Accurate rain drop size distribution models for the tropical region

Lakshmi Sutha Kumar

2011

Kumar, Lakshmi Sutha. (2011). Accurate rain drop size distribution models for the tropical region. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/49987>

<https://doi.org/10.32657/10356/49987>

Accurate Rain Drop Size Distribution Models for the Tropical Region

Lakshmi Sutha Kumar

School of Electrical and Electronic Engineering

**A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy**

2011

Statement of Originality

I hereby certify that the contents of this thesis are the result of work done by me and has not been submitted for a higher degree to any other University or Institution.

.....

Date

.....

G. Lakshmi Sutha

Acknowledgements

There are many people along the way who have contributed so much to my success that it is truly impossible to name them all and to adequately express my deepest gratitude to them in the written word. First, I am very fortunate to have Prof. Lee Yee Hui as my supervisor. I would like to thank her for giving me the opportunity to research under her guidance and supervision. I received motivation, strong encouragement, constructive comments and constant support from her during all my studies, even in personal issues. With her, I learned how to better share my ideas with others, visually and written. I deeply appreciate all of her efforts and I am extremely grateful to her.

I also want to thank the motivation and support I received from my co-supervisor, Prof. Jin Teong Ong. Professor Ong has very strong knowledge and experience in the field of rain and communication systems. With his enthusiasm, inspiration, and efforts to explain things clearly and simply, he made my research experience rich and rewarding.

I would also like to extend my thanks to my fellow team members Ms. Zhou Xiaoxiao, Mr. Yeo Jun Xiang, Mr. Miguel Nardy, Ms. Ma Ting, Dr. Meng Yu Song and Dr. Huang Shao Ying who have helped me over the past four years. Without their assistance, I would have experienced great difficulties. My special

thanks are given to Mr. Yeo Jun Xiang and Mr. Miguel Nardy who were always ready to tackle any computer question or coding problem I have.

I am also thankful to the school of Electrical and Electronics Engineering, NTU, for providing me with an opportunity and environment to research. I would also like to thank the laboratory executive of the Communication Research Laboratory, Mr. Lim Cheng Chye who installed the necessary softwares and attended various laboratory needs promptly.

This research was partially funded by Defence Science and Technology, DSTA, Singapore. I am thankful to the experts at DSTA who have provided useful discussions, suggestions. I would like to thank Dr. Merhala Thurai (Colorado State University, USA) for providing the T-Marix code used in this research and for her valuable suggestions. I am grateful to the journal reviewers who directed my research in the proper way with their constructive comments and suggestions.

I take this opportunity to offer my heartfelt and sincere thanks to all my teachers who shaped me. I am deeply thankful to my dear parents who were my first teachers in life and at a very early stage instilled in me a sense of curiosity and a thirst for knowledge. Their support, concern and unwavering encouragement have given me the courage to follow my dreams.

I extend my warmest gratitude to my husband. It was his constant encouragement, warmth and strength that kept me going through many days. I would like to thank him for his unconditional support and love. I extend my thankfulness to my children. They bear my utmost adoration and appreciation for their good understanding and patience throughout the research period. At last, but the most important, I would like to thank God for all His provisions, grace and guidance thus far.

Table of Contents

Acknowledgments.....	ii
Table of contents.....	iv
Summary.....	viii
List of Figures.....	x
List of Tables.....	xvi
List of Acronyms.....	xviii
1. Introduction.....	1
1.1. Motivation.....	1
1.2. Objectives.....	2
1.3. Organization of the Thesis.....	4
2. Literature Review.....	7
2.1. Introduction.....	7
2.2. Rain Drop Size Distribution Models.....	8
2. 2. 1. Limitations of Joss Distrometer.....	10
2. 2. 2. Previous DSD Models for Singapore.....	11
2. 2. 3. Selection of Central Moments.....	12
2.3. Rain Rate Retrieval from Single Polarized RADAR.....	14
2. 3. 1 Reflectivity-Rain rate Relations.....	14

2. 3. 2	Rain Classification.....	15
2. 3. 3	Rain Retrieval from DSD Retrieval.....	17
2.4.	Two parameter Gamma Models.....	18
2. 4. 1	Fixed μ Models.....	19
2. 4. 2	$\mu - A$ Relationship.....	19
2.5.	Summary.....	23
3.	Data Selection and Models.....	24
3.1.	Joss-Waldvogel Distrometer.....	24
3.2.	Selection of Rain Events.....	26
3.3.	RADAR Data.....	28
3.4.	Measured DSD.....	29
3.5.	Gamma Modeled DSD.....	30
3. 5. 1	MM234 Gamma Model.....	31
3. 5. 2	MM346 Gamma Model.....	32
3. 5. 3	MM246 Gamma Model.....	32
3.6.	Rain Integral Parameters.....	33
3.7.	Rain Attenuation Calculations.....	34
3. 7. 1	Specific Rain Attenuation using ITU-R P.838.3 Model.....	35
3. 7. 2	Specific Rain Attenuation using Forward Scattering Coefficients.....	36
3. 7. 3	ITU-R Rain Attenuation.....	41
3.8.	Summary.....	43
4.	Truncated Gamma Drop Size Distribution.....	44
4.1.	Analysis of Measured DSD.....	44
4.2.	Contribution of Individual Drop Diameters.....	48

4.3. Comparison of Gamma Models.....	51
4.4. Truncated Gamma Models.....	58
4.5. Summary.....	61
5. Specific and Slant-Path Rain Attenuation.....	63
5.1. Introduction.....	63
5.2. Specific Rain Attenuation.....	64
5. 2. 1. Specific Rain Attenuation at Different Elevation Angles.....	70
5. 2. 2. Specific Rain Attenuation Contribution for Individual Bins.....	72
5. 2. 3. Specific Rain Attenuation using Truncated Gamma Models.....	77
5.3. Slant Path Rain Attenuation.....	78
5.4. Summary.....	83
6. Rain Retrieval from Single Polarized RADAR.....	84
6.1. Reflectivity-Rain Rate Relations.....	84
6.2. Gamache-Houze method.....	85
6. 2. 1. Rain Classification.....	85
6. 2. 2. Z-R relations.....	86
6.3. Atlas-Ulbrich Method.....	88
6. 3. 1. Rain Classification.....	88
6. 3. 2. AU-SG Z-R relations.....	94
6.4. RADAR and Distrometer Reflectivities.....	97
6. 4. 1. RADAR Reflectivity.....	97
6. 4. 2. Calibrating the RADAR Data.....	99
6.5. Comparison of Rain Rates.....	103
6.6. Summary.....	105
7. Rain Rate Retrieval from Polarimetric RADAR Variables.....	106

7.1. Truncated and Un-truncated Moment Fitting.....	106
7.2. Gamma Model parameters.....	107
7.3. Fixed μ Models.....	108
7.4. μ - λ Relationship.....	111
7.4.1. Comparison of μ - λ Relations	113
7.4.2. μ - λ Relations for Different Rain Categories.....	115
7.5. Rain Retrieval.....	120
7.6. Summary.....	125
8. Conclusions and Recommendations.....	126
8.1. Conclusions.....	126
8.2. Recommendations for Future Work.....	132
Author's Publications.....	135
Bibliography.....	137
Appendix A.....	151
Appendix B	154
B.1. Truncated gamma models with actual gamma model and measured DSD.....	154
B.2. Distrometer and RADAR Reflectivity Data.....	158
Appendix C.....	161

Summary

In this thesis, a detail study on the modeling of rain drop size distribution is undertaken. The rain drop size data measured using the Joss distrometer during the years 1994 to 1995 and 1997 to 1998 and the RADAR data during the year 1998 are used in this study. Gamma model is found to be the preferred model for drop size distribution modeling in the Singapore climatic zone. The method of moments is used to retrieve the parameters of the gamma distribution. By studying the contribution of individual bins on rain rate estimation, it was found that the contributions of lower drop diameters are small as compared to the central drop diameters. Therefore, the lower drop diameters are removed from the drop size data before the gamma model is redesigned for its moments. The effects of this removal on the specific rain attenuation (in dB) and the slant-path rain attenuation calculations using ITU-R P.838-3 model and using forward scattering coefficients for vertical polarization are analyzed at Ku-band, Ka-band and Q-band frequencies. It is concluded that the sensitivity of the Joss distrometer although affects the rain rate estimation at low rain rates, does not affect the slant path rain attenuation on microwave links. Therefore, the small drop diameters can be ignored for slant path rain attenuation calculations in the tropical region.

The research work continues to find the suitable reflectivity to rain rate (Z-R) relations using a data set which consists of nine rain events selected from

Singapore's drop size distribution. The variability of the rain integral parameters R , Z , N_w , D_0 and gamma model parameter μ are used for the classification of rain into convective, stratiform and transition. Z-R relations are derived for each type of rain after classification. The Z-R relations for different rain types for the Singapore climatic zone are compared and analyzed. Reflectivities are extracted from RADAR data above NTU site for rain events and compared with the reflectivities derived from the distrometer data. Rain rates retrieved from RADAR data using the proposed Singapore Z-R relations are compared with the distrometer rain rates. It was found that the Singapore Z-R relations is able to extract the rain rate from RADAR data well although they are found to be constantly lower than the distrometer derived rain rates.

Finally the thesis examines the possibility of using a two parameter gamma models to retrieve the rain rates from dual polarized RADAR data. A two parameter gamma model can be found either by fixing μ or by deriving an appropriate shape-slope (μ - A) relation for the tropical region. In order to find an appropriate μ value, observed DSDs are fitted with different μ values to estimate the rain rates. In order to find an appropriate μ - A relation, different μ - A relations are fitted for different categories according to the rain rate and the number of drops. The derived μ - A relationships for the Singapore region are compared to published results from Gadanki and India. Two parameter gamma models are compared by retrieving the rain rate using the polarimetric RADAR variables found from the T-Matrix code. The use of the μ - A relation for rain retrieval is recommended for the tropical region.

List of Figures

3. 1.	Joss-Waldvogel Distrometer at Nanyang Technological University.....	25
3. 2.	Computed rain drop shapes for D_{eq} = 2, 3, 4 and 5 mm with origin at the center. Shown for comparison are dashed circles of diameter D_{eq}	38
4. 1.	Rain rate versus time for the rain event on 26 th February 1995.....	45
4. 2.	Drop size distributions for seven rain rates.....	46
4. 3.	Normalized Deviation (%) for each individual bin removal.....	49
4. 4.	Normalized Deviation (%) for consecutive bins removal.....	51
4. 5.	Gamma models, MM234, MM246 and MM346 with measured DSD (a) 10.45 mm/hr (b) 120.30 mm/hr.....	52
4. 6.	Gamma model parameters for the seven rain rates (a) Shape parameter, μ (b) Slope parameter, λ (c) Intercept parameter, N_0	54
4. 7.	Mean square errors in percentage of MM234, MM246 and MM346 gamma models with measured data.....	55
4. 8.	Total mean square errors in percentage of the gamma models (MM234, MM246 and MM346) as a function of the average rain rate for the 23 considered rain events (DSDs having rain drops greater than 100 only considered).....	56
4. 9.	Truncated gamma models with actual gamma model and measured DSD (a) 4.20 mm/hr (b) 66.54 mm/hr.....	59

5. 1.	ITU-R model coefficients, k and α , for both horizontal and vertical polarization (a) k values (b) α values.....	65
5. 2.	Specific rain attenuation using gamma model at different frequencies.....	67
5. 3.	Specific rain attenuation as a function of rain rate using gamma model at different frequencies.....	68
5. 4.	Difference in specific rain attenuation using forward scattering coefficients compared with the ITU-R model for horizontal and vertical polarizations.....	69
5. 5.	Specific rain attenuation using gamma model for four different frequencies at different elevation angles.....	71
5. 6.	Specific rain attenuation contributions at different frequencies using gamma model at 120.30 mm/hr (a) (a) Using ITU-R model (b) Using forward scattering coefficients	73
5. 7.	Specific rain attenuation contributions at different rain rates using gamma model at 11 and 38 GHz.....	75
5. 8.	Slant-path attenuation at the four frequencies using gamma model at $R_{0.01}$ (120.30 mm/hr)	80
5. 9.	Slant-Path rain attenuation changes for truncated gamma models compared to the actual gamma model (a) 28 GHz (b) 38 GHz	82
6. 1.	Scatter plot of reflectivity versus $\log_{10}$ (rain rate) (1514 minutes of data from year 98 rain events, DSDs having rain drops greater than 100 only considered)	86

6. 2.	Classification of distrometer data, recorded on 9 th May 1998. R (dBR), Z (dBZ), N_w (dBN), $10*D_0$ (mm) and μ are plotted for around 240 minutes.....	88
6. 3.	Dependence of $\log_{10}(N_w)$ and D_0 (mm) for the rain event on 9 th May 1998 where the unit of N_w is $\text{mm}^{-1}\text{m}^{-3}$. The separator line separates convective and stratiform points.....	91
6. 4.	Relations between the coefficient A and the exponent b in the Z-R relations of the nine rain events for different rain types and for the overall Z-R relations.....	95
6. 5.	Time – height plot of RADAR reflectivity in dBZ for the rain event on 9 th May 1998.....	98
6. 6.	Time series inter comparison between RADAR and distrometer reflectivity in dBZ for the rain event on 9 th May 1998.....	100
6. 7.	Time series inter comparison between RADAR and distrometer reflectivity in dBZ for the rain event on 12 th May 1998.....	101
6. 8.	Time series comparison between distrometer derived rain rates and RADAR rain rates using different Z-R relations from Singapore's data for the rain event on 9 th May 1998 (a) Using the MP, SG and GH-SG, Z-R relations (b) Using the W-SG, AU-SG1 and AU-SG2, Z-R relations.....	103
7. 1.	Scatter plots of gamma model parameters (932 minutes of data, DSDs having rain drops greater than 100 only considered)	108

7. 2.	Distribution of gamma fitted parameter μ (932 minutes of data, DSDs having rain drops greater than 100 only considered)	109
7. 3.	Root mean square error of rain rate, $RMSE-R$ calculated from measured data (932 minutes of data, DSDs having rain drops greater than 100 only considered).....	110
7. 4.	Scatter plots of $\mu-A$ values obtained from Singapore's DSD. The curves, obtained from distrometer measurements at Singapore, and distrometer measurements made at Florida (Florida) and Gadanki, India (Gadanki), are over laid in (c) and (d). (a) Un-truncated moment method - Without filtering (b) Truncated moment method - Without filtering (c) Un-truncated moment method - With filtering of rain rates ≥ 5 mm /hr and rain counts > 1000 drops; and (d) Truncated moment method - With filtering of rain rates ≥ 5 mm /hr and rain counts > 1000 drops.....	112
7. 5.	Scatter plot of $\mu-A$ values for different rain categories and their corresponding $\mu-A$ fits (a) Un-truncated moment method - $R \geq 1$ mm/hr (b) Truncated moment method - $R \geq 1$ mm/hr (c) Un-truncated moment method - $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$ and (d) Truncated moment method - $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$	116
7. 6.	Singapore's $\mu-A$ relationships for different rain categories, $R \geq 5$ mm /hr and rain counts > 1000 drops, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$, along with Florida and Gadanki (a) Un-truncated moment method (b) Truncated moment method.....	118
7. 7.	Dependence of Z_{dr} and $10\log_{10} (Z_{hh} / N_0)$ on A	121

7. 8.	Comparison of rain rates retrieved from polarimetric RADAR variables with measured rain rate for the rain event on 26/02/1995 (a) for $\mu = 4$ (b) using the μ - A relation for the rain category $R \geq 5$ mm/hr and rain counts ≥ 1000 drops (c) using the μ - A relations for the rain categories $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$	123
B. 1.	Truncated gamma models with actual gamma model and measured DSD at five rain rates (a) 1.96 mm/hr (b) 10.45 mm/hr (c) 22.80 mm/hr (d) 120.30 mm/hr (d) 141.27 mm/hr.....	156
B. 2.	Time – height plot of RADAR reflectivity in dBZ for the rain event on 18 th May 1998, Convective rain.....	158
B. 3.	Time series inter comparison between RADAR and distrometer reflectivity in dBZ for the rain event on 18 th May 1998, Convective rain.....	158
B. 4.	Time– height plot of RADAR reflectivity in dBZ for the rain event on 18 th May 1998, Stratiform rain.....	159
B. 5.	Time series inter comparison between RADAR and distrometer reflectivity in dBZ for the rain event on 18 th May 1998, Stratiform rain	159
B. 6.	Time– height plot of RADAR reflectivity in dBZ for the rain event on 10 th June 1998	160
B. 7.	Time series inter comparison between RADAR and distrometer reflectivity in dBZ for the rain event on 10 th June 1998.....	160
C. 1.	Comparison of rain rates retrieved from polarimetric RADAR variables with measured rain rate for the rain event occurred on 30/10/94 (a) for $\mu = 4$ (b) using the μ - A relation for the rain category $R \geq 5$ mm/hr and rain	

	counts ≥ 1000 drops (c) using the μ - A relations for the rain categories 1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr and $R \geq 25$ mm/hr.....	162
C. 2.	Comparison of rain rates retrieved from polarimetric RADAR variables with measured rain rate for the rain event occurred on 13/12/94 (a) for μ =4 (b) using the μ - A relation for the rain category $R \geq 5$ mm/hr and rain counts ≥ 1000 drops (c) using the μ - A relations for the rain categories 1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr and $R \geq 25$ mm/hr.....	164
C. 3.	Comparison of rain rates retrieved from polarimetric RADAR variables with measured rain rate for the rain event occurred on 21/02/95 (a) for μ =4 (b) using the μ - A relation for the rain category $R \geq 5$ mm/hr and rain counts ≥ 1000 drops (c) using the μ - A relations for the rain categories 1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr and $R \geq 25$ mm/hr.....	165
C. 4.	Comparison of rain rates retrieved from polarimetric RADAR variables with measured rain rate for the rain event occurred on 12/03/95 (a) for μ =4 (b) using the μ - A relation for the rain category $R \geq 5$ mm/hr and rain counts ≥ 1000 drops (c) using the μ - A relations for the rain categories 1 mm/hr $\leq R < 5$ mm/hr, 5 mm/hr $\leq R < 25$ mm/hr and $R \geq 25$ mm/hr.....	167

List of Tables

3. 1.	Selected rain events from year 1994-1995 (first 14 events) and from the year 1998 (last 9 events) from the distrometer data for analysis.....	27
3. 2.	Parameters for calculating scattering coefficients.....	40
4. 1.	Measured rain drops from JWD at seven rain rates	47
4. 2.	<i>MSE (%)</i> in terms of rain rate and specific rain attenuation values.....	57
4. 3.	Mean Square Error (%) for truncated gamma models.....	61
5. 1.	Real and imaginary parts of the complex scattering amplitudes (in m) for H and V polarizations at 11 GHz as a function of drop equivolumetric sphere diameter.....	66
5. 2.	Specific rain attenuation (dB/km) at different rain rates using forward scattering coefficients for horizontal and vertical polarizations compared with ITU-R model.....	68
5. 3.	Specific rain attenuation (dB/km) using forward scattering coefficients for vertical polarization using truncated gamma models.....	77
5. 4.	Elevation Angles at which Slant-Path Rain Attenuation (dB) starts to increase.....	80

6. 1.	Reflectivity - rain rate (Z-R) relationships for individual rain type and for the overall data set derived by linear regression of Z (dBZ) versus $\log_{10}(R)$	87
6. 2.	Reflectivity - rain rate (Z-R) relationships of the nine rain events for different rain types.....	93
7. 1.	Shape-slope relations for different category of rain rates fitted using the un-truncated moment method.....	117
7. 2.	Shape-slope relations for different category of rain rates fitted using the truncated moment method.....	117
A. 1.	Threshold of the Drop Size Bins.....	152
B. 1.	Parameters for the truncated and un-truncated gamma models.....	157

List of Acronyms

DSD	Drop Size Distribution
Z-R relation	Reflectivity-Rain rate relation
ITU-R	International Telecommunication Union – Radio communication Sector
$N(D)$	Number of drops per unit volume per unit interval of drop diameter D
N_o	Intercept parameter
Λ	Slope parameter
μ	Shape parameter
MP	Marshall and Palmer model
LP	Laws and Parsons model
N_T	Number concentration of drops
σ	Standard geometric deviation
D_g	Geometric mean diameter
D_0	Median volume diameter
D_m	Mass-weighted mean diameter
Σ	Optical extinction
W	Liquid water content
R	Rain rate
Z	Reflectivity
N_w	Generalized number concentration of an exponential DSD having the same liquid water content W and mass-weighted diameter D_m as the actual DSD
Z_{dr}	Differential reflectivity
C-G model	Constrained Gamma model
JWD	Joss-Waldvogel distrometer
UTC	Coordinated Universal Time

MDWR	Meteorological Doppler Weather RADAR
MSS	Meteorological Service of Singapore
D_i	Mean drop diameter or representative drop diameter
D_{il}	Lower drop diameter value of i^{th} bin
D_{iu}	Upper drop diameter value of i^{th} bin
ΔD_i	Drop size interval for i^{th} bin
D_{min}	Minimum drop diameter
D_{max}	Maximum drop diameter
n_i	Number of drops in the i^{th} bin
S	Sample area of distrometer
$v(D_i)$	Terminal velocity of rain drop
T	Integration time
M_k	k^{th} moment
A	Intercept of Z - R relation
b	Exponent of Z - R relation
F and η	Ratios
γ	Incomplete gamma function
Γ	Complete gamma function
ρ_w	Water density
γ_R	Specific rain attenuation
k and α	Frequency dependent coefficients of ITU-R P.838-3, power law
k_H or k_V	k for either horizontal or vertical polarization
α_H or α_V	α for either horizontal or vertical polarization
τ	Polarization tilt angle relative to the horizontal
γ_H	Specific rain attenuation for horizontal polarizations
γ_V	Specific rain attenuation for vertical polarizations
$f_H(D_i)$	Complex forward scattering coefficients for horizontal polarization
$f_V(D_i)$	Complex forward scattering coefficients vertical polarization
$R_{0.01}$	Point rainfall rate for the location for 0.01% of an average year
h_s	Height above mean sea level of the earth station
θ	Elevation angle

ϕ	Latitude of the earth station
f	Frequency
h_R	Rain height
L_s	Slant path length
L_G	Horizontal projection of the slant-path length
$r_{0.01}$	Horizontal reduction factor
$v_{0.01}$	Vertical reduction factor
L_E	Effective path length
ND_j	Normalized Deviation
MSE	Mean Square Error
C1	Convective rain stage 1
C2	Convective rain stage 2
T	Transition rain stage
ST	Stratiform rain stage
R_{DSD-T}	Total accumulated DSD rain rates
$R_{RADAR-T}$	Total accumulated RADAR rain rates
$RMSE-R$	Root Mean Square Error in rain rate estimation

Chapter 1

Introduction

1.1 Motivation

The ever increasing demand for new communication channels with higher data rate and bandwidth has led to the exploitation of higher and higher frequencies, extending well into the millimeter wave region of the electromagnetic spectrum. However, the performance of the microwave links at frequencies above 7 GHz in the tropical regions is constrained by the excess attenuation due to precipitation, especially rainfall.

One of the most complete descriptions of rain is given by its drop size distribution (DSD); it is defined as the number of drops per unit size interval (diameter) and per unit volume of space. The DSD and its moments define the most important integral parameters useful for the understanding of the microphysical mechanisms responsible for the precipitation formation. Several studies on the DSD, over different climatic regions and in different parts of the

world, have been encouraged in the past, from the early work of Laws and Parsons [1] and Marshall and Palmer [2] by the different applications of the DSD and variety of instruments related to the DSD applications.

Accurate DSD measurements are of great importance for a broad spectrum of applications: the quantitative estimate of rain from microwave remote sensing, active (weather RADAR) or passive (microwave radiometer), is subjected to the natural variability of the DSD characteristics; to design radio links for telecommunications and to evaluate the fading caused by rain, it is important to have good DSD models; moreover the distribution of drop sizes is of central importance in determining the most important parameters in radar-meteorology. DSD has the strong relationship with the precipitation. Therefore, it is a variable parameter.

In the tropical regions, models such as the lognormal, gamma and modified gamma distributions had been proposed to model rain drop size distribution. Thus, it is important to continue exploring this area of study in Singapore with its equatorial climate. There is also a need to study the effects of rain on the higher frequency band signals.

1.2 Objectives

The major objective of this research is to analyze and present the measured rain drop size distribution for Singapore and hence to establish the Singapore DSD models for rain attenuation calculations and for rain retrievals. In order to fulfill this objective, the current work done can be classified as such:

1. A comprehensive literature review of the DSD models and their

applications especially rain attenuation calculations and rain rate retrievals from RADAR data are presented. The exercise creates a strong foundation and background for further exploration.

2. Drop size data collected from a measuring instrument, the distrometer is processed to obtain the rainfall rates and DSD. Then different statistical models such as negative exponential, lognormal and gamma distributions are analyzed and gamma model is selected to fit the DSD of Singapore. Appropriate moments are selected to model the gamma DSD and the dead time effect of Joss distrometer is studied.

3. The contribution of individual bins is found in order to check the effect of the dead time problem at different rain rates. It is found that the error in rain rate calculations from the removal of drop diameters below 0.77 mm is least compared to the larger drop diameters. Therefore truncated gamma models are proposed with the removal of erroneous lower bins.

4. The contribution of particular rain drops on the specific rain attenuation of microwave signals using the ITU-R P.838-3 model and forward scattering coefficients for vertically polarized waves at frequencies 11 GHz, 20 GHz, 28 GHz and 38 GHz is found. The changes in gamma models due to the truncation of lower bins are analyzed along with the specific rain attenuation and slant-path rain attenuation calculations.

5. The rain is classified into different types using the DSD rain integral parameters and then Z-R relations are proposed for each rain type. The rain rates are retrieved from single polarized RADAR data and compared with the distrometer derived rain rates.

6. Two parameter gamma DSD models are analyzed. The appropriate fixed μ values for the tropical country of Singapore are found and μ - λ relations are proposed. The derived fixed μ values and μ - λ relationships for the Singapore region are compared to published results from two other regions. Rain rates are retrieved from polarimetric RADAR variables using the two parameter gamma DSD models and compared with the distrometer derived rain rates.

1.3 Organization of the Thesis

The remaining part of the thesis is organized as follows. This research work is divided into seven chapters. This chapter gives a brief introduction to the research motivation, objectives and organization of the research report.

Chapter 2 reviews the previous work on the theoretical models for drop size distribution, degraded sensitivity of Joss distrometer and the selection of proper moments for gamma DSD. The methods for calculating the attenuation of microwave signals due to rain are presented. From the literature survey, the progress and the status in these research areas are provided to further consolidate the current work.

Chapter 3 focuses on the analyses of the experiment data, which include the summary of data collected, selected rain events for analysis, and DSD modeling. The calculation methods used to find measured rain drop size distribution and gamma drop size distribution are explained. The specific rain attenuation and slant-path rain attenuation calculations are also included.

Chapter 4 examines the different moment combinations to model the gamma drop size distribution. Gamma model using 2nd, 3rd and 4th moments is selected based on the analysis. The contribution of individual bins in rain rate measurement is found using the measured data. The importance of critical drop diameters in DSD modeling is identified and truncated gamma models are designed.

Chapter 5 studies the specific rain attenuation values using ITU-R model and using forward scattering coefficients for vertical and horizontal polarization. Specific rain attenuation changes with rain rate, frequency and elevation angle and the specific rain attenuation contributions at different drop diameter sizes are analyzed. Specific rain attenuation and slant-path rain attenuation values are calculated for truncated gamma models using the forward scattering coefficients for vertically polarized waves and these values are compared with the specific rain attenuation and slant-path rain attenuation values calculated using the actual gamma DSD.

Chapter 6 classifies the rain into different types. The variability of the DSDs for different rain types and its influence on the Z-R relations are presented. Rain rates are calculated from Single polarized RADAR reflectivities using the proposed Z-R relations. These reflectivities are compared with the distrometer derived rain rates.

Chapter 7 deals with the different types of two parameter gamma models.

The calculation procedures for fixed μ models and the gamma models using μ - A relation are given. Root mean square deviations with measured data in rain rate estimation of the fixed μ models are presented. The comparison of two parameter models is given by retrieving the rain rates from polarimetric RADAR variables.

Chapter 8 provides a summary of the results presented in this thesis and suggests some recommends and proposals for further research work.

Chapter 2

Literature Review

2.1 Introduction

In this chapter, some introductory concepts about the theoretical models of the drop size distribution and the dead time problem of the measuring instrument, Joss distrometer are presented. Methods used to form models which are less sensitive to error in the extreme small and large drop diameters is included. The microwave signal attenuation due to rain is then discussed. The selection of proper DSD models will be useful for the further exploration to extract rain rates from RADAR data using the reflectivity-rain rate relationships developed from Singapore's data. Rain rates from Doppler Weather RADAR and distrometer data are derived and compared. A comprehensive review on the modeling of two parameter gamma models for extracting rain rates from dual polarized RADAR is presented.

2.2 Rain Drop Size Distribution Models

A detailed knowledge of the distribution of raindrop sizes is essential when considering the microwave properties of rain as these distributions, together with the fall velocities and the shapes of the drop, strongly influence the scattering and attenuation of microwaves.

In 1943, Laws and Parsons [1] examined the relationship between raindrop size and rain intensity. For a long time, the exponential DSD has been the most widely used analytical parameterization for the raindrop size distribution:

$$N(D) = N_0 \exp(-\Lambda D) \quad (2.1)$$

where $N(D)$ is the number of drops per unit volume per unit interval of drop diameter D and the parameters N_0 and Λ can be determined experimentally.

