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**NONLINEAR REGRESSION APPROACH FOR
GPS MULTIPATH MITIGATION: FROM CODE
TO CARRIER-PHASE MEASUREMENTS**

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2012

NONLINEAR REGRESSION APPROACH FOR GPS MULTIPATH MITIGATION: FROM CODE TO CARRIER-PHASE MEASUREMENTS

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Table of Contents

List of Figures	vii
------------------------	------------

List of Tables	x
-----------------------	----------

List of Publications	xiii
-----------------------------	-------------

1 Introduction	1
1.1 Global Positioning System Background	1
1.1.1 Space Segment	2
1.1.2 Control Segment	3
1.1.3 User Segment	3
1.2 GPS Observables	5
1.2.1 Pseudorange Observables	5
1.2.2 Carrier Phase Observables	7
1.3 GPS Error Sources	8
1.3.1 Satellite Clock Offset	8
1.3.2 Ephemeris Errors	8
1.3.3 Ionospheric Delay	9
1.3.4 Tropospheric Delay	10
1.3.5 Multipath	10
1.3.6 Receiver Noise	11
1.4 Previous Work	11
1.5 Problem Statement and Objectives	12
1.6 Contributions of the Research Work	12
1.7 Outline	14

2	Survey of GPS Multipath Mitigation Techniques	16
2.1	What is multipath and why is multipath mitigation necessary?	16
2.2	Properties of an Ideal Multipath Mitigation Approach	17
2.3	Multipath Mitigation Approaches in the Literature	19
2.3.1	Pre-receiver Approaches	20
2.3.2	In-receiver Approaches	22
2.3.3	Post-receiver Approaches	24
2.4	Discussion	34
2.4.1	Summary on the Multipath Mitigation Approaches	34
2.5	Conclusions	37
3	Principles of GPS Multipath and Multipath Geometrical Models	38
3.1	GPS Signal Acquisition and Tracking	38
3.2	Signal Tracking under Multipath	40
3.2.1	Code Multipath Error	41
3.2.2	Carrier-Phase Multipath Error	44
3.3	Geometrical Models of GPS Multipath Errors	46
3.4	Conclusions	51
4	Mitigation of Code Multipath with Nonlinear Regression	52
4.1	Introduction	52
4.2	Experimental Data Set	54
4.3	Repeatability of GPS Constellation's Geometry	55
4.4	Repeatability of Pseudorange Multipath Error	56
4.5	Pseudorange Multipath Mitigation with Nonlinear Regression	59
4.5.1	Linear regression problem formulation with ϵ -SVR	59
4.5.2	Nonlinear kernel mapping	63
4.5.3	Learning Pseudorange Multipath Estimators with ϵ -SVR	64
4.6	Experiments and Results	65
4.6.1	Extraction of Code Multipath Error	65
4.6.2	Training Multipath Estimators	67
4.6.3	Multipath Mitigation Performance	68
4.6.4	Discussions	71
4.7	Conclusions	74

5	Mitigation of Carrier-Phase Multipath with Nonlinear Regression	75
5.1	Introduction	75
5.2	Carrier-Phase Differential Combinations	77
5.2.1	Single-Differential Combination	77
5.2.2	Double-Differential Combination	79
5.3	Carrier-Phase Multipath Mitigation Estimation	80
5.4	Experiments and Preliminary Results	81
5.4.1	Experimental Data Set	81
5.4.2	Training Multipath Estimators	82
5.5	Discussion	87
5.6	Conclusions	88
6	Conclusions and Future Work	89
6.1	Conclusions	89
6.2	Future Work	90
6.2.1	Building evaluation framework for multipath mitigation problem	90
6.2.2	Further experiment on carrier-phase multipath mitigation	91
6.2.3	Reduce complexity of solution of regression problem	92
6.2.4	Incremental training for multipath estimators	92
6.2.5	Experiments on data recorded from the field	92
6.2.6	Integration of the new signals	93
	Appendices	94
A	Ionosphere-Free Combinations	94
A.1	Ionosphere-free Combinations	94
A.2	Difference of Ionosphere-free Combinations	95
B	Results of Other Satellites on Code Multipath Mitigation	96
B.1	Repeatability of Satellite's Geometry	96
B.2	Repeatability of Pseudorange Multipath Error	99
B.3	Code Multipath Estimation and Correction	110

Bibliography

110

List of Figures

1.1	GPS consists of three segments: space, control, and user. ¹	2
1.2	GPS receiver categories.	4
2.1	Domain-wise GPS signal processing.	19
2.2	Hierarchy of classification of multipath mitigation approaches.	20
2.3	Placement of electromagnetic fences at a hypothetical site.	21
2.4	Implementation of carrier-smoothing filter.	26
2.5	Adaptive FIR filter configuration.	26
2.6	Code multipath mitigation with FFT.	31
2.7	Wavelet analysis for code multipath mitigation.	32
2.8	Flow diagram for modeling SNR data to yield multipath corrections.	33
3.1	Generic digital GPS receiver block diagram.	39
3.2	Effect of multipath on code correlation function for different scenarios: (a) multipath-free scenario, (b) scenario with one multipath signal in-phase with the direct signal, (c) scenario with one multipath signal out-of-phase with direct signal.	42
3.3	Phasor diagram showing the relationship between I and Q components of the signal and the phase angle ϕ_d during carrier tracking loop operation when one multipath signal presents.	44
3.4	The direct signal from a GPS satellite and one multipath signal with simplified geometry.	47
3.5	Multipath reflections: (a) Forward-scatter, (b) Backscatter.	48
4.1	The observation site on the rooftop of N2 building (NTU campus).	54
4.2	Repeatable geometries of the satellites in the orbital plane B from DOY 306 to DOY 309 of 2010.	56

4.3	Multipath time series of PRN 12 of the plane B.	58
4.4	The soft margin loss setting corresponds for a linear SV machine.	61
4.5	Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 12 of the orbital plane B in DOY 310 of 2010.	69
4.6	Positioning solution on original data, CSF-smoothed data and SVR-corrected data of DOY 310 of 2010.	71
4.7	Simulation of event signal added to pseudorange measurements of PRN 12. It can be seen that the event signal is indeed left intact from correction of the PRN 12's SVR estimator.	73
5.1	Single difference.	78
5.2	Double difference.	79
5.3	IGS Observation Stations: (a) KIRU, (b) KIR0.	83
5.4	KIRU and KIR0 stations in IGS Tracking Network in Europe.	83
5.5	Geometries of PRN 8 and PRN 18 w.r.t. KIR0 and KIRU in DOY 050 and DOY 051 of 2011.	84
5.6	Carrier-phase multipath of DD combination of two satellites PRN 18 and PRN 8 in DOY 050 and 051 of 2011.	85
5.7	Original carrier-phase multipath (blue), response of the trained multipath estimator (red), and corrected carrier-phase multipath (black) sequences of the satellite pair PRN 18 - PRN 8.	86
B.1	Repeatable geometries of the satellites in the orbital plane A from DOY 306 to DOY 309 of 2010.	96
B.2	Repeatable geometries of the satellites in the orbital plane C from DOY 306 to DOY 309 of 2010.	97
B.3	Repeatable geometries of the satellites in the orbital plane D from DOY 306 to DOY 309 of 2010.	97
B.4	Repeatable geometries of the satellites in the orbital plane E from DOY 306 to DOY 309 of 2010.	98
B.5	Repeatable geometries of the satellites in the orbital plane F from DOY 306 to DOY 309 of 2010.	98

B.6	Multipath time series of PRN 9 of the plane A.	100
B.7	Multipath time series of PRN 3 of the plane C.	102
B.8	Multipath time series of PRN 21 of the plane D.	104
B.9	Multipath time series of PRN 10 of the plane E.	106
B.10	Multipath time series of PRN 13 of the plane F.	108
B.11	Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 9 of the orbital plane A in DOY 310 of 2010.	110
B.12	Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 3 of the orbital plane C in DOY 310 of 2010.	111
B.13	Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 21 of the orbital plane D in DOY 310 of 2010.	111
B.14	Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 10 of the orbital plane E in DOY 310 of 2010.	112
B.15	Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 13 of the orbital plane F in DOY 310 of 2010.	112

List of Tables

1.1	A summary of the errors in GPS measurements.	11
1.2	Taxonomy of multipath mitigation approaches.	13
2.1	Summary of multipath mitigation approaches.	35
4.1	GPS orbital plane assignments in 2011.	55
4.2	Normalized cross-correlation of PRN 12's multipath sequences.	57
4.3	Standard deviation (m) of noise before and after correction applied with CSF and SVR estimators.	70
4.4	Standard deviation (m) of coordinate time series.	70
4.5	Performance comparison of code multipath mitigation methods.	71
B.1	Normalized cross-correlation of PRN 9's multipath sequences.	101
B.2	Normalized cross-correlation of PRN 3's multipath sequences.	103
B.3	Normalized cross-correlation of PRN 21's multipath sequences.	105
B.4	Normalized cross-correlation of PRN 10's multipath sequences.	107
B.5	Normalized cross-correlation of PRN 13's multipath sequences.	109

Summary

Multipath, where GPS signals arrive by more than one path and thereby create a range error, has remained the long-lasting major error source in GPS solutions although having been a target of the GPS research community since the day the system started operating. Seeking to understand the GPS multipath errors, this report aims to efficiently mitigate multipath effects at observable levels to improve the accuracy and precision of GPS solutions.

Based on an analysis of the geometry of a multipath signal's reflections, and the principle of GPS receiver's tracking loop, geometrical models of multipath errors are developed at observable levels (both code and carrier-phase multipath). More specifically, multipath errors corresponding to a satellite are mathematically proven to be functions of the satellite's geometry with respect to a receiver which is parameterized by azimuth and elevation angles. Hence, the problem of multipath error estimation amounts to a regression problem where the multipath functions are approximated using training data. Whereas the theory behind regression problem is robust, the regression problem solution would be feasible as long as a priori data (e.g. training data) is available.

As code multipath errors can be easily isolated using a combination of GPS measurements and satellites' geometrical information can be computed using orbital information broadcast from the satellites, training data for the problem code multipath error estimation can be extracted. An experiment was conducted to demonstrate the performance of the proposed method. A real data set was recorded at 0.1 Hz during 5 days to use for the experiment. After training the multipath estimators with data from 31 visible satellites over four days using the ϵ -SVR algorithm, multipath estimation and correction have been performed on the data from the successive day. Results show approximately 80% reduction in term of code multipath error standard deviation, the proposed method shows great promise for understanding and removing multipath errors. Advantageously, while being scalable with data rate, the proposed method is shown to be not harmful to other signals (simulation).

An analogous experiment was also conducted to address carrier-phase multipath error. Unfortunately, for a standalone receiver, carrier-phase multipath error cannot be isolated by a combination of measurements as has been done with code multipath error, a carrier-phase double difference between two short-baselined receivers needs to be made. It results in combinatorial carrier-phase multipath error of 4 satellite-receiver pairs involved in double-difference. Mitigation of this error is beneficial for relative positioning applications. With the same rationale, the multipath error is modeled as a function of the geometries of 4 satellite-receiver pairs. Training the multipath estimation using 1-Hz data from two short-baselined stations of IGS network, preliminary experimental result showed 57% reduction in multipath error for a pair of satellites.

My next target is to unify an evaluation framework for multipath mitigation problems with well-defined tests, and evaluation criteria, as this is missing in the literature. Whereas further experiments on carrier-phase multipath mitigation as well as on data recorded from the field need to be done, there is also still room for improvement in multipath estimation algorithms with the focus on computational complexity reduction and incremental learning ability. Integration of the new signals in GPS modernization plans is also worth exploring.

List of Publications

1. Su-Lim Tan, Chee-Hui Lee, **Quoc-Huy Phan**, “A Comparative Study of Compression Techniques for GPS Data for Tectonic and Volcanic Observation,” in *Proceedings of the IEEE 9th Malaysia International Conference on Communications (MICC) 2009*, Kuala Lumpur, Malaysia, 15-17 December 2009.
2. **Q. H. Phan**, S. L. Tan, “Boosting Carrier Smoothing Performance with High-Rate GPS Data,” in *Proceedings of the AOTULE 2010 Graduate Student Conference*, Bandung, Indonesia, 2-3 November 2010.
3. **Quoc-Huy Phan**, Su-Lim Tan, “Mitigation of GPS Periodic Multipath Using Nonlinear Regression,” in *Proceedings of 19th European Signal Processing Conference (EU-SIPCO) 2011*, Barcelona, Spain, 29 August - 2 September 2011.
4. **Quoc-Huy Phan**, Su-Lim Tan, Ian McLoughlin “GPS Multipath Mitigation: A Nonlinear Regression Approach,” *GPS Solutions*, 2012, doi: 10.1007/s10291-012-0285-5.

Chapter 1

Introduction

Global Positioning System (GPS) has been the most widely used satellite-based navigation system since it became operational in 1992. In spite of being originally intended for military applications, it is well adapted to civilian applications such as survey, vehicle navigation, engineering applications like aviation, spacecraft attitude, orbit determination, remote sensing, and scientific applications such as oceanography, geodesy, and many others [26]. In this chapter, a background of GPS will be presented with the emphasis on the concepts which are required in the rest of this thesis, followed by an outline of the thesis.

1.1 Global Positioning System Background

GPS comprises three major segments: *Space*, *Control*, and *User Segments* illustrated in Figure 1.1 [64]. The Space Segment consists of a constellation of GPS satellites each of which broadcasts radio-frequency ranging codes and a navigation data message. The Control Segment consists of a Master Control Station (MCS) and a number of monitoring stations located around the world, which are responsible for tracking, monitoring, managing the satellite constellation and updating the navigation data messages. The User Segment consists of an unlimited number of users equipped with a variety of GPS receivers specifically designed to process the satellite

Figure 1.1: GPS consists of three segments: space, control, and user.¹

signals.

1.1.1 Space Segment

GPS's constellation encompasses 24 satellites arranged in 6 nearly-circular orbital planes with a radius of 26,560 km inclined at 55° relative to the equatorial plane, with 4 primary satellite slots distributed unevenly and a spare satellite slot in each orbit [59]. The GPS satellites orbit the Earth with an orbital period of approximately 12 hours. With this arrangement, it is likely that a user with a clear view of the sky would see 6 to 8 satellites with a minimum of 4 satellites guaranteed at every time and each satellite will be observable for approximately 5 hours at a time. The satellites broadcast ranging signals and navigation data, allowing the users to measure their ranges to the satellites and to estimate their position.

¹Image from <http://ucspace.canberra.edu.au/display/s651224/GPS>

GPS satellites continuously transmit signals on two L-band frequencies: L1 at 1575.42 MHz and L2 at 1227.60 MHz with their wavelengths being $\lambda_1 = 0.19029$ m and $\lambda_2 = 0.24421$ m respectively. Superimposed on these carriers are two coded signals unique to each satellite: a precision code (P-code) Pseudo Random Noise (PRN) signal with a 10.23 MHz chip rate and a coarse/acquisition code (C/A code) PRN signal with 1.023 MHz chip rate respectively. The L1 frequency contains both the P-code and C/A code while the L2 frequency contains only the P code. Furthermore, the navigation message is modulated on both carriers at a chipping rate of 50 Hz. According to the new requirements under the GPS modernization plan, the next generation of GPS satellites, called Block IIF, is planned to add C/A-code on L2. A new carrier with a frequency of 1176.45 MHz, which will be called L5 [59], will also be added.

1.1.2 Control Segment

The Control Segment consists of one MCS plus five monitor stations [59] [64]. The monitor stations passively track all visible GPS satellites, collecting ranging data from them. This information is passed on to the MCS where, after extensive computations, the satellite ephemeris and clock parameters are estimated and predicted. The ephemeris and clock data are uploaded via any of the ground antennae to the satellite, on a S-band link, for retransmission in the navigation message. The satellite clock drift is corrected so that all transmitted data are synchronized with GPS time. The ephemeris corrections are obtained through the estimation of the Cartesian coordinates of the satellites along the orbits by integrating their motion equations.

1.1.3 User Segment

The User Segment consists of a variety of military and civilian GPS receivers specifically designed to receive, decode and process the satellite signals. They include standalone receiver sets, as well as equipment that is integrated with or embedded into other systems. They serve a variety of user applications including navigation, positioning, time transfer, surveying and

Figure 1.2: GPS receiver categories.

attitude reference. Consequently, GPS receivers for different applications can vary significantly in design and function.

In general, GPS receivers can be divided into two major groups: those that can track four or more satellites simultaneously (multi-channel receivers), and those that can scan or sequence between all visible satellites. Sequencing receivers can be further divided into 3 additional categories: single-channel, fast-multiplexing single-channel, and dual-channel. Figure 1.2 illustrates the hierarchy of the receiver classification.

In particular, multi-channel receivers, available in a variety of forms, allow simultaneous reception of four, six, eight, ten or twelve channels. These devices are essential in dynamic applications that require accurate, instantaneous position and velocity information. Multi-channel devices have one strong advantage in being able to track as many satellites as the number of channels available, enabling them to choose the most appropriate satellites to obtain the lowest possible geometric dilution of precision (GDOP) figure [41] [82].

A GPS receiver uses the principle of trilateration to compute the receivers location. GPS positioning requires a minimum of four satellites to determine position: three for the Cartesian components of receiver position and a fourth satellite to solve for the GPS receiver clock [37] [59]. With the completed GPS constellation, most points on the Earths surface have more than

four GPS satellites visible at any one time, creating an over-determined problem.

1.2 GPS Observables

GPS provides two types of observables which are outputted by GPS receivers after processing within the receiver using sophisticated electronic and digital signal processing techniques. Both observables contain information on the distance between the in-view satellites and the receiver. Code measurement provides an estimate of apparent transit time of a signal. Carrier-phase measurement provides the receiver carrier phase relative to the phase of a sinusoidal signal generated by the receiver clock. GPS-based estimation of position, velocity and time rely on code measurements, carrier-phase measurements, or both depending on the application and required accuracy. Carrier-phase measurements are much more precise than code measurements and are usually used for high-precision applications such as geodesy. However, both are biased (and, in the case of carrier-phase, ambiguous) estimates of the instantaneous receiver-satellite range since they are affected by a number of error sources. In order to obtain a high-precision GPS-based solution, all of these errors need to be removed from the observables. Among these errors, multipath (which is the target of this thesis) still remains a major one. This section provides a review of the model equations of the GPS observables. GPS error sources will be also reviewed thereafter.

