Single-phase and two-phase heat transfer in microchannels

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**NANYANG
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**SINGLE-PHASE AND TWO-PHASE HEAT TRANSFER
IN MICROCHANNELS**

XU BIN

SCHOOL OF MECHANICAL AND AEROSPACE

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2012

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MICROCHANNELS**

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XU BIN

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School of Mechanical and Aerospace Engineering

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Abstract

Microfluidics has attracted great attention over the last decade due to its characteristic supreme miniaturisation and low energy consumption as compared to macroscale fluid mechanics. Heat transfer is a crucial process in microfluidic systems and has a significant effect on flow behaviour. This thesis presents analytical and numerical models of single and two-phase heat transfer in microfluidic systems. Experiments were performed to validate these models.

In a single-phase flow, experiments with different configurations including T-shaped, straight and H-shaped microchannels were performed. For the T-shaped microchannel, thermal mixing was investigated. A two dimensional analytical model was presented to describe the thermal mixing process in the microchannel. Experimental investigations on thermal mixing were performed to compare with the theoretical analysis. A three-dimensional numerical simulation was developed to investigate the thermal mixing process in the T-junction zone. The thermal effect on mass mixing was further studied. The effect of power input and flow rate were studied experimentally and numerically. For the straight microchannel, a thermal flow sensor was investigated experimentally and numerically. The operation mode of a constant heater temperature was considered in the experiment. A numerical simulation of conjugate forced convection-conduction heat transfer was employed to study fluid flow and heat transfer in the thermal flow sensor. For the H-shaped microchannel, a counter flow micro-heat exchanger (CFMHE) was investigated both theoretically and experimentally. A two-dimensional analytical model was

developed to study the heat transfer processes in the CFMHE. Experiments were conducted with hot and cold fluids entering the microchannel with the same heat capacity rate. A new correlation for Nusselt number is proposed based on the experimental Reynolds number and Prandtl number. Good agreements between analytical, experimental and numerical results were obtained.

In a two-phase flow, temperature-induced droplet coalescence in a microchannel was investigated. In the experiments, a resistive heater was embedded in the microchannel to provide heating function in a rectangular merging chamber. The temperature of the chamber was controlled by the voltage applied to the heater. The merging process of two neighbouring droplets was investigated under different applied voltages and flow rates. A two-dimensional code based on the Finite Volume method (FVM) was developed. The numerical modelling plays an important role in understanding of the merging process. In addition, a heat transfer model for plug trains under constant surface temperature condition was developed. Nusselt number and heat transfer index were employed to evaluate the heat transfer characteristics of plug trains. A high conductivity ratio (k_2/k_1) results in a lower Nusselt number and a higher heat transfer index in plug-2 compared to plug-1. A low plug length ratio ($\hat{L}_2/\hat{L}_1$) is effective in enhancing heat transfer in the microchannel, which results in a lower Nusselt number and a lower heat transfer index in plug-2 compared to plug-1.

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Nomenclature

A	Heat transfer area, [m ²]
c_p	Specific heat, [J/kg · K]
D_h	Hydraulic diameter, [m]
$\bar{F}$	External Force, [N]
H	Channel Height, [m]
h	Half Channel Height, [m]
k	Thermal conductivity, [W/m · K]
L	Channel length, [m]
Le	Lewis number, [-]
P	Wetted Perimeter, [m]
p	Pressure, [Pa]
r	Liquid fraction of the cold fluid, [-]
t	time, [s]
T	Temperature, [°C]
u	Velocity along x direction, [m/s]
U	Average flow velocity, [m/s]
$\bar{V}$	Velocity vector, [m/s]
v	Velocity along y direction, [m/s]
V	Voltage, [V]
W	Channel width, [m]
w	Velocity along z direction, [m/s]
x, y, z	Coordinates in Cartesian coordinates systems, [m]

Dimensionless parameters

$Br = \mu_f u_m^2 / (q_L D_h)$	Brinkman number, [-]
$Nu = h D_h / k$	Nusselt number, [-]
$Re = \rho u D_h / \mu$	Reynolds number, [-]
$Pr = \mu c_p / k$	Prandtl number, [-]

Nomenclature

$Po = f Re$	Poiseuille number, [-]
$Pe = Re Pr$	Peclet number, [-]

Subscripts

C	Cold fluid
eff	Effective value
f	fluid
H	Hot fluid
i	Initial
in	Inlet
o	Outlet
s	Solid
W	Wall

Overscript

$-$	Depth average value
$\wedge$	Dimensionless variable

Greek

ρ	Density, [kg/m ³]
ν	Kinematic viscosity, [m ² /s]
μ	Dynamic viscosity, [Pa·s]
α	Thermal diffusivity, [m ² /s]
ϕ	Level set function, [-]
σ	Surface tension, [kg/ s ²]
η	Heat transfer index, [-]
Θ	Dimensionless temperature, [-]
Γ	Interface, [-]

Abbreviations

CCD	Coupled Charge Device
CFMHE	Counter Flow Micro Heat Exchanger
DEP	Dielectrophoresis

Nomenclature

DC	Direct Current
DI	De-ionised
EC	Electrocoalescence
EDL	Electric Double Layer
EKI	Electro-Kinetic Instability
FFT	Finite Fourier Transform
FVM	Finite Volume Method
GPC	Generalised Phase Contrast
HS	Heaviside function
ITO	Indium Tin Oxide
LIF	Laser Induced Fluorescence
MEMS	Micro Electromechanical System
PAA	Poly(acrylic acid)
PC	Personal Computer
PCR	Polymerase Chain Reaction
PDMS	Poly(dimethyl siloxane)
PMMA	Polymethylmethacrylate
RMS	Root Mean Square
ROI	Region of Interest
SEM	Scanning Electron Microscope
SIMPLER	Semi-Implicit Method for Pressure Linked Equations Revised
TBL	Thermal Boundary Layer
TEC	Thermoelectric Module

Chapter 1

Introduction

1.1 Background

Microfluidics involves investigating flow behaviour, precise control and manipulation of fluids on the microscale, which is typically less than a millimetre. Due to the characteristic miniaturisation and low energy consumption compared to macroscale fluid mechanics, microfluidics has received immense interests over the past decades [1, 2]. Microfluidics has been widely used in many application areas such as biology, chemistry and electronics.

Mixing is very important especially for reaction in biological and chemical applications. When the fluid flow is confined in a microchannel, the Reynolds number is very low due to its relatively low velocity (less than 1 mm/s) and small dimension. Hence, fluid flow in microchannel is dominated by laminar flow. Mixing can only be achieved over a long time period which does not meet the requirement of fast mixing in many reactions. Mixing efficiency needs to be improved. In single-phase flow, T-shaped or Y-shaped microchannel with embedded obstacles or external energy such as pressure and electric field is used to investigate mass mixing on the microscale. Compared to mass mixing, there is little work on thermal mixing and thermal effect on mass mixing on the microscale. Investigation of thermal mixing can give us a better understanding of energy transport on the microscale. Temperature control can be realised

based on this investigation, which is useful for the biological application. Besides, the mass diffusion coefficient is temperature-dependent. Temperature control can also be employed to improve mixing efficiency. In a two-phase flow, mixing is realised using droplet coalescence. Electric field is widely used to merge droplets in the microchannel which may be harmful for the biological application. On the microscale, surface tension becomes dominant due the large surface-to-volume ratio. Temperature is an effective factor in affecting the surface tension due to the fact that surface tension is highly temperature dependent.

Cooling is another important process in the microelectronics application. With the development of microfluidics, electronics are miniaturised and integrated into a microchip. The heat flux in such a microchip becomes very large due to the higher packaging density of microelectronics. Liquid cooling is needed in such case. In a single-phase flow, micro heat exchanger is needed to remove the high heat flux. The heat transfer characteristics of such a micro heat exchanger need to be investigated. Due to the accuracy of temperature measurement induced by the thermocouple on the microscale, there is large diversity of heat transfer characteristics for micro heat exchangers. In a two-phase flow, droplets can potentially be used to remove heat on the microscale. The recirculating flow pattern in the droplet can enhance heat transfer significantly.

1.2 Objective and Scope

The transport phenomena in single and two-phase microfluidics play an important role in the applications of continuous-flow and droplet-based microfluidics. The objective of the present study is to study single and two-phase heat transfer in microfluidic systems without phase change. The scopes of this study are as follows:

- To study and characterise the thermal mixing and mass mixing processes in a T-shaped microchannel.
- To predict fluid flow velocity in thermal flow sensor based on the Laser Induced Fluorescence (LIF) technique.
- To systematically investigate the heat transfer processes in an H-shaped counter flow micro heat exchanger (CFMHE).
- To provide a new design to enable coalescence of droplets using thermal effect without phase change.
- To study the effects of the recirculating flow on heat transfer in plug trains without phase change.

1.3 Outline of the Thesis

There are nine chapters in this thesis. A brief introduction is presented in Chapter 1. Chapter 2 reviews the development of microfluidic mixing, thermal flow sensors, heat transfer in microchannels and droplet coalescence. Chapter 3 presents the modelling and experimental results of thermal mixing in a T-shaped microchannel. The experimental and numerical results of thermal effect on mass mixing are presented in Chapter 4. Chapter 5 presents the modelling

and experimental results of thermal flow sensor. The experimental and analytical studies of micro heat exchanger are presented in Chapter 6. Chapter 7 describes the experimental and numerical studies on temperature-induced droplet coalescence. Chapter 8 presents a numerical investigation of the heat transfer characteristics of plug trains based on an analytical two dimensional flow field. The thesis is concluded in Chapter 9 with suggestions for future works.

Chapter 2

Literature Review

This chapter presents the literature review on the different topics of the thesis, including mixing, thermal flow sensors, heat transfer and droplet coalescence in microchannels.

2.1 Microfluidic Mixing

Microfluidic mixing can be categorised as passive mixing and active mixing [3]. Passive micro-mixing only relies on diffusion or chaotic advection. According to the manipulation scheme of mixed fluids in microchannels, passive micro-mixing can be further categorised by the types of lamination, injection and chaotic advection. Active micro-mixing needs external energy such as pressure, temperature, electro-hydrodynamics, electro-kinetics, magneto-hydrodynamics and acoustics to generate disturbances in the microchannel to improve mixing efficiency. Active micromixers need an additional experimental setup in order to generate the external disturbances. As a result, the fabrication process of an active micromixer is complicated and time consumable. By contrast, the integration of a passive micromixer into a microfluidic system is easier and cheaper.

2.1.1 Passive micromixers

2.1.1.1 Lamination micromixers

Lamination micromixers can be categorised as parallel and serial lamination micromixers. The principle of lamination is to decrease the mixing path and increase the contacting area of the mixed fluids. Parallel lamination micromixers divide the inlet streams into many branches and then recombine them into one stream. The basic designs for the parallel micromixers are the T-shaped and Y-shaped flow configurations with two inlets as shown in Figs. 2.1(a) and 2.1(b) [4, 5]. This configuration is widely used for the investigation of basic transport phenomena on the microscale. Figure 2.1(c) is a schematic illustration of a concept of a parallel lamination mixer which realises mixing by introducing multiple streams into the microchannel [6]. Hydraulic focusing [7] is another type of parallel lamination micromixers. This configuration has a long microchannel with three inlets as shown in Fig. 2.1(d). The solute stream enters the middle inlet as the sample flow, while the solvent stream enters the two side inlets as the sheath flow. The mixing path is inherently decreased as the width of the solute stream is squeezed by the two side sheath flows. Serial lamination micromixers are a combination of many parallel lamination micromixers as shown in Fig. 2.2. The solute and solvent streams experience different combinations of dividing and combining stages in the microchannel [8-11].

Fig. 2.1 Parallel lamination micromixer [3].
 (a) T-mixer. (b) Y-mixer. (c) Parallel lamination mixer. (d) Hydraulic focusing.

Fig. 2.2 Serial lamination micromixer [3].
 (a) Combine-divide-combine. (b) Divide-combine.
 (c) Divide-divide-combine. (d) Multiple intersecting.

2.1.1.2 Injection micromixers

In a typical injection micromixer [12-15], a number of micro-nozzles are placed on the top of the micromixers as shown in Fig. 2.3. The solute stream is injected into the solvent stream through the micro-nozzles. The contacting area between solute and solvent streams is increased and the mixing path is reduced

due to the micro-nozzles. As a result, the mixing efficiency can be further improved.

Fig. 2.3 Schematic illustration of a injection micromixer [12].

2.1.1.3 Chaotic advection micromixers

For passive micromixer, a chaotic advection micromixer makes use of special channel geometries to induce chaotic advection in the microchannel to enhance mixing efficiency. Based on the concept, researchers have developed many channel geometries to realise chaotic advection. Embedded obstacles are employed to enhance mixing efficiency as shown in Fig. 2.4. Without the embedded obstacles, the solute and the solvent stream flow coaxially in the microchannel. With the embedded obstacles, a transverse velocity component will be induced to enhance mixing efficiency. Many investigations have been addressed to the arrangement of obstacles in the microchannel and the corresponding mixing efficiency in the microchannel [16-21].

Fig. 2.4 Schematic illustration of chaotic advection micromixers with embedded barriers [16].

An alternative method to generate chaotic advection in microchannels is to employ zigzag microchannels [22-24] or serpentine structures [25-30]. A representative of a serpentine micromixer is shown in Fig. 2.5 [30]. With the help of zigzag and serpentine structures, vortices will form at the corner of the micromixer, which will greatly enhance the mixing efficiency.

Fig. 2.5 A representative of serpentine micromixer [30].

Besides, researchers often employ twisted channels to induce flow oscillation as shown in Fig. 2.6 [31-33] to enhance mixing efficiency. A third degree freedom

offered to the fluid in the twisted microchannel enable the occurrence of chaotic advection in the microchannel. Mixing efficiency can be improved. For the twisted micromixer, the twisted microchannel is difficult to fabricate. Most of the research only remained on the numerical investigation stage.

Fig. 2.6 Schematic illustration of (a) T-mixer, (b) inclined mixer, (c) oblique mixer and (d) wavelike mixer [31].

2.1.2 Active micromixers

2.1.2.1 Pressure field perturbation

The pressure field perturbation micromixer is one of the earliest active micromixers. At the early stage, pressure field perturbation was induced by an integrated micro-pump in the microchannel [34]. At the later stage, pressure field perturbation was induced by a lateral pulsing velocity [35-37]. A typical pressure field perturbation micromixer consists of a main channel and two side channels as shown in Fig. 2.7(a), where liquid A (solute) and liquid B (solvent)

flow coaxially into the main channel. At the same time, fluids with periodic perturbation are driven into the side channels, which will affect the mixing process in the main channel. As a result, chaotic advection is induced to enhance the mixing efficiency.

Fig. 2.7 Model of a pressure perturbation micromixer.
(a) Experimental mixing result with one pair of side channel.
(b) Schematic illustration of micromixer with multiple side channels [35].

2.1.2.2 Acoustic actuation

Acoustic actuation is also an effective way to enhance mixing performance [38-41]. In a typical acoustic actuation micromixer, a piezoelectric disk is employed to provide acoustic actuation. Yang *et al.* [39] investigated an ultrasonic micromixer based on ultrasonic actuation with a high perturbation frequency. Complete mixing can be achieved with the help of ultrasonic actuation. Liu *et al.* [38] developed an acoustic micromixer to achieve complete mixing. In their

experiment, additional air bubbles were imported and trapped on the top surface of the mixing chamber. With the activation of an integrated piezoelectric disk, the air bubbles trapped on the top surface of the mixing chamber would be activated to induce transverse velocity component in the microchannel to enhance mixing. Compared to Yang *et al.*'s work, the frequency was much lower and the temperature rise could be omitted. Wang *et al.* [42] presented a technique to enhance mixing efficiency through the acoustically induced bubbles in a microchannel as shown in Fig. 2.8. Bubbles were found to be generated through the acoustic actuation when the applied frequency ranged from 1.0 to 5.0 kHz. The generated bubbles disturbed the flow field in the microchannel and enhanced mixing efficiency significantly.

Fig. 2.8 Schematic illustration of an acoustic micromixer.
 (a) Side view. (b) Top view. (c) Experimental demonstration of the acoustic micromixer [42].

2.1.2.3 Electric field actuation

Electric field actuation is also widely used in an active micromixer. Electric field actuation can be further categorised as electro-hydrodynamic actuation [43-45], dielectrophoretic actuation [46, 47] and electro-kinetic actuation. The layout of the electro-hydrodynamic actuation micromixer is similar to the pressure field perturbation micromixer as shown in Fig. 2.9. Mixing streams with different electric properties are introduced into a long main channel. Electrodes are placed perpendicularly to the main channel. Complete mixing can be achieved within a very short time by applying a desirable voltage and frequency on the electrodes. A dielectrophoretic actuation micromixer employs the force between a non-uniform electric field and polarised particles to induce a chaotic motion of the polarised particles. Electro-kinetic actuation micromixers often employ electro-kinetic instability (EKI) as an active way to enhance mixing. A schematic illustration of EKI is shown in Fig. 2.10. In such a configuration, two mixing streams with different electric conductivity flow coaxially into the microchannel. An electric field is applied along the flow direction in the microchannel. If the applied voltage is high enough, the flow instability will be observed along the interface of the two streams. This phenomena was first reported by Oddy *et al.* [48]. Since then, many researchers have focused on this type of micromixer to enhance mixing efficiency [49-55].

Fig. 2.9 Schematic illustration of a electro-hydrodynamic micromixer [43].

Fig. 2.10 Schematic illustration of electro-kinetic instability (EKI) [56].

2.1.2.4 Magneto hydrodynamic actuation

Magneto hydrodynamic actuation has also been employed to enhance mixing in the microscale [57, 58]. A combination of magnetic field and electric field will result in the generation of a Lorentz force which will agitate the mixing streams in the microchannel to enhance mixing. A schematic illustration of the micromixer is shown in Fig. 2.11 [59].

Fig. 2.11 Schematic illustration of a magneto-hydrodynamic micromixer.
 (a) Schematic diagram. (b) Current applied to the micromixer. (c) Mixing channel [59].

2.1.2.5 Thermal actuation

Mao *et al.* [60] designed a microfluidic device where hot and cold fluids flowed coaxially along the two outside walls to generate a linear temperature gradient

across the microchannel as shown in Fig. 2.12. This configuration could potentially be used for mixing in the microchannel as the mass diffusion coefficient is temperature-dependent. Tsai and Lin [61] developed a micromixer integrated with the generation of thermal bubbles to enhance mixing.

Fig. 2.12 Schematic illustration of microfluidics with linear temperature gradient across the microchannel [60].

2.2 Calorimetric Thermal Flow Sensor

A thermal flow sensor consists of a heater and temperature sensors. According to the working principle, the thermal flow sensors can be categorised as hot-film thermal flow sensor, calorimetric thermal flow sensor and time of flight thermal flow sensor [62]. The hot-film thermal flow sensor measures the amount of heat carried away by the fluid flow in the channel. The calorimetric thermal flow sensor detects the asymmetric temperature profile with respect to different fluid velocities. The thermal flow sensor detects the heat pulse generated from the heater. The flow velocity can be decided by the time

between the pulse and the response of the temperature sensor. The calorimetric thermal flow sensor had been demonstrated to be effective in the control of fluid flow on the microscale [63]. The calorimetric concept uses a miniaturised heat source and two temperature sensors located symmetrically on both sides of the heat source. The flow velocity can be calibrated by measuring the temperature difference (ΔT) between two positions located upstream and the downstream of the heat source. Its working principle is to measure the heat transfer from the heat source to the surrounding fluid, which is carried away by convection. The rate of heat transfer is proportional to the mass flow rate in the microchannel. A schematic illustration of the working principle of a thermal flow sensor is shown in Fig. 2.13, where two temperature sensing elements are located at both sides of a central heater.

Fig. 2.13 Schematic illustration of a thermal flow sensor [64].

Sabaté *et al.* [65, 66] designed a thermal flow sensor, which consisted of a heater and six temperature sensing element located symmetrically on both sides of the heater. The sensing substrate was made of silicon due to the relatively better thermal isolation property as compared to other materials. With the help of six temperature sensing elements, the flow range which could be detected by the thermal flow sensor became wider.

Fürjes *et al.* [64] studied a calorimetric thermal flow sensor which could be used to detect the fluid flow direction and velocity. The sensing substrate was made of porous silicon sacrificial layer. This thermal flow sensor consisted of a heater located at the intersection of two perpendicular microchannels and four temperature sensors arranged symmetrically around the central heater as shown in Fig. 2.14. An optimum channel design based on the experimental and numerical simulation results was proposed.

Fig. 2.14 SEM view of a two-dimensional thermal flow sensor [64].

Nguyen [67] proposed a novel thermal flow sensor for the detection of fluid flow direction and velocity. The proposed thermal flow sensor consisted of a cylindrical heater and an array of temperature sensing element arranged symmetrically around the central cylindrical heater as shown in Fig. 2.15. An average-temperature method was proposed for the evaluation of fluid flow direction and velocity based on the detection of temperature distribution around the heater.

Fig. 2.15 Schematic illustration of the proposed thermal flow sensor by Nguyen [67].

Shin and Besser [68] investigated a thermal flow sensor which was integrated in a micro-chemical reactor. Experiment and numerical simulations were performed to study the characteristics of the thermal flow sensor. The heating power of the heating resistor, the resistance of the sensing resistor and the distance between the heating resistor and the sensing resistor were found to play an important role in the design of a thermal flow sensor.

Kaltsas *et al.* [69] designed a novel system for the measurement of fluid flow direction and fluid velocity. This system consisted of an integrated thermal flow sensor and an electronic control circuit as shown in Fig. 2.16. This electronic circuit could manage the power applied to the heater and process the output signals of the thermopiles. This system could be implemented under three different modes: constant voltage, constant power and constant temperature.

Fig. 2.16 Top view of the integrated thermal flow sensor [69].

Dijkstra *et al.* [70] fabricated a calorimetric thermal flow sensor employing a versatile surface channel technological concept as shown in Fig. 2.17. In this concept, a freestanding surface microchannel was attached under the substrate surface. On the other hand, the thermal flow sensor structure could be attached on the top of the substrate surface. The performance of the sensor was tested experimentally and numerically. This concept allowed for the planar integration of sensor structures in close proximity to the fluid, while chemical-resistant fluidic connections could be made directly on top of the microchannel, without introducing large dead-volumes.

Fig. 2.17 Schematic illustration of the miniaturised thermal flow sensor [70].

Meng *et al.* [71] developed a biocompatible parylene thermal flow sensor. In this sensing device, a biocompatible parylene C membrane embedded with thin film platinum sensing elements was deposited on the microchannel to improve thermal isolation as shown in Fig. 2.18. In order to realise the usage of the thermal flow sensor in bio-microfluidics, electronic biasing conditions were employed. Two different operation modes (constant current and constant temperature) were investigated using three different sensing principles. This layout provided an efficient packaging scheme for making both electrical and fluidic connections.

Fig. 2.18 Structure of the sensing element [71].

Shen *et al.* [72, 73] designed a two-dimensional thermal flow sensor for the detection of wind flow direction and velocity as shown in Fig. 2.19. The thermal flow sensor consisted of a cross-type heater and four sensing elements arranged symmetrically around the heater. Based on this configuration, wind flow direction and velocity could be obtained through the two orthogonal temperature differences.

Fig. 2.19 Schematic illustration of the wind flow sensor [72].

Dijkstra *et al.* [70] tested a U-shaped thermal flow sensor as shown in Fig. 2.20. The U-shaped microchannel was embedded in the substrate. A gradient heater was deposited on top of the microchannel with two balancing heaters arranged symmetrically on both sides of the microchannel. When the heater was activated, a temperature gradient would occur. With the fluid flow, the U-shaped microchannel would be heated asymmetrically. One side of the U-shaped microchannel was cooled due to the incoming fluid flow, while another side was heated due to the coming hot fluid. The thermopile was employed to display the temperature differences between the two sides of the microchannel. The sensor could be deemed as a calorimetric thermal flow sensor.

Fig. 2.20 Schematic illustration of the U-shaped thermal flow sensor [70].

Petropoulos *et al.* [74-76] investigated a thermal flow sensor as shown in Fig. 2.21. Different from the previous thermal flow sensors, this sensor used polymer (SU8) as the sensing substrate. The use of SU8 as the substrate could provide better thermal isolation due to the low thermal conductivity.

Fig. 2.21 SEM image of the thermal flow sensor [74].

Talić *et al.* [77] reported a thermal flow sensor which assembled the advantage of the calorimetric and hot-film principles as shown in Fig. 2.22. Four germanium thermistors were employed as heater and temperature sensing element at the same time. These four thermistors were connected to form a Wheatstone bridge. The fluid flow direction and fluid velocity were obtained

based on the analysis of voltage of the bridge and the total dissipated power.

This sensor offered a wider measurement range as compared to other sensors.

Fig. 2.22 Experimental thermal flow sensor chip [77].

2.3 Heat Transfer in Rectangular Microchannels

Peng *et al.* [78] investigated fluid flow and heat transfer characteristics of water flowing in microchannels ($50 < Re < 4000$). The hydraulic diameter ranged from 133 to 367 μm , while the aspect ratio ranged from 0.333 to 1. The flow transition regime was found to be in the range of $200 < Re < 700$. The Nusselt number was found to be proportional to $Re^{0.62}$ in the laminar heat transfer regime. The channel geometry also played an important role on heat transfer in microchannels. Wang *et al.* [79] studied forced-convection heat transfer characteristics of fluid flow in microchannels. Water was used as the working fluid. The experimental Nusselt number in the turbulent regime ($Re > 3000$) could be predicted by a modified Dittus-Boelter equation where the empirical constant coefficient was modified from 0.023 to 0.00805. As a result, the experimental Nusselt number was lower than the prediction of the conventional

theory. Peng *et al.* [80] studied the effect of aspect ratio of microchannel on the Nusselt numbers in both laminar and turbulent regimes. The Nusselt number was correlated with microchannel dimensions. The Nusselt number was found to be lower than the conventional values. Peng *et al.* [81-83] investigated the effect of Reynolds number ($200 < Re < 1500$), Prandtl number, microchannel aspect ratio and microchannel dimensions on the heat transfer characteristics in microchannels. The variation of water property with temperature was taken into consideration. The heat transfer characteristics of the laminar heat transfer regimes were found to be strongly affected by the liquid temperature, liquid velocity and microchannel size.

