Recovering multiplexing loss and improving reliability in cooperative half-duplex relay networks

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**RECOVERING MULTIPLEXING LOSS AND IMPROVING
RELIABILITY IN COOPERATIVE HALF-DUPLEX RELAY
NETWORKS**

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List of Symbols

$\mathbf{A}$	matrix
$\mathbf{a}$	vector
$\mathbf{I}_m$	a $m \times m$ identity matrix.
$(\cdot)^T$	transpose
$(\cdot)^H$	Hermitian transpose
$\mathbb{E}\{\cdot\}$	statistical expectation operator
$\mathcal{CN}(\mu, \sigma^2)$	complex Gaussian random variable with mean μ and variance σ^2
$\text{diag}(\mathbf{a})$	a diagonal matrix with diagonal elements given by the vector $\mathbf{a}$
$\max$	maximization operator
$\max\{a, b\}$	the maximum of a and b
$\min\{a, b\}$	the minimum of a and b
$(x)^+$	$\max(0, x)$
$\mathbb{R}^{N+}$	the set of real non-negative N -tuples
$\lambda_i[\mathbf{A}]$	the i -th eigenvalue of matrix $\mathbf{A}$
$\mathbf{Q}(p, q)$	the element at p -th row and q -th column of matrix $\mathbf{Q}$
$\text{tr}(\mathbf{P})$	the trace of matrix $\mathbf{P}$
$\text{rank}(\mathbf{A})$	the rank of $\mathbf{A}$
$\mathbb{R}^{N+}$	the set of real non-negative N -tuples
$\mathbf{A}(i, j)$	the element in the i -th row and j -th column of $\mathbf{A}$

$\mathbf{a}(i)$ the i -th element of $\mathbf{a}$

$\mathbf{0}$ a zero matrix

$\mathcal{A}^c$ a complement of set $\mathcal{A}$

List of Abbreviations

AF	amplify-and-forward
AWGN	additive white Gaussian noise
BER	bit error rate
BLAST	Bell laboratories layered space-time
CF	compress-and-forward
CMA	cooperative multiple-access
CMA-SAFR	cooperative multiple-access with sequential amplify-and-forward relaying
CSI	channel state information
DF	decode-and-forward
DDF	dynamic decode-and-forward
DMT	diversity-multiplexing tradeoff
DPC	dirty paper coding
DSTC	distributed space-time coding
ISI	inter-symbol interference
SNR	signal-to-noise ratio
MAC	medium access control
MHRS	multi-hop relay selection
MIMO	multiple-input multiple-output
MISO	multiple-input single-output

ML	maximum likelihood
NAF	non-orthogonal amplify-and-forward
OSTBC	orthogonal space-time block codes
ORS	opportunistic relaying scheme
PEP	pairwise error probability
Q-OSTBC	quasi-orthogonal space-time block codes
RHS	right-hand-side
SAF	slotted amplify-and-forward
SIC	successive interference cancellation

To my parents

Abstract

To meet the rapid growth of wireless applications and multimedia services, the development of wireless communications that is able to provide higher data rates and more reliable communications while maintaining satisfactory quality of service is required. Multiple-input multiple-output (MIMO) system has shown tremendous potential to be an excellent approach for solving this problem by providing multiplexing and spatial diversity gains without requiring additional radio resources, such as bandwidth and power. However, the size and complexity limitations of wireless mobile devices mean that MIMO systems may possibly be unfeasible, especially for the uplink. For this reason, cooperative communications has been proposed and its potential benefits have attracted much research attention in the past few years. The idea of this technique is to utilize the broadcast nature of wireless signals by exploiting the fact that a signal intended for a particular destination node can be overheard at neighboring nodes, and thus these nodes can help each other to create a virtual MIMO system.

Spectral efficiency (multiplexing gain) and reliability (diversity gain) are the two most important features of cooperative communication protocols. Many cooperative communication protocols are able to achieve full spatial diversity gain, but fail to exploit the multiplexing gain. It is observed in many cases that the multiplexing gain is reduced to one-half due to the half-duplex constraint of the relay. Thus, in this thesis, we investigate cooperative communications protocols that are able to enhance this spectral efficiency (recover multiplexing loss) and improve the corresponding reliability in wireless networks.

Specifically, we first analyze the performance of amplify-and-forward (AF) two-path, also known as successive half-duplex, relaying protocol where two-relays are utilized to alternate between transmission and reception modes. This protocol is very interesting, but less studied in the literature due to inter-relay interference inherent in this protocol. We first show that the two-path relaying protocol is able to significantly recover the multiplexing loss of conventional half-duplex relaying. We then propose an inter-relay self-interference cancellation method to manage the interference. A precoder design is also proposed to achieve full diversity gain and thus improving the overall reliability.

Our next step is to characterize the diversity-multiplexing tradeoff (DMT) of

AF two-path relaying protocol and show that AF two-path relaying is able to achieve the MIMO DMT upper bound. Our contribution includes the derivation of general lower bounds for DMT of any arbitrary block lower triangular channel matrix. We also provide general result of the DMT when successive AF relaying with multiple relays is considered.

We then study the DMT of decode-and-forward (DF) two-path relaying protocol, a variant of AF two-path relaying. We relax the assumption of perfect decoding considered in the previous work and analyze the effect of inter-relay interference on DMT performance by utilizing a DMT framework with different relative distance between nodes. Finally, we consider a general multi-user scenario where multiple-source nodes attempt to communicate to a single destination node with the help of multiple relays. We propose a cooperative multiple-access system with sequential AF relaying protocol and analyze its DMT performance. Our results reveal that a protocol with better DMT performance, lower complexity, and lower CSI requirement at the relays than the existing protocols is obtained. Through this dissertation, we hope that we have shed some light on how to provide good spectral efficiency and reliability in general cooperative half-duplex relay networks.

Introduction

1.1 Overview

With the exponential growth of wireless applications and multimedia services such as high speed data access and video conferencing, we are witnessing a remarkable evolution of wireless communications which first started with an analog voice cellular system in the 1980's. This development has led to increasing demand for higher data rates and more sophisticated mobile devices while maintaining a satisfactory quality of service. Thus, many research efforts from different perspectives are geared towards achieving this objective.

1.1.1 MIMO system

Reliability in wireless communications is closely related with the impairment of wireless signals due to the unpredictable wireless channel. Wireless signals propagating from the transmitter to the receiver typically experience multiple propagation paths (multipath) due to signal reflection, diffraction, or scattering. This multipath propagation leads to the fading phenomenon where there is a rapid fluctuation of amplitudes and phases over a period of time, or over a distance, due to the constructive and destructive interference between the multipaths.

Multiple-input multiple-output (MIMO) system [1, 2, 3, 4, 5, 6, 7] has been widely accepted as a promising technique to mitigate fading and exploit the multipath propagation channel without requiring additional resources, such as transmit

power or bandwidth. The first work on MIMO system was presented in [1] where the idea is to provide spatial diversity to the wireless signals such that different signal replicas will experience independent fading conditions, thus achieving diversity gain. MIMO system also enables higher multiplexing gain [1, 2, 4] where independent information streams are transmitted in parallel through the spatial channels, thus increasing the data rates. It was shown in [3, 4] that the capacity of a MIMO channel with M_t transmit, M_r receive antennas, and i.i.d. Rayleigh-faded gains between each antenna pair in the high signal-to-noise ratio (SNR) regime is given by

$$C(\text{SNR}) \approx \min\{M_t, M_r\} \log \text{SNR} + O(1). \quad (1.1)$$

The channel capacity of MIMO system is thus linearly proportional to the minimum number of transmit and receive antennas with power and bandwidth constraint.

Later, various MIMO techniques were proposed to approach the capacity limits of MIMO channels in [3, 4]. A spatial multiplexing architecture, BLAST (Bell laboratories layered space-time), with equal number of multiple antennas at the transmitter and receiver was proposed in [2, 8, 9] to exploit the multipath signal components in a rich-scattering wireless environment. Space-time coding, one of the most vibrant research themes in recent times, was first proposed in [5, 6, 7] to improve the performance of MIMO system against fading conditions. Space-time coding was subsequently developed into orthogonal space-time block codes (OSTBC) [10] and quasi-orthogonal space-time block codes (Q-OSTBC) [11]. Differential space-time codes was then proposed in [12] that exploits diversity when neither the channel nor the statistics of the channel are available.

Despite its promises and extensive research, MIMO system has its own drawbacks that hinder its implementation in practical systems. MIMO may not be feasible, especially for the uplink due to size and complexity limitations of mobile devices. For example, MIMO antennas are required to be sufficiently separated to ensure that fading correlation among the antennas does not deteriorate its performance. It is also generally understood that the greater the number of antennas, the more complicated the signal processing is. Current wireless mobile devices can

Figure 1.1: Two-user cooperation where each user acts as both a user and a relay for its partner.

only afford small number of antennas. Current technologies standards, such as IEEE 802.11n standard for WLAN applications [13, 14] only advises the use of up to four antennas at the transmitter and receiver to provide data rate of 100 Mbps or higher. The IEEE 802.16 standard (WiMAX) [15, 16, 17] only integrates the use of Alamouti space-time coding [5] and spatial multiplexing up to four antennas.

1.1.2 Cooperative Communications

In view of the limitations of MIMO system, cooperative communications was proposed in [18] as a new technique that is able to reap the benefits of MIMO system within the practical constraints of mobile devices. Specifically, when the direct link between the source and destination node is too weak, surrounding nodes will act as a relay to assist the direct communication. With this arrangement, fading correlation effect can be significantly reduced and the signals from the source and relay nodes can be combined at the destination node to achieve spatial diversity gain.

The key building block of cooperative communications is the wireless relay channel, which originates from the seminal work by Van der Meulen in [19] where a three-terminal communication channel was introduced and its information characteristics were studied. Cover and El-Gamal [20] later developed capacity bounds for general single relay channel and proposed two basic relaying strategies, i.e.,

decode-and-forward (DF) scheme (based on block Markov encoding), in which the relay decodes the transmitted message, and compress-and-forward (CF) scheme, in which the relay sends the quantized received signal to the destination. Further analysis on the capacity relay channels was given in [21, 22, 23].

The pioneering work in [18] studied cooperative communications for code division multiple-access (CDMA) systems. Each user is assigned a partner such that the two-user will detect each other's transmitted signals and then cooperate to transmit the information as illustrated in Fig. 1.1. It was reported in [18] that the two-user cooperation is beneficial and can result in substantial gains over a non-cooperative strategy. These gains include higher data rate, higher throughput, less sensitivity towards channel variations (due to the spatial diversity gain), and higher coverage.

Cooperative diversity protocols based on repetition schemes were developed and analyzed in [24]. Amplify-and-forward (AF) scheme, in which the relays retransmit the noisy version of the received signal, was introduced. The work in [24] has also inspired the integration of space-time coding and cooperative schemes by utilizing two-phase transmissions to achieve the full spatial diversity corresponding to the number of cooperating terminals. Subsequently, cooperative communications based on distributed space-time coding (DSTC) was studied in [25, 26, 27]. DSTC was shown to achieve the same diversity and coding gain as the MISO system at very high SNR [26]. Performance comparison between DSTC with orthogonal and quasi-orthogonal space-time codes were given in [27]. Reduced complexity decoding for DSTC was studied in [28] and extension of DSTC to the multiple source scenario was presented in [29].

In contrast to DSTC, a relay selection protocol that selects the best relay between the source and destination based on instantaneous channel measurements, namely opportunistic relaying, was proposed in [30]. The opportunistic relaying protocol was shown to achieve similar performance to DSTC with much lower complexity. Its outage probability analysis was presented in [31]. Selection relaying was also extended to multi-source scenario in [32] and a more general selection relaying involving a greater number of relays was studied in [33]. Other works in [34, 35, 36] combine the use of channel coding in cooperative communications. This is usually regarded as coded cooperative diversity. Each user, instead of re-

Figure 1.2: Conventional half-duplex relaying.

peating the received bits (either by amplifying or decoding), transmits incremental redundancy for its partner [34]. The bounds for the bit error rate performance for single relay coded cooperative diversity was presented in [35]. Application of turbo codes and iterative diversity in coded cooperative diversity was studied in [37].

Multi-hop relaying systems were presented in [38, 39, 40]. In [38], end-to-end outage probability analysis for non-regenerative (amplify-and-forward) multi-hop relaying was derived for Nakagami- m fading channels. The bounds on the outage performance was derived in [40]. The capacity analysis for multi-hop relaying with adaptive transmission was analyzed in [41].

1.1.2.1 Full-Duplex and Half-Duplex Relaying

Relays in general can operate in either full-duplex or half-duplex mode. For full-duplex mode, the relay is assumed to be able to transmit and receive simultaneously on the same frequency band. This assumption was considered in the very early works of wireless relay channel [20]. Full-duplex relaying will enable the system to achieve higher sum rate or throughput. However, it is very difficult to implement in practice due to the high power level difference between the transmit and receive signals.

On the contrary, for the half-duplex mode, the relay is only able to either transmit or receive at any one time. This assumption is more realistic and has been the basic assumption in most works in cooperative communications. For conventional half-duplex relaying, a two-phase transmission [24] is typically required to transmit information from the source to the destination which leads to a spectral efficiency

Figure 1.3: Non-orthogonal amplify-and-forward protocol.

loss indicated by the pre-log factor of one-half in the sum rate expressions [42]. The first phase is for the source to send the information to the relays and the second phase is for the relays to forward the source's information to the destination as depicted in Fig. 1.2.

Non-Orthogonal Amplify-and-Forward

For single relay scenario, non-orthogonal amplify-and-forward (NAF) protocol was proposed in [25] and has been proven to be the optimal AF protocol for single half-duplex relay channel in [43]. The transmission protocol for NAF protocol is shown in Fig. 1.3. In the first transmission slot, the source communicates with the relay and the destination. In the second transmission slot, both source and relay communicates with the destination. As the name of the protocol implies, amplify-and-forward processing is performed at the relay. The NAF protocol realizes the maximum degree of broadcasting and receive collision for single relay network.

1.1.2.2 Spectrally-Efficient Cooperative Communication Protocols

The pre-log factor, induced in conventional half-duplex relaying, causes substantial loss in the spectral efficiency, particularly in the high SNR regime. Thus, to solve this inherent problem, two-way relaying and two-path relaying protocols were proposed in [42].

Figure 1.4: Two-way relaying protocol.

Two-Way Relaying

The idea of two-way communication was originally studied by Shannon in [44]. In two-way relaying protocol, the relay is used to assist the two-way communication between the source and destination as shown in Fig. 1.4. In the first phase, the source and the destination transmit simultaneously to the relay. The relay can either perform decoding (DF) or just amplify the received signals (AF). Then, in the second phase, the relay broadcasts the processed signal to the source and the destination. This signal contains back propagating self-interference to the receiving terminal. However, since both source and destination know exactly the information they had transmitted before, the back propagating interference can be subtracted away to retrieve the desired information.

For two-way relaying protocol, two-phase transmissions are still required. However, since two communications are involved during the transmissions, the spectral efficiency loss is recovered. The achievable rate for this two-way relaying has been studied in [42]. Two-way relaying protocol has also been extensively studied in [45, 46, 47, 48].

Two-Path Relaying

Quite different from two-way relaying, two-path relaying only considers one-way communication between the source and destination. The idea is to overcome the half-duplex constraints of the relay by utilizing two relays. The two relays alternate between transmission and reception modes to mimic the full-duplex relaying operation as shown in Fig. 1.5. Thus, the spectral efficiency loss can be recovered. The term “two-path” is derived from the two basic paths that are available due to

Figure 1.5: Two-path relaying protocol.

the two relays.

For a two-relay topology, also known as diamond relay channel [49], it was shown in [42] that two-path relaying protocol can recover a significant portion of the spectral efficiency loss in half duplex relaying schemes. Similar to two-way relaying, the processing at the relay can either be DF or AF where the AF relaying scheme has the advantage of being simpler to implement in practice. Compared to AF, decode-and-forward version of this two-path relaying (successive digital relaying) has been well studied. Achievable rates and capacity bounds for successive digital relaying were analyzed in [49, 50, 51]. Diversity multiplexing tradeoff analysis of this successive digital relaying was given in [52]. A space-time V-BLAST signalling and detection method that can approach the theoretical achievable rates under certain channel conditions was also developed in [52].

Slotted Amplify-and-Forward

Two-path relaying protocol has been further extended to the multiple relay scenario in [53, 54]. In [53], a class of AF two-path relaying for multiple relays called slotted amplify-and-forward (SAF) was analyzed. The transmission of a codeword from the source is divided into several slots, and for each slot several symbols are transmitted.

Figure 1.6: Slotted amplify-and-forward protocol.

The detail of SAF transmission protocol is shown in Fig. 1.6. The cooperation frame is composed of M slots of L_m symbols. The source tries to communicate with the destination through N relays. In each time slot, source and one of the relays transmit L_m symbols as shown in Fig. 1.6. A scheduler (that exists physically or logically) coordinates the transmission of the relays. Each of the relay will take turn to amplify-and-forward whatever it received in the previous time slot.

However, the work in [53] did not consider inter-relay interferences and it was shown that SAF significantly outperforms other schemes, especially in the high spectral efficiency and large network size region.

1.1.3 Diversity-Multiplexing Tradeoff

Diversity-Multiplexing Tradeoff (DMT) proposed by Zheng and Tse in [55] presents a theoretical framework to measure the fundamental performance of MIMO system in high SNR regime. The motivation behind the establishment of DMT is the difficulty in analytically characterizing the outage probability curve [56]. By characterizing the performance in high SNR regime, the reliability and the number of degrees of freedom of a communication system can be elegantly captured.

Reliability is measured by diversity gain, defined as the rate of decay of the error probability as the SNR increases, while the degrees of freedom is measured in terms of spatial multiplexing gain, defined as the rate of increase in the transmission rate with increasing SNR. Both gains can be simultaneously obtained, but there is a

fundamental tradeoff between how much of each type of gain a coding scheme can extract: higher spatial multiplexing gain comes at the expense of diversity gain [55].

Definition 1.1.1. *Mathematically, a coding scheme $\{\mathcal{C}(SNR)\}$ is said to achieve multiplexing gain r and diversity gain d if*

$$\lim_{SNR \rightarrow \infty} \frac{R(SNR)}{\log SNR} = r \quad (1.2)$$

and

$$\lim_{SNR \rightarrow \infty} \frac{\log P_e(SNR)}{\log SNR} = -d \quad (1.3)$$

where $P_e(SNR)$ is the average error probability with a maximum-likelihood (ML) receiver. $R(SNR)$ is the data rate measured by bits per channel use¹.

For a point-to-point MIMO system [55], the best DMT that can be achieved by any coding scheme, denoted as $d^*(r)$, is given by the piecewise-linear function connecting the points $(k, d^*(k))$, $k = 0, 1, \dots, \min\{m, n\}$,

$$d^*(k) = (M_t - M_r)(n - k) \quad (1.4)$$

where M_t and M_r are the number of transmit and receive antennas, respectively. k is the integer value of r . From (1.4), the maximum diversity gain is mn which is attained when $k \rightarrow 0$. The maximum multiplexing gain is $\min\{m, n\}$ which is attained when $d \rightarrow 0$.

DMT analysis for a point-to-point MIMO system in [55] was then extended to multiple-access channels in [57] where the tradeoff of diversity, multiplexing, and multiple-access gains was studied. In [58], a construction of approximately universal codes was presented to optimally tradeoff the diversity and multiplexing gains of MIMO channel. Other works in [59, 60, 61] have also been devoted to design codes that can achieve the desired diversity and multiplexing gain tradeoff of MIMO channel. Later works in [62, 63, 64] studied the finite-SNR DMT of MIMO channel where the tradeoff of the diversity and multiplexing gains was analyzed for practical range of SNR values.

¹For ease of presentation, we sometimes denote $R(SNR)$ as R .

On the other hand, DMT has also been a unified framework to analyze different cooperative communication protocols. The DMT analysis for cooperative communication systems was first studied in [24]. The DMT was derived for repetition based and space-time coded cooperative diversity protocols. Incremental relaying where the destination utilizes one-bit feedback to the source and the relay was proposed in [24] to reduce the multiplexing loss. In [43], DMT for non-orthogonal amplify-and-forward protocol [25] where the source is allowed to transmit simultaneously with the relay during the transmission in the second transmission phase was studied.

To improve the DMT of half-duplex single relay system, dynamic decode-and-forward (DDF) was also introduced in [43] where the time duration when the relay listens to the source is determined by the source-relay channel gain. The DMT of DDF protocol was shown to be optimal for the low multiplexing gain regime (up to half), but not for medium and high multiplexing gain regimes. An enhanced version of the DDF protocol was further proposed in [65] that slightly improved the DMT.

In [66], DMT of DF and CF protocols for single relay was studied. Multiple antennas at the nodes are considered. The CF protocol was shown to achieve the transmit diversity bound for a full-duplex and half-duplex relay with arbitrary number of antennas at the source, relay, and destination. This is achieved, however, with the requirement that the relay knows all the fading channel coefficients in the network.

1.2 Motivations and Objectives

As discussed earlier, the two important features in wireless communications are spectral efficiency (multiplexing gain) and reliability (diversity gain). In this thesis, we investigate cooperative communications protocols that are able to enhance this spectral efficiency and improve the reliability. Specifically, we address the following research issues concerning cooperative half-duplex relay networks:

- Many cooperative communication protocols [43, 30] are able to achieve full spatial diversity gain, but the multiplexing gain is reduced to one-half due

to the half-duplex constraint of the relay. Thus, we are motivated to study spectrally-efficient protocols that are able to recover multiplexing loss in cooperative half-duplex relay networks. We will focus on two-path relaying and sequential relaying protocols in this dissertation.

- Two-path relaying protocol suffers from inter-relay interference [42]. Whenever one of the relays receives an interference signal from the source, it will also receive a signal from the other relay. This makes the analysis for two-path relaying protocol relatively more difficult and less studied in the literature. The two-path protocol clearly has a great potential to offer higher spectral efficiency and better reliability for general cooperative communications if the interference can be managed well. This is the major motivation for the analysis of two-path relaying protocol in Chapter 2.
- Finding the optimal relaying protocol is rather difficult if we do not have a standard performance metric to compare different cooperative communication protocols. The two important gains, i.e., multiplexing and diversity gains are difficult to measure, unless we resort to the high SNR assumption. This is where DMT analysis for cooperative communications becomes very important. DMT is analytically tractable and powerful enough to provide a framework that is able to measure the tradeoff between the two gains. This has been the motivation of studying DMT of AF and DF two-path relaying in Chapter 3 and 4.
- Two-path relaying protocol only applies to a two-relay network scenario. Thus, the next question would be how to generalize the protocol to a more general problem where multiple relays and multiple sources are considered. Currently, only a few protocols are available in the literature. We are motivated to study this scenario in Chapter 5 and propose a protocol that is able to provide good spectral efficiency and reliability in general cooperative half-duplex relay networks.

1.3 Contributions and Outline of the Dissertation

The main contributions and outline of the dissertation are listed as follows:

Chapter 2

In Chapter 2, we study AF two-path relaying protocol and propose inter-relay self-interference cancellation at one of the relays to manage the interference. With the proposed inter-relay self interference cancellation, AF two-path relaying can be expressed as an equivalent MIMO space time code which allows us to derive an upper bound for the pairwise error probability of the two-path relaying scheme. A better performance than the AF scheme in [42], especially when the inter-relay channel gain is higher than the other channels, is also achieved.

From our analysis of pairwise error probability, we identify the reason to the limited diversity order of two in [67, 68] and propose a simple precoding scheme, which does not require any channel state information, to achieve a full diversity order of three while maintaining the spectral efficiency gain of the two-path relaying protocol.

Approaches from both coding gain and mutual information point of views for precoder design are also discussed to achieve full diversity and high coding gain. In particular, a class of precoding matrices that can be easily generated in practice and able to achieve full diversity order of three with high coding gain is proposed. Results for a low complexity receiver based on QRD-M algorithm [69, 70] are also studied. This work has been published in [68, 71].

H. Wicaksana, S. H. Ting, C. K. Ho, W. H. Chin, and Y. L. Guan, "On the diversity order of AF cooperative systems with inter-relay interference cancellation," in *Proc. IEEE Veh. Technol. Conf.*, May 2008.

H. Wicaksana, S. H. Ting, C. K. Ho, W. H. Chin, and Y. L. Guan, "AF two-path half duplex relaying with inter-relay self interference cancellation: Diversity analysis and its improvement," *IEEE Trans. Wireless Commun.*, vol. 8, no. 9, pp. 4720-4729, Sept 2009.

Chapter 3

Realizing the importance of DMT analysis in cooperative communications, we characterized the DMT of AF two-path relaying studied in Chapter 2. To reach our goal, we first generalize the result in [72, Theorem 2.12] and obtain lower bounds for the DMT of an arbitrary block lower triangular channel matrix. In [72], lower bounds can only be found by using the main and the last sub-diagonal matrix. However, with the generalized results, we are able to lower bound the DMT by using any sub-diagonal matrix.

We then derive the DMT of AF two-path relaying scheme, while taking into account inter-relay interference in the analysis. Inter-relay interference was not considered in [52, 53]. Our analysis indicates that a full diversity order of three can be achieved while maintaining a spatial multiplexing gain of one for sufficiently large frame/transmission length. It can also be seen that the DMT of this scheme asymptotically achieves the multiple-input single-output (MISO) upper bound. For the multiple-relay case, we provide a stronger result than [53] and derive the DMT for slotted amplify-and-forward scheme in [53] without the assumption that the relays are isolated, i.e. inter-relay interference is taken into consideration.

In [71], we have discovered that a full diversity order of three can be achieved easily by using precoding, which can be viewed as an equivalent linear space time code. Our analysis demonstrates that the same space time coding technique can be used as a simple and yet effective finite codeword length coding strategy to achieve the full DMT of this scheme.

From the DMT perspective, the proposed AF two-path relaying with the equivalent linear space time coding is optimal and outperforms existing schemes in [42, 67, 68, 52]. This study has been published in [73].

H. Wicaksana, S. H. Ting, M. Motani, and Y. L. Guan, "On the diversity-multiplexing tradeoff of amplify-and-forward half-duplex relaying," *IEEE Trans. Commun.*, vol. 58, no. 12, pp. 3621-3630, Dec 2010.

Chapter 4

In Chapter 4, we study DF two-path relaying, a variant protocol of AF two-path relaying.

With a distributed MISO approach, we first derive the DMT of DF two-path relaying, where perfect decoding at the relays is assumed [52], and show that the DMT approaches the 3×1 MISO DMT [55] with a sufficiently large frame length.

We then remove the assumption of perfect decoding at the relays and study the effects of inter-relay interference on the DMT performance of DF two-path relaying by utilizing a DMT framework with different relative distances between nodes [74]. In most cases, the relative distances between source, relay, and destination nodes do not affect DMT performance and is not easily reflected in the results. However for DF two-path relaying, there are certain conditions for the source-relay link where the DMT for the ideal condition, i.e., perfect decoding at the relays, can be achieved. A successive interference cancellation scheme at the relays is also proposed to further improve the DMT performance. The analysis and the derivation will appear in [75].

H. Wicaksana, S. H. Ting, Y. L. Guan, and X.-G. Xia, “Decode-and-forward two-path half-duplex relaying: Diversity-multiplexing tradeoff analysis,” *IEEE Trans. Commun.*, 2011, to appear.

Chapter 5

In Chapter 5, we consider a more general scenario where multiple sources are considered in the networks and propose a sequential amplify-and-forward relaying protocol for this system. The transmissions for both sources and relays are performed sequentially to the destination. We analyze the performance of this protocol in terms of DMT.

We first establish the upper bound for the sequential amplify-and-forward relaying protocol based on the DMT of MISO channel. We then derive the achievable DMT and show that the proposed transmission protocol overcomes the inherent spectral efficiency loss in half-duplex relaying and achieves the full diversity gain. Compared to the scheme in [76], which is based on decode-and-forward relaying, the proposed scheme has better DMT performance, lower complexity, and lower

CSI requirements at the relays. When the frame length is sufficiently large, the MISO DMT upper bound is achieved. This work is currently under review in [77].

H. Wicaksana, S. H. Ting, Y. L. Guan, and X.-G. Xia, “Cooperative multiple-access system with sequential amplify-and-forward relaying,” *IEEE Trans. Commun.*, 2011, in review.

Chapter 6

In Chapter 6, we draw conclusion for this dissertation where a brief summary of the work is given. We will also discuss some future research to extend the work presented in this dissertation.

Amplify-and-Forward Two-Path Half-Duplex Relaying with Inter-Relay Self Interference Cancellation

2.1 Introduction

The work in [42] highlighted the point that since AF two-path relaying alternates between transmission and reception, an inter-relay interference term will be additionally received at the destination. Successive decoding at destination with partial or full cancellation of this inter-relay interference term was proposed in [42]. However, this method only performs well for a weak to moderate inter-relay channel. Direct link from the source to the destination was not considered and a theoretical analysis of diversity order was also not given in [42].

As for successive digital relaying in [52], it was assumed that the interference between the relays is so strong that it can be perfectly cancelled through successive interference cancellation (SIC) at the relay. Otherwise, the relay will directly decode the signal while assuming the interference as Gaussian noise. This assumption of perfect SIC of the interference signal may not be applicable in practical scenarios and any interference between relays will definitely degrade the perfor-

mance and reduce the capacity. Direct link was considered in [52], but only a diversity order of two is achieved. In [50, 51], dirty paper coding (DPC) based on interference pre-subtraction at the source transmitter was proposed to solve the inter-relay interference problem. The DPC in [50, 51] requires full knowledge of all channel state information (CSI), in the two-path relaying channel at the source transmitter, which is very difficult to achieve in practice [78, 79]. Compared to AF relaying, the DF schemes differ significantly as there is no accumulation of inter-relay interference. This is achieved at the expense of extra decoding operations and SIC performed at the relays, and the performance is poor especially when the inter-relay channel gain is weak, as reported in [52].