1.2.1 Pseudorange Observables

The concept of *pseudorange* comes from the measurement of the time transmitting the code from the satellite to the receiver's antenna. This is achieved by correlating identical PRN codes generated by the satellite's clock with those internally generated by the receiver's clock. The time shift required to achieve correlation yields the transmission time between the satellite and the receiver, which is easily converted into a range by multiplying by the speed of propagation

of the radio wave. These range measurements are referred to as pseudorange as they differ from the true geometric distance corresponding to the epochs of emission and reception due to the fact that the satellite and receiver clocks are not synchronized. Pseudorange measurements are typically reported in meter units.

Denoting the true range from a satellite to a receiver to be r , the pseudorange measurement affected by clock biases from both the satellite and the receiver becomes $r + c(\delta_u - \delta_s)$ where c is the speed of light, δ denotes clock bias, and the subscripts u and s refer to the user (receiver) and the satellite, respectively. Beside clock biases, pseudorange observables are also affected by the troposphere delay T , ionosphere delay I , multipath M^p and random receiver noise ϵ^p . Taking all of these into account, the pseudoranges, ρ_1 and ρ_2 for L1 and L2 frequencies respectively, can be expressed as (1.1) and (1.2) [59] [64]:

$$\rho_1 = r + c(\delta_u - \delta_s) + I_1 + T + M_1^p + \epsilon_1^p \quad (1.1)$$

$$\rho_2 = r + c(\delta_u - \delta_s) + I_2 + T + M_2^p + \epsilon_2^p \quad (1.2)$$

As above-mentioned, there are two coded signals modulated on L1: P-code PRN signal and C/A PRN code signal while there is only P-code PRN signal modulated on L2. Hence, there are currently two types of pseudoranges available on L1 frequency: C/A and P1 whereas only P-code pseudorange, P2, is available on the L2 frequency. The precision of these pseudoranges depends on their chipping rates. The C/A-code has a chipping rate of 1.023 MHz while that for P-code is 10.23 MHz, resulting in their wavelength of $\simeq 300$ m and $\simeq 30$ m, respectively. In other words, P-code pseudoranges are ten times more precise than C/A code pseudoranges. If the receiver can track 1-2% of a wavelength, the accuracy of 3-5 m for C/A-code measurements and 30-50 cm for P-code measurements can be achieved [59] [64]. Unfortunately, civilian users do not have access to the P-codes as they have been encrypted.

1.2.2 Carrier Phase Observables

The carrier phase observables which are expressed in cycles are defined as the difference between the received satellite carrier phase and the phase of the carrier internally generated by the receiver. These observables are affected by the same error sources as the pseudorange observables. Furthermore, an extra parameter is induced in the carrier phase equation, named *integer ambiguity*, since the antenna cannot sense the number of whole cycles between the satellite and the receiver and this must be resolved in order to yield satellite-receiver range. The carrier phase observables are modeled as (1.3) and (1.4) [59] [64]:

$$\phi_1 \lambda_1 = r + c(\delta_u - \delta_s) - I_1 + T + N_1 \lambda_1 + M_1^\phi + \epsilon_1^\phi \quad (1.3)$$

$$\phi_2 \lambda_2 = r + c(\delta_u - \delta_s) - I_2 + T + N_2 \lambda_2 + M_2^\phi + \epsilon_2^\phi \quad (1.4)$$

where the symbols ϕ_1 and ϕ_2 represent the carrier phase of L1 and L2, respectively. The symbols λ_1 and λ_2 denote wavelengths of L1 and L2. The terms N_1 and N_2 are ambiguous integers of L1 and L2. The opposite signs of the ionospheric delays, I_1 and I_2 against the counterparts in pseudorange observables are due to the fact that the ionosphere affects code and carrier measurements equally but in opposite directions when the signals travel through dispersive ionospheric layers in the atmosphere [54]. Note that the *relativity* term [59] [64] is missing from the above formulations. Special and general relativity effect on the satellite orbit, satellite signal propagation, and satellite and receiver clocks [98]; all of these effects are grouped together into the *relativity* term. For the purposes of simplicity, these effects are grouped in with their respective clock terms. Most receiver tracking software accounts for the effects on the receiver clock, so that the portion of *relativity* is ignored here.

Regarding to the precision, the carrier phase measurements, with wavelengths of $\lambda_1 \simeq 19.02$ cm and $\lambda_2 \simeq 24.42$ cm, are much more precise than the P-code pseudoranges. Again, if the receiver can track 1-2% of a wavelength and the ambiguous integer is correctly resolved,

the accuracy of the phase measurement is about 2 mm [59] [64].

1.3 GPS Error Sources

There are various error sources contaminating the GPS observables, impairing the performance of GPS positioning solutions in terms of accuracy and precision. In order to achieve improvement, all these errors need to be removed. What follows will briefly discuss each of these errors.

1.3.1 Satellite Clock Offset

GPS satellites are equipped with atomic clocks which drift over time. The MCS is responsible to compute the corrections for clock drift that are broadcasted to the users in the navigation message to compensate for clock offset. Using the polynomial models with broadcasted coefficients, the clock error can be reduced down to 1 part per 10^{12} (1 ppt). As a result, the effect of satellite clock offset is theoretically negligible. However, due to the discontinuity of the corrections from the MCS (several times a day but the schedule is not fixed), this effect needs to be accounted for because the prediction error grows with the age of the data, defined as the time since the last correction happened. The clock error can be completely eliminated using differential techniques [59].

1.3.2 Ephemeris Errors

The satellite ephemeris is required for both pseudorange and phase computations. The MCS is responsible for estimating and updating the satellites ephemeris via broadcasted navigation data which can be used by real-time applications. Post-processing applications can access precise ephemeris which is calculated from actual observation to the satellites from the monitor stations by modeling of all forces acting on the satellites and is produced several days after

the observation period. Organizations such as International GNSS Service (IGS) [39] provide accurate orbital parameters to the public via anonymous FTP. The accuracies of broadcast ephemeris and precise ephemeris are about 1 m and 2.5 cm respectively. The ephemeris prediction error can be corrected using differential techniques but the efficiency of correction depends on the length of baseline between the reference station and user's receivers. Generally, for short (<10 km) and medium (10 - 50 km) baselines, satellite orbital errors become insignificant [59].

1.3.3 Ionospheric Delay

Ranging from 50 km to 1,000 km above the earth's surface, the ionosphere is the region of ionized gases of the earth's atmosphere, modifying the propagation of GPS signals (speed, direction and polarization). The ionospheric error is primarily dependent on the number of electrons that the travelling signal encounters along its propagation path. That number of electrons varies throughout the day and with seasons. Elevation and azimuth of the satellite, and receiver location also plays an important role. Because the ionosphere is a dispersive medium for radio waves, the PRN codes are delayed while the carrier phase is advanced as they pass through it. Furthermore, its effect is inversely proportional to the frequencies of the travelling signals; thus, the impact on L1 signals is smaller than that on L2 signals. The ionosphere can cause a range error of several tens of meters in zenith direction under extreme conditions [65].

The model of ionospheric delay with broadcast ionospheric delay coefficients in navigation messages [45] can remove approximately 50% of the delay. More interestingly, the ionospheric-free combination of dual-frequency measurements is able to cancel out the first order ionosphere delay. The higher-order ionospheric errors whose magnitude is about 1% of the first-order error [16] [46] can also be eliminated with improved models, one of which was developed by Brunner and Gu [12].

1.3.4 Tropospheric Delay

The troposphere is the lower part of the earth's atmosphere (up to 50 km). Unlike the ionosphere, the troposphere is a non-dispersive medium with respect to (w.r.t.) the GPS signals; hence the tropospheric delay is independent on the frequency and its characteristics depend on local meteorological parameters: atmospheric pressure, humidity, temperature. The tropospheric errors can amount to about 2.3 m at the zenith and about 20 m near the horizon [75]. This delay can be divided into a dry and a wet component which are responsible for 90% and 10% of the total tropospheric delay, respectively. Between the two, the dry component can be easily modeled while this is not the case for the wet component. Several models have been developed to estimate the tropospheric delay as a function of satellite elevation, receiver height and meteorological parameters [37], which can compensate up to 90% of the delay.

1.3.5 Multipath

Multipath is defined as one or more indirect replicas of line-of-sight signal received by the GPS antenna through reflecting from objects in the surrounding environment such as buildings, trees, water surface, etc. The reflected signal will have a different path length compared to the direct signal; resulting in biased distance measurements. Multipath error is dependent on environment and satellite geometry w.r.t. the receiver. In other words, it is not spatially-correlated. Thus, while atmospheric delays can be canceled out using differential GPS (DGPS) techniques [65], multipath errors are magnified instead of being removed due to no correlation between multipath errors at two ends of the baseline. Generally, code multipath is much greater than carrier phase multipath; and the C/A code multipath error is greater than that of the P code [85]. Multipath noise is also more severe for low-elevation satellites. More details of multipath error will be presented in the following chapters of this report.

1.3.6 Receiver Noise

Random measurement noise, called receiver noise, is introduced by the electronic components of a receiver like the antenna, amplifiers, cables, etc. The limited precision of software also contributes to the receiver noise errors. The receiver noise on code measurements is at the level of several decimeters for most modern receivers, while the receiver noise on carrier-phase measurements is at the level of a few millimeters. Although receiver noise is not spatially correlated; theoretically, it can be removed by averaging the measurements over fairly long periods of observation time.

A summary of the errors in GPS measurements are tabulated in Table 1.1 (from [59]) where FO is the slant factor defined as $FO = 1/\sin(elevation)$.

Source	Potential error size
Satellite clock offset	Clock modelling error: 2 m (rms)
Satellite ephemeris error	Along the line of sight: 2 m (rms)
Ionospheric delay	$\simeq([2-10 \text{ m}]) \times FO$
Tropospheric delay	$\simeq([2.3-2.5 \text{ m}]) \times FO$
Multipath	In typical environment: Code: 10 - 20 m Carrier: $\leq 5 \text{ cm}$
Receiver noise	Code: 0.25 - 0.5 m (rms) Carrier: 1 - 2 mm (rms)

Table 1.1: A summary of the errors in GPS measurements.

1.4 Previous Work

Before the work presented in this thesis, many approaches have been proposed to address GPS multipath mitigation. Below, we provide a taxonomical table of these approaches in Table 1.2 based on the lifetime stages of a multipath signal. Chapter 2 will provide a comprehensive literatur review on these approaches.

1.5 Problem Statement and Objectives

Here, we focus on a specific computational problem associated with GPS data processing known as multipath mitigation. So far, none of the multipath mitigation approaches has been satisfactory for widely use for high-precision applications such as earthquake monitoring, crustal deformation monitoring, etc., which require millimeter-level accuracy. While the pre-receiver and in-receiver approaches are limited by advances of hardware, it is worth exploring new post-receiver methods, especially focusing on processing in the observable domain. This is due to the vast amount of observable data in RINEX or compact RINEX standard format provided by various data archives, such as SOPAC, CDDIS, IGS, etc., that are available for free access. In addition, computational requirements for processing large amount of data became less of an issue due to continuing rapid advances in computer performance.

In spite of high efficiency frequency-domain approaches, their locality and scalability are still very limited, making them inappropriate for many applications in which important signals need to be reserved and a high rate data is involved. On the contrary, time-domain processing approaches with higher locality and scalability commonly score around 50% in term of efficiency. Recent work on carrier-phase multipath mitigation [97] had indicated that high efficiency can be achieved with time-domain processing techniques. This inspires further research in this direction.

This work will primarily focus on the following task: exploring and proposing alternative methods that efficiently address GPS multipath mitigation whilst being able to overcome the shortcomings experienced by the methods in the literature.

1.6 Contributions of the Research Work

This research work includes conceptual and algorithmic contributions to the field of high-precision GPS positioning. These contributions are:

Multipath mitigation approaches			
Pre-receiver	Multipath prevention		Involving in choosing installation site with less potential multipath surroundings.
	Multipath absorption		Absorbing multipath signals by using electromagnetic fence or placing microwave-absorbing materials underneath the antenna.
	Multipath rejection		Involving in antenna design techniques to reject multipath signals.
In-receiver			Involving in designing Delay Lock Loop and Phase Lock Loop (PLL) to reduce multipath effects.
Post-receiver	Time-domain processing	Filter-based	Employing filtering techniques to filter out multipath errors from measurements.
		Stacking-based	Stacking and time-shifting multipath sequences of one or more sidereal days to make multipath-correction for data of current day by differencing.
	Frequency-domain processing	Direct assessment	Cutting off estimated frequency content of multipath error after converting measurements into frequency domain.
		Indirect assessment	Using independent Signal-to-Noise (SNR) measurements to map multipath frequency content at each time window before estimating multipath errors.

Table 1.2: Taxonomy of multipath mitigation approaches.

1. Derivation of the geometrical models for GPS multipath errors.
2. Formulating multipath error (both code and carrier-phase) estimation problem as a non-linear regression problem.
3. Efficiently solving multipath error estimation by standard nonlinear regression algorithms.

1.7 Outline

We have overviewed the background of GPS with fundamental concepts, the observables, and the error sources contaminating the observables. The rest of the thesis is organized into five chapters. The main contents of each chapter are summarized below:

Chapter 2. Survey of GPS Multipath Mitigation Techniques. This chapter will present a literature review on the approaches in multipath mitigation and will discuss their strengths and weaknesses. This allows drawing a connection between the current approaches and the successive studying of multipath problem.

Chapter 3. Principles of GPS Multipath and Multipath Geometrical Models. What is presented in this chapter is to cover the background knowledge needed for later development of the report. It comprises the principle and properties of GPS multipath of which the works of literature have been taken advantage of to tackle GPS multipath errors. After that, geometrical models of multipath errors will be derived, where multipath errors corresponding to a satellite are viewed as functions of its repeatable geometry w.r.t. a fixed receiver. It turns out that the multipath estimation problem amounts to a regression problem which will be solved in successive chapters.

Chapter 4. Mitigation of Code Multipath with Nonlinear Regression. This chapter first discusses the repeatability of satellite geometry w.r.t. a ground fixed receiver and the high correlation of code multipath error on a sidereal daily basis under the assumption of a constant

environment. All of these facts makes solving the regression problem for multipath estimation feasible. After that a novel approach for code multipath modeling and estimation will be proposed. The multipath estimators are trained with historical data before being employed to estimate code multipath. The experimental results on real recorded data have demonstrated superior performance of the proposed approach.

Chapter 5. Mitigation of Carrier-Phase Multipath with Nonlinear Regression. For the applications requiring high-precision positioning, the carrier phase measurements must be used. Unfortunately, removing carrier phase multipath is not trivial because, unlike multipath of code measurements, there are no combinations of GPS measurements that can isolate it for standalone stations. Therefore, double differences of carrier-phase measurements between two short-baseline stations will be conducted to extract carrier-phase multipath. In the light of success in code multipath mitigation, the carrier-phase multipath can be arguably estimated likewise. The preliminary results from our experiments indicates the promise of the proposed approach in carrier-phase multipath mitigation.

Chapter 6. Conclusions and Future Directions. This last chapter of the report will summarize the results that have been achieved in this research work. Some future research directions for GPS multipath mitigation problems will also be drawn. These include further connection to additional measurements regarding the GPS modernization plan [59].

Chapter 2

Survey of GPS Multipath Mitigation Techniques

In this chapter, a comprehensive literature review on multipath mitigation approaches will be provided and their strengths and weaknesses will be discussed. The properties of an ideal multipath mitigation approach will be firstly discussed, followed by an overview of the literature organized into different categories in terms of multipath mitigation methods. This chapter will conclude with a summary and research motivation to further study the multipath mitigation problem.

2.1 What is multipath and why is multipath mitigation necessary?

GPS multipath is defined as one or more indirect replicas of line-of-sight signals received by the antenna from reflected objects in its surroundings [9]. After arriving at the receivers antenna, the multipath signals propagate into the receiver where they distort the correlation function of the tracking loop. Subsequently, they cause biases in range and carrier-phase estimations as

output by the tracking loop of the receiver, and finally induce errors in the resulting positions.

Among the variety of error sources contaminating receivers measurements, multipath disturbance remains a major error impairing the accuracy and precision of GPS positioning. Firstly, in the observable domain, multipath typically causes range errors of 10 - 20 m [37] in case of C/A code observations and up to 5 cm in carrier-phase observations [54]. These range errors are intolerant for high-precision applications, such as earthquake monitoring, crustal deformation monitoring, etc., which require millimeter-level precision. Secondly, multipath error is environment-dependent. Thus, the DGPS [65] that is employed to minimize the spatially correlated errors such as tropospheric delay, ionospheric delay, etc., does not work on reducing multipath but amplifies it instead, since the resulting multipath error is a combination of multipath errors from two stations. Therefore, reduction of multipath becomes essential for high-precision GPS applications.

2.2 Properties of an Ideal Multipath Mitigation Approach

Ideally, one would like to prevent all multipath signals from arriving at the receiver's antenna or cancel out the exact amount of errors induced by multipath signals contributing to GPS measurements. In practice, however, the unknown number of multipath reflections and the complexity of GPS signals with many kinds of errors make this unfeasible. Instead, a good multipath mitigation technique should have the following properties:

- *Efficiency*: It must be able to reduce multipath effects as much as possible. This is the key property determining its applicability. In the literature, the efficiency of a multipath mitigation technique has been commonly quantified by the percentage of reduced multipath error in term of standard deviation in the observable domain and coordinate domain. However, it is somehow ambiguous. In the observable domain, since various observable combinations, resulting in various levels of multipath error, are used in dif-

ferent methods. On another hand, in the coordinate domain it is difficult to determine the effect of multipath apart in isolation from the effects of other error sources on positioning solutions.

