Vibration energy harvesting using piezoelectric materials

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NANYANG
TECHNOLOGICAL
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**VIBRATION ENERGY HARVESTING USING
PIEZOELECTRIC MATERIALS**

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

2012

VIBRATION ENERGY HARVESTING USING PIEZOELECTRIC MATERIALS

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SUMMARY

Harvesting environmental energy around a system provides a promising solution to implement self-powered electronic devices, which currently rely on batteries as power supply. Vibration energy is a ubiquitous energy source in the environment which can be converted into electricity via different transduction mechanisms. Due to their high power density and ease of application, piezoelectric materials have been enthusiastically pursued for vibration-to-electricity transduction. The work in this thesis focuses on developing advanced modeling methods for piezoelectric energy harvesting systems and improving their efficiency from both sophisticated interface circuit and advanced mechanical configuration perspectives.

Piezoelectric energy harvesting can be achieved by directly bonding piezoelectric elements, or by installing harvesters as add-on systems (usually cantilever configurations with bonded piezoelectric elements) on a vibrating host structure. The former scheme is first investigated in this thesis. A prototype of vibration energy harvesting system using flexible and durable piezoelectric element, macro-fiber composite (MFC), is fabricated and tested. The bonded MFC elements are relatively tiny and the energy harvesting process barely affects the dynamics of the host structure. Thus, based on uncoupled assumption, finite element and circuit simulation models are established to predict the open circuit voltage and system performance of charging storage capacitor. The simulation models are then validated by experiment. Based on the validated models, the effects of various MFC parameters are discussed through parametric study. An optimal MFC configuration is thus proposed.

When piezoelectric energy harvesting is pursued with harvesters installed on a host structure as add-on systems, most of the reported literature either simplified the structure or employed a simple resistor as electric load. This thesis presents a novel generic equivalent circuit modeling method for piezoelectric energy harvesters. The

established equivalent circuit model (ECM) is more accurate than that based on uncoupled assumption. Equivalent circuit parameters of multiple modes of a harvester are identified by theoretical analysis for simple structures and finite element analysis for complicated structures. The identified parameters are then applied in system-level circuit simulation to evaluate the system performance. Numerical examples and experimental work are presented to validate the proposed method and to show its accuracy in accounting for the backward electromechanical coupling effect and its capability of handling complicated structures and sophisticated energy harvesting interface circuits.

To improve the efficiency of piezoelectric energy harvesting, one of the sophisticated interface circuits, namely, synchronized charge extraction technique (SCE), is pursued. The analytical solution is derived in this thesis for a system employing SCE technique, which is more accurate than those based on uncoupled and in-phase assumptions reported in the literature. With this solution, the applicability of the SCE technique is discussed. Circuit simulation is also conducted with the aforementioned ECM of piezoelectric harvesters to validate the analytical outcomes. Both analytical and simulation results show that the SCE technique cannot improve or even reduces the power output at resonance if the electromechanical coupling of a piezoelectric harvester is not negligible. The SCE technique is capable of significantly boosting the efficiency only for the harvester vibrating at off-resonance or with weak coupling.

Another aspect for efficiency improvement of piezoelectric energy harvesters involves increasing their bandwidth and functionality in practical scenarios. A novel multiple-DOF piezoelectric energy harvesting system is developed aiming at addressing the above issue. The 2DOF model is first analyzed and its two configurations are characterized. Based on the characterization, the 2DOF model is then generalized to an n -DOF system and its generic analytical solution is provided. With such n -DOF model for parametric study, a harvester can be designed to have close and effective multiple peaks, or to have one peak with significantly enhanced

magnitude, with only slight increase of system weight. The ECM of the proposed n -DOF harvester is also developed in order to evaluate the system performance when sophisticated interface circuits are attached.

This thesis also presents a comprehensive experimental work to investigate the use of magnets for improving the functionality of piezoelectric energy harvesters under various practical vibration scenarios. First, nonlinear configurations by magnets are exploited. Both monostable and bistable configurations are investigated under sinusoidal and random vibrations with various excitation levels. The optimal nonlinear configuration is determined to be near the monostable-to-bistable transition region. Near this region, both monostable and bistable nonlinear configurations can significantly outperform the linear harvester. Second, for ultra-low-frequency vibrations, a frequency up-conversion mechanism using magnets is proposed. Parametric study shows that the repulsive configuration of magnets is preferable, which is efficient and insensitive to excitation conditions when the magnets are placed sufficiently close.

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CHAPTER 1 INTRODUCTION

1.1 Background

To date, although the power consumption of wireless sensors and personal portable devices are drastically reduced, the reliance on electrochemical batteries as power supply remains a challenging issue for small electronic devices. The use of battery brings not only the troubles of recharging or costly replacement work, but also the pollution problem to the environment. Besides, the included battery also places certain limitation for miniaturization in micro electromechanical systems (MEMS). In some medical applications, such as pace makers, surgery is required for battery replacement which brings additional sufferings to patients. In such a context, the concept of harvesting energy from local environment around a system as power supply (self-powered devices) has aroused abundant interests from various research communities as well as industrial sectors.

There exist many candidate energy sources in the environment for harvesting. Solar energy and wind energy have been two widely utilized renewable energy sources for a long time. They are suitable for large-scale energy harvesting to supplement the electricity consumption of cities. Wave or current energy is another potential candidate for large-scale power generation. However, the maintenance and durability of wave energy converters in the marine and offshore environment is a critical issue. The waste heat energy, such as those in industrial plants, the heat pipes of vehicles and human bodies, can also be converted into useful electricity via two principles. The spatial temperature difference (temperature gradient) generates electricity via Seebeck effect by using thermoelectric materials, while the temporal temperature difference can generate electricity by using pyroelectric materials. Moreover, the ubiquitous mechanical vibration in daily life provides another option for energy harvesting. Vibration sources are not suitable for large-scale power generation to

address the nowadays energy crisis over the world. However, due to their pervasive existence, they are the most attractive option of small-scale energy harvesting and are envisioned for broad applications in small electronic components, which fit one goal of sustainable development, that is, to reduce the use of battery and create intelligent, autonomous and eco-friendly systems.

Vibration energy can be harvested via different materials and mechanisms. Among them, piezoelectric materials are commonly used for vibration-to-electricity transduction due to their high power density and ease of application. Piezoelectric energy harvesting can be achieved by directly bonding piezoelectric elements, or by installing harvesters as add-on systems (usually cantilever configurations with bonded piezoelectric elements) on a vibrating host structure. For these two different installing schemes, the modeling of the energy harvesting system is different. In the former scheme, the backward electromechanical coupling effect (in other words, electrical damping) induced by the energy harvesting process is neglected, which simplifies the modeling of the system. In the second scheme, the system has been frequently and wrongly modeled by neglecting the electrical damping or by regarding it as visous damping. Since the second scheme has been mostly used in the literature, Erturk and Inman (2008b) clarified the misleading electromechanical modeling issues for piezoelectric energy harvesters and an analytical distributed parameter model was developed (Erturk and Inman, 2008c). However, such analytical model could only take into account a simple structure (e.g., rectangular cantilever beam) and simple interface circuit (e.g., a resistive load). Once complex structure and sophisticated interface circuits are concerned, modeling of such piezoelectric energy harvesting systems remains a challenging task. A generic modeling approach is required to address this issue which should not only accurately account for the electrical damping induced by the energy harvesting process but also be capable of handling various complex structures and interface circuits.

In order to improve the efficiency of piezoelectric energy harvesting, vast methods have been attempted from the perspectives of different disciplines. Developing sophisticated interface circuits is one branch, which is pursued by many researchers from electrical and electronic engineering discipline. Although the reported interface circuits were claimed to be capable of significantly improving the power output from the harvester, many studies oversimplified the formulation by neglecting the electrical damping or making the in-phase assumption (Lefeuvre *et al.*, 2006). The outcomes based on these assumptions are definitely conditionally valid. The feasibility and applicability of these interface circuits require further investigation based on a more accurate modeling approach.

Another aspect for improving the efficiency of piezoelectric energy harvesting involves broadening the bandwidth of the harvesters. Different approaches have been reported to address the broadband energy harvesting issue in the literature. Tuning the resonant frequency of a harvester is one approach to make the harvester adaptive to the frequency-variant vibration circumstance. Complex control circuit and correspondent huge energy compensated from the harvested energy are still critical problems in the applications. Multimodal energy harvesting by exploiting multiple modes of a cantilever is another way of broadening the bandwidth. However, the high-order modes of a conventional cantilever beam are far away from the fundamental mode, reducing itself to be a single-mode harvester. A harvester with cantilever array configuration can be tuned to achieve multiple resonant frequencies close to each other in the given spectrum of a vibration source. However, the efficiency improvement compared to a single cantilever of such array configuration is doubtful if the performance is evaluated in terms of power volume density or power weight density. Moreover, such configuration is not applicable in some situations where weight or volume constraints exist. Hence, when multimodal energy harvesting is concerned, an excellent harvester is expected to be able to achieve multiple close and effective resonant peaks with slight increase of the volume or weight of the system. Exploiting nonlinearities is the third choice for wider bandwidth of a

harvester. Both monostable and bistable nonlinear configurations have been widely pursued in the very recent years. Regardless of the configuration adopted, the main challenge is to enable the harvester to persist on the large-amplitude oscillation orbit (high energy orbit). In a word, many challenges remain unaddressed properly in the aforementioned techniques, which limit their deployment in practical applications.

1.2 Research Objectives and Scope

The ultimate goal of vibration energy harvesting is to reduce or totally remove the reliance on battery as power supply for small electronic devices and thus to implement intelligent and eco-friendly systems. To achieve this goal, energy harvesting systems should be first properly modeled to predict their performances. Efforts should be also devoted to addressing the challenges in different approaches for efficiency enhancement so that the power output from the harvesters can be promoted to the expected useful levels. These two aspects compose the objectives of this research. The scope of this thesis includes the following issues.

Modeling Issues

Piezoelectric energy harvesting via piezoelectric elements directly bonded or add-on cantilever harvesters installed on a vibrating host structure is concerned. For the first scheme, uncoupled modeling and efficiency improvement of the system is studied. With determined dynamics of the host structure, the effects of the properties of MFC on energy harvesting require the most attention in optimizing the system performance. For the second scheme, a generic equivalent circuit modeling method for piezoelectric energy harvesters is pursued, which not only takes into account the backward electromechanical coupling effect (electrical damping) but also addresses the difficulty when complicated harvester structure and sophisticated interface circuits are involved.

Efficiency Enhancement Issues

The scope of this research also includes the pursuit of various approaches to improve the efficiency of piezoelectric energy harvesting systems. First, sophisticated interface circuits are considered for efficiency enhancement, with a focus on the synchronized charge extraction (SCE) technique. Efforts are devoted to deriving the analytical solution for the systems employing the SCE technique, which is more accurate than those based on the uncoupled and in-phase assumptions reported in the literature. With this solution and correspondent circuit simulation, the applicability of the SCE technique is thoroughly discussed. Second, multiple-DOF piezoelectric energy harvesting systems are investigated aiming at achieving broadband energy harvesting. Based on the analytical solution of the n -DOF model, a harvester can be designed to have close and effective multiple peaks, or to have one peak with significantly enhanced magnitude, with only slight increase of system weight. The equivalent circuit model of the n -DOF harvester is also developed. Finally, comprehensive experimental work is carried out to investigate the use of magnets for improving the functionality of piezoelectric energy harvesters under various practical vibration scenarios. Both monostable and bistable nonlinear configurations are investigated under sinusoidal and random vibrations with various excitation levels. For ultra-low-frequency vibration scenarios, a frequency up-conversion mechanism using magnets is proposed. For both nonlinear and frequency up-conversion schemes, optimal magnet configurations are studied.

1.3 Original Contributions

The original contributions of this research can be summarized as follows:

- ❖ The efficiency of energy harvesting using MFC transducers is investigated based on the parametric study on their various properties. The optimal configuration of MFC is proposed in this work, which has the potential to significantly increase the energy harvesting capability in the future version of MFC transducer.
- ❖ A novel equivalent circuit modeling method for piezoelectric energy harvester is developed. This method not only takes into account the backward

electromechanical coupling effect (electrical damping) but also addresses the difficulty when complicated harvester structure and sophisticated interface circuits are involved. This method bridges the mature finite element modeling techniques in the mechanical domain and powerful circuit simulation tools (e.g., SPICE simulators) in the electrical domain and exerts their respective advantages.

- ❖ An analytical solution is derived for a piezoelectric energy harvesting system employing an SCE interface circuit. Advantageous over the previous solutions based on uncoupled and in-phase assumptions, the derived solution is valid for harvesters with different electromechanical coupling and harvesters vibrating at resonance or off-resonance frequencies. Based on this solution, the applicability of the SCE interface is investigated for the first time.
- ❖ A novel multiple-DOF piezoelectric energy harvesting system is developed. With such n -DOF model for parametric study, a harvester can be designed to have close and effective multiple peaks, or to have one peak with significantly enhanced magnitude, with only slight increase of system weight. This is advantageous over previously reported harvesters in which multiple resonances were either separated far away from each other or only achievable with significant increase of system weight or volume (inefficient in terms of power density).
- ❖ Though the use of magnets for nonlinear oscillation and frequency up-conversion techniques is not a fresh idea, how to achieve optimal performance of these techniques has not been reported in the previous studies. This research conducts a comprehensive experimental work to determine the optimal configuration for these techniques and the outcomes from this work serve as important guidelines for designing harvesters exploiting nonlinearities or frequency up-conversion mechanism.

1.4 Organization of Thesis

This thesis consists of eight chapters, including this introductory chapter. Chapter 2 reviews the state-of-the-art vibration energy harvesting techniques with a focus on piezoelectric transduction. Different modeling methods and efficiency enhancement approaches are summarized.

Chapter 3 presents an uncoupled modeling of an energy harvesting system using MFC. Finite element modeling and circuit simulation are conducted and validated by experiment. Based on the validated models, parametric study on the effects of MFC properties is presented aiming at efficiency enhancement. Chapter 4 proposes a novel generic equivalent circuit modeling method for piezoelectric energy harvesters. This method is validated by numerical examples and experiment work. Chapter 5 derives an analytical solution for a piezoelectric energy harvesting system employing an SCE interface circuit. Subsequently, this solution is compared with those based on uncoupled and in-phase assumptions. The applicability of SCE interface is thus discussed with this analytical solution. Finally, electric simulation is carried out to validate the analytical outcomes.

Chapter 6 develops a novel multiple-DOF piezoelectric energy harvesting system. The 2DOF model is first analyzed and its two configurations are characterized. The favorable configuration is then generalized to an n -DOF system. With such n -DOF model for parametric study, a harvester can be designed to have close and effective multiple peaks, or to have one peak with significantly enhanced magnitude, with only slight increase of system weight. The equivalent circuit model of the proposed n -DOF harvester is also obtained. Chapter 7 presents an experimental investigation on the optimal configurations for nonlinear and frequency up-conversion techniques using magnets. For nonlinear oscillation technique, both monostable and bistable configurations are investigated and the optimal configuration is determined to be near the monostable-to-bistable transition region. For ultra-low frequency excitation scenarios, the repulsive configuration of magnets is found preferable for frequency

up-conversion. Chapter 8 summarizes the work accomplished in this research and recommends the future work.

CHAPTER 2 LITERATURE REVIEW

Vibrations pervasively existing in the environment have aroused the most interests in the energy harvesting research field from different disciplines in the past decade. Vibrations can present in periodic, frequency-variant or random forms, induced by human motions, machines, vehicles, wind, wave, etc. Different vibration energy harvesting schemes can be applied in different vibration scenarios. This chapter first briefly summarizes the basic vibration-to-electricity mechanisms with a focus on piezoelectric transduction. Subsequently, a comprehensive review of the state-of-the-art modeling methods and the efficiency enhancement approaches for piezoelectric energy harvesting systems is conducted.

2.1 Vibration-to-Electricity Transduction Mechanisms

Three basic vibration-to-electricity transduction mechanisms have been frequently utilized in the literature, namely, electrostatic, electromagnetic and piezoelectric transductions (Roundy *et al.*, 2003).

The basis of electrostatic transduction is the variable capacitor, which consists of two conductors separated by a dielectric material. If the charge on the capacitor is constrained, alternating voltage will be generated as the capacitance changes. If the voltage across the capacitor is constrained, charge will move from the capacitor as the capacitance changes. In either case, the movement of the conductors varies the capacitance and thus kinetic energy is converted to electricity. Roundy *et al.* (2003) reported three basic topologies of electrostatic generators, as shown in Figure 2.1. The advantage of electrostatic transduction is that the energy harvester with capacitor configuration is easy to scale down for integration into micro electromechanical systems (MEMS) using standard and mature manufacturing process (Mitcheson *et al.*,

2004; Suzuki *et al.*, 2010; Yang *et al.*, 2010). The major drawback of this conversion mechanism is that an external voltage or charge source is required to initiate the energy harvesting process.

Figure 2.1 Three topologies of electrostatic generators: (a) in-plane overlap converter (b) in-plane gap closing converter and (c) out-of-plane gap closing converter (figure reproduced from Roundy *et al.*, 2003, copyright: Elsevier)

The basic principle of electromagnetic transduction is based on the Faraday's law of electromagnetic induction. Current is generated when a coil within a magnetic field is moved. Since a permanent magnetic field is required, permanent magnets are usually used. However, magnets and pick-up coils are bulky and usually difficult to scale down. Although MEMS-scale electromagnetic generators were attempted recently (Arnold, 2007), the efficiency was considerably reduced during the scaling of the electromagnetic generator due to the poor properties of planar magnets and limited number of coil turns (Cook-Chennault *et al.*, 2008). Furthermore, the output voltage from electromagnetic transduction is normally small (0.1V~0.2V), requiring to be transformed to usable levels before practical usage by an electrical load (Cook-Chennault *et al.*, 2008).

Piezoelectric transduction is attributed to the constitutive behavior of piezoelectric materials. This intrinsic property enables voltage generation with no requirement of external voltage source or permanent magnetic field, and thus ensures the ease of

application in macro-scale, micro-scale (Jeon *et al.*, 2005) and even nano-scale devices (Wang, 2011). Besides, piezoelectric transduction can develop much higher voltage level than that in electromagnetic generator, which can be directly used for charging batteries or powering wireless electronics, thus voltage transformation in the post-processing circuit can be avoided. Moreover, among the three conventional transductions, harvesters using piezoelectric transduction provide the highest power density (Cook-Chennault, *et al.*, 2008). Due to the aforementioned advantages, piezoelectric transduction has received most interests for vibration energy harvesting in the past few years (Anton and Sodano, 2007; Cook-Chennault, *et al.*, 2008).

Besides the above common transductions, other mechanisms or materials were also reported in the literature for vibration energy harvesting, such as using dielectric elastomer (Jean-Mistral *et al.*, 2008; Czech *et al.*, 2010; Brochu *et al.*, 2010; Koh *et al.*, 2011) and magnetostrictive materials (Wang and Yuan, 2008). A dielectric elastomer sandwiched between two compliant electrodes in principle is also a variable capacitor. The polymer property enables the dielectric elastomers to withstand large mechanical deformation under quite low excitation forces (Czech *et al.*, 2010). Theoretically, these materials have very high energy density around 3.4J/g (Brochu *et al.*, 2010). However, the energy conversion capability of dielectric elastomers is restricted by four mechanisms of failure: material rupture, loss of tension, electrical breakdown and electromechanical instability (Koh *et al.*, 2011). Besides, although it is supposed to undergo large deformation, they are mostly suitable for ultra-low frequency conditions like human motion and ocean waves (Jean-Mistral *et al.*, 2008; Koh *et al.*, 2011). Magnetostrictive materials possess the capability of converting vibrations into an alternating magnetic flux, thus inducing the current in pick-up coils. Advantages of using magnetostrictive materials include their high electromechanical coupling coefficient, high flexibility and suitability for high-frequency vibrations (Wang and Yuan, 2008). However, similar to the conventional electromagnetic

generators, the bulky dimension due to the pick-up coils limits their compatibility in MEMS devices and the low voltage output requires voltage transformation in post-processing circuit.

2.2 Vibration Energy Harvesting Using Piezoelectric Materials

Due to the ease of application and high power density, vibration energy harvesting using piezoelectric materials is the focus throughout this thesis. Following a brief introduction of piezoelectricity and piezoelectric materials, various modeling approaches for piezoelectric energy harvesting systems are presented in this section.

2.2.1 Piezoelectricity and Piezoelectric Materials

Piezoelectricity and Constitutive Equations

Piezoelectricity is one type of coupling between the electrical and mechanical domains of certain materials. The phenomenon of piezoelectricity was first discovered in certain crystalline minerals in 1880 by Pierre and Paul-Jacques Curie. These crystals become electrically polarized when subjected to a mechanical force. Conversely, mechanical strain develops when they are subjected to an electric field. These behaviors were labeled as *direct* and *converse* piezoelectric effects, respectively.

The piezoelectric effect is induced by the dipoles in the piezoelectric ceramics. Usually, the dipole directions are random and no overall macroscopic piezoelectric behavior is observable. When the piezoelectric ceramics are exposed into an intense electric field ($\sim 1\text{kV/mm}$, Crawley and Anderson, 1990) at a temperature slightly below the Curie point, the random-oriented dipoles will be aligned and when the field is removed, most of the dipoles are locked and the dipoles stay roughly, but not completely in alignment. The material now has a strong permanent remnant

polarization and thus a strong piezoelectric coupling. This process (Figure 2.2) is termed ‘*poling*’.

Figure 2.2 Poling process of piezoelectric ceramics: electric dipoles (a) before (b) during and (c) after poling (figure from PI Ceramic)

The linear constitutive equations for piezoelectric materials can be expressed in four forms (Ikeda, 1990) as follows

$$\mathbf{d}\text{-form} \quad \begin{cases} \mathbf{S} = \mathbf{s}^E \mathbf{T} + \mathbf{d} \mathbf{E} \\ \mathbf{D} = \mathbf{d} \mathbf{T} + \boldsymbol{\varepsilon}^T \mathbf{E} \end{cases} \quad (2.1)$$

$$\mathbf{e}\text{-form} \quad \begin{cases} \mathbf{T} = \mathbf{c}^E \mathbf{S} - \mathbf{e} \mathbf{E} \\ \mathbf{D} = \mathbf{e} \mathbf{S} + \boldsymbol{\varepsilon}^S \mathbf{E} \end{cases} \quad (2.2)$$

$$\mathbf{h}\text{-form} \quad \begin{cases} \mathbf{T} = \mathbf{c}^D \mathbf{S} - \mathbf{h} \mathbf{D} \\ \mathbf{E} = -\mathbf{h} \mathbf{S} + \boldsymbol{\beta}^S \mathbf{D} \end{cases} \quad (2.3)$$

$$\mathbf{g}\text{-form} \quad \begin{cases} \mathbf{S} = \mathbf{s}^D \mathbf{T} + \mathbf{g} \mathbf{D} \\ \mathbf{E} = -\mathbf{g} \mathbf{T} + \boldsymbol{\beta}^T \mathbf{D} \end{cases} \quad (2.4)$$

where $\mathbf{D}$ (C/m²) is the electric displacement tensor; $\mathbf{S}$ is the strain tensor; $\mathbf{E}$ (V/m) is the applied electric field tensor; $\mathbf{T}$ (N/m²) is the stress tensor; $\boldsymbol{\varepsilon}$ (F/m) is the dielectric constant tensor; $\boldsymbol{\beta}$ (m/F) is the dielectric impermeability tensor; $\mathbf{d}$ (m/V), $\mathbf{e}$ (C/m²), $\mathbf{h}$ (V/m) and $\mathbf{g}$ (m²/C) are four forms of piezoelectric coefficient tensor; $\mathbf{c}$ (N/m²) is the elastic constant tensor; $\mathbf{s}$ (m²/N) is the elastic compliance tensor; and the superscripts $(\)^T$, $(\)^S$, $(\)^D$ and $(\)^E$ indicate that the quantity is measured at constant stress, constant strain, constant electric displacement and constant electric field, respectively. Among the four forms of constitutive equations, the $\mathbf{d}$ -form and $\mathbf{e}$ -form equations are mostly used in system formulation and analysis in the literature. The manufacturers of

piezoelectric material usually provide *d*-form or *e*-form piezoelectric coefficients in their datasheets. The coupling term in the second equation in constitutive equations of each form indicates the electrical output resulted from the mechanical domain, corresponding to the *direct piezoelectric effect*. The coupling effect is thus termed *forward electromechanical coupling*; While, the coupling term in the first equation in constitutive equations depicts the backward effects in the mechanical domain from the electrical domain (in other words, the electrical damping), corresponding to the *converse piezoelectric effect*. Such coupling effect is termed *backward electromechanical coupling*.

Piezoelectric Materials

Lead zirconate titanate (PZT) is the conventional piezoelectric material used for energy harvesting (Figure 2.3(a)). PZT transducers exhibit most of the characteristics of ceramics, including a high modulus of elasticity, brittleness and low tensile strength (Sirohi and Chopra, 2000). Extremely brittle nature of the PZT material requires extra attention during the handling and bonding procedures, and the conformability of PZT transducer to curved surfaces is extremely poor (Sodano *et al.*, 2004a). Polyvinylidene fluoride, or PVDF for short, is another option for energy harvesting. Despite of the low piezoelectric coefficient, PVDF is a polymer that possesses considerable flexibility compared to PZT, which enables itself to harvest energy from fluid-induced or wind-induced vibrations (Allen and Smits, 2001; Akaydin *et al.*, 2010; Ovejas and Cuadras, 2011; Li *et al.*, 2011). Another alternative to overcome the brittleness of PZTs is to use composite materials consisting of an active piezoceramic fibers embedded in a polymeric matrix phase (Figure 2.3(b)). The polymer matrix increases the flexibility of piezoceramic fibers to withstand large deformation in harsh environments and conformability to curved surfaces, and provides a protective layer around the piezoelectric material. Several piezofiber

technologies are commercially available, such as active fiber composites (AFC), and macro fiber composites (MFC).

Figure 2.3 (a) Conventional PZT ceramics (figure from PI Ceramic) and (b) PZT fiber composites (figure from Smart Material Corp.)

Other than the aforementioned piezoelectric materials which are frequently reported for energy harvesting, other materials also begin to arouse research interest in recent years, such as PMN-PT single crystals, AlN and ZnO nanowires. PMN-PT single crystals have outstanding piezoelectric coupling coefficients and their high quality factors enable very high power densities (Lefevre, *et al.*, 2009). Ren *et al.* (2006) reported that under the mechanical input level of around 40MPa, the single crystal PMN-PT 1-3 composites achieved the harvested power density of 22.1 mW/cm^3 , which is about twice of PZT ceramic 1-3 composite (12 mW/cm^3). Despite the lower piezoelectric coefficient, AlN is very attractive for micromechanical energy harvesting due to the simple manufacturing process and easy integration with complementary metal-oxide semiconductor (CMOS) (Massaro *et al.*, 2011). Besides, ZnO nanowires have been reported in application of nano-scale power generation devices in the very recent few years (Wang, 2011).

2.2.2 Mathematical Modeling of Piezoelectric Energy Harvesters

Piezoelectric energy harvesting can be achieved by two installing schemes: (a) directly bonding piezoelectric elements on a host structure undergoing large surface

deformation and (b) installing cantilever harvesters (with bonded piezoelectric elements) as add-on systems on a host structure undergoing large base excitation (Liang and Liao, 2010), as shown in Figure 2.4. During the design stage, it is advantageous to develop certain model of piezoelectric energy harvesting systems and then to improve the performance based on the established model. For the foregoing two different schemes, the modeling methods are definitely different. In the former scheme, the dynamics of the host structure will not be affected by the energy harvesting process (*backward electromechanical coupling* is negligible). Hence, with the given dynamics of the host structure, little can be done to optimize the energy harvesting efficiency expect adjusting the properties of the piezoelectric element and developing sophisticated interface circuits. In the latter scheme, since the piezoelectric energy harvester is installed as an add-on system to the host structure, one can further pursue the efficiency enhancement by tuning the dynamics of the harvester. Hence, the focus of this section is placed on summarizing the mathematical modeling issues of piezoelectric energy harvesters installed as add-on systems under base excitations.

Figure 2.4 Piezoelectric energy harvesting by (a) surface bonding of piezoelectric element and (b) piezoelectric cantilever installed as add-on system (figure from Liang and Liao, 2010, copyright: IEEE).

A common geometry of the piezoelectric energy harvester is a cantilever beam configuration, as shown in Figure 2.4(b). The mathematical analysis of harvester with cantilever configuration is relatively straightforward, based on vibration theory. Various mathematical models have been established in the past few years, which can be classified, from different perspectives, into single-degree-of-freedom (SDOF) and

distributed parameter models, uncoupled and coupled models, analytical and approximate models, etc. It is worth noting that “*uncoupled model*” here and afterwards means a model in which the *backward electromechanical coupling* is neglected in the mechanical domain. In such case, the system configuration is equivalent to the first installing scheme. The *forward electromechanical coupling* is definitely still considered in the electrical domain otherwise no electrical output is available.

2.2.2.1 SDOF Model

SDOF Model Assuming Viscous Electrical Damping

SDOF model (i.e. a mass-spring-damper system) is one common modeling approach to obtain fundamental insights of the dynamics of a piezoelectric system. Most researches do consider the backward coupling feedback, rather than neglect it, and treat it as viscous electrical damping. Such models were firstly used for electromagnetic generators by Willams and Yates (1996). They considered a magnetic seismic mass moving inside a coil as a microelectric generator (Figure 2.5). The motion of the seismic mass with respect to the generator housing is

$$m\ddot{w} + c\dot{w} + kw = -m\ddot{w}_b \quad (2.5)$$

where w_b is the base excitation; w is the displacement of the seismic mass m relative to the base excitation; k is the spring constant; the electrical damping induced by electromagnetic energy harvesting is treated as viscous damping c_e , included in the total damping c . The available power for energy harvesting is hence evaluated by the dissipated viscous electrical damping as

$$\left| \frac{P}{\ddot{w}_b^2} \right| = \left| \frac{c_e \dot{w}^2 / 2}{\ddot{w}_b^2} \right| = \frac{m}{\omega_n} \frac{\zeta_e \Omega^2}{[1 - \Omega^2]^2 + [2\zeta\Omega]^2} \quad (2.6)$$

where ζ_e is the electrically induced damping ratio; ζ is the total damping ratio; ω_n is the natural frequency; and $\Omega = \omega / \omega_n$ is the dimensionless frequency. This is a fairly accurate model for the electromagnetic generator since it is a reasonable assumption

that the backward coupling effect of electromagnetic energy harvesting in the mechanical domain is proportional to velocity only (Erturk and Inman, 2008b).

Later, this model was applied in modeling the SDOF piezoelectric energy harvesting system by some researchers (Jeon *et al.*, 2005). However, the piezoelectric transduction for energy harvesting uses different physics from electromagnetic transduction. The backward electromechanical coupling effect of piezoelectric energy harvesting in the mechanical domain is not necessarily proportional to velocity. Actually, one can anticipate this by examining the first equation of constitutive equations (2.1~2.4) (The backward electromechanical coupling term is related to electrical variables rather than velocity). This misrepresentation of backward piezoelectric coupling as viscous damping has been clarified in detail by Erturk and Inman (2008b).

Figure 2.5 Schematic of an electromagnetic generator (figure adapted from Willams and Yates, 1996, copyright: Elsevier).

Coupled SDOF Model

duToit *et al.* (2005) established an SDOF model of a piezoelectric energy harvester working in 33-mode, in which the backward electromechanical coupling effect in the mechanical domain was handled properly. The governing equations of their SDOF model were derived based on the *d*-form constitutive equations as

$$\ddot{w} + 2\zeta_m \omega_n \dot{w} + \omega_n^2 w - \omega_n^2 d_{33} v = -\ddot{w}_b \quad (2.7)$$

$$R_l C^S \dot{v} + v + m_{eff} R_l d_{33} \omega_n^2 \dot{w} = 0 \quad (2.8)$$

where w_b is the base displacement; w is the displacement of the harvester relative to the base; v is the voltage output across electric load R_l ; m_{eff} is the effective mass; ζ_m is the mechanical damping ratio; ω_n is the natural frequency; and C^S is the capacitance of the piezoelectric transducer. Equation (2.7) describes the dynamics of the harvester in the mechanical domain and thus is called *mechanical governing equation*, while Equation (2.8) describes the characteristics of the harvester in the electrical domain and thus is called *circuit governing equation*. According to the functions of the coupling terms in Equations (2.7) and (2.8), they are also called *actuator equation* and *sensor equation*, respectively (Saghafi and Meghdari, 2008). Since the backward coupling effect caused by the electrical output is treated as an additional term $\omega_n^2 d_{33} v$ in Equation (2.7), such model is termed as “*coupled model*” to differentiate it from the “*uncoupled model*” or the model assuming viscous electrical damping. The power response of the coupled SDOF model can be expressed as,

$$\left| \frac{P}{\ddot{w}_b^2} \right| = \frac{m_{eff}}{\omega_n} \frac{\alpha k_e^2 \Omega^2}{[1 - (1 + 2\zeta_m \alpha) \Omega^2]^2 + [(2\zeta_m + (1 + k_e^2) \alpha) \Omega - \alpha \Omega^3]^2} \quad (2.9)$$

where k_e is the alternative electromechanical coupling coefficient ($k_e^2 = d_{33}^2 \cdot c_{33}^E / \epsilon_{33}^S$) and $\alpha = \omega_n R_l C^S$ is the dimensionless electric load of the system. Comparing Equations (2.9) and (2.6), the backward piezoelectric coupling effect in power generation obviously acts in a more complicated way than viscous damping. Moreover, using the coupled model, dutToit *et al.* (2005) observed the difference of short circuit and open circuit resonances, which the uncoupled SDOF model failed to predict.

SDOF Correction Factors

A piezoelectric energy harvester is usually designed as a cantilever beam with proof mass. Although the coupled SDOF model (dutToit *et al.*, 2005) improved the prediction of system performance by properly handling the backward electromechanical coupling effect in Equation (2.7), Erturk and Inman (2008a) pointed out that it underestimated the system performance. Even though the natural

frequency is correct based on the equivalent mass and stiffness, the underestimation of voltage or power is due to the inaccurate consideration of the contribution of the distributed beam mass in the excitation amplitude. Based on the reduced single-mode distributed parameter solution, they proposed mode-shape dependent correction factors in the excitation term of the governing equations of the harvester. The correction factors approach unity (i.e., no need for correction) when the mass ratio of proof mass to bar mass is large enough. The SDOF correction factor κ_1 is applied to the mechanical equation of the coupled SDOF model (duToit *et al.*, 2005) as

$$\ddot{w} + 2\zeta_m \omega_n \dot{w} + \omega_n^2 w - \omega_n^2 d_{33} v = -\kappa_1 \ddot{w}_b \quad (2.10)$$

Then, κ_1^2 will appear in the numerator of Equation (2.9). In the numerical case study presented by duToit *et al.* (2005), the mass ratio of proof mass to bar mass is $M_p/mL=1.33$, which is not large enough to ignore the correction factor. The excitation amplitude hence should be corrected with $\kappa_1=1.0968$. Theoretically, this modification avoids an error of 8.83% in voltage output and an error of 16.9% in the power output (Erturk and Inman, 2008a).

2.2.2.2 Distributed Parameter Model

Uncoupled Distributed Parameter Model

The SDOF model only provides the fundamental insights of the behavior of a piezoelectric energy harvester. If the higher vibration modes of a harvester are concerned, distributed parameter modeling is desired. The uncoupled distributed parameter model is a simplified formulation in which the backward coupling effect in the mechanical domain is neglected. However, Erturk and Inman (2008b) clarified that ignoring the backward piezoelectric coupling not only fails to predict the resonant frequency shift but also gives an inaccurate estimation of optimal load and power, especially when the electromechanical coupling effect is strong. For example, the optimal load resistance of the bimorph harvester was predicted as about $35k\Omega$ for excitation at 45.6Hz (fundamental short circuit resonant frequency), validated by the

experimental results (Erturk and Inman, 2009). However, the expression $R_l = 1/\omega C^S$, which is derived from the uncoupled model, predicts a quite inaccurate optimal load resistance for excitation at 45.6 Hz as $84.6 k\Omega$, as shown in Figure 2.6.

Figure 2.6 (a) Resonant frequency shift of a bimorph cantilever harvester with the increase of electric load (b) comparison of optimal power and load in coupled and uncoupled distributed parameter models (figure from Erturk and Inman, 2009, copyright: IOP Publishing)

However, the uncoupled modeling method still provides an approximate solution for weakly coupled systems. One example is the first installing scheme for piezoelectric energy harvesting (Figure 2.4(a)). In such case, no significant difference between short circuit and open circuit resonant frequencies indicates that backward coupling induced by the piezoelectric element has little influence on the dynamics of the host structure and the uncoupled modeling method applies.

Coupled Distributed Parameter Model

The accurate estimation of piezoelectric energy harvester performance requires the backward coupling effect to be considered in the mechanical governing equation. Erturk and Inman (2008c) derived the closed-form solution of a rectangular piezoelectric cantilever energy harvester with a resistive load attached. Such solution set up a benchmark of coupled distributed parameter model of piezoelectric energy harvesters. Thus it is briefly reviewed at this point. In the absence of a proof mass, the coupled equation of the motion of a cantilever harvester relative to its base is

$$\begin{aligned}
& YI \frac{\partial^4 w_{rel}(x,t)}{\partial x^4} + c_s I \frac{\partial^5 w_{rel}(x,t)}{\partial x^4 \partial t} + c_a \frac{\partial w_{rel}(x,t)}{\partial t} + m \frac{\partial^2 w_{rel}(x,t)}{\partial t^2} \\
& + \theta V(t) \left[\frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] = -m \frac{\partial^2 w_b(x,t)}{\partial t^2} - c_a \frac{\partial w_b(x,t)}{\partial t}
\end{aligned} \quad (2.11)$$

where YI is the average bending stiffness; $w_b(x,t)$ and $w_{rel}(x,t)$ are the base excitation and the deflection relative to the base motion, respectively; c_s and c_a are the strain rate damping coefficient and viscous air damping coefficient, respectively; L is the length of the beam; m is the mass per unit length; θ is the system electromechanical coupling coefficient; $V(t)$ is the output voltage of the energy harvester; $\delta(x)$ is the Dirac delta function; and x is the longitudinal coordinate. The right-hand-side forcing function is a pressure distribution due to base excitation along with an external air damping term. The contribution from air damping to the forcing function is usually negligible, compared to the inertial excitation term for harvesters operating in air. In the presence of the tip mass M_t , m in the pressure distribution term should be replaced by $m+M_t\delta(x-L)$. The vibratory motion of the beam can be represented by an absolutely and uniformly convergent series of eigenfunctions as

$$w_{rel}(x,t) = \sum_{r=1}^{\infty} \phi_r(x) \eta_r(t) \quad (2.12)$$

In the presence of the tip mass, substituting (2.12) into (2.11) and applying the orthogonality conditions of the eigenfunctions $\phi_r(x)$ yield the electromechanically coupled ordinary differential equation for the modal response of the harvester as

$$\frac{d^2 \eta_r(t)}{dt^2} + 2\zeta_r \omega_r \frac{d\eta_r(t)}{dt} + \omega_r^2 \eta_r(t) + \chi_r V(t) = -\frac{\partial^2 w_b(x,t)}{\partial t^2} \left(m \int_0^L \phi_r(x) dx + M_t \phi_r(L) \right) \quad (2.13)$$

where ω_r and ζ_r are the natural frequency and damping ratio of the r^{th} mode; and χ_r is the modal electromechanical coupling coefficient. Here, only the mechanical equation is reviewed to show the treatment of the backward electromechanical coupling. Complete coupled modeling procedure will be presented in Chapter 4.

