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**DESIGN AND OPTIMIZATION OF MICROPHOTONIC
INTEGRATED DEVICES
FOR
COMMUNICATION APPLICATIONS**

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SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING

Year 2012

**DESIGN AND OPTIMIZATION OF MICROPHOTONIC
INTEGRATED DEVICES
FOR
COMMUNICATION APPLICATIONS**

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School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

Year 2012

“Don’t stop believing. Hold on to that feeling.”

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ABSTRACT

The communication bottleneck that currently limits the efficiencies of chip-to-chip and intra-chip data transfer could be eliminated if the electrical interconnects of the electronic chips inside computing devices could be replaced by photonics that can be interrogated by electronic means. In fact, in this Information Age where the digital data is expanding at an exponential rate, the synergy between integrated photonics and integrated electronics — known as integrated optoelectronics — is poised to be the most feasible approach to meet the information demand. However, whether such an approach can be fully implemented would depend on the rate of progress of integrated photonics as the diversities and complexities of photonic devices are larger than those of electronic devices currently supporting our digital world. Investigation into some of the basic component devices of integrated photonics, with emphasis on efficient communication applications, is the main thrust of this thesis.

In this thesis, we report on the optimizations and new design proposals of microphotonic integrated component devices based on silicon-on-insulator (SOI) waveguides and traveling wave microresonators (TWMRs). The results are split into two parts. The first part focuses on the basic functionalities of basic microphotonic component devices, while part two focuses on the generation of fast light (FL) and slow light (SL) effects in TWMR-based systems.

Waveguides are the most basic building blocks of photonic integrated circuits (PICs). We thus start the first part of the thesis by studying the modal characteristics of different waveguides. Attention was given to the single-mode (SM) and polarization-independent (PI) conditions at the communication wavelengths of 1.31 μm and 1.55 μm for SOI channel, rib and slab waveguides with height and width of less than 1 μm . This is to facilitate the ongoing trend of the scaling of SOI devices into the submicron regime. Our results show that submicron SOI waveguides exhibit strong dependency on their polarization states. Nevertheless, both SM and PI conditions in submicron SOI waveguides are possible under certain circumstances and are restricted by the fabrication process. We have identified a matrix of waveguide dimensions that allow both SM and PI operations at the submicron scale. This can serve as design rules in the fabrication of submicron SOI waveguides for communication applications, whereby SM and PI conditions are highly desirable.

Another basic component of PICs is the microring resonator (MRR). Like the straight waveguide, MRR must also be SM and PI for communication applications. As MRR is a bend-based device, it inherently has higher polarization sensitivity and different SM conditions in contrast to a straight waveguide. A fundamental question is if it is possible to shrink the size of the MRR and yet maintain its overall PI and SM conditions, while ensuring low round-trip losses. We have shown that it is possible to achieve this through the use of deeply etched submicron SOI rib waveguides. Our results demonstrate that, for a given cladding and core thickness, the radius of a SM and PI MRR

based on SOI rib waveguide can be as small as 3 μm , being limited chiefly by the residual birefringence of the components of the cavity of the MRR as well as the bend losses. A resonance mismatch of only about 0.22 nm between the TE and TM resonance spectra and a free spectral range (FSR) of 12 nm can be achieved. This is a large improvement as compared to conventional PI resonators that have much larger device size (height and width $> 1 \mu\text{m}$ and bend radius $> 30 \mu\text{m}$) and smaller FSR ($< 2 \text{ nm}$).

Having analyzed the SM and PI characteristics of the MRR, which is one particular class of TWMR, we then look into the use of periodic and aperiodic order to achieve photonic bandgap (PBG) engineering of coupled TWMRs, which is also known as coupled-resonator optical waveguides (CROWs). In the periodic order case, size-tuned defects are introduced at periodic locations among the regular resonators. This results in the formation of mini-defect bands and mini-PBGs within the wide PBG of the original defect-free CROW, with the position and number of such mini-defect bands and mini-PBGs depending on the size tuning of the defects. In the aperiodic-order case, a single high- Q localized state will appear and gradually transit to a mini-defect band within a wide photonic stop band as the number of unit cells increases. Interestingly, our results also illustrate that there is a strong periodic correlation even when the order in the CROW is aperiodic in nature.

Besides the above work on passive functionalities, we also look into active functionalities based on stimulated Raman scattering (SRS) in SOI waveguides. In this study, we have analyzed the temperature T effects of SRS in submicron SOI waveguides for $100 \text{ K} \leq T \leq 500 \text{ K}$ in regard to three specific types of SRS-based functionalities in the continuous-wave regime: Raman amplification and attenuation near the C-band wavelength of 1.55 μm and parametric Raman wavelength conversion between the C-band wavelength of 1.55 μm and O-band wavelength of 1.31 μm . Our results show that the effects of temperature variation can be harnessed to improve the Raman performances in the submicron SOI waveguides. In particular, the use of temperature variation provides an additional degree of freedom to tune the conversion efficiency (CE) or amplification efficiency (AE) of the SOI waveguides. Also, the CE or AE can be enhanced as compared to that at the room temperature.

In the second part of the thesis, the theme is on the use of degeneracy lifting of the resonator to improve the fast light (FL) and slow light (SL) effects. Degeneracy lifting is achieved by the excitation of contra-propagating cavity modes through various proposed schemes. The first such scheme is the coupler-induced localized backscattering (CILB). It is demonstrated that when there is a small net gain in the resonator, weak CILB will allow all three types of light propagation velocity — FL with negative group velocity, FL with positive group velocity and SL — to be generated at both the through and drop ports. This is in sharp contrast with the conventional TWMR (where CILB is negligible), in which only FL with negative group velocity and SL are present at the through port, while for the drop port, only SL is produced. Also, weak CILB aided by a small gain in the cavity can

produce FL with amplified transmission. This mitigates the constraint of FL with substantial attenuation in conventional TWMRs. As change in the strength of the CILB will translate into the modulation of the group delay, CILB can thus also provide an extra degree of freedom (besides using the physical parameters, such as evanescent coupling strengths) to tune the group delay.

Another source of contra-propagating cavity modes within the TWMR is the surface imperfections-induced distributed backscattering, which is simply termed as intracavity backscattering. In our work, the intracavity backscattering can be enhanced and controlled by the intentional addition of surface microstructures. It is found that, by adjusting the amplitude and/or phase differences between the dual contra-propagating inputs, which are inserted into the TWMR circuit, the interaction of the cavity modes with the enhanced intracavity backscattering can be controlled. This will then allow the structural dispersion and shape of the resonance spectrum to be tuned. Consequently, the transmission and dispersion of the output light can be easily manipulated, bringing about continuous tunability in the group delay and transmission. Switching between FL and SL can also be attained. This scheme allows tunable FL and SL to be achieved in a passive and linear cavity, unlike conventional TWMR systems, which require either the use of active tuning mechanisms or nonlinearity in the cavity so as to have tunable group delay and/or transmission.

Subsequently, we look into the use of intracavity backscattering in a system of twin-coupled TWMRs that has only one resonator coupled to the bus waveguide for enhanced coupled resonator induced transparency (CRIT) and optical Fano resonance (OFR) effects. In particular, the presence of intracavity backscattering in the twin-coupled TWMRs will generate a pair of CRIT peaks. This is in contrast to conventional twin-coupled TWMRs systems (intracavity backscattering is not utilized), whereby only a single CRIT peak can be realized within one FSR. In addition, FL and SL can be simultaneously produced, one at each output port. Note that in conventional twin-coupled TWMRs systems, switching between CRIT and OFR is commonly realized through the use of gain and/or phase tuning elements in the resonators. In our work, the CRIT peaks can be reshaped into asymmetric OFR lineshapes by suitable choice of the intracavity backscattering strength during fabrication. If dual inputs are employed, the dual CRIT symmetric peaks of our scheme can be reshaped into asymmetric OFR lineshapes simply by modulating the phase and/or amplitude differences between the dual inputs, given a fixed intracavity backscattering strength after fabrication. Active switching between CRIT and OFR can thus be achieved even in the absence of gain and/or phase tuning elements in the resonators, as compared to the conventional twin-coupled TWMRs.

Last but not least, we have utilized the contra-propagating cavity modes that are excited when both the resonators in the above-mentioned twin-coupled TWMRs system are evanescently coupled to the bus waveguide to generate flat-band SL. The presence of such contra-propagating cavity modes in the twin-coupled TWMRs system (with both of the resonators directly coupled to the bus waveguide) will generate multi-peaks in the resonance spectra. Flat-band SL can be generated if such multi-peaks

become undistinguishable and merge into one single broadened peak that is maximally flat in the resonance spectra. This will only happen if the inter-resonator coupling strength is optimized relative to the resonators-to-bus-waveguide coupling strengths. An important figure of merit to quantify the (flat-band) usable resonance bandwidth of SL systems is the delay-bandwidth product (DBP). It is shown that the DBP of the output light of our proposed scheme is 3- to 24-fold higher than that of conventional TWMR-based SL systems. This illustrates that our proposed system is capable of providing better SL performances in terms of the (flat-band) usable resonance bandwidth. Fabrication tolerance and cavity losses analyses have also revealed that our proposed scheme is rather robust to the fabrication errors and limitations of current state-of-the-arts semiconductor processing technology. Our proposed SL system will thus be suitable for practical applications in communication systems.

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LIST OF ACROYMNS

A

ADF	Add-drop filter
ADSL	Asymmetric Digital Subscriber Line
AE	Amplification efficiency
AOT	All-optical tuning
AP	Aperiodic
APF	All-pass filter

B

BPM	Beam propagation method
BW	Bandwidth

C

CARS	Coherent anti-Stokes Raman scattering
CAT	Conventional active tuning
CC	Critical coupling
CCW	Counterclockwise
CE	Conversion efficiency
CILB	Coupler-induced localized backscattering
CJs	Coupling junctions
CMOS	Complementary metal-oxide-semiconductor
CMT	Coupled mode theory
CO	Central exchange office of communication network
CPUs	Central processing units

CRIT Coupled-resonator-induced transparency

CROWs Coupled-resonator optical waveguides

CW Clockwise

D

DBP Delay-bandwidth product

E

EIM Effective Index Method

EIT Electromagnetically induced transparency

EMI Electromagnetic interference

F

F-ADSL Futuristic Asymmetric Digital Subscriber Line

FDTD Finite difference time domain

FL Fast light

FOM Figures-of-merit

FR Fano resonance

FSR Free spectral range

FTTB Fiber-to-the-building or Fiber-to-the-basement

FTTC Fiber-to-the-cabinet

FTTH Fiber-to-the-home

FTTN Fiber-to-the-node

FTTx Fiber-to-the- x , where x stands for B (building), C (cabinet), H (home), N (node) or any other FTTx networks

FWHM Full width at half maximum

FWM Four-wave mixing

G

GDD	Group delay dispersion
GVD	Group velocity dispersion

H

HDTV	High-definition television
------	----------------------------

I

ICs	Integrated circuits
IDBPM	Imaginary distance beam propagation method
IPTV	Internet protocol television

M

MBR	Minimum bend radius
MEMS	Micro-electrical-mechanical systems
MM	Multi-mode
MMC	Multi-mode condition
MOSFET	Metal-oxide-semiconductor field-effect transistor
MPFs	Minimum phase filters
MR	Microresonator
MRR	Microring resonator
MMI	Multi-mode interference

N

NoC	Network-on-Chip
-----	-----------------

O

OEICs Optoelectronic integrated circuits

OFR Optical Fano resonance

P

PBG Photonic bandgap

PC Polarization conversion

PDL Polarization dependent losses

PICs Photonic integrated circuits

PI Polarization-independent

PRWC Parametric Raman wavelength conversion

Q

QP Quasi-periodic

R

RS Raman scattering

RASS Raman anti-Stokes scattering

RSS Raman Stokes scattering

S

SL Slow light

SM Single-mode

SMC Single-mode condition

SOA Semimconductor optical amplifier

SOI Silicon-on-insulator

SPM Self-phase modulation

SRS Stimulated Raman scattering

SWG Slab waveguide

T

TE Transverse electric

TM Transverse magnetic

TWMR Traveling wave microresonator

V

VLSI Very-large-scale integration

VOD Video on demand

W

WG Waveguide

WGM Whispering gallery mode

X

XPM Cross-phase modulation

Z

ZBC Zero birefringence condition

LIST OF SYMBOLS

A

A	Amplitude modulation between the dual inputs S'_{+1} and S'_{+2} , i.e., $A = S'_{+2}/S'_{+1}$
A_I	Net cross-sectional area of on-chip electrical interconnects
A_0	Complex amplitude function of propagating pulse
A_{wg}	Effective modal area of waveguide
a	Complex amplitude function of monochromatic wave
$\propto$	Symbol of proportionality
α	Propagation losses; also the imaginary part of propagation constant β
α_{eff}	Effective propagation losses; also the imaginary part of propagation constant β_{eff}
α_a	Linear propagation losses (in dB) of anti-Stokes wave in Raman medium
α_p	Linear propagation losses (in dB) of input pump wave in Raman medium
α_s	Linear propagation losses (in dB) of Stokes wave in Raman medium
α_B	Pure bending loss per 90° bend (in dB)
α_R	Total round-trip losses of a racetrack microring resonator (in dB)
α_S	Transition loss due to scattering loss of waveguides (in dB)
α_T	Transition loss at each straight to bend WGs junction (in dB)
α_{TBL}	The total bend losses per 90° turn (in dB), equal to the sum of α_B and α_T

B

$B_1, B_2, B_3, \dots$	Coefficient constants for specific equations in each chapter
B'	Bit rate capacity of electrical interconnects
B'_0	Maximum bit rate capacity of electrical interconnects
$BW_{(max)}$	FWHM for the resonance centered at $\lambda_0 = 1.55 \mu\text{m}$ when $t_g(\theta) = t_{g(max)}$ and $T(\theta) = T_{(max)}$
$BW_{(min)}$	FWHM for the resonance centered at $\lambda_0 = 1.55 \mu\text{m}$ when $t_g(\theta) = t_{g(min)}$ and $T(\theta) = T_{(min)}$

b	Two-photon absorption coefficient
β	Propagation constant in ordinary optical medium, such as straight waveguide
β'	Real part of propagation constant β in ordinary optical medium
β_0	Propagation constant β at frequency ω_0
β_{eff}	Effective propagation constant in resonance optical medium, such as ring resonator
β'_{eff}	Real part of effective propagation constant β_{eff}
β_R	Propagation constant of resonator
$\beta_{R,1}$	Propagation constant of resonator 1
$\beta_{R,2}$	Propagation constant of resonator 2
β_w	Propagation constant of bus waveguide
C	
C'	Capacitance per unit length of electrical interconnects
C_n	CROW analog of Fibonacci number sequence ($n = 1, 2, 3 \dots$)
C_p	Critical turning point after which $ t_{g(min)} $ and $ t_{g(max)} $ start to decrease when Q_{int} increases
c	Speed of light in vacuum
$\chi^{(3)}$	Third-order nonlinear susceptibility of optical medium; equal to $\chi^{NR} + \chi^R$
χ^{NR}	Electronic-non-resonant susceptibility of optical medium
χ^R	Raman-resonant susceptibility of optical medium
D	
D	Differential operators of Maxwell equations
D_x	Differential operators of Maxwell equations for x component
D_y	Differential operators of Maxwell equations for y component
D_z	Differential operators of Maxwell equations for z component
D_{MAX}	Transmission to the drop port at on-resonance

Det	Determinant
d	Etch depth of waveguide
ΔBW	Tunable bandwidth (BW); equal to $ BW_{(max)} - BW_{(min)} $
ΔN_{eff}	Waveguide birefringence $ N_{eff (TE)} - N_{eff (TM)} $
ΔT	Tunable range of the transmission vs θ response; equal to $T_{(max)} - T_{(min)}$
ΔT_T	Tunable range of the transmission vs θ response at through port; equal to $T_{T(max)} - T_{T(min)}$
ΔT_R	Tunable range of the transmission vs θ response at reflection port; equal to $T_{T(max)} - T_{T(min)}$
Δf_p	FWHM of the pump beam spectrum used for the Raman medium
Δf_{FSR}	Free spectral range in frequency
Δf_u	Usable resonance bandwidth of resonator in which there is minimal GVD and high transmission
$\Delta f_{u, R}$	Usable resonance bandwidth of resonator at reflection port
$\Delta f_{u, T}$	Usable resonance bandwidth of resonator at through port
Δf_v	Change in resonance frequency of optical phonon of Raman medium
Δn_g	Waveguide birefringence $ n_{g(TE)} - n_{g(TM)} $, taking into account dispersion effects
Δt_g	Tunable range of the transmission vs θ response; equal to $ t_{g(max)} - t_{g(min)} $
$\Delta t_{g, T}$	Tunable range of the transmission vs θ response at through port; equal to $ t_{g, T(max)} - t_{g, T(min)} $
$\Delta t_{g, R}$	Tunable range of the transmission vs θ response at reflection port; equal to $ t_{g, R(max)} - t_{g, R(min)} $
$\Delta \beta$	Phase matching condition; equal to $2\beta_p - \beta - \beta_a$
$\Delta \beta_B$	Phase mismatch due to WG birefringence of Raman medium
$\Delta \beta_M$	Phase mismatch due to material dispersion of Raman medium
$\Delta \beta_W$	Phase mismatch due to WG dispersion
$\Delta \delta$	Difference in phase shift between the TE and TM modes
$\Delta \delta_u$	Normalized usable resonance bandwidth in which there is minimal GVD and high transmission
$\Delta \delta_{u, R}$	Normalized usable resonance bandwidth of reflection port
$\Delta \delta_{u, T}$	Normalized usable resonance bandwidth of through port
$\Delta \lambda_R$	Net shift in resonance wavelength due to net birefringence effects
$\Delta \lambda_{R, MMI}$	Shift in resonance wavelength due to birefringence effects in MMI

$\Delta\lambda_{R, \text{ straight}}$	Shift in resonance wavelength due to birefringence effects in straight waveguides
$\Delta\omega$	Bandwidth in angular frequency or detuning angular frequency $\omega - \omega_0$
$\Delta\omega_{ch}$	Channel spacing (in angular frequency) between two consecutive transmission channels
$\Delta\omega_{nor}$	Normalized detuning; equal to $(\omega - \omega_0)/\omega_0$
$\Delta\omega_0$	Detuning from degenerate resonance angular frequency ω_0 ; equal to $\Delta\omega_1 = \omega - \omega_0$
$\Delta\omega_1$	Detuning from degenerate resonance angular frequency ω_1 ; equal to $\Delta\omega_1 = \omega - \omega_1$
$\Delta\omega_2$	Detuning from degenerate resonance angular frequency ω_2 ; equal to $\Delta\omega_2 = \omega - \omega_2$
δ	Round-trip phase shift of the resonator or general phase shift of any optical medium
δ_A	Round-trip phase shift of resonator A in unit cell of CROW
δ_B	Round-trip phase shift of resonator B in unit cell of CROW
δ_0	Round-trip phase shift at degenerate resonance frequency
δ_1	Round-trip phase shift of resonator 1
δ_2	Round-trip phase shift of resonator 2
δ'_0	Complex eigenmode at $\delta = \delta_0$
δ'_1	Anti-symmetric split mode of δ'_0
δ'_2	Symmetric split mode of δ'_0
$\delta'_{1,1}$	Anti-symmetric split mode of δ'_1
$\delta'_{1,2}$	Symmetric split mode of δ'_1
$\delta'_{2,1}$	Anti-symmetric split mode of δ'_2
$\delta'_{2,2}$	Symmetric split mode of δ'_2

E

E	Electric field
E_a	Coefficient of the evanescent waves in the upper cladding
E_b	Coefficient of the evanescent waves in the lower cladding
E_{cw}	Electric field of clockwise cavity mode of resonator
$E_{cw, 1}$	Electric field of clockwise cavity mode of resonator 1

$E_{cw, 2}$	Electric field of clockwise cavity mode of resonator 2
E_{ccw}	Electric field of counterclockwise cavity mode of resonator
$E_{ccw, 1}$	Electric field of counterclockwise cavity mode of resonator 1
$E_{ccw, 2}$	Electric field of counterclockwise cavity mode of resonator 2
E_e	Coefficient of the even mode in the waveguide core
E_o	Coefficient of the odd mode in the waveguide core
E_x	Electric field along x coordinate of Cartesian coordinate system
E_y	Electric field along y coordinate of Cartesian coordinate system
E_z	Electric field along z coordinate of Cartesian coordinate system
E_0^N	Normal mode of a straight waveguide
E_0^R	Radial mode of a bend waveguide
E_+	Electric field of symmetric eigenmode of resonator
E_-	Electric field of anti-symmetric eigenmode of resonator
e_c	Electric charge density of Raman medium
ϵ_0	Permittivity of free space
ϵ_{clad}	Permittivity of the waveguide cladding
ϵ_{core}	Permittivity of the waveguide core
$\epsilon_{r, clad}$	Relative permittivity of the waveguide cladding
$\epsilon_{r, core}$	Relative permittivity of the waveguide core
η	Strength of coupler-induced backscattering at each coupling junction of resonator
η_1	Strength of coupler-induced backscattering at coupling junction 1 of resonator
η_2	Strength of coupler-induced backscattering at coupling junction 2 of resonator

F

F_d	Fractional delay of input pulse; equal to $ t_g(\omega) /t_{pw}$
F_n	Fibonacci number sequence ($n = 1, 2, 3 \dots$)

f	Frequency
f_a	Frequency of anti-Stokes wave
f_d	Detuning frequency
f_p	Frequency of input pump wave
f_s	Frequency of Stokes wave
f_v	Resonance frequency of optical phonon of Raman medium
f_0	Degenerate resonance frequency of resonator or centre wavelength of pulse
G	
g	Waveguide-to-resonator air gap separation distance
g_{eff}	Effective gain coefficient of Raman medium
Γ	Half of the full-width at half maximum (FWHM)
Γ_n	Half of the FWHM of the eigenmode δ'_n ($n = 1$ or 2)
$\Gamma_{n, m}$	Half of the FWHM of the eigenmode $\delta'_{n, m}$ ($n, m = 1$ or 2)
Γ_{ave}	Average decay rate or resonance linewidth; equal to $\Gamma_{ave} = (\Gamma_{net, 1} + \Gamma_{net, 2})/2$
Γ_{ext}	Decay rate of each cavity mode due to evanescent coupling between bus waveguide and cavity
$\Gamma_{eff, spc}$	Net effective loss rate of the twin-coupled TWMRs system at $\omega_{spc \pm 1}$ and $\omega_{spc \pm 2}$
$\Gamma_{eff, sps}$	Net effective loss rate of the twin-coupled TWMRs system at $\omega_{sps \pm}$
Γ_{int}	Decay rate of each cavity mode due to intrinsic losses of resonator
$\Gamma_{int, 1}$	Decay rate of each cavity mode due to intrinsic losses of resonator 1
$\Gamma_{int, 2}$	Decay rate of each cavity mode due to intrinsic losses of resonator 2
Γ_{net}	Net decay rate of each cavity mode in resonator
$\Gamma_{net, 1}$	Net decay rate of each cavity mode in resonator 1
$\Gamma_{net, 2}$	Net decay rate of each cavity mode in resonator 2
Γ_s	Decay rate due to the sum of effects of the intrinsic and extrinsic decay rates
$\Gamma_{s, 1}$	Decay rate due to the sum of effects of the intrinsic and extrinsic decay rates in cavity 1
$\Gamma_{s, 2}$	Decay rate due to the sum of effects of the intrinsic and extrinsic decay rates in cavity 2

γ Size tuning ratio (in terms of length L) of resonators A and B; equal to L_B/L_A

H

H Magnetic field

H_x Magnetic field along x coordinate of Cartesian coordinate system

H_y Magnetic field along y coordinate of Cartesian coordinate system

H_z Magnetic field along z coordinate of Cartesian coordinate system

H_D Transfer function of drop port

H_T Transfer function of through port

h Height of waveguide core

$h_{c,q}$ Cut off waveguide height at which mode of order q is cut off in the waveguide

$\hbar$ Reduced Planck's constant

I

I Identity matrix

I_a Intensity of anti-Stokes wave

I_c Stokes wave to anti-Stokes wave output intensity ratio

I_f Intensity of fundamental mode of waveguide

I_i Input intensity

I_p Intensity of input pump wave

I_s Intensity of Stokes wave

J

j Imaginary unit number; equal to $(-1)^{1/2}$

K

K	Bloch wave propagation vector
K_B	Boltzmann's constant
k_0	Wave number of free space
k_{clad}	Wave number of waveguide cladding
k_{core}	Wave number of waveguide core
κ	Forward cross-coupling coefficient at coupling junction of resonator
κ_1	Forward cross-coupling coefficient at coupling junction number 1 of resonator
κ_2	Forward cross-coupling coefficient at coupling junction number 2 of resonator
κ'_1	Backward cross-coupling coefficient at coupling junction number 1 of resonator
κ'_2	Backward cross coupling coefficient at coupling junction number 2 of resonator
κ_{aa}	Coupling coefficient between anti-Stokes waves in Raman medium
κ_{as}	Coupling coefficient between anti-Stokes and Stokes waves in Raman medium
κ_{sa}	Coupling coefficient between Stokes and anti-Stokes waves in Raman medium
κ_{ss}	Coupling coefficient of Stokes waves in Raman medium
$\kappa_{s, R}$	Value of κ_1 needed to realize flat-band slow light at the reflection port
$\kappa_{s, T}$	Value of κ_1 needed to realize flat-band slow light at the through port
κ_{intra}	Forward cross coupling coefficient between intra-resonators in each unit cell of CROW
κ_{inter}	Forward cross coupling coefficient between unit cells of CROW

L

L, ℓ	Length of waveguide components or device
L_A	Length of resonator A in each unit cell of coupled resonators
L_B	Length of resonator B in each unit cell of coupled resonators
L_{bends}	Length of bend waveguides in resonator cavity
L_{mmi}	Length of MMI in resonator cavity
L_c	Length of cavity

L_π	Beat length
$L_{straight}$	Length of straight waveguides in resonator cavity
L_{3dB}	3-dB coupler length
$L_{3dB, Type I}$	3-dB coupler length of Type-I MMI
$L_{3dB, Type II}$	3-dB coupler length of Type-II MMI
L_{rt}	Round-trip length of the resonator cavity
L'	Inductance per unit length of electrical interconnects
l_l	Thickness of waveguide upper cladding
l_l	Thickness of waveguide lower cladding
A	Lattice constant of CROW
λ	Input (free space) wavelength
λ_0	Degenerate resonance wavelength of resonator or centre wavelength of pulse
λ_a	Anti-Stokes wavelength
λ_p	Input Pump wavelength
λ_s	Stokes wavelength
λ_R	Resonance wavelength
$\lambda_{R, TE}$	Resonance wavelength of TE mode
$\lambda_{R, TM}$	Resonance wavelength of TM mode
M	
M	Modulation factor of dual-input scheme; equal to $A \exp(j\theta)$
MBR_{LOSS}	Minimum bend radius of resonator due solely to losses of bend waveguides
MBR_{NET}	Net minimum bend radius of resonator due to MBR_{LOSS} and MBR_{RB}
MBR_{RB}	Minimum bend radius of resonator due solely to residual birefringence in straight waveguide
m	Azimuthal mode number of the resonator
μ_0	Permeability of free space
μ_{clad}	Permeability of waveguide cladding

μ_{core} Permeability of waveguide core

N

N Number of unit cells of CROW or cascaded resonators

N' Number of intra-resonators in each unit cell of CROW or cascaded resonators

N_B Bose occupancy factor

N_{eff} Effective index of waveguide mode

$N_{eff(TE)}$ Effective index of TE waveguide mode

$N_{eff(TM)}$ Effective index of TM waveguide mode

$N_{eff(PI)}$ Effective index of waveguide mode, at which there is polarization independence (PI)

$N_{eff(bend)}$ Effective index of bend waveguide mode

$N_{eff(straight)}$ Effective index of straight waveguide mode

$N_{eff(mmi)}$ Effective index of waveguide mode in multi-mode interferometer (MMI)

$N_{eff(PI\ bend)}$ Effective index of bend waveguide mode, at which there is polarization independence

$N_{eff(PI\ straight)}$ Effective index of straight waveguide mode, at which there is polarization independence

n Numbering of unit cells and intra-resonators in CROW ($n = 1, 2, 3 \dots$)

n_{clad} Material refractive index of waveguide cladding

n_{core} Material refractive index of waveguide core

n_{kerr} Kerr coefficient of material

n_{si} Material refractive index of silicon waveguide core

n_g Group effective index of waveguide mode

$n_{g(TE)}$ Group effective index of waveguide TE mode

$n_{g(TM)}$ Group effective index of waveguide TM mode

O

Ω_d Detuning factor of Raman medium; equal to $f_p - f_d$

ω	Angular frequency $2\pi f$
ω_R	Angular resonance frequency
ω_{prob}	Probe field in electromagnetic induced transparency atomic medium
ω_{cou}	Coupling field in electromagnetic induced transparency atomic medium
ω_{carr}	Frequency of the carrier wave of the input pulse; equal to ω_0
$\omega_{c, q}$	Cut off angular frequency at which mode of order q is cut off in the waveguide
ω_a	Angular frequency $2\pi f_a$ of anti-Stokes wave
ω_p	Angular frequency $2\pi f_p$ of input pump wave
ω_s	Angular frequency $2\pi f_s$ of Stokes wave
ω_{s+}	Anti-symmetric mode of resonator
ω_{s-}	Symmetric mode of resonator
ω_{spc+}	Anti-symmetric split mode of ω_0 due to evanescent coupling between resonators 1 and 2
ω_{spc-}	Symmetric split mode of ω_0 due to evanescent coupling between resonators 1 and 2
ω_v	Angular resonance frequency $2\pi f_v$ of optical phonon of Raman medium
ω_0	Degenerate resonance angular frequency of resonator or centre wavelength of pulse
ω_n	Degenerate resonance angular frequency of resonator n ($n = 1$ or 2)
ω_1	Degenerate resonance angular frequency of resonator 1
ω_2	Degenerate resonance angular frequency of resonator 2
ω_{sps+}^n	Anti-symmetric split mode of ω_n due to distributed backscattering in resonator n
ω_{sps-}^n	Symmetric split mode of ω_n due to distributed backscattering in resonator n
ω_{sps+}^1	Anti-symmetric split mode of ω_1 due to distributed backscattering in resonator 1
ω_{sps+}^2	Anti-symmetric split mode of ω_2 due to distributed backscattering in resonator 2
ω_{sps-}^1	Symmetric split mode of ω_1 due to distributed backscattering in resonator 1
ω_{sps-}^2	Symmetric split mode of ω_2 due to distributed backscattering in resonator 2
ω_{spc+1}	Anti-symmetric split mode of ω_{spc+} due to distributed backscattering in resonator
ω_{spc-1}	Anti-symmetric split mode of ω_{spc-} due to distributed backscattering in resonator
ω_{spc+2}	Symmetric split mode of ω_{spc+} due to distributed backscattering in resonator

ω_{spc-2} Symmetric split mode of ω_{spc-} due to distributed backscattering in resonator

P

P_{mmi} Port pitch of multi-mode interferometer (MMI)

p Integer (i.e., $p = 0, 1, 2, 3 \dots$); can be either even and/or odd integer

Φ Effective phase shift of optical medium

Φ_D Effective phase shift of drop port of optical medium

Φ_R Effective phase shift of reflection port of optical medium

Φ_T Effective phase shift of through port of optical medium

Ψ Ratio of κ_I for flat-band SL at through port to κ_I for flat-band SL at through port; $\kappa_{s, T} / \kappa_{s, R}$

Q

Q Quality (Q)-factor

Q_{cou} Q -factor of waveguide (WG) mode due to evanescent coupling between resonators 1 and 2

Q_{cri} Q -factor that quantifies critical coupling; mode splitting occurs only if $|Q_{cri}| \gg Q_{mut}$

