Compact modeling of high-voltage (LDMOS/MISHEMT) devices

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**COMPACT MODELING OF HIGH-VOLTAGE
(LDMOS/MISHEMT) DEVICES**

**ZHANG JUNBIN
SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING
2012**

COMPACT MODELING OF HIGH-VOLTAGE (LDMOS/MISHEMT) DEVICES

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Abstract

In this thesis, physics-based analytical compact models are developed for the laterally diffused metal-oxide-semiconductor (LDMOS) transistor and the metal-insulator-semiconductor high electron mobility transistor (MISHEMT), respectively, in order to aid the microwave circuit simulation. The LDMOS is physically divided into two regions: the core channel and the drift channel. Surface potential based drain current models are developed for the core channel and the drift channel individually. Then a sub-circuit that consists of the core channel and the drift channel is used to model the current voltage characteristic of the LDMOS. Due to the lateral nonuniform doping in the core channel, there are peaks in the capacitances of the LDMOS. The “peaky” capacitances cannot be captured by the charge model formulated based on the Ward-Dutton (WD) partition scheme. In this regard, a new charge partition method that is applicable in the presence of lateral non-uniform doping is proposed. Based on the proposed method, for the first time, a compact charge model that is able to reproduce the peaky capacitance is derived. MISHEMT’s operation is based on the conduction of the two-dimensional-electron-gas (2DEG). A physical and explicit expression for the 2DEG density considering two lowest subbands, which is valid from subthreshold region to the active operation region, is derived for the first time. With the newly derived 2DEG density expression, an analytical and symmetrical drain current model for the MISHEMT is formulated based on the bulk MOS current model.

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List of Symbols

Symbols	Description (Unit)
ΔQ	Charge induced after $V_{gs} > V_{gc}$ (C/cm ²)
ΔQ_{D_A} , ΔQ_{D_Al} , ΔQ_{D_A2} , ΔQ_{D_An}	Charge deviation of Q_{D_A} (C/cm ²)
ΔQ_{D_B}	Charge deviation of Q_{D_B} (C/cm ²)
ΔQ_{D_h}	M1 drain terminal charge induced after $V_{gs} > V_{gc}$ (C/cm ²)
Δq_h , Δq_l	Fitting parameters for p_{em}
ΔQ_{S_A} , ΔQ_{S_Al} , ΔQ_{S_A2} , ΔQ_{S_An}	Charge deviation of Q_{S_A} (C/cm ²)
ΔQ_{S_B}	Charge deviation of Q_{S_B} (C/cm ²)
ΔQ_{S_l}	M2 source terminal charge induced after $V_{gs} > V_{gc}$ (C/cm ²)
ΔT	Temperature increase (K)
ΔV , ΔV_1 , ΔV_2 , ΔV_3	Voltage deviation (V)
$\Delta V_A(t)$	Real time voltage deviation at the point A (V)
$\Delta V_B(t)$	Real time voltage deviation at the point B (V)
μ	Electron mobility (cm ² V ⁻¹ s ⁻¹)
μ_0	Transverse field dependent mobility (cm ² V ⁻¹ s ⁻¹)
$\mu_{0,R}$	Mobility of the bulk carrier (cm ² V ⁻¹ s ⁻¹)
$\mu_{0,sf,s}$	Surface carrier transverse field dependent mobility of the drift channel (cm ² V ⁻¹ s ⁻¹)
μ_1 , μ_2 , μ_3 , v	Fitting parameters for the mobility (μ_l : cm ² V ⁻¹ s ⁻¹)
μ_b	Lateral field dependent mobility of the bulk carrier (cm ² V ⁻¹ s ⁻¹)
μ_{co}	columbic scattering mobility (cm ² V ⁻¹ s ⁻¹)
μ_{eff0} , μ_{neff}	Lateral field dependent mobility (cm ² V ⁻¹ s ⁻¹)
μ_{ph}	Phonon scattering mobility (cm ² V ⁻¹ s ⁻¹)
μ_{sf} , $\mu_{sf,s}$	Surface carrier lateral field dependent mobility of the drift channel (cm ² V ⁻¹ s ⁻¹)
μ_{sr}	Surface roughness scattering mobility (cm ² V ⁻¹ s ⁻¹)
A	One point at M1 channel (N.A)
B	One point at M2's channel (N.A)
C_d	Unified regional Fermi level for active region 1 and 2 (F/cm ²)
C_{dg}	Drain-gate capacitance (F)
C_{gd}	Gate-drain capacitances (F)
C_{gg}	Gate-gate capacitance (F)
C_{it}	Interface trap capacitance (F/cm ²)
C_{ox}	Oxide capacitance (F/cm ²)
$C_{ox,dr}$	The capacitance of the drift region (F/cm ²)
D	Density of states (cm ⁻² V ⁻¹)
d	Thickness of the barrier and the spacer layer (cm)
d_6	Fitting parameter for ΔQ_{D_h} (N.A)
d_7	Fitting parameter for ΔQ_{S_l} (N.A)
e	Thickness of the spacer layer (cm)

E_0	Energy of the first sub band (V)
E_1	Energy of the second sub band (V)
E_{eff}	Effective transverse field (V/cm)
$E_{eff,s}$	Effective transverse electric field at the source end (V/cm)
$E_{eff,sf}$	drift channel surface effective transverse field (V/cm)
E_f	Fermi level of the MISHEMT (V)
E_{f_B}	Regional Fermi level solution in the active region 1 (V)
E_{f_C}	Regional Fermi level solution in the active region 2 (V)
E_{f_op}	Active regions unified Fermi level solution (V)
E_{f_sub}	Regional Fermi level solution in the subthreshold region (V)
E_{f1}, E_B, E_C	horizontal coordinates in the n_s - E_f plane (V)
$E_{sar,R}$	Saturation electric field for bulk carrier (V/cm)
E_{sat}	Saturation electric field (V/cm)
$E_{sat,d}$	Saturation electric field evaluated at the drain side (V/cm)
$E_{sat,s}$	Saturation electric field evaluated at the source side (V/cm)
$E_{sat,sf}$	Saturation electric field for surface carrier (V/cm)
E_y	Lateral electric field (V/cm)
$E_{y,b}$	Lateral electric field of the bulk silicon (V/cm)
g_{ADA}	The conductance looking from A to the drain (A/V)
g_{ASA}	The conductance looking from A to the source (A/V)
g_{vo}	Channel length modulation gain factor (N.A)
I_{AD}	The steady state current flow from A to the drain (A)
I_{AS}	The steady state current flow from A to the source (A)
$i_{AS}(t)$	Real time current flow from A to the source (A)
i_{CD_A}	The charging current flow from drain to A (A)
i_{CS_A}	The charging current flow from source to A (A)
$i_D(\tau)$	The real time current goes into the channel from the drain (A)
$i_{DA}(t)$	Real time current flow from drain to the source (A)
$I_{dd,s}$	Drift diffusion current evaluated the source end (A)
I_{ds}	Drain source current (A)
$I_{ds,sf}$	Drift channel surface drift diffusion current (A)
I_{dsat}	Saturation current (A)
$i_{DX}(\tau)$	The charging current goes into the channel from the drain (A)
I_R	Bulk silicon current of the drift region (A)
$i_S(\tau)$	The real time current goes into the channel from the source (A)
$I_{sat,sf}$	Drift channel surface saturation current (A)
$i_{SX}(\tau)$	The charging current goes into the channel from the source (A)
$i_T(\tau)$	The steady state terminal current (A)
K	The value of M1 source charge density over M2 drain charge density (N.A)
K_B	Boltzmann constant = $1.3806488 \times 10^{-23}$ (JK ⁻¹)
$L\{\}$	Lambert W function

L_{dr}	Channel length of the drift channel (cm)
L_{eff}	Effective channel length (cm)
L_h	Channel length of M1 (cm)
L_l	Channel length of M2 (cm)
n	electrons concentration (cm^{-3})
$N(x)$	Doping profile in the barrier and the spacer layer
N_b	Doping of the barrier (cm^{-3})
N_d	Drift region donor doping (cm^{-3})
N_{epi}	Doping of the epi layer (C/cm^2)
n_i	Intrinsic carrier concentration in silicon $=1 \times 10^{10}$ (cm^{-3})
N_{pch}, N_a	Effective doping for lateral nonuniformly doped channel (cm^{-3})
n_s	2DEG density of the MISHEMT (cm^{-2})
n_{s_B}	Regional 2DEG density solution in the active region 1 (cm^{-2})
n_{s_C}	Regional 2DEG density solution in the active region 2 (cm^{-2})
n_{s_op}	Unified regional 2DEG density for active region 1 and 2 (cm^{-2})
n_{s_sub}	Regional 2DEG density solution in the subthreshold region (cm^{-2})
n_{s1}, n_B, n_C	vertical coordinates in the n_s - E_f plane (cm^{-2})
n_{step}	Number of steps (N.A)
n_{sth}	Subthreshold slop factor (N.A)
p	holes concentration (cm^{-3})
P_{em}	An empirical expression used to substitute pp (N.A)
pp	A constant value (N.A)
q	Elemental charge $=1.6 \times 10^{-19}$ (C)
Q_A	The charge density at the point A (C/cm^2)
Q_{acc}	Accumulation charge density of the drift channel (C/cm^2)
Q_b	Bulk charge density (C/cm^2)
Q_B	Terminal substrate charge for the p-channel (C/cm^2)
Q_B	The charge density at the point B (C/cm^2)
Q_{B_h}	Terminal substrate charge for M1 (C/cm^2)
Q_{B_l}	Terminal substrate charge for M2 (C/cm^2)
Q_D	Terminal drain charge for the p-channel (C/cm^2)
Q_{D_A}	The charge at the point A that belongs to Q_D (C/cm^2)
Q_{D_B}	The charge at the point B that belongs to Q_D (C/cm^2)
Q_{D_h}	Terminal drain charge for M1 (C/cm^2)
Q_{D_l}	Terminal drain charge for M2 (C/cm^2)
Q_g	Gate charge density (C/cm^2)
Q_g	Terminal gate charge for the p-channel (C/cm^2)
$Q_{g,dr}$	Gate charge density of the drift channel (C/cm^2)
Q_{G_h}	Terminal gate charge for M1 (C/cm^2)
Q_{G_l}	Terminal gate charge for M2 (C/cm^2)
Q_i	Inversion charge density (C/cm^2)
$q_i(y)$	Inversion charge density (C/cm^2)
$q_{i_h}(y)$	Normalized inversion charge density of M1 (C/cm^2)

$q_{i,l}(y)$	Normalized inversion charge density of M2 (C/cm ²)
Q_{ox}	oxide charge density (C/cm ²)
Q_{pn}	Space charge density of the p-n junction between the drift channel and the epi layer (C/cm ²)
Q_s	Terminal source charge for the p-channel (C/cm ²)
$Q_{s,A}$	The charge at the point A that belongs to Q_s (C/cm ²)
$Q_{s,B}$	The charge at the point B that belongs to Q_s (C/cm ²)
$Q_{s,h}$	Terminal source charge for M1 (C/cm ²)
$Q_{s,l}$	Terminal source charge for M2 (C/cm ²)
R_s	Source resistance (ohm)
R_{sd}	Total Source drain resistance (ohm)
R_{th}	Thermal resistance (K/W)
$sgn(x)$	Sign function =1 for $x>0$ and -1 for $x<0$
T	Absolute temperature (K)
T_0	Room temperature (=300) (K)
t_c	Thickness of the cap layer (cm)
t_{ox}	Insulator thickness (cm)
$t_{ox,dr}$	Oxide thickness of the drift region (cm)
$t_{si,dr}$	Silicon thickness of the drift channel (cm)
v_0	Electric field dependent velocity (cm/s)
V_A	Steady state voltage the point A (V)
$V_{Aeff,d}$	Effective early voltage (V)
V_b	Substrate voltage (V)
V_{ba}	Voltage drop across the barrier layer of the MISHEMT (V)
V_{bi}	Built in potential of the p-n junction (V)
V_c	Channel voltage of the MISHEMT (V)
V_{cap}	Voltage drop across the cap layer of the MISHEMT (V)
V_{cb}	Channel voltage (V)
V_d	Drain voltage (V)
$V_{d,eff}$	Effective drain voltage (V)
V_{db}	Voltage at the drain terminal (V)
V_{deff}	Effective drain voltage (V)
$V_{deff,sf}$	Drift channel effective drain source voltage (V)
V_{ds}	Drain source voltage (V)
$V_{ds,eff}$	Effective drain source voltage (V)
$V_{ds,eff}$	Effective drain-source voltage (V)
$V_{ds,sat}$	Saturation drain source voltage (V)
$V_{ds,sat}$	Saturation drain source voltage (V)
$V_{dsat,sf}$	Drift channel drain source saturation voltage (A)
V_{fb}	Flat band voltage (V)
$V_{fb,dr}$	Flat band voltage of the drift region (V)
V_g	Gate voltage (V)
V_{gb}	Gate bulk voltage difference (V)
V_{gc}	The gate voltage when M1 set in linear region from saturation region (V)
V_{gf}	Flat band shifted gate bulk voltage (V)
VGF	Forward interpolated flat band shifted gate bulk voltage

	(V)
$V_{gf,dr}$	Flat band shifted gate bulk voltage for the drift region (V)
V_{go}	V_{off} shifted gate voltage (V)
VGR	Reverse interpolated flat band shifted gate bulk voltage (V)
V_{gb}, q_i	Normalized inversion charge density (V)
$V_{gt,d}$	Normalized inversion charge density at the drain (V)
$V_{gt,s}$	Normalized inversion charge density at the source (V)
$V_{gt,sf,s}$	Drift channel normalized accumulation charge density (V)
V_i	Internal voltage at the joint point of M1 and M2 (V)
VOF	Interpolated V_{off} (V)
V_{off}	Off voltage of the MISHEMT, similar to the threshold voltage of MOSFET (V)
V_{ox}	Voltage drop across the insulator of the MISHEMT (V)
V_s	Source voltage (V)
$V_{s,eff}$	Effective source voltage (V)
v_{sat}	Saturation velocity (cm/s)
V_{sb}	Voltage at the source terminal (V)
$V_{sd,sat}$	Saturation source drain voltage (V)
V_t	Threshold voltage (V)
V_{thh}	Threshold voltage for M1 (V)
V_{thl}	Threshold voltage for M2 (V)
V_{tm}	Thermal voltage =0.0258 (V)
W	The width of the transistor (cm)
x_0	Integration dummy variable (cm)
x_A	The position of A (cm)
x_B	The position of B (cm)
β	Transistor gain factor (A/V ²)
γ	Bulk body factor (V ^{1/2})
γ_0	Physical parameters related to the first sub band of the MISHEMT (Vm ^{4/3})
γ_1	Physical parameters related to the second sub band of the MISHEMT (Vm ^{4/3})
δ_{dr}	Fitting parameters for unified inversion surface potential of the drift region (N.A)
$\delta_{eff}, \delta_{op}, \delta_{of}$	Smoothing parameters (N.A)
δ_L	Fitting parameters for Lateral field dependent mobility (N.A)
δ_R	Fitting parameters for the drift region bulk carrier mobility (N.A)
$\delta_{sat,sf}$	Fitting parameter for drift channel effective drain source voltage (N.A)
δ_{ss}	Smoothing parameters for ϕ_{DS} (N.A)
ϵ_b	Permittivity of the barrier (F/cm)
ϵ_c	Permittivity of the cap layer (F/cm)
ϵ_{ch}	Permittivity of the channel (F/cm)
ϵ_{ox}	Insulator permittivity (F/cm)
ϵ_{si}	Silicon permittivity (F/cm)

ζ_n, ζ_b	Fitting parameters for the effective transverse field (N.A)
$\zeta_{n,sf}$	Fitting parameters for drift channel transverse field (N.A)
ζ_{vo}	Fitting parameters for early voltage (V^{-1})
$\eta(), \mathcal{G}_f(),$ $\mathcal{G}_r(), \mathcal{G}_{eff}(), \mathcal{G}_f()$	Smoothing and interpolate functions (N.A)
$\theta, \Delta V_{fb}$	Fitting parameters of lateral nonuniform doping effect on the subthreshold behavior of p-channel
μ_{eff}	Effective lateral field dependent mobility ($cm^2V^{-1}s^{-1}$)
μ_n	Electron mobility ($cm^2V^{-1}s^{-1}$)
ξ	Electric field (V/cm)
ξ_{is}	Interface electric field (V/cm)
ξ_s	Channel surface transverse electric field (V/cm)
ρ	Net charge concentration in the semiconductor (C/cm^3)
ρ_{dr}	Net charge concentration of the drift region (C/cm^3)
σ_f	Fitting parameters for forward interpolate function (N.A)
$\sigma_{f,dr}$	Fitting parameters for the forward interpolate function (N.A)
σ_r	Fitting parameters for reverse interpolate function (N.A)
$\sigma_{r,dr}$	Fitting parameters for the reverse interpolate function (N.A)
ψ	Electron potential (band bending) in the semiconductor (V)
ψ_{dr}	Channel surface potential of the drift region (V)
ϕ_{Acc}	Interpolated accumulation regional surface potential solution (V)
ϕ_{CC}	Regional surface potential solution for the accumulation (V)
ϕ_{DD}	Regional surface potential solution for the depletion (V)
$\phi_{dr}(y)$	Drift channel surface potential (V)
$\phi_{dr,d}$	drift channel surface potential at the drain end (V)
$\phi_{dr,s}$	drift channel surface potential at the source end (V)
ϕ_{DS}	Unified surface potential for depletion and strong inversion (V)
ϕ_f	Bulk Fermi level (V)
$\phi_{f,dr}$	Bulk Fermi level of the drift region (V)
ϕ_{MB}	Metal semiconductor work function difference (V)
ϕ_s	Channel surface potential (V)
$\phi_{s,d}$	Surface potential at the drain side (V)
$\phi_{s,dr}$	Channel surface potential of the drift region (V)
$\phi_{s,s}$	Surface potential at the source side (V)
ϕ_{SB}	the difference between the insulator conduction band edge and the cap layer conduction band edge (V)
ϕ_{Seff}	Unified regional surface potential (V)
ϕ_{SS}	Regional surface potential solution for the strong inversion (V)
ϕ_{Str}	Interpolated strong inversion regional surface potential solution (V)
ϕ_{SUB}	Interpolated depletion regional surface potential solution

	(V)
ϕ_{MB}	difference between gate Fermi level and the insulator conduction band edge (V)

List of Acronyms

Acronyms	Description
(W-)CDMA	(Wideband) C ode d ivision m ultiple a ccess Network technology
2DEG	t wo- d imensional- e lectron- g as
3G/3.5G	3/3.5 generation mobile telecommunication
CDMA2000	C DMA version of IMT- 2000 standard Network technology
CLM	C hannel l ength m odulation
EB	E nergy b and
EDGE	E nhanced d ata rate for g lobal e volution Network technology
GPRS	G eneral p acket r adio s ervice
GPS	G lobal p ositioning s ystem
GSM	G roupe S pécial M obile: global system for mobile communication
HV	H igh v oltage
LDMOS	L ateral d iffused m etal o xide semiconductor
LHS	L eft h and s ide
MISHEMT	M etal i nsulator semiconductor h igh electron m obility transistor
MOSFET	M etal o xide semiconductor f ield e ffect t ransistor
NR	Newton R haphson
PA	P ower a mplifier
RF	R adio f requency
RHS	R ight h and s ide
SDP	S tep d oping p rofile
S-MOS	Step doping profile based MOS
STI	S hallow t rench i solation
TCAD	T echnology c omputer- a ided d esign
UID	U n-intentionally d oped
URM	U nified r egional m odeling
URSP	U nified r egional s urface p otential
VO	V elocity o vershoot
WD	W ard- D utton
Wi-Fi	W ireless f idelity Network technology
WiMax	W orldwide I nteroperability for m icrowave a ccess

Chapter 1 Introduction

Since the beginning of the 21st century, the wireless telecommunication systems and radio frequency (RF) consumer electronics have entered a new stage. One of the best examples is the cellular phone. Cellular data services have enabled phone users to access internet using their smart mobile phones, as the technology evolves from the early GSM, CDMA and GPRS networks to 3G networks, such as W-CDMA, EDGE or CDMA2000. It is recorded that there are more than 4.6 billion mobile cellular subscriptions worldwide by the end of 2010 [1]. Other examples include the Wi-Fi and GPS. The wireless telecommunications and RF consumer electronics have now become even more booming industries, where information plays a very important role in our life. To design, analyze, improve and optimize the relevant RF circuits in wireless systems, computer-aided simulations using *compact models* are highly demanded in order to save design time and lower the cost. In order to assist the RF circuit design, this work focuses on the compact modeling the high-voltage (HV) transistors, in particular, the lateral-diffused metal-oxide-silicon (LDMOS) transistor and the metal-insulator-semiconductor high electron mobility transistor (MISHEMT).

1.1 Motivation

LDMOS and HEMTs or MISHEMTs are the hot device choices for the wireless system design. LDMOS transistor is usually selected for the **power amplifier (PA)** designs in the base stations for wireless communication systems such as GSM/EDGE, WCDMA and WiMax [2-6], due to its high efficiency, low cost and good reliability. Its capability to operate at the s-band radar frequency (2 to 4GHz) has been recently demonstrated [7]. To optimize and enhance the transistor and circuit design, a compact model that faithfully describes the essential physics of the transistor would be very useful.

Due to the lateral nonuniformly doped channel, the capacitance of LDMOS behaves differently from that of the conventional **metal-oxide-semiconductor field effect transistor (MOSFET)**. Abnormal peaks show up in its gate-to-gate, gate-to-drain, and drain-to-gate capacitances (C_{gg} , C_{gd} and C_{dg}) [8-11]. Capturing these “peaky” behaviors in a compact model and developing the terminal gate-, drain-, source- and bulk-charge is not an easy work. The well-known **Ward-Dutton (WD)** [12] scheme for charge formulation is demonstrated to be inapplicable to the nonuniformly doped channel [13-18]. To the best of our knowledge, no charge model for LDMOS is currently available. Capacitances are important parameters in the RF design for wireless systems. Although there are some numerical methods or macro models [14, 16, 19, 20] available to calculate the capacitance, charge-based approach remains advantageous as it naturally guarantees charge conservation. Therefore, the

terminal charge model that can predict the peaky capacitances is highly demanded.

The HEMT is another hot device for RF design because it has high mobility, high power density and high breakdown voltage. The MISHEMT is a variation of HEMT, in which an insulator layer is inserted between the gate and the semiconductor. The MISHEMT has the advantages of a normal HEMT, meanwhile, it has improved gate leakage performance compared to the normal HEMT [21]. This makes it a good device candidate for the high frequency applications [22, 23]. To enable accurate circuit simulation of MISHEMT-based circuits, a good compact model that physically describes the device behaviors in all operation regions is highly demanded. Though the currently available HEMT models can be easily extended to describe the MISHEMT, most of them [24-29] are based on the semi-empirical expression for the **two-dimensional-electron-gas** (2DEG) density, n_s . Models based on the empirical expressions do not have the correct correlation to the physical parameters of the device. Some other models solve for n_s physically [30, 31], but they do not consider the subband splitting. In addition to the 2DEG model, the available HEMT models seldom accounts for the subthreshold region, which is also important for circuit simulations. Thus, it is desirable to derive a physical 2DEG density model that considers the subband splitting extends to subthreshold region for formulating the compact drain current model for MISHEMTs, which is valid in all operation regions.

1.2 Objectives

The objectives of this thesis are:

- i. Develop physics-based surface potential and drain current models for the LDMOS. Mobility degradation effect should be included in the drift region so that the quasi saturation behavior can be captured. The model should be scalable and accurate so as to predict the device behaviors.
- ii. Deduce the physical partition method that is applicable to the lateral nonuniformly doped channel and, hence, develop the terminal charge model for the LDMOS so as to predict the unique “peaky” capacitance behavior of LDMOS.
- iii. Develop a physics-based drain current model for the MISHEMT based on the unified regional modeling approach. The effects of mobility degradation, interface trap and self-heating should be included. The model should be scalable and accurate.

