Analysis and design of square waveguide ortho-mode transducers

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Analysis and Design of Square Waveguide Ortho-Mode Transducers

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Summary

Ortho-mode transducers (OMTs) are key components in antenna feed systems when dual polarizations are involved, such as satellite communication, remote-sensing applications, radio astronomy, and astrophysical observations of the cosmic microwave background. Typically, they are used to receive dual-polarized signals from a horn antenna and separate them into two independent single-polarized channels. With the development of modern communication systems, it is preferred to process dual-polarized signals over a wide band or multiple bands. However, the design theory of OMTs is not well-developed, and most OMTs are designed by experience and full-wave optimizations.

In this thesis, the mode-matching method is employed to efficiently analyze waveguide OMTs. An equivalent circuit model of a square waveguide T-junction is proposed as a key building block of waveguide OMTs, and its closed-form expressions are derived from the rigorous mode-matching analysis. A short-circuited branch waveguide OMT is then designed with this equivalent circuit model. Then, an improved equivalent circuit model for a square waveguide T-junction with coupling slot is proposed. Several useful formulas are derived from the circuit analysis to determine the

reflection zero of a short-circuited branch waveguide OMT. An accurate synthesis procedure is then established to determine the dimensions of the OMTs. Finally, two compact dual-band OMTs are designed. In the first OMT, two iris filters are incorporated to improve the isolation between two frequency bands; and the structure basically integrates a diplexer into the OMT. In the second OMT, an irregularly shaped diaphragm is employed as a compact band-rejection element to improve the isolation between two frequency bands.

Chapter 1

Introduction

1.1 Background

Since the first pair of antennas in the world was invented in 1886 by Heinrich Rudolf Hertz, various antenna systems have been deployed all over the world. In many receiving antenna systems, such as radio astronomy [1–3], remote sensing application [4], most received signals from space are unpolarized or partially polarized with a random polarization angle. A simple conversion of the electromagnetic signal from space into a guided wave in a single mode waveguide will approximately result in a polarization loss of 3 dB. To attain full efficiency and sensitivity as well as to measure the polarization state of an incident signal, these dual-polarized signals received from a dual-polarized antenna should be separated as two orthogonal polarizations and transferred into two independent single mode ports. This task is accomplished by the so-called ortho-mode transducer (OMT).

In the rapid developing wireless communications, such as the satellite telecommunications, the demand for high capacity is continuously increasing over the decades [5]. Therefore, it is also desirable to optimize the utilization of the available frequen-

1.1 Background

cy bands on a link to keep pace with the demand of continuously increasing data rate. A key component for this frequency reuse technique [6] is also the OMT, which can double the capacity of the link by transmitting and receiving two orthogonally polarized signals simultaneously over the same frequency band.

A block diagram of an antenna feed system for multi-frequency bands is given as an example and shown in Fig. 1.1 [7]. It can be seen that the OMTs play an important role in this system.

Figure 1.1: Communications antenna feed system for multifrequency band [7].

As a key component in dual-polarized antenna feed systems, narrow-band OMTs have been developed for many decades [8,9]. Although OMT structures vary in shape, in general, they are all a physical three-port device. A square or circular waveguide supporting dual-polarization is usually employed as a common port, which connects to a dual-polarized antenna, and two single-polarized ports are used to extract the orthogonal polarizations, respectively. Therefore, a diagram of a typical OMT can be described in Fig. 1.2.

1.1 Background

Figure 1.2: Diagram of a typical OMT.

Since the common port supports two orthogonally polarized signals, an OMT should be considered as an electrically four-port network, as shown in Fig. 1.3 [10], where Ports 1 and 2 are the two orthogonally polarized signal ports in the common physical port, Ports 3 and 4 are the two separated single-polarized signal ports. The solid lines denote the couplings within the same polarization, and the dash lines denote the couplings between orthogonal polarizations.

Although the electrical requirements considered in the OMT design depend on the specific application, in general, the following parameters, shown in Fig. 1.3, are usually mentioned when the performance of OMTs is evaluated [10, 11].

Figure 1.3: Scattering matrix representation of an OMT in receive mode [10].

- 1) Return loss ($-20\log|S_{ii}|$, $i = 1, 2, 3, 4$): a return loss should be no less than 15

1.2 Motivation

dB, with close to 20 dB over most of the operating band desirable, although the figure of some designs can be as low as 10 dB [12], which is unacceptable in many situations.

- 2) Insertion loss ($-20\log|S_{31}|$, $-20\log|S_{42}|$): an insertion loss must be kept to a minimum.
- 3) Isolation ($-20\log|S_{43}|$): the isolation between the output ports of the OMT is of importance and a figure of at least 30 dB is usually considered essential.
- 4) Cross-polarization ($-20\log|S_{21}|$): it is an important but often overlooked consideration. For the discrimination of polarizations, the couplings between two orthogonal polarizations should be kept as low as possible.

Ease of manufacturing [13, 14] should also be taken into consideration if mass production is required, especially in the millimeter wave. In addition, power handling capability will also be of importance under certain circumstances.

1.2 Motivation

There is always an ever-increasing demand for antenna systems to operate over large bandwidths in almost all applications. Therefore, it is also desirable to develop a single wide-band OMT to match the bandwidth of the entire system [11]. For the wide-band OMTs, the higher-order modes may propagate in the waveguide, which may deteriorate its bandwidth performance dramatically. To suppress the excitation

1.2 Motivation

of these unwanted higher-order modes, the symmetrical structures, such as quadruple ridged waveguide [11, 15], turnstile junction [16, 17], etc., are usually preferred.

Nowadays, for communication links of extreme high data rate, the capacity of one frequency band is insufficient even in the case of frequency reuse. Therefore, multiple frequency bands are exploited and multi-band OMTs are desired [12, 18, 19]. Different from the single-band OMT, the multi-band OMT has more than four electrical ports, and the isolation between the ports of the same polarization but operating at different frequency bands becomes an important concern and challenge. Therefore, the multiple operating bands are usually widely separated, and additional filters or diplexers are necessary to improve the isolation between them [7].

In summary, wide-band and multi-band OMTs require more sophisticated designs with more considerations, comparing to the narrow-band ones. However, it is always preferred to design an OMT in a compact size with good performance. Unfortunately, the waveguide OMTs are usually bulky and geometrically awkward, and are still one of the most difficult elements to integrate within a compact antenna feed system [20]. Therefore, the compact design of OMTs is still a big challenge, especially for the wide-band and multi-band cases.

On the other hand, despite the extensive application of the dual-polarized antenna systems for several decades, the design of the key component, OMT, is still treated very unsatisfactorily in the literature, even for a narrow-band one [7]. This is because an OMT is a dual-mode multi-port structure. Although some theories about the multi-port network have been presented in some papers [21, 22], there is still no

1.3 Objectives

proper circuit model to represent the dual-mode OMT structure. Therefore, most OMTs still have to be designed by experiments, cut-and-trial [11, 23], or full-wave optimization [24].

Recently, with the development of full-wave numerical methods, especially with the advent of commercial simulation software packages, the OMTs can be simulated accurately by these full-wave simulators, such as Ansoft's High Frequency Structure Simulator (HFSS) [25] and Computer Simulation Technology (CST) Studio Suite [26]. However, as the complexity of the structure increases, the efficiency of these commercial simulators degrades, and the full-wave optimization may not give satisfactory results in a timely manner without good initial values. Therefore, it is necessary to develop a synthesis method using circuit theory.

From the above discussions, it is understood that although many OMTs have been developed for various dual-polarized antenna feed systems, the design theory of the OMT is still not well established. There are still many challenges in the design of compact wide-band, multi-band OMTs, especially the integration of OMT within a compact antenna system. This thesis aims to develop some circuit analysis and synthesis theory to design waveguide OMTs, and implement some compact dual-band OMTs, which is the main motivation of this Ph.D study.

1.3 Objectives

In responding to the aforementioned issues, the goal of this study is to investigate the analysis and design methods of waveguide OMTs, and to design some compact

1.3 Objectives

dual-band OMTs with the proposed design theory. To fulfill the goal, the following tasks have to be envisaged.

- 1) A comprehensive literature review is conducted about structures and analysis methods of existing OMTs. This exercise provides a strong foundation and background for the exploration.
- 2) The square waveguide OMT is considered in this study. For the purpose of efficient analysis of OMTs, proper numerical method should be developed. Since the waveguide structure is mostly used in the design of OMTs, the full-wave mode-matching (MM) method is chosen for its high accuracy and high efficiency for this waveguide structure. The MM building blocks of rectangular waveguide structures are developed and the square waveguide OMT can be analyzed.
- 3) Based on the developed MM technique, we proceed to analyze a key building block of the square waveguide OMT, the square waveguide T-junction. Its equivalent circuit model is derived from the rigorous MM analysis. With this circuit model, a short-circuited waveguide OMT can be analyzed and designed by a circuit simulator efficiently.
- 4) Once the circuit model of the waveguide OMT is established, the synthesis method based on this circuit model can be investigated. The dimensions of the short-circuited square waveguide OMT can then be accurately determined.
- 5) We also extend our work on the design of compact dual-band OMTs. To implement a compact dual-band OMT with good performance, two filters can be

1.4 Major Contributions of the Thesis

integrated into the OMT. The whole structure can be analyzed efficiently based on the earlier developed circuit model. Some compact filtering elements are also investigated to realize a more compact design.

The outcomes of this research work have produced some original contributions and developments over the existing works. They are outlined in the next section.

1.4 Major Contributions of the Thesis

The major contributions of this PhD study are listed as follows.

- 1) The full-wave MM method has been developed for efficient analysis of square waveguide OMTs. A double-plane step in rectangular waveguide and a six-port waveguide junction are modeled as two building blocks for rectangular waveguide structures by the MM method. The cascade of these two building blocks are investigated. The cascade of two-step discontinuities are summarized as three types. It is found that proper choice of the generalized matrix representation for the cascade can improve the efficiency of our analysis.
- 2) As a key structure of the square waveguide OMT, the square waveguide T-junction has been carefully studied by our developed MM method. A five-port equivalent circuit model is proposed to represent this dual-mode multi-port structure. Based on the rigorous MM analysis, it is found that this five-port network can be divided into two separate sub-networks according to the polarizations. Therefore, the analysis of the original five-port network can be greatly simplified. More-

1.4 Major Contributions of the Thesis

over, closed-form expressions of this equivalent circuit model are derived from the MM analysis with proper approximations, which are very accurate over the entire dominant-mode operating band. With this circuit model, the short-circuited square waveguide OMT can be designed in the circuit domain accurately with high efficiency.

- 3) The synthesis method for short-circuited square waveguide OMT has been developed. A slot-coupled square waveguide T-junction is considered since it exhibits more flexibility and is more frequently used in practical design. An improved equivalent circuit model for a slot-coupled square waveguide T-junction is then proposed, which is more suitable for the circuit synthesis. Based on this improved circuit model, an accurate synthesis method is developed to determine the dimensions of the short-circuited square waveguide OMT. It is noted that the synthesized dimensions can be directly applied to the OMT without any full-wave optimization.
- 4) Two compact dual-band OMTs have been implemented, fabricated and tested. The first one is designed based on the earlier developed circuit analysis and synthesis method. To improve the isolation between two frequency bands, two iris filters are connected to the OMT. With the previously developed equivalent circuit model, the junction effect of the OMT is considered in the design of filters, which introduces additional transmission zeros. Therefore, the filters with sharp rejection can be realized and integrated as parts of the OMT, which results in a compact structure.

1.5 Organization of the Thesis

In the second dual-band OMT, a more compact design is implemented. A waveguide with reduced cross-section is incorporated into the common waveguide and an irregularly shaped diaphragm is proposed as a compact band-rejection element. In this way, the lengthy iris filters, which usually have to be fabricated as two pieces and recombined, can be removed. As a result, the whole structure can be fabricated as a monoblock with a stacked thin diaphragm. In this sense, the stability and power handling capability are greatly improved compared with the first one.

1.5 Organization of the Thesis

The remaining part of the thesis is organized as follows. Chapter 2 presents a literature review of OMTs. Existing designs of OMTs are classified and described. Different types of OMT analysis methods are also reviewed.

In Chapter 3, the MM analysis of rectangular waveguide junctions is presented. Two building blocks of waveguide structures are analyzed by the MM method: one is the double-plane step in rectangular waveguide, and the other is a six-port waveguide junction. The cascade of multi-port generalized scattering matrix (GSM) by these building blocks are discussed, and generalized matrix representations of the two-step cascaded waveguide junction are summarized. The waveguide OMT can then be modeled as the cascade of these building blocks.

In Chapter 4, a square waveguide T-junction is studied as a key building block of waveguide OMTs. A five-port equivalent circuit model is employed to represent

1.5 Organization of the Thesis

this square waveguide T-junction. It is then simplified as two separate subnetworks according to a rigorous mode-coupling analysis. Then closed-form expressions for these two subnetworks are derived separately with proper approximations. A short-circuited waveguide OMT is designed with this equivalent model and measured results verify our design.

In Chapter 5, an improved equivalent circuit model of a square waveguide T-junction with coupling-slot is proposed. The values of circuit elements can be extracted from the MM analysis. The short-circuited waveguide OMT is then analyzed with this circuit model. The reflection zero of the OMT can be determined by some simple formulas. An efficient synthesis procedure is presented to determine the dimensions of the OMT accurately.

In Chapter 6, two compact dual-band OMTs are designed. The first one is developed based on the short-circuited waveguide OMT synthesized in Chapter 5. Two filters are designed together with the OMT as a whole component to obtain good isolation between the two frequency bands. The equivalent circuit model is established to optimize this dual-band OMT efficiently. Finally, the full-wave MM method is used to optimize the whole structure. The second one is a more compact design of the dual-band OMT. To obtain good isolation between the transmit and receive signals of the same polarization with a compact size, a waveguide with reduced cross-section is incorporated into the common waveguide and an irregularly shaped diaphragm is proposed as a compact band-rejection element. This OMT can be fabricated as a monoblock with a stacked thin diaphragm, which is very stable and

1.5 Organization of the Thesis

suitable for mass production.

Finally, Chapter 7 concludes the work and gives recommendations for further research.

Chapter 2

Literature Review

2.1 Introduction

Ortho-mode transducers (OMTs) have been developed as a key component in dual-polarization antenna systems for many decades. Although they may also be referred as polarization diplexers, dual-mode transducers, ortho-mode tees, and ortho-mode junctions (OMJ) in the literature, their basic task is the same: to discriminate two independent signals of the orthogonal dominant modes provided at the common port, and supply them to the single mode ports. To serve this purpose, various types of OMTs have been designed.

In general, it must have one common port supporting dual-polarized signals and at least two orthogonal ports for single-polarized signals. Therefore, various structures supporting dual-polarization have been employed to implement different kinds of OMTs, such as square and circular waveguide T-junctions [27–29], finlines [30], quadruple ridged waveguides [11,31,32], turnstile junctions [3,33,34], etc. The choice of these structures is usually a tradeoff between the performance and complexity.

In this chapter, a thorough literature review of different types of OMTs will be

2.2 Existing OMT Designs

carried out first. After that, some of the numerical methods that have been employed in analyzing these OMTs are also summarized.

2.2 Existing OMT Designs

The design of OMTs is to maintain good match at all electrical ports and high cross-polarization discrimination between the independent signals. The impairment on the match and isolation of a practical OMT design usually results from the higher-order mode excitations. In the most commercial OMT applications that require narrow-band designs, all higher-order modes of the common waveguide are evanescent within the operating band. Therefore, these unwanted higher-order modes can only influence the propagating field of the dominant mode near the discontinuity region. However, in wide- or multi-band OMT applications, higher-order modes may propagate. To suppress these unwanted higher-order modes, symmetrical structures are usually preferred at the cost of increased complexity.

A suitable, although not complete, inventory of different OMT types was first presented according to the symmetrical property of the structures by Bøifot [35] in 1991. In this section, we will first introduce some asymmetrical narrow-band OMTs, and then some types of symmetrical structures for broadband OMT designs will be presented. Finally, some approaches to the design of multi-band OMTs will be described.

2.2 Existing OMT Designs

2.2.1 Asymmetrical Narrow-Band OMTs

Asymmetrical OMTs are most widely used in commercial communication systems since they can achieve good performance, such as low voltage standing wave ratio (VSWR) and high isolation with low fabrication expense. Usually, these asymmetrical structures are considered as narrow-band (usually less than 10%). However, as the development of accurate full-wave simulators, the bandwidth of these asymmetrical OMTs can be extended up to 30% with fine tuned geometry [13]. In general, there are four main types of narrow-band OMTs [7].

Taper/Branching OMT

As shown in Fig. 2.1, a longitudinal taper section is employed to provide a symmetrical or asymmetrical transition of the common square or circular waveguide cross-section to the standard waveguide interface of one fundamental mode. In the meanwhile, a waveguide branching perpendicular to the longitudinal axis is placed for the orthogonal signal.

There are different kinds of tapers including stepped taper, continuous taper, circular to rectangular waveguide taper and hybrid of coaxial port with associated probe. The performance of this type of OMT is mainly determined by the design of the taper and branching regions. Usually, multi-step taper is required to match the impedance between the common dual-mode port and the single-mode port, as shown in Fig. 2.1, which usually demands a full-wave optimization [24, 36].

An example of this type of OMT is shown in Fig. 2.2 [24]. The theoretical return

2.2 Existing OMT Designs

Figure 2.1: Structure of a typical taper/branching OMT.

loss at the in-line and side ports can be below -30 dB from 10.75 GHz to 14.5 GHz, which is about 30% bandwidth, as shown in Fig. 2.3.

Figure 2.2: An example of a taper/branching OMT operating at Ku band [24].

2.2 Existing OMT Designs

Figure 2.3: Theoretical return loss at the in-line input port and at side input port of a Ku-band OMT (30% bandwidth, 10.75-14.5 GHz) [24].

Septum/Branching OMT

A septum is inserted into the common waveguide [37], as shown in Fig. 2.4, which has little effect on the electrical field vertical to it, provide that the septum is sufficient thin. For the orthogonal mode, it is evanescent within the septum region and coupled to the side branch. So this provides more isolation between the two orthogonal modes. The shape, size and location can be varied, and double septum can be inserted [38], which provides more flexibility aiming to match the signal in the side arm better [13, 39]. Such designs provide excellent performance with up to 20% bandwidth.

2.2 Existing OMT Designs

Figure 2.4: Structure of a typical septum/branching OMT.

Acute Angle or Longitudinal Ortho-Mode Branching

Unlike the types mentioned above, whose branching is perpendicular to the common arm, the dual polarization signal ramifies under an acute angle from the longitudinal axis of the common waveguide, as a Y-junction [40] illustrated in Fig. 2.5, or bifurcated with a stepped septum along the axis [41, 42]. Also the septum can be inserted into both side arms to improve the isolation. It should be mentioned that a polarizer can be formed with this OMT, which separates the orthogonal signals with a differential phase shift, usually 90 degrees, so that circular polarization can be obtained from two orthogonal linearly polarized signals [41, 42]. The main advantage of this type is its simplicity, even analytical formulas have been provided [40]. The main drawback is that it is difficult to achieve good matching and high isolation at the same time, and its bandwidth is very limited, due to excitation of higher-order modes, and high manufacturing expense is another further disadvantage.

2.2 Existing OMT Designs

Figure 2.5: Structure of a typical Y-junction OMT.

Short-Circuited Common Waveguide

This is the simplest design, based on a short-circuited common waveguide that has two rectangular branching waveguides, where both are situated at perpendicular side walls of the common waveguide to serve the respective polarized signals, as shown in Fig. 2.6.

To achieve a higher isolation, the locations of l_1 and l_2 are usually determined by [43]

$$l_1 \approx \frac{\lambda_{g0}}{4} \quad (2.1a)$$

$$l_2 \approx \frac{3\lambda_{g0}}{4} \quad (2.1b)$$

where λ_{g0} represents the common waveguide wavelength of the respective dominant

2.2 Existing OMT Designs

mode at the center frequency (f_0) of the assigned operating band.

Figure 2.6: Structure of a typical short-circuited square waveguide OMT.: (a) 3D view and (b) side view.

This type of design is very simple; compact-size in the direction of the longitudinal axis. Since there is no septum inserted within the common waveguide, it is easy to fabricate even at millimeter wave [44]. Its drawback is the excitation of higher-order modes within the asymmetric branching region. Thus, adequate match and isolation properties can hardly be achieved for a bandwidth of more than 10%.

However, this type can be adapted to other OMT designs easily. For example, the rectangular waveguide ports can be replaced by coaxial ports with probe couplings. Moreover, the turnstile junction, which will be introduced in the broadband OMT part, can be viewed as a symmetrical version of this type.

It should be noted that only square waveguide narrow-band OMTs are illustrated in Figs. 2.1~2.6 for simplicity. The common port can also be circular, as seen in [28, 43, 45].

Besides these four types of asymmetrical narrow-band OMTs introduced above,

2.2 Existing OMT Designs

there are also some approaches to realizing wider bandwidth using an asymmetrical structure. One is the finline [30], which is an inherent broadband structure. About 40% bandwidth was achieved in [46]. Unfortunately, a thin gap is required in the design, which increases the fabrication expense and not suitable for millimeter wave applications [11, 13].