It is worth mentioning that the analytical coupled distributed model derived by Erturk and Inman (2008c) only accounts for a simple rectangular piezoelectric cantilever

beam with uniform material properties. In order to improve the efficiency of the harvester, the profile of the harvester can be more complicated with varying cross-section (Paquin and St-Amant, 2010; Dietl and Garcia, 2010) or irregular distribution of piezoelectric material on the beam after topology optimization (Rupp, *et al.*, 2009; Lee and Youn, 2011). In such cases, the derivation of analytical solutions is either more cumbersome or impossible. Hence, other than the accurate analytical distributed parameter model, approximate distributed parameter modeling approach such as the Rayleigh-Ritz formulation is an alternative method for modeling the piezoelectric energy harvesters. Hagood *et al.* (1990) first introduced the Rayleigh-Ritz formulation in modeling piezoelectric transducers for vibration control. Sodano *et al.* (2004b), Elvin and Elvin (2009a, 2009b) and duToit *et al.* (2005) reported some Rayleigh-Ritz solutions for piezoelectric energy harvesters. The Rayleigh-Ritz type coupled mechanical equation was written as (Elvin and Elvin, 2009a),

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} - \mathbf{\Theta}\mathbf{v} = -\ddot{w}_b \left(\int_0^L m\phi(x)dx + M_t\phi(L) \right) \quad (2.14)$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{\Theta}$ are the mass, damping, stiffness and piezoelectric coupling matrices after the Rayleigh-Ritz formulation; $\mathbf{r}$ denotes the temporal displacement vector; m is the mass per unit length (can be a function of x); M_t is the proof mass; w_b is the base excitation; ϕ is the vector of assumed mode shape functions; and $\mathbf{v}$ denotes the temporal voltage coordinate vector if multiple piezoelectric segments and electrode pairs exist. If only one piezoelectric segment exists, $\mathbf{v}$ is reduced to a scalar and $\mathbf{\Theta}$ to a vector. These mode shape functions are admissible functions which can be a set of any functions satisfying geometric boundary conditions. It is worth noting the similarities between Equations (2.13), (2.14) and (2.10). The difference in the Rayleigh-Ritz discrete model is that $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{\Theta}$ matrices are not diagonal (i.e., equations in (2.14) are not decoupled as Equation (2.13)). Besides, the accuracy of the Rayleigh-Ritz discrete model depends on the assumed mode shapes and the number of assumed modes. It is recommended to achieve adequate accuracy by including more

assumed modes (Elvin and Elvin, 2009a). It should be mentioned that though the approximate coupled distributed parameter model can handle complicated structures, they are still unable to evaluate the energy harvesting performance when practical interface circuits are considered.

2.2.3 Equivalent Circuit Modeling of Piezoelectric Energy Harvesters

Equivalent circuit modeling provides one unique way to study the characteristics of an electromechanical system from a quite different viewpoint. The advantages of equivalent circuit representations for electromechanical systems can be summarized as (Tilmans, 1997): (1) providing a single representation of the system operating in more than one energy domain, (2) giving a better understanding and visualization of the system, (3) facilitating further analyses of the system in order to investigate the effects of connecting subsystems, and (4) allowing system analysis using the powerful tools of electrical network theory, for instance, the circuit simulator SPICE (<http://en.wikipedia.org/wiki/SPICE>).

The development of equivalent circuit models of the piezoelectric energy harvesters is based on the analogies in the mathematical descriptions that exist between the mechanical and electrical domains. The analogies are a result of the formal similarities of the integrodifferential equations governing the behavior of a system. For instance, Newton's second law of motion $F = M d^2x/dt^2$ is mathematically analogous to the constitutive equation of an electric inductor $V = L d^2q/dt^2$. In this analogy, the force F plays the same role as the voltage V , the displacement x as the charge q and the mass M corresponds to the inductance L .

Equivalent Circuit Representation for Uncoupled Models

For a weakly coupled system (e.g., the configuration with piezoelectric element bonded on a host structure as mentioned in Section 2.2.2), the backward coupling

affects little on the dynamics of the system. In such case, the *forward electromechanical coupling* term of the circuit governing equation of the system can be represented as an ideal current source. For example, under constant excitation, if the backward coupling effect of distributed parameter model by Erturk and Inman (2008c) is negligible, the modal velocity $\frac{d\eta_r(t)}{dt}$ is constant and the circuit governing equation can be written as

$$\frac{V(t)}{R_l} + C^s \frac{dV(t)}{dt} - I = 0 \quad (2.15)$$

where $I = \sum_{r=1}^{\infty} \chi_r \frac{d\eta_r(t)}{dt}$ is the ideal current source. For open circuit condition,

$$C^s \frac{dV(t)}{dt} - I = 0 \quad (2.16)$$

Equation (2.16) indicates that the uncoupled piezoelectric energy harvesting system can be represented by an ideal current source in parallel with its internal capacitance (Norton's equivalent circuit) or, equivalently, by an ideal voltage source in series with its internal capacitance (Thevenin's equivalent circuit) (Park, 2001), as shown in Figure 2.7. Although such uncoupled equivalent circuit models are frequently used in the literature (Ng and Liao, 2005; Ottman *et al.*, 2002, 2003), the authors may have not realized its applicability.

Figure 2.7 (a) Norton's and (b) Thevenin's equivalent circuit models of piezoelectric energy harvesting systems

Equivalent Circuit Representation for Coupled Models

If the backward coupling term is not negligible and included in the mechanical equation, e.g., Equations (2.10), (2.13) or (2.14), the velocities of the generalized

coordinates change with the electrical load. Thus, the forward coupling term in the circuit governing equations cannot be represented by an ideal current or voltage source, and the equivalent circuit in Figure 2.7 is no longer applicable for coupled models. The accurate equivalent circuit representation should start from the coupled mechanical equations, based on the analogies between the mechanical and electrical domains. Tilmans (1997) proposed a general equivalent circuit representation for distributed parameter electromechanical transducers, which applies to various transduction mechanisms, e.g., electrostatic, electromagnetic and piezoelectric transductions. The flexural beam transducer with size of $l \times b \times h$, uniformly distributed total mass m and an additional uniformly covered mass layer m_0 is shown in Figure 2.8. The beam rests on an elastic Winkler foundation, described by $q(x) = k_0 w(x)$, where $q(x)$ denotes the foundation force per unit length, k_0 the spring constant and $w(x)$ the deflection of the foundation. The beam is further loaded by a point force F at $x = x_2$, a point moment M at $x = x_1$ and a uniform pressure p_0 . The beam has the stiffness K^* and damping coefficient c . The boundary conditions may be arbitrarily chosen.

Figure 2.8 Flexural beam transducer (figure reproduced from Tilmans, 1997,
copyright: IOP Publishing)

After modal analysis and derivation of the coupled governing differential equations, the general equivalent circuit representation for a distributed parameter electromechanical transducer was developed as shown in Figure 2.9. For the piezoelectric cantilever energy harvester subjected to base excitation, the external load due to the inertias of the beam is a uniformly distributed pressure load. In the presence of the proof mass, a point force should also be included, if the proof mass

can be regarded as a concentrated mass. Hence, recalling Equation (2.13), the equivalent circuit representation for the benchmark analytical coupled model of piezoelectric cantilever energy harvesters by Erturk and Inman (2008c) can be obtained as given in Figure 2.10. The equivalent parameters in Figure 2.10, i.e., C_n , R_n , L_n , γ_n and N_n , may change according to how the mode shape is normalized. However, the topology of the equivalent circuit in Figure 2.10 will not vary. For approximate distributed parameter models by the Rayleigh-Ritz discretization, $\mathbf{M}$ and $\mathbf{K}$ matrices in Equation (2.14) should be diagonalized with proportional damping assumption (fully decoupled equations) before constructing an equivalent circuit model, which still has the similar topology (Elvin and Elvin, 2009a). However, for non-proportional damping, a tridiagonalization procedure should be applied to the governing equations in state space (Saghafi and Meghdari, 2008) and then the equivalent circuit model can be constructed with a different topology from that shown in Figure 2.10.

Figure 2.9 General equivalent circuit representation for distributed parameter electromechanical transducer (figure reproduced from Tilmans, 1997, copyright: IOP Publishing)

Figure 2.10 Equivalent circuit representation for the benchmark coupled model of piezoelectric cantilever energy harvester by Erturk and Inman (2008c)

Equivalent Circuit of Piezoelectric Transducers Working in Actuator Mode

When the piezoelectric transducers are working in actuator mode, the mechanical ports (external forces) are short-circuited and only single electrical port (applying voltage or current) remains, as illustrated in Figure 2.11(a). Hence piezoelectric transducer working in actuator mode is defined as *one-port resonator* and the correspondent equivalent circuit is termed *one-port network* (Tilmans, 1997). After eliminating the ideal transformers (properties of ideal transformers can be referred to <http://en.wikipedia.org/wiki/Transformer>), the equivalent circuit can be further simplified as shown in Figure 2.11(b). It is observed that the equivalent circuit for piezoelectric transducer working in actuator mode is actually an RLC circuit network. Each RLC branch represents one vibration mode of the transducer. The circuit enclosed by the dashed box in Figure 2.11(b) provides an approximation of the equivalent circuit in the vicinity of the fundamental mode, which is frequently termed Van Dyke's model (Park, 2001). To achieve a more accurate approximation for low-frequency equivalent circuit, it is better to replace the clamped capacitance C^s by the damped capacitance C_d , which includes the motional capacitance from high-order branch (Ikeda, 1990; Tilmans, 1997). The equivalent circuit shown in Figure 2.11 is helpful to understand the properties of the admittance of the piezoelectric transducer

and its resonance phenomenon. Once the admittance is obtained from finite element analysis or experiment, one can derive the equivalent circuit parameters and construct the equivalent circuit model.

Figure 2.11 Equivalent circuit representation for the distributed piezoelectric transducer working in actuator mode (a) with and (b) without ideal transformer (figures reproduced from Tilmans, 1997, copyright: IOP Publishing)

2.3 Efficiency Enhancement of Vibration Energy Harvesting

Though the power consumption is dramatically decreasing with the advances in circuitry technologies, the efficiency improvement of vibration energy harvesting is still required to match the power requirements of current wireless sensing electronics. Considerable research attempts have been reported for efficiency enhancement, which can be classified into four categories: (1) advanced piezoelectric and structural configurations; (2) sophisticated energy harvesting interface circuits; (3) broadband energy harvesting; and (4) frequency up-conversion in ultra-low-frequency scenarios.

2.3.1 Advanced Piezoelectric and Structural Configurations

Effects of Proof Mass

The structural configuration is one important factor that influences the efficiency of piezoelectric energy harvesting. Anderson and Sexton (2006) studied the relationship between the system performance and physical and geometrical parameters of a

cantilever bimorph with a proof mass. By varying the proof mass, length and width of the bimorph, they found that changes of the proof mass had the largest effect on the energy harvesting performance. Kim *et al.* (2010) investigated the rotary inertia effect of the proof mass on the piezoelectric energy harvesting performance. A small change in proof mass geometry was found to result in a substantial change of performance due to the frequency shift and the change of strain distribution. Although a lower frequency and increase of power output are expected, a larger proof mass may cause incompatibility and fracture issues in micro-scale energy harvesting devices.

Evening Strain Distribution

A conventional unimorph and bimorph cantilever beam with regular profile (e.g., the one in Erturk and Inman (2008c, 2009)) only produces useful strain level for piezoelectric energy conversion at the root of the beam, which does not fully utilize the capacity of the piezoelectric beam. Some alternative configurations have been reported which could even the strain distribution along the piezoelectric layer, as shown in Figure 2.12. Roundy *et al.* (2005) suggested that, with a trapezoidal cantilever, the strain can be more evenly distributed along the beam length compared to a rectangular beam, as shown in Figure 2.12(a). It was concluded that a trapezoidal cantilever could supply more than twice the energy per unit volume of PZT than a rectangular one. Similarly, Dietl and Garcia (2010) conducted optimization of beam shape. Rectangular, tapered and reverse-tapered beams with non-uniform width were investigated. The tapered beam had a flatter strain distribution but the strain level near the root was much lower compared to the other two beams, while the largest strain near the root of the reversed-tapered beam was modulated by its smallest width for energy generation. Thus, even though the beam shape was further optimized, the power output had no substantial difference for the three beams. Paquin and St-Amant (2010) proposed a piezoelectric cantilever configuration tapered in thickness rather than in width and investigated the effect of the slope angle on the energy harvesting

performance. They showed that a slope angle of 0.94° was optimal, which increased the harvested power by a factor of 3.6 compared to the classical constant thickness harvester. The increased power was attributed to the more uniform strain distribution obtained by the tapered shape.

Figure 2.12 Some reported configurations to even strain distribution along piezoelectric layer: (a) trapezoidal piezoelectric cantilever (figure from Roundy *et al.*, 2005, copyright: IEEE) (b) piezoelectric cymbal (figure from Kim *et al.*, 2004, copyright: JSAP) (c) right-angle piezoelectric cantilever (figure from Xu *et al.*, 2010, copyright: AIP)

Rather than the conventional cantilever configuration, Kim *et al.* (2004) developed a novel circular piezoelectric cymbal transducer (Figure 2.12(b)), in which two dome-shaped metal end-caps were bonded on either side of a piezoelectric plate, inducing axial stress and making the strain distribution in piezoelectric material more evenly than that in a conventional configuration. The efficiency is thus increased since a larger part of the piezoelectric material was actively generating energy.

Xu *et al.* (2010) proposed a right-angle cantilever configuration (Figure 2.12(c)) in which a torque by the auxiliary beam and proof mass was applied on the main beam

with piezoelectric element. Theoretically, a torque rather than a force makes the strain distribution along the beam uniform. Their experiment showed that such configuration generated around two times the power output of the conventional cantilever.

Multilayer/Stack Configuration

Another potential way to improve the energy harvesting capabilities is to utilize multiple pieces of piezoelectric element or stack a large number of piezoceramic wafers together, termed *multilayer/stack configuration*. Such configuration is connected mechanically in series but electrically in parallel as shown in Figure 2.13. Platt *et al.* (2005) investigated the power harvesting performance from a PZT stack with approximately 145 PZT wafers stacked together. The tested PZT stack had a much high capacitance ($1\sim 10\mu\text{F}$) and a low open circuit voltage ($\sim 30\text{V}$), while the same scale monolithic PZT had an ultra low capacitance of 47pF and a very high open circuit voltage of 10kV . Experimental results revealed that the PZT stack and the monolithic PZT of the same geometry provided the same power if impedance matching was achieved, but the matching load for the stack configuration was in $k\Omega$ range, while for the monolithic in $\text{G}\Omega$ range. Thus the stack configuration is more attractive in terms of the manageability of output voltage and impedance matching.

Figure 2.13 PZT multilayer/stack configuration

Initially Curved Configuration

Danak *et al.* (2003) studied the optimal design for an initially curved PZT unimorph energy harvester. The effects were investigated of the initial dome height, substrate and PZT thickness, and substrate stiffness on the generated charge. It was found that

increasing the dome height gave the greatest increase for charge output, compared to increasing the substrate and PZT thickness, as well as the substrate stiffness.

Low-Resonant-Frequency Configuration in MEMS Scale

Energy harvesting at low frequency is a challenge facing MEMS-scale devices. A zigzag design (Figure 2.14(a)) was proposed by Karami and Inman (2011) to overcome this limitation without using an extremely bulky proof mass. An analytical electromechanical model was developed to predict the power output of such structure and a connection scheme was proposed to minimize the power cancellation by proper connection of electrodes of individual beams. The usefulness of the zigzag design was proved by the significant increase of the power output from the same base accelerations, providing a method of designing a micro-scale harvester with low natural frequency. Massaro *et al.* (2011) presented a novel piezoelectric (AlN) ring-type MEMS (RMEMS) energy harvester (Figure 2.14(b)) which showed low resonant frequency without adding a proof mass. Besides, high energy efficiency was achieved due to the additional torsional modes which are absent in the conventional planar cantilever systems. The realized RMEMS prototypes demonstrated the high power density of $30.20 \mu\text{W}\cdot\text{mm}^{-3}$ at 64Hz with an acceleration of 2g.

Figure 2.14 MEMS Piezoelectric energy harvesters with low resonant frequency: (a) Zigzag structure (figure from Karami and Inman, 2011, copyright: SAGE) and (b) RMEMS configuration (figure from Massaro *et al.*, 2011, copyright: AIP)

2.3.2 Sophisticated Energy Harvesting Interface Circuits

Other than the consideration of structural configuration, some research groups place more attention on developing advanced interface circuits for efficiency boost.

2.3.2.1 Impedance Adaptation Interface

The principle of load impedance adaptation is to tune the load impedance to get high power flow. For a pure resistive load and the uncoupled model of piezoelectric energy harvester (discussed in Section 2.2.2.2), the matching load impedance should be equal to the internal capacitive impedance of the piezoelectric energy harvester (i.e., $R_l^{\text{opt}} = 1/\omega C^S$). In practice, the energy harvesting process involves first rectifying the electrical output followed by energy storage in a battery (or usage by a load), rather than directly connecting a resistive load to the piezoelectric element and dissipating the energy. Such interface is termed *standard interface*, as shown in Figure 2.15. A resistor is usually used to emulate the battery but it is variable during battery charging process (Figure 2.15(b)). Shu and Lien (2006) presented a steady state analytical solution of standard interfaces while Wickenheiser *et al.* (2010) studied the transient behavior of charging a capacitor.

Figure 2.15 Standard interface with (a) a battery or (b) an emulating resistor.

Figure 2.16 Impedance adaptation interface

To achieve optimal power flow during battery charging process, some researchers proposed to use DC-DC converters to implement impedance adaptation interface

circuits (Ottman *et al.*, 2002,2003; Lefeuvre *et al.*, 2007a; Kong *et al.*, 2010). A typical configuration is shown in Figure 2.16. Ottman *et al.* (2002) proposed an adaptive circuit using a buck DC-DC converter to maximize the power flow from piezoelectric energy harvesters. An adaptive control algorithm was developed to continuously implement optimal power transfer. This active controller varied the duty cycle of the buck converter to raise the current to levels more acceptable for charging a battery. Figure 2.17 compares the maximum available power, the power to charge a battery with impedance adaptation circuit and the power of directly charging the battery. It was revealed that using the adaptive buck converter increased the power transfer by over 400% compared to directly charging a battery, with optimal duty cycle of 3.18% and the voltage on rectifier close to half open-circuit voltage. However, the buck converter only works when the input voltage is higher than the output voltage. Thus, the voltage output may not be directly used and the converter suffers significant power loss at low external excitation levels.

Figure 2.17 Comparison of (a) available power, (b) power to charge battery by impedance adaptation and (c) power of directly charging the battery (figure from Ottman *et al.*, 2002, copyright: IEEE)

A DC-DC converter which can step both up and down the input voltage definitely provides a flexible solution for piezoelectric energy harvester under different excitation levels. Lefeuvre *et al.* (2007a) proposed an interface using a buck-boost converter and a low consumption control circuit to realize impedance adaptation,

which could track the optimal power delivery for excitation RMS acceleration of 0.3~2.0g. The efficiency of the buck-boost converter reached 88% and remained 84% for the input voltage between 1.6~5.5V. Kong *et al.* (2010) also exerted a buck-boost converter in their proposed impedance adaptation interface but the large smoothing capacitor (C_R in Figure 2.16) was eliminated. Such interface was claimed to match the impedance of piezoelectric harvester successfully in the range of short-circuit to open-circuit resonant frequency with the efficiency of 58~72%. They applied a more accurate circuit representation of piezoelectric energy harvester derived from the distributed parameter model (refers to Section 2.2.3) rather than the uncoupled circuit model in previous impedance adaptation studies, which made their results more convincing.

2.3.2.2 Synchronized Switch Harvesting Interface

The *synchronized switch harvesting* interface (SSH) is derived from the *synchronized switch damping* (SSD) and *synchronized switch damping on inductor* (SSDI) concepts, which were previously developed to address the problem of vibration damping on structures (Guyomar *et al.*, 2005; Badel *et al.*, 2005; Lefeuvre *et al.*, 2010). In the SSD interface (Figure 2.18(a)), the switch is triggered each time the maxima or minima of the displacement (or voltage) of the piezoelectric transducer is reached (i.e., the instances when the maximal electrostatic energy is accumulated). The switch is closed for an extremely short time at these instances to dissipate the electrostatic energy into joule heat on the internal resistance and the voltage drops to zero. Except these instances, the piezoelectric transducer is always in open circuit condition. SSDI interface is obtained by just adding an inductor into the SSD interface in series with the switch. In such interface, the switch is still triggered in the same way as SSD, but the voltage of piezoelectric transducer will be inversed due to the electrical oscillation rather than dropping to zero. Instead of dissipating the energy into joule heat, the SSH interface rectifies the electrical output from piezoelectric transducer to store the

energy or to power an actual electrical load requiring DC power supply. There are two main branches of the SSH interfaces reported in the literature, i.e., the *synchronized charge extraction* (SCE) interface and the *synchronized switch harvesting on inductor* (SSHI) interface.

Figure 2.18 (a) SSD and (b) SSDI interfaces

SCE Interface

The SCE interface was first proposed by Lefeuvre *et al.* (2005). Figure 2.19 shows the schematic of the SCE circuit, in which a flyback switch-mode DC-DC converter is used. An additional control circuit is required to sense the voltage across the rectifier and switch the DC-DC converter at the maxima and minima of voltage. The electrostatic energy on the piezoelectric element is extracted and the voltage drops to zero at these instances. Thus the profile of voltage is similar to that of the foregoing SSD interfaces, as shown in Table 2.1. Compared with the impedance adaptation technique (Figure 2.16), the differences in the SCE interface includes: (1) The absence of the intermediate voltage smoothing capacitor C_R after the rectifier; (2) The DC-DC converter is only operating in short time intervals in one vibration period; (3) The control circuit focuses on synchronizing the charge extraction with the mechanical vibrations. While the impedance adaptation interface focuses on adjusting the duty cycle and the switching frequency of the converter is usually much higher than the frequency of mechanical vibrations; and (4) No impedance adaptation is required.

Lefeuvre *et al.* (2005) reported that the maximum available power by using SCE interface reached 17.5mW with a displacement amplitude of 2mm at 60Hz , nearly

400% of that by the standard interface with impedance adapted. Moreover, the DC-DC converter delivers the maximum useful harvested power of 12.3mW due to its efficiency of 70%. Considering the maximum efficiency of 70% in impedance adaptation (Ottman *et al.*, 2003), the ratio of final harvested power by these two interfaces remains 400%. Thus, Lefeuvre *et al.* (2005) claimed that the SCE technique allows a 70~75% reduction of piezoelectric material to reach the same maximum power by the standard impedance adaptation technique. It is worth mentioning that the study by Lefeuvre *et al.* (2005) was a steady-state analysis. Wu *et al.* (2009) further investigated the behavior of transient power delivery to a storage capacitor by the SCE technique. More details of the DC-DC converter architecture, its processing principle and the benefits of using SCE interface will be discussed in Chapter 5.

Figure 2.19 SCE interface

SSHI Interface

Besides the SCE interface, researchers (Badel *et al.*, 2005; Guyomar *et al.*, 2005; Lefeuvre *et al.*, 2006; Shu *et al.*, 2007; Lien *et al.*, 2010; Liang and Liao, 2011a, 2011b) reported another SSH technique, termed *synchronized switch harvesting on inductor* (SSHI), which is simply derived from SSDI by adding the energy storage components, as shown in Figure 2.20. According to the switch placed in parallel or in series with the piezoelectric element, SSHI has two configurations, i.e., Parallel-SSHI and Series-SSHI. The switch in SSHI is also triggered in the same way as that in the SCE interface, but the switch is placed before the rectifier.

For the Parallel-SSHI, if we compare it with the standard interface (Figure 2.15(b)), the only difference is the switch and inductor added between the piezoelectric element

and rectifier, which inverse the voltage of piezoelectric element by electrical oscillation at the maxima or minima of displacement. Thus the voltage profile of Parallel-SSHI is quite similar to that of the standard interface except the voltage inversion at these instances, as shown in Table 2.1. For the Series-SSHI, since the switch is only closed for extremely short time during a vibration period, the piezoelectric element is almost always in open circuit condition. At this point, the Series-SSHI is similar to SSD, SSDI and SCE but different from the Parallel-SSHI. Compared to SCE, the profile of Series-SSHI is quite similar except the voltage inversion at instances of the maxima and minima of displacement. But it should be emphasized that the inductor in Series-SSHI is used for voltage inversion, different from the inductor in the SCE interface (discussed in Chapter 5) which can isolate the impedances of source and load. Thus, the Series-SSHI together with the Parallel-SSHI still requires load impedance matching to achieve optimal power.

When the piezoelectric device is driven with constant displacement amplitude (the backward coupling effect by energy harvesting is neglected), Lefeuvre *et al.* (2006) reported that with a high electrical quality factor, the maximum harvested power by both SSHI interfaces can be increased impressively by nearly 17 times compared with the standard interface. The Series-SSHI achieved the optimal power with much lower matching resistive load compared to the Parallel-SSHI. Although the power used on the switch control circuit was not withdrawn from the harvested power in their analysis, they claimed that less than 5% of the extracted power for control circuit was experimented successfully.

Figure 2.20 (a) Parallel-SSHI and (b) Series-SSHI interfaces

Table 2.1 Summary of voltage and displacement profiles of piezoelectric energy harvesters with standard and synchronized switch harvesting interfaces

Energy harvesting interfaces	Schematic of interfaces	Voltage and displacement profile	References
SSD			Lefeuvre <i>et al.</i> (2010)
SSDI			Guyomar <i>et al.</i> (2005) Badel <i>et al.</i> (2005) Lefeuvre <i>et al.</i> (2010)
Standard			Shu and Lien (2006) Shu <i>et al.</i> (2007) Lien <i>et al.</i> (2010)
SCE			Lefeuvre <i>et al.</i> (2005, 2006, 2010)
Parallel-SSHI			Guyomar <i>et al.</i> (2005) Badel <i>et al.</i> (2005) Shu <i>et al.</i> (2007) Liang and Liao (2011a)
Series-SSHI			Lien <i>et al.</i> (2010) Lefeuvre <i>et al.</i> (2010) Liang and Liao (2011b)

2.3.2.3 Other Issues in Energy Harvesting Interface

The conventional standard, SCE and SSHI interfaces include full-wave bridges (four semiconductor diodes) as rectifiers. The intrinsic forward voltage drop of these rectifiers limits their voltage rectification when the voltage output from piezoelectric harvester is very low (e.g., when the harvester works under low external excitations or *off-resonance* conditions). To address this issue, Lallart and Guyomar (2008) proposed a new Series-SSHI technique in which two switches replaced two diodes in the full-wave rectifier and the switch between the piezoelectric element and rectifier

in the conventional SSHI was removed. The main benefit of this technique was that the removal of two diodes halved the voltage gap and thus the improved SSHI interface could operate below the threshold voltage of a conventional SSHI. Another research attempt was reported by Cheng *et al.* (2009), in which a bridge voltage doubler AC/DC converter was developed for low-voltage energy harvesting applications. Such interface used active components to implement ideal diode behavior and replace the conventional semiconductor diodes. It successfully rectified the input voltage as low as 20mV and the maximum efficiency of 92% was achieved for 1V input amplitude. However, the active components required extra energy consumption which was still sourced from external power source (Cheng *et al.*, 2009). This is another important issue in interface design, i.e., self-powered switching.

The energy harvesting system without the capability of self-powered switching is definitely not a ‘smart’ or ‘autonomous’ system. In most of the aforementioned literature of SCE and SSHI interfaces, researchers evaluated the power consumption for switching but did not implement the self-powered switching. Two implementations of self-powered switching interfaces were reported by Lallart and Guyomar (2008) and Liang and Liao (2012). In Liang and Liao (2012), the analysis of self-powered SSHI was carried out with the consideration of three aspects, the open circuit voltage, the switching phase lag and the intermediate voltage among different phases. Unlike the ideal SSHI, they found that the self-powered SSHI only outperformed the standard interface when the excitation was above certain level.

Besides, a brand-new concept of “*energy resonance*” was proposed to enhance energy harvesting efficiency in a very recent article by Lallart and Guyomar (2010). Different from the SSHI technique, the electrostatic energy was first extracted from the piezoelectric element and then a part of the harvested energy was injected back to it. The feedback from the output energy could arouse the increase of available

electrostatic energy on the piezoelectric element and therefore lead to an increase of harvested power. Such energy resonance due to the pulsed bidirectional energy flow between the source and the storage circuits was claimed to bring 40 times increase of harvested power compared to the standard interface.

Finally, when evaluating the aforementioned sophisticated interfaces (e.g., SCE and SSHI), many researchers made some assumptions to simplify the model formulation. One of them is the uncoupled assumption, i.e., the coupling is weak enough and the damping induced by the energy harvesting process is negligible. This assumption in mathematical and equivalent circuit modeling of piezoelectric energy harvesters has been discussed in Sections 2.2.2 and 2.2.3. Ottman *et al.* (2002) applied this assumption in their analysis of impedance adaptation interface. Lefeuvre *et al.* (2005) reported 400% power increase by the SCE interface based on the results for constant displacement excitation, which also implied the uncoupled assumption. The in-phase assumption was also frequently applied by researchers, in which the excitation force and velocity of the harvester was in phase (Guyomar *et al.*, 2005; Lefeuvre *et al.*, 2005, 2006; Wu *et al.*, 2009; Wickenheiser *et al.*, 2010). Unfortunately, the above two assumptions are only suitable for weakly coupled systems and for the harvesters vibrating at resonance.

To address more general cases (e.g., weak or strong coupling, resonance or off-resonance vibration), accurate prediction of the performance of various sophisticated interface circuits is necessary. To date, accurate analytical solutions have been derived to predict the performance of standard interface by Shu and Lien (2006) and SSHI interfaces by Shu *et al.* (2007). However, accurate analytical solution for the SCE interface and its applicability have not been reported, which will be derived and discussed in Chapter 5.

2.3.3 Broadband Energy Harvesting

Most of the vibration energy harvesters reported in the literature were designed as linear resonators, which were only efficient in a narrow bandwidth. A slight shift of the excitation frequency would cause a dramatic reduction in their performance. Unfortunately, the majority of the ambient vibrations are frequency-variant or random. Hence, efficiency enhancement of vibration energy harvesting also involves developing techniques to increase the operating bandwidth of harvesters. This section reviews the advances made in the past few years on this issue, including resonant frequency tuning, multimodal energy harvesting and nonlinear techniques with regard to their merits and applicability in different circumstances.

2.3.3.1 Resonant Frequency Tuning

When the excitation frequency is unknown or variant in different operational conditions, the vibration energy harvester with fixed resonant frequency may not achieve optimal power output. Hence, an energy harvester is expected to incorporate a resonance tuning mechanism to increase its functionality in practice. According to Roundy and Zhang (2005), the resonance can be tuned ‘*actively*’ or ‘*passively*’. Active mode requires continuous power input for resonance tuning. While in passive mode, intermittent power is input for tuning and no power required when frequency matching is completed, until the excitation frequency varies again. According to different tuning mechanism, the resonance tuning methods can also be categorized into mechanical, magnetic and piezoelectric methods. A short summary of reported resonance tuning methods are given in Table 2.2.

Mechanical Methods

The resonance of a system can be tuned by changing the stiffness or mass. Leland and Wright (2006), Eichhorn *et al.* (2008) and Hu *et al.* (2007) proposed to apply axial preload to alter the stiffness in their energy harvesting devices, thus tuning the

resonant frequencies. In Leland and Wright's work, their harvester prototype with an attached 7.1g proof mass achieved power output of 300~400 μ W under the excitation of 1g acceleration over the frequency range of 200~250Hz. However, the energy required for the tuning procedure was not addressed. Furthermore, the resonant frequency could only be tuned uni-directionally since only the compressive load was considered. Eichhorn *et al.* (2008) presented another tunable cantilever energy harvester with preload applied at the free end of the beam. Below the fracture limit, a resonance shift from 380Hz to 292Hz was achieved by applying preload from 7N to 22.75N. The quality factor was found reduced, which means damping arose with increased preload. Analytically, Hu *et al.* (2007) derived the governing equations of a piezoelectric cantilever bimorph with an axial preload. The resonance can be adjusted either higher or lower with a tensile or compressive load, respectively. In their model, it was reported that a tensile load of 50N increased the resonance from 129.3 to 169.4Hz while the same compressive load decreased the resonance from 129.3 to 58.1Hz. The forgoing three designs were intended to operate in 'passive' mode, where the device should be manually tuned.

Instead of considering bending mode, some researchers also investigated the tunable resonator working in extensional mode, termed XMR (Morris *et al.*, 2008; Youngsman *et al.*, 2010). The XMR presented by Morris *et al.* (2008) was formed of a seismic mass suspended with two piezoelectric membranes (PVDF). Pre-tension was applied to two membranes by a rigid link and the resonant frequency can be tuned by adjusting the link length. For the fabricated circular XMR prototype, the resonance shift between 80 and 235 Hz can be easily achieved with the change of pre-tension displacement of the rigid link around 1.25mm. However, the capability of self-tuning or sequential tuning during operation of the XMR was not investigated. A similar investigation was pursued by Loverich *et al.* (2008), in which the resonance could be tuned by adjusting the pre-deflection of the circular piezoelectric plate,

which was controlled by the boundary displacement. The resonant frequency could be experimentally varied between 56 and 62Hz in their prototype. More than its tunability, they also focused on the nonlinearities of the pre-deflected plate. For low vibration amplitudes, the system had a high quality factor; while for high vibration amplitudes, the quality factor was reduced due to the nonlinearity. This feature of nonlinear stiffness was claimed an auto protection mechanism, which is important for applications where high sensitivity is required for low vibration levels but mechanical robustness is required for high vibration levels.

Rather than applying the axial or in-plane preload, adjusting the gravity center of the tip mass is another idea to adjust the resonance of a cantilever. Wu *et al.* (2008) presented such a cantilever energy harvester by realizing a movable tip mass. The proof mass consisted of a fixed part and a movable part. The gravity center of the whole proof mass can be adjusted by driving the movable screw. In their prototype, the adjustable resonant frequency range could cover 130~180Hz by tuning the gravity center of the tip mass up to 21mm.

Magnetic Methods

Applying magnetic force to alter the effective stiffness of the harvester is another option for resonance tuning. Challa *et al.* (2008) developed a tunable cantilever harvester in which two magnets were fixed at the free end of beam, while the other two magnets were fixed at the top and bottom of the enclosure of the device. All magnets were vertically aligned so that attractive and repulsive magnetic forces could be generated on each side of the beam. By tuning the distance between the magnets using a spring-screw mechanism, the prototype with a volume of 50cm^3 was tunable over the frequency range of 22~32Hz with the power output of $240\sim 280\mu\text{W}$ operating at an acceleration of 0.8m/s^2 . Given the maximum tuning distance of 3cm, the required energy was 85mJ and it would take around 320s for each tuning procedure,

which means the harvester can only work where the excitation frequency changes slowly. Reissman *et al.* (2009) also demonstrated a tuning technique using variable attractive magnetic force. In this design, the resonance of the piezoelectric energy harvester could be tuned bi-directionally by adjusting a magnetic slider vertically near the free end of the cantilever harvester with another magnet. This is much simplified compared to the design of Challa *et al.* (2008). The total bandwidth of the harvester was 11.38Hz, including the resonant frequency shift from short-circuit to open-circuit condition due to the piezoelectric coupling. In previous two designs, however, no ‘smart’ controller for resonance tuning process was implemented.

Zhu *et al.* (2008) proposed a similar setup as Reissman *et al.* (2009) but they further implemented a smart controller for resonance tuning procedure. The microcontroller, woke up periodically, detected the output voltage of the generator and gave instructions to drive a linear actuator to adjust the distance between the two magnets. In their experimental test, the resonant frequency could be tuned from 67.6 to 98Hz when the distance between the two magnets was changed from 5 to 1.2mm but it could not be further increased when the distance was smaller than 1.2mm. At a constant acceleration of $0.588m/s^2$, the power output of 61.6~156.6 μ W over the tuning range could be achieved. In their design, the energy consumed for the tuning procedure was $2.04mJ \cdot mm^{-1}$. However, experimentally, the linear actuator and microcontroller were still powered by a separate power source for preliminary evaluation. Another drawback was that the control system detected the resonance by comparing the output voltage with a pre-defined threshold. Thus, such system could suffer from inefficient detection of frequency change direction and triggering by mistake if there was certain change in the excitation amplitude.

Piezoelectric Methods

A piezoelectric actuator can alter the stiffness of a system. In fact, the stiffness of the piezoelectric material itself can be varied with different attached shunt electrical load. Hence, piezoelectric transducers provide another option for resonance tuning. It should be emphasized that the notion “*piezoelectric methods*” refers to the methods for resonance tuning using piezoelectric transducers. The energy generation in these methods could be electrostatic, electromagnetic or piezoelectric conversion.

Wu *et al.* (2006) presented a piezoelectric bimorph generator in which the upper piezoelectric layer for tuning purpose by connecting various capacitive loads, while the lower layer for energy harvesting to charge a supercapacitor. The tunable bandwidth of the generator was 3Hz from 91.5 to 94.5Hz, which was much narrower than those achieved by the other aforementioned designs. The real-time tuning system achieved 13.4% and 27.4% average power increase when the device was excited under a chirp and random vibration from 80 to 115Hz, respectively. A microcontroller was utilized to sample the external frequency and adjusted the capacitive load to match the external vibration frequency in real-time, i.e., the device was tuned actively. The continuous power required by the microcontroller system was in μW level.

Peters *et al.* (2009) reported another tunable resonator in which two piezoelectric actuators were used to cause additional hinge moments and thus stiffen the harvester structure. For the large resonator they fabricated, a tuning range of over 30% from an initial frequency of 78 Hz was achieved by a tuning voltage of only $\pm 5\text{V}$. A discrete control circuit, which exploited the phase characteristic of the resonator, was implemented to actively control the tuning procedure. However, the power consumption of around 150mW was supplied externally, which significantly outweighed the harvested power (1.4mW). Thus, low-power CMOS integration of the control circuit was recommended for practical close-loop automatic tuning.

Roundy and Zhang (2005) investigated the feasibility of active tuning mechanism. Via analytical study, they demonstrated that an ‘active’ tuning actuator never resulted in a net increase in power output, i.e., the power required to continuously tune the natural frequency always exceeded the power increase resulting from the frequency tuning. In three experiments, the change in power output ($82\mu\text{W}$) as a result of tuning was significantly smaller than the power needed to continuously drive the piezoelectric actuator ($440\mu\text{W}$), which confirmed the conclusion of their analytical study. They suggested that passive tuning mechanism was worth more attention.

