Adaptive transmission schemes of cognitive radio for dynamic spectrum access using wavelets and multiwavelets

Manju Mathew

2012

Manju, M. (2012). Adaptive transmission schemes of cognitive radio for dynamic spectrum access using wavelets and multiwavelets. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/50779>

<https://doi.org/10.32657/10356/50779>

Adaptive Transmission Schemes of Cognitive Radio for Dynamic Spectrum Access using Wavelets and Multiwavelets

Manju Mathew

School of Computer Engineering
Nanyang Technological University

2012

Nanyang Technological University

Adaptive Transmission Schemes of Cognitive Radio for Dynamic Spectrum Access using Wavelets and Multiwavelets

A thesis submitted to the Nanyang Technological University in fulfillment
of the requirement for the degree of Doctor of Philosophy

by

Manju Mathew

School of Computer Engineering

2012

Abstract

Electromagnetic spectrum is a natural resource and at present its use is regulated by fixed spectrum assignment policy. That is, spectrum is assigned to license holders or services on a long term basis for large geographical regions. This fixed channel allocation served well in the past but the rapid and tremendous growth in wireless communication changed the entire scenario. Introduction of every new communication application, either military or civilian makes the spectrum more and more congested. Meanwhile, the spectrum allocated to specific users does not ensure it is being used effectively at all times. Measurement studies reveal that 15 to 85 percent of the licensed radio spectrum is relatively unutilized or under utilized in most of the geographical locations for a considerable amount of time. The limited available spectrum and its inefficient usage encouraged a new communication paradigm to exploit the radio frequency spectrum opportunistically. This new concept is termed Dynamic Spectrum Access (DSA). The technology that makes DSA possible is Cognitive Radio (CR) which was first suggested by Joseph Mitola. In fact, CR is an all-encompassing term for a wide variety of technologies that enable radios to achieve various levels of self configuration and different functionalities ranging from ubiquitous wireless access to DSA for a device centric interference management. DSA networks define the already existing licensed users as primary users or legacy users and CR as secondary user. CR, sits on top of Software Defined Radio (SDR) is an intelligent wireless system that adapts to its environments to utilize the spectrum by its secondary usage in a 'do no harm' fashion. In simple words, it senses the radio environment, detects vacant spectrum bands, decides the best spectrum and chooses the transmission scheme which insures minimum interference to adjacent legacy users. As DSA networks targets mainly the wideband spectrum, the transmission technology should mitigate multi-path frequency selective fading channel impairments and should have multi-user support. Thus DSA demands for CR transmission scheme of high flexibility, adaptivity, multi-user support and moderate error performance in fading channels.

The work presented in this dissertation focuses on adaptive transmission technologies for CR using wavelets and multiwavelets. The proposed methods insure minimum interference to adjacent users together with multi-user support to implement efficient DSA networks. A detailed review on the already proposed transmission schemes for CR available in literature is described in the first part. The proposed single carrier and multi carrier modulation techniques for CR with multiple access capability are outlined there onwards. The system models and simulation results are discussed in detail.

The first proposal is the system model of a multicarrier multiple access transmission scheme using standard wavelet packet transforms. The orthogonal subbands generated by the wavelet packet transform are used as data subcarriers of CR nodes. A modified carrier assignment scheme is proposed which acts as the spectrum decision and allocation block of CR. The system incorporates the flexibility and orthogonality of wavelet packet modulation and the multi-user support of Orthogonal Frequency Division Multiple Access (OFDMA). The error performance of the scheme is evaluated using existing wavelets and is compared with OFDMA.

Scalar wavelets for wireless communication have been explored well in the past and hence we focus more on multiwavelets in the second proposal. B spline based multiwavelets of multiplicity three are investigated. A multiuser CR network with spline multiwavelets of different orders are considered. Single carrier modulation is employed and the performance under different channel conditions are analyzed. The orthogonality of multiwavelets together with short support in time domain gives an efficient single carrier multi-user CR system for free spectrum access.

Finally, to further improve spectrum efficiency and performance, multifilter bank based multiuser network is proposed. Multiwavelet packet modulation is rarely considered in wireless communication but this multi dimensional modulation has adaptability and spectrum efficiency. The implementation can be done with vector extension of Mallat's algorithm which insures acceptable computational complexity. A preliminary spectrum sensing using multifilter bank is proposed which is possible at no additional computational cost. The spectrum sensing results are compared with that of scalar wavelets and error performances with those of scalar wavelets and OFDMA.

List of Abbreviations

A/D	Analog To Digital
AR	Auto Regressive
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BOI	Band of Interest
BS	Base Station
CAS	Carrier Allocation Scheme
CI	Carrier Interferometry
CP	Cyclic Prefix
CPE	Consumer Premise Equipment
CR	Cognitive Radio
CSK	Cyclic Shift Keying
CSI	Channel State Information
DFT	Discrete Fourier Transform
DMWPT	Discrete MultiWavelet Packet Transform
DSA	Dynamic Spectrum Access
DWMT	Discrete Wavelet MultiTone
DWPT	Discrete Wavelet Packet Transform
EIRP	Effective Isotropic Radiated Power
EWDCS	Enhanced Wavelet Domain Communication System
EWS	Evolutionary Wavelet Spectrum
FCC	Federal Communication Commission
FDM	Frequency Division Multiplexing
FFT	Fast Fourier Transform
FHSS	Frequency Hopping Spread Spectrum

FIR	Finite Impulse Response
FT	Fourier Transform
GSM	Global System for Mobile
ICI	Inter Carrier Interference
IDWPT	Inverse Discrete Wavelet Packet Transform
IDMWPT	Inverse Discrete MultiWavelet Packet Transform
IFFT	Inverse Fast Fourier Transform
IIR	Infinite Impulse Response
ISI	Inter Symbol Interference
MAC	Medium Access Control
MAI	Multiple Access Interference
MC CDMA	MultiCarrier Code Division Multiple Access
MRA	Multi Resolution Analysis
MUI	Multi User Interference
NC	Non Contiguous
NPRM	Notice of Proposed Rule Making
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
PAPR	Peak to Average Power Ratio
PSD	Power Spectral Density
QAM	Quadrature Amplitude Modulation
QMF	Quadrature Mirror Filter
QPSK	Quadrature Phase Shift Keying
QoS	Quality of Service
RF	Radio Frequency
RKRL	Radio Knowledge Representation Language
SDR	Software Defined Radio
SNR	Signal to Noise Ratio
SSF	Spectrum Sensing Function
TDCS	Transform Domain Communication System
TDM	Time Division Multiplexing
UHF	Ultra High Frequency
USRP	Universal Software Radio Peripheral
UWB	Ultra Wide Band

VHF	Very High Frequency
WDCS	Wavelet Domain Communication System
WG	Working Group
WPDM	Wavelet Packet Division Multiplexing
WPM	Wavelet Packet Modulation
WPMA	Wavelet Packet Multiple Access
WPMCM	Wavelet Packet MultiCarrier Modulation
WP-MC-MA	Wavelet Packet Multicarrier Multiple Access
WPT	Wavelet Packet Transform
WRAN	Wireless Regional Area Network

Contents

Abstract	ii
List of Abbreviations	iv
Table of Contents	vii
List of Figures	xii
List of Tables	xv
Acknowledgments	xvi
Dedication	xvii
1 Introduction	1
1.1 Motivation	3
1.2 Objectives	6
1.3 Outline of the Thesis	6
2 Literature Survey	8
2.1 Overview of CR	8
2.1.1 Evolution of CR	9
2.1.2 Definitions of CR	10
2.1.3 Cognition Cycle	11
2.1.4 Spectrum Holes	12
2.2 Physical Layer Challenges	13
2.2.1 Wideband Front end	13

2.2.2	Spectrum Sensing	14
2.2.3	Adaptive Transmission Scheme	14
2.3	Transmission scheme proposals for CR	14
2.3.1	OFDM Based Schemes	15
2.3.2	MC-CDMA based Schemes	20
2.3.3	TDCS Based Schemes	23
2.3.4	Wavelet Based Schemes	27
2.3.5	Conclusion	32
2.4	Spectrum Sensing Schemes	33
2.4.1	Energy Based Detection	33
2.4.2	Waveform Based Sensing	33
2.4.3	Cyclostationary Feature Detection	34
2.4.4	Matched Filtering	34
2.4.5	Filter Bank based Sensing	35
2.4.6	Cooperative Sensing	35
2.4.7	Beacon Based Sensing	36
2.4.8	Comparison among Various Schemes	36
2.5	Standardization of CR	37
2.6	Chapter Summary	38
3	Multicarrier Multiple Access Scheme for CR using Scalar Wavelet Packets	40
3.1	Introduction	41
3.2	WP-MC-MA System for CR	44
3.2.1	Uplink System Model	46
3.2.2	Modified CAS	47
3.2.3	Uplink Signal Model	50

3.2.4	The Downlink System	52
3.3	Performance Evaluations	54
3.3.1	Error performance in single path fading channels	54
3.3.2	Error performance in multi-path fading channels	56
3.3.3	Effect of phase offset	59
3.3.4	Effect of frequency and timing offset	61
3.3.5	Spectrum Efficiency and Sidelobe Suppression	63
3.3.6	Complexity Analysis	64
3.4	Chapter Summary	65
4	Spline Multiwavelets based multi-user CR network	66
4.1	Introduction	67
4.2	Single Carrier multi-user CR network with multiwavelets	68
4.2.1	B Spline Equations	71
4.2.2	Auto Correlation, Cross Correlation and Multi-user Interference	74
4.2.3	BER calculation in AWGN Channel	75
4.2.4	Cross correlation value under different cases	76
4.3	Simulation Results	80
4.3.1	Error performance under single path fading channels	84
4.3.2	Error performance under multi-path fading channels	88
4.4	Complexity Analysis	90
4.5	Chapter Summary	91
5	MultiWavelet Filter Bank based Cognitive Radio Transceiver	92
5.1	Introduction	93
5.2	Advantages of MWPM for CR	94
5.3	Spectrum Sensing with Multifilters	95

5.3.1	Related Work and Motivation	95
5.3.2	Multifilter based Sensing Methodology and Algorithm	96
5.4	MWP-MC-MA System for CR	99
5.4.1	Uplink System Model	99
5.4.2	Modified CAS of MWP-MC-MA System	100
5.4.3	Data Preprocessing and IDMWPT	101
5.4.4	Uplink Signal Model	102
5.4.5	The Downlink System	103
5.4.6	Downlink Signal Model	105
5.4.7	Choice of Multiwavelets	105
5.5	Performance Evaluation	106
5.5.1	Spectrum Measurements	106
5.5.2	Error Performance of the proposed MWP-MC-MA System	108
5.5.2.1	Error performance in single path fading channels	110
5.5.2.2	Error performance in multi-path fading channels	110
5.5.2.3	Effect of Phase Offset	112
5.5.2.4	Effect of Frequency Offset	113
5.5.3	Bandwidth Efficiency and Sidelobe Suppression	114
5.5.4	Complexity Analysis	115
5.6	Chapter Summary	116
6	Conclusions and Future Work	117
6.1	Introduction	117
6.2	Conclusions	118
6.2.1	Spectrum Efficiency	119
6.2.2	Flexibility	119
6.2.3	Error performances	120

6.2.4 Computational Complexity	120
6.3 Directions of Future Work	120
Bibliography	122
A Publication List	133
B Multiwavelet Filter Coefficients	135

List of Figures

1.1	Spectrum usage in Germany	3
1.2	Spectrum usage in U.S.	4
2.1	Cognition Cycle	11
2.2	OFDM Transceiver	15
2.3	OFDM based spectrum pooling	16
2.4	NC-OFDM Transmitter	18
2.5	NC-OFDM Receiver	18
2.6	Adaptive OFDMA Transceiver	20
2.7	MC-CDMA Transceiver	21
2.8	FFT based TDCS Transmitter	24
2.9	FFT based TDCS Receiver	24
2.10	OFDM based TDCS Transmitter	26
2.11	OFDM based TDCS Receiver	26
2.12	EWDCS transmitter	29
2.13	WPM Transceiver	31
3.1	Wavelet Packet Tree	43
3.2	Wavelet Packet Decomposition of signal 'S'	44
3.3	Basic multi user CR network	45

3.4	Uplink Schematic of WP-MC-MA system	46
3.5	Different Carrier Allocation Schemes, (a) Fixed CAS, (b) Interleaved CAS, (c) Generalized CAS	48
3.6	Flow chart of Modified CAS	49
3.7	Downlink transmitter of WP-MC-MA	53
3.8	Downlink receiver of WP-MC-MA	53
3.9	BER in flat fading channel	55
3.10	BER in Rayleigh channel with Doppler spread	55
3.11	Frequency Response of the channel with fixed fading coefficients	57
3.12	BER in multi-path fading channel with fixed fading coefficients	58
3.13	BER in multi-path fading channel with Rayleigh fading coefficients	58
3.14	BER with phase noise variance=0.001	60
3.15	BER with phase noise expressed in dBm	60
3.16	BER with phase noise variance (Uplink)	61
3.17	BER with relative frequency offset	62
3.18	BER relative frequency offset (Uplink)	62
3.19	BER with time offset	63
3.20	PSD of WPM and OFDM	64
4.1	Single carrier CR uplink transmitter	68
4.2	Single carrier CR uplink receiver	69
4.3	Single carrier CR downlink transmitter	70
4.4	Single carrier CR downlink receiver	70
4.5	Linear spline multiwavelets	81
4.6	Auto Correlation of linear, quadratic and cubic spline Scaling Functions . .	82
4.7	Intra Cross Correlation of linear, quadratic and cubic spline Scaling Functions	83
4.8	Inter Cross Correlation of linear, quadratic and cubic spline Scaling Functions	83

4.9	Variation of BER in flat fading Channel showing the effect of intra cross correlation	84
4.10	BER curves showing the effect of intra cross correlation (Frequency selective fading and AWGN channel)	85
4.11	BER curves showing the effect of inter cross correlation (Frequency selective fading and AWGN Channel)	85
4.12	BER comparison of different four user networks (Frequency selective fading and AWGN Channel)	86
4.13	BER performance for QPSK modulation (Frequency selective fading and AWGN Channel)	87
4.14	BER comparison of different four user networks (Multi-path fading Channel where FFC-Fixed Fading Coefficients and RFC-Rayleigh Fading Coefficients)	88
4.15	BER comparison of different four user networks (Multi-path Channel with Exponential power profile).	89
5.1	Signal decomposition using Multifilters	97
5.2	MWP-MC-MA Uplink Transmitter	100
5.3	Inverse Multiwavelet Packet Transform with two levels of iteration	102
5.4	MWP-MC-MA Downlink Transmitter	104
5.5	MWP-MC-MA Downlink Receiver	104
5.6	Spectrum sensing result of a partial band BPSK Signal	107
5.7	Spectrum sensing result of noise signal	107
5.8	Error performance in Flat Fading Channel	109
5.9	Error performance in Single path Rayleigh Fading Channel	109
5.10	Error performance in multi-path Channel with fixed fading coefficients	110
5.11	Error performance under multi-path Channel with exponentially decaying power profile	111
5.12	Effect of Phase Offset	112
5.13	Effect of Frequency Offset	113
5.14	Spectrum of 'GHM' and Daubechies Wavelets	114

List of Tables

3.1	Wavelet Specifications	47
3.2	Complexity Analysis	65
4.1	Complexity Analysis	91
5.1	Complexity Analysis	115

Acknowledgments

At this place, I would like to express my deepest gratitude to my supervisor Dr. Benjamin Premkumar for giving me this opportunity to do research and, for the valuable guidance. He guided me, instructed my research, and gave me strict academic training in presentation and publishing throughout the course of this work. His support, and considerable time spent in revising make this dissertation possible.

My sincere thanks to my co-supervisor Dr. Lau Chiew Tong, for his timely help and consistent support.

Very special thanks to Dr. A. P. Vinod and Dr. A. S. Madhukumar for all the discussions, advices and guidance. Their consistent support and concern has helped a lot in the completion of this dissertation.

Special thanks to my friends and research colleagues for their comradeship and their generous help throughout my study period, namely Kavitha P. Thomas, Mahesh R., Smitha K.G. and Sudhan Majhi.

I am especially grateful to my young brother Ashish not only for his care and concern, but for his hard work in the preparation of my research papers and this dissertation. He has shown immense patience and has spent his valuable time to clarify my doubts in Matlab and in various writing and drawing softwares.

My deep sense of gratitude goes to my parents, my husband and all other family members for their love, understanding and concern. Thanks to all my dear and near ones for their constant prayer support.

And above all, I am here only because of the merciful blessings from my Lord Jesus. It was His amazing love and great miracles that protected me in the time of struggles throughout this work.

Dedicated to my beloved parents.

Chapter 1

Introduction

Cognitive radio (CR) is the wireless technology that is envisioned to potentially optimize the usage of the most important resource of communication system - radio frequency (RF) spectrum. With the ability to learn from and adapt to both surrounding environment and user needs [1], CR based systems promote reliable communication and cater to almost all market interests such as military, public safety, government and commercial. The greater benefits of CR that makes it different from other radio devices has generated a lot of discussion and increasing attention from the wireless communications community. From the discussions and research works of the last decade, the main tasks of CR for dynamic spectrum access (DSA) can be concluded as

- Analysis of radio environment - this refers to the measurements and observation that CR needs to collect from the communication environment for its operation.
- Identification of vacant channels - CR occupies the free spectrum at different times and in different locations. So it is required to detect the empty bands within a short time duration.
- Adaptive transmission - According to the chosen band and application, CR needs to change its transmission power, modulation scheme and carrier waveform such that it insures minimum disturbance to primary users while giving maximum throughput.
- Dynamic spectrum management and spectrum mobility - Cognitive radio cannot adhere to a single spectrum and during its operation in a free channel it needs to share

the band with other CR users on mutual consent. Thus the spectrum is to be managed in real time. Besides, to have seamless connection, it needs to move from one band to another band when the licensed users occupy the channel which is termed spectrum mobility.

It is evident that a system which can implement the above tasks can provide numerous applications to wireless world. Primarily, it enhances the effective utilization of radio spectrum and provides viable communication. But a fully intelligent radio can do a lot more tasks such as increasing data rate and quality of service, providing seamless connection, safety and security of the cellular system and multimode operation using its sensors and adaptation techniques. A few CR applications are listed in [2]. According to [2], at its physical layer, opportunistic spectrum utilization is the main target of CR. A cognitive medium access control (MAC) and network layer can support improved link reliability and collaborative techniques. Use of smart antennas can provide intelligent beam forming. Finally, the unlicensed access to idle bands using CR technology may lead to the deployment of cheaper radios. But to achieve these goals, CR demands many changes from conventional radio systems. The physical layer architecture, MAC layer and network layer protocols need to support a highly flexible and adaptive wireless system. The physical layer challenges unique to CR systems can be classified into three categories [3].

- Wide band front end - the front end of CR needs to process a wide band of frequencies in the digital domain and hence front end circuit should support a high sampling rate at lower complexity.
- Spectrum Sensing - CR has to scan the whole RF spectrum and to locate the empty frequency bands. This is called spectrum sensing and it can be considered as a cross layer task between physical and MAC layer.
- Adaptive transmission scheme - Once the spectrum to operate is decided, CR has to design its carrier waveform to fit that spectrum mask.

The thesis mainly address the final issue by proposing flexible and spectrum efficient schemes based on scalar and multiwavelets. A preliminary spectrum measurement method using multiwavelet filter bank is also proposed which can be implemented in a multifilter bank based CR transceiver at no additional cost.

1.1 Motivation

Figure 1.1: Spectrum usage in Germany

Among the various applications of CR, opportunistic or dynamic spectrum access is the most attractive and important feature as it exploits the available spectrum in an efficient way and helps the deployment of new wireless applications. But to utilize the spectrum opportunistically, CR should have an adaptive transmission scheme. It is a challenging task and an ongoing research area as CR has to cope up with a changing environment by adaptively changing modulation, coding and frequency band to maintain quality of service and seamless connection. It is reported that majority of the frequency bands in the VHF/UHF range (300MHz-900MHz) is largely unoccupied most of the time [4]. This is true in most of the countries and has been verified by conducting various studies and are shown in Fig. 1.1 [5] and Fig. 1.2 [6]. At present, VHF/UHF range is used for television broadcasting and have good propagation characteristics. Utilizing these bands for CR operation will open up many applications in wireless internet access and public safety fields. But voice and data applications in CR context require novel transmission schemes. The transceiver has to support different frequency bands at different times in different geographical locations. These time durations can be varied as per TV broadcasting in different places. CR needs to change its carrier as and when required but has to keep the throughput maximum for

Figure 1.2: Spectrum usage in U.S.

reliable and seamless communication.

There are potential candidates already proposed for CR transmission. They are categorized into four different groups and are critically analyzed in the following chapter. In brief, the existing proposals could not guarantee the requirements of DSA completely, in terms of sidelobe suppression together with multi-user support or good error performance in multipath channels with high spectral efficiency. Hence, adaptive transmission technology for CR is still an active area of research.

Currently, CR is being extensively investigated and debated by regulatory bodies as the enabling technology for opportunistic access of TV white spaces (TVWS). In United States, Federal Communications Commission (FCC), has proposed a Notice of Proposed Rule Making in 2004 [8] which target the unlicensed access of TV bands for CR operation when the channels are idle. The United Kingdom regulatory body, Office of Communications (Ofcom), published a consultation providing details of its proposed cognitive access to TVWS in 2009 [9]. With both U.S. and U.K adapting the cognitive access model and the emerging 802.22 standard being at its final stage, it is expected that CR may become the mainstream technology worldwide in the near future. Hence, the proposal of a flexible transmission scheme which can insure minimum out of band interference, high spectral efficiency, adequate data rate together with multi-user support can enhance the ongoing research on CR and is the main emphasis of this work. Such a scheme can exploit the TVWS for wireless internet access in rural areas.

The work presented in the thesis focuses on single carrier and multicarrier modulation schemes for CR that possesses sidelobe suppression properties and multi-user capability. The efficacy of scalar and multiwavelets are explored to design adaptive transmission scheme for CR. Various definitions of CR share three concepts at the physical layer - flexibility, reconfigurability and adaptivity. Hence, it is required that the transmission scheme should possess these three features. The preliminary works on wavelet and multiwavelet based modulation schemes available in literature give an insight to the unique characteristics of these new techniques. In this dissertation, a detailed evaluation on various aspects of wavelets and multiwavelets is done and they are exploited to come up with spectrum efficient CR transceiver models. Most important feature of scalar and multiwavelet is the flexibility in choosing an orthogonal basis function based on a particular application. Moreover, the direct and inverse transforms are obtained by the successive iteration of the same filter

bank unit which insures better reconfigurability and moderate complexity compared to other conventional modulation techniques. Finally, due to the non rectangular waveforms in time domain, wavelet based systems show good sidelobe suppression capability and hence can adaptively transmit its data without causing interference to adjacent primary users.

1.2 Objectives

The inherent flexibility of wavelet bases with the additional advantage of spectrum efficiency and orthogonality catalyzed wavelet based transmission technologies in the recent past. Taking the application possibilities one step ahead, multiwavelets and multiwavelet packets are also explored in our research for implementing adaptive transmission scheme for CR. The main objectives of the research are

- To propose a novel multi carrier multiple access scheme for CR using standard wavelet filter banks and to study the effects of various channel distortions on its performance.
- To propose a single carrier multi-user CR network using spline multiwavelets and investigate the auto correlation and cross correlation features of different order splines.
- To propose multi carrier multi-user CR network using linear phase, orthogonal multiwavelet filter banks.
- To verify the efficacy of the proposed systems in terms of computational complexity, error performance and spectrum efficiency.

1.3 Outline of the Thesis

The literature survey and research contributions on adaptive transmission for CR are presented in this thesis. They are organized as follows.

Chapter 2 is a literature survey pertaining to the background, basic definitions and architecture of CR. A brief description on the physical layer challenges of CR is given. Different transmission scheme proposals of CR are explained. The merits and demerits of the existing schemes are analyzed. Some of the existing spectrum sensing techniques are highlighted

without pertaining into their theoretical assessment. Finally, an introduction to the first CR standard is given.

In Chapter 3, a novel multiple access scheme for CR is described. The scheme uses standard wavelet packets for generating data subcarriers and a modified carrier assignment scheme for data allocation of multiple users. System model, necessary equations and flowchart of the proposed algorithm are given. The existing orthogonal wavelet filter banks are used for performance evaluation. The results are compared with those of Orthogonal Frequency Division Multiple Access (OFDMA).

In Chapter 4, spline multiwavelets based single carrier multi user CR network is proposed. B spline multiwavelets of different orders are generated to modulate CR waveforms. The cross correlation features of splines are exploited to incorporate more users within a network without compromising on bit error rate (BER). The expressions for BER under additive white Gaussian noise (AWGN) channel and the cross correlation among different spline wavelets are derived. Performance analysis of the system in terms of BER, auto correlation, cross correlation and computational complexity are presented in detail.

Chapter 5 presents the use of multiwavelet packets for CR transmission. Multifilter banks that possess linear phase, compact support, orthogonality and symmetry (antisymmetry) are used to generate orthogonal subbands. A reliable spectrum sensing technique using multifilter bank is detailed and carrier allocation algorithm is provided for subband allocation among different users. Proposed transmission scheme opens up the possibility of large number of users within a network and efficient spectrum utilization.

Chapter 6 concludes the thesis by giving an overall comparison of the proposed systems in terms of spectrum efficiency, flexibility, error rate and computational complexity. Finally, an eye to the future directions of the research is provided.

Chapter 2

Literature Survey

By exploiting the existence of unused or free bands in the spectrum called 'spectrum holes' [10], CR promotes reliable communication and efficient utilization of radio spectrum. As RF spectrum is the most valuable commodity of the present day communication systems, the researchers and service provider industries all over the world are attracted by this new concept after its introduction by Joseph Mitola. Over the past decade, technical, economical and policy challenges that are to be addressed to make CR a reality have been the key research issues in the wireless communication area. In this chapter, an overview of CR, back ground of its evolution and various definitions available in literature are discussed. Major physical layer challenges in CR implementation are highlighted. The works related to adaptive transmission scheme for opportunistic spectrum access are explained with details of various modulation and multiple access techniques. A brief review on existing spectrum sensing schemes is provided. Finally, a brief introduction to different CR standards is given.

2.1 Overview of CR

It was in December 1901 that Guglielmo Marconi proved that wireless waves would not be affected by the curvature of the Earth and long distance wireless transmission is practical [11]. Within a century, wireless communication technology had advanced tremendously and a plethora of applications were deployed utilizing various RF bands. The introduction of mobile phones and notebook computers have brought a revolutionary change in the life and

culture of the modern society. To cope up with the requirements of the mobile and internet users, it was a necessity to insure seamless connection and Quality of Service (QoS). In tandem with the growth of mobile market, the research community started to focus more on customer satisfaction and new devices like reconfigurable and software defined radios have achieved prime importance. Novel wireless applications like video conferencing and internet browsing through mobile phones demand high data rate and hence wider bandwidth. The limited bandwidth availability became a severe issue in the deployment of such emerging applications. To resolve this challenge, the regulatory bodies across the world changed their spectrum allocation policies and researchers re-framed the concept of fixed communication parameters of coding and modulation which eventually lead to CR based intelligent and adaptive radio technologies.

