Performance study of efficient cooperative communication systems

Hu, Yongxu

2012

Hu, Y. (2012). Performance study of efficient cooperative communication systems. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/50625>

<https://doi.org/10.32657/10356/50625>

PERFORMANCE STUDY OF EFFICIENT COOPERATIVE COMMUNICATION SYSTEMS

HU YONGXU

School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2012

ACKNOWLEDGMENT

First of all, I would like to express my thanks to my supervisor, Prof. Li Kwok Hung and my co-supervisor, Prof. Teh Kah Chan. During the four years of my PhD study, they not only cultivate me the ability of independent thinking, but also give me invaluable guideline and support for my research. They are always very patient and encourage me to achieve the best.

I would also like to thank ex and current fellow students in Communications Lab III, who are very friendly and excellent. We often have discussions and share options on diverse research areas.

Last but not least, I would like to give my sincere thanks to my parents and family for their love, support and encouragement like always.

Table of Contents

Acknowledgment	i
Table of Contents	ii
Abstract	vi
List of Figures	viii
List of Tables	xi
1 Introduction	1
1.1 Motivation	1
1.2 Background of Cooperative Systems	2
1.3 Objectives	8
1.4 Organization of the Thesis	10
1.5 Conclusion	11
2 Performance Measure and Literature Survey	12

2.1	Channel Models and Performance Measure	12
2.1.1	Rayleigh and Nakagami- m fading channels	12
2.1.2	Error Probability	13
2.1.3	Outage Probability	16
2.1.4	Diversity Multiplexing Trade-off	17
2.2	Literature Survey	18
2.2.1	Single-user Cooperative Systems	18
2.2.2	Multiple-user Cooperative Systems	30
2.3	Conclusion	38
3	BER Performance of a Single Relay Cooperative System	39
3.1	Introduction	39
3.2	System Model and Transmission Protocol	41
3.3	Maximal Ratio Combining (MRC) and Cooperative MRC (C-MRC) .	42
3.4	BER Performance Analysis of MRC and C-MRC in Nakagami- m fading channels	45
3.4.1	MRC	46
3.4.2	C-MRC	52
3.5	Numerical Results and Discussions	58
3.6	Conclusion	63
4	BER Performance of a Two-user Cooperative Multiple-Access Sys-	

tem	64
4.1 Introduction	64
4.2 Three-time-slot TDBC Protocol and its BER Performance	67
4.3 Two-time-slot Superposition Modulation Protocol	70
4.4 Performance Analysis of the Superposition Modulated Protocol . . .	74
4.4.1 Occurrence Probabilities of Four Cases— $Pr(u)$	75
4.4.2 Average Error Probabilities for Four Cases— $P_{e,u}$	77
4.5 Numerical Results and Comparison	83
4.6 Conclusion	91
5 Performance of an Efficient Multiple-Relay Cooperative System	92
5.1 Introduction and Related Results	92
5.2 System Model and The Successive Relaying Protocol	95
5.3 Performance Analysis	99
5.3.1 The occurrence probability of the SR mode	100
5.3.2 The outage probability in the SR mode	103
5.3.3 The occurrence and outage probabilities of the OR mode . . .	106
5.4 Numerical Results and Discussions	109
5.5 Conclusion	113
6 Conclusions and Future Work	115

6.1	Conclusions	115
6.2	Future Work	117
	Author's Publications	119
	Bibliography	120
Appendix A	Derivation of decision regions for the first case of the superposition-based CMA scheme	131
Appendix B	Derivation of $P_{II,III I}$ and $P_{\overline{II},\overline{III} I}$	136

ABSTRACT

The cooperative relaying technique has gained a lot of research interest recently. It can obtain the spatial diversity and improve the system performance as the multiple-input multiple-output (MIMO) technique does, but without the requirement of installing multiple antennas in mobile devices. However, due to the half-duplex constraint that mobile terminals cannot transmit and receive at the same time over the same frequency band, some conventional cooperative protocols have to sacrifice the spectral efficiency to trade the accurate transmission. In the thesis, we concentrate on analyzing the performance of cooperative relaying systems with efficient and effective transmission protocols, which can achieve the full spatial diversity gain as well as improve the spectrum efficiency significantly.

Firstly, to achieve the full cooperative diversity gain of a single-relay system with fixed decode-and-forward (DF) relaying protocol, the cooperative maximal-ratio combining (C-MRC) scheme is employed. The bit-error rate (BER) performance of C-MRC and maximal-ratio combining (MRC) schemes is analyzed in the presence of independent and non-identical Nakagami- m fading channels. The BER upper bounds of both schemes are derived and the achievable diversity orders are compared. It is illustrated that by smartly designing weighting factors, the C-MRC scheme can achieve the full spatial diversity gain of the system. Additionally, C-MRC is shown to achieve higher diversity order than MRC in the fixed DF relaying protocol as well as outperform the adaptive DF relaying protocol.

To recover the spectrum efficiency of the simple three-node system, a superposition modulation based two-user cooperative multiple-access (CMA) transmission scheme is then employed, where two users alternatively transmit the superposition of its own and the other's messages. We develop a low-complexity near-optimal decoder at the receiver and investigate the BER performance. The superposition modulation scheme greatly improves the system efficiency and also achieves the full diversity. It even outperforms the three-time-slot and four-time-slot time division broadcast (TDBC) protocols under the same power and transmission rate constraints.

Next, we expand our analysis to a multiple-relay cooperative network and propose an efficient and effective successive relaying (SR) protocol. In each channel-static block, two relays are selected based on the instantaneous channel state information. The source keeps transmitting new messages. Two selected relays alternatively decode signals from the source and the other relay, and then transmit the superposition of decoded signals to the destination. The SR protocol is capable of achieving both full diversity gain and high multiplexing gain. Its spectrum efficiency is much higher than the conventional protocols and approaches one as the time slots go to infinity.

List of Figures

1.1	Four transmission schemes of a three-terminal cooperative system in the half-duplex mode: (a) no direct link; (b) direct transmission active in the first phase; (c) direct transmission active in the second phase; (d) direct transmission active in both phases.	5
1.2	Multiple-relay cooperative systems: (a) multi-hop relaying; (b) multi-branch relaying; (c) multi-branch multi-hop relaying.	7
3.1	Simulated BER and theoretical BER upper bound of the MRC scheme for a single-relay cooperative system with fixed DF relaying over i.n.i.d. Nakagami- m fading channels. © [2009] John Wiley & Sons, Ltd.	59
3.2	Simulated BER and theoretical BER upper bound of the C-MRC scheme for a single-relay cooperative system with fixed DF relaying over i.n.i.d. Nakagami- m fading channels. © [2009] John Wiley & Sons, Ltd.	60
3.3	BER performance comparison of MRC and C-MRC schemes for a single-relay cooperative system with fixed DF relaying over i.n.i.d. Nakagami- m fading channels.	61

3.4	BER performance comparison of the fixed DF relaying protocol with the C-MRC scheme and the adaptive DF relaying protocol with optimum threshold for a single-relay cooperative system over i.n.i.d. Nakagami- m fading channels.	62
4.1	A two-user cooperative multiple-access system.	68
4.2	Decision regions for the first case $f_A = 1$ and $f_B = 1$, when $s_A(k) = -1$. $O : (0, 0)$, $J : (-X_{th}, 0)$, $V : (X_{th}, 0)$, $K : (0, -Y_{th})$ and $H : (0, Y_{th})$, where $X_{th} = \gamma_{AD}\sqrt{1-\beta_A}$ and $Y_{th} = \gamma_{BD}\sqrt{1-\beta_B}$. © [2011] IEEE	80
4.3	BER of user A 's information vs the power allocation factor α_A for BPSK and 8PSK modulations in the three-time-slot TDBC protocol over i.i.d. Rayleigh fading channels.	84
4.4	User A 's BER performance for BPSK, QPSK and 8PSK modulations employing the three-time-slot TDBC protocol in i.i.d. Rayleigh fading channels.	85
4.5	BER of user A 's signal vs the superposition factor β_A , employing the superposition modulation protocol in i.i.d. Rayleigh channels. © [2011] IEEE	86
4.6	Simulated and theoretical BER performance of user A 's signal with different superposition factors in i.i.d. Rayleigh channels. © [2011] IEEE	88
4.7	Simulated and theoretical BER performance of user A 's signal, employing superposition modulation protocol over i.n.i.d. Rayleigh channels. © [2011] IEEE	89

4.8	BER performance comparison of superposition modulation, three-time-slot TDBC and four-time-slot TDBC protocols in i.i.d. Rayleigh fading channels.	90
5.1	The mulitple-relay network with the SR transmission protocol.	96
5.2	Outage performance comparison of the SR protocol and OR protocol, when $T = 7$ and $r_0 = 1$ BPCU.	110
5.3	Simulated and analytical outage probabilities of the SR protocol with $M = 4$ and $T = 10$	111
5.4	Outage probabilities of the SR protocol with various time slots T and transmission rates r_0 , when $M = 5$	112
5.5	DMT performance of the SR protocol, single relay selection protocol, DSTC protocol and MHRS protocol, when $M = 6$ and $T = 10$. © [2012] IEEE	113
A.1	The distributions and decision boundaries of X and Y , where $X_{th} = \gamma_{AD}\sqrt{1 - \beta_A}$, $Y_{th} = \gamma_{BD}\sqrt{1 - \beta_B}$ and $Y_{mid} = \gamma_{BD}\sqrt{\beta_B}$. © [2011] IEEE	133

List of Tables

2.1	The values of k and c for MPSK modulations in Equation (2.1.7) [1]	15
2.2	The values of a , b and q for BPSK, MPSK and MQAM modulations in Equation (2.1.9)	16
3.1	Comparison of the diversity orders achieved by MRC and C-MRC schemes	58
3.2	The optimum threshold used in the adaptive DF protocol [2]	63
5.1	The SR transmission schedule in one block	98
A.1	The decisions of $(1, b_B(k-1), b_B(k))$ and $(-1, \check{b}_B(k-1), \check{b}_B(k))$ minimizing $f(1, b_B(k-1), b_B(k))$ and $f(-1, \check{b}_B(k-1), \check{b}_B(k))$ in every region. © [2011] IEEE	135

Chapter 1

Introduction

1.1 Motivation

As the smart phones have become more and more popular recently, people not only satisfy with making calls or sending messages anytime and anywhere, but also wish to connect to the Internet via their phones anytime and anywhere. Just using the smart phones, people can conveniently check emails, post information and photos on social networks, download files and watch videos. These fast growing demands for the smart phones require the wireless communications to be much faster, more reliable but less expensive.

The qualities of wireless channels are diverse and significantly affected by the surrounding environments. To combat fading and multi-path propagation effects of the wireless environments, the multiple-input multiple-output (MIMO) technique was proposed and employed in many works [3–5]. Its basic principle is to add multiple transmit and receive antennas in order to construct multiple diverse channels between sources and destinations and thus increase the spatial diversity. The complexity is increased accordingly depending on the algorithms used at transceivers.

However, it is impractical to install many antennas within mobile devices due to the size restriction and the cost concern.

In this circumstance, the cooperative diversity technique was proposed, which is capable of achieving the spatial diversity without installing multiple antennas in one mobile device. The idea is to employ other mobile users as relay nodes to assist the communication from a source to a destination. Through this process, the destination receives signals experiencing diverse wireless channels from the source and different relays. A virtual antenna array is constructed and thus the spatial diversity can be obtained. At least two hops are required for signals to be transmitted from the source and forwarded by relays to the destination. The more hops required, the higher delay and the lower spectrum efficiency. The cooperative diversity technique sacrifices the efficiency to exchange the transmission accuracy. The trade-off between the diversity gain and the multiplexing gain also exists in cooperative networks as in multiple-antenna and multiple-access systems [6, 7]. Therefore, in our thesis, we are motivated to enhance the performance of cooperative communication systems in terms of accurate transmission and high transmission efficiency, and to obtain as good diversity-multiplexing trade-off (DMT) as in MIMO channels.

1.2 Background of Cooperative Systems

The idea of user cooperation diversity was first proposed in [8] and soon attracted a lot of attention and research interest due to its advantages of dynamically achieving diversity gains without the limit of terminal sizes, improving transmission performance and capacity, extending system coverage, etc. Therefore, cooperative diversity technique is applicable to many wireless scenarios, e.g., cellular systems and ad hoc networks. In the literature, researchers have developed various relaying

strategies and transmission schemes for different system models in diverse channel environments to approach the best performance as much as possible.

Based on how relays process the received signals, relaying protocols can be classified into three cases: amplify-and-forward (AF) relaying, decode-and-forward (DF) relaying and compress-and-forward (CF) relaying. The AF relaying protocol is also called non-regenerative relaying, where relays linearly process (or simply scale) the received signals from sources and forward to destinations under a power constraint. Compared with the other two relaying methods, the AF relaying protocol has the lowest computation complexity, but it amplifies and propagates relays' noise to destinations. In the DF relaying protocol, also named regenerative relaying, relays first decode the received signals from sources using some decoding methods, e.g., matched filter, minimum mean-square-error (MMSE) or maximum-likelihood (ML) decoding. After that, relays transmit the simply repeated or encoded signals to destinations. For the CF relaying protocol, also called estimate-and-forward or quantize-and-forward relaying, relays quantize and compress the received signals by using Wyner-Ziv source coding and send the compressed signals to destinations.

According to the ways that relays forward the processed signals, there are three relaying policies: fixed relaying, adaptive relaying and incremental relaying [9]. In the fixed relaying, also called deterministic relaying, relays process and retransmit all the received signals to destinations without considering the channel conditions, e.g., the signal-to-noise ratio (SNR) of incoming signals, whether relays can decode the signals correctly, whether the direct transmission is successful, etc. The adaptive relaying, sometimes called selection relaying, is defined as that relays only forward signals satisfying a certain criterion. The criterion might depend on whether the SNRs of the received signals at relays are above a fixed or an adaptive threshold, or whether relays could decode the signals correctly by using cyclic redundancy check

(CRC) codes in the case of DF relaying. In the incremental relaying protocol, relays assist the transmission only when the direct communication is in outage or fails. Therefore, it has higher spectral efficiency than the fixed and adaptive relaying protocols if the direct channel is in a good condition. If the direct link undergoes severe fading, the incremental relaying protocol becomes the fixed relaying or adaptive relaying protocol. Comparing the adaptive relaying and the incremental relaying, we can find that relays do not participate the cooperation all the time. The difference is that relays decide when to cooperate depending on the source-relay transmission in the adaptive relaying, and depending on the source-destination transmission in the incremental relaying.

The operating mode of relays can be divided into two cases: full-duplex and half-duplex. In the full-duplex mode, relays can transmit and receive at the same time over the same frequency band. In this case, the faded received signals would be affected by the strong transmitted signals. Hence, the full-duplex mode is difficult to implement and impractical. On the contrary, the half-duplex mode requires relays to transmit and receive either in different time slots using time division duplex (TDD) or in orthogonal frequency bands using frequency division duplex (FDD). Accordingly, the half-duplex constraint requires more time or frequency resources and causes spectral inefficiency. In the literature, the TDD half-duplex mode is more commonly employed.

The basic cooperative system is composed of a source, a relay and a destination. The relay is assumed to work in the TDD half-duplex mode. Therefore, a round of cooperative transmission is completed in two phases, namely, broadcast phase and cooperation phase. The source transmits its information to the relay in the broadcast phase, and the relay processes and forwards signals to the destination in the cooperation phase. Based on this basic operation, there are four transmission

Figure 1.1: Four transmission schemes of a three-terminal cooperative system in the half-duplex mode: (a) no direct link; (b) direct transmission active in the first phase; (c) direct transmission active in the second phase; (d) direct transmission active in both phases.

schemes depending on whether and when the direct transmission from the source to the destination is conducted, as depicted in Figure 1.1. Note that for the full-duplex mode, there are only two transmission schemes depending on whether the direct link exists or not.

In Figure 1.1(a), due to the long distance or severe channel fading between the source and the destination, the direct transmission is not available. With the assistance of the relay, the communication coverage can be enlarged. However, signals are only transmitted through the relay and thus cooperative diversity cannot be obtained. References [10–12] have studied the performance of this two-hop relay channel without the direct link.

In the second transmission scheme, as shown in Figure 1.1(b), the source can trans-

mit to the relay and the destination at the same time due to the broadcast nature of wireless communication. The destination receives both transmissions from the source and the relay in two orthogonal time slots. Hence, this transmission method belongs to the orthogonal relaying class. The outage behavior of this scheme was studied by Laneman *et al.* in [9] for AF and DF relaying protocols combined with fixed, adaptive and incremental relaying strategies. Based on the analysis in [9], many papers have investigated the error probability, outage probability and ergodic capacity of some modified relaying protocols in various environments.

For the third and fourth transmission schemes, as shown in Figure 1.1(c) and (d), the source transmits in both phases. In the cooperation phase, the destination receives signals from the source and the relay at the same time over the same frequency band. Therefore, these two schemes belong to the non-orthogonal relaying transmission. In [13], Nabar *et al.* studied and compared the mutual information, outage probabilities and diversity orders of three transmission schemes, shown in Figure 1.1(b), (c) and (d), based on the fixed AF and DF relaying protocols. Some papers have further analyzed the fourth transmission scheme in order to obtain better DMT performance.

When multiple relays are available to assist the transmission, they might be connected in series and the direct link is often ignored, which is called multi-hop relaying system as shown in Figure 1.2(a). Alternatively, relays might work in parallel to cooperate and the structure is called multi-branch relaying system as depicted in Figure 1.2(b). The combination of these two systems is the general multi-branch multi-hop relaying system, as shown in Figure 1.2(c). The multi-hop relaying system is applicable for a large network where the source and the destination are far away from each other or the direct transmission experiences severe path-loss and deep fading. However, since there is only one link between any two neighboring terminals,

Figure 1.2: Multiple-relay cooperative systems: (a) multi-hop relaying; (b) multi-branch relaying; (c) multi-branch multi-hop relaying.

the advantages of employing relay nodes, such as diversity gain, cannot be achieved. In addition, the system performance would be strongly affected by the worst one of all the links, especially when the qualities of individual channels are quite unbalanced. Different from the multi-hop relaying model, the multi-branch relaying system is much more preferred in that the source and the destination are connected by multiple paths. Even if some source-relay and relay-destination channels are blocked or in deep fading, the transmission from the source to the destination is still possible through other links and relays, and the diversity can be obtained.

Several cooperation schemes have been designed for the multi-branch cooperative systems operating in the TDD half-duplex mode. Among these schemes, the same process is that the source broadcasts signals to relays and the destination in the first time slot. In the all-participate cooperative scheme, each relay is allocated a unique

time slot to forward and the full diversity order can be achieved. However, $M + 1$ time slots are required for M relays to complete a round of cooperative transmission, and the bandwidth usage is very low as the number of relays increases. Alternatively, relays could use the distributed space-time codes (DSTC) to forward the scaled or decoded signals to the destination simultaneously in the second slot. Therefore, this DSTC-based relaying scheme needs only two phases and significantly improves the spectral efficiency compared to the all-participate cooperative scheme. Reference [14] studied and analyzed the outage probability and DMT performance of the DSTC-based relaying scheme with the adaptive DF protocol. The linear dispersion (LD) space-time codes were used in [15] and some other papers as well. The single relay selection scheme is an attractive low-complexity and easy-implementation cooperation scheme for the multiple-branch cooperative systems. In this scheme, one relay is selected depending on different criteria, e.g., max-min approach, harmonic mean approach or opportunistic relaying. The selected relay transmits processed signals to the destination in the second phase. The relay selection scheme has become popular since it was proposed in [16]. Many researchers have comprehensively investigated the performance of various relaying protocols in terms of outage probability, error probability, ergodic capacity and achievable DMT.

1.3 Objectives

In the thesis, we concentrate on analyzing the performance of efficient and effective transmission protocols for cooperative communication systems, which can achieve the full spatial diversity gain and significantly improve the spectrum efficiency. It was found in the literature that the deterministic DF relaying protocol cannot achieve the full diversity gain with the maximal-ratio combining (MRC) receiver [9, 10]. Hence, we decide to employ a cooperative MRC (C-MRC) detection

scheme for the destination, which is capable of fully exploring the available spatial diversity and achieving the full diversity order. In addition to analyzing the performance of MRC and C-MRC detectors, we aim to find the reasons leading to different performance results. We adopt the Nakagami- m faded channel model with different values of the fading parameter m to fully illustrate and compare the performance of both detectors.

Besides aiming to achieve the maximal available diversity gain, we are keen to improve the system spectrum efficiency as well. With the half-duplex and orthogonal transmission restrictions, the maximal multiplexing gain of a single user is only 0.5. However, we can apply the superposition modulation scheme and cooperation idea to the two-user multiple-access system, achieving the maximal multiplexing gain of one for the system as well as the full diversity order for each user. Since each user transmits the superposition of its own generated signal and the other user's relayed signal, the multiple-access interference (MAI) is inevitable. In this circumstance, we are motivated to design a high-performance and relatively low-complexity detector for the destination, which can achieve the bit-error rate (BER) performance as if MAI does not exist.

Next, we focus on improving the multiplexing gain for a single user. Although some non-orthogonal DF relaying transmission schemes in the literature are capable of achieving the maximal diversity gain and the unity multiplexing gain, their DMT performance still cannot approach the optimal DMT especially when the multiplexing gain is in the range of $(0.5, 1)$ [17, 18]. Therefore, we are motivated to propose an efficient and effective relaying scheme for the multiple-relay cooperative system to achieve the DMT upper bound.

1.4 Organization of the Thesis

The thesis is organized as follows. In Chapter 2, we will introduce some commonly employed performance measures of the cooperative systems, and review the works and development of cooperative communications.

In Chapter 3, we focus on analyzing the performance of a single-relay cooperative system over Nakagami- m fading channels. For the fixed DF relaying, both MRC and C-MRC detection methods are employed at the receiver. Their BERs are derived and the achievable diversity orders are compared.

A two-user cooperative multiple-access (CMA) system is considered in Chapter 4. Two efficient transmission schemes are employed, namely, three-phase time division multiple-access (TDMA) protocol and two-phase superposition-based protocol. For the latter one, we develop a low-complexity near-optimal decoder at the receiver. The BER performance of both protocols are investigated and compared with the four-phase TDMA transmission scheme.

In Chapter 5, we propose an efficient non-orthogonal successive relaying (SR) protocol for a multiple-relay cooperative network, which can achieve the optimal DMT performance if the number of time slots goes to infinity. In every channel block, two relays are selected based on the instantaneous channel state information. The source keeps transmitting new messages. Two relays alternatively decode signals from the source and the other relay, and then transmit the superposition of decoded signals to the destination. The destination decodes signals using the ML decoding after each block's transmission.

Finally, in Chapter 6, we conclude our work and contributions, and propose some directions for the future research of cooperative communications.

1.5 Conclusion

In this chapter, we have firstly presented the motivation of our work that why we choose to study on the cooperative techniques. We then introduced the principles and basic protocols of cooperative communications. After that, we have stated the specific objectives of the thesis and given the organization of the following chapters.

Chapter 2

Performance Measure and Literature Survey

2.1 Channel Models and Performance Measure

2.1.1 Rayleigh and Nakagami- m fading channels

Many mathematical models, including Rayleigh, Rician, Weibull and Nakagami- m distributions, have been employed to characterize diverse wireless fading channels and assist in analyzing communication systems. Here, we will introduce Nakagami- m distribution. With the assumption that X is a Nakagami- m distributed random variable with parameters Ω and m , its probability density function (PDF) is given by [19]

$$p_X(x) = \frac{2(m/\Omega)^m}{\Gamma(m)} x^{2m-1} \exp\left(-\frac{m}{\Omega}x^2\right) U(x), \quad (2.1.1)$$

where $\Gamma(\cdot)$ is the gamma function and $U(\cdot)$ is the unit step function. The parameters are given by

$$\Omega = E\{X^2\} \quad \text{and} \quad m = \frac{E^2\{X^2\}}{\text{Var}\{X^2\}}, \quad (2.1.2)$$

where $E\{\cdot\}$ and $\text{Var}\{\cdot\}$ compute the expectation and variance, respectively. By setting that $Y = X^2$, the PDF of Y can be expressed as

$$p_Y(y) = \frac{(m/\Omega)^m}{\Gamma(m)} y^{m-1} \exp\left(-\frac{m}{\Omega}y\right) U(y), \quad (2.1.3)$$

which is the gamma distribution.

If a channel follows Nakagami- m distribution, variable X can be used to denote the channel coefficient. Hence, Y represents the instantaneous channel power gain and Ω is the average channel power. The parameter m reflects the fading severity of the channel. As the value of m increases, the channel becomes less severely faded. It is noted that when $m = 1$, the PDFs of X and Y become Rayleigh and exponential distributions, respectively. Hence, Rayleigh distribution is a special case of Nakagami- m distribution, and the latter is more suitable and flexible to characterize different wireless channel environments.

2.1.2 Error Probability

Average error probability is an important measure for the performance of communication systems, transmission schemes, detection methods, etc. It is the expectation of the conditional error probability in the time-variant fading channel environment. Given the instantaneous received SNR per symbol γ_s , the conditional symbol-error rate (SER) of various modulation signals can normally be expressed in terms of the

Q -functions [1, 20]. The definition of the Q -function is given by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{u^2}{2}\right) du. \quad (2.1.4)$$

For $x \geq 0$, $Q(x)$ can also be defined by [20]

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta. \quad (2.1.5)$$

In addition, the Chernoff bound of $Q(x)$ is given by

$$Q(x) \leq \frac{1}{2} \exp\left(-\frac{x^2}{2}\right), \quad x \geq 0, \quad (2.1.6)$$

and $0 \leq Q(x) \leq 1$. According to these two definitions of $Q(x)$, two methods have been developed to calculate the average SER, namely, cumulative distribution function (CDF)-based method and moment generating function (MGF)-based method.

We first introduce the CDF-based approach. Let the conditional SER of a certain modulation scheme be $kQ(\sqrt{c\gamma_s})$, where k and c are constants and determined by the adopted modulation. For M -ary phase-shift keying (MPSK) modulations, the values of k and c are shown in Table 2.1. Note that M is the number of symbols in modulation constellation. In the CDF-based method, the average SER by using the definition in (2.1.4) is computed as [21, 22]

$$\begin{aligned} P_s &= k \int_0^\infty Q(\sqrt{c\gamma_s}) p_\Gamma(\gamma_s) d\gamma_s \\ &= \frac{k}{\sqrt{2\pi}} \int_0^\infty \left[\int_{\sqrt{c\gamma_s}}^\infty \exp\left(-\frac{u^2}{2}\right) du \right] p_\Gamma(\gamma_s) d\gamma_s \\ &= \frac{k}{\sqrt{2\pi}} \int_0^\infty \left[\int_0^{u^2/c} p_\Gamma(\gamma_s) d\gamma_s \right] \exp\left(-\frac{u^2}{2}\right) du \\ &= \frac{k}{\sqrt{2\pi}} \int_0^\infty F_\Gamma\left(\frac{u^2}{c}\right) \exp\left(-\frac{u^2}{2}\right) du, \end{aligned} \quad (2.1.7)$$

Table 2.1: The values of k and c for MPSK modulations in Equation (2.1.7) [1]

M	k	c
2	1	2
4	2	1
> 4	2	$2 \sin^2(\pi/M)$

where $p_{\Gamma_s}(\gamma_s)$ and $F_{\Gamma}(\gamma_s)$ are the PDF and CDF of γ_s , respectively. This CDF-based approach is not only suitable to compute the average SER of MPSK signals, but also applicable for those modulations whose conditional error probabilities can be expressed by the linear combinations of Q -functions.

In the MGF-based approach, according to the definition in (2.1.5), the conditional SER of some modulation schemes can be obtained by [20, 23]

$$P_s(\gamma_s) = a \int_0^{b\pi} \exp\left(-\frac{q\gamma_s}{\sin^2 \theta}\right) d\theta, \quad (2.1.8)$$

where a , b and q are determined by the type of modulation. The values of a , b and q for binary phase-shift keying (BPSK), MPSK and M -ary quadrature amplitude modulation (MQAM) modulations are given in Table 2.2 [20, 24]. Specifically, for MQAM modulation, the conditional SER is the sum of two parts, i.e., $P_s(\gamma_s) = P_{s,1}(\gamma_s) + P_{s,2}(\gamma_s)$. These two terms, $P_{s,1}(\gamma_s)$ and $P_{s,2}(\gamma_s)$, can be computed from (2.1.8) and the parameters have different values as given in Table 2.2. Therefore, based on (2.1.8), the average SER can be obtained as

$$\begin{aligned} P_s &= \int_0^\infty P_s(\gamma_s) p_{\Gamma}(\gamma_s) d\gamma_s \\ &= a \int_0^{b\pi} \left[\int_0^\infty \exp\left(-\frac{q\gamma_s}{\sin^2 \theta}\right) p_{\Gamma}(\gamma_s) d\gamma_s \right] d\theta \\ &= a \int_0^{b\pi} M_{\Gamma}\left(-\frac{q}{\sin^2 \theta}\right) d\theta, \end{aligned} \quad (2.1.9)$$

Table 2.2: The values of a , b and q for BPSK, MPSK and MQAM modulations in Equation (2.1.9)

		a	b	q
BPSK		$1/\pi$	$1/2$	1
MPSK		$1/\pi$	$(M-1)/M$	$\sin^2(\pi/M)$
MQAM	$P_{s,1}(\gamma_s)$	$4(1 - 1/\sqrt{M})/\pi$	$1/2$	$\frac{3}{2(M-1)}$
	$P_{s,2}(\gamma_s)$	$-4(1 - 1/\sqrt{M})^2/\pi$	$1/4$	$\frac{3}{2(M-1)}$

where $M_X(s)$ is the MGF of the random variable X and defined by $M_X(s) = \mathbb{E}\{e^{sx}\} = \int_{-\infty}^{\infty} e^{sx} p_X(x) dx$.