Q_{ext} Q -factor of WG mode due to evanescent coupling between resonator and bus WG

Q_{int} Q -factor of WG mode due to intrinsic losses of resonator

$Q_{int, 1}$ Q -factor of WG mode due to intrinsic losses of resonator 1

$Q_{int, 2}$ Q -factor of WG mode due to intrinsic losses of resonator 2

Q_{mut} Q -factor of WG mode due to mutual coupling between contra-propagating modes

$Q_{mut, 1}$ Q -factor of WG mode due to mutual coupling between contra-propagating modes in resonator 1

$Q_{mut, 2}$ Q -factor of WG mode due to mutual coupling between contra-propagating modes in resonator 2

Q_{net} Net Q -factor of WG mode of resonator

$Q_{net, 1}$ Net Q -factor of WG mode of resonator 1

$Q_{net, 2}$ Net Q -factor of WG mode of resonator 2

Q_s Sum of effects of the intrinsic and extrinsic Q -factor of WG mode

$Q_{s,1}$	Sum of effects of the intrinsic and extrinsic Q -factor of WG mode of resonator 1
$Q_{s,2}$	Sum of effects of the intrinsic and extrinsic Q -factor of WG mode of resonator 2
q	Mode number of waveguide ($q = 0, 1, 2, \dots$), with $q = 0$ being the fundamental mode
q_x	Mode number along x direction of Cartesian coordinate system
q_y	Mode number in y direction of Cartesian coordinate system
R	
R	Bend radius
R_1	Bend radius of resonator 1
R_2	Bend radius of resonator 2
R'	Resistance per unit length of electrical interconnects
R_L	Lower limit of polarization-independent bend radius due to whispering gallery mode
R_g	g -th Raman tensor of Raman medium, where $g = 1, 2$ or 3
r	Forward through coupling coefficient at coupling junction of resonator
r_1	Forward through coupling coefficient at coupling junction 1 of resonator
r_2	Forward through coupling coefficient at coupling junction 2 of resonator
r'_1	Backward through coupling coefficient at coupling junction 1 of resonator
r'_2	Backward through coupling coefficient at coupling junction 2 of resonator
r_{intra}	Forward through coupling coefficient between intra-resonators in each unit cell of CROW
r_{inter}	Forward through coupling coefficient between unit cells of CROW
S	
S	Spontaneous scattering efficiency of Raman medium
S_p	Pulse profile of optical medium in the spectral domain
S_s	Slowing factor
s	Non-integer number (for example, $2/3$, 2.33 , 2.7 , 0.11)

$\acute{s}$	Scaling factor of transistors
σ_c	Critical value of σ at which transmission is zero
σ_1	Coupler loss at coupling junction 1 of coupled resonators system
σ_2	Coupler loss at coupling junction 2 of coupled resonators system
σ_3	Coupler loss at coupling junction 3 of coupled resonators system

T

T	Temperature
T_T	Transmission of through port
T_D	Transmission of drop port
T_R	Transmission of reflection port
$T_{(max)}$	Maximum of the transmission vs θ response
$T_{T(max)}$	Maximum of the transmission vs θ response at through port
$T_{R(max)}$	Maximum of the transmission vs θ response at reflection port
$T_{(min)}$	Minimum of the transmission vs θ response
$T_{T(min)}$	Minimum of the transmission vs θ response at through port
$T_{R(min)}$	Minimum of the transmission vs θ response at reflection port
t	Time
t_0	Temporal location of the peak of the input pulse
t_{cav}	Photon life time of the resonator cavity
t_f	Pulse front delay
t_g	Group delay
$t_{g, D}$	Group delay at drop port
$t_{g, R}$	Group delay at reflection port
$t_{g, T}$	Group delay at through port
$t_{g(max)}$	Maximum of the group delay vs θ response
$t_{g, T(max)}$	Maximum of the group delay vs θ response at through port

$t_{g, R(max)}$	Maximum of the group delay vs θ response at reflection port
$t_{g(min)}$	Minimum of the group delay vs θ response
$t_{g, T(min)}$	Minimum of the group delay vs θ response at through port
$t_{g, R(max)}$	Minimum of the group delay vs θ response at reflection port
$t_{g(vac)}$	Group delay of light traveling in vacuum
t_{gm}	Maximum group delay t_g at resonance (absolute)
t'_{gm}	Maximum group delay t_g at resonance (normalized to t_{rt} , i.e., $t'_{gm} = t_{gm}/t_{rt}$)
t_p	Phase delay
t_{pw}	Temporal full width at half maximum (FWHM) of the input pulse
t_{rt}	Round-trip time of each resonator cavity
τ_c	Critical value of τ at which transmission is zero
τ_{mmi}	Amplitude attenuation constant of multi-mode interference (MMI) coupler
τ_n	Round-trip amplitude attenuation constant in the n^{th} intra-resonator of each unit cell of CROW
τ_r, τ	Round-trip amplitude attenuation constant of resonator cavity
θ	Phase modulation between dual inputs S'_{+1} and S'_{+2} , i.e., $S'_{+2} = \exp(j\theta)S'_{+1}$

U

U	Complex wave function of propagating pulse
u_0	Rate of energy coupling between the resonator and the bus waveguide
u_m	Rate of energy coupling between the contra-propagating modes in resonator
$u_{m, 1}$	Rate of energy coupling between the contra-propagating modes in resonator 1
$u_{m, 2}$	Rate of energy coupling between the contra-propagating modes in resonator 2
u_1	Rate of energy coupling between the resonator 1 and 2
Υ	Efficiency of the free-carrier absorption
ν	Free-carrier lifetime of Raman medium

V

	Modal volume
V	Pulse front velocity
v_f	Group velocity
v_g	Phase velocity
v_p	

W

w	Width of waveguide core
w_c	Critical width of waveguide, at which there is polarization independence
$w_{c(bend)}$	Critical width of bend waveguide, at which there is polarization independence
$w_{c(straight)}$	Critical width of straight waveguide, at which there is polarization independence
w_{eq}	Equivalent width of the MMI
w_{mmi}	Width of the MMI relative to the port pitch P_{mmi} of MMI

X

x	Cartesian coordinate; orientated along the height of the waveguide
ζ_T	Steady-state complex electric field transmittivity at the through port
ζ_R	Steady-state complex electric field transmittivity at the reflection port

Y

y	Cartesian coordinate; orientated along the width of the waveguide
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Z

z	Cartesian coordinate; orientated along the propagation direction of light in the waveguide
z^{-1}	Z-transform, where $z^{-1} = \exp(j\delta)$

$z_{p,T}$	Poles of through port of resonator
$z_{p,D}$	Poles of drop port of resonator
$z_{z,T}$	Zeros of through port of resonator
$z_{z,D}$	Zeros of drop port of resonator

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Chapter 1

An Overview of the Thesis – Research Motivations, Objectives, and Significant Contributions

1.1. Research Motivations

1.1.1. Moore's Law and Optic Fibers Empower Telecommunication

The semiconductor micro-fabrication technology first started as a method to create integrated circuits (IC) in the electronics industry. A typical semiconductor fabrication process consists of patterning a substrate, such as silicon, using lithography, followed by etching, doping, and depositing material on the exposed surface. By repeating these steps, many devices of various materials, which are normally layered with specific patterns, can be simultaneously fabricated on the same wafer with near identical behavior, and interconnected to build complex circuits. The power of such a method of semiconductor fabrication is that wafers can be printed with high yield: manufacturability of complex microelectronics can be realized with a high degree of reproducibility and at a rather low cost.

The manufacturability of complex microelectronics has allowed for rapid growth of the IC technology. This is best illustrated by the fact that Intel co-founder Gordon E. Moore's prediction [1], which states that the number of transistors would double approximately every two years, has proved to be considerably accurate to this date. Due to its validity, Moore's prediction is now commonly known as the "Moore's law" – a term that is coined by Caltech professor Carver Mead [2] and will also be used in this work in reference to Moore's prediction. As illustrated in Table 1.1 and Fig. 1.1, the number of transistors that can be packed onto an IC chip has increased from a mere 2,300 in 1971 to 2.6 billion by 2011 on a single silicon chip. This accomplishment is due to the shrinking of the transistors' gate lengths (from 10 μm in 1971 to 32 nm in 2011) through advancement in micro-fabrication process technology. Recently, on April 2012, Intel released its first third-generation core microprocessor chip, named Ivy Bridge [3]. Transistors on an Ivy Bridge processor are packed about twice as dense (with 1.4 billion on a 160 mm^2 die) as compared to its processor 10-Core Xeon. This is because Ivy Bridge uses the 22 nm silicon processing technology and tri-gate (or 3-D) transistors. Future silicon processing technologies of 16 nm and 11 nm, expected to be developed by 2013 and 2015, respectively, is predicted to further increase the transistor counts per chip to 10-15 billion [4, 5].

Table 1.1. The specifications of the different microprocessors used in commercially available central processing units (CPUs) of computers for year 1971 to 2012 [3-7]

Name of Processor	Transistor Count	[†] Year of Introduction	Manufacturer	Semi-conductor Process Node (or ~Transistor size)	Area
Intel 4004	2,300	1971	Intel	10 μm	12 mm ²
Intel 8008	3,500	1972	Intel	10 μm	14 mm ²
MOS Technology 6502	3,510	1975	MOS Technology	N/A	21 mm ²
Motorola 6800	4,100	1974	Motorola	N/A	16 mm ²
Intel 8080	4,500	1974	Intel	6 μm	20 mm ²
RCA 1802	5,000	1974	RCA	5 μm	27 mm ²
Intel 8085	6,500	1976	Intel	3 μm	20 mm ²
Zilog Z80	8,500	1976	Zilog	4 μm	18 mm ²
Motorola 6809	9,000	1978	Motorola	5 μm	21 mm ²
Intel 8086	29,000	1978	Intel	3 μm	33 mm ²
Intel 8088	29,000	1979	Intel	3 μm	33 mm ²
Intel 80186	55,000	1982	Intel	N/A	N/A
Motorola 68000	68,000	1979	Motorola	4 μm	44 mm ²
Intel 80286	134,000	1982	Intel	1.5 μm	49 mm ²
Intel 80386	275,000	1985	Intel	1.5 μm	104 mm ²
Intel 80486	1,180,000	1989	Intel	1 μm	N/A
Pentium	3,100,000	1993	Intel	0.8 μm	N/A
AMD K5	4,300,000	1996	AMD	0.5 μm	N/A
Pentium II	7,500,000	1997	Intel	0.35 μm	N/A
AMD K6	8,800,000	1997	AMD	0.35 μm	N/A
Pentium III	9,500,000	1999	Intel	0.25 μm	N/A
AMD K6-III	21,300,000	1999	AMD	0.25 μm	N/A
AMD K7	22,000,000	1999	AMD	0.25 μm	N/A
Pentium 4	42,000,000	2000	Intel	180 nm	N/A
Atom	47,000,000	2008	Intel	45 nm	N/A
Barton	54,300,000	2003	AMD	130 nm	N/A
AMD K8	105,900,000	2003	AMD	130 nm	N/A
Itanium 2	220,000,000	2003	Intel	130 nm	N/A
Cell	241,000,000	2006	Sony/IBM/Toshiba	90 nm	N/A
Core 2 Duo	291,000,000	2006	Intel	65 nm	N/A
AMD K10	463,000,000	2007	AMD	65 nm	N/A
AMD K10	758,000,000	2008	AMD	45 nm	N/A
Itanium 2 with 9MB cache	592,000,000	2004	Intel	130 nm	N/A
Core i7 (Quad)	731,000,000	2008	Intel	45 nm	263 mm ²
Six-Core Xeon 7400	1,900,000,000	2008	Intel	45 nm	N/A
POWER6	789,000,000	2007	IBM	65 nm	341 mm ²
Six-Core Opteron 2400	904,000,000	2009	AMD	45 nm	346 mm ²
16-Core SPARC T3	1,000,000,000	2010	Sun/Oracle	40 nm	377 mm ²
Six-Core Core i7	1,170,000,000	2010	Intel	32 nm	240 mm ²
8-core POWER7	1,200,000,000	2010	IBM	45 nm	567 mm ²
Quad-core z196	1,400,000,000	2010	IBM	45 nm	512 mm ²
Dual-Core Itanium 2	1,700,000,000	2006	Intel	90 nm	596 mm ²
Quad-Core Itanium Tukwila	2,000,000,000	2010	Intel	65 nm	699 mm ²
8-Core Xeon Nehalem-EX	2,300,000,000	2010	Intel	45 nm	684 mm ²
[†] 10-Core Xeon Westmere-EX	2,600,000,000	2011/2012	Intel	32 nm	512 mm ²

[†]On April 2012, Intel released its first third-generation core microprocessor chip, named Ivy Bridge [3]. Transistors on an Ivy Bridge processor are packed roughly twice as dense (with 1.4 billion on a 160 mm² die) as compared to the 10-Core Xeon because it uses the 22 nm silicon processing technology and tri-gate (or 3-D) transistors. However, Ivy Bridge is not listed in this table or in subsequent related graphs as most commercially available CPUs (as of September 2012, when this thesis was revised) still uses the 10-Core Xeon processor.

Fig. 1.1. Graphical representations of the transistor sizes and counts of Intel microprocessors to illustrate the validity of Moore's law to this date (August 2011). Data is based on Table 1.1. Note that the vertical scales are logarithmic.

Fig. 1.2. The on-chip local clock speed of the microprocessor and the various interconnection speeds in the electronic chip of the microprocessor [4]. Note that the vertical scales are logarithmic.

Pursuing Moore's law (i.e., the drive towards more transistors on a chip) is not an end to itself. Rather, it is only a means to an end – that end being better computing performances. Moore's law is useful only because we can transform its raw resource (more transistors) into a much more useful form (better computing performances). Exploiting Moore's law can be seen as mining a gold mine or any other raw resources for economic dividends. Thus far, as shown in Figs. 1.1-1.2, the dividends

(more transistors) from exploiting Moore's law has been transformed into better computing performances. This is because the capabilities of most digital electronic devices are strongly related to the chip density (i.e., the number of transistors per chip). Higher chip density means better processing speed, memory capacity as well as higher number and larger size of pixels for video processing in digital electronic devices. In particular, observe from Fig. 1.2 that the on-chip local clock speed of the microprocessor, which is strongly linked to the computing power, has increased from ~1 GHz to ~10 GHz within the period of 1999 to 2011, due to the increase in the transistor count per chip from 9.5 million to 2.6 billion in the same time frame. The exponential increment in the chip density has led to higher speed microprocessors, thereby enabling *high-density information processing*, which has played an important role in meeting the information demand for the past few decades.

Besides the *high-density information processing* of the microprocessors due primarily to Moore's law, another factor that has enabled the field of communications to keep pace with the ever increasing demand for information to this date is the *high-capability information transmission* provided by long-haul optic fiber communications networks [8, 9]. In particular, fiber to the x (or FTTx) technology are progressively being implemented nationwide in various countries [10-14] in recent years to replace existing telecommunication networks that still utilize electrical interconnections (typically copper cables) to link the optic fiber networks from the main transmission node or central exchange office (CO) to the end-users. Note that the x in the term FTTx is used to indicate how close the fiber end point comes to the actual users, before the last mile connection is continued by other means, such as electrical interconnections (typically copper cables) or wireless networks. In general, the main types of FTTx networks are [15, 16]:

- FTTN (or Fiber-to-the-node) — the optic fiber is terminated at a communication node in the street that is located up to several kilometers away from the premises of the end-users. The last mile connection from the node to the premises of the end-users is usually based on copper cables.
- FTTC (or Fiber-to-the-cabinet) — this is very similar to FTTN. However, in this case, the communication node in the street is closer to the premises of the end-users, which is usually within 300 m. The final connection to the end-users is based on copper cables.
- FTTB (or Fiber-to-the-building or Fiber-to-the-basement) — the optic fiber reaches the common boundary of the premises of a group of end-users, such as the basement in a multi-tenant building. The final connection to the premises of the individual end-user can then be made via copper, optic fiber or alternative means such as wireless connection.
- FTTH (or Fiber-to-the-home) — the optic fiber directly reaches the boundary of the premises of the individual end-user, such as a communication node on the outside wall of a house. The final connection from this communication node to the computers of the end-user can be made via copper, optic fiber or alternative means such as wireless connection

At present, FTTx technology can provide a bandwidth of up to 2.5 Gbps downstream and 1.2 Gbps upstream at an affordable price in most developed countries, such as Australia [10], Singapore [11], the United States [12] and Japan [13]. Note that even at this data transmission rate, the bandwidth usage is only a small fraction of the huge optical bandwidth of ~ 25 THz, if single-mode optical fiber is used. With the deployment of wavelength division multiplexing (WDM), in which a large number of transmission channels or wavelengths are being combined into one single fiber, the transmission capacity can be increased by ~ 32 times [8] or even by several hundreds of times [8, 9] if its more sophisticated counterparts — the dense wavelength division multiplexing (DWDM) and ultra-dense wavelength division multiplexing (UDWDM) — are being utilized. These will allow high data transmission rate on FTTx networks that is on the order of several Gbps and even Tbps.

1.1.2. The Limitations of Moore's Law and Optic Fibers in this Information Age

In this Information Age, digital explosion has become a reality, with digital data currently growing exponentially at a rate of tenfold every year. It is estimated that the amount of digital information being produced and processed will increase to 35,000 exabytes (i.e., 3.5×10^{22} bytes) in 2020, which is 44 times more than what was being produced in 2009 [17]. This is mainly driven by high-bandwidth consuming applications and services, such as high-definition television (HDTV), video on demand (VOD), video conferencing, Internet protocol television (IPTV) and file sharing, whose compressed bit rate can easily be on the order of Gbps in the future [17-20].

To cope with the increasing demand for information, it might seem logical to further improve on existing long-haul optic fiber networks and the widespread use of the above-mentioned FTTx technology as well as the further scaling of the size of the transistors in order to bring the microprocessors to greater information processing speed for higher computing power. However, it is not possible to continuously decrease the size of the transistors. The transistors would eventually reach the limits of miniaturization at atomic levels due to quantum tunneling effect. Then Moore's law will collapse. This is theoretically predicted to occur around 2020 [21].

It must be mentioned that the end of Moore's law does not mean that computing performances cannot be improved in the future; rather, it means that we will lose an important raw resource (i.e., Moore's law in terms of more transistors on a chip) for improving computing performances. In fact, when mainstream computing hits a wall in 2005 [22] because of the fall in the rate of increase of the number of transistors on a chip (which can be observed in Fig. 1.2), the computer industry begins to use other schemes, in conjunction with Moore's law, to improve the computing performances. These schemes, which have been rather successful to increase the computing speed (which can be observed in Fig. 1.2 for 2005 and subsequent years) include [3, 23-25]: (1) the transition from single-core to multi-core central processing units (CPUs) in mainstream computing systems (starting from 2005), (2) the use of heterogeneous cores (starting from 2009), (3) the use of elastic compute cloud cores

(starting from 2010), (4) the use of network-on-chip (NoC) that focuses on the design and optimization of network interfaces, topologies, routing, flow control, and router/memory micro-architectures, and (5) the use of tri-gate (or 3-D) transistors for third-generation chips (starting from 2012). Thus far, these schemes are used together with Moore's law. Whether they will continue to be effective in increasing the computing performances after the break down of Moore's law (i.e., whether they can take over Moore's law as the main resource for improving the computing performances) remain to be seen. In addition, alternative technologies, such as quantum computing [26-28], graphene and carbon-nanotubes based electronics [27], can also serve as new resources to replace the use of Moore's law to increase the computing performances, provided that these can be adequately developed and be matured for commercial applications.

However, the greatest hurdle in meeting the increasing demand for information lies not so much in the speed of the microprocessors that is highly dependent on Moore's law. In fact, even if Moore's law can be substained using current IC technology, it is still inadequate to effectively cope with the high-bandwidth consuming applications and services of the future. Neither will improvement in existing optic fiber networks nor the widespread use of FTTx fully solve the problem. This is the limitation of Moore's law and optic fiber networks. The real crux of the problem is in the electrical interconnections on the computer boards and in the electronic chips of the servers or computers of the end-users. These electrical interconnections are used for information transmission within the electronic chips, between electronic chips and between electronic chips and peripheral devices. Though optic fibers can be used in the last mile connection to the end-users and even to connect different computers of the end-users, the use of electrical interconnections in the computers gives rise to what is commonly known as the electrical interconnections bottleneck, which is the very root of the problem in meeting the increasing demand for information. This is elaborated in the next subsection.

1.1.3. Communication Bottleneck of Electrical Interconnections

The net computing time in the electronic chips is the sum of two parts: The switching time of the logic gates of the microprocessors and the delay time of the electrical interconnections. As the elements (i.e., the transistors and electrical interconnects) on an electronic chip are reduced in size so as to subtain Moore's law, only the switching time is scaled down while the electrical interconnection delay, which determines the number of bits per second (also known as the bit rate capacity, B') that we can get down the interconnects, does not scale down but stays constant. To understand this, first note that there are two types of electrical interconnection delay for the electronic chips: The $R'C'$ time delay, which is proportional to $R' \times C' \times L^2$, and the $R'L'$ time delay, which is proportional to $R' \times L' \times L^2$, where the symbols R' , C' , L' and L , respectively, denote the resistance per unit length, capacitance per unit length, inductance per unit length and length of the electrical interconnects. At the gigahertz frequencies of modern information processing systems, the $R'C'$ time

delay applies to data transmission for intra-chip (i.e., within chips), while the $R'L'$ time delay applies to data transmission for off-chip (i.e., chip-to-chip, chip-to-board or chip-to-peripherals) connections. Scaling down the transistors and electrical interconnects by a factor of s in all three dimensions shrinks the cross-sectional area by s^2 , reduces the length by $s \times L$ and increases the resistance per unit length to R'/s^2 . However, the capacitance per unit length and inductance per unit length remains relatively constant as they depend chiefly on the shape of the interconnects, not so much on the size [29]. As a result, with size scaling, the $R'C'$ and $R'L'$ time delays still remain fixed at $R'/s^2 \times C' \times (s \times L)^2 = R' \times C' \times L^2$ and $R'/s^2 \times L' \times (s \times L)^2 = R' \times L' \times L^2$, respectively. This is the size scaling limit issue: Though size scaling increases the speed of the microprocessor (as more transistors can be packed in a chip), the speed of the electrical interconnects stays somewhat constant. Consequently, the speed of the electrical interconnection cannot keep up with that of the microprocessor.

The above-mentioned $R'C'$ and $R'L'$ time constant constraints imply that the maximum speed of electronic chips will be limited by the electrical interconnection time delay, not by the speed of the microprocessors. As shown in Fig. 1.2, though the on-chip local clock speed of microprocessors has constantly increased to 10 GHz by 2011, the on-chip and off-chip interconnection speed have been expecting slower increasing rate since 2002. The National Technology Roadmap for Semiconductor (ITRS) [5] predicted that, even with the employment of copper and materials with lower dielectric constant and the optimization of the chip architectural structure, the electrical interconnection speed will remain relatively constant and eventually stagnate. It can be seen in Fig. 1.2 that by 2014, the on-chip and off-chip interconnection speeds are ~ 3 GHz and ~ 1 GHz, respectively, even though the microprocessor speed has already increased to 16 GHz in the same year. Such low interconnection speed is simply inadequate to handle the large information bandwidth in futuristic applications, which will easily consume bandwidth on the order of Gbps or even Tbps. This is the so-called electrical interconnection bottleneck of this Information Age. Computers will have all the capability to process and compute the received information based on Moore's law but not the adequate means to transmit it due to the limited bandwidth or transmission speed of the electrical interconnects in the electronic chips. In such a circumstance, the high optical bandwidth of existing optic fiber networks cannot be fully utilized.

It must also be mentioned that the optimization of the cross-sectional area A_I and length L of the electrical interconnects does not solve the $R'C'$ and $R'L'$ time constants problem as the bit rate capacity B' of electrical interconnection is estimated to be $B' \leq B'_0 A_I / L^2$, where B'_0 is the bit rate capacity constant while A_I and L are the cross-sectional area and length of the interconnect, respectively [30]. This relation of B' with A_I and length L can be derived by using: (i) $R' \propto L/A_I$, $C' \propto L$ and $L' \propto L$; and (ii) $B' \propto 1/(R'C')$ for on-chip interconnection and $B' \propto 1/(R'L')$ for off-chip interconnection so that successive bits do not overlap excessively with each other. Do note that $\propto$ is the symbol of proportionality. The bandwidth or bitrate of the electrical interconnects is thus limited

by the aspect ratio A_I/L^2 . In other words, B' or bandwidth of the electrical interconnects is fixed by the aspect ratio limit A_I/L^2 and cannot be simply altered by miniaturizing the size of the electronic devices or making them bigger. Do note that A_I refers not only to just one wire or electrical interconnect but also to the total cross-sectional area of all the used wires. This implies that once all the space on a chip has been filled up by the different wires and transistors, B' of the system will not change as the total area A_I remains the same regardless of whether a large number of small wires or a small number of large wires are used, assuming that L stays rather constant.

Apart from the constraint of a constant interconnection speed or bit rate capacity, there are also other issues related to the use of electrical interconnections. One of such issue is that each electrical interconnection path must be wired individually: Each path can only be used by one distinct data stream so as to avoid the mutual interaction of the electrons through the Coulomb force. This will imply that a large number of wires (one for each distinct data path) must be used in the electronic chip, leading to a complicated chip design. Also, in order to minimize crosstalk due to capacitive and inductive coupling between proximate electrical interconnects as well as to avoid electromagnetic interference (EMI) of the signals, the wires or electrical interconnects in the electronic chip must be evenly spaced out. This will result in a small interconnection density-bandwidth product or large packing density. Another issue is that there is an increase in joule heating of the electrical interconnects with smaller transistor size. In addition, it has been found that at data rate approaching 10 Gbps, microscopic imperfections in the electrical interconnects will begin to weaken and distort the signals [19, 20].

1.1.4. Photonics Integrated Circuits

The most feasible solution to eliminate the electrical interconnections bottleneck and other electrical interconnects related issues is to replace the electrical interconnections in the electronic chips of computing systems with its optical equivalents, i.e., optical interconnects, that transmit data through photons rather than electrons [18-20]. A network of optical interconnects and related optical component devices will form the photonic analogue of the electronic ICs, known as photonic integrated circuits (PICs). The employment of PICs as interconnections in computing system can also be seen as mimicking the use of long-haul optic fiber networks to replace cable-based networks. The greatest communication strength of using PICs as interconnections is that PICs operate at optical frequencies (THz regime) that will result in a much larger bandwidth, bit rate and transmission speed as compared to electrical interconnections that use signals at much lower frequencies (GHz regime). The size scaling and aspect ratio limits (mentioned in section 1.1.3) that result in a stagnation of the interconnection speed and bit rate capacity in electrical interconnects are eliminated when PICs are used. Moreover, as photons (at low input intensities) do not interact with each other or with other radiation, unlike electrons in electrical interconnections, several other major improved features can

also be found in PICs-based interconnections. First, optical interconnects can be packed closely with minimal crosstalk and EMI. This results in a much greater interconnection density-bandwidth product or packing density in PICs as compared to electronic chips that use electrical interconnects. Second, each optical interconnect or waveguide can be utilized by several data channels, unlike the case in electrical interconnections in which each data path must be wired individually. In fact, the signal bandwidth can be increased by combining or multiplexing a large number of data channels into one single optical interconnect, similar to the WDM technology in optic fiber. Such ability of several data streams to share one pathway will result in a simpler design in the interconnection layout when PICs are used.

In a PIC, all of the necessary optical component devices, such as photo-detectors, routers, switches, filters, buffers and amplifiers, to perform both passive and active functions (or signal processing) can be found. To perform active optical functions, such as light amplification and modulation, the PIC could subsequently be integrated seamlessly with current electrical circuits based on complementary metal-oxide-semiconductor (CMOS) technology or any other type of integrated electronic technologies, such as metal-oxide semiconductor field-effect transistor (MOSFET), so that the active optical devices on the PIC can be interrogated by electronic means and be responsive to any external electrical actuation-or-control signal. This marriage between integrated photonics and integrated electronics is commonly known as integrated optoelectronics and the resulting chip is known as the optoelectronic integrated circuit (OEIC). Such synergy of electronics and photonics will greatly boost the performances of current computing and communication systems, allowing the possibility to achieve high data transmission rate of > 10 Gbps [20] that will meet future information demand. The high bandwidth of optic fiber can then be better utilized. Computing and telecommunication will thus be revolutionized, ushering in the next Information Age. Note that in an active PIC (i.e., OEIC), the electrical drivers are connected internally to the photonic components (i.e., intra-chip) over a short path such that the $R' \times C'$ time delay is relatively low and rather negligible as compared to the electrical interconnects of conventional electronic ICs in the microprocessors.

The vision of a PIC integrating with electronic ICs to form the above-mentioned active PIC or OEIC was originally proposed by Abstreitter [31] in 1992 and refined by Soref [32] in 1993. In Soref's vision of such a superchip [32], which is shown in Fig. 1.3, optical fibers are butt-coupled to the waveguides on the PIC so that the optical signal can be transmitted into and out of the chip. The input optical signal is then detected by a photodiode, after which the data can be used for electrical interrogation or simply be transmitted into other devices on the PIC. To compensate for the waveguide losses, the input signal can be amplified, before it is re-shaped and/or re-encoded by the optical modulator on the PIC. New electronic data can also be encoded on the exiting optical signal by the same modulator. All the active functionalities are made possible in such a superchip due to the integration of photonics with integrated electronics technologies, such as CMOS, as shown in Fig. 1.3.

Fig. 1.3. Soref's vision of a photonics super chip [32], where photonics is integrated seamlessly with electronics so that the optical devices on the chip can be interrogated by electrical means. Note that as the electrical drivers are connected internally (i.e., intra-chip) to the photonic components over short paths, the resistance and parasitic capacitance are relatively low as compared to electrical interconnections in electronic chips and between electronic chips.

Thus far, we have only mentioned about replacing the electrical interconnections in existing electronic-based microprocessors chips with optical interconnects or OEICs. Do note that the computing of the received data from the CO still needs to be done by the electronic-based microprocessors, even if PICs or OEICs are used to replace the electrical interconnects in the chips. To do so, the optical information in the PICs must first be converted into the electronic domain before it is passed into the microprocessors for data computation. The computed data is then converted back into the optical domain before it is sent back to the PICs, where it will be transmitted out of the servers of the users and subsequently into the optic fiber of the CO. Of course, the ideal case would be to operate completely in the optical domain as the conversion of information between the optical and electrical domains not only causes OEIC devices to lose ~30% of their energy [26], but also slows down the transmission of data. However, the development of the integrated optical analogues of the microprocessors that can compute data in the optical (or even quantum) domain, i.e., all-optical transistors or computers, for viable commercial applications is still ongoing [26-28]. There are various challenges and limitations [33, 34] that must be overcome before all-optical or photonic computing can be fully realized on an IC chip. In particular, real-world logic systems require logic-level restoration, cascability, fan-out and input-output isolation, all of which currently cannot be adequately met by optical computers, but can be provided by electronic transistors at low cost, low power, and high speed [34]. Moreover, the fact that photons do not interact with each other or other radiation at low input intensities is a major drawback in the implementation of optical logic, though this characteristic is an advantage in the implementation of optical interconnects. For optical computers to be truly competitive and move from a few niche applications (which is the current situation) into real-world commercial applications would require major breakthroughs in non-linear

optic technology. In addition, note that electronic-based microprocessors can be easily programmable by software and their accuracy is practically unbounded. All-optical computing is still unable to achieve these. As such, for the time being, the best approach is to exploit and combine the communication strengths of photonics with the computation strengths of electronics in an optimal manner as further elaborated below.

