Viscoelastic behavior of rock mass and its effects on stress wave propagation

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AND ITS EFFECTS ON STRESS WAVE PROPAGATION

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PROPAGATION**

**FAN LIFENG
SCHOOL OF CIVIL AND ENVIRONMENTAL
ENGINEERING
2012**

**VISCOELASTIC BEHAVIOR OF ROCK
MASS AND ITS EFFECTS ON STRESS
WAVE PROPAGATION**

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SUMMARY

Rock mass is distinguished from other engineering materials by the existence of discontinuities. The discontinuities ranging in size from micro-defects to macro-joints greatly influence the dynamic behavior of rock mass. Efforts have been made to study the dynamic behavior of rock mass and the effects of discontinuities on wave propagation.

In order to evaluate the effects of macro-joint on dynamic behavior of rock mass, a series of modified split Hopkinson pressure bar (SHPB) tests are conducted for artificial filled macro-joints with different widths and water contents. The dynamic stress-strain relation of the filled macro-joints is calculated.

Based on the different loading conditions, a linear equivalent viscoelastic medium model and a nonlinear equivalent viscoelastic medium model are theoretically developed for the mechanical behavior of linear deformational jointed rock mass and nonlinear deformational jointed rock mass, respectively. The effects of macro-joints on the stress wave propagation are investigated.

In addition to macro-jointed rock mass, mechanical behavior of a micro-defected rock is investigated by a series of impact tests. A frequency-dependent viscoelastic model is proposed to describe the dynamic behavior of micro-defected rock and its effects on wave propagation. Numerical manifold method (NMM) is used to investigate the effects of macro-crack length, stiffness, density etc. on the viscoelastic behavior of micro-defected rock. The capability of NMM on modeling the dynamic mechanical behavior of micro-defected rock is verified.

Using a homogenization-discontinuity combination approach, the global mechanical behavior of rock mass containing double-scale discontinuities (macro-joints and micro-defects) is investigated.

Finally, an extended displacement discontinuity method is introduced to evaluate the effects of double-scale discontinuities on the wave propagation.

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LIST OF SYMBOLS

σ	stress
ε	strain
k_n	normal specific stiffness
k_{ni}	initial normal specific stiffness
d	closure of macro-joint
$d_{\max}$	maximum allowable closure of macro-joint
τ	shear stress
τ_{sp}	peak shear strength
τ_{sr}	residual shear strength
u_s	shear displacement
u_{sp}	peak shear displacement
u_{sr}	residual shear displacement
μ	shear modulus
S	joint spacing
ν	Poisson's ratio
k_t	transverse specific stiffness
θ	incident angle
C_{ijmn}	elastic constants
V	phase velocity
ρ	density
F	force
E	Young's modulus
$\sigma_I(t)$	incident stress
$\sigma_R(t)$	reflected stress
$\sigma_T(t)$	transmitted stress
$\varepsilon_I(t)$	incident strain

$\varepsilon_R(t)$	reflected strain
$\varepsilon_T(t)$	transmitted strain
C_0	elastic wave velocity in the rock
l_0	initial length of the specimen
A	cross-sectional area
A_0	initial cross-sectional area
A_s	cross area of the specimen
$H_i(t)$	Heaviside function
x	Cartesian coordinate along the wave propagation path
t	time
E_r	Young's modulus of rock
E_{eff}	effective modulus
V_{eff}	effective velocity
i	imaginary sign
ω	angle frequency
$\tilde{v}$	velocity frequency spectrum
$v_0(t)$	boundary condition of velocity
$\tilde{v}_0(\omega)$	frequency spectrum of $v_0(t)$
z	wave impedance
α	phase shift factor
β	attenuation factor
Re	real parts of the complex expression
Im	imaginary parts of the complex expression
f_0	frequency of the incident wave
λ	wavelength
E_a, E_v, η_v	parameters in the equivalent medium model
T_N	the transmission coefficient after N joints
T_1	transmission coefficient after a single joint

T	transmission coefficient
N	joint number
D	wave propagation distance
$\bullet$	by differentiation with respect to t
Γ	nondimensional joint spacing
$E^*(\omega)$	complex dynamic modulus
$E'(\omega)$	storage modulus
$E''(\omega)$	loss modulus
$\gamma(\omega)$	propagation coefficient
$\alpha(\omega)$	attenuation coefficient
$\beta(\omega)$	wave number
$\mathbb{F}$	treatment of applying the Fourier transformation
$E_v(\omega), \eta_v(\omega)$	frequency-dependent parameters of viscoelastic model
$\sim$	symbols after Fourier transformation
τ_v	retardation time of the Maxwell element
Q	number of micro-crack
T	periodic time
W_d	energy dissipation in one circle per unit volume
W	energy

LIST OF ABBREVIATIONS

DDM	displacement discontinuity method
CDDM	conventional displacement discontinuity method
EDDM	extended displacement discontinuity method
EMM	equivalent medium method
SEMM	static effective moduli method
DEMM	dynamic effective moduli method
EVMM	equivalent viscoelastic medium method
LEVMM	linear equivalent viscoelastic medium method
NEVMM	nonlinear equivalent viscoelastic medium method
REV	representative elementary volume
VRM	viscoelastic rock mass
SHPB	split Hopkinson pressure bar
SEM	scanning electron microscope
NMM	numerical manifold method
DFT	discrete Fourier transformation
IDFT	inverse discrete Fourier transformation
FFT	fast Fourier transformation
IFFT	inverse fast Fourier transformation

CHAPTER 1 INTRODUCTION

1.1 Background

Rock mass is a system of rock blocks and fragments separated by discontinuities as shown in Fig. 1.1. Generally, the discontinuities in the rock mass are divided into two groups. The primary discontinuities set with wider spacing relative to the stress wavelength is briefly called “macro-joint”, and the secondary discontinuities set characterized by high density and small spacing relative to the stress wavelength, briefly called “micro-defect” in this study. The mechanical behavior of the rock mass is characterized by the physical properties and geometrical dimensions of rock blocks as well as by characteristics of discontinuities. The presence of the discontinuities results in increasing overall mechanical compliance and decreasing overall strength of the rock mass.

Equation Chapter (Next) Section 1

Fig. 1.1 Illustration of typical rock mass

Due to the presence of the discontinuities, modeling of the dynamic behavior of rock mass becomes a difficult task and covers a wide variety of aspects such as mechanics, geology, hydrology and rheology. Stress wave propagation method is always employed for its easy operation, good repeatability and high accuracy. It is commonly recognized that when stress wave propagates through rock mass, the discontinuities result in the attenuation of stress wave amplitude, slowing of wave propagation velocity and transformation of waveforms.

To model the dynamic behavior of rock mass and consider the effect of discontinuities on the wave propagation, the discontinuities are sometimes taken into account implicitly when the engineering problems take material in quantities large enough so that the rock mass can be described by the averaged material properties, which are assumed to be continuous. Therefore, the rock mass properties are captured by the effective moduli of the equivalent continuum medium (Amadei and Goodman, 1981; Schoenberg and Muir, 1989; Sheshenin et al., 1997; Li et al., 2010a, b; and many others). These treatments can be used to avoid the difficulty in determining the complex multiple reflections between individual joints, so are widely used for analyzing the wave propagation across multiple parallel joints in a long wavelength limit, extremely effective for the micro-cracks or highly dense macro-joints cases (White, 1983; Schoenberg and Muir, 1989; Schoenberg and Sayers, 1995; Grechka and Tsvankin, 2003; Grechka et al., 2003). The effects of the rock and contained discontinuities (both macro-joints and micro-defects) are lumped into effective elastic modulus and quality factor of the equivalent medium. However, the rock mass is always discontinuous, inhomogeneous, anisotropic, non-uniform and nonlinear in engineering problems. The traditional equivalent simplification inherently results in some limitations. For example, the discreteness of wave attenuation at individual macro-joint is always ignored; the intrinsic frequency dependent property caused by the micro-defect cannot be considered; the wavelength should be larger than the joint spacing, etc. Viscoelastic equivalence has been introduced to investigate the mechanical behavior of rock mass and its effect on wave propagation to avoid the limitations introduced by traditional elastic equivalence.

However, up to now, no systematic study on the viscoelastic behavior of rock mass consisting of discontinuities and their effect on the wave propagation has been performed, especially when the rock mass containing both macro-joints and micro-defects. In addition, how to evaluate the effect of micro-defect and macro-joint on the stress wave propagation has yet to be explored. Therefore, it is necessary to conduct an investigation in proposing new models to simulate the dynamic behavior of rock mass and its effects on wave propagation.

1.2 Objectives and Scopes of the Thesis

The objective of this thesis is to study the viscoelastic behavior of rock mass and to investigate its effects on stress wave propagation. This study is focused on obtaining the viscoelastic behavior of macro-jointed rock mass, micro-defected rock and rock mass with double-scale discontinuities. The relationship among wave propagation (magnitude, frequency, energy, etc.), the mechanical properties of discontinuity (stiffness) and geometrical properties (spacing, discontinuity quantity, length of micro-crack, etc.) are investigated.

The detailed scopes of the present study are outlined as follows:

1. To carry out laboratory tests on normal constitutive relations of artificially filled joints under impact loadings, for determining the effect of macro-joint on wave propagation, such as the effect of joint width on transmitted and reflected waves, the effect of joint saturation (water content) on wave transmission, the effect of incident wave frequency and amplitude and the effect of sand pre-compaction on the wave transmission and for deriving a dynamic constitution model for filled joints.
2. To theoretically derive a linear equivalent viscoelastic medium model by analytically determining the parameters, for investigating stress wave propagation through rock mass with linear deformational behavior joints, and for evaluating the advantages and disadvantages of the existing methods by comparisons of results from different models.

3. To extend the linear equivalent viscoelastic medium model to the nonlinear equivalent viscoelastic medium model, for considering the wave propagation through rock mass when the joint is filled or the amplitude of stress wave is sufficient to mobilize the nonlinear deformation of the joint.
4. To perform the laboratory longitudinally impact test of the sedimentary rock bar using a pendulum, for determining the wave attenuation and dynamic moduli of the micro-defected rock blocks, and for building a viscoelastic constitutive model for the rock which can take the effects of micro-defect into account.
5. To perform the numerical simulation of wave propagation through rock mass using numerical manifold method (NMM), for completely investigating the effects of micro-defect properties (such as length, quantity and specific stiffness) on the viscoelastic behavior of rock and its interaction on the wave propagation.
6. To propose a equivalent viscoelastic medium method for investigating the global viscoelastic behavior of rock mass containing both micro-defects and macro-joints, and for considering the effects of such double-scale discontinuities on energy dissipation.
7. To extend the conventional displacement discontinuity method to the extended displacement discontinuity method, for considering the effects of both macro-joint and micro-defect on the wave propagation.

In the present thesis, the frequency of an inharmonic wave denotes the “indicative frequency” if no other special instructions.

1.3 Organization of the Thesis

The thesis consists of 10 chapters. Chapter 1 is the introduction to give an overview of mechanical behavior of rock mass and its effects on wave propagation, describes the problems to be solved, and outlines the objectives of the present study, together with the statement of the general approaches to achieve the objectives. In Chapter 2, a detailed literature review of the existing theories and previous studies

with respect to viscoelastic behavior of rock mass and its effects on the wave propagation are conducted, where the approaches are compared and discussed, and accordingly the methodology applicable for solving the problems in the present study is determined. Chapter 3 presents a laboratory test on the constitutive relation of a sand and water filled artificial joint. The geometrical effects and filled material percentage on the model are discussed. Chapter 4 presents an analytical equivalent viscoelastic medium method to investigate the mechanical behavior of rock mass and solve the wave propagation through jointed rock mass. The evaluation of the advantages and disadvantages of the equivalent viscoelastic medium method and conventional methods are performed. In Chapter 5, the linear equivalent viscoelastic medium method is extended to the nonlinear equivalent viscoelastic medium method to investigate the wave propagation through rock mass containing macro-joint with nonlinear deformation. A piece-wise linear model is used to describe the nonlinear deformation of the joint. A series of impact tests are conducted to evaluate the effect of rock micro-defects on the wave propagation in Chapter 6, and a frequency-dependent viscoelastic model is proposed to consider the inherent viscoelastic properties of the rock block. In Chapter 7, the numerical manifold method is used to evaluate the effect of micro-defects on the viscoelastic behavior of rock. The influences of defect length, defect quantity and defect stiffness are investigated. In Chapter 8, the overall mechanical behavior of rock mass containing double-scale discontinuities is investigated. A possible mechanism is proposed to explain the viscoelastic behavior of rock mass, and its effect on the stress wave propagation is discussed. The effects of double-scale discontinuities on the wave propagation are investigated in Chapter 9 by proposing an extended displacement discontinuity method. In the final chapter, Chapter 10, conclusions drawn from this study are summarized, and recommendations for further research are given.

CHAPTER 2 REVIEW OF PREVIOUS STUDIES

In this chapter, a comprehensive literature review of previous studies is presented in aspects of experimental observation, theoretical analysis and numerical modeling. For the experimental observation, descriptions of micro-/macro-discontinuities and the role of such discontinuities on rock physical properties are reviewed and a dynamic laboratory technology, SHPB is introduced. A detail review of the existing theories on dynamic mechanical behavior of rock/rock mass and the effects of micro-/macro- discontinuities on wave propagation is performed, and a brief review of numerical method to simulate the mechanical behavior of rock mass is also carried out. Finally, the limitations and challenges of current studies are brought forward. Equation Chapter (Next) Section 1

2.1 Discontinuities in Rock and Rock mass

Over the past decades, many approaches have been developed to study the mechanical behavior of rock/rock mass and the effects of its discontinuities on stress wave propagation. Generally, the effects of rock mass discontinuities on stress wave propagation can be attributed to two factors: (1) wave attenuation due to the existence of macro-joints. (2) internal fractional dissipation due to the grain boundaries, micro cracks, pores, fluid permeability and so on, briefly called “micro-defect”.

2.1.1 Macro-joints description

Macro-joint is defined as a break of geological origin in the continuity of a body of rock (ISRM, 1978), which commonly exists in the rock mass, and has significant effects on the mechanical behavior of the rock mass. Quantitative representation of rock joint dynamic properties has been one of the main tasks in the fields of rock mechanics and engineering. Joints can be non-filled or filled with fluids or solids such as sand, silt, breccias, gouge and mylonite and so on as shown in Fig.2.1. The physical properties of the filling materials influence the normal

stiffness of the joints. Due to the enormous variety of occurrences, joints display a wide range of physical behavior, in particular as regards their normal and shear strength, deformability, permeability, etc.

Fig. 2.1 Illustrate of non-filled joint and filled joint

There are two main approaches to the quantitative description of the mechanical properties of rock joint: (1) the theoretical approach, which adopts known theories (e.g. plasticity, contact theory, etc.) to simulate the observed behavior; (2) the empirical approach, in which physical data are analyzed to derive correlations between variables of influence and models are formulated according to the observed behaviors.

By using these two approaches, the studies on the constitutive relations of macro-joints can be divided into two groups.

Normal deformation

Fig. 2.2 shows a typical relation between the normal-stress and normal-closure of the macro-joint.

It can be observed from Fig. 2.2 that when the macro-joint is non-filled or the amplitude of the stress wave is insufficient to mobilize its nonlinear deformation, a linear constitutive relation can be adopted (Yi et al., 1997).

However, the complete stress-displacement relations of a macro-joint are commonly regarded as nonlinear (Goodman, 1976; Barton and Choubey, 1977; Swan, 1981; Bandis et al., 1983; Barton et al., 1985; Gens et al., 1990; Jing et al., 1993; Huang et al., 1993; Wang et al., 2003; Samadhiya et al., 2008; Pooyan and Fulvio, 2010).

Fig. 2.2 Normal stress-deformation relation of jointed granodiorite

(after Goodman 1976)

In order to describe the mechanical behavior of macro-joints clearly, an important concept of “normal specific stiffness” was introduced by Goodman et al. (1968) to define the rate of change of normal stress with respect to normal displacements.

Therefore, different from the traditional stress-strain constitution for engineering materials, the joint deformability was described by the stress-deformation curves as shown in Fig. 2.2. The contribution of the joints to the total displacement of the rock mass was computed by the above quantities, together with the values of maximum joint closure, which should be less than its aperture thickness defined as the maximum gap anywhere across the mated walls was introduced by Goodman (1974).

Different models are adopted to describe the basic mechanics of joint normal deformation, such as the power model (Sun, 1983), the hyperbolic model (Goodman, 1974), the modified hyperbolic model (Bandis et al., 1983), the exponential model

(Malama, 2003) and so on. A commonly accepted joint model in rock mechanics and engineering named B-B model was proposed by Bandis et al. (1983). This empirical hyperbolic model fitted well with test data for matched joints is expressed as

$$\sigma = \frac{k_{ni}d}{1 - (d/d_{\max})} \quad (2.1)$$

where d and $d_{\max}$ denote the closure and maximum closure of macro-joint, respectively. k_{ni} is the initial normal specific stiffness of joint. It can be obtained from Eq. (2.1) that the normal specific stiffness of the joint k_n is

$$k_n = \frac{\partial \sigma}{\partial d} = \frac{k_{ni}}{(1 - d/d_{\max})^2} \quad (2.2)$$

Eq. (2.2) reveals that when the normal stress approaches infinite, the joint closure approaches the maximum allowable joint closure, in that case, the normal stiffness of the joint becomes infinite. On the other hand, when the normal stress is zero, joint closure becomes zero, the corresponding normal stiffness of the joint approaches the initial specific stiffness. The practical cases are always between these two extremes.

Bandis et al. (1983) concluded that the factors which may influence the normal stiffness of joints are: 1) the initial actual contact area, relative amplitude and vertical distribution of the aperture between the joint walls; 2) the joint wall roughness; 3) the strength and deformability of asperities; and 4) the thickness, type and physical properties of the infilling material, if present.

Cai (2001) used cement mortar specimens with teeth contacts to simulate rock joints. The maximum allowable joint closure was found smaller than 1mm for most occasions. The small ratio of the maximum allowable joint closure to initial mechanical aperture was also recognized by other workers (Bandis et al., 1983). Therefore, the macro-joint is assumed to be large in extent and small in thickness relative to incident wavelength.

Ma et al. (2011a) investigated the normal behavior of artificial sand-water-filled joints using a modified split Hopkinson Pressure Bar (SHPB) system, and a three-phase model is proposed to evaluate the joint nonlinear behavior and its effects on the stress wave propagation through it. The transmitted waves predicted by the three-phase model agree well with the measured experimental data.

Shear deformation

The shear deformation of filled joints is dominated by joint surface characteristics and properties of the filled material, while the shear deformation of non-filled joints is mainly dominated by joint surface characteristics only. Efforts have been made to investigate the shear behavior of joint from both theoretical and experimental aspects (Jaeger, 1971; Barbero, 1996; Barla, 1990; Huang, 1993).

Kulhaway (1975) proposed a hyperbolic function to describe the nonlinear deformational behavior of a sheared fracture in pre-peak part

$$\tau = \frac{u_s}{m_1 + m_2 u_s} \quad (2.3)$$

where τ is the shear stress, u_s is the shear displacement, m_1 and m_2 are constants of the hyperbolic function.

Goodman (1976) presented a piecewise shear deformational behavior including both pre-peak part and post-peak part

$$\left\{ \begin{array}{ll} \tau = k_s u_s \quad \text{and} \quad k_s = \frac{\tau_{sp}}{u_{sp}} & u_s \leq u_{sp} \\ \tau = \left(\frac{\tau_{sp} - \tau_{sr}}{u_{sp} - u_{sr}} \right) u_s + \left(\frac{\tau_{sr} u_{sp} - \tau_{sp} u_{sr}}{u_{sp} - u_{sr}} \right) & u_{sp} < u_s < u_{sr} \\ \tau = \tau_{sr} & u_s \geq u_{sr} \end{array} \right. \quad (2.4)$$

where τ_{sp} and u_{sp} are peak shear strength and corresponding shear displacement, τ_{sr} and u_{sr} are residual shear strength and corresponding shear displacement.

Kulhaway (1975) reviewed the researches concerning the property of shear stiffness. The curved relationships between the shear strength of the joint and normal stress are obtained (Byerlee, 1975). On the other hand, the prediction of dilatancy phenomenon of regular or irregular joint subjected to direct shear loading has been addressed by different researchers (Jaeger, 1971; Barton, 1976; Homand et al., 2001) and the degradation of joint asperities have been investigated (Hutson and Dowding, 1990; Huang et al., 1993; Homan et al., 2001; Lee et al., 2001). Comparison between the pre-peak portions of shear stress-shear displacement curves reveals that the behavior of rock joint can be both linear and nonlinear according to different conditions (Bandis et al., 1983). Simple hyperbolic functions are often used to model the nonlinear shear constitution of joint.

2.1.2 Micro-defects description

Micro-defects commonly exist in rock, which are produced when the local stress exceeds the local strength. An enormous amount of work has been done on micro-defect effects, e.g. how defects affect physical rock properties such as compressibility, strength, elastic wave velocities, permeability and electrical conductivity; how they form, respond to various stress states, grow and interact, as well as their actual morphologies and population statistics and so on.

Simmons and Richter (1976) defined the micro-crack as an opening that occurs in rocks and has one or two dimensions smaller than the third. For flat micro-cracks, one dimension is much less than the other two and the crack aspect ratio defined as the width to length ratio, must be less than 10^{-2} and is typically 10^{-3} to 10^{-5} . The micro-cracks were subdivided into grain boundary cracks (associated and perhaps coincident with the grain boundary), intragranular or intracrystalline cracks (lying totally within the grain) and intergranular or intercrystalline cracks (extending from a grain boundary crossing into one or more other grains). The term transgranular crack is sometimes used for cracks running across a grain from grain boundary to grain boundary.

The scanning electron microscope (SEM) is usually used for detail descriptions of micro-defects due to the higher resolution. These investigations can be found in

Brace et al. (1972), Tapponnier and Brace (1976), Padovani et al. (1982) and He et al. (2010). Fig. 2.1 and Fig. 2.2 show the typical micro-cracks in the rock. Studies of micro-cracks in rock-forming minerals using transmission electron microscope (TEM) are illustrated and discussed by Boland and Hobbs(1973), Tullis and Yund (1977), Marshall and McLaren (1977) and Dunning et al. (1980).

(a) Intragranular cracks

(b) Intergranular cracks

Fig. 2.3 Illustration of micro-cracks of quartz grains

(after He et al. 2010)

The laboratory experiments for investigating the mechanical behavior of micro-defected rock can be categorized as three groups: Ultrasound (King et al. 2000); X-ray imaging-ultrasound combined (Toms-Stewart et al., 2009; Lei and Xue, 2009)

and low-frequency measurements (Batzle et al., 2006; Adam et al., 2009). A large body of experimental work addresses fluid-related wave attenuation and dispersion mechanisms. The attenuation is believed to be caused by local fluid flow (Toksöz et al., 1979; Sams et al., 1997; Adelinet et al., 2010), which is influenced by several factors, such as the presence of clays (Klimentos and McCann, 1990; Best and McCann, 1995).

The global mechanical behavior of material can always be explained by corresponding micromechanical responses based on micromechanics (Budiansky, 1983; Mura, 1987). Paterson (1978) discusses the role of micro-cracks in physical rock properties. Bruno (1991) investigated the inelastic material behavior of sedimentary rock with a two-dimensional element procedure. Basic mechanics principles were applied to the micro-structural constituents of the rock, simulating the physical behavior and failure of the material to model the process of micro-fracture in porous sedimentary rock and a micro-structural model is presented.

A detailed study of micro-cracks in the rock was reviewed by Kranz (1983). From both laboratory and field studies on dynamic behavior of rock and rock mass containing micro-defects, it can be seen that micro-defects cause a diminution in energy, a shift in phase (related to stress wave velocity) and a change in the frequency content of the stress wave signal.

2.2 Theoretical Methods for Rock Dynamic Analysis

2.2.1 Continuum equivalent methods: state-of-the-art

The equivalent medium methods treat the rock/rock mass in a view of entirety, predict the aggregate effects of macro-joints or micro-defects within a representative elementary volume, so as to make a continuum analysis of problems practical. The feature, scope, applicability and limitations of the equivalent medium methods for micro-defected rock and macro-jointed rock mass are reviewed separately in this section.

2.2.1.1 Continuum equivalence for micro-defected rock

Equivalent medium methods are originally used to investigate the global dynamic behavior of micro-defected rock and rock mass, and to analyze the effects of pores, impurities, grains or micro-cracks on stress wave propagation in micro-defected media. In these methods, the background material and the micro-defects together are approximated by an equivalent continuum and the effects of micro-defects on the rock properties are described by the effective moduli of the equivalent medium, which can be utilized to account for the wave propagation velocity. In addition, the attenuation coefficient is usually used to represent the attenuation characteristics.

Early efforts to study the overall behavior of micro-defected material mainly concentrated on the energy dissipation method. Eshelby (1957) studied the stress field and strain field inside and outside an ellipsoidal inclusion embedded in an isotropic solid. The energy dissipation was used to determine the effective elastic moduli. The method of Eshelby was extended by Nur and Simmon (1969) and Nur (1971) to determine the effective elastic moduli of micro-defected rocks. Pressure-dependent crack parameters were used to evaluate the anisotropy of wave velocity in a rock with an anisotropic micro-crack distribution. Walsh (1965a, b) presented an excess strain energy method to calculate the effective elastic moduli. The study was further proposed by Walsh (1966) for deriving seismic quality factor and attenuation coefficient from effective moduli and micro-crack surface friction coefficients in contact, respectively. These methods are under assumption that no stress-field interaction occurred between cracks, and there are therefore accurate only for sparse micro-cracks distribution.

To consider the stress-field interactions between highly concentrated micro-cracks, O'connell and Budianski (1974) developed a self-consistent method. The effective elastic moduli and bulk properties of a material with thin, randomly oriented ellipsoidal cracks and the wave propagation velocities under dry and saturation conditions were determined in their study. The results were further extended by O'connell and Budianski (1977) to determine effective viscoelastic moduli for fluid-saturated cracked solid. There are two characteristic frequencies

near which dissipation is the largest and the moduli change rapidly with frequency. The first corresponds to fluid flow between cracks, and its value can be estimated from the crack geometry or permeability. The second corresponds to the relaxation of shear stress in an isolated viscous fluid inclusion: its value may also be estimated.

The self-consistent method is a major advance in the effective moduli methods, it has been extensively utilized and further developed. Salganik (1973) presented a modified self-consistent method, and the associate effective moduli were derived as a function of crack density. The modified self-consistent method of Salganik (1973) was verified by Zimmerman (1984a, b) and Zimmerman and King (1985) based on the analytical solution and the field measurements of wave velocities measured by King (1966) and Johnston (1978), respectively.

Besides the energy methods, micro-mechanics was introduced to determine the effective elastic moduli from crack shapes, structures and distributions. Chatterjee and Mal (1978) presented a method for calculating effective elastic moduli of a micro-defected medium according to the crack shapes and distributions. Crampin (1984) derived anisotropic effective elastic moduli of rock masses containing preferred oriented cracks. Sayers and Kachanov (1991) developed a simple method to calculate the effective elastic constants of micro-defected solids based on arbitrary crack orientation statistics. Endres (1997) derived analogues for poroelastic parameters with explicit dependence on pore structure by using an inclusion-based model where inclusions represent individual pores. The equivalence in his work implies that pore geometry information can be consistently incorporated into poroelastic parameters with an inclusion-based model. Chapman et al. (2002) derived a microstructural poroelastic model for sandstone. The model was expressed through macroscopic parameters having physical interpretations that are independent of the specific microstructural geometry.

The developed approaches were applied for complicated cases. Fares (1993) studied the effective elastic stiffness of 3-D periodically cracked solids. A direct approach is used to solve the transition problem to obtain the effective stiffness from the details of the microstructure and compared with approximate methods. The results indicate that the crack density parameter is not sufficient to characterize a

damaged solid in general. Lyakhovsky et al. (1997) derived a macroscopic stress-strain relation for a 3-D elastic solid with non-interacting cracks embedded inside a homogeneous matrix following the self-consistent scheme of Budiansky and O'Connell (1977) to evaluate the damage effects of micro-cracks and voids on macroscopic elastic properties. The derived solution was used to analyse rock-mechanics experiments in which beams of Indiana limestone were deformed under four-point loading. The apparent moduli fit the effective elastic moduli calculated with the present damage model well, including the differences between tensile and compressive moduli. The effects of interaction between the cracks in the 3-D rock were also investigated by Sharma (2004) and Grechka and Kachanov (2006).

Considering the difficulty to obtain the analytical solutions for the complicated cases, Nishizawa (1982) proposed a numerical scheme applicable for seismic velocity anisotropy in transversely isotropic materials containing dense aligned micro-cracks based on self-consistent method. The scheme was followed by Xu et al. (1990) and Smyshlyaev et al. (1993) to study the wave propagation in micro-defected rock and Ma et al. (2004) to study the interaction and coalescence of numerous microcracks and Mogilevskaya et al. (2007) to evaluate the effective elastic moduli of micro-defected rocks.

Compare to dry micro-defected rock mass, the fluid-saturated rock mass exhibits complex mechanical behavior. Biot (1956a, b) built a framework to examine wave attenuation in poreelasticity and the effects of pores on wave propagation in a fluid-saturated porous solid were examined. Biot's theory was originally formulated for a single-fluid-saturated two-phase medium system. Several important assumptions were made to develop a theory studying the effects of saturated parallel cylindrical pores on wave attenuation aroused by microscopic flow in the pores. Frequency-dependent attenuation coefficient was derived based on these approximations. McCann and McCann (1985) extended Biot's theory to the cases of pores with a certain size distribution in liquid-saturated porous sediments, and the attenuation coefficient of linear variation with certain wave frequency range was derived.

