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STRENGTH AND STIFFNESS CHARACTERISTICS OF
UNSATURATED SOILS

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Strength and Stiffness Characteristics of Unsaturated Soils

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School of Civil and Environmental Engineering

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Abstract

Strength and stiffness of soils are the most important parameters for geotechnical engineering design. It is widely accepted that stiffness-strain behaviour of soil is highly non-linear and it is therefore important to understand stiffness degradation with strain amplitudes. For unsaturated soils, the relationships between stiffness and strain, and shear strength and strain rate are complicated by the presence of negative pore-water pressure. In Singapore two thirds of the land area is covered by residual soils where the groundwater table is found at some depth. Therefore the residual soils above the groundwater table are unsaturated. Not much research work has been done on stiffness-strain relationship and shear strength with strain rate of unsaturated soils. Laboratory measurement of stiffness at small strains requires local strain measurement device. In this study, local displacement transducers (LDTs) were fabricated in-house following the design of Goto et al. (1991) using a conventional full Wheatstone bridge circuit. An alternative circuit to the Wheatstone bridge circuit, the Anderson loop circuit, was introduced to improve the performance of LDT. LDTs were fabricated in identical fashion but with different circuits. Performances of the LDTs using the full Wheatstone bridge and the Anderson loop circuits were evaluated and discussed in detail. It was observed that the LDT with the Anderson loop circuit was twice as sensitive as the LDT with the Wheatstone circuit. In addition, the LDT with the Anderson loop circuit outperformed the LDT with the Wheatstone bridge circuit in terms of lower deviation, hysteresis and repeatability errors. Triaxial compression tests, bender element tests and resonant column test were performed using different soils (i.e. kaolin, sands and residual soils) to measure stiffness-strain relationship and small-strain stiffness ($G_{\max}$) of soils. Stiffness-strain relationships of consolidated kaolin were measured by performing constant water content test (CW) at different initial matric suctions and at different strain rates. Shear stiffness increased significantly with increasing initial matric suctions. The effect of strain

rate on stiffness-strain relationship was observed to be less significant compared to the effect of initial matric suction. With an aim to obtain a general bound of stiffness curve for sand, stiffness-strain relationships of dry sands at different initial void ratios and effective mean confining pressures were investigated by performing resonant column tests. A total of four sands (Ottawa 20-30, Ottawa Graded, Reclaimed sand A and Reclaimed sand B) were tested using resonant column apparatus. Shear stiffness increased with increasing effective mean confining pressure and decreasing void ratio. However, the effect of confining pressure on shear stiffness was observed to be more significant than the effect of initial void ratio. Stiffness-strain relationships and $G_{\max}$ of reconstituted unsaturated sand (Reclaimed sand B) were measured at different initial matric suctions under different effective mean or net confining pressure by performing multi-stage CW tests. Shear stiffness increased with increasing effective mean or net confining pressure. Non-linear variation of $G_{\max}$ with matric suction for unsaturated sand was observed from the experiments. Values of $G_{\max}$ measured from LDTs and bender element tests were also compared. It was observed that $G_{\max}$ measured from LDTs agreed well with $G_{\max}$ measured from bender element tests. Stiffness degradation curves for sands were also obtained by normalising stiffness-strain relationship with $G_{\max}$ at shear strain less than 0.0001%. These stiffness degradation curves were compared with the data from Seed and Idriss (1970). It was observed that the experimental data for sands plotted above the upper bound curve of Seed and Idriss (1970). Therefore, a modified upper bound curve was introduced to encompass all experimental data. This modified upper bound curve was also evaluated using data sets from the literature. Stiffness-strain relationships of undisturbed residual soils were measured at different initial matric suction and strain rates by performing constant water content (CW) and consolidated undrained (CU) tests. It was observed that the stiffness-strain relationships obtained from CW and CU tests were very close. Shear strength of unsaturated sand at different initial matric suctions under different effective

mean or net confining pressures were investigated by performing CW tests using multistage shearing tests. In addition, shear strength of unsaturated residual soils at different initial matric suctions and different strain rates were studied by performing CW and CU tests. It was observed from experiments that shear strength increased non-linearly with matric suction. The effect of strain rate on stress-strain relationship and variation of matric suction with strain was observed to be more significant for specimen with higher initial matric suctions. For unsaturated sand, values of ϕ_{app}^b (i.e. $\phi_{app}^b = \phi^b + \psi^b$ where ϕ^b and ψ^b are angles affecting change in shear strength with matric suction and ψ^b being dilation angle) in the near saturation zone were obtained to be greater than ϕ' . For unsaturated residual soils, values of ϕ_{app}^b in the near saturation zone were obtained to be equal to ϕ' . A sigmoid relationship of ϕ_{app}^b with matric suction was proposed in this research. Experimental shear strength data were fitted using an equation having similar form as the shear strength equation (Fredlund et al. 1978) with a sigmoid relationship of ϕ_{app}^b with matric suction. A total of 17 data sets from the experiments performed in this study and the literature was used for curve fitting. It was observed that the proposed equation fitted the experimental data well. From curve fitting, the upper and lower bound shear strength curves were proposed in terms of plasticity index, percent fines and effective friction angle to estimate unsaturated shear strength without requiring the SWCCs of the soils.

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List of Symbols

$(\phi^b_{app})_0$	apparent ϕ^b angle at $u_a - u_w = 0$ kPa
χ	a parameter related to the degree of saturation of the soil
AEV	air-entry value
ϕ^b	angle indicating the rate of change of shear strength with matric suction
ϕ^b_{app}	apparent ϕ^b angle equal to summation of ϕ^b and Ψ^b in which Ψ^b is dilation angle affecting change in shear strength with matric suction
$\overline{U}_f$	average degree of dissipation of the excess pore-water pressure at failure
ϵ_a	axial strain
β'	bonding factor which is a measure of the number of bonds under tension influencing the shear strength of the soil
ΔR	change in resistance
ω	circular frequency of the first torsional vibration mode
c^w_v	coefficient of consolidation with respect to the water phase
C_c	coefficient of curvature
k_d	coefficient of permeability of high air-entry ceramic disk
k_w	coefficient of permeability of unsaturated soil in terms of water phase
C_u	coefficient of uniformity
CU	consolidated undrained
CW	constant water content
I_o	current supply
S	degree of saturation
ρ	density of the soil specimen
$\sigma_1 - \sigma_3$	deviator stress
c'	effective cohesion
ϕ'	effective friction angle
σ'_m	effective mean confining pressure
σ'	effective stress
V_{in}	excitation voltage
λ	impedance factor
G_o	initial shear stiffness

J	mass polar moment of inertia of the specimen
J_o	mass polar moment of inertia the components mounted on top of the specimen
$(u_a - u_w)$	matric suction
$(u_a - u_w)_f$	matric suction at failure
e_{max}	maximum void ratio
e_{min}	minimum void ratio
$(\sigma_1 - u_a)_f$	net major normal stresses at failure
$(\sigma_3 - u_a)_f$	net minor normal stresses at failure
$(\sigma - u_a)$	net normal stress
Θ	normalised volumetric water content
OCR	over-consolidation ratio
$(\sigma_1 - \sigma_3)_f$	peak deviator stress
PI	plasticity index
u_a	pore-air pressure
u_w	pore-water pressure
σ_{1f}'	principal major stress at failure
σ_{3f}'	principal minor stress at failure
γ_r	reference shear strain
D_r	relative density
R	resistance
θ_s	saturated volumetric water content
ν_{sec}	secant Poisson's ratio
G_{sec}	secant shear stiffness
E_{sec}	secant Young's modulus
γ	shear strain amplitude
τ	shear strength
V_s	shear wave velocity
G_{max}	small-strain stiffness
ψ	soil suction
ψ_r	soil suction related to residual volumetric water content
SWCC	soil-water characteristic curve
G_s	specific gravity
ϵ_f	strain at failure

σ	total normal stress
E_u	undrained Young's modulus
e	void ratio
θ_w	volumetric water content
L_{tt}/λ	wave path length to wavelength ratio

Chapter 1 Introduction

1.1 Background

Soils are subjected to different climatic conditions and these different climatic conditions determine whether soil is saturated or unsaturated. Majority of soils especially near the ground surface exist in unsaturated conditions due to evaporation or evapotranspiration. Conventionally, most geotechnical engineering problems such as slope stability, excavation, and seepage through dams were analyzed using saturated soil mechanics. However, unsaturated soils possess negative pore-water pressures and the presence of negative pore-water pressures makes it difficult to apply saturated soil concept in predicting the mechanical behaviours of unsaturated soil. Historically, researchers attempted to express the mechanical behaviour of unsaturated soils using a single stress state variable similar to that of the saturated effective stress theory. Currently, it is commonly accepted that the mechanical behaviour of unsaturated soils is studied using two stress state variables, $(\sigma - u_a)$ and $(u_a - u_w)$, independently. Strength and stiffness of unsaturated soils were studied in terms of these two stress state variables in this study.

Shear strength of an unsaturated soil has been studied by many researchers in the past. Fredlund et al. (1978) proposed the shear strength equation for unsaturated soils in terms of the stress state variables using the extended Mohr-Coulomb failure envelope. In this equation a linear variation of shear strength with matric suction was assumed. However non-linear variations of shear strength with matric suction have been observed by many

researchers (Escario and Saez 1986; Gan et al. 1988; Thu et al. 2006; Hossain and Yin 2010; Likos et al. 2010). The slope of the variation of shear strength with matric suction is equivalent to $\tan \phi^b$ (Fredlund et al. 1978). Values of ϕ^b have been reported to be equal to or less than ϕ' for some soil types [e.g. compacted glacial till in Gan et al. (1988) and compacted silt in Thu et al. (2006)] and greater than ϕ' for other soil types [e.g. reconstituted Madrid clayey sand in Escario and Saez (1986), compacted sandy clay in Melinda et al. (2004), compacted silty sand (completely decomposed granite) in Hossain and Yin (2010) and reconstituted sandy colluviums in Likos et al. (2010)]. The reason for ϕ^b greater than ϕ' has been attributed to soil dilatancy. Most unsaturated soil tests are time-consuming and therefore many researchers have proposed a model to predict the shear strength of unsaturated soils. In the proposed models, soil-water characteristic curves together with effective shear strength parameters were used to predict the non-linear behaviour of unsaturated soils (Fredlund et al. 1996; Vanapalli et al. 1996; Oberg and Sallfors 1997; Bao et al. 1998; Khallili and Khabbaz 1998; and Goh et al. 2010). In this study, a sigmoid function was proposed for the relationship between $\tan \phi^b$ and matric suction for predicting shear strength.

Serviceability requirement for geotechnical structures such as foundation, retaining wall and tunnel is important. A typical range of ground movement due to working loads falls within 0.001% to 0.5% of shear strain (Jardine 1995). Stiffness-strain behaviour of soil is highly non-linear and soil stiffness can decrease with strain by several orders of magnitude (Atkinson 2000). The non-linear stiffness-strain relationship consists of the initial linear elastic region and the non-linear region ending at axial strain of about 0.1%. Measurement

of stiffness-strain relationship of soil over a wide strain range requires several test methods as indicated in Figure 1.1. Stiffness at very small strain ($G_{\max}$) plays an important role in characterizing the relationship of stiffness with strains. For saturated soils, small-strain stiffness ($G_{\max}$) has been studied by several researchers using wave velocity methods (i.e. resonant column and bender element tests). For unsaturated soils, limited measurement of $G_{\max}$ has been made and expressed using the stress states variables $[(\sigma - u_a), (u_a - u_w)]$. However, the experimental data on the effects of $(\sigma - u_a)$ and $(u_a - u_w)$ on $G_{\max}$ for unsaturated soils are inconclusive and therefore, requires further detailed investigation.

Figure 1.1 Typical stiffness-strain behaviour of soil with typical strain ranges for geotechnical structures and measurement methods for different strain regions (modified from Leong et al. 2006)

Many factors influence the stress-strain behaviour of soils. The factors include soil fabric or grain size distribution of sand, shear strain amplitude (γ), effective mean confining pressure (σ_m'), void ratio (e) or relative density (D_r), degree of saturation (S) or moisture content (w), stress history or over-consolidation ratio (OCR), strain rate or loading frequency, number of loading cycles and others. Non-linear stiffness-strain relationship in the small strain region for saturated soils has been obtained for various soil types. However, stiffness-strain relationships for unsaturated soils in terms of the stress state variables are very limited. Lack of experimental data on stiffness-strain behaviour of unsaturated soils is probably due to difficulties in conducting unsaturated soil test. In addition, it is usually time consuming and costly to conduct unsaturated soil tests in the laboratory as well as in the field.

1.2 Objectives and Scope

The main objective of this study is to investigate the strength and stiffness characteristics of unsaturated soils. The scope of this research consists of two main parts: (1) laboratory work, and (2) characterising stiffness characteristics and modelling of shear strength of soils. Strength and stiffness characteristics of unsaturated soils were studied in terms of the stress state variables by performing constant water content (CW) and consolidated undrained (CU) tests. Triaxial compression tests with local strain measurement were performed to measure stiffness of soil at different strain levels. The Anderson loop circuit was used in place of the conventional Wheatstone bridge to improve accuracy of the local strain measurement system. Bender element tests were carried out to obtain small-strain

stiffness of soils using a modified triaxial apparatus. Resonant column tests were also performed to obtain stiffness-strain relationship of dry sands. Shear strength of unsaturated soils were measured under different net confining pressure and matric suction for different soil types. Effect of strain rate on shear strength of unsaturated soil was investigated using CU and CW tests. Based on experimental results and unsaturated soil theory, a model for estimating shear strength was proposed. The proposed shear strength model was evaluated using data published in the literature.

1.3 Thesis Organisation

This thesis is organized as follows:

- Chapter 1 presents the background and the motivations of this study, the objectives and the scope of the research.
- Chapter 2 reviews the strength and stiffness characteristics of unsaturated soils with a brief description on the development of the stress state variables. A summary of triaxial tests for unsaturated soils is presented with description on constant water content and consolidated undrained tests. It also includes the reviews on parameters affecting the stress-strain behaviour of soils.
- Chapter 3 presents different types of experimental test set-ups used in this study and the procedure adopted for different tests. The modifications made to improve the accuracy of local strain measurement device are also described. The properties of the soils tested and specimen preparation procedures are also included.

- Chapter 4 presents the results and discussions for the tests performed using different soil types. The three tests performed in this study are triaxial compression tests with local strain measurement, bender element tests and resonant column tests.
- Chapter 5 describes the stiffness-strain relationship and curve fitting using Borden et al. (1996)'s equation. Relationship of $G_{\max}$ with effective mean confining pressure for dry sands was proposed. Evaluation of the proposed relationship of $G_{\max}$ with effective mean confining pressure using the published data is also included in this chapter.
- Chapter 6 describes shear strength modelling for unsaturated soils. Evaluations of the shear strength model are presented in this chapter.
- Chapter 7 summarises the experimental observations and conclusions made from the tests performed in this study. The major contributions of the research are highlighted. In addition, recommendations for future studies are also presented in this chapter.

Chapter 2 Literature Review

2.1 Introduction

Strength and stiffness of soils are the two most important parameters for geotechnical engineering design and analysis. Shear strength and deformation characteristics of unsaturated soils are reviewed with a brief description on the history of unsaturated soil mechanics in the following sections. Triaxial tests for unsaturated soils are also summarized and discussed. Parameters affecting the strength and stiffness behaviours of unsaturated soils are also reviewed. In addition, models for prediction of shear strength and stiffness of unsaturated soils are summarized and reviewed in this chapter.

2.2 Background on Shear Strength of Unsaturated Soils

2.2.1 History of Unsaturated Soils

In general, soil mechanics can be divided into two parts namely: saturated and unsaturated soil mechanics. The main difference between saturated and unsaturated soil mechanics is the consideration of air phase when studying the mechanical behaviour of soils. Soils near the ground surface are usually in a moist condition and therefore will be subjected to negative pore-water pressure. In addition, soils remain unsaturated in most field conditions (e.g. excavation, landfill and dam etc.). Problems associated with unsaturated soil cannot be solved using classical saturated soil mechanics as behaviour of

unsaturated soil can differ greatly from that of saturated soils. Thus, understanding engineering behaviour of unsaturated soil is important.

An unsaturated soil has commonly been viewed as a three-phase system consisting of solid, water and air phases (Lambe and Whitman 1979). Fredlund and Morgenstern (1977) introduced the air-water interface or the contractile skin as a fourth and an independent phase. Figure 2.1 shows a four-phase soil system for unsaturated soil. When the air phase is continuous, the contractile skin interacts with soil particles and influences the mechanical behaviour of soils. The thickness of the contractile skin is in the order of only a few molecular layers and it can also be considered as part of the water phase (i.e. soil can be considered as a three-phase system for simplification).

Figure 2.1 A four-phase unsaturated soil system (from Fredlund and Morgenstern 1977)

2.2.2 Stress States Variables

Terzaghi (1936) describes a single stress state variable which controls the behaviour of a saturated soil. The stress state variable is commonly referred to as the effective stress and is written as:

$$\sigma' = \sigma - u_w \quad (2.1)$$

where σ' is effective normal stress, σ is total normal stress and u_w is pore-water pressure. In the past, several researchers attempted to propose a single-value effective stress or stress state variable to describe the behaviour of unsaturated soils. Croney et al. (1952) proposed an effective stress equation for an unsaturated soil as:

$$\sigma' = \sigma - \beta' u_w \quad (2.2)$$

where β' is the holding or bonding factor which is a measure of the number of bonds under tension influencing the shear strength of the soil.

On the other hand, Bishop (1959) suggested the following effective stress equation:

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) \quad (2.3)$$

where u_a is the pore-air pressure and χ is a parameter related to the degree of saturation of the soil. The relationship between χ parameter and the degree of saturation was obtained experimentally and the value of χ ranges between unity and zero. The value of χ is unity for saturated soils and zero for dry soils.

Many other researchers (Aitchison 1961; Jennings 1961; Aitchison 1965; Richards 1966; Aitchison 1973) attempted to introduce the effective stress equation for an unsaturated soil based on a single-value stress state variable. However, (Burland 1964; Burland 1965), Matyas and Radhakrishna (1968), and Barden and Sides (1969) suggested that the effective stress equation for an unsaturated soil should be expressed with two independent stress state variables, $(\sigma-u_a)$ and (u_a-u_w) , rather than a single stress state variable. Fredlund and Morgenstern (1977) described a theoretical stress analysis of an unsaturated soil considering the soil as a four-phase system. From the analysis, it was concluded that there are three possible normal stress variables and any two of these stress variables can be used to describe the behaviour of an unsaturated soil. The three possible pairs of stress state variables which can be used as the stress states variables are:

- 1) $(\sigma-u_a)$ and (u_a-u_w) ,
- 2) $(\sigma-u_w)$ and (u_a-u_w) ,
- 3) $(\sigma-u_a)$ and $(\sigma-u_w)$.

It was suggested that the pair, $(\sigma-u_a)$ and (u_a-u_w) , referred to as the net normal stress and the matric suction, respectively, is the most satisfactory for engineering applications (Fredlund 1979; Fredlund and Rahardjo 1987). Bishop and Donald (1961) performed a triaxial test using unsaturated Braehead silt to support the validity of the proposed stress state variables. In the study, confining, pore-water and pore-air pressures were varied equally in order to keep the stress state variables, $(\sigma-u_a)$ and (u_a-u_w) , constant throughout the compression test. The shear strength of the soil should not vary significantly as the stress state variables were maintained at constant values. In other words, the stress-strain

relationship should maintain a monotonic behaviour. Figure 2.2 shows the controlled pressures during shearing and the stress-strain relationship obtained from the triaxial test. The test result showed a monotonic stress-strain relationship with little fluctuations at the end of the test. Bishop and Donald (1961) concluded that the fluctuations were due to the temporary fluctuations in the measurement system and from the test results, the stress state variables for an unsaturated soil were validated. Fredlund and Morgenstern (1977) tested the validity of the stress state variables by performing a null test. The parameters (i.e. σ , u_a , and u_w) were varied by equal amounts to maintain the stress state variables constant during the null test. Volume change and the degree of saturation of the soil were monitored. The idea of the null test is that there should be no change in volume or degree of saturation of the soil if there is no change in the stress state variables. From the null test, no volume change or little water flow was observed and in this way the stress state variables were validated for unsaturated soil.

Figure 2.2 Stress-strain relationship with variations of σ , u_a , and u_w while maintaining $(\sigma - u_a)$ and $(u_a - u_w)$ during shearing of drained test (from Bishop and Donald 1961)

2.2.3 Soil-water Characteristic Curve

Soil-water characteristic curve gives the relationship of gravimetric water content or volumetric water content of soil with matric suction. Figure 2.3 shows a typical soil-water characteristic curve with indication of air-entry value, saturated and residual volumetric water content. Before the air-entry value, soil remains essentially saturated and it is therefore expected that the mechanical behaviour of an unsaturated soil in the zone is similar to that of a saturated soil. The rate of desaturation with matric suction is the most significant between the air-entry value and the residual volumetric water content. Beyond the residual volumetric water content, the rate of desaturation becomes insignificant with matric suction. In the past, soil-water characteristic curve has been used to predict the saturated and unsaturated coefficient of permeability of soils (Marshall 1958; vanGenunchten. 1980; Mualem 1984). Similarly, a close correlation between shear

strength of unsaturated soil with SWCC has been observed. In other words, the change in shear strength with matric suction shows similar trend as that of water content with matric suction. (Lamborn 1986) observed that soil-water characteristic curve could be used to predict the shear strength of unsaturated soils. Escario and Juca (1989) also observed that the final water content of direct shear tests on unsaturated soil specimens were close to the respective soil-water characteristic curves. Therefore, many researchers have proposed equations to predict the shear strength of unsaturated soils using soil-water characteristic curves (Vanapalli et al. 1996, Fredlund et al. 1996, Oberg and Sallfors 1997, Khallili and Khabbaz 1998, Bao et al. 1998 and Goh et al. 2010). These shear strength equations for unsaturated soil using SWCC are reviewed in Section 2.2.6.

Figure 2.3 Typical soil-water characteristic curve

2.2.4 Failure Criteria

In general, shear strength of a soil can be determined using the following failure criteria:

- Peak deviator stress $[(\sigma_1' - \sigma_3')_{\max}]$
- Maximum principal stress ratio (σ_1' / σ_3')
- Limiting strain (ϵ_f)
- Critical state
- Residual state.

Traditionally, the maximum or peak deviator stress is used as a failure criterion for determination of failure stress conditions of soils (i.e. σ_{1f}' and σ_{3f}'). However, the maximum principal stress ratio criterion is preferred to the maximum deviator stress in most undrained tests. In undrained tests, the effective stress changes are not the same as the total stress changes due to the generation of excess pore-water pressure. Therefore the plot of peak deviator stress versus axial strain is different from that of principal stress ratio versus axial strain as the strain at which soil reaches the failure may not be the same in these two plots. Limiting strain criterion is not often used but sometimes preferred to the maximum deviator stress criterion in testing for soils which require a large deformation to mobilise the maximum shear resistance.

For unsaturated soil tests, maximum deviator stress is commonly taken as a failure criterion for determination of stress conditions at failure which are called the net major and net minor normal stresses at failure [i.e. $(\sigma_1 - u_a)_f$ and $(\sigma_3 - u_a)_f$]. Besides using the

maximum deviator stress as a failure criterion, the principal stress ratio, $\sigma_1 - \sigma_3 / \sigma_3 - u_w$, can be used as an alternative failure criterion in undrained tests (Bishop et al. 1960). The plot of deviator stress and principal stress ratio versus axial strain of the compacted shale from undrained triaxial tests are presented in Figure 2.4. The failure strain (ϵ_f) obtained using the maximum deviator stress and the maximum principal stress ratio criteria are different as shown in Figure 2.4. This is because excess pore-water pressure generation during shearing is not considered in obtaining deviator stress but in obtaining principal stress ratio. In this case, the maximum principal stress ratio is taken as a failure criterion as it occurred prior to the maximum deviator stress (Bishop et al. 1960).

Figure 2.4 Plot of deviator stress and principal stress ratio versus axial strain of a compacted shale from undrained triaxial test (from Bishop et al. 1960)

2.2.5 Extended Mohr-Coulomb Failure Envelope

The shear strength of a saturated soil is expressed using the Mohr-Coulomb failure criterion and the effective concept as Terzaghi (1936):

$$\tau = c' + (\sigma - u_w) \tan \phi' \quad (2.4)$$

where c' is effective cohesion, $(\sigma - u_w)$ is effective normal stress and ϕ' is effective friction angle. The failure envelope for saturated soil is obtained by plotting the Mohr circles at failure conditions in terms of effective normal stress as described in Equation (2.4). Figure 2.5 shows the typical Mohr-Coulomb failure envelope for a saturated soil.

Figure 2.5 Mohr-Coulomb failure envelope for a saturated soil (from Fredlund and Rahardjo 1993)

Fredlund et al. (1978) expressed the shear strength for an unsaturated soil using the stress state variables $(\sigma - u_a)$ and $(u_a - u_w)$ and the extended Mohr-Coulomb failure criterion as:

$$\tau = \underbrace{c' + (\sigma - u_a) \tan \phi'}_{\tau_{\text{sat}}} + \underbrace{(u_a - u_w) \tan \phi^b}_{\tau_{\text{unsat}}} \quad (2.5)$$

where ϕ^b is an angle describing the rate of change of shear strength with matric suction. Similar to saturated soils, the failure envelope for an unsaturated soil is obtained by plotting the Mohr circles of the unsaturated soil at failure condition with respect to the net normal stress. However, the location of the Mohr circles in the third dimension is in terms of matric suction. Figure 2.6 shows the extended Mohr-Coulomb failure envelope for an unsaturated soil with the frontal plane representing the failure envelope for the saturated soil. Equation 2.5 can be written in two-dimensional representation as follows:

$$\tau = c + (\sigma - u_a) \tan \phi', \text{ where} \quad (2.6)$$

$$c = c' + (u_a - u_w) \tan \phi^b \quad (2.7)$$

where c is total cohesion intercept. Figure 2.7 shows the graphical representation of Equation 2.6.

2.2.6 Shear Strength Characteristics of Unsaturated Soils

The shear strength equation for an unsaturated soil (as described in Equation 2.5) consists of two parts: the shear strength contribution from the saturated soil and that from the unsaturated soil. The shear strength contribution from the unsaturated soil as shown in Equation 2.5 was proposed based on the assumption that the increase in shear strength with matric suction was linear. However, non-linear variation of shear strength with matric suction has been observed by many researchers (Escario and Saez 1986; Gan et al. 1988; Melinda et al. 2004; Thu et al. 2006; Hossain and Yin 2010 and Likos et al. 2010).

Figure 2.6 Extended Mohr-Coulomb failure envelope for an unsaturated soil (from Fredlund and Rahardjo 1993)

Figure 2.7 Horizontal projection of failure envelope onto τ versus $\sigma - u_a$ plane (from Fredlund and Rahardjo 1993)

Escario and Saez (1986) performed a series of unsaturated direct shear tests on statically compacted specimens of three different soils (i.e. Madrid grey clay, red clay of Guadalix de la Sierra and Madrid clay sand). It was observed from the experimental results that the

shear strength increased with matric suction non-linearly for all the tested specimens. The variations of shear strength with matric suction for Madrid clayey sand are presented in Figure 2.8.

Figure 2.8 Shear strength versus matric suction of Madrid clayey sand under different net confining pressures (from Escario and Saez 1986)

Gan et al. (1988) also investigated the effect of matric suction on the shear strength of a compacted glacial till by performing multi-stage consolidated drained direct shear tests. From the study, non-linear variations of shear strength with matric suction were observed and the relationship between shear strength and matric suction is shown in Figure 2.9 (a). It was also noted that the relationship of ϕ^b angle with matric suction is a sigmoid similar to the soil-water characteristic curve and the initial value of ϕ^b angle obtained was the same as ϕ' as shown in Figure 2.9 (b).

Figure 2.9 Experimental results of compacted glacial till: (a) Shear strength versus matric suction and (b) ϕ^b angle versus matric suction (from Gan et al. 1988)

Thu et al. (2006) conducted a series of constant water content triaxial tests (CW) on the statically compacted kaolin specimens. The specimens were sheared monotonically at different initial matric suctions using constant water content triaxial tests. The variations of shear strength with matric suction under different net confining pressures are presented in Figure 2.10 (a). The respective relationships of ϕ^b angle with matric suction under different net confining pressures are also presented in Figure 2.10 (b). As shown in Figure 2.10, sigmoid relationships of ϕ^b angle with matric suction under different net

confining pressures were observed and the initial value of ϕ^b angle was observed to be the same as ϕ' .

Figure 2.10 Experimental results on a compacted kaolin under different net confining pressure: (a) shear strength versus matric suction, (b) ϕ^b angle versus matric suction (from Thu et al. 2006)

Hossain and Yin (2010) studied the effect of matric suction on a remoulded completely decomposed granite soil by performing a series of single-stage consolidated drained direct

shear test. The experimental results are presented in Figure 2.11. Non-linear variations of shear strength with matric suction were observed from the results and dilative behaviours of unsaturated completely decomposed soil were also reported in the study. It was noted that the dilation angle was significantly affected by matric suction and net confining pressure. Similarly, Likos et al. (2010) examined the shear strength behaviour of unsaturated sand by performing direct shear test in drained loading condition at relatively small matric suctions and net confining pressures by using the test set-up shown in Figure 2.12. A hanging column method comprising two reservoirs was used to control the required matric suction and net confining pressure (ASTM D6838-2008). In their study, the suction pressure was obtained by adjusting the relative elevations of the reservoirs. The suction pressure from the hanging column system was transferred to the bottom of the specimen through a saturated ceramic disk and air pressure on top of the specimen was kept at atmospheric pressure. The imposed suction on the soil specimen was taken as the difference in elevation of the two reservoirs plus the elevation equivalent to half of the specimen height or the elevation between the failure plane and the suction pressure inlet. The relationships of shear strength with matric suction under different net confining pressures ranging from 0.3 to 10 kPa are shown in Figure 2.13. The shear strength of unsaturated sandy colluvium varied non-linearly with matric suction under different net confining pressures. It was also noted that the variation of shear stress with matric suction under the higher net confining pressure of 9.12 Pa showed more non-linear behaviour than that under net confining pressures of 0.33 kPa and 0.19 kPa.

Figure 2.11 Shear strength versus matric suction of completely decomposed granite soil (from Hossian and Yin 2010)

Figure 2.12 Modified direct shear apparatus with the hanging column system (from Likos et al. 2010)

Figure 2.13 Shear strength versus matric suction of sandy colluvium (from Likos et al. 2010)

As reviewed above, non-linear variation of ϕ^b angle with matric suction has been observed by many researchers. Values of ϕ^b have been reported to be equal to or less than ϕ' from the experiments performed on some soil types [e.g. compacted glacial till in Gan et al. (1988) and compacted silt in Thu et al. (2006)] and greater than ϕ' for other soil types [e.g. reconstituted Madrid clayey sand in Escario and Saez (1986), compacted sandy clay in Melinda et al. (2004), compacted silty sand (completely decomposed granite) in Hossain and Yin (2010) and reconstituted sandy colluviums in Likos et al. (2010)].

