Numerical investigation on combustion instabilities and its passive control

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**NUMERICAL INVESTIGATION ON
COMBUSTION INSTABILITIES AND ITS
PASSIVE CONTROL**

ZHONG ZHIYUAN

**SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING**

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**NUMERICAL INVESTIGATION ON COMBUSTION
INSTABILITIES AND ITS PASSIVE CONTROL**

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Abstract

Combustion instabilities are caused by the interaction between unsteady heat releases and acoustic waves. Under certain conditions, the mutual interaction can grow to large oscillations damaging to structures. A lab-scale setup to demonstrate and investigate the combustion instabilities is Rijke tube which is a straight tube with heat source placed inside the tube. To gain insight of the generation mechanism of combustion instabilities, numerical simulation of combustion in a Rijke like tube is carried out in the present study. Different from the conventional Rijke tube, a new designed Rijke-Zhao tube, which has a mother tube (bottom stem) splitting into two or more bifurcating daughter tubes (i.e. upper branches) with different lengths, is considered. As a premixed laminar flame is placed inside the mother tube, it provides a mechanism to produce self-excited combustion oscillations (also known as combustion instabilities). To compare with our experimental results, 2D numerical simulation of the Rijke-Zhao tube by using ANSYSYS Fluent 13.0 is conducted. It was found that numerical model can capture the flow field and acoustic characteristics, predicting limit-cycle and its frequencies of the thermoacoustic oscillations very well. Moreover, the mode shape predicted by the simulation shows that the mode frequency is affected by the treatment on the boundary end. Two configurations are simulated: one case is considered as all acoustically open ends and another is considered as two acoustically closed and one acoustically open end. The effect of different treatment on the boundary end is presented.

To mitigate combustion instabilities, perforated liners, usually dependant on a low Mach number bias flow which is used to cool the combustor walls through the apertures of the liner, are installed along the bounding walls of a combustor. They dissipate the acoustic waves by generating vorticity at the rims of perforated apertures. To investigate the damping effect of perforated liner with bias flow on plane waves, a time domain numerical model of a cylindrical lined duct is developed. The liners' damping mechanism is characterized by using a time-domain 'compliance'. Numerical simulations of two different configurations of lined duct systems are performed by combining a 1D acoustic wave model with the compliance model. The results from present model are compared with those from the experiment and the frequency-domain model of previous investigation [Eldredge & Dowling, *J. Fluid Mech.* 485, 307-335(2003)]. Good agreement is observed. This confirms that the present model can be used to simulate the propagation and dissipation of acoustic plane waves in a lined duct in real-time.

List of Publications

1. Zhiyuan Zhong and Dan Zhao, *Time-domain characterization of the acoustic damping of a perforated liner with bias flow*, Journal of Acoustical Society of America, 2012.
2. Zhiyuan Zhong and Dan Zhao, *Self-excited Combustion Instability in Rijke-Zhao Tube with Bifurcating Branches*, 2012 World Conference on Science and Engineering, Hong Kong, 2012.

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Nomenclature

Roman letters

A :	Cross-sectional area
$\mathcal{A}_0$:	Coefficients of the z-domain compliance for outer liner $Y_2(z)$
a_i, a_l :	Parameters to determine compliance model in z-domain
$\mathcal{B}_1$:	Coefficients of the z-domain compliance for outer liner $Y_2(z)$
b_i :	Parameters to determine compliance model in z-domain
C :	Circumference of liner
$\bar{c}$:	Speed of sound
D :	Diameter of the liner
d_1, d_2 :	Parameter used in time-domain reflection coefficient
e :	Natural logarithm
f :	Frequency variable
H :	Enthalpy at the liner section
H_d :	Enthalpy downstream the liner
H_u :	Enthalpy upstream the liner
I_1 :	Modified Bessel function
i :	Integer number
j :	Complex number $\sqrt{-1}$ /Spatial point index as subscript
K_1 :	Modified Bessel function

K_a :	Rayleigh conductivity
K_a^s :	Simplified Rayleigh conductivity
k	Wave number
L :	Length of liner
L_d :	Length of downstream duct
l :	Integer number
M :	Mach number of men pipe flow in the liner section
M_u :	Mach number of men pipe flow upstream the liner section
M_h :	Mach number of bias flow
$\mathcal{M}$:	Total number of time steps
m :	Integer number
N :	Grid points for the liner
$\mathcal{N}$:	Total number of upstream incident signals
n :	The n^{th} time step
$\mathcal{P}_0$:	Parameter for compliance model in frequency-domain
p :	Pressure
Q_1 :	Parameter for compliance model in frequency-domain
q :	Heat release/flux
R :	Radius of duct in Chapter 3/Gas constant in Chapter 4
R_d :	Reflection coefficient
$\mathcal{R}$:	Radius of apertures of liner
r_p :	Radial direction of coordinate
S_t :	Stroul number
T :	Temperature
$\mathcal{T}$:	Time delay in $R_d(t)$

t :	Time
Δt :	Time step size
U_c :	Convection velocity of vortices shed from the aperture rim
u :	Axial velocity in the liner section
u_u :	Axial velocity upstream the liner section
v :	Bias flow velocity of the liner
v_h :	Bias flow velocity of the aperture
W :	Molecular weight
$\mathcal{X}$:	Contraction ratio
x :	Coordinate
Y :	Compliance
Y^s :	Simplified compliance
y :	Coordinate
z :	Variable in z-domain

Greek letters

α_1, α_2 :	Parameter used in time-domain reflection coefficient
β :	Transmission loss
γ :	Real part for Rayleigh conductivity
Δ :	Power absorption coefficient
δ :	Imaginary part for Rayleigh conductivity /Kronecker Symbol
ϵ :	Thickness of liner
ζ :	Decomposed enthalpy wave
η :	Dynamics viscosity
ϵ :	Thickness of liner

λ :	Thermal conductivity
μ, μ_0 :	Parameter used in reflection coefficient equation (4.28, 29)
ν :	Parameter used in reflection coefficient
π :	Circumference ratio
ρ :	density
σ :	Open ratio of liner
τ :	Shear stress
ϕ :	Represent variables in discrete form equations
ω :	Angular speed

Over hat

$\wedge$:	Denote frequency-domain quantity
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Superscript and subscript

$'$:	Denote fluctuating quantities
$^+$:	Denote incident wave
$^-$:	Denote reflected wave
$_1$:	Denote quantities related to inner liner
$_2$:	Denote quantities related to outer liner

Chapter 1

1 Introduction

1.1 Background

The drive for low NO_x emissions and increased fuel efficiency means that gas turbine and aero-engines combustors are operated under lean premixed conditions, which tend to cause combustion instabilities damaging to these combustors. The generation of these combustion instabilities is due to a coupling between unsteady combustion and acoustic waves. The unsteady heat release in combustor results in fluctuations in local conditions with unsteady density, velocity and pressure, and therefore generates acoustic waves propagating along the combustor. These acoustic waves partially reflect from the boundaries of the combustor back to further interact with the combustion. If the acoustic fluctuation and unsteady heat release are in phase, then this constructive feedback loop results in large amplitude oscillations in the combustion system, which may eventually damage the engine. To damp combustion instabilities, both active and passive control approaches can be applied in unstable combustion systems. Passive control involves using Helmholtz resonator and perforated liners. As a typical acoustic damper, perforated liners can be used to convert the acoustical energy into flow energy in the form of vorticity at the rim of apertures. Additionally, the bias flow through

perforated holes used for cooling purpose to protect the combustor walls, is found to further improve the effect of sound absorption of the liner.

An elementary and significant demonstration and investigation of these combustion instabilities is the Rijke tube combustor in which a heated metallic gauze placed at lower quarter of this vertical straight tube can trigger the combustion instabilities. Extensive investigations were carried out on Rijke tube and both experimental tests and numerical modeling provides resourceful information for combustion in Rijke tube including the relation between heat source locations and acoustic oscillations, the determination of amplitude and frequency of limit-cycles, the linear and nonlinear growth of acoustic oscillations, the modal and non-modal analysis of the oscillations and so on. Therefore, studying on heating or combustion in a confined geometry gains the insight for understanding combustion instabilities and paves the way to better control these instabilities, especially in gas turbine engines.

1.2 Motivations and Objectives

Since the combustion instabilities extensively exist such as in gas turbine engines, rockets and boilers, to understand the generation mechanism and the fundamentals of combustion instabilities is key to control it. However, due to its nonlinearity and complexity, analytical calculation is limited to explain combustion instabilities. Therefore, in the present study Computational Fluid Dynamics (CFD) techniques are applied to simulate the combustion in a Rijke-like tube since it is an elementary

demonstration for combustion instabilities. Distinguishing from the simple straight Rijke tube which results in a single direction flow field with combustion and heating inside it, the new designed tube, Rijke-Zhao tube, will result in a more complex flow field due to its multiple ends. Therefore, acoustic characteristics are expected to be different from the traditional Rijke tube. The aim of this work for combustion instabilities in Rijke-Zhao tube is to design experiments to fundamentally study the flow and acoustic characteristics resulted from the combustion and develop numerical model by using CFD software Fluent 13.0 to capture the combustion instabilities in Rijke-Zhao tube with comparisons to experimental results in details. The limit-cycle which is the saturation of acoustic oscillations to certain amplitude will be found in our numerical model and frequency spectrum is obtained to analyze the oscillation frequencies and its mode. Meanwhile, the mode shape in the new tube is also produced to validate which frequency mode is the easiest one to go unstable. Different type boundaries for open-open end and closed-open end are also studied separately and compared to each other.

Based on the understanding of combustion instabilities, control methods are of importance to mitigate combustion excited oscillation. Perforated liner is widely used as a passive control method in aero-engines due to its simplicity and high efficiency. Numerical modeling of perforated liners can provide insight to their acoustic damping behaviors, so that they can be optimized to be applied in real combustion systems to mitigate combustion instabilities. The majority of modeling work has been carried out in the frequency domain. However, numerical simulation of the liners' damping effect

in the time domain enjoys its own advantages despite its complexity and high computation cost. The motivation of the present work is to develop a model which can reduce the computation cost and provide a qualitative prediction of the liners' damping behaviors. With z-transform techniques successful applied in surface acoustic impedance condition this work is to use such technique to achieve real-time simulation of sound absorption of cylindrical perforated liner with a grazing flow tangential to it and a bias flow passing through it. The aim of this work for perforated liner is to develop a time-domain numerical model to simulate the acoustic damping effect of perforated liners. For this, a 'real-time' compliance is developed. It captures unsteady vortex shedding from the perforated holes which is the primary mechanism for acoustic wave dissipation. The liner's 'compliance' is formulated into a rational form in z-domain, which can be converted into the corresponding form in time-domain. The compliance can provide an explicit expression describing the relationship between the fluctuating volume flux through the perforated holes and the fluctuating stagnation enthalpy across the holes. Thus with flow conservation equations along the duct, the 1D lined-duct model is developed. Numerical results are obtained for different flow conditions and compared with those from the model of Eldredge & Dowling in frequency-domain. To solve the flow-conservation equations in the lined duct model, which are partial differential equations (PDEs), method of lines (MOL) is adopted. MOL can convert the PDEs to ordinary differential equations (ODEs) by retaining the time derivatives and approximating the special derivatives in a finite difference form, therefore allowing integrating the ODEs to obtain the solutions by using standard

integration routines or ODE solvers. MOL is an effective method to reduce the complexity of solving PDEs with low computational cost.

1.3 Organization

This thesis is organized in the following: Chapter 1 is the introduction which gives a brief description of the background knowledge and motivations as well as objectives to be fulfilled. Chapter 2 is the literature review of previous work carried out on perforated liner and combustion instabilities in Rijke tube. Chapter 3 is numerical simulation of combustion instabilities in a Rijke-Zhao tube, including experimental and numerical study process and results. Chapter 4 is time-domain characterizations of the acoustic damping of a perforated liner with bias flow, developing a time-domain model for computing the liner damping effect. Chapter 5 is the conclusions of the entire work and future work recommendations.

Chapter 2

2 Literature Review

2.1 Combustion Instabilities

The combustion instabilities were based on the so-called thermoacoustic oscillations which were first formally studied and explained by Lord Rayleigh[1] as early as in 1877 and he proposed a theory, which is now commonly referred as Rayleigh criterion, to explain the excitation of such instabilities. The criterion is expressed mathematically as the Rayleigh index:

$$R_a = \frac{1}{T} \int_0^T p'(t)q'(t)dt \quad (2.1)$$

where $p'(t)$ is the fluctuating pressure, $q'(t)$ is the fluctuating heat release and T is the period of the oscillation. A positive Rayleigh index means the amplification of oscillations while a negative value means the attenuation. A simple demonstration of combustion instabilities and widely studied system is the Rijke tube[2], in which the flame placed in the lower quarter of the tube can cause the pressure and velocity in phase and make a positive Rayleigh index. This will trigger the oscillations to grow in a self-excited limit cycle.

The combustion instabilities are widely observed and reported in aero-engines and gas-engines. The pursuit of reducing pollutant emissions requires combustors of those engines working under the lean premixed condition. Under such condition, the combustion is sensitive to the fluctuations due to the air-fuel equivalent ratio because the acoustic waves generated by unsteady flame heat release are reflected back to interact with the flame, resulting in deviation in fuel and air mixture and perhaps enhance the oscillations of the heat release to some damaging extent [3].