Table 2.2 Summary of reported resonance tuning methods and designs

Author	Methods	Tuning range	Tunability ($\Delta f / f_{ave}$)	Tuning load (force, distance, voltage)	Energy or power for tuning	Automatic controller
Leland and Wright (2006)	Mechanical (passive)	200~250Hz (7.1g tip mass)	22.22%	Up to 65N	—	×
Eichhorn <i>et al.</i> (2008)	Mechanical (passive)	292~380Hz	26.19%	Up to 22.75N	—	×
Hu <i>et al.</i> (2007)	Mechanical (passive)	58.1~169.4Hz	97.85%	-50~50N	—	×
Morris <i>et al.</i> (2008)	Mechanical (passive)	80~235Hz (can be wider)	$\geq 98.41\%$	$\approx 1.25\text{mm}$	—	×
Loverich <i>et al.</i> (2008)	Mechanical (passive)	56~62Hz	10.17%	0.5mm	—	×
Wu <i>et al.</i> (2008)	Mechanical (passive)	130~180Hz	32.26%	21mm	—	×
Challa <i>et al.</i> (2008)	Magnetic (passive)	22~32Hz	37.04%	3cm	85mJ	×
Reissman <i>et al.</i> (2009)	Magnetic (passive)	88~99.38Hz	12.15%	1.5cm	—	×
Zhu <i>et al.</i> (2008)	Magnetic (passive)	67.6~98Hz	36.71%	3.8mm	$2.04\text{mJ}\cdot\text{mm}^{-1}$	√
Wu <i>et al.</i> (2006)	Piezoelectric (active)	91.5~94.5Hz	3.23%	—	μW level (controller)	√
Peters <i>et al.</i> (2009)	Piezoelectric (active)	66~89Hz (actuator PL140)	29.68%	$\pm 5\text{V}$	150mW (discrete controller)	√
Roundy and Zhang (2005)	Piezoelectric (active)	64.5~67Hz	3.80%	5V	440 μW	×
Wischke <i>et al.</i> (2010)	Piezoelectric (semi-passive)	20Hz (10mm long electrode)	$\approx 6.7\%$	-65~+130V	200 μJ	×

Wischke *et al.* (2010) designed a tunable electromagnetic harvester in which the maximum tunable range covered 267~323Hz by applying the voltage -100V~+260V to the piezoelectric bimorph actuator, equivalent to 18% of the open circuit resonant frequency of 299Hz. However, once the control voltage was disconnected, the frequency drifted away from the initial adjusted value due to the leakage of the

piezoelectric actuator. This drifting was more intense for high control voltages ($>130\text{V}$). The charge had to be refreshed periodically to maintain the desired resonant frequency. Hence, the tuning mechanism was defined as '*semi-passive*' by Wischke *et al.* (2010), different from the '*passive*' principle in which the charge on the piezoceramics and accordingly the adjusted frequency would remain constant after disconnecting the control voltage. In order to reduce the frequency drifting and the energy required, the tuning range was suggested to be limited to 20Hz by applying a voltage of $-65\sim+130\text{V}$ with a shorter electrode of 10mm in length. Hence, given the power output of $50\mu\text{W}$, 20% circuit efficiency and $200\mu\text{J}$ required for each tuning procedure, the resonance of the harvester could be tuned across 20Hz in every 20s .

2.3.3.2 Multimodal Energy Harvesting

In practice, vibration energy harvester are multiple degree-of-freedom (DOF) systems or distributed parameter systems. If multiple vibration modes of the harvester are utilized, useful power can be harvested over multiple frequency spectra, i.e., wider bandwidth can be covered for efficient energy harvesting. Such techniques are termed as "*multimodal energy harvesting*". Some researchers have reported the theoretical investigation of exploiting the translation and rotation modes of a rigid body (Jang *et al.* 2010) or multiple translation modes of lumped mass systems (Aldraiheim and Baz, 2011) for multimodal energy harvesting. However, in practice, multimodal energy harvesters are usually implemented by exploiting multiple bending modes of a single beam or by exploiting a cantilever array configuration.

Exploiting Multiple Bending Modes of A Single Beam

Roundy *et al.* (2005) first proposed the idea of multiple-DOF system incorporating multiple proof masses attached on a beam to achieve wider bandwidth. In the reported research attempts to exploit multiple modes of a cantilever energy harvester, the first two modes were widely pursued for energy harvesting (in other words, a 2DOF

vibration energy harvester). Arafa *et al.* (2011) presented a 2DOF piezoelectric cantilever energy harvester in which one proof mass served as a dynamic magnifier. It magnified the power output but could not achieve two close working frequencies unless an unpractical huge magnifier was employed. Ou *et al.* (2010) theoretically modeled a two-mass cantilever beam for broadband energy harvesting. Although two useful modes were obtained, they were quite far apart at 26Hz and 174Hz, respectively. Tadesse *et al.* (2009) presented a cantilever harvester integrated with hybrid energy harvesting schemes. The electromagnetic scheme generated high output power for the first mode, while the piezoelectric scheme was efficient for the second mode. However, the first resonance and the second resonance of such prototype were 20Hz and 300Hz, respectively. Erturk *et al.* (2009b) introduced an L-shaped piezoelectric cantilever harvester in which the second natural frequency could approximately double the first and the response at both resonances with effective magnitudes was achieved.

When developing a multimodal energy harvester based on a single beam, having its multiple modes close to each other with significant magnitude is rationally preferable for practical random or frequency-variant vibrations. Unfortunately, most of the previous designs failed to achieve this. Besides, power density is also an important evaluation factor of a multimodal energy harvester. Although Arafa *et al.* (2011) achieved significant power increase and two close working frequencies, the huge magnifier added may deteriorate the system performance in terms of power density. Moreover, energy extraction and management interface circuit may be more complex for multimodal energy harvesters. For example, when exploiting multiple modes of a cantilever harvester, charge cancellation should be avoided, requiring discontinuous electrodes on the piezoelectric beam (Erturk and Inman, 2008a). In Tadesse *et al.* (2009), drastic difference in matching load for electromagnetic and piezoelectric schemes required separate energy extraction interfaces.

Exploiting Cantilever Array Configuration

Different from the discrete bandwidth corresponding to the multiple modes of a single beam, multiple cantilevers or cantilever array integrated in one energy harvesting device can easily achieve continuous wide bandwidth if the geometric parameters of the harvester are appropriately selected. Another advantage of array configuration is the enhanced flexibility of energy harvester design in some specific applications. For instance, given a limited area, a large piece of cantilever harvester can be divided into multiple pieces and arranged vertically to achieve the same power output.

Kim *et al.* (2011) developed a 2DOF piezoelectric energy harvesting device in which two piezoelectric cantilevers are connected by a rigid mass. Translation and rotation vibration modes of the rigid mass were exploited and the two natural frequencies could be designed to be very close to each other. The 2DOF device exhibited a 280% increase in bandwidth at a voltage level of 55V/g compared to a conventional single DOF device in their experiment. Yang and Yang (2009) conducted a theoretical study on wideband energy harvesting performance by using connected/coupled cantilever bimorphs. A spring was placed to connect the proof masses at the free end of two cantilever bimorphs. Two close modes and thus wider bandwidth could be achieved as compared to a single-beam harvester. They found the amplitude and location of the resonances were sensitive to the connection spring and the proof masses

Except for the previous two designs, most of the reported cantilever array configurations in the literature were composed of quasi-decoupled cantilevers. Since each cantilever is regarded as one substructure of the harvester, the first mode of each cantilever is one of the vibration modes of the harvester. Shahriz (2006), Xue *et al.* (2008) and Ferrari *et al.* (2008) reported similar cantilever array configurations, which comprised piezoelectric cantilevers with various system parameters (e.g., lengths, thickness, proof masses). They were capable of resonating at various frequencies

without the need of adjustment. By properly selecting the system parameters, the entire device could be designed to harvest energy over a wider frequency range. Xue *et al.* (2008) found that the bandwidth of their bimorph cantilever array configuration could be tailored by choosing an appropriate connection pattern (mixed series and parallel connections). Connecting multiple bimorphs in series could broaden the bandwidth. Comparing the single bimorph harvester and a 10-bimorph array configuration, their numerical results showed that not only the power magnitude of the energy harvesting system was increased but also the bandwidth (output power $> 10\mu\text{W}$) was widened from (97, 103)Hz to (87, 115)Hz. Besides, the bandwidth could be shifted to the dominant frequency range by changing the number of bimorphs in parallel. This shift was due to the change in the electrical boundary condition when increasing or decreasing bimorphs in parallel. Ferrari *et al.* (2008) devised a multi-frequency piezoelectric converter, which comprised three piezoelectric bimorph cantilevers with the same dimensions of $15\text{mm} \times 1.5\text{mm} \times 0.6\text{mm}$ but with different tip masses ($m_1=1.4\text{g}$, $m_2=0.7\text{g}$, $m_3=0.6\text{g}$). Capacitor charging behavior was investigated of such configuration at the excitation acceleration of around $1g$. When the voltage across the storage capacitor reached the upper and lower thresholds, the circuit would connect and disconnect to deliver the energy to power a wireless sensor that transmitted the measurement. It was observed that under resonant excitation, i.e., at either f_1 , f_2 , or f_3 (the fundamental resonant frequencies of three cantilevers), the corresponding single cantilever in the array could alone trigger the transmission, but a single cantilever could not do so at off-resonance frequency f_4 . Conversely, with the complete converter array, the converted energy was high enough to trigger the transmission for all the tested frequency, including f_4 . Besides, the shorter switching time was obtained using the converter array rather than a single cantilever. They claimed that the wider bandwidth and improved performance were worth the modest increase in size of the proposed array device.

Broadband energy harvesters with cantilever array configuration were also implemented compatibly with current standard MEMS fabrication techniques. Liu *et al.* (2008) implemented such a MEMS-based piezoelectric cantilever array harvester. In their experimental test, the phase difference of voltage output from each cantilever was observed, which impaired the electrical accumulation of three cantilevers. Thus, the DC voltage across the capacitor after rectification was only 2.51V, and the maximum DC power output was about 3.15mW. To address this problem, separate rectifier for each cantilever was required, which made the total DC voltage increased to 3.93V and the maximum DC power output to 3.98μW. With the wider bandwidth 226~234Hz of the prototype and the improved output from the generator array, such device was claimed to be promising in applications of ultra-low-power wireless sensor networks. However, the more complicated rectification circuit may consume significantly more electrical energy in these MEMS-scale devices especially at low-level excitations. Low-voltage-drop rectification techniques discussed in Section 2.3.2.3 may alleviate this problem in such cases.

2.3.3.3 Nonlinear Configurations

Section 2.3.3.1 presents several resonance tuning techniques using magnets (Challa *et al.*, 2008; Reissman *et al.*, 2009; Zhu *et al.*, 2008). Actually the magnets introduced not only a change in the linear stiffness, but also a nonlinear stiffness. The nonlinear behavior is obvious when the harvester experience oscillation with significant amplitude. Such nonlinearity can also benefit broadband energy harvesting.

As reported in the available literature, the nonlinearities of energy harvesters are considered from two perspectives, i.e., nonlinear stiffness (Ramlan *et al.*, 2010; Mann and Sims, 2009; Stanton *et al.*, 2009, 2010b; Lin *et al.*, 2010; Erturk *et al.*, 2009a; Marinkovic and Koser, 2009; Soliman *et al.*, 2009) and nonlinear piezoelectric coupling (Triplett and Quinn, 2009; Stanton *et al.*, 2010a). Compared to the nonlinear

piezoelectric coupling, which results from the manufacturing process of piezoelectric material, the nonlinear stiffness of a harvester is relatively easier to achieve and control. This section reviews recent advances in designing broadband energy harvesters with nonlinear stiffness.

The dynamics of a general oscillator can be described as

$$\ddot{x} = -\frac{dU(x)}{dx} - c\dot{x} + f(t) \quad (2.17)$$

where x represents the oscillator position; c represents the viscous damping; $f(t)$ is the ambient vibration force; and $U(x)$ is the potential function. If an electromagnetic generator is considered, c includes the viscous damping caused by electromagnetic coupling. For a piezoelectric generator, the damping caused by piezoelectricity cannot be modeled as a viscous damper (Erturk and Inman, 2008b) and Equation (2.17) should be modified by adding a coupling term as

$$\ddot{x} = -\frac{dU(x)}{dx} - c\dot{x} + \kappa V + f(t) \quad (2.18)$$

where κ represents the electromechanical coupling coefficient and V is the voltage on the electrical load. The circuit equations for the piezoelectric and the electromagnetic harvesters are quite different due to differences in their internal impedances. They are not given here but they can be readily found in the literature related to electromagnetic and piezoelectric transductions, such as El-Hami *et al.* (2001) and Erturk *et al.* (2009a). Usually, the potential energy function $U(x)$ can be considered in a quadratic form as (Cottone *et al.*, 2009; Gammaitoni *et al.*, 2009):

$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 \quad (2.19)$$

The potential function $U(x)$ is symmetric and bistable for $a > 0$, and monostable for $a \leq 0$. In the bistable case, two minima at $x_m = \pm\sqrt{a/b}$ are separated by a barrier at $x=0$. For both monostable and bistable cases, the benefits on improving the bandwidth of the vibration energy harvester are discussed in the following parts.

Monostable Nonlinear Configuration

Substituting Equation (2.19) into (2.17), we can obtain the forced Duffing's equation, which is widely used in modeling nonlinear energy harvester (Ramlan *et al.*, 2010; Mann and Sims, 2009; Moehlis *et al.*, 2009),

$$\ddot{x} + c\dot{x} - ax + bx^3 = f(t) \quad (2.20)$$

For $a \leq 0$, it can be used to describe a monostable system. $b > 0$ determines a hardening system response, while $a < 0$ a softening response.

Ramlan *et al.* (2010) investigated the hardening mechanism of monostable energy harvesters. By numerical and analytical studies, it was found that ideally, the maximum power harvested by a system with a hardening stiffness was the same as that by a linear system, regardless of the degree of nonlinearity. However, this might occur at a different frequency depending on the degree of nonlinearity. Mann and Sims (2009) presented a design for electromagnetic energy harvesting from the nonlinear oscillations of magnetic levitation. At low excitation level, the frequency response was found similar to the response of a linear system. While at high excitation level, the response curve was bent to the right. Thus, relatively large amplitudes persisted over a much wider frequency range. However, such a device using hardening mechanism could only broaden the frequency response in one direction (the peak response shifts to the right). Stanton *et al.* (2009) proposed another monostable piezoelectric harvester, which consisted of a piezoelectric beam with a magnetic end mass interacting with the field of oppositely poled stationary magnets. By tuning the nonlinear magnetic interactions around the end mass, both hardening and softening responses might occur, which allowed the frequency response to be broadened bi-directionally. In the experimental validation, a linearly decreasing frequency sweep was performed for the softening case. Different from Ramlan *et al.* (2010), their results showed that not only a wider bandwidth but also a better

performance could be obtained by the monostable configuration, as compared to the linear configuration (with stationary magnets removed).

Previous monostable designs have a larger bandwidth due to the shift in the resonant frequency. This advantage imparted in the nonlinearity depends on realizing the high-energy attractor (Stanton *et al.*, 2009). A linearly decreasing or increasing frequency sweep can capture the high-energy attractor, and hence improve the bandwidth for the softening and hardening cases, respectively. Unfortunately, such conditions cannot be guaranteed in practice. Certain means of mechanical or electrical disturbance or perturbation is required once the nonlinear devices enter low-energy orbits; otherwise little power can be harnessed. Previous work did not address the momentary perturbation if low-energy branch manifests and the requisite actuation energy. Daqaq (2010) demonstrated that under White Gaussian excitation environment, the hardening-type nonlinearity failed to provide any enhancement of output power over the typical linear harvesters. Under Colored Gaussian excitations, the power even decreased with hardening-type nonlinearity. This suggested that the monostable configuration may be only applicable for frequency sweep excitations.

Bistable Nonlinear Configuration

For $a > 0$, Equation (2.20) describes a bistable nonlinear system. Compared to the monostable configuration, more research efforts have been devoted to investigate the bistable configurations subjected to either periodic forcing or stochastic forcing.

A periodically forced oscillator can undergo various types of large-amplitude oscillations, including chaotic oscillation, large-amplitude periodic oscillation and large-amplitude quasi-periodic oscillation (Moehlis *et al.*, 2009). Moehlis *et al.* (2009) presented one example of bistable oscillator under periodic forcing with the following governing equation

$$\ddot{x} + 0.1\dot{x} - x + x^3 = \cos(\omega t) \quad (2.21)$$

In the bifurcation analysis (plotting the instantaneous x whenever $\dot{x} = 0$ for each ω by getting rid of transients), they found that large-amplitude response ($x > 1$ and $x < -1$) occurred over a wide frequency range and even extended to very low frequencies.

One physically realizable energy harvester with bistable configuration was proposed by Ramlan *et al.* (2010), termed *snap-through* mechanism. The setup consisted of two linear oblique springs connected to a mass and a damper, yielding a non-linear restoring force. This mechanism has the effect of steepening the displacement response of the mass as a function of time, resulting in a higher velocity for a given input excitation. Numerical results revealed that this mechanism could provide much better performance than the linear mechanism when the excitation frequency was much smaller than the natural frequency.

Bistable nonlinear stiffness can also be introduced by magnets. Erturk *et al.* (2009a) pursued such method in designing a broadband piezomagnetoelastic generator. The device consisted of a ferromagnetic cantilever beam with two piezoceramic layers attached at the root for energy generation and with two permanent magnets near the free end. For an initial deflection at one of the stable equilibrium, the voltage response could be chaotic strange attractor motion or large-amplitude periodic motion (limit cycle oscillation) under small or large harmonic base excitation. The large-amplitude periodic motion could also be obtained under small excitation level by simply applying a perturbation or equivalently an initial velocity condition. By such, the large-amplitude response could be obtained at *off-resonance* frequencies and thus produce broadband performance, advantageous over the linear piezoelectric configuration (with two magnets removed). However, for small excitation amplitude, actuation energy required to perturb the beam was not investigated in their work.

For a bistable system, stochastic forcing can also induce transitions between the stable equilibriums of the system, giving large-amplitude oscillations. Cottone *et al.* (2009) realized a piezoelectric inverted pendulum by using the bistable mechanism (polar opposing magnets with a small distance Δ). The random vibration made the pendulum swing with small oscillations around each equilibrium or large excursions from one to another. However, for extremely small Δ , the pronounced potential energy barrier confined the pendulum swing within one potential well. For specific Δ and noise level, the deflection of the pendulum x_{rms} reached a maximum and hence the maximum power could exceed by 4~6 times of the power obtained when the magnets were far away (quasi-linear). Ferrari *et al.* (2010), Lin and Alphenaar (2010), Andò *et al.* (2010) and Stanton *et al.* (2010b) extended this idea to study the energy harvesting performance of bistable cantilevers with repulsive magnets under wide-spectrum vibrations. From these studies, the critical issue for broadband energy harvesting involves how to enable the harvester to readily transit between the two stable states, which is dependent on the excitation amplitude, frequency and the extent of nonlinearity. For the bistable pendulum and a more general bistable dynamical system, Cottone *et al.* (2009) concluded that: (i) the raising of the response x_{rms} is mainly due to the growth of the separation between the two minima of the potential function; and (ii) the drop of x_{rms} is mainly due to decrease in the jump probability caused by the increase of the potential barrier height.

In order to increase the probability of transition between the potential wells and thus to further enhance the performance of a bistable system, some researchers (McInnes *et al.*, 2008; Formosa *et al.* 2009) proposed to exploit the phenomenon of stochastic resonance. The stochastic resonance can occur if the dynamics of the system is forced such that the potential barrier oscillates, and this forcing matches with the mean time between transitions — inverse Kramer's rate (Wellens *et al.*, 2004). For a beam clamped at both ends, the SDOF bistable model is similar to the *snap-through* setup

by Ramlan *et al.* (2010), except that boundaries can be modulated at frequency ω and hence the potential barrier is modulated. Thus the parametrically forced dynamics of the system is defined as (McInnes *et al.*, 2008)

$$\ddot{x} + c\dot{x} - \mu(1 - \eta \cos(\omega t))x + x^3 = Q(t) \quad (2.22)$$

where x is the non-dimensional coordinate; c is the damping coefficient; μ is a measure of the compressive load acting on the beam; η and ω are the magnitude and frequency of forcing for modulation, respectively; and $Q(t)$ is the external noise. With this model, McInnes *et al.* (2008) demonstrated that the properly tuned system in stochastic resonance by forcing (i.e. the forcing matched with inverse Kramer's rate) apparently experienced larger amplitude vibrations than those of the unforced mechanism, which was confined in a single potential well. Thus, significantly more energy was obtained. However, if the system was un-tuned, the net energy generated by the forcing mechanism could be less than the unforced mechanism, considering the energy consumed for forcing the boundaries to oscillate.

2.3.3.4 Comments on Broadband Energy Harvesting Techniques

This section provides the comments on the previously reviewed literature on broadband energy harvesting techniques and Table 2.3 gives a short summary.

- ❖ *Resonance tuning techniques.* As compared in Table 2.3, generally, the mechanical and magnetic tuning methods can achieve larger tunability as compared with the piezoelectric tuning methods. However, most of them are difficult to implement during operation and the system parameters should be adjusted manually. Resonance tuning by piezoelectric transducers is easier to implement by applying voltage to the actuators or operating the control circuit to vary the shunt damping load. Thus, the device can be designed for automatic tuning during operation. However, the tunability by piezoelectric methods is more limited. Another weakness is that the power required for tuning in most of the reported devices outweighed the power harvested in the active tuning mode.

The device in Wu *et al.* (2006) was an exception since the principle behind was piezoelectric shunt damping in which power is only required to continuously switch the shunt electrical load to change the system stiffness, rather than to apply the voltage to the actuator which usually consumes more power. All the methods can be further categorized into active and passive tuning. Passive tuning requires less power input to periodically detect and change the frequency, which is suitable when the excitation frequency varies infrequently. However, if the harvested power can afford the continuous power required for tuning, an active tuning harvester can work under the excitation with constantly changing frequency or random excitation.

- ❖ *Multimodal energy harvesting.* Multimodal energy harvesting does not require any tuning efforts and hence is much easier to implement. It can be implemented with a single beam by exploiting its multiple modes or with cantilever array integrated in one device (the first mode of each cantilever is regarded as one of the vibration modes of the entire device). The former provides a discrete bandwidth in the mostly reported literature while the latter can be conveniently designed to cover continuous bandwidth. Multimodal energy harvesting increases the bandwidth accompanied by the increased volume or weight of the device. Hence, with the known dominant spectrum of the ambient vibration, the harvester should be carefully designed with the proper parameters so that it can cover the targeted bandwidth with least sacrifice of the power volume density or power weight density. Additionally, in multimodal techniques, a more complex interface circuit is required to avoid mode-shape dependent voltage cancellation in a continuous beam or the cancellation due to the phase difference between cantilevers in array configurations.
- ❖ *Nonlinear techniques.* For the monostable nonlinear configuration, a linearly decreasing or increasing frequency sweep for softening or hardening case respectively can capture the high-energy attractor motion and hence acquire

large-amplitude response over a wide bandwidth. Since multi-value and jump phenomenon near resonance occur with the increased nonlinearity, a mechanism should be implemented to perturb and drive the system into high-energy orbits in case the system vibrates into a low-energy branch. The bistable nonlinear configurations conditionally achieve improved energy harvesting performance under both periodic and stochastic forcing. Under high level periodic forcing or low level forcing with proper perturbation, the bistable harvester can be driven into high-energy orbits. Hence, it can outperform the linear device over a wide bandwidth. Under stochastic forcing, the bistable system also shows significant performance improvement when the system parameters are properly selected. The performance of the bistable harvester can be further improved by exploiting stochastic resonance. However, the mechanisms for perturbation and stochastic resonance require external energy input and are quite difficult to implement.

Table 2.3 Summary on merits, weakness and applicability of various broadband energy harvesting techniques

Methods		Merits, weakness, and applicability
Resonant frequency tuning	Active tuning	<ul style="list-style-type: none"> • Usually implemented by piezoelectric methods. • Limited tunability. • Cost of tuning mostly outweighs the power harvested. • Easy to implement automatic and continuous tuning during operation. • Applicable when excitation frequency varies frequently or for random excitation, given affordable power consumption for tuning.
	Passive tuning	<ul style="list-style-type: none"> • Mostly by mechanical and magnetic methods. • Relatively large tunability. • Difficult to achieve automatically and during operation. • Applicable when excitation frequency varies infrequently.
Multimodal energy harvesting		<ul style="list-style-type: none"> • Much easier to implement than resonance tuning techniques. • Exploits multiple bending modes of a single beam or a cantilever array configuration. • Should be designed with proper parameters to cover the targeted frequency range with the least sacrifice of power density. • Requires complex interface circuit to avoid cancellation.
Nonlinear techniques	Monostable	<ul style="list-style-type: none"> • Applicable for excitations with slow and proper frequency sweep. • Requires perturbation when the harvester enters low-energy orbits.
	Bistable	<ul style="list-style-type: none"> • Applicable for high-level periodic excitation. • Applicable for low-level periodic excitation but with perturbation mechanism to drive the harvester into high-energy orbits. • Applicable for stochastic excitation with properly designed potential barrier at given excitation level. • Further performance enhancement achievable by implementing stochastic resonance, which requires proper actuation and extra power.

2.3.4 Frequency Up-Conversion in Ultra-Low-Frequency Scenarios

In many practical cases, ambient vibrations occur at ultra low frequency such as those caused by wave heaving motions or human motions, while the resonance of a harvester may possess high resonant frequency. Hence, another aspect of efficient energy harvesting involves amplifying the source vibration frequency.

One idea for frequency up-conversion is to use a mechanical gear, which can be found in the design of a windmill by Priya (2005). For a given rotation speed n of the windmill and m pieces of piezoelectric bimorph blades and stoppers, each blade can go through $(m \times n)$ Hz oscillation. Actually, in the piezoelectric windmill design, another frequency up-conversion mechanism was included, which was presented as the concept of two-stage energy harvesting design by Rastegar *et al.* (2006) or similarly termed as a “mechanical rectification approach” by Tieck *et al.* (2006). When a mass with the tooth passes and impacts on the cantilever tip, the cantilever will be excited and then freely vibrates at its natural frequency. Thus, the low-frequency vibration of the primary vibrating unit (i.e. the mass) can be transferred to high frequency vibrations of the secondary vibration units (i.e. the piezoelectric cantilevers), hence providing a frequency-robust energy harvesting solution in low-frequency excitation scenarios. The primary challenge in such a design is the mechanical loss from impact. To address this issue, pairs of attractive magnets (Rastegar and Murray, 2009; Murray and Rastegar, 2009) or ferromagnetic structures (Wickenheiser and Garcia, 2010) can be exerted, thus the mechanical energy could be transferred without mechanical impact. However, according to the literature, only attractive magnetic forces have been considered so far.

The frequency up-conversion technique is more significant and desirable in micro-scale energy harvesting devices, in which the resonant frequency of the structure can be at the level of 1kHz, while the ambient vibration frequency is

generally below 100Hz. In such cases, the resonance tuning or multimodal energy harvesting techniques fail to work due to the drastic difference between the excitation frequency and the MEMS resonant frequency.

Lee *et al.* (2007) developed one novel micro vibration energy harvester using frequency up-conversion technique. It consists of a sharp probe, micro ridges, a micro slider and a piezoelectric bimorph cantilever. The micro ridges were attached onto a micro slider mechanism. A probe tip attached on the edge of a piezoelectric cantilever traveled along the ridges to vibrate the cantilever. In their experiment, such device could generate $225\mu\text{W}/\text{cm}^2$ at 60Hz, which was substantially larger than the conventional resonance approaches. Kulah and Najafi (2008) presented another design of micro electromagnetic generator using frequency up-conversion. The top resonator with a large permanent magnet could easily deflect in the 1~100Hz range due to its large size and soft spring. The bottom resonator, a cantilever beam (or array of beams) with a magnetic tip, has a higher resonant frequency. A coil attached on the beam provided the electromagnetic current induction. As the top magnet resonated in response to external low-frequency vibration, the cantilever could be caught or released at certain points, thus resonating at its high natural frequency (1~20kHz). Kulah and Najafi (2008) expected that a maximum instantaneous power of $3.97\mu\text{W}$ could be generated from a single cantilever vibrating at 25Hz in vacuum. Within the overall generator area of 4mm^2 , using series-connected cantilevers could further increase the power level. For the same ambient vibration frequency, the generated power by using such up-conversion technique was claimed to be more than two orders larger than a conventional large mass/coil system with the same size (14nW).

2.4 Chapter Summary

This chapter reviews the state-of-the-art energy harvesting technologies with a focus on piezoelectric energy harvesting. First, the available mathematical and equivalent

circuit modeling methods for piezoelectric energy harvesters are reviewed. Moreover, the existing schemes for efficiency enhancement of piezoelectric energy harvesting are covered, including advanced piezoelectric and structural configurations, sophisticated energy harvesting interfaces, broadband energy harvesting techniques and frequency up-conversion techniques. However, these modeling methods and efficiency enhancement schemes have their respective limitations or applicable conditions. More efforts are definitely required in these areas before piezoelectric energy harvesting technologies can be widely deployed in practice for wireless sensing and portable electronic applications.

CHAPTER 3 UNCOUPLED MODELING AND EFFICIENCY ENHANCEMENT OF ENERGY HARVESTING SYSTEM USING MFC

3.1 Introduction

Compared to the traditional piezoelectric material PZT, the macro-fiber composite (MFC, Smart Material Corp.) is capable of withstanding large deformation in harsh environments and conformability to curved surfaces, which enables them to harvest energy from harsh vibration environment with long lifespan and little brittle risk. Although MFC was claimed less efficient compared to PZT (Sodano *et al.*, 2005; Shen *et al.*, 2007), its capability for energy harvesting is worth further investigating and efforts for efficiency improvement since MFC is expected to be more applicable than PZT in real applications due to its attractive flexibility and durability.

This chapter investigates the capability of energy generation by MFC in a weakly coupled system. A prototype of energy harvesting system with MFC patches bonded on a host structure (a large piece of aluminum cantilever beam) is developed. Due to the weak electromechanical coupling, the dynamics of the beam is nearly not affected by the energy harvesting procedure, corresponding to the uncoupled distributed parameter model discussed in Section 2.2.2. From the viewpoint of equivalent circuit representation, the MFC patches can be modeled as an ideal voltage source in series with the internal capacitance (uncoupled equivalent circuit model discussed in Section 2.2.3).

Firstly, the open circuit voltage from MFC patches is estimated using the finite element method (FEM) and validated by experiment. With the obtained open circuit voltage, the procedure of energy storage in a capacitor is simulated using the

electronic circuit simulator (Multisim 10.0, <http://en.wikipedia.org/wiki/Multisim>). The performances of a single piece of MFC and two pieces of MFC in parallel and in series are compared. These circuit simulation results are also validated by experiment. Subsequently, with the validated FEM and circuit simulation model, parametric study is conducted to evaluate the effects of various properties of MFC on energy harvesting and thus to optimize the MFC configuration.

3.2 Energy Harvesting System Using MFC

The energy harvesting system is composed of a large piece of aluminum cantilever beam, one piece of P1-type MFC (model:M2814-P1) as actuator and two pieces of P2-type MFCs (model:M2814-P2) as bonded harvesters, an energy harvesting and power management circuit EH300A (from Advanced Linear Devices, INC.), a small LED bulb and the power supply to the actuator. The entire system is shown in Figure 3.1. It should be mentioned that the P1-type MFC actuator is used for a good controllable excitation of the system. A harmonic actuation voltage $500\sin(\omega t)$ is applied to the actuator in the experiment.

Figure 3.1 Prototype of MFC energy harvesting system

The P1-type MFC utilizes d_{33} piezoelectric effect (Figure 3.2(a)) to serve as the actuator to excite the beam. The vibration amplitude of the beam can be tuned by altering the voltage input to the actuator. Two P2-type MFCs, which utilize the d_{31} effect (Figure 3.2(b)), are attached beside the P1-type MFC, serving as harvesters. The reasons for choosing P1-type MFC as actuator rather than harvester are: (1) the operational voltage of P1-type ranges from -500V to +1500V, referring to the

datasheet of MFC (<http://www.smart-material.com/MFC-product-main.html>), which enables it suitable for actuation, while the operational voltage of P2-type is only $-60V \sim +360V$; and (2) Although the piezoelectric coefficient d_{33} is larger than d_{31} , the significantly smaller capacitance of P1-type MFC ($C^S \approx 5nF$) compared to P2-type MFC ($C^S \approx 25nF$) due to the interdigital electrode configuration means much higher internal impedance and thus limits the current output. This is not favorable for charging capacitors or batteries. Two pieces of P2-type MFC harvesters are used to test the efficiency of the energy harvesting system for three cases, i.e., one single MFC, two MFCs electrically connected in series and in parallel.

Figure 3.2 (a) P1-type and (b) P2-type MFC (figure from Smart Material Corp.)

Besides, an EH300A module is used as energy harvesting circuit (Figure 3.3(a)). When an energy source starts to inject energy into the inputs of EH300A in the form of charge impulses, these charge packets are collected and accumulated in an internal storage capacitor bank (6.6mF). Figure 3.3(b) shows the typical waveform of voltage across the capacitor bank in EH300A (<http://www.aldinc.com/pdf/EH300ds.pdf>). Initially, the voltage across the capacitor bank, V_{DC} , starts at 0.0V. When V_{DC} reaches the upper threshold V_H (3.6V), the output of module (V_P) is enabled to deliver power to a load. In this study, the load is an LED bulb. As power is drawn by the LED from the capacitor bank during the time duration t_3 , V_{DC} decreases. When the lower threshold V_L (1.8V) is reached, V_P switches off and stops supplying any further power. The charging cycle will restart until V_H is reached again. During each charging cycle that V_{DC} increases from V_L to V_H or energy is delivered to the load, around 30mJ

energy (taken from the datasheet of EH300A) is accumulated or released. This level of V_H and energy accumulated is usable for most wireless electronics. In the experiment, the time t_2 of charging the capacitor bank in Figure 3.3(b) is our concern, as it reflects how fast the required energy is harvested.

Figure 3.3 (a) EH300A and (b) the voltage waveform across the inside capacitor bank (figure from Advanced Linear Devices, INC.)

3.3 Finite Element Modeling

The finite element model is established in the commercial code ABAQUS 6.7 to calculate the open circuit voltage from MFC. All the material and geometric parameters of the energy harvesting system are listed in Table 3.1. The geometric configuration is shown in Figure 3.4(a) and the mesh of finite element model is shown in Figure 3.4(b). The epoxy and aluminum beam are modeled using the reduced second order solid element C3D20R, and the MFC actuator and harvesters are modeled using the piezoelectric element C3D20RE. Besides, it should be mentioned that only the active volume of the MFC transducer is modeled in the finite element model (i.e. the embedded piezoelectric material PZT5A).

Figure 3.4 (a) Geometric configuration and (b) mesh of finite element model

Table 3.1 Material and geometric parameters of MFC energy harvesting system

Item	MFC (active volume with PZT5A)	Epoxy Adhesive Layer	Aluminum Substrate
Dimensions (mm×mm×mm)	28×14×0.2 (L _m ×W _m ×t _m)	28×14×0.1 (L _e ×W _e ×t _e)	350×62×1.5 (L _b ×W _b ×t _b)
Density (kg/m ³)	7750	2200	2700
Elastic constants (GPa)	E ₁ =E ₂ =60.98, E ₃ =53.19 ν ₁₂ =0.35, ν ₁₃ =ν ₂₃ =0.44, G ₁₂ =G ₁₃ =21.05, G ₂₃ =22.57	E=0.1 ν=0.38	E=70 ν=0.35
Dielectric constant (F/m)	ε ₃₃ ^s =1.327e-8	—	—
Piezoelectric constant (10 ⁻¹² m/V)	d ₃₃ = 440, d ₃₁ = -185	—	—

3.3.1 Electrode Modeling

The entire top and bottom surfaces of P2-type MFC are covered by electrode. In finite element analysis (FEA), the equation constraint (ABAQUS documentation, 2007) should be imposed on the electrical potential degrees of freedom (DOFs) of the surface nodes to ensure a uniform potential, which simulates the real electrodes. For the actuator P1-type MFC, it is difficult to directly model the interdigital electrodes. For the convenience of modeling and analysis, the P1-type MFC will be equivalently converted to the P2-type MFC so that the top surface and bottom surface are modeled as electrodes. The actuation voltage on the P1-type MFC actuator is applied by setting the potential on the bottom electrode to be zero and imposing the sinusoidal potential on the top electrode. The equivalent conversion should ensure that the piezoelectric and electrical properties of the P1-type MFC keep unchanged. For the piezoelectric property, when applying the same voltage, the strain generated should be the same:

$$d'_{31} \cdot \frac{V}{t_m} = d_{33} \cdot \frac{V}{e_m} \quad (3.1)$$

where $t_m=0.2\text{mm}$ and $e_m=0.5\text{mm}$ are the thickness and the distance between two neighboring interdigital electrodes of P1-type MFC, respectively. Hence, the equivalent piezoelectric coefficient d'_{31} applied is calculated as $1.76\text{e-}10\text{C/N}$. Besides,

the dielectric constant $1.327\text{e-}8\text{F/m}$ is changed to $0.265\text{e-}8\text{F/m}$ after conversion such that the capacitance is still $\sim 5\text{nF}$ as that of the original P1-type MFC.

3.3.2 Rayleigh Damping

A common type of damping used in structural analysis is to assume that the damping matrix is proportional to the mass and stiffness matrices. Due to the orthogonality properties of the mass and stiffness matrices, we can write the damping ratio as,

$$\xi_i = \frac{1}{2} \left(\frac{\alpha}{\omega_i} + \omega_i \beta \right) \quad (3.2)$$

where α and β are the unknown Rayleigh damping coefficients; and ω_i is the frequency of the i^{th} vibration mode. To obtain α and β , the damping ratios for the 1st and 2nd resonance modes should be experimentally determined. Usually, the log decrement method (Meirovitch, 2005) can be utilized to measure the damping ratio at resonant frequencies,

$$\xi = \frac{1}{2\pi \cdot n} \ln \frac{A_1}{A_{n+1}} \quad (3.3)$$

where A_1 and A_{n+1} are the 1st and the $(n+1)^{\text{th}}$ amplitudes in the attenuation voltage curve recorded by a digital multimeter (Agilent 34410A). In the experiment, the beam is tuned to vibrate at 1st and 2nd resonances to determine the two damping ratios. By shutting down the power supply for the actuator, we can record the free vibration responses of the system. Using the attenuation voltage curves and applying Equation (3.3), the damping ratios for the 1st and 2nd modes are obtained as $\zeta_1=0.0064$ and $\zeta_2=0.0077$. Applying ζ_1 and ζ_2 to Equation (3.2), the Rayleigh damping coefficients can be determined as $\alpha=0.66$ and $\beta=3.544\text{e-}5$.

3.3.3 Results and Discussion

Firstly, the modal analysis is conducted in FEA. The 1st resonant frequencies of the beam for short circuit and open circuit conditions are $f_r^{sc}=10.373\text{Hz}$ and $f_r^{oc}=10.377\text{Hz}$, respectively. The 2nd resonant frequencies of the beam for short circuit and open circuit conditions are $f_r^{sc}=64.429\text{Hz}$ and $f_r^{oc}=64.442\text{Hz}$, respectively. Obviously, the shift in frequency between the short circuit and open circuit conditions is not pronounced. Based on these values, the system electromechanical coupling coefficients ($k_e=((f_r^{oc}/f_r^{sc})^2-1)^{1/2}$, refers to Chapter 5) are 0.0278 and 0.02 at the 1st and 2nd modes, respectively. According to the criterion to classify the electromechanical coupling in Shu and Lien (2006), the fact of $k_e^2/\zeta \ll 1$ validates the weak coupling assumption for the developed energy harvesting system. In the later sections, we will not discriminate between the resonant frequency for short circuit and open circuit.