Unsaturated soil tests are time-consuming and a model that can predict the shear strength behaviour of an unsaturated soil will be beneficial. Many researchers have proposed shear strength equation for an unsaturated soil in terms of the stress state variables or the degree of saturation. Bishop (1959) introduced the shear strength

equation for unsaturated soil by extending the effective stress equation for saturated soils as:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) (\chi \tan \phi') \quad (2.8)$$

where $(\sigma - u_a)$ is net normal stress, $(u_a - u_w)$ is matric suction and χ is a parameter related to degree of saturation.

Fredlund et al. (1978) expressed the shear strength equation for unsaturated soil using the extended Mohr-Coulomb failure envelope as:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad (2.9)$$

Vanapalli et al. (1996) and Fredlund et al. (1996) proposed the shear strength equation for unsaturated soil by considering the non-linear shear strength behaviour along the matric suction plane as:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) (\Theta^\kappa \tan \phi') \quad (2.10)$$

where κ is a fitting parameter and Θ is normalised volumetric water content (θ_w / θ_s) in which θ_w is volumetric water content and θ_s is saturated volumetric water content. An entire soil-water characteristic curve (i.e. SWCC ranging from matric suction of 0 kPa to 10^6 kPa) with effective shear strength parameters is required when using Equation 2.10 to predict the shear strength of unsaturated soils. Fredlund and Xing (1994)'s SWCC equation is commonly used to describe the entire SWCC and it is expressed as:

$$\theta = c(\psi) \left\{ \frac{\theta_s}{\ln \left[\exp(1) + \left(\frac{\psi}{a} \right)^n \right]^m} \right\}, \text{where} \quad (2.11a)$$

$$c(\psi) = 1 - \frac{\ln \left(1 + \frac{\psi}{\psi_r} \right)}{\ln \left(1 + \frac{10^6}{\psi_r} \right)} \quad (2.11b)$$

where ψ is soil suction, a , n , and m are fitting parameters and ψ_r is soil suction related to residual volumetric water content.

Oberg and Sallfors (1997) proposed an equation for predicting shear strength of unsaturated soils especially for non-clayey soils such as sand and silt as:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w)(S \tan \phi') \quad (2.12)$$

where S is degree of saturation.

Khalili and Khabbaz (1998) proposed an equation similar to Bishop (1959)'s equation (Equation 2.8) as:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w)(\lambda' \tan \phi') \quad (2.13a)$$

$$\text{where, } \lambda' = \left\{ \frac{(u_a - u_w)_f}{(u_a - u_w)_b} \right\}^\eta \quad (2.13b)$$

where $(u_a - u_w)_f$ is matric suction at failure, $(u_a - u_w)_b$ is air-entry value and η is suggested as (-0.55).

Bao et al. (1998) suggested the following equation for prediction of shear strength for unsaturated soil as:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w)(\xi \tan \phi') \quad (2.14a)$$

$$\text{where, } \xi = \frac{\log(u_a - u_w)_r - \log(u_a - u_w)}{\log(u_a - u_w)_r - \log(u_a - u_w)_b} \quad (2.14b)$$

Recently Goh et al. (2010) suggested the following equations to predict the shear strength of unsaturated soil:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi', \quad (2.15a)$$

for $(u_a - u_w) < (u_a - u_w)_b$

$$\tau = c' + [(\sigma - u_a) + (u_a - u_w)_b] \tan \phi' + [(u_a - u_w) - (u_a - u_w)_b] b \Theta^\kappa \tan \phi', \quad (2.15b)$$

for $(u_a - u_w) \geq (u_a - u_w)_b$

$$\text{and } \kappa = [\log(u_a - u_w) - \log(u_a - u_w)_b]^y \quad (2.15c)$$

where y and b are fitting parameters. In most of the proposed equations for shear strength of unsaturated soil, soil-water characteristic curves were used either directly or indirectly along with the effective friction angle (ϕ') to account for the non-linear behaviour of ϕ^b angle with matric suction (Vanapalli et al. 1996, Fredlund et al. 1996, Oberg and Sallfors 1997, Khallili and Khabbaz 1998, Bao et al. 1998 and Goh et al. 2010). In this study, a sigmoid function was proposed for the relationship of $\tan \phi^b$ with matric suction for the calculation of increase in shear strength with matric suction.

2.2.7 Triaxial Tests for Unsaturated Soils

Triaxial test is one of the most commonly used tests to measure the shear strength of a soil in the laboratory (Fredlund and Rahardjo 1993). There are two main stages involved in the triaxial test, i.e., the application of confining pressure and the application of axial stress or the shearing stage. During application of confining pressure in the first stage, excess pore-water pressure will be generated. The excess pore-water pressure due to the applied confining pressure can be allowed to drain (consolidated condition) or prevented to drain (unconsolidated condition). During shearing, the excess pore-water pressure generated due to applied axial stress in the specimen can be allowed to drain (drained condition) and can also be prevented to drain (undrained condition) throughout the test. Table 2.1 summarises the various triaxial tests for unsaturated soil and the tests are differentiated based on the drainage conditions during the first and second stage of triaxial test.

Table 2.1 Various triaxial tests for unsaturated soils (from Fredlund and Rahardjo 1993)

Test methods	Consolidation prior to shearing	Drainage		Shearing process		
		u_a	u_w	u_w	u_a	ΔV
Consolidated drained (CD)	yes	yes	yes	C	C	M
Constant water content (CW)	yes	yes	no	C	M	M
Consolidated undrained (CU)	yes	no	no	M	M	
Undrained	no	no	no			
Unconfined compression (UC)	no	no	no			

C=control, M=measure and ΔV =volume change

2.2.7.1 Constant Water Content Test

In constant water content test, the specimen is first consolidated and then sheared with drained pore-air and undrained pore-water pressure conditions. Firstly, the specimen is isotropically consolidated under the required stress state $[(\sigma-u_a), (u_a-u_w)]$ by controlling pore-air and pore-water pressures. The required net confining pressure and the matric suction are obtained at the end of the consolidation. During shearing, the supply air pressure valve is opened to maintain the net confining pressure constant while the pore-water pressure valve is closed throughout the test. The pore-water pressure changes due to applied axial stress and therefore matric suction changes with loading during the test. Figure 2.14 shows the possible Mohr circles for constant water content test with I_0 indicating the initial stress condition at the end of consolidation and F_1 showing the failure stress condition. Satija (1978) performed constant water content triaxial tests using Dhanauri clay specimens at the same initial matric suction of 392 kPa and different net confining pressures of 96 kPa and 386 kPa. The plot of stress-strain curves obtained from CW tests is shown in Figure 2.15 (a). The variations of matric suction and volume change with axial strain are presented in Figures 2.15 (b) and (c), respectively. The values of matric suction decreased with axial strain for the specimens under different net confining pressures. However, the rate of matric suction reduction with axial strain was observed to be more significant for the specimen tested under the higher net confining pressure of 386 kPa. For the specimen tested under the low net confining pressure of 96 kPa, soil dilation with increasing matric suction was observed after reaching the peak deviator stress.

2.2.7.2 Consolidated Undrained Test

In consolidated undrained test, the specimen is first consolidated and then sheared with undrained pore pressures (i.e. pore-air and pore-water pressure) condition. Firstly, the specimen is isotropically consolidated under the required stress state $[(\sigma - u_a), (u_a - u_w)]$ by controlling pore-air and pore-water pressures. The required net confining pressure and the matric suction are obtained at the end of the consolidation. During shearing, the supply air pressure valve and the pore-water pressure valve are closed throughout the test.

Figure 2.14 Mohr circles for constant water content test

Figure 2.15 CW tests on Dhanauri clay, (a) stress-strain curves, (b) matrix suction change versus strain and (c) soil volume change versus strain (from Satija 1978)

Both pore-air and pore-water pressures change due to applied axial stress and therefore the net confining pressure and matrix suction are varying with loading during the test.

Figure 2.16 shows the possible Mohr circles for consolidated undrained test with I_0 indicating the initial stress condition at the end of consolidation and F_1 , F_2 , and F_3 showing the possible stress condition at failure. The change in pore pressures during shearing can be affected by strain rate used in the test. Satija (1978) performed constant water content triaxial tests using Dhanauri clay and reported the significant effect of strain rate on the variations of matric suction with axial strain. From the experimental results, matric suction decreased with axial strain more significantly when fast strain rate was used. In other words, more excess pore-water pressure generated when fast strain rate was used as pore-air pressure was kept constant throughout the tests. In consolidated undrained test, fully undrained condition for pore-air may be difficult to achieve as pore-air can diffuse into pore-water during shearing. The process of air diffusion into water is time-dependent and diffusion of air into water can therefore be more significant when slow strain rate is used in the test. Therefore, pore-air pressure can vary (i.e. increase, decrease or remain constant) with axial strain depending on the strain rate used in the test as described in Figure 2.16.

2.2.7.3 Strain Rate for Unsaturated Soils Tests

For triaxial test, an appropriate strain rate should be chosen to ensure equalization or dissipation of induced pore-water pressure for undrained and drained tests, respectively. Strain rate for unsaturated soils can be estimated using the theory of consolidation for the specimen under drained condition (Gibson and Henkel 1954 and Bishop and Henkel 1962). Generally, strain rate for testing of unsaturated soil in triaxial apparatus can be defined as:

$$\dot{\epsilon} = \frac{\epsilon_f}{t_f} \quad (2.16)$$

where ϵ_f is strain of the specimen at failure and t_f is time required for the specimen to fail. The strain at which the specimen reaches failure varies according to soil type. Table 2.2 summarizes the typical values of failure strain and strain rate for various soil types. For estimation of strain rate for unsaturated soil tests, it is necessary to consider the coefficient of permeability of high air-entry ceramic disk at the base of the specimen and the physical properties of unsaturated soil (Ho and Fredlund 1982). Figure 2.17 shows the properties and variables required to model pore pressure equalization for an unsaturated soil and a high air-entry ceramic disk.

Figure 2.16 Mohr circles for consolidated undrained test

Table 2.2 Typical values of strain rate and strain at failure for unsaturated soil tests (from Fredlund and Rahardjo 1993)

Soil Type	Triaxial test	Strain rate (%/s)	Approximate strain at failure ϵ_f (%)	References
Boulder clay; $w=11.6\%$ and $\% \text{ clay}=18\%$	CW	$3.5E-05$	15	Bishop et al. (1960)
Braehead silt	CW	$4.7E-05$	11	Bishop and Donald (1961)
	CD	$8.3E-06$	12	
Talybont boulder clay; $w=9.75\%$ and $\% \text{ clay}=6\%$	Undrained u_w^*	$4.7E-07$	$\sigma_3=83 \text{ kPa}$: 8.5 $\sigma_3=207 \text{ kPa}$: 11	Donald (1963)
Dhabauri clay; $w=22.2\%$ and $\% \text{ clay}=25\%$	CW	$6.7E-04$	20	Satija and Gulhata (1979)
	CD	$1.3E-04$	20	
Undisturbed decomposed granite and rhyolite	CD	$1.7E-05$	Stage 1: 3-5	Ho and Fredlund (1982)
	Multi-stage	$6.7E-05$	Stage 2: 1-3	
			Stage 3: 1-3	
Clayey sand; $w=14-17\%$ and $\% \text{ clay}=30\%$	Undrained and unconfined	$1.7E-03$	15-20	Chantawarangul (1983)

u_w^* : with pore-water pressure measurement

Figure 2.17 Properties and variables required to model pore pressure equalisation in unsaturated soil and high air-entry ceramic disk (from Fredlund and Rahardjo 1993)

The mathematical model for excess pore pressure dissipation with impeded flow through high air-entry ceramic disk is equivalent to that for heat conduction of a slab under a process of cooling from its initial state at uniform temperature using Newton's law (Carslaw and Jaeger 1959). The solution of heat conduction in slab can be obtaining using a Fourier's equation (Gray 1945). Using the Fourier's equation, the properties of unsaturated soils and high air-entry disk are expressed in the form of an impedance factor as follows:

$$\lambda = \frac{k_d d}{k_w L_d} \quad (2.17)$$

where λ is impedance factor, k_d is coefficient of permeability of high air-entry ceramic disk, k_w is coefficient of permeability of unsaturated soil in terms of water phase, d is length of drainage path of soil specimen and L_d is thickness of high air-entry ceramic disk. The length of the drainage path is equal to the height of the specimen for single drainage (i.e. pore-water drains only to the top or bottom of the soil specimen) and it becomes half of the specimen for double drainage (i.e. pore-water drains to top and bottom of the soil specimen). For unsaturated soil tests, the pore-water is usually allowed to drain to the bottom of the soil specimen through the high air-entry ceramic disk. The time required for the specimen to fail is expressed in terms of the degree of dissipation of excess pore-water pressure as follows (Bishop and Gibson 1963):

$$t_f = \frac{L^2}{\eta c_v^w (1 - \bar{U}_f)} \quad (2.18)$$

where L is distance equal to half of the specimen height for both single and double drainage, η is expressed as $0.75/(1+3/\lambda)$ and $3/(1+3/\lambda)$ for single and double drainage

respectively, c_v^w is average coefficient of consolidation with respect to the water phase [$= k_w / (\rho_w g m_2^w)$], g is gravitational acceleration and m_2^w is slope of plot of water volume change versus matric suction] and $\bar{U}_f$ is average degree of dissipation of the excess pore-water pressure at failure. The time required for the soil specimen to fail (t_f) can be plotted with the impedance factor (λ) for different values of coefficient of consolidation (c_v^w) using Equation (2.18). Figures 2.18 and 2.19 show the relationships of t_f with impedance factor for single drainage with different specimen heights of 140 mm and 76 mm for triaxial tests, respectively.

Figure 2.18 Time to failure versus impedance factor for the specimen height of 140 mm for a drained triaxial test (from Fredlund and Rahardjo 1993)

Figure 2.19 Time to failure versus impedance factor for the specimen height of 76 mm for a drained triaxial test (from Fredlund and Rahardjo 1993)

Several empirical equations for estimation of strain rate for unsaturated soil tests have been proposed by researchers (Bishop et al. 1960 and Lumb 1966; Ruddock 1966). Donald (1961) suggested that the strain rate should be selected by considering the effect of strain rate on the peak deviator stress. The experimental results from Gibson and Henkel (1954) and Bishop and Henkel (1962) showed variations in failure deviator stress with strain rate. The results also showed an upper strain limit for unsaturated soil tests. Below that limit shear strength was not affected by the strain rate. Satija and Gulhati (1979) concluded from their tests that strain rate did not significantly affect deviator stress. However, changes in matric suction and water content for constant water content and consolidated drained tests, respectively, are affected by the strain rate and therefore the variations of matric suction or water content with axial strain should be used as the criterion for selection of strain rate. Ong (1999) performed constant water content tests on compacted

residual soils using different strain rates to investigate the effect of strain rates. Identically compacted residual soil specimens were consolidated under net confining pressure of 250 kPa and matric suction of 400 kPa and tested using four different strain rates ranging from 0.009 mm/min to 0.081 mm/min. From the experimental results, relationship of deviator stress versus axial strain, matric suction versus axial strain and matric suction versus deviator stress were obtained. Ong (1999) observed that these relationships were not significantly affected by the strain rates used in the test. It was therefore concluded from the experimental results that 0.009 mm/min was slow enough to ensure pore pressure dissipation during shearing. It was also noted that the theoretical time required for the specimen to fail based on the dissipation of excess pore-water pressure for the compacted residual soils was unrealistically long. The selection of strain rate for unsaturated soil test depends on the stress condition and the properties of unsaturated soil. Thu et al. (2006) used strain rate of 0.009 mm/min for testing of compacted unsaturated silt specimens as unsaturated silt has similar properties as compacted unsaturated residual soils. In this study, three different strain rates of 0.006 mm/min, 0.1 mm/min and 0.5 mm/min were used to test undisturbed unsaturated residual soils under constant water content and consolidated undrained condition to investigate the effect of strain rates.

2.2.7.4 Multi-stage Triaxial Test

Shear strength parameters of soils can be studied by performing triaxial tests using single-stage or multi-stage testing. The main advantage of multi-stage testing is that it can provide maximum information on shear strength parameters using a single soil specimen and eliminate the effect of inconsistency in soil specimen on the results. However,

excessive deformation in earlier shearing stage and excessive strain accumulation due to repeated loadings can exist in multi-stage testing (Ho and Fredlund 1982). If the specimen undergoes excessive deformation in the earlier shearing stage, shear plane can form in the soil specimen and as a result, the subsequently measured shear strength will be reduced. Multi-stage testing can be performed using either the cyclic loading procedure (i.e. the deviator stress was released to zero when failure is imminent) or the sustained loading procedure (i.e. the deviator stress is sustained at maximum value before proceeding to the next shearing stage). The unloading process in multi-stage testing using the cyclic loading procedure restores part of the accumulated strain in the soil specimen through elastic recovery and therefore it is preferred to the sustained loading procedure (Ho and Fredlund 1982). Figures 2.20 and 2.21 show typical stress-strain relationships obtained from multi-stage testing using cyclic loading procedure and sustained loading procedure, respectively.

Figure 2.20 Stress-strain relationship of decomposed granite obtained from multi-stage testing using cyclic loading procedure (from Ho and Fredlund 1982)

Figure 2.21 Stress-strain relationship of decomposed granite obtained from multi-stage testing using sustained loading procedure (from Ho and Fredlund 1982)

2.2.8 Summary

Shear strength of soils is one of the important parameters for most geotechnical design applications. It is also important to understand the shear strength characteristics of unsaturated soils as soils in general remain unsaturated in most field situations. The stress state variables ($\sigma - u_a$) and ($u_a - u_w$) are commonly used to describe the shear strength behaviour of unsaturated soils. Non-linear relationship of shear strength with matric suction has been observed by many researchers. It is time-consuming to perform unsaturated soil tests and it is therefore useful to establish an analytical model for the prediction of shear strength of unsaturated soils. Many researchers have proposed models to predict the shear strength of unsaturated soils using SWCC. In this study, an equation for estimating shear strength of unsaturated soil will be proposed.

2.3 Background on Stiffness of Soils

2.3.1 General

It has been recognised that stress-strain behaviour of soil is highly non-linear and soil stiffness degrades greatly with strain. Stiffness of soils can decrease significantly with shear strain and the amount of stiffness reduction depends on the magnitude of shear strain. Stiffness-strain relationship with typical shear strain ranges for most geotechnical structures are shown in Figure 1.1. The stiffness degradation curve can be divided into three regions namely: the very small strain, the small strain and the large strain region as shown in Figure 1.1 (Atkinson and Sallfors 1991). The stiffness in the very small strain region can be obtained using dynamic test methods involving wave velocity and natural frequency measurements (e.g. ultrasonic pulse or piezoelectric bender element test and resonant-column test). The stiffness in the small strain region can be obtained by performing cyclic triaxial test and cyclic direct simple shear test. Jardine et al. (1984b) highlighted that reliable measurement of stiffness in the small strain region ($\approx$ from 0.001% to 0.1% strain) can only be obtained using local strain measuring devices. Figure 2.22 shows the typical stress-strain relationship with its stiffness degradation curve. The small-strain stiffness ($G_{\max}$) can be obtained from the slope at the origin of the stress-strain curve and it is also referred to as the initial shear stiffness (G_0). Stiffness of soil can be described in term of tangent shear stiffness ($G_{\tan}$) or secant shear stiffness ($G_{\sec}$) as shown in Figure 2.22 (a). Value of $G_{\tan}$ varies throughout a loading cycle when soil is subjected to cyclic loading and therefore $G_{\sec}$ is commonly used to represent an average shear stiffness of the entire loop.

Figure 2.22 Typical stress-strain relationship and normalised stiffness degradation curve of soil

2.3.2 Parameters Affecting Soil Stiffness

A comprehensive general stress-strain relationship for soils can be complex as there are many parameters that can affect the stress-strain behaviour of soils. These parameters can be listed as follows:

- Soil fabric or grain size distribution of sand
- Shear strain amplitude (γ)
- Effective mean confining pressure (σ'_m)
- Void ratio (e) or relative density (D_r)
- Degree of saturation (S) or moisture content (w)
- Stress history or over-consolidation ratio (OCR)
- Strain rate or loading frequency

- Number of cycles of loading and others.

Seed and Idriss 1970 and Seed et al. 1986 investigated the parameters affecting the shear stiffness of sand and gravelly soils by performing a series of laboratory tests as well as in situ tests. The conditions of sands and gravelly soils varied from saturated to dry conditions. The shear stiffness of sands and gravelly soils were measured at different relative densities. From the study, the relationship between shear stiffness and effective mean confining pressure was proposed with an empirical shear stiffness coefficient (K_2) which is function of void ratio (e) and shear strain amplitude (γ). It was observed that K_2 increased with relative density (D_r) as shown in Figure 2.23. The effect of relative density on K_2 was observed to be more significant at small shear strain amplitudes (i.e. $\gamma = 0.0001\%$ to 0.01%) than that at larger shear strain amplitudes (i.e. $\gamma = 0.01\%$ to 3%).

Hardin and Drnevich (1972) investigated the parameters that can affect the stiffness of soils by performing resonant column tests. In the study, parameters affecting the stiffness of clean sand and cohesive soils were categorized into three main categories namely: very important, relatively important and less important. Strain amplitude, effective mean confining pressure, void ratio and number of cycles of loading were categorized as the very important parameters affecting the shear stiffness of clean sands. From the experimental results, it was noted that shear stiffness generally decreased with increasing shear strain amplitude however the rate of shear stiffness degradation for clean sand and cohesive soils were rather different. The shear stiffness increased with increasing effective mean confining pressure for clean sand and cohesive soils. It was observed that for clean sands the shear stiffness increased with number of loading cycles. However, for cohesive soils, shear stiffness decreased with number of loading cycles. Shear stiffness increased

with decreasing void ratio but the effect of void ratio was more significant for the shear stiffness at small shear strain amplitudes. It was explained that at higher shear strain amplitudes, shear stiffness can also be affected by other parameters (e.g. strength and strain amplitude) besides void ratio.

Figure 2.23 Effect of relative density (D_r) on relationship of shear stiffness with shear strain of model-graded Pyramid material (from Seed et al. 1986)

Alarcon-Guzman et al. (1989) performed resonant column tests on dry Ottawa 20-30 sand using hollow specimen resonant column apparatus to investigate the effects of void ratio and effective mean confining pressures. The experimental shear stiffness versus shear strain relationships at different initial void ratios and effective mean confining pressures are presented in Figures 2.24 and 2.25. The values of shear stiffness increased with decreasing void ratios and increasing effective mean confining pressures. In addition, it

was also observed that the values of shear stiffness decreased with shear strain amplitude for all the tested specimens.

Figure 2.24 Shear stiffness versus shear strain of dry Ottawa 20-30 sand at different initial void ratios under the effective mean confining pressure of 100 kPa from hollow specimen resonant column tests (from Alarcon-Guzman et al. 1989)

Figure 2.25 Shear stiffness versus shear strain of dry Ottawa 20-30 sand at initial void ratio of 0.635 under different effective mean confining pressures from hollow specimen resonant column tests (from Alarcon-Guzman et al. 1989)

2.3.3 Non-linear Stress-strain Model

Vucetic and Dobry (1991) studied the effect of soil plasticity on undrained cyclic response of fine-grained soils. From the study, it was observed that shear stiffness behaved more linearly with increasing plasticity of soils at small shear strain amplitudes. In other words, shear modulus degraded at higher shear strain amplitude for soils with high plasticity index (PI) and the threshold shear strain increased with PI as indicated in Figure 2.26. The effect of PI on the relationship of shear modulus with shear strain is presented in term of normalized stiffness ($G/G_{\max}$) in Figure 2.26.

Figure 2.26 Effect of plasticity index on stiffness degradation curves of fine-grained soils (from Vucetic and Dorby 1991)

Ishibashi and Zhang (1993) investigated the effect of effective mean confining pressure on the stiffness degradation curves of soil. Figure 2.27 shows the effect of effective mean confining pressures on non-plastic soil (i.e. $PI=0$). It was observed that effective mean confining pressure has a very significant effect on the stiffness degradation curves of non-

plastic soils and the threshold shear strain increased with effective mean confining pressure.

Figure 2.27 Effect of effective mean confining pressure on non-plastic soils (from Ishibashi and Zhang 1990)

d'Onofrio et al. (1999) performed a series of resonant column and cyclic torsional shear tests on natural stiff clay. Figure 2.28 shows the normalised stiffness degradation curve using constant strain rates such as 0.0002 %/min, 0.002 %/min and 0.3 %/min. From the results, it was observed that the stiffness-strain relationship degraded at higher shear strain with faster strain rate. In other words, the linear threshold shear strain increased with strain rate as indicated in Figure 2.28. Similar observation was made by Santagata et al. (2007). Figure 2.29 shows the plots of undrained Young's modulus (E_u) normalised with the small-strain Young's modulus ($E_{u\max}$) in terms of axial strain (ϵ_a) at different strain rates. The values of $E_{u\max}$ were obtained at strains of less than 0.002% in the study.

Figure 2.28 Effect of loading rate on the normalized shear stiffness degradation curve of stiff Italian clay (modified from d' Onofrio et al. 1999)

Figure 2.29 Strain rate effect on the stiffness-strain relationship of normally consolidated clay (from Santagata et al. 2007)

Wang et al. (2007) performed a series of consolidated drained and constant suction test for saturated and unsaturated soils, respectively, to investigate the effect of matric suction on the stiffness-strain relationship of soils. In the study, triaxial compression tests were performed on reconstituted Yoneyama soil and Toyoura sand. The stiffness-strain relationships of saturated and unsaturated soil specimens of Yoneyama soil and Toyoura sand under different confining pressures are presented in Figures 2.30 and 2.31.

In Figures 2.30 and 2.31, saturated and unsaturated soil specimens were indicated as CD_x and CS_{y-z}, respectively, in which CD stands for consolidated drained test, x denotes the value of imposed effective mean confining pressure and CS stands for constant suction and y and z denote the value of imposed net confining pressure and matric suction, respectively. For Yoneyama soil, the stiffness-strain relationships of the unsaturated soils were observed to be higher than that of saturated soils under the respective confining pressures. For Toyoura sand, the stiffness-strain relationships of unsaturated soils were higher than that that of saturated soils. However the effect of matric suction on the relationship of stiffness with strain for Toyoura sand was not significant as compared to that for Yoneyama soil.

The parameters affecting the stiffness-strain relationship of dry and saturated soils have been investigated by many researchers in the past. However, the study on the parameters affecting the stiffness characteristics of unsaturated soils is limited. The general relationships of $G_{\max}$ and $G/G_{\max}$ with parameters affecting the stiffness of dry and saturated soils are summarized in Table 2.3.

Figure 2.30 Stiffness-strain relationships of saturated and unsaturated Yoneyama soil under different effective and net confining pressures (from Wang et al.2006)

Figure 2.31 Stiffness-strain relationship of saturated and unsaturated Toyoura sand under different effective and net confining pressures (from Wang et al. 2006)

Table 2.3 Relationship of G_{max} and G/G_{max} with effective mean confining pressure, void ratio, OCR, PI, and strain rate (from Kramer 1996)

Parameters	G_{max}	G/G_{max}

Confining pressure	↑	↑ but the effect decreases with increasing PI
Void ratio	↓	↓
Overconsolidation ratio, OCR	↑	Not affected
Plasticity index, PI	↑ if OCR > 1 and remains constant if OCR=1	↑
Strain rate	↑ for plastic soils. No effect for non-plastic soils	G increases with stain rate but $G/G_{\max}$ probably not affected if G and $G_{\max}$ are measured at the same rate

Stress-strain behaviour of soils is highly non-linear and the decrease in shear stiffness with shear strain has been well recognized in the past. Many researchers have proposed the relationships to predict non-linear stiffness-strain behaviour of soils. Seed and Idriss (1970) proposed the following relationship between shear stiffness (G) and effective confining pressure (σ_m') as:

$$G = 545 K_2 (\sigma_m')^{1/2} \quad (2.19a)$$

where the units for G and σ_m' are expressed in kPa and K_2 is empirical soil stiffness coefficient which is function of void ratio (e) and shear strain amplitude (γ). A better expression for Equation (2.19a) is:

$$G = 54.5 K_2 (\sigma_m' / P_a)^{1/2} \quad (2.19b)$$

where P_a is atmospheric pressure. The values of K_2 for sand and gravel at small strain were proposed in the study and the relationships between K_2 and shear strain for sands at different void ratios are presented in Figure 2.32. It was observed that the values of K_2

increased with decreasing void ratio. In addition, the lower and upper bound shear stiffness degradation curves for sands in general were proposed. However the approach to obtain the stiffness degradation curves was not presented. Figure 2.33 shows the upper and lower bound stiffness degradation curved for sands.

Figure 2.32 K_2 versus shear strain for sands at different void ratios (from Seed and Idriss 1970)

Figure 2.33 Variations of shear modulus with shear strain for sands (from Seed and Idriss 1970)

Hardin and Drnevich (1972) proposed the following hyperbolic relationship of shear stiffness (G) with shear strain (γ) as:

$$G = \frac{G_{\max}}{1 + \frac{\gamma}{\gamma_r}} \quad (2.20)$$

where γ_r is reference shear strain ($= \tau_{\max}/G_{\max}$) in which $\tau_{\max}$ is shear stress at failure. $G_{\max}$ is small-strain stiffness in kPa and it can be expressed as:

$$G_{\max} = 3230 \frac{(2.973 - e)^2}{1 + e} (\text{OCR})^a (\sigma'_m)^{1/2} \quad (2.21a)$$

$$\text{or } G_{\max} = 323 \frac{(2.973 - e)^2}{1 + e} (\text{OCR})^a (\sigma'_m / P_a)^{1/2} \quad (2.21b)$$

where OCR is overconsolidation ratio, a is a parameter that depends on the plasticity index of soils and the unit of σ'_m is kPa. Borden et al. (1996) conducted a series of resonant column tests on residuals soils. The effects of confining pressure, shear strain amplitude, particle size distribution, frequency and number of cycles on the stiffness-strain relationship were investigated in the study. Based on the experimental results, an empirical relationship of shear stiffness with shear strain was proposed as:

$$\frac{G}{G_{\max}} = \frac{1}{(1 + a\gamma^b)^c} \quad (2.22)$$

where a , b , and c are fitting parameters and $G_{\max}$ was obtained from the following relationship:

$$G_{\max} = \alpha (\sigma'_m)^n \quad (2.23)$$

where α and n are fitting parameters.