A photonic/electronic hybrid of the microprocessor chip, in which the transistors of the microprocessors are based on electronic ICs, while the data interconnections between the outside world and the microprocessor chips are based on photonic technologies that employ OEICs (or active PICs) can be deployed in the end-users' servers or computers, in conjunction with the use of FTTx technology. This will effectively meet the demand for information. A 3-D version of such a photonic/electronic hybrid of the microprocessor chip has been proposed by American multinational consulting and technology firm, International Business Machines or IBM [35]. This is shown in Fig. 1.4. In IBM's 3-D photonic/electronic hybrid microprocessor chip, which is a conceptual design of future computing networks, the lowest layer is filled with electronic-based microprocessors with

Fig. 1.4. A 3-D chip [35] proposed by IBM that integrates photonic integrated circuits (top layer) with the electronic memories (middle layer) and the electronic processors (bottom layer). The photonic integrated circuits (PICs) at the top layer not only connect the various cores but also process the incoming and outgoing data. Here, the PIC is shown performing optical routing. Other optical functions such as switching, modulating and amplification can also be implemented by the PICs before the data is passed to the electronic devices in the memory and processor layers or transmitted out of the 3-D chip.

thousands of individual cores, powered by Moore's law. The middle layer consists of memory units based on electronics while the top layer is the photonic layer, consisting of OEICs that connect the various cores of the chip. The optic fiber from the CO (assuming FTTH network is used) is then directly linked to the top photonic layer of the 3-D chip. Also, instead of converting the optical data (in the 3-D chip) that have just arrived from the external optical fiber into the electrical domain for various signal processing functions [36, 37], such as demodulation, differentiation, integration, filtering, routing, buffering and switching, the optical data can be directly processed in the optical domain, before passing it to the microprocessor for further data computation after it has been

converted into the electrical domain by the OEICs. This will greatly increase the transmission rate and processing speed as electrical interconnects are not used when performing the signal processing of the received data. Also, there is lower energy consumption as the conversion of information between the optical and electrical domains causes OEIC devices to lose ~30% of their energy, as mentioned earlier.

The most suitable material upon which PICs or OEICs could be built is silicon [38, 39]. This proposal of the use of silicon as an optical material, which is now known as silicon photonics, can be traced to the pioneering works of Soref and Petermann [40-43] in the late 1980s and early 1990s. The strongest argument in favor of silicon photonics is based on its compatibility with the mature silicon IC fabrication technology: the same standard silicon IC production process can be used to create optical waveguides onto the silicon surface. Presently, those working with microelectronic devices are seeking to move beyond the current state-of-the-art critical dimension of 22 nm. The semiconductor processing technologies of 13 nm and 11 nm are expected to be in use by 2013 and 2015, respectively [4, 5] (see also Table 1.1). However, most optical devices based on silicon are likely to have critical dimensions of ≥ 100 nm for the near future [18, 38]. This means that the infrastructure already exists for the next several generations of PICs, if silicon is used. Also, as mentioned earlier, the silicon IC production process is able to allow the manufacturability of complex microelectronics to be realized with a great degree of reproducibility and at a rather low cost, allowing them to be easily affordable to consumers. Hence, by exploiting the technologies of the mature electronic IC industry, low-cost photonics for mass-market applications will be possible. The economies of scale that we have seen for the electronics industry could easily be applied to the photonics industry.

Another motivation for the use of silicon is that it is unique from many other semiconductors in that it has a natural oxide (SiO_2), which is adhering and can thus be used as a passivation layer. This can be used to produce silicon-on-insulator (SOI) substrates [18-20]. SOI substrates consist of a thin layer of silicon dielectrically separated from the bulk silicon by SiO_2 . The strong optical confinement offered by the high index contrast between silicon (refractive index ≈ 3.45) and SiO_2 (refractive index ≈ 1.45) makes it possible to scale photonic devices to the hundreds of nanometer level. Many CMOS processes use an SOI substrate as a starting point for device fabrication. Hence, using SOI as a photonics platform will allow it to seamlessly merge with existing CMOS process technology, facilitating the realization of the earlier-mentioned OEICs as well as the 3-D chip. Such monolithic integration of optics and electronics onto a single substrate has the capacity to bring together the speed and bandwidth of optical systems with the complexity and well-established engineering background of current integrated electronic circuits. For this reason, the work in this thesis will focus on integrated optical devices based on the SOI platform.

With silicon photonics scheduled for high-volume market applications in the 2010-2015 window [20], products and advanced prototypes based on the SOI platform has already been

developed by various major industrial players, such as Intel [44], Luxtera [45], IBM [46] and BAE systems [47], just to name a few. In particular, Intel Corporation, the dominant market player in microprocessors, has demonstrated many breakthroughs in silicon photonics, including the demonstration of a silicon laser [48], hybrid silicon laser [49], a high-speed silicon modulator [50] and the recent silicon photonics 50 Gbps fiber-to-chip link [51], which link optic fiber directly into the electronic chips of the computers.

1.2. Research Objectives

To facilitate the development of PICs that can be used to alleviate the problem of electrical interconnection bottleneck as mentioned in section 1.1, this thesis will focus on the theoretical optimization and modeling of optical component devices that are typically being used on the PICs. In particular, we have focused on two optical devices — the straight waveguides and the travelling wave microresonators (TWMRs) [52] — in this thesis. The objectives of this thesis are further outlined as follows.

- The theoretical optimization of the straight waveguide and TWMRs for PICs based on the silicon-on-insulator (SOI) platform, with particular emphasis on efficient communications applications. Note that there are different types of TWMRs [52], such as the microspheres, microtoroids and microring resonators (MRRs). Our theoretical optimization for SOI devices in this part will only focus on the MRRs as it is the most commonly used TWMR due to their planar structure and relative ease of fabrication.
- New design proposals based on TWMRs to perform optical functionalities, such as photonic bandgap (PBG) engineering and the generation of fast and slow light effects. The proposed schemes are generic in nature and can be implemented on different types of TWMRs of different material platforms. For convenience, however, the MRR based on the SOI platform will be used for specific case studies (if any) in our design proposals.

1.3. Significant Contributions

The research works in this thesis, which are based on the above research objectives, are divided into two parts: (i) Part one (chapters 2 to 5); and (ii) Part two (chapters 6 to 9). These works have contributed 8 peer-reviewed journals (6 published and 2 submitted) and 2 conference proceedings. The main contributions of these works are outlined as follows.

1.3.1. Part One: Fundamental Building Blocks

Part one of the thesis focuses on the basic functionalities of two fundamental microphotonic components, namely, the straight waveguides and the traveling wave microresonator (with emphasis on the microring resonator).

(a) Single-Mode and Polarization-Independent Passive Silicon-on-Insulator Waveguides of Submicron Scale

Waveguides are the most fundamental building blocks of any PICs. We thus start by studying the modal characteristics of different types of waveguides. As there is a trend towards device miniaturization and the use of silicon-on-insulator (SOI) for the commercialization of PICs [53], we focus our study on the single-mode (SM) and polarization-independent (PI) conditions for SOI channel and rib waveguides with height and width of less than $1\text{ }\mu\text{m}$ using rigorous numerical modeling based on the 3-D imaginary distance beam propagation method (IDBPM) [54]. Our results show that submicron SOI waveguides exhibit strong dependency on their polarization states. Nevertheless, both SM and PI conditions are possible under certain circumstances and are restricted by the fabrication process, where birefringence is strongly dependent upon the waveguide dimensions. A matrix of waveguide parameters for the SOI rib, channel and slab waveguides that can satisfy SM and PI conditions simultaneously have been identified for the operating wavelengths of $1.31\text{ }\mu\text{m}$ and $1.55\text{ }\mu\text{m}$. This is useful in providing a general design rule for the fabrication of SOI waveguide of submicron dimensions, which is currently lacking in literature. In addition, we have also used analytical modeling to study the modal characteristics of SOI slab waveguides and their relation to those of the SOI channel and rib waveguides. This has been ignored in most of the literature. Also, we have demonstrated that the approximations provided by the analytical modeling matches well with the general trends of the results of the numerical modeling via the IDBPM. Such use of both analytical and numerical modeling gives a better physical insight into the stringent conditions needed for the realization of both SM and PI operations at the submicron level.

(b) How Small can a Passive Microring Resonator Be and Yet Be Single-Mode and Polarization-Independent?

Another fundamental block of PICs is the microring resonator (MRR). Like its straight waveguide counterpart as mentioned earlier, MRR must also be SM and PI in order for it to be suitable for communication applications. As MRRs are bend-based devices, they inherently have higher polarization sensitivity and different SM conditions in contrast to straight waveguide (WG) based devices. To accommodate the structural polarization independence, the design of SM microring resonators has been limited generally to relatively large bend radius ($> 30\text{ }\mu\text{m}$) [55-58]. However, there is an increasing need to scale the size of MRR into the submicron regime so as to facilitate high density integration. The ultimate question then is: How small can we shrink both the cross sectional dimensions and radius, and yet maintain its overall PI and SM conditions, while ensuring low round-trip losses? This will be objective of this work. We have shown theoretically in this research work of the feasibility of simultaneously maintaining the conditions of SM and structural polarization independence in a MRR while shrinking the size of both the bend radius and the waveguide cross

section via the more robust deeply etched submicron SOI rib waveguides, which are seldom used for MRR. It is found that, for a given cladding and core thickness, the radius of a SM and polarization-independent MRR based on the SOI platform can be as small as $3\text{ }\mu\text{m}$, being limited chiefly by the residual birefringence of the resonator and the bend losses. The results show that a resonance mismatch of only about 0.22 nm between the TE and TM resonance spectra and a free spectral range (FSR) of 12 nm can be achieved. This small polarization dependence in our submicron MRR (height and width $< 1\text{ }\mu\text{m}$, bend radius $< 10\text{ }\mu\text{m}$) represents a large improvement over what can usually be achieved with the conventional PI resonators [55-58] that have much larger device size (height and width $\geq 1\text{ }\mu\text{m}$, bend radius $\geq 30\text{ }\mu\text{m}$) and very small FSR (in the range of pico-meter).

(c) Traveling Wave Microresonators (TWMRs) Based Coupled-Resonator Optical Waveguides: Photonic Bandgap Engineering Using Periodic and Quasi-Periodic Orders

Having studied the properties of a single MRR as described in section 1.3.1(b), which is one particular type of traveling wave microresonators (TWMRs), we then proceed to the case of N -coupled TWMRs (where N is the number of resonators). If N is large, the coupled-resonators structure is known as coupled-resonator optical waveguides (CROWs). For this work, we focus on the photonic bandgap (PBG) engineering of TWMR-based CROWs using periodic and quasi-periodic orders. For the periodic-order case, size-tuned defects are introduced at periodic locations among the regular resonators, which are size-untuned, to form a periodic ordered CROW system. The periodic coupled defects result in multiple localization states that lead to the formation of mini-defect bands and mini-PBGs within the wide PBG of the original defect-free CROW. The position and number of such mini-defect bands and mini-PBGs depend on the size tuning of the defects. These features make the deployment of such schemes for PBG engineering very attractive in the absence of active elements to achieve tuning. Other possible applications include multi-passband filters. For the quasi-periodic order case, the arrangement of the defects and the regular rings is an intermediate between periodic order and randomness, forming a quasi-periodic ordered system. This interplay of order and disorder is illustrated using the CROW analog of the Fibonacci number sequence, which results in a single high- Q localized state to appear that gradually transits to a mini-defect band within a wide photonic stop band as N increases. If the total losses are sufficiently low, the high- Q localized state of the Fibonacci CROW is highly useful for bio-photonics and chemical sensing applications. Interestingly, our results also illustrate that there is a strong periodic correlation even when the order in the CROW is aperiodic in nature.

(d) Raman Scattering Processes in Submicron Silicon-on-Insulator Waveguides: Harnessing Temperature Effects

To realize active functionalities in SOI waveguides, stimulated Raman scattering (SRS) is

widely used. However, existing studies on SRS-based SOI waveguide devices generally assume a fixed operating temperature of $T = 298$ K (room temperature); the temperature effects of SRS in submicrometer scale SOI waveguides have generally been neglected. As such, in this work, we have studied the temperature T dependence of SRS in submicrometer scale SOI waveguides for $100 \text{ K} \leq T \leq 500 \text{ K}$ in regard to three specific types of SRS-based functionalities in the continuous-wave regime: Raman amplification and attenuation near the C-band wavelength of $1.55 \text{ } \mu\text{m}$ and parametric Raman wavelength conversion (PRWC) between the C-band wavelength of $1.55 \text{ } \mu\text{m}$ and O-band wavelength of $1.31 \text{ } \mu\text{m}$. It is found that the effects of temperature variation can be harnessed to improve the Raman performances in the submicron SOI waveguides. In particular, the use of temperature variation provides an additional degree of freedom to tune the conversion efficiency (CE) and amplification efficiency (AE) of the SOI waveguides, thereby allowing a more dynamical control of the Raman performances of the SOI waveguide, as compared to those working at a fixed room temperature. Furthermore, the CE and AE can be enhanced as compared to that at the room temperature. The detailed research findings in this work can be used as general design rules when engineering submicron SOI waveguides for efficient Raman amplification, Raman attenuation and PRWC at different temperatures. Most importantly, the results show the numerous advantages of harnessing the temperature effects of SRS, which have largely been overlooked in the literature.

1.3.2. Part Two: Generating Fast and Slow Light Effects in Photonic Integrated Circuits Using Traveling Wave Microresonators

Part two of the thesis focuses on the fast light (FL) and slow light (SL) effects in either a single traveling wave microresonator (TWMR) or a system of twin-coupled TWMRs. The main emphasis here is on the use of degeneracy lifting of the cavity via the excitation of contra-propagating cavity modes to improve the fast and slow light effects of TWMRs. A variety of schemes to generate contra-propagating cavity modes in TWMR-based photonic circuits for enhanced FL and SL effects have been proposed and looked into.

(a) Harnessing Coupler-Induced Localized Backscattering for Enhanced Fast and Slow Light Effects in a Traveling Wave Microresonator

Degeneracy lifting and mode splitting originating from localized backscattering at the coupling junction when a TWMR is coupled to an external port WG has recently been predicted [59] and experimentally demonstrated [60, 61]. We term such backscattering that will also excite contra-propagating cavity modes as coupler-induced localized backscattering (CILB). In our work, we look into the effects of CILB on the FL and SL performances of a single TWMR that is coupled to two bus waveguide (i.e., add-drop filter configuration). We have found that weak CILB that is aided by a small net optical gain in the cavity will enhance the FL and SL performances of a TWMR in the

following ways. First, all three types of light propagation velocity — FL with negative group velocity, FL with positive group velocity and SL — can be generated at both the through and drop ports at the degenerate resonance frequency ω_0 . This is in contrast to the conventional TWMR (where CILB is negligible), in which at the degenerate resonance frequency, only FL with negative group velocity and SL are present at the through port, while for the drop port, only SL is produced. Second, the possibility to generate stable FL with amplified transmission can be achieved, unlike the case of a conventional TWMR, where stable FL always has substantial attenuation. Third, there is an extra degree of freedom in addition to the parameters of κ and τ of the cavity (where κ and τ are, respectively, the cavity-to-bus-waveguide coupling coefficient and cavity round-trip attenuation constant) to control the group delay as any change in the strength of the CILB will change the resonance spectrum. However, in the regime of strong CILB, there will be some deleterious effects. In particular, the group delay enhancement at the through port is degraded and the width (i.e., the range of τ , given fixed κ and magnitude of the CILB) of the FL regime at the drop port is reduced.

(b) Tunable Fast and Slow Light via Interaction of Intracavity Backscattering with Dual Contra-propagating Inputs

Besides CILB, another type of backscattering that generates contra-propagating cavity modes is the surface imperfections-induced distributed backscattering within the cavity. We will term such backscattering simply as intracavity backscattering. In this work, the intracavity backscattering can be enhanced and controlled by the intentional addition of surface microstructures. To achieve tunable group delay, dual contra-propagating inputs are launched into the TWMR circuit to interact with the enhanced intracavity backscattering. It is found that by adjusting the amplitude and/or phase differences between the dual contra-propagating inputs, the interaction of the cavity modes with the enhanced intracavity backscattering can be controlled and the shape of the resonance spectrum will gradually evolve from a single Lorentzian to a two-split Lorentzian and vice versa. Such a change in the spectral shape will translate into changes in the transmission and group delay of the output light. Consequently, continuous tunability in the group delay and transmission can be achieved. Switching between FL and SL can also be realized if the change in the spectral shape is accompanied by a change in the dispersion response from anomalous dispersion to normal dispersions or vice versa. This proposed scheme offers a more dynamic control of the speed of light, as compared to systems that use only a single input. More importantly, it realizes the possibility to achieve tunable fast and slow light in a passive and linear cavity. This is in contrast to conventional TWMR systems [62-68], which require either the use of active tuning mechanisms, such as p-i-n diode or gain medium, or nonlinearity in the cavity so as to have tunable group delay. Also, as the active tunability of the group delay of this proposed scheme stems from the use of external dual inputs, the reconfigurability and

serviceability of the cavity system are also improved as the external dual inputs can be easily adjusted or replaced.

(c) Harnessing Intracavity Backscattering for Enhanced Tunable Coupled-Resonator-Induced Transparency and Optical Fano Resonance

Having exploited the intracavity backscattering in a single TWMR, which is described in section 1.3.2(b), we then look into the use of the intracavity backscattering in a system of twin-coupled TWMRs that has only one resonator coupled to the external bus waveguide. We have found that by intentionally introducing intracavity backscattering in the twin-coupled TWMRs, coupled-resonator-induced transparency (CRIT) [69] and optical Fano resonance (OFR) [70] effects can be enhanced, as compared to conventional twin-coupled TWMRs systems that do not utilize the intracavity backscattering effects. (Note: Due to the strong dispersive property of the CRIT and OFR lineshapes, CRIT and OFR are categorized as fast and slow light effects in some literature. We have adopted this convention and thus included the study of CRIT and OFR in part two of this thesis that deals with fast and slow light effects). In particular, the presence of intracavity backscattering in our modified twin-coupled TWMRs make it possible to either generate a single sharp CRIT peak or a pair of CRIT peaks in the transmission spectrum of the through port, in contrast to the case of the traditional twin-coupled TWMRs structure [69, 71], in which only a single sharp CRIT peak can be realized within one FSR. The distance between the twin-CRIT peaks can be controlled by adjusting the intracavity backscattering magnitude u_m . In addition, our proposed scheme makes it possible to simultaneously produce fast and slow light effects, one at each output port, from a single device. By using different u_m for the two resonators in our proposed modified twin-coupled TWMRs structure, the CRIT peaks can be reshaped into sharp asymmetric OFR lineshapes. Finally, if dual inputs are employed, the CRIT symmetric peaks of our proposed scheme can also be reshaped into asymmetric OFR lineshapes by modulating the phase and/or amplitude differences between the inputs. Active switching between CRIT and OFR in our proposed scheme can then be achieved in the absence of gain and/or phase tuning elements in the resonators. This is unlike the case of the conventional twin-coupled TWMRs, which require gain [72] and/or phase tuning elements [73] in the resonators to switch between CRIT and OFR. The enhanced CRIT effects can be utilized for fast and slow light applications, while the asymmetric OFR lineshape is highly useful for optical switching applications.

(d) Tunable Flat-Band Slow Light Using Contra-propagating Cavity Modes

Finally, we utilize the contra-propagating cavity modes that arise purely from the evanescent coupling (but not the surface perturbations-induced intracavity backscattering) of both the resonators to the bus waveguide in the twin-coupled TWMRs system to generate flat-band SL. The presence of such contra-propagating cavity modes in the twin-coupled TWMRs system will generate multi-peaks

in the resonance spectra. Flat-band SL can be generated if such multi-peaks become undistinguishable and merge into one single broaden peak that is maximally flat or close-to flat in the resonance spectra, which will only happen if the inter-resonator evanescent coupling strength is optimized relative to the resonators-to-bus-waveguide coupling strength. The generated flat-band SL from our proposed scheme has the characteristics of (1) maximal flat transmission spectrum, (2) near unity transmission and (3) minimal group delay dispersion. The bandwidth and the group delay of the region of flat-band SL can be tuned by adjusting the coupling coefficients of the TWMRs. A fundamental figure of merit to quantify the capability of the SL system to simultaneously fulfill the bandwidth and group delay enhancement requirement is the delay-bandwidth product (DBP). We have shown that the DBP of the output light at the through port and the reflection port are, respectively, 3- to 12-fold and 6- to 24-fold higher than that of conventional TWMR-based SL systems [74-79]. This illustrates that our proposed system is capable of providing better SL performances as compared to conventional TWMR-based SL systems. Fabrication tolerance and cavity losses analyses have also revealed that our proposed scheme is rather robust to the fabrication errors and limitations of current state-of-the-arts semiconductor processing technology. Therefore, our proposed device will be highly suitable for practical SL applications. It is also a significant contribution to the field of SL as a large usable resonance bandwidth with large delay-bandwidth product (i.e., flat-band SL) can be generated, without the need for complicated device design, external dispersion compensation devices or a large device footprint (as only two resonators are needed in our proposal).

1.4. Organization of Thesis

This thesis consists of 10 chapters. Chapter 1 gives an overview of the background, research motivations and major contributions of the completed research works of this thesis. Chapters 2 to 9 describe in detail of the completed research works as highlighted earlier in section 1.3. Finally, in chapter 10, we summarize and conclude all the major contributions of this thesis as well as outlining research directions for our future works.

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PART ONE

Fundamental Building Blocks of Photonic Integrated Circuits

Chapter 2

Single-Mode And Polarization-Independent Passive Silicon-On-Insulator Waveguides of Submicron Scale

2.1. Introduction: Research Motivations and Objectives

It is envisioned that silicon-on-insulator (SOI) would be one of the most suitable material platforms to commercially realize photonic integrated circuits (PICs). The most basic building blocks of any SOI-based PICs are the straight SOI passive waveguides. We thus begin the dissertation work by looking into the modal properties of such waveguides.

In general, three types of straight SOI waveguide configurations are possible: The slab (or planar) waveguide, the channel (or strip) waveguide and the rib (or ridge) waveguide, which are, respectively, shown in Figs. 2.1(a), 2.1(b) and 2.1(c). A SOI slab waveguide is formed by sandwiching silicon in between two cladding layers, while etching the core of the slab waveguide completely (partially) down to the lower cladding layer gives a channel (rib) waveguide. Evidently, slab waveguide is the simplest waveguide configuration as additional fabrication processes are involved for

Fig. 2.1. Schematic cross-sections of the three basic types of silicon-on-insulator (SOI) waveguides: (a) Slab waveguide, (b) channel waveguide and (c) rib waveguide, with the various design parameters shown.

Note that light propagates in the z direction.

channel and rib waveguides. In spite of this, channel and rib waveguides have been the primary building blocks for silicon photonics as they are able to provide a two dimensional confinement of light, in contrast to the slab waveguide, in which light can only be confined in one dimension.

For any waveguides to be commercial viable, they must preferably be single-mode (SM) as well as polarization-independent (PI) at the communication wavelengths [1]. SM means that only the lowest order mode, termed as the fundamental mode, is present in the waveguide. Ensuring SM propagation is highly desirable as SM waveguide avoids intermodal dispersion and also reduces coupling losses that occur when coupling light into a SM fiber, which is commonly used in long-haul data transmission. Moreover, among all the possible waveguide modes with different mode orders, the fundamental mode of a waveguide (which is the only mode present in a SM waveguide) carries the majority of the optical power and is the most well confined field distribution within the waveguide core [2]. On the other hand, PI operation means that the response of the optical circuit to any incoming optical signal should be the same (or nearly the same) regardless of the polarization state of that optical beam. Technically, this means that the propagation constant β and effective index N_{eff} of the transverse electric (TE) mode is the same (or nearly the same) as that of the transverse magnetic (TM) mode, i.e., $\Delta N_{eff} = |N_{eff(TE)} - N_{eff(TM)}|$, where $N_{eff} = \beta/k_0$ and k_0 is the free space wave number. Note that throughout this thesis, β and N_{eff} are taken to be purely real, unless otherwise stated. As it is highly challenging to design a waveguide to have zero waveguide birefringence (i.e., $\Delta N_{eff} \approx 0$) for all the different waveguide mode orders ($q = 0, 1, 2 \dots$), the term PI in this thesis applies only to the fundamental mode order ($q = 0$) due to the above-mentioned advantages of SM operation.

The design rules for single-mode SOI rib waveguides at the micron scale have been formulated by Soref et al. [3] and later refined by other researchers [4-7]. To achieve PI operation for these SM waveguides, polarization management schemes, such as polarization diversity [8, 9], can be employed. Alternatively, one can design several optical circuits that serve the same functionality, each having the ability to function with a specific polarization. In addition, active tuning [10] can also be used to achieve PI operation. We will term these schemes as extrinsic PI schemes. The main disadvantage of these extrinsic PI schemes is that additional device space and fabrication processes are required. For the case of using active tuning, the energy consumption of the optical circuit will inevitably be higher. These problems can essentially be mitigated if the SOI waveguides can be designed to be intrinsically PI, which means that polarization independence is attained merely by the use of just one single waveguide with its optimized geometrical settings, with no external device or aid. The issues of achieving both SM and PI operations at the micron scale for SOI rib waveguides have been successfully addressed by Chan et al. [11]. For channel waveguide, the conditions for SM and/or PI at the micron level can be easily deduced based on the above-mentioned research work on SOI rib by Soref et al. [3] and Chan et al. [11].

Recently, there has been a drive towards the miniaturization of silicon photonic devices toward the submicron regime (i.e., waveguide width and height $< 1 \mu\text{m}$) for high-density optical integration and improved device performances [11, 12]. Various components have been demonstrated at the submicron scale, such as ring resonators [13], modulators [14], arrayed waveguide gratings [15], and

one dimensional photonic bandgap microcavity [16]. The general trend (as observed in year 2007 when this research was started and completed) when designing SOI-based integrated photonic devices at the submicron level is that channel waveguides are commonly used, with little emphasis placed on submicron rib waveguide structures: SOI rib waveguides are more commonly used at the micron scale (i.e., waveguide width and height $\geq 1 \mu\text{m}$) while channel waveguides are used at the submicron scale (i.e., waveguide width and height $< 1 \mu\text{m}$). It must be noted that: (1) The use of submicron rib structures have several advantages, one of which is the implementation of a high-speed modulator based on depletion mechanism for high optical confinement and low losses [14]; and (2) even though at the submicron scale, channel waveguide-based devices have been widely demonstrated, they are usually designed only for single-mode operation at $1.55 \mu\text{m}$ [17-19]. In view of the above-mentioned points (1) and (2), we see the need to look into the conditions to simultaneously satisfy both SM and intrinsic PI operations at the communication wavelengths of $1.31 \mu\text{m}$ and $1.55 \mu\text{m}$ for the SOI channel and rib waveguides at the submicron scale and operating at the room temperature. This will be the main research objective of this chapter. In addition, for completeness, the modal characteristics of submicron slab waveguides at the communication wavelengths of $1.31 \mu\text{m}$ and $1.55 \mu\text{m}$, which generally have been neglected, will also be explored and be related to the properties of the SOI rib and channel waveguides. This will provide a much clearer picture of the modal characteristics of the SOI waveguides at the submicron level.

This chapter is organized as follows. We begin with the basic Maxwell equations to analytically solve for the dispersion curves and subsequently deduce the general trends of the modal characteristics of the SOI submicron slab waveguides in section 2.2 and the SOI submicron 2-D channel and rib waveguides in section 2.3. Though such analytical approach is highly useful to give a clear insight into the actual physical phenomena involved, it serves only as a general approximation for two-dimensional (2-D) waveguides like the channel and rib waveguides. Accurate analysis of the 2-D waveguides would require the use of numerical or computational modeling via electromagnetic algorithms. This is done in section 2.4. Our numerical modeling is based on the 3-D imaginary distance beam propagation method (IDBPM) [20]. This approach has been implemented on various silicon-on-insulator (SOI) devices [13-15] and showed good agreement with the experimental data in [21]. Finally, in section 2.5, we conclude and highlight important contributions of this research work.