2.1.1 Evolution of CR

The term CR was coined by Joseph Mitola in 1999 [1]. In his paper, he described how a CR can enhance the flexibility of mobile personal networks and services through a new language called radio knowledge representation language (RKRL). This idea was further extended in Mitola's doctoral dissertation [12], that was presented in 2000. The dissertation provides a detailed overview of CR as an exciting multidisciplinary and intelligent radio system. The suggestion of secondary usage of spectrum [12] encouraged FCC to rethink about static spectrum allocation which has inherent drawbacks. Thus in United States, CR research focused quickly on DSA and secondary use of spectrum. This was due to the fact that it was attracting a number of early research projects and due to the research by several vociferous researchers that pointed out the flaws of existing regulatory domain. FCC published a report in 2002 [13] that was aimed at the changes in technology and the profound impact that those changes would have on spectrum policy. The report catalyzed the workshop on CR held in Washington DC, May 2003 which was followed by a conference on cognitive radios in March 2004 in Las Vegas. In September 2004, FCC proposed to allow opportunistic access to TV bands. Prototype CRs operating in this mode were put forward to the FCC by many research groups like Adaptrum, I^2R , Microsoft, Motorola etc. in 2008 [14]. After extensive tests FCC adopted a Second Report and Order in November 2009 which describes rules to allow the operation of CRs in TVWS on a secondary basis [15]. Meanwhile, in Europe, Ofcom proposed to allow license exempt use of interleaved spectrum [9]. Thus, even though

the policies and economical strategies were not finalized, technical challenges of CR became an attractive and an ongoing research area all over the world.

2.1.2 Definitions of CR

Giving an exact definition for cognitive radio is still difficult as all of its goals and applications are yet to be unveiled completely. But experts have tried to say “what is cognitive radio” from different perspectives. A few of them are given below.

Simon Haykin [10] has described CR by defining it as *“An intelligent wireless communication system that is aware of its surroundings, learning and adaptively changing its operating parameters in real time with the objective of providing reliable anytime, anywhere and spectrally efficient communication”*.

The CR Working group in Virginia Technological University, has adopted the following definition of CR [16] *“An adaptive radio which is capable of awareness of its environment and its own capabilities, goal driven autonomous operation, understanding or learning how its actions impact its output and recalling and correlating past actions, environments and performance is called CR”*.

FCC uses a narrower definition for this concept and describes CR as *“A Cognitive Radio (CR) that can change its transmitter parameters based on interaction with the environment in which it operates”*.

While aiding the FCC in its efforts to define cognitive radio, IEEE USA offered the following definition [17] *“A radio frequency transmitter/receiver that is designed to intelligently detect whether a particular segment of the radio spectrum is currently in use, and to jump into (and out of, as necessary) the temporarily-unused spectrum very rapidly, without interfering with the transmissions of other authorized users.”*

Finally, Joseph Mitola, founder of CR defines it as *“A radio that employs model based reasoning to achieve a specified level of competence in radio related domains”*.

It is evident that the definitions of CR available in literature share the concepts of spectrum awareness, adaptation to parameter variations, continuous learning and decision making. These concepts together formulate the complete work cycle of CR known as **Cognition Cycle** [1].

Figure 2.1: Cognition Cycle

2.1.3 Cognition Cycle

The 'top level control loop' for CR is called cognition cycle which describes how a CR could achieve its various levels of functionalities. The cognition cycle is shown in Fig. 2.1 [1]. Outside world provides stimuli. CR receives or collects information about its operating environment (**outside world**) through direct observation or through signaling. This information is then evaluated (**orient**) to determine its importance. Based on its evaluation, the radio tries to find alternatives (**plan**) and chooses an alternative (**decide**) in a way that presumably would improve the performance. When the best alternative is decided, CR implements the alternative (**act**) by varying its parameters, adjusting its resources and performing suitable signaling. From this complete process, CR uses observation, orientation, decisions, planning and actions to learn about its surrounding environment and keep this as a database for future operations [1]. CR can create new modeling states, new evaluations based on its continuous learning process which will make a completely intelligent and an adaptive radio. In the context of DSA networks, this can be explained as follows. The main task of CR is free spectrum utilization. To achieve this, firstly CR has to scan the available RF spectrum (outside world) and has to locate free bands. Then it has to choose the best spectrum for its transmission if there are many vacant bands or has to remain idle based on the scanning results (**orient**). If a free band is located for transmission, CR has to design its

operating parameters like coding, modulation and transmitting filters based on the chosen spectrum (plan). Finally, it has to complete its transmission without causing any harmful interference to the in band and adjacent licensed users (act).

2.1.4 Spectrum Holes

In DSA networks, the cognition is essentially required for locating and **occupying** the unused spectrum in the electromagnetic environment. So an in depth understanding of the term spectrum hole is required. Haykin defines a spectrum hole as “a band of frequencies assigned to a primary user, but, at a particular time and specific geographic location, the band is not being utilized by that user” [10] . They represent the potential opportunities for non-interfering (safe) use of spectrum and can be considered as multidimensional regions within frequency, time, and space. In passively sensing the radio scene and thereby estimating the power spectra of incoming RF stimuli, there exists a basis for classifying the spectra into three broadly defined types, as summarized here.

- Black spaces, which are occupied by high-power local interferers some of the time.
- Grey spaces, which are partially occupied by low-power interferers.
- White spaces, which are free of RF interferers except for ambient noise, made up of natural and artificial forms of noise like broadband thermal noise, transient reflections from lightening and aircraft, impulsive noise produced by ignitions and microwave appliances.

White spaces are the sure candidates for use by unserved operators and grey space can be considered to a lesser extent. Black spaces are to be avoided whenever and wherever the RF emitters residing in them are switched ON. However, when at a particular geographic location those emitters are switched OFF and the black spaces assume the new role of “spectrum holes” cognitive radio provides the opportunity for creating significant white spaces by invoking its dynamic-coordination capability for spectrum sharing [10]. Later, Rahul Tandra et al. [18] have tried to give separate definitions of spectrum hole in time, frequency and space. In time domain, the definition of a spectrum hole is easy to understand - it is the period of time that the primary user is not transmitting. A spectrum hole in frequency is slightly ambiguous. If a secondary user finds a frequency band empty

(no primary user present in that band), its transmissions can still interfere with primary receivers operating in adjacent frequency bands (due to imperfect filters and analog front-ends). Hence, a spectrum hole in frequency is technically defined as a frequency band in which a secondary user can transmit without interfering with any primary users (across all frequencies). Spectrum hole in space can be identified as the region where the primary transmitter power is too low to cause an interference.

2.2 Physical Layer Challenges

Like any conventional communication system, CR framework is also based on ISO/OSI layering methodology. The advantage of this approach is to make CR design easier by enhancing the existing layers of conventional radios with unique cognitive functionalities. As mentioned before, the main physical layer issues comprise of wide band front end, spectrum sensing and adaptive transmission scheme [3].

2.2.1 Wideband Front end

CR needs to observe its radio environment throughout its operation. This implies it has to process a wide band of frequencies at its RF side. CR is built on SDR which aims at replacing all analog parts after antenna by digital circuits. Hence, it is evident that the RF front end of CR needs to have sufficient sensitivity to deal with wide band signals. One of the main limitations in a radio front end's ability to detect small signals is its dynamic range, which also indicates the requirement for number of bits in the A/D converter. The wideband sensing requires multi-GHz speed A/D converters which together with high resolution might be infeasible. Therefore, to receive and process weak signals, it is required to reduce the strong in band primary user signals, which are of no interest to detect. This reduction can be achieved by band stop filtering. But in wideband implementation, strong primary user signal can be located anywhere in the frequency band which demands tunable filters that are complex to implement in practice.

2.2.2 Spectrum Sensing

Spectrum sensing is finding vacant bands in the RF spectrum. The idle bands in the spectrum are termed as “spectrum holes” or white spaces. Spectrum sensing is vital in deployment of DSA in CR networks and is one of the most challenging issues in the physical layer. The key challenge in spectrum sensing is to detect the weak primary user signal in the presence of noise with minimum probability of miss detection [3]. The reliable detection of legacy users is important because

- i) it insures CR would not interfere with primary users
- ii) creates spectrum opportunities for capacity increase of CR networks

In order to achieve this, CR should have better sensitivity and wide band frequency agility than any existing conventional radio systems [3]. It is believed that a cross layer approach of both physical and MAC layer is required to have a good spectrum sensing scheme.

2.2.3 Adaptive Transmission Scheme

After reliable detection, CR has to use transmission schemes that should provide the best spectrum utilization and capacity. It has to change its modulation, coding etc as per the band chosen. The modulation technique of CR should be capable of occupying non contiguous bands spread over a wide frequency range with variable bandwidths and band separations. The spectrum for CR operation is a function of time and geographical location and has to be updated after every spectrum sensing period. Moreover, different applications may require different bands based on interference limit and propagation characteristics. Therefore, the transmission scheme of CR should support assignment of any frequency band at any instant of time and should be scalable with the number of users and bands.

2.3 Transmission scheme proposals for CR

The transmission scheme of CR should possess extreme flexibility and adaptivity as mentioned before. Researchers all over the world have tried to propose new transmission schemes for CR based on the conventional communication systems. These transmission methodolo-

gies can be classified as follows. Orthogonal Frequency Division Multiplexing (OFDM) based schemes, Multi Carrier Code Division Multiple Access (MC-CDMA) based schemes, Transform Domain Communication System (TDCS) based schemes and finally Wavelet based schemes. These schemes are described in the following subsections.

2.3.1 OFDM Based Schemes

Figure 2.2: OFDM Transceiver

OFDM is a multicarrier modulation scheme that has been in existence for several decades. The basic principle behind OFDM is the conversion of a high rate serial data stream into multiple parallel low rate streams. After the serial to parallel conversion, each sub stream is modulated onto a single subcarrier. The advantage of this parallelism is the decreased symbol rate on each subcarrier which results in lower sensitivity to inter symbol interference (ISI). Fig. 2.2 depicts the OFDM concept and a conventional OFDM transceiver. At the transmission side, the data is baseband modulated and the serial symbol stream is converted into parallel substreams that increases the symbol duration but reduces the data rate. These parallel substreams are mapped onto orthogonal subcarriers at the inverse fast Fourier transform (IFFT) block. To combat channel distortions, a cyclic extension of the OFDM symbol called cyclic prefix (CP) is added with each OFDM block. Finally, the parallel streams are converted into serial stream before transmission. At the receiver, reverse of each of these processes is performed to extract the data stream.

In practice, implementation of OFDM has a certain level of simplicity since modulation and demodulation can be easily done with IFFT-FFT operations on the data stream. Moreover, orthogonality of the subcarriers allows spectrum overlapping among individual subcarriers of the symbol to reduce overall bandwidth. The receiver structure allows relatively straightforward signal processing to combat channel delay spreads, thereby adopted as the modulation method in several standards [19]-[25].

The inherent flexibility of OFDM technique due to its subcarrier modulation scheme and the ease of implementation have made it a potential candidate for CR transmission and OFDM based CR is a well researched area [4],[26]-[37]. The following section discusses a few CR transmission technologies based on OFDM principle.

2.3.1.1 OFDM based Spectrum Pooling

Among the many proposals that use OFDM for CR, the first work was due to spectrum pooling strategy based on OFDM done by Weiss and Jondral [26]. Spectrum pooling concept was first introduced by Mitola and its details are given in [1]. It refers to the grouping of various frequency bands into a common pool based on their present applications. The spectrum within the pool can be made available for rental purpose and can be utilized by CR based devices whenever required. Weiss proposed OFDM as a good candidate for spectrum pooling. CR system is considered as a rental system that rents the licensed RF spectrum from its primary user when it is idle. The basic idea of OFDM based spectrum pooling is to match the bandwidth of one subband of the licensed user with an integer multiple of the carrier spacing used in the rental system. An example where one licensed subband is resolved by four subcarriers is shown in Fig. 2.3. The spectrum sensing, free

Figure 2.3: OFDM based spectrum pooling

spectrum signaling, channel assignment and synchronization issues related to spectrum pooling concept are discussed in [27] -[30]. Spectrum sensing method incorporates standard fast Fourier transform (FFT) processor of the OFDM transceiver to detect the vacant bands so that spectrum sensing requires no additional circuit [27] and information about the vacant bands is conveyed through a binary information vector with the help of a ‘boosting algorithm’ [28]. Controlled and uncontrolled channel assignments are proposed for spectrum pooling in [29]. These proposals make a complete framework for OFDM based spectrum pooling for CR. From literature, we can figure out the two advantages of OFDM based spectrum pooling.

- The spectral co existence of rental and licensed system is possible if rental system uses subcarriers that lie in idle subbands of the licensed system.
- In OFDM based spectrum pooling, the sensing of vacant bands is done with the help of FFT operation. So the FFT processing for modulation comes at no extra cost.

Even though the proposed work offers good flexibility and adaptivity, it suffers from two drawbacks. Firstly, spectrum leakage is caused by the FFT-IFFT operations. Rectangular pulses are used in the time domain which produce large sidelobes in the frequency domain. So the sidelobes of the rental system can cause interference to the licensed users. Secondly, for the successful reception of OFDM, perfect synchronization techniques are required. These issues are addressed in [30,31]. A frame detector together with cross correlator and adaptive prefilter is proposed to resolve the synchronization issues of the spectrum pooled OFDM [30]. To avoid spectrum leakage, two methods are suggested in [31]. First method is to use raised cosine filter with certain roll of factor which gives no significant improvement in sidelobe suppression. Second method is dynamic deactivation of the subcarriers at the vicinity of the legacy user and is observed as a powerful tool for the reduction of interference.

2.3.1.2 Non Contiguous OFDM

Spectrum pooling is an innovative strategy for enhancing the spectrum efficiency but the lease of licensed spectral resources to rental users brings forth many technological, juristical, economic and political questions concerning the regulatory aspects. A better way for opportunistic spectrum access is overlay sharing of the spectrum using non contiguous

OFDM (NC-OFDM) proposed by Rajesh Rajbanshi et al [32]. It refers to the use of non contiguous free bands of the RF spectrum by appropriate deactivation of OFDM subcarriers at the secondary user. The basic block diagrams of NC-OFDM transmitter and receiver are shown in Fig. 2.4 and Fig. 2.5. Basic difference from a conventional OFDM transceiver is the subcarrier ON/OFF information that is obtained from the spectrum sensing part of CR. This data is utilized to deactivate certain subcarriers during transmission to minimize the interference. The spectrum allocation details are passed to NC-OFDM CR receiver for proper synchronization and data extraction. Moreover, FFT pruning algorithm is used for saving hardware resources as the deactivated subcarriers carry no valuable information [7]. The qualitative and quantitative analysis of this scheme is done based on various

Figure 2.4: NC-OFDM Transmitter

Figure 2.5: NC-OFDM Receiver

parameters and can be summarized as follows.

- **Synchronization Issue:** In conventional OFDM, the synchronization is mainly done by pilot subcarrier to avoid frequency and timing offsets and this is not possible in a non contiguous system. Hence complex synchronization methods are required in NC-OFDM.
- **Frequency agility:** Conventional OFDM is contiguous and hence its frequency agility is very less compared to non contiguous schemes. But this agility is achieved in NC-OFDM at the cost of high transmission overhead and frequent updates on spectrum availability.
- **Error Robustness:** The BER results of NC-OFDM in AWGN and fading channel are discussed in [33,34]. It is observed that BER of the proposed system is comparable with conventional OFDM.
- **Peak to Average Power Ratio (PAPR):** As NC-OFDM is derived from conventional OFDM, it suffers from the PAPR problem and it is slightly different from that of OFDM due to the null subcarriers. Moreover, the existing PAPR reduction algorithm of OFDM should be modified for NC-OFDM systems by incorporating awareness about deactivated subcarriers and thereby avoiding interference.

2.3.1.3 Adaptive OFDMA

An adaptive OFDMA transmission scheme which satisfies the VHF/UHF band CR operation is discussed in [35]. This proposes flexible FFT size and supports fractional bandwidth use to avoid interference to incumbents. The channelization used in this scheme supports two methods - diversity subchannel and band selection subchannel and they are multiplexed in such a way that adaptive modulation and coding can be done more effectively. The basic system is as shown in Fig. 2.6. The method is mainly for TV band access of CR and the initial performance evaluations are reasonable. However, for the OFDM design, it should have 2048 subcarriers which make the implementation very complex. An adaptive subcarrier allocation strategy for OFDMA based CR is described in [36,37]. The conventional water filling algorithm is modified under the interference temperature constraint for an adaptive resource allocation of uplink OFDMA. The subcarrier allocation rules are obtained by theoretical deduction and the asymptotic performance analysis of the proposed system is provided.

Figure 2.6: Adaptive OFDMA Transceiver

2.3.2 MC-CDMA based Schemes

MC-CDMA combines the unique features of OFDM and CDMA. Like OFDM, the MC-CDMA signal is made up of a series of equal amplitude subcarriers. Unlike OFDM, where each subcarrier transmits a different symbol, MC-CDMA transmits the same data symbol over each N^{th} subcarrier. It applies spreading in the frequency domain by mapping a different chip of the spreading sequence to an individual OFDM subcarrier. The implementation of MC-CDMA is done by concatenating a Direct Sequence CDMA spreader and an OFDM transmitter. The schematic of an MC-CDMA transceiver is given in Fig. 2.7 and the various steps involved are described in the following paragraph.

The input high rate serial data sequence is first converted into L lower rate parallel data streams and the rate will be lower by a factor L . Following the conversion, each stream is replicated into N parallel copies using the copy function, with copy m of stream i being multiplied by chip m of spreading code C_i , for $i = 0, \dots, L-1$ and $m = 0, \dots, N-1$. The data in the spreading bits are modulated in the baseband by IFFT and converted back to serial data. The spreading sequence in MC-CDMA provides multiple access capability. A guard interval with cyclic extensions similar to OFDM is inserted between symbols to counter ISI caused by multi-path fading. At the receiver, a coherent detection method is employed to successfully despread the signal. Since MC-CDMA provides the multi-carrier feature of

OFDM and multi-user capability of CDMA, it is suggested as a promising candidate for CR transmission. Brief discussions on those proposals are given below.

Figure 2.7: MC-CDMA Transceiver

2.3.2.1 NC-MC-CDMA

The non contiguous version of MC-CDMA called NC-MC-CDMA can be considered as the simplest and straightforward approach to utilize MC-CDMA for CR applications. Such a system is proposed by Rajanshi et al and they have compared its performance with NC-OFDM [33, 34]. To implement NC-MC-CDMA, it is required to deactivate certain subcarriers to avoid interference with the incumbent user. So the methodology is similar to that in NC-OFDM and hence information about the free spectrum and deactivated subcarriers and their frequent updates are essential for NC-MC-CDMA also. The system provides frequency agility by filling in the gaps of the available spectrum and is equivalent to NC-OFDM. As it is a non contiguous version of MC-CDMA, PAPR problem will be lower compared to NC-OFDM. The use of spreading codes provide multi-user capability also. But NC-MC-CDMA system suffers from two major drawbacks.

- 1) To avoid interference and utilize the spectrum effectively, some of the subcarriers are to be turned off. Therefore, an increased transmission overhead is required for synchronization and data extraction. As MC-CDMA requires both spreading code generators and FFT processors, its complexity is always higher than OFDM system. Hence the transmission

overhead makes it a highly complex system.

2) The subcarriers of an MC-CDMA transceiver are not independent due to the spreading by Walsh Hadamard codes in the frequency domain. The orthogonality among the spreading codes will be lost when certain subcarriers are nulled. This will result in poor BER performance. In the case of NC-OFDM, subcarriers are independent and their orthogonality will be maintained even at the deactivation of subcarriers and hence BER will be almost equivalent to a conventional OFDM system.

It can be concluded that the proposed system needs to resolve these issues to become a strong candidate for CR transmission.

2.3.2.2 Carrier Interferometry NC-MC-CDMA

The main drawback of NC-MC-CDMA is that it employs Walsh Hadamard codes to spread the signal in the frequency domain thereby orthogonality can be maintained only when all the subcarriers are active. The simpler way to deal with is to use Walsh Hadamard codes of adaptive length depending on the active subcarriers. But Hadamard codes do not exist for all code length. Hence, a possible remedy is to use Carrier Interferometry (CI) codes which are orthogonal for any integer code length and is proposed in [38]. By introducing CI NC-MC-CDMA system, adaptive allocation of orthogonal CI spreading codes is done, totally eliminating the loss of orthogonality despite the number of deactivated subcarriers. With the support of theoretical analysis and performance evaluation, it is shown that the proposed system outperforms Walsh Hadamard code based NC-MC-CDMA in terms of BER.

2.3.2.3 Modified NC MC-CDMA

To resolve the issue of loss of orthogonality in NC-MC-CDMA, a restructuring method is discussed in [39]. The active subcarriers are grouped and it is assumed that there are K subcarriers serving maximum K users. A channel information matrix H of size $K \times K$ is formed whose rows will form the channel coefficients of each user. The inverse of the matrix obtained and each of its column is used as the spreading code. It is shown that the u^{th} row of matrix H is orthogonal to all columns of its inverse matrix except u^{th} column and is utilized

to extract the data of user u . The modified structure simplifies the equalization process and better error performance is achieved compared to NC-MC-CDMA. But certain assumptions are made to formulate the new structure such as the exact knowledge of channel coefficients is available, number of active subcarriers and active users are equivalent etc. Detailed mathematical analysis of the system is not presented in literature.

2.3.3 TDCS Based Schemes

TDCS is a transmission scheme that evolved mainly for military applications to avoid unknown interferences. The fundamental idea of TDCS is to avoid interference at the transmitter by notching waveform energy at corrupted spectrum locations. It assumes that transmitter and receiver are observing the same spectral environment and thus produce similar spectral estimates. In 1980s, L.B. Milstein [40] came up with TDCS filtering and in 1991, A. F. Andren proposed a Low probability of Intercept communication system[41] for military applications using TDCS concepts. Due to the inherent interference avoidance nature, TDCS is considered to be a good candidate for CR transmission. The adaptive transmission schemes based on TDCS that are proposed for CR are briefly explained in the following section.

2.3.3.1 FFT based TDCS

Chakravarthy describes FFT based TDCS and its application for CR in [42,43]. The transmitter and receiver are as shown in Fig. 2.8 and 2.9 respectively. The CR transmitter estimates its radio environment and calculates the spectrum magnitude based on an estimate. A hard limiting threshold is applied to generate a clean or interference free spectrum. That is, amplitude of interfering spectral components whose value exceeds the threshold are nulled and the rest of the frequency components are assigned a value of '1'. This spectrum magnitude is then multiplied element by element using random phase vector. This vector that has both magnitude and phase is then scaled to provide desired symbol energy in the signal spectrum. Multiplication with random phase provides multi-user capability. Finally, the transform domain vector is converted to the time domain fundamental modulation waveform by IFFT operation which is modulated and transmitted. At the receiver, coherent detection is performed to get back the original message.

Figure 2.8: FFT based TDCS Transmitter

Figure 2.9: FFT based TDCS Receiver

Based on the works mentioned in [42, 43], implementation of TDCS based CR is described in [44]. In this paper, Ruolin et al. implement and demonstrate an adaptive interference avoidance TDCS based cognitive radio via software defined radio implementation. The designed system applies GNU software radio [45] as the platform, and Universal Software Radio Peripheral (USRP) [46] as the hardware solution. This cognitive radio is capable of detecting primary users in real time and adaptively adjusting its transmission parameters to avoid interference to primary users. Additionally, it is demonstrated that when the primary user transmission changes, the cognitive radio dynamically adjusts its transmission accordingly.

The TDCS based system is quite useful in narrowband stationary interference mitigation. The narrow band user is adaptively notched while generating the CR waveform. But to have a low BER, the notch width provided is $2f$ if f is the bandwidth of the narrow band primary user. This results in lower spectrum utilization and can be ineffective in wideband primary user networks. Moreover, to avoid spectrally crowded regions, TDCS utilizes a 10th order Auto Regressive (AR) filter to estimate the Fourier periodogram (approximation of power spectrum) of the environment. It however fails to estimate non stationary interference. The random phase addition helps to have a noise like time domain waveform and multiple access capability. But this makes the conventional higher order modulation schemes impractical and the receiver structure complex. The modulation used is termed as Cyclic Shift Keying(CSK). Unlike OFDM based system, the waveform is directly generated in the RF domain.

2.3.3.2 OFDM Based TDCS

The TDCS scheme explained above has good interference property but uses a Rake receiver in multi-path fading environment. OFDM has good flexibility but has PAPR problem. To have a good transmission scheme for CR, C. Han [47] has proposed an OFDM based TDCS. The basic difference between a traditional TDCS and OFDM based TDCS can be explained with the help of block diagrams. After scaling the transform domain vector, it is data modulated using OFDM transmitter. That is instead of converting the waveform into time domain, the frequency domain signal is used to modulate the data. At the receiver, first the OFDM signal is demodulated to the transform domain signal and then it is correlated with the locally generated waveform. This system has certain advantages over the conventional

Figure 2.10: OFDM based TDCS Transmitter

Figure 2.11: OFDM based TDCS Receiver

FFT based TDCS system. Mainly, the transceiver complexity is reduced and the effective SNR is doubled. Also, such a system can be made compatible with existing transmission schemes. But apart from the disadvantages of traditional TDCS, this system suffers from higher transmission overhead.

2.3.4 Wavelet Based Schemes

Wavelets and wavelet packets are powerful tools in almost all aspects of digital communication systems such as signal denoising, data compression, source and channel coding and channel modeling. Recently, trends and developments in wavelet based research shows that flexible and adaptive modulation schemes are also possible. As the latter chapters focus on wavelet based transmission technologies, the unique features of wavelets that make them suitable for wireless communication application are outlined below.

- Multirate ability and semi arbitrary division of bandwidth - The wavelet transform can create subcarriers of different bandwidth and symbol length. Since wavelets have constant time bandwidth product, an increase of bandwidth is bound to a decrease of subcarrier symbol length and vice versa. The basis functions occupy different intervals in the time domain. This unique feature of wavelet transform can be exploited to create multirate systems. From a communication perspective, it is favorable for systems that must support multiple data streams with different transport delay requirements [48].
- Flexibility - The decomposition of a signal of length N generates a large number of wavelet packet basis functions, depending on the choice of the mother wavelet. The extensive selection of wavelet packet basis functions provides a great degree of flexibility in how they can be utilized in communication systems design. Another advantage of wavelets lies in their ability to arrange the time-frequency tiling in a flexible manner that minimizes the channel disturbances. Wavelet packet tree pruning allows a system to be adaptively reconfigured to best match the communication channel or system characteristics [49].
- Orthonormal basis - An orthonormal and complete set called an orthonormal basis set can be efficiently constructed for wavelet packets. In the context of multicarrier modulation, the wavelet packet transform has longer basis functions which are not

rectangular in shape and can offer a higher degree of side lobe suppression. The orthonormality of the wavelet packet basis makes them well suited to approximate piecewise smooth functions and to effectively separate signals from noise. Wavelet thresholding along with orthonormality is a powerful tool for the denoising and recovery of true signals from noisy data [50].

- Time and frequency localization - Wavelet packets are well localized in both the time and frequency domains. The orthogonality in both the time and frequency domains are maintained for overlapped wavelet packets even when the wavelet packet tree is pruned. This feature is especially attractive when using wavelet packets for modeling of frequency selective and time varying communication channels [50].
- Wavelet Signal diversity - Wavelets and wavelet packets give rise to a new dimension in diversity called waveform diversity. Waveform diversity or signal diversity, which is similar to spread spectrum systems, can be exploited in a cellular communication or radar system to differentiate users and to extract more information from a given target. Wavelet waveform diversity can be used in ultra wideband (UWB) communication systems to reduce interference between users by intelligently shaping the transmitted pulses [48].
- Low complexity - Wavelet packet based methods have the advantage of low computational complexity [51, 52]. A discrete signal can be decomposed into a wavelet packet basis with fast filter bank algorithms whose complexity is of the order of $N \log_2 N$. The iterative nature of the wavelet transform allows for a configurable transform size. The number of subcarriers in OFDM systems is usually fixed at design time, and it is especially difficult to implement an efficient FFT transform having a programmable size. This limitation does not exist in WPT since the transform size is determined by the number of iteration of the algorithm. From an implementation point of view, this can be made configurable without significantly increasing the overall complexity. This permits on the fly change of the transform size and can be used to reconfigure a transceiver according to a given communication protocol, the channel impulse response characteristics or link quality [48].
- Power Conservation - Wavelet based algorithms have long been used for data compression. In the context of mobile wireless devices, which are mostly energy starved, this has a valuable significance. Data compression facilitates reduction in the vol-

ume of data to be transmitted and thereby the communication power required for transmission is reduced [49].