- *Real-timeness*: It should be able to estimate and make corrections for multipath errors in an epoch-by-epoch fashion to be applicable for time-critical applications.
- *Scalability*: The multipath mitigation technique should be scalable with the data rate in terms of computational complexity and storage. Scalability also influences the real-timeness of the technique. Real-timeness and scalability are joint properties: the more scalable an approach, the more real-time it is. Due to the trend of using high-rate GPS data [49] [50] [31], in which a position is estimated at every epoch [11], these two properties become more and more critical.
- *Locality*: The technique should accurately localize and manipulate multipath errors only while ensuring other errors are not affected. Locality is especially important in applications where other modulating signals exist such as earthquake monitoring, deformation monitoring, etc. In these applications, the phenomena signals, i.e. earthquake signals, deformation signals, likely have their spectrum overlapping with the multipath signals spectrum and tend to be canceled out by existing multipath mitigation techniques [72]. In addition, it is required that other errors like ionospheric delay, tropospheric delay, etc. are not accidentally affected by the technique, leading to a deficiency of other models used to correct these errors [93]. Efficiency and locality are competing properties and generally cannot be fulfilled simultaneously.
- *Availability*: Preferably, the technique should be available to be used in different scenarios. Some methods become unavailable when the measurements they rely on are missing. For instance, the works in [6] [7] use correlation between signal-to-noise (SNR) measurements and multipath errors for multipath mapping. Unfortunately, SNR mea-

Figure 2.1: Domain-wise GPS signal processing.

measurements are not always reported by GPS receivers. In such scenarios, the multipath mitigation approach cannot be performed!

Clearly, the importance of these properties depends on the actual applications that they are designed for.

2.3 Multipath Mitigation Approaches in the Literature

In this section, an overview of multipath mitigation approaches proposed in the literature will be presented. Based on the lifetime stages of multipath, including pre-receiver, in-receiver, and post-receiver, as illustrated in Figure 2.1, the approaches can be classified accordingly. The hierarchy of classification is depicted in Figure 2.2. Further discussion for each category of techniques is elaborated with a more focused discussion on post-receiver approaches.

Figure 2.2: Hierarchy of classification of multipath mitigation approaches.

2.3.1 Pre-receiver Approaches

The first category of multipath mitigation approaches is based on reducing multipath signals from propagating into GPS receiver. These methods aim to avoid multipath signals from entering the the GPS receiver. Since multipath is caused by reflections of direct signals from surrounding objects, the idea of multipath avoidance is straightforward. These approaches can be further sub-classified into *multipath prevention*, *multipath absorption*, and *multipath rejection* approaches.

Multipath Prevention

These methods involve carefully choosing the observation sites for ground stations that have the least reflecting objects in their vicinities [17] [48] [79]. The evidence of multipath mitigation of these methods is shown based on data inspection; therefore, it is numerically unclear. However, these methods are of limited value as there are not always many possible choices for observation sites. Furthermore, no environment is completely multipath-free. The reflection

Figure 2.3: Placement of electromagnetic fences at a hypothetical site.

can even happen on land [91] [52] or water surfaces [79], etc.

Multipath Absorption

Zhang and Barton [92] proposed the implementation of an electromagnetic fence for GPS ground reference stations to absorb the ground bounce and reflection at different elevation angles and some specific azimuth angles. Figure 2.3 (image from [92]) illustrates an example of setting three electromagnetic fences for a hypothetical installation. Similarly, the method in [22] places microwave-absorbing materials underneath the antenna to absorb close-in GPS scattering signals. In particular, the method with microwave-absorbing material placement saw approximately 75% error reduction in the estimate of the vertical coordinates but insignificant effects on improving horizontal coordinate errors [22].

These methods are especially useful for ground-based reference receiver applications where the antenna is static and they can attenuate the low frequency multipath reflecting off the ground [9]. Nevertheless, they have no effect on multipath signals reflected from objects above the

antenna (e.g. tall buildings or trees). These techniques also require additional cost, for example, in the construction of special fences.

Multipath Rejection

Another trend is to reject multipath with novel antenna design technique, thanks to the advances of antenna design technology. Antennas with sharpen gain patterns [9] can reject or minimize signals coming from lower elevation angles. Thus, multipath signals will be down-weighted in receiver processing relative to the higher-gain direct signal. Antenna gain patterns for many common geodetic antennas are available in the literature [57] [68] [73]. With the observation that multipath signals are typically more rigorous at low elevation angles, improved multipath resistance can also be achieved with choke rings [54] which are designed to minimize signals coming from low elevation angles.

These techniques are claimed to be very effective to limit multipath at low elevations which is very difficult to tackle; however, it is unclear what are their performance. Nevertheless, these methods suffer similar drawbacks as multipath-absorption proposals since they cannot eliminate multipath signals from higher elevations. In addition, they typically require more costly antennas (a Zephyr choke ring antenna, for example, costs around US\$15,000).

2.3.2 In-receiver Approaches

These approaches mitigate multipath by processing incoming signals within the GPS receiver. In a receiver, the GPS incoming signal is beat with the locally generated in-phase and quadrature-phase replicas of the carrier signal. Next, it is correlated with the Prompt (P), Early (E) and Late (L) versions of the locally generated code to estimate the synchronization parameters (i.e. time-delays, and carrier-phases) [24]. The E and L correlations are generally used for code tracking using Delay Lock Loop (DLL), whereas the P correlation is used for carrier tracking with Phase Lock Loops (PLL).

Basically, DLLs in the literature have been designed to mitigate the effect of multipath propagation [81] [27] [40] [23] [5]. Historically, conventional GPS receivers have used 1.0 chip spacing, which is defined as the desired E/L correlator spacing, in the implementation of DLLs [81]. In order to reduce multipath signals, a narrower spacing is required because the multipath components of the E and L signals are correlated and tend to cancel out each other. With advances in CMOS technology, narrower-spacing can now be achieved. Numerous variations of DLL architecture have been proposed.

Narrow Correlator [81] with the spacing of 0.1 chips (or even 0.05 chips) increases the correlation between the error components in the E and L signals, and accordingly reduces the multipath effects in the DLL discriminator. It is reported to reduce 69% standard deviation of residual of P1 pseudorange and C/A pseudorange [81]. Some other designs, such as *Strobe & Edge Correlator* [27] and *Pulse Aperture Correlator* (PAC) [40] utilize the linear combination of Narrow Correlators to achieve better performance on short multipath delays [27] and on long multipath delays (>50 m) [40]. Another recent design is the *Vision Correlator* licensed to NovAtel [23]. It implements the Multipath Mitigation Technique (MMT algorithm) [88] and is able to reduce the standard deviation of multipath at low elevation angles (less than 10 degrees) 45% better than PAC.

Multipath Estimating Delay Lock Loop (MEDLL) [83] [78] is a robust statistical architecture where the Maximum Likelihood theory is applied to estimate the multipath signal component and eliminate it from the tracking loop. MEDLL receivers can reduce 10% - 40% standard deviation of real pseudorange double difference data whereas simulation results indicate 90% improvement of MEDLL over Narrow Correlator receivers on pseudorange measurement multipath [78]. This simulation performance is proven to be close to the theoretical performance but at the expense of increasing cost and complexity since it requires three correlators per direct and multipath signal path for each tracking channel theoretically. Reduction of computational complexity of MEDLL appears as an open research problem [71].

Similarly, carrier-phase multipath can also be mitigated with the advances of PLL [5]. The standard deviation of the carrier-phase multipath of double differenced phase measurements can be reduced by up to 30% [5].

Basically, these DLLs and PLLs are based on different combinations of E and L samples of the correlation function and considering different chip-spacings between the samples [15] [56]. Their performance is generally limited by chip spacing. Specifically, they are especially troublesome in the case of coherent multipath, which refers to replicas with relative delays shorter than the chip period, where timing synchronization may fail [87] [56] [15]. Most importantly, the license of these receiver tracking technologies belongs to individual receiver manufacturers. Furthermore, it is generally not possible for the end user to have access to the receiver hardware or modify the firmware. In order to overcome drawbacks as discussed above, a new trend named software receivers has been triggered to exploit statistical digital signal processing algorithms applied to fine synchronization [56] [15] with the advent of Software Defined Radio techniques devoted to navigation receivers [80].

2.3.3 Post-receiver Approaches

With hardware limitation, the techniques for multipath mitigation based on antenna design and receiver tracking loop perform well on long multipath, but not for close-by multipath [10] [89] [15]. In fact, the multipath effects on measurements produced by receivers are still large. In practice, the error caused by multipath can typically be in the range of 10 - 20 m [37] in case of C/A observations and up to 5 cm in case of carrier phase observations [54]. Such errors are considered large for applications where the accuracy requirements are often in the millimeter range. Hence, multipath error reduction needs to be addressed further.

Pseudorange (referring to both C/A and P code) and carrier-phase observables are usually reported by the receiver in the standard RINEX format [32] [33] [34], which are processed to produce the final coordinate time series. Thus far, various techniques have been proposed for

further reduction of the remaining multipath in observable and coordinate time series. However, processing on coordinates requires a full positioning solution. In addition, multipath error will be distributed among various parameters (e.g., receiver clock and atmospheric delay) on position solution, making it difficult to assign errors to a particular direction. Therefore, processing on observables is preferable.

Observable processing techniques can be further classified into *time-domain* and *frequency-domain processing* approaches as elaborated below.

Time-Domain Processing

There are two major classes of multipath mitigation based on time-domain processing: *filter-based* and *stacking-based* approaches. The former employs digital filtering techniques like carrier-smoothing filters (CSF) [38] [58], Finite-duration Impulse Response (FIR) filters [28] [55], etc., to process observables time series. Another line of research is to stack sidereal-daily repeatable measurement time series of one or more days and use them to calibrate multipath errors for measurements of successive days [14] [4].

Filter-Based Approaches: The pioneering work in filter-based approaches is credited to carrier-smoothing filter (CSF) [59], initially known as the Hatch filter [36]. The idea is to smooth absolute but noisy code measurements with precise carrier-phase measurements.

Firstly, the code-minus-carrier (CmC) quantity χ is formed by taking the difference of code measurement ρ and carrier-phase measurement ϕ . Next, the CmC sequence is fed to the low-pass filter F which is supposed to attenuate the multipath error, but preserves the true range between the receiver and satellites in code measurements. Eventually, the smoothed CmC $\bar{\chi}$ is combined with the carrier measurement ϕ to form the smoothed code measurement $\bar{\rho}$ as the output. Figure 2.4 illustrates a typical implementation of CSF.

CSF's performance heavily depends on the smoothing window width of the filter F . Specifically, the longer the smoothing window, the smoother code measurements can be achieved.

Figure 2.4: Implementation of carrier-smoothing filter.

Figure 2.5: Adaptive FIR filter configuration.

Unfortunately, expanding the smoothing window introduces an extra bias to the smoothed code measurements caused by the divergence of ionospheric delay remaining in CmC values over time [42]. 100 seconds is the recommended smoothing window size [20]. With dual-frequency measurement available, divergence-free and ionospheric-free CSF proposed in [38] [58] can overcome the ionospheric divergence. For single-frequency receivers where dual-frequency measurements are unavailable, the optimal smoothing windows can be adaptively chosen to maximize the performance of CSF [44] [63].

A 100-second smoothing filter can reduce the RMS error of raw pseudorange measurements from 4 m to 1.7 m or 58% [59]. Nevertheless, CSF is generally effective to reduce multipath error at higher frequencies (and receiver noise) but less effective for multipath error at lower frequencies [95]. Hence, it is usually employed as a preprocessing phase before applying other approaches [93]. Furthermore, it is only applicable for mitigation of code multipath.

Multipath error can also be eliminated using adaptive *Finite-duration Impulse Response* (FIR) filters [28] [55] [90]. The configuration of one such FIR filter is shown in Figure 2.5. It

might be useful to think of a signal corrupted by multipath as including two components: the signal itself and multipath. In this algorithm, the observable sequence of the previous day is used as a reference. It can be expressed as $s_0 + m_0$ where s_0 and m_0 are the multipath-free signal and multipath respectively. The input of the filter is the observable sequence of the current day. Similarly, it can be expressed as $s + m$ where s and m represent the multipath-free signal and multipath respectively. The rationale of this method is that m_0 and m are highly correlated while s_0 and s are not correlated [28] [55]; thus, the adaptive FIR filter can remove multipath m from the input observables. A Least-Mean-Square (LMS) algorithm can be employed to adaptively adjust the coefficients of the FIR filter during operation to minimize output errors. The length of the FIR filter can vary during operation to provide better performance at the expense of additional computational complexity [55].

The adaptive FIR filter can be used to mitigate either code multipath if CmC sequences are used [28] or carrier-phase multipath if a double difference residual is inputted [55] [90]. It is reported to reduce the standard deviation of code multipath errors by approximately 75% [28]. In case of carrier-phase double-difference multipath, the reduction ranges from 39.8% to 56.1% [55].

A negative aspect of the FIR filter is that it tends to filter out other important phenomena signals (e.g., crustal deformation induced by an earthquake) if they fall within the same frequency band as the targeted multipath error [28] [72]. Furthermore, performing FIR filtering on carrier-phase multipath requires one or more multipath-free station to form double differences [55] [90], which are rarely available in practice.

Stacking-Based Approaches: The GPS satellites orbit the Earth with an orbital period of half a sidereal day, giving rise to the same satellite configuration at the same time on successive sidereal days (nominally 23 hours 56 minutes 04 seconds or 236 seconds shorter than a solar day). For a fixed station, the geometry relating the GPS satellites, reflective surface and the antenna does not usually change significantly between consecutive sidereal days. Therefore, GPS

multipath signals also highly correlate themselves over the same time period. Taking advantage of the repeatability of multipath sequences and satellite constellation w.r.t. a fixed station on a daily basis, a multipath map can be built by stacking and time-shifting multipath sequences of one or more sidereal days. The stacked multipath is then subtracted from observations of subsequent days to correct for multipath errors [11] [66] [97].

The temporal stacked multipath is shifted by a whole number of sidereal periods (i.e. a multiple of 86,164 seconds) or by repeated periods determined by cross-correlation within the GPS data series [11] [30] [62] [66] (sidereal filtering). The mean of the orbit repeat periods of all the GPS satellites in view during the observation time period is also used [14] [4] (modified sidereal filtering). Study in [14] has indicated that the modified sidereal filtering gives more precise results than the standard sidereal filtering technique, especially for high-frequency multipath errors. The multipath model can also be shifted in a piecewise fashion. The aspect repeat time adjustment (ARTA) method developed in [51] estimates time-varying shift periods and piecewise shifts the multipath model to account for the variability of the GPS constellation.

Advantageously, these techniques can be applied to code measurements, carrier-phase measurements, and even coordinate time series of ground fixed stations. Furthermore, the study in [66] has shown that sidereal filtering using the coordinate residuals gives slightly better precision than using the double-differential carrier-phase residuals. Among the three time-shifting techniques, ARTA saw highest performance. Applying to coordinate time series, it reduces standard deviation of the north component from 8.2 to 5.1 mm (38%) and of the east component from 6.3 to 4.0 mm (37%) [51]. The ARTA method applied to single differential carrier-phase measurements experiences significant improvement; an 82% reduction in terms of RMS positioning variation [97], improving 13% compared to sidereal filtering.

However, it is not accurate to use nominal sidereal period in case of sidereal filtering whereas, for modified sidereal filtering, it is not obvious which time shift to use when different satellites are visible at different times of the day, resulting in the varying mean orbit

repeat time [96]. ARTA saw higher performance than sidereal and modified sidereal filtering; unfortunately, it cannot operate in real-time [96] [97]. Furthermore, stacking techniques would remove or modify part of the nonlinear signals which develops or persists over more than 1 day [11]. Lastly, as the multipath- stacking model is discrete in time, high resolution (i.e. high data rate) is required to ensure high performance [4].

Frequency-Domain Processing

Ionosphere and troposphere errors are of low frequency, and the receiver noise has high frequency components. Orbit errors are typically bias terms, and multipath errors have low/medium frequency components [93]. The frequency distribution of multipath errors is dependent on the sampling rate, antenna height, and obstruction environment, makes them very much site-specific. In the case of 1 Hz sampling frequency, the major multipath frequency spectrum occurs around 0.015 Hz when the antenna height is about 8.5 feet [93] [94]. Methods based on frequency-domain processing analyze the frequency spectrum of measurements to localize the frequency content of multipath errors. After that, multipath mitigation can be achieved by removal of the localized frequency content, resulting in multipath-corrected measurements after inverse transforming the modified frequency spectrum to time domain. The frequency spectrum of measurement time series can be obtained using the Fast Fourier Transform (FFT) [93] [6] or Wavelet Transform [7] [21] [72] [77] [94]. The Wavelet Transform can simultaneously provide time and frequency information of a signal at lower computational complexity, $O(n)$, compared to FFT, $O(n \log_2 n)$ [94].

The frequency content of multipath errors can be directly or indirectly obtained. The *direct assessment* approaches use code and carrier-phase measurements directly for analysis. Whereas the *indirect assessment* approaches utilize an intermediate measurement type, such as signal-to-noise ratio (SNR), to access the multipath error before correcting code and carrier-phase measurements.

Direct Assessment Approaches: In an attempt to mitigate code multipath [93], the CmC time series is firstly bias-corrected and filtered by CSF to remove receiver noise before being converted to frequency domain by FFT. In the sequel, the frequency spectrum analysis is performed to bound the frequency region of multipath error on frequency domain representation. Consequently, multipath errors can be eliminated by nulling out the spectrum in its corresponding bounded frequency region. The time series achieved by Inverse Fourier Transform is hence multipath-corrected and is utilized to correct code measurements. Figure 2.6 (image adapted from [93]) demonstrates this approach in a step-wise fashion. On a percentage basis, the standard deviation of the multipath-mitigated CmC is reduced by the order of 50% to 70% with FFT block sizes of 256 and 512 respectively. As expected, the block size needs to be carefully selected in order to leverage the tradeoff between the mitigation effect and the overlapping frequency spectrum of multipath and other measurement error components.

Alternatively, wavelet decomposition can also be applied to the CmC sequence to identify and localize low frequency content of multipath error in a time window [94]. In wavelet processing, a block of the unsmoothed CmC data can be decomposed into two parts: *approximation* and *details* at different levels. The key to multipath mitigation is to select appropriate decomposition level to isolate and discard the approximation which corresponds to multipath error at low frequencies. The decomposition level selection is dependent on the sampling rate, antenna height, obstruction environment, and data block size. The algorithm is as depicted in Figure 2.7 (image adapted from [94]). This technique is reported to reduce the standard deviation of CmC sequence between 55% to 65% with different visible satellites and a block size of 100 seconds. In comparison with CSF, this technique significantly reduces bias errors (up to 40%) due to ionospheric divergence when operating on single-frequency observables.