1.3 Major Contributions of the thesis

The major contributions of the thesis are:

- i. Developed a new drift-channel model for LDMOS, which considers both the surface and bulk current conductions, in which the surface conduction is modeled as an accumulation transistor and the bulk conduction is modeled as a bulk resistor. Mobility degradation is accounted for in modeling both conduction mechanisms. Then, the p-channel and drift-channel LDMOS models are coded into HSPICE, and a subcircuit consisting of the p-channel and drift-channel is used to simulate the I-V behavior of LDMOS.
- ii. Proposed a new physical charge partition method for the lateral nonuniformly doped channel, which is applicable to the LDMOS, and hence, formulated the source/drain terminal charge model for the intrinsic channel of the LDMOS, which is able to capture the “peaky” capacitance behaviors.
- iii. Derived a new physical model for the 2DEG density for heterojunction based transistors. The model is obtained by solving for the Fermi level regionally and combining the piecewise solutions into one single piece expression. It considers the subband splitting. Based on the new 2DEG

density model, a symmetric I-V model is developed for HEMTs and MISHEMTs.

1.4 Organization of the thesis

This thesis is written in five chapters. Chapter 1 introduces the motivations, objectives and the major contributions of this work.

Chapter 2 describes the formulation of LDMOS compact model. The **unified regional surface potential (URSP)** formulation is illustrated in detail for both n- and p- doped channel. The LDMOS is consisted of two regions: p-channel and drift channel. Based on the URSP, the p-channel model is developed according to the **unified regional modeling (URM)** [32-38] approach. The drift channel is modeled as a parallel combination of an accumulation transistor and a bulk resistor, both including the lateral mobility degradation effect. Finally, both the p-channel model and the drift channel model are coded into verilog-A and a subcircuit consisting of the p-channel and the drift channel connected in series is used to predict the I-V behavior of the LDMOS.

Chapter 3 discusses the charge formulation for transistors with lateral nonuniformly doped channel. A new definition for the source/drain terminal charge is introduced. A novel charge partition method is derived. Based on the partition method, terminal source and drain charge models for lateral

nonuniform channel are developed. The charge model is then shown to be able to predict the “peaky” capacitance behaviors.

Chapter 4 describes the I-V model derivation for the MISHEMT. A physical 2DEG density expression that considers the subband splitting is derived. Next, the 2DEG density model is used to develop the MISHEMT I-V model, which includes the effects of mobility degradation, interface trap and self-heating.

Finally, chapter 5 summarizes and concludes the compact modeling of high voltage transistors with suggestions and recommendations for future work.

Chapter 2 LDMOS Drain Current Model

2.1 Introduction

The LDMOS transistor is a modified version of the conventional MOSFET. It is specially designed for high-voltage applications in the power amplifier design, which requires the power transistor to be cost effective, with high breakdown voltage and good linearity. It is the technology of choice for base stations [2-6, 39-43]. The cross section of a typical n-type LDMOS is shown in Figure 2.1. Generally, the n-LDMOS consists of two main regions. The first region is the intrinsic channel, which is physically connected to the source and capped by a thin oxide layer. This region is referred to as the p-channel, as it is doped with p-type for n-LDMOS. The doping in the p-channel is achieved by diffusion from the source, so a decreasing doping profile from the source towards the drain in the p-channel can be expected. The second main region is the n-doped drift region, which physically and electrically connects the p-channel and the drain. This region is purposely inserted between the p-channel and the drain so that the LDMOS is able to stand high voltage. For the drift region design, different techniques, such as shallow trench isolation (STI) [44, 45], reduced surface field (RESUF) [46-49] and (drain side) metal shield plate at the gate end near the drain [50], are available to make the LDMOS more sophisticated for commercial use.

Figure 2.1 The cross section of a conceptual LDMOS structure

As the modern wireless communication system moving beyond the 3G/3.5G technology, the need for improving the transistor and relevant circuits to meet the requirement for future designs has been increasing. Accurate circuit simulations play an important role in the device and circuit optimization as it saves the design time and hence minimize the design cost. In order to enable accurate circuit simulation, the compact current/charge models for LDMOS are physically derived in this chapter based on the URM approach. Mobility degradation effect and channel length modulation effect are consistently included.

2.2 Modeling Strategy

The n-LDMOS transistor consists of two main regions: p-channel and drift channel. Both regions are very important to the I-V behavior of the LDMOS, but they cannot be modeled as one single channel; so these two regions are physically modeled separately. The LDMOS is therefore decomposed into two sub transistors connected in series as illustrated in Figure 2.2, in which one transistor is basically the p-channel and the other is the drift channel.

Figure 2.2 LDMOS decomposed into two sub transistors

These two sub transistors are separately modeled in the following sections. Then, a sub-circuit consisting of the p-channel and drift channel connected in series, as shown in bottom left of Figure 2.2, is used to describe the LDMOS.

2.3 P-channel Current Model

In this section, the I-V model for the p-channel is derived. The model is formulated based on the URM approach and is essentially surface-potential based. The cross section of the p-channels is shown in Figure 2.3, in which the doping level versus the lateral direction, y direction is qualitatively illustrated. In principle, as the p-channel is lateral nonuniformly doped, it is supposed to be ‘sliced’ into many sub sections so that each section can be treated and modeled as a uniformly doped MOSFET. However, this approach is computationally demanding and time consuming. Instead, the p-channel is modeled as a uniformly doped MOSFET with an effective doping in this work, with some modifications. Although this approach is not precise in physically describing the transistor, it is sufficient in modeling the I-V behavior of the p-channel transistor. Surface potential will be derived first, and then, the drain current model.

Figure 2.3: Schematic of the p-channel transistor. The inset shows the lateral doping versus y

2.3.1 Surface Potential Solution

Surface potential solution is the building block for a compact model. It can be derived by solving the Poisson equation with the relevant boundary conditions. The URSP solution is adopted from the previous work [51, 52]. The Poisson equation is given by,

$$\frac{\partial \psi^2}{\partial x^2} = -\frac{\rho}{\epsilon_{si}} \quad (2.1)$$

in which x (with origin at the oxide/silicon interface and positive below the interface) is the spatial variable, ψ is the electron static potential, ϵ_{si} is the silicon permittivity, and ρ is the net charge concentration in the channel and, for a n-type MOSFET, ρ is given by

$$\rho = q \cdot [p - n - N_a + N_d] \quad (2.2)$$

where $q=1.6 \times 10^{-19} \text{C}$ is the elemental charge, N_a is the p-type effective doping, N_d is the n-type doping, p and n are the hole and electron concentrations, respectively, given by

$$n = n_i \cdot e^{\frac{\psi - \phi_f - V_{cb}}{V_{tm}}} \quad (2.3)$$

$$p = n_i \cdot e^{-\frac{\psi - \phi_f}{V_{tm}}} \quad (2.4)$$

in which $V_{tm}=K_B T/q \approx 0.0258 \text{V}$ is the thermal voltage, $K_B=1.3806488 \times 10^{-23} \text{JK}^{-1}$ is the Boltzmann constant, T is the absolute temperature in K , V_{cb} is the channel voltage (which is V_{sb} at the source and V_{db} at the drain), and ϕ_f is the bulk Fermi level given by

$$\phi_f = V_{tm} \cdot \sinh^{-1} \left(\frac{N_a - N_d}{2n_i} \right) \quad (2.5)$$

in which $n_i=1 \times 10^{10} \text{cm}^{-3}$ is the intrinsic carrier concentration. The boundary conditions for (2.1) are written as

$$\begin{aligned} \psi|_{x=0} &= \phi_s & \frac{\partial \psi}{\partial x} \Big|_{x=0} &= -\xi_s \\ \psi|_{x=\infty} &= 0 & \frac{\partial \psi}{\partial x} \Big|_{x=\infty} &= 0 \end{aligned} \quad (2.6)$$

where ϕ_s is the channel surface potential and ξ_s is the surface electric field.

Applying Gauss law at the oxide/silicon interface, ξ_s can be obtained as

$$\xi_s = -\frac{\epsilon_{ox}}{\epsilon_{si}} \xi_{ox} = -\frac{V_{gf} - \phi_s}{t_{ox}} \cdot \frac{\epsilon_{ox}}{\epsilon_{si}} \quad (2.7)$$

$$V_{gf} = V_{gb} - V_{fb} \quad (2.8)$$

where ϵ_{ox} is the oxide permittivity, V_{gb} is the gate-bulk voltage, $V_{fb} = \phi_{MS} - Q_{ox}/C_{ox}$ is the flatband voltage, ϕ_{MS} is the metal-semiconductor work function difference, $C_{ox} = \epsilon_{ox}/t_{ox}$ is the gate capacitance, t_{ox} is the oxide thickness and Q_{ox} is the oxide charge. Substituting (2.3)-(2.4) into (2.1) and using the boundary conditions in (2.6)-(2.7), the Poisson equation (2.1) can be integrated as

$$V_{gf} - \phi_s = \gamma \operatorname{sgn}(\phi_s) \sqrt{f_{\phi_s}} \quad (2.9)$$

where $\operatorname{sgn}()$ is a sign function,

$$f_{\phi_s} = \underbrace{V_{tm} \cdot (e^{\frac{\phi_s}{V_{tm}}} - 1)}_{\text{holes}} + \underbrace{V_{tm} \cdot e^{\frac{2\phi_f + V_{cb}}{V_{tm}}} \cdot (e^{\frac{\phi_s}{V_{tm}}} - 1)}_{\text{electrons}} + \underbrace{\phi_s}_{\text{Acceptor+donor}} \quad (2.10)$$

and the bulk factor γ is written as

$$\gamma = \frac{\sqrt{2\epsilon_{si}qp_0}}{C_{ox}} \quad (2.11)$$

where $p_0 = ni \cdot \exp(\phi_f/V_{tm})$ is the equilibrium hole concentration. The contributions from electrons, holes and the ionized acceptors are shown in (2.10). Mathematically, (2.9) is an implicit function of ϕ_s . It can either be solved iteratively with the Newton-Raphson (NR) algorithm [53] or approximately [54, 55], which uses explicit and linearized-iteration approach to improve the accuracy of the surface potential. Explicit regional solutions exist when only considering the contributions of the dominant terms in strong inversion (electrons), depletion (ionized acceptor), and accumulation (holes),

respectively. These explicit regional solutions can be combined into one single piece expression through the use of smoothing and interpolation functions. The piece-wise regional explicit ϕ_s solutions for strong inversion, depletion and accumulation regions are derived first in the following sections, and then, they are unified by smoothing functions.

2.3.1.1 Strong Inversion

In strong inversion, for n-type MOSFETs, the surface potential is positive ($\text{sgn}(\phi_s)=1$). The electron charge is dominant. The contributions from acceptors and holes can be ignored and (2.10) can be approximated as

$$f_{\phi_s} \approx V_{tm} \cdot e^{\frac{\phi_s - 2\phi_f - V_{cb}}{V_{tm}}} \quad (2.12)$$

Substituting (2.12) into (2.9) and rearranging the equation, we have

$$\frac{(V_{gf} - \phi_s)}{2V_{tm}} e^{\frac{V_{gf} - \phi_s}{2V_{tm}}} = \frac{\gamma}{2\sqrt{V_{tm}}} \cdot e^{\frac{V_{gf} - 2\phi_f - V_{cb}}{2V_{tm}}} \quad (2.13)$$

Thus, solving the above equation for ϕ_s , the strong inversion regional surface potential solution ϕ_{ss} can be found as

$$\phi_{ss} = V_{gf} - 2V_{tm} L \left\{ \frac{\gamma}{2\sqrt{V_{tm}}} \cdot e^{\frac{V_{gf} - 2\phi_f - V_{cb}}{2V_{tm}}} \right\} \quad (2.14)$$

where $L\{\}$ is the Lambert W function.

2.3.1.2 Depletion

In depletion, for n-type MOSFETs, the surface potential is positive ($sgn(\phi_s)=1$). The acceptor is the dominant term, electrons and holes are ignored. Therefore, (2.10) can be approximated as

$$f_{\phi_s} \approx \phi_s \quad (2.15)$$

Substituting (2.15) into (2.9) and rearranging the equation, we have

$$\phi_s - \gamma \sqrt{\phi_s} - V_{gf} = 0 \quad (2.16)$$

Thus, solving the above equation for ϕ_s , the depletion regional surface potential solution ϕ_{DD} can be obtained as

$$\phi_{DD} = \left[-\frac{\gamma}{2} + \sqrt{\left(\frac{\gamma}{2}\right)^2 + V_{gf}} \right]^2 \quad (2.17)$$

2.3.1.3 Accumulation

In accumulation, for n-type MOSFETs, the surface potential is negative ($sgn(\phi_s)=-1$). Hole is the dominant factor, electrons and acceptors are ignored. Therefore, (2.10) can be approximated as

$$f_{\phi_s} \approx V_{tm} \cdot e^{\frac{\phi_s}{V_{tm}}} \quad (2.18)$$

Substituting (2.18) into (2.9) and rearranging the equation, we have

$$\frac{(\phi_s - V_{gf})}{2V_{tm}} e^{\frac{\phi_s - V_{gf}}{2V_{tm}}} = \frac{\gamma}{2\sqrt{V_{tm}}} \cdot e^{\frac{-V_{gf}}{2V_{tm}}} \quad (2.19)$$

Thus, solving the above equation for ϕ_s , the accumulation regional surface potential solution ϕ_{CC} is given by

$$\phi_{CC} = V_{gf} + 2V_{tm} L \left\{ \frac{\gamma}{2\sqrt{V_{tm}}} \cdot e^{\frac{-V_{gf}}{2V_{tm}}} \right\} \quad (2.20)$$

2.3.1.4 Unified Surface Potential Solution

The piecewise regional solutions derived in (2.14), (2.17) and (2.20) are only physically correct in the respective regions. To produce a surface potential solution that is usable across every region, the unified regional approach is adopted, in which the interpolation function and smoothing functions are used to join the regional solutions. In the unification of the regional solutions at the flat-band voltage ($V_{gb}=V_{fb}$ or $V_{gf}=0$), the complementary reverse and forward smoothing functions are used, which are written respectively as

$$\mathcal{G}_r(x, \sigma) \equiv \left(x - \sqrt{x^2 + 4 \cdot \sigma} \right) / 2 \quad (2.21)$$

$$\mathcal{G}_f(x, \sigma) \equiv \left(x + \sqrt{x^2 + 4 \cdot \sigma} \right) / 2 \quad (2.22)$$

The interpolated, flat-band shifted gate-bulk voltage ($V_{gb}-V_{fb}$) for below (VGR) and above V_{fb} (VGF) are expressed, respectively, as

$$VGR = \mathcal{G}_r(V_{gf}, \sigma_r) \quad (2.23)$$

$$VGF = \mathcal{G}_f(V_{gf}, \sigma_f) \quad (2.24)$$

where σ_r and σ_f are the smoothing parameters, which are used to ensure charge neutrality at flat-band ($\phi_s=0$ at $V_{gb}=V_{fb}$) and smoothness near V_{fb} . Then, for gate-bulk voltages below V_{fb} , the interpolated accumulation surface potential solution, ϕ_{Acc} is given as

$$\phi_{Acc} = VGR + 2V_{tm}L \left\{ \frac{\gamma}{2\sqrt{V_{tm}}} \cdot e^{\frac{-V_{gf}}{2V_{tm}}} \right\} \quad (2.25)$$

For gate voltages above V_{fb} , the interpolated depletion (or weak inversion) and strong inversion surface potential solutions, ϕ_{SUB} and ϕ_{Str} , are expressed respectively as

$$\phi_{SUB} = \left[-\frac{\gamma}{2} + \sqrt{\left(\frac{\gamma}{2}\right)^2 + VGF} \right]^2 \quad (2.26)$$

$$\phi_{Str} = VGF - 2V_{tm}L \left\{ \frac{\gamma}{2\sqrt{V_{tm}}} \cdot e^{\frac{V_{gf}-2\phi_f-V_{cb}}{2V_{tm}}} \right\} \quad (2.27)$$

In order to obtain a smooth and unified surface potential expression valid from depletion to strong inversion, another interpolation function is used, which is written as

$$\mathcal{G}_{eff}(x_{sat}, x, \delta) = 0.5 \cdot \left(x_{sat} + x + \delta - \sqrt{(x_{sat} - x - \delta)^2 + 4 \cdot \delta \cdot x_{sat}} \right) \quad (2.28)$$

The unified regional surface potential solution, ϕ_{DS} , from depletion to strong inversion, is therefore expressed as

$$\phi_{DS} = \mathcal{G}_{eff}(\phi_{Str}, \phi_{SUB}, \delta_{ss}) \quad (2.29)$$

where δ_{ss} is a smoothing parameter that controls the abruptness of the transition from depletion to strong inversion. (2.28) ensures that the depletion regional surface potential solution will smoothly join the regional solution in strong inversion. Finally, a single piece unified regional surface potential (URSP), ϕ_{Seff} , that is valid for all regions can be obtained as

$$\phi_{Seff} = \phi_{DS} + \phi_{Acc} \quad (2.30)$$

With (2.30), the surface potential in the channel can be evaluated. Figure 2.4 shows different pieces of the regional solutions, ϕ_{Acc} , ϕ_{Str} and ϕ_{SUB} for a channel doping $N_a=10^{17} \text{ cm}^{-3}$. It can be seen that the piecewise regional solutions are only valid in their intended regions. They can be combined into a single piece URSP through the procedure described above.

Figure 2.5 shows the comparison of single-piece URSP with iterative surface potential solutions for different body doping. The channel doping starts from the undoped body to highly doped body of 10^{18} cm^{-3} . The single-piece URSP solutions are able to follow the iterative solutions very well. The inset shows the absolute errors (in the mV range) for each doping concentration. Figure 2.6 shows the first-order derivatives of URSP for the corresponding body doping, with the second-order derivatives shown in the inset. The URSP solutions are smooth, continuous and matching all NR

solutions. Finally, the direct play-back of the single-piece URSP at different body bias (or channel voltage) is shown in Figure 2.7 and compared with the NR solutions with the first-order derivatives shown in the inset.

Figure 2.4: Components of unified regional surface potential at $N_a=10^{17} \text{ cm}^{-3}$: strong accumulation (ϕ_{Acc}), depletion (ϕ_{SUB}) and strong inversion (ϕ_{Str}).

Figure 2.5: Comparison of unified regional surface potential with respect to Newton-Raphson iterative solution of surface potential for different channel doping concentration. Inset: The error relative to Newton-Raphson solution.

Figure 2.6: Comparison of derivative of surface potential for unified regional surface potential derivative with Newton-Raphson solution for different channel doping concentration. Inset: 2nd derivative of corresponding surface potentials.

Figure 2.7: Direct playback of single-piece explicit surface potential, ϕ_{seff} and first order derivative (inset) for fermi level ($V_{cb}=V_{sb}$) variations, including positive bias condition, is compared with iterative solutions from Pao-Sah voltage equation (symbols).

2.3.1.5 Modification for Lateral Doping Effect

As mentioned, the p-channel is laterally nonuniformly doped. The aim is to model the p-channel with an effective doping. The value of the effective doping is actually extracted from the I-V data. However, the current conducting mechanisms are different for the subthreshold region and the strong inversion region. Thus, two different effective doping may be extracted for the two operating regions. All the previous formulations use N_a as the effective doping. N_a can be used as the effective doping for the strong inversion region. Then, some modification needs to be done for the subthreshold region. The modifications are done on the subthreshold regional

surface potential. The modified depletion regional surface potential is written as

$$\phi_{DDm} = \left[-\frac{\theta\gamma}{2} + \sqrt{\left(\frac{\theta\gamma}{2}\right)^2 + V_{gf} - \Delta V_{fb}} \right]^2 \quad (2.31)$$

where θ and ΔV_{fb} are fitting parameters. In this way, the term γ and V_{fb} are modified. This is because they are related to the doping value. Therefore, the modified interpolated depletion regional surface potential is expressed

$$\phi_{SUBm} = \left[-\frac{\theta\gamma}{2} + \sqrt{\left(\frac{\theta\gamma}{2}\right)^2 + VG Fm} \right]^2 \quad (2.32)$$

where

$$VG Fm = \mathcal{G}_f(V_{gf} - \Delta V_{fb}, \sigma_f) \quad (2.33)$$

The unified inversion surface potential can then expressed as

$$\phi_{DSm} = \mathcal{G}_{eff}(\phi_{SS}, \phi_{SUBm}, \delta_{ss}) \quad (2.34)$$

Note that, this modification is only used for I-V calculation.

2.3.2 I-V Model

In this section, the p-channel I-V model will be formulated based on the URSP derived in the above section. Although the URSP doesn't have perfect accuracy, it will become evident later that it is capable of correctly reproducing the current behavior of the transistor.

Most of the existing compact models [56, 57] are developed based on the charge-sheet approximation [58]. It is assumed that the depletion region under the gate is free of mobile carriers so that the depletion approximation is valid [53]. The bulk-charge density, Q_b is then approximated as

$$Q_b = -\gamma C_{ox} \sqrt{\phi_s(y)} \quad (2.35)$$

where $\phi_s(y)$ is the position dependent surface potential along the intrinsic channel between source and drain. Next, the inversion charge density, Q_i can be calculated from the charge conservation principle, which is written as

$$\begin{aligned} Q_i &= -Q_g - Q_b \\ &= -C_{ox} \left(V_{gf} - \phi_s(y) - \gamma \sqrt{\phi_s(y)} \right) \end{aligned} \quad (2.36)$$

in which $Q_g = C_{ox}(V_{gf} - \phi_s(y))$ is the gate charge density. From current continuity $I_{ds} = -\mu W Q_i \cdot \partial V / \partial y$, the drain current, I_{ds} can be found by integrating the inversion charge density from source $y=0$ to drain $y=L_{eff}$:

$$\int_0^{L_{eff}} I_{ds} dy = -\mu W \int_0^{V_{ds}} Q_i dV \quad (2.37)$$

in which W is the device width, L_{eff} is the effective channel length and μ is the electron mobility. If one performs a change of integration variable of the surface potential and quasi-Fermi potential to (2.37), while assuming charge-sheet approximation, (2.37) becomes

$$\int_0^{L_{eff}} I_{ds} dy = -\mu W \int (Q_i(y) d\phi(y) - V_{tm} dQ_i(y)) \quad (2.38)$$

Substituting (2.36) into (2.38) and integrating (2.38), the drain current is obtained as [53]

$$I_{ds} = \beta \left\{ V_{gf} (\phi_{s,d} - \phi_{s,s}) - \frac{1}{2} (\phi_{s,d}^2 - \phi_{s,s}^2) - \frac{2\gamma}{3} (\phi_{s,d}^{3/2} - \phi_{s,s}^{3/2}) \right. \\ \left. + V_{tm} \left[\gamma (\phi_{s,d}^{1/2} - \phi_{s,s}^{1/2}) + (\phi_{s,d} - \phi_{s,s}) \right] \right\} \quad (2.39)$$

where $\beta = W\mu C_{ox}/L_{eff}$ is the gain factor, $\phi_{s,d}$ and $\phi_{s,s}$ are surface potentials at drain and source end, respectively. The first line in (2.39) is usually referred to as the drift-component corresponding to the integration results of the first term in (2.38) while the second line is the diffusion-component corresponding to the second term in (2.38). This drain current model is accurate [59], but it results in a very complicated charge model [60]. Therefore, a compromise between complexity and physics is made by taking a Taylor expansion on (2.39). The Taylor expansion is done on the inversion charge density (2.36) with symmetric bulk-charge linearization [61] as follows

$$Q_i(x) \approx \bar{Q}_i \Big|_{\phi_s = \bar{\phi}_s} + \frac{d\bar{Q}_i}{d\bar{\phi}_s} \Big|_{\phi_s = \bar{\phi}_s} \cdot (\phi_s - \bar{\phi}_s) \\ = -C_{ox} \left(\bar{q}_i - \bar{A}_b (\phi_s - \bar{\phi}_s) \right) \quad (2.40)$$

where

$$\bar{q}_i = V_{gb} - V_{fb} - \bar{\phi}_s - \gamma \sqrt{\bar{\phi}_s} \quad (2.41)$$

is the average inversion charge density normalized to C_{ox}

$$\bar{A}_b = 1 + \frac{\gamma}{2\sqrt{\bar{\phi}_s}} \quad (2.42)$$

is the average bulk factor and

$$\bar{\phi}_s = \frac{\phi_{s,d} + \phi_{s,s}}{2} \quad (2.43)$$

is the average surface potential. Then the resulted drift-diffusion current expression is much simpler and it is given by

$$I_{ds} = \beta \left(\overline{q_i} + V_{tm} \overline{A_b} \right) \Delta \phi_s \quad (2.44)$$

where

$$\Delta \phi_s = \phi_{s,d} - \phi_{s,s} \quad (2.45)$$

Both models given by equation (2.39) and equation (2.44) are classified as the surface-potential based model. Both require accurate solution of the surface potential, in particular in the subthreshold region, due to the fact that I_{ds} is an exponential function of the surface potential difference, $\phi_{s,d} - \phi_{s,s}$, in the subthreshold region. However, $\phi_{s,s}$ in the subthreshold region is almost equal to $\phi_{s,d}$. A small error in the surface potential solution can result in a large error in I_{ds} .