Later, Lieuwen & Yang[4] generally identify several instabilities driven mechanisms in combustor systems which can cause fluctuations of heat release. Firstly, oscillations are generated by the pressure fluctuating and will affect the fuel injection rate which in turns affect the pressure. Secondly, equivalent ratio oscillations which are caused by pressure oscillations and affect the richness of fuel-air mixture can feed back to make the pressure fluctuations and even amplify the heat release oscillations. Thirdly, similar to equivalent ratio oscillations, the oscillatory atomization, vaporization and mixing coupled with the pressure oscillations vary the fuel spray shape, fuel droplet size and mixture richness. The variation leads to the unstable heat release. Fourthly, the variation of particle velocity of flame due to pressure oscillations leads to the change of heat addition to the acoustic field, resulting in further fluctuations of pressure. Last, hot pockets gas created by vortex shedding contains unreacted fuel. When it is ignited, additional heat will cause further turbulence.

2.2 Combustion Instabilities in a Rijke-Tube

In certain confined geometries, when the Rayleigh criterion is satisfied, the heat release will amplify the pressure fluctuations which in turn increase the instability of the heat

release and eventually a feedback loop between heat release and pressure fluctuations will be formed, causing the amplitude of fluctuating pressure to grow until it reaches the maximum referred as the limit cycle. An elementary and significant demonstration and investigation of these combustion instabilities is the Rijke tube combustor, named after its inventor Rijke[2] in 1859, in which a heated metallic gauze placed at lower quarter of this vertical straight tube can trigger the combustion instabilities. Due to its simple configuration and resourceful information, both experimental and analytical studies are widely carried out on the Rijke tube system to understand the fundamentals and details of combustion instabilities.

Zinn et al[38] used a burning bed of solid fuel instead of a metallic gauze in a Rijke tube, therefore modifying the heat source to be pulsating and change the Rijke tube to be a pulsating combustor. Their results show that the combustion performance in this type of combustion is superior because the pulsations can improve the mixing between fuel and oxidizer. Carvalho et al[39] defined the heater locations to drive maximum acoustic oscillation and identify the modes at each location. They applied mathematical formulae of Rayleigh criterion to Rijke tube, finding the region where heat release may drive the acoustic oscillations and calculating the locations of maximum amplitude oscillations. Simple experiments are implemented to examine their mathematical analysis and good agreement is observed.

Numerical methods are also applied to analyse these combustions instabilities in both linear and non-linear theory. Unlike the linear theory[40] which assumes the flow

perturbation is linear to the time dependence and provides no information about the amplitude of the oscillations occurring in practice, the non-linearity affects and determines the limit-cycle amplitude. Heckle[41] investigated the non-linearity in Rijke tube and gave a prediction for the amplitude of limit cycle by developing an empirical model for the heat transfer rate according to experimental results. Dowling[42] also developed a non-linear method to explore the pressure oscillation of a confined flame burning in the wake caused by a flame-holder and this work confirms the main non-linear effect resides in the heat release rate. More recently, Balasubramanian and Sujith[43] combined a heat release model suggested by Heckle with Galerkin method to solve the wave equation and investigated the non-normality and nonlinearity in a Rijke tube. They found the non-normality can turn on a quick growth of oscillations before they die down and the transient growth is capable of make large enough oscillations to trigger nonlinear driving.

Due to the development of computational fluid dynamics (CFD) techniques, more work for thermoacoustic instability based on CFD modelling is reported. Hantschk and Vortmeyer[44] applied CFD software Fluent to simulate heating bands in a Rijke tube and study the non-linearity effect on pressure oscillations. By creating an artificial temperature gradient in the tube and imposing an initial pressure disturbance at the boundary, the self-excited oscillations and higher harmonics frequencies of the oscillations can be observed and both the predicted frequencies and mode shape agrees with the experimental results well. Instead of using time-varying heat source or instability triggering method, Chatterjee et al[45] used Fluent to directly simulate the

reacting flow in a Rijke tube and studied the spectral characteristics of the combustion driving thermoacoustic instabilities in details. Validated by the experimental results, their study contributed to capture the reacting flow physics and provide a model for interaction between combustion and pressure oscillation.

Distinguishing from these CFD studies, Subramanian et al[46] performs a bifurcation analysis of the instability in a Rijke tube by using the numerical continuation method which can identify the bifurcation of a nonlinear system of equations with one or more parameters varying. They used software called DDE-BIFTOOL originally developed by Engelborghs and Roose[47, 48] to obtain the bifurcation plot of variations of heat power, damping coefficient and heat location for the different time lags and the linear and nonlinear stability boundary are also obtained for two parameters varying simultaneously. Their stability boundary and bifurcation behavior have the similar trends with the experimental data.

2.3 Active Control of Combustion Instabilities

Active control of thermoacoustic instability means to interfere with the combustion process and suppress the pressure oscillations by measuring the amplitude of the pressure oscillations, adding an appropriate phase, and generating a signal equivalent and opposite to the exact pressure oscillations. In general, active controller is fast-responding and effective over a large operating range of frequency. Typical active controller includes a sensor, a controller and an actuator. The sensor provides measurement for the perturbation of the actuator to the combustion systems. However, since the active controller injects energy into the system, there is a possibility that the

system become more unstable if the control implementation is not accurate or ill-designed.

Annaswamy & Ghoniem[5] reviewed the theory and application of active control methods and summarized several control strategies. A linear control strategy seeks to minimize a cost functions with the knowledge of the properties and response of the system already known. Examples are LQR-based controller which can minimize the modeling effect error and H_{∞} -optimal controller which achieve the desired measurement of instability robustness and the performance specified in frequency-domain. A time-delay controller is always used to cancel or minimize large delay effect. Adaptive controllers are capable of self-corrections to maintain the control performance in face of changes in operating conditions and parameters. One of them is LMS control which depends on adaptive filters that minimize the error representing the departure from a control variable of the system.

2.4 Passive Control of Combustion Instabilities

2.4.1 Helmholtz Resonator

To suppress combustion instabilities, an explicit way is to apply passive control methods including combination of baffles into a system and modification to geometries. However, simple modifications to geometries may not sufficiently stabilize a system with combustion instabilities excited by multiple means, while passive acoustic damping devices are more efficient for such cases.

One traditional damper is Helmholtz resonators for which considerable work and analysis related to design, implementation and transmission loss is done. A Helmholtz resonator mounted in a duct resists the propagation of incident waves by providing a high impedance to planar acoustic wave, resulting in large acoustic attenuation. However, this is only effective for a single, narrow frequency bandwidth near its resonator frequencies. Griffin *et al* [6, 7] demonstrated that mechanically coupled resonators can be used to specific transmission loss response with wider bandwidth of acoustic attenuation and by adjusting the mass or stiffness of the coupled member, the transmission loss can be adapted for varying frequencies. Although their work is remarkable, the effective frequency bandwidth for transmission is still narrow.

2.4.2 Perforated Liners

A solution is to utilize a perforated liner with bias flow because each orifice can work in isolation to dissipate acoustic energy and their distribution over a duct length enables the liner to work efficiently over a large range of frequencies and locations. Typically, perforated liners are metal sheet with arrays of orifices and are arranged in layers with rigid wall backed up. The orifice with its cavity formed by the backed rigid wall can be considered as a component 'cell', which works separately as a Helmholtz resonator for sound absorption. Crude theoretical model was developed by Lewis & Grrison[8], in which the perforated liner was modeled as a plate mounted many Helmholtz resonators without bias flow. The treatment of orifices and its cavity on the liner as simple

Helmholtz resonator provides basis for acoustic liner design by investigating a single Helmholtz resonator.

2.4.2.1 Frequency-Domain Method

The investigation of perforated liners for their explicit acoustic damping effect was started since 1970's. Ffowcs Williams[9] examined an infinite perforated screen without considering a mean flow, where he modeled the dynamics process of small-scale turbulence scattered by individual apertures into sound. His results showed the different acoustic properties of turbulence near a rigid perforated screen and near a baffle array of pistons.

Leppington and Levine[10] made refinements to Williams' results and additionally investigated a perforated screen in front of a rigid wall, solving for the reflection and transmission coefficients of the waves scattered by the screen with a use of an integral equation. Their results showed, unlike a simple Helmholtz resonator, the angle at which the wave is incident determines and affects the resonance frequency of perforated screen. The resonance frequency matches the Helmholtz resonance frequency of a cell only when the wave is at normal incidence. However, in their work, the reason for the absorption of sound energy was not explained. In Leppington's later work[11], he considered semi-infinite perforated screens which is more realistic and defined a compliance accounting for the behavior of acoustic wave jumping across the perforated screens. In general, the compliance is the ratio of the change of a quantity across the

surface at its normal direction. The estimation of the compliance was obtained by adopting a Wiener-Hopf approach.

Since holes in perforated screen can be regarded as working in isolation if their diameters are smaller enough compared to the distance between each holes, characterization of the perforated screens can depend on the work done for acoustic properties of single orifice. Howe[12] modeled the acoustic energy dissipated by the periodic shedding of vorticity for a single orifice in a high Reynolds-number flow using the Rayleigh conductivity with a theoretical expression derived for it. He determined the unsteady flow through an aperture by solving the Possion equations with the Kutta conditions imposed to obtain the vorticity. Since the unsteady sound and resultant vorticity are in a linear relationship, the Rayleigh conductivity is uncorrelated to the magnitude of the sound field. He applied the Rayleigh conductivity for the sound transmission through uniformly perforated screens with a low Mach number bias flow. His application shows that bias flow through the perforated liners brings in different sound damping effect from those liners without a mean-through flow.

Following Howe's research, Hughes and Dowling[13] studied the acoustic damping of plates with uniform lines of slits and circular holes at the presence of bias flow, showing that all impinging sound could be absorbed at a specific frequency in theory due to the reflection of the sound from the wall allows more interactions between the screen and the sound. These frequencies of largest sound absorption are identified by them as the

Helmholtz-type resonance of cavity behind the wall. Besides, they applied similar analysis to investigate the transverse-type resonance of an infinite long cylindrical duct.

Jing and Sun[14] experimentally investigated the effect of the screen thickness and the bias flow rate, showing that an appropriate bias flow rate can significantly increase damping and that the screen thickness is crucial. In their later work[15], numerical analysis of the effect of screen thickness was performed, showing that the screen thickness and bias flow velocity together affect the acoustic reactance and resistance of the perforated screens.

The above mentioned numerical and experimental investigations only considered the case that acoustic waves go through and impinge upon the perforated liners. However, many combustion instability problems, for instance, in a gas-turbine combustion chamber, rely on axial mode. Recently, Eldredge and Dowling[16] developed a 1D model using a homogeneous compliance derived from Rayleigh conductivity to predict the sound absorption ability of cylindrical perforated liner subjected to a bias-flow with planar acoustic waves tangential to its surface. They assumed that the fluctuating acoustic waves propagate along a duct with a perforated section and can be reflected back from the open end of the duct. When the geometry and the flow through the liner satisfy certain conditions, the maximum absorption of sound can reach 83%. Later, Eldredge[17] extended the previous 1D model to 3D by using Green function solution of Helmholtz equation, allowing calculation of the amplitude of existing wave modes and comparing this higher dimension results to the previous 1D model. With excellent agreement, it demonstrated that all modes that travels parallel to the duct with lined

section exhibit self-similar behavior, though the 1D model ignore the variation at the radial direction and evanescent waves at the duct/liner interface. Therefore, the previous 1D model is sufficiently accurate and can be used to determine the geometry to achieve maximum sound absorption of the liner. Eldredge also concluded that over wider frequencies, the transverse effect become dominant and there are mainly three classes of enhancement for incident wave absorption near associated frequencies, which are the Helmholtz-type, resonance-duct and cavity.

Rehman and Eldredge[18] combined the 1D model of perforated liners developed in [16] with a heuristic gain-delay flame model. They found varying the porosity and bias flow of the liner can achieve stability of the combustion system and there is a maximum in effective stability in double liners. Also based on Eldredge and Dowling's work, Zhao and Morgans[19] experimentally examined the perforated liners' damping effect and developed a multi-parameters tuning scheme, which can alter the length of downstream duct and bias flow, to broaden the effective frequency range of the liner and to achieve an optimization of liners damping. They pointed out that altering the downstream boundary condition can be implemented instead of tuning downstream length because the latter is difficult to be realized in a real combustion system.

In the axial mode, the downstream reflection coefficient is crucial to determine reflected wave. It was Lord Rayleigh[20] who first experimentally calculated the end correction for a flanged end in the absence of theoretical information. In the absence of mean flow, the expression of reflection coefficient for an open unflanged duct was derived by

Levine & Schiwnger[21]. They thought that when the acoustic wave reach the open end, the incident energy was partially carried by the reflected waves and the wave propagating to the external space take away the rest of the incident energy. They analytically solved for the expression of reflection coefficient which can be used to determine the amplitude and phase of the reflected waves. The exact value of end correction was found to be 0.6133. They also gave simple approximation to their full and complicated solution under certain restrictions.

For the presence of mean flow case, Cargill[22, 23] imposed Kutta condition at the edge to model dissipation of the acoustic energy due to jet instability wave and obtained the solution by using Wiener-Hopf technique. At a low mean duct flow Mach number, it is demonstrated that the influence of the Mach number on the reflection coefficient is not important and the difference between reflection coefficient with and without a mean duct flow is negligible.