Cuta *et al.* [84] studied the forced convection heat transfer characteristics in a microchannel heat exchanger ($100 < Re < 570$) which consisted of an array of 54 parallel microchannels. Refrigerant 124 was used as the working fluid. For single-phase flow in the micro-heat exchanger, the Nusselt number was found to be proportional to the Reynolds number. For two-phase flow in the micro-heat exchanger, the Nusselt number was found to be maintained at a constant value regardless of the Reynolds number. The Nusselt number in the microchannel was much larger than the corresponding conventional value. Ravigururajan and Drost [85] investigated the heat transfer characteristics of single-phase flow in a parallel microchannel heat exchanger ($220 < Re < 1250$). Their experimental results showed that the Nusselt number was 2 to 8 times larger than the corresponding conventional value. The pressure drop was also much larger than the conventional value.

Harms *et al.* [86] investigated the heat transfer characteristics of water flowing in a deep rectangular microchannel ($400 < Re < 1900$). The microchannel had a depth of $1030\ \mu\text{m}$ and a width of $251\ \mu\text{m}$. The critical Reynolds number was found to be 1500 which was much lower than the conventional value on the macroscale. The experimental results compared well with the theoretical results which indicated that the channel depth could influence heat transfer. Harms *et al.* [87] further reported the convective heat transfer characteristics in a deep rectangular microchannel was similar to the previously used geometry ($173 < Re < 12900$). A single channel system and a multiple channel system were tested to investigate heat transfer characteristics. The local Nusselt numbers were consistent with the prediction of conventional theory.

Mala *et al.* [88] investigated heat transfer and fluid flow in a microchannel using numerical simulation ($10 < Re < 500$). The effect of electrical double layer (EDL) on fluid flow and heat transfer characteristics in a microchannel between two parallel plates was taken into consideration. The heat transfer performance was worse due to the existence of the EDL in the microchannel.

Rahman [89] studied the heat transfer characteristics of water flowing in microchannel heat sinks ($200 < Re < 3300$). Two different channel patterns (I-channel and U-channel) were tested to investigate the effect of fluid flow and channel structure on the heat transfer performance of the heat sinks. The experimental local and average Nusselt numbers were higher than the conventional value on the corresponding macroscale.

Tso and Mahulikar [90] investigated the effect of viscous dissipation in terms of the Brinkman number on the convective heat transfer characteristics in rectangular microchannels ($10 < Re < 50$). An unusual behaviour was found based on the experimental results: Nu decreasing with increasing Re in the laminar regime. The Brinkman number was found to be responsible for this unusual relationship between Nusselt number and Reynolds number. A new correlation including Brinkman number was proposed. The authors concluded that there was no fully developed region in microchannels due to the axial variation of Brinkman number.

Jiang *et al.* [91] studied the heat transfer performance of fluid flow in a microchannel heat exchanger and a micro-porous heat exchanger ($60 < Re < 3000$). Numerical simulation was used to investigate the effect of channel dimensions on the heat transfer performance in the microchannel. A deep microchannel was fabricated based on the prediction of numerical results. The heat transfer performance of a micro-porous heat exchanger was better than that of the microchannel heat exchanger. Hence, the deep microchannel design was suggested in designing a micro heat exchanger.

Gao *et al.* [92] conducted experiments to study the scaling effect on the heat transfer performance in microchannels ($100 < Re < 8000$). Different channel heights, ranging from 0.1 mm to 1 mm, were tested. The author reported that for channel height which was larger than 0.4 mm, the experimental Nusselt number matched well with the conventional theory. For channel height which

was less than 0.4 mm, the experimental Nusselt number showed a large difference as compared to the conventional theory.

Kang *et al.* [93] fabricated a micro heat exchanger out of silicon ($80 < Re < 820$). Aluminium was used as the conduction layer in the micro heat exchanger. The experimental results showed a similar trend as compared to the conventional values.

Qu and Mudawar [94] investigated pressure drop and heat transfer of fluid flow in a single-phase microchannel heat sink through experiment and numerical simulation ($139 < Re < 1672$). The measured pressure drop and temperature distribution in the heat sink matched well with the numerical simulation results, which demonstrated the feasibility of using conventional equations for predicting the fluid flow and heat transfer on the microscale.

Lee *et al.* [95, 96] conducted experiments to investigate the heat transfer behaviour in microchannels ($300 < Re < 3500$). The experimental Nusselt number was used to compare with the theoretical Nusselt number predicted by the macroscale's correlation. The comparison between experimental and theoretical Nusselt numbers showed good agreement, which demonstrated the feasibility of using the conventional theory to predict the heat transfer behaviour on the microscale.

Bavière *et al.* [97] conducted experiments to study fluid flow and heat transfer characteristics in a rectangular microchannel ($200 < Re < 8000$). Bias effect in

the solid and fluid interface due to the temperature measurement using thermocouple was found to be responsible for the decrease of Nusselt number on the microscale.

Park and Punch [98] investigated the friction factor and heat transfer of fluid flow in microchannels with uniform flow distribution which was realised by a manifold configuration. The friction factor was found to be consistent with the conventional theory, while the Nusselt number was in contradiction. An empirical correlation was proposed based on the experiment results.

Jung and Kwak [99] proposed an empirical correlation to predict heat transfer behaviour in microchannels. The temperature-dependent fluid properties were taken into consideration. The proposed correlation provides a better understanding towards heat transfer characteristics in microchannels.

Jiang *et al.* [100] experimentally investigated fluid flow and heat transfer in a rectangular microchannel ($300 < Re < 6000$). The friction factor was 20% - 30% of the conventional value. At a low flow rate, the Nusselt number was lower than the conventional value. As a high flow rate, the Nusselt number was larger as compared to the conventional value.

Mokrani *et al.* [101] investigated the convective heat transfer characteristics in microchannels with a high aspect ratio which was equal to 10. The inner wall temperature was obtained from the outside wall temperature based on an

inverse heat conduction method. The heat transfer behaviour was consistent with the classic theory ($100 < Re < 5000$).

Dang *et al.* [102] investigated the heat transfer characteristics of fluid flow in a counter flow microchannel heat exchanger. The effectiveness of the micro heat exchanger and the effect of gravity on the microscale were investigated experimentally and numerically. It was found that the gravity effect was negligible for fluid flow in the microchannel.

Mohammed *et al.* [103, 104] conducted numerical simulations to study the heat transfer behaviour in a rectangular microchannel heat sink. DI-water with alumina nano-particles was used as the working fluid. The presence of nano-particles in DI-water were found to have enhanced heat transfer performance in the microchannel heat sink effectively.

2.4 Droplet Coalescence in Microfluidic Systems

In general, droplet coalescence techniques can be categorised as passive and active types. Active droplet coalescence technique employs energy generated by an external field to realise droplet fusion process. This can be achieved by applying electric, magnetic and temperature fields. Passive droplet coalescence technique only rely on the structure of microchannel [105] and the surface properties [106]. In this review, passive and active droplet coalescence techniques will be introduced. The basic theory and mechanism will also be discussed.

2.4.1 Passive droplet coalescence

2.4.1.1 Flow trapping

Droplet coalescence can be achieved by removing the continuous phase which separates a pair of droplets. Based on this concept, a passive trapping technique was proposed by Chokkalingam *et al.* [107, 108] to realise the fusion between two droplets. The simplest way is to introduce a fluid resistance element to trap and to slow down the droplet in the microchannel. There is a chance that the two neighbouring droplets would get close to each other due to the drainage of the continuous phase in the fluid resistance element. This process includes four steps: trapping, stopping of trapped droplet, fusion of two droplets and release of the fused droplets. Three different types of trap were demonstrated to show the fusion process, which includes an obstacle inside a channel, a locally enlarged diameter of fluid channel and a local asymmetry in the cross-section shape as shown in Fig. 2.23. The total flow rate must match with the fluid resistance element. Droplet fusion can also be achieved at channel junctions as shown in Fig. 2.24. The principle is similar to the fluid traps used in a microchannel. This design requires synchronisation of two subsequent droplets when they arrive at the Y-junction.

Fig. 2.23 Schematic illustration of fluid traps in a microchannel. Cross sections are shown in the right.

(A) Wall step. (B) Enlargement of cross section. (C) Elliptic cross section [107].

Fig. 2.24 Schematic illustration of fusion of droplets at a Y-junction [107].

Similar to the above principle, three different channel geometries including a rectangular expansion, a tapered expansion and a flow rectifying design were investigated by Tan *et al.* [105] as shown in Fig. 2.25. The rectangular expansion design enables the deceleration of the droplet in the rectangular expansion. The drawback of this design is that it only works for a limited range of total flow rates and flow rate ratios of dispersed phase and continuous phase which is determined by the length and the width of the expansion. Undesired multiple droplet coalescence and droplet break up would occur if the total flow rates and flow rate ratios do not match with the rectangular expansion. The

tapered expansion can be deemed as a series of rectangular expansions allowing a wider range of total flow rates and flow rate ratios. The droplet in the tapered expansion decelerates to a much slower speed due to the increasing drainage of the continuous phase. Due to the change in the flow field along the channel, subsequent droplets come closer to each other. As a result, multiple droplet coalescence occurs.

Among the three designs, the rectification design was demonstrated to be a more effective design for the fusion of subsequent droplets. Compared to the rectangular and tapered expansions, the rectification design can drain the continuous phase at a controllable rate. In the rectification design, equal amounts of the continuous phase are removed to the upper and lower channels with identical fluid resistance without the generation of a net force along the vertical axis. The droplet fusion process can be achieved by controlling the flow rate of the continuous phase.

Bremond *et al.* [109] studied the droplet fusion characteristics in a microchannel with a rectangular expansion. In the experiments, when two droplets encountered each other in the rectangular expansion, the droplet fusion process did not occur. The fusion process was found to occur at the separation stage when the former droplet began to leave the expansion. The formation of two face bulging portions significantly increased the surface area which led to droplet fusion.

Fig. 2.25 Three different channel geometries.

(a) Coalescence in channel with rectangular expansion. (b) Coalescence in channel with tapered expansion. (c) Coalescence in channel with rectification design [105].

Following the work of Bremond *et al.* [109], a multifunctional and highly efficient microfluidic device was proposed as shown in Fig. 2.26 [110]. This device consisted of three inlets, a double T-junction and a tapered merging chamber. Alternating droplet generation could be achieved within the double T-junction. Push-pull was the mechanism to generate the alternating droplet. Water from inlet 2 was pushed by the droplet from inlet 1. The increasing pressure pushed water from inlet 2 into the pinch junction, thus generating the

next droplet. These droplets were carried by the continuous phase to the tapered merging chamber. In the tapered merging chamber, there was a chance that these droplets would approach each other and resulted in continuous drainage of the oil film between them. The reduction of the channel width at the exit increased the flow pressure and evacuated the oil film. Droplet fusion could be achieved at the exit junction.

Fig. 2.26 Schematic illustration of the designed microchannel [110].

A droplet merging device with pillars was proposed to control the distance between subsequent droplets [111]. A schematic illustration of this device is shown in Fig. 2.27. Rows of pillars separated by a certain distance which were smaller than the droplet dimension were employed as fluid resistance element to assist droplet fusion. The merging mechanism utilised the difference between the hydrodynamic resistance of the continuous phase and the surface tension of the dispersed phase through the use of pillars incorporated in the microchannel. When a droplet entered the merging chamber, the fluid resistance elements (pillar) would slow down the movement of the droplet. This provided time for the subsequent droplet to arrive for merging. When the hydraulic pressure was

larger than the surface tension of the fused droplet, the droplet would move out of the merging chamber. This merging device could fuse two or multiple droplets through adjusting the mass flow rate and the volume ratio between the droplets and the merging chamber.

Fig. 2.27 Schematic illustration of pillar-induced droplet fusion device [111].

2.4.1.2 Surface modification

Beside the flow trapping method, another passive droplet fusion method was reported based on the surface energy pattern on the walls of a microfluidic device as shown in Fig. 2.28 [106]. In the experiment, hydrophilic poly(acrylic acid) (PAA) was patterned via UV photopolymerisation on the planar benzophenone-containing poly(dimethyl siloxane) (PDMS) substrates to trap the aqueous droplets. The droplet fusion could be divided into two steps including droplet trapping and droplet detachment. The flow mechanism was to make use of the differences between the viscous drag force and the surface energy. In a typical situation, as a droplet entered the surface modified fusion region, it slowed down and stopped. It would merge with the subsequent droplet until the surface energy stabilisation was overwhelmed by the viscous drag force.

Fig. 2.28 Schematic illustration of surface-induced droplet fusion device [106].

2.4.2 Active droplet coalescence

2.4.2.1 Electrocoalescence (EC)

Electrocoalescence is an active method which is widely used to realise droplet fusion in the presence of an alternating current electric field. Electrocoalescence of microfluidic droplets was firstly presented by Chabert *et al.* [112]. In order to realise electrocoalescence, the electrical conductivity of the droplets has to be higher than that of the continuous phase. In the presence of an electric field, the effect of the droplets can be deemed as dipolar disturbances situated at the droplet centre. When the two droplets approach each other, a Colombic force which is proportional to the square of the applied electric field, will attract the two droplets to achieve electrocoalescence.

Link *et al.* [113] demonstrated the concept of electrocoalescence by polarising droplets through applying a direct current (DC) field as shown in Fig. 2.29. The water stream behaved as a conductor, whereas the oil stream behaved as an insulator. Under the applied electric field, the upper and lower droplets carried opposite charges. As the two droplets came close at the T-junction, they were

attracted to each other. Subsequently, droplet fusion took place. Based on this experiment, Sarrazin *et al.* [114] studied the mixing characteristics with different angle bends along the microchannel. The forty-five degree angle bends was found to be the most effective for droplet mixing. Zagnoni *et al.* [115] investigated the electrocoalescence mechanism of micro-droplets using localised electric fields. The localised electric system was found to be effective in merging droplets regardless of the distance between them. The viscosity of the continuous phase was found to be a dominant factor in electrocoalescence.

Fig. 2.29 Schematic illustration of electrocoalescence of droplets by applying an electric field through the aqueous stream [113].

Based on the electrocoalescence microfluidic system, a novel merging element combining the advantages of the passive and active merging approaches was proposed by Niu *et al.* [116] as shown in Fig. 2.30. Pillar arrays and electrodes were employed as passive merging element and active merging element respectively. The pillar array played a role in slowing down and trapping droplets through the drainage of the continuous phase between them. As the droplets approached each other, an electric field was applied. Hence the thin oil film around the droplets would be removed.

Fig. 2.30 Schematic illustration of the merging chamber with pillar arrays and embedded electrodes [116].

The electrocoalescence of droplet population and their upstream cascade [117] were investigated as shown in Fig. 2.31. The upstream cascade could be described as a two-stage mechanism. In the first stage, droplet fusion caused by the electric field only occurred in the adjacent of electrodes. At higher applied voltage, droplet fusion occurred at the upstream of the electrodes. The fused large droplet caused the curvature along the droplet interface to be imbalanced. This imbalance led to the second stage which was described as the upstream cascade. Under the effect of the large droplet formed at the first stage, a localised decompression occurred between the adjacent droplets. As a result, the upstream coalescence took place. This phenomenon was found to be dependent on droplet size and distribution.

Fig. 2.31 Schematic illustration of the electrocoalescence behaviour of a stream of water droplets under the effect of an applied electric field [117].

In the above study, droplet generation and fusion were carried in two stages. Wang *et al.* [118] developed a microfluidic device to generate and merge two droplets at the same time. The schematic illustration of the microfluidic devices is shown in Fig. 2.32. A double T-junction with a pair of embedded Pt wire and indium tin oxide (ITO) electrodes was used as a microfluidic platform. Without an applied electric field between the ITO and Pt wire, an alternating droplet generation could be achieved. When a DC voltage was applied across the ITO electrode and the Pt wires, a non-uniform electric field was built around the tip of the Pt wire. As a result, the water-oil interface would be polarised. Incoming two aqueous streams were attracted at the tip of the Pt wire. Meanwhile, the two aqueous streams coalesce followed.

Fig. 2.32 Schematic illustration of microfluidic channel and electrode positions [118].

2.4.2.2 Dielectrophoresis (DEP) actuated droplet coalescence

Similar to electrocoalescence, dielectrophoresis is an effective way to realise droplet fusion. The key difference between DEP and EC is that DEP can only occur in a non-uniform applied electric field, while EC occurs in both uniform

and non-uniform applied electric fields [112]. DEP relies on the different dielectric constant between the droplet and the surrounding medium, while EC relies on the different in conductivity of droplet and the continuous phase. Droplet manipulation through DEP was demonstrated by Schwartz *et al.* [119], Singh and Aubry [120], and Wang *et al.* [121]. In a typical situation, an electric stress acts on a droplet surface which is subjected to a non-uniform electric field. A net electric force which is referred as DEP force causes the motion of the droplet.

2.4.2.3 Temperature actuated droplet coalescence

Beside electrically induced droplet coalescence, thermally induced droplet coalescence is also an effective method. The mechanism of this method is to exploit temperature-dependent viscosity and surface tension. On the one hand, increasing temperature leads to a decrease in viscosity of the continuous phase. As a result, the continuous phase flows faster due to the heating effect. On the other hand, the surface tension of the droplet decreases with the increase in temperature. Hence droplet fusion process can take place more easily.

Temperature actuated droplet coalescence was demonstrated by Kohler *et al.* [107]. Figure 2.33 shows the schematic illustration of thermally controlled droplet fusion in a microfluidic device. A high fluid resistance element with a thin film heater was incorporated in a long channel with reduced cross section. In operation, a small amount of continuous phase flowed through the channel with high fluid resistance. As the thin film heater was activated, the viscosity of the continuous phase decreased, hence allowed more fluid from the main

channel to flow through the small channel. With the drainage of continuous phase in the main channel, the subsequent droplets would approach each other. When droplet fusion occurred, the heater was turned off.

Fig. 2.33 Schematic illustration of thermally controlled droplet fusion in microfluidic device [107].

Instead of using a thin film heater, a focused laser was employed [122, 123] to provide heating function to fuse droplets as shown in Fig. 2.34. Without the activation of the laser, a train of droplets were neatly aligned in the microchannel. Although the droplets were in contact with each other, droplet coalescence could not be achieved due to a thin film between them. The presence of surfactant molecules on the oil-water interface was known to stabilise droplets against fusion [124]. With the activation of the laser, localised heating was induced. A spatial variation of temperature field built up resulting in a spatial imbalance of surface tension. A net force was produced on the droplet interface through the thermocapillary effect. Under this net force, the surfactant molecules and the lubrication film would be evacuated. Hence the merging of the two neighbouring droplets could be achieved. With the development of optical techniques, a generalised phase contrast (GPC) method was proposed to allow a single focused laser to be divided into many laser spots which could be independently manipulated [125]. A schematic illustration of a

GPC light shaping box is shown in Fig. 2.35. After the beam modulation and beam separation processes, a focused laser can be divided into many laser spots. This technique could potentially be used for droplet manipulation at different desired locations.

Fig. 2.34 Schematic illustration of localised fusion of droplets [122].

Fig. 2.35 Schematic illustration of a GPC light shaping box [125].

2.4.2.4 Pneumatically actuated droplet coalescence

Another method for active merging of droplets was pneumatically actuated droplet coalescence [126]. A pneumatically actuated membrane valve was constructed on the top of the merging chamber as shown in Fig. 2.36. As the

droplet entered the merging chamber, the membrane valve was activated and impeded the droplet motion. Droplet fusion took place as they approached each other. As the desired droplet fusion was achieved, the pneumatically actuated membrane valve would open to purge the fused droplet.

Fig. 2.36 Schematic illustration of pneumatically actuated droplet fusion device [126].

2.5 Remarks on Previous Works

The literature review shows the recent research related to microfluidic mixing, calorimetric thermal flow sensor, heat transfer in rectangular microchannels and droplet coalescence in microfluidic systems. The simple flow architectures assumed in the microfluidic systems are based on the constructal theory [127, 128], which provide the view that flow configuration (geometry, design) can be reasoned on the basis of a principle of configuration generation and evolution in time toward greater global flow access in systems that are free to morph. The generation of flow configuration is viewed as a physics phenomenon. For microfluidic mixing, researchers focused on mass mixing characteristics in T-shaped or Y-shaped microchannels which are the simplest class of tree-shaped

flows. There was no investigation on thermal mixing characteristics in the T-shaped or Y-shaped microchannels. For the calorimetric thermal flow sensor, the thermocouple-based temperature measurement was often used to record the specific points in microchannels. For the heat transfer in rectangular microchannels, there was no agreement on the variation of Nusselt number on the microscale. For droplet coalescence in microfluidic systems, coalescence of droplets of the same size and the same velocity was mainly realised by the electrostatic force from relatively high voltages. This was especially harmful in biological application.

The aim of this thesis is to study both the single and two-phase heat transfer characteristics in microfluidic systems using Laser Induced Fluorescence (LIF) to analyse the temperature field in microchannels. From the measured temperature field, the heat transfer behaviour of fluid flow in microchannels is being investigated.

Chapter 3

Thermal Mixing in a T-shaped Microchannel

Parallel lamination micromixer is one of the most commonly used passive micromixer. It is based on the decrease of molecular diffusion path by splitting the inlet streams into n substreams and then recombining them into a single stream. A basic design with two inlets is called as T-mixer or Y-mixer [3], which is an ideal configuration for investigation of transport phenomena on the microscale. Many experimental and numerical studies were conducted for mixing of miscible fluids in the T-shaped microchannels. Bokenkamp *et al.* [129] reported that better mixing efficiency could be achieved by connecting two T-shaped micromixers with a commercially available quench-flow instrument. Jacobson *et al.* [130] fabricated microfluidic devices for parallel mixing, which consisted of a series of independent T-shaped intersections. Gobby *et al.* [131] studied the mixing characteristics of gaseous flow in a T-shaped micromixer by using numerical simulation. T-shaped microchannels were also employed to measure the molecular diffusion coefficient for chemical analysis [132, 133]. Kockmann and co-workers [134, 135] performed numerical simulations and experimental studies on mixing in a T-shaped micromixer. Three flow regimes were distinguished, which were named as strictly laminar flow, symmetrical vortex flow and engulfment flow depending on the Reynolds number. Zhao *et al.* [136] investigated the mass transfer process in T-junction microchannels of immiscible fluids. Empirical correlations were proposed to predict the mass transfer coefficient in the microchannel. The diffusion

coefficient and flow rate ratios were also studied in a sandwich T-shaped micromixer [137].

The limitation of the previous experimental and numerical studies on the T-shaped micromixer was that they all focused on mass mixing of two miscible fluids. Little work has been carried out to study the thermal mixing characteristics in T-shaped microchannels. For the thermal mixing in the microchannel, a Y-shaped microchannel was investigated experimentally [138]. Temperature contours were reported to show the thermal mixing process. To the best of the knowledge of the author, theoretical analysis related to thermal mixing processes in a microchannel is yet to be reported. In this chapter, the main objective is to qualitatively study the thermal mixing characteristics of two miscible fluids in a T-shaped microchannel with different combination of flow rates. Two heat transfer zones (at the T-junction and the mixing channel) are analysed and investigated. An analytical model is proposed for the thermal mixing process in the mixing channel. Experiments are carried out to validate the analytical model. A three dimensional numerical simulation is carried out to provide a better understanding of the thermal mixing processes on the microscale.

3.1 Theoretical Investigation on Thermal Mixing

In this section, several analytical models are developed to model the thermal mixing phenomena in a basic mixer with rectangular cross section. A two-dimensional model of diluted mixing with 1:1 ratio is established and solved analytically. The model is further extended to arbitrary mixing ratios. Thermal

mixing with dispersion is also considered. The depth averaging method is employed to solve the three-dimensional case.

3.1.1 Problem definition and assumptions

The proposed analytical model considers a straight microchannel with a rectangular cross section (Fig. 3.1). The channel has two inlets for the hot and cold fluids. The width, height and length of the channel are W , H and L , respectively.

Fig. 3.1 The proposed analytical model and its parameters.

In the analysis, the following assumptions are made:

1. Continuum assumption;
2. Two fluids with the same properties;
3. Decoupled convection and diffusion processes. The process is assumed to occur in a fully developed flow field;
4. The viscosity and thermal diffusivity are independent of temperature;
5. Laminar, incompressible, Newtonian fluid, steady flow is assumed;
6. Convection is assumed to occur in one direction only, axially along the channel.