Another promising broadband technique is to introduce some “mirror impedance” or “image load” [47] within the asymmetrical branch region, which exhibits symmetrical properties to suppress higher-order modes with a single-branching compact size structure by proper discontinuities. It can achieve broader bandwidth (20% in [13]) with a reasonable complexity.

2.2.2 Symmetrical Broadband OMTs

In the design of broadband OMTs, symmetrical structures are usually employed since they can suppress the excitation of higher-order modes. A pair of identical waveguide junctions, called *dual junction*, are symmetrically situated with respect to the common waveguide. Each junction extract half of the signal energy of the dedicated polarization from the common waveguide. An appropriate power combiner is necessary to recover the total signal energy of the dedicated polarization at one interface port [7].

Accordingly, a complete symmetrical broadband OMT design comprises the common waveguide with two dual junctions and two power combiners. Obviously, it requires a much more complicated design, and the performance, such as return loss

2.2 Existing OMT Designs

and isolation may degrade, compared with its narrow-band counterpart. Therefore, it is often a trade-off between bandwidth, complexity and performance.

Here we will present some types of symmetrical broadband OMT designs available in the literature.

Quad-Ridged Waveguide OMT

The quad-ridged waveguide is an inherent symmetrical broadband structure supporting dual-polarization [48,49], which is very suitable for broadband OMT designs. The quad-ridges are tapered inside the waveguide and ended as a narrow gap with a shorting cap. Two orthogonal probes are placed perpendicular to the waveguide wall to extract the energy, as shown in Fig. 2.7 [11,32]. The geometry of the ridges may be modified to enhance the performance (e.g., bandwidth, power handling capability) [11,15].

It should be mentioned that only probes are employed to extract orthogonal signals. There is no dual junction in the design. Therefore, it is not a rigorous symmetrical design. The alignment of the four ridges inside the waveguide should be very careful. Otherwise, small asymmetry will excite higher-order modes and deteriorate the performance. An offset quadruple ridged waveguide OMT was reported in [31] to partially overcome this disadvantage.

Turnstile Junction OMT

The application of turnstile junction can be traced back to half a century ago [50]. It consists of four coplanar rectangular arms and a perpendicular circular arm, which

2.2 Existing OMT Designs

Figure 2.7: Cross-section of a quad-ridged waveguide OMT. Corners of the ridges are cut to enhance the bandwidth and power handling capability

contains two orthogonal TE_{11} modes, as shown in Fig. 2.8. The linearly polarized wave is naturally directed towards the arms having the same axis direction. Wide-band matching between the circular port and the four rectangular ports can be easily obtained by introducing some scattering element in the branching region, such as metallic pyramid [16, 33] and a tuneable stepped post [17, 51].

Each pair of opposite arms need to be recombined through a tee junction. But due to the particular geometry of the turnstile junction, these two pairs of arms have to cross over with each other, which usually leads to a large, asymmetrical network. In [51], four E-plane 180° bends and two E-plane combiners are used. Some similar designs with compact size were reported in [3, 33, 34]. An ultra-thin design is presented in [16]. An alternative design proposed by Hozuori [52] uses four magic tees to avoid the crossing between the adjacent arms. [53] reports a design

2.2 Existing OMT Designs

Figure 2.8: Turnstile junction

with only two magic tees used.

The fabrication of these turnstile junction is also an important issue. Due to its complicated branching and combining structure, it is usually divided into four blocks and fabricated separately [3, 34, 51]. Therefore, the alignment of these four blocks should be very careful. In [17], the OMT is fabricated as whole piece with electroforming technique by Forestal S.r.l. [54].

An example of turnstile junction OMT is shown in Fig. 2.9 [34]. Measured return losses at rectangular ports are better than 25 dB over the frequency band from 3.6 GHz to 7 GHz (64% bandwidth), as shown in Fig. 2.10.

Bϕifot Junction OMT

The Bϕifot junction OMT was first invented by A. M. Bϕifot in 1990 [23]. It can be considered as a folded turnstile junction, where two of the ports have been folded parallel to the common-port. The basic building block of this design is the symmetrical five-port branching– Bϕifot junction, as shown in Fig. 2.11. A stepped or taper

2.2 Existing OMT Designs

Figure 2.9: Example of a designed turnstile junction OMT : internal structure (left) and manufactured OMT (right) [34].

Figure 2.10: Measured and simulated reflection coefficients of the turnstile junction OMT at rectangular ports. [34].

septum is inserted in the common waveguide, two branches are symmetrically allocated at the sides with two posts. The septum and posts both aim at improving the isolation between the orthogonal modes. Several broadband Bøifot junction OMTs were reported in [55–59] with good performance.

2.2 Existing OMT Designs

Figure 2.11: Bøifot junction [7]

Compared with the turnstile junction OMT, the Bøifot junction OMT can be fabricated as two blocks, which improves the stability of the design. However, the inclusion of the septum and posts increases the fabrication expense.

Besides the above mentioned waveguide OMTs, several planar symmetrical OMTs were developed recently [60–63]. Although they are of low profile and easy to be integrated with other components, the performance of these planar OMTs are still limited compared with waveguide OMTs. Since the principle of these planar OMTs is similar to that of waveguide OMTs, we may not discuss them in details.

2.2 Existing OMT Designs

2.2.3 Multi-band OMT Designs

This kind of OMT allows the antenna to receive or transmit dual polarized signals in different frequency bands. Therefore, these OMTs are no longer physically three-port devices; they are four- or multi-port devices. It is noticed that a kind of so-called dual-band or double-band OMTs reported in [39, 45] should not belong to this group, for they are still physically three-port devices.

References about these multi-band OMTs are very limited in the public domain [64, 65], but it is desirable to equip the antenna system with multi-band OMTs, because it is convenient to receive or transmit signals from different frequency bands simultaneously and can improve performance of the entire system greatly. Several approaches for multi-band OMT designs are given below.

Modular Approach

This approach can be viewed as a combination of a broadband OMT and two broadband diplexers or multiplexers. It first separates the two orthogonal polarized signals of the complete band by a broadband OMT [66], and then broadband diplexers or multi-plexers are connected to single polarized ports. Thus, the design is performed independently, resulting in high design flexibility. But this design is limited by the restrictions of the broadband OMT, which may not cover all the separate frequency bands.

2.3 Existing OMT Designs

Asymmetrical Branching Approach

This approach is based on the longitudinally cascading of the asymmetrical OMT branching, as introduced for the narrow-band OMTs. Each branching is only responsible for the respective polarization and frequency band, and the rest signals go through until the next branching is met. Since these signals operating over different frequency bands may be of the same polarization, they cannot be isolated due to the orthogonality of polarization. Therefore, these frequency bands are usually widely separated, and some filters are usually employed to improve the isolation if necessary.

A dual-band OMT operating at 30/44 GHz was designed in [67], which employed a series of stacked irises as a compact filtering element to improve the isolation. A multi-frequency waveguide OMT operating at four frequency bands was implemented by this approach [12].

Symmetrical Branching Approach

This type cascades the symmetrical branching structures mentioned in broadband OMT designs to suppress the higher-order modes. A C/Ku dual frequency-band combiner was described as an example in [7]. Another C/Ku dual-band OMT was implemented using quadrature junction (QJ), which is a six-port junction [19]. Besides, a C/X/Ku tri-band OMJ was reported in [65], while its structure was not described in detail.

2.3 Analysis Methods of OMTs

The previous section reviews many kinds of OMT structures available in the literature. Another important problem is how to analyze and design the OMTs mentioned above. As most OMTs are implemented by rectangular or circular waveguide, the analysis method for waveguide structures can be applied. The early OMTs are analyzed largely based on experiments [11, 23], and later some numerical methods were applied. However, different from other waveguide components, such as filter and coupler, the design theory or synthesis method has not been fully developed.

2.3.1 The Mode-Matching Method

As a waveguide component, lots of OMTs have been analyzed by the mode-matching (MM) method for it is highly efficient. Because the normal MM technique can be applied if the geometry of the basic OMT section satisfies the following conditions:

- 1) no overlapping of septum and branching regions;
- 2) no change of the waveguide cross section within the branching region;
- 3) no shaped septum within the waveguide;
- 4) only stepped discontinuities.

The early mode matching method are primarily used in the analysis of a simple T-junction or Y-junction formed by square-rectangular [39] or circular-rectangular [43, 44, 68] waveguide junction. With more delicate matching procedure, the stepped septum structures were analyzed [41, 42] and the hybrid technique combining the

2.3 Analysis Methods of OMTs

generalized admittance matrix (GAM) and the generalized scattering matrix (GSM) was developed [39], which can be used to design a very compact OMT with double septum [38].

Later, by treating as two auxiliary boundary-value problems, the fields of a septum overlapping with the junction region can be computed [69]. A complicated wideband Bifot junction [70] and a compact OMT [24] were analyzed and optimized with CAD software tools based on the mode matching method [70]. A taper branching OMT, which cannot be solved with the normal MM technique, is also analyzed by dividing the branching region into more subregions [71]. Of course, they all requires very complicated derivation and matching procedures.

2.3.2 Other Numerical Methods

Compared to the MM method, the finite-element method (FEM) can analyze an arbitrarily shaped structure. With the development of commercial universal simulator, e.g. Ansoft's HFSS, a designer can readily analyze the model of an arbitrary OMT [12, 13, 46, 57].

The method of moments (MoM) was also used [10, 29], and a closed-form expression can be derived and only the source region must be meshed into subsections, rather than the whole geometry. In addition, the number of unknowns is extremely small, so the computational effort to obtain accurate solution is negligible compared to the FEM.

However, all these full-wave numerical methods are still less efficient compared

2.3 Analysis Methods of OMTs

with the MM method. Although modern computers are growing fast enough to analyze most of the OMT structures, it is still a challenge to analyze the broadband and multi-band OMT, or to optimize waveguide OMTs.

2.3.3 Hybrid Methods

It is known that the MM method is highly efficient, but limited to regular waveguide structures, and other universal methods, such as the FEM and the MoM, are flexible for arbitrary structures with low efficiency. Therefore, it is desirable to hybridize the efficient MM method with other universal methods.

Some hybrid methods were used in the design of OMT. In [65], design of the corrugated and conical structure was performed using the MM method, and the FEM analysis was used to design and optimize the coupling slot geometry and the OMT/filter sub-assembly. In [60], a circuit simulator was used to design the microstrip port and the region bounded by waveguide walls was modeled by HFSS.

More efficient hybrid method have been developed by combing the semi-analytical MM method with other universal full-wave numerical methods, such as the FEM, the boundary-element method (BEM), MoM, etc. In this area, many works have been done by Arndt's group [72–74]. The regular region is analyzed by the mode-matching method, while the irregular region, which is difficult or impossible to be modeled by MM method, is analyzed by the suitable full-wave method according to its geometry.

These hybrid methods can efficiently analyze most of the waveguide structures. A single-band OMT with discontinuities in branching region was designed [36]. They

2.3 Analysis Methods of OMTs

may receive more attentions in the design of multi-band OMTs, because the size of this type of OMT should be sufficiently large for the lower band, which results in more computational efforts for the upper band by full-wave methods. In some cases, it would be unacceptable to simulate the whole structure covering all the bands with these full-wave simulators, such as Ansoft's HFSS or CST Microwave Studio.

All the methods mentioned above have been employed in the analysis of OMT, However, the design theory has not been fully developed. Most of the designs are still largely carried out based on experience or try-and-error, useful synthesis method has not been proposed in the literature. Only an analytical formulas for a simple Y-junction OMT was presented in [40].

Chapter 3

Mode-Matching Analysis of OMT Building Blocks

3.1 Introduction

As introduced in Chapter 2, most of ortho-mode transducers (OMTs) are implemented with waveguide structures [7]. Although many types of numerical methods have been developed, it is well-known that for regular waveguide structures, the mode-matching technique (MMT) is most efficient compared with other full-wave methods. The mode-matching (MM) method was developed many years ago, and various waveguide structures were analyzed by different approaches of modal analysis [72, 75–77].

It is seen that most OMT structures can be modeled by a few building blocks [72]. For rectangular waveguides, only two building blocks are sufficient. One is the double-plane step [77, 78], and the other is a six-port junction [79, 80]. Although these two structures have been well studied by the MM method for many years, it is necessary to establish the in-house building blocks of the waveguide OMTs first in this chapter, which would facilitate the design of OMTs later.

3.2 Formulations of Building Blocks

In this chapter, the eigenmode functions in a rectangular waveguide are first presented. The electric and magnetic fields are then expanded by these eigenmode functions and matched at their interfaces of junctions. A mode coupling matrix can be established to relate the incident to reflected waves in different waveguide sections. The scattering matrix can be obtained thereafter.

To analyze the waveguide OMT structures with these two building blocks, the cascade of these blocks is investigated. The generalized matrix representations of two-step cascaded junctions are presented. The cascade of multi-port junctions used in the OMT structures is also studied.

Numerical results are compared with measured results available in the literature or those simulated by the commercial software Ansoft's High Frequency Structure Simulator (HFSS), which shows very good agreements.

3.2 Formulations of Building Blocks

3.2.1 Fields in a Rectangular Waveguide

In a rectangular waveguide shown in Fig. 3.1, the eigenfunctions are [7]

$$\begin{cases} \phi_{mn}^h(x, y) = N_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b}, & \text{for TE}_{mn} \text{ mode, } m, n = 0, 1, 2, \dots \\ \phi_{mn}^e(x, y) = N_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}, & \text{for TM}_{mn} \text{ mode, } m, n = 1, 2, 3, \dots \end{cases} \quad (3.1)$$

3.2 Formulations of Building Blocks

where the normalized factor is

$$N_{mn} = \frac{\sqrt{\epsilon_m \epsilon_n}}{k_{ci} \sqrt{ab}} \quad (3.2)$$

The cutoff wavenumber is $k_{ci} = \sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2}$, the subscript i is the combined

mode index for possible TE_{mn} , TM_{mn} modes, and $\epsilon_m = \begin{cases} 1, m = 0 \\ 2, m \neq 0 \end{cases}$.

Figure 3.1: A rectangular waveguide section.

The transverse electric and magnetic fields in a rectangular waveguide can be expressed by the eigenfunctions as follows.

$$\begin{cases} \vec{E}_t = \sum_{i=1}^N \frac{1}{\sqrt{Y_i}} (A_i^+ e^{-\gamma_i z} + A_i^- e^{\gamma_i z}) \vec{h}_i \times \hat{z} \\ \vec{H}_t = \sum_{i=1}^N \sqrt{Y_i} (A_i^+ e^{-\gamma_i z} - A_i^- e^{\gamma_i z}) \vec{h}_i \end{cases} \quad (3.3)$$

where N is the number of modes considered in the waveguide, A_i^+ and A_i^- are the amplitudes of incident and reflected wave of the i th mode, the propagation constant

3.2 Formulations of Building Blocks

is $\gamma_i = \sqrt{k_{ci}^2 - k_0^2}$, and

$$Y_i = \begin{cases} \frac{\gamma_i}{j\omega\mu}, & \text{for TE}_{mn} \text{ mode} \\ \frac{j\omega\epsilon}{\gamma_i}, & \text{for TM}_{mn} \text{ mode} \end{cases} \quad (3.4)$$

$$\vec{h}_i = \begin{cases} -\nabla_t \phi_{mn}^h(x, y), & \text{for TE}_{mn} \text{ mode} \\ \hat{z} \times \nabla_t \phi_{mn}^e(x, y), & \text{for TM}_{mn} \text{ mode} \end{cases} \quad (3.5)$$

3.2.2 Double-Plane Step in Rectangular Waveguides

A general structure of a double-plane step in rectangular waveguides is shown in Fig.

3.2, where a small rectangular waveguide is connected to a large one.

Figure 3.2: A double-plane step in rectangular waveguide.

The transverse electric and magnetic fields in these two waveguides can be described by (3.3). By matching the electric and magnetic fields at the junction plane,

3.2 Formulations of Building Blocks

the relationship of the incident and reflected wave amplitudes at each side of the junction can be described in the following equation [77, 78].

$$\begin{cases} \sqrt{\mathbf{Y}_L^{-1}}(\mathbf{A}_L^+ + \mathbf{A}_L^-) = \mathbf{M}\sqrt{\mathbf{Y}_S^{-1}}(\mathbf{A}_S^+ + \mathbf{A}_S^-) \\ \sqrt{\mathbf{Y}_S}(\mathbf{A}_S^+ - \mathbf{A}_S^-) = -\mathbf{M}^T\sqrt{\mathbf{Y}_L}(\mathbf{A}_L^+ - \mathbf{A}_L^-) \end{cases} \quad (3.6)$$

where $\mathbf{Y}$ is the diagonal matrix comprised of characteristic admittance of each mode, $\mathbf{A}$ is the column vector of mode amplitudes, the subscripts S and L denote the small and large waveguides, respectively, the superscripts $+$ and $-$ denote the incident and reflected waves, and the superscript T is the transpose operator. The mode coupling matrix $\mathbf{M}$ can be written as follows.

$$\mathbf{M} = \begin{bmatrix} [\mathbf{M}_{mn,ki}^{hh}] & 0 \\ [\mathbf{M}_{mn,ki}^{eh}] & [\mathbf{M}_{mn,ki}^{ee}] \end{bmatrix} \quad (3.7)$$

where

$$\mathbf{M}_{mn,ki}^{ee} = [(\frac{m\pi}{a_2})^2 + (\frac{n\pi}{b_2})^2]N_{1,ki}N_{2,mn}SS(m, k, a_1, a_2, c)SS(n, i, b_1, b_2, d) \quad (3.8)$$

$$\mathbf{M}_{mn,ki}^{hh} = [(\frac{k\pi}{a_1})^2 + (\frac{i\pi}{b_1})^2]N_{1,ki}N_{2,mn}CC(m, k, a_1, a_2, c)CC(n, i, b_1, b_2, d) \quad (3.9)$$

$$\begin{aligned} \mathbf{M}_{mn,ki}^{eh} = & N_{1,ki}N_{2,mn}[\frac{m\pi}{a_2}\frac{i\pi}{b_1}CC(m, k, a_1, a_2, c)SS(n, i, b_1, b_2, d) \\ & - \frac{n\pi}{b_2}\frac{k\pi}{a_1}SS(m, k, a_1, a_2, c)CC(n, i, b_1, b_2, d)] \end{aligned} \quad (3.10)$$

3.2 Formulations of Building Blocks

with

$$SS(m, n, a_1, a_2, b) = \int_0^{a_1} \sin \frac{m\pi(x+b)}{a_2} \sin \frac{n\pi x}{a_1} dx \quad (3.11)$$

$$CC(m, n, a_1, a_2, b) = \int_0^{a_1} \cos \frac{m\pi(x+b)}{a_2} \cos \frac{n\pi x}{a_1} dx \quad (3.12)$$

The normalized modal currents and voltages are defined as follows [81].

$$\begin{cases} \mathbf{I} = (\mathbf{A}^+ - \mathbf{A}^-) \\ \mathbf{V} = (\mathbf{A}^+ + \mathbf{A}^-) \end{cases}, \quad (3.13)$$

where the currents flowing into the reference plane are defined as positive. Then,

(3.6) can be rewritten as

$$\begin{cases} \mathbf{V}_L = \bar{\mathbf{M}}\mathbf{V}_S \\ \mathbf{I}_S = -\bar{\mathbf{M}}^T\mathbf{I}_L \end{cases}, \quad (3.14)$$

where the normalized coupling matrix is defined as

$$\bar{\mathbf{M}} = \sqrt{\mathbf{Y}_L}\mathbf{M}\frac{1}{\sqrt{\mathbf{Y}_S}}. \quad (3.15)$$

It is obviously seen that the waveguide step discontinuity can be modeled as an ideal transformer, as shown in Fig. 3.3. The scattering parameters of a double-plane step in rectangular waveguide can be then calculated as follows.

$$\mathbf{S}_{11} = 2(\mathbf{U} + \bar{\mathbf{M}}^T\bar{\mathbf{M}})^{-1} - \mathbf{U} \quad (3.16a)$$

$$\mathbf{S}_{21} = \bar{\mathbf{M}}(\mathbf{S}_{11} + \mathbf{U}) = \mathbf{S}_{12}^T \quad (3.16b)$$

$$\mathbf{S}_{22} = \bar{\mathbf{M}}\mathbf{S}_{12} - \mathbf{U} \quad (3.16c)$$

3.2 Formulations of Building Blocks

where $\mathbf{U}$ is the unitary matrix. It should be noted that the scattering parameters

Figure 3.3: (a) Waveguide step discontinuity, (b) equivalent transformer circuit.

calculated by (3.16) are in the form of matrix. Therefore, the entire scattering matrix is a generalized scattering matrix (GSM), which includes higher-order modes as well as the dominant mode in the waveguides.

3.2.3 Rectangular Waveguide Six-Port Junction

The rigorous analysis of the rectangular waveguide six-port junction was presented in [79]. It is found that this general six-port junction can be simplified as a cubic six-port junction [80] cascading with six double-plane steps. Therefore, the simplified six-port junction is a more fundamental building block, and the analysis is much simpler than the one in [79], which can be described only by (a, b, l) , as shown in Fig. 3.4.