Majority of work investigating the absorptive properties of perforated liner is carried out in frequency-domain because the acoustic response on absorptive material is functioned by wave frequency. Modeling the frequency-dependant characteristics of the liners is more explicit and easy in frequency domain. One disadvantage of frequency method is that it can only solve single frequency one at a time unlike the time domain method which is capable of handling broadband frequency problems. Mathematically, the time-domain application can be implemented by taking the inverse transform of the frequency model equations. However, this results in convolution integral, which is

computational expensive due to the requirement of a long time history of certain quantities involved and impractical especially for multi-dimensions problems.

2.4.2.2 Time-Domain Method

Despite the difficulties and complexity, some attempts are made to simulate the damping of perforated liners in time-domain. Inspired by the successful use of z-transform method in solving frequency-dependent electromagnetic problem[24], Özyörük, Long & Jones[25, 26] used a z-transform method to convert the frequency-domain Myers[27] impedance boundary condition which accounts for the characteristics of the acoustic treated wall to a finite difference form, and therefore incorporate this impedance boundary condition into 1D and 2D Euler equations. They proposed a nonlinear model of wall impedance with parameters fitted by using nonlinear least mean square method and then constructed a rational form of impedance in z-domain. By following the relations among frequency, z- and time-domain, they converted the impedance boundary condition to a time-domain discrete form. Their simulation results agree well with the experiments in both single and broadband frequency and showed that applying z-transform technique[28] can dramatically reduce the computation involved and so real-time simulations of liners damping is achieved.

Reichert & Biringen[29] constructed their liners model with an empirical continuous source term which modifies the inviscid flow momentum equation with additional term on right side of the equation. Moreover it captures the characteristics of frequency-dependent resistance and reactance of the liners in time-domain and therefore keeps the

liner effect inside the computational duct domain unlike applying extra impedance boundary condition as did in [25, 26]. The source term constants which determine the level of linearity and nonlinearity of resistance and the linearity reactance are obtained from frequency-domain impedance tube simulation. Their results confirmed that with an optimum bias flow rate, the liner damping efficiency is significantly improved than that with no bias flow.

Based on simplification of Reichert & Biringen's liner model, Sbardella, Tester & Imregun[30] developed a time-domain approach for perforated liners by assuming that the duct and cavity formed by the liner are separate areas. They modeled flow within the duct using nonlinear, unsteady Euler equations, while modeling the liner by using a frequency-independent resistive part and a reactive part determined by 1D Euler equations for the liner cavity. They solved the discontinuity of the velocity across the interface of duct and cavity by considering the continuity of the acoustic particle displacement, therefore deriving an expression for the relation of displacement and normal velocity of a particle. Since their liner model is captured by several constant parameters, their work leads to a possibility of parameter study for the liner acoustic response. Besides, their work satisfies the multi-frequency calculation at one time and the frequency-dependant impedance is not required in the time-domain model but its effect on the liner is incorporated in the reactive part.

Tam *et al*[31] implemented time-domain direct numerical simulation to a single liner cavity and showed that the vortex shedding is the dominant mechanism for acoustic energy

dissipation. They considered the single liner cavity as a resonator and flow visualization experiment confirms the occurrence of shed vortex near the mouth of the silts. The direct numerical simulation is to deal with compressible Navier-Stokes equations directly, which is a complete departure from conventional acoustic liner technologies because it does not require the modeling of the liner frequency-dependent acoustic properties.

Fung *et al*[32] developed an impulse method to measure the acoustic liner characteristic in a flow duct in order to obtain impedance boundary condition for time-domain use. They used a short-duration impulse to excite the liner response and therefore the unrelated boundaries such as the end of the duct would not affect the prediction and modeling. By assuming that the liner impedance can be represented by damped harmonic oscillators, the parameters describing the feature of the liner can be determined by matching the predicted-model and measured pressure response of the sensor located accordingly in the liner. Their work provides a time-domain applicable impedance boundary condition for acoustic computation.

In some above discussed time-domain simulations, the downstream flow boundary condition is assumed to be none reflection. However, in Eldredge and Dowling's model, the reflected wave can be seen to further interact with the upstream incident wave and affect the modeling of the sound absorption of the liner. Therefore, time-domain reflection function, the corresponding form of frequency-domain reflection coefficient, is crucial to perform a realistic time-domain simulation in a lined duct.

Polack[33] analytically calculated the reflection function for a plane acoustic wave propagating in cylindrical duct based on the reactive term of radiation impedance given by[21]. The reflection function, in fact, is the inverse Fourier transform of the reflection coefficient and is also the impulse response in time-domain. Their discussion of the reflection function properties showed that the function is stable through convolution and this allows a construction of reflection function for a series tubes connected with each other.

Fung *et al*[34] used time-domain impedance of the duct end to represent the reflection by extend the concept of impedance to an equivalent kernel of reflection. Their numerical analysis demonstrated that the inversion of impedance into time-domain boundary operator leads to unstable system, while direct inversion of corresponding reflection coefficient results in stable, simply applicable boundary operators of wave reflection. In their later work[35], they discussed some issues about implementing the time-domain impedance-equivalent boundary condition (TDIBC) and proposed a stable method to deal with physically unacceptable and mathematically feasible reflection condition. Various types of conventional defined impedance were demonstrated to correspond to reflection impulses whose identifications are important to broadband frequency applications.

Silva *et al*[36] provided several approximation formulae for reflection coefficient and its corresponding reflection function for both infinite flanged and unflanged rigid wall

of cylindrical ducts. These formulae satisfy basic, necessary physical and mathematical principles, say Hermitian symmetry for reflection coefficient and causality for time-domain impulse response. The formulae are compared to the reference data calculated from results of [21] for unflanged case and Zorumski's[37] work for flanged case. Well approximation is observed. They also discussed the relaxation of the causality of the reflection function in order to propose more accurate approximation models.

2.5 Numerical Simulation Techniques for Acoustic Problems

The time-domain simulation of the sound absorption in perforated liners and combustion instabilities are typical applications of computational aeroacoustic (CAA) technique. CAA deals with the numerical simulation of noise radiation generated by aeroacoustic sources and the propagation of sound in inhomogeneous flow field and has been beneficial from computational fluid dynamics (CFD), however, receiving its own challenge as stated by Tam[49]: aeroacoustic problems are always time dependent whereas the aerodynamics problems are generally time independent or low frequency unsteady; aeroacoustic problems involves a wide range of frequency, which lead to the difficulties for simulation accuracy of high frequencies; the numerical scheme must reduce the numerical noise to compute the sound waves accurately since the amplitude of the acoustic wave is often very small; minimal dispersion and dissipation are required for computational schemes because of the long distance propagation of acoustic waves; the numerical scheme is also required to be valid for different length scale in different parts of computational domain since the length scale of the acoustic

source can be very different from the flow far field. Various schemes and methods are established and developed for solving the above mentioned challenges.

The most widely used method is the finite volume method (FVM) which is well established in computational fluid dynamics (CFD). FVM is a discretization method for evaluating PDEs in the form of algebraic equations. A mesh is constructed for the computational domain and a finite volume is formed surrounding each node point. The volume integration of the PDEs over a finite volume is converted to surface integral by using the divergence theorem and then the terms in PDEs are evaluated as fluxes on the surfaces, which is conservative of each finite volume.

A revision of the traditional FVM is started by Jameson who combines a different scheme developed by Rizzi and Schmidt[50] into a code to solve the Euler equations. This modified method replaces the MacCormack scheme, which is the conventional scheme used in FVM for iteration calculations, by central difference scheme with three iterated stages while retain other feature of the FVM. The main difference of this modified method is to introduce a dissipative term by adding a filter stage at the end of each time step. Sensors concerning the local properties of the flow are used to determine the dissipative term. Compared to MacCormack scheme, the revised method remain stable when the Courant number is up to 2 for 1D problems and for multi-dimensional problems using a reduced time-step with no need of splitting. In addition, this scheme allows a variable time step subjected to the local Courant number used through the computational domain with its cell size varying largely. Later, Jameson *et*

al [51] extended the revised method to two-dimensional problems in order to find a more efficient and accurate scheme. They concluded that the time-step scheme is preferred to be the forth order Runge-Kutta scheme for their three-stage scheme; the dissipative term should be constructed as a combination of second and forth order differences; the boundary conditions in far field should be characterized by the variables of the flow; the convergence can be accelerated by adapting a variable time step at the maximum value allowed by the local Courant number and by adding a forcing term according to the difference of the local total enthalpy and the free stream. Jameson scheme, due to its efficiency and stability, now is widely used to solve the Euler equations and related CAA problems, including the simulation of perforated liners such as in the work done by[25, 26].

Finite element method (FEM) is an alternative approach for the discretization of differential equations. In FEM, a structure to be analyzed is divided into small elements with a certain shape. The loads and response at the nodes of the element is analyzed to account for the behavior of the element by using the Ritz-Rayleigh procedure. The analysis of the element results in a small matrix relating a vector of the nodal displacement to a vector of nodal force. A global matrix can be formed for the entire complex structure by combining each small matrix of an element. Although FEM has been studied and applied to CFD for long time, there are still difficulties[52] remaining in the application of FEM in acoustic problems. Firstly, the FEM is originally developed for bounded domains, whereas unbounded boundary condition often occurs in acoustic scattering problems. This is overcome by replacing the unbounded boundary

by an artificial bounded boundary and the boundary condition for the modified problem should be carefully chosen to approximate the original problem accurately. Absorption boundary condition is one choice of such boundary conditions which gives no reflection of scattered waves. Secondly, for large wave number the standard FEM solutions deteriorate due to the big oscillation of the error. An approach to get around this difficulty is to replace the piecewise linear basis function by higher order piecewise polynomials. An alternative is to use a special basis function tailored to the problem of high frequency, based on the generalized finite element method and partition of unity method. Despite these difficulties, FEM is a useful and promising method for aeroacoustic problems.

Notably, boundary element method (BEM) is especially suitable for computational acoustic problems because modeling of the fluid domain by BEM is much easier than modeling the fluid domain itself. BEM numerically solves a problem by reformulating the problem as a boundary integration equation. The general equations characterizing most acoustical problems provide a basis for BEM as an ideal method of control acoustics and noises. Generally, there are two types of BEM for acoustic analysis. The direct BEM solves the Helmholtz equations which are governing equations for most acoustic problems in either a bounded interior domain or an unbounded exterior domain at a time with primary variable chosen as pressure. Both continuous and discontinuous element can be applied in this method. On the other hand, the variational indirect BEM solves the Helmholtz equations in the interior domain and the exterior domain simultaneously, though the problem may only focus on one domain. The primary

variable is the pressure jump across the boundary and only continuous element can be used for this indirect method. Wu, in his book[53], describes the BEM in details and derives code for 2D and 3D simple duct flow problems. He also discussed the application of direct mixed-body boundary integral formulation in BEM and use this formulation to solve a concentric-tube resonator with a flow plug in the middle of the perforated tube. The numerical simulation was compared with the experimental transmission loss curve and the well agreement demonstrates that the BEM is sufficiently accurate for complex acoustic problems such as perforated liners.

Based on the above mentioned method, commercial CFD software is developed to facilitate the simulation of flow problems and they can be also applied to CAA problems. However, as mentioned above, the numerical scheme for CAA problems should be less dissipative and dispersive, while most robust CFD solvers (Star-CD, Fluent, and CFX) have inherent dissipation to enhance the stability. This hinders the application of most CFD solvers for the simulation of acoustic flow. Advanced schemes such as Dispersion Relation Preserving (DRP) and compact schemes are developed for reducing the dispersion, though the research of practical acoustic computation requires more effort due to the limitation of computational capability. The main code vendors decouple the CAA problems by solving for the acoustic sources in CFD codes and then coupling to acoustic propagation code to determine the scale of noise at some distant to the sources.

Chapter 3

3 Numerical Simulation of Combustion

Instability in a Rijke-Zhao Tube

3.1 Introduction

Thermoacoustic instabilities originate from interactions between unsteady heat release and acoustic waves generated by pressure fluctuations. Lord Rayleigh in 1887[1] has stated that combustion instabilities would be self-excited when the unsteady heat release is in phase with the pressure fluctuations, which is referred as the Rayleigh criterion. In certain confined geometries, when the criterion is satisfied, the heat release will amplify the pressure fluctuations which in turn increase the instability of the heat release and eventually a feedback loop between heat release and pressure fluctuations will be formed, causing the amplitude of fluctuating pressure to grow until it reaches the maximum referred as the limit cycle. An elementary and significant demonstration and investigation of these combustion instabilities is the Rijke tube combustor in which a heated metallic gauze placed at lower quarter of this vertical straight tube can trigger the combustion instabilities. Due to its simple configuration and resourceful information, both experimental and analytical studies are widely carried out on the Rijke tube system

to understand the fundamentals and details of combustion instabilities. Zinn et al[38] modified the original Rijke tube to be pulsating combustor by using a burning bed of solid fuel instead of the gauze. Carvalho et al[39] defined the heater locations to drive maximum acoustic oscillation and identify the modes at each location with comparisons between experiments and theoretical analysis. Numerical methods are also applied to analyse these combustions instabilities in both linear and non-linear theory. Unlike the linear theory[40] which assumes the flow perturbation is linear to the time dependence and provides no information about the amplitude of the oscillations occurring in practice, the non-linearity affects and determines the limit-cycle amplitude. Heckle[41] investigated the non-linearity in Rijke tube and gave a prediction for the amplitude of limit cycle by developing an empirical model for the heat transfer rate according to experimental results. Dowling[42] also developed a non-linear method to explore the pressure oscillation of a confined flame burning in the wake of a bluff-body flame-holder and this work confirms the main non-linear effect resides in the heat release rate. More recently, Balasubramanian and Sujith[43] combined a heat release model suggested by Heckle with Galerkin method to solve the wave equation and investigated the non-normality and nonlinearity in a Rijke tube. They found the non-normality can turn on a quick growth of oscillations before they die down and the transient growth is capable of make large enough oscillations to trigger nonlinear driving.