White (1975) and White et al. (1975) were the first to introduce the mesoscopic-loss mechanism based on approximations in Boit's theory, which was further refined and generalized for two-fluid systems by many authors (Brutsaert, 1964; Berryman et al., 1988; Santos et al., 1990a, b; Tuncay and Corapcioglu, 1996; Lo et al., 2005). Picotti et al. (2007) solved the Biot equations of motion for a two-fluid system. The results are in good agreement with the theoretical values predicted by the White model. However, this solution implies a very large computational effort, therefore the Zener model was recommended to approximate white's relaxation peaks in order to reduce the problem to the solution of equivalent single-phase viscoelastic differential equations (Picotti et al., 2010). White's models predict the dependence of wave velocity and attenuation as a function of frequency and dimension of the microscopic-scale heterogeneities of the medium. Different types of heterogeneities in the rock properties, such as porosity, grain and frame moduli, permeability and fluid properties were taken into consideration. The accuracy of White's results was confirmed by Dutta and Seriff (1979). However, they pointed out that the bulk modulus was instead of by P-wave modulus in deriving the complex bulk modulus in White's theory.

Both theoretical predictions (Mochizuki, 1982; LeRavalec and Gueguen, 1996) and experimental investigations (Johnston and Toksöz, 1980; Murphy 1982a, b; 1984) were performed to investigate the wave attenuation in fully saturated micro-cracked rocks. Some possible mechanisms, such as squirting flow (Mavko and Nur, 1975; O'Connell and Budianski, 1977), bubble movement, frictional sliding (Johnston, 1978) contact line movement (Miksis, 1988) and so on, are proposed to explain the attenuation phenomenon.

Hudson et al. (2001) preformed an evaluation of effective-medium theories for fluid-saturated materials with aligned cracks. The theoretical results are compared with the experimental data on synthetic sandstones. A reasonable fit between theory and observation can be shown. The results were further studied by Tod (2001), he chosen the distributions of aspect ratios to be in agreement with observation (Hay et al., 1988). Thomsen's parameters (Thomsen, 1986) and the attenuation coefficients are seen to be frequency-dependent via the non-dimensional parameter. By

reworking the model to allow the crack orientation and aspect ratios to vary, a more realistic model has the usual properties in which the cracks are isolated at high frequencies and undrained at low frequencies was obtained. A differential scheme for elastic properties of rocks with dry or saturated cracks was proposed by Berryman et al. (2002).

Dispersion and anisotropy of elastic waves in cracked rocks was investigated by Schubnel and Gueguen (2003). They focused on cracked rocks and develop a method to calculate both the elastic wave anisotropy and the dispersion in a fluid-saturated cracked rock. They showed that analytical expressions can be derived for both properties in terms of the crack density tensor components. These results point out important differences that could exist in fluid-saturated rocks between laboratory (high frequency) data and field (low frequency) results.

The global physical behavior of micro-defected rock was further studied and discussed by many other researchers using effective moduli methods. The real porous material is obviously more complex. Müller and Gurevich (2005) developed a theory for elastic wave propagation in a fluid-saturated porous medium with a random distribution of 3-D inhomogeneities by applying the method of statistical smoothing to Biot's equations of poroelasticity. These models show that the shape of the attenuation and dispersion curves is typical of a relaxation mechanism. The predicted dissipation factor for the random model is proportional to frequency and reciprocal value of square root of frequency at the low- and high-frequency limits, respectively.

Elastic properties of double-porosity rocks using the differential effective medium model were studied by Markov et al. (2005). Kozlov (2007) derived an effective model of a permeable dual-porosity layer enclosed by impermeable rocks. For such a model, the reflected wave train can be regarded as a sum of three components, one of which is related to acoustic-impedance contrasts, another to extra compliance caused by P-wave-induced fluid flows between fractures and intergranular pores with a high aspect ratio, and a third to the fluid-flow-induced, inelastic attenuation. And the Seismic signature of a permeable, dual-porosity layer

was studied. Velocity variations in carbonate rocks due to dual porosity and wave-induced fluid flow were investigated by Agersborg et al. (2009).

Lu et al. (2007) proposed a linear dynamic model for a saturated porous medium. The effect of microstructure and nonlinear stress on anisotropic seismic velocities was studied by Verdon et al. (2008). Gurevich et al. (2009) used a poroelasticity approach to study the P-wave dispersion and attenuation in fractured and porous reservoirs. Guéguen and Sarout (2009) studied anisotropy in the crustal rocks induced by crack. Thomsen's parameters for the dry and fluid-saturated rock were predicted. Frequency effects are discussed when fluid is present so that frequency dependence can be accounted for. A review about Seismic wave attenuation and dispersion resulting from wave-induced flow in porous rocks was did by Müller et al. (2010), numerous models for attenuation and velocity dispersion from wave-induced flow have been developed with varying degrees of rigor and complexity during the last four decades was reviewed and categorized. The global viscoelastic behavior of micro-defected rock is primarily caused by the viscosity of background material and filling fluid, the friction of grain boundaries and the existence of micro-defects.

2.2.1.2 Continuum equivalence for macro-jointed rock mass

The traditional methods for determining the dynamic behavior of rock mass are to derive its effective elastic moduli. In these methods, the additional compliance of a dilute population of joints is incorporated into an average strain, resulting in effective moduli for the rock as a whole, so as to make a continuum analysis of problems possible.

In the effective moduli model, a rock mass containing a single set of plane parallel joints can be represented by a transversely isotropic medium described by five elastic constants. Using an average strain method, the expressions for the five effective elastic constants can be derived in terms of the elastic properties of the intact rock, joint spacing, and joint specific stiffness (Amadei and Goodman, 1981; Schoenberg, 1983; Majer et al., 1985). An illustration of the transversely isotropic medium is as shown in Fig. 2.5.

Fig. 2.4 Coordinate system for equivalent transversely isotropic medium

(after Pyrak-Nolte et al., 1990b)

Therefore, the five elastic constants for the coordinate system shown in Fig.2.4 are

$$\begin{aligned}
 C_{1212} &= \mu C_{1111} = [1 + 4(1 - \gamma) E_n] C_{3333} \\
 C_{1133} &= (1 - 2\gamma) C_{3333} \\
 C_{2323} &= \mu / (1 + E_t) = C_{1313} \\
 C_{3333} &= \mu / (\gamma + E_n)
 \end{aligned} \tag{2.5}$$

where, it is defined that $E_T = \mu / (S k_t)$, $E_N = \mu / (S k_n)$, $\gamma = (1 - 2\nu) / (2 - 2\nu)$, μ is the shear modulus; S is the joint spacing, ν is the Poisson's ratio; and k_t and k_n are the transverse and normal specific stiffness of the joints, respectively. The ratios E_t and E_n express the contribution of the joints to the stiffness of the rock mass.

Define the angle of incidence θ as the angle between the direction of propagation of the phase velocity and the normal to the plane of the joints. The phase velocity V as a function of θ can be calculated as

$$\begin{aligned}
 \rho V_p^2(\theta) &= \frac{1}{2} [C_{3333} + C_{2323} + (C_{1111} - C_{3333}) \sin^2 \theta + D(\theta)] \\
 \rho V_{s2}^2(\theta) &= \frac{1}{2} [C_{3333} + C_{2323} + (C_{1111} - C_{3333}) \sin^2 \theta - D(\theta)] \\
 \rho V_{s1}^2(\theta) &= C_{1212} \sin^2 \theta + C_{2323} \cos^2 \theta
 \end{aligned} \tag{2.6}$$

where

$$\begin{aligned}
 D(\theta) &= \left\{ (C_{3333} - C_{2323})^2 + 2 \left[2(C_{1133} + C_{2323})^2 - (C_{3333} - C_{2323} (C_{1111} + C_{3333} - 2C_{2323})) \right] \sin^2 \theta \right. \\
 &\quad \left. + \left[(C_{1111} + C_{3333} - 2C_{2323})^2 - 4(C_{1133} + C_{2323})^2 \right] \sin^4 \theta \right\}^{\frac{1}{2}} \\
 &\text{(Thomsen, 1986).}
 \end{aligned}$$

Crampin et al. (1980) derived elastic constants for an equivalent purely elastic solid which has the same velocity variations as the jointed solid. The anisotropy in velocity is calculated and the joints are mapped by the measurements of velocity-anisotropy in underground refractors. For the purely elastic case, that is, no viscous loss (no complex moduli), the resulting velocities are nondispersive.

Hudson (1981) using a static approach to calculate the effect of joints on the displacement field, elastic constants were derived for rock mass containing a set of joints. From the elastic constants, the stress wave velocities of the jointed rock mass were determined. In his model, the size of joints was assumed small compared to the wavelengths and the concentration of the joints were small. To determine the effect of joints on attenuation, Hudson adapted a scattering formulation, using the velocities from the static model.

White (1983) derived the effective moduli of jointed rock mass in the explicit expression of the stiffness of individual joint in a set, as well as their spacing. These effective moduli were then used to calculate stress wave velocities.

Gerrard (1982a) considered a stratified rock mass consisting of a series of parallel layers each of which has orthorhombic elastic properties. Expressions for the properties of the equivalent medium are given in terms of the elastic properties of the constituent layers. Example results are plotted for a range of simple cases. The theory of the orthorhombic layer is then successively applied to analyses the

stress-deformation behavior of rock mass containing one, two and three sets of joints by Gerrard (1982b). The stress-deformation properties of a particular jointed rock mass are described in terms of an equivalent homogeneous orthorhombic material and these properties are defined as functions of nine ‘joint compliance’ terms, six relating to shear moduli and three relating to directing moduli and Poisson’s ratios.

The equivalent continuum method has advantages in modeling the highly jointed rock mass, where the joint spacing is small compared to the scale of the structure being analyzed and the wavelength of the propagating stress wave. Numerous work can be found, e.g., Gerrard (1982a, b), Amadei and Goodman (1981), Morland (1974), Chen (1989, etc. However, close-form expressions for elastic moduli are available only for rock mass with three sets of orthogonal intersecting joints. Huang et al. (1995) proposed a stress-strain model for an assemblage of intact rock blocks separated by joint planes. The stress-strain relationship accounts for spacing and orientations of the joint sets. Closed-form expressions of elastic moduli for a rock mass with three intersecting sets of joints are derived explicitly in terms of properties of joints and intact rock. Applicability of the derived expressions is evaluated by comparing the predicted results with experimental results from physical mode tests.

A completely equivalent medium representation of rock with different types of joints was presented by Liu et al. (2000). They directly simulated natural joint surfaces as planar distributions of small isolated areas of slip (model 1) or, conversely, as planar distributions of imperfect interfacial contacts (model 2) or, plane surfaces separated by thin continuous layers of viscous fluid or a soft material (model 3). The analytical expressions for the joint compliances for these three models were presented and, using these analytical results, the effective compliances and stiffnesses of the jointed rock mass are computed.

In summary, there are two main methods used to measure the elastic constants are: (1) the static method, which is based on the measurement of the deformation induced in a material by the application of a known force, and (2) the dynamic method, which implies the measuring the ultrasonic body wave velocity. The

differences between the static and dynamic effective elastic moduli of jointed rock mass were evaluated by King (1983), Eissa and Kazi (1988), Sheshenin et al. (1997), Ma et al. (2011b), etc. As a result of inelastic effects, it is not granted that these values coincide. The Laboratory measurements revealed the values obtained from static measurements can be smaller than those from dynamic estimates by as much as an order of magnitude in some cases (Eissa and Kazi, 1988).

Sitharam (2009) presented the summary of the work on jointed rock mass along with development of equivalent continuum model, validation and field case studies. It can be concluded from the various case studies that the equivalent continuum approach simplifies the field problem analysis to a large extent and gives a fair estimate of the behavior of the surrounding jointed rock with minimal input data. The equivalent continuum model developed works well for single-, multiple- and block-jointed rock with different joint fabric and joint orientation under a wide range confining pressures.

Li et al. (2010b) and Zhu et al. (2011a, 2011b) investigated the stress wave propagation through rock mass containing a parallel joint set, virtual wave sources were introduced to simulate the reflected wave caused by the discreteness of individual joints.

2.2.2 Discrete methods: state-of-the-art

Besides the continuum equivalence methods, discrete methods are also widely applied in analyzing stress wave propagation in rock/rock mass. There are mainly two theoretical methods in this category, wave scattering methods and displacement discontinuity methods. It should be noted that the initial purpose of the wave scattering methods is to study the effects of micro-defects on wave propagation, while the initial purpose of the displacement discontinuity methods is to study the effects of macro-joints on wave propagation.

2.2.2.1 Wave scattering methods

The wave scattering methods are used to study the effects of micro-defects on stress wave propagation velocity and amplitude attenuation in micro-defected media.

In these methods, the stress waves propagating through micro-cracks are considered to be uniformly scattered by the cracks, provided that the crack size is small compared to the wavelength and the crack concentration is dilute enough so that no interactions of wave scattering occur between individual cracks.

Early efforts made on wave scattering methods are mainly based on the linear model of scattering. That is, the wave scattering is modeled based on the spring contact condition of cracks. Some theories were developed to evaluate the scattering effects of micro-cracks on dynamic behaviors of rock mass, such as multiple scattering theory (Foldy, 1945; Lax, 1952), matrix theory (Waterman, 1976), damage mechanics-based theory (Han, 1995), reflection and transmission coefficients methods (Mal, 1970a; Angel and Achenbach, 1987; Zhang and Gross, 1993; Eriksson et al., 1995), and so on.

Hudson (1980, 1981) proposed a “method of smoothing” to calculate overall elastic constants and examined the wave velocity and attenuation in cracked materials by taking into account the wave scattering effects.), Hudson’s method (1981) predicts the frequency dependence of attenuation caused by wave scattering, which revealed that the attenuation coefficient due to wave scattering was proportional to the crack density and to the third power of ratio between the mean crack radius and the wavelength. The “method of smoothing” was extended by Hudson (1986, 1988, 1990), Hudson and Knopoff (1989), Peacock and Hudson (1990), and Hudson et al. (1996) to the cases of various orders of the crack density or multiple sets of cracks aligned in different directions. However, method of smoothing only considered wave scattering effects.

Besides the wave scattering effects, the effects of fluid-filled cracks on the wave propagation were investigated by Chatterjee et al. (1980), Garbin and Knopoff (1973 and 1975a, b), Craster (1998), Galvin and Gurevich (2009) and so on. The study of Chatterjee et al. (1980) concluded that effect of the fluid viscosity is more significant than that of scattering at low frequencies. Pointer et al. (2000) investigated seismic wave propagation in cracked porous media, three distinct mechanisms of wave-induced fluid flow: flow through connections between cracks in an otherwise non-porous material, fluid movement within partially saturated

cracks, and diffusion from the cracks into a porous matrix material were considered in their research.

On the other hand, laboratory experiments (Kuster and Toksöz, 1974a, b; Xu and King, 1990; Ass'ad et al., 1993; Fukushima et al., 2003) have been carried out by many researchers to measure the wave propagation velocity and amplitude attenuation in the cracked media, where comparisons between the experiments and scattering theory were made.

Besides linear model of scattering, studies on wave scattering at cracks with nonlinear contact conditions have been conducted (Morris et al., 1979; Achenbach and Norris, 1982; Hirose and Achenbach, 1993; Smyshlyaev and Willis, 1994; Capuani and Willis, 1997). In these studies, the nonlinear scattering is generally considered to arise from the nonlinear process of interaction of crack faces in contact. An important phenomenon of higher harmonics of scattered waves at cracks caused by the nonlinear relation between the crack face traction and crack closure was observed in the far field.

A comparative study of linear and nonlinear scattering at cracks was conducted by Smyshlyaev and Willis (1994), based on the quasi-static approximation for the displacement jump across the crack. In their study, the linear scattering results from the assumption that either the crack faces never come into contact, or they remain in permanent gliding contact; whereas, nonlinear scattering arises when a unilateral constrain is introduced to crack deformation, corresponding to the opening of the crack under tension, closure under compression and frictionless slip under shear.

Since analytical solutions for more complicated crack distributions are often difficult to derive, numerical methods such as the boundary element method have been used (Angel and Achenbach, 1985a, b, 1987; Achenbach and Li, 1986a, b; Coutant, 1989; Achenbach and Zhang, 1990; Hirose and Kitahara, 1991, Zeng et al., 2007). Iturraran-Viveros et al. (2008) performed a boundary element simulation of scattering of elastic waves by 3-D cracks. In order to validate the method, they compare results of displacements of a penny-shaped crack for a vertical incident P-wave with the classic results by Mal (1970b) obtaining excellent agreement.

To sum up, the wave scattering theories are suitable for analyzing the wave attenuation across the micro-cracks of smaller sizes compared to the wavelength. The crack shape, crack density and distribution are very important parameters in the wave scattering formulations. Apparent wave attenuation due to the scattering of energy at cracks is considered as the principal attenuation mechanism. The wave attenuation is evaluated in terms of the reflection and transmission coefficients of a distribution of cracks. If cracks are filled with liquid, intrinsic attenuation can be taken into account based on the real energy loss—viscous dissipation by the filling liquid. The quality factor (Q) and wave attenuation coefficient (α) can also be derived to quantify the wave attenuation due to both energy scattering and dissipation.

2.2.2.2 Displacement discontinuity methods

Displacement discontinuity methods treat the joint as individual boundary in the view of discrete medium. They are applicable to analyze the effects of macro-joint on the wave propagation, where the joints are large in extent and small in thickness relative to the incident wavelength. When stress waves propagate across such a joint, the stress field is continuous but the displacement field is discontinuous due to the joint deformation. These two assumptions can be mathematically represented as the discontinuity boundary conditions in the wave equations. The effects of a joint on wave attenuation are evaluated in terms of reflection and transmission coefficients of the joint, which are the ratio of the amplitudes of reflected and transmitted waves to the incident wave amplitude, respectively.

The displacement discontinuity method was originally developed by Mindlin (1960) in a study of the coupling between compressional and shears waves at the boundary of elastically restrained, elastic plates (Perino et al., 2010). The macro-joint can be treated as an incompletely welded interface or a non-welded interface. The fracture may have opening, closure or slip displacement under normal and shear stresses, resulting in the displacement discontinuity across the fracture. Studies on the effects of a single fracture on wave propagation have been conducted from various considerations of the fracture deformational behaviors (Jones and Whitter, 1967; Kendall and Tabor, 1971; Schoenberg, 1980).

Efforts have been made to establish the displacement discontinuity boundary conditions by considering the slip rate-dependent shear behaviour of a fracture (e.g. White, 1965; Miller, 1977 and 1978; Chen et al., 1993). In their studies, the shear stress was assumed to be a nonlinear function of frictional slip (discontinuity in shear displacement across the fracture) and slip rate of the fracture, which defines the displacement discontinuity boundary conditions as a nonlinear function of stress versus shear deformation.

Pyrak-Nolte (1988) and Pyrak-Nolte et al. (1987, 1990a) conducted a comprehensive study on the effects of single dry and saturated fracture on wave propagation. A generalized linear displacement discontinuity model was established by considering the linear elastic and linear rheologic behavior of fracture, respectively.

Besides the linear displacement discontinuity methods, studies have been conducted to establish the displacement discontinuity boundary conditions by considering the nonlinear normal behaviour (closure) of the fracture (Miller 1977, 1978).

Zhao and Cai (2001) presented a theoretical study on normally incident elastic P-wave transmission across single dry joints with a nonlinear normal deformational behavior. The effects of nonlinear fracture normal behavior on P-wave transmission are examined without the mixture of fracture shear behavior. A hyperbolic elastic model (B-B model) was used and the numerical solutions of magnitudes of transmission and reflection coefficients were obtained. Comparisons between the linear and nonlinear models shown that the transmission and reflection coefficients for the linear model are special solutions of those for the nonlinear model, when the incident wave amplitude is so low that the current maximum closure of joint incurred during the wave transmission is much small, relative to the maximum allowable closure.

Li et al. (2009) presented an analytical and experimental study on a longitudinal wave transmission normally across a filled rock joint. The dynamic property of the filling material for the artificial joints is derived from a series of

modified split Hopkinson sure bar tests and simulated using a three-phase material (Ma et al., 2011a). An extended displacement method is used to consider the deformation of the joints. The derived wave transmission coefficients across the filled joint agree very well with those from the test results.

2.3 Numerical Methods for Rock Dynamic Analysis

In the cases when the rock mass is with high density discontinuities, such as highly damaged by micro-cracks, numerical modeling is a powerful tool to character the mechanical behavior of rock mass and quantitatively evaluate the effects of discontinuities on wave propagation, sometimes irreplaceable. Various efforts, either explicitly or implicitly, have been dedicated by researchers. These researches can be categorized as continuum-based numerical methods and discrete element method.

In the continuum-based numerical methods, it is assumed that The discontinuities are not taken into account explicitly when the engineering problems take material in quantities large enough so that the material can be described by the averaged material properties, which are assumed to be continuous. During the last a few decades, various continuum-based numerical methods have been developed, such as the finite difference method (FDM), the finite element method (FEM), the boundary element method (BEM), and various meshless (or meshfree) methods. However, limitations of the continuum-based assumption restrict their applications in discontinuous problems. For example, when the representative volume is either much larger than or of a similar order of the engineering problem, the continuum hypothesis is violated and an explicit representation of the discontinuities becomes essential.

To overcome the disadvantages in the continuum-based numerical methods, another class of numerical techniques called the discrete element method (DEM) has been developed. The key concept of the DEM is that the problem domain is treated as an assemblage of rigid or deformable blocks and the contacts between them need to be identified and continuously updated during the entire

deformation/motion process, and represented by proper constitutive models (Jing, 2003). The DEM is especially suitable for simulation of large-scale displacements of individual blocks, block rotations, and complete detachment. Typical examples of the DEM include the distinct element method (DEM) originated by Cundall (1971) and the discontinuous deformation analysis (DDA) pioneered by Shi (1988). The recently developed numerical manifold method (NMM) (Shi, 1991; 1992; 1995; 1996; 1997) is such a hybrid method.

NMM is extremely suitable for discontinuity rock/rock mass modeling for the features of: (1) its mathematical covers do not need to conform to either the external boundaries or the internal discontinuities, which makes the meshing task very convenient and re-meshing be completely avoided for discontinuity evolution; (2) the NMM models the discontinuities in a straightforward manner by splitting each mathematical cover which is completely cut by the crack surfaces into several physical covers; each physical cover is assigned an independent local function; by such a way, complex cracks with arbitrary number of branches can be modeled in an exactly same way as the modeling of a single crack; (3) its simplex integration method makes the integration of the weak form over elements intersected by internal discontinuities and/or external boundaries be evaluated conveniently, without the need of element partitioning which is usually adopted by the extended finite element method (XFEM); (4) its contact logic makes the frictional contact boundary conditions between the two sides of a crack surface or between two discrete blocks be satisfied accurately (An, 2010).

2.4 Experimental Techniques for Rock Dynamic Analysis

Split Hopkinson Pressure Bar was invented in the 19th century due to awareness of the fact that material behaves different under dynamic loading and static loading. It is commonly adopted to build the dynamic constitutive laws of material for its easy operation, good repeatability and high accuracy.

The origin of this technique was from the work of Hopkinson (1914) who used it to measure a pressure-pulse profile with a long thin bar due to an impact produced

by a bullet or explosive. The main components of Hopkinson's apparatus were means of developing impacts such as pressures, a long steel rod, a short steel billet, and a ballistic pendulum. By exerting impact on one end of the rod, a compressive pressure wave of finite length was generated within the rod while at the other far end of the rod, a short steel billet attached was held by only a thin layer of grease. Hopkinson's idea was that as the compressive wave traversed down the bar, through the greased joint, and into the billet the wave would be reflected at the far end as a pulse of tension. And since the grease could not withstand any appreciable tensile loads, the billet would fly off with a definite momentum, measured by a ballistic pendulum. The time over which this momentum acted was the round trip time of the longitudinal wave in the billet. By running several test of identical magnitude but with varying length of cylindrical billets, a series of pressure-time curves were generated describing the impact event. Hopkinson was always capable of determining the maximum pressure and total duration of these impact events, but exact pressure-time curves were sketchy. Thus, the shape and evolution of the stress pulses that propagated down the long metallic rods as a function of time with the use of momentum traps of differing length was studied by him and based on his study, the experimental apparatus using elastic stress-wave propagation in long rods to study dynamic processes in materials was named as the Hopkinson bar.

Bancroft (1941) solves bar frequency equation of Pochhammer for the velocities of longitudinal waves in cylindrical bars. The velocity of longitudinal waves in cylindrical bars is expressed in terms of a wave of infinite wavelength, Poisson's ratio, and the ratio of the bar diameter to the wavelength of interest. The usefulness of Bancroft's work applied towards Hopkinson bar testing was not fully recognized until much later when the technology of the computer sped up the data processing. Hence, it was Bancroft's work which fueled significant advancements in the area of dispersion correction.

The Split Hopkinson Pressure Bar technique has also been well established since the critical study made by Davies (1948) and Kolsky (1949). The practical configuration with two long bars in series and a short specimen between the bars

was setup to measure the dynamic stress-strain response of the material based on the one dimensional wave propagation theory in the pressure bars.

Davies (1948) developed the technique using condensers to measure the strains existing in the pressure bar. Assuming that the pressures in the bars were under the elastic limit of the material, the output from the condenser was thus said to be proportional to the displacement–time relations that were proportional to the pressure-time relations. Thereafter, the accuracy of Hopkinson’s original apparatus was greatly improved with the aid of using the condensers to measure strains, which relied on measuring the momentum of a steel billet flying off at the end of the pressure bar. However, recalling that Hopkinson had attached the billets with a film of grease, which had introduced a major unknown in relation to the forces required to break the greased joint.

Kolsky (1949) included a second pressure bar to Hopkinson’s original apparatus, therefore naming it as the split Hopkinson bar. In addition, instead of attaching a billet at the far end of a bar, Kolsky sandwiched a specimen between the two bars. He presented the expressions used for calculating specimen properties based on strain histories in the bars as the strains were measured using similar condensers as those used by Davies. Of course the new two bar apparatus required measurements in both bars and this two bar technique had become the most widely used testing procedure today and in some literature the split Hopkinson bar may be referred to as the Kolsky bar. At the later stage, this technique which is developed for compression tests had been used to apply to tensile loading and to torsional loading by Harding et al. (1960) and Duffy et al. (1971) respectively. To improve its precision, wave dispersion effects, three-dimensional effects and transient effects in the specimen had been studied by the researchers in the past decades. For the wave dispersion correction method, it was used to reduce the oscillation.

Li et al. (2000) proposed an improved experimental approach for eliminating the oscillation that exists in the dynamic stress-strain response of rocks and other brittle materials. While Meng and Li (2003) proposed another modification of SHPB test method based on wave separation technique, which could be used to reduce the time shifting distance and the pressure bar length in SHPB test. Incident

and reflected strains were separated through two-point strain measurement and associated iterative algorithm. Nevertheless, the two-point strain measurement intended to collect signals from two very close points might introduce some inevitable errors. Furthermore, as the SHPB test method was established based on one-dimensional elastic wave propagation theory, there were some constraints on the length of the pressure bars and the dimension of the specimen. Thus, a parameter identification method was used to overcome these constraints.

2.5 Comparisons of Theoretical Methods and Discussions

Based on the foregoing review, wave scattering theories are used to study the effects of micro-defect on wave propagation. In the theories, individual defect and contact characteristics are explicitly considered, and the reflection and transmission coefficients are used to examine the effects of wave scattering by micro-defect. It will be difficult to obtain the analytical solutions in complicated cases, and then, the numerical method (e.g. boundary element method) is implemented. One major limitation of these theories is that the wave scattering theories are inappropriate to analyze the effects of joints, which are large in the extent relative to incident wavelength. Therefore, the wave scattering theories cannot be used to analyze the global mechanical behavior of rock mass containing two-scale discontinuities (macro-joint/micro-crack) (Cai, 2001; Zhao, 2004).

The displacement discontinuity theories treat problems from a viewpoint of individuality. The effects of discontinuities on wave propagation are evaluated in term of reflection and transmission coefficients at individual discontinuities. The displacement discontinuity theories are valid only when the discontinuities are large in extent and small in thickness relative to incident wavelength and asperity separations are much smaller than incident wavelength Nolte et al. (2000). Furthermore, for the complex problems, the method of characteristic lines is commonly used in the wave propagation equations based on the DDM, however, the results obtained by this method is general in a differential form, which is limited by giving an explicit expression of the solutions and thus it is not realistic for practical applications for rock mass (Li et al., 2010b; Perino et al., 2010; Ma et al.,

2011b). Therefore, the displacement discontinuity theories are unsuitable for analyzing the global mechanical behavior of rock mass containing two-scale discontinuities neither.

Equivalent medium theories treat problems from a viewpoint of entirety. The effects of discontinuities (micro-defect/macro-joint) are lumped into effective moduli and quality factor of the equivalent medium and no discreteness need to be considered in such continuum equivalence analysis. Moreover, the results obtained from the equivalent medium theories are common explicit for its continuous characters (Cai, 2001; Zhao, 2004; Li et al., 2010b; Ma et al., 2011b; Fan et al., 2012). Therefore, the equivalent medium theories are applicable to analyze the global mechanical behavior of rock mass containing double-scale discontinuities, and to evaluate the effects of double-scale discontinuities on the stress wave propagation. The comparisons of these theoretical methods are as shown in Table 2.1.

Table 2.1 Comparisons of theoretical methods

	Discrete methods	Conventional elastic continuum equivalent methods
Application	Macro-jointed rock mass or micro-defected rock	Macro-jointed rock mass and micro-defected rock
Basic principle	Treat the joint as individual boundary in the view of discrete medium	Treat the rock/rock mass in a view of entirety
Merit	Explicit discontinuity model	Continuum mechanics is used
	Easy numerical calculation	Easy theoretical analysis
Demerit	Result is general in a differential form	Frequency dependence always ignored
	Become complex when joint number is large	Wave attenuation cannot be taken into consideration

2.6 Limitations of Previous Work and Remaining Problems

Natural joints can be filled with sand, water, clay and so on. Extensive studies have been performed on the constitution of rock joint. However, most studies are concentrated in the static or low-frequency loading (cyclic loading) category. No studies on influences of joint components (e.g. water percentage, sand thickness etc.) on its interaction under stress wave propagation have been performed systemically.