3.1.2 Governing equations

The two-fluid flow in a rectangular microchannel can be described by the following three governing equations:

The continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \quad (3.1)$$

The momentum equation:

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} = \vec{F} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{V} \quad (3.2)$$

The energy equation:

$$\frac{\partial T}{\partial t} + \vec{V} \cdot \nabla T = \alpha \nabla^2 T \quad (3.3)$$

3.1.3 Scale analysis

Before presenting the analytical solution of thermal mixing in the T-shaped microchannel, a scale analysis [128] is conducted to determine the time scales of the problem. The objective of scale analysis is to use the basic principles of heat and fluid flow to produce order of magnitude estimates for the prediction of residence time of thermal mixing in the mixing channel. The governing equation for heat and fluid flow in mixing channel is as follows:

$$\frac{\partial T}{\partial t} + U_x \frac{\partial T}{\partial x} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (3.4)$$

where α is the thermal diffusivity of the fluid $\left[\alpha = k / (\rho c_p) \right]$. The order of magnitude of each of the terms appearing in Eq. (3.4) will be estimated. On the left-hand side of Eq. (3.4),

$$\frac{\partial T}{\partial t} \sim \frac{\Delta T}{t} \quad (3.5)$$

$$U_x \frac{\partial T}{\partial x} \sim U \frac{\Delta T}{L} \quad (3.6)$$

In other words, the scale of the temperature change (in the chosen space and in a time of order t) is ΔT . The scale of the velocity along the fluid flow direction is U . On the right-hand side of Eq. (3.4),

$$\alpha \frac{\partial^2 T}{\partial x^2} = \alpha \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) \sim \frac{\alpha}{L} \frac{\Delta T}{L} = \frac{\alpha \Delta T}{L^2} \quad (3.7)$$

$$\alpha \frac{\partial^2 T}{\partial y^2} = \alpha \frac{\partial}{\partial y} \left(\frac{\partial T}{\partial y} \right) \sim \frac{\alpha}{W} \frac{\Delta T}{W} = \frac{\alpha \Delta T}{W^2} \quad (3.8)$$

In the mixing channel, $L \gg W$. The two orders of magnitude (3.6) and (3.7) can be neglected. Equating the two orders of magnitude (3.5) and (3.8), as required by the energy equation (3.4), the residence time to the thermal mixing in the mixing channel is:

$$t \sim \frac{W^2}{\alpha} \quad (3.9)$$

3.1.4 Analytical solution

The temperature distribution is obtained by solving the energy equation (3.3). This equation cannot be solved analytically in the three-dimensional form even with the assumption of a fully developed flow. A simplification for the model is needed. For a rectangular channel having a high aspect ratio, the streamwise velocity in Eq. (3.3) can be approximated by the channel averaged streamwise velocity [139]. Under these assumptions, the energy equation can be reduced to a two-dimensional form.

3.1.4.1 Thermal mixing with 1:1 ratio

The two-dimensional model of two-fluid flow in a rectangular channel is depicted in Fig. 3.2. The microchannel is a long channel with a width of W and two inlet streams. One stream is a hot fluid with high temperature T_H , while the other stream is a cold fluid with low temperature T_C . The flow in the rectangular channel is assumed to have a constant velocity of U . The boundary conditions are depicted in Fig. 3.2.

Fig. 3.2 Two-dimensional model of heat transfer with two-fluid flow.
(a) The actual model. (b) The dimensionless model.

Equation (3.3) becomes:

$$\alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = U \frac{\partial T}{\partial x} \quad (3.10)$$

where α is the thermal diffusivity of the fluid $\left[\alpha = k / (\rho c_p) \right]$. By introducing

the dimensionless coordinate system $\hat{x} = x / W$, $\hat{y} = y / W$, the normalised

temperature $\Theta = \frac{T - T_C}{T_H - T_C}$ and the Peclet number

$$Pe = \frac{UW}{\alpha} \quad (3.11)$$

where U , W and α are the uniform velocity, channel width and thermal diffusivity, respectively. Equation (3.10) can be formulated in dimensionless form as:

$$\frac{\partial^2 \Theta}{\partial \hat{x}^2} + \frac{\partial^2 \Theta}{\partial \hat{y}^2} = \text{Pe} \frac{\partial \Theta}{\partial \hat{x}} \quad (3.12)$$

The boundary conditions for Eq. (3.12) are:

$$\begin{aligned} \Theta|_{(\hat{x}=0, 0 < \hat{y} < 1/2)} &= 0 \\ \Theta|_{(\hat{x}=0, 1/2 < \hat{y} < 1)} &= 1 \\ \frac{\partial \Theta}{\partial \hat{x}} \Big|_{(\hat{x} \rightarrow \infty)} &= 0 \end{aligned} \quad (3.13)$$

The channel wall is assumed to be adiabatic; therefore the zero flux condition can be applied to the channel walls:

$$\frac{\partial \Theta}{\partial \hat{y}} \Big|_{\hat{y}=0} = \frac{\partial \Theta}{\partial \hat{y}} \Big|_{\hat{y}=1} = 0 \quad (3.14)$$

The solution can be written in a series expansion as:

$$\Theta(\hat{x}, \hat{y}) = \sum_{n=0}^{\infty} \Theta_n(\hat{x}) \Phi_n(\hat{y}) \quad (3.15)$$

$$\Phi_0(\hat{y}) = 1, \quad \Phi_n(\hat{y}) = \sqrt{2} \cos n\pi \hat{y}, \quad n = 1, 2, \dots \quad (3.16)$$

The Finite Fourier Transform (FFT) of the temperature is defined as

$$\Theta_n(\hat{x}) = \int_0^1 \Phi_n(\hat{y}) \Theta(\hat{x}, \hat{y}) d\hat{y} \quad (3.17)$$

Applying the FFT to both sides of Eq. (3.12) gives:

$$\int_0^1 \Phi_n(\hat{y}) \left[\frac{\partial^2 \Theta}{\partial \hat{x}^2} + \frac{\partial^2 \Theta}{\partial \hat{y}^2} \right] d\hat{y} = \int_0^1 \Phi_n(\hat{y}) \text{Pe} \frac{\partial \Theta}{\partial \hat{x}} d\hat{y} \quad (3.18)$$

The first term of Eq. (3.18) is rewritten in terms of the transformed temperature as:

$$\int_0^1 \Phi_n(\hat{y}) \frac{\partial^2 \Theta}{\partial \hat{x}^2} d\hat{y} = \frac{d^2}{d\hat{x}^2} \int_0^1 \Phi_n(\hat{y}) \Theta d\hat{y} = \frac{d^2 \Theta_n}{d\hat{x}^2} \quad (3.19)$$

The second term of Eq. (3.18) is rearranged using two integrations by parts to give:

$$\begin{aligned} \int_0^1 \Phi_n(\hat{y}) \frac{\partial^2 \Theta}{\partial \hat{y}^2} d\hat{y} &= \left(\Phi_n \frac{\partial \Theta}{\partial \hat{y}} \right) \bigg|_{\hat{y}=0}^{\hat{y}=1} - \left(\Theta \frac{\partial \Phi_n}{\partial \hat{y}} \right) \bigg|_{\hat{y}=0}^{\hat{y}=1} + \int_0^1 \Theta \frac{\partial^2 \Phi_n(\hat{y})}{\partial \hat{y}^2} d\hat{y} \\ &= -(n\pi)^2 \Theta_n \end{aligned} \quad (3.20)$$

The right hand side term of Eq. (3.18) is rewritten in terms of the transformed temperature as:

$$\int_0^1 \Phi_n(\hat{y}) \text{Pe} \frac{\partial \Theta}{\partial \hat{x}} d\hat{y} = \text{Pe} \frac{d}{d\hat{x}} \int_0^1 \Phi_n(\hat{y}) \Theta d\hat{y} = \text{Pe} \frac{d \Theta_n}{d\hat{x}} \quad (3.21)$$

The results are combined to give the differential equations $\Theta_0(\hat{x})$ and $\Theta_n(\hat{x})$

$$\frac{d^2 \Theta_0}{d\hat{x}^2} - \text{Pe} \frac{d \Theta_0}{d\hat{x}} = 0 \quad (3.22)$$

$$\frac{d^2 \Theta_n}{d\hat{x}^2} - \text{Pe} \frac{d \Theta_n}{d\hat{x}} - (n\pi)^2 \Theta_n = 0 \quad (3.23)$$

Equations (3.22) and (3.23) require two boundary conditions which are obtained by transforming the original boundary conditions (Eqs. (3.13) and (3.14)). The dimensionless temperature distribution in the microchannel can be obtained:

$$\Theta(\hat{x}, \hat{y}) = \frac{1}{2} - \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right) \exp\left(\frac{\text{Pe} - \sqrt{\text{Pe}^2 + 4n^2\pi^2}}{2} \hat{x}\right) \cos(n\pi \hat{y}) \quad (3.24)$$

Figure 3.3 shows typical results of the temperature distribution. Obviously, thermal mixing efficiency decreases as Peclet number increases. The

temperature distribution at the outlet becomes more uniform at lower Peclet number. Figure 3.4 shows the temperature profiles at the outlet for different Peclet numbers. r is defined as the liquid fraction of the cold fluid in the microchannel. The thermal diffusion process is less pronounced as the Peclet number increases. This is because the residence time of thermal mixing for a small Peclet number is longer than that for a larger Peclet number.

Fig. 3.3 Normalised temperature distribution in the microchannel with 1:1 ratio.
(a) $Pe = 2$. (b) $Pe = 4$. (c) $Pe = 6$. (d) $Pe = 8$.

Fig. 3.4 Evolution of thermal mixing at the outlet for different Peclet numbers.
 $(\hat{x} = 1, r = 0.5)$

Figure 3.5 shows the evolution of temperature profile of thermal mixing at three different sections along the axial direction. At the inlet region ($\hat{x} = 0$), the temperature distribution for the hot fluid side ($\Theta = 1$) and cold fluid side ($\Theta = 0$) are assumed to be uniform. At the one quarter position along the axial direction ($\hat{x} = 0.25$), the evolution of temperature can be clearly seen. This is due to the heat transfer effect between the hot and cold fluids. At the outlet ($\hat{x} = 1$), the thermal diffusion is more pronounced. The cross-stream thermal diffusion is found to be proportional to the distance along the axial direction.

Fig. 3.5 Evolution of thermal mixing at three different sections along the axial direction.
($Pe = 4$, $r = 0.5$)

3.1.4.2 Thermal mixing with arbitrary ratio

In Section 3.1.3.1, mixing with 1:1 ratio is investigated. For actual situations, thermal mixing with arbitrary ratio is required. In the analytical model, r is defined as the liquid fraction of the cold fluid in the microchannel. The mixing ratio of the hot and cold streams is then $(1-r:r)$. The boundary conditions are depicted in Fig. 3.6.

Fig. 3.6 Two-dimensional model of heat transfer with two-fluid flow.

(a) The actual model. (b) The dimensionless model.

The governing equation is:

$$\frac{\partial^2 \Theta}{\partial \hat{x}^2} + \frac{\partial^2 \Theta}{\partial \hat{y}^2} = \text{Pe} \frac{\partial \Theta}{\partial \hat{x}} \quad (3.25)$$

The boundary conditions for Eq. (3.25) are:

$$\begin{aligned} \Theta|_{(\hat{x}=0, 0 < \hat{y} < r)} &= 0 \\ \Theta|_{(\hat{x}=0, r < \hat{y} < 1-r)} &= 1 \\ \frac{\partial \Theta}{\partial \hat{x}} \Big|_{(\hat{x} \rightarrow \infty)} &= 0 \end{aligned} \quad (3.26)$$

The channel wall is assumed to be adiabatic. Therefore the zero flux condition can be applied to the channel walls, which is the same as Eq. (3.14). Using the Finite Fourier Transform (FFT) method, and applying corresponding boundary conditions, the dimensionless temperature distribution in the microchannel can be obtained as follows,

$$\Theta(\hat{x}, \hat{y}) = 1 - r - \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin(n\pi r) \exp\left(\frac{\text{Pe} - \sqrt{\text{Pe}^2 + 4n^2\pi^2}}{2} \hat{x}\right) \cos(n\pi \hat{y}) \quad (3.27)$$

Figure 3.7 shows some typical results of the temperature distribution. Obviously, it is a more general solution of thermal mixing in microchannels. $r = 0$ and $r = 1$ equal to the single fluid flow in microchannels, so there is no thermal mixing. $r = 1/2$ corresponds to the thermal mixing with 1:1 ratio (Fig. 3.3).

Fig. 3.7 Normalised temperature distribution in the microchannel with different ratios for $Pe = 4$.
(a) $r = 0.2$. (b) $r = 0.4$. (c) $r = 0.6$. (d) $r = 0.8$.

Figure 3.8 shows the temperature profiles at the outlet for different mixing ratios. As the mixing ratio increases, which means the cold fluid occupies a larger fraction in the microchannel, the temperature at the outlet decreases.

Fig. 3.8 Evolution of thermal mixing at the outlet for different mixing ratios.
($\hat{x} = 1$, $Pe = 4$)

3.1.4.3 Thermal mixing with dispersion

For the large aspect ratio (the ratio of width w to height h), the flow in the microchannel can be treated generally as if it were flow between infinite parallel plates. The x component of the velocity can be approximated as:

$$u_x(z) = \frac{h^2}{2\mu} \frac{dp}{dx} \left[1 - \left(\frac{z}{h} \right)^2 \right] \quad (3.28)$$

Based on the assumption introduced in Section 3.1.1, the energy equation in three-dimensions can be expressed as follows:

$$\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (3.29)$$

To investigate the effect of channel depth on thermal mixing in the microchannel, the depth averaging method is employed which can eliminate the depthwise dependence in the energy equation by introducing an effective heat

transfer coefficient. This effective heat transfer coefficient characterises the convective-diffusive heat transfer in the microfluidic channel. In the depth averaging method, the depthwise average $\bar{f} \equiv (2h)^{-1} \int_{-h}^h f dz$ is used to eliminate the effect of some quantity f in the depthwise direction. Depth averaging the velocity profile can lead to $\bar{u} = \left(\frac{dp}{dx}\right) \left(\frac{h^2}{3\mu}\right)$, $u'(z) = \frac{dp}{dx} \frac{h^2}{6\mu} \left[1 - 3\left(\frac{z}{h}\right)^2\right]$. This report defines $u_x(z) = \bar{u} + u'(z)$ and $T(x, y, z, t) = \bar{T}(x, y, t) + T'(x, y, z, t)$, and substituting them into Eq. (3.29) to obtain:

$$\begin{aligned} & \frac{\partial \bar{T}}{\partial t} + \frac{\partial T'}{\partial t} + \bar{u} \frac{\partial \bar{T}}{\partial x} + u' \frac{\partial \bar{T}}{\partial x} + \bar{u} \frac{\partial T'}{\partial x} + u' \frac{\partial T'}{\partial x} \\ & = \alpha \left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2} \right) + \alpha \left(\frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2} + \frac{\partial^2 T'}{\partial z^2} \right) \end{aligned} \quad (3.30)$$

Depthwise averaging of Eq. (3.29) leads to the mean convective-diffusion equation:

$$\frac{\partial \bar{T}}{\partial t} + \bar{u} \frac{\partial \bar{T}}{\partial x} = \alpha \left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2} \right) - \overline{u' \frac{\partial T'}{\partial x}} \quad (3.31)$$

In order to obtain the governing equation of T' , Eq. (3.31) is subtracted from Eq. (3.30) to obtain:

$$\frac{\partial T'}{\partial t} + u' \frac{\partial \bar{T}}{\partial x} + \bar{u} \frac{\partial T'}{\partial x} + u' \frac{\partial T'}{\partial x} = \alpha \left(\frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2} + \frac{\partial^2 T'}{\partial z^2} \right) + \overline{u' \frac{\partial T'}{\partial x}} \quad (3.32)$$

Due to the long duration characteristic of diffusion, the effect will only be significant when the flow length is relatively large, and because $u' = O(\bar{u})$ while $T' \ll \bar{T}$, then Eq. (3.32) becomes:

$$u' \frac{\partial \bar{T}}{\partial x} \approx \alpha \frac{\partial^2 T'}{\partial z^2} \quad (3.33)$$

Integrating Eq. (3.33) twice with respect to z direction, coupled with the boundary condition $z=0$, $T'=0$, $\frac{\partial T'}{\partial z}=0$, the equation obtained is:

$$T' = \frac{1}{\alpha} \frac{dp}{dx} \frac{h^2}{6\mu} \left(\frac{z^2}{2} - \frac{z^4}{4h^2} \right) \frac{\partial \bar{T}}{\partial x} \quad (3.34)$$

Since u' is known, the additional contribution due to velocity variation from the average velocity can be expressed as:

$$\overline{u' \frac{\partial T'}{\partial x}} = -\frac{2}{105\alpha} \left(\frac{dp}{dx} \right)^2 \frac{h^6}{9\mu^2} \frac{\partial^2 \bar{T}}{\partial x^2} \quad (3.35)$$

The steady state energy equation, Eq. (3.31), by considering dispersion is reduced to:

$$\bar{u} \frac{\partial \bar{T}}{\partial x} = \alpha_{eff} \frac{\partial^2 \bar{T}}{\partial x^2} + \alpha \frac{\partial^2 \bar{T}}{\partial y^2} \quad (3.36)$$

The effective thermal diffusivity coefficient is defined as:

$$\alpha_{eff} = \alpha \left[1 + \frac{Pe^2}{210} \left(\frac{h}{W} \right)^2 \right] \quad (3.37)$$

The dimensionless form of Eq. (3.36) can be written as:

$$\frac{\alpha_{eff}}{\alpha} \frac{\partial^2 \Theta}{\partial \hat{x}^2} + \frac{\partial^2 \Theta}{\partial \hat{y}^2} = Pe \frac{\partial \Theta}{\partial \hat{x}} \quad (3.38)$$

The boundary conditions are the same as Eq. (3.26). The dimensionless temperature distribution in the microchannel is:

$$\Theta(\hat{x}, \hat{y}) = 1 - r - \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin(n\pi r) \exp \left(\frac{Pe \frac{\alpha}{\alpha_{eff}} - \sqrt{Pe^2 \frac{\alpha^2}{\alpha_{eff}^2} + 4n^2 \pi^2 \frac{\alpha}{\alpha_{eff}}}}{2} \hat{x} \right) \cos(n\pi \hat{y}) \quad (3.39)$$

Figure 3.9 shows typical results of the temperature distribution. Figure 3.10 shows a comparison of temperature profiles at the outlet for different aspect

ratios. $H/W = 0$ corresponds to the case without dispersion. From Fig. 3.10, the thermal diffusion between the hot and cold fluids decreases with consideration of dispersion can be observed. The presence of dispersion enhances the thermal diffusion in the axial direction. However, dispersion reduces the cross-stream thermal diffusion between the hot and cold fluids. Thus, it will result a sharp transition of temperature profile between the hot and cold fluids.

Fig. 3.9 Normalised temperature distribution in the microchannel with different ratios for $Pe = 4$.

(a) $H/W = 0$. (b) $H/W = 0.4$. (c) $H/W = 0.6$. (d) $H/W = 0.8$.

Fig. 3.10 Evolution of thermal mixing at the outlet for different aspect ratios.
 $(\hat{x} = 1, \text{Pe} = 4, r = 0.5)$

3.1.5 Theoretical investigation on the mixing channel heat transfer zone

In the experiment, a T-shaped microchannel is employed to investigate the thermal mixing process. To accurately describe the heat transfer process in the opposed T-junction micromixer, the thermal mixing process in the T-shaped microchannel is divided into T-junction heat transfer zone (a-b-d-c) and mixing channel heat transfer zone as shown in Fig. 3.11. In the T-junction, hot and cold DI-water flow coaxially into the two inlet arms. Thermal mixing occurs as soon as they contact each other. Thermal diffusion is dominant in the T-junction heat transfer zone. Meanwhile, low energy dissipation arises due to that the kinetic energy is converted into a motion through collision and redirection of the fluid flow at the T-junction. In this study, Reynolds number is less than 1. The flow is laminar and the streamlines are scarcely bent and follow the channel walls [134]. In the mixing channel heat transfer zone, thermal diffusion and

convection dominate in the thermal mixing process. Thermal mixing is greatly enhanced due to the long residence time provided by the mixing channel.

Fig. 3.11 (a) Schematic illustration of T-shaped microchannel. (b) The opposed T-shaped microchannel.

Fig. 3.12 Two-dimensional model of thermal mixing in the mixing channel. (a) The actual model. (b) The dimensionless model.

An analytical model is developed to describe the heat transfer processes in the mixing channel heat transfer zone (Fig. 3.11). Figure 3.12 illustrates the two-dimensional model for the thermal mixing processes in the mixing channel. The microchannel is a long channel with a width of W . The flow in the rectangular

channel is assumed to have a constant velocity of U . For the low Reynolds number flow ($Re < 1$) [140], due to the strong thermal diffusion occurs at the T-junction heat transfer zone, a linear temperature distribution is specified at the inlet of the mixing channel ($\hat{x} = 0$). At the upper wall ($\hat{y} = 1$), constant wall temperature (T_w) is specified due to the thermal conduction from the thermal electric module. As under an applied voltage, the thermal electric module will provide a constant heating temperature continuously. At steady state, the upper wall temperature which is adjacent to the thermal electric module will achieve a constant surface temperature. At the bottom wall ($\hat{y} = 0$), adiabatic condition is specified due to the negligible heat loss to the ambient environment. For a very long channel and in particular as $\hat{x} \rightarrow \infty$, full thermal mixing will be achieved. Hence there will be no temperature gradient in the flow direction in the mixing channel.

The governing equation can be formulated in the dimensionless form as:

$$\frac{\partial^2 \Theta}{\partial \hat{x}^2} + \frac{\partial^2 \Theta}{\partial \hat{y}^2} = Pe \frac{\partial \Theta}{\partial \hat{x}} \quad (3.40)$$

The boundary conditions for Eq. (3.40) are:

$$\Theta|_{(\hat{x}=0)} = \hat{y}, \quad \left. \frac{\partial \Theta}{\partial \hat{x}} \right|_{(\hat{x} \rightarrow \infty)} = 0 \quad (3.41)$$

The boundary condition for the wall is set as follows:

$$\Theta|_{\hat{y}=1} = (T_w - T_c) / (T_H - T_c), \quad \left. \frac{\partial \Theta}{\partial \hat{y}} \right|_{\hat{y}=0} = 0 \quad (3.42)$$

Using the Finite Fourier Transform (FFT) method, and applying the corresponding boundary conditions (Eqs. (3.41) and (3.42)), the dimensionless temperature distribution in the microchannel can be obtained as:

$$\Theta(\hat{x}, \hat{y}) = \sum_{n=1}^{\infty} \left\{ A \exp \left(\frac{\text{Pe}^2 - \sqrt{\text{Pe}^2 + 4\lambda_n^2}}{2} \hat{x} \right) + \frac{2c(-1)^n}{\lambda_n} \right\} \cos(\lambda_n \hat{y}) \quad (3.43)$$

where

$$A = \frac{2(-1)^n}{\lambda_n} (1 - c) - \frac{2}{\lambda_n^2},$$

$$c = (T_w - T_c) / (T_H - T_c), \lambda_n = \sqrt{2} \cos(n + 1/2) \pi \hat{y}.$$

3.2 Experimental Investigation on Thermal Mixing

In this section, a calibration setup to obtain the relationship between the fluorescence intensity and temperature is shown in Section 3.2.1. In Section 3.2.2, the experimental investigation on thermal mixing is presented.

3.2.1 Design of microchannel for calibration of temperature

Figure 3.13 shows the layout of the microchannel for the calibration of temperature measurement. The thermal bonding technique was used to bond three polymethylmethacrylate (PMMA) plates (55 mm × 25 mm) together. The channel height is determined by the middle PMMA plate. The channel structures are cut into the middle PMMA plate using a CO₂ laser. A type K thermocouple ($D = 0.1930$ mm) was inserted into the vertical slot in the middle PMMA plate to measure the temperature in the microchannel. The dimensions of the square groove on the top PMMA plate are 15 mm × 15 mm.

Fig. 3.13 A layout of a chip for the calibration of the temperature measurement.
(a) Top PMMA plate. (b) Middle PMMA plate. (c) Bottom PMMA plate.

3.2.1.1 Calibration of the thermocouple

Thermocouples of smaller diameter are more sensitive. The thermocouple is calibrated using Thermal Calibration Cool/Heat Source Model 18B (Fig. 3.14). The thermocouple is inserted into the wells of the Thermal Calibration Model, which can be represented by the equation of $y = ax + b$ and compared with the measured temperature data. In Fig. 3.15, $a = 0.9866$, $b = -0.2780$.

Fig. 3.14 Thermocouple calibrator (Model 18B).

Fig. 3.15 Calibration of thermocouple.

3.2.1.2 Fluorescence imaging measurement

The schematic of the experimental setup is shown in Fig. 3.16. Fluorescence imaging technique is used in the experiments. The setup consists of four main components: an illumination system, an optical system, a coupled charge device (CCD) camera and a personal computer (PC) based control system. A mercury lamp is used as the illumination source for the fluorescence measurement. The optical system is a Nikon inverted microscope (Model ECLIPSE TE2000-S) with a set of epi-fluorescent attachments. The filter cube consists of three optical elements: excitation filter, dichroic mirror and emission filter. Emission filters are used in the measurements to select the specific emission wavelength of the sample and to remove traces of excitation light. The image acquisition device consists of an interline transfer CCD camera (HiSense MkII), a Flow Map system hub with last-in-first-out (LIFO) capability. The resolution of the CCD camera is 1344×1024 pixels with 12 bits greyscale.

Fig. 3.16 Schematic illustration of experimental setup for fluorescence imaging.

3.2.1.3 Calibration of Temperature Measurement

The schematic illustration of the experimental setup for the calibration of temperature measurement is shown in Fig. 3.17(a) and 3.17(b). The calibration is used to determine the relationship between the fluorescence intensity and temperature. The microchannel has a cross section of $1 \text{ mm} \times 0.5 \text{ mm}$. Rhodamine B solution ($0.025 \text{ g}/100 \text{ mL}$) is introduced into the inlet port A by a single syringe pump (Cole-Parmer, 74900-05, $0.2 \text{ }\mu\text{L}/\text{h}$ to $500 \text{ L}/\text{h}$, accuracy of 0.5%). A low voltage power supply (GW Model GPC- 30300) is used to supply voltage to a thermoelectric module (TEC) to heat up the Rhodamine B solution in the microchannel. A thermoelectric module is a solid-state method of heat transfer by means of different semiconductor materials,

which is also commonly known as the Peltier effect. The voltage is supplied to enable heating/cooling of liquid solution in the microchannel.