A few transmission schemes are already proposed for CR based on wavelets and wavelet packets. They are described in the following sections.

2.3.4.1 Enhanced Wavelet Domain Communication System

Figure 2.12: EWDCS transmitter

Incorporating the concept of fundamental waveform generation of TDCS and accurate spectrum estimation using wavelets, Enhanced Wavelet Domain Communication System (EWDCS) is proposed in [53]. The basic idea in this system is to synthesize an intelligent adaptive waveform in the wavelet domain at both the transmitter and receiver to avoid spectral crowded regions. This approach offers better bit error performance in contrast to other conventional interference mitigation techniques that process the signal at the receiver only. The EWDCS exploits both transform domain processing and the EWS estimation technique to extend interference avoidance capability from stationary interference to non stationary interference. The transmitter of EWDCS is shown in Fig.2.12. The major difference from FFT based TDCS is in spectrum estimation and in the time domain waveform generation. Estimation technique used is called Evolutionary Wavelet Spectrum (EWS) which is more powerful and is capable of notching non stationary interference. The fun-

damental idea is to extend the Wiener-Kintchine theorem, which relates the variance of a stationary random signal to its Power Spectral Density (PSD), to the non stationary case. The EWS is related to the local auto-covariance through the auto-correlation of a wavelet. Unlike the PSD, which is only frequency dependent, the EWS is both time and scale dependent, and this scale dependence is related to frequencies. The fundamental modulation waveform generated is a wavelet based function.

Pseudo-random sequence plays a different role in the EWDCS. In contrast to the TDCS, a pseudo-random sequence controls which scale is used to transmit signal instead of mapping a random phase to a basis function. There are two reasons for doing this. Firstly, the meaning of mapping a random phase to a basis function in the wavelet domain is ambiguous. Secondly, experimental results show that the effect of mapping phase in the EWDCS results in poor bit error performance. The EWDCS is therefore a “frequency hopping” spread spectrum system, or more accurately, a “scale hopping” spread spectrum system [53].

2.3.4.2 Discrete Wavelet Multitone Modulation

A novel CR system is proposed by Kuan-Hung Chen and Tzi-Dar Chiueh [54] to operate in the GSM band and the discrete wavelet multitone (DWMT) is adopted as the modulation scheme. This modulation has stronger side-lobe attenuation than the popular OFDM modulation and thus generates much lower adjacent channel leakage, making it suitable for applications with dynamically allocated spectrum. Signal and interference analysis of the adopted DWMT modulation is presented and signal processing techniques that tackle synchronization, equalization, and detection are developed. A DWMT baseband receiver is designed and verified by functional simulation. A digital baseband receiver is implemented on chip for the proposed DWMT-based CR system. When the whole GSM band is available, the receiver circuit can provide up to 153.6 Mbps of data transmission.

The DWMT modulation is a multicarrier modulation scheme similar to OFDM. Both the OFDM and the DWMT systems are based on M band filter banks. The OFDM system uses FFT and its inverse that can be interpreted as discrete-time filtering operation with an M band filter bank composed of Fourier filters. On the other hand, the DWMT system is realized using an M band wavelet filter bank. The M band wavelet filter bank is composed of a set of wavelet filters with longer impulse responses than its counterpart in FFT. It

is the much longer time span that makes the DWMT signal spectrum have sharper side-lobe fall-off. Three types of maximally decimated filter banks are presented in the paper with necessary block diagrams and equations. They are cosine-modulated filter bank, sine-modulated filter bank, and exponentially modulated filter bank. The third type is adopted to realize the DWMT modulation and its details are discussed.

2.3.4.3 Wavelet Packet Modulation

Figure 2.13: WPM Transceiver

Lindsey and Dill were among the first to propose wavelet packet modulation (WPM) [56]. The theoretical foundation of this orthogonal multicarrier modulation technique and its interesting possibility of leading to an arbitrary time-frequency plane tiling are underlined in their paper. Lindsey has further completed the work in this area by presenting additional results [57]. In particular, it is shown that power spectral density and bandwidth efficiency are equal for WPM and standard quadrature amplitude modulation (QAM) modulation. The basic block diagram of WPM is shown in Fig. 2.13. The key idea is to use the wavelet subbands generated by the successive iteration of two channel filter banks as data subcarriers and exploit the orthogonality of the basis function for data retrieval. The Inverse discrete wavelet packet transform (IDWPT) block or synthesis block in the transmission part corresponds to successive iterations each consisting of up-sampling and filtering operation. Similarly, at the receiver, the discrete wavelet packet transform (DWPT) block (analysis) corresponds to successive iterations of filtering and down sampling.

Jamin and Mahonen outlined the salient features of wavelet and pointed out its significance in adaptive systems like CR [48] and are summarized in section 2.3.4. Based on these

findings, wavelet packet multicarrier modulation (WPMCM) for CR was first proposed by Lakshmanan et al [58]. They have evaluated the performance of existing orthogonal and biorthogonal wavelets and have designed a new set of wavelets called maximally frequency selective wavelets based on Remez exchange algorithm. Various research results of this team on WPM for CR are given in [59]-[63]. Effects of phase noise, frequency offset and time offset are analyzed and plotted in [59]-[61]. A wavelet based spectrum estimation is proposed by Ariananda et al in [62]. Finally, combining these results, a wavelet based spectrum pooling is proposed in [63]. From the results given in [59]-[63], it can be concluded that WPM is a promising candidate for future wireless systems. Inspired by these results, its application is extended into multi-user CR environment in Chapter 3.

2.3.5 Conclusion

Various transmission schemes discussed above show research challenges in adaptively occupying a spectrum without causing interference. OFDM based schemes are predominantly considered for CR transmission owing to its flexibility and multi-user capability. But it fails in interference mitigation and sidelobe suppression. Moreover, it is prone to PAPR problem and the presence of CP is useful only when the channel delay spread is within certain limits. Thus spectrum efficiency is compromised since CP uses about 25 percent of the available spectrum.

TDCS can be a good scheme while the BOI is limited and RF domain waveform generation is practical. As it uses only CSK modulation at baseband, the transmitter and receiver architecture is complex. MC-CDMA based schemes suffers from high transmission overhead, poor BER and high PAPR.

Wavelet based scheme offer good flexibility and spectrum efficiency. Single carrier and multicarrier schemes are possible with wavelets. The sidelobes can be controlled by properly choosing the basis function and PAPR problem is lower in a wavelet based system compared to that in other multicarrier systems. Among different wavelet based schemes, WPM incorporates the advantages of a typical filter bank based modulation technique. As filter bank based modulation schemes attains more attraction of the wireless communication research community and filter bank based spectrum sensing for CR results in good performance, it would be interesting to explore the practical benefits and limits of this scheme in a detailed

fashion. Hence, the latter chapters focus mainly on wavelet based modulation and multiple access schemes for CR.

2.4 Spectrum Sensing Schemes

Spectrum sensing is the task of obtaining awareness about spectrum holes and is the most important component for the establishment of CR. This is not common in any conventional system and hence its successful implementation is a crucial task [64]. Various schemes proposed and a brief comparison of the proposed methods are outlined in the subsequent subsections.

2.4.1 Energy Based Detection

This is the simplest detection method and is also known as radiometry or periodogram. Compared to other methods available in literature, this is more generic and involves lower computational complexity as receivers do not need any knowledge of the primary user signal. The signal is detected by comparing the output of the energy detector with a threshold. It can be implemented similar to a spectrum analyzer by averaging frequency bins of an FFT. The main parameters in this detection method are the FFT of size N and an averaging time T . When N is increased, it results in good frequency resolution and thus narrow band signal detection. Large averaging time T helps to reduce noise power and thus improve Signal to Noise ratio (SNR) [64]. Finding a threshold which results in maximum probability of detection is very crucial in energy detection. Even when threshold is set adaptively, the presence of in band interference will degrade the performance of an energy detector. Energy detector is a suboptimal detector and as it involves non coherent processing, more samples are required to meet a probability of detection constraint. Various sensing algorithms using this principle are listed and analyzed in [65].

2.4.2 Waveform Based Sensing

To support synchronization and other tasks, in a wireless system, the transmitted signal usually includes preambles, pilots, spreading sequences etc. These known patterns can be

used for sensing by correlating the received signal with a known copy of itself. This method is applicable to systems with known signal patterns and is termed coherent sensing or waveform based sensing [66]. The accuracy of this sensing algorithm increases as the length of the known pattern increases. According to [66], waveform based detection outperforms energy detection in terms of convergence time and reliability.

2.4.3 Cyclostationary Feature Detection

In this method, instead of PSD, cyclic correlation function (CCF) is used for detecting the signal in a given spectrum. CCF is a measure of the cyclostationary features of the signal which are caused by the periodicity either in the signal or in its statistics like mean and autocorrelation [65]. This periodicity can be used for detection of a random signal with a particular modulation in a noisy background. The implementation of cyclostationary detection is outlined in [64]. The basic difference between an energy detector and feature extraction lies in using the spectral correlation of the received signal instead of applying a threshold. As the spectral correlation of different types of signals are distinct, reliable detection is possible in the presence of interference and noise. But it is computationally complex and requires long observation time.

2.4.4 Matched Filtering

Matched-filtering is known as the optimum method for detection of primary users when the transmitted signal is known [67]. The main advantage of matched filtering is the short time to achieve a certain probability of false alarm or probability of miss detection [68] as compared to other methods that are discussed in this section. The received signal is matched with any of the known signal. The CR has prior knowledge of the primary user signal at physical (PHY) and MAC layers. So a dedicated receiver for each primary user is required at the CR terminal. Required number of samples grows only as $O(1/SNR)$ to meet a given probability of detection. But architecture complexity involved in matched filtering is too high [64].

2.4.5 Filter Bank based Sensing

Locating the vacant bands using filter banks is already addressed in literature [69] and the merits and demerits are discussed. It has been shown that if the filter banks that are employed for multicarrier communication of CR networks is utilized for spectrum sensing, the sensing circuit can be implemented at no additional cost. The detailed performance analysis of various filter bank based spectrum sensing is presented in [69]. From the results and discussions, it can be concluded that multi taper based spectral estimation is not suitable for CR applications but filter bank based techniques give promising results. Wavelet filter bank based sensing is discussed in [70] and [62]. A wavelet packet based energy detector using Infinite Impulse Response (IIR) scalar wavelet filters is described in [70]. IIR filters are used for sensing purpose to reduce the complexity of the sensing circuit. But this demands additional circuitry in the CR transceiver for sensing the spectrum. The use of FIR wavelet filter bank for spectrum sensing is outlined in [62] and it has been shown that the results are comparable with existing power estimation techniques. Since the sensing circuit proposed in [62] uses scalar wavelet FIR filters, its implementation is possible by utilizing wavelet based CR receiver (demodulator) at no additional hardware circuitry [63].

2.4.6 Cooperative Sensing

As the title indicates, this detection approach consists of using the collective information from a set of CR users within the network for free spectrum identification. Co operation of individual CR users is encouraged for a reliable detection. The information about the spectrum vacancy from each CR user is collected at the base station in most of the cases and based on that final decision on free band is taken. Challenges of cooperative sensing include developing efficient information sharing algorithms and increased complexity [71]. Cooperative sensing is further classified into centralized and distributed sensing based on whether the final decision is taken with the help of a central Base Station (BS) or not. A centralized spectrum sensing scheme based on credibility and evidence theory is described in [72]. Each CR performs spectrum detection using energy measurement or feature extraction and the final decision is taken by an Access Point. Distributed sensing for ad hoc wireless networks is described in [73]. At the sensing node, the received power levels of neighboring CR nodes are weighted according to their distances from the sensing CR node and collective information is used to decide the presence or absence of a legacy user. Cooperative sensing

is more accurate and reliable than the detection methods discussed earlier since the final decision is based on the collective information from various CR nodes. But the complexity involved is higher and require proper control channel for each CR node to transmit its sensing information.

2.4.7 Beacon Based Sensing

In all sensing methods discussed above, CR scans the entire spectrum, measures the features, and tries to come to a conclusion about the occupancy of the spectrum. So probability of miss detection is comparatively high as fading and noisy environment may give wrong information about the channels. Moreover, as CR needs to scan the entire spectrum, these methods require large amount of power and are time consuming. Therefore, an operator assisted spectrum sensing is also considered as a good approach for reliable detection. If the legacy user can send a particular beacon signal intimating its idle or active state, CRs within the network can listen to it and can do the spectrum detection based on the beacon. J. Mitola [1] proposed an initial framework for formal radio etiquette in the secondary use of spectrum. The need for a simple protocol to determine which legacy user can communicate with the secondary user to determine a spectrum hole is discussed. A dual beacon structure for vacant band detection and its detailed evaluation is given in [74]. The beacon is the emission of a single electromagnetic harmonic wave and it requires line of sight (LOS) for successful reception. Here, two types of beacons- grant and denial beacons are proposed for identifying a vacant band. But this results in complex beacon structure and in some situations additional beacons from primary base stations are necessary. A reliable power efficient beacon structure is suggested in [75] but is considered for wireless microphone and other low power devices. The beacon is transmitted to signal the presence of primary user rather than its vacancy [75].

2.4.8 Comparison among Various Schemes

Sensing methodologies can be compared based on three performance metrics- accuracy, speed and complexity. Among the various methods discussed above, energy detector has lowest complexity and moderate speed but has good accuracy only when noise is stationary. Therefore, energy detector can be used only at a preliminary level of detection process.

Cyclostationary based detection has good accuracy but has higher complexity and lower speed compared to energy detector. Moreover, this type of sensing may completely fail when the system undergoes fading. Waveform based sensing is more accurate than both energy and cyclostationary sensing. But it works only when there is a priory knowledge of the primary user and is applicable for signals having known patterns or pilots. Matched filtering has good reliability and fast convergence time but architectural complexity is too high. Filter bank based sensing share the properties of energy detection and can be implemented in multicarrier systems without additional complexity. Cooperative sensing is reliable than single node sensing but the topology and MAC layer of CR network should support the extra task of collecting information from other nodes. Hence, this method is usually considered at the final stage of detection to reach to a final decision regarding the given spectrum. Beacon based sensing reduces the task of CR nodes but the application is limited to bands of licensed users who sends a beacon during their idle or active periods.

In short, efficient and reliable spectrum sensing can be implemented in two stages. Energy detection, filter bank based detection or waveform based detection can be used in the preliminary stage. The collective information of the preliminary sensing from different CR nodes can be used at the final stage to make accurate decision.

2.5 Standardization of CR

To enable new technologies to flourish and reach maturity, tight and efficient interaction between research and standardization is required. In the area of CR, IEEE SCC41 is a coordination body within the standardization organization for dynamic spectrum access, interference management, advanced spectrum management and policy languages. The efforts of SCC41 was preceded by the IEEE P1900 Standards Committee which is a joint venture of IEEE Communications Society and IEEE Electromagnetic Compatibility Society. The objective of P1900 was to develop supporting standards dealing with innovative technologies for next generation radio and advanced spectrum management. As SCC41 is a direct sponsor of standards and reports directly to IEEE, the P1900 Committee ceased to exist at the inaugural meeting of SCC41 in April 2007. The work of all P1900 working groups(WG) continued under SCC41 retaining the same names as before. There are six WGs until 2010. A related effort from IEEE 802 working group brought out an emerging

standard for TV white space access using CR and the standard is termed IEEE 802.22 or wireless regional area network (WRAN). The scope of different P1900 working groups and that of 802.22 available in literature. [7], [76]- [78]. Among the different WGs, WRAN is considered as the first IEEE standard for CR.

WRAN specifies a fixed point to multipoint wireless air interface whereby a BS manages its own cell and all associated consumer premise equipments (CPE). In addition to the traditional role of BS, WRAN base station also co ordinate a unique feature of distributed spectrum sensing. This will help to have reliable spectrum detection and its periodic up-dation among all CPEs. The physical layer data rate is 18 Mbps in a 6 MHz channel with a spectral efficiency of 3 bits/sec/Hz. The base station has coverage of 33 Km for an EIRP of 4 watts for CPE. The coverage range can go upto 100 km if power is not an issue. Physical air interface is suggested as OFDMA with an FFT size of 2048 and contiguous band is considered for CR use. Higher data rate can be achieved by channel bonding where more than one channel is used for CR operation. The CPEs are fixed wireless systems with antenna height of more than 10 meters with high power. The low power mobile systems are expected to be in use in the near future. Every 802.22 CPE is required to implement a spectrum sensing function (SSF), which is defined in the standard as a “black box” that performs spectrum sensing. The standard specifies inputs and outputs for the SSF, as well as the performance requirements for the sensing algorithm implemented (e.g., probability of detection, incumbent detection threshold, and probability of false alarm). The specific sensing algorithm is implementation dependent but description and performance results of several sensing algorithms are included in the standard. Sensing algorithms can be broadly classified as blind and nonblind. In the former, no assumption is made on signals of the incumbents, while in the latter the characteristics of the incumbent signal is used to detect it. Various blind and nonblind algorithms considered for TV white space detection are detailed in [7].

2.6 Chapter Summary

An attempt to provide a comprehensive review on CR physical layer research efforts is made in this chapter with main emphasis on adaptive transmission schemes and sensing methods. The ongoing research and developments shows that CR can be one of the most promising

technologies of the near future. Transmission schemes proposed already in literature have contributed well in the formation of the first CR standard. But a good spectrum sensing technique and transmission scheme for CR in general are still not finalized. The interference minimization and complexity of the transceiver design are very crucial in making DSA networks possible. OFDM can be a transmission candidate in CR networks but it is to be modified to reduce its sidelobes and PAPR. There are works based on transform domain waveforms but the generation of orthogonal basis functions with good spectral efficiency is still to be explored. MC-CDMA based schemes are proposed for CR transmission but the results are not promising in terms of BER, flexibility and computational complexity. Wavelet based schemes open up the possibility of a spectrum efficient and flexible transmission technique with good reconfigurability and moderate complexity. The multi resolution property of wavelets can be used for detecting the presence and absence of a primary user in a given spectrum. Therefore, the work presented in this dissertation in the following chapters explore wavelet based transmission and sensing technologies.

Chapter 3

Multicarrier Multiple Access Scheme for CR using Scalar Wavelet Packets

A detailed review on existing transmission scheme proposals for DSA has been given in Chapter 2. OFDM based system incorporates flexibility and ease of receiver design under multipath fading but suffer from high sidelobes and lower degree of reconfigurability. MC-CDMA based systems possess higher complexity and lower flexibility. TDCS based systems have lower error robustness and high transmission overhead. All these schemes have multi-user capability at the physical layer. Published wavelet based schemes in the literature indicate that a filter bank based multicarrier scheme with wavelets that ensures lower sidelobes and high degree of reconfigurability can be used. However, it has been limited to single user environments. As WPM is a generalization of time division multiplexing (TDM) and frequency division multiplexing (FDM), it can be considered in a multiuser network with necessary modifications. Hence, a Wavelet Packet based Multi Carrier Multiple Access scheme called WP-MC-MA for CR applications is proposed in this work. Uplink and downlink system models and signal representations are explained. The performance of the system is evaluated using standard orthogonal wavelet basis functions and is compared with OFDMA. The error rate, spectrum efficiency and computational complexity of the proposed system are analyzed.

3.1 Introduction

The theory of wavelets and wavelet packets are well developed and the details are presented in numerous research papers and textbooks [79]-[84]. However, a brief introduction to wavelet packets and the unique characteristics of wavelet waveforms are given in this section. Since wavelet packets are a generalization of multi resolution analysis (MRA), it is begun with MRA.

A scaling function generates MRA if

- The translates (time shifted versions) $\phi(t - k)$ are linearly independent and produce a basis of the subspace V_0 [84].
- Dilates (scaled versions) $\phi(2^j - k)$ generate V_j , $j \in \mathbb{Z}$ such that

$$\dots \subset V_{-1} \subset V_0 \subset V_1 \subset \dots \subset V_j \subset \dots \quad (3.1)$$

$$\overline{\bigcup_{j=-\infty}^{\infty} V_j} = L_2(R), \quad \bigcap_{j=-\infty}^{\infty} V_j = 0 \quad (3.2)$$

- There is a wavelet $\psi(t)$ such that translates $\psi(t - k)$ are linearly independent and produce a basis of the subspace W_0 . This subspace is obtained from the difference between V_1 and V_0 . Mathematically,

$$V_1 = V_0 \oplus W_0 \quad (3.3)$$

The concept of MRA leads to the two scale relation of scaling function and wavelet. This validates a special feature of wavelet analysis - Wavelets are not generated from differential equations but of two scale equations [84]. In other words, $\phi(t)$ is the dilated translate of itself and can be expressed as

$$\phi(t) = \sum_n h(n)\phi(2t - n) \quad (3.4)$$

This is called the “refinement equation”. In a similar manner there exists a “wavelet equation” and wavelet $\psi(t)$ is given by

$$\psi(t) = \sum_n g(n)\psi(2t - n) \quad (3.5)$$

where $h(n)$ and $g(n)$ are the impulse responses of low pass and high pass (band pass) filters and solely determine the characteristics of the wavelets. The unique properties of wavelets determined by the filter coefficients are

- **Compact Support** - The scaling function is compactly supported if and only if the filter h has a finite support, and the support of both scaling function and filter are the same. The wavelet would be of compact support if h and g are FIR filters. If the support of the scaling function is $[N_1, N_2]$, then the wavelet support is $[(N_1 - N_2 + 1)/2, (N_2 - N_1 + 1)/2]$.
- **Orthonormality** - The wavelets and scaling waveforms will be orthogonal if the inherent filters are orthogonal. Orthonormality of the filters implies

$$\sum_n h(n)h(n - 2k) = \delta(k); \quad (3.6)$$

where $\delta(k)$ is the Kronecker delta function. Scaling function and wavelet are orthogonal to each other if

$$\sum_n g(n)h(n - 2k) = 0 \quad (3.7)$$

The basis wavelet obtained using orthonormal filter coefficients is called orthonormal basis.

- **Vanishing Moments** - A wavelet has m vanishing moments if and only if its scaling function can generate polynomials of degree smaller than or equal to m . While this property is used to describe the approximating power of scaling functions, in the wavelet case it has a “dual” usage, e.g. the possibility to characterize the order of isolated singularities. The number of vanishing moments is entirely determined by the coefficients $h[n]$ of the filter h . Daubechies has proved that, to generate an orthogonal wavelet with p vanishing moment, a filter h with minimum length $2p$ has to be used. Daubechies filters, which generate Daubechies wavelets, have a length of $2p$.
- **Symmetry** - Symmetric scaling functions and wavelets are important because they are used to build bases of regular wavelets over an interval, rather than the real axis. Daubechies has proved that, for a wavelet to be symmetric or antisymmetric, its filter must have a linear complex phase and the only scalar symmetric compactly supported conjugate mirror filter is Harr filter.

Figure 3.1: Wavelet Packet Tree

Equations (3.3) and (3.4) show the existence of a set of derived waveforms orthogonal to respective subspaces and are the dilated and translated forms of the parent scaling function and wavelet. This mutually orthogonal set of derived waveforms form a tree which is called the “wavelet packet tree”. The depth of the tree denotes the branches and is determined by the value of transform size “ j ”. There will be 2^j leaves or wavelet bins in the tree. Fig. 3.1 shows the part of a wavelet packet tree [86] showing the indexing of each derived waveform based on the level (dilation) and the frequency. Fig. 3.2 shows the basic wavelet packet decomposition of a signal [81] into its approximations (low pass) and details (High pass). Both diagrams portray the characteristics of wavelet packet transform to generate 2^j subbands or derived basis functions that are mutually orthogonal. The derived bases are exploited for modulating multi-user signals in Wavelet Packet Multiple Access (WPMA) scheme [85] and in Wavelet Packet Division Multiplexing (WPDM) [86]. The fundamental difference between WPMA and WPDM is in the receiver structure. WPMA uses recursive joint detector and WPDM uses coherent matched filter detector at the receiver. Both systems show that scalar wavelet packets can be employed for multi-user applications.

Both WPMA and WPDM are single carrier systems using scalar wavelet packets and hence are more applicable to flat fading wireless links. Wideband systems like CR have to mitigate the frequency selective nature of the channel and a spectrally efficient multicarrier multiple access scheme is needed. Hence a scalar wavelet based multicarrier multiple access scheme is proposed in this chapter. It can be viewed as a WPMA or WPDM system where each user is allotted a set of orthogonal basis or a WPM system with its subbands are allocated

Figure 3.2: Wavelet Packet Decomposition of signal 'S'

among various users. So it can provide the bandwidth efficiency and multi-carrier feature of WPM together with multi-user support of WPDM. The multiple signal transmission in parallel combat frequency selective fading channel distortions and the mutual orthogonality of derived waveforms insures lower multiple access interference (MAI). The use of wavelet base provides more flexibility in the design of transmitting waveform and the implementation of wavelet packets with fast Mallat's algorithm [79] provides a transceiver with reconfigurable transform size at moderate complexity. These features imply that the proposed system can be a promising candidate for CR based systems where spectrum efficiency, adaptivity and flexibility are mandatory. The system model and performance analysis of the proposed WP-MC-MA system are outlined in the following sections.

3.2 WP-MC-MA System for CR

The basic difference between OFDM and WPM is the replacement of IFFT and FFT blocks in the OFDM scheme by IDWPT and DWPT blocks respectively [48] as depicted in Fig.2.13. As in OFDM, the inverse transform (Synthesis) is used for generating the transmitted signal and the direct transform (Analysis) for retrieving the symbol [56]. The transmitted signal in discrete time domain $x[k]$, is the sum of successive modulated symbols, each of which is

constructed as the sum of M waveforms $\psi[k]$ individually amplitude modulated. This can be expressed as

$$x[k] = \sum_s \sum_{m=0}^{M-1} a_{s,m} \psi_m[k - sM] \quad (3.8)$$

where “ $a_{s,m}$ ” is the s^{th} baseband data symbol modulating the m^{th} waveform “ ψ_m ”. Better error performance is achieved in this scheme by choosing the waveforms $\psi_m[k]$ that are mutually orthogonal. Such a system can be extended to a multi-user network since the waveforms are orthogonal which is exploited in our proposed scheme. The basic multi-user CR network is shown in Fig. 3.3. It is assumed that a number of CR nodes (CPE) communicate with a central BS. The concept is adopted from the existing IEEE 802.22 standard of CR. This simplifies the downlink transmission and insures reliable spectrum sensing of the RF environment.

Figure 3.3: Basic multi user CR network

3.2.1 Uplink System Model

The basic uplink model of the proposed WP-MC-MA system can be described with the help of the block diagram shown in Fig. 3.4. It is based on two assumptions:

- 1) The spectrum sensing block in each CR node gives the exact information regarding the free bands.
- 2) CR nodes within a cell have Channel State Information (CSI) of their neighboring nodes.

The spectrum sensing scheme considered is a wavelet packet based spectrum sensing as

Figure 3.4: Uplink Schematic of WP-MC-MA system

given in [87]. The CR node scans its BOI and generates a wavelet packet coefficient vector $C_v(n)$. Based on the value of $C_v(n)$ a preliminary decision of free spectrum is made. The final spectrum decision and allocation block for CR in this scheme is a modified form of the generalized Carrier Assignment Scheme (CAS) and is called the modified CAS.

There are 3 different types of CAS that allocate subcarriers among active users in OFDMA based networks as detailed in [88]. In CR context, these schemes cannot be used as DSA network demand more flexibility and adaptivity. Firstly, the spectrum sensing information is vital in assigning subcarriers among different users. Secondly, CR nodes will have better environmental awareness than conventional transceiver models that can be used to allocate carriers more effectively to get better error performance and link quality compared to that

of existing schemes. The details of existing and modified CAS algorithms are described in Section 3.2.2.