Subsequently, steady state analysis is carried out to obtain the open circuit voltage response (V_{oc}). Figure 3.5 shows the root mean square (RMS) electrical potential distribution of an MFC harvester. It is noted that the potential on the top surface is uniform as the real electrode, by using the equation constraint technique in FEA. Since the potential on the bottom electrode is assigned to be zero, the potential distribution on the top electrode is actually the voltage developed in the harvester.

Figure 3.5 Electrical potential distribution at first resonant frequency

Figure 3.6 illustrates the RMS V_{oc} from single MFC harvester. It is noted the FEA results match well with the experimental results except for a minor shift on the frequency axis. This could be attributed to the clamping condition in the experiment.

In FEA, a perfect clamping condition is applied to the cantilever beam. While in experiment, the cantilever beam is not perfectly clamped, as shown in Figure 3.1, which may have softened the structure. For the 1st mode, the natural frequencies obtained from the experiment and simulation results are 9.75Hz and 10.373Hz, respectively, and the RMS V_{oc} are 11.5V and 11.0V, respectively. The differences between the experiment and simulation results are in the acceptable range of around 5%. Hence, the finite element model is validated and the V_{oc} predicted by FEA will be used to construct the uncoupled equivalent circuit model of the system for circuit simulation.

Figure 3.6 RMS open circuit voltage of single MFC harvester

Moreover, it is worth mentioning that in energy harvesting applications with ultra-low frequency (around 1Hz) impulse excitation such as human motions, the cantilever beam will experience free vibration with damping at the 1st resonant frequency (similar to the concept of frequency up-conversion discussed in Section 2.3.4). As a result, the efficiency of energy harvesting at 1st resonance is of great importance. In the later sections, we only concern the energy harvesting performance of MFC harvesters at the 1st resonant frequency.

3.4 Circuit Simulation

Since the developed energy harvesting system is confirmed as weakly coupled, uncoupled equivalent circuit model (Figure 2.7(b)) is adopted in circuit simulation.

3.4.1 System Schematic

The circuit simulator Multisim10.0 (National Instruments) is utilized to simulate the energy extraction and storing procedures from the MFC harvesters. Figure 3.7 shows the whole circuit schematic of the energy harvesting system. The EH300A module is simplified as a full-wave rectifier and a capacitor bank. The voltage increase from V_L to V_H , i.e., the procedure for the energy accumulation of 30mJ is simulated and the power management module in EH300A is neglected (V_H measured in experiment is 3.53V rather than 3.6V given in the datasheet). The equivalent circuit model of MFC harvester is shown in the dashed box in Figure 3.7.

Figure 3.7 Circuit schematic of energy harvesting system

3.4.2 Efficiency at First Resonant Frequency

The efficiency of the energy harvesting system at the 1st resonant frequency is tested for three cases, namely, one single MFC, two MFCs electrically connected in series and in parallel. Figure 3.8 illustrates the charging current I_{rms} and the voltage across the storage capacitor (V_{DC}) in EH300A (6.6mF). Again, the simulation results are consistent with the experimental ones. Table 3.2 lists the time t_2 (refers to Figure 3.3(b)) and the average power (P_{ave}) for 30mJ accumulated in EH300A in the three cases at 1st resonant frequency. In Figure 3.8 and Table 3.2, it is noted that MFCs connected in parallel are most efficient, while the series connection has no substantial increase of the average power of charging the storage capacitor in EH300A. The reason can be explained from the viewpoint of the system impedance as follows.

For uncoupled model, the internal impedance of MFC harvester(s) is simplified as $1/\omega C^S$. Besides, making the quasi-static assumption, the effective resistance during the period of charging the storage capacitor is given by $R_{\text{eff}} = V_{\text{DC}}/I_{\text{rms}}$. Figure 3.9 illustrates R_{eff} and the internal impedance of MFC harvesters. In all the three cases, it is observed that R_{eff} is much smaller than the internal impedance during the period of charging the capacitor from V_L to V_H . Hence, the voltages across the capacitor (Figure 3.8(b)) are nearly linear and the charging current curves (Figure 3.8(a)) slightly decrease. In the parallel connection, although V_{oc} does not increase as compared to the single MFC (Table 3.2), the doubled internal capacitance C^S and thus halved internal impedance enables the charging current to be doubled as shown in Figure 3.8(a), which contributes to the nearly doubled P_{ave} . While in the series connection, V_{oc} is doubled compared to the single MFC (Table 3.2) but at the same time the internal impedance is also doubled due to the halved capacitance, which results in nearly the same charging current I_{rms} as the single MFC (Figure 3.8(a)). Thus the series connection of 2 MFCs only has slight increase of P_{ave} . Since the R_{eff} is similar but the internal impedance is doubled as compared to the single MFC, R_{eff} in series connection has less effect on the charging current I_{rms} and thus a straighter curve of V_{DC} is obtained in Figure 3.8(b). This also explains why the series connection of 2 MFCs has some contribution to the increase of P_{ave} .

Figure 3.8 (a) Charging current I_{rms} and (b) voltage across storage capacitor in EH300A V_{DC} for three cases at first resonant frequency

Table 3.2 Summary of energy harvesting performance for three cases

			Experiment		Simulation	
MFC connection	RMS V_{oc} (V)	C^S (nF)	t_2 (s)	P_{ave} (μ W)	t_2 (s)	P_{ave} (μ W)
single	11.5	25	978	30.67	942	31.85
in series	21.5	12.5	845	35.50	835	35.93
in parallel	11.85	50	450	66.67	471	63.69

Figure 3.9 R_{eff} during capacitor charging and internal impedance of MFC harvesters for three cases at first resonant frequency

3.5 Efficiency Enhancement of Energy Harvesting

As validated in previous sections, the FEA and circuit simulations are able to accurately evaluate the voltage output of MFC harvesters and the energy storage in the EH300A. Hence, combining the two simulations, we can perform a parametric study on the effects of MFC properties for efficiency enhancement. In Table 3.2, we have already noted that the parallel and series connections alter both the capacitance and the MFC output voltage, which influence the charging current I_{rms} (Figure 3.8(a)) and thus the average power for capacitor charging. These results suggest that both the capacitance and voltage output of MFC harvesters are worth attention for improving the efficiency of the energy harvesting system.

3.5.1 Varying MFC Dielectric Constant

Besides the PZT5A, other piezoelectric material such as PZT5H and PMN-PT are potential candidates to be processed as piezoelectric fibers and embedded inside the polymer matrix to form the MFC transducer in the future. Different piezoelectric materials definitely have various dielectric constants. Here, 6 values of the dielectric constant are considered to study its effect on the energy harvesting performance. The results are shown in Table 3.3. The values with ‘*’ in Tables 3.3, 3.4 and 3.5 are the actual values of MFC harvester in the experiment. Only slight increase of P_{ave} is observed with the decrease of the dielectric constant. The reason may be similar to the case of 2 MFCs connected in series discussed in previous section. The increase of V_{oc} together with the decrease of C^S (increase of internal impedance) results in the charging current unchanged. Thus, the dielectric constant fails to show significant influence on the average power for capacitor charging.

Table 3.3 Energy harvesting performance of MFC with various dielectric constants

MFC dielectric constant (F/m)	RMS V_{oc} (V)	C^S (nF)	t_2 (s)	P_{ave} (μ W)
8.00e-09	17.82	15.07	833.2	36.01
1.00e-08	14.43	18.84	853.1	35.17
1.20e-08	12.11	22.61	879.5	34.11
1.327e-08 (*)	11.00	25	897.3	33.43
1.40e-08	10.44	26.38	909.2	33.00
1.60e-08	9.17	30.14	943.9	31.78

3.5.2 Varying MFC Thickness

The change in MFC thickness also induces the change in its internal capacitance. Here, we consider 7 values of thickness. To our disappointment, varying the MFC thickness also fails to show significant influence on the efficiency of MFC harvester, as shown in Table 3.4. In fact, increasing the thickness is equivalent to electrically connecting multiple pieces of MFC in series as discussed in Section 3.4.2. Hence it is no surprise that varying the thickness has little effect on P_{ave} . However, in Section 3.4.2, the

series connection does have slight contribution to the increase of power. But in Table 3.4, increasing the thickness slightly decreases the power. This is due to the fact that the thicker MFC makes the structure stiffer and restrains the increase of voltage output, although the system electromechanical coupling is still extremely small (e.g., for MFC thickness of 0.4mm , the open circuit and short circuit resonant frequencies are 10.461 and 10.455, respectively, corresponding to $k_e^2/\zeta=0.18$). This is reflected from the slight increase of resonant frequency and the minor decrease of tip deflection of the beam.

Fortunately, although P_{ave} is not sensitive to MFC thickness, this can help reduce the usage of piezoelectric material to generate the same level of power. In other words, a thinner MFC has higher power density and thus the piezoelectric material is used more efficiently, as long as the V_{oc} maintains a sufficient level to charge the capacitor from V_L to V_H .

Table 3.4 Energy harvesting performance of MFC with various thicknesses

MFC thickness (mm)	First resonance (Hz)	RMS V_{oc} (V)	C^s (nF)	RMS tip deflection (mm)	t_2 (s)	P_{ave} (μW)
0.1	10.331	6.44	50	9.42	955.5	31.40
0.15	10.355	8.89	33.33	9.26	892.2	33.62
0.2 (*)	10.377	11.00	25	9.13	897.3	33.43
0.25	10.399	12.83	20	8.99	924	32.47
0.3	10.42	14.36	16.67	8.85	965.7	31.07
0.35	10.44	15.76	14.29	8.73	1007.1	29.79
0.4	10.461	16.98	12.5	8.6	1053.5	28.48

3.5.3 MFC Stack Configuration

In Section 3.4.2, two MFC harvesters electrically connected in parallel have been proven to significantly increase the power output. Such configuration equivalently increases the area of MFC, resulting in the increase of internal capacitance. But increasing MFC area may be not suitable for certain applications where the area

allowed for installing MFC patches is limited. Thus, we try another way to equivalently increase the area of MFC, i.e., using stack configuration where multiple MFCs are geometrically stacked in series while electrically connected in parallel, as shown in Figure 3.10. An adhesive epoxy layer is also created between neighboring layers of MFCs in FEA.

Figure 3.10 MFC stack configuration

In Table 3.5, it is observed that for the 2-layer MFC stack configuration, P_{ave} can achieve an increase of 35.4% compared to the non-stack configuration, but this is less than our expectation of 100% increase as that achieved in Section 3.4.2. Although the capacitance doubles with the vertically stacked two patches of MFC, the voltage output significantly decreases, resulting in the undermined power increase, as shown in Table 3.5. While, in Section 3.4.2, both MFC patches in parallel are bonded directly on the beam and the voltage output V_{oc} is almost the same as a single MFC patch, as shown in Table 3.2. The reason for the decrease of V_{oc} is similar to that for the thicker MFC in the previous section, i.e., the increase of vertically stacked MFC layers stiffens the structure and suppress the vibrations. Actually, although globally the change of stiffness is slight, the local strain distribution in the MFC layers significantly declines, especially in the lower MFC layer, as compared in Figure 3.11. With more layers stacked, V_{oc} and thus P_{ave} decreases further. Considering the increased usage of piezoelectric material, the capability of the stacked MFC layers is actually reduced in terms of the power density.

Table 3.5 Energy harvesting performance of MFC stack configurations

Multilayer number	First resonance (Hz)	RMS V_{oc} (V)	C^S (nF)	RMS tip deflection (mm)	t_2 (s)	P_{ave} (μ W)
1 (*)	10.377	11.00	25	9.13	897.3	33.43
2	10.462	8.17	50	8.59	662.7	45.27
3	10.525	6.12	75	8.19	681.1	44.05
4	10.573	4.68	100	7.91	834.2	35.96

Figure 3.11 Comparison of strain distribution at the middle of MFC layer at first resonance. (a) single-layer MFC (b) two-layer MFC stack configuration

3.5.4 Optimized MFC Configuration

Based on the discussion in Sections 3.5.2 and 3.5.3, an optimal MFC configuration can be achieved by dividing in thickness the original MFC into two pieces and then stacking them together (electrically connected in parallel). By doing so, the V_{oc} is halved but the internal capacitance C^S is increased by four times, as shown in Table 3.6. The former is harmful while the latter is favorable for charging the capacitor. When we divide the original MFC into two pieces, the influence of the increase of C^S is dominant. Thus, the combined influences of V_{oc} and C^S result in the P_{ave} of 47.21μ W without increase of the usage of piezoelectric material. This corresponds to a 41.2% increase compared to the original MFC transducer. If the original MFC is

divided into more pieces in thickness (>2), intuitively, this can further increase the average power for charging EH300A module. However, this induces the further decrease of V_{oc} and the negative influence of the reduced V_{oc} will be dominant, which smashes the benefit brought by the increased capacitance and decreases the P_{ave} .

It should be mentioned that the impedance matching module is not included in the EH300A. The performance of the proposed optimized MFC configuration is expected to be further improved by implementing the impedance matching interface circuit, which is out of the scope of this chapter.

Table 3.6 Energy harvesting performance of optimized MFC configuration

Multilayer number	Layer thickness (mm)	First resonance (Hz)	RMS V_{oc} (V)	C^s (nF)	RMS tip deflection (mm)	t_2 (s)	P_{ave} (μ W)
2	0.1	10.382	5.44	100	9.105	635.4	47.21

3.6 Chapter Summary

This chapter presents an uncoupled modeling of an energy harvesting system using MFC transducers. A prototype is developed and its efficiency is tested. Finite element modeling and circuit simulation are conducted and validated by experiment. Based on the validated models, parametric study on the effects of MFC properties is presented aiming at efficiency enhancement. It is demonstrated that the MFC configuration by dividing in thickness the original MFC into two pieces and then stacking them together can provide a 41.2% increase of average power for capacitor charging in EH300A. The work in chapter has potential impact on manufacturing more cost-effective future version of MFC transducers.

CHAPTER 4 A GENERIC MODELING METHOD OF PIEZOELECTRIC ENERGY HARVESTERS

4.1 Introduction

The analytical models of piezoelectric energy harvesters available in the literature, as discussed in Section 2.2.2, focused on the maximum power that can be achieved by simplifying the energy harvesting interface circuit as a resistor. In reality, the interface circuit attached to an energy harvester is more complicated than a resistor, in which case, the theoretical analysis is either cumbersome or impossible. On the other hand, researchers, usually from electrical engineering discipline, focused on developing sophisticated interface circuits to improve the efficiency of piezoelectric energy harvesting, as discussed in Section 2.3.2. However, they usually made simplification in the mechanical domain of the energy harvester for the ease of model formulation, such as single-degree-of-freedom (SDOF) models. The performances of their sophisticated interfaces are unable to be predicted once multimodal energy harvesters are concerned. The challenge remains to establish a more general modeling approach to account for both complex mechanical conditions and sophisticated energy harvesting interface circuits.

Finite element analysis (FEA) and SPICE software are powerful tools in structural and electrical engineering, respectively. Using FEA, it is easy to estimate the open-circuit voltage or short-circuit current of the piezoelectric energy harvester. Unfortunately, these calculated results could only be applied in the SPICE simulation for weakly coupled energy harvesting system as we did in Chapter 3 (uncoupled modeling). When the electromechanical coupling is strong, the backward coupling effect (i.e., electrical damping due to energy harvesting) will affect the dynamics of the harvester (refers to Section 2.2.2) and hence the voltage or current source is variant with different electric loads. Elvin and Elvin (2009a) proposed one general

equivalent circuit model for piezoelectric energy harvesters, which considered the backward coupling effect in the mechanical domain and multiple vibration modes. The parameters of equivalent circuit were derived using the Rayleigh-Ritz approach. However, this approach required proper assumption of mode shape functions and enough number of assumed modes to ensure accuracy, which would be cumbersome and challenging for complicated structures. To avoid this difficulty and utilize the available powerful tools in structural and electrical modeling, Elvin and Elvin (2009b) developed a coupled FEA-SPICE simulation model considering the backward coupling effect in the mechanical domain. However, in their algorithm, an automation program is needed to extract nodal displacements from the FEA output and coupling voltage from the SPICE output, and then transfer these data between the FEA and SPICE solvers at each iteration. However, a new difficulty accompanied is that for a complicated model, the post-process of data extraction and transfer is quite difficult to implement and tedious even for one iteration, which limits the applicability of this simulation model. The challenge now is to enable efficient collaboration of robust FEA and SPICE solvers and exert their respective advantages, and meanwhile take into account the backward coupling.

In this chapter, an efficient method based on the equivalent circuit model (ECM) is proposed to bridge the FEA and SPICE simulation for accurate estimation of performance of piezoelectric energy harvesters. According to different mechanical conditions, the system parameters used in the equivalent circuit model are determined by theoretical analysis or FEA. Numerical examples and experimental work are presented to validate the accuracy of the developed equivalent circuit model and to illustrate its capability of dealing with complicated structures and sophisticated interface circuits. Together with the proposed ECM-based modeling method, other modeling techniques for piezoelectric energy harvesters and their applicability are summarized.

4.2 Analytical Modeling

Mathematical models of piezoelectric energy harvesters have been reviewed in Section 2.2.2. Here we give a detailed introduction of the benchmark model for a simple cantilever unimorph harvester (Figure 4.1), established by Erturk and Inman (2008c). The harvester is attached with a resistive load (the simplest interface). The profile of the energy harvester is rectangular and the density of the beam along longitudinal direction is uniform.

Figure 4.1 Rectangular unimorph energy harvester subjected to base excitation

The analytical model is derived based on the reduced constitutive equations of piezoelectricity and the following assumptions: (a) Euler-Bernouli beam assumption; (b) negligible external excitation from air damping; (c) proportional damping (i.e., the strain rate damping and viscous air damping are assumed to be proportional to the bending stiffness and mass per length of the beam); and (d) uniform electric field through piezoelectric thickness. The *e*-form piezoelectric constitutive relations are

$$\begin{aligned} T_1 &= Y_1^E S_1 - e_{31} E_3 \\ D_3 &= e_{31} S_1 + \epsilon_{33}^S E_3 \end{aligned} \quad (4.1)$$

where T_1 and S_1 are the stress and strain components in the piezoelectric layer along 1-direction (longitudinal direction), respectively; D_3 and E_3 are the electric displacement and electric field along 3-direction (thickness direction), respectively; Y_1^E and ϵ_{33}^S are the Young's modulus and electric permittivity of the piezoelectric layer, respectively; the superscripts $()^E$ and $()^S$ indicate that the parameters are measured at constant electric field (short-circuit) and at constant strain (beam is totally clamped), respectively; and e_{31} is the piezoelectric constant. Based on assumptions (a) and (b), the governing equation of mechanical motion is,

$$\frac{\partial^2 M(x,t)}{\partial x^2} + c_s I \frac{\partial^5 w_{rel}(x,t)}{\partial x^4 \partial t} + c_a \frac{\partial w_{rel}(x,t)}{\partial t} + m \frac{\partial^2 w_{rel}(x,t)}{\partial t^2} = -m \frac{\partial^2 w_b(x,t)}{\partial t^2} \quad (4.2)$$

where $M(x,t)$ is the internal bending moment of the beam; I is the equivalent area moment of inertia of the composite cross section; $w_b(x,t)$ and $w_{rel}(x,t)$ are the base excitation and the deflection relative to the base motion, respectively; c_s and c_a are the strain rate damping coefficient and viscous air damping coefficient, respectively; m is the mass per unit length; x is the longitudinal coordinate and t is time. Applying the first constitutive equation, expressing the moment by integrating stress distribution over composite cross section of the beam and considering assumption (d), we obtain

$$\begin{aligned}
& YI \frac{\partial^4 w_{rel}(x,t)}{\partial x^4} + c_s I \frac{\partial^5 w_{rel}(x,t)}{\partial x^4 \partial t} + c_a \frac{\partial w_{rel}(x,t)}{\partial t} + m \frac{\partial^2 w_{rel}(x,t)}{\partial t^2} \\
& + \theta V(t) \left[\frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] = -m \frac{\partial^2 w_b(x,t)}{\partial t^2}
\end{aligned} \quad (4.3)$$

where YI is the average bending stiffness; θ is the electromechanical coupling coefficient; $V(t)$ is the voltage across the resistive load; and $\delta(x)$ is the Dirac delta function. Integrating the second constitutive equation over the area of piezoelectric layer and differentiating it with respect to t , we obtain

$$\frac{V(t)}{R_l} + C^S \frac{dV(t)}{dt} + \int_0^L e_{31} h_{pc} b \frac{\partial^3 w_{rel}(x,t)}{\partial x^2 \partial t} dx = 0 \quad (4.4)$$

where C^S is the capacitance measured at constant strain, corresponding to the clamped electric permittivity ϵ_{33}^S ; R_l is the attached resistive load; b is the width of the beam; h_{pc} is the distance from the center of the piezoelectric layer to the neutral axis of the beam. Based on assumption (c), the vibration response relative to the base can be represented as an absolutely and uniformly convergent series of the eigenfunctions as

$$w_{rel}(x,t) = \sum_{r=1}^{\infty} \phi_r(x) \eta_r(t) \quad (4.5)$$

where $\phi_r(x)$ and $\eta_r(t)$ are the mass normalized eigenfunctions and the modal coordinate of the r^{th} mode, respectively. Substituting Equation (4.5) into Equations (4.3) and (4.4), the electromechanical coupled differential equations for the modal response of the harvester is obtained as

$$\frac{d^2\eta_r(t)}{dt^2} + 2\zeta_r\omega_r \frac{d\eta_r(t)}{dt} + \omega_r^2\eta_r(t) + \chi_r V(t) = -f_r \ddot{u}_g(t) \quad (4.6)$$

$$\frac{V(t)}{R_l} + C^s \frac{dV(t)}{dt} - \sum_{r=1}^{\infty} \chi_r \frac{d\eta_r(t)}{dt} = 0 \quad (4.7)$$

where ω_r and ζ_r are the natural frequency and damping ratio of the r^{th} mode; χ_r is the modal electromechanical coupling coefficient; and $-f_r \ddot{u}_g(t)$ is the modal forcing function.

$$\chi_r = \int_0^L \theta \left[\frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] \phi_r(x) dx = \int_0^L \theta \frac{d^2\phi_r(x)}{dx^2} dx = \theta \frac{d\phi_r(x)}{dx} \Big|_{x=L} \quad (4.8)$$

$$f_r = \int_0^L m \phi_r(x) dx \quad (4.9)$$

When the harmonic base excitation $u_g(t) = A e^{j\omega t}$ is applied (j is the unit imaginary number and ω is the excitation frequency), the steady state voltage response across the resistive load can be expressed as,

$$V(t) = i(t) \cdot R_l = \frac{\sum_{r=1}^{\infty} \frac{j\omega\chi_r f_r}{\omega_r^2 - \omega^2 + j2\zeta_r\omega_r\omega}}{\sum_{r=1}^{\infty} \frac{j\omega\chi_r^2}{\omega_r^2 - \omega^2 + j2\zeta_r\omega_r\omega} + j\omega C^s + \frac{1}{R_l}} A \omega^2 e^{j\omega t} \quad (4.10)$$

where $i(t)$ is the current through the resistive load.

4.3 System-level Finite Element Modeling

To improve system performance, the profile of energy harvester is not necessarily designed as conventional rectangular cantilevers, as discussed in Section 2.3.1. For example, a trapezoidal cantilever beam was claimed to supply more than twice the amount of energy than a rectangular beam (Roundy *et al.*, 2005). In such case, the parameters of beam, i.e., YI , m , θ , h_{pc} and b in Equations (4.3) and (4.4) should be replaced by $YI(x)$, $m(x)$, $\theta(x)$, $h_{pc}(x)$ and $b(x)$, respectively. Hence, χ_r and f_r should be written as

$$\chi_r = \int_0^L \theta(x) \frac{d^2\phi_r(x)}{dx^2} dx \quad (4.11)$$

$$f_r = \int_0^L m(x) \phi_r(x) dx \quad (4.12)$$

However, in such case, theoretical derivation of mode shape functions and parameters χ_r and f_r at each resonance is a tough task. To avoid this difficulty, FEA can be used to estimate the response of the piezoelectric energy harvester. Several robust commercial codes such as ABAQUS and ANSYS allow for the modeling of piezoelectric transducer as both actuator and generator (energy harvester). Although various circuit elements from resistors to diodes are available in the element library of ANSYS, only the basic linear circuit elements such as capacitor, inductor and resistor are compatible with the piezoelectric elements. This limits the applicability of system-level FEA. However, if one's concern is to estimate the maximum achievable power from the energy harvester, linear circuit elements are adequate for system-level simulation in ANSYS.

A finite element model for system-level simulation is shown in Figure 4.2. In this model, the voltage degrees of freedom on the top and bottom surfaces of the piezoelectric layer are coupled separately to implement uniform electrical potentials on the top and bottom electrodes. It should be mentioned that Figure 4.1 is just an illustration on how to model a piezoelectric energy harvester in ANSYS. In the later part of this chapter, the mesh of the finite element model is further refined.

Figure 4.2 System-level finite element model of piezoelectric energy harvester

4.4 Equivalent Circuit Modeling

Generally, the analytical models and system-level FEA are only applicable to estimate the maximum power available to be extracted from a piezoelectric energy harvester by connecting a resistive load. In practice, however, the energy harvesting interface circuit is required to include nonlinear electric components such as rectifier and power conditioning modules. Neither the analytical models nor the system-level FEA is able to address this issue. Besides, if the geometry of energy harvester is complicated so as to achieve optimal system performance (as mentioned in Section 2.3.1), modeling of the energy harvesting system will be more challenging. A novel modeling method based on equivalent circuit model of piezoelectric energy harvesters is thus proposed in this chapter to address these problems. However, different from the approach of Elvin and Elvin (2009a), the parameters of equivalent circuit model in the current method can be conveniently extracted from FEA results for complicated structures, avoiding the tedious calculation by the Rayleigh-Ritz approach.

In Equations (4.6) and (4.7), analogies exist between the mechanical and electrical domains of the piezoelectric coupled system, as listed in Table 4.1. Based on equivalent circuit representations of piezoelectric transducer (Section 2.2.3), a generic modeling method is developed. Once the equivalent circuit parameters of the system are determined, energy harvesting performance can be evaluated by the SPICE simulation. The remaining issue is thus identification of the required parameters.

Table 4.1 Analogies between electrical and mechanical domains

Equivalent circuit parameters at r^{th} mode	Mechanical counterparts
Charge $q_r(t)$	Modal coordinate $\eta_r(t)$
Current $i_r(t)$	Modal velocity $d\eta_r(t)/dt$
Inductance L_r	1
Resistance R_r	$2\zeta_r\omega_r$
Capacitance C_r	$1/\omega_r^2$
Voltage source $v_r(t)$	$-f_r\ddot{u}_g(t)$
Ideal transformer ratio N_r	Electromechanical coupling χ_r

Here, we consider two cases, i.e., a simple and a complex mechanical condition of the energy harvester. The simple mechanical condition refers to simple geometric configuration and simple mechanical boundaries. The complex mechanical condition refers to non-uniform beam configuration and complex mechanical boundaries. For the first case, the parameters in the equivalent circuit model can be directly obtained through the analogies shown in Table 4.1 after theoretical modal analysis. For the second case, a parameter identification method by FEA will be proposed. It should be mentioned that no matter in which case, the basis for parameter identification is the analogy between the electrical and mechanical domains. It should be mentioned that Equations (4.6) and (4.7) represents a linear system and thus nonlinear structural behavior cannot be handled by the proposed equivalent circuit modeling method.

4.4.1 Equivalent Circuit Model with Theoretically Derived Parameters

After theoretical modal analysis, we can determine the corresponding parameters C_r , L_r , R_r , N_r and $v_r(t)$ by the analogizing Equations (4.6) and (4.7) with the constitutive differential equations of a circuit network. The circuit network consists of infinite branches, each composed of an inductor, a capacitor, a resistor, an ideal voltage source and an ideal transformer, as shown in Figure 4.3. The r^{th} circuit branch represents the r^{th} vibration mode of the system. After equivalent circuit representation, it is noted that Equations (4.6) and (4.7) satisfy Kirchoff's voltage and current laws, respectively. In Equations (4.8) and (4.9), the integrals may give positive or negative values. Since the SPICE software only accepts positive input value, we can change the wire connection pattern when the ideal voltage magnitude or transformer ratio is negative from the integrals in Equations (4.8) and (4.9). For example, if the transformer ratio N_2 is negative in Figure 4.3, we can swap the terminals of the transformer connected to capacitor C^S .

The procedure to evaluate the system performance includes the following steps:

- (A) Theoretical modal analysis to obtain modal information such as the natural frequencies and mode shape functions of the energy harvester;
- (B) Determining the equivalent circuit parameters through analogy; and
- (C) System modeling in the SPICE software with the determined parameters.

Figure 4.3 Multi-mode equivalent circuit model of piezoelectric energy harvester

4.4.2 Equivalent Circuit Model with Parameters Identified by FEA

As mentioned in Section 4.3, to achieve optimal system performance, the geometry of the energy harvester could be more complicated and the harvester may have complex mechanical boundaries. In such case, it is difficult to theoretically derive the parameters of equivalent circuit model. Here we propose a parameter identification method based on FEA. The parameters to be identified are C_r , L_r , R_r , N_r and $v_r(t)$. The first four parameters will be identified from the admittance of the piezoelectric energy harvester, and subsequently, the last parameter $v_r(t)$ will be determined from the short-circuit charge response of the harvester under base excitation.

Identification of C_r , L_r , R_r and N_r

To obtain the admittance of piezoelectric transducer, a harmonic voltage $v_r(t)$ is applied to the energy harvester. Hence, replacing $V(t)/R_l$ with $-i(t)$ in Equation (4.7), short-circuiting the mechanical port (i.e., setting the forcing term $-f_r \ddot{u}_g(t)$ to zero in

Equation (4.6)) and transforming the two equations into frequency domain, we can obtain the admittance Y as

$$Y = \frac{i}{V} = \frac{j\omega Q}{V} = j\omega \left(C^S + \sum_{r=1}^{\infty} \frac{\chi_r^2}{\omega_r^2 - \omega^2 + j2\zeta_r \omega_r \omega} \right) \quad (4.13)$$

Here, the negative sign of $-i(t)$ means that the current flows into the system, as shown in Figure 4.4, since now the piezoelectric transducer works as an actuator. This is different from the current flow direction in Figure 4.3, in which the transducer works as a generator. Applying the analogies in Table 4.1 to Equation (4.13), we obtain

$$Y = j\omega \left(C^S + \sum_{r=1}^{\infty} \frac{N_r^2}{\frac{1}{C_r} - L_r \omega^2 + jR_r \omega} \right) \quad (4.14)$$

According to the properties of an ideal transformer, the impedance in the circuit can be transformed by the square of turn ratio as shown in Figure 4.4, that is,

$$L_{mr} = \frac{L_r}{N_r^2}, \quad R_{mr} = \frac{R_r}{N_r^2}, \quad C_{mr} = C_r N_r^2 \quad (4.15)$$

Figure 4.4 r^{th} branch of equivalent circuit model of piezoelectric transducer in actuator mode

Rearranging Equation (4.14), we obtain

$$Y = j\omega \left(C^S + \sum_{r=1}^{\infty} \frac{C_{mr}}{1 - L_{mr} C_{mr} \omega^2 + jR_{mr} C_{mr} \omega} \right) \quad (4.16)$$

The resonant frequency and quality factor (Ikeda, 1990) can be written as

$$\omega_r = \frac{1}{\sqrt{L_{mr}C_{mr}}}, \quad Q_{mr} = \frac{\omega_r}{\omega_1 - \omega_2} = \frac{1}{R_{mr}} \sqrt{\frac{L_{mr}}{C_{mr}}} = \frac{1}{R_{mr}C_{mr}\omega_r} = \frac{\omega_r L_{mr}}{R_{mr}} \quad (4.17)$$

where ω_1 and ω_2 are the half-width frequencies near the r^{th} resonance.

Consider the following three cases:

(1) $\omega \rightarrow 0$, from Equation (4.16), we obtain

$$Y \rightarrow j\omega \left(C^S + \sum_{r=1}^{\infty} C_{mr} \right) = j\omega C^T \quad (4.18)$$

where C^T is the static capacitance measured when no external mechanical force is applied on the energy harvester (i.e., capacitance at zero stress).

(2) $\omega \rightarrow \infty$, from Equation (4.16), we obtain

$$Y \rightarrow j\omega C^S \quad (4.19)$$

where C^S is the static clamped capacitance. The result is understandable because when the frequency of applied voltage approaches infinity, the structure deformation cannot accompany with the alternating electric field. Hence the boundary condition is similar to the case that the harvester is totally clamped.

(3) $\omega \rightarrow \omega_r$, we can derive the admittance as follows

$$\begin{aligned} Y &= j\omega \left(C^S + \frac{C_{m1}}{1 - L_{m1}C_{m1}\omega^2 + jR_{m1}C_{m1}\omega} + \frac{C_{m2}}{1 - L_{m2}C_{m2}\omega^2 + jR_{m2}C_{m2}\omega} + \dots \right. \\ &\quad \left. \dots + \frac{C_{mr}}{1 - L_{mr}C_{mr}\omega^2 + jR_{mr}C_{mr}\omega} + \dots \right) \\ &\approx j\omega \left(C^S + \underbrace{0 \dots 0}_{r-1} + \frac{C_{mr}}{1 - L_{mr}C_{mr}\omega^2 + jR_{mr}C_{mr}\omega} + \sum_{i=r+1}^{\infty} C_{mi} \right) \\ &= j\omega \left(C^T - \sum_{i=1}^r C_{mi} + \frac{C_{mr}}{1 - L_{mr}C_{mr}\omega^2 + jR_{mr}C_{mr}\omega} \right) \\ &= j\omega \left(C_{dr} + \frac{C_{mr}}{1 - L_{mr}C_{mr}\omega^2 + jR_{mr}C_{mr}\omega} \right) \\ &= Y_d + Y_{mot} \end{aligned} \quad (4.20)$$

where Y_d and Y_{mot} are termed damped admittance and motional admittance (Ikeda, 1990), respectively.

$$Y_d = j\omega C_{dr} \quad (4.21)$$

$$Y_{mot} = \frac{j\omega C_{mr}}{1 - L_{mr} C_{mr} \omega^2 + jR_{mr} C_{mr} \omega} \quad (4.22)$$

The approximation in Equation (4.20) can be understood by substituting Equation (4.17) into it and examining the denominators term by term. The locus of Y_{mot} in the complex plane near the r^{th} resonance is an approximate circle whose equation can be written as

$$\left[\text{Re}(Y_{mot}) - \frac{1}{2R_{mr}} \right]^2 + [\text{Im}(Y_{mot})]^2 = \left(\frac{1}{2R_{mr}} \right)^2 \quad (4.23)$$

C_{dr} is termed damped capacitance (Ikeda, 1990) at the r^{th} vibration mode. It should be noted that C_{dr} is not equal to C^S as it also includes the contribution from the higher modes ($r+1$ and above). Hence, when we draw the approximate circle of Y_{mot} of each mode, different Y_d should be excluded from the total admittance Y , according to different C_{dr}

$$C_{dr} = C^T - \sum_{i=1}^r C_{mi} \quad (4.24)$$

From Equation (4.20), we know that C_{dr} can be directly extracted from FEA results according to

$$C_{dr} = \text{Im}[Y(\omega_r)] \quad (4.25)$$

Then drawing the admittance locus of Y_{mot} of each vibration mode, which is an approximate circle as described by Equation (4.23), and considering Equation (4.17), we can identify parameters C_{mr} , L_{mr} , R_{mr} and N_r as

$$\begin{cases} R_{mr} = \frac{1}{2 \cdot \text{raduis}} = \frac{1}{\max[\text{Re}(Y_{mot})]} \\ L_{mr} = \frac{R_{mr}}{\omega[\min[\text{Im}(Y_{mot})]] - \omega[\max[\text{Im}(Y_{mot})]]} \\ C_{mr} = \frac{1}{\omega_r^2 L_{mr}} \\ N_r = \sqrt{\frac{1}{L_{mr}}} \end{cases} \quad (4.26)$$

Parameter N_r is obtained in the above equation. According to Equation (4.15), we can determine the other three parameters C_r , L_r and R_r . Here, we have no concern for the sign of N_r , which will be clarified in the following section.

Identification of $v_r(t)$

The last undetermined parameter in the equivalent circuit model is $v_r(t)$. If a unit harmonic base acceleration is applied to the system, the magnitude of $v_r(t)$ is f_r . Given short-circuit condition, according to Equation (4.10), we can obtain from FEA the charge response at each resonant frequency ω_r as

$$Q(j\omega_r) = \chi_r \eta_r(j\omega_r) = \frac{\chi_r f_r}{\omega_r^2 - \omega^2 + j2\zeta_r \omega_r \omega} \Big|_{\omega=\omega_r} = \frac{\chi_r f_r}{\chi_r^2 R_{mr} j\omega_r} = \frac{f_r}{N_r R_{mr} j\omega_r} \quad (4.27)$$

Hence, with the obtained N_r and R_{mr} , we can calculate f_r as

$$f_r = N_r R_{mr} j\omega_r Q(j\omega_r) \quad (4.28)$$

The sign of $\chi_r f_r$ can be determined according to the phase angle of the charge response by FEA. Furthermore, in Equation (4.10), it should be noted that it is the sign of $\chi_r f_r$ that affects the response of the system rather than the sign of χ_r or f_r alone. Hence, we need not determine the sign of χ_r or f_r alone. Actually, from the viewpoint of circuit, there is no change in the flow direction of the output current if we change at the same time the signs of the terminals of the ideal transformer and the ideal voltage source of one circuit branch in Figure 4.3.

General Procedure of Equivalent Circuit Modeling

In summary, the general procedure of equivalent circuit modeling to evaluate the performance of the energy harvester is as follows:

- (A) Finite element static analysis to determine the static capacitance C^T and C^S ;
- (B) Finite element modal analysis to determine the short-circuit resonant frequency of each vibration mode ω_r ;

- (C) Finite element harmonic analysis to obtain the charge response and then the admittance with a harmonic alternating voltage input. Identify the parameters C_r , L_r , R_r and N_r from each admittance circle of Y_{mot} ;
- (D) Finite element harmonic analysis to obtain the charge response at each resonance, with base excitation applied, to determine $v_r(t)$;
- (E) Circuit modeling and simulation in the SPICE software with the parameters identified from FEA to evaluate the performance of energy harvester.

4.5 Model Validation

4.5.1 Validation Example 1

In the first validation example, we consider the same rectangular unimorph harvester as Erturk and Inman (2008c) with a resistor attached, as shown in Figure 4.1. Due to the simple geometry of the energy harvester, the parameters of the equivalent circuit model are derived by theoretical analysis, as discussed in Section 4.4.1. Although the equivalent circuit model can account for more complicated energy harvesting interface, only a resistor is considered here so that the results by equivalent circuit modeling approach can be compared with those obtained by the analytical model and system-level FEA. The properties of the energy harvester are listed in Table 4.2.