In the URM approach, the drain-current model (2.44) is further approximated with a “pinned surface potential” concept where $\phi_s \approx \phi_{s,s} + V_{cb}$, therefore, (2.44) can be rewritten as,

$$I_{ds} = \beta \left(\overline{q_i} + v_{th} \overline{A_b} \right) V_{ds} \quad (2.46)$$

with the approximation

$$\Delta \phi_s \approx \left(\phi_{s,s} + V_{db} \right) - \left(\phi_{s,s} + V_{sb} \right) = V_{ds} = V_{ds,eff} \quad (2.47)$$

in which V_{ds} is to be replaced by the effective drain-source voltage, $V_{ds,eff}$, in the current evaluation for correct linear (and strong inversion) to saturation (and subthreshold) behavior. This form of drain current equation does not

require accurate surface potential solution to produce accurate exponential subthreshold behavior as long as it has the correct subthreshold slope in $V_{ds,eff}$, as shown in Figure 2.8.

Figure 2.8 The drain current as a function of V_{gs} based on (2.46): drift current (I_{drift}), diffusion current (I_{diff}) and combined total current I_{ds} (solid line)

2.3.2.1 Effective Mobility and Transverse Field Degradation

The carrier in the surface inversion layer suffers from phonon scattering, surface roughness scattering, and Columbic scattering. These scattering effects cause the mobility of the surface carrier to decrease. This is usually referred as transverse field mobility degradation as the scattering

effects are related to the transverse electric field. The transverse field-dependent mobility, μ_0 is usually expressed using the well-known Matthiessen's rule for the scattering mechanisms [62],

$$\begin{aligned}\mu_0 &= \left(\frac{1}{\mu_{co}} + \frac{1}{\mu_{ph}} + \frac{1}{\mu_{sr}} \right)^{-1} \\ &= \frac{\mu_1}{1 + \left(\frac{\mu_1}{\mu_2} \right) E_{eff}^{\frac{1}{3}} + \left(\frac{\mu_1}{\mu_3} \right) E_{eff}^\nu}\end{aligned}\tag{2.48}$$

in which semi-empirical model for phonon scattering μ_{ph} , surface roughness scattering μ_{sr} , and Columbic scattering μ_{co} are related to the effective transverse field, E_{eff} [53] by

$$\mu_{ph} = \frac{\mu_2}{E_{eff}^{1/3}}\tag{2.49}$$

$$\mu_{sr} = \frac{\mu_3}{E_{eff}^\nu}\tag{2.50}$$

and

$$\mu_{co} = \mu_1\tag{2.51}$$

respectively. μ_1 is assumed as a constant with respect to the effective transverse field, μ_2 and μ_3 are related to acceptor density and the absolute temperature, and $\nu=2$ for NMOS and $\nu=1$ for PMOS. Nevertheless, μ_1 , μ_2 , μ_3 , and ν are usually used as fitting parameters to match the measurement data. The effective transverse field is related to the inversion charge, Q_i and bulk charge, Q_b as [53]

$$E_{eff}(y) = [\zeta_b Q_b(y) + \zeta_n Q_i(y)] / \epsilon_{si}\tag{2.52}$$

where ζ_n and ζ_b are 0.5 and 1, respectively, for $\langle 100 \rangle$ electrons. Both ζ_n and ζ_b are fitting parameters related to inversion charge, and bulk charge respectively.

In this work, average E_{eff} according to [63] is used. It can be derived as:

$$\begin{aligned}
 E_{eff} &= \frac{1}{\phi_{s,d} - \phi_{s,s}} \int_{\phi_{s,s}}^{\phi_{s,d}} E_{eff}(y) d\phi_s \\
 &= \frac{1}{\Delta\phi\epsilon_{si}} \left[\int_{\phi_{s,s}}^{\phi_{s,d}} \zeta_n \bar{Q}_i d\phi_s + \zeta_b \int_{\phi_{s,s}}^{\phi_{s,d}} \bar{Q}_b d\phi_s \right] \\
 &= \frac{C_{ox}}{\Delta\phi\epsilon_{si}} \left[\int_{\phi_{s,s}}^{\phi_{s,d}} \zeta_n \left(\bar{q}_i - \bar{A}_b (\phi_s - \bar{\phi}_s) \right) d\phi_s + \zeta_b \gamma \int_{\phi_{s,s}}^{\phi_{s,d}} \left(\sqrt{\bar{\phi}_s} + \frac{1}{2\sqrt{\bar{\phi}_s}} (\phi_s - \bar{\phi}_s) \right) d\phi_s \right] \\
 &= \frac{\zeta_n C_{ox}}{\epsilon_{si}} \left(\frac{\bar{q}_i}{\zeta_n} + \frac{\zeta_b}{\zeta_n} \gamma \sqrt{\bar{\phi}_s} \right)
 \end{aligned} \tag{2.53}$$

$\bar{q}_i$ actually is the average normalized charge density, which is equal to $0.5(Q_i(0) + Q_i(L_{eff}))/C_{ox}$, in which $Q_i(y)$ is given in (2.36) and $\bar{q}_i$ is given in (2.41). Theoretically speaking, based on (2.9), $Q_i(y)$ can either be obtained by using the left-hand-side (LHS) of the input-voltage equation (2.9), or the right-hand-side (RHS) of (2.9), thus yielding

$$Q_i(y) = C_{ox} V_{gt}(y) \tag{2.54}$$

with

$$V_{gt}(y) = V_{gf} - \phi_s(y) - \gamma \sqrt{\phi_s(y)} \quad LHS \tag{2.55}$$

$$V_{gt}(y) = \gamma \sqrt{\phi_s(y) + V_{tm} e^{\frac{\phi_s(y) - 2\phi_F - V_{cb}}{V_{tm}}}} - \gamma \sqrt{\phi_s(y)} \quad RHS \tag{2.56}$$

(2.36) and (2.41) are actually using the LHS form. The RHS form is distinct since when the exponential term is small (subthreshold region), it can be approximated as

$$\begin{aligned}
 V_{gt}(y) &= \gamma \sqrt{\phi_s(y) + V_{tm} e^{\frac{\phi_s(y) - 2\phi_F - V_{cb}}{V_{tm}}}} - \gamma \sqrt{\phi_s(y)} \\
 &\approx \gamma \sqrt{\phi_s(y)} \left(1 + \frac{1}{2\phi_s(y)} V_{tm} e^{\frac{\phi_s(y) - 2\phi_F - V_{cb}}{V_{tm}}} \right) - \gamma \sqrt{\phi_s(y)} \quad (2.57) \\
 &\approx \frac{\gamma}{2\sqrt{\phi_s(y)}} V_{tm} e^{\frac{\phi_s(y) - 2\phi_F - V_{cb}}{V_{tm}}}
 \end{aligned}$$

Thus, the RHS form naturally shows exponential behavior when it is in subthreshold region and it doesn't require accurate surface potential. In contrast, the LHS form doesn't have such a functional (exponential functionality) behavior in the subthreshold region. As expected, it is shown in Figure 2.9 that the RHS form shows the correct subthreshold-slope trend as compared to the LHS form, which becomes negative below threshold voltage that cannot be plotted on logarithm scale. Thus, in the URM/URSP approach, in which strict requirement on the surface potential accuracy is released, the RHS form is adopted. Using the RHS form is the key for the URM/URSP approach without requiring very accurate surface-potential solutions.

Figure 2.9. Comparison of normalized inversion charge using the Left-hand-side, q_i and Right-hand-side, V_{gt} of the input voltage equation (2.9).

In the later context of this chapter, the symbol V_{gt} is dedicated to represent the normalized inversion charge density evaluated using the RHS form, which is given in (2.56). Therefore, (2.53) is rewritten as

$$E_{eff} = \frac{\zeta_n C_{ox}}{\epsilon_{si}} \left(\overline{V_{gt}} + \frac{\zeta_b}{\zeta_n} \gamma \sqrt{\phi_s} \right) \quad (2.58)$$

in which

$$\overline{V_{gt}} = 0.5 (V_{gt,s} + V_{gt,d}) \quad (2.59)$$

where $V_{gt,s}$ and $V_{gt,d}$ are the normalized charge density evaluated using (2.56) at the source and drain end, respectively.

2.3.2.2 Lateral Field Mobility Degradation

The mobility degradation due to lateral field, $E_y = d\phi/dy$ begins to play a more significant role at large transverse-field when V_{ds} increases beyond the gradual-channel approximation limit. The velocity of carriers, v_0 in the channel is proportional to E_y at low lateral field (equivalently, low V_{ds}), and begins to saturate when the lateral field is larger than the saturation field, E_{sat} . The two-region piecewise velocity-field relation is given by [64]

$$v_0 = \begin{cases} \frac{\mu_0 E_y}{1 + E_y/E_{sat}} & E_y < E_{sat} \\ v_{sat} & E_y \geq E_{sat} \end{cases} \quad (2.60)$$

with

$$E_{sat} = \frac{2v_{sat}}{\mu_0} \quad (2.61)$$

where v_{sat} is the saturation velocity. When $E_y < E_{sat}$, E_y is approximately a linear function of V_{ds} and, therefore, the velocity, v_0 is given as

$$v_0 = \frac{\mu_0 E_y}{1 + \frac{\delta_L V_{ds}}{L_{eff} E_{sat}}} \quad (2.62)$$

and the effective mobility, μ_{eff0} with lateral field degradation can be deducted

$$\mu_{eff0} = \frac{\mu_0}{1 + \frac{\delta_L V_{ds}}{L_{eff} E_{sat}}} \quad (2.63)$$

where δ_L is a fitting parameter for the lateral-field mobility degradation.

2.3.2.3 Saturation Voltage

The saturation voltage is defined as the voltage at which the current reaches its saturated value (I_{dsat}). The saturation voltage can be determined from the intercept point of the saturation currents according to the V_t -based approach [35]. The saturation current, I_{dsat} with a Taylor expansion at the source end is given as

$$I_{dsat} = v_{sat} WC_{ox} (V_{gt,s} - A_{b,s} \Delta\phi) \quad (2.64)$$

where

$$V_{gt,s} = \gamma \sqrt{\phi_{s,s}} + V_{tm} e^{\frac{[\phi_{s,s} - 2\phi_F - V_{sb}]}{V_{tm}}} - \gamma \sqrt{\phi_{s,s}} \quad (2.65)$$

$$A_{b,s} = 1 + \gamma / (2\sqrt{\phi_{s,s}}) \quad (2.66)$$

and the linear drift-diffusion current, $I_{dd,s}$ (after taking Taylor expansion on (2.39) at the source end and including the mobility degradation effect) can be rewritten as

$$I_{dd,s} = \mu_{eff0} C_{ox} \frac{W}{L_{eff}} \left(V_{gt,s} - \frac{A_{b,s} \Delta\phi}{2} + A_{b,s} V_{tm} \right) \Delta\phi \quad (2.67)$$

where

$$\mu_{eff0,s} = \frac{\mu_{0,s}}{1 + \frac{\delta_L V_{ds}}{L_{eff} E_{sat}}} \quad (2.68)$$

and

$$\mu_{0,s} = \mu_0 \big|_{E_{eff} = E_{eff,s}} \quad (2.69)$$

where $E_{eff,s}$ is the transverse e-field at the source end, evaluated using (2.52) at $y=0$. Solving $I_{dsat}=I_{dd,s}$ at $V_{ds}=V_{ds,sat}$ with (2.47) and (2.61), the drain-source saturation voltage, $V_{ds,sat}$ is obtained as,

$$V_{ds,sat} = \frac{V_{gt,s} L_{eff} E_{sat}}{V_{gt,s} + A_{b,s} L_{eff} E_{sat} + 2A_{b,s} V_{tm}} \quad (2.70)$$

For short-channel devices, the series-resistance effect is included in the saturation voltage. Similar to [35], the saturation current considering series-resistance (the source resistance, R_s and the total drain-source resistance, R_{sd}) can be derived as

$$I_{dsat} = \frac{v_{sat} WC_{ox} (V_{gt,s} - A_{b,s} V_{ds,sat})}{1 - v_{sat} WC_{ox} (A_{b,s} R_{sd} - R_s)} \quad (2.71)$$

According to [35], the linear drift-diffusion current including R_{sd} and evaluated at $V_{ds}=V_{ds,sat}$, can be obtained by modifying (2.67) as

$$I_{dsat} = \frac{\mu_{eff0,s} C_{ox} \frac{W}{L_{eff}} \left(V_{gt,s} - \frac{A_{b,s} V_{ds,sat}}{2} + A_{b,s} V_{tm} \right) V_{ds,sat}}{1 + \mu_{eff0,s} C_{ox} \frac{W}{L_{eff}} \left(V_{gt,s} - \frac{A_{b,s} V_{ds,sat}}{2} + A_{b,s} V_{tm} \right) R_{sd}} \quad (2.72)$$

Combining and solving (2.71) and (2.72) for $V_{ds,sat}$, we obtain

$$V_{ds,sat} = \frac{-b_s - \sqrt{b_s^2 - 4a_s c_s}}{2a_s} \quad (2.73)$$

in which

$$a_s = v_{sat} WC_{ox} A_{b,s} R_s \quad (2.74)$$

$$b_s = -\left[V_{gt,s} + v_{sat} WC_{ox} V_{gt,s} (2R_s + A_{b,s} R_{sd}) + A_{b,s} E_{sat} L_{eff} + 2A_{b,s} V_{tm} (1 + v_{sat} WC_{ox} R_s) \right] \quad (2.75)$$

$$c_s = V_{gt,s} E_{sat} L_{eff} + 2v_{sat} WC_{ox} R_{sd} V_{gt,s} (V_{gt,s} + A_{b,s} V_{tm}) \quad (2.76)$$

The effective drain voltage, V_{deff} and the effective drain-source voltage, $V_{ds,eff}$ is then given by

$$V_{deff} = \mathcal{G}_{eff}(V_{ds,sat}, V_{ds}, \delta_{dssat}) + V_s \quad (2.77)$$

$$V_{ds,eff} = V_{deff} - V_s = \mathcal{G}_{eff}(V_{ds,sat}, V_{ds}, \delta_{dssat}) \quad (2.78)$$

The drain current is rewritten in a unified expression with average lateral field mobility in (2.63) and effective drain voltage in (2.77)

$$I_{ds0} = \frac{WC_{ox}}{L_{eff}} \mu_{eff} (\overline{q_{ieff}} + V_{tm} \overline{A_{beff}}) V_{ds,eff} \quad (2.79)$$

where

$$\mu_{eff} = \mu_{eff0} \Big|_{V_d=V_{deff}} \quad (2.80)$$

$$\overline{q_{ieff}} = \overline{q_i} \Big|_{V_d=V_{deff}} \quad (2.81)$$

$$\overline{A_{beff}} = \overline{A_b} \Big|_{V_d=V_{deff}} \quad (2.82)$$

$\overline{q_i}$ is evaluated using (2.41), which is using the LHS form. This is because this term doesn't affect the model's subthreshold behavior. For the short channel devices, the unified drain current expression is then written as

$$I_{ds} = \frac{I_{ds0}}{1 + R_{sd} I_{ds0} / V_{ds,eff}} \quad (2.83)$$

2.3.2.4 Channel Length Modulation

Channel-length modulation (CLM) and velocity overshoot (VO) have been modeled based on energy-balance formulation [65] through an “effective Early voltage”. The final short-channel current equation including series resistance and CLM/VO effects is obtained by modifying (2.83) as

$$I_{ds} = \frac{g_{vo} I_{ds0}}{1 + R_{sd} I_{ds0} / V_{ds,eff}} \quad (2.84)$$

where g_{vo} is the CLM/VO effect factor. The VO factor is derived as

$$g_{vo,d} = 1 + \left(\frac{V_d - V_{deff}}{V_{Aeff,d}} \right) \quad (2.85)$$

with the effective Early voltage, $V_{Aeff,d}$ given by

$$V_{Aeff,d} = \frac{\left(L_{eff} E_{sat} \left[1 + h_d (V_d - V_{deff}) \right] \right) + \delta_L V_{deff}}{\delta_L h_d V_{deff}} \quad (2.86)$$

$$h_d = \frac{\frac{\xi_{vo}}{L_{eff}^2} \left[1 + \sqrt{1 + \left(\frac{V_d - V_{deff}}{L_{eff} E_{sat}} \right)^2} \right]}{\sqrt{1 + \left(\frac{V_d - V_{deff}}{L_{eff} E_{sat}} \right)^2} - \frac{\xi_{vo}}{L_{eff}^2} (V_d - V_{deff})} \quad (2.87)$$

where ξ_{vo} is a fitting parameter. Numerical data are generated with Medici [66] simulation for verification of the URM I_{ds} model.

2.3.2.5 Model Validation

To verify the model, the p-channel transistor in Figure 2.3 is simulated in using 2D technology computer-aided design (TCAD) simulation tool (Medici) [66]. The simulated device has $t_{ox}=5\text{nm}$ and $L_{eff}=0.6\mu\text{m}$. The lateral doping, N_{pch} is

$$N_{pch} = 10^{18} e^{-(y/0.18\mu\text{m})^2} + 10^{15} \text{ cm}^{-3} \quad (2.88)$$

I-V data are extracted from the TCAD simulation for comparison with the model. The comparison results are shown in Figure 2.10, Figure 2.11 and Figure 2.12 (lines: model; Symbols: numerical data). Figure 2.10 compares the I_d - V_{gs} behavior for different V_{ds} . Figure 2.11 compares the I_d - V_{gs} for different V_{sb} . Figure 2.12 compares the I_d - V_{ds} for different V_{gs} . Although the p-channel model is essentially based on uniform doping, it is able to follow the numerical data well.

Figure 2.10 comparison of modeled drain current (lines) with the numerical data from TCAD (symbols) for V_{gs} variation at different V_{ds} ($=0.3, 0.6, 0.9, 1.2\text{V}$) with fixed $V_{sb}=0$.

Figure 2.11 comparison of modeled drain current (lines) with the numerical data from TCAD (symbols) for V_{gs} variation at different V_{sb} ($=0, 0.2, 0.4V$) with fixed $V_{ds}=0.3V$.

Figure 2.12 comparison of modeled drain current (lines) with the numerical data from TCAD (symbols) for V_{ds} variation at different V_{gs} ($=0.5, 1, 1.5, 2V$) with fixed $V_{sb}=0V$.

2.4 Drift Channel Current Model

For the drift region, there are two main electron sources. The first is surface accumulation electrons and the second is the intrinsic electrons in the silicon body. Thus, the drift region is basically a parallel combination of an n-type accumulation transistor and an n-type bulk silicon resistor. A conceptual cross section of the drift region transistor is illustrated in Figure 2.13.

Figure 2.13 The cross section of the conceptual drift region transistor

2.4.1 Surface Potential Solution

In order to derive the compact model for the drift region, surface potential is needed first. Similarly to section 2.3.1, the Poisson equation can be established in the channel of the drift region as

$$\frac{\partial \psi_{dr}^2}{\partial x^2} = -\frac{\rho_{dr}}{\epsilon_{si}} \quad (2.89)$$

Where ψ_{dr} is the electron potential and the ρ_{dr} is the net charge concentration in the channel, given as

$$\rho_{dr} = q \cdot [p - n + N_d] \quad (2.90)$$

in which N_d is the donor concentration. The electrons concentration, n and the hole concentration, p are expressed as

$$n = N_d \cdot \exp\left(\frac{\psi_{dr}}{V_{tm}}\right) \quad (2.91)$$

$$p = N_d \cdot \exp\left(-\frac{\psi_{dr} + 2\phi_{f,dr}}{V_{tm}}\right) \quad (2.92)$$

where the bulk Fermi level of the drift region, $\phi_{f,dr}$ is given as

$$\phi_{f,dr} = V_{tm} \cdot \ln\left(\frac{N_d}{n_i}\right) \quad (2.93)$$

The boundary conditions for (2.89) can be written as

$$\begin{aligned} \psi_{dr}|_{x=0} &= \phi_{s,dr} & \left. \frac{\partial \psi_{dr}}{\partial y} \right|_{x=0} &= \frac{V_{gf,dr} - \phi_{s,dr}}{t_{ox,dr}} \cdot \frac{\epsilon_{ox}}{\epsilon_{si}} \\ \psi_{dr}|_{x=\infty} &= 0 & \left. \frac{\partial \psi_{dr}}{\partial y} \right|_{x=\infty} &= 0 \end{aligned} \quad (2.94)$$

where $\phi_{s,dr}$ is the drift region surface potential, $V_{gf,dr}=V_{gb}-V_{fb,dr}$, $V_{fb,dr}$ is the flatband voltage of the drift region and $t_{ox,dr}$ is the oxide thickness. Integrating (2.89) once and applying the boundary conditions in (2.94), we have,

$$V_{gf,dr} - \phi_{s,dr} = \gamma_{dr} \cdot \text{sgn}(\phi_{s,dr}) \cdot \sqrt{f_{\phi_{s,dr}}} \quad (2.95)$$

$$f_{\phi_{s,dr}} = \sqrt{\underbrace{V_{tm} \cdot \exp(-\frac{2\phi_{f,dr}}{V_{tm}}) \cdot (\exp(-\frac{\phi_{s,dr}}{V_{tm}}) - 1)}_{\text{holes}} + \underbrace{V_{tm} \cdot (\exp(\frac{\phi_{s,dr}}{V_{tm}}) - 1) - \phi_{s,dr}}_{\text{electrons}}}_{\text{donor}} \quad (2.96)$$

$$\gamma_{dr} = \frac{\sqrt{2\epsilon_{si}qN_d}}{C_{ox,dr}} \quad (2.97)$$

where $C_{ox,dr}=\epsilon_{ox}/t_{ox,dr}$ is the gate capacitance. Similar to section 2.3.1, (2.95) can be explicitly solved regionally for accumulation, depletion, and the strong inversion. The interpolated regional solutions for accumulation, depletion, and strong inversion can be derived, respectively, as

$$\phi_{Acc,dr} = VGF_{dr} - 2 \cdot V_{tm} \cdot L \left\{ \frac{\gamma_{dr}}{2\sqrt{V_{tm}}} \exp\left(\frac{V_{gf,dr}}{2 \cdot V_{tm}}\right) \right\} \quad (2.98)$$

$$\phi_{SUB,dr} = - \left[-\frac{\gamma_{dr}}{2} + \sqrt{\left(\frac{\gamma_{dr}}{2}\right)^2 - VGR_{dr}} \right]^2 \quad (2.99)$$

$$\phi_{Str,dr} = VGR_{dr} + 2 \cdot V_{tm} \cdot L \left\{ \frac{\gamma_{dr}}{2\sqrt{V_{tm}}} \exp\left(-\frac{V_{gf,dr} + 2\phi_{f,dr}}{2 \cdot V_{tm}}\right) \right\} \quad (2.100)$$

where the interpolated forward and reverse flatband shifted gate-bulk voltage are given, respectively as

$$VGF_{dr} = \mathcal{G}_f(V_{gf,dr}, \sigma_{f,dr}) \quad (2.101)$$

$$VGR_{dr} = \mathcal{G}_r(V_{gf,dr}, \sigma_{r,dr}) \quad (2.102)$$

in which $\sigma_{r,dr}$ and $\sigma_{f,dr}$ are the smoothing parameters and the forward and reverse functions $\mathcal{G}_f()$ and $\mathcal{G}_r()$ are given in (2.22) and (2.21), respectively. In order to smoothly ‘join’ the two pieces of inversion solutions (depletion and strong inversion), a new smoothing function is introduced. It is given by

$$\mathcal{G}_{n_eff}(x_{sat}, x, \delta) = 0.5 \cdot [x_{sat} + x + \delta + \sqrt{(x_{sat} - x - \delta)^2 - 4 \cdot \delta \cdot x_{sat}}] \quad (2.103)$$

Then, the unified single-piece expression for inversion is derived as

$$\phi_{DS,dr} = \mathcal{G}_{n_eff}(\phi_{Str,dr}, \phi_{SUB,dr}, \delta_{dr}) \quad (2.104)$$

where δ_{dr} is a smoothing parameter that controls the abruptness of the transition. Finally, the unified surface potential for the drift channel is given as

$$\phi_{Seff,dr} = \phi_{DS,dr} + \phi_{Acc,dr} \quad (2.105)$$

With (2.105), the surface potential in the drift channel can be evaluated. Figure 2.14 and Figure 2.15 show the comparison between the unified surface potential and its derivative with the numerical data from the TCAD at different doping, N_d . As the field oxide above the drift region is usually thick, $t_{ox,dr}$ is set to 100nm and the model results match the numerical data nicely.