The user's data is baseband encoded, channeled through P parallel streams where P is the number of subcarriers allotted for each user. The data stream is fed into modified CAS block, which will allocate subbands to the selected user based on a set of parameters. The symbol stream is allocated to P subbands within available R subcarriers and by inserting (R-P) zeros an R dimensional vector is obtained as in OFDMA [88]. Mathematically, this can be expressed as

$$d_{m,i}(n) = \begin{cases} c_{m,i}(n) & \text{if } n \in I_m; \\ 0 & \text{otherwise.} \end{cases} \quad (3.9)$$

where $c_{m,i}(n)$ represents the i^{th} constellation block of encoded data stream of the m^{th} user and I_m is the set of the indices of m^{th} subchannel assigned to m^{th} user. The vector d_m generated is input to the IDWPT block for waveform modulation and the serial data stream s_m is obtained. Unlike OFDM, the waveforms of wavelet packet are longer and overlapping and hence there is no need to insert CP [48].

The subbands generated by wavelet packet transform in a multiple access scheme should be orthogonal so as to ensure minimum MAI and hence orthogonal wavelet bases are to be chosen. In addition to orthogonality, wavelet bases should have compact support, regularity and frequency localization. Hence, an arbitrary choice of wavelet bases for multi-user CR applications will lead to undesirable results. The standard wavelets used in performance evaluation are tabulated below.

Name	Filter Length	Compact support	Orthogonality	Notation
Daubechies	20	Yes	Yes	db10
Symlet	20	Yes	Yes	sym10
Coiflet	18	Yes	Yes	coif3
Discrete Meyer	102	Yes	Yes	dmey

Table 3.1: Wavelet Specifications

3.2.2 Modified CAS

For a multicarrier scheme, the uplink process requires a carrier assignment scheme to be performed. According to [88], there exist three different types of CAS to allocate subcarriers

Figure 3.5: Different Carrier Allocation Schemes, (a) Fixed CAS, (b) Interleaved CAS, (c) Generalized CAS

among active users in OFDMA networks as shown in Fig.3.5. They are termed fixed CAS, interleaved CAS and generalized CAS. With fixed CAS, each subchannel is assigned a group of R/M adjacent subcarriers where R is the total number of free subbands and M is the total number of users. The method is simple but because of the lack of flexibility, this scheme is rarely used. The second method is the interleaved CAS where the subcarriers within a subchannel are spaced by frequency bands equal to the number of subchannels. This scheme is more flexible compared to fixed CAS and the retrieval of the information at the receiver is possible with moderate complexity. To completely exploit the frequency diversity of available subcarriers and to incorporate dynamic frequency allocation, the generalized CAS is used in most of the applications where each user is assigned subcarriers with the highest SNR that are currently available [88]. As generalized CAS provides high flexibility due to dynamic frequency allocation, its modified form is implemented within CR context to allocate free subbands among various users. Modified CAS in the uplink generates the subcarrier index set for each user based on the spectrum measurement vector and CSI. The proposed CAS functions are shown in the flowchart in Fig.3.6.

The basic inputs to the proposed CAS unit are the base band data stream, spectrum information vector and CSI. The spectrum information is the most important parameter in this algorithm which is not present in the generalized CAS algorithm. If the spectrum information indicates the availability of a free subband, CAS unit maps data stream to that subband. To fulfill this task, CAS unit checks for the CSI of various CR nodes of the network. SNR of the subband and that of various CR nodes is an important parameter for carrier allocation. If there are many CR nodes for the same subband with maximum SNR of that particular subband, other channel parameters will be taken into account for subband allocation. The CR node that can give best link quality will be allowed to map its

data stream to that subband. There are two iteration parameters- n and k . Incrementing n allows to check for availability of free subbands in the scanned spectrum. The parameter k is additionally given to allocate data stream only within subbands that satisfy the specific requirements of the algorithm. All other subbands will be padded with zeros so as to ensure no harmful interference to other users. Various steps involved in modified CAS are described by the accompanying algorithm below.

Figure 3.6: Flow chart of Modified CAS

Algorithm:

- 1: The received spectrum measurement vector $C_v(n)$ is compared with a predefined threshold to determine the available free bands.
- 2: CSI of each free subband available within the BOI is input to CAS.
- 3: CSI of the CR node under consideration along with the neighboring nodes are obtained.
- 4: Based on the information obtained, CAS assigns subbands for the CR node considered. The subband with maximum SNR is the primary choice.
- 5: If the specified CR as well as a competing neighbor has maximum SNR at the k^{th} subband, other CSI factors such as channel delay, phase offset will be taken into consideration to make the final decision.
- 6: Once the subband is fixed, final decision is made and that subband index is included in the spectrum index vector I_m and the n^{th} data stream of the user is mapped onto that subband.
- 7: The process is repeated until all the P data streams of the user are mapped accordingly. The remaining subcarriers carry no data and are padded with zeros.

Even though the proposed CAS algorithm allocates subcarriers effectively among CR nodes, it suffers from two drawbacks. Firstly, its performance depends on the probability of detection/probability of false alarm of the spectrum sensing method. Secondly, imperfect CSI collected from neighbouring CR nodes can cause incorrect allocation of free subbands. The remedy for the first problem is to use sensing method of minimum probability of missed detection. In the proposed system model, WP sensing with predefined threshold is used for detection. The threshold can be set adaptively for each subband using the technique proposed by David Donoho in [89]. Moreover, spectrum sensing can be done in two stages where the preliminary decision based on energy detection is verified by any feature extraction technique and is proposed in Chapter 5. Imperfect CSI is a common problem that exists in most of the CAS schemes. In CR context, this issue can be minimized by using more than one channel parameter for subband allocation. But in deep fades, this method also fail to give good results.

3.2.3 Uplink Signal Model

The signal transmitted by the m^{th} user can be written as

$$s_{wpm}(k) = \sum_{p=0}^{R-1} \sum_n d_{p,m}(n) \psi_m^{syn}(k - nR) \quad (3.10)$$

where $d_{p,m}(n)$ is the symbol stream mapped onto the subchannel of the m^{th} user as given before and ψ_m^{syn} is the wavelet packet synthesis waveform for the m^{th} user subchannels.

Signal received at the base station will be the composite signal of N_u user waveforms summed up which are subjected to individual frequency, phase and timing offset along with their corresponding fading components. The received signal can be expressed as

$$r(k) = \sum_{m=1}^{N_u} s'_{wpm}(k)h_m(k) + n(k) \quad (3.11)$$

where $n(k)$ is the noise and $s'_{wpm}(k)$ is wavelet packet modulated component of the m^{th} user and $h_m(k)$ is the impulse response (IR) of the corresponding fading channel and N_u is the total number of users. The signal component can be expressed as

$$s'_{wpm}(k) = e^{j\phi_m(k)} \sum_{p=0}^{R-1} \sum_n d_{p,m}(n) \psi_m^{syn}(k - nR - \tau_m) \quad (3.12)$$

The term $e^{j\phi_m}$ corresponds to the frequency and phase offset of the m^{th} user and is given by

$$\phi_m(k) = \frac{2\pi\epsilon_mk}{R} + \theta_m(k) \quad (3.13)$$

where θ_m is the phase noise component and ϵ_m is the relative frequency offset of the m^{th} user. R is the total number of subcarriers. The frequency offset is normalized to the inter carrier spacing. The integer timing offset τ_m is expressed in sampling periods. After channel equalization, the signal is retrieved through DWPT and the demodulated signal will be

$$\begin{aligned} \tilde{r}(k) &= \left(\sum_{m=1}^{N_u} s'_{wpm}(k) + n(k) \right) \psi_m^{anal}(k - nR) \\ &= \left(\sum_{m=1}^{N_u} e^{j\phi_m(k)} \sum_{p=0}^{R-1} \sum_n d_{p,m}(n) \psi_m^{syn}(k - nR - \tau_m) + n(k) \right) \psi_m^{anal}(k - nR) \end{aligned} \quad (3.14)$$

where ψ_m^{anal} represents the DWPT (analysis) waveforms. When the frequency, phase and timing offsets are nulled this expression can be simplified as

$$\begin{aligned} \tilde{r}(k) &= \left(\sum_{m=1}^{N_u} \sum_{p=0}^{R-1} \sum_n d_{p,m}(n) \psi_m^{syn}(k - nR) \right) \psi_m^{anal}(k - nR) + n(k) \psi_m^{anal}(k - nR) \\ &= \sum_{m=1}^{N_u} \sum_{p=0}^{R-1} \sum_n d_{p,m}(n) + n(k) \psi_m^{anal}(k - nR) \end{aligned} \quad (3.15)$$

since the wavelet basis used is orthonormal. It is evident that to maintain orthogonality among subcarriers during the detection process, proper timing and frequency estimation are required at the base station. The channel index set I_m is received at the BS through the control channel. This will help channel assignment for downlink.

3.2.4 The Downlink System

The downlink transmitter and receiver of the proposed system is shown in Fig. 3.7 and Fig. 3.8 respectively. The downlink transmission is almost equivalent to the basic wavelet packet modulation scheme but each transmitted block carries data stream of multiple users. The data stream of each user is divided into blocks after symbol mapping. The i^{th} block of the m^{th} user is denoted as $b_{m,i}$. Similar to the uplink, the CAS unit maps the P data symbols of each block onto subcarriers assigned to the corresponding user. The final R dimensional vector will have data stream of all the N_u users combined and hence the i^{th} block is given by

$$d_i = \sum_{m=1}^{N_u} d_{m,i} \quad (3.16)$$

Each data block is fed to the IDWPT modulator. The serial data stream obtained is input to the D/A converter and then to the RF modulator. At the receiver, the A/D output will be the composition of data blocks of all users. Frequency and timing errors are present in the signal received at the user terminal, similar to that in the BS. Hence the coarse frequency and timing estimation units employ the received sequence $r(k)$ to compute estimates of frequency and timing error. The frequency error estimate can be used to counter rotate $r(k)$ and the timing error estimate for correct positioning of the DWPT window. After the correction process, the data stream is input to DWPT block where the serial stream is decomposed into R subbands. The channel equalization block will compensate for channel impairments and fractional timing errors. For data detection, P subbands of the particular user is considered. For brevity, the estimation techniques at the receiver for uplink and downlink are excluded in the simulation study. In practice, frequency and phase estimation can be done using higher order statistics [90].

Figure 3.7: Downlink transmitter of WP-MC-MA

Figure 3.8: Downlink receiver of WP-MC-MA

3.3 Performance Evaluations

To evaluate the performance of the proposed system, a multi-user CR environment with four users is considered. It is assumed that the BOI is scanned with wavelet packet transform and a frequency band of sixteen subcarriers is found vacant. Equal number of subcarriers for all users is considered. Hence there are four subchannels with four subcarriers for each user in the simulated system. QPSK and 16-QAM are the baseband modulation schemes adopted in the simulation.

The user data stream is base band encoded, converted from serial to four parallel streams and is mapped onto sixteen subcarriers at the CAS. In IDWPT block, the data stream undergoes four iterations of upsampling and filtering to generate the serial data stream. It is assumed that there is perfect synchronization between the users and base station (BS). The channel index set I_m generated is known at the user end in order to separate the required data streams. The different user signals embedded in noise is demodulated, passed through the DWPT block for successive iterations of filtering and downsampling. Thus sixteen parallel streams are recovered and using the knowledge of subband index vector, the user data stream is extracted and detected. Orthogonal wavelet functions given in Table 3.1 are used to evaluate the performance of the proposed WP-MC-MA system. The notations of the wavelet based systems used in the plots are as given in Table 3.1. The results obtained for WP-MC-MA are compared with those of OFDMA system of four users and sixteen subcarriers. The BER curves in this work correspond to downlink scenario of WP-MC-MA and OFDMA. The effect of phase and frequency offset are considered for both uplink and downlink.

3.3.1 Error performance in single path fading channels

To incorporate the effects of flat fading, the channel gain is modeled as complex normal distribution $CN(0, 1)$, with variance 1 such that its real and imaginary parts are normally distributed with half the variance. Fig. 3.9 shows the BER in flat fading channel for downlink scenario of both wavelet and OFDMA system. It is assumed that the channel information is known at the receiver and is compensated. The different wavelets simulated in the work show similar performance. For both QPSK and 16-QAM modulation, the BER of the proposed system and that of OFDM are comparable.

Figure 3.9: BER in flat fading channel

Figure 3.10: BER in Rayleigh channel with Doppler spread

In Fig. 3.10 the effect of a single path Rayleigh fading channel with a maximum Doppler shift of 55 rad/sec is plotted. The channel model is generated based on Jakes model as given in [91]. It is assumed that the scatterers are uniformly distributed around a circle at angles α_n with k rays emerging from each scatterer. The doppler shift of ray n is given by

$$f_n = f_d \cos \alpha_n \quad (3.17)$$

where f_d is the maximum doppler shift. With M such scatterers, Rayleigh fading of k^{th} ray over time t can be given as

$$h(t, k) = J \left[\sum_{n=1}^M (\cos \beta_n + j \sin \beta_n) \cos (2\pi f_n t + \theta_{n,k}) + \cos 2\pi f_d t (\cos \alpha + j \sin \alpha) \right] \quad (3.18)$$

where the variables α_n , β_n and $\theta_{n,k}$ are model parameters and α_n and $\theta_{n,k}$ are set to zero to simulate the effect of single path Rayleigh fading. The value of β_n is chosen so that there is no cross-correlation between the real and imaginary parts of the fading component. J is a constant and according to [91] it is equivalent to $2\sqrt{2}$. From Fig. 3.10, it is evident that error performance does not show significant change with different wavelets used. The BER of OFDMA and WP-MC-MA is almost similar.

3.3.2 Error performance in multi-path fading channels

To evaluate the capability of the system to mitigate multi-path fading channel impairments, three path fading channels with fixed and Gaussian fading coefficients are considered. OFDMA system with and without CP is plotted as the latter system will have spectrum efficiency equivalent to that of the proposed wavelet based system. The CP of the simulated OFDMA system is taken as 25 percent of the total number of subcarriers and is equal to four. At the receiver, the wavelet based scheme uses a time domain zero forcing equalizer with three taps per sample as given in [92] and OFDMA uses a frequency domain zero forcing equalizer with one tap per subcarrier since the demodulated OFDM signal is in the frequency domain.

The fixed channel impulse response used for simulation is $h = [0.407 \ 1 \ 0.407]$. The coefficients are chosen such that the channel is frequency selective in nature and nulls are not present in the channel. The frequency response of the channel is as shown in Fig. 3.12 [92].

Figure 3.11: Frequency Response of the channel with fixed fading coefficients

It is evident that the channel gain decreases gradually without any nulls. The Rayleigh fading channel impulse response is given as $h = [r_1 + jp_1 \quad r_2 + jp_2 \quad r_3 + jp_3]$ where r and p are Gaussian random variables.

The error curves of both systems are plotted in Fig. 3.12 and 3.13. In both cases, OFDMA without CP gives high error rate as one tap frequency domain equalizer is not sufficient for nullifying multi-path channel distortions. When the channel coefficients are fixed, the wavelet based system performs nearly as OFDMA system with CP since the equalizer weights of the wavelet scheme can be designed such that the combined channel and equalizer impulse response will be zero at all sample points except the desired sample point. But under multi-path Rayleigh fading channel conditions, the proposed scheme gives a very poor performance since the time domain equalizer fails to combat the random amplitude and phase distortions. OFDMA system with CP gives a fairly good performance. As CP cannot be incorporated into wavelet systems, it is required to design a better equalization technique in order to improve the error performance in a multi-path Rayleigh fading channel.

Figure 3.12: BER in multi-path fading channel with fixed fading coefficients

Figure 3.13: BER in multi-path fading channel with Rayleigh fading coefficients

3.3.3 Effect of phase offset

In any multi carrier modulation, it is important to consider the effect of frequency, phase and timing offset. A brief study of the effects of frequency, phase and timing offset in wavelet packet modulation is given in [59]- [61]. In this work, the effect of phase, frequency and time offset on error performance in a multi-user environment is considered and for the simulation study, each one is considered independently.

The presence of phase noise will affect the multicarrier modulation in two ways. Firstly, it will rotate all the constellation symbols by equal shift which is approximately equal to the average phase noise. Secondly, it will introduce inter carrier interference (ICI) due to the spread of subcarriers to a larger bandwidth around the carrier frequency [59]. To simulate the effects of phase noise in a multi-user environment, the phase noise is expressed as a zero mean Gaussian noise. If the channel is assumed to be an AWGN channel and relative frequency error ϵ_m to be zero, the received signal at the base station can be written as

$$r(k) = \sum_{m=1}^N s'_{wpm}(k) e^{j\theta(k)} + n(k) \quad (3.19)$$

where $e^{j\theta(k)}$ is the phase noise component. When the phase noise is too small, the expression of $e^{j\theta(k)}$ can be simplified as

$$e^{j\theta(k)} = 1 + j\theta(k) \quad (3.20)$$

Fig. 3.14 and 3.15 show the effect of phase offset on BER in WP-MC-MA system and OFDMA. In Fig. 3.14 the BER of the system using different wavelets is plotted as a function of SNR, corresponding to a small phase noise with a variance of 0.001 so that equation (3.20) will be satisfied. In Fig. 3.15, the phase noise power is expressed in dB_m and its effect on BER is plotted corresponding to SNR of 15dB. Results are obtained for both QPSK and 16-QAM modulation. It is shown that irrespective of the type of wavelet used, both WP-MC-MA and OFDMA give equal performance for these modulation levels.

Fig. 3.16 shows the BER for uplink channel when users are subjected to distinct phase offsets but all users employ QPSK modulation. BER is plotted for user 1 affected by a phase noise variance of 0.01 and user 4 with 0.05. It can be seen that each user performance is unaffected by the phase alterations of neighboring users. The results are comparable for both OFDMA and WP-MC-MA system.

Figure 3.14: BER with phase noise variance=0.001

Figure 3.15: BER with phase noise expressed in dBm

Figure 3.16: BER with phase noise variance (Uplink)

3.3.4 Effect of frequency and timing offset

Frequency offset can occur due to Doppler shift or by misalignment between the transmitted carrier frequency and the locally generated carrier frequency at the receiver. It destroys orthogonality among subcarriers and hence causes severe ICI. To simulate the effect of frequency offset, the channel is assumed to be AWGN and the phase and timing offsets are assumed to be zero. The error curves are plotted for both WP-MC-MA and OFDMA.

Fig. 3.17 shows the effect of frequency error on performance. BER is plotted as a function of relative frequency offset for a constant SNR of 15dB and no error compensation is done at the receiver. From the plots it is evident that both systems are severely affected when the relative frequency offset is increased to 0.2 or more. OFDMA and WP-MC-MA results are comparable and the type of wavelet has no impact on performance. The BER for 16-QAM modulation is more severe and is not shown in the diagram. Fig. 3.18 shows the uplink channel with frequency offset. Each user experiences different frequency offsets and no compensation is done at BS. Relative frequency offset considered for user 1 is 0.05 and that for user 3 is 0.1. QPSK modulation is adopted for all users. Similar to downlink, as frequency offset increases, performance of the system degrades drastically. So frequency estimation and correction techniques are essential at the receiver to improve the performance

Figure 3.17: BER with relative frequency offset

Figure 3.18: BER relative frequency offset (Uplink)

of the system.

Figure 3.19: BER with time offset

Multicarrier systems are always vulnerable to time synchronization errors. Latency in timing will appear as phase shift and will rotate the constellation symbols at an angular speed of $\frac{-2\pi\tau k}{R}$ where k is the subcarrier index and τ is the timing offset. That is, the subcarriers of higher indices will be subjected to greater phase shifts [61]. Besides phase shift, timing offset can cause ICI and ISI. Fig. 3.19 shows the effect of timing offset on BER. Timing offset is simulated as a phase shift proportional to subcarrier index and both systems show similar degradation in performance. In practice, use of CP in OFDMA system can reduce ICI and ISI effects of timing offset.

3.3.5 Spectrum Efficiency and Sidelobe Suppression

CR targets secondary usage of spectrum and hence its transmitting waveform should possess maximum spectrum efficiency and minimum sidelobes. So it is crucial to check the spectrum efficiency and interference mitigation property of the proposed transmission system. It is well known that OFDM is more spectrally efficient than conventional FDM because of overlap of orthogonal subcarriers. But as mentioned in Chapter 2, to mitigate multi-path distortions of wireless channels, OFDM signals incorporate CP. The length of

CP is usually determined by the channel delay spread and as a thumb rule, it should be 25 percent of the total symbol duration. The insertion of CP will result in poor bandwidth efficiency compared to WP-MC-MA system where CP is not included. Regarding sidelobes, OFDMA inherits large sidelobes for its transmitting waveform due to rectangular pulse shape. Fig. 3.20 gives the PSD of WPM and OFDM and verifies that the sidelobe suppression capability of wavelet based systems. For “coif3” wavelet with filter length 18 the sidelobes are much lower compared to OFDM and for “discrete Meyer” with filter length 102 sidelobes are almost absent. It is evident that in WP-MC-MA systems the sidelobes can be controlled by adjusting the filter length. Thus the proposed system insures lower harmful interference to adjacent primary users and higher spectral efficiency by eliminating CP.

Figure 3.20: PSD of WPM and OFDM

3.3.6 Complexity Analysis

The FWT is asymptotically faster than the FFT requiring only $O(N)$ steps instead of $O(N \log N)$ where N is the transform size. In general, for a transform size N and filter length L , the total number of multiplications required for wavelet packet transform is always less than $2NL$ [84]. The required number of multiplications for the different wavelet systems and OFDM system for a transform size of 64 is tabulated in Table 3.2. The number of multiplications for smaller transform size is higher for wavelet based system. But due to

the iterative nature of WPT, the overall complexity can be reduced. To implement N stage WPT, it is only required to run a single occurrence of the elementary block at a clock speed of $N2^{N-1}$ times the symbol rate. Moreover, the transform size in WPT can be changed by changing the number of iterations and hence adaptive change in number of subbands is possible without increasing the complexity. As the filter length of wavelet base does not affect error performance, the complexity can be reduced by using filters of fewer number of coefficients. It can be concluded that for CR type of applications where flexibility and reconfigurability are crucial, WP-MC-MA can be a promising scheme than OFDMA.

Name	Filter Length	Transform Size	Number of Multiplications
Daubechies	20	64	2560
Symlet	20	64	2560
Coiflet	18	64	2304
Discrete Meyer	102	64	12816
OFDM	-	64	1024

Table 3.2: Complexity Analysis

3.4 Chapter Summary

In this work, a new multiple access scheme for CR using wavelet packet modulation is considered. The WP-MC-MA system incorporates both the flexibility and orthogonality of wavelet packet modulation and the multi-user support of OFDMA. A modified CAS unit is described for proper subcarrier assignment in CR. The proposed system outperforms OFDMA in terms of flexibility, spectrum efficiency, sidelobe suppression and reconfigurable transform size. The error performance in single path flat and frequency selective fading are equivalent to that of OFDMA. But the proposed system fails to mitigate multi-path channel impairments effectively. Hence, low complexity equalizer is essential for the proposed system.

Chapter 4

Spline Multiwavelets based multi-user CR network

In the previous chapter, the merits and demerits of scalar wavelet based multicarrier multiple access scheme have been analyzed. It is evident that WP-MC-MA system outperforms OFDMA by providing better sidelobe suppression, high spectral efficiency, flexibility and reconfigurability. But the proposed system suffers from high bit error rate in multi-path channels and higher computational complexity at lower transform size. It is required to explore additional features of wavelets which make them suitable for CR transmission. Hence, the new approach in wavelet theory called multiwavelets are explored in this chapter. The basic system models of spline multiwavelet based multi-user CR network for single carrier transmission scheme is first proposed in this work. The single carrier multi-user CR network using B-splines is described in detail by investigating the application of orthogonal B-spline multiwavelets of different orders. Linear, quadratic and cubic spline multiwavelets are used for modulating CR waveforms. The expression for BER in AWGN channel is derived. Auto correlation and cross correlation features of different splines and their impact on BER are briefly described. Simulations are done to examine the error performance and correlation characteristics. Results show that due to orthogonal and symmetric waveforms, spline multiwavelets could be an alternative to existing modulation schemes but with restricted number of users.

4.1 Introduction

Multiwavelets can be considered as the vector extension of scalar wavelet theory or as the special case of scalar wavelets [93]. That is, multiwavelets occur when single wavelet function cannot compose the basis of $L^2(R)$ by its dilates and translates but a set of functions is required. Therefore, unlike the standard scalar wavelets, multiwavelet families have more than one scaling and wavelet function and the number of wavelet functions is dubbed as their multiplicity. A set of functions $\psi_1, \psi_2, \dots, \psi_r \in L^2(R)$ are called orthogonal multiwavelets of multiplicity r if $\psi_1(2^j x - k), \psi_2(2^j x - k), \dots, \psi_r(2^j x - k), j, k \in Z$ form an orthonormal basis of $L^2(R)$ [94]. A multiwavelet system can simultaneously provide perfect reconstruction while preserving length (orthogonality), good performance at the boundaries (linear-phase symmetry) and a high order of approximation (vanishing moments) together with short support. It is practically difficult for a scalar wavelet to possess these features simultaneously. Therefore, multiwavelets include more degrees of freedom and additional advantages than scalar wavelets [95].

To explore the use of multiwavelets in CR based communication engineering, B-spline multiwavelets are considered in this chapter to implement a multi-user CR network. B-splines belong to the class of piecewise polynomials and can be generated by repeated convolution of the box function [96]. It is possible to obtain scaling and wavelet functions from B-splines of any order [97]. In the past, spline wavelets were not considered for major applications due to the high computational complexity involved in their formulation. Recently, Okkyung Cho and M. J. Lai [98] have proposed a simple method to obtain the spline scaling functions by using numerical approximation for the factorization of Laurent polynomial matrices and associated wavelets. Hence in the proposed work, spline wavelets using analytical method described in [98] is discussed and their performance in a multi-user CR network is evaluated. Multicarrier systems can be implemented for CR with multiwavelets by incorporating orthogonal and linear phase multifilter banks. This scheme can be advantageous for CR based systems in terms of bandwidth efficiency, data rate and throughput improvisation and the performance evaluation of such a system is discussed in the next chapter.

4.2 Single Carrier multi-user CR network with multiwavelets

The basic multi-user CR network similar to that discussed in Chapter 3 is considered in this section too. It is assumed that a number of CR nodes are communicating with a central BS. Basic block diagrams of uplink and downlink single carrier multi-user system using multiwavelets are shown in Fig. 4.1 - 4.4 . For uplink transmission, each CR node within a network performs identical spectrum scanning. The best free band is chosen based on the CSI obtained. The number of users that can be accommodated in a network depends on the type of wavelet and the modulation scheme used. In general, for single carrier networks, the maximum number of users is w where w is the number of wavelet bases used in the network. CR nodes within the same network share the same vacant band but the wavelet base used for modulation will be different and due to inherent orthogonality minimum interference is insured. At the receiver, each CR node generates its own unique wavelet base and is used

Figure 4.1: Single carrier CR uplink transmitter

for detecting the received signal.

The signal transmitted from the node is

$$s_m(k) = \sum_n \chi_m(n) e^{j2\pi k f_c} \quad (4.1)$$

where $\chi_m(n)$ is the m^{th} user baseband data stream modulated with the m^{th} orthogonal

Figure 4.2: Single carrier CR uplink receiver

Figure 4.3: Single carrier CR downlink transmitter

Figure 4.4: Single carrier CR downlink receiver

wavelet basis. The signal received at the BS will be the sum of all user waveforms and hence the composite signal $\sum_{m=0}^{N_u} s_m(k)$ embedded in noise will be obtained at BS where N_u is the total number of CR nodes in a network. The signal is demodulated, compensated for channel impairments and correlated with the wavelet base corresponding to each user to extract the information. BPSK or QPSK scheme can be used to map the baseband data stream to the wavelet base. In this work, it is assumed that spectrum sensing is already performed and a contiguous free band is available for transmission. The basic spline wavelet equations, their correlation features, multi-user interference and the steps to derive the BER of a spline based network are outlined briefly in the following sections.