In this chapter, we consider an AF two-path relaying with inter-relay self interference cancellation where the cancellation is performed at *one of the relays*. The direct link between source and destination is considered. The relay that performs interference cancellation is required to have knowledge of the average channel gain from the source to the other relay and perfect CSI is assumed to be available at all receiving nodes, but no CSI is available at the transmitting nodes. This scheme was first proposed in [67] and numerical simulations under various channel conditions using a suboptimal sequence estimator were given. However, achievable rate and diversity gain of the scheme were not studied and similar to [52], only a diversity order of two was reported in the simulation results of [67]. We will first provide a comprehensive analysis for the achievable rate and diversity order of this AF two-path relaying with inter-relay self interference cancellation. After which, we propose a simple precoding scheme that achieves a diversity order of three and thus betters the schemes in [52, 67].

Our main contributions can be summarized as follows:

- With the proposed inter-relay self interference cancellation, we can express AF two-path relaying as an equivalent MIMO space time code which allows us to derive an upper bound for the pairwise error probability (PEP) of two-path relaying scheme.
- Our analysis indicates that by using inter-relay self interference cancellation, we can achieve a better performance than the AF scheme in [42] especially when the inter-relay channel gain is higher than the other channels.

- In [67], only a diversity order of two can be achieved even though physically there are three possibly independent spatial paths to the destination. From our analysis of PEP, we identify the reason to the limited diversity order of two and propose a simple precoding scheme, which does not require any channel state information, to achieve a full diversity order of three while maintaining the spectral efficiency gain of the two-path relaying protocol.
- Approaches from both coding gain and mutual information point of views for precoder design are also discussed to achieve full diversity and high coding gain. We then propose a class of precoding matrices that can be easily generated in practice and able to achieve full diversity order of three with high coding gain. Simulation results show that the proposed precoder design is robust and gives a significant improvement over the scheme without precoding. Results for a low complexity receiver based on QRD-M algorithm [69, 70] are also presented.

2.2 Two-Path Relaying with Inter-Relay Self Interference Cancellation

Consider a cooperative network system with one source node (S), two relay nodes (R1, R2) and one destination node (D). To gain insights, we will presume all the nodes have a single antenna and no precoding (except for power scaling) is performed. In Section 2.3.2, we will extend this work with precoding at S to improve the diversity gain of the system.

The source transmission is divided into frames, each containing L symbols. The φ -th frame is denoted as $\mathbf{x}_\varphi = [x(1)x(2)\cdots x(L)]^T$ with $\mathbb{E}\{\mathbf{x}_\varphi\mathbf{x}_\varphi^H\} = \mathbf{I}_L$. The destination will perform detection on a frame-by-frame basis. Without loss of generality, we assume L is even. Since the analysis is the same for all frames, the subscript φ is removed, except when there is a need to differentiate between frames. We will also presume that the channel is static within a frame.

Figure 2.1: AF two-path relaying scheme.

2.2.1 Transmission Protocol

The transmission protocol of AF two-path relaying scheme within two consecutive time slots is depicted in Fig. 2.1. Let $y_{rj}(i)$ and $y_d(i)$ denote the received signal at j -th relay, $j \in \{1, 2\}$, and received signal at D, at time slot i respectively. At time slot 1, $x(1)$ is sent from S. Thus at R1 and D,

$$y_{r1}(1) = \sqrt{\rho} h_{s,r1} x(1) + n_{r1}(1), \quad (2.1)$$

$$y_d(1) = \sqrt{\rho} h_{s,d} x(1) + n_d(1), \quad (2.2)$$

where ρ , $h_{s,d} \sim \mathcal{CN}(0, \gamma_{s,d}^2)$, and $h_{s,rj} \sim \mathcal{CN}(0, \gamma_{s,rj}^2)$ are the average transmit power, the channel coefficients from S to D and S to j -th relay respectively. $n_{rj}(i) \sim \mathcal{CN}(0, \sigma^2)$ and $n_d(i) \sim \mathcal{CN}(0, \sigma^2)$ are the additive white Gaussian noise (AWGN) at the j -th relay and D respectively at i -th time slot where we have presumed that R1, R2, and D have the same noise variance. Notice that $y_d(1)$ contains only $x(1)$ as R2 does not transmit in the first time slot.

Let $x_{rj}(i)$ denote the transmit signal at the j -th relay at time slot i . Then, at time slot 2, $x(2)$ is sent from S and $x_{r1}(2)$ is sent from R1. Thus at R2 and D,

$$y_{r2}(2) = \sqrt{\rho} h_{s,r2} x(2) + h_r x_{r1}(2) + n_{r2}(2), \quad (2.3)$$

$$y_d(2) = \sqrt{\rho} h_{s,d}x(2) + h_{r1,d}x_{r1}(2) + n_d(2), \quad (2.4)$$

where $x_{r1}(2) = g_{r1}(2)y_{r1}(1)$. Here, $g_{rj}(i)$, $h_{rj,d} \sim \mathcal{CN}(0, \gamma_{rj,d}^2)$ and $h_r \sim \mathcal{CN}(0, \gamma_r^2)$ are the power scaling factor at j -th relay in time slot i which we will define later, the channel coefficients from j -th relay to D and channel coefficient between the two relays respectively. We will assume that the channel between relays is reciprocal. Notice that $y_d(2)$ contains both $x(2)$ and $x(1)$ (which is contained in $x_{r1}(2)$).

At time slot 3, $x(3)$ is sent from S, while $x_{r2}(3)$ is sent from R2. Thus at R1 and D,

$$\begin{aligned} y_{r1}(3) &= \sqrt{\rho} h_{s,r1}x(3) + h_r x_{r2}(3) + n_{r1}(3) \\ &= \sqrt{\rho} h_{s,r1}x(3) + h_r g_{r2}(3) \left[\sqrt{\rho} h_{s,r2}x(2) \right. \\ &\quad \left. + h_r x_{r1}(2) + n_{r2}(2) \right] + n_{r1}(3), \end{aligned} \quad (2.5)$$

$$y_d(3) = \sqrt{\rho} h_{s,d}x(3) + h_{r2,d}x_{r2}(3) + n_d(3), \quad (2.6)$$

where $x_{r2}(3) = g_{r2}(3)y_{r2}(2)$. Notice that $y_d(3)$ contains $x(3)$, $x(2)$, and $x(1)$. We observe that $y_{r1}(3)$ contains the inter-relay self interference term, $h_r g_{r2}(3)h_r x_{r1}(2)$, which is known perfectly at R1 and thus it can be cancelled away. Hence, the transmit signal at time slot 4 from R1 will be

$$\begin{aligned} x_{r1}(4) &= g_{r1}(4) \left[y_{r1}(3) - h_r g_{r2}(3)h_r x_{r1}(2) \right] \\ &= g_{r1}(4) \left\{ \sqrt{\rho} h_{s,r1}x(3) + h_r g_{r2}(3) \left[\sqrt{\rho} h_{s,r2}x(2) \right. \right. \\ &\quad \left. \left. + n_{r2}(2) \right] + n_{r1}(3) \right\}. \end{aligned} \quad (2.7)$$

At time slot 4, $x(4)$ is sent from S, while $x_{r1}(4)$ is sent from R1. Thus at R2 and D,

$$y_{r2}(4) = \sqrt{\rho} h_{s,r2}x(4) + h_r x_{r1}(4) + n_{r2}(4), \quad (2.8)$$

$$y_d(4) = \sqrt{\rho} h_{s,d}x(4) + h_{r1,d}x_{r1}(4) + n_d(4). \quad (2.9)$$

Notice that $y_d(4)$ now contains $x(4)$, $x(3)$, and $x(2)$.

The transmit signal at time slot 5 from R2 is $x_{r2}(5) = g_{r2}(5)y_{r2}(4)$. No cancel-

lation of inter-relay interference is done at R2 because $x_{r2}(3)$ does not exist in the received signals due to the cancellation in R1 as shown in (2.7). These transmission steps are then continuously repeated until L symbols have been transmitted from the source. Two additional time slots are required at the end as the terminating sequence and thus a multiplexing gain of $\frac{L}{L+2}$ is achieved. This slight overhead is also reported in [52] where the multiplexing gain is given by $\frac{L}{L+1}$. However, this slight loss in rate is asymptotically zero for large values of L . It is also necessary for D to wait for all $L + 2$ transmissions to occur before performing decoding.

From the series of equations above, we can observe that the maximum memory of the transmission protocol is only 4 symbols. The transmitted signals at R1 and R2 in the i -th time slot can be generalized as

$$x_{r1}(i) = \begin{cases} g_{r1}(i)y_{r1}(i-1) & \text{for } i = 2 \\ g_{r1}(i) \left[y_{r1}(i-1) - h_r g_{r2}(i-1) h_r x_{r1}(i-2) \right] & \text{for } i = 4, 6, 8, \dots, L, \end{cases}$$

$$x_{r2}(i) = g_{r2}(i)y_{r2}(i-1) \quad \text{for } i = 3, 5, 7, \dots, L-1,$$

respectively, where $g_{rj}(i)$ is expressed as

$$g_{r1}(i) = \begin{cases} \sqrt{\frac{\rho}{\gamma_{s,r1}^2 \rho + \sigma^2}} & i = 2 \\ \sqrt{\frac{\rho}{\gamma_{s,r1}^2 \rho + \gamma_r^2 g_{r2}^2(i-1) (\gamma_{s,r2}^2 \rho + \sigma^2) + \sigma^2}} & i = 4, 6, 8, \dots, L \end{cases}$$

$$g_{r2}(i) = \sqrt{\frac{\rho}{\gamma_{s,r2}^2 \rho + \gamma_r^2 \rho + \sigma^2}}, \quad i = 3, 5, 7, \dots, L-1.$$

Here, we presume that R1 and R2 have the same average transmit power as S. Notice that $g_{r2}(i)$ is constant for $i = 3, 5, \dots, L-1$ as we use the average channel gains instead of the instantaneous ones. This assumption is also being used in [25, 80]. The use of average channel gains is more practical and its performance is comparable to that using instantaneous channel gains as shown in [15]. Thus, $g_{r1}(i)$ which depends on $g_{r2}(i-1)$ is also constant for $i = 4, 6, \dots, L$. Hence, we can see that both $g_{r1}(i)$ and $g_{r2}(i)$ are constants for $i \geq 3$. Only the expression for $g_{r1}(2)$ is different, but this will not affect our analysis later where we consider the steady state performance of the system. Thus, we will drop the time index i

hereafter.

For the transmission protocol described above, we can see that a few assumptions have been made. Firstly, perfect CSI is presumed to be available at all receivers, i.e. perfect channel estimation is achieved. In practice, robust channel estimation can be performed through the transmission of training sequences. Furthermore, by ensuring that the training sequences transmitted by S and relays¹ (R1, R2) are orthogonal, we will be able to achieve simultaneous estimation of $h_{s,rj}$, $j \in \{1, 2\}$ and h_r by R1, R2, and simultaneous estimation of $h_{rj,d}$, $j \in \{1, 2\}$ and $h_{s,d}$ at D.

We have also assumed that R1 has been informed of its responsibility to perform interference cancellation and it has the knowledge of $\gamma_{s,r2}^2$. In practice, both assumptions can be easily realized through the use of control messages carried in the header of each frame.

2.2.2 Equivalent MIMO Space Time Code

Consolidating all the equations in Section 2.2.1, the equivalent channel matrix $\mathbf{H}$ is a lower triangular matrix with at most 4 non-zero entries in each row and

$$\mathbf{y}_d = \sqrt{\rho} \mathbf{H} \mathbf{x} + \mathbf{B} \mathbf{n}_r + \mathbf{n}_d \quad (2.10)$$

where

$$\begin{aligned} \mathbf{y}_d &= [y_d(1) \ y_d(2) \ \cdots \ y_d(L+2)]^T, \quad \mathbf{x} = [x(1) \ x(2) \ \cdots \ x(L)]^T, \\ \mathbf{n}_r &= [n_{r1}(1) \ n_{r2}(2) \ n_{r1}(3) \ \cdots \ n_{r2}(L)]^T, \text{ and } \mathbf{n}_d = [n_d(1) \ n_d(2) \ \cdots \ n_d(L+2)]^T. \end{aligned}$$

Let $K(p, q)$ denotes the elements at p -th row and q -th column of $\mathbf{K}$. Then, the non-zero elements of $\mathbf{H} \in \mathcal{C}^{(L+2) \times L}$ and $\mathbf{B} \in \mathcal{C}^{(L+2) \times L}$ are given as

$$\begin{aligned} \mathbf{H}(m, m) &= h_{s,d}, \quad m = 1, 2, 3, \dots, L, \\ \mathbf{H}(m, m-1) &= \begin{cases} h_{s,r1} g_{r1} h_{r1,d}, & m = 2, 4, \dots, L \\ h_{s,r2} g_{r2} h_{r2,d}, & m = 3, 5, \dots, L+1, \end{cases} \end{aligned}$$

¹R1 and R2 can use the same training sequence because, according to the two-path relaying protocol, they do not transmit at the same time.

$$\mathbf{H}(m, m-2) = \begin{cases} h_{s,r1}g_{r1}h_rg_{r2}h_{r2,d}, & m = 3, 5, \dots, L+1 \\ h_{s,r2}g_{r2}h_rg_{r1}h_{r1,d}, & m = 4, 6, \dots, L+2, \end{cases}$$

$$\mathbf{H}(m, m-3) = h_{s,r2}g_{r2}^2h_r^2g_{r1}h_{r2,d}, \quad m = 5, 7, \dots, L+1,$$

furthermore,

$$\mathbf{B}(m, m-1) = \begin{cases} g_{r1}h_{r1,d}, & m = 2, 4, \dots, L \\ g_{r2}h_{r2,d}, & m = 3, 5, \dots, L+1, \end{cases}$$

$$\mathbf{B}(m, m-2) = \begin{cases} g_{r1}h_rg_{r2}h_{r2,d}, & m = 3, 5, \dots, L+1 \\ g_{r2}h_rg_{r1}h_{r1,d}, & m = 4, 6, \dots, L+2, \end{cases}$$

$$\mathbf{B}(m, m-3) = g_{r2}^2h_r^2g_{r1}h_{r2,d}, \quad m = 5, 7, \dots, L+1,$$

Notice that $\mathbf{B}\mathbf{n}_r$ is the residual noise from the relays and it is in general not white.

We can further write (2.10) as

$$\mathbf{y}_d = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{B}'\mathbf{n} \quad (2.11)$$

where $\mathbf{B}' = [\mathbf{B} \ \mathbf{I}_{L+2}]$, and $\mathbf{n} = [\mathbf{n}_r^T \ \mathbf{n}_d^T]^T$.

From the standard MIMO channel model in (2.11), we obtain

$$\begin{aligned} I(\mathbf{x}; \mathbf{y}_d) &= \frac{L}{L+2} \log_2 \det \left(\mathbf{I}_{L+2} + \rho \mathbf{H}\mathbf{R}_x\mathbf{H}^H (\mathbf{B}'\mathbf{R}_n\mathbf{B}'^H)^{-1} \right) \\ &= \frac{L}{L+2} \log_2 \det \left(\mathbf{I}_{L+2} + \frac{\rho}{\sigma^2} \mathbf{H}\mathbf{H}^H (\mathbf{B}\mathbf{B}^H + \mathbf{I}_{L+2})^{-1} \right) \end{aligned} \quad (2.12)$$

where $\mathbf{R}_x = \mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_L$, $\mathbf{R}_n = \mathbb{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma^2\mathbf{I}_{2L+2}$. The derivation for the mutual information with correlated noise in (2.12) can be found in [81]. Equation (2.12) will be used to compute the achievable rate of the proposed protocol in Section 2.5. We will use $\frac{\rho}{\sigma^2}$ and SNR interchangeably in the analysis hereafter.

2.3 Diversity Order Analysis and Precoding

In this section, we will derive the upper bound of PEP for two-path relaying to analyze the diversity order of the proposed scheme. First, we express the received

signal in (2.10) equivalently as

$$\mathbf{y}_d = \sqrt{\rho} \mathbf{X}_k \mathbf{h} + \mathbf{w} \quad (2.13)$$

where $\mathbf{X}_k$ is a $(L+2) \times 6$ matrix consisting of the transmitted symbols $\mathbf{x}_k$,

$$\mathbf{h} = \begin{bmatrix} h_{s,d} \\ h_{r1,d} \ g_{r1} \ h_{s,r1} \\ h_{r2,d} \ g_{r2} \ h_r \ g_{r1} \ h_{s,r1} \\ h_{r2,d} \ g_{r2} \ h_{s,r2} \\ h_{r1,d} \ g_{r1} \ h_r \ g_{r2} \ h_{s,r2} \\ h_{r2,d} \ g_{r2} \ h_r \ g_{r1} \ h_r \ g_{r2} \ h_{s,r2} \end{bmatrix} \quad (2.14)$$

is the equivalent channel vector, and $\mathbf{w} \triangleq \mathbf{B}\mathbf{n}_r + \mathbf{n}_d$ conditioned on $\mathbf{h}$ is the correlated Gaussian noise with covariance matrix $\mathbf{\Sigma}_W = \sigma^2 (\mathbf{B}\mathbf{B}^H + \mathbf{I}_L)$. The non-zero elements of $\mathbf{X}_k$ are given as

$$\begin{aligned} X_k(m, 1) &= X_k(m+1, 2) = X_k(m+2, 3) = x_k(m) \\ &\text{for } m = 1, 3, 5, 7, \dots, L-1, \\ X_k(m, 1) &= X_k(m+1, 4) = X_k(m+2, 5) = X_k(m+3, 6) \\ &= x_k(m) \quad \text{for } m = 2, 4, 6, 8, \dots, L. \end{aligned} \quad (2.15)$$

Thus, we can view $\mathbf{X}_k$ as the equivalent space time code for $\mathbf{x}_k$. The structure of $\mathbf{X}_k$ can be shown as

$$\mathbf{X}_k = \begin{bmatrix} x_k(1) & 0 & 0 & 0 & 0 & 0 \\ x_k(2) & x_k(1) & 0 & 0 & 0 & 0 \\ x_k(3) & 0 & x_k(1) & x_k(2) & 0 & 0 \\ x_k(4) & x_k(3) & 0 & 0 & x_k(2) & 0 \\ x_k(5) & 0 & x_k(3) & x_k(4) & 0 & x_k(2) \\ \vdots & \ddots & \dots & \dots & \ddots & \ddots \\ \vdots & \dots & \dots & \dots & x_k(L) & \dots \end{bmatrix}. \quad (2.16)$$

We employ maximum likelihood (ML) decoding at the receiver. Suppose $\mathbf{X}_k$ is

transmitted, then from [82], the PEP of mistaking $\mathbf{X}_k$ with $\mathbf{X}_l$ has the following Chernoff upper bound

$$P(\mathbf{X}_k \rightarrow \mathbf{X}_l) \leq \mathbb{E}_{\mathbf{h}} \left\{ e^{-\frac{1}{4} \rho \mathbf{h}^H (\mathbf{X}_k - \mathbf{X}_l)^H \Sigma_W^{-1} (\mathbf{X}_k - \mathbf{X}_l) \mathbf{h}} \right\}. \quad (2.17)$$

The covariance matrix Σ_W is not a diagonal matrix. Hence, (2.17) cannot be easily analyzed. Since Σ_W is a positive semi-definite matrix, we can further upper bound the PEP by upperbounding² $\Sigma_W \leq \text{tr}(\Sigma_W) \mathbf{I}_L$ where $\text{tr}(\Sigma_W)$ is the trace of Σ_W . With this, we obtain

$$P(\mathbf{X}_k \rightarrow \mathbf{X}_l) \leq \mathbb{E}_{\mathbf{h}} \left\{ e^{-\frac{\rho}{4 \text{tr}(\Sigma_W)} \mathbf{h}^H (\mathbf{X}_k - \mathbf{X}_l)^H (\mathbf{X}_k - \mathbf{X}_l) \mathbf{h}} \right\}. \quad (2.18)$$

To calculate the expectation in (2.18), we express $\mathbf{h} = \mathbf{T}\mathbf{C}\mathbf{v}$ where

$$\mathbf{T} = \text{diag} \left(\begin{bmatrix} 1 \\ h_{r1,d} g_{r1} \\ h_{r2,d} h_r g_{r1} g_{r2} \\ h_{r2,d} g_{r2} \\ h_{r1,d} h_r g_{r1} g_{r2} \\ h_{r2,d} h_r^2 g_{r1} g_{r2}^2 \end{bmatrix} \right), \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad (2.19)$$

and $\mathbf{v} = [h_{s,d}, h_{s,r1}, h_{s,r2}]^T \sim \mathcal{CN}(0, \Sigma_v)$ where $\Sigma_v = \text{diag}(\gamma_{s,d}^2, \gamma_{s,r1}^2, \gamma_{s,r2}^2)$.

Thus, by taking expectation over $\mathbf{v}$, we obtain

$$\begin{aligned} P(\mathbf{X}_k \rightarrow \mathbf{X}_l) &\leq \mathbb{E}_{\mathbf{T}} \left\{ \int \frac{1}{(2\pi)^{3/2} |\Sigma_v|^{1/2}} e^{-\frac{\rho}{4 \text{tr}(\Sigma_W)} \mathbf{v}^H \mathbf{C}^H \mathbf{T}^H (\mathbf{X}_k - \mathbf{X}_l)^H (\mathbf{X}_k - \mathbf{X}_l) \mathbf{T} \mathbf{C} \mathbf{v}} e^{-\frac{1}{2} \mathbf{v}^H \Sigma_v^{-1} \mathbf{v}} d\mathbf{v} \right\} \\ &= \mathbb{E}_{\mathbf{T}} \left\{ \det^{-1} \left(\mathbf{I} + \frac{\rho}{4 \text{tr}(\Sigma_W)} \Sigma_v \mathbf{A}^H \mathbf{A} \right) \right\} \end{aligned} \quad (2.20)$$

$$\leq \mathbb{E}_{\mathbf{T}} \left\{ \left(\prod_{i=1}^{\mu} \lambda_i \right)^{-1} \left(\frac{\rho}{4 \text{tr}(\Sigma_W)} \right)^{-\mu} \right\} \quad (2.21)$$

²The matrix inequality $\mathbf{N} \leq \mathbf{M}$ means that $\mathbf{M} - \mathbf{N}$ is a positive semi-definite matrix. An alternative for the upper bound is $\Sigma_W \leq \mu_{\max}(\Sigma_W) \mathbf{I}_L$ where $\mu_{\max}(\Sigma_W)$ denotes the maximum eigenvalue of Σ_W .

where $\mathbf{A} = (\mathbf{X}_k - \mathbf{X}_l) \mathbf{T} \mathbf{C}$, λ_i is the i -th non-zero eigenvalue of $\mathbf{\Sigma}_v \mathbf{A}^H \mathbf{A}$, and μ is the rank of $\mathbf{A}$.

2.3.1 Diversity Order Analysis

Proposition 2.3.1. *AF two-path relaying scheme with inter-relay self interference cancellation achieves a diversity order of two.*

Proof. Given $\mathbf{T}$, based on the diversity criterion [6] in Appendix B, the diversity order of this system can be defined as

$$d \triangleq \min_{\substack{\mathbf{X}_k, \mathbf{X}_l \\ \mathbf{X}_k \neq \mathbf{X}_l}} \text{rank}(\mathbf{A}) = \min_{\substack{\mathbf{X}_k, \mathbf{X}_l \\ \mathbf{X}_k \neq \mathbf{X}_l}} \text{rank}[(\mathbf{X}_k - \mathbf{X}_l) \mathbf{C}] \quad (2.22)$$

as $\mathbf{\Sigma}_v$ and $\mathbf{T}$ are full rank and diagonal matrices. It follows that the diversity order is given by d for any $\mathbf{\Sigma}_v$ and $\mathbf{T}$.

To obtain the minimum rank in (2.22), let us consider the following three cases for the space time code difference matrix $\Delta \mathbf{X} = (\mathbf{X}_k - \mathbf{X}_l)$.

1. Only odd-numbered symbols are different.

Suppose that $p \geq 1$ odd-numbered symbols are different between $\mathbf{X}_k$ and $\mathbf{X}_l$, i.e. $x_k(m) \neq x_l(m)$ for $m \in \mathcal{S}$, where $\mathcal{S} \subset \{1, 3, 5, \dots, L-1\}$ and $|\mathcal{S}| = p$. Then, in the equivalent space time code matrix (2.15), we have three linearly independent columns (1st, 2nd, and 3rd) in $\Delta \mathbf{X}$ as all the non-zero terms are always in the 1st, 2nd, and 3rd columns. For example, if $p = 2$ and $\mathcal{S} = \{1, 3\}$, we have

$$\Delta \mathbf{X} = (\mathbf{X}_k - \mathbf{X}_l) = \begin{bmatrix} \Delta x_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta x_1 & 0 & 0 & 0 & 0 \\ \Delta x_3 & 0 & \Delta x_1 & \vdots & \vdots & \vdots \\ 0 & \Delta x_3 & 0 & \vdots & \vdots & \vdots \\ 0 & \ddots & \Delta x_3 & \vdots & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \end{bmatrix} \quad (2.23)$$

where $\Delta x_m = x_k(m) - x_l(m)$. However, $\mathbf{C}$ only has two corresponding linearly independent rows (in the 1st, 2nd, and 3rd rows). Thus, the product has a limited rank of two.

2. Only even-numbered symbols are different.

Similarly, suppose that $q \geq 1$ even-numbered symbols are different between $\mathbf{X}_k$ and $\mathbf{X}_l$, i.e. $x_k(m) \neq x_l(m)$ for $m \in \mathcal{V}$, where $\mathcal{V} \subset \{2, 4, 6, \dots, L\}$ and $|\mathcal{V}| = q$. Then, in the equivalent space time code matrix (2.15), we have four linearly independent columns (1st, 4th, 5th, and 6th) in $\Delta\mathbf{X}$ as all the non-zero terms are always in the 1st, 4th, 5th, and 6th columns. For example, if $q = 2$ and $\mathcal{V} = \{2, 4\}$, we have

$$\Delta\mathbf{X} = \begin{bmatrix} 0 & 0 & 0 & \cdots & \cdots & 0 \\ \Delta x_2 & 0 & 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \Delta x_2 & 0 & \ddots \\ \Delta x_4 & 0 & 0 & 0 & \Delta x_2 & 0 \\ 0 & 0 & 0 & \Delta x_4 & 0 & \Delta x_2 \\ \vdots & \vdots & \vdots & 0 & \Delta x_4 & 0 \\ 0 & \vdots & \vdots & \cdots & 0 & \Delta x_4 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix}. \quad (2.24)$$

However, again the rank is limited to two since there are only two corresponding linearly independent rows in $\mathbf{C}$ (in the 1st, 4th, 5th, and 6th rows). Hence, the rank of the product matrix is two.

3. Some odd and some even-numbered symbols are different.

For $p \geq 1$ and $q \geq 1$, we have:

- 1st column of $\Delta\mathbf{X}$ is independent to the 2nd-6th columns. 1st row of $\mathbf{C}$ is independent to the 2nd-6th rows. This contributes 1 rank to the product matrix $\Delta\mathbf{X}\mathbf{C}$.
- 2nd-6th columns of $\Delta\mathbf{X}$ and 2nd-6th rows of $\mathbf{C}$ will contribute at least 1 rank to the product matrix $\Delta\mathbf{X}\mathbf{C}$.

Thus, the rank of the product matrix is at least two.

Based on the above three cases, the minimum rank of the product $(\mathbf{X}_k - \mathbf{X}_l)\mathbf{C}$ is two. Thus, AF two-path relaying scheme with inter-relay interference cancellation achieves a diversity order of two.

□

Moreover, the following observations can be deduced from the PEP upper bound derived in (2.21).

- If there is no direct path from the source to destination ($h_{s,d} = 0$) [42], then $\mathbf{C}$ will become a 6×2 matrix (removal of 1st column) and only a diversity order of one can be achieved.
- If h_r does not exist ($h_r = 0$), then the 3rd, 5th, and 6th diagonal elements of $\mathbf{T}$ will be 0. Thus, if one symbol is different between $\mathbf{X}_k$ and $\mathbf{X}_l$, we still have two independent rows in $(\mathbf{X}_k - \mathbf{X}_l) \mathbf{T}$ for both odd and even-numbered symbol cases. Hence, the rank of $\mathbf{A}$ is two. Therefore, without h_r in the equivalent channel matrix, diversity order of two can still be achieved. The diversity order achieved using AF is thus the same as the DF two-path scheme [52], where inter-relay channel does not exist in the equivalent channel matrix due to the decoding operation at the relay.
- Likewise, if either $h_{r1,d}$ or $h_{r2,d}$ is 0, diversity order of two is still achieved. However, if both $h_{r1,d}$ and $h_{r2,d}$ are 0, then only diversity of one is achieved.

2.3.2 Precoding to Improve Diversity Gain

So far, the proposed scheme only achieves diversity order of two even though there are actually three independent spatial paths available from S to D, namely $h_{s,d}$, $h_{r1,d}$, and $h_{r2,d}$. From Section 2.3.1, it is apparent that the problem lies in the rank of the product $(\mathbf{X}_k - \mathbf{X}_l) \mathbf{C}$ which determines the overall rank of $\mathbf{A}$. As discussed before, if one symbol is different, the rows in $\mathbf{C}$ only have two independent rows which causes the overall rank to be two. This also means that the diversity gain of the system can be improved if we can create a condition such that there are three independent rows in the corresponding rows of $\mathbf{C}$.