The equivalent medium methods are applicable in rock dynamics. However, traditional equivalent medium theories inherently results in two limitations. One is the loss of discreteness of individual joint for its continuous characteristic. The other is the loss of intrinsic frequency-dependent property of rock mass for its elastic characteristic. Therefore, it is necessary to develop a new equivalent medium to overcome these limitations. A continuum medium can be used to consider both stress wave attenuation and frequency-dependence should be introduced.

Micro-defects commonly exist in the natural rock, which will result in phase shift and amplitude attenuation in stress wave propagation. It is of significant interest to rock engineering that the effects of micro-defects can be quantitatively described.

In addition, it is extremely complicated for theoretical methods to evaluate the effects of micro-defects characteristics (e.g. length, quantity, specific stiffness etc.) on mechanical properties of micro-defected rock, and its influence on the wave propagation. As an alternative of theoretical analysis, numerical methods, such as NMM, can be adopted to solve such complicated problems.

Most of the current methods to consider the dynamic behavior of rock mass are all scale-dependent, effective either for the macro-joint or for the micro-defect. No studies on the dynamic behavior of rock mass consists of double-scale discontinuities (micro-defects/macro-joints) have been preformed.

CHAPTER 3 DYNAMIC BEHAVIOR OF MACRO-JOINT

3.1 Introduction

This chapter presents a study of using a series of modified split Hopkinson press bar (SHPB) tests to investigate the dynamic properties of artificial sand filled macro-joints. Two granitic pressure bars with a sandwiched sand or clay layer are used to represent the filled joint to simulate longitudinal stress wave propagation across such geological discontinuities. The stress-closure relationship of the filled macro-joint under dynamic loading is obtained experimentally, and the geometrical effects on the model are discussed.

Macro-joints exist in many rock mass, and have significant effects on the mechanical behavior of the rock mass. Quantitative representation of rock joint dynamic properties and their effects on rock mass behavior has been one of the main tasks in the rock mechanics and engineering. Joints can be filled with sand, silt, breccia, gouge and mylonite, etc. Due to the enormous variety of occurrences, joints display a wide range of physical behavior, in particular as regards their normal and shear strength, deformability, permeability, etc. The physical properties of the filling materials influence the normal stiffness of the joints. The influence of the width on the strength of the filled rock joint has been investigated through experimental studies. The test results of Pyrak-Nolte et al. (1992) showed that the dynamic stiffness of rock joints was always higher than the quasi-static stiffness. Due to the complex nature of dynamic tests for rock joints, the dynamic properties of filled rock joint are still not well understood. Equation Chapter (Next) Section 1

3.2 Experimental Test

3.2.1 Test set-up: Modified SHPB

A series of modified SHPB tests to study the dynamic properties of the filled macro-joint has been carried out.

Based on the fact that stress wave behavior differently when propagates across steel/sand interface and rock/sand interface (Kolsky, 1963), in order to simulate the stress wave propagation through rock mass similarly to which in the natural situation, granitic bars are used to replace the traditional steel bar in the SHPB tests. In addition, one-dimensional wave propagation theory is used to obtain the mechanical parameters of the macro-joint, thus geometrical spreading should be avoided which requires small ratio of diameter to length. Therefore, the granitic bars cored from an underground construction site are selected with the same diameters of 50mm, and lengths of 0.97 and 1.005m, respectively.

The density of the granitic bar was measured as 2650 kg/m^3 . The Young's modulus of the granitic bars was calculated based on the wave propagation velocity and the mass density of the granitic bars. The integrity and homogeneity of the granitic bars were examined carefully. The end surfaces of the two granitic bars were ground by a grinding machine before the SHPB tests.

A pendulum hammer was used to generate a longitudinal pulse wave which was applied to the incident pressure bar. The swing-angle of the hammer indicates different intensity of the pulse load.

During the test, the sand layer was contained in a plastic tube to prevent out flow of the sand. The two granitic-bar/sand-layer interfaces were contacted firmly. Four gauges were used to get the strain wave recordings, where two of them were glued on the incident bar and the other two on the transmitter bar.

Fig. 3.1(a) shows the test setup and Figs. 3.1(b) and (c) show the granitic bars and the sand layer in the test, respectively.

Fig. 3.1 Scheme of modified SHPB

3.2.2 Testing programs

In order to understand the dynamic stress-strain relation of filled rock joints, twenty-seven groups of tests were carried out with respect to different widths and water contents of sand layers, and different intensities of the pulse load. The width of the sand layer varied from 3 mm, 4 mm to 5 mm. And for each width, the sand layer had three different water contents, i.e., dry, 5% and 10% in weight ratio. During the test, swing-angles of 20°, 30° and 40° of the pendulum hammer were adopted for joints filled with 3 mm and 4 mm sand layers. Since the 5 mm width sand layer absorbed larger impact energy, the swing-angles used were 30°, 35° and 40°, respectively. Table 3.1 shows the program of the modified SHPB tests.

Table 3.1 Program of SHPB tests

No.	Joint thickness	Water content	Swing angle	Test times	No.	Joint thickness	Water content	Swing angle	Test times	No.	Joint thickness	Water content	Swing angle	Test times
1	3mm	dry	20 °	5	10	4mm	dry	20 °	5	19	5mm	dry	30 °	5
2			30 °	5	11			30 °	5	20			35 °	5
3			40 °	5	12			40 °	5	21			40 °	5
4		5%	20 °	5	13		5%	20 °	5	22		5%	30 °	5
5			30 °	5	14			30 °	5	23			35 °	5
6			40 °	5	15			40 °	5	24			40 °	5
7		10%	20 °	5	16		10%	20 °	5	25		10%	30 °	5
8			30 °	5	17			30 °	5	26			35 °	5
9			40 °	5	18			40 °	5	27			40 °	5

3.2.3 Data processing method

Based on one-dimensional wave propagation theory, two waves propagating in opposite directions exist in a finite-length bar due to wave reflections. Since the granitic pressure bars used in the present tests were relatively short, the incident wave and the reflected wave in both the incident and transmitter bars overlapped, a wave separation method was proposed by Li and Ma (2009) for data processing. The method was based on the characteristic diagram shown in Fig. 3.2, where x_A and x_B were respectively the distances from gauge A and gauge B to the upstream end O where the incident pulse wave was generated. The other end D represents the down-stream end where the incident wave propagated to and reflected backward. The length of the bar was denoted as l . The wave separation method was valid for both the incident and the transmitter bars. Assuming that the test recordings of these two strain gauges are $\varepsilon_A(t)$ and $\varepsilon_B(t)$ which are superposed by the two opposite strain waves $\varepsilon_{AP}(x_A, t)$ and $\varepsilon_{AN}(x_A, t)$ for gauge A, and $\varepsilon_{BP}(x_B, t)$ and $\varepsilon_{BN}(x_B, t)$ for gauge B, respectively, where the symbols P and N represent the positive and negative directions, the relations between $\varepsilon_{AP}(x_A, t)$ and $\varepsilon_{BP}(x_B, t)$, $\varepsilon_{AN}(x_A, t)$ and $\varepsilon_{BN}(x_B, t)$ can be obtained by using the time shifting function (Zhao and Gary, 1997)

Fig. 3.2 Characteristic diagram in a pressure bar

Define $t_A = x_A / c$, $t_0 = (x_B - x_A) / c$, $t_B = (l - x_B) / c$, the two opposite strain waves shown in Fig. 3.3 can be separated and expressed as

$$\begin{cases} \varepsilon_{AP}(x_A, t) = [\varepsilon_A(t) - \varepsilon_{BN}(x_B, t - t_0)] \times H_1(t - t_1) \\ \varepsilon_{BP}(x_B, t) = \varepsilon_{AP}(x_A, t - t_0) \times H_2(t - t_2) \\ \varepsilon_{BN}(x_B, t) = [\varepsilon_B(x_B, t) - \varepsilon_{BP}(x_B, t)] \times H_3(t - t_3) \\ \varepsilon_{AN}(x_A, t) = \varepsilon_{BN}(x_B, t - t_0) \times H_4(t - t_4) \end{cases} \quad (3.1)$$

where $H_i(t)$ is the Heaviside function and

$$H_i(t - a_i) = \begin{cases} 1 & \text{when } t \geq a_i \\ 0 & \text{when } t < a_i \end{cases} \quad (i = 1, 2, 3, 4) \quad (3.2)$$

and $t_1 = t_A$, $t_2 = t_A + t_0$, $t_3 = t_A + t_0 + 2t_B$, $t_4 = t_A + 2t_0 + 2t_B$; $\varepsilon_{BN}(x_B, t - t_0) = 0$ when $t < t_4$; C is the wave propagation velocity in the granitic bar.

For an arbitrary point on the bar, the two opposite strain waves can be obtained by time shifting the positive wave at the gauge A and negative wave at the gauge B , that is

$$\varepsilon_P(x, t) = \varepsilon_{AP}(x_A, t - \frac{x - x_A}{c}) \quad , \quad \varepsilon_N(x, t) = \varepsilon_{BN}(x_B, t - \frac{x_B - x}{c}) \quad (3.3)$$

So, the sum of these two opposite waves derives the strain $\varepsilon(x, t)$ of this point,

$$\varepsilon(x, t) = \varepsilon_P(x, t) + \varepsilon_N(x, t) \quad (3.4)$$

and the particle velocity $v(x, t)$ of the point is proportional to their difference

$$v(x, t) = c \cdot [\varepsilon_P(x, t) - \varepsilon_N(x, t)] \quad (3.5)$$

If the arbitrary point is happened at the up-stream end or the down-stream end, the two opposite waves at this end can be calculated from Eq. (3.3), and the strain and the particle velocity of the end can be obtained from Eqs. (3.4) and (3.5). According to the fundamental theory of the SHPB test, the stress σ , strain rate $\dot{\varepsilon}$ and strain ε of the specimen (the sand layer in the present study) can be expressed as

$$\sigma = \frac{E}{2}(\varepsilon_I + \varepsilon_R + \varepsilon_T) = \frac{E}{2}[(\varepsilon_{IP} + \varepsilon_{IN}) + (\varepsilon_{TP} + \varepsilon_{TN})] \quad (3.6)$$

$$\dot{\varepsilon} = \frac{1}{L}(v_I - v_R - v_T) = \frac{c}{L}[(\varepsilon_{IP} - \varepsilon_{IN}) - (\varepsilon_{TP} - \varepsilon_{TN})] \quad (3.7)$$

$$\varepsilon = \int_0^t \dot{\varepsilon} dt = \frac{c}{L} \int_0^t [(\varepsilon_{IP} - \varepsilon_{IN}) - (\varepsilon_{TP} - \varepsilon_{TN})] dt \quad (3.8)$$

where ε_{IP} and ε_{IN} , ε_{TP} and ε_{TN} are the two opposite strain waves at the downstream end of the incident bar and the up-stream end of the transmitter bar which are also the incident-bar/sand-layer and transmitter-bar/sand-layer interfaces; L is the width of the sand layer.

With the signals at the four strain gauges recorded for different cases, Eqs. (3.6) ~ (3.8) were adopted to obtain the dynamic stress-strain relations of the sand layers. The positive and negative waves for each strain gauge can be separated from Eq. (3.1). On the two interfaces of the incident-bar/sand-layer and sand-layer/transmitter-bar, the strain and particle velocities can be calculated from Eqs. (3.3) ~ (3.5).

3.3 Experimental Results

The strains at the four gauge locations were recorded. Fig. 3.3 shows the average strain waves from testing results.

(1a) Incident bar recordings

(1b) transmitter bar recordings

(1) 3mm width and dry sand layer with swing-angle = 20°

(2a) Incident bar recordings

(2b) transmitter bar recordings

(2) 3mm width and dry sand layer with swing-angle = 30°

(3a) Incident bar recordings

(3b) transmitter bar recordings

(3) 3mm width and dry sand layer with swing-angle = 40°

(4a) Incident bar recordings

(4b) transmitter bar recordings

(4) 3mm width and 5% water content with swing-angle = 20°

(5a) Incident bar recordings

(5b) transmitter bar recordings

(5) 3mm width and 5% water content with swing-angle = 30°

(6a) Incident bar recordings

(6b) transmitter bar recordings

(6) 3mm width and 5% water content with swing-angle = 40°

(7a) Incident bar recordings

(7b) transmitter bar recordings

(7) 3mm width and 10% water content with swing-angle = 20°

(8a) Incident bar recordings

(8b) transmitter bar recordings

(8) 3mm width and 10% water content with swing-angle = 30°

(9a) Incident bar recordings

(9b) transmitter bar recordings

(9) 3mm width and 10% water content with swing-angle = 40°

(10a) Incident bar recordings

(10b) transmitter bar recordings

(10) 4mm width and dry sand layer with swing-angle = 20°

(11a) Incident bar recordings

(11b) transmitter bar recordings

(11) 4mm width and dry sand layer with swing-angle = 30°

(12a) Incident bar recordings

(12b) transmitter bar recordings

(12) 4mm width and dry sand layer with swing-angle = 40°

(13a) Incident bar recordings

(13b) transmitter bar recordings

(13) 4mm width and 5% water content with swing-angle = 20°

(14a) Incident bar recordings

(14b) transmitter bar recordings

(14) 4mm width and 5% water content with swing-angle = 30°

(15a) Incident bar recordings

(15b) transmitter bar recordings

(15) 4mm width and 5% water content with swing-angle = 40°

(16a) Incident bar recordings

(16b) transmitter bar recordings

(16) 4mm width and 10% water content with swing-angle = 20°

(17a) Incident bar recordings

(17b) transmitter bar recordings

(17) 4mm width and 10% water content with swing-angle = 30°

(18a) Incident bar recordings

(18b) transmitter bar recordings

(18) 4mm width and 10% water content with swing-angle = 40°

(19a) Incident bar recordings

(19b) transmitter bar recordings

(19) 5mm width and dry sand layer with swing-angle = 30°

(20a) Incident bar recordings

(20b) transmitter bar recordings

(20) 5mm width and dry sand layer with swing-angle = 35°

(21a) Incident bar recordings

(21b) transmitter bar recordings

(21) 5mm width and dry sand layer with swing-angle = 40°

(22a) Incident bar recordings

(22b) transmitter bar recordings

(22) 5mm width and 5% water content with swing-angle = 30°

(23a) Incident bar recordings

(23b) transmitter bar recordings

(23) 5mm width and 5% water content with swing-angle = 35°

(24a) Incident bar recordings

(24b) transmitter bar recordings

(24) 5mm width and 5% water content with swing-angle = 40°

Fig. 3.3 Four strain wave recordings on the incident and transmitter bars

Fig. 3.4 shows the strain waves at the incident-bar/sand-layer interface and the sand-layer/transmitter-bar interface based on the strain wave measurements given in Fig. 3.3. These waves were calculated by using the wave separation method. From Figs. 3.4.1(a) ~ 3.4.27(a), it can be seen that the strain wave at the sand-layer/transmitter-bar interface consists of two pulses, the first one was transmitted from the sand layer while the second one superposed the reflected wave from the free end. The strain at the sand-layer/transmitter bar was released after the two pulse waves since the positive wave and the negative wave canceled each other and it behaved as a free end afterwards.

(1a) incident-bar/sand-layer interface

(1b) sand-layer/transmitter-bar interface

(1) 3mm width and dry sand layer with swing-angle = 20°

(2a) incident-bar/sand-layer interface

(2b) sand-layer/transmitter-bar interface

(2) 3mm width and dry sand layer with swing-angle = 30°

(3a) incident-bar/sand-layer interface

(3b) sand-layer/transmitter-bar interface

(3) 3mm width and dry sand layer with swing-angle = 40°

(4a) incident-bar/sand-layer interface

(4b) sand-layer/transmitter-bar interface

(4) 3mm width and 5% water content with swing-angle = 20°

(5a) incident-bar/sand-layer interface

(5b) sand-layer/transmitter-bar interface

 (5) 3mm width and 5% water content with swing-angle = 30°

(6a) incident-bar/sand-layer interface

(6b) sand-layer/transmitter-bar interface

 (6) 3mm width and 5% water content with swing-angle = 40°

(7a) incident-bar/sand-layer interface

(7b) sand-layer/transmitter-bar interface

 (7) 3mm width and 10% water content with swing-angle = 20°

(8a) incident-bar/sand-layer interface

(8b) sand-layer/transmitter-bar interface

 (8) 3mm width and 10% water content with swing-angle = 30°

(9a) incident-bar/sand-layer interface

(9b) sand-layer/transmitter-bar interface

 (9) 3mm width and 10% water content with swing-angle = 40°

(10a) incident-bar/sand-layer interface

(10b) sand-layer/transmitter-bar interface

 (10) 4mm width and dry sand layer with swing-angle = 20°

(11a) incident-bar/sand-layer interface

(11b) sand-layer/transmitter-bar interface

 (11) 4mm width and dry sand layer with swing-angle = 30°

(12a) incident-bar/sand-layer interface

(12b) sand-layer/transmitter-bar interface

 (12) 4mm width and dry sand layer with swing-angle = 40°

(13a) incident-bar/sand-layer interface

(13b) sand-layer/transmitter-bar interface

(13) 4mm width and 5% water content with swing-angle = 20°

(14a) incident-bar/sand-layer interface

(14b) sand-layer/transmitter-bar interface

(14) 4mm width and 5% water content with swing-angle = 30°

(15a) incident-bar/sand-layer interface

(15b) sand-layer/transmitter-bar interface

(15) 4mm width and 5% water content with swing-angle = 40°

(16a) incident-bar/sand-layer interface

(16b) sand-layer/transmitter-bar interface

(16) 4mm width and 10% water content with swing-angle = 20°

(17a) incident-bar/sand-layer interface (17b) sand-layer/transmitter-bar interface
(17) 4mm width and 10% water content with swing-angle = 30°

(18a) incident-bar/sand-layer interface (18b) sand-layer/transmitter-bar interface
(18) 4mm width and 10% water content with swing-angle = 40°

(19a) incident-bar/sand-layer interface (19b) sand-layer/transmitter-bar interface
(19) 5mm width and dry sand layer with swing-angle = 30°

(20a) incident-bar/sand-layer interface (20b) sand-layer/transmitter-bar interface
(20) 5mm width and dry sand layer with swing-angle = 35°

(21a) incident-bar/sand-layer interface

(21b) sand-layer/transmitter-bar interface

(21) 5mm width and dry sand layer with swing-angle = 40°

(22a) incident-bar/sand-layer interface

(22b) sand-layer/transmitter-bar interface

(22) 5mm width and 5% water content with swing-angle = 30°

(23a) incident-bar/sand-layer interface

(23b) sand-layer/transmitter-bar interface

(23) 5mm width and 5% water content with swing-angle = 35°

(24a) incident-bar/sand-layer interface

(24b) sand-layer/transmitter-bar interface

(24) 5mm width and 5% water content with swing-angle = 40°

Fig. 3.4 Strains after wave separation

When the strain waves for two interfaces between the pressure bars and the sand-layer were obtained, the stress acted on the sand-layer was obtained from Eq. (3.6). And the strain of the sand-layer was calculated by integrating Eq. (3.8). Subsequently, the stress-strain relations with respect to different joint conditions were obtained and plotted, as shown in the following section.

3.4 Discussions

3.4.1 Effect of filled joint width

The effect of the width of the sand layers on the dynamic stress-strain relation can be seen from Fig. 3.5 in which the water content and swing-angle are given. Fig. 3.5 shows that the dynamic stress-strain relations of the filled joints are nonlinear. For a given strain, the stress of the 3 mm width joint is the largest and the stress of the 5 mm width joint is the lowest when the swing-angle and the water content of the sand layer were maintained the same. In other words, the width of the filled joint affects significantly its dynamic behavior. When the width of the filled joint is larger, its corresponding stress becomes smaller. This phenomenon is accordant to the results of Sinha and Singh (2000) whose test was carried out under quasi-static loading and the strength of joint infilling degraded quickly when the width of the infilling became larger, such as 20 mm and 30 mm.

It is also found from Fig. 3.5 that there were two or three unloading returns on the stress-strain curves. For most cases, two pulse waves were applied to the sand layer which was generated by the pendulum and superposed with the free end reflections. For the sand layer with 5 mm width and when the water content was 5% and 10% in the filling joint, three unloading returns were observed as seen in Fig. 3.5(b) and (c). This is because the 5 mm width sand layer experienced larger compaction which took longer response time and three pulse waves were exerted to the sand layer. The compaction of the sand was from 8% to 14% for different cases. Higher swing-angle of the pendulum resulted in higher maximum deformation of the sand layer. The 30° swing-angle resulted in 8% to 9% maximum strain, however, the 40° swing-angle and 10% water content gave 12% to 14% maximum strain. The stress level was about 1 MPa to 5 MPa for different cases. It should be mentioned the existing joint models developed for unfilled joint cannot reflect the joint width's effect.

(c) 10% water content

Fig. 3.5 Width effect on the stress-strain relation of the sand layers

(swing-angle = 40 °)

3.4.2 Effect of water content

Fig. 3.6 reveals the influence of water content of the sand layers on the dynamic stress-strain relation. The water saturation of the filling material made significant difference of the dynamic stress-strain curve. This is another special feature of the filled joint and the existing joint model developed for unfilled joint cannot reflect the water content effect as well. The highest water content, i.e. 10% water in the sand-layer, gave the lowest stress-strain curve. It is seen again that the pulse loading included two or three waves such that two or three unloading returns were resulted.

(a) 3 mm, swing-angle= 30 °

(b) 3 mm, swing-angle= 40 °

(c) 4 mm, swing-angle= 40 °

(d) 5 mm, swing-angle= 30 °

Fig. 3.6 Water content effect on the stress-strain relation of the sand layers

From Fig. 3.6, it is clear that a mechanical model for filled rock joint should consider the water content effect. Corresponding to a strain level of 8% in Fig. 3.6(a)

to (d), the sand layer with 10% water content resulted in a compressive stress less than half of that for the dry sand layer (Li and Ma, 2009; Ma et al., 2011).

3.4.3 Effect of input energy

During the tests, the input energy produced by the hammer was controlled by the swing-angle. The stress-strain relations with different swing-angles but the same water content and width of the sand layers were shown in Fig. 3.7. The stress increased with the increase of the strain with the same tendency. The larger the swing-angle, the larger the maximum stress and strain. Since they had the same tendency, it can be observed that the dynamic stress-strain relation is less sensitive to the input energy even the sand layer was under repeated load. However, the stress-strain relation of the filled joint was highly nonlinear and it highly depended on the joint width and the water content. The fitted curves for different joint conditions will be discussed below.

(a) 3 mm, dry

(b) 3 mm, 5% water content

(c) 4 mm, dry

(d) 5 mm, dry

Fig. 3.7 Input energy effect on the dynamic stress-strain relation of the sand layers

3.5 Conclusions

This chapter presents an experimental technique to characterize the dynamic stress-strain relation of filled rock joints. A wave separation method was used to obtain the strain waves at the interfaces between the granitic pressure bar and the artificial sand filled joint. From the test results, it can be concluded

The complete dynamic stress-strain relation of the filled rock joints is highly nonlinear. However, if the magnitude of stress wave is insufficient to mobilize the nonlinear deformation of the joint, the linear model can be used to describe the normal deformational behavior of the joint

Both the width and the water content of the sand layer have significant effect on its dynamic stress-strain relation. The higher water content and the larger joint width result in lower stress-strain curve of the filled joint. The results can be used for the natural rock joint filled with water and sand.

However, the pulse loading intensity is not sensitive to the tendency of the stress-strain curve. Quantitative dynamic stress-strain relations have been derived with respect to different joint conditions.

CHAPTER 4 LINEAR VISCOELASTIC EQUIVALENCE FOR MACRO-JOINTED ROCK MASS

4.1 Introduction

This chapter further develops the equivalent viscoelastic medium method. The advantages and limitations of different equivalences to the discontinuous rock mass are discussed. Theoretical derivation of stress wave propagation through the equivalent viscoelastic medium is carried out by adopting the Fourier transformation method and the parameters of the equivalent viscoelastic medium method are determined analytically. The frequency dependence and the wave attenuation phenomenon can be properly described when the imaginary terms of the complex moduli of the rock mass are included. The results show that the equivalent viscoelastic medium method is able to predict the effective velocity and the stress wave transmission coefficient in a rock mass more accurately than the conventional effective elastic moduli methods. An example of the stress wave propagation through rock mass with parallel joints shows that the equivalent viscoelastic medium method is promising and worthy to be further explored for application in practical rock engineering problems.

4.1.1 Background

Macro-joints commonly exist in natural rock mass, which occur as nearly parallel groups or sets. Such set (or sets) of joints dominate the mechanical behavior of the rock mass (Jaeger et al., 2007; Li and Ma, 2009). It is of significance to investigate dynamic behavior and wave propagation across jointed rock mass to assess the stability and damage of rock structures. However, because of the blockiness of rock mass induced by the presence of joint, analysis of wave propagation across jointed rock mass is a complicated process. A few different methods have been used to analyze the wave propagation in jointed rock mass. However, most of these are limited by their various fundamental assumptions.

Therefore, a detailed review of the joint effects on the stress wave propagation in rock mass and a comparison of the available methods are necessary. Equation Chapter (Next) Section 1

Wave propagation analysis in jointed rock mass can be classified into two categories. One is the “displacement discontinuity method (DDM)”, in which the joint is treated as non-welded interfaces (also as incompletely welded interfaces or slip interfaces) and the stresses across the joint are assumed continuous, whereas the displacement maybe discontinuous (Miller, 1977; Schoenberg, 1980; Pyrak-Nolte, 1988).

The other category is the “effective moduli method (EMM)”, in which the jointed rock mass is treated as a continuous medium and the effects of joints on the rock mass properties are reflected by the effective moduli of the equivalent medium (Amadei and Goodman, 1981; Li et al., 2010b; Schoenberg and Muir, 1989; Sheshemin et al., 1997). The effective elastic moduli method can be used to derive an explicit expression of the transmitted wave. However, the conventional methods simplify the discontinuous rock mass to an elastic continuous medium, which inherently results in two limitations. One is that the wave attenuation caused by imperfect elastic properties of rock mass cannot be described in the individual representative element since the models are purely elastic. The other is that the intrinsic frequency dependency of a rock joint model is ignored (Pyrak-Nolte et al., 1990; Zhao et al., 2006).

4.1.2 Conventional effective elastic moduli method

Effective elastic moduli methods are commonly used to determine the overall properties of the jointed rock mass. Fig. 4.1 illustrates the definition of the equivalent medium.

(a) Jointed rock mass

(b) Representative elementary volume

(c) Equivalent medium

Fig. 4.1 Illustration of equivalent medium method

The scheme of the rock mass with a set of equally-spaced parallel joints is shown in Fig. 4.1(a). And the dash frame in Fig. 4.1(a) is selected to be a representative elementary volume (REV) for the jointed rock mass, as shown in Fig. 4.1(b). The effective length of rock mass REV S is defined as the sum of the joint thickness and the joint spacing. Hence, the associated equivalent medium is defined as the continuum medium with the same length of the representative elementary volume as plotted in Fig. 4.1(c).

There are two main conventional effective elastic moduli methods used to derive the elastic constants of the rock mass.

In the static effective elastic moduli methods, the rock mass with a set of parallel joints is replaced by a transversely isotropic medium model, which can be expressed by five elastic constants. And these five elastic constants are described in terms of the properties of joints and rock (Pyrak-Nolte et al., 1990).

Therefore, if an incident wave normally propagates across parallel and linear deformational rock joints, the propagating velocity of the wave is expressed as (Pyrak-Nolte et al., 1990; Zhao et al., 2006).

$$\rho V^2 = C_{3333} \quad (4.1)$$

where V is the magnitude of the wave velocity, ρ is the density of the rock and C_{3333} is the elastic constant of the transversely isotropic medium model, which can be obtained from

$$\frac{1}{C_{3333}} = \frac{1}{E_r} + \frac{1}{S_0 k} \quad (4.2)$$

where E_r is the Young's modulus of the intact rock, S_0 is the joint spacing, and k is the normal specific stiffness of the joints.

In the dynamic effective elastic moduli method (Crampin et al., 1980; Ciccotti and Mulargia, 2004) the effective modulus of the rock mass is calculated by

$$E_{eff} = \rho V_e^2 \quad (4.3)$$

where the subscript '*eff*' denotes the effective property and '*e*' denotes the value obtained by experimental or site measurement.

The effective modulus E_{eff} of the representative element is then extended to the entire rock mass. Therefore, the effective velocity of the stress wave propagating in the jointed rock mass is the same as given in Eq. (4.3).

As already mentioned, the material constants can be obtained by using the static and dynamic effective elastic moduli method in Eqs. (4.2) and (4.3), respectively. However, as a result of the presence of the joint, the jointed rock mass will no longer be purely elastic, accordingly, their dynamic effective moduli will not be the same as their static effective moduli. Therefore, it is not possible that the effective velocities obtained by these two methods coincide with each other. Several laboratory measurements showed that the static effective elastic modulus appeared

to be lower than the dynamic effective modulus (Ciccotti and Mulargia, 2004; Eissa and Kazi, 1988).

4.2 Theoretical Formulation

In order to avoid the essential limitations of the conventional effective elastic moduli methods, the equivalent viscoelastic medium method has been put forward in describing the stress wave propagation in a jointed rock mass (Li et al., 2010b).

4.2.1 Equivalent viscoelastic medium model

The equivalent viscoelastic medium model is shown in Fig. 4.2, which is known as the model for the standard linear solid. The model can be regarded as an auxiliary spring placed in series with the Voigt model, which will be used to represent both the attenuation of the propagating stress wave and the frequency dependence of the jointed rock mass.