4.2.1 B Spline Equations

Orthogonal B spline wavelets with compact support are considered in this work for simulation purpose. The steps in [98] are followed to generate B splines of three different degrees starting from 1. Order of a basis spline is one higher than its degree. As B-splines are generated by repeated convolution of box function, it is convenient to represent them in the Fourier domain. The Fourier transform (FT) of the B-spline of order m can be obtained by multiplying the lowest order B-spline FT, m times. The normalized spline function of order m is denoted by $N_m(x)$ and its FT is given by

$$N_m(f) = \left(\frac{1 - e^{-jf}}{jf} \right)^m \quad (4.2)$$

There are three scaling functions and three wavelet functions for spline of any order. The scaling and wavelet equations are given in [98]. For making this chapter self-contained, they are discussed in this section.

4.2.1.1 Linear Spline Equations

Linear spline wavelets are the simplest and of the lowest order waveforms among the B-spline multiwavelets. The basis spline function is a triangular function and hence called as linear splines. The complexity involved in generating the scaling and wavelet functions are

lower compared to other spline functions described in this work. The scaling functions are

$$\phi_1(x) = \sqrt{3}N_2(2x) \quad (4.3)$$

$$\begin{aligned} \phi_2(x) = \frac{\sqrt{165}}{11}N_2(2x) - \frac{4(2 + \sqrt{5})}{\sqrt{33}}N_2(4x) \\ + \frac{4(2 - \sqrt{5})}{\sqrt{33}}N_2(4x - 2) \end{aligned} \quad (4.4)$$

$$\phi_3(x) = \sum_{j=0}^2 \alpha_j N_2(2x - j) + \sum_{k=0}^3 \beta_k N_2(2x - k) \quad (4.5)$$

where $N_2(x)$ is the time domain basis spline of order 2 and is given by

$$N_2(x) = \frac{1}{2} \sum_{j=0}^{m+1} (-1)^j \binom{m+1}{j} [x + m/2 - j]_+^m \quad (4.6)$$

where $\binom{m+1}{j} = {}^{m+1}C_j$, $[x]_+^m = [\max(0, x)]^m$ and $m=2$. The three associated wavelets of linear B-spline are the linear combinations of scaling functions and hence the coefficients for each will be different. The equations are as given below.

$$\psi_1(x) = \sum_{j=0}^3 \alpha_j \sqrt{2} \phi_1(2x - j) + \sum_{k=0}^3 \beta_k \sqrt{2} \phi_2(2x - k) + \sum_{l=0}^3 \gamma_l \sqrt{2} \phi_3(2x - l) \quad (4.7)$$

$$\psi_2(x) = \sum_{j=0}^3 \alpha'_j \sqrt{2} \phi_1(2x - j) + \sum_{k=0}^3 \beta'_k \sqrt{2} \phi_2(2x - k) + \sum_{l=0}^3 \gamma'_l \sqrt{2} \phi_3(2x - l) \quad (4.8)$$

$$\psi_3(x) = \sum_{j=0}^3 \alpha''_j \sqrt{2} \phi_1(2x - j) + \sum_{k=0}^3 \beta''_k \sqrt{2} \phi_2(2x - k) + \sum_{l=0}^3 \gamma''_l \sqrt{2} \phi_3(2x - l) \quad (4.9)$$

The values of the coefficients α , β and γ for both scaling and wavelet functions are equivalent to that in [98].

4.2.1.2 Quadratic Spline Equations

Quadratic splines are one order higher than linear splines. They are smooth waveforms but of higher complexity. The basis spline is a Gaussian shaped waveform and is given by

$$N_3(x) = \frac{1}{6} \sum_{j=0}^{m+1} (-1)^j \binom{m+1}{j} [x + (m+1)/2 - j]_+^m \quad (4.10)$$

where $m=3$. Scaling functions and the associated wavelet functions are given as

$$\phi_1(x) = \sum_{j=0}^6 \alpha_j N_3(2x - j) + \sum_{k=4}^{15} \beta_k N_3(4x - k) \quad (4.11)$$

$$\phi_2(x) = \sum_{j=0}^6 \alpha'_j N_3(2x - j) + \sum_{k=4}^{15} \beta'_k N_3(4x - k) \quad (4.12)$$

$$\phi_3(x) = \sum_{j=0}^6 \alpha''_j N_3(2x - j) + \sum_{k=4}^{15} \beta''_k N_3(4x - k) \quad (4.13)$$

$$\psi_1(x) = \sum_{j=0}^7 \alpha_j \sqrt{2} \phi_1(2x - j) + \sum_{k=0}^7 \sqrt{2} \beta_k \phi_2(2x - k) + \sum_{l=0}^7 \gamma_l \sqrt{2} \phi_3(2x - l) \quad (4.14)$$

$$\psi_2(x) = \sum_{j=0}^7 \alpha'_j \sqrt{2} \phi_1(2x - j) + \sum_{k=0}^7 \beta'_k \sqrt{2} \phi_2(2x - k) + \sum_{l=0}^7 \gamma'_l \sqrt{2} \phi_3(2x - l) \quad (4.15)$$

$$\psi_3(x) = \sum_{j=0}^7 \alpha''_j \sqrt{2} \phi_1(2x - j) + \sum_{k=0}^7 \beta''_k \sqrt{2} \phi_2(2x - k) + \sum_{l=0}^7 \gamma''_l \sqrt{2} \phi_3(2x - l) \quad (4.16)$$

4.2.1.3 Cubic Spline Equations

The basis spline of cubic wavelets is of order four and can be given as

$$N_4(x) = \frac{1}{24} \sum_{j=0}^{m+1} (-1)^j \binom{m+1}{j} [x + m/2 - j]_+^m \quad (4.17)$$

and $m=4$. Scaling function equations are

$$\phi_1(x) = \sum_{j=0}^8 \alpha_j N_4(2x - j) + \sum_{k=4}^{20} \beta_k N_4(4x - k) \quad (4.18)$$

$$\phi_2(x) = \sum_{j=0}^9 \alpha'_j N_4(2x - j) + \sum_{k=4}^{20} \beta'_k N_4(4x - k) \quad (4.19)$$

$$\phi_3(x) = \sum_{j=0}^8 \alpha''_j N_4(2x - j) + \sum_{k=4}^{20} \beta''_k N_4(4x - k) \quad (4.20)$$

The associated wavelet functions are

$$\psi_1(x) = \sum_{j=1}^9 \alpha_j \sqrt{2} \phi_1(2x - j) + \sum_{k=1}^9 \beta_k \sqrt{2} \phi_2(2x - k) + \sum_{l=1}^9 \gamma_l \sqrt{2} \phi_3(2x - l) \quad (4.21)$$

$$\psi_2(x) = \sum_{j=1}^9 \alpha'_j \sqrt{2} \phi_1(2x - j) + \sum_{k=1}^9 \beta'_k \sqrt{2} \phi_2(2x - k) + \sum_{l=1}^9 \gamma'_l \sqrt{2} \phi_3(2x - l) \quad (4.22)$$

$$\psi_3(x) = \sum_{j=1}^9 \alpha''_j \sqrt{2} \phi_1(2x - j) + \sum_{k=1}^9 \beta''_k \sqrt{2} \phi_2(2x - k) + \sum_{l=1}^9 \gamma''_l \sqrt{2} \phi_3(2x - l) \quad (4.23)$$

4.2.2 Auto Correlation, Cross Correlation and Multi-user Interference

The error performance of a multi user CR system depends on the interference caused by neighbouring users who share the same spectrum. In any single carrier orthogonal waveform based system, this Multi User Interference (MUI) is directly proportional to cross correlation between the waveforms. In the proposed spline based system, the data is mapped on the orthogonal wavelets and it is retrieved by generating the exact wavelet base at the receiver. Hence, better error performance can be achieved only if the auto correlation is maximum and cross correlation is minimum at the sampling instants. It is well known that the auto correlation function of a basis spline of degree n is a higher degree spline of degree $2n + 1$ [99]. Moreover, the orthonormal feature of the wavelets are invariant for integer shifts and hence even when user signals are nearly synchronized autocorrelation value will be fairly good.

Cross correlation among spline multiwavelets can be of two types. The cross correlation between wavelets of same order can be termed as intra cross correlation and that between wavelets of different order can be called inter cross correlation. If the inter cross correlation is good enough, splines of different order can be used within a network to increase the number of users without compromising on the performance. Inter cross correlation has three different cases: (i) between quadratic and linear, (ii) between cubic and quadratic and (iii) between linear and cubic. In each case, nine different combinations are possible. In this work, auto correlation, intra and inter cross correlation of different spline scaling functions are simulated and plotted.

4.2.3 BER calculation in AWGN Channel

To evaluate the BER of the proposed system, the uplink channel of a BPSK modulated CR network is considered. The signal transmitted from the m^{th} user can be written as

$$s^{(m)}(t) = \sqrt{\frac{2E_b}{T}} \sum_k a^{(k)} p_m(t - kT) \quad (4.24)$$

where E_b is the bit energy, T is the symbol period, $p_m(t)$ is the wavelet base of the m^{th} CR node, ' a ' is the symbol variable that takes the values +1 or -1 depending on the transmitted symbol and k denotes the symbol sampling instant. Received signal at the BS is a composite signal of all the N_u users present in the network along with noise and channel impairments. For analytical simplicity, AWGN channel is considered. However, in the performance evaluation section, the effect of single path and multi-path frequency selective fading channels are analyzed and various graphs are plotted.

The signal at BS can be given as

$$r(t) = \sum_{m=1}^{N_u} s^{(m)}(t) + n(t) \quad (4.25)$$

The correlator extracts the k^{th} symbol of the desired user data and let $m=1$ be the desired user. Hence the output of the correlator is

$$R = \int_0^T r(t) p_1(t - kT) dt \quad (4.26)$$

$$R = \int_0^T \left[\sum_{m=1}^{N_u} s^{(m)}(t) + n(t) \right] p_1(t - kT) dt \quad (4.27)$$

This can be expanded as shown below

$$\begin{aligned} R = & \int_0^T \sum_{m=2}^{N_u} s^{(m)}(t) p_1(t - kT) dt + \int_0^T n(t) p_1(t - kT) dt \\ & + \int_0^T \left[\sqrt{\frac{2E_b}{T}} \sum_k a^{(k)} p_1(t - kT) \right] p_1(t - kT) dt \end{aligned} \quad (4.28)$$

The first term of (4.25) is the multi-user interference (MUI) caused by $N_u - 1$ users. The second term is the noise component and last term is the useful signal component. Using

(4.21), the MUI can be expressed as

$$\begin{aligned}
 MUI &= \sum_{m=2}^{N_u} \int_0^T \sqrt{\frac{2E_b}{T}} \sum_{k'} a^{(k')} p_m(t - k'T) p_1(t - kT) dt \\
 &= \sum_{m=2}^{N_u} \sqrt{\frac{2E_b}{T}} \sum_{k'} a^{(k')} \int_0^T p_m(t - k'T) p_1(t - kT) dt \\
 &= \sum_{m=2}^{N_u} \sqrt{\frac{2E_b}{T}} \sum_{k'} a^{(k')} C_{m,d}
 \end{aligned} \tag{4.29}$$

where $C_{m,d}$ represents the cross correlation between the wavelet base of the desired user and the m^{th} user. It is evident that the interference is a measure of the cross correlation between the wavelet bases used in the network. Hence, the interference and BER will be minimum in a network where the cross correlation between wavelets is the lowest. As the correlator output expression is similar to that of a BPSK based Direct Sequence Spread Spectrum (DS-SS) system, BER of the proposed system with BPSK modulation can be written as

$$P_B = Q \left(\sqrt{\frac{2E_b}{(MAI + N_0)}} \right) \tag{4.30}$$

where MAI denotes the multiple access interference or MUI which is given by equation (4.29) and N_0 is the two sided noise power spectral density.

4.2.4 Cross correlation value under different cases

As the value of BER mainly depends on the cross correlation between the wavelets, it is required to derive the expression for cross correlation in different conditions and two major cases are discussed below.

4.2.4.1 Case (I) When spline wavelets of the same order are used-Intra Cross Correlation

If each user modulates the waveform with multiwavelets of the same order, the cross correlation will be zero with perfect synchronization. As splines are shift orthogonal for all integer translates up to its support length, the orthogonality will be maintained for limited number of integer shifts. Hence, in the absence of noise the MAI will be zero. Under fading

channel conditions, the time shifts would not be integers and the hence orthogonality could be lost even in the absence of white noise. The amount of cross correlation depends on the type of wavelet. As cubic wavelets are the highest order wavelet used in our simulation, steps to calculate cross correlation between two cubic spline wavelets is discussed first.

Let $C_{m,n}$ denotes cross correlation between m^{th} and n^{th} user waveforms that employs two cubic wavelets for modulation. Then

$$C_{m,n}^{cubic} = \int \left(\sum_{j=1}^9 \alpha_j \sqrt{2} \phi_1(2(x-\tau)-j) + \sum_{k=1}^9 \beta_k \sqrt{2} \phi_2(2(x-\tau)-k) \right. \\ \left. \sum_{l=1}^9 \gamma_l \sqrt{2} \phi_3(2(x-\tau)-l) \right) \left(\sum_{p=1}^9 \alpha'_p \sqrt{2} \phi_1(2(x-\tau')-p) \right. \\ \left. \sum_{q=1}^9 \beta'_q \sqrt{2} \phi_2(2(x-\tau')-q) + \sum_{r=1}^9 \gamma'_r \sqrt{2} \phi_3(2(x-\tau')-r) \right) d(\tau - \tau') \quad (4.31)$$

where τ and τ' are the time shifts of users m and n respectively. The final expression obtained is

$$C_{m,n}^{cubic} = \sum_{j=1}^9 \sum_{p=1}^9 2\alpha_j \alpha'_p R_{\phi_1}(2(\tau - \tau') + j - p) + \sum_{j=1}^9 \sum_{q=1}^9 2\alpha_j \beta'_q R_{\phi_{1,2}}(2(\tau - \tau') + j - q) \\ + \sum_{k=1}^9 \sum_{q=1}^9 2\beta_k \beta'_q R_{\phi_2}(2(\tau - \tau') + k - q) + \sum_{k=1}^9 \sum_{r=1}^9 2\beta_k \gamma'_r R_{\phi_{2,3}}(2(\tau - \tau') + k - r) \\ + \sum_{l=1}^9 \sum_{r=1}^9 2\gamma_l \gamma'_r R_{\phi_3}(2(\tau - \tau') + l - r) + \sum_{j=1}^9 \sum_{r=1}^9 2\alpha_j \gamma'_r R_{\phi_{1,3}}(2(\tau - \tau') + j - r) \\ + \sum_{k=1}^9 \sum_{p=1}^9 2\alpha'_p \beta_k R_{\phi_{2,1}}(2(\tau - \tau') + k - p) + \sum_{l=1}^9 \sum_{p=1}^9 2\alpha'_p \gamma_l R_{\phi_{3,1}}(2(\tau - \tau') + l - p) \\ + \sum_{l=1}^9 \sum_{q=1}^9 2\gamma_l \beta'_q R_{\phi_{3,2}}(2(\tau - \tau') + l - q) \quad (4.32)$$

where R_{ϕ_m} denotes the auto correlation of the m^{th} scaling function and $R_{\phi_{m,n}}$ denotes the cross correlation between m^{th} and n^{th} scaling function. The equation implies that the intra cross correlation between wavelets is a measure of the cross correlation and autocorrelation of the scaling functions.

In a similar fashion, the cross correlation between two quadratic splines can also be obtained

as follows.

$$C_{m,n}^{quadra} = \int \left(\sum_{j=0}^7 \alpha_j \sqrt{2} \phi_1(2(x-\tau)-j) + \sum_{k=0}^7 \beta_k \sqrt{2} \phi_2(2(x-\tau)-k) \right. \\ \left. \sum_{l=0}^7 \gamma_l \sqrt{2} \phi_3(2(x-\tau)-l) \right) \left(\sum_{p=0}^7 \alpha'_p \sqrt{2} \phi_1(2(x-\tau')-p) \right. \\ \left. \sum_{q=0}^7 \beta'_q \sqrt{2} \phi_2(2(x-\tau')-q) + \sum_{r=0}^7 \gamma'_r \sqrt{2} \phi_3(2(x-\tau')-r) \right) d(\tau - \tau') \quad (4.33)$$

The final expression will be

$$C_{m,n}^{quadra} = \sum_{j=0}^7 \sum_{p=0}^7 2\alpha_j \alpha'_p R_{\phi_1}(2(\tau - \tau') + j - p) + \sum_{j=0}^7 \sum_{q=0}^7 2\alpha_j \beta'_q R_{\phi_{1,2}}(2(\tau - \tau') + j - q) \\ + \sum_{k=0}^7 \sum_{q=0}^7 2\beta_k \beta'_q R_{\phi_2}(2(\tau - \tau') + k - q) + \sum_{k=0}^7 \sum_{r=0}^7 2\beta_k \gamma'_r R_{\phi_{2,3}}(2(\tau - \tau') + k - r) \\ + \sum_{l=0}^7 \sum_{r=0}^7 2\gamma_l \gamma'_r R_{\phi_3}(2(\tau - \tau') + l - r) + \sum_{j=0}^7 \sum_{r=0}^7 2\alpha_j \gamma'_r R_{\phi_{1,3}}(2(\tau - \tau') + j - r) \\ + \sum_{k=0}^7 \sum_{p=0}^7 2\alpha'_p \beta_k R_{\phi_{2,1}}(2(\tau - \tau') + k - p) + \sum_{l=0}^7 \sum_{p=0}^7 2\alpha'_p \gamma_l R_{\phi_{3,1}}(2(\tau - \tau') + l - p) \\ + \sum_{l=0}^7 \sum_{q=0}^7 2\gamma_l \beta'_q R_{\phi_{3,2}}(2(\tau - \tau') + l - q) \quad (4.34)$$

Finally, the cross correlation between two linear wavelets can be given as

$$C_{m,n}^{linear} = \int \left(\sum_{j=0}^3 \alpha_j \sqrt{2} \phi_1(2(x-\tau)-j) + \sum_{k=0}^3 \beta_k \sqrt{2} \phi_2(2(x-\tau)-k) \right. \\ \left. \sum_{l=0}^3 \gamma_l \sqrt{2} \phi_3(2(x-\tau)-l) \right) \left(\sum_{p=0}^3 \alpha'_p \sqrt{2} \phi_1(2(x-\tau')-p) \right. \\ \left. \sum_{q=0}^3 \beta'_q \sqrt{2} \phi_2(2(x-\tau')-q) + \sum_{r=0}^3 \gamma'_r \sqrt{2} \phi_3(2(x-\tau')-r) \right) d(\tau - \tau') \quad (4.35)$$

Similar to cubic and quadratic splines, the final expression will have nine terms and can be given as

$$\begin{aligned}
C_{m,n}^{linear} = & \sum_{j=0}^3 \sum_{p=0}^3 2\alpha_j \alpha'_p R_{\phi_1}(2(\tau - \tau') + j - p) + \sum_{j=0}^3 \sum_{q=0}^3 2\alpha_j \beta'_q R_{\phi_{1,2}}(2(\tau - \tau') + j - q) \\
& + \sum_{k=0}^3 \sum_{q=0}^3 2\beta_k \beta'_q R_{\phi_2}(2(\tau - \tau') + k - q) + \sum_{k=0}^3 \sum_{r=0}^3 2\beta_k \gamma'_r R_{\phi_{2,3}}(2(\tau - \tau') + k - r) \\
& + \sum_{l=0}^3 \sum_{r=0}^3 2\gamma_l \gamma'_r R_{\phi_3}(2(\tau - \tau') + l - r) + \sum_{j=0}^3 \sum_{r=0}^3 2\alpha_j \gamma'_r R_{\phi_{1,3}}(2(\tau - \tau') + j - r) \\
& + \sum_{k=0}^3 \sum_{p=0}^3 2\alpha'_p \beta_k R_{\phi_{2,1}}(2(\tau - \tau') + k - p) + \sum_{l=0}^3 \sum_{p=0}^3 2\alpha'_p \gamma_l R_{\phi_{3,1}}(2(\tau - \tau') + l - p) \\
& + \sum_{l=0}^3 \sum_{q=0}^3 2\gamma_l \beta'_q R_{\phi_{3,2}}(2(\tau - \tau') + l - q) \quad (4.36)
\end{aligned}$$

4.2.4.2 Case (II) When spline wavelets of different orders are used - Inter Cross Correlation

In this case, even when the user signals are perfectly synchronized, cross correlation will obtain a non zero value since the wavelets of different orders do not possess inherent orthogonality. The value of cross correlation between wavelets will be the measure of cross correlation between scaling functions of different orders. Derivation of this expression for different splines would follow similar behaviour as basis splines of different orders are obtained from the repeated convolution process. The final expression for cross correlation of three different cases are given - (i) between cubic and quadratic, (ii) between cubic and linear and (iii) between quadratic and linear.

The cross correlation between cubic and quadratic splines can be given as

$$\begin{aligned}
C_{m,n}(c, q) = & \int \left(\sum_{j=1}^9 \alpha_j \sqrt{2} \phi_1^c(2(x - \tau) - j) + \sum_{k=1}^9 \beta_k \sqrt{2} \phi_2^c(2(x - \tau) - k) \right. \\
& \left. \sum_{l=1}^9 \gamma_l \sqrt{2} \phi_3^c(2(x - \tau) - l) \right) \left(\sum_{j'=0}^7 \alpha_{j'} \sqrt{2} \phi_1^q(2(x - \tau') - j') \right. \\
& \left. + \sum_{k'=0}^7 \beta_{k'} \sqrt{2} \phi_2^q(2(x - \tau') - k') \sum_{l'=0}^7 \gamma_{l'} \sqrt{2} \phi_3^q(2(x - \tau') - l') \right) \quad (4.37)
\end{aligned}$$

The final expression will comprise of nine terms and its simplified form is given as

$$\begin{aligned}
C_{m,n}^{(c,q)} = & \sum_{m=1}^3 \sum_{j=1}^9 \sum_{j'=0}^7 2A_j^c B_{j'}^q R_{\phi_{m,m}}^{c,q} (2(\tau - \tau') + j - j') \\
& + \sum_{m=1, n=1, m \neq n}^3 \sum_{j=1}^9 \sum_{k'=0}^7 2A_j^c B_{k'}^q R_{\phi_{m,n}}^{c,q} (2(\tau - \tau') + j - k') \quad (4.38)
\end{aligned}$$

where $R_{\phi_{m,m}}^{c,q}$ denotes the cross correlation between ϕ_m of cubic and quadratic splines and $R_{\phi_{m,n}}^{c,q}$ represents that between ϕ_m and ϕ_n of cubic and quadratic. A^c and B^q corresponds to the respective coefficient values of each wavelet function of cubic and quadratic splines. It is evident that both inter and intra cross correlations depend on wavelet coefficients and cross correlation of scaling functions. Hence, MUI can be minimized by using the waveforms with minimum inter and intra cross correlation.

Following the same steps, cross correlation between cubic and linear wavelets can be found. The general expression will be

$$\begin{aligned}
C_{m,n}^{(c,l)} = & \sum_{m=1}^3 \sum_{j=1}^9 \sum_{j'=0}^3 2A_j^c B_{j'}^l R_{\phi_{m,m}}^{c,l} (2(\tau - \tau') + j - j') \\
& + \sum_{m=1, n=1, m \neq n}^3 \sum_{j=1}^9 \sum_{k'=0}^3 2A_j^c B_{k'}^l R_{\phi_{m,n}}^{c,l} (2(\tau - \tau') + j - k') \quad (4.39)
\end{aligned}$$

Similarly, the cross correlation between quadratic and linear splines can be given as

$$\begin{aligned}
C_{m,n}^{(q,l)} = & \sum_{m=1}^3 \sum_{j=0}^7 \sum_{j'=0}^3 2A_j^q B_{j'}^l R_{\phi_{m,m}}^{q,l} (2(\tau - \tau') + j - j') \\
& + \sum_{m=1, n=1, m \neq n}^3 \sum_{j=0}^7 \sum_{k'=0}^3 2A_j^q B_{k'}^l R_{\phi_{m,n}}^{q,l} (2(\tau - \tau') + j - k') \quad (4.40)
\end{aligned}$$

4.3 Simulation Results

Spline multiwavelets of order two (linear), three (quadratic) and four (cubic) are generated using equations (4.5) to (4.23) given in section 4.2.1. Linear multiwavelets are shown in Fig. 4.5 as an example. The unit of time is taken in seconds and is marked on X axis and the normalized amplitude of the waveform is given on Y axis. After generating the waveforms, auto correlation and cross correlation properties are analyzed.

Figure 4.5: Linear spline multiwavelets

Fig. 4.6 portrays the auto correlation of all spline scaling functions. The lag range or shifts of the waveform in terms of number of samples is given on X axis and the normalized amplitude of the auto correlation function result in Y axis. Upon perfect synchronization, all splines give good auto correlation values at the sampling instant. Fig. 4.7 shows the intra cross correlation property of linear, quadratic and cubic spline scaling functions with same axes as that of auto correlation whereas Fig. 4.8 shows a few of the inter cross correlation graphs. Cubic splines have better intra cross correlation characteristics than quadratic and linear wavelets as the amplitude at zero (sampling instant where autocorrelation is maximum) are lower for cubic splines. So it is expected that cubic based system can produce better error performance compared to lower order spline based systems in a multi-user network.

To plot the error curves, BPSK modulated data stream is considered and baseband signal is of the form

$$\chi_m(n) = \begin{cases} \psi_m(n) & \text{for bit} = 1; \\ -\psi_m(n) & \text{for bit} = 0. \end{cases} \quad (4.41)$$

The signal is then up converted and transmitted and each user is identified by his own unique wavelet base. Simple correlator receiver is used for detection. All error curves represent the downlink channel of the CR network.

Figure 4.6: Auto Correlation of linear, quadratic and cubic spline Scaling Functions

Figure 4.7: Intra Cross Correlation of linear, quadratic and cubic spline Scaling Functions

Figure 4.8: Inter Cross Correlation of linear, quadratic and cubic spline Scaling Functions

Figure 4.9: Variation of BER in flat fading Channel showing the effect of intra cross correlation

4.3.1 Error performance under single path fading channels

In Section 4.2.3, it is shown that the BER of the proposed system depends on MUI and hence the cross correlation properties of wavelets. Performance curves showing the effect of intra and inter cross correlation in different channel conditions are obtained. Fig. 4.9 shows the effect of intra cross correlation of different spline wavelets on BER in a flat fading channel. The maximum number of nodes is limited to three as wavelet bases of same order are used within the network. The BER curves of single user splines of any order are almost similar. With increasing order, as the intra cross correlation of spline improves, the BER also improves in a multi-user environment. Cubic spline based multi-user system shows lower BER compared to quadratic and linear based spline systems in the case of two user and three user networks.

As CR mainly targets wide band RF spectrum, the channel will be of frequency selective nature rather than simple AWGN. Hence to obtain the error performance in a more realistic channel environment, frequency selective Rayleigh fading together with AWGN channel is simulated for a Doppler shift of 60Hz and the results are plotted in Figs. 4.10- 4.12. The channel impulse response coefficients are generated such that baseband symbols of one signal

Figure 4.10: BER curves showing the effect of intra cross correlation (Frequency selective fading and AWGN channel)

Figure 4.11: BER curves showing the effect of inter cross correlation (Frequency selective fading and AWGN Channel)

block undergoes different fades. Fig.4.10 shows the effect of intra cross correlation in the frequency selective Rayleigh fading channel with AWGN. The error curves validates the cross correlation properties of splines as cubic wavelets gives the minimum BER and linear wavelets produces the maximum BER.