2.1.3 Outage Probability

The mutual information measures the amount of transmitted information that can be obtained by observing the received one. Given a slow-fading channel, denoted as h , the instantaneous mutual information between transmitter and receiver in the presence of AWGN noise is given by

$$I = \log_2 (1 + P|h|^2/N_0), \quad (2.1.10)$$

where P and N_0 denote the transmitted power and noise power levels, respectively. Hence, the instantaneous received SNR is given by $\gamma = P|h|^2/N_0$. The outage event occurs if the instantaneous mutual information is lower than a target transmission rate r_0 , i.e., $I < r_0$. The outage probability is defined as the probability of the total outage events over all the channel realizations, and is expressed as

$$\begin{aligned} P_{out} &= Pr(I < r_0) \\ &= \int_0^{2^{r_0}-1} p_{\Gamma}(\gamma) d\gamma \end{aligned}$$

$$= F_{\Gamma}(\gamma < 2^{r_0} - 1). \quad (2.1.11)$$

The calculation of the outage probability is equivalent to compute the CDF of the distribution that the received SNR obeys.

2.1.4 Diversity Multiplexing Trade-off

According to the definitions in [6, 7], the diversity gain d and the multiplexing gain r can be obtained as

$$d = - \lim_{\text{SNR} \rightarrow \infty} \frac{\log P_e(\text{SNR})}{\log \text{SNR}} \quad (2.1.12)$$

and

$$r = \lim_{\text{SNR} \rightarrow \infty} \frac{r_0(\text{SNR})}{\log \text{SNR}}, \quad (2.1.13)$$

respectively, where $P_e(\text{SNR})$ denotes the error probability at a certain SNR value and $r_0(\text{SNR})$ is the transmission rate in bits per channel use (BPCU) unit. It can be seen from (2.1.12) and (2.1.13) that the diversity gain is a measure for the system accuracy in high SNR regime, and the multiplexing gain is a measure for the system capacity increase at high SNR values. High diversity gain means low error probability and good transmission quality, while high multiplexing gain indicates large transmission capability. The diversity gain can also be obtained from the outage probability as

$$d = - \lim_{\text{SNR} \rightarrow \infty} \frac{\log P_{out}(\text{SNR})}{\log \text{SNR}}. \quad (2.1.14)$$

Note that maximizing one type of gain may not necessarily maximize the other.

Higher spatial multiplexing gain comes at the price of sacrificing the diversity gain. In [6], Zheng and Tse proposed the concept of diversity and multiplexing trade-off (DMT), $d(r)$, to characterize the fundamental relationship between these two contradictory gains. DMT has soon become a well-recognized and important measure to evaluate various systems and schemes. The achievable DMT performance of the multiple-antenna channels and multiple-access channels was investigated in [6] and [7], respectively. The DMT performance of various cooperative systems has been studied in some papers as well [17, 18, 25, 26].

2.2 Literature Survey

2.2.1 Single-user Cooperative Systems

Amplify-and-forward relaying

For the two-hop single-relay system without the direct channel, Hasna *et al.* studied the performance of the CSI-based and the fixed gain based AF relaying protocols over independent and non-identically distributed (i.n.i.d.) Rayleigh fading channels in [11] and [12], respectively. Reference [11] firstly derived the PDF, CDF and MGF of the harmonic mean of two exponentially distributed random variables, and then applied these statistical results to compute the average BER and the outage probability of the CSI-based AF relaying system. Reference [12] analyzed the error probability and outage probability of the fixed gain AF relaying approach and compared these results with those of the CSI-based relaying scheme. Employing the CSI-assisted AF relaying, [21] assumed that multiple Rayleigh faded interference was present at the relay node and derived the exact outage probability and accurate bound of the BER. Moreover, the outage performance was investigated for the

dual-hop AF and DF relaying systems in the presence of multiple interferers with arbitrary power levels at both relay and destination [27].

Considering the direct channel in the single-relay cooperative system, Boyer *et al.* in [10] investigated the BER performance of AF relaying protocol and proved that a full diversity order of two is achievable by combining the direct transmission, which is higher than the diversity gain of the multi-hop relaying systems without direct channel. The outage probability of the fixed AF relaying protocol for the single-relay cooperative diversity model was analyzed by Laneman *et al.* in [9], which also showed that its achievable diversity gain is two and emphasized the advantages of the cooperative technique.

Instead of the deterministic relaying scheme, also called fixed relaying in [9], an efficient opportunistic cooperation scheme was proposed and employed for the single-relay cooperative system [28, 29]. The relay amplifies and forwards received signals from the source only if no outage occurs in the source-relay transmission; otherwise, the relay is silent and the source retransmits the signal in the second time slot. Its closed-form BER expression was derived in [28] and its BER performance was shown to outperform that of coded cooperation used in [30, 31]. The authors further developed the selection diversity combining (SDC)-based and the MRC-based opportunistic cooperation schemes and analyzed their outage probabilities in [29]. It can be concluded from numerical results that both opportunistic relaying schemes achieve better outage performance than the deterministic relaying scheme and the MRC-based scheme outperforms the SDC-based one.

The orthogonal all-participate (AP) AF relaying protocol was employed in the multiple-relay cooperative networks in [22, 23, 32–34], where all the relays amplify and forward signals successively in orthogonal time slots. In [32], the asymptotic SER of the CSI-based AF relaying protocol at sufficient large SNR values was de-

rived for the system consisting of an arbitrary number of hops in each branch and an arbitrary number of branches. Still for the CSI-based AF relaying, the exact, upper and lower bounds of SER valid for all SNR regimes were derived by the MGF-based approach in [23]. The outage probability at high SNR and the achievable diversity were analyzed in [33]. The error probability and outage probability of the multi-hop, multi-branch and multi-hop multi-branch systems with fixed gain relays were investigated and their bounds were evaluated for large SNR values [22]. Furthermore, assuming multiple antennas installed at the destination, [34] derived the exact and asymptotic SERs for MPSK signals for both fixed gain and CSI-based AF relaying schemes.

Although the orthogonal AP AF relaying protocol achieves the full cooperative diversity gain, it wastes time and channel resources, especially when systems consist of a large amount of relays. To increase the bandwidth usage, the best relay selection protocol was proposed and widely adopted in the literature. In this efficient protocol, only the best relay that maximizes the instantaneous end-to-end SNR is selected to forward signals to the destination. The outage performance of relay selection combined with CSI-assisted and fixed gain based AF relaying schemes was investigated in [16] and [35], respectively. It was shown that the relay selection achieves the same diversity order and lower outage probability than the AP relaying protocol. Focusing on the CSI-based AF relaying, Yi Zhao *et al.* analyzed the asymptotic SER of the relay selection scheme and compared it with that of the AP relaying approach in [36]. Moreover, they investigated the optimal power allocation (OPA) for both AP and relay selection schemes to minimize their outage probabilities and presented the simulated results to illustrate that the relay selection scheme with OPA significantly outperforms the AP scheme with OPA in terms of throughput and outage probability [37]. Using a simplified max-min relay selection method, the authors in [38] derived the upper and lower bounds of the outage probability,

average transmission rate and the SER for MPSK modulation.

In order to further improve the spectrum efficiency, an incremental relaying protocol was developed in [9]. According to this protocol, relays assist in forwarding the transmission from the source to the destination only when the received SNR of the direct link is less than a threshold, e.g., the direct transmission is in outage or the destination cannot decode the source's signals successfully. The outage probability of incremental AF (IAF) relaying protocol was analyzed in [9] and shown to outperform the AF relaying and the adaptive DF relaying protocols with 1.5-dB SNR. The IAF relaying also outperforms the other schemes in terms of DMT and achieves the optimal DMT as the 2×1 multiple-input single-output (MISO) channel does. Reference [39] derived the asymptotic error probability, outage probability and average achievable rate of the IAF relaying for the single-relay cooperative system and provided the performance with different values of the threshold. If the threshold is equal to zero, the incremental relaying protocol is identical to the direct transmission. If the threshold goes to infinity, it becomes the regular relaying protocols. Numerical results in [39] showed that the larger the threshold, the better the BER performance but the lower the throughput is, which reflects the trade-off relationship between the transmission accuracy and transmission rate. In the system composed of multiple relay nodes, the best relay selection scheme was applied with the IAF relaying in [40] and two extended protocols, namely, opportunistic incremental selection AF (OISAF) and opportunistic joint incremental selection relaying (OJISR), were proposed as well. The asymptotic outage probabilities of these three protocols were evaluated and compared via numerical results. It was shown that all of these protocols achieve the identical diversity order and the OJISR protocol outperforms the others since it considers both AF and DF relaying methods. However, the incremental relaying protocol is more suitable for the case that the direct channel is in a good condition; otherwise, it loses its advantages of achieving both high diversity

and multiplexing gains.

In this circumstance, non-orthogonal AF (NAF) relaying protocols have been widely used in references [13, 17, 25, 41–43]. Firstly, an NAF relaying protocol for a single-relay cooperative system was proposed by Nabar *et al.* in [13]. It differs from the previous orthogonal relaying protocols in that the source is allowed to transmit new messages in the second time slot while the relay forwards to the destination, which significantly improves the throughput and efficiency of the system. The authors investigated and compared the achievable mutual information and outage probabilities of this NAF protocol and two orthogonal AF relaying schemes, and concluded that the NAF protocol is superior to the others in terms of both capacity and diversity. Later, in [17], Azarian *et al.* established the DMT upper bound of the single-relay system and also proved that the NAF scheme achieves this upper bound and is the optimal AF relaying scheme for the half-duplex single-relay channel. The authors then generalized the NAF protocol to a large system with arbitrary number of relays and characterized the achievable DMT curve. It was shown that the NAF protocol outperforms the DSTC-based protocol [14] and relay selection scheme [16] for the multiple-relay cooperative networks in all multiplexing gain regime. However, it was found that when the multiplexing gain is higher than 0.5, the DMT curve of the NAF protocol is identical to that of the direct transmission and the achievable diversity gain does not improve as the number of relays increases.

To fully exploit the diversity in the high multiplexing gain regime, Yang and Belfiore proposed the slotted AF (SAF) scheme and further developed the sequential SAF scheme in [25], in which the source keeps transmitting new data in every time slot and from the second time slot, in each slot, one and only one relay is selected to forward the linear combination of its received signals in previous time slots. Reference [25] established the upper bound of the DMT for the multiple-relay system

operating in arbitrary number of time slots and proved that the sequential SAF scheme is capable of achieving the DMT upper bound with the assumption that relays are isolated. Moreover, for the two-relay system considering the inter-relay transmission, the three-slot sequential SAF protocol still can obtain the DMT upper bound and outperforms the NAF scheme of [17] in the multiplexing gain region $r \leq 2/3$. Specifically, [41] considered the inter-relay interference between two relays and assumed that one of the relays is assigned to perform interference cancellation. The achievable DMT was shown to approach that of an MISO channel as the number of time slots is very large. Assuming that the inter-relay interference is absent or present, a modified random sequential (RS) SAF protocol was proposed for the cooperative system with arbitrary number of multiple-antenna relays and its achievable DMT was investigated in [42]. A generalized sequential SAF (GSSAF) scheme was proposed and applied to cooperative multiple-relay channels, cooperative broadcast channels (CBC) and cooperative multiple-access channels (CMAC) in [43]. More importantly, it was shown that the GSSAF strategy can achieve the DMT upper bound of each channel model.

Decode-and-forward relaying

Unlike in the AF relaying protocol, relays do not forward the noise signals in the DF relaying protocol. However, the incorrectly decoded signals are forwarded by relays as well as the correctly decoded ones in the fixed DF relaying protocol, which will cause error propagation and degrade the BER and outage performance. This phenomenon has been illustrated in the early papers [9, 10]. To solve this problem, a piece-wise linear (PL) decoder was adopted by the destination and its achievable BER performance of the cooperative network was studied in [44]. It was shown that the PL decoder only achieves the diversity order of $\lceil M/2 \rceil + 1$ in the system with

M parallel relays and the direct channel. In order to achieve the full cooperative diversity, the authors in [45] developed a low-complexity high-performance C-MRC demodulator for the destination. The C-MRC scheme fully considers the channel quality of each hop in each path, and thus it requires the destination to have the perfect CSI of each link. Instead of employing C-MRC at the destination, the same group of people then proposed a link-adaptive regeneration (LAR) strategy to explore the full spatial diversity in [46], where relays smartly determine complex power scaling factors based on the source-relay and relay-destination CSI. The instantaneous and average channel gains of relay-destination links are obtained by the relays via the feedback from the destination. Additionally, in [47], Yi and Kim combined the C-MRC and the LAR with the relay selection scheme, and derived their BER upper bounds of a multiple-relay DF cooperative system. It was shown that the full diversity order is also achievable when the relay selection is adopted.

Apart from the C-MRC and LAR methods employed with the fixed DF relaying protocol, the adaptive DF relaying protocol is often adopted to achieve the full diversity. In [48, 49], the threshold-based selective relaying scheme was employed for the single-relay DF relaying network, the optimal threshold to minimize the end-to-end BER was derived and the relationship between threshold and achievable BER was further analyzed.

The adaptive DF relaying protocol is more widely employed in multiple-relay cooperative networks. In [50], relays forward the correctly decoded signals in orthogonal time slots. The exact and approximate SER expressions of MPSK and MQAM signals for a multiple-node network were derived. The results showed that the full diversity order equal to the number of relays is achievable. For the same system model, [51] studied when to cooperate and how to select a relay to cooperate. The authors in [51] adopted the relay selection method that a relay with the maximum

harmonic mean of instantaneous source-relay and relay-destination channel gains is selected to decode and forward if necessary, which is capable of achieving the full diversity as [50] and higher bandwidth efficiency than [50]. In [52], the authors derived the error probability and outage probability of two single relay selection schemes, namely, max-min relay selection and opportunistic relay selection, and compared these two selection schemes in terms of the BER and outage performance for the multiple-relay system with the direct channel. A modified max-min relay selection approach was proposed in [53], where the relay selected based on the max-min manner forwards only if the minimum of its source-relay and relay-destination channel SNRs is larger than the SNR of the direct link. Its closed-form SER expression was derived and its performance was shown to be better than that in [47, 51]. An improved opportunistic relay selection scheme was developed in [54], in which the source also participates the opportunistic selection in the second time slot. Specifically, the relays will keep silent and the source will retransmit in the second time slot if the channel gain of the direct link is higher than the relay-destination channel gains of those relays in the decoding set. Reference [54] analyzed the SER for both MPSK and MQAM modulations and also presented that the improved opportunistic relaying scheme slightly outperforms the conventional one with less than 1-dB SNR gain.

In addition to the error probability, many papers have concentrated on analyzing the outage performance of multiple-relay cooperative diversity systems with the single relay selection schemes. In [55, 56], the max-min relay selection was employed. Reference [55] firstly provided the CDF, PDF and MGF of the output SNR and then derived the upper bound and lower bound of the outage probability, which were proved to be tight with the simulation results. Moreover, the exact expression and the tight approximation of the outage probability were presented in [56]. The opportunistic relaying protocol was adopted in [35, 57, 58]. The exact outage prob-

ability expression of the multiple-relay cooperative system without direct link was provided in [35]. Taking into account of the direct transmission, [57] derived the approximate outage probability at high SNR regimes and presented that the opportunistic relaying protocol performs better than the DSTC protocol in the networks with more than three relays. The exact outage probability and the average capacity in close-forms and valid over all SNR regimes were derived in [58]. For the system that relays have multiple antennas, [59] adopted an antenna processing scheme for the relays that MRC was employed for reception and transmit beamforming was used for retransmission, which was proved to achieve the full spatial diversity order equal to the total number of antennas of all relays, regardless of the number of relays and antennas in each relay.

As the incremental relaying strategy was proposed and used with the AF relaying protocol in [9], it can also be employed for the cooperative DF relaying systems to improve the spectral efficiency. For the single-relay case, the average error probability, outage probability and transmission rate of the incremental DF (IDF) relaying scheme were analyzed and derived in [39]. For the multiple-relay system, IDF was combined with the best relay selection schemes in [60–62]. In [60], the best relay selected via the max-min method will forward the source's signals only when the direct transmission fails to satisfy a certain requirement. The accurate close-form expression of the outage probability was derived and validated by the simulation results. Using the opportunistic relaying strategy that a relay with the largest instantaneous relay-destination channel gain is chosen from a decoding set, [61] derived the approximate outage probability of the opportunistic IDF scheme at high SNR values and showed that the achievable diversity order is identical to the MISO and single-input multiple-output (SIMO) systems. Ikki and Ahmed also considered the opportunistic IDF scheme in [62] and derived the BER, outage probability and average channel capacity in close-form expressions.

In the case that the direct link is in a very poor condition, the incremental relaying protocol is not an ideal choice to achieve the best DMT performance. Next, we review the DMT analysis of some static DF relaying protocols, where the time duration of broadcast and cooperation phases is fixed and independent of the channel fading coefficients. Reference [13] proposed an efficient nonorthogonal selection DF (NSDF) relaying protocol for the single-relay cooperative system, which differs from the orthogonal selection DF (OSDF) protocol of [9] in that the source continues to transmit a new independent message in the second time slot. The maximal multiplexing gain is accordingly improved from 0.5 to 1. However, the message transmitted in the second slot does not have the benefit of cooperation and the overall diversity gain is limited to one, as shown in Figure of [17]. Prasad and Varanasi proposed an enhanced static DF (E-SDF) protocol for the single-relay scenario [63], where the source transmits an independent codeword of another codebook during the cooperation phase. The durations of two phases in a transmission block are fixed, irrelevant of the multiplexing gain but may not be equal as the NSDF protocol of [13]. The authors obtained the optimal time allocation and characterized the corresponding DMT upper bound of the E-SDF protocol. It was shown that the maximal achievable diversity gain is two and the E-SDF protocol has the same DMT performance as the NAF protocol [17] for $r \in [0, 1/3]$ and is superior to NAF for $r \in [1/3, 1]$. Furthermore, based on whether the ratio of two phases' durations independent of the multiplexing gain or not, Elia *et al.* considered fixed-NSDF and variable-NSDF protocols for the multiple-relay systems and determined their DMT [18]. When the time allocation ratio falls within a certain range, the fixed-NSDF protocol dominates the NAF protocol [17] in a certain multiplexing gain range. For the single relay case, the variable-NSDF protocol has better DMT performance than the E-SDF protocol for $r \in [0, 1/2]$ but worse DMT for $r \in [1/2, 1]$. In the case of more than two relays, the variable-NSDF protocol was proved to achieve

the best DMT performance compared to its previously known static protocols.

Apart from the static protocols, the class of dynamic DF (DDF) protocols was investigated in some papers to achieve better DMT performance. In DDF protocols, the durations of broadcast and cooperation phases are dynamic and dependent on the source-relay channel coefficients, specifically determined by when the source-relay mutual information exceeds a certain threshold or when relays can decode messages correctly. In [17], Azarian *et al.* developed a DDF protocol and evaluated its DMT for both single relay and multiple-relay scenarios. It was shown that the DDF protocol achieves the optimal DMT for $r \leq 1/M$, where $M - 1$ is the number of relays. Moreover, the DDF protocol was presented to be superior to the NAF protocol, since relays transmit independent Gaussian codebook in DDF but they forward linearly amplified noisy signals in AF schemes. However, in the range of $r \geq 1/2$, its DMT performance is identical for any value of M , which is still very poor for multiple-relay systems. Prasad and Varanasi also established an enhanced DDF (E-DDF) protocol for the single-relay case [63], which differs from the DDF protocol in that the source transmits two independent codewords from two different codebooks and only the first codeword is decoded and forwarded by the relay. The DMT upper bound of the E-DDF protocol was characterized and shown to achieve the optimal DMT bound for $r \in [0, 1/2]$ and outperform that of the DDF protocol for $r \in (1/2, 1)$. It is the best achievable DMT performance of the single-relay case to our best knowledge.

Nakagami- m fading channels

Previous papers have evaluated the performance of kinds of relaying schemes for different cooperative networks over Rayleigh-fading channels. It is well known that Rayleigh distribution is a simple and commonly used channel model, while

Nakagami- m distribution is a more general, flexible and practical model to characterize the variable wireless channels. Therefore, many papers have also studied the performance of different cooperative networks in the presence of Nakagami- m fading channels. For the class of AF relaying protocols, in [64], Hasna *et al.* first presented the statistics of the harmonic mean of two independent and identically distributed (i.i.d.) gamma random variables, and then applied these results to study the error probability by the MGF-based method as well as the outage probability of a simple two-hop single-relay system without direct link. Karagiannidis expanded the analysis to a multihop AF relaying network with i.n.i.d Nakagami- m faded channels and derived the lower and upper bounds of BER and outage probability in [65] and [66], respectively. With the aid of the MGF-based method, the error probability of a system with multiple parallel AF relays was analyzed in [24, 67] in the i.n.i.d Nakagami- m fading environment. The CDF-based method was also employed in [68] to derive the error probability for the cooperative system with a fixed gain based AF relay, and the results are suitable for many modulation schemes and i.n.i.d. Nakagami- m fading channels.

Employing the adaptive DF relaying protocol, [2, 69] investigated the error probability of a single-relay cooperative diversity system experiencing i.n.i.d. Nakagami- m fading channels. In [2], Ikki *et al.* assumed that the relay only forwards if its received SNR is higher than a threshold and derived the optimal threshold to obtain the best BER performance. In [69], the relay only retransmitted the correctly decoded signals with the aid of CRC codes and the optimal power allocation between the source and the relay was further studied to improve the SER performance of both MPSK and MQAM modulations. For a multiple-relay cooperative system, by assuming that all the relays in the decoding set orthogonally decode and forward signals to the destination, [70] and [71] analyzed the outage probability in the presence of identical and non-identical Nakagami- m fading channels, respectively. In addition,

the SER and outage probability of the multiple-relay DF cooperative system were analyzed in [72], where a single relay is determined by the max-min selection as in [16]. By using the opportunistic DF relaying protocol [35], the outage performance was provided and the optimal power allocation issue was addressed in [73] as well. In addition, the opportunistic relay selections based on partial CSI and full CSI were employed in [74], and the corresponding error probabilities were derived by the CDF-based approach.

2.2.2 Multiple-user Cooperative Systems

Two-way Relaying Networks

In two-way relaying networks, at least a pair of terminals not only work as sources to transmit signals but also act as destinations to receive incoming signals, and the bidirectional transmission is assisted by relays. Following the conventional relaying schemes in [9] and satisfying the half-duplex constraint, the transmission for a simple two-source system can be conducted as that one user transmits first and the other user transmits after it received its desired signals. Hence, this two-way relaying scheme requires four phases to exchange messages between two users, which is spectrum inefficient. Depending on the broadcast nature of wireless channels and the fact that each user has the knowledge of its own information, Rankov and Wittneben proposed an efficient transmission protocol using only two phases for two-way relaying networks in [75]. In the first phase, two users transmit their signals simultaneously and the relay receives the sum of signals from both users. In the second phase, the relay broadcasts to both users the scaled signal under the transmit power constraint in the AF protocol or decodes both signals and forwards the superposed signal in the DF relaying scheme. Although the efficiency for each user

is not changed, the total efficiency for two users is significantly improved. In [75], the sum-rates of this efficient two-way relaying scheme under AF and DF protocols were investigated and shown to outperform that of the one-way relaying scheme.

Focusing on the AF relaying, [76] derived the tight upper and lower bounds of the average sum-rate for the two-way relaying system. However, it was noted that for the simple three-node system operating in the two-phase two-way relaying strategies, the direct transmission cannot be realized and the cooperative diversity cannot be achieved. Motivated by this, [77] analyzed the error exponent of the two-way AF relaying protocol to characterize the fundamental trade-off between the transmission reliability and the rate. In addition, optimal rate allocation and power allocation schemes were proposed to maximize the bottleneck error exponent and proved to improve the transmission reliability by numerical results. In [78], the two-way AF relaying transmissions over two, three and four phases were all investigated. For a fair comparison, the direct channel was not considered in every transmission scheme. Specially, in the three-phase transmission scheme, two users transmit to the relay successively in the first two time slots and the relay transmits a function of the received signals to both users in the third time slot. The exact outage probability, the closed-form sum-BER and the tight upper bound of the maximum sum-rate for a single-relay system were derived and compared. It was shown that the two-phase scheme achieves a higher maximum sum-rate but a lower sum-BER than the four-phase scheme due to the self-interference and that the three-phase scheme obtains a balance between the sum-BER and the maximum sum-rate. Furthermore, a large system of multiple relays was also considered in [78] and a single relay which maximizes the sum-rate or minimizes the sum-BER is selected for relaying. The performance and the diversity order of the two-way multiple-relay system were significantly improved, since more available paths are constructed by multiple relays. For the system consisting of two single-antenna sources and a multiple-antenna

relay, [79] analyzed the capacity region of the two-phase AF relaying protocol and designed the optimal as well as suboptimal relay beamforming schemes for the relay node.

Adopting the DF relaying strategy, [80] analyzed the rate regions of coded bidirectional transmission protocols operating in two, three and four phases, namely, multiple-access broadcast (MABC) protocol, time division broadcast (TDBC) protocol and hybrid broadcast (HDBC) protocol, respectively. Numerical results showed that the HDBC protocol can even achieve rates outside the outer bounds of MABC and TDBC protocols in some cases. The outage performance of DF-based MABC, TDBC and opportunistic source selection (OSS) protocols for the single-relay two-way relay system was studied in [81]. In the OSS protocol, only a single user with the larger mutual information to the other user is allowed to transmit in each channel block, which exploits the multiuser diversity in the time-varying channels. Moreover, [81] analyzed the optimal power allocation and relay location based on the asymptotic outage probabilities, and derived the DMT of each protocol in finite and infinite SNR values. Unlike reference [80] assuming perfect channel state information at the transmitters (CSIT), [82] considered a two-way single-relay system in a practical scenario, and only quantized CSIT can be obtained by dedicated low-rate feedback links. It studied and compared the achievable DMT of different adaptive two-way DF protocols in many cases, i.e., no CSIT or partial CSIT, with power control or not, the direct link present or not. It was shown that without the direct link, power control with partial CSIT at the relay can achieve the optimal DMT in certain cases. When the direct link was considered, the best DMT can be achieved if every terminal adapts its transmit power based on the partial CSI from sources. Reference [80] assumed that two sources transmit at the same rate, while [83] considered the more general case that both users are possible to communicate at different rates. It first derived an outer bound of the distortion exponent region for the three-node

two-way relay system and then obtained the optimized distortion exponent pairs of the conventional four-phase protocol, three-phase TDBC and two-phase MABC protocols based on both AF and DF relaying. Additionally, [83] derived the achievable DMT of each user under two-way AF and DF relaying protocols, rather than the DMT of the system as its previous work.

For the system that multiple intermediate relays are available to assist the bidirectional transmission between two users, [84] considered both AF and DF relaying protocols over two, three and four phases and employed the DSTC for the signal processing at relays. The pairwise error probabilities of all schemes were analyzed and the full diversity gain was achievable if the number of symbols in a time slot is no less than the number of relays. However, the space-time codes have to be orthogonal at different relays to achieve better performance, and it is not adaptive to change the system size of relays. Therefore, the relay selection strategy was adopted for the multiple-relay two-way relay networks in [85–88] due to its low-complexity and easy-implement properties. In [85], a relay selection AF protocol operating in two phases was proposed, and a single relay which minimizes the sum SER of both users is selected to broadcast in the second phase according to the optimal relay selection scheme. The paper also proposed a suboptimal min-max relay selection, where a relay minimizing the maximum SER of two users is chosen. The asymptotic SER was derived and thus the optimal power allocation between users and the selected relay was determined, which was shown to outperform the equal power allocation scheme via simulation results. Two DF-based relay selection schemes were considered in [86] for the two-way relay channels, namely, max-min scheme and max-sum scheme, which maximize the minimum instantaneous rate of two sources and the sum rate, respectively. The outage probabilities of both selection methods were analyzed and showed that the max-min relay selection scheme achieves better performance in the high SNR regime and the max-sum scheme has superiority in the

low SNR range. Thus, a hybrid relay selection protocol switching between max-min and max-sum schemes was proposed as well to achieve better outage performance in all SNR regime. In [86], a best relay which maximizes the harmonic mean of both source-relay channel coefficients was selected from a decoding set. The error probability of the DF-based two-way relaying system with this opportunistic relay selection scheme was analyzed and the error performance was shown to outperform that of the DSTC-based two-way relaying protocol [84] through numerical results. Furthermore, [88] proposed two relay selection protocols for the two-way DF relaying system and analyzed the BER performance operating in three time slots. One relay selection scheme is to select a single relay which minimizes the worse instantaneous BER of two users as the min-max criterion, which was shown to achieve almost exactly the same performance as the optimal one that minimizes the sum BER of both users. The other selection scheme is double relay selection, which selects one best relay for each user. The exact BER expressions of both schemes were derived and verified by simulation results. It was shown that both relay selection schemes are capable of achieving the full diversity order, and the double relay selection scheme achieves even better performance than the single relay selection.

Some papers have considered the system that multiple pairs of users wish to exchange information with their preassigned partners through an intermediate relay [89–91]. Therefore, not only the self-interference signal but also the multi-user interference signal might be present at each user when it receives from the relay, and the interference management and cancellation becomes an important design issue for the multiuser two-way relay channels. In [89], the code division multiple access (CDMA) technique was employed to differentiate many pairs of users, and each pair of users share a common spreading code. The two-way relay transmission operates in two phases, and the relay demodulates the received signals and transmits the XOR superposed signal to all users in the second phase. Reference [89] derived the

decision rules and the BERs at the relay and users, and further investigated the power control and receiver optimization issues for both phases. The proposed two-way relaying scheme was shown to outperform the one-way scheme in terms of power saving and capacity. In [90], each pair of users are assumed to occupy an orthogonal channel, and thus the co-channel interference from other pairs is avoided. For three-phase and two-phase DF as well as two-phase AF and CF relaying schemes, the authors studied the power allocation among different pairs to maximize the arbitrary weighted sum rate of all users. Reference [91] focused on studying efficient transmission schemes over two phases for the multiuser two-way AF relay system without using the orthogonal code, space, frequency or time resources. Assuming that multiple antennas are installed at each user and the relay, the authors designed the transceiver processing for the relay to eliminate both self-interference and co-channel interference based on zero-forcing and minimum mean-square-error criteria and various predetermined transmit and receive beamforming methods. Local and global power control problems were also investigated to obtain the fairness among all users and to maximize the SNR. Numerical results about the system SNR, sum rate and BER were provided to verify the effectiveness of the proposed relay processing and power control methods.

Cooperative Multiple-Access Channels (CMAC)

In CMAC, multiple users not only transmit their own information but also forward other users' information to a common destination, obtaining the cooperative diversity and better performance. Some papers assigned orthogonal time slots [30, 31, 92, 93], frequency bands [94], spreading codes [95, 96] or space-time codes [97] for multiple users to differentiate their transmission and eliminate the MAI. Instead of occupying so many channel resources, some papers designed the

intelligent, spectrally efficient signal processing and decoding methods at sources and destination.

