_2}S_-$ (Fig.

4.7) and the flow rate ratio can be formulated as:

$$\frac{Q_{\text{core}}}{Q_{\text{cladding B}}} = \frac{\arctan\left(\frac{y_{B_2} + 0.5R}{\sqrt{3}R/2}\right)}{\pi/3 - \arctan\left(\frac{y_{B_2} + 0.5R}{\sqrt{3}R/2}\right)}. \quad (4.39)$$

The position of intersections between the interface and y -axis is calculated as:

$$y_{B_2} = y_0 + r - \sqrt{r^2 - x_0^2}, \quad (4.40)$$

where r is the radius of the curvature of the upper interface $S_+ \widehat{B_2} S_-$ (Fig. 4.7). Combining Eqn. (4.39) and (4.40), a function of the flow rate ratio dependent on radius of the curvature is retrieved:

$$\frac{Q_{\text{core}}}{Q_{\text{cladding B}}} = \frac{\arctan\left(\frac{y_0 + r - \sqrt{r^2 - x_0^2} + 0.5R}{\sqrt{3}R/2}\right)}{\pi/3 - \arctan\left(\frac{y_0 + r - \sqrt{r^2 - x_0^2} + 0.5R}{\sqrt{3}R/2}\right)} \quad (4.41)$$

where $(-x_0, y_0)$ and (x_0, y_0) are the positions of source and sink respectively. $R = 2y_0$ is the radius of the circular chamber, and r is the radius of the upper interface.

In the characterization, the total flow rate was fixed at 2.4 mL/h, and the corresponding Reynolds number and Peclet number were 1.33 and 1333 respectively. The flow rate of the cladding A was kept at a constant value of 0.8mL/h, whereas the flow rate ratio between the core and the cladding B was tuned in a range from 0.33 to 11. Fig. 4.18 shows the relationship between the flow rate ratio ($\phi_{\text{core}}/\phi_{\text{cladding B}}$) and the curvature of the upper interface theoretically and experimentally. As the flow rate ratio increased, the upper interface became more intensively curved. However its shape remained an arc even when the interface approached the wall of the chamber. One highlighted point is that the maximum curvature. According to Eqn. (4.41), the curvature of the upper interface reaches its maximum value of $R/x_0 = 1.15$, when the flow rate of the core stream is 3 times of that of cladding B (the curvature of the wall of

Figure 4.18: The relationship between the curvature of interface and the flow rate ratio. The insets illustrate the fluorescence images of the lenses. The dimensions are normalized by the radius of circular chamber. The total flow rate was fixed at 2.4 mL/h, and the corresponding Reynolds number and Peclet number were 1.33 and 1333 respectively. The flow rate of lower cladding was kept at a constant 0.8mL/h, whereas the flow rate ratio between core and cladding B was tuned in a range from 0.33 to 11.

circular chamber normalized by itself is equal to 1). This was verified by the experiment illustrated in Fig. 4.18. This enhancement of curvature due to the shift of lens-injecting channel makes possible shortening the focal length and therefore reinforce the focusing ability. For comparison, the theoretical curvature of the interface of a plano-convex lens developed in symmetric geometry is depicted as the dotted line in Fig. 4.18. The limit of the curvature finally goes to 1, when the flow rate ratio becomes infinitely large.

To demonstrate the focusing ability of the lens, benzyl alcohol ($n = 1.538$) was

used to shape the lens and converge the light beam, and ethylene glycol ($n = 1.429$) was used to match the refractive index of PDMS. To strongly focus the light beam, the inlet A (Fig. 4.7) was blocked, and let the core stream took up one of the cladding streams. In this way, one of the lens interfaces is fixed with a curvature equal to that of the circular chamber; the other is tuned by changing the flow rate ratio between the lower (benzyl alcohol) and upper (ethylene glycol) streams (Fig. 4.19). The lower fixed chamber wall and the upper reconfigurable interface form the optofluidic lens. Because the refractive index ethylene glycol and PDMS almost match each other, thus the upper stream has a negligible impact on beam focusing. In the experiment, the light emitting and detection fibers were slotted into a pre-defined channels, and aligned stably along the optical axis of the lens. Fig. 4.19 shows the experimental result of the signal variation. By varying the flow rate ratio, we can change the curvature of the upper interface, and therefore tune the focal length of the optofluidic lens. As the flow rate ratio increases, the upper interface becomes more intensively curved, resulting in a shorter focal length. Therefore the focal point is drawn to approach the circular chamber from far away. When the flow rate ratio between lower and upper streams reached 4 times, the signal output was maximum. At this maximum, the divergence of the focal spot created may match the NA of the fiber. Increasing the flow rate ratio further causes a drop in the output signal. In this experiment, the signal variation proves that the single lens has a tunable focal length. Also noted that the laser provide a power of 1.5mW and the received power by the photodiode is about 10^{-3} mW. The power loss may happen at the laser-fiber and fiber-photodiode couplings as well as within the optofluidic system.

Figure 4.19: The variation of the signal output by changing the flow rate ratio. Emission and detection fibres are aligned and positioned along the optical axis of the optofluidic lens to test its focusing ability. The upper interface of the lens is tuned by changing the flow rate ratio of lower to upper streams, which results in a shifting of the focal point along the optical axis. The signal output is normalized by its maximum.

4.3.3 Two pairs of source-sink in symmetric geometry

In this part, an optofluidic concave lens is developed in a circular chamber by placing two source-sink pairs on the boundary (Fig. 4.8 and 4.9). The lens is demonstrated to perform light focusing with a low-RI liquid. The liquids to form the lens are ethanol ($n=1.36$ $\rho=0.789$ g/ml $\mu=1.074 \times 10^{-3}$ Pa·s) and benzyl alcohol ($n=1.54$ $\rho=1.044$ g/ml $\mu=8.0 \times 10^{-3}$ Pa·s), which are relatively safe to use. Figure 4.20 depicts the concept of the liquid-core liquid-cladding optofluidic biconcave lens in a circular chamber to focus light. In traditional geometric optics, lenses are fabricated with glass, which has a higher RI than the surrounding medium. Thus a concave lens cannot be used to focus light.

However in optofluidic systems, the properties of optical components can be adjusted by simply changing the liquids inside the system. In the micro optofluidic lens depicted in Figure 4.20(a), ethanol and benzyl alcohol were used as the core and cladding liquids, respectively. The lower RI of the core enables the concave lens to have light focusing ability.

Figure 4.20: (a) Schematic diagram of an optofluidic system with a tunable biconcave lens. The system includes a telescope system, which collimates and expands the light beam, and an optofluidic biconcave lens to focus the rays. The multi-mode optical fiber has a numerical aperture of 0.22 and a diameter of $125\ \mu\text{m}$. The radii of the interface of the PDMS solid lenses are $700\ \mu\text{m}$ and $400\ \mu\text{m}$. The diameter of the circular lens chamber is 1 mm. (b) The telescope system consists of two PDMS lenses that collimate and expand the divergent beam from the multi-mode optical fiber ($\text{NA} = 0.22$). The lens chamber is filled with stationary fluorescence liquid to visualize the light rays. The outline depicts the ray propagation.

To shape a parallel beam for the characterization of the micro optofluidic lens, A

telescope system consisting of two solid lenses made of PDMS was used to collimate and expand the divergent green beam (532 nm) emitted from an optical fiber. This parallel beam with a relatively large width incidents on the optofluidic biconcave lens formed in the circular chamber. A rays-visualization chamber is located behind the lens chamber along the light propagation direction. Rhodamine B diluted in a mixture of glycerol (60% by weight) and water (40% by weight) which has matched RI with PDMS was used to visualize the beam. The incident light excites the fluorescent dye and so visualizes the ray path. The cladding flow rates were kept constant at 1 ml/hr each, while the flow rate of the core stream is adjusted to get the different flow rate ratios ($\Phi = Q_{\text{core}}/Q_{\text{cladding}}$). Adjusting the flow rate ratio allows tuning the focused beam.

The liquid-core liquid-cladding concave lens was developed in a circular chamber (Fig. 4.21) to demonstrate its light focusing ability. The fluorescence dye solution with RI matched to PDMS was introduced into the visualization chamber to trace the rays coming out from the concave lens. Figure 4.21 shows that the parallel beam was focused to the optical axis after passing the concave lens at flow rate ratios of $\Phi = 2.0$ and $\Phi = 3.0$. The variable Φ is defined as the ratio between the flow rates of the core stream and one cladding stream. As a comparison benzyl alcohol flow was stopped, and subsequently ethanol fully filled the circular chamber. Since the chamber represents a biconvex lens with lower RI, the light beam become divergent in the rays-visualization chamber (Fig. 4.21(c)).

For a flow rate ratio of $\Phi = 3.0$, the intensity profiles at different lines (Fig. 4.22) were extracted and fitted with Gaussian curves. All the intensity values were normalized by the peak value of intensity along line A. We measured the intensity distributions at

Figure 4.21: A liquid-core liquid-cladding biconcave lens formed in the circular chamber at different flow rate ratios: (a) $\Phi = 2.0$; (b) $\Phi = 3.0$. Ethanol ($n = 1.36$) works as core stream and benzyl alcohol ($n = 1.54$) serves as cladding streams. The parallel beam leaving the telescope system passes through the biconcave lens and is focused to the optical axis. (c) For comparison, the circular chamber was fully filled with ethanol, which causes the parallel beam to diverge.

lines located $810\ \mu\text{m}$, $1100\ \mu\text{m}$, and $1330\ \mu\text{m}$ away from the biconcave lens. The beam width is defined as the full width at half maximum (FWHM) of the fitting curves. Figure 4.22 shows that the beam width decreases along the light propagation direction, which verifies that the light beam is focused by the optofluidic biconcave lens. We found that the narrowest beam width of $370\ \mu\text{m}$ is at line C. After passing line C, the beam width remained almost constant. This phenomenon may be caused by the uneven curvature distribution along the interface of the concave lens. Due to the limit of the streamline inside the chamber and the surface tension between the two fluids, the curvature at the paraxial region is very small, and therefore the bundle of rays passing this paraxial region will not be bent, but propagate straightly. The non-paraxial rays will be bent due to the large curvature at off-axis region. After the beam waist, the light rays was observed to be parallel. This is because the mount of paraxial rays is larger than that of the off-axis rays which are bent un-evenly and finally scattered and absorbed by the fluorescence. So after a long optical path of propagation, only the un-bent paraxial rays can be observed, which look like collimated. The light beam waist was found to be located at line C in the inset of Figure 4.22. This experiment verifies that our optofluidic biconcave lens can perform light focusing.