Fig. 4.11 gives the effect of inter cross correlation of quadratic and cubic spline wavelets on BER for the frequency selective fading condition. To achieve any specific BER, slight SNR enhancement is required with increase in number of users. In order to examine the cost of expanding the network from three users to six users, the three user cubic spline graph is also plotted. For a BER of 10^{-2} , the SNR enhancement required for a six user network compared to three user is approximately 3dB. Moreover, the required SNR enhancement level decreases as the number of users increases. The SNR enhancement required to increase the number of users from three to four is approximately 2 dB, four to five is less than 1 dB and from five to six is nearly 0 dB. Hence, simulation results support the concept of exploiting inter cross correlation of spline wavelets for the expansion of network.

Figure 4.12: BER comparison of different four user networks (Frequency selective fading and AWGN Channel)

To compare the performance of the proposed system with an existing scheme, a sixteen subcarrier OFDMA system with four users is simulated. To obtain a four user network with multiwavelets, it is required to consider B-spline wavelets of different orders since there are only three wavelet bases for spline of any order. Quadratic and cubic wavelets

are used for implementing four user CR network. BER comparison of different four user networks of spline based system incorporating quadratic and cubic splines with a four user OFDMA system in a single path frequency selective fading channel is given in Fig. 4.12. It is evident that the single carrier spline based system gives a moderate error performance compared to the multicarrier OFDMA scheme in a frequency selective fading channel. In the worst case scenario, the spline based system requires an SNR enhancement of 1dB to attain the same BER characteristics of OFDMA.

Figure 4.13: BER performance for QPSK modulation (Frequency selective fading and AWGN Channel)

To examine the error performance of the proposed system with higher order baseband modulation schemes, four user network employing QPSK modulation is simulated. The error performance under single path Rayleigh fading channel for a maximum Doppler shift of 20 rad/sec is evaluated and is shown in Fig. 4.13. To achieve a specific BER of 10^{-2} , the SNR enhancement required for a six user network compared to three user is less than 3dB. It is shown that for higher order modulation schemes also, expansion of network is possible with an acceptable increment in power.

Figure 4.14: BER comparison of different four user networks (Multi-path fading Channel where FFC-Fixed Fading Coefficients and RFC-Rayleigh Fading Coefficients)

4.3.2 Error performance under multi-path fading channels

To evaluate the capability of the system to mitigate multi-path fading channel impairments, three path fading channel with fixed and Gaussian fading coefficients are considered. CP of the simulated OFDMA system is 25 percent of the total number of subcarriers which is equal to four. The channel delay is kept lower than CP of the OFDMA system, thereby reducing the equalizer complexity at the receiver side. The wavelet based receiver uses a time domain zero forcing equalizer with three taps per sample as given in [92] and OFDMA uses a frequency domain equalizer with one tap per subcarrier to compensate for channel distortions. It is assumed that the channel impulse response is known at the receiver.

The fixed channel impulse response used for simulation is $h = [0.407 \ 1 \ 0.407]$. These coefficients are chosen such that the frequency response of the channel represents low pass filter characteristics. The Rayleigh fading channel impulse response can be given as $h = [r_1 + jp_1 \ r_2 + jp_2 \ r_3 + jp_3]$ where each r and p are normally distributed Gaussian random variables. The error curves of both systems are plotted in Fig. 4.14. When the channel coefficients are fixed the wavelet based system outperforms OFDMA system since the equalizer weights of the wavelet scheme can be designed such that the combined channel and equalizer impulse response will be zero at all sample points except the desired sample

point. All four user networks using the proposed wavelet system show better performance than OFDMA. Among the simulated wavelet systems, the network incorporating three cubic splines and one quadratic spline gives the best performance since the inter cross correlation is minimum in that typical network. But when the channel coefficients generate Rayleigh fading multi-paths, the proposed scheme gives a very poor performance due to the random changes in amplitude and phase of the received signal components. In the absence of line of sight (LOS) component, the time domain equalizer fail to combat the channel distortions. Only wavelet based network with minimum inter cross correlation is plotted. Other spline networks also behave in a similar manner are not shown in the diagram. OFDMA system gives a fairly good performance because of the presence of CP. As CP cannot be incorporated with spline wavelet systems, it is required to design a better equalization technique in order to improve the error performance in a multi-path Rayleigh fading channel.

Figure 4.15: BER comparison of different four user networks (Multi-path Channel with Exponential power profile).

To illustrate the effects of multi-path Rayleigh fading channel with exponential power decaying profile, a block fading channel model with attenuation factor of 2 is considered as described in [100]. The channel coefficients are taken as normalized Gaussian random variables and the variance is chosen such that the attenuation follows exponentially decaying profile. Fig.4.15 shows the BER for a three path Rayleigh channel with exponentially decaying profile. It is shown that zero forcing equalizer fail to combat random changes in amplitude and phase and hence the proposed system give poor performance similar to

scalar wavelet based system.

4.4 Complexity Analysis

In this section, the computational complexity in generating different spline wavelets is described. As B-spline wavelets have explicit expressions, calculating the number of multiplications and additions in each case is an easy task. Generating spline wavelets involve only real multiplications and additions. The equations given in Section 4.2.1 show that multiwavelets of the same order requires the same number of computations. Mathematical operations involved in the final stage of wavelet generation is considered to obtain the values given in this section. Linear, quadratic and cubic splines are obtained by the linear combinations of their respective scaling functions and hence there will be three groups of summation for all wavelet equations.

In the case of linear wavelets, each summation includes 4 multiplications and 3 additions. Therefore to generate the wavelet function total number of multiplications will be $12(3 \times 4)$ and total number of additions will be $11(3 \times 3 + 2)$. For quadratic wavelets in each group there are 8 multiplications and 7 additions. Hence to generate each quadratic wavelet 24 multiplications and 23 additions are required. Lastly cubic wavelets require 9 multiplications and 8 additions in each group. Therefore 27 multiplications and 26 additions are required for cubic wavelets formulation. In general, number of multiplications is equivalent to N_c which corresponds to the total number of coefficients involved and the number of additions is equal to $N_c - 1$. It is evident that linear wavelets have the lowest complexity. As the order of the spline wavelet increases, complexity also increases. Different wavelets of the same order can be generated by changing the coefficient values of the same equation which demands a simple and reconfigurable hardware circuitry. Hence, the overall complexity of spline based CR system is comparable with existing schemes.

The number of computations required for different four user multiwavelet systems is tabulated and is compared with OFDMA system of four users and 16 subcarriers. From Table 4.1, it can be seen that number of additions required are lower for spline based system but number of multiplications are higher. But the main advantage of spline system is that it is a single carrier system and hence the computations required remains constant for a given network. But in the case of OFDM, as the number of subcarriers increases, the complexity

also increases at a scale of $N \log N$. In short, overall computational complexity of spline based system is comparable to that of OFDMA.

Network type	Number of real multiplications	Number of real additions
3 cubic + 1 quadratic	105	101
2 cubic + 2 quadratic	102	98
1 cubic + 3 quadratic	99	95
OFDMA	96	128

Table 4.1: Complexity Analysis

4.5 Chapter Summary

In this work, a novel multi-user CR network based on B-spline multiwavelets is described in detail and the auto correlation and cross correlation features of spline waveforms are analyzed. Exploiting the adaptivity and orthogonality of B-spline multiwavelets of different orders for a multi-user CR system is considered for the first time in literature. The single carrier spline based CR system is simulated and various performance curves are plotted. The computational complexity of the proposed system is briefly discussed. The BER of the proposed system is compared with OFDMA. It is shown that comparable error performance can be achieved with the proposed system in single path fading channels. But the proposed system fail to combat multi-path fading when the amplitude and phase changes in random. Moreover the number of users within a network is limited by the multiplicity of splines. Lastly, the proposed system require an additional circuit to find out the vacant bands. These issues are tried to resolve by proposing multifilter bank based multicarrier CR transceiver which is discussed in the next Chapter.

Chapter 5

MultiWavelet Filter Bank based Cognitive Radio Transceiver

The efficacy of spline multiwavelets to implement a multi-user CR network has been explored in the previous chapter. The major advantages of spline multiwavelet based systems are the simplicity of transmission and the low peak to average power ratio due to single carrier waveform modulation. The network can incorporate as many users equivalent to the total number of spline wavelets. But the system fails to offer flexibility in occupying non contiguous bands. Moreover it requires additional circuitry for sensing the spectrum. Hence it is desirable to implement a flexible and adaptive multi-user system together with spectrum sensing capability using multiwavelets. In this chapter, we propose a multicarrier CR transceiver with orthogonal multiwavelet filter banks that outperforms existing scalar WPM and OFDM in terms of flexibility, spectrum efficiency and adaptability. A new spectrum sensing technique using multifilter bank is proposed that can be implemented at virtually no additional computational cost. Performance is analyzed by simulating existing orthonormal multiwavelet filter banks. The results are compared with that of scalar wavelet based system and OFDMA.

5.1 Introduction

As mentioned in Chapter 4, multiwavelets are the generalization of multi resolution analysis (MRA). Similar to scalar wavelets, there exist two scale relations that lead to scaling functions and wavelet functions. These can be represented by two scale matrix equations [93]

$$\Phi(t) = \sum_n \mathbf{H}(n)\Phi(2t - n) \quad (5.1)$$

$$\Psi(t) = \sum_n \mathbf{G}(n)\Psi(2t - n) \quad (5.2)$$

where $\Phi(t)=[\phi_1(t) \ \phi_2(t) \ \dots \phi_r(t)]^T$ and $\Psi(t)=[\psi_1(t) \ \psi_2(t) \ \dots \psi_r(t)]^T$ form the set of scaling functions and corresponding wavelets. The suffix r denotes the number of wavelets and is known as the multiplicity of the wavelet system. Each $\mathbf{H}(n)$ and $\mathbf{G}(n)$ are square matrices of size $r \times r$ and are termed as multifilters [93]. This matrix structure of the filter bank provides more degrees of freedom for multiwavelets. Hence, multiwavelets result in superior performance in signal denoising and in image compression applications compared to scalar wavelets [95]. To implement a multicarrier modulation using multiwavelets, it is required to prove the existence of orthogonal multiwavelet packet subspaces and this is discussed in [101]. According to [101], multiwavelet packets can be generated from r scaling functions and their corresponding wavelet functions as follows.

$$\mathbf{W}_{2l}(t) = \sqrt{2}\mathbf{H}(n)\mathbf{W}_l(2t - n) \quad (5.3)$$

$$\mathbf{W}_{2l+1}(t) = \sqrt{2}\mathbf{G}(n)\mathbf{W}_l(2t - n) \quad (5.4)$$

where $\mathbf{W}_0(t)=\Phi(t)$ and $\mathbf{W}_1(t)=\Psi(t)$. $\mathbf{H}(n)$ and $\mathbf{G}(n)$ are the orthonormal low pass and high pass filter coefficients respectively. Unlike scalar wavelets, high pass filter coefficients of the multifilter cannot be obtained by alternating flip of the low pass filter but have to be designed [102]. Design of various orthogonal multifilter banks is described in [103] - [106].

Based on these designs, application of multiwavelet packets for digital communications was first proposed by Mingli You and Jacek Ilow [107]. They have proposed a vector extension of Mallat's algorithm to implement multiwavelet packet modulation (MWPM) and have proved that spectrum efficiency increases by r times compared to OFDM and scalar WPM. Besides, additional results are presented in [108] to verify the adaptability of MWPM to

mitigate strong narrow band interference with its flexible time frequency tiling. The symbol overlapped multiwavelet OFDM for system capacity improvement is discussed in [109]. A simpler method to obtain direct and inverse multiwavelet transform using toeplitz matrix formation is described in [110].

All of the results discussed above show that multifilter banks result in enormous advantages in next generation wireless networks. But the use of multiwavelet filter banks to implement a spectrum sensing circuit and adaptive modulation scheme within CR context is still to be explored. The multiple access capability of MWPM has not been addressed in the literature. Performance evaluation of multiwavelets in the literature is limited to the filter banks proposed in [103] and [104] and hence a fair comparison among different orthogonal multiwavelets in terms of error performance is unavailable. These issues are addressed in the following sections.

5.2 Advantages of MWPM for CR

Features of orthonormal multiwavelet packets that make them suitable for CR applications are listed below.

- In the case of multiwavelets each filter coefficient is an $r \times r$ matrix and hence larger number of subcarriers are possible for a given bandwidth. For a transform size of N and multiplicity r there can be as many as $r \cdot 2^N$ orthogonal subbands and hence r times bandwidth efficiency. This helps in allocating more subbands among different users compared to scalar wavelets and OFDM.
- The discrete multiwavelet packet transform (DMWPT) implemented for data reception can be used for spectrum sensing at no additional cost. As multiwavelets provide better results in feature extraction and signal denoising compared to scalar wavelets, the multiwavelet spectrum estimator can outperform scalar wavelet based spectrum detectors described in [62, 70]
- Multiwavelets maintain orthogonality even with certain integer shifts. If the support length of multiwavelet base is $(0, L)$ the orthogonality condition is satisfied for integer shifts of the wavelet base up to L . These shifted waveforms can be used to modulate different data of the same user. Thus the capacity of the system can be improved L

times for a given transform size or the number of subcarriers can be reduced for a given data rate [109]. Thus, multiwavelet based system offers high degree of flexibility and adaptivity in terms of data rate, modulation and number of subcarriers. OFDM subcarriers overlap only in frequency domain and hence this kind of flexibility cannot be achieved in OFDM based systems.

5.3 Spectrum Sensing with Multifilters

Spectrum sensing is the major task of CR as it is restricted to operate in unused bands. This is not common in any conventional system and hence its successful implementation is a crucial task [64]. Considerable research has been done in this area and numerous algorithms and architectures available in the literature are outlined in Chapter 2. The existing research results prove that spectrum sensing can be implemented in two stages - preliminary or coarse sensing at the physical layer and fine sensing at the MAC layer [65]. An energy or power detector is suggested as the preliminary sensing technique as it does not require prior knowledge of the signal characteristics. In this chapter, multifilter banks are exploited to calculate energy of the given spectrum and thus to locate the vacant bands. The related work, motivation and the proposed sensing algorithm are given in the subsequent subsections.

5.3.1 Related Work and Motivation

Locating the vacant bands using filter banks is already addressed in the literature [69] and the merits and demerits are discussed. The main advantage of filter bank based sensing is the lower computational overhead associated with its implementation in a multicarrier transmission scheme. The detailed performance analysis of various filter bank based spectrum sensing is presented in [69]. Wavelet filter bank based sensing is discussed in [62, 70]. A wavelet packet based energy detector using IIR scalar wavelet filters is described in [70]. IIR filters are used for sensing purpose to reduce the complexity of the sensing circuit. But this demands additional circuitry in the CR transceiver for sensing the spectrum. The use of FIR wavelet filter bank for spectrum sensing is outlined in [62] and it has been shown that the results are comparable with existing power estimation techniques. Since the sensing circuit proposed in [62] uses scalar wavelet FIR filters, its implementation is

possible by utilizing wavelet based CR receiver (demodulator) at no additional hardware circuitry. Multiwavelet FIR filter bank based spectrum sensing is proposed in this section which has not been addressed in the previous works. The motivation behind this proposal is the added advantage of multifilter based spectrum sensing compared to existing filter bank based sensing and are listed below.

- **Reliable Feature Extraction** - In multiwavelet filter banks, each filter coefficient is a square matrix of size $r \times r$. Hence, the filtering operation using multiwavelets involves matrix convolution. This demands r input rows in multifilter based operations. The additional input rows are obtained by repeated row pre-processing which increases the reliability of feature extraction [93].
- **Lower Number of Iterations** - Even though each step of multiwavelet transform is computationally complex than scalar wavelet transform, the required frequency (time) resolution can be achieved with reduced number of iterations (at lower transform size). Hence the overall complexity of multiwavelet transform will be equivalent to scalar wavelet transform. That is, for multiwavelets of multiplicity r each iteration generates $r \cdot 2^j$ subbands where j is the iteration level. If r is a power of 2, $(N - a)$ iterations are sufficient to generate 2^N subbands where $r = 2^a$.
- **Lower Number of Filter Taps** - According to [70], the complexity of wavelet filter bank based spectrum sensing is $L \cdot N \log_2 N$ where L is the filter length and N is the transform size. Multiwavelet FIR filters have fewer number of filter coefficients compared to scalar FIR filters within the same support length. In the proposed work, a symmetric multiwavelet filter bank is used and the number of filter coefficients is only three. Hence, the complexity of sensing is comparable to that of IIR filter based spectrum sensing and is lower than that of FIR filter based scalar wavelet spectrum sensing.

5.3.2 Multifilter based Sensing Methodology and Algorithm

In this work, only multifilter banks of multiplicity two are considered as their computational complexity is comparable to that of scalar wavelets and OFDM. Since multiplicity is two, spectrum sensing circuit needs two input rows which can be easily obtained by single repetition of the signal stream under consideration. Generating the multiwavelet coefficients of

Figure 5.1: Signal decomposition using Multifilters

the signal stream is shown in Fig. 5.1. The input stream is replicated and the 2×2 matrix filters operate on both streams to generate four output streams which are down sampled by a factor of 2. The decomposition process is repeated until the desired level of frequency-time resolution is achieved. Each row of the multifilter is a combination of two ordinary filters, one operating on the first data stream and the other operating on the second stream. For a given transform size, the output of the DMWPT will contain 2.2^N multiwavelet packet coefficients. It is shown that at level two, there are eight wavelet packet coefficients or eight frequency subbands. Successive iterations of filtering and downsampling can also be obtained using a toeplitz matrix [93].

Let $\mathbf{H}[0]$, $\mathbf{H}[1]$ and $\mathbf{H}[2]$ be the low pass filter coefficients. The doubly infinite toeplitz matrix corresponding to low pass filter will be

$$\begin{bmatrix} \mathbf{H}[0] & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots \\ \mathbf{H}[2] & \mathbf{H}[1] & \mathbf{H}[0] & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots \\ \mathbf{0} & \mathbf{0} & \mathbf{H}[2] & \mathbf{H}[1] & \mathbf{H}[0] & \mathbf{0} & \mathbf{0} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{H}[2] & \mathbf{H}[1] \end{bmatrix} \quad (5.5)$$

where each element in the matrix is a 2×2 matrix. In practice, the size of the toeplitz matrix depends on the length of the signal sequence under consideration. In a similar manner, a toeplitz matrix can be obtained for the high pass filter. Thus the calculation of wavelet packet coefficients in each level of the signal stream can be done by simple matrix

multiplication and is given as

$$\mathbf{C}_L^{(j)} = \mathbf{T}_L^{(j)} \mathbf{X}^{(j)} \quad (5.6)$$

where $\mathbf{C}_L^{(j)}$ are the multiwavelet coefficients, $\mathbf{T}_L^{(j)}$ is the low pass filter toeplitz matrix and $\mathbf{X}^{(j)}$ is the input vector at level j . It is important to note that each element of $\mathbf{C}_L^{(j)}$ and $\mathbf{X}$ is a vector instead of scalar with size 2×1 . Hence $\mathbf{C}_L^{(j)}$ corresponds to coefficient value of two subbands. Similarly, $\mathbf{C}_H^{(j)}$ can also be calculated corresponding to high pass filter and thus completes the process of obtaining coefficients of level j .

Once the multiwavelet packet coefficients are obtained, the energy contained in a certain band can be found from the inner product of coefficient array of the corresponding band with itself. The relationship between the energy E_{MWP} and power P_{MWP} for a node m and total number of samples K_{sample} is given as [62]

$$P_{MWP}^{(m)} = \frac{E_{MWP}^{(m)}}{K_{sample}}. \quad (5.7)$$

Power Spectral Density (PSD) of each subband can be calculated using the relation

$$PSD_{MWP}^{(m)} = \frac{P_{MWP}^{(m)}}{f_{MWP}^{(m)}} \quad (5.8)$$

where $f_{MWP}^{(m)}$ denotes the frequency range of a single subband. The PSD calculated for each subband can be compared with a predefined threshold to determine vacant bands. A simple algorithm for spectrum sensing using multiwavelets is given below.

Algorithm:

- 1: The iteration parameter j is initialized to zero. The input signal is converted into a matrix of two rows by repeating the signal.
 - 2: The low pass and high pass filter toeplitz matrices of the corresponding iteration is calculated.
 - 3: The Multiwavelet packet coefficients are obtained.
 - 4: Check whether the required iteration level is achieved. If yes go to next step. Otherwise increase the iteration parameter and repeat step 2 and 3.
 - 5: If the required iteration level is reached the wavelet packet coefficients are saved and energy of the subbands are calculated.
 - 6: The power spectral density is obtained and is compared with the predefined threshold.
 - 7: Preliminary decision on vacant spectrum is taken based on step 6.
-

5.4 MWP-MC-MA System for CR

To exploit multiwavelet filter banks in a multi-user CR environment, MWP-MC-MA system is proposed in this section. It is the multiwavelet extension of WP-MC-MA system described in Chapter 3. Multifilters offer higher spectrum efficiency and high degree of flexibility but the architecture is complex as described in the following subsections.

5.4.1 Uplink System Model

The basic uplink schematic diagram of the proposed system is shown in Fig. 5.2. The spectrum sensing circuit is the multiwavelet packet based spectrum estimator described in the previous section. The CR node scans its BOI and generates multiwavelet packet coefficient PSD vector $C_v(n, i)$ where n represents the subband index and i is the corresponding multiwavelet base. It is evident that based on the number of wavelet functions (multiplicity) the scanned spectrum can be sub divided into a large number of uniform sub bands. Based on the value of $C_v(n, i)$ and comparing it with predefined threshold, the preliminary decision on vacant spectrum is made. Final decision on vacant frequency band is done by spectrum decision block. The spectrum decision block is a cross layer approach which involves fine spectrum sensing at physical and MAC layers. Fine sensing at the physical layer incorporates feature extraction techniques to verify the presence of primary user (PU). Finally MAC layer takes the decision after considering certain link layer characteristics of the scanned frequency band and generates a binary information vector regarding spectrum occupancy.

Spectrum allocation is made by modified CAS similar to that in Chapter 3. The purpose of the CAS algorithm is to obtain the spectrum index vector I_m and proper allocation of the subbands among CR users. The user's data are baseband encoded, preprocessed to obtain $r \times 1$ vector points instead of scalars, channeled through P parallel streams where P is the number of subcarriers (subbands) allotted for each user. The data stream is fed into CAS, which will allocate subbands to the selected user based on the spectrum information vector $SI_v(n, i)$ and CSI of the users within the given cell. The symbol stream is allocated to P subbands within available R subbands and by inserting $(R-P)$ zeros an R dimensional

vector is obtained. Mathematically it can be expressed as

$$d_{n,i}^{(m)} = \begin{cases} c_{n,i}^{(m)} & \text{if } n, i \in I_m; \\ 0 & \text{otherwise.} \end{cases} \quad (5.9)$$

where $c_{n,i}^{(m)}$ represents the constellation of encoded data stream of the user and I_m is the set of the indices of m^{th} subchannel assigned to m^{th} user. The two variable suffix of the data stream is due to multiwavelet bases and the additional subbands obtained due to them. The vector $d^{(m)}$ generated is input to the Inverse Discrete Multi Wavelet Packet Transform (IDMWPT) block for waveform modulation and the serial data stream $s^{(m)}$ is obtained. Due to the time domain overlapping of multi wavelet bases, CP is excluded. The details of modified CAS and IDMWPT process are outlined in the following subsections.

Figure 5.2: MWP-MC-MA Uplink Transmitter

5.4.2 Modified CAS of MWP-MC-MA System

The carrier allocation among different users is a difficult task in multifilter based system compared to scalar wavelet based system. This is due to the vector nature of data processing. When the multiplicity is two, each data vector should be a 2×1 matrix and each filtering in IDMWPT operates on two subbands of the available bandwidth. Hence for a transform size N the total available subbands will be $r \cdot 2^N$. However, dynamic allocation of subcarriers is

possible only among 2^N bands. Modified CAS of multiwavelet based system functions as described by the accompanying algorithm.

Algorithm:

- 1: The binary spectrum information vector is obtained.
 - 2: CAS tries to determine CSI of each free subband available within the BOI.
 - 3: CSI of the CR node under consideration along with the neighboring nodes are obtained.
 - 4: Based on the information obtained, CAS assigns subbands for the considered CR node. two consecutive subbands will be allocated for the same user. The subbands with maximum SNR is the primary choice.
 - 5: If the specified CR as well as a competing neighbor has maximum SNR at the k^{th} consecutive subbands, other CSI factors such as channel delay, phase offset will be taken into consideration to make the final decision.
 - 6: Once the user and band is fixed, final decision is made and that subband index is included in the spectrum index vector I_m and the n^{th} vector data stream of the user is mapped onto those subbands.
 - 7: The process is repeated until all the P data streams of the user are mapped accordingly. The remaining subcarriers carry no data and are padded with zeros.
-

It is clear that the CAS algorithm depends on the SNR estimation and spectrum sensing information. Hence, it is required to use an adequate estimation technique to avoid incorrect allocation of subcarriers. Various SNR estimation techniques are available in the literature and are outlined in [111]. The estimation techniques are mainly classified as data aided (DA) and non-data aided (NDA). In DA techniques, knowledge of the received signal is essential which is not required for NDA estimators. In CR context, any existing NDA estimator can be used such as envelope based (EVB) estimator or signal to variance based (SVR) method. In the proposed work, EVB estimator could be considered as it uses only the signal magnitude.

5.4.3 Data Preprocessing and IDMWPT

Due to the matrix structure of multifilters, multiwavelet based transformation requires vectors instead of scalars. Preprocessing of data results scalar to vector transformation and serial to parallel conversion. The IDMWPT is performed on the parallel data stream using vector extension of Mallat's algorithm by using transpose of the low pass and high pass matrices H and G . The process involved in IDMWPT block is shown in Fig. 5.3. Each data symbol $\mathbf{X}[0]$, $\mathbf{X}[1]$ etc are $r \times 1$ vector points where r is the multiplicity [107]. $\mathbf{H}$ and $\mathbf{G}$ correspond to low pass and high pass multiwavelet filters. From Fig. 5.3, it is

Figure 5.3: Inverse Multiwavelet Packet Transform with two levels of iteration

evident that $r \cdot 2^2$ data symbols can be transformed into a serial stream by two levels of iteration. Successive iterations of upsampling and filtering operation can also be done by multiplication with corresponding toeplitz matrix. The details of single level multiwavelet packet transform and inverse transform using toeplitz matrix are discussed in [110].

5.4.4 Uplink Signal Model

The signal transmitted from the m^{th} CR node is

$$s_{MWPm}(k) = \frac{1}{\sqrt{r2^D}} \sum_{i=1}^r \sum_{n=0}^{2^D-1} d_{n,i}^m \Phi_{n,i}(k) \quad (5.10)$$

where D is the decomposition level and $\Phi_{n,i}(k)$ corresponds to the multiwavelet synthesis waveform. The signal received at the base station is written as

$$r(k) = \sum_{m=1}^M s'_{MWPm}(k) h_m(k) + n(k) \quad (5.11)$$

where $n(k)$ is the noise and $s'_{MWPm}(k)$ is the multiwavelet packet modulated component of the m^{th} user and $h_m(k)$ the corresponding fading channel. The signal component can be written as

$$s'_{MWPm}(k) = e^{j\phi_m(k)} \sum_{n=0}^{R-1} \sum_{i=0}^r d_{n,i}^m \Phi_{n,i}^{syn_m}(k - nR - \tau_m) \quad (5.12)$$

where $d_{n,i}^m$ is the symbol stream mapped onto the sub channel of the m^{th} user as in equation (3.12) and $\Phi_{n,i}^{syn_m}$ are multiwavelet packet synthesis waveforms for the m^{th} user subchannels. R is the number of subcarriers. The term $e^{j\phi_m}$ corresponds to the frequency and phase offset of the m^{th} user and is defined as

$$\phi_m(k) = \frac{2\pi\epsilon_m k}{rR} + \theta_m(k) \quad (5.13)$$

where θ_m is the phase noise component and ϵ_m is the relative frequency offset of the m^{th} user. The product rR gives the total number of subbands. The frequency offset is normalized to the inter carrier spacing. The integer timing offset τ_m is expressed in sampling periods. It is evident that the signal model differ from scalar wavelet system only in terms of vector data processing.