Consider a precoding matrix $\mathbf{U} \in \mathcal{C}^{L \times L}$ to disperse the symbols in the equivalent space time code. The transmit vector from S is then given as $\mathbf{U}\mathbf{x}$. Clearly, the precoding matrix has to be a full rank matrix to achieve full diversity order of

three. Thus, (2.11) and (2.13) can be expressed as

$$\mathbf{y}_d = \sqrt{\rho} \mathbf{H} \mathbf{U} \mathbf{x} + \mathbf{w} = \sqrt{\rho} \hat{\mathbf{X}} \mathbf{h} + \mathbf{w} \quad (2.25)$$

where

$$\hat{\mathbf{X}} = \begin{bmatrix} \mathbf{U}(1,:) \mathbf{x} & 0 & 0 & \dots & \dots & 0 \\ \mathbf{U}(2,:) \mathbf{x} & \mathbf{U}(1,:) \mathbf{x} & 0 & 0 & \ddots & \vdots \\ \mathbf{U}(3,:) \mathbf{x} & 0 & \mathbf{U}(1,:) \mathbf{x} & \mathbf{U}(2,:) \mathbf{x} & 0 & \vdots \\ \mathbf{U}(4,:) \mathbf{x} & \mathbf{U}(3,:) \mathbf{x} & 0 & 0 & \mathbf{U}(2,:) \mathbf{x} & 0 \\ \mathbf{U}(5,:) \mathbf{x} & 0 & \mathbf{U}(3,:) \mathbf{x} & \mathbf{U}(4,:) \mathbf{x} & 0 & \mathbf{U}(2,:) \mathbf{x} \\ \vdots & \ddots & \dots & \dots & \ddots & \ddots \\ \vdots & \dots & \dots & \dots & \mathbf{U}(L,:) \mathbf{x} & \dots \end{bmatrix}. \quad (2.26)$$

Here, $\mathbf{U}(i,:)$ is the i^{th} row of precoding matrix $\mathbf{U}$ where $\|\mathbf{U}(i,:)\| = 1$ to satisfy the power constraint and the i^{th} column $\|\mathbf{U}(:,i)\| = 1$ to ensure that each symbol has equal transmitted power.

2.4 Precoder Design

Theorem 2.4.1. *AF two-path relaying with inter-relay self interference cancellation with a precoding matrix $\mathbf{U}$ achieves full diversity order of three, if the space time code difference matrix $\Delta \hat{\mathbf{X}} = (\hat{\mathbf{X}}_k - \hat{\mathbf{X}}_l)$ is a full rank matrix for all $\hat{\mathbf{X}}_k$ and $\hat{\mathbf{X}}_l$. In other words, it is necessary that*

$$\Delta \hat{\mathbf{X}} = \begin{bmatrix} \mathbf{U}(1,:) \Delta \mathbf{x} & 0 & 0 & \dots & \dots & 0 \\ \mathbf{U}(2,:) \Delta \mathbf{x} & \mathbf{U}(1,:) \Delta \mathbf{x} & 0 & 0 & \ddots & \vdots \\ \mathbf{U}(3,:) \Delta \mathbf{x} & 0 & \mathbf{U}(1,:) \Delta \mathbf{x} & \mathbf{U}(2,:) \Delta \mathbf{x} & 0 & \vdots \\ \mathbf{U}(4,:) \Delta \mathbf{x} & \mathbf{U}(3,:) \Delta \mathbf{x} & 0 & 0 & \mathbf{U}(2,:) \Delta \mathbf{x} & 0 \\ \mathbf{U}(5,:) \Delta \mathbf{x} & 0 & \mathbf{U}(3,:) \Delta \mathbf{x} & \mathbf{U}(4,:) \Delta \mathbf{x} & 0 & \mathbf{U}(2,:) \Delta \mathbf{x} \\ \vdots & \ddots & \dots & \dots & \ddots & \ddots \\ \vdots & \dots & \dots & \dots & \mathbf{U}(L,:) \Delta \mathbf{x} & \dots \end{bmatrix} \quad (2.27)$$

where $\Delta \mathbf{x} = \mathbf{x}_k - \mathbf{x}_l$, is full rank in order to achieve the full diversity of three. Do note that there is no requirement for channel knowledge at S .

Proof. Following a similar diversity order analysis in Section 2.3.1 by replacing $\mathbf{X}$ with $\hat{\mathbf{X}}$, we have

$$d \triangleq \min_{\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_l} \text{rank} [(\hat{\mathbf{X}}_k - \hat{\mathbf{X}}_l)\mathbf{C}] \quad (2.28)$$

where the rank of $\mathbf{C}$ is three.

Thus, it is clear that since rank of $\mathbf{C}$ is 3, $\Delta \hat{\mathbf{X}} = (\hat{\mathbf{X}}_k - \hat{\mathbf{X}}_l)$ must always be full rank in order to ensure the rank of $(\hat{\mathbf{X}}_k - \hat{\mathbf{X}}_l)\mathbf{C}$ to be 3. $\square$

2.4.1 Diversity Criterion

In order to achieve the full diversity of three, the precoding matrix must satisfy Theorem 2.4.1. Thus, a straightforward precoder design is by having two non-zero elements in each row of the precoding matrix $\mathbf{U}$ such that when one symbol is different (odd or even), $\Delta \hat{\mathbf{X}}$ is full rank.

A specific example of the precoding matrix using two non-zero elements in each row (TNZ) for $L = 6$ is given as

$$\mathbf{U} = \begin{bmatrix} x & 0 & 0 & y & 0 & 0 \\ 0 & x & 0 & 0 & y & 0 \\ 0 & 0 & x & 0 & 0 & y \\ y & 0 & 0 & x & 0 & 0 \\ 0 & y & 0 & 0 & x & 0 \\ 0 & 0 & y & 0 & 0 & x \end{bmatrix} \quad (2.29)$$

where $x = \cos \theta$, $y = \sin \theta$, and $\theta = 15^\circ$. We will use this precoding matrix as a comparison for our simulation results in Section 2.5.

2.4.2 Coding Gain Criterion

From Theorem 2.4.1, we can see that there are essentially an infinite number of precoding matrices which will result in full diversity order of three, but the coding gains will be different. Thus, it is desirable to select precoding matrices that achieve full diversity and at the same time optimize the coding gain.

From (2.20), we define the coding gain³ as

$$G_c \triangleq \det \left(\mathbf{I} + \frac{\rho}{4 \text{tr}(\mathbf{\Sigma}_W)} \mathbf{\Sigma}_v \mathbf{A}^H \mathbf{A} \right). \quad (2.30)$$

To optimize PEP upper bound in (2.20), our objective is to obtain

$$\arg \min_{\mathbf{U}} \det \left(\mathbf{I} + \frac{\rho}{4 \text{tr}(\mathbf{\Sigma}_W)} \mathbf{\Sigma}_v \mathbf{A}^H \mathbf{A} \right) \quad (2.31)$$

where $\mathbf{U}$ satisfies the power constraint of $\|\mathbf{U}(i, :)\| = \|\mathbf{U}(i, :)\| = 1$.

Direct optimization of (2.31) is intractable and we will have to resort to exhaustive computer searches. However, a computer search is not practical as the size of the matrix $\mathbf{U}$, given by L , is arbitrary and can be very large.

Since the code design criteria are different for different ranges of SNR [83], we simplify the maximization problem by considering the low and high SNR regions separately. For the low SNR region, we will first derive a lower bound for G_c , denoted as G_c^{lb} , and then calculate $\mathbb{E}_{\Delta \mathbf{x}} \{G_c^{lb}\}$, the average coding gain across all possible error patterns, denoted by $\Delta \mathbf{x} = \mathbf{x}_k - \mathbf{x}_l$. In this low SNR region, $\Delta \mathbf{x}$ is assumed to be i.i.d., i.e., $\mathbb{E}_{\Delta \mathbf{x}} \{\Delta \mathbf{x} \Delta \mathbf{x}^H\} = \delta_e \mathbf{I}_L$ where δ_e is the error variance. For the high SNR region, we will seek a precoding matrix that will maximize the average mutual information. With these approaches, we are able to obtain a class of precoding matrices that can be easily generated in practice and is able to achieve full diversity order of three with high coding gains.

Proposition 2.4.1. *In the low SNR region, a lower bound of the coding gain G_c is given by*

$$G_c^{lb} = 1 + \beta \mu_n(\mathbf{T} \mathbf{C} \mathbf{\Sigma}_v \mathbf{C}^H \mathbf{T}^H) \text{tr}(\Delta \hat{\mathbf{X}}^H \Delta \hat{\mathbf{X}}) \quad (2.32)$$

where

$$\beta = \frac{\rho}{4 \text{tr}(\mathbf{\Sigma}_W)}$$

and $\mu_n(\mathbf{T} \mathbf{C} \mathbf{\Sigma}_v \mathbf{C}^H \mathbf{T}^H)$ is the smallest eigenvalue of $\mathbf{T} \mathbf{C} \mathbf{\Sigma}_v \mathbf{C}^H \mathbf{T}^H$. Any precoding matrix $\mathbf{U}$, for AF two-path relaying with inter-relay self interference cancellation,

³Here, coding gain is defined as the gain relative to the system that does not apply any precoding matrices.

that satisfies the power constraint of $\|\mathbf{U}(i, :)\| = \|\mathbf{U}(i, :)\| = 1$ has the same expected lower bound $\mathbb{E}_{\Delta\mathbf{x}} \{G_c^{lb}\}$.

Proof. From (2.30), we obtain

$$G_c = \prod_i (1 + \beta \lambda_i) \geq 1 + \beta \sum_i \lambda_i \quad (2.33)$$

$$\begin{aligned} &= 1 + \beta \operatorname{tr}(\mathbf{T}^H \Delta \hat{\mathbf{X}}^H \Delta \hat{\mathbf{X}} \mathbf{T} \mathbf{C} \Sigma_v \mathbf{C}^H) \\ &= 1 + \beta \operatorname{tr}(\Delta \hat{\mathbf{X}}^H \Delta \hat{\mathbf{X}} \mathbf{T} \mathbf{C} \Sigma_v \mathbf{C}^H \mathbf{T}^H) \\ &\geq 1 + \beta \mu_n(\mathbf{T} \mathbf{C} \Sigma_v \mathbf{C}^H \mathbf{T}^H) \operatorname{tr}(\Delta \hat{\mathbf{X}}^H \Delta \hat{\mathbf{X}}) = G_c^{lb} \end{aligned} \quad (2.34)$$

where we assume that $\beta^k \approx 0$ for $k \geq 2$ at low SNR to obtain the inequality in (2.33). The inequality in (2.34) is due to the following [84],

$$\begin{aligned} \operatorname{tr}(\mathbf{X}\mathbf{Y}) &\geq \sum_{i=1}^n \mu_{n-i+1}(\mathbf{X}) \mu_i(\mathbf{Y}) \geq \mu_n(\mathbf{Y}) \sum_{i=1}^n \mu_i(\mathbf{X}) \\ &= \mu_n(\mathbf{Y}) \operatorname{tr}(\mathbf{X}) \end{aligned} \quad (2.35)$$

where $\mathbf{X}$ and $\mathbf{Y}$ are $n \times n$ Hermitian matrices and the eigenvalues are arranged in decreasing order $\mu_1(\mathbf{X}) \geq \dots \geq \mu_n(\mathbf{X})$ and $\mu_1(\mathbf{Y}) \geq \dots \geq \mu_n(\mathbf{Y})$.

We then average G_c^{lb} across all possible error patterns,

$$\begin{aligned} \mathbb{E}_{\Delta\mathbf{x}} \{G_c^{lb}\} &= 1 + \beta \mu_n(\mathbf{T} \mathbf{C} \Sigma_v \mathbf{C}^H \mathbf{T}^H) \mathbb{E}_{\Delta\mathbf{x}} \left\{ \operatorname{tr}(\Delta \hat{\mathbf{X}}^H \Delta \hat{\mathbf{X}}) \right\} \\ &= 1 + \beta \mu_n(\mathbf{T} \mathbf{C} \Sigma_v \mathbf{C}^H \mathbf{T}^H) \\ &\quad \times \mathbb{E}_{\Delta\mathbf{x}} \left\{ 3 \sum_{i=1,2,3,\dots} \Delta \mathbf{x}^H \mathbf{U}(i, :)^H \mathbf{U}(i, :) \Delta \mathbf{x} \right. \\ &\quad \left. + \sum_{i=2,4,6,\dots} \Delta \mathbf{x}^H \mathbf{U}(i, :)^H \mathbf{U}(i, :) \Delta \mathbf{x} \right\} \\ &= 1 + \beta \mu_n(\mathbf{T} \mathbf{C} \Sigma_v \mathbf{C}^H \mathbf{T}^H) \\ &\quad \times \left\{ 3 \sum_{i=1,2,3,\dots} \mathbf{U}(i, :) \mathbb{E}_{\Delta\mathbf{x}} \{ \Delta \mathbf{x} \Delta \mathbf{x}^H \} \mathbf{U}(i, :)^H \right. \\ &\quad \left. + \sum_{i=2,4,6,\dots} \mathbf{U}(i, :) \mathbb{E}_{\Delta\mathbf{x}} \{ \Delta \mathbf{x} \Delta \mathbf{x}^H \} \mathbf{U}(i, :)^H \right\} \end{aligned}$$

$$= 1 + \beta \mu_n(\mathbf{T}\mathbf{C}\Sigma_v\mathbf{C}^H\mathbf{T}^H)7L \delta_e/2 \quad (2.36)$$

as $\mathbb{E}_{\Delta\mathbf{x}} \{\Delta\mathbf{x}\Delta\mathbf{x}^H\} = \delta_e\mathbf{I}_L$. Hence, we can observe that G_c^{lb} averaged across all possible error patterns is independent of $\mathbf{U}$ and thus all precoding matrices satisfying the power constraint will achieve the same $\mathbb{E}_{\Delta\mathbf{x}} \{G_c^{lb}\}$. $\square$

From Proposition 2.4.1, we can conclude that any $\mathbf{U}$ that satisfies the power constraint and Theorem 2.4.1 performs equally well in the low SNR region, in the average sense, as they achieve the same $\mathbb{E}_{\Delta\mathbf{x}} \{G_c^{lb}\}$.

For the high SNR region, a direct optimization of G_c is again intractable. Thus, as mentioned earlier, we will attempt to define a class of precoding matrices that maximizes the mutual information instead [85].

It has been shown in [3] that for i.i.d. Gaussian channels the mutual information maximizing precoding matrix in a MIMO system is a unitary matrix. We will prove that this result is also true in our case.

Proposition 2.4.2. *In the high SNR region, AF two-path relaying with inter-relay self interference cancellation achieves maximum average mutual information when the precoding matrix $\mathbf{U}$ is a unitary matrix.*

Proof. From (2.12), with precoding matrix $\mathbf{U}$ and at high SNR, our objective is to obtain

$$\begin{aligned} & \arg \max_{\mathbf{U}} \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\frac{\rho}{\sigma^2} \mathbf{H} \hat{\mathbf{R}}_x \mathbf{H}^H (\mathbf{B}\mathbf{B}^H + \mathbf{I}_L)^{-1} \right) \right\} \\ &= \arg \max_{\mathbf{U}} \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\frac{\rho}{\sigma^2} \mathbf{H} \hat{\mathbf{R}}_x \mathbf{H}^H \right) \right. \\ & \quad \left. + \log_2 \det (\mathbf{B}\mathbf{B}^H + \mathbf{I}_L)^{-1} \right\} \\ &= \arg \max_{\mathbf{U}} \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\frac{\rho}{\sigma^2} \mathbf{H} \hat{\mathbf{R}}_x \mathbf{H}^H \right) \right\} \end{aligned} \quad (2.37)$$

where $\hat{\mathbf{R}}_x = \mathbb{E}_{\mathbf{x}} \{\mathbf{U}\mathbf{x}\mathbf{x}^H\mathbf{U}^H\} = \mathbf{U}\mathbf{U}^H$.

Since $\hat{\mathbf{R}}_x$ is non-negative definite, we can express it as $\hat{\mathbf{R}}_x = \mathbf{V}\mathbf{D}\mathbf{V}^H$ where $\mathbf{V}$ is unitary and $\mathbf{D}$ is a non-negative diagonal matrix. Denoting

$$\Psi(\hat{\mathbf{R}}_x) = \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\frac{\rho}{\sigma^2} \mathbf{H} \hat{\mathbf{R}}_x \mathbf{H}^H \right) \right\}, \quad (2.38)$$

we have

$$\begin{aligned}
\Psi(\widehat{\mathbf{R}}_x) &= \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\frac{\rho}{\sigma^2} \mathbf{H} \widehat{\mathbf{R}}_x \mathbf{H}^H \right) \right\} \\
&\approx \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\mathbf{I}_{L+2} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{V} \mathbf{D} \mathbf{V}^H \mathbf{H}^H \right) \right\} \\
&= \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\mathbf{I}_L + \frac{\rho}{\sigma^2} \mathbf{V} \mathbf{D} \mathbf{V}^H \mathbf{H}^H \mathbf{H} \right) \right\} \\
&\approx \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\frac{\rho}{\sigma^2} \mathbf{D} \mathbf{H}^H \mathbf{H} \right) \det(\mathbf{V}) \det(\mathbf{V}^H) \right\} \\
&= \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\frac{\rho}{\sigma^2} \mathbf{D} \mathbf{H}^H \mathbf{H} \right) \right\} \\
&\approx \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\mathbf{I}_L + \frac{\rho}{\sigma^2} \mathbf{D} \mathbf{H}^H \mathbf{H} \right) \right\} \\
&= \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\mathbf{I}_{L+2} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{D} \mathbf{H}^H \right) \right\} \\
&\approx \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\frac{\rho}{\sigma^2} \mathbf{H} \mathbf{D} \mathbf{H}^H \right) \right\} \\
&= \Psi(\mathbf{D})
\end{aligned} \tag{2.39}$$

as $\det(\mathbf{V}) \det(\mathbf{V}^H) = 1$. Thus, we can restrict our attention to a non-negative diagonal $\widehat{\mathbf{R}}_x$. For such $\widehat{\mathbf{R}}_x$, if we let $\widehat{\mathbf{R}}_x^\Pi = \Pi \widehat{\mathbf{R}}_x \Pi^H$ where Π is a permutation matrix, we again obtain $\Psi(\widehat{\mathbf{R}}_x^\Pi) = \Psi(\widehat{\mathbf{R}}_x)$ as $\det(\Pi) \det(\Pi^H) = 1$. Since $\log \det$ is concave, $\Psi(\widehat{\mathbf{R}}_x)$ is also concave. From Jensen's inequality, we have

$$\begin{aligned}
\Psi(\tilde{\mathbf{R}}_x) &= \mathbb{E}_{\mathbf{H}} \left\{ \log_2 \det \left(\frac{\rho}{\sigma^2 L!} \sum_{\Pi} \mathbf{H} \widehat{\mathbf{R}}_x^\Pi \mathbf{H}^H \right) \right\} \\
&\geq \mathbb{E}_{\mathbf{H}} \left\{ \frac{1}{L!} \sum_{\Pi} \log_2 \det \left(\frac{\rho}{\sigma^2} \mathbf{H} \widehat{\mathbf{R}}_x^\Pi \mathbf{H}^H \right) \right\} \\
&= \mathbb{E}_{\mathbf{H}} \left\{ \frac{1}{L!} \sum_{\Pi} \log_2 \det \left(\frac{\rho}{\sigma^2} \mathbf{H} \widehat{\mathbf{R}}_x \mathbf{H}^H \right) \det(\Pi) \det(\Pi^H) \right\} \\
&= \Psi(\widehat{\mathbf{R}}_x)
\end{aligned} \tag{2.40}$$

where $\tilde{\mathbf{R}}_x = \frac{1}{L!} \sum_{\Pi} \widehat{\mathbf{R}}_x^\Pi$ and the summation is across all possible $L!$ permutations. Do note that $\tilde{\mathbf{R}}_x$ is a multiple of identity matrix.

Hence, we can conclude that the optimal $\widehat{\mathbf{R}}_x$ given by $\tilde{\mathbf{R}}_x$ must be of the form $\alpha \mathbf{I}$ which occurs when $\mathbf{U}$ is a unitary matrix. $\square$

The significance of the conclusions in Proposition 2.4.1 and 2.4.2 for the low

and high SNR regions will be discussed in our simulation results in the next section.

2.4.3 Proposed Precoder Design

From Theorem 2.4.1, Proposition 2.4.1 and 2.4.2, we have the following approaches to design the precoding matrix:

- The precoding matrix $\mathbf{U}$ is an arbitrary complex unitary matrix.
- It must satisfy Theorem 2.4.1 in order to achieve full diversity order of three.

Denoting this set of unitary matrices as $\mathbb{V}_p$, we will be able to achieve full diversity order of three and a potentially high coding gain. Although the proposed precoder design may not achieve the maximum coding gain, it serves as a good and practical solution to the code design problem. The example on how to generate the arbitrary unitary matrices using MATLAB codes is shown in Appendix C.

2.5 Simulation Results

In Fig. 2.2, we compare the performance of the proposed scheme i.e. AF two-path relaying with inter-relay self interference cancellation (Proposed), AF two-path relaying with no inter-relay interference cancellation at the relay (NC)⁴, and non-orthogonal amplify-and-forward single relaying (NAF)⁵[25]. Although the proposed scheme utilizes two relays, only one relay is active at each transmission slot. Thus, it will be interesting to see how the performance is compared to NAF, which is a half duplex single-relay protocol with the same spectral efficiency as two-path relaying. All schemes are compared under the same total power constraint in the network and $\text{SNR} \triangleq \frac{\rho}{\sigma^2}$. All average channel gains are normalized to 0 dB unless otherwise stated. In Fig. 2.2 and Fig. 2.3, the rate is normalized with respect to L . Thus, we are presenting the asymptotic achievable rate where L is sufficiently large and $\frac{L}{L+2} \rightarrow 1$.

From Fig. 2.2, we can observe that when $h_{s,d}$ has the same average channel gain as other channels, all schemes will have similar average rates. For the proposed

⁴Here, we consider the case where S-D link exists. This generalizes the scheme in [5] where the S-D link is non-existent.

⁵Please refer to Section 1.1.2.1.

Figure 2.2: Average rates for the proposed scheme (without precoding), NC and NAF.

scheme, this average rate is calculated from (2.12). The difference in average rate occurs when $\gamma_{s,d}^2$ is weak, e.g. when $\gamma_{s,d}^2$ is 20 dB lower than other channels. It can be seen that AF two-path relaying schemes (proposed and NC) achieve higher average rate than NAF. Do note that a weak $\gamma_{s,d}^2$ is the most realistic scenario where the advantages of cooperative communications can be fully exploited in practice.

As mentioned before, AF two-path relaying scheme suffers from inter-relay interference. From Fig. 2.3, we can see that the proposed scheme achieves higher average rates than NC for moderate to high γ_r^2 . The value of γ_r^2 is generally proportional to the distance between the two relays. Hence in a practical scenario, the proposed scheme is more robust to an arbitrary selection of relays for cooperation. There is no need to ensure a weak γ_r^2 between the two selected relays.

Similar conclusion can be obtained from the bit error rate (BER) point of view.

Figure 2.3: Average rates for the proposed scheme (without precoding) and NC with varying γ_r^2 . Here, SNR = 15 dB.

In Fig. 2.4 and Fig. 2.5, $L = 6$ and BPSK is used. Although not shown here, similar results are also obtained for other values of L . As shown in Fig. 2.4 where a ML receiver is used, when γ_r^2 is the same as other channels, both the proposed scheme and NC have similar BER. However, as γ_r^2 increases, the diversity order of NC degrades significantly. We can see that although the BER of the proposed scheme increases slightly, the diversity order remains the same.

Next, in Fig. 2.5 we compare the BER of the proposed scheme without precoding matrix (No precoding), the proposed scheme with precoding matrix using two non-zero elements in each row (TNZ), and with precoding using arbitrary $\mathbf{U} \in \mathbb{V}_p$ (Proposed precoding). To show the robustness of our design approach for the proposed precoding that any $\mathbf{U} \in \mathbb{V}_p$ is as good, $\mathbf{U}$ was chosen randomly from $\mathbb{V}_p$ for every channel realization. The precoding matrix for TNZ $\theta = 15^\circ$ is given in

Figure 2.4: Bit error rates for proposed scheme (without precoding) and NC with different γ_r^2 .

(2.29). We can see from Fig. 2.5 that the BER of the proposed scheme without precoding achieves only a diversity of two. However with precoding, a full diversity order of three is achieved, which is consistent with Theorem 2.4.1. In Fig. 2.5, we can also observe that although TNZ $\theta = 15^\circ$ has approximately the same coding gain as $\mathbf{U} \in \mathbb{V}_p$ in the low SNR region, the latter performs better in the high SNR region. This is exactly the conclusion that we have arrived at in Proposition 2.4.1 and 2.4.2: any precoding matrix that satisfies the power constraint performs equally well in the low SNR region (Proposition 2.4.1) and in the high SNR region, $\mathbf{U} \in \mathbb{V}_p$ is the best in terms of maximizing the mutual information (Proposition 2.4.2). This in turn translates to a better BER performance as shown in Fig. 2.5.

Despite the fact that the proposed scheme using precoding matrix is able to achieve full diversity with high coding gain, a ML receiver suffers from a high

Figure 2.5: Bit error rate of the proposed scheme with and without precoding.

decoding complexity. To this end, we use the QRD-M algorithm [69, 70] to solve this issue. From Fig. 2.5, the performance of the proposed precoding scheme with the QRD-M receiver depends on a parameter M , which determines the maximum number of replicas to be searched. As M increases, the performance becomes better but at the expense of higher decoding complexity. However from Fig. 2.5, we can deduce that with just $M = 16$, we are able to approach very closely the performance of a ML receiver which corresponds to $M = 2^L = 64$. Hence, it can be observed that joint decoding for the proposed precoding scheme using ML receiver can also be achieved by using a low complexity decoder, such as QRD-M algorithm.

2.6 Summary

In this chapter, we have analyzed AF two-path relaying with inter-relay self interference cancellation at the relay. This scheme significantly reduces the degradation in performance of the system when the inter-relay channel gain is higher than other channels. From the PEP analysis, we have identified the reason to the limited diversity order of two and showed that full diversity order of three can be achieved using a simple precoding matrix while maintaining the spectral efficiency gain of the two-path relaying protocol. We proposed a precoder design where an arbitrary complex unitary matrix is used as the precoding matrix. The proposed precoder is robust in the sense that any arbitrary unitary matrix, which satisfies the diversity criterion, is as good and full diversity order with high coding gain can be achieved.

Diversity-Multiplexing Tradeoff of Amplify-and-Forward Half-Duplex Relaying

3.1 Introduction

A fundamental performance measure to evaluate different cooperative schemes is the diversity-multiplexing tradeoff introduced in [55] in the high SNR regime. The DMT will enable us to characterize the two main advantages of cooperative schemes, i.e., reliability and the number of degrees of freedom. Reliability is measured by diversity gain, defined as the rate of decay of the error probability as the SNR increases, while the degrees of freedom is measured in terms of spatial multiplexing gain, defined as the rate of increase in the transmission rate with increasing SNR.

For a two-relay topology, also known as the diamond relay channel, we have shown in Chapter 2 that AF two-path relaying scheme can recover a significant portion of the spatial multiplexing loss in conventional half-duplex relaying. To overcome this constraint, the AF two-path relaying scheme requires the two relays to alternate between transmission and reception modes. The accumulation of the inter-relay interference and the noise can be reduced by introducing inter-relay self-interference cancellation at one of the relays. The inter-relay interference can be

achieved easily because the relay has perfect knowledge of the signal it transmitted in the previous time slot. We have also shown that the diversity order three can be achieved by using precoding at the source. However, the important tradeoff between the diversity and multiplexing gains for AF two-path relaying has not been discussed.

In [24], repetition-based and space-time-coded cooperative diversity schemes were proposed for half-duplex single relay system. Non-orthogonal amplify-and-forward (NAF) protocol was proposed in [43] where the source is allowed to transmit simultaneously with the relay during the transmission in the second half. However, both protocols in [24, 43] do not achieve the multiple-input single-output (MISO) DMT bound and are not optimal in the high multiplexing gain regime.

In [53], slotted amplify-and-forward (SAF) scheme, which is a class of AF two-path relaying for multiple relays, was analyzed. Under the assumption that the relays are isolated¹, [53] has shown that the DMT of SAF scheme is close to optimal in the high multiplexing gain regime and will approach the MISO DMT when the number of slots goes to infinity. Thus, the work in [53] gave a fundamental result towards achieving the optimal DMT. However, with the relay isolation assumption, inter-relay interference was presumed to be non-existent and thus the effects of inter-relay interference was not captured in the DMT analysis.

The recent work in [52] studied the DMT of DF two-path relaying. However, the DMT analysis of this successive digital relaying was derived based on the assumption that the relays are always able to correctly decode the signals. The DMT curve derived is a straight line with maximum diversity of two and a multiplexing gain that approaches one as the source frame/transmission length goes to infinity.

In this chapter, we will characterize the DMT of the AF two-path relaying and also touch on the DMT for multiple-relay case. Our main contributions are summarized as follows:

- We first generalize the result in [72, Theorem 2.12] and obtain lower bounds for the DMT of an arbitrary block lower triangular channel matrix. In [72], lower bounds can only be found by using the main and the last sub-diagonal matrix. However, with the generalized results, we are able to lower bound the DMT by using any sub-diagonal matrix.