Due to the development of computational fluid dynamics (CFD) techniques, more work for thermoacoustic instability based on CFD modeling is reported. Unlike solving the wave equations by the above mentioned work, CFD solves for the conservation

equations of mass, momentum and energy directly for the flow field and the solution can also be used to analysis the perturbations for the acoustic field. A 1D CFD simulation is conducted by Ishii et al[54] who create a heat source and sink with a steep temperature gradient which cause the change of flow velocity and in turns affect the heat supply. They explained this change converts the thermal energy into mechanical energy. Similarly, Entezam et al[55] implemented a CFD software called Flow 3D to perform a two dimensional verification of acoustic oscillation in a Rijke tube combustor. By using a porous solid obstacle as heat source and varying the its locations and power supply, they confirmed that the maximum pressure oscillation amplitude occurs when the heat source is placed at lower one-quarter length of the tube and there is a minimum threshold value for the power supply to generate the acoustic oscillation, which according to their numerical calculation is 376 W. Song et al[56] combined the CFD and CAA method to analyze the stability of combustion oscillation inside the Rijke tube. In their work, the impulse heat release function and distributions of density and speed of sound predicted by CFD are input into Euler equations to determine the stable boundary for the acoustic oscillation in a Rijke-tube.

Hantschk and Vortmeyer[44] applied CFD software Fluent to simulate heating bands in a Rijke tube and study the non-linearity effect on pressure oscillations. By creating an artificial temperature gradient in the tube and imposing an initial pressure disturbance at the boundary, the self-excited oscillations and higher harmonics frequencies of the oscillations can be observed and both the predicted frequencies and mode shape agrees with the experimental results well. Instead of using time-varying heat source or instability triggering method, Chatterjee et al[45] used Fluent to directly simulate the

reacting flow in a Rijke tube and studied the spectral characteristics of the combustion driving combustion instabilities in details. Validated by the experimental results, their study contributed to capture the reacting flow physics and provide a model for interaction between combustion and pressure oscillation. A similar application to Rijke tube is thermoacoustic engine. Unlike the Rijke tube in which only a simple heat source is placed, the structure thermoacoustic engine includes heat stack, heat block and heat exchanger which are used to convert the thermal energy to mechanical energy more efficiently. CFD numerical method are also implemented in these acoustic engines to help in understanding and design and such as the work of Yu et al[57].

In this present study, a new designed Rijke-Zhao tube which contains a mother tube joint with two daughter tubes with different length is considered as the combustor for a premixed laminar flame placed in it. Distinguishing from the conventional straight Rijke tube, combustion in this Rijke-Zhao tube reveals different flow field and combustion instabilities. Experiments are first conducted to examine the flow characteristics and the amplitude and frequency of the combustion oscillations. Since its 2D geometry features, traditional linear and non-linear numerical analysis for 1D Rijke tube is not applicable to this new combustor while a 2D CFD modelling by using Fluent 13.0[58] is implemented with a higher temperature heating bands included instead for combustion for simulation purpose like in the work of Hantschk and Vortmeyer[44].

3.2 Experiments

3.2.1 Experimental Setup

The Rijke-Zhao tube, as shown in Figure 3.1 (a), is a Y-shape tube made of quartz, containing a mother tube which splits into two bifurcating daughter tubes with different length. For the selected test tube, the mother tube (straight part) is 300 mm and the daughter tubes (branch parts) are 500 mm and 900 mm respectively with a 30 degree angle between them. On each daughter tube, a T section connecting to the tube is designed for placing the microphone which is applied to acquire the pressure signature and a rubber tube is connected to the end of the T section in order to provide an ambient condition at the end of the T section. A bunsen burner is placed in the mother tube with its gauze at the height of half-length of the tube with LPG gas fed into the bottom of the burner. The gauze was covered by a metal wire net which increases the heat transfer from the flame to the gauze. Therefore the thermal-diffusive instability defined by Margolis[59] is enhanced, moving the flame sheet forth and back from the gauze and resulting in a change of heat release rate which is the cause of combustion driven instabilities and generates the sound in the tube[45].

Figure 3.1 The experimental setup (a) and schematic (b) of Rijke-Zhao tube

3.2.2 Flow Characteristics

Unlike straight Rijke tube which only contains two ends and single flow direction, this triple ends tube results in a different flow field. The experimental test was conducted to determine the flow direction due to convection in each tube when the mixture is burning at an equivalent ratio of 1.0 and a volume flow rate of 100 cc/s. The experimental results show that the flow fluxes from the bottom end into the tube and goes outward at the long branch tube, whereas the air is sucked into the tube at the short branch end, as depicted in the schematic by Figure 3.1 (b). This flow field caused by the special geometry of the tube means that the hot flow heated by the flame will mix with the cool air sucked from ambient by the short branch and together go out through the long branch tube. The surface temperature distribution captured by infrared camera shows

the above mentioned flow condition in Figure 3.2. In the mother tube, the area near flame is at a high temperature around 330 °C. The average surface temperature is about 60 °C on the surface of the long branch in which hot flow passes by, while the temperature is almost 27 °C everywhere in the short branch in which the cool air is sucked in from the ambient. The table 1 display the temperature measured at different T section locations and the axis of each tube is denoted by x_1 , x_2 and x_3 respectively as shown in Figure 3.1 (b).

Table 1 Measured temperatures in branch tubes

Long Branch	Temperature (°C)
Position 1	190.3
Position 2	180.1
Position 3	169.6
Position 4	161.1
Short Branch	Temperature (°C)
Position 5	28.5
Position 6	27.7
Position 7	27.7

3.2.3 Acoustic Characteristics

Like a straight Rijke tube combustor, this new designed Rijke-Zhao tube also can generate combustion instabilities with large oscillating pressure amplitude when Rayleigh criterion is satisfied. These acoustic signatures are recorded by microphones at certain positions as shown in Figure 3.1 (b) and Fourier transform is taken to these signatures in order to find the pressure frequency spectrum which exhibits useful

information for the combustion instabilities. In the experiments, the fundamental frequency of the oscillation is at 175Hz which is also the first

Figure 3.2 Infrared image for tube surface temperature distribution

acoustic mode to go unstable. As shown in Figure 3.7, the mode shapes processed from the microphone data[19, 60] in different branch tubes confirm the fundamental frequency is the first mode, showing a three-quarter mode shape in long branch whose length is just three-quarter of the total length of mother tube and the long branch tube. Similarly, an almost half mode shape is displayed in the short branch tube which matches the ratio of short branch length to the total length of short branch and mother tube.

3.3 CFD Modeling

3.3.1 Open-Inlet and Open-Outlet

In order to verify the experimental results and numerically capture the flow and acoustic characteristics of Rijke-Zhao tube, CFD techniques are applied. Since there is no forcing flow in our experiments, the three ends of the Rijke-Zhao tube are all regarded as open end and corresponding boundary conditions will be applied in our numerical simulation. The Rijke-Zhao tube geometry is generated with structural mesh in Pointwise and for simulation purpose, the metallic gauze used in experiments is replaced by several heating bands in the CFD modeling as shown in Figure 3.3.

Before starting the unsteady solution to capture the limit-cycle generated in the tube, a steady solution is first performed as the initial condition for unsteady run. Three different boundary configurations was considered: (a) the mother tube end as inlet and the two branch ends as outlet; (b) the mother tube end and long branch end as inlet and the short branch end as outlet; (c) the mother tube end and short branch end as inlet and the long branch end as outlet. The heating bands are assigned a surface temperature at 700 K which approximates the temperature at the metallic gauze in the experiments. For the three configurations, the mother end is set as the pressure inlet with a small amplitude of 0.07 pa for the gauge pressure which results in a Reynolds number of 1180 remaining in the laminar flow, whereas the long branch end and short branch end is set as pressure outlet with an ambient pressure of 1 bar ($p' = 0$) in the first configuration. In the second configuration the long and short branch ends are set as pressure inlet with a

gauge pressure 0.02 pa and pressure outlet at 1bar respectively, while the long Branch and short branch ends are set as pressure outlet at 1 bar and pressure inlet with gauge pressure of 0.02 pa respectively in the third configuration. The small gauge pressure for the inlet is used to cause a flow flux towards outlet, simulating the convection caused by combustion in our experiments. The tube wall is assumed to have a convection heat loss to the ambient with a heat transfer coefficient of 20 W/m²K assigned. The air is considered to be ideal gas with constant values for all air properties.

The steady conservation equations of mass, momentum and energy with equation of state for idea gas from Eq. (3.1)-(3.4) are solved in Fluent 13.0 iteratively for a laminar, two-dimensional flow. Second-order schemes are chosen for pressure interpolation and special discretization, while the SIMPLE method is used for velocity-pressure coupling.

Conservation of mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0. \quad (3.1)$$

Conservation of momentum

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} = -\frac{\partial p}{\partial x} - \frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}. \quad (3.2)$$

Conservation of Energy

$$\begin{aligned} & c_p \frac{\partial(\rho T)}{\partial t} + c_p \frac{\partial(\rho u T)}{\partial x} + c_p \frac{\partial(\rho v T)}{\partial y} - \frac{1}{\rho} \left(\frac{\partial(\rho P)}{\partial t} + \frac{\partial(\rho u P)}{\partial x} + \frac{\partial(\rho v P)}{\partial y} \right) \\ & = \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \tau_{xy} \frac{\partial v}{\partial x} + \tau_{yx} \frac{\partial u}{\partial y} \end{aligned} \quad (3.3)$$

Equation of ideal gas

$$\frac{p}{\rho} = \frac{R}{W} T. \quad (3.4)$$

where $\tau_{xy} = \tau_{yx} = \eta \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$, and ρ , p , u , v and T are the density, pressure, x -direction velocity, y -direction velocity and temperature respectively. λ is the thermal conductivity. $R = 8.31451$ (J/mol K) is gas constant. W is the molecular weight. η is the dynamic viscosity and δ is the Kronecker symbol.

Figure 3.4 displaying the steady solution results for 3 different configurations shows that only the third configuration where the mother tube end short branch tube end are inlet and long branch end is outlet matches the experimental results shown in Figure 3.2. The temperatures calculated from the simulation at four positions along the long branch are 198.5K, 190.6K, 175.1K and 168.3K, approximating the values listed in Table 2. Therefore, the steady result of third configuration is chosen as the starting point for the unsteady simulation to capture the limit-cycle in the Rijke-Zhao tube.

3.3.1.1 Acoustic Characteristics

Figure 3.3 Geometry and mesh grid for the Rijke-Zhao tube

Since the combustion instabilities are natural time dependant, unsteady calculation is needed to capture the acoustic characteristics in Fluent 13.0. By using steady solution of the third configuration as the initial condition, a second-order implicit formulation in time integration is performed with a time step size $1e-4$ s for unsteady simulation. The other conditions and schemes are kept the same, while a small initial disturbance $\Delta p = 100\text{ pa}$ is assigned to the pressure outlet at long tube end for 10 time steps and keep ambient pressure after that in order to trigger the combustion instabilities. In Figure 3.5 (a) the unsteady pressure recorded at the location of microphone 1 shown in Figure 3.2 is displayed and the transition process in which the amplitude of the fluctuation pressure grows initially and saturate at a fixed value finally is observed. In Figure 3.5 (b), the frequency spectrum of the unsteady pressure after reaching the limit-cycle was displayed for both numerical and experimental results at the same location.

Figure 3.4 Temperature (K) Contour and velocity vector for different boundary condition configurations: (a) mother tube end as inlet and two branch ends as outlet; (b) mother tube end and long branch end as inlet and short branch end as outlet; (c) mother tube end and short branch end as inlet and long branch end as outlet

Since the energy of heat release and heat loss is not measured in the experiments, the heat release and loss which is only approximations to the experimental setup causes the difference of the sound pressure level (SPL) of the limit-cycle for numerical and experimental results. Our focus, however, is to identify the frequency mode of this all open end type tube. From the frequency spectrum, frequency of both results agrees very well with 168 Hz for numerical result and 175Hz for experimental result which is the first mode for the combustion instability in the Rijke-Zhao tube.

Figure 3.5 (a) unsteady pressure signal and (b) frequency spectrum of limit-cycle

In Figure 3.6, the unsteady pressure p' over time is compared to the unsteady velocity u' and the total surface heat flux q' calculated from Fluent at the position of microphone 1. The velocity oscillation which precedes the pressure oscillation by 90° , a condition for standing wave shown in Figure 3.6(a), causes a unsteady heat flux from the heating bands to the passing air and since the unsteady heat flux q' and unsteady p' are almost in the same phase with a slightly difference at 7° .