Marshall and Palmer [2] suggested that $N_0 = 0.08 \text{ cm}^{-4} = 8000 \text{ m}^{-3} \text{ mm}^{-1}$ and $\Lambda = 41R^{-0.21} \text{ cm}^{-1} = 4.1R^{-0.21} \text{ mm}^{-1}$ where R is the rainfall rate in mm/hr. In temperate climates, rain drop size distribution is usually characterized by the Laws and Parsons model [1] or by the negative exponential model as proposed by Marshall and Palmer (MP) [2]. The MP model is not sufficiently general to describe all the DSD cases; it applies mainly to cases where sufficient averaging in time is performed [3]; in particular, N_0 and Λ has been found to vary considerably within each rain event [4], and from one rain event to another. Subsequent DSD measurements have shown that the exponential distribution does not capture instantaneous rain DSDs especially for tropical climates and a more general function is necessary.

Other attempts to account for distribution shape involve the use of specific mathematical forms different from exponential. One of the earliest of these is a lognormal function suggested by Feingold, 1986 [5] of the form

$$N(D) = \frac{N_T}{\sqrt{2\pi}(\ln\sigma)D} \exp \left[-\frac{\ln^2(D/D_g)}{2\ln^2\sigma} \right] \quad (2.2)$$

where $\ln$ is the natural logarithm, N_T , σ and D_g are parameters. This form has been proposed in the past for the analysis of cloud droplet and raindrop distributions by many investigators including Mueller and Sims [6], Levin [7]. An alternative function proposed by Ulbrich [8] and Willis [9], which has come into widespread use is the gamma function having the form

$$N(D) = N_0 D^\mu \exp(-\Lambda D) \quad (2.3)$$

with N_0 , μ and Λ as parameters. The advantage of this distribution is that it allows for distributions with a wide variety of shapes including those which are either concave upward or downward on a plot of $\log(N(D))$ versus D .

DSDs are usually fitted with any one of the above two models, gamma (complete or modified) [10, 11-14] and the lognormal forms [11, 15-17]. Another three parameter distribution proposed is the Weibull distribution, however, this is not considered in this thesis. A very useful method for measuring the DSD was the flour method first utilized by Laws and Parsons in 1943 [1] and Marshall and Palmer in 1948 [2].

At ground level, the study of drop size distributions was revolutionized with the introduction of the electromechanical impact distrometer by Joss and Waldvogel (JWD) in 1967 [18], used in a number of fundamental studies [3-4, 10, 19]. Before considering the models of DSD, we need to understand the limitations of the Joss distrometer (JWD), which is the instrument used to measure DSD in this study.

2. 2. 1   Limitations of the Joss Distrometer

JWD tends to underestimate the number of small drops during a heavy rain event because of ringing of the styrofoam cone when it is hit by the rain drops. This is known as the distrometers' dead time. This was first pointed out by Sheppard [19]. To correct for it, the correction matrix, supplied by the manufacturer is used [19]. In the presence of numerous large rain drops during intense tropical rain events ($R \geq 20$ mm/hr), drop sizes smaller than 1.0 mm are underrepresented [10]. This problem is due to an automatic threshold circuitry that monitors the ambient noise level to reject spurious pulses. However, under intense rain, the high noise level of the drops themselves is interpreted as ambient noise and small-drop signals are rejected. The larger drops produce longer dead times and therefore, requires greater correction. However, if there are no drops in a given bin, the correction matrix does not add any drops to the bin. Rather, it modifies the DSD and increases the high moments of the drop size such as rain rate significantly.

This is a problem of the correction matrix, and thus, many users choose not to implement it [10, 20]. At the large drop end, drops larger than 5.0-5.5 mm diameter cannot be resolved at their true size; rather, they are assigned to the largest size bin. The sensitivity degradation of the JWD has been discussed in the comparative studies carried out between the JWD and other drop size measurement instruments in [19] and [20]. In the present study, the dead-time correction has been applied using the DOS software provided by Distromet. Inc. The correction is intended to correct up to 10% of the error. Another drawback associated with the JW distrometer is its sensitivity to the noise in the environment. In order to avoid the noise, the JW distrometer has a variable noise threshold, which in effect will mask the counts of small drops. The dead-time correction is not intended to correct

this effect. However, in the present study, the distrometer was installed on the rooftop of a 50 m high building where the environmental noise is low.

2. 2. 2 Previous DSD Models for Singapore

The DSD over Singapore was studied previously by Li *et al.* [21] and Ong and Shan [12, 22]. Li *et al.* [21], Ong and Shan [12] have proposed a modified gamma model for DSD (using the moment estimators). Ong and Shan have also studied the DSD properties from JWD measurements during the years 1994 and 1995. This research uses the distrometer data of the years 1994, 1995 and 1998 to analyze the DSD of Singapore. Ong and Shan [22] also modeled the rain drop size distributions using the lognormal model (using maximum likelihood estimators and moment estimators). In their paper, Singapore lognormal and gamma models have been compared with the results [22] from different regions in the world. Ong and Shan used 0th, 1st and 2nd moments and 0th, 2nd and 3rd moments to represent modified gamma and lognormal models.

Comparative studies show that these three distributions (gamma, modified gamma and lognormal) lead to very similar results to fit the observed tropical DSDs [17]. That is why, in the present research, only one of the three is considered, the gamma model. This distribution is preferred because it can give reasonable approximations to the observed spectra [8]. Deviations from the exponential are expressed in terms of the curvature parameter μ . Furthermore, it reduces to the exponential DSD for $\mu = 0$ and this DSD model is popularly used for retrieval of rain rate in RADAR remote sensing. The rain drop size distributions of Singapore are modeled by lognormal [16] and gamma [22] models. Since these two models represent well the DSD of Singapore, the Weibull distribution is not considered in this thesis.

2. 2. 3 Selection of Central Moments

The use of the low order moments are highly affected by wind, splashing, and instrumentation limits [14]. For distrometer data, the use of the 0th moment is impossible and the use of the 1st moment is not advisable, since the number of drops with diameters less than D_{min} is not known [14]. Many authors [10, 11, 13-14, 23-25] preferred to work with central moments, since JWD has degraded sensitivity at small drop diameters. Kozu and Nakamura [11], and Tokay and Short [10] used 3rd, 4th and 6th moments. Smith [23, 24] suggested 2nd, 3rd and 4th moments to model gamma DSD. Ulbrich and Atlas [25] took into account the maximum value for drop diameters, D_{max} , and used the 2rd, 4th and 6th moments. Their method allows for truncation of the DSD at the large-diameter end of the spectrum due in part to instrumental effects. Timothy [16] used 3rd, 4th and 6th moments to represent Singapore's DSD using lognormal model. He observed a significant decrease in drop density in small drop bins from visual inspection of Singapore's data (year 1998 data).

The sixth moment which is proportional to reflectivity is usually selected for fitting the gamma model [8, 10-11, 26]. However, the sampling distribution of the sixth moment of DSD is skewed [24, 27], especially so for small samples, which can lead to substantial errors. Smith et al. [28] explained that for raindrop observations involving small sample sizes, but including the full range of drop sizes, reflectivity values based on MM234 estimators generally give superior results to those from the sample sixth moments themselves. They also suggested the estimation of reflectivity using the maximum likelihood and L-moment methods. Unfortunately, the absence of small drops from the samples (a common circumstance with ground-based distrometer instruments, like Joss-Waldvogel

distrometer) causes difficulties when using the maximum likelihood and L-moment methods.

In this research, different moment combinations are analyzed to fit the gamma drop size distributions. Mean square error in percentage is calculated to compare the gamma DSD with measured data. Based on the analysis, the MM234 and the MM246 gamma models are selected to model the rain drop size distribution.

Caracciolo [13] modeled the gamma DSD using higher order moments of 4th, 5th and 6th to form a model that is less sensitive to small drop diameters. But the bias is stronger when higher order moments are used [24]. Furthermore, this procedure deviates more from the measured data [14]. Brawn [14] suggested an interesting alternative to form less sensitive model by completely ignoring the counts in the lower order bins. Therefore, in this study an investigation is performed to study the validity of ignoring the counts in the lower order bin as proposed by Brawn for rain attenuation calculations. This is done by removing the small drop size bins consecutively starting from the first bin and redesigning the gamma models using the remaining bins for each bin removal.

The deviations in the redesigned gamma models as compared to the actual gamma model are studied. This will aid in the understanding of the importance of small rain drop sizes to the rain attenuation of the terrestrial and earth-satellite communication links. Based on these redesigned models, the importance of small rain drop sizes on communication links at different frequencies is examined.

Therefore, truncated gamma DSD (truncating the first four bins of distrometer data) with the 2nd, 3rd and 4th moments (MM234) by considering Joss distrometer's degraded sensitivity at small drop diameters is used in this study.

These gamma models are used to fit the rain drop size distribution and the importance of small diameters is analyzed by calculating rain attenuation at different frequencies. Gamma model using 2nd, 3rd and 4th moments is used to analyze reflectivity-rain rate relations and the gamma model using 2rd, 4th and 6th moments is used to check the shape-slope relationships using the truncated moment method and un-truncated moment method.

2.3 Rain rate Retrieval from Single Polarized RADAR

Weather RADAR provides measurements of rainfall over large spatial domains at high spatial resolution and relatively frequent time intervals. These characteristics make weather RADAR a useful instrument for rainfall measurement.

2.3.1 Reflectivity-Rain Rate relations

Rain rate estimation from RADAR measurements is based on empirical models such as the reflectivity and rain rate, the Z-R relation, which have been studied for more than 60 years [2]. In RADAR meteorology, accurate measurement of rainfall is important, and variations in reflectivity-rain rate (Z-R) relationships are strongly dependent on DSD variations [8]. The Z-R relationships relate the value of the measured reflectivity to the value of the rain rate according to the general formula [2] by Marshall and Palmer,

$$Z = AR^b \quad (2.4)$$

where the RADAR reflectivity factor Z (mm⁶/m³) and the rain rate R (mm/hr), depend on the DSD [5, 8, 29-30].

Marshall and Palmer [2] published the Z-R relation using the exponential DSD with a set of generic parameters of $A=200$ and $b=1.6$. Battan [29] presented a

list with 69 different Z-R relationships for different climatic conditions in different parts of the world. Although MP-DSD is very popular in computing rainfall rates derived from RADAR reflectivities measurements, actual drop sizes change significantly by geographic location, type of storm, season, and region within the storm. Regarding tropical climates, MP model produce huge overestimations of rainfall rates [31]. This is due to the fact that they overestimate the number of big drops; compared to what is measured by the 2DVD. There can be dramatic changes in Z-R law parameters within an individual storm as well as between storms [10, 30, 32-35]. These changes are clearly identified with the physical processes acting to form the rain event. In the past, research work has been done to improve the accuracy of Z-R relations by classifying the rain into different types.

2. 3. 2   Rain Classification

Many studies have demonstrated that stratiform rain is characterized by larger raindrop diameters relative to convective type rain for the same liquid water content [10, 30]. In recent years [10, 13, 32-34], variations in the gamma DSD parameters are used for rain type classification. Tokay and Short [10] observed a significant change in the gamma parameter, the intercept parameter, N_0 , during the transition from convective to stratiform rain. Later, many researchers reported the existence of a transition region between the convective and stratiform regimes [32-35]. Bringi et al. [36] used a simple scheme to separate stratiform and convective rain types based on the standard deviation of rain rate over 5 consecutive DSD samples. If standard deviation is ≤ 1.5 mm/hr then the rain type is classified as stratiform, otherwise it is assumed to be convective.

Atlas et al. in [32] and Ulbrich and Atlas in [37] studied the DSDs during the three regimes (stratiform, convective and transition) and determined the Z-R

relations for each of these regimes. They pointed out that there is a systematic variation of the Z-R relations for these three types of rain [37]. They identified a rain event to consist of all three regimes. A rain event initially starts off as convective, where the rain rate, R , rises sharply and peak in excess at about 10-15 mm/hr while the median volume diameter, D_0 , does not vary greatly. When D_0 and R decrease simultaneously, following the initial convective period, the rain is classified as transition. This is followed by the stratiform rain which is characterized by its approximately steady rain rate of $R \leq 10$ mm/hr and its higher median volume diameter, D_0 , values. In [37-38], they also studied the variations of N_w , the normalizing constant defined as the intercept of an equivalent exponential DSD with the same water content W and variations of μ , the shape parameter for different types of rain. They concluded that the coefficient A is smaller for stratiform rain and increases for increasing convective activity, while b behaves in the opposite manner (smaller for convective rain and larger for stratiform systems) [32,37].

Motopoli et al. [39] used the D_m and μ variations for rain classification along with the classification used in [36]. The mean diameter D_m evolves around the values of 1.5 mm, and the shape parameter μ is nearly zero (lower values) for their convective samples. For the transition and stratiform samples, both D_m and μ are found to slightly decrease and then increase, respectively, with respect to the convective zone. On the other hand, in the stratiform time slots, D_m oscillates around 1 mm, whereas μ oscillates around five.

Wilson [35] used Singapore's RADAR (Installed by the Rutherford Appleton Laboratory, UK at Nanyang Technological University, Singapore) data and distrometer data and found the Z-R relationships. She also used the variations

in the integral parameters to classify the rain types. In this research, the RADAR data from the RADAR installed by the Meteorological Service of Singapore (MSS) at Changi airport, Singapore, is used for the determination and classification of rain types. The results are compared with those reported by Wilson in [35].

This research intends to study the characteristics of tropical DSDs in terms of bulk rain integral parameters during convective, transition, and stratiform rain and to develop the improved Z - R relations for rain rate retrieval during these three stages. Nine rain events from the nine months data are considered, different Z - R relationships are derived. The variations in the coefficients, A and b , of Z - R relations (and thus in the raindrop size distributions) between different kinds of events are analyzed. The derived Z - R relations are compared with those reported in [35]. This study performs an inter comparison between distrometer derived rain rates and RADAR derived rain rates over Singapore. Z - R relations suitable for the tropical country of Singapore are selected from the DSD. Rain rates retrieved from RADAR data for the rain event is compared with distrometer derived rain rates.

2.3.3 Rain Retrieval from DSD Retrieval

With the development of DSD modeling, much effort has been put into DSD retrieval. The DSD retrieval is obviously superior to the retrieval of integral parameters because DSD is more informative. Much progress has been made in retrieving DSD parameters from polarimetric RADAR measurements recently [26, 36, 40-44]. Dual polarized RADAR has two remote measurements which can be used to estimate the rain rate more accurately than the RADAR which has only a single measurement.

The gamma model has three parameters. In order to retrieve the three DSD parameters, three remote measurements are required from RADAR. But

polarimetric RADAR has only two remote measurements, reflectivity (Z) and differential reflectivity (Z_{DR}). Therefore, for the purpose of retrieving the DSD parameters from remote sensing measurements, it is necessary to reduce the number of parameters of the gamma distribution to two instead of three. Rain DSD parameters can be retrieved from two remote measurements and rain rate can be retrieved from these DSD parameters more accurately [26]. Literature review about the two parameter gamma models is presented next.

2.4 Two parameter Gamma Models

Two parameter gamma models can be formed by assuming a fixed μ or using an empirical constraining relationship between the DSD parameters. As explained in section 2.2.3, the dead time problem of JWD is addressed in [45] and the truncated gamma models can be used to model the drop size distribution. The rain drops in the first four JWD bins which are likely to have error due to the distrometer's dead time, are removed to calculate the observed moments and the equations for the un-truncated moment method are used in [45].

Recent studies [43] have shown that a large error might be introduced and cause a notable bias of μ or Λ estimation if truncated observations are assumed as the un-truncated moments. Therefore, this research uses the iterative truncated moment fitting using 2nd, 4th and 6th moments between the drop diameter ranges from 0.3 mm to 5 mm proposed by Vivekanandan et al. [43]. Un-truncated moment method is used to find the appropriate μ for the Singapore region and both un-truncated and truncated moment methods are used to form the μ - Λ relations.

2. 4. 1 Fixed μ Models

Fixed μ models are previously used by many researchers [10-11, 46-47] for the retrieval of rain rate. Bringi et al. [46] fixed $\mu=3$ in the gamma DSD to estimate rain rate from differential reflectivity (Z_{DR}). Kozu and Nakamura [11] retrieved the rain rate from rain attenuation and reflectivity using the fixed μ model. Their fixed μ range is from 4 to 6 for the gamma model using the 3rd, 4th and 6th moments. The moments 3rd and 4th and 6th was reapplied for fixed μ values by Tokay and Short [10]. The mode of the shape parameter, μ , was close to 6 from their data. Rincon [47] used the fixed μ value of 4 for the gamma DSD to estimate path average DSD and rain rate. Lakshmi et al. [48] tried to find the suitable fixed μ using ten average rain rates from Singapore' drop size data. It is found that $\mu=3$ produce less error in the modeled rain rate with the measured rain rate at the middle rain rates (from 23.29 mm/hr to 76.15 mm/hr) and $\mu=4.58$ or 5 produce less error at the higher rain rates (99.55 mm/hr to 147.70 mm/hr).

2. 4. 2 μ - Λ Relationship

An empirical relation relating any two of the gamma parameters reduces the three-parameter gamma DSD to a two-parameter function. Two dimensional video distrometer (2DVD), the more recent type of distrometer [31], measurements made in Florida, shows a high correlation between μ and Λ [26] indicating that these parameters are related. Their gamma DSD is fitted by using the 2nd, 4th and 6th moments and a second order polynomial constraining relation between μ and Λ is derived. Later, Brandes et al. [41] redefined the constraining relation to accommodate heavy rainfalls using 2DVD measurements made in Florida. He suggested that prior to curve fitting in the μ - Λ scatter plot, the database should be filtered to exclude rain rates smaller than 5 mm/hr, and drop counts smaller than

1000. Zhang et al. [42] have used the same μ - A relation and shown that the relation contains useful information and characterizes the natural rain DSD variations quite well. He noted that the coefficients of the μ - A relation might change with location and season. He has also shown that the μ - A relation might be valid for convective precipitation but may fail in weak, stratiform rain.

Seifert [49] examined the validity of μ - A relation and concluded that this relation is the fundamental property of rain DSD. He found a good agreement between the empirical μ - A relations that he has derived with that by Zhang et al. [26]. This is especially true for the convective and stratiform DSDs except at leading edges of convective storms and drizzle rains. He also found that for strong rain events, μ is much larger in increasing rain than in decreasing rain, resulting in the data points lying above the empirical μ - A relation derived by [26]. The weakest precipitation events having rain rates less than 10 mm/hr shows lower μ values below the empirical μ - A relation derived by [26].

Moisseev and Chandrasekar [50] attributed the μ - A relation to the effect of moment errors. Although it is shown in [42] that the μ - A relation is related to physics as well as moment error, Moisseev and Chandrasekar [50] proposed that the correlation was mainly due to the truncation of small raindrops (≤ 0.6 mm) and data filtering. But the μ - A relation has been successfully applied for rain retrievals using polarimetric RADAR measurements of reflectivity (Z_H) and differential reflectivity (Z_{DR}) [41, 44, 51-52]. Cao and Zhang [53] found that the μ - A relation is practically equivalent to the mean function of normalized DSDs proposed by Testud. He concluded that the equivalence between the μ - A relation and Testud's function indicates the physical information in the μ - A relation of the constrained-gamma (C-G) model proposed by [26]. Narayana et al. [54] studied the variability

of the shape-slope (μ - A) relation using the impact type, Joss-Waldvogel distrometer data measured at Gadanki, India. He used 3rd, 4th and 6th moments to model DSD. He reported that the μ - A relation as a function of height.

Recently, Brawn and Upton [55] fitted the μ - A relation for rain rates greater than 1 mm/hr based on their drop size data measured at Dumfries and Galloway, Scotland, using the Thies and Parsivel distrometers. The procedure described by Brawn and Upton [55] to estimate the values of gamma parameters was used. They filtered all the rain rates less than 1 mm/hr from their drop size data to fit the relation. It was concluded that the relation appears to vary with the type of distrometer used. Munchak and Tokay [56] fitted the μ - A relation for different reflectivity ranges for nine different regions and concluded that the μ - A relations perform best at high reflectivity (> 35 dBZ). In general, the use of μ - A relation is shown by [41, 52-53] to perform well for DSD retrieval.

Atlas and Ulbrich [57] explained that the μ - A correlations proposed by [26, 41] appear to be limited to rainfall events which do not include convective rain; they are biased toward stratiform and transition rains. Therefore, in this research, three different categories of rain rates are considered to fit μ - A relations. Tokay and Short [33] have found using the distrometer and wind profiler measurements that values of $R < 2$ mm/hr are representative of stratiform spectra (which lead to significant rain accumulation) while values of $R > 10$ mm/hr are representative of convective ones (relatively short in duration and highly fluctuating).

Therefore, three different categories of rain rates, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$ (stratiform and transition rain), $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ (stratiform, transition and convective type rain) and $R \geq 25 \text{ mm/hr}$ (convective type rain), are considered in order to fit the μ - A relations for the 996 minutes of drop size data for Singapore.

The rain categorizations used by [41, 54-55] are also considered for the μ - λ relations fitting purposes so as to provide a fair comparison of the different types of rains over different climatic zones.

In this research, we aim to find a suitable fixed μ value and derive an appropriate μ - λ relation for the tropical country of Singapore. Gamma model parameters are calculated for 14 major rain events using the gamma model with 2nd, 4th and 6th moments. The shape parameter, μ , is fixed at different constant values while the DSD is fitted by the gamma model.

An analysis of DSD observations also indicated the existence of a μ - λ relation. Therefore, in order to find a two parameter model, a shape slope relation will also be proposed from the gamma model parameters for Singapore. Rain rates are calculated from fixed μ models. These rain rates are then compared with rain rates from measured data in Singapore. The μ - λ relationship found using Singapore's filtered data is compared with those found in other region.

The fixed μ model and the gamma model using μ - λ relation are compared to find the best two parameter gamma models for the Singapore region. T-matrix calculations are performed for the 1-minute integrated DSDs for the 14 major rain events. Polarimetric RADAR variables from the T-Matrix calculations along with either fixed μ or best μ - λ relations are used to find gamma DSD parameters. The rain rates retrieved using the calculated gamma DSD are compared with the measured rain rate. The fixed μ model and the gamma model using μ - λ relation are compared to find the best two parameter gamma models for the Singapore region.

2.5 Summary

This chapter has been designed to provide a general introduction to the rain drop size distribution models. The three most popular DSD models are discussed and gamma drop size distribution is selected for modeling. The method of moments is used to retrieve the parameters of a gamma distribution. Review about the selection of proper moments for modeling the DSD is presented. The limitations of the Joss-Waldvogel distrometer, measuring instrument of the DSD data have been discussed. Based on the analysis, gamma model with 2nd, 3rd and 4th moments is selected to model the DSD of Singapore. Starting with a brief introduction about the DSD models, the literature review about the applications of DSD data is discussed. It can be found that a proper classification of rain into three rain types is needed before forming the Z-R relations. The ways to form two parameter gamma models are discussed for the retrieval of rain rate from polarimetric rain variables. Gamma model using 2nd, 4th and 6th moments is selected to fit truncated moment and un-truncated moment gamma models. Therefore, in the following chapters, the research is focused on the modeling of the DSD, and then using it to calculate the rain attenuation and to retrieve the rain rates from weather RADAR and polarimetric rain variables.

Chapter 3

Data selection and Models

This chapter describes the Joss-Waldvogel distrometer setup, the data measurement and the rain events selection from distrometer data for analysis. It also describes the Meteorological Doppler Weather RADAR data which is available for the year 1998. Then, the theoretical models for the drop size distribution are reviewed. Finally microwave signal attenuation due to rain is discussed.

3.1 Joss-Waldvogel Distrometer

The measurement of rain drop size distribution has been conducted at an open space outside experimental station of Civil and Environmental School, Nanyang Technological University (1°21'N, 103°41'E) [12, 16, 22]. The distrometer RD-69 installation is illustrated in Figure 3.1. The details about the JWD are attached at Appendix A. The data recorded were from September 1994 to December 1998. During the 5 years of data measurement, the data on June 1995, July 1995, the whole year 1996 and May 1997 to September 1997 were not recorded. Therefore, a

total 31 months of data were recorded. For the 31 months data, a total of around 350 rain events were recorded. A rain event is defined as the start of the rain to the end until it completely stops regardless of several minutes break of rain in between.

Figure 3.1 Joss-Waldvogel Distrometer at Nanyang Technological University

The distrometer was available during the years 1994-1998 at Singapore. Because of the equipment difficulties, the distrometer was removed afterwards. Therefore, there is no recent data from Singapore to verify the accuracy and applicability of the proposed models. But, the DSD data in this region [58] is available for the past years (from the year 1997 to 2001 at Gadanki, India and from the year 2001 to 2003 at Kototabang, West Sumatra). Gamma rain drop size distribution is able to model well all the above mentioned data including the Singapore's DSD data [58]. The DSD data measured at Malaysia in the year 1993 is compared with Singapore data in [59] and it also matches well with Singapore data. This shows that the DSD measured over 10 years (from 1993 to 2003) does not have significant variations within the same climatic zone. The DSD data in this region from the year 2004 to till date are not available for comparison. This is a limitation in this study. Therefore, selected rain events, from the available 31

months of DSD data, are used for analyzing DSD models, attenuation calculations and for rain retrievals.

3.2 Selection of Rain Events

Rain events from the year 1994-1995 are used for rain attenuation calculations and for analyzing the two parameter gamma models. Seven individual DSD minutes from the rain event recorded on the 26th February 1995 are selected for analyzing the degraded sensitivity of JWD to measure small drops and to calculate rain attenuation. Fourteen major rain events are selected from the year 1994-1995 for analyzing the two parameter gamma models. Similarly nine rain events are selected from the year 1998 to analyze the Z-R relations.

Most of the rain events in Singapore reach high intensity very rapidly; remains heavy for a few minutes; then decreases slowly before increasing again. Lower rain rates usually occur during the decreasing period. The highest rain rate recorded was 194.22 mm/hr, on the 3rd July 1998 at 13:36 hour. The corresponding rain event started at 13:13 hr and ended at 14:03 hr. The rain event lasted for 50 minutes and is a convective rain event. Table 3.1 shows the list of rain events, duration of the rain events, number of DSD minutes and the highest rain rates of the events involved in the study. Column 3 of Table 3.1 shows the duration the rain events in Coordinated Universal Time, UTC.

The rain events for analysis are selected based on the rain rates. The selected rain events generally have high rain rates and most of these selected rain events last for long durations without discontinuities. The rain events which include all the three rain types, convective, transition and stratiform stages are selected in order to study different types of rain. Two rain events, one convective

(rain event 19) and one stratiform (rain event 20), are also included for the analysis of the Z-R relations.

Table 3.1 Selected rain events from year 1994-1995 (first 14 events) and from the year 1998 (last 9 events) from the distrometer data for analysis

Number	Date of event	Time (UTC) in hours	No. of samples	Maximum rain rate (mm/hr)
1	5/10/1994	1300-1458	118	69.64
2	30/10/1994	1500-1639	100	153.02
3	6/11/1994	1330-1459	90	191.59
4	13/12/1994	1600-1659	60	111.85
5	18/12/1994	316-359	44	87.71
6	1/01/1995	1628-1659	32	70.44
7	11/01/1995	1600-1659	60	29.66
8	18/01/1995	1715-1733	19	108.93
9	6/02/1995	1100-1159	60	28.04
10	21/02/1995	1600-1659	60	111.66
11	26/02/1995	1704-1950	167	162.06
12	12/03/1995	323-459	97	54.23
13	18/03/1995	308-359	52	96.13
14	18/03/1995	1905-1959	55	139.25
15	9/01/1998	1353-1854	127	85.76
16	5/04/1998	1700-1754	55	114.71
17	9/05/1998	500-959	300	123.37
18	12/05/1998	1353-1859	307	107.40
19	18/05/98	1654-1757	143	97.40
20	18/05/98	1825-1959	43	5.92
21	7/06/1998	1900-2359	300	38.47
22	10/06/1998	135-520	223	64.41
23	25/09/98	515-759	165	90.69

As explained in the Section 3.1, for the 31 months data, a total of around 350 rain events were recorded. By analyzing a few rain events from the region, it is possible to identify the rain structure of the region. First 14 rain events in Table 3. 1 are selected for analyzing the two parameter gamma models. The selection of last 9 rain events (15-23, in Table 3. 1 in the year 1998) for analyzing reflectivity - rain

rate, Z-R relations is also dependent on the availability of the RADAR data which are limited to days within the year 1998.

3.3 RADAR Data

The RADAR data which is utilized in this study is produced by the S-band Meteorological Doppler Weather RADAR (MDWR) system from Meteorological Service of Singapore (MSS). The antenna of the MDWR system is located at 1.3512N, 103.97E, which is adjacent to the Changi airport of Singapore. The location of Nanyang Technological University (NTU) (Block S2) is 1.3423N, 103.68E. The distance between the RADAR and NTU is 32.21 km and the bearing angle is 268.24°.