To analyze this simplified six-port junction, the whole structure is first divided into seven regions [7]. The transverse electromagnetic fields in Regions I-VI can be expanded in the same way as in (3.3). For example, the transverse fields in Regions

3.2 Formulations of Building Blocks

Figure 3.4: A simplified six-port junction.

I and II can be expressed as

$$\begin{cases} \vec{E}_t^I = \sum_{i=1}^{N_1} \frac{1}{\sqrt{Y_i^z}} (A_{1i}^+ e^{-\gamma_i^z z} + A_{1i}^- e^{\gamma_i^z z}) \vec{h}_i^z \times \hat{z} \\ \vec{H}_t^I = \sum_{i=1}^{N_1} \sqrt{Y_i^z} (A_{1i}^+ e^{-\gamma_i^z z} - A_{1i}^- e^{\gamma_i^z z}) \vec{h}_i^z \end{cases} \quad (3.17)$$

$$\begin{cases} \vec{E}_t^{II} = \sum_{i=1}^{N_1} \frac{1}{\sqrt{Y_i^z}} [A_{2i}^+ e^{\gamma_i^z (z-l)} + A_{2i}^- e^{-\gamma_i^z (z-l)}] \vec{h}_i^z \times \hat{z} \\ \vec{H}_t^{II} = \sum_{i=1}^{N_1} \sqrt{Y_i^z} [-A_{2i}^+ e^{\gamma_i^z (z-l)} + A_{2i}^- e^{-\gamma_i^z (z-l)}] \vec{h}_i^z \end{cases} \quad (3.18)$$

where N_1 is the mode number in Regions I and II, and the superscript z denotes the propagation direction.

In Region VII, the fields can be expanded by the resonant method [7], which treats the resonant Region VII as a superposition of three finite length rectangular waveguides with short-circuited side walls. For example, in a finite length rectan-

3.2 Formulations of Building Blocks

gular waveguide along the z direction, the transverse electromagnetic fields can be expressed as

$$\begin{cases} \vec{E}_t^z = \sum_{i=1}^{N_1} \frac{1}{\sqrt{Y_i^z}} [A_{7i}^{z+} \sinh \gamma_i^z z + A_{7i}^{z-} \sinh \gamma_i^z (l - z)] \vec{h}_i^z \times \hat{z} \\ \vec{H}_t^z = \sum_{i=1}^{N_1} \sqrt{Y_i^z} [-A_{7i}^{z+} \cosh \gamma_i^z z + A_{7i}^{z-} \cosh \gamma_i^z (l - z)] \vec{h}_i^z \end{cases} \quad (3.19)$$

and the magnetic fields along the z direction is

$$\vec{H}_z = \sum_{i=1}^{N_1} \sqrt{Y_i^z} \frac{(k_{ci}^z)^2}{\gamma_i^z} [A_{7i}^{z+} \sinh \gamma_i^z z + A_{7i}^{z-} \sinh \gamma_i^z (l - z)] \begin{pmatrix} \phi_{mn}^{hz} \\ 0 \end{pmatrix} \quad (3.20)$$

The electrical fields in the propagating direction is not listed because it would be zero automatically along the respective matching plane. The fields in other two finite length waveguides can be expressed in the same way.

Once the electromagnetic fields in all these regions are expanded by their eigenmodes, the field continuity condition can be applied along the interface between these regions, which yields a matrix to relate the incident and reflected waves at both sides of the reference plane. For example, by matching the electric fields at plane $z = 0$, it yields

$$\mathbf{A}_1^+ + \mathbf{A}_1^- = \mathbf{D}_E^z \mathbf{A}_7^{z-} \quad (3.21)$$

where $\mathbf{A}_1^+$ and $\mathbf{A}_1^-$ are column vectors composed of mode amplitudes for the incident and reflected waves in Region I, $\mathbf{A}_7^{z-}$ is a column vector Region VII, and the diagonal

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matrix $\mathbf{D}_E^z = \text{diag}\{\sinh \gamma_i^z l\}$.

By matching the magnetic fields at plane $z = 0$, it yields

$$\mathbf{A}_1^+ - \mathbf{A}_1^- = \mathbf{M}_1 \mathbf{A}_7 \quad (3.22)$$

where column vector $\mathbf{A}_7 = [\mathbf{A}_7^{z-}, \mathbf{A}_7^{z+}, \mathbf{A}_7^{x-}, \mathbf{A}_7^{x+}, \mathbf{A}_7^{y-}, \mathbf{A}_7^{y+}]^T$, and

$$\mathbf{M}_1 = [\mathbf{M}_1^{z-}, \mathbf{M}_1^{z+}, \mathbf{M}_1^{x-}, \mathbf{M}_1^{x+}, \mathbf{M}_1^{y-}, \mathbf{M}_1^{y+}] \quad (3.23)$$

The detailed expressions of the sub-matrices in (3.23) are presented in Appendix A.

With the same procedure, the electromagnetic fields can be matched along other five planes, and corresponding mode coupling matrices can be obtained, respectively. Combining all these matrices, the relationship between incident and reflected waves in all the regions can be described by the following matrix.

$$\begin{cases} (\mathbf{A}^+ + \mathbf{A}^-) = \mathbf{D}_E \mathbf{A}_7 \\ (\mathbf{A}^+ - \mathbf{A}^-) = \mathbf{M}_H \mathbf{A}_7 \end{cases} \quad (3.24)$$

where $\mathbf{A}^\pm = [\mathbf{A}_1^\pm, \mathbf{A}_2^\pm, \mathbf{A}_3^\pm, \mathbf{A}_4^\pm, \mathbf{A}_5^\pm, \mathbf{A}_6^\pm]^T$.

$$\mathbf{D}_E = \begin{bmatrix} \mathbf{D}_E^z & & & & & \\ & \mathbf{D}_E^z & & & & \\ & & \mathbf{D}_E^x & & & \\ & & & \mathbf{D}_E^x & & \\ & & & & \mathbf{D}_E^y & \\ & & & & & \mathbf{D}_E^y \end{bmatrix} \quad (3.25)$$

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$$\mathbf{M}_H = \begin{bmatrix} \mathbf{M}_1^{z-} & \mathbf{M}_1^{z+} & \mathbf{M}_1^{x-} & \mathbf{M}_1^{x+} & \mathbf{M}_1^{y-} & \mathbf{M}_1^{y+} \\ \mathbf{M}_2^{z-} & \mathbf{M}_2^{z+} & \mathbf{M}_2^{x-} & \mathbf{M}_2^{x+} & \mathbf{M}_2^{y-} & \mathbf{M}_2^{y+} \\ \mathbf{M}_3^{z-} & \mathbf{M}_3^{z+} & \mathbf{M}_3^{x-} & \mathbf{M}_3^{x+} & \mathbf{M}_3^{y-} & \mathbf{M}_3^{y+} \\ \mathbf{M}_4^{z-} & \mathbf{M}_4^{z+} & \mathbf{M}_4^{x-} & \mathbf{M}_4^{x+} & \mathbf{M}_4^{y-} & \mathbf{M}_4^{y+} \\ \mathbf{M}_5^{z-} & \mathbf{M}_5^{z+} & \mathbf{M}_5^{x-} & \mathbf{M}_5^{x+} & \mathbf{M}_5^{y-} & \mathbf{M}_5^{y+} \\ \mathbf{M}_6^{z-} & \mathbf{M}_6^{z+} & \mathbf{M}_6^{x-} & \mathbf{M}_6^{x+} & \mathbf{M}_6^{y-} & \mathbf{M}_6^{y+} \end{bmatrix} \quad (3.26)$$

The detailed expressions of the sub-matrices in (3.26) can be found in Appendix A.

From (3.24), the amplitudes of incident and reflected waves in Regions I-VI can be expressed as

$$\begin{cases} \mathbf{A}^+ = \frac{1}{2}(\mathbf{D}_E + \mathbf{M}_H)\mathbf{A}_7 \\ \mathbf{A}^- = \frac{1}{2}(\mathbf{D}_E - \mathbf{M}_H)\mathbf{A}_7 \end{cases} \quad (3.27)$$

The GSM of a simplified six-port junction can be then calculated as

$$\mathbf{S} = (\mathbf{D}_E - \mathbf{M}_H)(\mathbf{D}_E + \mathbf{M}_H)^{-1} \quad (3.28)$$

Once the mode-matching analysis of this simplified rectangular waveguide six-port junction is done, the scattering matrices of the five-port turnstile junction, four-port planar cross junction, or magic T-junction [82], three-port H-plane or E-plane T-junctions [83, 84] can be easily obtained by terminating the corresponding ports by short-circuited planes. In fact, from the derivation procedure, they can be directly obtained by removing the corresponding sub-matrices in $\mathbf{M}_H$ and $\mathbf{D}_E$. Taking the three-port T-junction as an example, the corresponding mode coupling matrix can

3.3 Cascade of Building Blocks

be written as

$$\mathbf{D}_E = \begin{bmatrix} \mathbf{D}_E^z & & \\ & \mathbf{D}_E^z & \\ & & \mathbf{D}_E^x \end{bmatrix} \quad (3.29)$$

$$\mathbf{M}_H = \begin{bmatrix} \mathbf{M}_1^{z-} & \mathbf{M}_1^{z+} & \mathbf{M}_1^{x-} \\ \mathbf{M}_2^{z-} & \mathbf{M}_2^{z+} & \mathbf{M}_2^{x-} \\ \mathbf{M}_3^{z-} & \mathbf{M}_3^{z+} & \mathbf{M}_3^{x-} \end{bmatrix} \quad (3.30)$$

The GSM of the three-port T-junction can be calculated by (3.28) thereafter.

3.3 Cascade of Building Blocks

Once the building blocks of the rectangular waveguide structures have been developed, complicated waveguide components, such as waveguide OMTs, can be analyzed by cascading these building blocks.

Since the higher-order modes are considered in the analysis, the cascade should be carried out in the form of matrix, which usually requires time-consuming matrix inversion. In this section, three types of two-port network cascades are discussed first, proper generalized matrix representations are employed to avoid the matrix inversion. Since the OMT is a multi-port device, the multi-port cascade modules are also presented. Therefore, the waveguide OMTs can be analyzed by the mode-matching method efficiently.

3.3 Cascade of Building Blocks

3.3.1 Cascade of Two-Port Networks

In general, two two-port networks can be cascaded by their GSMs, as shown in Fig.

3.5. The resultant cascaded GSM is [7]

$$\mathbf{S}_{11} = \mathbf{S}_{11}^I + \mathbf{S}_{12}^I \mathbf{S}_{11}^{II} \mathbf{W} \mathbf{S}_{21}^I \quad (3.31a)$$

$$\mathbf{S}_{21} = \mathbf{S}_{21}^{II} \mathbf{W} \mathbf{S}_{21}^I = \mathbf{S}_{12}^T \quad (3.31b)$$

$$\mathbf{S}_{22} = \mathbf{S}_{22}^{II} + \mathbf{S}_{21}^{II} \mathbf{W} \mathbf{S}_{22}^I \mathbf{S}_{12}^{II} \quad (3.31c)$$

where $\mathbf{W} = (\mathbf{U} - \mathbf{S}_{22}^I \mathbf{S}_{11}^{II})^{-1}$. In this case, the cascade requires one matrix inversion.

Figure 3.5: Diagram of two cascaded two-port networks.

In practice, two step discontinuities are usually connected by an intermediate uniform waveguide, as shown in Fig. 3.6, where $\mathbf{Y}_0$ is the multimode characteristic admittance and l is the length of the intermediate waveguide, the arrows indicate the current directions. Depending upon the size of the intermediate waveguide, they can be classified into three types:

- 1) Iris type, the intermediate waveguide is of the smallest size;
- 2) Stub type, the intermediate waveguide is of the largest size;

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3) Transformer type, the intermediate waveguide is of medium size.

Figure 3.6: Two-step cascaded waveguide discontinuities (a) iris, (b) stub and (c) transformer.

It is known that the iris and stub discontinuities can be represented by the generalized impedance matrix (GIM) [85] and the generalized admittance matrix (GAM) [86] without matrix inversion. The key to derive the GIM of an iris and the GAM of a stub is to choose an appropriate generalized matrix representation of the intermediate waveguide, which connects two coupling matrices on both sides.

Iris Discontinuity

For an iris-type discontinuity, the GIM representation of the intermediate waveguide is adopted, which is

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix} = \begin{bmatrix} \text{diag}\{\coth\theta_n\} & \text{diag}\{\text{csch}\theta_n\} \\ \text{diag}\{\text{csch}\theta_n\} & \text{diag}\{\coth\theta_n\} \end{bmatrix} \begin{bmatrix} -\mathbf{I}_2 \\ -\mathbf{I}_3 \end{bmatrix}. \quad (3.32)$$

where $\theta_n = \gamma_n l$ with γ_n being the propagation constant of the n -th mode in the intermediate waveguide section. Then the resultant GIM of the iris-type discontinuity

3.3 Cascade of Building Blocks

is

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_4 \end{bmatrix}, \quad (3.33)$$

where

$$\begin{cases} \mathbf{Z}_{11} = \bar{\mathbf{M}}_1 \text{diag}\{\coth \theta_n\} \bar{\mathbf{M}}_1^T \\ \mathbf{Z}_{12} = \mathbf{Z}_{21}^T = \bar{\mathbf{M}}_1 \text{diag}\{\text{csch} \theta_n\} \bar{\mathbf{M}}_2^T \\ \mathbf{Z}_{22} = \bar{\mathbf{M}}_2 \text{diag}\{\coth \theta_n\} \bar{\mathbf{M}}_2^T \end{cases} \quad (3.34)$$

with being $\bar{\mathbf{M}}_1$ and $\bar{\mathbf{M}}_2$ the normalized coupling matrices for the two step discontinuities.

Stub Discontinuity

In the similar way, the GAM representation of the intermediate waveguide is adopted for the stub-type discontinuity, which is

$$\begin{bmatrix} -\mathbf{I}_2 \\ -\mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} \text{diag}\{\coth \theta_n\} & -\text{diag}\{\text{csch} \theta_n\} \\ -\text{diag}\{\text{csch} \theta_n\} & \text{diag}\{\coth \theta_n\} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix}. \quad (3.35)$$

Then the resultant GAM of stub-type discontinuity is

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_4 \end{bmatrix}, \quad (3.36)$$

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where

$$\begin{cases} \mathbf{Y}_{11} = \bar{\mathbf{M}}_1 \text{diag}\{\coth \theta_n\} \bar{\mathbf{M}}_1^T \\ \mathbf{Y}_{12} = \mathbf{Y}_{21}^T = -\bar{\mathbf{M}}_1 \text{diag}\{\text{csch} \theta_n\} \bar{\mathbf{M}}_2^T \\ \mathbf{Y}_{22} = \bar{\mathbf{M}}_2 \text{diag}\{\coth \theta_n\} \bar{\mathbf{M}}_2^T \end{cases} \quad (3.37)$$

From (3.32), (3.34), (3.35) and (3.37), we can see that a singularity in both GIM and GAM representations will occur when θ_n is close to $m\pi$, ($m = 0, 1, 2, \dots$). Especially for the case of the iris with zero thickness, the elements of GIM tend to be infinite at the same time. Therefore, the GSM has to be adopted under the zero thickness. However, although the computational efforts to obtain the scattering parameters of a single iris or stub with GIM or GAM are similar to the GSM [72], for the cascade of N irises or stubs separated by $N - 1$ uniform waveguide sections, the GIM and GAM is highly efficient, where the total number of matrix inversions is N , while the GSM requires a total number of $4N - 1$ [86].

Transformer Discontinuity

For the transformer-type discontinuity, the relationship between the normalized modal currents and voltages at two discontinuities are

$$\begin{cases} \mathbf{V}_1 = \bar{\mathbf{M}}_1 \mathbf{V}_2 \\ \mathbf{I}_2 = \bar{\mathbf{M}}_1^T \mathbf{I}_1 \end{cases}, \quad (3.38)$$

$$\begin{cases} \mathbf{V}_3 = \bar{\mathbf{M}}_2 \mathbf{V}_4 \\ \mathbf{I}_4 = \bar{\mathbf{M}}_2^T \mathbf{I}_3 \end{cases}. \quad (3.39)$$

3.3 Cascade of Building Blocks

Both the GIM and GAM representations fail to model this type of cascaded waveguide discontinuity. However, with the experience from the derivation of GIM and GAM for iris and stub, the generalized hybrid matrix can be employed [87]. Therefore, the generalized hybrid matrix of a finite length uniform waveguide in Fig. 3.6(c) can be written as

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} \text{diag}\{\tanh\theta_n\} & \text{diag}\{\text{sech}\theta_n\} \\ \text{diag}\{\text{sech}\theta_n\} & -\text{diag}\{\tanh\theta_n\} \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{V}_3 \end{bmatrix}. \quad (3.40)$$

It should be mentioned that, different from the GIM and GAM, the current direction in the generalized hybrid matrix is defined along the forward direction, as shown in Fig. 3.6(c).

Combining (3.38), (3.39) and (3.40), we can obtain the generalized hybrid matrix representation of a transformer-type discontinuity as follows.

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_4 \end{bmatrix}, \quad (3.41)$$

where

$$\begin{cases} \mathbf{H}_{12} = \mathbf{H}_{21}^T = \bar{\mathbf{M}}_1 \text{diag}\{\text{sech}\theta_n\} \bar{\mathbf{M}}_2 \\ \mathbf{H}_{11} = \bar{\mathbf{M}}_1 \text{diag}\{\tanh\theta_n\} \bar{\mathbf{M}}_1^T \\ \mathbf{H}_{22} = -\bar{\mathbf{M}}_2^T \text{diag}\{\tanh\theta_n\} \bar{\mathbf{M}}_2 \end{cases}. \quad (3.42)$$

From (3.42), we can see that a singularity of the generalized hybrid matrix occurs

3.3 Cascade of Building Blocks

when $\theta_n = \frac{2m+1}{2}\pi$, ($m = 0, 1, 2, \dots$), which is different from the one of GIM and GAM.

Figure 3.7: Cascading of two generalized hybrid matrices.

With the above generalized hybrid matrix, the multistep transformer can be analyzed as the cascading of the generalized hybrid matrices. The cascading of two generalized hybrid matrices are shown in Fig. 3.7, and the cascaded generalized hybrid matrix can be obtained as

$$\begin{cases} \mathbf{H}_{12} = \mathbf{H}_{21}^T = \mathbf{H}_{12}' \mathbf{W} \mathbf{H}_{12}'' \\ \mathbf{H}_{11} = \mathbf{H}_{11}' + \mathbf{H}_{12}' \mathbf{W} \mathbf{H}_{11}'' \mathbf{H}_{21}' , \\ \mathbf{H}_{22} = \mathbf{H}_{22}'' + \mathbf{H}_{21}'' \mathbf{H}_{22}' \mathbf{W} \mathbf{H}_{12}' \end{cases} \quad (3.43)$$

where $\mathbf{W} = (\mathbf{U} - \mathbf{H}_{11}'' \mathbf{H}_{22}')^{-1}$. We can see that the cascading of more than two steps will require matrix inversion because $\mathbf{H}_{11}'' \mathbf{H}_{22}'$ may not be a diagonal matrix in general case.

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Finally, the GSM can be calculated from the generalized hybrid matrix as follows.

$$\begin{cases} \mathbf{S}_{11} = -2(\mathbf{H}_{12}\mathbf{W}\mathbf{H}_{21} + \mathbf{H}_{11} + \mathbf{U})^{-1} + \mathbf{U} \\ \mathbf{S}_{21} = \mathbf{S}_{12}^T = \mathbf{W}\mathbf{H}_{21}(\mathbf{U} - \mathbf{S}_{11}) \\ \mathbf{S}_{22} = \mathbf{W}(\mathbf{H}_{22} - \mathbf{H}_{21}\mathbf{S}_{12} + \mathbf{U}) \end{cases}, \quad (3.44)$$

where $\mathbf{W} = (\mathbf{U} - \mathbf{H}_{22})^{-1}$.

From the above equations, we can see that, for an M -step transformer, the generalized hybrid matrix approach requires only M matrix inversions to obtain the total scattering parameters, while the GSM needs $2M - 1$ matrix inversions. Therefore, the generalized hybrid matrix for cascaded transformers is of higher efficiency than the GSM.

3.3.2 Cascade of Multi-Port Junctions

In the design of waveguide OMTs, two types of multi-port junction cascades are usually encountered. One is a three-port network cascade with a two-port network, the other is the cascade of two three-port networks, as shown in Fig. 3.8.

Figure 3.8: Diagram of (a) a three-port network cascading with a two-port network and (b) two cascaded three-port networks.