2.2. Analytical Modeling of Submicron Silicon-on-Insulator Slab Waveguides

2.2.1. Theoretical Formulation

Consider the SOI slab waveguide in Fig. 2.1(a), in which the silicon core with a thickness of h is sandwiched between two semi-infinite media of silica (SiO_2), which act as the upper and lower cladding layers. This renders the slab waveguide to be a symmetric structure. Note that some literatures, such as [21], use air as the upper cladding instead of SiO_2 . We have, however, used SiO_2

as the cladding layer as this will act as a passivation layer for the silicon core that is needed in most practical applications. Assuming that the input electromagnetic fields are time-harmonic, i.e.,

$$\begin{aligned}\tilde{E}(x, y, z, t) &= E(x, y) \exp(j[\omega t - \beta z]) \equiv E \exp(j[\omega t - \beta z]), \quad \tilde{E}(x, y, z, t) \equiv \tilde{E}, \quad E \equiv E(x, y) \\ \tilde{H}(x, y, z, t) &= H(x, y) \exp(j[\omega t - \beta z]) \equiv H \exp(j[\omega t - \beta z]), \quad \tilde{H}(x, y, z, t) \equiv \tilde{H}, \quad H \equiv H(x, y),\end{aligned}\quad (2.1)$$

where E is the electric field amplitude, H is the magnetic field amplitude, β is the propagation constant, which is defined as the vectorial component of the k -vector along the z direction in the optical medium throughout this thesis, ω is the angular frequency, t denotes time and $j = \sqrt{-1}$, while (x, y, z) is the Cartesian coordinate system employed in Fig. 2.1. Then Maxwell's equations [22] for a source free (i.e., electric charge density and conduction current density are negligible, which is the usual case for optical media) and linear (i.e., ε and μ are independent of E and H) medium can be recasted as

$$\begin{aligned}\nabla \cdot \varepsilon(x, y, z, t) E &= 0, & \nabla \times E &= -j\omega\mu(x, y, z, t) H, \\ \nabla \cdot \mu(x, y, z, t) H &= 0, & \nabla \times H &= j\omega\varepsilon(x, y, z, t) E,\end{aligned}\quad (2.2)$$

where ε and μ are the permittivity and permeability, respectively, while $\nabla = \partial/\partial x + \partial/\partial y + \partial/\partial z$. Note that the SI units (or MKS units), instead of the Gaussian units (or CGS units), are used when dealing with the Maxwell's equations in this thesis. Also, with reference to Fig. 2.1, TE (TM) polarization means that the electric (magnetic) field exists only in the y direction. Using Eq. (2.2) and assuming that ε and μ are independent of time and position in each type of medium (i.e., either waveguide core or claddings) and that the waveguide claddings are effectively infinite in thickness, the wave equation for the TE mode of the slab waveguide can be expressed as

$$\frac{\partial^2 E_y}{\partial x^2} + (k_o^2 \varepsilon_i \mu_i - \beta^2) E_y \equiv \frac{\partial^2 E_y}{\partial x^2} + (k_o^2 n_i^2 - \beta^2) E_y = 0, \quad (2.3a)$$

$$\text{with} \quad H_x = -\frac{\beta}{\omega\mu_i} E_y, \quad H_z = \frac{j}{\omega\mu_i} \frac{\partial E_y}{\partial x}, \quad E_x = E_z = H_y = 0, \quad (2.3b)$$

where E_x, E_y, E_z, H_x, H_y and H_z are, respectively, the x, y and z vectorial components of either $\tilde{E}$ or $\tilde{H}$, c is the speed of light in vacuum, $k_0 = \omega/c$ is the wave number in vacuum, $k_i = k_{core}$ ($k_i = k_{clad}$) is the wave number in the waveguide core (claddings), $\varepsilon_i = \varepsilon_{core}$ ($\varepsilon_i = \varepsilon_{clad}$) is the permittivity in the waveguide core (claddings) and $\mu_i = \mu_{core}$ ($\mu_i = \mu_{clad}$) is the permeability in the waveguide core (claddings). Likewise, the wave equation for the TM mode of the slab waveguide can be written as

$$\frac{\partial^2 H_y}{\partial x^2} + (k_o^2 \varepsilon_i \mu_i - \beta^2) H_y \equiv \frac{\partial^2 H_y}{\partial x^2} + (k_o^2 n_i^2 - \beta^2) H_y = 0, \quad (2.4a)$$

$$\text{with} \quad E_x = -\frac{\beta}{\omega \varepsilon_i} H_y, \quad E_z = \frac{j}{\omega \varepsilon_i} \frac{\partial H_y}{\partial x}, \quad E_y = H_x = H_z = 0. \quad (2.4b)$$

Note that the refractive index can be expressed as $n_i = (\varepsilon_{r,i} \mu_{r,i})^{1/2}$, where $\varepsilon_{r,i} = \varepsilon_0 / \varepsilon_i$ and $\mu_{r,i} = \mu_0 / \mu_i$, in which ε_0 and μ_0 are, respectively, the permittivity and permeability in vacuum while $\varepsilon_{r,i}$ and $\mu_{r,i}$ are, respectively, the relative permittivity and relative permeability of the waveguide core (if $i = \text{core}$) and claddings (if $i = \text{clad}$). The general solutions to the wave equation (cf. Eq. (2.3a)) of the TE mode for the waveguide core and claddings have the form of

$$E_y(x, z) \equiv E_y = \exp[j\beta z] \begin{cases} E_a \exp[-k_{\text{clad}}(x - h/2)] & (\text{for } x > h/2) \\ E_e \cos[k_{\text{core}} x h/2] + E_o \sin[k_{\text{core}} x h/2] & (\text{for } -h/2 < x < h/2), \\ E_b \exp[k_{\text{clad}}(x + h/2)] & (\text{for } x < -h/2) \end{cases} \quad (2.5a)$$

$$\text{with } k_{\text{clad}} = \sqrt{\beta^2 - \varepsilon_{\text{clad}} \mu_{\text{clad}} \omega^2} \text{ and } k_{\text{core}} = \sqrt{\varepsilon_{\text{core}} \mu_{\text{core}} \omega^2 - \beta^2}, \quad (2.5b)$$

where E_a is the coefficient constant of the evanescent waves in the upper cladding region, E_b is the coefficient constant of the evanescent waves in the lower cladding region, while E_e and E_o are, respectively, the coefficient constants of the even and odd modes in the waveguide core. The fact that the propagating modes in the waveguide core can be split into even and odd modes is a natural consequence of the symmetry of the slab waveguide structure in Fig. 2.1 (as the upper cladding and lower claddings have the same refractive index). Even (odd) mode simply implies that the field distributions in the waveguide core is a cosine (sine) function. We now solve for the even and odd modes separately by starting with the TE mode. Even TE modes are the solutions for which $E_o = 0$ so that $E_y(x, z) = E_e \exp(j\beta z) \cos(k_{\text{core}} x h/2)$ for $-h/2 < x < h/2$. Making use of the fact that E_y is continuous across the boundary at $x = \pm h/2$, we express E_e , E_o and E_b in terms of E_a for the even TE modes, with the final E_y fields being expressed as

$$E_y^{\text{even}}(x, z) \equiv E_y^{\text{even}} = E_a \exp[j\beta z] \begin{cases} \exp[-k_{\text{clad}}(x - h/2)] & (\text{for } x \geq h/2) \\ \cos[k_{\text{core}} x h_{\text{core}}/2] / \cos[k_{\text{core}} h/2] & (\text{for } -h/2 \leq x \leq h/2) \\ \exp[k_{\text{clad}}(x + h/2)] & (\text{for } x \leq -h/2) \end{cases} \quad (2.6)$$

On the other hand, the odd TE modes are the solutions for which $E_e = 0$ such that $E_y(x, z) = E_o \exp(j\beta z) \sin(k_{\text{core}} x h/2)$ for $-h/2 < x < h/2$. This results in the following E_y fields:

$$E_y^{\text{odd}}(x, z) \equiv E_y^{\text{odd}} = E_a \exp[j\beta z] \begin{cases} \exp[-k_{\text{clad}}(x - h/2)] & (\text{for } x \geq h/2) \\ \sin[k_{\text{core}} x h/2] / \sin[k_{\text{core}} h/2] & (\text{for } -h/2 \leq x \leq h/2) \\ -\exp[k_{\text{clad}}(x + h/2)] & (\text{for } x \leq -h/2) \end{cases} \quad (2.7)$$

The corresponding magnetic field H components of the even and odd TE modes can be easily obtained from Eq. (2.2) and thus will not be presented here. From Eqs. (2.6) and (2.7), and making use of the boundary condition that $H_z = j / (\omega \mu_i) \times \partial E_y / \partial x$ (cf. Eq. (2.3b)) should be continuous at the boundaries at $x = \pm h / 2$, the dispersion relations (or the eigenvalue equation) for the even and odd TE modes can be written as

$$\begin{aligned} k_{clad} h &= \frac{\mu_{clad}}{\mu_{core}} k_{core} h \tan\left(\frac{k_{core} h}{2}\right) \quad (\text{for even modes}), \\ k_{clad} h &= -\frac{\mu_{clad}}{\mu_{core}} k_{core} h \cot\left(\frac{k_{core} h}{2}\right) \quad (\text{for odd modes}). \end{aligned} \quad (2.8)$$

The above two equations can be combined into one single eigenvalue equation:

$$k_{clad} h_{core} = \frac{\mu_{clad}}{\mu_{core}} k_{core} h \tan\left(\frac{k_{core} h_{core}}{2} - \frac{q\pi}{2}\right), \quad (2.9)$$

where an even q (i.e., $q = 0, 2, 4, \dots$) gives an even mode while an odd q (i.e., $q = 1, 3, 5, \dots$) gives an odd mode. Note that q also represents the number of nodes in the field distribution $E_y(x)$ in the slab waveguide core. For instance, the fundamental mode $TE_{q=0}$ has no nodes in its $E_y(x)$ profile while the first higher-order mode $TE_{q=1}$ has one node in its $E_y(x)$ profile. Since $\mu_{r, core} = \mu_{r, clad} = \mu_0$ for conventional optical media, Eq. (2.9) reduces to

$$k_{clad} h = k_{core} h \tan\left(\frac{k_{core} h}{2} - \frac{q\pi}{2}\right). \quad (2.10)$$

The above procedures can be repeated in a similar manner for the TM modes from duality by interchanging (E_y, ε_i) and (E_a, E_b, E_e, E_o) , respectively, with (H_y, μ_i) and (H_a, H_b, H_e, H_o) in Eqs. (2.5)-(2.7). Subsequently, the dispersion relations for the even and odd TM modes can be written as

$$k_{clad} h = \frac{\varepsilon_{clad}}{\varepsilon_{core}} k_{core} h \tan\left(\frac{k_{core} h}{2} - \frac{q\pi}{2}\right), \quad (2.11)$$

where an even q (i.e., $q = 0, 2, 4, \dots$) gives an even mode while an odd q (i.e., $q = 1, 3, 5, \dots$). As mentioned earlier, $\mu_{core} = \mu_{clad} = \mu_0$ for conventional optical media. Also, note that $n_i = (\varepsilon_{r, i} \mu_{r, i})^{1/2}$. Then it is convenient to present Eq. (2.11) in the following form:

$$k_{clad} h = \frac{n_{clad}^2}{n_{core}^2} k_{core} h \tan\left(\frac{k_{core} h}{2} - \frac{q\pi}{2}\right). \quad (2.12)$$

Evidently, the above dispersion relations of the TE and TM modes of the slab waveguide are transcendental equations, which can be solved either by graphical or numerical approaches. Both approaches will later be illustrated. These transcendental equations can easily be solved by using the earlier Eq. (2.5b) to obtain a direct relation between the parameters k_{clad} and k_{core} , which is

$$k_{clad}^2 h^2 + k_{core}^2 h^2 = \omega^2 h^2 (\epsilon_{core} \mu_{core} - \epsilon_{clad} \mu_{clad}) \equiv \rho. \quad (2.13)$$

Having obtained the dispersion relations of the symmetric slab waveguide structure, we now proceed to derive the cut off conditions in terms of the waveguide dimensions and propagating frequency for the different mode orders, including the fundamental mode. To do so, first note that in order for a conventional dielectric waveguide to satisfy the guidance condition, which means that the wave energy is confined within the waveguide core and that the propagating waves in the waveguide claddings decay exponentially with distance from the waveguide core, k_{clad} and k_{core} must both be real and positive. Applying this constraint on Eq. (2.5b) will yield the following inequality for β :

$$\omega \sqrt{\epsilon_{clad} \mu_{clad}} < \beta < \omega \sqrt{\epsilon_{core} \mu_{core}} \Rightarrow n_{clad} < N_{eff} < n_{core}, \quad (2.14)$$

where $N_{eff} = \beta/k_0$ is known as the effective index of the propagating modes in the waveguide core. As N_{eff} approaches n_{clad} (i.e., β approaches $\omega(\epsilon_{clad} \mu_{clad})^{1/2}$), observe that k_{clad} will tend to zero. This means that the propagating wave outside the waveguide core is no longer evanescent and the wave energy will no longer be guided by the waveguide core. In other words, the condition $k_{clad} = 0$ will define the cut off conditions for each mode order. The cut off frequencies ω_c can be obtained by substituting $k_{clad} = 0$ into Eq. (2.13), which gives

$$k_{core} = \omega_c \sqrt{\epsilon_{core} \mu_{core} - \epsilon_{clad} \mu_{clad}}. \quad (2.15)$$

This means that the wave energy can only be guided in the waveguide if the input light with angular frequency ω fulfills the condition of $\omega > \omega_c$. The specific cut off frequencies for a given mode can be determined by substituting Eq. (2.15) and the cut off condition of $k_{clad} = 0$ into Eq. (2.10) for the TE mode and Eq. (2.12) for the TM mode. Consequently, we obtain

$$\omega_{c,q} = \frac{q\pi}{h \sqrt{\epsilon_{core} \mu_{core} - \epsilon_{clad} \mu_{clad}}} \equiv \frac{q\pi c}{h \sqrt{n_{core}^2 - n_{clad}^2}} \Rightarrow h_{c,q} = \frac{q\lambda}{2 \sqrt{n_{core}^2 - n_{clad}^2}}, \quad (2.16)$$

for both the TE and TM modes, where $h_{c,q}$ denotes the waveguide thickness that leads to cut off for the mode of order q and λ is the input wavelength. It can be observed that, for $q = 0$, which is the lowest order mode or fundamental mode, $\omega_{c,q} = 0$. This means that the fundamental mode can propagate in the symmetric slab waveguide of any thickness for both the TE and TM modes. In other words, for a symmetric slab waveguide, there will always be at least one guided mode, regardless of the waveguide thickness. This is in contrast to the case of the asymmetric waveguide (i.e., the upper and lower claddings have different refractive indices) [23], in which it is possible to achieve cut off for all the modes if the waveguide is thin enough. For waveguides based on SOI, an asymmetric waveguide typically consists of air as the upper cladding while SiO₂ forms the lower cladding. We will not be looking into the properties of such asymmetric slab waveguide because a symmetric waveguide structure with SiO₂ as the upper cladding layer has the advantage of passivating the waveguide core, as

mentioned earlier. Given a specific input waveguide, single-mode propagation occurs for the symmetric waveguide if its thickness h is smaller than $h_c(q, \lambda)|_{q=1}$. Based on Eq. (2.16), this means

$$h < h_c(q, \lambda)|_{q=1} \equiv h_{c, q=1} = \frac{\lambda}{2\sqrt{n_{core}^2 - n_{clad}^2}}, \quad (2.17)$$

for the symmetric slab waveguide.

2.2.2. Modal Characteristics

Having derived the dispersion relations, we are now ready to analyze the modal characteristics of the symmetric SOI slab waveguide at the submicron scale and at the communication wavelengths. We first employ the graphical approach to determine the eigensolutions of the transcendental dispersion equations of Eqs. (2.10) and (2.12). To analyze all the possible eigensolutions, we followed a standard waveguide analysis approach (cf. Ref. [24]) and plotted the Riemann sheets for the SOI slab waveguide. In each Riemann sheet, the vertical axis represents the real part of the normalized wave number in the waveguide cladding, $k_{clad}h$, while the left-side (right-side) of the horizontal axis represents the imaginary (real) part of the normalized wave number in the waveguide core, $k_{core}h$. The TE and TM dispersion relations are plotted in the Riemann sheet in Fig. 2.2(a) for $\lambda = 1.31 \mu\text{m}$ and in Fig. 2.2(b) for $\lambda = 1.55 \mu\text{m}$ and for three different values of $\rho(h)$ (cf. Eq. 2.13) that correspond to three different submicron waveguide dimensions of $h = 0.2 \mu\text{m}$, $0.6 \mu\text{m}$ and $1.0 \mu\text{m}$. The dashed and solid lines in the Riemann sheets correspond, respectively, to the left- and right-hand sides of the TE (TM) dispersion relations in Eq. (2.10) (Eq. (2.12)), with the left-hand side of the dispersion relation being determined by Eq. (2.13). Note that the points of intersection of the dashed and solid lines will give the eigensolutions or roots of the characteristic dispersion equations of the waveguide.

We now explain the physical significance of the roots found in the Riemann sheets. As mentioned earlier, in order for the waveguide to satisfy the guidance condition, the propagating waves in the claddings must decay exponentially with distance from the waveguide core. Thus, only the roots located in the half top of the Riemann sheet shown in Fig. 2.2(a) and 2.2(b) would give the actual physical solutions of the slab waveguide because $k_{clad} > 0$ in this region, which then brings about exponentially decay fields in the claddings. On the other hand, the roots in the half bottom of the Riemann sheets are unphysical as the fields in the claddings grow exponentially since $k_{clad} < 0$ for this region. Note that for the SOI slab waveguide in this chapter, the physical roots that are found in the half top of the Riemann sheet, as shown in Fig. 2.2(a) and 2.2(b), are confined to the right half region, in which k_{core} is purely real. This is the case for conventional dielectric waveguide structures like the SOI slab waveguide, which can only support oscillatory modes (with real k_{core}) in the waveguide core. The left half of the top region of the Riemann sheets has imaginary k_{core} that will correspond to evanescent or surface plasmon polariton (SPP) modes, which can only be supported if left-hand media

Fig. 2.2. Graphical solutions of the dispersion relation of the submicron SOI slab waveguide in Fig. 2.1(a) for the communication wavelengths of (a) 1.31 μm and (b) 1.55 μm . The vertical axis represents the real part of $k_{\text{clad}} h$ while the left (right) segment of the horizontal axis represents the imaginary (real) part of $k_{\text{core}} h$. The dashed and solid lines correspond, respectively, to the left- and right-hand sides of the dispersion relations in Eqs. (2.10) and (2.12)). The three dashed lines in each graph correspond to the SOI waveguide with different parameters of $\rho(h)$ (cf. Eq. (2.13) for definition of $\rho(h)$), which are evaluated at three different submicron waveguide dimensions of $h = 0.2 \mu\text{m}$, $0.6 \mu\text{m}$ and $1.0 \mu\text{m}$. The points of intersections of the dashed and solid lines give the modes of the slab waveguide.

(i.e. media with negative refractive index) or metal is used for the waveguide core and/or claddings [25, 26]. Since the SOI slab waveguide for our case is made up entirely of dielectric materials, no such physical solutions to such SPP modes will exist in the Riemann sheets in Figs. 2.2(a) and 2.2(b).

From the Riemann sheets in Figs. 2.2(a) and 2.2(b), we are able to observe the number of oscillatory modes that can be supported by the core of the SOI slab waveguide at the submicron scale of $h < 1 \mu\text{m}$. Up to a total of around five oscillatory modes ($q = 0$ to 4) is possible at $h \approx 1 \mu\text{m}$ for both the TE and TM modes. The number of possible modes that can be supported by the waveguide progressively decreases as h is scaled down to the submicron scale. Evidently, single-mode propagation is only possible at around $h = 0.2 \mu\text{m}$ for both $\lambda = 1.31 \mu\text{m}$ and $1.55 \mu\text{m}$. To better quantify the dimensions of single-mode condition, we have used Eq. (2.17) to calculate the cut off waveguide core height $h_{c, q=1}$ for the first higher-order mode for the entire communication wavelength span of $1.3 \mu\text{m}$ to $1.6 \mu\text{m}$. These will form the upper limit of h that gives single-mode propagation in the SOI slab waveguide. This is shown in Fig. 2.3. Note that single-mode propagation can only be

Fig. 2.3. Calculated upper limit of the waveguide thickness $h_{c, q=1}$ for single-mode propagation in the symmetric SOI slab waveguide for the communication wavelength window of $\lambda = 1.3 \mu\text{m}$ to $1.6 \mu\text{m}$.

Single-mode condition is fulfilled if the waveguide thickness h is in the range of $0 < h < h_{c, q=1}$.

achieved at a given wavelength λ for the waveguide dimension range of $0 < h < h_{c, q=1}$. As shown in Fig. 2.3, $0.20 \mu\text{m} \leq h_{c, q=1} \leq 0.25 \mu\text{m}$ for the communication wavelength span of $1.3 \mu\text{m} \leq \lambda \leq 1.6 \mu\text{m}$. This implies that, at the submicron scale, the range of h dimensions for single-mode operation for the symmetric SOI slab waveguide structure is rather limited.

In order to understand the SM condition of the SOI slab waveguide in conjunction with its polarization-dependence behaviors, we need to analyze the effective index N_{eff} or propagation constant

β for both the TE and TM modes for the different mode orders at different h , given a specific fixed wavelength. We chose to use $N_{eff} = \beta/k_0$ for this purpose for convenience as β is a large quantity. To solve for N_{eff} , the transcendental characteristic equations in Eqs. (2.10) and (2.12) must be solved numerically, which is a relatively straightforward procedure that can be carried out using established root-finding algorithms like the Newton-Raphson or the bisection methods [23], with the aid of Eqs. (2.5b) and (2.13). The N_{eff} of the TE and TM modes in the SOI slab waveguide at submicron scale are subsequently presented in Figs. 2.4(a) and 2.4(b), respectively, for $\lambda = 1.31 \mu\text{m}$ and $1.55 \mu\text{m}$. It can be

Fig. 2.4. Calculated effective index N_{eff} as a function of waveguide thickness h for the symmetric submicron SOI slab waveguide in Fig. 2.1(a) at the communication wavelengths of (a) $1.31 \mu\text{m}$ and (b) $1.55 \mu\text{m}$. For both wavelengths, the single-mode regime in the SOI slab waveguide is limited to a small region of $0 < h < \sim 0.2 \mu\text{m}$, in which polarization independent behavior is absent.

observed that the waveguide birefringence, $\Delta N_{eff} = |N_{eff(TE)} - N_{eff(TM)}|$, at any mode increases with decreasing h , where $N_{eff(TE)}$ and $N_{eff(TM)}$ are the effective indices N_{eff} of the TE and TM modes, respectively. In particular, Fig. 2.4 illustrates that as h is scaled into the submicron regime (i.e., $h < 1 \mu\text{m}$), the N_{eff} curves of the TE and TM modes of any given mode order q move away from each other, resulting in an increase of the waveguide birefringence ΔN_{eff} . Conversely, as h expands into the micron regime (i.e., $h \geq 1 \mu\text{m}$), the N_{eff} curves of the TE and TM modes approach each other such that ΔN_{eff} tends to zero at large h for any given mode order q . In other words, it is not possible to achieve PI operation (i.e., $\Delta N_{eff} = |N_{eff(TE)} - N_{eff(TM)}| = 0$) in the submicron regime, while close to PI operation is possible in the micron regime. It can also be seen that, consistent with earlier conclusions (based on Fig. 2.2), at both $\lambda = 1.31 \mu\text{m}$ and $1.55 \mu\text{m}$, the SM regime is limited to a small region of $0 < h < \sim 0.2 \mu\text{m}$, in which no PI behavior can be seen. Though we have only plotted the N_{eff} curves of $\lambda = 1.31 \mu\text{m}$ and $1.55 \mu\text{m}$, the above trends also apply to other communication wavelengths of $1.3 \mu\text{m} \leq \lambda \leq 1.6 \mu\text{m}$. Thus it is not necessary to show the N_{eff} curves of the entire communication wavelength span.

We thus conclude that it is not possible to simultaneously achieve single-mode and PI (or close to PI) operations at the submicron scale at the communication wavelengths using the symmetric SOI slab waveguide structure as the waveguide dimensions for PI operation (which has dimensions of the micron scale) do not fall within the single-mode regime (which has dimensions of the submicron scale). This motivates the use of channel and rib waveguides in the next section.

2.3. Analytical Modeling of Submicron Silicon-on-Insulator Channel and Rib Waveguides

We now look into the SOI channel and rib waveguides. Unlike the slab waveguide, which is a one-dimensional (1-D) structure, channel and rib waveguides are two-dimensional (2-D) structures. For reasons to be explained later, an accurate analysis of the modal characteristics of 2-D waveguides would require the use of rigorous numerical or computational modeling using electromagnetic algorithms, such as the beam propagation method (BPM) [20]. The use of analytical modeling via the basic Maxwell's equations can only be used as a general approximation for these 2-D waveguides. However, numerical modeling often does not give a clear physical insight as the bulk of the work in any accurate numerical modeling is about the optimization of the simulation parameter settings like the mesh size so as to ensure the stability and convergence of the numerical solutions. To address this fundamental short-coming of numerical modeling, we first employ analytical modeling to take a preliminary look into SOI channel and rib waveguides at the submicron scale. Physically speaking, the 2-D channel and rib waveguides can generally be decomposed into constituent 1-D slab waveguides. The use of analytical modeling will allow us to clearly see this relation of the channel and rib waveguides with the earlier discussed slab waveguide. In addition, the analytical results obtained will give the general trends of the modal characteristics of the SOI channel and rib waveguides. This would serve as the impetus for subsequent numerical modeling in later sections.

2.3.1. Theoretical Formulation

We employed the Effective Index Method (EIM) [12, 23] to provide an analytical modeling of the channel and rib waveguides. In the EIM, a 2-D waveguide is converted into two 1-D slab waveguides. Consider the case of the SOI channel waveguide, shown earlier in Fig. 2.1(b). By employing the EIM, as illustrated in Fig. 2.5, the SOI channel waveguide is decomposed into two slab waveguides. The first slab waveguide, denoted as SWG-1, is oriented along the x direction so that the height or thickness h of the channel waveguide is also the thickness of the slab waveguide. On the other hand, the second slab waveguide, denoted as SWG-2, is oriented along the y direction so that the width w of the channel waveguide becomes the thickness of the slab waveguide. Assuming that the input light into the channel waveguide has its electric field orientated in the y direction (i.e., TE), then the incoming mode can also be interpreted as a TE mode for the first slab waveguide SWG-1. We can then apply the characteristic dispersion equation found earlier for the TE mode for the slab waveguide in Eq. (2.10) on the first slab waveguide SWG-1. For convenience, such a dispersion equation is recasted as

$$\frac{h_{core} k_0 \sqrt{n_{core}^2 - N_{eff(1)}^2}}{2} = \arctan \left(\frac{\sqrt{N_{eff(1)}^2 - n_{clad}^2}}{\sqrt{n_{core}^2 - N_{eff(1)}^2}} \right) + \frac{q_x \pi}{2}, \quad (2.18)$$

where an even (odd) q_x gives an even (odd) mode in the x direction and $N_{eff(1)}$ represents the effective index of the TE mode of the first slab waveguide SWG-1. This newly obtained $N_{eff(1)}$ can be seen as the

Fig. 2.5. The Effective Index Method (EIM) for the channel waveguide. The channel waveguide is decomposed into two constituent slab waveguides — SWG-1 and SWG-2. The effective index $N_{eff(1)}$ of SWG-1 is first calculated, which will form the refractive index of the waveguide core for SWG-2. Solving the effective index $N_{eff(2)}$ of SWG-2 gives the final effective index of the channel waveguide.

refractive index of the waveguide core of the second slab waveguide SWG-2. By so doing, the interaction between SWG-1 and SWG-2 is taken into account. For the second slab waveguide SWG-2, the input light is interpreted as a TM mode. We thus use the TM dispersion equation for the slab waveguide in Eq. (2.10) on SWG-2, which is re-casted here as

$$\frac{h_{core} k_0 \sqrt{N_{eff(1)}^2 - N_{eff(2)}^2}}{2} = \arctan \left(\frac{N_{eff(1)}^2 \sqrt{N_{eff(2)}^2 - n_{clad}^2}}{n_{clad}^2 \sqrt{N_{eff(1)}^2 - N_{eff(2)}^2}} \right) + \frac{q_y \pi}{2}, \quad (2.19)$$

where an even (odd) q_y gives an even (odd) mode in the y direction. Note that the refractive index of the waveguide core of the second slab is equivalent to $N_{eff(1)}$ of SWG-1 while $N_{eff(2)}$ is used to represent the effective index of the TM mode of SWG-2. This parameter of $N_{eff(2)}$ is also the *net effective index* of the incoming TE mode of the original channel waveguide under consideration. For the case in which the incoming light into the channel waveguide is a TM mode, the above procedure can also be repeated, except that the incoming light is viewed as TM for the first slab waveguide SWG-1 and TE for the second slab waveguide SWG-2.

The EIM can be applied on the rib waveguide in a similar manner as the channel waveguide. As illustrated in Fig. 2.6, we decompose the rib waveguide into two groups of slab waveguides. The first

Fig. 2.6. The Effective Index Method (EIM) for the rib waveguide. The rib waveguide is first decomposed into three constituent slab waveguides — SWG-1A, SWG-1B and SWG-1C. The effective indices of SWG-1A, SWG-1B and SWG-1C denoted, respectively, as $N_{eff(1A)}$, $N_{eff(1B)}$ and $N_{eff(1C)}$ are then calculated, which will form the refractive indices of the waveguide core and claddings for SWG-2. Solving the effective index $N_{eff(2)}$ of SWG-2 gives the final effective index of the rib waveguide.

group is composed of three slab waveguides that are all oriented along the x direction. These waveguides are labeled as SWG-1A, SWG-1B and SWG-1C. On the other hand, there is only one slab waveguide in the second group. This slab waveguide is oriented along the y direction and is labeled as SWG-2. We then calculate the effective indices of the first set of slab waveguides SWG-1A, SWG-1B and SWG-1C. Assuming that the input light is TE polarized for the rib waveguide, the dispersion equations for SWG-1A, SWG-1B and SWG-1C can be expressed as follows using the TE dispersion equation for a typical slab waveguide in Eq. (2.10):

$$\frac{(h_{core} - d)k_0\sqrt{n_{core}^2 - N_{eff(1A)}^2}}{2} = \arctan\left(\frac{\sqrt{N_{eff(1A)}^2 - n_{clad}^2}}{\sqrt{n_{core}^2 - N_{eff(1A)}^2}}\right) + \frac{q_x\pi}{2}, \quad (2.20)$$

$$\frac{h_{core}k_0\sqrt{n_{core}^2 - N_{eff(1B)}^2}}{2} = \arctan\left(\frac{\sqrt{N_{eff(1B)}^2 - n_{clad}^2}}{\sqrt{n_{core}^2 - N_{eff(1B)}^2}}\right) + \frac{q_x\pi}{2}, \quad (2.21)$$

$$\frac{(h_{core} - d)k_0\sqrt{n_{core}^2 - N_{eff(1C)}^2}}{2} = \arctan\left(\frac{\sqrt{N_{eff(1C)}^2 - n_{clad}^2}}{\sqrt{n_{core}^2 - N_{eff(1C)}^2}}\right) + \frac{q_x\pi}{2}, \quad (2.22)$$

where we have used $N_{eff(1A)}$, $N_{eff(1B)}$ and $N_{eff(1C)}$ to represent the effective indices of the TE mode of the slab waveguides SWG-1A, SWG-1B and SWG-1C, respectively. Note that Eqs. (2.20) to (2.22) are solved independently of one another by numerical methods, such as the Newton-Raphson or the bisection methods [23]. The dimension $(h - d)$ of the rib waveguide forms the thickness of the slab waveguides SWG-1A and SWG-1C, while the dimension h of the rib waveguide will form the thickness of the slab waveguide SWG-1B. The above obtained parameters of $N_{eff(1A)}$, $N_{eff(1B)}$ and $N_{eff(1C)}$ will subsequently form the refractive indices of the upper waveguide cladding, waveguide core and lower waveguide cladding, respectively, of the second set of slab waveguide SWG-2, where $N_{eff(1A)} = N_{eff(1C)}$, since the two claddings are composed of the same material in our SOI waveguides. Using this condition, we then utilize the TM dispersion equation for a typical slab waveguide in Eq. (2.12), which is expressed as

$$\frac{h_{core}k_0\sqrt{N_{eff(1B)}^2 - N_{eff(2)}^2}}{2} = \arctan\left(\frac{N_{eff(1B)}\sqrt{N_{eff(2)}^2 - N_{eff(0)}^2}}{N_{eff(0)}\sqrt{N_{eff(1B)}^2 - N_{eff(2)}^2}}\right) + \frac{q_y\pi}{2}, \quad (2.23)$$

where $N_{eff(0)} = N_{eff(1A)} = N_{eff(1C)}$ and $N_{eff(2)}$ is the effective index of the slab waveguide SWG-2. The *final effective index* of the TE mode of the original rib waveguide will be given by $N_{eff(2)}$. For the case in which the input light is TM polarized, the above procedures can also be repeated by using the TM

dispersion equation for the slab waveguide on SWG-1A, SWG-1B and SWG-1C and the TE dispersion equation for the slab waveguide on SWG-2.

2.3.2. Modal Characteristics

We now proceed to use the analytical models established by the EIM to deduce the general trends of the modal characteristics of SOI channel and rib waveguides at the submicron scale for $\lambda = 1.55 \mu\text{m}$. These trends also apply to other communication wavelengths of $1.3 \mu\text{m} \leq \lambda \leq 1.6 \mu\text{m}$ and thus the results for other wavelengths will not be shown for brevity.

(a) Channel Waveguide

Using Eqs. (2.18) and (2.19), the effective indices of the TE and TM modes of the channel waveguide are plotted in Fig. 2.7 as a function of the waveguide height h for four different waveguide width w of $0.3 \mu\text{m}$, $0.4 \mu\text{m}$, $0.5 \mu\text{m}$ and $0.6 \mu\text{m}$, all for the case of $\lambda = 1.55 \mu\text{m}$. Note that we will use the notations TE_0 and TM_0 for the TE and TM fundamental modes and the notations TE_1 and TM_1 for the first higher-order TE and TM modes throughout this thesis. Some general observations based on these results are summarized below.

Fig. 2.7. Calculated effective index N_{eff} as a function of waveguide thickness h for the symmetric submicron SOI channel waveguide in Fig. 2.1(b) at the communication wavelength of $1.55 \mu\text{m}$ at fixed waveguide width of (a) $0.3 \mu\text{m}$, (b) $0.4 \mu\text{m}$, (c) $0.5 \mu\text{m}$ and (d) $0.6 \mu\text{m}$. The points of intersection of the N_{eff} curves mark the polarization-independent (PI) waveguide dimensions as $\Delta N_{\text{eff}} = |N_{\text{eff}}(\text{TE}) - N_{\text{eff}}(\text{TM})| = 0$ gives PI operation.