Reference [17] employed the AF relaying protocol for the multi-user CMA system. In every channel-static block, each source successively transmits a linear combination of its own signal and the signal it overheard in the last time slot. This proposed schemes was proved to achieve the optimal DMT performance even under the half-duplex constraint. The DF relaying protocol was considered for the multi-user CMA system in [26], where each user transmits a superposition of its own information as well as the others' decoded information. The achievable DMT of the superposition CMA system was also characterized and shown to outperform those of the non-cooperative MA system and the MA system with selective relaying, furthermore, it is the optimal DMT performance for the MA system. Specifically, [98] considered a CMA system of two single-antenna sources and a two-antenna destination. The automatic repeat request-DDF (ARQ-DDF) protocol was adopted and its DMT upper bound as well as achievable DMT were investigated. It was shown that the ARQ-DDF is capable of obtaining the optimal diversity order of three. Some papers have further studied the BER and outage performance of the CMA systems with two sources, and will be introduced in Section 4.1.

Multiple-Access Relay Channels (MARC)

In the multi-user MARC model, multiple users wish to send information to a common destination with the assistance of one or multiple relays. The MARC is distinguished from the multiple-access channel (MAC) in that one or more relays exist and solely intend to facilitate the communication. The intermediate relays not only provide diverse paths to achieve the spatial diversity, but also transmit the combination of multiple users' signals in order to save the bandwidth usage. References [98–100]

focused on the simple system of two users and a single relay. In [99], the CF relaying protocol was employed and sources keep transmitting all the time. The relay listens for a fraction of time, employs source coding with side information to compress received signals, and forwards to the destination in the rest fraction of time. The achievable DMT of the CF strategy was derived in [99], and it is optimal for the high multiplexing gains. In addition, the DDF protocol was adopted for the MARC model in [98], and proved to achieve the optimal DMT for low and medium multiplexing gains. Similar to the CF protocol, in the DDF strategy, two users transmit the full codeword, while the relay listens to the sources until it collects enough information to decode both error free. The relay then jointly encode two signals via an independent codebook and transmits for the rest of time. Furthermore, [100] applied the AF relaying protocol to the MARC system, which was proved to achieve better DMT performance than the CF protocol at low multiplexing gains and outperforms the DDF protocol at high multiplexing gains.

An MARC system consisting of more than two sources and a multiple-antenna relay was investigated in [101]. The relay employed the DSTC to linearly process the received signals, while the multiple-antenna destination performed zero-forcing interference cancellation to detect signals from different sources. Two transmission schemes were proposed for this MARC system and their achievable diversity orders were analyzed accordingly. Besides, [102–104] considered the MARC networks composed of multiple sources, multiple relays and a destination. In [102], K sources transmit their signals simultaneously in the first time slot, and $K - 1$ relays are selected to successively forward the scaled combination to the destination in the following $K - 1$ time slots. Its theoretical and numerical results of the ergodic capacity and the outage probability were provided to show that the proposed protocol can achieve both large ergodic capacity and full diversity gain. Reference [103] employed the DF protocol for the relays, and designed a transmission scheme that a source

with a poor connection to the destination has a higher priority to obtain help from a relay with better relay-destination channel condition. The full diversity gain was shown to be achievable for each user, and the DMT of the proposed scheme was derived and approaching the optimal DMT upper bound if the number of relays is very large. The authors in [104] also considered the DF relaying strategy and proposed a novel protocol that divides sources and relays into clusters. The nodes within one cluster transmit non-orthogonally, and different clusters transmit in orthogonal channels. The DMT of this protocol was characterized, which can be optimized by properly clustering.

2.3 Conclusion

In this chapter, the commonly used performance measures of communication systems were introduced, such as BER, outage probability and DMT. The definitions and the general derivations of these measures were given. We have explicitly reviewed the works and development of cooperative communications in recent years, from AF relaying to DF relaying, from single-user communication to multiple-user communication.

Chapter 3

BER Performance of a Single Relay Cooperative System

3.1 Introduction

In this chapter, we consider a simple three-terminal cooperative system, which consists of a source, a relay and a destination. The BER performance of the single relay system employing the fixed AF relaying protocol was investigated in [9, 10]. It is demonstrated that the system can achieve the full diversity order of two for both CSI-based and fixed gain based AF relaying schemes. However, from the error probability and the outage probability analysis and numerical results in [9, 10], it is seen that the fixed DF relaying protocol cannot achieve the full diversity as the fixed AF relaying protocol does if the MRC scheme is employed at the destination. The reason is that in the AF relaying protocol, the destination is assumed to have the full channel knowledge of all the links and use them to recover the transmitted signals by MRC. Hence, the full diversity is achievable by using the AF protocol. On the contrary, in the fixed DF relaying protocol, the relay node forwards every decoded

signal to the destination without considering whether its decoding is correct or not. The MRC detection only uses source-destination and relay-destination channel information to decode. Thus the full diversity cannot be achieved by the fixed DF protocol. In addition, the adaptive DF relaying protocol, proposed in [9], is proved to be capable of achieving the higher diversity gain and better performance than the fixed DF protocol. Although only the source-destination and relay-destination channel information is used for MRC at the destination, the relay may or may not forward the decoded signal to the destination based on a threshold and the quality of received signals.

If the fixed DF relaying protocol is still employed rather than the adaptive DF relaying, is it possible to achieve the full diversity? The answer is yes. In this chapter, we will study the C-MRC scheme, which can fully explore the available spatial diversity and obtain even better BER performance than the adaptive DF relaying. Unlike MRC, C-MRC considers all the channel information of source-relay, source-destination and relay-destination links to decode. Wang et al. analyzed the BER performance of the C-MRC scheme over Rayleigh fading channels and concluded that C-MRC can achieve the diversity order equal to the number of paths between the source and the destination [45]. Changing the channel assumption to a more general model, we analyze and compare the BER performance of the MRC and C-MRC schemes over Nakagami- m fading channels. It is well-known that Nakagami- m distribution is more suitable and flexible to model various wireless channel environments [19]. More importantly, the results in Nakagami- m fading channels can more explicitly illustrate the factors which affect the performance of the fixed DF relaying protocol. In addition, C-MRC is proved to provide better BER performance than the adaptive DF relaying protocol with the optimal threshold.

3.2 System Model and Transmission Protocol

The system model is shown in Figure 1.1(b). We use S , R and D to represent the source, relay and destination nodes, respectively. Each node is assumed to have one antenna and work in the time division half-duplex mode. The channel from node i to j , denoted as h_{ij} , is independent flat-faded channel and keeps stationary within a block duration. The block duration is equally divided into two time slots. In the first time slot, S broadcasts signal x_s to D and R with the average power P_S . The received signals at D and R are respectively given by

$$y_{SD} = \sqrt{P_S} h_{SD} x_S + n_{D1} \quad (3.2.1)$$

and

$$y_{SR} = \sqrt{P_S} h_{SR} x_S + n_{R1}, \quad (3.2.2)$$

where n_{D1} and n_{R1} are the corresponding noise terms of D and R in the first time slot. In this chapter, we only consider that x_S is a binary phase-shift keying (BPSK) signal and $E\{|x_S|^2\} = 1$. In the second time slot, according to the fixed DF relaying protocol, S does not transmit and R forwards the decoded signal x_R to D with the transmitted power P_R . The transmitted signal from R is obtained by

$$x_R = \text{sgn} \left\{ \Re \left(\sqrt{P_S} h_{SR}^* y_{SR} \right) \right\}, \quad (3.2.3)$$

where $\text{sgn}\{\cdot\}$ is the signum function, $\Re(\cdot)$ denotes the real part of a complex number, and $(\cdot)^*$ denotes conjugation. The received signal at D is given by

$$y_{RD} = \sqrt{P_R} h_{RD} x_R + n_{D2}, \quad (3.2.4)$$

where n_{D2} is the noise term at D in the second time slot. Without loss of generality, we assume n_{D1} , n_{R1} and n_{D2} are independent complex Gaussian random variables with zero mean and variance N_0 . In this chapter, we assume that the channel coefficient h_{ij} follows the Nakagami- m distribution with the corresponding fading parameter m_{ij} and the average channel gain $\sigma_{ij}^2 = E\{|h_{ij}|^2\}$. The PDF of $|h_{ij}|$ is given by

$$p(|h_{ij}|) = \frac{2(m_{ij}/\sigma_{ij}^2)^{m_{ij}}}{\Gamma(m_{ij})} |h_{ij}|^{2m_{ij}-1} \exp\left(-\frac{m_{ij}}{\sigma_{ij}^2} |h_{ij}|^2\right). \quad (3.2.5)$$

Therefore, the instantaneous received SNR for the channel from i to j , defined as $\gamma_{ij} = P_i |h_{ij}|^2 / N_0$, is gamma distributed with PDF given by [70]

$$p(\gamma_{ij}) = \frac{(m_{ij}/\bar{\gamma}_{ij})^{m_{ij}}}{\Gamma(m_{ij})} \gamma_{ij}^{m_{ij}-1} \exp\left(-\frac{m_{ij}}{\bar{\gamma}_{ij}} \gamma_{ij}\right), \quad (3.2.6)$$

where $\bar{\gamma}_{ij}$ is the average received SNR for the link from i to j and $\bar{\gamma}_{ij} = P_i \sigma_{ij}^2 / N_0$.

3.3 Maximal Ratio Combining (MRC) and Cooperative MRC (C-MRC)

At the end of the second time slot, the destination uses two weighting factors, w_{SD} and w_{RD} , to combine the received signals from S and R , respectively. Depending on whether R decodes correctly or not, the signal after combining is expressed as

$$y_{co} = w_{SD} y_{SD} + w_{RD} y_{RD}$$

$$= \begin{cases} (w_{SD}\sqrt{P_S}h_{SD} + w_{RD}\sqrt{P_R}h_{RD})x_S + w_{SD}n_{D1} + w_{RD}n_{D2}, & \text{if } x_R = x_S, \\ (w_{SD}\sqrt{P_S}h_{SD} - w_{RD}\sqrt{P_R}h_{RD})x_S + w_{SD}n_{D1} + w_{RD}n_{D2}, & \text{if } x_R = -x_S. \end{cases} \quad (3.3.1)$$

Hence, the end-to-end conditional error probability can be computed as

$$P_e(\gamma_{SD}, \gamma_{SR}, \gamma_{RD}) = \left(1 - P_{R,e}(\gamma_{SR})\right) P_{D,c}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD}) + P_{R,e}(\gamma_{SR}) P_{D,e}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD}), \quad (3.3.2)$$

where $P_{R,e}(\gamma_{SR})$ represents the conditional error probability that R decodes signals from S . For BPSK signals, $P_{R,e}(\gamma_{SR}) = Q(\sqrt{2\gamma_{SR}})$ [1]. Provided that R decodes the signal correctly or incorrectly, the conditional error probabilities that D decodes the combined signals are denoted as $P_{D,c}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD})$ and $P_{D,e}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD})$, respectively. Since MRC and C-MRC schemes set different values for the weighting factor w_{RD} , we will analyze them separately in the following.

In the MRC scheme, D is assumed to have perfect channel knowledge of S - D and R - D links, but it does not require the CSI of the S - R link. The weighting factors are set to be $w_{SD} = \sqrt{P_S}h_{SD}^*$ and $w_{RD} = \sqrt{P_R}h_{RD}^*$ [1]. Hence, the received SNRs from direct and relay links are γ_{SD} and γ_{RD} , respectively. Substituting w_{SD} and w_{RD} into (3.3.1), we obtain the instantaneous SNR of the combined signal as

$$\gamma_{co}^{MRC} = \begin{cases} \gamma_{SD} + \gamma_{RD}, & \text{if } R \text{ decodes correctly,} \\ \frac{(\gamma_{SD} - \gamma_{RD})^2}{\gamma_{SD} + \gamma_{RD}}, & \text{if } R \text{ decodes incorrectly.} \end{cases} \quad (3.3.3)$$

Therefore, for BPSK signals, we have

$$P_{D,c}^{MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD}) = Q\left[\sqrt{2(\gamma_{SD} + \gamma_{RD})}\right] \quad (3.3.4)$$

and

$$P_{D,e}^{MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD}) = Q \left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{RD})}{\sqrt{\gamma_{SD} + \gamma_{RD}}} \right]. \quad (3.3.5)$$

The conditional error probability for the MRC scheme is accordingly obtained by substituting (3.3.4) and (3.3.5) into (3.3.2). Furthermore, we have

$$P_e^{MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD}) \leq Q \left[\sqrt{2(\gamma_{SD} + \gamma_{RD})} \right] + Q \left[\sqrt{2\gamma_{SR}} \right] Q \left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{RD})}{\sqrt{\gamma_{SD} + \gamma_{RD}}} \right]. \quad (3.3.6)$$

In the C-MRC scheme, D uses the CSI of S - R , S - D and R - D links. As in the MRC scheme, the weighting factor of the S - D link is set to be $w_{SD} = \sqrt{P_S} h_{SD}^*$. For the S - R - D link, the transmission quality is dominated by the worse one of S - R and R - D channels. Hence, the equivalent SNR of the S - R - D link is $\gamma_{\min} = \min\{\gamma_{SR}, \gamma_{RD}\}$. The weighting factor of this link is subsequently defined as

$$\begin{aligned} w_{RD} &= \frac{\gamma_{\min}}{\sqrt{P_R} h_{RD} / N_0} \\ &= \frac{\min\{P_S |h_{SR}|^2, P_R |h_{RD}|^2\}}{\sqrt{P_R} h_{RD}}. \end{aligned} \quad (3.3.7)$$

It can be seen that the denominator in (3.3.7) is used to normalize the amplitude of the received signal as well as to eliminate the phase shift effect. The numerator assigns the minimal power to the signal. The instantaneous SNR of the combined signal is given by

$$\gamma_{co}^{C-MRC} = \frac{(P_S |h_{SD}|^2 \pm w_{RD} \sqrt{P_R} h_{RD})^2}{(P_S |h_{SD}|^2 + w_{RD}^2) N_0}, \quad (3.3.8)$$

where $\pm$ is used as $+$ or $-$ depending on whether R can decode the source's transmission correctly or not. When $P_S |h_{SR}|^2 < P_R |h_{RD}|^2$, $w_{RD}^2 = P_S |h_{SR}|^2 \frac{P_S |h_{SR}|^2}{P_R |h_{RD}|^2} <$

$P_S|h_{SR}|^2$. When $P_S|h_{SR}|^2 \geq P_R|h_{RD}|^2$, $w_{RD}^2 = P_R|h_{RD}|^2$. Hence, w_{RD}^2 can be expressed as

$$w_{RD}^2 \leq \min \left\{ P_S|h_{SR}|^2, P_R|h_{RD}|^2 \right\}. \quad (3.3.9)$$

Substituting w_{RD} and (3.3.9) into (3.3.8), we obtain the lower bound of γ_{co}^{C-MRC} that

$$\gamma_{co}^{C-MRC} \geq \begin{cases} \gamma_{SD} + \gamma_{\min}, & \text{if } R \text{ decodes correctly,} \\ \frac{(\gamma_{SD} - \gamma_{\min})^2}{\gamma_{SD} + \gamma_{\min}}, & \text{if } R \text{ decodes incorrectly,} \end{cases} \quad (3.3.10)$$

where $\gamma_{\min} = \min \{\gamma_{SR}, \gamma_{RD}\}$. Similar to (3.3.6), the conditional BER performance for C-MRC is bounded as

$$P_e^{C-MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD}) \leq Q \left[\sqrt{2(\gamma_{SD} + \gamma_{\min})} \right] + Q \left[\sqrt{2\gamma_{SR}} \right] Q \left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{\min})}{\sqrt{\gamma_{SD} + \gamma_{\min}}} \right]. \quad (3.3.11)$$

3.4 BER Performance Analysis of MRC and C-MRC in Nakagami- m fading channels

In this section, we derive the upper bound of the average error probability and analyze the diversity gain of the fixed DF relaying cooperative system with MRC and C-MRC schemes over Nakagami- m fading channels.

3.4.1 MRC

To derive the average BER of the MRC scheme, we will start from its conditional BER shown in (3.3.6). The first and the second terms on the right-hand side of (3.3.6) are defined as $P_{e1}^{MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD})$ and $P_{e2}^{MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD})$, respectively. Hence, the conditional error probability of the MRC scheme can be expressed as

$$P_e^{MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD}) = P_{e1}^{MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD}) + P_{e2}^{MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD}) \quad (3.4.1)$$

Referring to the Chernoff bound of Q -function given in (2.1.6), the expectation of $P_{e1}^{MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD})$ is upper bounded by

$$\begin{aligned} P_{e1}^{MRC} &= \int_0^\infty \int_0^\infty Q\left[\sqrt{2(\gamma_{SD} + \gamma_{RD})}\right] p(\gamma_{SD}) p(\gamma_{RD}) d\gamma_{SD} d\gamma_{RD} \\ &\leq \frac{1}{2} \int_0^\infty \int_0^\infty \exp\left[-(\gamma_{SD} + \gamma_{RD})\right] p(\gamma_{SD}) p(\gamma_{RD}) d\gamma_{SD} d\gamma_{RD} \\ &= \frac{1}{2} \int_0^\infty \exp(-\gamma_{SD}) p(\gamma_{SD}) d\gamma_{SD} \int_0^\infty \exp(-\gamma_{RD}) p(\gamma_{RD}) d\gamma_{RD}. \end{aligned} \quad (3.4.2)$$

We first introduce one useful integration that [105, Eq. (3.381.4)]

$$\int_0^\infty x^{\nu-1} \exp(-\mu x) dx = \mu^{-\nu} \Gamma(\nu). \quad (3.4.3)$$

With the help of (3.4.3), the first integral in (3.4.2) is computed as

$$\begin{aligned} &\int_0^\infty \exp(-\gamma_{SD}) p(\gamma_{SD}) d\gamma_{SD} \\ &= \frac{(m_{SD}/\bar{\gamma}_{SD})^{m_{SD}}}{\Gamma(m_{SD})} \int_0^\infty \gamma_{SD}^{m_{SD}-1} \exp\left[-(1 + m_{SD}/\bar{\gamma}_{SD})\gamma_{SD}\right] d\gamma_{SD} \\ &= \left(\frac{m_{SD}}{\bar{\gamma}_{SD} + m_{SD}}\right)^{m_{SD}}. \end{aligned} \quad (3.4.4)$$

Similarly, the second integral in (3.4.2) can be obtained. Finally, the upper bound of P_{e1}^{MRC} is given by

$$P_{e1}^{MRC} \leq \left(\frac{m_{SD}}{\bar{\gamma}_{SD} + m_{SD}} \right)^{m_{SD}} \left(\frac{m_{RD}}{\bar{\gamma}_{RD} + m_{RD}} \right)^{m_{RD}}. \quad (3.4.5)$$

When $\bar{\gamma}_{SD}$ is exponentially equal to $\bar{\gamma}_{RD}$ and goes to infinity, i.e., $\bar{\gamma}_{SD} \doteq \bar{\gamma}_{RD} \rightarrow \infty$, the slope that P_{e1}^{MRC} decays can be observed from (3.4.5). For the purpose of easy expression in the following analysis, we assume that $P_S = P_R$ and the transmitted SNR is given by $\rho = P_S/N_0 = P_R/N_0$. Accordingly, the average received SNR is $\bar{\gamma}_{ij} = \rho \sigma_{ij}^2$. In high SNR regime, the upper bound of P_{e1}^{MRC} in (3.4.5) can be expressed as

$$\begin{aligned} P_{e1}^{MRC} &\stackrel{\rho \rightarrow \infty}{\leq} \left(\frac{m_{SD}}{\sigma_{SD}^2} \right)^{m_{SD}} \left(\frac{m_{RD}}{\sigma_{RD}^2} \right)^{m_{RD}} \rho^{-(m_{SD}+m_{RD})} \\ &= c_1 \rho^{-(m_{SD}+m_{RD})}, \end{aligned} \quad (3.4.6)$$

which indicates that the MRC scheme can obtain the diversity order of $(m_{SD}+m_{RD})$ under the condition that R decodes the source's signals correctly. The coefficient c_l , for $l = 1, 2, 3, 4$, is a constant and independent of the transmitted SNR ρ . It only depends on the fading parameters m_{ij} and average channel gains σ_{ij}^2 of corresponding channels.

Next, we will derive the BER of the MRC scheme in the case that R cannot correctly decode signals from S .

Proposition 3.4.1. *The upper bound of the expectation of $P_{e2}^{MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD})$ decays with the exponent equal to m_{SR} , which can be represented as*

$$P_{e2}^{MRC} = E \left\{ Q \left[\sqrt{2\gamma_{SR}} \right] Q \left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{RD})}{\sqrt{\gamma_{SD} + \gamma_{RD}}} \right] \right\} \stackrel{\rho \rightarrow \infty}{\leq} c_2 \rho^{-m_{SR}}. \quad (3.4.7)$$

Proof.

$$\begin{aligned}
 P_{e2}^{MRC} &= \int_0^\infty \int_0^\infty \int_0^\infty Q\left[\sqrt{2\gamma_{SR}}\right] Q\left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{RD})}{\sqrt{\gamma_{SD} + \gamma_{RD}}}\right] \\
 &\quad \times p(\gamma_{SD}) p(\gamma_{SR}) p(\gamma_{RD}) d\gamma_{SD} d\gamma_{SR} d\gamma_{RD} \\
 &\leq \underbrace{\frac{1}{2} \int_0^\infty \exp(-\gamma_{SR}) p(\gamma_{SR}) d\gamma_{SR}}_{\mathcal{I}_A} \\
 &\quad \times \underbrace{\int_0^\infty \int_0^\infty Q\left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{RD})}{\sqrt{\gamma_{SD} + \gamma_{RD}}}\right] p(\gamma_{SD}) p(\gamma_{RD}) d\gamma_{SD} d\gamma_{RD}}_{\mathcal{I}_B}. \quad (3.4.8)
 \end{aligned}$$

Similar to (3.4.4), we can obtain

$$\mathcal{I}_A = \left(\frac{m_{SR}}{\bar{\gamma}_{SR} + m_{SR}}\right)^{m_{SR}} \stackrel{\rho \rightarrow \infty}{=} \left(\frac{m_{SR}}{\sigma_{SR}^2}\right)^{m_{SR}} \rho^{-m_{SR}} = c_A \rho^{-m_{SR}}. \quad (3.4.9)$$

For the derivation of $\mathcal{I}_B$, we set $\mathcal{I}_B = \mathcal{I}_{B_1} + \mathcal{I}_{B_2}$. $\mathcal{I}_{B_1}$ and $\mathcal{I}_{B_2}$ are defined by

$$\mathcal{I}_{B_1} = \int_0^\infty \left\{ \int_0^{\gamma_{RD}} Q\left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{RD})}{\sqrt{\gamma_{SD} + \gamma_{RD}}}\right] p(\gamma_{SD}) d\gamma_{SD} \right\} p(\gamma_{RD}) d\gamma_{RD} \quad (3.4.10)$$

and

$$\mathcal{I}_{B_2} = \int_0^\infty \left\{ \int_{\gamma_{RD}}^\infty Q\left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{RD})}{\sqrt{\gamma_{SD} + \gamma_{RD}}}\right] p(\gamma_{SD}) d\gamma_{SD} \right\} p(\gamma_{RD}) d\gamma_{RD}. \quad (3.4.11)$$

Firstly, $\mathcal{I}_{B_1}$ is derived as

$$\begin{aligned}
 \mathcal{I}_{B_1} &\stackrel{(a)}{\leq} \int_0^\infty \left\{ \int_0^{\gamma_{RD}} p(\gamma_{SD}) d\gamma_{SD} \right\} p(\gamma_{RD}) d\gamma_{RD} \\
 &\stackrel{(b)}{=} \int_0^\infty \frac{\gamma \left(m_{SD}, \frac{m_{SD}}{\bar{\gamma}_{SD}} \gamma_{RD}\right)}{\Gamma(m_{SD})} p(\gamma_{RD}) d\gamma_{RD} \\
 &\stackrel{(c)}{=} \sum_{v=0}^\infty \frac{(-1)^v}{v! (m_{SD} + v)} \frac{\Gamma(m_{SD} + m_{RD} + v)}{\Gamma(m_{SD}) \Gamma(m_{RD})} \left(\frac{m_{SD} \sigma_{RD}^2}{m_{RD} \sigma_{SD}^2}\right)^{m_{SD} + v}
 \end{aligned}$$

$$= c_{B_1} \rho^0. \quad (3.4.12)$$

It is observed from (3.4.12) that the upper bound of $\mathcal{I}_{B_1}$ is a constant and independent of ρ . The derivation of (a) is achieved by that $1/2 < Q(x) \leq 1$ if $x < 0$. Step (b) is obtained based on the definition of the lower incomplete gamma function, which is given by

$$\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt. \quad (3.4.13)$$

Using the series representation of $\gamma(s, x)$ [105, Eq. (8.354.1)] that

$$\gamma(s, x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{s+n}}{n! (s+n)} \quad (3.4.14)$$

and (3.4.3), we can achieve the expression in step (c).

Next, the upper bound of $\mathcal{I}_{B_2}$ can be calculated as

$$\begin{aligned} \mathcal{I}_{B_2} &\stackrel{(a)}{\leq} \frac{1}{2} \int_0^\infty \left\{ \int_{\gamma_{RD}}^\infty \exp \left[-\frac{(\gamma_{SD} - \gamma_{RD})^2}{\gamma_{SD} + \gamma_{RD}} \right] p(\gamma_{SD}) d\gamma_{SD} \right\} p(\gamma_{RD}) d\gamma_{RD} \\ &\stackrel{(b)}{\leq} \frac{1}{2} \int_0^\infty \left\{ \int_{\gamma_{RD}}^\infty \exp \left[-(\sqrt{\gamma_{SD}} - \sqrt{\gamma_{RD}})^2 \right] p(\gamma_{SD}) d\gamma_{SD} \right\} p(\gamma_{RD}) d\gamma_{RD} \\ &\stackrel{(c)}{\leq} \frac{(m_{SD}/\bar{\gamma}_{SD})^{m_{SD}}}{2\Gamma(m_{SD})} \\ &\quad \times \underbrace{\int_0^\infty \left\{ \int_{\gamma_{RD}}^\infty \gamma_{SD}^{m_{SD}-1} \exp \left[-(\sqrt{\gamma_{SD}} - \sqrt{\gamma_{RD}})^2 \right] d\gamma_{SD} \right\} p(\gamma_{RD}) d\gamma_{RD}}_{\mathcal{I}_0}. \end{aligned} \quad (3.4.15)$$

Step (a) is obtained with the aid of Chernoff bound. In the derivation of (b), since

$$\frac{(\gamma_{SD} - \gamma_{RD})^2}{\gamma_{SD} + \gamma_{RD}} > \frac{(\gamma_{SD} - \gamma_{RD})^2}{(\sqrt{\gamma_{SD}} + \sqrt{\gamma_{RD}})^2} = (\sqrt{\gamma_{SD}} - \sqrt{\gamma_{RD}})^2, \quad (3.4.16)$$

we have

$$\exp \left[-\frac{(\gamma_{SD} - \gamma_{RD})^2}{\gamma_{SD} + \gamma_{RD}} \right] < \exp \left[-(\sqrt{\gamma_{SD}} - \sqrt{\gamma_{RD}})^2 \right]. \quad (3.4.17)$$

In step (c), we use the fact that $e^{-x} < 1$ for $x > 0$. To compute $\mathcal{I}_0$ in (3.4.15), we set $z = \sqrt{\gamma_{SD}} - \sqrt{\gamma_{RD}}$. Hence, by referring to [105, Eq. (3.326.2)] that

$$\int_0^\infty x^m \exp(-\beta x^n) dx = \frac{\Gamma(\gamma)}{n \beta^\gamma}, \quad \text{where } \gamma = \frac{m+1}{n}, \quad (3.4.18)$$

$\mathcal{I}_0$ is computed as

$$\begin{aligned} \mathcal{I}_0 &= 2 \int_0^\infty (z + \sqrt{\gamma_{RD}})^{2m_{SD}-1} \exp(-z^2) dz \\ &= 2 \sum_{k=0}^{2m_{SD}-1} \binom{2m_{SD}-1}{k} \gamma_{RD}^{m_{SD}-\frac{k+1}{2}} \int_0^\infty z^k \exp(-z^2) dz \\ &= \sum_{k=0}^{2m_{SD}-1} \binom{2m_{SD}-1}{k} \Gamma\left(\frac{k+1}{2}\right) \gamma_{RD}^{m_{SD}-\frac{k+1}{2}}. \end{aligned} \quad (3.4.19)$$

Substituting (3.4.19) into (3.4.15) and employing (3.4.3), we obtain the upper bound of $\mathcal{I}_{B_2}$ as

$$\begin{aligned} \mathcal{I}_{B_2} &\leq \frac{1}{2 \Gamma(m_{SD}) \Gamma(m_{RD})} \left(\frac{m_{SD} \sigma_{RD}^2}{m_{RD} \sigma_{SD}^2} \right)^{m_{SD}} \\ &\quad \times \sum_{k=0}^{2m_{SD}-1} \binom{2m_{SD}-1}{k} \Gamma\left(\frac{k+1}{2}\right) \left(\frac{m_{RD}}{\sigma_{RD}^2 \rho} \right)^{\frac{k+1}{2}} \Gamma\left(m_{SD} + m_{RD} - \frac{k+1}{2}\right) \\ &\stackrel{\rho \rightarrow \infty}{\approx} c_{B_2} \rho^{-\frac{1}{2}}. \end{aligned} \quad (3.4.20)$$

From the results in (3.4.12) and (3.4.20), it can be seen that $\mathcal{I}_B$ is mainly dominated by the larger term $\mathcal{I}_{B_1}$ and is not affected by ρ at high SNR. Therefore, substituting

(3.4.9), (3.4.12) and (3.4.20) into (3.4.8), we can obtain

$$P_{e2}^{MRC} \leq \frac{1}{2} \mathcal{I}_A (\mathcal{I}_{B1} + \mathcal{I}_{B2}) \stackrel{\rho \rightarrow \infty}{\approx} c_2 \rho^{-m_{SR}}. \quad (3.4.21)$$

□

Proposition 3.4.1 can be explained as follows. When R decodes the source's signal incorrectly, S and R transmit different signals to D . In this case, the average error probability at D is given by

$$P_{D,e}^{MRC} = \mathbb{E} \left\{ P_{D,e}^{MRC} (\gamma_{SD}, \gamma_{SR}, \gamma_{RD}) \right\} \stackrel{\rho \rightarrow \infty}{=} c_B \rho^0, \quad (3.4.22)$$

which equals the term $\mathcal{I}_B$ in (3.4.8). It is proved to converge to a constant when ρ increases to infinity. Thus, the diversity order of P_{e2}^{MRC} is determined by that of $\mathbb{E} \left\{ P_R(\gamma_{SR}) \right\}$ and equal to m_{SR} .