The RADAR system is programmed to operate in two scanning modes, namely, the “Aerial Mode” and the “Airport Mode.” Each mode takes around 4 minutes for a full-volume scan. Both modes cover the entire land area of Singapore, parts of Malaysia to the north and Indonesia to the east, west and south. The normal operation of the RADAR system is in the “Aerial Mode.” The switching from “Aerial Mode” to “Airport Mode” is triggered when rainfall is detected within a 40X40 km² region centered at the Changi weather station. Once the RADAR system is switched to the “Airport Mode,” it will maintain in this mode for at least 20 minutes before switching back to the “Aerial Mode.” The elevation angles of the rays for the aerial mode are: 0.1°, 1°, 1.5°, 2°, 3°, 5°, 7.5°, 10°, 15° and 20°. The elevation angles of the rays for airport mode are: 1°, 1.5°, 1.7°, 15°, 20°, 30° and 40°. The reflectivity data above NTU can be extracted from the ray of the RADAR using the distance, elevation angle and bearing angle. There are

limited rain events (around 30) available from the year 1998 from Doppler Weather RADAR. Rain rates are calculated using the proposed Z-R relations using the DSD data of Singapore and using M-P Z-R relation and these rain rates are compared with the distrometer derived rain rates.

3.4 Measured DSD

The Joss-Waldvogel Distrometer is capable of measuring the drop diameter ranging from 0.3 mm to >5 mm with an accuracy of $\pm 5\%$. It distinguishes between drops with time interval of about 1ms. The total number of drops of diameters ranging from 0.3 mm to > 5 mm is divided into 20 different bins with 1 minute integration time [60]. The thresholds for the drop size bins are listed in Appendix A. The diameter D_i is a representative diameter for the i^{th} bin. Generally, the mean value of the range is calculated in (3.1).

$$D_i = D_{il} + \frac{D_{iu} - D_{il}}{2} = D_{il} + \frac{\Delta D_i}{2} \quad (3.1)$$

where D_{il} and D_{iu} are the lower and upper diameter value of that i^{th} bin respectively and ΔD_i is the drop size interval for i^{th} bin. The representative diameter is used for computing DSD, rain rate, reflectivity and for all other calculations. The number of raindrops, n_i , in the i^{th} channel with diameters, D_i , in the range of $D_i \pm \Delta D_i/2$ are collected over a sample area of $S=5000 \text{ mm}^2$ with an integration time of $T=60 \text{ sec}$ determined by the JWD. The rain rate (in mm/hr) and reflectivity in mm^6/mm^3 can be calculated from the measured data [60] by

$$R = \frac{3600\pi}{65T} \sum_{i=1}^{20} D_i^3 n_i \quad (3.2)$$

$$Z = \frac{10^6}{ST} \sum_{i=1}^{20} \frac{D_i^6 n_i}{v(D_i)} \quad (3.3)$$

The measured rain drop size distribution $N(D_i)$ ($\text{m}^{-3}\text{mm}^{-1}$) can be expressed by

$$N(D_i) = \frac{n_i \times 10^6}{v(D_i) \times S \times T \times \Delta D_i} \quad (3.4)$$

where $v(D_i)$ is the terminal velocity of rain drop in m/s from Gunn and Kinzer [61].

The terminal velocity of rain drops are usually described using empirical formulas derived directly from experimental data [61, 62]. Use of the terminal velocity measured in stagnant air fitted by Gunn and Kinzer [61], which exhibits a one-to-one relationship with drop sizes, has been a common practice in rain related studies [62]. Therefore, Gunn and Kinzer's terminal velocity values, measured in the subtropical climate, are used in this research.

3.5 Gamma Modeled DSD

Rain drop size distribution can be modeled using gamma model and is expressed using (2.3). The gamma model parameters N_0 , μ and λ are to be determined through measured data. The estimates from the method of moments are obtained by equating a sufficient number of measured moments to the corresponding theoretical moments.

It is possible to define the most important rain integral parameters using DSD and its moments. All the rain integral rain parameters of interest can be represented by the form [8]

$$P = a_p \int_0^\infty D^p N(D_i) dD \quad (3.5)$$

where P is the rain integral parameter, and a_p and p are constants. The values of a_p and p are listed in [8] for several parameters P . For example, the rain integral parameter for $p=2$ is optical extinction, Σ , which is proportional to 2nd moment, M_2 . Similarly, Liquid water content, W , is proportional to 3rd moment, M_3 ; Rain rate, R ,

is proportional to 3.67th moment, $M_{3.67}$ (assuming that the terminal fall velocity, V_t , is proportional to D); Reflectivity, Z , is proportional to 6th moment, M_6 , and the rain attenuation coefficients are proportional to 3rd to 4.5th moments, $M_3 - M_{4.5}$ depending on frequency [8, 11].

As explained in the introduction, the low order moments are highly affected by wind, splashing, and instrumentation limits. The bias is stronger when higher moments are used for fitting the rain DSD [24]. Therefore, the gamma DSDs are fitted using the central moment combinations MM234 using 2nd, 3rd and 4th moments, MM346 using 3rd, 4th and 6th moments and MM246 using MM234 using 2nd, 4th and 6th moments next. The selection of proper moment combination improves the accuracy of the modeled rain rate and therefore gives the accurate rain attenuation.

The k^{th} experimental moment is expressed by M_k

$$M_k = \frac{N_t}{n} \sum_{i=1}^n D_i^k \quad (3.6)$$

where n is the number of samples and N_t is the particle number concentration; M_k is obtained through the experimental data. The k^{th} theoretical moments can be written as

$$\Omega_k = \int_0^\infty D^k N(D) dD \quad (3.7)$$

By equating experimental (3.6) moments and theoretical (3.7) moments, it is derived that

$$M_k = \frac{N_0}{\Lambda^{\mu+k+1}} \Gamma(\mu + k + 1) \quad (3.8)$$

3.5.1 MM234 gamma model

By solving the three simultaneous equations that are obtained by substituting $k=2, 3$ and 4 respectively, into (3.8), the expression for the estimators of DSD obtained is as follows

$$\mu = \frac{4M_3^2 - 3M_2M_4}{M_2M_4 - M_3^2} \quad (3.9)$$

$$\Lambda = (\mu + 4) M_2 / M_3 \quad [\text{mm}^{-1}] \quad (3.10)$$

$$N_0 = \Lambda^{\mu+3} M_2 / \Gamma(\mu + 3) \quad [\text{mm}^{-1-\mu} \text{m}^{-3}] \quad (3.11)$$

3.5.2 MM346 gamma model

By solving the three simultaneous equations that are obtained by substituting $k=3$, 4 and 6 respectively, into (3.8), the expression for the estimators of DSD obtained in [11] is as follows

$$\mu = \frac{11F - 8 + \sqrt{F(F+8)}}{2(1-F)}, \quad \text{with} \quad F = \frac{M_4^3}{M_3^2 M_6} \quad (3.12)$$

$$\Lambda = (\mu + 4) M_3 / M_4 \quad [\text{mm}^{-1}] \quad (3.13)$$

$$N_0 = \Lambda^{\mu+4} M_3 / \Gamma(\mu + 4) \quad [\text{mm}^{-1-\mu} \text{m}^{-3}] \quad (3.14)$$

3.5.3 MM246 gamma model

The three gamma model parameters (N_0 , μ or Λ) can be solved from the 2nd, 4th and 6th moments as proposed by [25]. To eliminate Λ and find μ , a ratio is defined as

$$\eta = \frac{M_4^2}{M_2 M_6} \quad (3.15)$$

Then μ can be solved using

$$\mu = \frac{(7-11\eta) - [(7-11\eta)^2 - 4(\eta-1)(30\eta-12)]^{1/2}}{2(\eta-1)} \quad (3.16)$$

Λ and N_0 can be calculated

$$\Lambda = \left[\frac{M_2(\mu+3)(\mu+4)}{M_4} \right]^{1/2} \quad [\text{mm}^{-1}] \quad (3.17)$$

$$N_0 = \Lambda^{\mu+3} M_2 / \Gamma(\mu + 3) \quad [\text{mm}^{-1-\mu} \text{m}^{-3}] \quad (3.18)$$

The above method for estimating DSD parameters is applicable only for un-truncated DSD. The DSD parameters are overestimated if the truncated size data is

used in the un-truncated moment method [43]. For a gamma distribution with a truncated size range, the statistical moments are calculated [43] as

$$M_k = \int_{D_{min}}^{D_{max}} D^k N(D) dD = N_0 \Lambda^{-(\mu+k+1)} [\gamma(\mu+k+1, \Lambda D_{max}) - \gamma(\mu+k+1, \Lambda D_{min})] \quad (3.19)$$

where $\gamma(. . .)$ is an incomplete gamma function, D_{min} and D_{max} are 0.3 mm and 5 mm for Joss distrometer data.

Using the truncated moments shown in (3.19), moments consistent with truncation can be calculated, and then the corresponding expressions for DSD parameters can be derived as follows:

$$\eta = \frac{[\gamma(\mu+5, \Lambda D_{max}) - \gamma(\mu+5, \Lambda D_{min})]^2}{[\gamma(\mu+3, \Lambda D_{max}) - \gamma(\mu+3, \Lambda D_{min})] \times [\gamma(\mu+7, \Lambda D_{max}) - \gamma(\mu+7, \Lambda D_{min})]} \quad (3.20)$$

$$\Lambda = \left[\frac{M_2 [\gamma(\mu+5, \Lambda D_{max}) - \gamma(\mu+5, \Lambda D_{min})]}{M_4 [\gamma(\mu+3, \Lambda D_{max}) - \gamma(\mu+3, \Lambda D_{min})]} \right]^{1/2} \quad (3.21)$$

Equations (3.20) and (3.21) constitute joint equations for μ and Λ for the truncated moments that are difficult to separate from each other. An iterative approach is used as explained in [43] to estimate the DSD parameters for the truncated moment method.

3.6 Rain integral Parameters

The important parameters of precipitation can be calculated using DSD and its parameters. The rain integral parameters such as rain rate in mm/hr and reflectivity in mm^6/mm^3 are calculated by the following expressions:

$$R = 6\pi \times 10^{-4} \sum_{i=1}^{20} D_i^3 v(D_i) N(D_i) \Delta D_i \quad (3.22)$$

$$Z = \sum_{i=1}^{20} D_i^6 N(D_i) \Delta D_i \quad (3.23)$$

where $N(D)$ is gamma drop size distribution using (2.3). The gamma model parameters from MM234 using (3.9) to (3.11) are used to find $N(D)$ and rain

integral parameters. The median volume diameter (D_0) is calculated using the gamma model parameters μ and Λ using

$$D_0 = (3.67 + \mu)/\Lambda \quad [\text{mm}] \quad (3.24)$$

N_w is generalized number concentration of an exponential DSD having the same liquid water content W and mass-weighted diameter D_m as the actual DSD [63] and it is calculated using

$$N_w = \frac{4^4}{\pi \rho_w} \left[\frac{1000W}{D_m^4} \right] \quad [\text{mm}^{-1}\text{m}^{-3}] \quad (3.25)$$

where W is in gm^{-3} , proportional to the third moment of the drop size distribution $N(D)$; $\rho_w = 1$ is the water density in g cm^{-3} ; D_m is the mass weighted mean diameter and defined as the ratio of the fourth to the third moments of the DSD.

$$D_m = \frac{\int_0^\infty N(D)D^4 dD}{\int_0^\infty N(D)D^3 dD} \quad [\text{mm}] \quad (3.26)$$

These rain integral parameter and gamma model parameter variations are used for rain classification in chapter 7.

3.7 Rain Attenuation Calculations

Rain attenuation of microwave signals is caused by the effect of absorption and scattering on microwaves by rain drops of various sizes. Mathematically, the signal attenuation on a path is the algebraic sum of the components due to scattering and absorption. The relative importance of scattering and absorption is a function of the complex index of refraction of the absorbing / scattering particle. If the particle is very small compared to the wavelength of the radio wave, Rayleigh scattering theory can be applied. In these situations, the attenuation will be due mainly to

absorption. This condition generally holds for signals below 10 GHz propagating through an ensemble of hydrometeors.

As the frequency increases, the size of the rain drop becomes an appreciable fraction of the wavelength. The signal is scattered and absorbed by the raindrop. The rain can no longer be considered a lossless dielectric at or above 10 GHz for lower rainfall rates (above 7 GHz for tropical climates) [64]. For these cases, Mie Scattering Theory is used. The calculation methods to find specific rain attenuation and slant-path rain attenuation are presented next.

3.7.1 Specific Rain Attenuation using ITU-R P.838.3 Model

The specific rain attenuation γ_R (dB/Km) is obtained from the rain rate R (mm/hr) using the power law relationship by ITU-R P.838-3 [65]:

$$\gamma_R = kR^\alpha \quad (3.27)$$

where k and α are frequency dependent coefficients, rain temperature, refractive index of raindrops, DSD and the polarization state of the system. Empirical relations obtained from experimental measurements more or less support the form of the above relation.

The theoretical background of the above relation had been examined by Olsen et al [66] and found to be accurate over a large range of frequencies including the millimeter range. The values of k and α were obtained from logarithmic regression fitted to the exact specific attenuation values computed using Mie Scattering Theory for spherical raindrops in the frequency range of 1-1000 GHz for the Laws and Parsons (LP), Marshall-Palmer (MP), Thunderstorm (J-T), Drizzle (J-D) drop size distribution functions. The values for the refractive index of water required in the calculations were obtained at temperatures of 20°C, 0°C and –10°C from the

equations given by Ray [67]. The calculations for the J-T and J-D distributions are applied to convective and widespread rain respectively.

Values for the coefficients k and α are determined as functions of frequency, f (GHz), in the range from 1 to 1000 GHz, from the following equations, which have been developed from curve-fitting to power-law coefficients derived from scattering calculations:

$$\log_{10}k = \sum_{j=1}^4 a_j \exp \left[- \left(\frac{\log_{10}f - b_j}{c_j} \right)^2 \right] + m_k \log_{10}f + c_k \quad (3.28)$$

$$\alpha = \sum_{j=1}^5 a_j \exp \left[- \left(\frac{\log_{10}f - b_j}{c_j} \right)^2 \right] + m_\alpha \log_{10}f + c_\alpha \quad (3.29)$$

where f is the frequency (GHz), k is either k_H or k_V and α is either α_H or α_V . Values for the constants for the coefficients k_H for horizontal polarization, k_V for vertical polarization, α_H for horizontal polarization and α_V for vertical polarization are given in ITU-R P.838-3 [65]. For linear and circular polarization and for all path geometries, the coefficients k and α can be calculated by the following equations as recommended by [65].

$$k = [k_H + k_V + (k_H - k_V)\cos^2\theta\cos 2\tau]/2 \quad (3.30)$$

$$\alpha = k_H\alpha_H + k_V\alpha_V + (k_H\alpha_H - k_V\alpha_V)\cos^2\theta\cos 2\tau/2k \quad (3.31)$$

where θ is the experimental angle of elevation 40° and τ is the polarization tilt angle relative to the horizontal ($\tau=0$ and 90 degrees for linear polarization). The rain rates, calculated from (3.22) are used to calculate the specific rain attenuation.

3.7.2 Specific Rain Attenuation using Forward Scattering

Coefficients

Specific attenuation due to a rain path depends on the rain rate, shape of the rain drops, distribution of the rain drops size, operating frequency and wave polarization. Rain drops range in size from very small to fairly large ones. It is

known from photographic observations that water drops falling at terminal velocity in air have the shape of asymmetric oblate spheroids with a flattened base when their sizes exceed about 1.0 mm in radius [68]. It has generally been recognized that a water drop of diameter less than 1 mm has nearly spherical shape due to the strong surface tension effect at the water–air interface, with the relative air flow outside and liquid circulation inside of the drop being basically laminar, steady, and axisymmetric.

Some of the initial work on drop shapes is done by [69]. Pruppacher and Pitter [69] developed a refined vertical wind tunnel to freely suspend water drops in the upward air stream and to obtain detailed information on axis ratio with high-quality photographs as a function of rain drop size. A water drop falling under gravity in a viscous medium such as air takes the form such that the internal and external pressures at the surface balance with each other. The pressure balance equation is solved in [69] to find the real shape of raindrops and is shown that water drops larger than about 1.0 mm in radius are of oblate shape with a flattened base.

In addition, theoretical work has been done, the most quoted being the numerical model of Beard and Chuang [70] for equilibrium shapes. Beard and Chuang [70] iteratively solved the full nonlinear Young–Laplace equation by numerical means, using an internal hydrostatic pressure with an external aerodynamic pressure based on measurements for a sphere but adjusted for the effect of free surface distortion and drag force magnitude. Physically, the numerical model of Beard and Chuang contained basically the same ingredients as that of Pruppacher and Pitter. But mathematically it became applicable to very large raindrops with substantial deformations. Past studies [69-70] have

approximated the drop shapes to oblate spheroids, even though it is well known that larger drops tend to deviate from such an approximation. Since then, many publications have appeared containing empirically derived formulas to relate the mean axis ratio, defined by the ratio between the maximum vertical and horizontal chords. An oblate spheroid model with the same axis ratio is used in place of the “true” shape to measure the polarimetric Radar variables in the T-Matrix code.

Figure 3.2 Computed rain drop shapes for $D_{eq}=2, 3, 4$ and 5 mm with origin at the center. Shown for comparison are dashed circles of diameter D_{eq}

More recently, Thurai and Bringi [71, 73] measured the mean axis ratios of water drops falling from an 80 m bridge using a 2-dimensional video distrometer. Each drop was categorized into various equivolumetric sphere diameter, the diameter of a sphere with the same volume as the deformed drop, ranging from 1.5 to 8 mm. Figure 3.2 represents their estimate of the best fitted equation to the most probable drop shape, given by

$$x = c_1 \sqrt{1 - \left(\frac{y}{c_2}\right)^2} \left[\cos^{-1} \left(\frac{y}{c_3 c_2} \right) \right] \left[c_4 \left(\frac{y}{c_2} \right)^2 + 1 \right] \quad (3.32)$$

where x and y are the Cartesian coordinates and the parameters c_1 , c_2 , c_3 , and c_4 were fitted to obtain the mean dependence on the equivolumetric sphere diameter (D_{eq} in mm), given by

$$c_1 = \frac{1}{\pi} (0.02914 D_{eq}^2 + 0.926 D_{eq} + 0.07791)$$

$$c_2 = -0.01938 D_{eq}^2 + 0.4698 D_{eq} + 0.09538$$

$$c_3 = -0.06123 D_{eq}^3 + 1.3880 D_{eq}^2 - 10.41 D_{eq} + 28.34$$

$$c_4 = -0.01352 D_{eq}^3 + 0.2014 D_{eq}^2 - 0.8964 D_{eq} + 1.226 \text{ for } D_{eq} > 4 \text{ mm}$$

$$c_4 = 0 \text{ for } 1.5 \text{ mm} \leq D_{eq} \leq 4 \text{ mm}$$

It is clear from Figure 3.2 that the larger drops have more deviation from oblate spheroid shapes.

The scattering calculations were performed using the T-Matrix method [72,74] which enables the computation of the complex forward scattering coefficients, assuming oblate spheroidal drops at 20°C with the shape according to the Beard and Chung model [70] for H and V polarizations. T-Matrix code is initially developed to study the polarimetric radar observable structures during the cloud and precipitation evolution to give better understanding and estimation of cloud microphysics by making use of multi parameter polarimetric radars. A Mueller-matrix-based approach is adopted for calculating radar parameters using the T-matrix method [74]. The computation of the Mueller matrix requires the following information: the particle size relative to the wavelength of the incident wave, eccentricity of the scattering particle, complex refractive index, relative to the surrounding medium, orientation of the particle relative to the direction of the incident wave, and the polarization state of the incident wave. The values for the

refractive index of water required in the calculations were obtained at temperatures of 20°C from the equations given by Ray [67].

Table 3.2 Parameters for calculating scattering coefficients

Parameters	Specifications
Drop shapes	Oblate spheroidal drops
Drop Shape model	Beard and Chung model
Temperature	20°C
Refractive index of water	Ray's equations [67]
Polarization	H and V polarizations
Approach	Mueller-matrix-based approach
Scattering theory used	<u>Rayleigh scattering theory</u> : Small rain drops compared with wavelength of the radio waves <u>Mie scattering theory</u> : The size of the rain drop becomes an appreciable fraction of the wavelength
DSD	Gamma DSD
Canting angle distribution	Gaussian distribution for polar angle and uniform distribution for azimuth angle

The backward and forward scattering coefficients are calculated for rotationally and equatorially symmetric dielectric particles at an arbitrary incidence. At frequencies below 6 GHz, $\lambda > 50$ mm, rain drop sizes satisfy the condition that radius of the rain drops, r , (0.1 mm to 3 mm) are less than one tenth of the wavelength, and therefore the Rayleigh scattering theory is applicable. For frequencies below 10 GHz ($\lambda > 30$ mm), $r < \lambda/10$, except for heavy rain, and Rayleigh scattering gives a good approximation. In T-Matrix code, Rayleigh scattering theory is used for a spheroid (oblate/ spherical) with small sizes compared with wavelength. The sizes of the rain drops are compared with the wavelength and if $r < \lambda/10$, then Rayleigh scattering theory is used; otherwise Mie scattering theory is used. At higher frequencies where the sizes of the rain drop become an appreciable fraction of the wavelength, $r > \lambda/10$, Mie scattering theory

is used. The calculations should be accurate up to frequencies around 35 GHz. Single scattering between rain drops are assumed in calculations.

Gaussian distribution (zero mean with standard deviation from 0 to 10°) for polar angle and uniform distribution (0 to 2 π) for azimuth angle (orientation of the raindrop in terms of polar and azimuth angles, usually called canting angle distribution) is considered. Table 3.2 summarizes the parameters chosen for calculating the scattering coefficients for any DSD minute.

Gamma rain drop size distribution parameters are used for calculating the scattering coefficients. The specific rain attenuation for H and V polarizations γ_H and γ_V in dB/km are given by

$$\gamma_{H,V} = 8.686 \times 10^3 \times \lambda \times \sum_{i=1}^{20} \text{Im} \left(f_{H,V}(D_i) \right) N(D_i) dD_i \quad (3.33)$$

where $f_H(D_i)$ and $f_V(D_i)$ are complex forward scattering coefficients (in units of m) for horizontal and vertical polarization respectively, $N(D_i)$ is the number of drops per unit volume per unit drop diameter in $\text{m}^{-3}\text{mm}^{-1}$, dD_i is the drop size interval in mm and λ is the wavelength.

3. 7. 3 ITU-R Rain Attenuation

The ITU-R P.618-9 [75] gives the following procedure to estimate the long-term statistics of the slant-path rain attenuation at a given location for frequencies up to 55 GHz. The following parameters are required:

$R_{0.01}$: the point rainfall rate for the location for 0.01% of an average year
(mm/hr)

h_s : the height above mean sea level of the earth station (km)

θ : the elevation angle (degrees)

φ : the latitude of the earth station (1.3426 degrees)

f : the frequency

The earth station height above mean sea level, h_s , is approximately 56 m, measured at the roof-top of a building in NTU, Singapore using Symmetricom XL-GPS receiver (1°21'N, 103°41'E).

The steps are:

1. Determine the rain height h_R by (3.34) using the 0° isotherm height data h_o , given by ITU-R P.839-3 [76]

$$h_R = h_o + 0.36 \quad [\text{km}] \quad (3.34)$$

2. For $\theta \geq 5^\circ$, compute the slant path length, L_s by the following formula,

$$L_s = \frac{(h_R - h_s)}{\sin \theta} \quad [\text{km}] \quad (3.35)$$

where θ , the elevation angle, is considered from 10° to 90° in steps of 10°. For $\theta < 5^\circ$, equation 2 of [75] can be used.

3. Calculate the horizontal projection, L_G , of the slant-path length from:

$$L_G = L_s \cos \theta \quad [\text{km}] \quad (3.36)$$

4. As stated in the introduction, $R_{0.01}=120.30$ mm/hr is used from the rain statistics measured at NTU, Singapore.
5. Compute the specific attenuation, γ_R (dB/km) for the rainfall rate of 120.30 mm/hr at the frequency and polarization of interest for the transmission. The specific rain attenuation calculated using (3.27) and (3.33) is used as γ_R in this research.
6. Calculate the horizontal reduction factor, $r_{0.01}$, for 0.01% of the time;

$$r_{0.01} = \frac{1}{1 + 0.78 \sqrt{\frac{L_G \gamma_R}{f}} - 0.38(1 - e^{-2L_G})} \quad (3.37)$$

7. Calculate the vertical reduction factor, $v_{0.01}$, for 0.01% of the time:

$$\zeta = \tan^{-1} \left[\frac{(h_R - h_s)}{L_G r_{0.01}} \right] \text{ [degrees]} \quad (3.38)$$

$$\text{For } \zeta > \theta, \quad L_R = \frac{L_G r_{0.01}}{\cos \theta} \text{ [km]}$$

$$\text{Else,} \quad L_R = \frac{(h_R - h_s)}{\sin \theta} \text{ [km]} \quad (3.39)$$

$$\text{Since } |\varphi| = 1.3426^\circ < 36^\circ, \quad \chi = 36 - |\varphi| \text{ [degrees]} \quad (3.40)$$

$$v_{0.01} = \frac{1}{1 + \sqrt{\sin \theta} \left(31(1 - e^{-(\theta/(1+\chi))}) \frac{\sqrt{L_R \gamma_R}}{f^2} - 0.45 \right)} \quad (3.41)$$

8. The effective path length is;

$$L_E = L_R v_{0.01} \text{ [km]} \quad (3.42)$$

9. The predicted attenuation exceeding 0.01% of the time is obtained from:

$$A_{0.01} = \gamma_R L_E \text{ [dB]} \quad (3.43)$$

3.8 Summary

This chapter describes the Joss distrometer which is set up at NTU during the year 1994. It also describes the S-band Meteorological Doppler Weather RADAR (MDWR) data from Meteorological Service of Singapore (MSS). The experimental data analyses, which include the summary of the collected data, selection of rain events for analysis, DSD modeling methods and calculation of rain integral parameters from DSD models are also explained. Then the methods to calculate specific rain attenuation and slant path rain attenuation from DSD are presented.

Chapter 4

Truncated Gamma Drop Size Distribution

This chapter presents the comparison of different gamma drop size distribution models. Mean Square Error in percentage is calculated to compare the gamma DSD models. Measured drop size distribution is examined for seven instantaneous rain rates. The contribution of individual bins in rain rate measurements is found for different rain rates. Truncated gamma models are designed by removing the lower drop size bins. Truncated gamma models are compared with the actual gamma model. The importance of truncated gamma models is explained in our paper [77-79].

4.1 Analysis of Measured DSD

Figure 4.1 illustrates the rain event which occurred on 26 February 1995. This event started at 1024 min (17:04 hr). In 10 minutes, it reached its maximum intensity of 162.06 mm/hr. The rain remained heavy for a few minutes. Then it decreased. This event had two rain peaks. The second peak had a rain rate of

136.22 mm/hr, occurred at 1105 min (18:25 hr). The event which lasted over two hours stopped at 1190 min (19:50 hr). The total amount of rain was 112.01 mm. This is a heavy and longer rain event. This rain event has a broad range of rain rates from lower to higher values. Seven different DSD minutes are selected from this rain event for analysis as there is no discontinuity in the rain in this rain event. The DSD minutes are selected in such a way to represent the structure of DSD at different rain rates from lower to higher values.

Figure 4.1 Rain rate versus time for the rain event on 26th February 1995

Figure 4.2 illustrates the DSD obtained from the measured data using (3.4) for seven one minute rain rates 1.96 mm/hr, 4.20 mm/hr, 10.45 mm/hr, 22.80 mm/hr, 66.54 mm/hr, 120.30 mm/hr and 141.27 mm/hr. Logarithmic scale is used to represent $N(D_i)$ in the vertical axis. The DSD increases initially with the diameter of the rain drop and then decreases for all the rain rates. For example, the maximum diameter for a rain rate of 4.20 mm/hr is 2.58 mm whereas for a rain

rate of 141.27 mm/hr is 5.30 mm.

It is also observed that the drop diameter corresponding to the maximum $N(D_i)$, usually called mode diameter increases with the rain rate. For example, the mode diameter for 4.20 mm/hr is 0.46 mm while for 120.30 mm/hr is 1.12 mm. The measured drop counts at different bins from the JWD for those seven minutes are listed in Table 4.1. It is possible to calculate the rain rate from the measured number of drops for a minute.

Figure 4.2 Drop size distributions for seven rain rates

As can be seen in Table 4.1, as the rain rate increases, the dead time problem can be observed from the lack of drops in the lower bins (labeled as 'X'). The sensitivity of the distrometer at different bins can be checked by the number of rain drops in the measured data at different rain rates. The lower bins which are marked as 'X' in Table 4.1 have no rain drops in the measured data. It is clear that the number of lower bins which are affected by dead time problem increase with

rain rate. There are zero rain drops in the first bin at 22.80 mm/hr and 66.54 mm/hr and in the first two bins at 120.30 mm/hr.