¹The relays are located very far from each other, i.e., inter-relay channel does not exist.

- We then derive the DMT of AF two-path relaying scheme, while taking into account inter-relay interference in the analysis. Inter-relay interference was not considered in [52, 53]. We prove that the DMT of the AF two-path relaying scheme with inter-relay interference cancellation at one of the relays in a slow fading channel, where the channel coefficients remain constant for $L + 2$ time slots, is given by

$$d(r) = 3 \left(1 - \frac{L+2}{L} r \right)^+. \quad (3.1)$$

This indicates that a full diversity order of three can be achieved while maintaining a spatial multiplexing gain of one for $L \rightarrow \infty$, where L is the frame length. It can also be seen that the DMT of this scheme asymptotically achieves the MISO upper bound as $L \rightarrow \infty$.

- For the multiple-relay case, we provide a stronger result than [53] and prove that the DMT of the N -relay M -slot SAF scheme in [53] is indeed given as

$$d_{\text{SAF}}(r) = (1 - r)^+ + N \left(1 - \frac{M}{M-1} r \right)^+ \quad (3.2)$$

without the assumption that the relays are isolated, i.e. inter-relay interference is taken into consideration.

- In [71], we have discovered that a full diversity order of three can be achieved easily by using precoding, which can be viewed as an equivalent linear space time code. Our analysis in this chapter demonstrates that the same space time coding technique can be used as a simple and yet effective finite code-word length coding strategy to achieve the full DMT of this scheme. The precoding matrix for this linear space time code does not require any form of CSI. From the DMT perspective, the proposed AF two-path relaying with the equivalent linear space time coding is optimal and outperforms existing schemes in [42, 67, 68, 52].

The dot equal operator denotes asymptotic equality in the high SNR regime

Table 3.1: Transmission Protocol of AF Two-Path Relaying

Node \ Time	1	2	3	4	5	...
S	Tx : $x(1)$	Tx : $x(2)$	Tx : $x(3)$	Tx : $x(4)$	Tx : $x(5)$	
R1	Rx : $x(1)$	Tx : $x_{r1}(2)$	Rx : $x(3), x_{r2}(3)$	Tx : $x_{r1}(4)$	Rx : $x(5), x_{r2}(5)$	
R2		Rx : $x(2), x_{r1}(2)$	Tx : $x_{r2}(3)$	Rx : $x(4), x_{r1}(4)$	Tx : $x_{r2}(5)$	
D	Rx : $x(1)$	Rx : $x(2), x_{r1}(2)$	Rx : $x(3), x_{r2}(3)$	Rx : $x(4), x_{r1}(4)$	Rx : $x(5), x_{r2}(5)$	

i.e., $p_1 \doteq p_2$ denotes

$$\lim_{\text{SNR} \rightarrow \infty} \frac{\log p_1}{\log \text{SNR}} = \lim_{\text{SNR} \rightarrow \infty} \frac{\log p_2}{\log \text{SNR}}$$

and $\leq, \geq$ are similarly defined. For any set $O \subset \mathbb{R}^N$, $O^+ = O \cap \mathbb{R}^{N+}$.

3.2 System Model

The system model considered in this chapter is similar to Chapter 2. We will provide a quick review for ease of following the analysis.

A cooperative network system with one source node (S), two relay nodes (R1, R2) and one destination node (D) is considered. We assume that all nodes have a single antenna. The source transmission is divided into frames, each containing L symbols denoted as $x(l)$, $l \in \{1, 2, \dots, L\}$. The destination will perform detection on a frame-by-frame basis. Perfect receive CSI at all receiving nodes is assumed². R1 performs interference cancellation and has the knowledge of $\gamma_{s,r2}^2$. Without loss of generality we assume L is even. We will also assume that the channel remains static within a frame.

3.2.1 Transmission Protocol

The transmission protocol of the AF two-path relaying scheme in two consecutive time slots is depicted in Fig. 2.1 and described in detail in Chapter 2. The summary is given in Table 3.1.

In time slot 1, $x(1)$ is sent from S. R1 and D receive $x(1)$. In time slot 2, $x(2)$ is sent from S and $x_{r1}(2)$ is sent from R1, where $x_{rj}(i)$ is the AF transmitted signal

²Transmit CSI is not available at the transmitters.

from j -th relay in i -th time slot. Both R2 and D receive a linear combination of $x_{r1}(2)$ and $x(2)$. In time slot 3, $x(3)$ is sent from S. $x_{r2}(3)$ is sent from R2. Both R1 and D receive a linear combination of $x_{r2}(3)$ and $x(3)$. Taking note that this is an AF protocol, we realize that $x_{r2}(3)$ contains $x_{r1}(2)$ which is known perfectly at R1 as $x_{r1}(2)$ is the transmitted signal from R1 in time slot 2, and thus it can be cancelled off at R1. These transmission steps are continuously repeated, with R1 performing the cancellation, until L symbols have been transmitted from the source. Two additional time slots are required at the end as the terminating sequence.

3.2.2 Equivalent Channel Model

The equivalent channel model is given as

$$\mathbf{y}_d = \sqrt{\rho} \mathbf{H} \mathbf{x} + \mathbf{B} \mathbf{n}_r + \mathbf{n}_d \quad (3.3)$$

where $\mathbf{y}_d \in \mathcal{C}^{L+2}$ is the received signal vector at D, ρ is the average transmit power, $\mathbf{H} \in \mathcal{C}^{(L+2) \times L}$ is the equivalent channel matrix, $\mathbf{x} \in \mathcal{C}^L$ is the transmitted signal vector, $\mathbf{B} \in \mathcal{C}^{(L+2) \times L}$ is the channel $\mathbf{n}_r \in \mathcal{C}^{L+2}$ and $\mathbf{n}_d \in \mathcal{C}^{L+2}$ are the additive white Gaussian noise vectors at the relays and destination respectively.

The equivalent channel matrix $\mathbf{H}$, as shown in Appendix A, is a matrix with at most 4 non-zero entries in each row. As shown in Chapter 2, the non-zero elements of $\mathbf{H} \in \mathcal{C}^{(L+2) \times L}$ and $\mathbf{B} \in \mathcal{C}^{(L+2) \times L}$ are given as

$$\begin{aligned} \mathbf{H}(m, m) &= h_{s,d}, \quad m = 1, 2, 3, \dots, L, \\ \mathbf{H}(m, m-1) &= \begin{cases} h_{s,r1} g_{r1} h_{r1,d}, & m = 2, 4, \dots, L \\ h_{s,r2} g_{r2} h_{r2,d}, & m = 3, 5, \dots, L+1, \end{cases} \\ \mathbf{H}(m, m-2) &= \begin{cases} h_{s,r1} g_{r1} h_r g_{r2} h_{r2,d}, & m = 3, 5, \dots, L+1 \\ h_{s,r2} g_{r2} h_r g_{r1} h_{r1,d}, & m = 4, 6, \dots, L+2, \end{cases} \\ \mathbf{H}(m, m-3) &= h_{s,r2} g_{r2}^2 h_r^2 g_{r1} h_{r2,d}, \quad m = 5, 7, \dots, L+1, \end{aligned}$$

and,

$$\mathbf{B}(m, m-1) = \begin{cases} g_{r1}h_{r1,d}, & m = 2, 4, \dots, L \\ g_{r2}h_{r2,d}, & m = 3, 5, \dots, L+1, \end{cases}$$

$$\mathbf{B}(m, m-2) = \begin{cases} g_{r1}h_r g_{r2}h_{r2,d}, & m = 3, 5, \dots, L+1 \\ g_{r2}h_r g_{r1}h_{r1,d}, & m = 4, 6, \dots, L+2, \end{cases}$$

$$\mathbf{B}(m, m-3) = g_{r2}^2 h_r^2 g_{r1} h_{r2,d}, \quad m = 5, 7, \dots, L+1,$$

where $h_{s,d} \sim \mathcal{CN}(0, \gamma_{s,d}^2)$, $h_{s,rj} \sim \mathcal{CN}(0, \gamma_{s,rj}^2)$, $h_{rj,d} \sim \mathcal{CN}(0, \gamma_{rj,d}^2)$ and $h_r \sim \mathcal{CN}(0, \gamma_r^2)$ are the channel coefficients from S to D, S to j -th relay, j -th relay to D and R1 (R2) to R2 (R1) respectively where $j \in \{1, 2\}$. $g_{r1}(i)$ and $g_{r1}(i)$ are the scaling factors which are assumed constant.

From (2.12), we obtain the mutual information as

$$I(\mathbf{x}; \mathbf{y}_d) = \log \det \left(\mathbf{I}_{L+2} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{H}^H (\mathbf{B} \mathbf{B}^H + \mathbf{I}_{L+2})^{-1} \right) \quad (3.4)$$

where $\mathbf{R}_x = \mathbb{E} \{ \mathbf{x} \mathbf{x}^H \} = \mathbf{I}_L$, $\mathbf{R}_n = \mathbb{E} \{ \mathbf{n} \mathbf{n}^H \} = \sigma^2 \mathbf{I}_{2L+4}$.

3.2.3 Motivation

In [67, 68], the bit error rate of AF two-path relaying scheme with inter-relay interference cancellation only shows a diversity of two, even though physically there are three independent paths from S to D. This observation motivates us to look into the DMT to analyze the potential diversity gains and investigate what kind of strategies will enable us to achieve the full DMT offered by the scheme.

3.3 Diversity-Multiplexing Tradeoff

We first generalize the result in [72, Theorem 2.12] to find lower bounds for the DMT of an arbitrary block lower triangular channel matrix.

3.3.1 DMT Lower Bound for an Arbitrary Block Lower Triangular Channel Matrix

Definition 3.3.1. Let $\mathbf{A}$ be a block lower triangular matrix comprising of block matrices $\mathbf{A}_{ij} \in \mathcal{C}^{N_i \times N_j}$ where $i \geq j$. The l -th sub-diagonal matrix $\mathbf{A}^{(l)}$ of matrix $\mathbf{A}$ is defined as the matrix where

$$\mathbf{A}_{ij}^{(l)} = \begin{cases} \mathbf{A}_{ij} & \text{if } i - j = l, \\ \mathbf{0} & \text{otherwise.} \end{cases} \quad (3.5)$$

The last sub-diagonal matrix is defined as $\mathbf{A}^{(k)}$ where k is the largest integer for which $\mathbf{A}^{(k)}$ is a non-zero matrix. We denote the DMT of channel matrix $\mathbf{A}$ as $d_A(r)$.

Theorem 3.3.1. Consider a random block lower triangular channel matrix $\mathbf{H}$ comprising of block matrices $\mathbf{H}_{ij} \in \mathcal{C}^{N_i \times N_j}$ where $i \geq j$ and $i, j \in \{1, 2, \dots, N\}$. Then, we have

$$d_H(r) \geq d_{H^{(l)}}(r) \quad (3.6)$$

for $0 \leq l \leq k$, where $\mathbf{H}^{(k)}$ is the last sub-diagonal matrix.

Proof. In [72], the special cases where $l = 0$ and $l = k$ have been proven. Here, we will use a similar method to prove (3.6) for any $0 \leq l \leq k$.

In general, the channel model is given by $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$. Without loss of generality, we assume that $\mathbf{w} \sim \mathcal{CN}(0, \mathbf{I})$ since the noise is white in the scale of interest [72, Theorem 2.3]. Let $\mathbf{x} = [\mathbf{x}_1^T \mathbf{x}_2^T \dots \mathbf{x}_N^T]^T$, $\mathbf{y} = [\mathbf{y}_1^T \mathbf{y}_2^T \dots \mathbf{y}_N^T]^T$, and $\mathbf{w} = [\mathbf{w}_1^T \mathbf{w}_2^T \dots \mathbf{w}_N^T]^T$ such that $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$ is expressed as

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{pmatrix} = \begin{pmatrix} \mathbf{H}_{11} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \ddots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{H}_{N1} & \mathbf{H}_{N1} & \dots & \mathbf{H}_{NN} \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{pmatrix} + \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_N \end{pmatrix}. \quad (3.7)$$

We further define $\mathbf{x}_i^N \triangleq [\mathbf{x}_i^T \mathbf{x}_{i+1}^T \dots \mathbf{x}_N^T]^T$.

For any instantaneous channel matrix H , the outage probability exponent is given by

$$\text{SNR}^{-d_H(r)} \doteq \Pr \{I(\mathbf{x}; \mathbf{y} : \mathbf{H} = H) \leq r \log \text{SNR}\}. \quad (3.8)$$

Hence, to obtain the lower bound on the exponent, we will begin by identifying the lower bound for the mutual information.

Thus,

$$\begin{aligned} I(\mathbf{x}; \mathbf{y} | \mathbf{H} = H) &\stackrel{(a)}{=} \sum_{i=1}^N I(\mathbf{x}_i; \mathbf{y} | \mathbf{H} = H, \mathbf{x}_{i+1}^N) \\ &\stackrel{(b)}{\geq} \sum_{i=1}^{N-l} I(\mathbf{x}_i; \mathbf{y} | \mathbf{H} = H, \mathbf{x}_{i+1}^N) \\ &= \sum_{i=1}^{N-l} I(\mathbf{x}_i; \mathbf{H}\mathbf{x} + \mathbf{w} | \mathbf{H} = H, \mathbf{x}_{i+1}^N) \end{aligned} \quad (3.9)$$

where (a) is from chain rule and (b) is from the fact that $I \geq 0$.

Let $\hat{\mathbf{H}} = \mathbf{H}^{(l)} + \mathbf{H}^{(l+1)} + \dots + \mathbf{H}^{(k)}$; $\hat{\mathbf{H}}$ is formed by removing the first $\sum_{i=1}^l N_i$ all-zero rows and the last $\sum_{i=N-l+1}^N N_i$ all-zero columns of $\hat{\mathbf{H}}$, $\tilde{\mathbf{w}} = \mathbf{w}_{l+1}^N$, and $\tilde{\mathbf{x}} = \mathbf{x}_1^{N-l}$. Then from (3.9),

$$\begin{aligned} I(\mathbf{x}; \mathbf{y} | \mathbf{H} = H) &\geq \sum_{i=1}^{N-l} I(\mathbf{x}_i; \mathbf{H}\mathbf{x} + \mathbf{w} | \mathbf{H} = H, \mathbf{x}_{i+1}^N) \\ &\stackrel{(c)}{\geq} \sum_{i=1}^{N-l} I(\mathbf{x}_i; \hat{\mathbf{H}}\mathbf{x} + \mathbf{w} | \mathbf{H} = H, \mathbf{x}_{i+1}^N) \\ &\stackrel{(d)}{=} I(\tilde{\mathbf{x}}; \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{w}}_i | \tilde{\mathbf{H}} = \tilde{H}) \\ &\stackrel{(e)}{\geq} I(\tilde{\mathbf{x}}; \tilde{\mathbf{H}}^{(0)}\tilde{\mathbf{x}} + \tilde{\mathbf{w}}_i | \tilde{\mathbf{H}}^{(0)} = \tilde{H}^{(0)}) \\ &\stackrel{(f)}{=} I(\mathbf{x}; \mathbf{H}^{(l)}\mathbf{x} + \mathbf{w}_i | \mathbf{H}^{(l)} = H^{(l)}) \end{aligned} \quad (3.10)$$

where (c) is from the fact that $\hat{\mathbf{H}}\mathbf{x}$ contains less information than $\mathbf{H}\mathbf{x}$, (d) is from chain rule, (e) is from [72, Theorem 2.12], and (f) is obvious since the non-zero block matrices of $\tilde{\mathbf{H}}^{(0)}$ are the same as $\mathbf{H}^{(l)}$.

Hence,

$$\begin{aligned}
\text{SNR}^{-d_H(r)} &\doteq \Pr \{I(\mathbf{x}; \mathbf{y} | \mathbf{H} = H) \leq r \log \text{SNR}\} \\
&\leq \Pr \{I(\mathbf{x}; \mathbf{H}^{(l)} \mathbf{x} + \mathbf{w}_i | \mathbf{H}^{(l)} = H^{(l)}) \leq r \log \text{SNR}\} \\
&\doteq \text{SNR}^{-d_{H^{(l)}}(r)} \\
\implies d_H(r) &\geq d_{H^{(l)}}(r).
\end{aligned} \tag{3.11}$$

□

Remark:

- Although Theorem 3.3.1 is only derived for block lower triangular matrices, we can also apply the theorem to arbitrary banded matrices which are not square in general. This can be proven by adding an appropriate number of all-zero rows to the top and all-zero columns to the right of the matrix.

Proposition 3.3.1. *If the entries of $\mathbf{H}^{(l)}$ and $\mathbf{H}^{(l')}$ are independent where $l \neq l'$, $0 \leq l \leq k$, and $0 \leq l' \leq k$, then we have*

$$d_H(r) \geq d_{H^{(l)}}(r) + d_{H^{(l')}}(r). \tag{3.12}$$

Proof. The proof is similar to the one in [72] for the case $l = 0$ and $l' = k$. Since the entries of $\mathbf{H}^{(l)}$ and $\mathbf{H}^{(l')}$ are independent, we have

$$\begin{aligned}
\text{SNR}^{-d_H(r)} &\doteq \Pr \{I(\mathbf{x}; \mathbf{y} | \mathbf{H} = H) \leq r \log \text{SNR}\} \\
&\leq \Pr \left\{ \max \left(I(\mathbf{x}; \mathbf{H}^{(l)} \mathbf{x} + \mathbf{w} | \mathbf{H} = H), \right. \right. \\
&\quad \left. \left. I(\mathbf{x}; \mathbf{H}^{(l')} \mathbf{x} + \mathbf{w} | \mathbf{H} = H) \right) \leq r \log \text{SNR} \right\} \\
&= \Pr \{I(\mathbf{x}; \mathbf{H}^{(l)} \mathbf{x} + \mathbf{w} | \mathbf{H}^{(l)} = H^{(l)}) \leq r \log \text{SNR}\} \\
&\quad \times \Pr \left\{ I(\mathbf{x}; \mathbf{H}^{(l')} \mathbf{x} + \mathbf{w} | \mathbf{H}^{(l')} = H^{(l')}) \leq r \log \text{SNR} \right\} \\
&\doteq \text{SNR}^{-d_{H^{(l)}}(r)} \text{SNR}^{-d_{H^{(l')}}(r)}.
\end{aligned} \tag{3.13}$$

Thus, $d(r) \geq d_{H^{(l)}}(r) + d_{H^{(l')}}(r)$.

□

3.3.2 DMT of AF Two-Path Relaying with Inter-Relay Self Interference Cancellation

From (3.4), the outage probability can be expressed as

$$\begin{aligned} P_{\text{out}}(R) &= P_r \{I(\mathbf{x}; \mathbf{y}_d) < (L+2)R\} \\ &= P_r \left\{ \log \det \left(\mathbf{I}_{L+2} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{H}^H (\mathbf{B} \mathbf{B}^H + \mathbf{I}_{L+2})^{-1} \right) < (L+2)R \right\} \\ &\doteq P_r \left\{ \log \det \left(\mathbf{I}_{L+2} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{H}^H \right) < (L+2)R \right\} \end{aligned} \quad (3.14)$$

$$\doteq \text{SNR}^{-d(r)} \quad (3.15)$$

since we are only interested in the exponent function of $P_{\text{out}}(R)$ and the DMT is white in the scale of interest³[72]. The factor $L+2$ is to account for the fact that $L+2$ transmission slots are required to transmit L symbols.

Theorem 3.3.2. *The diversity-multiplexing tradeoff for the AF two-path relaying scheme with one of the relays additionally performing inter-relay interference cancellation in a slow fading channel, where the channel coefficients remain constant for $L+2$ time slots, is given by*

$$d(r) = 3 \left(1 - \frac{L+2}{L} r \right)^+. \quad (3.16)$$

Proof. We prove by showing in Appendix D that RHS of (3.16) is both an upper and a lower bound for the DMT of the AF two-path relaying scheme, and thus the DMT is given by (3.16). $\square$

From DMT expression in (3.16), it is clear to see that AF two-path relaying with inter-relay self interference cancellation achieves maximum diversity gain of three and maximum multiplexing gain of $\frac{L}{L+2}$.

For a N -relay cooperative diversity scheme, the DMT is upper bounded by the MISO transmit diversity upper bound which is given by [24]

$$d^*(r) = (N+1)(1-r)^+. \quad (3.17)$$

³The colored noise encountered in cooperative networks can be treated as a white noise for DMT computations.

Thus, we can see that for $N = 2$, AF two-path relaying scheme asymptotically achieves this upper bound as $L \rightarrow \infty$.

3.3.3 Achievability of the DMT

The outage formulation used to derive the DMT is based on the existence of *universal codes* [56] that can achieve arbitrarily small error probability whenever the channel is not in outage, i.e. $P_e \approx P_{out}$. However, to achieve such performance, arbitrarily long block lengths and powerful codes are usually required. In this subsection, we will show that the DMT of the AF two-path relaying scheme with inter-relay interference cancellation in Theorem 3.3.2 can indeed be achieved with a finite codeword length. Practical coding strategies to achieve the DMT have not been discussed before in previous works [52, 53, 54].

In [67, 68], it was shown that AF two-path relaying with inter-relay interference cancellation only achieves a maximum diversity order of two. However, Theorem 3.3.2 suggests that potentially, a diversity order of three can be achieved while maintaining a spatial multiplexing gain of one for large L . To provide insights to this phenomenon, we will first express (3.3) as

$$\mathbf{y}_d = \sqrt{\rho} \mathbf{X} \mathbf{h} + \mathbf{w} \quad (3.18)$$

where

$$\mathbf{X} = \begin{bmatrix} x(1) & 0 & 0 & 0 & 0 & 0 \\ x(2) & x(1) & 0 & 0 & 0 & 0 \\ x(3) & 0 & x(1) & x(2) & 0 & 0 \\ x(4) & x(3) & 0 & 0 & x(2) & 0 \\ x(5) & 0 & x(3) & x(4) & 0 & x(2) \\ \vdots & \ddots & \dots & \dots & \ddots & \ddots \\ \vdots & \dots & \dots & \dots & x(L) & \vdots \end{bmatrix}, \quad (3.19)$$

$$\mathbf{h} = \begin{bmatrix} h_{s,d} \\ h_{r1,d} \ g_{r1} \ h_{s,r1} \\ h_{r2,d} \ g_{r2} \ h_r \ g_{r1} \ h_{s,r1} \\ h_{r2,d} \ g_{r2} \ h_{s,r2} \\ h_{r1,d} \ g_{r1} \ h_r \ g_{r2} \ h_{s,r2} \\ h_{r2,d} \ g_{r2} \ h_r \ g_{r1} \ h_r \ g_{r2} \ h_{s,r2} \end{bmatrix}, \quad (3.20)$$

and $\mathbf{w} \triangleq \mathbf{B}\mathbf{n}_r + \mathbf{n}_d$, conditioned on $\mathbf{h}$, is the correlated Gaussian noise with covariance matrix $\Sigma_W = \sigma^2 (\mathbf{B}\mathbf{B}^H + \mathbf{I})$. To be precise, the non-zero elements of $\mathbf{X}$ are given as

$$\begin{aligned} \mathbf{X}(m, 1) &= \mathbf{X}(m + 1, 2) = \mathbf{X}(m + 2, 3) = x(m) \\ &\text{for } m = 1, 3, 5, \dots, L - 1, \\ \mathbf{X}(m, 1) &= \mathbf{X}(m + 1, 4) = \mathbf{X}(m + 2, 5) \\ &= \mathbf{X}(m + 3, 6) = x(m) \text{ for } m = 2, 4, 6, \dots, L. \end{aligned} \quad (3.21)$$

Thus, we can view $\mathbf{X}$ as the equivalent space time code for $\mathbf{x}$. With this equivalent space time code expression, the reason for failing to achieve the full diversity order of three can then be attributed to the limitations in the code design resulting from the transmission scheme. However, by using a simple linear coding strategy at S, we shall show that the full DMT of the AF two-path relaying scheme can indeed be achieved.

Consider at data rate $R = r \log \text{SNR}$ (bits/symbol), for finite codeword lengths we have⁴ [55],

$$\begin{aligned} P_e(\text{SNR}) &= P_{\text{out}}(R)P(\text{error}|\text{outage}) + P(\text{error, no outage}) \\ &\leq P_{\text{out}}(R) + P(\text{error, no outage}) \\ &\approx P_{\text{out}}(R) + P^*(\text{error, no outage}) \\ &\leq P_{\text{out}}(R) + P^*(\text{error}) \\ &\leq \text{SNR}^{-d(r)} + \text{SNR}^{-d_G(r)} \end{aligned} \quad (3.22)$$

where $P^*(\text{error}) \doteq \text{SNR}^{-d_G(r)}$ is the probability of the dominant error in the high

⁴ $P(\text{error, no outage})$ cannot be ignored for finite codeword lengths.

SNR regime and $d_G(r)$ is the negative of its exponential order. This dominant error event occurs when only one data symbol is different between two space time codewords.

$P^*(\text{error})$ can be determined from the pairwise error probability (PEP) in [68, 71], where we have

$$\begin{aligned}
P(\mathbf{X}_k \rightarrow \mathbf{X}_l) &\leq \mathbb{E}_{\mathbf{h}} \left\{ e^{-\frac{\rho}{4 \text{tr}(\Sigma_W)} \mathbf{h}^H (\mathbf{X}_k - \mathbf{X}_l)^H (\mathbf{X}_k - \mathbf{X}_l) \mathbf{h}} \right\} \\
&= \mathbb{E}_{\mathbf{T}} \left\{ \int \frac{1}{\pi^3} e^{-\frac{\rho}{4 \text{tr}(\Sigma_W)} \mathbf{v}^H \mathbf{C}^H \mathbf{T}^H (\mathbf{X}_k - \mathbf{X}_l)^H (\mathbf{X}_k - \mathbf{X}_l) \mathbf{T} \mathbf{C} \mathbf{v}} \right. \\
&\quad \left. \times e^{-\mathbf{v}^H \mathbf{v}} d\mathbf{v} \right\} \\
&= \mathbb{E}_{\mathbf{T}} \left\{ \det^{-1} \left(\mathbf{I} + \frac{\rho}{4 \text{tr}(\Sigma_W)} \Sigma_v \mathbf{A}^H \mathbf{A} \right) \right\} \\
&\leq \mathbb{E}_{\mathbf{T}} \left\{ \left(\prod_{i=1}^{\alpha} \lambda_i[\Sigma_v \mathbf{A}^H \mathbf{A}] \right)^{-1} \left(\frac{\rho}{4 \text{tr}(\Sigma_W)} \right)^{-\alpha} \right\} \\
&\doteq \text{SNR}^{-\min_{\mathbf{X}_k - \mathbf{X}_l} (\text{rank}(\Sigma_v \mathbf{A}^H \mathbf{A}))} \\
&\doteq \text{SNR}^{-\min_{\mathbf{X}_k - \mathbf{X}_l} (\text{rank}(\mathbf{A}))}. \tag{3.23}
\end{aligned}$$

Here, $\mathbf{h} = \mathbf{T} \mathbf{C} \mathbf{v}$, $\mathbf{v} = [h_{s,d}, h_{s,r1}, h_{s,r2}]^T$, $\mathbf{T} = \text{diag} \left(\begin{bmatrix} 1 & & & & \\ & h_{r1,d} & g_{r1} & & \\ & h_{r2,d} & h_r & g_{r1} & g_{r2} \\ & & h_{r2,d} & g_{r2} & \\ & h_{r1,d} & h_r & g_{r1} & g_{r2} \\ & h_{r2,d} & h_r^2 & g_{r1} & g_{r2}^2 \end{bmatrix} \right)$ is a full rank

matrix, $\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}^T$ is a rank 3 matrix, $\mathbf{A} = (\mathbf{X}_k - \mathbf{X}_l) \mathbf{T} \mathbf{C}$, and α is the minimum rank of $\mathbf{A}$.

From (3.23) and by taking the union bound across all space time codewords which differ by one symbol, we have

$$\begin{aligned}
P^*(\text{error}) &\leq (L 2^{R_s} - 1) P(\mathbf{X}_k \rightarrow \mathbf{X}_l) \\
&\leq 2^{R_s} P(\mathbf{X}_k \rightarrow \mathbf{X}_l) \\
&\leq \text{SNR}^{\frac{L+2}{L}r} \text{SNR}^{-\min_{\mathbf{X}_k - \mathbf{X}_l} (\text{rank}(\mathbf{A}))} \\
&\doteq \text{SNR}^{-\left(\min_{\mathbf{X}_k - \mathbf{X}_l} (\text{rank}(\mathbf{A})) - \frac{L+2}{L}r \right)} \tag{3.24}
\end{aligned}$$

where each $x(l)$ is transmitted with data rate $R_s = \frac{L+2}{L}R$ bits in each transmission

slot.

Thus, we obtain

$$d_G(r) = \left(\min_{\mathbf{x}_k - \mathbf{x}_l} (\text{rank}(\mathbf{A})) - \frac{L+2}{L}r \right)^+ \quad (3.25)$$

and from (3.22) we have

$$\begin{aligned} P_e(\text{SNR}) &\leq \text{SNR}^{-d(r)} + \text{SNR}^{-d_G(r)} \\ &= \text{SNR}^{-d(r)} + \text{SNR}^{-\left(\min_{\mathbf{x}_k - \mathbf{x}_l} (\text{rank}(\mathbf{A})) - \frac{L+2}{L}r \right)^+}. \end{aligned} \quad (3.26)$$

As explained in [67, 68], the transmission protocol causes $\min_{\mathbf{x}_k - \mathbf{x}_l} (\text{rank}(\mathbf{A}))$ in (3.26) to be limited to 2, which will cause $\text{SNR}^{-d_G(r)}$ to dominate over $\text{SNR}^{-d(r)}$. Thus, only a maximum diversity order of two is observed as shown in [67, 68].