Figure 2.14 comparison of the modeled surface potential (lines) with the numerical solution from TCAD (symbols) for V_{gs} variation at $V_d=V_s=V_b=0$ for different doping level.

Figure 2.15 comparison of the modeled surface potential derivatives (lines) with the numerical solution from TCAD (symbols) for V_{gs} variation at $V_d=V_s=V_b=0$ for different doping level.

2.4.2 Drift I-V Model

As mentioned before, there are two main electron sources: the surface accumulation electrons and the bulk electrons. In accordance, there are two carrier conduction paths: the surface from source to the drain and the silicon body.

2.4.2.1 Surface Conduction Current

For the surface conduction current, it can be modeled as the current in an accumulation transistor. The charge density of the accumulated surface carriers (electrons), Q_{acc} can be derived based on the charge neutrality principle, which is written as,

$$Q_{acc} + Q_{g,dr} = 0 \quad (2.106)$$

in which the gate charge density $Q_{g,dr}$ is expressed as

$$Q_{g,dr} = C_{ox,dr} (V_{gf,dr} - \phi_{dr}(y)) \quad (2.107)$$

where $\phi_{dr}(y)$ is the position dependent channel surface potential of the drift region. Therefore, based on (2.106)-(2.107), Q_{acc} can be deduced

$$Q_{acc} = -C_{ox,dr} (V_{gf,dr} - \phi_{dr}(y)) \quad (2.108)$$

Using the RHS of (2.95), Q_{acc} can also be written as

$$Q_{acc} = C_{ox,dr} \gamma_{dr} \cdot \sqrt{V_t \cdot \exp\left(\frac{\phi_{dr} - V_{cb}}{V_{tm}}\right)} \quad (2.109)$$

According to (2.38), the surface conduction current can be expressed as

$$I_{sf} = -\mu_{sf} \frac{W}{L_{dr}} \int (Q_{acc}(y) d\phi_{dr}(y) - V_{tm} dQ_{acc}(y)) \quad (2.110)$$

in which μ_{sf} is the electron mobility in the surface and L_{dr} is the channel length of the drift region. Substituting (2.108) into (2.110), (2.110) can be integrated out as

$$I_{sf} = \mu_{sf} \frac{C_{ox,dr} W}{L_{dr}} (q_{acc} + V_{tm}) \Delta\phi_s \quad (2.111)$$

where

$$q_{acc} = V_{gf,dr} - \frac{\phi_{dr,s} + \phi_{dr,d}}{2} \quad (2.112)$$

$$\Delta\phi_s = \phi_{dr,d} - \phi_{dr,s} \quad (2.113)$$

and $\phi_{dr,s}$ and $\phi_{dr,d}$ are the surface potentials evaluated at the source and drain of the drift channel, respectively.

2.4.2.1.1 Transverse and Lateral Mobility Degradation Effects

Similar to the surface inversion electrons in the p-channel, the surface accumulated carriers (electrons) will suffer from phonon scattering, surface roughness scattering, and Columbic scattering. Therefore, the low field mobility in the drift surface can be accordingly expressed as

$$\begin{aligned}\mu_{0,sf} &= \left(\frac{1}{\mu_{co}} + \frac{1}{\mu_{ph}} + \frac{1}{\mu_{sr}} \right)^{-1} \\ &= \frac{\mu_1}{1 + \left(\frac{\mu_1}{\mu_2} \right) E_{eff,sf}^{\frac{1}{3}} + \left(\frac{\mu_1}{\mu_3} \right) E_{eff,sf}^{\nu}}\end{aligned}\quad (2.114)$$

Following the symmetric linearization concept similar to (2.53), the effective transverse electric field $E_{eff,sf}$ of the drift channel is deduced

$$E_{eff,sf} = \frac{\zeta_{n,sf} C_{ox,dr} \overline{V_{gt,sf}}}{\epsilon_{si}} \quad (2.115)$$

in which $\zeta_{n,sf}$ is a fitting parameter and $\overline{V_{gt,sf}}$ is the average accumulation charge density normalized to the $C_{ox,dr}$ (RHS of (2.95)), and it is given by

$$\begin{aligned}\overline{V_{gt,sf}} &= V_{gf,dr} - \frac{\phi_{dr,s} + \phi_{dr,d}}{2} \\ &= 0.5 \left(\gamma_{dr} \cdot \sqrt{V_t \cdot \exp\left(\frac{\phi_{dr,s} - V_s}{V_{tm}}\right)} + \gamma_{dr} \cdot \sqrt{V_t \cdot \exp\left(\frac{\phi_{dr,d} - V_d}{V_{tm}}\right)} \right)\end{aligned}\quad (2.116)$$

Besides the transverse field mobility degradation effect modeled by (2.114), the surface carrier will also suffer from the high lateral field degradation. The two region piecewise high lateral field dependent velocity is used. It is written as

$$v_0 = \begin{cases} \frac{\mu_{0,sf} E_y}{1 + E_y / E_{sat,sf}} & E_y < E_{sat,sf} \\ v_{sat,sf} & E_y \geq E_{sat,sf} \end{cases} \quad (2.117)$$

where the saturation electric field $E_{sat,sf}$ is expressed as

$$E_{sat,sf} = \frac{2v_{sat,sf}}{\mu_{0,sf}} \quad (2.118)$$

E_y for $E_y < E_{sat,sf}$ is approximately a linear function of V_{ds} . Thus, the lateral field degraded velocity and mobility can be obtained as

$$v_{0,sf} = \frac{\mu_{0,sf} E_y}{1 + \frac{\delta_{sf} V_{ds}}{L_{dr} E_{sat,sf}}} \quad (2.119)$$

$$\mu_{sf} = \frac{\mu_{0,sf}}{1 + \frac{\delta_{sf} V_{ds}}{L_{dr} E_{sat,sf}}} \quad (2.120)$$

2.4.2.1.2 Saturation Voltage

Due to the lateral field mobility degradation effect, the surface carrier will also experience velocity saturation at the drain end. The saturation effect is modeled by the saturation voltage according to [56]. The linear drift diffusion current expression (2.111), following the ‘pinned surface’ concept ($\phi_{dr,d} - \phi_{dr,s} = V_{ds}$), is simplified (Taylor expansion at the source side) as

$$I_{ds,sf} = \mu_{sf,s} \frac{C_{ox,dr} W}{L_{dr}} (V_{gt,sf,s} - \frac{V_{ds}}{2} + V_{tm}) V_{ds} \quad (2.121)$$

in which $V_{gt,sf,s}$ and $\mu_{sf,s}$ are given by

$$V_{gt,sf,s} = \gamma_{dr} \cdot \sqrt{V_t \cdot \exp(\frac{\phi_{dr,s} - V_s}{V_{tm}})} \quad (2.122)$$

$$\mu_{sf} = \frac{\mu_{0,sf,s}}{1 + \frac{\delta_{sf} V_{ds}}{L_{dr} E_{sat,sf}}} \quad (2.123)$$

where $\mu_{0,sf,s}$ is the low field mobility evaluated by substituting the source transverse field into (2.114). The saturation current, $I_{sat,sf}$ in the surface conducting path is given by

$$\begin{aligned} I_{sat,sf} &= v_{sat,sf} WC_{ox,dr} (V_{gt,sf,s} - \Delta\phi_{sat,sf}) \\ &= v_{sat,sf} WC_{ox,dr} (V_{gt,sf,s} - V_{dsat,sf}) \end{aligned} \quad (2.124)$$

The saturation voltage can be derived by solving $I_{sat,sf} = I_{ds,sf}$ and $V_{ds} = V_{dsat,sf}$ with (2.118) and (2.120). It is deduced

$$V_{dsat,sf} = \frac{V_{gt,sf,s} L_{dr} E_{sat,sf}}{V_{gt,sf,s} + L_{dr} E_{sat,sf} + 2V_{tm}} \quad (2.125)$$

2.4.2.1.3 Unified Current Model

As (2.121) is valid only for the linear operation region, the effective drain source voltage, $V_{dseff,sf}$ and effective drain voltage $V_{deff,sf}$ are introduced to include the saturation effect. They are expressed as

$$V_{dseff,sf} = \mathcal{G}_{eff}(V_{dsat,sf}, V_{ds}, \delta_{sat,sf}) \quad (2.126)$$

$$V_{deff,sf} = \mathcal{G}_{eff}(V_{dsat,sf}, V_{ds}, \delta_{sat,sf}) + V_s \quad (2.127)$$

in which the smoothing function $\mathcal{G}_{eff}()$ is given in (2.28) and $\delta_{sat,sf}$ is a smoothing parameter used to tune the transition from linear region to the saturation region. Then, a unified surface current expression can be obtained by replacing the V_d and V_{ds} in (2.111) with $V_{deff,sf}$ and $V_{dseff,sf}$, respectively. The final expression is given by

$$I_{ds,dr} = \mu_{sfeff} \frac{C_{ox,dr} W}{L_{dr}} (q_{acc,eff} + V_{tm}) V_{dseff,sf} \quad (2.128)$$

in which

$$\mu_{sfeff} = \mu_{sf} \Big|_{V_d=V_{dseff,sf}} \quad (2.129)$$

$$q_{acc,eff} = q_{acc} \Big|_{V_d=V_{dseff,sf}} \quad (2.130)$$

2.4.2.2 Silicon Bulk Current

As mentioned, the second main electron conduction path is in the silicon body. This current is modeled as a uniform resistor with lateral field mobility degradation effect. The effective electron charge in the bulk (silicon body) can be calculated by subtracting the p-n junction charge and the depletion charge from the total bulk electron density ($-qN_d t_{si,dr}$). It is expressed as

$$Q_{bulk} = \underbrace{-qN_d t_{si,dr}}_{\text{bulk electron}} + \underbrace{Q_{pn}}_{\text{pn charge}} + \underbrace{C_{ox,dr} \gamma_{dr} \sqrt{-\phi_{dr}}}_{\text{depletion charge}} \quad (2.131)$$

in which $t_{si,dr}$ is the thickness of the drift layer and Q_{pn} represents the electron charge depleted by the p-n junction formed between the n-type drift channel and the p-type epi layer. According to the p-n junction theory, Q_{pn} is given by

$$Q_{pn} = \sqrt{\epsilon_{si} q N_d (V_{bi} + V_{sb})} \quad (2.132)$$

where V_{bi} is the built in barrier of the p-n junction and it is expressed as

$$V_{bi} = V_{tm} \ln \left(\frac{N_d N_{epi}}{n_i^2} \right) \quad (2.133)$$

where N_{epi} is the doping in the p-epi layer. In practice, using (2.131) may cause problems. This is because Q_{bulk} is supposed to be negative, but if the value t_{si} is not selected carefully (t_{si} too small) or if V_{sb} is too large, Q_{bulk} evaluated by (2.131) would become positive and the model will produce unphysical behavior. So, (2.131) is modified as

$$\begin{aligned}
 Q_{bulk} &= -\left(qN_d t_{si,dr} - Q_{pn} - C_{ox,dr} \gamma_{dr} \sqrt{-\phi_{dr}}\right) \\
 &= -qN_d t_{si,dr} \left(1 - \frac{Q_{pn}}{qN_d t_{si,dr}} - \frac{C_{ox,dr} \gamma_{dr} \sqrt{-\phi_{dr}}}{qN_d t_{si,dr}}\right) \\
 &\approx -qN_d t_{si,dr} \frac{1}{\left(1 + \frac{Q_{pn}}{qN_d t_{si,dr}} + \delta_{del} \frac{C_{ox,dr} \gamma_{dr} \sqrt{-\phi_{dr}}}{qN_d t_{si,dr}}\right)} \\
 &= -\frac{(qN_d t_{si,dr})^2}{\left(qN_d t_{si,dr} + \sqrt{\epsilon_{si} qN_d (V_{bi} + \delta_{pn} V_{sb})} + \delta_{del} C_{ox,dr} \gamma_{dr} \sqrt{-\phi_{dr}}\right)}
 \end{aligned} \tag{2.134}$$

in which δ_{pn} and δ_{del} are added for better fitting. Mathematically, the above approximation is like the reverse process of Taylor expansion [first order Taylor expansion on the final result of (2.134) gives (2.131)]. For a uniform resistor, only drift current is necessary to be considered. Therefore, the bulk resistor current I_R is written as

$$I_R = -W \mu_b Q_{bulk} E_{y,R} \tag{2.135}$$

where μ_b is the mobility in the bulk and $E_{y,b}$ is the bulk electric field. The lateral-field degraded mobility is given as

$$\mu_b = \frac{\mu_{0,R}}{\sqrt{1 + \delta_R \left(\frac{E_{y,R}}{E_{sat,R}}\right)^2}} \tag{2.136}$$

where δ_R is a fitting parameter and $E_{sat,R}$ is the saturation e-field in the bulk silicon, which is given by

$$E_{sat,R} = \frac{V_{sat,dr}}{\mu_{0,R}} \quad (2.137)$$

where $\mu_{0,R}$ is the low field mobility, extracted from the experiment data. It is noticed that in the saturation e-field expressions, (2.118) and (2.137), there is a single difference in the form of expression, i.e., there is ‘2’ in the nominator of (2.118). This is due the use of different form of the lateral-field dependent mobility models. In fact, the difference can be absorbed by in fitting parameters δ_{sf} and δ_R — δ_{sf} and δ_R can be tuned to obtain consistency between both the surface current and the bulk current on the saturation behavior. So the difference will not cause any potential problems in the model. The electric field in the resistor $E_{y,R}$ is actually equal to V_{ds}/L_{dr} . Thus, the final bulk resistor current is obtained by rewriting (2.135) as

$$I_R = \frac{W}{L_{dr}} \frac{\mu_{0,R}}{\sqrt{1 + \delta_R \left(\frac{V_{ds}}{L_{dr} E_{sat,R}} \right)^2}} (-Q_{bulk}) V_{ds} \quad (2.138)$$

2.4.2.3 Drift I-V Model

As mentioned, the surface current and the bulk silicon current actually are present simultaneously in parallel. So the total current in the drift channel can be obtained by summing the two currents, and the final result is given by

$$I_{dr} = I_{sf} + I_R \quad (2.139)$$

In order to validate the drift channel model, the device in Figure 2.13 is simulated in TCAD. The simulated transistor has $t_{ox,dr}=100\text{nm}$, $L_{dr}=2\mu\text{m}$ and $N_d=10^{15}\text{cm}^{-3}$. I-V data are extracted from the simulation for comparison with the model, which are shown in Figure 2.16-Figure 2.17. Figure 2.16 compares the I_d - V_{ds} for different V_{gs} . Figure 2.17 compares I_d - V_{gs} for different V_{ds} . Figure 2.18 and Figure 2.19 show the model's behaviors for different L_{dr} and $t_{ox,dr}$, respectively. It can be seen that the model is able to reproduce the I-V data well.

Figure 2.16 comparison of modelled drain current (lines) with the numerical data from TCAD (symbols) for V_{ds} variation at different V_{gs} ($=0.5, 1, 1.5, 2\text{V}$) with fixed $V_{sb}=0$

Figure 2.17 comparison of modelled drain current (lines) with the numerical data from TCAD (symbols) for V_{gs} variation at different V_{ds} ($=0.5, 1, 2\text{V}$) with fixed $V_{sb}=0$

Figure 2.18 comparison of modelled drain current (lines) with the numerical data from TCAD (symbols) for L_{dr} variation at different $V_{ds}=1\text{V}$ with fixed $V_{sb}=0$

Figure 2.19 comparison of modelled drain current (lines) with the numerical data from TCAD (symbols) for $t_{ox,dr}$ variation at different $V_{ds}=1\text{V}$ with fixed $V_{sb}=0$

2.5 LDMOS I-V Model and Validation

Figure 2.20 Subcircuit representation of the LDMOS

Physically, LDMOS can be modeled by a subcircuit consisting of p-channel and drift channel in series. A schematic of the subcircuit is shown in

Figure 2.20. The LDMOS transistor in Figure 2.1 is simulated by 2D simulator (Medici). The configuration of the simulated LDMOS is as follows. The p-channel length: $L_{eff}=0.6\mu\text{m}$; oxide thickness: $t_{ox}=5\text{nm}$; lateral doping: $N_{pch}=10^{18}e^{-(y/0.18\mu\text{m})^2}+10^{15}\text{cm}^{-3}$; the drift channel length: $L_{dr}=2\mu\text{m}$; field oxide thickness: $t_{ox,dr}=100\text{nm}$ and the doping: $N_d=15\text{cm}^{-3}$. Numerical data are extracted for comparison in Figure 2.21--Figure 2.25. The p-channel and drift channel models are coded in Verilog-A, and the LDMOS is simulated in HSPICE. Figure 2.21 compares the I_d - V_{gs} for different V_{ds} . Figure 2.22 compares the transconductance, g_m - V_{gs} for different V_{ds} . Figure 2.23 compares the I_d - V_{ds} for different V_{gs} . Figure 2.24 compares the conductance, g_{ds} - V_{ds} for different V_{gs} . Figure 2.25 compares I_d - V_{gs} for different V_{sb} . Figure 2.26-Figure 2.27 show the model's behaviors for different t_{ox} . Excellent agreements between the model results and the numerical data are obtained.

Figure 2.21 comparison of modeled drain current (lines) with the numerical data from TCAD (symbols) for V_{gs} variation at different V_{ds} ($=0.3, 0.6, 0.9, 1.2\text{V}$) with fixed $V_{sb}=0$

Figure 2.22 comparison of model transconductance, g_m (lines) with the numerical data from TCAD (symbols) for V_{gs} variation at different V_{ds} ($=0.3, 0.6, 0.9, 1.2\text{V}$) with fixed $V_{sb}=0$

Figure 2.23 comparison of modelled drain current (lines) with the numerical data from TCAD (symbols) for V_{ds} variation at different V_{gs} ($=1, 1.5, 2\text{V}$) with fixed $V_{sb}=0$

Figure 2.24 comparison of model conductance, g_{ds} (lines) with the numerical data from TCAD (symbols) for V_{ds} variation at different V_{gs} ($=1, 1.5, 2V$) with fixed $V_{sb}=0$

Figure 2.25 comparison of modeled drain current (lines) with the numerical data from TCAD (symbols) for V_{gs} variation at different V_{sb} ($=0, 0.2, 0.4$) with fixed $V_{ds}=0.6V$

Figure 2.26 comparison of modeled drain current (lines) with the numerical data from TCAD (symbols) for t_{ox} variation at different $V_{sb}=0$ with fixed $V_{ds}=1V$

Figure 2.27 comparison of model transconductance, g_m (lines) with the numerical data from TCAD (symbols) for t_{ox} variation at different $V_{ds}=1V$ with fixed $V_{sb}=0$

2.6 Chapter 2 summary

A LDMOS subcircuit I-V model that consists of the p-channel and the drift channel in series is demonstrated in this chapter. The p-channel model is surface potential based. The surface potential is obtained by solving the Poisson equation analytically using URM approach. The drift channel is modeled as a parallel combination of an accumulation transistor and the mobility degraded bulk resistor. Both the p-channel model and the drift channel model are proved to match the respective numerical data very well. Finally, the subcircuit model is compared with the LDMOS numerical simulation data, it can be seen that the model match the data very well.

Chapter 3 LDMOS Charge Model

3.1 Introduction

LDMOS is extensively used in many RF circuits. It's important for a compact model to correctly predict the capacitance of LDMOS so as to enable accurate RF circuit simulation. The most preferred approach in modeling the MOS transistor capacitance is charge-based modeling [12, 67] as it guarantees the charge conservation of the transistor. The charge model associates the terminals of the transistor with the electrical charge inside the channel. The charge associated with the source terminal is referred to as the source-charge; so do the charges for the drain, gate, and bulk terminals. Generally, the depletion and accumulation charges are assigned to the bulk terminal and the inversion charge inside the channel is partitioned into two parts and being assigned to the source and drain terminals. Finally, the gate charge is equal to the sum of the source, drain and bulk charges based on charge neutrality. The most challenging part in deriving the charge model is to find a suitable method to partition the channel inversion charge to the source and drain. For the conventional MOSFET, the well-known Ward-Dutton (WD) partition scheme [12] is used. According to the WD scheme, the source charge, Q_S and the drain charge, Q_D can be expressed as, respectively,

$$Q_S = -C_{ox} W \int_0^L \frac{L-y}{L} q_i(y) dy \quad (3.1)$$

$$Q_D = -C_{ox} W \int_0^L \frac{y}{L} q_i(y) dy \quad (3.2)$$

where W is the device width, L is the channel length and $q_i(y)$ is the normalized inversion charge density inside the channel at the distance y from the source. (3.1) and (3.2) also implies that

$$Q_S + Q_D = -C_{ox} \int_0^L q_i(y) dy \quad (3.3)$$

However, the WD scheme is derived for the conventional MOSFET with lateral uniform channel [12]. In LDMOS, the doping in the channel is laterally non-uniform. Therefore, the WD scheme is not applicable to LDMOS, as demonstrated in [13-17]. In this regard, a new charge partition method is introduced and discussed in this chapter. Based on some reasonable simplifications and with the new partition method, the charge model for LDMOS is then formulated.