To demonstrate the tunability of the optofluidic biconcave lens, the fluorescence intensities profile along line C in Figure 4.22 were measured at different flow rate ratios Φ . The results are shown in Figure 4.23. The three insets in Figure 4.23 illustrate the intensity distributions generated with flow rate ratios of 2.0, 3.0, and 3.6, respectively. The peak values of the intensity profiles of $\Phi = 2$ (4.23(a)) and $\Phi = 3$ (4.23(b)) are approximately the same, but the beam width of $\Phi = 2$ is larger than that of $\Phi = 3$.

Figure 4.22: Under a flow rate ratio of 3.0, the intensity profiles along different lines (dotted lines in the inset) were measured and fitted with Gaussian curves. All the intensity values are normalized by the peak value of intensity along line A. l_A , l_B , and l_C are the distances from the dotted lines to the concave lens, respectively.

The light beam of $\Phi = 3.6$ (4.23(c)) was measured to have the same width as the one of $\Phi = 3$, but the peak of intensity value is lower. The comparison shows that the beam waist of $\Phi = 2$ is further away from the micro optofluidic lens as that of $\Phi = 3.6$. These results show that the beam characteristics can be tuned by adjusting the flow rate ratio between the core and the cladding streams.

Figure 4.23: Characteristics of the beam formed by the optofluidic lens at different flow rate ratios. The intensity profiles were measured along the dotted line in the insets. Gaussian curves are fitted to intensity profiles respectively. All the intensity values are normalized by the peak value of intensity profile in inset (a). The full width at half maximums for the cases depicted in insets (a), (b) and (c) are $430\mu\text{m}$, $370\mu\text{m}$ and $370\mu\text{m}$, respectively.

4.3.4 Two-fluid flow in symmetric geometry

Figure 4.24 (a) depicts the schematic configuration of a liquid-core liquid-cladding (L^2) optofluidic bi-convex lens developed in a circular chamber. If the core liquid is sandwiched by two cladding liquids entering the circular chamber, the core follows the arc-shape streamlines inside the chamber and therefore forms the typical shape of a biconvex lens. In this experiment, cinnamaldehyde (Sigma Aldrich, USA) with a viscosity of $\mu = 5.7 \times 10^{-3} \text{ Ns/m}^2$ at 25°C and a refractive index of $n = 1.62$ serves as the core stream to shape the lens. A mixture of 73.5% ethylene glycol and 26.5% ethanol by weight with a viscosity of $\mu = 9.8 \times 10^{-3} \text{ Ns/m}^2$ at 25°C and refractive index of $n = 1.412$ matching

that of polydimethylsioxane (PDMS) is used as the cladding stream. The curved interface and the mismatched refractive indices between the core stream and the cladding stream allow this configuration to focus the in-plane light beam. By adjusting the flow rate ratio ($Q_{\text{core}}/Q_{\text{cladding}}$), the interface between core and cladding liquids can be tuned accordingly. As a result, the curvature of the interface as well as the focal length of the lens can be manipulated. To further investigate the effect of viscosity mismatch, two other liquid systems were used for verifying the model. The first liquid system consists of DI water with a viscosity of $\mu = 1.0 \times 10^{-3}$ Ns/m² as the core, and the mixture of 55% glycerol and 45% DI water with a viscosity of $\mu = 7.6 \times 10^{-3}$ Ns/m² as the cladding. The second liquid system comprises the mixture of 20% glycerol and 80% DI water with a viscosity of $\mu = 1.8 \times 10^{-3}$ Ns/m² as the core, and DI water as the cladding.

A multi-mode optical fiber (AFS105/125Y, THORLABS Inc.) with a numerical aperture of NA=0.22 was inserted into a pre-fabricated microchannel with a width of 130 μm to work approximately as a point light source. The optical fiber was located at a distance of 4 mm from the center of the circular chamber which has a diameter of 1 mm. An aperture was formed by filling black ink into two channels with a width of 50 μm . The size of the aperture is 350 μm . The purpose of the aperture is to confine the divergence of the light beam, and to allow only the paraxial rays to incident on the optofluidic lens. Therefore, the spherical aberration can be reduced and the quality of the focused light beam can be improved. Between the optical fiber and the aperture stands a hollow chamber for positioning the tip of the optical fiber. Before the experiment, a mixture of 73.5% ethylene glycol and 26.5% ethanol was injected into this hollow chamber to form an index-matching medium between the optical fiber and the lens chamber. To

Figure 4.24: (a) Schematic configuration of the experimental setup for focusing the light beam emitted from an optical fiber by an L^2 optofluidic bi-convex lens developed in a circular chamber. (b) Bright field image of the L^2 optofluidic bi-convex lens focusing a light beam. Cinnamaldehyde is used as the core stream, and a mixture of 73.5% ethylene glycol and 26.5% ethanol served as the cladding stream. The flow rate ratio between core and cladding streams is 7.

visualize the light beam focused by the lens, a ray-tracing chamber was placed behind the circular chamber along the light propagation direction. Fluorescence dye Rhodamine B (Sigma Aldrich, excitation wavelength of 540 nm, emission wave length of 625 nm) diluted in a mixture of glycerol (60% by weight) and water (40% by weight) which has a matching refractive index of $n = 1.412$ was injected into the ray-tracing chamber to visualize the light rays. The color bright field images were recorded with a digital camera (DCRDVD803E, SONY), while the grey-scale fluorescent images was recorded with a interline transfer CCD camera (Sony ICX 084).

To verify the model based on hydrodynamic spreading of two-fluid flow in a

Figure 4.25: Relationship between the curvature of the interface and the flow rate ratio ($Q_{core}/Q_{cladding}$). The solid line represents the analytical result. The discrete points denotes the experimental results. The flow rates of two cladding streams are kept the same as 1 mL/h. The flow rate of the core stream varies to achieve symmetrical biconvex lenses with different interface curvatures. Different combination of liquids are used in the test: (1) cinnamaldehyde with a viscosity $\mu = 5.7 \times 10^{-3}$ Ns/m² as core stream and the mixture of 73.5% ethylene glycol and 26.5% ethanol with a viscosity $\mu = 9.8 \times 10^{-3}$ Ns/m² as cladding stream (viscosity ratio $\mu_{core}/\mu_{cladding} = 0.58$); (2) DI water with a viscosity of $\mu = 1.0 \times 10^{-3}$ Ns/m² as core stream, and the mixture of 55% glycerol and 45% DI water with a viscosity of $\mu = 7.6 \times 10^{-3}$ Ns/m² as cladding (viscosity ratio $\mu_{core}/\mu_{cladding} = 0.13$); (3) the mixture of 20% glycerol and 80% DI water with a viscosity of $\mu = 1.8 \times 10^{-3}$ Ns/m² as core, and DI water as cladding (viscosity ratio $\mu_{core}/\mu_{cladding} = 1.8$). The insets illustrate the images of the lens at different flow rate ratios by using cinnamaldehyde as core stream and the mixture of 73.5% ethylene glycol and 26.5% ethanol as cladding stream. The dimensions are normalized by the radius of the circular chamber. The error bars show the fluctuation of flow inside the circular chamber.

circular chamber, liquids with different viscosities were tested. In the experiment, the flow rates of the two cladding streams were fixed at 1 mL/h. The flow rate of the core stream was varied from 1 mL/h to 13 mL/h. By increasing the flow rate of the core stream, the opening angle of the core at the entrance of the circular chamber can be enlarged. Since all the streamlines inside a circular chamber follow arc-shape paths, the expansion of the opening angle of the core stream makes the interface between core and cladding streams more curved. Therefore, the curvature of the interface increases with increasing flow rate ratio ($Q_{\text{core}}/Q_{\text{cladding}}$), as shown in Figure 4.25. Due to the geometric advantage of the circular-shape boundary, the curvature of the interface does not decrease when the interface approaches the side wall of the chamber, but gets closer to 1, which is the normalized curvature of the boundary of the circular chamber. This trend was also observed previously using the same liquid for core and cladding streams. The experimental results agree well with the theoretical analysis through the whole tuning range (Fig. 4.25) indicating that our theory can be properly applied to the optical design of a lens with a specific curvature of the interface. Further more, the test of different liquid systems also validates that this theory can be used to address the problem of mismatched viscosities.

Ray-tracing method was carried out experimentally and numerically to verify the analytical model. A green laser with a wavelength of $\lambda = 532$ nm was used to excite the fluorescence dye in the ray-tracing chamber and therefore experimentally trace the light rays refracted by the optofluidic bi-convex lens. Numerical simulation was carried out with a programme written in MATLAB (MathWorks, Inc.). In the simulation, light was assumed to emit from a point source, and the refraction at each interface during

Figure 4.26: Comparisons between experimental and numerical ray-tracing under conditions of different flow rate ratios.

the propagation of light was calculated using Snell's law. The interface involved in the simulation is the theoretical arc calculated from the corresponding flow rate ratios used in the experiments. Figure 4.26 compares the experimental with numerical results. These results were obtained at the core stream flow rates of 1mL/h, 5mL/h, 9mL/h, 13mL/h and 17mL/h, while keeping the two cladding streams at a fixed flow rate of 1mL/h. By increasing the flow rate of the core stream in the experiment, the light beam was manipulated from being slightly divergent to well-focused. It can be found that even when the flow rate ratio ($Q_{\text{core}}/Q_{\text{cladding}}$) is increased to a very large value ($\phi = 17$), the image distance is not elongated but keeping on being shortened. This is different from the case of using a square chamber to develop the optofluidic lens [54]. In that previous case, the paraxial region of the lens interface becomes flat when the flow rate ratio ($Q_{\text{core}}/Q_{\text{cladding}}$) reaches a high value due to the geometric limit. Therefore, the focal length will be elongated.

Figure 4.27(a) shows the ray-tracing image in grey scale, and the corresponding result of quantitative analysis is shown in Figure 4.27(b). The light beam was focused under the condition of 13mL/h core stream and 1mL/h cladding streams. The intensity profiles at different positions (Figure 4.27 (a)) were extracted and fitted with Gaussian curves. All the intensity values were normalized by the maximum value of intensity along line A. The intensity distributions at 870 μm , 3100 μm , and 3600 μm away from the center of the circular chamber were investigated. The full width at half maximum (FWHM) of the fitting curve is defined as the width of light beam. The beam widths were measured as 185, 52, and 74 μm for line C, A, and B respectively. From this quantitative analysis, we found that the light beam becomes convergent after being refracted by the

Figure 4.27: The intensity distribution of fluorescence at different locations (A, B, and C) in the ray-tracing chamber. The light beam was focused by an optofluidic biconvex lens developed with a flow rate ratio of 13. We measured the intensity profiles of the beam before (C), at (A), and after (B) the focal point. The beam widths are defined as the full width at half maximum of each fitting Gaussian curve and measured as 185, 52, and 74 μm at line C, A, and B, respectively. The solid lines denote the experimental results, and the discrete marks represent the fitting Gaussian curves.