Similarly, wavelet decomposition has been applied to GPS double-differenced residuals of carrier-phase measurements to address carrier-phase multipath [72] [21]. In order to improve the efficiency of multipath mitigation, the work in [21] further used thresholding techniques to

Figure 2.6: Code multipath mitigation with FFT.

Figure 2.7: Wavelet analysis for code multipath mitigation.

truncate not only the *approximation* but also the *details* at high levels of wavelet decomposition. By choosing appropriate levels of wavelet decomposition and threshold values, a reduction of 84% in term of standard deviation of double difference residual is achieved.

These techniques probably outperform most approaches that have been discussed so far in term of reduction of multipath noises. However, the high performance is achieved at the expense of reducing their locality, since other important phenomena signals and other measurement errors are also likely to be affected [93] [94] [72].

Indirect Assessment Approaches: Side effects of direct assessment on code and carrier measurements give rise to the need for an independent measurement type to assess the multipath error without involving the carrier phase or pseudorange observables. Signal quality measurements (e.g., SNR), which measure signal amplitude of composites of direct and reflected signals that are reported by GPS receivers, appears to be a good candidate. This known sensitivity of SNR to carrier phase multipath makes it desirable for multipath analysis [3] [6] [7] [53] [69].

Figure 2.8: Flow diagram for modeling SNR data to yield multipath corrections.

The SNR measurements are firstly separated into two components: the SNR of the direct signal SNR_d , and the SNR of multipath SNR_m . SNR_d is determined by fitting a low-degree polynomial to the raw SNR sequence on the entire satellite arc [6] [7]. It provides an estimate of the amplitude of the direct signal A_d . Subtracting SNR_d from SNR measurements yields SNR caused by multipath SNR_m . Spectral analysis via sliding-window FFT [6] or wavelet analysis [7] on SNR_m is then performed to provide the time-frequency localization of the dominant frequencies which coincide with dominant multipath frequencies. Each localized estimate of the multipath frequency is then used to estimate the multipath relative phase ψ and multipath amplitude A_m . Once the parameters A_d , A_m , and ψ have been determined, pseudorange multipath and carrier-phase multipath errors can be estimated given carrier-phase multipath model [6] [7]. Applying this technique can reduce 20% RMS of postfit carrier-phase residuals at low elevation angles ($< 20^\circ$) [7] and standard deviation of coordinates up to 13% [70]. These steps are outlined in the flowchart in Figure 2.8.

Because SNR is generally not affected by the signals of phenomena (e.g. earthquakes) [7],

it provides a valuable data type to independently assess multipath errors without any consideration of the signals of phenomena. This is a significant advantage of this method compared to the two previous methods which directly process the carrier phase or pseudorange observables. Another discriminant advantage is that it does not require differential measurements from multiple stations to address carrier-phase multipath. However, signal quality measurements are not always available in RINEX observation file and different GPS receiver types may report signal quality measurements in different units and resolution which make it inapplicable in many situations. Finally, these algorithms are appropriate only for post-processing applications as they cannot run in a real-time fashion [72].

2.4 Discussion

In this section, a summary of all the methods previously discussed will be presented with their strengths and weakness highlighted and compared. From these discussion, some open issues will be discussed and motivations for future studying multipath mitigation problem will be derived.

2.4.1 Summary on the Multipath Mitigation Approaches

Table 2.1 gives an overview of the most important properties defined in Section 2.2 for the multipath mitigation approaches described in Sections 2.3. The approaches are organized into three groups according to the stage of processing. The properties of the approaches within each group will be evaluated and comparisons made between different groups.

For pre-receiver approaches, real-timeness, scalability and locality are maximized because the radio-frequency signals are involved. The multipath rejection and multipath absorption approaches claim to be very effective especially for multipath at low elevation angles [9] [54] but they are less operative to multipath signals reflected from objects above the antenna. The mul-

Multipath Mitigation Approaches		Efficiency		Real-timeness	Scalability	Locality	Availability	Cost
		Code	Carrier					
Pre-receiver Domain	Multipath Prevention Multipath Absorption Multipath Rejection	N/A N/A N/A	N/A N/A N/A	N/A	N/A	N/A	Low	N/A
				N/A	N/A	N/A	High	Fences
				N/A	N/A	N/A	High	Antennae
In-receiver Domain	Tracking Loop	10%-40% [78]	~30% [5]	Yes	Low	High	High	High-end receivers
Post-receiver Domain	Filter-based Stacking-based Direct Assessment Indirect Assessment	~75% [28] 18%-50% [4] 50%-70% [93] N/A	40%-56% [55] ~38% [51] ~84% [21] ~20% [7]	Yes	High	Medium	Medium	N/A
				Yes	Medium	Medium	Medium	N/A
				Yes	Low	Low	High	N/A
				No	Low	High	Low	N/A

Table 2.1: Summary of multipath mitigation approaches.

tipath prevention approaches are of limited value as no environment is completely multipath-free. Furthermore, appropriate spatial locations are required, resulting in its limited availability which is not the case for the other two approaches in the pre-receiver category.

Within the in-receiver category, the shortcoming of the DLL architecture is that performance is limited by chip spacing [15], but chip spacing reduction is limited by hardware issues. Specifically speaking, receiver tracking loops cannot handle multipath delay that is shorter than the chip period. As a consequence, they perform well on long multipath, but not for close-by multipath. Furthermore, performance on multipath mitigation is achieved at the expense of increasing complexity (e.g. MEDLL [83]) as it requires more correlators. This drawback limits the scalability of the technique, implying the maximum sampling rate of a receiver. On the positive side, these approaches ensure real-timeness, locality and availability.

Concerning time-domain approaches in the post-receiver category, their locality is limited as they can unintentionally modify wanted phenomena signals [72] [11]. However, these effects can be diminished if data without displacement is used as a reference, at the expense of decreasing availability as data with displacement cannot be used. Their efficiency and availability also decrease when satellite's orbits change significantly (e.g. satellites being maneuvered) [97] because it results in satellite geometry and multipath errors no longer daily being correlated on a day-to-day basis. On the other hand, filter-based approaches are especially fast and real-time. In the case of FIR filter, the computational complexity depends on the length of the filter [72] [55] which is small, whilst CSF can be accumulatively performed with just a few calculations [38] [58]. Likewise, temporal shifting the stacked multipath is also simple if the time shift period is known.

However, ARTA methods [51] cannot run in real-time as piece-wise shifting requires sufficient amounts of data to be available [97]. Regarding efficiency, stacking-based approaches surpass filter-based counterparts in reduction of multipath errors; however, they requires high resolution data to achieve high performance [4], resulting in storage un-scalability.

Frequency-domain approaches require transforming, both forward and backward. The transformation of signals between time domain and frequency domain also results in high computation cost. Hence, they are not scalable although they can operate in real-time [93] [94]. The Wavelet Transform [72] [94] [21] is more computationally efficient than the FFT [93] [6]. For direct assessment approaches, efficiency and locality will be compromised. More specifically, the trade-off between the efficiency of multipath mitigation and the accidental modification of other measurement error components as well as other signals like earthquake signals, deformation signals, etc. [93] [94] will be leveraged. It is accomplished by carefully choosing the window size in case of FFT and wavelet decomposition level in case of the Wavelet Transform. In contrast, indirect assessment approaches saw high locality since they rely on intermediate measurements (e.g. SNR) which are independent to other kinds of measurement errors and signals. However, SNR measurements are not always available for use. Furthermore, indirect assessment approaches cannot work in real-time [72]. In terms of efficiency, direct assessment approaches, overall, far outperform indirect assessment approaches.

2.5 Conclusions

A comprehensive literature review has been presented with the taxonomies of multipath mitigation approaches. Their strengths and weaknesses has also been discussed. This discussion motivates further studying GPS multipath mitigation. In the next chapter, the principle of the proposed approach in this thesis to address multipath mitigation problem will be incrementally derived. Firstly, the questions how multipath introduces errors to GPS measurements and how satellite-specific multipath errors relate to satellite-receiver geometrical parameters will be answered. As a result, estimation of multipath errors amounts to a regression problem which will be solved by standard regression algorithm thereafter.

Chapter 3

Principles of GPS Multipath and Multipath Geometrical Models

In this chapter, a review of the effects of multipath-delayed signals on common GPS observables: code and carrier phase measurements in the context of GPS tracking loops will be conducted to clarify how multipath introduces errors to the pseudorange and carrier phase measurements. In the sequel, the relationship between satellite-specific multipath errors and geometry of the satellite w.r.t. a receiver will be explored to explain the geometrical models of multipath errors. Taking advantage of these models, the multipath errors can be viewed as functions of the satellite's geometry that are characterized by azimuth and elevation angles. Multipath estimation then amounts to approximation of these functions, i.e. a regression problem.

3.1 GPS Signal Acquisition and Tracking

Most modern GPS receiver designs are digital receivers whose generic architecture is as shown in Figure 3.1 (image from [86]). Briefly, the process of GPS acquisition and tracking for a single GPS satellite can be explained as follows. The GPS radio-frequency (RF) signals received by the Right-Hand Circular Polarized (RHCP) antenna are amplified by a low noise

Figure 3.1: Generic digital GPS receiver block diagram.

preamplifier. These amplified signals are then down-converted to an intermediate frequency (IF) by mixing with a locally-generated reference signal. Next, the A/D conversion process takes place at IF. At this point the digitized IF signals are ready to be processed by each of the N digital receiver channels.

In each digital receiver channel, the digital IF signal is then separately mixed with cosine and sine versions of a locally-generated carrier signal, yielding two output data channels: in-phase (I) channel and quadrature (Q) channel. These I and Q channels are 90 degrees out-of-phase with respect to each other. The I and Q signals are then correlated with early, prompt, and late replica codes (plus code Doppler) of locally-generated version of code. As a result, there are three replica code phases designated as Early (E), Prompt (P), and Late (L). E and L are typically spaced in phase by 1 chip, and P is in the middle of E and L.

For code tracking, the code tracking loop seeks to shift the local-generated code in time until maximum correlation is achieved between local and received signals. At this point, the time shift required for maximum correlation is used to compute the signal transmission time and the pseudorange measurement is obtained by multiplying the transmission time by the signal propagation constant.

By correlating with the local generated code, the received code is stripped off from the

incoming signal and converted to baseband, leaving only the received carrier in I and Q channels. This recovered carrier signal is then fed to a carrier tracking loop that seeks to match the recovered carrier with a locally-generated version by minimizing their phase difference. Once the locally-generated carrier's frequency has been altered to achieve lock with the incoming carrier, the tracking loop outputs the change in phase of the signal over time, i.e. the carrier phase measurement. More comprehensive GPS signal acquisition and tracking can be found in [59] [86].

3.2 Signal Tracking under Multipath

Ideally, in a multipath-free environment, only one direct signal is received by the antenna from each satellite. However, no environment is completely multipath-free in practice. Receiver antennas receive one or more replicas of the direct signal reflected from objects in its vicinity. As a result, the receiver will track the composite signal that is a combination of the direct signal and the multipath replicas. Many references have developed expressions for the measurement errors produced by multipath [9] [35] [67]. The most complete models include features of the tracking loops. This section will present a basic model that is sufficient for this chapter's purpose.

For clarity, the simplified case of one multipath signal will be considered before generalizing for multiple multipath signals. A_d and A_m are defined as the amplitudes of the direct signal and the multipath signal respectively and $\alpha = A_m/A_d$ as the ratio of the multipath and direct amplitudes. When a replica of the direct signal reflected by a surface from the surrounding environment is introduced, its amplitude is typically diminished [59]. Therefore, $A_m \leq A_d$ which leads to $\alpha \leq 1$. Since a multipath signal arrives by an indirect path which is longer than the direct path, the additional path length introduces a range error in the measurements. We denote the path delay as δ . Furthermore, the reflected signal's carrier wave, upon reception by the GPS antenna, will likely have a different phase compared to the direct signal. The differ-

ence in the phases of the two signals is described as the multipath relative phase, ψ , in radians. The pseudorange multipath error and the carrier-phase multipath error are denoted by ρ_{MP} and $\delta\phi$ in units of meters and cycles, respectively. The relationship between code, carrier-phase multipath errors, the path delay δ and the relative phase ψ during operation of tracking loops of a receiver can be established.

3.2.1 Code Multipath Error

The effect of code multipath on code tracking can be summarized in Figure 3.2. In the diagrams, $2T_{E/L}$ represents the correlator spacing where $T_{E/L}$ is the chip length, and zero is given for reference as the correct Prompt code correlation. The horizontal double-tipped arrow shows the spacing between Early and Late correlation values.

The output of any code tracking loop can be represented in terms of the triangle correlation function which is represented in Figure 3.2a. When no multipath is present, the correlation function is a perfect triangle with peak correlation achieved at the position of the Prompt correlator (zero time offset). When multipath occurs, one or more smaller correlation peaks are introduced [59]. Summing these multipath correlation functions with the direct signal correlation leads to a composite correlation function which is a distorted triangle as in Figure 3.2b and Figure 3.2c. Note that the effect of code multipath may be either positive (longer range) or negative (shorter range), depending on whether the multipath correlation function sums additively (Figure 3.2b) or subtractively (Figure 3.2c) with the direct signal's correlation peak [4] [59]. Eventually, the distorted correlation peak leads to erroneous values in correlator outputs. Since the position of the Prompt correlator determines the pseudorange observation, the offset of the new Prompt correlation from the true (undistorted) Prompt correlation is the code tracking error due to multipath ρ_{MP} .

By denoting $R(\cdot)$ to be the autocorrelation function of the code and assuming the carrier tracking error to be zero, the pseudorange error due to multipath, ρ_{MP} , is implicit in the ideal

Figure 3.2: Effect of multipath on code correlation function for different scenarios: (a) multipath-free scenario, (b) scenario with one multipath signal in-phase with the direct signal, (c) scenario with one multipath signal out-of-phase with direct signal.

code tracking loop discriminator equation as (3.1) [4]:

$$[R(\rho_{MP} + T_{E/L}) - R(\rho_{MP} - T_{E/L})] + \alpha[R(\rho_{MP} + T_{E/L} - \delta) - R(\rho_{MP} - T_{E/L} - \delta)] \cos \psi = 0 \quad (3.1)$$

If the multipath delay δ is long compared to the chip length $T_{E/L}$ (approximately 300 m or 1 μ s for the C/A code), the multipath will not cause any pseudorange errors [59]. Therefore, it is reasonable to assume that the additional path length traveled by the multipath signal is shorter than the chip length, i.e. $\delta < T_{E/L}$. In addition, if an ideal triangular autocorrelation function is assumed:

$$R(\rho) = 1 - \frac{|\rho|}{T} \text{ for } |\rho| < T \quad (3.2)$$

the code multipath error can be solved explicitly:

$$\rho_{MP} = \frac{\alpha \delta \cos \psi}{1 + \alpha \cos \psi} \quad (3.3)$$

The actual multipath error varies as the relative phase changes. The upper bound for code multipath error corresponds to additive interference, and the lower bound corresponds to subtractive interference. If the path delay is small, then the error bounds are independent of the correlator spacing. For these short delays, error bounds are functions of relative multipath amplitude and path delay [59]. As the path delay increases, the narrow correlator spacing will help filter out long path delay. Smaller correlator spacing has good effect in causing the error bounds to be smaller [59]. Therefore, narrow correlators are preferred for modern receivers.

In practice, code multipath errors can be isolated by analyzing experimental data without the necessity of accessing the receiver hardware and correlation function. By taking advantage of dual-frequency observables, it is common to estimate the code multipath error based on the difference between the code and carrier-phase observables, corrected for ionospheric delay. The linear combinations named the pseudorange multipath combinations of the GPS carrier

Figure 3.3: Phasor diagram showing the relationship between I and Q components of the signal and the phase angle ϕ_d during carrier tracking loop operation when one multipath signal presents.

phase and pseudorange observables can be formed to effectively isolate the pseudorange multipath error (more details in Chapter 4).

3.2.2 Carrier-Phase Multipath Error

Similarly, multipath signals also affect the carrier-phase signal because the receiver tracks a composite signal which is the sum of the direct and one or more multipath signals. A carrier tracking loop can be represented in terms of a phasor diagram [4] [86] in Figure 3.3 which shows the phase relationship between the I and Q channels. A_c denotes the amplitude of the composite signal which is the combination of the multipath signal with amplitude A_m and the direct signal with amplitude A_d . ϕ_d , ϕ_c , ψ are the direct signal's phase, the composite signal's phase and the multipath relative phase with w.r.t. the direct signal respectively. $\delta\phi$ denotes the phase error due to multipath.

When no multipath signal is present, the phasor diagram would contain a single phasor of amplitude A_d ; any misalignment of the local and incoming carriers result in a non-zero phase angle ϕ_d . The GPS carrier phase is measured by keeping track of ϕ_d . In case when one or more

multipath signals are introduced, additional phasors will appear in the phasor diagram. The carrier tracking loop attempts to track a composite signal which is a summing vector of direct phasor and multipath phasors. As a result, the carrier tracking loop reports an incorrect phase measurement with phase error $\delta\phi$. From the phasor diagram, the phase error due to multipath is easily derived in terms of multipath parameters as follows:

$$\begin{aligned}\tan(\delta\phi) &= \frac{A_m \sin(\psi + \phi_0)}{A_d + A_m \cos(\psi + \phi_0)} \\ &= \frac{\alpha \sin(\psi + \phi_0)}{1 + \alpha \cos(\psi + \phi_0)}\end{aligned}\quad (3.4)$$

where ϕ_0 is a possible phase offset at time 0. Therefore,

$$\delta\phi = \arctan\left(\frac{\alpha \sin(\psi + \phi_0)}{1 + \alpha \cos(\psi + \phi_0)}\right) \quad (3.5)$$

The magnitude of the phase error changes over time as the phase difference between direct and multipath signals changes. In terms of the phasor diagram, changes in ψ cause the multipath phasor to spin around the end of the direct phasor. The phase errors then oscillate between an absolute maximum $\delta\phi$ when $\psi = 90^\circ$ or 270° and a minimum (no phase error) when $\psi = 0^\circ$ or 180° . The maximum possible phase multipath error is substantially smaller than possible pseudorange multipath errors. If the signal is unattenuated at the reflection interface; that is, the direct signal amplitude is equal to the reflected signal amplitude and $\alpha = 1$. From this maximum value and equation (3.5), it is apparent that carrier-phase errors can reach a maximum of $\pm\frac{1}{8}$ of a cycle or a total phase error of 0.25λ where λ is signal wavelength, i.e. 4.7 cm for L1 and 6.1 cm for L2 phase measurements. When phase data are combined in common linear observable combinations for analysis, multipath errors can be amplified by 2 to 9 times of their single-frequency values [59]. High-precision applications of GPS rely heavily on the accuracy of phase measurements, so even centimeter-level phase errors albeit small are still of concern.