3.3 Cascade of Building Blocks

The resultant GSM of a three-port network cascading a two-port network is

$$\mathbf{S}_{11} = \mathbf{S}_{11}^I + \mathbf{S}_{13}^I \mathbf{W} \mathbf{S}_{11}^{II} \mathbf{S}_{31}^I \quad (3.45)$$

$$\mathbf{S}_{12} = \mathbf{S}_{12}^I + \mathbf{S}_{13}^I \mathbf{W} \mathbf{S}_{11}^{II} \mathbf{S}_{32}^I \quad (3.46)$$

$$\mathbf{S}_{13} = \mathbf{S}_{13}^I \mathbf{W} \mathbf{S}_{12}^{II} \quad (3.47)$$

$$\mathbf{S}_{21} = \mathbf{S}_{21}^I + \mathbf{S}_{23}^I \mathbf{W} \mathbf{S}_{11}^{II} \mathbf{S}_{31}^I \quad (3.48)$$

$$\mathbf{S}_{22} = \mathbf{S}_{22}^I + \mathbf{S}_{23}^I \mathbf{W} \mathbf{S}_{11}^{II} \mathbf{S}_{32}^I \quad (3.49)$$

$$\mathbf{S}_{23} = \mathbf{S}_{23}^I \mathbf{W} \mathbf{S}_{12}^{II} \quad (3.50)$$

$$\mathbf{S}_{31} = \mathbf{S}_{21}^{II} (\mathbf{U} + \mathbf{S}_{33}^I \mathbf{W} \mathbf{S}_{11}^{II}) \mathbf{S}_{31}^I \quad (3.51)$$

$$\mathbf{S}_{32} = \mathbf{S}_{21}^{II} (\mathbf{U} + \mathbf{S}_{33}^I \mathbf{W} \mathbf{S}_{11}^{II}) \mathbf{S}_{32}^I \quad (3.52)$$

$$\mathbf{S}_{33} = \mathbf{S}_{22}^{II} + \mathbf{S}_{21}^{II} \mathbf{S}_{33}^I \mathbf{W} \mathbf{S}_{12}^{II} \quad (3.53)$$

where $\mathbf{W} = (\mathbf{U} - \mathbf{S}_{11}^{II} \mathbf{S}_{33}^I)^{-1}$.

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The resultant GSM of two cascaded three-port networks is

$$\mathbf{S}_{11} = \mathbf{S}_{11}^I + \mathbf{S}_{13}^I \mathbf{W} \mathbf{S}_{11}^{II} \mathbf{S}_{31}^I \quad (3.54)$$

$$\mathbf{S}_{12} = \mathbf{S}_{12}^I + \mathbf{S}_{13}^I \mathbf{W} \mathbf{S}_{11}^{II} \mathbf{S}_{32}^I \quad (3.55)$$

$$\mathbf{S}_{13} = \mathbf{S}_{13}^I \mathbf{W} \mathbf{S}_{12}^{II} \quad (3.56)$$

$$\mathbf{S}_{14} = \mathbf{S}_{13}^I \mathbf{W} \mathbf{S}_{13}^{II} \quad (3.57)$$

$$\mathbf{S}_{21} = \mathbf{S}_{21}^I + \mathbf{S}_{23}^I \mathbf{W} \mathbf{S}_{11}^{II} \mathbf{S}_{31}^I \quad (3.58)$$

$$\mathbf{S}_{22} = \mathbf{S}_{22}^I + \mathbf{S}_{23}^I \mathbf{W} \mathbf{S}_{11}^{II} \mathbf{S}_{32}^I \quad (3.59)$$

$$\mathbf{S}_{24} = \mathbf{S}_{23}^I \mathbf{W} \mathbf{S}_{13}^{II} \quad (3.60)$$

$$\mathbf{S}_{31} = \mathbf{S}_{21}^{II} (\mathbf{U} + \mathbf{S}_{33}^I \mathbf{W} \mathbf{S}_{11}^{II}) \mathbf{S}_{31}^I \quad (3.61)$$

$$\mathbf{S}_{32} = \mathbf{S}_{21}^{II} (\mathbf{U} + \mathbf{S}_{33}^I \mathbf{W} \mathbf{S}_{11}^{II}) \mathbf{S}_{32}^I \quad (3.62)$$

$$\mathbf{S}_{33} = \mathbf{S}_{22}^{II} + \mathbf{S}_{21}^{II} \mathbf{S}_{33}^I \mathbf{W} \mathbf{S}_{12}^{II} \quad (3.63)$$

$$\mathbf{S}_{34} = \mathbf{S}_{23}^{II} + \mathbf{S}_{21}^{II} \mathbf{S}_{33}^I \mathbf{W} \mathbf{S}_{13}^{II} \quad (3.64)$$

$$\mathbf{S}_{41} = \mathbf{S}_{31}^{II} (\mathbf{U} + \mathbf{S}_{33}^I \mathbf{W} \mathbf{S}_{11}^{II}) \mathbf{S}_{31}^I \quad (3.65)$$

$$\mathbf{S}_{42} = \mathbf{S}_{31}^{II} (\mathbf{U} + \mathbf{S}_{33}^I \mathbf{W} \mathbf{S}_{11}^{II}) \mathbf{S}_{32}^I \quad (3.66)$$

$$\mathbf{S}_{43} = \mathbf{S}_{32}^{II} + \mathbf{S}_{31}^{II} \mathbf{S}_{33}^I \mathbf{W} \mathbf{S}_{12}^{II} \quad (3.67)$$

$$\mathbf{S}_{44} = \mathbf{S}_{33}^{II} + \mathbf{S}_{31}^{II} \mathbf{S}_{33}^I \mathbf{W} \mathbf{S}_{13}^{II} \quad (3.68)$$

where $\mathbf{W} = (\mathbf{U} - \mathbf{S}_{11}^{II} \mathbf{S}_{33}^I)^{-1}$.

3.4 Numerical Results

3.4 Numerical Results

To verify the building blocks and the cascade modules, some numerical examples are examined in this section.

3.4.1 Verification of Building Blocks

First, scattering parameters of the two building blocks: double-plane step and six-port waveguide junction, are calculated by the MM method. The mode number in each waveguide section is determined by the area of its cross-section to satisfy the convergence condition [88].

The dimensions of the examined double-plane step are $a_1 = 14$ mm, $b_1 = 6$ mm, $a_2 = 15.9$ mm, $b_2 = 7.9$ mm, $c_1 = d_1 = 0.95$ mm, and those of the six-port waveguide junction are $a = b = 50$ mm, $l = 25$ mm. With different mode numbers, scattering parameters are first calculated at a single frequency point and compared with those from the commercial simulator HFSS. Calculated results with corresponding CPU time are listed in Table 3.1. It is seen that the MMT is well convergent as the mode number increases, and requires much less computational time than HFSS. Frequency response of these two building blocks are then calculated and compared with those simulated by HFSS, as shown in Figs. 3.9 and 3.10, which shows very good agreement over the interested frequency band.

Then, an H-plane T-junction is calculated using (3.28)-(3.30), where Ports IV-VI in Fig. 3.4 are short-circuited. Calculated results are compared with those from HFSS, which agree well with each other, as shown in Fig. 3.11.

3.4 Numerical Results

Table 3.1: Calculated scattering parameter and CPU time by the MMT with different mode numbers and HFSS.

	N_1	N_2	N_3	$ S_{11} $	CPU time (s)
Double-plane step	8	14	NaN	0.2519	0.0045
	23	33	NaN	0.2585	0.016
	59	87	NaN	0.2592	0.075
	83	128	NaN	0.2595	0.1224
	HFSS			0.2588	1.4553
Six-port junction	50	25	25	0.3404	0.0426
	100	50	50	0.3426	0.1881
	150	75	75	0.3422	0.4884
	200	100	100	0.3428	0.9709
	HFSS			0.3427	28

Figure 3.9: Scattering parameters of a double-plane step. ($a_1 = 14$ mm, $b_1 = 6$ mm, $a_2 = 15.9$ mm, $b_2 = 7.9$ mm, $c_1 = d_1 = 0.95$ mm, $N_1 = 75$, $N_2 = 100$.)

3.4.2 Verification of Cascaded Modules

Three types of cascaded waveguide step junctions are first examined.

Scattering parameters of two rectangular iris structures with finite thickness are

3.4 Numerical Results

Figure 3.10: Scattering parameters of a six-port waveguide junction. ($a = b = 50$ mm, $l = 25$ mm, $N_1 = 100$, $N_2 = N_3 = 50$. Solid line: HFSS)

calculated by the GIM formulation (3.34) and compared with measured ones in [77], as shown in Fig. 3.12. 40 modes are considered in the rectangular waveguide section. The CPU time used by the GIM is about 0.14 seconds, while that used by the GSM is about 0.15 seconds. An E-plane stub is calculated by the GAM formulation (3.37) and compared with the results given in [89], as shown in Fig. 3.13. The mode number considered in the rectangular waveguide section is 40. The CPU time used by the GAM is about 0.20 seconds, while that used by the GSM is about 0.32 seconds. Two-step double-plane transformers between a WR62 and a WR90 waveguide are calculated by the generalized hybrid matrix with different lengths. Calculated scattering parameters are compared with those obtained by the GSM and Ansoft's High Frequency Structure Simulator (HFSS), as shown in Fig. 3.14.

3.4 Numerical Results

Figure 3.11: Scattering parameters of an H-plane T-junction. ($a = l = 22.86$ mm, $b = 10.16$ mm, $N_1 = N_2 = 50$. Solid line: HFSS)

The mode numbers considered in the WR62 and WR90 waveguides are 40 and 74. The CPU time used by the generalized hybrid matrix is about 0.23 seconds, while that used by the GSM is about 0.31 seconds. We can see that our results for all three cascaded discontinuities are numerically identical to those obtained by the GSM and agree very well with those from the literature and HFSS.

In order to demonstrate the cascade of generalized hybrid matrix, a 24-step double-plane transformer with equal section length reported in [77] is considered. The mode number in each section is proportional to the size of its corresponding cross-section. Comparing with the literature [77] and HFSS, very good agreements are observed from Fig. 3.15. The efficiency of our generalized hybrid matrix approach is also compared with that of GSM. When the mode number in WR62 waveguide is

3.4 Numerical Results

Figure 3.12: Scattering parameters of rectangular iris structures with finite thickness $t = 2$ mm (waveguide dimensions: $a = 15.8$ mm, $b = 7.9$ mm, iris dimensions: A), $a' = a/\sqrt{2}$, $b' = b/\sqrt{2}$; B), $a' = a/2$, $b' = b/2$. Measured results from [77].)

Figure 3.13: Scattering parameters of an E-plane stub in a WR75 waveguide ($h = 9.52$ mm, $s = 3.46$ mm, $l = 7.68$ mm).

3.4 Numerical Results

Figure 3.14: Scattering parameters of two-step transformers between a WR62 and a WR90 waveguide. The dimensions are $a_0 = 15.8$ mm, $b_0 = 7.9$ mm, $a_2 = 22.86$ mm, $b_2 = 10.16$ mm, $a_1 = \sqrt{a_0 a_2}$, $b_1 = \sqrt{b_0 b_2}$.

chosen to be 100, to calculate a single frequency point, the computational time using the generalized hybrid matrix is 14.46 seconds, while using the GSM requires 18.12 seconds on the same computer.

A typical example of the three-port network cascading with a two-port network is a square waveguide T-junction, which is a key component in the design of OMTs [90], as shown in Fig. 3.16(a). The mode number used in the square waveguide section and the branch rectangular waveguide section are 21 and 15. Calculated results are compared with those obtained by HFSS, as shown in Fig. 3.17. The CPU time used by the MMT is 0.11 seconds, while that used by HFSS is 10 seconds.

In the design of OMTs, two square waveguide T-junctions are usually orthogonally cascaded as shown in Fig. 3.16(b). Using the same mode numbers in the square

3.4 Numerical Results

Figure 3.15: Magnitude of the reflection coefficient of double-plane transformer with 24 steps of equal length between a WR62 and a WR90 waveguide. Measured results from [77]

Figure 3.16: Structure of (a) a square waveguide T-junction and (b) two orthogonally cascaded square waveguide T-junctions.

and branch waveguide sections, calculated results are compared with those obtained by HFSS, as shown in Fig. 3.18. The CPU time used by the MMT and 0.24 seconds, while that used by HFSS is 15 seconds. It is seen that results obtained from the MMT agree very well with those from HFSS, while the CPU time required by it is

3.5 Numerical Results

much less than HFSS.

Figure 3.17: Scattering parameters of a square waveguide T-junction. ($a = b = 18$ mm, $l = 22.86$ mm, $h = 10.16$ mm. Solid line: HFSS.)

Figure 3.18: Scattering parameters of two orthogonal cascaded square waveguide T-junctions. ($a = b = 18$ mm, $l = 22.86$ mm, $h = 10.16$ mm. Solid line: HFSS.)

3.5 Conclusion

In this chapter, the double-plane rectangular waveguide step and the waveguide six-port junction have been analyzed first as two building blocks of rectangular waveguides by the MM method for the efficient analysis of waveguide OMTs.

Then the cascade of these building blocks has been investigated. The two-step cascaded discontinuities have been summarized into three types and proper generalized matrix representations have been introduced for each type. The iris and stub structures are modeled as a GIM and a GAM, respectively, while the transformer structure is modeled as a generalized hybrid matrix. Compared with the conventional GSM cascade approach, all these generalized matrix representations requires no matrix inversion. Two multi-port cascade modules, which would be used in the design of OMTs, have also been developed.

Numerical examples have been presented and compared with those available in the literature or obtained by a commercial simulator to verify our formulations.

Chapter 4

Closed-Form Expressions for Square Waveguide T-junctions

4.1 Introduction

Waveguide T-junctions have been studied extensively for many decades because they are an essential component in numerous microwave applications [91, 92]. Equivalent circuits of a rectangular waveguide T-junction were initially derived based on the electrostatic approximations [91]. Later on, some improved methods were developed to provide more accurate models [93–97].

In the design of ortho-mode transducers (OMTs), the square waveguide T-junction is of most interest. Although many numerical methods [98] can successfully analyze this kind of structure, the mode-matching (MM) method appears to be the most efficient one to obtain either the generalized scattering matrix (GSM) [84] or generalized admittance matrix (GAM) [24, 99] for this regular structure. However, they still cannot lead to an equivalent circuit model as the one available for rectangular E-plane or H-plane T-junction in [91]. This is because two degenerated TE_{10} and TE_{01} modes

4.1 Introduction

are propagating simultaneously in the square waveguide, which results in an electrical five-port network, rather than the conventional three-port one. The synthesis method for this multi-port network is still not fully established [7, 40]. Therefore, although many OMTs have been designed in different antenna feed systems, the design of OMTs still relies mostly on experience [11], cut and try, or full-wave optimization [24]. For the purpose of efficient design of OMTs, a good equivalent circuit model of this multi-port network is most desired.

In [100], the equivalent circuit model of square waveguide T-junctions was developed numerically from the GSM [101, 102]. In this chapter, this work would be extended as follows. First, the GAM of a square waveguide T-junction is employed instead of the GSM, which avoids matrix inversions. Second, the GAM is reduced to a general five-port network by terminating all the localized modes with their respective characteristic impedances [86, 103], from which the equivalent circuit model is obtained. Third, the resultant five-port equivalent circuit model is separated into two de-coupled two-port and three-port sub-networks theoretically based on the knowledge of mode coupling mechanism rather than numerical observations in [100]. Finally, accurate closed-form expressions are derived to calculate the circuit parameters of these two sub-networks with proper approximations in the matrix reduction process. These closed-form expressions are more useful than numerical results given in [100]. An X -band short-circuited branch OMT is designed based on this equivalent circuit model with circuit simulator, such as Agilent's Advanced Design System (ADS), which is much faster than the original electromagnetic field simulator and

4.2 Theoretical Formulation

provides the possibility of efficient synthesis. The designed OMT is fabricated and tested to verify the validity of our equivalent circuit model.

Figure 4.1: Square waveguide T-junction: (a) 3-D view and (b) top view.

4.2 Theoretical Formulation

A square waveguide T-junction is shown in Fig. 4.1, where a rectangular waveguide is connected to the side wall of a square waveguide. Because the square waveguide supports two degenerated TE_{10} and TE_{01} modes simultaneously, this T-junction should be electrically represented by a five-port network. The general equivalent circuit model of a five-port network is shown in Fig. 4.2. The series and shunt admittances can be calculated from its admittance matrix [22].

$$\begin{cases} Y_{sij} = -Y_{ij}, i \neq j, \\ Y_{pi} = \sum_{j=1}^5 Y_{ij} \end{cases} \quad (4.1)$$

4.2 Theoretical Formulation

where Y_{ij} is the (i, j) element in the admittance matrix, $i, j = 1, 2, \dots, 5$. It should be noted that the value of calculated circuit elements in (4.1) may be slightly different from [91] due to the different definitions for the direction of the equivalent currents [91].

Figure 4.2: General equivalent circuit model of a five-port network.

4.2.1 Reduced Admittance Matrix of T-Junction

Although the admittance matrix may be calculated from the scattering matrix obtained by any numerical method [101,102], it can be directly derived from the GAM by terminating the localized modes with their characteristic impedances [86,103].

To obtain its GAM, the T-junction is considered as an open T-junction cascading a waveguide step discontinuity with zero length, as shown in Fig. 4.3. Matching the fields at all the reference planes T and T', the normalized GAM of an open T-junction

4.2 Theoretical Formulation

can be readily obtained as follows [81]:

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}'_3 \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} & \mathbf{M}_{13} \\ \mathbf{M}_{21} & \mathbf{M}_{22} & \mathbf{M}_{23} \\ \mathbf{M}_{31} & \mathbf{M}_{32} & \mathbf{M}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}'_3 \end{bmatrix} \quad (4.2)$$

where the normalized mode currents and voltages are defined as

$$\begin{cases} \mathbf{V} = \mathbf{A}^+ + \mathbf{A}^- \\ \mathbf{I} = \mathbf{A}^+ - \mathbf{A}^- \end{cases} \quad (4.3)$$

and

$$\mathbf{M}_{11} = \mathbf{M}_{22} = \text{diag}\{\coth(\gamma_{mn}^A l)\} \quad (4.4)$$

$$\mathbf{M}_{12} = \mathbf{M}_{21} = \text{diag}\{-\text{csch}(\gamma_{mn}^A l)\} \quad (4.5)$$

$$\mathbf{M}_{33} = \text{diag}\{\coth(\gamma_{mn}^B a)\} \quad (4.6)$$

$$M_{13}(i, j) = (-1)^{m_A + 1} M_1(i, j) \quad (4.7)$$

Figure 4.3: Open T-junction and step discontinuity.

4.2 Theoretical Formulation

$$M_{23}(i, j) = (-1)^{m_A + n_B} M_1(i, j) \quad (4.8)$$

$$M_{31}(j, i) = (-1)^{m_A + 1} M_3(j, i) \quad (4.9)$$

$$M_{32}(j, i) = (-1)^{m_A + n_B} M_3(j, i) \quad (4.10)$$

where $\mathbf{A}^+$ and $\mathbf{A}^-$ are the column vectors for the amplitudes of incident and reflected waves, γ_{mn}^A and γ_{mn}^B are propagation constants of the (mn) th mode in Regions A and B, i and j correspond to the mode indices (m_A, n_A) in Region A and (m_B, n_B) in Region B, respectively, and the elements of $\mathbf{M}_1$ and $\mathbf{M}_3$ are given in Appendix B.

For the step discontinuity, the normalized mode currents and voltages can be related to each other by the following equations [81]

$$\begin{cases} \mathbf{V}'_3 = \tilde{\mathbf{M}} \mathbf{V}_3 \\ \mathbf{I}'_3 = \tilde{\mathbf{M}}^T \mathbf{I}_3 \end{cases}, \quad (4.11)$$

where the normalized coupling matrix is

$$\tilde{\mathbf{M}} = \sqrt{\mathbf{Y}^B} \mathbf{M} (\sqrt{\mathbf{Y}^C})^{-1} \quad (4.12)$$

where $\mathbf{Y}^B$ and $\mathbf{Y}^C$ are diagonal matrices consisting of respective characteristic impedances of Regions B and C [81], $\mathbf{M}$ is the frequency-independent coupling matrix, which is defined in Appendix B.

Substituting (4.11) into (4.2), the complete GAM of the waveguide T-junction can be written as

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$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \mathbf{Y}_{13} \\ \mathbf{Y}_{12} & \mathbf{Y}_{11} & \mathbf{Y}_{23} \\ \mathbf{Y}_{31} & \mathbf{Y}_{32} & \mathbf{Y}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix} \quad (4.13)$$

where $\mathbf{Y}_{11} = \mathbf{M}_{11}$, $\mathbf{Y}_{12} = \mathbf{M}_{12}$, $\mathbf{Y}_{13} = \mathbf{M}_{13}\tilde{\mathbf{M}}$, $\mathbf{Y}_{23} = \mathbf{M}_{23}\tilde{\mathbf{M}}$, $\mathbf{Y}_{31} = \tilde{\mathbf{M}}^T\mathbf{M}_{31}$, $\mathbf{Y}_{32} = \tilde{\mathbf{M}}^T\mathbf{M}_{32}$, and $\mathbf{Y}_{33} = \tilde{\mathbf{M}}^T\mathbf{M}_{33}\tilde{\mathbf{M}}$, where the superscript T denotes the transpose of matrix.