In Theorem 2.4.1, we have shown that $\min_{\mathbf{x}_k - \mathbf{x}_l} (\text{rank}(\mathbf{A}))$ can be improved to 3 by using an equivalent linear space time code at S. With this linear space time coding, (3.3) and (3.18) can be expressed as

$$\mathbf{y}_d = \sqrt{\rho} \mathbf{H} \mathbf{U} \mathbf{x} + \mathbf{w} = \sqrt{\rho} \hat{\mathbf{X}} \mathbf{h} + \mathbf{w} \quad (3.27)$$

where $\mathbf{U}$ is the precoding matrix and $\hat{\mathbf{X}}$ is the corresponding equivalent space time codeword. The precoding matrix $\mathbf{U}$ only has to satisfy the following two conditions:

- It is an arbitrary complex unitary matrix.
- It must satisfy Theorem 2.4.1 in order to ensure $\min_{\mathbf{x}_k - \mathbf{x}_l} (\text{rank}(\mathbf{A})) = 3$.

By using this linear space time code at S [71], we have $d_G(r) = \left(3 - \frac{L+2}{L}r \right)^+$ and

$$\begin{aligned} P_e(\text{SNR}) &\leq \text{SNR}^{-d(r)} + \text{SNR}^{-d_G(r)} \\ &\leq \text{SNR}^{-d(r)} \end{aligned} \quad (3.28)$$

as $d_G(r) \geq d(r)$. Hence, the full DMT of the AF two-path relaying scheme in Theorem 3.3.2 is achieved.

Figure 3.1: Diversity-multiplexing tradeoff for two-relay topology. Here, $L = M = 40$.

3.3.4 Discussion

1. In Fig. 3.1, we compare the DMT of the proposed AF two-path relaying scheme with other cooperative schemes, such as NAF scheme⁵[25, 43], successive digital relaying⁶[52], and DMT upper bound of SAF scheme [53] for the two-relay topology. For the two-relay case, DMT for NAF scheme is given in [43] as

$$d_{\text{NAF}}(r) = (1 - r) + 2(1 - 2r)^+ . \quad (3.29)$$

It can be observed that AF two-path relaying outperforms both successive

⁵Please refer to Section 1.1.2.1.

⁶Please refer to Section 1.1.2.2.

digital relaying and NAF scheme. AF two-path relaying is also very close to the DMT upper bound of SAF scheme. The slight difference is due to the fact that we have an additional two time slots for the finite codeword length design. For large values of L , the DMT of AF two-path relaying will approach the MISO upper bound.

2. Both AF two-path relaying with and without interference cancellation scheme achieve the same DMT.

Proposition 3.3.2. *The DMT of AF two-path relaying without interference cancellation at one of the relays is also given by*

$$d(r) = 3 \left(1 - \frac{L+2}{L} r \right)^+. \quad (3.30)$$

Proof. Please refer to Appendix E. □

This result is intuitively satisfying since DMT is computed in the high SNR regime and thus the effects of inter-relay interference is negligible⁷. However, this does not mean that inter-relay interference cancellation is not necessary. With practical SNR values, the accumulation of interference will degrade the overall performance of two-path relaying without cancellation, as shown in [68, 71]. Interference cancellation at one of the relays is required to reduce the accumulation of inter-relay interference at the destination.

3. AF two-path relaying *without* interference cancellation at one of the relays can also be considered as the two-relay case of SAF scheme in [53]. We can extend the result in Proposition 3.3.2 to the multiple-relay case considered in [53].

Proposition 3.3.3. *For the multiple-relay case, the DMT of N -relay M -slot SAF scheme in [53], is indeed given as*

$$d_{SAF}(r) = (1-r)^+ + N \left(1 - \frac{M}{M-1} r \right)^+. \quad (3.31)$$

⁷Unlike decode-and-forward protocol, in amplify-and-forward protocol the relay does not treat the interference as 'interference'.

Proof. First, consider $\check{\mathbf{H}}$ as the channel matrix of the SAF scheme in [53]. With the assumption that the relays are isolated, $\check{\mathbf{H}}$ will have non-zero elements only at the main and first sub-diagonal. With the relay isolation assumption, [53] has shown that the DMT in (3.31) is achievable. Now, if we remove the assumption of relay isolation, i.e. inter-relay interference is considered, the only change in $\check{\mathbf{H}}$ is that all the entries in the lower triangular part of $\check{\mathbf{H}}$ will be non-zeros. However, the main and first sub-diagonal elements will remain the same as the case with the relay isolation assumption.

Hence, by using Theorem 3.3.1, the lower bounds for the DMT are given by the DMT of $\check{\mathbf{H}}^{(0)}$ and $\check{\mathbf{H}}^{(1)}$, which are taken from the main and the first-sub diagonal of $\check{\mathbf{H}}$ respectively. It is easy to compute that $d_{\check{H}^{(0)}}(r) = (1 - r)^+$ and $d_{\check{H}^{(1)}}(r) = N \left(1 - \frac{M}{M-1}r\right)^+$. Since $\check{\mathbf{H}}^{(0)}$ and $\check{\mathbf{H}}^{(1)}$ are independent, from Proposition 3.3.1, we can see that the sum of the lower bounds $d_{\check{H}^{(0)}}(r) + d_{\check{H}^{(1)}}(r)$ coincides with the SAF DMT upper bound in [53], and this completes the proof.

□

In [53], the DMT of SAF scheme is derived under the assumption that the relays are isolated, i.e. there is no inter-relay interference. By using Theorem 3.3.1, we have proven a stronger result in Proposition 3.3.3 that the DMT of SAF scheme meets the genie-aided upper bound in [53], while taking into account the inter-relay interference. The assumption of relay isolation in [53] is not necessary to achieve the same DMT performance.

3.4 Summary

In this chapter, an amplify-and-forward two-path relaying scheme is considered in which one of the relays additionally performs inter-relay interference cancellation. We first generalized lower bounds for the DMT of an arbitrary block lower triangular channel matrix. The DMT was then derived and it was shown that the DMT of this scheme is achievable for finite codeword lengths by using a simple equivalent linear space time code at the source. The precoding matrix of this linear space time code does not require any form of channel state information. From the

DMT perspective, we have shown that the proposed AF two-path relaying scheme is optimal as the DMT achieves the MISO upper bound. We then extended our analysis to the SAF scheme [53] with multiple relays, where we provided a stronger result by deriving the DMT while taking into account inter-relay interference.

Decode-and-Forward Two-Path Half-Duplex Relaying

4.1 Introduction

The relaying operation for a cooperative communication system can be broadly classified into two types, namely amplify-and-forward and decode-and-forward relaying. In Chapter 2 and 3, we have studied AF two-path relaying in great details and shown that AF two-path relaying is a very attractive protocol to solve the multiplexing loss problem in half-duplex relaying. There have been some works on both AF [42, 67, 68, 71, 53] and DF [42, 52]. We will focus on DF relaying in this chapter and study its DMT characteristics.

In [52], a successive digital relaying or DF two-path relaying was studied. A simple repetition coding scheme, where the relays decode and re-encode each received codeword, was shown to be able to recover the multiplexing loss of half-duplex relaying. Each codeword is received twice at the destination node, i.e., one from the source and another from one of the relays. Thus, the DMT curve in [52] indicates a maximum diversity of two and a multiplexing gain that approaches one as the source frame length goes to infinity. However, the DMT was derived under the assumption that the relays are always able to correctly decode the source signals. This assumption simplifies the problem a lot and does not provide a complete picture of the DMT performance of DF two-path relaying.

Further extension of DF two-path relaying to multiple relay scenario was given in [54] where multihop relay selection (MHRS) scheme was proposed. MHRS utilizes relay selection methods with limited feedback to recover the multiplexing loss in DF half-duplex relay networks. Inter-relay interference was considered and the DMT of MHRS was analyzed in [54]. MHRS performs well for a large number of relays, but is not suitable for the two-relay scenario since a relay will not be able to contribute to the diversity gain once it has been selected for transmission, totally defeating the purpose of the relaying protocol.

In this chapter, we revisit the DMT analysis of DF two-path relaying protocol. In contrast to the DMT analysis in [52], which is based on a multiple-access channel, our work is based on a distributed MISO approach (similar to [53, 86]). For the multiple-access channel approach [52], each transmitted symbol¹ from the source is viewed as a symbol from different users and thus joint encoding across different symbols is not considered. However, for the more general distributed MISO approach, joint encoding across different symbols is allowed². As shown later in the chapter, a distributed MISO approach allows us to achieve a better DMT. To the best of our knowledge, the DMT of DF two-path relaying with a distributed MISO approach has not been reported in the literature.

Our contributions can be summarized as follows:

- First, with a distributed MISO approach, we derive the DMT of DF two-path relaying, where perfect decoding at the relays is assumed [52], and show that the DMT is given as

$$d(r) = 3 \left(1 - \frac{L+1}{L} r \right)^+ \quad (4.1)$$

where $(x)^+$ denotes $\max\{0, x\}$, L is the frame length, and r is the multiplexing gain. It can be seen that the DMT approaches the 3×1 MISO DMT [55] with large L .

- We then remove the assumption of perfect decoding at the relays and study

¹In [52], the term 'codeword' is used instead. The source transmits a codeword in each transmission slot and thus a total of L codewords are received at the destination. Joint decoding across all codewords is then performed to retrieve the information from the source.

²In DF two-path relaying protocol, there is only one information source and thus joint encoding is possible.

Table 4.1: Transmission Protocol of DF Two-Path Relaying

Node \ Time	1	2	3	4	5	...
S	Tx : $x(1)$	Tx : $x(2)$	Tx : $x(3)$	Tx : $x(4)$	Tx : $x(5)$	
R1	Rx : $x(1)$	Tx : $x(1)$	Rx : $x(3), x(2)$	Tx : $x(3)$	Rx : $x(5), x(4)$	
R2		Rx : $x(2), x(1)$	Tx : $x(2)$	Rx : $x(4), x(3)$	Tx : $x(4)$	
D	Rx : $x(1)$	Rx : $x(2), x(1)$	Rx : $x(3), x(2)$	Rx : $x(4), x(3)$	Rx : $x(5), x(4)$	

the effects of inter-relay interference on the DMT performance of DF two-path relaying by utilizing a DMT framework with different relative distances between nodes [74]. In most cases, the relative distances between source, relay, and destination nodes do not affect DMT performance and is not easily reflected in the results. However for DF two-path relaying, there are certain conditions for the source-relay link where the DMT for the ideal condition, i.e., perfect decoding at the relays, can be achieved.

- A successive interference cancellation scheme at the relays is also proposed to further improve the DMT performance.

4.2 System Model

A cooperative network system with one source node (S), two relay nodes (R1, R2) and one destination node (D) is considered in this chapter. The source transmission is divided into frames, each containing L symbols denoted as $\mathbf{x} = [x(1)x(2)\cdots x(L)]^T$ with $\mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_L$. Each symbol $x(i)$ is transmitted at rate R . The destination performs detection on a frame-by-frame basis. Without loss of generality we assume L is even. We assume perfect CSI at the receiving nodes (but not at the transmitting nodes). We also assume that the channel is constant within a frame.

4.2.1 Transmission Protocol

The transmission protocol of DF two-path relaying in two consecutive time slots is depicted in Fig. 4.1. We denote $h_{s,d} \sim \mathcal{CN}(0, \gamma_{s,d})$, $h_{s,rj} \sim \mathcal{CN}(0, \gamma_{s,rj})$, $h_{rj,d} \sim \mathcal{CN}(0, \gamma_{rj,d})$ and $h_r \sim \mathcal{CN}(0, \gamma_r)$ as the channel coefficients from S to D, S to j -th relay, j -th relay to D and R1 (R2) to R2 (R1) respectively, where $j \in \{1, 2\}$. We

Figure 4.1: Transmission protocol for DF two-path relaying.

have implicitly assumed that the inter-relay channel h_r is reciprocal. The details of the transmission steps are shown in Table 4.1.

- In odd time slot i , $i \in \{1, 3, 5, \dots\}$, S transmits $x(i)$ and R2 forwards $x(i-1)$. R1 and D will receive both $x(i)$ and $x(i-1)$, and R1 decodes for $x(i)$.
- In even time slot i' , $i' \in \{2, 4, 6, \dots\}$, S transmits $x(i')$ and R1 forwards $x(i'-1)$. R2 and D will receive both $x(i')$ and $x(i'-1)$, and R2 decodes for $x(i')$.

The destination will perform joint decoding of L symbols after $(L+1)$ time slots. The additional time slot is needed at the the end to complete the transmission steps. Let $y_{rj}(i)$, $j \in \{1, 2\}$, and $y_d(i)$ be the received signals in time slot i at relay j and D respectively. Then, we have

$$y_{rj}(i) = \sqrt{\rho} h_{s,rj} x(i) + \sqrt{\rho} h_{r,j} x(i-1) + n_{rj}(i), \quad (4.2)$$

$$y_d(i) = \sqrt{\rho} h_{s,d} x(i) + \sqrt{\rho} h_{r,d} x(i-1) + n_d(i). \quad (4.3)$$

The basic idea behind the two-path relaying protocol is that R1 and R2 will take turns to relay for S, thus mitigating the multiplexing loss due to the half duplex constraint. Do also note that inter-relay interference exists in this protocol [71].

4.2.2 Equivalent MISO Channel Model

From the transmission protocol described above, the received signals at D can be expressed as

$$\mathbf{y}_d = \sqrt{\rho} \mathcal{H} \mathbf{x} + \mathbf{n}_d \quad (4.4)$$

where $\mathbf{y}_d \in \mathcal{C}^{L+1}$ is the received signal vector at D, $\mathbf{x} \in \mathcal{C}^L$ is the transmitted signal vector, $\mathcal{H}$ is the equivalent $(L+1) \times L$ channel matrix with at most two non-zero entries in each column, and $\mathbf{n}_d \in \mathcal{C}^{L+1}$ is the additive white Gaussian noise vector at D where $\mathbf{n}_d \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{L+1})$. Both R1 and R2 are assumed to have the same average transmit power ρ , as S. The channel matrix can be shown to be in the following form:

$$\mathcal{H} = \begin{bmatrix} g_0 \mathbf{I}_L \\ \mathbf{0}^T \end{bmatrix} + \begin{bmatrix} \mathbf{0}^T \\ \text{diag}(\mathbf{g}) \end{bmatrix} \quad (4.5)$$

where $\mathbf{g} = [g_1 \ g_2 \ \dots \ g_L]$ and g_i is the channel coefficient. Here, g_i will be zero if the relay does not transmit at the corresponding time slot.

The mutual information with the equivalent MISO channel model in (4.4) can be expressed as

$$\begin{aligned} I(\mathbf{x}; \mathbf{y}_d) &= \log \det (\mathbf{I}_{L+1} + \rho \mathcal{H} \mathbf{R}_{\mathbf{x}} \mathcal{H}^H \mathbf{R}_{\mathbf{n}}^{-1}) \\ &= \log \det \left(\mathbf{I}_{L+1} + \frac{\rho}{\sigma^2} \mathcal{H} \mathcal{H}^H \right) \end{aligned} \quad (4.6)$$

where $\mathbf{R}_{\mathbf{x}} = \mathbb{E} \{ \mathbf{x} \mathbf{x}^H \} = \mathbf{I}_L$, $\mathbf{R}_{\mathbf{n}} = \mathbb{E} \{ \mathbf{n}_d \mathbf{n}_d^H \} = \sigma^2 \mathbf{I}_{L+1}$.

Thus, the outage probability of DF two-path relaying protocol is given by

$$\begin{aligned} P_{\text{out}}(R) &= P_r \{ I(\mathbf{x}; \mathbf{y}_d) < (L+1)R \} \\ &= P_r \left\{ \log \det \left(\mathbf{I}_{L+1} + \frac{\rho}{\sigma^2} \mathcal{H} \mathcal{H}^H \right) < (L+1)R \right\}. \end{aligned} \quad (4.7)$$

The factor $L+1$ is to account for the fact that $L+1$ transmission slots are required to transmit L symbols.

4.3 Diversity-Multiplexing Tradeoff

We first establish the following definitions and lemmas.

Lemma 4.3.1. *Let $\mathbf{G}$ be a $(k+1) \times k$ bi-diagonal matrix defined by*

$$\mathbf{G} \triangleq \begin{bmatrix} g_0 \mathbf{I}_k \\ \mathbf{0}^T \end{bmatrix} + \begin{bmatrix} \mathbf{0}^T \\ \text{diag}(\mathbf{g}) \end{bmatrix} \quad (4.8)$$

where $\mathbf{g} = [g_1 \ g_2 \ \dots \ g_k]^T$. Then,

$$\det(\mathbf{I}_{k+1} + \mathbf{G}\mathbf{G}^H) \geq (|g_0|^2)^k + \prod_{i=1}^k (1 + |g_i|^2). \quad (4.9)$$

Proof. Let $\hat{\mathbf{G}}$ be the $(k+1) \times (k+1)$ bi-diagonal matrix defined in [53, Lemma 4]. Notice that as compared to [53, Lemma 4], we have $(|g_0|^2)^k$ instead of $(|g_0|^2)^{k+1}$. This is because $\mathbf{G}$ is a $(k+1) \times k$ matrix without the last column of $\hat{\mathbf{G}}$. As shown in [53, Lemma 4], $\mathbf{I}_{k+1} + \hat{\mathbf{G}}\hat{\mathbf{G}}^H$ is in the following form:

$$\begin{bmatrix} 1 + |g_0|^2 & g_0 g_1^* & \dots & 0 \\ g_0^* g_1 & 1 + |g_0|^2 + |g_1|^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & g_0 g_k^* \\ 0 & \dots & g_0^* g_k & 1 + |g_0|^2 + |g_k|^2 \end{bmatrix}, \quad (4.10)$$

whereas without the last column, $\mathbf{I}_{k+1} + \mathbf{G}\mathbf{G}^H$ is in the following form:

$$\begin{bmatrix} 1 + |g_0|^2 & g_0 g_1^* & \dots & 0 \\ g_0^* g_1 & 1 + |g_0|^2 + |g_1|^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & g_0 g_k^* \\ 0 & \dots & g_0^* g_k & 1 + |g_k|^2 \end{bmatrix}. \quad (4.11)$$

We can see that the only difference between (4.10) and (4.11) is the number of $|g_0|^2$ in the diagonal elements. Thus, by following the proof of Lemma 4 in [53], the power of $|g_0|^2$ is reduced from $k+1$ to k . $\square$

Lemma 4.3.2. *If X and Y are i.i.d. exponential distributed random variables*

with unit mean, the cumulative distribution function of $Z = \frac{X}{Y}$ is given by

$$F_Z(z) = 1 - \frac{1}{1+z}. \quad (4.12)$$

4.3.1 Perfect Decoding at the Relays

We will first assume that the relays are able to perfectly decode the source signals³. In the next subsection, we will remove this assumption and consider a more realistic scenario where the relays may not always successfully decode the source signals.

Let $\mathbf{H}$ be the equivalent channel matrix when the relays are able to decode the source signal perfectly. Similar to [52], if the relays are able to decode the source signal perfectly, $\mathbf{H}$ can be expressed as

$$\mathbf{H} = \begin{pmatrix} h_{s,d} & 0 & \cdots & \cdots & 0 \\ h_{r1,d} & h_{s,d} & & 0 & \vdots \\ 0 & h_{r2,d} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & h_{s,d} & 0 \\ \vdots & \vdots & & h_{r1,d} & h_{s,d} \\ 0 & \cdots & \cdots & 0 & h_{r2,d} \end{pmatrix}. \quad (4.13)$$

From (4.7), the outage probability is then given by

$$\begin{aligned} P_{\text{out}}(R) &= P_r \{I(\mathbf{x}; \mathbf{y}_d) < (L+1)R\} \\ &= P_r \left\{ \log \det \left(\mathbf{I}_{L+1} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{H}^H \right) < (L+1)R \right\}. \end{aligned} \quad (4.14)$$

Theorem 4.3.1. *Conditioned on the perfect decoding at the relays, the diversity-multiplexing tradeoff of DF two-path relaying protocol in a slow fading channel, where the channel coefficients remain constant for $L+1$ time slots, is given by*

$$d(r) = 3 \left(1 - \frac{L+1}{L} r \right)^+. \quad (4.15)$$

³In practice, it is reasonable to assume that the source will select relays that have strong channel links with itself. Thus, the relays will most probably be able to decode the source signals [52].

Proof. We prove by showing that the RHS of (4.15) is both an upper and lower bound for the DMT of DF two-path relaying protocol, and thus the DMT is given by (4.15).

1. DMT Upper Bound

For a two-relay cooperative diversity protocol, the DMT is upper bounded by the 3×1 MISO DMT [55], which is given by

$$d_{\text{UB}}(r) = 3 \left(1 - \frac{L+1}{L} r \right)^+ \quad (4.16)$$

where the factor $\frac{L+1}{L}$ is to take into account the extra time slot needed to complete the transmission in the two-path relaying protocol.

2. DMT Lower Bound

By using Lemma 4.3.1 and observing the structure of $\mathbf{H}$ in (4.13), we have

$$\begin{aligned}
\det \left(\mathbf{I}_{L+1} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{H}^H \right) &\geq (|g_0|^2)^k + \prod_{i=1}^k (1 + |g_i|^2) \\
&\geq \left(\frac{\rho}{\sigma^2} |h_{s,d}|^2 \right)^L + \left(1 + \frac{\rho}{\sigma^2} |h_{r1,d}|^2 \right)^{\frac{L}{2}} \\
&\quad \times \left(1 + \frac{\rho}{\sigma^2} |h_{r2,d}|^2 \right)^{\frac{L}{2}} \\
&\doteq \left(\frac{\rho}{\sigma^2} |h_{s,d}|^2 \right)^L + \left(\frac{\rho}{\sigma^2} |h_{r1,d}|^2 \right)^{\frac{L}{2}} \left(\frac{\rho}{\sigma^2} |h_{r2,d}|^2 \right)^{\frac{L}{2}}. \tag{4.17}
\end{aligned}$$

From (4.14) and (4.17), we obtain

$$\begin{aligned}
P_{\text{out}}(R) &\doteq \text{SNR}^{-d(r)} \\
&\doteq P_r \left\{ \log \det \left(\mathbf{I}_{L+1} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{H}^H \right) < (L+1)R \right\} \\
&\leq P_r \left\{ \log \left((\text{SNR} |h_{s,d}|^2)^L \right. \right. \\
&\quad \left. \left. + (\text{SNR}^2 |h_{r1,d}|^2 |h_{r2,d}|^2)^{\frac{L}{2}} \right) < (L+1)R \right\} \tag{4.18}
\end{aligned}$$

$$\doteq \text{SNR}^{-d_{\text{LB}}(r)} \tag{4.19}$$

where $R = r \log \text{SNR}$ and $d_{\text{LB}}(r)$ is the negative exponential order of the upper bound.

In high SNR regime, the outage probability in (4.18) is dominated by two conditions, i.e.,

$$\begin{aligned}
&P_r \left\{ \log (\text{SNR} |h_{s,d}|^2)^L < (L+1)R \right\} \\
&= P_r \left\{ \frac{\log (\text{SNR} |h_{s,d}|^2)^L}{\log \text{SNR}} < \frac{\log \text{SNR}^{(L+1)r}}{\log \text{SNR}} \right\} \tag{4.20}
\end{aligned}$$

and

$$P_r \left\{ \log (\text{SNR}^2 |h_{r1,d}|^2 |h_{r2,d}|^2)^{\frac{L}{2}} < (L+1)R \right\}$$

$$= P_r \left\{ \frac{\log (\text{SNR}^2 |h_{r1,d}|^2 |h_{r2,d}|^2)^{\frac{L}{2}}}{\log \text{SNR}} < \frac{\log \text{SNR}^{(L+1)r}}{\log \text{SNR}} \right\}. \quad (4.21)$$

Thus, the outage events in high SNR regime will be dominated by

$$O^+ = \left\{ \left(v_{s,d}, v_{r1,d}, v_{r2,d} \right) \in \mathbb{R}^{3+} \right. \\ \left. \begin{array}{l} (1 - v_{s,d})^+ < \frac{L+1}{L}r, \\ (2 - v_{r1,d} - v_{r2,d})^+ < \frac{L+1}{L}2r \end{array} \right\} \quad (4.22)$$

where $v_{s,d}$, $v_{r1,d}$, and $v_{r2,d}$ are the exponential orders of $h_{s,d}$, $h_{r1,d}$, and $h_{r2,d}$ respectively.

From [43, Definition 5] in Lemma D.1.1, the DMT lower bound can thus be expressed as

$$d_{\text{LB}}(r) = \inf_{O^+} (v_{s,d} + v_{r1,d} + v_{r2,d}) \quad (4.23)$$

which can be solved by summing the two inequalities in (4.22), and obtained in closed-form as

$$d_{\text{LB}}(r) = 3 \left(1 - \frac{L+1}{L}r \right)^+ = d_{\text{UB}}(r). \quad (4.24)$$

This completes the proof.

□

Remark:

- Theorem 4.3.1 indicates that DF two-path relaying protocol achieves a full diversity order of three and approaches the 3×1 MISO DMT for sufficiently large L . This result is in fact intuitively satisfying as we can observe from the equivalent channel matrix $\mathbf{H}$ that three independent spatial paths are indeed available, i.e., $h_{s,d}$, $h_{r1,d}$ and $h_{r2,d}$.
- Although Theorem 4.3.1 gives us an insight on the achievable DMT for DF two-path relaying under the ideal condition of perfect decoding at the relays,

it does not capture the effects of inter-relay interference⁴. One then may ask under what kind of conditions will this ideal DMT be achieved in practice when inter-relay interference cannot be neglected and decoding at the relays is not always successful. The discussion in the next subsection is devoted to answering this question.

4.3.2 Imperfect Decoding and Inter-Relay Interference

In practice, perfect decoding at the relays may not always be attainable since sufficiently strong source to relay links are required and the inter-relay interference has to be managed [71]. In this subsection, we relax the assumption of perfect decoding at the relays and analyze the effects of inter-relay interference on the DMT. We analyze the DMT performance when the relays simply decode the source signal directly and treat the inter-relay interference as noise. We also assume that if the relays fail to decode correctly (outage occurs), it will not transmit in the next time slot.

We utilize the DMT framework proposed in [74] with different relative distances between nodes. Specifically, it is assumed that the average SNR of the source to relay links are η dB higher than the other links. In [74], η is defined as the proximity gain between the source and the relays. Intuitively, if the average SNR of the source to relay links are the same as the inter-relay interference link, then as SNR goes to infinity, there will always be an outage at the relays since both links are equally strong. Thus, in order to capture the effects of inter-relay interference and realistic decoding at the relays, we need to introduce the notion of proximity gain.

Theorem 4.3.2. *Under a more realistic scenario where decoding at the relays is not always successful and the relays simply try to decode the source signal directly, i.e., treating the inter-relay interference as noise, the achievable DMT of DF two-*

⁴Since the decoding at the relays is assumed to be perfect.

path relaying protocol in a slow fading channel is given by

$$d(r) = \begin{cases} (1 - \frac{L+1}{L}r)^+ & \text{for } 0 \leq \eta \leq 1, \\ (1 - \frac{L+1}{L}r)^+ + (\eta - 1 - r)^+ + \min \left\{ (\eta - 1 - r)^+, (1 - \frac{L+1}{L}2r)^+ \right\} & \text{for } 1 < \eta < 2, \\ (1 - \frac{L+1}{L}r)^+ + \min \left\{ (1 - \frac{L+1}{L}2r)^+ + (\eta - 1 - r)^+, 2(1 - \frac{L+1}{L}r)^+ \right\} & \text{for } 2 \leq \eta < 2 + \frac{L}{2(L+1)}, \\ 3(1 - \frac{L+1}{L}r)^+ & \text{for } \eta \geq 2 + \frac{L}{2(L+1)} \end{cases} \quad (4.25)$$

where η is the proximity gain and it represents the scaling factor (in dB) on the average SNR between the source and the relays.

Proof. First, for a slow fading scenario, the relays will either be in outage or a non-outage condition during the whole duration L as the channels are static. By the law of total probability, we can express the outage probability as

$$\begin{aligned} P_{out} &= P_{out|R1\ out, R2\ non} P_r \{R1\ out, R2\ non\} \\ &\quad + P_{out|R1\ non, R2\ out} P_r \{R1\ non, R2\ out\} \\ &\quad + P_{out|R1\ out, R2\ out} P_r \{R1\ out, R2\ out\} \\ &\quad + P_{out|R1\ non, R2\ non} P_r \{R1\ non, R2\ non\} \doteq \text{SNR}^{-d(r)} \end{aligned} \quad (4.26)$$

where $\{Ri\ out\}$ and $\{Ri\ non\}$ denote the events when relay i , $i \in \{1, 2\}$, is in outage and non-outage condition, respectively. $P_r \{\cdot, \cdot\}$ is the joint probability of the events.

Due to symmetry, the SNR exponent of the first and second term in (4.26) will be the same. Since $h_{s,r1}$ and $h_{s,r2}$ are independent, the outage probability at R1 and R2 will also be independent. Thus, (4.26) can be expressed as

$$\begin{aligned} P_{out} &\doteq P_{out|E_1} P_r \{R1\ out\} P_r \{R2\ non\} \\ &\quad + P_{out|E_2} P_r \{R1\ out\} P_r \{R2\ out\} \\ &\quad + P_{out|E_3} P_r \{R1\ non\} P_r \{R2\ non\} \doteq \text{SNR}^{-d(r)} \end{aligned} \quad (4.27)$$

where $E_1 \in \{R1\ out, R2\ non\}$, $E_2 \in \{R1\ out, R2\ out\}$, and $E_3 \in \{R1\ non, R2\ non\}$.