Table 4.2 Properties of unimorph energy harvester (Erturk and Inman, 2008c)

Item	Value
Dimensions of piezoelectric layer $L \times b \times h_p$	100mm×20mm×0.4mm
Dimensions of substrate layer $L \times b \times h_p$	100mm×20mm×0.5mm
Young's Modulus of piezoelectric layer	66GPa
Young's Modulus of substrate layer	100GPa
Density of piezoelectric layer	7800kg/m ³
Density of substrate layer	7165kg/m ³
Clamped permittivity ϵ_{33}^S	1.593e-8F/m
Piezoelectric constant e_{31}	-12.54C/m ²
Rayleigh damping coefficients	$\alpha=4.894$, $\beta=1.2349\text{e-}5$

Table 4.3 Parameters of equivalent circuit model determined from theoretical analysis

mode	1	2	3
N_r	0.017563	-0.06010	0.10014
L_r (H)	1	1	1
R_r (Ω)	6.00833	48.6582	348.013
C_r (F)	1.1082e-05	2.82171e-07	3.59905e-08
f_r (V)	0.090655	0.050241	0.029457

We consider the first three vibration modes of the system. The multi-mode equivalent circuit model is established with the parameters determined from theoretical analysis as listed in Table 4.3. Figures 4.5, 4.6 and 4.7 compare the magnitude and phase of the output voltage and power of the energy harvester from the equivalent circuit model, analytical model and system-level FEA for five different resistive loads. It is observed that the results of equivalent circuit model match perfectly with those from the analytical model and system-level FEA. Besides, it is noted in Figure 4.7 that the maximum power is obtained at different frequency for different electric load, due to the backward electromechanical coupling effect. Hence, it validates that the proposed equivalent circuit modeling method is capable of accounting for backward coupling as the system-level FEA and analytical model (Erturk and Inman, 2008c) do.

Figure 4.5 Magnitude of output voltage from analytical model, system-level FEA and equivalent circuit model with various resistive loads

Figure 4.6 Phase of output voltage from analytical model, system-level FEA and equivalent circuit model with various resistive loads

Figure 4.7 Output power from analytical model, system-level FEA and equivalent circuit model with various resistive loads

4.5.2 Validation Example 2

The second validation example considers an isosceles trapezoidal piezoelectric beam with unimorph configuration, as shown in Figure 4.8(a). Again, a single resistor of $1\text{M}\Omega$ is attached. For this example, the accurate analytical model is not available.

Some approximate analytical method such as Rayleigh-Ritz formulation may apply but it is quite tedious. However, since a pure resistor is attached, the system-level FEA still applies in this case and will be used to validate the proposed equivalent circuit modeling method.

Figure 4.8 Isosceles trapezoidal unimorph energy harvester subjected to base excitation with a resistive load

Figure 4.9 Locus of motional admittance Y_{mot}

The two parallel sides of the isosceles trapezoid of the beam are of length $15mm$ and $5mm$, respectively. The other geometric and material parameters are the same as those listed in Table 4.2. Since the geometry of the energy harvester is more complicated than that in the first validation example, the equivalent circuit parameters of the harvester should be identified by the FEA method proposed in Section 4.4.2. Again, we consider the first three modes of the system. Figure 4.9 shows the locus of

motional admittance Y_{mot} near resonances obtained from FEA. Applying Equations (4.15) and (4.26), we can obtain C_r , L_r , R_r and N_r from each admittance circle of Y_{mot} in Figure 4.9. According to Equation (4.28), f_r (the magnitude of $v_r(t)$) is determined from the charge response at each resonant frequency from FEA. Table 4.4 lists all the parameters identified from FEA.

Table 4.4 Parameters of equivalent circuit model identified from FEA

mode	1	2	3
C_{dr} (F)	8.513e-08	8.431e-08	8.413e-08
L_{mr} (H)	1400.662	340.8209	149.5977
R_{mr} (Ω)	9715.873	19722.66	51396.43
C_{mr} (F)	4.19e-9	6.67e-10	2.203e-10
N_r	0.02672	0.05417	0.08176
L_r (H)	1	1	1
R_r (Ω)	6.9366	57.868	343.56
C_r (F)	5.874e-6	2.274e-7	3.296e-08
f_r (V)	8.30e-2	-5.23e-2	3.05e-2

Finally, we model the equivalent circuit of the energy harvesting system in the SPICE software, as shown in Figure 4.10. The magnitude of the voltage and power on the electric load by equivalent circuit modeling and system-level FEA are shown in Figures 4.11 and 4.12, respectively. Again, excellent agreement in the results of equivalent circuit modeling and system-level FEA is observed, validating the accuracy of the equivalent circuit model with the parameters identified from FEA. Here, FEA is a pre-process to determine the parameters of SPICE simulation. Compared with the coupled FEA-SPICE model given by Elvin and Elvin (2009b), which involves the alternate FEA and SPICE simulation till convergence in each time increment and the computationally expensive pre-process and post-process at each iteration, the proposed approach based on the equivalent circuit model is much simpler and more efficient. It should be emphasized that although the FEA and SPICE simulation are separately conducted, the constructed equivalent circuit model here accounts for the backward coupling effect in the mechanical domain. In Chapter 3,

although we also conduct separate FEA and SPICE simulation to evaluate the energy harvesting performance, the circuit model is based on uncoupled assumption and only valid for weakly coupled system.

Figure 4.10 Multi-mode equivalent circuit model with parameters identified from FEA

Figure 4.11 Magnitude of voltage on the resistor $R_l=1\text{M}\Omega$ by system-level FEA and equivalent circuit model

Figure 4.12 Magnitude of power on the resistor $R_l=1\text{M}\Omega$ by system-level FEA and equivalent circuit model

4.5.3 Validation Example 3

In this example, a practical energy storage circuit is considered as shown in Figure 4.13(a). Meanwhile, the structure of the harvester is more complicated than a rectangular uniform cantilever. Analytical model and system-level FEA cannot work in such a general case with both complicated mechanical condition and sophisticated interface circuit. Hence, a prototype of such kind of energy harvesting system is developed to further validate the proposed generic ECM-based method.

Figure 4.13 (a) PEH attached with energy storage circuit and (b) prototype and experimental apparatus

Figure 4.13(b) illustrates the prototype of the fabricated PEH and the experimental apparatus. Two piezoelectric transducers (P2-type MFC, from Smart Material Corp., model: M8528-P2), electrically connected in parallel, are bonded to each side of an aluminum substrate (the beam) by epoxy adhesive. A steel proof mass is attached at the beam tip. The beam is clamped on an electromagnetic shaker. An accelerometer is attached to the shaker to monitor the input acceleration of the base excitation. Table 4.5 lists the properties of the fabricated PEH prototype, which are required in FEA to derive the parameters of the circuit model of the PEH. Following the same procedure (discussed in Section 4.4.2) as previous validation example, the equivalent circuit parameters are derived from FEA, as shown in Figure 4.14. Here, we only consider the frequency range near the first mode and thus only one branch is included in the equivalent circuit model.

Figure 4.14 Equivalent circuit model of fabricated PEH prototype

Table 4.5 Properties of fabricated PEH prototype

Item	MFC (active volume with PZT5A)	Epoxy Adhesive Layer	Aluminum Substrate	Steel Proof Mass
Dimensions (mm×mm×mm)	85×28×0.2	85×28×0.1	178×32×1.5	18.5×32×14
Density (kg/m ³)	Refers to Table 3.1			7850
Elastic constants (GPa)				E=200 ν=0.29
Dielectric constant (F/m)				—
Piezoelectric constant (10 ⁻¹² m/V)				—
Rayleigh damping coefficients	α=1.7, β=2.25e-5 (calculated by experimentally measured 1 st and 2 nd mode damping ratio ζ ₁ =0.011, ζ ₂ =0.0114)			

A unity root mean square (RMS) acceleration input is kept in the experiment. The frequency responses of open circuit voltage V_{oc} and short circuit current I_{sc} in the range dominated by the first mode from simulation and experiment are compared in Figure 4.15. It is notable that the derived circuit model predicts the shift between the open circuit and short circuit resonant frequencies as observed in the experiment, which is one of the prominent characteristics of the electromechanical coupling effect. Additionally, the magnitudes of V_{oc} and I_{sc} from simulation with the derived circuit model are comparable with those from the experiment.

Figure 4.15 Frequency responses of V_{oc} and I_{sc} : (a) experiment and (b) simulation

Figure 4.16 (a) Voltage development and (b) energy accumulated in various storage capacitors

Subsequently, the energy storage circuit is considered (Figure 4.13(a)). The harvester is tuned to vibrate at its open circuit resonant frequency (this value is slightly different between experiment measurement and simulation prediction, as shown in Figure 4.15).

The voltage development (V_{cap}) and energy accumulated (E_{cap}) across the five capacitors in the time domain is illustrated in Figure 4.16. Again, good consistency is observed between experiment and circuit simulation results, which further validate the proposed ECM-based modeling method. Besides, it is noted in Figure 4.16(b) that a smaller storage capacitor can accumulate energy faster at the beginning (higher instant power) but it is also quickly charged to saturation. Hence, a small capacitor is favorable if the energy is required to be used instantly. While a large capacitor such as $330\mu\text{F}$ is favorable to store the energy for later use because of its larger energy capacity but smaller instant power compared with small capacitors.

Table 4.6 Methodologies for modeling of piezoelectric energy harvesters

Mechanical conditions Interface circuit	Simple (e.g. uniform beam with unimorph / bimorph configuration, simple boundaries)	Complex (e.g. non-uniform beam with complex profile, complex boundaries)
only linear elements included (resistor, capacitor and inductor)	Analytical modeling	System-level FEA
Sophisticated interfaces including nonlinear circuit element (e.g. rectifier, DC-DC converter, ...)	ECM-based modeling (with theoretically determined parameters)	ECM-based modeling (with parameters identified by FEA)

4.6 Chapter Summary

In this chapter, a generic ECM-based modeling method for piezoelectric energy harvesters is developed. The capability and applicability of the proposed model, together with other modeling techniques are compared in Table 4.6. If the maximum achievable power of the system is the concern, the analytical models and system-level FEA can be employed for system evaluation. When the practical energy harvesting interface circuit is considered, the proposed ECM-based method is recommended to address the challenges of accurate modeling of the electromechanical coupling system. The parameters used in the equivalent circuit model can be identified by theoretical analysis or FEA according to different mechanical conditions. Three validation examples are presented and the results demonstrate the accuracy of the proposed ECM-based method and its capability of handling complex structures and sophisticated interface circuits. By exerting the respective advantages of FEA and

SPICE simulations, the ECM-based method supplies a generic solution for the accurate modeling of piezoelectric energy harvesters.

CHAPTER 5 ANALYSIS OF SYNCHRONIZED CHARGE EXTRACTION FOR PIEZOELECTRIC ENERGY HARVESTING

5.1 Introduction

In the past few years, considerable research efforts have been devoted into developing sophisticated energy harvesting interface circuits to improve power transfer from piezoelectric energy harvester (PEH) to electrical loads or energy storage media, such as impedance adaptation interfaces, SSHI and SCE, as summarized in Section 2.3.2. Among them, the SCE technique has been enthusiastically pursued since no effort of load impedance matching is required (Lefeuvre *et al.*, 2007b). In the literature, the SCE technique is investigated based on the uncoupled and in-phase assumptions (Lefeuvre *et al.*, 2007b; Wu *et al.*, 2009). The uncoupled assumption is only valid for weak electromechanical coupling and the in-phase assumption is not applicable for energy harvesting at off-resonance, as discussed in Section 2.3.2.3. As far as the author knows, no accurate analytical solution is available in the literature for the SCE technique. It is thus valuable to derive an accurate solution to further investigate the applicability of the SCE circuit.

On the other hand, electronic simulators such as SPICE are powerful tools for circuit design and optimization. If certain equivalent circuit model of a PEH can be established, the whole energy harvesting system can be modeled in an electronic simulator to predict its performance. Under the uncoupled assumption, a circuit model of a PEH is established as an ideal voltage source in series or an ideal current source in parallel with the internal capacitance of the transducer (Ottman *et al.*, 2002), but such models ignored backward coupling effect on the dynamics of a PEH, as emphasized in Section 2.2.3. The accurate circuit model of a PEH should account for the backward electromechanical coupling effect, which can be derived by the

approach proposed in Chapter 4. With the derived circuit model, the power output from a PEH with different interfaces can be predicted by system-level circuit simulation.

This chapter presents an accurate analytical solution and a circuit simulation method to predict the performance of piezoelectric energy harvesting systems with the SCE technique. The solutions based on the uncoupled and in-phase assumptions are first presented, followed by the derivation of the accurate analytical solution. Based on the accurate solution, the applicability of the SCE technique is investigated for different cases, i.e., the PEH with various degrees of electromechanical coupling and the PEH excited at various frequencies. Circuit simulation is also conducted with an accurate circuit model derived for PEHs and the results validate the analytical outcomes. Both the accurate analytical solution and the circuit simulation show that the SCE technique cannot improve or even reduces the power output at resonance if the coupling of the PEH is not negligible. The SCE technique is found capable of significantly boosting the efficiency of energy harvesting only for the PEH vibrating at off-resonance or with weak coupling.

5.2 Analytical Solutions

In the literature, a PEH is usually designed as a unimorph cantilever, a bimorph cantilever or a cantilever beam bonded with piezoelectric transducers. In the frequency range dominated by the first vibration mode, the system can be simplified as a single-degree-of-freedom (SDOF) system as (Shu and Lien, 2006; Lien *et al.*, 2010),

$$M\ddot{u} + \eta\dot{u} + Ku + \theta V = F \quad (5.1a)$$

$$I + C^S \dot{V} - \theta u = 0 \quad (5.1b)$$

where M , η and K denotes effective mass, damping and stiffness; θ denotes the electromechanical coupling coefficient of the system; F is excitation force which

should properly include the contribution of the distributed mass of the cantilever (Erturk and Inman, 2008a); I is the current from the PEH and V is the voltage across the PEH; C^S is the clamped capacitance and u is the displacement of the PEH.

A standard energy harvesting circuit comprises an AC-DC rectifier, a filtering capacitor C_R and an electrical load R_l in parallel, as shown in Figure 5.1(a). In order to ensure a nearly constant DC voltage V_{DC} , C_R should be large enough such that the time constant $R_l C_R$ is much larger than the oscillating period of the harvester (Shu and Lien, 2006). At the steady state, the rectifier will be blocked when $V < V_{DC}$ and will conduct and transfer energy when V reaches V_{DC} . The waveform of the voltage V across the PEH is shown in Figure 5.1(b) (Lefeuvre *et al.*, 2005).

The main components of the SCE circuit comprise an inductor L_{SCE} , a diode D_{SCE} and a switch S , as shown in Figure 5.2(a). The switch S remains open in most time of one vibration period except for the two instances (t_1 and t_2 in Figure 5.2(b)) when the deflection of the PEH reaches maximum. Once the PEH bends to its maximum deflection, S will be closed for a short time for energy transfer. Hence, the voltage across the PEH at these instances quickly drops to zero. The power consumption for the control circuit to turn on and off the switch can be compensated from less than 5% of the harvested power (Lefeuvre *et al.*, 2006), thus it is not taken into consideration in this Chapter. At the steady state, the voltage waveform is assumed to have the profile shown in Figure 5.2(b) (Lefeuvre *et al.*, 2005). This assumed profile will be verified in later circuit simulation.

Figure 5.1 (a) Standard energy harvesting interface; (b) corresponding voltage and displacement waveform of PEH (figure from Lefeuvre *et al.*, 2005, copyright: SAGE)

Figure 5.2 (a) SCE energy harvesting interface; (b) corresponding voltage and displacement waveform of PEH (figure from Lefeuvre *et al.*, 2005, copyright: SAGE)

5.2.1 SCE Interface Circuit Analysis

To simplify the derivation procedure, the analysis of piezoelectric energy harvesting is usually carried out based on some assumptions. In this section, the power extracted from a PEH using the SCE technique under various assumptions will be summarized and an accurate analytical solution will be derived.

5.2.1.1 Solution Based on Uncoupled Assumption

The uncoupled assumption implies that the electromechanical coupling is extremely weak in the system, i.e., the backward coupling force will not affect the dynamics of the cantilever PEH. Hence, the coupling term in the governing Equation (5.1a) is dropped out as

$$M\ddot{u} + \eta\dot{u} + Ku = F \quad (5.2)$$

Referring to Shu and Lien (2006), the following parameters are introduced to simplify the analysis,

$$\omega_n = \sqrt{\frac{K}{M}}, \quad k_e^2 = \frac{\theta^2}{KC^S}, \quad \zeta = \frac{\eta}{2\sqrt{KM}}, \quad \Omega = \frac{\omega}{\omega_n}, \quad r = C^S \omega_n R_l \quad (5.3)$$

where k_e is the alternative electromechanical coupling coefficient of the PEH; and Ω and r are the normalized frequency and electrical resistance, respectively. For the open circuit condition, from Equations (5.2) and (5.1b), we can easily obtain the magnitude of displacement u_{moc} and voltage V_{moc} as,

$$u_{moc} = \frac{C^S}{\theta} V_{moc} = \left(\frac{F_m}{K} \right) \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \quad (5.4)$$

$$V_{moc} = \left(\frac{F_m}{\theta} \right) k_e^2 \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \quad (5.5)$$

where F_m is the magnitude of the base excitation force. With the uncoupled assumption, the vibration amplitude will not be affected by the electrical load (or energy extraction procedure). Hence, for any circuit interface attached to the PEH, the magnitude of vibration u_m remains constant, i.e., $u_m = u_{moc}$. The charge accumulated from t_1^+ to t_2^- (the switch S is open in this duration.) on the piezoelectric transducer to be extracted at t_2 is

$$Q = \int_{t_1^+}^{t_2^-} \theta \dot{u} dt = \int_{t_1^+}^{t_2^-} C^S \dot{V} dt \quad (5.6)$$

Based on the profile of u and V shown in Figure 5.2(b), the magnitude of voltage across the PEH V_m can be obtained from Equation (5.6) as

$$V_m = 2 \frac{\theta}{C^S} u_m \quad (5.7)$$

It should be mentioned that Equations (5.6) and (5.7) are independent of the uncoupled assumption and hence valid for the in-phase and accurate solutions presented in the later sections. Considering $u_m = u_{moc}$ and Equations (5.4) and (5.5), Equation (5.7) gives

$$V_m = 2V_{moc} = \left(\frac{F_m}{\theta} \right) 2k_e^2 \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \quad (5.8)$$

Obviously, the magnitude of voltage V_m is double of the magnitude of the open circuit voltage V_{moc} . The average power during the half vibration period can be calculated by

$$P_{ave} = \frac{\frac{1}{2} C^S V_m^2}{\frac{T}{2}} = \frac{2\omega C^S}{\pi} V_{moc}^2 = \left(\frac{F_m^2}{M\omega_n} \right) \frac{2k_e^2 \Omega}{\pi} \frac{1}{(1-\Omega^2)^2 + (2\zeta\Omega)^2} \quad (5.9)$$

Written in the non-dimensional form as,

$$\tilde{V}_m = \frac{V_m}{\left(\frac{F_m}{\theta} \right)} \quad \tilde{V}_{moc} = \frac{V_{moc}}{\left(\frac{F_m}{\theta} \right)} \quad \tilde{P}_{ave} = \frac{P_{ave}}{\left(\frac{F_m^2}{M\omega_n} \right)} \quad (5.10)$$

the voltage magnitude and average power finally become

$$\begin{cases} \tilde{V}_m = 2\tilde{V}_{moc} = 2k_e^2 \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \\ \tilde{P}_{ave} = \frac{2k_e^2\Omega}{\pi} \frac{1}{(1-\Omega^2)^2 + (2\zeta\Omega)^2} \end{cases} \quad (5.11)$$

5.2.1.2 Solution Based on In-Phase Assumption

In a more accurate analysis, the coupling term θV should be kept in Equation (5.1a).

For the open circuit condition, from Equation (5.1b), we obtain

$$u_{moc} = \frac{C^S}{\theta} V_{moc} \quad (5.12)$$

Substituting it into Equation (5.1a) gives

$$V_{moc} = \left(\frac{F_m}{\theta} \right) k_e^2 \frac{1}{\sqrt{(1+k_e^2-\Omega^2)^2 + (2\zeta\Omega)^2}} \quad (5.13)$$

The charge Q accumulated from t_1^+ to t_2^- and the magnitude of voltage across the PEH V_m have the same form as Equations (5.6) and (5.7), respectively. However, since the coupling term is kept, the vibration amplitude u_m is affected by the charge extraction procedure, i.e., $u_m \neq u_{moc}$ and hence $V_m \neq 2V_{moc}$. In fact, if the term θV in Equation (5.1a) is moved to the right-hand side, it acts as an additional excitation force induced by the electromechanical coupling. Obviously, when the coupling coefficient θ or V is not negligible, this term will affect the vibration of the PEH. Hence, we cannot simply derive the voltage magnitude V_m and the power output P_{ave} from the open circuit information (u_{moc} , V_{moc}) as we did for the uncoupled solution.

Multiplying Equation (5.1a) by $\dot{u}$ and Equation (5.1b) by V and integrating with respect to time t , we obtain the energy balance equations of the system as follows,

$$\int F \dot{u} dt = \frac{1}{2} M \dot{u}^2 + \frac{1}{2} K u^2 + \int \eta \dot{u}^2 dt + \int \theta V \dot{u} dt \quad (5.14a)$$

$$\int \theta V \dot{u} dt = \frac{1}{2} C^S V^2 + \int V I dt \quad (5.14b)$$

where the five terms in Equation (5.14a) from left to right are the external input energy, kinetic energy, potential energy, mechanical dissipating energy and converted electrical energy; and the two terms at the right-hand side of equation (5.14b) are the energy accumulated in the PEH and energy consumed by the electrical load. At the steady state, during t_1^+ to t_2^- , there is no variation in kinetic and potential energy and no energy consumed by the load. Hence, combining Equations (5.14a) and (5.14b), the energy balance during this half period of vibration (setting $t_1=0$ and $t_2=T/2$) can be expressed as

$$\int_0^{\frac{T}{2}} F \dot{u} dt = \int_0^{\frac{T}{2}} \eta \dot{u}^2 dt + \frac{1}{2} C^S V_m^2 \quad (5.15)$$

At resonance, Lefeuvre *et al.* (2006) assumed that F and $\dot{u}$ are in phase, i.e., the phase between F and u is 90° ,

$$\begin{cases} F = F_m \sin(\omega t) \\ u = -u_m \cos(\omega t) \end{cases} \quad (5.16)$$

This is a relatively accurate assumption for a pure mechanical system. However, for an electromechanical coupled system like PEH vibrates at both resonance and off-resonance frequencies, the applicability of the in-phase assumption requires further investigation. Equation (5.16) is given in such a form that at $t_1=0$ and $t_2=T/2$, the displacement reaches its minimum and maximum, respectively, as shown in Figure 5.2(b). Considering Equation (5.7), substituting Equation (5.16) to Equation (5.15) and integrating over the half period of vibration,

$$F_m u_m \omega \frac{\pi}{2\omega} = \eta \omega^2 u_m^2 \frac{\pi}{2\omega} + \frac{2\theta^2}{C^S} u_m^2 \quad (5.17)$$

we obtain the relationship between the magnitudes of displacement u_m and force F_m as

$$u_m = \frac{F_m}{\eta \omega + \frac{4\theta^2}{C^S \pi}} \quad (5.18)$$

Hence, considering Equation (5.7), the magnitude of voltage and the average power can be obtained as

$$V_m = 2 \frac{\theta}{C^s} \frac{F_m}{\eta\omega + \frac{4\theta^2}{C^s\pi}} = \frac{F_m}{\theta} 2k_e^2 \frac{1}{2\zeta\Omega + \frac{4k_e^2}{\pi}} \quad (5.19)$$

$$P_{ave} = \frac{\frac{1}{2} C^s V_m^2}{\frac{T}{2}} = \frac{2\theta^2}{C^s} \frac{\omega}{\pi} \frac{F_m^2}{\left(\eta\omega + \frac{4\theta^2}{C^s\pi}\right)^2} = \frac{F_m^2}{M\omega_n} \frac{2k_e^2\Omega}{\pi} \frac{1}{\left(2\zeta\Omega + \frac{4k_e^2}{\pi}\right)^2} \quad (5.20)$$

which can be written in the non-dimensional form as

$$\begin{cases} \tilde{V}_m = 2k_e^2 \frac{1}{2\zeta\Omega + \frac{4k_e^2}{\pi}} \\ \tilde{P}_{ave} = \frac{2k_e^2\Omega}{\pi} \frac{1}{\left(2\zeta\Omega + \frac{4k_e^2}{\pi}\right)^2} \end{cases} \quad (5.21)$$

5.2.1.3 Accurate Solution

Even though the result of in-phase analysis is valid for energy harvesting at resonant frequency, they cannot take into account the response of the harvester at off-resonance frequency (detailed discussion is given later). A more accurate procedure is to set an unknown phase angle φ between the force and displacement, that is,

$$\begin{cases} F = -F_m \cos(\omega t + \varphi) \\ u = -u_m \cos(\omega t) \end{cases} \quad (5.22)$$

Equation (5.22) is given in such a form that at $t_1=0$ and $t_2=T/2$, the displacement reaches its minimum and maximum, respectively, as illustrated in Figure 5.2(b). Considering Equation (5.7) and substituting (5.22) into Equation (5.15), we have

$$\int_0^{\frac{T}{2}} -F_m \cos(\omega t + \varphi) u_m \omega \sin(\omega t) dt = \int_0^{\frac{T}{2}} \eta u_m^2 \omega^2 \sin^2(\omega t) dt + \frac{2\theta^2}{C^s} u_m^2 \quad (5.23)$$

The integration over the half period gives

$$F_m \sin \varphi = \eta u_m \omega + \frac{4\theta^2}{C^s\pi} u_m \quad (5.24)$$

There are two unknowns, i.e., u_m and φ , in Equation (5.24), thus we need another equation to solve for them. We follow a similar procedure in Shu and Lien (2006) to establish this equation. Differentiating Equation (5.1a) with respect to t , we obtain

$$M \frac{d}{dt} \ddot{u} + \eta \frac{d}{dt} \dot{u} + K \frac{d}{dt} u + \theta \dot{V} = \frac{d}{dt} F \quad (5.25)$$

From t_1^+ to t_2^- , the switch S is open and the circuit is an open circuit. Hence, Equation (5.1b) is written as

$$\dot{V} = \frac{1}{C^S} (\theta \dot{u}) \quad (5.26)$$

Substituting it into Equation (5.25), we obtain

$$M \frac{d}{dt} \ddot{u} + \eta \frac{d}{dt} \dot{u} + \left(K + \frac{\theta^2}{C^S} \right) \frac{d}{dt} u = \frac{d}{dt} F \quad (5.27)$$

Integrating Equation (5.27) from $t_1=0$ to $t_2=T/2$,

$$Mu_m \omega^2 \cos(\omega t) \Big|_0^{\frac{T}{2}} + \eta u_m \omega \sin(\omega t) \Big|_0^{\frac{T}{2}} - \left(K + \frac{\theta^2}{C^S} \right) u_m \cos(\omega t) \Big|_0^{\frac{T}{2}} = -F_m \cos(\omega t + \varphi) \Big|_0^{\frac{T}{2}} \quad (5.28)$$

we find

$$F_m \cos \varphi = \left(K - M\omega^2 + \frac{\theta^2}{C^S} \right) u_m \quad (5.29)$$

Hence, the sum of square of Equations (5.24) and (5.29) gives

$$\left[\left(K - M\omega^2 + \frac{\theta^2}{C^S} \right) u_m \right]^2 + \left[\eta u_m \omega + \frac{4\theta^2}{C^S \pi} u_m \right]^2 = F_m^2 \quad (5.30)$$

Solving for u_m , we obtain

$$u_m = \frac{F_m}{\sqrt{\left(K - M\omega^2 + \frac{\theta^2}{C^S} \right)^2 + \left(\eta\omega + \frac{4\theta^2}{C^S \pi} \right)^2}} \quad (5.31)$$

Using Equation (5.7), we obtain the magnitude of voltage across the PEH as

$$\begin{aligned} V_m &= 2 \frac{\theta}{C^S} \frac{F_m}{\sqrt{\left(K - M\omega^2 + \frac{\theta^2}{C^S} \right)^2 + \left(\eta\omega + \frac{4\theta^2}{C^S \pi} \right)^2}} \\ &= \frac{F_m}{\theta} 2k_e^2 \frac{1}{\sqrt{\left(1 - \Omega^2 + k_e^2 \right)^2 + \left(2\zeta\Omega + \frac{4k_e^2}{\pi} \right)^2}} \end{aligned} \quad (5.32)$$

and the power harvested as

$$\begin{aligned}
 P_{ave} &= \frac{\frac{1}{2}C^S V_m^2}{\frac{T}{2}} = 2 \frac{\theta^2}{C^S} \frac{\omega}{\pi} \frac{F_m^2}{\left(K - M\omega^2 + \frac{\theta^2}{C^S}\right)^2 + \left(\eta\omega + \frac{4\theta^2}{C^S\pi}\right)^2} \\
 &= \frac{F_m^2}{M\omega_n} \frac{2k_e^2\Omega}{\pi} \frac{1}{(1 - \Omega^2 + k_e^2)^2 + \left(2\zeta\Omega + \frac{4k_e^2}{\pi}\right)^2}
 \end{aligned} \tag{5.33}$$

Finally, Equations (5.32) and (5.33) can be written in non-dimensional forms as

$$\begin{cases} \tilde{V}_m = 2k_e^2 \frac{1}{\sqrt{(1 - \Omega^2 + k_e^2)^2 + \left(2\zeta\Omega + \frac{4k_e^2}{\pi}\right)^2}} \\ \tilde{P}_{ave} = \frac{2k_e^2\Omega}{\pi} \frac{1}{(1 - \Omega^2 + k_e^2)^2 + \left(2\zeta\Omega + \frac{4k_e^2}{\pi}\right)^2} \end{cases} \tag{5.34}$$

Equation (5.34) provides the accurate analytical prediction for the performance of PEHs using the SCE technique.

5.2.2 Standard Interface Circuit

The performance of PEHs with the standard interface under different assumptions has been investigated in the literature (Lefeuvre *et al.*, 2006; Shu and Lien, 2006). These findings are summarized to compare with the performance of the SCE interface.

Solution Based on Uncoupled Assumption

The non-dimensional power delivered on the load with a standard circuit can be found in (Shu and Lien, 2006),

$$\tilde{P} = \frac{k_e^2 r \Omega^2}{\left(r\Omega + \frac{\pi}{2}\right)^2} \frac{1}{(1 - \Omega^2)^2 + (2\zeta\Omega)^2} \tag{5.35}$$

The optimal load and power at the steady state can be obtained by $\partial \tilde{P} / \partial r = 0$ as

$$\begin{cases} r_{opt} = \frac{\pi}{2\Omega} \\ \tilde{P}_{opt} = \frac{k_e^2 \Omega}{2\pi} \frac{1}{(1 - \Omega^2)^2 + (2\zeta\Omega)^2} \end{cases} \quad (5.36)$$

Solution Based on In-Phase Assumption

With the in-phase assumption, the non-dimensional power is described as (Lefeuvre *et al.*, 2005),

$$\tilde{P} = \frac{k_e^2 r \Omega^2}{\left(r\Omega + \frac{\pi}{2}\right)^2} \frac{1}{\left(2\zeta\Omega + \frac{2k_e^2 r \Omega}{\left(r\Omega + \frac{\pi}{2}\right)^2}\right)^2} \quad (5.37)$$

The optimal load and power at the steady state are determined by two cases.

Case (1): $\frac{k_e^2}{\zeta} - 2\pi\Omega \leq 0$. The optimal load and power are,

$$\begin{cases} r_1^{opt} = \frac{\pi}{2\Omega} \\ \tilde{P}_{opt} = \frac{k_e^2 \Omega}{2\pi} \frac{1}{\left(2\zeta\Omega + \frac{k_e^2}{\pi}\right)^2} \end{cases} \quad (5.38)$$

Case (2): $\frac{k_e^2}{\zeta} - 2\pi\Omega > 0$. The optimal load and power are,

$$\begin{cases} r_{2,3}^{opt} = \frac{\left(\frac{k_e^2}{\zeta} - \pi\Omega\right) \pm \sqrt{\left(\frac{k_e^2}{\zeta} - \pi\Omega\right)^2 - (\pi\Omega)^2}}{2\Omega^2} \\ \tilde{P}_{opt} = \frac{1}{16\zeta} \end{cases} \quad (5.39)$$

In Case (2), there exist two optimal loads to achieve the equal optimal power at each frequency.

Accurate Solution

The accurate solution of the power for the PEH using a standard circuit is given in (Shu and Lien, 2006) as,

$$\tilde{P} = \frac{k_e^2 r \Omega^2}{\left(r\Omega + \frac{\pi}{2}\right)^2} \frac{1}{\left(1 - \Omega^2 + \frac{r\Omega k_e^2}{r\Omega + \frac{\pi}{2}}\right)^2 + \left(2\zeta\Omega + \frac{2r\Omega k_e^2}{\left(r\Omega + \frac{\pi}{2}\right)^2}\right)^2} \quad (5.40)$$

Obviously, it is difficult to derive the explicit solution for the optimal load from Equation (5.40). The optimal load and power can be determined numerically. For the strong coupling case, Shu and Lien (2006) also found two equal optimal powers. The two optimal powers are achieved at $\Omega_1^{opt} \approx \Omega_{sc}$ for $r_1^{opt} \ll 1$ and at $\Omega_2^{opt} \approx \Omega_{oc}$ for $r_2^{opt} \gg 1$, where $\Omega_{sc}=1$, $\Omega_{oc}=\sqrt{1+k_e^2}$ are the normalized short circuit and open circuit resonant frequencies, respectively. This is different from Equation (5.39), in which, for each frequency, equal optimal power can be achieved at two optimal loads.

5.2.3 Discussion

We have shown various analytical solutions of the power achieved with the SCE circuit under different assumptions in Section 5.2.1. Considering various degree of electromechanical coupling, we will examine the applicability of the SCE technique for efficiency improvement of PEHs.

5.2.3.1 Weak Coupling

In this case, we assume the alternative coupling coefficient $k_e=0.05$ and damping ratio $\zeta=0.04$. In the literature, k_e^2/ζ has been proposed as an indicator of coupling effect (Shu and Lien, 2006; Shu *et al.*, 2007; Lien *et al.*, 2010) and some criterions have been suggested based on this indicator for different circuit interfaces. For the standard circuit interface, Shu and Lien (2006) defined the strong coupling by $k_e^2/\zeta > 10$ for the appearance of two optimal pairs (electric load and frequency). For the SCE circuit, we propose a criterion for the extent of coupling effect, which will be discussed in

Section 5.2.3.4. Here, $k_e^2/\zeta=0.0625$ corresponds to a weak coupling. Figures 5.3(a)~5.3(c) compare the magnitude of voltage across the PEH V_m when the SCE circuit works with the magnitude of the open circuit voltage V_{moc} . And figures 5.3(d)~5.3(f) compare the power harvested with the SCE and standard circuits from the uncoupled, in-phase and accurate solutions, respectively.

Comparison of Uncoupled and Accurate Solutions

Firstly, since the coupling is weak, V_{moc} and V_m from the uncoupled and accurate solutions are similar, as shown in Figures 5.3(a) and 5.3(c). V_m approaches two times of V_{moc} , and the power harvested with the SCE circuit increases by nearly four times of the optimal power harvested with the standard circuit at various frequencies from the accurate solution (Figure 5.3(c) and 5.3(f)), similar to the uncoupled solution (Figure 5.3(a) and 5.3(d)). These observations show that the SCE circuit is useful to boost the power output if the PEH has a weak electromechanical coupling. In such case, the uncoupled and accurate solutions give close predictions.

Comparison of In-Phase and Accurate Solutions

In Figures 5.3(b) and 5.3(e), V_m and the power harvested obtained by the in-phase solution are totally different from the accurate solution. However, at open circuit resonant frequency $\Omega_{oc} = \sqrt{1+k_e^2}$, it is shown that V_m and the power with the SCE circuit obtained by the in-phase solution are exactly the same as the accurate solution. If we revisit Equations (5.21) and (5.34), the above conclusion is more obvious and can be extended for any degree of coupling. Since the in-phase solution can only be applied to predict the system performance at open circuit resonance, it will not be considered in later discussion.

Figure 5.3 Weak coupling case: (a)~(c) comparison of V_m with SCE interface and V_{moc} ; (d)~(f) comparison of power harvested with SCE and standard interfaces. (a)(d): uncoupled solution; (b)(e): in-phase solution; (c)(f): accurate solution

5.2.3.2 Medium Coupling

In this case, we fix the damping ratio $\zeta=0.04$ but change the coupling coefficient to $k_e=0.3$. Hence, $k_e^2/\zeta=2.25$ is obtained, which corresponds to a medium coupling effect according to the criterion discussed in Section 5.2.3.4. Since the coupling is not weak, there is a frequency shift between the short circuit and open circuit resonances in the accurate solution, as shown in Figure 5.4(b). However, there is no difference between the Ω_{sc} and Ω_{oc} in the uncoupled solution (Figure 5.4(a)) since the coupling term is dropped out in Equation (5.2). Figures 5.4(a) and 5.4(b) compare V_m (the magnitude of voltage across the PEH when the SCE circuit works) and V_{moc} (the magnitude of the open circuit voltage), and Figures 5.4(c) and 5.4(d) compare the power harvested with the SCE and standard circuits from the uncoupled and accurate solutions.

Comparison of Uncoupled and Accurate Solutions

Firstly, consider the PEH vibrating at open circuit resonant frequency. From the accurate solution, V_m cannot be increased to the double of V_{moc} . Actually, it is even less than V_{moc} , as shown in Figure 5.4(b). In terms of power harvested, the SCE circuit achieves a normalized power of 1.5204, failing to significantly outperform the standard circuit, as shown in Figure 5.4(d). However, the uncoupled solution gives the wrong estimation that V_m with the SCE circuit still can reach two times of V_{moc} and the corresponding power is improved by four times compared to the standard circuit, as shown in Figures 5.4(a) and 5.4(c). In fact, by merely examining the performance of standard circuit predicted by the uncoupled and accurate solutions, we can see that the uncoupled solution gives wrong estimation (the optimal powers from uncoupled and accurate solutions are 2.2381 and 1.1992, respectively, as shown in Figures 5.4(c) and 5.4(d)). It is concluded that the uncoupled solution is not applicable when the electromechanical coupling is not negligible. Hence it will not be considered in the later discussion of strong coupling.

Merits of SCE Interface at Off-Resonance

It is observed from Figure 5.4(d) that although the SCE circuit cannot improve the power output at resonance, the performance of PEH vibrating at off-resonance is significantly enhanced. For example, at $\Omega=0.7$, the power with the SCE and the optimal power with the standard circuit are 0.1031 and 0.032, respectively, indicating a power increase of 322% by the SCE. Such phenomenon shows the merits of the SCE circuit for broadening the useful frequency bandwidth of piezoelectric energy harvesting, despite that it does not prominently increase the maximum power output from the PEH.

Figure 5.4 Medium coupling case: (a)~(b) comparison of V_m with SCE interface and V_{moc} ; (c)~(d) comparison of power harvested with SCE and standard interfaces. (a)(c): uncoupled solution; (b)(d): accurate solution

5.2.3.3 Strong Coupling

In this case, we fix the damping ratio $\zeta=0.04$ but further increase the coupling coefficient to $k_e=0.9$. Hence, $k_e^2/\zeta=20.25$ is obtained, which corresponds to a strong coupling effect. Figure 5.5(a) compares V_m and V_{moc} , and Figure 5.5(b) compares the power harvested with the SCE and standard circuits from the accurate solution. With a strong coupling of the PEH, V_m is drastically reduced to a level much smaller than V_{moc} . And it is quite uniform over a wide frequency range, as shown in Figure 5.5(a). Interestingly, although V_m approaches nearly two times of V_{moc} at off-resonance frequency $\Omega=0.7$ (Figure 5.5(a)), the SCE circuit does not show significant improvement in power output compared to the standard circuit, as shown in Figure 5.5(b). Obviously, when compared with the medium coupling case, the frequency at which the SCE circuit significantly outperforms the standard circuit should be further away from the resonances (Ω_{sc} and Ω_{oc}).