By partitioning the sub-matrices in the GAM in (4.13) according to the accessible and localized modes as [86]

$$[\mathbf{Y}] = \begin{bmatrix} [\mathbf{Y}^{aa}] & [\mathbf{Y}^{al}] \\ [\mathbf{Y}^{la}] & [\mathbf{Y}^{ll}] \end{bmatrix} \quad (4.14)$$

the admittance matrix consisting of only accessible modes can be obtained as

$$[\mathbf{Y}^{\text{red}}] = [\mathbf{Y}^{aa}] - [\mathbf{Y}^{al}] ([\mathbf{Y}^{ll}] + \mathbf{U})^{-1} [\mathbf{Y}^{la}] \quad (4.15)$$

where the indices a and l denote the accessible and localized modes, respectively, and $\mathbf{U}$ is the identity matrix.

For the waveguide T-junction, because $\mathbf{Y}_{11}$, $\mathbf{Y}_{12}$, $\mathbf{Y}_{21}$ and $\mathbf{Y}_{22}$ in (4.13) are all diagonal matrices, more explicit expressions for the reduced admittance matrix

4.2 Theoretical Formulation

(RAM) of the T-junction can be derived as

$$\begin{cases} \mathbf{Y}_{11}^{\text{red}} = \mathbf{Y}_{11}^{aa} - \mathbf{Y}_{13}^{al} \mathbf{W} \mathbf{Y}_{31}^{la} \\ \mathbf{Y}_{12}^{\text{red}} = \mathbf{Y}_{12}^{aa} - \mathbf{Y}_{13}^{al} \mathbf{W} \mathbf{Y}_{32}^{la} \\ \mathbf{Y}_{13}^{\text{red}} = \mathbf{Y}_{13}^{aa} - \mathbf{Y}_{13}^{al} \mathbf{W} \mathbf{W}_2 \\ \mathbf{Y}_{33}^{\text{red}} = \mathbf{W}_4 - \mathbf{W}_3 \mathbf{W} \mathbf{W}_2 \end{cases} \quad (4.16)$$

where

$$\mathbf{W} = (\mathbf{U} + \mathbf{W}_1)^{-1} \quad (4.17)$$

$$\begin{cases} \mathbf{W}_1 = \mathbf{Y}_{33}^{ll} - [\mathbf{Y}_{31}^{ll} \quad \mathbf{Y}_{32}^{ll}] \mathbf{D} [\mathbf{Y}_{13}^{ll} \quad \mathbf{Y}_{23}^{ll}]^T \\ \mathbf{W}_2 = \mathbf{Y}_{33}^{la} - [\mathbf{Y}_{31}^{ll} \quad \mathbf{Y}_{32}^{ll}] \mathbf{D} [\mathbf{Y}_{13}^{la} \quad \mathbf{Y}_{23}^{la}]^T \\ \mathbf{W}_3 = \mathbf{Y}_{33}^{al} - [\mathbf{Y}_{31}^{al} \quad \mathbf{Y}_{32}^{al}] \mathbf{D} [\mathbf{Y}_{13}^{ll} \quad \mathbf{Y}_{23}^{la}]^T \\ \mathbf{W}_4 = \mathbf{Y}_{33}^{aa} - [\mathbf{Y}_{31}^{al} \quad \mathbf{Y}_{32}^{al}] \mathbf{D} [\mathbf{Y}_{13}^{la} \quad \mathbf{Y}_{23}^{la}]^T \end{cases} \quad (4.18)$$

$$\mathbf{D} = \frac{1}{2} \begin{bmatrix} \mathbf{U} & e^{-\gamma_l^A l} \\ e^{-\gamma_l^A l} & \mathbf{I} \end{bmatrix} \quad (4.19)$$

where γ_l^A is the propagation constant of localized modes in Region A.

From (4.16), we can see that the RAM is composed of two parts: one is contributed by accessible modes, which is simple and analytic, and the other is contributed by localized modes, which interact with accessible ones. The second part requires matrix inversion of order N_3^l , where N_3^l is the number of localized modes in Region C. Compared with the GSM approach [78, 84], which requires three matrix inversions, the RAM requires much less computational efforts and reveals more physical insight.

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4.2.2 Determination of Equivalent Circuit

From the above formulation, the circuit element values of any waveguide T-junction can be numerically obtained. Here, we will focus on the square case, which is represented by a five-port network shown in Fig. 4.2.

From the expressions of the derived GAM given in Appendix B, we notice that the vertically polarized TE_{10} and horizontally polarized TE_{01} modes in the square waveguide can only be coupled with the TE_{0i} and $\{TE_{1i}, TM_{1i}\}$ modes in the branch waveguide, respectively. There is no cross-coupling between them, and the complicated five-port network can therefore be divided into two independent sub-networks. For the vertically polarized modes, it is a three-port sub-network composed of Ports 1, 3, and 5. For the horizontally polarized modes, it is a two-port sub-network composed of Ports 2 and 4. The above finding is also confirmed by numerical computations of (4.16), where the admittances Y_{s12} , Y_{s14} and Y_{s25} are almost zero, which correspond to an open circuit. The simplified equivalent network of a square waveguide T-junction is shown in Fig. 4.4.

It is interesting to note that if we simply replace the RAM $\mathbf{Y}^{\text{red}}$ by the accessible mode matrix $\mathbf{Y}^{aa}$, the resultant admittance and scattering parameters of three-port sub-network calculated accordingly present a good approximation to the exact one in (4.16), except Y_{11} and S_{11} . While for the two-port sub-network, we cannot obtain a good approximation from this simplification.

This phenomenon can be justified by looking into the field distributions, as shown in Fig. 4.5. The vertically polarized modes can propagate in the T-junction without

4.2 Theoretical Formulation

Figure 4.4: Simplified equivalent circuit of a square waveguide T-junction.

4.2 Theoretical Formulation

much distortion because their electric fields are parallel to the discontinuity caused by the longitudinal branch waveguide. While for the horizontally polarized modes, the electric fields are distorted greatly. Also, as mentioned previously, the TE_{10} mode in the square waveguide can only excite the TE_{0i} modes in the branch waveguide, while the TE_{01} mode can excite both TE_{1i} and TM_{1i} modes. The above observations imply that the 3-port sub-network can be accurately described by accessible modes including a few localized modes, while more localized modes should be considered for the two-port sub-network.

Figure 4.5: Electric field patterns in a square waveguide T-junction: (a) vertically polarized mode and (b) horizontally polarized mode.

In fact, by only considering the matrix element corresponding to the first localized TE_{02} mode in (4.18), matrix $\mathbf{W}_1$ is reduced to a single number, and $\mathbf{W}_2$ and $\mathbf{W}_3$ become zero. Therefore, a set of accurate closed-form expressions for the three-port sub-network can be found explicitly, which avoids the most time-consuming matrix

4.2 Theoretical Formulation

inversion in (4.17).

$$\begin{cases} Y_{11}^V = -j \cot(\beta^A l) - A_{12}^2 W \\ Y_{12}^V = j \csc(\beta^A l) + A_{12}^2 W \\ Y_{13}^V = A_{11} \\ Y_{33}^V = -j \frac{b}{a} \cot(\beta^B a) + L_1 - A_{21}^2 \end{cases}, \quad (4.20)$$

where

$$L_i = 8 \sum_{m=2}^{\text{even}} \frac{\gamma_{mi}^B \sin^2 \frac{m\pi b}{a}}{\gamma_{0i}^C (k_{cmi}^B)^2 ab} \left[\left(\frac{ia}{ml} \right)^2 - \left(\frac{k_0}{\gamma_{mi}^B} \right)^2 \right] \coth(\gamma_{mi}^B a), \quad (4.21)$$

$$A_{ij} = 2 \left(\frac{i\pi}{a} \right) \left(\frac{j\pi}{l} \right) \frac{1}{a} \sqrt{\frac{b}{\gamma_{i0}^A \gamma_{0j}^B l}} \frac{1}{(\gamma_{0j}^B)^2 + \left(\frac{i\pi}{a} \right)^2}, \quad (4.22)$$

$$W^{-1} = \mathbf{1} + \frac{b}{a} \coth(\gamma_{02}^B a) + L_2 - A_{22}^2, \quad (4.23)$$

k_{cmi}^B is the cutoff wavenumber of the (mi) th mode in Region B, β^A and β^B are phase constants of accessible modes in the corresponding regions. It should be noted that, although only the first localized TE₀₂ mode is considered in (4.17), the contribution of other localized modes is still included in (4.21), where m is even and $i, j = 1, 2$. It is also observed that a few terms in (4.21) are enough for convergent results.

For the two-port sub-network, the above approximation cannot be made because it has to consider more localized modes in (4.17). However, we notice that the real part of the diagonal elements in $\mathbf{W}_1$ is always positive, and most of the off-diagonal elements in $\mathbf{W}_1$ are relatively small compared to the diagonal ones, except the ones corresponding to the mutual couplings between the two degenerated TE_{1i} and TM_{1i} modes. Therefore, the diagonal elements in the matrix $(\mathbf{U} + \mathbf{W}_1)$ contribute more in the matrix inversion than others. It suggests that it can be approximated by ignoring

4.2 Theoretical Formulation

the off-diagonal elements, except the ones between the TE_{1i} and TM_{1i} modes.

$$[\mathbf{I} + \mathbf{W}_1] \cong \begin{bmatrix} W^0 & 0 & 0 \\ 0 & [\mathbf{W}^{hh}] & [\mathbf{W}^{he}] \\ 0 & [\mathbf{W}^{eh}] & [\mathbf{W}^{ee}] \end{bmatrix}, \quad (4.24)$$

where W^0 is the element corresponding to the TE_{10} mode in the branch waveguide, $\mathbf{W}^{hh}$ and $\mathbf{W}^{ee}$ are diagonal matrices corresponding to the TE_{1i} and TM_{1i} modes, and $\mathbf{W}^{he}$ and $\mathbf{W}^{eh}$ are the diagonal matrices corresponding to the mutual couplings between them.

By invoking the above approximation in (4.17), the matrix can be inverted analytically and the closed-form expressions can be obtained as

$$\begin{cases} Y_{11}^H = -j \cot \beta_{01}^A l - Y_{r0} - \sum_{i=1}^{\infty} Y_{ri} \\ Y_{12}^H = j \csc \beta_{10}^A l + Y_{r0} - \sum_{i=1}^{\infty} (-1)^{i+1} Y_{ri} \end{cases}, \quad (4.25)$$

where

$$Y_{r0} = \frac{4}{b^3 l \gamma_{01}^A \gamma_{10}^C W^0} C C_{11}^2, \quad (4.26)$$

$$Y_{ri} = \frac{(Y_{ri}^h)^2 W_i^{ee} + (Y_{ri}^e)^2 W_i^{hh} - Y_{ri}^h Y_{ri}^e (W_i^{eh} + W_i^{he})}{D_{Wi}}. \quad (4.27)$$

The variables used in (4.26) and (4.27) are given in Appendix C.

From (4.1) and (4.15), the physical interpretation of this approximation can be revealed. The series admittances between the localized modes are negligible because the shunt admittances are enhanced by terminating with corresponding characteristic

4.2 Theoretical Formulation

impedances. It should be pointed that the closed-form expressions of other waveguide structures may also be obtained if similar approximation is applied.

4.2.3 Numerical Verification

Figure 4.6: Admittance parameters of the three-port sub-network.

As an example, we consider an X-band square waveguide T-junction here. The dimension of the square waveguide is $a = 18$ mm, while the side rectangular waveguide is WR90 ($l = 22.86$ mm, $b = 10.16$ mm). For the three-port sub-network, the admittance parameters are calculated using our closed-form equations (4.20), where $m = 2, 4, 6, 8, 10$ in (4.21). For the two-port one, they are calculated using (4.25), where $i = 1, 2, 3$, which means that the TE_{10} , TE_{11} , TM_{11} , TE_{12} , TM_{12} , TE_{13} and TM_{13} modes are considered in (24). Numerical results are compared with those obtained from the RAM and the formally exact GSM, and a very good agreement is

4.2 Theoretical Formulation

observed as shown in Figs. 4.6 and 4.7.

It is noticed that the admittance parameters of the three-port sub-network intend to be resonant at several frequencies at the same time. This phenomenon can be explained directly from our closed-form expressions, which contain item of cotangent or cosecant function. Therefore, these frequencies can be easily estimated by

$$\beta^A l = 0, \pm n\pi. \quad (4.28)$$

Obviously, the first resonant frequency is the cutoff frequency of the dominant mode in the square waveguide and the following ones are the resonant frequencies of the cavity region (Region D in Fig. 4.3).

For the two-port sub-network, similar resonances are observed, but shifted to a little higher frequency due to the effect of localized modes. The scattering matrix calculated from the above admittance parameters are also compared with those from the RAM and the GSM, as shown in Figs. 4.8 and 4.9, which also exhibit a very good agreement. It is noticed that in the scattering parameters, the resonance occurred in the admittance parameters disappears.

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Figure 4.7: Admittance parameters of the two-port sub-network.

Figure 4.8: Scattering parameters of the three-port sub-network.

4.2 Theoretical Formulation

Figure 4.9: Scattering parameters of the two-port sub-network.

Figure 4.10: Short-circuited branch OMT.

4.2 Theoretical Formulation

Figure 4.11: Equivalent circuit model of short-circuited branch OMT.

4.3 Design of Short-Circuited Branch OMT

A short-circuited branch OMT can be implemented by orthogonally cascading two square waveguide T-junctions with one end terminated by a short-circuit, as shown in Fig. 4.10. With the equivalent circuit model obtained above, this structure can be efficiently analyzed in a circuit simulator rather than the full-wave one [43], as shown in Fig. 4.11. The lengths of the two transmission lines l_1 and l_2 can be easily optimized in the circuit simulator to obtain the desired scattering parameters.

It is noticed that in this circuit model, the cross-polarization between the orthogonal port is ignored because this structure can intrinsically suppress it [43].

When $l_1 = 1.07$ mm and $l_2 = 5.64$ mm, the calculated S-parameters of the designed OMT are shown in Fig. 4.12. Results calculated by the equivalent circuit model are still in good agreement with those obtained by the GSM. The center frequency is located at about 9.75 GHz. For the vertically polarized Port 1, the 15dB return loss bandwidth is about 10 %, while for the horizontally polarized Port 2, it is about 5 %. A good agreement is observed from 8 GHz to 11.7 GHz, which covers most of the entire X band. The discrepancy at the high frequencies is due to the fact that only the accessible modes are considered in our equivalent circuit model and higher-order modes start to propagate near 11.7 GHz.

To verify our design derived from the proposed equivalent circuit model, two identically designed OMTs are fabricated and connected in the back-to-back configuration, and the port numbers of the back-to-back configuration are redefined accordingly, as shown in Fig. 4.13.

4.3 Design of Short-Circuited Branch OMT

Figure 4.12: Calculated S-parameters of a short-circuited branch OMT ($a = 18$ mm, $l = 22.86$ mm, $b = 10.16$ mm, $l_1 = 1.07$ mm, $l_2 = 5.64$ mm).

Figure 4.13: Back-to-back configuration of the designed short-circuited branch OMT.

Measured results [100] are compared with those calculated by our closed-form expressions, which exhibits a good agreement over the frequency band of interest, as shown in Fig. 4.14. The discrepancy at the high frequency is due to the effect of higher-order modes and the resonances caused by the back-to-back configuration.

4.3 Design of Short-Circuited Branch OMT

Because the cutoff frequency of the dominant mode is 8.3 GHz, the discrepancy below this frequency observed in Fig. 4.14(a) is also expected. Measured isolation between the orthogonally polarized Ports 1 & 2 and 1 & 4 is better than 30 dB up to 11.5 GHz, as shown in Fig. 4.15, which indicates the correctness of our early observation that two polarizations are isolated. These results verify the validity of our equivalent circuit model.

4.3 Design of Short-Circuited Branch OMT

(a)

(b)

Figure 4.14: Calculated and measured S-parameters of the designed OMT in the back-to-back configuration: (a) $|S_{11}|$ and $|S_{13}|$ and (b) $|S_{22}|$ and $|S_{24}|$ ($a = 18$ mm, $l = 22.86$ mm, $b = 10.16$ mm, $l_1 = 1.07$ mm, $l_2 = 5.64$ mm).

4.4 Conclusion

Figure 4.15: Measured isolation between orthogonally polarized ports in the back-to-back configuration.

4.4 Conclusion

An equivalent circuit model for a square waveguide T-junction has been derived from the GAM obtained by the mode-matching analysis. The original five-port network is separated into two individual three-port and two-port sub-networks. A set of accurate closed-form expressions is obtained to calculate the admittance parameters of these two sub-networks.

With this circuit model, it is possible to analyze the ortho-mode transducers in a very efficient circuit simulator and can avoid the complex treatment of multi-port networks. A short-circuited branch OMT has been designed and fabricated as an example. Measured results agree very well with those from the circuit simulation, which verifies our circuit model approach.

Chapter 5

Synthesis of Short-Circuited Waveguide OMTs

5.1 Introduction

With the advent of commercial modeling tools and availability of computing facilities, complicated ortho-mode transducers (OMTs) can be analyzed accurately now [10, 24, 104]. However, the design theory of OMTs is still very limited, and most OMTs are still designed by experience, or cut-and-trial. To my best knowledge, very few literature was available until now on the synthesis of OMTs. The only analytical formulas found in the open literature are derived for a simple Y-junction OMT [40]. For the widely used short-circuited waveguide OMTs, there is only a rough guideline to estimate the distance between the short-circuited plane and two branches [43].

The main difficulty for the synthesis of OMTs is due to the fact that two orthogonally polarized waves propagate in the common waveguide simultaneously, which results in a dual-mode circuit. Different from many single-mode circuits, conventional single-mode H- or E-plane circuit models [91] cannot be adopted directly. In

5.1 Introduction

general, it is physically a three-port network, electrically exhibiting properties of a four-port one. Therefore, the well-developed synthesis theory for two-port networks cannot be applied directly.

In Chapter 4, an equivalent circuit model for a square waveguide T-junction has been proposed and a short-circuited square waveguide ortho-mode transducer (OMT) has been designed in the circuit domain thereafter. However, this OMT is over-simplified, where two branch waveguides are directly connected to the main waveguide. Therefore, only the locations of the two branches can be modified in the design. In practice, a slot is usually employed in the T-junction to control the couplings between the main and branch waveguides [84] for better performance.

In this chapter, this square waveguide T-junction with a rectangular coupling slot is considered as a key building block of the short-circuited square waveguide OMT. The design of this slot-coupled short-circuited square waveguide OMT becomes to determine the locations of the two branches as well as the sizes of the corresponding coupling slots. An accurate synthesis method will be presented to determine the dimensions of this slot-coupled short-circuited square waveguide OMT efficiently. Our synthesis approach can be divided into three steps.

First, an improved equivalent circuit model of a slot-coupled square waveguide T-junction is developed as a key building block of the OMT. Compared to [90], a coupling slot is employed between the main and branch waveguides, which provides more flexibility and is more useful in the design of OMTs [43]. Moreover, the physical insight gained for the improved circuit model is clearer than [90]. The coupling slot

5.2 Introduction

is modeled as an impedance inverter and the junction effect is modeled by the shunt reactance. Additional transmission lines are employed to compensate the phase shift, which can be easily absorbed by the adjacent waveguide sections. Its circuit element values can be extracted from full-wave simulations accurately, e.g. the mode-matching (MM) analysis.

Second, the circuit model of a slot-coupled short-circuited OMT is established using the key building block. Since the cross-couplings are very weak and can be ignored, the whole circuit can be considered as two separate subnetworks. Therefore, the ABCD transfer matrix can be applied to analyze them separately.

Third, useful analytical formulas are derived from the circuit analysis for the purpose of synthesis, and the dimensions of the OMT can be synthesized thereafter. The whole synthesis procedures can be efficiently carried out simply by looking up some reference charts, or running a simple program. Since the circuit parameters are extracted rigorously from the mode-matching method, calculated results from our circuit model are very accurate compared with those from the full-wave simulator. Therefore, the synthesized dimensions can be directly employed in the fabrication without any full-wave optimizations.

Finally, a slot-coupled short-circuited OMT operating at X-band is designed by our proposed synthesis method. Measured results of the designed OMT validate our synthesis method.

5.2 Circuit Model of Slot-Coupled Square Waveguide T-Junction

The square waveguide T-junction is a typical structure often used in the design of OMTs, which supports dual-polarization in the main waveguide and extracts single-polarized wave through a branch waveguide, as shown in Fig. 5.1. Its dimensions and reference planes are depicted in Fig. 5.2.

Figure 5.1: Structure of a slot-coupled square waveguide T-junction.

Figure 5.2: Detailed description of a slot-coupled square waveguide T-junction: (a) top and (b) side view.

It has been shown in [90] that there is no cross-coupling between orthogonal modes

5.2 Circuit Model of Slot-Coupled Square Waveguide T-Junction

in a square waveguide T-junction without coupling slot. Although the slot would introduce some cross-couplings between orthogonal modes, we can still ignore them and divide the whole five-port network into one two-port subnetwork and another three-port subnetwork, as in [90], since the cross-couplings in any practical OMTs can be suppressed at a very low level.

Figure 5.3: Equivalent circuit model of a square waveguide T-junction.