Figure 3.6 (a) unsteady pressure and velocity over time; (b) unsteady pressure and heat flux over time

Figure 3.7 Mode shape along tube axis: (a) in mother and short tube; (b) in mother tube and long branch tube

In Figure 3.7, the mode shape along each tube axis is calculated from the simulation and the tube axis is normalized by the total length of mother tube axis x_1 and long branch tube axis x_2 . Both the mode shape from mother tube to short branch (Figure 3.7 (a)) and from mother tube to long tube are all full length mode. This confirms that the frequency spectrum analysis which shows the acoustic oscillation is dominated by the first frequency mode at this open-end configuration. Besides, the numerical results present a half-length mode and three-quarter length mode in short tube and long tube respectively, again agreeing well in our experiments shown in Figure 3.7.

Figure 3.8 Temperature contour and velocity vector of steady solution for closed-inlet and open-outlet

3.3.2 Closed-Inlet and Open-Outlet

As demonstrated and validated by the experimental results in previous sections, the numerical simulation can predict the combustion instabilities in Rijke-Zhao tube very well. In this section, we will discuss effect of forcing flow at inlet on the acoustic instabilities and although the experiments do not provide data for such situation, we can obtain significant insight from the numerical simulations. Unlike traditional Rijke-tube which has just one inlet and one outlet, the multiple ends configuration grants a multiple inputs combustor system. Here, we set the mother tube and short branch end as velocity inlet at 0.1 m/s and 0.05 m/s respectively at a Reynolds Number smaller than 1000 and

forms an acoustically closed boundary condition at each end. For the long branch tube end, the pressure outlet at the ambient pressure is remained to keep it open. The surface temperature for the heating bands is set as 1000 K. For steady calculation, the second order scheme for pressure interpolation and spacial discretization is chosen and PISO is used for pressure-velocity coupling. The air is considered as ideal gas, while the thermal conductivity is chosen as 0.242, 10 times larger than the default value in order to use coarse mesh and reduce computational cost[44] and other value of air properties keep constant. In Figure 3.9 showing temperature distribution and flow direction, the cold air is sucked from both inlets, where the cold air in mother tube is heated and accelerated when passing the heating bands and then mixing with the cold air from the short tube inlet. As shown in the Figure, this multiple input combustor system enables cooling the hot air and control of temperature downstream of the heating band by adjusting the velocity of the forcing flow at the short tube end, which makes this new designed combustor more attractive and grants more potential uses.

Figure 3.9 (a) acoustic signal at position of microphone 1 and (b) frequency spectrum of limit-cycle

3.3.2.1 Acoustic Characteristics

Based on the steady solution, the unsteady simulation for this closed-inlet and open-outlet configuration is performed to examine the effect of closed end on combustion instabilities. The time step size is chosen as $2e-5$ and second order implicit method is used for time integration. An initial pressure disturbance, $\Delta p' = 100$, is added to the outlet for 10 time steps and kept to 0 after that in order to trigger the oscillation. In Figure 3.9(a) the acoustic signal is displayed in which the oscillation of the pressure is onset after the unsteady calculation started for 2 seconds and reaches the limit-cycle after another 2 seconds. However, different from the open-inlet case, the frequency of the oscillation shown in Figure 3.9 (b) is much larger at 781 Hz. Considering the experimental results which reveal the fundamental mode for the Rijke-Zhao tube is around 175Hz, the frequency of oscillation for closed-inlet is the $4\frac{1}{2}$ mode. Usually in traditional Rijke tube combustor, the higher modes are not easy to go unstable due to larger damping to them, whereas in this new Rijke-Zhao tube with two forcing inlets, the 4th mode is triggered to oscillate and dominate.

In Figure 3.10, the phase diagram between unsteady heat flux and pressure is shown. In Figure 3.10 (a), initially in the transition process, the diagram is disordered and irregular with some disturbance. However, after the onset of oscillations, the diagram gradually becomes an ellipse and after reaching the limit-cycle, the ellipse becomes stable with fixed radius as shown in Figure 3.10 (b) for one cycle in the limit-cycle. Moreover, the phase diagram between unsteady heat flux and pressure also shows that they are within

90° phase difference which again satisfies the Rayleigh criterion for generation of self-excited acoustic instabilities.

Figure 3.10 phase diagram between unsteady heat flux and pressure: (a) over all time for entire process; (b) one cycle after reaching limit cycle.

Figure 3.11 Mode shape along tube axis: (a) mother tube and short branch; (b) mother tube and long branch tube.

In Figure 3.11, the mode shape in each tube is drawn and there is one and half mode shape in mother tube and three times length mode shape in long tube (Figure 3.11 (a)). Therefore, the mother tube and long branch tube in total confirms that the $4\frac{1}{2}$ mode dominates the oscillation in the closed-inlet and open-outlet case and the frequency mode is main dependant on the length of long branch and the mother tube, whereas the length of short branch has less effect on that.

3.4 Summary

The present work preliminarily studies a newly designed combustor originated from Rijke tube. However, distinguishing from the traditional Rijke tube which only has a simple flow field, this Rijke-Zhao structured tube has a special flow field with hot flow going out in long branch tube and ambient air sucked in through the short branch and mother tube when combustion is taken in the mother tube. This special geometry and flow characteristics in the tube can enjoy substantial merits and potential applications in the future. Besides, CFD numerical simulation by using fluent is performed to confirm the flow field tube and capture the combustion instability in the Rijke-Zhao tube. Two cases, the open-inlet and open-outlet and closed-inlet and open-outlet, are studied. In the numerical simulations for both cases, the limit-cycle of self-excited oscillation can be found. Comparing the first case to the experimental result, the frequency and mode shape predicted by the simulation agrees experiment very well, which proves the capability of Fluent to capture the nature of combustion in confined geometry. In the numerical simulation for second case, the higher frequency oscillation is observed. This

interesting result reveals that the multiple input design of Rijke-Zhao tube can trigger higher mode to go unstable even they are usually thought to be prevented by large damping. From the mode shape analysis, the total length of mother tube and long branch tube determines the mode for acoustic oscillations.

Chapter 4

4 Acoustic Damping Effect of Perforated Liner

4.1 Introduction

In order to meet more stringent requirements on emissions, combustors in ground based gas turbines and aero-engines tend to operate under lean premixed conditions to achieve low NO_x. However, under lean premixed conditions, combustors are more sensitive to combustion instabilities[4]. Combustion instabilities are caused by a mutual influence between acoustic waves and unsteady combustion. Unsteady heat release is an efficient acoustic source and generates acoustic waves. These pressure waves travel along the combustor and partially bounce back from boundaries to reach the combustion zone, where they cause more unsteady heat release. Under certain circumstances, this feedback mechanism can generate large and damaging self-excited oscillations.

To suppress combustion instabilities, the coupling between unsteady heat release and the pressure perturbation must somehow be interrupted[61]. Perforated liners[14, 16, 62] are widely used as acoustic dampers to dissipate acoustic waves. They are usually metal sheets, frequently arranged in layers, which have tiny perforated holes in them. In practice, a cooling air flow through the perforated holes (i.e. a bias flow) is needed to

prevent the liners from being damaged under high temperature. Remarkably, the bias flow is found to increase liners damping performance. The main mechanism of acoustic damping involves vortex shedding generated over the rims of the perforated holes.

The majority of modeling work of perforated liners has been carried out in frequency domain. Howe[12] modeled the acoustic energy dissipated by the periodic shedding of vorticity for a single orifice in a high Reynolds-number flow using Rayleigh conductivity. Following Howe's research, Hughes and Dowling[13] studied the acoustic damping of plates with uniform lines of slits and circular holes with mean bias flow, showing that all impinging sound could be absorbed at a particular frequency in theory. Jing & Sun[14] experimentally investigated the effect of the screen thickness and the bias flow rate, showing that an appropriate bias flow rate can significantly increase damping and that the screen thickness is crucial. Eldredge and Dowling[16] recently developed a 1D duct model in frequency domain to simulate the absorption of axial plane wave by a double-liner with a bias flow. The damping mechanism of vortex shedding was embodied using a homogeneous liner compliance adapted from the Rayleigh conductivity. In addition, the effect of liners' thickness was considered by the compliance model. Their numerical results are found to agree very well with their experimental ones.

In comparison with frequency-domain modeling, time-domain modeling is more challenging.

This is most likely due to the fact that either the perforated holes are too tiny to accurately simulate or higher computational cost is involved with the studies of entire liners. However, when air flow through perforated holes do not interact significantly, a single hole with its vicinity can be analyzed, which behaves like a Helmholtz resonator[63]. To investigate the acoustic damping mechanism of perforated liners, Tam et al[31] carried out direct numerical simulation (DNS) of a single aperture. It is showed that vortex shedding is the dominant mechanism of absorption for incident waves of high amplitude. Recently, Mendez & Eldredge[64] conducted compressible large-eddy simulations in time domain to study the flow through a single perforated hole or multiple holes. Reichert & Biringen[29] proposed a time-domain approach which introduced a source term to the momentum equations, and assessed the effect of a bias flow numerically. They found that with an optimum bias flow rate, the liners damping efficiency was significantly improved over that with no bias flow. Sbardella et al[30] proposed a liner model, which combines a frequency independent resistive part and a reactive part. The liner model is then coupled with the acoustic wave model in the duct. Ozyoruk et al[26] proposed a time-domain impedance to describe the damping of a constant depth ceramic tubular liner, which was attached to the NASA Langley flow-impedance tube. The time-domain impedance was converted from the frequency-domain one[27] by using z-transform method. Excellent agreement between their numerical results and the experimental data was observed. They also found that applying z-transform technique resulted in much less computational cost.

In this work, a 1D numerical time-domain model was developed to simulate the acoustic waves propagation and dissipation along a cylindrical lined duct. To characterize the liner's damping, a time-domain liner model based on the homogeneous compliance model derived by Eldredge & Dowling[16] was developed. The compliance model is associated with frequency-domain Rayleigh conductivity[12]. It was firstly formulated in a simplified rational form in z-domain, and then converted into the time-domain one. For different flow conditions, curve-fitting was involved in estimating the compliance. To develop a more general time-domain compliance model, a simplified Rayleigh conductivity as proposed by Luong et al[65] was used to replace the unsimplified one in our compliance model. In Sect.4.2, the model equations of a cylindrical lined duct with a single-liner surrounded by a large annular cavity were developed. Emphasis is being placed on characterizing the liners damping over a broad frequency range by simply setting the incident wave consisting of multiple tones (non-harmonic). The numerical results by using either the simplified compliance model or unsimplified one were then compared with those from the frequency-domain model of Eldredge & Dowling[16]. To solve the partial differential equations (PDEs) of the system, method of lines (MOL) [66, 67] was applied. Further validation of the model was conducted by investigating the acoustic damping of a lined duct with a double-liner encased by a large annular cavity, as typically found within a combustion system. This is described in Sect.4.3. Comparison was then made between the numerical results and the experimental ones³.

Finally, in Sect. 4.4, transmission loss analysis of perforated liners is conducted.

4.2 Model of a Lined Duct with a Single-Layer Liner

The acoustic damping of perforated liners is generally characterized by power absorption, which describes the fraction of incident waves being absorbed. Following the work of Eldredge & Dowling³, power absorption is defined as

$$\Delta = 1 - \frac{\|H_d'^+(L, t)\|^2 + \|H_u'^-(0, t)\|^2}{\|H_u'^+(0, t)\|^2 + \|H_d'^-(L, t)\|^2} \quad (4.1)$$

where $H_u'^+(0, t)$ and $H_u'^-(0, t)$ denote the incident and reflected enthalpy fluctuations upstream the lined section at $x = 0$. $H_d'^+(L, t)$ and $H_d'^-(L, t)$ are downstream ones at $x = L$, as shown schematically in Fig. 4.1. A single-layer liner was encased in a large annular cavity formed by a large rigid cylinder and attached to a cylindrical duct. An imposed mean flow $\bar{u}_u$ pass through the upstream duct. A steady supply of air is fed into the cavity to produce bias flow through the liners, and $\bar{v}_1$ denote the homogeneous bias flow velocity through the liner. The geometry of the lined duct is summarized in table I. Here L is the liner length, D_1 , C_1 , A_1 and σ_1 are diameters, circumferences, cross-sectional areas and open-area ratio of the liner respectively. $\bar{M}_{h1}$ is the bias flow Mach number through the liner holes.

Figure 4.1 Schematic of acoustic waves and flow quantities in a cylindrical duct with a single liner attached.