The thermoelectric module is placed on the square slot on the top PMMA plate to heat the Rhodamine B solution by varying the voltages. The temperature of the solution increases with increasing the voltage. The image of the Rhodamine B solution in the microchannel for different temperature is captured with the microscope. Each captured image is generated by 30 sequential video frames. The fluorescence intensity for the image at each temperature is determined by averaging the intensity value (after background subtraction) of all the pixels of the corresponding image. A calibration curve is generated to accurately describe the dependence of Rhodamine B fluorescence on temperature. The resulting intensity (normalised by the intensity at 25°C) versus temperature is plotted in Fig. 3.17(c).

(a) Fabricated device used in the calibration

(b) Schematic representation of the calibration of temperature measurement

(c) Normalised fluorescence intensity as a function of temperature

Fig. 3.17 Calibration of temperature measurement.

3.2.2 Experimental setup for thermal mixing

The configuration of an opposed T-shaped microchannel is shown in Fig. 3.11. The microchannel through which the two opposing inlet streams flow horizontally along the y -direction is a rectangular section ($3.0 \text{ mm} \times 0.5 \text{ mm}$). A vertical rectangle bisects the horizontal channel and forms two short rectangular inlet arms, each with a length of 1.25 mm and a width of 0.5 mm. The fabrication is based on the lamination thermal bonding technique [141]. In this method, three polymethylmethacrylate (PMMA) plates ($50 \text{ mm} \times 50 \text{ mm}$) (Fig. 3.18) were bonded together to form a closed microfluidic channel with inlet and outlet holes. Thus, the middle PMMA defines the depth of the channel. The two fluids flow side by side in the T-shaped straight microchannel in the direction from left to right by two identical syringe pumps. The straight portion of the T-shaped channel has a cross section of $1000 \text{ } \mu\text{m} \times 500 \text{ } \mu\text{m}$ and the length of 3 cm. Hot DI-water with Rhodamine B solution ($0.025 \text{ g}/100 \text{ mL}$) enters the upper inlet arm and cold DI-water with Rhodamine B solution enters the lower inlet arm. Two low voltage power supplies were used to supply voltage to the thermoelectric module to enable heating/cooling of the working fluids. The images of the Rhodamine B solution at the inlet arms and the main channel for different flow rates (Q_H / Q_C) were captured by the CCD camera and microscope. The corresponding experimental setup is shown in Fig. 3.19.

Fig. 3.18 Thermal mixing chip design.
(a) Top PMMA plate. (b) Middle PMMA plate. (c) Bottom PMMA plate.

(a) The fabricated device used in the experiments

(b) Schematic representation of the two-fluid T-shaped microchannel

Fig. 3.19 Schematic illustration of experimental setup for thermal mixing.

3.3 Numerical Simulation of Thermal Mixing

Thermal mixing within the T-shaped microchannel depends on the transport phenomena in the T-junction heat transfer zone and the mixing channel heat transfer zone. The proposed analytical model describes heat transfer within the mixing channel heat transfer zone only. Therefore, a 3-D numerical simulation is needed to investigate the two heat transfer zones, especially the mixing channel heat transfer zone, is needed. Furthermore, the flow fields obtained from the numerical simulation can be used to confirm the analytical model. The schematic illustration of the computational domain is shown in Fig. 3.20. The temperature and transverse velocity along the y direction at the inlet are specified. Both the temperature and velocity at the inlet are assumed to be uniform. The non-slip and adiabatic boundary conditions are used for the walls. At the outlet, zero gradient conditions are imposed on the transverse velocity and temperature. Axial velocity at the outlet is calculated to ensure mass conservation over the whole computational domain. The density, specific heat, thermal conductivity and viscosity of DI-water are allowed to vary with temperature [142].

3.3.1 Governing equations

The fluid flow and heat transfer is assumed to be in steady state, incompressible and laminar. The continuity, momentum and energy equations can be written in the Cartesian tensor notation as follows:

Continuity equation

$$\frac{\partial(\rho u_i)}{\partial x_i} = 0 \quad (3.44)$$

Momentum equation

$$\frac{\partial(\rho u_j u_i)}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \frac{\partial}{\partial x_i} \left(\mu \frac{\partial u_k}{\partial x_k} \right) - \frac{\partial p}{\partial x_i} \quad (3.45)$$

Energy equation

$$\frac{\partial(\rho c_p u_j T)}{\partial x_j} = \frac{\partial}{\partial x_j} \left(k \frac{\partial T}{\partial x_j} \right) + S \quad (3.46)$$

where ρ , μ , c_p and k are the fluid density, viscosity, specific heat and thermal conductivity respectively, whose unit are kg/m^3 , $\text{Pa} \cdot \text{s}$, $\text{J/kg} \cdot \text{K}$ and $\text{W/m} \cdot \text{K}$, respectively. Their dependence on temperature [142] are given as follows, the unit of temperature (T) is $^{\circ}\text{C}$.

$$\rho = \frac{999.8 + 18.2 \times T - 7.9 \times 10^{-3} \times T^2 - 5.5 \times 10^{-5} \times T^3 + 1.5 \times 10^{-7} \times T^4 - 3.9 \times 10^{-10} T^5}{1 + 1.8 \times 10^{-2} \times T} \quad (3.47)$$

$$\mu = 2.414 \times 10^{-5} \times 10^{247.8/(T+133)} \quad (3.48)$$

$$c_p = 8958.9 - 40.535 \times (T + 273) + 0.11243 \times (T + 273)^2 - 1.0138 \times 10^{-4} \times (T + 273)^3 \quad (3.49)$$

$$k = -0.58166 + 6.3555 \times 10^{-3} \times (T + 273) - 7.9643 \times 10^{-6} \times (T + 273)^2 \quad (3.50)$$

3.3.2 Numerical procedure

The Finite Volume Method (FVM) is employed to solve the continuity, momentum and energy equations. A stagger grid is used to store the scalar and velocities at the nodes and faces of each control volume, respectively. The convection-diffusion effect is manipulated by using the power-law scheme.

SIMPLER algorithm is used to treat the velocity-pressure coupled problem [143].

A grid independence test is conducted by using different numbers of unstructured grid. Three different grid numbers as $41 \times 18 \times 21$, $62 \times 27 \times 32$ and $93 \times 41 \times 48$ are employed. The differences in u , v , w and T at all grid points are less than 1%, which means the numerical solution is grid independent. Therefore, the numerical method is considered appropriate for the simulation of conjugate convection-conduction heat transfer problem.

Fig. 3.20 Computational domain.

3.4 Results and Discussion

3.4.1 Experimental results and discussion

Figure 3.21(a) to 3.21(c) shows the measured temperature profiles for three flow rate ratios (Q_H/Q_C): 1, 0.66 and 0.5, respectively, of which Q_H is

maintained at 1000 $\mu\text{L/h}$. Hot and cold DI-water with flow rates Q_H and Q_C flow coaxially into the respective inlet arms. Formation of temperature contours was observed, indicating that extensive thermal mixing has already taken place in the inlet arms. The striation in the temperature contours indicates that diffusion is dominant in this area. This phenomenon is due to the fact that the thermal diffusivity is three orders of magnitude larger than the mass diffusivity, which corresponds to Lewis number $Le = \frac{\alpha}{D} = 1000$. As a result, the efficiency of thermal mixing is much larger than the mass mixing in laminar flow. At the inlet of the mixing channel ($\hat{x} = 0$), a linear temperature distribution is observed due to thermal diffusion in the inlet arms. The two injected streams flow into the mixing channel which is perpendicular to the inlet arms. Two streams flow side by side down the mixing channel. Thermal mixing occurs during the parallel flow through the combined diffusion-convection mechanism. The number of striations in the temperature contours decrease in the mixing channel. A more uniform temperature distribution is observed in the mixing channel. When $Q_H / Q_C = 1$, each stream occupies half of the cross section in the mixing channel. At flow rate ratios of 0.66 (Fig. 3.21(b)) and 0.5 (Fig. 3.21(c)), the cold stream occupies a larger fraction of the channel as Q_C increases. Figure 3.21(d) to 3.21(f) shows the measured temperature profiles for three flow rate ratios (Q_H / Q_C): 1.5, 1 and 0.75, respectively, of which Q_H is maintained at 1500 $\mu\text{L/h}$, while Fig. 3.21(g) to 3.21(i) shows the measured temperature profiles for three flow rate ratios (Q_H / Q_C): 2, 1.33 and 1,

respectively, of which Q_H is maintained at 2000 $\mu\text{L}/\text{h}$. These figures show the same trend as Fig. 3.21(a) to 3.21(c).

Fig. 3.21 Experimental temperature contours plot of thermal mixing in the T-shaped microchannel.

3.4.2 Comparison between analytical model and experimental results

Comparison between theoretical model and experimental results is shown in Fig. 3.22. The analytical model agrees well with the experimental results, which demonstrate the usability of the boundary conditions setting for the heat transfer processes in the mixing channel heat transfer zone. At the entrance region ($\hat{x}=0$) of the mixing channel, a linear temperature distribution boundary

condition confirms the thermal mixing processes in the inlet arms. Striation in the temperature contours forms due to the thermal diffusion in the inlet arms. The evolution of temperature profiles at four different sections ($\hat{x} = 0, \hat{x} = 0.25, \hat{x} = 0.5, \hat{x} = 0.75$) along the axial direction can be clearly seen in Fig. 3.22. As the mixing channel length increases, a more uniform temperature distribution is observed due the combined convection and diffusion. Thermal mixing is found to be proportional to the mixing channel length.

3.4.3 Comparison between numerical and experimental results

Three-dimensional temperature and velocity distributions in the T-shaped microchannel are shown in Fig. 3.23(a) and 3.23(b), respectively. In order to compare the numerical result with the experimental result, the depthwise averaging approach was employed to average the temperature distributions along the z direction. Based on this method, the three-dimensional temperature distribution was converted to a two-dimensional temperature distribution. The comparison between numerical and experimental temperature profiles in the T-junction heat transfer zone is shown in Fig. 3.24. The numerical model agrees well with the experimental results. Striation temperature contours in the T-junction are confirmed by the numerical simulation.

Fig. 3.22 Comparison between analytical and experimental temperature profiles at four different sections in the mixing channel
 $(Q_H = 1500 \text{ } \mu\text{L/h}, Q_H / Q_C = 1).$

Fig. 3.23 Numerical result
(a) Temperature distribution in the T-shaped microchannel. (b) Velocity distribution in the T-shaped microchannel. ($Q_H = 1500 \mu\text{L/h}$, $Q_H / Q_C = 1$)

Fig. 3.24 Comparison between experimental and numerical temperature profiles in the T-junction heat transfer zone.
($Q_H = 1500 \mu\text{L/h}$, $Q_H / Q_C = 1$)

3.5 Summary

In this chapter, the Laser Induced Fluorescence (LIF) technique allows the measurement of the temperature field in microchannels. This measurement

would be impossible with conventional techniques using thermocouples. The thermal mixing characteristics of two fluids in the T-shaped microchannel were investigated theoretically, experimentally and numerically. Some significant results obtained are listed as follows:

- (1) The heat transfer process in the T-shaped microchannel can be divided into two heat transfer zones: the T-junction and the mixing channel.
- (2) Striations in the temperature contours in the T-junction are experimentally and numerically revealed.
- (3) The flow rate ratio plays an important role in the thermal mixing process. This deserves to be developed for design guidance.

Chapter 4

Thermal Effect on Mass Mixing in a T-shaped Microchannel

Due to the dominating laminar flow on the microscale, mixing in a microfluidic system relies mainly on molecular diffusion, which results in low mixing efficiency. Coupling of mass mixing with thermal effect has been attracting more attention from the research community. The motivation of this study is due to the significant difference between mass transfer rate and heat transfer rate. The thermal diffusivity is three orders of magnitude higher compared to the mass diffusivity. Heat transfer will have significant impact on the advection-diffusion problem [144, 145]. Mokrani *et al.* [146] studied the effect of chaotic advection on heat transfer at low Reynolds number. Chagny *et al.* [147] investigated the chaotic heat transfer characteristics of heat exchanger. Chaotic heat transfer was found to be effective at low Reynolds number. Acharya *et al.* [148] analysed the phenomena of heat transfer enhancement due to chaotic advection. Lefèvre *et al.* [149] studied chaotic advection and heat transfer enhancement in stokes flow experimentally. The heat transfer rate between the solid boundary and the fluid was found to be enhanced in the chaotic configuration. Kumar and Nigam [150] investigated the laminar convective heat transfer in chaotic configuration numerically. Chaotic mixing was found to enhance the heat transfer by about 25-36%. Yamagishi *et al.* [151] numerically studied the mixing performance and heat transfer in multiple pipe bends for different switching angles.

Based on the discussion above, it can be seen that researchers had focused on the heat transfer enhancement which is due to chaotic advection. Little literature exists on the mass mixing enhancement due to the heat transfer in the chaotic configuration. The main objective of this chapter is to study the effect of heat transfer on mixing performance in a T-shaped micromixer with heated cylinder experimentally and numerically. Experiments were carried out to show the feasibility of utilising heat transfer to enhance mixing performance in a chaotic configuration. The proposed numerical simulation provides a better understanding of the enhanced mixing performance due to the effect of heat transfer.

4.1 Experimental Investigation on Thermal Effect on Mass Mixing in a T-shaped Microchannel

The T-shaped microchannel used in the experiment is shown in Fig. 4.1(a), while the schematic illustration of the focused area captured by the CCD camera is shown in Fig. 4.1(b). The fabrication is based on the lamination thermal bonding technique. In this method, three polymethylmethacrylate (PMMA) plates (50 mm $\times$ 25 mm) are bonded together to form a closed microfluidic channel with the inlet and outlet holes. Thus, the middle PMMA defines the depth of the channel. A copper cylinder was inserted into the microchannel from the hole of the top PMMA plate. The length and diameter of the cylinder are 10 mm and 0.67 mm, respectively. An electric resistance wire (Nickel/Chromium, 80% Ni, 20% Cr, 0.125 mm diameter, 200 Ω /m) was wrapped around the copper cylinder. The number of windings is 20. The

vertical portion of the T-shaped microchannel has a cross section of $1000\ \mu\text{m} \times 2000\ \mu\text{m}$ and a length of 3 cm, while the straight portion has a cross section of $1000\ \mu\text{m} \times 2000\ \mu\text{m}$ and a length of 3 cm.

Fig. 4.1 Schematic illustration of the experimental setup for thermal effect on mass mixing. (a) The fabricated device. (b) Schematic illustration of the focused area captured by CCD camera. (c) Computational domain.

DI-water with fluorescent dye (fluorescein disodium salt $\text{C}_{20}\text{H}_{10}\text{Na}_2\text{O}_5$, Acid Yellow 73) and DI-water is introduced into inlet A and inlet B, respectively by two identical syringes. The flows are driven by a syringe pump (Cole-Parmer,

74900-05, 0.2 - 500 mL/h, accuracy of 0.5%) to supply the same volumetric flow rates. A low voltage power supply (GW Model GPC- 30300) is used to supply voltage to the electric resistance wire to enable heating of the copper cylinder. The images of the mixing process around the cylinder for different flow rates are captured by the CCD camera. The images of the mixing process are determined by a customised MATLAB program. This program is used to remove the noise of the collected images, and to calculate concentration distribution in the microchannel. A linear relationship is assumed between the fluorescence intensity and the concentration of the fluorescent dye [152].

To quantify the mixing efficiency of a micromixer, a region of interest (ROI) is needed in the captured images. The average intensity value of each row i in the ROI was calculated as I_i . This value was normalised by the maximum and minimum fluorescence intensity values:

$$\hat{I}_i = \frac{I_i - I_{\min}}{I_{\max} - I_{\min}} \quad (4.1)$$

where $\hat{I}_i$ is the normalised average intensity value of each row i . $I_{\max}$ and $I_{\min}$ are the maximum and minimum fluorescence intensity value in the unmixed states. The mixing efficiency is defined as:

$$\eta = 1 - \frac{\sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{I}_i - \hat{I}_{\infty})^2}}{\sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{I}_{oi} - \hat{I}_{\infty})^2}} \quad (4.2)$$

where N is the number of rows in the ROI, $\hat{I}_{oi}$ is the normalised average intensity value of each row i without mixing taking place (e.g., value 0 or 1),

$\hat{I}_{\infty}$ is the normalised average intensity value of each row i with complete mixing taking place (e.g., value 0.5).

To quantify the heating effect of the copper cylinder, DI-water and DI-water with fluorescent dye (Rhodamine B) are introduced into inlet A and inlet B, respectively by two identical syringes, which are driven by a syringe pump. A proportional relationship is assumed between fluorescence intensity and temperature [153]. The resulting intensity (normalised by the intensity at 25°C) versus temperature is the same as shown in Section 3.2.1.3.

4.2 Numerical Investigation on Thermal Effect on Mass Mixing in a T-shaped Microchannel

The schematic of the computational domain is shown in Fig. 4.1(c). The grey colour parts are the PMMA plates. A copper cylinder was placed in the T-shaped microchannel. The combined heat and mass transfer process of the thermal effect on mass mixing in the T-shaped microchannel is a conjugate forced convection-conduction problem. Therefore, it is needed to consider heat conduction in the PMMA plates and convective heat and mass transfer in the fluid. The temperature distribution in the PMMA plates is governed by the two-dimensional heat conduction equation while the temperature and concentration fields in the fluid region is governed by the two-dimensional momentum, energy and concentration equations. In the numerical simulation, laminar, incompressible, Newtonian fluid and steady flow are assumed. The density,

specific heat, thermal conductivity and viscosity of DI-water are allowed to vary with temperature as shown in Section 3.3.

4.2.1 Governing equations

4.2.1.1 Momentum equation

The governing equations can be written in a Cartesian coordinate system as follows. With the assumption of the incompressible, Newtonian fluid, the velocity field is described by the continuity and Navier-Stokes equations.

$$\nabla \cdot \vec{V} = 0 \quad (4.3)$$

$$\rho \left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} \right) = -\nabla p + \nabla (\mu \nabla \vec{V}) \quad (4.4)$$

where $\vec{V}$ is the velocity vector, p is the hydrodynamic pressure. ρ and μ are the fluid density and dynamic viscosity, respectively, whose dependence on temperature are given as shown in Section 3.3.

4.2.1.2 Energy equation

The temperature field in the microchannel is expressed as

$$\rho_f c_{pf} \left(\frac{\partial T_f}{\partial t} + \vec{V} \cdot \nabla T_f \right) = \nabla (k_f \nabla T_f) \quad (4.5)$$

where c_{pf} and k_f are specific heat and thermal conductivity of the fluid, respectively, whose dependence on temperature are given as shown in Section 3.3. In addition, since the thermal boundary condition on the channel wall is

unknown, a conjugate heat transfer analysis has to be modelled. The temperature field of the channel wall is expressed as

$$\rho_s c_{ps} \frac{\partial T_s}{\partial t} = k_s \nabla^2 T_s \quad (4.6)$$

where ρ_s, c_{ps}, k_s are the specific heat, density, and thermal conductivity of the microchannel wall (PMMA plate), respectively.

4.2.1.3 Concentration equation

The concentration field in the microchannel is governed by the mass transport equation expressed as:

$$\frac{\partial C}{\partial t} + \vec{V} \cdot \nabla C = \nabla (D \nabla C) \quad (4.7)$$

where C is the concentration of the fluorescent dye, and D is the temperature dependent mass diffusion coefficient of fluorescent dye [154], given by $D = D_0 [1 + 0.02(T - T_0)]$. D_0 is the mass diffusion coefficient of the fluorescent dye at room temperature T_0 .

4.2.2 Initial and boundary conditions

The temperature, concentration and transverse velocity along the y direction at the inlet are specified. The temperature, concentration and velocity at the inlet are assumed uniform. The non-slip and zero mass flux boundary conditions are specified at the walls. Constant surface temperature condition is set for the left and right walls. The top and bottom walls are subjected to natural convection. At the outlet, zero gradient conditions are imposed on the transverse velocity, temperature and concentration.

4.2.3 Numerical method

Since the governing equations for the velocity, temperature and concentration fields are strongly coupled in the temperature dependent parameters, a numerical method is used to solve this coupled problem. COMSOL MultiphysicsTM was employed to numerically solve Eqs. (4.3) to (4.7) to obtain the velocity field, temperature field and concentration field. A grid independence test was conducted to ensure the accuracy of the numerical simulation results.

4.3 Results and Discussion

The parameters investigated in the experiments and numerical simulations are the heating power applied to the copper cylinder and the Reynolds number. For different operating conditions, the concentration and temperature fields obtained from the simulation are validated by experiments.

4.3.1 Effect of heating power

Figure 4.2 shows the images taken in the experiments under the effect of heating power to the cylinder. The Reynolds number was maintained at 0.2. When no heating power was applied on the resistance wire, the mixing behaviour is simply mass mixing around a cylinder. Since the copper cylinder is positioned in the middle of the microchannel, the concentration distribution around the cylinder is almost symmetrical. There is almost no occurrence of mixing, as shown by a clear interface between the two fluids (Fig. 4.2(a)). This is expected for low Reynolds number flows in the microchannel where mixing is dominated by molecular diffusion. Once a heating power is applied on the

resistance wire, an enhancement in the mixing can be observed in Figs. 4.2(b) to 4.2(d), which depicts the thermal effect on the mixing behaviour. When a heating power 0.5 W is applied on the resistance wire, the copper cylinder will be heated. The interface between the two fluids in the microfluidic structure becomes obscure, suggesting the occurrence of mixing between the two fluids as shown in Fig. 4.2(b). At the applied heating power of 0.5 W mixing is poor (indicated by the large dark area in the ROI). However, mixing enhancement can be observed when a heating power 1.125 W and 2 W are applied on the resistance wire as shown in Figs. 4.2(c) and 4.2(d), respectively. This is due to the increase in surface temperature of the copper cylinder, resulting in the increase of the mass diffusivity.

Fig. 4.2 Thermal effect on mass mixing around a cylinder at $Re = 0.2$. The dotted lines denote the ROI.

A close examination of the temperature distribution at different heating power is shown in Fig. 4.3. The greyscale intensity images are shown in Figs. 4.3(a)

and 4.3(b) with the heating power of 0.5 W and 2 W, respectively. Based on the relationship between the fluorescent intensity and temperature, the colour-coded intensity images are shown in Figs. 4.3(c) and 4.3(d). From these figures, as the heating power applied on the resistance wire increases from 0.5 W to 2 W, the heating temperature increases from 30 °C to 40 °C, which confirms the above analysis.

Fig. 4.3 Measured temperature distribution under the effect of heating power at $Re=0.2$.
 (a), (c) Original and colour-coded intensity images for $P = 0.5$ W, respectively.
 (b), (d) Original and colour-coded intensity images for $P=2$ W, respectively.

4.3.2 Effect of Reynolds number

Figure 4.4 shows the images taken in the experiments under the effect of Reynolds number. The applied heating power is maintained at 2 W. When $Re = 0.2$, mixing enhancement can be observed as discussed in Section 4.3.1. As the Reynolds number increases from 0.2 to 0.8, the mixing behaviour between the two fluids becomes poorer (indicated by the increasingly dark region in the

ROI). This is expected as the residence time of the two fluids decreases with the increase of flow rate. The fluid temperature around the cylinder decreases as shown in Fig. 4.5.

Fig. 4.4 Thermal effect on mass mixing around a cylinder at $P = 2$ W. The dotted lines denote the ROI.

Fig. 4.5 Measured temperature field under the effect of flow rate at $P = 2$ W.
 (a), (c) Original and colour-coded intensity images for $Re = 0.4$, respectively.
 (b), (d) Original and colour-coded intensity images for $Re = 0.8$, respectively.

4.3.3 Comparison of numerical results with experimental results

Figure 4.6 shows the numerical results under the effect of heating power at the fixed Reynolds number ($Re = 0.2$). The concentration field under the effect of heating power is shown in Figs. 4.6(a) and 4.6(b), while the corresponding temperature field is shown in Figs. 4.6(c) and 4.6(d) respectively. It is observed that as temperature increases, the concentration distribution at the ROI becomes more uniform which indicate thermal effect enhances mixing. Figure 4.7 shows the experimental results under the effect of Reynolds number, where the heating power is maintained at 2 W. Figure 4.7(a) and 4.7(c) show the concentration and temperature fields for $Re = 0.4$, respectively. Figure 4.7(b) and 4.7(d) show the concentration and temperature fields for $Re = 0.8$, respectively. It is noted that the temperature around the cylinder decreases with the increase of flow rate. Figure 4.8(a) presents a comparison between the measured and the predicted

mixing efficiency variations with applied heating power. The results show that the mixing efficiency increases with increasing applied heating power. Since the thermal effect can enhance mixing efficiency, the limiting value is observed to be 0.4. Figure 4.8(b) presents a comparison between the measured and the predicted mixing efficiency variations with Reynolds number. The mixing efficiency decreases with increasing Reynolds number.

Fig. 4.6 Numerical results of concentration and temperature fields under the effect of applied voltage at $Re = 0.2$.

- (a), (c) Concentration and temperature fields for $P = 0.5$ W, respectively.
 (b), (d) Concentration and temperature fields for $P = 2$ W, respectively.

Fig. 4.7 Numerical results of concentration and temperature fields under the effect of flow rate at $P = 2$ W.
 (a), (c) Concentration and temperature fields for $Re = 0.4$, respectively.
 (b), (d) Concentration and temperature fields for $Re = 0.8$, respectively.

Fig. 4.8 Comparison between numerical and experimental results of the mixing efficiency.
 (a) Effect of heating power. (b) Effect of Reynolds number.