3.2 Modeling Goal and Strategy

A conceptual n-type LDMOS structure is shown in Figure 3.1 (a). The doping level in the p-channel gradually decreases towards the drain side, as shown in Figure 3.1 (b). It is aimed to derive a charge model for the p-channel but directly modeling the channel with gradually decreased doping is very complicated and hardly possible. Therefore, the laterally-diffused channel doping profile is approximated by a **step doping profile (SDP)**, as illustrated in Figure 3.1 (b). Then, modeling the p-channel can be decomposed into two sub

problems. The first sub problem obviously is to deduce the charge model for the channel with the SDP. The diffused doping profile may introduce more subtle effects than the SDP. Therefore, the second sub problem is to add on those subtle effects to the SDP based modeling in order to compensate the error introduced by the SDP approximation. This chapter will solve the first sub problem, that is, to formulate the charge model for the p-channel with the SDP.

Figure 3.1 (a): typical cross-section of a typical LDMOS. (b): diffused doping profile and approximated step doping profile for the p-channel. (c): cross-section of a transistor (S-MOS) with step doping profile. (d): equivalent circuit representation for the S-MOS

3.3 Charge Partition Principle

The p-channel with the SDP can be perceived as a MOS transistor shown in Figure 3.1(c). This transistor will be referred to as “S-MOS” in the following context. The S-MOS is actually two conventional MOSFETs, M1 on the left and M2 on the right, connected in series. The doping of M1 and M2 are laterally uniform, and the doping of M1 should be higher than that of M2. Hence, terminal charges can be formulated for M1 and M2 based on the WD scheme [12, 56, 61]. Denoting Q_{D_h} (Q_{D_l}), Q_{S_h} (Q_{S_l}), Q_{G_h} (Q_{G_l}) and Q_{B_h} (Q_{B_l}) as M1’s (M2’s) terminal drain-, source-, gate- and bulk-charge, respectively, they are readily available in the explicit forms. The definitions of the respective source and drain charges are written as

$$Q_{S_h} = -C_{ox} W \int_0^{L_h} \frac{L_h - y}{L_h} q_{i_h}(y) \partial y \quad (3.4)$$

$$Q_{D_h} = -C_{ox} W \int_0^{L_h} \frac{y}{L_h} q_{i_h}(y) \partial y \quad (3.5)$$

$$Q_{S_l} = -C_{ox} W \int_{L_h}^{L_h+L_l} \frac{L_l - (y - L_h)}{L_l} q_{i_l}(y) \partial y \quad (3.6)$$

$$Q_{D_l} = -C_{ox} W \int_{L_h}^{L_h+L_l} \frac{(y - L_h)}{L_l} q_{i_l}(y) \partial y \quad (3.7)$$

It is noted that

$$Q_{S_h} + Q_{D_h} = -C_{ox} W \int_0^{L_h} q_{i_h}(y) \partial y \quad (3.8)$$

$$Q_{S_l} + Q_{D_l} = -C_{ox} W \int_{L_h}^{L_h+L_l} q_{i_l}(y) dy \quad (3.9)$$

The explicit expressions for the charge based on [68] are written below for the sake of completeness,

$$Q_{S_h} = -\frac{WC_{ox}L_h}{2} \left(\frac{\overline{q_{i_h}}}{q_{i_h}} + \frac{\overline{A_{b_h}} \Delta \phi_{s_h}}{6} \left(1 + \frac{\Delta \phi_{s_h}}{2N_h} - \frac{\Delta \phi_{s_h}^2}{20N_h^2} \right) \right) \quad (3.10)$$

$$Q_{D_h} = -\frac{WC_{ox}L_h}{2} \left(\frac{\overline{q_{i_h}}}{q_{i_h}} - \frac{\overline{A_{b_h}} \Delta \phi_{s_h}}{6} \left(1 - \frac{\Delta \phi_{s_h}}{2N_h} - \frac{\Delta \phi_{s_h}^2}{20N_h^2} \right) \right) \quad (3.11)$$

$$Q_{S_l} = -\frac{WC_{ox}L_l}{2} \left(\frac{\overline{q_{i_l}}}{q_{i_l}} + \frac{\overline{A_{b_l}} \Delta \phi_{s_l}}{6} \left(1 + \frac{\Delta \phi_{s_l}}{2N_l} - \frac{\Delta \phi_{s_l}^2}{20N_l^2} \right) \right) \quad (3.12)$$

$$Q_{D_l} = -\frac{WC_{ox}L_l}{2} \left(\frac{\overline{q_{i_l}}}{q_{i_l}} - \frac{\overline{A_{b_l}} \Delta \phi_{s_l}}{6} \left(1 - \frac{\Delta \phi_{s_l}}{2N_l} - \frac{\Delta \phi_{s_l}^2}{20N_l^2} \right) \right) \quad (3.13)$$

$$Q_{B_h} = WC_{ox}L \left(\gamma_h \sqrt{\phi_{s_h}} - \frac{(\overline{A_{b_h}} - 1) \Delta \phi_{s_h}^2}{12N_h} + (VGR_h - \phi_{CC_h}) \right) \quad (3.14)$$

$$Q_{B_l} = WC_{ox}L \left(\gamma_l \sqrt{\phi_{s_l}} - \frac{(\overline{A_{b_l}} - 1) \Delta \phi_{s_l}^2}{12N_l} + (VGR_l - \phi_{CC_l}) \right) \quad (3.15)$$

$$Q_{G_h} = -Q_{S_h} - Q_{D_h} - Q_{B_h} \quad (3.16)$$

$$Q_{G_l} = -Q_{S_l} - Q_{D_l} - Q_{B_l} \quad (3.17)$$

where

$$N_h = \frac{\overline{q_{i_h}}}{A_{b_h}} + V_{tm} \quad (3.18)$$

$$N_l = \frac{\overline{q_{i_l}}}{A_{b_l}} + V_{im} \quad (3.19)$$

The definition of the quantities in the above equations can be found in section 2.3. Quantities with subscript ‘*h*’ are the parameters for M1 and those with subscript ‘*l*’ are for M2. For examples, $\overline{q_{i_h}}$ is the $\overline{q_i}$ (average normalized inversion charge density) of M1 and $\Delta\phi_{s_l}$ is the $\Delta\phi_s$ (drain source surface potential difference) of M2. Based on geometry relations, the S-MOS terminal gate- and bulk-charge, Q_G and Q_B , can be obtained as, respectively,

$$Q_G = Q_{G_h} + Q_{G_l} \quad (3.20)$$

$$Q_B = Q_{B_h} + Q_{B_l} \quad (3.21)$$

The drain- and source-charge are essentially the inversion charge in the channel. The inversion charge in the channel of S-MOS is the sum of the inversion charge in M1’s and M2’s channel. From this perspective, the terminal drain- and source-charge of the S-MOS, Q_D and Q_S can be related to Q_{S_h} , Q_{D_h} , Q_{S_l} and Q_{D_l} by the following relationship:

$$Q_S + Q_D = Q_{S_h} + Q_{D_h} + Q_{S_l} + Q_{D_l} \quad (3.22)$$

Once Q_S is obtained, Q_D is automatically known, or vice versa. To obtain Q_S and Q_D , however, it is not trial as mentioned before. A proper partition method is required. Before discussing that, two important voltages are introduced. The first is V_i , the internal voltage at the joint point of M1 and M2. The second voltage is V_{gs} , the S-MOS’s gate-source voltage at which V_i is equal to M1’s drain saturation voltage for a given S-MOS drain-source voltage, V_{ds} .

Naturally, when the gate-source voltage of the S-MOS, V_{gs} , is smaller than V_{gc} , M1 operates in saturation; when V_{gs} is greater than V_{gc} , M1 operates in linear region. Both V_i and V_{gc} can be obtained by numerically equating the current of M1 and the current of M2 with appropriate bias conditions.

Regarding the partition method, an alternative form of defining the source- and drain-charge [69] is used here, according to which Q_S and Q_D conform with the following conditions:

$$i_D(t) = i_T(t) + \frac{\partial Q_D}{\partial t} \quad (3.23)$$

$$i_S(t) = -i_T(t) + \frac{\partial Q_S}{\partial t} \quad (3.24)$$

which implies

$$Q_D = \int_{-\infty}^t [i_D(\tau) - i_T(\tau)] d\tau = \int_{-\infty}^t i_{DX}(\tau) d\tau \quad (3.25)$$

$$Q_S = \int_{-\infty}^t [i_S(\tau) + i_T(\tau)] d\tau = \int_{-\infty}^t i_{SX}(\tau) d\tau \quad (3.26)$$

where i_D and i_S are the real time current that flow into the channel through S-MOS's drain and source terminal, respectively, as indicated in Figure 3.1(d); i_T is the static state terminal current calculated using the DC formula. $i_{DX}(\tau)$ and $i_{SX}(\tau)$ can be interpreted as the charging current. This definition means Q_S or Q_D is actually equal to the time accumulation of the charging current that goes through the source or drain into the channel. Please note the defined positive direction for the charging current: i_{DX} flows from drain to the channel

or source while i_{SX} flows from source to the channel or drain. So, if there is a positive charge flowing from the channel to the drain, it is considered as a negative valued i_{DX} . Similar concept applies to i_{SX} .

To illustrate the new partition method, let A be one point in M1's channel and B in M2's, shown in as Figure 3.1(c). There is some amount of inversion charge 'stored' at A (and B). According to the concept of charge partition, this amount of inversion charge can be divided into two parts. The first part belongs to Q_S , while the second part belongs to Q_D . Denote the first part at A as Q_{S_A} and the second part as Q_{D_A} . (accordingly, Q_{S_B} , the first part and Q_{D_B} , the second part for B). So, in fact, the mission of a partition method is to find out Q_{S_A} and Q_{D_A} (Q_{S_B} and Q_{D_B}).

Figure 3.2 the voltage (V_A) at the point A and the current go through A versus time

Imagine a scenario when there is an infinitesimal voltage deviation, ΔV , applied to any of the external terminal (S-MOS's drain, source, gate or bulk). This infinitesimal ΔV can also be perceived as the change to the terminal bias and the value of ΔV is so small that the steady state terminal bias remains

practically unchanged (DC biases do not change). Then there will be induced voltage deviation $\Delta V_A(t)$ at the point A and $\Delta V_B(t)$ at the point B . A descriptive plot of the current/voltage deviation for this scenario at A is shown in Figure 3.2, where V_A is the steady state voltage at the point A . As there is a voltage deviation, ΔV_A at A , there will be induced small signal current flow from A to the source and drain. Because ΔV is extremely small, ΔV_A , and ΔV_B can be considered as small signals. The small signal current flowing from A to the source can be calculated as $\Delta i_{AS} = g_{ASA} \Delta V_A$ ($\Delta i_{AD} = g_{ADA} \Delta V_A$, from A to drain), where g_{ASA} (g_{ADA}) is the conductance looking from A to the source (drain). Reversely speaking, Δi_{AS} and Δi_{AD} can be considered as the small signal current flowing from source and drain to A , respectively, (direction difference can be absorbed by the positive or negative sign). These small signal currents can actually be perceived as the charging current from drain or source based on (3.25)-(3.26) (charging current is equal to real time current minus DC state current), as the steady state current remains practically unchanged with the terminal bias. Thus, the charging current, ic_{D_A} , that comes from the drain to the point A and ic_{S_A} that comes from the source to the point A can be calculated as

$$ic_{D_A} \equiv i_{DA}(t) - i_T = -g_{ADA} \cdot \Delta V_A(t) \quad (3.27)$$

$$ic_{S_A} \equiv i_T - i_{AS}(t) = -g_{ASA} \cdot \Delta V_A(t) \quad (3.28)$$

where i_{AS} (i_{DA}) is the real time current from A (drain) to the source (A). Denote the deviation of Q_{S_A} and Q_{D_A} due to ΔV as ΔQ_{S_A} and ΔQ_{D_A} , respectively. ΔQ_{S_A} (ΔQ_{D_A}) contributes to the change of Q_S (Q_D) in this event. Correspondingly, ic_{S_A} (ic_{D_A}) contributes to the time accumulation of the total

charging current that goes through the source (drain) in this event. Thus, according to (3.25)-(3.26), we can relate the charging current going to A with A 's contribution to the deviation of Q_S and Q_D :

$$\Delta Q_{S_A} = \int_{t_0}^{t_1} i_{c_{S_A}} \partial t \quad (3.29)$$

$$\Delta Q_{D_A} = \int_{t_0}^{t_1} i_{c_{D_A}} \partial t \quad (3.30)$$

Under small signal assumption, g_{ASA} and g_{ADA} can be considered to be constant. Substituting (3.27)-(3.28) into (3.29)-(3.30), the following approximation can be derived

$$\Delta Q_{S_A} : \Delta Q_{D_A} \approx g_{ASA} : g_{ADA} \quad (3.31)$$

in which g_{ASA} and g_{ADA} are defined as

$$g_{ASA} \equiv \frac{\partial I_{AS}}{\partial V_A} \quad (3.32)$$

$$g_{ADA} \equiv \frac{\partial I_{AD}}{\partial V_A} \quad (3.33)$$

where I_{AS} and I_{AD} are the steady state currents from A to source and drain, respectively, which are expressed as

$$I_{AS} = \frac{C_{ox} W}{y_A} \int_{V_A}^{V_S} u_n q_{i_h}(V_{cb}) \partial V_{cb} \quad (3.34)$$

$$I_{AD} = \frac{C_{ox} W}{L_h - y_A} \int_{V_A}^{V_i} u_n q_{i_h}(V_{cb}) \partial V_{cb} = \frac{C_{ox} W}{L_l} \int_{V_i}^{V_D} u_n q_{i_l}(V_{cb}) \partial V_{cb} \quad (3.35)$$

where y_A is the distance between A and the source of the S-MOS, μ_n is the electron mobility and V_{cb} is the channel voltage. For simplicity, constant mobility is assumed. Based on (3.32)-(3.35), we obtain

$$g_{ASA} = -\frac{C_{ox} W u_n q_{i_h}(V_A)}{y_A} \quad (3.36)$$

$$g_{ADA} = -\frac{C_{ox} W u_n q_{i_h}(V_A)}{L_h - y_A + L_l K} \quad (3.37)$$

where $K = q_{i_h}(V_i)/q_{i_l}(V_i)$. $q_{i_h}(V_i)$ ($q_{i_l}(V_i)$) is actually the normalized inversion charge density to the left (to the right) of the joint point, i.e., M1's drain (M2's source). Substituting (3.36) and (3.37) into (3.31), (3.31) becomes

$$\frac{\Delta Q_{S_A}}{\Delta Q_{D_A}} = \frac{L_h - y_A + L_l K}{y_A} \quad (3.38)$$

Similar to (3.27)-(3.38), same analyses can be performed to the point B . Then, we have

$$\frac{\Delta Q_{S_B}}{\Delta Q_{D_B}} = \frac{L_h + L_l - y_B}{y_B - L_h + L_l / K} \quad (3.39)$$

where y_B is the distance between the point B and the source end. (3.38) and (3.39) actually represents the partition ratio of the change in the charge at A and B . Ideally, K equals to zero when $V_{gs} \leq V_{gc}$ and unity when $V_{gs} > V_{gc}$.

3.3.1 Charge Partition Ratio for $V_{gs} \leq V_{gc}$

For $V_{gs} \leq V_{gc}$, M1 operates in saturation that the inversion charge density at M1's drain is much less than that at the M2's source, which means $q_{i_h}(V_i) \ll q_{i_l}(V_i)$. Thus, $K \approx 0$. (3.38) and (3.39) can be simplified to

$$\frac{\Delta Q_{S_A}}{\Delta Q_{D_A}} \approx \frac{L_h - y_A}{y_A} \quad (3.40)$$

$$\frac{\Delta Q_{S_B}}{\Delta Q_{D_B}} \approx 0 \quad (3.41)$$

Remember that, ΔQ_{S_A} , ΔQ_{D_A} , ΔQ_{S_B} , and ΔQ_{D_B} are charge deviations due to ΔV , an infinitesimal signal change at the external bias. (3.40) and (3.41) are supposed to be only valid when ΔV is very small. But it is noticed that, the ratio in (3.40) and (3.41) are constant. This means after many small voltage deviations ($\Delta V_1, \Delta V_2, \Delta V_3 \dots$) applied to one terminal, the total ΔQ_{S_A} , ΔQ_{D_A} , ΔQ_{S_B} and ΔQ_{D_B} will still follow (3.40) and (3.41). For example, if the S-MOS's gate-source voltage increases from 0 to V_{gs} , the increase of the gate-source voltage is divided into ' n_{step} ' steps such that the voltage increment at each step can be considered as an infinitesimal signal. Each step will induce small deviation to the channel inversion charge. For the point A, the deviation will be ΔQ_{S_A1} and ΔQ_{D_A1} , ΔQ_{S_A2} and $\Delta Q_{D_A2} \dots \Delta Q_{S_An}$ and ΔQ_{D_An} . Then, according to (3.40), we have

$$\begin{aligned} \frac{\Delta Q_{S_A1}}{\Delta Q_{D_A1}} &= \frac{\Delta Q_{S_A2}}{\Delta Q_{D_A2}} = \dots = \frac{\Delta Q_{S_An}}{\Delta Q_{D_An}} \\ &= \frac{\Delta Q_{S_A1} + \Delta Q_{S_A2} + \dots + \Delta Q_{S_An}}{\Delta Q_{D_A1} + \Delta Q_{D_A2} + \dots + \Delta Q_{D_An}} \approx \frac{L_h - y_A}{y_A} \end{aligned} \quad (3.42)$$

In another form, (3.42) can be rewritten as

$$\frac{Q_{S_A}(V_{gs}, V_{ds}) - Q_{S_A}(0, V_{ds})}{Q_{D_A}(V_{gs}, V_{ds}) - Q_{D_A}(0, V_{ds})} \approx \frac{L_h - y_A}{y_A} \quad (3.43)$$

and similarly for B ,

$$\frac{Q_{S_B}(V_{gs}, V_{ds}) - Q_{S_B}(0, V_{ds})}{Q_{D_B}(V_{gs}, V_{ds}) - Q_{D_B}(0, V_{ds})} \approx 0 \quad (3.44)$$

$Q_{D_A}(0, V_{ds})$, $Q_{S_B}(0, V_{ds})$, and $Q_{D_B}(0, V_{ds})$ in (3.43) and (3.44) are in fact negligible. Thus (3.43) and (3.44) are further simplified to

$$\frac{Q_{S_A}(V_{gs}, V_{ds})}{Q_{D_A}(V_{gs}, V_{ds})} \approx \frac{L_h - y_A}{y_A} \quad (3.45)$$

$$\frac{Q_{S_B}(V_{gs}, V_{ds})}{Q_{D_B}(V_{gs}, V_{ds})} \approx 0 \quad (3.46)$$

What (3.45) and (3.46) indicate is actually the charge partition ratio at A and B , similar to that of the WD. Note that A or B is randomly picked in M1 or M2. So, (3.45) and (3.46) are applicable to all the points in M1 and M2, respectively.

3.3.2 Charge Partition Ratio for $V_{gs} > V_{gc}$

For V_{gs} well above V_{gc} , both M1 and M2 operate in the linear region that the inversion charge density at M1's drain is approximately equal to that

at the M2's source so that $q_{i_h}(V_i) \approx q_{i_l}(V_i)$. Ideally, K can be taken as a constant, pp (defaults to 1). Then, (3.38) and (3.39) can be rewritten as

$$\frac{\Delta Q_{S-A}}{\Delta Q_{D-A}} = \frac{L_h - y_A + L_l \cdot pp}{y_A} \quad (3.47)$$

$$\frac{\Delta Q_{S-B}}{\Delta Q_{D-B}} = \frac{L_h + L_l - y_B}{y_B - L_h + L_l / pp} \quad (3.48)$$

Similar to (3.40)-(3.44), (3.47) and (3.48) lead to

$$\frac{Q_{S-A}(V_{gs}, V_{ds}) - Q_{S-A}(V_{gc}, V_{ds})}{Q_{D-A}(V_{gs}, V_{ds}) - Q_{D-A}(V_{gc}, V_{ds})} = \frac{L_h - y_A + L_l \cdot pp}{y_A} \quad (3.49)$$

$$\frac{Q_{S-B}(V_{gs}, V_{ds}) - Q_{S-B}(V_{gc}, V_{ds})}{Q_{D-B}(V_{gs}, V_{ds}) - Q_{D-B}(V_{gc}, V_{ds})} = \frac{L_h + L_l - y_B}{y_B - L_h + L_l / pp} \quad (3.50)$$

With (3.49)-(3.50), we know how to partition point A's inversion charge induced after V_{gs} goes beyond V_{gc} . Finally, with (3.45), (3.46), (3.49) and (3.50), we know how to partition point A's inversion charge for any bias.