Table 4.1 Measured Rain Drops from JWD at Seven Rain Rates

Bin i	Date : 26-02-1995 / Time (hrs)						
	1934	1801	1747	1856	1846	1828	1711
	Rain rate (mm/hr)						
	1.96	4.20	10.45	22.80	66.54	120.30	141.27
1	2	5	5	X	X	X	X
2	8	35	26	26	23	X	X
3	38	36	43	59	62	7	X
4	68	55	66	50	144	31	5
5	49	41	52	56	123	49	23
6	88	87	99	88	253	169	112
7	72	100	158	110	364	446	308
8	24	63	72	114	260	342	277
9	9	29	41	75	145	231	217
10	2	17	40	89	176	311	214
11		6	67	148	236	471	417
12		3	24	63	168	315	347
13		1	3	20	86	157	168
14			1	10	37	86	91
15				1	41	56	89
16				1	7	20	52
17					3	5	30
18					1	6	4
19						0	1
20						1	1

It is also observed that the first three bins shows low number of counts compared to the higher bins at all the rain rates even after the dead time correction is applied. At 141.27 mm/hr, there are no rain drops in the first 3 bins and only five rain drops at the 4th bin. However, common amongst all rain rates is that, the fifth bin always has a reasonable number of rain drop counts. As stated in the introduction, the numbers of rain drops in the lower 4 bins are severely affected by the dead time problem at high rain rates. In order to study the significance of lower bins which are more erroneous due to the dead time problem, the rain rate estimation is taken. The contribution of individual bins in rain rate estimation is

found using measured data next.

4.2 Contribution of Individual Drop Diameters

The rain rates are calculated using (3.2) from the measured DSD. Then the rain rate contribution of individual bin is removed one by one, starting from the first bin to the last bin (containing rain drops) for the measured DSD. The rain rate with individual bins removed is calculated for each case. The difference between the measured rain rate and the rain rate with the j^{th} bin removed is then calculated in order to study the significance of individual bins with its corresponding range of rain drop size diameters. This will provide information on the importance and contribution of each drop diameter range to the overall rain rate.

The normalized deviation (%) is calculated using the true rain rate and the rain rate with the j^{th} bin removed using the following equation

$$ND_j(\%) = \frac{[R(true) - R(j^{th} \text{ bin removed})]}{R(true)} \times 100 \quad (4.1)$$

where $R(true)$ is found from equation (3.2) and $R(j^{th} \text{ bin removed})$ is also found from equation (3.2) but with $i=j^{th}$ term removed from the summation.

Figure 4.3 shows a normalized deviation (%) using (4.1) for each bin removal at the seven rain rates considered. As seen from Figure 4.3, the contribution of bins in rain rate measurement increases gradually with the removal of bin 1 to the middle bins and then decreases for all rain rates. As the rain rate increases, the bin that has the major contribution increases. At higher rain rates, the contribution of larger drops is more significant in rain rate estimation. The lower bins have null data or underestimated at higher rain rates, then, removing them will not make much difference to the total rain rate calculated from the modified DSD

(R_j^{th} bin removed)). The low deviations in Figure 4.3 can be due to this false removal, as $R(true)$ in (4.1) will also have errors due to the null data.

Figure 4.3 Normalized Deviation (%) for each individual bin removal

At the lower rain rate of 4.20 mm/hr, from Table 4.1, the highest number of rain drop count appears at bin 7 ($D_i = 1.12$ mm), however, the most significant bin in rain rate estimation is bin 8 ($D_i = 1.33$ mm). Even though the highest number of rain drop count appears at bin 7, the most significant bin in rain rate estimation is bin 11 at 10.45 mm/hr. This shows that the contribution of rain drops to the overall rain fall rate is not only dependent on the number of drop counts and drop diameter but also the distribution. At the low rain rate of 1.96 mm/hr, most drop counts are in bin 6, with the number of drop counts distributed between bins 4 to 8 (Table 4.1). The most significant bin at this rain rate is bin 7 (Figure 4.3). At the high rain rate of 141.27 mm/hr, the drops are quite equally distributed over bins 6 to 18. This results in a number of bins (above bin 10) showing significant contribution to

the overall rain rate. This confirms that the contribution to the overall rain rate is dependent on both the drop diameter and drop count distribution.

The $ND_j(\%)$ is very small at rain rates from 1.96 mm/hr to 10.45 mm/hr for the first 2 bins. The contributions of 3rd and 4th bins are more significant at these rain rates. The total $ND_j(\%)$ produced by the first 4 bins is 8.45, 3.75 and 1.71 at 1.96 mm/hr, 4.20 mm/hr and 10.45 mm/hr respectively. Afterwards, the $ND_j(\%)$ increases with the removal of the successive bins. The reduction in the total normalized deviation produced by the first 4 bins at higher rain rates from 22.80 mm/hr to 141.27 mm/hr is due the underestimation of drop counts at these bins because of the distrometer's dead time.

In Figure 4.4, the percentage of normalized deviation is calculated, but this time, instead of removing one bin at a time, consecutive bins are removed starting from the smallest drop size diameter. This enables us to study the contribution of the range of the rain drop size diameters to the overall rain rate. As expected from Figure 4.4, when the number of bins removed increases, the deviation increases correspondingly. The point of this study is to find the bin after which there is a sharp increase in deviation. When the first 1 to 2 bins are removed, the deviation is minimal for all the considered rain rates. However, when bins 1 to 3 are removed, there is a sudden increase in the deviation for rain rates 1.96 mm/hr and 4.20 mm/hr. After that, with the consecutive removal of bins 1 to 4, 1 to 5 and so on, the change in deviation is significant for all the rain rates considered. This gives a clear indication that the first 2 bins are affected severely by the distrometers' dead time problem at rain rates from 1.96 mm/hr to 10.45 mm/hr. For higher rain rates of 22.80 mm/hr and above, the significant increase in deviation starts from the consecutive removal of bins 1 to 5.

Figure 4.4 Normalized Deviation (%) for consecutive bins removal

Therefore, it is concluded from $ND_j(\%)$ calculations that the first two bins can be neglected for lower rain rates (≤ 22.8 mm/hr) and the first 4 bins can be neglected for higher rain rates (> 22.80 mm/hr). From the above analysis, it can be assumed that the dead time problem is severe at the lower four bins. Therefore, taking into account of both the lower and higher rain rates, the first four bins are removed consecutively from bin 1 to bin 4 to redesign the gamma models in section 4.4. Gamma drop size distribution models using different moment combinations are compared next and then truncated gamma models are designed.

4.3 Comparison of Gamma Models

Figures 4.5a and 4.5b show the measured DSD and gamma modeled DSDs using the three different moment combinations, MM234, MM346 and MM246 at rain

rates 10.45 mm/hr and 120.30 mm/hr respectively. It is clear from Figure 4.5 that all the three gamma models fit well with the measured data at both 10.45 mm/hr and 120.30 mm/hr.

(a) 10.45 mm/hr

(b). 120.30 mm/hr

Figure 4.5 Gamma models, MM234, MM246 and MM346 with measured DSD

MM346 gamma model deviates furthest at lower drop diameters (fit slightly inwards) at 10.45 mm/hr followed by MM246 and MM234 gamma model. However, at the higher rain rate of 120.30 mm/hr, all the three models fit in a similar way. The difference between the three gamma model fits is higher at lower rain rates of less than 66.54 mm/hr. This difference decreases at higher rain rates. The differences in the fits results from the differences in the gamma model parameter estimations. Therefore, the three gamma model parameters are plotted for seven rain rates in Figure 4.6.

From Figure 4.6, it can be seen that the gamma model parameters, N_0 , μ and λ , calculated from MM346 model are highest followed by MM246 and MM234 except at 120.30 mm/hr. Even though the gamma model parameter values increase in the reverse order at 120.30 mm/hr, the difference between the parameter values are insignificant compared to the lower rain rates. At high rain rates from 66.54 mm/hr, the gamma model parameter values from the three gamma models are nearer to each other as the errors in calculations reduce at high rain rates.

Figure 4.6 Gamma model parameters for the seven rain rates (a) Shape parameter, μ (b) Slope parameter, Λ (c) Intercept parameter, N_0

This is because of the reduction in the dynamic range of the DSD parameters. The gamma models, MM346 and MM246, have higher estimation of the gamma model parameters as the higher-order 6th moment is used in these models. The accuracy of the gamma models can be evaluated by calculating mean square error as a percentage using the formula in (4.2)

$$MSE (\%) = \sum_{i=1}^n \frac{[\log(\text{Measured } N(D_i)) - \log(\text{Modeled } N(D_i))]^2}{n} \times 100 \quad (4.2)$$

where n is the number of bins in the measured DSD with data. $MSE (\%)$ is calculated using (4.2) for all the rain rates.

Figure 4.7 shows the difference between the mean square errors of MM234, MM246 and MM346 gamma models to the measured data for the seven rain rates. It can be seen that the overall fit in which the three gamma DSD parameters are computed with MM234 performs better than MM246 and MM346 gamma models. At 147.27 mm/hr, MM346 model produces lower error than MM246 and MM234 models. However, the difference in error between the three models is small.

Figure 4.7 Mean square errors in percentage of MM234, MM246 and MM346 gamma models with measured data

A data set of the 23 rain events listed in Table 3.1, is used for comparing the gamma models. Only DSDs having number of rain drops greater than 100 are considered (2443 minutes of data) from the 23 rain events. The average rain rate of each event is found by adding the rain rates of the DSD minutes of the rain event.

The sum is then divided by the total sample minutes of the same rain event. Similarly, MSE (%) of each rain event is obtained by averaging the sum of the MSE (%) of the individual DSD minutes of the rain event. Figure 4.8 shows the average mean square errors in percentage of the gamma models (MM234, MM246 and MM346) as a function of the average rain rate for the 23 considered rain events. From Figure 4.8, it is clear that MM234 model produces smaller error with measured data compared to the other two models MM246 and MM346.

Figure 4.8 Total mean square errors in percentage of the gamma models (MM234, MM246 and MM346) as a function of the average rain rate for the 23 considered rain events (DSDs having rain drops greater than 100 only considered)

A data set of the 23 rain events listed in Table 3.1, is used for comparing the gamma models in terms of rain rate and specific rain attenuation calculations. Only DSDs having number of rain drops greater than 100 are considered (2443 minutes of data) from the 23 rain events. Gamma model DSDs using different moment combinations are used in (3.22) to calculate rain rates and these rain rates are compared with the JWD measured rain rates. Similarly, measured DSD,

MM234, MM246 and MM346 gamma models are used in (3.33) for calculating specific rain attenuation using forward scattering coefficients for horizontal and vertical polarization. The accuracy of the gamma models in terms of rain rate and specific rain attenuation calculations can be evaluated by finding mean square error as a percentage using the formulas in (4.3) and (4.4) respectively.

$$MSE_R (\%) = \sum_{i=1}^n \frac{[Measured\ R - Modeled\ R]^2}{n} \times 100 \quad (4.3)$$

$$MSE_{\gamma} (\%) = \sum_{i=1}^n \frac{[\gamma_{measured\ DSD} - \gamma_{Modeled\ DSD}]^2}{n} \times 100 \quad (4.4)$$

where n=2443, Measured R is the JWD measured rain rate, modeled R is the rain rate found from either MM234 or MM246 or MM346, $\gamma_{measured\ DSD}$ and $\gamma_{modeled\ DSD}$ are the specific rain attenuation at 11 GHz, 20 GHz, 28 GHz and 38 GHz for vertical and horizontal polarization found using the measured and modeled DSDs (either MM234 or MM246 or MM346) respectively. Table 4.2 shows the mean square errors in percentage in terms of rain rate and specific rain attenuation values.

Table 4.2 $MSE (\%)$ in terms of rain rate and specific rain attenuation values

Gamma DSD	$MSE_R(\%)$	$MSE_{\gamma-scatter}(\%)$							
		11 GHz		20 GHz		28 GHz		38 GHz	
		HP	VP	HP	VP	HP	VP	HP	VP
MM234	26.83	0.29	0.23	0.86	0.30	0.98	0.46	3.31	2.17
MM246	5.07	0.15	0.13	0.09	0.23	1.15	1.04	8.63	6.62
MM346	3.26	0.09	0.09	0.06	0.23	1.08	0.85	6.62	4.72

As can be seen from Table 4.2, mean square error in percentage in terms of rain rate calculations, $MSE_R(\%)$, is higher for the MM234 model followed by MM246 and MM346. But mean square error in percentage in terms of specific rain attenuation calculations at 11 GHz and 20 GHz, $MSE_{\gamma-scatter}(\%)$, is slightly higher for the MM234 model followed by MM246 and MM346 for both vertical and horizontal polarizations.

As rain rate and specific attenuation have larger weight to intermediate to larger drop sizes, MM346 and MM246 models produce less mean square error. But at higher frequencies, 28 GHz and 38 GHz, MM234 model produces least mean square error compared to MM346 and MM246 models. For the design of truncated gamma models, MM234 model is selected based on the smallest error compared with the measured DSD and its least error in terms of specific rain attenuation calculations at high frequencies. The truncated gamma models using 2nd, 3rd and 4th moments which are less sensitive to smaller drop diameters are explained next.

4.4 Truncated Gamma Models

In order to obtain the gamma model less sensitive to lower bins; the first bin is removed and the moments are calculated from the remaining bins using (3.7). Then, the gamma model is redesigned using (3.9) to (3.11); next, the first two bins are removed and the moments are calculated using the remaining bins and using these moments the gamma model is redesigned; similarly, gamma models are redesigned for the removal of the first 3 bins and the first 4 bins.

Figure 4.9 shows the redesigned gamma models at 4.20 mm/hr and 66.54 mm/hr. From Figure 4.9, it can be seen that, in general, all the gamma models fit well with the measured data at both 4.20 mm/hr and 66.54 mm/hr with the exception of one redesigned gamma model. This is where 4 bins are removed before the gamma model is redesigned at the rain rate of 4.20 mm/hr as shown in Figure 4.9a. There is slight deviation at the lower and higher drop diameters. The redesigned gamma models show the same trend as 4.20 mm/hr at the rain rates 1.96 mm/hr, 10.45 mm/hr and 22.80 mm/hr, where the removal of the first 4 bins

shows a slight deviation at the lower and higher drop diameters. The deviation at the large diameter end is minimal at 66.54 mm/hr for the 4 bins removed model, however this model also deviates with the measured data at the lower diameters as shown in Figure 4.9b.

(a) 4.20 mm/hr

(b) 66.54 mm/hr

Figure 4.9 Truncated gamma models with actual gamma model and measured DSD

The truncated gamma models for the other five rain rates are plotted in Figure B.1.a to Figure B.1.e and attached in Appendix B.1. The parameters μ , D_0 in (mm) and N_w , in ($\text{mm}^{-1}\text{m}^{-3}$) corresponding to the removal of the first few bins compared with the un truncated case, are included in Appendix B.1, Table B.1.

All the gamma models fit well with the measured data at 120.30 mm/hr and 141.27 mm/hr at all drop diameters. The accuracy of the gamma models can be evaluated by calculating mean square error as a percentage using (4.2) for all the rain rates.

Table 4.3 shows the *MSE (%)* for all the rain rates with different number of bins removed. The second column shows the *MSE (%)* of the actual MM234 model. The cells which are marked as ‘X’ in the Table 4.3 have zero rain drops in the measured data. Therefore, the removal of that bins does not change the *MSE (%)*. From Table 4.3, the removal of bins from 1 to 4 will not make much difference for all the higher rain rates of 66.54 mm/hr and above. At 22.80 mm/hr, from Table 4.3, the *MSE (%)* of the MM234 gamma model itself is high at 8.70%. This high *MSE (%)* is because the measured DSD at 22.80 mm/hr deviates more from the gamma model as compared to the DSD at other rain rates, especially at the lower drop size bins.

For the lower rain rates of 1.96 mm/hr, 4.20 mm/hr and 10.45 mm/hr, the removal of the first 3 and 4 bins introduce higher deviations in *MSE (%)*. It can be concluded that at higher rain rates (> 22.80 mm/hr), the first 4 bins have erroneous drop counts because of the dead time effect of distrometer and therefore the removal of mean drop diameters of less than 0.77 mm can be done for the higher rain rates whereas only lower 2 bins which are affected by dead time effect with mean drop diameters less than 0.55 mm can be removed at lower rain rates ($\leq$

22.80 mm/hr) and yet the accuracy of the redesigned model is not affected.

Table 4.3 Mean Square Error (%) for Truncated Gamma Models

MSE (%)					
Rain rate (mm/hr)/ Bins Removed	MM234	1	2	3	4
1.96	1.44	1.33	1.15	6.82	83.32
4.20	2.93	3.03	5.08	10.73	32.67
10.45	2.21	2.23	2.68	4.91	14.50
22.80	8.70	X	10.41	16.66	25.94
66.54	1.58	X	1.56	1.61	2.46
120.30	4.60	X	X	4.58	4.46
141.27	6.05	X	X	X	5.92

However, for terrestrial and earth-satellite communication links, the contributions of different rain drop diameters at different frequencies are important. The importance of small drop diameters increases at lower rain rates, it is necessary to check their contribution to the rain attenuation calculations especially at high frequencies. This is because, as frequency increases, the drop diameters relative to the wavelength become comparable and their complex permittivity [80] is different at different frequencies, therefore, the attenuation caused by these small rain drops becomes significant. As frequency increases, the drop diameters relative to the wavelength becomes comparable, therefore, the attenuation caused by these small rain drops becomes significant. The rain attenuation contribution of the rain drop diameters is studied using truncated gamma models in the next Chapter.

4.5 Summary

This chapter finds the contribution of drop size diameters from the calculation of rain rate using measured data for seven one minute rain rates. The drop diameters

which are affected due to dead time problem are identified. The three appropriate gamma models, MM234, MM246 and MM346 are fitted to model the measured data at seven rain rates. *MSE* (%) is calculated to compare the gamma models. Based on the results, MM234 model is selected to model the drop size distribution of Singapore. Four truncated gamma models (redesigned gamma model) are designed with removal of first bin, first two bins, first three bins and first four bins. These truncated models are compared with measured data. Specific rain attenuation values are calculated using MM234 gamma DSD and then truncated gamma DSD models are used to find the attenuation contributions of Joss distrometer bins.

Chapter 5

Specific and Slant-Path Rain Attenuation

The specific rain attenuation values at four frequencies using both the ITU-R model and forward scattering coefficients are discussed. The changes in the specific rain attenuation with rain rate, frequency, elevation angle and polarization are analyzed. The specific rain attenuation calculated using the ITU-R model and using the forward scattering coefficients are compared. The changes in specific rain attenuation and slant-path rain attenuation with rain rate, frequency and elevation angle are presented in our paper [81]. Specific rain attenuation and slant-path rain attenuation calculations using the truncated gamma models are found and these values are compared with the calculated rain attenuation values using the actual gamma models. The use of truncated gamma models for rain attenuation calculations are explained in our paper [79].

5.1 Introduction

The study continues by examining the contribution of particular rain drops on the specific rain attenuation of microwave signals using ITU-R P.838.3 model and

using forward scattering coefficients for a vertically polarized wave at frequencies 11 GHz, 20 GHz, 28 GHz and 38 GHz. T-Matrix code [72-74] is used to calculate the forward scattering coefficients which are more accurate over various drop shapes and radio frequencies.

The Ku-band frequency of 11 GHz [16, 82], representative of the INTELSAT 602 satellite and the Ka-band frequencies of 20 GHz and 28 GHz [82], representative of the IPSTAR satellite is used in this study. A number of sources [83] have identified 38 GHz as the largest growth area in the supply of fixed radio-relay systems as lower frequency bands become congested. Therefore, in the Q-band, 38 GHz is selected for this study. A description about the specific rain attenuation at the four frequencies for vertical and horizontal polarization at various elevation angles is discussed first.

5.2 Specific Rain Attenuation

Rain rates are calculated using (3.22) for MM234 gamma model. These rain rates are used to calculate specific rain attenuation using (3.27) for all the frequencies. Figure 5.1 shows the k and α values for both horizontal (denoted as 'HP') and vertical polarization (denoted as 'VP'), calculated from (3.30) and (3.31) at the four frequencies 11 GHz, 20 GHz, 28 GHz and 38 GHz.

The coefficients, k and α , as shown in Figure 5.1, increases with the increase in elevation angle for vertically polarized waves whereas the reverse is true for horizontally polarized waves at three of the four frequencies, 11 GHz, 28 GHz and 38 GHz. The k values decrease with the increase of elevation angle for vertically polarized waves whereas the reverse is true for horizontally polarized

waves at 20 GHz.

(a) k values

b. α values

Figure 5.1 ITU-R model coefficients, k and α , for both horizontal and vertical polarization

This is due to the decrease in the difference, $k_h - k_v$, between the frequencies 11.2 GHz to 22.8 GHz since the k values using (3.30) depend on $k_h - k_v$. The specific rain attenuation using forward scattering coefficients is calculated using (3.33) for both

horizontal and vertical polarizations. Real and imaginary parts of the complex scattering amplitudes (in m) for H and V polarizations at 11 GHz are listed in Table 5.1.

Table 5.1 Real and imaginary parts of the complex scattering amplitudes (in m) for H and V polarizations at 11 GHz as a function of drop equivolumetric sphere diameter

D _{eq} (mm)	Real, forward for V	Imaginary, forward for V	Real, forward for H	Imaginary, forward for H	Real, backward for V	Imaginary, backward for V	Real, backward for H	Imaginary, backward for H
0.359	2.97E-07	-7.37E-09	2.96E-07	-7.34E-09	2.95E-07	-6.14E-09	-2.94E-07	6.12E-09
0.455	6.06E-07	-1.58E-08	6.06E-07	-1.58E-08	5.99E-07	-1.17E-08	-5.99E-07	1.17E-08
0.55	1.07E-06	-2.97E-08	1.08E-06	-2.99E-08	1.06E-06	-1.91E-08	-1.06E-06	1.92E-08
0.655	1.82E-06	-5.43E-08	1.83E-06	-5.48E-08	1.78E-06	-2.84E-08	-1.79E-06	2.88E-08
0.771	2.97E-06	-9.74E-08	3.00E-06	-9.90E-08	2.87E-06	-3.79E-08	-2.90E-06	3.90E-08
0.913	4.95E-06	-1.85E-07	5.03E-06	-1.90E-07	4.72E-06	-4.18E-08	-4.80E-06	4.45E-08
1.115	9.10E-06	-4.18E-07	9.34E-06	-4.35E-07	8.48E-06	-4.74E-09	-8.71E-06	1.11E-08
1.331	1.56E-05	-9.10E-07	1.62E-05	-9.61E-07	1.41E-05	1.67E-07	-1.47E-05	-1.57E-07
1.551	2.50E-05	-1.89E-06	2.64E-05	-2.03E-06	2.17E-05	6.53E-07	-2.29E-05	-6.53E-07
1.665	3.13E-05	-2.71E-06	3.33E-05	-2.95E-06	2.65E-05	1.13E-06	-2.82E-05	-1.15E-06
1.913	4.83E-05	-5.72E-06	5.24E-05	-6.36E-06	3.87E-05	3.10E-06	-4.21E-05	-3.27E-06
2.259	8.16E-05	-1.52E-05	9.11E-05	-1.76E-05	6.05E-05	1.01E-05	-6.82E-05	-1.13E-05
2.584	1.23E-04	-3.55E-05	1.40E-04	-4.28E-05	8.81E-05	2.64E-05	-1.04E-04	-3.06E-05
2.869	1.61E-04	-6.81E-05	1.86E-04	-8.34E-05	1.25E-04	5.30E-05	-1.58E-04	-6.21E-05
3.198	1.93E-04	-1.20E-04	2.24E-04	-1.42E-04	2.00E-04	9.30E-05	-2.67E-04	-1.01E-04
3.545	2.09E-04	-1.67E-04	2.60E-04	-1.87E-04	3.20E-04	1.18E-04	-4.32E-04	-1.07E-04
3.915	2.34E-04	-1.99E-04	3.33E-04	-2.29E-04	4.64E-04	1.09E-04	-6.27E-04	-6.81E-05
4.35	2.88E-04	-2.39E-04	4.63E-04	-3.14E-04	6.25E-04	6.93E-05	-8.63E-04	1.53E-05
4.859	3.66E-04	-3.04E-04	6.49E-04	-4.75E-04	7.99E-04	1.95E-05	-1.15E-03	1.37E-04
5.3	4.46E-04	-3.74E-04	8.35E-04	-6.81E-04	9.52E-04	-1.25E-05	-1.44E-03	2.63E-04

Figure 5.2 shows the specific attenuation in dB/km at different frequencies for both horizontal and vertical polarization using the MM234 gamma model at 0° elevation angle for the rain event occurred on 26th February 1995. Figure 5.3 shows the same specific attenuation in dB/km as a function of rain rate. The legend in Figure 5.3 for 11 GHz is applicable to the calculated specific attenuation values in Figure 5.3 for the frequencies 20 GHz, 28 GHz and 38 GHz. It is clear from Figures 5.2 and 5.3 that ITU-R model overestimates the specific rain attenuation

using scattering coefficients at the lower frequency 11 GHz. The amount of overestimation reduces at 20 GHz for horizontal polarization. But for vertical polarization, the ITU-R model slightly underestimates at the higher rates. However, at the other 2 higher frequencies, the ITU-R model underestimates the specific rain attenuation as compared to the specific rain attenuation using scattering coefficients especially at the higher rain rates.

Figure 5.2 Specific rain attenuation using gamma model at different frequencies

Table 5.2 compares the specific rain attenuation values calculated using the forward scattering coefficients with Recommendation P. 838 derived values, for selected rain rates. The percentage difference of specific rain attenuation using forward scattering coefficients compared with the specific rain attenuation using ITU-R model, for different polarizations at the 4 frequencies is plotted in Figure 5.4, for the selected rain rates.

Figure 5.3 Specific rain attenuation as a function rain rate using gamma model at different frequencies

Table 5.2 Specific rain attenuation (dB/km) at different rain rates using forward scattering coefficients for horizontal and vertical polarizations compared with ITU-R model

Frequency - 11 GHz					Frequency - 20 GHz				
Rain rate (mm/hr)	ITU-R HP	Using f_h	ITU-R VP	Using f_v	Rain rate (mm/hr)	ITU-R HP	Using f_h	ITU-R VP	Using f_v
1.96	0.040	0.020	0.038	0.019	1.96	0.188	0.112	0.187	0.107
4.20	0.101	0.054	0.092	0.050	4.20	0.417	0.312	0.395	0.293
10.45	0.304	0.191	0.263	0.170	10.45	1.089	0.933	0.965	0.853
22.80	0.785	0.551	0.651	0.479	22.80	2.484	2.280	2.080	2.028
66.54	2.891	2.086	2.267	1.781	66.54	7.729	7.102	5.989	6.019
120.30	5.953	4.129	4.526	3.513	120.30	14.496	13.361	10.762	11.199
141.27	7.236	5.636	5.456	4.739	141.27	17.180	16.426	12.608	13.195
Frequency - 28 GHz					Frequency - 38 GHz				
Rain rate (mm/hr)	ITU-R HP	Using f_h	ITU-R VP	Using f_v	Rain rate (mm/hr)	ITU-R HP	Using f_h	ITU-R VP	Using f_v
1.96	0.395	0.260	0.368	0.250	1.96	0.727	0.524	0.686	0.501
4.20	0.822	0.657	0.743	0.615	4.20	1.417	1.255	1.310	1.164
10.45	1.979	1.826	1.725	1.644	10.45	3.154	3.269	2.849	2.943
22.80	4.213	4.268	3.558	3.711	22.80	6.277	7.164	5.553	6.308
66.54	11.911	12.701	9.635	10.660	66.54	16.176	19.626	13.912	17.036
120.30	21.190	23.576	16.736	19.500	120.30	27.336	35.309	23.143	30.358
141.27	24.758	27.728	19.428	22.336	141.27	31.499	38.729	26.554	32.895

As can be seen from Table 5.2 and Figure 5.4, specific rain attenuation values using Recommendation P. 838-3 overestimate by 103%, for 1.96 mm/hr,

reducing to 15% for 141.27 mm/hr for vertical polarization at 11 GHz. The corresponding differences for horizontal polarization are significantly bigger, ranging from 106–28%. The specific rain attenuation for vertical polarization is smaller than horizontal polarization at the four frequencies as expected.

Figure 5.4 Difference in specific rain attenuation using forward scattering coefficients compared with the ITU-R model for horizontal and vertical polarizations

This is because, as the size of the rain drops increase, their shape tends to change from spherical to oblate spheroids. Furthermore [84], rain drops may also be inclined (canted) to the horizontal because of vertical wind gradients. Thus, the depolarization due to rain can significantly depends on canting and tilt angle with drop vibration effects. The rain induced depolarization has been studied previously [85] by calculating the differential attenuation and the differential phase.

In the above analysis, forward scattering coefficients are calculated using the DSD of Singapore for the corresponding rain rates. But Rec. P. 838-3 ITU-R model is a statistical model, which includes a statistically-representative

generalized DSD model for all the climates. Therefore, the calculations using forward scattering coefficients are more accurate compared to ITU-R model. Both these models are compared in [72] and noticeable differences were found between the two models. It is stated in [72] that the assumed drop size distributions in the P. 838, as well as the upper diameter limit over which the integration is performed, are probably the causes for these discrepancies. The specific rain attenuation at different elevation angles using both models are analyzed next.

5. 2. 1 Specific Rain Attenuation at Different Elevation

Angles

Figure 5.5 shows the specific rain attenuation calculated using the ITU-R model (denoted as ‘ITU- R’ in the legend) and using forward scattering coefficients (denoted as ‘Using f_v ’ in the legend) at four rain rates around 120 mm/hr for four different frequencies at different elevation angles. Four DSD minutes around 120 mm/hr from 30 October 1994, 952 minutes, 3 November 1994, 1122 minutes, 12 November 1994, 85 minutes and 26 February 1995, 1108 minutes are selected. The corresponding day, time and rain rates are indicated in the titles of Figure 5.5a, 5.5b, 5.5c and 5.5d respectively. The legend in Figure 5.5a is applicable to Figures 5.5b, 5.5c and 5.5d.