Finally, we have

$$P_e^{MRC} = P_{e1}^{MRC} + P_{e2}^{MRC} \stackrel{\rho \rightarrow \infty}{\leq} c_1 \rho^{-(m_{SD}+m_{RD})} + c_2 \rho^{-m_{SR}}. \quad (3.4.23)$$

It shows that the average error probability is dominated by the larger term and decays with the exponent of $\min(m_{SR}, m_{SD} + m_{RD})$ in the high SNR regime. Therefore, we can conclude that a single-relay cooperative system employing the fixed DF relaying protocol and the MRC scheme is capable of achieving the diversity order of $\min(m_{SR}, m_{SD} + m_{RD})$ in the presence of Nakagami- m faded channels. This also indicates that the first hop should be more reliable than the second hop when the fixed DF relaying and MRC method are employed.

3.4.2 C-MRC

In this section, we will derive the upper bound of the average BER when the C-MRC scheme is employed. We define the first and the second terms on the right-hand side of (3.3.11) as $P_{e1}^{C-MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD})$ and $P_{e2}^{C-MRC}(\gamma_{SD}, \gamma_{SR}, \gamma_{RD})$, and their expectations as P_{e1}^{C-MRC} and P_{e2}^{C-MRC} , respectively.

Proposition 3.4.2. *The upper bound of P_{e1}^{C-MRC} decays with an exponent of $m_{SD} + \min(m_{SR}, m_{RD})$ at high SNR values, which can be expressed as*

$$P_{e1}^{C-MRC} \stackrel{\rho \rightarrow \infty}{\leq} c_3 \rho^{-(m_{SD} + m_{\min})}, \quad (3.4.24)$$

where $m_{\min} = \min\{m_{SR}, m_{RD}\}$.

Proof.

$$\begin{aligned} P_{e1}^{C-MRC} &= \int_0^\infty \int_0^\infty \int_0^\infty Q\left[\sqrt{2(\gamma_{SD} + \gamma_{\min})}\right] p(\gamma_{SD}) p(\gamma_{SR}) p(\gamma_{RD}) d\gamma_{SD} d\gamma_{SR} d\gamma_{RD} \\ &\leq \int_0^\infty \int_0^\infty \int_0^\infty \frac{1}{2} \exp[-(\gamma_{SD} + \gamma_{\min})] \\ &\quad \times p(\gamma_{SD}) p(\gamma_{SR}) p(\gamma_{RD}) d\gamma_{SD} d\gamma_{SR} d\gamma_{RD} \\ &= \frac{1}{2} \underbrace{\int_0^\infty \exp(-\gamma_{SD}) p(\gamma_{SD}) d\gamma_{SD}}_{\mathcal{I}_C} \\ &\quad \times \underbrace{\int_0^\infty \int_0^\infty \exp(-\gamma_{\min}) p(\gamma_{SR}) p(\gamma_{RD}) d\gamma_{SR} d\gamma_{RD}}_{\mathcal{I}_D}. \end{aligned} \quad (3.4.25)$$

Similar to (3.4.4), the term $\mathcal{I}_C$ is obtained as

$$\mathcal{I}_C = \left(\frac{m_{SD}}{\bar{\gamma}_{SD} + m_{SD}} \right)^{m_{SD}} \stackrel{\rho \rightarrow \infty}{\approx} c_C \rho^{-m_{SD}}. \quad (3.4.26)$$

To solve the integration of $\mathcal{I}_D$, we first assume that $\gamma_{SR} \leq \gamma_{RD}$ and thus $\gamma_{\min} = \gamma_{SR}$.

By using the definition of incomplete upper gamma function that

$$\Gamma(s, x) = \int_x^\infty t^{s-1} e^{-t} dt \quad (3.4.27)$$

and its series representation [105, Eq. (8.354.2)] that

$$\Gamma(s, x) = \Gamma(s) - \sum_{n=0}^{\infty} \frac{(-1)^n x^{s+n}}{n! (s+n)}, \quad (3.4.28)$$

$\mathcal{I}_D$ is obtained as

$$\begin{aligned} \mathcal{I}_{D_1} &= \int_0^\infty \exp(-\gamma_{SR}) p(\gamma_{SR}) \left\{ \int_{\gamma_{SR}}^\infty p(\gamma_{RD}) d\gamma_{RD} \right\} d\gamma_{SR} \\ &= \left(\frac{m_{SR}}{\sigma_{SR}^2 \rho + m_{SR}} \right)^{m_{SR}} \left\{ 1 - \sum_{v=0}^{\infty} \frac{(-1)^v}{v! (m_{RD} + v)} \frac{\Gamma(m_{SR} + m_{RD} + v)}{\Gamma(m_{SR}) \Gamma(m_{RD})} \right. \\ &\quad \times \left. \left[\frac{m_{RD} \sigma_{SR}^2}{\sigma_{RD}^2 (\sigma_{SR}^2 \rho + m_{SR})} \right]^{m_{RD} + v} \right\} \\ &\stackrel{\rho \rightarrow \infty}{\approx} c_{D_1} \rho^{-m_{SR}}. \end{aligned} \quad (3.4.29)$$

Similarly, when $\gamma_{SR} \geq \gamma_{RD}$, $\gamma_{\min} = \gamma_{RD}$, and

$$\begin{aligned} \mathcal{I}_{D_2} &= \int_0^\infty \exp(-\gamma_{RD}) p(\gamma_{RD}) \left\{ \int_{\gamma_{RD}}^\infty p(\gamma_{SR}) d\gamma_{SR} \right\} d\gamma_{RD} \\ &\stackrel{\rho \rightarrow \infty}{\approx} c_{D_2} \rho^{-m_{RD}}. \end{aligned} \quad (3.4.30)$$

Therefore, P_{e1}^{C-MRC} and its upper bound can be obtained by substituting (3.4.26), (3.4.29) and (3.4.30) into (3.4.25), resulting in

$$P_{e1}^{C-MRC} \leq \frac{1}{2} \mathcal{I}_C (\mathcal{I}_{D_1} + \mathcal{I}_{D_2}) \stackrel{\rho \rightarrow \infty}{\approx} c_3 \rho^{-(m_{SD} + \min\{m_{SR}, m_{RD}\})}. \quad (3.4.31)$$

□

Proposition 3.4.3. *The upper bound of P_{e2}^{C-MRC} has an exponent of $-(m_{SD}+m_{SR})$ based on the transmitted SNR, which can be represented as*

$$P_{e2}^{C-MRC} = E \left\{ Q \left[\sqrt{2\gamma_{SR}} \right] Q \left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{\min})}{\sqrt{\gamma_{SD} + \gamma_{\min}}} \right] \right\} \stackrel{\rho \rightarrow \infty}{\leq} c_4 \rho^{-(m_{SD}+m_{SR})}. \quad (3.4.32)$$

Proof.

$$\begin{aligned} P_{e2}^{C-MRC} &\leq \frac{1}{2} \int_0^\infty \int_0^\infty \exp(-\gamma_{SR}) \underbrace{\left\{ \int_0^\infty Q \left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{\min})}{\sqrt{\gamma_{SD} + \gamma_{\min}}} \right] p(\gamma_{SD}) d\gamma_{SD} \right\}}_{f(\gamma_{\min})} \\ &\quad \times p(\gamma_{SR}) p(\gamma_{RD}) d\gamma_{SR} d\gamma_{RD}. \end{aligned} \quad (3.4.33)$$

Firstly, similar to the derivation of term $\mathcal{I}_B$ in the proof of Proposition 3.4.1, we set $f(\gamma_{\min}) = f_1(\gamma_{\min}) + f_2(\gamma_{\min})$, where

$$\begin{aligned} f_1(\gamma_{\min}) &= \int_0^{\gamma_{\min}} Q \left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{\min})}{\sqrt{\gamma_{SD} + \gamma_{\min}}} \right] p(\gamma_{SD}) d\gamma_{SD} \\ &\leq \int_0^{\gamma_{\min}} p(\gamma_{SD}) d\gamma_{SD} \\ &= \frac{1}{\Gamma(m_{SD})} \gamma \left(m_{SD}, \frac{m_{SD} \gamma_{\min}}{\bar{\gamma}_{SD}} \right) \\ &= \frac{1}{\Gamma(m_{SD})} \sum_{v=0}^{\infty} \frac{(-1)^v}{v! (m_{SD} + v)} \left(\frac{m_{SD} \gamma_{\min}}{\bar{\gamma}_{SD}} \right)^{m_{SD}+v} \end{aligned} \quad (3.4.34)$$

and

$$\begin{aligned} f_2(\gamma_{\min}) &= \int_{\gamma_{\min}}^\infty Q \left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{\min})}{\sqrt{\gamma_{SD} + \gamma_{\min}}} \right] p(\gamma_{SD}) d\gamma_{SD} \\ &\leq \frac{1}{2} \int_{\gamma_{\min}}^\infty \exp \left[-(\sqrt{\gamma_{SD}} - \sqrt{\gamma_{\min}})^2 \right] p(\gamma_{SD}) d\gamma_{SD} \\ &\leq \frac{(m_{SD}/\bar{\gamma}_{SD})^{m_{SD}}}{2\Gamma(m_{SD})} \int_{\gamma_{\min}}^\infty \gamma_{SD}^{m_{SD}-1} \exp \left[-(\sqrt{\gamma_{SD}} - \sqrt{\gamma_{\min}})^2 \right] d\gamma_{SD}. \end{aligned} \quad (3.4.35)$$

Note that the integration in (3.4.35) has the same form with $\mathcal{I}_0$ in (3.4.15). Hence, using the result in (3.4.19), we can obtain

$$f_2(\gamma_{\min}) = \frac{(m_{SD}/\bar{\gamma}_{SD})^{m_{SD}}}{2\Gamma(m_{SD})} \sum_{k=0}^{2m_{SD}-1} \binom{2m_{SD}-1}{k} \Gamma\left(\frac{k+1}{2}\right) \gamma_{\min}^{m_{SD}-\frac{k+1}{2}}. \quad (3.4.36)$$

Then we define

$$P_{e2,1}^{C-MRC} = \frac{1}{2} \int_0^\infty \int_0^\infty \exp(-\gamma_{SR}) f_1(\gamma_{\min}) p(\gamma_{SR}) p(\gamma_{RD}) d\gamma_{SR} d\gamma_{RD} \quad (3.4.37)$$

and

$$P_{e2,2}^{C-MRC} = \frac{1}{2} \int_0^\infty \int_0^\infty \exp(-\gamma_{SR}) f_2(\gamma_{\min}) p(\gamma_{SR}) p(\gamma_{RD}) d\gamma_{SR} d\gamma_{RD}. \quad (3.4.38)$$

When $\gamma_{SR} \leq \gamma_{RD}$, (3.4.37) and (3.4.38) are derived as follows

$$\begin{aligned} P_{e2,1a}^{C-MRC} &\leq \frac{1}{2\Gamma(m_{SD})\Gamma(m_{SR})} \left(\frac{m_{SR}}{\sigma_{SR}^2 \rho + m_{SR}} \right)^{m_{SR}} \\ &\quad \times \sum_{v=0}^{\infty} \frac{(-1)^v}{v! (m_{SD} + v)} \left[\frac{m_{SD} \sigma_{SR}^2}{\sigma_{SD}^2 (\sigma_{SR}^2 \rho + m_{SR})} \right]^{m_{SD}+v} \\ &\quad \times \left\{ \Gamma(m_{SD} + m_{SR} + v) - \sum_{u=0}^{\infty} \frac{(-1)^u}{u! (m_{RD} + u)} \right. \\ &\quad \times \left. \frac{\Gamma(m_{SD} + m_{SR} + m_{RD} + v + u)}{\Gamma(m_{RD})} \left[\frac{m_{RD} \sigma_{SR}^2}{\sigma_{RD}^2 (\sigma_{SR}^2 \rho + m_{SR})} \right]^{m_{RD}+u} \right\} \\ &\stackrel{\rho \rightarrow \infty}{\approx} c_{4,1a} \rho^{-(m_{SD}+m_{SR})} \end{aligned} \quad (3.4.39)$$

and

$$P_{e2,2a}^{C-MRC} \leq \frac{1}{4\Gamma(m_{SD})\Gamma(m_{SR})} \left[\frac{m_{SD} \sigma_{SR}^2}{\sigma_{SD}^2 (\sigma_{SR}^2 \rho + m_{SR})} \right]^{m_{SD}} \left(\frac{m_{SR}}{\sigma_{SR}^2 \rho + m_{SR}} \right)^{m_{SR}}$$

$$\begin{aligned}
 & \times \sum_{k=0}^{2m_{SD}-1} \binom{2m_{SD}-1}{k} \Gamma\left(\frac{k+1}{2}\right) \left(1 + \frac{m_{SR}}{\sigma_{SR}^2 \rho}\right)^{\frac{k+1}{2}} \\
 & \times \left\{ \Gamma\left(m_{SD} + m_{SR} - \frac{k+1}{2}\right) - \sum_{u=0}^{\infty} \frac{(-1)^u}{u! (m_{RD} + u)} \right. \\
 & \times \left. \frac{\Gamma\left(m_{SD} + m_{SR} + m_{RD} + u - \frac{k+1}{2}\right)}{\Gamma(m_{RD})} \left[\frac{m_{RD} \sigma_{SR}^2}{\sigma_{RD}^2 (\sigma_{SR}^2 \rho + m_{SR})} \right]^{m_{RD}+u} \right\} \\
 & \stackrel{\rho \rightarrow \infty}{\approx} c_{4,2a} \rho^{-(m_{SD}+m_{SR})}.
 \end{aligned} \tag{3.4.40}$$

when $\gamma_{SR} > \gamma_{RD}$, the derivations of (3.4.37) and (3.4.38) are respectively given by

$$\begin{aligned}
 P_{e2,1b}^{C-MRC} & \leq \frac{1}{2 \Gamma(m_{SD}) \Gamma(m_{SR}) \Gamma(m_{RD})} \left(\frac{m_{SR}}{\sigma_{SR}^2 \rho + m_{SR}} \right)^{m_{SR}} \\
 & \times \sum_{v=0}^{\infty} \frac{(-1)^v}{v! (m_{SD} + v)} \left[\frac{m_{SD} \sigma_{SR}^2}{\sigma_{SD}^2 (\sigma_{SR}^2 \rho + m_{SR})} \right]^{m_{SD}+v} \\
 & \times \sum_{u=0}^{\infty} \frac{(-1)^u}{u! (m_{SD} + m_{RD} + v + u)} \Gamma(m_{SD} + m_{SR} + m_{RD} + v + u) \\
 & \times \left[\frac{m_{RD} \sigma_{SR}^2}{\sigma_{RD}^2 (\sigma_{SR}^2 \rho + m_{SR})} \right]^{m_{RD}+u} \\
 & \stackrel{\rho \rightarrow \infty}{\approx} c_{4,1b} \rho^{-(m_{SD}+m_{SR}+m_{RD})}
 \end{aligned} \tag{3.4.41}$$

and

$$\begin{aligned}
 P_{e2,2b}^{C-MRC} & \leq \frac{1}{4 \Gamma(m_{SD}) \Gamma(m_{SR}) \Gamma(m_{RD})} \left[\frac{m_{SD} \sigma_{SR}^2}{\sigma_{SD}^2 (\sigma_{SR}^2 \rho + m_{SR})} \right]^{m_{SD}} \left(\frac{m_{SR}}{\sigma_{SR}^2 \rho + m_{SR}} \right)^{m_{SR}} \\
 & \times \sum_{k=0}^{2m_{SD}-1} \binom{2m_{SD}-1}{k} \Gamma\left(\frac{k+1}{2}\right) \left(1 + \frac{m_{SR}}{\sigma_{SR}^2 \rho}\right)^{\frac{k+1}{2}} \\
 & \times \sum_{u=0}^{\infty} \frac{(-1)^u}{u! (m_{SD} + m_{RD} + u - \frac{k+1}{2})} \Gamma\left(m_{SD} + m_{SR} + m_{RD} + u - \frac{k+1}{2}\right) \\
 & \times \left[\frac{m_{RD} \sigma_{SR}^2}{\sigma_{RD}^2 (\sigma_{SR}^2 \rho + m_{SR})} \right]^{m_{RD}+u} \\
 & \stackrel{\rho \rightarrow \infty}{\approx} c_{4,2b} \rho^{-(m_{SD}+m_{SR}+m_{RD})}.
 \end{aligned} \tag{3.4.42}$$

From (3.4.39)-(3.4.42), the end-to-end average BER of the C-MRC scheme in the case that R cannot successfully decode the source's information is obtained as

$$\begin{aligned}
 P_{e2}^{C-MRC} &\leq P_{e2,1}^{C-MRC} + P_{e2,2}^{C-MRC} \\
 &= P_{e2,1a}^{C-MRC} + P_{e2,1b}^{C-MRC} + P_{e2,2a}^{C-MRC} + P_{e2,2b}^{C-MRC} \\
 &\stackrel{\rho \rightarrow \infty}{\leq} c_4 \rho^{-(m_{SD}+m_{SR})}.
 \end{aligned} \tag{3.4.43}$$

□

Finally, the error probability of a single-relay cooperative system employing the C-MRC scheme is given by

$$\begin{aligned}
 P_e^{C-MRC} &= P_{e1}^{C-MRC} + P_{e2}^{C-MRC} \\
 &\stackrel{\rho \rightarrow \infty}{\leq} c_3 \rho^{-(m_{SD}+m_{\min})} + c_4 \rho^{-(m_{SD}+m_{SR})}.
 \end{aligned} \tag{3.4.44}$$

We can observe that the fixed DF relaying protocol is capable of achieving the diversity order of $\min\{m_{SD}+m_{SR}, m_{SD}+m_{RD}\}$ when the C-MRC scheme is adopted at D .

We compare the achievable diversity orders of both MRC and C-MRC schemes in Table 3.1. It can be found that when m_{SR} is relatively small, the diversity gain of the MRC scheme is determined by m_{SR} and the diversity difference between both schemes is the largest. When m_{SR} grows, the channel from S to R becomes less faded and the diversity gap gets smaller. When m_{SR} is comparatively large, R can make correct decoding at a very high probability. Thus, C-MRC loses its advantage over MRC, and both schemes achieve the identical diversity order. In summary, C-MRC always obtains a higher or at least the same diversity gain as MRC. The reason is that C-MRC fully considers the fading effect of each link and maximizes

Table 3.1: Comparison of the diversity orders achieved by MRC and C-MRC schemes

m_{SR}	MRC	C-MRC	Diversity difference
	$\min\{m_{SR}, m_{SD} + m_{RD}\}$	$\min\{m_{SD} + m_{SR}, m_{SD} + m_{RD}\}$	
$(0, m_{RD})$	m_{SR}	$m_{SD} + m_{SR}$	m_{SD}
$[m_{RD}, m_{SD} + m_{RD}]$	m_{SR}	$m_{SD} + m_{RD}$	$< m_{SD}$
$(m_{SD} + m_{RD}, \infty)$	$m_{SD} + m_{RD}$	$m_{SD} + m_{RD}$	0

the received SNR of the relay channel through channel-dependent weighting factors. Therefore, C-MRC can achieve better performance and higher diversity gain than MRC.

3.5 Numerical Results and Discussions

In this section, we first provide the theoretical and simulated BER performance of a single-relay DF relaying cooperative system with MRC and C-MRC schemes. Next, we compare the performance of MRC, C-MRC and the adaptive DF relaying protocol. We assume that S and R transmit BPSK signals with the same power level and all three channels experience independent and non-identically distributed (i.n.i.d.) Nakagami- m fading.

In Figure 3.1-3.3, to emphasize the effect of the fading parameters m_{ij} , we assume that the average channel gains of three links are identical and equal to one, i.e., $\sigma_{ij}^2 = 1$. Actually, the derived BER upper bounds in Section 3.4 are suitable for any m_{ij} and σ_{ij}^2 values. Firstly, we provide the simulated BER and the theoretical BER upper bound of MRC and C-MRC schemes in Figure 3.1 and Figure 3.2, respectively. Both figures show that the simulated BER is upper bounded by the theoretical results. As the SNR increases, the theoretical upper bounds get closer to the simulated BER curves. Both simulated and theoretical results have the identical diversity order under the same channel conditions. Figure 3.1 verifies that the MRC scheme can

Figure 3.1: Simulated BER and theoretical BER upper bound of the MRC scheme for a single-relay cooperative system with fixed DF relaying over i.n.i.d. Nakagami- m fading channels. © [2009] John Wiley & Sons, Ltd.

obtain the diversity gain of $\min(m_{SR}, m_{SD} + m_{RD})$ and Figure 3.2 proves that the C-MRC scheme can achieve the diversity order of $\min(m_{SD} + m_{SR}, m_{SD} + m_{RD})$.

In Figure 3.3, we compare the simulated BER performance of both MRC and C-MRC schemes for a fixed DF relaying cooperative system experiencing various channel conditions. As seen from Table 3.1, the comparison of the diversity orders achieved by MRC and C-MRC schemes can be classified into three situations. Therefore, we provide one example for each scenario to compare the achievable diversity gains in Figure 3.3. We fix the values of m_{SD} and m_{RD} , i.e., $m_{SD} = m_{RD} = 1$, and only adopt different values of m_{SR} . For the first case, when the average channel condition of the S - R link is worse than the R - D link, i.e., $m_{SR} \leq m_{RD}$, the diversity order of MRC is always less than that of C-MRC by m_{SD} . It is illustrated by the example $(m_{SD}, m_{SR}, m_{RD}) = (1, 0.5, 1)$. We can see from Figure 3.3 that MRC only

Figure 3.2: Simulated BER and theoretical BER upper bound of the C-MRC scheme for a single-relay cooperative system with fixed DF relaying over i.n.i.d. Nakagami- m fading channels. © [2009] John Wiley & Sons, Ltd.

achieves a diversity order of 0.5, while C-MRC achieves a diversity gain of 1.5. For the second case, as the S - R channel condition improves and satisfies the condition that $m_{RD} < m_{SR} \leq m_{SD} + m_{RD}$, we can find that the gap between the diversity orders of m_{SR} and $m_{SD} + m_{RD}$ achieved by MRC and C-MRC, respectively, is less than m_{SD} . As shown by the case $(m_{SD}, m_{SR}, m_{RD}) = (1, 1.5, 1)$, MRC achieves a diversity gain of 1.5 and C-MRC achieves a diversity of 2. For the third case, only when the S - R condition becomes even better than both S - D and R - D channels, i.e., $m_{SR} > m_{SD} + m_{RD}$, MRC can obtain the same diversity gain as C-MRC equal to $m_{SD} + m_{RD}$ and as good BER performance as C-MRC. It is shown by the case $(m_{SD}, m_{SR}, m_{RD}) = (1, 4, 1)$ and both schemes achieve the identical diversity order of 2. In addition, it can also be noticed from Figure 3.3 that C-MRC is capable of achieving the same BER performance in the case of $m_{SR} = 0.5$ as the MRC does in

Figure 3.3: BER performance comparison of MRC and C-MRC schemes for a single-relay cooperative system with fixed DF relaying over i.n.i.d. Nakagami- m fading channels.

a better channel condition than $m_{SR} = 1.5$. Therefore, it is concluded that C-MRC always achieves better BER performance than MRC does, especially when the S - R channel is in a relatively poor condition.

Furthermore, in Figure 3.4, we compare the BER performance of the fixed DF protocol employing the C-MRC scheme with that of the adaptive DF relaying protocol with optimum threshold [2] in different channel conditions. It was concluded in [2] that the optimal threshold is determined by the fading parameters and average channel gains of the S - D and R - D links and is independent of the channel statistics of the S - R link. For a fair comparison, we adopt the same channel gain assumption as that in [2]. The path-loss exponent β is equal to 3 and the lengths of S - D , S - R and R - D links are set to be $d_{SD} = 1$, $d_{SR} = 0.6$ and $d_{RD} = 0.4$, respectively. The average channel gain is given by $\sigma_{ij}^2 = d_{ij}^{-\beta}$. We also set the same fading parameters

Figure 3.4: BER performance comparison of the fixed DF relaying protocol with the C-MRC scheme and the adaptive DF relaying protocol with optimum threshold for a single-relay cooperative system over i.n.i.d. Nakagami- m fading channels.

of the S - D and R - D channels as that in [2], i.e., $m_{SD} = m_{RD} = 1$. Three scenarios with different values of m_{SR} are considered in Figure 3.4 that the fading severity of the S - R link is less, equal or larger than the R - D link. The optimum threshold of the adaptive DF protocol is given in Table 3.2. Note that BER curves are plotted in terms of the total transmitted SNR in Figure 3.4. It can be observed that the fixed DF protocol with the C-MRC scheme and the adaptive DF protocol with optimum threshold can achieve the identical diversity gain equal to $m_{SD} + \min\{m_{SR}, m_{RD}\}$. Moreover, it is also noticed that the C-MRC scheme achieves even better BER performance than the adaptive DF protocol employed in [2] under the same channel conditions.

The above analysis and numerical results help us to design the appropriate transmission and detection scheme for single-relay cooperative systems based on the practical

Table 3.2: The optimum threshold used in the adaptive DF protocol [2]

2ρ	5 dB	10 dB	15 dB	20 dB	25 dB	30 dB
γ_{opt} (dB)	0.350	2.760	4.507	5.842	6.886	7.7422

channel conditions and system requirements. If the channel quality of the source-relay link is much better than that of the source-destination and relay-destination links or the system has a low requirement of accuracy, the low-complexity MRC detector is preferred. Otherwise, the C-MRC scheme and the adaptive DF protocol with optimum threshold are optional to achieve the full diversity order in any channel environments. In particular, the adaptive relaying protocol is adopted in the case that the CSI of the source-relay link cannot be known at the destination. The C-MRC scheme might be employed to achieve even better performance, but it takes some resources to transmit the CSI of the source-relay channel to the destination.

3.6 Conclusion

In this chapter, we have analyzed the BER performance of a single-relay cooperative system over Nakagami- m fading channels. The relay works in the fixed DF relaying mode. The destination uses either MRC without source-relay channel knowledge or C-MRC which requires perfect instantaneous source-relay channel information. We have derived the BER upper bounds and diversity gains for both MRC and C-MRC schemes over Nakagami- m fading channels. Numerical results were provided to show that the C-MRC scheme always achieves better performance than the MRC scheme does. Part of the work in this chapter has been published in Wireless Communications and Mobile Computing as a journal paper, which is the second paper in Author's Publications.

Chapter 4

BER Performance of a Two-user Cooperative Multiple-Access System

4.1 Introduction

In Chapter 3, we have employed the C-MRC scheme for the fixed DF relaying cooperative system to enhance the available spatial diversity and to further improve the achievable BER performance. It has been shown that C-MRC can obtain higher diversity than the MRC scheme does in the fixed DF relaying. Moreover, it can achieve better BER performance than the adaptive DF relaying protocol with optimal threshold. However, the source only transmits in the first time slot in both AF and DF relaying protocols, which wastes half of the time resource. Therefore, the efficiency is lower than the direct transmission. This spectral inefficiency issue also exists in [9, 10, 45]. In this circumstance, a non-orthogonal relaying protocol [13] was proposed for the single-relay cooperative system to improve the bandwidth us-

age by allowing the source to transmit a new message in the second time slot, as shown in Figure 1.1(d). Due to the half-duplex constraint, the relay cannot assist the transmission of this message. Therefore, its achievable average BER is not as good as the orthogonal relaying protocol.

Different from the non-orthogonal relaying protocol, other methods are adopted to improve the spectral efficiency in this chapter. A simple three-terminal system is still considered but it is composed of two sources, also acting as relays of each other, and a destination. Two sources can transmit not only their own information to the same destination but also cooperate with each other to increase the diversity. It is called two-user cooperative multiple-access (CMA) system. It is applicable to the scenario that both users are willing to transmit to a common base station and also wish to get help from each other to obtain better performance in up-link cellular networks. In this chapter, we will analyze and compare the performance of the two-user CMA system employing different transmission protocols.

Similar to those orthogonal transmission strategies [9], the authors in [30, 31, 92, 93] assumed that each user's information is transmitted and relayed in orthogonal time or frequency slots in this two-user CMA system. Hence, four time slots or equivalent channel resources are required to fulfill a cooperative transmission of both users' signals. We name it as a four-time-slot time division broadcast (TDBC) protocol. In [30] and [31], both the error rate and outage probability of coded cooperation were presented. The joint power and channel resource allocations for the orthogonal AF and DF cooperation were discussed in [92] and [93], respectively. However, this transmission scheme still has the disadvantage of spectrum inefficiency. To recover the efficiency loss and keep half-duplex constraint, a three-time-slot TDBC protocol can be applied for the two-user CMA system. Both users transmit their own information orthogonally in the first two time slots, and then forward each

other's information simultaneously in the third time slot. We will analyze the BER performance of this protocol with AF relaying and provide the simulated results to validate the derivation.

Comparing to the above two TDBC schemes, a more efficient transmission protocol based on superposition modulation was first proposed in [106]. Two users take turns to transmit the superposition of both users' information in successive time slots, and thus only two time slots are required. Inspired by [106], several extended studies have been done based on the algebraic superposition and superposition in Euclidean space, also called Galois field superposition coding (GFSC) and complex field superposition coding (CFSC) [107], respectively. The GFSC approach, used in [107] and [108], combines one user's original information and the other user's relayed information by the bit-wise XOR operation. In the CFSC-based method, relays combine both information linearly depending on a superposition factor [106, 109–111]. The simulated performance of the two-user CMA system was presented in [106], and the error probability of coded cooperation was further analyzed and presented in an integral form in [109]. Both works considered DF relaying and employed an maximum *a posteriori* (MAP) demodulator at the destination. The approximate capacity region and outage probability of both AF and DF relaying protocols were analyzed, and the optimal superposition factor was derived in [110]. The works in [107] and [111] employed interference cancellation and iterative detection at the destination, and investigated the performance for this two-phase transmission scheme over multipath and flat fading channels, respectively. However, a larger number of iterations is required to improve the accuracy of the interference cancellation and signal detection. Hence, the complexity and processing delay increase linearly with the number of iterations.