Fig. 4.2 Equivalent viscoelastic medium model

The governing equation for one-dimensional longitudinal wave propagation in the equivalent viscoelastic medium can be obtained as follows (Li et al., 2010b; Kolsky, 1964)

$$\rho\eta_v \frac{\partial^3 v}{\partial t^3} + \rho(E_a + E_v) \frac{\partial^2 v}{\partial t^2} - \eta_v E_a \frac{\partial^3 v}{\partial x^2 \partial t} - E_v E_a \frac{\partial^2 v}{\partial x^2} = 0 \quad (4.4)$$

where σ is stress, ε is strain, v denotes the particle velocity. E_a and E_v are the elastic modulus of the springs, respectively, η_v is the viscosity ratio, t denotes the time and x denotes the length of the rock mass along the wave propagation path.

For an arbitrary incident wave, the boundary condition is given as follows,

$$v(x, t)|_{x=0} = v_0(t) \quad (4.5)$$

where $v_0(t)$ is the incident wave at the boundary where $x = 0$.

In order to obtain the transmitted wave, the application of the Fourier transformation to the motion Eq. (4.4) and the boundary Eq. (4.5) yields,

$$\left[-\omega^2 \rho \eta_v + i \omega \rho (E_a + E_v) \right] \tilde{v}(\omega) - \left[\eta_v E_a - i E_v E_a / \omega \right] \tilde{v}_{,xx}(\omega) = 0 \quad (4.6)$$

and

$$\tilde{v}(x, \omega)|_{x=0} = \tilde{v}_0(\omega) \quad (4.7)$$

respectively, where we use the velocity $\tilde{v}(x, \omega)$ in the frequency domain to describe Fourier transformed time domain velocity, ω is the angle frequency in the transform domain, i denotes the imaginary sign, the comma subscript in the Eq. (4.6) denotes the differential to x and $\tilde{v}_0(\omega)$ is the boundary condition of $v_0(t)$ after Fourier transformation.

Note that Eq. (4.6) is an ordinary differential equation, which has the solution of

$$\tilde{v}(x, \omega) = C_1 \exp(-\sqrt{B/A}x) + C_2 \exp(+\sqrt{B/A}x) \quad (4.8)$$

where $B = -\omega^2 \rho \eta_v + i \omega \rho (E_a + E_v)$ and $A = \eta_v E_a - i E_v E_a / \omega$, C_1 and C_2 are integral constants.

Eq. (4.8) can be used to describe the right-running and left-running waves. For the case that an incident wave applied on the left end of the equivalent viscoelastic medium (Fig. 4.1) and consider the boundary condition, it can be obtained that

$$\tilde{v}(x, \omega) = C_1 \exp(-\sqrt{B/A}x) \quad (4.9)$$

Applying the boundary condition Eq. (4.7) to Eq. (4.9) gives the velocity expression in the frequency domain as,

$$\tilde{v}(x, \omega) = \tilde{v}_0(\omega) \exp(-\sqrt{B/A}x) \quad (4.10)$$

The inverse Fourier transformation of Eq. (4.10) for the velocity in the time domain is,

$$v(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{v}_0(\omega) \exp(-\sqrt{B/A}x) \exp(i\omega t) d\omega \quad (4.11)$$

where $\tilde{v}_0(\omega)$ is obtained from the transformation of the arbitrary function $v_0(t)$, so the method can be applied to solve the motion problems with any arbitrary incident wave. Furthermore, no additional assumption is introduced during the derivation, so no additional numerical errors will be encountered. It can also be derived from Eq. (4.11) that the transmitted wave $v(x, t)$ can be regarded as the sum of the harmonic waves with frequencies from negative infinity to positive infinity.

4.2.2 Parameter determination

In the following analysis, the joint closure (opening) and compression (tension) are denoted to be positive (negative). If there is a harmonic incident wave propagating through a rock mass with a single joint, the transmitted wave was given by (Pyrak-Nolte et al., 1990; Cook, 1992)

$$v_{T1} = \frac{2k/z}{i\omega + 2k/z} V_0 \exp[i(\omega t - x\omega/C)] \quad (4.12)$$

where k is the normal joint stiffness as defined in Eq. (4.2), z is the wave impedance and $z = \rho C$, C is the wave velocity in the intact rock and V_0 is the amplitude of the incident wave. When the representative elementary element in Fig. 4.1(b) is considered and $x = S$, the phase shift factor and the attenuation factor of the amplitude given by Eq. (4.12) are respectively

$$\begin{cases} \alpha_r = \frac{\omega}{C} - \frac{1}{S} \arctan\left(\frac{\omega}{2k/z}\right) \\ \beta_r = \frac{1}{S} \ln \left[\frac{2k/z}{\sqrt{\omega^2 + (2k/z)^2}} \right] \end{cases} \quad (4.13)$$

When the same harmonic incident wave is applied to the representative element of the equivalent viscoelastic medium as shown in Fig 4.1(c), the phase shift and the attenuation factor are determined from Eq. (4.11),

$$\begin{cases} \alpha_e = -\text{Im}\left(-\sqrt{B(E_a, E_v, \eta, \omega)/A(E_a, E_v, \eta, \omega)}\right) \\ \beta_e = \text{Re}\left(-\sqrt{B(E_a, E_v, \eta, \omega)/A(E_a, E_v, \eta, \omega)}\right) \end{cases} \quad (4.14)$$

where α_e and β_e are the phase shift factor and the attenuation factor for the equivalent viscoelastic medium, respectively. *Re* and *Im* denotes real and imaginary parts of the complex expression, respectively.

When $x = S$, the transmitted wave of the harmonic incident wave is obtained from Eq. (4.12) for the representative element and from Eq. (4.11) for the corresponding equivalent medium. The parameters E_a , E_v , and η in the equivalent viscoelastic model can be obtained by equating Eq. (4.13) and Eq. (4.14),

$$\begin{aligned} \text{Im}\left(\sqrt{B(E_a, E_v, \eta, \omega)/A(E_a, E_v, \eta, \omega)}\right) &= \frac{\omega}{C} - \frac{1}{S} \arctan\left(\frac{\omega}{2k/z}\right) \\ \text{Re}\left(\sqrt{B(E_a, E_v, \eta, \omega)/A(E_a, E_v, \eta, \omega)}\right) &= -\frac{1}{S} \ln \left[\frac{2k/z}{\sqrt{\omega^2 + (2k/z)^2}} \right] \end{aligned} \quad (4.15)$$

It has been recommended by Li et al. (2010b) that the Young's modulus of the intact rock can be used to replace E_a in the viscoelastic model. Therefore, the relations between E_v , η and ω are obtained by the implicit expressions of Eq. (4.15). A Newton-like method is recommended to obtain the numerical results of Eq. (4.15) (Li et al., 2010b; Kelley, 2003).

4.3 Results and Discussions

A displacement discontinuity method is adopted here as a validation to the equivalent medium methods. If a harmonic incident P-wave with the form of $v = V_0 \exp(-i\omega t)$ is applied at the left boundary ($x=0$) of rock mass, the transmitted wave at $x=S$ is derived as written in Eq. (4.12) (Pyrak-Nolte, 1988; Pyrak-Nolte et al., 1990; Cook, 1992).

Treats the incident wave as the sum of a series of harmonic waves in the frequency domain, then the components of the transmitted wave in the frequency domain are obtained by Eq. (4.12). The transmitted wave is thus determined by the sum of all the wave components.

To verify the equivalent viscoelastic medium model, and to describe the frequency dependent property of the equivalent viscoelastic medium, an analysis of the wave propagation through a rock mass with one to four parallel joints using the four methods, i.e., the static effective moduli method, the dynamic effective moduli method, the displacement discontinuity method and the equivalent viscoelastic medium method, is carried out.

It is assumed that the density of the intact rock is 2650 kg/m^3 , the stress wave velocity in the intact rock is 5830 m/s , and the stiffness of the joint 3.5 GPa/m , as typical properties of granite and the joint. The incident P-wave has the form of a half sinusoidal wave which is described as

$$v_I(t,0) = \begin{cases} 0.02 \sin(2\pi \times f_0 \times t), & \text{when } 0 \leq t \leq \frac{1}{2f_0} \\ 0 & \text{others} \end{cases} \quad (4.16)$$

where the amplitude of the incident wave is 0.02, which is not large enough to mobilize nonlinear deformation of the joints. And f_0 is the fundamental frequency of the incident wave.

It can be obtained from the Fourier transformation that the wave in the time domain is treated as the sum of a series of harmonic waves. The parametric determination for all these harmonic component waves is carried out.

 (a) $E_v \sim \omega$

 (b) $\eta_v \sim \omega$

 Fig. 4.3 Relation among (a) elastic modulus E_v and (b) damping ratio η_v , with frequency ω for different joint spacing S

Fig. 4.3(a) and Fig. 4.3(b) respectively show the relationships between E_v and ω , η_v and ω , when S is λ , 5λ , and 8λ , respectively, ω is the frequency of the harmonic component wave and λ is the wavelength of the harmonic component wave.

It is observed from the two figures that both E_v and η_v depend on the incident wave frequency and the joint spacing S . For a given S , E_v and η_v decrease with increase of the ω . And for a given ω , E_v and η_v increase with the increasing S .

It is also shown from Fig. 4.3 that the equivalent viscoelastic medium model is frequency dependent and the transmitted wave is derived as the sum of each transmitted harmonic component waves from Eq. (4.11).

4.3.1 Comparison of transmitted waveform

In order to focus on the mechanical effect of the joint on the wave propagation rather than the geometrical effect, the multiple reflections induced by the parallel joints which are highly dependent on the geometrical location of the joints are not considered in the present numerical example. Therefore, only the effect of the joint on the directly transmitted wave is investigated. Consideration of the multiple reflections has been discussed by Li et al. (2010b) by using a virtual wave source model which is not the focus of the present analysis although it can also be incorporated.

In this example, the incident wave applied at $x=0$ has the form of Eq. (4.16), $f_0 = 50$ Hz and the joint spacing S is $8\lambda_0$, where λ_0 is the elastic wave length and $\lambda_0=2\pi C/\omega_0$. The transmitted waves through parallel jointed rock mass with a joint spacing of S are compared. Fig. 4.4 shows the results for different joint numbers.

Fig. 4.4 Comparison of transmitted waves

In Fig. 4.4, the incident waves, the transmitted wave through an intact elastic rock block, and the transmitted waves passing through jointed rock masses based on the displacement discontinuity method, the static effective moduli method, the dynamic effective moduli method and the equivalent viscoelastic medium method are denoted as the “Incident”, “Intact rock”, DDM, SEMM, DEMM and EVMM, respectively.

It can be seen from Fig. 4.4 that there is a distinct time delay between the transmitted wave through intact rock and those through jointed rock mass. The time delay is caused by the presences of the joints, and it increases with the increase of the joint number. The existence of the time delay results the effective velocity in the rock mass smaller than the effective velocity in the intact rock. It is also observed that the dynamic effective moduli method and the equivalent viscoelastic medium method give more accurate estimation of the time delay (effective velocity) when

comparing to the results from the displacement discontinuity method. The static effective moduli method can only be used to obtain an approximated time delay (effective velocity) while the error increases with increasing joint numbers as shown in Fig. 4.4(a-d).

It is commonly known that the stress wave amplitude attenuates when the wave propagates across a jointed rock mass. It is observed from Fig. 4.4 that the static effective moduli method and the dynamic effective moduli method cannot be used to describe the attenuation phenomenon of the transmitted waves. The equivalent viscoelastic medium method however is able to express the wave attenuation and dispersion phenomenon as the displacement discontinuity method does. The comparison also shows that the equivalent viscoelastic medium method is an effective method with a high accuracy. It has advantages comparing to the displacement discontinuity method and the traditional effective elastic moduli method in analyzing the stress wave propagation in jointed rock mass.

4.3.2 Transmission coefficients

In rock engineering, the damage criteria of rock structures are generally regulated according to the threshold values of wave amplitudes, such as the peak particle velocity. Therefore, the transmission coefficients for the waves propagating through the rock mass with multiple joints are important.

It has been concluded from the previous sections that in the frequency domain, the incident wave is treated as the sum of a series of harmonic waves with different frequencies. Therefore, the transmitted wave across a rock mass with parallel joints can be treated as a wave superposition of transmitted component waves arriving at different times. To explicitly determine the wave superposition, a simplified method used in practice is referred as T^N -method. The transmission coefficient across parallel joints is calculated as the product of transmission coefficients of individual joints, if these joints have the identical property. Therefore,

$$|T_N| = |T_1|^N \quad (4.17)$$

where T_N denotes the transmission coefficient after N joints and T_1 denotes the transmission coefficient after a single joint. Laboratory experiments preformed verified that the T^N -method is valid when the first arriving wave is not contaminated by the multiple reflections, which will be satisfied only when the joint spacing is relatively large than the incident wavelength. However, the laboratory researches by Hopkins et al. (1988) and Myer et al. (1995) also showed that $|T^N|$ was larger than $|T_1|^N$.

To study the relationship between the transmission coefficient and the frequency of the incident wave, the incident wave in the form of Eq. (4.16) with $f_0 = 10.0$ Hz, 25.0 Hz, 50.0 Hz, 100.0 Hz and 150.0 Hz is adopted. The joint stiffness is $k_n = 3.5$ GPa, and the joint space is $8\lambda_0$. Similarly, to investigate the relationship between the transmission coefficient and the joint stiffness, it is assumed that f_0 is 50 Hz for the incident wave in Eq. (4.16), and the joint stiffness k_n varies from 1.0 GPa, to 3.5 GPa, 6.0 GPa, 8.5 GPa and 11.0 GPa, respectively.

The effects of the incident wave frequency and the joint stiffness on the transmission coefficients are shown in Fig. 4.5 and 4.6, respectively, where the solid marks represent the transmission coefficients obtained from the equivalent viscoelastic medium method and the hollow marks represent the results from the T^N -method.

Fig. 4.5 Effect of incident wave frequency on transmission coefficient
($k_n = 3.5$ GPa/m and $S = 8\lambda_0$)

Fig. 4.6 Effect of joint normal stiffness on transmission coefficient
($f_0 = 50$ Hz and $S = 8\lambda_0$)

It can be seen from Fig. 4.5 that the transmission coefficients decrease when the joint number increases, while for the wave propagation through the same jointed rock mass, the transmission coefficient decreases as the frequency of the incident wave increases. The similar tendency is observed from Fig. 4.6 that the transmission coefficient decreases as the joint number increases, and for the same incident wave, the transmission coefficient increases as the stiffness of the joint increases.

It is also seen from Figs. 4.5 and 4.6 that the transmission coefficients obtained from T^N -method are close to the transmission coefficient from the equivalent medium method. However, the difference increases as the frequency of the incident wave increases or the stiffness of the joint decreases. And the value T_N obtained from the equivalent viscoelastic medium method is larger than that from the T^N -method.

In order to explain the above phenomenon, the frequency spectrum of the incident waves and transmitted waves are plotted in Fig. 4.7, 4.8 and 4.9.

Fig. 4.7 Frequency spectrum of incident and transmitted wave

$$(f_0 = 50 \text{ Hz}, S = 8\lambda_0 \text{ and } k_n = 3.5 \text{ GPa/m})$$

, where $N = 0$ denotes the frequency spectrum of the incident wave, and $N = 1, 2, 3$ and 4 denotes the frequency spectra of the transmitted waves passing through 1, 2, 3 and 4 joints, respectively.

Fig. 4.8 Effect of incident wave frequency on frequency spectrum of transmitted wave

($k_n = 3.5$ GPa/m and $S = 8\lambda_0$)

Fig. 4.9 Effect of joint stiffness on frequency spectrum of transmitted wave

$$(f_0 = 50 \text{ Hz and } S = 8\lambda_0)$$

It is seen from Figs. 4.7, 4.8 and 4.9 that the equivalent viscoelastic medium are frequency dependent, and as the wave propagates through the rock mass, the percentage of its component waves changes.

Figs. 4.7, 4.8 and 4.9 also show that the frequency spectrum decreases faster for the incident wave with a higher frequency or the wave across a joint with a smaller stiffness. Furthermore, from Figs. 4.7, 4.8 and 4.9, the frequency spectrum becomes narrower when the joint number increases and the stress wave attenuate more significantly when it passes the first joint. However, the T^N -method assumed that the stress wave transmission through any individual joint is T_1 , therefore, the final transmission coefficient obtained by the T^N -method will be less than the real results.

4.3.3 Effective velocity

The effective velocity for the stress wave propagation in an equivalent medium is defined as a function of the length to the time difference between the peak velocity of the incident and transmitted waves. Assume the incident wave is in the form of Eq. (4.16) with $f_0 = 50$ Hz, the joints spacing is $8\lambda_0$ and the joint stiffness is 3.5 GPa/m. From the waveforms in Fig. 4.4, the effective velocities of wave propagation through the rock mass with parallel linear deformational joint can be obtained. The effective velocity of the rock mass is defined as

$$V_{eff} = \frac{1}{N} \sum_j^N V_j \quad (4.18)$$

where the subscript *eff* denotes the average effective value, and subscript number *j* denotes the effective velocity of the wave propagating through *j*th joint.

V_j based on the DDM is given by (Pyrak-Nolte et al., 1990; Cook, 1992)

$$V_j = C \left[\frac{1 + \left(\frac{z\omega}{2k} \right)^2}{1 + \left(\frac{z\omega}{2k} \right)^2 + \frac{jC}{L} \frac{z}{2k}} \right] \quad (4.19)$$

where L is the total path length and j is the number of joint, other symbols are defined the same as in Eq. (4.12).

According to Eqs. (4.18) and (4.19), it can be calculated that the average effective velocity for the representative elementary element in Fig. 4.1(b) based on the DDM is $V_{eff (DDM)} = 5765.25$ m/s, it is $V_{eff (EVMM)} = 5765.45$ m/s based on the equivalent viscoelastic medium method. By using the static effective moduli method, the effective velocity is $V_{eff (SEMM)} = 5751.21$ m/s, however, the effective velocity based on the dynamic effective elastic moduli method highly depends on the representative element selection. Assume that the rock mass with single joint is selected as the representative element, from Fig. 4.4(a), the dynamic effective elastic modulus is obtained as $E_{eff (DEMM)} = 88.10$ GPa. Therefore, the effective

velocity from the dynamic effective moduli method is determined as $V_{eff (DEMM)} = 5765.85$ m/s. When the rock mass with two to four joints are selected as the representative elements, respectively, the associated effective velocities become 5765.49 m/s, 5765.14 m/s and 5764.52 m/s, respectively.

The effective velocity obtained by the equivalent viscoelastic medium method has very high accuracy when comparing with the result from the DDM method. While the effective velocity obtained by the static effective moduli method has the poorest accuracy, which is caused by the essential limitation of the static effective moduli method, its application to wave propagation analysis is limited. The results also show that the accuracy of the effective velocity obtained by the dynamic effective moduli method is affected by the representative element selection. Although the effective velocities from different methods are very much close to each other, it has been misleading to simplify the jointed rock mass to an equivalent elastic medium as the conventional methods did since they cannot consider the frequency dependence and stress wave attenuation in rock mass. It is the advantage of the equivalent viscoelastic medium method which is able to describe the change of both the amplitude and the waveform for different frequency waves propagating in a jointed rock mass.

4.3.4 Frequency dependence of equivalent viscoelastic medium method

To investigate the frequency dependence of the equivalent medium methods, the frequency spectra of the incident wave and the transmitted waves passing through a rock mass with 1 to 4 joints in an equivalent viscoelastic medium respectively (Fig. 4.4) are calculated by using FFT, as shown in Fig. 4.7.

It is shown in Fig. 4.7 that the frequency spectra of waves (incident wave and transmitted waves passing through 1 to 4 joints) are different. For a component wave with a fixed frequency (e.g. $f = 100$ Hz), its amplitude decreases rapidly with the increasing joint number, however, the decreasing rate slows down as the number of the joints increases. For the component waves with different frequencies, such as $f_1 = 50$ Hz and $f_2 = 100$ Hz, Fig. 4.7 and Fig. 4.8 showed that the amplitude of the component wave with a larger frequency attenuate more significantly than that with

a smaller frequency. This observation can be used to explain that the joints filter out the waves with higher frequencies, which agrees with the common understanding of wave propagation in discontinuous media.

4.4 Numerical Examples

An application of the equivalent viscoelastic medium method is given to study the stress wave propagation through rock mass with parallel joints. The incident waves are in the form of Eq. (4.16), where the amplitude of the incident waves is assumed to be 0.02 m/s. A series of frequencies of the incident wave, i.e., $f_0 = 100$ Hz, 200 Hz and 500 Hz are adopted respectively in the analysis. The joints have the stiffness of 12.0 GPa/m and the spacing of 2.9 m. The Young's modulus of the intact rock is 90.0 GPa.

Fig. 4.10(a-c) shows the incident wave and transmitted waves when the incident wave has frequencies of 100 Hz, 200 Hz and 500 Hz, respectively, where the dashed curve denotes the incident wave, and the curve 2,3,4,5 denote the transmitted wave passing through 5, 10, 15, 20 representative elements (joints), respectively.

(a) $f_0 = 100$ Hz

(b) $f_0 = 200$ Hz

(c) $f_0 = 500$ Hz

Fig. 4.10 Transmitted wave for different incident frequency

The wave attenuation along the propagating distance is shown in Fig. 4.11, where the normalized amplitude of the stress wave is defined as the ratio of the amplitudes of the transmitted wave to that of the incident wave.

Fig. 4.11 Wave attenuation along the distance

By using the equivalent viscoelastic medium method, analysis of wave transmission through jointed rock mass is much more efficient with satisfactory accuracy in terms of the transmitted wave form, transmission coefficient, effective velocity, and frequency dependence of the transmitted wave. When the displacement discontinuity method is adopted, the complexity of the calculation highly depends on the joint numbers included in the rock mass. The equivalent medium method is more effective for the analysis of wave propagation through rock mass with a large numbers of joints, which is common in practice. On the other hand, as can be seen from Fig. 4.11 that, the static and dynamic effective moduli methods are not applicable in describing the attenuation of the stress wave. The equivalent viscoelastic medium method has obvious advantage in considering the stress wave attenuation and the viscous loss (complex moduli) in jointed rock masses.

4.5 Conclusions

In this chapter, comparative studies between the equivalent viscoelastic medium method, the static effective elastic moduli method, the dynamic effective elastic moduli method and the displacement discontinuity method for jointed rock mass are carried out. The frequency dependent properties of a stress wave propagating in jointed rock mass which are not reflected by the conventional effective elastic moduli method are discussed. An application is conducted to show the advantage of using the equivalent viscoelastic medium method.

It is shown from the comparison that the static effective moduli method can only be used to obtain an approximate effective velocity, and the accuracy is acceptable only for a rock mass with few joints. The dynamic effective moduli method can be used to obtain a relative accurate effective velocity, but the accuracy highly depends on the representative element selection. Both of these two methods cannot consider the stress wave attenuation behavior in jointed rock mass due to their elastic deformation assumptions. By comparison with the results from the DDM method, the equivalent viscoelastic medium method is able to give satisfactory results in terms of transmitted wave form, transmission coefficients, effective velocity, and frequency dependence of the transmitted wave.

When the incident wave propagates through an equivalent viscoelastic medium, the wave component with higher frequencies is filtered out, however, the component wave with lower frequencies is less affected which coincides with the phenomenon for wave propagation through a jointed rock mass in geophysical seismic exploration.

The equivalent viscoelastic medium method is worthy to be further explored in order to be used in practical rock engineering and its advantages over conventional equivalent medium methods have been demonstrated.

CHAPTER 5 NONLINEAR VISCOELASTIC EQUIVALENCE FOR MACRO-JOINTED ROCK MASS

5.1 Introduction

This chapter proposes a nonlinear equivalent viscoelastic model for the study of longitudinal stress wave propagation in a rock mass with equally-spaced parallel rock joints. The deformation of the rock joints satisfies the B-B model. The present equivalent medium model is an extended standard solid model with three piecewise-linear parameter sets. In order to derive the wave propagation equation in the viscoelastic medium efficiently, a method of characteristic lines is used. To theoretically verify the present model, a comparison with the results obtained from the displacement discontinuity method is performed. The amplitude dependence, which only exist in the wave propagation through a nonlinear medium, and the frequency dependence of the present model are discussed.

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The joint deformation can be described by a linear model if the joint is dry and the magnitude of the stress wave is insufficient to mobilize the nonlinear deformation. However, if a joint is filled or the magnitude of the stress wave is sufficient large, it has been found that the complete deformational behavior of a rock joint is generally nonlinear (Bandis et al., 1983; Cook, 1992; Ma et al., 2011a). Li and Ma (2009) performed an experimental study on constitution of filled joints using modified split Hopkinson pressure bar, and a nonlinear model for filled joint was proposed. By using DDM, a series of analysis of stress wave propagation through unfilled/filled nonlinear deformational jointed rock mass were carried out (Zhao and Cai, 2001; Zhao et al., 2006; Li et al., 2011). However, how to use the equivalent medium method to analyze stress wave propagation in rock mass consisting of nonlinear deformational joints has yet to be explored.

5.2 Theoretical Formulation

5.2.1 Joint models

Fig. 5.1 illustrates an REV of a jointed rock mass and the definition of corresponding equivalent medium. In the present study, the joint is assumed to be planar, large in extent and small in thickness, thus the length of the REV of the equivalent medium is defined as the joint spacing S as shown in Fig. 5.1.

Fig. 5.1 Illustration of equivalent medium method

Efforts have been made to study the normal deformational behavior of the joints under quasi-static or cyclic loading conditions (Bandis et al., 1983; Li and Ma, 2009). Fig. 5.2 illustrates a piecewise linear model for the nonlinear deformational behavior of the joint, in comparison with the linear joint model and the B-B Model (Bandis et al., 1983). If the closure (opening) of the joint and the compression (tension) stress are assumed to be positive (negative), the normal constitutive relations between the stress and the closure can be described by the B-B model as,

$$d = \frac{\sigma}{k_n + \frac{\sigma}{d_{\max}}} \quad (5.1)$$

where d is the closure of the joint, σ is the normal stress, $d_{\max}$ is the maximum allowable closure of the joint and k_n is the normal stiffness of the joint at the initial state. The dashed curve in Fig. 5.2 gives a typical hyperbolic fitting of the B-B model.

Fig. 5.2 Piecewise linear model, B-B model and linear model of a rock joint

In the present study, a piecewise linear model is proposed to approach such hyperbolic B-B model. The present piecewise linear model is a piecewise linear function whose pieces are linear. It can be observed from Fig. 5.2 that the graph of the B-B model is a continuous curve, while the present piecewise linear model can be regarded as a polygonal curve consists of a series of line segments by a line fitting with reference to different end points.

The values of closure and stress at the two ends of each segment are assumed located on the hyperbolic B-B model relation yields

$$d_i = \frac{\sigma_i}{k_n + \frac{\sigma_i}{d_{\max}}}, \quad i = 1, 2, 3, \dots \quad (5.2)$$

where i is the segment number, d_i and σ_i denote the discrete threshold closure and stress at the two ends of each segment, respectively.

The difference of stress and closure for every individual segment can be obtained as

$$\Delta\sigma_i = \sigma_i - \sigma_{i-1}, \quad i = 1, 2, 3, \dots \quad (5.3)$$

and

$$\Delta d_i = d_i - d_{i-1}, \quad i = 1, 2, 3, \dots \quad (5.4)$$

respectively.

Therefore, the stiffness of the joint for segment i can be obtained from Eqs. (5.2), (5.3) and (5.4) as

$$k_i = \frac{\Delta\sigma_i}{\Delta d_i} = \frac{k_n}{d_i - d_{i-1}} \left(\frac{d_i}{1 - \frac{d_i}{d_{\max}}} - \frac{d_{i-1}}{1 - \frac{d_{i-1}}{d_{\max}}} \right), \quad i = 1, 2, 3, \dots \quad (5.5)$$

It can be obtained from Fig. 5.2 that the present piecewise linear model and the B-B model involves in joint stiffness that increases with the increasing normal stress, while the stiffness of the joint in the linear model keeps constant. When the difference of the closure Δd_i is small enough, the present piecewise linear model will approximate to the B-B model. The value of $d_{\max}$ and k_n for the piecewise linear model and B-B model can be determined from the laboratory measurements as obtained by Bandis et al. (1983) and Barton et al. (1985).

5.2.2 Nonlinear equivalent viscoelastic medium method

Investigations show that the conventional purely elastic equivalent medium model has no viscous loss (complex moduli are not involved), the resulting stress wave propagation velocities are thus nondispersive and frequency-independent. However, this limitation can be solved by considering the jointed rock mass as a viscoelastic medium (Li et al., 2010b; Pyrak-Nolte et al., 1990).

A nonlinear equivalent viscoelastic model is proposed to describe the overall dynamic constitutive relation of the rock mass with nonlinear deformational joints, which is shown in Fig. 5.3.

Fig. 5.3 Piecewise linear equivalent viscoelastic medium model

A piecewise linear spring with a modulus of E_{vi} is in parallel with a piecewise linear dashpot η_{vi} , then in series with another piecewise linear spring E_{ai} . The piecewise linear quantities of E_{ai} , E_{vi} and η_{vi} are corresponding to the stress-closure relation of the joint at segment i . Therefore, for the i th-segment, E_{ai} , E_{vi} and η_{vi} can be obtained from the phase shift coefficient and attenuation coefficient of wave propagation (Li et al., 2010b).

The constitutive equation of the nonlinear model shown in Fig. 5.3 can be obtained as

$$(E_{ai} + E_{vi})\sigma + \eta_{vi} \frac{\partial \sigma}{\partial t} - \eta_{vi} E_{ai} \frac{\partial \varepsilon}{\partial t} - E_{ai} E_{vi} \varepsilon = 0 \quad (5.6)$$

where ε is strain, E_{ai} and E_{vi} are piecewise elastic modulus of the springs, respectively. η_{vi} is the piecewise viscosity of the dashpot and t denotes the time. E_{ai} , E_{vi} and η_{vi} vary as the magnitude of joint stiffness k_i varies.