3.4 Terminal Charge Model

With (3.45), (3.46), (3.49), and (3.50), we can partition the inversion charge at the point A for all biases, and thus, derive the terminal charge. The inversion charge Q_A 'stored' at the point A and Q_B at the point B can be expressed as

$$Q_A = Q_{S-A} + Q_{D-A} = -C_{ox} \Delta y \cdot W q_{i_h}(y_A, V_{gs}) \quad (3.51)$$

$$Q_B = Q_{S_B} + Q_{D_B} = -C_{ox}\Delta y \cdot W q_{i_l}(y_B, V_{gs}) \quad (3.52)$$

where Δy is an infinitesimal number. As mentioned, the points A and B are randomly selected. Summing up the contribution of all the possible ‘ A ’ in M1 and the contribution of all possible ‘ B ’ in M2, we can obtain the source and drain charges. So, based on the definition of Q_{S_A} , Q_{D_A} , Q_{S_B} , and Q_{D_B} , the terminal source and drain charges can be written as

$$Q_S(V_{gs}, V_{ds}) = \int_{A \in M1} Q_{S_A} + \int_{B \in M2} Q_{S_B} \quad (3.53)$$

$$Q_D(V_{gs}, V_{ds}) = \int_{A \in M1} Q_{D_A} + \int_{B \in M2} Q_{D_B} \quad (3.54)$$

3.4.1 Source and Drain Charges for $V_{gs} \leq V_{gc}$

When $V_{gs} \leq V_{gc}$, combining with (3.45)-(3.46) and (3.51)-(3.52), Q_{S_A} , Q_{D_A} , Q_{S_B} , and Q_{D_B} can be obtained as

$$Q_{S_A}(V_{gs}, V_{ds}) \approx -C_{ox}\Delta y \cdot W \frac{L_h - y_A}{L_h} q_{i_h}(y_A) \quad (3.55)$$

$$Q_{D_A}(V_{gs}, V_{ds}) \approx -C_{ox}\Delta y \cdot W \frac{y_A}{L_h} q_{i_h}(y_A) \quad (3.56)$$

$$Q_{S_B}(V_{gs}, V_{ds}) \approx 0 \quad (3.57)$$

$$Q_{D_B}(V_{gs}, V_{ds}) \approx -C_{ox}\Delta y \cdot W q_{i_l}(y_B) \quad (3.58)$$

Thus, according to (3.53)-(3.58), the terminal source- and drain-charge for the S-MOS when $V_{gs} \leq V_{gc}$ can be obtained by summing up the contributions from all the points in M1 and M2 as (with Δy changed to ∂y),

$$\begin{aligned} Q_S(V_{gs}, V_{ds}) &= \int_{A \in M1} Q_{S_A} + \int_{B \in M2} Q_{S_B} \\ &= -C_{ox} W \int_0^{L_h} \frac{L_h - y_A}{L_h} q_{i_h}(y_A) \partial y_A - C_{ox} W \int_{L_h}^{L_h+L_l} 0 \cdot \partial y_A \end{aligned} \quad (3.59)$$

$$\begin{aligned} Q_D(V_{gs}, V_{ds}) &= \int_{A \in M1} Q_{D_A} + \int_{B \in M2} Q_{D_B} \\ &= -C_{ox} W \int_0^{L_h} \frac{y_A}{L_h} q_{i_h}(y_A) \partial y_A - C_{ox} W \int_{L_h}^{L_h+L_l} q_{i_l}(y_B) \partial y_B \end{aligned} \quad (3.60)$$

Using (3.4)-(3.9), (3.59) and (3.60) can be rewritten as

$$Q_S(V_{gs}, V_{ds}) = Q_{S_h}(V_{gs}, V_{ds}) \quad (3.61)$$

$$Q_D(V_{gs}, V_{ds}) = Q_{D_h}(V_{gs}, V_{ds}) + Q_{S_l}(V_{gs}, V_{ds}) + Q_{D_l}(V_{gs}, V_{ds}) \quad (3.62)$$

3.4.2 Source and Drain Charges for $V_{gs} > V_{gc}$

The inversion charge induced after V_{gs} goes beyond V_{gc} at the points A and B can be expressed as

$$\begin{aligned} &Q_A(V_{gs}, V_{ds}) - Q_A(V_{gc}, V_{ds}) \\ &= Q_{S_A}(V_{gs}, V_{ds}) + Q_{D_A}(V_{gs}, V_{ds}) \\ &\quad - (Q_{S_A}(V_{gc}, V_{ds}) + Q_{D_A}(V_{gc}, V_{ds})) \\ &= -C_{ox} \Delta y \cdot W (q_{i_h}(y_A, V_{gs}) - q_{i_h}(y_A, V_{gc})) \end{aligned} \quad (3.63)$$

$$\begin{aligned}
 & Q_B(V_{gs}, V_{ds}) - Q_B(V_{gc}, V_{ds}) \\
 &= Q_{S_B}(V_{gs}, V_{ds}) + Q_{D_B}(V_{gs}, V_{ds}) \\
 & - (Q_{S_B}(V_{gc}, V_{ds}) + Q_{D_B}(V_{gc}, V_{ds})) \\
 &= -C_{ox}\Delta y \cdot W (q_{i_l}(y_B, V_{gs}) - q_{i_l}(y_B, V_{gc}))
 \end{aligned} \tag{3.64}$$

In the following section, Q_S will be first derived and followed by Q_D . Based on (3.49)-(3.50) and (3.63)-(3.64), Q_{S_A} and Q_{S_B} can be obtained as

$$\begin{aligned}
 & Q_{S_A}(V_{gs}, V_{ds}) - Q_{S_A}(V_{gc}, V_{ds}) \\
 &= -C_{ox}\Delta y \cdot W \frac{L_h - y_A + L_l \cdot pp}{L_h + L_l \cdot pp} (q_{i_h}(y_A, V_{gs}) - q_{i_h}(y_A, V_{gc}))
 \end{aligned} \tag{3.65}$$

$$\begin{aligned}
 & Q_{S_B}(V_{gs}, V_{ds}) - Q_{S_B}(V_{gc}, V_{ds}) \\
 &= -C_{ox}\Delta y \cdot W \frac{L_h + L_l - y_B}{L_l + L_h / pp} (q_{i_l}(y_B, V_{gs}) - q_{i_l}(y_B, V_{gc}))
 \end{aligned} \tag{3.66}$$

According to (3.53), we have

$$\begin{aligned}
 & Q_S(V_{gs}, V_{ds}) - Q_S(V_{gc}, V_{ds}) \\
 &= \int_{A \in M1} Q_{S_A}(V_{gs}) + \int_{B \in M2} Q_{S_B}(V_{gs}) - \left(\int_{A \in M1} Q_{S_A}(V_{gc}) + \int_{B \in M2} Q_{S_B}(V_{gc}) \right) \\
 &= \int_{A \in M1} (Q_{S_A}(V_{gs}) - Q_{S_A}(V_{gc})) + \int_{A \in M2} (Q_{S_B}(V_{gs}) - Q_{S_B}(V_{gc}))
 \end{aligned} \tag{3.67}$$

Substituting (3.65) into the first integral of (3.67) and using (3.4)-(3.9) to simplify the integral, the first integral of (3.67) can be evaluated as

$$\begin{aligned}
& \int_{A \in M1} (Q_{S_A}(V_{gs}) - Q_{S_A}(V_{gc})) \\
&= -C_{ox} W \int_0^{L_h} \frac{L_h - y_A + L_l \cdot pp}{L_h + L_l \cdot pp} (q_{i_h}(y_A, V_{gs}) - q_{i_h}(y_A, V_{gc})) \partial y_A \\
&= -C_{ox} W \int_0^{L_h} \frac{L_l \cdot pp}{L_h + L_l \cdot pp} (q_{i_h}(y_A, V_{gs}) - q_{i_h}(y_A, V_{gc})) \partial y_A \\
&\quad - C_{ox} W \int_0^{L_h} \frac{L_h - y_A}{L_h + L_l \cdot pp} (q_{i_h}(y_A, V_{gs}) - q_{i_h}(y_A, V_{gc})) \partial y_A \\
&= -C_{ox} W \frac{L_l \cdot pp}{L_h + L_l \cdot pp} \int_0^{L_h} (q_{i_h}(y_A, V_{gs}) - q_{i_h}(y_A, V_{gc})) \partial y_A \\
&\quad - C_{ox} W \frac{L_h}{L_h + L_l \cdot pp} \int_0^{L_h} \frac{L_h - y_A}{L_h} (q_{i_h}(y_A, V_{gs}) - q_{i_h}(y_A, V_{gc})) \partial y_A \\
&= \frac{L_l \cdot pp}{L_h + L_l \cdot pp} (Q_{S_h}(V_{gs}, V_{ds}) + Q_{D_h}(V_{gs}, V_{ds}) - Q_{S_h}(V_{gc}, V_{ds}) - Q_{D_h}(V_{gc}, V_{ds})) \quad (3.68) \\
&\quad + \frac{L_h}{L_h + L_l \cdot pp} (Q_{S_h}(V_{gs}, V_{ds}) - Q_{S_h}(V_{gc}, V_{ds}))
\end{aligned}$$

Substituting (3.66) into the second integral of (3.67) and using (3.4)-(3.9) to simplify the integral, the second integral of (3.67) becomes

$$\begin{aligned}
& \int_{A \in M2} (Q_{S_B}(V_{gs}) - Q_{S_B}(V_{gc})) \\
&= -C_{ox} W \int_{L_h}^{L_h+L_l} \frac{L_h + L_l - y_B}{L_l + L_h / pp} (q_{i_l}(y_B, V_{gs}) - q_{i_l}(y_B, V_{gc})) \partial y_B \quad (3.69) \\
&= -C_{ox} W \frac{L_l}{L_l + L_h / pp} \int_{L_h}^{L_h+L_l} \frac{L_l - (y_B - L_h)}{L_l} (q_{i_l}(y_B, V_{gs}) - q_{i_l}(y_B, V_{gc})) \partial y_B \\
&= \frac{L_l}{L_l + L_h / pp} (Q_{S_l}(V_{gs}, V_{ds}) - Q_{S_l}(V_{gc}, V_{ds}))
\end{aligned}$$

According to (3.61), $Q_S(V_{gc}, V_{ds})$ can be expressed as

$$Q_S(V_{gc}, V_{ds}) = Q_{S_h}(V_{gc}, V_{ds}) \quad (3.70)$$

Substituting (3.68)-(3.70) in (3.67), (3.67) is rearranged as

$$\begin{aligned}
& Q_S(V_{gs}, V_{ds}) = Q_{S_h}(V_{gs}, V_{ds}) \\
&+ \frac{L_l}{L_h / pp + L_l} \left\{ [Q_{D_h}(V_{gs}, V_{ds}) - Q_{D_h}(V_{gc}, V_{ds})] \right. \quad (3.71) \\
&\left. + [Q_{S_l}(V_{gs}, V_{ds}) - Q_{S_l}(V_{gc}, V_{ds})] \right\}
\end{aligned}$$

Therefore, Q_S for $V_{gs} > V_{gc}$ is obtained in (3.71).

Next, Q_D will be derived. Based on (3.49)-(3.50) and (3.63)-(3.64), Q_{D_A} and Q_{D_B} can be obtained as

$$\begin{aligned} & Q_{D_A}(V_{gs}, V_{ds}) - Q_{D_A}(V_{gc}, V_{ds}) \\ &= -C_{ox} \Delta y \cdot W \frac{y_A}{L_h + L_l \cdot pp} (q_{i_h}(y_A, V_{gs}) - q_{i_h}(y_A, V_{gc})) \end{aligned} \quad (3.72)$$

$$\begin{aligned} & Q_{D_B}(V_{gs}, V_{ds}) - Q_{D_B}(V_{gc}, V_{ds}) \\ &= -C_{ox} \Delta y \cdot W \frac{(1/pp - 1)L_h + y_B}{L_l + L_h/pp} (q_{i_l}(y_B, V_{gs}) - q_{i_l}(y_B, V_{gc})) \end{aligned} \quad (3.73)$$

According to (3.54), we have

$$\begin{aligned} & Q_D(V_{gs}, V_{ds}) - Q_D(V_{gc}, V_{ds}) \\ &= \int_{A \in M1} (Q_{D_A}(V_{gs}) - Q_{D_A}(V_{gc})) + \int_{A \in M2} (Q_{D_B}(V_{gs}) - Q_{D_B}(V_{gc})) \end{aligned} \quad (3.74)$$

Substituting (3.72) into the first integral of (3.74) and using (3.4)-(3.9) to simplify the integral, the first integral of (3.74) becomes

$$\begin{aligned} & \int_{A \in M1} (Q_{D_A}(V_{gs}) - Q_{D_A}(V_{gc})) \\ &= -C_{ox} W \int_0^{L_h} \frac{y_A}{L_h + L_l \cdot pp} (q_{i_h}(y_A, V_{gs}) - q_{i_h}(y_A, V_{gc})) \partial y_A \\ &= \frac{-C_{ox} W L_h}{L_h + L_l \cdot pp} \int_0^{L_h} \frac{y_A}{L_h} (q_{i_h}(y_A, V_{gs}) - q_{i_h}(y_A, V_{gc})) \partial y_A \\ &= \frac{L_h}{L_h + L_l \cdot pp} (Q_{D_h}(V_{gs}, V_{ds}) - Q_{D_h}(V_{gc}, V_{ds})) \end{aligned} \quad (3.75)$$

Substituting (3.73) into the second integral of (3.74) and using (3.4)-(3.9) to simplify the integral, the second integral of (3.74) can be evaluated as

$$\begin{aligned}
 & \int_{A \in M2} (Q_{D-B}(V_{gs}) - Q_{D-B}(V_{gc})) \\
 &= -C_{ox} W \int_{L_h}^{L_l+L_h} \frac{(1/pp-1)L_h + y_B}{L_l + L_h/pp} (q_{i-l}(y_B, V_{gs}) - q_{i-l}(y_B, V_{gc})) dy_B \\
 &= \frac{-C_{ox} L_l W}{L_l + L_h/pp} \int_{L_h}^{L_l+L_h} \frac{L_h/pp + (y_B - L_h)}{L_l} (q_{i-l}(y_B, V_{gs}) - q_{i-l}(y_B, V_{gc})) dy_B \\
 &= \frac{L_h/pp}{L_l + L_h/pp} (Q_{D-l}(V_{gs}, V_{ds}) + Q_{S-l}(V_{gs}, V_{ds}) - Q_{D-l}(V_{gc}, V_{ds}) - Q_{S-l}(V_{gc}, V_{ds})) \\
 &+ \frac{L_l}{L_l + L_h/pp} (Q_{D-l}(V_{gs}, V_{ds}) - Q_{D-l}(V_{gc}, V_{ds}))
 \end{aligned} \tag{3.76}$$

According to (3.62), $Q_D(V_{gc}, V_{ds})$ can be expressed as

$$Q_D(V_{gc}, V_{ds}) = Q_{D-h}(V_{gc}, V_{ds}) + Q_{S-l}(V_{gc}, V_{ds}) + Q_{D-l}(V_{gc}, V_{ds}) \tag{3.77}$$

Using (3.75)-(3.77), (3.74) is rearranged as

$$\begin{aligned}
 Q_D(V_{gs}, V_{ds}) &= Q_{D-l}(V_{gs}, V_{ds}) + Q_{S-l}(V_{gs}, V_{ds}) + Q_{D-h}(V_{gs}, V_{ds}) \\
 &- \frac{L_l}{L_h/pp + L_l} \left\{ [Q_{D-h}(V_{gs}, V_{ds}) - Q_{D-h}(V_{gc}, V_{ds})] \right. \\
 &\left. + [Q_{S-l}(V_{gs}, V_{ds}) - Q_{S-l}(V_{gc}, V_{ds})] \right\}
 \end{aligned} \tag{3.78}$$

Therefore, Q_D is for $V_{gs} > V_{gc}$ is obtained in (3.78). Finally, Q_S and Q_D can be evaluated for all biases with (3.61)-(3.62), (3.71), and (3.78). A plot of the drain-gate capacitance using the regional drain charge model [(3.71) and (3.78)] (lines) is shown in Figure 3.3, compared with TCAD data (symbols).

Figure 3.3 a plot of $|C_{dg}|$ derived from regional drain charged model (Eq (3.62) and (3.78)) and the unified drain charged model (Eq (3.80)) against the numerical $|C_{dg}|$ from TCAD, for the device with M1 doping $5 \times 10^{18} \text{ cm}^{-3}$ and M2 doping $1 \times 10^{15} \text{ cm}^{-3}$, $t_{ox}=5\text{nm}$ and $L_h:L_l=1:10$, $L_h+L_l=2\mu\text{m}$ at $V_{ds}=1.5\text{V}$. The plotted capacitances are normalized to $C_{ox}W(L_h+L_l)$.

3.5 Unified Source and Drain Charge Model

Now, Q_S and Q_D can be evaluated for all biases. However, (3.61) and (3.62) are valid when $V_{gs} \leq V_{gc}$, while (3.71) and (3.78) are valid when $V_{gs} > V_{gc}$. It is preferred to have a continuous model for circuit simulation. Notice that (3.61) & (3.71), or (3.62) & (3.78), have common terms and that $Q_{D_h}(V_{gs}, V_{ds})$ and $Q_{S_l}(V_{gs}, V_{ds})$ are increasing functions of V_{gs} . Therefore, (3.61) & (3.71), or (3.62) & (3.78), can be unified into one single expression as

$$Q_S(V_{gs}, V_{ds}) = Q_{S_h}(V_{gs}, V_{ds}) + \Delta Q \quad (3.79)$$

$$Q_D(V_{gs}, V_{ds}) = Q_{D_h}(V_{gs}, V_{ds}) + Q_{S_l}(V_{gs}, V_{ds}) + Q_{D_l}(V_{gs}, V_{ds}) - \Delta Q \quad (3.80)$$

$$\Delta Q = (\Delta Q_{D_h} + \Delta Q_{S_l}) \cdot \frac{L_l}{L_h / pp + L_l} \quad (3.81)$$

$$\Delta Q_{D_h} = \eta \left[Q_{D_h}(V_{gs}, V_{ds}) - Q_{D_h}(V_{gc}, V_{ds}), d_6 \right] \quad (3.82)$$

$$\Delta Q_{S_l} = \eta \left[Q_{S_l}(V_{gs}, V_{ds}) - Q_{S_l}(V_{gc}, V_{ds}), d_7 \right] \quad (3.83)$$

$$\eta(x, d) = 0.5 \cdot \left(x + \sqrt{x^2 + 4d} \right) \quad (3.84)$$

in which d_6 and d_7 are smoothing parameters. $\eta(x)$ equals zero when $x < 0$ and equals x when $x > 0$. As $Q_{D_h}(V_{gs}, V_{ds})$ and $Q_{S_l}(V_{gs}, V_{ds})$ are increasing functions of V_{gs} , when $V_{gs} \leq V_{gc}$, $Q_{D_h}(V_{gs}, V_{ds}) - Q_{D_h}(V_{gc}, V_{ds})$ and $Q_{S_l}(V_{gs}, V_{ds}) - Q_{S_l}(V_{gc}, V_{ds})$ are negative. Then, ΔQ_{D_h} and ΔQ_{S_l} both equal 0; so does ΔQ , and (3.79) automatically becomes (3.61). When $V_{gs} > V_{gc}$, ΔQ_{D_h} and ΔQ_{S_l} equal $Q_{D_h}(V_{gs}, V_{ds}) - Q_{D_h}(V_{gc}, V_{ds})$ and $Q_{S_l}(V_{gs}, V_{ds}) - Q_{S_l}(V_{gc}, V_{ds})$, respectively, as they are positive, and (3.79) becomes (3.71). It is similar for (3.80). Thus, (3.79) and (3.80) are the unified expressions for the source and drain charges. In Figure 3.3, it can be seen that the unified model (3.80) is able to produce the correct capacitance for all biases.

3.6 Source and Drain Charge Model Refinement

In section 3.3.2, K is assumed to be constant ($=pp=1$). However, when $V_{gs} > V_{gc}$, the value of K changes with the bias (V_{gs} , V_{ds}). So, this assumption will cause some inaccuracy in the final charge model. To remedy this inaccuracy, an empirical expression ' p_{em} ' is proposed to replace ' pp ' in the Q_s and Q_D expressions; or, exactly in (3.81). The empirical expression p_{em} is given by

$$p_{em} = \frac{q_{i_h}(V_i) + \Delta q_h}{q_{i_l}(V_i) + \Delta q_l} \quad (3.85)$$

where Δq_h and Δq_l are fitting parameters. The normalized inversion charge density, q_{i_h} and q_{i_l} , can be expressed in different compact forms [56, 70, 71]. For simplicity, the following expression is adopted in the model. It is written as

$$q_{i_x}(V_{cb}) = \frac{2 \cdot n_x \cdot V_t \cdot \ln \left[1 + \exp \left(\frac{V_{gs} - V_{thx} - A_{bx} V_{cb}}{2 \cdot n_x \cdot V_t} \right) \right]}{1 + 2 \cdot n_x \cdot c_{ox} \cdot \sqrt{\frac{4\phi_{B_x}}{q\epsilon_{si}N_{a_x}} \exp \left(-\frac{V_{gs} - V_{thx} - A_{bx} V_{cb} - 2 \cdot V_{off_x}}{2 \cdot n_x \cdot V_t} \right)}} \quad (3.86)$$

in which ' x ' in the subscript stands for ' h ' or ' l '. Parameters subscript with ' h ' or ' l ' are the device parameters for M1 or M2. The definitions of all these parameters can be found in [56]. Then, we obtain the final terminal charge model for the S-MOS—(3.20), (3.21) and (3.79)-(3.80). Figure 3.4 shows the comparison between the numerical C_{dg} data and the model with $pp=1$ and $pp=p_{em}$. Although the model with $pp=1$ has reproduced the main C_{dg} behavior,

it doesn't fit the data well as expected. The model with $pp=p_{em}$ instead is able to follow the numerical data more closely.

Figure 3.4 comparison of the $|C_{dg}|$ derived from the model with $pp=1$ and $pp=p_{em}$ against the numerical simulation for V_{gs} variation at $V_{ds}=1.5\text{V}$. The plotted capacitances are normalized to $C_{ox}W(L_h+L_l)$.

3.7 Results and Discussion

The difference between the new partition method and the WD scheme lies in the term ‘ K ’ ($=q_{i_h}(V_i)/q_{i_l}(V_i)$). In a uniformly doped MOSFET, K is always equal to 1; then through the analysis from section 3.3 to 3.4, we will get back to the WD scheme. However, in the context of lateral nonuniform doping, K becomes bias dependent, especially when saturation occurs inside the channel that electrically divides the physical channel into two linear

channels (channels that operate in linear region). So, the WD scheme is not applicable to lateral nonuniformly doped channel.

Note that the final Q_S and Q_D expressions in (3.79)-(3.80) are consistent with (3.22), although they are formulated separately from (3.22). The terminal charges of M1 and M2 are formulated based on the WD scheme, which obeys the charge conservation principle. Thus, we have $Q_{G_h}+Q_{D_h}+Q_{S_h}+Q_{B_h}=0$ and $Q_{G_l}+Q_{D_l}+Q_{S_l}+Q_{B_l}=0$. Then, for the S-MOS, we will also have $Q_G+Q_D+Q_S+Q_B=0$, which means the charge model derived here also obeys the charge conservation principle. In order to verify the charge model, the S-MOS with different M1 and M2 doping or channel length is simulated using Medici, in which the capacitance of the device is calculated using ac small-signal analysis [72]. The numerical capacitance data are then used to compare with the capacitance derived from the proposed charge model. The total channel length of the S-MOS is always $2\mu\text{m}$ unless otherwise stated.

Figure 3.5 Comparison of the capacitances: (a): C_{gg} (b) $|C_{gd}|$ (c) $|C_{dg}|$ derived from the charge model with the numerical data for V_{gs} variation at $V_{ds}=0.5$ to $2.5V$ in step of $0.5V$.

The plotted capacitances are normalized to $C_{ox}W(L_h+L_l)$.

The transistor capacitances can be calculated according to the following equation

$$C_{ij} = \begin{cases} -\frac{\partial Q_i}{\partial V_j} & i \neq j & i, j = g, d, s, b \\ \frac{\partial Q_i}{\partial V_j} & i = j & i = j = g, d, s, b \end{cases} \quad (3.87)$$

in which g, d, s, b stands for gate, drain, source, and bulk. Figure 3.5 shows the normalized C_{gg} , C_{dg} and C_{gd} of the S-MOS, in which the doping of M1 and M2 are $5 \times 10^{18} \text{ cm}^{-3}$ and $1 \times 10^{15} \text{ cm}^{-3}$, respectively, and the channel length ratio is $L_h:L_l=1:10$. Although the S-MOS adopts a simplified doping structure and the drift region is not included in the simulation, the capacitance of the S-MOS does possess the basic characteristic behavior of a typical LDMOS capacitance [8, 10, 13-16]: 1) the rising edge of C_{gg} shifts to the right with increased V_{ds} ; 2) peaks in the capacitances; These behaviors are qualitatively explained below.

It's relatively easy to understand the shift of C_{gg} rising edge. The rising edge of C_{gg} can be attributed to the onset of strong inversion of M2 or M1. As M2's threshold voltage (V_{thl}) is smaller than M1's (V_{thh}), M2 will enter strong inversion before M1 when M1 is still in the subthreshold region on the condition that V_{ds} is small. In this case, most of the voltage drops across M1 and V_i will be approximately equal to V_{ds} . Meanwhile, the condition for M2 to be strongly inverted is $V_{gs}-V_i > V_{thl}$. So, the condition for M2 being set in strong inversion region before M1 is approximately given by $V_{gs}-V_{ds} > V_{thl}$, that is, $V_{gs} > V_{ds} + V_{thl}$. For a larger V_{ds} , a larger V_{gs} is needed to strongly invert M2. Once M2 becomes strongly inverted, C_{gg} rises. This is why the C_{gg} rising edge

shifts to the right with increased V_{ds} . However, if V_{ds} is too large that $V_{ds}+V_{thl}>V_{thh}$, M2 will not enter strong inversion before M1, as M1 enters strong inversion as soon as $V_{gs}=V_{thh}$, which will also cause C_{gg} to rise and the rising edge is no longer V_{ds} dependent. This is why in Figure 3.5 (a), the C_{gg} rising edge shifts to the right for $V_{ds}=0.5\sim 2V$, but for $V_{ds}=2.5$ it doesn't shift to the right any more.