2.3.4 Behaviour of Small-strain Stiffness

Small-strain stiffness ($G_{\max}$) is an important parameter for characterizing the stiffness degradation of soil with shear strain amplitude. In the past, many researchers have observed the relationship of small-strain stiffness with void ratio and effective mean confining pressure (Seed and Idriss 1970; Drnevich and Richart 1970 and Hardin and Drnevich 1972). The effects of capillary and degree of saturation of partially saturated soils have been studied by many researchers using resonant column tests and bender element tests (Wu et al. 1989; Qian et al. 1991; Marinho et al. 1995; Picornell and Nazarian 1998; Cabarkapa et al. 1999; Cho and Santamarina 2001; Leong et al. 2006). Qian et al. (1991)

performed a series of resonant column tests on partially saturated sands with different grain diameters. In the tests, remoulded specimens prepared at predetermined degree of saturation and void ratio were used to study the effects of void ratio and degree of saturation on the stiffness. The values of small-strain stiffness of partially saturated sand were normalised with the small-strain stiffness at dry condition to obtain the normalised small-strain stiffness (G/G_{dry}). Two distinct variations of G/G_{dry} with degree of saturation were observed from the experimental results (Figure 2.34). The value of G/G_{dry} reached a maximum at the optimum degree of saturation. Beyond the optimum degree of saturation, G/G_{dry} decreased gradually and approached unity. Qian et al. (1991) reported that the effect of degree of saturation on G/G_{dry} was more significant under higher confining pressures.

Figure 2.34 G/G_{dry} versus degree of saturation for the sand specimen at initial void ratio of 0.83 under different confining pressures (from Qian et al. 1991)

Cho and Santamarina (2001) also studied the effect of degree of saturation for four different materials (i.e. Clean Glass Beads, Kaolinite, Granite Powder and Sandboil Sand)

by measuring the shear wave velocity using bender element mounted in oedometer cell. The effect of degree of saturation was studied by allowing the fully saturated specimens to dry gradually to dry state. The shear wave velocity generally increased with decreasing degree of saturation for all the tested specimens. The shear wave velocity increased with decreasing degree of saturation gradually in the zone near saturation. However the shear wave velocity increased abruptly with decreasing degree of saturation in the zone near the residual degree of saturation (i.e. degree of saturation ≈ 0). Figure 2.35 shows the relationship of shear wave velocity with degree of saturation for four different specimens.

Figure 2.35 Shear wave velocity versus degree of saturation (a) Clean Glass Beads, (b) Mixture of Kaolinite and Glass Beads, (c) Granite Powder, (d) Sandboil Sand (from Cho and Satamarina 2001)

Leong et al. (2006) investigated the small-strain stiffness behaviour of unsaturated residual soils by measuring the shear wave velocity using bender elements mounted in the

triaxial apparatus. Identical specimens were prepared by compacting the residual soils at the same initial water content. The compacted specimen were equilibrated in the pressure plate apparatus under the required matric suctions before testing the specimens in undrained condition under different confining pressures. In other words, unconsolidated undrained tests were performed to study the effects of initial matric suctions and confining pressures on the compacted residual soils. It was observed that $G_{\max}$ increased with initial matric suctions and net confining pressures as shown in Figure 2.36.

Figure 2.36 Small-strain stiffness of compacted unsaturated residual soils (a) G_{max} versus matric suction and (b) G_{max} versus confining pressure (from Leong et al. 2006)

Mendoza and Colmenares (2006) studied the small-strain stiffness of unsaturated kaolin by performing bender element tests. Kaolin specimens were prepared using static compaction method at different initial degree of saturations. The effect of matric suction on small-strain stiffness of unsaturated kaolin specimens were investigated by measuring shear wave velocity at different water contents. The specimens were air-dried gradually to obtain different degrees of saturation or water contents. Figure 2.37 shows the relationship of G_{max} with matric suction for the unsaturated kaolin specimens.

As discussed above, the small-strain stiffness of unsaturated soils were measured in terms of either degree of saturation or water content without controlling or measuring the pore-air and pore-water pressure during the tests. The study of small-strain stiffness of unsaturated soil in terms of stress-state variables (i.e. $u_a - u_w$ and $\sigma - u_a$) is beneficial as it can

provide better understanding on the mechanical behaviour of unsaturated soils. Recently, small-strain stiffness of unsaturated soil was studied in terms of stress state variables by controlling the pore-air and pore-water pressure separately (Mancuso et al. 2002; Khosravi et al. 2010 and Hoyos et al. 2011).

Mancuso et al. (2002) conducted resonant column tests using compacted unsaturated silty sand to investigate the effects of matric suction and net confining pressure. Small-strain stiffness was measured under constant matric suction condition by controlling pore-air and pore-water pressure separately. From the experiments, the variation of $G_{\max}$ with matric suction showed a sigmoid trend similar to soil-water characteristic curve. The relationship of $G_{\max}$ with matric suction is shown in Figure 2.38.

Figure 2.37 $G_{\max}$ versus suction of kaolin specimens prepared at different initial water contents (from Mendoza and Colmenares 2006)

Figure 2.38 Variations of $G_{\max}$ with net confining pressure and matric suction of compacted silty sand (from Mancuso et al. 2002)

Khosravi et al. (2010) studied small-strain stiffness of unsaturated silica sand by performing resonant column tests. Hanging column technique was used to control the negative pore-water pressure below the sand specimens. The relationships of $G_{\max}$ with matric suctions at low net confining pressures are shown in Figure 2.39. It was observed that $G_{\max}$ reached a maximum value at matric suction near the residual volumetric water content.

Figure 2.39 Experimental results for unsaturated silica sand at low net confining pressures ranging from 3.5 kPa to 20 kPa: (a) $G_{\max}$ versus matric and (b) SWCCs (from Khosravi et al. 2010)

2.3.5 Models for Small-strain Stiffness of Unsaturated Soils

It is widely known that $G_{\max}$ is a function of void ratio and effective confining pressure for saturated or dry sand. Hardin and Black (1968) proposed the following general relationship of $G_{\max}$ with void ratio (e) and effective confining pressure (σ):

$$G_{\max} = A f(e) \sigma^n \quad (2.24)$$

where $f(e)$ is a function of void ratio and n is an empirical fitting parameter. The relationship of $G_{\max}$ for saturated soil can be expanded for unsaturated soil using the two stress state variables, net confining pressure and matric suction.

Leroueil and Hight (2003) proposed the following relationship of $G_{\max}$ for unsaturated soils:

$$G_{\max\text{unsat}} = G_{\max\text{sat}} + f(u_a - u_w) \quad (2.25)$$

where $G_{\max\text{unsat}}$ and $G_{\max\text{sat}}$ are unsaturated and saturated small-strain stiffness, respectively, and $f(u_a - u_w)$ is a function of matric suction in which the effect of matric suction on soil particles is additional to the effect of applied stress resulting from resistance to slippage at a contact point between soil particles. The stress resulting from resistance to slippage between soils particles can be written as:

$$\sigma = \sigma_e + \Delta\sigma_{\text{st}} + \Delta\sigma_{\text{suction}} + \Delta\sigma_{\text{st+suction}} \quad (2.26)$$

where σ_e , $\Delta\sigma_{\text{st}}$, $\Delta\sigma_{\text{suction}}$ and $\Delta\sigma_{\text{st+suction}}$ are stresses associated with void ratio, bonding between particles, matric suction, and combined effect of bonding between particles and matric suction, respectively.

Mendoza et al. (2005) investigated the effect of matric suction on $G_{\max}$ for compacted kaolin and proposed the following equation of $G_{\max}$ as a function of void ratio and matric suction ($u_a - u_w$):

$$G_{\max} = 13300 \frac{(2.3 - e)^2}{1 + e} [\ln(u_a - u_w)]^{1.35} \quad (2.27)$$

Leong et al. (2006) studied the small-strain stiffness of unsaturated residual soil and proposed the following relationship of $G_{\max}$ in terms of net confining pressure ($\sigma_3 - u_a$) and matric suction:

$$G_{\max} = (G_{\max})_o \left[1 + \left(\frac{\sigma - u_a}{P_a} \right)^n \right] \left[1 + \left\{ \ln \left(1 + \frac{u_a - u_w}{P_a} \right) \right\}^m \right] \quad (2.28)$$

where $(G_{\max})_o$ is small-strain shear stiffness at zero net confining pressure and matric suction, P_a is atmospheric pressure (≈ 100 kPa), and n and m are empirical fitting parameters.

Khosravi and McCartney (2009) proposed the following relationship of $G_{\max}$ with net confining pressure and matric suction for clayey soils:

$$G_{\max} = A \frac{P_a^n}{0.3 + 0.7e^2} \left[\left(\frac{\sigma - u_a}{P_a} \right) + \left(\frac{(u_a - u_w)_b}{P_a} + 1 \right) \ln \left(\frac{u_a - u_w}{P_a} + 1 \right) \right]^n \quad (2.29)$$

where A and n are fitting parameters, $(u_a - u_w)_b$ is the air-entry value of the soil.

2.3.6 Summary

Stiffness-strain behaviours of saturated and dry soils (saturated cohesive soils and saturated or dry sands) have been studied extensively by many researchers in the past (Hardin and Drnevich 1972; Seed and Idriss 1970; Tatsuoka et al. 1978; Borden et al. 1996). Many models incorporating the important parameters affecting characteristics of stiffness have been proposed to predict the non-linear stiffness-strain behaviour for those soils. However, studies on stiffness-strain behaviour of unsaturated soil in terms of the stress state variables are rather limited. Therefore, in this study, non-linear stress-strain behaviours of unsaturated soils were investigated in terms of the stress state variables.

Chapter 3 Test Set-up and Material

3.1 Introduction

The experimental test set-ups used in this study (a modified triaxial apparatus equipped with bender element test set-up and resonant column apparatus) are described in the following sections. The resonant column apparatus belongs to a commercial soil testing laboratory in Singapore. The apparatus was made available for the author's use for a limited time. Soils tested in this study include kaolin, four different types of sands (i.e. Ottawa 20-30, Ottawa graded sand, reclaimed sands from Changi and Jurong reclamation sites in Singapore) and the residual soils from Bukit Timah Granite. The sands from Changi and Jurong reclamation sites are denoted as Reclaimed sand A and Reclaimed sand B, respectively in this thesis. Consolidated kaolin specimens were used to evaluate the triaxial apparatus for the measurement of stress-strain behaviour of unsaturated soils. Four sands (Ottawa 20-30, Ottawa graded sand, Reclaimed sand A and Reclaimed sand B) were tested to obtain the stiffness-strain relationships of dry sands in a resonant column apparatus. Ottawa 20-30 and Ottawa graded sands were tested as they are standard sands. The stiffness-strain behaviours of Ottawa sands have been studied by many researchers using resonant column apparatus in the past and these soils provide a good basis for verifying the resonant column apparatus. Stiffness and strength characteristics of unsaturated reclaimed sand B were studied by performing triaxial compression tests. Undisturbed residual soils from Bukit Timah Formation were also tested in the triaxial

apparatus to measure stress-strain relationships and shear strength of unsaturated soils using different strain rates. The properties of these tested materials, the specimen preparation methods and the experimental procedures are presented in the following sections.

3.2 Triaxial Compression Test

Figure 3.1 shows the schematic diagram of the modified triaxial apparatus used in this study. The triaxial apparatus was equipped with the following instrumentations:

- A 5 kN submersible load cell for measurement of axial load.
- A pore-water pressure transducer attached at the base plate for monitoring of pore-water pressure in the soil specimen.
- A pore-air pressure transducer attached to the top platen for monitoring of pore-air pressure in the soil specimen.
- A pair of local displacement transducers (LDTs) for measurement of local axial strain in small strain region (range $\approx$ 0-0.4 mm)
- A submersible linear variable differential transformer (LVDT) attached to one of the internal tie rods with its spindle resting on the load cell for axial strain measurement (range = $\pm$ 2.5 mm).
- An external LVDT for large axial strain measurement (range = $\pm$ 15 mm)
- A pair of proximity transducers for radial strain measurement.

All the instruments were connected to a control and data acquisition system (CDAS) and a personal computer. All the instruments (i.e., load cell, LDT, proximity transducer, pore pressure transducer and LVDT) need to be calibrated before commencing the experiment. Cahyadi (2007) performed a series of experiments using the same triaxial apparatus and

the calibrations for the instruments were done prior to the experiments. The same calibration data were used for the tests mentioned in this study except for the LDTs. The local displacement transducers were fabricated in-house following the design mentioned in Goto et al. (1991). In this study, a new circuit, the Anderson loop (Anderson 1998) was used for the local strain measurement and its performance was compared with the conventional LDT using the Wheatstone bridge circuit. The detailed fabrication and calibration of the modified LDT are described in the next section.

Figure 3.1 Modified triaxial test set-up (from Leong et al. 2006)

3.2.1 Proximity Transducer for Radial Strain Measurement

Radial deformation was measured using a pair of proximity transducers model 6U KDM-8200 from KAMAN, USA, which has a measurement range up to 6 mm. The proximity transducers were mounted diametrically opposite of the specimen and an

aluminium sheet was used as a target. The proximity transducer was used with amplifiers for different measurement ranges. Figure 3.2 shows the typical calibrations curves of the proximity transducers using different amplifications. In this study, the amplification factor of 5 was used as it provides a suitable measurement range for this study. The calibration curves were therefore repeated to verify the previously measured curves shown in Figure 3.2. Figure 3.3 shows the typical loading and unloading curves of proximity transducer using amplification factor of 5.

Figure 3.2 Calibration curves of proximity transducers at different amplifications (from Cahyadi 2007)

Figure 3.3 Loading and unloading calibration curves of proximity transducers using amplification factor of 5

3.2.2 Modification of Local Strain Measurement System

3.2.2.1 Overview of Local Strain Measurement Devices

Stress-strain behaviour of soil is generally studied in the laboratory by performing triaxial test. Conventionally, soil stiffness was measured using external strain measurement devices mounted in the triaxial apparatus. Figure 3.4 depicts the various locations of strain measurements for triaxial tests. It has been recognized that soil stiffness obtained conventionally from laboratory test can be smaller than in-situ stiffness value. The soil deformation using conventional laboratory measurement system is overestimated due to the seating, alignment, bedding and compliance errors. These errors are illustrated in Figure 3.5. Seating error is caused by gaps between load cell and top platen or platens and

porous stones. Alignment error is due to equipment and specimen nonconformity (i.e. tilt of specimen, non-horizontality of platen surfaces, non-verticality and eccentricity of loading ram and porous stone of non-uniform thickness). Bedding error is caused by surface irregularity between the specimen and porous stone. Compliance error may occur due to the following situations: deflection of load cell, compressibility of lubricant used in the system, and compressibility of filter paper and tie bar.

Figure 3.4 Triaxial apparatus with various local and external strain measurement devices (from Scholey et al. 1995)

Lo Presti et al. (1993) performed triaxial tests in which strains were measured internally between the top cap and bottom pedestal using non-contacting proximity transducers. Based on the results, it was concluded that the effect of bedding and seating errors can be eliminated by measuring strain internally or locally. The works described by (Rowe and Barden 1964; Jardine et al. 1984a and Jamiolkowski et al. 1994) have also highlighted the

importance of local strain measurement in triaxial testing for determination of soil stiffness at strain less than 0.001%.

Figure 3.5 Error sources associated with the strain measurement system in a triaxial apparatus (from Baldi et al. 1998)

Yimsiri et al. (2005) performed triaxial tests on London clay specimens using self-releasing cantilever type local displacement transducers (LDTs) modified from the LDT design of Goto et al. (1991). The working principles of LDTs are illustrated in Figure 3.6 and the strain of the soil specimens can be calculated as the ratio of the difference between the change in displacements of the top and bottom LDTs over the initial distance between the two LDTs.

Figure 3.6 Working principle of the self-releasing LDTs (from Yimsiri et al. 2005)

The lateral strain measurement device which was first introduced by Bishop and Henkel (1957) formed the basis for many later developments for strain measurements. Lateral strain was then measured by a mechanical device which relied on a direct contact between the soil and the measurement device. In the early 1970s, Atkinson introduced internal axial strain measurements using submersible dial gauges attached to the platens. Later submersible linear variable differential transformers (LVDTs) were commonly used for the axial strain measurements. There are several axial strain measurement devices such as: (Burland and Symes 1982)'s inclinometer level which was further developed by Jardine et al. (1984a; 1985a; 1985b) and Ackerly et al. (1987), Hall effect gauges introduced by Clayton and Khatrush (1986) and Clayton et al. (1989) and the local displacement transducer (LDT) developed by Goto et al. (1991). Figures 3.7 to 3.10 illustrate the different types of local strain measurement devices.

Figure 3.7 LVDT on a fixed support (from Costa -Filho 1985)

Figure 3.8 Working principle of inclinometer level (electrolevel) (from Jardine et al. 1984)

Figure 3.9 Hall effect gauge for axial strain measurement (from Clayton and Khatrush 1986)

Figure 3.10 Local displacement transducer (from Tatsuoka 1988)

Each local strain measurement device has both merits and limitations which are reviewed and discussed in detail in Scholey et al. (1995). Among the above-mentioned local strain measurement devices, the local displacement transducer, LDT, developed by Goto et al.

(1991) was chosen to measure axial deformation of soils in this study. The LDT was selected in this study due to its simplicity in fabrication and low cost. The other attractiveness of LDT is that it requires less space in the triaxial cell due to its slim size which provides more working space for other measurement devices in the cell.

Local displacement transducer (LDT) consists of a thin phosphor bronze strip and strain gauges pasted on both sides of the strip. Hoque et al. (1997) tested the LDTs by varying the dimensions of the strips for different specimens' sizes. Table 3.1 shows a summary of the dimensions of the LDT strips. The LDTs can be mounted on the specimen by means of pseudo-hinges as shown in Figure 3.10 to measure the axial deformation of the specimen. The LDTs are positioned in the hinges by its own elastic forces against the hinges glued on the specimen's membrane. Axial deformation causes relative displacement of the hinge and the bending of the LDT. Strains in the strips due to bending are measured by the strain gauges pasted on each side of the strip using a Wheatstone bridge circuit connection. The deformation of the specimen or the relative displacement of the LDT can then be related to the output voltage of the strain gauges. The relative displacement of the LDT can be expressed in term of the output voltage of the gauges using a quadratic equation. A typical relationship of displacement versus output voltage of the LDT is shown in Figure 3.11.

Table 3.1 Dimensions of the LDT strip for specimens having various sizes (from Hoque et al. 1997)

Diameter x Height of specimen (Units in mm)	Width x Length x Thickness of LDT (Units in mm)
35 x 80	3.0 x 60 x 0.20
50 x 100	3.5 x 80 x 0.20
75 x 150	3.5 x 105 x 0.30
	4.0 x 120 x 0.30
100 x 200	4.0 x 170 x 0.30
	4.5 x 170 x 0.30
300 x 600	5.0 x 500 x 1.0
	6.0 x 500 x 1.0

Figure 3.11 Typical calibration characteristics of LDT (from Goto et al. 1991)

3.2.2.2 Differences between Wheatstone Bridge and Anderson Loop

The Wheatstone bridge is an electrical bridge circuit that has been used to measure resistance, inductance and capacitance. It can be found in many instruments which use a change in resistance to indicate a change in some physical quantity such as strain, temperature or moisture. Essentially the Wheatstone bridge is a circuit composed of four resistances R_1 , R_2 , R_3 and R_4 as shown in Figure 3.12. The output voltage, V_{out} , can be expressed as a function of the resistances R_1 , R_2 , R_3 and R_4 and the input voltage V_{in} as follows:

$$V_{out} = V_{in} \left(\frac{R_2}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right) \quad (3.1)$$

V_{out} will be zero when the bridge is perfectly balanced, i.e.

$$R_2 R_4 = R_1 R_3 \quad (3.2)$$

Normally R_1 , R_2 , R_3 and R_4 are selected such that they are the same i.e.

$$R = R_1 = R_2 = R_3 = R_4 \quad (3.3)$$

If there is a change in one of the resistances, say $R' = R + \Delta R$, then

$$V_{out} = \frac{\Delta R}{4R} V_{in} \quad (3.4)$$

For circuit that uses a long lead wire, the resistance of the wire will be part of Equation 3.4 i.e. $R = R + R_{wire}$. The Anderson loop was originally developed to deal with the variation in lead-wire resistance when connecting high temperature strain gauges to conventional Wheatstone bridge signal conditioning (Anderson 1998 and Anderson 1995). In contrast to the Wheatstone bridge which uses a voltage source and a passive subtractor, the

Anderson loop uses a current source and an active, dual-differential subtractor. The Anderson loop circuit is shown in Figure 3.13. From the figure, if $R_{\text{ref}} = R$ and the gains G_1 and G_2 are unity:

$$V_{\text{out}} = i\Delta R \quad (3.5)$$

Equation 3.5 shows that V_{out} is directly proportional to the change in resistance ΔR . It is not affected by the resistances of the lead wires.

Figure 3.12 Schematic diagram of full Wheatstone bridge circuit

Figure 3.13 Schematic diagram of Anderson loop circuit

3.2.2.3 Fabrication of Local Displacement Transducer (LDT)

The local displacement transducer (LDT) consists of a rectangular strip of thin phosphor bronze with strain gauges pasted on both sides of the strip to form a Wheatstone bridge (Goto et al. 1991). The LDT was developed to measure local strain on a triaxial specimen. The LDT is mounted on the middle two-thirds of the specimen by means of pseudo-hinges glued to the rubber membrane as shown in Figure 3.14 to measure axial deformation, δ , of the specimen. As the specimen deforms during shearing (Figure 3.14), the LDT bends due to the relative displacement of the hinges. The strains in the strips due to bending are measured by the strain gauges pasted on each side of the strip. The deformation of the specimen can then be related to the output voltage of the strain gauges. In this study, LDTs (3.5 mm width x 80 mm length x 0.2 mm thick) were fabricated with the Wheatstone bridge circuit and the Anderson loop circuit. In both circuits, $350\ \Omega$ strain gauges (TML FLA-2-350-11) with gauge length of 2 mm and gauge factor of 2.1 were used. For the Wheatstone bridge circuit, the LDT was fabricated according to Goto et al. (1991) using four strain gauges. Referring to Figure 3.12 for the Wheatstone bridge circuit, resistances R_1 and R_3 were on the tension side of the strip and resistances R_2 and R_4 were on the compression side of the strip. For the Anderson loop circuit, a low cost high accuracy instrumentation amplifier AD620 was used as a voltage amplifier and also as a differential subtractor. It requires one external resistor to set a gain ranging from 1 to 1000. The gain for the amplifier can be computed as:

$$G = \frac{49.4\ \text{k}\Omega}{R_G} + 1 \quad (3.6)$$

where R_G is the resistance of the resistor to set the required gain. The AD620 has a common mode rejection ratio, CMRR, of 73 dB. An amplifier with high CMRR is important

when the signal of interest is represented by a small voltage change. A dual current source, REF200 Burr Brown Corporation (1988), was chosen to provide the constant current supply for the Anderson loop circuit. REF200 can provide current source ranging from 500 μA to 10 mA with an external operational amplifier, OPA602 Texas Instrument (2002), as shown in Figure 3.15.

Figure 3.14 Bending behaviour of LDT under loading

Figure 3.15 Adjustable current source using OPA602 (from Burr Brown Corporation 1988)

The output voltage from the Anderson loop circuit can be increased by varying the number of strain gauges on the LDT strip, increasing the supply current, I_o , and increasing the amplifier gain. To investigate the improvement in output voltage by increasing the

number of strain gauges, the Anderson loop circuit was fabricated using one, two and four strain gauges as shown in Table 3.2 while maintaining the same gain and current supply. REF200 was set up according to the configuration shown in Figure 3.16 to provide a constant current of $400\ \mu\text{A}$ for this investigation.

Figure 3.16 Configuration for current source of $400\ \mu\text{A}$ (from Burr Brown Corporation 1988)

Table 3.2 Block diagrams for one, two and four strain gauges in Anderson loop circuit

<p>(a)</p> <p>One gauge</p>		$V = (R + \Delta R)I_o$ $V_{ref} = R_{ref} I_o = RI_o$ $V_{out} = \Delta RI_o$
<p>(b)</p> <p>Two gauges</p>		$V_1 = (R + \Delta R)I_o$ $V_2 = (R - \Delta R)I_o$ $V_{out} = 2\Delta RI_o$
<p>(c)</p> <p>Four gauges</p>		$V_1 = (2R + 2\Delta R)I_o$ $V_2 = (2R - 2\Delta R)I_o$ $V_{out} = 4\Delta RI_o$

Figures 3.17 and 3.18 show the schematic diagrams of one and two strain gauges pasted on the LDT, respectively, for testing of the Anderson loop circuit. For four strain gauges,

two strain gauges were pasted on each side of the LDT similar to that for the Wheatstone bridge circuit. Figure 3.19 shows the output voltage using one, two and four strain gauges. From the figure, it clearly shows that the output voltage increases with the number of strain gauges. This result is expected as given in the expressions in Table 3.2. The four-gauge Anderson loop circuit produced the largest output voltage and therefore this configuration was used for subsequent evaluation of the Anderson loop circuit.

Figure 3.17 Single gauge LDT in Anderson loop circuit

Figure 3.18 Two gauges in series connection in Anderson loop circuit

Figure 3.19 Output voltage using one, two and four strain gauges in the Anderson loop circuit

The magnitude of the output voltage can be increased by increasing the current supply I_o .

The current supply, I_o , from REF200 can be varied according to the following relationship:

$$I_o = (N+1) 100 \mu A, \quad N = NR/R \quad (3.7)$$

where NR and R are the resistors. Using the above relationship, an appropriate N value can be set to obtain the required current supply I_o . Tests were conducted using different I_o from $500 \mu A$ to 10 mA by setting different N value. A digital multimeter was used to measure the actual I_o . From these tests, it was found that when N value was set to obtain I_o larger than 3 mA , self-heating of the strain gauge occurred leading to unstable readings. Therefore, current supply values of 1 mA , 2 mA , and 3 mA were used to study the effects of I_o on the response of LDT with Anderson loop circuit.

Amplifiers indicated in Table 3.2 can be selected to gain up the output voltage. However output voltage saturation can occur if a large gain is used. It was observed that the four-gauge Anderson loop circuit shown in Table 3.2 was unstable when a large gain of 2000 was used. Therefore the circuit was modified by adding another stage of differential subtraction as shown in Figure 3.20. The gains of Amplifier1 and Amplifier2 were set at unity. The gains of Amplifier3 and Amplifier4 were adjusted to optimize the resolution and measurable range of LDT. The final gains for Amplifier3 and Amplifier4 were 500 and 4.5, respectively. The overall gain employed for the Anderson loop circuit was 2250. A low-pass filter of 10 Hz was added to filter out electrical noise. The second differential subtractor amplifier, Amplifier4, was powered from a separate power source as shown in Figure 3.20. The final Anderson loop circuit for the LDT is illustrated in Figure 3.20. The final output voltage, V_4 , can be calculated as follows:

$$V_4 = 4G\Delta RI_o - G_4 V_{\text{offset}} \quad (3.8)$$

where G is the overall gain (=2250), G_4 is the gain for Amplifier4 (=4.5), ΔR is the change in resistance of the resistor, I_o is the supply current and V_{offset} is the fixed voltage to adjust the output voltage of LDT at initial displacement to a required value.

3.2.2.4 Response of LDT

The responses of the LDTs using four strain gauges with Anderson loop circuit and Wheatstone bridge circuit were studied using the set-up shown in Figure 3.21. The digital micrometer shown in Figure 3.21 has a resolution of 1 μm . The gain employed in both Anderson loop and Wheatstone bridge circuits was identical, i.e. 2250. In the LDT with Wheatstone bridge circuit, the instrumentation amplifier AD620 and a 10 Hz filter was similarly added to amplify the signal and filter out electrical noise, respectively.

Figure 3.20 Block diagram of LDT with Anderson loop circuit

Figure 3.21 Set-up to measure response of LDT

For ease of discussion, the LDT with Anderson loop circuit is denoted as LDT_A and the LDT with Wheatstone bridge circuit is denoted as LDT_W hereinafter. As the Anderson loop uses a current source and the Wheatstone bridge uses a voltage source, an

equivalent input needs to be established for fair comparison. As LDT_A was evaluated for I_o of 1 mA, 2 mA and 3 mA, the excitation voltage V_{in} for LDT_W were set as 0.7 V, 1.4 V and 2.1V, respectively, to obtain the same current flowing through the Wheatstone bridge circuit. Sensitivity of the LDT can be adjusted by changing the resolution of the data logger, voltage range and input voltage. Sensitivity of the LDT was recorded with a 12-bit resolution data logger for a voltage range of ± 10 volts. Therefore, the resolution voltage (RV) is 0.00488 V/bit. Figure 3.22 shows the responses of LDT_A and LDT_W at different I_o or V_{in} . The relationship between the output voltage and displacement of LDT is nonlinear and can be represented by a polynomial equation of order 2 (Hoque et al. 1997). As shown in Figure 3.22, resolution of the LDT increases significantly with increases in I_o or V_{in} . Comparison of the responses of LDT_A with LDT_W with equal current flowing in the circuit shows that the output voltage of LDT_A is about twice of LDT_W.

The performance of the LDT depends on its components and manufacturing method (Hoque et al. 1997). For the LDT_A and LDT_W, the same components and manufacturing method were used. When used in a triaxial apparatus, the LDT is mounted between a pair of pseudo hinges. Slippage at the hinges may lead to “apparent” inaccuracy of the LDT. To evaluate the effect of slippage at the pseudo hinges, the responses of LDT_A and LDT_W were tested in a triaxial cell by mounting them in the pseudo hinges attached to the top and bottom platens of the triaxial apparatus. The top platen was rigidly connected to the piston and a submersible linear variable differential transformer (LVDT) was used to check the displacement of the LDTs.

Figure 3.22 Comparison of responses of LDT_A with LDT_W under different excitations

Figures 3.23 (a) and (b) show the quasi-static loading and unloading curves for LDT_A and LDT_W, respectively. Figures 3.24 (a) and (b) show the loading and unloading curves for LDT_A and LDT_W, respectively, at different rates. The slopes of the loading and unloading curves in Figure 3.24 are tabulated in Table 3.3. Also shown in Table 3.3 are the coefficients of determination r^2 and average slopes of the loading and unloading curves. Two observations can be made from Table 3.3: (1) Slopes of the loading curves and unloading curves for LDT_A and LDT_W are comparable with values of r^2 greater than 0.9997 indicating that slippage at the pseudo hinges during loading or unloading was negligible; and (2) There is a difference in the slopes of the loading and unloading curves. The difference in slopes of the loading and unloading curves is attributed to hysteresis and movement of the LDT at the pseudo hinges when the loading direction was reversed. Further improvement to the pseudo hinges is needed when using LDT in situation where there is a reversal in loading condition, e.g. cyclic loading.