5.4.5 The Downlink System

The downlink transmitter and receiver are shown in Fig. 5.4 and 5.5 respectively. After symbol mapping, the data stream of each user is divided into blocks. Similar to uplink, CAS unit maps the P data symbols of each block onto subcarriers assigned to the corresponding user. The resultant vector contains data stream of all M users. It is modulated by the IDMWPT modulator and the data stream obtained serially is input to Digital to Analog Converter (D/A) and upconversion. At the receiver, the A/D output is the combination of data blocks of all users. Similar to the signal received at BS, frequency and timing errors are likely to be present. Hence the coarse frequency and timing estimation units are required to compute estimates of frequency and timing error. The frequency error estimate can be used to counter rotate the received sequence and the timing error estimate is used for positioning of the DMWPT window correctly. The channel equalization block corrects the channel impairments, if any, as well as the fractional timing errors. After the correction process, the data stream is input to DMWPT block where the serial stream is divided into subbands. For data detection, P subbands of the particular user is considered from the available subbands.

Figure 5.4: MWP-MC-MA Downlink Transmitter

Figure 5.5: MWP-MC-MA Downlink Receiver

5.4.6 Downlink Signal Model

The signal transmitted from the BS is

$$\mathbf{s}_{MWP}(k) = \frac{1}{\sqrt{r2^D}} \sum_{i=1}^r \sum_{n=0}^{2^D-1} \mathbf{d}_{n,i} \Phi_{n,i}(k) \quad (5.14)$$

where $\mathbf{d}$ denotes the vector representation of the summed data stream. The expression is similar to that of single user case but $\mathbf{d}$ includes data of all M users. By adopting matrix representation for data stream and for multiwavelet modulation/demodulation, the signal transmitted from the BS can be written as

$$\mathbf{s}_{MWP}(k) = \mathbf{W}^T(k) \mathbf{X}(k) \quad (5.15)$$

where $\mathbf{W}^T(k)$ is the transpose of the toeplitz matrix. Assumig AWGN channel at the receiver, the demodulated signal is

$$\tilde{\mathbf{s}}_{MWP}(k) = \mathbf{W}(k)(\mathbf{W}^T(k) \mathbf{X}(k) + \mathbf{N}(k)) \quad (5.16)$$

where $\tilde{\mathbf{s}}_{MWP}(k)$ represents the demodulated signal component. This can be elaborated as

$$\tilde{\mathbf{s}}_{MWP}(k) = \mathbf{W}(k) \mathbf{W}^T(k) \mathbf{X}(k) + \mathbf{W}(k) \mathbf{N}(k) \quad (5.17)$$

When the underlying multifilter banks are orthonormal, this can be simplified as

$$\tilde{\mathbf{s}}_{MWP}(k) = \mathbf{I}(k) \mathbf{X}(k) + \mathbf{W}(k) \mathbf{N}(k) \quad (5.18)$$

where $\mathbf{I}(k)$ is the identity matrix. From the demodulated signal, the corresponding user's subbands can be extracted using the spectrum index vector and then data bits are obtained using symbol demapper.

5.4.7 Choice of Multiwavelets

From the details of the IDMWPT process and downlink signal model, the constraints to obtain error free transmission under noiseless condition with multifilters are:

- 1) Each filter coefficient matrix should be a square matrix.
- 2) For perfect reconstruction, the toeplitz matrix formed using filter coefficient matrices should be an orthogonal matrix.

Hence the selection of multiwavelets depends on the above constraints. Moreover, the computational complexity involved in multiwavelet transform is to be considered in CR type of applications. Multiwavelets of multiplicity 3 and above are not used in the simulation due to the high complexity associated with their transform formulation. In the proposed work, all orthogonal multifilters of multiplicity 2 available in the literature are considered for performance evaluation. Four different set of multifilters discussed in [103] - [106] are used in the simulation and are named after the authors. The filter bank mentioned in [103] is termed as “GHM” (Gerenimo, Hardin, Massopust) and that in [104] as “ChuiLian” (C. K. Chui and J. Lian). The orthogonal multiwavelet with optimum time frequency resolution described in [105] is termed “Quintang” (Quintang Jiang) and the orthogonal filter bank described in [106] is denoted as “Shouzhi” (Yang Shouzhi). All multiwavelet families have multiplicity 2 and hence each filter coefficient is a 2×2 matrix. The number of filter coefficients and support length of each set is different and the details are given in Appendix B of this dissertation.

5.5 Performance Evaluation

In this section the performance of the proposed system is analyzed using MATLAB. Results obtained are categorized into two parts - spectrum measurements and error performances.

5.5.1 Spectrum Measurements

A partial band BPSK signal with 1000 symbols embedded in additive white Gaussian noise is considered as the signal to be detected. The carrier frequency is 300MHz and SNR is kept at 10dB. For simulation purpose, threshold is determined based on the principle of scalar wavelet based detection technique explained in [62, 63]. In [63], the threshold for detecting vacant band is kept as -7dB as it gives probability of detection equal to 1 for a minimum probability of false alarm. The same threshold is adopted for multiwavelet based

Figure 5.6: Spectrum sensing result of a partial band BPSK Signal

Figure 5.7: Spectrum sensing result of noise signal

system also due to the inherent similarity of both the systems and for making a fair comparison. Fig. 5.6 shows the useful signal detection using both “Shouzhi” multiwavelet and Symlet scalar wavelet. The given signal is decomposed into eight subbands with two levels of iteration using multiwavelets where scalar wavelet requires three levels of iteration. The subband index is given on X axis and power in dB is plotted on Y axis. It is evident that in the proposed system all subband power measurements are well above the threshold giving the presence of the signal. In the case of scalar wavelet, power measurements in only two subbands are above the threshold value. Hence the multiwavelet system is more powerful in detecting the presence of a primary user. Fig. 5.7 shows the spectrum measurements corresponding to the noise signal detection. Both systems give accurate information about the free spectrum as subband power measurements are well below the threshold.

5.5.2 Error Performance of the proposed MWP-MC-MA System

To evaluate the performance of the proposed system a multi-user CR environment with 4 users is considered. It is assumed that the BOI is scanned with multiwavelet packet transform and a total frequency band of 64 subbands found vacant. Equal number of subcarriers for all users is considered. Hence there are 4 subchannels with 16 subcarriers for each user in the simulated system. This implies that each MWPM block corresponds to 32 multifilter subcarriers which handle 2×1 vector data. The data of each user is converted into 2×1 vector points and are input to IDMWPT block. It is assumed that there is perfect synchronization between users and base station (BS). The channel index set I_m generated is known at the user end to separate the required data streams. The simulations correspond to downlink channel of MWP-MC-MA. It is assumed that QPSK modulation is the baseband symbol mapping scheme adopted by all users. Multifilters discussed in the previous section are used for simulation. The inverse and direct MWPT is performed using toeplitz matrix formulation. The results obtained are compared with those of scalar WP-MC-MA using Daubechies filter of length 20 (db10) and OFDMA with same number of users and subcarriers.

Figure 5.8: Error performance in Flat Fading Channel

Figure 5.9: Error performance in Single path Rayleigh Fading Channel

5.5.2.1 Error performance in single path fading channels

To incorporate the effects of flat fading, the channel gain is modeled as complex normal distribution $CN(0, 1)$, with variance 1 similar to that used in Chapter 3. Fig. 5.8 shows the BER in flat fading channel of different multicarrier schemes. It is assumed that the channel information is known at the receiver and is compensated. The different multiwavelets simulated in the work show similar performance except Quintang multiwavelet with optimum time frequency resolution. It is evident that the BER of other multiwavelets are comparable with those of scalar wavelet based system and OFDMA. In Fig. 5.9 the effect of a single path Rayleigh fading channel with a maximum Doppler shift of 55 rad/sec is plotted. The channel model is generated based on Jakes model and zero forcing equalization algorithm is used at the receiver. The proposed system is highly sensitive to Doppler shifts compared to OFDMA and scalar WP-MC-MA.

5.5.2.2 Error performance in multi-path fading channels

Figure 5.10: Error performance in multi-path Channel with fixed fading coefficients

To evaluate the capability of the system to mitigate multi-path fading channel impairments, three path fading channel with fixed and Rayleigh fading coefficients are considered. CP of the simulated OFDMA system is higher than the channel delay. At the receiver, the

Figure 5.11: Error performance under multi-path Channel with exponentially decaying power profile

wavelet based scheme uses a time domain zero forcing equalizer with three taps per sample as given in [92] and OFDMA uses a frequency domain zero forcing equalizer with one tap per subcarrier since the demodulated OFDM signal is in the frequency domain. The fixed channel impulse response used for simulation is $h = [0.407 \ 1 \ 0.407]$. When the channel coefficients are fixed, the proposed multiwavelet based system using symmetric pair multiwavelets termed "Shouzhi" performs better than that of OFDMA and scalar wavelet system.

To illustrate the effects of multi-path Rayleigh fading channel with exponential power decaying profile, a block fading channel model with attenuation factor of 2 is considered, similar to that in Chapter 4. The channel coefficients are taken as normalized Gaussian random variables and the variance is chosen such that the attenuation follows exponentially decaying profile. Under multi-path Rayleigh fading channel condition, zero forcing equalizer fail to combat random changes in amplitude and phase and hence the proposed system give poor performance similar to scalar wavelet based system.

Figure 5.12: Effect of Phase Offset

5.5.2.3 Effect of Phase Offset

As in any multi carrier modulation, it is important to consider the effect of frequency, phase and timing offset on error performance of the proposed system. Firstly the effect of phase noise on MWPM is considered. It is known that the presence of phase noise will affect the multicarrier modulation in two ways - (1) it will rotate all the constellation symbols by the same angle which is approximately equal to the average phase noise, (2) it will introduce ICI due to the spread of subcarriers with a larger bandwidth around the carrier frequency. To simulate the effects of phase noise in a multi-user environment the phase noise is expressed as a zero mean Gaussian noise and the BER is calculated for different values of noise variance. The channel is assumed to be an AWGN channel and relative frequency error ϵ_m and timing offset to be zero. The signal received at the user terminal can be written as

$$\mathbf{s}_{MWP}(k) = \mathbf{W}^T(k)\mathbf{X}(k)e^{j\Theta(k)} + \mathbf{N}(k) \quad (5.19)$$

where Θ denotes the phase offset vector and $\mathbf{N}(k)$ the noise vector. The demodulated signal would be

$$\begin{aligned}
 \tilde{\mathbf{s}}_{MWP}(k) &= \mathbf{W}(k)\mathbf{W}^T(k)\mathbf{X}(k)e^{j\Theta(k)} + \mathbf{W}(k)\mathbf{N}(k) \\
 &= \mathbf{I}(k)\mathbf{X}(k)e^{j\Theta(k)} + \mathbf{W}(k)\mathbf{N}(k) \\
 &= \mathbf{X}(k)e^{j\Theta(k)} + \mathbf{W}(k)\mathbf{N}(k) \\
 &= \frac{1}{\sqrt{r2^D}} \sum_{i=1}^r \sum_{n=0}^{2^D-1} \mathbf{d}_{n,i} e^{j\Theta(k)} + \mathbf{W}(k)\mathbf{N}(k)
 \end{aligned} \tag{5.20}$$

It is evident that the signal constellations are rotated by the corresponding phase angle and the noise component is altered by the respective multifilter coefficients. Fig. 5.12 shows the effect of phase offset on error performance when the signal to noise ratio is kept constant at 15 dB. The phase noise variance is expressed in dB_m and is given on X axis. The bit error rate is depicted on Y axis. Figure shows that error increases considerably as the phase noise approaches zero and the type of multifilter does not improve the performance. OFDMA and scalar WPM perform comparatively better than multifilter bank based system.

5.5.2.4 Effect of Frequency Offset

Figure 5.13: Effect of Frequency Offset

Frequency offset can occur due to Doppler shift or by misalignment between the transmitted carrier frequency and the locally generated carrier frequency at the receiver. It destroys orthogonality among subcarriers and hence causes severe ICI. To simulate the effect of frequency offset, AWGN channel is considered and the phase and timing offsets are assumed to be zero. Fig. 5.13 shows the effect of frequency error on performance. BER is plotted as a function of relative frequency offset when the SNR is kept constant as 15dB. No error compensation is done at the receiver. Other than “Quintang” multifilter all other systems have comparable error performance. The error degradation of Quintang based systems is due to optimization of its time frequency resolution.

Figure 5.14: Spectrum of 'GHM' and Daubechies Wavelets

5.5.3 Bandwidth Efficiency and Sidelobe Suppression

Due to matrix structured filter bank, multiwavelet based systems possess higher bandwidth efficiency compared to those of scalar wavelet and OFDMA system. In the proposed system, multifilters of multiplicity two are considered. Hence, there can be twice the number of

subbands compared to those of OFDM and WPM for a given iteration. The spectrum of multiwavelet of multiplicity two and Daubechies scalar wavelet is illustrated in Fig. 5.14 [107]. It is evident that there are only two subcarriers for scalar wavelet (red curves) but four subcarriers for multiwavelet system (blue dotted curves) for a given bandwidth.

Thus bandwidth efficiency will be twice that of scalar WPM and OFDM. Moreover, similar to scalar WPM, multiwavelet system does not include CP along with the signal and hence the spectral efficiency will be always higher than that of OFDM. Lastly, the spectral leakage is also minimum in multiwavelet based system due to non-rectangular waveforms in time domain.

Name	Filter Length	Number of Subbands	Number of Multiplications
Quintang	7	128	1792
Shouzhi	3	128	768
GHM	4	128	1024
ChuiLian	4	128	1024
Daubechies	20	64	2560
OFDM	-	64	1024

Table 5.1: Complexity Analysis

5.5.4 Complexity Analysis

The main disadvantage of multiwavelet packet transform is the higher complexity involved in each level of its computation. When the filter coefficients are 2x2 matrices, the computations required in one level of iteration will be approximately twice to that of scalar wavelet transform. But as multiwavelet transform can provide twice the number of subbands in a given iteration compared to scalar WPM, it is expected that for a fixed number of subbands, the computational complexity would be almost similar. Theoretically, for an N stage WPT, the number of real multiplications required to calculate the scalar wavelet coefficients is always less than $2LT$ where $T = 2^N$ and L is the filter length. In the case of multiwavelets, this expression can be approximated as $r.2LT$ since the additional computations in each level of the transform depends on multiplicity. The total number of real multiplications required for different orthonormal multiwavelet systems, Daubechies wavelet (db10) and OFDM for transform stage N= 6 are given in Table 5.1. It is shown that due to lower number of filter taps, “Shouzhi”, “GHM” and “ChuiLian” multiwavelet based systems have

computational complexity comparable or lower than that of OFDM for a given N.

5.6 Chapter Summary

In this work, a novel multicarrier multiple access scheme for CR using multiwavelet packet modulation has been proposed. The MWP-MC-MA system provides spectrum efficiency, flexibility, orthogonality and multi-user support. A new spectrum sensing method using multifilter bank is proposed. Modified CAS unit has been exploited for adaptive subcarrier allocation. The simulation results show that error performance of the proposed system under fixed fading channel is comparable with that of OFDM system. The spectral efficiency of the system is higher compared to existing scheme. The computational complexity of the proposed system is comparable to that of OFDM since the number of filter taps are lower for existing orthonormal multiwavelets.

Chapter 6

Conclusions and Future Work

6.1 Introduction

The prolific growth of wireless devices and services have resulted in a high demand for bandwidth and new solutions are sought for this apparent spectrum scarcity. As the spectrum scarcity is not due to maximum spectrum utilization by existing incumbents but by an inefficient allocation strategy of the RF spectrum, the most viable solution is to alter the current static spectrum assignment policies. If dynamic or opportunistic access of spectrum is possible the existing spectrum can be utilized effectively and can provide room for upcoming wireless applications and services. CR based DSA is expected to achieve this goal by its cognitive and flexible physical and MAC layer architecture. But implementation of such cognitive and flexible communication layers are open issues. The key research challenges of adaptive transmission at CR physical layer has been addressed and probable solutions have been provided in this thesis using scalar and multiwavelet based techniques. Specifically, the challenges in system and signal modeling of the multi-user CR network using wavelets have been addressed and modified CAS algorithms have been described for both scalar and multiwavelet based CR system. This chapter provides an overview of the main contributions described in the thesis and some of the possible future directions for this work.

6.2 Conclusions

The work discussed in this thesis proposes flexible and adaptive transmission schemes for CR to realize DSA. To clearly investigate the research challenges involved in realizing an adaptive transmission technique, a survey on existing CR transmission scheme proposals are given in the beginning. The motivation for the present work is also clearly defined. Three transmission techniques are described in Chapter 3, 4 and 5. First proposal is based on scalar wavelet packet transform. Second method utilizes spline multiwavelets with explicit expressions and final proposal exploits existing orthonormal multiwavelet filters.

A novel multicarrier multiple access scheme using scalar wavelet packet based transceiver is proposed in Chapter 3. New carrier allocation scheme inspired from generalized CAS of OFDMA is detailed and this is called modified CAS. The system model and signal model of WP-MC-MA are outlined and its uplink and downlink performances are simulated using existing standard orthogonal wavelets of different filter lengths. The results are compared with those of OFDMA. The proposed system outperforms OFDMA, in terms of spectrum efficiency and reconfigurability. But it fails to mitigate multi-path distortions effectively and imposes higher computational complexity compared to those of OFDM.

As multiwavelets are least explored in CR based systems compared to scalar wavelets, the potential of spline multiwavelets to implement a multiuser CR network with lower MAI is investigated in the second proposal. The time domain single carrier waveform modulation using linear, quadratic and cubic spline multiwavelets is described and various performance metrics are analyzed. The expression for multi-user interference in terms of cross correlation among different splines are derived. Due to orthogonal and symmetric waveforms spline multiwavelets is proposed as an alternative to existing OFDM based networks but with restricted number of users.

Finally, to further improve spectrum efficiency and throughput, multifilter bank based multicarrier multi-user network is proposed in Chapter 5. Multiwavelet packet modulation is rarely considered in wireless communication but this multi-dimensional modulation has flexibility and adaptivity. A novel energy detection technique using multifilter bank is proposed for preliminary detection of vacant bands. The computational complexity of each level of multiwavelet transform is shown to be higher compared to that of scalar wavelet and OFDM but the proposed system gives higher bandwidth efficiency. The spectrum sensing results are compared with that of scalar wavelets and error performances are compared

with scalar wavelets and OFDMA. A comparison among the three proposals in terms of different performance metrics are given in the following subsections.

6.2.1 Spectrum Efficiency

As CR was proposed to improve spectral utilization, transmission scheme of CR should possess good bandwidth efficiency. All wavelet based systems possess inherent spectrum efficiency mainly due to two important facts

- 1) Wavelet based systems never incorporate CP since the waveforms overlap in time domain.
- 2) The spectral leakage or sidelobes are minimum in wavelets due to the non rectangular shape of their transmitting pulse.

Hence it can be concluded that wavelet based transmission schemes will have higher spectral efficiency and lower sidelobes than its counterpart OFDM. Among the three transmission schemes described in this dissertation multiwavelet filter based multicarrier system has maximum bandwidth efficiency. It is observed that by using multifilters of multiplicity two for waveform modulation, spectral efficiency will be increased to twice compared to that of scalar wavelets and OFDM. Sidelobes are also minimum in multiwavelet based transmission scheme.

6.2.2 Flexibility

The transmission scheme of CR should have good flexibility as it has to change its modulation, coding, power scaling and frequency as and when required. Hence multicarrier systems are supposed to be promising candidates for CR transmission. The proposals in Chapter 3 and 5 are multicarrier systems and hence possess good flexibility. Spline multiwavelet based CR system described in Chapter 4 has lower degree of flexibility as it can operate only in contiguous frequency bands. Multifilter based multicarrier system proposed in Chapter 5 has maximum flexibility as it can adaptively change the data rate and capacity by its symbol overlapping methodology.

6.2.3 Error performances

Wavelet based systems will show good error performance in the absence of Doppler shift and timing offset. In single path fading channels all wavelet based schemes give comparable error performance with OFDM. But in multi-path Rayleigh fading channel, the proposed schemes fail to maintain orthogonality and give very poor performance compared to conventional OFDM systems. This can be regarded as the main drawback of wavelet based systems.

6.2.4 Computational Complexity

The computational complexity of wavelet based multicarrier systems depends on the required transform size and filter length. For a given transform size multifilter based multicarrier scheme has higher complexity compared to that of scalar wavelet system since MWPM can generate more subbands due to its matrix structured filter banks. Spline multiwavelets are generated from linear combination of its scaling functions and the complexity involved in each stage is lower compared to that of multicarrier systems. In general the complexity of wavelet based schemes are higher compared to that of OFDM for a fixed transform size. But the iterative nature of wavelet based modulation scheme insures 'on the fly' changes in transform size at moderate complexity.

6.3 Directions of Future Work

From the concluding remarks given in the previous section, it is evident that the major drawback of wavelet based schemes is their inability to combat multi-path Rayleigh fading channel distortions. Hence a remedy to this issue is relevant. Another important facet of CR physical layer is a flexible and fast spectrum sensing algorithm to locate free bands with high accuracy. A novel spectrum sensing using multiwavelet filters is described in Chapter 5 but the description is limited to the preliminary stage of free spectrum identification employing energy detection. It would be meaningful to work towards the modification of the multifilter bank based spectrum sensing to make the proposed transceiver be a promising candidate for CR physical layer.

The possible future works can be summarized as follows;

- Design of a low complex equalizer to nullify the multi-path random amplitude and phase change: The time domain zero forcing and minimum mean square error equalizers perform sufficiently well when the channel impulse response are fixed but they cannot combat Rayleigh fading effects of the channel. As CR mainly occupies wide-band frequency spectrum, multi-path fading effects will be naturally present in the received signal. OFDM based systems use their cyclic extension feature to compensate for these distortions. Wavelet waveforms are longer than their transform size and CP cannot be included with their transmitting signal. Hence a novel equalizer technique is required to resolve this issue. It is expected that a wavelet based equalizer that maintain orthogonality of the subbands even in the presence of Rayleigh fading could be an answer to this issue.
- Reliable spectrum sensing using multiwavelet filter banks: In Chapter 5 an energy detection based spectrum sensing using multifilters is proposed. Energy detection is the simplest spectrum sensing technique available but it suffers from lower accuracy and precision. Hence it is required to modify the spectrum sensing algorithm using multiwavelets with additional feature extraction techniques. In multiwavelet based sensing the signal stream to be detected is oversampled (repeated) to generate the multiwavelet packet coefficients. Oversampling of a given one dimensional signal is quite useful in extracting its features accurately. If the scanning is done over a range of frequencies whose signal characteristics are known and if it is possible to obtain some of these features from the multiwavelet packet coefficients, a reliable sensing scheme can be designed using multifilters with moderate computational complexity. As multiwavelet based sensing has not been addressed previously in literature, a detailed research analysis is required to obtain substantial results in this area.
- Design of new multiwavelet filters : The existing orthonormal multiwavelets have been designed for signal and image processing applications. Until now there has been little research done to design orthonormal compact support multifilters that are entirely useful for communication based applications. Even though design steps of multifilters are slightly complex than that of scalar wavelet design, it is expected that a novel multifilter bank might give better error performance together with good free band detection capability. Optimizing both sensing accuracy and error performance would be a key issue in the design process.

Bibliography

- [1] J.Mitola, “Cognitive radio for flexible mobile multimedia Communications”, *IEEE international Conference on mobile multimedia Communications*, San Diego, CA, November 1999, pp. 3-10.
- [2] www.mprg.org/people/gametheory/files/chapter201.pdf
- [3] D. Cabric, R. W. Brodersen, “Physical layer design issues unique to cognitive radio systems”, *IEEE International Symposium on Personal, indoor and mobile radio communications*, Berlin, September 2005, pp. 759-763.
- [4] I. F. Akyldiz, W. Y. Lee, M. C. Vuran, S. Mohanty, “Next Generation/dynamic spectrum access/cognitive radio wireless networks: A Survey”, *Computer Networks Journal (Elsevier) Vol. 50, Issue 13*, September 2006, pp. 2127-2159.
- [5] F. Jondral, “Spectrum pooling - An Efficient Strategy for Radio Resource Sharing,” *Blacksburg (VA)*, June 8, 2004.
- [6] <http://www.sharespectrum.com/papers/spectrum-reports/>
- [7] A. M. Wygliski, M. Nekovee and Y. T. Hou, “Cognitive radio Communications and Networks, Principle and Practice,” *Academic Press*, 2010
- [8] Federal Communications Commission, “Notice of proposed rule making”, *ET Docket No. 04-113*, May 25, 2004.
- [9] Office of Communications (Ofcom), “Digital Dividend: Cognitive Access”, Available Online www.ofcom.org.uk/consult/condocs/cognitive, February 2009
- [10] S. Haykin, “Cognitive radio: Brain Empowered Wireless Communications”, *IEEE*

- Journal on selected areas in Communications, Vol.23, Issue 2*, February 2005, pp. 201-220.
- [11] J. E. Brittain, "Electrical engineering hall of fame: Guglielmo Marconi" *Proceedings of the IEEE, Vol. 92, Issue 9*, September 2004, pp. 1501-1504.
- [12] J. Mitola, "Cognitive radio : An Integrated Agent architecture for Software defined Radio", *PhD dissertation, Royal Institute of Technology, Stockholm, Sweden*, May 2000.
- [13] Federal Communications Commission, "Spectrum Policy Task Force", *Rep. ET Docket No. 02-135*, November 2002.
- [14] Federal Communications commission, "The FCC's Office of Engineering and Technology release report on tests of prototype white space devices", *ET Docket No.04-186*, October 2008.
- [15] Federal Communications commission, "FCC adopts rules for unlicensed use of television white space," *Official FCC announcement*, November 2009.
- [16] "Cognitive radio Definition", *Virginia Technology Cognitive Radio Work group*
- [17] "Improving Spectrum Usage through Cognitive Radio Technology," *Available online: <http://www.ieeeusa.org/policy/positions/cognitiveradio.asp>*
- [18] Rahul Tandra, Shridhar Mubaraq and Mishra Anant Sahai, "What is a spectrum hole and what does it take to recognize one? ", *In Proceedings of the IEEE, Vol. 97, Issue 5*, May 2009, pp. 824-848.
- [19] R.V. Nee, R. Prasad, "OFDM for Wireless Multimedia Communications", *Artech House, London*, 2000.
- [20] IEEE, Part 11, "Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications: High-speed physical layer in the 5 GHz band", *IEEE Std 802.11a*, 1999.
- [21] J. Foerster, "Channel modeling sub-committee report final, Technical report", *IEEE 802.15-02/490*, 2003.
- [22] J. Balakrishnan, A. Batra, A. Dabak, "A multi-band OFDM system for UWB communication", *IEEE Conference on Ultra Wideband Systems and Technologies*, Reston, VA, November 2003, pp. 354-358.