4.4 Summary

This chapter reports a study of thermal effect on mass mixing in a T-shaped microchannel with a copper cylinder embedded at the T-junction. Fluorescent dye Acid Yellow and Rhodamine B were used to characterise the thermal effect

on mass mixing behaviour. The effects of heating power and Reynolds number were experimentally investigated. The experimental results show that a higher heating power and a lower Reynolds number enhance mixing efficiency. The proposed numerical model gives reasonable agreement with the experimental data.

Chapter 5

Thermal Flow Sensor

Flow measurement is of great importance in many applications of microfluidics such as pharmaceutical industry, medical and (bio)chemical analysis. Many kinds of micro flow sensors based on different measuring principles have been proposed [155-159]. Conventional micro flow sensors concepts such as pyroelectric elements, pn-junctions and resonating microbridges require complex fabrication processes and extensive measurement setup [160].

Thermal flow sensors usually employ a temperature field to measure the fluid velocity and direction in a microfluidic system. Thermal flow sensing is popular due to its low cost and high-specification performance characteristics [67]. Hot wires and hot films are the two simplest forms of thermal flow sensor, but they can only provide fixed point measurements. The more sensitive calorimetric concept uses a miniaturised heat source and two temperature sensors located symmetrically on both side of the heat source [77]. The flow can be calibrated by measuring the temperature difference (ΔT) between two positions, one upstream and one downstream of the heat source. Its working principle is to measure the heat transfer from the heat source to the surrounding fluid, which is then carried away by convection [63]. The rate of heat transfer is proportional to the mass flow in the microchannel.

Many researchers in the past had focused on the calorimetric type of thermal flow sensor due to its high sensitivity [64, 65, 68, 69, 161]. The accuracy of the

calorimetric concept is limited by the drift in the electrical resistance of the thin film layers and the influence of external temperature gradients across the sensor element [162]. In order to overcome the limitations of the sensor, Dijkstra *et al.* [162] designed a highly sensitive micro thermal flow sensor using thermopiles combined with a controlled heater resistor which eliminated the drift in the electrical resistance. The special layout of the microchannel compensated for the influence of varying external temperature gradients on the temperature sensor. Wiegerink *et al.* [163] developed a micro thermal and Coriolis flow sensor based on the measurement of Coriolis forces, which were independent of temperature and fluid properties.

This chapter presents a micro thermal flow sensor for liquid flows. The flow sensor consists of a heating element embedded in a straight microchannel with a rectangular cross section. The Laser Induced Fluorescence (LIF) method was employed to measure the temperature field in the microchannel. Compared to conventional temperature measurements based on sensing elements located at fixed points in the flow, the LIF method provides isotherm contours of the fluid in the microchannel. Experiments were performed to show the feasibility of the approach for thermal flow sensing. In addition the author had developed a two-dimensional numerical model for comparison with the measured results.

5.1 Experimental Investigation on Thermal Flow Sensor

The thermal flow sensor used in the experiment is shown in Fig. 5.1(a), while the schematic illustration of the focused area captured by the CCD camera is shown in Fig. 5.1(b). The fabrication is based on the lamination thermal bonding technique. In this method, three polymethylmethacrylate (PMMA)

plates (50 mm $\times$ 25 mm) are bonded together to form a closed microfluidic channel with inlet and outlet holes. Thus, the middle PMMA defines the depth of the channel. A copper cylinder is inserted into the microchannel from the hole of the top PMMA plate. The diameter of the cylinder is 0.67 mm. An electric resistance wire is wrapped on the copper cylinder circumference. The straight channel has a cross section of 1000 μm $\times$ 2000 μm and a length of 3 cm. DI-water with Rhodamine B solution (0.025 g/100 mL) is introduced into inlet port by a syringe pump. A low voltage power supply is used to supply voltage to the electric resistance wire to enable heating of the copper cylinder. The images of the Rhodamine B solution around the heater for different flow rates are captured by the CCD camera. The Laser Induced Fluorescence method presented in Section 3.2.1.2 is used to measure the full temperature field of the fluid in a microchannel.

Fig. 5.1 Schematic illustration of the experimental setup of a thermal flow sensor.
 (a) The fabricated device. (b) Schematic illustration of the focused area captured by a CCD camera. (c) Computational domain (including two PMMA plates, microchannel and a heating element).

5.2 Numerical Investigation on Thermal Flow Sensor

The heat transfer process of a micro thermal flow sensor is a conjugate forced convection-conduction problem. The heat transfer process of the thermal flow sensor (Fig. 5.1(c)) can be divided into two regions; a solid region (PMMA plates) and a fluid region in the microchannel. The temperature distribution in the solid region is predominantly governed by the two-dimensional heat conduction equation while the temperature and velocity distributions in the fluid region are governed by the energy and momentum equations. In the model, laminar, incompressible, Newtonian fluid and steady flow are assumed. The density, specific heat, thermal conductivity and viscosity of DI-water are allowed to vary with temperature [142].

The dimensions of the microchannel, $2L$, $2H$ and $2W$ are specified in Fig. 5.1(c). The cylinder is located in the centre of the microchannel. A Cartesian coordinate system (x, y) is used where the origin point, O , is set at the centre of the cylinder. DI-water at room temperature enters the microchannel. A constant surface temperature (T_h) is maintained at the circumference of the cylinder.

5.2.1 Governing equations

The governing equations can be written in a Cartesian coordinate system as follows.

5.2.1.1 Fluid velocity field

The fluid velocity field is described by the continuity and Navier-Stokes equations.

$$\nabla \cdot \vec{V} = 0 \quad (5.1)$$

$$\rho \vec{V} \cdot \nabla \vec{V} = -\nabla p + \nabla (\mu \nabla \vec{V}) \quad (5.2)$$

where $\vec{V}$ is the fluid velocity vector and p is the hydrodynamic pressure. ρ and μ are the fluid density and viscosity, respectively. Their units are kg/m^3 , $\text{Pa} \cdot \text{s}$, respectively. Their dependence on temperature (T) are given as follows [142] :

$$\rho = \frac{999.8 + 18.2 \times T - 7.9 \times 10^{-3} \times T^2 - 5.5 \times 10^{-5} \times T^3 + 1.5 \times 10^{-7} \times T^4 - 3.9 \times 10^{-10} T^5}{1 + 1.8 \times 10^{-2} \times T} \quad (5.3)$$

$$\mu = 2.414 \times 10^{-5} \times 10^{247.8/(T+133)} \quad (5.4)$$

5.2.1.2 Fluid and channel wall temperature fields

The temperature field in the microchannel is governed by the energy equation as follows:

$$\rho_f c_{pf} \vec{V} \cdot \nabla T_f = \nabla (k_f \nabla T_f) \quad (5.5)$$

where c_{pf} and k_f are specific heat and the thermal conductivity of the working fluid, respectively. Their units are $\text{J/kg} \cdot \text{K}$ and $\text{W/m} \cdot \text{K}$, respectively. Their dependence on temperature (T) are given as follows [142]:

$$c_{pf} = 8958.9 - 40.535 \times (T + 273) + 0.11243 \times (T + 273)^2 - 1.0138 \times 10^{-4} \times (T + 273)^3 \quad (5.6)$$

$$k_f = -0.58166 + 6.3555 \times 10^{-3} \times (T + 273) - 7.9643 \times 10^{-6} \times (T + 273)^2 \quad (5.7)$$

Since the thermal boundary condition on the channel walls is unknown, a conjugate heat transfer analysis has to be performed. The temperature field of the channel wall is expressed as

$$\frac{k_s}{\rho_s c_{ps}} \nabla^2 T_s = 0 \quad (5.8)$$

where ρ_s, c_{ps}, k_s are the specific heat, density, and thermal conductivity of the microchannel wall (PMMA plate), respectively.

5.2.2 Initial and boundary conditions

The initial conditions in the fluid region and the solid region are:

$$T_i = 25^\circ\text{C}, \quad u_i = 0, \quad v_i = 0 \quad (5.9)$$

At the inlet ($x = -L, -H < y < H$),

$$T_{in} = 25^\circ\text{C}, \quad u = u_{in}, \quad v = 0 \quad (5.10)$$

At the outlet ($x = L, -H < y < H$),

$$\frac{\partial T_f}{\partial x} = 0, \quad \frac{\partial u}{\partial x} = 0, \quad \frac{\partial v}{\partial x} = 0 \quad (5.11)$$

The boundary conditions in the solid region are:

At the left wall ($x = -L, -W < y < -H, H < y < W$),

$$T_s = 25^\circ\text{C} \quad (5.12)$$

At the right wall ($x = L, -W < y < -H, H < y < W$),

$$T_s = 25^\circ\text{C} \quad (5.13)$$

At the top and bottom walls ($y = W, y = -W$),

$$\frac{\partial T_s}{\partial y} = 0 \quad (5.14)$$

The boundary conditions at the interface between the solid and the fluid regions are governed by the continuity of heat flux and temperature:

At the upper interface ($y = H$),

$$T_s = T_f, \quad k_s \frac{\partial T_s}{\partial y} = k_f \frac{\partial T_f}{\partial y} \quad (5.15)$$

At the lower interface ($y = -H$),

$$T_s = T_f, \quad k_s \frac{\partial T_s}{\partial y} = k_f \frac{\partial T_f}{\partial y} \quad (5.16)$$

At the channel walls, the no-slip boundary condition is applied to the momentum equation. A constant surface temperature (T_h) is applied at the circumference of the cylinder.

5.2.3 Numerical method

Since the governing equations for the velocity and temperature fields are strongly coupled in the temperature dependent parameters, COMSOL MultiphysicsTM was employed to solve Eqs. (5.1) - (5.8) to obtain the velocity field and the temperature field of the fluid.

A grid independence test was conducted by using different numbers of unstructured grids. Three different grid numbers, 1284, 5136 and 20544, were tested. The conditions are $L = 0.015$ m, $H = 0.001$ m, $W = 0.012$ m, $Q = 0$ $\mu\text{L/h}$, $k_s = 0.25$ W/m·K, $k_f = 0.6$ W/m·K and $T_h = 50$ °C. Grid

independence was ensured as the differences in u , v and T at all grid points are less than 1%.

5.3 Results and Discussion

Figure 5.2 shows the isotherms at different flow rates. The experimental results are presented on the left column, while the numerical results are presented on the right column. The numerical results agree with the experimental isotherms. At zero flow ($Q = 0 \text{ } \mu\text{L/h}$), a symmetrical temperature distribution (Figs. 5.2(a) and 5.2(d)) is obtained. The heat transfer is governed by pure heat conduction. All the isotherms are concentric and exhibit no effect of convection. Figure 5.2(b) to 5.2(f) show representative isotherms in the vicinity of the cylinder for different flow rates. At low flow rate, heat transfer occurs primarily by conduction and is seen as concentric circles. With increasing flow rate, the isotherms behind the cylinder expand in the downstream direction.

To gain a better understanding of the thermal characteristics of the thermal flow sensor, the temperature distribution along the centre axis of the microchannel with different flow rates is plotted in Fig. 5.3. Figure 5.3(a) shows the temperature profile along the centre axis of the microchannel with flow rate $Q = 0 \text{ } \mu\text{L/h}$. As expected, a symmetrical temperature is obtained by heat conduction. At higher flow rate, the temperature distribution exhibits an asymmetric profile as shown in Figs. 5.3(b) and 5.3(c). The results show that the temperature at the downstream of the heated cylinder is higher as compared

to the upstream temperature. The model agrees well with the experimental results.

Fig. 5.2 Experimental and numerical isotherms for different flow rates, experiment (left) and numerical (right). ($T_h = 46^\circ\text{C}$, $T_{in} = 25^\circ\text{C}$)

Fig. 5.3 Comparison of experimental result with numerical model for different flow rates
 $(T_h = 46\text{ }^{\circ}\text{C}, T_{in} = 25\text{ }^{\circ}\text{C})$, where R_h is the radius of the cylinder.

To investigate the difference of the temperatures (ΔT) between two sensing positions (in the upstream and downstream of the heated cylinder) with increasing flow rate, the following sensing positions: 2d, 3d, 4d and 5d are selected as shown in Fig. 5.1(b). Figure 5.4 presents a comparison between the measured and the predicted temperature difference variations with flow rate at various sensing positions. The model agrees with the experimental results. The results show that temperature difference increases as the flow rate increases.

Fig. 5.4 Comparison of experimental result with numerical model for various sensing positions ($T_h = 46^\circ\text{C}$, $T_{in} = 25^\circ\text{C}$).

5.4 Summary

The heat transfer characteristics of a thermal flow sensor were investigated experimentally and numerically. The important results obtained are:

- (1) The heat transfer process of a thermal flow sensor is a conjugate forced convection-conduction heat transfer problem. The heat conduction in the channel wall should be taken into consideration.
- (2) The full isotherm contours of the fluid within the thermal flow sensor can be obtained from the Laser Induced Fluorescence (LIF) method.

Chapter 6

Counter Flow Micro-Heat Exchanger (CFMHE)

In the last decade, due to the rapid development of micro-electronics and biotechnology, micro heat exchangers (MHE) have attracted increasing attention from the research community [164]. Compared with conventional heat sinks and heat pipes, micro heat exchangers are proved to be effective in removing high heat fluxes which take advantage of the large surface-to-volume ratio and short transport path [136]. Therefore, there is a need for understanding the fluid flow and heat transfer characteristics in a microchannel [165].

The process of heat exchange between two fluids that are at different temperatures and separated by a solid wall occurs in many engineering applications. With the development of MEMS, micro heat exchangers are widely used in many fields such as biological engineering and temperature control of superconductors. Compared to the macro heat exchanger, the main advantage of the micro heat exchanger is its extremely high heat transfer coefficient. In this chapter, the counter flow micro heat exchanger will be introduced to analyse the local Nusselt number in microchannels.

6.1 Analytical Investigation on CFMHE

The hot and cold fluid temperature distributions associated with a counter flow heat exchanger are shown in Fig. 6.1. The fluids flow in opposite directions. For the two fluids with equivalent heat capacity rates $\left(\dot{m}c_p\right)_h = \left(\dot{m}c_p\right)_c$, the

temperature difference between the two fluids, $\Delta T = T_h - T_c$, is a constant value with respect to x . The endpoint temperature differences are defined as follows:

$$\begin{bmatrix} \Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i} \end{bmatrix} \quad (6.1)$$

Fig. 6.1 Temperature distribution for a counter flow heat exchanger with the same heat capacity rates.

6.1.1 Assumptions

The micro heat exchanger has two inlets for the hot and cold fluids and two outlets. The width, height and length of the channel are W , H and L , respectively.

In the analysis, the following assumptions are made:

1. Continuum assumption;
2. Decoupled convection and diffusion processes. The process is assumed to occur in a fully developed flow field;
3. The viscosity and thermal diffusivity are independent of temperature;
4. Laminar, incompressible, Newtonian fluid, steady flow are assumed;
5. Convection is assumed to occur in one direction only, axially along the channel.

6.1.2 Governing equations

Under the above assumptions, the behaviour of the two-fluid flow in rectangular microchannels can be described by the governing equations as shown in section 3.1.2. For this special case, the heat flux between the hot and the cold fluids is constant. Based on this condition, an analytical model is proposed to analyse the heat transfer process.

6.1.3 Analytical solution

Fig. 6.2 Two-dimensional model of heat transfer in a counter flow heat exchanger.
 (a) The actual model (hot stream). (b) The dimensionless model (hot stream).
 (c) The actual model (cold stream). (d) The dimensionless model (cold stream).

An analytical model is developed to describe the heat transfer process in the micro heat exchanger as shown in Fig. 6.2. Figures 6.2(a) and 6.2(b) illustrate a two-dimensional model for the heat transfer process of the hot stream. The microchannel has a width of W and a length of L . The flow in the rectangular channel is assumed to have a constant velocity of U . The hot stream at a high temperature $T_{h,i}$ enters the upper region of the micro heat exchanger, while leaving the channel with a low temperature $T_{h,o}$. At the bottom wall ($\hat{y}=0$), a constant heat flux boundary condition is specified due to the heat exchange

between the hot and cold streams with the same heat capacity rates. At the upper wall ($\hat{y} = 1$), a constant heat flux boundary condition is specified due to the heat loss from the upper wall of the hot stream to the surrounding.

The governing equation can be formulated in the dimensional form as:

$$\alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = U \frac{\partial T}{\partial x} \quad (6.2)$$

where α is the thermal diffusivity of the fluid ($\alpha = k / (\rho c_p)$). By introducing the dimensionless coordinate system $\hat{x} = x / W$, $\hat{y} = y / W$, the normalised temperature $\Theta = \frac{T - T_{h,o}}{T_{h,i} - T_{h,o}}$ and the Peclet number,

$$\text{Pe} = \frac{UW}{\alpha} \quad (6.3)$$

Equation (6.2) can be formulated in the dimensionless form as:

$$\frac{\partial^2 \Theta}{\partial \hat{x}^2} + \frac{\partial^2 \Theta}{\partial \hat{y}^2} = \text{Pe} \frac{\partial \Theta}{\partial \hat{x}} \quad (6.4)$$

The inlet and outlet boundary conditions for Eq. (6.4) are:

$$\Theta|_{(\hat{x}=0)} = 1, \quad \Theta|_{(\hat{x}=1)} = 0 \quad (6.5)$$

The constant heat flux boundary conditions are applied to the upper and bottom walls:

$$\left. \frac{\partial \Theta}{\partial \hat{y}} \right|_{\hat{y}=0} = E_2 W / (T_{h,i} - T_{h,o}), \quad \left. \frac{\partial \Theta}{\partial \hat{y}} \right|_{\hat{y}=1} = E_1 W / (T_{h,i} - T_{h,o}) \quad (6.6)$$

Assuming the solution can be written in a series expansion as:

$$\Theta(\hat{x}, \hat{y}) = \sum_{n=0}^{\infty} \Theta_n(\hat{x}) \Phi_n(\hat{y}) \quad (6.7)$$

$$\Phi_0(\hat{y}) = 1, \quad \Phi_n(\hat{y}) = \sqrt{2} \cos n\pi \hat{y}, \quad n = 1, 2, \dots \quad (6.8)$$

The Finite Fourier Transform (FFT) of the temperature is defined as

$$\Theta_n(\hat{x}) = \int_0^1 \Phi_n(\hat{y}) \Theta(\hat{x}, \hat{y}) d\hat{y} \quad (6.9)$$

Applying the FFT to both sides of Eq. (6.4) gives:

$$\int_0^1 \Phi_n(\hat{y}) \left[\frac{\partial^2 \Theta}{\partial \hat{x}^2} + \frac{\partial^2 \Theta}{\partial \hat{y}^2} \right] d\hat{y} = \int_0^1 \Phi_n(\hat{y}) \text{Pe} \frac{\partial \Theta}{\partial \hat{x}} d\hat{y} \quad (6.10)$$

The first term is rewritten in terms of the transformed temperature as:

$$\int_0^1 \Phi_n(\hat{y}) \frac{\partial^2 \Theta}{\partial \hat{x}^2} d\hat{y} = \frac{d^2}{d\hat{x}^2} \int_0^1 \Phi_n(\hat{y}) \Theta d\hat{y} = \frac{d^2 \Theta_n}{d\hat{x}^2} \quad (6.11)$$

The second term is rearranged using two integrations by parts to give:

$$\begin{aligned} \int_0^1 \Phi_n(\hat{y}) \frac{\partial^2 \Theta}{\partial \hat{y}^2} d\hat{y} &= \left(\Phi_n \frac{\partial \Theta}{\partial \hat{y}} \right) \bigg|_{\hat{y}=0}^{\hat{y}=1} - \left(\Theta \frac{\partial \Phi_n}{\partial \hat{y}} \right) \bigg|_{\hat{y}=0}^{\hat{y}=1} + \int_0^1 \Theta \frac{\partial^2 \Phi_n}{\partial \hat{y}^2} d\hat{y} \\ &= E_1 \Phi_n(1) - E_2 \Phi_n(0) - (n\pi)^2 \Theta_n \end{aligned} \quad (6.12)$$

The right term is rewritten in terms of the transformed temperature as:

$$\int_0^1 \Phi_n(\hat{y}) \text{Pe} \frac{\partial \Theta}{\partial \hat{x}} d\hat{y} = \text{Pe} \frac{d}{d\hat{x}} \int_0^1 \Phi_n(\hat{y}) \Theta d\hat{y} = \text{Pe} \frac{d\Theta_n}{d\hat{x}} \quad (6.13)$$

The results are combined to give the differential equation for $\Theta_0(\hat{x})$ and

$$\Theta_n(\hat{x})$$

$$\frac{d^2 \Theta_0}{d\hat{x}^2} - \text{Pe} \frac{d\Theta_0}{d\hat{x}} = E_2 - E_1 \quad (6.14)$$

$$\frac{d^2 \Theta_n}{d\hat{x}^2} - \text{Pe} \frac{d\Theta_n}{d\hat{x}} - (n\pi)^2 \Theta_n = \sqrt{2} \left[E_2 - (-1)^n E_1 \right] \quad (6.15)$$

Equations (6.14) and (6.15) require two boundary conditions, which are obtained by transforming the original boundary conditions. The overall solution is:

$$\begin{aligned}\Theta(\hat{x}, \hat{y}) = & A_1 + A_2 \exp(\text{Pe} * \hat{x}) - (E_2 - E_1) * \hat{x} / \text{Pe} \\ & + \sqrt{2} \cos(n\pi \hat{y}) \sum_{n=1}^{\infty} B_1 \exp(\lambda_1 * \hat{x}) + B_2 \exp(\lambda_2 * \hat{x}) - A\end{aligned}\quad (6.16)$$

where

$$A = \frac{\sqrt{2} [E_2 - (-1)^n E_1]}{(n\pi)^2} \quad (6.17)$$

$$A_2 = \left(1 - \frac{E_2 - E_1}{\text{Pe}} L\right) / (1 - \exp(\text{Pe} * L)), \quad A_1 = 1 - A_2 \quad (6.18)$$

$$\lambda_{1,2} = \left(\text{Pe} \pm \sqrt{\text{Pe}^2 + 4(n\pi)^2}\right) / 2 \quad (6.19)$$

$$B_2 = \frac{A(1 - \exp(\lambda_1 * L))}{\exp(\lambda_2 * L) - \exp(\lambda_1 * L)}, \quad B_1 = A - B_2 \quad (6.20)$$

A two-dimensional model for fluid flow in the lower region of the micro heat exchanger is depicted in Figs. 6.2(c) and 6.2(d). The cold stream with a low temperature $T_{c,i}$ enters the lower region of the micro heat exchanger, while leaving the channel with a high temperature $T_{c,o}$. Similar with the upper region of the micro heat exchanger, the solution for the lower region of the micro heat exchanger can be obtained as follows:

$$\begin{aligned}\Theta(\hat{x}, \hat{y}) = & C_1 + C_2 \exp(\text{Pe} * \hat{x}) - (E_3 - E_4) * \hat{x} / \text{Pe} \\ & + \sqrt{2} \cos(n\pi \hat{y}) \sum_{n=1}^{\infty} D_1 \exp(\lambda_1 * \hat{x}) + D_2 \exp(\lambda_2 * \hat{x}) - B\end{aligned}\quad (6.21)$$

where

$$B = \frac{\sqrt{2} [E_3 - (-1)^n E_4]}{(n\pi)^2} \quad (6.22)$$

$$C_2 = \left(1 + \frac{E_3 - E_4}{\text{Pe}} L\right) / (\exp(\text{Pe} * L) - 1), \quad C_1 = -C_2 \quad (6.23)$$

$$D_2 = \frac{B(1 - \exp(\lambda_1 * L))}{\exp(\lambda_2 * L) - \exp(\lambda_1 * L)}, \quad D_1 = B - D_2 \quad (6.24)$$

The values of the heat flux (E_1, E_2, E_3, E_4) at the walls were calculated from the experimental data.

6.2 Experimental Investigation on CFMHE

6.2.1 Materials and fabrications

The setup of the experiment is shown in Fig. 6.3(a). The H-shaped microfluidic device used in the experiment is shown in Fig. 6.3(b), while the schematic illustration of the focused area captured by the CCD camera is shown in Fig. 6.3(c). The fabrication is based on the lamination thermal bonding technique. In this method, three polymethylmethacrylate (PMMA) plates ($50 \text{ mm} \times 50 \text{ mm}$) were bonded together to form a closed microfluidic channel with inlet and outlet holes. After the thermal bonding process, a micro-heat exchanger with a cross section of $500 \text{ } \mu\text{m} \times 1000 \text{ } \mu\text{m}$ is fabricated. The Laser Induced Fluorescence (LIF) method presented in Section 3.2.1.2 was used to measure the fluid temperature field in the microchannel.