- The upper limit of the SM regime (in terms of h) can be adjusted by the waveguide width w . In general, the upper limit for SM condition increases with decreasing w . This can be observed in Fig. 2.7, in which the cut off waveguide thickness h for the first higher-order mode ($q_x, q_y = 0, 1$) increases from $\sim 0.34 \mu\text{m}$ to $\sim 0.37 \mu\text{m}$, $\sim 0.44 \mu\text{m}$ and $\sim 0.69 \mu\text{m}$ when w decreases from $0.6 \mu\text{m}$ to $0.5 \mu\text{m}$, $0.4 \mu\text{m}$ and $0.3 \mu\text{m}$. The channel waveguide is thus a more feasible candidate for integrated-optics applications at the submicron scale as compared to the slab waveguides (cf. section 2.2.2, Fig. 2.4), in which the upper limit of single-mode condition is fixed at $\sim 0.20 \mu\text{m}$ to $0.25 \mu\text{m}$ for the communication wavelengths.
- The polarization dependence of the waveguide for different mode orders depends on its aspect ratio (i.e., ratio of w to h). In general, for the fundamental order mode, polarization-independent (PI) operation (i.e. $\Delta N_{\text{eff}} = |N_{\text{eff}(\text{TE})} - N_{\text{eff}(\text{TM})}| = 0$) occurs when the channel waveguide is square-like in appearance, i.e., when the aspect ratio is $\sim$ one (i.e., $w = h$). As the channel waveguide becomes rectangular-like, the polarization dependence (ΔN_{eff}) increases.
- In contrast to the SOI slab waveguide, it is now possible to simultaneously achieve both SM and PI conditions at the submicron scale for the SOI channel waveguide by manipulating its aspect ratio to engineer the dimensions for PI operation to fall within the SM regime. As observed from Fig. 2.7, both SM and PI operations are achieved when the waveguide height h that results in cut off for the first higher-order mode is larger than the width w of the waveguide. This occurs when the waveguide width w is of $w \leq 0.4 \mu\text{m}$ for $\lambda = 1.55 \mu\text{m}$.

(b) Rib Waveguide

Next, we analyze the modal characteristics of the rib waveguide. Using Eqs. (2.20) to (2.23), the effective indices of the TE and TM modes of the fundamental order ($q_x, q_y = 0, 0$) and the first highest order ($q_x, q_y = 0, 1$) of the rib waveguides are plotted in Fig. 2.8 for the case of $\lambda = 1.55 \mu\text{m}$. Unlike the channel waveguide in Fig. 2.7, here the N_{eff} plots are presented as a function of the waveguide width w for four different combinations of waveguide height h and etch depth d so as to better illustrate the use of the etch depth as an additional control variable. It can be seen from Fig. 2.8 that, with the rib waveguide, it is more flexible to control the single-mode condition as there are now three geometrical settings — the etch depth, height and width — that can be used to control the overall design of the waveguide as compared to the slab and channel waveguides. However, the results of the analytical modeling of the rib waveguide in Fig. 2.8 shows that PI operation is not possible at any dimensions. This is a fundamental difference from that of the channel waveguide in section 2.3.2(a), whereby PI operation is possible when the waveguide height and width are equal or close to each other. In fact, we can observe that the rib waveguide behaves somewhat like a slab waveguide: The N_{eff} of the TE and TM modes approach each other as the dimensions of the rib waveguide increases.

Fig. 2.8. Calculated effective index N_{eff} as a function of waveguide width w for the symmetric submicron SOI rib waveguide in Fig. 2.1(c) at the communication wavelength of $1.55 \mu\text{m}$ at fixed waveguide height h and etch depth d of (a) $(h, d) = (0.4 \mu\text{m}, 0.2 \mu\text{m})$ (b) $(h, d) = (0.6 \mu\text{m}, 0.3 \mu\text{m})$, (c) $(h, d) = (0.8 \mu\text{m}, 0.4 \mu\text{m})$ and (d) $(h, d) = (1.0 \mu\text{m}, 0.5 \mu\text{m})$.

2.3.3. Basic Limitations of the Analytical Modeling

The reason why PI operation is not observed for the above 2-D rib waveguide is due to the inherent limitations of the analytical modeling. These limitations will also affect the modal characteristics of the channel and slab waveguides, although the impact is much smaller as compared to the rib waveguides. In general, the limitations of the analytical modeling can be summarized below.

- The finite thickness of the claddings is not taken into consideration when using the Maxwell's equations to derive the dispersion relations for the different SOI waveguides in the analytical modeling of sections 2.2 and 2.3. The assumption that the claddings is of infinite thickness is only valid if the evanescent waves do not penetrate very far into the claddings level. However, at the submicron scale, the evanescent waves do extent considerably into the cladding layers. In fact, it has been shown that the thickness of the waveguide claddings can be used to adjust the waveguide dimensions for SM condition [27] and PI behavior [28].
- For the 2-D waveguides (rib and channel), the propagation losses (in terms of the substrate mode leakage losses and lateral mode leakage losses) can determine the final modal characteristics (in terms of whether they are single-mode (SM) or multi-mode (MM)) of the waveguides. In particular, for the rib waveguide, the higher-order modes under the rib can leak into the slab region as the light propagates, leaving only the fundamental mode in the rib waveguide core. This is not taken into account in the analytical modeling.

- For pure TE (TM) mode, the field component $E_z = 0$ ($H_z = 0$); only the E_y (H_y) field component is present. However, strictly speaking, for 2-D waveguides, the modes are not pure but are hybrid [22, 23] as all 6 vectorial components of the electric (E_x , E_y , E_z) and magnetic (H_x , H_y , H_z) fields are always present. Thus, each hybrid mode inherently exhibits both E_x and E_y electric field components and is described as quasi-TE or quasi-TM, depending on which E field component is dominant. For the fundamental quasi-TE (TM) mode, the E_y (E_x) field is the dominant or majority field, while the E_x (E_y) field is the minority field component. The ratio of the minority field to the majority field is defined as the degree of modal hybridness. The hybrid nature of such mode is not taken into account in the analytical modeling, which we have assumed pure TE or TM mode. For convenience, the terms quasi-TE and quasi-TM modes will be used interchangeably with the terms TE and TM modes when dealing with submicron 2-D waveguides throughout this thesis.

In the next section, rigorous numerical modelings, which take into consideration of the above shortcomings, are used to determine the single-mode condition and polarization-dependence behavior of the SOI submicron waveguides. We will only be focusing on the SOI channel and rib waveguides as we have found that the above-mentioned limitations of analytical modeling will not severely affect the results and conclusion obtained earlier (in section 2.2.2) for the SOI slab waveguides at the submicron scale. Moreover, slab waveguides, being 1-D in nature, has less applications as compared to 2-D channel and rib waveguides and are thus not widely used in photonic integrated circuits. Finally, note that despite of the above-mentioned limitations, the analytical modeling is still useful in that it serves to provide a physical insight of the waveguides, which is not possible if numerical modeling is directly employed. Also, if we later compare the results of the numerical modeling in section 2.4 with the results of the analytical modeling in this section, one can observe similarities in the general trends of the modal characteristics of the 2-D SOI waveguides, particularly for the channel waveguide. Thus, the analytical modeling in this section is useful as an approximation tool-kit.

2.4. Numerical Modeling of Submicron SOI Channel and Rib Waveguides

2.4.1. Methodology in Numerical Modeling

(a) General Simulation Set Up

Our numerical modeling or simulation is performed using the 3-D imaginary distance beam propagation method (IDBPM) [20]. This is an alternative and significantly faster approach than the correlation method used in [11]. The IDBPM simulation is set up for both the SOI channel and rib waveguides with the configurations shown earlier in Fig. 2.1. The waveguide core is based on silicon (Si) whereas the upper and lower claddings are based on silica (SiO_2). SOI channel waveguides with height $h = 0.2 \mu\text{m}$ to $0.6 \mu\text{m}$ are analyzed, while rib waveguides are only analyzed at $h = 0.3 \mu\text{m}$ and

0.4 μm . Such choice of h is typically used for submicron optical devices and would also match the Si core with existing modulator dimension for a homogeneous integration [14] in photonic integrated circuits. The thicknesses of the upper and lower claddings, which are termed, respectively, as l_1 and l_2 , affect the modal characteristics in SOI waveguides. In our simulation, we define our cladding and substrate SiO_2 layers for the SOI waveguides to be symmetrical and set $l_1 = l_2 = l$. It is worth mentioning that the waveguide loss is strongly dependent on l_2 due to substrate leakage and has been reported elsewhere [29]. Thus, in our simulation, we only consider $l = l_2 \geq 0.7 \mu\text{m}$ for its low substrate leakage loss. We have mentioned earlier that in 2-D waveguides, hybrid modes (quasi-TE or quasi-TM), instead of pure modes (TE or TM) are present. Each such hybrid mode can be resolved into its constituent TE and TM components. This hybrid nature can be described using the following full-vectorial Helmholtz equations [22]:

$$\begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = q^2 \begin{bmatrix} E_x \\ E_y \end{bmatrix}, \quad (2.24)$$

where q^2 is the eigenvalue and $D_{xx} \dots D_{xy}$ are the differential operators. From Eq. (2.24), it can be seen that each hybrid mode is polarization dependent ($D_{xx} \neq D_{yy}$) and polarization coupled ($D_{xy}, D_{yx} \neq 0$). The degree of modal hybridness (cf. section 2.3.3 for definition) generally increases with decreasing waveguide dimensions. For large cross-sectional rib structures, the degree of hybridness is substantially small such that the minority field components are generally diminutive and can be neglected, giving $D_{xy} = D_{yx} = 0$. This allows us to decouple the transverse field components and approximate Eq. (2.24) as two semi-vectorial equations: $D_{xx}E_x = q_x^2E_x$ and $D_{yy}E_y = q_y^2E_y$. Solving these equations yields semi-vectorial TE and TM modes, each consisting of only one field component in the plane transverse to the z axis. For submicron 2-D waveguides, in which the dimensions shrink below the operating wavelength ($\lambda = 1.31 \mu\text{m}$ or $1.55 \mu\text{m}$), the degree of hybridness is significantly high and so the minority components cannot be considered as negligible as in the case of large rib structures, i.e., D_{xy} and $D_{yx} \neq 0$. The smaller waveguide cross-section also implies the increased influence of the boundary conditions. This results in the hybrid modes to be strongly polarization dependent [1, 11]. Therefore, for accuracy in results, the full-vectorial method was employed in our numerical simulation, though this has substantially increased the computational resources and time. In addition to this, computational parameters in the x , y and z directions were carefully optimized to include the entire waveguide structure into the simulation domain and to allow the mode field to be sufficiently close to zero at the boundaries. Also, the simulation parameters, especially the mesh size, were carefully set to ensure that the stringent boundary conditions in submicron waveguides do not affect the accuracy of the simulated results. Upon a rigorous mesh optimization exercise, in which different mesh size settings are tested for stability and convergence of the numerical modal solutions, we have employed a grid mesh of $0.005 \mu\text{m}$ uniformly in all three directions (x, y, z) so that numerical

errors are minimal. To ensure further accuracy in the simulation results, we have also used the finite difference method (FDM) [30] to verify the results obtained from the above-mentioned IDBPM. Results calculated using the IDBPM are shown to be in good agreement with those using the FDM.

(b) General Simulation Procedures

In the simulation, the effective indices N_{eff} are computed for both the TE and TM modes at different waveguide dimensions for $\lambda = 1.31\mu\text{m}$ and $\lambda = 1.55\mu\text{m}$. To do so, a Gaussian field was chosen to approximate an optical fiber mode and it is launched off-centre at one third of the waveguide width w and height h in order to excite all the possible modes supported by the structure. The propagation length was chosen to be 10 mm, which is long enough to model the effects of substrate leakage loss and lateral slab leakage loss of all the modes. Whether the waveguide is SM or multi-mode will depend on the number of waveguide modes that remain in the waveguide core after

Fig. 2.9. Illustration of our modal simulation in small SOI rib waveguides using IDBPM, which is based on Soref et al.'s model [3]. In our simulation, at $z = 0$, a Gaussian mode is launched off centre to excite the propagation of fundamental and higher order modes. If the designed waveguide is single-mode (SM), the higher-order modes under the rib will leak into the slab regions as the light propagates, leaving only the fundamental mode remaining in the rib waveguide at $z = 10\ 000\ \mu\text{m}$.

the propagation length. This concept is illustrated in Fig. 2.9 for a SM rib waveguide. Note that mode-solving in the IDBPM is iterative in nature; the method will automatically stop if the modal solutions converge before the specified length of $L = 10$ mm. To expedite the computation time, the number of computed modes is restricted to two modes — the fundamental mode, with order $(q_x, q_y) = (0, 0)$ and the first higher order mode, with order $(q_x, q_y) = (0, 1)$ — for both polarizations (TE and TM). Note that for simplicity, we will use the notations TE_0 and TM_0 to represent TE_{00} and TM_{00} while the notations TE_1 and TM_1 will be used to represent TE_{01} and TM_{01} throughout this thesis.

2.4.2. Submicron Channel Waveguides

(a) Single-Mode Condition

The single-mode (SM) analysis for submicron channel waveguides (cf. Fig. 2.1(b)) is carried out for Si overlayer or core of $h = 0.2$ μm to 0.6 μm , with $l_1 = l_2 = l = 0.75$ μm . The simulation is carried out at both wavelengths at 1.31 μm and 1.55 μm . In the simulation, whereby the field shape and effective index of the propagating modes are computed, the waveguide height (h) is fixed while the width (w) is increased in steps of 0.01 μm until the next higher-order mode is supported by the waveguide structure. When no effective index is being computed by the IDBPM, it indicates single-mode cut off condition. If the waveguide structure is single-mode, only the effective index of the fundamental mode ($q = 0$) will be computed by the IDBPM. Otherwise, a second higher order mode ($q = 1$) will be present and its effective index will be computed, indicative of multi-mode. Thus, the transition boundary between the single-mode cut off and single-mode regimes can be computed by determining the minimum waveguide width, in which no mode is allowed to guide in the waveguide. Likewise, the transition boundary between the single-mode and multi-mode regimes can be computed by determining the minimum waveguide width, whereby the first-order mode is allowed to guide in the waveguide. In these ways, a matrix of waveguide dimensions that fulfill the single-mode condition (SMC) can be obtained for the submicron SOI channel waveguides.

Fig. 2.10 shows the simulated results for SMC at the wavelengths of 1.31 μm and 1.55 μm for the submicron SOI channel waveguide. The single-mode region is governed by two distinct conditions; (1) TE_1 cut off (upper limit) and (2) TM fundamental mode (lower limit) due to the differing mode shape of different polarizations. Both TE and TM polarizations are considered because the analysis will be later extended to solve for zero birefringence (i.e., PI operation, with $\Delta N_{eff} \approx 0$). If we compare the waveguide dimension of $w \times h = 0.3$ $\mu\text{m} \times 0.35$ μm , the result indicates that the waveguide is single-mode at 1.55 μm , with a cut off wavelength longer than 1.31 μm ; this implies that SMC is more relaxed at a longer wavelength. To verify the accuracy of our simulation, we compared our result with existing experiment data, and plot these data as circles indicated in Fig. 2.10(b). We can see that the experiment results fall into their respective region as proposed by various authors [17-19]. Also, if we compare the results in [21] at $h = 0.3$ μm and $\lambda = 1.55$ μm (indicated as squares in

(a)

(b)

Fig. 2.10. Boundary lines for single-mode condition cut off dimensions as a function of SOI channel waveguide dimensions for operating wavelengths of (a) $1.31 \mu\text{m}$ and (b) $1.55 \mu\text{m}$. Circles represent experimental data from [17-19] and squares represent results from [21].

Fig. 2.10(b)) with our simulated results, we can note a high degree of similarity. Their results only specify single-mode condition (SMC) in TE polarization. From Fig. 2.10, it can be noted that the TE polarization is of a more stringent criterion. This is due to the TE polarization having two close

boundaries where the tangential E field must be met. If the boundaries of TM_0 and TE_1 cut offs are rigorously fitted, we can obtain an equation that describes the SMC at wavelength $1.55 \mu\text{m}$ as follows:

$$\lambda = 1550 \text{ nm}$$

$$0.2 + 162e^{-(\text{Height}/0.03)} \leq \text{Width} \leq 0.3 + 5.9e^{-(\text{Height}/0.08)} \quad (2.25)$$

Eq. (2.25) describes the width (in nm) and height (in nm) for submicron SOI channel waveguides to operate in single-mode condition at $\lambda = 1.55 \mu\text{m}$. Different wavelengths can be solved using the same approach.

(b) Waveguide Birefringence

In this section, we look into the waveguide birefringence of the submicron SOI channel waveguides to determine the zero birefringence condition (ZBC), which will give PI operation. The waveguide birefringence of the modes is due to different alignment of the E -fields of the TE and TM modes. The index difference between the guiding and cladding regions in the vertical x direction is much smaller than the lateral y direction. As a result, the TE and TM modes have difference effective indices. The effective indices of the fundamental TE and TM modes for different width and height are first computed. Subsequently, the waveguide birefringence ΔN_{eff} can be obtained from the difference between the fundamental TE and TM modes ($\Delta N_{\text{eff}} = N_{\text{eff(TE)}} - N_{\text{eff(TM)}}$). Graphical representations of ΔN_{eff} for different waveguide dimensions for $\lambda = 1.31 \mu\text{m}$ and $\lambda = 1.55 \mu\text{m}$ are shown in Fig. 2.11.

The results in Fig. 2.11 show that the condition for $N_{\text{eff(TE)}} = N_{\text{eff(TM)}}$ is possible by using an appropriate combination of waveguide height and width. The points at which the birefringence curves intersect the horizontal axis indicate the zero birefringence condition (ZBC). For each height h , there is a possible width w for the waveguide to fulfill ZBC. In particular, observe that the dimensions that give ZBC occur when the aspect ratio of the channel waveguide is close to one ($w \approx h$). This matches closely with our earlier analytical modeling of the channel waveguide (cf. Fig. 2.7), in which ZBC or PI operation occurs at $\sim w = h$ for the fundamental modes. We can thus conclude that the trends based on the approximations given by the earlier analytical modeling of the channel waveguides are valid. Also, it can be deduced that the effects of finite cladding thickness and propagation losses, which are only taken into account in this section but not in the earlier analytical modeling, serve to only slightly alter the aspect ratio, whereby ZBC occurs in the submicron SOI channel waveguides.

From the birefringence curves in Fig. 2.11, it can be observed that small variations in the channel waveguide dimensions result in a considerable change in the birefringence. For example, a $\pm 10 \text{ nm}$ uncertainty on waveguide width gives a birefringence of 0.02 for channel waveguide height h of 300 nm. If we compare the birefringence at this height of 300 nm to a larger height of 600 nm, we notice that the birefringence increases by an order of magnitude to 0.002. This clearly demonstrates the strong polarization dependency of the submicron SOI channel waveguides.

Fig. 2.11. Effective index difference between TE and TM polarized modes for different SOI channel waveguide dimensions with operating wavelengths of (a) 1.31 μm and (b) 1.55 μm . Note that the H values are in nm. Here, H also means h , which is the symbol used to denote the height of the silicon core of the channel waveguide in the discussions.

(c) Satisfying Both Single-Mode and Zero-Birefringence Conditions

We have just illustrated that SMC and ZBC can be achieved individually in submicron SOI channel waveguides. Can we achieve both conditions simultaneously? If we extract the ZBC points in Fig. 2.11 and form a locus of ZBC for different waveguide dimensions, we effectively obtain a matrix of SOI channel waveguide dimensions that can satisfy this condition. This ZBC locus is superimposed on the SMC in Fig. 2.10 for both 1.31 μm and 1.55 μm wavelengths and shown in Fig. 2.12. Hence, we now acquire a set of device parameters that signify both SMC and ZBC operations.

Fig. 2.12 show plots of both SMC and ZBC loci at wavelengths of 1.31 μm and 1.55 μm . In order to satisfy both requirements of SMC and ZBC, we need to choose a point on the ZBC locus where it intersects within the single-mode region. These results give us a clear indication of the device dimensions where SMC and ZBC are found. At longer wavelengths, SOI channel waveguides that are able to satisfy both conditions simultaneously can occur at larger dimensions, thus, relaxing the fabrication constraints. Moreover, coupling of light at the submicron scale into small waveguides may prove to be difficult. It is interesting to note that the intersection between the birefringence loci and the TE_1 cut off boundary line determines the maximum SOI channel waveguide height that allows both conditions to be achieved.

Fig. 2.12. Single-mode condition (SMC) boundaries and zero-birefringence condition (ZBC) loci as a function of SOI channel waveguide dimensions at operating wavelengths of (a) 1.31 μm and (b) 1.55 μm .

2.4.3. Submicron Rib Waveguides

(a) Single-Mode Condition

Single-mode analysis for the SOI rib waveguide (cf. Fig. 2.1(c)) uses the same approach as described in section 2.4.1. The single-mode analysis for the submicron rib waveguide is restricted to h

$= 0.3 \mu\text{m}$ and $0.4 \mu\text{m}$ for the Si core and are carried out at both wavelengths of $1.31 \mu\text{m}$ and $1.55 \mu\text{m}$. In our work, the etch depth d is used as a control parameter to eliminate the presence of higher-order modes via differential slab leakage losses between the fundamental mode and higher-order modes. This is based on the fact that the slab leakage losses for the higher-order mode are higher than that of the fundamental mode with decreasing d due to the coupling of higher-order modes into the fundamental slab mode of the rib lateral regions. The loss of each computed mode is associated with $\text{Im}(N_{\text{eff}})$, which is used to determine whether it is considered as guided or cut off. A waveguide is considered as single-mode (SM) if there is less than 1 dB/cm in loss for the fundamental mode and at least ten times larger loss for the first higher-order mode so that at the end of the propagation length, only the fundamental mode remains (cf. Fig. 2.9). If not, it would be considered multimode (MM) as the loss of the higher-order mode would then be too low for it to be considered as well-suppressed.

In the simulation, for each waveguide height (h) and etch depth (d), the width (w) is increased in steps of $0.01 \mu\text{m}$ until the next higher-order mode is supported by the waveguide structure. Through this process, the minimum w dimension at which the first higher-order mode starts to be supported by the waveguide is obtained. This sets the SM/MM boundary for different d and h . Our simulation results for rib waveguides with h of $0.3 \mu\text{m}$ and $0.4 \mu\text{m}$ at $1.31 \mu\text{m}$ and $1.55 \mu\text{m}$ are presented in Figs. 2.13 and 2.14. It can be noted that a gradual increase of the etch depth d at $0.3 \mu\text{m}$ at a wavelength of $1.31 \mu\text{m}$ creates a wider region for single-mode condition. For $h = 0.3 \mu\text{m}$, $d = 0.28 \mu\text{m}$, the range of w increases from $0.18 \mu\text{m}$ to $0.31 \mu\text{m}$. If we compare with d at $0.2 \mu\text{m}$, this is a significant increase of w by 325%. On the contrary, for $h = 0.4 \mu\text{m}$, the range of w needed to satisfy the single-mode condition decreases by 25%, implying that the single-mode condition becomes more stringent with increasing d at larger h . A similar trend is observed at both $h = 0.3 \mu\text{m}$ and $h = 0.4 \mu\text{m}$ for wavelength $1.55 \mu\text{m}$ as shown in Fig. 2.14. This suggests the increase of the etch depth to achieve a broader region of single-mode operation at $1.55 \mu\text{m}$. The differences between SMC for different polarizations at submicron waveguides are significant as shown in Figs. 2.13 and 2.14 as compared to [11] due to the dominance of the boundary conditions for both TE and TM polarizations at the submicron scale. However, the propagation loss increases at these waveguides. It is well-known that the mode-field intensity is strongly enhanced at the waveguide edges with increasing waveguide dimensions, causing increased light scattering due to sidewall roughness. However, with advances in fabrication technology, it is now possible to reduce the sidewall roughness [31, 32]. This will enhance the viability of using submicron waveguides for practical applications.

(a)

(b)

Fig. 2.13. Boundary lines for single-mode condition cut off dimensions for submicron SOI rib waveguides with (a) $h = 0.3 \mu\text{m}$ and (b) $h = 0.4 \mu\text{m}$, where h is the height of the silicon core. The input wavelength is $1.31 \mu\text{m}$. The shaded region indicates single-mode condition for both TE and TM polarizations.

(a)

(b)

Fig. 2.14. Boundary lines for single-mode condition cut off dimensions for submicron SOI rib waveguides with (a) $h = 0.3 \mu\text{m}$ and (b) $h = 0.4 \mu\text{m}$, where h is the height of the silicon core. The input wavelength is $1.55 \mu\text{m}$. The shaded region indicates single-mode condition for both TE and TM polarizations.

(b) Waveguide Birefringence

The waveguide birefringence for the rib waveguides is computed in a similar manner as those of the channel waveguides described earlier. Figs. 2.15(a) to (d) are the graphical representation of the final birefringence curves. It can be observed that, for $h = 0.3 \mu\text{m}$, the birefringent curves do not cross the horizontal axis. The discontinuities of the birefringence curves suggest that beyond the critical dimension, no mode is allowed to propagate; hence there will be no equivalent effective indices. Therefore, at $h = 0.3 \mu\text{m}$, the graphs clearly indicate that no possible zero birefringence condition (ZBC) is present for this structure for both wavelengths of $1.31 \mu\text{m}$ and $1.55 \mu\text{m}$. Similarly, for $h = 0.4 \mu\text{m}$, ZBC is not possible at wavelength $1.31 \mu\text{m}$ as shown in Fig. 2.15 (c).

Fig. 2.15 (d) shows the birefringence curves for $h = 0.4 \mu\text{m}$ at an input wavelength of $1.55 \mu\text{m}$. Here, a critical width w_c is defined where the effective indices of TE and TM polarizations are equal. At $d = 0.36 \mu\text{m}$ and $d = 0.38 \mu\text{m}$, the birefringence curves cross the horizontal axis and continue to extend into the negative region. Above the critical width, the TE mode has a higher effective index as most power is confined under the rib region. This allows the higher modes to couple to the slab region. If the width is reduced, the effective indices of the TE and TM modes become similar. Further reducing the width beyond the critical width increases the effective index of TM modes, thus causing the effective index difference to become negative. The critical width corresponds to the point where the mode profile is nearly symmetrical to both TE and TM polarizations, thereby leading to a ZBC waveguide. Therefore, we can see that, at the submicron scale, realizing ZBC in SOI rib waveguides is more difficult than that of SOI channel waveguides (compare Fig. 2.15 with Fig. 2.11). This has been also shown in the analytical modeling of the SOI rib waveguides earlier in section 2.3.2(b) (cf. Fig. 2.8). Also, comparing the results in this section to that based on analytical modeling (cf. Fig. 2.8), in which ZBC was not possible at any waveguide dimensions, one can also conclude that the effects of finite cladding thickness and propagation losses (which are modeled in this section, but not in the analytical modeling) change the birefringence of the SOI rib waveguide such that ZBC can be achieved at least in a limited range of waveguide etch depth d for $h = 0.4 \mu\text{m}$ at $\lambda = 1.55 \mu\text{m}$.

Fig. 2.15. Birefringence curves as a function of waveguide width and depth. The effective index difference of the TE and TM fundamental modes at various etch depth d or D for different wavelengths and silicon core thickness h : (a) Wavelength $1.31 \mu\text{m}$, $h = 0.3 \mu\text{m}$, (b) wavelength $1.31 \mu\text{m}$, $h = 0.4 \mu\text{m}$, (c) wavelength $1.55 \mu\text{m}$, $h = 0.3 \mu\text{m}$, and (d) wavelength $1.55 \mu\text{m}$, $h = 0.4 \mu\text{m}$.

(c) Satisfying Both Single-Mode and Zero Birefringence Conditions

To identify SOI rib waveguides which are able to fulfill SMC and ZBC simultaneously, the zero birefringence points in Fig. 2.15(d) have to be located first. If we now plot the loci of the points that cross the ZBC axis and combine it with the SMC mode-map in Fig. 2.14(b), we can demonstrate that both conditions can be met under certain circumstances. It is to be noted that this exercise is only applicable for rib waveguides with $h = 0.4 \mu\text{m}$ at $1.55 \mu\text{m}$ wavelength. For analyses of other rib structures done in sections 2.4.3(a) and 2.4.3(b), even though SMC exists in most cases, ZBC is largely absent.

Fig. 2.16 shows the plots of both SMC and the birefringence-free locus for $h = 0.4 \mu\text{m}$ at a wavelength of $1.55 \mu\text{m}$. Two dimensions that satisfy both SMC and PI operation are identified at $d = 0.36 \mu\text{m}$, $w = 0.277 \mu\text{m}$ and $d = 0.38 \mu\text{m}$, $w = 0.318 \mu\text{m}$. These figures suggest that it is possible to achieve single-mode operation at both polarizations and maintaining polarization independence at fixed waveguide height. Theoretically, possible points in the multi-mode region may also exhibit ZBC (or polarization independence), hence a point on the birefringence-free locus that is below the single-mode boundary for both TE and TM modes should be chosen. However, for this particular case, all the identified ZBC dimensions are single-mode in both polarizations.

Fig. 2.16. Trend and boundary lines for single-mode cut off dimensions and zero-birefringence condition (ZBC) as a function of waveguide dimensions for silicon core thickness $h = 0.4 \mu\text{m}$ and an operating wavelength of $1.55 \mu\text{m}$. Single-mode and ZBC are simultaneously fulfilled at $d = 0.36 \mu\text{m}$, $w = 0.277 \mu\text{m}$ and $d = 0.38 \mu\text{m}$, $w = 0.318 \mu\text{m}$.

(d) Varying the Cladding Thickness to Relax the PI conditions

In the earlier section, it is found that at $\lambda = 1.31 \mu\text{m}$, PI operation is not possible at any waveguide dimensions for $h = 0.3 \mu\text{m}$ and $h = 0.4 \mu\text{m}$. On the other hand, it is found that at $\lambda = 1.55 \mu\text{m}$, PI operation is possible at $h = 0.4 \mu\text{m}$, although the range of dimensions that allows for such PI operation is limited to a narrow range of dimensions of etch depth $d = 0.36 \mu\text{m} - 0.38 \mu\text{m}$. Here, we illustrate that, by changing the cladding thickness, the range of d that allows for PI operation can be significantly extended for $h = 0.4 \mu\text{m}$ when using $\lambda = 1.55 \mu\text{m}$.

It has been shown in [28] that the birefringence of the waveguide can be tuned by stress engineering. In this mechanism, a change in the thickness of the upper cladding alter the stress exerted on the waveguide core, which in turn will influence the effective indices of the TE and TM modes, thereby manipulating the waveguide birefringence. Thus far, for all the sections in this chapter, we

have used an upper cladding thickness of $l_1 = 0.75 \mu\text{m}$. To demonstrate the effect of stress engineering on the waveguide core, we change l_1 to $2 \mu\text{m}$ in this section. The thickness of the lower cladding l_2 is also changed to $2 \mu\text{m}$ so that we have a symmetric waveguide structure, similar to all the other subsections in this chapter. Thereafter, the same simulation procedures described in section 2.4.1 are employed to the new SOI rib waveguides. The results are shown in Fig. 2.17.

Fig. 2.17. Boundary lines for single-mode condition (SMC) cut off dimensions of submicron SOI rib waveguides for wavelength $1.55 \mu\text{m}$ and at $h = 0.4 \mu\text{m}$, with a different cladding thickness of $l_1 = l_2 = l = 2 \mu\text{m}$. The polarization independent (PI) locus that falls into the single-mode (SM) region signifies dimensions that allow both SM and PI operations. Note that the SM region is bounded by the TE_1 cut off and TM_0 cut off boundaries.

It can be seen in Fig. 2.17 that the change in the upper cladding thickness effectively relaxes the conditions to realize PI conditions. Previously, at $\lambda = 1.55 \mu\text{m}$, SM and PI operations at $h = 0.4 \mu\text{m}$, which is shown in Fig. 2.16, are possible only in a narrow range of dimensions of etch depth $d = 0.36 \mu\text{m}$ - $0.38 \mu\text{m}$. With the use of a thicker upper cladding, the range of etch depth dimensions that gives PI operation is considerably increased to $d = 0.28 \mu\text{m}$ - $0.38 \mu\text{m}$. This is an increment of fivefold. Also, the overall SM region widens as the use of a thicker lower cladding results in: (i) A lower substrate leakage losses for the TE and TM fundamental modes; and (ii) the increased in differential substrate leakage losses between the fundamental and higher-order modes (with the higher-order modes having greater losses than the fundamental modes). Consequently, there is more effective suppression of the higher-order modes, resulting in the waveguide to exhibit stronger single-mode behaviors. These imply that the cladding thickness can be used to improve the SM as well as the PI conditions. To

expediate computational time and resources, we have only demonstrated this method for $h = 0.4 \mu\text{m}$ and $\lambda = 1.55 \mu\text{m}$. Future work will look into the systematic mapping of the SM and PI conditions at different h and λ values for both the submicron SOI rib and channel waveguides.