Figure 3.23 Responses of LDT_A and LDT_W in a triaxial apparatus

(a) LDT_A

(b) LDT_W

Figure 3.24 Responses of LDT_A and LDT_W at different loading rates in a triaxial apparatus

Table 3.3 Slopes and coefficient of determination r^2 of loading and unloading curves for LDT_A and LDT_W in a triaxial apparatus

LDT	Loading rate (mm/s)	Load		Unload	
		Slope	r^2	Slope	r^2
LDT_A	0.003	0.9642	0.9999	1.0050	1.0000
	0.005	0.9890	0.9999	1.0066	1.0000
	0.01	0.9685	0.9999	0.9889	0.9999
	Average	0.9739	-	1.0002	-
LDT_W	0.003	1.0007	0.9997	1.0215	0.9999
	0.005	0.9788	0.9998	1.0187	0.9999
	0.01	0.9649	0.9998	1.0054	0.9998
	Average	0.9815	-	1.0152	-

Testing duration of some triaxial tests may be long (several hours to several days) and therefore the stability of the LDT over a long test period is important to obtain reliable measurements. The stability of LDT_A and LDT_W were investigated by mounting the LDTs on a dummy steel cylindrical specimen. The pseudo hinges were glued to the surface of the dummy specimen and readings of the LDTs were recorded over a long period of 36 hours. It was observed that both LDT_A and LDT_W experienced an output voltage drift when large I_o (> 3 mA) or V_{in} (> 2.1 V), was used. The output voltage drift due to large I_o or V_{in} is a result of self-heating caused by power dissipation in the strain gauge. Self-heating is difficult to calculate as it is dependent on strain gauge grid area, gauge resistance, heat-sink properties of the mounting surface, environmental operating temperature range of the gauge installation, installation and wiring technique and waterproofing materials but can be observed experimentally. The effects of self-heating on the output voltage for LDT_A at various I_o (2, 3, 4, 5 and 7 mA) are shown in Figure 3.25. The monitoring period was for 50 minutes except for $I_o = 5$ mA and $I_o = 7$ mA which were monitored for 15 and 10

minutes, respectively, to avoid damaging LDT_A. When I_o is greater than 3 mA, the output voltage gradually increases. In general, self-heating can be avoided by using the lowest input current or voltage in conjunction with the overall amplifier gains that will provide the required sensitivity. It was found that I_o should not exceed 3 mA for LDT_A and V_{in} should not exceed 2.1 V for LDT_W to avoid self-heating effect in the strain gauge for this particular experimental setup. Figure 3.26 shows the output voltage for LDT_A ($I_o = 3$ mA) and LDT_W ($V_{in} = 2.1$ V) over a period of 36 hours. It was observed that the overall drift of the LDTs' output voltage was within $1 \mu\text{m}$ ($\pm 0.5 \mu\text{m}$) corresponding to the resolution of the LDT. For a LDT of 80 mm span length, this will correspond to a strain resolution of 6.25×10^{-6} or 0.000625% similar to that reported by Goto et al. (1991). The typical strain range for dynamic tests (hollow cylinder, cyclic triaxial, simple shear and torsional shear) ranges from 0.001% to 1%.

Figure 3.25 Output voltage drift due to self-heating effect in strain gauge

Figure 3.26 Stability of output voltage of LDT_A and LDT_W over 36 hours

The performances of the LDT can be improved following the recommendations of Hoque et al. (1997). However the purpose in this study is to evaluate the improvement in performance of the LDT using Anderson loop circuit instead of the Wheatstone bridge circuit. Both LDT_A and LDT_W were manufactured in an identical fashion with the same material, strain gages, adhesive, gage terminals, water proofing coatings and lead wires. The data acquisition system used for the experiment was identical. Therefore the differences in performance of LDT_A and LDT_W could be attributed to the circuitry. Accuracy of transducers is usually quantified in terms of nonlinearity, hysteresis and repeatability errors (e.g. Fraden 2004). However the calibration equations for LDT_A and LDT_W are nonlinear. The least-squares calibration equations for LDT_A and LDT_W are, respectively,

$$\delta = 0.0031V_{\text{out}}^2 + 0.0645V_{\text{out}} + 0.3080 \quad (3.9a)$$

$$\delta = 0.0048V_{\text{out}}^2 + 0.0451V_{\text{out}} + 0.0931 \quad (3.9b)$$

The coefficients of determination, R^2 , for Equations 3.9 (a) and (b) are 1 and 0.995, respectively. The nonlinearity error is replaced with deviation error given as the maximum deviation between the actual displacement and the computed displacement using Equation 3.9 divided by the calibration range. Therefore, comparison of the accuracy of LDT_A ($I_o = 3 \text{ mA}$) and LDT_W ($V_{\text{in}} = 2.1 \text{ V}$) is made in terms of deviation, hysteresis and repeatability errors. Two sets of loading and unloading curves for LDT_A and LDT_W are shown in Figure 3.27 (a) and (b), respectively. The deviation, hysteresis and repeatability errors as a percentage of full-scale output (FSO) where FSO is 0.4 mm are summarized in Table 3.4. Table 3.4 shows that LDT_A has lower non-linearity, hysteresis and repeatability errors compared with LDT_W.

Table 3.4 Accuracy of LDT_A and LDT_W as a percentage of full-scale output

LDT	Deviation error	Hysteresis error	Repeatability error
LDT_A	$\pm 0.92\%$	$\pm 1.00\%$	$\pm 0.60\%$
LDT_W	$\pm 4.02\%$	$\pm 1.30\%$	$\pm 1.98\%$

Figure 3.27 Two sets of loading curves for LDT_A and LDT_W

3.2.2.5 Summary

The Wheatstone bridge has been successfully employed in many strain-gauged based measuring instruments. In this study, the Anderson loop which uses a current source was investigated as an alternative to the Wheatstone bridge which uses a voltage source in the Goto et al. (1991) local displacement transducer. Caution should be exercised in the selection of input current for the local displacement transducer with Anderson loop or input voltage for the local displacement transducer with Wheatstone bridge as self-heating in the strain gauges can occur when the input current or voltage is too high. Self-heating in strain gauges is only evident in long duration tests. With proper selection of input current or voltage, the local displacement transducer is stable with minimum output voltage drift of about 1 μm . For similar current flowing through the circuits, the local displacement transducer with Anderson loop was found to be twice as sensitive as the Wheatstone bridge. The local displacement transducer with Anderson loop offers better performance in terms of lower deviation, hysteresis and repeatability errors than the Wheatstone bridge circuit with the addition of few circuitries.

3.2.3 Specimen Preparations and Procedure

Different types of soils (kaolin, reclaimed sand B and undisturbed residual soils) were tested in the triaxial apparatus to measure the strength and stiffness characteristics of the soils. Table 3.5 shows the summary of triaxial tests performed using different soils in this study.

Table 3.5 Summary of triaxial compression tests performed for different types of soils

Soil Types	Specimen	Parameters studied	Saturated Test	Unsaturated test	
			CU	CU	CW
Kaolin	Consolidated	Matric suction and strain rate	--	--	Yes
Reclaimed sand B	Remoulded	Matric suction and net confining pressure	Yes	--	Yes
Residual soil	Undisturbed	Matric suction and strain rate	Yes	Yes	Yes

3.2.3.1 Kaolin

Commercially available refined kaolin was used. Table 3.6 summarises the basic properties of the kaolin and Figure 3.28 shows the grain size distribution curve of the kaolin. The kaolin was classified as silt with high plasticity (MH) according to the Unified Soil Classification System (USCS). Kaolin was mixed with water in a weight ratio of 1:1 to form a slurry and consolidated using a consolidation tank under an axial pressure of 100 kPa. The consolidation curve of the kaolin is shown in Figure 3.29. Specimens of 50 mm diameter and 100 mm height were trimmed from the consolidated sample. The trimmed specimens were placed in the pressure plate apparatus to impose the required matric suction in order to reduce the consolidation time in the triaxial cell. After obtaining matric suction equalization in the pressure plate apparatus, the specimen was transferred to the triaxial apparatus for testing. The trimmed specimen was placed on the 5-bar high air-entry ceramic disk which was fixed to the base pedestal using epoxy for unsaturated soil tests. The specimen was enclosed in the rubber membrane and the top cap with a fixed porous disk was lowered onto the specimen. After securing the rubber membrane on the

top cap and the pedestal with O-rings, local displacement transducers (LDTs) were attached on the specimen for axial strain measurements and proximity sensors were positioned for radial strain measurements.

Figure 3.28 Grain size distribution of kaolin

Table 3.6 Summary of basic properties of kaolin

Properties	Value
Specific Gravity, G_s	2.65
Liquid Limit, LL (%)	73
Plastic Limit, PL (%)	41
Plastic Index, PI (%)	32
Grain Size Distribution	
-Silt	82
-Clay	18
Unified Soil Classification System (USCS)	MH

Figure 3.29 Consolidation curve of kaolin

Prior to shearing, the specimen was isotropically consolidated under a net confining pressure of 100 kPa and the required matric suction. After obtaining matric suction equalization in the cell, the specimen was sheared with drained pore-air and undrained pore-water condition. The specimens were tested at three different initial matric suctions (20 kPa, 200 kPa and 400 kPa) and using three different strain rates (0.06 mm/min, 0.1 mm/min and 1 mm/min). Figure 3.30 shows the soil-water characteristic curve of kaolin plotted with the initial water contents of all the tested specimens. The specimens shown in Figure 3.30 were labelled as CW-a-b in which CW denotes constant water content test, a is the value of imposed matric suction and b is the number representing the strain rate used (i.e. strain rates of 1 mm/min, 0.1 mm/min and 0.06 mm/min were represented using 1, 2, and 3, respectively).

Figure 3.30 Soil-water characteristic curve of kaolin with tested specimens

3.2.3.2 Reclaimed Sand B

Reclaimed sand from Jurong reclamation site (Reclaimed sand B) is medium sand and classified as poorly graded sand (SP) according to Unified Soil Classification System. The basic properties and the grain size distribution curve of Reclaimed sand B are presented together with the other tested sands in Section 3.4.3.

Reconstituted sand specimens were prepared by moist tamping method (Ladd 1978) at a water content of 10%. The specimens were prepared in a split mould with internal dimensions of 50 mm diameter and 100 mm height. Required amount of dry sand was weighed and mixed thoroughly with predetermined amount of de-aired water in a container. A thin plastic sheet was used to line the inner surface of the mould to reduce adhesion between the sand particles and the mould for ease of removal of the mould. To

obtain a more uniform sand specimen, moist tamping was done by dividing the specimen height into approximately equal layers of 25 mm. The energy of tamping was increased from the lowest layer to the topmost layer. After moist tamping, the specimen was placed on the pedestal of the triaxial apparatus. The split mould was then removed carefully followed by removal of the plastic sheet. The specimen was enclosed in a rubber membrane and the top cap was lowered onto the sand specimen. The top cap has a fixed coarse porous stone while the pedestal has a fixed 1-bar high air-entry ceramic disk. After securing the rubber membrane on the top cap and the pedestal with O-rings, local displacement transducers (LDTs) were attached on the specimen for axial strain measurements and proximity sensors were positioned for radial strain measurements.

Reconstituted unsaturated sand specimens were either saturated or left unsaturated (as compacted condition) prior to consolidation depending on the initial matric suction. For the specimens tested at initial matric suction of 3 kPa, saturation was performed to ensure that the specimen was fully saturated prior to consolidation. Skempton's pore-pressure parameter B obtained was greater than 0.7 for the saturated specimens. From back-calculation of degree of saturation from the specimen's final water content at the end of the test, the specimen had initial degree of saturation greater than 100%. Therefore, it can be assumed that the specimen has reached full saturation at B value greater than 0.7 (Black and Lee 1973 and Head 1980). After saturation, the specimen was consolidated under the required net confining pressure. At the end of the consolidation stage, matric suction equalization was commenced using the axis-translation technique (Hilf 1956). The specimens tested at initial matric suctions of 5 kPa were consolidated to the required net

confining pressure without saturation. Matric suction equalization was then performed after the consolidation stage was complete. In this manner, the time required to reach matric suction equilibrium was reduced. The water that drained out from the specimen during matric suction equalization was monitored via DPVC. Matric suction equilibrium was assumed to be attained when the amount of water that drained out had either leveled off or the change in water volume with time had become negligibly small. Generally, it took about 3 to 4 days to reach matric suction equilibrium for all the tested specimens. The measured water content and the soil-water characteristic curve (SWCC) of the sand obtained using the Tempe pressure cell apparatus are shown in Figure 3.31. This figure shows that the water contents of the specimens beyond initial matric suctions of 5 kPa under different net confining pressures are close to the SWCC. The air-entry value of the sand from the SWCC is about 2 kPa. It is expected that the air-entry value of the sand under net confining pressure will be higher than 2 kPa. The void ratio of the sand specimen under net confining pressure will be smaller as compared with the one under zero net confining pressure and therefore the rate of water draining out from the sand specimen under net confining pressure will be slower, resulting in the higher air-entry value of the sand (Ng and Pang 2000).

The sand specimen was sheared monotonically under the applied initial matric suction and net confining pressure to obtain shear strength parameters. During shearing the valve connecting to the air pressure supply was kept open while the valve connecting to the water compartment below the ceramic disk was closed. In other words, air was allowed to drain but no drainage of water was allowed during shearing, i.e. constant water content test (Fredlund and Rahardjo 1993). Strain rate of 1 mm/min was used and multi-stage shearing was performed to study the effect of initial matric suctions using the same

specimen. Five initial matric suction values $[(u_a - u_w)_i = 3, 5, 10, 15, \text{ and } 20 \text{ kPa}]$ which span the entire SWCC were chosen to study the effect of initial matric suction on the mechanical behaviour of the sand. Mainly 2-stage and 3-stage shearing tests were carried out at three different net confining pressures. An additional 5-stage shearing test at net confining pressure of 200 kPa was performed to compare with the results obtained from 2-stage and 3-stage shearing tests. The test was stopped at the end of each shearing stage when failure of the specimen was imminent. The specimen was then unloaded using the same strain rate. The specimen was reconsolidated to the next required initial matric suction under the same net confining pressure for the next shearing stage.

Figure 3.31 Measured initial water contents for all the tested specimens with SWCC from Tempe pressure cell test

Table 3.7 summarizes the test conditions used in this study. Air pressure was fixed at 50 kPa for all the tests and the required initial matric suction was imposed by changing the

pore-water pressure via the DPVC. In Table 3.7, saturated and unsaturated sand specimens were annotated as CUa and CWa-b, in which CU and CW denote consolidated undrained and constant water content tests, respectively, a is the value of imposed effective or net confining pressure in kPa and b is the value of imposed initial matric suction in kPa. For multi-stage constant water content tests, the sand specimens were annotated as CWa-b-c-d... where c-d... denotes the subsequent imposed initial matric suction.

Table 3.7 Summary of test conditions

Saturated Tests																
Test specimen	*	Shearing stage														
		1			2			3								
		σ_3 (kPa)	u_a (kPa)	u_w (kPa)	σ_3 (kPa)	u_a (kPa)	u_w (kPa)	σ_3 (kPa)	u_a (kPa)	u_w (kPa)						
CU50-100-200	Y	250	200	200	250	150	150	250	50	20						
Unsaturated Tests																
Test specimen	*	Shearing stage														
		1			2			3			4			5		
		σ_3 (kPa)	u_a (kPa)	u_w (kPa)	σ_3 (kPa)	u_a (kPa)	u_w (kPa)	σ_3 (kPa)	u_a (kPa)	u_w (kPa)	σ_3 (kPa)	u_a (kPa)	u_w (kPa)	σ_3 (kPa)	u_a (kPa)	u_w (kPa)
CW50-3-15	Y	100	50	47	100	50	35									
CW50-5-10-20	N	100	50	45	100	50	40	100	50	30						
CW100-3-15	Y	150	50	47	150	50	35									
CW100-5-10-20	N	150	50	45	150	50	40	150	50	30						
CW200-3-15	Y	250	50	47	250	50	35									
CW200-5-10-20	N	250	50	45	250	50	40	250	50	30						
CW200-3-5-10-15-20	Y	250	50	47	250	50	45	250	50	40	250	50	35	250	50	30

*Saturation of sand specimen prior to consolidation

3.2.3.3 Undisturbed Residual Soil from Bukit Timah Granite

Residual soils cover about two-thirds of the land area of Singapore. The groundwater tables in these residual soils are well below the ground surface and thus the bulk of the residual soils are unsaturated (Leong et al. 2003). The residual soils are derived from two formations, namely: Bukit Timah Granite and Jurong Formation. Bukit Timah Granite is one of the oldest formations in Singapore and was formed approximately 200 to 250 million years ago. Heavily weathered residual soils formed the top portions of the Bukit Timah Granite up to depths of 60 m. Generally, residual soils from Bukit Timah Granite consist of reddish to yellow brown clayey soils. The Jurong Formation was formed about 100 to 200 million years ago. The upper portions of the Jurong Formation were heavily weathered and the residual soils can be found to depths of 30 m. The residuals soils of Jurong Formation consist mainly of silty clays.

Figure 3.32 shows the geological map of Singapore with the location of the sampling site. The undisturbed residual soils from Bukit Timah Granite were tested in the triaxial apparatus to measure the shear strength of unsaturated soils. The average initial water content and the average void ratio of the undisturbed residual soil sample were 42% and 1.3, respectively. The residual soil was classified as silt with low plasticity (ML) according to USCS with LL=45% and PL=30%. The sand and fines content of the residual soils were 45% and 55%, respectively. Figure 3.33 shows the grain size distribution and Table 3.8 summarizes the basic properties of the residual soils.

Figure 3.32 Geological map of Singapore with location of sampling site

Figure 3.33 Grain size distribution of the residual soil from Bukit Timah Granite

Table 3.8 Summary of basic properties of undisturbed residual soil

Soil properties	Value
Depth (m)	2.0-3.0
Liquid limit (%)	45
Plastic limit (%)	30
Plasticity index (%)	15
Sand (%)	45
Silt (%)	35
Clay (%)	20
Specific gravity	2.74
Unified Soil Classification System (USCS)	ML (Low plasticity silt)

The undisturbed residual soil samples were obtained using Mazier sampling (Brand and Philipson 1985). The residual soil specimens tested in this study were from depths of 2 to 3 m. Soil specimens of 50 mm diameter and 100 mm height were trimmed from 70 mm diameter undisturbed Mazier residual soil samples. The specimen after trimming is shown Figure 3.34. The trimmed specimen was placed on the base pedestal of the triaxial apparatus with a fixed 5-bar high air entry ceramic disk. The specimen was then enclosed in a rubber membrane and the top cap with a fixed porous disk was lowered onto the specimen. The triaxial apparatus top cap was rigidly connected to the piston. After securing the rubber membrane on the top cap and the pedestal with O-rings, local displacement transducers (LDT) were attached on the specimen for axial strain measurements and proximity sensors were positioned for radial strain measurements. The specimens were saturated in the triaxial cell prior to consolidation to ensure identical initial water content in the specimens. The pore-water parameter B obtained was greater than 0.96 for the soil specimens during saturation. Near or full saturation was assumed when pore-water pressure parameter B was greater than 0.96 (Head 1980).

Figure 3.34 Residual soil specimen after trimming

After saturation, the soil specimens were isotropically consolidated to the in situ effective confining pressure and subsequently at the required net confining pressure ($\sigma_3 - u_a$) and matric suction ($u_a - u_w$) via the axis-translation technique (Hilf 1956). For the application of matric suction, a constant air pressure of 450 kPa was supplied at the top of the soil specimen through a porous disk and water pressure was controlled via a digital pressure volume controller (DPVC) through a 5-bar high air entry ceramic disk. The amount of water flowing out of the soil specimen during matric suction equalization was monitored via DPVC. Matric suction equalization was assumed to have been completed when the amount of water flowing out of the soil specimen had levelled off. Multi-stage shearing test using cyclic loading procedure was adopted to study the stress-strain behaviour and shear strength of the undisturbed residual soil at different initial matric suctions. Shearing was carried out in four stages at initial matric suctions of 50 kPa, 100 kPa, 200 kPa and 400 kPa under net confining pressure of 50 kPa. At the end of each matric suction equalization stage, the specimen was sheared until failure was imminent and unloaded before

imposing the next required matric suction. The soil specimens were tested under two different test conditions: consolidated undrained test (CU) and constant water content test (CW). Three different strain rates (0.5 mm/min, 0.1 mm/min and 0.006 mm/min) were used for both CU and CW tests. In CU test, both pore-air and pore-water pressures were used for both CU and CW tests. In CU test, both pore-air and pore-water pressures were not allowed to drain during shearing. In CW test, air was allowed to drain freely, but, pore-water pressure was undrained during shearing. The variations of pore-air and pore-water pressures during shearing were recorded. At the end of the test, water content was measured to obtain the variation of water content with respect to the imposed matric suction. Figure 3.35 shows the water contents at different matric suctions for CU and CW tests tested at different loading rates.

Figure 3.35 Water content versus matric suction for all the tested specimens for CU and CW tests at different loading rates

3.3 Bender Element Test

3.3.1 Overview of Bender Element Test

Small-strain stiffness ($G_{\max}$) of soil is an important parameter for a variety of geotechnical design applications. The strain level associated with $G_{\max}$ is usually about 0.001% and below. Bender elements have been used in triaxial cell, resonant column, oedometer and direct simple shear apparatuses to measure $G_{\max}$ (Bates 1989; Dyvik and Madshus 1985; Brignoli et al. 1996; Pennington 1999). Bender element consists of two sheets of piezoelectric ceramic material such as lead zirconate titanate, barium titanate, or lead titanate sandwiching a centre shim of brass, stainless steel, or other ferrous nickel alloys to add strength to it. The bender element is an electromechanical transducer capable of converting mechanical energy into electrical energy and vice versa. When a driving voltage is applied to the bender element, the polarization will cause a bending displacement and therefore the bender element acts as a signal generator. When the element is forced to bend, a voltage is generated and thus the bender element can also act as a signal receiver. In this study, a pair of bender elements was used for the wave velocity measurement.

3.3.1.1 Characteristics of Bender Element

There are two types of piezoceramic elements, x-poled and y-poled piezoceramic elements, and their directions of polarization are shown in Figure 3.36. For x-poled piezoceramic element, the directions of polarization for each of the two layers are either pointing in opposite directions or towards each other. For y-poled piezoceramic element, the directions of the polarization for each of the two layers are pointing in the same direction.

Figure 3.36 Types of piezoceramic element (from Piezo 2003)

These piezoceramic elements can be wired in series and parallel configurations. For series configuration, the excitation voltage is applied across the two piezoceramic layers at once, as illustrated in Figures 3.37 (a) and (d). For parallel configuration, the excitation voltage is applied to the piezoceramic layers and the centre shim as shown in Figures 3.37 (b) and (c). To transmit and/or receive shear wave (S-wave), the x-poled and y-poled piezoceramic elements are connected in series and parallel configurations as shown in Figure 3.37 (a) and (b). To transmit and/or receive a compression wave (P-wave), the x-poled and y-poled piezoceramic elements are connected in parallel and series configuration as shown in Figure 4.37 (c) and (d).

As the excitation voltage is applied to each of the piezoceramic layers, the motion sensitivity of the parallel configuration in terms of deflection per unit of applied voltage is greater by a factor of two as compared with the series configuration. In other words, for the same motion, the parallel configuration requires only half of the voltage needed for the series operation. Therefore, the parallel configuration is better suited as a transmitter as it gives a stronger mechanical motion for a given excitation voltage. However, the series

configuration is better suited as a receiver as it gives a higher output voltage for a given mechanical excitation.

Figure 3.37 Wiring configuration using x-poled and y-poled elements (from Piezo System, 2003)

In general, a pair of shear wave piezoceramic elements and a pair of compression wave piezoceramic elements are mounted in the same apparatus to measure both shear wave and compression wave velocities (Schultheiss 1981; Bates 1989; de Alba and Baldwin 1991; Brignoli et al. 1996; Nakagawa et al. 1996; Fioravante and Capoferri 2001;). However, recently, Lings and Greening (2001) introduced a single hybrid element called “bender-extender element”. A simple modification is made to the bender element to produce an extender element, capable of transmitting and receiving compression wave.

The main modification is by changing the bender element wiring configuration from a two-wire configuration to a three-wire configuration and matching it with the corresponding type of polarization. Lings and Greening (2001) further suggested that using a single pair of bender-extender element enables both shear wave and compression wave velocities measurement to be made over exactly the same soil path, which facilitates the determination of the arrival of the shear wave, especially when it is obscured by the near-field effect.

3.3.1.2 Waveform

The two commonly used waveforms to excite the bender element are the square wave and the sinusoidal wave. Leong et al. (2005) used both square wave and sinusoidal wave to investigate the effect of wave on the arrival time determination. Square wave made it harder to determine the arrival time of the receiver signal (Blewett et al. 2000). Leong et al. (2005) highlighted that sinusoidal wave produced lesser distortion at the beginning of the receiver signal. The transmitter and receiver signals were similar in shape if sinusoidal wave is used.

3.3.1.3 Excitation Voltage and Frequency

The magnitude of the excitation voltage does not affect the shape or appearance of the receiver signal but it increases the receiver signal intensity (Leong et al. 2005). It is also important that the applied voltage should be limited to a value which would not cause depolarization of the bender element. Near-field effects affect the shape of the receiver

signal and cause difficulty in determination of arrival time for the receiver S-wave in bender tests. Excitation frequencies were varied to eliminate the near-field effects on S-wave. Wave path length to wavelength ratio (L_{tt}/λ) can be used as a guide to choose the input frequency to minimize the near-field effect on shear wave (Sanchez-Salinero et al. 1986; Arroyo et al. 2002 and Leong et al. 2005). The magnitude of near-field effects were studied by many researchers in terms of wave path length to wavelength ratio. Table 3.9 summarises the reported L_{tt}/λ values at which near-field effects become negligible.

Table 3.9 Summary of wave path length to wavelength ratio

Reference	L_{tt}/λ
Sanchez-Salinero et al. (1986)	>2
Gajo et al. (1997)	Between 6 and 9
Arulnathan et al. (1998)	>1
Pennington (1999)	Between 2 and 10
Arroyo et al. (2002)	> 1.6
Leong et al. (2005)	> 3.33

3.3.2 Test Set-up

The bender element test set-up consists of a function generator (Hewlett Packard Model 33120A), a power amplifier (Piezo EPA-104) and a digital oscilloscope (Yokogawa Model DL750P). The excitation voltage of 10 V_{pp} was generated from the function generator and it was further amplified by the power amplifier. The power amplifier (Piezo EPA-104) can amplify the excitation voltage ranging from 20 V_{pp} to 200 V_{pp}. Sinusoidal waveform was

used to excite the bender elements for S-wave measurements. The transmitter and receiver signals were recorded by the oscilloscope and the recorded wave data were transferred to a personal computer for further data processing. A three-wire configuration was adopted for the bender-extender elements. The x-poled piezoceramic element was installed inside the bottom pedestal, whereas the y-poled piezoceramic element was installed inside the top cap. The piezoceramic elements used were from Piezo System, Inc, USA.

Figure 3.38 Bender element test set-up

The x-poled and y-poled were cut from PTZ5A T220-A4-103X and PTZ5A T220-A4-303Y into sizes of 15.9 x 3.2 x 0.51 mm and 15.9 x 9.5 x 0.51 mm, respectively (Leong et al. 2009). To generate shear wave, the y-poled piezoceramic element was connected in

parallel configuration as a transmitter and the x-poled piezoceramic element was connected in series configuration as a receiver. Figure 3.38 shows the schematic diagram of the bender element test set-up.

3.3.3 Measurement Procedure

The bender element tests for measurement of shear wave velocity were performed using reconstituted saturated and unsaturated sand specimens and undisturbed residual soil specimens. Generally, tests were carried out at different initial matric suctions and confining pressures. After consolidation of the specimen, shear wave velocities were measured using three different input frequencies. Determination of arrival time for shear wave can be ambiguous due to near-field effects. Near-field effects in shear wave become negligible when high excitation frequency is used as shown in Figure 3.39. Figure 3.39 shows the arrival time determined at the zero crossing as suggested in Leong et al. (2009). The near-field effect on shear wave can be identified and the shear wave arrival time can be determined more reliably when L_{tt}/λ was greater than 3.33 as suggested in Leong et al. (2005).

(a)

$$f \text{ (kHz)} = 8$$

$$\Delta t \text{ (}\mu\text{s)} = 320$$

$$L_{tt}/\lambda = 2.6$$

(b)

$$f \text{ (kHz)} = 10$$

$$\Delta t \text{ (}\mu\text{s)} = 310$$

$$L_{tt}/\lambda = 3.1$$

(c)

$$f \text{ (kHz)} = 25$$

$$\Delta t \text{ (}\mu\text{s)} = 365$$

$$L_{tt}/\lambda = 9.1$$

Figure 3.39 Typical shear waves arrivals at different excitation frequencies

3.4 Resonant Column Test

Resonant column test is one of the most commonly used laboratory test for measuring stiffness-strain relationship at low strains. The resonant column test consists of a solid or hollow cylindrical specimen that is excited to vibrate in one of its natural modes. Wave velocity of the specimen can be determined when the frequency at resonance is known. The cylindrical specimen can be excited in longitudinal and torsional mode to obtain longitudinal and shear wave velocity. Dynamic soil properties (e.g. shear stiffness, Young's modulus and material damping) can be obtained by performing resonant column test.

3.4.1 Resonant Column Apparatus

There are two types of resonant column apparatus. The main difference between the two types is the end conditions of the specimen mounted in the apparatus. Figure 3.40 shows the schematic diagram of the test set-up for a free-free end condition resonant column apparatus used by (Hardin and Richart 1963). The other commonly used resonant column apparatus is a fixed-free type. Figure 3.41 shows the typical fixed-free solid cylindrical resonant column apparatus with a plan view of drive plate and the elevation view of the test set-up. The schematic diagram of a fixed-free hollow cylindrical resonant column apparatus is shown in Figure 3.42. When a cylindrical solid specimen is excited in torsional mode, maximum shear strain occurs at the circumference and minimum shear strain (i.e. shear strain = 0) occurs at the centre. It is therefore difficult to determine a representative shear strain when using solid cylindrical specimen. Thus hollow cylindrical specimen is often used in resonant column apparatus to measure dynamic properties of soils at large

strain as the maximum and minimum shear strain difference is insignificant in hollow specimen (Drnevich et al. 1966 and Drnevich et al. 1967).

Figure 3.40 Schematic diagram of resonant column device, free-free end condition (from Das 1993)

Figure 3.41 Typical fixed-free resonant column apparatus: (a) plan view of drive plate and (b) elevation view of device with solid cylindrical specimen (from Kramer 1996)

Figure 3.42 Typical fixed-free hollow specimen resonant column apparatus (from Das 1993)

3.4.2 Test Set-up and Calibration

The resonant column apparatus (RCA) used in this study is manufactured by (GDS Instruments 2008) as shown in Figure 3.43. The apparatus is a fixed-free type resonant column. The RCA belongs to a commercial soil testing laboratory. For a fixed-free resonant column, the solution for the torsional vibration mode is given by Richart et al. (1970):

$$\frac{J}{J_0} = \frac{\omega L}{V_s} \tan \frac{\omega L}{V_s} \quad \text{or} \quad \frac{J}{J_0} = \beta \tan \beta \quad (3.10)$$

where J and J_0 are mass polar moment of inertia of the specimen and the components mounted on top of the specimen (drive plate, top platen and accelerometers), respectively, J is $md^2/8$ for a solid cylinder (where m and d are mass and diameter), ω is the circular frequency of the first torsional vibration mode ($\omega = 2\pi f$ where f is the measured resonant frequency), V_s is shear wave velocity and L is length of the specimen. Due to manufacturing tolerances, each RCA should be calibrated using calibration bars of known stiffness (Figure 3.43 c).