Figure 4.28: Relationship between focal length of the optofluidic biconvex lens and flow rate ratio ($Q_{\text{core}}/Q_{\text{cladding}}$). In the measurement of focal length, the flow rates of the two cladding streams were kept at 1 mL/h, and the flow rate of the core stream was varied from 2 mL/h to 18 mL/h. By increasing the flow rate ratio, the focal length can be tuned as the result of the change of the curvature of the lens interface. The solid line in the figure represents the analytical result, the cross marks denote the results obtained from numerical simulation. The inset in the figure illustrates the axial aberration retrieved from the computation of numerical simulation.

lens, and propagated divergently after the focus.

For the measurement of focal lengths under different flow rate ratios ($Q_{\text{core}}/Q_{\text{cladding}}$), the flow rates of two cladding stream (a mixture of 73.5% ethylene glycol and 26.5% ethanol with a refractive index of $n = 1.412$) were fixed at 1 mL/h. The flow rate of core stream (cinnamaldehyde with a refractive index of $n = 1.62$) was varied from 2 mL/h to 18 mL/h. At each flow rate ratio condition, the grey-scale image of the light rays was captured. The ray with the angular aperture in the image space was extracted. The

intersection position of the ray and the optical axis was calculated to retrieve the image distance. By applying the conjugate relationship between the object and the image, the focal length of the optofluidic lens for each corresponding flow rate ratio can be obtained. The experimental measurement of the focal length is shown as circles in Figure 4.28. The solid line in Figure 4.28 represents the curve of equation (4.36). The cross marks denote the results computed by numerical simulation of the ray propagation. The experimental results agree well with the analytical analysis as well as the numerical simulation. Therefore, our theoretical model allows the design of an optofluidic lens with a specific focal length by properly choosing a corresponding flow rate ratio. For reference, the inset in Figure 4.28 shows the axial aberration (δL) retrieved from numerical ray tracing.

An interesting observation during our experiment was light scattering at the chamber wall. If the circular chamber is fully filled with cinnamaldehyde ($n = 1.62$), the light rays behind the optofluidic lens becomes scattered and not sharply focused, Figure 4.29(b). If the chamber is filled with the mixture of ethylene glycol and ethanol ($n = 1.412$), the light rays propagate without refraction or scattering, Figure 4.29(a). This two cases reveal that the mismatch of refractive indices between cinnamaldehyde ($n = 1.62$) and PDMS ($n = 1.412$) and the surface roughness of the lens chamber cause the diffusive reflection and refraction of light rays at the liquid-solid interface leading to strong scattering. This phenomenon confirms the significance of cladding liquid with a refractive index matched to that of PDMS. Besides the roles of shaping the lens and forming an atomically smooth interface, a cladding liquid with a matched refractive index reduces scattering of light at the liquid-solid interface and thus improves the beam quality.

Figure 4.29: (a) The light rays propagated without scattering when the circular chamber was fully filled with a liquid with a refractive index matched to that of PDMS ($n = 1.412$). (b) The light rays were diffusively scattered when the circular chamber was fully filled with cinnamaldehyde ($n = 1.62$). In both cases, bright field images and fluorescence images are shown.

4.4 Conclusions

Firstly, this chapter described the application of dipole flow theory to address the modelling of optofluidic lenses developed in a circularly bounded domain. The advantage of using dipole flow theory is the convenience of obtaining the velocity field and the streamline shapes even when there are multiple inlets and outlets of the circular domain. Optofluidic lenses with different geometries, such as bi-convex, plano-convex, meniscus and bi-concave configurations, can be realized hydrodynamically and described mathematically in circularly bounded domain. Experiments were carried out to verify the analytical modelling. However, a common case is that different fluids are required

to generate a difference in refractive index. Consequently the viscosities of the fluids are not identical either, which makes the dipole flow theory not applicable to this situation. Hydrodynamic spreading theory is further proposed to address the two-fluid flow with different viscosities. The opening angles of core and cladding streams at the entrance of the circular chamber can be mathematically formulated and thereafter the corresponding curvature of the interface for each flow rate ratio can be retrieved. Combining with Gaussian optics model, the focal length of the liquid-core liquid-cladding optofluidic lens can be mathematically formulated. Experiments were carried out to validate the theory. Ray-tracing method was used experimentally and numerically to measure the focal length of the optofluidic biconcave lens. The good agreement between the theoretical, experimental, and numeric results shows that this model can be used to design a microlens system with a mathematically predefined focal length for each lens. This can help to simplify the overall design of optical components in a lab-on-chip system.

Notes

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Chapter 5

Tunable optofluidic lens in a rectangular chamber ²

5.1 Motivation and device concept

Fluorescence has been widely used in biochemistry and analytical chemistry as a non-destructive means of tracking or analysis of bio-molecules or chemicals. Traditionally, the excitation of fluorescence is introduced by employing an external mercury lamp with a filter set or a laser with a specific wavelength. Recently, the emergence of microfluidics has led to the realization of on-chip biological/chemical assays. Fluorescence excitation and detection have also been integrated into a chip to make the whole system cost-effective and portable [121, 154, 155, 156, 147]. To improve the performance of the fluorescence excitation and to modify the light properties, a single solid-interface lens was integrated into the microfluidic network [12]. A system of lenses was used to correct the optical aberration of the single lens [157]. However, the drawback of the solid-based

lens is the roughness of interface, which can cause strong light scattering. Following this concept of in-plane light modulation, a number of tuneable configurations have been demonstrated, including the two-fluid controlled cylindrical microlens [74], liquid core liquid cladding (L^2) optofluidic lenses [55, 54], and pressure controlled air-liquid-interface lens [69]. These in-plane lenses can dynamically modify the light introduced from an optical fibre. Moreover, their L^2 configuration has an atomically smooth interface as opposed to the rough interface of solid based lenses. Currently, most of the in-plane light manipulation methods only involve a tuneable light beam that is focused along the optical axis. A gradient-refractive-index lens has been reported to swing a focused light beam [42]. This work provides different ways of modifying the in-plane light, by which the area out of the optical axis can also be selectively illuminated, thus exciting the local fluorescence dye.

This chapter describes the design and demonstration of an optofluidic bi-concave lens that can perform both light focusing and diverging. In traditional optics, a concave lens is fabricated with glass, whose refractive index (RI) is higher than that of surrounding medium, and thus cannot be used to focus light. The properties of an optofluidic component can be changed simply by replacing the liquid inside the component[1]. In the focusing mode of the optofluidic concave lens, a liquid with low refractive index is used as the core stream and a liquid with higher refractive index serves as the cladding stream, Figure 5.1 (a). It is found that for a L^2 lens system the mis-match of refractive index between cladding liquid and PDMS result in light scattering [54]. An auxiliary cladding liquid with a very low flow rate and a refractive index matching that of PDMS is introduced to prevent the light beam from scattering. The light emitted from an optical fibre

can be well focused and the focal length can be tuned by adjusting the flow rate ratio between the core and the cladding streams. If the core inlet is blocked, the liquid from cladding inlets will converge into a new core stream and the liquid from the auxiliary cladding inlet will serve as cladding streams. Therefore, the device can switch from the focusing mode to diverging mode. In the diverging mode, a liquid with high refractive index works as core stream, and a liquid with refractive index matching PDMS is used as cladding streams, Figure 5.1 (b). Owing to the higher refractive index of the core stream and the tuneable lens interface, the divergence of the light beam can be expanded and adjusted. Combining the performance of focusing and diverging, the device can greatly enhance the tunability of the focal length of an optofluidic lens. Moreover, the width of light beam can be tuned from on the order of $10\ \mu\text{m}$ (focusing mode) to $1\ \text{mm}$ (diverging mode). Overall, the multiple lens-effects to modify the in-plane light make this bi-concave lens more adaptive to lab-on-chip applications such as on-chip fluorescence detection and micro-particle manipulation [158], and flow cytometry [124].

5.2 Numerical model using level set method

Figure 5.1 illustrates the configuration of the device. The optofluidic lens is hydrodynamically formed in a rectangular chamber measuring $1\ \text{mm} \times 1\ \text{mm}$, which has one core inlet, two pairs of cladding inlets and two outlets. The widths of the inlets and outlets are $50\ \mu\text{m}$. To characterize the performance of the concave lens by ray tracing, an optical fibre is introduced by a pre-defined channel with a width of $130\ \mu\text{m}$. The distance between the tip of the fibre and the center of the rectangular chamber is $4\ \text{mm}$. An aperture with a size of $400\ \mu\text{m}$ is built by filling ink into two channels. A ray-tracing

Figure 5.1: Schematics: (a) Illustration of the optofluidic concave lens in the focusing mode; (b) Bright field image of the concave lens focusing a light beam. The liquids for the core, cladding, and auxiliary cladding streams are ethanol, cinnamaldehyde, and the mixture of 73.5% ethylene glycol and 26.5% ethanol respectively. The flow rate ratio between core and cladding stream is 6. The flow rate of the auxiliary cladding stream is set as one-tenth of that of the cladding stream; (c) Illustration of the concave lens in the diverging mode; (d) Bright field image of the concave lens spreading a light beam. The liquids for the core and cladding streams are cinnamaldehyde and the mixture of 73.5% ethylene glycol and 26.5% ethanol respectively. The flow rate ratio between core and cladding stream is 2.8.

chamber filled with fluorescence dye is placed behind the rectangular chamber to visualize the propagation of light rays.

A two-dimensional numerical model is built based on the above geometry of the test device. The level-set method is applied to treat the interfaces between different fluids [131]. The details about the computation of tracking the interface can be referred to Chapter 3. The computed positions of the interface were transferred into a custom MATLAB program to implement the numerical ray-tracing based on Snell's law.