2.5. Summary and Significant Contributions of this Research Work

The possibility of achieving single-mode and polarization independent (PI) conditions based on submicron SOI channel and rib waveguides has been clearly demonstrated by rigorous numerical modeling via the imaginary distance beam propagation method (IDBPM) in this research work. It has shown that, three distinctive regions, namely, no modes, single-mode and multimode regimes exist for SOI channel and rib waveguides at the submicron scale. The conditions that govern single-mode condition is determined by the stringent boundaries of TM_0 and TE_1 . The polarized modes are strongly dependent upon waveguide dimensions. This results in an increase in birefringence by an order of magnitude if the waveguide height is reduced from $0.6 \mu\text{m}$ to $0.3 \mu\text{m}$ in channel waveguide. In the rib structure, single-mode and polarization-independent conditions only co-exist at $h > 0.3 \mu\text{m}$. These results clearly distinguish the dimension of channel and rib waveguides required to satisfy both single-mode and polarization independent conditions simultaneously at the submicron scale for the communication wavelengths of $1.31 \mu\text{m}$ and $1.55 \mu\text{m}$. In addition, by using analytical modeling, we have clearly shown the modal characteristics in SOI slab waveguide and its relation to the SOI channel and rib waveguides. This has often been ignored in most literatures. Also, we have demonstrated that the approximations provided by the analytical modeling for the submicron SOI channel and rib waveguides match well with the general trends of the results of the numerical modeling via the IDBPM. Such use of both analytical and numerical modelings in this research work has provided a better physical insight into the stringent conditions needed for the realization of both single-mode and PI operations at the submicron level. The results in this chapter will be employed in later chapters, in which the single-mode and/or polarization-independent dimensions are used to design SOI microring resonator and the SOI Raman waveguide.

Lastly, we mention that part of this research work has been published in the following journal and conference proceeding:

- Soon Thor Lim, **Thomas Y.L. Ang**, Ching Eng Png and Yong Ann Ong, “Single-mode, polarization-independent submicron waveguides using geometrical adjustment,” *Optics Express* 15, 11061–11072 (2007).
- Soon Thor Lim, **Thomas Y.L. Ang**, and Ching Eng Png, “Single-mode, polarization-independent submicron silicon waveguides,” 7th Pacific Rim Conference on Lasers and Electro-Optics, Seoul, Korea, Technical Digest, pp. 1–2 (2007).

To date, the above published journal has a total citation count of sixteen (listed in Refs. [33] – [48]), excluding self-citations. Previously, SOI submicron rib waveguides are seldom used for photonic devices as the SM and PI conditions at the submicron scale are not very well understood. However, since the publication of our research work, Ref. [33] has further looked into the modal characteristics of submicron SOI rib waveguides via numerical analysis while Refs. [34] and [35] have used submicron SOI rib waveguides to fabricate microring resonators. In addition, our work has also inspired the use of SOI waveguides at the submicron scale in the design and fabrication of polarization independent (PI) couplers [36-40], PI demultiplexer [41], PI chalcogenide glass nanowires [42], a slow light device [43], microbends [44], microring resonators based optical interconnects [45], SOI interleaver [46], polarization splitter and rotator [47], and last but not least, plasmonic waveguide devices [39, 40, 48]. In many of these works, our design rules in this chapter have been used to optimize the device performances. All the above-mentioned points highlight the fact that our research work in this chapter is a significant contribution to the field of photonics, in particular, silicon photonics.

2.6. References for Chapter

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Chapter 3

How Small can a Passive Microring Resonator be and yet be Single-Mode and Polarization-Independent?

3.1. Introduction: Research Motivations and Objectives

In the previous chapter, we have looked into the conditions needed to achieve single-mode (SM) and polarization independent (PI) operations in submicron silicon-on-insulator (SOI) waveguides. In this chapter, we will utilize some of these results to design a SOI waveguide (WG) based photonic device – the microring resonator (MRR) – that is suitable to be deployed in commercial applications. This would be of great research significance as the MRR has been proposed by many researchers as one of the elemental building blocks of very large scale integrated (VLSI) photonics.

For microring resonators to be commercially viable, they must be capable of PI operation while maintaining SM and low round-trip losses. Without the use of extrinsic polarization management schemes, such as polarization diversity [1, 2], which would increase the fabrication cost and complexity, the microring resonators must be designed to be intrinsically PI in order to handle input optical signals of random polarizations. As microring resonators are bend-based devices, they inherently have higher polarization sensitivity in contrast to straight waveguide (WG) based devices. Consequently, to accommodate the structural polarization independence, the design of SM microring resonators based on SOI has been limited generally to relatively large bend radius ($\geq 100 \mu\text{m}$) [3, 4] and large WG dimensions (width and height $\geq 1 \mu\text{m}$) as it is difficult to maintain both the SM and PI behaviours when shrinking the size of the microrings. In addition, we also notice that channel WGs, instead of rib WGs, are commonly used when designing small SOI microrings (i.e., bend radius $< 10 \mu\text{m}$) [5-8]; rib WGs tend to be used when designing larger microrings (i.e., bend radius $\geq 10 \mu\text{m}$) [3, 4, 9]. For microring resonators based on other material platforms, such as InP and GaAs, similar trends also apply, although for such cases, SM and PI operations have been demonstrated in rings with slightly smaller bend radii of around $30 \mu\text{m}$ [10, 11]. The above-mentioned trends were first observed in 2008 when we started and completed our research in this chapter. To clearly illustrate this, we have summarized some of the key experimental results in Table 3.1 for SOI based microrings in the literature for the period 2004 to 2008. Note that such trends (as mentioned above and can be

Table 3.1. [†]Representative works from year 2004 to 2008 on microring resonators (MRRs) based on silicon-on-insulator waveguides

Published Research	Type of Waveguide Used	Waveguide Dimensions (in μm)			Device Performances	
		Height	Width	Bend Radius	Single-Mode?	Polarization-Independent?
Dumon et al. [5], February 2004.	Channel Waveguide	0.22	0.50	5.0	Yes	No
Isa Kiyat et al. [9], October 2005.	Rib Waveguide	1.0	1.0	20	Yes	No
Headley et al. [3], December 2005.	Rib Waveguide	1.35	1	400	Yes	Yes
Xu et al. [4], January 2006.	Rib Waveguide	1.5	1.5	200	No	Yes
Nawrocka et al. [6], September 2006.	Channel Waveguide	0.25	0.45	2.0	Yes	No
Chen et al. [7], November 2007.	Channel Waveguide	0.24	0.52	10	No	No
Xu et al. [8], March 2008.	Channel Waveguide	0.25	0.45	1.5	No	No

[†]The general trend is that large waveguide cross-sectional dimensions (height and width $\geq 1 \mu\text{m}$) and large bend radius (radius $\geq 100 \mu\text{m}$) are used when designing a MRR for single-mode (SM) and polarization independent (PI) conditions. In the literature, MRRs with smaller dimensions (height and width $< 1 \mu\text{m}$, radius $< 10 \mu\text{m}$) are typically not simultaneously SM and PI. The question to be addressed in this chapter then is: Can we realize both SM and PI operations in a MRR at small waveguide dimensions and bend radius? How small can a MMR be and yet fulfills both SM and PI conditions?

seen in Table 3.1) in the design of microring resonators have remained largely valid till this date.

It is highly critical to design intrinsic PI MRR at smaller device footprint in order to minimize valuable real estate on photonic integrated circuits (PICs) and to maximize high-density integration for increased optical functions per optical chip [12]. This miniaturization of MRR, requires the WG cross-sectional dimensions to be reduced to the submicron scale (width and height $< 1 \mu\text{m}$) and the bend radii to be shrunk into the ultrasmall regime (bend radii below $10 \mu\text{m}$). However, the smaller WG cross section results in enhanced optical confinement that generally exacerbates the polarization

sensitivity of the WG, leading to increased difficulty in controlling the polarization properties of the submicron device while maintaining SM and low-loss operations. It is challenging to simultaneously fulfill both the SM and PI conditions when the WG is scaled down into the submicron regime because both birefringence and WG dispersion increases [13]. This is further complicated by the need to use ultrasmall bends to miniaturize the MRR for highly dense PICs. At smaller radii, bends have significantly different and more stringent PI conditions from the straight WGs. It is therefore generally not possible to fulfill the PI conditions for both straight and bend WGs at the same WG cross section, without resulting in residual birefringence (or polarization dependence) in the MRR intended for PI operation. With reducing bend radius, the residual birefringence becomes increasingly significant and cannot be ignored when designing PI MRRs unlike earlier works [3, 4, 10, 11] that are based on larger bends. In addition, the reduction of the bend radius is further limited by the bend losses that increase exponentially with reducing radius. A legitimate question, then, is: *How small can a ring resonator be, i.e., how small can we shrink both the cross sectional dimensions and radius, and yet maintain its overall polarization-independence and single-mode conditions while ensuring low round-trip losses?* The objective of this work is to demonstrate theoretically the possibility of achieving this and to show that the extent of size miniaturization in MRR while achieving both PI and SM conditions is limited primarily by the residual birefringence in the cavity and bend losses. However, we do not attempt to find an exact theoretical lower size limit. Only a heuristic formulation that provides an approximate but yet reasonably accurate guidelines on the extent of size miniaturization of MRR while maintaining PI is the interest of this work.

The realization of PI side-coupled resonator requires that the resonance wavelength, amplitude, linewidth, and even the spectral shape to be the same for both input TE and TM polarizations. PI also implies SM, since it is not possible to make all the modes PI simultaneously. These impose very stringent constraints on design, requiring identical effective index N_{eff} as well as identical coupling κ^2 between the WG and the ring for all polarizations for a perfect PI design. Due to the presence of polarization-dependent losses (PDL), N_{eff} is inherently complex: $N_{eff} = \text{Re}[N_{eff}] + j\text{Im}[N_{eff}]$. The real part of the N_{eff} is related to the real propagation constant β of the WG mode as $\beta = N_{eff}(2\pi/\lambda)$, where λ is the free-space wavelength, while the imaginary part describes the round trip losses. It is generally not possible to simultaneously maintain a similar β and round-trip losses for both TE and TM modes when designing PI devices. A more feasible approach, which is used in previous PI MRR designs [3, 4, 10, 11] and also in this work, would be to match $N_{eff(TE)}$ and $N_{eff(TM)}$, while maintaining low PDL. Hence, a perfectly PI design is not possible and a PI design is understood to meet the criteria only within certain specified tolerance limits. Nevertheless, even theoretical demonstration of nearly PI design is of practical significance as it is the most compact and low-cost when compared with extrinsic polarization management schemes [1, 2].

It must be noted that several assumptions are made in this work for a more meaningful discussion of the extent of PI MRR miniaturization. First, we only focus on MRR that is based on conventional dielectric WGs. The use of novel waveguides, such as slot waveguides, to design PI microrings has been demonstrated in [14] and will not be the focus of this work. To relax the stringent operating conditions of PI and SM in submicron WGs when designing small-size MRRs, we have focused on deeply etched rib WGs, instead of the commonly used channel WG. This is in contrast to the conventional practice (cf. Table. 4.1) of using channel WG when designing small-size microring resonators. The advantage of a rib over a channel WG is that it is more robust for device miniaturization as both the etch depth and width can be carefully tailored to achieve both SM and PI performances. Also, we are only concerned with the extent of PI ring miniaturization (bend radius and WG cross section), given a fixed upper cladding thickness. The use of stress birefringence to balance the geometrical birefringence has already been demonstrated [4, 15] and thus will not be the scope of this work. In addition, we only consider WG with vertical sidewalls. Finally, we ignore scattering losses due to sidewall roughness as it depends on the fabrication process. Moreover, it has been shown that such losses can be kept to a very low level [16, 17].

A modular approach is used for the analysis of the theme of this work, which considers the polarization properties of every individual component (namely the straight WG, bend WG and couplers) in the MRR. Each component is analyzed to obtain the single-mode (SM) optimum design with minimum birefringence. In section 3.2, we first study how PI and SM conditions can be simultaneously achieved for straight WGs using deeply etched ribs, which determine the range of smallest possible cross-sectional dimensions of the MRR at a specific upper cladding thickness. Using this range of WG cross-sectional dimensions, we then analyze how the residual birefringence and the bend losses dictate the minimum bend radius of the PI MRR. In section 3.3, we look into the design issues of the cavity of the MRR circuit. Subsequently, in section 3.4, we discuss how compact PI coupling can be attained by the use of a 2×2 multi-mode interference (MMI) coupler. In section 3.5, all the PI components are integrated into a single bus-coupled MRR filter configuration and the transmittance spectra are simulated for both TE and TM polarizations. Finally, in section 3.6, we analyze how fabrication tolerances might affect the PI performance of our modeled device.

3.2. Single-Mode and Polarization-Independent Waveguides in Microring Resonator

The fundamental parameter that determines the polarization sensitivity of the microring resonator is the resonance wavelengths λ_R , which is described by [10, 11]

$$L_{rt} N_{eff(TE, TM)} = m \lambda_{R(TE, TM)}, \quad (3.1)$$

where L_{rt} is the round-trip length of the resonator cavity, N_{eff} is the effective index of the modes of the ring cavity and m ($= 1, 2, 3, \dots$) is the azimuthal mode number of the resonator. In principle, PI implies that the resonant wavelength is the same for both the quasi-TE and quasi-TM modes for an

arbitrary value of m . This in turn strictly requires the WGs of the ring cavity as well as the input bus to satisfy the zero birefringence condition (ZBC), i.e., $\Delta N_{eff} = |N_{eff(TE)} - N_{eff(TM)}| = 0$, not just at the resonance wavelength λ_R but within a certain range of wavelength centered around λ_R . The ZBC implies the TE and TM modes have the same phase velocity although not necessarily the same modal profile, leading to a PI WG in the absence of PDL. In general, the straight and cavity bend WGs have different ZBCs. This results in residual birefringence that increases with reducing bend radius. We will thus consider the PI as well as single-mode conditions separately for the straight and bend WGs of the microring resonator circuit. Deeply etched SOI rib WGs with silicon waveguide height h of $0.4 \mu\text{m}$ and a thickness of $2 \mu\text{m}$ for both the upper and lower cladding of silicon dioxide ($l_1 = l_2 = 2 \mu\text{m}$) are employed for both the straight and bend WGs of the microring resonator cavity. This is the configuration that has been earlier studied in section 2.4.3(d) and is shown again below in Fig. 3.1.

Fig. 3.1. Cross-sectional view of the straight and bend rib waveguides that are used in the microring resonator circuit. To fulfill the single-mode condition, the silicon waveguide is set at height $h = 0.4 \mu\text{m}$ and a thickness of $2 \mu\text{m}$ for both the upper and lower claddings of silicon dioxide (i.e., $l_1 = l_2 = 2 \mu\text{m}$).

3.2.1. Straight Waveguides

The conditions to simultaneously achieve the SM and PI operations for deeply etched SOI rib WGs at $\lambda = 1.55 \mu\text{m}$ has already been determined in chapter 2.4. We will make use of these results for our design work of the straight WGs. The SM and PI waveguide dimensions loci for a SOI rib waveguide with $h = 0.4 \mu\text{m}$, $l_1 = l_2 = 2 \mu\text{m}$ (cf. section 2.4.3(d)) is reproduced here in Fig. 3.2 for convenience. Both PI and single-mode condition (SMC) can be achieved along the locus within the proper bounds of WG width indicated by the dotted lines. We will term the width w at which PI operation is possible for each etch depth d setting of the straight WG as the critical width $w_{c(straight)}$. At the low end of the PI locus in Fig. 3.2, where both w and d are small, the TM_0 mode is cut off and hence the PI locus is terminated. At the high end where $w_{c(straight)}$ and d both approach h , the deeply etched rib WG becomes a channel WG, which tends to be multi-mode, hence violating the single-mode condition (SMC). Note also that, for our rib WG design with an oxide cladding thickness $l_1 = l_2 = 2 \mu\text{m}$, no SM and PI modes co-exist at $h = 0.3 \mu\text{m}$. This has been found through simulations in

Fig. 3.2. Birefringence-free (or polarization independent) waveguide dimensions of fundamental modes for waveguide height $h = 0.4 \mu\text{m}$ at a wavelength of $1.55 \mu\text{m}$. As the rib transforms to a channel WG, polarization independent (PI) and single-mode (SM) conditions cannot be fulfilled simultaneously.

section 2.4.3(b) of chapter 2. Hence, for this specific oxide cladding thickness, the smallest possible cross sectional WG dimensions for both SM and PI performances can be approximately obtained at $h = 0.4 \mu\text{m}$ from Fig. 3.2.

3.2.2. Bend Waveguides

In curved WGs, modal propagation depends additionally on the bend radius R , and hence at the same WG cross section, straight and bend WGs have different β and therefore different PI conditions. The PI condition for bends is more stringent as the WG curvature distorts the modal symmetry. The propagation constant of the bend is complex, with the imaginary part corresponding to radiation losses. We will first ignore this PDL, assuming it to be small.

The solution of effective index $N_{\text{eff}(\text{bend})}$ for bend is solved using BeamPROP [18], which first applies a coordinate transformation to map a straight WG onto a curved WG with the desired bend radius and then employing the 3-D full-vectorial BPM for solving the radial modes. In our simulations, the bend radius is defined as the physical radius, R , to the centre of the WG. The $N_{\text{eff}(\text{bend})}$ plots for the quasi-TE and quasi-TM fundamental radial modes as a function of w at a fixed d of $0.37 \mu\text{m}$, for two bend radii $2 \mu\text{m}$ and $4 \mu\text{m}$ are shown in Fig. 3.3, along with the $N_{\text{eff}(\text{straight})}$ of the straight WG with a similar cross section. At the critical width that occurs at the intersection, the curve WG is PI in the absence of PDL. Similar N_{eff} plots of Fig. 3.3 are also computed for other R and d values. It can be observed that, at the same d , curved and straight WG have different critical widths w_c , which

Fig. 3.3. Calculated effective indices for the quasi-TE and quasi-TM modes as a function of width for straight and curved WGs at bend radii (R) of 2 and 4 μm , all at etch depth of 0.37 μm . The dotted line points to the various critical widths w_c for each R . The critical width w_c of the bend decreases and tends to that of the straight WG (marked by a thickened dotted line to the extreme left) with increasing R .

are termed as $w_{c(bend)}$ and $w_{c(straight)}$, respectively, for the bend and straight WGs. For a fixed d and R , using $w_{c(bend)}$ for the WG design gives ZBC for the bend but results in a birefringent straight WG. With reducing R , the birefringence in the straight WG, with $w = w_{c(bend)}$, increases. Likewise, if $w_{c(straight)}$ is used to design the bend, birefringence will be present for all R and will tend to zero as R increases, as illustrated in Fig. 3.4. Hence, strictly speaking, for a fixed d , it is not possible to achieve PI for both straight and curved WGs by using a common WG width. However, the difference in $w_{c(bend)}$ and $w_{c(straight)}$ is negligible at large bend radii as shown in Fig. 3.4. In the limit of $R = \infty$, $w_{c(bend)}$ will approach that of $w_{c(straight)}$ and thus at sufficiently large R , both the straight and bend WGs in a MRR can be designed to be near-PI, in the absence of PDL, by using a common w_c . Any residual birefringence present in the MRR is too small to induce any significant misalignment between the TE and TM resonance spectra. However, in shrinking R , the difference between $w_{c(straight)}$ and $w_{c(bend)}$ widens, resulting in rising residual birefringence if all components of the cavity have the same cross section. Then the best approach is to choose an optimized cross-sectional dimensions at a suitably small R , which will minimize the residual birefringence. In the absence of bend loss, the minimum bend radius (MBR) that can be used is thus limited by the acceptable level of residual birefringence.

Using the computed $w_{c(bend)}$ data for different bend and WG cross sections, the PI loci as a function of R for various d are then plotted in Fig. 3.5. The effective indices of the radial modes generally increases with decreasing R because $w_{c(bend)}$ becomes larger at smaller bend curvature, which better confines the optical modes in the WG. It is also clearly shown that as R increases for a fixed d ,

Fig. 3.4. Plot of effective index of 90° bend as a function of bend radius R at a fixed cross section that is based on the polarization independent (PI) dimension ($d = 0.37 \mu\text{m}$ and $w_c(\text{straight}) = 0.39 \mu\text{m}$) of the straight WG, whose effective index is also plotted for comparison. Using the PI dimension ($d = 0.37 \mu\text{m}$ and $w_c(\text{straight}) = 0.39 \mu\text{m}$), the straight WG satisfies zero birefringence condition (ZBC) but the bend is birefringent. The bend birefringence reduces and approaches zero as R increases.

the effective indices of the PI radial modes tend asymptotically to that of the straight WG, which forms the horizontal asymptote. Conversely, reducing R results in an increasing difference between $N_{\text{eff}}(\text{PI bend})$ and $N_{\text{eff}}(\text{PI straight})$. Achieving PI in bends requires a judicious combination of w , d and R , as illustrated in Fig. 3.5. PI bends can be achieved for radius R down to about $1 \mu\text{m}$ for the given h and cladding thickness. For even smaller R , PI operation becomes impossible even with increasing d and w due to the transition of the radial mode into a microdisk-like whispering gallery mode (WGM), which is highly polarization dependent. The WGM is guided only by the outer WG boundary and is insensitive to the inner boundary, as illustrated in the inset in Fig. 3.5. This transition to WGM is marked by a sharp increase in $w_c(\text{bend})$ that will approach the vertical asymptote, which is the theoretical size limit to achieving PI performance in bends. Once the mode is in the WGM regime at sufficiently large w , both the loci of the TE and TM bend effective indices become flattened with increasing w in Fig. 3.3 and $\Delta N_{\text{eff}}(\text{bend})$ remains unaffected for any change in w . For smaller R , this trend occurs without any intersection of the $N_{\text{eff}}(\text{bend})$ locus for the TE and TM modes. Hence, PI cannot be achieved for smaller R . Without considering bend loss, the WGM regime will result in a lower PI bend radius limit, R_L , of about $1 \mu\text{m}$ at our considered h and cladding thickness. For larger h and at the same cladding thickness, R_L will become larger. For smaller h , R_L is not considered as PI and SM modes do not co-exist as mentioned earlier.

Fig. 3.5. The loci of PI critical widths w_c at varying bend radius for different etch depths for fixed silicon height of 0.4 μm at an input wavelength of 1.55 μm . The inset shows the transformation of the radial mode into a WGM as the width w is increased while keeping the bend radius fixed at 2 μm and the etch depth at 0.37 μm .

In addition, d must be increased in bend WG (as compared to the straight WGs) in order to ensure SM operation, as the differential loss between the fundamental mode and the higher-order mode tends to decrease due to the additional slab leakage loss that depends on d and that increases as R is reduced. Based on simulations, it was found that as long as $d > 0.3$ μm , our modeled PI bends will be SM, while for the straight WG, the required d must be smaller than 0.39 μm as concluded earlier in sections 2.4.3(d) and 3.2.1.

3.3. Design Issues of PI Microring Resonator with Small Bend Radius

3.3.1. Racetrack Cavity with Low Birefringence

Two types of commonly used architectures when designing laterally-coupled compact MRR filters are shown in Figs. 3.6(a) and 3.6(b), which are based on directional coupling and point coupling, respectively. In the design of MRRs with submicron cross sections, we adhere to the principle that the design should be *simple* and within the limits of *practical* fabrication tolerances. Hence, we will focus on the *race-track* MRR since it is easier to control the coupling factor simply by

varying the coupler length without resort to very small gap dimensions [19], although this will result in a larger cavity and hence smaller free spectral range (FSR). In addition, we assume all the contiguous WGs have the same cross-sectional dimensions and etch depths, hence the combination of

Fig. 3.6. Two basic architectures of ring resonator with submicron dimensions. (a) Type 1: Directional-coupled race-track resonator. (b) Type 2: Point-coupled circular micro-ring resonator.

straight and curved WGs in a race-track MRR as well as the use of the use of a MMI coupler will inevitably result in residual birefringence as well as PDL. The difference in phase shift $\Delta\delta$ between the TE and TM modes after propagating through a sub-component of the racetrack cavity is: $\Delta\delta = 2\pi\Delta N_{eff}L/\lambda$ where ΔN_{eff} and length L are, respectively, the geometrical birefringence and length of the specific component. This $\Delta\delta$ induced by each birefringent sub-component would translate to a resonant wavelength shift $\Delta\lambda_R$ between the TE and TM resonance spectra, which can be described as

$$\Delta\lambda_R = \lambda \left(\Delta N_{eff} / N_{eff(PI)} \right) (L / L_{rt}), \quad (3.2)$$

where $N_{eff(PI)}$ is the effective index of the polarization independent (PI) bus WG and L_{rt} is the total cavity length. To minimize $\Delta\lambda_R$, the PI dimensions of the bend instead of that the straight WG is used for our design of the WG cross section of the cavity as the four 90° -bends are much longer in length than the access straight WGs and the multi-mode interference (MMI) coupler. By using PI 90° bends, the residual birefringence in the resonator is *minimized* to that remaining in the MMI coupler and the access straight WGs. The total resonant wavelength shift, $\Delta\lambda_R$, is then: $\Delta\lambda_R = \Delta\lambda_{R, MMI} + \Delta\lambda_{R, straight WGs}$. The residual birefringence present in each birefringent constituent component of the racetrack cavity must be within a specific tolerance limit so as to minimize $\Delta\lambda_R$. Without seriously impacting the PI response in an actual device, $\Delta\lambda_R$ should be within 15% of the full-width at half maximum (FWHM) [11]. The actual birefringence tolerance limit to meet such $\Delta\lambda_R$ criteria depends on the actual length of each birefringent component as well as the FWHM. For our MRR design, the FWHM is on the order of 1-3 nm. Thus, the residual birefringence should be below the tolerance limit of ~ 0.009 . We have shown in Fig. 3.7 the birefringence that will be present in the straight WG component of a racetrack cavity as a function of R . For each R , the ring cavity is designed at $w = w_{c(bend)}$, giving PI bends but birefringent straight WG. Note that $w_{c(bend)}$ is the critical width that gives PI bend for a given R and etch depth, as mentioned earlier in section 3.2.2 and Fig. 3.5. The birefringence level in the straight WG generally rises with reducing R but stabilizes at around 0.006 for $R \geq 3 \mu\text{m}$ for all etch depths d

considered in Fig. 3.7. Thus, without considering bend losses, the minimum bend radius (MBR) that can be used for PI operation within our specific tolerance limit for $\Delta\lambda_R$ is $3\ \mu\text{m}$. We label this MBR of $3\ \mu\text{m}$ that is determined solely by the residual birefringence as MBR_{RB} . In other words, $MBR_{RB} = 3\ \mu\text{m}$ for our waveguide design of $h = 0.4\ \mu\text{m}$ and different etch depths d (shown in Fig. 3.7).

Fig. 3.7. Birefringence of straight WGs as a function of the bend radius for various etch depths d and fixed $h = 0.4\ \mu\text{m}$ and $l_1 = l_2 = 2\ \mu\text{m}$ (cf. Fig. 3.1). The straight WG is designed using the dimensions of the PI bend for each bend radius, resulting in birefringence in the straight WG. The minimum bend radius (determined solely by birefringence) that can be used is about $3\ \mu\text{m}$.

3.3.2. Racetrack Cavity with Acceptable Round Trip Losses

The total round trip losses α_R of a *racetrack* MRR is defined as: $\alpha_R [\text{dB}] = 4\alpha_B + 4\alpha_T + \alpha_S$, where α_B is the pure bending loss per 90° bend and α_T is the transition loss at each straight to bend WG junction. The scattering losses due to side-wall roughness α_s is ignored in this work. The pure bending loss α_B occurs because the radial mode is shifted towards the outer edge of the curvature, resulting in the radial mode to couple to the radiation modes. α_B is mathematically defined as [20]

$$\alpha_B [\text{dB} / 90^\circ] = -20 \log_{10} \left(\exp \left[-\text{Im} \{ N_{\text{eff}}(\text{bend}) \} k_0 R \pi / 2 \right] \right), \quad (3.3)$$

where $k_0 = 2\pi/\lambda$ is the propagation constant in free space and $\text{Im}(N_{\text{eff}}(\text{bend}))$ is the imaginary component of $N_{\text{eff}}(\text{bend})$. The transition loss α_T at each straight-bend WG junction is due to the coupling loss induced by the modal mismatch as the propagating mode couples from the normal mode in the straight WG to the radial mode in the bend WG. The magnitude of α_T can be computed by employing the overlap integrals [21]:

$$\alpha_T[dB] = -\log_{10} \frac{\left| \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_0^N(x,y) E_0^{R*}(x,y) dx dy \right|^2}{\left| \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_0^N(x,y) E_0^{N*}(x,y) dx dy \right| \left| \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_0^R(x,y) E_0^{R*}(x,y) dx dy \right|}, \quad (3.4)$$

where E_0^N and E_0^R are the normal mode of a straight WG and the radial mode of a bend, respectively, and the symbol $*$ denotes complex conjugation. The sum of α_B and α_T gives α_{TBL} , which is the total bend losses per 90° turn. Both α_B and α_T can be computed accurately using the finite-difference time-domain (FDTD) method [18].

We now analyze if the earlier defined $MBR_{RB} = 3 \mu\text{m}$ for the racetrack cavity will be affected by α_{TBL} and the PDL. There are different sets of PI bend cross sections that correspond to $R = MBR_{RB} = 3 \mu\text{m}$ as seen from Fig. 3.7. The objective of this section is to determine if the bend with $R = 3 \mu\text{m}$ at the respective PI width dimensions for each d will give low α_{TBL} and PDL. If $R = 3 \mu\text{m}$ satisfies these conditions, then $MBR_{RB} = 3 \mu\text{m}$ can be used and will thus be the overall MBR, which is denoted as MBR_{NET} , for a particular d . Otherwise, $MBR_{RB} = 3 \mu\text{m}$ cannot be used and MBR_{NET} will then be larger than the MBR_{RB} for the specific d concerned. Here, we define a maximum acceptable level of α_{TBL} as 0.2 dB, giving a bend loss tolerance range of $\alpha_{TBL} \leq 0.2 \text{ dB}$.

The α_{TBL} for both quasi-TE and quasi-TM as a function of R are computed and shown in Fig. 3.8

Fig. 3.8. Total bend loss α_{TBL} in a 90° bend as a function of the bend radius for the fundamental mode at a wavelength of $1.55 \mu\text{m}$ ($d = 0.37 \mu\text{m}$ and $w_{c(bend)} = 0.394 \mu\text{m}$).

for one particular bend dimension of $w_{c(bend)} = 0.394 \mu\text{m}$, $d = 0.37 \mu\text{m}$ that corresponds to $MBR_{RB} = 3 \mu\text{m}$. The bend losses are generally dependent on the optical confinement, which vary with R and the polarization state. As shown in Fig. 3.8, α_{TBL} increases with reducing R , with the quasi-TE mode having more losses than the quasi-TM mode as it experiences more slab leakages owing to the larger

extent of the electric field in the horizontal direction. Note that for the race-track resonator, α_T is the dominant component of α_{TBL} for all R . For the case in Fig. 3.8, $\alpha_{TBL} \leq 0.2$ dB when $R \geq 2.5$ μm , in which low PDL is also observed. We denote the MBR that only considers the bend losses as MBR_{LOSS} . Thus, for this WG design of $h = 0.4$ μm and $d = 0.37$ μm , $MBR_{LOSS} = 2.5$ μm . Comparing this value of MBR_{LOSS} with the earlier defined MBR_{RB} of 3 μm in section 3.3.1, we can conclude that $MBR_{LOSS} < MBR_{RB}$. Thus, MBR_{RB} of 3 μm can be used for PI design (i.e., $MBR_{NET} = MBR_{RB}$) for the bends.