Figure 3.43 Resonant column set-up

Considering the drive plate of the resonant column apparatus as a torsional pendulum moving in simple harmonic motion and the calibration bar as a torsional spring, ω can be described as:

$$\omega = \sqrt{\frac{k}{J_0}} \quad (3.11)$$

where k is torsional stiffness of the calibration bar. The resonant frequency of the bar can be altered by adding weights on top of the drive plate (Figure 3.43 b) to obtain J_{aw} :

$$J_{aw} = \frac{k}{\omega^2} - J_0 \quad (3.12)$$

The value of J_0 can be obtained by plotting J_{aw} , versus $1/\omega^2$ as shown in Figure 3.44.

Figure 3.44 Calibration of resonant column apparatus

The resonant frequency is identified and Equation 3.10 is solved to obtain V_s . From elastic wave theory, shear stiffness (G) is calculated as:

$$G = \rho V_s^2 \quad (3.13)$$

where ρ is the density of the soil specimen.

3.4.3 Specimen Preparation and Procedure

Four different sands were used. All four sands are medium sand and classified as poorly graded sand (SP) according to Unified Soil Classification System. The maximum and minimum void ratios and the basic properties of the sands are presented in Table 3.10. The reclaimed sands from Changi and Jurong reclamation sites are denoted as Reclaimed sand A and Reclaimed sand B, respectively. Figure 3.45 shows the grain size distribution curves of the sands. The sands were tested in the resonant column apparatus to measure stiffness-strain relationship of dry sands. The specimens, 70 mm in diameter and 140 mm in height, were prepared in dry state by air-pluviation and tamping method. Split mould with rubber membrane was secured on the base pedestal of the resonant column device by O-rings. The required amount of sand was weighed. The dry sand was divided into five equal parts by weight to form 25 mm layers in the sand specimen. The energy of tamping was increased from the lowest layer to topmost layer to ensure uniform sand specimen density (Ladd 1978). After tamping, the top platen was aligned and positioned on top of the specimen and then the split mould was carefully removed. Figure 3.46 shows the sand specimen prior to the resonant column tests.

Table 3.10 Summary of basic properties of four sands

	Ottawa 20-30	Ottawa Graded	Reclaimed sand A	Reclaimed sand B
Specific gravity, G_s	2.65	2.65	2.6	2.67

$e_{\max}$	0.69	0.82	0.92	0.81
$e_{\min}$	0.48	0.5	0.53	0.39
D_{60}	0.75	0.4	0.35	0.6
D_{30}	0.68	0.31	0.23	0.36
D_{10}	0.62	0.18	0.17	0.18
Coefficient of uniformity, C_u	1.21	2.22	2.06	3.33
Coefficient of curvature, C_c	0.99	1.33	0.89	1.20
Unified Soil Classification System (USCS)	SP	SP	SP	SP

Figure 3.45 Grain size distribution curves of the sands

Figure 3.46 Sand specimens before resonant column tests

3.5 Summary

The test set-ups (triaxial apparatus, bender elements and resonant column apparatus) used to measure the stiffness and strength of soils were presented with experimental procedures. A new circuit, the Anderson loop, was incorporated in the local strain measurement system to improve the axial strain measurement accuracy. Comparisons between the local strain measurement system using the conventional Wheatstone bridge and the Anderson loop circuits were also discussed.

Various soils (kaolin, four sands and undisturbed residual soils) were tested. Kaolin was consolidated from a slurry. The kaolin specimens were tested in the triaxial apparatus to measure the shear strength and stiffness of unsaturated soil. Four sands prepared by air-

pluviation and tamping method were tested in the resonant apparatus to measure the stiffness-strain relationship of dry sands. Reconstituted unsaturated sand specimens from Reclaimed sand B were prepared by wet tamping method and tested in the triaxial apparatus to measure the strength and stiffness of unsaturated sand. Bender element tests were also performed to investigate the effects of initial matric suctions and net confining pressures on unsaturated soils. In addition, undisturbed residual soils from Bukit Timah Granite were tested to measure the shear strength of unsaturated soils using CW and CU tests. The experimental results on the strength and stiffness characteristics of all the tested soils are presented in Chapter 4.

Chapter 4 Results and Discussions

4.1 Kaolin

4.1.1 Effects of Initial Matric Suction and Strain Rate

Consolidated kaolin specimens were used to investigate the effects of matric suction and strain rate on the stress-strain relationship and shear strength in a triaxial apparatus. Constant water content tests were performed at three different initial matric suctions (20 kPa, 200 kPa and 400 kPa) to investigate the effect of matric suction and using three different strain rates (1 mm/min, 0.1 mm/min and 0.06 mm/min) to study the effect of strain rates. Tests were performed under net confining pressure of 100 kPa. Results obtained from the tests are presented in the following sections. The triaxial compression test results obtained from all the tested specimens were presented by plotting the following graphs:

- (a) Deviator stress versus axial strain for large strain range
- (b) Deviator stress versus axial strain for small strain range
- (c) Secant Young's modulus versus axial strain
- (d) Secant shear modulus versus shear strain
- (e) Radial strain versus axial strain
- (f) Secant Poisson's ratio versus axial strain

Secant Young's modulus (E_{sec}) was obtained from the stress-strain relationship as:

$$E_{\text{sec}} = \frac{(\sigma_1 - \sigma_3) - (\sigma_1 - \sigma_3)_0}{\epsilon_a - \epsilon_{a0}}, \quad \epsilon_a = \frac{\Delta L}{L_0} \quad (4.1)$$

where $\sigma_1 - \sigma_3$ and ϵ_a are deviator stress and axial strain, respectively, $(\sigma_1 - \sigma_3)_0$ and ϵ_{a0} are deviator stress and axial strain at initial position, ΔL is change in specimen length and L_0 is initial specimen length. Secant shear stiffness (G_{sec}) was computed from secant Young's modulus (E_{sec}) by the following equation:

$$G_{\text{sec}} = \frac{E_{\text{sec}}}{2(1 + \nu_{\text{sec}})} \quad (4.2)$$

where ν_{sec} is secant Poisson's ratio and it can be expressed as:

$$\nu_{\text{sec}} = \frac{\epsilon_r - \epsilon_{r0}}{\epsilon_a - \epsilon_{a0}}, \quad \epsilon_r = \frac{\Delta r}{r_0} \quad (4.3)$$

where ϵ_r is radial strain, ϵ_{r0} is radial strain at initial position, Δr is change in radius of specimen and r_0 is initial radius of specimen. In this study, secant shear stiffness was plotted with shear strain (γ) and it was obtained using the following equation:

$$\gamma = \frac{\epsilon_a}{(1 + \nu_{\text{sec}})} \quad (4.4)$$

Figure 4.1 shows the typical triaxial compression test results obtained from CW-400-2. The triaxial compression test results for all the tested kaolin specimens are presented in Appendix A. From the test results, it was observed that the secant Young's moduli measured by LDTs were higher than that measured by LVDT. However at larger strain

range ($\approx 0.1\%$ to 0.5%), secant Young's moduli measured by LDT and LVDT showed a good agreement. Secant Poisson's ratios were obtained by plotting radial strain measured by the proximity transducers and axial strains measured by LDTs and LVDT, respectively. The relationships of radial strain with axial strain measured by LDT and LVDT were observed to be linear. Therefore, the experimental data for radial strain versus axial strain measured by LDTs and LVDT were best-fitted using linear equation. The fitted equations and the respective r^2 values were summarized in Table 4.1. For the specimens, CW-20-2 and CW-200-2, radial strain measurements were not available and therefore secant Poisson's ratio of 0.3 and 0.25, respectively, were assumed to obtain the secant shear stiffness.

Table 4.1 Summary of best-fit equations and r^2 values for relationship of radial strain with axial strain measured by LDT and LVDT, respectively

Specimen	Radial strain measurement	LDT		LVDT	
		Equation	r^2	Equation	r^2
CW-20-1	Yes	$\varepsilon_r = 0.33\varepsilon_a$	0.99	$\varepsilon_r = 0.25\varepsilon_a$	1.00
CW-20-2	No	—		—	
CW-20-3	Yes	$\varepsilon_r = 0.23\varepsilon_a$	0.91	$\varepsilon_r = 0.24\varepsilon_a$	0.98
CW-200-2	No	—		—	
CW-400-2	Yes	$\varepsilon_r = 0.18\varepsilon_a$	0.99	$\varepsilon_r = 0.17\varepsilon_a$	1.00

Figure 4.1 Typical triaxial compression test results of the specimen CW-400-2

4.1.1.1 Stress-strain Relationship and Variation of Matric Suction with Strain

Stress-strain relationships for the specimens, CW-20-2, CW-200-2 and CW-400-2, are presented in Figure 4.2. The specimens, CW-200-2 and CW-400-2, exhibited brittle failures with distinct peak akin to the strain-softening behaviour of overconsolidated soils. Similar observation was made by Rahardjo et al. (2004) for specimens tested at initial matric suctions higher than the air-entry value. The specimens at the end of shearing are shown in Figure 4.3. Similarly, the stress-strain relationship of the specimens, CW-20-1, CW-20-2 and CW-20-3, are shown in Figure 4.4. The specimen failed in a similar behaviour as normally consolidated soils except for specimen CW-20-1. Figure 4.5 shows the specimens at the end of shearing using different strain rates.

Figure 4.2 Deviator stress versus axial strain of the specimens, CW-20-2, CW-200-2 and CW-400-2

Figure 4.3 Specimens at the end of shearing

Figure 4.4 Deviator stress versus axial strain of the specimens, CW-20-1, CW-20-2, and CW-20-3

Figure 4.5 Specimens at the end of shearing

Figure 4.6 shows the effect of initial matric suctions on the change in matric suction with axial strain. Matric suction decreased with axial strain and reached an equilibrium near the failure strain. It was observed that the rate of matric suction reduction with axial strain was more significant for specimen CW-400-2. Similarly, variations of matric suction with axial strain were also plotted for the specimens, CW-20-1, CW-20-2 and CW-20-3, in Figure 4.7. The change in matric suction with axial strain was observed to be similar for specimens, CW-20-2 and CW-20-3, except for specimen CW-20-1. For specimens CW-20-2 and CW-20-3, a positive pore-water pressure (i.e. $u_{wf} > u_a$) was observed. No positive pore-water pressure was measured for specimen CW-20-1. Specimen CW-20-1 was sheared using strain rate of 1 mm/min and the shearing was completed within half an hour. It was suspected that there was not enough time to measure the actual pore-water pressure build-up in the specimen by the pressure transducer due to compliance of the ceramic disk.

Figure 4.6 Variation of matric suction with axial strain for specimens, CW-20-2, CW-200-2 and CW-400-2

Figure 4.7 Variation of matric suction with axial strain for specimens, CW-20-1, CW-20-2 and CW-20-3

4.1.1.2 Stiffness-strain Relationship

Relationships of secant shear stiffness with shear strain for the specimens tested at different initial matric suctions of 20 kPa, 200 kPa and 400 kPa (CW-20-2, CW-200-2 and CW-400-2) are presented in Figure 4.8. Secant shear stiffness increased significantly with initial matric suctions. The values of secant shear stiffness measured by LDTs were higher than that measured by LVDT in small shear strain range ($\gamma \approx 0.005\%$ to 0.05%). However, the values of secant shear stiffness measured by LDTs and LVDT showed a good agreement in medium shear strain range ($\gamma \approx 0.05\%$ to 5%). Relationships of secant shear stiffness with shear strain for the specimens tested using different strain rates of 1 mm/min, 0.1 mm/min and 0.06 mm/min (CW-20-1, CW-20-2 and CW-20-3) are presented in Figure 4.9. It was observed that relationships of shear stiffness with shear strain were not significantly affected by strain rates.

Figure 4.8 Relationships of secant shear modulus with shear strain of the specimens, CW-20-2, CW-200-2 and CW-400-2

Figure 4.9 Relationships of secant shear modulus with shear strain of the specimens, CW-20-1, CW-20-2 and CW-20-3

4.1.1.3 Shear Strength

Figure 4.10 shows the Mohr circles for the specimens, CW-20-2 and CW-20-3. The specimens, CW20-2 and CW20-3, failed at positive pore-water pressure and therefore the effective internal friction angle was obtained using the specimens. The effective friction angle was obtained as $\phi' = 23^\circ$ as shown in Figure 4.10. The Mohr-Coulomb failure envelope of CW20-1 was not plotted as the pore-water pressure reading of the specimen did not response properly due to the compliance of the ceramic disk. The cohesion intercepts for the specimens, CW200-2 and CW400-2, were obtained by drawing the extended Mohr-Coulomb failure envelopes with $\phi' = 23^\circ$ tangential to the Mohr circles of the respective specimens. Figure 4.11 shows the extended Mohr-Coulomb failure envelopes with the values of total cohesion intercept.

Figure 4.10 Mohr circles for specimens CW-20-2 and CW-20-3

Figure 4.11 Mohr circles with total cohesion intercepts for specimens CW-200-2 and CW-400-2

4.2 Sands (Ottawa 20-30, Ottawa Graded, Reclaimed sand A and Reclaimed sand B)

A total of four different sands (Ottawa 20-30, Ottawa Graded, Reclaimed sand A and Reclaimed sand B) in dry condition was tested using resonant column apparatus to measure shear stiffness of sands at different initial void ratios and effective mean confining pressures. In addition, reconstituted unsaturated sand specimens using Reclaimed sand B were also tested in a triaxial apparatus to measure the stress-strain relationship and shear strength at different initial matric suctions and net confining pressures. The results obtained from the resonant column tests and triaxial compression tests are presented in the following sections.

4.2.1 Effects of Void Ratio and Effective Mean Confining Pressure for Dry Sands

Shear stiffness of four sands were measured at three different shear strain amplitudes under drained condition. Multi-stage testing was adopted to study the effect of effective mean confining pressure. Relationships of shear stiffness with shear strain for all the tested sands at different initial void ratios under different effective mean confining pressures are shown in Figures 4.12 through 4.15. Values of shear stiffness increased with increasing effective mean confining pressures and decreasing void ratios. It was also observed that shear stiffness decreased with shear strain amplitudes for all the tested sand specimens. Table 4.2 summarises the values of average initial void ratios for the tested sands specimens.

Figure 4.12 Shear stiffness versus shear strain for Ottawa 20-30 and Ottawa Graded sand at confining pressures of 50 kPa and 200 kPa

Figure 4.13 Shear stiffness versus shear strain for Ottawa 20-30 and Ottawa Graded sand at confining pressures of 100 kPa and 400 kPa

Figure 4.14 Shear stiffness versus shear strain for Reclaimed sand A and Reclaimed sand B at confining pressures of 50 kPa and 200 kPa

Figure 4.15 Shear stiffness versus shear strain for Reclaimed sand A and Reclaimed sand B at confining pressures of 100 kPa and 400 kPa

Table 4.2 Summary of initial void ratios for all the tested sands

Average initial void ratio	Ottawa 20-30	Ottawa Graded	Reclaimed sand A	Reclaimed sand B
e1	0.47	0.62	0.57	0.63
e2	0.50	0.64	0.67	0.66

4.2.2 Effects of Initial Matric Suction and Net Confining Pressure for Reclaimed Sand B

A series of constant water content test was performed to investigate the effects of initial matric suction and net confining pressure on the stress-strain relationship of unsaturated reclaimed sand. Multi-stage shearing tests were performed to investigate the effect of initial matric suctions using the same specimen. Mainly, 2-stage and 3-stage shearing tests were performed in this study. A 5-stage shearing test was also performed to compare with the results obtained from 2-stage and 3-stage shearing test. In addition, consolidated undrained test were performed to measure the stress-strain relationship and shear strength of saturated sand. The sand specimens were annotated as CUa and CWa-b, in which CU and CW denote consolidated undrained and constant water content tests, respectively, a is the value of imposed effective or net confining pressure in kPa and b is the value of imposed initial matric suction in kPa. For multi-stage constant water content tests, the sand specimens were annotated as CWa-b-c-d... where c-d... denotes the subsequent imposed initial matric suction. Change in axial strain of the specimen was measured by three different transducers: local displacement transducers (LDT), an internal submersible linear variable differential transformer (LVDT) and an external LVDT in every first shearing stage of multi-stage tests. For subsequent shearing stage, change in axial strain was measured only by the external LVDT. The triaxial compression test results of reconstituted saturated and unsaturated sand specimens obtained from the first shearing

stage were summarised by plotting similar graphs as described in Section 4.1.1. Figure 4.16 shows the typical triaxial compression test results for the specimen CW200-3. The triaxial compression test results obtained from first shearing stage for all the tested sand specimens are presented in Appendix B1. As shown in Figure 4.16, the secant Young's moduli measured by LDTs were higher than that measured by LVDT. According to Equation 4.2, secant Poisson's ratios were required to obtain secant shear stiffness. Values of secant Poisson's ratio were obtained by best-fitting the experimental relationship of radial strain with axial strain measured by LDTs and LVDT. Table 4.3 summarises the fitted equations and the respective r^2 values obtained from curve fitting. The slope of the relationship of radial strain with axial strain is equivalent to Poisson's ratio. The values of secant Poisson's ratio measured by LDT ranged from 0.1 to 0.2 as shown in Table 4.3. It was observed that the values of Poisson's ratio obtained using axial strain measurement from LDTs and LVDT showed a good agreement.

Table 4.3 Summary of best-fit equations and r^2 values for the relationship of radial strain with axial strain measured by LDT and LVDT, respectively

Specimen	LDT		LVDT	
	Equation	r^2	Equation	r^2
CU50	$\epsilon_r = 0.2 \epsilon_a$	0.99	$\epsilon_r = 0.04 \epsilon_a^2 + 0.1 \epsilon_a$	1.00
CU200	$\epsilon_r = 0.2 \epsilon_a$	0.88	$\epsilon_r = 0.03 \epsilon_a^2 + 0.1 \epsilon_a$	1.00
CW50-3	$\epsilon_r = 0.2 \epsilon_a$	1.00	$\epsilon_r = 0.03 \epsilon_a^2 + 0.1 \epsilon_a$	1.00
CW100-3	$\epsilon_r = 0.1 \epsilon_a$	0.99	$\epsilon_r = 0.05 \epsilon_a^2 + 0.1 \epsilon_a$	1.00
CW200-3	$\epsilon_r = 0.1 \epsilon_a$	0.99	$\epsilon_r = 0.02 \epsilon_a^2 + 0.1 \epsilon_a$	1.00
CW50-5	$\epsilon_r = 0.1 \epsilon_a$	0.99	$\epsilon_r = 0.03 \epsilon_a^2 + 0.1 \epsilon_a$	1.00
CW100-5	$\epsilon_r = 0.1 \epsilon_a$	0.94	$\epsilon_r = 0.03 \epsilon_a^2 + 0.1 \epsilon_a$	1.00
CW200-5	$\epsilon_r = 0.1 \epsilon_a$	0.94	$\epsilon_r = 0.02 \epsilon_a^2 + 0.01 \epsilon_a$	0.99

Figure 4.16 Triaxial compression test results for the specimen CW200-3

4.2.2.1 Stress-strain Relationship and Change in Excess Pore-water Pressure with Strain

Stress-strain relationship from a 5-stage shearing test is presented in Figure 4.17. It was observed that deviator stress increased with initial matric suctions. The excess pore-water pressure generated during shearing was also plotted with axial strain in Figure 4.18. It was observed that the pore-water pressure increased to a maximum value and then decreased gradually. Figure 4.18 also shows that the excess pore-water pressure decreased during unloading. The stress-strain relationship obtained from multi-stage tests and the variation of excess pore-water pressure with axial for all the sand specimens are presented in Appendix B2.

Stress-strain relationships obtained from different transducers (LDTs, internal LVDT and external LVDT) for saturated and unsaturated sand specimens under different confining pressures are compared in Figures 4.19 (a), 4.20 (a), and 4.21 (a) for large axial strain range (0% to 10%) and in Figures 4.19 (b), 4.20 (b), and 4.21 (b) for small axial strain range (0% to 0.5%). Stress-strain relationship increases with confining pressure for both saturated and unsaturated sand specimens. In addition the stress-strain relationships measured by local displacement transducer (LDT) were stiffer than those measured by the submersible internal LVDT and external LVDT. The axial strain measured using external measurement devices can be affected by bedding error and as a result, lower stiffness or gentler stress-strain relationship is obtained from the external strain measurement devices (Goto et al. 1991 and Yimsiri et al. 2005).

Figure 4.17 Stress-strain relationship of the specimen CW200-3-5-10-15-20

Figure 4.18 Excess pore-water pressure versus axial strain of the specimen CW200-3-5-10-15-20

(a)

(b)

Figure 4.19 Stress-strain relationships of CU50 and CU200 (a) large strain amplitudes and (b) small strain amplitudes

(a)

(b)

Figure 4.20 Stress-strain relationships of CW50-3, CW100-3 and CW200-3 (a) large strain amplitudes and (b) small strain amplitudes

(a)

(b)

Figure 4.21 Stress-strain relationships of CW50-5, CW100-5 and CW200-5 (a) large strain amplitudes and (b) small strain amplitudes

4.2.2.2 Stiffness-strain Relationship

Relationships of secant shear stiffness with shear strain for all the tested sand specimens are presented in Figures 4.22 through 4.24. As shown in Figures 4.22 through 4.24, shear stiffness of saturated and unsaturated sand degraded significantly with shear strain amplitudes. It was observed that the stiffness-strain relationship of saturated and unsaturated specimens increased significantly with effective mean confining pressure or net confining pressures.

Figure 4.22 Relationships of shear stiffness with shear strain of the specimens, CU50 and CU200

Figure 4.23 Relationships of shear stiffness with shear strain of the specimens, CW50-3, CW100-3 and CW200-3

Figure 4.24 Relationships of shear stiffness with shear strain of the specimens, CW50-5, CW100-5 and CW200-5

4.2.2.3 Shear Strength

Consolidated undrained (CU) tests were performed according to ASTM D4767 (2002) to obtain the effective shear strength parameters. Multi-stage shearing test was performed at three different effective confining pressures, 50 kPa, 100 kPa and 200 kPa, respectively. Stress conditions at failure can be determined according to the peak deviator stress, or the effective stress ratio, σ_1'/σ_3' (ASTM D4767-2002). Peak deviator stress, $(\sigma_1 - \sigma_3)_f$, was used to determine the failure stresses for reconstituted sand specimens. The shear strength parameters (ϕ' and c') were obtained by plotting the Mohr-Coulomb envelope at failure and $q - p'$, respectively and these plots are shown in Figures 4.25 and 4.26, respectively. The stresses at failure condition were calculated using the following equations:

$$q = \frac{\sigma_1 - \sigma_3}{2} \quad (4.5)$$

where is $\sigma_1 - \sigma_3$ deviator stress.

$$p' = \frac{\sigma_1' + \sigma_3'}{2} \quad (4.6)$$

where σ_1' and σ_3' are effective major and minor principal stresses. The shear strength parameters for reconstituted sand specimen were obtained as $\phi' = 38^\circ$ and $c' = 0$.

The extended Mohr-Coulomb failure envelopes were plotted by drawing the tangent line to the Mohr circles of the unsaturated sand specimens with slope of ϕ' where $\phi' = 38^\circ$ to obtain the total cohesion intercepts. The Mohr circles for the constant water content tests were plotted using peak deviator stress. The plot for peak deviator stress versus matric suction is presented in Figure 4.27.

Figure 4.25 Mohr circles at different effective mean confining pressures of the specimen CU50-100-200

Figure 4.26 Stress paths for sand specimen from CU tests

Figure 4.27 Peak deviator stress versus matric suction

Figures 4.28 to 4.31 shows the plot for the determination of the total cohesion intercept for the unsaturated sand specimens at different initial matric suctions under different net confining pressures. Table 4.4 summarizes the values of total cohesion intercepts obtained from all the tested specimens. Alternatively, the total cohesion intercept can be obtained by drawing the best-fit line to the Mohr circles having similar matric suction at failure as shown in Figures 4.32 to 4.35. The total cohesion intercepts with matric suction at failures are plotted in Figure 4.36. The shear strength for different net confining pressures (50 kPa, 100 kPa, and 200 kPa) are also plotted with matric suction at failures in Figure 4.36. The total cohesion intercepts are the shear strength on the $(\sigma - u_a) = 0$ kPa plane and is labelled as such in Figure 4.36. The total cohesion intercepts obtained from Figures 4.28 to 4.31 are shown as empty symbols while those obtained from Figures 4.32 to 4.35 are shown as

filled symbols. It was observed that the total cohesion intercepts obtained from Figures 4.32 to 4.35 were very close to the mean values of the total cohesion intercept obtained from Figures 4.28 to 4.31. Table 4.5 summarizes the values of peak deviator stress and matric suction at failure for all the tested specimens.

Table 4.4 Summary of total cohesion intercepts of the sand specimens

Specimen	$\sigma_3 - u_a$ (kPa)	Initial matric suction (kPa)				
		3	5	10	15	20
CW50-3-15	50	0	8	4	12	23
CW50-5-10-20						
CW100-3-15	100	--	12	22	19	31
CW100-5-10-20						
CW200-3-15	200	--	8	22	13	35
CW200-5-10-20						
CW200-3-5-10-15-20	200	0	10	20	23	32

Figure 4.28 Mohr circles at different initial matric suctions under the net confining pressure 50 kPa

Figure 4.29 Mohr circles at different initial matrix suctions under the net confining pressure 100 kPa

Figure 4.30 Mohr circles at different initial matrix suctions under the net confining pressure 200 kPa

Figure 4.31 Mohr circles at different initial matrix suctions under the net confining pressure 200 kPa from CW200-3-5-10-15-20

Figure 4.32 Mohr circles at average matrix suction at failure of about 3 kPa under net confining pressures of 50 kPa, 100 kPa, and 200 kPa

Figure 4.33 Mohr circles at average matric suction at failure of about 10 kPa under net confining pressures of 50 kPa, 100 kPa, and 200 kPa

Figure 4.34 Mohr circles at average matric suction at failure of about 13 kPa under net confining pressures of 50 kPa, 100 kPa, and 200 kPa

Figure 4.35 Mohr circles at average matrix suction at failure of about 19 kPa under net confining pressures of 50 kPa, 100 kPa, and 200 kPa

Figure 4.36 Shear stress versus matric suction for all the tested specimens

Table 4.5 Summary of peak deviator stress and matric suction at failure

Test specimen	$\sigma_3 - u_a$ (kPa)	Initial matric suction (kPa)									
		3		5		10		15		20	
		$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$
CW50-3-15	50	133.65	3.34	183.77	4.32	210	9.66	202.27	13.29	242.51	18.63
CW50-5-10-20											
CW100-3-15	100	256.71	1.97	355.43	3.63	390.81	9.322	371.05	12.61	417.71	18.29
CW100-5-10-20											
CW200-3-15	200	530.94	0.42*	670.72	2.95	771.43	8.63	685.22	12.95	830.224	17.95
CW200-5-10-20											
CW200-3-5-10-15-20	200	598.25	0.61	680.80	3.29	729.55	4.87	749.26	9.87	776.10	12.80

*Positive pore-water pressure

4.2.2.4 Small-strain Stiffness

Shear wave velocity (V_s) is obtained by dividing the distance between the tips of the bender elements or wave path length (L_{tt}) to the wave travel time (Δt) and it is expressed as:

$$V_s = \frac{L_{tt}}{\Delta t} \quad (4.7)$$

From elastic wave theory, small-strain stiffness, G_{max} , is obtained from the shear wave velocity as follows:

$$G_{max} = \rho V_s^2 \quad (4.8)$$

where ρ is density of the soil. Typical receiver signals that were recorded at different initial matric suctions under a net confining pressure of 200 kPa are shown in Figure 4.37. The receiver shear wave signals recorded at different initial matric suctions for all the tested specimens are presented in Appendix B3. The relationships of G_{max} with matric suction under different net confining pressures are plotted in Figure 4.38. Nonlinear variations of G_{max} with matric suction were observed. The small-strain stiffness of unsaturated sand increases at a faster rate with matric suction in the zone below the air-entry value (AEV) of the sand. The change in G_{max} with matric suction becomes more gradual beyond AEV. From the relationship of G_{max} with matric suction, the rates of change of G_{max} with matric suction are 40% below the AEV and 15 % above the AEV. Similar nonlinear relationship of G_{max} with matric suction was obtained by Mancuso et al. (2002) for compacted silty sand whose G_{max} was measured using the resonant column apparatus under constant matric suction conditions. The relationship of G_{max} with matric suction can be explained using the bulk water effect below the AEV and the matric suction effect above the AEV. Below the

AEV, bulk water controls and the variation of $G_{\max}$ with matric suction is expected to be similar to the effect of effective or net confining pressure on $G_{\max}$. Above the AEV, matric suction effect becomes dominant and $G_{\max}$ increases with matric suction but at a relatively slower rate.

Figure 4.37 Shear waves arrival at different initial matric suctions under net confining pressure of 200 kPa

Figure 4.38 Variation of G_{max} with matric suctions

The effect of confining pressure on both saturated and unsaturated sand specimens was studied by plotting G_{max} versus effective or net confining pressure as shown in Figure 4.39. The relationships of G_{max} with confining pressure for both saturated and unsaturated sand were observed to be of similar trend. The values of G_{max} increase with effective or net confining pressure for both saturated and unsaturated sand specimens.

Figure 4.39 G_{max} versus effective mean confining pressure or net confining pressure

4.3 Residual Soil

A total of six multi-stage tests were performed on unsaturated undisturbed residual soils. Consolidated undrained (CU) tests and constant water content (CW) test were carried out using three different strain rates of 0.5 mm/min, 0.1 mm/min and 0.006 mm/min to investigate the effect of strain rate on the stress-strain relationship and strength of the soils. The effect of initial matric suctions on stress-strain relationship and shear strength was studied by performing multi-stage tests using four different matric suctions of 50 kPa, 100 kPa, 200 kPa and 400 kPa. The specimens tested using CU test and CW test are labelled as CU-a-b.. and CW-a-b., respectively, in which a and b represent the imposed matric suction in multi-stage tests.

4.3.1 Effects of Initial Matric Suction and Strain Rate

4.3.1.1 Stress-strain Relationship and Variation of Matric Suction with Strain

Stress-strain relationship obtained from a multi-stage test and variation of matric suction with strain of the specimen CU-50-100-200-400 tested using strain rate of 0.1 mm/min are presented in Figures 4.40 and 4.41. As shown in Figure 4.40, deviator stress increased significantly with initial matric suction. Matric suction decreased with axial strain and reached an equilibrium near the failure strain. The stress-strain relationships and variations of matric suction with strain for all the tested specimens are shown in Appendix C2. Stress-strain relationships obtained from CU and CW tests using different strain rates and at different initial matric suctions are compared in Figures 4.42 and 4.43.