Based on the literature review, an electrical resistance wire was often employed to wrap around a microchannel to provide the constant-heat-flux boundary

condition. The main drawback of this method is that the heat loss to the surroundings, which will lead to a non-uniform distribution of heat flux at the boundary. This section attempts to use the concept of a counter flow micro-heat exchanger with equivalent heat capacity rates of the fluids to realise a constant-heat-flux boundary condition. In this configuration, a constant-heat-flux will be obtained due to the heat transfer between the hot and cold streams in the micro-heat exchanger. The fabricated device used in the experiment is shown in Fig. 6.3(b). The enlarged schematic illustration of the counter flow micro-heat exchanger is shown in Fig. 6.3(c). In this model, the heat transfer process is sub-divided into the upper and lower fluid channels. A hot stream with a higher temperature $T_{h,i}$ flows into the inlet arms of the upper fluid channel (a-b-d-c), while a cold stream with a lower temperature $T_{c,i}$ flows into the inlet arms of lower fluid channel (l-m-o-n). Subsequently, the hot stream flows in the horizontal part of upper fluid channel (A-B-D-C) from left to right, while the cold stream flows in the horizontal part of lower fluid channel (E-F-H-G) from right to left. During the counter flow process, heat transfer from the hot stream to the cold stream through the wall of the PMMA plate. Hence, for the hot stream $T_{h,o} < T_{h,i}$, while for the cold stream $T_{c,o} > T_{c,i}$. The hot and cold streams experience little or no change in their velocities and elevations, and thus the kinetic and potential energy changes are negligible. For an ideal circumstance where the outer surface of the heat exchanger is perfectly insulated, there is no heat loss to the surrounding. Hence, the rate of heat transfer from the hot stream is equal to the rate of heat transfer to the cold one. That is,

$$\dot{Q} = (\dot{m}c_p)_h (T_{h,i} - T_{h,o}) \quad (6.25)$$

and,

$$\dot{Q} = (\dot{m}c_p)_c (T_{c,o} - T_{c,i}) \quad (6.26)$$

where the subscripts h and c stand for hot and cold streams, respectively. For the hot and cold streams with equivalent heat capacity rates $(\dot{m}c_p)_h = (\dot{m}c_p)_c$, the temperature difference between the two fluids is a constant value respect to x . However, it is difficult to perfectly insulate the outer surface of the heat exchanger. Two rectangular slots were introduced near the upper and lower fluid channels to avoid heat loss by conduction.

6.2.3 Local Nusselt number

The local convective heat transfer coefficient and the corresponding local Nusselt number along the microchannel were obtained from the Newton's cooling law,

$$h = \frac{k_l \partial T / \partial y}{T_m - T_s} \quad (6.27)$$

$$\text{Nu} = \frac{hD_h}{k_l} \quad (6.28)$$

where T_m is the mean temperature across the microchannel.

The hydraulic diameter, D_h

$$D_h = \frac{2WH}{W + H} \quad (6.29)$$

The Reynolds number, Re

$$\text{Re} = \frac{\rho U D_h}{\mu} \quad (6.30)$$

The Brinkman number, Br

$$\text{Br} = \frac{\mu_l u_m^2}{k_s \partial T / \partial y \cdot D_h} \quad (6.31)$$

The Prandtl number, Pr

$$Pr = \frac{\mu_l c_p}{k_l} \quad (6.32)$$

U , k_s and k_l are the fluid velocity, PMMA wall thermal conductivity and fluid thermal conductivity respectively. ρ_l , μ_l and c_p are the local fluid density, fluid dynamic viscosity and fluid specific heat, respectively. The local fluid properties are functions of the mean fluid temperature.

6.3 Results and Discussion

6.3.1 Experimental results and discussion

The Laser Induced Fluorescence (LIF) technique is used to measure the fluid temperature in the microchannel. A typical grey scale image for the counter flow micro heat exchanger is shown in Fig. 6.4. The resulting intensity (normalised by the intensity at 25°C) versus temperature is the same as shown in Section 3.2.1.3. Figure 6.5(a) and 6.5(b) shows the measured temperature profiles for the hot and cold streams of which the flow rate was maintained at 1000 $\mu\text{L/h}$. The formation of temperature contours is observed, indicating that heat transfer between hot and cold streams occurred in the counter flow micro-heat exchanger. The hot stream temperature decreases from 41°C to 36°C, while the cold stream increases from 32°C to 36°C. The resulting mean temperature distribution of the hot and cold streams and the surface temperature are shown in Fig. 6.6(a). As noted, a constant temperature gradient is maintained at the hot PMMA wall (CD) and the cold PMMA wall (EF), hence a

constant heat flux boundary condition is obtained in the experiment. The experimental temperature gradient is shown in Fig. 6.6(b). A straight fitting line is plotted to show an almost constant temperature gradient obtained in the experiment.

Fig. 6.4 A typical grey scale image for the counter flow microchannel heat exchanger.

Fig. 6.5 Experimental temperature distribution ($Q = 1000 \mu\text{L/h}$).
(a) Temperature contours of hot stream. (b) Temperature contours of cold stream.

Fig. 6.6 Experimental temperature distribution in micro-heat exchanger.
 (a) Mean temperature and surface temperature versus channel length.
 (b) Temperature gradient versus channel length.

The variation of the experimental local Nusselt number (Nu) with the channel length for the hot stream and cold stream is depicted in Figs. 6.7(a) and 6.7(b).

The decrease of local Nusselt number with the increase of channel length represents the existence of the entrance region, which is important for fluid flow and heat transfer in the micro heat exchanger. The experimental local Nusselt number obtained in the microchannel can be correlated by Eq. (6.33) for the Reynolds number which is in the order of 1 (i.e. $Re < 1$),

$$Nu = 1.58 \times Re^{1.23} Pr^{0.39} \left(\frac{x}{L} \right)^{-0.08} \quad (6.33)$$

Equation (6.33) indicates that the Nusselt number for fluid flow and heat transfer in rectangular microchannels is significantly dependent on the Reynolds number (Re) and the Prandtl number (Pr). Compared to other dimensionless numbers, the Brinkman number (Br) is too low (of the order of 10^{-11}) to influence the convective heat transfer characteristics in the microchannel. Therefore, when $Re < 1$, the Brinkman number is not included in Eq. (6.33). The comparison between the correlated Nusselt number and experimental Nusselt number are presented in terms of Root Mean Square error (RMS), which is defined as follows,

$$RMS \text{ error} = \left[\frac{1}{N} \sum_1^N \left(\frac{S_{pred} - S_{exp}}{S_{exp}} \right)^2 \right]^{1/2} \quad (6.34)$$

where S is the quantity being assessed such as the Nusselt number. The RMS error gives the arithmetic mean absolute error of the data. Estimation of the RMS error in the experimental results is 0.02. From the error analysis, it can be concluded that there is a small discrepancy between the correlated and experimental Nusselt numbers. Figure 6.7(c) presents comparison between measured Nusselt number and the correlated Nusselt number. The Nusselt number versus the Reynolds number at three different sections ($x/L = 0.1$,

$x/L = 0.5$ and $x/L = 1$) is shown in Fig. 6.7(d). It can be seen that the Nusselt number increases as the Reynolds number increases. Near the channel entrance ($x/L = 0.1$), the Nusselt number increases rapidly. In the middle section of micro-heat exchanger ($x/L = 0.5$), the Nusselt number increases at a slower rate initially before speeding up with the increase of the Reynolds number. At the channel outlet ($x/L = 1$), the Nusselt number will undergo a slower increase as compared to the middle section. The variation of the Nusselt number with the Reynolds number shows that as the distance to the entrance decreases, the sensitivity of the variation increases.

6.3.2 Comparison between analytical analysis and experimental results

Analytical temperature distributions for the hot and cold streams are shown in Figs. 6.8(a) and 6.8(b), respectively. Compared to the measured temperature profiles (Figs. 6.5(a) and 6.5(b)), the analytical results agree with the experimental results. The mean temperature distribution along the channel length is shown in Fig. 6.8(c). As the channel length increases, the temperature difference between the hot and cold streams is maintained at a nearly constant value.

Fig. 6.7 Experimental Nusselt number.

- (a) Local Nu versus channel length for the hot stream.**
- (b) Local Nu versus channel length for the cold stream.**
- (c) Experimental Nu versus correlated Nu.**
- (d) Variation of Nu versus Re at different sections.**

Fig. 6.8 Comparison between analytical and experimental temperature distributions ($Q = 1000 \mu\text{L/h}$).

(a) Analytical temperature contours for hot stream.

(b) Analytical temperature contours for cold stream.

(c) Comparison of experimental mean temperature distributions with analytical model.

6.3.3 Uncertainty analysis

Uncertainty analysis of the experimental system was carried out to determine the total errors that will exist in the results. Propagation of individual measure uncertainties will be used to estimate the final uncertainty of Reynolds number, Nusselt number, Brinkman number and Prandtl number. Details of calculations are listed in Appendix B.

6.4 Summary

In this chapter, the heat transfer characteristics of fluid flow in a micro-heat exchanger (MHE) are investigated theoretically and experimentally. With the help of the Laser-Induced Fluorescence (LIF) method (Section 3.2.1.2), DI-water with temperature-dependent fluorescent Rhodamine B is employed to show the temperature field in the MHE. The measured mean temperature distributions match well with the analytical model. Based on the measured temperature field, local Reynolds number (Re) and Prandtl number (Pr) can be obtained. A correlation for the calculation of Nusselt number (Nu) is developed. The convective heat transfer in microchannels is found to be significantly dependent on the Reynolds number (Re) and Prandtl number (Pr). The Nusselt number is found to increase with the Reynolds number and the closer distance to the entrance, the more sensitive of the variation of Nusselt number versus Reynolds number.

Chapter 7

Temperature-Induced Droplet Coalescence

In recent years, droplet-based microfluidics has emerged as an alternative for continuous-flow microfluidics. Droplet-based microfluidics has advantages such as fast mixing, short residence time and low cross contamination [166, 167] when compared to continuous phase based microfluidics. This new technology has been used in various applications such as micro reactor [167], on-chip polymerase chain reaction (PCR) [168] and protein crystallisations [169]. Mixing of fluids in a droplet requires merging droplets that initially are separated. Droplet coalescence can be achieved by different schemes. Olbricht and Kung [170] realised coalescence utilising the differences in velocity and size of the droplets. However, coalescence of droplet with the same size and the same velocity is difficult.

In order to achieve complete mixing within a droplet, numerous droplet coalescence techniques have been developed which can be categorised as passive and active techniques. Passive techniques utilise structure of microchannel to trap droplet in a microchannel. With the help of the structures, the trapped droplet will wait for the subsequent droplet for coalescence. Two neighbouring droplets have a chance to get close to each other due to the drainage of the continuous phase in the fluid resistance element. Kohler *et al.* [107, 108] proposed a locally enlarged diameter of fluid channel to realise droplet coalescence. Tan *et al.* [105] and Bremond *et al.* [109] developed a trapping method that employs a rectangular expansion, a tapered expansion and a

flow rectifying structure. Hung *et al.* [110] developed a multifunctional and highly efficient microfluidic device to achieve droplet coalescence. This device consisted of three inlets, a double T-junction and a tapered merging chamber. Niu *et al.* [111] employed pillars in the microchannel to initiate droplet coalescence. Active droplet coalescence technique employs energy generated by an external field to realise droplet coalescence. Examples are the droplet coalescence using electro-coalescence [112-114, 116-118], dielectrophoresis actuated droplet coalescence [119-121] and pneumatically actuated droplet coalescence [126]. Most of these previous concepts for droplet coalescence were based on electrostatic forces that require relatively high voltages which may be harmful in biological applications. Temperature is a parameter that can be more easily controlled using integrated heaters and temperature sensors [171]. Baroud *et al.* [122], Lorenz *et al.* [123] and Perch-Nielsen *et al.* [125] employed a focused laser to enable droplet coalescence. However, this technique requires extensive setup for the generation of the focused laser.

In this chapter, a technique using temperature-induced coalescence of droplets in a microchannel is reported. Experiments were carried out to investigate the effect of the applied temperature, the flow rate ratio and the total flow rate. The combination of thermocapillary force, lower viscosity and lower interfacial tension at high temperature allows controlled coalescence of two droplets.

7.1 Experimental Investigation on Temperature-Induced Droplet Coalescence

7.1.1 Design and fabrication of microfluidic devices

Figures 7.1(a) and 7.1(b) show the schematic layout of the microfluidic device used in the experiments. Droplets generated at the flow-focusing junction are directed to merging chamber. The microchannel has a rectangular cross section of $200\ \mu\text{m} \times 400\ \mu\text{m}$. The square chamber has a width of $1000\ \mu\text{m}$. The heating wire is inserted into the slot as shown in Fig. 7.1(c).

Fig. 7.1 Schematic illustration of experimental setup.

- (a) Schematic illustration of the microfluidic device for droplet merging. Droplets are formed at the first junction (droplet formation junction) and then merge at the second junction (droplet coalescence junction).
- (b) Enlarged schematics of the merging chamber ($W_2 = 400\ \mu\text{m}$, $W_3 = 1000\ \mu\text{m}$).
- (c) Exploded view of the top and bottom parts to form the merging chamber ($W_1 = 200\ \mu\text{m}$, $H = 400\ \mu\text{m}$).

The structure of the microchannel was cut by using a CO_2 laser cutting system (Universal Laser Systems Inc., laser power of 25 W, and a maximum beam speed of about 640 mm/s). The width of the microchannel was determined by the width of the drawn lines, where the depth of the microchannel is controlled

by the power and the speed of the laser beam. The fabrication was based on the lamination thermal bonding technique. In this method, two PMMA plates (60 mm $\times$ 25 mm) were bonded together to form a closed microfluidic channel with inlet and outlet holes. The two PMMA parts were thermally bonded using a hot press (CARVER manual press 4386).

7.1.2 Materials

De-ionised (DI) water and mineral oil (Sigma M5904, St. Louis, USA) were used as the dispersed phase for the droplet and the continuous carrying phase, respectively. Fluorescent dye (Fluorescein disodium salt $C_{20}H_{10}Na_2O_5$) was added into DI-water for the flow visualisation of the droplets. Non-ionic surfactant Span 80 (Sigma S6760) 2% by weight was added into the mineral oil to assist and stabilise the formation process of droplets. The liquids were delivered by a syringe pump (KDS230, KD Scientific Inc, USA, 0.2 - 500 μ L/h , accuracy of 0.5%). A DC power supply (GW, Model GPC-30300) provided the voltage for the heating wire (Newport NIC80, 80% Ni, 20% Cr, 0.2 mm diameter, 80 Ω /m). An inverted microscope (Nikon, ECLIPSE TE2000-S) with a set of epi-fluorescent attachments and a CCD camera (Sony ICX 084, 1324 $\times$ 1024 pixels, 16 bits greyscale) were used for recording the fluorescence images.

7.1.3 Experimental setup

A schematic of the experimental setup is shown in Fig. 7.2. In the experiment, a soft Cole Parmer tube (Vernon Hills, Illinois 60061) was employed to connect

the syringe and the microfluidic device. The DI-water and the mineral oil in the syringes were introduced into the inlets A and B, respectively by using the syringe pumps. The DC power supply was used to provide an electric field to the nickel chrome wire.

Fig. 7.2 Schematic illustration of the experimental setup.

7.2 Numerical Investigation on Temperature-Induced Droplet Coalescence

7.2.1 Problem description

Figure 7.3 shows the computational domain used in the numerical investigation on temperature-induced droplet coalescence which consists of a straight microchannel with an enlarged merging chamber. Two different immiscible

fluids (oil and DI-water) are denoted as the –ve fluid and the +ve fluid. Two droplets (+ve fluid) denoted as D1 and D2 flow in the microchannel as shown in Fig. 7.3. The level-set method is employed to track interfaces and shapes of the droplets [172].

Fig. 7.3 Computational domain.

7.2.2 Governing equations

The level set function is defined as

$$\phi = \begin{cases} -d, & \vec{x} \in \Omega_- \\ 0, & \vec{x} \in \Gamma \\ +d, & \vec{x} \in \Omega_+ \end{cases} \quad (7.1)$$

where $\vec{x}$ is the position vector in the computational domain and d is the shortest normal distance from the interface Γ . The computational domain consists of a single special fluid with variable properties. The properties of the special fluid can be expressed as follows:

$$\alpha(\phi) = (1 - HS(\phi))\alpha_- + HS(\phi)\alpha_+ \quad (7.2)$$

where the smeared Heaviside function $HS(\phi)$ is defined as

$$HS(\phi) = \begin{cases} 0, & \phi < -\varepsilon \\ \frac{\phi + \varepsilon}{2\varepsilon} + \frac{1}{2\pi} \sin\left(\frac{\pi\phi}{\varepsilon}\right), & |\phi| \leq \varepsilon \\ 1, & \phi > +\varepsilon \end{cases} \quad (7.3)$$

The conservation of mass, momentum and energy of the special fluid can be expressed as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \quad (7.4)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V} \vec{V}) = -\nabla p + \nabla \cdot [\mu (\nabla \vec{V} + \nabla \vec{V}^T)] + \vec{f}_F \quad (7.5)$$

$$\frac{\partial (\rho c_p T)}{\partial t} + \nabla \cdot (\rho c_p \vec{V} T) = \nabla \cdot (k \nabla T) \quad (7.6)$$

In Eq. (7.5), $\vec{f}_F$ is the interfacial force given by,

$$\vec{f}_F = -\kappa \sigma \hat{N}_F D(\vec{x} - \vec{x}_F) + (\hat{N}_F \times \nabla \sigma) \times \hat{N}_F D(\vec{x} - \vec{x}_F) \quad (7.7)$$

where

$$D(\vec{x} - \vec{x}_F) = D(\phi) = \begin{cases} \frac{1 + \cos(\pi(\phi)/\varepsilon)}{2\varepsilon} & |(\phi)| < \varepsilon \\ 0 & \text{otherwise} \end{cases} \quad (7.8)$$

$$\hat{N}_F = \frac{\nabla(\phi)}{|\nabla(\phi)|} \quad (7.9)$$

$$\kappa = \nabla \cdot \hat{N}_F \quad (7.10)$$

The first and second terms in Eq. (7.7) account for capillary and thermo-capillary effects, respectively. The evolution of the interface is governed by

$$\phi_t + \vec{V} \cdot \nabla \phi = 0 \quad (7.11)$$

To maintain ϕ as a distance function, ϕ is set to the steady state solution of the following equation,

$$\frac{\partial \phi'}{\partial \bar{t}} = \overline{\text{sign}}(\phi)(1 - |\nabla \phi'|) \quad (7.12)$$

where $\bar{t}$ is a pseudo time for ϕ' and $\overline{\text{sign}}(\phi)$ is given by

$$\overline{\text{sign}}(\phi) = \frac{\phi}{\sqrt{\phi^2 + |\nabla \phi|^2 (\Delta x^2)}} \quad (7.13)$$

and is subjected to the following initial condition,

$$\phi'(\vec{x}, 0) = \phi(x) \quad (7.14)$$

To avoid the mass loss in the level-set method, a particle correction procedure is employed. The radius of each particle is set to

$$r_p = \begin{cases} r_{\max}, & \text{if } S_p > r_{\max} \\ S_p, & \text{if } r_{\min} < S_p < r_{\max} \\ r_{\min}, & \text{if } S_p < r_{\min} \end{cases} \quad (7.15)$$

where

$$S_p = \text{sign}[\phi(\vec{x}_p)]\phi(\vec{x}_p) \quad (7.16)$$

These particles are advected via

$$\frac{d\vec{x}_p}{dt} = \vec{V}(\vec{x}_p) \quad (7.17)$$

The sets of positive and negative particles that happen to be in the opposite side of the interface in the advection process are identified respectively as E^+ and E^- . Two new level-set functions each associated with the -ve and the +ve particles, ϕ^+ and ϕ^- , are first initialised as ϕ and then updated via

$$\phi^+ = \begin{cases} \max_{\forall p \in E^+} (|\phi^+|, |\phi_p|), & \phi^+ > 0 \\ -\min_{\forall p \in E^+} (|\phi^+|, |\phi_p|), & \phi^+ < 0 \end{cases} \quad (7.18)$$

$$\phi^- = \begin{cases} \min_{\forall p \in E^+} (|\phi^-|, |\phi_p|), & \phi^- > 0 \\ -\max_{\forall p \in E^+} (|\phi^-|, |\phi_p|), & \phi^- < 0 \end{cases} \quad (7.19)$$

where

$$\phi_p = \text{sign}[\phi(\vec{x}_p)](r_p - |\vec{x} - \vec{x}_p|) \quad (7.20)$$

Particle correction on the level-set function ϕ is performed via

$$\phi = \begin{cases} \phi^+, & |\phi^+| \leq |\phi^-| \\ \phi^-, & |\phi^+| > |\phi^-| \end{cases} \quad (7.21)$$

7.2.3 Boundary conditions

A constant velocity boundary condition is assumed at the inlet. Non-slip boundary condition is enforced at channel walls. Outflow boundary condition is employed at the outlet. The temperature of fluid at the inlet is set to room temperature. At the merging channel adjacent to the heater, the temperature is maintained at T_h . The remaining portion of the wall is set to room temperature.

At the outlet, zero-gradient is employed.

7.3 Results and Discussion

The three parameters investigated are the applied voltage, the flow rate ratio between the dispersed phase (DI-water) and the continuous phase (mineral oil)

$\beta = Q_{\text{water}} / Q_{\text{oil}}$ and the total flow rate. A calibrated K-type fine wire gauge

thermocouple ($D=0.1930$ mm) is used to measure the temperature of the merging chamber filled with mineral oil. Figure 7.4 shows a second-order relationship between the temperature and the applied voltage, because the temperature is proportional to the heating power which in turn is a square function of the applied voltage. Since inserting the thermocouple into the merging chamber during the experiment would affect the coalescence process and block the view of the camera, the temperature values in Fig. 7.4 are used to estimate the actual temperature in the chamber. The later results are discussed in term of heating voltage and not the temperature, which cannot be monitored during the experiment.

Fig. 7.4 Temperature of the merging chamber as a function of applied voltage.

7.3.1 Effect of applied voltage

Figure 7.5 shows the behaviour of two droplets in the microchannel with merging chamber when the heater is switched off. The temperature in the microchannel is maintained at room temperature. The snapshots of the two droplets flow in the merging chamber obtained experimentally and numerically are shown in Figs. 7.5(a) and 7.5(b), respectively. Without heating, the first

droplet (D1) moves in the microchannel and prepares to enter the chamber at $t = 0$ s. In the chamber, droplet D1 decelerates due to the lower flow velocity in the expanded chamber. At $t = 2$ s, droplet D1 is completely inside the chamber. At $t = 4$ s, droplet D1 approaches the exit of the chamber, while the second droplet D2 just prepare to enter the inlet of the chamber. At $t = 6$ s, a portion of droplet D1 leaves the chamber, while a portion of the second droplet D2 enters the chamber. When D2 is completely inside the chamber, D1 has left the chamber ($t = 8$ s). The two droplets have no opportunity to come in close contact and cannot be merged because of the continuous oil phase separating them.

Figure 7.6 shows the velocity field that a droplet moves through the chamber. Before entering the chamber, the droplet is carried forward by the main stream, where u velocity component is dominant as shown in Fig. 7.6(a). Upon entering the chamber, the flow expands due to the sudden increase in the cross section area. In this stage, the flow field is dominated by u and v velocity components. The emerging v velocity component will swell the head of the droplet. Mass in the tail is transferred to the head of the droplet which result in the shrinkage of the tail of the droplet. The droplet will deform from a capsule shape to a mushroom shape as shown in Fig. 7.6(b). As the whole portion of droplet stays in the chamber, the surface tension attempts to maintain the droplet to its original circular shape as shown in Fig. 7.6(c). Upon leaving the chamber, the flow shrinks due to the sudden decrease in the cross section area. A low pressure is developed in the narrowed main channel which sucks droplet into the narrowed main channel. This creates a cusp in the leading edge of the

droplet as shown in Fig. 7.6(d). In middle of the main channel, the fluid flows faster compared to that near the wall. The cusp of the droplet is driven by a larger velocity at the middle of the channel. The head of the droplet becomes longer. Mass is transferred from the tail to the head of the droplet with the shrinkage of the tail of the droplet as shown in Fig. 7.6(e).

Figure 7.7 shows the behaviour of temperature-induced droplet coalescence in the microchannel with merging chamber when the heater was switched on. The temperature in the microchannel was controlled by adjusting the applied voltage to the heater. The snapshots of the droplet coalescence in the merging chamber obtained experimentally and numerically are shown in Figs. 7.7(a) and 7.7(b), respectively. With heating, the first droplet (D1) moves into the microchannel and prepares to enter at $t = 0$ s. At $t = 2$ s, droplet D1 is completely inside the chamber. At $t = 4$ s, instead of moving forward D1 remains trapped at the exit and waits for the subsequent droplet D2 to enter the chamber. When D1 and D2 are in close contact, D2 pushes D1 to initiate a slight forward motion ($t = 6$ s). The two droplets subsequently merge at $t = 8$ s. After turning off the applied voltage, it will take three seconds to cool down to room temperature.

The physics behind the observed coalescence can be explained by a two-step process of competition between thermocapillary force and pressure as well as hydrodynamic force. The first step occurs at the entrance of the merging chamber, while the second step happens at the exit. Before entering the chamber, the droplets move at a constant velocity within the continuous phase (mineral oil). Once the droplet is inside the merging chamber, the average flow

velocity of droplet and oil will decrease due to the expansion of the cross section area. At a higher temperature induced by the wire heater, the viscosity of the continuous phase decreases. As a result, the oil in the chamber flows faster to escape from the chamber, allowing the droplet to stay longer in the chamber for a longer period of time. Moreover, a gradient of the surface tension between the front and the tail of the droplet emerges due to the non-uniform temperature distribution around the heater. The front of the droplet has a smaller surface tension than the tail due to its higher temperature. Thermocapillary force pushes the droplet to move forward. However, when the front of droplet reaches the exit of the chamber, it is colder than the tail since the tail is being heated. Thermocapillary force counters the hydrodynamic drag force and decelerates the droplet. The balance between these two forces allows the first droplet to remain at the exit of the merging chamber while waiting for the next droplet. With the second droplet D2 inside the chamber, thermocapillary forces push D1 backward and D2 forward initiating a merge. As the D1 and D2 come into close contact after the drainage of the mineral oil between them, a singularity occurs at the joint of the two droplets. As time goes by, a connecting tiny bridge will be formed between the two droplets. Evidently, the meniscus around the bridge is a region of very high curvature. The breakage of this bridge results from a thermocapillary force. After that, the merged droplet is big enough to block the exit of the chamber, increasing the pressure inside the chamber. As a result, despite of the thermocapillary force, the merged droplet can be released from the chamber and allowing space for the next two droplets.