4.2.1 Governing Equations

According to Eq. (4.1), acoustic plane waves either side of the lined section need to be characterized so that the power absorption is estimated. This means that the lined duct can be divided into 3 regions (i.e. upstream $x \leq 0$, lined segment $0 < x < L$ and downstream region $x \geq 0$), and they need to be coupled in terms of flow properties. Thus the stagnation enthalpy and velocity perturbations need to match at $x = 0$ and $x = L$:

$$H'(t, 0) = H_u'^+(t, 0) + H_u'^-(t, 0), \quad u'(t, 0) = \frac{H_u'^+(t, 0)}{\bar{c} + \bar{u}(0)} - \frac{H_u'^-(t, 0)}{\bar{c} - \bar{u}(0)} \quad (4.2a)$$

$$H'(t, L) = H_d'^+(t, 0) + H_d'^-(t, 0), \quad u'(t, L) = \frac{H_d'^+(t, L)}{\bar{c} + \bar{u}(L)} - \frac{H_d'^-(t, L)}{\bar{c} - \bar{u}(L)} \quad (4.2b)$$

where the fluctuating stagnation enthalpy, $H'(t, 0) = p'(t, x)/\bar{\rho} + \bar{u}(x)u'(t, x)$. p' is the fluctuating pressure and u' is the fluctuating axial velocity. In the lined region $0 < x < L$, the instantaneous flow properties consisting of both mean and fluctuating parts are given by

$$\begin{aligned} H'(t, L) &= \bar{H}(x) + H'(t, x), \\ u(t, x) &= \bar{u}(x) + u'(t, x), \\ \rho(x, t) &= \bar{\rho}(x) + \rho'(x, t) \end{aligned} \quad (4.3)$$

The axial mean flow velocity $\bar{u}(x)$ in the duct is varied with along the liner due to the presence of bias flow, and can be expressed as

$$\bar{\rho}A_1\bar{u}_1(x) = \bar{\rho}A_1\bar{u}_u + \bar{\rho}C_1\bar{v}_1x \quad (4.4)$$

where $\bar{v}_1$ the mean bias flow velocity through the liner and $\bar{\rho}$ is the mean density. Since both the mean axial pipe and bias flow are small with the same stagnation temperature

associated with them, the mean density $\bar{\rho}$ and mean sound speed $\bar{c}$ is uniform in the duct.

In addition, the density fluctuation ρ' can be written in terms of H' as

$$\rho'(t, x) = \frac{\bar{\rho}H'(t, x)}{\bar{c}^2} - \frac{\bar{\rho}\bar{u}u'(t, x)}{\bar{c}^2} \quad (4.5)$$

The equation of mass conservation over an annular control volume of length Δx in the lined section is

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = \frac{C_1}{A_1} \rho v_1 \quad (4.6)$$

where ρ , u and v are the instantaneous density, axial flow velocity and bias flow velocity respectively. The motion equation of the air flow through the duct, i.e. the momentum equation, can be written as:

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = 0 \quad (4.7)$$

where p is the instantaneous pressure.

Linearizing Eqs. (4.6) and (4.7) gives

$$\frac{\partial \rho'(t, x)}{\partial t} + \bar{\rho} \frac{\partial u'(t, x)}{\partial x} + \bar{u}(x) \frac{\partial p'(t, x)}{\partial x} = \frac{C_1}{A_1} \bar{\rho} v_1'(t, x) \quad (4.8)$$

$$\bar{\rho} \frac{\partial u'(t, x)}{\partial t} + \bar{\rho} \bar{u}(x) \frac{\partial u'(t, x)}{\partial x} + \frac{\partial p'(t, x)}{\partial x} = 0 \quad (4.9)$$

Substituting Eq. (4.5) into Eqs. (4.8) and (4.9) gives

$$\frac{\partial H'(t, x)}{\partial t} - \bar{u}(x) \frac{\partial u'(t, x)}{\partial t} + (\bar{c}^2 - \bar{u}(x)^2) \frac{\partial u'(t, x)}{\partial x} + \bar{u}(x) \frac{\partial H'(t, x)}{\partial x} = \frac{C_1}{A_1} \bar{c}^2 v_1'(t, x) \quad (4.10)$$

$$\frac{\partial u'(t, x)}{\partial t} + \frac{\partial H'(t, x)}{\partial x} = 0 \quad (4.11)$$

Manipulating Eqs. (4.10) and (4.11) leads to

$$\frac{\partial H'(t,x)}{\partial t} + 2\bar{u}(x) \frac{\partial H'(t,x)}{\partial x} + (\bar{c}^2 - \bar{u}(x)^2) \frac{\partial u'(t,x)}{\partial x} = \frac{C_1}{A_1} \bar{c}^2 v_1'(t,x) \quad (4.12)$$

If we decompose the stagnation enthalpy fluctuation $H'(t, x)$ in the lined section into the incident ζ^+ and reflected one [16] ζ^- , then they can be shown as

$$\zeta^+(t, x) = \frac{1}{2} [H'(t, x) + \bar{c}u'(t, x) + H'(t, x)\bar{M}(x) - \bar{c}u'(t, x)\bar{M}^2(x)] \quad (4.13a)$$

$$\zeta^-(t, x) = \frac{1}{2} [H'(t, x) - \bar{c}u'(t, x) - H'(t, x)\bar{M}(x) + \bar{c}u'(t, x)\bar{M}^2(x)] \quad (4.13b)$$

Note that $H'(t, x) = \zeta^+(t, x) + \zeta^-(t, x)$. $\bar{M}(x) = \bar{u}(x)/\bar{c}$ is the Mach number of axial mean flow through the duct.

Substituting Eqs. (4.13a) and (4.13b) into Eq. (4.12) gives rise to two coupled PDEs as

$$\frac{\partial \zeta^+(t, x)}{\partial t} = -[\bar{c} + \bar{u}(x)] \frac{\partial \zeta^+(t, x)}{\partial x} + \frac{C_1}{2A_1} \bar{c}[\bar{c} + \bar{u}(x)]v_1'(t, x) \quad (4.14a)$$

$$\frac{\partial \zeta^-(t, x)}{\partial t} = [\bar{c} - \bar{u}(x)] \frac{\partial \zeta^-(t, x)}{\partial x} + \frac{C_1}{2A_1} \bar{c}[\bar{c} - \bar{u}(x)]v_1'(t, x) \quad (4.14b)$$

The boundary values of ζ^+ and ζ^- at $x = 0$ and $x = L$ are

$$\begin{aligned} \zeta^+(0, t) &= H_u'^+(0, t), \zeta^-(0, t) = H_u'^-(0, t), \\ \zeta^+(L, t) &= H_d'^+(L, t), \zeta^-(L, t) = H_d'^-(L, t) \end{aligned} \quad (4.15)$$

Note that Eqs. (4.14a), (4.14b) and (4.15) are corresponding to the frequency-domain Eqs.(2.14) and (2.15)[16]. It can be seen that the time evolution of acoustics waves in the lined section can be determined by solving Eqs. (4.14a) and (4.14b) with the boundary values and the bias flow $v_1'(t, x)$.

Following the work of Eldredge & Dowling[16], we assume $H_u'^+(0, t)$ is known. In addition as described in Eq. (2.16b) in [16], the incident wave $H_d'^+(L, t)$ is coupled to reflected one $H_d'^-(L, t)$ via a downstream reflection coefficient $R_d(t)$ as

$$\hat{H}_d^-(L, \omega) = \hat{R}_d(L, \omega) \hat{H}_d^+(L, \omega) \quad (4.16)$$

where $\hat{R}_d$, $\hat{H}_d^-(L, \omega)$ and $\hat{H}_d^+$ are Fourier transform of $R_d(t)$, $H_d'^-(L, t)$ and $H_d'^+(L, t)$ respectively. The bias flow through the liner $v_1'(t, x)$ is shown to related to the difference of stagnation enthalpy fluctuations across the liner through a homogeneous compliance[16] as

$$\hat{v}_1(\omega, x) = \hat{Y}_1(\omega) [\hat{\zeta}^+(\omega, x) + \hat{\zeta}^-(\omega, x) - \hat{H}_1(\omega, x)] \quad (4.17)$$

where $\hat{v}_1(\omega, x)$ denotes Fourier transform of $v_1'(t, x)$. $\hat{Y}_1(\omega)$ is the frequency-domain compliance and is given as

$$\hat{Y}_1(\omega) = \frac{-1}{j\omega} \frac{\sigma_1 K_a}{(\pi \mathcal{R}_1^2 + \epsilon_1 K_a)} \quad (4.18)$$

where $\mathcal{R}_1$ is the radius of the perforated aperture, σ_1 and ϵ_1 are the open area ratio and thickness of the liner. K_a is the Rayleigh conductivity, which is a function of Strouhal number, $S_t = \omega \mathcal{R}_1 \sigma_1 / \bar{v}_1$, as derived by Howe[12] (See Appendix A).

Eq.(4.17) indicates that the following equation in z-domain holds

$$v_1'(z, x) = Y_1(z) [H'(z, x) - H_1(z, x)] \quad (4.19)$$

where $Y_1(z)$ is the compliance in z-domain, which is given in a general form of

$$Y_1(z) = \frac{a_0 + \sum_{l=1}^h a_l z^{-l}}{1 - \sum_{i=1}^m b_i z^{-i}} \quad (4.20)$$

Here $a's$ and $b's$ are real constants. Substituting Eq. (4.20) into Eq. (4.19), taking the inverse z-transform and using the shift property, a time evolution description of v_1' can be shown as

$$v_1'^{(n)}(x) = \sum_{i=1}^m b_i v_1'^{(n-i)} + a_0 (H'^{(n)}(x) - H_1'^{(n)}(x)) + \sum_{l=1}^h a_l (H'^{(n-l)}(x) - H_1'^{(n-l)}(x)) \quad (4.21)$$

The superscript n denotes the n^{th} time-step. It is obvious that the local relationship between v_1' and the difference in stagnation enthalpy fluctuations across the liner, as given in Eq. (4.21) enables the system equations ready to solve. However, the z-domain compliance involves many unknown constants $a's$ and $b's$, which might be associated a huge computational cost. Furthermore, to ensure convergence and stability, the poles of the compliance should stay inside the unit circle. These requirements limit the application of such compliance formulation.

4.2.2 Low-Pass-Filter Type Compliance Model

In order to achieve robustness and low-cost computation, we propose a simple but robust low-pass-filter type compliance in z-domain as

$$Y_1^s(z) = \frac{a_0}{1 - b_1 z^{-1}} \quad (4.22)$$

where a_0 and b_1 are the parameters to be estimated. The local relationship between v_1' and the difference in stagnation enthalpy fluctuations across the liner is now given as simply as

$$v_1'^{(n)}(x) = b_1 v_1'^{(n-1)}(x) + a_0 [H'^{(n)}(x) - H_1'^{(n)}]$$

(4.23)

It is important to note that using the proposed compliance is associated much less computational cost than using the full general form of Eq. (4.21) or applying inverse Fourier transform directly to Eq. (4.18).

4.2.2.1 Compliance Model with Rayleigh Conductivity

Frequency-domain compliance in corresponding to Eq. (4.22) can be shown as

$$\hat{Y}_1^s(\omega) = \frac{1}{\mathcal{P}_0 + j\omega Q_1} = \frac{\mathcal{P}_0}{\mathcal{P}_0^2 + \omega^2 Q_1^2} - \frac{j\omega Q_1}{\mathcal{P}_0^2 + \omega^2 Q_1^2} \quad (4.24)$$

where $\mathcal{P}_0$ and Q_1 are real constant, which can be estimated by equating Eq. (4.18) and (4.24). A nonlinear least square (NLS) curve fitting procedure based on Levenberg-Marquardt method[68] is used to match $\hat{Y}_1^s(\omega)$ with $\hat{Y}_1(\omega)$ for different frequency and mean bias flow velocity. Fig. 4.2 shows the curve fitting results of $\hat{Y}_1^s(\omega)$ and $\hat{Y}_1(\omega)$ under two different flow conditions. Excellent curve fitting is observed.

With the curve fitting, $\mathcal{P}_0$ and Q_1 can be estimated and so a_0 and b_1 in Eq. (4.23) as given as

$$a_0 = \frac{\Delta t}{\mathcal{P}_0 \Delta t + Q_1}, \quad b_1 = \frac{Q_1}{\mathcal{P}_0 \Delta t + Q_1} \quad (4.25)$$

Thus, Eq. (4.23) provides a simple and explicit expression between the bias flow v_1' and the difference in stagnation enthalpy fluctuations across the liner. Since the liner is encased by a large annular cavity, the stagnation enthalpy fluctuation inside the large cavity is approximately zero, i.e. $H_1'(t, x) = 0$.

Figure 4.2 The real and imaginary parts of frequency-domain compliance $\hat{Y}_1(\omega)$ with simplified

and unsimplified Rayleigh conductivity implemented: (a) and (b) $\bar{v}_{h,1} = 3.09m/s$, (c) and (d)

$\bar{v}_{h,1} = 10.26m/s$.

4.2.2.2 Compliance Model with Simplified Rayleigh Conductivity

As illustrated in previous section, curve-fitting is needed to estimate the compliance components, i.e. a_0 and b_1 , which are related to Rayleigh conductivity[12]. When different flow conditions are considered, this curve-fitting needs to be repeated, which is time-consuming and not economical in terms of computation. In order to develop a

more general compliance model, a simplified Rayleigh conductivity as proposed by Luong et al[65] is then used to replace the unsimplified one[12, 16] in the compliance model, i.e. Eq. (4.24). The simplified Rayleigh conductivity K_a^s is derived by applying the Cummings equation[69] to a perforated aperture with bias flow, and solving the linearized form of this modified Cummings equation. And it is given as

$$K_a^s = 2\mathcal{R}_i \left(\frac{\omega \mathcal{R}_i / \bar{v}_{hi}}{\omega \mathcal{R}_i / \bar{v}_{hi} - 2j/\pi \mathcal{X}^2} \right) \quad (4.26)$$

where $\mathcal{R}_i$ denotes the aperture radius. Subscript i denotes inner or outer liner. $\bar{v}_{hi}$ is the aperture mean velocity, which is related to the mean flow through the inner or outer liner as $\bar{v}_{hi} = \bar{v}_i / \sigma_i$. The contraction ratio $\mathcal{X}$ is set to 0.71. Substituting Eq. (4.26) into Eq. (4.18) and recasting into Eq. (4.24) leads to

$$\mathcal{P}_0 = -\frac{\bar{v}_{hi}}{\sigma_i \mathcal{X}^2} \quad (4.27a)$$

$$\mathcal{Q}_1 = -\frac{(\epsilon_1 + \pi \mathcal{R}_i / 2)}{\sigma_i} \quad (4.27b)$$

Now the compliance components a_0 and b_1 can be analytically determined by using Eq.(4.25). In order to evaluate its performance, such compliance model with simplified Rayleigh conductivity is compared with unsimplified one, i.e. Eq.(4.18) and shown in Fig. 4.2. Excellent agreement between the simplified and unsimplified compliance models is observed. Further evaluation is conducted later, when these compliance models are combined with the lined duct model.