The motion equation for the one dimensional longitudinal wave propagation problem can be obtained as

$$\rho \frac{\partial v}{\partial t} = \frac{\partial \sigma}{\partial x} \quad (5.7)$$

where ρ is the density of the material, v is the particle velocity and x is the Cartesian coordinate along the wave propagation path.

In addition to constitutive Eq. (5.6) and motion Eq. (5.7), the continuity equation is known as

$$\frac{\partial v}{\partial x} = \frac{\partial \varepsilon}{\partial t} \quad (5.8)$$

Eqs. (5.6), (5.7) and (5.8) construct the governing equations.

In order to solve the wave propagation problem in the viscoelastic medium, a characteristic line method is adopted by multiplying Eqs. (5.6), (5.7) and (5.8) by undetermined coefficients N , M and L , respectively (Wang et al., 1994). The three equations are then added together, it gives

$$\begin{aligned} & \left[(-N\eta_v E_{ai} - L) \frac{\partial}{\partial t} + 0 \frac{\partial}{\partial x} \right] \varepsilon + \left(M\rho \frac{\partial}{\partial t} + L \frac{\partial}{\partial x} \right) v \\ & + \left(N\eta_{vi} \frac{\partial}{\partial t} - M \frac{\partial}{\partial x} \right) \sigma + N(E_{ai} + E_{vi})\sigma - NE_{ai}E_{vi}\varepsilon = 0 \end{aligned} \quad (5.9)$$

Therefore, along a characteristic line $C(x, t)$, the undetermined coefficient L , M and N should satisfy the following relations

$$\left. \frac{dx}{dt} \right|_{C(x,t)} = -\frac{0}{L + N\eta_{vi}E_{ai}} = \frac{L}{M\rho} = -\frac{M}{N\eta_{vi}} \quad (5.10)$$

Two solutions can be obtained from Eq. (5.10).

The first solution is

$$\begin{aligned} L + N\eta_{vi}E_{ai} &= 0 \\ \rho M^2 &= -LN\eta_{vi} \end{aligned} \quad (5.11)$$

Based on Eq. (5.11), two sets of characteristic lines can be obtained as

$$\frac{dx}{dt} = \pm \sqrt{\frac{E_{ai}}{\rho}} = \pm C_{vi} \quad (5.12)$$

where C_{vi} is defined as the wave velocity along the characteristic lines. And the corresponding two sets of compatibility conditions along the characteristic lines are

$$\begin{aligned} dv &= \pm \frac{1}{\rho C_{vi}} d\sigma \pm \frac{(E_{ai} + E_{vi})\sigma - E_{ai}E_{vi}\varepsilon}{\rho\eta_{vi}C_{vi}} dt \\ &= \pm \frac{1}{\rho C_{vi}} d\sigma + \frac{(E_{ai} + E_{vi})\sigma - E_{ai}E_{vi}\varepsilon}{\rho\eta_{vi}C_{vi}^2} dx \end{aligned} \quad (5.13)$$

where the sign “+” means the rightward running waves and “−” denotes the leftward running waves.

The second solution of Eq. (5.10) is obtained as:

$$\begin{aligned} L &= M = 0 \\ N &\neq 0 \end{aligned} \quad (5.14)$$

Therefore, the characteristic line of the second solution is

$$dx = 0 \quad (5.15)$$

And the compatibility condition along the characteristic line of Eq. (5.15) is

$$d\varepsilon - \frac{d\sigma}{\rho C_{vi}^2} - \frac{(E_{ai} + E_{vi})\sigma - E_{ai}E_{vi}\varepsilon}{\rho\eta_{vi}C_{vi}^2} dt = 0 \quad (5.16)$$

From Eqs. (5.12) and (5.15), the wave propagation in the viscoelastic medium can be described by the x - t plane with three sets of characteristic lines as Fig. 5.4 shows.

Fig. 5.4 Characteristic lines in x - t plane for wave propagation through viscoelastic medium

When the initial condition is given, it can be obtained from the so-called Hugoniot relations that across the front of strong discontinuity stress wave propagates in a material should satisfy (Wang, 1994; Kolsky, 1963; Wang, 1985; Kelley, 2003)

$$\sigma = -\rho C_{vi} v = E_{ai} \varepsilon \quad (5.17)$$

Furthermore, since the locus of the strong discontinuity wave should coincide with the rightward running characteristic line $P(0,0)$ - $P(i,i)$, therefore, it can be obtained from Eq. (13) that

$$dv = \frac{1}{\rho C_{vi}} d\sigma + \frac{(E_{ai} + E_{vi})\sigma - E_{ai} E_{vi} \varepsilon}{\rho \eta_{vi} C_{vi}^2} dx \quad (5.18)$$

Together with Eq. (5.17), the ordinary differential equation of the stress is obtained,

$$\frac{d\sigma}{dx} + \frac{E_{ai}\sigma}{2\eta_{vi} C_{vi}} = 0 \quad (5.19)$$

Thus, along the characteristic line $P(0,0)$ - $P(i,i)$, the stress is determined by,

$$\sigma = \sigma(0, t) \exp\left(-\frac{E_{ai}}{2\eta_{vi}C_{vi}}x\right) \quad (5.20)$$

where $\sigma(0, t)$ denotes the stress at boundary $x = 0$.

With similar derivation, the velocity and strain along the characteristic lines $P(0,0)$ - $P(i,i)$ can be obtained as

$$v = v(0, t) \exp\left(-\frac{E_{ai}}{2\eta_{vi}C_{vi}}x\right) \quad (5.21)$$

and

$$\varepsilon = \varepsilon(0, t) \exp\left(-\frac{E_{ai}}{2\eta_{vi}C_{vi}}x\right) \quad (5.22)$$

where $v(0, t)$ and $\varepsilon(0, t)$ are the velocity and strain at boundary $x = 0$, respectively.

Based on the above derivation, it is found that there are two cases to determine the mechanical behavior of all the points on the plane x - t . One is the triangle case, such as $P(0,j)P(0,j-2)P(1,j-1)$ in Fig. 5.4, and the other is the diamond case, such as $P(i,j)P(i-1,j-1)P(i,j-2)P(i+1,j-1)$ in Fig. 5.4.

For the triangle case, the velocity $v(0, j)$ on the boundary line $P(0, j)$ is derived from the incident wave. Furthermore, along the leftward running characteristic line $P(1,j-1)P(0,j)$, if Δt is small enough, it can be determined from the differential part of Eq. (5.13) that

$$\begin{aligned} \sigma(0, j) = \sigma(1, j-1) - \rho C_{vi} [v(0, j) - v(1, j-1)] \\ - \frac{(E_{ai} + E_{vi})\sigma(1, j-1) - E_{ai}E_{vi}\varepsilon(1, j-1)}{\eta_{vi}} \Delta t \end{aligned} \quad (5.23)$$

And for the upward running characteristic line $P(0,j-2)P(0,j)$, the differential part of Eq. (5.16) can be expressed as

$$\begin{aligned} \varepsilon(0, j) = \varepsilon(0, j-2) &+ \frac{\sigma(0, j) - \sigma(0, j-2)}{\rho C_{vi}^2} \\ &+ \frac{(E_{ai} + E_{vi})\sigma(0, j-2) - E_{ai}E_{vi}\varepsilon(0, j-2)}{\rho \eta_{vi} C_{vi}^2} 2\Delta t \end{aligned} \quad (5.24)$$

For the diamond case, such as $P(i, j)P(i-1, j-1)P(i, j-2)P(i+1, j-1)$, if Δt is small enough, it is derived from the differential part of the rightward running wave compatibility conditions of Eq. (5.13) that

$$\begin{aligned} v(i, j) - v(i-1, j-1) &= \frac{1}{\rho C_{vi}} [\sigma(i, j) - \sigma(i-1, j-1)] \\ &+ \frac{(E_{ai} + E_{vi})\sigma(i-1, j-1) - E_{ai}E_{vi}\varepsilon(i-1, j-1)}{\rho \eta_{vi} C_{vi}} \Delta t \end{aligned} \quad (5.25)$$

The differential part of the compatibility conditions of the leftward running wave of Eq. (5.13) and the compatibility conditions of the upward running wave of Eq. (5.16) are expressed as

$$\begin{aligned} v(i, j) - v(i+1, j-1) &= -\frac{1}{\rho C_{vi}} [\sigma(i, j) - \sigma(i+1, j-1)] \\ &- \frac{(E_{ai} + E_{vi})\sigma(i+1, j-1) - E_{ai}E_{vi}\varepsilon(i+1, j-1)}{\rho \eta_{vi} C_{vi}} \Delta t \end{aligned} \quad (5.26)$$

and

$$\begin{aligned} \varepsilon(i, j) - \varepsilon(i, j-2) &= \frac{\sigma(i, j) - \sigma(i, j-2)}{\rho C_{vi}^2} \\ &+ \frac{(E_{ai} + E_{vi})\sigma(i, j-2) - E_{ai}E_{vi}\varepsilon(i, j-2)}{\rho \eta_{vi} C_{vi}^2} 2\Delta t \end{aligned} \quad (5.27)$$

respectively.

Eqs. (5.25), (5.26) and (5.27) can be rewritten in an explicit form, i.e.,

$$\begin{aligned}
 \sigma(i, j) = & \frac{1}{2} \rho C_{vi} [v(i+1, j-1) - v(i-1, j-1)] \\
 & + \frac{1}{2} [\sigma(i+1, j-1) + \sigma(i-1, j-1)] \\
 & - \frac{1}{2} \frac{(E_{ai} + E_{vi})}{\eta_{vi}} [\sigma(i+1, j-1) + \sigma(i-1, j-1)] \Delta t \\
 & + \frac{1}{2} \frac{E_{ai} E_{vi}}{\eta_{vi}} [\varepsilon(i+1, j-1) + \varepsilon(i-1, j-1)] \Delta t
 \end{aligned} \tag{5.28}$$

$$\begin{aligned}
 v(i, j) = & \frac{1}{2} \frac{1}{\rho C_{vi}} [\sigma(i+1, j-1) - \sigma(i-1, j-1)] \\
 & + \frac{1}{2} [v(i+1, j-1) + v(i-1, j-1)] \\
 & + \frac{1}{2} \frac{E_{ai} E_{vi}}{\rho \eta_{vi} C_{vi}} [\varepsilon(i+1, j-1) - \varepsilon(i-1, j-1)] \Delta t \\
 & - \frac{1}{2} \frac{E_{ai} + E_{vi}}{\rho \eta_{vi} C_{vi}} [\sigma(i+1, j-1) - \sigma(i-1, j-1)] \Delta t
 \end{aligned} \tag{5.29}$$

$$\begin{aligned}
 \varepsilon(i, j) = & \varepsilon(i, j-2) + \frac{\sigma(i, j) - \sigma(i, j-2)}{\rho C_{vi}^2} \\
 & + \frac{(E_{ai} + E_{vi}) \sigma(i, j-2) - E_{ai} E_{vi} \varepsilon(i, j-2)}{\rho \eta_{vi} C_{vi}^2} 2 \Delta t
 \end{aligned} \tag{5.30}$$

Eqs. (5.23), (5.24), (5.28), (5.29) and (5.30) show that the response at time j can be calculated from those at time $j-1$ and $j-2$. Therefore, considering the boundary Eqs. (5.20), (5.21) and (5.22), and the initial conditions $v(0, j)$, any particle velocity, stress and strain at x - t plane can be obtained by an iterative computation using the differential Eqs. (5.23), (5.24), (5.28), (5.29) and (5.30).

5.2.3 Parameter determination

Fig. 5.1 illustrates the REV of a jointed rock mass, in which S is the initial length of the representative element. When a single joint was considered, the transmitted wave was derived as (Li et al., 2010b; Pyrak-Nolte et al., 1990)

$$v_T(x, t) = \frac{2k_i / z}{i\omega + 2k_i / z} A \exp[i(\omega t - x\omega / C_0)] \tag{5.31}$$

where k_i is defined in Eq. (5.1) as the normal joint stiffness, z is the wave impedance and $z = \rho C_0$, C_0 is wave velocity in an intact rock and ω denotes the angular frequency of the incident wave.

When $x = S$, the phase shift coefficient α_r and the attenuation coefficient β_r in Eq. (5.31) are respectively equal to (Li et al., 2010b)

$$\begin{cases} \alpha_r = \frac{\omega}{C_0} + \frac{1}{S} \arctan\left(\frac{\omega}{2k_i / z}\right) \\ \beta_r = \frac{1}{S} \ln\left[\frac{2k_i / z}{\sqrt{\omega^2 + (2k_i / z)^2}}\right] \end{cases} \quad (5.32)$$

When the same incident wave is applied to the REV of an equivalent viscoelastic medium, the phase shift coefficient α_e and the attenuation coefficient β_e can be obtained as (Kolsky, 1963)

$$\begin{cases} \alpha_e = \left\{ \frac{\rho \omega^2}{2E_{ci}E_{ai}} \left[\left(\frac{E_{ai}^2 + E_{ci}^2 \omega^2 \tau_i^2}{1 + \omega^2 \tau_i^2} \right)^{\frac{1}{2}} + \frac{E_{ai} + E_{ci} \omega^2 \tau_i^2}{1 + \omega^2 \tau_i^2} \right] \right\}^{\frac{1}{2}} \\ \beta_e = - \left\{ \frac{\rho \omega^2}{2E_{ci}E_{ai}} \left[\left(\frac{E_{ai}^2 + E_{ci}^2 \omega^2 \tau_i^2}{1 + \omega^2 \tau_i^2} \right)^{\frac{1}{2}} - \frac{E_{ai} + E_{ci} \omega^2 \tau_i^2}{1 + \omega^2 \tau_i^2} \right] \right\}^{\frac{1}{2}} \end{cases} \quad (5.33)$$

Define $E_{ci} = E_{ai}E_{vi}/(E_{ai}+E_{vi})$ and $\tau_i = \eta_{vi}/E_{vi}$ as the time of retardation of the nonlinear Voigt element. When $x = S$, the transmitted wave is obtained from Eq. (5.32) for jointed rock mass or from Eq. (5.33) for the corresponding equivalent medium. Thus, equating Eq. (5.32) and Eq. (5.33), the parameters of the nonlinear viscoelastic model are determined by,

$$\begin{aligned} \left\{ \frac{\rho\omega^2}{2E_{ci}E_{ai}} \left[\left(\frac{E_{ai}^2 + E_{ci}^2\omega^2\tau_i^2}{1 + \omega^2\tau_i^2} \right)^{\frac{1}{2}} + \frac{E_{ai} + E_{ci}\omega^2\tau_i^2}{1 + \omega^2\tau_i^2} \right] \right\}^{\frac{1}{2}} &= \frac{\omega}{C_0} + \frac{1}{S} \arctan\left(\frac{\omega}{2k_i/z}\right) \\ \left\{ \frac{\rho\omega^2}{2E_{ci}E_{ai}} \left[\left(\frac{E_{ai}^2 + E_{ci}^2\omega^2\tau_i^2}{1 + \omega^2\tau_i^2} \right)^{\frac{1}{2}} - \frac{E_{ai} + E_{ci}\omega^2\tau_i^2}{1 + \omega^2\tau_i^2} \right] \right\}^{\frac{1}{2}} &= -\frac{1}{S} \ln\left[\frac{2k_i/z}{\sqrt{\omega^2 + (2k_i/z)^2}}\right] \end{aligned} \quad (5.34)$$

It is recommended by Li et al. (2010b), that the Young's modulus of the intact rock can be used to give E_a in the viscoelastic model. Therefore, the relations between E_{vi} , η_{vi} and ω can be obtained by the implicit expressions given in Eq. (5.34) (Kelley, 2003). If k_i keeps constant, E_{vi} , η_{vi} degrade to the linear model as proposed by Li et al. (2010b).

In the following, it is assumed that rock density ρ is 2650 kg/m^3 , and wave velocity C_0 in the intact rock material is 5830 m/s . The difference of closure for each segment is set equally to be $\Delta d = 0.005 \text{ mm}$, the initial stiffness of the joint in Eq. (5.1) is $k_n = 3.5 \text{ GPa/m}$ and the maximum closure of the joint is $d_{max} = 1 \text{ mm}$. The concept of nondimensional joint spacing Γ is defined as the ratio of the joint spacing S to the wavelength of the incident wave λ .

E_{vi} and η_{vi} relies on the incident wave frequency, the segment number and the joint spacing. Fig. 5.5 shows such relations when $\Gamma = 1$, where f denotes the frequency of the incident wave, and $f = \omega/(2\pi)$.

It is determined from Fig. 5.5 that with a certain segment number, E_{vi} and η_{vi} increases as the increase of the frequency of the incident wave. And for a certain frequency of the incident wave, E_{vi} and η_{vi} increases as the segment number increasing.

Fig. 5.5 Relations of parameters E_{vi} , η_{vi} and frequency f for segment i
($\Gamma = 1$)

5.3 Results and Discussions

5.3.1 Results verification

A displacement discontinuity method is used to verify the proposed nonlinear equivalent viscoelastic medium method. The transmitted velocity after an individual joint is derived as (Zhao and Cai, 2001)

$$v_T(x_1, t_{j+1}) = \frac{2v_i(0, t - x_1 / C_0) - 2v_i(x_1, t_j)}{z/k_i} \Delta t \quad i = 1, 2, 3 \dots \quad (5.35)$$

$$+ v_T(x_1, t_j), \sigma_i \leq \sigma < \sigma_{i+1} \quad ,$$

where v_i and v_T denotes the incident and transmitted waves, respectively. x_1 is the location of the joint and σ is the normal stress applied onto the joint, which can be obtained as $\sigma = \rho C_0 v_i$ (Zhao and Cai, 2001) and k_i is the piecewise linear stiffness of the joint as defined in Eq. (5.1). It is different from the linear displacement method that k_i increases (decreases) as the magnitude of stress wave increasing (decreasing). It is also different from the results obtained by Zhao and Cai (Zhao and Cai, 2001) because k_i varies due to the use of the piecewise linear model in the present study.

Eq. (5.35) shows that the response at time step $j+1$ can be calculated from those at time step j . Therefore, considering the boundary and initial conditions, any particle velocity just passing through the piecewise linear deformational joint can be calculated by an iterative computation using Eq. (5.35).

To verify the present nonlinear equivalent viscoelastic medium method, an incident wave propagates through the RVE of a jointed rock mass and the corresponding nonlinear equivalent viscoelastic medium is studied, and the transmitted waves are compared as shown in Fig. 5.6. Assume the incident wave applied on boundary a in Fig. 5.1 is in a form of

$$v_i(0, t) = \begin{cases} A \sin(2\pi f_0 t) & , \text{ when } 0 \leq t \leq 1/(2f_0) \\ 0 & \text{others} \end{cases} \quad (5.36)$$

where A is the amplitude and f_0 the frequency of the incident wave, which are as shown in Fig. 5.6(a-d) with $f_0 = 50$ Hz and amplitude $A = 0.057, 0.151, 0.340$ and 0.529 m/s, respectively.

Fig. 5.6 Comparing transmitted waves based on DDM, NEVMM and LEVMM with different amplitude incident waves

The transmitted waves obtained based on the displacement discontinuity method, linear equivalent viscoelastic medium method and nonlinear equivalent viscoelastic medium method with respect to different incident wave amplitudes are shown in Fig. 5.6(a-d), respectively. “LEVMM” and “NEVMM” denote the linear equivalent viscoelastic medium method and the present nonlinear equivalent viscoelastic medium method, respectively. Fig. 5.6 shows that the transmitted waves obtained using DDM and NEVMM agree very well, which proves that the present NEVMM can efficiently describe the dynamic property of the jointed rock mass. It is also obtained from Fig. 5.6 that for the incident wave with small

amplitude, the transmitted wave determined by the NEVMM approximates the transmitted wave by the LEVMM. However, the difference of the transmitted waves obtained by these two methods increases as the incident wave amplitude increases.

5.3.2 Wave attenuation

The transmission coefficient is used to assess the attenuation of the transmitted wave. When an incident wave given in the form of Eq. (5.36) propagates across rock mass with a single joint of joint spacing $S = 5.83$ m, the relations of the transmission coefficient versus the incident wave amplitude and the incident wave frequency are investigated, respectively.

The relation between the transmission coefficient and the amplitude of the incident wave based on the LEVMM and the present NEVMM for frequencies of $f_0 = 100, 200, 300$ and 400 Hz are shown in Fig. 5.7.

Fig. 5.7 Effect of incident wave amplitude on transmission coefficient

It can be seen from Fig. 5.7 that the transmission coefficient obtained using LEVMM is independent of the incident wave amplitude, while those obtained by

the NEVMM is amplitude dependent. This is reasonable because a nonlinear model is adopted in the NEVMM.

For the incident wave with a certain frequency, the transmission coefficient increases with the increase of the incident wave amplitude based on the present NEVMM. And all the transmission coefficients will converge to 1 if the amplitude of the incident wave is sufficiently large. This phenomenon can be explained that as the incident wave amplitude increases, the joint is compacted nonlinearly and thus the joint stiffness increases accordingly. The stiffness of the joint approaches the stiffness of the rock material when the joint is fully compacted. From Fig. 5.7, the transmission coefficient for the incident wave with a larger frequency is smaller than that with a smaller frequency, which coincides the geological investigation that joint filters out waves with higher frequencies, but has less effect on the waves with lower frequencies (Pyrak-Nolte, 1988).

Fig. 5.8 illustrates the relation between the transmission coefficient and the incident wave frequency with respect to different incident wave amplitudes, i.e., $A = 0.05, 0.1, 0.3, 0.5$ and 0.8 m, respectively.

Fig. 5.8 Effect of incident wave frequency on transmission coefficient

It is seen from Fig. 5.8 that there is a flat low frequency region within which the transmission coefficient is nearly a constant or reduces slightly with frequency increasing. When the frequency is sufficiently large, the transmission coefficient becomes zero. Between these two extremes, there is a rapid decreasing region in the transmission coefficient. For a fixed incident wave amplitude, the incident waves with a larger frequency result in less transmission. For the same frequency, the incident waves of large amplitudes result in a larger transmission coefficient.

It is also found from Fig. 5.8 that as the amplitude of the incident wave decreases, the relationship between the transmission coefficient and the incident wave frequency obtained by the NEVMM approaches to that given by the LEVMM.

5.3.3 Effective velocity

The effective wave propagation velocity in an equivalent medium is defined as the wave travelling length over the corresponding wave traveling time. The wave traveling time is measured as the time difference between the peak velocity of the incident and transmitted waves. Considering the joint spacing $S = 5.83$ m, the effective velocity versus the incident wave amplitude for different frequencies of $f_0 = 100, 200, 300$ and 400 Hz are shown in Fig. 5.9.

Fig. 5.9 Effect of incident wave amplitude on effective wave propagation velocity

It is seen with a fixed wave frequency, the larger the incident wave amplitude, the larger the effective velocity. The effective velocity approaches the maximum value when the stiffness of the joint is close to the stiffness of the intact rock. In the LEVMM, however, the effective velocity is independent to the incident wave amplitude.

The relations between the effective velocity and the incident wave frequency for different incident wave amplitudes, i.e., $A = 0.05, 0.1, 0.3, 0.5$ and 0.8 m are shown in Fig. 5.10.

Fig. 5.10 Effect of incident wave frequency on effective wave propagation velocity

There is a low frequency region for which the effective velocity is the minimum and it varies slightly in the region. And the effective velocity approaches the wave propagation velocity of the intact rock material at high frequencies, which is $v = 5830$ m/s in the present analysis. Between these two regions, there is a rapid increase in the effective velocity as the frequency increases. Similar results have been obtained by the displacement discontinuity method (Pyrak-Nolte et al., 1990a).

It is also observed from Fig. 5.10 that as the incident wave amplitude decreases, the relationship between the effective velocity and the incident wave frequency obtained by the NEVMM approaches to that by the LEVMM. And the LEVMM can be used to replace the NEVMM approximately if the amplitude of the incident wave is insufficient to cause nonlinear deformation of the joint.

5.4 Conclusions

A nonlinear equivalent viscoelastic medium model is proposed in the present study for investigation of stress wave propagation in rock masses which consists of equally-spaced parallel joints with nonlinear deformational behavior. The nonlinear

behavior of jointed rock mass is approached by a piecewise linear continuum. A method of characteristic line is adopted to solve the wave propagation in the equivalent viscoelastic continuum. The parameters of the present equivalent viscoelastic model are theoretically derived and the frequency-dependence and amplitude-dependence is clearly presented.

It is concluded that wave propagation through nonlinear jointed rock mass is amplitude-dependent. The conventional LEVMM, as limited by the linear assumption, are not able to describe the property of amplitude-dependence. Therefore, cannot accurately predict the transmitted wave when the wave amplitude is sufficient large. On the basis of this study, the present NEVMM not only predicts accurately the attenuation coefficient and the effective velocity when the amplitude of stress wave is small, but also in the condition when the amplitude of stress wave is sufficient large. The conventional LEVMM can be regarded as a special case of the NEVMM when the amplitude of stress wave is insufficient to mobilize the nonlinear deformation of the joints.

The proposed NEVMM has advantage over DDM. The comparison of the results obtained by DDM and present NEVMM shows that NEVMM can be used to study the wave propagation in jointed rock mass as accuracy as DDM. However, because of its continuous characters, the proposed NEVMM is much more efficient in analyzing wave propagation in jointed rock mass with equally-spaced parallel joints.

Above all, this study can serve as a principle work of stress wave propagation through rock mass. Further exploration is underway to extend the current model for multi-reflections of waves among the joints using VWS, and for more complicated joint set configurations and other forms of incident waves, such as S-wave.

CHAPTER 6 EXPERIMENTAL STUDY ON VISCOELASTIC BEHAVIOR OF MICRO-DEFECTED ROCK

6.1 Introduction

The present chapter performed an experimental study to determine the viscoelastic properties of a sedimentary rock. The attenuation coefficient, the wave number and the phase velocity of the sedimentary rock with respect to the component wave frequencies were derived. The corresponding dynamic storage modulus and dynamic loss modulus were calculated from the experimentally determined wave propagation coefficients. A modified 3-element viscoelastic model was proposed, in which the frequency dependence had been taken into account. Two exponential forms were used to describe the parameters of the present model. An analysis of stress wave propagation in the sedimentary rock was carried out in order to investigate the effect of the material viscosity. It is concluded that the sedimentary rock has obvious viscoelastic behavior in stress wave propagation and the effect of rock viscosity on stress wave attenuation is substantial. The viscoelastic behavior of the sedimentary rock depends on the frequency of the incident stress wave. Equation Chapter (Next) Section 1

6.2 Experimental Test

6.2.1 Experimental set-up

To investigate the viscoelastic property of the sedimentary rock bar, a series of impact tests were carried out. The experimental set-up is shown in Fig. 6.1.

(a) Scheme for strain wave measurement

(b) Experimental setup

Fig. 6.1 Experimental setup for impact test

The sedimentary rock bar was cored from an underground cavern construction site. The inherent defects in the sedimentary rock, such as micro-fracture and voids are too small to be visible to the naked eyes. However, the existence of these defects will cause stress wave reflections and diffractions when a stress wave passing through it, which results in attenuation and dispersion of stress waves and make the mechanical response of the sedimentary rock viscoelastic. The sedimentary rock bar used in the present test had a length of 129.80 cm and a diameter of 4.49 cm. The density of the sedimentary rock bar was measured to be 2680.98 kg/m^3 . The integrity and homogeneity of the sedimentary rock bar were carefully examined and the end surface of the bar was ground by a grinding machine before the experiment to make a flat free end. A steel pendulum hammer was used

to generate a longitudinal pulse and the intensity of the pulse load was adjusted by changing the swinging angle of the hammer. A pair of rollers was designed to support the sedimentary rock bar in order to eliminate the friction effect from the supports. The longitudinal strain of the sedimentary rock bar was measured by using a pair of diametrically opposite strain gauges located at the middle of the bar as shown in Fig. 6.1.

6.2.2. Viscoelastic behavior of the sedimentary rock bar

Fig. 6.2 shows typical strain pulses measured at the middle of the rock bar. The strain history was recorded at a time rate of $1.00 \times 10^7 \text{ s}^{-1}$ (time resolution of $1.00 \times 10^{-7} \text{ s}$) which gave sufficient data points with sufficient accuracy to perform a discrete Fourier transformation.

Fig. 6.2 Measured strain pulses at the middle point of the rock bar

As seen in Fig. 6.2, $\varepsilon_1(t)$ is the rightward running strain pulse in the direction of increasing x , and $\varepsilon_2(t)$ is the leftward running strain pulse in the direction of decreasing x produced by the pendulum. The amplitude of $\varepsilon_1(t)$ was 0.863×10^{-4} and that of $\varepsilon_2(t)$ reduced to 0.786×10^{-4} , from which stress wave attenuation can thus be determined. Previous investigations (Kolsky, 1963; Bacon, 1998; Lundberg and

Blanc, 1988; Blanc, 1993) showed that when the dimension of the bar's cross-section is much smaller in comparison with the wavelengths involved in the disturbance, the one-dimensional wave propagation theory will then be valid. In the present study, the pressure bar has a uniform diameter of 4.49 cm, much smaller than the wavelength which was about 1.76 m. Therefore, the wave attenuation was caused by the viscosity of the material rather than the lateral geometrical effect.

The discrete Fourier transform was applied to the measured strain pulses $\varepsilon_1(t)$ and $\varepsilon_2(t)$, respectively. The absolute values of $\tilde{\varepsilon}_1(\omega)$ and $\tilde{\varepsilon}_2(\omega)$ are shown in Fig. 6.3 which are the frequency spectrum of the stress pulses.

Fig. 6.3 Frequency spectrum of experimental pulses

Since the Fourier transformed strain pulses are even functions of the frequency ω , the frequency spectrum of the strains are symmetric to the axis of $\omega = 0$, therefore only the positive part is plotted in Fig. 6.3. Quantitative attenuation is obtained for every component wave with different frequencies as shown in Fig. 6.3.