Different from previous works which also employed the CFSC, here we propose

a near-optimal detector at the destination, which requires no iteration and costs much less complex than the detector in [106, 107, 111] and the maximum-likelihood sequence detector (MLSD). More importantly, this detector can achieve the same performance as that in [106] and approach the optimal performance obtained by the MLSD. We derive the corresponding decision regions and obtain the exact BER expressions with integrations and BER bound, in the presence of independent, and non-identically distributed (i.n.i.d.) user-destination channels. We also provide simulation results to validate the theoretical analysis. Finally, we compare the BER performance of the two-user CMA system employing various transmission schemes, namely, superposition modulation scheme, three-time-slot TDBC protocol and four-time-slot TDBC protocol under the identical transmitted power and data rate conditions. It is demonstrated that the superposition modulation is capable of achieving both full diversity gain and high coding gain.

4.2 Three-time-slot TDBC Protocol and its BER Performance

The two-user CMA system model is illustrated in Figure 4.1. Two sources and the destination are denoted as A , B and D , respectively. Each terminal has only one antenna and operates in the half-duplex mode. In this section, we first introduce the three-time-slot AF-based TDBC transmission protocol for the two-user CMA system, and then analyze its corresponding BER performance. The cooperative transmission of both users' information is conducted within three time slots. User i allocates a fraction of its power, denoted as α_i , to forward the other user's information. User A and user B transmit their own signals to the destination in the first and second time slots separately. The received signals at user i and D in the first

Figure 4.1: A two-user cooperative multiple-access system.

two slots are, respectively, given by

$$y_i = \sqrt{1 - \alpha_j} h_{AB} s_j + n_i, \quad (4.2.1)$$

$$y_{D1} = \sqrt{1 - \alpha_A} h_{AD} s_A + n_{D1}, \quad (4.2.2)$$

and

$$y_{D2} = \sqrt{1 - \alpha_B} h_{BD} s_B + n_{D2}, \quad (4.2.3)$$

where h_{ij} denotes the channel coefficient from node i to j and follows the Rayleigh distribution. In addition, the inter-user channels are assumed to be reciprocal, i.e., $h_{AB} = h_{BA}$. User i 's original signal is represented by s_i , and we denote the symbol power as $P = E\{|s_i|^2\}$. The noise terms, n_i and n_{Dt} ($t = 1, 2, 3$), represent the noise at node i and D in the t -th time slot, respectively. They are independent complex Gaussian random variables with zero mean and unit variance. We define that $\gamma_{ij} = P|h_{ij}|^2$ and $\bar{\gamma}_{ij} = P E\{|h_{ij}|^2\}$. For the CSI-based AF relaying, user i 's scaling factor is given by $g_i = 1/\sqrt{(1 - \alpha_j)\gamma_{AB} + 1}$. In the third time slot, both

users forward their scaled signals to D under the power allocation constraint. The received signal can be expressed as

$$\begin{aligned} y_{D3} = & g_B \sqrt{\alpha_B(1 - \alpha_A)} h_{AB} h_{BD} s_A + g_A \sqrt{\alpha_A(1 - \alpha_B)} h_{AB} h_{AD} s_B \\ & + g_A \sqrt{\alpha_A} h_{AD} n_A + g_B \sqrt{\alpha_B} h_{BD} n_B + n_{D3}. \end{aligned} \quad (4.2.4)$$

The variance of the noise terms in (4.2.4) is denoted as σ_N^2 and $\sigma_N^2 = g_A^2 \alpha_A \gamma_{AD} + g_B^2 \alpha_B \gamma_{BD} + 1$. The detection of user A 's signal is based on the received signals in the first and third time slots, and the decision rule is given by

$$\begin{aligned} \hat{s}_A = \operatorname{argmin}_{b_A} & \left\{ \left| y_{D1} - \sqrt{1 - \alpha_A} h_{AD} b_A \right|^2 \right. \\ & \left. + \frac{1}{\sigma_N^2} \left| y_{D3} - h_{AB} (g_B \sqrt{\alpha_B(1 - \alpha_A)} h_{BD} b_A + g_A \sqrt{\alpha_A(1 - \alpha_B)} h_{AD} b_B) \right|^2 \right\}, \end{aligned} \quad (4.2.5)$$

where b_i denotes the candidate of user i 's information. User B 's signal can be detected in a similar way and thus the detail is omitted here.

To derive the average BER of user A , we consider the ideal case that user B 's signal can be perfectly detected by the destination from the direct transmission. Hence, the inter-user interference, the second term in (4.2.4), can be completely removed from y_{D3} . The sum of received SNRs of the direct and relay channels is upper bounded by

$$\gamma_{sum,A} = (1 - \alpha_A) \gamma_{AD} + \frac{\alpha_B(1 - \alpha_A) \gamma_{AB} \gamma_{BD}}{\alpha_A \gamma_{AD} + \alpha_B \gamma_{BD} + (1 - \alpha_A) \gamma_{AB} + 1}. \quad (4.2.6)$$

The PDF and CDF of user A 's combined SNR, denoted as $p_{\gamma_{sum,A}}(\gamma)$ and $F_{\gamma_{sum,A}}(\gamma)$, can be obtained numerically. Here, we adopt the CDF-based approach to derive the error probability. As introduced in Section 2.1.2, the conditional SER for MPSK

modulations with high M and at high SNR can be approximated as

$$P_s(\gamma) \approx kQ(\sqrt{c\gamma}). \quad (4.2.7)$$

The values of k and c are given in Table 2.1. Therefore, the approximation of the average SER under the three-time-slot TDBC transmission scheme can be computed by substituting $F_{\gamma_{sum,A}}(\gamma)$ into

$$P_s = \frac{k}{\sqrt{2\pi}} \int_0^\infty F_\Gamma\left(\frac{u^2}{c}\right) \exp\left(-\frac{u^2}{2}\right) du. \quad (4.2.8)$$

The BER is accordingly obtained by

$$P_b \approx P_s / \log_2 M, \quad (4.2.9)$$

where M is the number of possible symbols in modulation constellation. Note that the above derivation is applicable for any modulation whose conditional SER can be expressed by linear combinations of Q -functions.

4.3 Two-time-slot Superposition Modulation Protocol

In this section, we introduce the superposition modulation protocol for the two-user CMA system and propose a near-optimal, low-complexity detector for the destination. The adaptive repetition-based decode-and-forward (DF) relaying is used. We assume that users can detect whether they decode each other's signal correctly by using CRC codes [112]. In this superposition modulation protocol, each time frame is divided into two equal slots. In the first slot of the k th time frame, user A

transmits the linear combination of its own signal in the current time frame and B 's signal in the last time frame if A can decode B 's data correctly. Otherwise, user A transmits its own signal with full power to B and D . Similarly, in the second slot of the k th time frame, user B transmits either the combination of its own signal and correctly decoded A 's signal or its own signal, depending on the outcome of the CRC code. Hence, the transmitted signals from A and B are given by

$$x_A(k) = \sqrt{1 - \beta_A f_A} s_A(k) + \sqrt{\beta_A f_A} s_B(k-1) \quad (4.3.1)$$

and

$$x_B(k) = \sqrt{1 - \beta_B f_B} s_B(k) + \sqrt{\beta_B f_B} s_A(k). \quad (4.3.2)$$

The constant, $\beta_i \in [0, 1]$, denotes user i 's superposition factor and controls the fraction of power user i allocating to forward the other user's signal. The values of superposition factors can be computed by a central controller, users or the destination before the data transmission, and then informed to other terminals. Both users might have different superposition factors, depending on the system requirements and channel conditions. The flag bit, f_i , varies with the instantaneous channel condition and is defined as

$$f_i = \begin{cases} 1, & \text{if user } i \text{ decodes the other user's signal correctly,} \\ 0, & \text{otherwise.} \end{cases} \quad (4.3.3)$$

The value of f_i is determined by user i and then forwarded to the other user and the destination together with the transmitted data. The signal $s_i(k)$ is user i 's own signal in the k th time frame. Without loss of generality, we assume that $s_A(k)$ and $s_B(k)$ have the same average power level equal to P , i.e., $P = \mathbb{E}\{|s_i(k)|^2\}$. The

signal $x_i(k)$ is the combined signal transmitted by user i and its power is also equal to P . For instance, if $s_A(k)$ and $s_B(k)$ are quadrature phase-shift keying (QPSK) signals, $x_i(k)$ is a non-uniform 16-ary quadrature amplitude modulation (QAM) signal if $f_i = 1$. For the sake of simplicity, in the analysis, we assume that $s_A(k)$ and $s_B(k)$ are BPSK signals. Hence, $x_i(k)$ is a non-uniform 4-ary pulse-amplitude modulation (PAM) signal when $f_i = 1$.

The received signal at B in the first time slot of the k -th frame is expressed as

$$\begin{aligned} y_{AB}(k) &= h_{AB}(k)x_A(k) + n_{AB}(k) \\ &= h_{AB}(k)\sqrt{1 - \beta_A f_A} s_A(k) + h_{AB}(k)\sqrt{\beta_A f_A} s_B(k-1) + n_{AB}(k), \end{aligned} \quad (4.3.4)$$

where $h_{ij}(k)$ denotes the channel coefficient from node i to j in the k th time frame and is also assumed to follow the Rayleigh distribution. The noise term, $n_{ij}(k)$, represents the noise at node j during the transmission from node i to j in the k th frame, and it is a complex Gaussian random variable with zero mean and unit variance. The values of β_i and f_i are known to every terminal and perfect CSI is only available at receivers. Since B has the knowledge of $s_B(k-1)$, it can first subtract the self-interference term (the second term in (4.3.4)) from the received signal, and then decode A 's signal. User B 's signal is detected by user A in the same way. The received signals at D from A and B are given by

$$y_{AD}(k) = h_{AD}(k)x_A(k) + n_{AD}(k) \quad (4.3.5)$$

and

$$y_{BD}(k) = h_{BD}(k)x_B(k) + n_{BD}(k). \quad (4.3.6)$$

The instantaneous SNR in the k th frame and the average SNR of the channel from

i to j are expressed as $\gamma_{ij}(k) = P |h_{ij}(k)|^2$ and $\bar{\gamma}_{ij} = P E\{|h_{ij}(k)|^2\}$, respectively.

Since the inter-user interference cannot be perfectly canceled at the destination, we accordingly propose a near-optimal detector for the superposition modulation based CMA system. Although MLSD can obtain the optimal detection based on at least five received signals, its complexity and delay increase significantly [1]. Our proposed near-optimal detector achieves the BER performance very close to that of MLSD but with much less complexity, shown in Section 4.5. Since both users play the same role in the system, we focus on the detection and analysis of user A 's signal, and that can be similarly obtained for user B . According to the MAP demodulator used in [106], the log-likelihood ratio (LLR) for $s_A(k)$ is computed as

$$\ell(s_A(k)) = \ln \left\{ \frac{\sum_{b_B(k-1)} p(y_{AD}(k)|b_A(k) = 1, b_B(k-1))}{\sum_{b_B(k-1)} p(y_{AD}(k)|b_A(k) = -1, b_B(k-1))} \times \frac{\sum_{b_B(k)} [p(y_{BD}(k)|b_A(k) = 1, b_B(k))]^{f_B}}{\sum_{b_B(k)} [p(y_{BD}(k)|b_A(k) = -1, b_B(k))]^{f_B}} \right\}, \quad (4.3.7)$$

where $b_i(t)$ denotes the candidate of user i 's information in the t th time frame, i.e., $s_i(t)$, and $p(y_{iD}|b_A, b_B)$ is the conditional PDF of a Gaussian distribution. To simplify the computation, our proposed detector only considers the maximal term rather than the sum of all probabilities in the numerator and denominator in (4.3.7).

Hence, the approximate LLR is expressed as

$$\tilde{\ell}(s_A(k)) = \ln \left\{ \frac{\max p(y_{AD}(k)|b_A(k) = 1, b_B(k-1))}{\max p(y_{AD}(k)|b_A(k) = -1, b_B(k-1))} \times \frac{\max [p(y_{BD}(k)|b_A(k) = 1, b_B(k))]^{f_B}}{\max [p(y_{BD}(k)|b_A(k) = -1, b_B(k))]^{f_B}} \right\}, \quad (4.3.8)$$

which is equivalent to the following decision rule:

$$\hat{s}_A(k) = \underset{b_A(k)}{\operatorname{argmin}} \left\{ \left| y_{AD}(k) - h_{AD}(k) \left[\sqrt{1 - \beta_A f_A} b_A(k) + \sqrt{\beta_A f_A} b_B(k-1) \right] \right|^2 + f_B \cdot \left| y_{BD}(k) - h_{BD}(k) \left[\sqrt{1 - \beta_B f_B} b_B(k) + \sqrt{\beta_B f_B} b_A(k) \right] \right|^2 \right\}. \quad (4.3.9)$$

Note that D has the knowledge of f_i and β_i . If B cannot detect A 's signal correctly, i.e., $f_B = 0$, the destination only needs to decode the signal from the original source A . Hence, the computation complexity can be further reduced. The detection of A 's signal involves the received signals from A and B in the current time frame. The detection of B 's signal is similar except that D combines the received signals containing B 's information $s_B(k)$.

4.4 Performance Analysis of the Superposition Modulated Protocol

In this section, we analyze the BER performance of the two-user CMA system employing the superposition modulation scheme and our proposed detector over i.n.i.d. Rayleigh fading channels. As seen from (4.3.9), under the conditions that A and B may or may not decode each other's signal correctly, the detection of A 's information can be divided into the following four cases.

- (i) The first case: $f_A = 1$ and $f_B = 1$. Both users can decode each other's signal correctly, and provide cooperation for each other.
- (ii) The second case: $f_A = 0$ and $f_B = 1$. User B helps to transmit A 's signal, while A does not cooperate for B .
- (iii) The third case: $f_A = 1$ and $f_B = 0$. User A helps to transmit B 's signal, while

B does not cooperate for A .

- (iv) The fourth case: $f_A = 0$ and $f_B = 0$. There is no cooperation, and both users transmit their own data to D with full power.

Therefore, the average BER can be computed as the sum of the error probability in each case weighted by the probability of that case, which is given by

$$P_e = \sum_{u=1}^4 P_{e,u} \cdot Pr(u). \quad (4.4.1)$$

In the following section, we derive $Pr(u)$ and $P_{e,u}$ separately.

4.4.1 Occurrence Probabilities of Four Cases— $Pr(u)$

In this part, we derive the occurrence probability of each of these four cases, denoted by $Pr(u)$, where $u = 1, 2, 3, 4$. Similar to [109], we use η_A and η_B to represent the probabilities that users A and B cooperate to transmit each other's information, respectively. Hence,

$$1 - \eta_A = \eta_B P_{BA}^C + (1 - \eta_B) P_{BA}^N \quad (4.4.2)$$

and

$$1 - \eta_B = \eta_A P_{AB}^C + (1 - \eta_A) P_{AB}^N, \quad (4.4.3)$$

where P_{ij}^C and P_{ij}^N are the average error probabilities of user i 's data transmitted from i to j when i works in the cooperation mode or non-cooperation mode, respectively. Note that the inter-user channels are reciprocal and Rayleigh faded. Therefore, P_{AB}^C , P_{BA}^C and $P_{AB}^N (=P_{BA}^N)$, depending on the corresponding average

received SNRs $\bar{\gamma}_{AB}(1 - \beta_A)$, $\bar{\gamma}_{AB}(1 - \beta_B)$ and $\bar{\gamma}_{AB}$, are, respectively, given by [1]

$$P_{AB}^C = \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_{AB}(1 - \beta_A)}{\bar{\gamma}_{AB}(1 - \beta_A) + 1}} \right), \quad (4.4.4)$$

$$P_{BA}^C = \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_{AB}(1 - \beta_B)}{\bar{\gamma}_{AB}(1 - \beta_B) + 1}} \right) \quad (4.4.5)$$

and

$$P_{AB}^N = P_{BA}^N = \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_{AB}}{\bar{\gamma}_{AB} + 1}} \right). \quad (4.4.6)$$

From (4.4.2) and (4.4.3), η_A and η_B can be calculated as

$$\eta_A = \frac{(1 - P_{BA}^N) - (P_{BA}^C - P_{BA}^N)(1 - P_{AB}^N)}{1 - (P_{AB}^C - P_{AB}^N)(P_{BA}^C - P_{BA}^N)} \quad (4.4.7)$$

and

$$\eta_B = \frac{(1 - P_{AB}^N) - (P_{AB}^C - P_{AB}^N)(1 - P_{BA}^N)}{1 - (P_{AB}^C - P_{AB}^N)(P_{BA}^C - P_{BA}^N)}. \quad (4.4.8)$$

The occurrence probabilities of those four cases are given by

$$Pr(1) = \eta_A \cdot (1 - P_{AB}^C), \quad (4.4.9)$$

$$Pr(2) = (1 - \eta_A) \cdot (1 - P_{AB}^N), \quad (4.4.10)$$

$$Pr(3) = \eta_A \cdot P_{AB}^C \quad (4.4.11)$$

and

$$Pr(4) = (1 - \eta_A) \cdot P_{AB}^N. \quad (4.4.12)$$

By substituting (4.4.4)-(4.4.8) into (4.4.9)-(4.4.12), we can obtain the complete expressions of $Pr(u)$.

4.4.2 Average Error Probabilities for Four Cases— $P_{e,u}$

In this part, we first derive the decision regions according to our proposed detection rule (4.3.9) and then calculate the average BER for each of the four cases.

For the first case, $f_A = 1$ and $f_B = 1$. The information of user A is transmitted by itself with power $(1 - \beta_A)P$ and also relayed by user B with power $\beta_B P$. The performance is improved by the other user's cooperation and also affected by the other user's interference. By expanding the square terms in (4.3.9) and omitting the time frame notation of the received signals and instantaneous channel SNRs, the detection of user A 's information can be rewritten as

$$\begin{aligned} \hat{s}_A(k) = \arg\min_{b_A(k)} & \left\{ |y_{AD}|^2 + |y_{BD}|^2 + \gamma_{AD} + \gamma_{BD} + 2c_A b_A(k)b_B(k-1) \right. \\ & + 2c_B b_A(k)b_B(k) - 2X \left(\sqrt{1 - \beta_A} b_A(k) + \sqrt{\beta_A} b_B(k-1) \right) \\ & \left. - 2Y \left(\sqrt{1 - \beta_B} b_B(k) + \sqrt{\beta_B} b_A(k) \right) \right\}. \end{aligned} \quad (4.4.13)$$

Two coefficients in (4.4.13) are defined by

$$c_A = \gamma_{AD} \sqrt{(1 - \beta_A)\beta_A} \quad \text{and} \quad c_B = \gamma_{BD} \sqrt{(1 - \beta_B)\beta_B}. \quad (4.4.14)$$

Two variables X and Y are respectively given by

$$X = \Re\{h_{AD}^* y_{AD}\} \quad \text{and} \quad Y = \Re\{h_{BD}^* y_{BD}\}. \quad (4.4.15)$$

We assume that h_{AD} , h_{BD} , x_A and x_B are known, and only the noise terms are vari-

ables. Hence, X and Y obey the Gaussian distribution, i.e., $X \sim \mathcal{N}(\gamma_{AD}x_A, \gamma_{AD}/2)$ and $Y \sim \mathcal{N}(\gamma_{BD}x_B, \gamma_{BD}/2)$. We then remove the common term $|y_{AD}|^2 + |y_{BD}|^2 + \gamma_{AD} + \gamma_{BD}$ in (4.4.13), and simplify (4.4.13) to

$$\hat{s}_A(k) = \underset{b_A(k)}{\operatorname{argmin}} f(b_A(k), b_B(k-1), b_B(k)), \quad (4.4.16)$$

where the cost function is given by

$$\begin{aligned} f(b_A(k), b_B(k-1), b_B(k)) = & (c_A b_B(k-1) + c_B b_B(k) - X\sqrt{1-\beta_A} - Y\sqrt{\beta_B})b_A(k) \\ & - X\sqrt{\beta_A} b_B(k-1) - Y\sqrt{1-\beta_B} b_B(k). \end{aligned} \quad (4.4.17)$$

Since $b_A(k), b_B(k-1)$ and $b_B(k) \in \{1, -1\}$, the cost function has eight possible values. Equation (4.4.16) demonstrates that $\hat{s}_A(k)$ takes the value of $b_A(k)$ when the cost function reaches its minimum value. Therefore, the decoding of user A 's information is equivalent to

$$\hat{s}_A(k) = \begin{cases} 1, & \text{if } \min f(1, b_B(k-1), b_B(k)) \leq \min f(-1, \check{b}_B(k-1), \check{b}_B(k)) \\ -1, & \text{otherwise.} \end{cases} \quad (4.4.18)$$

The signals $\check{b}_B(k-1)$ and $\check{b}_B(k)$ also denote the candidates of user B 's signals in the $(k-1)$ th and k th time frames. Hence, $\check{b}_B(k-1)$ and $\check{b}_B(k) \in \{1, -1\}$.

Depending on the specific ranges of X and Y , we find pairs of $(b_B(k-1), b_B(k))$ and $(\check{b}_B(k-1), \check{b}_B(k))$ which can minimize $f(1, b_B(k-1), b_B(k))$ and $f(-1, \check{b}_B(k-1), \check{b}_B(k))$, respectively. Assuming $s_A(k) = -1$, an error occurs if

$$\min f(1, b_B(k-1), b_B(k)) \leq \min f(-1, \check{b}_B(k-1), \check{b}_B(k)). \quad (4.4.19)$$

The regions of error detection can be illustrated in the two-dimensional space (X, Y) shown in Figure 4.2. The detailed derivation is given in Appendix A. When $s_A(k) = -1$, x_A has two possible values, $x_A = -\sqrt{1 - \beta_A} - \sqrt{\beta_A}$ or $x_A = -\sqrt{1 - \beta_A} + \sqrt{\beta_A}$, with the same probability. Similarly, $x_B = -\sqrt{1 - \beta_B} - \sqrt{\beta_B}$ or $x_B = \sqrt{1 - \beta_B} - \sqrt{\beta_B}$ with the equal probability. Hence, the PDFs of X and Y are, respectively, given by

$$p_X(X|s_A(k) = -1) = \frac{1}{2\sqrt{\pi\gamma_{AD}}} \left\{ \exp \left[-\frac{\left(X + \gamma_{AD}(\sqrt{1 - \beta_A} + \sqrt{\beta_A})\right)^2}{\gamma_{AD}} \right] + \exp \left[-\frac{\left(X + \gamma_{AD}(\sqrt{1 - \beta_A} - \sqrt{\beta_A})\right)^2}{\gamma_{AD}} \right] \right\} \quad (4.4.20)$$

and

$$p_Y(Y|s_A(k) = -1) = \frac{1}{2\sqrt{\pi\gamma_{BD}}} \left\{ \exp \left[-\frac{\left(Y + \gamma_{BD}(\sqrt{1 - \beta_B} + \sqrt{\beta_B})\right)^2}{\gamma_{BD}} \right] + \exp \left[-\frac{\left(Y - \gamma_{BD}(\sqrt{1 - \beta_B} - \sqrt{\beta_B})\right)^2}{\gamma_{BD}} \right] \right\}. \quad (4.4.21)$$

By integrating the PDFs of X and Y over the error decision regions, the conditional error probability is

$$P_{e,1}(\gamma_{AD}, \gamma_{BD}) = \iint_{R_I + R_{II} + R_{III}} p_X(X|s_A(k) = -1) p_Y(Y|s_A(k) = -1) dX dY. \quad (4.4.22)$$

Therefore, the average BER for the first case is given by

$$P_{e,1} = \int_0^\infty \int_0^\infty P_{e,1}(\gamma_{AD}, \gamma_{BD}) p_{\gamma_{AD}}(\gamma_{AD}) p_{\gamma_{BD}}(\gamma_{BD}) d\gamma_{AD} d\gamma_{BD}, \quad (4.4.23)$$

where $p_{\gamma_{iD}}(\gamma_{iD})$ is the PDF of the exponentially distributed variable γ_{iD} with parameter $\bar{\gamma}_{iD}$. This exact BER can be computed by numerical integration and the

Figure 4.2: Decision regions for the first case $f_A = 1$ and $f_B = 1$, when $s_A(k) = -1$. $O : (0, 0)$, $J : (-X_{th}, 0)$, $V : (X_{th}, 0)$, $K : (0, -Y_{th})$ and $H : (0, Y_{th})$, where $X_{th} = \gamma_{AD}\sqrt{1 - \beta_A}$ and $Y_{th} = \gamma_{BD}\sqrt{1 - \beta_B}$. © [2011] IEEE

theoretical results are provided in Section 4.5 under different channel conditions.

To derive the lower bound of BER, we replace the limits of integration in (4.4.22) by R_I , which is expressed as $\{X \geq 0\} \cap \{-Y_{th} < Y \leq 0 \cup Y > Y_{th}\}$. It represents the situation that errors occur in both transmissions if D detects the received signals

from A and B separately. Hence, the lower bound of BER is given by

$$\begin{aligned}
 P_{e,1,lb} = & \underbrace{\int_0^\infty \int_0^\infty p_X(X|s_A(k) = -1) p_{\gamma_{AD}}(\gamma_{AD}) dX d\gamma_{AD}}_{P_{e,1,AD}} \\
 & \times \underbrace{\int_0^\infty \left(\int_{-Y_{th}}^0 p_Y(Y|s_A(k) = -1) dY + \int_{Y_{th}}^\infty p_Y(Y|s_A(k) = -1) dY \right) p_{\gamma_{BD}}(\gamma_{BD}) d\gamma_{BD}}_{P_{e,1,BD}}.
 \end{aligned} \tag{4.4.24}$$

Note that $P_{e,1,AD}$ and $P_{e,1,BD}$ are given by

$$P_{e,1,AD} = \frac{1}{2} \left[\frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_{AD,1}}{\bar{\gamma}_{AD,1} + 1}} \right) + \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_{AD,2}}{\bar{\gamma}_{AD,2} + 1}} \right) \right] \tag{4.4.25}$$

and

$$P_{e,1,BD} = \frac{1}{2} \left[\left(1 - \sqrt{\frac{\bar{\gamma}_{BD,1}}{\bar{\gamma}_{BD,1} + 1}} \right) + \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_{BD,2}}{\bar{\gamma}_{BD,2} + 1}} \right) \right], \tag{4.4.26}$$

where $\bar{\gamma}_{AD,1} = \bar{\gamma}_{AD} (\sqrt{1 - \beta_A} + \sqrt{\beta_A})^2$, $\bar{\gamma}_{AD,2} = \bar{\gamma}_{AD} (\sqrt{1 - \beta_A} - \sqrt{\beta_A})^2$, $\bar{\gamma}_{BD,1} = \bar{\gamma}_{BD} \beta_B$ and $\bar{\gamma}_{BD,2} = \bar{\gamma}_{BD} (\sqrt{1 - \beta_B} - \sqrt{\beta_B})^2$.

In the second case, $f_A = 0$ and $f_B = 1$. The information of user A is transmitted by itself with full power of P and also relayed by user B with power $\beta_B P$. The decision region, the exact average BER and the lower bound of BER in this case can be derived in the same way as in the first case. The error probability of transmission from A to D in this case is given by

$$P_{e,2,AD} = \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_{AD}}{\bar{\gamma}_{AD} + 1}} \right). \tag{4.4.27}$$

The error probability of transmission from B to D is $P_{e,2,BD} = P_{e,1,BD}$. Therefore,

the lower bound of BER in the second case is given by

$$P_{e,2,lb} = P_{e,2,AD} \cdot P_{e,2,BD}. \quad (4.4.28)$$

For the third case, $f_A = 1$ and $f_B = 0$. User A transmits its own information with power $(1 - \beta_A)P$ and also helps to forward B 's information with power $\beta_A P$, i.e., $x_A(k) = \sqrt{1 - \beta_A}s_A(k) + \sqrt{\beta_A}s_B(k - 1)$. But user B does not help to relay user A 's information. Hence, the detection of $s_A(k)$ becomes

$$\hat{s}_A(k) = \underset{b_A(k)}{\operatorname{argmin}} \left| y_{AD}(k) - h_{AD}(k) \left[\sqrt{1 - \beta_A} b_A(k) + \sqrt{\beta_A} b_B(k - 1) \right] \right|^2. \quad (4.4.29)$$

The conditional error probability of user A 's information in the third case is given by

$$P_{e,3}(\gamma_{AD}) = \begin{cases} Q\left(\sqrt{2\gamma_{AD}(\sqrt{1 - \beta_A} + \sqrt{\beta_A})^2}\right), & \text{if } s_A(k) = s_B(k - 1), \\ Q\left(\sqrt{2\gamma_{AD}(\sqrt{1 - \beta_A} - \sqrt{\beta_A})^2}\right), & \text{if } s_A(k) = -s_B(k - 1). \end{cases} \quad (4.4.30)$$

Therefore, the exact average BER in this case is

$$P_{e,3} = P_{e,1,AD} = \frac{1}{2} \left[\frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_{AD,1}}{\bar{\gamma}_{AD,1} + 1}} \right) + \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_{AD,2}}{\bar{\gamma}_{AD,2} + 1}} \right) \right]. \quad (4.4.31)$$

In the fourth case, $f_A = 0$ and $f_B = 0$. The information of user A is only transmitted by itself with full power of P and $x_A(k) = s_A(k)$. The detection of A 's signal in (4.3.9) is simplified as

$$\hat{s}_A(k) = \underset{b_A(k)}{\operatorname{argmin}} \left| y_{AD}(k) - h_{AD}(k) b_A(k) \right|^2. \quad (4.4.32)$$

It is the same with direct transmission of BPSK signals. Hence, the conditional error

probability is $Q(\sqrt{2\gamma_{AD}})$ and the exact average BER in the fourth case is given by

$$P_{e,4} = \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_{AD}}{\bar{\gamma}_{AD} + 1}} \right). \quad (4.4.33)$$

Finally, the average BER and its lower bound can be obtained by substituting (4.4.9)-(4.4.12) and $P_{e,u}$ ($P_{e,u,lb}$) into (4.4.1).