5.3 Experimental results and discussions

In the focusing mode, Ethanol (RI $n = 1.36$; viscosity $\mu = 1.2 \times 10^{-3}$ Ns/m²) was used as the core liquid, cinnamaldehyde (RI $n = 1.62$; viscosity $\mu = 5.7 \times 10^{-3}$ Ns/m²) as the cladding liquid, and a mixture of 73.5% ethylene glycol and 26.5% ethanol (viscosity $\mu = 9.8 \times 10^{-3}$ Ns/m²) with a refractive index matching PDMS ($n=1.412$) worked as the auxiliary cladding liquid. During the test, the flow rate of the auxiliary cladding stream was fixed at one-tenth that of the cladding stream. In the diverging mode, the pumping of core liquid was stopped to enable the liquid from the cladding inlets to converge into a new core stream (cinnamaldehyde), and therefore changing the role of the auxiliary cladding liquid from light-scattering protection to providing a mis-match of refractive index with that of the core stream to refract the light rays. Laser light (output = 200mW, wavelength $\lambda = 532\text{nm}$) was coupled into a multi-mode optical fibre (AFS105/125Y, THORLABS Inc.) with a numerical aperture NA=0.22, which was inserted into a pre-fabricated microchannel with a width of 130 μm to act as a point light source. The light rays refracted by the concave lens are visualized in a ray-tracing chamber filled

with fluorescence dye Rhodamine B (Sigma-Aldrich, excitation wavelength of 540 nm, emission wavelength of 625 nm).

5.3.1 Focusing mode

Figure 5.2: Relationship between the flow rate ratio and the curvature of the interface in the focusing mode of the optofluidic bi-concave lens. Ethanol (RI $n = 1.36$; viscosity $\mu = 1.2 \times 10^{-3}$ Ns/m²) is used as the core liquid, cinnamaldehyde (RI $n = 1.62$; viscosity $\mu = 5.7 \times 10^{-3}$ Ns/m²) serves as cladding liquid, and a mixture of 73.5% ethylene glycol and 26.5% ethanol (viscosity $\mu = 9.8 \times 10^{-3}$ Ns/m²) with a refractive index matching PDMS ($n=1.412$) works as auxiliary cladding liquid. The flow rate of the cladding streams and the auxiliary cladding stream were fixed at 1 mL/h and 0.1 mL/h, respectively. The flow rate of the core stream was increased from 2 mL/h to 14 mL/h. The thin layer close to the side-wall of the chamber in the inset is the auxiliary cladding stream.

Figure 5.2 shows the manipulation of the lens interface by adjusting the flow rate ratio between core and cladding streams for the focusing mode of the optofluidic

bi-concave lens. The error bars in Figure 5.2, as well as in other figures, show the effect of flow fluctuation inside the rectangular chambers. During the test, the flow rate of the cladding streams and the auxiliary cladding stream were fixed at 1 mL/h and 0.1 mL/h, respectively. The flow rate of the core stream was increased from 2 mL/h to 14 mL/h. At low flow rate ratio ($Q_{\text{core}}/Q_{\text{cladding}}$), the curvature of the interface between the core and cladding streams is small due to the relatively straight streamlines in the center part of the rectangular chamber. When the interface approaches the side-wall of the chamber at high flow rate ratio, the value of the curvature decreases owing to the less curved streamlines near the boundary. When the position of the interface locates between the center and side-wall of the chamber, the curvature can achieve a maximum value. As a result of the significantly lower flow rate of the auxiliary cladding stream compared to the cladding streams, a thin layer liquid with a refractive index matching that of PDMS forms on the side-wall of the chamber. This thin layer prevents the scattering of the incident light on the concave lens that is due to the roughness of the PDMS surface, and the small thickness of the layer is supposed to reduce the refraction effect by this layer itself.

Ray tracing method was carried out experimentally and numerically to investigate the optical performance of the optofluidic concave lens in the focusing mode. In the numerical simulation, the shape of the interface between different streams was computed with level-set method and imported into a custom MATLAB program which implements the ray tracing based on Snell's law. In the experiment, a green laser with a wavelength of $\lambda = 532$ nm was used to excite the fluorescence dye in the ray-tracing chamber, and the visualized light rays were recorded in gray-scale images. Due to the refractive in-

Figure 5.3: Comparison between experimental and numerical ray-tracing results under conditions of different flow rate ratios between core and cladding streams in the focusing mode.

Since the refractive index of the core liquid (ethanol $n = 1.36$) being lower than that of the cladding liquid (cinnamaldehyde $n = 1.62$), the optofluidic concave lens possesses light focusing effect. The simulation and experimental results, which were obtained at flow rate ratio of 2, 6, and 10, are compared and shown in Figure 5.3. At the low flow rate ratio $\phi = 2$, the concave lens has a longer focal length owing to the small curvature when the interface is positioned around the center part of the chamber, Fig. 5.3(a). By increasing the flow rate ratio, the interface becomes more curved, which results in a shorter focal length, Fig. 5.3(b). When the interface approaches the side-wall of the rectangular chamber, the curvature of the interface decreases, thus elongating the focal length, Fig. 5.3(c).

To measure the focal length of the optofluidic concave lens, the gray-scale images

Figure 5.4: Relationship between the focal length of the optofluidic bi-concave lens in the focusing mode and the flow rate ratio between core and cladding streams. The core liquid is ethanol, cladding liquid is cinnamaldehyde, and the auxiliary cladding liquid is the mixture of 73.5% ethylene glycol and 26.5% ethanol. The flow rates of the cladding and auxiliary cladding streams were fixed at 1 mL/h and 0.1 mL/h respectively, while adjusting the flow rate of core stream to tune the focal length.

of the light rays in the ray-tracing chamber were captured, and rays with the angular aperture in the image space were extracted to calculate the intersection position of the rays and the optical axis to obtain the image distance. The focal length of the concave lens at each corresponding flow rate ratio can be calculated by applying the conjugate relationship between the object and the image. The experimental results are compared with our numerical analysis in Fig. 5.4.

To demonstrate the importance of the auxiliary cladding stream in the focusing mode of the optofluidic bi-concave lens, we compare the optical performance of the

Figure 5.5: Comparison between the optofluidic bi-concave lens with and without the auxiliary cladding streams in the focusing mode. The light beam is focused under a condition of flow rate ratio 6. (a) A well focused light beam by a concave lens with the auxiliary cladding stream; (b) a focused and scattered light beam by a concave lens without the auxiliary cladding stream; (c) intensity profiles extracted from the gray-scale images (a) and (b).

concave lens with and without this stream. We carried out the testing at a flow rate ratio of 6. The ray-tracing images were captured and shown in Fig. 5.5 (a) and (b). The intensity distributions along the focal plane were extracted and compared in Fig. 5.5 (c). The comparison shows that the concave lens with auxiliary cladding stream has a narrower beam width at the focal plane, and its intensity profile is more smoothly and tightly distributed. However, the beam width of the concave lens without the auxiliary cladding stream is wider, and the intensity profile is not continuously distributed. This phenomenon is due to the rough PDMS surface, which would diffusively refract the light rays when the refractive index of the cladding stream is different from that of PDMS.

5.3.2 Diverging mode

The focusing mode can be simply converted to the diverging mode by stopping the pumping of liquid (ethanol) from the core inlet. By this way, the previous cladding liquid (cinnamaldehyde) in focusing mode will converge in the rectangular chamber and become a core stream in the diverging mode, and the role of the auxiliary cladding liquid (mixture of ethylene glycol and ethanol) changes from preventing light-scattering to refracting light rays, Fig. 5.1 (c). Therefore, the core and cladding liquids involved in the diverging mode testing are cinnamaldehyde and the mixture of 73.5% ethylene glycol and 26.5% ethanol respectively. Figure 5.6 shows the relationship between the flow rate ratio ($Q_{\text{core}}/Q_{\text{cladding}}$) and the curvature of the interface for the diverging mode. In the test, the core stream was pumped with a fixed flow rate of 2 mL/h, and the flow rate of the cladding stream was varied from 0.2 mL/h to 5 mL/h. Because the streamlines at the center and near the side-wall of the rectangular chamber have relatively straight

Figure 5.6: Relationship between the flow rate ratio and the curvature of the interface in the diverging mode of the optofluidic bi-concave lens. Cinnamaldehyde (RI $n = 1.62$; viscosity $\mu = 5.7 \times 10^{-3} \text{ Ns/m}^2$) serves as core liquid, and a mixture of 73.5% ethylene glycol and 26.5% ethanol (viscosity $\mu = 9.8 \times 10^{-3} \text{ Ns/m}^2$) with a refractive index matching PDMS ($n = 1.412$) works as cladding liquid. The flow rate of the core stream was fixed at 2mL/h. The flow rate of the cladding stream was tuned from 0.2mL/h to 5mL/h.

line shape, the values of the curvature of the interface at low and high flow rate ratios are smaller. When the interface is positioned between the center and the side-wall of the chamber, the curvature can achieve a maximum value. This trend of the curve is similar to that in the focusing mode.

Due to the higher refractive index of the core liquid in the diverging mode, the optofluidic bi-concave lens spreads the light rays thus expanding the divergent angle of the light beam originating from the optical fiber. To investigate the optical performance of this concave lens, experimental and numerical ray-tracing methods were carried out

Figure 5.7: Comparisons between experimental and numerical ray-tracing results under conditions of different flow rate ratios between core and cladding streams in the diverging mode. The shape of the interfaces between different streams involved in the numerical simulation are computed with level-set method and imported in Matlab environment for ray-tracing. The experimental results were retrieved by using an optical fibre to introduce an incident green laser light (wavelength $\lambda = 532 \text{ nm}$) on the concave lens. The refracted light rays were visualized by the fluorescence dye in the ray-tracing chamber and recorded by CCD camera in gray-scale images.

in the same way as for the focusing mode. The comparison between experimental results and numerical simulation is illustrated in Figure 5.7. The results were obtained at flow rate ratios of 20, 10, 2 and 0.5. At the highest flow rate ratio of 20, the interface is close to the side-wall of the chamber resulting in the small curvature of the interface having little impact on the refraction of light rays. Therefore the divergent angle of the light beam from the fiber remains almost unchanged after passing the concave lens. With a decrease in the flow rate ratio, the curvature of the interface is increased, so the concave lens greatly expands the divergent angle of the light beam. However, as the interface approaches the center of the chamber, its curvature drops, and the divergent angle of the light beam reduces as well.