For the sake of completeness, the composite amplitude A_c is also reported as signal-to-noise

ratio (SNR) measurements. Due to the link of SNR measurement and the multipath error, this measurement type can be used to obtain multipath parameters [3] [6] [7]. Unfortunately, it is not always reported by receivers.

Because carrier phase multipath is a potential source of error in high-precision applications, a measurement of phase multipath derived from common GPS observables is desired. Unfortunately unlike code multipath, no linear combination of the GPS observable equations exists that isolates the effects of carrier-phase multipath for a standalone receiver. However, on a positive note, combination of carrier-phase multipath errors from multiple receivers can be extracted in relative positioning using double-differential techniques (more details in Chapter 5).

3.3 Geometrical Models of GPS Multipath Errors

In the previous section, code and carrier phase multipath errors have been linked to the path delay δ and the multipath relative phase ψ . However, computing multipath errors based on these variables is difficult or even infeasible because the surrounding environment and the number of multipath signals have to be modeled. Hence, it is necessary to relate multipath errors to variables that can be evaluated. Intuitively, under the assumption that the environment is constant, changes of multipath errors would be caused by motion of a satellite w.r.t. a receiver and multipath error should repeat when the satellites geometry w.r.t. a receiver repeats. In other words, there should be an underlying relationship between multipath errors and the satellite-receiver respective geometry which is characterized by azimuth and elevation angles. In the literature, it is common that multipath errors were considered as functions of satellite geometry [4] [8] [14]. However, to the best of our knowledge, this has not been mathematically proven. This section aims to reveal the multipath functions of geometrical parameters of a satellite w.r.t. a receiver assuming that the multipath environment is constant. This important evident is the basis for the study of multipath mitigation problem in the successive chapters of this report.

Figure 3.4: The direct signal from a GPS satellite and one multipath signal with simplified geometry.

The position of any reflecting object is described as a planar surface tilted relative to the local level with a tilt angle γ at a distance h from the antenna center. Figure 3.4 graphically depicts the simplified case of one planar surface and one multipath signal, where the multipath signal reflects off the surface with the angle of incidence β w.r.t. the reflecting surface.

Based on the reflecting geometry, multipath reflections fall into two categories: *forward-scatter* and *backscatter* [6] [29] as illustrated in Figure 3.5. Forward-scattering occurs when the reflection point of the multipath is located somewhere between the satellite and the antenna on the horizontal direction (Figure 3.5a). Conversely, backscattering occurs when the distance between the satellite and multipath reflection point is larger than the distance between the satellite and antenna (Figure 3.5b). The difference between forward-scatter and backscatter reflections is the magnitude of the path delay. The path delay of a backscatter reflection includes the travelling path after reflecting off the surface and the additional delay relative to the direct signal. Whereas, the forward scatter reflections produce a single component of path delay, the additional path length traveled after reflection. The path delays in the two scenarios are highlight in red in Figure 3.5.

In order to derive the mathematical relationship between the multipath errors and the geometry of a satellite w.r.t. a receiver, path delay δ and the relative phase of the multipath signal

Figure 3.5: Multipath reflections: (a) Forward-scatter, (b) Backscatter.

and the direct signal ψ appearing in equations (3.3) and (3.5) need to be related to the geometrical parameters: azimuth and elevation angles denoted by θ and φ respectively. For simplicity, considered the simplified geometry in Figure 3.5 with assumption that the satellite, antenna, and normal vector to the reflecting surface are all coplanar (this is why the azimuth angle does not present in the figure).

In the forward-scatter scenario, it is easily to obtain the path delay:

$$\delta = 2h \sin \beta \quad (3.6)$$

For backscatter scenario, the path delay comprises of two segments δ_1 , δ_2 but it is still mathe-

atically identical to the forward-scatter case since:

$$\begin{aligned}
 \delta &= \delta_1 + \delta_2 \\
 &= \delta_2 \cos(\pi - 2\beta) + \delta_2 \\
 &= \delta_2(1 - \cos 2\beta) \\
 &= \frac{h}{\sin \beta} 2 \sin^2 \beta \\
 \delta &= 2h \sin \beta
 \end{aligned}$$

Turning to multipath relative phase ψ , due to angular nature, path delay from (3.6) (in distance units) is easily converted to a quantity with units of radians by 2π -modulo:

$$\begin{aligned}
 \psi &= \frac{2\pi}{\lambda} \delta \\
 &= \frac{2\pi}{\lambda} 2h \sin \beta
 \end{aligned} \tag{3.7}$$

where λ is the wavelength of the selected GPS frequency.

From equations (3.6) and (3.7), it turns out that the time-varying code and carrier-phase multipath errors depend only on evolution of the incident angle β because for a fixed receiver, the distance from the antenna to the reflector is constant under the assumption of constant environment. So what need to be done is to relate the angle of reflection β to the satellites elevation and azimuth angles.

Snell's Law dictates that the angle of reflection is equal to the angle of incidence for specular reflections, so the incident and reflected signals have equivalent angles of β relative to the reflecting surface. Using the convention of angles measured counter-clockwise over the interval $[0^\circ 180^\circ]$, $\beta + \gamma = \theta$ as shown in Figure 3.5a whereas $\beta + \theta = \gamma$ in Figure 3.5b. To generalize, the reflection angle can be determined in terms of any satellite elevation angle θ

and any reflector tilt angle γ :

$$\beta = |\theta - \gamma| \quad (3.8)$$

In most cases, the reflection angle β cannot be determined because the exact multipath environment is unknown. However, when the surface orientation γ is held constant, the rate of change of β is generally equivalent to the rate of change of the satellite's elevation angle θ (and azimuth angle φ involved in the general case). For a single reflector, γ is a constant and does not vary with time. In most physical environments, no single reflector orientation can describe the entire multipath environment but it is reasonable to assume that the reflectors remained fixed for some periods [6]. Eventually, substituting (3.6) and (3.8) into (3.3) and (3.5), the code and carrier-phase multipath equations corresponding to one reflecting signal can be re-written as:

$$\rho_{MP} = \frac{\alpha 2h \sin(|\theta - \gamma|) \cos(\frac{4\pi h}{\lambda} \sin(|\theta - \gamma|))}{1 + \alpha \cos(\frac{4\pi h}{\lambda} \sin(|\theta - \gamma|))} \quad (3.9)$$

$$\delta\phi = \arctan\left(\frac{\alpha \sin(\frac{4\pi h}{\lambda} \sin(|\theta - \gamma|) + \phi_0)}{1 + \alpha \cos(\frac{4\pi h}{\lambda} \sin(|\theta - \gamma|) + \phi_0)}\right) \quad (3.10)$$

In the general case of m reflecting signals, the total code multipath and carrier phase multipath is the sum of the individual code multipath and carrier-phase multipath:

$$\rho_{MP} = \frac{\sum_{i=1}^m \alpha_i 2h_i \sin(|\theta - \gamma_i|) \cos(\frac{4\pi h_i}{\lambda} \sin(|\theta - \gamma_i|))}{1 + \sum_{i=1}^m \alpha_i \cos(\frac{4\pi h_i}{\lambda} \sin(|\theta - \gamma_i|))} \quad (3.11)$$

$$\delta\phi = \arctan\left(\frac{\sum_{i=1}^m \alpha_i \sin(\frac{4\pi h_i}{\lambda} \sin(|\theta - \gamma_i|) + \phi_0^i)}{1 + \sum_{i=1}^m \alpha_i \cos(\frac{4\pi h_i}{\lambda} \sin(|\theta - \gamma_i|) + \phi_0^i)}\right) \quad (3.12)$$

With the assumption that the multipath environment is held fixed, the environmental parameters for every multipath signal i , including α_i , h_i , and γ_i , are constant. That is, code multipath ρ_{MP} and carrier-phase multipath $\delta\phi$ only depends on changes of the relative elevation θ (and the relative azimuth φ in the general case) which describe the motion of the satellite

w.r.t. the receiver. In other words, the code multipath error ρ_{MP} and carrier-phase multipath $\delta\phi$ are functions of the elevation θ and the azimuth φ expressed in the equations (3.11) and (3.12) respectively. Given these functions, in order to compute multipath errors, it is necessary to somehow evaluate the derived functions given an azimuth/elevation pair. The satellite elevation and azimuth angles as viewed from a GPS station can be easily determined using readily-available orbital information and a reasonable estimate of the GPS station's location. However, the multipath functions look complicated and they are actually difficult to evaluate because the unknown number of multipath signals m and unknown environmental-dependent constants α_i , h_i , γ_i . Hence, these functions need to be somehow approximated. And approximation of the functions in the equations (3.11) and (3.12) is equivalent to a regression problem given training data. Viewed as a regression problem, these functions will be approximated by learning from the historical multipath data and satellite orbital information. The successive chapters will incrementally develop this approach for code and carrier-phase multipath estimation.

3.4 Conclusions

The operation of a GPS receiver's tracking loops have been briefly reviewed. By analyzing the operation of a GPS receiver's tracking loops under multipath conditions and geometries of multipath reflections, this chapter has linked satellite-specific multipath errors induced in code and carrier-phase measurements to respective geometry of the satellite with a receiver which is featured by azimuth and elevation angles. With this established relationship, multipath errors can be estimated given an azimuth/elevation pair if the derived functions can be evaluated. By posing multipath estimation as a regression problem, the multipath functions can be approximated using historical data. In the next chapter, formulation and experiments on estimation of code multipath error will be conducted using the proposed approach presented in this chapter. Estimation of carrier-phase multipath error will be addressed in Chapter 5.

Chapter 4

Mitigation of Code Multipath with Nonlinear Regression

4.1 Introduction

Since the day GPS started operations, there has been a continuing interest to mitigate the effects of multipath in GPS code and carrier phase measurements. Although many positioning algorithms rely mainly on the carrier-phase observations, accurate pseudorange observations are important to a variety of applications such as ionospheric monitoring, geodesy and navigation applications [8]. In those applications, pseudorange multipath can be the dominating error, seriously degrading the positioning accuracy that relies on accurate measurements of the pseudorange observable.

For ionospheric monitoring applications, efficient multipath mitigation techniques could extend the monitoring capability for ionospheric delay to lower elevation angles, where multipath is typically large [8]. Extending elevation coverage down toward the horizon from 15 degrees could double the radius of coverage at ionospheric altitudes. This means a GPS-based ionospheric monitor could, in the limit, quadruple the area of the monitored ionosphere. Furthermore, extending elevation coverage toward the horizon would significantly increase the

visibility of GPS satellites and improve the geometric dilution of precision (GDOP).

For applications dependent on pseudorange DGPS [65], a reference station is located over a known benchmark, thus the pseudorange measurements can be compared to the true satellite ranges to generate range corrections for broadcast to users. This range correction is composed of errors from the atmosphere, satellite orbits and multipath. Of these errors, only multipath is uncorrelated between the reference station and the user, thus acting as a limiting factor in DGPS accuracy.

Code multipath mitigation also aids carrier-phase ambiguity resolution techniques for applications requiring centimeter-level accuracy [47]. If the multipath error in pseudorange fluctuates largely, the initial ambiguity for the satellite is biased and its search space can be enlarged. As a result, a longer time is needed to resolve the ambiguity and the probability of successful integer ambiguity estimation decreases. Pseudorange multipath reduction would help minimize searching for the correct carrier phase ambiguity set [47] [95].

In this chapter, a novel method to efficiently mitigate code multipath error of a fixed receiver is proposed. With the developed geometrical model of code multipath error in Chapter 3, the code multipath estimation is posed as a regression problem where the multipath function can be approximated through learning from historical data. Based on the analysis of the repetition of GPS satellites' geometries with respect to a fixed ground station and the repetition of multipath errors, training data for the regression problem can be extracted. The geometrical information is computed from broadcast navigation data whereas the code multipath error can be isolated with the aid of carrier-phase measurements. The ability to extract training data makes the regression problem solution feasible. As a result, a multipath estimator can be created for each specific satellite. This multipath estimator can be applied to successive days of data to correct pseudorange observables. Experimental results based on logged GPS data show that the proposed method achieves state-of-the-art performance in code multipath mitigation.

Figure 4.1: The observation site on the rooftop of N2 building (NTU campus).

4.2 Experimental Data Set

Throughout this chapter, a recorded data set will be used for analysis. This data set was recorded at 0.1 Hz at the station equipped with Trimble NetRS receiver on the rooftop of N2 building in Nanyang Technological University (NTU) campus during five consecutive days: from the day of year (DOY) 306 to 310 of 2010. The nominal position of the observation site is $(-1507932.6167, 6195587.6757, 148897.9990)$ in earth-centered, earth-fixed (ECEF) Cartesian coordinate system [59]. Rooftops are usually bad multipath environments since there are often many vents and other reflective objects within the GPS antenna field of view. In the photos shown in Figure 4.1, the observation site is surrounded by many buildings and reflectors which make multipath potentially more severe.

Over the course of the experiment, 31 visible satellites ranging from PRN 2 to PRN 32 were observed. The GPS constellation is organized as six orbital planes with the satellites assigned to each plane tabulated in Table 4.1 [60].

Plane	PRNs					
A	9	8	7	27	31	
B	16	25	28	12	30	
C	29	3	19	17	6	
D	2	11	21	4	24	
E	20	22	18	32	5	10
F	14	15	13	23	26	

Table 4.1: GPS orbital plane assignments in 2011.

4.3 Repeatability of GPS Constellation's Geometry

It is well-known that the GPS satellite orbits were selected to have a period of half a sidereal day (23 hours 56 minutes 4 seconds) with a daily repeating ground track [11] [30] [62]. Because of this, satellite visibility from any point on earth is the same from day to day, with the satellites appearing in their positions approximately 4 minutes or 236 seconds earlier each day due to the time difference between the sidereal and solar day.

To illustrate the repeatability of GPS satellites' geometries, Figure 4.2 plots the geometries of visible satellites of orbital plane B with respect to the observation site, extracted from the recorded data set during 4 consecutive days from DOY 306 to DOY 309. More plots for satellites on other orbital planes are also provided in the Appendix where the satellites are grouped according to all six orbital planes. Each satellite's geometry is characterized by its azimuth and elevation angles respective to the observation station. As seen from these plots, the footprints of the day-to-day repeated geometries of the satellites are obviously exposed.

There are various ways to compute orbital repeatable periods of GPS satellites [4] [1]. In reality, the orbital period is not exactly half of the nominal sidereal day [19] [74]. The perturbations of the orbits are caused by non-central gravity field of the earth, the gravitational attraction of the sun, moon, and other planets, solar radiation pressure and atmospheric drag [4]. As a consequence, repeatable periods vary with time and determining their values becomes difficult and complicated as it requires models to account for various affective forces.

Figure 4.2: Repeatable geometries of the satellites in the orbital plane B from DOY 306 to DOY 309 of 2010.

4.4 Repeatability of Pseudorange Multipath Error

For a fixed station, its surrounding environment and the antenna usually do not change significantly across consecutive days. Therefore, GPS multipath signals are expected to repeat over the same time period [30] [54] although variations do occur under certain conditions such as when the surface moisture content changes [43] or when the satellite orbits are significantly altered [25].

To investigate multipath repeatability for a fixed ground site, six satellites from six orbital planes: PRN 9 (plane A), PRN 12 (plane B), PRN 3 (plane C), PRN 21 (plane D), PRN 10 (plane E), and PRN 13 (plane F) are chosen for exploration. For each satellite, four pseudorange multipath sequences are extracted from the recorded data set during four observation days from DOY 306 to DOY 309 of 2010. In order to reveal the repeatability of multipath sequences, the sequences are smoothed with 50-second CSF to remove high-frequency noise.

The CSF is implemented in recursive form [93] [59] as show in equation (4.1):

$$\bar{\chi}(t+1) = \frac{\tau-1}{\tau}\bar{\chi}(t) + \frac{1}{\tau}\chi(t+1) \quad (4.1)$$

where τ , χ , $\bar{\chi}$ and t represent the smoothing window, CmC, smoothed CmC and epoch index respectively. Day-to-day correlation of the two multipath sequences can be numerically evaluated by their normalized cross-correlation.

For purpose of demonstration, the plots for the PRN 12 are shown in Figure 4.3a and Figure 4.3b where the original multipath sequences are plotted in blue and CSF-smoothed multipath sequences are plotted in red. Pair-wise normalized cross-correlation values of the multipath sequences are tabulated in Table 4.2 with blue and red figures representing the original and smoothed multipath sequences respectively. The results for other satellites can be found in the Appendix. All satellites exhibit very high day-to-day correlation, in case of PRN 12 it is around 89%, between two consecutive days with the correlation slowly degrading with time. This is understandable since accumulative environment changes become noticeable when the time span increases.

	DOY 306	DOY 307	DOY 308	DOY 309
DOY 306	N/A	0.6668	0.5215	0.5215
DOY 307	0.9055	N/A	0.6513	0.5254
DOY 308	0.8775	0.8988	N/A	0.6260
DOY 309	0.8649	0.8630	0.8851	N/A

Table 4.2: Normalized cross-correlation of PRN 12's multipath sequences.

In the literature, the sidereal day-to-day correlation of GPS constellation's geometry and multipath has been taken advantage of for multipath research [11] [62] [66] [97]. In those works, the stacked multipath of multiple reference days is shifted in time by the orbital repeat period to correct multipath for observation of successive days. However, it is difficult to accurately determine orbital repeated periods of GPS satellites [4] [14] [51] as discussed in Chapter 2. In addition, multipath correction by time-shifting methods requires high temporal resolution

(a) Original

(b) 50-second CSF-smoothed

Figure 4.3: Multipath time series of PRN 12 of the plane B.