Similar procedures as mentioned above are also repeated for other PI bend cross-sectional dimensions that correspond to $MBR_{RB} = 3$ μm to determine their MBR_{LOSS} . The locus of MBR_{LOSS} for the various etch depths at their respective $w_{c(bend)}$ that correspond to MBR_{RB} is plotted in Fig. 3.9. It is

Fig. 3.9. Plot of MBR_{LOSS} as a function of the etch depth. At each etch depth, the bend width $w_{c(bend)}$ for PI operation at $R = 3$ μm is used to compute this plot.

observed that MBR_{LOSS} generally increases as etch depth d is being decreased. This is because a reduction in d , which also corresponds to smaller $w_{c(bend)}$, will bring about a rise in bend losses. Hence, in order to minimize MBR_{LOSS} , d must be sufficiently deep such that MBR_{LOSS} is smaller than MBR_{RB} so that $R = 3$ μm can be used to design the PI racetrack MRR. In Fig. 3.9, the dotted-line marks R where $MBR_{LOSS} = MBR_{RB}$ and the locus above (below) it marks $MBR_{LOSS} > MBR_{RB}$ ($MBR_{LOSS} < MBR_{RB}$). Etch depths can be used for PI bend operation at MBR_{RB} with their respective $w_{c(bend)}$ only if $MBR_{LOSS} \leq MBR_{RB}$, which also implies $MBR_{NET} = MBR_{RB}$. This is fulfilled only at larger etch depths, which is ≥ 0.35 μm for our case of $h = 400$ nm. Unless bend loss optimization schemes, such as [20], are used to minimize the MBR_{LOSS} so that it matches MBR_{RB} , the MBR_{NET} for shallower d will generally be larger than MBR_{RB} .

3.3.3. Polarization Conversion

Besides taking into consideration of residual birefringence and bend losses when designing PI MRR, it is also important to consider polarization conversion (PC), which will partially or fully convert an input polarization state into another polarization state. In general, PC takes place in the presence of geometric asymmetry [22-24], which will provide a means for the polarization state to be rotated from one orthogonal polarization state to another. Hence, bends and straight WGs (with slanted sidewalls) are highly prone to PC. The presence of PC has two main deleterious effects on the performance of MRR [25]. First, it would result in a secondary set of resonance minima to appear in both the TE and TM spectra. This is due to the extra phase shift accumulated by the converted light when it rotates from one polarization state to another and also of its extra propagation distance. Secondary, PC will result in a lower Q factor as the resonator will be more lossy. This is because the converted light cannot interfere with the unconverted light circulating in the cavity and will thus leak out at the coupling region.

The polarization of an incoming TE or TM mode will be rotated by 90° after a beat length L_π , which is defined as

$$L_\pi = \frac{\pi}{\beta_{TE} - \beta_{TM}} = \frac{\lambda}{2(N_{eff(TE)} - N_{eff(TM)})} \quad (3.5)$$

In order for PC to be significant, the length of the WG must be much larger than L_π [23, 24]. Generally, PC in bends increases with decreasing R and is further enhanced in the presence of slanted sidewalls. In our design of MRR, the PI dimensions of the bend instead that of the straight WG is used for our design of the WG cross section of the cavity because the four 90° bends are much longer in length than the sub-components of the straight WGs, as mentioned earlier. As such, PC is not an issue in our designed bends as the two polarization states are degenerate when the bends are tailored to be PI [23]. Also, only vertical sidewall WGs have been our focus in all our modeling work to avoid PC. However, actual fabrication generally results in dimensional deviations in the WG from its optimal PI value, throwing the birefringence off balance in the bend WG, which will affect its beat length. Then for PC to be of negligible effect, the total length of each bend must be much smaller than its L_π . We have found that as long as the uncertainties in width w and etch depth d of the bends at $h = 400$ nm and $R \geq 3$ μm fall within the maximum possible tolerance of $\pm 2.5\%$, PC can be ignored. In addition, using state-of-the-art technology [16, 17], it is possible to fabricate WGs with almost vertical sidewalls, and hence PC can be considered as negligible in straight and bend WGs in our work.

3.4. Polarization-Independent (PI) Coupling

The design of a PI waveguide-coupled micro-resonator requires an additional condition: The coupling must be PI in order for the TE and TM resonance spectra to have the same finesse. In

general, for a racetrack MRR, it is not easy to achieve a PI coupling length using a directional coupler that is insensitive to wavelength and dimensions. Without compromising on the small device footprint, a better alternative is the 2×2 multi-mode interference (MMI) coupler which has better fabrication tolerance and exhibits weaker polarization and wavelength dependences. Hence, the final step in realizing a PI resonator is to design the polarization insensitive MMI coupler. The MMI coupler is shown together with the racetrack resonator circuit in Fig. 3.10(a).

Essentially, the split ratios between the output ports 3 and 4 can be controlled and varied by adjusting the width w_{mmi} of the MMI coupler relative to the centre-to-centre separation (port pitch) P_{mmi} of the access waveguides. The length of the MMI coupler, L , is then determined depending on the split ratio. As found in [10, 11], except for cross coupling (0:100) and 3-dB (50:50) splitting, other split ratios are generally sensitive to polarization as well as the MMI coupler's length. Hence, we will design the MMI coupler to be a 3-dB (50:50) coupler, giving a coupling factor of 50%. This relatively high coupling factor will be justified after tallying all the round-trip losses in the cavity.

Fig. 3.10. (a) The proposed submicron PI ring resonator coupled with one bus WG via a 2×2 Type I MMI coupler. (b) The Type I MMI coupler used for the racetrack resonator design showing the various parameters and the ports labeling. Close up views of the Type I and Type II MMI couplers are shown, respectively, in (c) and (d).

The two types of MMI couplers that can be used for PI coupling are illustrated in Figs. 3.10(c) and 3.10(d). Type I has the smallest possible width given by $w_{mmi} = (P_{mmi} + w)$, where w is the width of the access waveguides. The restricted interference Type II MMI coupler can be formed by expanding the MMI coupler's width to $w_{mmi} = 3P_{mmi}$. The 3-dB coupler length L_{3dB} is given by $(3L_\pi)/2$ for Type I, and $L_\pi/2$ for Type II, where L_π is the beat length of the MMI coupler [11] given as

$$L_\pi = \frac{4}{3} N_{eff} \frac{w_{eq}^2}{\lambda_R}, \text{ in which } N_{eff} \text{ is the effective index of the slab WG, } \lambda_R \text{ is the resonance wavelength and}$$

w_{eq} is the equivalent MMI coupler's width, which for the present case is essentially the same as the physical width of the rib slab WG. By substituting the above expression of L_π and the respective w_{mmi} into L_{3dB} for Type I and II, it can be easily derived that the ratio of the 3-dB coupling length of Type I to that of Type II is

$$\frac{L_{3dB, Type I}}{L_{3dB, Type II}} = \frac{3L_{\pi, Type I} / 2}{L_{\pi, Type II} / 2} = 3 \frac{\frac{4}{3} n_{r, Type I} \frac{(w_{eq, Type I})^2}{\lambda_R}}{\frac{4}{3} n_{r, Type II} \frac{(w_{eq, Type II})^2}{\lambda_R}} \bigg/ 2 = 3 \frac{(P_{mmi} + w)^2}{(3P_{mmi})^2} \quad (3.6)$$

In our analysis, the port pitch P_{mmi} is set at 1.0 μm , which is large enough to keep the port WG separate from the resonator so that the coupling effect is negligible, and the width of the access WGs is fixed at 0.394 μm , corresponding to the PI dimension for a 3 μm radius cavity. Hence, $L_{3dB, Type I} / L_{3dB, Type II} = 0.65$, implying a shorter L_{3dB} if Type I MMI coupler is used. To verify this theoretical calculation, we use simulation [18, 26] to determine the actual 3-dB coupling length L_{3dB} for the TE and TM modes and the final PI coupling length. Other device performances such as the insertion and coupling losses are also computed. These results are summarized in Table 3.2, in which $\tau_{mmi}^2 = \kappa^2 + r^2$ is the power attenuation constant of the MMI coupler for the TE or TM mode that quantifies the coupler loss of the MMI coupler ($\tau_{mmi}^2 = 1$ for a lossless coupler), where κ^2 and r^2 are,

Table 3.2. Computed coupling length, losses and birefringence for Type I and Type II MMI couplers

	Type I MMI		Type II MMI	
w_{mmi} (in μm)	1.394		3	
L_{3dB} (in μm)	TE	TM	TE	TM
	7.76	8.55	12.25	13.27
PI coupling length (in μm)	8.2		12.4	
Power splitting ratio r^2 at PI coupling length	0.493		0.497	
Attenuation constant τ_{mmi}^2 of the MMI at PI coupling Length	TE	TM	TE	TM
	0.982	0.987	0.992	0.995
Insertion loss (dB) at PI coupling length based on τ_{mmi}^2	TE	TM	TE	TM
	0.0789 (~1.3%)	0.0568 (~1.9%)	0.0349 (~0.6%)	0.0218 (~0.4%)
PDL (dB) at PI coupling length	0.0221 (~0.6%)		0.0131 (~0.2%)	
Birefringence ΔN_{eff} of MMI at PI coupling length	4.7×10^{-5}		5.0×10^{-5}	

respectively, the power splitting ratio at port 3 and port 4. For Type I, $L_{3dB} = 7.76 \mu\text{m}$ for TE and $8.55 \mu\text{m}$ for TM, while for Type II, it is $12.25 \mu\text{m}$ for TE and $13.27 \mu\text{m}$ for TM. These computed figures of L_{3dB} are in close agreement with the above theoretical calculated ratio of $L_{3dB,TypeI} / L_{3dB,TypeII} = 0.65$ in Eq. (3.6). In between the above two different values of L_{3dB} for the TE and TM modes for each type of MMI coupler, one can find a PI coupling length that gives close to 50:50 splitting ratio. As highlighted in Table 3.2, these PI coupling lengths are found to be $8.2 \mu\text{m}$ and $12.4 \mu\text{m}$ for Type I and Type II MMI coupler, respectively. The corresponding insertion losses (i.e., $10 \log(1/\tau_{mmi}^2)$), the PDL (i.e., difference between the insertion losses of the TE and TM modes), and the birefringence ($\Delta N_{eff} = |N_{eff(TE)} - N_{eff(TM)}|$) of both types of MMI couplers are also given in the table. Note also, from Table 3.2 that the Type I MMI coupler has a somewhat lower birefringence than Type II MMI coupler although its PDL is slightly higher. The Type I MMI coupler is subsequently employed as the 3-dB coupler in our PI MRR design as it has a shorter PI coupling length and slightly lower birefringence.

3.5. Towards a Polarization-Independent Small Size SOI Microring Resonator

The ring is normally coupled evanescently to one or two bus WGs. A single ring coupled to two bus WGs is analogous to a Fabry-Perot cavity and can serve as an add-drop filter. The major performance characteristics of the filter are the finesse and the maximum drop efficiency (D_{MAX}). Here, D_{MAX} is defined as the transmission to the drop port at on-resonance. The finesse is independent of the ring size and depends only on the intrinsic and coupling losses. In the simplest case where the two coupling coefficients are identical, D_{MAX} is less than one due to the loss-induced imbalance. There is an intrinsic trade-off between finesse and D_{MAX} , as the higher the finesse, the higher will also be the loss-induced imbalance. To achieve both high finesse and high D_{MAX} , both the round-trip loss and coupling factor must be kept small, with the latter several times larger than the former. The actual coupling factor employed must be large enough to compensate the ring losses and yet small enough to maintain a reasonably high finesse.

The actual final design of a PI MRR will depend on the silicon core h . At our specified cladding thickness (cf. Fig. 3.1), $h = 400 \text{ nm}$ is the minimum possible WG height in which both SM and PI conditions can be fulfilled, as concluded in section 3.2.1. For simplicity, we have focused in this work on demonstrating PI operation in a single bus MRR as shown earlier in Fig. 3.10. The design methodology in this work can be easily extended to coupled-MRR circuits. For concreteness, we assume the resonator to have a PI bend at radius of $3 \mu\text{m}$, which is the lower limit with an acceptable bend loss and residual birefringence in the straight WGs, and Type I MMI is employed with a coupling length of $8.2 \mu\text{m}$, having a straight section of $2 \mu\text{m}$ added to the port at each side of the coupler. The total cavity length is thus about $43 \mu\text{m}$. The bend losses α_{TBL} are 0.140 dB and 0.110 dB per 90° bend for TE and TM modes, respectively. For a racetrack MRR coupled to a single bus, the total losses per round trip including that in the MMI coupler's will be 0.639 dB ($\sim 11\%$) for TE and

0.497 dB ($\sim 8\%$) for TM, which implies a total PDL of 3%. This significant round-trip loss means that the finesse will be quite small, but does justify the use of 3-dB MMI coupler to give 50% coupling coefficient, since the coupling must be larger than the round-trip loss as discussed earlier. To minimize residual birefringence, the racetrack cavity is designed using the PI bend's critical width of $0.394 \mu\text{m}$ with the corresponding etch depth of $0.37 \mu\text{m}$. The straight WG segments in the cavity ($2 \mu\text{m}$ added to each port of the MMI coupler) with the same width as the bends, will then have a birefringence of around 0.006. The birefringence in the MMI coupler is slightly smaller, at 0.0047. The straight bus WG that connects the input and exit ports to the MMI coupler (cf. Fig. 3.10(a)) must also be PI and this requires a slightly different w of $0.39 \mu\text{m}$. This may taper to $0.394 \mu\text{m}$ before entering the MMI coupler with a negligible effect.

Achieving PI operation requires close-matching of the output transmission response for both quasi-TE and quasi-TM polarizations. The normalized transmittance power or transmission T in port 4 (cf. Fig. 3.10b) can be easily obtained using the coupling matrix formalism presented in [27]:

$$T = \tau_{mmi}^2 \left(\frac{\tau_{mmi}^2 \tau_{ring}^2 + r^2 - 2\sqrt{\alpha_{mmi} \tau_r} r \cos \delta}{1 + \alpha_{mmi}^2 \tau_{ring}^2 r^2 - 2\sqrt{\alpha_{mmi} \tau_r} r \cos \delta} \right), \quad (3.7)$$

where τ_{mmi} , τ_r , r and δ are the coupling attenuation constant of the MMI, round-trip amplitude transmission factor of the cavity, the reflectivity of the coupler and the round trip phase shift, respectively. For the racetrack cavity that consists of the MMI, bends and straight WGs, δ can be expressed as the sum of the phase shift induced by each subcomponent:

$$\delta = \frac{2\pi}{\lambda} \left(N_{eff(bend)} L_{bends} + N_{eff(straight)} L_{straightWG} + N_{eff(mmi)} L_{mmi} \right), \quad (3.8)$$

where L is the total length of each sub-component. PI operation, without considering PDL, thus requires similar δ for both TE and TM at all input wavelengths. For our proposed design where contiguous WGs have the same cross-sectional dimensions, this is generally not possible due to the inherent presence of residual birefringence in the straight WGs and the MMI coupler.

Note that all the modeled WGs thus far are for SM and PI operations at the communication wavelength of $\lambda = \lambda_0 = 1.55 \mu\text{m}$. However, it is important for the device to be PI not just at λ_0 but also in the vicinity of the centre wavelength $\lambda_0 = 1.55 \mu\text{m}$, especially when the FSR extends over a wide wavelength range for MRR with an ultrasmall radius. Moreover, as the WG dimensions are scaled down into the submicron regime, WG dispersion increases and must be accounted for in our design. The effect of WG dispersion, which is the sum of material and geometrical dispersions around λ_0 , is considered by using the group effective index n_g in place of N_{eff} when computing δ . Group effective index n_g is related to the WG dispersion by $n_g(\lambda) = N_{eff(PI Bend)} - \lambda(dN_{eff}/d\lambda + dn_{si}/d\lambda)$, where n_{si} is the wavelength-dependent material refractive index of silicon. The consideration of the group index is a more accurate representation of the polarization-dependence response as $n_g(\lambda)$ of the bends may no

longer be PI though it is PI for the N_{eff} at $\lambda = 1.55 \mu\text{m}$ when radius $R = 3 \mu\text{m}$ for our MRR design above. Using n_g , the round-trip phase shift can then be more accurately represented as

$$\delta = \frac{2\pi}{\lambda} \left(n_{g(bend)} L_{bends} + n_{g(straight)} L_{straightWG} + n_{g(mmi)} L_{mmi} \right). \quad (3.9)$$

To determine n_g for our SOI rib, the material dispersion ($dn_{si}/d\lambda$) of silicon and the WG geometric dispersion ($dN_{eff}/d\lambda$) must be determined. The material dispersion of silicon $dn_{si}/d\lambda$ follows the Sellmeier equation and the Sellmeier coefficients for silicon [28] which was found to be $-0.0759 \mu\text{m}^{-1}$ at ambient room temperature (298 K). To calculate the geometric dispersion ($dN_{eff}/d\lambda$), the N_{eff} of the fundamental modes for the TE and TM polarizations as a function of wavelength were simulated using the full-vectorial BPM mode solver for both the straight and the bend WGs with $w = 0.394 \mu\text{m}$ and $d = 0.37 \mu\text{m}$. The results for the $3 \mu\text{m}$ -radius bend and straight WGs are both shown in a single plot in Fig. 3.11. Geometric dispersion for each respective component can be calculated from the slope of the graph of N_{eff} against wavelength. For the bend WG, the geometric dispersion is calculated to be $-1.0135 \mu\text{m}^{-1}$ for TE and $-0.9426 \mu\text{m}^{-1}$ for TM whereas for the straight WG, it is $-0.9665 \mu\text{m}^{-1}$ for TE and $-0.9915 \mu\text{m}^{-1}$ for TM. It is clearly seen that even though the bends is PI at $\lambda = 1.55 \mu\text{m}$, as indicated in Fig. 3.11, there is, however, a $\sim 7\%$ difference in its geometric dispersion between the TE and TM fundamental modes, resulting in slight polarization dependence in its group index. For the straight WG, note the presence of birefringence of around 0.006 even at $\lambda = 1.55 \mu\text{m}$ in

Fig. 3.11. The effective index as a function of wavelength, for straight WG and bend of $R = 3 \mu\text{m}$, both with $d = 0.37 \mu\text{m}$ and $w = w_{c(bend)} = 0.394 \mu\text{m}$. At a wavelength of $1.55 \mu\text{m}$, bend is PI while residual birefringence exists in the straight WG. However, the geometric dispersion (gradient of graph) is polarization dependent for both bend and straight WG.

Fig. 3.11. This agrees with our earlier results in section 3.3.1. The polarization dependence in geometric dispersion of $\sim 3\%$ for the straight WGs is slightly lower than that of the bends WG due to its better optical confinement. The geometric dispersion in the MMI coupler is generally quite small and can be neglected [10, 11]. Hence, after taking into account of the polarization dependence of the group index, the bends will also contribute to the overall residual birefringence in the cavity, besides the straight WGs and MMI. The computed birefringence in terms of group index: $\Delta n_g = n_{g(TE)} - n_{g(TM)}$ for the various sub-components of the cavity is 0.1099 for the bend, 0.0328 for the straight WG and 0.0047 for the MMI coupler. The sum of the effects of these residual birefringences results in a round-trip phase shift difference $\Delta\delta_R$ that induces a misalignment $\Delta\lambda_R$ in the resonance spectra of the TE and TM. To determine the extent of $\Delta\lambda_R$ as well as the effects of the small PDL present, Eq. (3.7) can be used after substituting in Eq. (3.9).

To verify the effects of residual birefringence and PDL, numerical simulation was also carried out for our PI MRR design using Apollo APSS2 circuit simulator [26] that shows good agreement with the bench mark FDTD method [18]. The APSS2 circuit simulator generally breaks the device into its constituent sub-components, which are then simulated individually by suitable numerical methods. Each building block segment is then assigned a unique transfer function. These sub-components are then re-assembled and their unique input-output relations are then used to calculate the overall transfer function T . The resulting simulated transmittance or transmission (T) spectra at port 4 are shown in Fig. 3.12. The effects of birefringence and dispersion in the whole device are taken into consideration in the simulation, giving the overall transmittance mismatch that verifies well with our analytic calculations based on Eq. (3.7). Important features of the simulated resonance spectra for our proposed PI racetrack resonator are highlighted below.

First, the resonance spectra are close to being PI, being nearly aligned. Near $\lambda = 1.55\ \mu\text{m}$, the total mismatch $\Delta\lambda_R$ in the spectra response of about 0.22 nm is due to the sum of the effects of the residual birefringence of each component. The net misalignment of 0.22 nm between the TE and TM spectra represents only 11% of the full-width half-maximum resonance linewidth, which is about 2 nm. This small fractional mismatch implies that our proposed MRR design is adequate for PI operation. Note that no significant polarization conversion occurs for our PI MRR design as the modes remain as either quasi-TE or quasi-TM in the simulation.

Secondly, the relatively wide FSR of 14 nm is made possible by keeping the cavity length as small as possible. The use of submicron high-index contrast WGs has allowed the bend radius to be shrunk while keeping the round-trip loss within a tolerable level. The coupling length has also been considerably shortened by utilizing a MMI coupler instead of the conventional waveguide-based directional coupler. As the FSR must be on the order of 10 nm for it to be effective as a filter in most coarse wavelength division multiplexing (CWDM) systems, the proposed ultra-compact device can thus be used for SM and intrinsic PI operations in such applications.

Fig. 3.12. Simulated transmission (T) spectra of the proposed single-port racetrack resonator design. The inset shows the proposed submicron racetrack ring resonator coupled to a 2×2 Type I MMI coupler that has a coupling length of $8.2 \mu\text{m}$. A straight section of $2 \mu\text{m}$ is added to the port at each side of the MMI coupler. The straight and bend WGs in the ring cavity as well as the port WGs are all based on silicon-on-insulator rib WG with silicon core $h = 0.4 \mu\text{m}$, a thickness of $2 \mu\text{m}$ for both the upper and lower claddings of silicon dioxide (cf. Fig. 3.1), etch depth $d = 0.37 \mu\text{m}$ and width $w = 0.394 \mu\text{m}$. The bend radius of the ring cavity is $3 \mu\text{m}$.

Thirdly, as the total round-trip loss is quite considerable for a submicron rib based resonator as compared to a channel-based resonator, the finesse and Q factor is only around 8 and 1000 respectively, for both polarizations. Improvement to the finesse and thus the Q factor may be achieved by reducing the round-trip losses by introducing lateral offsets at the straight-bend junctions [20] and adding a series of anti-resonant reflecting optical waveguides [29] on the outside of the bends. However, no attempt on such loss minimization is made as our focus is to demonstrate the feasibility of PI operation in a MRR that is based on deeply etched SOI submicron rib waveguides.

A final remark is that although the total PDL is around 3% in this work, which results in the TE and TM spectra to have slightly different T , actual device fabrication that causes sidewall roughness may increase the total PDL to a level high enough to impact PI operation as the TE mode will generally have higher sidewall roughness loss. However, such sidewall roughness loss can generally be reduced as demonstrated in [16, 17]. Also, other schemes such as WG width optimization and mirror bends [30] can be used to further reduce the PDL, which will improve the PI operation. In addition, the total PDL could be eliminated by balancing the bend losses with the coupling into the cavity. For instance, the higher TE bend losses can be compensated for by using a slightly higher TE coupling factor than the TM. We will not further reduce the PDL in this work as the focus of this work is to achieve PI in terms of the phase response.

3.6. Fabrication Tolerance Analysis

The miniaturization of MRR is not only restricted by the stringent PI conditions and bend losses but also relies heavily on the process technology. Fabrication imperfection generally produces dimensional uncertainties in the actual device. A primary concern, then, is to determine how wide the fabrication tolerances may be, without severely affecting the desired operating performance. In this section, the fabrication tolerance of our proposed PI MRR design is looked into. We will, in particular, investigate the effects of small changes in the geometrical parameters, etch depth d and rib width w , of the WGs of the racetrack resonator from their optimal values on the performance variation. We will not look into the fabrication issues of MMI as it has been shown that it generally has good fabrication tolerance [31] and low birefringence [11].

We have computed the birefringence of the fundamental modes of the straight WG as a function of w for different d in Fig. 3.13 for our WG design of $h = 0.4 \mu\text{m}$, where only the SM dimensions are shown. The results in Fig. 3.13 show that it might be more advantageous to utilize wider WGs as they will result in greater flexibility during fabrication. For example, at $d = 0.39 \mu\text{m}$, introducing an etch depth uncertainty of $\pm 20 \text{ nm}$ gives a change in birefringence ΔN_{eff} of 2.09×10^{-2} for $w = 0.4 \mu\text{m}$. However, for a larger WG width of $w = 0.9 \mu\text{m}$ at the same d and uncertainty, ΔN_{eff} is reduced to 3.02×10^{-3} . In addition, the effect of width uncertainty at a fixed d is also smaller for wider WGs as it is evident in Fig. 3.13 that the gradient of the curves decreases and stabilizes as w increases. If we consider $d = 0.39 \mu\text{m}$ again, introducing an width uncertainty of $\pm 20 \text{ nm}$ gives a change in birefringence ΔN_{eff} of 2.32×10^{-2} for $w = 0.4 \mu\text{m}$ whereas ΔN_{eff} is reduced to 9.29×10^{-4} for the larger w of $0.9 \mu\text{m}$. Any uncertainties in d and w will have little impact on the modal birefringence for wider WG widths because the TE mode is well-confined under the rib region [12]. Thus, wider WGs have better fabrication tolerance.

The points of intersection with the x axis are the critical widths $w_{c(\text{straight})}$ that indicated the ZBC, which can be used for our PI MRR. However, these critical widths are rather narrow and it might be difficult to use them as WGs with narrower width have smaller fabrication tolerances, as concluded above. This suggests that our design for PI WGs might be difficult to implement. However, in general, any fabrication related dimensional uncertainties will meet the criteria for PI ring resonator operation if the resultant mismatch $\Delta\lambda_R$ is still within 15% of the 3-dB resonance bandwidth [11]. To fulfill the above criteria for $\Delta\lambda_R$, the allowed uncertainty in birefringence ΔN_{eff} for the ZBC dimensions is found to be around ± 0.009 . For our modeled MRR, the PI input bus WG is designed at $w_{c(\text{straight})} = 0.39 \mu\text{m}$ at the corresponding etch depth of $0.37 \mu\text{m}$. Using Fig. 3.13, we can determine that the largest possible fabrication tolerances for this PI WG design are about $d \pm 3\%$ and $w \pm 4\%$, in order for the birefringence uncertainty to be within ± 0.009 . The straight WG in the racetrack cavity has w of $0.394 \mu\text{m}$, which can be adjusted to $w_{c(\text{straight})} = 0.39 \mu\text{m}$ with a negligible effect and thus has similar fabrication tolerances as the input bus straight WG. Similarly, the bends with a PI width of

Fig. 3.13. Birefringence of the fundamental modes of the straight WG as a function of width w for various etch depths d , at fixed $h = 400$ nm.

$0.394 \mu\text{m}$ can be adjusted to $w_{c(\text{straight})} = 0.39 \mu\text{m}$, giving a residual birefringence of 0.005. The fabrication tolerances of the bends at $R = 3 \mu\text{m}$ with $w_{c(\text{straight})} = 0.39 \mu\text{m}$ can also be determined by a similar approach of plotting the birefringence curves and are found to be slightly smaller at about $\pm 2\%$ for both d and w in order to satisfy our defined PI criteria. To date, low-loss SOI rib fabrication using deep UV lithography has made it possible to control the WG dimensions within $\pm 1\%$ [32]. This would suggest the possibility of realizing our PI MRR design. With better process controllability and improved accuracy in fabrication technology, better PI performance can be achieved.

It must also be mentioned that the bend losses are dependent on the cross-sectional dimensions of the WG. Hence, any fabrication induced dimensional uncertainties in the cross section of the WG will also affect the bend losses at any fixed bend radius R , which in turn might increase the minimum bend radius (MBR) that could be used. However, we have found that the MBR generally increases by a factor of ~ 2 when d is decreased by $0.1 \mu\text{m}$ and by a factor of ~ 3 for a similar decrement in w for our WG design. This would imply that with an uncertainty of $\pm 1\%$ in d and w , if deep UV lithography is used, the MBR would be relatively unaffected.

3.7. Summary and Significant Contributions of this Research Work

In summary, we have demonstrated the feasibility of realizing MRRs of sub-micron cross-sectional dimensions with a small radius of a few microns that have substantially reduced polarization sensitivity while maintaining the essential SM and low-loss conditions. We have also provided an approximate but reasonably accurate theoretical lower size limit of the MRR for simultaneous SM and PI operations, at our considered upper oxide cladding thickness and using the more robust deeply etched rib WG. To achieve greater size miniaturization, stress engineering [4, 15] could be employed

in conjunction to our suggested proposal. Also, slanted WG sidewalls with varying slant angles can be used to alter the SM and PI conditions [15]. These schemes are not considered in our work as our focus is to determine the extent of PI MRR size miniaturization in an approximate but yet reasonably accurate and realistic manner, in the absence of advanced external birefringence compensation schemes. It is generally not possible to determine an exact theoretical lower size limit as many factors need to be taken into account and unforeseen contingencies might also arise. Our proposed heuristic formulation would thus be sufficient for most practical applications. In addition, if further size miniaturization is required, novel waveguides, like the slot waveguides [14, 33], can be used in the design of the PI MRR.

Our design approach to achieve SM and PI operations for compact resonators is modular and hierarchical. We first show how PI and SM conditions can be achieved for straight waveguides using deeply etched ribs, followed by how birefringence-free and low-loss bends and PI MMI coupler can be designed as key components for a bus-coupled resonator. All the PI components are then integrated into a single-bus filter configuration and the transmittance spectra are simulated for both TE and TM polarizations. The results for a specific design where the core thickness is $0.4\text{ }\mu\text{m}$ show that a resonance mismatch of about 0.22 nm , which represents only 11% of the FWHM linewidth of 2 nm , and a FSR of 12 nm can be achieved. This small polarization dependence represents a large improvement over what can usually be achieved with the conventional PI resonators [3, 4, 10, 11] that have much larger device size (height and width $\geq 1\text{ }\mu\text{m}$, bend radius $\geq 30\text{ }\mu\text{m}$) and very small FSR (in the range of pico-meter). The general conclusion is that compact and intrinsically PI resonator devices can be achieved as long as the components have very small birefringence or are PI. The intrinsic PI device is relatively simple and compact compared with the *extrinsic* PI device that requires more complex circuit designs based on polarization diversity [1, 2] or polarization management architecture [34]. The main limitations of the intrinsic PI design are: (i) The minimum bend radius for PI operation is limited to about $3\text{ }\mu\text{m}$ (for the given core thickness and cladding thickness) as the total bend loss and residual birefringence would increase exponentially for smaller bend radius; (ii) the dimensions of the straight and bend waveguides require very stringent fabrication control as any deviation will throw the birefringence off balance; and (iii) the coupling factor between the bus and the resonator is restricted to 50% as the 3-dB MMI coupler, or for that matter any other coupler, is generally not PI for an arbitrary coupling coefficient. These limitations imply that the resonator device may have a low finesse due to a relatively high loss, and a small FSR due to relatively large cavity size. An approach to enhance the FSR of a filter is to use the vernier effect by cascading several MRRs together [35]. With these extensions, our theoretical design approach could be applied to many more applications using resonators with bend radius $\geq 3\text{ }\mu\text{m}$ that satisfy the PI, SM and low-loss requirements simultaneously.