The divergent angle at each corresponding flow rate ratio was measured and compared with the numerical analysis in Fig. 5.8. In the measurement, the experimental ray-tracing results were recorded in gray-scale images, and the rays with the angular aperture in the image space were extracted. The included angle of the rays is defined as the divergent angle. Owing to the aperture, the opening angle of the light beam originating from the fiber is 6 degree before the refraction by the concave lens. After passing the lens, the divergent angle can be expanded to more than 20 degree. Since this tunable expansion of the divergent angle can help to control the radiant flux and the size of the illumination area, it may make the way of fluorescence excitation more flexible. Moreover, this tunable diverging lens may be very useful to correct spherical aberration in an optofluidic lens system.

Figure 5.8: Relationship between the divergent angle of light beam and the flow rate ratio between core and cladding streams in the diverging mode. The core liquid is cinnamaldehyde, and the cladding liquid is the mixture of 73.5% ethylene glycol and 26.5% ethanol. The flow rates of the core streams were fixed at 2 mL/h, while adjusting the flow rate of cladding stream to tune the divergent angle.

5.4 Conclusions

In this chapter, the design and characterization of an optofluidic bi-concave lens were introduced, which can perform both light focusing and diverging. The new design was modelled numerically. Level set method was applied to compute the fluid dynamics and capture the fluid interface. The computation results of interface positions were transferred to MATLAB, where numerical ray tracing was carried out using Snell's law based on the interface positions. Experiment was conducted to verify our modelling. The experimental results agree well with the numerical analysis, which strengthens the

validity of the method of numerical simulation. Therefore this numerical method can be used to analyze similar design of optofluidics in the future with good reliability. In the focusing mode of the concave lens, a liquid (ethanol) with a low refractive index is used as the core stream and a liquid (cinnamaldehyde) with a higher refractive index serves as cladding stream. The relatively lower refractive index of the core and the concave shape of the interface conjunctly enable the lens to focus light. An auxiliary cladding stream with low flow rate was introduced to effectively minimize the light scattering at the rough surface of the chamber wall. This concept of auxiliary cladding stream can be further applied to other optofluidic designs, in which the light may scatter at the optically-rough contact surface between refractive index mis-matching substances. The focusing mode of the optofluidic bi-concave lens can be simply switched to diverging mode by stopping pumping the liquid from the core inlet. The previous cladding liquid (cinnamaldehyde) would join together in the chamber and become the core stream for the diverging mode. The auxiliary cladding liquid would change its function from avoiding light-scattering to becoming the cladding stream for the diverging mode. Combining the performance of focusing and diverging, the tunability of the focal length of the optofluidic lens and the width of the light beam can be greatly enhanced. This improvement of tunability will make this optofluidic bi-concave lens more adaptive to the lab-on-chip applications.

Notes

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Chapter 6

Tunable optofluidic prism in a sector-shaped chamber ³

6.1 Motivation and device concept

The combination of microfluidics and optics has enabled the miniaturization and tunability of optofluidic devices for lab-on-chip systems. The miniaturization and integration of optical components promise the potential of reducing the amount of bulky external devices and the alignment of multiple components in a single chip. One challenging problem is the alignment of optical path among multiple integrated components. Moreover, precise adjustment of optical path is critically important for interferometer sensor [159] and multi-prism laser pulse compressor [160], especially when they are integrated into a chip. Laminar flow based liquid core liquid cladding configuration has widely been applied to configure optical components in versatile ways. On the one hand, the interface between two immiscible fluids is atomically smooth, which can be utilized

Figure 6.1: Schematic configuration of the optofluidic prism. The core and cladding streams are driven by syringe pumps into a sector-shape chamber. The arc-shape distributed pillars work as pressure barrier to make the interface between streams extend as a straight line. An optical fiber is inserted to guide the laser beam into the device. The divergence of the emitting light beam is reduced by a build-in aperture. The refracted light beam is traced by the rays-visualization chamber locating beside the prism chamber.

in reflection or refraction of light. On the other hand, two miscible fluids can create an index-gradient distribution, which can modulate the propagation of light inside the fluids. Recently, optical waveguides [73, 46], reconfigurable lenses [54, 42], and optical switches [76] based on this configuration have been demonstrated.

In this chapter, a tunable micro optofluidic prism is proposed, which is hydrodynamically formed by one core and two cladding streams inside a sector-shape chamber (Fig. 6.1). Benzyl alcohol (viscosity $\mu = 5 \times 10^{-3}$ Pa s at 25°C) with a refractive index of 1.536 is employed as the core stream, which forms the geometry of a triangular prism. A

mixture (viscosity $\mu = 9 \times 10^{-3}$ Pa s at 25°C) of glycerol (60% by weight) and water (40% by weight) with a refractive index of $n = 1.412$ matching that of polydimethylsiloxane PDMS serves as cladding streams. Due to the higher refractive index of the core stream, this configuration can perform a prism function. The apex angle of this optofluidic prism can be tuned by adjusting the flow rate ratio between core and cladding streams, and therefore the deviation angle of an incident light beam can be changed accordingly. Since the propagation direction of refracted light beam can be accurately controlled by choosing a proper flow rate ratio, this tunable prism can continuously scan the light beam, and therefore can be engaged in the alignment of optical path, or in the development of optical switches.

6.2 Modelling, experiment and results

The device was fabricated in PDMS using standard soft lithography technique. The configuration of our tunable prism is based on three streams of laminar flows in a sector-shape chamber, which has a 90° opening angle (Fig. 6.1) and a channel depth of $150\mu\text{m}$. In the chamber, an array of pillars distributed in an arc and located at a distance of 3.5 mm from the inlet serves as pressure barriers. The angular interval between each pillar is 1.5° . The entrance of the sector-shape chamber with a width of $300\mu\text{m}$ is much smaller than the radius of the chamber. The small gaps between the pillars are supposed to make the pressure drop equally distributed along the arc where the pillars stand. Therefore, the entrance of the chamber and the arc consisting of the gaps act approximately as a source-sink pair bounded within a divergent boundary with a 90° opening angle. The streamlines inside a source-sink pair domain have a similarity

in shape with the boundary according to the theory introduced in Chapter 3. Thus the interfaces between the core and cladding streams are theoretically radiating lines from the source. This fact was confirmed experimentally (insets in Fig. 6.2). Due to the even distribution of the pressure at the pillars, the flow flux through the gaps is assumed to be equally distributed. Therefore, the apex angle can be derived as a function of flow rate ratio based on the theory of the liquid core liquid cladding system reported in [76]:

$$\theta = 90^\circ \times \frac{\phi}{\phi + 2\beta} \quad (6.1)$$

where ϕ and β are the flow rate ratio and viscosity ratio between core and cladding streams, respectively. The relationship between the angle θ and flow rate ratio ϕ is depicted as the solid line in Fig. 6.2. In the experiment, the flow rate of cladding streams was fixed at 1mL/h, while the flow rate of core stream varied from 1mL/h to 11mL/h. By increasing the flow rate ratio between core and cladding streams, the apex angle of our optofluidic prism can be tuned accordingly.

To demonstrate the tunability of the refracted light beam by our optofluidic prism, we inserted an optical fiber (multi-mode, NA = 0.22) into a pre-defined micro channel with a width of 130 μm . An aperture was built by filling black ink into two channels with a width of 50 μm , and the size of the aperture is 130 μm . To trace the light beam refracted by the prism, a visualization chamber was placed beside the prism chamber. Rhodamine B diluted in a mixture of glycerol (60% by weight) and water (40% by weight) was injected into the visualization chamber to trace the rays. When the prism chamber was filled only with the mixture of glycerol and water, the rays emitted from the optical fiber propagated horizontally without refraction, which is shown by the inset (a) of Figure 6.2. If the chamber is fully occupied by the core liquid (benzyl alcohol),

Figure 6.2: The relationship between the apex angle of the prism and the flow rate ratio between core and cladding streams. The apex angle of the prism can be tuned by adjusting the flow rate ratio.

the total deviation angle can achieve a maximum value of 11.4° , which is depicted as the inset (c) in Fig. 6.3. During the tuning process of flow rate ratio, the total deviation angle can be adjusted continuously from 0° to 11.4° .

Assuming that a light beam horizontally incident on a prism with an apex angle of θ (Fig. 6.1), the total deviation angle can be calculated based on Snell's law, which is denoted in equation (6.2).

$$\zeta = \arcsin[n_1/n_0 \cdot \sin(\theta - \alpha)] - \theta/2 \quad (6.2)$$

where the n_1 and n_0 are the refractive indices of core and cladding streams respectively, and the angle α is as calculated as:

$$\alpha = \arcsin[n_0/n_1 \cdot \sin(\theta/2)] \quad (6.3)$$

By substituting equation (6.1) into (6.2), the relationship between the total deviation

Figure 6.3: The relationship between the deviation angle of the refracted light beam by the prism and the flow rate ratio between core and cladding streams. The blue lines in the insets depict the boundaries of refracted light beams. By adjusting the flow rate ratio, the refracted light beam can be changed in a scanning mode. ($n_1=1.536$, $n_0=1.412$ at the wavelength of $\lambda=532\text{nm}$).

angle and the flow rate ratio is:

$$\zeta = \arcsin \left[\frac{n_1}{n_0} \sin \left(\frac{90^\circ \cdot \phi}{\phi + 2\beta} - \alpha \right) \right] - \frac{45^\circ \cdot \phi}{\phi + 2\beta} \quad (6.4)$$

which is represented by the solid line in Fig. 6.3. The experimental result agrees well with the theoretical analysis, which indicates that the control of the refracted light beam by our optofluidic prism can be mathematically predicted.

Figure 6.4: The deviation angle of the refracted light beam as a function of the refractive index of the core liquid and the flow rate ratio between core and cladding streams, provided the light beam horizontally incidents on the prism. It is assumed that the refractive index of the surrounding medium is 1.412, which matches the PDMS.

6.3 Discussion and conclusion

Equation (6.4) indicates that the flow rate ratio and the refractive index of core liquid are the two main parameters to influence the performance of the prism. Figure 6.4 shows the impact of these two parameters on the deviation angle. It is found that the deviation angle is highly dependent on the refractive index of the prism material. A higher refractive index of the prism material can ensure a larger deviation angle and broaden the beam scanning range.

In conclusion, this chapter introduced a reconfigurable micro optofluidic prism based on the liquid core liquid cladding configuration in a sector-shape chamber. The pillars distribute in an arc were built to ensure the streamlines inside the chamber to be

along the radial direction, and therefore to shape a straight optical interface. An optical fiber was induced as a light source, whose beam divergence was reduced by a build-in aperture. The apex angle of the prism can be tuned and predicted mathematically by adjusting the flow rate ratio between core and cladding streams. As a consequence, the deviation angle of refracted light by the prism can be controlled accurately. This beam scanning function of our optofluidic prism can be utilized in the alignment of optical path among integrated optical components inside a lab-on-a-chip system, or for optical switches.