Let $\hat{\mathbf{H}}$ be the equivalent channel matrix when R1 is in outage and R2 is in non-outage condition, i.e., event E_1 . We notice that whenever R1 is in outage, the term $h_{r1,d}$ in the equivalent channel matrix $\mathbf{H}$ in (4.13) will be zero since the destination will not receive any signal from R1. Thus, $\hat{\mathbf{H}}$ can be shown to be

$$\hat{\mathbf{H}} = \begin{pmatrix} h_{s,d} & 0 & \cdots & \cdots & 0 \\ 0 & h_{s,d} & & 0 & \vdots \\ 0 & h_{r2,d} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & h_{s,d} & 0 \\ \vdots & \vdots & & 0 & h_{s,d} \\ 0 & \cdots & \cdots & 0 & h_{r2,d} \end{pmatrix}. \quad (4.28)$$

By using Lemma 4.3.1 and observing the structure of $\hat{\mathbf{H}}$ in (4.28), we have

$$\begin{aligned} \det \left(\mathbf{I}_{L+1} + \frac{\rho}{\sigma^2} \hat{\mathbf{H}} \hat{\mathbf{H}}^H \right) &\geq (|g_0|^2)^k + \prod_{i=1}^k (1 + |g_i|^2) \\ &\geq \left(\frac{\rho}{\sigma^2} |h_{s,d}|^2 \right)^L + \left(1 + \frac{\rho}{\sigma^2} |h_{r2,d}|^2 \right)^{\frac{L}{2}} \\ &\doteq \left(\frac{\rho}{\sigma^2} |h_{s,d}|^2 \right)^L + \left(\frac{\rho}{\sigma^2} |h_{r2,d}|^2 \right)^{\frac{L}{2}}. \end{aligned}$$

Then, similar to (D.4), we obtain

$$P_{out|E_1}(R) \leq P_r \left\{ \log \left(\text{SNR}^L |h_{s,d}|^{2L} + \text{SNR}^{\frac{L}{2}} |h_{r2,d}|^{\frac{L}{2}} \right) < (L+1)R \right\}$$

and the outage events in high SNR regime will be dominated by

$$O^+ = \left\{ \left(v_{s,d}, v_{r2,d} \right) \in \mathbb{R}^{2+} \left| \begin{array}{l} (1 - v_{s,d})^+ < \frac{L+1}{L}r, \\ (1 - v_{r2,d})^+ < \frac{L+1}{L}2r \end{array} \right. \right\}.$$

Thus, we obtain

$$P_{out|E_1} \doteq \text{SNR}^{-\left[\left(1 - \frac{L+1}{L}r\right)^+ + \left(1 - \frac{L+1}{L}2r\right)^+ \right]}. \quad (4.29)$$

If both R1 and R2 are in outage, the terms $h_{r1,d}$ and $h_{r2,d}$ will be zero and we

have

$$P_{out|E_2} \doteq \text{SNR}^{-(1-\frac{L+1}{L}r)^+}. \quad (4.30)$$

Next, when both R1 and R2 are in non-outage condition, from Theorem 4.3.1, we have

$$P_{out|E_3} \doteq \text{SNR}^{-3(1-\frac{L+1}{L}r)^+}. \quad (4.31)$$

For relay i , $i \in \{1, 2\}$, the outage probability is given by

$$P_r \{Ri \text{ out}\} = P_r \left\{ \log \left(1 + \frac{\rho^\eta |h_{s,ri}|^2}{\rho |h_r|^2 + \sigma^2} \right) < R \right\}.$$

As $\rho \rightarrow \infty$,

$$\begin{aligned} P_r \{Ri \text{ out}\} &\doteq P_r \left\{ \log \left(\text{SNR}^{\eta-1} \frac{|h_{s,ri}|^2}{|h_r|^2} \right) < r \log \text{SNR} \right\} \\ &\doteq P_r \left\{ \frac{|h_{s,ri}|^2}{|h_r|^2} < \text{SNR}^{-(\eta-1-r)} \right\}. \end{aligned}$$

From Lemma 4.3.2, we obtain

$$\begin{aligned} P_r \{Ri \text{ out}\} &\doteq 1 - \frac{1}{1 + \text{SNR}^{-(\eta-1-r)}} \\ &\doteq 1 - \left(1 - \text{SNR}^{-(\eta-1-r)^+} + \text{SNR}^{-2(\eta-1-r)^+} + \dots \right) \\ &\doteq \text{SNR}^{-(\eta-1-r)^+}. \end{aligned} \quad (4.32)$$

The non-outage probability is thus given as

$$P_r \{Ri \text{ non}\} = 1 - P_r \{Ri \text{ out}\} = 1 - \text{SNR}^{-(\eta-1-r)^+}. \quad (4.33)$$

Hence, substituting (4.29)-(4.33) to (4.27), we obtain

$$\begin{aligned} P_{out} &\doteq \text{SNR}^{-[(1-\frac{L+1}{L}r)^+ + (1-\frac{L+1}{L}2r)^+]} \text{SNR}^{-(\eta-1-r)^+} \\ &\quad \times \left(1 - \text{SNR}^{-(\eta-1-r)^+} \right) \end{aligned}$$

$$\begin{aligned}
& + \text{SNR}^{-(1-\frac{L+1}{L}r)^+} \text{SNR}^{-2(\eta-1-r)^+} \\
& + \text{SNR}^{-3(1-\frac{L+1}{L}r)^+} \left(1 - \text{SNR}^{-(\eta-1-r)^+} \right) \\
& \times \left(1 - \text{SNR}^{-(\eta-1-r)^+} \right). \tag{4.34}
\end{aligned}$$

Since $\text{SNR}^{-\alpha} - \text{SNR}^{-(\alpha+\beta)} \doteq \text{SNR}^{-\alpha}$ for $\alpha, \beta > 0$, we have

$$\begin{aligned}
P_{out} & \doteq \text{SNR}^{-[(1-\frac{L+1}{L}r)^+ + (1-\frac{L+1}{L}2r)^+ + (\eta-1-r)^+]} \\
& + \text{SNR}^{-((1-\frac{L+1}{L}r)^+ + 2(\eta-1-r)^+)} + \text{SNR}^{-3(1-\frac{L+1}{L}r)^+} \\
& \doteq \text{SNR}^{-d(r)} \tag{4.35}
\end{aligned}$$

where $d(r)$ is given by the absolute value of the largest exponent term and this completes the proof. $\square$

Observation:

- The DMT of DF two-path relaying in the presence of inter-relay interference and realistic decoding at the relays is a function of η and is fully characterized by Theorem 4.3.2. The DMT curves with different η values are shown in Fig 4.2. The higher the proximity gain η , the better the DMT that can be achieved.
- From Theorem 4.3.2, the ideal DMT with perfect decoding at the relays can be achieved whenever $\eta \geq 2 + \frac{L}{2(L+2)}$. In other words, for sufficiently large L , if the average SNR between the source and the relays is at least 2.5 times (in dB) higher than other links, the 3×1 MISO DMT is achieved. This result also gives us some guidelines on the network geometry that will optimize the DMT in practical deployments.
- The worst DMT is obtained when $\eta \leq 1$. The DMT is reduced to the non-cooperative case where $d(r) = (1 - \frac{L+1}{L}r)^+$. This result is not surprising since $\eta \leq 1$ corresponds to the situation where the average inter-relay channel SNR (the interference signal) is stronger than the average source to relay SNR (the source signal) and Theorem 4.3.2 is derived based on the assumption that

Figure 4.2: Diversity-multiplexing tradeoff for DF two-path relaying protocol. Here, $L = 50$.

the relays decode directly for the source signal. Thus, decoding at the relays will always fail and the network is as good as one without relays.

4.3.3 Further Improvement

In order to improve the DMT performance when $\eta \leq 1$, the relays can perform successive interference cancellation. If the inter-relay interference between relays is stronger, the relay will decode the interference first and then subtract it away from the received signals before decoding for the desired source signal. Otherwise, the relay will just decode directly for the source signal and treat the interference as noise.

Theorem 4.3.3. *Under a more realistic scenario, where decoding at the relays is not always successful and the relays perform successive interference cancellation, the achievable DMT of DF two-path relaying protocol in a slow fading channel is given by*

$$d(r) = \begin{cases} (1 - \frac{L+1}{L}r)^+ + 2 \min \{(\eta - r)^+, (1 - \eta - r)^+\} & \text{for } 0 \leq \eta \leq 1, \\ (1 - \frac{L+1}{L}r)^+ + (\eta - 1 - r)^+ + \min \{(\eta - 1 - r)^+, (1 - \frac{L+1}{L}2r)^+\} & \text{for } 1 < \eta < 2, \\ (1 - \frac{L+1}{L}r)^+ + \min \left\{ (1 - \frac{L+1}{L}2r)^+ + (\eta - 1 - r)^+, 2(1 - \frac{L+1}{L}r)^+ \right\} & \text{for } 2 \leq \eta < 2 + \frac{L}{2(L+1)}, \\ 3(1 - \frac{L+1}{L}r)^+ & \text{for } \eta \geq 2 + \frac{L}{2(L+1)}. \end{cases} \quad (4.36)$$

Proof. Please refer to Appendix F. $\square$

Remark:

- Theorem 4.3.3 differs from Theorem 4.3.2 only when $\eta \leq 1$. For $\eta \leq 1$, we can observe from Theorem 4.3.2 that the DMT of DF two-path relaying is reduced to the non-cooperative case. However, by using successive interference cancellation at the relays, the DMT performance of DF two-path relaying for $\eta \leq 1$ can be improved. The additional DMT gain is

$$2 \min \{(\eta - r)^+, (1 - \eta - r)^+\}. \quad (4.37)$$

As shown in Fig. 4.2, when $\eta = 0.5$, we can see that instead of achieving only a diversity gain of one, a diversity gain of two (at $r = 0$) can be achieved.

4.4 Achievability of the DMT

The outage probability used to derive the DMT is also a benchmark to evaluate the performance of space time code and cooperative communication systems. It is based on the existence of *universal codes* [56] that achieve arbitrarily small error probability whenever the channel is not in outage, i.e., $P_e \approx P_{out}$. It is also

naturally used for computation of the DMT where $\rho \rightarrow \infty$. For direct transmission with single antenna (for example, transmission from the source to relay), given a target symbol rate R , the achievability of the outage exponent is obvious. However, the analysis at the destination for two-path relaying is not so straightforward.

Consider at data rate $R = r \log \text{SNR}$ (bits/symbol), the error probability at the destination node is expressed as

$$\begin{aligned} P_e(\text{SNR}) &= P_{\text{out}}(R)P(\text{error}|\text{outage}) + P(\text{error, no outage}) \\ &\leq P_{\text{out}}(R) + P(\text{error, no outage}) \end{aligned} \quad (4.38)$$

where $P(\text{error, no outage})$ is the error probability when there is no outage at the destination node. $P(\text{error, no outage})$ can be upper bounded by the union bound of the pairwise error probability (PEP) [55]. Similar to $P_{\text{out}}(R)$ in (4.27), $P(\text{error, no outage})$ depends on the outage conditions at the relays.

To compute the PEP, we first need to express (4.4) as $\mathbf{y}_d = \sqrt{\rho}\mathbf{X}\mathbf{h} + \mathbf{n}_d$ where $\mathbf{X}$ is the equivalent space time code matrix for $\mathbf{x}$ and $\mathbf{h}$ is a vector consisting of the channel coefficients. The structure of $\mathbf{X}$ and $\mathbf{h}$ can take on the following forms:

- If one of the relays is in outage condition, e.g., R2 in outage:

$$\begin{aligned} \mathbf{X} &= \begin{bmatrix} x(1) & x(2) & x(3) & x(4) & \dots & x(L) & 0 \\ 0 & x(1) & 0 & x(3) & \dots & x(L-1) & 0 \end{bmatrix}^T \\ \text{and } \mathbf{h} &= \begin{bmatrix} h_{s,d} \\ h_{r1,d} \end{bmatrix}. \end{aligned} \quad (4.39)$$

- If both relays are in outage condition, it will reduce to a direct transmission:

$$\mathbf{X} = [x(1) \ x(2) \ \dots \ x(L)]^T \quad \text{and} \quad \mathbf{h} = [h_{s,d}]. \quad (4.40)$$

- If both relays are in non-outage condition:

$$\mathbf{X} = \begin{bmatrix} x(1) & x(2) & x(3) & x(4) & \dots & x(L) & 0 \\ 0 & x(1) & 0 & x(3) & \dots & x(L-1) & 0 \\ 0 & 0 & x(2) & 0 & x(4) & \dots & x(L) \end{bmatrix}^T$$

$$\text{and } \mathbf{h} = \begin{bmatrix} h_{s,d} \\ h_{r1,d} \\ h_{r2,d} \end{bmatrix}. \quad (4.41)$$

Similar to P_{out} in (4.27), we have

$$\begin{aligned} P(\text{error, no outage}) &= P(\text{error, no outage}|E_1)P_r\{R1 \text{ out}\}P_r\{R2 \text{ non}\} \\ &\quad + P(\text{error, no outage}|E_2)P_r\{R1 \text{ out}\}P_r\{R2 \text{ out}\} \\ &\quad + P(\text{error, no outage}|E_3)P_r\{R1 \text{ non}\}P_r\{R2 \text{ non}\}. \end{aligned} \quad (4.42)$$

By taking the union bound of the PEP across all space time codewords that differ by one symbol, which is the dominant error event in the high SNR regime, we have

$$\begin{aligned} P(\text{error, no outage}|E_i) &\leq (L2^{R_s} - 1)P(\mathbf{X}_k \rightarrow \mathbf{X}_l) \\ &\leq 2^{R_s} \int \frac{1}{\pi^3} e^{-\frac{\rho}{4\text{tr}(\Sigma_{\mathbf{n}_d)}} \mathbf{h}^H \Delta \mathbf{X}^H \Delta \mathbf{X} \mathbf{h}} e^{-\mathbf{h}^H \mathbf{h}} d\mathbf{h} \\ &\leq 2^{R_s} \left(\prod_{j=1}^{\beta_i} \lambda_j[\Delta \mathbf{X}^H \Delta \mathbf{X}] \right)^{-1} \left(\frac{\rho}{4(L+1)\sigma^2} \right)^{-\beta_i} \\ &\leq \text{SNR}^{\frac{L+1}{L}r} \text{SNR}^{-\beta_i} = \text{SNR}^{-(\beta_i - \frac{L+1}{L}r)} \end{aligned} \quad (4.43)$$

where $i \in \{1, 2, 3\}$, $\Delta \mathbf{X} = \mathbf{X}_k - \mathbf{X}_l$, $\beta_i = \min_{\mathbf{X}_k - \mathbf{X}_l} (\text{rank}(\Delta \mathbf{X}))$, and each $x(l)$ is transmitted with data rate $R_s = \frac{L+1}{L}R$ bits in each transmission slot. Depending on the outage conditions at the relays, i.e., E_i , the structure of $\mathbf{X}_k$ or $\mathbf{X}_l$ is given in (4.39)-(4.41).

From (4.38), we have

$$P_e(\text{SNR}) \leq \text{SNR}^{-d(r)} + \text{SNR}^{-d_G(r)} \quad (4.44)$$

where $d(r)$ is given in (4.15), (4.25), or (4.36). Here, $d_G(r)$ is the exponent of $P(\text{error, no outage})$.

If $\Delta \mathbf{X}$ is full rank⁵ across all space time code difference matrices, from (4.39)-(4.41), (4.43), we obtain

$$P(\text{error, no outage}|E_1) \doteq \text{SNR}^{-(2-\frac{L+1}{L}r)^+}, \quad (4.45)$$

$$P(\text{error, no outage}|E_2) \doteq \text{SNR}^{-(1-\frac{L+1}{L}r)^+}, \quad (4.46)$$

$$P(\text{error, no outage}|E_3) \doteq \text{SNR}^{-(3-\frac{L+1}{L}r)^+}. \quad (4.47)$$

Thus, we can observe that each exponent term of $P_{\text{out}|E_i}$ in (4.29)-(4.31) will always dominate over that of $P(\text{error, no outage}|E_i)$ in (4.45)-(4.47). In other words, $\text{SNR}^{-d(r)}$ will dominate in $P_e(\text{SNR})$. Hence, the DMT in Theorem 4.3.1, 4.3.2, and 4.3.3 is achieved.

4.5 Numerical Results

Fig. 4.3 shows the outage probability performance of DF two-path relaying protocol under different scenarios. The diversity gain of the protocol can be seen from the slope of the outage probability curve. We can see that the higher the value of η , the better the performance. When $\eta = 2.5$, the full diversity order of three is achieved. It can also be observed that the SIC scheme improves the performance of DF two-path relaying for $\eta \leq 1$.

4.6 Summary

In this chapter, we analyzed the DMT for DF two-path half duplex relaying protocol with a distributed MISO approach. We first derived the DMT under the assumption of perfect decoding at the relays and proved that this protocol achieves a full diversity gain of three and approaches the 3×1 MISO DMT. We then removed the assumption of perfect decoding at the relays and derived the achievable DMT based on the relative distance between the nodes. For sufficiently large frame

⁵This full rank criterion can be satisfied for example by transmitting linear combinations of the symbols at the source, which is similar to the precoding method in [71] for the AF case. In each time slot, the source transmits a linear combination of the data symbols, i.e., $\hat{\mathbf{x}} = \mathbf{U}\mathbf{x}$ where the precoding matrix $\mathbf{U}$ is an arbitrary complex unitary matrix.

Figure 4.3: Outage probability of DF two-path relaying. Here, $L = 20$.

length, if the average source-relay SNR is at least 2.5 times (measured in dB) of other links, we showed that the 3×1 MISO DMT can still be achieved. Further improvement of the DMT is also achieved when successive interference cancellation is used at the relays.

Cooperative Multiple-Access System with Sequential Amplify-and-Forward Relaying

5.1 Introduction

In Chapter 2-4, we studied cooperative communications as an effective method to combat fading in wireless networks by exploiting spatial diversity of the multiple independent paths between source and destination via relay nodes. Early works [18, 24, 25, 43, 87, 88] focused on small number of nodes to gain insights on the performance of the system. However, recently more attention is given to larger networks where multiple sources and multiple relays scenarios are considered. These scenarios, although more complex, are more relevant to practical wireless communication systems.

In [43], a transmission protocol for cooperative multiple-access (CMA) channels where multiple sources try to communicate with a single destination node was proposed. The basic idea of the protocol presented in [43] is to create an artificial inter-symbol interference (ISI) channel where each source transmits a linear combination of its own signal and the noisy signal it previously received. In [89], a similar scheme was proposed for CMA channels where each source node applies superposition coding [88] to transmit other sources' information in a se-

quential manner. Both CMA schemes in [43, 89] show promising results. However, these CMA schemes only exploit the cooperation among the source nodes and thus the achievable diversity gain is constrained by the number of source nodes. For the scheme in [89], it is also necessary for all source nodes to share their own information with one another, which may only be applicable for specific wireless applications.

The works in [43, 89] spur the interest to study more general cases where relays are also available to assist the transmission in CMA channels. A simple opportunistic relaying scheme based on network path selection in [30, 90] can be readily extended to multiple sources scenario. However, this opportunistic relaying based CMA scheme will not be optimal due to the half-duplex constraint of the relays. The relays need to wait the transmission from the sources before the chosen relay is able to forward the information to the destination. Full diversity gain can be achieved, but the multiplexing gain will only be half. Thus, more efforts have been given to obtain more effective and efficient schemes for CMA channels.

In [76], a relay assisted transmission protocol based on decode-and-forward relaying was proposed for CMA channels. Transmissions from the source nodes are scheduled such that the source with the poorest connection to the destination transmits first. A subset of relays that are able to decode the transmission from all sources is then selected. Relay transmission is then scheduled in such a way that the relay with better connection to the destination transmits first. It was shown in [76] that this CMA scheme is able to achieve the maximum diversity gain from all relays. However, the selection and scheduling procedures will require both transmit and receive channel state information (CSI) at each relay.

In this chapter, we propose a sequential amplify-and-forward relaying protocol for cooperative multiple-access systems. This protocol was inspired by the two-path relaying protocol [42, 67, 71, 75] for single-source two-relay nodes scenario. We analyze the fundamental performance of this protocol in terms of DMT [55]. DMT was first studied for multiple-input multiple-output channels, but now it has been widely used as a fundamental performance measure to evaluate different cooperative communication protocols. Our contributions can be summarized as follows:

- We first establish the upper bound for the sequential amplify-and-forward

relaying protocol based on the DMT of multiple-input single-output channel. The upper bound is given by

$$d(r) = (N + 1)(1 - r)^+ \quad (5.1)$$

where $(x)^+$ denotes $\max\{0, x\}$, N is the number of relays, and r is the multiplexing gain.

- We then derive the achievable DMT and show that the DMT is given by

$$d(r) = (N + 1) \left(1 - \frac{KL + 1}{KL} r \right)^+ \quad (5.2)$$

where K is the number of source nodes and L is the number of transmitted symbols for each source node. We show that the proposed transmission protocol overcomes the inherent spectral efficiency loss in half-duplex relaying¹ and achieves the full diversity gain. Compared to the scheme in [76], which is based on decode-and-forward relaying, the proposed scheme has a better DMT performance, lower complexity, and lower CSI requirements at the relays. When the frame length is sufficiently large, the MISO DMT upper bound is achieved.

5.2 System Model

A cooperative wireless multiple-access system with K source nodes (S_m , $m = 1, 2, \dots, K$), N relay nodes (R_i , $i = 1, 2, \dots, N$) and one destination node (D) is considered in this chapter. Each source node transmits a codeword each containing L symbols where, without loss of generality, we assume L is an integer multiple of N . The codeword from m -th source is denoted as $\mathbf{x}_{s_m} = [x_{s_m}(1)x_{s_m}(2) \cdots x_{s_m}(L)]^T$ with $\mathbb{E}\{\mathbf{x}_{s_m}\mathbf{x}_{s_m}^H\} = \mathbf{I}_N$. Each symbol $x_{s_m}(i)$ is allocated a time slot for transmission at rate R bits/s/Hz. Each source is assumed to be ignorant of other source nodes' information. All relays are assumed to be half-duplex relays where they can only either transmit or receive at any time instant. The destination performs detection after all codewords from all source nodes have been received. We assume

¹The multiplexing gain achieved is $\frac{KL}{KL+1}$, which is approximately equal to 1 for large KL .

perfect CSI at the receiving nodes (but not at the transmitting nodes). We also assume quasi-static Rayleigh fading channels.

5.2.1 Transmission Protocol

We denote $h_{s_md} \sim \mathcal{CN}(0, \gamma_{s_md})$, $h_{s_mr_i} \sim \mathcal{CN}(0, \gamma_{s_mr_i})$, $h_{r_id} \sim \mathcal{CN}(0, \gamma_{r_id})$ and $h_{r_ir_j} \sim \mathcal{CN}(0, \gamma_{r_ir_j})$ as the channel coefficients from S_m to D , S_m to R_i , R_i to D , and R_i (R_j) to R_j (R_i), respectively, where $i, j \in \{1, 2, \dots, N\}$. We have implicitly assumed that the inter-relay channel $h_{r_ir_j}$ is reciprocal.

The proposed cooperative multiple-access with sequential amplify-and-forward relaying (CMA-SAFR) scheme can be described as follows:

- The source nodes transmit in a round-robin manner, symbol-by-symbol. The sequence of transmission is arbitrary and is repeated after every K time slots. We define one cycle as the event where each source has transmitted one symbol. Thus, if each source has L symbols, there will be a total of L cycles where each cycle consists of K time slots.
- Similarly, the relay nodes also transmit in a round-robin manner. The transmitting relay node will amplify-and-forward whatever it has received in the immediate previous time slot. Although there is no specific rule in the sequencing of the relay transmissions, the relay nodes have to decide on a round-robin sequence such that after every N cycles each relay has directly² relayed one symbol from all source nodes. We shall denote this as the full diversity requirement. The purpose is to allow each source node to be directly assisted by all relay nodes and thus achieving the maximum diversity gain. This sequencing is dependent on the sequence of the source transmission and is always possible, except when $K = N = 2$.
- At any time instant, there will be one source and one relay node transmitting simultaneously, except for the first time slot where only a source node will transmit and the last time slot where only a relay node will transmit. The total number of time slots required for all source nodes to transmit one codeword is $KL + 1$.

²There is one to one link between source and relay.

Pseudo-code of the transmission protocol:

Let $\mathcal{V}_{m,R}$ denote a set of relays that have been selected by S_m . $\mathcal{U}_R$ denotes the set of all relays in the network. $\mathcal{R}_c$ denotes the relay that is selected in the previous immediate time slot. The pseudo-code of the transmission protocol is shown below:

```

Initialize  $\mathcal{U}_R = \{R_1, R_2, \dots, R_N\}$ ,  $\mathcal{V}_{m,R} = \emptyset$ , and  $\mathcal{R}_c = \emptyset$ 
for  $j = 1$  to  $L$ 
  for  $m = 1$  to  $K$ 
     $S_m$  transmits the information to  $R_k \in \mathcal{U}_R - \mathcal{V}_{m,R} - \mathcal{R}_c$  and to  $D$ 
    At the same time,  $\mathcal{R}_c$  transmits to  $D$ 
    The selected relay  $R_k$  is then added to  $\mathcal{V}_{m,R}$ 
    Set  $\mathcal{R}_c = \{R_k\}$ 
  end
end

```

Example of the transmission protocol:

To provide a clearer exposition of the transmission protocol, let us consider a specific example where $K = 3$, $N = 3$, and $L = 6$ as shown in Fig.5.1. We first need to determine the sequence of the source and relay transmissions. For the source transmission, we can always set it as $(S_1, S_2, S_3, S_1, S_2, S_3, \dots)$. However, for the relay transmission, we will require that after $N(= 3)$ cycles (9 time slots) each relay has directly relayed one symbol from all source nodes. An example of a possible sequence for relay transmission is

$$(R_1, R_2, R_3, R_2, R_3, R_1, R_3, R_2, R_1, \dots). \quad (5.3)$$

Notice that for this specific example, a simple sequence $(R_1, R_2, R_3, R_1, R_2, R_3, \dots)$ will not satisfy the full diversity requirement. Each source will only be directly connected to one of the relays, and thus the diversity will be limited by a single link between a source and a relay.

The details of the transmission steps are as follows:

Figure 5.1: Transmission protocol of CMA-SAFR for $K = 3$ and $N = 3$.

Table 5.1: Transmission Sequence for CMA-SAFR for $K = 3$ and $N = 3$

Time Slot	Transmitting nodes	Receiving nodes
1	S_1	R_1, D
2	S_2, R_1	R_2, D
3	S_3, R_2	R_3, D
4	S_1, R_3	R_2, D
5	S_2, R_2	R_3, D
6	S_3, R_3	R_1, D
7	S_1, R_1	R_3, D
8	S_2, R_3	R_1, D
9	S_3, R_1	R_2, D
10	S_1, R_2	R_1, D
11	S_2, R_1	R_2, D
12	S_3, R_2	R_3, D
13	S_1, R_3	R_2, D
14	S_2, R_2	R_3, D
15	S_3, R_3	R_1, D
16	S_1, R_1	R_3, D
17	S_2, R_3	R_1, D
18	S_3, R_1	R_2, D
19	R_2	D

- In time slot 1, $x_{s_1}(1)$ is sent from S_1 . R_1 and D listen and receive $x_{s_1}(1)$.
- In time slot 2, $x_{s_2}(1)$ is sent from S_2 . R_1 amplifies-and-forwards whatever it received in time slot 1. Both R_2 and D receive a linear combination of the signals transmitted from S_2 and R_1 .
- In time slot 3, $x_{s_3}(1)$ is sent from S_3 . R_2 amplifies-and-forwards whatever it received in time slot 2. Both R_3 and D receive a linear combination of the signals transmitted from S_1 and R_2 .
- In time slot 4, it is now S_1 's turn to transmit again. Thus, $x_{s_1}(2)$ is sent

from S_1 . At the same time, R_3 amplifies-and-forwards whatever it received in time slot 3. Following the sequence of the relay transmission in (5.3), R_2 will receive a linear combination of the signals transmitted from S_1 and R_3 .

- In time slot 5, $x_{s_2}(2)$ is sent from S_2 . R_2 amplifies-and-forwards whatever it received in time slot 3. R_3 will receive a linear combination of the signals transmitted from S_2 and R_2 .
- As shown in Fig.5.1 and Table 5.1, these transmission steps are then continued according to the predetermined sequence for the source and relay transmissions. There is a total of 6 cycles where each cycle consists of 3 time slots. Do note that the sequence for relay transmission in (5.3) is not unique. As long as within $N(=3)$ cycles the transmission from each source goes through all the relays, other sequencing is valid too. An alternative transmission sequence is shown in Table 5.2.