Figure 4.40 Stress-strain relationship of the specimen CU-50-100-200-400 tested using strain rate of 0.1 mm/min

Figure 4.41 Variation of matric suction with axial strain of the specimen CU-50-100-200-400 tested using strain rate of 0.1 mm/min

Figure 4.42 Stress-strain relationships obtained using different strain rates and at different initial matrix suctions from CU tests

Figure 4.43 Stress-strain relationships obtained using different strain rates and at different initial matrix suctions from CW tests

Generally, the stress-strain relationship obtained from the test using slow strain rate was observed to be stiffer than that using fast strain rate in both CU and CW tests. In constant water content tests, the effect of strain rate on the stress-strain relationship of unsaturated soil specimens was observed to be more significant for specimens with higher initial matric suctions. For CU test, pore-water pressure increased with axial strain for all the tests using different strain rates. However, different behaviours of pore-air pressure changes with axial strain were observed during shearing. For the specimen tested using strain rate of 0.5 mm/min, pore-air pressure increased during shearing for all shearing stages at different initial matric suctions. For the specimen tested using strain rate of 0.1 mm/min, no change in pore-air pressure was measured during shearing. For the specimen test using strain rate of 0.006 mm/min, pore-air pressure decreased with axial strain. This is because when the specimen was sheared under undrained pore pressures condition at a slow strain rate, pore-air could diffuse into pore-water and as a result, decreased pore-air pressured was measured during shearing. For CW test, pore-water pressure increased and pore-air pressure remained constant with axial strain during shearing. Figures 4.44 and 4.45 show the variations of matric suction with axial strain for CU and CW tests using different strain rates at different initial matric suctions. As shown in Figures 4.44 and 4.45, the variations of matric suction with axial strain from the tests using different strain rates were observed to be of similar trend except for the specimen tested at initial matric suction of 400 kPa.

Figure 4.44 Variations of matric suction with axial strain from CU tests using different strain rates

Figure 4.45 Variations of matric suction with axial strain from CW tests using different strain rates

4.3.1.2 Stiffness-strain Relationship

The triaxial compression tests results obtained from first shearing stage are presented by plotting similar graphs as described in Section 4.1.1. Figure 4.46 shows the triaxial compression test results of the specimen CU-50 tested using strain rate of 0.1 mm/min. The results obtained from the first stage shearing of the tested residual soil specimens are presented in Appendix C1. Stiffness-strain relationship measured from LDT and LVDT were compared and presented in Figures 4.47 and 4.48 for CU and CW tests, respectively. The effect of strain rate on relationship of shear stiffness with shear strain was insignificant (Figures 4.47 and 4.48). Relationships of shear stiffness with shear strain obtained using different strain rates showed relatively consistent trend except for specimen CW-50 tested using strain rate of 0.006 mm/min. Lag between the LDTs and hinges was observed for the specimen and as a result much higher values of secant shear stiffness measured by LDT were obtained as indicated in Figure 4.48.

4.3.1.3 Shear Strength

Principal stress ratios ($\sigma_1 - \sigma_3 / \sigma_3 - u_w$) were plotted with axial strain as shown in Figures 4.49 to 4.51 for undisturbed residual soils from CU and CW tests at different initial matric suctions using different strain rates. It was observed that the principal stress ratios decrease with increased initial matric suctions. Similar behaviours of principal stress ratio with axial strain were observed for all the tested specimens. Deviator stress, axial strain and matric suction at failure obtained from maximum deviator stress and maximum principal stress ratio criteria were compared and the comparisons are shown Figures 4.52 to 4.54.

Figure 4.46 Triaxial compression test results of the specimen CU-50 tested using strain rate of 0.1 mm/min

Figure 4.47 Relationship of secant shear stiffness with shear strain of the specimens CU-50 tested using different strain rates

Figure 4.48 Relationship of secant shear stiffness with shear strain of the specimens CW-50 tested using different strain rates

In Figures 4.52 to 4.54, the parameters at failure (deviator stress, axial strain and matric suction) determined using the maximum principal stress ratio criterion were annotated as $(\sigma_1 - \sigma_3)_{f-PSR}$, ϵ_{af-PSR} , and $(u_a - u_w)_{f-PSR}$, respectively, and that using the maximum deviator stress

criterion were annotated as $(\sigma_1 - \sigma_3)_{f-max}$, ϵ_{af-max} , and $(u_a - u_w)_{f-max}$, respectively. It was observed that the values of axial strain obtained using maximum principal stress ratio criterion were smaller than that obtained using maximum deviator stress as failure criterion. In other words, failure occurred earlier when using maximum deviator stress as failure criterion. Values of deviator stress and matric suction at failure obtained from the two different failure criteria were very close as shown in Figures 4.52 and 4.54. This shows that maximum deviator stress is the most suitable failure criterion to determine shear strength for the tested unsaturated soil specimens. Tables 4.6 to 4.9 summarize the values of deviator stress, axial strain and matric suction at failure obtained using the two different approaches for CU and CW tests.

Figure 4.49 Principal stress ratio versus axial strain for CW and CU tests at different initial matric suctions using strain rate of 0.5 mm/min

Figure 4.50 Principal stress ratio versus axial strain for CW and CU tests at different initial matric suctions using strain rate of 0.1 mm/min

Figure 4.51 Principal stress ratio versus axial strain for CW and CU tests at different initial matric suctions using strain rate of 0.006 mm/min

Figure 4.52 Comparison of deviator stress at failure using principal stress ratio and maximum deviator stress as failure criteria

Figure 4.53 Comparison of axial strain at failure using principal stress ratio and maximum deviator stress as failure criteria

Figure 4.54 Comparison of matric suctions at failure using principal stress ratio and maximum deviator stress as failure criteria

Table 4.6 Summary of deviator stress, $(\sigma_1 - \sigma_3)_{f-max}$, axial strain, ϵ_{af-max} , and matric suction, $(u_a - u_w)_{f-max}$, at failure from CU tests using maximum deviator stress as failure criteria

Strain rate (mm/min)	Specimen	$(\sigma_1 - \sigma_3)_{f-max}$	ϵ_{af-max}	$(u_a - u_w)_{f-max}$
0.5	CU-50	156.3	8.1	4.9
	CU-50-100	225.6	6.0	38.5
	CU-50-100-200	319.0	4.4	109.6
	CU-50-100-200-400	468.9	5.0	376.8
0.1	CU-50	153.1	6.1	2.7*
	CU-50-100	235.9	5.7	28.6
	CU-50-100-200	306.5	5.0	117.3
	CU-50-100-200-400	508.3	5.8	297.4
0.006	CU-50	144.8	6.8	2.6*
	CU-50-100	231.7	6.0	28.9
	CU-50-100-200	391.1	4.3	92.7
	CU-50-100-200-400	625.8	5.3	272.2

*Positive pore-water pressure

Table 4.7 Summary of deviator stress, $(\sigma_1 - \sigma_3)_{f-max}$, axial strain, ϵ_{af-max} , and matric suction, $(u_a - u_w)_{f-max}$, at failure from CW tests using maximum deviator stress as failure criteria

Strain rate (mm/min)	Specimen	$(\sigma_1 - \sigma_3)_{f-max}$	ϵ_{af-max}	$(u_a - u_w)_{f-max}$
0.5	CW-50	159.5	7.0	0.3*
	CW-50-100	230.6	4.6	23.4
	CW-50-100-200	334.3	4.6	86.8
	CW-50-100-200-400	518.0	4.6	357.6
0.1	CW-50	156.8	6.1	4.7*
	CW-50-100	229.8	6.0	31.3
	CW-50-100-200	331.9	4.4	100.2
	CW-50-100-200-400	528.6	5.0	346.0
0.006	CW-50	155.8	6.7	5.9
	CW-50-100	232.3	5.2	35.4
	CW-50-100-200	348.3	4.0	108.7
	CW-50-100-200-400	550.7	3.2	295.7

Table 4.8 Summary of deviator stress, $(\sigma_1 - \sigma_3)_{f-PSR}$, axial strain, ϵ_{af-PSR} , and matric suction, $(u_a - u_w)_{f-PSR}$, at failure from CU tests using principal stress ratio, $\sigma_1 - \sigma_3 / \sigma_3 - u_w$, as failure criteria

Strain rate (mm/min)	Specimen	$(\sigma_1 - \sigma_3)_{f-PSR}$	ϵ_{af-PSR}	$(u_a - u_w)_{f-PSR}$
0.5	CU-50	124.9	4.5	24.5*
	CU-50-100	217.0	5.0	35.1
	CU-50-100-200	319.0	4.4	109.5
	CU-50-100-200-400	457.6	6.0	358.4
0.1	CU-50	143.0	3.4	5.9*
	CU-50-100	233.5	3.0	22.6
	CU-50-100-200	305.9	1.8	116.9
	CU-50-100-200-400	498.9	1.5	280.0
0.006	CU-50	140.9	2.3	1.6*
	CU-50-100	229.5	2.2	30.2
	CU-50-100-200	391.1	2.3	92.7
	CU-50-100-200-400	625.3	1.7	274.2

*Positive pore-water pressure

Table 4.9 Summary of deviator stress, $(\sigma_1 - \sigma_3)_{f-PSR}$, axial strain, ϵ_{af-PSR} , and matric suction, $(u_a - u_w)_{f-PSR}$, at failure from CW tests using principal stress ratio, $\sigma_1 - \sigma_3 / \sigma_3 - u_w$, as failure criteria

Strain rate (mm/min)	Specimen	$(\sigma_1 - \sigma_3)_{f-PSR}$	ϵ_{af-PSR}	$(u_a - u_w)_{f-PSR}$
0.5	CW-50	135.9	3.7	13.9*
	CW-50-100	225.6	4.1	19.7
	CW-50-100-200	334.3	4.6	86.8
	CW-50-100-200-400	516.9	4.7	356.2
0.1	CW-50	116.8	4.2	22.1*
	CW-50-100	229.8	2.8	31.3
	CW-50-100-200	331.8	2.2	99.8
	CW-50-100-200-400	527.9	1.3	341.2
0.006	CW-50	149.8	2.8	2.8
	CW-50-100	232.3	2.7	35.1
	CW-50-100-200	348.3	2.2	108.7
	CW-50-100-200-400	546.3	1.6	301.1

*Positive pore-water pressure

Figures 4.55 and 4.56 shows the variations of peak deviator stress or maximum deviator stress, $(\sigma_1 - \sigma_3)_f$, obtained from CU and CW tests using different strain rates with matric suctions at failure, $(u_a - u_w)_f$. The peak deviator stress increased with matric suction non-linearly. As shown in Figure 4.55 and 4.56, the effect of strain rate on the peak deviator stress was insignificant below matric suction of 100 kPa in both CU and CW tests. However, peak deviator stress decreased with strain rate for the specimens tested at initial matric suction of 400 kPa in both CU and CW tests. In other words, the effect of strain rate on the peak deviator stress was observed to be more significant for the specimens tested at high initial matric suctions. In addition, it was observed that below matric suctions of 100 kPa, the effect of difference in drainage condition of pore-air pressure did not significantly affect the peak deviator stress. However, beyond matric suction of 100 kPa, differences in peak deviator stress were observed between CU and CW

tests. Tables 4.10 and 4.11 summarize the values of peak deviator stress and matric suction at failure for CU and CW tests.

Figure 4.55 Peak deviator versus matric suction at failure for CU tests using different strain rates

Figure 4.56 Peak deviator versus matric suction at failure for CW tests using different strain rates

Table 4.10 Summary of peak deviator stress and matric suction at failure for CU tests

Strain rate (mm/min)	Specimen							
	CU-50		CU-50-100		CU-50-100-200		CU-50-100-20-400	
	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$
0.5	156.3	5	225.6	39	320.5	110	468.9	377
0.1	153.1	0	236.0	29	306.5	117	508.3	297
0.006	119.9	0	191.8	29	323.8	93	518.1	272

Table 4.11 Summary of peak deviator stress and matric suction at failure for CW tests

Strain rate (mm/min)	Specimen							
	CW-50		CW-50-100		CW-50-100-200		CW-50-100-20-400	
	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$	$(\sigma_1 - \sigma_3)_f$	$(u_a - u_w)_f$
0.5	159.5	0	230.6	23	334.3	87	518.0	358
0.1	156.8	0	230.4	31	331.9	100	528.6	346
0.006	155.8	6	236.1	35	352.5	109	550.8	296

The shear strength parameters (ϕ' and c') for saturated specimen were obtained by plotting the Mohr-Coulomb envelope at failure and $q - p'$, respectively. These plots are shown in Figures 4.57 and 4.58, respectively. The extended Mohr-Coulomb failure envelopes for CU and CW tests were plotted by drawing the tangent line to the Mohr circles of the unsaturated sand specimens with slope of ϕ' where $\phi' = 33^\circ$. The Mohr circles for CU and CW tests were plotted using peak deviator stress. Figures 4.59 and 4.60 show the plots for the determination of the total cohesion intercept for CU and CW tests using different strain rates. Table 4.12 summarizes the values of total cohesion intercepts obtained from the tests. The total cohesion intercepts obtained from CU and CW tests were compared and plotted with matric suctions at failure as shown in Figure 4.61. The relationship of total cohesion intercept with matric suction is the failure envelope along zero net confining pressure. Non-linear failure envelope was observed from Figure 4.61. This figure also shows that failure envelopes obtained from CU and CW were similar except for the specimens tested using strain rate of 0.006 mm/min.

Figure 4.57 Mohr circles at different effective mean confining pressures of saturated residual soil specimen

Figure 4.58 Stress paths for saturated residual soil specimen from CU tests

Figure 4.59 Mohr circles at different initial matrix suctions from CU tests using different strain rates

Figure 4.60 Mohr circles at different initial matric suctions from CW tests using different strain rates

Table 4.12 Summary of total cohesion intercepts from CU and CW tests using different strain rate for undisturbed residual soils

Strain rate (mm/min)	Specimen for CU test	c (kPa)	Specimen for CW test	c (kPa)
0.5	CU-50	25	CW-50	14
	CU-50-100	39	CW-50-100	30
	CU-50-100-200	69	CW-50-100-200	60
	CU-50-100-200-400	109	CW-50-100-200-400	108
0.1	CU-50	14	CW-50	12
	CU-50-100	32	CW-50-100	32
	CU-50-100-200	54	CW-50-100-200	61
	CU-50-100-200-400	109	CW-50-100-200-400	114
0.006	CU-50	-	CW-50	12
	CU-50-100	6	CW-50-100	32
	CU-50-100-200	44	CW-50-100-200	64
	CU-50-100-200-400	88	CW-50-100-200-400	118

Figure 4.61 Shear stress versus matric suction at net confining pressure of 0 kPa

4.3.1.4 Small-strain Stiffness

Shear wave velocities were measured to obtain small-strain stiffness of residual soil under net confining pressure of 50 kPa. Figure 4.62 shows the arrival of shear waves at different matric suctions for specimen CU-50 which was loaded at a strain rate of 0.1 mm/min. As shown in Figure 4.62, shear wave velocity increases with matric suction. Similar plots of shear wave arrival at different matric suctions under net confining pressure of 50 kPa for all the tested specimens are presented in Appendix C3. Small-strain stiffness at different matric suctions were calculated from the shear wave velocities obtained. Variations of small-strain stiffness with matric suction for all the tested specimens are plotted in Figure 4.63. Figure 4.63 shows that small-strain stiffness increased almost linearly with matric suction.

4.4 Summary

Experimental results obtained from triaxial compression tests, bender element tests and resonant column tests are presented. The effects of initial matric suctions and strain rate on the stress-strain relationship and shear strength were studied by performing constant water content tests using consolidated kaolin specimens. The effect of initial matric suction on stress-strain relationship was observed to be significant. However, the effect of strain rate on stress-strain relationship was insignificant. The shear strength of unsaturated kaolin increased with initial matric suction. Positive pore-water pressures were observed for the specimens tested using strain rates of 0.1 and 0.06 mm/min.

Figure 4.62 Shear waves arrival at different initial matrix suctions of the specimen CU-50-100-200-400 (from multi-stage shearing test using strain rate of 0.1 mm/min)

Figure 4.63 Variation of G_{max} with matric suction obtained from all the tested specimens CU-50-100-200-400 and CW-50-100-200-400

Relationships of shear stiffness with shear strain for four different sands were obtained by performing resonant column tests at different initial void ratios and effective mean confining pressures. Values of shear stiffness increased with increasing effective mean confining pressure and decreased with increasing void ratio. The stress-strain relationship and shear strength of unsaturated sand were also studied by performing constant water content tests. Multi-stage shearing tests were carried out to investigate the effect of initial matric suctions using the same specimen. Shear wave velocities of unsaturated sand were also measured to obtain small-strain stiffness at different initial matric suctions and net confining pressures. The maximum deviator stress of unsaturated sand increased with initial matric suctions. Variations of shear strength and small-strain stiffness with matric suction showed non-linear behaviour.

Effects of initial matric suction and strain rate on the stress-strain relationship and shear strength of undisturbed residual soils were investigated by performing CU and CW tests. Multi-stage shearing tests were carried out at different initial matric suctions and using different strain rates. The maximum deviator stress increased significantly with initial matric suction. Non-linear variations of shear strength with matric suction were observed from CU and CW tests using different strain rates. In addition, it was observed that relationships of shear strength with matric suction from CU and CW tests using different strain rates were observed to be similar.

Chapter 5 Comparisons of Stiffness-strain Relationship and G_{max} with Other Studies

5.1 Stiffness-strain relationship and Stiffness Degradation Curve

Stiffness-strain relationships were obtained by performing triaxial compression and resonant column tests using different soils (kaolin, sands, and residual soil). The experimental stiffness-strain relationships were fitted using Borden et al. (1996)'s equation to obtain the small-strain stiffness (G_{max}) at shear strain of 0.0001 %. The equation used for curve fitting is written as:

$$\frac{G}{G_{max}} = \frac{1}{(1 + a\gamma^b)^c} \quad (5.1)$$

where γ is shear strain and a , b , and c are the fitting parameters. In this study, the parameter c was taken as 1 to fit the experimental data as suggested by Leong et al. (2003). Values of G_{max} obtained from curve fitting were used for normalising experimental stiffness-strain relationship to obtain stiffness degradation curves. Table 5.1 summarises soil type used for measurement of stiffness-strain relationship, test method employed, and test conditions such as effective confining pressure (σ') or net confining pressure ($\sigma - u_a$) and matric suction ($u_a - u_w$). Values of G_{max} obtained from curve fitting were compared with that of G_{max} obtained from bender element test for reconstituted unsaturated sand at

different matric suctions and under different net confining pressure in the following section.

Table 5.1 Summary of soil type, test method, effective or net confining pressure, matric suction for measurement of stiffness-strain relationship

Soil type	Test method	Strain rate (mm/min)	σ' or $\sigma-u_a$ (kPa)	u_a-u_w (kPa)	Fitted G_{max} using Eq. (5.1) (MPa)	Measured G_{max} using BE test (MPa)
Kaolin	TC	0.1	50	20, 200, 400	Yes	NA
Dry sands	RC	NA	50, 100, 200, 400	NA	Yes	NA
Unsaturated sand	TC	1	50, 100, 200	0, 3, 5	Yes	Yes
Residual soil	TC	0.5, 0.1, 0.006	50	50	Yes	Yes

Note: TC and RC are triaxial compression and resonant column test, respectively.

5.1.1 Kaolin

The experimental relationship of shear stiffness with shear strain of consolidated kaolin at different initial matric suctions and tested using different strain rates were fitted using Equation 5.1 to obtain small-strain stiffness (G_{max}). Figure 5.1 shows the fitted and measured shear stiffness with shear strain at different initial matric suctions. The parameters a and b were obtained as $a=18$ and $b=0.9$, respectively. The values of a and b are the same for the specimens at different initial matric suctions. In other words, matric suction affects the small-strain stiffness of kaolin but not the stiffness degradation curve. The fitted and measured shear stiffness with shear strain at initial matric suction of 20 kPa and different strain rates are presented in Figure 5.2.

Figure 5.1 Comparison of calculated and measured shear stiffness with shear strain for the specimens CW-20-2, CW-200-2 and CW-400-2

Figure 5.2 Comparison of calculated and measured shear stiffness with shear strain for the specimens CW-20-1, CW-20-2 and CW-20-3

Figure 5.3 Effect of strain rate on fitting parameters a and b

Values of small-strain stiffness obtained from curve fitting were observed to be very close regardless of strain rate used. The fitting parameter a was not affected by strain rate however the parameter b increased slightly with strain rate. Figure 5.3 shows the effect of strain rate on fitting parameters a and b. Table 5.2 summarises values of G_{max} obtained from curve fitting and the fitting parameters, a and b, for kaolin specimens.

Table 5.2 Summary of G_{max} and fitting parameters a and b for kaolin

Specimen	Strain rate (mm/min)	G_{max} (MPa)	a	B
CW-20-1	1.00	30	18	1.05
CW-20-2	0.10	30	18	0.90
CW-20-3	0.06	30	18	0.75
CW-200-2	0.10	80	18	0.90
CW-400-2	0.10	175	18	0.90

Stiffness-strain relationships of all the tested kaolin specimens were normalised with G_{max} shown in Table 5.2 to obtain stiffness degradation curve. Figure 5.4 shows the stiffness degradation curves of the specimens, CW-20-2, CW-200-2, and CW-400-2, respectively. Figure 5.5 shows the stiffness degradation curve of the specimens, CW-20-1, CW-20-2 and CW-30-2. The shear stiffness at the faster strain rate started to degrade at a higher shear amplitude compared with that at the slower strain rate as shown in Figure 5.5. Similar observation of strain rate effect on stiffness degradation curves was made by other researchers (e.g. d' Onofrio et al. 1999 and Santagata et al. 2007).

Figure 5.4 Stiffness degradation curves of the specimens CW-20-2, CW-200-2 and CW-400-2

Figure 5.5 Stiffness degradation curves of the specimens CW-20-1, CW-20-2, and CW-20-3

5.1.2 Sands

5.1.2.1 Dry sands

Relationships of shear stiffness with shear strain of four sands were measured using a resonant column apparatus. The resonant column apparatus was calibrated by using the standard sands, Ottawa 20-30 and Ottawa Graded sand at two different initial void ratios under different effective mean confining pressures. Alarcon-Guzman et al. (1989) performed the resonant column tests using Ottawa 20-30 sand at different initial void ratios under effective confining pressure of 100 kPa. The shear stiffness of Ottawa 20-30 sand were compared with the data reported in Alarcon-Guzman et al. (1989) in Figure 5.6. The shear stiffness obtained from the experiments are indicated using filled symbols and those data from Alarcon-Guzman et al. (1989) are indicated using empty symbols in Figure 5.6. Figure 5.6 shows that shear stiffness increased with decreased void ratio. It was

observed that the measured shear stiffness for $e=0.50$ and $\sigma'_m=100$ kPa is higher than the corresponding shear stiffness from Alarcon-Guzman et al. (1989) obtained for $e=0.513$ and $\sigma'_m=100$ kPa by approximately 10 %. The values of shear stiffness at different void ratios from Alarcon-Guzman et al. (1989) were obtained by performing single-stage tests using dry hollow specimens. This comparison highlights the effect of pre-straining in the multi-stage tests. The effect of pre-straining due to repeated cycles can alter the soil fabric and density of specimens. Chung et al. (1984) performed the single- and multi-stage tests for Monterey No.0 sand using resonant column apparatus. The experimental results show that values of shear stiffness obtained from multi-stage tests were higher than that from single-stage tests. Moreover, it was reported that the effect of pre-straining in sand specimen became more significant under higher confining pressure. The variations of shear stiffness due to pre-straining can range from 30% to 60 % depending on the strain levels at which shear stiffness are measured (Alarcon-Guzman et al. 1989).

Figure 5.6 Comparison of shear stiffness with shear strain at different void ratios under effective confining pressure of 100 kPa (modified from Alarcon-Guzman et al. 1989)

Relationships of shear stiffness between shear strain for all the tested specimens under different effective mean confining pressures were fitted using Equation 5.1 with $c=1$ to obtain the small-strain stiffness (G_{max}). A typical plot for the fitted and measured shear stiffness with shear strain under different effective mean confining pressures is presented in Figure 5.7. It was observed that Equation 5.1 with $c=1$ fitted the experimental data well. A summary of fitting parameters and small-strain stiffness obtained from curve fitting is presented in Table 5.3. Values of G_{max} increased with effective mean confining pressure significantly. However, the effect of effective mean confining pressure on G_{max} was observed to be less significant. Fitting parameter a decreased with σ'_v but no significantly affected by initial void ratio. Values of parameter b ranged from 0.97 from 1.64 for all the tested specimens. The plots of fitted and measured stiffness-strain relationship under different effective mean confining pressures for all the tested specimens are shown in Appendix D.

Figure 5.7 Comparison of calculated and measured shear stiffness with shear strain for Ottawa 20-30_e1 under different effective mean confining pressures

Table 5.3 Summary of fitted G_{max} and fitting parameters a and b

Initial void ratio, e		Ottawa 20-30		Ottawa Graded		Reclaimed sand A		Reclaimed sand B	
		$e_1=0.47$	$e_2=0.50$	$e_1=0.62$	$e_2=0.64$	$e_1=0.57$	$e_2=0.67$	$e_1=0.63$	$e_2=0.66$
$\sigma' = 50$ (kPa)	G_{max} (MPa)	108	100	89	82	109	94	61	76
	a	40	35	35	45	47	28	40	30
	b	1.17	1.17	1.18	1.27	1.26	1.10	1.64	1.09
$\sigma' = 100$ (kPa)	G_{max} (MPa)	150	140	124	117	151	128	127	107
	a	40	27	30	29	40	25	24	19
	b	1.30	1.20	1.23	1.23	1.27	1.12	1.20	1.02
$\sigma' = 200$ (kPa)	G_{max} (MPa)	205	197	169	158	203	170	187	145
	a	20	22	25	24	15	20	10	15
	b	1.23	1.25	1.30	1.32	1.08	1.14	0.97	1.04
$\sigma' = 400$ (kPa)	G_{max} (MPa)	276	268	236	219	273	226	246	200
	a	10	10	25	20	7	13	9	10
	b	1.13	1.16	1.25	1.31	0.97	1.10	1.05	1.00

The small-strain stiffness (G_{max}) shown in Table 5.3 was used to normalize the respective measured shear stiffness under different effective mean confining pressures to obtain the stiffness degradation curves. Figure 5.8 shows the typical stiffness degradation curves under different effective mean confining pressures for Ottawa 20-30_e1. It was observed that the stiffness degrades more at higher shear strain amplitude and at higher confining pressure. These normalized stiffness degradation curves were compared with the upper and lower bound stiffness degradation curves of Seed et al. (1986). As shown in Figure 5.8, the experimental stiffness degradation data are above the upper bound stiffness degradation curve of Seed et al. (1986). Therefore, a new upper bound was determined using Equation 5.1 to envelope the experimental data. The proposed upper bound curve was then compared with the stiffness degradation data of the other tested sands. It was observed that all the measured stiffness degradation data are now below the proposed upper bound curve.

Figure 5.8 Comparison of stiffness degradation curves for specimen Ottawa 20-30_e1

Figure 5.9 Comparison of stiffness degradation curves

Table 5.4 Summary of test conditions for the results extracted from the literatures

Reference	Test apparatus	Test method	Specimen type	Effective mean or vertical effective confining pressures (kPa)	Type of sand
Chung et al. (1984)	Resonant column	Single- and Multi- stage	Hollow and solid specimens	100 and 300	Dry Monterey No.0 sand
Alarcon-Guzman et al. (1989)	Resonant column	Single-stage	Hollow and solid specimens	28.6, 68.3, 103, and 202	Dry and saturated Ottawa 20-30 sand
Lanzo and D'Elia (2003)	Double specimen direct simple shear	Multi-stage	Cylindrical specimens	90, 180, 420, and 900	Dry Toyoura sand
Khan et al. (2008)	Resonant column	Multi-stage	Solid specimen	40, 100, and 200	Dry Round grain sand

The modified upper bound stiffness degradation curve were evaluated using the data reported in Chung et al. (1984), Alarcon-Guzman et al. (1989), Lanzo and D'Elia (2003) and Khan et al. (2008). Table 5.4 summarizes the test apparatus, test method, specimen type, effective mean or vertical effective confining pressure and type of sand used to determine the dynamic properties of sand. The stiffness-strain relationships from the references except for Lanzo and D'Elia (2003) were fitted using Equation 5.1 to obtain the small-strain stiffness at strain amplitude of less than 0.0001%. The stiffness-strain relationships were normalized with the respective fitted G_{max} at shear strain of 0.0001% in Figure 5.9. Figure 5.9 also shows the stiffness degradation curves from Seed and Idriss (1970) and proposed upper bound stiffness degradation curve in this study. It was observed that the data from Alarcon-Guzman et al. (1989) plot within the upper and lower bound stiffness degradation curves of Seed et al. (1989) whereas the data from Chung et al. (1984), Lanzo and D'Elia (2003), and Khan et al. (2008) plot above the upper bound stiffness degradation curve of Seed et al. (1986) but below the proposed upper bound curve. The proposed upper bound curve provides a better upper bound to the degradation curves for sand.

5.1.2.2 Saturated and Unsaturated Sand

Stiffness-strain relationships of reconstituted saturated and unsaturated sand specimen were fitted using Equation 5.1 with the fitting parameter $c=1$. The plots of fitted and measured shear stiffness with shear strain for all the tested specimens are presented in Figures 5.10 to 5.12. It was observed that Equation 5.1 with $c=1$ fitted the experimental data well. A summary of small-strain stiffness obtained from curve fitting and the fitting parameters is presented in Table 5.5. Generally, the small-strain stiffness increased with

effective mean confining pressure or net confining pressure except CW100-5. These stiffness-strain relationships were normalised with G_{max} shown in Table 5.5 for the respective specimens. The plots of fitted and measured normalised stiffness degradation curves are presented in Figures 5.13 to 5.15. The normalised stiffness degradation curves were then compared with the data from Seed and Idriss (1970) and the modified upper bound curve proposed in Section 5.1.2.1. Figure 5.16 shows the comparisons of measured stiffness degradation curves with the data from Seed and Idriss (1970) and the modified upper bound curve. It was observed that all the experimental data plotted below the modified upper bound curve obtained in this study.