(e.g. high data rate) to boost performance [4]. For example, if the sampling rate of the data is 10 seconds per epoch and the time shift modulus the sampling interval is smaller than 10, let say 5 seconds, how can one shift multipath sequences for stacking and for multipath correction? This gives rise to a need for alternative methods which can take advantage of repeatable properties of satellite's geometry and multipath error.

4.5 Pseudorange Multipath Mitigation with Nonlinear Regression

Using the code multipath geometrical model in equation (3.11) in Chapter 3, code multipath estimation can arguably be posed as a regression problem. This section will present how the code multipath estimation function can be approximated for each satellite by solving the regression problem. At first, a Support Vector Machine (SVM) learning algorithm named ϵ -SVR for linear regression problems will be presented. After that, nonlinearity of the regression problem will be handled by introducing kernel mapping. Finally, the code multipath estimation will be formulated as a regression problem and solved by ϵ -SVR algorithm.

4.5.1 Linear regression problem formulation with ϵ -SVR

SVM is a popular “off-the-shelf” supervised learning algorithm that is widely considered to perform well. As it would be too ambitious to cover the whole SVM story, more details and comprehensive material for SVM can be found in [76].

Denoted by $x \in \mathcal{X}$, an input vector where $\mathcal{X}$ represents the space of the input patterns, for instance $\mathbb{R}^d$. $y \in \mathfrak{R}$ denotes the target output corresponding to the input vector x . The goal is to learn a function $f : \mathcal{X} \mapsto \mathfrak{R}$ that maps from the input vector x to the target output y . Formally, this can be accomplished by first choosing a set of N training samples $\{(x_1, y_1), \dots, (x_N, y_N)\} \subset \mathcal{X} \times \mathfrak{R}$. Due to noise in the training data, it is unlikely that $f(x_i)$

will be equal to y_i for all x_i , so a loss function $L(f(x); y)$ must also be chosen to quantify the penalty for $f(x_i)$ differing from y_i . The estimator f can be found by minimizing the total loss over the training data. In ϵ -SVR [76] [84], the function $f(x)$ has at most ϵ deviation from the actually target y_i for all training data, meaning that one does not care about errors as long as they are less than ϵ , but will not accept any deviation larger than this. At the same time, the function $f(x)$ does not overfit the data.

Denote the linear regression function as:

$$f(x) = \langle w, x \rangle + b \quad (4.2)$$

Here, $w \in \mathcal{X}$ is the weight vector in the input space, $b \in \mathfrak{R}$ is a bias term, $\langle \cdot, \cdot \rangle$ denotes the dot product. One way to ensure non-overfitting is to minimize the Euclidean norm $\|w\|^2$ [76]. Formally, this regression problem can be written as a convex optimization problem as:

$$\begin{aligned} & \text{minimize} && \frac{1}{2} \|w\|^2 \\ & \text{subject to} && \begin{cases} y_i - \langle w, x_i \rangle - b \leq \epsilon \\ \langle w, x_i \rangle + b - y_i \leq \epsilon \end{cases} \end{aligned} \quad (4.3)$$

With equation (4.3), if the convex optimization is feasible, the function f approximates all pairs (x_i, y_i) with ϵ precision. Sometimes, however, this may not be the case when some outputs are outside the ϵ -tube. In order to overcome this, one can introduce slack variables ξ_i, ξ_i^* to cope with otherwise infeasible constraints of the optimization problem (4.3). Hence, it arrives at the

Figure 4.4: The soft margin loss setting corresponds for a linear SV machine.

formulation stated in [84]:

$$\begin{aligned}
 & \text{minimize} && \frac{1}{2} \|w\|^2 + C \sum_{i=1}^N (\xi_i + \xi_i^*) \\
 & \text{subject to} && \begin{cases} y_i - \langle w, x_i \rangle - b \leq \epsilon + \xi_i \\ \langle w, x_i \rangle + b - y_i \leq \epsilon + \xi_i^* \\ \xi_i, \xi_i^* \geq 0 \end{cases}
 \end{aligned} \tag{4.4}$$

The constant $C > 0$ controls the trade-off between the slack variable penalty and the Euclidean norm $\|w\|^2$. Any error smaller than ϵ does not require a nonzero ξ_i or ξ_i^* and hence does not enter the objective function. The formulation above corresponds to dealing with the so-called *ϵ -insensitive loss function* [84]:

$$L(f(x), y) = |\xi|_\epsilon = \begin{cases} 0 & \text{if } |f(x) - y| < \epsilon \\ |f(x) - y| - \epsilon & \text{otherwise} \end{cases} \tag{4.5}$$

Figure 4.4 (image from [76]) depicts equation (4.4) graphically. Only the points outside the shaded region contribute to the objective function whilst the deviations of the points outside the tube are linearly penalized as in (4.4).

By introducing Lagrange multipliers [76] to the optimization problem (4.4), the following

formulization can be obtained:

$$\begin{aligned}
 & \text{maximize} && -\epsilon \sum_{i=1}^N (\alpha_i + \alpha_i^*) + \sum_{i=1}^N (\alpha_i - \alpha_i^*) y_i \\
 & && -\frac{1}{2} \sum_{i,j=1}^N (\alpha_i - \alpha_i^*) (\alpha_j - \alpha_j^*) \langle x_i, x_j \rangle \\
 & \text{subject to} && \begin{cases} \alpha_i, \alpha_j \in [0, C] \\ \sum_{i=1}^N (\alpha_i - \alpha_i^*) = 0 \end{cases}
 \end{aligned} \tag{4.6}$$

here $C, \epsilon > 0$ are chosen *a priori* and α_i, α_i^* are Lagrange multipliers. This leads to the regression solution, that is so-called *Support Vector* expansion, taking the form:

$$f(x) = \sum_{i=1}^{N_{SV}} (\alpha_i - \alpha_i^*) \langle x_i, x \rangle + b \tag{4.7}$$

where w is implicitly contained in the solution without the need to compute explicitly when evaluating the function. It can be completely described as a linear combination of the training patterns x_i :

$$w = \sum_{i=1}^{N_{SV}} (\alpha_i - \alpha_i^*) x_i \tag{4.8}$$

and b can be computed as follows:

$$\begin{aligned}
 b &= y_i - \langle w, x_i \rangle - \epsilon \text{ for } \alpha_i \in (0, C) \\
 b &= y_i - \langle w, x_i \rangle + \epsilon \text{ for } \alpha_i^* \in (0, C)
 \end{aligned} \tag{4.9}$$

An important property of the SV expansion is *sparsity* [76]. That is, only for $|f(x_i) - y_i| \geq \epsilon$ the Lagrange multipliers may be nonzero. In other words, for all samples inside ϵ -tube (i.e. $|f(x_i) - y_i| < \epsilon$), the α_i, α_i^* vanish. Therefore, expansion of w is sparse in terms of x_i (i.e. one does not need all x_i to describe w). The samples that come with non-vanishing coefficients are called Support Vectors. Complexity of a function which is represented by SVs is independent on the dimensionality of the input space, and depends only on the number of SVs. In addition,

the complete algorithm can be described in terms of dot products between the data points. Even when evaluating $f(x)$, there is no need to compute w explicitly.

4.5.2 Nonlinear kernel mapping

The algorithm above converges only for linearly dependent data. If the data set is not linearly dependent, one can map the samples into a feature space of higher dimensions:

$$x \mapsto \Psi(x) \quad (4.10)$$

in which the dependency between inputs and outputs is linear. This, for instance, could be achieved by simply preprocessing the training patterns x_i by a map $\Psi : \mathcal{X} \mapsto \mathcal{F}$ into a feature space $\mathcal{F}$ [2] [61] and then applying the standard SV regression algorithm. However, this approach becomes computationally infeasible if the feature space $\mathcal{F}$ has high, or even infinite, dimensionality [76]. A computationally cheaper alternative has to be found.

Fortunately, the mapping function $\Psi(x)$ does not need to be explicitly specified. Instead, only the dot product of the vectors in the new space since the vectors x_i appear only in dot products in both regression solution and learning procedure. The function $\Psi(x)$ is a kernel-induced implicit mapping [76]. A kernel is defined as a function that takes two vectors x_i and x_j as arguments and returns the value of the dot product of $\Psi(x_i)$ and $\Psi(x_j)$ in feature space:

$$\kappa(x_i, x_j) = \langle \Psi(x_i), \Psi(x_j) \rangle \quad (4.11)$$

Therefore, using kernel functions, the dimensionality of the new space becomes unimportant. Properties of a kernel function can be found in [76].

The learning algorithm in the kernel space can be obtained by replacing all the dot products of the learning algorithm in the original space with kernels. Thus, this allows rewriting the

Support Vector algorithm as follows:

$$\begin{aligned}
 &\text{maximize} && -\epsilon \sum_{i=1}^N (\alpha_i + \alpha_i^*) + \sum_{i=1}^N (\alpha_i - \alpha_i^*) y_i \\
 &&& -\frac{1}{2} \sum_{i,j=1}^N (\alpha_i - \alpha_i^*) (\alpha_j - \alpha_j^*) \kappa(x_i, x_j) \\
 &\text{subject to} && \begin{cases} \alpha_i, \alpha_j \in [0, C] \\ \sum_{i=1}^N (\alpha_i - \alpha_i^*) = 0 \end{cases}
 \end{aligned} \tag{4.12}$$

The expansion of f may be written as:

$$f(x) = \sum_{i=1}^{N_{SV}} (\alpha_i - \alpha_i^*) \kappa(x_i, x) + b \tag{4.13}$$

and w are implicitly contained in (4.13) as:

$$w = \sum_{i=1}^{N_{SV}} (\alpha_i - \alpha_i^*) \Psi(x_i) \tag{4.14}$$

The difference to the linear case is that the regression function is in feature space, not in input space anymore.

4.5.3 Learning Pseudorange Multipath Estimators with ϵ -SVR

In Chapter 3, the geometrical model for code multipath is given as equation (3.11) where the multipath error is viewed as a function of receiver-satellite geometry parameterized by azimuth and elevation angles of the satellite w.r.t. the receiver. Therefore, code multipath mitigation can be posed as regression problem and the complicated function in (3.11) can be approximated for each satellite by learning from historical data. Specifically, estimating code multipath is a 2-dimensional regression setting:

$$\text{multipath} = f(\text{azimuth}, \text{elevation}) \tag{4.15}$$

Given a nominal position of a receiver, azimuth and elevation angles of each satellite w.r.t. the receiver is easily computed using the broadcast navigation information [59]. Furthermore, pseudorange code multipath error can be isolated by combination of measurements which will be described in Section 4.6. For a fixed receiver, repeatability of satellite's geometry and code multipath error, and the ability to extract training data makes the regression problem solvable.

For each satellite, the multipath estimator is trained using ϵ -SVR. The input vector $x \in \mathfrak{R}^2$ consists of two components: azimuth and elevation angles and the target output $y \in \mathfrak{R}$ is the multipath error expressed in terms of the azimuth and elevation angles. The commonly used Gaussian kernel [76] given by (4.16) is reasonably chosen for kernel mapping due to its ability to handle nonlinearity.

$$\kappa(x_i, x_j) = \exp(-\eta \|x_i - x_j\|^2) \quad (4.16)$$

where η is kernel bandwidth. Given N training samples $\{(x_1, y_1), \dots, (x_N, y_N)\} \subset \mathfrak{R}^2 \times \mathfrak{R}$, the multipath estimation function $f : \mathfrak{R}^2 \mapsto \mathfrak{R}$ will be learnt to minimize ϵ -insensitive loss function.

4.6 Experiments and Results

This section will describe the experiments conducted to train the code multipath estimators and subsequently use them for multipath correction. The experiments are conducted on L1 measurements. L2 measurements can be done likewise. It will be demonstrated that the proposed method outperforms state-of-the-art results in code multipath mitigation in term of standard deviation. The advantages of the proposed method will also be discussed.

4.6.1 Extraction of Code Multipath Error

Isolated code multipath errors are needed to train multipath estimators. The code multipath error can be extracted by firstly by subtracting code measurements from carrier-phase mea-

measurements to eliminate the common terms including the true geometric range, clock errors and tropospheric delay. As a consequence, the CmC quantity for L1 channel χ_1 is produced as shown in equation (4.17) by taking residual of equations (1.1) and (1.3):

$$\begin{aligned}\chi_1 &= \rho_1 - \phi_1 \lambda_1 \\ &= 2I_1 - N_1 \lambda_1 + (M_1^\rho - M_1^\phi) + (\epsilon_1^\rho - \epsilon_1^\phi)\end{aligned}\quad (4.17)$$

Practically, the phase multipath and receiver noise are far smaller than those of code measurement, i.e. $M_1^\rho \gg M_1^\phi \approx 0$ and $\epsilon_1^\rho \gg \epsilon_1^\phi \approx 0$. Hence, the phase multipath and receiver noise can be ignored for simplicity. Then, the CmC equation can be re-written:

$$\chi_1 = 2I_1 - N_1 \lambda_1 + M_1^\rho + \epsilon_1^\rho \quad (4.18)$$

Furthermore, in order to isolate multipath error, the ionospheric delay I_1 and the integer ambiguity N_1 need to be removed from χ_1 . Notice that the ambiguity N_1 is a constant bias as long as no cycle slips occur. In addition, since $(M_1^\rho + \epsilon_1^\rho)$ is zero-mean [59], N_1 can be computed by averaging over a whole orbit arc, and then subtract it from the χ_1 value at each epoch. The averaging should be restarted if any cycle slip occurs. Pertaining to ionospheric delay, there are several techniques that can be used for removal. In case of dual-frequency receivers where dual-frequency measurements are available, it is removed by forming ionosphere-free combinations before calculating the residuals. Ionosphere-free combinations are presented in Appendix A. Otherwise, in case of single-frequency receiver, it can be removed by fitting a low frequency polynomial to the CmC sequence. It is guaranteed that no multipath error is incidentally removed as long as the polynomial has lower frequency than that of the multipath fading frequency [93]. Eventually, only code multipath error and receiver noise remain in CmC χ_1 .

4.6.2 Training Multipath Estimators

For each satellite, the training data is prepared using data during 4 days from DOY 306 to DOY 309 of the experimental data set, which we have found to be redundant enough to capture distribution of multipath sequences. Azimuth and elevation angles (in degrees) of the satellites w.r.t. the receiver, which are inputs for training, are computed from the broadcast navigation data [59]. For the desired multipath outputs, after being detached from observation data, the Cmc sequences containing multipath errors are filtered with CSF as per equation (4.1) to remove high-frequency noise. The effect of this smoothing operation to the multipath errors is negligible as long as the smoothing window is shorter than the highest rate multipath. The smoothing window is set to 50 seconds (equivalent to 5 epochs) which is only a fraction of the shortest anticipated multipath fading period of 200 seconds [18] [93]. Thus, the receiver noise is significantly reduced without removing the multipath which is to be quantified. This smoothing operation helps to clearly expose multipath patterns; as a result, to enhance the estimators' generalization.

Scaling is applied to the training data before training. Azimuth angles, elevation angles, and code multipath values are scaled to the range $[-1; +1]$ as recommended for general SVM applications [13]. The main advantage of scaling is not only to avoid numerical difficulties during the calculation but also to prevent domination of values in greater numeric ranges over those in smaller numeric ranges.

The *libSVM* package [13] which implements ϵ -SVR was used to find the support vectors and coefficients of each satellite-specific code multipath estimator. ϵ , the kernel parameter η and the penalty parameter of the error term C must be chosen. It is not known beforehand which parameter values are best for a given problem; consequently, some kind of model selection (parameter search) must be done.

For η and C , grid search and cross-validation was applied for parameter search. Firstly, the coarse grid-search was performed on exponentially growing sequences of η and C (e.g.

$\eta = 2^{-15}, 2^{-13}, \dots, 2^3$ and $C = 2^{-5}, 2^{-3}, \dots, 2^{15}$) followed by cross-validation for each pair of (η, C) with a fixed ϵ . The result with the best 6-fold cross-validation accuracy was picked. With this strategy, better regions of the grid can be identified and finer grid-search conducted around that region. ϵ was searched separately in the range $[0.0050.1]$ with a step size of 0.005 after the best pair (η, C) has been chosen.

Learning from the training data set, the support vectors and coefficients in (4.13) are found for multipath estimation function f of each satellite. Each trained estimator f should estimate multipath error when presented with a new observation of azimuth/elevation angles thereafter.

4.6.3 Multipath Mitigation Performance

In order to evaluate the performance of code multipath estimators, the proper multipath correction is directly applied to code measurements of DOY 310. Note that the inputs need to be scaled as they have been during the training phase, and the estimated multipath values need to be de-scaled.

Data from 6 satellites (PRN 9, PRN 12, PRN 3, PRN 21, PRN 10 and PRN 13) picked up from six orbital planes is again used for demonstration. For the sake of concreteness, Figure 4.5 presents pseudorange multipath errors of DOY 310 of 2010 and responses of the multipath corresponding to PRN 12. The results of other satellites can be found in Appendix B. As observed, the errors are significantly reduced after being corrected with the responses of the SVR estimators.

Performance of all multipath estimators corresponding to the visible satellites are tabulated in Table 4.3 where multipath reduction is measured in term of the reduced standard deviation of multipath errors. The percentages of reduction range from 68% to 91%. On average, calibrating the data with CSF followed by the SVR estimators gains improvement from 36.68% to 78.99%. With the assumption about unchanged surroundings of this approach, performance of the multipath estimators would depends on how fast the reflecting surfaces along the propa-

Figure 4.5: Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 12 of the orbital plane B in DOY 310 of 2010.

gation direction changes. The environmental changes are expected to be different for different propagation directions of the GPS satellites. Therefore, the variation in performance of the estimators in Table 4.3 is expected.

The goodness of the corrections is additionally illustrated in the position domain. Figure 4.6 shows the variation of the solved positions from the known nominal position of the receiver. Weighted Least Mean Square single-point positioning [59] with broadcast navigation data was applied to the data of DOY 310. The measurements were only multipath-corrected while other noises and biases (e.g. atmosphere delays, etc.) were not calibrated. The plot reveals noticeably higher centralization of the solution on the data corrected with SVR multipath estimators over those obtained from the original data and the 50-second CSF-smoothed data. The reduction of standard deviation of coordinate time series North, East and Up is tabulated in Table 4.4.