To develop its equivalent circuit model, an impedance inverter is employed between the through and branch ports to evaluate the couplings between them. The junction effect is modeled as a shunt reactance for each subnetwork. To compensate the phase shift, transmission line sections are connected to each port. Therefore, the whole equivalent circuit model of the square waveguide T-junction is established, as shown Fig. 5.3, which can be considered as a building block or unit cell in the

5.2 Circuit Model of Slot-Coupled Square Waveguide T-Junction

synthesis of OMTs.

To extract its circuit elements, the whole structure is initially analyzed by the full-wave mode-matching method. The impedance matrix $\mathbf{Z}$ with reference planes T_1, T_2, T_3 can be calculated from the simulated scattering matrix as [101]

$$\mathbf{Z} = (\mathbf{U} - \mathbf{S})^{-1}(\mathbf{U} + \mathbf{S}) \quad (5.1)$$

where $\mathbf{U}$ is the identity matrix. The resultant impedance matrix can be arranged as

$$\mathbf{Z}_{5 \times 5} = \begin{bmatrix} \mathbf{Z}_{3 \times 3}^v & \mathbf{Z}_{3 \times 2}^{vh} \\ \mathbf{Z}_{2 \times 3}^{hv} & \mathbf{Z}_{2 \times 2}^h \end{bmatrix} \quad (5.2)$$

where $\mathbf{Z}_{3 \times 3}^v$ and $\mathbf{Z}_{2 \times 2}^h$ denote the impedance matrices of the three-port subnetwork and the two-port subnetwork, and $\mathbf{Z}_{3 \times 2}^{vh}$, $\mathbf{Z}_{2 \times 3}^{hv}$ represent the interactions between them.

Numerical simulations show that elements in $\mathbf{Z}_{3 \times 2}^{vh}$ and $\mathbf{Z}_{2 \times 3}^{hv}$ are very small compared with those in $\mathbf{Z}_{3 \times 3}^v$ and $\mathbf{Z}_{2 \times 2}^h$. It therefore justifies our assumption of weak cross-couplings between the two orthogonal modes.

For the three-port subnetwork, we can first establish the relationships between the voltages and the currents defined in Fig. 5.3 as follows.

$$V_1^{v'} = V_2^{v'} = jk^2 b_v I_3^{v'} \quad (5.3a)$$

$$V_3^{v'} = -jk(I_1^{v'} + I_2^{v'} - kb_v I_3^{v'}) \quad (5.3b)$$

$$V_{1,2}^{v'} = \cos \phi_v V_{1,2}^v - j \sin \phi_v I_{1,2}^v \quad (5.4a)$$

$$-I_{1,2}^{v'} = j \sin \phi_v V_{1,2}^v - \cos \phi_v I_{1,2}^v \quad (5.4b)$$

5.2 Circuit Model of Slot-Coupled Square Waveguide T-Junction

$$V_3^{v'} = \cos \phi_0 V_3^v - j \sin \phi_0 I_3^v \quad (5.5a)$$

$$-I_3^{v'} = j \sin \phi_0 V_3^v - \cos \phi_0 I_3^v \quad (5.5b)$$

According to the definition of the impedance parameters [105],

$$z_{ij} = \left. \frac{V_i}{I_j} \right|_{I_k=0 \text{ for } k \neq j}, \quad (5.6)$$

we can obtain the transmission line ϕ_v by substituting (5.4a) into (5.3a),

$$\tan \phi_v = j(z_{12}^v - z_{11}^v) \quad (5.7)$$

where z_{11}^v and z_{12}^v are elements in $\mathbf{Z}_{3 \times 3}^v$. After ϕ_v is extracted, the reference planes can be shifted from T_1, T_2 to $T_1^{v'}, T_2^{v'}$, and the remaining impedance matrix is updated as $\mathbf{Z}_{3 \times 3}^{v'}$.

Similarly, substituting (5.5b) into (5.3a), we can have

$$z_{13}^{v'} = -jk^2 b_v (j \sin \phi_0 z_{33}^{v'} - \cos \phi_0) \quad (5.8)$$

when $I_1^{v'} = I_2^{v'} = 0$, and

$$z_{11}^{v'} = k^2 b_v \sin \phi_0 z_{13}^{v'} \quad (5.9)$$

when $I_2^{v'} = I_3^{v'} = 0$.

Dividing (5.8) by (5.9), the transmission line ϕ_0 can be extracted as

$$\tan \phi_0 = -j \frac{z_{11}^{v'}}{z_{11}^{v'} z_{33}^{v'} - z_{13}^{v'2}} \quad (5.10)$$

5.2 Circuit Model of Slot-Coupled Square Waveguide T-Junction

where $z_{11}^{v'}$, $z_{13}^{v'}$ and $z_{33}^{v'}$ are elements in $\mathbf{Z}_{3 \times 3}^{v'}$.

By removing ϕ_0 , the reference plane is shifted from T_3 to $T_3^{v'}$, and the remaining impedance matrix is updated as $\mathbf{Z}_{3 \times 3}^{v''}$.

Then the impedance inverter and the shunt reactance in the three-port subnetwork can be determined from (5.3) as

$$k = j z_{13}^{v''} \quad (5.11)$$

$$b_v = -j \frac{z_{33}^{v''}}{z_{13}^{v''2}} \quad (5.12)$$

where $z_{13}^{v''}$ and $z_{33}^{v''}$ are elements in $\mathbf{Z}_{3 \times 3}^{v''}$.

For the two-port subnetwork, the shunt reactance can be determined in a similar way as

$$b_h = j \frac{1 - z_{11}^2 + z_{12}^2}{z_{12}} \quad (5.13)$$

where z_{11}^h and z_{12}^h are elements in $\mathbf{Z}_{2 \times 2}^h$. Then the transmission line ϕ_h can be determined by comparing the phase difference between reference planes T_1 and $T_1^{h'}$.

It should be mentioned that the extracted impedance inverter and shunt reactance have been normalized to their corresponding characteristic impedances. The unnormalized ones can be calculated as follows.

$$\begin{cases} B_{v,h} = b_{v,h}/Z_{01} \\ K = k\sqrt{Z_{01}Z_{02}} \end{cases} \quad (5.14)$$

where Z_{01} and Z_{02} are the characteristic impedances of the main and branch wave-

5.3 Analysis of Short-Circuited Waveguide OMT

guides, respectively.

As an example, a slot-coupled square waveguide T-junction with the following dimensions is analyzed by the mode-matching method: $a = 22.86$ mm, $b = 10.16$ mm, $s = 18$ mm, $t = 2$ mm, $w = 20$ mm, $h = 6$ mm. Its circuit parameters are extracted, as shown in Fig. 5.4. The phase of a uniform square waveguide of length $w/2$ is compared with θ_v and θ_h . It is found that both θ_v and θ_h are close to $\theta_w/2$, and $\theta_v > \theta_h > \theta_w/2$.

Figure 5.4: Extracted circuit parameters. θ_w is the phase of a uniform square waveguide with length w .

5.3 Analysis of Short-Circuited Waveguide OMT

A typical short-circuited square waveguide OMT is shown in Fig. 5.5, where two square waveguide T-junctions are cascaded orthogonally and the through port is

5.3 Analysis of Short-Circuited Waveguide OMT

terminated by a short-circuited plane.

Figure 5.5: Structure of a slot-coupled short-circuited waveguide OMT: (a) 3D view and (b) side view.

With the equivalent circuit model proposed in the previous section, the circuit model of this OMT can be established, as shown in Fig. 5.6, where the θ_v and θ_h in the T-junction model are absorbed into the adjacent waveguide sections.

5.3 Analysis of Short-Circuited Waveguide OMT

Since the cross-couplings between orthogonal modes are assumed to be very small, the four-port OMT can be separated as two two-port subnetworks. Therefore, the classic circuit analysis method using the ABCD transfer matrix can be employed.

Figure 5.6: Equivalent circuit model of the short-circuited OMT.

For the vertically polarized subnetwork, the whole transfer matrix can be derived as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & j k^v \\ j/k^v & -k^v(b_1^v + b_{in}^v) \end{bmatrix} \quad (5.15)$$

where

$$b_{in}^v = \frac{b_2^v + \tan \theta_1^v - \cot \theta_2^v}{1 - (b_2^v - \cot \theta_2^v) \tan \theta_1^v} \quad (5.16)$$

For the horizontally polarized subnetwork, the elements of the whole transfer

5.4 Synthesis of Short-Circuited Waveguide OMT

matrix are

$$A = -\frac{\sin \theta_1^h}{k^h} \quad (5.17a)$$

$$B = jk^h(\cos \theta_1^h - \sin \theta_1^h b_{in}^h) \quad (5.17b)$$

$$C = j\frac{\cos \theta_1^h - b_1^h \sin \theta_1^h}{k^h} \quad (5.17c)$$

$$D = -k^h[\sin \theta_1^h(1 - b_{in}^h b_1^h) + \cos \theta_1^h(b_1^h + b_{in}^h)] \quad (5.17d)$$

where $b_{in}^h = b_2^h - \cot \theta_2^h$.

The scattering parameters can be calculated from its transfer matrix as [105]

$$S_{11} = \frac{A + B - C - D}{A + B + C + D} \quad (5.18a)$$

$$S_{21} = \frac{2}{A + B + C + D} \quad (5.18b)$$

5.4 Synthesis of Short-Circuited Waveguide OMT

For a waveguide OMT, the main waveguide is usually connected to a horn antenna and two branch waveguides are connected to standard rectangular waveguides. The synthesis of this short-circuited waveguide OMT is to determine the size of two slots and their positions along the main waveguide.

In theory, one branch should be placed a quarter-wavelength away from the short plane to maximize the coupling between the main and branch waveguides, and the other should be kept half a wavelength away from the first one to maintain good isolation. In practice, the location of the first branch should be less than a quarter-

5.4 Synthesis of Short-Circuited Waveguide OMT

wavelength to compensate the junction reactance [43]. Therefore, the slots' size and their positions have to be optimized to meet given specifications.

In this section, a rigorous synthesis method is presented. The slot size can be chosen from a reference chart and their positions can also be determined accurately thereafter.

When the two branches are separated by half a wavelength, that means $\theta_1^{v,h} = \pi$. For both vertically and horizontally polarized subnetworks, their transfer matrices derived in (5.15-5.17) can be simplified in the same form as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & jk \\ j/k & k(\cot \theta_2 - b_1 - b_2) \end{bmatrix} \quad (5.19)$$

where the superscript v and h are omitted for simplicity. It also implies that the two slots are identical and indistinguishable for both subnetworks when $\theta_1^{v,h} = \pi$. Therefore, we have $b_1 + b_2 = b_v + b_h$ in this case.

Substituting (5.19) into (5.18), we have

$$S_{11} = \frac{j(k - 1/k) - k(\cot \theta_2 - b_v - b_h)}{j(k + 1/k) + k(\cot \theta_2 - b_v - b_h)} \quad (5.20)$$

The reflection zero occurs when

$$k = 1 \quad (5.21a)$$

$$\cot \theta_2 = b_v + b_h \quad (5.21b)$$

5.4 Synthesis of Short-Circuited Waveguide OMT

The physical insight of (5.21) is clear. The slot size can then be selected to satisfy (5.21a), and θ_2 can be calculated by (5.21b). It can also explain why the branch close to the short plane is usually placed less than quarter-wavelength.

In practice, we only need to satisfy a prescribed return loss level RL over a frequency range of interest.

$$|S_{11}| < \epsilon \quad (5.22)$$

where $\epsilon = 10^{-RL/20}$. According to the following inequality

$$\left| \frac{j(k - 1/k) + k(b_v + b_h - \cot \theta_2)}{j(k + 1/k) - k(b_v + b_h - \cot \theta_2)} \right| \geq \left| \frac{k - 1/k}{k + 1/k} \right|, \quad (5.23)$$

we can derive the necessary condition for (5.22) as

$$\sqrt{\frac{1 - \epsilon}{1 + \epsilon}} < k < \sqrt{\frac{1 + \epsilon}{1 - \epsilon}} \quad (5.24)$$

For a given k satisfying (5.24), we can further obtain

$$|\cot \theta_2 - b_v - b_h| \leq \frac{\sqrt{\epsilon^2(k^2 + 1)^2 - (k^2 - 1)^2}}{k^2 \sqrt{1 - \epsilon^2}} \quad (5.25)$$

The dimensions of the coupling slots and their positions along the main waveguide can then be synthesized. The synthesis procedures of the slot-coupled short-circuited waveguide OMT are summarized as follows.

1) The dimensions of the main waveguide are usually prescribed according to the size of a horn antenna, which connects to the OMT. The branch waveguides are

5.5 Synthesis of Short-Circuited Waveguide OMT

usually selected as standard waveguides covering the operating frequency band of the OMT.

2) Circuit parameters of the slot-coupled square waveguide T-junction of different slot dimensions are extracted. Three charts with different slot dimensions are plotted as a reference, as shown in Figs. 5.7-5.9: (a) the inverter impedance k , (b) the sum of shunt reactance $b_v + b_h$, (c) the average electrical length of two equivalent transmission lines $\theta_a = (\theta_v + \theta_h)/2$.

3) The dimensions of the slots are selected by looking up Reference chart (a) to satisfy (5.21a) at the center frequency.

4) Looking at Reference chart (b), the value of $b_v + b_h$ can be obtained for the selected slot dimensions, and θ_2 can be calculated by (5.21b) at the center frequency.

5) Looking at Reference chart (c), θ_a is obtained for the selected slot dimensions.

The positions of two slots can be determined as

$$l_1 = \left(\frac{1}{2} - \frac{\theta_a}{\pi}\right)\lambda_{g0} + w \quad (5.26a)$$

$$l_2 = \frac{\theta_2 - \theta_a}{2\pi}\lambda_{g0} + \frac{w}{2} \quad (5.26b)$$

where λ_{g0} is the guided wavelength in the square waveguide at the center frequency.

The reference charts can be obtained by any full-wave simulations accurately. For our case, the mode-matching method is employed for its high efficiency and accuracy.

It should be mentioned that this synthesis procedure is also applicable to other short-circuited waveguide OMT, such as circular waveguide OMTs.

5.5 Design Example

In this section, we will design an X-band slot-coupled short-circuited square waveguide OMT using our proposed synthesis method. The center frequency of this OMT is 10 GHz.

First, the dimensions of the square waveguide T-junction are determined. The branch waveguides are the standard waveguide WR90, where $a = 22.86$ mm and $b = 10.16$ mm. The size of the square waveguide is chosen to be $s = 18$ mm, whose TE_{10}/TE_{01} modes start propagating from 8.3 GHz and the first higher-order TE_{11}/TM_{11} modes start at 11.7 GHz. Therefore, our equivalent circuit model would be valid from 8.3 GHz to 11.7 GHz. To simplify the design, the thickness of the slots is fixed to be $t = 2$ mm, though this is not essential. The coupling between the square and branch waveguides is mainly controlled by the slot width w and slot height h . Then, circuit parameters of different slot dimensions are extracted at 10 GHz and reference charts are plotted, as shown in Figs. 5.7~5.9. If the prescribed return loss level is 20 dB, we have $0.9045 < k < 1.1055$. It is observed that there are multiple sets of the slot dimensions satisfying (5.21a) and (5.24). For example, when the slot dimensions $w = 18.8$ mm, $h = 7$ mm are selected from Reference chart (a), we can have $\cot \theta_2 = b_v + b_h = 0.2199$ from Reference chart (b), which means $\theta_2 = 1.3544$. We then have $\theta_a = 1.134$ from Reference chart (c). The positions of the two slots can be calculated using (5.26) as $l_1 = 26.3$ mm and $l_2 = 11.3$ mm. To demonstrate the simplicity and versatility, a simple programme is written to calculate the dimensions automatically starting from these satisfied sets following our synthesis

5.5 Design Example

Figure 5.7: Reference chart (a): impedance inverter k .

Figure 5.8: Reference chart (b): shunt reactance $b_v + b_h$.

5.5 Design Example

Figure 5.9: Reference chart (c): $\theta_a = (\theta_v + \theta_h)/2$.

procedure. The scattering parameters can be calculated for the circuit model using the extracted circuit parameters corresponding to the selected dimensions, as shown in Fig. 5.10. It is seen that the reflection zeros can be accurately located at the center frequency for both vertically and horizontally polarized subnetworks using our synthesis method.

Scattering parameters obtained from the circuit model are also compared with those from the full-wave mode-matching method, as shown in Fig. 5.11. The dimensions are $w = 20.6$ mm, $h = 8.6$ mm, $l_1 = 25.8$ mm, $l_2 = 13$ mm. The reflection zeros for both polarizations are still accurately located at 10 GHz, and very good agreements are observed between our circuit model and the full-wave mode-matching method within the dominant mode operation region.

5.5 Design Example

Figure 5.10: Scattering parameters of different synthesized slot dimensions.

Figure 5.11: Scattering parameters of synthesized OMT (Solid line: full-wave simulation, $\times$: circuit model.).

5.5 Design Example

It is interesting to find that there are two transmission zeros at about 9.25 GHz and 11.25 GHz, respectively, for the vertical subnetwork. This can be explained from the circuit analysis. From the field distributions, we find that it satisfies $\theta_1^v + \theta_2^v = \frac{n\pi}{2}$, $n = 1, 2$, at these two frequency points. Substituting this into (5.15) we find $b_{in}^v \rightarrow \infty$, which results in $A + B + C + D \rightarrow \infty$. According to (5.18b), this produces a transmission zero.

In order to verify the proposed synthesis method, the designed OMT is fabricated in a back-to-back configuration. The photo of the final fabricated structure is shown in Fig. 5.12. Measured results of the fabricated OMT are compared with those obtained by the full-wave simulation, and they are in good agreement, as shown in Fig. 5.13. The isolation between the orthogonal ports are better than 40 dB below 11 GHz, as shown in Fig. 5.14.

Figure 5.12: Photo of our fabricated OMT in the back-to-back configuration.

It should be emphasized that all the dimensions we mentioned above: w , h ,

5.5 Design Example

Figure 5.13: Comparison between measured and circuit simulated results of the designed OMT in the back-to-back configuration.

Figure 5.14: Isolation of the designed OMT in the back-to-back configuration.

5.6 Conclusion

l_1 , l_2 , are rigorously calculated by our synthesis procedure without any full-wave optimizations. Therefore, our synthesis method is very accurate and efficient in the design of OMTs.

5.6 Conclusion

An accurate and efficient synthesis method for short-circuited square waveguide OMTs has been presented. An improved equivalent circuit model of the slot-coupled square waveguide T-junction has been proposed as a key building block of the waveguide OMT. Circuit parameters have been extracted accurately from the full-wave simulations. The circuit model of a slot-coupled short-circuited waveguide OMT has been established with this building block and can be analyzed by the classic transfer matrix method. Some useful formulas have been derived from this circuit model, which can be used to synthesize the dimensions of OMTs. The synthesis procedure has been summarized and implemented by a simple program. One designed OMT has been fabricated and tested to verify the validity of our synthesis method.

Chapter 6

Design of Compact Dual-Band OMTs

6.1 Introduction

In the previous chapters, the mode-matching (MM) method has been developed for the efficient analysis of waveguide OMTs, equivalent circuit models have been proposed for the circuit analysis and synthesis of ortho-mode transducers (OMTs).

With the explosive growth of wireless communication applications, it is preferred to receive and transmit orthogonally polarized signals simultaneously through a single antenna, which may cover more than one frequency band. Therefore, dual- or multi-band OMTs are desirable.

Several dual-band, even multi-band OMTs were reported in the literature [7, 12, 15, 45, 65, 106, 107]. In [45] and [106], two orthogonally polarized signals were separated at two different frequency bands. Therefore, they are still physically three-port networks. In practical dual-band dual-polarized antenna feed systems [7, 65, 107], it is more common to receive and separate dual-polarized signals at one frequency

6.1 Introduction

band, and transmit single-polarized signals at another frequency band. Therefore, it is physically a four-port network [7]. The design of these dual-band OMTs is usually complicated because they should separate multiple channels both in polarizations and frequency bands. Although there are usually wide guard bands between the operating frequency channels [7], additional filtering functions are commonly required to achieve good isolation between them, especially for those operating in transmit and receive (Tx/Rx) modes simultaneously. In 2006, a dual-band OMT operating at 30/44 GHz was designed in [67], which employed a series of stacked irises as a compact filtering element to improve the isolation.

In this chapter, two compact dual-band OMTs are presented. The first one is developed from the single-band short-circuited waveguide OMT. One more T-junction is connected to transmit single-polarized signals at another frequency band. Two filters are employed to achieve good isolation between these two frequency bands. With the equivalent circuit model developed in Chapter 5, the junction effect of the OMT can be considered in the design of filters, which introduces additional transmission zeros. Therefore, elliptical filters can be realized and integrated as parts of the OMT, which results in a compact structure. The equivalent circuit model is adopted to pre-optimize the structure efficiently, and then the full-wave mode-matching method is used to finalize the whole design. An example of this dual-band OMT is designed to operate at 9.4~9.6 GHz and 10.4~10.6 GHz, and measured results exhibit good performance as expected.