4.5 Numerical Results and Comparison

In this section, we first present the simulated and theoretical BER performance when the three-time-slot TDBC protocol is employed by a two-user CMA system. Next, we provide the BER performance achieved by the superposition modulation protocol with different superposition factors and channel conditions. Finally, we compare the BER performance of superposition modulation, three-time-slot TDBC and four-time-slot TDBC protocols under the same power and data rate constraints.

In Figures 4.3-4.4, the three-time-slot TDBC protocol with the CSI-based AF relaying is employed. We assume that three channels in the two-user CMA system are i.i.d. Rayleigh faded and $P_A = P_B$. Therefore, we have $\bar{\gamma}_{AD} = \bar{\gamma}_{BD} = \bar{\gamma}_{AB}$. In Figure 4.3, we provide the simulated BER performance in terms of the power allocation factor for BPSK and 8PSK modulations at different SNR values. It can be seen that $\alpha_A = \alpha_B = 0.2$ provides the best BER performance for both BPSK and 8PSK modulations. Therefore, we will adopt this optimal power allocation factor, $\alpha_{opt} = 0.2$, for the three-time-slot TDBC protocol in Figures 4.4 and 4.8. In Figure 4.4, we present both simulated and theoretical BER curves of BPSK, QPSK and 8PSK modulation schemes. The simulated results are obtained based on the detection rule given in (4.2.5) and the theoretical results are obtained by the

(a) BPSK

(b) 8PSK

Figure 4.3: BER of user A's information vs the power allocation factor α_A for BPSK and 8PSK modulations in the three-time-slot TDBC protocol over i.i.d. Rayleigh fading channels.

Figure 4.4: User A 's BER performance for BPSK, QPSK and 8PSK modulations employing the three-time-slot TDBC protocol in i.i.d. Rayleigh fading channels.

numerical integration of (4.2.8). It can be observed that the simulated results are lower bounded by the theoretical results for BPSK and QPSK signals. However, the theoretical BER curve of 8PSK is slightly higher than the actual BER in the low and medium SNR region, since the approximation in (4.2.7) to compute the conditional SER is more accurate for high order PSK and high SNR values. But the theoretical result of 8PSK coincides with its simulated BER at high SNRs. It is also illustrated by Figure 4.4 that the three-time-slot TDBC protocol is capable of achieving a diversity order of two.

In Figures 4.5-4.7, the two-time-slot superposition modulation protocol is employed. All of its simulated results are obtained according to the decision rule in (4.3.9). We firstly provide the simulated BER performance of user A 's data in terms of the superposition factor β_A at different SNR values in Figure 4.5. It is assumed that

Figure 4.5: BER of user A 's signal vs the superposition factor β_A , employing the superposition modulation protocol in i.i.d. Rayleigh channels. © [2011] IEEE

channels are i.i.d. Rayleigh faded and that $\beta_A = \beta_B$. Therefore, the error probability of user B 's information is identical to that of user A 's. As seen from Figure 4.5, the system achieves the best performance when $\beta_A = 0.1$. If $\beta_A = 0$, the two users only transmit their own signals alternatively. It becomes a time-division multiple-access channel without source cooperation. Hence, the achievable diversity gain is only one and the BER is non-optimal. As the value of β_A increases, each user allocates a larger fraction of power to assist in the other user's transmission and thus the BER performance of the other user is improved. However, the more power allocated to forward the other user's information, the less power is used for its own transmission and the higher interference is caused to each other. This is the trade-off between cooperation and interference. Until $\beta_A = 0.5$, both original and relayed signals are transmitted at the same power level. In the case that user A can decode B 's signal

correctly, the transmitted signal from A becomes

$$x_A(k) = \sqrt{0.5}s_A(k) + \sqrt{0.5}s_B(k-1). \quad (4.5.1)$$

Thus it has a high error probability to detect $s_A(k)$ from $x_A(k)$ when $s_A(k) = -s_B(k-1)$. Moreover, it can also be observed that the local optimal value of β_A in the range of $[0.5, 1]$ is 0.8. However, the BER achieved at $\beta_A = 0.8$ is not as good as the BER at $\beta_A = 0.1$. The reason is that when $0.5 \leq \beta_A \leq 1$, user A spends higher power to transmit user B 's signals than its own signals. The probability that user B can correctly decode A 's information is decreased. Thus, f_B has a higher chance to be equal to 0. In addition, user A 's information is received by D with higher power from the relay channel rather than the direct link. According to the decision rule of user A 's information in (4.3.9), it can be understood why the minimal BER achieved in $[0.5, 1]$ is not globally optimal. This also explains why the BER curves in $[0, 0.5]$ and $[0.5, 1]$ are not symmetric with respect to $\beta_A = 0.5$. For the case that $\beta_A = 1$, user A transmits either its own signal or user B 's signal with its full power. It is not suitable to use (4.3.9) to obtain the corresponding BER in this special case. Hence, the BER at $\beta_A = 1$ is the worst.

Figure 4.6 provides the simulated and theoretical BER performance of user A 's information with different superposition factors. The three channels are also assumed to be i.i.d. Rayleigh faded. Based on the results in Figure 4.5, we set $\beta_A = \beta_B = 0.1$ to achieve the best and identical performance for both users. In this case, a diversity gain of two can be achieved as seen from both the theoretical and simulated BER curves. When $\beta_A = \beta_B = 0$, the system becomes direct transmission. There is no cooperation between two users and diversity gain is only one. When $\beta_A = 0$ and $\beta_B = 1$, the system turns into the conventional cooperation for user A . Although it achieves better performance than the superposition modulation-based CMA system,

Figure 4.6: Simulated and theoretical BER performance of user A 's signal with different superposition factors in i.i.d. Rayleigh channels. © [2011] IEEE

user B 's signal has not been transmitted at all, which is spectrally inefficient. We also provide the performance of the MAP detector [106] and the MLSD for comparison. It is shown that our proposed detector can obtain the same performance as the MAP detector and close to that of the MLSD with low complexity.

In Figure 4.7, we consider two kinds of non-identical channel scenarios. The theoretical and simulated BER performance of user A 's data in terms of $\bar{\gamma}_{AD}$ is provided. The BER curves in the cases of $\beta_A = \beta_B = 0.1$ and i.i.d. channels are also plotted for comparison. Firstly, the channel condition from B to D is better than that from A to D . We assume that $\bar{\gamma}_{BD} = \bar{\gamma}_{AD} + 10 \text{ dB} = \bar{\gamma}_{AB} + 10 \text{ dB}$, $\beta_A = 0.1$ and $\beta_B = 0.3$. Compared with user A , user B allocates more power to forward the other's signal in order to take advantage of the better channel condition. Therefore, the performance of user A 's signal improves significantly as compared to that in the

Figure 4.7: Simulated and theoretical BER performance of user A 's signal, employing superposition modulation protocol over i.n.i.d. Rayleigh channels. © [2011] IEEE

i.i.d. case. Contrarily, in the other circumstance, the channel condition from B to D is worse than that from A to D , e.g., $\bar{\gamma}_{BD} = \bar{\gamma}_{AD} - 10$ dB and $\bar{\gamma}_{AB} = \bar{\gamma}_{AD}$. The superposition factors are still set to be $\beta_A = \beta_B = 0.1$. Apparently, the BER performance is worse than that in the i.i.d. case due to the poor channel condition from user B to D . It is noticed that the theoretical results obtained by numerical integration in these three cases are exactly consistent with the simulated BER curves, which confirms the derivation of error decision regions. Furthermore, it is shown that the full diversity of two can be achieved by the superposition modulation protocol in any channel conditions with appropriate superposition factors.

In Figure 4.8, we compare the simulated BER performance of the superposition modulation protocol, three-time-slot TDBC and four-time-slot TDBC protocols over i.i.d. Rayleigh fading channels. For a fair comparison, we assume that the average

Figure 4.8: BER performance comparison of superposition modulation, three-time-slot TDBC and four-time-slot TDBC protocols in i.i.d. Rayleigh fading channels.

transmitted power in each time slot is equal to one, and the average data rate is set to be 2 bits per channel use in these three transmission schemes. For the superposition modulation protocol, each user's signal is modulated to QPSK signal and the adaptive DF relaying strategy is employed. Each user allocates 10% ($\beta_A = \beta_B = 0.1$) of its own power for relaying. For the three-time-slot TDBC protocol, we assume that both users transmit 8PSK signals. To satisfy the average power constraint, each user transmits with power $3(1 - \alpha)/2$ in the first two time slots, and with power $3\alpha/2$ in the third time slot. To achieve the best performance, $\alpha_A = \alpha_B = 0.2$ is required. For the four-time-slot TDBC protocol, we assume that users transmit 16PSK signals. The adaptive DF relaying protocol and the MRC detection at the destination are adopted. It is observed from Figure 4.8 that these three schemes achieve the full diversity gain of two, and the superposition modulation protocol outperforms the TDBC protocols under the same transmission

rate and power constraints. Therefore, the superposition modulation scheme is a superior strategy for CMA systems, maximizing the system spectral efficiency as well as achieving the full diversity order for each user.

4.6 Conclusion

In this chapter, we have analyzed the BER performance of two-user CMA systems with efficient transmission schemes, namely, the AF-based three-time-slot TDBC scheme and the DF-based superposition modulation scheme. In the superposition modulation scheme, we have proposed a low-complexity, near-optimal detector. The achievable BER results of both schemes have been derived and validated by simulation results. We also compared the BER performance of these two protocols and the four-time-slot TDBC scheme. It has been shown that the superposition modulation scheme outperforms the TDBC protocols and can achieve both good BER performance and high spectral efficiency with less complexity. Part of the work in this chapter has appeared in our journal paper “Performance analysis of two-user multiple access systems with DF relaying and superposition modulation” on IEEE Transactions on Vehicular Technology, the third one in Author’s Publications.

Chapter 5

Performance of an Efficient Multiple-Relay Cooperative System

5.1 Introduction and Related Results

In this chapter, we will expand our analysis to a large-scale cooperative system, which consists of a source, a destination and multiple parallel relays. Similar to the assumption in previous chapters, all the terminals are under the half-duplex constraint. One simple transmission protocol is to assign orthogonal time slots or frequency bands for relays to forward the same source signal to the destination separately [22, 23, 33, 50]. However, the spectral efficiency is quite low and is reversely proportional to the number of relays. In this situation, a DSTC protocol was proposed and employed in [14, 15], which takes only two time slots to transmit the source's messages. In the first time slot, the source broadcasts to all the relays. In the second time slot, all the relays or a selected group of relays are allowed to trans-

mit simultaneously to the destination by employing different space-time codes. The maximal multiplexing gain is increased to 0.5 and the full spatial diversity gain can be achieved as well. In addition to the DSTC protocol, several single relay selection protocols are also proposed for the multiple-relay cooperative system and proved to achieve the same DMT performance as the DSTC protocol does. In [16, 72], the max-min relay selection approach was adopted. Each relay finds the minimum of its corresponding instantaneous source-relay and relay-destination channel gains. A single relay which has the largest channel gain among these minimal ones is selected to forward the source's information to the destination in the second phase. Another kind of relay selection method, adopted in [35, 57, 58], is called the opportunistic relaying (OR) protocol. It selects a relay with the best relay-destination channel quality from a decoding set, and the decoding set is composed of relays whose source-relay transmission is not in outage. However, due to the half-duplex constraint and orthogonal transmission, the maximal achievable multiplexing gain of either single relay selection scheme is only 0.5.

In order to improve the spectral efficiency but still satisfy the half-duplex constraint, a two-path relaying protocol [75], also called successive relaying (SR) protocol [113], was proposed for a two-relay cooperative system. The source keeps transmitting new messages and two relays mimic a full-duplex relay to alternatively forward messages to the destination. The achievable rates for both AF and DF relaying methods were analyzed when the direct channel between source and destination is unavailable [75]. The capacity for DF relaying was further investigated in [113] under different channel and interference conditions while the direct link is considered. Additionally, from the information theoretic aspect, [114] and [115] studied the achievable rate of the SR protocol in a two-relay DF network without direct link, while the inter-relay interference is considered or ignored, respectively. The authors in [116] proposed the shifted successive DF relaying (SSDFR) protocol for the multi-antenna two-

relay network and investigated its asymptotically achievable DMT performance. Although all these studies in [75, 113–116] made the efficient use of the spectrum, they only investigated the systems with two relays but did not consider large systems with a sizable number of prospective relays.

Aiming to achieve both high diversity and multiplexing gains, the sequential slotted AF scheme and the random sequential AF scheme were proposed for the multi-relay networks in [25] and [42], respectively. The DMT performance was investigated for the two cases that the inter-relay interference is ignored or considered. However, due to the AF relaying, the noise terms are accumulated at relays and propagated to the destination, which degrades the transmission accuracy as time slots increases. Reference [117] employed the quantize-and-map scheme for the multi-relay system and proved that it can approach the optimal DMT bound as the multiple-input single-output (MISO) channel. However, this good result was based on an impractical assumption that the relays are isolated and not interfering with each other. In [118], the inter-relay transmission was considered as interference and relays employed the hybrid of successive cancellation and joint decoding to decode the signals transmitted from the source. The maximal achievable diversity order is one less than the full diversity, as one relay cannot receive the continuous source transmission while it is occupied to transmit to the destination.

In this chapter, we propose an effective and efficient relaying protocol for the multi-relay DF cooperative system and the inter-relay transmission is considered. In our proposed SR protocol, two selected relays alternatively and successively receive and forward to the destination during each constant fading block. The receiving relay jointly decodes both signals transmitted from the source and the other selected relay in a certain time slot. Thus the relay not only acquires the knowledge of the source's transmitted signal in this time slot, but also the source's previous transmitted signal

that it cannot obtain from the source directly. The ML decoding is adopted by the destination after a block transmission. We make use of the inter-relay interference and let the relay transmit the superposition of these two signals in the next time slot. Hence, each signal is transmitted twice in total by the selected relays and the full diversity is achievable. On the other hand, since the source keeps transmitting newly generated signals, the maximal multiplexing gain is larger than 0.5 and approaching 1 as the number of time slots in one channel block increases. The lower bound of the outage probability and the optimal achievable DMT are derived in Section 5.3. Numerical results are provided in Section 5.4 to validate the analysis and illustrate the effects of the transmission rate, the number of relays and the time slot on the outage performance.

5.2 System Model and The Successive Relaying Protocol

The multiple-relay cooperative system is shown in Figure 5.1. The source, destination and M parallel relay nodes are denoted as S , D , and R_m ($m = 1, 2, \dots, M$), respectively. Due to the long distance between S and D , there is no direct transmission. Except the direct link, we assume that all the other channels connecting any two nodes experience independent Rayleigh flat-fading and keep stationary within a block duration. The channel coefficient from nodes i to j can be denoted as h_{ij} , which is a complex Gaussian random variable with zero mean and variance σ_{ij}^2 . Furthermore, the channels are assumed to be reciprocal, i.e., $h_{ij} = h_{ji}$. Without loss of generality, we assume that the noise at every receiving node is independent Gaussian noise with zero mean and unit power. We denote the transmitted power of node i as P_i . Hence, the instantaneous received SNR from i to j is given by

Figure 5.1: The multiple-relay network with the SR transmission protocol.

$\gamma_{ij} = P_i |h_{ij}|^2$, which is exponentially distributed and its average value is given by $\bar{\gamma}_{ij} = P_i \sigma_{ij}^2$.

We assume that only the receiver has the full CSI of each incoming channel. In our proposed SR protocol, the relay selection process is conducted before the data transmission in each block as follows. Firstly, S broadcasts a bit to all the relays. Secondly, each relay successively transmits a bit to other relays and D . In this case, every relay acquires the channel conditions from S and all the other relays. Next, each relay continues to successively transmit a sequence of $M - 1$ bits indicating whether it can jointly decode the concurrent transmission from S and other $M - 1$ relays. After receiving all the information, D determines the decoding set for SR, $\mathcal{A}_{SR}$, in which every relay is eligible to decode the transmission from S and any other relay not in outage. Two relays with the largest and second largest $\gamma_{R_m D}$ are selected from $\mathcal{A}_{SR}$ and denoted as R_1^* and R_2^* , respectively. D feeds back $2 \log_2 M$ bits to

inform the selected relays. Therefore, the SR protocol requires $1 + M^2 + 2 \log_2 M$ bits of overhead in each block, which diminish for a large amount of data transmission in each block or in high SNR regions [118]. In the special case that $\mathcal{A}_{SR}$ contains less than two relays, a different decoding set for single relay selection, $\mathcal{A}_{OR}$, is formed as in the OR protocol [58]. It involves those relays that the transmission from S to any of them is not in outage. A single relay with the largest $\gamma_{R_m D}$, denoted as $\tilde{R}$, is selected from $\mathcal{A}_{OR}$ for data transmission. For the worst case that $\mathcal{A}_{OR}$ is empty, no relay is eligible to forward in this block. Note that $\mathcal{A}_{SR}$, $\mathcal{A}_{OR}$, R_1^* , R_2^* and $\tilde{R}$ are determined in every block as channel conditions change.

During the data transmission, two transmission modes might be adopted, namely, SR mode and OR mode. The OR mode is identical to the conventional OR protocol in [58]. In the SR transmission mode, two relays are selected. The block duration is equally divided into $T + 2$ time slots ($T \geq 3$). S transmits new message x_t in the t -th time slot ($1 \leq t \leq T$) and $\mathbb{E}\{|x_t|^2\} = 1$. Two selected relays successively decode and forward signals to D from the second to the $(T + 2)$ -th time slot. Two extra time slots are used for the relays to complete the forwarding process. Hence, the spectrum efficiency of the SR protocol is $T/(T + 2)$, which is larger than that of the OR protocol and approaches the optimal value of one as T goes to infinity. In reality, the value of T is determined based on the channel coherence time, decoding complexity, system requirement, etc. Specifically, as shown in Table 5.1, in the first time slot, S broadcasts x_1 to all the relays, and R_1^* decodes it as $\hat{x}_1$. In the second time slot, S continues to transmit x_2 . R_1^* forwards $\hat{x}_1$ to D so that it cannot receive x_2 at the same time. All the other relays receive x_2 from S and also overhear $\hat{x}_1$ from R_1^* . The selected R_2^* jointly decodes x_2 and $\hat{x}_1$, and the decoded signals are expressed as $\hat{x}_2$ and $\hat{\hat{x}}_1$, respectively. In the third time slot, S transmits x_3 and R_2^* broadcasts $x_{3,R}$, the superposition of $\hat{x}_2$ and $\hat{\hat{x}}_1$, to other relays and the destination. At the same time, R_1^* decodes x_3 from S and $x_{3,R}$ from R_2^* as $\hat{x}_3$ and $\hat{x}_{3,R}$, respectively.

Table 5.1: The SR transmission schedule in one block

Time Slot	1	2	3	4	$\dots$	$t + 1$	$\dots$	$T + 1$	$T + 2$
S	x_1	x_2	x_3	x_4	$\dots$	x_{t+1}	$\dots$		
R_1^*		$\hat{x}_1$		$x_{4,R} :$ $(\hat{x}_3, \hat{x}_2)$	$\dots$	$x_{t+1,R} :$ $(\hat{x}_t, \hat{x}_{t-1})$	$\dots$		$\hat{x}_T$
R_2^*			$x_{3,R} :$ $(\hat{x}_2, \hat{x}_1)$		$\dots$		$\dots$	$x_{T+1,R} :$ $(\hat{x}_T, \hat{x}_{T-1})$	

Since $\hat{x}_1$ is already known by R_1^* in the first slot, $\hat{x}_2$ can be consequently obtained from the decoded superposition $\hat{x}_{3,R}$. In the fourth time slot, R_1^* transmits $x_{4,R}$, the superposition of $\hat{x}_3$ and $\hat{x}_2$ accordingly. The superimposed signal transmitted in the t -th time slot can be generally expressed as

$$x_{t,R} = \sqrt{\alpha_t} \hat{x}_{t-1} + \sqrt{\beta_t} \hat{x}_{t-2}, \quad (5.2.1)$$

where α_t and β_t are the superposition factors to control the transmitted power levels of $\hat{x}_{t-1}$ and $\hat{x}_{t-2}$ in the t -th time slot, respectively. Hence, $E\{|x_{t,R}|^2\} = E\{\alpha_t + \beta_t\}$. Note that $\hat{x}_t$ and $\hat{\hat{x}}_t$ represent signals originally transmitted from S in the t -th time slot but decoded by different relays in successive time slots. For example, as shown in Table 5.1, $\hat{x}_t$ denotes the decoded signal by R_1^* from the source transmission in the t -th time slot, while $\hat{\hat{x}}_t$ is the decoded signal by R_2^* based on the inter-relay transmission in the $(t + 1)$ -th time slot. The process continues until the $(T + 1)$ -th time slot, and the source stops transmitting messages. One relay (R_1^* or R_2^* , depending on T is odd or even) transmits the superposition of $\hat{x}_T$ and $\hat{\hat{x}}_{T-1}$. In the $(T + 2)$ -th time slot, the other relay forwards $\hat{\hat{x}}_T$ to the destination.

The successive transmission from relays to the destination in one block can be expressed as

$$\mathbf{y} = \text{diag}\{\sqrt{\mathbf{p}}\} \mathbf{H} \mathbf{x} + \mathbf{n}, \quad (5.2.2)$$

where $\mathbf{y}$ and $\mathbf{n}$ are the $(T+1) \times 1$ received signal and noise vectors at D , respectively.

The $(T+1) \times 1$ power vector is given by

$$\mathbf{p} = [P_{R_1^*}, P_{R_2^*}, P_{R_1^*}, P_{R_2^*}, \dots]^T. \quad (5.2.3)$$

We assume that signals can be decoded correctly if the transmission is not in outage.

Hence, we have

$$\mathbf{x} = [x_1, x_2, \dots, x_T]^T. \quad (5.2.4)$$

The $(T+1) \times T$ equivalent channel matrix $\mathbf{H}$ is expressed as

$$\mathbf{H} = \begin{pmatrix} h_{R_1^*D} & 0 & 0 & \cdots & 0 & 0 \\ \sqrt{\beta_3}h_{R_2^*D} & \sqrt{\alpha_3}h_{R_2^*D} & 0 & \cdots & 0 & 0 \\ 0 & \sqrt{\beta_4}h_{R_1^*D} & \sqrt{\alpha_4}h_{R_1^*D} & \cdots & 0 & 0 \\ 0 & 0 & \sqrt{\beta_5}h_{R_2^*D} & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \sqrt{\beta_{T+1}}h_{R_2^*D} & \sqrt{\alpha_{T+1}}h_{R_2^*D} \\ 0 & 0 & 0 & \cdots & 0 & h_{R_1^*D} \end{pmatrix}. \quad (5.2.5)$$

The destination jointly decodes all the messages in one block.

5.3 Performance Analysis

In this section, we analyze the outage probability of our proposed SR protocol. Since it includes both SR and OR transmission modes, the total outage probability is the

sum of the product of each mode's occurrence probability and the respective outage probability. It can be expressed as

$$P_{out} = P_{SR} P_{out,SR} + P_{OR} P_{out,OR}. \quad (5.3.1)$$

We will derive the occurrence probability and outage probability of the SR mode, denoted as P_{SR} and $P_{out,SR}$, in Section 5.3.1 and Section 5.3.2, respectively. The occurrence and outage probabilities for the OR mode, P_{OR} and $P_{out,OR}$, will be derived in 5.3.3.

5.3.1 The occurrence probability of the SR mode

In this part, we first investigate the requirements for the occurrence of SR and then derive the corresponding probability. The concurrent transmission of the desired signals from S and R_v to R_m is not in outage if the following conditions are satisfied

$$I_m : \quad \frac{T}{T+2} \log_2(1 + \gamma_{SR_m}) \geq r_0 \quad (5.3.2)$$

$$II_{vm} : \quad \frac{T}{T+2} \log_2(1 + \alpha \gamma_{R_v R_m}) \geq r_0 \quad (5.3.3)$$

$$III_{vm} : \quad \frac{T}{T+2} \log_2(1 + \gamma_{SR_m} + \alpha \gamma_{R_v R_m}) \geq 2r_0, \quad (5.3.4)$$

where $\alpha = \min\{\alpha_t\}$ and r_0 is the required transmission rate in bits per channel use (BPCU) unit. If any condition is not satisfied, the concurrent transmission from S and R_v to R_m is in outage. Condition I_m requires that the transmission from S to R_m is not in outage. Condition II_{vm} represents that the inter-relay transmission of the desired signal from R_v to R_m is not in outage. Condition III_{vm} depicts that the multiple-access transmission from S and R_v to R_m is not in outage. Similarly, the

requirements for the transmission from S and R_m to R_v not in outage are given by

$$I_v : \quad \frac{T}{T+2} \log_2(1 + \gamma_{SR_v}) \geq r_0 \quad (5.3.5)$$

$$II_{mv} : \quad \frac{T}{T+2} \log_2(1 + \alpha \gamma_{R_m R_v}) \geq r_0 \quad (5.3.6)$$

$$III_{mv} : \quad \frac{T}{T+2} \log_2(1 + \gamma_{SR_v} + \alpha \gamma_{R_m R_v}) \geq 2r_0. \quad (5.3.7)$$

If (5.3.2)-(5.3.7) are all satisfied, R_m and R_v can jointly decode the concurrent transmission from the source and each other. We call that the pair of R_m and R_v is successful. The decoding set $\mathcal{A}_{SR}$ is assumed to include N relays and any two of them can construct a successful pair. Hence, any two relays in $\mathcal{A}_{SR}$ are probably selected to forward. The requirement for operating under the SR mode is that $\mathcal{A}_{SR}$ exists, i.e., $N \geq 2$. Since the channels are reciprocal, Conditions II_{vm} and II_{mv} are identical. Additionally, since we focus on the outage probability and DMT performance at high SNR values, different settings of α_t will not affect the analysis and results. For simplicity, we assume that $\alpha_t = \beta_t = 1$, P_i is identical and the variance of each channel is identical. Hence, we can define that $\bar{\gamma} = \bar{\gamma}_{ij}$.

We use $\mathcal{A}_{SR,I}$ to denote the set composed of all the relays satisfying Condition I_m , thus $\mathcal{A}_{SR,I} \supseteq \mathcal{A}_{SR}$. The probability that R_m satisfies Condition I_m is computed as

$$P_I = \Pr(\gamma_{SR_m} \geq 2^{\frac{T+2}{T}r_0} - 1) = e^{-z_I/\bar{\gamma}}, \quad (5.3.8)$$

where we define that

$$z_I = 2^{r'_0} - 1 \quad \text{and} \quad r'_0 = \frac{T+2}{T}r_0. \quad (5.3.9)$$

The probability that $\mathcal{A}_{SR,I}$ includes N_I relays ($2 \leq N_I \leq M$) is given by

$$P_I(N_I) = \binom{M}{N_I} P_I^{N_I} (1 - P_I)^{(M-N_I)}. \quad (5.3.10)$$

There are $N_p = N_I(N_I-1)/2$ inter-relay channels in $\mathcal{A}_{SR,I}$ and each channel connects a pair of relays. Given $\mathcal{A}_{SR,I}$, the probability that a pair of relays within $\mathcal{A}_{SR,I}$ is successful is given by

$$P_{II,III|I} = 2 e^{-(z_{III}-z_I)/\bar{\gamma}} - e^{-(2z_{III}-3z_I)/\bar{\gamma}}, \quad (5.3.11)$$

where $z_{III} = 2^{2r'_0} - 1$. The probability of any pair of relays not successful, provided that they are within $\mathcal{A}_{SR,I}$, is obtained as

$$P_{\bar{II},\bar{III}|I} = 1 + e^{-(2z_{III}-3z_I)/\bar{\gamma}} - 2 e^{-(z_{III}-z_I)/\bar{\gamma}}. \quad (5.3.12)$$

The derivation of (5.3.11) and (5.3.12) is provided in Appendix B. Therefore, the probability that there are k unsuccessful pairs of relays within $\mathcal{A}_{SR,I}$ is given by

$$P_{\bar{II},\bar{III}|I}(k) = \binom{N_p}{k} P_{II,III|I}^{N_p-k} P_{\bar{II},\bar{III}|I}^k. \quad (5.3.13)$$

Since $\mathcal{A}_{SR} \subseteq \mathcal{A}_{SR,I}$, we assume the worst case that one relay in $\mathcal{A}_{SR,I}$ is removed to be involved in $\mathcal{A}_{SR}$ if one pair of relays is not successful. Additionally, there are always more than two relays in $\mathcal{A}_{SR}$ as long as at least one pair is successful. The $\mathcal{A}_{SR}$ is empty if no pair can satisfy all the conditions. Hence, the number of relays within $\mathcal{A}_{SR}$ is expressed as

$$N = \begin{cases} N_I - k, & 0 \leq k \leq N_I - 3 \\ 2, & N_I - 2 \leq k \leq N_p - 1 \\ 0, & k = N_p. \end{cases} \quad (5.3.14)$$

The occurrence probability of the SR mode is given by

$$P_{SR} = \begin{cases} \sum_{N_I=2}^M P_I(N_I) \sum_{k=0}^{N_I-3} P_{\overline{II}, \overline{III}|I}(k), & 3 \leq N \leq M \\ \sum_{N_I=2}^M P_I(N_I) \sum_{k=N_I-2}^{N_p-1} P_{\overline{II}, \overline{III}|I}(k), & N = 2. \end{cases} \quad (5.3.15)$$

5.3.2 The outage probability in the SR mode

In the SR mode, once the SR decoding set is determined in each block, two relays with the largest and second largest $\gamma_{R_m D}$ are selected among N relays in $\mathcal{A}_{SR}$ and alternatively transmit to D according to (5.2.2). The channel shown in (5.2.5) is equivalent to an inter-symbol interference (ISI) channel. Hence, during the successive transmission from relays to D , the outage occurs if

$$\frac{T}{T+2} \log_2 \det(\mathbf{I} + \mathbf{F}_t \mathbf{F}_t^H) < t r_0, \quad \forall t = \{1, 2, \dots, T\}. \quad (5.3.16)$$

The matrix $\mathbf{F}_t$ is a $(t+1) \times t$ matrix and identical to the first $t+1$ rows and first t columns of the matrix product $\text{diag}\{\sqrt{\mathbf{p}}\} \mathbf{H}$ in (5.2.2). Therefore, the outage probability of (5.2.2) is dominated by the highest one among T probabilities of (5.3.16). Recall Sylvester's determinant theorem that

$$\det(\mathbf{I} + \mathbf{F}_t \mathbf{F}_t^H) = \det(\mathbf{I} + \mathbf{F}_t^H \mathbf{F}_t). \quad (5.3.17)$$

We define that $\mathbf{G}_t = \mathbf{I} + \mathbf{F}_t^H \mathbf{F}_t$. Hence, $\mathbf{G}_t$ is a $t \times t$ matrix and given by

$$\mathbf{G}_t = \begin{pmatrix} 1 + \gamma_{sum} & \gamma_{R_2^*D} & 0 & \cdots & \cdots & 0 \\ \gamma_{R_2^*D} & 1 + \gamma_{sum} & \gamma_{R_1^*D} & \cdots & \cdots & 0 \\ 0 & \gamma_{R_1^*D} & 1 + \gamma_{sum} & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 + \gamma_{sum} & \gamma_{R_2^*D} \\ 0 & 0 & 0 & \cdots & \gamma_{R_2^*D} & 1 + \gamma_{sum} \end{pmatrix}, \quad (5.3.18)$$

where $\gamma_{sum} = \gamma_{R_1^*D} + \gamma_{R_2^*D}$. Therefore, we have $\det(\mathbf{G}_t) < (1 + \gamma_{sum})^t$. Given the size of $\mathcal{A}_{SR}$ equal to N , the outage probability of the SR transmission is lower bounded by

$$\begin{aligned} P_{out,SR|N} &\approx Pr \left(\frac{T}{T+2} \log_2 \det(\mathbf{I} + \mathbf{F}_t \mathbf{F}_t^H) < t r_0 \right) \\ &> Pr \left(\frac{T}{T+2} \log_2 (1 + \gamma_{sum})^t < t r_0 \right) \\ &= Pr(\gamma_{sum} < z_I). \end{aligned} \quad (5.3.19)$$

We know that $\gamma_{R_1^*D}$ and $\gamma_{R_2^*D}$ are the largest and the second largest random variables among N independent and identically distributed variables. According to the order statistics, the joint probability of $\gamma_{R_1^*D}$ and $\gamma_{R_2^*D}$ is given by

$$p(\gamma_{R_1^*D}, \gamma_{R_2^*D}) = N(N-1) [F(\gamma_{R_2^*D})]^{N-2} p(\gamma_{R_2^*D}) p(\gamma_{R_1^*D}), \quad (5.3.20)$$

where $p(\cdot)$ and $F(\cdot)$ are the PDF and CDF of an exponential distribution with mean value of $\bar{\gamma}$, respectively.