PRN	Original	CSF-corrected	SVR-corrected
02	1.4927	0.9992 (33.06%)	0.3671 (75.41%)
03	1.1479	0.6741 (41.28%)	0.2478 (78.41%)
04	1.4009	0.8913 (36.38%)	0.2201 (84.29%)
05	1.2474	0.7722 (38.10%)	0.2325 (81.36%)
06	1.3194	0.6957 (47.28%)	0.2946 (77.67%)
07	1.2561	0.8768 (30.20%)	0.2711 (78.42%)
08	1.2806	0.8680 (32.22%)	0.2409 (81.19%)
09	1.3406	0.7955 (40.66%)	0.3599 (73.15%)
10	1.2084	0.7954 (34.18%)	0.2193 (81.85%)
11	1.4290	0.9993 (30.07%)	0.2995 (79.04%)
12	1.4813	0.9943 (32.87%)	0.2509 (83.06%)
13	1.5451	0.9960 (35.54%)	0.4969 (67.84%)
14	1.4514	0.8575 (40.92%)	0.2880 (80.16%)
15	1.3496	0.9539 (29.32%)	0.3034 (77.52%)
16	1.2200	0.7730 (36.64%)	0.3902 (68.01%)
17	1.4946	0.9515 (36.34%)	0.2763 (81.51%)
18	1.7889	1.1555 (35.40%)	0.3425 (80.85%)
19	1.1336	0.6832 (39.73%)	0.2506 (77.90%)
20	1.3501	0.8263 (38.80%)	0.2020 (85.04%)
21	1.1886	0.7329 (38.34%)	0.3492 (70.62%)
22	1.2568	0.8065 (35.83%)	0.3058 (75.67%)
23	1.2730	0.8483 (33.36%)	0.3378 (73.47%)
24	1.4779	0.9000 (39.11%)	0.1706 (88.46%)
25	1.4986	0.9260 (38.21%)	0.2914 (80.55%)
26	1.4292	0.9721 (31.99%)	0.2940 (79.43%)
27	1.9437	1.1705 (39.78%)	0.2504 (87.12%)
28	1.2519	0.8063 (35.59%)	0.2662 (78.74%)
29	1.6434	1.0105 (38.51%)	0.5116 (68.87%)
30	1.3446	0.8137 (39.48%)	0.1207 (91.02%)
31	1.3725	0.8317 (39.40%)	0.2569 (81.28%)
32	1.3985	0.8607 (38.46%)	0.2675 (80.87%)
Average reduction		36.68%	78.99%

Table 4.3: Standard deviation (m) of noise before and after correction applied with CSF and SVR estimators.

	Original	CSF-corrected	SVR-corrected
North	0.9136	0.7223 (20.94%)	0.5902 (35.40%)
East	1.2180	0.9997 (17.92%)	0.9033 (25.94%)
Up	2.6069	2.1715 (16.70%)	1.9496 (25.21%)

Table 4.4: Standard deviation (m) of coordinate time series.

Figure 4.6: Positioning solution on original data, CSF-smoothed data and SVR-corrected data of DOY 310 of 2010.

4.6.4 Discussions

For comparison with other methods in code multipath mitigation, Table 4.5 numerically tabulates performance of different methods in terms of percentage of reduced multipath error. The performance of CSF is reported with a 100-second smoothing window [59]. The performance range of frequency analysis methods with FFT [93] is reported with block sizes of 256 and 512 respectively whilst the block size of frequency analysis method with wavelet analysis [94] is 100 seconds. In particular, the performance of the FIR filter method [28] is unreliable as only one satellite PRN 9 was used for analysis. It is clear that SVR estimators significantly outperform other methods with a large margin.

CSF [59]	FIR Filter [28]	FFT [93]	Wavelet [94]	SVR estimator
58%	75%	50% - 70%	55% - 65%	68% - 91%

Table 4.5: Performance comparison of code multipath mitigation methods.

The state-of-the-art performance of SVR estimators on pseudorange multipath reduction emphasizes the *efficiency* of the proposed method. However, as the accumulative environmental changes get more and more significant over time, performance of the estimators would temporally degrade. Therefore, the multipath estimators need to be equipped with adaptability, which has not been addressed so far. The *availability* of the approach is also ensured as

it directly relies on GPS code, carrier-phase measurements, and relative orbital information of the constellation which will be available as long as the receiver and GPS satellites are still operating.

Unlike the multipath stacking-based approaches, modeling multipath errors as functions of continuous variables (i.e. azimuth and elevation angles) does not experience the difficulty in determination of time-shifting period. In addition, the interpolation ability of the trained estimators makes them applicable for different data-rates providing that training data is adequate to capture the underlying distribution of multipath errors. Furthermore, with the nature of sparsity, the multipath estimators just count on a subset of training data, being simpler while requiring less storage. All of these imply better *scalability*.

The algorithm can also execute in *real-time*. Given a new azimuth/elevation pair, a multipath estimator will evaluate the kernel function of the new input and every predetermined support vectors to produce an estimate of pseudorange multipath error. Therefore, it may require large amounts of computation. Compared to other approaches, CSF for example, SVR is intuitively much slower since CSF can be done with a few numerical operators. Due to the low computational requirement of real-time algorithms, computational complexity of SVR estimators need to be evaluated in the future work.

Intuitively, CSF is much faster than SVR because CSF can be done with a few numerical operators while SVR requires evaluation of a kernel function on every support vector, which may require large amounts of computation.

One distinct advantage of the proposed approach is that it is able to preserve other signals of studied phenomena such as deformation caused by earthquakes. This is achieved by training the models with data on normal days without displacement before using them to correct data on the subsequent day where a phenomenon occurs. For the purpose of demonstration, we simulated an event with the signal given by (4.19) to add to PRN 12's pseudorange measurements

Figure 4.7: Simulation of event signal added to pseudorange measurements of PRN 12. It can be seen that the event signal is indeed left intact from correction of the PRN 12's SVR estimator.

of DOY 310.

$$e(t) = 2 \cos\left(\frac{\pi}{10}t + \pi\right) + \cos\left(\frac{\pi}{15}t\right) \quad (4.19)$$

Since signals of phenomena are usually of low frequency [28] [11], the frequencies of the simulated event were chosen to exhibit diminishing effects of CSF which is a low-pass filter [38] [93]. The PRN 12's pseudorange multipath sequence was smoothed by CSF with 50-second smoothing window and then corrected by the trained PRN 12's SVR estimator. As shown in Figure 4.7, the corrected multipath sequence aligns very well with the event signal; that is, the event signal is not affected.

4.7 Conclusions

In this chapter, a nonlinear regression approach has been proposed to address GPS pseudorange multipath mitigation problem for fixed stations with the abilities to learn using previous days' data to correct measurements for successive day. The proposed method demonstrates state-of-the-art performance with high availability and scalability. Furthermore, the multipath estimators do not affect the simulated signal of phenomena. Motivated by these results, the problem of estimation of carrier-phase multipath error will be discussed and preliminarily experiments will be further conducted in the next chapter. The same approach will be used as in estimation of code multipath error in this chapter.

Chapter 5

Mitigation of Carrier-Phase Multipath with Nonlinear Regression

5.1 Introduction

In high-precision applications of GPS such as deformation monitoring, geodesy, etc., carrier-phase measurement plays a crucial role in position solutions to provide centimeter-, or even millimeter-level accuracy. Generally, as high precision of the solution is required, tackling all the biases and errors in carrier-phase measurements as shown in equations (1.3) and (1.4) is essential.

Firstly, the feasibility of processing carrier-phase measurements heavily depends on the ability to solve for integer ambiguity. Fortunately, nowadays the integer ambiguity in the carrier-phase measurement can be reliably solved either in an epoch-by-epoch manner [11] or in a batch-processing fashion. The batch-processing solution appears to be more reliable than the epoch-by-epoch counterpart [59].

Secondly, the atmospheric delays (e.g. ionospheric and tropospheric delays) must be calibrated. Two scenarios need to be considered: standalone receivers and networked receivers. For standalone receivers, although multipath mitigation can be performed by analyzing the

correlation of carrier-phase multipath and signal-to-noise measurements like the works of Billich [6] [7], calibration of atmosphere delays relies on the models or combination of dual-frequency measurements. The ionospheric models can compensate for 50% of ionospheric delay [45] whereas the ionospheric-free combination of dual-frequency measurements is able to cancel out the first order ionosphere delay which is approximately 99% of the delay [59]. The higher-order ionospheric errors whose magnitude is about 1% of the first-order error [16] [46] can be further eliminated with improved models [12]. Models for tropospheric delay can compensate 90% of the delay [37]. However, since the atmosphere delays are in the order of meters, the remaining of errors is still large for centimeter-level accuracy. Advantageously, processing carrier-phase measurements in networked receivers can almost completely remove atmospheric delays by forming carrier-phase differential (that is why the technique named differential GPS [65]) of two short-baseline receivers. This is achieved because the atmosphere delays in carrier-phase measurements of the two receivers in short baseline (< 10 km) are almost the same. Unfortunately, carrier-phase multipath errors are likely magnified because multipath errors are site-specific, implying no correlation of multipath errors at two ends of the baseline. Once the integer ambiguity is solved, multipath error remains the major source in carrier-phase residuals. The majority of proposed techniques in the literature tackle carrier-phase multipath error using this scenario [21] [51] [55] [72] [97].

However, no carrier-phase multipath mitigation methods have, to date, been satisfactory for wide-scale use. It is partly because some of them rely on very strong assumption about multipath environment, specifically one of the ends of baseline is multipath-free, that is impractical, such as [55]. Others enjoy superior performance in multipath reduction but are harmful to other important signals such as deformation signals induced by earthquake, etc. [72] [21]. The rest employ stacking multipath sequences of multiple days and time-shifting to calibrate multipath for the subsequent days but they require high resolution data to ensure the performance [4] [51] [97]. These methods and their strengths and weaknesses had been discussed in Chapter 2. In

the light of success of nonlinear regression methods for code multipath mitigation that were described in Chapter 4, these can be extended for mitigation of carrier-phase multipath errors in this chapter. Firstly, the carrier-phase differential combinations will be presented, followed by the formulation to pose carrier-phase multipath estimation as a novel regression problem. Similar to code multipath mitigation, the regression problem will be cast in an ϵ -SVR framework. Although more experiments have to be conducted to confirm the performance, initial results indicate the potential of the proposed method. On the other hand, for this proposed method, no multipath-free station is assumed. It is also scalable to different data rates and potentially harmless to other signals.

5.2 Carrier-Phase Differential Combinations

When multiple stations and/or multiple satellites are used together, it is possible to form observable combinations which eliminate common clock terms and atmosphere delays [37] [59]. These observable combinations are formed with one observable type (carrier phase or pseudo-range) of one GPS frequency (L1 or L2) from multiple stations and satellites. In this section, the carrier phase on L1 will be used, but these combinations can be easily extended to other GPS observables.

5.2.1 Single-Differential Combination

A single difference (SD) is formed by differencing the observables from two different stations observing the same GPS satellite, indicated by a superscript s as shown in Figure 5.1.

Re-writing the L1 phase observable equations where some terms are now specific to stations

Figure 5.1: Single difference.

A and B:

$$\phi_A^s \lambda = r_A^s + c(\delta_A - \delta_s) + T_A - I_A + M_A^\phi + N_A \lambda + \epsilon_A^\phi \quad (5.1)$$

$$\phi_B^s \lambda = r_B^s + c(\delta_B - \delta_s) + T_B - I_B + M_B^\phi + N_B \lambda + \epsilon_B^\phi \quad (5.2)$$

The SD is formed by obtaining the difference between equations (5.1) and (5.2):

$$\phi_{AB}^s \lambda = \Delta r^s + c \Delta \delta_u + \Delta T - \Delta I + \Delta M^\phi + \lambda \Delta N + \Delta \epsilon^\phi \quad (5.3)$$

The Δ notation indicates the difference between A and B, e.g. $\Delta \rho^s = \rho_A^s - \rho_B^s$. With the SD, the common satellite clock term δ^s has been eliminated.

The above equation can be simplified further when the distance between stations A and B is small, i.e. a short baseline (< 10 km) [59] [65]. In this case, the tropospheric effects T at both stations are approximately equal. Similarly, the two stations will have identical ionospheric delays on their incoming signals, and the difference in ionospheric delay can be approximated by a low-order polynomial for very short baselines. Finally, when both stations observe the same satellite at every epoch, the change in geometric range at any one instant is equivalent to

Figure 5.2: Double difference.

the baseline length between the two stations, $\Delta \rho^s = \rho_A^s - \rho_B^s = |\vec{A} - \vec{B}| = AB$. With these assumptions, the SD equation can be simplified to:

$$\phi_{AB}^s \lambda = \Delta r^s + c \Delta \delta_u + \Delta M^\phi + \lambda \Delta N + \Delta \epsilon^\phi \quad (5.4)$$

5.2.2 Double-Differential Combination

The scenario of double-differential combination is illustrated in Figure 5.2 where a double difference (DD) is formed by subtracting two single-differenced observables. Continuing with the single difference and rewriting for two different satellites a and b :

$$\phi_{AB}^a \lambda = \Delta r^a + c \Delta \delta_u + \Delta T - \Delta I + \Delta M^\phi + \lambda \Delta N + \Delta \epsilon^\phi \quad (5.5)$$

$$\phi_{AB}^b \lambda = \Delta r^b + c \Delta \delta_u + \Delta T - \Delta I + \Delta M^\phi + \lambda \Delta N + \Delta \epsilon^\phi \quad (5.6)$$

Differencing these equations leads to DD, which further eliminates the receiver clock term $\Delta \delta_u$:

$$\phi_{AB}^{ab} \lambda = \Delta \rho^{ab} + \Delta T^{ab} - \Delta I^{ab} + \Delta M^{\phi,ab} + \lambda \Delta N^{ab} + \Delta \epsilon^{\phi,ab} \quad (5.7)$$

With the same short-baseline simplifications to the above equation, the DD is reduced further to include only receiver noise, the integer phase ambiguity, and multipath terms:

$$\phi_{AB}^{ab}\lambda = \Delta M^{\phi,ab} + \lambda \Delta N^{ab} + \Delta \epsilon^{\phi,ab} \quad (5.8)$$

5.3 Carrier-Phase Multipath Mitigation Estimation

After forming carrier-phase DD, only the phase receiver noise, the integer phase ambiguity, and the carrier-phase multipath error remain. The integer ambiguity term ΔN^{ab} can be computed and removed from the carrier-phase DD data. Note that the integer ambiguity N_j^i ($i = a, b$ and $j = A, B$) of each carrier-phase measurement can be computed as:

$$N_j^i = \lfloor \left(\frac{\rho_j^i}{\lambda} \right) \rfloor \quad (5.9)$$

where $\lfloor \cdot \rfloor$ means *floor* function [55]. The term ΔN^{ab} is a linear combination of N_j^i can then be computed and removed. The term ΔN^{ab} can also be de-biased over the entire DD sequence as long as there is no cycle slips. The multipath effect is now the dominant error component in the DD measurements which can now be written as:

$$\phi_{AB}^{ab}\lambda_1 = \Delta M^{\phi,ab} + \Delta \epsilon^{\phi,ab} \quad (5.10)$$

With the same assumption that the surrounding environment is constant as described in code multipath mitigation, one should still see sidereal repeatability of the GPS constellation geometry. Furthermore, carrier-phase multipath of each receiver also correlate in sidereal daily basis as it is a function of satellite geometry as given in equation (3.12). However, the DD now is the combination of four multipath errors of the four carrier-phase components, how can we proceed at this point? Actually, the rationale is still the same. We use f_j^i , θ_j^i , and φ_j^i to denote

the function of carrier-phase multipath error, azimuth and elevation of the satellite i w.r.t. the receiver j , expanding the DD equation to:

$$DD = [f_A^a(\theta_A^a, \varphi_A^a) - f_B^a(\theta_B^a, \varphi_B^a)] - [f_A^b(\theta_A^b, \varphi_A^b) - f_B^b(\theta_B^b, \varphi_B^b)] + \Delta\epsilon^{\phi,ab} \quad (5.11)$$

The function above depends on eight geometrical variables θ_j^i and φ_j^i . Therefore, it can be generalized as a combinatorial function $f_\phi : \Re^8 \mapsto \Re$:

$$DD = f_\phi(\theta_A^a, \varphi_A^a, \theta_B^a, \varphi_B^a, \theta_A^b, \varphi_A^b, \theta_B^b, \varphi_B^b) + \Delta\epsilon^{\phi,ab} \quad (5.12)$$

Fortunately, at each epoch eight variable parameters can be calculated given the orbital information of the satellites broadcast to the receivers and nominal positions of the receivers which are known beforehand. Therefore, the combinatorial carrier-phase multipath error can be estimated if the complicated function f_ϕ can be evaluated. Similar to estimation of code multipath error, the function f_ϕ can be approximated using nonlinear regression approach.

5.4 Experiments and Preliminary Results

This section describes an experiment to train carrier-phase multipath estimators. Although this experiment is simple and more rigorous exploration may be required, these preliminary results demonstrate the potential of the proposed method. In the sequel, the advantages of the proposed method will be also discussed.

5.4.1 Experimental Data Set

In order to conduct the experiment, a data set from two stations with a short baseline is needed. There are many GPS data archives available online but most of them usually provide data with low sampling rate (e.g. 30 seconds/ sample). These data sets are unsuitable because the data

density has to be increased to capture the underlying distribution of multipath; therefore, more data samples across multiple days are needed. As a result, the time span will be large enough so that the assumption about the constant surrounding environment would be violated.

There are a few selected stations with high data rate (e.g. 1 second/sample) available in the IGS network [39]. Their data are provided in a hourly manner; unfortunately, discontinuity of data is quite common with data missing during one or a few hours (perhaps on purpose). In this experiment, two short-baseline stations from IGS monitoring network were chosen for the experiment. Their continuous 1-second data during two consecutive days (DOY 050 and 051 of 2011) were downloaded from IGS data archive [39]. The stations named KIRO and KIRU whose corresponding geodetic coordinates are $(21.0602^\circ, 67.8776^\circ, 497.9000 \text{ m})$ and $(20.9684^\circ, 67.8573^\circ, 391.1000 \text{ m})$ in (longitude, latitude, height) triplet or equivalent to $(2242.624 \text{ km}, 5516.729 \text{ km}, 2277.795 \text{ km})$ and $(2245.915 \text{ km}, 5519.218 \text{ km}, 2268.268 \text{ km})$ in ECEF coordinate system. They are located at Kiruna, Sweden with a distance apart of approximately 10 km. Figure 5.3 shows photographs of the locations and surroundings of the two stations. As seen in Figure 5.3 (image from [39]), the stations' vicinity is covered by snow; therefore, the multipath environment is expected to change quickly even in a short time span as the weather changes. If so, the performance of the multipath estimator would degrade. The baseline of the two stations can be seen through their relative positions in IGS tracking network in Europe as shown in Figure 5.4 (image from [39]).