The rain rate in (3.27) ITU-R P.838.3 model is the point rain fall rate, $R_{0.01}$, the rainfall intensity for the area of interest, integrated over one minute, which is exceeded for 0.01% of the time. The averaged $R_{0.01}$ of 122 mm/hr is used in [86] for the NTU site from year 1990 to 1996. $R_{0.01}$ is 126.65 mm/hr for the year 2000, 105.58 mm/hr for 2005 to 2006 and 111.18 mm/hr for 2007 to 2008 also at NTU site. Therefore, 120.30 mm/hr is taken as the $R_{0.01}$ in this research since it is one of the rain rates considered.

It is clear from Figure 5.5, the specific rain attenuation increases with the increase in both the frequency and the elevation angle. Even though at 20 GHz, as can be seen from Figure 5.1, the power law coefficient, k , in (3.30) decreases with the increase in elevation angle, whereas, α , the other power law coefficient, in (3.31) increases with the increase in elevation angle. Since the specific rain attenuation depends on both the coefficients, k and α , when the specific rain attenuation is calculated using both k and α in (3.27), it increases with the elevation angle at 20 GHz similar to the other three considered frequencies as shown in Figure 5.5.

Figure 5.5 Specific rain attenuation using gamma model for four different frequencies at different elevation angles

The ITU-R model overestimates the specific rain attenuation at the lower frequency of 11 GHz at all the elevation angles. However, at the other 3 higher frequencies, the ITU-R model underestimates the specific rain attenuation as compared to the specific rain attenuation using scattering coefficients at all the

elevation angles. The rate of increase of specific rain attenuation calculated using forward scattering coefficients is higher than the rate of increase of specific rain attenuation calculated using the ITU-R model especially at higher frequencies for higher elevation angles. The above analysis is applicable to the four considered rain rates around 120 mm/hr as shown in Figure 5.5a, 5.5b, 5.5c and 5.5d respectively. Specific rain attenuation contributions for individual bins are calculated using both the ITU-R model and the forward scattering coefficients next.

5.2.2 Specific Rain Attenuation Contribution for Individual Bins

It is interesting to study the contribution of particular rain drops on the attenuation. The specific rain attenuation contributions due to individual drop diameters are calculated using both the ITU-R model and the forward scattering coefficients at all the rain rates for all the frequencies using vertical polarization. The rain contribution of individual bin is used in these calculations. Figure 5.6 shows the specific rain attenuation contribution of each bin for the four frequencies at 120.30 mm/hr (rain rate observed at 1108 minutes occurred on 26th February 1995). Rain rates can be calculated using (3.22), for ITU-R model. In (3.22), the summation is used for adding the individual bins' rain contributions and rain rate is found. But for finding the rain contribution of individual bins, the summation in (3.22) is removed and the rain contribution of individual bins are calculated using

$$R_i = 6\pi \times 10^{-4} \times D_i^3 v(D_i) N(D_i) \Delta D_i \quad (5.1)$$

where $V(D_i)$, $N(D_i)$ and ΔD_i are terminal velocity, gamma DSD and drop size interval for the i^{th} bin. $i=1$ is considered while calculating the first bin's rain contribution and similarly for the other bins.

a. Using ITU-R model

b. Using forward scattering coefficients

Figure 5.6 Specific rain attenuation contributions at different frequencies using gamma model at 120.30 mm/hr

The specific rain attenuation contribution for the individual channel/drop diameter is calculated using

$$\gamma_{R_i} = k R_i^\alpha \quad (5.2)$$

where k and α are the power law coefficients from ITU-R P.838-3 and R_i is the rain contribution of individual bin from (5.1). Similarly the specific rain attenuation using forward scattering coefficients is calculated using (3.33). The summation in (3.33) is for adding the specific rain attenuation contribution of individual bins to find the total specific rain attenuation for the corresponding DSD minute. The summation in (3.33) is removed and the specific rain attenuation contribution using forward scattering coefficients for the individual channel/drop diameter is calculated by considering only one bin at a time using

$$\gamma_{H_i,V_i} = 8.686 \times 10^3 \times \lambda \times \text{Im} \left(f_{H,V}(D_i) \right) N(D_i) dD_i \quad (5.3)$$

The trend of specific rain attenuation components is the same in Figure 5.6a and 5.6b, where the specific rain attenuation found from the ITU-R model is higher than the forward scattering coefficients at 11 GHz and the specific rain attenuation found from the ITU-R model is lower than the forward scattering coefficients at all the other 3 higher frequencies.

It is clear from Figure 5.6 that specific rain attenuation contributions increase from the lower drop diameters. It reaches a threshold around the middle drop diameters and then decrease for larger drop diameters. As can be seen from Figure 5.6a, the highest specific rain attenuation contribution is at 2.26 mm regardless of frequency at 120.30 mm/hr. But in practice, the highest rain attenuation component for a given rain rate should decrease in drop diameters with an increase in frequency since the wavelength becomes comparable with the smaller drop diameters as frequency increases. The absolute value of complex permittivity decreases with frequency from 0 Hz to 38 GHz. Water relaxation frequencies is in the 18-20 GHz region at normal atmospheric incidence. Therefore, the highest specific rain attenuation contributions should move to

higher drop diameters when the rain rate increases at all the frequencies.

Figure 5.7 shows the specific rain attenuation contributions at the seven rain rates for 11 GHz and 38 GHz using both the ITU-R model and the forward scattering coefficients. It is clear from Figure 5.7 that regardless of frequency, when rain rate increases, the highest attenuation contribution moves to the same bins if the ITU-R model is used. For example the highest attenuation contribution is at bin 7 (1.12 mm) for 1.96 mm/hr; it moves to bin 11 (1.91 mm) for 22.80 mm/hr and to bin 12 (2.26 mm) for 120.30 mm/hr respectively.

Figure 5.7 Specific rain attenuation contributions at different rain rates using gamma model at 11 and 38 GHz

But, if the forward scattering coefficients are used, then the highest attenuation contribution moves to different bins at different frequencies. For example, at 11 GHz, when the rain rate is 1.96 mm/hr, the highest attenuation contribution is at bin 7 (1.12 mm) and it moves to bin 12 (2.26 mm) and to bin 15 (3.20 mm), when the rain rate increases to 22.80 mm/hr and 120.30 mm/hr respectively. However, at 38 GHz, when the rain rate is 1.96 mm/hr, the highest

attenuation contribution is at bin 7 (1.12 mm) and it moves to bin 11 (1.91 mm) and to bin 12 (2.26 mm), when the rain rate increases to 22.80 mm/hr and 120.30 mm/hr respectively. It is clear from Figure 5.6b and Figure 5.7 that the highest specific rain attenuation contribution moves to lower drop diameters as the frequency increases.

The smaller wavelengths of higher frequencies are comparable with the rain drop diameters, therefore, results in more attenuation at smaller drop diameters as frequency increases. Small drops tend to be Rayleigh scatterers. Larger drops due to small number and reduced increase of scattering coefficients become less significant [85-86, 88]. Therefore, contribution is so a question of number of drops and wavelength.

Most striking effects of bigger drops at high frequency are the reduction of differential phase that can be observed in depolarization measurements [85, 87]. The reversal in sign of the differential phase at high frequencies is a purely resonance phenomenon in which large drops produce negative differential phase outweighing the positive contribution from smaller ones. Major drops contribute mostly for differential attenuation. As a whole, less deformed smaller drops make a greater relative contribution to the total specific rain attenuation at high frequencies.

In Figure 5.6a, regardless of frequency, the highest rain attenuation contribution is always at the same drop diameter for a rain rate (2.26 mm at 120.30 mm/hr) using the ITU-R model, in Figure 5.6b, as frequency increases, the highest rain attenuation contribution decreases with the drop diameter using the forward scattering coefficients correspondingly. Therefore, the use of forward scattering coefficients is considered more accurate for specific rain attenuation calculations

using truncated gamma models.

5.2.3 Specific Rain Attenuation using Truncated Gamma Models

Table 5.3 shows the calculated specific rain attenuation (dB/km) for the truncated gamma models with 1, 2, 3 and 4 bins removed using (3.33) at the Ka-band and Q-band frequencies, 28 GHz and 38 GHz, at 0° elevation angle respectively. Only specific rain attenuation for the two higher frequencies is shown since they have the maximum change in attenuation. The cells which are marked as ‘X’ in Table 4.3 have no rain drops in the measured DSD.

The specific rain attenuation increases with both the rain rate and the frequency increase for all the truncated gamma models. The point to note from Table 5.3 is that there is not much deviation in attenuation values if any of the lower 4 bins are removed for the truncated gamma models at any rain rate. For example, at 38 GHz, rain rate of 120.30 mm/hr, the specific rain attenuation varies from 30.34 dB/km to 30.35 dB/km. Although this change in specific rain attenuation is small, it is worthwhile to evaluate the slant-path rain attenuation values using (3.43), which are considered next.

Table 5.3 Specific Rain Attenuation (dB/km) using Forward Scattering Coefficients for Vertical Polarization using Truncated Gamma Models

Frequency (GHz)	28					38				
Rain rate (mm/hr)/ Bins Removed	MM234	1	2	3	4	MM234	1	2	3	4
1.96	0.250	0.250	0.249	0.247	0.238	0.50	0.50	0.50	0.50	0.48
4.2	0.615	0.615	0.616	0.616	0.611	1.16	1.16	1.17	1.17	1.15
10.45	1.644	1.645	1.648	1.651	1.650	2.94	2.94	2.95	2.96	2.96
22.8	3.711	X	3.719	3.734	3.742	6.31	X	6.33	6.36	6.38
66.54	10.654	X	10.662	10.679	10.701	17.03	X	17.04	17.07	17.09
120.3	19.495	X	X	19.498	19.508	30.34	X	X	30.34	30.35
141.27	22.305	X	X	X	22.308	32.85	X	X	X	32.85

For slant-path rain attenuation calculations, the specific rain attenuation at different elevation angles are calculated at the four frequencies using forward scattering coefficients for vertically polarized waves using the truncated gamma models. It is important to note that the specific rain attenuation increases with the increase of elevation angle for vertically polarized waves.

5.3 Slant-Path Rain Attenuation

The slant-path rain attenuation A_I (in dB) exceeding 1% of the time is calculated using (3.34) to (3.43) from Recommendations ITU-R P. 618-9, with $R_{0.01}=120.30$ mm/hr, for all the truncated gamma models at the four frequencies. This is for practical application purposes. The rain height, h_R , is determined using the 0° isotherm height data h_o , given by ITU-R P.839-3 [76] using (3.34). The value h_o corresponding to Singapore is 4.604 km. Therefore rain height of 4.97 km is used. Ong and Zhu [89] has calculated the slant-path rain attenuation using the horizontally polarized beacon receiver data and compared it with ITU-R Rec. 618-3 at 11.198 GHz. The elevation and azimuth angles of the antenna of the beacon receiver, used in their studies, are 42.9° and 268.6° respectively. The calculated slant-path rain attenuation exceeded for 0.001% time to 1% for the two models showed that for the percentages of the time between 0.01 and 1 %, ITU-R Rec. 618-3 [90] predictions underestimate beacon measured rain attenuation.

Recently, Yeo et al. [82] used the measured beacon signal data at 20.199 GHz (Ka band), seven months from March to September 2008, to analyze the rain attenuation of the earth-space link. The elevation and azimuth angles of the antenna of the beacon receiver, used in their studies, are 71.35° and 94.73° .

respectively and the receiving polarization is linear. They concluded that ITU-R P.618-9 Rec. tends to underestimate the rain attenuation for the earth-space link in tropical areas like Singapore.

From [88], the calculated slant-path rain attenuation (in dB) $A_{0.01}$ exceeding 0.01% of the time and $A_{0.1}$ exceeding 0.1% of the time for a coastal region like Calabar in Nigeria, where the highest average annual accumulation results in a $R_{0.01}=130$ mm/hr, the $A_{0.01}$ and $A_{0.1}$ are as high as 37.8 dB and 17 dB for 19.45 GHz (Ka-band), and 19.6 dB and 8.4 dB for 12.675 GHz (Ku-band) respectively. These rain attenuation values may exceed the fade margins of practical systems.

Similarly, the calculated slant-path rain attenuation values for Singapore at these percentages of time are well above the feasible fade margins especially at high frequencies. Therefore, the slant-path rain attenuation A_1 (in dB) exceeding 1% of the time is selected for the study of the truncated gamma models. Figure 5.8 shows the calculated slant-path rain attenuation using the gamma model at the four frequencies.

As frequency increases, slant-path rain attenuation also increases. The increase in elevation angle decreases the slant-path rain attenuation since the slant-path length decreases. This is clear from Figure 5.8 that attenuation decreases with the decrease in path length (increase in elevation angle) from the elevation angle 10° to 50° at the three higher frequencies. But at the high elevation angles, the slant-path rain attenuation again increases. From Figure 5.8, it can be seen that the slant-path rain attenuation A_1 (in dB) exceeding 1% of the time at 20 GHz with an elevation angle of 40° is 5.19 which is less than 5.34 for the same frequency with an elevation angle of 80° . This despite the fact that the former has a path through the rain which is considerably longer than the path of the latter.

Figure 5.8 Slant-path rain attenuation at the four frequencies using gamma model at $R_{0.01}$ (120.30 mm/hr)

Table 5.4 shows the elevation angles at which the slant-path rain attenuation starts to increase at the four frequencies. The increase starts at the lower elevation angles for high frequencies where the rain attenuation is maximal.

Table 5.4 Elevation Angles at which Slant-Path Rain Attenuation (dB) starts to increase

Frequency (GHz)	Elevation at which A_l (dB) starts to increase
11	80°
20	70°
30	60°
38	60°

The Recommendation ITU-R P. 618-9 uses a single-rain cell model. The increase in slant-path rain attenuation at higher elevation angles is primarily a geometrical effect due to rain cell model used in P. 618-9 (which is an empirically-based model). The variations in the specific rain attenuation and slant-path rain

attenuation with different elevation angles are explained in our paper [81].

The calculated slant-path rain attenuation values show that there are insignificant changes for the truncated gamma models with bins removed from actual gamma model at the four frequencies. The absolute difference between the actual gamma modeled and the redesigned gamma modeled slant-path rain attenuation is calculated by (5.1).

$$\text{Change in slant-path rain attenuation} = \text{ABS} \left((A_{0.01}(MM234)) - A_{0.01}(\text{other } ga) \right) \quad (5.1)$$

where $A_I(MM234)$ is the slant-path rain attenuation exceeding 1% of time calculated from actual gamma model and $A_I(\text{other } ga)$ is the slant-path rain attenuation exceeding 1% of time calculated from redesigned gamma models with bins removed. There are no rain drops in the first two bins at 120 mm/hr; therefore, their slant-path attenuation changes are not plotted. Figure 5.9 shows the slant-path rain attenuation changes for the redesigned gamma models with bins 3 and 4 removed as compared to the actual gamma model at 28 GHz and 38 GHz.

a. 28 GHz

b. 38 GHz

Figure 5.9 Slant-path rain attenuation changes for truncated gamma models compared to the actual gamma model

As seen in Figure 5.9, the changes in slant-path rain attenuation increase at both the frequencies especially at the lower and higher elevation angles for the redesigned gamma models with 4 bins removed from actual gamma model. From Figure 5.9a, it can also be seen that the removal of the first 4 bins results in less than 0.055 dB change in slant-path rain attenuation at 28 GHz. However, from Figure 5.9b, at 38 GHz, the removal of the first 4 bins results in a high change in slant-path rain attenuation of around 0.096 dB at the elevation angle of 90°.

This shows that, as frequency increases, the relative contribution from the smaller drop size increases. At 120.30 mm/hr, there are few counts in bin 4; however, as the wavelength of the higher frequency is comparable with the drop diameter, their contribution to the slant-path rain attenuation increases. At the drop diameter 0.66 mm, the drop sizes are one sixteenth of λ and one twelfth of λ at 28 GHz and 38 GHz respectively.

Since the change in slant-path rain attenuation are below 0.096 dB at all the

frequencies for the redesigned gamma model with 4 bins removed, it can be concluded that, since the dynamic range of a satellite system is generally above 1 dB, the removal of the first 4 bins will not affect the satellite communication system for all frequencies within the Ku-band, Ka-band and Q-band. This validates Brawn's recommendations [14] of ignoring the counts in erroneous bins and the truncated gamma models can be used for DSD modeling and rain attenuation calculations. The analysis of DSD's will hold for rain attenuation calculations, however, other uncertainties such as DSD variations along the path, temperature, bright band are also important factors to be considered.

5.4 Summary

Specific rain attenuation calculations are done at 11 GHz, 20 GHz, 28 GHz and 38 GHz using ITU-R and forward scattering coefficients for horizontal and vertical polarization using gamma model. The changes in the specific attenuation values with rain rate, frequency and elevation angle are analyzed for both polarizations. Based on the analysis, the use of forward scattering coefficients for vertically polarized waves are selected for slant-path rain attenuation calculations to study the truncated gamma models. Specific rain attenuation and slant-path rain attenuation changes shows that the redesigned gamma models with the first 4 bins removed can be used at Ku-band, Ka-band and Q-band frequencies in Singapore.

Chapter 6

Rain Rate Retrieval from Single Polarized RADAR

This chapter describes the classification of rain into different types based on the drop size distribution data. The variability of the DSDs for different rain types and its influence on the Z-R relations are presented. The coefficients of the Z-R relation for different rain types are compared and analysis is presented. Rain rates are calculated from single polarized RADAR reflectivities using the proposed Z-R relations. These rain rates are compared with the distrometer derived rain rates. The coefficients of Z-R relations for different rain types and the comparison of retrieved rain rates from single polarized RADAR data with distrometer derived rain rates are explained in [91].

6.1 Reflectivity-Rain Rate Relations

The rain events from 15 to 23 listed in Table 3.1 are used for analyzing the Z-R relations. As shown in Table 3.1, there are 1663 minutes of data in total for the 9 rain events. Of the 1663 minutes of data, only DSDs having number of rain drops

greater than 100 (1514, one minute samples) are considered. The rain events which have higher rain rates are selected for rain classification. One convective rain event, numbered as 19, and one stratiform rain event, numbered as 20, are also included for analysis. The selection of rain events is dependent on the availability of the RADAR data which are limited to days within the year 1998.

This research work uses the variations in the rain integral parameters to classify the rain before deriving the Z-R relations. Rain events are separated into convective and stratiform using two methods; the Gamache-Houze method, a simple threshold technique; and the Atlas-Ulbrich method. In Gamache-House method, one of the rain integral parameter, reflectivity, is used to classify the rain into convective and stratiform types whereas Albrich-Utlas uses the variations in the rain integral parameters, rain rate, reflectivity, the parameter N_w , median volume diameter and the gamma model parameter μ to classify the rain into convective (C1, C2), transition (T) and stratiform (ST) types.

6.2 Gamache-Houze Method

6.2.1 Rain Classification

The rain integral parameters rain rate, R in mm/hr and reflectivity, Z in mm^6/mm^3 , median volume diameter, D_0 in mm, N_w in $\text{mm}^{-1}\text{m}^{-3}$ and gamma model parameter μ are calculated using (3.22), (3.23), (3.24), (3.25) and (3.9) respectively. Reflectivity in dBZ, rain rate in dBR and in N_w in dBN can be calculated using

$$Z(\text{dBZ}) = 10\log_{10}(Z) \quad (6.1)$$

$$R(\text{dBR}) = 10\log_{10}(R) \quad (6.2)$$

$$N_w(\text{dBN}) = 10\log_{10}(N_w) \quad (6.3)$$

Figure 6.1 shows the Z (dBZ) versus $\log(R)$ plot for 1514 DSD minutes that contains greater than 100 rain drops from the 9 rain events.

Reflectivity value of 38 dBZ is used as the threshold as explained by Gamache and Houze [92] to distinguish the precipitation types such that all the DSD minutes having reflectivity values above this threshold are assumed to be convective while those below 38 dBZ are assumed stratiform. After splitting the DSD minutes based on this threshold, Z - R relations are found separately for the convective rain type, the stratiform rain type and all the DSD minutes (including both the convective and stratiform rain type). Of the total 1514 DSD minutes, the number of stratiform and convective points are 1078 and 436 respectively.

Figure 6.1 Scatter plot of reflectivity versus $\log_{10}(\text{rain rate})$ (1514 minutes of data from year 98 rain events, DSDs having rain drops greater than 100 only considered)

6. 2. 2 Z-R relations

The Z - R relations are shown in Figure 6.1. It is clear from the values of the coefficients derived from (2.4) that A is larger and b is smaller for convective rain

type whereas A value is reduced and b value is increased for stratiform rain type. The coefficients of the overall Z-R relation are closer to the coefficients of the stratiform rain type than the convective rain type. This indicates that when fitting the overall Z-R relation to both the convective and stratiform rain types, the stratiform rain points dominate. Therefore, it is better to use a separate relation for the different rain types so that a more accurate estimate of the rain rates can be obtained from the RADAR reflectivities.

The coefficients of the Z-R relations, A and b , proposed by Marshall and Palmer (MP) [2], from the 1514 DSD minutes of Singapore (SG) as shown in Figure 6.1, from the Gamache and Houze method (GH-SG) also shown in Figure 6.1 and reported by Wilson (W-SG) [35] are tabulated in Table 6.1. In W-SG, the A coefficient for the convective rain is less than the A coefficient of stratiform rain and b values are in the reverse order.

Table 6.1 Reflectivity - rain rate (Z-R) relationships for individual rain type and for the overall data set derived by linear regression of Z (dBZ) versus $\log_{10}(R)$

Type	General	C	T	ST
MP	$Z=200 R^{1.6}$	-	-	-
SG	$Z=287.10 R^{1.35}$	-	-	-
W-SG	-	$Z=139 R^{1.5}$	$Z=271 R^{1.25}$	$Z=330 R^{1.35}$
GH-SG	-	$Z=557.80 R^{1.17}$	-	$Z=290.84 R^{1.31}$

Wilson used the method proposed by Atlas-Ulbrich [32] which includes the transition stage in the convective rain during his derivation of the Z-R relations. As explained in the introduction, in general, a transition stage is present in between the convective and stratiform rain stages within each rain event. Therefore, in order to study the method proposed by Atlas and Ulbrich [32], the Atlas and Ulbrich method is used to separate one of the nine rain events into different rain stages. The event that occurred on the 9th May 1998 which has of all rain types is

used in this analysis.

6.3 Atlas-Ubrich Method

6.3.1 Rain Classification

The rain event recorded on 9th May 1998 is selected for the analysis. It is a long convective rain event that lasts for around 4 hours. This rain event is chosen for analysis because it is one of the long rain events and it consists of all types of rain. It can be classified into three different rain types, convective, stratiform and transition, using the variations in the integral parameters.

Figure 6.2 Classification of distrometer data, recorded on 9th May 1998. R (dBR), Z (dBZ), N_w (dBN), $10 \cdot D_0$ (mm) and μ are plotted for around 240 minutes

The variability of the rain integral parameters rain rate R (dBR), reflectivity Z (dBZ), the parameter N_w (dBN), the median volume diameter D_0 (mm) as $10 \cdot D_0$

and the shape parameter μ for this rain event is plotted in subplot 1 of Figure 6.2. Subplot 2 of Figure 6.2 shows the rain rate of the same rain event as a function of time. The time in x-axis is in Coordinated Universal Time, abbreviated as UTC in minutes. The rain event has two convective peaks (C1 and C2) followed by transition (T) and stratiform (ST) stages. Stage C1, from 347 to 382 minutes, 36 minutes of duration, is characterized by DSDs of higher Z and R values, D_0 values above 1.6 mm and with μ values less than 4. All the rain integral parameters have stable values with very little variations during the C1 stage.

Stage C2, from 383 to 395 minutes, 13 minutes of duration, has a sharp convective peak. There is a short transition at the end of stage C1 and at the beginning of C2. Afterwards, there is the increase in D_0 , Z and R values. The highest rain rate for the whole rain event occurs within stage C2 at 392 minutes and is 123.37 mm/hr with a corresponding reflectivity of 52.64 dBZ. The μ values start to reduce during the increase of the convective peak. After reaching the highest rain rate, the convective peak starts to decrease. The point at which the rain rate starts to decrease to less than 60 % of the maximum rain rate is identified as the beginning of the transition stage [32]. This stage can clearly be distinguished from the convective stages by the reduction in all the integral parameters. After 49 minutes of convective rain, three of the integral parameters, R , Z and D_0 decrease continuously and reaches a minimum point where $D_0 = 1.00$ mm, $Z = 25.77$ dBZ and $R = 1.97$ mm/hr. The T stage lasted for 36 minutes. During the T stage, the N_w shows a slight decrease and the gamma parameter μ increases continuously and reaches a higher value of 9.75.

From 432 minutes onwards, R , Z and D_0 values again starts to increase. This indicates the beginning of the stratiform (ST) stage. Over the steady

stratiform duration of 165 minutes, around 120 minutes of rain have D_0 values less than 1.6 mm and R has values less than 10 mm/hr except for the short peaks one at the beginning of the stratiform stage and other one at around 485 to 500 minutes. The other two variables Z and D_0 have the same trend as R . N_w values which have almost constant values around 40 dBN throughout the C1, C2 and T stage, have a clear decrease during the stratiform stage and settle around 32 dBN for the last 50 minutes of the stratiform stage. The gamma parameter μ has ups and downs in the ST stage.

From Figure 6.2, it is observed that the time during which there is a change in D_0 value is closely related to the changes in the values of R and Z . D_0 tends to be constant when the variation of R is small, whereas D_0 tends to have a sharp peak when R has a sharp peak. This observation is similar to the pattern reported by Maki et al. in [34]. As can be seen from Figure 6.2, sharp peaks in D_0 are accompanied by sharp minimums in N_w according to (3.25). Sharp minimums in N_w are followed by the drop of R and Z values for the consecutive minutes. For example, the sharp peak in stage C2 with $D_0 = 2.26$ mm at 389 minutes is accompanied by a sharp minimum in N_w of 37.98 dBN. The decrease in N_w from 40.17 dBN at 385 minutes to 37.98 dBN at 389 minutes produces drop in Z and R from 393 minutes. This is the similar type of rain structure as explained by [38].

After the rain classification, classified C (C1, C2), T and ST points are compared in the $\log N_w$ - D_0 plot with the separator line derived by Bringi et al. [93]. This separator line separates the three rain types well for the Australian, C-POL-retrieved DSD parameters (N_w and D_0) in [93]. Bringi et al. [93] derived the separator line using the profiler-retrieved N_w and D_0 data in the rain region which showed a clear separation of convective and stratiform rain types. Then they

applied the separator line to C-POL-retrieved DSD parameters (N_w and D_0). In their paper, the deviation from the separator line for a given D_0 was used to define a smoothly varying index value representing the likelihood of stratiform or convective rain type, including a continuous “transition” region between the two.

Figure 6.3 shows a very clear separation of convective and stratiform rain types in the N_w - D_0 domain. The straight line in Figure 6.3 represents the separator, in [93], between the two rain types, given by

$$\log_{10}(N_w) = -1.6D_0 + 6.3 \quad (6.4)$$

As can be seen from Figure 6.3, all the convective points lie above the separator line with high values of D_0 and $\log_{10}(N_w)$.

Figure 6.3 Dependence of $\log_{10}(N_w)$ and D_0 (mm) for the rain event on 9th May 1998 where the unit of N_w is $\text{mm}^{-1}\text{m}^{-3}$. The separator line separates convective and stratiform points

The stratiform points lie below the separator line with low values of D_0 and $\log_{10}(N_w)$. The values of $\log_{10}(N_w)$ seem to remain constant during transition stage

and the transition points lie on both sides of the separator line. But the transition points which lie below the line has relatively higher values of $\log_{10}(N_w)$ compared to the stratiform points. Classified rain types for the other rain events (15 to 23 listed in Table 3.1) are also compared with the separator line and it is found that the classification works well in the $\log N_w - D_0$ domain.

After classifying the rain into different types, reflectivity-rain rate relationships are calculated for each type of rain and also for the entire rain event in the dBR-dBZ diagram using the log-log fitting. The relations between reflectivity and rainfall-rate are traditionally derived from a linear regression in which the logarithm of rainfall rate is an independent variable. Campos and Zawadski [94] found that the Z-R relationship depends on the regression technique and is therefore highly method dependent. Atlas et. al. [95] found that the Z-R relationship derived from linear regression produces an accurate representation of the relationship at low rain rates but not necessarily at high rain rates. Therefore, in this research, a linear regression of $10 \cdot \log(Z)$ versus $\log(R)$ is used. The empirical relation of reflectivity (Z) and rain rate (R) has the form of (2.4), where A is the intercept and b is the exponent. According to the above procedure, 9 rain events in the year 1998 is classified and the A and b parameters for the Z-R relation is derived for each stage of the rain event and also for the entire rain event. The Z-R relation parameters are tabulated in Table 6.2.

According to Ulbrich and Atlas [37], the coefficient A increases when the median volume diameter D_0 is large and constant or when the shape parameter μ is small. It is clear from the Singapore's Z-R relations that the large and constant value of D_0 produces higher A values. For the rain event on the 9th May 1998, A values in descending order is from ST→C1→C2→T and b values in descending

order is from T→C2→ST→C1 reverse order. As shown in Figure 6.2, the D_0 is the largest during the C2 stage for a very short duration, larger as well as constant during the C1 stage, small for most of the time except for a short peak at ST stage and smallest during the transition stage. During the stratiform stage, the D_0 is large but smaller than that of the two convective stages. Therefore, the A value should be the largest for C1 and C2, followed by ST, with T having the smallest A values. However, the A value of C1 and C2 stages are less than the A value of ST. This is because of the large transition stage at the end of C1 and at the beginning of the C2 which are included into the convective stages during classification causing the A values for stages C1 and C2 to be less than that of stage ST.