- [23] I. Koffman, V. Roman, "Broadband wireless access solutions based on OFDM access in IEEE 802.16", *IEEE Communications Magazine, Volume 40*, 2002, pp. 96-103
- [24] ETSI EN 300 401 V1.3.3 (2001-05), "Radio Broadcasting Systems; Digital Audio Broadcasting (DAB) to mobile, portable and fixed receivers", 2001.
- [25] R. Prasad, "OFDM for Wireless Communications Systems", *Artech House, London*, 2004.
- [26] T. A. Weiss and F. K. Jondral, "Spectrum Pooling: An Innovative strategy for the enhancement of spectrum efficiency", *IEEE Radio Communications Magazine*, March 2004, pp. S8-S14.
- [27] T. A. Weiss, J. Hillenbrand and F. K. Jondral, "A diversity approach for the detection of idle spectral resources in spectrum pooling systems", *48th International Science Colloquium*, Ilmenau, Germany, September 2003.
- [28] T. A. Weiss, "Efficient Signaling of spectral resources in spectrum pooling systems", *Proceedings of the 10th Symposium on Communications and Vehicular Technology*, Eindhoven, Netherlands, November 2003.
- [29] F. Capar, "Comparison of bandwidth utilization for controlled and uncontrolled channel assignment in a spectrum pooling system", *Vehicular Technology Conference*, Birmingham, Alabama, March 2002, pp. 1069-1073.
- [30] T. Weiss, A. Kohn and F. K. Jondral "Synchronization algorithm and preamble concepts in spectrum pooling systems", *IST Mobile and wireless Telecommunication Summit*, Aveiro, Portugal, June 2003.
- [31] T. Weiss, A. Kohn and F. K. Jondral "Mutual Interference in OFDM based spectrum pooling systems", *IEEE Vehicular Technology Conference*, Milan, Italy, May 2004, pp. 1873-1877.
- [32] R. Rajabanshi, A. M. Wyglinski and J. M Gary, "An efficient implementation of NC-OFDM transceivers for cognitive radio", *First International conference on Cognitive Radio Oriented Wireless Networks and Communication*, Mykonos Island, June 2006, pp. 1-5.
- [33] R. Rajabanshi, A. M. Wyglinski, J. B. Evans and J. M Gary, "Comparative Study of Frequency agile data transmission schemes for Cognitive radio transceivers", *Proceedings*

- of the first international workshop on technology and policy for accessing spectrum*, New York, July 2006, Article No.15.
- [34] R. Rajabanshi, A. M. Wyglinski, J. B. Evans and J. M Gary, "Qualitative comparison of agile modulation techniques for CR transceivers", *Proceedings of the consumer communications and networking Conference, Workshop on Cognitive radio networks*, Las Vegas, USA, January 2007, pp. 1144-1148.
- [35] J. Y. Won, S. B. Shim, Y.H. Kim, S. H. Hwang, C. J. Kim, "An Adaptive OFDMA platform for IEEE 802.22 based on cognitive radio", *Asia pacific Conference on Communications*, Busan, Korea, August 2006, pp. 1-5.
- [36] Tao Peng, Wei Wang, Qianxi Lu and Wenbo Wang, "Subcarrier Allocation Based on Water-Filling Level in OFDMA-Based Cognitive Radio Networks," *International Conference on Wireless Communications, Networking and Mobile Computing*, Shanghai, China, September 2007, pp. 196-199.
- [37] Wei Wang, Wenbo Wang, Qianxi Lu, Tao Peng, "An uplink resource allocation scheme for OFDMA-based cognitive radio networks", *Wileys' International Journal of Communication Systems*, Volume 22, Issue 5, 2009 pp. 603-623.
- [38] Zhiqiang Wu, Paul Ratazzi, Vasu D. Chakravarthy, and Lang Hong, "Performance Evaluation of Adaptive Non-contiguous MC-CDMA and Non-contiguous CI/MC-CDMA for Dynamic Spectrum Access", *IEEE International Conference on Cognitive Radio Oriented Wireless Networks and Communications*, Singapore, May 15-17, 2008, pp. 1-6.
- [39] H. Shahrokh and K. Mohamed, "A New Structure for NC-MC-CDMA in Cognitive Radio Networks", *9th International Symposium on Communications and Information Technology*, 2009, pp. 653-657.
- [40] L. B. Milstein, "An analysis of real time transform domain filtering digital communication systems - wide band interference rejection", *IEEE Transactions on Communications*, Vol.31 Issue 1, January 1983, pp. 21-27.
- [41] A. F. Andren, "Low Probability of intercept Communication System" *Harris Corp. U. S. patent 5029184*, July 1991

- [42] V.D. Chakravarthy, A. K. Shaw and M. A. Temple, "Cognitive radio- An adaptive waveform with Spectral sharing capability", *IEEE Wireless Communications and Networking Conference*, New Orleans, USA, March 2005.
- [43] V. D. Chakravarthy, A. K. Shaw and M. A. Temple, "TDCS, OFDM and MC-CDMA : A brief tutorial", *IEEE Radio Communications Magazine*, Volume 43, Issue 9, September 2005, pp. S11-S16.
- [44] Ruolin Zhou, Qian Han, R. Cooper, V. Chakravarthy, and Zhiqiang Wu, "A Software Defined Radio Based Adaptive Interference Avoidance TDCS Cognitive Radio", *IEEE International Conference on Communications*, Capetown, SA, June 2010, pp. 1-5.
- [45] E. Blossom, "GNU radio: Tools for exploring the radio frequency spectrum" *Linux Journal*, No. 122, June 2004.
- [46] M. Ettus, "Universal software radio peripheral." [Online]. Available: www.ettus.com
- [47] C. Han, J. Wang, S. Gong and S. Li, "Detection and Performance of an OFDM based Transform domain Communication system", *International Conference on Communications, Circuits and Systems*, Guilin, June 2006, pp. 1332-1336.
- [48] A. Jamin, P. Mahonen, "Wavelet packet modulation for wireless communications," *Wireless Communications and Mobile Computing Journal*, Vol. 5, Issue 2, March 2005, pp. 123-137.
- [49] M.K. Lakshmanan and H. Nikookar, "A review of wavelets for Digital Wireless Communications," *Wireless personal Communications*, Vol. 37, 2006, pp. 387-420.
- [50] Mohan Chandra Baro, "Wavelet based Modulation Techniques for Agile Wireless Communication Systems", *PhD Thesis, Dalhousie University*, July 2008
- [51] D. Daly, C. Heneghan, A. Fagan, and M. Vetterli, "Optimal wavelet packet modulation under finite complexity constraint", *Proceedings of ICASP*, Vol. 3, 2002, pp. 2789-2792.
- [52] H. Zhang, "Wavelet packet based multicarrier cdma wireless communication systems", *Ph.D. dissertation, University of Cincinnati, Cincinnati, Ohio*, March 2004.
- [53] Kefeng Tan, Jean Andrian, Frank M. Candocia, Chi Zhou, "An Enhanced Wavelet Domain Communication System (EWDCS) with Nonstationary Interference Avoidance Capability", *IEEE Vehicular Technology Conference, Fall*, Montreal, September 2006, pp. 1-6.

- [54] Kuan-Hung Chen and Tzi-Dar Chiueh, "A Cognitive Radio System Using Discrete Wavelet Multitone Modulation ", *IEEE Transactions on Circuits and Systems - I*, Vol. 55, Issue 10, November 2008, pp. 3246-3258.
- [55] F.Daneshgaran and M. Mondin, "Wavelets and scaling functions as envelope waveforms for Modulation," *IEEE SP International Symposium on Time-Frequency and Time-Scale Analysis*, October 1994, pp. 504-507.
- [56] A. R. Lindsey and J. C. Dill, "Wavelet packet modulation: a generalized method for orthogonally multiplexed communications", *IEEE 27th Southeastern Symposium on System Theory*, 1995, pp. 392-396.
- [57] A. R. Lindsey, "Wavelet packet modulation for orthogonally multiplexed communication", *IEEE Transaction on Signal Processing*, Vol. 45, Issue 5, May 1997, pp. 1336-1339.
- [58] M. K. Lakshmanan, I. Budiarjo and H. Nikookar, "Maximally selective wavelet packet based Multi carrier modulation scheme for Cognitive Radio systems," *Global Telecommunications Conference*, Washington, DC, November 2007, pp. 4185-4189.
- [59] D.Karamehmedovic, M.K.Lakshmanan and H.Nikookar, "Performance of Wavelet packet Modulation and OFDM in the presence Carrier frequency and phase noise," *Proceedings of the first European Wireless Technology Conference*, Amsterdam, Netherlands, October 2008,
- [60] H. Nikookar and M.K. Lakshmanan , "Performance evaluation of WPMCM with carrier frequency offset and phase noise," *Journal of Communications*, Vol.4, Issue 7, August 2009.
- [61] H. Nikookar and M.K. Lakshmanan, "Comparison of sensitivity of OFDM and Wavelet Packet Modulation to Time Synchronization error," *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, Cannes, France, September 2008, pp. 1-6.
- [62] D.D.Ariananda, M.K.Lakshmanan, H.Nikookar, "A Study on the Application of Wavelet Packet Transforms to Cognitive Radio Spectrum Estimation," *IEEE Proceedings of the 4th International Conference on CROWNCOM*, Hannover, June 2009, pp. 1-6.

- [63] Lakshmanan, M.K. Ariananda, D.D. Nikookar, H. "Cognitive Radio transmission and spectrum sensing using a wavelet packet transceiver ", *IEEE 20th International Symposium on Personal, Indoor and Mobile Radio Communications*, Tokyo, Japan, 2009, pp. 142-146.
- [64] D. Cabric, S. M. Mishra, R. W. Brodersen, "Implementation issues in spectrum sensing for Cognitive radio", *Thirty Eighth conference on Signals, Systems and Computers*, November 2004, pp. 772-776.
- [65] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Communications Surveys and Tutorials*, Vol. 11, Issue 1, First Quarter, 2009, pp. 116-130.
- [66] H. Tang, "Some physical layer issues of wide-band cognitive radio systems", *IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks*, Baltimore, Maryland, USA, November 2005, pp. 151-159.
- [67] J. G. Proakis, "Digital Communications", 4th ed. McGraw-Hill, 2001.
- [68] R. Tandra and A. Sahai, "Fundamental limits on detection in low SNR under noise uncertainty", *IEEE International Conference on Wireless Networks, Communications and Mobile Computing*, Vol. 1, Maui, HI, June 2005, pp. 464-469.
- [69] Behrouz Farhang-Boroujeny, "Filter Bank Spectrum Sensing for Cognitive Radios", *IEEE Transactions on Signal Processing*, Vol. 56, Issue 5, May 2008, pp. 1801-1811.
- [70] Youngwoo Youn, Hyoungsuk Jeon, Hoiyoon Jung and Hyuckjae Lee, "Discrete Wavelet Packet Transform based Energy Detector for Cognitive Radios", *Proceedings of IEEE Vehicular Technology conference Spring*, Dublin, Ireland May 2007, pp. 2641 - 2645.
- [71] C. Guo, T. Zhang, Z. Zeng, and C. Feng, "Investigation on spectrum sharing technology based on Cognitive radio", in *Proceedings of IEEE International Conference on Communications and Networking*, Beijing, China, October 2006, pp. 1-5.
- [72] P. Qihang, Z. Kim, W. Jun, and Li Shaoqian, " A distributed spectrum sensing scheme based on credibility and evidence theory in cognitive radio context", *IEEE Symposium on Personal , Indoor and Mobile Radio Communications*, Helsinki, Greece, 2006, pp. 1-5.

- [73] M. Matsui, H. Shiba, K. Akabane and K. Uehara, “ A Novel Co operative Sensing Technique for Cognitive radio”, *IEEE Symposium on Personal , Indoor and Mobile Radio communications*, Athens, September 2007, pp. 1-5.
- [74] S. Mangold, A. Janosch, C. Monney, “Operator assisted Cognitive radio and Dynamic spectrum assignment with dual beacon- detailed evaluation”, *First international Conference on Communication system software and middleware*, Zurich, Switzerland, January 2006, pp. 54-57.
- [75] Z. Lei, F. Chin, “ A reliable and power efficient beacon structure for cognitive radio systems”, *International Conference on Communications*, Beijing, China, May 2008, pp. 2038-2042.
- [76] Institute of Electrical and Electronics Engineering, Inc., “IEEE Communications Society Technical Committee on Cognitive Networks”, [Online]: www.eecs.ucf.edu/tccn/, 2008.
- [77] J. Hoffmeyer, D. Stewart, S. Berger, B. Eydt, F. Frantz, F. Granelli, K. Kontson, K. Nolan, P. Paweczak, R. V. Prasad, R. Roy, D. Swain, and P. Tenhula, “Standard definitions and concepts for dynamic spectrum access Terminology relating to emerging wireless networks, system functionality, and spectrum management”, *Technical Report, IEEE 1900.1 Standard*, October 2008.
- [78] C. Carlos, K. Challapali, D. Birru, S. Shankar, “IEEE 802.22: An Introduction to the first Wireless Standard based on Cognitive radios,” *Journal of Communications, Vol.1, Issue 1*, April 2006, pp. 38-47.
- [79] S. Mallat, “ A theory for multiresolution signal decomposition: The wavelet transform”, *IEEE Transactions on Pattern Analysis and Machine Intelligence, Issue 11*, July 1989, pp. 674-693.
- [80] I. Daubechies , “Ten lectures on Wavelets”, *SIAM*, 1992
- [81] R. Coifman and M.Wickerhauser, “Entropy-based algorithms for best basis selection”, *IEEE Transactions on Information Theory, (Special Issue on Wavelet Transforms and Multiresolution Signal Analysis)*, 38, February 1992, pp. 713-718.
- [82] P.P Vaidyanathan, “Multirate Systems and Filter banks”, *Prentice Hall*, 1993
- [83] M. Vitterli and Kovacevic, “Wavelets and subband coding”, *Prentice Hall*, 1995

- [84] G. Strang, T. Nguyen, "Wavelets and Filter Banks", *Wellesley Cambridge Press*, 1996
- [85] E.L. Rachel, K. Hamid, C. Bernhard, S. W. Alan and W. C. Karl, "Wavelet Packet Multiple Access", *SPIE International Symposium on Optics, Imaging and Instrumentation*, July 1994.
- [86] Jiangfeng Wu, "Wavelet packet Division Multiplexing", *PhD Thesis, McMaster University*, June 1998
- [87] M. Baro, Jacek Ilow, "Multi-band wavelet based spectrum agile Communications for Cognitive radio secondary user Communications," *IEEE International Symposium on broadband multimedia systems and broadcasting*, Las Vegas, USA, March 2008, pp. 1-5.
- [88] M. Morelli, C. J. Kuo, Man-On Pun, "Synchronization Techniques for orthogonal Frequency Division Multiple Access (OFDMA): A tutorial Review," *Proceedings of the IEEE, Vol. 95, No. 7*, July 2007, pp. 1394-1427.
- [89] David Donoho and Iain Johnstone, "Adapting to Unknown Smoothness via Wavelet Shrinkage", *Journal of the American Statistical Association Vol. 90, No. 432*, Dec., 1995, pp. 1200-1224
- [90] Georgios B. Giannakis and Guotong Zhou, "Harmonics in Multiplicative and Additive Noise: Parameter Estimation Using Cyclic Statistics" *IEEE transactions on Signal Processing, Vol. 43, No. 9*, September 1995, pp. 2217-2221.
- [91] William C. Jakes, (Editor), "Microwave Mobile Communications", *New York: John Wiley and Sons Inc. ISBN 0-471-43720-4*, February 1, 1975.
- [92] U. Khan, S. Baig and M. J. Mughal, "Performance Comparison of Wavelet Packet Modulation and OFDM for Multipath Wireless Channels", *2nd International Conference on Computer, Control and Communication*, Karachi, Pakistan, February 2009, pp. 1-4.
- [93] Vasily Strela, "Multiwavelets: Theory and Applications", *PhD Thesis, Massachusetts Institute of Technology*, June 1996.
- [94] Q.T. Jiang, "On the design of multifilter banks and orthonormal multiwavelet bases," *IEEE Transactions on Signal Processing, Vol. 46, No. 12*, December 1998, pp. 3292-3303.
- [95] V. Strela, P. Heller, and G. Strang. "The application of multiwavelet filterbanks to image processing", *IEEE Transactions on Image Processing, Vol. 8, Issue 4*, April 1999, pp. 548-563.

- [96] G. Strang, T. Nguyen, "Wavelets and Filter Banks," *Wellesley Cambridge Press*, 1996
- [97] Tim. N. T. Goodman, "A Class of orthogonal refinable functions and Wavelets," *Constr. Approximations.19*, 2003, pp.525-540.
- [98] Okkyung Cho and Ming Jun Lai, "A Class of compactly supported Orthonormal B Spline Wavelets," *Splines and Wavelets*, Athens 2005
- [99] Michael Unser, "Shift Orthogonal Wavelet Bases," *IEEE Transactions on Singal Processing*, Vol.46, No.7, July 1998, pp.1827-1836.
- [100] <http://www.ece.ubc.ca/janm/Lectures/matlabworkshop.pdf>
- [101] B. Behera, "Multiwavelet packets and frame packets", *Proceedings of Indian Acad. Sci.(Math sci.) vol.111, No.4*, Nov. 2001, pp.439-463.
- [102] G. Strang and V. Strela, "Short wavelets and matrix dilation equations", *IEEE Transactions on Signal Processing*, vol.43, No.1, 1995, pp.108-115.
- [103] J. Geronimo, D. Hardin and P. Massopust, " Fractal function and wavelet expansion based on several scaling functions", *Journal of Approximation Theory*, Vol.78, No.3 , September 1994, pp.373-401.
- [104] C. Chui and J. Lian, " A study of orthonormal multiwavelets", *Centre for Approximation Theory*, Vol.20, No.3, 1995, pp.273-298.
- [105] Q.T.Jiang, "Orthogonal multiwavelets with optimum time frequency resolution," *IEEE Transactions on Signal Processing Vol.46, Issue 4*, April 1998, pp. 830-844.
- [106] Y. Shouzhi, "A fast Algorithm for Constructing Orthogonal Multiwavelets", *ANZIAM J. Vol. 46* , 2004, pp. 185-201
- [107] M.L. You and J. Ilow, "An application of multi-wavelet packets in digital communications" *Proceedings of Second Annual Conference on Communication Networks and Services Research*, Fredericton, N.B., Canada, May 2004, pp. 10-18.
- [108] M.L. You and J. Ilow, "Adaptive Multi-wavelet Packet Modulation for Channels with Time-Frequency Localized Interference", *IEEE 6th CAS Symposium on Emerging Technologies: Mobile and Wireless Communications*, Shanghai, China, May 31-June 2,2004, pp. 441-444.

-
- [109] X.H. Yan and G.Z. Liu, "Multiwavelet packet based OFDM system," *IEEE Asia-Pacific Conference on Circuits and Systems Proceedings*, , Dec. 6-9, 2004, pp. 681-684.
- [110] Abbas Hasan Kattoush , Waleed A. Mahmoud, S. Nihad, "The performance of multiwavelets based OFDM system under different channel conditions", *Digital Signal Processing*, Vol. 20, Issue 2, March 2010, pp. 472-482.
- [111] Seon Ae Kim, Dong Geon An, Heung-Gyoon Ryu and Jin-up Kim, "Efficient SNR Estimation in OFDM System" *IEEE International Conference on Radio and Wireless Symposium*, Pheonix, January 2011, pp. 182-185.

Appendix A

Publication List

Journals

[1] M. Manju, A. B. Premkumar and A. S. Madhukumar, "Spectrum Efficient Cognitive Radio Transceiver using Multiwavelet Filters", *ISRN Journal of Communications And Networking*, Vol.2012, Article ID 607825, 13 pages, 2012.

[2] M. Manju, A. B. Premkumar and C. T. Lau, "Multi-user Interference and Cross Correlation Effects of spline multiwavelets based Cognitive radio Network", *Journal Of Communications: Special Issue on Interference Management in 4G Wireless Networks*, Vol.7, No.9, 2012.

[3] M. Manju, A. B. Premkumar and C. T. Lau, "Scalar wavelet packet based multi-user Cognitive Radio Transceiver", *Accepted in the International Journal on Engineering and Technology (IJET)*.

Conferences

[1] Manju Mathew and A.B. Premkumar and A.S. Madhukumar, "Spectrum Sensing and Adaptive Transmission using Symmetric Pair Multiwavelet Filter Bank", *Accepted in the IEEE International Conference on Communication Systems*, November 2012, Singapore.

[2] Manju Mathew, A.B. Premkumar and C.T. Lau, "Multiwavelets based multi-user Cognitive Radio Network," *Proceedings of the IEEE International Conference on Communication*

Systems, Singapore, November 17-20, 2010.

[3] Manju Mathew, A.B. Premkumar and C.T. Lau, “Multiple Access Scheme for Multi User Cognitive Radio based on Wavelet Transforms,” *Proceedings of the IEEE Vehicular Technology Conference, Spring*, Taipei, Taiwan, May 16-19, 2010.

[4] Manju Mathew, A.B. Premkumar and C.T. Lau, “Pulse Based Adaptive Carrier Waveform Generation for Cognitive Radio Applications” *Proceedings of the IEEE International Conference on Cognitive Radio Oriented Wireless Networks and Communications*, May 15-17, 2008.

[5] M. Manju, A. B. Premkumar and C. T. Lau, “An Adaptive Waveform Generation Technique for Cognitive Radio”, *Proceedings of the IEEE Vehicular Technology Conference Spring*, Singapore, May 11-14 , 2008.

Appendix B

Multiwavelet Filter Coefficients

The filter coefficient values of multiwavelets used in Chapter 5 are tabulated.

Shouzhi Multiwavelet

Low Pass Filter Coefficients

$$\begin{aligned} H[0] &= \begin{bmatrix} (2 - \sqrt{7})/4 & 0 \\ (2 + \sqrt{7})/4 & 0 \end{bmatrix} \\ H[1] &= \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix} \\ H[2] &= \begin{bmatrix} 0 & (2 + \sqrt{7})/4 \\ 0 & (2 - \sqrt{7})/4 \end{bmatrix} \end{aligned} \tag{B.1}$$

Low Pass Filter Coefficients

$$\begin{aligned} G[0] &= \begin{bmatrix} 0 & 3/4 \\ 0 & 1/4 \end{bmatrix} \\ G[1] &= \begin{bmatrix} -(2 + \sqrt{7})/4 & -(2 - \sqrt{7})/4 \\ -(2 - \sqrt{7})/4 & -(2 + \sqrt{7})/4 \end{bmatrix} \\ G[2] &= \begin{bmatrix} 1/4 & 0 \\ 3/4 & 0 \end{bmatrix} \end{aligned} \tag{B.2}$$

ChuiLian MultiwaveletLow Pass Filter Coefficients

$$\begin{aligned}
H[0] &= \begin{bmatrix} (10 - 3\sqrt{10})/40 & (5\sqrt{6} - 2\sqrt{15})/40 \\ (5\sqrt{6} - 3\sqrt{15})/40 & (5 - 3\sqrt{10})/40 \end{bmatrix} \\
H[1] &= \begin{bmatrix} (30 + 3\sqrt{10})/40 & (5\sqrt{6} - 2\sqrt{15})/40 \\ (-5\sqrt{6} + 7\sqrt{15})/40 & (15 - 3\sqrt{10})/40 \end{bmatrix} \\
H[2] &= \begin{bmatrix} (30 + 3\sqrt{10})/40 & (-5\sqrt{6} - 2\sqrt{15})/40 \\ (5\sqrt{6} + 7\sqrt{15})/40 & (5 - 3\sqrt{10})/40 \end{bmatrix} \\
H[3] &= \begin{bmatrix} (10 - 3\sqrt{10})/40 & (-5\sqrt{6} - 2\sqrt{15})/40 \\ (-5\sqrt{6} - 3\sqrt{15})/40 & (5 - 3\sqrt{10})/40 \end{bmatrix}
\end{aligned} \tag{B.3}$$

High Pass Filter Coefficients

$$\begin{aligned}
G[0] &= \begin{bmatrix} (5\sqrt{6} - 2\sqrt{15})/40 & (10 + 3\sqrt{10})/40 \\ (-5 + 3\sqrt{10})/40 & (5\sqrt{6} - 3\sqrt{15})/40 \end{bmatrix} \\
G[1] &= \begin{bmatrix} (-5\sqrt{6} - 2\sqrt{15})/40 & (30 + 3\sqrt{10})/40 \\ (15 - 3\sqrt{10})/40 & (5\sqrt{6} + 7\sqrt{15})/40 \end{bmatrix} \\
G[2] &= \begin{bmatrix} -(5\sqrt{6} - 2\sqrt{15})/40 & -(30 + 3\sqrt{10})/40 \\ -(15 - 3\sqrt{10})/40 & (5\sqrt{6} + 7\sqrt{15})/40 \end{bmatrix} \\
G[3] &= \begin{bmatrix} (5\sqrt{6} - 2\sqrt{15})/40 & (10 - 3\sqrt{10})/40 \\ (5 - 3\sqrt{10})/40 & (5\sqrt{6} - 3\sqrt{15})/40 \end{bmatrix}
\end{aligned} \tag{B.4}$$

GHM MultiwaveletLow Pass Filter Coefficients

$$\begin{aligned}
H[0] &= \begin{bmatrix} 3/(5\sqrt{2}) & 4/5 \\ -1/20 & -3/(10\sqrt{2}) \end{bmatrix} \\
H[1] &= \begin{bmatrix} 3/(5\sqrt{2}) & 0 \\ 9/20 & 1/\sqrt{2} \end{bmatrix} \\
H[2] &= \begin{bmatrix} 0 & 0 \\ 9/20 & -3/(10\sqrt{2}) \end{bmatrix} \\
H[3] &= \begin{bmatrix} 0 & 0 \\ -1/20 & 0 \end{bmatrix}
\end{aligned} \tag{B.5}$$

High Pass Filter Coefficients

$$\begin{aligned}
G[0] &= \begin{bmatrix} -1/20 & -3/(10\sqrt{2}) \\ 1/10\sqrt{2} & 3/10 \end{bmatrix} \\
G[1] &= \begin{bmatrix} 9/20 & -1/\sqrt{2} \\ -9/10\sqrt{2} & 0 \end{bmatrix} \\
G[2] &= \begin{bmatrix} 9/20 & -3/10\sqrt{2} \\ 9/10\sqrt{2} & 3/10 \end{bmatrix} \\
G[3] &= \begin{bmatrix} -1/20 & 0 \\ -1/10\sqrt{2} & 0 \end{bmatrix}
\end{aligned} \tag{B.6}$$

Quintang MultiwaveletLow Pass Filter Coefficients

$$\begin{aligned}
H[0] &= \begin{bmatrix} 0.0117 & 0.0117 \\ -0.0071 & -0.0071 \end{bmatrix} \\
H[1] &= \begin{bmatrix} -0.0212 & -0.0339 \\ 0.0057 & 0.0280 \end{bmatrix} \\
H[2] &= \begin{bmatrix} 0.2383 & 0.2092 \\ -0.3161 & -0.2632 \end{bmatrix} \\
H[3] &= \begin{bmatrix} 0.5433 & 0 \\ 0 & 0.3997 \end{bmatrix} \\
H[4] &= \begin{bmatrix} 0.2632 & -0.3161 \\ 0.2092 & -0.2383 \end{bmatrix} \\
H[5] &= \begin{bmatrix} -0.0280 & 0.0057 \\ -0.0339 & 0.0212 \end{bmatrix} \\
H[6] &= \begin{bmatrix} 0.0071 & -0.0071 \\ 0.0117 & -0.0117 \end{bmatrix}
\end{aligned} \tag{B.7}$$

High Pass Filter Coefficients

$$\begin{aligned}
G[0] &= \begin{bmatrix} 0.0061 & 0.0061 \\ -0.0111 & -0.0112 \end{bmatrix} \\
G[1] &= \begin{bmatrix} -0.0262 & -0.0028 \\ 0.0335 & 0.0193 \end{bmatrix} \\
G[2] &= \begin{bmatrix} 0.2438 & 0.2994 \\ -0.1829 & -0.2167 \end{bmatrix} \\
G[3] &= \begin{bmatrix} -0.4470 & 0 \\ 0 & -0.5793 \end{bmatrix} \\
G[4] &= \begin{bmatrix} -0.0262 & -0.0028 \\ 0.0335 & 0.0193 \end{bmatrix} \\
G[5] &= \begin{bmatrix} -0.0193 & 0.0335 \\ -0.0028 & 0.0262 \end{bmatrix} \\
G[6] &= \begin{bmatrix} 0.0112 & -0.0111 \\ 0.0061 & -0.0061 \end{bmatrix}
\end{aligned} \tag{B.8}$$