As mentioned earlier, the key idea in achieving full diversity is in the sequencing of the relay transmission such that after every N cycles each relay has directly relayed one symbol from all sources. Thus, for the analysis, we will introduce $h_{s_m r_{c,m}}$ to denote the channel coefficient between S_m and the corresponding relay in c -th cycle, $R_{c,m} \in \{R_1, R_2, \dots, R_N\}$. Here, $r_{c,m}$ denotes the index of $R_{c,m}$. The actual index is not important since through the full diversity requirement the protocol has ensured that the transmissions from each source will go through all the relays. We will also denote

$$\mathbf{r} = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,K} & r_{2,1} & \cdots & r_{2,K} & \cdots & r_{L,K} \\ r_{1,1} & r_{1,2} & \cdots & r_{1,K} & r_{2,1} & \cdots & r_{2,K} & \cdots & r_{L,K} & \cdots \end{bmatrix}$$

as the vector denoting the sequence of relay transmission. We will also denote the channel coefficient between $\mathbf{r}(i)$ and $\mathbf{r}(j)$ by $h_{\mathbf{r}(i)\mathbf{r}(j)}$, and the power scaling factor at $R_{c,m}$ by $g_{r_{c,m}}$.

5.2.2 Equivalent Multiple-Access Channel Model

Let $y_{r_j}(i)$ and $y_d(i)$ be the received signals in time slot i at R_j and D, respectively. Suppose S_m and R_q are the transmitting nodes at time slot i as shown in Fig. 5.2.

Table 5.2: Alternative Transmission Sequence for CMA-SAFR for $K = 3$ and $N = 3$

Time Slot	Transmitting nodes	Receiving nodes
1	S_1	R_3, D
2	S_2, R_3	R_2, D
3	S_3, R_2	R_1, D
4	S_1, R_1	R_2, D
5	S_2, R_2	R_1, D
6	S_3, R_1	R_3, D
7	S_1, R_3	R_1, D
8	S_2, R_1	R_3, D
9	S_3, R_3	R_2, D
10	S_1, R_2	R_3, D
11	S_2, R_3	R_2, D
12	S_3, R_2	R_1, D
13	S_1, R_1	R_2, D
14	S_2, R_2	R_1, D
15	S_3, R_1	R_3, D
16	S_1, R_3	R_1, D
17	S_2, R_1	R_3, D
18	S_3, R_3	R_2, D
19	R_2	D

Then, we have

$$y_{r_j}(i) = \sqrt{\rho} h_{s_m r_j} x_{s_m} + \sqrt{\rho} h_{r_q r_j} x_{r_q} + n_{r_j}(i), \quad (5.4)$$

$$y_d(i) = \sqrt{\rho} h_{s_m d} x_{s_m} + \sqrt{\rho} h_{r_q d} x_{r_q} + n_d(i) \quad (5.5)$$

where x_{s_m} and $x_{r_q} = g_{r_q} y_{r_q}(i-1)$ are the transmitted signals from S_m and R_q at time slot i , respectively. The average transmit power at all nodes are assumed to be the same and denoted as ρ . $n_{r_j}(i) \sim \mathcal{CN}(0, 1)$ and $n_d(i) \sim \mathcal{CN}(0, 1)$ are the additive white Gaussian noise in time slot i at R_j and D , respectively. g_{r_q} is the

Figure 5.2: Simultaneous transmission in CMA-SAFR at any time instant.

power scaling factor at R_q which is a constant since we use average channel gains instead of the instantaneous ones for normalization [25, 80, 71].

Based on the transmission protocol described earlier, the received signals at D can be expressed as

$$\mathbf{y}_d = \sqrt{\rho} \mathbf{H} \mathbf{x} + \mathbf{B} \mathbf{n}_r + \mathbf{n}_d = \sqrt{\rho} \mathbf{H} \mathbf{x} + \mathbf{B}' \mathbf{n} \quad (5.6)$$

where $\mathbf{y}_d \in \mathcal{C}^{KL+1}$ is the received signal vector at D , $\mathbf{H}$ is the equivalent channel matrix with a lower triangular matrix structure, $\mathbf{x} = [x_{s_1}(1) \ x_{s_2}(1) \ \cdots \ x_{s_K}(L)]^T \in \mathcal{C}^{KL}$ is the transmitted signal vector consisting of all symbols from all sources, $\mathbf{B}' = [\mathbf{B} \ \mathbf{I}_{KL+1}]$, and $\mathbf{n} = [\mathbf{n}_r \ \mathbf{n}_d]^H$. $\mathbf{n}_r \in \mathcal{C}^{KL}$ and $\mathbf{n}_d \in \mathcal{C}^{KL+1}$ are the additive white Gaussian noise vectors at the relays and D , respectively. Notice that $\mathbf{B} \mathbf{n}_r$ is the forwarded noise from the relays and it is not white in general.

The non-zero elements of $\mathbf{H} \in \mathcal{C}^{(KL+1) \times KL}$ and $\mathbf{B} \in \mathcal{C}^{(KL+1) \times KL}$ are given as:

$$\begin{aligned} \mathbf{H}(K(c-1) + m, K(c-1) + m) &= h_{s_m d} \\ \text{for } c &= 1, 2, 3, \dots, L, \quad m = 1, 2, 3, \dots, K. \\ \mathbf{H}(K(c-1) + m + 1, K(c-1) + m) &= h_{s_m r_{c,m}} g_{r_{c,m}} h_{r_{c,m} d} \\ \text{for } c &= 1, 2, 3, \dots, L, \quad m = 1, 2, 3, \dots, K. \\ \mathbf{H}(K(c-1) + m + 1 + p, K(c-1) + m) & \end{aligned}$$

$$= h_{s_m r_c, m} g_{r_c, m} \left(\prod_{i=1}^p h_{\mathbf{r}(K(c-1)+m+i-1)\mathbf{r}(K(c-1)+m+i)} g_{\mathbf{r}(K(c-1)+m+i)} \right) h_{\mathbf{r}(K(c-1)+m+p), d}$$

for $c = 1, 2, 3, \dots, L$ $m = 1, 2, 3, \dots, K$ $p = 1, 2, 3, \dots$

And,

$$\mathbf{B}(K(c-1) + m + 1, K(c-1) + m) = h_{r_c, m} g_{r_c, m}$$

for $c = 1, 2, 3, \dots, L$ $m = 1, 2, 3, \dots, K$.

$$\mathbf{B}(K(c-1) + m + 1 + p, K(c-1) + m)$$

$$= \left(\prod_{i=1}^p h_{\mathbf{r}(K(c-1)+m+i-1)\mathbf{r}(K(c-1)+m+i)} g_{\mathbf{r}(K(c-1)+m+i)} \right) h_{\mathbf{r}(K(c-1)+m+p), d},$$

for $c = 1, 2, 3, \dots, L$ $m = 1, 2, 3, \dots, K$ $p = 1, 2, 3, \dots$

Example of the channel matrix:

For the CMA-SAFR scheme in Table 5.1 where $K = 3$ and $N = 3$, the channel matrix is given by

$$\mathbf{H} = \begin{pmatrix} h_{s_1 d} & 0 & \dots & \dots & 0 \\ h_{s_1 r_1} g_{r_1} h_{r_1 d} & h_{s_2 d} & 0 & \ddots & \vdots \\ h_{s_1 r_1} g_{r_1} h_{r_1 r_2} g_{r_2} h_{r_2 d} & h_{s_2 r_2} g_{r_2} h_{r_2 d} & h_{s_3 d} & \ddots & \vdots \\ h_{s_1 r_1} g_{r_1} h_{r_1 r_2} g_{r_2} h_{r_2 r_3} g_{r_3} h_{r_3 d} & h_{s_2 r_2} g_{r_2} h_{r_2 r_3} g_{r_3} h_{r_3 d} & h_{s_3 r_3} g_{r_3} h_{r_3 d} & h_{s_1 d} & 0 \\ h_{s_1 r_1} g_{r_1} h_{r_1 r_2} g_{r_2}^2 h_{r_2 r_3} g_{r_3} h_{r_3 r_2} h_{r_2 d} & \vdots & h_{s_3 r_3} g_{r_3} h_{r_3 r_2} g_{r_2} h_{r_2 d} & h_{s_1 r_1} g_{r_1} h_{r_1 r_2} h_{r_2 d} & \ddots \\ \vdots & \dots & \vdots & \dots & \vdots \end{pmatrix} \quad (5.7)$$

where in the above matrix we have replaced the indices in $\mathbf{r}(i)$ with the indices of the actual relay for clarity, i.e., $\mathbf{r} = [r_{1,1} \ r_{1,2} \ r_{1,3} \ r_{2,1} \ r_{2,2} \ r_{2,3} \ r_{3,1} \ r_{3,2} \ r_{3,3} \ \dots] = [r_1 \ r_2 \ r_3 \ r_2 \ r_3 \ r_1 \ r_3 \ r_1 \ r_2 \ \dots]$.

5.3 Diversity-Multiplexing Tradeoff

From Section 5.2.2, the general structure of the equivalent channel matrix $\mathbf{H}$ is a lower triangular matrix and can be expressed as

$$\mathbf{H} = \begin{pmatrix} h_{s_1 d} & 0 & \dots & \dots & 0 \\ h_{s_1 r_{1,1}} g_{r_{1,1}} h_{r_{1,1} d} & \ddots & & 0 & \\ h_{s_1 r_{1,1}} g_{r_{1,1}} h_{r_{1,1} r_{1,2}} g_{r_{1,2}} h_{r_{1,2} d} & \ddots & h_{s_K d} & \ddots & \\ \vdots & \vdots & h_{s_K r_{1,K}} g_{r_{1,K}} h_{r_{1,K} d} & \ddots & 0 \\ \vdots & \vdots & h_{s_K r_{1,K}} g_{r_{1,K}} h_{r_{1,K} r_{2,1}} g_{r_{2,1}} h_{r_{2,1} d} & \ddots & h_{s_1 d} & \ddots \\ \dots & \dots & \vdots & \dots & h_{s_1 r_{N,1}} g_{r_{N,1}} h_{r_{N,1} d} & \\ \vdots & \ddots & & \dots & \vdots & \ddots \end{pmatrix} \quad (5.8)$$

where we will make use of the diagonal and first sub-diagonal to derive the DMT.

5.3.1 DMT of the Proposed CMA-SAFR

Proposition 5.3.1. *The DMT upper bound for cooperative multiple-access system with sequential amplify-and-forward relaying in a quasi-static fading channel is given by*

$$d(r) = (N + 1) (1 - r)^+. \quad (5.9)$$

Proof. Notice that for the proposed CMA-SAFR protocol, each source node does not have information about the transmitted signals from other source nodes. Thus, the DMT is naturally upper bounded by the DMT of a MISO system with $(L + 1)$ transmit antennas and 1 receive antenna. $\square$

Theorem 5.3.1. *The achievable diversity-multiplexing tradeoff of cooperative multiple -access system with sequential amplify-and-forward relaying in a quasi-static fading channel, where the channel coefficients remain constant for $MN + 1$ time*

slots, is given by

$$d(r) = (N + 1) \left(1 - \frac{KL + 1}{KL} r \right)^+ . \quad (5.10)$$

Proof. We proof by using the fact that error probability of the maximum likelihood detector can be tightly bounded by the outage probability at high SNR. The proof is given in Section 5.4. $\square$

5.3.2 Discussion

- From Theorem 5.3.1, we can see that when the frame length, KL , is sufficiently large, we have

$$d(r) = (N + 1) \left(1 - \frac{KL + 1}{KL} r \right)^+ \approx (N + 1) (1 - r)^+ \quad (5.11)$$

which is the MISO DMT. From (5.11), multiplexing gain of $\frac{KL}{KL+1} \approx 1$ and full diversity gain of $N + 1$ are achieved. This proves the optimality (in terms of DMT) of the proposed CMA-SAFR protocol when KL is large.

- The derived DMT in Theorem 5.3.1 for CMA-SAFR is very general in the sense that it applies to arbitrary number of sources and relays when $N > 2$.
- In [30, 90], CMA with opportunistic relaying scheme (ORS) is proposed where the best relay is selected to assist the transmission of the multiple sources. Compared to this CMA with ORS in [30, 90], our proposed scheme achieves a better DMT. Opportunistic relaying scheme for multiple-access scenario achieves a DMT of $N(1 - 2r)$ where $1 \leq r \leq 0.5$, which is below the DMT achieved by the proposed CMA-SAFR scheme.
- The DMT for CMA with scheduling scheme in [76] is given as

$$d(r) = \begin{cases} (N + 1) \left(1 - r - \frac{K}{N+1} r \right) & \text{for } 0 \leq r \leq \frac{N}{N+K} \\ 1 - r & \text{for } \frac{N}{N+K} \leq r \leq 1 \end{cases} , \quad (5.12)$$

Figure 5.3: Diversity-multiplexing tradeoff for different CMA schemes. Here, $K = 3$ and $L = 30$.

which is derived based on the assumption that the number of qualified relays³ plus one, $Q + 1$, is an integer multiple of the number of source nodes K . The proposed CMA-SAFR scheme does not require this assumption.

- For the CMA with scheduling scheme in [76], we can also see that when $Q \ll K$ there will be some source nodes that are not assigned to any relays and have to rely only on the direct link. This is different from our proposed scheme where all sources will be served equally by the relays to achieve the

³A subset of relays that experience good channel conditions and are able to decode signal from any source node.

same level of diversity gain for all source nodes.

- Numerical comparison of the DMT for different CMA schemes is shown in Fig. 5.3 where we set $K = 3$ and $L = 30$. We can see that CMA-SAFR outperforms other schemes. Do note that in [76] each source transmits a codeword in every transmission slot, whereas for CMA-SAFR each source transmits a symbol (of a codeword) in every transmission slot⁴. Practically, the codeword length for the CMA scheme in [76] and the frame length for CMA-SAFR are assumed to be sufficiently large.

5.4 Proof of Theorem 5.3.1

For a multiple-access channel with K source nodes, the outage events can be decomposed into a collection of disjoint outage events [57, 76], i.e.,

$$\mathcal{O} \triangleq \bigcup_{\mathcal{A}} \mathcal{O}_{\mathcal{A}} \quad (5.13)$$

where

$$\mathcal{O}_{\mathcal{A}} \triangleq \left\{ I(\mathbf{x}_{\mathcal{A}}; \mathbf{y} | \mathbf{x}_{\mathcal{A}^c}, \mathbf{H} = H) < L \sum_{i \in \mathcal{A}} R_i \right\}, \quad (5.14)$$

$R_i = r_i \log \text{SNR}$ denotes the target data rate for source i , $\mathcal{A} \subseteq \{1, 2, \dots, K\}$ contains the indexes of the set of source nodes that are in outage, and $|\mathcal{A}|$ denotes cardinality of $\mathcal{A}$. Here, $|\mathcal{A}|R = \sum_{i \in \mathcal{A}} R_i$ since the symmetric tradeoff is of interest in this chapter.

The outage probability of CMA-SAFR can thus be expressed as

$$P(\mathcal{O}) = P\left(\bigcup_{\mathcal{A}} \mathcal{O}_{\mathcal{A}}\right) \leq \sum_{\mathcal{A}} P(\mathcal{O}_{\mathcal{A}}). \quad (5.15)$$

The mutual information for each outage event with the equivalent multiple-

⁴This is possible since CMA-SAFR is an amplify-and-forward relaying scheme which does not require any decoding at the relays.

access channel model in (5.6) can be written as

$$\begin{aligned}
I_{\mathcal{A}} &= \log \det \left(\mathbf{I}_{KL+1} + \rho \mathbf{H} \mathbf{\Gamma}_{\mathcal{A}} \mathbf{R}_{\mathbf{x}} \mathbf{H}^H \left(\mathbf{B}' \mathbf{R}_{\mathbf{n}} \mathbf{B}'^H \right)^{-1} \right) \\
&= \log \det \left(\mathbf{I}_{KL+1} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{\Gamma}_{\mathcal{A}} \mathbf{H}^H (\mathbf{B} \mathbf{B}^H + \mathbf{I})^{-1} \right) \\
&\doteq \log \det \left(\mathbf{I}_{KL+1} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{\Gamma}_{\mathcal{A}} \mathbf{H}^H \right)
\end{aligned} \tag{5.16}$$

where $\mathbf{R}_{\mathbf{x}} = \mathbb{E} \{ \mathbf{x} \mathbf{x}^H \} = \mathbf{I}_{KL}$, $\mathbf{R}_{\mathbf{n}} = \mathbb{E} \{ \mathbf{n} \mathbf{n}^H \} = \sigma^2 \mathbf{I}_{2KL+1}$, and $\mathbf{\Gamma}_{\mathcal{A}}$ is constructed by setting the columns of identity matrix $\mathbf{I}_{MN}$ corresponding to columns of $\mathbf{H}$ associated with the source nodes in $\mathcal{A}^c$ to be zero columns. The non-zero elements of $\mathbf{\Gamma}_{\mathcal{A}}$ in (5.16) are

$$\mathbf{\Gamma}_{\mathcal{A}}(K(i-1) + m, K(i-1) + m) = 1 \quad \text{for } i \in \{1, 2, \dots, L\}, m \in \mathcal{A}. \tag{5.17}$$

The last dot equality in (5.16) is because the DMT is white in the scale of interest [72], i.e., the colored noise encountered in AF cooperative networks can be treated as white noise for DMT computations.

Depending on $\mathcal{A}$, the outage events can be classified into two major events.

1. *Only one source node is in outage, $|\mathcal{A}| = 1$*

We denote this event as $\mathcal{O}_1$. $\mathcal{O}_1$ corresponds to the scenario where all source nodes' information can be decoded correctly except for the i -th source node's. Let $\mathbf{H}_1 = \mathbf{H} \mathbf{\Gamma}_{\mathcal{A}}$ be the equivalent channel matrix for outage event $\mathcal{O}_1$.

From (5.16), the outage probability for $\mathcal{O}_1$ can be expressed as

$$\begin{aligned}
P(\mathcal{O}_1) &= P\{I_{\mathcal{A}} < LR'\} = P\left\{ \log \det \left(\mathbf{I} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{\Gamma}_{\mathcal{A}} \mathbf{H}^H \right) < LR' \right\} \\
&= P\left\{ \log \det \left(\mathbf{I} + \frac{\rho}{\sigma^2} \mathbf{H}_1 \mathbf{H}_1^H \right) < LR' \right\} \doteq \text{SNR}^{-d_{\mathcal{O}_1}(r)}
\end{aligned} \tag{5.18}$$

where $R' = r' \log \text{SNR} = \frac{KL+1}{KL} R$ is the normalized target data rate.

By using Theorem 3.3.1, we can obtain DMT lower bounds for $d_{\mathcal{O}_1}(r)$. We first compute the DMT of diagonal channel matrices whose diagonal elements are taken from the main and the first sub-diagonal of $\mathbf{H}_1$ denoted by $d_{\mathbf{H}_1^{(0)}}(r)$ and $d_{\mathbf{H}_1^{(1)}}(r)$, respectively.

For $d_{\mathbf{H}_1^{(0)}}(r)$, we have

$$\begin{aligned}
\text{SNR}^{-d_{\mathbf{H}_1^{(0)}}(r)} &\doteq P \left\{ I_{\mathbf{H}_1^{(0)}(R)} < LR' \right\} = P \left\{ \log \det \left(\mathbf{I} + \frac{\rho}{\sigma^2} \mathbf{H}_1^{(0)} \mathbf{H}_1^{(0)\text{H}} \right) < LR' \right\} \\
&= P \left\{ \log \prod_{c=1}^L \frac{\rho}{\sigma^2} |h_{s_id}|^2 \leq Lr' \log \text{SNR} \right\} \\
&= P \left\{ \log \text{SNR}^L (|h_{s_id}|^2)^L \leq Lr' \log \text{SNR} \right\} \\
&= P \left\{ \text{SNR} |h_{s_id}|^2 \leq \text{SNR}^{r'} \right\}. \tag{5.19}
\end{aligned}$$

Thus, for large SNR the outage event in (5.19) is dominated by

$$O^+ = \left\{ v_{s_id} \in \mathbb{R}^+ \mid (1 - v_{s_id})^+ < r' \right\} \tag{5.20}$$

where v_{s_id} is the exponential order of h_{s_id} .

From [43, Definition 5], the DMT lower bound can be expressed as

$$d_{\mathbf{H}_1^{(0)}}(r) = \inf_{O^+} v_{s_id} \tag{5.21}$$

which can be solved easily and obtained as

$$d_{\mathbf{H}_1^{(0)}}(r) = (1 - r')^+. \tag{5.22}$$

For $d_{\mathbf{H}_1^{(1)}}(r)$, we have

$$\begin{aligned}
\text{SNR}^{-d_{\mathbf{H}_1^{(1)}}(r)} &\doteq P \left\{ I_{\mathbf{H}_1^{(1)}(R)} < LR' \right\} = P \left\{ \log \det \left(\mathbf{I} + \frac{\rho}{\sigma^2} \mathbf{H}_1^{(1)} \mathbf{H}_1^{(1)\text{H}} \right) < LR' \right\} \\
&= P \left\{ \log \prod_{c=1}^L \frac{\rho}{\sigma^2} |h_{s_ir_{c,i}}|^2 |g_{r_{c,i}}|^2 |h_{r_{c,i}d}|^2 \leq Lr' \log \text{SNR} \right\}. \tag{5.23}
\end{aligned}$$

Recall that in the transmission protocol, the full diversity requirement guarantees that in every N cycles all relay nodes would have served all the source nodes. Since there are only N relays, only N different coefficients are available for $h_{s_ir_{c,i}}$ and $h_{r_{c,i}d}$ for each source node i , i.e., $h_{s_ir_j}$ and h_{r_jd} where

$j \in \{1, 2, \dots, L\}$. Thus, from (5.23) we have

$$\begin{aligned} \text{SNR}^{-d_{\mathbf{H}_1^{(1)}}(r)} &\doteq P \left\{ \log \prod_{j=1}^N \left(\frac{\rho}{\sigma^2} |h_{s_i r_j}|^2 |g_{r_j}|^2 |h_{r_j d}|^2 \right)^{\frac{L}{N}} \leq Lr' \log \text{SNR} \right\} \\ &= P \left\{ \text{SNR}^L (|h_{s_i r_1}|^2 |g_{r_1}|^2 |h_{r_1 d}|^2)^{\frac{L}{N}} (|h_{s_i r_2}|^2 |g_{r_2}|^2 |h_{r_2 d}|^2)^{\frac{L}{N}} \right. \\ &\quad \left. \dots (|h_{s_i r_L}|^2 |g_{r_L}|^2 |h_{r_L d}|^2)^{\frac{L}{N}} \leq \text{SNR}^{Lr'} \right\}. \end{aligned} \quad (5.24)$$

Notice that the power scaling factor, g_{r_j} is constant and thus $|g_{r_j}|^2$ will not affect the exponential order analysis, i.e., $\lim_{\text{SNR} \rightarrow \infty} \frac{\log |g_{r_j}|^2}{\log \text{SNR}} = 0$.

Thus, for large SNR the outage events in (5.24) are dominated by

$$O^+ = \left\{ \begin{pmatrix} v_{s_i r_1}, \dots, v_{s_i r_N}, \\ v_{r_1 d}, \dots, v_{r_N d} \end{pmatrix} \in \mathbb{R}^{2N+} \mid \left(L - \frac{L}{N} \left(\sum_{j=1}^N v_{s_i r_j} + v_{r_j d} \right) \right)^+ < Lr' \right\} \quad (5.25)$$

where $v_{s_i r_j}$ and $v_{r_j d}$ are the exponential orders of $h_{s_i r_j}$ and $h_{r_j d}$, respectively.

From [43, Definition 5], the DMT lower bound can thus be expressed as

$$d_{\mathbf{H}_1^{(1)}}(r) = \inf_{O^+} \sum_{j=1}^N v_{s_i r_j} + v_{r_j d} \quad (5.26)$$

which can also be solved easily and obtained as

$$d_{\mathbf{H}_1^{(1)}}(r) = N(1 - r')^+. \quad (5.27)$$

Hence, from Proposition 3.3.1, (5.22), (5.27), we have

$$\begin{aligned} d_{\mathcal{O}_1}(r) &\geq d_{\mathbf{H}_1^{(1)}}(r) + d_{\mathbf{H}_1^{(1)}}(r) = (N+1)(1 - r')^+ \\ &= (N+1) \left(1 - \frac{KL+1}{KL} r \right)^+. \end{aligned} \quad (5.28)$$

2. More than one source nodes are in outage, $|\mathcal{A}| = \Psi$, $1 < \Psi \leq K$

We denote this event as $\mathcal{O}_\Psi$. Let $\mathbf{H}_\Psi = \mathbf{H}\mathbf{\Gamma}_\mathcal{A}$ be the equivalent channel matrix for outage event $\mathcal{O}_\Psi$. The outage probability for $\mathcal{O}_\Psi$ can be written

as

$$P(\mathcal{O}_\Psi) = P \left\{ \log \det \left(\mathbf{I} + \frac{\rho}{\sigma^2} \mathbf{H}_\Psi \mathbf{H}_\Psi^H \right) < L\Psi R' \right\} \doteq \text{SNR}^{-d_{\mathcal{O}_\Psi}(r)}. \quad (5.29)$$

Similar to the case for $\mathcal{O}_1$, we can also compute the DMT for $\mathbf{H}_\Psi^{(0)}$ and $\mathbf{H}_\Psi^{(1)}$.

For $d_{\mathbf{H}_\Psi^{(0)}}(r)$, we have

$$\begin{aligned} \text{SNR}^{-d_{\mathbf{H}_\Psi^{(0)}}(r)} &\doteq P \left\{ I_{\mathbf{H}_\Psi^{(0)}(R)} < L\Psi R' \right\} = P \left\{ \log \det \left(\mathbf{I} + \frac{\rho}{\sigma^2} \mathbf{H}_\Psi^{(0)} \mathbf{H}_\Psi^{(0)H} \right) < L\Psi R' \right\} \\ &= P \left\{ \log \prod_{c=1}^L \prod_{i=1}^\Psi \frac{\rho}{\sigma^2} |h_{s_i d}|^2 \leq L\Psi r' \log \text{SNR} \right\} \\ &= P \left\{ \log \text{SNR}^{L\Psi} \prod_{i=1}^\Psi (|h_{s_i d}|^2)^L \leq L\Psi r' \log \text{SNR} \right\} \\ &= P \left\{ \text{SNR}^{L\Psi} \prod_{i=1}^\Psi (|h_{s_i d}|^2)^L \leq \text{SNR}^{L\Psi r'} \right\}. \end{aligned} \quad (5.30)$$

Thus, for large SNR the outage events in (5.30) is dominated by

$$O^+ = \left\{ (v_{s_1 d}, \dots, v_{s_\Psi d}) \in \mathbb{R}^+ \mid (L\Psi - L \sum_{i=1}^\Psi v_{s_i d})^+ < L\Psi r' \right\}. \quad (5.31)$$

The DMT lower bound can thus be expressed as

$$d_{\mathbf{H}_\Psi^{(0)}}(r) = \inf_{O^+} \sum_{i=1}^\Psi v_{s_i d} = \Psi (1 - r')^+. \quad (5.32)$$

For $d_{\mathbf{H}_\Psi^{(1)}}(r)$, we have

$$\begin{aligned} \text{SNR}^{-d_{\mathbf{H}_\Psi^{(1)}}(r)} &\doteq P \left\{ I_{\mathbf{H}_\Psi^{(1)}(R)} < L\Psi R' \right\} = P \left\{ \log \det \left(\mathbf{I} + \frac{\rho}{\sigma^2} \mathbf{H}_\Psi^{(1)} \mathbf{H}_\Psi^{(1)H} \right) < L\Psi R' \right\} \\ &= P \left\{ \log \prod_{i=1}^\Psi \prod_{c=1}^L \frac{\rho}{\sigma^2} |h_{s_i r_{c,i}}|^2 |g_{r_{c,i}}|^2 |h_{r_{c,i} d}|^2 \leq L\Psi r' \log \text{SNR} \right\}. \end{aligned} \quad (5.33)$$

As mentioned earlier, since there are only N relays, only N different coeffi-

cients are available for $h_{s_i r_{c,i}}$ and $h_{r_{c,i} d}$ for each source node i , i.e., $h_{s_i r_j}$ and $h_{r_j d}$ where $j \in \{1, 2, \dots, N\}$. Thus, from (5.33) we have

$$\begin{aligned} \text{SNR}^{-d_{\mathbf{H}_\Psi^{(1)}}(r)} &\doteq P \left\{ \log \prod_{i=1}^{\Psi} \prod_{c=1}^L \rho |h_{s_i r_{c,i}}|^2 |g_{r_{c,i}}|^2 \prod_{j=1}^N (|h_{r_j d}|^2)^{\frac{L\Psi}{N}} \leq L\Psi r' \log \text{SNR} \right\} \\ &= P \left\{ \text{SNR}^{L\Psi} \prod_{i=1}^{\Psi} \prod_{j=1}^N (|h_{s_i r_j}|^2 |g_{r_j}|^2)^{\frac{L}{N}} \prod_{j=1}^N (|h_{r_j d}|^2)^{\frac{L\Psi}{N}} \leq \text{SNR}^{L\Psi r'} \right\}. \end{aligned} \quad (5.34)$$

For large SNR the outage events in (5.34) are dominated by

$$O^+ = \left\{ \begin{array}{l} (v_{s_1 r_1}, \dots, v_{s_1 r_N}, \dots, v_{s_\Psi r_1}, \dots, v_{s_\Psi r_N}, v_{r_1 d}, \dots, v_{r_N d}) \in \mathbb{R}^{(\Psi N + N)^+} \\ \left| \left(L\Psi - \frac{L}{N} \left(\sum_{i=1}^{\Psi} \sum_{j=1}^N v_{s_i r_j} + \Psi \sum_{j=1}^N v_{r_j d} \right) \right)^+ < L\Psi r' \right| \end{array} \right\}. \quad (5.35)$$

Notice that there are ΨN independent channel coefficients for $v_{s_i r_j}$, but only N independent channel coefficients for $v_{r_j d}$. This N independent channel coefficients will limit the overall DMT.