6.3 Theoretical Formulation

6.3.1 Wave propagation theory

Considering the propagation of a harmonic wave in the straight, cylindrical and slender bar with density ρ and complex modulus $E^*(\omega)$. The one-dimensional axial motion equation of the bar in the frequency domain can then be expressed as

$$\frac{\partial^2 \tilde{\varepsilon}}{\partial x^2} + \frac{\rho \omega^2}{E^*} \tilde{\varepsilon} = 0 \quad (6.1)$$

where $\tilde{\varepsilon}(\omega)$ denotes the strain in the frequency domain, $E^*(\omega)$ is the complex dynamic modulus and can be expressed as $E^*(\omega) = E'(\omega) + iE''(\omega)$, $E'(\omega)$ is the storage modulus and $E''(\omega)$ is the loss modulus, i denotes the imaginary sign and ω is the angular frequency, which can be obtained from wave frequency f by $\omega = 2\pi \cdot f$.

Eq. (6.1) was solved with the general solution of strain as

$$\tilde{\varepsilon}(x, \omega) = \tilde{P}(\omega) e^{-\gamma(\omega)x} + \tilde{N}(\omega) e^{\gamma(\omega)x} \quad (6.2)$$

where the propagation coefficient $\gamma^2(\omega) = -\rho \omega^2 / E^*$, which can be obtained from $\gamma(\omega) = \alpha(\omega) + ik(\omega)$, the coefficient $\alpha(\omega)$ is the attenuation coefficient and $k(\omega)$ is the wave number. $\tilde{P}(\omega)$ and $\tilde{N}(\omega)$ are amplitudes of harmonic component wave traveling in the directions of increasing and decreasing x , respectively. And $\tilde{P}(\omega)$ and $\tilde{N}(\omega)$ can be determined by initial and boundary conditions. Since the stress on the free end surface of the rock is zero, it can be obtained that $\tilde{P}(\omega) = -\tilde{N}(\omega)$ on the free end of the rock bar (Lundberg and Blanc, 1988).

Assuming there is an impact on the boundary at $x = 0$, the Fourier transform of the measured strain as a function of x and ω can be used to obtain the attenuation coefficient and wave number from the following relations (Bacon, 1998; Sogabe et al., 1982; Benatar et al., 2003; Lundberg and Blanc, 1988).

$$\alpha(\omega) = -\operatorname{Re} \left[\ln \left(\frac{\mathbb{F}(|\varepsilon_2|)}{\mathbb{F}(|\varepsilon_1|)} \right) / l \right] \quad (6.3)$$

and

$$k(\omega) = -\operatorname{Im} \left[\ln \left(\frac{\mathbb{F}(|\varepsilon_2|)}{\mathbb{F}(|\varepsilon_1|)} \right) / l \right] \quad (6.4)$$

respectively, where $\mathbb{F}$ denotes the treatment of applying the Fourier transformation, Re and Im denote real and imaginary parts of the complex expression, respectively. The subscript of ε_1 and ε_2 denotes the sequence of the strain pulses measured on the strain gauge, and l denotes the wave traveling distance between these pulses.

It can be observed from the definition of the wave propagation coefficient that the relations between the real and imaginary parts of the propagation coefficient $\gamma(\omega)$ and dynamic complex moduli $E^*(\omega)$ yields

$$E'(\omega) = \frac{\rho\omega^2(k^2 - \alpha^2)}{(k^2 + \alpha^2)^2} \quad (6.5)$$

and

$$E''(\omega) = \frac{2\rho\omega^2 k\alpha}{(k^2 + \alpha^2)^2} \quad (6.6)$$

respectively.

Observing Eqs. (6.5) and (6.6), if the propagation coefficient of the rock is obtained from the experimental tests, the dynamic storage modulus and dynamic loss modulus of the rock can be calculated, respectively.

6.3.2 Modified 3-element viscoelastic model

A modified 3-element viscoelastic model as shown in Fig. 6.4 is used to investigate the mechanical properties of the rock.

Fig. 6.4 Modified 3-element viscoelastic model

The model is an auxiliary spring in parallel with a modified Maxwell element. The spring with elastic constant E_a is used to describe the modulus of rock under fully relaxation condition. If the strain-rate is small enough, the Maxwell element term can be neglected for the completely relaxed rock. The modified Maxwell element consists of a spring and a dashpot with frequency-dependent elastic constants $E_v(\omega)$ and frequency dependent damping ratio $\eta_v(\omega)$, which is used to describe the frequency-dependent property of the rock.

The constitutive equation of the present model can be obtained as

$$E_v \sigma(x, t) + \eta_v \frac{\partial \sigma(x, t)}{\partial t} = E_a E_v \varepsilon(x, t) + (E_a + E_v) \eta_v \frac{\partial \varepsilon(x, t)}{\partial t} \quad (6.7)$$

where $\sigma(x, t)$ denotes the stress, $\varepsilon(x, t)$ is the strain, E_a and E_v denote the elastic modulus of auxiliary spring and the Maxwell element, respectively. η_v expresses the viscosity and t is the time (Christensen, 1982).

Fourier transformation was applied to Eq. (6.7) to obtain the constitutive equation in the frequency domain as

$$(E_v + i\omega\eta_v)\tilde{\sigma} = [E_a E_v + i\omega(E_a + E_v)\eta_v]\tilde{\varepsilon} \quad (6.8)$$

where $\tilde{\sigma}(x, \omega)$ denotes the Fourier transform of the stress.

Therefore, the complex dynamic modulus of the present model is derived from Eq. (6.8) as

$$E^*(\omega) = \frac{\tilde{\sigma}(x, \omega)}{\tilde{\varepsilon}(x, \omega)} = E_a + \frac{\omega^2 \tau_v^2}{1 + \omega^2 \tau_v^2} E_v + i \frac{\omega \tau_v}{1 + \omega^2 \tau_v^2} E_v \quad (6.9)$$

The retardation time of the Maxwell element is defined as $\tau_v = \eta_v / E_v$.

Thus, the relations between real and imaginary parts of the rock dynamic modulus versus material constants are given by

$$E'(\omega) = E_a + \frac{\omega^2 \tau_v^2}{1 + \omega^2 \tau_v^2} E_v \quad (6.10)$$

and

$$E''(\omega) = \frac{\omega \tau_v}{1 + \omega^2 \tau_v^2} E_v \quad (6.11)$$

respectively, where $E'(\omega)$ and $E''(\omega)$ are the dynamic storage modulus and dynamic loss modulus of the rock model and they should equate to the dynamic moduli obtained by experimental tests as given in Eqs. (6.5) and (6.6), respectively.

It is noticed from Eqs. (6.5), (6.6), (6.10) and (6.11) that $\alpha(\omega)$, $k(\omega)$, $E'(\omega)$ and $E''(\omega)$ all depend on frequency ω as well as material constants E_a , E_v , η_v . The wave number $k(\omega)$ is a monotonic continuous function of the frequency. However, the wave number $k(\omega)$ obtained from the phase shift of the transfer

function is generally calculated in the range from 0 to 2π as shown in Eq. (6.4), which makes the function discontinuous to the frequency during numerical calculation. This error can be corrected by adding multiples of $\pm 2\pi$ when absolute jumps occur (Bacon, 1998).

6.4 Results and Discussions

6.4.1 Propagation coefficient

In order to quantitatively describe the dynamic mechanical properties of the rock, the wave propagation coefficient $\gamma(\omega)$, later the attenuation coefficient $\alpha(\omega)$ and wave number $k(\omega)$ were derived by Eq. (3) and Eq. (4) from the experimental data, as shown in Fig. 6.5, respectively.

Fig. 6.5 Relation of propagation coefficients (α and k) with frequency f

It is seen from Fig. 6.5 that the wave number is approximately proportional to the frequency of the component wave. However, they are not absolutely linear, a slightly difference can be seen which leads to variations of phase velocity with

respect to the wave frequency. For the wave with a larger frequency, a larger wave number will be generated.

The relations of attenuation coefficient and component wave frequency are shown in Fig. 6.5. It can be obtained that the attenuation coefficient increases with the increase of wave frequency. The stress wave with a higher frequency will attenuate much faster than those with lower frequencies, which is one of the causes of stress wave dispersion when it propagates through viscoelastic sedimentary rock bar.

6.4.2 Wave phase velocity and 3-D effect

When a longitudinal stress wave propagating through a slender bar, the geometric effect on the wave dispersion has been investigated (Kolsky, 1963; Bacon, 1998; Benatar et al., 2003; Li and Ma, 2009; Li et al., 2010a). Results show that only if the wavelengths are less than or in the same order of the bar diameter, the geometric effects due to the radial inertia become important and in that cases the 3-D effect needs to be taken into account. In that case, the phase velocity decreases as the frequency of the component wave increases.

From the wave number given in Fig. 6.5, the phase velocity of stress wave propagating through the sedimentary rock is determined as shown in Fig. 6.6.

Fig. 6.6 Relation of phase velocity V with frequency f

It can be obtained from Fig. 6.6 that the phase velocity is in the range from 5133.30 m/s to 5169.00 m/s. The phase velocity increases as the frequency of the component wave increasing in the frequency ranges from -6.5 kHz to $+6.5$ kHz, which also verifies that the 3-D effect on the wave propagation can be neglected in the present study.

6.4.3 Dynamic complex modulus

The dynamic complex modulus of the rock was calculated, and the storage modulus and loss modulus determined by Eq. (6.5) and Eq. (6.6) are plotted in Fig. 6.7. The storage modulus describes the elastic properties of the sedimentary rock and the loss modulus relates to the viscosity of the sedimentary rock. Therefore, the storage modulus is understood as the static elastic modulus in rock engineering for approximate calculation.

Fig. 6.7 Dynamic complex modulus of sedimentary rock

It can be obtained from Fig. 6.7 that in the high frequency range, the storage modulus and loss modulus remain constants where E_v is approximately 75.0 GPa and η_v is 2.1 GPa·s. The storage modulus is much larger than the loss modulus. It also can be seen from Fig. 6.7 that the storage modulus is relatively small in the smaller frequency range, which increases rapidly with the increase of the component wave frequency. The storage modulus approaches a constant if the frequency is sufficiently large.

Fig. 6.7 also shows that there are three stages for the loss modulus changing with the component wave frequency. The loss modulus increases rapidly to a maximum value with the frequency increase in the first stage and then decreases from the maximum value with increasing frequency in the second stage. In the third stage, the loss modulus becomes a constant with the further increase of the wave frequency. Although the dynamic modulus in the frequency range from -6.5 kHz to $+6.5$ kHz can only be obtained by the present experiments, their values in the higher frequency range can be extrapolated based on the varying trend as shown in Fig. 6.7.

6.4.4 Parameters of modified 3-element viscoelastic model

The parameters of the modified 3-element model were determined as a function of the component wave frequency ω . Several methods were developed for determining the parameter values of a viscoelastic model from the variation of the dynamic complex modulus (Sogabe et al., 1982; Bland and Lee, 1956; Yin and Wang, 2008). However, all these methods were based on the assumption that the parameters of the model were constants and independent of the frequency variable. Li et al. (2010b) proposed a numerical method for data fitting, and a Newton-like method (Kelley, 2003) was adopted in their investigation.

In the present study, the experimental results of $E'(\omega)$ and $E''(\omega)$ shown in Fig. 6.7 are used to derive the parameters of the modified 3-element model. The elastic modulus of the spring E_a in the modified 3-element model is assumed to be a constant, while η_v and E_v are frequency dependent. Trial values of E_a , η_v and E_v were used to search for the best fitting to the experiment results, the final values of E_a , η_v and E_v were determined when a negligible error was calculated.

Fig. 6.8 Relations among parameters $E_v(\omega)$, $\eta_v(\omega)$ with frequency ω

Fig. 6.8 shows the relations among the parameters of $E_v(\omega)$, $\eta_v(\omega)$ and ω , in which the elastic modulus of the spring is $E_a = 55.77$ GPa. As seen from Fig. 6.8, both $E_v(\omega)$ and $\eta_v(\omega)$ are highly frequency dependent, and they decrease with increasing component wave frequency ω . Similar to the dynamic complex modulus, it can be observed from the varying tendency of $E_v(\omega)$, $\eta_v(\omega)$ that although only the frequency range of -6.5 kHz to $+6.5$ kHz was calculated in the present experimental results, the parameters of the present model can be extrapolated reasonably in the higher frequency range.

To make the present model more practicable, the relation between $E_v(\omega)$ and ω , $\eta_v(\omega)$ and ω is respectively derived by the least square regression method as two exponential forms of

$$y(\omega) = A_0 + A_1 \exp(-\omega/B_1) + A_2 \exp(-\omega/B_2) \quad (6.12)$$

where A_i and B_j ($i = 0, 1, 2, j = 1, 2$) are coefficients determined by curve fitting, and they are listed in Table 6.1. The fitting results are shown in Fig. 6.8.

Table 6.1 Coefficients for curves fittings of $E_v(\omega) \sim \omega$, $\eta_v(\omega) \sim \omega$

	A_0	A_1	A_2	B_1	B_2
$E_v(\omega)$ (Pa)	15.54×10^9	13.99×10^{10}	10.05×10^{13}	60.77	9.23
$\eta_v(\omega)$ (Pa·s)	24.70×10^5	20.45×10^6	27.89×10^7	1868.11	37.38

6.4.5 Effect of Viscoelastic Properties on Wave Transmission

A numerical example is given to illustrate the application of the present model in practice. In principle, a measured signal of incident waveform (e.g. rectangle wave, triangle wave, seismic wave, blast wave) can always be regarded as the sum of periodical harmonic component waves in the form of Eq. (6.2) using Fourier transformation. The frequency-dependent parameters in the present modified 3-

element viscoelastic model are used to determine the complex moduli by Eqs. (6.10) and (6.11), then to determine the propagation coefficient $\gamma(x, \omega)$. Therefore, the corresponding propagating harmonic component waves can be calculated by Eq. (6.2). Inverse discrete Fourier transformation (IDFT) is applied to the sum of the propagating component waves in order to obtain the propagating group wave. A transmission coefficient is defined as the ratio of the maximum magnitude of the propagating wave over the magnitude of the incident wave.

Assume that a half-cycle sinusoidal incident wave is applied, it has the form of

$$\varepsilon_{inc} = \begin{cases} I_0 \sin(2\pi \times f_0 \times t) & , \text{ when } 0 \leq t \leq 1/(2f_0) \\ 0 & \text{others} \end{cases} \quad (6.13)$$

where f_0 is the frequency of the incident wave. I_0 is the magnitude of the incident wave.

The relations of the transmission coefficient versus the incident wave frequency f_0 and propagation distance D , which is defined as the length difference between the two peak particle velocities of incident and propagating waves, are plotted in Fig. 6.9.

Fig. 6.9 Effects of frequency f_0 and propagation distance D on the transmission coefficient T

It is seen that wave transmission in the sedimentary rock not only relates to the wave propagation distance, but also highly depends on the wave frequency.

Fig. 6.10(a) and (b) show the effect of the incident wave frequency f_0 on the transmission coefficient for fixed wave propagation distances and the effect of wave propagation distance on the transmission coefficient for fixed incident wave frequencies, respectively.

Fig. 6.10 Relations of transmission coefficient T versus frequency f_0 and propagation distance D

As depicted in Fig. 6.10(a), the transmission coefficient decreases as the incident wave frequency increasing for a given wave propagation distance. The wave with higher frequency attenuates faster than those with lower frequencies. The transmission coefficient in Fig. 6.10(a) shows the present modified 3-element

viscoelastic model describes accurately the frequency dependent property of the sedimentary rock.

Fig. 6.10(b) shows that the transmission coefficient decreases with the increase of the wave propagation distance for an incident wave with given frequency. However, the decreasing rate of the transmission coefficient becomes smaller with the increase of the wave propagation distance. The transmission coefficient approaches to 1 when the propagation distance becomes zero. It is also noticed that the incident waves with higher frequencies attenuate faster. When the incident wave frequency f_0 approaches zero, the transmission coefficient approaches one, in which condition, the present 3-element viscoelastic model degenerates to the traditional static elastic model. It is worth mentioning that the traditional static elastic model is unable to describe stress wave attenuation in a one-dimensional pressure bar.

6.5 Conclusions

The present study experimentally characterized the dynamic viscoelastic property of a sedimentary rock. Impact testes on a slender rock bar showed that the sedimentary rock exhibits obvious viscoelastic behavior under dynamic loading. By applying one-dimensional stress wave propagation theory, wave propagation coefficients such as the attenuation coefficient and the wave number of the sedimentary rock are determined. The wave propagation coefficients are frequency dependent, which is different from those traditional viscoelastic model used for creep and stress relaxation. The modified 3-element viscoelastic model used in the present study is capable in describing the frequency dependence of the viscoelasticity under dynamic loading.

The dynamic moduli of the sedimentary rock (both storage modulus and loss modulus) are also highly frequency dependent. The viscoelastic parameters of the modified 3-element model can be regarded as a constant only if the frequency is sufficiently large, in which case, the modified 3-element viscoelastic model degrades to the traditional frequency-independent 3-element model.

In addition, the traditional elastic rock model is a special case of the present modified 3-element viscoelastic model when the incident wave frequency is sufficiently small. Although only the compressive stress wave with half sinusoidal form was analyzed, other kinds of waves (e.g. shear wave) with other forms (e.g. triangle wave form, rectangle wave form) can also be applied for similar analysis.

CHAPTER 7 NUMERICAL STUDY ON VISCOELASTIC BEHAVIOR OF MICRO-DEFECTED ROCK

7.1 Introduction

Numerical simulations of stress wave propagation in a one dimensional defected rock bar are performed in this Chapter. The NMM is validated by studying longitudinal (or P-) wave propagation through intact and cracked rock bars. Stress wave passing through a heavily defected rock bar with numerous randomly distributed micro-cracks is modeled to quantitatively investigate the rock viscoelasticity in terms of crack length, quantity, and specific stiffness. The relation of the frequency dependent dynamic complex modulus which is the main index of rock viscoelasticity and the parameters of the micro-cracks (i.e. crack length, quantity, specific stiffness) are also discussed through a parametric study.

7.1.1 Background

Micro-defects are one of the most distinguishing features of rock materials, which affect both the static and dynamic behavior of rock materials remarkably. These micro-defects (i.e. pores, open cracks, veins, etc.) may stem from earth crust motion, weathering, high pore pressure, intrinsic inhomogeneous mixtures and minerals that form the rock mass. Due to the existence of various defects, rock materials behave viscoelastically when stress wave propagation is taken into consideration. It is significant to quantitatively reveal the effects of micro-defects in rock materials on the rock viscoelasticity. Equation Chapter (Next) Section 1

The numerical manifold method (NMM) (Shi, 1991; Ma and An, 2010) provides a natural bridge between the continuum based and discrete element based numerical methods, in which mathematical covers and physical covers are incorporated to make the description of cracks arbitrary and much more convenient. The mathematical covers of NMM used in the present study are regular hexagons, and they are independent and completely cover the physical domain in an

overlapped manner. The intersection of the mathematical covers and physical domain generates physical covers, which can be regarded as the subdivision of the mathematical covers by the physical domain. Finally, overlapped parts of these physical covers form the manifold elements, which can have an arbitrary geometry. The cracks in the NMM are treated as contacts of physical boundaries. The normal specific stiffness and tangential specific stiffness of the cracks are modeled by normal and tangential springs with certain specific stiffness of the contact respectively. And the tangential specific stiffness is designated 0.4 times of the normal specific stiffness in this study. Therefore, the crack model in the NMM coincides with the assumption of the displacement discontinuity method (DDM) (Miller, 1977; Schoenberg, 1980; Pyrak-Nolte et al., 1990a, b), whose cracks are treated as imperfectly welded interfaces (displacement discontinuous boundaries) across which stresses are continuous, however, the displacements before and after the interface are discontinuous. Since its applicability and the specific feature in modeling discontinuous rock material, it is employed in the present study to study stress wave propagation in the highly cracked rock material.

7.1.2 Validation of NMM modeling on stress wave propagation

Although the NMM has been extensively applied in the simulation of discontinuous deformation analysis of jointed rock mass, its application on stress wave propagation has been seldom reported. It is necessary to validate the suitability and accuracy of the NMM for the simulation of stress wave propagation and attenuation across the fractured rock.

NMM incorporates the implicit algorithm and establishes the simultaneous equilibrium equations by minimizing the total potential energy of the entire physical system. When a stress wave propagates in an elastic material without damping, the total energy (summation of the potential energy and kinetic energy) will not decay providing that the system has no energy exchange with the environment. Furthermore, if the stress wave is traveling in a one-dimensional bar (when the wave length is more than ten times of the transverse dimension of the bar, the geometrical dissipation is small enough to be ignored), the amplitude of the stress

wave in the longitudinal direction will not attenuate. A test (case 1) is carried out to confirm the energy conservation and invariance of the wave amplitude when a P-wave propagates in a one-dimensional elastic bar without defects as plotted in Fig. 7.1(a). The NMM model is illustrated in Fig. 7.1(b).

It is the advantage of the NMM that the mathematical covers do not need to conform to the boundaries of the physical domain. It has the flexibility to simulation different density micro-cracks in the rock bar using consistent a mathematical cover system. Detail description of NMM models and advantages can be found in (Shi, 1991, Ma and An, 2010).

In order to investigate the stress wave property exclusively, no gravity force is applied. The dimensions of the rock bar are 2 m of length and 0.05 m of width. A half sinusoidal impacting wave with a frequency of 2500 Hz is applied at the left side of the rock bar to introduce a 1 MPa in amplitude compression stress wave. A measuring point (point A) is located at 1.0 m away from the left side of the bar as seen in Fig. 7.1(a). Other parameters are $E=75$ GPa, $\nu=0.25$, $\rho=2680.98$ kg/m³.

Fig. 7.1(b) Illustration of concept of cover and element in NMM

The result of the test is presented in Fig. 7.2, from which three stress waves passing through point A are recorded, they are subsequently the incident wave, the reflected wave from the free end, and the re-reflected wave from the impact end. The amplitudes of the three stress waves are -0.999 MPa, 0.995 MPa and -0.995 MPa (minus represents compression) respectively at point A, which demonstrates that the NMM model has excellent energy conservative property and accuracy in simulating stress wave propagation. The phase velocity of the stress wave is also obtained based on the time difference of the three peaks, and it is 5281.23m/s (the error is -0.15% compared with the theoretical result, which is 5289.13m/s). This test shows NMM has the ability in simulation of stress wave propagation in a homogeneous elastic material with sufficient accuracy.

Fig. 7.2 Stress wave propagation across homogeneous elastic rock mass with free boundary

Case 2 is carried out to validate the NMM by simulating a stress wave passing through a rock joint. In order to avoid superposition of the stress waves, the one-dimensional bar is scaled up to 200 m by 5 m. And the incident wave is a half sinusoidal compression wave with a frequency of 50 Hz. A single linear elastic run-

through joint is positioned in the center of the elastic bar (at 100 m from the impact end) with a specific stiffness of 5.0 GPa/m. A measuring point B is placed at 50 m from the impact end and another measuring point C is placed adjacent to the right of the joint in order to obtain the waves just before and after the joint (see Fig. 7.3(a)). Other parameters are the same as those in case 1.

Fig. 7.3(a) Scheme of NMM simulation of P-wave propagation across a run-through joint

Fig. 7.3(b) Comparison of particle velocity time histories

Analytical solution of the particle velocity time histories at point B and the joint is derived in order to comparing with the numerical results obtained from the

NMM. The particle velocity time history immediately before and after the joint can be derived based on the displacement discontinuity method and the characteristic line theory (Miller, 1977; Schoenberg, 1980; Zhao and Cai, 2001). They are given as follows:

$$\frac{\partial v^+(x_J, t - x_J / \alpha)}{\partial t} = \frac{-2v^+(x_J, t - x_J / \alpha) + 2v_I(x_B, t - x_B / \alpha)}{z / K_n} \quad (7.1)$$

$$v^+(x_J, t - x_J / \alpha) + v^-(x_J, t - x_J / \alpha) = 2v_I(x_B, t - x_B / \alpha) \quad (7.2)$$

$$v_R(x_B, t - (2x_J - x_B) / \alpha) = v^-(x_J, t - x_J / \alpha) - v_I(x_J, t - x_J / \alpha) \quad (7.3)$$

where $v^-(x_J, t - x_J / \alpha)$ is the particle velocity time history just before the joint at the coordinate x_J , which is superposed by the incident and reflected waves. $v^+(x_J, t - x_J / \alpha)$ is the particle velocity time history just after the joint, i.e., the transmitted particle velocity. $v_I(x_B, t - x_B / \alpha)$ is the particle velocity time history of incident wave passing through point B. α, z, K_n represent the wave propagation velocity, mechanical impedance, joint specific stiffness, respectively. $v_R(x_B, t - (2x_J - x_B) / \alpha)$ is the particle velocity time history of reflected wave at point B.

Given the initial condition $v_I(t)$ and $v^+(x_J, 0)$, the particle velocity at the joint position x_J just before and after the joint can be derived numerically based on Eqs. (7.1) and (7.2). Fig. 7.3(b) compares the NMM result with the theoretical solution. As seen from Fig. 7.3(b), the numerical results by using the NMM agree well with the analytical solution not only the peak particle velocities but also the waveforms of the transmitted and reflected waves. The time delay phenomenon caused by the frequency dependent phase lag of the harmonic waves is also simulated. In the time domain, the square of the area enwrapped by stress time history and the time axis indicates the wave energy. When a stress wave travels across a joint, the incident wave energy is divided into two parts, one part of which is reflected and the other is transmitted. The sum of square of the areas enwrapped by transmitted and reflected

stress time history equates to the total incident wave energy, which also proves the correctness and precision of the NMM. In fact, in consideration of the reflected wave, the run-through joint acts as a viscoelastic boundary which behaves between a free boundary and a rigid boundary. When the joint stiffness decreases to zero, the joint acts as a free boundary without viscosity, from which the incident energy is completely reflected back, however, when the joint stiffness increases to infinity, the joint acts as a rigid boundary with no elasticity and the incident wave energy is totally absorbed by the downstream material. The results show that the NMM adopted in the present study has satisfactory ability to model the stress wave propagation across jointed rock mass.

As a general form in the natural environment, non-through rock joints with a finite length widely exist in a rock mass. It is also necessary to validate the NMM in simulation of stress wave passing through a non-through joint. A P-wave traveling across a non-through joint in the one-dimensional rock bar is subsequently modeled. It is worth mentioning that the NMM has a special feature that mathematical covers do not necessarily conform to the physical domain boundaries, the validation of the NMM in simulation of stress wave propagation in fractured rock is necessary.

Fig. 7.4(a) Scheme of NMM simulation of P-wave propagation
across a non-through crack

Fig. 7.4(b) Effects of non-through crack of different lengths on stress wave propagation

Different from case 2, a non-through joint with finite length as shown in Fig. 7.4(a) is further simulated. The measuring points B, D and the non-through joint are positioned at 40 m, 120 m, and 80 m respectively. And the joint specific stiffness is reduced to 2.5 GPa/m to amplify the attenuation in order to observe the results more obviously. The length of the joint is set to be 0.4, 0.6 and 0.8 times of the width of the slender bar as marked 0.4H, 0.6H, 0.8H (H denotes the width of the rock bar) respectively, and the joint is set in the center of cross section. Fig. 7.4(b) gives the stress waves recorded at point B and point D, which represents the incident wave, reflected wave passing B and transmitted wave passing D. From Fig. 7.4(b), the incident waves are identical and the transmitted waves are also almost the same, according to which the transmission coefficient is about 99%, while that caused by the run-through joint was 78.6%. The effect of the joint length on the reflected and transmitted waves is minor. Most of the incident wave can pass through the non-through crack by diffractions from the two intact sides of the rock bar. When the stress wave arrives at the joint, the stress distribution is altered due to the stiffness

difference between the joint and the intact rock. In order to achieve equilibrium at the cross section, a larger stress is concentrated in the two sides beyond the non-through joint. The larger the joint length, the higher the stress of the two sides is. The average stress of the two intact sides can reach to about 1.6, 1.9 and 2.2 times of the incident peak stress in cases of $0.4H$, $0.6H$ and $0.8H$ of the joint lengths, respectively.

7.2 Numerical Simulation

7.2.1 NMM simulation

A rock mass is a highly defected material in a microscopic view, which significantly affects the mechanical behavior, especially the dynamic behavior of the rock mass. The complicated effects of the multiple irregular cracks on the rock mass dynamic properties can hardly be described analytically. The NMM, on the other hand, has the advantage in modeling statistically and heavily crack material using a consistent mathematical cover system.

Fig. 7.5 NMM model for simulation the highly defected rock mass

A half sinusoidal compression wave is applied onto a slender rock bar, which has the same dimension as in the case 1 example. A series of random micro-cracks

are generated in the rock bar to simulate the highly defected rock, which is illustrated in Fig. 7.5. The whole rock bar is inhomogeneous, in which micro-cracks are randomly distributed with their lengths varying with a certain fraction of the bar width, and with the uniform distribution of their orientations and center positions. In the NMM, the cracks is modeled by simply splitting a mathematical cover into several physical covers and assigning each physical cover an independent local function, which makes the arbitrarily complex cracks be modeled in a consistent manner. The crack stiffness is described by a contact model whose specific normal and tangential stiffness are modeled by normal and tangential springs, respectively (Fig. 7.5). Due to the independence of the mathematical covers to the physical domain, the mesh of the NMM is always regular and is not required to coincide with neither the external boundary nor the internal cracks. If singular physical covers are enriched with the asymptotic crack tip functions, crack tips that partially cut the mathematical covers can be modeled (Ma and An, 2010; An, 2010). In summary, the NMM is suitable to model a highly cracked rock mass in a convenient way.