4.2.3 Time-Domain Downstream Reflection Coefficient

In the boundary condition Eq.(4.16), the reflection coefficient, R_d , appears and we now discuss its form in time domain. In the downstream region of the duct, $H_d'^+$ propagate

along the duct to the open end and partially reflects back due to the boundary condition change. The reflected wave $H_d'^{-}$ is related to $H_d'^{+}$ via R_d , and it depends on the duct specific geometry and mean flow condition. In the case of semi-infinite downstream section, no acoustic waves are reflected and $R_d = 0$. For a finite-length pipe with an unflanged open end, which is more likely to be encountered in practice, many research works have been carried out to determine the reflection coefficient in frequency domain. Levine & Schwinger[21] provided an exact analytical solution of the reflection coefficient using the Wiener-Hopf method for a duct with no mean flow. Based on this solution, Eldredge & Dowling[16] provided a second-order reflection coefficient for low-frequency plane waves as

$$\hat{R}_d(\omega) = -\left(1 - \frac{1}{2}(kR^2)\right) \exp(-j2k(L + L_d + \mu)) \quad (4.28)$$

where R is the duct radius, $k = \omega/\bar{c}$ is the wave number, μ is the end correction $\mu = 0.6133R$, L_d is the length of downstream pipe. The corresponding time-domain reflection coefficient can be calculated by taking IFT to Eq. (4.28). Silva et al[36] also proposed to approximate the reflection coefficient by using

$$\hat{R}_d(\omega) = -\frac{1 - \nu jkR}{1 - d_1 jkR + d_2 (jkR)^2} \exp(-j2k(L + L_d + \mu_0)) \quad (4.29)$$

where ν , d_1 and d_2 are real constant parameters given in Table I and $\mu_0 = 0.0779$. This expression is similar to Eq. (4.28). However an explicit form of $R_d(t)$ can be derived by using the residue method[70], as given as

$$R_d(t) = \begin{cases} \frac{\bar{c}}{d_2 R(\alpha_1 - \alpha_2)} [(v\alpha_2 - 1) \exp\left(\frac{-\alpha_2 \bar{c}(t - \mathcal{T})}{R}\right) - (v\alpha_1 - 1) \exp\left(\frac{-\alpha_1 \bar{c}(t - \mathcal{T})}{R}\right)] & t \geq \mathcal{T} \\ 0 & t < \mathcal{T} \end{cases} \quad (4.30)$$

where $\alpha_{1,2} = (1/2d_2)(d_1 \pm \sqrt{d_1^2 - 4d_2})$ and $\mathcal{T} = 2(L + L_d + \mu_0)/\bar{c}$. Before we choose Eq. (4.28) or Eq. (4.29) to couple $H_d'^+(t, L)$ and $H_d'^-(t, L)$, it is necessary to see their differences. Fig. 4.3 shows the comparison of the reflection coefficients estimated from Eq. (4.28) and Eq. (4.29). It can be seen that the agreement is excellent. This means either model can be used. However, for convenience, we coupled $\zeta^-(t, L)$ with $\zeta^+(t, L)$ via Eq. (4.30) as

$$\zeta^-(t, L) = R_d(t) * \zeta^+(t, L) = \int_0^t R_d(\tau) \zeta^+(t - \tau, L) d\tau = \Delta t \sum_{m=0}^{\mathcal{M}} R_d(m\Delta t) \zeta^+((\mathcal{M} - m)\Delta t, L) \quad (4.31)$$

In our work, $\mathcal{M}$ is set to 3×10^4 with a sampling rate of 1.0×10^5 Hz.

Figure 4.3 The real (a) and imaginary (b) part of downstream reflection coefficient $\hat{R}_d$.

4.2.4 Governing Equations in Discrete Time

Forgoing analysis shows that the governing equations for the lined duct are PDEs. To solve these equations, method of lines (MOL)[66] is used. It converts the PDEs into ODEs by discretizing them in all but one dimension and leaving time variables continuous. The spatial derivatives are always approximated algebraically by finite differences (FDs). Therefore, there is only one independent variable, time, remaining and resulting in an initial-value integration problem, which can be easily solved by well-established standard routines or ODE solvers.

In the lined section $0 < x < L$, the discrete-time form of Eqs. (4.14a), (4.14b) and (4.23) are given as

$$\frac{\partial \zeta_j^{+(n)}}{\partial t} = -(\bar{c} + \bar{u}_j) \frac{\zeta_{j+1}^{+(n-1)} - \zeta_{j-1}^{+(n-1)}}{2\Delta x} + \frac{C_1}{2A_1} \bar{c}(\bar{c} + \bar{u}_j) v_{1,j}'^{(n-1)} \quad (4.32a)$$

$$\frac{\partial \zeta_j^{-(n)}}{\partial t} = (\bar{c} - \bar{u}_j) \frac{\zeta_{j+1}^{-(n-1)} - \zeta_{j-1}^{-(n-1)}}{2\Delta x} + \frac{C_1}{2A_1} \bar{c}(\bar{c} - \bar{u}_j) v_{1,j}'^{(n-1)} \quad (4.32b)$$

$$\frac{\partial v_{1,j}'^{(n)}}{\partial t} = \frac{v_{1,j}'^{(n)} - v_{1,j}'^{(n-1)}}{\Delta t} = \frac{b_1 - 1}{\Delta t} v_{1,j}'^{(n-1)}(x) + \frac{a_0}{\Delta t} (\zeta_j^{+(n)} + \zeta_j^{-(n)} - H_{1,j}'^{(n)}) \quad (4.32c)$$

In our calculation, the flow properties are evaluated at $N = 55$ locations across the linedsection spacing (i.e. $\Delta x = L/(N - 1)$). And the time-step increment is sufficiently small, $\Delta t = \Delta x/2\bar{c}$ to satisfy the requirement of Courant-Friedrichs-Lewy number. In addition, the spatial derivatives are discretized by using a three-point centered difference approximation, which results in second-order accuracy. Moreover, the spatial derivative of a given flow variable at the boundaries $x = 0$ and $x = L$, are approximated as

$$\frac{\partial \phi_1^{(n-1)}}{\partial x} = \frac{-3\phi_1^{(n-1)} + 4\phi_2^{(n-1)} - \phi_3^{(n-1)}}{2\Delta x} \quad (4.33a)$$

$$\frac{\partial \phi_N^{(n-1)}}{\partial x} = \frac{3\phi_N^{(n-1)} - 4\phi_{N-2}^{(n-1)} + \phi_{N-3}^{(n-1)}}{2\Delta x} \quad (4.33b)$$

where ϕ represents the flow variable. Now Eqs.(4.32a-4.32c) can be systematically solved for the cylindrical lined duct with a single-layer liner attached.

4.2.5 Numerical Results

Since our numerical simulation is conducted in time-domain, we can assume that the incident wave consists of multiple tones with unit magnitude

$$H_u'^+(0, t) = \sum_{i=1}^{\mathcal{N}} 1.0 \cos(\omega_i t) \quad (4.34)$$

where ω_i is the tone frequency, and $\mathcal{N}$ is an integer and denotes the tones number. By choosing

Figure 4.4 Time evolution of the acoustic waves consisting of 70 different tones in the lined duct with a single-liner, as $M_{h,1} = 0.023$ and $M_u = 0.0$.

a large $\mathcal{N}$, the liners' damping can be estimated over a broad frequency range all at once. This is different from the frequency-domain models [13, 16], which calculate the liners' damping one frequency at a time.

Figure 4.5 Frequency spectrum of $\hat{H}_u^+(0, \omega)$.

Fig. 4.4 shows the time evolution of incident and reflected enthalpy waves at the inlet and outlet of the lined section of the cylindrical duct with a single-liner attached, as $M_{h,1} = 0.023$, $M_u = 0.0$ and $\mathcal{N} = 70$. The frequencies of the tones are illustrated in the frequency spectrum of $H_u'^+(0, t)$ in Fig. 4.5.

The liner's damping characterized by using power absorption is shown in Fig. 4.6(a). Comparison is then made between the results from the compliance models with

simplified or unsimplified Rayleigh conductivity implemented, and those from the frequency-model of Eldredge & Dowling[16]. Good agreement is observed. In order to evaluate the models performance, it is then applied to the lined duct under another 3 different flow conditions. Fig. 4.6 (b) show the variation of power absorption with the forcing frequency, as the bias flow Mach number is increased to $M_{h,1} = 0.041$ but M_u remains 0. It can be seen that from 6(a) and (b) that the general trend of the maximum damping is to increase with increased frequency. In addition, as the bias flow Mach number is increased from 0.023 to 0.041, the maximum power absorption is increased slightly.

Figure 4.6 The variation of power absorption with forcing frequency in a lined duct with a single-liner for: (a) $M_{h,1} = 0.023$ and $M_u = 0.0$; (b) $M_{h,1} = 0.041$ and $M_u = 0.0$; (c) $M_{h,1} = 0.03$ and $M_u = 0.046$; (d) $M_{h,1} = 0.009$ and $M_u = 0.057$.

Fig. 4.6(c) and (d) show the variation of the liner's damping with forcing frequency, as there is a mean axial flow through the duct, i.e. $M_u \neq 0$. The maximum power absorption has a similar trend as the case of $M_u = 0$, as shown in Fig. 4.6(a) and (b). However, as the bias flow $M_{h,1}$ is reduced from 0.03 to 0.009, the liner's damping is dramatically reduced. This reveals that the bias flow plays an important role in improving liner's damping. In general, our model, especially the one with simplified Rayleigh conductivity implemented, was shown to be able to simulate the acoustic damping of a lined duct with a single-liner attached.

4.3 Model of a Lined Duct with a Double-Layer Liner

4.3.1 Description of the Lined Duct with a Double-Liner

Figure 4.7 Schematic of acoustic waves and flow quantities in a cylindrical duct with a double- liner attached.

In order to further validate our model, numerical investigation of a cylindrical lined duct with a double-liner attached is conducted. The lined duct configuration is very similar to the single-liner one, as shown schematically in Fig. 4.7. We assign circumference C_2 , compliance $Y_2(z)$ cross-sectional area A_2 and open area ratio σ_2 to the outer liner. Since the cavity/region between inner liner and outer liner is sufficiently shallow, the acoustic equations in this region are 1D, as in the duct. Thus mass and momentum conservation equations through an annular control volume of length Δx hold as

$$\frac{\partial H'_1(t, x)}{\partial t} = -\bar{c}^2 \frac{\partial u'_1(t, x)}{\partial x} - \bar{c}^2 \frac{C_1}{A_1} v'_1(t, x) + \bar{c}^2 \frac{C_2}{A_2} v'_2(t, x) \quad (4.35)$$

$$\frac{\partial u'_1(t, x)}{\partial t} + \frac{\partial H'_1(t, x)}{\partial x} = 0 \quad (4.36)$$

where $v'_2(t, x)$ is the inward fluctuating flow through the outer liner, related to the stagnation enthalpy difference across the outer liner by

$$v'_2{}^{(n)}(x) = \mathcal{B}_1 v'^{(n-1)}_2(x) + \mathcal{A}_0 H'^{(n)}_1(x) - H'^{(n)}_2 \quad (4.37)$$

where $\mathcal{A}_0$ and $\mathcal{B}_1$ are coefficients of the z-domain compliance for outer liner $Y_2(z)$, which is given as

$$Y_2(z) = \frac{\mathcal{A}_0}{1 - \mathcal{B}_1 z^{-1}} \quad (4.38)$$

It is worth noting that Eqs. (4.35) and (4.36) are corresponding to the frequency-domain ones as given in Eq. (2.21) of Eldridge & Dowling[16]. Since the outer liner in this configuration is encased by a larger annular cavity. This indicates that $H'_2(t, x) = 0$. In addition, the presence of the rigid wall at the each end of the lined section gives rise to $u'_1(t, 0) = 0$ and $u'_1(t, L) = 0$. H'_1 can be computed in the same way as Eqs. (4.10) and (4.11) are obtained.

The discrete-time forms of Eqs. (4.35)-(4.37) can be shown as

$$\frac{\partial H'_{1,j}}{\partial t} = -\bar{c}^2 \frac{u'_{1,j+1} - u'_{1,j-1}}{2\Delta x} - \bar{c}^2 \frac{C_1}{A_1} v'_{1,j}{}^{(n-1)} + \bar{c}^2 \frac{C_2}{A_2} v'_{2,j}{}^{(n-1)} \quad (4.39a)$$

$$\frac{\partial u'_{1,j}{}^{(n)}}{\partial t} = -\frac{H'_{1,j+1}{}^{(n-1)} - H'_{1,j-1}{}^{(n-1)}}{2\Delta x} \quad (4.39b)$$

$$\frac{\partial v'_{2,j}{}^{(n)}}{\partial t} = \frac{v'_{2,j}{}^{(n)} - v'_{2,j}{}^{(n-1)}}{\Delta t} = \frac{\mathcal{B}_1 - 1}{\Delta t} v'_{2,j}{}^{(n-1)}(x) + \frac{\mathcal{A}_0}{\Delta t} (H'_{1,j}{}^{(n)} - H'_{2,j}{}^{(n)}) \quad (4.39c)$$

With the boundary values at $x = 0$ and $x = L$, systematically solving Eqs. (4.32a)-(4.32c) and (4.39a)-(4.39c) yields the time evolution of the acoustic waves and so the acoustic damping of the double-liner.