Figure 5.5 Strong coupling case: (a) comparison of V_m with SCE interface and V_{moc} ; (b) comparison of power harvested with SCE and standard interfaces from accurate solution

5.2.3.4 Applicable Region of SCE Circuit

In the previous three case studies, the accurate analytical solution shows that the applicability of the SCE circuit becomes much more limited with the increase in the degree of electromechanical coupling. We define the *applicable region* of the SCE circuit as a region where the power harvested by the SCE circuit surpasses that

obtained by the standard circuit. Figures 6.6(a) to 6.6(c) show the contour plots of the power harvested by the SCE and standard circuits versus frequency Ω and coupling coefficient k_e for three given damping ratios $\zeta=0.04, 0.02$ and 0.01 , respectively. The contours with mesh indicate the applicable regions of the SCE circuit, which are projected in the Ω - k_e plane in Figures 5.6(d)~5.6(f). If we define G as the ratio of power harvested by the SCE circuit to the optimal power by the standard circuit, the regions with different blue colors in Figures 5.6(d)~5.6(f) represent $4>G>3$, $3>G>2$ and $2>G>1$.

It is noted that the applicable region ($G>1$) bifurcates at a critical point k_e^* (for example, $k_e^*=0.33$ in Figure 5.6(d)). For $k_e<k_e^*$, the SCE circuit outperforms the standard circuit at any frequency. Conversely, for $k_e>k_e^*$, the standard circuit can outperform the SCE circuit especially near resonance and for larger k_e . Similarly, the region for $4>G>3$ bifurcates at a critical point k_e^{**} (for example, $k_e^{**}=0.12$ in Figure 5.6(d)). For $k_e<k_e^{**}$, at any frequency, the SCE circuit significantly improves the performance of PEH. These critical points shift to left with the decrease of damping ratio, as shown in Figures 5.6(d)~5.6(f). However, we have $(k_e^*)^2/\zeta \approx 2.7$ and $(k_e^{**})^2/\zeta \approx 0.36$ for any damping ratio. Based on these critical points, we propose a criterion to classify the coupling effect of a system using the SCE circuit as follows,

- 1) Strong coupling: $k_e^2/\zeta > 2.7$, i.e., the standard circuit can outperform the SCE circuit around the resonance.
- 2) Weak coupling: $k_e^2/\zeta < 0.36$, i.e., the SCE circuit always significantly outperforms the standard circuit ($4>G>3$) at any frequency.

The values of k_e and ζ have been selected based on this criterion in the previous Sections 5.2.3.1~5.2.3.3 for case studies.

Another phenomenon observed in Figures 5.6(a)~5.6(c) is that, the maximum powers achieved by the SCE and standard circuits are equal, and the SCE circuit can achieve this with a smaller coupling k_e . Besides, with the decrease of damping ratio from 0.04

to 0.02 to 0.01, the maximum power output increases from 1.562 to 3.12 to 6.22, as shown in Figures 5.6(a)~5.6(c).

Figure 5.6 (a)~(c) Power harvested by SCE and standard interfaces versus Ω and k_e ; (d)~(f) applicable region of SCE interface for three damping ratio $\zeta=0.04, 0.02, 0.01$

5.2.3.5 Comments on Analytical Outcomes

The accurate solution shows that (a) for a PEH with weak electromechanical coupling, the SCE technique is able to boost the power output by nearly four times compared to the standard circuit. (b) For medium coupling case, the SCE circuit fails to significantly outperform the standard circuit for PEH vibrating at resonance. However, the performance of PEH at off-resonance is improved prominently by the SCE circuit. This characteristic of SCE circuit can be utilized to broaden the bandwidth of piezoelectric energy harvesting. (c) With further increase of the coupling, the SCE technique dramatically undermines the power output compared to the standard circuit. Although it still can improve the performance when the PEH vibrates at off-resonance, the working frequency should be further away from the resonance.

In addition, the uncoupled solution can provide approximate predictions only when the PEH has a weak electromechanical coupling. Furthermore, for various degrees of coupling, the in-phase solution cannot provide good estimation of the performance of the SCE circuit except for the PEH vibrating at the open circuit resonant frequency.

5.3 Circuit Simulation and Validation

The SPICE simulators (such as Multisim 10.0, <http://en.wikipedia.org/wiki/Multisim>) are powerful tools for circuit design and analysis. Once the equivalent circuit model of PEH is obtained, the whole energy harvesting system with standard or more sophisticated interface can be evaluated by circuit simulation. In Chapter 3, uncoupled equivalent circuit model has been used for electric simulation due to the fact of weakly coupled system. However, since various degree of coupling is concerned in this section, we adopt the accurate equivalent circuit model of PEH derived by the method proposed in Chapter 4. With such circuit model, together with the standard or SCE interface, we conduct system-level circuit simulation to validate the analytical outcomes obtained in Section 5.2.

5.3.1 System Schematic

The validated equivalent circuit model of one fabricated PEH (Figure 4.14) in Section 4.5.3 is reused in the following simulations. The overall circuit diagram consists of the circuit model of PEH, a full-wave rectifier D , the SCE circuit, a filter capacitor C_R and an electrical load R_l , as shown in Figure 5.7. If the SCE part is removed, the system is degraded to a standard energy harvesting circuit. In the SCE circuit, L_{SCE} is used to temporarily store the energy extracted. Each time the beam reaches the maximum deflection, the switch S , controlled by the pulse generator, will conduct and transfer the energy from the PEH to the inductor L_{SCE} . Hence, the period of pulse generator is set to be half of the excitation period, i.e., $T/2$. When S is conducted, the internal capacitance C^S and the L_{SCE} form an LC circuit. Hence, the conduction duration of S should be set to be $T_0/4$ to discharge C^S , where

$$T_0 = 2\pi\sqrt{C^S L_{SCE}} \quad (5.41)$$

We assume $L_{SCE} = 22.5\text{mH}$ in the following simulation to ensure $T_0 \ll T$, such that the energy can be instantly transferred from the PEH. A capacitor of $100\mu\text{F}$ is selected as the filter capacitor C_R to smoothen the voltage across the load R_l . An oscilloscope is used to measure the voltage across the PEH and the load R_l . A wattmeter is used to directly measure the power delivered on the load R_l .

Figure 5.7 Overall circuit diagram of piezoelectric energy harvesting system with SCE technique

5.3.2 Simulation and Validation

5.3.2.1 SCE at Resonance

The system performance with SCE and standard circuits is first investigated at resonance. With the derived parameters, $C^S=366.483\text{nF}$, $N=0.0085$ (analogy of θ) and $C=124\mu\text{F}$ (analogy of $1/K$), the alternative coupling coefficient k_e is obtained as 0.1564. With damping ratio $\zeta=0.011$, we can obtain $k_e^2/\zeta=2.22$, which corresponds to a medium degree of coupling according to the proposed criterion in Section 5.2.3.4. Figure 5.8(a) shows the transient response of the voltage across the PEH when it vibrates at the open circuit resonant frequency f_r^{oc} . The dashed line is the open circuit voltage magnitude V_{moc} obtained from the simulation. The dash-dotted line indicates the accurate analytical solution of V_m (the magnitude of voltage across the PEH with the SCE circuit at the steady state). Similar to the accurate analytical solution, when the PEH vibrates at the steady state, V_m cannot approach two times of V_{moc} and even smaller than V_{moc} . However, if we replace the accurate circuit model of the PEH in the system diagram (Figure 5.7) with the uncoupled circuit model (Figure 2.7(b)), we observe erroneous prediction that V_m approaches two times of V_{moc} , as shown in Figure 5.9. These observations are consistent with the findings obtained from the accurate solution in Section 5.2.3. With different values of R_l , we keep running the simulation until the PEH enters the steady state, and then acquire the power delivered on R_l from the wattmeter. The magnitudes of power by the SCE and standard circuits are shown in Figure 5.8(b). The conjecture in Lefeuvre *et al.* (2006) that the power output by SCE is independent of load is still valid. Besides, like the accurate analytical solution, the simulation results show that the SCE circuit fails to improve the system performance by four times compared to the standard circuit, though it increases the power by 45.2%. Moreover, in Figure 5.8, both V_m and power at the steady state from simulation agree well with the accurate analytical solution, verifying the analytical derivation in Section 5.2.1. In addition, when the SCE circuit works, the enlarged view of voltage profile across the PEH shown at the bottom of Figure 5.8(a) depicts the same pattern as that assumed in Figure 5.2(b).

Figure 5.8 Performance of PEH at f_r^{oc} : (a) transient response of voltage across PEH when SCE circuit works; (b) comparison of power delivered by SCE and standard circuits at steady state

Figure 5.9 Voltage across PEH at f_r^{oc} under uncoupled assumption

5.3.2.2 SCE at Off-Resonance

In Figure 4.15(b), it is noted that at 13Hz, the voltage response dramatically decreases, which is regarded as ‘off-resonance’ status. In this case, the open circuit voltage V_{moc} is obtained as 5.832V from the simulation. The simulation predicts that V_m nearly doubles V_{moc} when SCE circuit works at the steady state, as shown in Figure 5.10(a). Besides, the simulation predicts that the power with the SCE circuit is around 0.517mW and the optimal power with the standard circuit is 0.13724mW, as shown in Figure 5.10(b). This means a nearly four times increase of power by using the SCE circuit. Furthermore, in Figure 5.10(b), it is noted that the predictions by simulation

are smaller than the accurate analytical solution. This is probably due to the loss in the full-wave rectifier. Figure 5.11 indicates that there is a voltage drop of around 0.8V after the rectifier, which is not considered in analytical solution. Since V_m at off-resonance is much smaller than that at resonance, such voltage drop is significant for V_m , which results in the loss of power. The loss of power for SCE circuit is around 15.7% while the loss of optimal power for the standard circuit is around 24.8%. This is reasonable since the SCE circuit enhances the magnitude of voltage across the PEH V_m and hence reduces the loss of voltage as well as the loss of power.

Figure 5.10 Performance of PEH at off-resonance (13Hz): (a) transient response of voltage across PEH when SCE circuit works; (b) comparison of power delivered by SCE and standard circuits at steady state

Figure 5.11 Voltage loss in rectifier

5.3.2.3 SCE for PEH with Weak and Strong Coupling

In Sections 5.3.2.1 and 5.3.2.2, $N=0.0085$ is derived from the fabricated PEH, which corresponds to a medium degree of coupling. To further investigate the performance of SCE interface for weak and strong coupling, N is set to be 0.001 and 0.04, which corresponds to $k_e^2/\zeta=0.0308$ and 49.22, respectively. The other derived parameters in Figure 4.14 are kept unchanged.

For the weak coupling case $k_e^2/\zeta=0.0308$, when the PEH vibrates at the open circuit resonant frequency f_r^{oc} , V_m approaches two times of V_{moc} at the steady state, as shown in Figure 5.12(a). This is similar to what we observed in the accurate analytical solution. In Figure 5.12(b), nearly four times increase of power can be achieved by using the SCE circuit compared to the optimal power by using the standard circuit. For the strong coupling case $k_e^2/\zeta=49.22$, V_m and the power with the SCE circuit are dramatically smaller than those with the standard circuit, as shown in Figure 5.13. Overall, these simulation results are not only consistent with the analytical solution but also validate the conclusions drawn in Section 5.2.3. The reason for the slight discrepancy between the simulation and analytical solution for the weak coupling case (figure 5.12(b)) is the same as what we discussed in Section 5.3.2.2.

Figure 5.12 Performance of PEH with weak coupling at f_r^{oc} : (a) transient response of voltage across PEH when SCE circuit works; (b) comparison of power delivered by SCE and standard circuits at steady state

Figure 5.13 Performance of PEH with strong coupling at f_r^{oc} : (a) transient response of voltage across PEH when SCE circuit works; (b) comparison of power delivered by SCE and standard circuits at steady state

5.4 Chapter Summary

This chapter presented an investigation on the applicability of the SCE circuit for improving the efficiency of piezoelectric energy harvesting. An accurate analytical solution of the PEH with the SCE circuit has been derived and compared with the uncoupled and in-phase solutions. The applicability of the SCE circuit has been investigated for different cases, i.e., the PEH with various degrees of coupling and the PEH excited at resonance or off-resonance frequencies. With the derived accurate circuit model of the PEH in Chapter 4, circuit modeling and simulation have also been conducted to validate the analytical solution. The main findings in this chapter can be concluded as follows:

- ❖ For the PEH with weak coupling, the SCE technique can enhance the power output by nearly four times compared to the standard circuit.
- ❖ For the PEH with medium coupling, the SCE technique cannot significantly improve the performance of the PEH vibrating at resonance. However, it significantly improves the performance of the PEH vibrating at off-resonance. This characteristic of SCE circuit would be useful to broaden the bandwidth for piezoelectric energy harvesting.

- ❖ For the PEH with strong coupling, the SCE technique dramatically undermines the performance of PEH. Although it still can improve the performance of the PEH vibrating at off-resonance, the working frequency should be further away from the resonant frequency.
- ❖ The uncoupled solution of SCE technique can provide approximate predictions only when the PEH has a weak electromechanical coupling. Furthermore, for various degrees of coupling, the in-phase solution cannot provide accurate estimation of the performance of the SCE circuit except when the PEH vibrates at the open circuit resonant frequency.
- ❖ The applicable region of the SCE technique is given for various damping ratios. These plots further confirm that the SCE circuit is useful for prominent efficiency improvement when the PEH's coupling is weak ($k_e^2/\zeta < 0.36$). With the increase of coupling, the applicable region bifurcates for $k_e^2/\zeta > 2.7$ and the applicable frequency range decreases. Based on the applicable region and the critical values of k_e^2/ζ , a criterion is proposed to classify the coupling effect of a system using the SCE circuit. Besides, the maximum power with the SCE and standard circuits is equal but with the SCE circuit, the system can achieve it with a smaller coupling k_e .
- ❖ In general, the results of circuit simulation on the performance of PEH with both the SCE and standard circuits agree well with the accurate analytical solution. The simulations for the PEH with various coupling and vibrating at resonance or off-resonance frequencies validate the outcomes of theoretical analysis on the applicability of the SCE interface circuit.

CHAPTER 6 A MULTIPLE-DOF PIEZOELECTRIC ENERGY HARVESTING SYSTEM

6.1 Introduction

Conventional vibration energy harvesters with cantilever configurations have been usually designed as single-degree-of-freedom (1DOF) systems by exploiting the fundamental mode. The fact that such harvesters are only efficient near sole resonance limits their applicability in frequency-variant or broadband random vibration scenarios. Hence, a critical issue in vibration energy harvesting research involves broadening the operating bandwidth of the harvesters. Various broadband energy harvesting techniques have been reviewed and summarized in Section 2.3.3, among which multimodal energy harvesting schemes are easy to implement. However, the challenges in designing multimodal energy harvesters include: (1) to achieve each mode with effective peak magnitude; (2) to ensure the multiple modes close to each other; (3) to obtain multiple modes with as slight increase as possible in system weight or volume. Unfortunately, most of the reported literature failed to satisfy all the requirements. Some configurations of achieving broad bandwidth, such as cantilever array, significantly sacrifices the power density by increasing weight and volume of the system, making it infeasible in many practical applications.

This chapter presents a novel multiple-DOF piezoelectric energy harvesting system (PEHS) that can satisfy above requirements. First, a 2DOF model is analyzed and its two configurations are characterized. In the first configuration, the piezoelectric element is placed between one mass and the base, and in the second configuration it is placed between the two masses. The advantage of the former is that with slight increase of overall weight to the 1DOF system, we can achieve two close and effective peaks in power response, or one effective peak with significantly enhanced magnitude. The latter can provide two close and enhanced peaks with a huge increase

of system weight, which may not improve or even deteriorate the performance in terms of power density. Even if the power density is not concerned, it can only significantly outperform the 1DOF model with unpractical and unfavorable damping. Based on the characterization of the 2DOF model, the first configuration is generalized to an n -DOF system and its analytical solution is derived. This solution provides a convenient tool for parametric study and design of a multiple-DOF PEHS. Finally, the equivalent circuit model (ECM) of the proposed n -DOF PEHS is developed via the analogies between the mechanical and electric domains. With the ECM, system-level electric simulation can be performed to evaluate the system performance when sophisticated interface circuits are attached, which is usually difficult to address in analytical modeling.

6.2 Analytical Modeling

6.2.1 Conventional 1DOF Model

A PEHS is conventionally modeled as a 1DOF lumped parameter model to simplify the formulation procedure (DuToit *et al.*, 2005; Guyomar *et al.*, 2005; Shu and Lien, 2006; Lien *et al.*, 2010; Tang and Yang, 2011).

Figure 6.1 1DOF model of PEHS

The schematic of 1DOF PEHS under base excitation is depicted in Figure 6.1. Here, a resistor R_l is connected to the electrodes of the piezoelectric element to represent the electric load. The governing equations of the system can be written as,

$$\begin{cases} m_1 \ddot{u}_1 = -k_1(u_1 - u_0) - \eta_1(\dot{u}_1 - \dot{u}_0) - \theta V \\ -\theta(\dot{u}_1 - \dot{u}_0) + C^s \dot{V} + V/R_l = 0 \end{cases} \quad (6.1)$$

where m_1 , k_1 , η_1 are the mass, stiffness and mechanical damping of the system, respectively; C^s is the clamped capacitance of the piezoelectric element; θ is the electromechanical coupling coefficient of the system; V is the voltage across R_l ; thus, $-\theta V$ is the force induced by backward electromechanical coupling in the mechanical equation; u_0 and u_1 are the displacements of base and mass m_1 , respectively. Setting $x=u_1-u_0$ and applying the Laplace transform for (6.1), we have

$$\begin{cases} m_1 s^2 \hat{X} + \eta_1 s \hat{X} + k_1 \hat{X} + \theta \hat{V} = -s^2 m_1 \hat{U}_0 \\ -\theta s \hat{X} + C^s s \hat{V} + \hat{V}/R_l = 0 \end{cases} \quad (6.2)$$

where s is the Laplace variable; the cap denotes a variable or function in the Laplace domain. Solving (6.2) and setting $s=j\omega$, we obtain the dimensionless voltage across R_l as,

$$|\tilde{V}| = \left| \frac{\hat{V}}{\frac{m_1 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{1}{(1 - \Omega^2 + j2\zeta_1 \Omega) \frac{j r \Omega + 1}{j r k_e^2 \Omega} + 1} \right| \quad (6.3)$$

in which the dimensionless parameters are

$$\omega_1 = \sqrt{\frac{k_1}{m_1}}, \zeta_1 = \frac{\eta_1}{2\sqrt{k_1 m_1}}, \Omega = \frac{\omega}{\omega_1}, r = \omega_1 C^s R_l, k_e^2 = \frac{\theta^2}{C^s k_1} \quad (6.4)$$

where r is the dimensional electric load and k_e is the alternative coupling coefficient.

6.2.2 2DOF Model

A conventional 1DOF PEHS is only efficient to harness energy near its sole resonant frequency. This intrinsic drawback of 1DOF systems limits their applicability in the majority of practical vibration scenarios. Though a distributed parameter system such as a cantilever beam with proof mass carries multiple modes, the contribution to energy harvesting from the high-order modes is usually neglected as they are far away from the fundamental one. Hence, a conventional distributed parameter system can be reduced to a single-mode harvester, which is actually an improved 1DOF model with some correction factor introduced to forcing amplitude (Erturk and Inman, 2008a). Obviously, it is preferable not only that an energy harvesting system has multiple

modes or multiple DOFs but also that the corresponding peaks in the response are close to each other and with effective magnitudes to contribute to energy harvesting. To design such a multiple-DOF PEHS is the motivation of this chapter.

We begin the modeling and analysis of multiple-DOF PEHS from the 2DOF lumped parameter model. There are two configurations of the 2DOF model according to the location of the piezoelectric element, as shown in Figure 6.2. The performance of the two configurations will be characterized in the following sections.

Figure 6.2 Two configurations of 2DOF PEHS model

6.2.2.1 Configuration A of 2DOF Model

For Configuration A of the 2DOF model (Figure 6.2(a)), the piezoelectric element is placed between the base and mass m_1 . Such system comprises two subsystems. The primary subsystem, composed of the primary mass m_1 , spring k_1 , damper η_1 and piezoelectric element, is actually the same as the 1DOF model shown in Section 6.2.1. The parasitic subsystem is composed of the parasitic mass m_2 , spring k_2 and damper η_2 . Thus, we can regard the 2DOF model as the combination of original 1DOF model with an attached parasitic subsystem. The governing equations of the system is

$$\begin{cases} m_2 \ddot{u}_2 = -\eta_2 (\dot{u}_2 - \dot{u}_1) - k_2 (u_2 - u_1) \\ m_1 \ddot{u}_1 = -\eta_1 (\dot{u}_1 - \dot{u}_0) - k_1 (u_1 - u_0) - \theta V + \eta_2 (\dot{u}_2 - \dot{u}_1) + k_2 (u_2 - u_1) \\ -\theta (\dot{u}_1 - \dot{u}_0) + C^s \dot{V} + V/R_l = 0 \end{cases} \quad (6.5)$$

Rearranging (6.5) and setting $y = u_2 - u_1$, $x = u_1 - u_0$, we have

$$\begin{cases} m_2 \ddot{y} + \eta_2 \dot{y} + k_2 y = -m_2 \ddot{x} - m_2 \ddot{u}_0 \\ (m_1 + m_2) \ddot{x} + \eta_1 \dot{x} + k_1 x + \theta V + m_2 \ddot{y} + (m_1 + m_2) \ddot{u}_0 = 0 \\ -\theta \dot{x} + C^s \dot{V} + V/R_l = 0 \end{cases} \quad (6.6)$$

Let

$$\omega_1 = \sqrt{\frac{k_1}{m_1}}, \omega_2 = \sqrt{\frac{k_2}{m_2}}, \zeta_1 = \frac{\eta_1}{2\sqrt{k_1 m_1}}, \zeta_2 = \frac{\eta_2}{2\sqrt{k_2 m_2}}, \mu = \frac{m_2}{m_1} \quad (6.7)$$

It should be mentioned that ω_1 and ω_2 are the natural frequencies when the primary and parasitic subsystems work separately. Applying the Laplace transform for (6.6), we obtain

$$\begin{cases} s^2 \hat{Y} + 2\zeta_2 \omega_2 s \hat{Y} + \omega_2^2 \hat{Y} = -s^2 \hat{X} - s^2 \hat{U}_0 \\ (1 + \mu)s^2 \hat{X} + 2\zeta_1 \omega_1 s \hat{X} + \omega_1^2 \hat{X} + (\theta/m_1) \hat{V} + \mu s^2 \hat{Y} + (1 + \mu)s^2 \hat{U}_0 = 0 \\ -\theta s \hat{X} + C^s s \hat{V} + \hat{V}/R_l = 0 \end{cases} \quad (6.8)$$

Solving (6.8), we have

$$\hat{V} = \frac{\left(\frac{\mu s^2}{s^2 + 2\zeta_2 \omega_2 s + \omega_2^2} - (1 + \mu) \right) s^2 \hat{U}_0}{\left((1 + \mu)s^2 + 2\zeta_1 \omega_1 s + \omega_1^2 - \frac{\mu s^4}{s^2 + 2\zeta_2 \omega_2 s + \omega_2^2} \right) \frac{R_l C^s s + 1}{R_l \theta s} + \frac{\theta}{m_1}} \quad (6.9)$$

Setting $s = j\omega$, the dimensionless voltage across the R_l is written as

$$|\tilde{V}| = \left| \frac{\hat{V}}{\frac{m_1 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{(1 + \mu) + \frac{\mu \Omega^2}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)}}{\left(1 - (1 + \mu)\Omega^2 + j2\zeta_1 \Omega - \frac{\mu \Omega^4}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)} \right) \frac{jr\Omega + 1}{jrk_e^2 \Omega} + 1} \right| \quad (6.10)$$

and the dimensionless power is

$$|\tilde{P}| = \left| \frac{\hat{P}}{\frac{m_1 (\omega^2 \hat{U}_0)^2}{\omega_1}} \right| = \frac{rk_e^2 \Omega^2}{(r\Omega)^2 + 1} \left| \frac{(1 + \mu) + \frac{\mu \Omega^2}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)}}{\left(1 - (1 + \mu)\Omega^2 + j2\zeta_1 \Omega - \frac{\mu \Omega^4}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)} \right) + \frac{jr\Omega + 1}{jrk_e^2 \Omega}} \right|^2 \quad (6.11)$$

where the dimensionless parameters are

$$\alpha = \frac{\omega_2}{\omega_1}, \Omega = \frac{\omega}{\omega_1}, r = \omega_1 C^s R_l, k_e^2 = \frac{\theta^2}{C^s k_1} \quad (6.12)$$

Given $\mu \rightarrow 0$, we can obtain the same expression of $|\tilde{V}|$ as (6.3), which implies that the 2DOF model degrades to the 1DOF model.

Two Resonant Frequencies

To be versatile in frequency-variant or random vibrations with known dominant bandwidth, the two resonant frequencies of the 2DOF model are preferred to be close to each other. Thus, we need to understand how to tune the system parameters to achieve this. For open circuit condition (i.e., $r \rightarrow \infty$), the dimensionless open circuit voltage of Configuration A can be derived from (6.10) as,

$$|\tilde{V}_{oc}| = \left| \frac{k_e^2 \left((1+\mu) + \frac{\mu \Omega^2}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)} \right)}{\left(1 - (1+\mu)\Omega^2 + j2\zeta_1 \Omega - \frac{\mu \Omega^4}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)} \right) + k_e^2} \right| \quad (6.13)$$

For undamped condition, the following equation from (6.13)

$$(1 - \Omega^2)(\alpha^2 - \Omega^2) - \mu \alpha^2 \Omega^2 + k_e^2(\alpha^2 - \Omega^2) = 0 \quad (6.14)$$

yields the two undamped open circuit resonant frequencies as

$$\Omega_{1,2} = \sqrt{\frac{(1+\mu)\alpha^2 + (1+k_e^2)}{2} \pm \frac{\sqrt{((\mu+1)\alpha^2 + 1+k_e^2)^2 - 4\alpha^2(1+k_e^2)}}{2}} \quad (6.15)$$

Figure 6.3 $\Delta\Omega_{1,2}$ versus α and μ for different k_e : (a) $k_e=0.02$ (b) $k_e=0.3$ (c) $k_e=0.6$

Figure 6.3 shows the difference of the two resonant frequencies $\Delta\Omega_{1,2}$ versus α and μ for different coupling coefficient k_e . Though $\Delta\Omega_{1,2}$ tends to increase with μ , we can tune α to an optimal value to minimize $\Delta\Omega_{1,2}$ for each specific μ . For $\mu=0$, the 2DOF model is degraded to 1DOF one and thus optimal α approaches $\sqrt{1+k_e^2}$ for $\Delta\Omega_{1,2}=0$. (i.e., one sole resonant frequency). Besides, we note that the optimal α (for minimal $\Delta\Omega_{1,2}$) decreases with the increase of μ and shifts to the right of α axis with the increase of alternative coupling coefficient k_e . According to these results, when designing a 2DOF PEHS, we should carefully select α according to μ and k_e to reduce $\Delta\Omega_{1,2}$. Moreover, with a relatively larger μ , the region near optimal α is much flatter.

Thus, even α is not accurately tuned to its optimum, $\Delta\Omega_{1,2}$ is still very close to its minimum (i.e., the selection of α can be relatively flexible. For example, with $\mu=1$ in Figure 6.3(a), we can freely select α from 0.4 to 0.6).

Effects of α and μ on Peak Magnitude

For a 2DOF model, tuning the two resonant frequencies to be close to each other is only one aspect in system design. To ensure advantageous performance, another aspect is to ensure that both peaks in the power response have significant magnitudes for energy harvesting. Thus, it is equally important to understand how the system parameters affect the magnitudes of the two peaks.

In the following case studies, we select $k_e=0.02$, $\zeta_1=0.02$ and $\zeta_2=0.004$. The dimensionless optimal power (impedance matching is achieved at each frequency) versus Ω and μ for different α is illustrated in Figure 6.4. For the optimal $\alpha\approx 1$ when $\mu\rightarrow 0$, we note in Figure 6.4(b) that the two peaks gradually approach each other and merge into one peak in the 1DOF response. With the increase of μ , the magnitude of the first peak initially decreases then constantly increases, while the magnitude of the second peak monotonically decreases. For $\alpha>1$ (Figure 6.4(a)), the first peak of the 2DOF model degrades to the 1DOF peak when $\mu\rightarrow 0$. With the increase of μ , the magnitude of the first peak initially decreases then constantly increases, while the second peak appears, increases and finally decreases. The trend of the second peak may be difficult to discern in the contour plot (Figure 6.4(a)). However, due to its much smaller magnitude compared to the first peak, the second peak is not concerned. For $\alpha<1$ (Figures 6.4(c) and 6.4(d)), the second peak of the 2DOF model degrades to the 1DOF peak when $\mu\rightarrow 0$. With the increase of μ , the first peak of power appears and monotonically increases, while the second peak monotonically decreases. In general, we can conclude that except for the initial limited range, the magnitude of the first peak increases and the magnitude of the second peak decreases with the increase of μ , regardless of α . Besides, based on these observations, we can conclude that

given $\alpha < 1$ and certain small μ , it is possible to achieve both peaks with equally significant magnitudes. This is one important aspect for the design of a 2DOF system.

Figure 6.4 Dimensionless optimal power output versus Ω and μ for different α . (a) $\alpha=1.05$ (b) $\alpha=1$ (c) $\alpha=0.95$ (d) $\alpha=0.9$

Given specific μ , Figure 6.5 further depicts how the two peaks in power response are affected by α . For a very small μ (e.g. 0.04), it is observed that the magnitudes of both peaks are sensitive to α . Besides, the two peaks can be close and have significant contribution to energy harvesting if α is carefully tuned (e.g., $\alpha = 0.85$ can give two peaks with nearly equal magnitudes). While, for a relative larger μ (e.g. 0.2), the magnitude of the first peak barely varies with α . Though the first peak has much larger magnitude as compared to that for $\mu=0.04$, the second peak is drastically lower than the first peak and thus its contribution to energy harvesting is negligible. In a word, if one's intension is to have one larger peak response, a relatively larger μ can

be selected (i.e., use bigger parasitic mass) and α can be flexibly chosen. While, if one's intension is to have two close peaks with effective magnitudes to contribute to broadband energy harvesting, a relatively small μ should be selected and α should be carefully tuned.

Figure 6.5 Dimensionless optimal power output versus Ω for different α . (a) $\mu=0.04$ and (b) $\mu=0.2$

Besides, compared to the dimensionless power response of the 1DOF model, for $\mu=0.04$, the two peaks of the 2DOF model have the magnitude at the same order, while for $\mu=0.2$, the first peak is significantly enhanced, as shown in Figure 6.5. Here, we should emphasize that these dimensionless power responses are normalized with respect to the mass of the primary subsystem of Configuration A (i.e., m_1). When the mass ratio is not extremely small, a more fair comparison should be based on the power density. Therefore, we provide a further example for quantitative comparison. We assume that the parameters in Configuration A (Figure 6.2(a)) are $m_1=0.04\text{kg}$, $k_1=100\text{Nm}^{-1}$, $\eta_1=0.08\text{Nsm}^{-1}$, $m_2=0.008\text{kg}$, $k_2=14.45\text{Nm}^{-1}$, $\eta_2=0.00272\text{Nsm}^{-1}$, $\theta=3.1623\text{e-}5\text{NV}^{-1}$ and $C^S=25\text{nF}$. The corresponding dimensionless parameters are $\mu=0.2$, $\alpha=0.85$, $k_e=0.02$ and $\zeta_1=0.02$, $\zeta_2=0.004$. Under the excitation level of 1m/s^2 , the first peak in the power response has the magnitude of $250.4\mu\text{W}$ and the power density as 5.22mW/kg . While, for the 1DOF model (m_2 , k_2 and η_2 are removed), the power magnitude and power density at the peak are obtained as $99.04\mu\text{W}$ and 2.48mW/kg , respectively. Obviously, for a relatively large $\mu=0.2$, the power density of the 2DOF

model is still significantly larger than that of the 1DOF model. Furthermore, the above observations of the peak magnitudes of the 1DOF model and Configuration A are only valid with the given damping ratios of $\zeta_1=0.02$ and $\zeta_2=0.004$. Whether they can be extended to other cases with different damping ratios should be further studied, which is given in the following section.

In addition, it should be mentioned that conditions for small $\Delta\Omega_{1,2}$ and for two effective peaks or one enhanced peak may conflict. For relatively large μ , fortunately, the selection of α can be flexible to achieve both small $\Delta\Omega_{1,2}$ and one enhanced peak. For relatively small μ , the values of α to achieve small $\Delta\Omega_{1,2}$ and two effective peaks may be inconsistent. For example, with $\mu=0.04$ and $k_e=0.02$, the minimal $\Delta\Omega_{1,2}$ requires $\alpha=0.96$ while two peaks with equal magnitudes requires $\alpha=0.85$. In such case, certain tradeoff should be made.

Effects of Damping on Peak Magnitude

Other than α and μ , the damping in the system is another factor affecting the magnitudes of the two peaks in power response. We assume $k_e=0.02$, $\alpha=1$. Two cases are studied: (1) $\zeta_2=0.004$ and ζ_1 varies and (2) $\zeta_1=0.02$ and ζ_2 varies.

For Case (1), it is noted in Figure 6.6 that ζ_1 significantly affects the performance of both the 2DOF and the original 1DOF system. Fortunately, the influence of ζ_1 is almost equivalent to both systems. Hence, with various ζ_1 , for $\mu=0.04$, the 2DOF system always exhibits two peaks that have the same level magnitude as the 1DOF system; and for $\mu=0.2$, the first peak of the 2DOF system always has significantly larger magnitude than the 1DOF system. Even in terms of the power density, these conclusions still hold. For Case (2), Figure 6.7 shows that ζ_2 has very minor influence on the performance of the 2DOF system. The 1DOF system does not include damper η_2 thus is not affected by ζ_2 . Based on these results, when designing a 2DOF PEHS, one should devote efforts to reducing ζ_1 to improve the performance of the harvester rather than ζ_2 .

Figure 6.6 Dimensionless optimal power output versus Ω for different ζ_1 . (a) $\mu=0.04$ and (b) $\mu=0.2$

Figure 6.7 Dimensionless optimal power output versus Ω for different ζ_2 . (a) $\mu=0.04$ and (b) $\mu=0.2$

6.2.2.2 Configuration B of 2DOF Model

For Configuration B of the 2DOF model shown in Figure 6.2(b), the piezoelectric element is placed between m_1 and m_2 . Now the primary subsystem of the PEHS comprises the primary mass m_2 , spring k_2 , damper η_2 and piezoelectric element, and the parasitic subsystem comprises parasitic mass m_1 , damper η_1 and spring k_1 . If the parasitic subsystem is removed and the primary subsystem is directly excited by the base, the system is equivalent to the 1DOF model. Configuration B is similar to the 2DOF model introduced by Aldraihem and Baz (2011). The difference is that the

damping η_1 between the base and the parasitic mass m_1 was not included in their model, but it will be considered in our model. In addition, the parasitic subsystem is termed as “dynamic magnifier” by Aldraihem and Baz.

The governing equations of Configuration B are written as,

$$\begin{cases} m_2 \ddot{u}_2 = -\eta_2 (\dot{u}_2 - \dot{u}_1) - k_2 (u_2 - u_1) - \theta V \\ m_1 \ddot{u}_1 = -\eta_1 (\dot{u}_1 - \dot{u}_0) - k_1 (u_1 - u_0) + \theta V + \eta_2 (\dot{u}_2 - \dot{u}_1) + k_2 (u_2 - u_1) \\ -\theta (\dot{u}_2 - \dot{u}_1) + C^s \dot{V} + V/R_l = 0 \end{cases} \quad (6.16)$$

Solving the above equations in a similar way to Configuration A in Section 6.2.2.1, we obtain

$$\hat{V} = \frac{-\frac{2\zeta_1 \omega_1 s + \omega_1^2}{(1+\mu)s^2 + 2\zeta_1 \omega_1 s + \omega_1^2} s^2 \hat{U}_0}{\left(s^2 + 2\zeta_2 \omega_2 s + \omega_2^2 - \frac{\mu s^4}{(1+\mu)s^2 + 2\zeta_1 \omega_1 s + \omega_1^2} \right) \frac{R_l C^s s + 1}{R_l \theta s} + \frac{\theta}{m_2}} \quad (6.17)$$

The dimensionless voltage can be written as

$$|\tilde{V}| = \left| \frac{\hat{V}}{\frac{m_2 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{\frac{\alpha'^2 + j2\zeta_1 \alpha' \Omega'}{\alpha'^2 - (1+\mu)\Omega'^2 + j2\zeta_1 \alpha' \Omega'}}{\left(1 - \Omega'^2 + j2\zeta_2 \Omega' - \frac{\mu \Omega'^4}{\alpha'^2 - (1+\mu)\Omega'^2 + j2\zeta_1 \alpha' \Omega'} \right) \frac{j r' \Omega' + 1}{j r' k_e'^2 \Omega'} + 1} \right| \quad (6.18)$$

and the dimensionless power is

$$|\tilde{P}| = \left| \frac{\hat{P}}{\frac{m_2 (\omega^2 \hat{U}_0)^2}{\omega_2}} \right| = \left| \frac{r' k_e'^2 \Omega'^2}{(r' \Omega')^2 + 1} \frac{\frac{\alpha'^2 + j2\zeta_1 \alpha' \Omega'}{\alpha'^2 - (1+\mu)\Omega'^2 + j2\zeta_1 \alpha' \Omega'}}{\left(1 - \Omega'^2 + j2\zeta_2 \Omega' - \frac{\mu \Omega'^4}{\alpha'^2 - (1+\mu)\Omega'^2 + j2\zeta_1 \alpha' \Omega'} \right) + \frac{j r' k_e'^2 \Omega'}{j r' \Omega' + 1}} \right|^2 \quad (6.19)$$

where the dimensionless parameters are

$$\alpha' = \frac{1}{\alpha} = \frac{\omega_1}{\omega_2}, \Omega' = \frac{\omega}{\omega_2}, r' = \omega_2 C^s R_l, k_e'^2 = \frac{\theta^2}{C^s k_2} \quad (6.20)$$

Given $\mu \rightarrow 0$, we have

$$\left| \frac{\hat{V}}{\frac{m_2 \omega^2 \hat{U}_0}{\theta}} \right| \rightarrow \psi \left| \frac{1}{\left(1 - \Omega'^2 + j2\zeta_2 \Omega' \right) \frac{j r' \Omega' + 1}{j r' k_e'^2 \Omega'} + 1} \right| \quad (6.21)$$

where $\psi = \left| \frac{\alpha'^2 + j2\zeta_1 \alpha' \Omega'}{\alpha'^2 - \Omega'^2 + j2\zeta_1 \alpha' \Omega'} \right|$.