Notes

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Chapter 7

A tunable optofluidic aperture configured by liquid core liquid cladding structure ⁴

7.1 Motivation and device concept

Recent research has been intensively focused on the development of optofluidic components for the miniaturization of lab-on-chip systems [161]. Optical components configured by flow of liquids such as waveguides [73], lenses [74] and prisms [127] have been realized in microfluidic networks. One of the motivation to integrate optical components into microfluidics is to miniaturize the whole system and to reduce the overall cost. Instead of using a bulky external optical system, optofluidic systems can be made compact and portable for field use [162]. Further more, the liquid/liquid interface can be deformed by shear stress making optofluidic components dynamically tunable. Recent works have

shown that the optical properties of the optofluidic components can be well pre-defined mathematically [126, 149].

Aperture stop or entrance pupil is an important optical component in imaging and detection systems. The component determines the amount of light passing through the optical system as well as the angular aperture on the object side. Previous works have demonstrated tunable out-of-plane apertures using micro-fabrication technology [163, 164, 165]. The problem of these apertures is the difficulty to integrate them into microfluidics. Recently, a simple aperture in microfluidic network built by filling ink into micro-channel was reported [54]. But the size of this aperture remains fixed after fabrication. This chapter proposes and demonstrates a tunable optofluidic aperture stop, which can be dynamically re-configured according to the flow condition.

The schematic configuration of the optofluidic aperture is illustrated in Fig. 7.1 (a). A liquid core liquid cladding structure was used to form this aperture. The core liquid is optically transparent allowing light rays to pass through, while the ink with negligible optical transmittance ($\approx 0\%$) works as the cladding liquid blocking the propagation of light. To test the optical performance of the aperture, an optical fiber inserted in an alignment groove mimics an axial point light source; an optofluidic lens is positioned after the aperture to represent a lens system. A ray-tracing chamber filled with fluorescent liquid is placed on the image side to visualize the light rays, which have gone through the aperture as well as the lens. When the aperture is tuned with a given flow rate ratio ($\phi = Q_{\text{cladding}}/Q_{\text{core}}$), the amount of light which reaches the image space as well as the angular aperture on the image side can be adjusted accordingly.

Figure 7.1: An illustration of the optical system to test the performance of the optofluidic aperture. For the test, the focal length of the optofluidic lens is fixed, while the flow rate ratio of the optofluidic aperture is adjusted to control the radiant flux and angular aperture of the light beam. (a) Schematic configuration; (b) Bright field image.

7.2 Experimental results and discussions

To test this concept, the device was fabricated by molding poly dimethylsiloxane (PDMS) on a master made of a thick resist called SU-8[166]. The channel depth was defined to be 150 μm by choosing a proper spin coating speed for SU-8. The distance between the fiber tip and the center of aperture is 3 mm. The lens is positioned 1 mm away from the aperture. The dimension of the rectangular chamber for forming the optofluidic aperture is 1 mm $\times$ 0.6 mm, and the diameter of the circular chamber for the optofluidic lens is 1 mm. Figure 7.1(b) shows a typical situation where a point light source is projected to the image space by the lens with its light cone restricted by the optofluidic aperture. The core liquid of the aperture is a mixture (viscosity $\mu = 9 \times 10^{-3}$ Pa s at 25°C) of glycerol (60% by weight) and water (40% by weight) with a refractive index of $n = 1.412$ matching that of PDMS. The light blocking cladding is black ink (viscosity $\mu = 2.0 \times 10^{-3}$ Ns/m² at 25°C). The core and cladding liquids of the optofluidic lens are cinnamaldehyde (viscosity $\mu = 5.7 \times 10^{-3}$ Ns/m² at 25°C and refractive index $n = 1.62$) and a mixture of 73.5% ethylene glycol and 26.5% ethanol (viscosity $\mu = 9.8 \times 10^{-3}$ Ns/m² at 25°C and refractive index $n = 1.412$) respectively.

The relationship between the aperture size and the flow rate ratio ($\phi = Q_{\text{cladding}}/Q_{\text{core}}$) was first investigated. The flow can dynamically change the aperture size using phenomenon of hydrodynamic spreading [105]:

$$W_0/W_1 = 1/(2\beta\phi + 1), \quad (7.1)$$

where W_0 and W_1 are the widths of the core stream and the channel respectively (Figure 7.1). β and ϕ are the viscosity ratio and flow rate ratio between cladding and core streams respectively. The low aspect ratio of 0.6 between the chamber length and width leads to

curved streamlines, which cause the discrepancy between the experimental results and the theoretical prediction for a ratio of infinity, Fig. 7.2.

In the test, the flow rate of core stream (mixture of glycerol and water) was fixed at 200 $\mu\text{L}/\text{h}$. The flow rate of the cladding stream (black ink) was varied from 200 $\mu\text{L}/\text{h}$ to 2600 $\mu\text{L}/\text{h}$. Owing to the low Reynolds number, flow inside the rectangular chamber is laminar. The streamlines are smooth and fluid motion is steady. Consequently, the aperture size is set once the flow condition is fixed. According to the theory of hydrodynamic spreading, the width of the ink flow (cladding stream) was narrower than that of the transparent flow (core stream) at a low flow rate ratio. Thus, an aperture with a large diameter can be realized. When the flow rate ratio was increased, the cladding stream squeezed the core stream to be narrower leading to a smaller aperture. Figure 7.2 shows the experimental results of the flow rate ratio dependence changing the aperture size from 700 μm to 309 μm . The insets of Fig. 7.2 depict the optofluidic apertures at flow rate ratios of 1, 7 and 13 respectively. Although not shown here, if a large flow rate ratio is applied, this optofluidic aperture can be adjusted to have a very small size, which is only limited by the diffusion distance between fluids of about 60 μm under the flow condition reported in this letter chapter.

To assess the optical performance of this aperture, fluorescence dye Rhodamine B (Sigma Aldrich, excitation wavelength of 540 nm, emission wave length of 625 nm) was diluted in a mixture of glycerol (60% by weight) and water (40% by weight) was injected into the ray-tracing chamber to visualize the light rays coming out from the aperture and the optofluidic lens. The flow rate ratio ($\phi = Q_{\text{cladding}}/Q_{\text{core}}$) for the lens was fixed at 6. A multi-mode optical fiber (AFS105/125Y, THORLABS Inc.) with

Figure 7.2: The relationship between the aperture size (normalized by 1 mm) and the flow rate ratio. The flow rate of the core stream (transparent liquid) is fixed at $200 \mu\text{L/h}$, while the flow rate of the cladding stream (black ink) is varied from $200 \mu\text{L/h}$ to $2600 \mu\text{L/h}$.

a numerical aperture of $\text{NA}=0.22$ was inserted into a pre-fabricated microchannel with a width of $130 \mu\text{m}$ to mimic a point light source. The optical fiber introduced the light generated from a green laser with a wavelength of $\lambda = 532 \text{ nm}$. When the light rays were incident on the rectangular chamber where the aperture was developed, the light cone was partially blocked while the paraxial part was allowed to pass through and be focused by the optofluidic lens. The light rays that finally reach the image space were visualized and recorded by a CCD camera. Figure 7.3 shows the light rays on the image side restricted by the apertures formed by flow rate ratios of 1, 7 and 13 respectively. A qualitative inspection of the ray-tracing results shows that the amount of light rays and the angular aperture on the image can be tuned by the flow rate ratio.

Furthermore we quantitatively investigate how the optofluidic aperture controls

Figure 7.3: Light beam on the image side visualized by fluorescence dye Rhodamine B can be dynamically controlled by tuning the optofluidic aperture size: flow rate ratio of (a) 1, (b) 7 and (c) 13.

the amount of light by regarding light as the flow of luminous energy along the geometrical rays. Although the quantum yields of Rhodamine B are different for individual solutions, the amount of photons emitted can be considered to be proportional to the amount of photons absorbed by Rhodamine B in a specific solution [167]. The intensity of each pixel in the ray-tracing images can represent the local energy flux that the passing light ray carries. The fluorescence intensity value was integrated over the dotted line shown in Fig. 7.3), and considered it as the radiant flux (the amount of light) through the optofluidic system consisting of the aperture and the lens. Figure 7.4(a) shows the experimental result of the radiant flux as a function of the flow rate ratio. Increasing the flow rate ratio between cladding and core stream squeezes the transparent liquid resulting in a smaller aperture size and therefore causing less radiant flux to reach the image side. Previous research reported the integration of lens into microfluidic network to enhance

the excitation of light for biochemical applications [162, 12]. The proposed optofluidic aperture may be combined with this lens system to generate a more flexible and robust optical system to control the amount of light. Another interesting usage of this device is to control the angular aperture on the object side as well as the image side. By confining the light cone within the paraxial region with a small angular aperture, the spherical aberrations can be reduced significantly. Figure 7.4(b) shows the quantitative result of the angular aperture on the image side as a function of the flow rate ratio.

7.3 Conclusion

In conclusion, this chapter presents the design and the demonstration of an optofluidic aperture stop. The device can be easily integrated into microfluidic networks and combined with previously proposed optofluidic components. The tunable aperture stop was realized by adjusting the flow rate ratio between an opaque cladding and a transparent core. The test results showed that the radiant flux through the optical system as well as the angular aperture on the image side can be precisely controlled by changing the flow rate ratio between cladding and core.

Notes

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Figure 7.4: (a) The radiant flux through the optical system as a function of the flow rate ratio; (b) The angular aperture on the image side as a function of the flow rate ratio.

Chapter 8

Optofluidic flow cytometer⁵

8.1 Motivation and device concept

Flow cytometer provides an efficient way for the detection of suspended micro particles or biological cells and the analysis of their physical and chemical properties. The applications of cytometers can be widely found in environmental monitoring [168], industrial testing [169] and clinical analysis [170]. Generally, the suspended particles are directed by the streaming flow to pass through a light beam. The scattering or fluorescent light can be collected by optoelectronic systems and transformed into an electric signal. The interpretation of the signal reveals the biochemical and physical characteristics of the particles. Recently, many efforts have been devoted to integrate the flow cytometer into a microfluidic system. The integration allows the miniaturization of the whole system and makes it portable for field use. Another advantage is that the microfluidic system possesses the ability to easily align the particles into a single line by hydrodynamic focusing using sheath flows. This alignment is critical during the analysis since clogging

or agglomerating of particles would result in a misinterpretation.