Regarding the peak in C_{gg} , actually, there may or may not be peaks in the C_{gg} of LDMOS seen from the published literatures [13, 16]. To investigate the C_{gg} behavior, C_{gg} of the S-MOS is plotted against V_{gs} in Figure 3.6 (a) along with the internal voltage solution, V_i for two S-MOS configurations. One device has M1 and M2's doping combination as 5×10^{18} and $1\times 10^{15} \text{ cm}^{-3}$, the channel length ratio as $l_h:l_l=1:10$. The other device has M1 and M2's doping combination as 5×10^{16} and $1\times 10^{15} \text{ cm}^{-3}$, the channel length ratio as $l_h:l_l=1:10$. Generally, V_i first rises, then declines and eventually becomes relatively flat. The decline of V_i is due to M1's conductivity being drastically increased during the stage when M1 enters strong inversion from subthreshold region as V_{gs} increases. Although V_i is the voltage at only one point in the channel, it actually reflects the behavior of part of the channel voltage, $V_{cb}(y)$ —for some range of y , $V_{cb}(y)$ will also decline and become flat. The formation of the peak in C_{gg} is related to the decline of V_i or $V_{cb}(y)$. If the decline of V_i or $V_{cb}(y)$ stops before C_{gg} rises, the peak will not be formed as shown in Figure 3.6 (a) by the triangles. But if the decline of V_i or $V_{cb}(y)$ continues after C_{gg} rises, the peak will be formed, as the circles in Figure 3.6(a) indicate. This is because when C_{gg} rises it means M1 or M2's channel, or both

channels, are strongly inverted. For the strongly inverted channel, the channel surface potential, $\phi_s(y)$, will follow $V_{cb}(y) - \partial\phi_s(y)/\partial V_{gs} \approx \partial V_{cb}(y)/\partial V_{gs}$. On the other hand, the normalized C_{gg} is related to $\phi_s(y)$ by

Figure 3.6 (a): Left axis: comparison of normalized C_{gg} for different devices, in which one shows the peak and the other does not. Right axis: comparison of the internal voltage, V_i for the two devices. **(b):** normalized $|C_{dg}|$ of the S-MOS. The plotted capacitances are normalized to $C_{ox}W(L_h+L_l)$.

$$C_{gg} \propto \frac{\partial(V_{gs} - \phi_s(y))}{\partial V_{gs}} = 1 - \frac{\partial\phi_s(y)}{\partial V_{gs}} \quad (3.88)$$

Thus, if $\partial\phi(y)/\partial V_{gs} \approx \partial V_{cb}(y)/\partial V_{gs} < 0$, the normalized C_{gg} will go above 1. When $V_{cb}(y)$ becomes flat, $\partial V_{cb}(y)/\partial V_{gs} \approx 0$, C_{gg} falls back to 1. This is the formation of the peak in C_{gg} . However, before C_{gg} rises, both M1 and M2 are not fully strongly inverted, and $\partial\phi_s(y)/\partial V_{gs} \neq \partial V_{cb}(y)/\partial V_{gs}$; hence, no peak is formed. This can be understood in a simpler way if the gate charge is expressed in an intuitive form as $Q_g = C_{ox}WL_h(V_{gs} - V_i - V_{th}) + C_{ox}WL_l(V_{gs} - V_i - V_{tl})$, then the normalized C_{gg} can be calculated as $C_{gg} = 1 - \partial V_i / \partial V_{gs}$. Thus, C_{gg} will go above 1 if $\partial V_i / \partial V_{gs}$ is negative and fall back to 1 when $\partial V_i / \partial V_{gs} \approx 0$, as shown by the circles in Figure 3.6(a). However, if the V_{gs} range, for which $\partial V_i / \partial V_{gs}$ is negative, is relatively small, no peak is formed, as shown by the triangles in Figure 3.6(a). For the peak in C_{dg} , not only the decline of V_i or $V_{cb}(y)$ but the term ΔQ in (3.79) and (3.80) is also responsible. Figure 3.6(b) shows the comparison between the numerical data and the model results calculated with or without ΔQ . It can be seen that without ΔQ , C_{dg} calculated from the model decreases a bit and with ΔQ , it decreases even more.

On the whole, from Figure 3.5(a)-(c), the model has matched the numerical data fairly well. The peaks of C_{gg} and C_{dg} as well as the shift in the C_{gg} rising edge are well captured by the model.

To further verify the model, S-MOS with different M1 and M2 doping combination and L_h/L_l ratio are simulated and its capacitances are extracted for comparison with the model in Figure 3.7 and Figure 3.8. In Figure 3.7, it can be seen that M1's doping has a strong effect on the location of the peak in C_{gg}

or C_{dg} . This can be understood as the peak forms when M1 enters strong inversion from subthreshold region, which is related to M1's threshold voltage that is doping dependent. Different L_h/L_l ratio, however, only affect the peaks' magnitudes, as shown in Figure 3.8. The model's behavior for different t_{ox} is shown in Figure 3.9. Overall, the model is able to capture the effects due to different doping or L_h/L_l ratio and has matched the numerical data very well.

Figure 3.7 plots of (a): C_{gg} (b) $|C_{gd}|$ (c) $|C_{dg}|$ for different channel doping combination along with the numerical data for V_{gs} variation at $V_{ds}=1.5V$. The plotted capacitances are normalized to $C_{ox}W(L_h+L_l)$.

Figure 3.8 plots of (a): C_{gg} (b) $|C_{gd}|$ (c) $|C_{dg}|$, derived from the charge model for different L_h/L_l ratio along with numerical data for V_{gs} variation at $V_{ds} = 1.5V$. The plotted capacitances are normalized to $C_{ox}W(L_h + L_l)$.

Figure 3.9 plots of (a) C_{gg} and (b) $|C_{gd}|$, derived from the charge model for $L_h/L_l=1:10$ along with numerical data for V_{gs} variation at $V_{ds}=1V$ for different t_{ox} . The plotted capacitances are normalized to $C_{ox}W(L_h+L_l)$.

3.8 Chapter 3 summary

As the WD partition scheme is not applicable in the presence of lateral nonuniformity, which may produce peaks above 1 in the normalized capacitance, a new physical charge partition method is proposed in this chapter. The partition method is derived based on the step doping profile.

Based on the new method, terminal source/drain charges are formulated regionally for $V_{gs} \leq V_{gc}$ and $V_{gs} > V_{gc}$. The regional source/drain charges are unified into one single-piece expression that is valid for all bias with the use of the smoothing function. Then the charges model is used to calculate the trans-capacitances, which are compared to the numerical simulation data. It is shown that the peaky capacitances are well reproduced by the charge model.

Chapter 4 MISHEMT Compact Drain Current Model

4.1 Introduction

HEMT and MISHEMT are heterojunction based field effect transistors whose operation relies on the conduction of the 2DEG accumulated near the heterojunction interface. The detailed device can be referred to [73-77]. The typical device structures of the HEMT and MISHEMT are shown in Figure 4.1.

Figure 4.1 the cross sections of the HEMT (on the left) and the MISHEMT (on the right).

Generally, the structure of a HEMT under the gate consists of barrier, spacer, and the **un-intentionally-doped** (UID) body/channel layers. Metal gate and the barrier form a Schottky junction, which enables the gate to control the channel. The MISHEMT is an altered version of HEMT. An insulator layer is inserted between the gate and the semiconductor. A cap layer can be added between

the insulator and the barrier to suppress the current collapse effect [18]. The HEMT is a hot device for RF designs because it has high mobility, high power density and high breakdown voltage [22, 23]. The MISHEMT has the advantages over a normal HEMT. At the same time, it has improved gate leakage performance compared to the normal HEMT. This makes it an excellent device candidate for high frequency applications. To enable accurate circuit simulation of MISHEMT based circuits, a good compact model that physically describes the transistor behavior in all operation regions is highly demanded.

However, there is no available compact model for MISHEMT. Though the currently available HEMT models can be easily extended to describe MISHEMT, most of the available HEMT models are formulated based on the semi-empirical n_s functions. Models based on the semi-empirical expressions do not have the correct physical correlation to the device parameters, hence, they are poor in providing physical insights for further study and process engineering improvement. In [30, 31], n_s is solved physically, but considering only the lowest subband, i.e., it is not taken into the account the electrons occupied in the second subband in the triangle quantum well of the hetero interface. In fact, electron occupation in the two lowest subbands has been observed across different heterojunction systems, including AlGaAs/GaAs and AlGaN/GaN [77-79], and most of the electrons are in the two lowest subbands while there are much less electrons above the third subband. So, the effect of third and above subbands can be safely ignored, but the second subband is needed to be included. Besides the n_s expression, the subthreshold behavior of

a compact model is also very important for accurate circuit simulation, yet, most of the present models do not consider this region.

In this chapter, based on the unified regional modeling approach, a single-piece explicit and physical approximate solution for n_s is derived. The derived n_s expression considers the effect of subband splitting and is valid over a wide range of gate bias. With this n_s expression, a physics-based and symmetric drain-current compact model for the MISHEMT is formulated. The model is continuous from the subthreshold region to the active region. It takes into consideration the mobility degradation effect and self-heating effect. It is then validated with the measured AlGaIn/GaN MISHEMT data, as shown in the end of this chapter.

4.2 2DEG density Model

The 2DEG lies at the heart of MISHEMT's operation. In order to calculate the current-voltage characteristic of the device, it is important to obtain the relationship between n_s and the external gate bias V_{gs} , and drain bias V_{ds} . This can be started by analyzing the energy band (EB) diagram of the MISHEMT. The EB diagram under the gate of a typical MISHEMT is shown in Figure 4.2.

Figure 4.2 Energy band diagram of a typical MISHEMT

If the insulator and the cap layer are removed, the EB in Figure 4.2 is automatically reduced to the EB for HEMT. The MISHEMT EB is used for analysis in this work. n_s can be calculated by assuming that the 2DEG only occupies the two lowest subbands in the triangular quantum well at the heterojunction interface [77]. The self-consistent solution of the Schrödinger's and Poisson's equations gives n_s as

$$n_s = DV_{tm} \left[\ln(1 + \exp(\frac{E_f - E_0}{V_{tm}})) + \ln(1 + \exp(\frac{E_f - E_1}{V_{tm}})) \right] \quad (4.1)$$

where

$$E_0 = \gamma_0 n_s^{2/3} \quad (4.2)$$

$$E_1 = \gamma_1 n_s^{2/3} \quad (4.3)$$

in which D is the density of states, V_m is the thermal voltage, E_f is the Fermi level referenced to the bottom point of the triangular quantum well, and γ_0 and γ_1 are the physical parameters related to the ground and first excited energy levels, respectively, which are usually used as fitting parameters.

4.2.1 Charge Control Equation

(4.1), (4.2) and (4.3) relate n_s to the Fermi level, E_f . Actually, E_f is dependent on V_g , V_d and V_s . By analyzing the EB diagram in Figure 4.2, the relationship between E_f and the external bias can be expressed as

$$V_g - V_c - \phi_{MB} + \phi_{SB} + V_{ox} + V_{cap} + V_{ba} - E_f = 0 \quad (4.4)$$

where ϕ_{MB} is the difference between gate Fermi level and the insulator conduction band edge, ϕ_{SB} is the difference between the insulator conduction band edge and the cap layer conduction band edge, V_{ox} , V_{cap} , and V_{ba} are the voltage drops at the insulator layer, cap layer, and barrier layer, respectively, and V_c is channel voltage, which is equal to V_s at the source and V_d at the drain side. In (4.4), ϕ_{MB} and ϕ_{SB} are known physical constant but V_{ox} , V_{cap} , and V_{ba} are unknown. They can be obtained by solving the Poisson equation in the barrier region. If the hetero interface between the spacer and UID body is chosen as the origin, as shown in Figure 4.2, the doping profile, $N(x)$ in the barrier and spacer layer can be expressed as

$$N(x) = \begin{cases} 0 & -e < x < 0 \\ N_b & -d < x < -e \end{cases} \quad (4.5)$$

where e is the spacer layer thickness, d is the thickness of the barrier and spacer layer, and N_b is the doping in the barrier layer. Taking the spacer's conduction band edge at $x=0^-$ as the voltage reference and denoting the electric field at $x=0^-$ as ξ_{is} , the Poisson equation and the boundary equations for the barrier and the spacer are written as

$$\frac{\partial^2 V(x)}{\partial x^2} = -\frac{q}{\varepsilon_b} N(x) \quad (4.6)$$

$$V(0^-) = 0 \quad (4.7)$$

$$\xi(0^-) = -\left. \frac{\partial V(x)}{\partial x} \right|_{x=0^-} = \xi_{is} \quad (4.8)$$

in which ε_b is the permittivity of the barrier layer. If the barrier layer is fully ionized and the electron charge in the barrier and spacer layer is negligible compared to the space charge, then, substituting (4.5) into (4.6), integrating (4.6) once with respect to x from 0^- to $-x_0$ and using (4.8) for the e-field boundary conditions, we have

$$\left. \frac{\partial V(x)}{\partial x} \right|_{x=-x_0} = \begin{cases} \frac{q}{\varepsilon_b} N_b \cdot (x_0 - e) - \xi_{is} & -x_0 < -e \\ -\xi_{is} & -x_0 \geq -e \end{cases} \quad (4.9)$$

where x_0 is an integration dummy variable. Integrating (4.9) with respect to $-x_0$ from 0^- to $-d$ and using (4.7) as the voltage boundary condition, we have

$$V_{ba} = V(-d) = \frac{qN_b}{2\varepsilon_b} (d - e)^2 - \xi_{is} d \quad (4.10)$$

Applying the Gauss law to the Gaussian box (the dotted rectangle) in Figure 4.2 which encloses the insulator/cap interface and the cap/barrier interface, and using (4.9) to calculate the electric field at the interface of the barrier layer ($x=-d^+$), we have

$$\frac{\varepsilon_{ox}}{\varepsilon_b} \frac{V_{ox}}{t_{ox}} = -\xi(-d) = \frac{qN_b}{\varepsilon_b} (d - e) - \xi_{is} \quad (4.11)$$

in which ε_{ox} is the permittivity of the insulator and t_{ox} is the insulator thickness.

Applying the Gauss law at the insulator/cap interface, we have

$$\frac{\varepsilon_{ox}}{\varepsilon_c} \frac{V_{ox}}{t_{ox}} = \frac{V_{cap}}{t_c} \quad (4.12)$$

where ε_c is the permittivity and t_c is the thickness of the cap layer. Applying the Gauss law from $x=0^+$ to $x=\infty$, the relationship between the e-field, ξ_{is} and the 2DEG density, n_s can be obtained as

$$-\varepsilon_b \xi_{is} = -qn_s \quad (4.13)$$

Combining (4.4), (4.10), (4.11), (4.12), and (4.13) and eliminating ξ_{is} , V_{ox} , V_{cap} , and V_{ba} , the relationship between V_g , V_c , E_f , and n_s , which is also referred to as the voltage control equation, can be deduced to be

$$n_s = \frac{C_d}{q} (V_g - E_f - V_c - V_{off}) \quad (4.14)$$

where

$$C_d = \frac{\varepsilon_b}{d \left(1 + \frac{t_c}{\varepsilon_c} \frac{\varepsilon_b}{d} + \frac{\varepsilon_b t_{ox}}{d \varepsilon_{ox}} \right)} \quad (4.15)$$

$$V_{off} = \phi_{MB} - \phi_{SB} - \left(\frac{t_c}{\epsilon_c} + \frac{t_{ox}}{\epsilon_{ox}} \right) \cdot qN_b(d - e) - \frac{qN_b}{2\epsilon_b}(d - e)^2 \quad (4.16)$$

4.2.2 Piecewise 2DEG Density Solution

Mathematically, based on (4.1) and (4.14), n_s and E_f can be calculated once the input voltages, V_g and V_c are known. However, due to the complicated nature of (4.1), numerical method is required to solve for E_f and n_s . In circuit simulation, compact models using numerical solutions will slow down the simulation speed. It is, therefore, desired to use the analytical expressions for compact models. But analytically solving (4.1) and (4.14) is hardly possible. Through careful study, it is found that (4.1) can be reduced to a simple form under different conditions in different regions of operation: A) the subthreshold region when $E_f < E_0$; B) the active region 1, when $E_0 < E_f < E_1$; C) the active region 2, when $E_1 < E_f$. Then, approximate solutions for E_f and n_s can still be solved regionally.

4.2.2.1 A) Subthreshold Region: $E_f < E_0$

In the subthreshold region, both $E_f - E_0$ and $E_f - E_1$ are negative. The exponential terms, $\exp((E_f - E_0)/V_{tm})$ and $\exp((E_f - E_1)/V_{tm})$ are very small. Making use of the approximation that $\log(1+x) \approx x$ when x is very small and using (4.2) and (4.3) to substitute E_0 and E_1 , (4.1) can be approximated as

$$\begin{aligned}
 n_s &\approx DV_{tm} \left(\exp\left(\frac{E_f - \gamma_0 n_s^{2/3}}{V_{tm}}\right) + \exp\left(\frac{E_f - \gamma_1 n_s^{2/3}}{V_{tm}}\right) \right) \\
 &\approx 2DV_{tm} \exp\left(\frac{E_f}{V_{tm}}\right)
 \end{aligned} \tag{4.17}$$

In (4.17), $\gamma_0 n_s^{2/3}$ and $\gamma_1 n_s^{2/3}$ are ignored as n_s is very small in this region. Combining (4.17) with (4.14), the approximate subthreshold regional Fermi level solution, E_{f_sub} can be obtained as

$$\begin{aligned}
 E_{f_sub} &= V_{go} - V_{tm} \cdot L \left\{ \frac{2qD}{C_d} e^{\frac{V_{go}}{V_{tm}}} \right\} \\
 &\approx V_{go} - \frac{2V_{tm}qD}{C_d} \exp\left(\frac{V_{go}}{V_{tm}}\right)
 \end{aligned} \tag{4.18}$$

where $V_{go} = V_g - V_{off} \sim V_c$. The regional 2DEG density solution, n_{s_sub} can be obtained by substituting (4.18) into (4.14). The exponential term in the second line of (4.18) is actually very small compared to the first term and is negligible, but it is a crucial term that ensures the correct exponential subthreshold behavior, so it is retained in the formula.

4.2.2.2 B) Active Region 1: $E_0 < E_f < E_1$

In this region, $E_f - E_0$ is positive and $E_f - E_1$ is negative. Ideally, it can be assumed that $\exp((E_f - E_0)/V_{tm}) \gg 1 \gg \exp((E_f - E_1)/V_{tm})$. Ignoring the small terms, (4.1) can be approximated as

$$\frac{n_s}{D} = E_f - \gamma_0 \cdot n_s^{2/3} \tag{4.19}$$

The term $n_s^{2/3}$ can be simplified by using Taylor expansion as

$$\begin{aligned}
 n_s^{2/3} &= \left(\frac{C_d}{q} (V_{go} - E_f) \right)^{2/3} \\
 &\approx \left(\frac{C_d}{q} V_{go} \right)^{2/3} \left(1 - \frac{2 E_f}{3 V_{go}} \right)
 \end{aligned} \tag{4.20}$$

Using (4.20) to replace the $n_s^{2/3}$, (4.19) is further approximated as

$$\frac{n_s}{D} = E_f - \gamma_0 \cdot \left(\frac{C_d}{q} V_{go} \right)^{2/3} \left(1 - \frac{2 E_f}{3 V_{go}} \right) \tag{4.21}$$

Combining (4.21) with (4.14), the regional approximate Fermi level solution, E_{f_B} can be obtained as

$$E_{f_B} = M_0 \cdot V_{go} \tag{4.22}$$

where

$$M_0 = \frac{\frac{C_d}{qD} (V_{go})^{1/3} + \gamma_0 \left(\frac{C_d}{q} \right)^{2/3}}{\left(1 + \frac{C_d}{qD} \right) (V_{go})^{1/3} + \frac{2}{3} \gamma_0 \left(\frac{C_d}{q} \right)^{2/3}} \tag{4.23}$$

The regional 2DEG density solution, n_{s_B} can be derived by substituting (4.22) into (4.14).

4.2.2.3 C) Active Region 2: $E_I < E_f$

In this region, both $E_f - E_0$ and $E_f - E_I$ are positive. Ideally, it can be assumed that $\exp((E_f - E_0)/V_{tm}), \exp((E_f - E_I)/V_{tm}) \gg 1$. (4.1) can be approximated as

$$\frac{n_s}{D} \approx E_f - \gamma_0 \cdot n_s^{2/3} + E_f - \gamma_1 \cdot n_s^{2/3} \tag{4.24}$$

Using (4.20) to replace the $n_s^{2/3}$, (4.24) is further simplified as

$$\frac{n_s}{D} \approx 2E_f - (\gamma_0 + \gamma_1) \cdot \left(\frac{C_d}{q} V_{go}\right)^{2/3} \left(1 - \frac{2}{3} \frac{E_f}{V_{go}}\right) \quad (4.25)$$

Then, similar to the active region 1, the approximate regional Fermi level solution, E_{f_C} can be obtained by solving (4.25) and (4.14)

$$E_{f_C} = M_1 \cdot V_{go} \quad (4.26)$$

where

$$M_1 = \frac{\frac{C_d}{2qD} (V_{go})^{1/3} + \frac{\gamma_1 + \gamma_0}{2} \left(\frac{C_d}{q}\right)^{2/3}}{\left(1 + \frac{C_d}{2qD}\right) (V_{go})^{1/3} + \frac{\gamma_0 + \gamma_1}{3} \left(\frac{C_d}{q}\right)^{2/3}} \quad (4.27)$$

The regional 2DEG density solution, n_{s_C} can be derived by substituting (4.26) into (4.14). Figure 4.3 shows the three regional pieces of the Fermi level solution compared to the exact numerical solution of (4.1) and (4.14). Although, it is not easy to find the boundary of different regions, it can be seen that the regional solutions fit the numerical data best in their intended regions.

Figure 4.3 different pieces of regional solutions E_{f_sub} , E_{f_B} and E_{f_C} versus gate-source voltage along with the exact numerical E_f solution

4.2.3 Unified n_s Expression

All the three pieces of solutions derived above are only valid in their intended regions. To develop a continuous compact model, it is desired to have unified E_f and n_s expressions that are valid for all biases. However, it is found that the hetero system may or may not enter the active region 2. To demonstrate this, three different sets of physical parameters listed in Table 4.1 are used to generate the numerical solutions of (4.1) and (4.14) at the source ($V_c=V_s=0$). Each set of parameters corresponds to one hetero system. The relative energy levels E_f-E_0 and E_f-E_I are plotted in Figure 4.4. As can be seen, for the parameter sets 1 and 3, E_f-E_I becomes positive as V_{gs} increases. Thus, systems 1 and 3 enter the active region 2.

Table 4.1 Three different sets of parameters from the published literature

Parameters	γ_0 ($10^{-12}\text{Vm}^{4/3}$)	γ_1 ($10^{-12}\text{Vm}^{4/3}$)	D ($10^{17}\text{m}^{-2}\text{V}^{-1}$)
Set 1	2.5	3.5	3.24
Set 2	2.12	3.73	10.01
Set 3	2.26	4	3.24

Figure 4.4 (a) $E_f - E_0$ (b) $E_f - E_1$ for three different sets of parameters

On the other hand, for the parameter set 2, $E_f - E_I$ is always negative. The corresponding hetero system never enters the active region 2. In accordance, all the three pieces of regional solutions, E_{f_sub} , E_{f_B} and E_{f_C} should be used for the systems 1 and 3. But for the system 2, only E_{f_sub} and E_{f_B} should be used. This makes it difficult to find a general unified expression for E_f as it remains a question whether E_{f_C} should be used or not without assuming the specific values of the physical parameters so as to judge whether the system enters active region 2 or not. Fortunately, after careful mathematical reasoning, it can be found out that the smaller one between E_{f_B} and E_{f_C} will definitely give the correct approximate estimation to E_f in both active regions. The physical reason is described as follows.