This is similar to the 1DOF response in (6.3) (without parasitic subsystem) except the coefficient ψ . This coefficient is dependent on the dynamics of the parasitic subsystem (dynamic magnifier). In fact, when $\mu \rightarrow 0$, the displacement of mass m_1 is

$$\hat{U}_1 \rightarrow \frac{1 + j2\zeta_1\Omega}{1 - \Omega^2 + j2\zeta_1\Omega} \hat{U}_0 = \frac{\alpha'^2 + j2\zeta_1\alpha'\Omega'}{\alpha'^2 - \Omega'^2 + j2\zeta_1\alpha'\Omega'} \hat{U}_0 \quad (6.22)$$

Hence, if we regard mass m_1 as the base and u_1 as the base displacement, we have

$$\left| \frac{\frac{\hat{V}}{m_2\omega^2\hat{U}_1}}{\theta} \right| \rightarrow \left| \frac{1}{(1 - \Omega'^2 + j2\zeta_2\Omega') \frac{jr'\Omega' + 1}{jr'k_e'^2\Omega'} + 1} \right| \quad (6.23)$$

Thus, Configuration B of the 2DOF system is also degraded to 1DOF model as $\mu \rightarrow 0$.

Two Resonant Frequencies

Similar to Configuration A, for open circuit condition (i.e., $r' \rightarrow \infty$), the dimensionless open circuit voltage of Configuration B can be obtained from (6.18) as,

$$|\tilde{V}_{oc}| = \left| \frac{k_e'^2 \left(\frac{\alpha'^2 + j2\zeta_1\alpha'\Omega'}{\alpha'^2 - (1 + \mu)\Omega'^2 + j2\zeta_1\alpha'\Omega'} \right)}{\left(1 - \Omega'^2 + j2\zeta_2\Omega' - \frac{\mu\Omega'^4}{\alpha'^2 - (1 + \mu)\Omega'^2 + j2\zeta_1\alpha'\Omega'} \right) + k_e'^2} \right| \quad (6.24)$$

For undamped condition, the following equation from (6.24)

$$(1 - \Omega'^2)(\alpha'^2 - \Omega'^2) - \mu\Omega'^2 + k_e'^2(\alpha'^2 - (1 + \mu)\Omega'^2) = 0 \quad (6.25)$$

yields two undamped open circuit resonant frequencies as

$$\Omega'_{1,2} = \sqrt{\frac{(1 + \mu)(1 + k_e'^2) + \alpha'^2}{2}} \pm \sqrt{\frac{((\mu + 1)(1 + k_e'^2) + \alpha'^2)^2 - 4\alpha'^2(1 + k_e'^2)}{2}} \quad (6.26)$$

Figure 6.8 shows the difference of the two resonant frequencies $\Delta\Omega'_{1,2}$ versus α' and μ for different coupling coefficient k_e' . Compared to $\Delta\Omega_{1,2}$ of Configuration A (Figure 6.3), we note that $\Delta\Omega'_{1,2}$ increases faster with μ . Besides, the optimal α' for minimum $\Delta\Omega'_{1,2}$ shifts to the right of axis α' with the increase of k_e' but does not vary with μ , as shown in Figure 6.8. Moreover, the two resonant frequencies can also be tuned close to each other with a small μ . However, contrary to Configuration A, this means that a much heavier parasitic mass m_1 has to be added to the original 1DOF system to achieve this purpose, which not only sacrifices the system performance in terms of

power density but also limits its application. Even we select a practical μ , for example $\mu=1$, the difference of the two resonant frequencies is much larger than that of Configuration A, as compared in Figures 6.3 and 6.8. In a word, Configuration A is preferred to have two close resonant frequencies as compared to Configuration B.

Figure 6.8 $\Delta\Omega'_{1,2}$ versus α' and μ for different k'_e : (a) $k'_e=0.02$ (b) $k'_e=0.3$ (c) $k'_e=0.6$

Effects of α' and μ on Peak Magnitude

Similar to Configuration A, the magnitudes of the two peaks are also concerned. In the following case studies, we assume $k'_e=0.02$, $\zeta_1=0.02$ and $\zeta_2=0.004$. The dimensionless optimal power (impedance matching is achieved at each frequency) for different α' and μ are illustrated in Figure 6.9. Since an extremely small μ is impractical (infinite large mass added to original 1DOF harvester), only $\mu \geq 0.1$ are considered. Different from Configuration A, we note in Figure 6.9 that regardless of α' , the magnitudes of both peaks monotonically decrease with the increase of μ . In other words, a smaller μ can always provide better performance, but the apparent larger power output may be depreciated in terms of the power density. Additionally, to illustrate how α' affects the performance, the power for various α' at specific mass ratios $\mu=0.2$ and $\mu=1$ is shown in Figure 6.10. For both $\mu=0.2$ and $\mu=1$, the magnitude of the first peak is quite sensitive to α' and increases with α' , while the second is insensitive to α' . Since the magnitude of the second peak is substantially smaller than the first one, the overall performance is attributed to the first peak and thus sensitive to α' . Hence, Configuration B always requires a larger α' to ensure a better performance. However, similar to Configuration A, a tradeoff may be needed to ensure a small $\Delta\Omega'_{1,2}$ at the same time.

Figure 6.9 Dimensionless optimal power output versus Ω' and μ for different α' . (a) $\alpha'=1.1$ (b) $\alpha'=1$ (c) $\alpha'=0.9$ (d) $\alpha'=0.8$

Figure 6.10 Dimensionless optimal power output versus Ω' for different α' . (a) $\mu=0.2$ and (b) $\mu=1$

Effects of Damping on Peak Magnitude

Other than α' and μ , the damping effect on the magnitudes of the two peaks in the power response is also our concern. In the following case studies, we assume $k_e'=0.02$, $\alpha'=1$. Two cases are studied: (1) $\zeta_2=0.004$ and ζ_1 varies and (2) $\zeta_1=0.02$ and ζ_2 varies.

For Case (1), Figure 6.11 illustrates the significant influence of ζ_1 on the performance of Configuration B. As compared to the 1DOF response (under base excitation with parasitic subsystem removed), Configuration B cannot guarantee to provide advantageous performance. Neglecting ζ_1 in the modeling may greatly overestimate the performance of Configuration B. For Case (2), since the 1DOF model also includes damper η_2 , we observe in Figure 6.12 that ζ_2 has pronounced influence on the performances of both Configuration B and 1DOF model. However, ζ_2 obviously affects the latter more significantly. In other words, if a small ζ_2 is feasible (a small ζ_2 is always preferred to have large output in practice), Configuration B has no obvious advantage or even worse performance than the 1DOF model, as depicted in Figure 6.12. Configuration B can only provide significantly advantageous performance when a large ζ_2 exists. It should be mentioned that these dimensionless power responses are normalized with respect to the mass of the primary subsystem of Configuration B (i.e., m_2). In terms of the power density, the superiority of Configuration B for amplitude magnification will be further depreciated. In the analytical model of Aldrahem and Baz (2011), the harvester with dynamic magnifier improved the power of 1DOF system by more than 20 times for $\mu=1/5.8$ when ζ_1 is neglected and a very large ζ_2 is applied. However, a negligible ζ_1 could be very difficult to achieve and a large ζ_2 is not preferred for large output. These are probably the reasons that their experimental implementation (Arafa *et al.*, 2011) could only achieve the improvement by around 5 times with the cost of increasing system weight by more than 5 times.

Figure 6.11 Dimensionless optimal power output versus Ω' for different ζ_1 . (a) $\mu=0.2$ and (b) $\mu=1$

Figure 6.12 Dimensionless optimal power output versus Ω' for different ζ_2 . (a) $\mu=0.2$ and (b) $\mu=1$

6.2.2.3 Summary of 2DOF Model

In Sections 6.2.2.1 and 6.2.2.2, we have discussed the characteristics of two configurations of 2DOF model in terms of $\Delta\Omega_{1,2}$ (or $\Delta\Omega'_{1,2}$) and magnitudes of two peaks in their power responses, as summarized in Table 6.1. These characteristics provide important guidelines for designing a 2DOF PEHS. In general, the advantage of Configuration A is that with slight increase of overall weight (parasitic mass added to original 1DOF model), the system can be tuned to provide: (a) two close and effective peaks (though their magnitudes may not be greater that of the 1DOF model),

or (b) one effective peak with significantly larger magnitude than the 1DOF model. Configuration B can only have two close and effective peaks with huge increase of overall weight, which may not help improve and even deteriorate the performance in terms of power density. Even if the power density is not concerned, it can only significantly outperform the 1DOF model when ζ_1 is very small (difficult to achieve) and ζ_2 is very large (not desirable for large power output).

Table 6.1 Summary of two configurations of 2DOF model

	Configuration A	Configuration B
$\Delta\Omega_{1,2}$ (or $\Delta\Omega'_{1,2}$)	<ul style="list-style-type: none"> ◆ A small $\Delta\Omega_{1,2}$ requires a small μ (slight increase of weight to original 1DOF model provides two close resonances) ◆ Slowly increases with μ at optimal α ◆ Optimal α for minimum $\Delta\Omega_{1,2}$ increases with k_e and decreases with μ ◆ For larger μ, $\Delta\Omega_{1,2}$ versus α is flat around optimal α (minor mistuning of α still ensures $\Delta\Omega_{1,2}$ close to minimum) 	<ul style="list-style-type: none"> ◆ A small $\Delta\Omega'_{1,2}$ requires a small μ (huge weight should be added to original 1DOF model for two close resonances) ◆ Fast increases with μ even at optimal α' ◆ Optimal α' for minimum $\Delta\Omega_{1,2}$ increases with k_e' but does not change with μ
Magnitudes of two peaks	<ul style="list-style-type: none"> ◆ Except for the limited range close to $\mu \rightarrow 0$, the magnitude of first peak increases while the second peak decreases with μ. ◆ For a very small μ, the performance is sensitive to α and both peaks can have significant contribution to energy harvesting (though the magnitudes may not be larger than that of the 1DOF model). While for a relatively large μ, the performance is insensitive to α and only the first peak has significant contribution to energy harvesting and can outperform the 1DOF model. ◆ The performance is sensitive to ζ_1 but insensitive to ζ_2. Besides, ζ_1 has nearly equivalent influence on Configuration A and the 1DOF model. 	<ul style="list-style-type: none"> ◆ Generally, the magnitudes of both peaks decrease with μ. ◆ For different μ, the performance is always sensitive to α' (thus α' should be always carefully tuned) and only the first peak significantly contributes to energy harvesting. ◆ The performance is sensitive to both ζ_1 and ζ_2. ζ_2 has more substantial influence on the 1DOF model than that on Configuration B. Hence, Configuration B cannot guarantee an advantageous performance over the 1DOF model unless ζ_1 is negligible and ζ_2 is large. However, in practice, a negligible ζ_1 is very difficult to achieve and small ζ_2 is always preferred for large output.

6.2.3 Generalized n -DOF Model

Based on the characterization of the 2DOF model, Configuration A is preferred in this chapter due to the fact that two close and effective peaks can be achieved with slight increase of overall weight as compared to the original 1DOF system. With more small parasitic masses attached, we can generalize Configuration A of the 2DOF model to

an n -DOF system, as shown in Figure 6.13. It is expected that more effective peaks can be obtained with minor increase of overall weight.

Figure 6.13 n -DOF model of PEHS

The governing equations of the n -DOF PEHS can be written as

$$\begin{cases} m_1 \ddot{u}_1 = -\eta_1 (\dot{u}_1 - \dot{u}_0) - k_1 (u_1 - u_0) - \theta V \\ \quad + \eta_2 (\dot{u}_2 - \dot{u}_1) + k_2 (u_2 - u_1) + \eta_3 (\dot{u}_3 - \dot{u}_1) + k_3 (u_3 - u_1) + \dots + \eta_n (\dot{u}_n - \dot{u}_1) + k_n (u_n - u_1) \\ m_2 \ddot{u}_2 = -\eta_2 (\dot{u}_2 - \dot{u}_1) - k_2 (u_2 - u_1) \\ m_3 \ddot{u}_3 = -\eta_3 (\dot{u}_3 - \dot{u}_1) - k_3 (u_3 - u_1) \\ \vdots \\ m_n \ddot{u}_n = -\eta_n (\dot{u}_n - \dot{u}_1) - k_n (u_n - u_1) \\ -\theta (\dot{u}_1 - \dot{u}_0) + C^S \dot{V} + V/R_l = 0 \end{cases} \quad (6.27)$$

Setting $x = u_1 - u_0$, $y_1 = u_2 - u_1$, $y_2 = u_3 - u_1$, $\dots$, $y_{n-1} = u_n - u_1$ and rearranging (6.27), we have

$$\begin{cases} \sum_{p=1}^n m_p \ddot{x} + \eta_1 \dot{x} + k_1 x + \theta V + \sum_{p=2}^n m_p \ddot{y}_{p-1} + \sum_{p=1}^n m_p \ddot{u}_0 = 0 \\ m_2 \ddot{y}_1 + \eta_2 \dot{y}_1 + k_2 y_1 = -m_2 \ddot{x} - m_2 \ddot{u}_0 \\ m_3 \ddot{y}_2 + \eta_3 \dot{y}_2 + k_3 y_2 = -m_3 \ddot{x} - m_3 \ddot{u}_0 \\ \vdots \\ m_n \ddot{y}_{n-1} + \eta_n \dot{y}_{n-1} + k_n y_{n-1} = -m_n \ddot{x} - m_n \ddot{u}_0 \\ -\theta \dot{x} + C^S \dot{V} + V/R_l = 0 \end{cases} \quad (6.28)$$

Assuming that all parasitic masses have same weight, i.e., $m_2 = m_3 = \dots = m_n$ and setting,

$$\mu = \frac{m_2}{m_1}, \omega_1 = \sqrt{\frac{k_1}{m_1}}, \omega_2 = \sqrt{\frac{k_2}{m_2}}, \dots, \omega_n = \sqrt{\frac{k_n}{m_n}}, \zeta_1 = \frac{\eta_1}{2\sqrt{k_1 m_1}}, \zeta_2 = \frac{\eta_2}{2\sqrt{k_2 m_2}}, \dots, \zeta_n = \frac{\eta_n}{2\sqrt{k_n m_n}} \quad (6.29)$$

we can write

$$\begin{cases} (1 + (n-1)\mu) \ddot{x} + 2\zeta_1 \omega_1 \dot{x} + \omega_1^2 x + (\theta/m_1) V + \mu \sum_{p=2}^n \ddot{y}_{p-1} + (1 + (n-1)\mu) \ddot{u}_0 = 0 \\ \ddot{y}_1 + 2\zeta_2 \omega_2 \dot{y}_1 + \omega_2^2 y_1 = -\ddot{x} - \ddot{u}_0 \\ \ddot{y}_2 + 2\zeta_3 \omega_3 \dot{y}_2 + \omega_3^2 y_2 = -\ddot{x} - \ddot{u}_0 \\ \vdots \\ \ddot{y}_{n-1} + 2\zeta_n \omega_n \dot{y}_{n-1} + \omega_n^2 y_{n-1} = -\ddot{x} - \ddot{u}_0 \\ -\theta \dot{x} + C^S \dot{V} + V/R_l = 0 \end{cases} \quad (6.30)$$

Applying the Laplace transform for (6.30), it is not difficult to find

$$\hat{V} = \frac{\left(-(1+(n-1)\mu) + \mu s^2 \sum_{p=2}^n \frac{1}{s^2 + 2\zeta_p \omega_p s + \omega_p^2} \right) s^2 \hat{U}_0}{\left((1+(n-1)\mu)s^2 + 2\zeta_1 \omega_1 s + \omega_1^2 - \mu s^4 \sum_{p=2}^n \frac{1}{s^2 + 2\zeta_p \omega_p s + \omega_p^2} \right) \frac{R_l C^s s + 1}{R_l \theta} + \frac{\theta}{m_1}} \quad (6.31)$$

Setting $s = j\omega$, the dimensionless voltage across the R_l is obtained as

$$|\tilde{V}| = \left| \frac{\hat{V}}{\frac{m_1 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{(1+(n-1)\mu) + \mu \Omega^2 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega}}{\left(1 - (1+(n-1)\mu)\Omega^2 + j2\zeta_1 \Omega - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right) \frac{j r \Omega + 1}{j r k_e^2 \Omega} + 1} \right| \quad (6.32)$$

and the dimensionless power is

$$|\tilde{P}| = \left| \frac{\hat{P}}{\frac{m_1 (\omega^2 \hat{U}_0)^2}{\omega_1}} \right| = \frac{r k_e^2 \Omega^2}{(r \Omega)^2 + 1} \left| \frac{(1+(n-1)\mu) + \mu \Omega^2 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega}}{\left(1 - (1+(n-1)\mu)\Omega^2 + j2\zeta_1 \Omega - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right) + \frac{j r k_e^2 \Omega}{j r \Omega + 1}} \right|^2 \quad (6.33)$$

where the dimensionless parameters are

$$\alpha_1 = \frac{\omega_2}{\omega_1}, \alpha_2 = \frac{\omega_3}{\omega_1}, \dots, \alpha_{n-1} = \frac{\omega_n}{\omega_1}, \Omega = \frac{\omega}{\omega_1}, r = \omega_1 C^s R_l, k_e^2 = \frac{\theta^2}{C^s k_1} \quad (6.34)$$

Multiple Resonant Frequencies

For open circuit condition (i.e., $r \rightarrow \infty$), the voltage is obtained as,

$$|\tilde{V}_{oc}| = \left| \frac{\hat{V}_1}{\frac{m_1 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{k_e^2 \left((1+(n-1)\mu) + \mu \Omega^2 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right)}{\left(1 - (1+(n-1)\mu)\Omega^2 + j2\zeta_1 \Omega - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right) + k_e^2} \right| \quad (6.35)$$

The undamped open circuit resonant frequencies can be determined with the following equation from (6.35),

$$\left(1 - (1+(n-1)\mu)\Omega^2 - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2} \right) + k_e^2 = 0 \quad (6.36)$$

It is difficult to express the multiple resonant frequencies explicitly from the above equation, but they can be determined with numerical method.

Case Study

In the following case studies, a 3DOF model is analyzed. Firstly, we assume $k_e=0.02$, $\zeta_1=0.02$ and $\zeta_2=\zeta_3=0.004$. In this case and the case studies in Section 6.2.2, we have $k_e^2/\zeta \ll 1$, corresponding to a weak coupling system (Shu and Lien, 2006; Shu *et al.*, 2007). The dimensionless optimal power versus Ω and μ with various combinations of α_1 and α_2 is shown in Figure 6.14. Obviously, the effects of μ , α_1 and α_2 on the three peaks in the power response are more complicated than the 2DOF model. First, for $\alpha_1=1$ or $\alpha_2=1$, two of the three peaks will merge when $\mu \rightarrow 0$, as shown in Figures 6.14(a) and 6.14(b). If μ is not extremely small, for all cases the magnitude of first peak increases while the magnitudes of other two decreases with increase of μ . Moreover, among the four combinations of α_1 and α_2 , only for $\alpha_1=1$ and $\alpha_2 < 1$ (Figure 6.14(b)), all three peaks have significant magnitudes at certain μ (swap between α_1 and α_2 , i.e., $\alpha_2=1$ and $\alpha_1 < 1$ does not affect this conclusion).

With fixed $\alpha_1=1$, Figure 6.15 illustrates the dimensionless optimal power for various α_2 at specific mass ratios $\mu=0.04$ and $\mu=0.2$. The damping is still assumed as $\zeta_1=0.02$ and $\zeta_2=\zeta_3=0.004$. It is observed that, for both $\mu=0.04$ and $\mu=0.2$, only the second peak in power response is quite sensitive to α_2 . When α_2 decreases from 1.1 to 1 then further to 0.8, the second peak declines, disappears (for $\alpha_1=\alpha_2$), reappears and increases. For $\mu=0.04$, though the magnitudes of the three peaks may not be larger than that of the 1DOF model, they can be tuned to have significant values that contribute to energy harvesting (for example, $\alpha_2=0.8$), as shown in Figure 6.15(a). For a relatively large $\mu=0.2$, the first peak contributes most to energy harvesting and its magnitude can drastically surpass that of the 1DOF model. Thus, we can design a multiple-DOF system to have either multiple effective peaks with reasonable magnitudes, or one peak with significantly enhanced magnitude. The former will improve the energy harvesting by broadening the frequency bandwidth and the latter by enhancing the power output.

Figure 6.14 Dimensionless optimal power output of 3DOF model with weak coupling.

 (a) $\alpha_1=1.1, \alpha_2=1$ (b) $\alpha_1=1, \alpha_2=0.8$ (c) $\alpha_1=0.9, \alpha_2=0.8$ (d) $\alpha_1=1.2, \alpha_2=0.8$

Figure 6.15 Dimensionless optimal power output of 3DOF model with weak coupling

 versus Ω for different α_2 . (a) $\mu=0.04$ and (b) $\mu=0.2$

We present a further example regarding strong coupling. All the parameters are the same as those in the previous 3DOF model except $k_e=0.45$. Hence, we have $k_e^2/\zeta > 10$, corresponding to a strong coupling system (Shu and Lien, 2006; Shu *et al.*, 2007). The power response at specific mass ratios $\mu=0.04$ and $\mu=0.2$ are shown in Figure 6.16. Obviously, the main trends of power with different α_2 and μ are similar to those of weak coupling system. The only difference is that the power response near each mode may appear as a flat peak, or split into two peaks as the 1DOF system does in the case of strong coupling.

Figure 6.16 Dimensionless optimal power output of 3DOF model with strong coupling versus Ω for different α_2 . (a) $\mu=0.04$ and (b) $\mu=0.2$

Besides, it should be noted in Figures 6.15 and 6.16 that there are deep valleys between the two neighboring peaks, which are harmful for broadband performance of the system. These valleys correspond to the anti-resonance points of the system. To overcome this limitation, the power reduction near the valleys can be improved by certain sophisticated interface circuits, including SSHI (Shu *et al.*, 2007; Lien and Shu, 2011) and SCE (Tang and Yang, 2011). For instance, in Lien and Shu (2011), for an array configuration of piezoelectric cantilevers with medium coupling, the parallel-SSHI interface could significantly widen the bandwidth by enhancing the responses in off-resonance frequency range.

Finally, it should be mentioned that with more DOFs introduced, the effects of parameters on system behavior become more complicated. Fortunately, the analytical solution given in (6.33) provides a way to investigate the system performance through parametric study, which would greatly assist in designing a multiple-DOF PEHS.

6.3 Equivalent Circuit Modeling

In previous analytical modeling of a multiple-DOF PEHS, a resistor R_l is attached to represent the electric load. In practice, however, the energy harvesting interface circuits can be much more complicated than a resistor. To improve the energy harvesting efficiency, interface circuits such as SSHI (Lien *et al.*, 2010) and SCE (Tang and Yang, 2011) have been enthusiastically pursued. When these sophisticated interfaces are considered, it becomes very challenging to obtain the analytical solution of the multiple-DOF system. System-level circuit simulation is thus resorted to. The equivalent circuit model of the proposed n -DOF PEHS is presented in this section to address this issue.

In (6.27), the current through the resistor is represented by V/R_l . With the consideration of a sophisticated interface, we use I to replace V/R_l . Setting $u_1^* = u_1 - u_0$, $u_2^* = u_2 - u_0, \dots, u_n^* = u_n - u_0$, we rearrange (6.27) as

$$\begin{cases} m_1 \ddot{u}_1^* + \eta_1 \dot{u}_1^* + k_1 u_1^* + \sum_{p=2}^n (\eta_p (\dot{u}_1^* - \dot{u}_p^*) + k_p (u_1^* - u_p^*)) + \theta V + m_1 \ddot{u}_0 = 0 \\ m_2 \ddot{u}_2^* + \eta_2 (\dot{u}_2^* - \dot{u}_1^*) + k_2 (u_2^* - u_1^*) + m_2 \ddot{u}_0 = 0 \\ m_3 \ddot{u}_3^* + \eta_3 (\dot{u}_3^* - \dot{u}_1^*) + k_3 (u_3^* - u_1^*) + m_3 \ddot{u}_0 = 0 \\ \vdots \\ m_n \ddot{u}_n^* + \eta_n (\dot{u}_n^* - \dot{u}_1^*) + k_n (u_n^* - u_1^*) + m_n \ddot{u}_0 = 0 \\ -\theta \dot{u}_1^* + C^S \dot{V} + I = 0 \end{cases} \quad (6.37)$$

Considering the analogies between the mechanical and electric domains as listed in Table 6.2, (6.37) can be further written as

$$\begin{cases} L_1 \ddot{q}_1 + R_1 \dot{q}_1 + q/C_1 + \sum_{p=2}^n (R_p (\dot{q}_1 - \dot{q}_p) + (q_1 - q_p)/C_p) + NV + v_1 = 0 \\ L_2 \ddot{q}_2 + R_2 (\dot{q}_2 - \dot{q}_1) + (q_2 - q_1)/C_2 + v_1 = 0 \\ L_3 \ddot{q}_3 + R_3 (\dot{q}_3 - \dot{q}_1) + (q_3 - q_1)/C_3 + v_3 = 0 \\ \vdots \\ L_n \ddot{q}_n + R_n (\dot{q}_n - \dot{q}_1) + (q_n - q_1)/C_n + v_n = 0 \\ -N \dot{q}_1 + C^s \dot{V} + I = 0 \end{cases} \quad (6.38)$$

which represents the constitutive equations of the circuit network shown in Figure 6.17. The first equation in (6.38) represents the main loop of the equivalent circuit model, satisfying the Kirchhoff voltage law. The second to n^{th} equations in (6.38) represent the $n-1$ parasitic circuit loops (equivalent to $n-1$ parasitic subsystems in Figure 6.13), independent of each other but coupled to the main circuit loop. They also satisfy the Kirchhoff voltage law. The last equation in (6.38) represents the circuit part at the right-hand-side of the transformer (Figure 6.17), satisfying the Kirchhoff current law. With this equivalent circuit model shown in Figure 6.17, the system performance with any type of energy harvesting interface circuits attached can be evaluated through system-level electric simulation.

Table 6.2 Analogies between mechanical and electrical domains

Mechanical Parameters	Equivalent Circuit Parameters
Relative coordinate $u_1^*, u_2^*, \dots, u_n^*$	Charge $q_1, q_2, \dots, q_n$
Relative velocity $\dot{u}_1^*, \dot{u}_2^*, \dots, \dot{u}_n^*$	Current $i_1, i_2, \dots, i_n$
Mass $m_1, m_2, \dots, m_n$	Inductor $L_1, L_2, \dots, L_n$
Damper $\eta_1, \eta_2, \dots, \eta_n$	Resistor $R_1, R_2, \dots, R_n$
Reciprocal of spring stiffness $1/k_1, 1/k_2, \dots, 1/k_n$	Capacitor $C_1, C_2, \dots, C_n$
Inertia force on mass $m_1 \ddot{u}_0, m_2 \ddot{u}_0, \dots, m_n \ddot{u}_0$	Ideal voltage source $v_1, v_2, \dots, v_n$
Electromechanical coupling θ	Turn ratio of ideal transformer N

Figure 6.17 Equivalent circuit model of proposed n -DOF PEHS with sophisticated interfaces

Validation Case

A 3DOF PEHS is again taken as example to validate the proposed equivalent circuit modeling method, with the parameters $m_1=1\text{kg}$, $m_2=m_3=0.04\text{kg}$, $k_1=400\text{Nm}^{-1}$, $k_2=16\text{Nm}^{-1}$, $k_3=10.24\text{Nm}^{-1}$, $\eta_1=0.8\text{Nsm}^{-1}$, $\eta_2=0.0064\text{Nsm}^{-1}$, $\eta_3=0.00512\text{Nsm}^{-1}$, $\theta=6.325\text{e-}5\text{NV}^{-1}$ and $C^S=25\text{nF}$. The corresponding dimensionless parameters are obtained as $\mu=0.04$, $\alpha_1=1$, $\alpha_2=0.8$, $\zeta_1=0.02$, $\zeta_2=\zeta_3=0.004$, $k_e=0.02$. A unity root mean square (RMS) base acceleration is assumed. Recalling the analogies between the mechanical and electric domains, the equivalent circuit parameters and the circuit model of the 3DOF PEHS are depicted in Figure 6.18(a). By electric simulation in SPICE software, the open circuit voltage is evaluated and compared with the analytical result, as shown in Figure 6.18(b). The simulation result is observed exactly consistent with the analytical solution. Hence, with the proposed equivalent circuit model, the system performance with AD/DC standard (Shu and Lien, 2006), SCE (Tang and Yang, 2011) or SSHI interfaces (Lien *et al.*, 2010) can be evaluated.

Figure 6.18 Evaluation of dimensionless open circuit voltage of a 3DOF PEHS: (a) equivalent circuit model and (b) comparison between equivalent circuit simulation and analytical solution.

6.4 Chapter Summary

In this chapter, a multiple-DOF PEHS under base excitations is proposed and analyzed. The main findings of this chapter can be summarized as follows:

- ❖ First, a 2DOF model is analyzed and its two configurations are characterized. For each configuration, parametric study is conducted to evaluate and characterize the

system performance in terms of $\Delta\Omega_{1,2}$ (or $\Delta\Omega'_{1,2}$) and magnitudes of two peaks in their power response, as summarized in Table 6.1. In general, the advantage of Configuration A is that with slight increase of overall weight to conventional 1DOF system, it can be tuned to provide two close and effective peaks, or one peak with significant enhanced magnitude. Configuration B can only have two close and enhanced peaks with a huge increase of overall weight, which may not help improve or even deteriorate the performance in terms of power density. Even if the power density is not concerned, it can only significantly outperform the 1DOF model when ζ_1 is very small (which is unable or difficult to achieve in practice) and ζ_2 is very large (which is not preferred for efficient power output). Based on the characterization of the 2DOF model, Configuration A is preferred and generalized to an n -DOF PEHS and its analytical solution of power output is derived. With this n -DOF model for parametric study, we can design a harvester to have multiple close and effective peaks, or to have one peak with significantly enhanced magnitude, with only slight increase of system weight.

- ❖ The equivalent circuit model of the proposed n -DOF PEHS is also developed via the analogies between the mechanical and electric domains. With such model, system-level electric simulation can be performed to evaluate the system performance when sophisticated interface circuits are attached, which is usually difficult to address by analytical modeling.

In the future, a prototype of the proposed multiple-DOF piezoelectric energy harvester will be devised and tested for practical vibration scenarios.

CHAPTER 7 IMPROVING FUNCTIONALITY OF VIBRATION ENERGY HARVESTERS USING MAGNETS

7.1 Introduction

Vibration energy harvesters designed as linear mechanical resonators are only robust in a narrow bandwidth near their natural frequencies. With a slight drift from the resonance, their performances will decrease drastically. Unfortunately, the majority of practical vibration sources are frequency-variant or totally random with energy distributed over wide bandwidth. In some other scenarios, such as human movements and wave heave motions, it is not feasible to match the frequency of harvester with these ultra-low-frequency vibration sources. This chapter presents a comprehensive experimental study on the use of magnets to improve the functionality of energy harvesters under these practical vibration scenarios. Different from the methods summarized in Sections 2.3.3 and 2.3.4, the work in this chapter further investigates the optimal configurations in nonlinear energy harvesting and frequency up-conversion techniques, and at the same time addresses some limitations in previously reported studies, such as the high energy barrier issue in bistable nonlinear energy harvesters.

Firstly, nonlinearities introduced by the magnets are exploited to improve the performance of energy harvesting. Both monostable and bistable configurations are investigated under sinusoidal and random vibrations with various excitation levels. The nonlinear energy harvester is demonstrated to be able to outperform its linear counterpart except for the monostable softening configuration and the bistable configuration with too closely placed magnets. Under practical random vibrations, the optimal nonlinear configuration is found near the monostable-to-bistable transition region. Results show that both monostable and bistable nonlinear configurations can significantly outperform the linear harvester near this transition region. Such

performance enhancement of nonlinear configurations is attributed to two aspects: the increased output magnitude in the frequency domain for low-level excitations and the enlarged bandwidth for high-level excitations. For the bistable configuration, this finding helps avoid the difficulty of overcoming the high energy barrier.

Secondly, for ultra-low-frequency scenarios such as wave heave motions, a frequency up-conversion mechanism using magnets is proposed. Different configurations under various wave conditions are investigated by parametric study. The repulsive magnetic configuration has not been reported in the literature for frequency up-conversion devices. In this work, the repulsive configuration of magnets is found to be favorable in the frequency up-conversion technique, and its capability can be saturated and insensitive to wave conditions when the magnets are placed sufficiently close.

7.2 Exploiting Nonlinearities Using Magnets

At the very beginning when permanent magnets were used in energy harvesting devices, they played the role of vibration-to-electricity transduction obeying Faraday's law (electromagnetic transduction). Later, in order to enable the energy harvesters to adapt in frequency-variant circumstances, magnetic forces were introduced to tune the structural resonance of the energy harvesters (refers to Section 2.3.3.1 in Chapter 2). A typical resonance-tunable energy harvester and the layout of experiment apparatus are shown in Figure 7.1. Two piezoelectric transducers are bonded at the root of a cantilever aluminum beam to form a bimorph harvester. Two NdFeB permanent magnets with diameter of 10mm, thickness of 5mm and surface flux of 3500 gauss are held in plastic holders and spanned at a distance D . With the arrangement of attractive or repulsive magnets at various distances, the stiffness of the bimorph cantilever can be altered accordingly and thus its resonant frequency tuned.

Figure 7.1 (a) Resonance-tunable energy harvester and (b) layout of experiment apparatus

The sweep responses of open circuit voltage V_{OC} under low-level sinusoidal excitations ($a_{RMS}=0.2\sim0.5m/s^2$) are shown in Figure 7.2. The resonance can be tuned bi-directionally with repulsive or attractive magnets. Since the off-resonance responses of some configurations are quite small, different excitation levels are applied for different configurations to ensure accurate reading and V_{OC} in Figure 7.2 are normalized by acceleration. It is noted that the harvester coupled with magnetic forces still behaves like a linear one under low-level excitations.

Figure 7.2 Resonance tuning under low-level excitations

Here the resonance is manually tuned. Although the feedback-loop control has been successfully implemented for automatic tuning as reported by Zhu *et al.* (2008) and Challa *et al.* (2011), the huge energy consumption for actuators requires the harvester to accumulate enough energy for a long time before starting one tuning process,

which is unfeasible if the excitations vary from time to time or present in the form of random vibrations. Hence, it is preferable if broadband energy harvesting can be achieved in a passive mode (no extra energy injection is required).

With the increase of excitation level, nonlinear behaviors will appear. The experiment setup for the nonlinear harvester is the same as that shown in Figure 7.1. The system can be modeled as a nonlinear pendulum with dipole-dipole magnetic interaction. The potential energy of the system can be expressed as follows (Cottone *et al.*, 2009; Andò *et al.*, 2010):

$$U(x) = \frac{1}{2}K_0x^2 + \frac{\mu_0 m_1 m_2}{2\pi d} (d^2x^2 + D^2)^{-\frac{3}{2}} \quad (7.1)$$

where x represents the deflection of the harvester; K_0 ($K_0 > 0$) is the effective stiffness of the initial linear configuration (without magnets); $\mu_0 = 4\pi \times 10^{-7} \text{NA}^{-2}$ is the permeability constant; m_1 and m_2 are the effective magnetic moment; d is a geometrical parameter related to the point of measuring the displacement x and the cantilever length; and D is the distance between the two magnets. For the prototype in our experiment, $d=1$, $m_1=m_2=m=0.2\text{Am}^2$ for repulsive arrangement, $m_1=-m_2=m=0.2\text{Am}^2$ for attractive arrangement and $K_0=226\text{Nm}^{-1}$ (calculated from the static deflection due to a concentrated transverse force at the free end of the cantilever beam by finite element analysis). Thus, the potential energy for different magnet configurations is shown in Figure 7.3.

Figure 7.3 Theoretical prediction of potential energy $U(x)$ for different magnet configurations

To further understand the nonlinearity introduced by the magnets, we expand the second term in Equation (7.1) as a Maclaurin series,

$$\frac{\mu_0 m_1 m_2}{2\pi d} (d^2 x^2 + D^2)^{-\frac{3}{2}} = \frac{\mu_0 m_1 m_2}{2\pi d} \left(D^{-3} - \frac{3}{2} d^2 D^{-5} x^2 + \frac{15}{8} d^4 D^{-7} x^4 \right) + O(x^4) \quad (7.2)$$

Ignoring the high-order terms, the potential energy function can be approximated as

$$U(x) \approx \frac{1}{2} (K_0 + K_1) x^2 + \frac{1}{4} a x^4 + C \quad (7.3)$$

where

$$K_1 = -\left(\frac{\mu_0 m_1 m_2}{2\pi} 3dD^{-5} \right), \quad a = \frac{\mu_0 m_1 m_2}{2\pi} \frac{15}{2} d^3 D^{-7}, \quad C = \frac{\mu_0 m_1 m_2}{2\pi d} D^{-3} \quad (7.4)$$

and the spring force of the system can be approximated as

$$F(x) \approx (K_0 + K_1)x + ax^3 \quad (7.5)$$

where $K_1 x$ and ax^3 are the linear and nonlinear components of the spring force introduced by the magnets, respectively. For the two attractive magnets, we have $K_1 = (\mu_0 m^2 / 2\pi) 3dD^{-5} > 0$ and $a = -(\mu_0 m^2 / 2\pi) \frac{15}{2} d^3 D^{-7} < 0$, that is, the linear part of the magnetic force stiffens the harvester but the cubic nonlinear part induces a “softening” nonlinear behavior. While for two repulsive magnets, we have $K_1 = -(\mu_0 m^2 / 2\pi) 3dD^{-5} < 0$ and $a = (\mu_0 m^2 / 2\pi) \frac{15}{2} d^3 D^{-7} > 0$. For a large D , $K_0 + K_1 > 0$ still holds, thus the harvester has a reduced linear stiffness and a “hardening” nonlinear behavior. In these cases, only one stable equilibrium exists at $x=0$ in the potential energy (Figure 7.3). Hence, the configuration providing softening or hardening behavior is termed “monostable configuration”. To avoid any confusion, from here onwards in this chapter, “softening” and “hardening” refer to the nonlinear behavior of the harvester rather than the change in linear stiffness. This is consistent with the definition given in the literature (Ferrari *et al.*, 2010; Stanton *et al.*, 2009; Ramlan *et al.*, 2010). When D decreases and passes a critical value, negative linear stiffness ($K_0 + K_1 < 0$) appears, resulting in two stable equilibriums in the potential energy, separated by the unstable equilibrium at $x=0$ (Figure 7.3). Such nonlinear configuration is termed “bistable configuration”. By letting $K_0 + K_1 = 0$, this critical distance between two repulsive magnets D^* can be expressed as $D^* = (3d\mu_0 m^2 / 2\pi K_0)^{\frac{1}{5}}$.

Using the parameters given previously, the theoretical value of D^* is obtained as $\approx 10\text{mm}$ for the prototype in our experiment. While in the experiment, D^* is achieved around 9.0 to 9.5mm. The discrepancy between the theoretical and experimental values may result from the assumption of dipoles for magnets.

7.2.1 Monostable Configuration

This section discusses the nonlinear softening and hardening responses introduced by magnets. It can achieve broadband energy harvesting in a totally passive mode, different from the active resonance tuning method. Tests under both sinusoidal and random excitations are performed.