In the second case, a more compact dual-band OMT is presented. Two orthogo-

6.2 Dual-Band OMT with Integrated Filters

nally polarized signals are received and separated at 10.8~12.8 GHz and 10.2~11.9 GHz, respectively, and a single-polarized signal is transmitted at 13.8~15 GHz. To obtain good isolation between the Tx and Rx signals of the same polarization with a compact size, a waveguide with reduced cross-section is incorporated into the common waveguide and an irregularly shaped diaphragm is proposed as a compact band-rejection element. Moreover, the designed OMT can be fabricated as a monoblock with a thin stacked diaphragm, which is very stable and suitable for mass production. Two identical OMTs are fabricated and measured in a back-to-back configuration to verify the design concept.

6.2 Dual-Band OMT with Integrated Filters

The single-band short-circuited waveguide OMT has been synthesized in Chapter 5. To transmit a single-polarized signal at another frequency band, one more branch could be connected to the main square waveguide. Since the polarization of this additional branch must be the same as one of the other two branches, two filters are necessary to achieve good isolation between these two branches [7]. Usually, the OMT and the channel filters are considered as two different components in the system and designed separately.

However, in Chapter 5, it is found that the short-circuited waveguide T-junction in the OMT can introduce some transmission zeros. Therefore, it implies that quasi-elliptical filters can be realized if the T-junction effect is considered in the design of the channel filters.

6.2 Dual-Band OMT with Integrated Filters

In this section, a dual-band OMT is designed together with the two channel filters as an integrated component. In this way, good isolation between the two frequency bands can be achieved in a compact size.

6.2.1 Proposed Structure and Its Equivalent Circuit Model

The proposed dual-band OMT with two integrated channel filters is shown in Fig. 6.1, where Port 1 is the common square waveguide port, Port 2 is to transmit a vertical-polarized signal at 9.4-9.6 GHz, and Ports 3 and 4 are to receive two orthogonal-polarized signals at 10.4-10.6 GHz. Two iris filters are connected to Ports 2 and 4 to achieve good isolation between these two frequency bands.

Figure 6.1: Structure of the proposed dual-band OMT.

Based on the equivalent circuit model presented in Chapter 5, the equivalent circuit model of the proposed dual-band OMT with integrated channel filters can be established, as shown in Fig. 6.2. The circuit model of the two iris filters can be

6.2 Dual-Band OMT with Integrated Filters

represented in the same form, as shown in Fig. 6.3.

Figure 6.2: Equivalent circuit model of the dual-band OMT.

Figure 6.3: Circuit model of the iris filter.

6.2.2 Design Procedure

The dimensions of the proposed dual-band OMT can be described as shown in Fig. 6.4. The square waveguide of a side length is $s = 18$ mm, and the three branch rectangular waveguides are $a = 22.86$ mm, $b = 10.16$ mm, which are the same with the ones used in Chapter 5. The thickness of all the slots and the irises are 2 mm for the ease of manufacture. The slot size of these three T-junctions can be firstly

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determined by the synthesis method proposed for the single-band OMT in Chapter 5. The distances between these T-junctions can be then determined accordingly.

Figure 6.4: Dimensional description of the proposed dual-band OMT.

If the coupling slot in the T-junction is considered as the first impedance inverter of the channel filters, the two iris filters can be first synthesized as third-order Tchebychev filters at 9.4-9.6 GHz and 10.4-10.6 GHz, respectively [108, 109].

In this way, the initial dimensions of the dual-band OMT can be determined. It is found that for the horizontally polarized Port 3, this initial value can give very good return loss at the desired frequency band. However, for the vertically polarized

6.2 Dual-Band OMT with Integrated Filters

Ports 2 and 4, the return loss would be distorted significantly due to the junction effect introduced by the T-junctions [110, 111]. Therefore, the coupling slot size (W_i, H_i) and the first iris width W_{i1} , $i = 1, 3$, should be modified to compensate the junction effect. The waveguide length L_{i1} and L_{i2} , $i = 1, 3$, should also be modified accordingly. Since the equivalent circuit model has been established, these dimensions can be optimized very efficiently in the circuit domain. Once the circuit optimization is finished, the whole structure can be further refined by tuning all the dimensions with the full-wave mode-matching method.

The design procedure of the proposed dual-band OMT can be summarized as follows.

- 1) The initial values of the coupling slots and the T-junctions' positions are determined by synthesizing two single band short-circuited waveguide OMTs at two specified frequency bands separately using the method proposed in Chapter 5;
- 2) Two filters are synthesized independently using the filter synthesis theory [108, 109];
- 3) The performance of the OMT is optimized in the circuit domain with the initial values obtained in the previous two steps;
- 4) The final structure is fine tuned in our full-wave mode-matching simulator and the optimal performance is obtained.

6.2 Dual-Band OMT with Integrated Filters

6.2.3 Simulated and Measured Results

The initial dimensions of the dual-band OMT are given as follows. $W_{11} = 8.92$ mm, $W_{12} = 8.92$ mm, $W_{13} = 12.59$ mm, $L_{11} = 17.08$ mm, $L_{12} = 18.95$ mm, $L_{13} = 17.08$ mm, $W_{31} = 8.2$ mm, $W_{32} = 8.2$ mm, $W_{33} = 12.04$ mm, $L_{31} = 14.11$ mm, $L_{32} = 15.85$ mm, $L_{33} = 14.11$ mm, $W_1 = 17.8$ mm, $H_1 = 7$ mm, $W_2 = 18$ mm, $H_2 = 8$ mm, $W_3 = 18$ mm, $H_3 = 8$ mm, $L_1 = 32$ mm, $L_2 = 20$ mm, $L_3 = 13$ mm. With these initial values, the performance of the dual-band OMT can be optimized efficiently in the circuit domain. The optimization function *fmincon* in Matlab [112] is used to minimize the return loss at Port 1 over frequency bands 9.4-9.6 GHz and 10.4-10.6 GHz for both polarizations, as shown in Figs. 6.5 and 6.6. It is seen that the return loss for the vertical polarization improves significantly at the two frequency bands, while the one for the horizontal polarization remains good.

Once the circuit optimization is completed, the full-wave MM method is employed to refine the whole structure. The final dimensions of the OMT are given as follows. $W_{11} = 10.85$ mm, $W_{12} = 7.75$ mm, $W_{13} = 11.48$ mm, $L_{11} = 20.83$ mm, $L_{12} = 18.80$ mm, $L_{13} = 18.14$ mm, $W_{31} = 8.81$ mm, $W_{32} = 7.26$ mm, $W_{33} = 10.87$ mm, $L_{31} = 10.9$ mm, $L_{32} = 16.15$ mm, $L_{33} = 15.1$ mm, $W_1 = 18.85$ mm, $H_1 = 10.16$ mm, $W_2 = 17.8$ mm, $H_2 = 8$ mm, $W_3 = 16.72$ mm, $H_3 = 10.16$ mm, $L_1 = 32$ mm, $L_2 = 20$ mm, $L_3 = 13$ mm.

It should be mentioned that due to the circuit-model optimization, one is able to quickly obtain very good initial values that can be used in the full-wave MM optimization. Using these good initial values, the physical dimensions can be fine

6.2 Dual-Band OMT with Integrated Filters

Figure 6.5: Scattering parameters for the vertical polarization calculated by equivalent circuit model.

Figure 6.6: Scattering parameters for the horizontal polarization calculated by equivalent circuit model.

6.2 Dual-Band OMT with Integrated Filters

tuned using the full-wave MM simulator after several iterations using the full-wave MM simulator after several iterations. Without the synthesized values and circuit-level optimization, one would need much more CPU time to obtain the optimized value if it is not impossible.

The full-wave simulated results of the final structure are shown in Figs. 6.7-6.9. It is seen that the return loss is better than 15 dB for both frequency bands and polarizations. Moreover, for the S_{12}^V , three transmission zeros can be observed at 9, 10.1 and 11.2 GHz, its rejection level is better than 30 dB over 10.4-10.6 GHz. The rejection level for the S_{14}^V is better than 40 dB over 9.4-9.6 GHz.

Figure 6.7: Full-wave refined results of the dual-band OMT for the vertical polarization.

Also, the cross-polarization at the square waveguide port and the isolation between the rectangular branches can be simulated with the mode-matching method,

6.2 Dual-Band OMT with Integrated Filters

Figure 6.8: Full-wave refined results of the dual-band OMT for the horizontal polarization.

Figure 6.9: Simulated cross-polarization and isolations.

6.2 Dual-Band OMT with Integrated Filters

as shown in Fig. 6.9. The cross-polarization is better than 35 dB and the isolation between Ports 2 and 4 is better than 40 dB at the lower band and better than 30 dB at the upper band. The isolation between the vertical-polarized Ports 2 and 4 and horizontal-polarized Port 3 is better than 35 dB.

Figure 6.10: Structure of square-to-rectangular waveguide transition.

For the purpose of measurement, a square-to-rectangular waveguide transition is designed, as shown in Fig. 6.10. This transition can be efficiently analyzed by the MM program developed in Chapter 3. The optimization function *fmincon* in Matlab [112] is employed to optimize the dimensions of the transitions. The optimized scattering parameters are plotted in Fig. 6.11, where the return loss is about 30 dB over the band of interest.

The mechanical model of the designed dual-band OMT connected with the square-to-rectangular waveguide transition is shown in Fig. 6.12. The designed OMT is divided into two parts and fabricated separately, as shown in Fig. 6.13. Then the two parts are combined and assembled with the square-to-rectangular waveguide transition, as shown in Fig. 6.14.

6.2 Dual-Band OMT with Integrated Filters

Figure 6.11: Scattering parameters of the designed square-to-rectangular waveguide transition.

Figure 6.12: Mechanical model of the designed OMT with transistion.

6.2 Dual-Band OMT with Integrated Filters

Figure 6.13: Two fabricated parts of the designed OMT.

Figure 6.14: Assembled OMT with square-to-rectangular transition.

The fabricated OMT connecting with the square-to-rectangular waveguide transition is measured and compared with simulated results from MM method, as shown in Figs. 6.15-6.17. It should be noted that the transition is considered in the simulation to make a fair comparison. Measured insertion loss is about 0.2 dB at 10.5

6.2 Dual-Band OMT with Integrated Filters

GHz for the horizontal polarization, and about 0.5 dB at both 9.5 and 10.5 GHz for the vertical one. Measured cross-polarization and isolation are better than 30 dB over the two operating bands. Good agreement is observed between simulated and measured results, which verifies our equivalent circuit model and design procedure.

Figure 6.15: Comparison between measured and simulated results of the dual-band OMT for the vertical polarization.

6.2 Dual-Band OMT with Integrated Filters

Figure 6.16: Comparison between measured and simulated results of the dual-band OMT for the horizontal polarization.

Figure 6.17: Measured cross-polarization and isolations.

6.3 Dual-Band OMT with an Irregularly Shaped Diaphragm

In the previous section, a dual-band OMT has been designed together with two channel filters, which realizes good performance with a compact size. In this section, a more compact dual-band OMT is presented, which employs compact filtering elements within the OMT.

6.3.1 Description of the Proposed Structure

The structure of the proposed compact dual-band OMT is shown in Fig. 6.18. Two rectangular branch waveguides are still connected to the common waveguide to extract the received dual-polarized signals, respectively. Different from the previous short-circuited OMTs, the short-circuited plane is removed and a rectangular waveguide port is connected instead to transmit the single-polarized signal at a higher frequency band. Some step transitions are employed to connected this rectangular waveguide with the common waveguide. The size of the square waveguide port is $16 \times 16 \text{ mm}^2$, which is usually chosen according to the size of the horn antenna to which it is connected. The three rectangular ports are the standard waveguide WR75 with the cross-section dimensions of $19.1 \times 9.5 \text{ mm}^2$.

Port 3 is intrinsically isolated with Ports 2 and 4 due to the orthogonality of polarization. The main problem is to obtain good isolation between Ports 2 and 4.

A waveguide section with reduced cross-section is usually used after the extraction

6.3 Dual-Band OMT with an Irregularly Shaped Diaphragm

of signals at the lower frequency band, which isolates them with the port operating at the higher frequency band [67]. It works well when the two frequency bands are separated by a wide guard band. However, when the two frequency bands are close to each other, it may require a lengthy waveguide section with reduced cross-section to suppress the vertically polarized Rx signals. In this design, the common waveguide is narrowed in the horizontal direction immediately after the vertically polarized Rx signals are extracted into Port 4, as shown in Fig. 6.18. In this way, the propagation of the vertically polarized Rx signals can be stopped before it reaches Port 2, while the horizontally polarized Rx signals can still propagate and reach Port 3. This way results in a more compact structure because the waveguide with reduced cross-section is incorporated as a part of the common waveguide.

Figure 6.18: Structure of the proposed dual band Tx/Rx OMT with an irregularly shaped diaphragm inserted.

However, the Tx signals can still leak into Port 4. In [113], a sharp step bended

6.3 Dual-Band OMT with an Irregularly Shaped Diaphragm

bar is employed in a waveguide as a band-rejection element. Here an irregularly shaped diaphragm is inserted into Port 4 as a compact band-stop element to reject the Tx signals. The geometry and band-stop response of the diaphragm are plotted in Fig. 6.19. It is seen that a transmission zero is located at about 14.25 GHz. The band-stop phenomenon can be explained as follows. It is well-known that the rectangular diaphragm and slot in the waveguide can be inductive or capacitive [91] and diaphragms or posts of partial height in the waveguide can be resonant structures [114]. The irregularly shaped diaphragm can be considered as a combination of the regular inductive and capacitive ones. With a particularly designed irregular diaphragm, a series of inductor and capacitor can be obtained, which forms a transmission zero at the desired frequency.

6.3.2 Simulated and Measured Results

The designed OMT is fabricated, and one piece of the irregularly shaped diaphragm is mounted on the top of the vertically polarized Rx port, as shown in Fig. 6.20. The thickness of the diaphragm is 0.5 mm, and the total length of the OMT is 89.8 mm. Simulated results are plotted in Fig. 6.21. For the Tx Port 2, the return loss is better than 15 dB from 13.8 GHz to 15 GHz. For the horizontal-polarized Rx Port 3, the return loss is better than 15 dB from 10.8 GHz to 12.8 GHz. For the vertical-polarized Rx Port 4, the return loss is better than 15 dB from 10.2 GHz to 11.9 GHz.

For the ease of measurement, two identical OMTs are back-to-back connected at

6.3 Dual-Band OMT with an Irregularly Shaped Diaphragm

Figure 6.19: The band-stop response of the proposed irregularly shaped diaphragm.

Figure 6.20: Photo of the fabricated OMT with an irregularly shaped diaphragm.

6.3 Dual-Band OMT with an Irregularly Shaped Diaphragm

Figure 6.21: Simulated results of the designed compact dual-band OMT.

their common square waveguide ports, as shown in Fig. 6.22. Measured results of our fabricated OMT exhibit good performance and agree very well with simulated ones, as shown in Figs. 6.23-6.26. For the Tx band, the back-to-back insertion loss is less than 0.2 dB. For the Rx signals, the back-to-back insertion loss between the horizontally polarized ports is less than 0.55 dB from 10.8 GHz to 12.8 GHz, the back-to-back insertion loss between the vertically polarized ports is less than 0.62 dB from 10.2 GHz to 11.8 GHz.

The isolation between Ports 2 and 4 is measured, as shown in Fig. 6.26. It is improved from 15 dB to 30 dB with the presence of our irregularly shaped diaphragm over the Tx band from 14 GHz to 14.5 GHz. It proves that our proposed irregularly shaped diaphragm indeed acts as a band-stop element.

It should be mentioned that the whole OMT can be fabricated as a single block

6.3 Dual-Band OMT with an Irregularly Shaped Diaphragm

Figure 6.22: Back-to-back configuration of the designed OMT.

Figure 6.23: Back-to-back transmission and reflection between the Tx ports.

and the thin diaphragm is stacked on the top of the port. Therefore, compared to the dual-band OMT designed in previous section, which has to be split into two pieces,

6.3 Dual-Band OMT with an Irregularly Shaped Diaphragm

Figure 6.24: Back-to-back transmission and reflection between the horizontally polarized Rx ports.

Figure 6.25: Back-to-back transmission and reflection between the vertically polarized Rx ports.

6.4 Conclusion

the stability and power handling capability are greatly improved and the fabrication cost is reduced.

Figure 6.26: Isolation between the Tx and Rx ports of the same polarization with and without diaphragm.

6.4 Conclusion

In this chapter, two dual-band OMTs have been presented to receive two orthogonally polarized signals and transmits a single-polarized signal simultaneously .

The first dual-band has been developed based on the equivalent circuit model presented in Chapter 5. Two iris channel filters have been connected to the OMT to obtain good isolation between the two frequency bands. The synthesis method proposed in Chapter 5 has been applied to obtain the initial dimensions of the OMT. The two iris filters have been synthesized by the classic filter theory. The OMT and

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integrated filters have been optimized as a whole in the circuit domain. Based on the circuit optimized results, the whole structure has been tuned by the full-wave mode-matching analysis to finalize the design. Since the filters have been optimized as a part of the OMT, additional transmission zeros can be obtained. In this way, good isolation can be achieved by low order filters, which results a compact structure.

For the second dual-band OMT, a more compact structure has been presented. A waveguide with reduced cross-section has been incorporated into the common waveguide, which reduces the length of the OMT. An irregularly shaped diaphragm has been proposed as a compact band-rejection element, which forms a transmission zero at the Tx band. Therefore, good isolation between the Tx and Rx port of the same polarization has been obtained with a compact size.

The two dual-band OMTs have been fabricated and measured results have verified our design.

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

The work presented in this thesis focuses on the analysis and design of square waveguide OMTs.

A comprehensive literature review on the design of OMTs and relevant numerical methods has been given in Chapter 2. Based on this review, the MM method is selected from different numerical methods as the analyzing method in this thesis. The double-plane step in rectangular waveguide and six-port rectangular waveguide junction have been developed as two building blocks of waveguide OMT in Chapter 3. For the efficient analysis, the cascade of these two building blocks have also been investigated and proper generalized matrix representations for different types of cascade have been summarized. With this work, the square waveguide OMT can be analyzed by this efficient MM method later.

As a key building block in the design of OMT, a square waveguide T-junction has been studied by the previously developed MM method in Chapter 4. This square waveguide T-junction has first been modeled as a five-port network. After a rigorous

7.1 Conclusions

analysis of coupling modes between two orthogonal dominant modes, it has been found that there is no cross-coupling between them. Therefore, the complicated five-port network can be divided into two separate subnetworks: one three-port network and one two-port network. The equivalent circuit models of these two subnetworks have also been determined. Further study of the coupling modes has suggested that a set of closed-form expressions for these two equivalent circuit models can be derived from the MM analysis by proper approximation. These closed-form expressions exhibit very good accuracy over the dominant-mode frequency band. Then a short-circuited square waveguide OMT has been designed by these closed-form expressions in the circuit domain. The designed OMT has been fabricated and measured. Very good agreement has been observed between our circuit-simulated results and measured ones.

After the circuit analysis of short-circuited square waveguide OMT in Chapter 4, the circuit synthesis method has been studied in Chapter 5. First, a slot-coupled square waveguide T-junction has been considered as a key building block of the OMT. An improved equivalent circuit model has been proposed by employing impedance inverter and transmission lines. The values of these circuit elements can be extracted from simulated results by the MM method. Then, the equivalent circuit model of the slot-coupled short-circuit square waveguide OMT has been established with this key building block. The cross-coupling between two orthogonal polarizations are still ignored, therefore, the whole OMT can be considered as two separate two-port subnetworks. The ABCD matrix can be used to analyze these two subnetworks

7.2 Conclusions

separately. Based on the analysis of the two ABCD matrices, the reflection zero of these two subnetworks can be calculated by some simple formulas. Finally, an efficient synthesis procedure has been proposed based on these formulas, and the synthesized results can be directly used in the fabrication without any additional tuning.

Based on the equivalent circuit model proposed in Chapter 5, a dual-band OMT integrated with two iris filters has been designed in Chapter 6. One more T-junction is connected to transmit single-polarized signals at another frequency band. Two filters are employed to achieve good isolation between these two frequency bands. With this equivalent circuit model, the OMT and the two iris filters can be considered as a whole network, and the junction effect of the OMT can be utilized to introduce additional transmission zeros for the filters, which results in a compact design. The designed OMT is divided into two parts for the ease of fabrication and a square-to-rectangular waveguide transition is designed for the purpose of measurement. Measured results exhibit good agreement with our simulated ones. Moreover, a more compact dual-band OMT has also been designed in this chapter, using an irregularly shaped diaphragm as a compact band-rejection element. This design can be fabricated as a monoblock with a stacked thin diaphragm mounted on its interface. Therefore, it is more stable compared to the previous designed OMTs and very suitable for mass production.

7.2 Recommendations for Future Work

Although many efforts have been made on the analysis and design of square waveguide OMTs, there is further work on this topic that deserves investigation. Some of them are recommended as follows.