The DMT lower bound can be expressed as

$$d_{\mathbf{H}_\Psi^{(1)}}(r) = \inf_{O^+} \sum_{i=1}^{\Psi} \sum_{j=1}^N v_{s_i r_j} + \sum_{j=1}^N v_{r_j d} = N(1 - r')^+. \quad (5.36)$$

From Lemma 3.3.1, (5.32), and (5.36), we have

$$\begin{aligned} d_{\mathcal{O}_\Psi}(r) &\geq d_{\mathbf{H}_\Psi^{(0)}}(r) + d_{\mathbf{H}_\Psi^{(1)}}(r) = (\Psi + N)(1 - r')^+ \\ &= (\Psi + N) \left(1 - \frac{KL + 1}{KL} r \right)^+. \end{aligned} \quad (5.37)$$

Hence, for any $\Psi > 1$, $d_{\mathcal{O}_\Psi}(r) \geq d_{\mathcal{O}_1}(r)$. From (5.15), (5.28), and (5.37), we obtain

$$P(\mathcal{O}) \leq \sum_{\mathcal{A}} P(\mathcal{O}_{\mathcal{A}}) \doteq \text{SNR}^{-d_{\mathcal{O}_1}(r)} + \text{SNR}^{-d_{\mathcal{O}_2}(r)} + \dots + \text{SNR}^{-d_{\mathcal{O}_K}(r)} \doteq \text{SNR}^{-d_{\mathcal{O}_1}(r)}$$

$$\doteq \text{SNR}^{-(N+1)\left(1-\frac{KL+1}{KL}r\right)^+} \quad (5.38)$$

and the proof is complete.

5.5 Summary

In this chapter, we have proposed a sequential amplify-and-forward relaying protocol for wireless multiple-access systems. The DMT is used as a performance metric. The upper bound based on the DMT of MISO channel was first established and then the achievable DMT was derived. The proposed transmission protocol was shown to overcome the inherent spectral efficiency loss in half-duplex relaying and achieves the full diversity gain. The DMT achieved by this protocol approaches the optimal MISO DMT when the frame length is sufficiently large. Compared to existing schemes, the proposed protocol has a better DMT performance.

Conclusion

Cooperative communications has been a promising technique to improve spatial multiplexing and reliability in wireless networks. This dissertation is focused on investigating cooperative communication protocols that are able to recover the multiplexing loss and improve reliability in half-duplex relay networks.

We began by studying AF two-path relaying for a two-relay network. This protocol is shown to recover the spectral multiplexing loss in half-duplex relaying by alternating the transmission and reception modes of the two relays. We presented both the simulation and theoretical analysis in this dissertation.

For AF two-path relaying protocol, we proposed inter-relay self-interference cancellation at one of the relays to manage the interference in Chapter 2. A better performance is achieved compared to the scheme without cancellation, especially when the inter-relay channel gain is higher than the other channels. From our analysis of the PEP, we theoretically derived the diversity order achieved by AF two-path relaying protocol and proposed a simple precoding scheme, which does not require any channel state information, to achieve a full diversity order of three while maintaining the spectral efficiency gain of the two-path relaying protocol. Approaches from both coding gain and mutual information point of views for precoder design are also discussed to achieve full diversity and high coding gain. In particular, a class of precoding matrices that can be easily generated in practice and able to achieve full diversity order of three with high coding gain is proposed. Results for a low complexity receiver based on QRD-M algorithm are also presented.

To establish a benchmark performance for AF two-path relaying protocol, we characterized its diversity-multiplexing tradeoff in Chapter 3. To reach our goal, we first derived lower bounds for the DMT of an arbitrary block lower triangular channel matrix. With the generalized results, we are able to lower bound the DMT of an arbitrary block lower triangular channel matrix by using any of its sub-diagonal matrices. The DMT of AF two-path relaying scheme was then computed while taking into account inter-relay interference in the analysis. Our analysis indicates that a full diversity order of three can be achieved while maintaining its high spatial multiplexing gain. Asymptotically, the DMT achieves the MISO upper bound and outperforms existing schemes. Our analysis also demonstrates that the precoding technique proposed in Chapter 2 can be used as a simple and yet effective finite codeword length coding strategy to achieve the full DMT of this scheme. For the multiple-relay scenario, we provided a stronger result and derived the DMT for N -relay M -slot SAF scheme without the relay isolation assumption.

Cooperative communications based on DF two-path relaying was then studied in Chapter 4. The DF two-path relaying is a variant of AF two-path relaying where the relays decode and re-encode the information received from the source. With a distributed MISO approach, we first derive the DMT of DF two-path relaying, where perfect decoding at the relays is assumed, and show that the DMT approaches the 3×1 MISO DMT with large frame length. We then removed the assumption of perfect decoding at the relays and derived the achievable DMT based on the relative distances between the nodes. For sufficiently large frame length, if the average source-relay SNR is at least 2.5 times (measured in dB) of other links, we showed that the 3×1 MISO DMT can still be achieved. Further improvement of the DMT is also achieved when successive interference cancellation is used at the relays.

Finally, we studied a more general scenario of cooperative communication system where multiple sources are considered. We presented a sequential amplify-and-forward relaying protocol for this system. We analyzed the performance of this protocol in terms of DMT. The upper bound based on the DMT of MISO channel was first established and then the achievable DMT was derived. The proposed transmission protocol was shown to overcome the inherent spectral efficiency loss in half-duplex relaying and achieve the full diversity gain. The DMT achieved by

this protocol approaches the optimal MISO DMT when the frame length is sufficiently large. Compared to the existing schemes, the proposed protocol was shown to have a better DMT performance, lower complexity, and lower CSI requirements at the relays.

6.1 Practical Implications

There are several practical issues and implications for the cooperative communication protocols proposed in this dissertation.

- Time synchronization

For the cooperative communication protocols described in Chaptre 2-5, it can be observed that time synchronization is required for coordination among the nodes. At each time slot, the relay and destination nodes receive two simultaneous signals from the source and one of the relay nodes. Thus, in practice, the source and relay nodes must be first synchronized, in both time and frequency domains, which generally can be achieved by periodically inserting known training symbols to the transmitted frame. Another alternative is to combine the protocol with delay tolerant codes that are insensitive to synchronization errors.

- MAC layer coordination

In practical cooperative communication system, it is important for the relay and destination nodes to be aware of the cooperation that is going to occur. For example, the destination node needs to know whether the relay has cooperated. This is required to perform optimal detection method at the destination to retrieve the information transmitted from the source node. Another example is the scheduling of transmission among the relays. Each relay node must be aware of the transmission and reception modes required at each transmission slot. This normally can be done by using distributed or centralized control mechanisms [91].

- Inclusion in wireless standards

Relay technology is introduced to WiMAX through IEEE 802.16j where relay nodes are utilized to assist the direct communication between mobile devices and base stations [92]. Opportunistic and cooperative forwarding are proposed for the cooperative protocols. However, these two protocols are conventional half-duplex relaying protocols that suffer from multiplexing loss. Thus, the two-path relaying or sequential relaying protocols proposed in this dissertation would be good candidates to improve the performance of cooperative communications in current and future wireless standards.

6.2 Future Work

In this dissertation, we have studied two-path relaying (AF and DF) and sequential AF relaying protocols to enhance spectral efficiency (recover multiplexing loss) and improve reliability in cooperative communication system. The following are potential areas for future research:

- Optimal power allocation for AF two-path relaying

Power efficiency can be a critical concern for a cooperative communication system, especially when the nodes are powered by battery and frequent battery replacement is to be avoided. The analysis in Chapter 2 is based on the assumption that equal average power ρ is allocated at the relays and the source. Depending on the condition of the channels between the nodes, this equal power allocation may not be power efficient. Thus, it is interesting to study the optimal power allocation of the AF two-path relaying protocols. Both centralized and distributed power allocation can be considered.

- Extension to multi-hop scenario

One of the benefits of cooperative communication system is its ability for coverage extension. By deploying a larger number of relays in a multi-hop fashion, the transmission can be extended for longer distances. However, a conventional multi-hop relaying will incur more delay and thus reduce the spectral efficiency. The AF two-path relaying which has been shown to be

a promising protocol to provide higher spectral efficiency and reliability can be extended to the multi-hop scenario. It would be interesting to study the optimal scheduling mechanism of the relays and efficient methods to reduce the interference between the nodes.

- Explicit coding strategy to achieve DMT of sequential AF relaying

Chapter 5 has provided us an insight to the performance of sequential AF relaying based on the diversity-multiplexing tradeoff. However, the explicit coding strategy or modulation scheme required to achieve this DMT has not been studied. The available DMT achieving codes in [60, 61] may not be directly applicable to this protocol. Thus, the study of the explicit coding strategy will also be a good future research direction.

Example of Equivalent Channel Matrix

For $L = 6$, the channel matrix $\mathbf{H}$ for AF two-path relaying with inter-relay self-interference cancellation is expressed as

$$\mathbf{H} = \begin{pmatrix} h_1 & 0 & 0 & 0 & 0 & 0 \\ h_2 & h_1 & 0 & 0 & 0 & 0 \\ h_3 & h_4 & h_1 & 0 & 0 & 0 \\ 0 & h_5 & h_2 & h_1 & 0 & 0 \\ 0 & h_6 & h_3 & h_4 & h_1 & 0 \\ 0 & 0 & 0 & h_5 & h_2 & h_1 \\ 0 & 0 & 0 & h_6 & h_3 & h_4 \\ 0 & 0 & 0 & 0 & 0 & h_5 \end{pmatrix} \quad (\text{A.1})$$

where

$$\begin{aligned} h_1 &= h_{s,d}, \\ h_2 &= h_{s,r1}g_{r1}h_{r1,d}, \\ h_3 &= h_{s,r1}g_{r2}h_rg_{r1}h_{r2,d}, \\ h_4 &= h_{s,r2}g_{r2}h_{r2,d}, \\ h_5 &= h_{s,r2}g_{r2}h_rg_{r1}h_{r1,d}, \\ h_6 &= h_{s,r2}g_{r2}^2h_r^2g_{r1}h_{r2,d}. \end{aligned} \quad (\text{A.2})$$

Diversity Criterion for Rayleigh Space Time Code

Consider a wireless communication system where the transmitter is equipped with M_t and the receiver is equipped with M_r antennas. At each time slot t , signals c_t^i , $i = 1, 2, 3, \dots, M_r$, are transmitted simultaneously from the M_t transmit antennas.

At time t , the signal r_j^t is received at antenna j is given by

$$r_j^t = \sigma_{i=1}^{M_t} \alpha_{i,j} c_t^i + n_t^j \quad (\text{B.1})$$

where the coefficient $\alpha_{i,j}$ is the path gain from transmit antenna i to receive antenna j and n_t^j is the AWGN noise.

The diversity criterion for Rayleigh space-time code: In order to achieve the maximum diversity order of $M_t M_r$, the matrix

$$\Delta \mathbf{X}(\mathbf{c}, \mathbf{e}) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & \cdots & e_l^1 - c_l^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & \cdots & e_l^2 - c_l^2 \\ e_1^3 - c_1^3 & e_2^3 - c_2^3 & \ddots & \vdots & e_l^3 - c_l^3 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ e_1^{M_t} - c_1^{M_t} & e_2^{M_t} - c_2^{M_t} & \cdots & \cdots & e_l^{M_t} - c_l^{M_t} \end{bmatrix}. \quad (\text{B.2})$$

has to be full rank for any pair of distinct codewords $\mathbf{c}$ and $\mathbf{e}$. If $\Delta \mathbf{X}(\mathbf{c}, \mathbf{e})$ has minimum rank r over the set of pairs of distinct codewords, then a diversity of

rM_r is achieved.

Example of Generating the Arbitrary Complex Unitary Matrices in MATLAB codes

Function `generate_precoder` is the Matlab function to generate unitary matrix U

```
% L is the matrix size
% U is the generated unitary matrix
function U = generate_precoder(L)
% generate complex matrix with size L
H = 1/sqrt(2) * ( randn(L,L) + j* randn(L,L) ) ;
% generate the unitary matrix
[ U S V] = svd( H ) ;
```

Proof of Theorem 3.3.2

D.1 Upper Bound for the DMT

Lemma D.1.1. *[43, Definition 5] Let $v = -\lim_{\rho \rightarrow \infty} \frac{\log(|g|^2)}{\log(\rho)}$ be the exponential order of $1/|g|^2$, where $g \sim \mathcal{CN}(0, 1)$. Then the probability density function (pdf) of v can be shown to be*

$$\begin{aligned}
p_v &= \lim_{\rho \rightarrow \infty} \ln(\rho) \rho^{-v} \exp(-\rho^{-v}) \\
&\doteq \begin{cases} \rho^{-\infty} = 0 & \text{for } v < 0 \\ \rho^{-v} & \text{for } v \geq 0 \end{cases}.
\end{aligned} \tag{D.1}$$

Thus, the probability P_O that $(v_1, v_2, \dots, v_N)$ belongs to set O can be characterized by

$$P_O \doteq \rho^{-d_o}, \quad \text{for } d_o = \inf_{(v_1, v_2, \dots, v_N) \in O^+} \sum_{i=1}^N v_i \tag{D.2}$$

where O^+ denotes the set of nonnegative O . $\{v_i\}_{i=1}^N$ are independent and identically distributed random variables and provided that O^+ is non-empty.

Proof. This is true because $\{v_i\}_{i=1}^N$ are independent and the outage exponent will be determined by the largest SNR exponent. □

Lemma D.1.2. *Given a positive semidefinite matrix $\mathbf{H}^H\mathbf{H}$, we have*

$$\det\left(\frac{\rho}{\sigma^2}\mathbf{H}^H\mathbf{H}\right) \leq \det\left(\frac{\rho}{\sigma^2}\text{tr}(\mathbf{H}^H\mathbf{H})\mathbf{I}_L\right). \quad (\text{D.3})$$

Proof. The proof is straightforward and is thus omitted. $\square$

From (3.14), we have

$$P_{\text{out}}(R) \doteq P_r \left\{ \log \det \left(\mathbf{I}_L + \frac{\rho}{\sigma^2} \mathbf{H}^H \mathbf{H} \right) < (L+2)R \right\}$$

since $\det(\mathbf{I}_{L+2} + \frac{\rho}{\sigma^2} \mathbf{H} \mathbf{H}^H) = \det(\mathbf{I}_L + \frac{\rho}{\sigma^2} \mathbf{H}^H \mathbf{H})$.

As $\rho \rightarrow \infty$,

$$P_{\text{out}}(R) \doteq P_r \left\{ \log \det \left(\frac{\rho}{\sigma^2} \mathbf{H}^H \mathbf{H} \right) < (L+2)R \right\}.$$

Using Lemma D.1.2, we obtain

$$\begin{aligned} P_{\text{out}}(R) &\doteq \text{SNR}^{-d(r)} \\ &\doteq P_r \left\{ \log \det \left(\frac{\rho}{\sigma^2} \mathbf{H}^H \mathbf{H} \right) < (L+2)R \right\} \\ &\geq P_r \left\{ \log \det \left(\frac{\rho}{\sigma^2} \text{tr}(\mathbf{H}^H \mathbf{H}) \mathbf{I}_L \right) < (L+2)R \right\} \\ &\doteq \text{SNR}^{-d_{\text{UB}}(r)} \end{aligned} \quad (\text{D.4})$$

where $d_{\text{UB}}(r)$ is the DMT upper bound.

We denote h_i , $1 \leq i \leq 6$, as the elements of $\mathbf{H}$, as shown in Appendix A. Continuing from the RHS of (D.4), we have

$$\begin{aligned} &P_r \left\{ \log \det \left(\frac{\rho}{\sigma^2} \text{tr}(\mathbf{H}^H \mathbf{H}) \mathbf{I}_L \right) < (L+2)R \right\} \\ &\doteq P_r \left\{ \log \left(\frac{\rho L}{2\sigma^2} \left(|h_1|^2 + \sum_{i=1}^6 |h_i|^2 \right) \right)^L \right. \\ &\quad \left. < (L+2)R \right\} \\ &\doteq P_r \left\{ \log \left(\text{SNR} \sum_{i=1}^6 |h_i|^2 \right) < \frac{L+2}{L} r \log \text{SNR} \right\} \end{aligned} \quad (\text{D.5})$$

$$\doteq \text{SNR}^{-d_{\text{UB}}(r)}. \quad (\text{D.6})$$

$$O^+ = \left\{ \left(\begin{array}{c} v_{s,d}, v_{s,r_1}, v_{s,r_2}, \\ v_{r_1,d}, v_{r_2,d}, v_r \end{array} \right) \in \mathbb{R}^{6+} \mid \max \left(\begin{array}{c} (1 - v_{s,d})^+, \\ (1 - v_{r_1,d} - v_{s,r_1})^+, \\ (1 - v_{r_2,d} - v_r - v_{s,r_1})^+, \\ (1 - v_{r_2,d} - v_{s,r_2})^+, \\ (1 - v_{r_1,d} - v_r - v_{s,r_2})^+, \\ (1 - v_{r_2,d} - 2v_r - v_{s,r_2})^+ \end{array} \right) < \frac{L+2}{L}r \right\} \quad (\text{D.9})$$

Thus for large SNR, the outage events will be dominated by (D.9) where v_x is the exponential order of the corresponding channel coefficient. Here, the six inequalities in (D.9) can be obtained by considering the scenario when each of h_j , $j = 1, 2, \dots, 6$, in (D.5) dominates over the outage probability, i.e.,

$$P_r \left\{ \log \left(\text{SNR} \sum_{i=1}^6 |h_i|^2 \right) < \frac{L+2}{L}r \log \text{SNR} \right\} \quad (\text{D.7})$$

$$\doteq P_r \left\{ \log (\text{SNR} |h_j|^2) < \frac{L+2}{L}r \log \text{SNR} \right\} \quad (\text{D.8})$$

where $j = 1, 2, \dots, 6$.

From Lemma D.1.1, the DMT upper bound can be expressed as

$$d_{\text{UB}}(r) = \inf_{O^+} (v_{s,d} + v_{s,r_1} + v_{s,r_2} + v_{r_1,d} + v_{r_2,d} + v_r) \quad (\text{D.10})$$

which can be solved from the first inequality in (D.9) and summing the second to the fifth inequalities¹ in (D.9). By simplifying the expression, we obtain

$$d_{\text{UB}}(r) = 3 \left(1 - \frac{L+2}{L}r \right)^+. \quad (\text{D.11})$$

Hence, we have

$$d(r) \leq d_{\text{UB}}(r) = 3 \left(1 - \frac{L+2}{L}r \right)^+. \quad (\text{D.12})$$

¹ $\max(a, b) < c \implies a < c, b < c$

D.2 Lower Bound for the DMT

For AF two-path relaying scheme with inter-relay interference cancellation, we can add two all-zero columns to the right of the channel matrix $\mathbf{H}$ to form a block lower triangular matrix. From Theorem 3.3.1 and Proposition 3.3.1, we obtain $d(r) \geq d_{H^{(0)}}(r) + d_{H^{(1)}}(r) = d_{\text{LB}}(r)$ where $d_{\text{LB}}(r)$ is the DMT lower bound. The DMT of the matrices $\mathbf{H}^{(0)}$ and $\mathbf{H}^{(1)}$ can be easily derived as $d_{H^{(0)}}(r) = (1 - \frac{L+2}{L}r)^+$ and $d_{H^{(1)}}(r) = 2(1 - \frac{L+2}{L}r)^+$ respectively. Thus, we obtain

$$d_{\text{LB}}(r) = 3 \left(1 - \frac{L+2}{L}r \right)^+ = d_{\text{UB}}(r) \quad (\text{D.13})$$

and this completes the proof for Theorem 3.3.2.

Proof of Proposition 3.3.2

Let $\bar{\mathbf{H}} \in \mathcal{C}^{(L+2) \times L}$ be the equivalent MIMO channel matrix of AF two-path relaying *without* interference cancellation at one of the relays. First, notice that the elements in the columns of $\bar{\mathbf{H}}$ are just the repeated elements in the first two columns. The first two columns of $\bar{\mathbf{H}}$ can be shown as

$$\bar{\mathbf{H}} = \begin{pmatrix} h_1 = h_{s,d} & 0 & \cdots \\ h_2 = h_{s,r1}h_{r1,d} & h_1 & \cdots \\ h_3 = h_{s,r1}h_rh_{r2,d} & h_4 = h_{s,r2}g_{r2}h_{r2,d} & \cdots \\ h_7 = h_{s,r1}h_r^2h_{r1,d} & h_5 = h_{s,r2}h_rh_{r1,d} & \cdots \\ h_8 = h_{s,r1}h_r^3h_{r2,d} & h_6 = h_{s,r2}h_r^2h_{r2,d} & \cdots \\ h_9 = h_{s,r1}h_r^4h_{r1,d} & h_{11} = h_{s,r2}h_r^3h_{r1,d} & \cdots \\ h_{10} = h_{s,r1}h_r^5h_{r2,d} & h_{12} = h_{s,r2}h_r^4h_{r2,d} & \cdots \\ \vdots & \vdots & \cdots \end{pmatrix}$$

where g_{r1} and g_{r2} terms are ignored in the above matrix because they are constants and will not affect the exponential order analysis i.e. $\lim_{\text{SNR} \rightarrow \infty} \frac{\log g_{ri}}{\log \text{SNR}} = 0$, $i \in \{1, 2\}$.

The DMT can be derived easily by following closely the DMT derivation in Appendix D. The upper bound of the outage probability can be derived as $\text{SNR}^{-3(1-\frac{L+2}{L}r)^+}$ by observing that the exponent of $\text{SNR} \sum_{i=1}^6 |h_i|^2$ will dominate

in $\text{SNRtr} \{ \bar{\mathbf{H}}^H \bar{\mathbf{H}} \}$, i.e.

$$\frac{\log \{ \text{SNRtr} \{ \bar{\mathbf{H}}^H \bar{\mathbf{H}} \} \}}{\log \text{SNR}} = \frac{\log \{ \text{SNR} \sum_{i=1}^6 |h_i|^2 \}}{\log \text{SNR}}$$

which is a similar step involved in (D.6), Appendix D.

Notice that the main and the first sub-diagonal elements of $\bar{\mathbf{H}}$ is exactly the same as $\mathbf{H}$, which corresponds to the case with interference cancellation. Thus, for the lower bound, we have

$$\begin{aligned} d_{\bar{H}}(r) &\geq d_{\bar{H}^{(0)}}(r) + d_{\bar{H}^{(1)}}(r) = d_{H^{(0)}}(r) + d_{H^{(1)}}(r) \\ &= 3 \left(1 - \frac{L+2}{L} r \right)^+. \end{aligned} \tag{E.1}$$

Hence, the DMT is given by $3 \left(1 - \frac{L+2}{L} r \right)^+$.

Proof of Theorem 4.3.3

Theorem 4.3.3 can be proven by observing the fact that successive interference cancellation at the relays will only change the expressions for $P_r\{Ri\ out\}$ and $P_r\{Ri\ non\}$ in (4.27).

By the law of total probability, we have

$$\begin{aligned} P_r\{Ri\ out\} &= P_r\{Ri\ out|Direct\} P_r\{Direct\} \\ &\quad + P_r\{Ri\ out|SIC\} P_r\{SIC\}. \end{aligned} \tag{F.1}$$

Here, $\{Direct\}$ is the event where the relays decode the source signal directly and treat the interference as noise, and $\{SIC\}$ is the event where the relays decode the interference signal first and then perform SIC.

Event $\{Direct\}$ happens when the source to relay link is stronger than the inter-relay link. Thus, we have

$$\begin{aligned} P_r\{Direct\} &= P_r\{\rho^\eta |h_{s,ri}|^2 > \rho |h_r|^2\} \\ &= P_r\left\{\frac{|h_r|^2}{|h_{s,ri}|^2} > \text{SNR}^{-(1-\eta)}\right\} \\ &\doteq \text{SNR}^{-(1-\eta)^+}. \end{aligned} \tag{F.2}$$

Since $\{SIC\}$ is the complementary event of $\{Direct\}$, we have

$$P_r\{SIC\} = 1 - P_r\{Direct\} \doteq 1 - \text{SNR}^{-(1-\eta)^+}. \tag{F.3}$$

For event $\{Direct\}$, the outage probability at the relay can be expressed as

$$\begin{aligned} P_r\{Ri\ out|Direct\} &= P_r\left\{\log\left(1 + \frac{\rho^\eta|h_{s,ri}|^2}{\rho|h_r|^2 + \sigma^2}\right) < R\right\} \\ &\doteq \text{SNR}^{-(\eta-1-r)^+}. \end{aligned} \quad (\text{F.4})$$

For event $\{SIC\}$, the outage probability at the relay is given as

$$P_r\{Ri\ out|SIC\} = P_r\{Int\ fail\} + P_r\{Int\ ok\ but\ S\ fail\} \quad (\text{F.5})$$

where $\{Int\ fail\}$ is the event where the relay fails to decode the interference signal, and $\{Int\ ok\ but\ S\ fail\}$ is the event where the relay succeeds in decoding the interference signal, but fails to decode the desired source signal.

From (F.5), we have

$$\begin{aligned} P_r\{Ri\ out|SIC\} &= P_r\{Int\ fail\} + P_r\{Int\ ok\ but\ S\ fail\} \\ &= P_r\left\{\log\left(1 + \frac{\rho|h_r|^2}{\rho^\eta|h_{s,ri}|^2 + \sigma^2}\right) < R\right\} \\ &\quad + P_r\left\{\log\left(1 + \frac{\rho|h_r|^2}{\rho^\eta|h_{s,ri}|^2 + \sigma^2}\right) > R\right\} \\ &\quad \times P_r\left\{\log\left(1 + \frac{\rho^\eta|h_{s,ri}|^2}{\sigma^2}\right) < R\right\} \\ &\doteq P_r\left\{\frac{|h_r|^2}{|h_{s,ri}|^2} < \text{SNR}^{-(1-\eta-r)}\right\} \\ &\quad + \left(1 - P_r\left\{\frac{|h_r|^2}{|h_{s,ri}|^2} < \text{SNR}^{-(1-\eta-r)}\right\}\right) \\ &\quad \times P_r\left\{|h_{s,ri}|^2 < \text{SNR}^{-(\eta-r)}\right\} \\ &\doteq \text{SNR}^{-(1-\eta-r)^+} + \left(1 - \text{SNR}^{-(1-\eta-r)^+}\right) \text{SNR}^{-(\eta-r)^+} \\ &\doteq \text{SNR}^{-(1-\eta-r)^+} + \text{SNR}^{-(\eta-r)^+}. \end{aligned} \quad (\text{F.6})$$

Thus, from (F.1)-(F.6), we obtain

$$\begin{aligned} P_r\{Ri\ out\} &= P_r\{Ri\ out|Direct\} P_r\{Direct\} + P_r\{Ri\ out|SIC\} P_r\{SIC\} \\ &\doteq \text{SNR}^{-(\eta-1-r)^+} \text{SNR}^{-(1-\eta)^+} + \left(\text{SNR}^{-(1-\eta-r)^+} + \text{SNR}^{-(\eta-r)^+}\right) \end{aligned}$$

$$\begin{aligned}
& \times \left(1 - \text{SNR}^{-(1-\eta)^+}\right) \\
& \doteq \text{SNR}^{-(\eta-1-r)^+} \text{SNR}^{-(1-\eta)^+} + \text{SNR}^{-(1-\eta-r)^+} + \text{SNR}^{-(\eta-r)^+} \\
& \doteq \begin{cases} \text{SNR}^{-(\eta-1-r)^+} & \text{for } \eta > 1, \\ \text{SNR}^{-\min\{(1-\eta-r)^+, (\eta-r)^+\}} & \text{for } \eta \leq 1. \end{cases} \tag{F.7}
\end{aligned}$$

For $\eta > 1$: $P_r\{Ri \text{ out}\}$ has the same expression as the case without successive interference cancellation. Thus, for $\eta > 1$, $d(r)$ will be the same as in Theorem 4.3.2.

For $\eta \leq 1$: From (4.27), (4.29)-(4.31), (F.7), we have

$$\begin{aligned}
P_{out} & \doteq P_{out|E_1} P_r\{R1 \text{ out}\} P_r\{R2 \text{ non}\} \\
& + P_{out|E_2} P_r\{R1 \text{ out}\} P_r\{R2 \text{ out}\} \\
& + P_{out|E_3} P_r\{R1 \text{ non}\} P_r\{R2 \text{ non}\} \\
& \doteq \text{SNR}^{-\left[\left(1-\frac{L+1}{L}r\right)^+ + \left(1-\frac{L+1}{L}2r\right)^+\right]} \\
& \quad \times \text{SNR}^{-\min\{(1-\eta-r)^+, (\eta-r)^+\}} \\
& \quad \times \left(1 - \text{SNR}^{-\min\{(1-\eta-r)^+, (\eta-r)^+\}}\right) \\
& + \text{SNR}^{-\left(1-\frac{L+1}{L}r\right)^+} \text{SNR}^{-2\min\{(1-\eta-r)^+, (\eta-r)^+\}} \\
& + \text{SNR}^{-3\left(1-\frac{L+1}{L}r\right)^+} \left(1 - \text{SNR}^{-\min\{(1-\eta-r)^+, (\eta-r)^+\}}\right) \\
& \quad \times \left(1 - \text{SNR}^{-\min\{(1-\eta-r)^+, (\eta-r)^+\}}\right) \\
& \doteq \text{SNR}^{-\left(1-\frac{L+1}{L}r\right)^+} \text{SNR}^{-2\min\{(1-\eta-r)^+, (\eta-r)^+\}} \tag{F.8}
\end{aligned}$$

and this completes the proof.

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