7.2.1.1 Monostable Configuration under Sinusoidal Excitations

The voltage outputs for a monostable hardening configuration (repulsive magnets with a gap $D=9.5\text{mm}$) and a monostable softening configuration (attractive magnets with a gap $D=7.5\text{mm}$) under three sinusoidal excitation levels are illustrated in Figure 7.4. Both upward and downward sinusoidal sweeps are conducted for each nonlinear case and compared with that from the linear configuration.

In the monostable hardening case (Figure 7.4(a)), for low-level excitation ($a_{\text{RMS}}=0.5\text{m/s}^2$) and thus small response x , the nonlinear part of the spring force induced by magnets (Equation (7.5)) is negligible. Only the linear part affects the behavior of harvester, which can be used for resonance tuning (Figure 7.2). Hence, the peak of response curve is slightly shifted (i.e., the harvester behaves like a linear one) and the upward and downward sweeps do not create remarkable difference in responses (top of Figure 7.4(a)). With the increase of excitations to $a_{\text{RMS}}=1.4\text{m/s}^2$ (bottom of Figure 7.4(a)), the response curve is significantly bent to right, providing useful voltage up to 15Hz by upward sweep. After 15Hz, the harvester cannot maintain the large-amplitude oscillations and jumps to the stable low-energy orbit. At the same excitation level ($a_{\text{RMS}}=1.4\text{m/s}^2$), downward sweep can re-capture the

high-energy orbit at 14Hz. In other words, between 14Hz to 15Hz, high-energy and low-energy orbits co-exist. The high energy orbit is unstable, and can only be captured by upward sweep (or by some disturbance).

In the monostable softening case (Figure 7.4(b)), similar to the hardening case, no difference is observed between upward and downward sweep at low-level excitations, and the response curves are slightly bent. With the increase of excitation level to $a_{\text{RMS}}=1.4\text{m/s}^2$, opposite to the hardening case, the peak of response shifts to the left and can reach as far as 40Hz by downward sweep. The upward sweep can re-capture the large-amplitude oscillations at around 40.5Hz. Hence 40 to 40.5Hz is the range where high-energy and low-energy orbits co-exist.

Figure 7.4 Sinusoidal sweep responses of monostable configurations versus linear configuration for various excitation levels: (a) monostable hardening case ($D=9.5\text{mm}$);
(b) monostable softening case ($D=7.5\text{mm}$)

Obviously, since the response curves of monostable configurations are bent with the increase of excitation level, useful voltage output covers wider frequency ranges as compared to a linear configuration. For example, at the voltage level of 3V under $a_{\text{RMS}}=1.4\text{m/s}^2$, the monostable hardening configuration by upward sweep provides a bandwidth of 11.2Hz~15Hz, while the linear configuration only provides a bandwidth of 25Hz~26.3Hz (bottom of Figure 7.4(a)). However, in terms of the voltage

magnitude under sinusoidal excitation, both hardening and softening configurations show no superiority to the linear configuration. Moreover, the wider bandwidth must be achieved with proper sinusoidal sweep. Hence, it cannot be concluded from the results of sinusoidal excitations that a monostable configuration can outperform its linear counterpart. Further study on its performance under random excitations is needed, and is presented in the next section.

7.2.1.2 Monostable Configuration under Random Excitations

The majority of the vibration sources in our daily life presents in random patterns with energy distributed mostly in low frequency range (<200Hz) and with a relatively flat acceleration spectrum (Roundy *et al.*, 2003).

Figure 7.5 A sample of random control signal input in shaker controller

$$(a_{\text{RMS}}=2.5\text{m/s}^2)$$

In this section, the acceleration amplitude of random excitations is assumed to be uniformly distributed over a bandwidth of 7Hz~100Hz, as shown in Figure 7.5. Such profile is close to the vibration spectra for office windows next to a busy street (refers to Figure 2 in Roundy *et al.* (2003)). The efficiencies of different monostable configurations are investigated at three random excitation levels, namely, $a_{\text{RMS}}=2.5\text{m/s}^2$, 5m/s^2 and 10m/s^2 . Figure 7.6 compares the open circuit voltage V_{OC} in time domain from various monostable configurations and linear harvester. The responses of the softening configuration (attractive magnets) are observed to be always smaller than those of the linear configuration at various excitation levels. On

the other hand, the responses of hardening configurations can be significantly larger than that of the linear harvester, especially when the magnets is closely placed ($D=9.5mm$).

Figure 7.6 V_{OC} of linear and monostable configurations under random excitations

The energy storage circuit is also considered here to evaluate from another perspective the energy harvesting performance of different configurations. Rectifying the output from the harvester by an AC/DC full-wave bridge and charging a storage capacitor of $33\mu F$, the voltages developed across the capacitor for various configurations and excitation levels are compared in Figure 7.7. The time duration of charging is set to be sufficiently long (45 seconds) to ensure that the ultimate voltage developed across the storage capacitor under random excitations is nearly repeatable. From here onwards, we refer to charging the $33\mu F$ capacitor for 45 seconds when considering the energy storage circuit. In Figure 7.7, the softening configuration is found to be unable to outperform while the monostable configurations successfully outperform the linear configuration. For the hardening configuration with repulsive magnets arranged closely ($D=9.5mm$), the capacitor is charged fastest among the four configurations. If the magnet span is larger ($D=10.5mm$), the hardening configuration has no substantial advantage over the linear configuration under large excitation level, as shown in Figure 7.7(c). These conclusions drawn from Figure 7.7 are consistent with the observations of V_{OC} responses in Figure 7.6.

Figure 7.7 Voltage developed across storage capacitor for linear and monostable configurations under random excitations

To investigate the bandwidth of monostable configuration, fast Fourier transform (FFT) is conducted and the V_{OC} spectra of different configurations are shown in Figure 7.8. From Figure 7.8, we can explain why the hardening configuration (especially for $D=9.5mm$) can outperform its linear counterpart. Applying the Plancherel theorem (Bracewell, 1978),

$$\int_0^{+\infty} |V_{oc}(t)|^2 dt = \int_0^{+\infty} |\hat{V}_{oc}(\omega)|^2 d\omega \quad (7.6)$$

the Fourier transform preserves the energy of the original quantity. Under low-level excitations, the hardening configuration has no obvious wider bandwidth than the linear configuration, but the larger magnitude is achieved in the frequency domain (Figure 7.8(a)), which means that $\int_0^{+\infty} |\hat{V}_{oc}(\omega)|^2 d\omega$ for the hardening configuration is larger than that for the linear configuration. Hence, according to the Plancherel theorem, it is no surprise that the hardening configuration has a larger $\int_0^{+\infty} |V_{oc}(t)|^2 dt$ and thus larger magnitude of V_{OC} in the time domain (comparing Figures 7.6(d) and

7.6(a)). Under high-level excitations, the hardening configuration (especially for $D=9.5\text{mm}$) has no obvious larger magnitude in the frequency domain, but the wider bandwidth is observed (Figure 7.8(c)). This also means that $\int_0^{+\infty} |\hat{V}_{OC}(\omega)|^2 d\omega$ for the hardening configuration is larger than that of the linear configuration, and thus a larger $\int_0^{+\infty} |V_{OC}(t)|^2 dt$ and V_{OC} magnitude in the time domain (comparing Figures 7.6(f) and 7.6(c)). Finally, the higher magnitude of V_{OC} in the time domain contributes to the higher energy harvesting rate when charging the capacitor (Figure 7.7(c)).

Figure 7.8 V_{OC} spectra (FFT) of linear and monostable configurations under random excitations

7.2.1.3 Comments on Monostable Configuration

Monostable nonlinear energy harvester can be implemented by using twin repulsive or attractive magnets. Under low-level sinusoidal excitations, the nonlinear part of the magnetic force is negligible while the linear part causes the change of linear stiffness of the harvester that can be used for resonance tuning. Under high-level sinusoidal excitations, high-energy and low-energy orbits co-exist in certain frequency range. Wider bandwidth for energy harvesting can be achieved by proper sweep or by disturbance, especially for the hardening configuration. Under practical broadband random excitations, monostable hardening configuration is demonstrated to be able to

significantly outperform the linear configuration, especially when the repulsive magnets are closely placed ($D=9.5mm$). The improved performance can be attributed to two aspects: the increased output magnitude in the frequency domain under low-level excitations and the enlarged bandwidth under high-level excitations.

7.2.2 Bistable Configuration

The experiment setup for bistable configuration is the same as that for the monostable hardening configuration. By decreasing the gap D between the repulsive magnets to a certain threshold D^* , negative linear stiffness ($K_1+K_0<0$) and double stable equilibriums appear as shown in Figure 7.3. That is, by reducing the gap D , the harvester can move from a monostable configuration to a bistable one. If the harvester can oscillate between the two stable equilibriums under specific excitations, the voltage output can be remarkably improved. With the decrease of D , the two potential wells can be separated further, which intuitively can further help improve the performance of bistable configuration. However, the decrease of D also increases the energy barrier δ between the two potential wells as shown in Figure 7.3, causing difficulty for the harvester to jump between the two equilibrium states. Hence, reducing the gap D enables the harvester to reach a bistable configuration but does not guarantee its performance improvement if D is further reduced. In this section, various bistable configurations are investigated and compared with the linear counterpart under different levels of sinusoidal and random excitations.

7.2.2.1 Bistable Configuration under Sinusoidal Excitations

In this section, the gap between the repulsive magnets is fixed at $D=8.5mm$. Firstly, for different excitation levels, the dynamics of this bistable configuration in terms of open circuit voltage output under sinusoidal excitation (12Hz) is investigated (Figure 7.9). Given low-level excitations ($a_{RMS}=1m/s^2$), initial small-amplitude oscillation is observed (Figure 7.9(a)). With a disturbance applied (Throughout this chapter, the disturbance mentioned is applied manually by using a small plastic hammer in the

experiment), oscillations with larger amplitude are achieved. These oscillations, however, are still confined in one potential well (also observed in the experiment). Such oscillation is termed as “*small limit cycle oscillation*” (or “*small LCO*” in short). In Figure 7.9(b), the excitation level increases to $a_{\text{RMS}}=1.4\text{ m/s}^2$, but it is still not strong enough to drive the harvester to jump between the two potential wells. Even with a disturbance applied, the harvester restores the small LCO around one of the two stable equilibriums after some transients. With further increase of a_{RMS} (Figure 7.9(c)), the harvester can be driven to jump between two potential wells by disturbance. Such large-amplitude oscillation is termed “*large limit cycle oscillation*” (or “*large LCO*” in short). Under significantly large excitations, chaotic oscillations are initially captured and then followed by large LCO with some disturbance (Figure 7.9(d)), or even without disturbance (Figure 7.9(e)).

Figure 7.9 Dynamics of a bistable configuration under sinusoidal excitations (12Hz)

Obviously, if the large LCO can be captured and maintained at various frequencies, the bistable nonlinear energy harvester will be more efficient and functional. Figure

7.10 shows the sinusoidal upward sweep responses of the bistable and linear configurations with different excitation levels. The solid line depicts the performance of the bistable harvester by upward sweep without any disturbance applied. When disturbances are applied during upward sweep, multi-value response may exist. The dashed line depicts the maximum response achievable at each frequency, which can be regarded as the best performance of the bistable harvester. Under low-level excitations ($a_{\text{RMS}}=0.7\text{m/s}^2$, Figure 7.10(a)), an important finding is that the response has the same pattern as that of monostable softening harvester (Figure 7.4(b)). The small LCO is captured by disturbance. With increased excitation level ($a_{\text{RMS}}=1.4\text{m/s}^2$), the large LCO is firstly captured in low-frequency range (7Hz~10Hz) by disturbance. With further increase in the excitation level ($a_{\text{RMS}}=2.1\text{m/s}^2$, Figure 7.10(c)), not only the frequency range for large LCO is extended (7Hz~16Hz), but also the small LCO is captured by disturbance in high-frequency range (22Hz~24Hz). In addition, within certain range (12Hz~14Hz), either small oscillation or small LCO or large LCO can occur depending on whether and how the disturbance is applied. Under strong excitations ($a_{\text{RMS}}=2.8\text{m/s}^2$), the response is similar to Figure 7.10(c) except that in some ranges, the bistable harvester captures frequency-lowered large LCO with disturbance (17Hz~19Hz) or chaotic oscillations (10Hz~14Hz).

In addition, the maximum responses (large LCO captured) of the bistable harvester are comparable with those of the linear configuration (Figures 7.10(b) to 7.10(d)). On the other hand, if the large LCO is maintained, wider bandwidth is achieved and it increases with the excitation level. Thus, bistable configuration is definitely able to improve the performance of the harvester. However, under practical random excitations, the large LCO may not be captured and maintained. This is because the probability for the harvester to jump between two potential wells depends on the energy barrier δ (Figure 7.3). If the small LCO or chaotic oscillations are captured, due to their smaller magnitudes compared to the linear configuration, the bistable configuration may not be able to warrant better performance.

Figure 7.10 Sinusoidal sweep responses of bistable configurations versus linear configuration for various excitation levels

7.2.2.2 Bistable Configuration under Random Excitations

The V_{OC} responses in the time domain for various bistable and linear configurations, under three different levels of random excitations, are compared in Figure 7.11. When the repulsive magnets are placed at $D=9mm$, due to the two close and shallow potential wells separated by a low energy barrier δ , the bistable harvester can easily jump between the potential wells and capture the large LCO at all three excitation levels. Thus the bistable configuration provides much larger voltage responses than the linear configuration, as compared in Figures 7.11(a) to 7.11(f). Reducing D between the repulsive magnets separates further the two potential wells but meanwhile increases the energy barrier δ (Figure 7.3). Therefore, the probability for the harvester to jump between two potential wells under random excitations will be reduced. For $D=8.5mm$, only 2~3 jumps are observed (Figure 7.11(g)) under low-level excitations ($a_{RMS}=2.5m/s^2$), and the large LCO can be captured under larger excitations (Figures 7.11(h) to 7.11(i)). For $D=8.2mm$, under low-level excitations ($a_{RMS}=2.5m/s^2$), oscillations are totally confined in one potential well (Figure 7.11(j)) and the magnitude is much smaller compared to Figure 7.11(d). When the excitation level increases to $5m/s^2$, only a few jumps appear (Figure 7.11(k)). When the

excitation level further increases to 10m/s^2 , frequent jumps are observed (Figure 7.11(l)), implying that the large LCO is captured. With further decrease of the gap ($D=8\text{mm}$), only infrequent jumps occur even though a_{RMS} is increased to 10m/s^2 . Once the oscillations are confined in one potential well, there will be no guarantee that the bistable configuration will outperform the linear one.

Figure 7.11 V_{OC} of linear and bistable configurations under random excitations. The eclipses in (g) and (k) highlight the infrequent jumps between the two potential wells.

To further evaluate the performance of various bistable configurations, an energy storage circuit is connected to the harvester to charge the $33\mu\text{F}$ capacitor. The voltage developed across the capacitor for various configurations and excitation levels is compared in Figure 7.12. It is observed that the bistable configurations outperform the linear configuration at all three excitation levels except for $D=8\text{mm}$, in which case the increased energy barrier firmly confines the harvester to oscillate in one potential well, thus deteriorating its performance. Besides, it is observed that when $D=9\text{mm}$, the harvester achieves the best performance of all three excitation levels. In addition, at high excitation level ($a_{\text{RMS}}=10\text{m/s}^2$) and except for $D=8\text{mm}$, there is no significant difference in the performance of bistable configurations due to the fact that the large LCO is captured when $D=9, 8.5$ and 8.2mm . These results are consistent with the observation of V_{OC} in Figure 7.11.

Figure 7.12 Voltage developed across storage capacitor for linear and bistable configurations under random excitations

To investigate the bandwidth of bistable configurations under random excitations, fast Fourier transform (FFT) is conducted on the V_{OC} signal shown in Figure 7.11. The results are shown in Figure 7.13. Under low-level excitations ($a_{RMS}=2.5m/s^2$), since the oscillations of the bistable harvester are mostly confined in one potential well, obvious peaks appear in the V_{OC} spectra, especially for $D=8.2$ and $8mm$. These peaks correspond to the post-buckled fundamental resonant frequency of the cantilever beam (Erturk *et al.*, 2009a). For $D=8.5mm$, the peak is not obvious and the V_{OC} amplitude is significantly increased at low frequency range due to the infrequent jumps between two potential wells (Figure 7.11(g)). For $D=9mm$, the useful bandwidth is obtained at low frequency range and the V_{OC} amplitude is much larger than the other configurations because the large LCO can always be captured. When the excitation level increases, the peaks in the spectra of bistable configurations disappear, indicating that the oscillations of the harvester are dominated by the large LCO. For example, for $D=8.5mm$, a flat peak exists when $a_{RMS}=2.5m/s^2$ but disappears when $a_{RMS}=5$ and $10m/s^2$. Besides, under high-level excitations ($a_{RMS}=10m/s^2$), the V_{OC} magnitude of linear configuration in the frequency domain is comparable with those of bistable configurations ($D=9, 8.5, 8.2mm$) (Figure 7.13(c)). However, similar to the discussion in Section 7.2.1.2, the significantly increased

bandwidth of the bistable configurations (e.g., $D=9\text{mm}$) results in the increased V_{OC} outputs in the time domain (e.g., Figures 7.11(f), 7.11(i) and 7.11(l)) and thus the advantageous performance for charging the capacitor over the linear configuration (Figure 7.12(c)). The observation that the bandwidth increases with the excitation level is consistent with the best performances shown in Figures 7.10(b) to 7.10(d). In addition, the V_{OC} spectra for $D=9$, 8.5 and 8.2mm almost overlap (Figure 7.13(c)), which explains why the performances of these configurations are almost the same (Figure 7.12(c)).

Figure 7.13 V_{OC} spectra (FFT) of linear and bistable configurations under random excitations

7.2.2.3 Comments on Bistable Configuration

A bistable nonlinear energy harvester can be implemented using repulsive magnets. Under low-level sinusoidal excitations, the oscillations of the bistable harvester are confined in one potential well and its frequency sweep response shows the same pattern as that of a monostable softening configuration. Under high-level sinusoidal excitations, oscillations of different types, including small oscillation, small LCO, large LCO, frequency-lowered large LCO and chaotic oscillation, can show up with proper disturbance. Once the large LCO is captured, useful wider bandwidth can be achieved compared to the linear configuration. Under practical broadband random

excitations, the bistable configuration is demonstrated to always be capable of outperforming the linear configuration except when the repulsive magnets are too closely placed, in which case oscillations are mostly confined in one potential well. With a large gap D between the magnets ($D=9mm$), the large LCO can always be captured and the harvester performs best at various excitation levels. Similar to the monostable configuration, the improved performance of a bistable configuration can be attributed to two aspects: the increased output magnitude in the frequency domain for low-level excitations and the significantly enlarged bandwidth for high-level excitations.

7.2.3 Optimal Nonlinear Configuration

Reducing the gap D between two repulsive magnets can change a nonlinear harvester from a monostable configuration to a bistable configuration. Figure 7.7 in Section 7.2.1 shows that the performance of a monostable hardening configuration increases when D decreases. While, in Section 7.2.2, Figure 7.12 indicates that generally, the performance of a bistable configuration declines when D decreases. These results suggest that the optimal performance of nonlinear configuration could be achieved around the region where a monostable configuration transforms into a bistable one (i.e., D approaches D^*).

To verify the above conjecture, Figure 7.14 compares the energy accumulated in the storage capacitor within 45 seconds for various nonlinear configurations under random excitations. Although the energy accumulated for each case is not exactly repeatable due to the nature of random excitations, the tendency in Figure 7.14 is clear, that is, the optimal performance of nonlinear configuration at various excitation levels is achieved around the monostable-to-bistable transition region. This important finding reveals two anti-intuitive facts: (1) Not only bistable but also monostable configuration can achieve optimal performance, when their repulsive magnets are arranged close to the monostable-to-bistable transition region, though the bistable

configuration has been more enthusiastically pursued and reported in the literature due to its larger bandwidth (we can see this when comparing Figure 7.8(c) and Figure 7.13(c)); and (2) It is unnecessary to design a bistable energy harvester with two potential wells separated far away. Intuitively, by reducing D , larger voltage output is expected when the harvester oscillates between further separated potential wells. However, certain mechanism such as exploiting stochastic resonance (McInnes *et al.*, 2008) should be implemented to overcome the high energy barrier, which not only induces the complexity in harvester design but also consumes some amount of the harvested energy. Fortunately, the findings in this study show that a bistable configuration with large D (meaning shallow potential wells separated by low energy barrier) close to the monostable-to-bistable transition region will provide better performance (Figure 7.14). Thus, we can avoid the high energy barrier issue. These results provide important guidelines for designing a nonlinear energy harvester.

Figure 7.14 Accumulated energies in storage capacitor with various configurations

7.3 Frequency Up-Conversion Using Magnets

In some ultra-low-frequency vibration scenarios such as human movements ($\sim 1\text{Hz}$) and wave heave motions ($< 1\text{Hz}$), tuning resonance or exploiting nonlinearity is obviously infeasible. Hence, certain frequency up-conversion mechanism should be implemented to improve the functionality of the energy harvesters. Pairs of magnets (Rastegar and Murray, 2009) or ferromagnetic structures (Wickenheiser and Garcia,

2010) can be exerted in frequency up-conversion mechanism. However, according to the literature, only attractive magnetic forces have been considered so far. In this section, both repulsive and attractive magnetic configurations for frequency up-conversion are investigated. Wave heave motion is taken as an example of low-frequency vibration source and parametric study is performed to compare the efficiency of various configurations under various wave conditions.

7.3.1 Frequency Up-Conversion Mechanism

The proposed frequency up-conversion mechanism, using both repulsive and attractive magnets, is illustrated in Figure 7.15(a). The piezoelectric bimorph cantilever is the same as that used in the frequency tuning or nonlinear energy harvester configuration for vibration-to-electricity transduction. However, the difference between them is the way excitations are applied. In the nonlinear energy harvester configurations, piezoelectric cantilever and magnets are applied with base excitations as a whole. While, in the frequency up-conversion mechanism, the piezoelectric cantilever stands still and is excited by magnetic force applied at the free end. The magnet, which follows motions from the low-frequency vibration source, regularly passes by and interacts with another magnet fixed at the free end of the piezoelectric cantilever and thus excites the cantilever to vibrate at its high natural frequency. Buoy-based sensors can be powered by efficient wave energy harvester using such frequency-up conversion mechanism as shown in Figure 7.16. The experimental implementation of frequency up-conversion is shown in Figure 7.15(b). The cantilever is clamped and stands still at the rim of the shaker, and one magnet is fixed at the shaker shaft undergoing low-frequency sinusoidal motions to mimic wave heave motions. Since significant noise appears in the signals of the accelerometer under ultra-low-frequency vibrations, the magnitude of motion generated by the shaker is directly recorded by a pen. In later sections, parametric study is conducted to compare the efficiency of various repulsive and attractive magnetic configurations under various wave conditions.

Figure 7.15 Frequency up-conversion mechanism and experimental implementation

Figure 7.16 Conceptual drawing of buoy-based sensors powered by wave energy harvester using frequency up-conversion technique

7.3.2 Repulsive Magnets for Frequency Up-Conversion

The repulsive magnetic configuration is first investigated. The gap between the repulsive magnets is set to be $D=12\text{mm}$, 8mm and 6mm , respectively. For each case, the performance of the harvester is studied for three wave heights $H=15\text{mm}$, 30mm and 50mm , and three frequencies $f=0.4\text{Hz}$, 0.6Hz and 0.8Hz .

For a large gap between the repulsive magnets ($D=12mm$), the open circuit voltage V_{OC} under various wave conditions is shown in Figure 7.17. It can be observed that the harvester is not effectively excited by the weak magnetic forces. Here, “*effectively excited*” refers to that the piezoelectric cantilever is excited to vibrate at its high natural frequency. For lower H and f , the deflection of the harvester just follows the wave heave motion. With the increase of H and f , the excited high-frequency vibrations appear, though the magnitude is small, as shown in Figures 7.17(h) and 7.17(i). When the energy storage circuit is connected, the voltage developed across the $33\mu F$ capacitor is as illustrated in Figure 7.18. It is noted that generally, the energy accumulation is very slow for $D=12mm$. The energy harvesting performance is improved with the increase of H and f , in accordance with the observations of V_{OC} .

Figure 7.17 V_{OC} for different H and f . Repulsive magnets with $D=12mm$

Figure 7.18 Voltage across storage capacitor for different H and f . Repulsive magnets with $D=12mm$

For a medium gap between the repulsive magnets ($D=8mm$), V_{OC} under different wave conditions is shown in Figure 7.19. In this case, the piezoelectric cantilever is effectively excited and experiences quasi damped free vibrations between two impulses induced by magnetic forces. Substantial increase in the magnitude of V_{OC} is observed as compared to Figure 7.17. The significant efficiency improvement is also reflected in the voltage developed when the harvester charges the storage capacitor (Figure 7.20). However, the performance is still sensitive to the wave conditions. More vigorous wave motions (larger H and f) generate more electricity.

Figure 7.19 V_{OC} for different H and f . Repulsive magnets with $D=8mm$

Figure 7.20 Voltage across storage capacitor for different H and f . Repulsive magnets with $D=8mm$

With further decrease of the gap ($D=6mm$), no matter under which wave condition, the piezoelectric cantilever can be effectively excited and the peak-to-peak magnitude

of V_{OC} after one impulse is around 45V, as shown in Figure 7.21. The only difference is that the excited large-magnitude output is divided into different number of segments for different wave frequency. Considering the energy storage circuit, Figure 7.22 clearly shows that when the repulsive magnets are sufficiently close ($D=6mm$), the energy harvested is relatively insensitive to the wave conditions (H and f). This characteristic is preferable for wave energy harvesting in the uncertain marine environment. Besides, for $D=6mm$, the energy harvesting capability approaches saturation. For instance, under wave motion of $H=50mm$ and $f=0.8Hz$, only 3V (~20%) increase is observed in the voltage developed across the $33\mu F$ capacitor for 45 seconds, as compared to that for $D=8mm$ (Figure 7.22(c) and Figure 7.20(c)).

Figure 7.21 V_{OC} for different H and f . Repulsive magnets with $D=6mm$

Figure 7.22 Voltage across storage capacitor for different H and f . Repulsive magnets with $D=6mm$

7.3.3 Attractive Magnets for Frequency Up-Conversion

Attractive magnetic configuration is also investigated for frequency up-conversion mechanism to compare with the repulsive configuration. For a large gap between the attractive magnets ($D=12mm$), it is observed in Figure 7.23 that no matter under which wave condition, the harvester just follows the heave motion rather than vibrates at its natural frequency. Even though the gap is decreased to $D=6mm$, the harvester is still unable to be effectively excited by the attractive magnetic forces (Figure 7.24). The high-frequency vibrations of the harvester only appear under extremely vigorous wave motion ($H=50mm$, $f=0.8Hz$, Figure 7.24(d)). However, compared to the repulsive case (Figure 7.21(i)), the magnitude of the excited high-frequency vibrations and thus the energy harvesting efficiency by attractive configuration is drastically lower. Hence, given same parameters and wave conditions, the repulsive magnetic configuration is preferable in frequency up-conversion mechanism.

Figure 7.23 V_{OC} for different H and f . Attractive magnets with $D=12mm$

Figure 7.24 V_{OC} for different H and f . Attractive magnets with $D=6mm$

7.4 Chapter Summary

This chapter presents an experimental investigation on the merits of using magnets to improve the functionality of vibration energy harvesters under various scenarios. The nonlinearities introduced by magnets and the frequency up-conversion technique using magnets have been given the most attention. Main findings of this work can be summarized as follows:

Exploiting Nonlinearities Using Magnets

- ❖ *Monostable configuration.* Under low-level sinusoidal excitations, the nonlinear part of the magnetic force is negligible, while the linear part of the magnetic force contributes to changing the stiffness of the harvester and thus can be used for resonance tuning. Under high-level sinusoidal excitations, high-energy and low-energy orbits co-exist in certain frequency range. Wider bandwidth for efficient energy harvesting can be achieved by proper sweep or by disturbance especially for monostable hardening configurations. Under practical random excitations, monostable hardening configuration can significantly outperform the linear configuration, especially when the repulsive magnets are closely placed ($D=9.5mm$). The improved performance can be attributed to two aspects: the increased output magnitude in the frequency domain for low-level excitations and the enlarged bandwidth for high-level excitations.
- ❖ *Bistable configuration.* Under low-level sinusoidal excitations, oscillations of a bistable configuration may be confined in one potential well and the frequency sweep response shows the same pattern as that of a monostable softening configuration. Under high-level sinusoidal excitations, oscillations of different types, including small oscillation, small LCO, large LCO, frequency-lowered large LCO and chaotic oscillations can show up with proper disturbance. Once the large LCO is captured, useful wider bandwidth can be achieved as compared to the linear configuration. Under practical random excitations, bistable configurations can always outperform the linear configuration except when the repulsive magnets are too closely placed, in which case, oscillations of the harvester are firmly confined in one potential well. For a large gap between the

magnets ($D=9mm$) at various excitation levels, the large LCO is always easily captured, which gives the best performance among all bistable configurations. In addition, similar to monostable configuration, the improved performance of bistable configuration can be attributed to two aspects: the increased output magnitude in the frequency domain for low-level excitations and the significantly enlarged bandwidth for high-level excitations.

- ❖ *Optimal nonlinear configuration.* The configuration for optimal performance of a nonlinear energy harvester is found near the monostable-to-bistable transition region (i.e., $D \approx D^*$). This is an important finding in this work which tells us two anti-intuitive things: (1) Not only bistable but also monostable configurations can achieve optimal performance when their repulsive magnets are arranged close to the monostable-to-bistable transition region; and (2) It is unnecessary to design a bistable energy harvester with two potential wells separated far away. A bistable configuration with larger D close to the monostable-to-bistable transition region can provide better performance. Thus, we can also avoid the difficulty in implementing certain mechanism such as exploiting stochastic resonance (McInnes *et al.*, 2008) to overcome the high energy barrier issue.

Frequency Up-Conversion Using Magnets

- ❖ In ultra-low-frequency vibration scenarios, such as wave heave motions, the frequency up-conversion technique using magnets is proposed. Different configurations under various wave conditions are investigated by parametric study. The repulsive configuration of magnets is found preferable in the frequency up-conversion technique, which can efficiently excite the cantilever energy harvester to vibrate at its high fundamental natural frequency. When the repulsive magnets are placed sufficiently close, the energy harvesting capability can be saturated and relatively insensitive to various wave conditions. Hence, the energy harvester can be functional and adaptive in the uncertain marine environment.

The findings presented in this chapter serve as useful design guidelines when one seeks nonlinearity or frequency up-conversion techniques using magnets to improve the functionality of vibration energy harvesters.

CHAPTER 8 CONCLUSIONS AND FUTURE WORK

8.1 Conclusions

The thesis presented the research work on vibration energy harvesting using piezoelectric materials, which can be classified into two categories, the modeling issues and the efficiency enhancement issues in the piezoelectric energy harvesting systems. The work carried out can be summarized as follows:

- (1) Macro-fiber composites (MFC) are featured in their flexibility of large deformation compared to PZT. The uncoupled modeling and efficiency enhancement of an energy harvesting system using MFC transducers are presented. Finite element modeling and circuit simulation are conducted and validated by experiment. Based on the validated models, parametric study on the effects of MFC properties is conducted aiming at efficiency enhancement. It is demonstrated that the MFC configuration by dividing in thickness the original MFC into two pieces and stacking them together can provide a 41.2% increase of average power for charging a storage capacitor. The research attempt has potential impact on manufacturing more cost-effective future version of MFC transducers.
- (2) A novel equivalent circuit modeling method for piezoelectric energy harvesters is developed. This method not only takes into account the backward electromechanical coupling effect (electrical damping) but also addresses the difficulty when complicated harvester structure and sophisticated interface circuits are concerned. It bridges the mature finite element modeling techniques in the mechanical domain and the powerful circuit simulation tools (e.g., SPICE simulators) in the electrical domain, and exerts their respective advantages. Firstly, the parameters in the equivalent circuit model (ECM) are identified by theoretical analysis and finite element analysis for simple and complex structures,

respectively. Subsequently, the ECM considering multiple modes of the system is established and simulated in the circuit simulator. Numerical examples and experimental work are presented to show the accuracy of the proposed method and its capability of dealing with complicated structures and sophisticated energy harvesting interface circuits.

- (3) An accurate analytical solution is derived to predict the performance of piezoelectric energy harvesting systems (PEHSs) with the synchronized charge extraction (SCE) technique. The solutions based on the uncoupled and in-phase assumptions are first presented, followed by the derivation of the accurate analytical solution. Based on the accurate solution, the applicability of the SCE technique is studied for different cases, i.e., the piezoelectric energy harvester (PEH) with various degrees of electromechanical coupling and the PEH excited at various frequencies. Circuit simulation is also conducted based on the proposed accurate equivalent circuit modeling method. The simulation results validate the analytical outcomes. Both the accurate analytical solution and the circuit simulation show that the SCE technique cannot improve or even reduces the power output at resonance if the coupling of the PEH is not negligible. The SCE technique is found capable of significantly boosting the efficiency of energy harvesting only for the PEH vibrating at off-resonance frequencies or with weak coupling.
- (4) To address the bandwidth issue of vibration energy harvesters, this thesis presents a novel multiple-DOF PEHS. A 2DOF model is first analyzed and its two configurations are characterized. In the first configuration, the piezoelectric element is placed between one mass and the base, and in the second configuration it is placed between the two masses. The advantage of the former is that with slight increase of overall weight to the 1DOF system, we can achieve two close and effective peaks in power response, or one effective peak with significantly enhanced magnitude. The latter can provide two close and enhanced peaks with a huge increase of system weight, which may not improve or even deteriorate the performance in terms of power density. Even if the power density is not

concerned, it can only significantly outperform the 1DOF model with unpractical and unfavorable damping. Based on the characterization of the 2DOF model, the first configuration is generalized to an n -DOF system and its analytical solution is derived. This solution provides a convenient tool for parametric study and design of a multiple-DOF PEHS. The ECM of the proposed n -DOF PEHS is also developed via the analogies between the mechanical and electric domains. With the ECM, system-level electric simulation can be performed to supplement the analytical solution to evaluate the system performance when sophisticated interfaces are attached.

- (5) This thesis also reports an experimental study on improving the functionality of vibration energy harvesting using magnets. Firstly, for nonlinear oscillation technique, both monostable and bistable configurations are investigated and the optimal configuration is determined to be near the monostable-to-bistable transition region. Both monostable and bistable nonlinear configurations can significantly outperform the linear harvester near this transition region. For the bistable configuration, this finding helps avoid the difficulty of overcoming the high energy barrier. Secondly, for frequency up-conversion technique in ultra-low-frequency vibration scenarios, the repulsive configuration of magnets is found preferable, which is efficient and insensitive to excitations when the magnets are placed sufficiently close. The findings serve as useful design guidelines when one seeks using magnets to improve the functionality of vibration energy harvesters.

8.2 Recommendations for Future Work

Although vibration energy harvesting explored in this thesis are not suitable for large-scale power generation to address the nowadays energy crisis concerned all over the world, they do have the impact on small-scale electricity usage for broad applications to reduce the reliance on chemical batteries and create autonomous, eco-friendly and sustainable systems. Although considerable research attempts have

been made in the past few years, including the contributions in this thesis, there are still many huddles ahead before the vibration energy harvesting techniques can be widely deployed in practice. Further efforts are required to match the power consumption of specific electronic devices with the capability and functionality of harvesters in specific vibration scenarios. The work presented in this thesis can be further extended as follows:

- (1) In the last validation example in Chapter 4, a single-mode ECM of PEH is derived to predict its performance by assuming that the behavior of the PEH is dominated by the fundamental mode in a specific range and the influence of high order modes are negligible. This model is also adopted in the analysis and simulation of SCE interface in Chapter 5. However, one should be careful when using the single-mode ECM. In some specifically designed beam configurations, e.g., the novel 2-DOF cut-out beam configuration reported in a recent work by the author and his colleagues (Wu *et al.*, 2012), the first two modes of the beam can be tuned close to each other. In such case, the single-mode ECM of PEH is not suitable. The analysis of the energy harvesting performance with the SCE interface should be based on the multiple-mode ECM of PEH. Fortunately, the ECM-based modeling method proposed in Chapter 4 does have the capability of accounting for multiple modes. Hence, for some specific beam configuration with close multiple modes, the SCE interface as well as other sophisticated interfaces such SSHI requires further investigation based on the multiple-mode ECM of PEH. Analytical solutions for SCE and SSHI in such case are cumbersome and even impossible, making the multiple-mode ECM the only option for system evaluation.
- (2) The SSHI interfaces (Parallel-SSHI or Series SSHI) are frequently reported in the literature. They are usually regarded as more powerful than the SCE interface to boost the efficiency of piezoelectric energy harvesting. However, they still require the impedance matching efforts to achieve the claimed significant improvement of power output. The SCE interface, on the contrary, has no

requirement of impedance matching. Hence, an improved SSHI is desired which does not require impedance matching. Such interface may be developed by including the topology of isolating load and source in the SCE interface.

- (3) As discussed in Chapter 5, when a PEH vibrates under low-level excitations or at off-resonance, the performance of SCE interface will be dramatically undermined due to the loss in the rectifier. Advanced topology of rectifier without semiconductor diodes such as that proposed by Cheng *et al.* (2009) can be considered in the SCE as well as other interface circuits to lower the threshold for voltage rectifying and energy storage. Besides, in Chapter 5, the synchronized switching is still implemented by a preset pulse generator. Since no peak detector is considered, the simulation can only handle a simple excitation such as harmonic vibration. Moreover, the power required for switching is not considered. An ideal SCE interface is required to incorporate the self-powered detecting (required in uncertain or random vibrations) and self-powered switch triggering modules, which is a very challenging issue definitely worth more research efforts.
- (4) Chapter 6 proposes a concept of multiple-DOF piezoelectric energy harvesting system. Theoretically, such system is able to achieve effective multiple resonant peaks close to each other with a slight increase of system weight. In the future, a prototype should be designed and tested to validate this concept.
- (5) In Chapter 7, the optimal nonlinear configuration is experimentally investigated by parametric study. Further efforts are demanded to develop a theoretical model to determine the optimal configuration. Besides, in the bistable configuration, the increase of the extent of nonlinearity brings the increased energy barrier and reduces the possibility of transition between the two stable states. One possible solution for the energy barrier issue is to introduce additional stable states. For example, in the strong bistable nonlinear configuration (Figure 8.1(a)), an additional magnet at $x=0$ changes the original unstable equilibrium to a stable one (Figure 8.1(b) and 8.1(c)), which is believed to be able to significantly reduce the energy barrier. In the future, the performance of these nonlinear configurations with more than two stable states will be investigated. Furthermore, as mentioned

previously, a recent work by the author and his colleagues (Wu *et al.*, 2012) reported a novel 2-DOF cut-out beam configuration (not included in this thesis), in which the first two modes of the beam can be tuned close to each other. The broadband energy harvesting performance by such configuration can be further improved by incorporating nonlinear technique using magnets.

Figure 8.1 Nonlinear configurations with (a) two stable states and (b) triple stable states and (c) their potential functions

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APPENDIX: AUTHOR'S PUBLICATIONS

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- Wu H., **Tang L.H.**, Yang Y.W. and Soh C.K., 2012, "A Novel 2-DOF Piezoelectric Energy Harvester," *Journal of Intelligent Material Systems and Structures* (tentatively accepted).
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