Figure 7.8 shows the velocity field of droplet coalescence in the chamber. The velocity field is complex with vortices formed within the droplet when the two droplets are in close contact in the chamber. To have a better understanding of the velocity field, a schematic illustration of the thermocapillary forces acting on the two droplets during this merging process and vortices formed within the droplet is shown in Fig. 7.8(e). When D1 and D2 come into close contact, the continuous phase (mineral oil) between them is being squeezed out as shown in Fig. 7.8(a). The thermocapillary forces acting on the two droplets induce a secondary flow within each droplet which are demonstrated by a pair of vortices formed within the droplets. The thermocapillary forces drive the fluid particles towards the interface of the two droplets. The strength and direction of the thermocapillary force is clearly manifested in the velocity field at the interface of the two droplets. The vortices in the second droplet is stronger compared to the vortices in the first droplet due to the larger curvature. The thermocapillary force pulls the two droplets towards each other accompanied by the two cusps in the interface as shown in Fig. 7.8(b). As the two droplets merge together, the surface tension attempts to minimise the interfacial area of the fused droplet which resulted in the vanishment of the two cusps. The two pair of vortices will decay as shown in Figs. 7.8(c) and 7.8(d).

Fig. 7.5 Two droplets flow in a microchannel without the heating effect.
(a) Experimental snapshots. (b) Simulation results.

Fig. 7.6 Velocity field around a droplet moving through the merging chamber.

Fig. 7.7 Droplet coalescence in a microchannel with the heating effect.
(a) Experimental snapshots. (b) Simulation results.

Fig. 7.8 Velocity field of droplet coalescence in the merging chamber.

7.3.2 Effect of flow rate ratio ($\beta = Q_{water} / Q_{oil}$)

The behaviour of the two droplets in the merging chamber is subsequently investigated for different combination of flow rate ratios. Figure 7.9(a) shows

the schematic illustration of the distance (d) between the two droplets in the merging chamber. Figure 7.9(b) shows the variation of the distance (d) as a function of the applied voltage to the heater. The solid line for each is a linear fitting function. Zero distance ($d = 0$) means that the two droplets are able to coalesce. At a given flow rate ratio, the distance between two neighbouring droplets decreases with increasing applied voltage. At a given applied voltage, d decreases as the flow rate ratio increases. At a threshold voltage, d approaches zero and the droplets are merged. The relationship between the flow rate ratio and the corresponding threshold voltage is shown in Fig. 7.10. A large flow rate ratio results in a small d and only requires a low voltage for droplet coalescence.

7.3.3 Effect of total flow rate

The behaviour of the two droplets in the merging chamber was studied for different total flow rates. Figure 7.11 shows the relationship between total flow rate and the corresponding threshold voltage at different flow rate ratios. The solid line is a second-order fitting function. At a given flow rate ratio, the threshold voltage increases as the total flow rate increases.

Fig. 7.9 Droplet behaviour in the merging chamber.

(a) Schematic illustration of the distance between two droplets in the merging chamber.

(b) Distance between two neighbouring droplets versus heating voltage.

Fig. 7.10 Flow rate ratio versus the threshold voltage required for merging.

Fig. 7.11 Total flow rate versus the threshold voltage required for merging at different flow rate ratio.

7.4 Summary

This chapter reports on a concept to merge two droplets in a microchannel. The droplets were trapped and manipulated in a rectangular merging chamber. The merging process was controlled by the voltage applied to a wire heater in the

merging chamber. This method is effective for high flow rate ratio, low total flow rate and high temperature. The high flow rate ratio ensures that the droplets are in reasonable close contact and remain in the chamber at the same time for merging. The low total flow rate ensures the low velocity of droplets in the chamber. The high temperature reduces the viscosities and the interfacial tension, and at the same time increases the thermocapillary forces. The concept presented in this chapter could potentially be used for merging and mixing in droplet-based lab-on-a-chip replacing the continuous-flow counter parts [173]. One of the most important applications for droplet coalescence in such a device is the mixing of reagents. Reagent mixing can be achieved in the merging chamber when droplets containing different reagents are generated subsequently.

Chapter 8

Heat Transfer Model for Plug Trains under Constant Surface Temperature Condition

Heat transfer process is governed by thermal diffusion and thermal advection. For fluid flow in microchannels, the Reynolds number (Re) is very small which is of the order of 1. Thus, the contribution of thermal advection to the transverse heat transfer process is limited due to the fact that thermal diffusion is parallel to the main flow direction. Hence, the heat transfer process in the microchannels is mainly governed by thermal diffusion. In order to enhance heat transfer by utilising thermal advection, curved microchannels and embedded obstacle structures are employed to induce vortices in microchannels. Vortices can also be induced in the microchannel by means of formed interfaces in the flow field. When a miscible and an immiscible fluid are introduced into a T junction or a flow-focusing junction, the formation of droplets/bubbles will result in the generation of plug flow [174-176]. A pair of vortices will form automatically in the liquid plugs which can be employed to enhance heat transfer in microchannels. Investigation on heat transfer for plug trains has not been investigated systematically due to the complex flow pattern as compared to the single-phase flow [177].

Theoretical and numerical investigations on the heat transfer process in the plug flow have been conducted. Muzychka *et al.* [178] studied the heat transfer process of plug flow in microchannels with non-circular cross section

theoretically. The flow field in the liquid plug was assumed to be uniformly distributed. Followed by the previous research, Muzychka *et al.* [179] investigated laminar gas-liquid segmented plug flows which was based on Graetz theory. In the actual situation, the flow field in the plug flow is dominated by recirculating vortices. It is necessary to take the recirculating vortices into consideration. Due to the complex flow pattern in the plug flow, many numerical methods such as the front tracking method [180], the volume of fluid method [181], the level set method [182, 183], the Lattice Boltzmann method [184, 185] and the moving grid method [186] are employed to predict the flow field in the plug flow. The use of these numerical methods would be time consuming. In this chapter, heat transfer model for plug trains under constant surface temperature condition will be investigated based on the velocity field derived by Che *et al.* [187]. This approach can investigate the heat transfer characteristics for plug trains efficiently with considering the recirculating vortices.

8.1 Theoretical Investigation

8.1.1 Modelling of flow field in plug trains

The governing equation for Stokes flow in plug trains can be simplified as a biharmonic equation. Using the Finite Fourier Transform (FFT), Che *et al.* [187] found the dimensionless velocity field as follows:

The dimensionless form of the velocity field in plug trains is

$$\begin{aligned}\hat{u}_{x,i} = & \sum_{m=1}^{\infty} \frac{(-1)^m}{\alpha_m} \left[p_m^{o'}(\hat{y}_i) X_m^o + p_m^{e'}(\hat{y}_i) X_m^e \right] \cos(\alpha_m \hat{x}_i) \\ & + \hat{L}_i \sum_{l=1}^{\infty} (-1)^l \left[q_l^e(\hat{x}_i) Y_l^e + q_l^o(\hat{x}_i) Y_l^o \right] \cos(\beta_l \hat{y}_i)\end{aligned}\quad (8.1)$$

$$\begin{aligned}\hat{u}_{y,i} = & \sum_{m=1}^{\infty} (-1)^m \left[p_m^o(\hat{y}_i) X_m^o + p_m^e(\hat{y}_i) X_m^e \right] \sin(\alpha_m \hat{x}_i) \\ & - \hat{L}_i \sum_{l=1}^{\infty} \frac{(-1)^l}{\beta_l} \left[q_l^{e'}(\hat{x}_i) Y_l^e + q_l^{o'}(\hat{x}_i) Y_l^o \right] \sin(\beta_l \hat{y}_i)\end{aligned}\quad (8.2)$$

where

$$p_m^{o'}(\hat{y}_i) = \left[\alpha_m - \tanh(\alpha_m) \right] \frac{\cosh(\alpha_m \hat{y}_i)}{\cosh(\alpha_m)} - \hat{y}_i \alpha_m \tanh(\alpha_m) \frac{\sinh(\alpha_m \hat{y}_i)}{\cosh(\alpha_m)} \quad (8.3)$$

$$p_m^{e'}(\hat{y}_i) = \left[\alpha_m \tanh(\alpha_m) - 1 \right] \frac{\sinh(\alpha_m \hat{y}_i)}{\cosh(\alpha_m)} - \hat{y}_i \alpha_m \frac{\cosh(\alpha_m \hat{y}_i)}{\cosh(\alpha_m)} \quad (8.4)$$

$$q_l^{o'}(\hat{x}_i) = \left[\beta_l \hat{L}_i - \tanh(\beta_l \hat{L}_i) \right] \frac{\cosh(\beta_l \hat{x}_i)}{\cosh(\beta_l \hat{L}_i)} - \beta_l \hat{x}_i \tanh(\beta_l \hat{L}_i) \frac{\sinh(\beta_l \hat{x}_i)}{\cosh(\beta_l \hat{L}_i)} \quad (8.5)$$

$$q_l^{e'}(\hat{x}_i) = \left[\beta_l \hat{L}_i \tanh(\beta_l \hat{L}_i) - 1 \right] \frac{\sinh(\beta_l \hat{x}_i)}{\cosh(\beta_l \hat{L}_i)} - \beta_l \hat{x}_i \frac{\cosh(\beta_l \hat{x}_i)}{\cosh(\beta_l \hat{L}_i)} \quad (8.6)$$

X_m^e , X_m^o , Y_l^e and Y_l^o are obtained from the following set of infinite algebra

linear equations

$$\frac{4\varepsilon_1(\hat{t})\alpha_m}{\pi(2m-1)} = -\left[\Delta_1^o(\alpha_m) X_m^o + \Delta_1^e(\alpha_m) X_m^e \right] + \sum_{l=1}^{\infty} \frac{4\alpha_m^2 \beta_l Y_l^e}{(\alpha_m^2 + \beta_l^2)^2} \quad (8.7)$$

$$\frac{4\varepsilon_2(\hat{t})\alpha_m}{\pi(2m-1)} = -\left[\Delta_1^o(\alpha_m) X_m^o - \Delta_1^e(\alpha_m) X_m^e \right] + \sum_{l=1}^{\infty} \frac{4\alpha_m^2 \beta_l Y_l^e}{(\alpha_m^2 + \beta_l^2)^2} \quad (8.8)$$

$$\begin{aligned} & \left[-\sum_{m=1}^{\infty} X_m^o \frac{4\alpha_m \beta_l^2 \tanh^2(\alpha_m)}{(\alpha_m^2 + \beta_l^2)^2} + \hat{L} \left[\Delta_2^e(\beta_l \hat{L}_i) Y_l^e + \Delta_2^o(\beta_l \hat{L}_i) Y_l^o \right] \right]_{\text{Plug}-i} \\ &= \left[\sum_{m=1}^{\infty} X_m^o \frac{4\alpha_m \beta_l^2 \tanh^2(\alpha_m)}{(\alpha_m^2 + \beta_l^2)^2} + \hat{L} \left[-\Delta_2^e(\beta_l \hat{L}_i) Y_l^e + \Delta_2^o(\beta_l \hat{L}_i) Y_l^o \right] \right]_{\text{Plug}-(i+1)} \end{aligned} \quad (8.9)$$

$$\left[\mu \hat{L} \left[\Delta_4 Y_l^e + \Delta_3^o(\beta_l \hat{L}_i) Y_l^o \right] \right]_{\text{Plug}-i} = \left[\mu \hat{L} \left[\Delta_4 Y_l^e - \Delta_3^o(\beta_l \hat{L}_i) Y_l^o \right] \right]_{\text{Plug}-(i+1)} \quad (8.10)$$

$$\begin{aligned} & \left[-\sum_{m=1}^{\infty} X_m^o \frac{4\alpha_m \beta_l^2 \tanh^2(\alpha_m)}{(\alpha_m^2 + \beta_l^2)^2} + \hat{L} \left[\Delta_2^e(\beta_l \hat{L}_i) Y_l^e + \Delta_2^o(\beta_l \hat{L}_i) Y_l^o \right] \right]_{\text{Plug}-N} \\ &= \left[\sum_{m=1}^{\infty} X_m^o \frac{4\alpha_m \beta_l^2 \tanh^2(\alpha_m)}{(\alpha_m^2 + \beta_l^2)^2} + \hat{L} \left[-\Delta_2^e(\beta_l \hat{L}_i) Y_l^e + \Delta_2^o(\beta_l \hat{L}_i) Y_l^o \right] \right]_{\text{Plug}-1} \end{aligned} \quad (8.11)$$

$$\left[\mu \hat{L} \left[\Delta_4 Y_l^e + \Delta_3^o(\beta_l \hat{L}_i) Y_l^o \right] \right]_{\text{Plug}-N} = \left[\mu \hat{L} \left[\Delta_4 Y_l^e - \Delta_3^o(\beta_l \hat{L}_i) Y_l^o \right] \right]_{\text{Plug}-1} \quad (8.12)$$

where

$$\Delta_1^o(\alpha_m) \equiv p_m^{o'}(\hat{y}_i = 1) = \frac{\alpha_m}{\cosh^2(\alpha_m)} - \tanh(\alpha_m) \quad (8.13)$$

$$\Delta_1^e(\alpha_m) \equiv p_m^{e'}(\hat{y}_i = 1) = \frac{-\alpha_m}{\cosh^2(\alpha_m)} - \tanh(\alpha_m) \quad (8.14)$$

$$\Delta_2^o(\beta_l \hat{L}_i) = \beta_l \hat{L}_i - \tanh(\beta_l \hat{L}_i) - (\beta_l \hat{L}_i) \tanh^2(\beta_l \hat{L}_i) \quad (8.15)$$

$$\Delta_2^e(\beta_l \hat{L}_i) = -\beta_l \hat{L}_i - \tanh(\beta_l \hat{L}_i) + (\beta_l \hat{L}_i) \tanh^2(\beta_l \hat{L}_i) \quad (8.16)$$

$$\Delta_3^o(\beta_l \hat{L}_i) = 2\beta_l \tanh^2(\beta_l \hat{L}_i) \quad (8.17)$$

$$\Delta_4 = -2\beta_l \quad (8.18)$$

8.1.2 Analysis of heat transfer

To analyse the heat transfer in plug trains, the energy conservation equation without the heat source term is employed as follows:

$$\frac{\partial T_i}{\partial t} + \frac{\partial(u_{x,i} T_i)}{\partial x_i} + \frac{\partial(u_{y,i} T_i)}{\partial y_i} = \frac{k_i}{\rho_i c_{pi}} \left(\frac{\partial^2 T_i}{\partial x_i^2} + \frac{\partial^2 T_i}{\partial y_i^2} \right) \quad (8.19)$$

where T_i is the temperature of plug- i , ρ_i is the density of plug- i , k_i is the thermal conductivity of plug- i , and c_{pi} is the specific heat capacity of plug- i .

Using the following dimensionless variables,

$$\hat{t} = \frac{t}{\tau}, \quad \text{Pe} = \frac{2hV}{\alpha}, \quad \hat{T}_i = \frac{T_i - T_0}{T_c} \quad (8.20)$$

where $\tau = 2h/V$, $\alpha = k/(\rho c_p)$, and T_c is the characteristic temperature. The dimensionless form of the energy equation is

$$\frac{\partial \hat{T}_i}{\partial \hat{t}} + \frac{\partial(\hat{u}_{x,i} \hat{T}_i)}{\partial \hat{x}_i} + \frac{\partial(\hat{u}_{y,i} \hat{T}_i)}{\partial \hat{y}_i} = \frac{1}{\text{Pe}} \left(\frac{\partial^2 \hat{T}_i}{\partial \hat{x}_i^2} + \frac{\partial^2 \hat{T}_i}{\partial \hat{y}_i^2} \right) \quad (8.21)$$

With this normalisation, the dimensionless time $\hat{t}$ and the dimensionless axial location of the plug in the microchannel $\hat{X}$ is related as

$$\hat{t} = \hat{X} \quad (8.22)$$

The analytical velocity field obtained in Eqs. (8.1) and (8.2) was substituted into Eq. (8.21) for heat transfer analyses. The Finite Volume Method (FVM) was employed to solve the energy equation. A stagger grid was used to store the scalar and velocities at the nodes and faces of each control volume respectively [143].

Before entering the microchannel, the plug trains were assumed to have an initial uniform temperature field with $\hat{T}_i = 0$ at $\hat{t} = 0$. Constant surface temperature boundary condition was considered as shown in Fig. 8.1. The temperature distribution in plug trains can be normalised as

$$\hat{T}_i = \frac{T_i - T_0}{T_w - T_0} \quad (8.23)$$

where T_w is the wall temperature. The boundary condition is

$$T_i = T_w \quad \text{at} \quad y_i = \pm h \quad (8.24)$$

and in dimensionless form

$$\hat{T}_i = 1 \quad \text{at} \quad \hat{y}_i = \pm 1 \quad (8.25)$$

Fig. 8.1 Schematic illustration of constant surface temperature boundary condition for heat transfer in plug trains.

8.1.3 Evaluation of heat transfer

The Nusselt number (Nu) (Section 8.1.3.1) and heat transfer index η (Section 8.1.3.2) are employed to evaluate the heat transfer characteristics of plug trains in the microchannel. The heat transfer process of two plug units, the effect of thermal conductivity ratio and plug length ratio on the heat transfer characteristics on each plug unit will be investigated.

Fig. 8.2 Heat transfer is evaluated at the axial location $\hat{X}$ for each plug unit.

8.1.3.1 Nusselt number

The Nusselt number is defined as

$$Nu = \frac{hD_H}{k} \quad (8.26)$$

where D_H is the hydraulic diameter of the microchannel ($D_H = 2h$). The mean fluid temperature ($\bar{T}_{plug-i}$) of plug- i at the axial location X as shown in Fig. 8.2 was used as the reference temperature

$$\bar{T}_{plug-i}(X) = \frac{\int_{A_{plug-i}} \rho_i c_{pi} u_{x,i} T_i dA_{plug-i}}{\int_{A_{plug-i}} \rho_i c_{pi} u_{x,i} dA_{plug-i}} \quad (8.27)$$

The mean fluid temperature of the plug- i in dimensionless form is

$$\hat{\bar{T}}_{plug-i}(\hat{X}) = \frac{\int_{-1}^1 d\hat{y}_i \int_{-\hat{L}_i}^{\hat{L}_i} \hat{u}_{x,i} \hat{T}_i d\hat{x}_i}{\int_{-1}^1 d\hat{y}_i \int_{-\hat{L}_i}^{\hat{L}_i} \hat{u}_{x,i} d\hat{x}_i} \quad (8.28)$$

For the constant surface temperature boundary condition, the convective heat transfer coefficient for plug- i can be calculated as follows:

$$h_{plug-i} = \frac{\int_{-L_i}^{L_i} (-k_i \partial T_i / \partial y_i)_{y_i=-h} dx_i}{\int_{-L_i}^{L_i} (T_w - \bar{T}_i) dx_i} \quad (8.29)$$

and the Nusselt number with the dimensionless variable is

$$\text{Nu}_{\text{plug}-i} = \frac{2 \int_{-\hat{L}_i}^{\hat{L}_i} \left(-\partial \hat{T}_i / \partial \hat{y}_i \right)_{\hat{y}_i=-1} d\hat{x}_i}{\int_{-\hat{L}_i}^{\hat{L}_i} \left(1 - \hat{T}_i \right) d\hat{x}_i} \quad (8.30)$$

8.1.3.2 Heat transfer index

For the constant surface temperature boundary condition, the heat transfer index is defined as follows:

$$\eta(\hat{X}) = \frac{Q(\hat{t})}{Q_{\max}} = \frac{\int_{-h}^h dy_i \int_{-L_i}^{L_i} \rho_i c_{pi} (T_i - T_0) dx_i}{\int_{-h}^h dy_i \int_{-L_i}^{L_i} \rho_i c_{pi} (T_w - T_0) dx_i} = \frac{1}{2\hat{L}_i} \int_{-1}^1 d\hat{y}_i \int_{-\hat{L}_i}^{\hat{L}_i} \hat{T}_i d\hat{x}_i \quad (8.31)$$

where $Q(\hat{t})$ is the total energy transferred from the wall to the plug unit over the time interval 0 to $\hat{t}$. $Q_{\max}$ is the maximum amount of energy that the whole plug unit needs to achieve an uniform temperature distribution which is the same as the wall temperature. The heat transfer index, indicating the heat transfer progress for the plug flow in the microchannel, varies from 0 to 1. $\eta=0$ indicates that there is no heat transfer, while $\eta=1$ indicates that the whole plug has achieved an uniform temperature distribution which is the same as the wall temperature.

8.2 Results and Discussion

8.2.1 Flow field in two liquid plugs

The flow field in the two liquid plugs is shown in Fig. 8.3. In each plug, two symmetrical vortices can be clearly seen in Fig. 8.3. Before encountering the front interface of the plug, the liquid moves along the centre path in the plug.

After encountering the front interface, the liquid is blocked by the front interface of the plug. The liquid will change flow direction which is along the front interface of the plug. When the liquid encounters the microchannel wall, the flow direction will be directed from the front interface of the plug towards the microchannel wall. After encountering the rear interface of the plug, the liquid will be blocked by the rear interface of the plug. The liquid will change flow direction which is along the rear interface of the plug. As a result, two symmetrical vortices form in the plug due to the existence of the front and rear interfaces. The two symmetrical recirculating vortices will greatly enhance heat transfer in the microchannel as shown in Section 8.2.2.

Fig. 8.3 Flow field in two liquid plugs. The dimensionless plug length for each plug is $\hat{L} = 1$.

8.2.2 Heat transfer analysis

8.2.2.1 Heat transfer process in two liquid plugs

As the two liquid plugs (plugs 1 and 2) with identical dimensionless plug lengths ($\hat{L} = 1$) and Peclet number ($Pe = 50$) move through a 2D microchannel, they are subjected to a dimensionless constant wall temperature $\bar{T}_w = 1$. The variation of the isotherms with time for the two liquid plugs is shown in Fig. 8.4.

The variation of the Nusselt number (Nu), the wall temperature ($\hat{T}_w$), the mean fluid temperature of the plug ($\hat{T}_{\text{plug}}$) and the heat transfer index of the plug (η) with time is shown in Fig. 8.5. Based on the evolution of temperature contours, the heat transfer process can be divided into three stages including (I) development of thermal boundary layer, (II) advection of heated/fresh fluid in the plug and (III) thermally fully developed flow.

Stage I: Development of thermal boundary layer (TBL), [Figs. 8.4(B) and 8.4(C)]: Before entering the microchannel, plugs 1 and 2 are assumed to have a uniformly temperature distribution ($\hat{T} = 0$). When the two liquid plugs enter the microchannel, the two liquid plugs are subjected to the microchannel wall with relatively high temperature ($\hat{T}_w = 1$). Heat transfer will take place immediately due to the presence of the large temperature difference between the hot microchannel wall and the liquid plugs. The formation of stratified temperature contours is observed in Fig. 8.4(B), indicating that a thin TBL has formed at the vicinity of the microchannel wall. Due to the existence of the recirculating vortices in the liquid plugs, the fluid heated by the microchannel wall will be transported to the rear interface of the plug continuously. Consequently, the TBL is thinner at the vicinity of the front interface and thicker at the vicinity of the rear interface. At the interface between plugs 1 and 2, heat exchange between the plugs take places as shown in Fig. 8.6. The fresh fluid from the central region of plug-1 moves towards the front interface while the heated fluid of plug-2 moves towards the rear interface. The effect of heat exchange

between the plugs causes the TBL of plug-1 to increase in thickness at the interface region, while it becomes thinner in plug-2. As time progresses, heat will be transferred towards the central region of the liquid plug and the thickness of the TBL will increase as shown in Fig. 8.4(C).

Stage II: Advection of heated/fresh fluid in the plug, [Figs. 8.4(D) to 8.4(H)]: In stage I, and as the fluid in the TBL moves along the heated microchannel wall, it becomes heated fluid. Because of the recirculating vortices in the plug, the heated fluid will be transferred to the central region of the plug. Thermal mixing occurs between the heated fluid and the fresh fluid which is in the central region. As the flow direction is from left to right, temperature will decrease in the flow direction. Accompanied with the thermal mixing process, the heated fluid and the fresh fluid are transported towards the front interface.

Stage III: Thermally fully developed flow, [Figs. 8.4(I) to 8.4(J)]: In this stage, the pattern of the temperature contours remains unvaried with the time. The temperature of the plug trains will approach the microchannel wall temperature. Besides, the Nusselt number approaches a constant value for each of the plug

$(Nu_{asympt,plug1} = 31.4, Nu_{asympt,plug2} = 27)$ as shown in Fig. 8.5(a).

To investigate the oscillation of the Nusselt number (Fig. 8.5(a)), the variation of the mean temperature gradient along the microchannel wall

$$\left(\overline{-\partial \hat{T}_i / \partial \hat{y}_i} = \frac{1}{2\hat{L}_i} \int_{-\hat{L}_i}^{\hat{L}_i} \left(-\partial \hat{T}_i / \partial \hat{y}_i \right)_{\hat{y}_i=-1} d\hat{x}_i \right) \text{ and the mean fluid temperature of the}$$

plug unit ($\hat{T}_{\text{plug-i}}$) with time is plotted in Fig. 8.5(b). The evolution of $\overline{-\partial\hat{T}_i/\partial\hat{y}_i}$ and $\hat{T}_{\text{plug-i}}$ from stage I to stage III is found to be responsible for the oscillation of the Nusselt number. In stage I, an extremely large $\overline{-\partial\hat{T}_i/\partial\hat{y}_i}$ is formed due to the large temperature difference between the wall and the plug at the instant the plugs contact the heated wall, which result in an extremely large Nusselt number (Eq. (8.30)). As time goes by, more heat is transferred to the fluid in the plug. The temperature difference between the wall and the plug becomes much smaller and the mean fluid temperature becomes higher which result in $\overline{-\partial\hat{T}_i/\partial\hat{y}_i}$ decreases rapidly while $\hat{T}_{\text{plug-i}}$ increases gradually. As a result, the Nusselt number decreases dramatically in stage I [Figs. 8.4(B) and 8.4(C)]. In stage II [Figs. 8.4(D) to 8.4(H)], the fresh fluid in the central region is continuously transported to the heated wall and the heated fluid in the vicinity of the wall is continuously transported to the central region by the recirculation vortices which result in $\overline{-\partial\hat{T}_i/\partial\hat{y}_i}$ decreases at a slower rate. Meanwhile, $\hat{T}_{\text{plug-i}}$ increase smoothly due to the thermal mixing process of heated and fresh fluids in the plug unit. As a result, the Nusselt number increases in this period. In stage III [Figs. 8.4(I) to 8.4(J)], Nusselt number remains a constant value due to the unchanged temperature contours in this stage.