Table 6.2 Reflectivity - rain rate (Z-R) relationships of the nine rain events for different rain types

Rain event	Date	C		T	ST	ALL
1	09/01/98	$Z=330.74 R^{1.25}$		$Z=149.44 R^{1.55}$	$Z=182.61 R^{1.43}$	$Z=192.13 R^{1.40}$
2	05/04/98	$Z=426.56 R^{1.26}$		$Z=384.03 R^{1.22}$	$Z=128.49 R^{1.58}$	$Z=158.46 R^{1.51}$
5	18/05/98	$Z=737.45 R^{1.24}$				
6	18/05/98				$Z=445.89 R^{1.29}$	
8	10/06/98	$Z=645.09 R^{1.12}$		$Z=139.83 R^{1.43}$	$Z=309.47 R^{1.57}$	$Z=325.63 R^{1.30}$
9	25/09/98	$Z=318.10 R^{1.29}$		$Z=232.33 R^{1.31}$	$Z=271.04 R^{1.34}$	$Z=256.87 R^{1.33}$
AU-SG1		$Z=613.76 R^{1.17}$		$Z=198.60 R^{1.37}$	$Z=269.73 R^{1.42}$	-
Rain event	Date	C1	C2	T	ST	
3	09/05/98	$Z=243.06 R^{1.30}$	$Z=122.43 R^{1.53}$	$Z=112.58 R^{1.60}$	$Z=277.21 R^{1.43}$	$Z=285.33 R^{1.31}$
4	12/05/98	$Z=359.06 R^{1.30}$	$Z=209.70 R^{1.47}$	$Z=166.18 R^{1.63}$	$Z=352.04 R^{1.43}$	$Z=311.92 R^{1.34}$
7	07/06/98	$Z=177.02 R^{1.37}$	$Z=164.79 R^{1.47}$	$Z=349.71 R^{1.13}$	$Z=321.65 R^{1.52}$	$Z=380.69 R^{1.19}$
AU-SG2		$Z=294.07 R^{1.30}$	$Z=239.44 R^{1.38}$	$Z=182.71 R^{1.42}$	$Z=334.44 R^{1.37}$	-

As can be seen from Table 6.2, there are two forms of rain events. The rain events (1, 2, 8 and 9) have convective (C), transition (T) and stratiform (ST) rain stages. In the other three events (3, 4 and 7), there is another one convective peak named as C2, following the first convective peak C1. Transition (T) and stratiform

(ST) stages follow the C1 and C2 stages in these three rain events. There are two rain events (5 and 6) considered from 18th May 1998. One is a convective rain event and another one is a stratiform rain event. AU-SG1 Z-R relation is fitted by considering all the C, T and ST points of first 6 rain events in Table 6.2 respectively. Similarly, AU-SG2 Z-R relation is fitted by considering all the C1, C2, T and ST points of the last 3 rain events in Table 6.2 respectively.

After analyzing the nine rain events, it is found that convective stages have higher rain rates (>10 mm/hr for most of the points) and higher reflectivities (> 37 dBZ for most of the points). Few exceptional points are either present during the initial convective period or during the intermediate transition stage in between the two convective stages. The N_w values are 40 dBN and above for the convective points and 35 dBN to 40 dBN for the transition points and spreads from less than 30 dBN to 40 dBN for the stratiform points. In the stratiform regime, D_0 is mainly between 0.50 mm to 1.90 mm, which is smaller than the convective center where D_0 is in the range of 1.10 mm to 3.46 mm, for the analyzed rain events. The transition regions have D_0 values from 0.90 mm to 1.96 mm. The gamma parameter, μ , is very least, < 8 , for the convective stage followed by transition stage for which μ , is less than 10. But the value of μ is larger in the stratiform region and its spread is also large with abrupt changes in the consecutive minutes.

6.3.2 AU-SG Z-R relations

Figure 6.4 shows the A and b values of different rain events for the convective, transition and stratiform stages. Subplot 1 and subplot 2 of Figure 6.4 show the coefficients of Z-R relations for the stages C, T and ST and C1, C2, T and ST respectively. It is clear from Table 6.2 and subplot 1 of Figure 6.4 that convective stages, C, have high A values of greater than 318. The b values are low for the C

stages varying from 1.12 to 1.29. As expected, the stratiform stages have the next highest A values after the C convective stages. The A value for the ST stages ranges from 128.47 to 445.89. The b values in the ST stages are higher than those in the C stages ranging from 1.29 to 1.58. The transition stages have generally low A values compared to the convective and stratiform stages with moderate b values between C and ST stages. The AU-SG1 Z-R relation follows this trend that the A values for the three rain types are in the order $C > ST > T$ and the b values are in the reverse order $ST > T > C$. The decrease of the coefficient A (with the corresponding increase of the coefficient b) from convective to stratiform rain type is clear from the two separate rain events on 18th May 1998.

Figure 6.4 Relations between the coefficient A and the exponent b in the Z-R relations of the nine rain events for different rain types and for the overall Z-R relations

As can be seen from subplot 2 of Figure 6.4, the A values of C1 and ST stages are comparable whereas the b values are lower from 1.30 to 1.37 for the C1 stage and higher from 1.43 to 1.52 for the ST stage. Whenever there are two

convective peaks, the A values of C1 and C2 stages reduce because of the inclusion of the transition stage between the two convective peaks into the convective classification. The C2 stage has a different trend from the C1 stage. C2 stages have lower A values and higher b values of around 1.5. The reason for this as explained before is due to the transition stages which appears between the C1 and C2 stages and is mostly included in the C2 stage during classification. For the T stage, the spread of the A and b values is large and covers a wide range from 112.58 to 349.71 and 1.13 to 1.63 respectively. The reason for the wider spread of A and b value is due to the change of rain type from convective to stratiform during this transition period. During this transition stage, both the high and low rain rates exist.

As explained by Ulbrich and Atlas [37], it is not easy to identify sharp boundaries between the various stages. The trend of b is consistent for the convective to stratiform stages, i.e. it increases from C1→ST. However, the trend of b from C1 or ST→T varies for different rain events. This is due to the broad range of rain rates present in the transition stages. From Table 6.2, AU-SG2 Z-R relation has the decreasing trend of the A values from ST→C1→ C2→T and the b values from T→C2→ST→C1. Several studies [96] have shown how the DSD, and therefore the Z–R relationship, varies geographically, with rainfall intensity. It is also stated in the review that the coefficient A is smaller for stratiform rain and increases for increasing convective activity, while b behaves in the opposite manner (smaller for convective rain and larger for stratiform systems). As expected, the coefficient A is larger for the tropical convective rain (Singapore) followed by the stratiform stage because of the presence of larger drops in these stages.

Clearly visible from Table 6.2, the coefficients of the overall Z-R relation for the 7 rain events (excluding the purely convective and purely stratiform rain events on 18th May 1998) are closer to the stratiform rain types than the convective or transition rain types as concluded from Figure 6.1. According to Wilson in [35], the A values for the three rain types are in the order $C < T < ST$ and the b values are in the order $T < ST < C$ which is in a reverse order to that obtained in this paper. This is due to the analysis of only one rain event for the rain classification in [35]. The Z-R relations proposed in this section are used to derive the rain rates from RADAR data and the resultant rain rates are compared with the distrometer derived rain rates next.

6.4 RADAR and Distrometer Reflectivities

6.4.1 RADAR Reflectivity

The RADAR dataset [91, 97] used in this study is collected at the Changi weather station (1.3512 N, 103.97 E) on the east coast of Singapore for the year 1998. As explained in Section 3.3, the reflectivity data above NTU can be extracted from the ray of the S-band Meteorological Doppler Weather RADAR (MDWR) system from Meteorological Service of Singapore (MSS), using the distance, elevation angle and bearing angle. Figure 6.5 shows the reflectivity values from RADAR at NTU for the rain event occurrence on the 9th May 1998. There are a total of 72 scans displayed in Figure 6.5. The first scan's starting time is 302.03 minutes and the last scan's ending time is 597.77 minutes. Each scan is approximately 4 minutes of duration. Highest reflectivity of 48.39 dBZ was recorded during the 22nd scan between 390.08 minutes to 394.30 minutes.

The rain event has two convective peaks C1 and C2 similar to that of the distrometer data. C1 stage is in between the 11th to the 20th scan (from 344 minutes to 381.70 minutes) and C2 stage is in between the 20th to the 23rd scan, from 381.70 minutes to 394.30 minutes. The convective event started around 344 minutes, decreased for a small transition at 381.70 before increasing again and ending at around 394.30 minutes.

Figure 6.5 Time - height plot of RADAR reflectivity in dBZ for the rain event on 9th May 1998

The minimum elevation angle for the airport mode is 1°; the beam width of the RADAR ray is 1°; therefore, the minimum height of the RADAR ray above NTU is 281 m. The DSD measurements are taken at NTU, 50 m above ground level. A RADAR beam broadens and increases in height as it propagates with range. Thus, the measured reflectivity is a weighted average within the beam volume with the radar beam pattern centered at a certain height above the ground. Convective precipitation has strong horizontal gradients of reflectivity, whereas

stratiform precipitation has an enhanced layer of reflectivity - the bright band - caused by the melting of frozen precipitation particles. The differences between the radar measurements and precipitation intensity at the ground may be caused by the height of the radar measurement coupled with the vertical profile of reflectivity, contamination by non-meteorological target, etc.

Therefore the lowest height above 500 m with non-clutter contaminated radar data is selected in [98] for analysis. It is found that the melting layer is found between 4 km to 5 km using the stratiform rain events. Height variation in terms of the correlation between time series of reflectivity as well as rain rate estimates of radar data and distrometer data is analyzed in [99]. Steady stratiform rain shows much slower decay in correlation from melting layer to the selected lower height than convective events. Ladd et al. [100] used RADAR reflectivities at a distance of 1 km to compare with distrometer reflectivities for the data from Papua New Guinea. The distance 1.2 km is selected to compare with distrometer data in this research since at this distance RADAR reflectivity is nearer to the distrometer reflectivities. It is also clear from Figure 6.5 that the reflectivity values are higher at distances greater than 1 km. Therefore, RADAR reflectivity at a height of 1.2 km is compared with the distrometer data for calibrating the RADAR data.

6. 4. 2   Calibrating the RADAR Data

Figure 6.6 shows the RADAR reflectivities in dBZ derived from the RADAR data at a height of 1.2 km and the distrometer data derived reflectivity at NTU. The reflectivity comparison in Figure 6.6 between the RADAR reflectivity and the distrometer derived reflectivity is in very good qualitative agreement with deviations well in the rain event within expected uncertainties. The correlation

between the RADAR reflectivities and distrometer reflectivities for the whole rain event is 85 %. The deviations are higher at the peak present in the stratiform portion of the rain event. As seen from the Figure 6.6, the RADAR derived reflectivities are always less than the DSD derived reflectivities around 92 percentage of the time within the rain event. Even though, the difference between the two reflectivities is from -2 dB to 8 dB, for 80 % of the minutes. Since the deviations are larger, 5 dB of reflectivity can be added with the RADAR data for calibration.

Figure 6.6 Time series inter comparison between RADAR and distrometer reflectivity in dBZ for the rain event on 9th May 1998

One of the possible reasons for the difference between the RADAR reflectivities and distrometer reflectivities is due to the differences in the sampling volumes of the two measuring instruments. As is well known, Joss distrometer data have a problem of limited number of drop sampling within an integration time. The integration time is related to the sampling volume of the DSD. The sampling

volume is also related to the simulation of radar rain rate estimation and path-attenuation estimation in which the volumes involved in the problems are much larger than the sampling volume of distrometer. Some uncertainty in radar-distrometer comparison due to the difference in sampling volumes is minimized by averaging the DSD for several minutes, especially by applying a sequential intensity filtering technique (SIFT) [101]. But averaging the DSD data is not considered in this thesis. It can be done in future for RADAR and distrometer data comparisons.

The reflectivity comparison between the RADAR and the distrometer derived reflectivities are plotted in Figure 6.7 for another rain event, the 18th event in Table 3.1, occurring on the 12th May 1998. This is to show that the reflectivity addition of 5 dB derived from the rain event 9th May is suitable for the year 1998 data.

Figure 6.7 Time series inter comparison between RADAR and distrometer reflectivity in dBZ for the rain event on 12th May 1998

This rain event is also a long convective event lasting four hours. From Figure 6.7, it can be seen that it has two convective peaks (C1 and C2) followed by transition (T) and stratiform (ST) stages. The reflectivity comparison in Figure 6.7 is in very good qualitative agreement between the RADAR and distrometer with deviations well in the stratiform stage within the expected uncertainties.

The correlation between the RADAR reflectivities and distrometer reflectivities for the whole rain event from 864 minutes to 1096 minutes is 61 %. The correlation between the two parameters increases to 76 % for the duration of 870 minutes to 1096 minutes. This is because the deviation between the two reflectivities is higher during the initial minutes (864 minutes – 869 minutes) of the rain event. The deviations are higher at the convective portion of the rain event especially at the troughs. As seen from Figure 6.7, the RADAR derived reflectivities are always less than the DSD derived reflectivities this is true for around 87 percentage of the time for this rain event.

Even though, the difference between the two reflectivities is from -25 dB to 7 dB for this rain event, for 80 % of the minutes, the absolute difference is between 0 dB to 9 dB. Since the deviations are larger, 5 dB of reflectivity is added with the RADAR data for calibration. Based on the analysis of few of the rain events from the year 1998, listed in Table 3.1, a calibration reflectivity of 5 dB with the radar data is found. The figures for time - height plot of RADAR reflectivity in dBZ and comparison of RADAR Reflectivities with the distrometer reflectivities in dBZ are added in the Appendix B.2 for three more rain events.

6.5 Comparison of Rain rates

Figure 6.8 shows the rain rates calculated from the distrometer data and the rain rates calculated from the RADAR data, after the addition of 5 dB at a height of 1.2 km, using the RADAR derived ZR relations for the rain event on 9th May 1998.

(a) Using the MP, SG and GH-SG, Z-R relations

(b) Using the W-SG, AU-SG1 and AU-SG2, Z-R relations

Figure 6.8 Time series comparison between distrometer derived rain rates and RADAR rain rates using different Z-R relations from Singapore's data for the rain event on 9th May 1998

Figure 6.8a shows the rain rates from RADAR data using the MP, SG and GH-SG, Z-R relations along with the distrometer derived rain rates. Figure 6.8b shows the rain rates from RADAR data using the W-SG, AU-SG1 and AU-SG2, Z-R relations along with the distrometer derived rain rates. The rain rates are calculated for the RADAR reflectivity at a height of 1.2 km using

$$R = \left(\frac{Z}{A} \right)^{\left(\frac{1}{b} \right)} \quad (6.5)$$

where A and b are the derived coefficients of the Z-R relation and Z is the reflectivity in mm^6/mm^3 . As shown in Figure 6.8, the MP Z-R relation works well at the ST stage of rain and tends to under estimate the C1, C2, and T rain stages.

As can be seen from Figure 6.8, the RADAR reflectivity of C1 stage has two peaks which are closer to the distrometer reflectivities. Therefore the calculated rain rates after the addition of 5 dB with the RADAR data at these two peaks are closer to the distrometer derived rain rates. All the other Z-R relations, derived from Singapore data, estimates the rain rates higher as compared to the MP Z-R relations especially in the C1, C2 and ST stages. The SG and GH-SG Z-R relations slightly over estimate the rain rates. The estimation of the rain rates by the other three Z-R relations, W-SG, AU-SG1 and AU-SG2, are almost similar. The SG Z-R relation does not use any rain classification method whereas the GH-SG Z-R relation uses simple rain threshold technique to classify the rain. But the other three Z-R relations, W-SG, AU-SG1 and AU-SG2, use the Atlas-Ulbrich method for rain classification. Even though all the Z-R relations almost estimate the rain rates in the same manner, the correlation between the RADAR and distrometer rain rates is higher using the MP, AU-SG1 and AU-SG2 Z-R relations.

6.6 Summary

This chapter uses the nine rain events from distrometer data to find Z-R relations. Rain is classified into different types based on the rain integral parameters. Z-R relations proposed after rain type classification (convective, stratiform, transition) and Z-R relations proposed without rain type classification for Singapore are compared. It was found that the variation in A and b values for the Z-R relations differs depending on the model and also the rain type classification technique.

Next, the reflectivity data from single polarized RADAR data is extracted for one of the rain event occurred on 9th May 1998 for comparison with the distrometer derived reflectivities. A calibration value of 5 dB has to be added in order for it to compare well with the RADAR data. With the proposed Z-R relations and the calibration factor, rain rate can be retrieved accurately from RADAR data.

Chapter 7

Rain Rate Retrieval from Polarimetric RADAR Variables

The methods for the retrieval of rain rates from polarimetric RADAR variables are examined in this chapter. Two parameter gamma models which are required to retrieve rain drop size distribution from dual polarized RADAR data are discussed. A fixed value of μ is found after analyzing the drop size data and a relationship is found between two of the gamma model parameters. Different filtering of rain categories are tried to find μ - λ relations. Rain rates are retrieved from the polarimetric RADAR variables for one of the rain event using the proposed two parameter gamma models and the results are compared. The proposed two parameter gamma models for the tropical country of Singapore are explained in our papers [102-103].

7.1 Truncated and Un-truncated Moment Fitting

The dead time problem of JWD is addressed in Chapter 3 and the truncated gamma models are used for modeling the drop size distribution. The first four bins which

are likely to have error due to the distrometer's dead time, are removed to calculate the observed moments. However, the equations for the truncated moments are not used in [45]. Recent studies have shown that a large error might be introduced and can cause a notable bias of μ or Λ estimation if truncated observations are assumed as the un-truncated moments [25, 43]. Therefore, the analysis of μ - Λ relations is performed using the gamma model with 2nd, 4th and 6th moments proposed by Ulbrich et al. [25] and using the iterative truncated moment fitting between the drop diameter ranging from 0.3 mm to 5 mm given in [43].

The other two parameter gamma models using fixed μ with the 2nd, 4th and 6th moments un-truncated moment fitting are analyzed and compared. The truncated moment fitting proposed by [43] is not used since as it is not possible to adjust the μ value once it is fixed. This combination of moments is selected in this Chapter for the fair comparison of two parameter gamma models found using both truncated moment fitting and un-truncated moment fitting.

7.2 Gamma Model Parameters

The rain events from 1 to 14, listed in Table 3.1, are used for analyzing the two parameter gamma models. Gamma model parameters, μ , Λ and N_0 , are calculated using (3.16) to (3.18), from the distrometer data for the year 1994 to 1995. Only DSDs having number of rain drops greater than 100 are considered (932 minutes of data) from 14 rain events.

Figure 7.1 illustrates scatter plots of the gamma model parameters versus rain rate. It is clear from Figure 7.1 that even though the parameters show large variability at lower rain rates, their variability reduces at higher rain rates. The

reduction in DSD parameters' variability is mainly attributed with the reduction of mathematical dynamic range of DSD parameters. Of the three parameters plotted in Figure 7.1, since the variation in intercept parameter, N_0 in $\text{mm}^{-1-\mu}\text{m}^{-3}$ is large and in powers of 10, it cannot be kept constant. The other two gamma model parameters can be retrieved from the polarimetric RADAR variables if the shape parameter is kept constant. As can be seen from Figure 7.1, 85 % of shape parameter values are in the range from 1 to 9. The value of μ is to be adjusted in the iteration process of truncated moment method. Therefore fixed μ values are analyzed only for the un-truncated moment method.

Figure 7.1 Scatter plots of gamma model parameters (932 minutes of data, DSDs having rain drops greater than 100 only considered)

7.3 Fixed μ Models

The number of gamma DSD minutes for the particular μ is calculated and the

distribution plotted in Figure 7.2 to study the range of fixed μ values. In Figure 7.2, the number of gamma DSD minutes for the particular μ is plotted in terms of counts. For example, the number of gamma DSD minutes for which $\mu=4$ is 149. From Figure 7.2, μ values of 3, 4 and 5 show higher counts and the peak appears at $\mu=4$. Although the high counts are at $\mu=4$, it does not necessarily mean that this is the best value for the fixed μ model. In order to investigate the use of fixed μ gamma models, the shape parameter μ is fixed at -2 to 16 in steps of one. For fixed μ models, μ is fixed at a constant value, while Λ and N_0 are calculated using (3.17) and (3.18). Rain rate in dBR is then calculated using (3.22) and (6.2), where $N(D_i)$ is the fixed μ gamma DSD and ΔD_i is the drop size interval in mm.

Figure 7.2 Distribution of gamma fitted parameter μ (932 minutes of data, DSDs having rain drops greater than 100 only considered)

To evaluate the accuracy of the fixed μ gamma models, the root mean square error in rain rate estimation ($RMSE-R$) is used. It is calculated by using,

$$RMSE - R = \sqrt{\frac{1}{c} \sum_{i=1}^c (R_{calci} - R_{measi})^2} \quad (7.1)$$

where c is the number of data points. In this paper, $c=932$. R_{calci} is the calculated rain rate using (3.22) and (6.2) from the fixed μ gamma modeled DSD. R_{measi} is the rain rate calculated from the measured data using (3.2) and (6.2). Figure 7.3 shows $RMSE-R$ for fixed μ gamma models.

Figure 7.3 Root mean square error of rain rate, $RMSE-R$ calculated from measured data (932 minutes of data, DSDs having rain drops greater than 100 only considered)

As can be seen from Figure 7.3, the $RMSE$ values are higher for the fixed μ values less than two. $RMSE$ values are decreasing rapidly for the increase of μ from -2 to 2. From the μ values of 2 to 4, the $RMSE$ is still decreasing, but at a slower rate. The minimum $RMSE-R=0.116$ appears for the fixed μ value of 4. Since the $RMSE-R$ for the fixed μ values of 3 and 5 is also near the minimum, the fixed μ values in the range of 3 to 5 are found to be suitable for Singapore's tropical DSD and fixed μ value of 4 which have the maximum DSD minute counts and produce the minimum $RMSE-R$ is more appropriate. The choice of fixed μ range from 3 to 5 for Singapore is close to Kozu and Nakamura [11] and Tokay

and Short [10] where their fixed μ range is from 4 to 6.

By fixing the shape parameter μ , the three parameter gamma model now becomes a two parameter model and can be used for the retrieval of rain rate from RADAR data. The other two parameter model, the μ - Λ relationship which also shows potential [41, 51] for rain rate and reflectivity retrieval with the distrometer estimates, is discussed next.

7.4 μ - Λ Relationship

Figure 7.4 shows the scatter plots of the fitted DSD parameters (μ versus Λ) for the DSD of 932 minutes of data. Figures 7.4a and 7.4b are obtained from the un-truncated moment method and truncated moment method respectively. As can be seen from Figures 7.4a and 7.4b, the variation between the μ and Λ values is large and there is low correlation between the two parameters. However, the estimated values of μ and Λ obtained using the truncated moment method show better correlation than the estimated values using the un-truncated moment method. The Pearson correlation coefficients between the μ and Λ values for the un-truncated and truncated moment methods are 0.830 and 0.861 respectively.

As given in Figure 7.1, the large values of μ and Λ correspond to low rain rates of less than 5 mm/hr. It is reported in [41] that the retrieval of rain rate using μ - Λ relationship agrees well with the measured rain rate in strong convection and higher rain rates. Therefore, the data points are filtered similar to [41-42], and only the DSDs which have rain rates greater than 5 mm/hr and rain counts greater than 1000 drops are selected.

Figures 7.4c and 7.4d show the shape and slope parameters using the filtered data for the un-truncated and truncated moment method. It is noted that the

scatter plot between μ and Λ for the filtered rain cases in Figures 7.4c and 7.4d have higher correlation than that without filtering as shown in Figures 7.4a and 7.4b. Figures 7.4c and 7.4d contain only 337 data points but captured 81% of the rainfall amount in Figures 7.4a and 7.4b. The scatter plots shown in Figures 7.4c and 7.4d show less scatter, and the correlation between μ and Λ is higher, having the Pearson correlation coefficients 0.904 and 0.918 respectively. Filtering makes the correlation coefficients between the μ and Λ values to increase from 0.830 to 0.904 and 0.861 to 0.918 for the un-truncated and truncated moment methods respectively.

Figure 7.4 Scatter plots of μ - Λ values obtained from Singapore's DSD. The curves, obtained from distrometer measurements at Singapore, and distrometer measurements made at Florida (Florida) and Gadanki, India (Gadanki), are overlaid in (c) and (d).

(a) Un-truncated moment method - Without filtering (b) Truncated moment method - Without filtering (c) Un-truncated moment method - With filtering of rain rates ≥ 5 mm /hr and rain counts > 1000 drops; and (d) Truncated moment method - With filtering of rain rates ≥ 5 mm /hr and rain counts > 1000 drops.

A relation between μ and Λ is estimated using a polynomial least squares fit, and it is given as

$$\Lambda = 0.041\mu^2 + 0.362\mu + 1.644 \quad (7.2)$$

when un-truncated moments are used. In the case of the truncated moment method, the corresponding equation is

$$\Lambda = 0.036\mu^2 + 0.432\mu + 1.507 \quad (7.3)$$

Similar to [43], the μ - Λ relations do not change much but the mean values of μ and Λ change from 4.40 and 4.21 in Figure 7.4c for the un-truncated moment method to 4.09 and 4.06 in Figure 7.4d for the truncated moment method. The μ - Λ relationships proposed by [41] and [54], denoted as Florida and Gadanki respectively are also plotted in Figures 7.4c and 7.4d for comparison purposes.

7.4.1 Comparison of μ - Λ Relations

It is clear from Figures 7.4c and 7.4d that the trend of the μ - Λ fit of Singapore follows the Florida curve for $\mu > 4$. Even though, both curves started near the same points, given the same lambda value, Florida's μ values are lower as compared to the μ values of Singapore. The distance between the two curves increases as the rain rate decreases. The differences in μ - Λ relationship between Singapore and Florida could arise from the use of different type of distrometers at these locations. Another reason for the higher μ values of Singapore given the same lambda value may be due to the type of rain events in both the countries. Most of Singapore's rain events are convective. The precipitations used to fit the μ - Λ relationship of Florida may be weaker than the precipitations in Singapore.

Seifert [49] compared Florida's μ - Λ relation with the μ and Λ values of different rain events. He stated that μ is much larger in increasing rain than in decreasing rain, resulting in the data points lying above the Florida's μ - Λ relation

for the strongest events. He also stated that the values lying somewhat below Florida's μ - λ relation correspond to the weakest precipitation events with maximum rain rates below 10 mm/hr. This may be the reason for the lower μ values of Florida's fit as compared to Singapore's μ values given the same lambda value.

The Gadanki curve has higher μ values than the Singapore fits given the same lambda value, which indicates that their data consists mainly of convective rain events. Gadanki's μ - λ relationship in [54] considers a total of 16 rain events of which 5 events are stratiform, 4 events are convective and 7 events are mesoscale convective systems. Both Gadanki and Singapore curves follows the same trend at most of the higher rain rates and this is because both regions falls in the tropical climate, most of the rain events considered are convective in both data sets. Furthermore, both data sets are collected using the same Joss-Waldvogel distrometer for the measurements of DSD. However, the Gadanki μ - λ relationship has slightly higher λ values for the same μ values than Singapore's curve at higher rain rates.

The μ - λ relationship obtained for Singapore shows that the data set contains a significant amount of convective rain events. The curve of Florida has very low μ values compared to Singapore and Gadanki fits given the same lambda value indicating weak events in Florida compared to the two regions. This strongly agrees with Zhang's statement [42] that μ - λ relations vary with the location since each location has different types of rain.

Another reason for Florida's fit to have a low μ value given the same lambda value is the use of different distrometers. Florida uses the video distrometer, whereas Gadanki and Singapore use the JWD. Simultaneous

observations with impact and video disdrometers have indicated that impact disdrometers like JWD seriously under sample raindrops with diameters less than 1.5 mm [104]. This may be the reason for the over estimation of μ values given the same lambda value at Gadanki and Singapore curves compared to Florida's fit. Different ways of rain filtering is done to check the variations in the μ - λ relationships next.

7.4.2 μ - λ Relations for Different Rain categories

In order to examine the appropriate μ - λ relationship for gamma DSD modeling, at all the rain rates, the μ - λ relationship is fitted for the category $R \geq 1$ mm/hr. Figures 7.5a and 7.5b show the scatter plot of μ and λ values and polynomial fit of the filtered rain rates greater than 1 mm/hr for the un-truncated and truncated moment methods.

The spread of the μ and λ values is still large. Therefore, the μ - λ relationship is fitted for the categories, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$ (stratiform and transition rain), $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ (stratiform and convective rain) and $R \geq 25 \text{ mm/hr}$ (convective rain). Figures 7.5c and 7.5d show the scatters of μ and λ values and polynomial fits of the above three rain categories for the un-truncated and truncated moment methods respectively. It is clear from Figures 7.5c and 7.5d that by splitting the category $R \geq 1 \text{ mm/hr}$; into three different rain categories, an improvement to the fitting can be achieved.