- 1) Efficient mode-matching programs have been developed to analyze rectangular waveguide OMTs with regular cross-section. However, in practice, high performance OMTs usually require very complicated structures, such as Bøifot junctions and turnstile junctions. Discontinuities within the junction regions are commonly employed as tuning elements to obtain better impedance matching condition in a wider band. Normal mode-matching method cannot analyze these structures of irregular cross-section. To overcome this limitation, these irregular structures can be divided into several regular regions, which can be analyzed by the mode-matching procedure. However, for complicated structures, this procedure becomes very tedious and the efficiency of the mode-matching method suffers thereafter. A promising solution to analyzing these complicated structures is to use hybrid methods, which divide the structures into regular and irregular parts. The regular part is still analyzed by the efficient mode-matching method, while the irregular part is analyzed by other numerical methods, such as the finite element method (FEM), the method of moment (MoM), etc. The two parts are then combined together in a proper way. More studies on these hybrid methods should be conducted to design more complicated OMTs with better performance.
- 2) The equivalent circuit model of a square waveguide T-junction has been devel-

7.2 Recommendations for Future Work

oped, and accurate closed-form expressions have been derived by rigorous mode-matching analysis to determine the values of the circuit elements. In Chapter 4, it has been seen that the method used to derive these closed-form expressions are more accurate over a wider band compared with those derived from the traditional quasi-static methods. It is possible to use this method to derive the closed-form expressions of other waveguide structures, such as H-plane waveguide T-junctions, waveguide corners, waveguide turnstile junctions, etc. More studies on this method can be carried out.

- 3) The synthesis method for a waveguide OMT has been proposed in this thesis. However, the cross coupling between orthogonal polarizations are ignored during the synthesis, and this synthesis method is also limited to the simple short-circuited waveguide type. A more general synthesis method should be studied.
- 4) An irregularly shaped thin diaphragm has been proposed as a compact filtering element, which is used to implement a compact dual-band OMT. However, the electrical property of this diaphragm is not fully investigated. Further investigation can be performed on this topic to obtain an even more compact filtering element.

Author's Publications

Journal Papers

1. Y. Tao, Z. Shen, and G. Liu, "Analysis of quadruple corner-cut ridged circular waveguide by hybrid mode-matching boundary-element method," *IEEE Transactions on Magnetics*, vol. 45, no. 3, pp. 1076–1079, Mar. 2009.
2. Y. Tao and Z. Shen, "Broadband substrate integrated waveguide orthomode transducers," *Journal of Electromagnetic Waves and Applications*, vol. 23, no. 16, pp. 2099–2108, 2009.
3. Y. Tao, Z. Shen, and G. Liu, "Closed-form expressions for the equivalent circuit model of square-waveguide T-junctions and its application in orthomode transducer design," *IEEE Trans. Microwave Theory Tech.*, vol. 58, no. 5, pp. 1167–1174, May 2010.
4. Y. Tao, Z. Shen, and B. Zheng, "Analysis of quadruple corner-cut ridged square waveguide by hybrid mode-matching boundary-element method," *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*,

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5. Y. Tao and Z. Shen, “Closed-form expressions for the equivalent circuit model of H-plane waveguide T-junctions,” *IET Microw. Antennas Propag.*, vol. 4, no. 12, pp. 2008 – 2013, Dec. 2010.

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1. Y. Tao, Z. Shen, and G. Liu, “Hybrid mode-matching boundary-element method for quadruple corner-cut ridged circular waveguides,” in *13th Biennial IEEE Conference on Electromagnetic Field Computation*, Athens, May 2008.
2. Y. Tao and Z. Shen, “Design of compact ortho-mode transducers,” in *IEEE MTT-S International Microwave Workshop Series on Art of Miniaturizing RF and Microwave Passive Components*, Dec. 2008, pp. 38–42.
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6. Y. Tao and Z. Shen, “Coupling matrix synthesis of orthonode transducers,” in *IEEE AP-S International Symposium*, Toronto, Jul. 2010, pp. 1–4
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APPENDIX A

Elements of Matrices in Eq. (3.26)

The detailed expressions of Eq. (3.26) are given in this appendix. The submatrices in the first row of matrix $\mathbf{M}_H$ in (3.26) can be expressed as follows.

$$\mathbf{M}_1^{z-} = [\cosh \gamma_i^z l] \quad (\text{A.1})$$

$$\mathbf{M}_1^{z+} = -\mathbf{U}_{N_1} \quad (\text{A.2})$$

$$\mathbf{M}_1^{x-} = \begin{bmatrix} \mathbf{M}_1^{xhh} & \mathbf{M}_1^{xhe} \\ \mathbf{M}_1^{xeh} & \mathbf{M}_1^{xee} \end{bmatrix} \quad (\text{A.3})$$

$$\mathbf{M}_1^{y-} = \begin{bmatrix} \mathbf{M}_1^{yhh} & \mathbf{M}_1^{yhe} \\ \mathbf{M}_1^{yeh} & \mathbf{M}_1^{yee} \end{bmatrix} \quad (\text{A.4})$$

$$\mathbf{M}_1^{x+}(i, j) = (-1)^{m_z+1} \mathbf{M}_1^{x-}(i, j) \quad (\text{A.5})$$

$$\mathbf{M}_1^{y+}(i, k) = (-1)^{n_z+1} \mathbf{M}_1^{y-}(i, k) \quad (\text{A.6})$$

$$\mathbf{M}_1^{xhh}(i, j) = \begin{cases} c_{ij}^{zx} \left(\frac{m_z \pi}{a} \right)^2 \frac{k_{cx}^2}{\gamma_j^x} + \left(\frac{n_z \pi}{b} \right)^2 \gamma_j^x, & m_x = n_z \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.7})$$

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$$\mathbf{M}_1^{xhc}(i, j) = \begin{cases} -c_{ij}^{zx} \frac{n_z \pi}{b} \frac{n_x \pi}{l} \gamma_j^x, & m_x = n_z \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.8})$$

$$\mathbf{M}_1^{xch}(i, j) = \begin{cases} -c_{ij}^{zx} \frac{m_z \pi}{a} \frac{n_z \pi}{b} \left(\frac{k_{cx}^2}{\gamma_j^x} - \gamma_j^x \right), & m_x = n_z \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.9})$$

$$\mathbf{M}_1^{xce}(i, j) = \begin{cases} -c_{ij}^{zx} \frac{m_z \pi}{a} \frac{n_x \pi}{l} \gamma_j^x, & m_x = n_z \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.10})$$

$$\mathbf{M}_1^{yhh}(i, k) = \begin{cases} c_{ik}^{zy} \left(\frac{n_z \pi}{b} \right)^2 \frac{k_{cy}^2}{\gamma_k^y} + \left(\frac{m_z \pi}{a} \right)^2 \gamma_k^y, & n_y = m_z \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.11})$$

$$\mathbf{M}_1^{yhc}(i, k) = \begin{cases} c_{ik}^{zy} \frac{m_z \pi}{a} \frac{m_y \pi}{l} \gamma_k^y, & n_y = m_z \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.12})$$

$$\mathbf{M}_1^{ych}(i, k) = \begin{cases} c_{ik}^{zy} \frac{m_z \pi}{a} \frac{n_z \pi}{b} \left(\frac{k_{cy}^2}{\gamma_k^y} - \gamma_k^y \right), & n_y = m_z \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.13})$$

$$\mathbf{M}_1^{ycc}(i, k) = \begin{cases} -c_{ik}^{zy} \frac{n_z \pi}{b} \frac{m_y \pi}{l} \gamma_k^y, & n_y = m_z \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.14})$$

where $c_{ij}^{zx} = \sqrt{\frac{Y_j^x}{Y_i^z}} N_i^z N_j^x \frac{b}{\varepsilon_{n_z}} \frac{\sinh \gamma_j^x a}{(\gamma_j^x)^2 + \left(\frac{m_z \pi}{a}\right)^2}$, $c_{ik}^{zy} = \sqrt{\frac{Y_k^y}{Y_i^z}} N_i^z N_k^y \frac{a}{\varepsilon_{m_z}} \frac{\sinh \gamma_k^y b}{(\gamma_k^y)^2 + \left(\frac{n_z \pi}{b}\right)^2}$,
 $i = 1, 2, \dots, N_1$, $j = 1, 2, \dots, N_2$, $k = 1, 2, \dots, N_3$.

i , j and k are the combined mode indices for possible TE_{mn} and TM_{mn} modes in the z , x and y directions, respectively. N_1 , N_2 and N_3 are the mode number

Appendix A

considered in the corresponding cross-sections, respectively, and N_i^z , N_j^x and N_k^y are the normalized factors defined in (3.2) accordingly. The subscripts z , x and y in the mode indices m and n denote the direction of the waveguide section.

For the submatrices in the the second row of matrix $\mathbf{M}_H$, they can be expressed as follows.

$$\begin{pmatrix} \mathbf{M}_2^{x-} \\ \mathbf{M}_2^{x+} \end{pmatrix} (i, j) = (-1)^{n_x+1} \begin{pmatrix} \mathbf{M}_1^{x-} \\ \mathbf{M}_1^{x+} \end{pmatrix} (i, j) \quad (\text{A.15})$$

$$\begin{pmatrix} \mathbf{M}_2^{y-} \\ \mathbf{M}_2^{y+} \end{pmatrix} (i, k) = (-1)^{m_y+1} \begin{pmatrix} \mathbf{M}_1^{y-} \\ \mathbf{M}_1^{y+} \end{pmatrix} (i, k) \quad (\text{A.16})$$

$$\mathbf{M}_2^{z-} = -\mathbf{U}_{N_1} \quad (\text{A.17})$$

$$\mathbf{M}_2^{z+} = \mathbf{M}_1^{z-} \quad (\text{A.18})$$

The remaining submatrices can be defined in a similar way due to the rotating symmetry.

APPENDIX B

Elements of Matrices $\mathbf{M}_1$ in Eq. (4.7), $\mathbf{M}_3$ in Eq. (4.10) and $\mathbf{M}$ in Eq. (4.12)

The matrices $\mathbf{M}_1$ in (4.7) and $\mathbf{M}_3$ in (4.10) can be written as

$$\mathbf{M}_i = \begin{bmatrix} \mathbf{M}_i^{hh} & \mathbf{M}_i^{he} \\ \mathbf{M}_i^{eh} & \mathbf{M}_i^{ee} \end{bmatrix}, \quad (\text{B.1})$$

where $i = 1$ and 3 , pq and mn are used to represent the mode indices in Regions A and B, respectively. The non-zero elements of both matrices can be obtained only when $q = m$.

$$M_1^{hh}(pq, mn) = c_0 c_1 \left[\left(\frac{p\pi}{a} \right)^2 \frac{(k_{cmn}^B)^2}{\gamma_{mn}^B} + \left(\frac{q\pi}{a} \right)^2 \gamma_{mn}^B \right] \quad (\text{B.2})$$

$$M_1^{he}(pq, mn) = -c_0 c_1 \frac{q\pi}{a} \frac{n\pi}{l} \gamma_{mn}^B \quad (\text{B.3})$$

$$M_1^{eh}(pq, mn) = -c_0 c_1 \frac{p\pi}{a} \frac{q\pi}{a} \left[\frac{(k_{cmn}^B)^2}{\gamma_{mn}^B} - \gamma_{mn}^B \right] \quad (\text{B.4})$$

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$$M_1^{ee}(pq, mn) = -c_0 c_1 \frac{p\pi}{a} \frac{n\pi}{l} \gamma_{mn}^B \quad (\text{B.5})$$

$$M_3^{hh}(mn, pq) = c_0 c_1^{-1} \left[\left(\frac{n\pi}{l} \right)^2 \frac{(k_{cpq}^A)^2}{\gamma_{pq}^A} + \left(\frac{m\pi}{a} \right)^2 \gamma_{pq}^A \right] \quad (\text{B.6})$$

$$M_3^{he}(mn, pq) = c_0 c_1^{-1} \frac{p\pi}{a} \frac{m\pi}{a} \gamma_{pq}^A \quad (\text{B.7})$$

$$M_3^{eh}(mn, pq) = c_0 c_1^{-1} \frac{m\pi}{a} \frac{n\pi}{l} \left[\frac{(k_{cpq}^A)^2}{\gamma_{pq}^A} - \gamma_{pq}^A \right] \quad (\text{B.8})$$

$$M_3^{ee}(mn, pq) = -c_0 c_1^{-1} \frac{p\pi}{a} \frac{n\pi}{l} \gamma_{pq}^A \quad (\text{B.9})$$

where $c_0 = \frac{\sqrt{\varepsilon_p \varepsilon_n}}{k_{cpq}^A k_{cmn}^B \sqrt{al}} \frac{1}{\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{l} \right)^2 + \left(\frac{p\pi}{a} \right)^2 - k_0^2}$, $c_1 = \sqrt{Y_{mn}^B / Y_{pq}^A}$.

The matrix $\mathbf{M}$ in (4.12) can be written as

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}^{hh} & 0 \\ \mathbf{M}^{eh} & \mathbf{M}^{ee} \end{bmatrix}, \quad (\text{B.10})$$

where the superscript h and e represent the TE and TM modes, respectively, mn and ki are the mode indices in Regions B and C. For each sub-matrix, the non-zero elements can be found only when $n = i$.

$$M^{hh}(mn, ki) = N_{mn,ki} \left[\left(\frac{k\pi}{b} \right)^2 + \left(\frac{i\pi}{l} \right)^2 \right] CC_{mk} \quad (\text{B.11})$$

$$M^{ee}(mn, ki) = N_{mn,ki} \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{l} \right)^2 \right] SS_{mk} \quad (\text{B.12})$$

$$M^{eh}(mn, ki) = N_{mn,ki} \frac{n\pi}{l} \left(\frac{m\pi}{a} CC_{mk} - \frac{k\pi}{b} SS_{mk} \right) \quad (\text{B.13})$$

where $N_{mn,ki} = \frac{\sqrt{\varepsilon_m \varepsilon_k}}{k_{cmn}^B k_{cki}^C \sqrt{ab}}$, $\varepsilon_m = 1$ for $m = 0$; $\varepsilon_m = 2$ for $m = 1$, and

$$CC_{mk} = \int_0^b \cos \left[\frac{m\pi}{a} \left(x + \frac{a-b}{2} \right) \right] \cos \frac{k\pi x}{b} dx \quad (\text{B.14})$$

$$SS_{mk} = \int_0^b \sin \left[\frac{m\pi}{a} \left(x + \frac{a-b}{2} \right) \right] \sin \frac{k\pi x}{b} dx \quad (\text{B.15})$$

APPENDIX C

Expressions for Variables in Eqs. (4.26) and (4.27)

The expressions for variables used in (4.26) and (4.27) are given as follows.

$$W^0 = 1 + Y_{33}^0 - H_0 + \frac{4(1 - e^{-\gamma_{01}^A l})}{\gamma_{01}^A \gamma_{10}^C b^3 l} C C_{11}^2 \quad (\text{C.1})$$

$$W_i^{hh} = 1 + Y_{33i}^{hh} - H H_i^h - H H_i^e \quad (\text{C.2})$$

$$W_i^{he} = Y_{33i}^{he} - H E_i^h - H E_i^e \quad (\text{C.3})$$

$$W_i^{eh} = Y_{33i}^{eh} - E H_i^h - E H_i^e \quad (\text{C.4})$$

$$W_i^{ee} = 1 + Y_{33i}^{ee} - E E_i^h - E E_i^e \quad (\text{C.5})$$

$$Y_{ri}^h = \frac{-k_{c1i}^C}{k_{c1i}^B} \sqrt{\frac{K^a}{\gamma_{01}^A \gamma_{1i}^C}} \left[\frac{\pi}{a} C C_{11} + \left(\frac{i\pi}{l} \frac{k_0}{\gamma_{1i}^B k_{c1i}^C} \right)^2 C_1 \right] \quad (\text{C.6})$$

$$Y_{ri}^e = -j \frac{k_0 k_{c1i}^B i\pi}{\gamma_{1i}^B k_{c1i}^C l} \sqrt{\frac{K_{1i}^a}{\gamma_{01}^A \gamma_{1i}^B}} S S_{11} \quad (\text{C.7})$$

$$D_{Wi} = W_i^{hh} W_i^{ee} - W_i^{he} W_i^{eh} \quad (\text{C.8})$$

Appendix C

where

$$Y_{33}^0 = \sum_{m=1}^{\infty} 4 \frac{\gamma_{m0}^B}{\gamma_{10}^C} \coth(\gamma_{m0}^B a) \frac{a}{m^2 b^3} C C_{m1}^2 \quad (C.9)$$

$$H_0 = \sum_{m=1}^{\infty} \sum_{p=0}^{\infty} \frac{4(1 - e^{-\gamma_{pm}^A l}) C C_{m1}^2}{\gamma_{pm}^A \gamma_{10}^C b^3 l (p^2 + m^2)} [\varepsilon_p - \frac{2k_0^2 p^2}{(\gamma_{pm}^A)^2 m^2}] \quad (C.10)$$

$$Y_{33i}^{hh} = \sum_{m=1}^{\infty} \frac{4 \coth(\gamma_{mi}^B a) k_{cli}^{C2}}{ab \gamma_{1i}^C k_{cmi}^{B2} \gamma_{mi}^B} [\gamma_{mi}^{B2} C C_{m1}^2 - (\frac{k_0 C_m i \pi}{k_{cli}^{C2} l})^2] \quad (C.11)$$

$$Y_{33i}^{ce} = \sum_{m=1}^{\infty} \frac{4 \coth(\gamma_{mi}^B a)}{ab} \frac{\gamma_{1i}^C}{\gamma_{mi}^B} (\frac{k_{cmi}^B}{k_{cli}^C})^2 S S_{m1}^2 \quad (C.12)$$

$$Y_{33i}^{eh} = Y_{33i}^{he} = \sum_{m=1}^{\infty} \frac{4}{ab} \frac{j k_0 \coth(\gamma_{mi}^B a)}{\gamma_{mi}^B (k_{cli}^C)^2} \frac{i \pi}{l} C_m S S_{m1} \quad (C.13)$$

$$H H_i^h = \sum_{m=1}^{\infty} \sum_{p=0}^{\infty} \frac{\gamma_{pm}^A K^a F_i R}{\gamma_{1i}^C k_{cpm}^{A2} k_{cmi}^{B2}} [(K^r C C_{m1} k_{cli}^C)^2 - (\frac{m i \pi^2 k_0^2 C_m}{a l \gamma_{pm}^A k_{cli}^C})^2] \quad (C.14)$$

$$H H_i^c = \sum_{m=1}^{\infty} \sum_{p=0}^{\infty} \frac{i \pi k_0^2 K^a F_i R P_{pm}}{k_{cli}^C l} S S_{m1} Q \quad (C.15)$$

$$H E_i^h = \sum_{m=1}^{\infty} \sum_{p=0}^{\infty} \frac{j k_0 p}{k_{cpm}^A} \frac{\gamma_{pm}^A}{\gamma_{1i}^C} \frac{k_{cli}^C}{k_{cmi}^{B2}} K^a F_i P R Q (C C_{m1} K^r - \frac{m i^2 \pi^3 k_0^2 C_m}{a l^2 \gamma_{pm}^A k_{cli}^{C2}}) \quad (C.16)$$

$$H E_i^c = - \sum_{m=1}^{\infty} \sum_{p=0}^{\infty} \frac{j k_0 \gamma_{pm}^A p^2 i \pi}{k_{cli}^C l} K^a F_i P^2 R Q S S_{m1} \quad (C.17)$$

$$E H_i^h = - \sum_{m=1}^{\infty} \sum_{p=0}^{\infty} \frac{j m i a k_0}{k_{cli}^C l} K^a F_i P^2 R S S_{m1} (C C_{m1} K^r k_{cli}^C - \frac{m i^2 \pi^3 C_m k_0^2}{a l^2 \gamma_{pm}^A k_{cli}^{C2}}) \quad (C.18)$$

$$E H_i^c = \sum_{m=1}^{\infty} \sum_{p=0}^{\infty} j k_0 \gamma_{1i}^C \frac{p m i^2 \pi^2 k_{cmi}^{B2}}{l^2 k_{cli}^{C2}} K^a F_i P^2 R S S_{m1}^2 \quad (C.19)$$

$$E E_i^h = \sum_{m=1}^{\infty} \sum_{p=0}^{\infty} \frac{p m i \pi k_0^2}{k_{cli}^C l} K^a F_i P^2 R Q S S_{m1} \quad (C.20)$$

$$E E_i^c = \sum_{m=1}^{\infty} \sum_{p=0}^{\infty} p^2 \gamma_{pm}^A \gamma_{1i}^C \frac{k_{cmi}^{B2} i^2 \pi^2}{k_{cli}^{C2} l^2} K^a P^2 F_i R S S_{m1}^2 \quad (C.21)$$

and $F_i = 1 + (-1)^{i+1} e^{-\gamma_{1i}^A l}$, $P = (p^2 + m^2)^{-1/2}$, $R = [(\gamma_{pm}^A)^2 + (\frac{i\pi}{l})^2]^{-2}$, $C_m = \frac{m\pi}{a} C C_{m1} - \frac{\pi}{b} S S_{m1}$, $K^a = \frac{8\varepsilon_p}{a^2 b l (k_{cmi}^B)^2}$, $K^r = (\frac{i\pi}{l})^2 \frac{(k_{cpm}^A)^2}{\gamma_{pm}^A} + (\frac{m\pi}{a})^2 \gamma_{pm}^A$, $Q = \frac{m\pi}{a} C C_{m1} k_{cli}^C - \frac{i^2 \pi^2 C_m}{l^2 k_{cli}^C}$.