Figure 5.10 Comparison of calculated and measured shear stiffness with shear strain for specimens CU50 and CU200

Figure 5.11 Comparison of calculated and measured shear stiffness with shear strain for specimens CW50-3, CW100-3 and CW200-3

Figure 5.12 Comparison of calculated and measured shear stiffness with shear strain for specimens CW50-5, CW100-5 and CW200-5

Table 5.5 Summary of G_{max} and fitting parameters a and b for saturated and unsaturated sand

Specimen	Strain rate (mm/min)	G_{max} (MPa)	a	b
CU50	1.0	43	35.0	1.2
CU200	1.0	103	16.0	1.1
CW50-3	1.0	48	35.0	1.0
CW100-3	1.0	80	18.0	1.0
CW200-3	1.0	138	10.0	1.0
CW50-5	1.0	69	21.0	1.0
CW100-5	1.0	70	10.0	0.9
CW200-5	1.0	122	8.0	0.9

Figure 5.13 Stiffness degradation curves for specimens CU50 and CU200

Figure 5.14 Stiffness degradation curves for specimens CW50-3, CW100-3 and CW200-3

Figure 5.15 Stiffness degradation curves for specimens CW50-5, CW100-5 and CW200-5

Figure 5.16 Comparisons of stiffness degradation curves with measured data

The modified upper bound curve was also evaluated using the experimental data reported in Wang et al. (2007). A series of saturated and unsaturated monotonic triaxial compression was performed using Toyoura sand in the studies. Consolidated drained and constant suction tests were performed for saturated and unsaturated Toyoura sand specimens, respectively. Specimens were prepared in loose and dense conditions and tested at different matric suctions and under different net confining pressures. Figures 5.17 and 5.18 show the relationships of secant Young's moduli obtained from the tests. In Figures 5.17 and 5.18, the specimens were annotated as CDa and CSa-b, in which CD and CS denote consolidated drained and constant suction tests, respectively, a is the value of effective mean or net confining pressure in kPa and b is the value of matric suction in kPa. As shown in Figures 5.17 and 5.18, secant Young's moduli increased with effective mean confining pressure and net confining pressure. Relationships of secant Young's modulus

with axial strain obtained from CD and CS tests were observed to be similar. Values of secant Young's modulus obtained from CS tests were observed to be slightly higher than that from CD tests for some specimens. Secant shear stiffness of Toyoura sand were calculated from the secant Young's moduli shown in Figures 5.17 and 5.18 using Equation 4.2 with Poisson's ratio of 0.15. The calculated secant shear stiffness under different net confining pressure were normalized with the respective small-strain stiffness at $\gamma=0.001\%$. Figures 5.19 and 5.20 show the stiffness degradation curves under different effective or net confining pressures of Toyoura sand specimen plotted together with the upper and lower bound curves from Seed et al. (1986) and the modified upper bound curve. Generally, the stiffness degradation curves plotted between the upper bound curve of Seed et al. (1986) and the modified upper bound curve obtained in this study.

Figure 5.17 Relationships of secant Young's modulus with axial strain for loose Toyoura sand under different effective and net confining pressures (from Wang et al. 2007)

Figure 5.18 Relationships of secant Young's modulus with axial strain for dense Toyoura sand under different effective and net confining pressures (from Wang et al. 2007)

Figure 5.19 Comparisons of stiffness degradation curves for loose Toyoura sand (modified from Wang et al. 2007)

Figure 5.20 Comparisons of stiffness degradation curves for dense Toyoura sand (modified from Wang et al. 2007)

5.1.3 Residual Soil

Relationships of secant shear stiffness with shear strain for the specimens, CU-50 and CW-50, tested using different strain rate of 0.5 mm/min, 0.1 mm/min and 0.006 mm/min were fitted using Equation 5.1 to obtain small-strain stiffness (G_{max}). The experimental relationship of secant shear stiffness with shear strain of the specimens, CU-50 and CW-50, were not significantly affected by the strain rate. The plots of fitted and measured stiffness-strain relationship of unsaturated residual soil are presented in Figures 5.21 and 5.22. Values of G_{max} from curve fitting for all the tested specimens were obtained as 25 MPa. The parameter a was not affected by strain rate and it was obtained as 18 for all the tested specimens. The parameter b of the specimens, CU-50 and CW-50, at strain rate of 0.5 mm/min was slightly higher than that at strain rates of 0.1 mm/min and 0.006 mm/min. A summary of small-strain stiffness obtained from curve fitting and the fitting

parameters a and b is presented in Table 5.6. Values of G_{max} obtained from bender element test are also summarised in Table 5.6. As shown in Table 5.6, the values of G_{max} obtained from bender element tests are close to that of G_{max} obtained from curve fitting of experimental stiffness-strain relationships.

Table 5.6 Summary of G_{max} and fitting parameters

Specimen	Strain rate (mm/min)	G_{max} (MPa)	a	b	G_{max} from bender element test (MPa)
CU-50	0.5	25	18	0.85	25.9
	0.1	25	18	0.75	30.7
	0.006	25	18	0.75	--
CW-50	0.5	25	18	0.85	34.0
	0.1	25	18	0.75	28.6
	0.006	25	18	0.75	32.0

Figure 5.21 Comparisons of calculated and measured secant shear stiffness with shear strain for specimens CW-50 at different strain rates

Figure 5.22 Comparisons of calculated and measured secant shear stiffness with shear strain for specimens CW-50 at different strain rates

5.2 Small-strain Stiffness

5.2.1 Dry Sands

A commonly used equation proposed by Hardin (1965) was used to calculate the small-strain stiffness (G_{max}). The effective confining pressure in the equation was normalized with atmospheric pressure to make it dimensionally consistent. The equation used to fit the experimental data can be expressed as:

$$G_{max} = A f(e) \left(\frac{\sigma'_m}{P_a} \right)^n, \text{ where} \quad (5.2a)$$

$$f(e) = \frac{(a-e)^2}{1+e} \quad (5.2b)$$

where A , a , and n , are constants, σ'_m is effective mean confining pressure, P_a is atmospheric pressure (≈ 100 kPa) and A have the same units as G_{max} .

The values of small-strain stiffness shown in Table 5.3 were plotted with respect to void ratios and compared with the calculated values using Equation 5.2 in Figure 5.23. From the curve-fitting, $A = 50$ MPa, $f(e) = 5.7\exp(-1.36e)$ and $n = 0.48$ were obtained. Figure 5.24 show the relationships between G_{max} and effective mean confining pressures for all the tested sands. Equation 5.2 with the fitted values of $A = 50$ (MPa), $f(e) = 5.7\exp(-1.36e)$ and $n = 0.48$ was evaluated using the G_{max} data from Chung et al. (1984), Alarcon-Guzman et al. (1989), Lanzo and D' Elia (2003), and Khan et al. (2008). The goodness of fit for the G_{max} obtained from multi-stage tests for dry sands in this study is shown in Figure 5.25. The comparison of the calculated and measured G_{max} from Chung et al. (1984), Alarcon-Guzman et al. (1989), Lanzo and D' Elia (2003), and Khan et al. (2008) are shown in Figure 5.26. It is noted that values of G_{max} from Chung et al. (1984) were obtained from single-stage tests for dry Monterey No. 0 sand and that from Alarcon-Guzman et al. (1989) were obtained from single-stage tests using saturated Ottawa 20-30 sand. The G_{max} from Lanzo and D' Elia (2003) were obtained from multi-stage tests of dry Toyoura sand using Double Specimen Direct Simple Shear Device. The G_{max} from Khan et al. (2008) were obtained by extrapolating the measured relationship of shear stiffness with shear strain from multi-stage tests for dry round grain sand. Figure 5.26 shows that the fitted values of $A = 50$ (MPa), $f(e) = 5.7\exp(-1.36e)$ and $n = 0.48$ for Equation 5.2 generally give values of G_{max} within $\pm 30\%$ except for Lanzo and D' Elia (2003). The Double Specimen Direct Simple Shear Device gave lower values of G_{max} .

Figure 5.23 G_{max} versus void ratio for dry sand specimens under different effective mean confining pressure

Figure 5.24 G_{max} versus effective confining pressure for dry sand specimens at different initial void ratios

Figure 5.25 Goodness of fit between calculated and measured G_{max} for dry sand specimens

Figure 5.26 Comparison of measured G_{max} and calculated G_{max} for sands from Equation 5.2

5.2.2 Unsaturated Sand-Reclaimed Sand B

Leong et al. (2006) proposed the following relationship of G_{max} in terms of net confining pressure ($\sigma_3 - u_a$) and matric suction:

$$G_{max} = (G_{max})_o \left[1 + \left(\frac{\sigma - u_a}{P_a} \right)^n \right] \left[1 + \left\{ \ln \left(1 + \frac{u_a - u_w}{P_a} \right) \right\}^m \right] \quad (5.3)$$

where $(G_{max})_o$ is small-strain shear stiffness at zero net confining pressure and matric suction, P_a is atmospheric pressure (≈ 100 kPa), and n and m are empirical fitting parameters. Equation 5.3 was used to fit the small-strain stiffness measured by bender element tests for the reconstituted unsaturated sand specimens. Sum of the squared residuals (SSR) as defined by Leong and Rahardjo (1997) was used to minimize the errors between the measured G_{max} and the calculated G_{max} using Equation 5.3. From the curve fitting, the values of $(G_{max})_o$, n and m were obtained as 27 MPa, 1.2 and 0.3, respectively. Figure 5.27 shows the calculated G_{max} using Equation 5.3 as dashed curves. The effect of confining pressure on both saturated and unsaturated sand specimens was studied by plotting G_{max} versus effective mean or net confining pressure as shown in Figure 5.28. The relationships of G_{max} with confining pressure for both saturated and unsaturated sand were observed to be of similar trend. The values of G_{max} increase with effective or net confining pressure for both saturated and unsaturated sand specimens.

The small-strain stiffness of unsaturated sand specimens measured from the bender element tests were compared with the small-strain stiffness obtained from the triaxial compression tests shown in Table 5.5. Figure 5.29 shows the comparison of the small-

strain stiffness obtained from the bender element and the triaxial compression tests. In Figure 5.29, G_{max} obtained from bender element test was denoted as $G_{max-bender}$ and that from curve fitting of stiffness-strain relationship obtained from triaxial compression test was denoted as $G_{max-triaxial}$. It was observed that G_{max} measured from the bender element and triaxial compression generally show a good agreement with accuracy of 0- 20%.

Figure 5.27 Comparison of calculated and measured G_{max} with matric suctions

Figure 5.28 Comparison of calculated and measured G_{max} with effective mean or net confining pressure

Figure 5.29 Comparison of G_{max} obtained from bender element and triaxial compression tests

Loading rate effect on shear stiffness has been recognized by many researchers (Dorby and Vucetic 1987 and Tatsuoka and Shibuya 1992). However, at small strain behaviour of soil is essentially elastic and therefore similar stiffness at small strain is expected from

dynamic and static (e.g. cyclic and monotonic tests) tests (Tatsuoka and Shibuya 1992). Lo Presti et al. (1996) performed a series of monotonic and cyclic loading torsional shear tests on dry sand specimens at different void ratios and effective mean confining pressures. Table 5.7 summarises the basic properties of sand tested in Lo Presti et al. (1996)'s study. The small-strain stiffness, G_{max} , at shear strain less than 0.001% were obtained from the monotonic and cyclic loading torsional shear tests. Figure 5.30 show the comparison of G_{max} obtained from the monotonic and cyclic loading torsional shear tests. The small-strain stiffness obtained from monotonic loading torsional shear test was indicated as $G_{max-monotonic}$ and that from cyclic loading torsional shear test was indicated as $G_{max-cyclic}$ in Figure 5.30. It was observed that values of $G_{max-cyclic}$ and $G_{max-monotonic}$ agreed well as shown in Figure 5.30.

Figure 5.30 Comparison of G_{max} obtained from monotonic and cyclic loading torsional shear tests (modified from LoPresti et al. 1997)

Table 5.7 Summary of sand properties (from Lo Presti et al. 1997)

Soil type	FC (%)	G_s	D_{50} (mm)	C_u	e_{max}	e_{min}
Toyoura sand	-	2.650	0.22	1.35	0.985	0.611
Quiou sand	2	2.716	0.75	4.4	1.281	0.831

FC: fines content

5.3 Summary

The stiffness-strain relationships obtained from resonant column and triaxial compression tests were fitted using Borden et al. (1996) equation with $c=1$ to obtained small-strain stiffness (G_{max}) at $\gamma=0.001\%$. It was observed that Equation 5.1 with $c=1$ fitted the experimental data well. The stiffness degradation curves were also obtained by normalising the stiffness-strain relationship with G_{max} . A modified upper bound stiffness degradation curve for sands was obtained to encompass all the experimental stiffness degradation curves. This modified upper bound stiffness degradation curve was also evaluated using the published stiffness degradation curves of different sands. It was observed that most stiffness degradation curves obtained from the literature plot below the modified upper bound stiffness degradation curve proposed in this study. G_{max} obtained from resonant column tests using dry sands was also fitted using Hardin (1965) equation and the fitted values, $A = 50$ (MPa), $f(e) = 5.7 \exp(-1.36e)$ and $n = 0.48$, were obtained. These fitted values were evaluated using G_{max} from the literature. It was observed that the fitted values obtained from the resonant column tests using dry sand generally overestimated G_{max} of sands obtained from the literature. $G_{max-bender}$ and $G_{max-triaxial}$ of saturated and unsaturated sand obtained from the same specimen were also

compared. It was observed that $G_{max-bender}$ and $G_{max-triaxial}$ showed reasonably good agreement within 0-20 %.

Chapter 6 Modelling for Shear Strength of Unsaturated Soils

6.1 Equations for Shear Strength of Unsaturated Soils

Fredlund et al. (1978) proposed the following shear strength equation for unsaturated soil in terms of the stress states variables, $(\sigma - u_a)$ and $(u_a - u_w)$:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad (6.1)$$

A number of equations have been proposed for estimating unsaturated shear strength in terms of the stress state variables. Vanapalli and Fredlund (2000) and Goh et al. (2010) provided reviews of these equations. In most of the proposed equations, soil-water characteristic curve along with the saturated shear strength parameters were used to replace $\tan \phi^b$ when describing non-linear change in shear strength with matric suction (Abramento and Carvalho 1989; Vanapalli et al. 1996; Lee et al. 2005;) and Houston et al. (2008) proposed equations having similar form as Equation 6.1 in which exponential (Equation 6.2) and hyperbolic functions (Equation 6.3), respectively, were used to describe the non-linear relationship of ϕ^b with matric suction.

$$\tau = c' + (\sigma - u_a) \tan \phi' + A_1 (u_a - u_w)^{B_1} \quad (6.2)$$

where A_1 and B_1 are fitting parameters.

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad (6.3)$$

$$\text{where, } \phi^b = \phi' - \left[\frac{(u_a - u_w)^*}{A_2 + B_2 (u_a - u_w)^*} \right]$$

where A_2 and B_2 are fitting parameters, $(u_a - u_w)^* = (u_a - u_w) - \text{AEV}$ and AEV = air-entry value.

Goh et al. (2010) proposed the following equations for estimation of unsaturated shear strength under drying and wetting stress history incorporating the SWCC:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi', \quad (6.4a)$$

for $(u_a - u_w) < \text{AEV}$

$$\tau = c' + [(\sigma - u_a) + \text{AEV}] \tan \phi' + [(u_a - u_w) - \text{AEV}] A_3 \Theta^\kappa \tan \phi', \quad (6.4b)$$

for $(u_a - u_w) \geq \text{AEV}$

$$\text{and } \kappa = [\log(u_a - u_w) - \log(\text{AEV})]^{B_3} \quad (6.4c)$$

where A_3 and B_3 are fitting parameters. In most of the proposed equations for shear strength of unsaturated soils, the rate of change in shear strength with matric suction near the air-entry value of soil was assumed to be equal to ϕ' . However, experimental data have shown that the initial slopes of shear strength with matric suction can be greater than ϕ' (i.e. $\phi^b > \phi'$) (e.g. Escario and Saez 1986; Melinda et al. 2004; Hossain and Yin 2010; Likos et al. 2010). In this study, the relationship of ϕ^b with matric suction was examined for a wide range of soils. For this purpose, 15 soils from the literature and two soils tested in this study were investigated. A summary of the basic properties of the 17 soils is given in Table 6.1.

Table 6.1 Summary of basic properties of soils used for curve fitting

Reference	Soil type	LL (%)	PI (%)	P ₂₀₀ (%)	φ' (degrees)	Notation
Escario and Saez (1987)	Madrid clayey sand	32	15	39.2	39.2	S1
Gan et al. (1988)	Glacial till	35.5	18.7	72	25.5	S2
Abramento and Carvalho (1989)	Residual soil from natural slopes	39	14	26	40	S3
Vanapalli et al. (1996)	Glacial till	35.5	18.7	72	23	S4
Miao et al. (2002)	Nanyang expansive soil	58.3	31.8	93.3	21.3	S5
Lee et al. (2005)	Residual soil (Weathered granite soil)	0	0	12.4	41.4	S6
Reis and Vilar (2005)	Mature residual soil	68	29	90*	31	S7
Thu et al. (2006)	Kaolin	51	15.4	100	32	S8
Kayadelen et al. (2007)	Diyarbakir residual soil	77	45	95	21.9	S9
Houston et al. (2008)	Price club	25.6	7.6	54.8	27.1	S10
	Sheely clay	41.1	17.4	94.8	33.1	S11
Hossian and Yin (2010)	Completely decomposed granitic soil	32.8	10.1	50.1	29.9	S12
Likos et al. (2010)	Sandy colluviums	0	0	0	40.1	S13
Nam et al. (2011)	SM	0	0	30.5	35	S14
	CL1	43.5	20.5	82	32.4	S15
This study	Reclaimed sand B	0	0	0	38	S16
	Residual soil from Bukit Timah Granite	45	15	55	33	S17

* Assumed value

6.2 Variation of ϕ^b with Matric Suction

The incremental change in shear strength due to matric suction can be obtained by differentiating Equation 6.1 as:

$$\frac{d\tau}{d(u_a - u_w)} = \tan \phi^b = f(u_a - u_w) \quad (6.5)$$

where $f(u_a - u_w)$ is a function of matric suction and may not be constant. Therefore, shear strength at any matric suction is obtained as:

$$\tau = c' + (\sigma - u_a) \tan \phi' + \int_0^{(u_a - u_w)} f(u_a - u_w) d(u_a - u_w) \quad (6.6)$$

For the case where $f(u_a - u_w)$ is a constant (i.e. $\tan \phi^b$ is constant), Equation 6.6 reverts to Equation 6.1. Figure 6.1 (a) shows the experimental shear strength data from Vanapalli et al. (1996) curve fitted with a third order polynomial equation. Values of ϕ^b for different matric suctions were backcalculated according to Equation 6.1 and shown in Figure 6.1 (b). The relationships of ϕ^b with matric suction were obtained for all the data sets summarised in Table 6.1 and presented in Figure 6.2. As shown in Figure 6.2, relationships of ϕ^b with matric suction were of sigmoid shape similar to the soil-water characteristic curve. Initial values of ϕ^b (ϕ^b_o) ranged approximately from 20° to 80°. All the data sets have ϕ^b_o values less than 45° except for three data sets. As shown in Figure 6.2, value of ϕ^b is constant at low matric suction and decreases with matric suction at higher matric suctions. The matric suction at which ϕ^b decreases rapidly is different for different soil types. This is because variation of shear strength with matric suction showed different behaviour for different soil types. From the curve fitting exercise, it was observed that variations of shear strength with matric suction were significantly affected by plasticity index. Therefore, general behaviour of shear strength with matric suction and the respective relationship of ϕ^b with matric suction were categorised according to soil types: namely, high and low plasticity soils in the following section.

Figure 6.1 Experimental shear strength data and fitted curve of compacted Glacial Till under net confining pressure of 25 kPa (data obtained from Vanapalli et al. 1996)

Figure 6.2 Relationships of ϕ^b with matric suction for a wide range of soils shown in Table 6.1

Table 6.2 Soil classification according to plasticity index (modified from Burmister 1949)

PI	Description	This Study
0	Nonplastic	Low plasticity
1-5	Slightly plastic	
5-10	Low plasticity	
10-20	Medium plasticity	High plasticity
20-40	High plasticity	
>40	Very high plasticity	

Table 6.2 shows the soil classification according to Burmister (1949). In this study, $PI = 0-10$ and $PI > 10$ were categorized as low plasticity and high plasticity soils, respectively, as shown in Table 6.2. The typical relationships of shear strength with matric suction for a high plasticity soil and low plasticity soil are presented in Figure 6.3.

Figure 6.3 Typical τ , ϕ^b_{max}/ϕ' , and ψ versus matric suction for soils with high plasticity and sandy soils

The relationship of shear strength with matric suction can be divided into three zones, namely, near saturation zone, transition zone and residual zone. In the near saturation zone, ϕ^b remains constant. Initial values of ϕ^b can be equal to or greater than ϕ' depending on the soil type. Values of ϕ^b in the near saturation zone have been reported to be equal to ϕ' from the experiments performed on some soil types [e.g. compacted glacial till in Gan et al. (1988) and compacted silt in Thu et al. (2006)] and greater than ϕ' for other soil types [e.g. reconstituted Madrid clayey sand in Escario and Saez (1986), compacted sandy clay in Melinda et al. (2004), compacted silty sand (completely decomposed granite) in Hossain and Yin (2010) and reconstituted sandy colluviums in Likos et al. (2010)]. The values of ϕ^b greater than ϕ' in most low plasticity soils are attributed to soil dilatancy. Therefore from here onwards, ϕ^b is described as ϕ^b_{app} having two components (i.e. $\phi^b_{app} = \phi^b + \psi^b$). The operative dilation angle ψ^b may be different from the dilation angle ψ experienced in ϕ' .

In the near saturation zone for high plasticity soils, value of ϕ^b_{app} is equal to ϕ' [i.e. $\phi^b = \phi'$ and $\psi^b = 0$]. In other words, the initial value of ϕ^b in the near saturation zone is equal to ϕ' and any value of ϕ^b greater than ϕ' is attributed to soil dilation, ψ^b . In the transition zone, value of ϕ^b_{app} is less than ϕ' [i.e. $\phi^b < \phi'$ and $\psi^b = 0$] for both high and low plasticity soils. In the residual zone, $\phi^b_{app} = 0$ for high plasticity soil [i.e. $\phi^b = 0$ and $\psi^b = 0$] whereas $\phi^b_{app} < 0$ for low plasticity soil [i.e. $\phi^b = 0$ and $\psi^b < 0$]. Simplified relationships of ϕ^b , ψ^b and ϕ^b_{app} with matric suction are presented in Figures 6.3(b), 6.3(c) and 6.3 (d).

In the transition zone, soils start to desaturate rapidly and therefore the effect of matric suction becomes less significant. Shear strength increases with matric suction in the transition zone however the change in shear strength with respect to matric suction is relatively slower than that with respect to effective confining pressure. Therefore, values of ϕ^b become smaller than that of ϕ' in the transition zone. This decreasing trend of ϕ^b with matric suction in the transition zone is expected to be similar for all types of soil. However, the matric suction at which ϕ^b enters into the transition zone can be different for different soil types. For low plasticity soils, ϕ^b enters into the transition zone at relatively smaller matric suctions similar to the trend observed in the SWCCs of the soils. Low plasticity soils desaturate much faster than high plasticity soils as air-entry value of low plasticity soils is much lower than that of high plasticity soils.

In the residual zone, shear strength of soil can continue to increase ($\phi_{app}^b = \text{constant} > 0$), decrease ($\phi_{app}^b < 0$) or remain essentially constant ($\phi_{app}^b = 0$) with matric suction depending on soil type and range of matric suction (Vanapalli et al. 1996). Donald (1956) performed experiments using four different types of sands and showed that the decrease in shear strength occurred near matric suctions of 10 kPa to 15 kPa which can be considered to be the residual zone. Escario and Juca (1989) measured the shear strength of three different soils over a wide range of matric suction (i.e. matric suction up to 15 MPa). The decrease in shear strength started at matric suction value of 4 MPa for Madrid clay sand, i.e., the start of the residual zone was at 4 MPa. The shear strength remained essentially constant with matric suction in the residual zone for the other two soils, Madrid gray clay and red silty clay (Escario and Juca 1989). In the residual zone, possible changes in shear strength

with matric suction and the respective relationship of ϕ^b with matric suction can be listed as:

- When shear strength increases with matric suction, value of ϕ_{app}^b remains constant, i.e. $\phi^b = \text{constant} > 0$, $\Psi^b = 0$.
- When shear strength decreases with matric suction, value of ϕ_{app}^b becomes negative, i.e. $\phi^b = 0$, $\Psi^b < 0$.
- When shear strength remains constant with matric suction, value of ϕ_{app}^b tends to zero, i.e. $\phi^b = 0$, $\Psi^b = 0$.

For high plasticity soil, Ψ^b is zero for all the three zones (near saturation, transition and residual) as shown in Figure 6.3 (c). In the residual zone, ϕ_{app}^b can be negative for low plasticity soil due to volume compression taking place at high matric suction. In this case, the value of ϕ^b becomes essentially zero at high matric suction and the value of Ψ^b becomes negative. The simplified variation of dilation angle with matric suction for low plasticity soils in the residual zone is shown in Figure 6.3(c).

6.3 Parametric Study

For non-linear variation of shear strength with matric suction, the sigmoid relationship of ϕ_{app}^b with matric suction was assumed as an error function for mathematical expediency as the error function can be easily integrated unlike the other sigmoid functions (e.g.

hyperbolic function, logistic function and etc..). The error function representing non-linear relationship of ϕ_{app}^b with matric suction is expressed as:

$$\tan \phi_{app}^b = f(u_a - u_w) = A + \left(B \operatorname{erf} \left(\frac{u_a - u_w}{CP_a} \right) \right) \quad (6.7)$$

where A, B and C are fitting parameters, and P_a is atmospheric pressure (= 100 kPa) introduced for consistency in units. The integrand for Equation 6.4 is expressed as:

$$\int_0^{u_a - u_w} \tan \phi_{app}^b d(u_a - u_w) = A(u_a - u_w) + \frac{BCP_a \exp \left(\frac{-(u_a - u_w)^2}{(CP_a)^2} \right)}{\sqrt{\pi}} + B(u_a - u_w) \operatorname{erf} \left(\frac{u_a - u_w}{CP_a} \right) - \frac{BC}{\sqrt{\pi}} P_a \quad (6.8)$$

By substituting Equation 6.8 into 6.6, shear strength at any matric suction is obtained as:

$$\tau = c' + (\sigma - u_a) \tan \phi' + A(u_a - u_w) + \frac{BCP_a \exp \left(\frac{-(u_a - u_w)^2}{(CP_a)^2} \right)}{\sqrt{\pi}} + B(u_a - u_w) \operatorname{erf} \left(\frac{u_a - u_w}{CP_a} \right) - \frac{BC}{\sqrt{\pi}} P_a \quad (6.9)$$

The initial slope of shear strength with matric suction or the incremental change in shear strength with matric suction in the near saturation zone can be controlled by setting the parameter A as $\tan (\phi_{app}^b)_0$ (Equation 6.7) as shown in Figure 6.4(a). The curve of ϕ_{app}^b versus matric suction shifts up and down with different values of parameter A as shown in Figure 6.4 (b). The parameter B mainly controls the relationship of shear strength with

matric suction in the residual zone as shown in Figure 6.5a. As discussed earlier, the shear strength of unsaturated soil can increase, decrease or remain constant with matric suction in the residual zone. These three possible behaviours of shear strength with matric suction can be obtained by adjusting the parameter B along with parameter A. The effect of parameter B on the relationship of ϕ_{app}^b with matric suction is presented in Figure 6.5b. The fitting parameter C influences the change point where ϕ^b enters into the transition zone. Figure 6.6 shows the effect of parameter C on the relationships of shear strength and ϕ_{app}^b with matric suction.

(a) τ versus $u_a - u_w$

Figure 6.4 Effect of parameter A with $B=-0.5$ and $C=200$ on shear strength and ϕ_{app}^b with matric suction

Figure 6.5 Effect of parameter B with $A=0.5$ and $C=200$ on shear strength and ϕ_{app}^b with matric suction

Figure 6.6 Effect of parameter C with $A=0.5$ and $B=-0.5$ on shear strength and ϕ_{app}^b with matric suction

6.4 Evaluation of Equation 6.9

The 17 shear strength data summarised in Table 6.1 was curve fitted using Equation 6.9 to determine the relationship between fitting parameters (A , B , and C) and soil properties (e.g. plasticity index, fine content, grain diameters, etc.). The data sets shown in Table 6.1 cover matric suctions ranging from 0 kPa up to 800 kPa under net confining pressures ranging from 0 kPa to 588 kPa. For curve fitting, the error between experimental and fitted shear strength data was minimised using the normalised sum of squared error (SSE_{norm}) given below:

$$SSE_{norm} = \frac{1}{n} \cdot \sum_{i=1}^n \left(\frac{\tau - \tau_i}{\tau} \right)^2 \quad (6.10)$$

where τ is experimental shear strength and τ_i is fitted shear strength and n is number of data point. A smaller SSE_{norm} denotes a better fit. The goodness of fit between experimental and fitted shear strength data was also assessed by mean of average relative error (ARE). The average relative error is written as:

$$ARE = \frac{1}{n} \cdot \sum_{i=1}^n \left(\frac{\tau - \tau_i}{\tau} \right) \cdot 100 \% \quad (6.11)$$

A smaller ARE denotes a better fit. The experimental results obtained from the multi-stage tests under constant water content condition for the residual soils from Bukit Timah Granite (S17) were fitted using Equation 6.9 as shown in Figure 6.7. Value of $(\phi^b_{app})_0$ equal to ϕ' gave a good fit for the variation of shear strength with matric suction in the near saturation zone. The gradual increment of shear strength between matric suctions of 200 kPa to 400 kPa was obtained by setting $IBI = 0.59 < A$. The value of $C = 225$ provided a good fit for the shear strength. Examples of $\phi^b_{app} > \phi'$ are shown in Figures 6.8 and 6.9 for Madrid clayey sand (S1) from Escario and Saez (1987) and sandy colluviums (S13) from Likos et al. (2010), respectively. In Figure 6.8, the value of $A = 1.15$ equivalent to $(\phi^b_{app})_0 = 49^\circ$ with $C = 160$ fitted well the experimental data under the net confining pressure of 441 kPa. The gradual increment of shear strength between matric suction of 200 kPa to 800 kPa was obtained by setting $IBI = 1.08 < A$ as shown in Figure 6.8. In Figure 6.9, the value of $A = 3.36$ equivalent to $(\phi^b_{app})_0 = 73.4^\circ$ with $C = 2$ produced a good fit for the relationship of shear strength with matric suction in the near saturation zone. The gradual decreasing trend of shear strength with matric suction beyond matric suction of approximately 5 kPa was obtained by setting $IBI > A$ as shown in Figure 6.9.

Figure 6.7 Shear strength versus matric suction under net confining pressure of 0 kPa of residual soil from Bukit Timah Granite

Figure 6.8 Shear strength versus matric suction under net confining pressure of 441 kPa of Madrid clayey sand (data obtained from Escario and Saez 1987)

Figure 6.9 Shear strength versus matric suction under net confining pressure of 0.33 kPa of sandy colluvium (data obtained from Likos et al. 2010)

The effect of confining pressure on the fitting parameters was also studied by fitting the experimental data under different net confining pressures. Figure 6.10 shows the experimental data and fitted curves under different net confining pressures of glacial till (S4) from Vanapalli et al. (1996). The value of $(\phi^b_{app})_o = \phi'$ provided a good fit for the experimental data under different net confining pressures. The values of parameter A and B were not affected by the net confining pressures however the parameter C increased slightly with net confining pressure.