Figure 4.5 is an $n_s - E_f$ plane with n_s as the vertical axis and E_f as the horizontal axis. Then, each of (4.1), (4.19), and (4.24) corresponds to a curve in this plane. It can be judged that, for (4.1), (4.19), and (4.24), n_s is an increasing function of E_f , while for (4.14), n_s is a decreasing function of E_f . Without loss of generality, let the line 1 represent (4.1) and the points on the line 1 are denoted as (n_{s1}, E_{f1}) . Denote the points of (4.19) as (n_B, E_B) and the point of (4.24) as (n_C, E_C) . Assume the system is in the active region 1. Then, the curve representing (4.19) will be close to line 1, such as line 2. So, let line 2 represent (4.19). In this region, $E_{f1} - \gamma_I n_{s1}^{2/3}$ is negative. So is $E_B - \gamma_I n_B^{2/3}$. If (n_B, E_B) is substituted into (4.24), (4.24) will not be a balanced equation (both sides of (4.24) are not equal) and from the substitution result, it can be judged that if n_B and n_C are equal, E_C has to be larger than E_B so that both sides of (4.24) can be equal. Therefore, the curve representing (4.24) must be on the right side of

line 2 (same n_s , larger E_f), such as line 3. So, let line 3 represent (4.24). (4.14) can be represented by the line 4. The E_f value of the intersect points between line 4 and line 1, line 2 and line 3 are E_{fi} the exact solution of (4.1) and (4.14), E_{f_B} and E_{f_C} , the regional solutions, respectively for the same V_g . It is easily seen from Figure 4.5 that, in the active region 1, E_{f_B} is smaller than E_{f_C} if both are calculated with the same V_g .

The same logic applies if the system is in the active region 2. It can be similarly deduced that E_{f_C} is smaller than E_{f_B} for the same V_g . Thus, on the whole, the smaller one between E_{f_B} and E_{f_C} gives the correct estimation to E_{fi} in both active regions. In another word, the smaller one of E_{f_C} and E_{f_B} is the correct approximation no matter the system is in active region 1 or 2.

Figure 4.5 A conceptual plot for (1) (4) (9) and (13).

Therefore, a unified single-piece E_f solution for the active region can be obtained by selecting the smaller one of E_{f_B} and E_{f_C} with the help of the interpolation function. The unified expression is given as

$$E_{f_op} = M \cdot V_{go} \quad (4.28)$$

$$M = \mathcal{G}(M_1, M_0, \delta_{op}) \quad (4.29)$$

$$\mathcal{G}(x, y, \alpha) = 0.5 \cdot (x + y + \alpha - \sqrt{(x - y - \alpha)^2 + 4\alpha x}) \quad (4.30)$$

where δ_{op} is a smoothing parameter. The regional 2DEG density solution, n_{s_op} can be derived by substituting (4.28) into (4.14). As (4.28) is only valid in the active region, the term ' V_{go} ' in M_0 and M_1 is replaced by VOF , which is given by

$$VOF = 0.5 \cdot (V_{go} + \sqrt{V_{go}^2 + 4\delta_{of}}) \quad (4.31)$$

where δ_{of} is a smoothing parameters and, therefore, E_{f_op} is rewritten as

$$E_{f_op} = M' \cdot V_{go} \quad (4.32)$$

where

$$M' = M|_{V_{go}=VOF} \quad (4.33)$$

VOF equals V_{go} when V_{go} is positive and equals zero when V_{go} is negative.

Using VOF helps avoid unpredicted model behavior in the subthreshold region.

Finally, a single-piece E_f expression that is continuous from the subthreshold region to the active region can be obtained by joining E_{f_sub} and E_{f_op}' into one unified equation using the interpolation function as

$$dE_f = \frac{2(1 - M')V_{tm} \ln(1 + \exp(\frac{V_{go}}{2V_{tm}}))}{1 + \frac{C_d(1 - M')}{qD} \exp(-\frac{V_{go}}{2V_{tm}})} \quad (4.34)$$

$$E_f(V_c) = V_{go} - dE_f \quad (4.35)$$

When $V_{go} > 0$, (4.35) becomes (4.32) and when $V_{go} < 0$, (4.35) becomes (4.18). Finally, a single-piece expression for the 2DEG sheet density, n_s and the channel charge density normalized to C_d can be obtained by substituting $E_f(V_c)$ into (4.14).

$$n_s = \frac{C_d}{q} (V_g - E_f(V_c) - V_c - V_{off}) \quad (4.36)$$

$$V_{gt}(V_c) = \frac{qn_s}{C_d} = V_g - E_f(V_c) - V_c - V_{off} \quad (4.37)$$

4.2.4 Unified E_f and n_s Model Validation

In this section, the unified E_f and n_s models are compared to the numerical solution of (4.1) and (4.14). The numerical data are generated using the three sets of parameters listed in Table 4.1, with $\phi_{MB} = \phi_{SB} = N_b = 0$, $t_c = 2\text{nm}$, $t_{ox} = 5\text{nm}$, $e = 2\text{nm}$, and $d = 20\text{nm}$. Figure 4.6 shows the Fermi level E_f and $\partial E_f / \partial V_{gs}$ versus gate-source voltage, V_{gs} .

Figure 4.6 (a) numerical E_f data versus the gate source voltage for the different device parameters along with the model results at $T=300\text{K}$. (b) Derivative of E_f with respect to the gate source voltage at $T=300\text{K}$.

Figure 4.7 presents the 2DEG density n_s and $\partial n_s / \partial V_{gs}$ versus V_{gs} . As mentioned before, systems 1 and 3 will go through subthreshold region, active region 1 and 2 while system 2 will only go through the subthreshold region and the active region 1. It can be seen from Figure 4.6 and Figure 4.7 that the unified

model is able to cover both cases, produce the correct results and agree well to the numerical data of systems 1, 2, and 3.

Figure 4.7 numerical n_s data versus the gate source voltage for the different device parameters along with the model results at $T=300\text{K}$. (b) Derivative of n_s with respect to the gate source voltage at $T=300\text{K}$.

It is also of interest to evaluate the temperature scalability of the model. Numerical data of n_s at different temperatures, $T=300\text{K}$, 400K and 500K , are

generated for comparison with the model, which is shown in Figure 4.8. It can be seen that temperature seems have negligible effect in the active region but it strongly affects the subthreshold behavior. The higher the temperature, the larger the subthreshold swing. In the transition from subthreshold region to the active region, at around $V_{gs}=0$, there is some discrepancy between the model and the numerical data. This discrepancy arises from the interpolation function in (4.34). This interpolation function is used to join the subthreshold exponential behavior and the (approximately) linear behavior in the active region. The active region behavior and subthreshold behavior in the final unified model [i.e., (4.35) and (4.36)] are determined by the approximate solution (E_{f_sub} , E_{f_op} , n_{s_sub} , and n_{s_op}). But the transition behavior of the unified model is determined by the interpolation function itself, which is not able to follow the exact transition from the subthreshold region to the active region. In spite of this, the model is capable of fitting the overall numerical data nicely. The model's behavior for different t_{ox} is shown in Figure 4.9 and the model well reproduces the curves.

Figure 4.8 n_s versus gate-source voltage at different temperature

Figure 4.9 normalized n_s versus gate-source voltage for different t_{ox}

4.3 MISHEMT Drain Current Model

The 2DEG transport takes place in the UID body. When the transistor operates in the linear region, the drain current I_{ds} can be expressed as a function of the external bias V_g , V_d , V_s , and V_b as

$$I_{ds} = C_d \mu_{eff} \frac{W}{L} (V_{gt}(V_b) - \frac{V_d + V_s}{2} + 2V_{tm})(V_d - V_s) \quad (4.38)$$

where μ_{eff} is the effective mobility that includes all degradation effects, and W is the width of the transistor.

4.3.1 Mobility Models for MISHEMT

The electron velocity in the III-V semiconductor can be well described by the model proposed by Schwierz [80], which is given as

$$v_0 = \frac{\mu_0 E_y + \left(\frac{E_y}{E_{sat}} \right)^4}{1 + \left(\frac{E_y}{E_{sat}} \right)^4 + aa \left(\frac{E_y}{E_{sat}} \right)} \quad (4.39)$$

in which E_{sat} is the saturation electric field, μ_0 is the low field mobility and aa is a fitting parameter. However, using (4.39) is not convenient for compact modeling because of the 4th order term. Instead, a simpler velocity model is used, which is given by

$$v_0 = \frac{\mu_0 E_y}{\sqrt{1 + \delta_h \cdot \left(\frac{E_y}{E_{sat}}\right)^2}} \quad (4.40)$$

where δ_h is a fitting parameter. In fact, it is shown in [81] that (4.40) does not cause behavioral change in the calculated drain current. In accordance, combining with the approximation $E_y = V_{ds}/L$, the electron mobility in III-V semiconductor can be expressed as

$$\mu_{eff} = \frac{\mu_0}{\sqrt{1 + \delta_h \cdot \left(\frac{V_d - V_s}{E_{sat} L}\right)^2}} \quad (4.41)$$

where L is the channel length and the saturation electric field is written as

$$E_{sat} = \frac{2v_{sat}}{\mu_0} \quad (4.42)$$

and the low field mobility μ_0 is calculated using a phenomenological model, given as

$$\mu_0 = \frac{\mu_1}{1 + \frac{\mu_2}{\mu_1} \xi^{1/3} + \frac{\mu_3}{\mu_1} \xi^2} \quad (4.43)$$

in which μ_1 , μ_2 and μ_3 are the fitting parameters that can be extracted from the measured data and ξ is the average transverse electric field, given by

$$\xi = \frac{qn_s}{2\epsilon_{ch}} = \frac{C_d(V_{gt}(V_b) - \frac{V_d + V_s}{2})}{2\epsilon_{ch}} \quad (4.44)$$

where V_b is the substrate bias.

4.3.2 Effective Source/Drain Voltage

The MISHEMT I-V behavior resembles that of the conventional MOSFET. The drain current of the MISHEMT first rises with increased drain-source voltage and then saturates when the drain-source voltage difference reaches some critical value, which is usually referred to as the saturation voltage. However, (4.38) does not account for the saturation effect. In order to include the saturation effect, effective drain and source voltages, which are formulated according the symmetric modeling concept [82, 83], are introduced. The effective drain, $V_{d,eff}$ and source voltage, $V_{s,eff}$ are written by

$$V_{d,eff} = \mathcal{G}(V_{ds,sat} + V_s, V_d, \delta_{eff}) \quad (4.45)$$

$$V_{s,eff} = \mathcal{G}(V_{sd,sat} + V_d, V_s, \delta_{eff}) \quad (4.46)$$

in which δ_{eff} is a smoothing parameter, $V_{ds,sat}$ and $V_{sd,sat}$ are the saturation voltages elevated at the source and drain respectively. They can be similarly derived as [83]

$$V_{ds,sat} = \frac{V_{gt}(V_s)E_{sat,s}L}{V_{gt}(V_s) + 4V_{tm} + E_{sat,s}L} \quad (4.47)$$

$$V_{sd,sat} = \frac{V_{gt}(V_d)E_{sat,d}L}{V_{gt}(V_d) + 4V_{tm} + E_{sat,d}L} \quad (4.48)$$

where $E_{sat,s}$ and $E_{sat,d}$ are the saturation electric fields at the source and the drain, respectively. They are expressed as

$$E_{sat,x} = \frac{2v_{sat}}{\mu_{0,x}} \quad (4.49)$$

$$\mu_{0,x} = \frac{\mu_1}{1 + \frac{\mu_2}{\mu_1} \xi_x^{1/3} + \frac{\mu_3}{\mu_1} \xi_x^2} \quad (4.50)$$

$$\xi_x = \frac{C_d V_{gt}(V_x)}{2\epsilon_{ch}} \quad (4.51)$$

where the subscript ‘ x ’ stands for ‘ s ’ at the source and ‘ d ’ at the drain side, and ϵ_{ch} is the permittivity of the channel.

4.3.3 Unified Drain Current Model

With the effective drain and source voltages in (4.45) and (4.46), a unified drain current model including the saturation effect can be obtained by replacing V_d and V_s in (4.38) with $V_{d,eff}$ and $V_{s,eff}$ respectively. The unified drain current expression is written as

$$I_{ds} = C_d \mu_{neff} \cdot \frac{W}{L} (V_{gt}(V_b) - \frac{V_{d,eff} + V_{s,eff}}{2} + 2V_{tm})(V_{d,eff} - V_{s,eff}) \quad (4.52)$$

in which

$$\mu_{neff} = \mu_{neff} \Big|_{V_s=V_{s,eff}, V_d=V_{d,eff}} \quad (4.53)$$

4.3.4 The Effect of Interface Traps on the Subthreshold Slope

Studies [84, 85] show that interface traps have significant effects on the subthreshold slope behavior of the heterojunction based transistors. The effect of interface traps can be accounted for by modifying (4.34) as

$$dE_f = \frac{2n_{sth}(1-M')V_{tm} \ln(1 + \exp(\frac{V_{go}}{2n_{sth}V_{tm}}))}{1 + \frac{n_{sth}C_d(1-M')}{qD} \exp(-\frac{V_{go}}{2n_{sth}V_{tm}})} \quad (4.54)$$

where $n_{sth}=1+C_{it}/C_d$ is the subthreshold slope factor and C_{it} is the interface trap capacitance. C_{it} is used as a fitting parameter and extracted from the measurement data.

4.3.5 Self-Heating Effect

Figure 4.10 Self heating network for DC operation

As MISHEMT/HEMT devices are usually used in high power applications, self-heating effect plays an important role in the device operation. It can be modeled according to the thermal circuit concept [86], which is described in Figure 4.10. The temperature dependent mobility and saturation velocity models are given as

$$\mu_1(T_0 + \Delta T) = \mu_1(T_0) \left(\frac{T_0}{T_0 + \Delta T} \right)^2 \quad (4.55)$$

$$v_{sat}(T_0 + \Delta T) = v_{sat}(T_0) \frac{1 + 0.8 \cdot \exp(\frac{T_0}{600})}{1 + 0.8 \cdot \exp(\frac{T_0 + \Delta T}{600})} \quad (4.56)$$

where $T_0=300\text{K}$ is the room temperature. $\mu_I(T_0)$ and $v_{sat}(T_0)$ are the physical parameters extracted from the measurement data at the room temperature. The increase in temperature, ΔT is related to the thermal resistance, R_{th} , and is expressed as

$$\Delta T = R_{th} \cdot |I_{ds}(V_d - V_s)| \quad (4.57)$$

where R_{th} is the thermal resistance and used as a fitting parameter to be extracted from measurements.

4.3.6 Model Verification

In this section, the MISHEMT model is verified by running **Gummel symmetry test (GST)** and compared to the experiment data. In the GST, the source and drain terminals are to be biased at certain potential together with a varying voltage V_x , and it is required that at least the second-order derivatives of I_{ds} with respect to (w.r.t.) V_x must be smoothly passing through $V_x=0$. In addition, it is also necessary that $d^{(n)}I_{ds}(0) \equiv d^n I_{ds}(V_x)/dV_x^n|_{V_x \rightarrow 0}$ exist (i.e., no singularity at $V_x=0$) [87] in order to pass GST with n th-order derivative. A plot of GST is shown in Figure 4.11, in which it is easily seen that perfect symmetry is obtained up to third order. The model is also compared with the experiment data from two GaN/AlGaN MISHEMTs. The structures of both

devices are the same as the MISHEMT depicted in Figure 4.1. Both devices have $L_g=0.25\mu\text{m}$, $W=50\mu\text{m}$, $t_{ox}=25\text{nm}$ (Hf_2O), $t_c=2.5\text{nm}$ (GaN), $d=18\text{nm}$ (AlGaN), $N_b=0$ and $L_{gd}=0.8\mu\text{m}$. The first device has $L_{gs}=1.6\mu\text{m}$, while the second device has $L_{gs}=2.15\mu\text{m}$. The L_{gs} and L_{gd} sections are modeled as series resistors in the model (R_S for the L_{gs} section and R_D for the L_{gd} section). The data of the first device are shown in Figure 4.12 and Figure 4.13 along with the fitted model results. The data from the second device are shown in Figure 4.14 and Figure 4.15. All the structural and physical parameters in the model that can be found in the process (W , L_g , N_b ...) use the process values mentioned previously. From the fitting results, $V_{off} = -3.35\text{ V}$. Based on (4.16), if $N_b = 0$, V_{off} should be equal to $\phi_{MB} - \phi_{SB}$, which is the difference between the gate metal (Ni/Au) workfunction and the cap (GaN) layer's electron affinity. However, it can be checked [88, 89] that $\phi_{MB} - \phi_{SB}$ is a positive number. There may be two possible reasons: N_b is not perfectly zero and the traps' effect on V_{off} is not considered. The traps' effect should be included in the model in the future. Another two important parameters are the low-field mobility and saturation velocity, μ_1 and v_{sat} , with $\mu_1 = 800\text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ and $v_{sat} = 10^7\text{ cm s}^{-1}$. These values are reasonable for heterostructure-based devices. From Figure 4.12 to Figure 4.15, it can be seen that the model does not match quite well in the peak g_m , further improvement on the model is needed, but overall, good agreement is obtained in I_d - V_{ds} and I_d - V_{gs} .

Figure 4.11 GST results of the drain current model.

Figure 4.12 I_{ds} - V_{ds} model results compared to the experiment data.

Figure 4.13 (a) I_{ds} - V_{gs} model results compared to the experiment data. (b) g_m model results compared with the experiment results.

Figure 4.14 Comparison of modeled drain current with the experiment data for V_{ds} variation at different $V_{gs}=-5V$ to $3V$ in step of $1V$

Figure 4.15 Comparison of modeled drain current and transconductance with experiment data for V_{gs} variation at $V_{ds}=2.5V$

4.4 Chapter 4 summary

In this chapter, the analytical expression for V_{off} is derived for the generic MISHEMT. Next, the physical expressions for the Fermi level and the 2DEG density are obtained by solving the self-consistent solution of the Schrödinger's and Poisson's equations regionally, in which the lowest two sub-bands are considered. The regional solutions are unified into one single-piece solutions using proper smoothing and interpolate functions. The final Fermi level and 2DEG density models are compared with the exact numerical solutions. Good agreements are obtained from subthreshold region to active regions and for varying temperature. Based on the Fermi level and 2DEG density models, a surface-potential based, continuous and symmetric, drain-current model for MIS-HEMTs is formulated and is verified with the measured AlGaIn/GaN MIS-HEMT data.

Chapter 5 Conclusion and Recommendation

5.1 Conclusion

In summary, this work demonstrates the compact model formulations for the LDMOS and MISHEMT.

A subcircuit approach is proposed to model the LDMOS. Surface potentials are solved regionally. The piecewise regional surface potential solutions are combined as one single piece URSP. Based on the URSP, the p-channel model is formulated according to the URM approach. Although the p-channel is modeled as a uniform MOSFET and the lateral nonuniform doping is not considered in the model, the model results are shown to be able to follow closely the numerical simulation data, which simulates the lateral nonuniformly doped channel. Regarding the drift region, unlike most of the existing models, which describe the drift region as a resistor with empirical gate bias dependence, it is modeled as a parallel combination of an accumulation transistor and a bulk resistor, both including the lateral mobility degradation effect. The drift region model agrees with the corresponding numerical results very well. Both the p-channel model and drift channel model are coded into verilog-A and a subcircuit of p-channel and the drift channel connected in series is used to describe the I-V behavior of the LDMOS. It is shown that the subcircuit model results are in good agreement with the numerical data.

The charge model formulation for LDMOS transistors with lateral nonuniform doped channel has been presented. As the conventional WD scheme fails due to lateral nonuniformity, an alternative definition of the source/drain terminal charge is introduced. In the meanwhile, the lateral diffused doping profile is simplified to a step doping profile and the charge modeling strategy is discussed. Based on the introduced charge definition, partition ratio for small charge deviation is derived. By making some reasonable and appropriate approximations, the small signal based partition ratio is extended to the large signal regime. Regional source and drain terminal charges are then obtained based on the partition ratio. Finally, smoothing functions are used to unify the regional charge expressions. The final unified charge model is used to calculate the capacitances, which are compared with the numerical results from TCAD. It is shown that the model results are able to match the simulation results nicely and the “peaky” behaviors in capacitance are well captured.

By analyzing the energy band of the MISHEMT, a set of equations that describe the relationship between the 2DEG density, Fermi level and the input voltages are obtained. These equations are solved regionally and the piecewise Fermi level regional solutions are obtained. Through careful mathematical reasoning, the piecewise regional Fermi level solutions are unified into one single-piece expression with proper smoothing and interpolation functions. The final unified Fermi level expression is validated in all regions and independent of the actual physical device parameters. The 2DEG density can

be calculated based on the unified Fermi level expression. It is demonstrated that the unified Fermi level and 2DEG density model are able to fit the numerical solutions excellently. Next, the 2DEG density model is used to develop the MISHEMT I-V model, which includes the effects of mobility degradation, interface-trap and self-heating effects. The I-V model adopts the symmetric modeling concept [82, 83]. It shows perfect symmetry in high order derivatives. Last, the I-V model is validated by comparison with the experimental data, in which good agreement is obtained.

5.2 Recommendation

The following topics are recommended for future research work.

- i. The p-channel LDMOS model is formulated using lateral uniform doping. However, the real diffused doping profile should have an influence on the transistor I-V behaviors. As mentioned in chapter 3, saturation may occur inside the channel, which is in fact originating from the nonuniform doping. Thus, although the uniform channel I-V model is able to match the numerical results well, the effect of the lateral nonuniform doping on the I-V behavior, including the subthreshold and active regions, should be studied in order to describe the transistor more precisely. In the meantime, deeper understanding of the transistor can be gained and, thus, possibly helps improve the design of the transistor.

- ii. The substrate current model is yet to be derived for the LDMOS. Due to the lateral nonuniform doping, the substrate current may also behave differently from the uniform MOSFET.
- iii. Regarding the charge model for the p-channel, as mentioned in chapter 3, the subtle effects of the lateral nonuniform doping on the C-V behavior are not yet studied. These effects can be investigated through numerical simulation first and then added in to the SDP based charge model.
- iv. The p-channel charge model is derived by assuming quasi-static condition. For very high frequency operation, quasi-static condition will fail. Non-quasi static model is therefore needed to be developed for time domain analysis for very high frequency applications.
- v. In modeling the drain current of the MISHEMT, a simplified mobility model is adopted, which can not account for the decreased velocity and transferred electron effects in the high electric field region. It is hence desirable to study the influence of the transferred electron effect and add this effect into the model.

- vi. In this thesis, only the MISHEMT I-V relationship is modeled. The capacitance behavior is equally important. It is therefore desirable to develop the charge model for the MISHEMT.
- vii. Parameter extraction is a very important step for compact model implementation and deployment. However, the extraction procedure for all the models discussed in this thesis is not yet standardized. So, it is recommended to develop the standardized “technology based” [90] procedure for parameter extraction.
- viii. It may be interesting to study the substrate current of the MISHEMT, since the transistor is based on a different operation mechanism compared to the conventional MOSFET.
- ix. The models described in this thesis are only core models. Many secondary effects, like the short channel effects, should be included in the model so as to improve the models’ scalability.

Publication

Main-author

1. J. B. Zhang, *et al.*, "Partition Scheme and Charge Model for Transistors with Lateral Nonuniform Doping," to be submitted to *IEEE Trans. Electron Devices*.
2. J. B. Zhang, *et al.*, "A Compact MIS-HEMT Drain-Current Model Based on the Unified 2DEG Density Expression," to be submitted to *IEEE Trans. Electron Devices*.
3. J. B. Zhang and X. Zhou, "An Analytical 2DEG Model Considering the Two Lowest Subbands," *Proc. Nanotech (WCM-Nanotech2012)*, Santa Clara, CA, Jun. 2012, vol. 2, pp. 734-737.
4. J. B. Zhang, X. Zhou, G. J. Zhu, and S. H. Lin, "Charge Partition in Lateral Nonuniformly-Doped Transistor," *Proc. Nanotech (WCM-Nanotech2011)*, Boston, MA, Jun. 2011, vol. 2, pp. 784-787.

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