Different amount of cracks are randomly distributed in the rock bar to simulate the crack quantity effect on the rock dynamic behavior, which is divided into four groups marked Q100, Q200, Q400 and Q600, whose total number of cracks is 100, 200, 400 and 600 respectively. In each crack quantity group, four different crack lengths are simulated; they are 0.2, 0.3, 0.4 and 0.5 times of the width of the rock bar and are marked with 0.2H, 0.3H, 0.4H and 0.5H respectively. Due to the highly discontinuity of the rock bar, the stress caused by the P-wave in the bar is extremely fluctuant and usually is under-represented in a particular area. So, three transverse sections of the bar are selected as measuring sections (0.4 m, 0.8 m, 1.2 m away from the left side respectively) to measure the average stress of the section when the stress wave propagates over it. It should be mentioned that the cracks should not be too close to the measuring section to prevent the stress re-disturbance caused by the cracks.

7.2.2 Viscoelastic behavior of micro-defected rock

According to the one-dimensional harmonic wave propagation theory, the axial motion equation and the constitutive equation of a stress wave propagating across a straight, slender bar with density ρ and dynamic complex modulus $E^*(\omega)$ can be expressed as

$$\frac{\partial^2 \tilde{\varepsilon}(x, \omega)}{\partial x^2} + \frac{\rho \omega^2}{E^*(\omega)} \tilde{\varepsilon}(x, \omega) = 0 \quad (7.4)$$

$$\tilde{\sigma}(x, \omega) = E^*(\omega) \cdot \tilde{\varepsilon}(x, \omega) \quad (7.5)$$

where ω is the angular frequency of the component harmonic wave that can be obtained from wave frequency f by $\omega = 2\pi \cdot f$. $\tilde{\varepsilon}(x, \omega)$ and $\tilde{\sigma}(x, \omega)$ are strain and stress variables in the frequency domain. The rock dynamic complex modulus $E^*(\omega)$ can be expressed as $E^*(\omega) = E'(\omega) + iE''(\omega)$, in which i denotes the imaginary sign. And the real part $E'(\omega)$ of the dynamic complex modulus $E^*(\omega)$ is the dynamic storage modulus which measures the stored energy, representing the elastic portion. While the imaginary part $E''(\omega)$ measures the energy dissipated, representing the viscous portion.

In order to calculate the storage modulus and loss modulus of the micro-cracked rock mass, the propagation coefficient should be introduced, which can be defined as $\gamma(\omega) = \alpha(\omega) + ik(\omega)$, where the coefficient $\alpha(\omega)$ is the attenuation coefficient, and $k(\omega)$ is the wave number. And the propagation coefficient $\gamma(\omega)$ and complex dynamic modulus have the relation as $E^*(\omega) = -\rho\omega^2 / \gamma^2(\omega)$. Comparing the real and imaginary parts of the above equation, the storage modulus and loss modulus can be obtained as follows,

$$E'(\omega) = \frac{\rho\omega^2 (k^2 - \alpha^2)}{(k^2 + \alpha^2)^2} \quad (7.6)$$

$$E''(\omega) = \frac{2\rho\omega^2 k\alpha}{(k^2 + \alpha^2)^2} \quad (7.7)$$

When the incident wave and the attenuated wave are available, the attenuation coefficient $\alpha(\omega)$ and the wave number $k(\omega)$ are obtained from the following equations (Bacon, 1998):

$$\alpha(\omega) = -\text{Re} \left[\ln \left(\frac{\mathbb{F}(|\varepsilon_2|)}{\mathbb{F}(|\varepsilon_1|)} \right) / l \right] \quad (7.8)$$

$$k(\omega) = -\text{Im} \left[\ln \left(\frac{\mathbb{F}(|\varepsilon_2|)}{\mathbb{F}(|\varepsilon_1|)} \right) / l \right] \quad (7.9)$$

where $\mathbb{F}$ denotes the treatment of applying the Fourier transformation, Re and Im denote real and imaginary parts of the complex expression, respectively. ε_1 and ε_2 denote the incident strain pulse and the attenuated pulse, and l denotes the wave traveling distance between these pulses.

7.3 Results and Discussions

Fig. 7.6 shows a typical result from the numerical simulation, from which obvious stress wave attenuation as well as time delay could be observed. This phenomenon reveal that the heavily micro-cracked rock mass has viscoelasticity to the stress wave due to the effect of multiple reflection, diffraction and scattering on the incident wave when there are intrinsic micro-cracks in the rock mass. The viscosity that is caused by these micro-cracks will dissipate the wave energy and slow down the wave velocity as well.

Fig. 7.6 Effects of microcracks on waveforms

($Q=200$, $L=0.5H$, $f=2500\text{Hz}$, $k_n=1\text{GP/m}$)

From a serial numerical results, the storage modulus and loss modulus which are the main indicators of the viscoelasticity of the micro-cracked rock mass can be calculated, and their relations with micro-cracks and incident wave frequency are shown in Fig. 7.7 ~ Fig. 7.10. From these Figures, the common trend can be revealed that the storage modulus increases sharply with the increasing of angular frequency at the initial stage when the angular frequency is relatively small and arrives at a peak value then keeps stable when the frequency keeps increasing. While the loss modulus also rockets to a peak value at the first stage, however, it plunges after it reaching its peak value at the second stage, then goes to a stable value at the third stage when the frequency increases further. Both the storage modulus and loss modulus are frequency dependent.

Fig. 7.7 Effect of length of microcracks on dynamic complex modulus
($Q = 400, f = 2500\text{Hz}, k_n = 1\text{GP/m}$)

Fig. 7.7 shows that the dynamical complex modulus of the micro-cracked rock mass can be affected by the micro-crack length in the rock mass significantly. The storage modulus will reach a much larger stable value as well as a much steeper slope at the initial stage with the decreasing of the crack length. While the shorter the cracks are, the higher peak value the loss modulus reaches at its first stage and lower stable value it holds at the stable stage. And the decay of the loss modulus is much faster when the crack length is relatively short. These indicate that the increase of the crack length will degrade the overall elasticity of the micro-cracked rock mass but enhances the viscosity, which will dissipate much more wave energy and cause much time delay when wave propagates. In fact, the wave time delay relates to the phase lag $\theta(\omega)$ of the component harmonic waves which can be calculated from $\tan \theta(\omega) = E''(\omega) / E'(\omega)$. Theoretically, when the crack length approaches to zero, the viscosity in the rock will decrease gradually and the storage modulus will approach to the Young's modulus E , while the loss modulus at the stable stage will approach to zero.

Fig. 7.8 Effect of quantity of micro-cracks on dynamic complex modulus
($L = 0.4H$, $f = 2500\text{Hz}$, $k_n = 1\text{GP/m}$)

The relation between dynamic complex modulus and the quantity of the cracks as presented in Fig. 7.8 shows the similar tendency as the effects of the length of the cracks on the dynamic complex modulus. The increase of the quantity of the micro-cracks will increase the loss modulus and decrease the storage modulus. When the crack quantity is 100, the storage modulus reaches to about 68 GPa, which is a little smaller than the intact rock Young's modulus (75 GPa). In comparison, when the crack quantity rises to 600, the storage modulus can only approach about 28 GPa. The overall elasticity can be greatly degraded by the presence of the micro-cracks. The loss modulus also increase from about 1 GPa to about 2.5 GPa as the quantity increases from 100 to 600, which shows that although the increase of the absolute value of E'' is smaller than the decrease of the absolute value of E' , the changing ratios of them are similar.

Fig. 7.9 Effect of specific stiffness of micro-cracks on dynamic complex modulus
($L = 0.4H$, $Q = 400$, $f = 2500\text{Hz}$)

Fig. 7.9 shows influence of variation of crack specific stiffness on the viscoelasticity of the micro-cracked rock. When the specific stiffness of the cracks increases, less energy will be reflected. Therefore, the increase of the specific stiffness rehabilitates the elasticity of the micro-cracked rock mass. The changing of storage modulus as well as loss modulus is within a relative small range.

Fig. 7.10 Effect of incident wave frequency on dynamic complex modulus

$$(L = 0.3H, Q = 600, k_d = 1\text{GP/m})$$

Fig. 7.10 illustrates that the material viscoelasticity of the micro-cracked rock mass is independent of the incident wave frequency. Although much wave attenuation is observed as the incident wave frequency increases, the viscoelasticity does not vary according to the incident wave frequency. The dynamic complex modulus only changes with the component harmonic wave frequency. When the incident wave frequency is relatively low, most parts of component harmonic wave are mainly in the low frequency range. While the incident wave frequency is increased, the component harmonic wave of the incident wave extends to a broader range. According to the theory of wave propagation across cracks, the transmission coefficient will decrease as the harmonic wave frequency increases. Therefore, much energy will be dissipated when the stress wave propagates across the micro-cracked rock mass if the incident wave frequency increases. However, the dynamic modulus does not alter with the incident wave frequency. It is the intrinsic property of the heavily micro-cracked rock mass.

7.4 Conclusions

NMM is suitable in solving static and dynamic problems in both continuous and discontinuous materials. Its description of discontinuity is convenient due to the conception of dual cover systems (mathematical and physical covers). And the validated NMM has shown sufficient accuracy in the simulation of stress wave propagation.

The simulation of P-wave propagation across a non-through crack is conducted to investigate the transmission, reflection and diffraction of a stress wave. The diffraction phenomenon is the main contribution to the transmission coefficient as the stress wave passing the non-through joint. The transmission coefficient of the wave propagation through a non-through joint is much larger than that of a run-through crack.

In addition, the present study uses NMM to investigate the dynamic stress wave behavior when P-wave transmits highly micro-cracked rock mass. It shows that the highly micro-cracked rock mass has remarkable viscoelastic property to the stress wave propagation, which can be verified by the attenuation of the wave amplitude and wave time delay when it propagates through the defected rock mass with multiple uniformly randomly distributed micro-cracks.

The dynamic complex modulus which indicates the viscoelasticity is significantly affected by the crack length, quantity, and stiffness. However, it is independent of the incident wave frequency. When either the length or quantity of the micro cracks increases, they remarkably degrades the overall elasticity of the micro-cracked rock mass; meanwhile, enhances its viscosity. That is because the increasing discontinuity will make the diffraction much more difficult. Thus, when the length and the quantity of cracks increase, the viscosity will be intensified due to the multiple reflection and dispersion. The decrease of the crack stiffness also intensifies the discontinuity of the rock mass, which makes viscosity more significant. Nevertheless the incident wave frequency has no influence on viscosity of the micro-cracked rock mass, which is the intrinsic property of a material, although the incident wave could be attenuated more greatly when the incident

wave frequency increases. In conclusion, the micro-cracks in the elastic rock mass will cause viscoelasticity of the overall micro-cracked rock mass, which would significantly attenuate the stress wave and make a travel time delay when the stress wave propagates.

CHAPTER 8 VISCOELASTIC BEHAVIOR OF ROCK MASS DUE TO DOUBLE-SCALE DISCONTINUITIES

8.1 Introduction

This chapter investigates the global viscoelastic mechanical properties of a rock mass containing micro-defects and macro-joints. Experiments are carried out to reveal the double-scale discontinuities in the sedimentary rock mass. Theoretical solutions of stress wave propagation coefficients for the rock mass containing double-scale discontinuities are derived, and the analytical expressions of global viscoelastic moduli are obtained. The typical two-phase viscoelastic phenomenon can be observed when the rock mass is under dynamic loading. The energy dissipation for the stress wave propagation is also discussed.

8.1.1 Background

The global mechanical behavior of rock mass is difficult to describe or model when both macro-joints and micro-defects are considered. The complexity comes mainly from two difficulties. First, there is problem to evaluate the effect of micro-defects and macro-joint on the global dynamic mechanical response quantitatively, suitable models are required for the micro-defects and macro-joints under dynamic loading. Second, the mechanism of micro-defects and macro-joints under dynamic loading are different, it is difficult to combine them together. It is essentially necessary to conduct a uniform approach which can be used to describe the global mechanical behavior of defected rock mass. Equation Chapter (Next) Section 1.

8.1.2 Observation of double-scale discontinuities

Natural sedimentary rock samples collected from the Jurong Island, Singapore are used in the present study. The density of the sedimentary rock was measured to be 2680.98 kg/m³.

Fig. 8.1 shows a typical macro-discontinuity set in the sedimentary rock mass. A 21.4 mm diameter coin is also shown for the scale reference. It can be observed that macro-joints are nearly planar and parallel located. The extent of the joint is large compared to its thickness, so these visible macro-joints are described as physical discontinuities. The macro-joints are unfilled-joints and firmly contacted in a dry condition, therefore a linear deformational model is employed according to Chapter 3 (Zhao & Cai 2001).

Fig. 8.1 Illustration of macro-joints

Micro-defects are inherently present in rocks, which are too small to be visible by naked eyes. The presence of these micro-defects will cause stress wave diffractions, which result in attenuation and dispersion of stress waves, and make the mechanical behavior of the rock block viscoelastic (Jaeger et al. 2007). In the present study, the scanning electronic microscopy (SEM) technology is used to observe the microscopic mechanical characteristics of the micro-defected sedimentary rock.

The samples used in SEM tests were cut from the same sedimentary rock bar as in Fig. 8.1. Fig. 8.2 shows the SEM images of the sedimentary rock.

Fig. 8.2 Illustration of micro-defects

Fig. 8.2(a) shows the microstructure of sedimentary rock with a magnification power of 1500 times. A defect band can be observed as the inner area bounded by the white lines in Fig. 8.2(a). A close-up view of the defect band at a magnification power of 6500 times is shown in Fig. 8.2(b). It can be found in Fig. 8.2(b) that an individual micro-crack has a high length-to-width ratio. Rough surfaces are also observed in the defect band. When the sedimentary rock is under transient conditions, such as stress wave propagation, the existence of these defects will largely increase the internal friction, which causes the kinetic energy of the stress waves to be transformed into internal energy. This energy is not lost, but rather serves to raise the temperature of the rock slightly. From a purely mechanical point of view, this energy appears to be dissipated. This may account for the global viscoelastic mechanical behavior of rock (Jaeger et al. 2007).

8.2 A Homogenization-discontinuity Combination Approach

Fig. 8.3 shows a scheme of the homogenization-discontinuity combined approach. A representative elementary volume (REV) of a rock mass containing double-scale discontinuities is shown in Fig. 8.3(a) (Li et al. 2010). According to the geometrical characteristics described in Section 8.1.2, the micro-defects in the rock block are uniformly distributed while the macro-joints in the rock mass are assumed to be parallel separated at a equal joint spacing S . Therefore, the REV of the homogenization-discontinuity combined model can be defined as shown in

Fig.8.3(b). The micro-defected rock is modeled as an equivalent viscoelastic medium, while the macro-joint is modeled explicitly as a physical discontinuity. The REV of the present model has the same length S as the REV of the rock mass.

(a) Double-scale discontinuities

(b) Homogenization-discontinuity combined model

Fig. 8.3 Representative elementary volume of rock mass

with double-scale discontinuities

Using the trial solutions that satisfy the wave propagation equations in the form of a harmonic wave as

$$\varepsilon = \varepsilon_0 \exp(i\omega t) \exp(-ikx - \alpha x) = \varepsilon_0 \exp(i\omega t) \exp(-\gamma x) \quad (8.1)$$

where t and x denote time and distance along the wave propagation path, respectively. i is the imaginary sign, ε_0 is amplitude of strain wave ε at the boundary $x = 0$. The propagation coefficient γ can be obtained from wave number k and attenuation coefficient α as $\gamma = \alpha + ik$ (Kolsky, 1963).

Using the linear deformation model of macro-joint, the transmitted wave of the harmonic wave ε_{tran} given in Eq. (8.1) propagating through the representative elementary volume can be obtained as (Fan et al. 2011)

$$\begin{aligned}\varepsilon_{tran} &= \frac{2}{(2 + \gamma_d E_d^*/k_n)} \varepsilon_0 \exp(i\omega t) \exp(-\gamma_d S) \\ &= \frac{2}{(2 + \gamma_d E_d^*/k_n)} \varepsilon_0 \exp(i\omega t) \exp(-ik_d S - \alpha_d S)\end{aligned}\quad (8.2)$$

where, k_n is the normal stiffness of the macro-joint, E^* is the dynamic viscoelastic modulus and S is the length of the representative elementary volume. The subscript d denotes the parameters of micro-defected rock. Eq. (8.2) is simplified to the transmitted wave when the micro-defects are ignored (Pyrak-Nolte et al. 1990b; Cook 1992).

Defining $A = \ln \left[2 / (2 + \gamma_d E_d^*/k_n) \right]$, Eq. (8.2) can be rewritten as

$$\begin{aligned}\varepsilon_{tran} &= \varepsilon_0 \exp(i\omega t) \exp \left[i(\text{Im}(A) - k_d S) + (\text{Re}(A) - \alpha_d S) \right] \\ &= \varepsilon_0 \exp(i\omega t) \exp(-ik_g S - \alpha_g S)\end{aligned}\quad (8.3)$$

where Re and Im denote the real- and imaginary- part of the complex expression of A , respectively. The subscript g denotes the parameters of the global behavior of the rock mass.

Therefore, the global wave attenuation coefficient α_g and the global wave number k_g are respectively derived to be,

$$\alpha_g = \frac{\alpha_d S - \text{Re}(A)}{S} \quad (8.4)$$

and

$$k_g = \frac{k_d S - \text{Im}(A)}{S} \quad (8.5)$$

It is noticed that the global wave number k_g is a monotonic continuous function of the frequency. However, it is observed from Eq. (8.5) that the imaginary part of the complex expression is generally calculated in the range from 0 to 2π , which makes the function discontinuous to the frequency during numerical calculation. This error can be corrected by adding multiples of $\pm 2\pi$ when absolute jumps occur (Bacon 1998).

Considering the propagation of a harmonic wave in the straight, cylindrical and slender rock bar containing double-scale discontinuities, the one-dimensional axial motion equation can be expressed as

$$\frac{\partial^2 \sigma_g(t)}{\partial x^2} = \rho \frac{\partial^2 \varepsilon_g(t)}{\partial t^2} \quad (8.6)$$

where σ_g is the global stress, $\varepsilon_g(t)$ is the global strain and ρ is the density of the bar.

To investigate the mechanical behavior of rock in the frequency domain, Fourier transformation is applied to Eq. (8.6) which yields

$$\frac{\partial^2 \sigma_g(\omega)}{\partial x^2} + \rho \omega^2 \varepsilon_g(\omega) = 0 \quad (8.7)$$

where $\sigma_g(\omega)$ and $\varepsilon_g(\omega)$ denote the global stress σ_g and global strain ε_g after Fourier transformation, respectively.

The constitutive equation of rock in the frequency domain can be expressed as

$$\sigma_g(\omega) = E_g^*(\omega) \varepsilon_g(\omega) \quad (8.8)$$

where $E_g^*(\omega)$ is the dynamic complex viscoelastic modulus, which is expressed as

$$E_g^*(\omega) = E_g'(\omega) + iE_g''(\omega) \quad (8.9)$$

where $E_g'(\omega)$ denotes the global storage modulus, $E_g''(\omega)$ denotes the global loss modulus of the rock containing double-scale discontinuities. It can be found from Eqs. (8.7) and (8.8) that the complex viscoelastic modulus is frequency-dependent.

Together with Eq. (8.8), the motion equation Eq. (8.7) can be expressed as

$$\frac{\partial^2 \varepsilon_g(\omega)}{\partial x^2} + \frac{\rho \omega^2}{E_g^*} \varepsilon_g(\omega) = 0 \quad (8.10)$$

Note that Eq. (10) is an ordinary differential equation with respect to x , which has the solution of

$$\varepsilon_g(\omega) = P(\omega) \exp\left(+\sqrt{-\rho \omega^2 / E_g^*} x\right) + N(\omega) \exp\left(-\sqrt{-\rho \omega^2 / E_g^*} x\right) \quad (8.11)$$

where $P(\omega)$ and $N(\omega)$ can be determined by the boundary values of $\varepsilon_g(\omega)$. The signs $+$ and $-$ in exponential functions of Eq. (8.11) can be used to describe the

right-running and left-running waves, respectively. Without loss of generality, for the case that an incident wave is applied at one end, the solution can be expressed as

$$\varepsilon_g(\omega) = P(\omega) \exp\left(+\sqrt{-\rho\omega^2/E_g^*}x\right) \quad (8.12)$$

Comparing Eqs. (8.1) and (8.12), the relation between the global dynamic complex modulus and global propagation coefficient is obtained as

$$(-\alpha_g - ik_g)^2 = -\frac{\rho\omega^2}{E_g^*} \quad (8.13)$$

From the equilibria of the real-part and imaginary-part of Eq. (8.13), considering eqs (8.4) and (8.5), the global dynamic viscoelastic moduli of rock mass are obtained as

$$E'_g(\omega) = \frac{\rho\omega^2(k_g^2 - a_g^2)}{(k_g^2 + a_g^2)^2} \quad (8.14)$$

and

$$E''_g(\omega) = \frac{2\rho\omega^2 k_g a_g}{(k_g^2 + a_g^2)^2} \quad (8.15)$$

where E'_g and E''_g denote the global storage modulus and global loss modulus, respectively.

8.3 Results and Discussions

In this Chapter, the macro-joint is assumed to be planar, large in extent, small in thickness as compared to the wave-length as shown in Fig. 8.3. The viscoelastic behavior of micro-defected rock is referenced to the experimental research performed by Fan et al. (2011).

8.2.1 Global attenuation coefficient

Fig. 8.4 plots the frequency spectrum of global attenuation coefficient for different macro-joint stiffness of $k_n = 1.25, 2.5, 5.0, 7.5$ and 12.5 GPa/m.

Fig. 8.4 Frequency spectrum of global attenuation coefficient

It can be obtained from Fig. 8.4 that wave attenuation occurs when the stress wave propagates through a rock mass containing double-scale discontinuities. The attenuation coefficient increases as the component wave frequency increases, while the extent of increase becomes smaller as the component wave frequency increases. The results reveal that the rock mass has a larger scattering effect on the harmonic waves with a larger frequency than those with a smaller frequency. Fig. 8.4 also shows that for the same micro-defected rock, the wave propagation through a rock mass with a larger macro-joint stiffness will attenuate less than that through a rock mass with a smaller macro-joint stiffness, which is consistent with the geological investigations (Pyrak-Nolte 1990b). This phenomenon can be explained in the complex material mechanics that wave scattering occurs when a stress wave propagates across the interface of two different materials. A more significant difference between the properties of the two materials will cause a larger wave attenuation.

8.3.2 Effective wave velocity

The wave number of stress wave propagation through rock mass contains double-scale discontinuities can be obtained from Eq. (8.5). Therefore, the effective phase velocity for the harmonic component wave can be obtained as

$$v(\omega) = \frac{\omega}{k(\omega)} \quad (8.16)$$

Fig. 8.5 shows the relationship between the phase velocity and component wave frequency.

Fig. 8.5 Frequency spectrum of effective velocity

The results reveal that there is a range of low frequencies for which the effective phase velocity is a minimum and changes only slightly with the component wave frequency. When the component wave frequency is large enough, such as in the 10⁴ Hz order, the phase velocity approaches the wave velocity of the micro-defected rock, which is 5,030 m/s. Between the two extremes, there is a rapid increase in effective phase velocity with the increase of the component wave

frequency. For the harmonic component wave with the same frequency, the effective phase velocity increases as the stiffness of the macro-joint increases. However, the difference decreases as the frequency increases.

8.3.3 Global complex viscoelastic moduli

Based on the obtained global attenuation coefficient and effective velocities, the storage modulus and the loss modulus for the global viscoelastic properties of rock mass containing double-scale discontinuities can be obtained by Eqs. (8.14) and (8.15), respectively.

Fig. 8.6 and Fig. 8.7 show the relationships of the global storage and loss modulus versus the harmonic component wave frequency, respectively.

Fig. 8.6 Frequency spectrum of global storage modulus

Fig. 8.7 Frequency spectrum of global loss modulus

It is seen that the rock mass containing double-scale discontinuities exhibits similar typical viscoelastic behavior as two phase polymer materials (Yin et al., 2008). Different sensitive ranges of the harmonic component wave frequency to the global complex viscoelastic moduli can be found for the micro-defects and macro-defects, as shown in Fig. 8.6 and Fig 8.7.

8.3.3.1 Global storage modulus

The variations of the global storage modulus $E'(\omega)$ with respect to different macro-joint stiffness of $k_n = 1.25, 2.5, 5.0, 7.5$ and 12.5 GPa/m are compared as shown in Fig. 8.6. It is seen that the global storage modulus increases sharply with the increase of the harmonic component wave frequency at the initial stage when the harmonic wave frequency is relatively small. It arrives at the first peak and then remains stable before it increases again at a higher frequency. This is different from the viscoelastic behavior of a rock mass containing micro-defects only, in which the second rising slope does not exist when the frequency further increases (Fan et al.

2011; Ren et al. 2011). The global storage modulus with double-scale discontinuities increases again sharply and then reaches the second peak. Fig. 8.6 also shows that the rock mass containing macro-joints with a higher stiffness results in a higher storage modulus $E'(\omega)$.

8.3.3.2 Global loss modulus

Fig. 8.7 compares the frequency spectrums of global loss modulus of rock mass respect to different macro-joint stiffness of $k_n = 1.25, 2.5, 5.0, 7.5$ and 12.5 GPa/m. It is observed from Fig. 7 that two peak values of global loss modulus are obtained from its frequency spectrum. The first and second peak values of $E''(\omega)$ of rock mass are located at approximately 3 Hz and 1,300 Hz, respectively, which correspond to the fastest increasing ranges of the global storage modulus $E'(\omega)$. Besides, the first peak value of $E''(\omega)$ increases with the increase of the macro-joint stiffness, while the second peak value of $E''(\omega)$ decreases with the increase of the macro-joint stiffness. The phenomenon indicates that the behavior of the rock mass containing double-scale discontinuities approaches that of the rock block containing micro-defects only when the stiffness of the macro-joint is large enough, in which the peak value of $E''(\omega)$ occurs in the initial frequency range spanning from 10^0 Hz to about 10^1 Hz (Fan et al. 2011; Ren et al. 2011). If the stiffness of the macro-joint is small enough, the viscoelastic behavior of the rock mass containing double-scale discontinuities is close to that of a rock mass with linear elastic rock joints, for which the peak value of the loss modulus occurs in the frequency range from 10^2 Hz to 10^4 Hz (Pyrak-Nolte 1990b; Zhao et al. 2006). The rock mass containing double-scale discontinuities behaves between these two extremes, just like a typical two-phase viscoelastic material does (Yin et al. 2008).

8.3.4 Average energy dissipation rate

A dynamic strain loading in the form of $\varepsilon(t) = \varepsilon_0 \exp(i\omega t)$ was applied on the present rock mass, the stress response therefore can be obtained as

$$\sigma(t) = E^*(\omega)\varepsilon(t) = E'(\omega)\varepsilon(t) + \frac{E''(\omega)}{\omega}\dot{\varepsilon}(t) \quad (8.17)$$

where $\dot{\varepsilon}(t)$ denotes the strain rate. It can be observed from Eq. (8.17) that the dynamic complex stress contains two parts, the elastic part $E'_g(\omega)\varepsilon(t)$ and the viscous part $\dot{\varepsilon}(t)E''_g(\omega)/\omega$. The elastic part is related to the storage modulus and the amplitude of the strain signal, while the viscous part is related to the loss modulus, strain rate and component wave frequency.

The work generated by the strain wave in a periodic time can be calculated by

$$\begin{aligned} W &= \oint dw = \oint \sigma(t) d\varepsilon(t) \\ &= \oint E' \varepsilon(t) d\varepsilon(t) + \oint \frac{E''}{\omega} \dot{\varepsilon}(t) d\varepsilon(t) \\ &= \frac{1}{2} E' \varepsilon^2(t) \Big|_0^T + \int_0^T \frac{E''}{\omega} \dot{\varepsilon}^2(t) dt \end{aligned} \quad (8.18)$$

where $T = 2\pi/\omega$ denotes the periodic time. Eq. (8.18) denotes the work generated by every periodic strain loading per unit volume. It can be observed that the first part is the reversible elastic potential energy, and the second part relating to $E''_g(\omega)$ denotes the energy dissipation by the material viscosity.

Therefore, for the strain in the form of $\varepsilon(t) = \varepsilon_0 \cos(\omega t)$, the corresponding energy dissipation is obtained as

$$W_d = \frac{E''}{\omega} \int_0^{2\pi/\omega} \omega^2 \varepsilon_0^2 \sin^2(\omega t) dt = \pi \varepsilon_0^2 E''(\omega) \quad (8.19)$$

where W_d denotes the energy dissipation in one circle per unit volume.

The average energy dissipation in one circle per unit volume yields

$$D = \frac{W_d}{T} = \frac{1}{2} \varepsilon_0^2 \omega E''(\omega) \quad (8.20)$$

Fig. 8.8 illustrates the relationship between the frequency of component harmonic wave and the average energy dissipation rate, the amplitude of the strain was assumed to be $\varepsilon_0 = 0.02\%$.

Fig. 8.8 Relationship of average energy dissipation rate and dynamic loading frequency

It can be observed that the average energy dissipation increases as the frequency increases. In addition, the increase of the joint stiffness can effectively decrease the energy dissipation. For frequency $\omega=0$, the energy dissipation vanishes. In such a condition, the rock mass containing double-scale discontinuities behaves as an elastic solid. No attenuation occurs when a stress wave propagates through it, as can be verified by Fig. 8.4 (relation of $\alpha \sim \omega$).

8.4 Conclusions

The global mechanical behavior of rock mass containing double-scale discontinuities is investigated. A homogenization-discontinuity combined approach is proposed to investigate the global viscoelastic behavior of rock mass.

The SEM examination shows that micro-defects inherently exist in the rock blocks, which increase the internal friction and cause energy dissipation when a stress wave propagates through the rock mass. The results obtained by the homogenization-discontinuity combined approach show that the global mechanical behavior of the rock mass containing double-scale discontinuities is also viscoelastic. However, there are two frequency sensitive ranges with respect to the micro-defects and macro-joints. The rock mass behaves as a two-phase viscoelastic material, with two transition frequency ranges for the global storage modulus and two peak values for the global loss modulus.

The energy analysis shows that the rock mass containing double-scale discontinuities absorbs mechanical energy remarkably under dynamic loading. Increase of the joint stiffness can effectively reduce the energy dissipation.