To further improve the level of integration of the system, optical fiber, which can be inserted into the microfluidic networks, was used to perform both excitation and collection of light instead of using external bulky objectives [121, 122]. The problem of merely using optical fiber for excitation include the low intensity in illumination and inaccuracy in detection owing to the divergent light beam emitted from the fiber. Therefore, some on-chip lenses have been proposed to focus the light beam from the fiber. Wang *et al.* designed a flow cytometer with a PDMS-based lens [123] and Godin *et al.* demonstrated a fluidic lens for flow cytometry [124]. Barat *et al.* built waveguide and lens both based on polymer for a flow cytometer [125]. However, due to the solid-based lens interfaces, all of these designs suffer from light scattering, which degrades the quality of the focused light beam. Divergence or scattering of the light beam results in a larger beam width. An obvious disadvantage resulting from the large beam width is that the cytometer would miscount if multiple particles enter the large illumination area because the multiple particles would give only one burst in the collected signal. Normally this burst owing to multiple particles has a larger peak value than that of one particle. Thus the other consequent problem is that this burst of the signal would be misleading as it would be wrongly interpreted as a larger particle.

Therefore, a well-focused and non-scattered light beam is highly demanded for improving the efficiency and accuracy of microfluidic flow cytometer. Recently, optofluidic lenses [54] have been proposed to perform light focusing in microfluidic systems. In this chapter, we design a microfluidic flow cytometer with an optofluidic lens formed in a circular chamber. This optofluidic lens has a mathematically predictable focal length,

and is immune to light scattering (refer to chapter 4). A well-focused light beam is achieved with a much smaller beam width of $23\ \mu\text{m}$ compared to all previous flow cytometers. This flow cytometer presents a good performance on particle counting, and the signal intensities shows small aberrations and strong dependence on the particle size.

Figure 8.1: Schematic configuration of microfluidic flow cytometer using an optofluidic lens to focus a Gaussian beam. Single-mode fiber (SMF) is used to introduce illumination. Particles are aligned along center line of the channel by hydrodynamic focusing. The scattering light by particles is collected by a multi-mode fiber (MMF) placed with an angle 45° to the optical axis.

Figure 8.1 illustrates the schematic configuration of the flow cytometer integrated into a microfluidic network. The device was fabricated in PDMS with standard soft lithography technique with a channel depth of $150\ \mu\text{m}$. A pre-defined channel was positioned along the optical axis for introducing of a single-mode fiber (SMF), whose tip was at a distance of 4 mm from the center of the optofluidic lens. Two symmetric channels filled with ink acted as the aperture with a width of $350\ \mu\text{m}$. A liquid core liquid cladding optofluidic lens was developed in the circular chamber with a diameter of 1 mm

to focus the diverging Gaussian beam emitted from the fiber tip. Another fluidic channel with a width of $300\ \mu\text{m}$ was built for hydrodynamic focusing of the particles. The center of this channel was at a distance of 4 mm from the lens. The light scattered by the flowing particle was collected by a multi-mode fiber (MMF) (AFS105/125Y, THORLABS Inc.) placed with an angle of 45° to the optical axis. The optical signal was transferred to an avalanche photodiode module (APD, C5460-01, Hamamatsu, Japan) to generate the electric signal, which was recorded by an oscilloscope (Tektronix TDS320) connected to a personal computer.

8.2 Experimental results and discussions

A preliminary test was carried out to optimize the Gaussian beam profile along the channel where particles flow. In the optimization, a flow rate ratio ($\phi_{\text{core}}/\phi_{\text{cladding}}$) between core and cladding streams of 6 was chosen to form the optofluidic lens (inset (b) in Figure 8.2). In the experiment, cinnamaldehyde (Sigma Aldrich, USA) with a viscosity of $\mu = 5.7 \times 10^{-3}\ \text{Ns/m}^2$ at 25°C and a refractive index of $n = 1.62$ serves as the core stream to shape the lens. A mixture of 73.5% ethylene glycol and 26.5% ethanol by weight with a viscosity of $\mu = 9.8 \times 10^{-3}\ \text{Ns/m}^2$ at 25°C and refractive index of $n = 1.412$ matching that of polydimethylsioxane (PDMS) was used as the cladding stream. The significance of refractive-index-matching liquid is the effective prevention of light beam from scattering, which was explained in detail in our previous works [149, 148]. Fluorescence dye Rhodamine B (Sigma Aldrich, excitation wavelength of 540 nm, emission wave length of 625 nm) diluted in a mixture of glycerol (60% by weight) and water (40% by weight) which has a matching refractive index of $n = 1.412$

Figure 8.2: The intensity profile along the beam waist (blue line in the inset (a)) of a focused beam is extracted and plotted. The beam was focused by an optofluidic lens with a flow rate ratio of 6, and was visualized by fluorescence dye Rhodamine B diluted in a mixture of glycerol (60% by weight) and water (40% by weight). Inset (a): a well-focused Gaussian beam across the channel; inset (b): an optofluidic lens with a flow rate ratio ($\phi_{\text{core}}/\phi_{\text{cladding}}$) of 6.

was injected into the channel for particle focusing to visualize the beam profile. Figure 8.2 shows the intensity profile extracted along the center line (blue color line depicted in the inset (a)). The experimental data can be well-fitted to a Gaussian curve, whose full width at half maximum (FWHM) is defined as the beam width. Under the optimized condition, the beam width along the center line of the can reach $23 \mu\text{m}$.

Polymer microspheres of three different sizes were used in the experiment: $20 \mu\text{m}$ (concentration: $C = 8.6 \times 10^4/\text{mL}$; standard deviation: $0.28\mu\text{m}$), $10 \mu\text{m}$ (concentration: $C = 1.9 \times 10^5/\text{mL}$; standard deviation: $0.09\mu\text{m}$), $5 \mu\text{m}$ (concentration: $C = 2.8 \times 10^5/\text{mL}$;

Figure 8.3: Scattered light signals for three different sizes of microsphere: (a) 20 μm , (b) 10 μm , (c) 5 μm respectively. The signal peak values are caused by the scattering light of the particles passing through the illuminated volume.

standard deviation: $0.05\mu\text{m}$) respectively. The particles were diluted in the mixture of glycerol (60% by weight) and water (40% by weight) with a refractive index matching that of PDMS. This mixture was also used as cladding streams to hydrodynamically focus the particles (Figure 8.1). During the experiment, a flow rate ratio between core ($50\text{ }\mu\text{L/h}$) and cladding ($500\text{ }\mu\text{L/h}$) streams of 0.1 was chosen to properly align the particles along the center line of the channel. A flow rate ratio of 6 was used for the optimized lens shape for the detection of particles. When the illuminated space was free of particles, the light beam propagated without scattering, and the detection fiber with an angle of 45° to the optical axis received no light signal. When a flowing particle entered the illuminated space, the mismatch in refractive index between polystyrene ($n = 1.59$) and liquid ($n = 1.41$) caused the light to refract and reflect. Due to the movement of the particle, the light refraction and reflection was changing resulting in dynamic scattering of light. Part of the scattered light enters the detection fiber leading to a peak in the signal (Figure 8.3). Each peak in Figure 8.3 represents a particle passing by, which is the basic concept of particle counting.

The scattered light signals for different sizes of particles are plotted as histograms of peak values (Figure 8.4 (a)). The diagram shows separated distributions of the signal peak values for each size of particle, which indicates small variations of the scattered light signals. The coefficient of variation (CV), defined as the standard deviation divided by the signal mean value, are 25%, 10%, and 11% for 5, 10, 20 μm particles, respectively. Under the condition of a small illuminated volume (small beam width at the beam waist), the particles passed through the illuminated space one by one, and the situation of multiple particles simultaneously entering that space was prevented. Multi-

Figure 8.4: (a) Distributions of scattering light intensities (peak values) for three different sizes of particles; (b) scattering light intensities (peak values) as a function of particle size.

ple particles entering would lead to a larger peak compared to that of a single particle. Another reason is that a well-focused and non-scattered light beam helps to increase the intensity of illumination and therefore improve the signal-noise-ratio (SNR) of the light received by the detection fiber. Figure 8.4(b) shows the scattered light intensity as a function of particle size. Basically, a larger particle has a larger surface to interact with the incident light, and thus results in stronger scattered light to be received by the detection fiber. Achieving lower coefficients of variation using our flow cytometer can help to identify the size of the particle more accurately than the other previously reported on-chip cytometers [123, 124]. However, the CV is still considered high compared to commercial flow cytometers, especially when the particle size is much smaller than the beam width (the experimental CV of $5\text{ }\mu\text{m}$ particle is 25%). Although in this work we achieved a focused light beam with a beam waist of $23\text{ }\mu\text{m}$, by using higher refractive-index liquid to develop the optofluidic lens we can foresee a more tightly focused beam and a narrower beam waist, and a better performance of the micro-chip flow cytometer.

8.3 Conclusions

In conclusion, this chapter presents and demonstrates a disposable and cost-efficient flow cytometer with integrated optical components, such as lens and optical fibers, instead of using external bulky optics. Owing to the well-focused and non-scattered light beam coming from the optofluidic lens, the flow cytometer shows an more efficient and accurate performance on particle counting and sizing compared to the previous on-chip flow cytometers [123]. Liquids with a refractive index matching that of PDMS are used for both cladding streams of the optofluidic lens and hydrodynamic focusing of the

particle, which is critical for protecting the light beam from being scattered by rough solid interfaces. Using a single-mode fiber (SMF) is another important factor to achieve a small width of the beam waist in the imaged space due to the small core diameter of $\approx 5\mu\text{m}$.

Notes

⁵This chapter was published in: Song *et al.*, *Opt. Lett.* **36**(5) 657 2011.