The μ - λ relation of the lower rain rate category, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, which has mainly stratiform and transition type rain has lower μ values given the same lambda value whereas the μ - λ relation of convective rain category, $R \geq 25 \text{ mm/hr}$ has higher μ values. The μ - λ relation of the middle rain category has moderate μ values given the same lambda value in between the other two rain

categories. The upward increase of fits from lower rain category to higher rain category clearly indicate that the lower and higher μ values given the same lambda value correspond to stratiform and convective type rains.

Figure 7.5 Scatter plot of μ - λ values for different rain categories and their corresponding μ - λ fits

(a) Un-truncated moment method - $R \geq 1$ mm/hr (b) Truncated moment method - $R \geq 1$ mm/hr (c) Un-truncated moment method - $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$ and (d) Truncated moment method - $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$.

It is clear from Figures 7.5c and 7.5d that the μ - λ relations of the rain categories $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$ and $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ have the same trend and looks similar for the un-truncated and truncated moment methods. However, when truncation is considered for designing the moment method, the scattering of the μ and λ values is found to reduce for the rain category, $R \geq 25 \text{ mm/hr}$. Therefore, the fitting for the truncated moment method has reduced μ values given the same lambda value as compared to the un-truncated moment method for this rain category. Table 7.1 and Table 7.2 show the rain categories, the

type of rain, the polynomial fit coefficients of the μ - Λ relations and Pearson correlation coefficients between the μ and Λ values of the un-truncated and truncated moment methods.

Table 7.1 Shape-slope relations for different category of rain rates fitted using the un-truncated moment method

Splitting criteria	$\Lambda = C\mu^2 + B\mu + A$			Pearson correlation coefficient
	C	B	A	
$R \geq 1$ mm/hr	0.0012	0.813	1.155	0.875
$1 \text{ mm/hr} \leq R < 5$ mm/hr	-0.00098	0.881	2.574	0.919
$5 \text{ mm/hr} \leq R < 25$ mm/hr	-0.011	0.827	1.520	0.899
$R \geq 25$ mm/hr	-0.0057	0.587	1.286	0.825
$R \geq 5$ mm/hr & drops ≥ 1000	0.041	0.362	1.644	0.904
* $1 \text{ mm/hr} \leq R < 5$ mm/hr	X	0.862	2.635	

* - Corresponds to linear fit

Table 7.2 Shape-slope relations for different category of rain rates fitted using the truncated moment method

Splitting criteria	$\Lambda = C\mu^2 + B\mu + A$			Pearson correlation coefficient
	C	B	A	
$R \geq 1$ mm/hr	-0.0014	0.874	0.984	0.890
$1 \text{ mm/hr} \leq R < 5$ mm/hr	-0.00078	0.875	2.596	0.921
$5 \text{ mm/hr} \leq R < 25$ mm/hr	-0.011	0.816	1.593	0.907
$R \geq 25$ mm/hr	0.0178	0.428	1.512	0.887
$R \geq 5$ mm/hr & drops ≥ 1000	0.0356	0.432	1.507	0.918
* $1 \text{ mm/hr} \leq R < 5$ mm/hr	X	0.859	2.650	

* - Corresponds to linear fit

As can be seen in Tables 7.1 and 7.2, for the rain category, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, the coefficient 'C' of the polynomial fits are small and the values are -0.00098 and -0.00078 for the un-truncated and truncated moment methods respectively. Therefore, this coefficient has minimal effect and instead of a polynomial fit, a linear fit is proposed for this rain category, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$ for both the methods. The resultant coefficients from the linear fit are also added at the last row in Tables 7.1 and 7.2 respectively. The linear fit has the

advantage of being less complex compared to the polynomial fit. For the remaining categories, polynomial fit is preferred.

(a) Un-truncated moment method

(b) Truncated moment method

Figure 7.6 Singapore's μ - Λ relationships for different rain categories, $R \geq 5$ mm/hr and rain counts > 1000 drops, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$, along with Florida and Gadanki.

As can be seen from Table 7.1 and 7.2, the correlation between the μ and λ values is improved for truncated cases for all the considered rain categories. The rain categories $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$ and $R \geq 5 \text{ mm/hr}$ & drops ≥ 1000 have higher correlation coefficients for both truncated and un-truncated cases.

Figures 7.6a and 7.6b show the μ - λ relations of Singapore for the rain categories, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$, $R \geq 25 \text{ mm/hr}$ and $R \geq 5 \text{ mm/hr}$ and rain counts ≥ 1000 for the un-truncated and truncated moment methods. The μ - λ relationships, Florida and Gadanki are also plotted in Figure 7.6 for comparison purposes. All the μ - λ relations in the Tables 7.1 and 7.2, use one minute sampling time. The μ - λ relation derived for the rain category, $R \geq 25 \text{ mm/hr}$ consists of only convective type rain and it is above Gadanki curve in Figure 7.6a. But when truncation is considered, the μ - λ relation for the rain category, $R \geq 25 \text{ mm/hr}$, is closer and follows the same trend of Gadanki curve.

As can be seen from Figure 7.6, the μ - λ relations of the lower rain categories, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$ and $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ are closer to Florida curve. This indicates that the rain events used by [41] are weaker precipitation events. It is also clear that even though there are convective points in the Florida curve, most of their points are stratiform and transition points, therefore results in a fit that has lower μ values given the same lambda value.

It is observed from Singapore's data that the fit for the rain category, $R \geq 5 \text{ mm/hr}$ and rain counts ≥ 1000 drops, has 301 DSD minutes. In those 301 DSD minutes, 43 DSD minutes have rain rates less than 25 mm/hr. These DSD minutes results in a fit with lower μ values given the same lambda value as compared to the fit for the category, $R \geq 25 \text{ mm/hr}$.

Polarimetric RADAR variables are calculated using the T-Matrix code for the rain event. The fixed μ value of 4 which is appropriate for the Singapore DSD and the μ - Λ relations are used with the polarimetric RADAR variables to estimate the rain rate. The retrieved rain rates are compared with measured rain rates in order to find the best two parameter models in the next section. The μ - Λ relationships found using the MM234 gamma model for different rain filtering are also used to retrieve the rain rates and the results are compared with the rain rates retrieved from the μ - Λ relationships using truncated and un-truncated moment fitted MM246 gamma models.

7.5 Rain Retrieval

T-Matrix calculations are performed at S band, 2.72 GHz for the Beard and Chung drop shape model [70]. The calculations are done at the elevation angle of 1° for the water temperature of 20°C . The canting angle distribution with zero mean and 10° standard deviation is used for Singapore's tropical climate. Gamma DSD calculated from the Singapore's drop size data is used as an input to the T-Matrix code. The calculated polarimetric RADAR variables, differential reflectivity (Z_{dr}) in dB and horizontal reflectivity (Z_{hh}) in mm^6/mm^3 are then used as explained in [26] to find the gamma model parameters.

The rain event on 26th February 1995 is used as an example for the retrieval of rain from polarimetric RADAR variables. Figure 7.7 shows the relations of Z_{dr} versus Λ and Z_{hh}/N_0 versus Λ for the rain event occurred on 26 February 1995 where the Λ and N_0 are the parameters found from DSD. The following relationships are found from Figure 7.7.

$$\Lambda = 7.700Z_{dr}^2 - 26.094Z_{dr} + 25.119 \quad (7.4)$$

$$N_0 = \frac{Z_{hh}}{10^{(Z_{hh}-norm/10)}} \text{ where } Z_{hh-norm} = 0.070\Lambda^2 - 7.309\Lambda + 30.66 \quad (7.5)$$

The gamma model parameter, Λ can be inferred for a specified Z_{dr} using (7.4) and then using the inferred Λ and Z_{hh} , the parameter N_0 can be obtained using (7.5). Shape parameter, μ is either fixed as 4 or is calculated using the μ - Λ relations for the filtered rain categories.

Figure 7.7 Dependence of Z_{dr} and $10\log_{10} (Z_{hh}/N_0)$ on Λ

In this thesis, polarimetric variables, Z_{dr} and Z_{hh} , found from T-Matrix code are used for finding the DSD parameters as there is no dual polarized RADAR data available during this research. But if there is the dual polarized RADAR data, rain rate is first guessed by using a Z-R relation and Z_{hh} , and then rain-rate categorized μ - Λ relation can be used to estimate rain rate from Z_{dr} and Z_{hh} .

Figures 7.8a and 7.8b and 7.8c show the distrometer measured rain rates and retrieved rain rates for $\mu=4$, using the μ - Λ relation for the rain category $R \geq 5$

mm/hr and rain counts ≥ 1000 drops and using the μ - λ relations for the rain categories $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$ respectively. Figure 7.8b shows the rain rates which are retrieved using the μ - λ relations in (7.2) and (7.3) for the un-truncated and truncated moment methods along with the distrometer measured rain rates.

Figure 7.8 Comparison of rain rates retrieved from polarimetric RADAR variables with measured rain rate for the rain event on 26/02/1995

(a) for $\mu = 4$ (b) using the μ - λ relation for the rain category $R \geq 5$ mm/hr and rain counts ≥ 1000 drops (c) using the μ - λ relations for the rain categories $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$

Similarly, Figure 7.8c shows the retrieved rain rates which are retrieved using the μ - λ relations for the rain categories, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$ of the un-truncated and truncated moment methods along with the distrometer measured rain rates. Untr- μ - λ fit and Tr- μ - λ fit in the legends of Figure 7.8 represent the μ - λ relations derived using the un-truncated and truncated moment methods. The rain rates retrieved using the μ - λ relations from MM234 gamma model is also included in the Figures 7.8b and 7.8c. As the gamma DSD using 2nd, 3rd and 4th moments is used throughout the thesis, the μ - λ relations derived using MM234 model is also included for comparison.

As can be seen from Figure 7.8a, retrieved rain rates using the fixed value of $\mu = 4$ are over estimated at rain rates greater than around 50 mm/hr and under estimated at the lower rain rates. Rain rates retrieved using the μ - λ relations are

closer to the measured rain rates compared to the rain rates retrieved using the fixed μ value of 4. It is clear from Figure 7.8b that the rain rates retrieved using the truncated moment method μ - A relation for the rain category $R \geq 5$ mm/hr and rain counts ≥ 1000 drops matches well with the distrometer measured rain rates at all the rain rates except for the few points above around 140 mm/hr. However, the same rain filtering μ - A relations from MM246 and MM234 models using the un-truncated moment method slightly over estimates at rain rates greater than around 70 mm/hr and the over estimation is larger especially at the convective peaks.

The μ - A relations fitted for the rain categories, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$ and $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$, estimate the rain rates similarly for both the un-truncated and truncated moment methods. As can be seen from Figure 7.8c, the truncated moment method μ - A relation fitted for the rain category, $R \geq 25 \text{ mm/hr}$, indicated by the square markers, estimates the rain rates closer to the measured rain rates than the un-truncated moment methods. The un-truncated moment models using the MM234, indicated by the circle markers in Figure 7.8c, and MM246, indicated by the star markers in Figure 7.8c, overestimate at the convective peaks. The distrometer measured rain rates and retrieved rain rates for $\mu = 4$, using the μ - A relation for the rain category $R \geq 5 \text{ mm/hr}$ and rain counts ≥ 1000 drops and using the μ - A relations for the rain categories $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$ are attached for 4 rain events in Appendix C.

It is concluded from Figure 7.8 that the use of truncated moment method makes the rain retrieval more accurate. Furthermore, the two parameter model using the μ - A relation retrieves the rain rates better than the two parameter model which uses the constant value of μ . Although the rain filtering, $1 \text{ mm/hr} \leq R < 5$

mm/hr and $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$, for rain rate retrieval produce accurate results, the rain filtering $R \geq 5 \text{ mm/hr}$ and rain counts ≥ 1000 drops is recommended since it produces the most accurate results for rain rate retrieval. The μ - λ relations fitted for different rain categories, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$ and $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 5 \text{ mm/hr}$, are lack of data points. This may be the reason for the less accurate retrieval of rain rates by these fits as compared to the μ - λ relation fitted for the rain category $R \geq 5 \text{ mm/hr}$ and rain counts ≥ 1000 drops.

7.6 Summary

This chapter analyzes the two parameter gamma models which are used to retrieve the rain rates using polarimetric RADAR variables. The range of μ values from 3 to 5 is suitable to fit gamma DSD and $\mu=4$ is more appropriate. The other two gamma model parameters are calculated from the polarimetric RADAR variables. Truncated moment fitting between the drop diameter values 0.3 mm to 5 mm is used to find μ - λ relations along with un-truncated moment fitting. The relation between the shape and slope parameters is found for different rain categories. Rain rates are retrieved by finding two of the gamma model parameters from the polarimetric RADAR variables and the μ - λ relation is used to find the third gamma model parameter. The μ - λ relation fitted using the truncated moment method for the rain filtering $R \geq 5 \text{ mm/hr}$ and rain counts ≥ 1000 drops is recommended since it produces the most accurate results for rain rate retrieval.

Chapter 8

Conclusions and Recommendations

8.1 Conclusions

In this thesis, the drop size data measured using the Joss-Waldvogel distrometer is processed and drop size distribution is found from the measured data. Rain drop size distribution models are analyzed and gamma drop size distribution is selected to model the DSD of Singapore. The work is divided basically in two parts: the first part concerns a comprehensive review and reconsideration of past works on DSDs; and the second part focuses on their application to the Singapore database. Based on the investigation and analysis, the thesis is summarized with the following four categorized conclusions.

1. The rain drop size distribution measuring instrument JWD underestimates the number of small drops in heavy rain because of dead time problem. Therefore, the contribution of drop size diameters are found through the analysis of the rain rate calculated using the measured data for seven one minute

rain rates. The contribution of lower drop diameters in rain rate measurement shows that the dead time problem is severe at the lower four bins (small drop sizes). As the JWD has degraded sensitivity at the extreme small and large drop diameters, central moment combinations 2nd, 3rd and 4th and 2nd, 4th and 6th are selected to model the gamma DSD. The smaller drop diameter bins which have negligible drop counts are removed and the truncated gamma models are designed. Consecutive bins are removed, meaning, first, bin 1 is removed, and then bins 1 and 2 are removed, followed by the removal of bins 1 to 3 and 1 to 4 starting from the smallest drop size diameter. Moments are calculated from the remaining bins in each case. Measured data from 2 to 20, 3 to 20, 4 to 20 and 5 to 20 bins are used to calculate the moments. In this way, four gamma models which are less sensitive to small drop diameters are designed with the removal of bins.

Mean square errors (%) are calculated for all the truncated gamma models. There are not many changes in *MSE (%)* for the first 2 models for all the rain rates. The removal including the third bin introduces slight deviation at the lower rain rates and the deviation increases for the fourth bin removed model, this is especially so at the lower rain rate of 1.96 mm/hr. It can therefore be concluded that the removal of the first 4 bins with mean drop diameters of less than 0.77 mm can be done for the higher rain rates (> 22.80 mm/hr) whereas only the lower 2 bins with mean drop diameters less than 0.55 mm can be removed at the lower rain rates ($\leq$ 22.80 mm/hr). This can be done with minimum loss in accuracy of the truncated gamma model.

For terrestrial and earth-satellite communication links, the contributions of different rain drop diameters at different frequencies are important. The importance of small drop diameters increases at lower rain rates as concluded. It is

necessary to check their contribution to the rain attenuation calculations especially at high frequencies. This is because, as frequency increases, the drop diameters relative to the wavelength becomes comparable, therefore, the attenuation caused by these small rain drops becomes significant. Therefore, truncated gamma models will be used in the specific rain attenuation and slant-path rain attenuation calculations at different frequencies in order to find the contribution of lower drop diameters.

2. Specific rain attenuation calculations are done at 11 GHz, 20 GHz, 28 GHz and 38 GHz using the ITU-R model and the forward scattering coefficients for horizontal and vertical polarization using gamma drop size distribution. The specific rain attenuation values are found for different elevation angles from 0° to 90° in steps of 10° . It is shown that the ITU-R model overestimates the specific rain attenuation at 11 GHz and underestimates at higher frequencies at all the elevation angles. Vertical polarized waves suffer less attenuation than horizontal polarized waves. Therefore, vertically polarized waves are selected for further analysis.

The specific rain attenuation contributions due to individual drop diameters are calculated using the ITU-R model and the forward scattering coefficients at all the rain rates for all the frequencies. Regardless of frequency, the highest rain attenuation contribution is always at 2.26 mm when the ITU-R model is used to find the specific rain attenuation contributions. However, when the forward scattering coefficients are used to find the specific rain attenuation contributions, the highest rain attenuation contribution decreases to smaller drop diameters with the increase in frequency. Thus, it was shown that the use of forward scattering coefficients is considered more accurate for Singapore as compared to the ITU-R

model. Therefore, slant-path rain attenuation A_I (in dB) exceeding 1% of the time are calculated for the truncated gamma models using vertically polarized forward scattering coefficients.

The changes in the specific and slant-path rain attenuation values of the truncated gamma models with the actual gamma model show that the truncated gamma models with the first 4 bins removed can be used at Ku-band, Ka-band and Q-band frequencies in Singapore ($R_{0.01}=120.30$ mm/hr). This concludes that Brawn's recommendation to ignore the small drop diameters due to the dead time problem is valid for the calculation of slant-path rain attenuation of microwave links in Singapore. Therefore, the truncated gamma models, with the first 4 bins removed can be used for DSD modeling and rain attenuation calculations in Singapore.

3. The study continues to find reflectivity to rain rate relations from Singapore's DSD to retrieve the rain rate from single polarized RADAR. Z-R relations are derived using the 1514 minutes of DSD data from the year 1998 rain events. Rain is classified into different types based on the rain integral parameters. In Gamache-House method, one of the rain integral parameter, reflectivity, is used to classify the rain into convective and stratiform types whereas Albrich-Utias uses the variations in the rain integral parameters, rain rate, reflectivity, the parameter N_w , median volume diameter and the gamma model parameter μ to classify the rain into convective (C1, C2), transition (T) and stratiform (ST) types.

It is found that convective stages have higher rain rates (>10 mm/hr for most of the points) and higher reflectivities (> 37 dBZ for most of the points). Few exceptional points are either present during the initial convective period or during

the intermediate transition stage in between the two convective stages. The N_w values are higher for the convective points followed by the transition points and spreads a lot in the stratiform stage with lower values than C and T stages. In the stratiform regime, D_0 is smallest followed by transition and convective stage. The gamma parameter, μ , is very least in the convective stage followed by transition stage. But the value of μ is larger in the stratiform region and its spread is also large with abrupt changes in the consecutive minutes.

Four Z-R relations, SG without classifying the rain, using the 1514 DSD points, GH-SG for C and ST type rains, using a simple threshold technique, AU-SG1 for C, T and ST type rains and AUSG2 for C1, C2, T and ST type rains, using the variations in the rain integral parameters are derived from the DSD of Singapore. GH-SG relation has the higher value of A and lower value of b for the convective rain and the reverse is true for stratiform rain. The AU-SG1 Z-R relation has the trend that the A values for the three rain types are in the order $C > T > ST$ and the b values are in the reverse order $ST > T > C$. The AU-SG2 Z-R relation has the decreasing trend of the A values from $C1 \rightarrow ST \rightarrow C2 \rightarrow T$ and the trend of b is consistent for the convective to stratiform stages, i.e. it increases from $C1 \rightarrow ST$. However, the trend of b from C1 or $ST \rightarrow T$ varies for different rain events. After the rain classification, classified C (C1, C2), T and ST points are compared in the $\log N_w - D_0$ plot with the separator line derived by Bringi et al. [93]. This separator line separates the three rain types well for the Australian, C-POL-retrieved DSD parameters (N_w and D_0) in [93]. The comparisons in the $\log N_w - D_0$ domain in this research show that the separations by the separator line matches well with the Atlas-Ulbrich rain classification method. Four Z-R relations, MP Z-R relation derived by Marshall and Palmer for stratiform rain and W-SG, derived by

Wilson are used to calculate rain rates from RADAR data. The rain rates calculated using the RADAR data are compared with the distrometer derived rain rates. The DSD derived rain rates are always higher than the RADAR derived rain rates. Therefore 5 dB of reflectivity is added with the RADAR data as calibration. The Z-R relations, MP, W-SG, AU-SG1 and AU-SG2 relations estimate the rain rates well in the stratiform region and the correlation between the RADAR and distrometer rain rates are higher for the MP, AU-SG1 and AU-SG2 Z-R relations.

4. The two parameter gamma models are analyzed using the measured drop size data of Singapore. The μ value of 4 is found to be most appropriate and the range of μ values from 3 to 5 can be used to form the two parameter gamma model.

The μ - A relationship is derived for Singapore for rain rates greater than 5 mm/hr and rain counts greater than 1000 drops. It is found to be closer to the curve derived for Gadanki, India since the rain rates in both countries are high. The curve for Florida is further away from the curve of Singapore at lower rain rates. Florida curve has lower μ values given the same lambda value as compared to Singapore's μ - A relationship. However, the trend of Singapore curve follows Florida curve for μ values greater than 4. The differences in fit may be due to the location, instrument (Singapore and India-Joss-Waldvogel distrometer, Florida-Video distrometer) used for measuring DSD, the selected events used to fit the μ - A relationship and the type of gamma model to fit DSD.

The μ - A relationship is fitted for the categories, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$. Lower rain rate categories, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$, are closer to Florida's curve and Gadanki curve is almost similar to the Singapore's μ - A relation for $R > 25 \text{ mm/hr}$. This

indicates that the rain events selected for fitting Florida curve have more stratiform and transition type rain and the rain events selected for fitting Gadanki curve have more convective type rain. The comparison of retrieved rain rates with measured rain rates show that the use of the μ - A relationship is more accurate than the use of the fixed μ value for the rain retrievals. Although the rain filtering, $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$ and $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$, for rain rate retrieval produce accurate results, the μ - A relation fitted using the truncated moment method for the rain filtering $R \geq 5 \text{ mm/hr}$ and rain counts ≥ 1000 drops is recommended since it produces the most accurate results for rain rate retrieval.

8.2 Recommendations for Future Work

Based on the studies examined in this thesis, the scope of present work can be further extended and categorized as below.

1. In April 1998, a Joss distrometer and 3 GHz S-band polarisation-diversity Doppler RADAR, the latter designed and built at Rutherford Appleton Laboratory, RAL, were installed at Nanyang Technological University (NTU) in Singapore. The rain events measured from RAL RADAR and distrometer can be compared and rain can be classified using both the data. Based on the comparison, RADAR data can be calibrated. Rain rates can be found using the Z-R relations from the calibrated RAL RADAR reflectivities. This rain rate can then be compared with the RAL distrometer derived rain rates.

2. Dual polarized RADAR is installed by the Meteorological Service of Singapore (MSS) in the year 2010. The dual polarized data can be processed and the two parameter gamma models proposed in this thesis will be used to retrieve rain rate from dual polarized RADAR measurements. The retrieved rain rates can be compared and calibrated with the rain gauge data. K_{dp} from the filtered Φ_{dp} provided by the RADAR software output can be used to calibrate the RADAR data.

3. From the dual polarized RADAR, the forward scattering and back scattering from rain drops and water vapor in cloud cells can be estimated and compared to the theoretically derived and modeled propagation loss. The effects of cloud and rain drop depolarization of electromagnetic wave propagation can also be analyzed and determined.

4. It is found in literature that DSDs are affected by diurnal convective cycles and seasonal variations in precipitation characteristics. Therefore, diurnal and seasonal variations of raindrop size distribution (DSD) at Singapore can be studied to elucidate characteristics of DSD in monsoon versus non-monsoon rain.

5. There are large uncertainties associated with radar based estimates of rainfall. Numerous sources of these errors are due to parameter estimation, the observational system and measurement principles, and not fully understood physical processes. The error analyses and/or simulations including anticipated random and bias errors, especially for the radar estimation of rain rate can be done for both single polarized and dual polarized RADAR data. In particular, it is found

from literature that Z_{dr} is sensitive to received signal level fluctuation due to fading, and estimation error might be large in moderate to light rain rate. The validity of the distrometer derived Z-R relations and μ - A relations can be evaluated through the analyses and/or simulations incorporating such errors.

Author's Publications

International Journal Publications

1. J. X. Yeo, Y. H. Lee, L. S. Kumar and J. T. Ong, "Comparison of S-Band RADAR Attenuation Prediction with Beacon Measurements," Accepted with revision for IEEE Transactions on Antennas and Propagation.
2. L. S. Kumar, Y. H. Lee, and J. T. Ong, "Tropical Rain classification and Estimation of Rain from Z-R (Reflectivity-Rain rate) Relationships," Progress In Electromagnetics Research B, Vol. 32, pp. 107-127, 2011.
3. L. S. Kumar, Y. H. Lee, and J. T. Ong, "Two-Parameter gamma drop size distribution models for Singapore," IEEE Transactions on Geoscience and Remote Sensing, Vol. 49, No. 9, pp. 3371-3380, 2011.
4. L. S. Kumar, Y. H. Lee, and J. T. Ong, "Shape slope parameter distribution modeling of electromagnetic scattering by rain drops," Progress In Electromagnetics Research B, Vol. 25, pp. 191-209, Sept. 2010.
5. L. S. Kumar, Y. H. Lee, and J. T. Ong, "Truncated gamma drop size distribution models for rain attenuation in Singapore," IEEE Transactions on Antennas and Propagation, Vol. 58, No. 4, pp. 1325-1335, Apr. 2010.

International Conference Publications

1. M. Thurai, V. N. Bringi, Y. H. Lee, L. S. Kumar, J. D. Eastment, and D. Ladd, "Height- Correlation analysis of data from an S-band Zenith-pointing RADAR in Singapore," 35th Conference on Radar Meteorology, 26 September 2011.
2. L. S. Kumar, Y. H. Lee, J. T. Ong, "Slant path rain attenuation at different elevation angles for tropical region," Seventh International Conference on Information, Communications and Signal Processing (ICICS 2009), Art. No. 5397706, Macau, 8-10 Dec'2009.
3. S. Lakshmi, Y. H. Lee, J. T. Ong, "Comparative study of Singapore's drop size distribution for fixed μ ", General Assembly of the International Union of Radio Science , Conference Publication No. F05p3, Chicago, Illinois, USA on August 07-16, 2008.
4. S. Lakshmi, Y. H. Lee, J. T. Ong, "The Role of Particular Rain Drop Size on Rain Attenuation at 11 GHz, Sixth International Conference on Information, Communications and Signal Processing (ICICS 2007), Conference Publication No. 0598, Singapore, 10-13 Dec'2007.
5. Y. H. Lee, S. Lakshmi, J. T. Ong, "Rain drop size distribution modelling in Singapore-Critical diameters," The Second European Conference on Antenna and Propagation (EuCAP 2007), Conference Publication No. 0678, Edinburgh, UK, November 2007.
6. Y. H. Lee, S. Lakshmi, J. T. Ong, "Critical drop size on rain attenuation of Ku-band signal in Singapore," Proc. of 11th URSI Commision F Triennial Open Symposium on Radio Wave Propagation Remote Sensing, Rio de Janeiro, Brazil, October 2007.

Bibliography

- [1] J. O. Laws & D. A. Parsons, "The relation of raindrop-size to intensity," Trans. Amer. Geophys. Union, Vol. 24, pp.452-460, 1943.
- [2] J. S. Marshall & W. M. Palmer, "The distribution of raindrops with size," J. Meteorology, Vol. 5, pp.165-166, 1948.
- [3] J. Joss, E. G. Gori, "Shapes of raindrop size distributions," Journal of Applied Meteorology, Vol. 17, pp. 1054-1061, 1978.
- [4] A. Waldvogel, "The N_0 jump of raindrop spectra," J. Atmos. Sci., Vol. 31, pp. 1067-1078, 1978.
- [5] G. Feingold, Z. Levin, "The lognormal fit to raindrop spectra from frontal convective clouds in Israel," J. Clim. Appl. Meteorology, Vol. 25, pp. 1346-1363, 1986.
- [6] E. A. Mueller and A. L. Sims, "Radar cross sections from drop size spectra," Tech. Rep. ECOM-00032-F, Contract DA-28-043AMC-00032(E), Illinois State Water Survey, Urbana, 110 pp. [AD-645218], 1966
- [7] Z. Levin, "Charge Separation by splashing of naturally falling rain drops," J. Atmos. Sci., Vol. 28, pp. 543-548, 1971

- [8] C. W. Ulbrich, "Natural variation in the analytical form of the raindrop size distribution," *J. Climate Appl. Meteor.*, Vol 22, pp. 1764-1775, 1983.
- [9] P. T. Wills, "Functional fits to some observed drop size distributions and parameterization of rain," *J. Atmos. Sci.*, Vol. 41, pp. 1648-1661, 1984.
- [10] A. Tokay, D. A. Short, "Evidence from tropical raindrop spectra of the origin of rain from stratiform versus convective clouds," *Journal of Applied Meteorology*, Vol. 35, pp. 355–371, 1996.
- [11] T. Kozu, K. Nakamura, "Rainfall parameter estimation from dual-radar measurements combining reflectivity profile and path-integrated attenuation," *Journal of Atmospheric and Oceanic Technology*, Vol. 8, pp. 259-271, 1991.
- [12] J. T. Ong, Y. Y. Shan, "Modified Gamma Model for Singapore Raindrop Size Distribution," *IGARSS'97-1997 International Geoscience and Remote Sensing Symposium*, Vol. 4, pp.1757-1759, Singapore, 3-8 August 1997.
- [13] C. Caracciolo, F. Prodi, A. Battaglia, F. Porcu, "Analysis of the moments and parameters of a gamma DSD to inferprecipitation properties: a convective stratiform discrimination algorithm," *Atmospheric Research*, Vol. 80, pp. 165–186, 2008.
- [14] D. Brawn, G. Upton, "Estimation of an atmospheric gamma drop size distribution using disdrometer data," *Atmospheric Research*, Vol. 87, pp. 66–79, 2008.
- [15] G. O. Ajayi & R. L. Olsen, "Modelling of a tropical raindrop size distribution for microwave and millimetre wave applications," *Radio Science*, Vol. 20, No.2, pp.193-202, March-April 1985.