CHAPTER 9 EFFECTS OF DOUBLE-SCALE DISCONTINUITIES ON WAVE PROPAGATION

9.1 Introduction

A rock mass is composed of a system of rock blocks separated by the discontinuities in different forms, geometries, and sizes. The dynamic properties of a rock mass are always dominated by mechanical and geometrical properties of the discontinuities characteristics. The discontinuities (macro-joints and micro-defects) in a rock mass as shown in Fig. 9.1(a).

(a) Wave propagation through jointed rock mass

(b) Conventional displacement discontinuity method (CDDM) for elastic rock mass (ERM)

(c) Extended displacement discontinuity method (EDDM) for viscoelastic rock mass (VRM)

Fig. 9.1 Displacement discontinuity methods for wave propagation through a rock mass

Investigating the mechanical effect of discontinuities on stress wave propagation is important in solving problems of seismology, mining, dynamic stability of rock slopes and tunnels, etc. However, the conventional displacement discontinuity method can only be used to consider the wave propagation through rock mass with macro-joint. Therefore, an extended displacement discontinuity method can be used to consider the effects of macro-joint and micro-defect on the wave propagation through rock mass is necessary.

The present chapter extends the displacement discontinuity method (EDDM) to analyze the effects of macro-joints on stress wave propagation through a viscoelastic rock mass, the transmission and reflection coefficients based on viscoelastic rock mass (VRM) model using the present EDDM are analytically derived. An equivalent viscoelastic medium model is then introduced to investigate the effects of micro-defects on the wave propagation as shown in Fig. 9.1(c). The propagation coefficients (attenuation coefficient and wave number) and the dynamic viscoelastic moduli (storage modulus and loss modulus) of the equivalent viscoelastic medium were obtained by longitudinally impacting a cored long defected sedimentary rock bar using a pendulum. The transmitted waveforms for the same identical incident wave using present EDDM based on VRM model and CDDM based on elastic rock mass (ERM) model were compared. Discussions of the rock viscosity influence on the stress wave dispersion, transmission and reflection are also carried out.

9.2 Theoretical Formulation

9.2.1 Extended displacement discontinuity method for viscoelastic rock mass

Among many methods to study the effects of joints on stress wave propagation across a rock joint, displacement discontinuity method (DDM) has been widely adopted. DDM is valid when the joints are planar, large in extent and small in thickness related to the wavelength. Thus, the stresses across a rock joint are

assumed continuous, but the particle displacements across the joint are discontinuous.

Considering one-dimensional stress wave propagation, when the magnitude of stress wave is insufficient to mobilize the nonlinear deformation of the joint, the linear model can be used to describe the normal deformational behavior of the joint, i.e.,

$$\Delta u = \sigma / k_n \quad \Delta u = \sigma / k_n \quad (9.1)$$

where Δu is the closure of the joint, σ is the normal stress and k_n is the normal stiffness of the rock joint.

Assuming the two opposite sides of the joint are viscoelastic, homogeneous and isotropic, the DDM can be expressed as boundary conditions in one-dimensional longitudinal wave propagation equations as

$$\begin{cases} u_I - u_{II} = \Delta u \\ \sigma_I = \sigma_{II} \end{cases} \quad (9.2)$$

where u is particle displacement with subscripts I and II referring to the two opposite sides of the joint as shown in Fig. 9.2, respectively.

The first term of Eq. (9.2) indicates that the difference of the displacement between the two sides of the rock joint equals the closure (opening) and the second term of Eq. (9.2) requires that the stresses across the joint are continuous.

Using the trial solutions that satisfied the wave equations in the forms of harmonic wave (Li et al. 2009), the strain wave has the form of

$$\varepsilon = \varepsilon_0 \exp(i\omega t) \exp(-ikx - \alpha x) \quad (9.3)$$

where ε denotes the one-dimensional harmonic particle strain in the x -direction with the amplitude of ε_0 , t is the time. k is the wave number and α denotes the attenuation coefficient; i denotes the imaginary sign and ω is the angular

frequency, which can be obtained from frequency f by $\omega = 2\pi f$. The strain waves in the incident, reflected and transmitted waves are shown in Fig. 9.2, respectively.

For the wave propagation in a viscoelastic medium, the dynamic constitutive relation can be obtained as

$$\sigma(\omega) = E_{(I, II)}^* \varepsilon(\omega) \quad (9.4)$$

where $E_{(I, II)}^*$ denotes the dynamic viscoelastic modulus and it can be expressed as $E^*(\omega) = E'(\omega) + iE''(\omega)$, $E'(\omega)$ is the storage modulus and $E''(\omega)$ is the loss modulus.

Defining $\gamma^2(\omega) = -\rho\omega^2 / E^*$, in which γ is a wave propagation coefficient, the solution for longitudinal stress wave propagation in a one-dimensional bar can be solved, and the transmitted and reflected waves for a P-wave at a normal incidence are given by

$$\left. \begin{aligned} \varepsilon_{\text{tran}} &= 2 / (2 + \gamma E^* / k_n) \varepsilon_{\text{inc}} \\ \varepsilon_{\text{ref}} &= \gamma E^* / [k(2 + \gamma E^* / k_n)] \varepsilon_{\text{inc}} \end{aligned} \right\} \quad (9.5)$$

where subscripts “inc”, “ref”, and “tran” denote the incident wave, reflected wave and transmitted wave, respectively, as shown in Fig. 9.2.

Fig. 9.2 Scheme of extended displacement discontinuity method

It is noticed that for a viscoelastic medium, the material parameter E^* and wave propagation coefficient γ are both frequency-dependent. And for the conditions when the viscosity of the rock material is ignored, the loss modulus becomes zero. In such cases, the transmitted and reflected waves can be degenerated to the solutions for the wave propagation in elastic rock mass (Pyrak-Nolte, 1988; Pyrak-Nolte et al. 1990b). Similar analytical process can be used to obtain the transmission and reflection coefficients for a shear wave.

9.2.2 Viscoelastic behaviors of micro-defected rocks

A series of impact tests were carried out to investigate the viscoelastic property of a micro-defected sedimentary rock. The experimental set-up is shown in Chapter 6. A sedimentary rock bar with a length of 129.80 cm and a diameter of 4.49 cm was used. The density of the sedimentary rock bar was measured to be 2 680.98 kg/m³. The integrity and homogeneity of the sedimentary rock bar were carefully examined. The end surface of the bar was ground by a grinding machine before the experiment to make sure a free stress condition at the free end. A pendulum steel hammer was used to generate a longitudinal pulse. The intensity of the pulse load can be adjusted by changing the swinging-angle of the hammer. A pair of

diametrically opposite strain gauges located at the middle of the sedimentary rock bar as shown in Fig. 6.1 was used to measure the longitudinal strain. The history of the strain pulses in the rock bar was recorded at a time rate of $1.00 \times 10^7 \text{ s}^{-1}$ (time resolution of $1.00 \times 10^7 \text{ s}$), which gives sufficient data points with sufficient accuracy to carry out the discrete Fourier transformation (DFT).

Considering the propagation of a harmonic wave in the straight, cylindrical and slender bar with density ρ and complex modulus $E^*(\omega)$, the one-dimensional axial motion equation of the bar in the frequency domain can then be expressed as

$$\frac{\partial^2 \tilde{\varepsilon}(\omega)}{\partial x^2} + \frac{\rho \omega^2}{E^*} \tilde{\varepsilon}(\omega) = 0 \quad (9.6)$$

where $\tilde{\varepsilon}(\omega)$ denotes the strain in the frequency domain, $E^*(\omega)$ is the complex dynamic modulus as in Eq. (9.4) and ρ is density of the sedimentary rock.

Eq. (9.6) can be solved with a general solution of strain as,

$$\tilde{\varepsilon}(x, \omega) = \tilde{P}(\omega) e^{-\gamma(\omega)x} + \tilde{N}(\omega) e^{\gamma(\omega)x} \quad (9.7)$$

where $\tilde{P}(\omega)$ and $\tilde{N}(\omega)$ are the amplitudes of harmonic component wave traveling in the directions of increasing and decreasing x , respectively. And $\tilde{P}(\omega)$ and $\tilde{N}(\omega)$ can be determined by initial and boundary conditions. Since the stress on the free end surface of the rock is zero, it can be obtained that $\tilde{P}(\omega) = -\tilde{N}(\omega)$ on the free end of the rock bar (Lundberg and Blanc, 1988) $\gamma(\omega)$ is expressed as

$$\gamma(\omega) = ik(\omega) + \alpha(\omega) \quad (9.8)$$

where $\alpha(\omega)$ is the attenuation coefficient and $k(\omega)$ denotes wave number as given in Eq. (9.3), respectively. The relations of $\alpha(\omega)$ and $k(\omega)$ versus ω are obtained as shown in Chapter 6, respectively.

The dynamic complex modulus of the rock is then calculated, and the storage modulus $E'(\omega)$ and loss modulus $E''(\omega)$ are determined by Bacon et al (Bacon,

1998; Benatar, 2003; Lundberg and Blanc, 1988; Sogabe, 1982), which can be written as follows:

$$E'(\omega) = \rho\omega^2(k^2 - a^2) / (k^2 + a^2)^2 \quad (9.9)$$

and

$$E''(\omega) = 2\rho\omega^2 ka / (k^2 + a^2)^2 \quad (9.10)$$

The relations of the storage modulus and loss modulus against the component wave frequency were obtained by Chapter 6. It can be obtained from Chapter 6 that the mechanical properties of the sedimentary rock are highly frequency-dependent. The magnitude comparison of storage modulus and loss modulus shows the viscosity of the sedimentary rock cannot be ignored during the impact analysis. In the high frequency range, the storage modulus and loss modulus both become constant.

9.3 Results and Discussions

9.3.1 Numerical examples

Using the discrete Fourier transformation (DFT), any arbitrary incident waveform can be expressed as the sum of periodical harmonic component wave in the form of Eq. (9.7). The experimentally determined moduli are applied to calculating the stress wave propagation in the viscoelastic medium, and the EDDM will then be employed when a set of macro-joints are included in the viscoelastic medium.

In the following study, assuming a half-cycle sinusoidal wave is applied as the incident wave in the form:

$$\varepsilon_{\text{inc}} = \begin{cases} I_0 \sin(\omega_0 t) & (0 \leq t \leq \pi/\omega_0) \\ 0 & (t > \pi/\omega_0) \end{cases} \quad (9.11)$$

where I_0 is the amplitude of the incident wave, ω_0 is the angular frequency of the incident wave and $\omega_0 = 2\pi f_0$, where f_0 is the incident wave frequency. It is assumed that the defected rock material is with the wave propagation coefficients as shown in Fig. 6.5 and the dynamic viscoelastic moduli as shown in Fig. 6.7. The joints are assumed to be equally spaced and with the stiffness of $k_n = 12.5$ GPa/m.

The transmitted waves and reflected waves based on the present VRM using EDDM and those based on the conventional ERM using CEEM (Lundberg and Blanc, 1988) are compared in the Figs. 9.3 and 9.4 for different joint numbers and different joint spacings, respectively. To clearly evaluate the effects of micro-defects and macro-joints on the stress wave propagation, the transmitted wave across multi-joint can be divided into two components. The first one is the directly transmitted wave and the second one is the catching-up wave caused by multi-wave reflections among joints. And the transmission coefficient in the present study is defined as the ratio of directly transmitted wave amplitude to the incident wave amplitude.

Transmitted wave Reflected wave
(a) One joint (transmitted and reflected waves)

Transmitted wave Reflected wave
(b) Two joints (transmitted and reflected waves)

(c) Three joints (transmitted and Reflected waves)

(d) Four joints (transmitted and reflected waves)

Fig. 9.3 Comparisons of transmitted waves and reflected waves by using ERM and VRM for different joint numbers ($S = 1/20 \lambda_0$)

In Fig. 9.3, the joint spacing $S = 1/20 \lambda_0$, where λ_0 is the wavelength of the incident wave, while the joint number varies from one to four to investigate the effect of joint number and the distance of wave propagation on the transmitted waves. It can be observed from Fig. 9.3 that the viscoelastic properties of defected rock material have distinct effects on the wave attenuation, and the difference of the transmitted waves and reflected waves based on ERM and VRM will increase as the joint number increases for a rock mass with a fixed joint spacing.

Table 9.1 shows the transmission coefficients for the directly transmitted waves obtained based on the present VRM and conventional ERM. The difference of the transmission coefficient is defined as

$$\Delta_T = |T_{\text{ERM}} - T_{\text{VRM}}| / T_{\text{VRM}} \quad (9.12)$$

Table 9.1 Transmission coefficients for different joint numbers ($S = 1/20 \lambda_0$)

Joint number(N)	1	2	3	4
T_{ERM}	0.95	0.90	0.85	0.81
T_{VRM}	0.91	0.82	0.75	0.68
Δ_T (%)	4.40	9.76	13.33	19.12

It can be obtained from Table 9.1 that the wave propagation in the viscoelastic rock mass attenuates faster than that based on the conventional ERM, and the difference increases as the joint number increases, which arises to 19.12% when stress wave propagates through a rock mass containing four joints with the joint spacing of $S = 1/20 \lambda_0$.

Fig. 9.4 Comparisons of transmitted waves and reflected waves by using ERM and VRM for different joint spacings ($N = 2$)

Fig. 9.4 shows the effect of joint spacing on the transmitted waves and reflected waves. Two joints are analyzed, and the joint spacings are $S = 1/200\lambda_0$, $1/50\lambda_0$, $1/10\lambda_0$ and $1/5\lambda_0$, respectively. It can be obtained from Fig. 9.4 that the difference of transmitted waves and reflected waves increases as the joint spacing increases for a rock mass with a fixed joint number.

Table 9.2 shows the comparison of the transmission coefficients of directly transmitted waves obtained based on the VRM and ERM for different joint spacings with a given joint number of $N = 2$. The difference of the transmission coefficients are the same as that defined in Eq. (9.12).

Table 9.2 Transmission coefficients for different joint spacings ($N = 2$)

Joint spacing (S)	$0.005\lambda_0$	$0.02\lambda_0$	$0.05\lambda_0$	$0.08\lambda_0$	$0.10\lambda_0$	$0.20\lambda_0$
T_{ERM}	0.90	0.90	0.90	0.90	0.90	0.90
T_{VRM}	0.89	0.87	0.82	0.78	0.75	0.63
Δ_T (%)	1.12	3.45	9.76	15.38	20.00	42.86

From Table 9.2, the wave propagation distance has a significant effect on wave attenuation. When the joint spacing is small enough, the transmission coefficients obtained using VRM can be replaced by those obtained using ERM for an approximate investigation, such as $\Delta_T = 1.12\%$ when $S = 1/200\lambda_0$. However, the difference increases as the joint spacing increases. When the wave propagation through a rock mass with two joints and a joint spacing of $S = 1/5\lambda_0$, Δ_T can be as large as 42.86%. In that case, the conventional ERM is no longer valid.

9.3.2 Discussions

9.3.2.1 Effects of rock viscosity on the transmitted waveform

Assuming the stress wave in the form of Eq. (9.11) is applied on the boundary of the rock mass with joint spacings of $S = 1/100\lambda_0$ and $S = 15/100\lambda_0$, respectively, the transmitted waves through a rock mass containing one joint based on the present VRM, conventional ERM, and the transmitted wave across an integrated rock with micro-defects are denoted as “VRM”, “ERM” and “VR” respectively, as shown in Fig. 9.5. “ERM” shows the effect of macro-joints on stress wave propagation and “VR” gives the effect of micro-defects. “VRM”, on the other hand, combines both effects.

(a) Transmitted wave ($S = 1/100 \lambda_0$, $N = 1$) (b) Transmitted wave ($S = 15/100 \lambda_0$, $N = 1$)

Fig. 9.5 Effects of rock viscosity on transmitted waveforms

From the comparison of the peak magnitudes of the transmitted waves, it can be observed that when the joint spacing is small, such as $S = 1/100 \lambda_0$, the attenuation of transmitted wave is mainly caused by the macro-joints. However, when the joint spacing is sufficiently large, such as $S = 15/100 \lambda_0$, the attenuation caused by the rock viscoelastic properties or micro-defects is larger than that caused by the macro-joint.

The effective propagation time for the stress wave is defined as the difference between the two time spots corresponding to the peaks of the incident and transmitted waves. Therefore, it can be obtained from Fig. 9.5 that the existence of the joint will cause a distinct time delay when a stress wave passes through it. Both the present VRM and conventional ERM can be used to consider the time delay caused by the rock joint.

9.3.2.2 Effects of rock viscosity on the transmission coefficient

When an incident wave in the form of Eq. (9.11) propagating through a rock mass contains one to four joints, the relations between the transmission coefficient and joint spacing based on the present VRM and conventional ERM is shown in Fig. 9.6.

Fig. 9.6 Transmission coefficients for wave propagation through a rock mass with different joint spacings by using ERM and VRM

It is seen from Fig. 9.6 that the transmission coefficient obtained by the ERM is independent of the joint spacing, while the transmission coefficient obtained by the VRM decreases as the joint spacing increases. The decreasing tendency of the transmission coefficient to the joint spacing becomes smaller as the joint spacing increases. It can also be observed from Fig. 9.6 that the difference between the transmission coefficients obtained by ERM and VRM increase as the joint spacing

increases. The transmission coefficient obtained by the present VRM approaches to the transmission coefficient obtained by the conventional ERM as the joint spacing approaches zero when the total joint number is fixed.

The transmission coefficients for wave propagation through joints with joints $N = 1, 2, 3$ and 4 and a joint spacing S of $1/200 \lambda_0$, $1/20 \lambda_0$, $1/10 \lambda_0$, $3/20 \lambda_0$, $1/5 \lambda_0$ and $1/4 \lambda_0$ are shown in Fig. 9.7.

Fig. 9.7 Transmission coefficient for wave propagation through a rock mass with different joint numbers by using ERM and VRM

It is seen from Fig. 9.7 that the transmission coefficients decrease as the joint number increases both for the VRM and ERM. However the wave attenuates faster when the viscoelastic properties are considered. Same as Fig. 9.6, it is also observed from Fig. 9.7 that the conventional ERM is the special cases of the present VRM when the joint spacing approaches zero for a given joint number.

From Figs. 9.6 and 9.7, it can be concluded that the practical wave attenuation phenomenon in a rock mass is caused by two effects: one is the existence of the macro-joints and the other is due to the viscoelastic properties caused by essential

micro-defects. The conventional ERM can only reflect the effects of the macro-joints, therefore, the transmission coefficient keeps consistent for different joint spacings as shown in Fig. 9.6 and varies in the form of T^N to the joint number as shown in Fig. 9.7 (Zhao et al. 2006). However, the VRM can be used to consider the wave attenuation caused by both effects, which is able to represent more accurately the wave propagation distance effect, defined as joint spacing S multiplied by a joint number N .

9.4 Conclusions

Stress wave propagation through jointed rock mass is experimentally and theoretically investigated. An extended displacement discontinuity method for stress wave propagation through jointed viscoelastic rock mass is proposed. The dynamic properties of the sedimentary rock are obtained from a series of impact tests. Frequency-dependent storage modulus, loss modulus and wave propagation coefficients are determined through the analysis of the experimental results. The transmitted and reflected waves are analytically derived by using the proposed extended displacement discontinuity method. The effects of micro-defects and macro-joints on stress wave propagation attenuation are evaluated quantitatively. The following conclusions can be drawn based on the analysis results:

- (1) The impact tests showed that the dynamic viscoelastic moduli (storage modulus and loss modulus) and the wave propagation coefficients (wave number and attenuation coefficient) of the sedimentary rocks were frequency-dependent. The influence of viscosity on the wave propagation through such rock (e.g. sedimentary rock) cannot be simply ignored.
- (2) The analytical solutions of transmitted and reflected waves for a harmonic wave propagates across a rock joint in the viscoelastic rock mass were deduced. The results showed that the transmission and reflection coefficients based on the conventional ERM using CDDM

were the special cases of that based on VRM using the present EDDM when the loss modulus of the rock material can be neglected.

- (3) The comparisons of the reflected and transmitted waves based on the ERM using CDDM and the VRM using EDDM show that the essential rock viscosity has significant effects on stress wave attenuation, while it has less effect on the effective propagation velocity.
- (4) The comparisons of transmission coefficients for a rock mass with different joint spacings and different joint numbers based on the ERM and VRM show that the conventional ERM could only be used to consider the effects of macro-joints on the stress wave attenuation, while the VRM combines both effects of macro-joints and micro-defects.
- (5) When a stress wave propagates at a short distance, the ERM can replace VRM to obtain an approximated solution. However, when the wave propagates for a sufficient distance related to the wavelength, the rock viscosity on the wave attenuation cannot be ignored.
- (6) Although this study involves only limited cases for compressive waves, it demonstrates that the effects of essential rock viscosity on the wave propagation should be considered and the present viscoelastic model can be used to investigate the effects of micro-defects. The same analytical process can be used to analyze other kinds of waves, e.g. shear wave and surface wave, propagating through a rock mass, and similar results can be expected.

CHAPTER 10 CONCLUSIONS AND RECOMMENDATIONS

10.1 Conclusions

Viscoelastic behavior of rock and rock mass and its effects on the stress wave propagation have been investigated analytically and numerically.

Main conclusions drawn from these studies are summarized as follows:

1. Complete dynamic stress-closure relation of the filled rock joints is highly nonlinear. However, if magnitude of stress wave is insufficient to mobilize the nonlinear deformation of the joint, the linear model can be used to describe the normal deformational behavior of the joint. Otherwise, the nonlinear properties need be considered. Chapters 4 and 5 described separately linear and nonlinear viscoelastic models for jointed rock mass.
2. Both the width and the water content of the sand layer have significant effects on dynamic stress-closure relations of filled macro-joint. The higher water content and the larger joint width result in lower specific stiffness of the filled joint.
3. The modified SHPB test method recommends using short pressure bars which is very effective for testing heterogeneous materials. The present study has potential applications in characterizing the mechanical behavior of rock mass with filled joints and wave propagation analysis through discontinuous media.
4. An evaluation between the equivalent viscoelastic medium method, the static effective elastic moduli method, the dynamic effective elastic moduli method and the displacement discontinuity method for jointed rock mass are carried out. It is shown from the comparison that the static effective moduli method can only be used to obtain an approximate effective velocity, and the accuracy is acceptable only for a rock mass with fewer joints. The dynamic effective moduli method can be used to

obtain a relative accurate effective velocity, but the accuracy highly depends on the representative element selection. Both of these two methods cannot consider the stress wave attenuation behavior in jointed rock mass due to their elastic deformation assumptions. By comparison with the results from the displacement discontinuity method, the equivalent viscoelastic medium method is able to give satisfactory results in terms of transmitted wave form, transmission coefficients, effective velocity, and frequency dependence of the transmitted wave.

5. When the incident wave propagates through an equivalent viscoelastic medium, the component wave with higher frequencies is filtered out, however, the component wave with lower frequencies is less affected which coincides with the phenomenon for wave propagation through a jointed rock mass in geophysical seismic exploration. The equivalent viscoelastic medium method is worthy to be further explored in order to be used in practical rock engineering and its advantages over conventional equivalent medium methods have been demonstrated.
6. The dependence of the attenuation coefficient and the effective velocity on the incident wave amplitude is also investigated. The results show that both the NEVMM and LEVMM are able to describe the attenuation of stress wave propagating in jointed rock mass. However, the NEVMM has the advantage to include the incident wave amplitude effect. The present NEVMM is more reasonable to analyze stress wave propagation in jointed rock mass, especially when the incident wave amplitude is relatively large so that to mobilize the nonlinear deformation of the joints.
7. The attenuation coefficient and the effective velocity versus the incident wave frequency are also investigated by the NEVMM and the LEVMM. The results show that both the NEVMM and LEVMM are frequency dependent and the relations obtained by the NEVMM converges to the results by the LEVMM as the incident wave amplitude decreases. The LEVMM can be regarded as a special case of the NEVMM when the

amplitude of the incident wave is insufficient to mobilize the nonlinear deformation of the joint.

8. Impact testes combined with one-dimensional stress wave propagation theory are applied to characterize the dynamic viscoelastic property of a sedimentary rock containing micro-defects. Wave propagation coefficients such as the attenuation coefficient and the wave number of the sedimentary rock are determined. The wave propagation coefficients are frequency dependent, which is different from those traditional viscoelastic model used for creep and stress relaxation. The modified 3-element viscoelastic model used in the present study is capable in describing the frequency dependence of the viscoelasticity under dynamic loading.
9. The dynamic moduli of the sedimentary rock (both storage modulus and loss modulus) are also highly frequency dependent. The viscoelastic parameters of the modified 3-element model can be regarded as a constant only if the frequency is sufficiently large, in which case, the modified 3-element viscoelastic model degrades to the traditional frequency-independent 3-element model. The traditional elastic rock model is a special case of the present modified 3-element viscoelastic model when the incident wave frequency is sufficiently small. Although only the compressive stress wave with half sinusoidal form was analyzed, other kinds of waves (e.g. shear wave) with other forms (e.g. triangle wave form, rectangle wave form) can also be applied for similar analysis.
10. Complete investigation of viscoelastic behavior of micro-defected rock is performed using numerical manifold method. The dynamic complex modulus which indicates the viscoelasticity is significantly affected by the crack length, quantity, and stiffness. When either the length or quantity of the micro cracks increases, they remarkably degrades the overall elasticity of the microcracked rock mass; meanwhile, enhances its viscosity. The decrease of the crack stiffness also intensifies the discontinuity of the rock mass, which makes viscosity more significant. the microcracks in the elastic rock mass will cause viscoelasticity of the

overall microcracked rock mass, which would significantly attenuate the stress wave and make a travel time delay when the stress wave propagates.

11. A homogenization-discontinuity combination approach is proposed to study the global viscoelastic behavior of rock mass containing double-scale discontinuities. The results obtained by homogenization-discontinuity approach show the global mechanical behavior of rock mass containing double-scale discontinuities is also viscoelastic. However, the frequency sensitive range for the micro- and macro-discontinuities are different, therefore, the rock mass will appear as two-phase viscoelastic material, with two transition frequency ranges for the global storage modulus and two peak values for the global loss modulus. The energy analysis shows that the rock mass containing double-scale discontinuities will absorb mechanical energy markedly under dynamic loading. Increasing the stiffness of macro-joint can effectively reduce the energy dissipation.

10.2 Recommendations for Further Research

1. A theoretical approach is developed to investigate the dynamic behavior of defected rock and jointed rock mass. The present study focused on one-dimensional wave propagation only. Further study should be conducted for oblique incidence of stress waves, in which, the effects of arbitrary incident angles, the coupling between the normal and shear stiffness, and energy conservation must be considered.
2. The laboratory test in the present study tested only longitudinal wave propagation in jointed rock. It is suggested that the shear constitutive relation of macro-joints and the effects of shear behavior of macro-joints on stress wave propagation should also be investigated.
3. Two-dimensional or three-dimensional wave propagation theory in rock mass containing double-scale discontinuities can be extended. The

geometrical attenuation of a stress wave can be then considered together with the stress wave attenuation caused by viscosity.

4. Based on the experimental data, there is a hysteresis between the loading/unloading cycles for the nonlinear behavior of a macro rock joint. However, only the loading period is considered in the present study using the equivalent viscoelastic medium method. Further efforts can be made to develop a nonlinear equivalent viscoelastic medium method to consider the complete loading/unloading cycles.
5. Further development of NMM are required to modeling the global viscoelastic behavior of rock mass containing double-scale discontinuities, and its effects on the stress wave propagation.
6. Furthermore, NMM can be improved by combining with other numerical methods (e.g. finite element method, discontinuous deformation analysis, finite difference method and boundary element method). With further verification and improvement, NMM will be hopefully become a powerful and convenient tool to study dynamic problems of rock mass.

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PUBLICATIONS

Journal Papers

1. **Fan LF**, Ma GW, Wong LNY (2012), “Effective viscoelastic behavior of a rock mass with double-scale discontinuities”, *Geophysical Journal International* (*accepted*).
2. **Fan LF**, Ma GW, Li JC (2012), “Nonlinear viscoelastic medium equivalence for stress wave propagation in jointed rock mass”, *International Journal of Rock Mechanics and Mining Science*, 50, 11-18.
3. **Fan LF**, Ren F, Ma GW (2011), “Experimental Study on Viscoelastic Property of Sedimentary Rock under Dynamic Loading”, *Rock Mechanics and Rock Engineering*, DOI: 10.1007/s00603-011-0197-7.
4. Ma GW, **Fan LF**, Li JC (2011), “Evaluation of equivalent medium methods for stress wave propagation in jointed rock mass”, *International Journal for Numerical and Analytical Methods in Geomechanics*, DOI: 10.1002/nag.1118.
5. **Fan LF**, Ren F, Ma GW (2011), “An extended displacement discontinuity method for analysis of stress wave propagation in viscoelastic rock mass”, *Journal of Rock Mechanics and Geotechnical Engineering*, 3(1), 73-81.
6. Ren F, **Fan LF**, Ma GW (2011), “Simulation of viscoelastic behavior of defected rock by using numerical manifold method”, *Frontiers of Architecture and Civil Engineering in China*, 5(2), 199-207.

Conference Papers

1. **Fan LF**, Ren F, Ma GW (2011) “Effect of micro-defect and macro-joint on wave propagation through rock mass”, *ISRM the 12th International Congress on Rock Mechanics*, October 19-22, 2011, Beijing, China (Accepted).
2. **Fan LF**, Li JC, Ma GW (2010), “Analysis of longitudinal stress wave propagation through viscoelastic rock mass”, *ISRM International Symposium 2010 & 6th Asian Rock Mechanics Symposium*, October 23-27, 2010, New Delhi, India.
3. Ma GW, **Fan LF**, Li JC (2010), “A new equivalent medium model for P-wave propagation through rock mass with parallel joint”. *Analysis of Discontinuous Deformation: New Developments and Applications*, 563-571.