It is important to note that the mean bias flow velocities through both liners are related due to the mass conservation as shown:

$$\bar{v}_1 = \frac{C_2 \sigma_2}{C_1 \sigma_1} \bar{v}_2 \quad (4.40)$$

where $\bar{v}_1$ and $\bar{v}_2$ are the mean bias flow velocity through inner and outer liner respectively.

4.3.2 Numerical Results

Effect of Forcing Frequency on Liners' Damping

Figs.8 shows the variation of power absorption with the forcing frequency, as $M_u = 0$ and the bias flow Mach number $M_{h,1}$ is set to 2 different values. It can be seen that the acoustic damping of the double-liners varied dramatically with the frequency. The maximum damping reveals that about 82% of incident waves being absorbed, whereas the minimum damping is as little as 10%. Furthermore, with bias flow Mach number increased from 0.023 to 0.041, the damping effect of the liners is slightly reduced in general. The maximum power absorption is reduced from 82% to 73%. Moreover, our results agree very well with the experimental ones from Eldredge & Dowling. This confirms that our model is able to simulate the liners damping in time domain. It is important to note that the biggest discrepancy between the experimental result and our simulation occurs at the frequency, $f \approx 535Hz$. This is most likely due to the fact that a pressure node is located in the lined section and the power absorption of the liners depends strongly on the pressure fluctuations across the liners.

Figure 4.8 The variation of power absorption with forcing frequency in a lined duct with a double-liner for: (a) $M_{h,1} = 0.023$ and $M_u = 0$, (b) $M_{h,1} = 0.041$ and $M_u = 0$.

Fig. 4.9(a) shows the variation of power absorption with forcing frequency, as the mean pipe flow is set to $M_u = 0.046$ and bias flow $M_{h,1} = 0.03$. Similar damping effect is observed as in the absence of mean pipe flow. The maximum power absorption is approximately 80%. Again our results agree favorably with those from Eldredg & Dowling. Fig. 4.9(b) illustrates the liners damping varied with the forcing frequency under the condition of a small bias flow $M_{h,1} = 0.009$ and a large mean pipe flow $M_u = 0.057$. It can be seen that there are several damping peaks over the frequency range of 20 to 700 Hz. However, the maximum power absorption occurs at the 3rd peak, which is different from Fig. 4.8. Moreover, the power absorption peaks become more flattened. This indicates that the frequency range corresponding to the maximum damping is broadened.

Figure 4.9 The variation of power absorption with forcing frequency in a lined duct with a double-liner for: (a) $M_{h,1} = 0.03$ and $M_{h,1} = 0.046$, (b) $M_{h,1} = 0.009$ and $M_u = 0.057$.

Effect of Bias Flow Mach Number on Liners' Damping

The forgoing simulations showed that the model performed well in predicting the absorption of the liners over a range of frequencies, with constant flow conditions. Now we present the results when the frequency remained unchanged, $M_u = 0$, and the mean liner bias flow was varied. The power absorption of the double-liner at four different frequencies is illustrated in Fig 10. It can be seen that the general trend of the power absorption variation at each given frequency is that it increased and reached the maximum and then decreased, as the bias flow is increased. In addition, there is an optimum bias flow Mach number $M_{h,1}$, which gives rise to the maximum liners damping. The model predicts the liner damping well at the lowest frequency, but

performs poorly with increased frequency. This is most likely due to the increased excitation frequency and the presence of pressure nodes at the lined section. Finally, the compliance model with simplified Rayleigh conductivity implemented performs as well as the unsimplified curve-fitting. The difference between the simplified and unsimplified compliance models is negligible.

Figure 4.10 The variation of power absorption with bias flow Mach number at 4 different frequencies, as there is no mean flow through the lined duct with a double-liner (i.e. $M_u = 0.0$).

4.4 Transmission Loss

In some cases, perforated liners are implemented as acoustic dampers to prevent reflected acoustic waves in the pipe/combustor from transmitting back to the noise

source (e.g. flame), thus alleviating the growth of acoustically driven instabilities. For this, the fraction of the acoustic wave energy that is transmitted downstream of the lined section (i.e. the transmission loss, β) can be used as an alternative to access the liners' damping effect. The variation of the transmission loss β , as defined as

$$\beta = 1 - \|H_u'^+(0, t)\|^2 / \|H_d'^-(L, t)\|^2 \quad (4.14)$$

The variation of transmission loss with the forcing frequency for the double-liner is also investigated as shown in Fig. 4.11. It can be seen that there are several transmission loss peaks, which is corresponding to maximum transmission loss of approximately 100%. The local minimum transmission loss is reduced with increased forcing frequency. In general, our model captures the main damping characteristics of the double-liner in terms of transmission loss.

Figure 4.11 The variations of transmission loss with forcing frequency in a lined duct with a double-liner for $M_u = 0.0$.

Furthermore, the compliance model with simplified Rayleigh conductivity implemented performs as well as the unsimplified curve-fitting one with discrepancy less than 1%. This indicates the potential wide application of such time-domain compliance model in simulating liner's damping.

4.5 Summary

Perforated liners are widely applied in aero-engines and gas turbine combustors to suppress combustion instabilities. Before implementing such liners, the acoustic damping of the liners needs to be characterized and estimated in terms of power absorption or transmission loss. For this, we developed a 1D numerical time-domain model to simulate the acoustic plane waves propagation and dissipation in a cylindrical lined duct. It performs in time-domain and involves the development of a compliance model based on the principle that unsteady vortex shedding from the perforated holes is the primary mechanism for acoustic waves damping. The compliance model with Rayleigh conductivity⁶ implemented is formulated in a simplified rational form in z -domain. However, curve fitting is involved with estimating the compliance. For different flow conditions, curve fitting needs to be repeated. These limit the application of such compliance model.

In order to make our model more general and applicable, a simplified compliance formulation is developed. It is based on the simplified Rayleigh conductivity as proposed by Luong et al[65]. Both simplified and unsimplified compliance models are then incorporated into partial differential equations (PDEs) which describe the lined-duct system. To solve the system PDEs, method of lines (MOL) is applied. The

numerical results of two lined duct configurations, single- and double-liner encased in a large cavity, are evaluated and compared with those from the experiment and the frequency-domain model of Eldredge and Dowling[16]. Good agreement is observed. Furthermore, the compliance model with simplified Rayleigh conductivity implemented performs as well as the unsimplified one, with discrepancy around 1%. Since the simplified compliance does not involve curve-fitting, this indicates it has great potential to be applied in time-domain model to simulate liner's damping performance. In general, the numerical model developed is shown to be able to predict the liners' damping in time domain, of which the acoustic waves either side of the lined section need to be decomposed and characterized[71].

It is important to note that the liners' damping is estimated over a broad frequency range all at once in our simulation. This is different from the frequency-domain models, which calculate the liners' damping one frequency at a time. Furthermore, the numerical time-domain liner model can be integrated with other CFD models to simulate the flow features. For example, it can be combined with combustion model of lean premixed pre-vaporized combustors with perforated liners confined to study time evolution of flow disturbances in triggering combustion instability²⁶, and the interaction between flow disturbances and flame. The time-domain model can also be applied in computational aeroacoustics (CAA)[31]. Finally, time-dependent boundary conditions or variables can be easily implemented in our time-domain model. These features indicate that developing such a time-domain liner model is useful for understanding the effect of acoustic liners on the flow field and noise damping.

Chapter 5

5 Conclusions and Future Work

5.1 Conclusions

To understand the mechanism and effect of acoustical inlet on combustion instabilities, a newly designed Rijke-Zhao tube combustor is examined both experimentally and numerically. Like the conventional straight Rijke tube, the Rijke-Zhao tube can generate acoustic sound with large oscillating pressure amplitude of combustion instabilities. However, unlike the Rijke tube, this Rijke-Zhao tube has multiple ends which cause a different flow field when combustion is taken place inside the tube other than a single flow direction inside straight tube. CFD numerical study reveals the first acoustic mode dominates the combustion instabilities for the open inlet at mother and short branch tube end and open outlet at long branch tube end. The numerically predicted frequency and mode shape in the tube matches the experimental results very well. Besides, the simulation for the closed inlet and open outlet shows a higher mode, four and half modes, is triggered and goes unstable in the Rijke-Zhao tube. This is an interesting discovery since higher frequency mode for combustion instabilities is not

usually reported in straight rijke tubes, showing the dual input for the Rijke-Zhao tube can affect the frequency of the combustion instabilities.

In order to suppress the combustion instabilities, a numerical model characterizing the acoustic damping effect of perforated liner is developed. In the present model, the liner compliance originated from Rayleigh conductivity in a form in frequency-domain is converted to time domain and therefore the bias flow condition at the liner can be resolved in time-domain, together with incident and reflected enthalpy wave equations derived from conservation equations of mass and momentum to form a closed system of partial differential equations (PDEs). By using Method of Lines (MOL) which convert the PDEs to DOEs and solving them, the incident and reflected enthalpy waves are obtained at both upstream and downstream of the liner. The damping effect of liner can be determined by comparing the enthalpy waves propagating before and after the liner to calculate the absorption of these waves by perforated liner which in fact dissipates the mechanical energy to thermal energy in vorticities at the rim of its apertures. The predictions of the absorption of acoustic waves by present model for both single-layer liner and double-layer liners are compared to the numerical results of frequency domain and experimental data in existing literature and good agreement is observed, proving the validity and accuracy of the present time-domain model.

5.2 Future Work

Based on the present CFD modeling for combustion instabilities in Rijke-Zhao tube and the time-domain lined duct model, more details of experimental and numerical study

can be implemented to thoroughly understand the effect of multiple forcing inlets on combustion instabilities and improvements and more applications of lined duct can be proposed for future work. Here several suggestions are listed as below:

- Combustion process can be simulated as the heat source instead of the heating bands in the present CFD modeling to examine the difference in combustion instabilities.
- Modified Rijke-Zhao tube such as mother tube splitting into more than two branches can be studied and since the Rijke-Zhao tube can be used to control the downstream temperature after combustion by inputting cool air from a branch end, new acoustic engine might be designed based on it.
- In the present model for liner damping effect, the combustion oscillation is replaced by an artificial cosine wave at upstream because the focus is on the liner characteristics. However, the oscillations can be generated by using a real combustion model and then the present liner model can be combined with it to simulate a more realistic situation in combustors in time-domain. Unlike the frequency-domain, the effect of perforated liner on transition process of combustion instabilities can be captured, therefore better understanding triggering mechanism of combustion instabilities.
- The first order compliance model in the present work can be further improved to higher order model. Although it will increase the computational cost and numerical instabilities, the accuracy of the liner model can be improved. Moreover, the time-domain equation for bias flow injecting via perforated liner

can be incorporated into computational aeroacoustic (CAA) with more applications for perforated liner in time-domain.

APPENDIX A: Rayleigh Conductivity

From Howe[12], Rayleigh conductivity is given as

$$K_a = 2a(\gamma + j\delta) \quad (\text{A1})$$

where γ and σ are given as

$$\gamma = \frac{I_1^2(S_t) \left[1 + \frac{1}{S_t}\right] + \frac{4}{\pi^2} e^{2S_t} \cosh(S_t) K_1^2(S_t) [\cosh(S_t) - \frac{\sinh(S_t)}{S_t}]}{I_1^2(S_t) + \frac{4}{\pi^2} e^{2S_t} \cosh^2(S_t) K_1^2(S_t)}$$

$$\sigma = \frac{[\frac{2}{\pi S_t}] I_1(S_t) K_1(S_t) e^{2S_t}}{I_1^2(S_t) + \frac{4}{\pi^2} e^{2S_t} \cosh^2(S_t) K_1^2(S_t)} \quad (\text{A2})$$

where I_1 and K_1 are modified Bessel functions, S_t is the Strouhal number $S_t = \omega R / \bar{U}_c$.

APPENDIX B: Parameters Used In the Numerical Model

The parameters used in our time-domain simulations are shown in table I.

Table I Parameters used in the numerical model.

Parameter	Value(unit)	Parameter	Value(unit)
A_1	0.0128 (m^2)	R	0.0635 (m)
$\mathcal{R}_1$	0.375×10^{-3} (m)	A_2	0.0181 (m^2)
C_1	0.399 (m)	$\mathcal{R}_2$	1.35×10^{-3} (m)
σ_1	0.04	C_2	0.478 (m)
$\bar{c}$	343 (m/s)	σ_2	0.02
d_1	1.493	d_2	0.457
ϵ_1	3×10^{-3} (m)	ϵ_2	3×10^{-3} (m)
L	0.178 (m)	v	0.167 (m)
L_d	0.8 (m)	μ	0.039 (m)
D_1	0.127 (m)	D_2	0.152 (m)

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