Fig. 8.4 Variation of temperature contours in two liquid plugs subjected to constant surface temperature boundary condition.

Fig. 8.5 Variation of the Nusselt number, the temperature gradient at the wall $-\partial \hat{T}_i / \partial \hat{y}_i$ and the fluid temperature in the plug $\hat{T}_{\text{plug}}$ in two liquid plugs.

Fig. 8.6 Schematic illustration of heat exchange between plug-1 and plug-2 at the interface.

8.2.2.2 Effect of thermal conductivity ratio (k_2 / k_1)

The effect of thermal conductivity ratio on the Nusselt number and the heat transfer index in each plug unit is shown in Figs. 8.7(a) and 8.7(b), respectively. In order to investigate the effect of thermal conductivity ratio, the dimensionless plug length for plugs 1 and 2 is fixed at $\hat{L}_1 = \hat{L}_2 = 1$ and the thermal conductivity ratio is varied from 1 to 4. The Nusselt number for plugs 1 and 2 decreases with the increase of the thermal conductivity ratio between the two plugs. When $k_2 / k_1 = 1$, the asymptotic limits of Nusselt number for plugs 1 and 2 are 32 and 27, respectively. When $k_2 / k_1 = 4$, the asymptotic limits of Nusselt number for plugs 1 and 2 are 24.5 and 9.5, respectively. When the thermal conductivity ratio increases, the heat transfer in plug-2 is dominated by thermal diffusion and heat exchange between plugs 1 and 2. Higher thermal conductivity of plug-2 will lead to faster thermal equilibrium in plug-2, and a higher heat transfer index in plug-2 as compared to plug-1.

Fig. 8.7 Variation of Nusselt number (Nu) and heat transfer index (η) with thermal conductivity ratio (k_2/k_1).

8.2.2.3 Effect of plug length ratio ($\hat{L}_2/\hat{L}_1$)

The effect of plug length ratio on the flow field and transverse velocity in two liquid plugs at a typical cross section $\hat{y} = 3/4$ inside the plug is shown in Fig.

8.8. When $\hat{L}_2 / \hat{L}_1 = 1$, the flow field in each liquid plug is symmetric due the symmetrical boundary conditions imposed on each plug [Fig. 8.8(a)]. The transverse velocity in plugs 1 and 2 is effective except at the front/rear interface ($\hat{u}_y = 0$). As $\hat{L}_2 / \hat{L}_1$ increases to 2 ($\hat{L}_2 / \hat{L}_1 = 2$), the length of plug-2 is two times longer than plug-1 as shown in Fig. 8.8(b). The transverse velocity in plug-2 becomes less effective in the mid region of the plug. When $\hat{L}_2$ increases (eg, $\hat{L}_2 = 3\hat{L}_1$, $\hat{L}_2 = 4\hat{L}_1$), the transverse velocity is almost zero at most of the axial distance of plug-2 [Figs. 8.8(c) and 8.8(d)]. The results indicate that shorter plug length ratio is effective in enhancing heat transfer due to the non-zero transverse velocity component in the axial distance of plugs.

Fig. 8.8 Transverse velocity profiles in two liquid plugs with different plug length ratio at the cross section $\hat{y} = 3/4$.

The effect of plug length ratio on the Nusselt number and the heat transfer index is shown in Figs. 8.9(a) and 8.9(b), respectively. In the simulation, the thermal conductivity ratio is fixed at $k_2/k_1 = 1$ and $\hat{L}_2/\hat{L}_1$ is varied from 1 to 4.

The Nusselt number for plugs 1 and 2 decreases with the increase of the plug

length ratio. When $\hat{L}_2 / \hat{L}_1 = 1$, the asymptotic limits of Nusselt number for plugs 1 and 2 are 32 and 27, respectively. When $\hat{L}_2 / \hat{L}_1 = 4$, the asymptotic limits of Nusselt number for plugs 1 and 2 are 32 and 25.4, respectively. The decrease in $Nu_{asymp,2}$ from 27 to 25.4 is due to the decrease in the transverse flow as explained in Fig. 8.8(d). Figure 8.9(b) shows that the heat transfer index of plug-2 decreases with the increase of the plug length ratio which is due to the lower Nusselt number in plug-2.

Fig. 8.9 Variation of Nusselt number (Nu) and heat transfer index (η) with plug length ratio ($\hat{L}_2 / \hat{L}_1 = 1$).

8.3 Summary

This chapter investigates the heat transfer characteristics of two liquid plugs moving in a microchannel under the constant surface temperature boundary condition. The heat transfer processes in two liquid plugs were studied numerically by incorporating the analytical flow field. Nusselt number (Nu) and

heat transfer index (η) were employed to evaluate the heat transfer characteristics of two liquid plugs. In this study, it can be concluded that:

- (1) Heat transfer in plugs can be divided into three stages: (I) development of thermal boundary layer; (II) advection of heated/fresh fluid in the plug; and (III) thermally fully developed flow.
- (2) A high conductivity ratio (k_2 / k_1) results in a lower Nusselt number and a higher heat transfer index in plug-2 compared to plug-1.
- (3) A low plug length ratio is effective in enhancing heat transfer in the microchannel, which results in a lower Nusselt number and a lower heat transfer index in plug-2 compared to plug-1.

Chapter 9

Conclusions and Recommendations for Future Work

9.1 Conclusions

This thesis focuses on the investigation of single and two-phase heat transfer in microfluidic systems. Following the objective and scope of the thesis, extensive and rigorous studies on the single and two-phase heat transfer in microfluidic systems have been conducted using theoretical, experimental and numerical approaches. Reasonable agreement between the experimental results and the numerical predictions is obtained. An in-depth understanding of physical principles and influencing factors in single and two-phase heat transfer has been achieved.

Theoretical investigation on the thermal mixing process is presented. Thermal mixing with the same ratio, arbitrary ratio and dispersion were taken into consideration. Theoretical investigation on the mixing channel heat transfer zone was performed. Experiments were carried out to validate the analytical model. A three dimensional numerical simulation was subsequently carried out to give a better understanding of the thermal mixing processes on the microscale. The heat transfer process in the T-shaped microchannel can be divided into two heat transfer zones: the T-junction and the mixing channel. Striations in the temperature contours in the T-junction were experimentally

and numerically determined. The results show that flow rate ratio plays an important role in the thermal mixing process.

Experimental and numerical investigation of mass mixing in a T-shaped microchannel was also carried out. The Laser Induced Fluorescence method was used to characterise the thermal effect on mass mixing. The proposed numerical model gives reasonable agreement with the experiment data. A higher heating power applied to the heater and a lower flow rate is useful for the improvement of the mass mixing behaviour.

A thermal flow sensor was investigated experimentally and numerically. Experiments were carried out to show the feasibility of using the Laser Induced Fluorescence method for micro thermal flow sensing. In addition, the two-dimensional numerical simulation gives a better understanding of the conjugate forced convection-conduction heat transfer process of a micro thermal flow sensor. The results show that the heat conduction in the channel wall should be taken into consideration. The magnitude of the flow rate plays an important role in the heat transfer process of a thermal flow sensor.

An analytical model of a counter flow micro-heat exchanger is developed. Experiments are performed to compare with the theoretical analysis. Based on the measured temperature field, the local Reynolds number (Re) and Prandtl number (Pr) can be obtained. A new correlation for Nusselt number is proposed. The convective heat transfer in microchannels is found to be significantly dependent on the Reynolds number (Re) and Prandtl number (Pr). The Nusselt number increases with increasing Reynolds number and the distance to the

entrance. The correlation developed in this thesis can be used as guidelines for the design of micro heat exchangers.

A novel method called the temperature-induced droplet coalescence in microchannels has been proposed and demonstrated. The droplets are trapped and manipulated in a rectangular merging chamber. The merging process is controlled by the voltage applied to a wire heater in the merging chamber. This method is effective for a high flow rate ratio, low total flow rate and high temperature. The high flow rate ratio makes sure that the droplets are in reasonably close contact and can remain in the chamber at the same time for merging. The low total flow rate ensures the low velocity of droplets in the chamber. The high temperature reduces the viscosities and the interfacial tension, and at the same time increases the thermocapillary forces. A two-dimensional numerical simulation is carried out to provide a better understanding of the droplet merging process.

Heat transfer characteristics in plug trains have been investigated theoretically and numerically. Heat transfer in two liquid plugs can be divided into three stages: (I) development of thermal boundary layer; (II) advection of heated/fresh fluid in the plug; and (III) thermally fully developed flow. A high thermal conductivity ratio (k_2 / k_1) results in a lower Nusselt number and a higher heat transfer index in the second plug compared to the first plug. Low plug length ratio is effective in enhancing heat transfer in the microchannel. A low plug length ratio ($\hat{L}_2 / \hat{L}_1$) results in a lower Nusselt number and a lower heat transfer index in the second plug compared to the first plug.

9.2 Recommendations for Future Work

The following is a list of possible extensions of the present work:

1. For the CFMHE, further experimental studies are needed to investigate the heat transfer characteristic. Different liquids with various velocities could be taken into consideration. A general correlation can be implemented based on the experimental results. A three-dimensional numerical model can also be developed to investigate the fluid flow and heat transfer characteristics in details.
2. For the temperature-induced droplet coalescence, further experimental studies are needed to investigate to realise multiple droplet coalescence based on the combination of various chamber sizes and voltage applied to the heater as shown in Fig. 9.1. With the increase of chamber size, multiple droplets will enter the merging chamber. On the activation of the heater, multiple droplets coalescence can be achieved. Scale analysis can be employed to reveal the mechanism of droplet coalescence. A three-dimensional numerical simulation is to be conducted especially in the depthwise direction.

Fig. 9.1 Schematic illustration for multiple droplet coalescence

3. The numerical simulation of plug trains and convective heat transfer in microchannel have proved to be convenient and effective. Further experimental investigations (Fig. 9.2) are needed to investigate the heat

transfer characteristics in plug trains under a constant surface heat flux boundary condition.

Fig. 9.2 Schematic illustration of constant heat flux boundary condition for heat transfer in plug trains

Appendix A: Finite Fourier Transform (FFT)

Method

The Finite Fourier Transform (FFT) method involves:

- (1) Expand the solution in terms of known functions (basis functions);
- (2) Applying the FFT method to PDEs to reduce the number of spatial variables until only a two-point boundary value problem or initial-value problem remains;
- (3) Determine the unknown coefficients in the expansion.

The solution form is expressed as a sum of basis functions

$$\Theta(x, y) = \sum_{n=1}^{\infty} C_n(y) \Phi_n(x) \quad (\text{A.1})$$

where Φ_n is the basis function (orthonormal in most cases) satisfying orthogonality: the inner product equals the Kronecker delta.

$$\int_a^b w(x) \Phi_m(x) \Phi_n(x) dx = \delta_{mn} \quad (\text{A.2})$$

where w is the weighting function and the Fourier transform of Θ is defined as follows:

$$C_n(y) = \int_a^b w(x) \Theta(x, y) \Phi_n(x) dx \quad (\text{A.3})$$

The basis functions are solutions to eigenvalue problem

$$\frac{1}{w} \left[\frac{d}{dx} \left(p \frac{d\Phi}{dx} \right) + q\Phi \right] = -\lambda^2 \Phi \quad (\text{A.4})$$

where p and q are functions of x , depending on the coordinate system.

Appendix B: Uncertainty Analysis

An in-depth uncertainty analysis of the experimental system was carried out to explore the total errors that will exist in the results. Propagation of individual measured uncertainties will be used to estimate the final uncertainty of the Reynolds number, the Nusselt number, the Brinkman number and the Prandtl number.

B.1 Uncertainty Analysis

B.1.1 Uncertainty propagation

Consider the result R as a function of independent variables $x_1, x_2, \dots, x_n$. i.e $R = R(x_1, x_2, \dots, x_n)$. Let u_R be the uncertainty in the result and $u_1, u_2, \dots, u_n$ be the uncertainties in the independent variables. The propagation of the overall uncertainty can be expressed as

$$u_R = \pm \left[\left(\frac{\partial R}{\partial x_1} u_1 \right)^2 + \left(\frac{\partial R}{\partial x_2} u_2 \right)^2 + \dots + \left(\frac{\partial R}{\partial x_n} u_n \right)^2 \right]^{1/2} \quad (\text{B.1})$$

The functional equations for the Reynolds number, the Nusselt number, the Brinkman number and the Prandtl number are as follows

$$\text{Re} = f(\rho, Q, \mu_l, P) \quad (\text{B.2})$$

$$\text{Nu} = (D_h, \Delta y, \Delta T / (T_m - T_s)) \quad (\text{B.3})$$

$$\text{Br} = f(Q, \mu_l, \Delta T, \Delta y, A, D_h) \quad (\text{B.4})$$

$$\text{Pr} = f(\mu_l, c_{pl}, k_l) \quad (\text{B.5})$$

Appendix B: Uncertainty Analysis

Applying equation (B.1) to the above functional relationships, the uncertainties for the Reynolds number, the Nusselt number, the Brinkman number and the Prandtl number can be obtained.

B.1.2 Independent uncertainty

(A) Microchannel width and height

The uncertainty of the measurement of the microchannel width and height is taken as $\pm 1/2$ of the measuring instrument resolution, i.e.

$$u_W = u_H = 0.0000005 \text{ m.}$$

(B) Microchannel length

The uncertainty of the measurement of the microchannel of the tube length is taken as $\pm 1/2$ of the standard ruler, i.e. $u_L = 0.0005 \text{ m.}$

(C) 'K' type thermocouple and calibration set

The uncertainties of the thermocouple and the calibration set are given by the supplier to be 0.1°C and 0.05°C , respectively, i.e. $u_{Thermocouple} = 0.1^\circ\text{C}$ and

$$u_{ThermCalSet} = 0.05^\circ\text{C}.$$

(D) Data acquisition system

The thermometer (VC-01) has the following specifications:

Resolution: 0.05°C

Accuracy: within 0.05% of reading

The uncertainty of the data Thermal Calibration Cool/Heat Source Model 18B using the Root-Sum-Square (RSS) is given by:

Appendix B: Uncertainty Analysis

$$u_{Thermometer} = \pm \sqrt{(u_1)_{tm}^2 + (u_2)_{tm}^2} \quad (B.6)$$

where $u_{Thermometer}$ is the uncertainty of the thermometer, $(u_1)_{tm}$ and $(u_2)_{tm}$ are the uncertainties due to the resolution and accuracy components.

$$(u_1)_{tm} = \pm 0.0025^\circ \text{C} \quad (B.7)$$

$$(u_2)_{tm} = 0.0005 \times T_{Meas.Temp} \quad (B.8)$$

(E) Measured temperature

The uncertainty of measured temperature, $u_{Meas.Temp}$ can be obtained from the following formulation:

$$u_{Meas.Temp} = \pm \sqrt{u_{Thermocouple}^2 + u_{Thermometer}^2} \quad (B.9)$$

(F) Calibrated temperature

A linear curve fitted line has been obtained from the calibration and it is presented below as:

$$T_{Cal.Temp} = 0.9866T_{Meas.Temp} - 0.2780 \quad (B.10)$$

The uncertainty of the temperature due to calibration, $u_{Cal.Temp}$ is then described as:

$$u_{Cal.Temp} = \pm \sqrt{\left[\left(\frac{\partial T_{Cal.Temp}}{\partial T_{Meas.Temp}} u_{Meas.Temp} \right)^2 \right]} = 0.9866 u_{Meas.Temp} \quad (B.11)$$

Finally, the uncertainty of the actual temperature, $u_{Actual.Temp}$ is :

$$u_{Actual.Temp} = \pm \sqrt{u_{Therm.Cal.Set}^2 + u_{Cal.Temp}^2} \quad (B.12)$$

(G) Dynamic viscosity

Appendix B: Uncertainty Analysis

The dynamic viscosity is a function of temperature and it has the form as follows:

$$\mu = 2.414 \times 10^{-5} \times 10^{247.8/(T+133)} \quad (\text{B.13})$$

Therefore, the uncertainty for dynamic viscosity is given as:

$$u_{\mu} = \pm \sqrt{\left(\frac{\partial \mu}{\partial T} u_{\text{Actual.Temp}} \right)^2} = \left[\pm 0.01377 \times \frac{10^{247.8/(T+133)}}{(T+133)^2} \right] u_{\text{Actual.Temp}} \quad (\text{B.14})$$

(H) Density

The density is also a function of temperature and it has the form as follows:

$$\rho = \frac{999.8 + 18.2 \times T - 7.9 \times 10^{-3} \times T^2 - 5.5 \times 10^{-5} \times T^3 + 1.5 \times 10^{-7} \times T^4 - 3.9 \times 10^{-10} T^5}{1 + 1.8 \times 10^{-2} \times T} \quad (\text{B.15})$$

The uncertainty for density gives:

$$u_{\rho} = \pm \sqrt{\left(\frac{\partial \rho}{\partial T} u_{\text{Actual.Temp}} \right)^2} = \left[\pm \left(\frac{0.2036}{(1 + 0.018T)^2} - \frac{0.0158T}{(1 + 0.018T)^2} - \frac{0.000372T^2}{(1 + 0.018T)^2} - \frac{1.38 \times 10^{-6} T^3}{(1 + 0.018T)^2} + \frac{6.15 \times 10^{-9} T^4}{(1 + 0.018T)^2} - \frac{2.808 \times 10^{-11} T^5}{(1 + 0.018T)^2} \right) \right] u_{\text{Actual.Temp}} \quad (\text{B.16})$$

(I) Heat capacity

The heat capacity is also a function of temperature and it has the form as follows:

$$c_p = 8958.9 - 40.535 \times (T + 273) + 0.11243 \times (T + 273)^2 - 1.0138 \times 10^{-4} \times (T + 273)^3 \quad (\text{B.17})$$

The uncertainty for heat capacity is:

$$\begin{aligned}
 u_{c_p} &= \pm \sqrt{\left(\frac{\partial c_p}{\partial T} u_{Actual.Temp} \right)^2} \\
 &= \left[\pm \left(-40.535 + 0.22486(T + 273) - 3.0414 \times 10^{-4} (T + 273)^2 \right) \right] u_{Actual.Temp}
 \end{aligned}
 \tag{B.18}$$

(J) Thermal conductivity

The thermal conductivity is also a function of temperature and it has the form as follows:

$$k = -0.58166 + 6.3555 \times 10^{-3} \times (T + 273) - 7.9643 \times 10^{-6} \times (T + 273)^2 \tag{B.19}$$

The uncertainty for thermal conductivity is:

$$\begin{aligned}
 u_k &= \pm \sqrt{\left(\frac{\partial k}{\partial T} u_{Actual.Temp} \right)^2} \\
 &= \left[\pm \left(6.3555 \times 10^{-3} - 7.9643 \times 10^{-6} (T + 273) \right) \right] u_{Actual.Temp}
 \end{aligned}
 \tag{B.20}$$

(K) Flow rate

The flow rate is provided by The Cole-Parmer syringe pump (74900-05) possesses the below specifications:

Syringe size accepted: 10 μ L to 60 mL

Syringe travel time: 9.5×10^{-6} mm/min

Reproducibility: $\pm 0.2\%$

Accuracy: $\pm 0.5\%$

Brand: Cole-Parmer

Dimensions: 9" $\times$ 4" $\times$ 6"

The uncertainty of the flow rate is given as:

$$u_Q = \pm \sqrt{(u_1)_{pump}^2 + (u_2)_{pump}^2} \quad (B.21)$$

where $(u_1)_{pump}$ is the uncertainty due to the reproducibility and $(u_2)_{pump}$ is uncertainty due to the accuracy. They are given by:

$$(u_1)_{pump} = 0.002 \times Q_{Meas} \quad (B.22)$$

$$(u_2)_{pump} = 0.005 \times Q_{Meas} \quad (B.23)$$

B.1.3 Reynolds number

The Reynolds number is defined as:

$$Re = \frac{\rho u D_h}{\mu} = \frac{4 \rho Q}{\mu P} \quad (B.24)$$

The propagated uncertainty is:

$$u_{Re} = \pm \left[\left(\frac{\partial Re}{\partial Q} u_Q \right)^2 + \left(\frac{\partial Re}{\partial \rho} u_\rho \right)^2 + \left(\frac{\partial Re}{\partial P} u_P \right)^2 + \left(\frac{\partial Re}{\partial \mu} u_\mu \right)^2 \right]^{1/2} \quad (B.25)$$

$$\frac{u_{Re}}{Re} = \pm \left[\left(\frac{u_Q}{Q} \right)^2 + \left(\frac{u_\rho}{\rho} \right)^2 + \left(\frac{u_P}{P} \right)^2 + \left(\frac{u_\mu}{\mu} \right)^2 \right]^{1/2} \quad (B.26)$$

B.1.4 Nusselt number

The Nusselt number is defined as:

$$Nu = \frac{h D_h}{k_l} = \frac{4 k_l A}{k_l P (T_m - T_s)} \frac{\Delta T}{\Delta y} = 4 \frac{A}{P \Delta y} \cdot \frac{\Delta T}{(T_m - T_s)} \quad (B.27)$$

The propagated uncertainty is:

$$\frac{u_{Nu}}{Nu} = \pm \left[\left(\frac{\partial Nu}{\partial A} u_A \right)^2 + \left(\frac{\partial Nu}{\partial P} u_P \right)^2 + \left(\frac{\partial Nu}{\partial \Delta y} u_{\Delta y} \right)^2 + \left(\frac{\partial Nu}{\partial [\Delta T / (T_m - T_s)]} u_{[\Delta T / (T_m - T_s)]} \right)^2 \right]^{1/2} \quad (B.28)$$

B.1.4 Brinkman number

The Brinkman number is defined as:

$$\text{Br} = \frac{\mu_l u_m^2}{k_s \Delta T / \Delta y \cdot D_h} = \frac{\mu_l \cdot \Delta y \cdot Q^2}{k_s \cdot \Delta T \cdot A^2 \cdot D_h} \quad (\text{B.29})$$

The propagated uncertainty is:

$$u_{\text{Br}} = \pm \left[\left(\frac{\partial \text{Br}}{\partial \mu_l} u_{\mu_l} \right)^2 + \left(\frac{\partial \text{Br}}{\partial \Delta y} u_{\Delta y} \right)^2 + \left(\frac{\partial \text{Br}}{\partial Q} u_Q \right)^2 + \left(\frac{\partial \text{Br}}{\partial k_s} u_{k_s} \right)^2 + \left(\frac{\partial \text{Br}}{\partial A} u_A \right)^2 + \left(\frac{\partial \text{Br}}{\partial \Delta T} u_{\Delta T} \right)^2 + \left(\frac{\partial \text{Br}}{\partial D_h} u_{D_h} \right)^2 \right]^{1/2} \quad (\text{B.30})$$

$$\frac{u_{\text{Br}}}{\text{Br}} = \pm \left[\left(\frac{u_{\mu_l}}{\mu_l} \right)^2 + \left(\frac{u_{\Delta y}}{\Delta y} \right)^2 + 4 \left(\frac{u_Q}{Q} \right)^2 + \left(\frac{u_{k_s}}{k_s} \right)^2 + 4 \left(\frac{u_A}{A} \right)^2 + \left(\frac{u_{\Delta T}}{\Delta T} \right)^2 + \left(\frac{u_{D_h}}{\Delta D_h} \right)^2 \right]^{1/2} \quad (\text{B.31})$$

B.1.5 Prandtl number

The Prandtl number is defined as:

$$\text{Pr} = \frac{\mu_l c_p}{k_l} \quad (\text{B.32})$$

The propagated uncertainty is:

$$u_{\text{Pr}} = \pm \left[\left(\frac{\partial \text{Pr}}{\partial \mu_l} u_{\mu_l} \right)^2 + \left(\frac{\partial \text{Pr}}{\partial c_p} u_{c_p} \right)^2 + \left(\frac{\partial \text{Pr}}{\partial k_l} u_{k_l} \right)^2 \right]^{1/2} \quad (\text{B.33})$$

$$\frac{u_{\text{Pr}}}{\text{Pr}} = \pm \left[\left(\frac{u_{\mu_l}}{\mu_l} \right)^2 + \left(\frac{u_{c_p}}{c_p} \right)^2 + \left(\frac{u_{k_l}}{k_l} \right)^2 \right]^{1/2} \quad (\text{B.34})$$

B.2 Uncertainty Results

Based on the above uncertainty estimation, the average uncertainties for the Reynolds number, the Nusselt number, the Brinkman number and the Prandtl number were calculated as 3.5%, 4.2%, 6.4% and 2.7%, respectively.

Publications arising from this thesis

1. Xu, B., T.N. Wong, N.T. Nguyen, Z. Che and J.C.K. Chai, “Thermal mixing of two miscible fluids in a T-shaped microchannel”, *Biomicrofluidics*, 2010. **4**(4): pp. 044102/1-044102/13.
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