Following the derivation in (5.3.19), we have

$$\begin{aligned}
 Pr(\gamma_{sum} < z_I) &= Pr(\gamma_{R_1^*D} + \gamma_{R_2^*D} < z_I, \gamma_{R_1^*D} \geq \gamma_{R_2^*D}) \\
 &= \underbrace{\int_0^{z_I/2} \int_0^{\gamma_{R_1^*D}} p(\gamma_{R_1^*D}, \gamma_{R_2^*D}) d\gamma_{R_2^*D} d\gamma_{R_1^*D}}_{p_1} \\
 &\quad + \underbrace{\int_{z_I/2}^{z_I} \int_0^{z_I - \gamma_{R_1^*D}} p(\gamma_{R_1^*D}, \gamma_{R_2^*D}) d\gamma_{R_2^*D} d\gamma_{R_1^*D}}_{p_2}. \tag{5.3.21}
 \end{aligned}$$

Substituting (5.3.20) into (5.3.21), we can obtain

$$\begin{aligned}
 p_1 &= \int_0^{z_I/2} N p(\gamma_{R_1^*D}) \int_0^{\gamma_{R_1^*D}} (N-1) [F(\gamma_{R_2^*D})]^{N-2} p(\gamma_{R_2^*D}) d\gamma_{R_2^*D} d\gamma_{R_1^*D} \\
 &= \int_0^{z_I/2} N p(\gamma_{R_1^*D}) [F(\gamma_{R_1^*D})]^{N-1} d\gamma_{R_1^*D} \\
 &= [F(z_I/2)]^N \\
 &= (1 - e^{-z_I/2\bar{\gamma}})^N. \tag{5.3.22}
 \end{aligned}$$

Similarly, it is derived that

$$\begin{aligned}
 p_2 &= \int_{z_I/2}^{z_I} N p(\gamma_{R_1^*D}) \int_0^{z_I - \gamma_{R_1^*D}} (N-1) [F(\gamma_{R_2^*D})]^{N-2} p(\gamma_{R_2^*D}) d\gamma_{R_2^*D} d\gamma_{R_1^*D} \\
 &= \int_{z_I/2}^{z_I} N p(\gamma_{R_1^*D}) [F(z_I - \gamma_{R_1^*D})]^{N-1} d\gamma_{R_1^*D}. \tag{5.3.23}
 \end{aligned}$$

We assume that $u = z_I - \gamma_{R_1^*D}$ and then

$$\begin{aligned}
 p_2 &= \int_0^{z_I/2} N p(z_I - u) [F(u)]^{N-1} du \\
 &= \int_0^{z_I/2} N \frac{e^{-(z_I-u)/\bar{\gamma}}}{\bar{\gamma}} [1 - e^{-u/\bar{\gamma}}]^{N-1} du \\
 &\approx \int_0^{z_I/2} N \frac{e^{-u/\bar{\gamma}}}{\bar{\gamma}} [1 - e^{-u/\bar{\gamma}}]^{N-1} du \\
 &= (1 - e^{-z_I/2\bar{\gamma}})^N. \tag{5.3.24}
 \end{aligned}$$

Finally, substituting (5.3.22) and (5.3.24) into (5.3.21) and (5.3.19), we can express the outage probability of the relay-destination SR transmission, given that two best relays are selected from N relays, as

$$P_{out,SR|N} \geq 2 \left(1 - e^{-z_I/2\bar{\gamma}}\right)^N. \quad (5.3.25)$$

It is observed that the conditional outage probability decays with an exponent of N at high SNR region.

5.3.3 The occurrence and outage probabilities of the OR mode

The transmission mode can be adaptively switched to the OR mode if $\mathcal{A}_{SR}$ is empty. It occurs in two situations. One case is that more than two relays meet the requirement of (5.3.2) but no pair of relays satisfies (5.3.2)-(5.3.7), i.e., $N_I \geq 2$ but $N = 0$. The other situation is that $\mathcal{A}_{SR,I}$ includes one or no relay, i.e., $N_I \leq 1$. Therefore, the occurrence probability of the OR mode is given by

$$P_{OR} = \sum_{N_I=2}^M P_I(N_I) P_{\bar{II},\bar{III}|I}(N_p) + \sum_{N_I=0}^1 P_I(N_I). \quad (5.3.26)$$

In the OR mode, signals are transmitted from S to relays and from a single relay to D in two orthogonal time slots. Hence, the inter-relay channels are not necessary to be considered. The OR decoding set includes relays if $\gamma_{SR_m} \geq 2^{2r_0} - 1$. Since $2^{2r_0} - 1 > 2^{\frac{T+2}{T}r_0} - 1$, $\mathcal{A}_{OR} \subseteq \mathcal{A}_{SR,I}$. Therefore, if a relay is not involved in $\mathcal{A}_{SR,I}$, it is not included in $\mathcal{A}_{OR}$ either. Provided that $R_m \in \mathcal{A}_{SR,I}$, the probability that $R_m \in \mathcal{A}_{OR}$ is given by

$$P_{OR|I} = Pr \left(\gamma_{SR_m} \geq 2^{2r_0} - 1 | \gamma_{SR_m} \geq 2^{\frac{T+2}{T}r_0} - 1 \right)$$

$$= e^{-(z_{OR}-z_I)/\bar{\gamma}}, \quad (5.3.27)$$

where $z_{OR} = 2^{2r_0} - 1$. Therefore, for a certain $\mathcal{A}_{SR,I}$, the probability that N_{OR} relays are involved in $\mathcal{A}_{OR}$ is

$$P_{N_{OR}|N_I} = \binom{N_I}{N_{OR}} P_{OR|SR,I}^{N_{OR}} (1 - P_{OR|SR,I})^{N_I - N_{OR}}. \quad (5.3.28)$$

In the OR transmission mode, a relay with the largest γ_{R_mD} is selected among N_{OR} relays within $\mathcal{A}_{OR}$ to forward. The outage probability of the relay-destination transmission is given by

$$P_{out,OR|N_{OR}} = [Pr(\gamma_{R_mD} < z_{OR})]^{N_{OR}} = (1 - e^{-z_{OR}/\bar{\gamma}})^{N_{OR}}. \quad (5.3.29)$$

Therefore, the outage probability of the OR mode is obtained as

$$\begin{aligned} P_{out,OR|N_I} &= \sum_{N_{OR}=0}^{N_I} P_{N_{OR}|N_I} P_{out,OR|N_{OR}} \\ &= \sum_{N_{OR}=0}^{N_I} \binom{N_I}{N_{OR}} e^{-N_{OR}(z_{OR}-z_I)/\bar{\gamma}} \\ &\quad \times (1 - e^{-(z_{OR}-z_I)/\bar{\gamma}})^{N_I - N_{OR}} (1 - e^{-z_{OR}/\bar{\gamma}})^{N_{OR}}, \end{aligned} \quad (5.3.30)$$

which illustrates that the outage probability decays with an exponent of N_I at high SNR levels.

Finally, substituting (5.3.15), (5.3.25), (5.3.26) and (5.3.30) into (5.3.1), we obtain the lower bound of the outage probability for the SR protocol, including both SR and OR transmission modes, as

$$P_{out} \geq \sum_{N_I=2}^M P_I(N_I) \left(\sum_{k=0}^{N_I-3} P_{\bar{II},\bar{III}|I}(k) P_{out|N=N_I-k} + \sum_{k=N_I-2}^{N_p-1} P_{\bar{II},\bar{III}|I}(k) P_{out|N=2} \right)$$

$$+ P_{\overline{II}, \overline{III}|I}(N_p) P_{out, OR|N_I} \Big) + \sum_{N_I=0}^1 P_I(N_I) P_{out, OR|N_I}. \quad (5.3.31)$$

In the high SNR regime, the lower bound can be approximated as

$$\begin{aligned} P_{out} &\geq \sum_{N_I=2}^M \binom{M}{N_I} \left(\frac{z_I}{\bar{\gamma}}\right)^{(M-N_I)} \left[\sum_{k=0}^{N_I-3} \frac{1}{2^{N_I-k-1}} \left(\frac{z_I}{\bar{\gamma}}\right)^{N_I} + \sum_{k=N_I-2}^{N_p-1} \frac{1}{2} \left(\frac{z_I}{\bar{\gamma}}\right)^{k+2} \right. \\ &\quad \left. + \left(\frac{z_I}{\bar{\gamma}}\right)^{N_p} \sum_{N_{OR}=0}^{N_I} \binom{N_I}{N_{OR}} \left(\frac{z_{OR} - z_I}{\bar{\gamma}}\right)^{N_I} \right] \\ &\quad + 2M \left(\frac{z_I}{\bar{\gamma}}\right)^{M-1} \left(\frac{z_{OR} - z_I}{\bar{\gamma}}\right) + \left(\frac{z_I}{\bar{\gamma}}\right)^M \\ &\doteq (z_I/\bar{\gamma})^M. \end{aligned} \quad (5.3.32)$$

From (5.3.32), it is observed that the outage probability is exponentially equal to $(z_I/\bar{\gamma})^M$. Note that $z_I = 2^{\frac{T+2}{T}r_0} - 1$. According to the definitions of the diversity gain d and the multiplexing gain r that [6]

$$d = - \lim_{SNR \rightarrow \infty} \frac{\log P_{out}(SNR)}{\log SNR} \quad \text{and} \quad r = \lim_{SNR \rightarrow \infty} \frac{r_0(SNR)}{\log SNR}, \quad (5.3.33)$$

the transmission rate can be expressed as $r_0 = r \log \bar{\gamma}$ in the high SNR regime.

Substituting r_0 into z_I , we can obtain the DMT of the SR protocol as

$$d_{SR}(r) = - \lim_{\bar{\gamma} \rightarrow \infty} \frac{\log P_{out}}{\log \bar{\gamma}} = M \left(1 - \frac{T+2}{T} r\right)^+, \quad (5.3.34)$$

where x^+ means $\max\{x, 0\}$. It is shown that the SR protocol can achieve the full diversity gain of M and the maximal multiplexing gain is $T/(T+2)$. Recall that the achievable DMT of the single relay selection scheme is given by [16]

$$d_{OR}(r) = M (1 - 2r)^+, \quad (5.3.35)$$

which also characterizes the DMT of the OR protocol [58] and DSTC based relay protocol [14]. Additionally, for the same system model, the DMT upper bound of the multi-hop with relay selection (MHRS) protocol proposed in [118] is given by

$$d_{MHRS}(r) = (M - 1) \left(1 - \frac{T + 1}{T} r \right)^+. \quad (5.3.36)$$

Comparing among (5.3.34), (5.3.35) and (5.3.36), we can observe that the proposed SR protocol is capable of achieving higher diversity order than the MHRS protocol [118]. Furthermore, the SR protocol is superior to the single relay selection schemes in all multiplexing gain region.

5.4 Numerical Results and Discussions

In this section, we first present the superiority of our proposed SR protocol over the OR protocol. Next, we provide the simulated outage performance to validate the theoretical derivation. We also show many numerical results of the SR protocol in different situations and discuss the effects of number of relays, transmission rate, and time slot on the outage performance.

Firstly, we compare the simulated outage probabilities of our proposed SR protocol and the conventional OR protocol over i.i.d. Rayleigh fading channels in Figure 5.2. We fix the transmission rate and the time slot for the SR protocol, i.e., $r_0 = 1$ BPCU and $T = 7$. For a fair comparison, the total energy consumptions of both protocols are assumed to be identical in one block. In the SR protocol, source and relays transmit approximately for the whole duration of one block when T is large. In the OR protocol, source and the selected relay transmit only for the half length of one block. Therefore, the transmitted power level of each node under the OR protocol is twice that of the SR protocol. Figure 5.2 shows that both protocols achieve

Figure 5.2: Outage performance comparison of the SR protocol and OR protocol, when $T = 7$ and $r_0 = 1$ BPCU.

the identical diversity order, which is equal to the number of relays. Thus, every message is forwarded once by each selected relay and full diversity can be obtained. Additionally, it is noticed that the SR protocol obtains higher coding gain than the OR protocol. Therefore, the SR protocol is an ideal choice in minimizing the outage probability of a multiple-relay network.

Next, we compare the simulated outage probability with its analytical lower bound for various transmission rates in Figure 5.3. We fix the number of relays and the time slot, i.e., $M = 4$ and $T = 10$. It is observed that the simulated curves are lower bounded by the theoretical results given in (5.3.31), which validate our derivation. Although the gap between the simulated outage probability and the analytical lower bound is enlarged when the required transmission rate and SNR increase as seen in (5.3.19), both results achieve the identical diversity gain. Additionally, Figure 5.3

Figure 5.3: Simulated and analytical outage probabilities of the SR protocol with $M = 4$ and $T = 10$.

illustrates the impact of transmission rate on the outage performance. For a higher rate requirement, the outage probability increases but its curve still keeps the same slope for the fixed number of relays and time slot.

In Figure 5.4, we present the influence of the time slot on the outage performance for a fixed number of relays, i.e., $M = 5$. It can be observed that the achievable diversity gain is identical to the number of relays for different transmission rates and time slots. It is known that in the SR protocol the larger the time slot, the higher the multiplexing gain $T/(T+2)$ is. Comparing the simulated results for the identical value of r_0 , we can observe that the outage performance becomes better as T increases, but the improvement is less for the same increase of T . The outage probability will converge to an optimal performance as T goes to infinity. As a trade-off, the decoding complexity of the destination is consequently increased with the increase of T . Furthermore, we can compare the improvement of the outage

Figure 5.4: Outage probabilities of the SR protocol with various time slots T and transmission rates r_0 , when $M = 5$.

probability by the same amount increase of T with different transmission rates. It is seen from Figure 5.4 that the effect of T is more significant on the system with higher transmission rate.

Finally, the DMT performance of the single relay selection scheme [16], DSTC based relay protocol [14], MHRS protocol [118] and the proposed SR protocol is compared in Figure 5.5. We assume that $M = 6$ for all schemes and $T = 10$ for the MHRS protocol and the SR protocol. It is shown that the SR protocol dominates the performance of the single relay selection scheme and the DSTC based relay protocol in all the multiplexing gain regions. It is also observed that the SR protocol outperforms the MHRS protocol in terms of the maximal achievable diversity gain. The reason is that in our proposed SR protocol, relays jointly decode the signals from S and the other selected relay rather than treating the inter-relay transmission as

Figure 5.5: DMT performance of the SR protocol, single relay selection protocol, DSTC protocol and MHRs protocol, when $M = 6$ and $T = 10$. © [2012] IEEE

interference. Hence, relays can still obtain the source's information while they are occupied to forward information. Moreover, relays transmit the superposition of two decoded signals from the source and the other relay. Thus, every message is forwarded once by each selected relay and the full diversity can be obtained.

5.5 Conclusion

In this chapter, we have developed an efficient SR transmission protocol for the multiple-relay cooperative system. Each terminal has only one antenna and satisfies the half-duplex constraint. In this protocol, two selected relays, mimicking a full-duplex relay, successively receive and forward to the destination. The jointly decoding is performed by the destination after a block transmission. We have de-

---

rived the closed-form lower bound of outage probability and evaluated its achievable DMT. Numerical results showed that the proposed SR protocol is capable of achieving both full diversity gain and high multiplexing gain. Part of the work in this chapter has been published on IEEE Transactions on Wireless Communications, the fourth one in Author's Publications.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis, we have studied some useful and important cooperative systems employing efficient and high-performance protocols. Through smartly designing the transmission schemes and decoding strategies, higher transmission accuracy and spectrum efficiency can be achieved without requiring more channel resources, compared to the previous works in the literature. For the practical application, it was assumed that every terminal has only one antenna and satisfies the half-duplex constraint.

First of all, we considered a three-node cooperative system with one fixed DF relay. The C-MRC receiver, which exploits not only the CSI of source-destination and relay-destination links but also source-relay channel information, was employed to achieve the full cooperative diversity. The BERs of both MRC and C-MRC schemes were analyzed and derived in the presence of Nakagami- m fading channels, and the asymptotic upper bounds were further provided in closed-form expressions. Numerical results were presented to validate the derivations. In addition, we compared

the performance and the diversity orders of MRC and C-MRC in different fading conditions. It was concluded that C-MRC always achieves better BER performance than MRC does, especially when the source-relay channel is in a relatively poor condition. As the source-relay channel condition improves, the gap between MRC and C-MRC decreases. We also obtained the requirement of the source-relay link that MRC can achieve as good performance as C-MRC does. Moreover, the fixed DF relaying protocol with the C-MRC scheme was shown to outperform the adaptive DF protocol with optimum threshold.

Next, we studied a two-user CMA system, where two users not only transmit their own signals but also help to forward each other's signals to a common destination. A three-phase TDBC protocol and a two-phase superposition modulation based protocol were employed to gain better system spectrum efficiency than the conventional four-phase TDBC protocol. For the three-phase TDBC protocol, the ML decoding was adopted at the destination and the BER was derived by the CDF-based method. For the superposition modulation based protocol, a low-complexity near-optimal decoder was designed for the destination, the error decision regions were derived and the BER performance was investigated. Numerical results showed that under the identical power and transmission rate requirements, the superposition modulation scheme achieves the best BER performance, and the three-phase TDMA protocol outperforms the four-phase TDMA protocol.

Finally, we proposed an efficient non-orthogonal DF relaying protocol for multiple-relay cooperative networks, which is capable of achieving superior DMT performance than the literature and approaching the optimal DMT upper bound. In the SR protocol, the source keeps broadcasting new messages. Two selected relays, mimicking a full-duplex relay, successively decode messages from the source and the other relay and then forward the superposition of both decoded signals to the destination. The

multiplexing gain is thus significantly improved and the superposition makes the full diversity order achievable. The lower bound of outage probability was derived in a closed-form expression and the DMT was obtained as well. Numerical results were provided to validate the derivation and illustrate the influence of transmission rate, number of relays and time slots on the outage performance of the SR protocol. Our proposed efficient SR protocol was also shown to achieve higher coding gains and better outage performance than the conventional OR protocol.

6.2 Future Work

It is possible to expand the analysis to some more complicated cooperative systems in the future research, e.g., multi-user MARC with source cooperation, multi-user interference channel (IFC) and multi-user interference relay channel (IFRC). To our best knowledge, the multi-user MARC in the literature assumed that no direct communication between any two sources. The source cooperation has only been considered in the CMA systems. Therefore, the MARC with source cooperation could be researched in the future work, which would improve the diversity gain and enhance the performance further. In the multi-user IFC, there are multiple source-destination pairs, and each source is required to transmit to its preassigned partner. Source cooperation, receiver cooperation or both strategies could be adopted to achieve higher DMT performance. Different from the IFC, relay nodes are present in the IFRC, where they do not have their own messages to transmit and solely assist the transmission between source-destination pairs.

In order to achieve high spectral efficiency for these multi-user cooperative systems, transmission schemes which allow multiple nodes' concurrent transmission are expected to be intelligently designed. However, MAI and ISI are inevitable to degrade

the transmission accuracy. Therefore, some precoding strategies, relaying schemes, decoding methods and interference management are required to be investigated to diminish the interference in practical situations.

To analyze the performance of a channel model, we can firstly determine the diversity and capacity upper bounds via the channel cut-set upper bounds. Next, we evaluate the achievable performance of different proposed transmission protocols or detection techniques. Through comparison, we will find the schemes which can achieve the optimal performance approaching or even reaching the performance upper bounds of the channel. In addition to focusing on the performance at the high SNR regime, we will further consider the resource allocation issues at the moderate SNR regime, and investigate the system performance with imperfect channel knowledge or limited CSI feedback.

Author's Publications

1. Y. Hu, K. H. Li, and K. C. Teh, "Performance of two-way amplify-and-forward relay networks over asymmetric fading channels," in *Proc. IEEE MILCOM*, Oct. 2009, pp. 1-6.
2. Y. Hu, K. H. Li, and K. C. Teh, "Performance and diversity analysis of decode-and-forward cooperative system over Nakagami- m fading channels," *Wireless Commun. and Mobile Computing*, vol. 11, pp. 742-749, June 2011.
3. Y. Hu, K. H. Li, and K. C. Teh, "Performance analysis of two-user multiple access systems with DF relaying and superposition modulation," *IEEE Trans. Veh. Technol.*, vol. 60, no. 7, pp. 3118-3126, Sep. 2011.
4. Y. Hu, K. H. Li, and K. C. Teh, "An efficient successive relaying protocol for multiple-relay cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 5, pp. 1892-1899, May 2012.

Bibliography

- [1] J. Proakis, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2003.
- [2] S. Ikki and M. Ahmed, "Performance of decode-and-forward cooperative diversity networks over Nakagami- m fading channels," in *Proc. IEEE GLOBE-COM*, 2007, pp. 4328–4333.
- [3] D. Gesbert, M. Shafi, D. shan Shiu, P. Smith, and A. Naguib, "From theory to practice: An overview of MIMO space-time coded wireless systems," *IEEE J. Select. Areas. Commun.*, vol. 21, no. 3, pp. 281–302, Apr. 2003.
- [4] A. Goldsmith, S. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. Select. Areas. Commun.*, vol. 21, no. 5, pp. 684–702, June 2003.
- [5] A. Paulraj, D. Gore, R. Nabar, and H. Bolcskei, "An overview of MIMO communications - A key to gigabit wireless," *Proceedings of the IEEE*, vol. 92, no. 2, pp. 198–218, Feb. 2004.
- [6] L. Zheng and D. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Info. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.
- [7] D. Tse, P. Viswanath, and L. Zheng, "Diversity-multiplexing tradeoff in multiple-access channels," *IEEE Trans. Info. Theory*, vol. 50, no. 9, pp. 1859–1874, Sep. 2004.
- [8] A. Sendonaris, E. Erkip, and B. Aazhan, "User cooperation diversity–Part I and II," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1948, Nov. 2003.
- [9] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Info. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

- [10] J. Boyer, D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. Commun.*, vol. 52, no. 10, pp. 1820–1830, Oct. 2004.
- [11] M. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126–1131, Nov. 2003.
- [12] ———, "A performance study of dual-hop transmissions with fixed gain relays," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1963–1968, Nov. 2004.
- [13] R. Nabar, H. Bolcskei, and F. Kneubuhler, "Fading relay channels: Performance limits and space-time signal design," *IEEE J. Select. Areas. Commun.*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.
- [14] J. Laneman and G. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Info. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.
- [15] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524–3536, Dec. 2006.
- [16] A. Bletsas, A. Khisti, D. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Select. Areas. Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.
- [17] K. Azarian, H. El Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Info. Theory*, vol. 51, no. 12, pp. 4152–4172, Dec. 2005.
- [18] P. Elia, K. Vinodh, M. Anand, and P. Kumar, "D-MG tradeoff and optimal codes for a class of AF and DF cooperative communication protocols," *IEEE Trans. Info. Theory*, vol. 55, no. 7, pp. 3161–3185, July 2009.
- [19] M. Nakagami, "The m -distribution, a general formula of intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation*. Oxford, U.K.: Pergamon, 1960.
- [20] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. Wiley Interscience, 2005.
- [21] H. Suraweera, H. Garg, and A. Nallanathan, "Performance analysis of two hop amplify-and-forward systems with interference at the relay," *IEEE Commun. Lett.*, vol. 14, no. 8, pp. 692–694, Aug. 2010.

- [22] G. Farhadi and N. Beaulieu, "On the performance of amplify-and-forward cooperative systems with fixed gain relays," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1851–1856, May 2008.
- [23] P. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a Rayleigh-fading environment," *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, pp. 1416–1421, Sep. 2004.
- [24] L.-L. Yang and H.-H. Chen, "Error probability of digital communications using relay diversity over Nakagami- m fading channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1806–1811, May 2008.
- [25] S. Yang and J.-C. Belfiore, "Towards the optimal amplify-and-forward cooperative diversity scheme," *IEEE Trans. Info. Theory*, vol. 53, no. 9, pp. 3114–3126, Sep. 2007.
- [26] Z. Ding, T. Ratnarajah, and C. Cowan, "On the diversity-multiplexing trade-off for wireless cooperative multiple access systems," *IEEE Trans. Signal Processing*, vol. 55, no. 9, pp. 4627–4638, Sep. 2007.
- [27] D. Lee and J. H. Lee, "Outage probability for dual-hop relaying systems with multiple interferers over Rayleigh fading channels," *IEEE Trans. Veh. Technol.*, vol. 60, no. 1, pp. 333–338, Jan. 2011.
- [28] Y. Zou, B. Zheng, and W.-P. Zhu, "An opportunistic cooperation scheme and its BER analysis," *IEEE Trans. Wireless Commun.*, vol. 8, no. 9, pp. 4492–4497, Sep. 2009.
- [29] Y. Zou, B. Zheng, and J. Zhu, "Outage analysis of opportunistic cooperation over Rayleigh fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 3077–3085, June 2009.
- [30] T. Hunter and A. Nosratinia, "Diversity through coded cooperation," *IEEE Trans. Wireless Commun.*, vol. 5, no. 2, pp. 283–289, Feb. 2006.
- [31] T. Hunter, S. Sanayei, and A. Nosratinia, "Outage analysis of coded cooperation," *IEEE Trans. Info. Theory*, vol. 52, no. 2, pp. 375–391, Feb. 2006.
- [32] A. Ribeiro, X. Cai, and G. Giannakis, "Symbol error probabilities for general cooperative links," *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 1264–1273, May 2005.
- [33] S. Savazzi and U. Spagnolini, "Design criteria of two-hop based wireless networks with non-regenerative relays in arbitrary fading channels," *IEEE Trans. Commun.*, vol. 57, no. 5, pp. 1463–1473, May 2009.

- [34] M. Elkashlan, P. L. Yeoh, R. Louie, and I. Collings, "On the exact and asymptotic ser of receive diversity with multiple amplify-and-forward relays," *IEEE Trans. Veh. Technol.*, vol. 59, no. 9, pp. 4602–4608, Nov. 2010.
- [35] A. Bletsas, H. Shin, and M. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Wireless Commun.*, vol. 6, no. 9, pp. 3450–3460, Sep. 2007.
- [36] Y. Zhao, R. Adve, and T. J. Lim, "Symbol error rate of selection amplify-and-forward relay systems," *IEEE Commun. Lett.*, vol. 10, no. 11, pp. 757–759, Nov. 2006.
- [37] —, "Improving amplify-and-forward relay networks: Optimal power allocation versus selection," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 3114–3123, Aug. 2007.
- [38] Q. Zhou and F. Lau, "Performance bounds of opportunistic cooperative communications with CSI-assisted amplify-and-forward relaying and MRC reception," *IEEE Trans. Veh. Technol.*, vol. 59, no. 5, pp. 2159–2165, June 2010.
- [39] S. Ikki and M. Ahmed, "Performance analysis of incremental-relaying cooperative-diversity networks over Rayleigh fading channels," *Communications, IET*, vol. 5, no. 3, pp. 337–349, Nov. 2011.
- [40] Q. Zhou, F. Lau, and S. Hau, "Asymptotic analysis of opportunistic relaying protocols," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 3915–3920, Aug. 2009.
- [41] H. Wicaksana, S. Ting, M. Motani, and Y. Guan, "On the diversity-multiplexing tradeoff of amplify-and-forward half-duplex relaying," *IEEE Trans. Commun.*, vol. 58, no. 12, pp. 3621–3630, Dec. 2010.
- [42] S. Gharan, A. Bayesteh, and A. Khandani, "On the diversity-multiplexing tradeoff in multiple-relay network," *IEEE Trans. Info. Theory*, vol. 55, no. 12, pp. 5423–5444, Dec. 2009.
- [43] H. Ning, C. Ling, and K. Leung, "Generalized sequential slotted amplify and forward strategy in cooperative communications," *IEEE Trans. Info. Theory*, vol. 57, no. 4, pp. 1968–1974, Apr. 2011.
- [44] D. Chen and J. Laneman, "Modulation and demodulation for cooperative diversity in wireless systems," *IEEE Trans. Wireless Commun.*, vol. 5, no. 7, pp. 1785–1794, Jul. 2006.