The plots of experimental data with the fitted curves for compacted completely decomposed granite soil (S12) under different net confining pressures from Hossian and Yin (2010) are presented in Figure 6.11. As shown in Figure 6.11, the parameters A, B,

and C increased with net confining pressures. The fitting parameters (A, B, and C) are not independent of net confining pressure. Therefore an attempt was made to obtain correlations between fitting parameters and net confining pressure. However, a good correlation between fitting parameters and net confining pressure could not be obtained based on the limited data sets used for curve fitting. Table 6.3 summarises the values of fitting parameters, SSE_{norm} , and ARE obtained from curve fitting. Generally, Equation 6.6 fitted the experimental data well with ARE less than 20% except for the experimental data of Price Club soil (S10) from Houston et al. (2008). A relatively high value of ARE= 26.9% was obtained for the soil due to the scattered experimental data of shear strength with matric suction.

Figure 6.10 Shear strength versus matric suction under different net confining pressure of compacted glacial till at initial water content of 13% and dry density of 1.73 Mg/m^3 (data obtained from Vanapalli et al. 1996)

Figure 6.11 Shear strength versus matric suction under different net confining pressures of compacted completely decomposed granite soil (data obtained from Hossian and Yin 2010)

6.5 Correlations between Fitting Parameters and Properties of Soils

A number of unsaturated shear strength equations were proposed to predict the behaviour of shear strength with matric suction (Vanapalli et al. 1996; Oberg and Sallfors 1997; Bao et al. 1998; Khalili and Khabbaz 1998; Houston et al. 2008; and Goh et al. 2011). In the equations, either the entire soil-water characteristic curve (i.e. SWCC up to matric suction of 1000 MPa) or the air-entry value was needed along with the effective shear strength parameters for the prediction of unsaturated shear strength. Unsaturated soil testing including determination of SWCC is time consuming and also difficult to perform as it requires specialised testing equipment. Therefore, an equation that can estimate the variation of shear strength with matric suction without SWCC is beneficial.

Table 6.3 Summary of soil types, net confining pressures, fitting parameters, SSE_{norm} , and ARE

Reference	Soil type	$\sigma-u_a$	A	B	C	SSE_{norm}	ARE
Escario and Saez (1987)	Madrid clayey sand	118	0.82	-0.74	1.80	0.0018	3.6
		294	0.82	-0.74	2.20	0.0005	1.7
		441	1.20	-1.08	1.50	0.0009	1.9
		588	1.20	-1.08	1.60	0.0006	1.6
Gan et al. (1988)	Glacial till	72.6	0.48	-0.43	3.50	0.0110	9.9
Abramento and Carvalho (1989)	Residual soil from natural slopes	0	0.84	-0.76	0.30	0.0404	14.3
Vanapalli et al. (1996)	Glacial till	25	0.43	-0.39	1.80	0.0032	5.3
		100	0.43	-0.39	2.20	0.0022	3.6
		200	0.43	-0.39	2.50	0.0050	5.3
Miao et al. (2001)	Nanyang expansive soil	0	0.39	-0.35	3.50	0.0002	1.1
Lee et al. (2005)	Residual soil (Weathered granite soil)	0	1.30	-1.17	0.10	0.0037	5.0
Reis and Vilar (2005)	Mature residual soil	0	0.60	-0.54	2.80	0.0039	5.7
Thu et al. (2006)	Kaolin	50	0.63	-0.56	3.50	0.0126	10.4
Kayadelen et al. (2007)	Diyarbakir residual soil	0	0.40	-0.36	3.49	0.0021	4.0
Houston et al. (2008)	Price club	0	0.65	-0.59	2.30	0.1420	26.9
	Sheely clay	0	0.65	-0.59	0.90	0.0551	16.7
Hossian and Yin (2010)	Completely decomposed granitic soil	50	0.58	-0.52	2.20	0.0103	8.1
		100	0.70	-0.63	2.50	0.0009	2.7
		200	0.80	-0.72	3.00	0.0038	5.0
		300	0.95	-0.86	3.00	0.0010	2.8
Likos et al. (2010)	Sandy colluvium	0.33	3.63	-3.71	0.02	0.0006	1.3
		1.9	3.30	-3.40	0.03	0.0070	7.7
Nam et al. (2011)	SM	43.3	1.00	-0.90	1.20	0.0018	3.4
	CL1	43.3	0.66	-0.59	2.50	0.0301	14.4
This study	Reclaimed sand B	50	11.00	-9.90	0.10	0.0114	8.3
	Residual soil from Bukit Timah Granite	0	0.65	-0.59	2.25	0.0011	2.8
Smallest value		--	0.39	-0.35	0.02	--	--
Largest value		--	11	-9.90	3.50	--	--

In this section, an attempt was made to estimate unsaturated shear strength by using soil properties such as ϕ' , PI and fines content only. The correlation of fitting parameters (A, B and C) with various soil properties (PI, P_{200} , LL, PL, ϕ' , D_{50} , D_{10} , D_{30} , D_{60} , C_u , C_c) were examined. The best correlations between fitting parameters and soil properties are shown in Figures 6.12 to 6.14. Ratios of $(\phi_{app}^b)_o / \phi'$ were correlated to basic soil properties (PI and P_{200}) using a power function as shown in Figure 6.12. The values of $(\phi_{app}^b)_o / \phi'$ for sandy

colluviums from Likos et al. (2010) and reclaimed sand from this study were obtained as 1.9 and 2.2, respectively, from curve fitting. These two soils are non-plastic with zero fines content. It is not possible to describe the data in Figure 6.12 with a single curve; therefore an upper bound curve and a lower bound curve are used to describe the relationship:

$$\text{Lower bound: } (\phi_{\text{app}}^b)_o / \phi' = 1.75 (PI + P_{200} + 1)^{-0.15} \geq 1 \quad (6.12a)$$

$$\text{Upper bound: } (\phi_{\text{app}}^b)_o / \phi' = 2.25 (PI + P_{200} + 1)^{-0.15} \geq 1 \quad (6.12b)$$

where PI is plasticity index and P_{200} is fines content or percent passing of No. 200 sieve.

Note that $A = \tan (\phi_{\text{app}}^b)_o$.

Figure 6.12 Relationship between $(\phi_{\text{max}}^b)_o / \phi'$ with PI and P_{200}

A good correlation between fitting parameters B and A was obtained as shown in Figure 6.13. A value of 0.9966 was obtained for R^2 from curve fitting. For most soils, a gradual

increase in shear strength with matric suction was observed beyond the near saturation zone except for the experimental data of sandy colluviums (S13) from Likos et al. (2010).

The relationship between parameters A and B is:

$$B = -0.92(A) \quad (6.13)$$

The relationship between fitting parameter C and $(\phi^b_{app})_o$ is shown in Figure 6.14. Upper and lower bound relationships of fitting parameter C and $(\phi^b_{app})_o$ were introduced to best-fit the data. The upper and lower bound relationships are:

$$\text{Lower bound: } C = [(\phi^b_{app})_o / \phi']^{-5.5} \quad (6.14a)$$

$$\text{Upper bound: } C = 7[(\phi^b_{app})_o / \phi']^{-5.5} \quad (6.14b)$$

Figure 6.13 Relationship between parameter B and parameter A

Figure 6.14 Relationship between parameter C and $(\phi^b_{app})_o / \phi'$

The value of $(\phi^b_{app})_o$ or fitting parameter A is an important parameter for estimation of unsaturated shear strength using Equation 6.9 as it guides the rate of change of shear strength with matric suction in the near saturation zone. In addition, the fitting parameters B and C are also functions of parameter A (Equations 6.13 and 6.14).

The proposed equations, Equation 6.12 to 6.14, were verified using the data sets shown in Table 6.1. The correlation of parameter A calculated using the proposed upper and lower bound relationships ($A_{calculated}$) with parameter A obtained from curve fitting ($A_{experiment}$) is presented in Figure 6.15. A magnified plot for the correlation between $A_{calculated}$ and $A_{experiment}$ was also plotted for all the data except sandy colluviums (S13) and reclaimed sand B (S16) in Figure 6.15. A good correlation ($R^2=0.89$) between $A_{calculated}$ and $A_{experiment}$ was observed as shown in the magnified plot of Figure 6.15.

Figure 6.15 Correlation of fitting parameter A with ϕ' , PI , and P_{200} for all the soils shown in Table 6.2

The proposed equations, Equations 6.12 to 6.14, were firstly verified using the three sets of data that showed the least agreement between $A_{\text{calculated}}$ and $A_{\text{experiment}}$. For evaluation of shear strength estimation, the lower bound shear strength with matric suction was obtained using Equations 6.12, 6.13 (a), and 6.14 (a), and the upper bound shear strength with matric suction curve was obtained using Equations 6.12, 6.13 (b), and 6.14 (b). The comparisons of the experimental and estimated upper and lower bound shear strength with matric suction for the three soils are presented in Figures 6.16 to 6.18. The experimental data from Lee et al. (2005) and Likos et al. (2010) are closer to the lower bound curve and that of Reclaimed sand B obtained in this study are closer to the upper bound curve.

The proposed equations were further evaluated using three other sets of data which were not in Table 6.1. Table 6.4 shows the soil types and properties of soils used for the evaluations. These three soils have different values of $(PI + P_{200} + 1)$ covering the entire range of the proposed relationship between $(\phi_{app}^b)_o / \phi'$ and basic soil properties. The comparisons of experimental and estimated shear strength with matric suction of these three soils are shown in Figures 6.19 to 6.21. The values of A obtained from the proposed equations [Equations 6.12 (a) and (b)] estimated reasonably well for the rate of change of shear strength with matric suction in the near saturation zone. According to Equation 6.13, the estimation of gradual increment of shear strength with matric suction beyond the near saturation zone also provided an acceptable trend of shear strength with matric suction. The upper bound value of C fitted well for the experimental data of residual soil from Rahardjo et al. (2004) as shown in Figure 6.19. The lower bound values of C fitted well for the experimental data of ASU East soil from Houston et al. (2008) and MH soil from Nam et al. (2011) as shown in Figures 6.20 and 6.21.

Table 6.4 Summary of soil types, net confining pressure, PI, P_{200} and ϕ' used for evaluation of the proposed equations

Reference	Soil type	$\sigma - u_a$ (kPa)	PI	P_{200} (%)	ϕ' (degrees)
Rahardjo et al. (2004)	Residual soil from Jurong Formation	0	14.8	66	31.5
Houston et al. (2008)	ASU east	0	0	27	36.1
Nam et al. (2011)	MH	21.6	21	90.5	32

Figure 6.16 Comparison of experimental and estimated shear strength with matric suction of residual soil (weathered granite soil) at net confining pressure of 0 kPa (data from Lee et al. 2005)

Figure 6.17 Comparison of experimental and estimated shear strength with matric suction of sandy colluvium at net confining pressure of 0.33 kPa (data from Likos et al. 2010)

Figure 6.18 Comparison of experimental and estimated shear strength with matric suction of reclaimed sand B at net confining pressure of 50 kPa

Figure 6.19 Comparison of experimental and estimated shear strength with matric suction of residual soil from Jurong Formation at net confining pressure of 0 kPa (data from Rahardjo et al. 2004)

Figure 6.20 Comparison of experimental and estimated shear strength with matric suction of ASU east soil at net confining pressure of 0 kPa (data from Houston et al. 2008)

Figure 6.21 Comparison of experimental and estimated shear strength with matric suction of MH soil at net confining pressure of 21.6 kPa (data from Nam et al. 2011)

6.6 Procedure and Limitations for Estimation of Unsaturated Shear Strength

The procedure for estimation of unsaturated shear strength can be listed as follows:

- Firstly, using soils properties (ϕ' , PI and P_{200}), the lower and upper bound values of $(\phi^b_{app})_o$ are calculated according to Equations 6.12 (a) and 6.12 (b), respectively.
- Secondly, the lower and upper bound values of B are obtained using Equation 6.13 in which $A = \tan [(\phi^b_{app})_o]$.
- Lastly, the lower and upper bound values of C are obtained using Equations 6.14 (a) and 6.14(b), respectively.

Shear strength of soil at any matric suction can be calculated using Equation 6.9 after obtaining the fitting parameters, A, B and C. The proposed equations for estimation of unsaturated shear strength were obtained from the data sets tested at matric suction up to 800 kPa under different net confining pressures. Therefore, the values of fitting parameters obtained from Equations 6.12 to 6.14 will provide a reasonable estimation for similar ranges of matric suction. The smallest and largest values of fitting parameters A, B, and C are shown in Table 6.3 and these values should be taken as limiting values when using Equations 6.12 and 6.14 for estimation of unsaturated shear strength. The effect of confining pressure on variation of shear strength with matric suction was not considered in the proposed equations. In other words, it was assumed that the rate of change in shear strength with matric suction was the same under different net confining pressures. As shown in Equation 6.13, it was also assumed that shear strength increased with matric suction in the residual zone.

6.7 Summary

To date, most equations for estimation of shear strength of unsaturated soils require the soil-water characteristic curve along with the saturated shear strength parameters. In this study, the equation proposed to estimate unsaturated shear strength do not require SWCC but only soil properties ϕ' , PI and P_{200} . The proposed equations have similar form as Fredlund et al. (1978)'s unsaturated shear strength equation. In the proposed equation, an error function with three fitting parameters (A, B, and C) was used to describe non-linear variation of ϕ_{app}^b with matric suction. Relationship of ϕ_{app}^b with matric suction was studied by curve fitting a total of 17 sets of shear strength data for a wide range of soil types. From curve fitting, it was observed that values of ϕ_{app}^b were greater than ϕ' for most low plasticity soils and equal to ϕ' for high plasticity soils in the near saturation zone. The correlations between fitting parameters (i.e. A, B and C) and soil properties (i.e. ϕ' , PI and P_{200}) were obtained from curve fitting. The proposed equations were also verified using independent published data from the literature.

Chapter 7 Conclusion and Recommendations

7.1 Summary

The objective of this study is to investigate the stiffness and strength characteristics of unsaturated soils. Triaxial compression tests, bender element tests and resonant column tests were performed to investigate the stiffness and strength behaviours of different soil types (i.e. kaolin, sands and undisturbed residual soils) under different stress conditions (i.e. σ'_m , $u_a - u_w$ and $\sigma - u_a$). Test set-up, materials and findings related to experimental works conducted in this study were presented in Chapter 3 and Chapter 4. Strain rate is one of the parameters that can affect the stress-strain behaviour of soils. Therefore, the effects of strain rate on stiffness and strength behaviours of unsaturated soils were investigated using consolidated undrained (CU) and constant water content (CW) tests by performing multi-stage shearing tests. It was observed that the effect of strain rate on pore-water pressure generation was more significant than that on deviator stress. In summary, stiffness and strength characteristics of unsaturated soils were investigated in terms of matric suction, net confining pressure and strain rate. In addition to experiments, the existing Borden et al. (1996) equation was modified for estimation of stiffness-strain relationships and equations were proposed for estimation of unsaturated shear strength with matric suction. In Chapter 5, experimental stiffness-strain relationships were fitted using Borden et al. (1996) equation to characterise the stiffness degradation curves in terms of matric suction, net confining pressure and strain rate. Small-strain stiffness ($G_{\max}$)

was observed to be important in characterising stiffness degradation of soils. It was observed that the effects of matric suction and net confining pressure were more significant on $G_{\max}$ than stiffness degradation curve. In Chapter 6, experimental shear strength data were fitted using a sigmoid equation proposed in this research. It was observed that the proposed equation fitted the experimental data well

7.2 Conclusion

The more significant findings of this research are listed as follows:

7.2.1 Improved Local Displacement Transducers (LDTs)

- Wheatstone bridge circuit is conventionally used for strain measurement devices such as LDTs. In this research, the use of Anderson loop circuit was introduced. Performances of LDTs using the Wheatstone bridge and the Anderson loop circuit were investigated. The local displacement transducer with Anderson loop was found to be twice as sensitive as that with Wheatstone bridge for similar input current. Furthermore, the local displacement transducer with Anderson loop outperformed the local displacement transducer with Wheatstone bridge in terms of lower deviation, hysteresis and repeatability errors.

7.2.2 Stiffness

- For dry sands, shear stiffness increases with increasing effective mean confining pressure (σ_m') and decreasing void ratio (e). However, the effect of σ_m' was observed to be more significant than that of e . For saturated and unsaturated

sands, shear stiffness increases significantly with increasing effective or net confining pressure.

- For dry sands, stiffness degradation curves degraded at higher strain level with higher σ'_m . However, for unsaturated sand, net confining pressure showed less influence on stiffness degradation curves. Matric suction showed no influence on stiffness degradation curves for unsaturated sands.
- All experimental stiffness degradation curves of sands plotted within a band. The upper bound curve proposed by Seed and Idriss (1970) plotted below all the experimental stiffness degradation curves. Therefore, a modified upper bound curve was proposed based on the experimental data obtained in this research. The modified upper bound curve was evaluated using experimental data obtained from the literature. It was observed that the experimental data from the literature plotted below the modified upper bound curve suggesting that the modified upper bound curve is more appropriate than the upper bound curve of Seed and Idriss (1970).
- For unsaturated kaolin and residual soils, the shear stiffness increased significantly with matric suction. However, matric suction did not affect the strain level at which stiffness degraded as well as the rate of degradation. Thus, the effect of matric suction could not be observed in stiffness degradation curves. On the other hand, the shear stiffness was not significantly affected by strain rate. However, shear stiffness degraded at higher strain levels with faster strain rate. Thus, the effect of strain rate showed up in stiffness degradation curves in that stiffness degradation curves shifted to the right with faster strain rate.

- Values of $G_{\max}$ increase significantly with matric suction for kaolin and residual soils, and effective or net confining pressure for sands. The effect of strain rate on $G_{\max}$ obtained from LDTs measurement showed little or no influence. There is good agreement between $G_{\max}$ obtained from bender element tests and LDTs measurements.

7.2.3 Shear Strength

- Shear strength increased with matric suction and net confining pressure. Variation of shear strength with matric suction was observed to be non-linear for both sand and residual soils.
- From curve fittings a range of data sets obtained from this research and the literature, equations for estimation of unsaturated shear strength were proposed with basic soil parameters and effective friction angle alone (i.e. PI , P_{200} and ϕ') without requiring the SWCC of the soils were proposed. The proposed unsaturated shear strength equations were able to predict an upper and a lower bound of the unsaturated shear strength of soils. This will be useful for preliminary design or parametric study purposes.
- Initial values of ϕ^b angle were obtained to be higher than ϕ' for sand and equal to ϕ' for residual soils. It appeared that the change in shear strength with matric suction can be described using ϕ^b_{app} which is a summation of ϕ^b and Ψ^b due to soil dilatancy. Values of ϕ^b and Ψ^b for high ($PI > 10$) and low plasticity ($PI \leq 10$) soils, respectively, can be summarised as follows:

- In near saturation zone, $\phi_{app}^b = \phi'$ [i.e. $\phi^b = \phi'$ and $\Psi^b = 0$] for high plasticity soils and $\phi_{app}^b > \phi'$ [i.e. $\phi^b = \phi'$ and $\Psi^b > 0$] for low plasticity soil
 - In transition zone, $\phi_{app}^b < \phi'$ [i.e. $\phi^b < \phi'$ and $\Psi^b = 0$] for high plasticity soils and $\phi_{app}^b < \phi'$ [i.e. $\phi^b < \phi'$ and $\Psi^b = 0$] for low plasticity soils.
 - In residual zone, $\phi_{app}^b = 0$ [i.e. $\phi^b = 0$ and $\Psi^b = 0$] for high plasticity soils and $\phi_{app}^b < 0$ [i.e. $\phi^b = 0$ and $\Psi^b < 0$] for low plasticity soil.
- The effect of strain rate on stress-strain relationship was observed to be more significant for specimens with higher initial matric suction. Similarly, the effect of strain rate on the change in matric suction with axial strain was also more significant for specimens with higher initial matric suction.
 - Variations of shear strength with matric suction obtained using CU and CW tests and at different strain rates were observed to be very close. It is more convenient to perform CW test as true undrained behaviour of pore-air pressure can be sometimes difficult to maintain due to dissolution of pore-air into pore-water.

7.3 Recommendations

7.3.1 Testing Equipment

- In this research, hinges for local displacement transducers (LDTs) were positioned using super glue on the rubber membrane by marking its position prior to positioning. LDT is mounted between two hinges and held in place by its own elastic force and therefore proper contacts between LDT and hinges are obtained by means of initial bending. Initial bending of LDT depends on the distance

between the two hinges. It was difficult to control the initial bending of LDTs due to the difficulty in obtaining an accurate positioning of hinges. Excessive bending of LDT can cause an out-of-range problem. A device that can position the two hinges at a fixed distance would be beneficial. Using such device, improper or loose contact between hinges and LDT or over bending of LDT can be avoided.

- The highest confining pressure used in this research was limited to 700 kPa as higher confining pressure were found to intrude the water proofing system of the bender element. It will be desirable to improve the water proofing system of the bender element to withstand higher confining pressures. In this way, effect of effective or net confining pressure on $G_{\max}$ can be investigated for a wider range of pressures.
- Resonant column apparatus used in this research belongs to a commercial laboratory and thus, the equipment was made available for the author's use for a limited time. It will be good if the base pedestal of the resonant column apparatus could be modified with a high air-entry ceramic disk for measurement of unsaturated soil tests. In this way, stiffness-strain relationships of unsaturated soils under similar stress conditions using identical specimens can be measured using resonant column and triaxial apparatus. In addition, it is desirable to compare shear stiffness measured by bender element and resonant column apparatus using the same specimen and under same stress conditions. Such comparisons can be achieved by incorporating the bender element measurement system in the resonant column apparatus.

7.3.2 Experimental Work and Proposed Equation

- It was observed that the effect of strain rate on stress-strain relationship and change in pore-water pressure with axial strain became more significant for the residual soil specimen tested at high initial matric suction of 500 kPa. Therefore, it would be useful to investigate the effect of strain rate using unsaturated soil specimens with initial matric suctions greater than 500 kPa.
- The fastest strain rate used in this research was 0.5 mm/min. It was observed that the measurement of pore-water pressure below a high air-entry ceramic disk was inaccurate when the specimen was tested at a strain rate greater than 0.5 mm/min. Low coefficient of permeability of a high air-entry ceramic disk contributed to the inaccurate pore-water pressure measurement. Pore-water pressure measurement at mid-height of the specimen should be used to give a more representative pore-water pressure measurement when the test is carried out at a strain rate faster than 0.5 mm/min.
- Due to time and resources constraints, shear stiffness measured using different equipments (i.e. bender element and resonant column apparatus) could not be compared as tests were performed under different stress conditions. In an ideal situation, experiments on different soil types should be performed at same stress conditions using different test equipments (i.e. triaxial apparatus, bender element and resonant column apparatus) so that results obtained using different equipment can be compared and discussed.
- From the results obtained from unsaturated sand tests performed under constant water content condition at different matric suctions and net confining pressures,

it was observed that ϕ^b angles are much higher than ϕ' obtained from saturated tests. This is attributed to soil dilatancy. More research should be performed for different sandy soils to determine factors affecting the ϕ^b angles.

- The modified upper bound stiffness degradation curve were proposed based on the experimental data of sands performed using triaxial apparatus with local strain measurement and resonant column apparatus. This proposed upper bound curve were evaluated using available stiffness degradation curves for dry, saturated and unsaturated sands from the literature. To date, available shear stiffness-strain relationship data for unsaturated sands are rather scarce. It will be beneficial if the proposed upper bound curve could be further evaluated using a wider range of unsaturated sand data sets.

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Kaolin

Appendix A Summary of Triaxial Compression Test Results

Figure A1 Triaxial compression test results of specimen CW-20-1

Figure A2 Triaxial compression test results of specimen CW-20-2

Figure A3 Triaxial compression test results of specimen CW-20-3

Not Available

Not Available

Figure A4 Triaxial compression test results of specimen CW-200-2

Figure A5 Triaxial compression test results of specimen CW-400-2

Reconstituted Reclaimed Sand

Appendix B1 Summary of Triaxial Compression Test Results

Figure B1-1 Triaxial compression test results of specimen CU50

Figure B1-2 Triaxial compression test results of specimen CU200

Figure B1-3 Triaxial compression test results of specimen CW200-3

Figure B1-4 Triaxial compression test results of specimen CW100-3

Figure B1-5 Triaxial compression test results of specimen CW50-3

Figure B1-6 Triaxial compression test results of specimen CW200-5

Figure B1-7 Triaxial compression test results of specimen CW100-5

Figure B1-8 Triaxial compression test results of specimen CW50-5

Reconstituted Reclaimed Sand

Appendix B2 Stress-strain relationship and Excess Pore-water Pressure Versus Axial Strain

Figure B2-1 Stress-strain relationship of specimen CU50-100-200

Figure B2-2 Excess pore-water pressure versus axial strain of specimen CU50-100-200

Figure B2-3 Stress-strain relationship of specimen CW200-5-10-20

Figure B2-4 Excess pore-water pressure versus axial strain of specimen CW200-5-10-20

Figure B2-5 Stress-strain relationship of specimen CW100-5-10-20

Figure B2-6 Excess pore-water pressure versus axial strain of specimen CW100-5-10-20

Figure B2-7 Stress-strain relationship of specimen CW50-5-10-20

Figure B2-8 Excess pore-water pressure versus axial strain of specimen CW50-5-10-20

Figure B2-9 Stress-strain relationship of specimen CW200-3-15

Figure B2-10 Excess pore-water pressure versus axial strain of specimen CW200-3-15

Figure B2-11 Stress-strain relationship of specimen CW100-3-15

Figure B2-12 Excess pore-water pressure versus axial strain of specimen CW100-3-15

Figure B2-13 Stress-strain relationship of specimen CW50-3-15

Figure B2-14 Excess pore-water pressure versus axial strain of specimen CW50-3-15

Figure B2-15 Stress-strain relationship of specimen CW200-3-5-10-15-20

Figure B2-16 Excess pore-water pressure versus axial strain of specimen CW200-0-3-5-10-15-20

Reconstituted Reclaimed Sand

Appendix B3 Shear Waves Arrival at Different Initial Matric Suctions

Figure B3-1 Shear waves arrival at different initial matric suctions of specimen CW200-5-10-20

Figure B3-2 Shear waves arrival at different initial matric suctions of specimen CW100-5-10-20

Figure B3-3 Shear waves arrival at different initial matric suctions of specimen CW50-5-10-20

Figure B3-4 Shear waves arrival at different initial matric suctions of specimen CW200-0-3-15

Figure B3-5 Shear waves arrival at different initial matric suctions of specimen CW100-3-15

Figure B3-6 Shear waves arrival at different initial matrix suctions of specimen CW50-3-15

Figure B3-7 Shear waves arrival at different initial matric suctions of specimen CW200-0-3-5-10-15-20

Figure B3-8 Shear waves arrival at different initial matric suctions of specimen CU50-100-200

Undisturbed Residual Soil from Bukit Timah Granite

Appendix C1 Summary of Triaxial Compression Test Results

Figure C1-1 Triaxial compression test results of specimen CU-50 tested using strain rate of 0.5 mm/min

Figure C1-2 Triaxial compression test results of specimen CU-50 tested using strain rate of 0.1 mm/min

Figure C1-3 Triaxial compression test results of specimen CU-50 tested using strain rate of 0.006 mm/min

Figure C1-4 Triaxial compression test results of specimen CW-50 tested using strain rate of 0.5 mm/min

Figure C1-5 Triaxial compression test results of specimen CW-50 tested using strain rate of 0.1 mm/min

Figure C1-6 Triaxial compression test results of specimen CW-50 tested using strain rate of 0.006 mm/min

Undisturbed Residual Soil from Bukit Timah Granite

Appendix C2 Stress-strain Relationship and Variation of Matric Suction with Strain

Figure C2-1 Stress-strain relationship of specimen CU-50-100-200-400 tested using strain rate of 0.5 mm/min

Figure C2-2 Variation of matric suction with axial strain of specimen CU-50-100-200-400 tested using strain rate of 0.5 mm/min

Figure C2-3 Stress-strain relationship of specimen CU-50-100-200-400 tested using strain rate of 0.1 mm/min

Figure C2-4 Variation of matric suction with axial strain of specimen CU-50-100-200-400 tested using strain rate of 0.1 mm/min

Figure C2-5 Stress-strain relationship of specimen CU-50-100-200-400 tested using strain rate of 0.006 mm/min

Figure C2-6 Variation of matric suction with axial strain of specimen CU-50-100-200-400 tested using strain rate of 0.006 mm/min

Figure C2-7 Stress-strain relationship of specimen CW-50-100-200-400 tested using strain rate of 0.5 mm/min

Figure C2-8 Variation of matric suction with axial strain of specimen CW-50-100-200-400 tested using strain rate of 0.5 mm/min

Figure C2-9 Stress-strain relationship of specimen CW-50-100-200-400 tested using strain rate of 0.1 mm/min

Figure C2-10 Variation of matric suction with axial strain of specimen CW-50-100-200-400 tested using strain rate of 0.1 mm/min

Figure C2-11 Stress-strain relationship of specimen CW-50-100-200-400 tested using strain rate of 0.006 mm/min

Figure C2-12 Variation of matrix suction with axial strain of specimen CW-50-100-200-400 tested using strain rate of 0.006 mm/min

Undisturbed Residual Soil from Bukit Timah Granite

Appendix C3 Shear Waves Arrival at Different Initial Matric Suctions

Figure C3-1 Shear waves arrival at different initial matrix suctions of specimen CU-50-100-200-400 (from multi-stage shearing test using strain rate of 0.5 mm/min)

Figure C3-2 Shear waves arrival at different initial matrix suctions of specimen CU-50-100-200-400 (from multi-stage shearing test using strain rate of 0.1 mm/min)

Figure C3-3 Shear waves arrival at different initial matrix suctions of specimen CU-50-100-200-400 (from multi-stage shearing test using strain rate of 0.006 mm/min)

Figure C3-4 Shear waves arrival at different initial matrix suctions of specimen CW-50-100-200-400 (from multi-stage shearing test using strain rate of 0.5 mm/min)

Figure C3-5 Shear waves arrival at different initial matrix suctions of specimen CW-50-100-200-400 (from multi-stage shearing test using strain rate of 0.1 mm/min)

Figure C3-6 Shear waves arrival at different initial matrix suctions of specimen CW-50-100-200-400 (from multi-stage shearing test using strain rate of 0.006 mm/min)

Dry Sands (Ottawa 20-30, Ottawa Graded, Reclaimed sand A, and Reclaimed sand B)

Appendix D Comparison of Calculated and Measured Shear Stiffness with Strain

Figure D-1 Comparison of calculated and measured shear stiffness with strain of Ottawa 20-30_e1

Figure D-2 Comparison of calculated and measured shear stiffness with strain of Ottawa 20-30_e2

Figure D-3 Comparison of calculated and measured shear stiffness with strain of Ottawa Graded_e1

Figure D-4 Comparison of calculated and measured shear stiffness with strain of Ottawa Graded_e2

Figure D-5 Comparison of calculated and measured shear stiffness with strain of Reclaimed sand A_e1

Figure D-6 Comparison of calculated and measured shear stiffness with strain of Reclaimed sand A_e2

Figure D-7 Comparison of calculated and measured shear stiffness with strain of Reclaimed sand B_e1

Figure D-8 Comparison of calculated and measured shear stiffness with strain of Reclaimed sand B_e2