- [16] K. I. Timothy, J. T. Ong, E. B. L. Choo, "Raindrop size distribution using method of moments for terrestrial and satellite communication," IEEE Transactions on Antennas and Propagation, Vol. 50, No. 10, 2002.
- [17] A. Nzeukou, H. Sauvageot, A. D. Ochou, C. Mouhamed and F. Kebe, "Raindrop Size Distribution and Radar Parameters at Cape Verde," Journal of Applied Meteorology, Vol. 43, pp. 90-105, 2004.
- [18] J. Joss and A. Waldvogel, "Ein Spektrograph fuer Niederschlagstropfen mit automatischer Auswertung," Pure Appl. Geophys, 68, pp. 240-246, 1967.
- [19] B. E. Sheppard, P. I. Joe, "Comparison of raindrop size distribution measurements by a Joss–Waldvogel disdrometer, a PMS 2DG spectrometer, and a POSS Doppler radar ,"Journal of Atmospheric and Oceanic Technology, Vol. 11, pp. 874-887, 1994.
- [20] C. Caracciolo, F. Prodi, R. Uijlenhoet, "Comparison between Pludix and impact/optical disdrometers during rainfall measurement campaigns," Atmospheric Research, 82, pp. 137–163, 2006.
- [21] L. W. Li, P. S. Kooi, M. S. Leong, and T. S. Yeo, "A gamma distribution of raindrop sizes and its application to Singapore's tropical environment," Microwave Opt. Tech. Lett., Vol. 7, pp. 253–257, 1994.
- [22] J. T. Ong, Y. Y Shan, "Rain drop size distribution models for Singapore– Comparison with results from different regions," 10th International Conference on Antennas and Propagation, Conference Publication No.436, pp. 2.281-2.285, 14-17 April 1997.
- [23] P. L. Smith, "Raindrop size distributions: exponential or gamma— does the difference matter?," Journal of Applied Meteorology, Vol. 42, pp. 1031-1034, 2003.

- [24] P. L. Smith, D. V. Kliche and R. W. Johnson, "The bias and error in moment estimators for parameters of drop size distribution functions: sampling from gamma distributions," *Journal of Applied Meteorology and Climatology*, vol. 48, no. 10, pp. 2118-2126, 2009.
- [25] C. W. Ulbrich, D. Atlas, "Rain microphysics and radar properties: analysis methods for drop size spectra," *Journal of Applied Meteorology*, Vol. 37, pp. 912-923, 1998.
- [26] G. Zhang, J. Vivekanandan, E. Brandes, "A method for estimating rain rate and drop size distribution from polarimetric Radar measurements," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 39, no. 4, pp. 830-840, April 2001.
- [27] R. Uijlenhoet, J. M. Porra, D. S. Torres and J. D. Creutin, "Analytical solutions to sampling effects in drop size distribution measurements during stationary rainfall: estimation of bulk rainfall variables," *J. Hydrology*, 328, 65-82. 2006.
- [28] P. L. Smith, D. V. Kliche and R. W. Johnson, "Estimating radar reflectivity from observations of raindrop size distributions," *The Fifth European Conference on RADAR Meteorology and Hydrology, ERAD 2008*, Helsinki, Finland, 2008.
- [29] L. J. Battan, "Radar Observations of the Atmosphere," Univ. of Chicago Press, pp. 323, 1973.
- [30] M. Fujiwara, "Raindrop-size distribution from individual storms," *J. Atmos. Sci.*, vol. 22, pp. 585-591, 1965.
- [31] M. Schönhuber, H. Urban, W. Randeu, and J. P. Baptista, "Distrometer results obtained in various climates and their applications to weather radar data

inversion,” in ESA SP-444 Proceedings, Millenium Conference on Antennas and Propagation, Davos (CH), 9-14 April 2000.

- [32] D. Atlas, C.W. Ulbrich, F.D. Marks, E. Amitai, and C.R. Williams, "Systematic variation of drop size and radar – rainfall relations," J. Geophysical Research, vol. 104, pp. 6155-6169, 1999.
- [33] A. Tokay, D. A. Short, C. R. Williams, W. L. Ecklund and K. S. Gage, "Tropical rainfall associated with convective and stratiform clouds: intercomparison of disdrometer and profiler measurements," J. Appl. Met., vol. 38, no. 3, pp. 302-320, 1999.
- [34] M. Maki, T. D. Keenan, Y. Sasaki, and K. Nakamura, "Characteristics of the raindrop size distribution in tropical continental squall lines observed in Darwin, Australia," J. Appl. Meteor., vol. 40, pp. 1393-1412, 2001.
- [35] C. L. Wilson and J. Tan, "The characteristics of rainfall and melting layer in Singapore: Experimental results from radar and ground instruments," 11th international Conference on Antennas and Propagation, Conference Publication No. 480, pp. 852-856, 17-20 April 2001.
- [36] V. N. Bringi, V. Chandrasekar, J. Hubbert, E. Gorgucci, W. L. Randeu, and M. Schoenhuber, "Raindrop size distribution in different climatic regimes from disdrometer and dual-polarized radar analysis," J. Appl. Meteorol., vol. 60, no. 2, pp. 354–365, 2003.
- [37] C.W. Ulbrich, and D. Atlas, "Microphysics of raindrop size spectra: Tropical continental and maritime storms," J. Appl. Meteor. Climatol., vol. 46, pp. 1777–1791, 2007.
- [38] C.W. Ulbrich, and D. Atlas, "Radar measurement of rainfall with and without polarimetry," J. Appl. Meteor. Climatol., vol. 47, pp. 1929-1939, 2008.

- [39] M. Montopoli, F. S. Marzano, and G. Vulpiani, "Analysis and synthesis of raindrop size distribution time series from disdrometer data," *IEEE Trans. Geosci. Remote Sens.*, vol. 46, no. 2, pp. 466-478, 2008.
- [40] V. N. Bringi and V. Chandrasekar, "Polarimetric Doppler Weather Radar: Principles and Applications," Cambridge University Press, 2001.
- [41] E. A. Brandes, G. Zhang and J. Vivekanandan, "An evaluation of a drop distribution based polarimetric radar rainfall estimator," *J. Appl. Meteorol.*, Vol. 42, No. 5, 652–660, 2003.
- [42] G. Zhang, J. Vivekanandan, E. Brandes, R. Menegini and T. Kozu, "The shape-slope relation in observed gamma raindrop size distributions: Statistical error or useful information?" *J. Atmos. Oceanic Technol.*, Vol. 20, No. 8, 1106–1119, 2003.
- [43] J. Vivekanandan, G. Zhang, and E. Brandes, "Polarimetric radar rain estimators based on constrained gamma drop size distribution model," *J. Appl. Meteor.*, vol. 43, pp. 217–230. 2004.
- [44] G. Zhang, J. Sun and E. Brandes, "Improving parameterization of rain microphysics with disdrometer and radar observations," *Journal of Atmospheric Sciences*, Vol. 63, pp. 1273-1290, 2006.
- [45] L. S. Kumar, Y. H. Lee, and J. T. Ong, "Truncated gamma drop size distribution models for rain attenuation in Singapore," *IEEE Transactions on Antennas and Propagation*, Vol. 58, No. 4, pp. 1325-1335, Apr. 2010.
- [46] V. N. Bringi, T.A. Selgia and W. A. Cooper, "Analysis of aircraft hydrometeor spectra and differential reflectivity (Z_{DR}) radar measurements during the Cooperative Convective Precipitation Experiment," *Radio Science*, Vol. 19, pp. 157-167, 1984.

- [47] R. F. Rincon and R. H. Lang, "Microwave Link Dual-Wavelength measurements of Path-Average Attenuation for the Estimation of Drop Size Distributions and Rainfall," IEEE Transactions on Geoscience and Remote Sensing, Vol. 40, no. 4, pp. 760-770, April 2002.
- [48] S. Lakshmi, Y. H. Lee and J. T. Ong, "Comparative study of Singapore's drop size distribution for fixed μ ," General Assembly of the International Union of Radio Science, Publication No. F05p3, Chicago, Illinois, USA, 2008.
- [49] A. K. Seifert, "On the shape-slope relation of drop size distributions in convective rain," J. Appl. Meteor., Vol. 44, No. 7, 1146-1151, 2005.
- [50] D. N. Moisseev and V. Chandrasekar, "Examination of μ - Λ relation suggested for drop size distribution parameters," J. Atmos. Oceanic Tech., Vol. 24, No. 5, 847-855, 2007.
- [51] M. N. Anagnostou, E. N. Anagnostou, J. Vivekanandan, and F. L. Ogden, "Comparison of raindrop size distribution estimates from X-band and S-band polarimetric observations," IEEE Geosci. Remote Sens. Lett., Vol. 4, No. 4, 601-605, Oct. 2007.
- [52] Q. Cao, G. Zhang, E. Brandes, T. Schuur, A. Ryzhkov, and K. Ikeda, "Analysis of video disdrometer and polarimetric radar data to characterize rain microphysics in Oklahoma," J. Appl. Meteor., Vol. 47, No. 8, 2238-2255, 2008.
- [53] Q. Cao, and G. Zhang, "Errors in estimating raindrop size distribution parameters employing disdrometer and simulated raindrop spectra," J. Appl. Meteor. Climat, Vol. 48, No. 2, 406-425, 2009.
- [54] T. Narayana Rao, N. V. P. Kirankumar, B. Radhakrishna, and D. Narayana Rao, "On the variability of the shape-slope parameter relations of the gamma

- raindrop size distribution model,” *Geophysical Research Letters*, 33, L22809, doi: 10.1029/2006GL028440, 2006.
- [55] D. Brawn, G. Upton, “On the measurement of atmospheric gamma drop-size Distributions,” *Atmos. Sci. Lett.*, Vol. 9, No. 4, 245–247, 2008.
- [56] S. J. Munchak and A. Tokay, “Retrieval of raindrop size distribution from simulated dual-frequency radar measurements,” *Journal of Applied Meteorology and Climatology*, Vol. 47, No. 1, 223-239, 2008.
- [57] D. Atlas, and C. W. Ulbrich, “Drop size spectra and integral remote sensing parameters in the transition from convective to stratiform rain,” *Geophys. Res. Lett.*, 33, L16803, doi: 10.1029/ 2006GL026824, 2006.
- [58] T. Kozu, K. K. Reddy, S. Mori, M. Thurai, J. T. Ong, D. N. Rao and T. Shimomai, “Seasonal and Diurnal Variations of Raindrop Size Distribution in Asian Monsoon Region,” *Journal of the Meteorological Society of Japan*, Vol. 84A, pp. 195-209, 2006.
- [59] H.Y. Lam, Din.J, L Luini, A. D.Panagopoulos, C.Capsoni, “Analysis of Raindrop Size Distribution Characteristics in Malaysia for Rain Attenuation Prediction,” *General Assembly and Scientific Symposium, 2011 XXXth URSI, Conference Publication No. F02-7, Istanbul, Turkey, 13-20 Aug. 2011.*
- [60] Distromet Ltd, Distrometer RD-69 Instruction manual, 1993.
- [61] R. Gunn and K.D. Kinzer, “The terminal velocity of fall for water droplets in stagnant air,” *J. Metero.*, Vol. 6, No. 4, pp. 243-248, 1949.
- [62] S. Niu,d X. Jia, J. Sang, X. Liu and C. Lu and Y. Liu, “Distributions of Raindrop Sizes and Fall Velocities in a Semiarid Plateau Climate: Convective versus Stratiform Rains”, *J. Appl. Meteor.*, Vol. 49, pp. 632–645, 2010.

- [63] J. Testud, S. Oury, R. A. Black, P. Amayenc, and X. Dou, “The concept of “normalized” distribution to describe raindrop spectra: A tool for cloud physics and remote sensing,” *J. Appl. Meteor.*, Vol. 40, pp. 1118–1140, 2001.
- [64] M.O Odedina and T.J.O. Afullo, “Rain attenuation prediction along terrestrial paths in south Africa using existing attenuation models,” *IEEE AFRICON conference (AFRICON 2007)*, p4401596, Africa, 2007.
- [65] Recommendation ITU-R P.838-3, Specific attenuation model for rain for use in prediction methods.
- [66] R.L. Olsen, D.V. Rogers, and D.B. Hodge, The aR^b relation in the calculation of rain attenuation, *IEEE Trans. Antennas and Propagation-AP* 26, pp. 318-329, 1978.
- [67] P. S. Ray, “Broadband complex refractive indices of ice and water,” *Applied Optics*, vol. 11, no. 8, pp. 1836-1844, Aug. 1972.
- [68] T. Oguchi, “Summary of studies on scattering of centimeter and millimetre waves due to rain and hail,” *Annals of telecommunications*, Vol. 36, No. 7-8, pp. 383-399, 1981.
- [69] H.R. Pruppacher and K.V. Beard, “A wind tunnel investigation of the internal circulation and shape of water drops fall at terminal velocity in air,” *Quart. J. Roy. Meteor. Soc.*, Vol. 96, pp. 247-256, 1970.
- [70] K. V. Beard and C. Chuang, “A new model for the equilibrium shape of raindrops,” *J. Atmos. Sci.*, Vol. 44, No. 11, 1509–1524, 1987.
- [71] M. Thurai and V.N. Bringi, “Drop Axis Ratios from a 2D Video Disdrometer,” *J. Atmos. Oceanic Tech.*, Vol. 22, No. 7, pp. 963-975, 2005.

- [72] M. Thurai, V. N. Bringi and A. Rocha, "Specific attenuation and depolarization in rain from 2-dimensional video distrometer data," *IET Microw. Antennas Propagation*, Vol. 1, Issue2, pp. 373-380, 2007.
- [73] M. Thurai, G. J. Huang, V. N. Bringi, W. L. Randeu and M. Schönhuber, "Drop Shapes, Model Comparisons, and Calculations of Polarimetric Radar Parameters in Rain," *J. Atmos. Oceanic Technol.*, Vol. 24, pp. 1019–1032, 2007.
- [74] J. Vivekanandan, W. M. Adams, and V. N. Bringi, "Rigorous approach to polarimetric radar modeling of hydrometeor orientation distributions," *J. Appl. Meteor.*, 30, 1053–1063, 1991.
- [75] Recommendation ITU-R P.618-9, Propagation data and prediction methods required for the design of Earth-Space Telecommunication systems, 2007.
- [76] Recommendation ITU-R P.839-3, Rain height model for prediction methods, 2001.
- [77] S. Lakshmi, Y. H. Lee, J. T. Ong, "The Role of Particular Rain Drop Size on Rain Attenuation at 11 GHz, Sixth International Conference on Information, Communications and Signal Processing (ICICS 2007), Conference Publication No. 0598, Singapore, 10-13 Dec'2007.
- [78] Y. H. Lee, S. Lakshmi, J. T. Ong, "Rain drop size distribution modelling in Singapore-Critical diameters," *The Second European Conference on Antenna and Propagation (EuCAP 2007)*, Conference Publication No. 0678, Edinburgh, UK, November 2007.
- [79] L. S. Kumar, Y. H. Lee, and J. T. Ong, "Truncated gamma drop size distribution models for rain attenuation in Singapore," *IEEE Transactions on Antennas and Propagation*, Vol. 58, No. 4, pp. 1325-1335, Apr. 2010.

- [80] M. Marzuki, W. L. Randeu, F. Teschl, M. Schonhuber, W. Harjupa, "Complex Permittivity Measurements of Rainwater in the 0.5–26.5 GHz Frequency Range," *IEEE Geoscience and Remote Sensing Letters*, Vol. 7, No. 3, July 2010.
- [81] L. S. Kumar, Y. H. Lee, J. T. Ong, "Slant path rain attenuation at different elevation angles for tropical region," *Seventh International Conference on Information, Communications and Signal Processing (ICICS 2009)*, Art. No. 5397706, Macau, 8-10 Dec'2009.
- [82] J. X. Yeo, Y. H. Lee, and J. T. Ong, "Ka-band satellite beacon attenuation and rain rate measurements in Singapore-Comparison with ITU-R models," *IEEE AP-S International Symposium on Antennas and Propagation*, June 2009.
- [83] S. Dynes, T. Gordon, "38 GHz fixed links in telecommunications networks," *Exploiting the Millimetric Wavebands*, IEE Colloquium on, pp.5/1-5/4, London, 7 Jan 1994.
- [84] L. J. Ippolito, "Radio Propagation for Space Communications Systems," *Proceedings of the IEEE*, vol. 69, no. 6, pp. 697–727, June 1981.
- [85] G. O. Ajayi, I. E. Owolabi and A. Adimula, "Rain induced depolarization from 1 GHz to 300 GHz in a tropical environment," *International Journal of Infrared and Millimeter Waves*, vol. 8, no. 2, pp. 177–197, 1987.
- [86] J. T. Ong, C. N. Zhu, "Rain rate measurements by a rain gauge network in Singapore," *Electronics Letters*, Vol. 33, Issue. 3, pp. 240-242, 1997.
- [87] K. Aydin and S. Daisley, "Rainfall rate relationships with propagation parameters (attenuation and Phase) at centimeter and millimeter wavelengths," *Int. Geosc. Remote Sensing Symp. (IGARSS 2000)*, vol. 1, pp. 177–179, 2000.

- [88] J. S. Ojo, M. O. Ajewole and S. K. Sarkar, "Rain rate and rain attenuation prediction for satellite communication in Ku and Ka bands over Nigeria," *Progress in Electromagnetics Research B*, vol. 5, pp. 207–223, 2008.
- [89] J. T. Ong, C. N. Zhu, "Slant path attenuation at 11 GHz in Singapore." *Electronic Letters*, Vol. 33. No.13, 1997.
- [90] ITU-R, "Propagation in non-ionized media," Rec. ITU-R PN.618-3, 1995, 1994 PN Series.
- [91] L. S. Kumar, Y. H. Lee, and J. T. Ong, "Tropical Rain classification and Estimation of Rain from Z-R (Reflectivity-Rain rate) Relationships," *Progress In Electromagnetics Research B*, Vol. 32, pp. 107-127, 2011.
- [92] J. F. Gamache and A. R Houze, "Mesoscale air motions associated with a tropical squall line," *Monthly Weather Review*, Vol. 110, pp. 118-135, 1982.
- [93] V. N. Bringi, C. R. Williams, M. Thurai and P. T. May, "Using dual-polarized Radar and dual-frequency profiler for DSD characterization: A case study from Darwin, Australia," *J. Atmos. Oceanic Technol.* Vol. 26, pp. 2107-2122, 2009.
- [94] E. Campos and I. Zawadski, "Instrumental uncertainties in Z-R relations," *J. Appl. Met.*, Vol. 39, pp. 1088-1102, 2000.
- [95] D. Atlas, C. Ulbrich, F. D. Marks, R. A. Black, E. Amitai, P. T. Willis, and C. E. Samsury, "Partitioning tropical oceanic convective and stratiform rains by draft strength," *J. Geoph. Res.*, 105, D2, pp. 2259-2267, 2000.
- [96] G. Villarini, W. F. Krajewski, "Review of the different sources of uncertainty in single polarization Radar-based estimates of rainfall," *Surveys in Geophysics*, 31, pp. 107-129, 2009.

- [97] J. X. Yeo, Y. H. Lee and J. T. Ong, "Performance of Site Diversity Investigated Through RADAR Derived Results," IEEE Transactions on Antennas and Propagation, Vol. 59, No. 10, pp. 3890-3898, 2011.
- [98] M. Thurai, T. Iguchi, T. Kozu, J. D. Eastment, C. L. Wilson, and J. T. Ong, "Radar observations in Singapore and their implications for TRMM Precipitation radar retrieval algorithms," Radio Science, Vol. 38, No. 5, 2003.
- [99] M. Thurai, V. N. Bringi, Y. H. Lee, L. S. Kumar, J. D. Eastment, and D. Ladd, "Height- Correlation analysis of data from an S-band Zenith-pointing RADAR in Singapore," 35th Conference on Radar Meteorology, 26 September 2011.
- [100] D. N. Ladd, C.L. Wilson, M. Thurai, "Radar Measurements from Papua New Guinea and Their Implications for TRMM PR retrieval algorithms," Geoscience and Remote Sensing, IGARSS '97, Remote Sensing - A Scientific Vision for Sustainable Development, Vol. 4, pp. 1648-1650, 1997.
- [101] G. Lee, I. Zawadzki, "Radar calibration by gage, disdrometer, and polarimetry: Theoretical limit caused by the variability of drop size distribution and application to fast scanning operational radar data," Journal of Hydrology, Vol. 328, No.1-2, pp. 83-97, 2006.
- [102] L. S. Kumar, Y. H. Lee, and J. T. Ong, "Two parameter gamma drop size distribution models for Singapore," accepted for IEEE Transactions on Geoscience and Remote Sensing.
- [103] L. S. Kumar, Y. H. Lee, and J. T. Ong, "Shape slope parameter distribution modeling of electromagnetic scattering by rain drops," Progress In Electromagnetics Research B, Vol. 25, pp. 191-209, Sept. 2010.

- [104] A. Tokay, A. Kruger and W. F. Krajewski, "Comparison of drop size distribution measurements by impact and optical disdrometers," J. Appl. Met., Vol. 40, No. 11, 2083-2097, 2001.

Appendix A

RD 69 Distrometer

The RD-69 Distrometer is measuring instrument [60] for measuring rain drop size distributions continuously and automatically. The distrometer transforms the vertical momentum of impacting drop into an electrical pulse, the amplitude of which is a function of the drop diameter. It is capable of measuring the drop diameter ranging from 0.3mm to >5mm with an accuracy of about $\pm 5\%$. Two constituents of this equipment are the transducer and the processor. A cable, 10 meter long is used to connect the two units.

The transducer consists of an electromechanical unit and a feedback amplifier housed in a cylindrical case. Exposed to rain, the transducer transforms the mechanical momentum of an impacting drop into an electric pulse. The amplitude of the pulse is roughly proportional to this momentum. The transducer has a sampling area of 5000 mm^2 and an integration time of 1 min. The roles of the processor are to supply power to the transducer, to process the transducer signal i.e. to reduce its dynamic range and lastly, to eliminate unwanted signals due mainly to the presence of acoustic noise.

Before the input of the processor, the signal of the transducer has amplitude, U_{lin} , which is a function of drop diameter:

$$U_{lin} \propto v(D) \times D^3 \quad (A.1)$$

where $v(D)$ is the terminal velocity of the impacting raindrop and D is the diameter. The terminal velocity measurements [61, 62] exhibit a one to correspondence with drop diameter and it is almost proportional to D . Since $v(D)$ is almost proportional to D , then:

$$U_{lin} \propto D^4 \quad (A.2)$$

At the processor's output, the relationship between the pulse amplitude, U_{comp} , and the diameter, D is much simpler.

$$U_{comp} = 0.94 \times D^{1.47} \quad (A.3)$$

Under normal circumstances, the output of the processor is used to derive D .

Table A.1 Thresholds for the Drop Size Bins

Bin i	Lower bound D_{li} (mm)	Upper bound D_{ui} (mm)	Mean value D_i (mm)
1	0.31	0.41	0.36
2	0.41	0.51	0.46
3	0.51	0.60	0.55
4	0.60	0.72	0.66
5	0.72	0.83	0.77
6	0.83	1.00	0.91
7	1.00	1.23	1.12
8	1.23	1.43	1.33
9	1.43	1.58	1.51
10	1.58	1.75	1.67
11	1.75	2.08	1.91
12	2.08	2.44	2.26
13	2.44	2.73	2.58
14	2.73	3.01	2.87
15	3.01	3.39	3.20
16	3.39	3.70	3.54
17	3.70	4.13	3.92
18	4.13	4.57	4.35
19	4.57	5.15	4.86
20	>5.15		5.30

The analyzer ADA-90 is designed to be used as an interface between the RD-69 Distrometer and a computer. The ADA-90 accepts the drop size pulses from the RD-69 Distrometer, converts their peak amplitudes into a digital code, which is then transmitted in serial form to the computer. In the analyzer ADA-90, the pulses are divided into 127 classes of drop diameter, which are further reduced to 20 classes by a computer program provided with the system. This program can be used to put the data in a suitable format for recording on a diskette. The Distrometer produces an output of raindrop size for 1 minute integration time and 20 different channels of size from 0.3 mm to >5 mm. The thresholds for drop size bins are listed in Table A.1.

Besides allocating the 20 bins, the software mentioned above can be used to compute parameters like rainfall rate, water content, reflectivity factor, total amount of rain, etc.

Appendix B

B.1. Truncated gamma models with actual gamma model and measured DSD

(a) 1.96 mm/hr

(d) 120.30 mm/hr

(e) 147.27 mm/hr

Figure B.1 Truncated gamma models with actual gamma model and measured DSD at five rain rates

Table B.1 Parameters for the truncated and un-truncated gamma models

Rain rate (mm/hr)/ Bins Removed	μ					D_0 (mm)					N_w (mm ⁻¹ m ⁻³)				
	MM 234	1	2	3	4	MM 234	1	2	3	4	MM 234	1	2	3	4
1.96	10.13	10.29	10.88	13.69	20.96	1.01	1.02	1.02	1.04	1.09	9613	9586	9428.1	8504.3	6638.6
4.20	5.51	5.65	6.71	7.98	10.39	1.24	1.24	1.25	1.27	1.30	7554.6	7524.3	7203.3	6776.9	5973.1
10.45	4.47	4.54	4.95	5.68	7.07	1.51	1.51	1.52	1.54	1.56	7051.2	7036.5	6919.6	6663.1	6158.4
22.80	5.46	5.46	5.81	6.76	7.75	1.80	1.80	1.80	1.82	1.83	7035.3	7035.3	6970.9	6775.2	6557.4
66.54	3.15	3.15	3.22	3.42	3.96	2.01	2.01	2.02	2.02	2.04	11687	11687	11652	11521	11122
120.30	5.06	5.06	5.06	5.09	5.20	2.18	2.18	2.18	2.19	2.19	15202	15202	15202	15190	15123
141.27	5.01	5.01	5.01	5.01	5.03	2.43	2.43	2.43	2.43	2.43	11034	11034	11034	11034	11026

B. 2. Distrometer and RADAR Reflectivity data

Time-height plot of RADAR reflectivity in dBZ and comparison of RADAR Reflectivities with the distrometer reflectivities in dBZ are plotted in this section for four rain events, 19, 20 and 22 listed in Table 3.1.

Figure B.2 Time - height plot of RADAR reflectivity in dBZ for the rain event on 18th May 1998, convective rain

Figure B.3 Time series inter comparison between RADAR and distrometer reflectivity in dBZ for the rain event on 18th May 1998, convective rain

Figure B.4 Time - height plot of RADAR reflectivity in dBZ for the rain event on 18th May 1998, Stratiform rain

Figure B.5 Time series inter comparison between RADAR and distrometer reflectivity in dBZ for the rain event on 18th May 1998, stratiform rain

As can be seen from Figure B.4, the stratiform rain is present at heights from 0 1.5 km. Therefore, in Figure B.5 the RADAR reflectivities at heights 600 m and 1.2 km are plotted.

Figure B.6 Time - height plot of RADAR reflectivity in dBZ for the rain event on 10th June 1998

Figure B.7 Time series inter comparison between RADAR and distrometer reflectivity in dBZ for the rain event on 10th June 1998

Appendix C

Retrieved rain rates using polarimetric variables

(a)

(b)

(c)

Figure C.1 Comparison of rain rates retrieved from polarimetric RADAR variables with measured rain rate for the rain event occurred on 30/10/94

(a) for $\mu=4$ (b) using the $\mu-A$ relation for the rain category $R \geq 5$ mm/hr and rain counts ≥ 1000 drops (c) using the $\mu-A$ relations for the rain categories $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$

(a)

(b)

(c)

Figure C.2 Comparison of rain rates retrieved from polarimetric RADAR variables with measured rain rate for the rain event occurred on 13/12/94
 (a) for $\mu=4$ (b) using the μ - A relation for the rain category $R \geq 5$ mm/hr and rain counts ≥ 1000 drops (c) using the μ - A relations for the rain categories $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$

(c)

Figure C.3 Comparison of rain rates retrieved from polarimetric RADAR variables with measured rain rate for the rain event occurred on 21/02/95

(a) for $\mu = 4$ (b) using the μ - A relation for the rain category $R \geq 5$ mm/hr and rain counts ≥ 1000 drops (c) using the μ - A relations for the rain categories $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$

(a)

(b)

Figure C.4 Comparison of rain rates retrieved from polarimetric RADAR variables with measured rain rate for the rain event occurred on 12/03/95
 (a) for $\mu = 4$ (b) using the μ - A relation for the rain category $R \geq 5$ mm/hr and rain counts ≥ 1000 drops (c) using the μ - A relations for the rain categories $1 \text{ mm/hr} \leq R < 5 \text{ mm/hr}$, $5 \text{ mm/hr} \leq R < 25 \text{ mm/hr}$ and $R \geq 25 \text{ mm/hr}$