Chapter 9

Conclusions and future work

9.1 Conclusions

Liquid replacing, liquid refractive index control and interface control are the three main methods to make the optofluidic components tunable. According to the functionality, optofluidic components can be categorized into two groups: out-of-plane and in-plane. The advantage of in-plane lenses over out-of-plane lenses is the possibility of integrating them into microfluidic networks which perform the sample analysis, and thus they are suitable for lab-on-chip applications. This thesis focuses on the manipulation of interface between different flows through hydrodynamic spreading to develop in-plane optofluidic components. The thesis also presents the improvement on lab-on-chip applications such as flow cytometer using micro optofluidic technology

Firstly, two-dimensional dipole flow theory is applied to address the single-fluid flow in a circular chamber. The advantage of this method is the convenience of obtaining the velocity field as well as the streamlines even when there are multiple inlets

and outlets of the circular chamber. Bi-convex, plano-convex, meniscus and bi-concave lenses can be realized hydrodynamically and described mathematically in the circular chamber. But the common case is that different fluids are needed to generate a difference in refractive index to refract the light. Therefore the viscosities of the fluids are different correspondingly, which makes the dipole flow theory not applicable to this situation. To solve this problem, hydrodynamic spreading theory is used to address the two-fluid flow with different viscosities. The opening angle of streams at the entrance of the circular chamber can be calculated analytically. Based on this fact, the curvature of the interface as well as the focal length can be derived according to the flow rate ratio between the core and the cladding streams.

Secondly, the problem of light scattering is discussed here. Owing to the rough surface of PDMS and the mismatch of refractive index between liquid and PDMS, the light will diffusively refract and reflect upon the incidence on the surface. The surface roughness greatly depends on the mask and fabrication process, and to reduce the roughness always means the increase of the fabrication cost. The other solution is to match the refractive index of liquid with PDMS, which is demonstrated here to be an effective and cost-efficient way to prevent light scattering for optofluidic applications. The non-scattered light beam can be focused by the optofluidic lens with an optically smooth interface between streams to achieve a very narrow beam width at the focal point. This method can significantly improve the efficiency of optofluidic flow cytometer in particle counting and sizing, which is presented in Chapter 8.

With a purpose to flexibly manipulate a light beam, an optofluidic bi-concave lens is developed in rectangular chamber with liquid core liquid cladding (L^2) config-

uration, which can perform both light focusing and diverging without changing liquid system. Level-set method is used to numerically compute the interface position. Based on the computation, a numerical ray-tracing can be implemented in MATLAB. Another optofluidic component based on L^2 structure is a prism formed in a sector-shaped chamber. An arc-shape distributed pillars are used as pressure barrier to make the interface to be straight so as not to deform the wave front of a light beam. The light beam can be deflected with different angles according to the applied flow rate ratio between the core and cladding streams. Of great interest is the control of radiant flux and numerical aperture of a light beam in an optical system. An optofluidic aperture stop is built with ink as cladding stream and transparent liquid as core stream. Adjusting the flow rate ratio can tune the size of the aperture. Therefore the radiant flux as well as the numerical aperture of a light beam can be manipulated accordingly.

In conclusion, this thesis presents the works to develop tunable micro optofluidic components to manipulate in-plane light beam for biochemical analysis. Theories are proposed to mathematically describe the fluidic characteristics of these components. Applications with good performance can be developed based on the components reported in this thesis.

9.2 Future work

For fully understanding of the fluidic characteristics of the flow field to develop optofluidic components, PIV should be carried out to obtain the velocity field, which determines the interface shape and position. Velocity is also an important factor to affect the diffusion between different fluids. Thus the measurement of velocity field becomes

critical, if diffusion-based components, such as refractive index gradient lens [42], are to be developed.

Although the light beam can be well focused using a bi-convex optofluidic lens, the numerical aperture (NA) on the image side is still low compared to a commercial objective lens. A lens with high NA is critical for the applications, such as optical tweezers and fluorescence correlation spectroscopy. of great interests is the enhancement of NA for the optofluidic lens. Compound lenses can be combined to generate a strong focusing ability. Another method is to design new configurations of streams to create a large difference of refractive index between liquids. PDMS/air interface can be used to form the lens, if the surface of PDMS can be polished using some techniques such as etching. After developing an optofluidic lens with high NA, functions, such as particle manipulation, can be integrated for lab-on-chip applications.

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Appendix A

Ray-tracing simulation code in MATLAB

```
clear;

clc;

close all;

Viscocore=5.7; %Define viscosity of the core stream

Viscocladding=9.8; %Define viscosity of the cladding stream

RIcladding=1.412; %Define refractive index of the cladding stream

RIcore=1.62; %Define refractive index of the core stream

Radiuschamber=1; %Define the radius of the circular chamber

FlowRateRatio=6; %Define the flow rate ratio

Belta=Viscocore/Viscocladding;

h=tan(Belta*FlowRateRatio*pi/4./(2+Belta*FlowRateRatio));

Radiusinterface=(h.^2+Radiuschamber^2)/2./h;
% plot(FlowRateRatio,1./Radius_interface)
```



```

xA=0;

yA=h-Radiusinterface;

xupperInterface=
-sqrt(Radiusinterface^2-(Radiusinterface-h)^2):0.001
:sqrt(Radiusinterface^2-(Radiusinterface-h)^2);
%Calculation of the x coordinate of the upper interface

yupperInterface=
yA+sqrt(Radiusinterface^2-(xupperInterface-xA).^2);
%Calculation of the y coordinate of the upper interface

xB=0;

yB=Radiusinterface-h;

xlowerInterface=
-sqrt(Radiusinterface^2-(Radiusinterface-h)^2):0.001
:sqrt(Radiusinterface^2-(Radiusinterface-h)^2);
%Calculation of the x coordinate of the lower interface

ylowerInterface=
yB-sqrt(Radiusinterface^2-(xlowerInterface-xB).^2);
%Calculation of the y coordinate of the lower interface

plot(xlowerInterface,ylowerInterface)

hold on

plot(xupperInterface,yupperInterface)

axis equal

lightsource=[0,-8];
plot([lightsource(1),lightsource(1)],[lightsource(2),15]);
%Plot the optical axis

nodex=zeros(1,3);

nodey=zeros(1,3); %Define the coordinates of the nodes where ray intersect with interf

Angle1=[1.5/180*pi,2.1/180*pi,2.78/180*pi]; %Define the angular aperture of each ray

```

```

for i=1:3

    slot1=tan(Angle1(i)+pi/2);

    nodex(1)=lightsource(1);

    nodey(1)=lightsource(2); %The first node is the point light source position
    yline=nodey(1):0.005:0;

    xline=(yline-node_y(1))/slot1+nodex(1);

    [nodex(2),nodey(2)]=polyxpoly(xline,yline,xlowerInterface,ylowerInterface);
    %Find out the intersect

    d=abs(xlowerInterface-nodex(2));

    Mind=min(d);

    index=find(Mind==d);

    tangentslotnode2=
    (ylowerInterface(index+1)-ylowerInterface(index))/
    (xlowerInterface(index+1)-xlowerInterface(index)); %Find out the tangent slot at n

    normslotnode2=-1/tangentslotnode2;

    Angle2=atan(abs((normslotnode2-slot1)/(1+normslotnode2*slot1)));

    Angle3=asin(RIcladding/RIcore*sin(Angle2)); %Snell's law

    slot2=tan(Angle3+atan(normslotnode2));

    yline=nodey(2):0.005:1;

    xline=(yline-nodey(2))/slot2+nodex(2);

    [nodex(3),nodey(3)]=polyxpoly(xline,yline,xupperInterface,yupperInterface);

    d=abs(xupperInterface-nodex(3));

    Mind=min(d);

    index=find(Mind==d);

    tangentslotnode3=

```

```

(yupperInterface(index+1)-yupperInterface(index))/
(xupperInterface(index+1)-xupperInterface(index));

normslotnode3=-1/tangentslotnode3;

Angle4=atan(abs((normslotnode3-slot2)/(1+normslotnode3*slot2)));

Angle5=asin(RIcore/RIcladding*sin(Angle4));

slot3=tan(pi+atan(normslotnode3)-Angle5);

yline=nodey(3):0.001:20;

xline=(yline-node_y(3))/slot3+nodex(3);

f1(i)=nodey(3)-nodex(3)*slot3;

plot(nodex,nodey)

plot(xline,yline)
end

Angle1=[-1.5/180*pi,-2.1/180*pi,-2.78/180*pi]

for i=1:3

    slot1=tan(Angle1(i)+pi/2);

    node_x(1)=lightsource(1);

    node_y(1)=lightsource(2);

    yline=nodey(1):0.005:0;

    xline=(yline-nodey(1))/slot1+nodex(1);

    [nodex(2),nodey(2)]=polyxpoly(xline,yline,xlowerInterface,ylowerInterface);

    %find out the tangent slot at node2
    d=abs(x_lowerInterface-nodex(2));

    Mind=min(d);

    index=find(Mind==d);

    tangentslotnode2=

```

```

(ylowerInterface(index+1)-ylowerInterface(index))/
(xlowerInterface(index+1)-xlowerInterface(index));

normslotnode2=-1/tangentslotnode2;

Angle2=atan(abs((normslotnode2-slot1)/(1+normslotnode2*slot1)));

Angle3=asin(RIcladding/RIcore*sin(Angle2));

slot2=tan(-Angle3+pi+atan(normslotnode2));

yline=nodey(2):0.005:1;

xline=(yline-nodey(2))/slot2+nodex(2);

[nodex(3),nodey(3)]=polyxpoly(xline,yline,xupperInterface,yupperInterface);

d=abs(xupperInterface-nodex(3));

Mind=min(d);

index=find(Mind==d);

tangentslotnode3=
(yupperInterface(index+1)-yupperInterface(index))/
(xupperInterface(index+1)-xupperInterface(index));

normslotnode3=-1/tangentslotnode3;

Angle4=atan(abs((normslotnode3-slot2)/(1+normslotnode3*slot2)));

Angle5=asin(RIcore/RIcladding*sin(Angle4));

slot3=tan(atan(normslotnode3)+Angle5);

yline=nodey(3):0.001:20;

xline=(yline-nodey(3))/slot3+nodex(3);

f2(i)=nodey(3)-nodex(3)*slot3;

plot(nodex,nodey)

plot(xline,yline)
end

```

Figure A.1: Procedures of ray tracing implementation. The blue square denotes the point light source. The yellow circles represent the nodes where light rays intersect with lens interfaces.

A.1 Procedures of ray-tracing implementation

Step 1: Define the viscosities and refractive indices of the core and cladding streams.

Step 2: Calculate the coordinates of the interface positions under corresponding flow rate ratio based on the theory presented in Chapter 4, and plot out the interfaces depicted as Figure A.1 (a).

Step 3: Define the optical axis and plot it out, Figure A.1 (b).

Step 4: Based on the Snell's law, calculate the propagation direction of the ray after each refraction and find out the locations where the ray intersects with interfaces as well as the optical axis, Figure A.1 (c).

Step 5: Repeat step 4 to complete the tracing for other rays with different angular position, Figure A.1 (d).