- [45] T. Wang, A. Cano, G. Giannakis, and J. Laneman, "High-performance cooperative demodulation with decode-and-forward relays," *IEEE Trans. Commun.*, vol. 55, no. 7, pp. 1427–1438, July 2007.
- [46] T. Wang, G. Giannakis, and R. Wang, "Smart regenerative relays for link-adaptive cooperative communications," *IEEE Trans. Commun.*, vol. 56, no. 11, pp. 1950–1960, Nov. 2008.
- [47] Z. Yi and I.-M. Kim, "Diversity order analysis of the decode-and-forward cooperative networks with relay selection," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1792–1799, May 2008.
- [48] F. Onat, A. Adinoyi, Y. Fan, H. Yanikomeroglu, J. Thompson, and I. Marsland, "Threshold selection for SNR-based selective digital relaying in cooperative wireless networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4226–4237, Nov. 2008.
- [49] F. Onat, Y. Fan, H. Yanikomeroglu, and J. Thompson, "Asymptotic BER analysis of threshold digital relaying schemes in cooperative wireless systems," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 4938–4947, Dec. 2008.
- [50] A. K. Sadek, W. Su, and K. J. R. Liu, "Multinode cooperative communications in wireless networks," *IEEE Trans. Signal Processing*, vol. 55, no. 1, pp. 341–355, Jan. 2007.
- [51] A. Ibrahim, A. Sadek, W. Su, and K. Liu, "Cooperative communications with relay-selection: When to cooperate and whom to cooperate with?" *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2814–2827, July 2008.
- [52] D. Michalopoulos and G. Karagiannidis, "Performance analysis of single relay selection in Rayleigh fading," *IEEE Trans. Wireless Commun.*, vol. 7, no. 10, pp. 3718–3724, Oct. 2008.
- [53] M. Fareed and M. Uysal, "On relay selection for decode-and-forward relaying," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3341–3346, July 2009.
- [54] H. Chen, J. Liu, L. Zheng, C. Zhai, and Y. Zhou, "An improved selection cooperation scheme for decode-and-forward relaying," *IEEE Commun. Lett.*, vol. 14, no. 12, pp. 1143–1145, Dec. 2010.
- [55] K.-S. Hwang, Y.-C. Ko, and M.-S. Alouini, "Outage probability of cooperative diversity systems with opportunistic relaying based on decode-and-forward," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5100–5107, Dec. 2008.

- [56] A. Adinoyi, Y. Fan, H. Yanikomeroglu, H. Poor, and F. Al-Shaalan, "Performance of selection relaying and cooperative diversity," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, pp. 5790–5795, Dec. 2009.
- [57] E. Beres and R. Adve, "Selection cooperation in multi-source cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 1, pp. 118–127, Jan. 2008.
- [58] S. Ikki and M. Ahmed, "Performance analysis of adaptive decode-and-forward cooperative diversity networks with best-relay selection," *IEEE Trans. Commun.*, vol. 58, no. 1, pp. 68–72, Jan. 2010.
- [59] Y. Fan, A. Adinoyi, J. Thompson, H. Yanikomeroglu, and H. Poor, "A simple distributed antenna processing scheme for cooperative diversity," *IEEE Trans. Commun.*, vol. 57, no. 3, pp. 626–629, Mar. 2009.
- [60] K. Tourki, H.-C. Yang, and M.-S. Alouini, "Accurate outage analysis of incremental decode-and-forward opportunistic relaying," *IEEE Trans. Wireless Commun.*, vol. 10, no. 4, pp. 1021–1025, Apr. 2011.
- [61] J. Chen and J. Wang, "Cooperative transmission in wireless networks using incremental opportunistic relaying strategy," *Communications, IET*, vol. 3, no. 12, pp. 1948–1957, Dec. 2009.
- [62] S. S. Ikki and M. H. Ahmed, "Performance analysis of cooperative diversity with incremental-best-relay technique over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 59, no. 8, pp. 2152–2161, Aug. 2011.
- [63] N. Prasad and M. Varanasi, "High performance static and dynamic cooperative communication protocols for the half duplex fading relay channel," *IEEE Trans. Wireless Commun.*, vol. 9, no. 1, pp. 328–337, Jan. 2010.
- [64] M. Hasna and M.-S. Alouini, "Harmonic mean and end-to-end performance of transmission systems with relays," *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 130–135, Jan. 2004.
- [65] G. Karagiannidis, T. Tsiftsis, and R. Mallik, "Bounds for multihop relayed communications in Nakagami- m fading," *IEEE Trans. Commun.*, vol. 54, no. 1, pp. 18–22, Jan. 2006.
- [66] G. Karagiannidis, "Performance bounds of multihop wireless communications with blind relays over generalized fading channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 498–503, Mar. 2006.

- [67] S. Ikki and M. Ahmed, "Performance analysis of cooperative diversity wireless networks over Nakagami- m fading channel," *IEEE Commun. Lett.*, vol. 11, no. 4, pp. 334–336, Apr. 2007.
- [68] H. Suraweera and G. Karagiannidis, "Closed-form error analysis of the non-identical Nakagami- m relay fading channel," *IEEE Commun. Lett.*, vol. 12, no. 4, pp. 259–261, Apr. 2008.
- [69] Y. Lee and M.-H. Tsai, "Performance of decode-and-forward cooperative communications over Nakagami- m fading channels," *IEEE Trans. Veh. Technol.*, vol. 58, no. 3, pp. 1218–1228, Mar. 2009.
- [70] H. A. Suraweera, P. J. Smith, and J. Armstrong, "Outage probability of cooperative relay networks in Nakagami- m fading channels," *IEEE Commun. Lett.*, vol. 10, no. 12, pp. 834–836, Dec. 2006.
- [71] N. Sagias, F. Lazarakis, G. Tombras, and C. Datsikas, "Outage analysis of decode-and-forward relaying over Nakagami- m fading channels," *IEEE Signal Process. Lett.*, vol. 15, pp. 41–44, 2008.
- [72] T. Duong, V. N. Q. Bao, and H.-j. Zepernick, "On the performance of selection decode-and-forward relay networks over Nakagami- m fading channels," *IEEE Commun. Lett.*, vol. 13, no. 3, pp. 172–174, Mar. 2009.
- [73] F. Xu, F. Lau, Q. Zhou, and D.-W. Yue, "Outage performance of cooperative communication systems using opportunistic relaying and selection combining receiver," *IEEE Signal Process. Lett.*, vol. 16, no. 4, pp. 237–240, Apr. 2009.
- [74] H. Cui, G. Wei, and Y. Wang, "Effects of CSI on ASEP based opportunistic DF relaying systems," *IEEE Trans. Veh. Technol.*, vol. 60, no. 4, pp. 1898–1904, May 2011.
- [75] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Select. Areas. Commun.*, vol. 25, pp. 379–389, Feb. 2007.
- [76] Y. Han, S. H. Ting, C. K. Ho, and W. H. Chin, "Performance bounds for two-way amplify-and-forward relaying," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 432–439, Jan. 2009.
- [77] H. Q. Ngo, T. Quek, and H. Shin, "Amplify-and-forward two-way relay networks: Error exponents and resource allocation," *IEEE Trans. Commun.*, vol. 58, no. 9, pp. 2653–2666, Sep. 2010.

- [78] R. Louie, Y. Li, and B. Vucetic, "Practical physical layer network coding for two-way relay channels: Performance analysis and comparison," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 764–777, Feb. 2010.
- [79] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE J. Select. Areas Commun.*, vol. 27, no. 5, pp. 699–712, June 2009.
- [80] S. J. Kim, P. Mitran, and V. Tarokh, "Performance bounds for bidirectional coded cooperation protocols," *IEEE Trans. Info. Theory*, vol. 54, no. 11, pp. 5235–5241, Nov. 2008.
- [81] P. Liu and I.-M. Kim, "Performance analysis of bidirectional communication protocols based on decode-and-forward relaying," *IEEE Trans. Commun.*, vol. 58, no. 9, pp. 2683–2696, Sep. 2010.
- [82] T. Kim and H. Poor, "Diversity-multiplexing trade-off in adaptive two-way relaying," *IEEE Trans. Info. Theory*, vol. 57, no. 7, pp. 4235–4254, July 2011.
- [83] J. Wang and J. Liang, "Distortion exponents of two-way relay networks," *IEEE Trans. Signal Proc.*, vol. 59, no. 9, pp. 4424–4437, Sep. 2011.
- [84] T. Cui, F. Gao, T. Ho, and A. Nallanathan, "Distributed space-time coding for two-way wireless relay networks," *IEEE Trans. Signal Proc.*, vol. 57, no. 2, pp. 658–671, Feb. 2009.
- [85] L. Song, "Relay selection for two-way relaying with amplify-and-forward protocols," *IEEE Trans. Veh. Technol.*, vol. 60, no. 4, pp. 1954–1959, May 2011.
- [86] Q. Zhou, Y. Li, F. Lau, and B. Vucetic, "Decode-and-forward two-way relaying with network coding and opportunistic relay selection," *IEEE Trans. Commun.*, vol. 58, no. 11, pp. 3070–3076, Nov. 2010.
- [87] I. Krikidis, "Relay selection for two-way relay channels with MABC DF: A diversity perspective," *IEEE Trans. Veh. Technol.*, vol. 59, no. 9, pp. 4620–4628, Nov. 2010.
- [88] Y. Li, R. Louie, and B. Vucetic, "Relay selection with network coding in two-way relay channels," *IEEE Trans. Veh. Technol.*, vol. 59, no. 9, pp. 4489–4499, Nov. 2010.
- [89] M. Chen and A. Yener, "Multiuser two-way relaying: Detection and interference management strategies," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4296–4305, Aug. 2009.

- [90] —, “Power allocation for F/TDMA multiuser two-way relay networks,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 546–551, Feb. 2010.
- [91] J. Joung and A. Sayed, “Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems,” *IEEE Trans. Signal Proc.*, vol. 58, no. 3, pp. 1833–1846, Mar. 2010.
- [92] W. Mesbah and T. Davidson, “Joint power and channel resource allocation for two-user orthogonal amplify-and-forward cooperation,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4681–4691, Nov. 2008.
- [93] —, “Optimized power allocation for pairwise cooperative multiple access,” *IEEE Trans. Signal Proc.*, vol. 56, no. 7, pp. 2994–3008, July 2008.
- [94] L. Weng and R. Murch, “Cooperation strategies and resource allocations in multiuser OFDMA systems,” *IEEE Trans. Veh. Technol.*, vol. 58, no. 5, pp. 2331–2342, June 2009.
- [95] W. J. Huang, Y. Hong, and C. Kuo, “Relay-assisted decorrelating multiuser detector (RAD-MUD) for cooperative cdma networks,” *IEEE J. Select. Areas. Commun.*, vol. 26, no. 3, pp. 550–560, Apr. 2008.
- [96] K. Vardhe, D. Reynolds, and M. Valenti, “The performance of multi-user cooperative diversity in an asynchronous CDMA uplink,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1930–1940, May 2008.
- [97] P. Dayal and M. Varanasi, “Distributed QAM-based space-time block codes for efficient cooperative multiple-access communication,” *IEEE Trans. Info. Theory*, vol. 54, no. 9, pp. 4342–4354, Sep. 2008.
- [98] K. Azarian, H. El Gamal, and P. Schniter, “On the optimality of the ARQ-DDF protocol,” *IEEE Trans. Info. Theory*, vol. 54, no. 4, pp. 1718–1724, Apr. 2008.
- [99] M. Yuksel and E. Erkip, “Multiple-antenna cooperative wireless systems: A diversity-multiplexing tradeoff perspective,” *IEEE Trans. Info. Theory*, vol. 53, no. 10, pp. 3371–3393, Oct. 2007.
- [100] D. Chen, K. Azarian, and J. Laneman, “A case for amplify-forward relaying in the block-fading multiple-access channel,” *IEEE Trans. Info. Theory*, vol. 54, no. 8, pp. 3728–3733, Aug. 2008.
- [101] L. Li, Y. Jing, and H. Jafarkhani, “Multisource transmission for wireless relay networks with linear complexity,” *IEEE Trans. Signal Processing*, vol. 59, no. 6, pp. 2898–2912, June 2011.

- [102] Z. Ding, T. Ratnarajah, and K. Leung, "On the study of network coded AF transmission protocol for wireless multiple access channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 118–123, Jan. 2009.
- [103] Z. Ding, K. Leung, D. Goeckel, and D. Towsley, "A relay assisted cooperative transmission protocol for wireless multiple access systems," *IEEE Trans. Commun.*, vol. 58, no. 8, pp. 2425–2435, Aug. 2010.
- [104] C. Wang, M. Xiao, and M. Skoglund, "Diversity-multiplexing tradeoff analysis of coded multi-user relay networks," *IEEE Trans. Commun.*, vol. 59, no. 7, pp. 1995–2005, July 2011.
- [105] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Oxford: Academic, 2007.
- [106] E. Larsson and B. Vojcic, "Cooperative transmit diversity based on superposition modulation," *IEEE Commun. Lett.*, vol. 9, no. 9, pp. 778–780, Sep. 2005.
- [107] T.-W. Yune, D. Kim, and G.-H. Im, "Iterative detection for spectral efficient user cooperative transmissions over multipath fading channels," *IEEE Trans. Commun.*, vol. 58, no. 4, pp. 1121–1128, Apr. 2010.
- [108] L. Xiao, T. Fuja, J. Kliewer, and D. Costello, "A network coding approach to cooperative diversity," *IEEE Trans. Info. Theory*, vol. 53, no. 10, pp. 3714–3722, Oct. 2007.
- [109] —, "Error performance analysis of signal superposition coded cooperative diversity," *IEEE Trans. Commun.*, vol. 57, no. 10, pp. 3123–3131, Oct. 2009.
- [110] I. Krikidis, "Analysis and optimization issues for superposition modulation in cooperative networks," *IEEE Trans. Veh. Technol.*, vol. 58, no. 9, pp. 4837–4847, Nov. 2009.
- [111] T. Yang and J. Yuan, "Performance of iterative decoding for superposition modulation-based cooperative transmission," *IEEE Trans. Wireless Commun.*, vol. 9, no. 1, pp. 51–59, Jan. 2010.
- [112] J. Liu, K. Lu, X. Cai, and M. Murthi, "Regenerative cooperative diversity with path selection and equal power consumption in wireless networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 3926–3932, Aug. 2009.
- [113] Y. Fan, C. Wang, J. Thompson, and H. Poor, "Recovering multiplexing loss through successive relaying using repetition coding," *IEEE Trans. Wireless Commun.*, vol. 6, no. 12, pp. 4484–4493, Dec. 2007.

- [114] S. Rezaei, S. Gharan, and A. Khandani, "Relay scheduling in the half-duplex gaussian parallel relay channel," *IEEE Trans. Info. Theory*, vol. 56, no. 6, pp. 2668–2687, Jun. 2010.
- [115] F. Xue and S. Sandhu, "Cooperation in a half-duplex Gaussian diamond relay channel," *IEEE Trans. Info. Theory*, vol. 53, no. 10, pp. 3806–3814, Oct. 2007.
- [116] C. Wang, Y. Fan, J. Thompson, M. Skoglund, and H. Poor, "Approaching the optimal diversity-multiplexing tradeoff in a four-node cooperative network," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3690–3700, Dec. 2010.
- [117] S. Pawar, A. Avestimehr, and D. Tse, "Diversity-multiplexing tradeoff of the half-duplex relay channel," in *46th Annual Allerton Conference on Communication, Control, and Computing*, 2008, pp. 27–33.
- [118] R. Tannious and A. Nosratinia, "Spectrally-efficient relay selection with limited feedback," *IEEE J. Select. Areas Commun.*, vol. 26, no. 8, pp. 1419–1428, Oct. 2008.

Appendix A

Derivation of decision regions for the first case of the superposition-based CMA scheme

Recall that during the BER derivation of the superposition-based CMA scheme in Section 4.4.2, two variables X and Y are respectively defined as

$$X = \Re\{h_{AD}^* y_{AD}\} \quad \text{and} \quad Y = \Re\{h_{BD}^* y_{BD}\}. \quad (\text{A.1})$$

They are Gaussian distributed and can be expressed by

$$X \sim \mathcal{N}(\gamma_{AD}x_A, \gamma_{AD}/2) \quad \text{and} \quad Y \sim \mathcal{N}(\gamma_{BD}x_B, \gamma_{BD}/2). \quad (\text{A.2})$$

According to (4.3.1) and (4.3.2), both x_A and x_B have four possible values. The expectations of X and Y , denoted as X_l and Y_m ($l, m = 1, 2, 3, 4$), are respectively given by

$$X_l = \gamma_{AD}(\sqrt{1 - \beta_A}s_A(k) + \sqrt{\beta_A}s_B(k - 1)) \quad (\text{A.3})$$

and

$$Y_m = \gamma_{BD}(\sqrt{1 - \beta_B} s_B(k) + \sqrt{\beta_B} s_A(k)), \quad (\text{A.4})$$

where $s_A(k)$, $s_B(k-1)$ and $s_B(k)$ take values as given in Figure A.1. We define the midpoint of X_3 and X_4 as X_{th} , yielding

$$X_{th} = \frac{X_3 + X_4}{2} = \gamma_{AD} \sqrt{1 - \beta_A}. \quad (\text{A.5})$$

Thus $-X_{th}$ is the midpoint of X_1 and X_2 . Similarly, we set Y_{th} as the midpoint of Y_3 and Y_4 , so that

$$Y_{th} = \frac{Y_3 + Y_4}{2} = \gamma_{BD} \sqrt{1 - \beta_B}. \quad (\text{A.6})$$

We also use Y_{mid} to denote the midpoint of Y_2 and Y_4 , which is given by

$$Y_{mid} = \frac{Y_2 + Y_4}{2} = \gamma_{BD} \sqrt{\beta_B}. \quad (\text{A.7})$$

We first find pairs of $(b_B(k-1), b_B(k))$ and $(\check{b}_B(k-1), \check{b}_B(k))$ to minimize $f(1, b_B(k-1), b_B(k))$ and $f(-1, \check{b}_B(k-1), \check{b}_B(k))$, respectively. Since $b_B(k-1)$ and $b_B(k)$ are independent, we can determine any one of them by assuming the other as a constant to minimize the corresponding cost function $f(1, b_B(k-1), b_B(k))$ in different ranges of X and Y . Similarly, the optimal $(\check{b}_B(k-1), \check{b}_B(k))$ is obtained to minimize $f(-1, \check{b}_B(k-1), \check{b}_B(k))$. For example, when $X \leq -X_{th}$ and $Y \geq Y_{mid}$, we obtain

$$\min f(1, b_B(k-1), b_B(k)) = f(1, -1, 1) \quad (\text{A.8})$$

Figure A.1: The distributions and decision boundaries of X and Y , where $X_{th} = \gamma_{AD}\sqrt{1 - \beta_A}$, $Y_{th} = \gamma_{BD}\sqrt{1 - \beta_B}$ and $Y_{mid} = \gamma_{BD}\sqrt{\beta_B}$. © [2011] IEEE

and

$$\min f(-1, \check{b}_B(k-1), \check{b}_B(k)) = f(-1, -1, 1). \quad (\text{A.9})$$

The optimal groups of $(1, b_B(k-1), b_B(k))$ and $(-1, \check{b}_B(k-1), \check{b}_B(k))$ for all regions are listed in Table A.1.

Here we only derive the error decision regions when $s_A(k) = -1$. The decision regions for $s_A(k) = 1$ can be obtained in a similar way. Hence, an error occurs if the following condition is satisfied:

$$\min f(1, b_B(k-1), b_B(k)) \leq \min f(-1, \check{b}_B(k-1), \check{b}_B(k)). \quad (\text{A.10})$$

By substituting the groups of $(1, b_B(k-1), b_B(k))$ and $(-1, \check{b}_B(k-1), \check{b}_B(k))$ listed in Table A.1 into (A.10), the error detection conditions can be derived in every

region. For instance, in the region of $X \leq -X_{th}$ and $Y \geq Y_{mid}$, errors occur if

$$Y > -\frac{\sqrt{1-\beta_A}}{\sqrt{\beta_B}}X - \frac{c_A}{\sqrt{\beta_B}} + \gamma_{BD}\sqrt{1-\beta_B}. \quad (\text{A.11})$$

The error decision region is shown by grid, and the boundary between correct and error decision is denoted by the line CD in Figure 4.2. Finally, the error decision regions are shown by bias, grid and shadow, denoted as R_I , R_{II} and R_{III} , and the boundaries are given by

$$\text{CD:} \quad Y = -\frac{\sqrt{1-\beta_A}}{\sqrt{\beta_B}}X - \frac{c_A}{\sqrt{\beta_B}} + \gamma_{BD}\sqrt{1-\beta_B}, \quad (\text{A.12})$$

$$\text{EF:} \quad Y = \frac{\sqrt{1-\beta_A}}{\sqrt{1-\beta_B}-\sqrt{\beta_B}}X + \frac{c_A}{\sqrt{1-\beta_B}-\sqrt{\beta_B}}, \quad (\text{A.13})$$

$$\text{FG:} \quad Y = -\frac{\sqrt{1-\beta_A}}{\sqrt{\beta_B}}X - \frac{c_A}{\sqrt{\beta_B}} - \gamma_{BD}\sqrt{1-\beta_B}, \quad (\text{A.14})$$

$$\text{DL:} \quad Y = -\frac{\sqrt{1-\beta_A}-\sqrt{\beta_A}}{\sqrt{\beta_B}}X + \gamma_{BD}\sqrt{1-\beta_B}, \quad (\text{A.15})$$

$$\text{LJ:} \quad Y = \frac{\sqrt{1-\beta_A}-\sqrt{\beta_A}}{\sqrt{1-\beta_B}-\sqrt{\beta_B}}X, \quad (\text{A.16})$$

$$\text{JM:} \quad Y = -\frac{\sqrt{1-\beta_A}-\sqrt{\beta_A}}{\sqrt{\beta_B}}X - \gamma_{BD}\sqrt{1-\beta_B}, \quad (\text{A.17})$$

$$\text{QR:} \quad Y = -\frac{\sqrt{1-\beta_A}}{\sqrt{\beta_B}}X + \frac{c_A}{\sqrt{\beta_B}} + \gamma_{BD}\sqrt{1-\beta_B}, \quad (\text{A.18})$$

$$\text{RS:} \quad Y = \frac{\sqrt{1-\beta_A}}{\sqrt{1-\beta_B}-\sqrt{\beta_B}}X - \frac{c_A}{\sqrt{1-\beta_B}-\sqrt{\beta_B}}, \quad (\text{A.19})$$

and

$$\text{MT:} \quad Y = -\frac{\sqrt{1-\beta_A}}{\sqrt{\beta_B}}X + \frac{c_A}{\sqrt{\beta_B}} - \gamma_{BD}\sqrt{1-\beta_B}. \quad (\text{A.20})$$

Table A.1: The decisions of $(1, b_B(k-1), b_B(k))$ and $(-1, \check{b}_B(k-1), \check{b}_B(k))$ minimizing $f(1, b_B(k-1), b_B(k))$ and $f(-1, \check{b}_B(k-1), \check{b}_B(k))$ in every region. © [2011] IEEE

$(1, b_B(k-1), b_B(k))$ $(-1, \check{b}_B(k-1), \check{b}_B(k))$	$X \leq -X_{th}$	$-X_{th} < X \leq X_{th}$	$X > X_{th}$
$Y > Y_{mid}$	$(1, -1, 1)$ $(-1, -1, 1)$	$(1, -1, 1)$ $(-1, 1, 1)$	$(1, 1, 1)$ $(-1, 1, 1)$
$-Y_{mid} < Y \leq Y_{mid}$	$(1, -1, -1)$ $(-1, -1, 1)$	$(1, -1, -1)$ $(-1, 1, 1)$	$(1, 1, -1)$ $(-1, 1, 1)$
$Y \leq -Y_{mid}$	$(1, -1, -1)$ $(-1, -1, -1)$	$(1, -1, -1)$ $(-1, 1, -1)$	$(1, 1, -1)$ $(-1, 1, -1)$

Appendix B

Derivation of $P_{II,III|I}$ and $P_{\overline{II},\overline{III}|I}$

We denote the complement of I_m as $\overline{I}_m$. The probability that a pair of relays is not successful, given that they are within $\mathcal{A}_{SR,I}$, is computed as

$$\begin{aligned}
 P_{\overline{II},\overline{III}|I} &= Pr(\overline{II}_{mv} \cup \overline{III}_{mv} \cup \overline{III}_{vm} | I_m, I_v) \\
 &= Pr(\overline{II}_{mv} | I_m, I_v) + Pr(\overline{III}_{mv} \cup \overline{III}_{vm} | I_m, I_v) \\
 &= Pr(\overline{II}_{mv}) + Pr(\overline{III}_{mv} | I_v) + Pr(\overline{III}_{vm} | I_m) \\
 &\quad - Pr(\overline{III}_{mv} \cap \overline{III}_{vm} | I_m, I_v) \\
 &= Pr(\overline{II}_{mv}) + 2Pr(\overline{III}_{mv} | I_v) - Pr(\overline{III}_{mv} \cap \overline{III}_{vm} | I_m, I_v). \quad (B.1)
 \end{aligned}$$

The last expression in (B.1) is obtained based on that II_{mv} is independent of I_m and I_v , and III_{mv} is independent of I_m . The probability of $Pr(\overline{II}_{mv})$ is given by

$$Pr(\overline{II}_{mv}) = Pr(\gamma_{R_m R_v} \leq z_I) = 1 - e^{-z_I/\tilde{\gamma}}. \quad (B.2)$$

The joint probability of $Pr(I_v, II_{mv}, \overline{III}_{mv})$ is computed as

$$\begin{aligned}
 &Pr(I_v, II_{mv}, \overline{III}_{mv}) \\
 &= Pr(\gamma_{SR_v} \geq z_I, \gamma_{R_m R_v} \geq z_I, \gamma_{SR_v} + \gamma_{R_m R_v} < z_{III})
 \end{aligned}$$

$$\begin{aligned}
&= \int_{z_I}^{z_{III}-z_I} \int_{z_I}^{z_{III}-\gamma_{R_m R_v}} f(\gamma_{SR_v}) d\gamma_{SR_v} f(\gamma_{R_m R_v}) d\gamma_{R_m R_v} \\
&= e^{-z_I/\bar{\gamma}} (e^{-z_I/\bar{\gamma}} - e^{-(z_{III}-z_I)/\bar{\gamma}}) - \frac{z_{III}-2z_I}{\bar{\gamma}} e^{-z_{III}/\bar{\gamma}}.
\end{aligned} \tag{B.3}$$

Hence, the conditional probability of $Pr(I_{mv}, \overline{III}_{mv} | I_v)$ is given by

$$\begin{aligned}
&Pr(I_{mv}, \overline{III}_{mv} | I_v) \\
&= \frac{Pr(I_v, I_{mv}, \overline{III}_{mv})}{Pr(I_v)} \\
&= e^{-z_I/\bar{\gamma}} - \left(1 + \frac{z_{III}-2z_I}{\bar{\gamma}}\right) e^{-(z_{III}-z_I)/\bar{\gamma}}.
\end{aligned} \tag{B.4}$$

Similarly, to obtain $Pr(I_{mv}, \overline{III}_{mv}, \overline{III}_{vm} | I_m, I_v)$, the joint probability of $Pr(I_m, I_v, I_{mv}, \overline{III}_{mv}, \overline{III}_{vm})$ is firstly computed as

$$\begin{aligned}
&Pr(I_m, I_v, I_{mv}, \overline{III}_{mv}, \overline{III}_{vm}) \\
&= Pr(\gamma_{SR_m} \geq z_I, \gamma_{SR_v} \geq z_I, \gamma_{R_m R_v} \geq z_I, \gamma_{SR_v} + \gamma_{R_m R_v} < z_{III}, \gamma_{SR_m} + \gamma_{R_m R_v} < z_{III}) \\
&= \int_{z_I}^{z_{III}-z_I} f(\gamma_{R_m R_v}) \left(\int_{z_I}^{z_{III}-\gamma_{R_m R_v}} f(\gamma_{SR_v}) d\gamma_{SR_v} \int_{z_I}^{z_{III}-\gamma_{R_m R_v}} f(\gamma_{SR_m}) d\gamma_{SR_m} \right) d\gamma_{R_m R_v} \\
&= e^{-3z_I/\bar{\gamma}} - e^{-(2z_{III}-z_I)/\bar{\gamma}} - 2 \frac{z_{III}-2z_I}{\bar{\gamma}} e^{-(z_{III}+z_I)/\bar{\gamma}}.
\end{aligned} \tag{B.5}$$

The conditional probability of $Pr(I_{mv}, \overline{III}_{mv}, \overline{III}_{vm} | I_m, I_v)$ is subsequently obtained as

$$\begin{aligned}
&Pr(I_{mv}, \overline{III}_{mv}, \overline{III}_{vm} | I_m, I_v) \\
&= \frac{Pr(I_m, I_v, I_{mv}, \overline{III}_{mv}, \overline{III}_{vm})}{Pr(I_m)Pr(I_v)} \\
&= e^{-z_I/\bar{\gamma}} - e^{-(2z_{III}-3z_I)/\bar{\gamma}} - 2 \frac{z_{III}-2z_I}{\bar{\gamma}} e^{-(z_{III}-z_I)/\bar{\gamma}}.
\end{aligned} \tag{B.6}$$

Finally, substituting (B.2), (B.4) and (B.6) into (B.1), we obtain $P_{\bar{II},\bar{III}|I}$ as

$$P_{\bar{II},\bar{III}|I} = 1 + e^{-(2z_{III}-3z_I)/\bar{\gamma}} - 2e^{-(z_{III}-z_I)/\bar{\gamma}}. \quad (\text{B.7})$$

The probability that a pair of relays is successful, provided that they are within $\mathcal{A}_{SR,I}$, denoted as $P_{II,III|I}$, is accordingly obtained as

$$P_{II,III|I} = 1 - P_{\bar{II},\bar{III}|I} = 2e^{-(z_{III}-z_I)/\bar{\gamma}} - e^{-(2z_{III}-3z_I)/\bar{\gamma}}. \quad (\text{B.8})$$