5.4.2 Training Multipath Estimators

In this experiment, the station KIRU is selected as the reference station. With predetermined nominal positions of the stations, the geometrical parameters of the satellites w.r.t. the station can be computed easily at every epoch given the broadcast navigation data. In order to extract DD multipath as described earlier, a satellite with high elevation is usually selected as a reference to form the DD combination [37] [59]. Due to time limitation, only one pair of satellites,

(a)

(b)

Figure 5.3: IGS Observation Stations: (a) KIRU, (b) KIR0.

Figure 5.4: KIRU and KIR0 stations in IGS Tracking Network in Europe.

Figure 5.5: Geometries of PRN 8 and PRN 18 w.r.t. KIR0 and KIRU in DOY 050 and DOY 051 of 2011.

PRN 8 and PRN 18, with the longest DD sequence is selected for analysis. PRN 8 with higher elevation peak is used as the reference satellite. Their geometries w.r.t. the two stations are illustrated in Figure 5.5. The carrier-phase multipath sequences extracted from carrier-phase DD combinations of the two satellites in DOY 050 and 051 of 2011 are also plotted in Figure 5.6.

Although carrier-phase DD multipath in the second DOY 051 is shorter than that of the first DOY 050 due to data discontinuity, the repetition of satellites geometries and multipath errors are still visible in the same time span. In order to train DD multipath estimator for the satellite pair, the steps that have been done previously for code multipath estimation will be carried out in this experiment. The geometrical data and multipath errors in DOY 050 are scaled to $[-1; +1]$ before feeding to the training program. The *libSVM* package that implements ϵ -SVR

Figure 5.6: Carrier-phase multipath of DD combination of two satellites PRN 18 and PRN 8 in DOY 050 and 051 of 2011.

algorithm is employed to train the multipath estimators. The values of ϵ , the Gaussian kernel parameter η , and the penalty parameter of the error term C are again chosen using grid search and cross-validation with the strategy described in code multipath mitigation.

Learning from the training data set, the support vectors and coefficients of the DD multipath estimator are found. Finally, the multipath estimator f_ϕ should estimate DD multipath error when presented with a new observation of azimuth/elevation angles thereafter. The estimated multipath error will be used to correct carrier-phase DD of the satellite pair in the successive days.

In order to demonstrate the ability of the proposed method, the trained estimator is used to estimate the carrier-phase multipath error of the following DOY 051. From top to bottom,

Figure 5.7 shows the original multipath sequence, the response of the trained estimator and the corrected multipath sequence with the response. The standard deviations of the original multipath sequence and the corrected multipath sequence are 2.15 cm and 0.92 cm respectively. In other words, the multipath error has been reduced by 57.2% using the trained multipath estimator. Although only data from one satellite pair is explored, this preliminary result gives a strong indication that the proposed technique can be applied to the carrier-phase multipath mitigation problem.

Figure 5.7: Original carrier-phase multipath (blue), response of the trained multipath estimator (red), and corrected carrier-phase multipath (black) sequences of the satellite pair PRN 18 - PRN 8.

5.5 Discussion

As expected, the response of the multipath estimator can capture the dominant trend of the multipath sequence which is the dominant component of multipath error [94] [21]. However, the result of the proposed method when applied to carrier-phase multipath mitigation seems to be lower than what was achieved in code multipath mitigation. It is explainable as there is no equivalent smoothing algorithm like CSF to smooth the multipath sequence to attenuate high-frequency noise during training and correction. Although it is not convincing to conclude firmly since the experimental result is only for one pair of satellites, the proposed method, for the sake of comparison, outperforms FIR filter method (39.8% - 56.1%) [55] and the frequency-domain processing methods relying on analysis of SNR measurements (20%) [6] [7]. It gives a motivation for further exploration of the proposed method on carrier-phase multipath mitigation problem.

However, this result is incomparable to that of the direct frequency-domain processing methods using wavelets like [21], which can reduce carrier-phase DD multipath up to 84%. It is because these methods can attenuate both errors from low to high frequencies to achieve better mitigation effects at the cost of filtering out other signals if their frequency content overlaps with multipath's frequency content. Although it has not been proven and more comprehensive experiments need to be conducted in future work, the proposed method should share similar advantages of code multipath estimators that it, intuitively, does not affect other phenomena signals. This could be fulfilled if data without displacement is used for training before estimating multipath error for data with displacement. Therefore, for applications where additional important signals such as seismic signals exist, the proposed method is still promising. Furthermore, better real-timeness and availability are the advantages of the SVR estimator compared to the frequency-domain processing methods.

As carrier-phase multipath is much more sensitive to environmental changes than pseudo-range multipath, the data training over a short time span is more suitable for training SVR

estimators under the constant environment assumption. It leads to higher data rates being required for training in order to retain data density. However, it does not strictly require high rate data ($\geq 1\text{Hz}$) like stacking-based approaches [14] [51] but data rates that can capture the underlying distribution of multipath error. Fortunately, the proposed method is also scalable to the data rate due to its sparsity. In other words, only a subset of training data which are support vectors are kept and involved in the computation. However, if the number of support vectors is significantly large, computational complexity may still be a concern for real-time operation. This issue needs further exploration.

5.6 Conclusions

In the light of success on code multipath mitigation, the nonlinear regression method has been applied to estimate carrier-phase multipath of double difference measurements. The double-differential multipath is viewed as a function of geometries of the four satellite-receiver pairs in a double-differential combination. The preliminarily experimental results on real data show the potential of the proposed method in carrier-phase multipath mitigation. In the following chapter, the work in this thesis will be concluded followed by addressing the future work.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

Multipath mitigation is an important problem in GPS data processing for high-precision applications. Despite the recent progress towards multipath mitigation, obtaining a good multipath mitigation technique remains an open research issue. In this thesis, a novel approach has been proposed which is applicable to both code and carrier-phase multipath errors. On analyzing the effects of multipath-delayed signals on GPS measurements in the context of tracking loops and understanding reflecting geometries of multipath signals, the geometrical models of multipath are firstly derived so that the code and carrier-phase multipath errors of each satellite are viewed as functions of the satellites geometry with respect to a fixed receiver. It turns out that the multipath mitigation problem can be expressed as a general regression problem which is then solved by ϵ -SVR. Under the observation that constellation geometry with respect to a fixed receiver repeats on a sidereal daily basis, and multipath errors are highly correlated between consecutive days, solving the regression problem is feasible due to the ability to extract training data.

For the code multipath mitigation problem, it was experimentally verified that the proposed method can improve the performance of state-of-the-art code multipath mitigation methods

while avoiding the drawbacks that other approaches suffer. Training data extracted from 4 consecutive days was used to train satellite-specific multipath estimators. The trained multipath estimators were then employed to estimate and correct multipath errors for measurements on subsequent days. It was empirically shown that the proposed method can reduce the standard deviation of code multipath error from 68% to 91% specific to satellite with an average of about 79% reduction. This performance significantly outperforms other code multipath mitigation methods. In addition, the SVR multipath estimators are real-time and scalable to data rate. More interestingly, it was shown with simulations that the proposed method is able to preserve other phenomena signals.

The proposed method can also be applied to carrier-phase multipath mitigation. It was argued that the double-different carrier phase multipath error can be viewed as a function of 8 geometrical parameters of the two satellites with respect to the two receivers involved in double difference. Hence, carrier-phase multipath mitigation can be tackled likewise as a regression problem. Measurements of the satellite pair PRN 8 and PRN 18 observed by 2 short-baseline receivers in the IGS monitoring network were employed for the experiment. It had been shown that the double-different carrier phase multipath error had been reduced 57.2% by correction with the estimation of the trained multipath estimator. Although applying this method to mitigate carrier-phase multipath faces a few obstacles and has not been fully exploited yet, the preliminarily experimental results in Chapter 5 have clearly shown its promise.

6.2 Future Work

6.2.1 Building evaluation framework for multipath mitigation problem

So far, different multipath mitigation approaches have used different data set with different evaluation measures for evaluation which potentially leads to bias. In general, a more principled and statistical way of evaluating a multipath mitigation method which would give a

good performance in various application scenarios would be valuable. Furthermore, there is insufficient amount of benchmark data explicitly created for to evaluate multipath mitigation methods. These benchmark data sets should be able to emphasize various aspects of a multipath mitigation method. A unified framework with well-defined tests and criteria would be of great use and worth for future work.

6.2.2 Further experiment on carrier-phase multipath mitigation

The proposed method shows potential for carrier-phase multipath mitigation but further experiment is needed. Different satellite pairs should be taken into account to confirm consistency of the proposed method in multipath mitigation. Furthermore the effect of multipath reduction on positioning solution should also be investigated.

So far, only carrier-phase DD multipath has been considered in this research work. However, it is preferable that the carrier-phase multipath is addressed for standalone station. In literature, some works [6] [7] [69] has been proposed for standalone stations. Although their performance in multipath mitigation is limited, they gave initial motivation for this research direction. They utilized correlation between SNR measurements and multipath to build a map from SNR measurements to both code and carrier-phase multipath errors for standalone stations. Unfortunately, SNR measurements are not always available to use in RINEX data files reported by receivers. Furthermore, SNR measurements also differ in resolution and unit from this receiver to another, degrading availability of these methods. If both code and carrier-phase multipath errors can be mapped from SNR measurements, there could potentially exist a mapping from code multipath to carrier-phase multipath. While carrier-phase multipath is difficult to addressed, code multipath can be estimated quite accurately in real-time. Therefore, it is worth investigating mapping from code multipath to carrier-phase multipath for standalone stations.

6.2.3 Reduce complexity of solution of regression problem

Due to the low computational requirement for real-time operation of multipath estimation, the complexity of the proposed method needs to be evaluated. It is well-known that computational complexity of SVR algorithm depends on the number of support vectors which is tricky to control in ϵ -SVR algorithm. In order to estimate multipath for a new observation, evaluation of the kernel function of the new input with all support vectors is required. Therefore, if the number of support vectors is large, a cheaper alternative algorithm would be valuable. Study computational complexity of the proposed method is necessary in future work.

6.2.4 Incremental training for multipath estimators

The proposed method strongly relies on the assumption of unchanged multipath environment for a few days. In fact, in normal condition the environment does change, small though. And the environmental changes would accumulate more and more with time to become significant. It turns out that performance of multipath estimators would temporally degrade. The question is how performance of multipath estimators degrade with age of data? If so, how this could be overcome? Intuitively, the estimators need to adapt with the environmental changes. It can be achieved if there is a mechanism to incrementally train the multipath estimator with the new data available.

6.2.5 Experiments on data recorded from the field

In real application scenarios, a GPS monitoring network consists of many GPS stations located in harsh environment, such as under snow coverage. In those cases, their surrounding environment may change faster than normal conditions even for a short time span; therefore, the constant multipath environment between multiple days may be violated. In addition, data discontinuity and cycle slip also happens more frequently. It is necessary to further evaluate

performance of the proposed method in these harsh conditions.

6.2.6 Integration of the new signals

According to GPS modernization plan [59], beside two channels L1 and L2 retained, a new channel L5 with more measurements will be added to GPS system. With additional measurements, more promising combinations can be formed for different tasks in GPS data processing. It is worth exploring new algorithms for multipath mitigation taking advantage of integration of new signals.

Appendix A

Ionosphere-Free Combinations

A.1 Ionosphere-free Combinations

The ionospheric delay contributing to the signal is linearly dependent on the total electron content (TEC) of the ionospheric region ranging from the attitude of ≈ 50 to $\approx 1,000$ kilometers (km) [59]. Note that the ionosphere is dispersive for GPS frequencies, meaning the group velocity and carrier phase measurements are delayed while the phase velocity and pseudorange measurements are advanced. Consequently, the measured pseudorange and carrier phase ranges are too short or too long, respectively, compared to the true satellite-receiver distance by an equal but opposite amount [37], i.e. $I^\rho = -I^\phi$. This relationship applies to both GPS frequencies. The dispersive nature of the ionosphere also leads to I terms which have an inverse dependence on frequency squared [59]:

$$\begin{aligned} I_1^\rho &\approx 40.3 \frac{TEC}{f_1^2} \\ I_2^\rho &\approx 40.3 \frac{TEC}{f_2^2} \\ I_1^\phi &\approx -40.3 \frac{TEC}{f_1^2} \\ I_2^\phi &\approx -40.3 \frac{TEC}{f_2^2} \end{aligned}$$

Using these relationships, the delay can be removed by combinations of dual-frequency measurements to yield ionosphere-free pseudorange (PC) and carrier phase (LC).

$$\begin{aligned} PC &= \frac{f_1^2}{f_1^2 - f_2^2} \rho_1 - \frac{f_2^2}{f_1^2 - f_2^2} \rho_2 \\ &= r + c(\delta_u - \delta_s) + T + M_{LC}^\rho + \epsilon_{LC}^\rho \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} LC &= \frac{f_1^2}{f_1^2 - f_2^2} \phi_1 \lambda_1 - \frac{f_2^2}{f_1^2 - f_2^2} \phi_2 \lambda_2 \\ &= r + c(\delta_u - \delta_s) + T + N_{LC} \lambda_{LC} + M_{LC}^\phi + \epsilon_{LC}^\phi \end{aligned} \quad (\text{A.2})$$

The ionosphere-free combination has the advantage of removing ionospheric delays in the observables. However, the cost is the higher noise level. The noise is ≈ 3 times larger than each observable [59].

A.2 Difference of Ionosphere-free Combinations

The code-minus-carrier for ionosphere-free measurements can be formed to remove any dependency on troposphere, clocks, or geometric range and leaves only multipath, noise and LC phase ambiguity:

$$\rho_{LC} - \phi_{LC} \lambda_{LC} = (M_{LC}^\rho - M_{LC}^\phi) - N_{LC} \lambda_{LC} + (\epsilon_{LC}^\rho - \epsilon_{LC}^\phi) \quad (\text{A.3})$$

Appendix B

Results of Other Satellites on Code Multipath Mitigation

B.1 Repeatability of Satellite's Geometry

Figure B.1: Repeatable geometries of the satellites in the orbital plane A from DOY 306 to DOY 309 of 2010.

Figure B.2: Repeating geometries of the satellites in the orbital plane C from DOY 306 to DOY 309 of 2010.

Figure B.3: Repeating geometries of the satellites in the orbital plane D from DOY 306 to DOY 309 of 2010.

Figure B.4: Repeating geometries of the satellites in the orbital plane E from DOY 306 to DOY 309 of 2010.

Figure B.5: Repeating geometries of the satellites in the orbital plane F from DOY 306 to DOY 309 of 2010.

B.2 Repeatability of Pseudorange Multipath Error

(a) Original

(b) 50-second CSF-smoothed

Figure B.6: Multipath time series of PRN 9 of the plane A.

	DOY 306	DOY 307	DOY 308	DOY 309
DOY 306	N/A	0.582	0.5573	0.7163
DOY 307	0.8485	N/A	0.5844	0.5368
DOY 308	0.8625	0.8502	N/A	0.5916
DOY 309	0.8758	0.8229	0.8574	N/A

Table B.1: Normalized cross-correlation of PRN 9's multipath sequences.

(a) Original

(b) 50-second CSF-smoothed

Figure B.7: Multipath time series of PRN 3 of the plane C.

	DOY 306	DOY 307	DOY 308	DOY 309
DOY 306	N/A	0.5601	0.6529	0.4651
DOY 307	0.8645	N/A	0.5806	0.6131
DOY 308	0.8579	0.8714	N/A	0.5101
DOY 309	0.8121	0.8362	0.8362	N/A

Table B.2: Normalized cross-correlation of PRN 3's multipath sequences.

(a) Original

(b) 50-second CSF-smoothed

Figure B.8: Multipath time series of PRN 21 of the plane D.

	DOY 306	DOY 307	DOY 308	DOY 309
DOY 306	N/A	0.7000	0.6371	0.5849
DOY 307	0.8947	N/A	0.7203	0.6467
DOY 308	0.8886	0.8954	N/A	0.7030
DOY 309	0.8654	0.8798	0.8959	N/A

Table B.3: Normalized cross-correlation of PRN 21's multipath sequences.

(a) Original

(b) 50-second CSF-smoothed

Figure B.9: Multipath time series of PRN 10 of the plane E.

	DOY 306	DOY 307	DOY 308	DOY 309
DOY 306	N/A	0.5192	0.5672	0.5335
DOY 307	0.8408	N/A	0.4783	0.5753
DOY 308	0.8402	0.8447	N/A	0.5296
DOY 309	0.8320	0.8377	0.8505	N/A

Table B.4: Normalized cross-correlation of PRN 10's multipath sequences.

(a) Original

(b) 50-second CSF-smoothed

Figure B.10: Multipath time series of PRN 13 of the plane F.

	DOY 306	DOY 307	DOY 308	DOY 309
DOY 306	N/A	0.5310	0.5990	0.5523
DOY 307	0.8123	N/A	0.5142	0.6342
DOY 308	0.8674	0.7985	N/A	0.5226
DOY 309	0.7980	0.8112	0.7710	N/A

Table B.5: Normalized cross-correlation of PRN 13's multipath sequences.

B.3 Code Multipath Estimation and Correction

Figure B.11: Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 9 of the orbital plane A in DOY 310 of 2010.

Figure B.12: Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 3 of the orbital plane C in DOY 310 of 2010.

Figure B.13: Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 21 of the orbital plane D in DOY 310 of 2010.

Figure B.14: Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 10 of the orbital plane E in DOY 310 of 2010.

Figure B.15: Original pseudorange multipath, CSF-smoothed pseudorange multipath, SVR-estimator response, and SVR-corrected pseudorange multipath of PRN 13 of the orbital plane F in DOY 310 of 2010.

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