Finally, note that this research work has been published in the following journal and conference proceeding:

- **Thomas Y.L. Ang**, Soon Thor Lim, Shuh Ying Lee, Ching Eng Png and Mee Koy Chin, “How small can a micro-ring resonator be and yet be polarization independent?” Appl. Opt. 48, 2821-2835 (May 2009).
- **Thomas Y.L. Ang**, Soon Thor Lim, Shuh Ying Lee, Ching Eng Png and Mee Koy Chin, “Polarization Independent micro-ring resonators on submicron Silicon-on-Insulator (SOI) rib waveguides,” IEEE 5th International Conference on Group IV Photonics, September 2008.

Recently, in July 2011, the above published journal paper has been cited in [36], which looked into the design and fabrication of a TE-TM polarization convertor based on the SOI platform. This highlights the significance of our research work in this chapter.

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Chapter 4

Traveling Wave Microresonators (TWMRs) Based Coupled-Resonator Optical Waveguides: Photonic Bandgap Engineering Using Periodic and Quasi- Periodic Orders

4.1. Introduction and Research Objectives

We have in chapter 3 looked into the single-mode and polarization-independent behaviours in a single microring resonator, which is one particular class of traveling wave microresonators (TWMRs). Essentially, a TWMR is a compact waveguide realization of a Fabry-Perot cavity, with the capacity of possessing whispering gallery modes (WGMs) with high-quality Q factor and small modal volume. By coupling various TWMRs together, many interesting phenomena can occur [1-3] and have been used in various applications such as optical filtering [4-7], delay lines [8] and optical buffering [9]. If the number of such coupled cavities is large, it can be regarded as a new type of waveguide known as coupled-resonator optical waveguide (CROW) [1], in which light propagates via photons hopping from one tightly confined mode to the neighboring one due to the weak interaction between them. Studies on TWMR-based CROWs have focused mainly on the properties of the photonic bands for slowing light [1-3] and optical filtering [4-7], with only some coverage [5, 7, 10, 11] on the characteristics of the photonic bandgap (PBG) structures. The PBG structures can generally be modified by breaking the lattice periodicity through the introduction of defects to form *disorder* in the periodic system and this will generate localized states within the PBG. Such a process is commonly known as PBG engineering. The introduced defect allows light to be localized at the defect cavity, forming a localized mode or defect mode. When such a defect mode is coupled to the propagating mode in a finite structure, a high- Q transmission appears at the corresponding resonant frequency. The nature of light localization via implementing disorder in a periodic system has been widely

explored in photonic crystals [12, 13] and has been recently looked into in ring resonator arrays [10], where the effect of a single defect on the PBG structures was investigated.

In this work, we provide an analysis of the effects of PBG structures of direct TWMR-based CROWs [3], whereby all the cavities are directly coupled to one another (cf. Fig. 4.1) in the presence of multiple defects of similar size. Using such defects, which are size-tuned cavities, we will illustrate how PBG engineering can be achieved by implementing either periodic or quasi-periodic order in TWMR-based CROW systems in contrast to using disorder [10-13]. The defects are introduced at specific locations in the CROW so that together with the regular resonators, which are size-untuned cavities, there will be either periodic or quasi-periodic order in the final resonator arrangement of the cascade of TWMRs. This is in contrast to traditional TWMR-based CROWs [14] whereby periodic order is theoretically implemented by cascade of cavities of similar size, without defects. Note that strictly speaking, the normal distribution of defects in real systems is generally neither periodic nor quasi-periodic but random [15]. However, for ease of illustration, we loosely use the term “defects” throughout this theoretical work to refer to the size-tuned cavities or resonators intentionally introduced in the TWMR-based CROW to form either a periodic or quasi-periodic order in the resonator arrangement. Such intentional defects can be created either by physically fabricating resonators of different sizes or by using active tuning, such as thermal heating [16], to phase tune the resonators. A combination of both schemes can also be employed. For the periodic order case, we explore how size tuning the defect in each unit cell affects the PBG structures via modes localization. It will be shown that (1) the coupling between the periodic size-tuned defects results in mode splitting of the single cavity defect mode into multiple distinct localization states that lead to the formation of mini-defect bands and mini-PBGs in what was formerly a single PBG in the original defect-free CROW, and (2) varying the coupling strengths creates a PBG at each location of resonance in the photonic bands.

In addition, deterministic aperiodic (AP) order in a TWMR-based CROW system is also looked into in this work. Traditionally, implementing disorder in a periodic system has been employed to achieve light localization, having its origin in the Anderson localization of electron waves in solid-state physics. However, it has been found that wave localization can occur not only in disordered or absolutely random systems but also in deterministic AP ordered ones [17]. Such AP systems behave much like disordered ones but are constructed based on a deterministic procedure and thus possess order without periodicity. In photonics, deterministic AP order has already been looked into in optical multi-layers [18, 19] and photonic crystals [20-22]. Here, we study the nature of localization using one dimensional deterministic AP order in TWMR-based CROWs. There are different classes of deterministic AP order, mainly quasi-periodic (QP) and fractal. The main difference between QP and fractal-ordered lattices is that the former exhibits two or more incommensurate periods while the latter does not [17-19, 23]. To achieve a strong connection with the theme of the work in the first part where

the effects of periodic order are studied, we will only analyze deterministic QP order as mathematically, quasi-periodic functions can be classified as almost periodic functions [23]. In our work, a QP TWMR-based CROW is designed by cascading together resonators of two different sizes, namely the defect and regular resonators, according to simple rules based on the Fibonacci series, thereby encoding a one dimensional QP order in the resonator arrangement. Note that QP order of higher dimensionality can be generated by using photonic crystals in which *identical* dielectric cylinders are arranged according to specific rules in the absence of defects, as demonstrated in [19, 20], unlike the case of our TWMR-based Fibonacci CROW. It will be shown here that the quasi-periodicity of the Fibonacci CROWs can also generate mini-PBGs and localized states in what was formerly a single PBG of a CROW with identical resonators. This usage of Fibonacci sequences in the design of TWMR-based CROW results in the appearance of a single high- Q localized state that transits to a mini-band within a wide PBG as the number of unit cells increases.

We use the transfer matrix formalism [14] in our analysis, which is physically more convenient over the tight binding approximation [1] and the coupled mode theory [4, 24] as it can better represent the physical parameters such as cavity size and can be easily coupled with the Bloch theorem to study the PBG structures of periodic systems. In section 4.2, we use the transfer matrix formalism to analyze the PBG structures of periodic CROWs in the presence of defect size tuning as well as coupling tuning. The Fibonacci class of quasi-periodic TWMR-based CROWs is then studied in section 4.3. Throughout this work, all the resonators are assumed to be lossless so as to better focus on the transfer matrix analysis. We only discuss the effects of losses in section 4.4. The transfer matrix method is then compared with the finite-difference time domain (FDTD) simulation in section 4.5. Finally, we conclude, highlighting important results and possible extension of this work as well as discussing possible applications of the proposed ideas in section 4.6.

4.2. CROWs with Periodic Order

4.2.1. Coupling Matrix Formalism

The schematic of the CROW with periodic defects is shown in Fig 4.1. We label the layout of the unit cells as $n = 1, 2, 3 \dots N$ and the component resonators in each cell as $n = 1, 2, 3 \dots N'$. In this case in Fig. 4.1, N is taken to be arbitrarily large while $N' = 2$. The component resonators in each unit cell are also termed as *intra-resonators* (labeled **A** and **B**) and the ratio of their cavity lengths is defined as the size tuning factor $\gamma = L_B/L_A = \delta_B/\delta_A$, where δ_A and δ_B are, respectively, the round-trip phase shift in resonators **A** and **B**. Here, resonator **A** is considered as the regular resonator and **B** as the “defect” resonator. The input and output fields to the n^{th} lattice are denoted, respectively, as $(a_{n-1, 2}, d_{n+1, 1})$ and $(a_{n, 2}, d_{n, 1})$, where the first number in the subscript refers to the unit cell while the second number refers to the intra-resonators (1 for resonator **A** and 2 for resonator **B**). The fields in the bus waveguides follow the conventions as that of the unit cell as shown in Fig. 4.1. Within each unit cell,

Fig. 4.1. A traveling wave microresonator-based CROW with N mutually coupled unit cells, consisting of defect **B** at periodic locations.

the input field to the 1st intra-resonator is denoted as $d_{n,2}$, while that for the 2nd intra resonator is $a_{n,1}$. Following the analysis in [24], each coupler is treated as a partially transmitting mirror with real transmittivity (or amplitude cross-coupling coefficient) κ and reflectivity (or amplitude through-coupling coefficient) r . We then denote $(r_{\text{intra}}, \kappa_{\text{intra}})$ as the coupling scheme between **A** and **B** in each unit cell, and $(r_{\text{inter}}, \kappa_{\text{inter}})$ as the inter-unit cell as well as the bus-to-unit cell coupling. Using the nomenclature in Fig. 4.1, the coupling mechanism from $(a_{0,2}, d_{1,1})$ to $(b_{0,2}, c_{1,1})$, is represented as

$$\begin{bmatrix} b_{0,2} & c_{1,1} \end{bmatrix}^T = [g] \begin{bmatrix} a_{0,2} & d_{1,1} \end{bmatrix}^T, \quad (4.1)$$

where $g_{11} = g_{22} = -r_{\text{inter}}$, $g_{12} = g_{21} = j\kappa_{\text{inter}}$ and $j = (-1)^{1/2}$. The fields in the first intra-resonator in the first unit cell can be written as

$$a_{1,1} = c_{1,1}(\tau_1)^{1/2} \exp(-j\delta_1/2), \quad d_{1,1} = b_{1,1}(\tau_1)^{1/2} \exp(-j\delta_1/2), \quad (4.2)$$

where τ_n is the intensity attenuation factor due to losses for one round trip in the n^{th} intra-resonator ($n = 1$ for intra-resonator **A** and 2 for intra-resonator **B**) of each unit cell. Using (4.1) and (4.2), it can be derived that

$$\begin{aligned} \begin{bmatrix} a_{n,2} & b_{n,2} \end{bmatrix}^T &= [S2][S1] \begin{bmatrix} a_{n-1,2} & b_{n-1,2} \end{bmatrix}^T = [U] \begin{bmatrix} a_{n-1,2} & b_{n-1,2} \end{bmatrix}^T, \\ S_{n11} &= -B_1/j\kappa_n, \quad S_{n12} = -r_n B_1/j\kappa_n, \quad S_{n21} = r_n B_2/j\kappa_n, \quad S_{n22} = B_2/j\kappa_n, \\ B_1 &= \sqrt{\tau_n} \exp(-j\delta_n/2), \quad B_2 = \sqrt{\tau_n} \exp(j\delta_n/2). \end{aligned} \quad (4.3)$$

Note that $S_{nlq} = S1_{lq}$ or $S2_{lq}$ denotes for the individual entry of either the matrix $[S1]$ or $[S2]$, where the subscripts l ($= 1$ or 2) and q ($= 1$ or 2) are used to represent the rows and columns of each matrix, respectively. For the reflectivity r_n , when $n = 1$, then $r_n = r_{\text{inter}}$ and if $n = 2$, $r_n = r_{\text{intra}}$ and likewise for transmittivity κ_n . Matrices $[S1]$ and $[S2]$ are the intra-resonator translation matrices that relate the complex amplitudes (a, b) between each intra-resonator, while matrix $[U]$ is the unit cell translation matrix that relates the complex amplitudes (a, b) in one resonator of a unit cell to those of the equivalent resonator in the next unit cell. Matrix U is unitary if the intra-resonators and the couplers are lossless. For N unit cells, the field amplitudes $(a_{N,2}, b_{N,2})$ at the last resonator **B** in the N^{th} lattice can be obtained by iterating the unit cell translation matrix N times via the Chebyshev polynomials [25]:

$$\begin{bmatrix} a_{N,2} \\ b_{N,2} \end{bmatrix} = \begin{bmatrix} U_{11}C'_{N-1}(x) - C'_{N-2}(x) & U_{12}C'_{N-1}(x) \\ U_{21}C'_{N-1}(x) & U_{22}C'_{N-1}(x) - C'_{N-2}(x) \end{bmatrix} \begin{bmatrix} a_{0,1} \\ b_{0,1} \end{bmatrix} = [G] \begin{bmatrix} a_{0,2} \\ b_{0,2} \end{bmatrix}^T \quad (4.4)$$

where $x = \cos \phi = \frac{1}{2}(U_{11} + U_{22})$, U_{lq} is the element or entry of the unit cell translation matrix $[U]$ and $C'_N(x) = \sin[(N+1)\phi]/\sin \phi$ are the Chebyshev polynomials of the second kind. Note that l ($= 1$ or 2) and q ($= 1$ or 2) denotes the rows and columns of the matrix $[U]$.

4.2.2. Band Structures of Infinite CROWs with Periodic Defects

The photonic band of an infinite CROW with periodic defects can be derived by implementing the Bloch theorem to the unit cell transfer matrix in Eq. (4.3). For a periodic structure of infinite length, the fields are periodic at the lattice constant A :

$$\begin{bmatrix} a_{n,2} \\ b_{n,2} \end{bmatrix}^T = \exp(jK\Lambda) \begin{bmatrix} a_{n-1,2} \\ b_{n-1,2} \end{bmatrix}^T, \quad (4.5)$$

where K is the Bloch wave propagation vector. Combining (4.5) with (4.3) gives $\text{Det}[[U] - \exp(iK\Lambda)I] = 0$, where I is the identity matrix and Det is the determinant. Noting that $U_{11}U_{22} - U_{12}U_{21} = 1$ (for unimodular matrix $[U]$) and $\cos \phi = \frac{1}{2}(U_{11} + U_{22})$, it can be evaluated that

$$\exp(jK\Lambda) = 0.5(U_{11} + U_{12}) \pm j\sqrt{1 - [0.5(U_{11} + U_{12})]^2} = \cos \theta \pm j \sin \theta. \quad (4.6)$$

The characteristic equation of the eigenvalue problem of Eq. (4.6) for the lossless case can be expressed as

$$\cos(K\Lambda) = 0.5(U_{11} + U_{12}) = \left\{ r_u \cos[0.5(\delta_1 - \delta_2)] - \cos[0.5(\delta_1 + \delta_2)] \right\} k_u^{-1}, \quad (4.7)$$

where $r_u = r_{\text{inter}} \cdot r_{\text{intra}}$ and $k_u = \kappa_{\text{inter}} \cdot \kappa_{\text{intra}}$. Eq. (4.7) gives the dispersion relation for a CROW of infinite length, with size-tuned periodic defects and is only satisfied if the left most cosine term is between -1 and 1 . For values outside this range, K is purely imaginary and the corresponding frequency range forms the PBG. We now use Eq. (4.7) to analyze the effects of periodic defects on the PBG structures of CROWs in the absence of losses.

4.2.3. Results and Discussions

(a) Effects of Size Tuning the Periodic Defects

In this section, the effects of size tuning the periodic defects on the photonic bands and PBG structures of an infinite CROW with unit cell (**AB**) are looked into. For simplicity, we first consider only small integer values of the tuning factor, $\gamma = 1, 2, 3 \dots$ (i.e., resonator **B** is an integer multiple larger than resonator **A**), and subsequently non-integer values. Displayed in Fig. 4.2 are the dispersion diagrams for an infinite CROW of different integer size tuning γ of the defect. All the resonators have identical coupling and are assumed to be lossless. As an example, we use $\kappa_{\text{inter}}^2 = \kappa_{\text{intra}}^2 = 0.7$ for Fig.

4.2. The resonance order m is defined as $m = f / \Delta f_{\text{FSR}} = \delta / 2\pi$, where f is the eigen-frequency, Δf_{FSR} denotes the free spectral range and we refer to δ_A simply as δ for the round-trip phase shift for the size-untuned (or regular) resonator **A** throughout this work. In the absence of losses, K is purely real at the photonic bands, which is indicated by the solid curves, and is strictly imaginary at the PBGs, represented by the shaded regions in the dispersion diagrams. Initially, when $\gamma = 1$ that corresponds to $L_A = L_B$, the wave propagation takes place within the photonic bands centered at the resonance order of integer m . The PBGs, where wave propagation is forbidden, are centered at half-integer m . Tuning γ to higher integer values simply increases the number of PBGs. For integer values of γ , the number of PBGs within the vicinity of $\delta = \pi(2m + 1)$ is also γ . In Fig. 4.2, we show the full sequence of PBG

Fig. 4.2. The dispersion diagrams of infinite CROW, showing the regions of PBG (shaded) and photonic bands (solid curves) for different integral value of the defect size tuning factor γ : (a) $\gamma = 1$, (b) $\gamma = 2$, (c) $\gamma = 3$, and (d) $\gamma = 4$. Note that PBG stands for photonic bandgap.

for $\gamma = 1$ to 4 within the range $m = 0.5$ to 2.5. Observation of the behaviors of these PBGs shows the following general principles. First, due to the equal number of resonators **A** and **B**, the sequence of PBG is always symmetrically centered at $m = 1.5$, the half-integer value being required by the *anti*-resonance condition for resonator **A**. Then within the sequence, each of the PBG is roughly located at a value of m such that $m\gamma$ is a half-integer. This is the anti-resonance condition for resonator **B**. For example, for $\gamma = 3$, the possible m values within the range 1 to 2 are $m = 7/6, 9/6 = 1.5$, and $11/6$. Similarly, for $\gamma = 4$, these values are $m = 9/8, 11/8, 13/8$ and $15/8$. However, due to the mutual interaction between resonators **A** and **B**, the PBGs are not exactly centered at these values but “pulled in” slightly towards the center ($m = 1.5$). These PBGs are not of equal widths, with those near the center being much broader.

This general principle applies also for non-integer γ , such as when γ ranges between 1.5 and 2.5 (given the way we have defined resonators **A** and **B**, it is not necessary to consider the case of $\gamma < 1$). We would find that the most obvious difference is that the PBGs are no longer distributed evenly about the half-integer m points $\delta = \pi(2m + 1)$, but may shift up or down as γ decreases or increases from the integer value. The location of each PBG is still given roughly by the anti-resonance condition $m\gamma = p + 1/2$ (where p is an integer). The physical picture of the formation of these PBGs will be more clearly illustrated using the case of finite CROW in section 4.2.4.

(b) Effects of Tuning the Intra and Inter-Lattice Coupling Strengths

In this section, we look into the effects of tuning the coupling strengths such that $\kappa_{intra}^2 \neq \kappa_{inter}^2$. For a CROW with resonators that have identical coupling ($\kappa_{intra}^2 = \kappa_{inter}^2 = \kappa^2$), the width of the primary bands centered at $\delta = 2m\pi$ is proportional to the interaction, κ^2 . When the intra-resonator and inter-unit cell coupling strengths are different, i.e. $\kappa_{intra}^2 \neq \kappa_{inter}^2$, it is found that a gap opens up, forming a PBG in the middle of each primary band at integer resonance order m , even without size tuning, i.e. $\gamma = 1$. As shown in Fig 4.3, when $|\kappa_{inter}^2 - \kappa_{intra}^2| = |\Delta\kappa|$ increases, the photonic bands shift

Fig. 4.3. The dispersion diagrams of infinite CROW for $\gamma = 1$: (a) Weaker inter-unit cell coupling ($\kappa_{inter}^2 < \kappa_{intra}^2$): κ_{inter}^2 is fixed at 0.3 while varying κ_{intra}^2 from 0.3 to 0.5, 0.7 and 0.9. (b) Stronger inter-unit cell coupling ($\kappa_{inter}^2 > \kappa_{intra}^2$): κ_{inter}^2 is fixed at 0.9 while reducing κ_{intra}^2 from 0.9 to 0.7, 0.5 and 0.3. Arrows indicate how photonic bands move as $|\Delta\kappa|$ increases, where $\Delta\kappa = \kappa_{inter}^2 - \kappa_{intra}^2$.

in the direction of the arrows and PBG forms and widens within each resonance location of $\delta = 2\pi m$. The width of the PBG formed is proportional to the absolute magnitude of $\Delta\kappa$ and is independent of its sign (i.e. whether κ_{intra}^2 or κ_{inter}^2 is larger). The sign only dictates how the photonic bands move as

$|\Delta\kappa|$ changes. A negative (positive) sign signifies a weaker (stronger) κ_{inter}^2 relative to κ_{intra}^2 and is related to an outward (inward) movement of the photonic bands from the locations of the integer resonance order m as $|\Delta\kappa|$ increases. Note that in Fig. 4.3 we have varied κ_{intra}^2 while keeping κ_{inter}^2 fixed. Similar band diagrams are obtained if κ_{inter}^2 is instead varied with a fixed κ_{intra}^2 but with the sign of $\Delta\kappa$ reversed. An intuitive insight on the dependence of the movement of the photonic bands on $\Delta\kappa$ and the formation and subsequent widening of PBG at resonance when $|\Delta\kappa|$ changes are given in the next section in terms of the spectral properties of a single unit cell of **(AB)**.

4.2.4. Band Structures of Finite CROWs with Periodic Order

(a) Matrix Analysis of Finite CROWs

The dispersion diagrams shown in Figs. 4.2 to 4.3 based on the dispersion relation of Eq. (4.7) strictly apply only for Bloch-wave propagation in an infinite periodic CROW. Any actual physical realization of CROW is of finite length. The exact dispersion relation for a finite CROW has been derived using the tight binding approximation in [26] and the transfer matrix method in [27]. However, it is not necessary to use the exact dispersion relation when modeling finite CROW as the general features of the transmission and phase responses of a finite CROW approximately satisfy the photonic band properties of the dispersion diagrams for an infinite one. The transmissivity T_T of the drop port of the finite CROW is defined as $T_D = c_{N+1,1}/a_{0,2}$ and can be derived by employing Eq. (4.3) and the identity $c_{N+1,1} = j\kappa a_{N,2}$, where $\kappa = \kappa_{inter}$, and assuming that $d_{N+1,1} = 0$, which gives

$$T_D = |c_{N+1,1}/a_{0,2}| \exp(j\Phi_D) = j\kappa_{inter} / (r_{inter} G_{12} + G_{22}). \quad (4.8)$$

Making use of the Chebyshev polynomials in Eq. (4.4), the transmissivity expression of Eq. (4.8) for the general case of N unit cells, each consisting of dual asymmetrical resonators, can be expressed as

$$T_D = -j\kappa_{inter}\kappa_u B_3 \sin(\varphi) / \left[(r_{inter}^2 B_4 - r_u [B_5 + B_6] + B_7) \sin(N\varphi) + \kappa_u B_3 \sin([N-1]\varphi) \right], \quad (4.9)$$

$$\begin{aligned} B_3 &= \exp(j\delta_2), & B_4 &= \exp(-j0.5[\delta_1 - \delta_2]), \\ B_5 &= \exp(j0.5[\delta_1 + \delta_2]), & B_6 &= \exp(-j0.5[\delta_1 - 3\delta_2]), \\ B_7 &= \exp(j0.5[\delta_1 + 3\delta_2]). \end{aligned}$$

We use Eqs. (4.8) and (4.9) to analyze the transmissivity and corresponding phase response of a finite CROW for different defect size tuning in section 4.3.3.

(b) Formation of Localized Defect Bands in Finite CROWs

We first give a physical picture of the formation of mini-passbands or defect bands due to the presence of multiple periodic defects by using a CROW with $7\frac{1}{2}$ unit cells, i.e., $((\mathbf{AB})^7\mathbf{A})$, with each cell having 2 resonators, all with identical coupling strength $\kappa^2 = 0.7$. The defect resonator, denoted as

D, has twice the cavity size of regular resonators (i.e., $\gamma = 2$) for this illustration. Initially, when the CROW contains only 1 defect, a single high- Q defect mode is localized in the PBG as shown in Fig. 4.4(a).

As the number of periodic defects in the CROW increases progressively, the localized states on separated defects interact with one another and their mutual coupling causes the splitting of the original degenerate eigenmodes. The initial splitting of the single cavity defect mode into symmetric and anti-symmetric modes is analogous to the splitting of degenerate atomic levels into bonding and anti-bonding orbitals when 2 atoms interact in a diatomic molecule. The extent of this splitting is proportional to the overlap of the localized eigenmodes of the periodic defects. The overlapping of

Fig. 4.4. The transformation of the state localization within the PBG from a single high- Q resonant state to a mini-defect band as the number of periodic defects in the CROW is increased in the following sequence: (a) $\{0\ 0\ 0\ 0\ 0\ 0\ \mathbf{D}\ 0\ 0\ 0\ 0\ 0\ 0\}$, (b) $\{0\ 0\ 0\ 0\ \mathbf{D}\ 0\ \mathbf{D}\ 0\ \mathbf{D}\ 0\ 0\ 0\ 0\}$, (c) $\{0\ 0\ 0\ \mathbf{D}\ 0\ \mathbf{D}\ 0\ \mathbf{D}\ 0\ \mathbf{D}\ 0\ 0\}$ and (d) $\{0\ \mathbf{D}\ 0\ \mathbf{D}\ 0\ \mathbf{D}\ 0\ \mathbf{D}\ 0\ \mathbf{D}\ 0\ \mathbf{D}\ 0\}$. The resonator defect is represented as **D** with twice the cavity size as the size-untuned or regular resonator 0, i.e., $\gamma = 2$. Note that δ is the round-trip phase shift for the size-untuned resonator **A**.

the localized states will induce the formation of a defect mini-band in what was formerly a PBG in the original defect-free CROW. The final defect band in Fig 4.4(d) is thus the result of the N -fold splitting of the frequency f_0 (which corresponds to $\delta/2\pi = 1.5$ in Fig. 4.4) of the defect resonator into a band of frequencies f . The frequencies $f < f_0$ correspond to the symmetric defect modes while $f > f_0$ are the anti-symmetric defect modes. Note that all the defects in Fig 4.4 have $\gamma = 2$. If the defects have other integer sizes, such as $\gamma = 3, 4 \dots$, then there will be $(\gamma - 1)$ mini defect bands, each with N -fold splitting, as we shall see in the next section. This physical picture of the formation of defect bands via coupled defect states can generally also be applied to the case of quasi-periodic CROWs in the subsequent

section, as it will then be observed that there is *strong periodic correlation* in the quasi-periodic resonator sequence. Besides this work, recent works on quasi-periodic structures in [19-22] and references therein have also given much detailed treatment on the physical origins of the localized states in deterministic aperiodic-ordered systems.

4.2.5. Results and Discussions

(a) Effects of Size Tuning the Periodic Defects in Finite CROWs

We consider the effect of size-tuned defects on the case of a periodic CROW structure with $7\frac{1}{2}$ unit cells: $\{(\mathbf{AB})(\mathbf{AB}) \dots \mathbf{A}\}$ for $\kappa_{intra}^2 = \kappa_{inter}^2 = \kappa^2 = 0.7$ and $r_{intra}^2 = r_{inter}^2 = r^2 = 0.3$. Fig. 4.5 shows the transmission and phase responses of this CROW configuration for each specific value of γ at 1, 2 and 3. It can clearly be seen that the general features of the amplitude responses of this finite CROW agree well with the dispersion diagrams for an infinite CROW as shown in Fig 4.2. Size tuning the defects γ such that γ is an integer introduces an equal number γ of mini-PBGs instead of a single wide PBG. In between the mini-PBGs are passbands with N -fold splitting because of mutual coupling between the N defects (where $N = 7$ for the example in Fig. 4.5). Likewise, there are N -fold ripples, each of π phase shift, in the corresponding phase response at $\delta = \pi(2m+1)$. Note that δ is the round-

Fig. 4.5. Transmissions and corresponding phase responses for a finite CROW with $7\frac{1}{2}$ unit cells for different defect size tuning factor of $\gamma = 1$ (leftmost plots), 2 (central plots) and 3 (rightmost plots) at $\kappa_{intra}^2 = \kappa_{inter}^2 = \kappa^2 = 0.7$ and $r_{intra}^2 = r_{inter}^2 = r^2 = 0.3$. At $\gamma = 1$, there is no defect and thus only 1 wide PBG is formed. Tuning γ above 1 forms mini-PBGs and mini-defect bands in what was formerly a single wide PBG of the original CROW with $\gamma = 1$. Note that δ is the round-trip phase shift for the size-untuned resonator $\mathbf{A}$.

trip phase shift for the size-untuned resonator **A**. Strictly speaking, the stopbands in a finite CROW are only approximately PBG as they exhibit only near-zero amplitude. They occur near where the anti-resonance condition is satisfied by resonators **B**, i.e., $m\gamma = (p + 1/2)$ (where p is an integer). For example, if $\gamma = 3$, then $m = 7/6, 9/6$ (1.5), and $11/6$. However, only at $m = 1.5$ are resonators **A** also at anti-resonance, hence this PBG is the widest and “deepest”.

Similarly, the passbands in between the PBGs occur where some or all of the resonators resonate, i.e., at frequencies where $m\gamma$ is an integer. For example, for $\gamma = 3$, they occur at $m = 1, 4/3$ (1.33), $5/3$ (1.67) and 2. However, only at $m = 1$ and 2 are all resonators in resonance and hence these passbands are broadest and have $2N$ resonant peaks, while the other two passbands are associated mainly with resonators **B** and thus narrower with only N peaks. They may be called the “defect” or secondary passbands and together with the passbands at $\delta = 2m\pi$, which we term as primary passbands, give a multi-passband transmission spectrum.

The transmission of a $7\frac{1}{2}$ unit cells is also illustrated for non-integer γ at $\gamma = 2.3$ and 2.7, as shown in Fig. 4.6. The primary passbands and defect bands are no longer fixed at $\delta = 2m\pi$ and $\delta = \pi(2m+1)$ but generally move *down* the wide PBG when γ is tuned *up* from 2.3 to 2.7. This is in agreement with the characteristics of the dispersion graph for an infinite CROW shown below the transmission graph in Fig 4.6. Mini-PBGs are also formed within the wide PBG. The width of these mini-PBGs can generally be tuned by adjusting the non-integer γ that controls the number and position of the defect bands. Non-integer γ thus provides a more dynamic tuning scheme in contrast to the integer γ .

Fig. 4.6. Transmission (top graph) for a CROW with $7\frac{1}{2}$ unit cells at $\kappa_{intra}^2 = \kappa_{inter}^2 = \kappa^2 = 0.7$ and $r_{intra}^2 = r_{inter}^2 = r^2 = 0.3$ for non-integer size tuning factor $\gamma = 2.3$ and 2.7, which is in good agreement with the dispersion diagram (bottom graph) of an infinite CROW. Arrows indicate directions of movements of *defect bands* with increasing γ . Similar leftward translations also apply to the *primary bands*. Note that δ is the round-trip phase shift for the size-untuned resonator **A**.

(b) Effects of Tuning the Coupling Strengths on the Transmission at Resonance

The effect of tuning the coupling strength in a finite CROW is presented here for two cases: $\kappa_{intra}^2 = \kappa_{inter}^2$ and $\kappa_{intra}^2 \neq \kappa_{inter}^2$. To focus only on coupling tuning without the complication of size tuning, we first assume $\gamma = 1$ in Fig. 4.7. If $\kappa_{intra}^2 = \kappa_{inter}^2$, then all the resonators are identical and also coupled identically and the whole array behaves as a single CROW. The passband at resonance order

Fig. 4.7. The effects of varying coupling strengths on the primary passbands centered at integer resonance order $\delta/2\pi$ (at resonance) for a CROW with 7 unit cells for 2 different cases: (a) and (b). In (a), all the resonators have similar coupling strength, i.e., $\kappa_{intra}^2 = \kappa_{inter}^2 = \kappa^2$; κ^2 is tuned from 0.7 to 0.5 and 0.1. In (b), $\kappa_{inter}^2 \neq \kappa_{intra}^2$; κ_{inter}^2 is fixed at 0.3 while tuning κ_{intra}^2 from 0.3 to 0.5 and 0.9. Note that δ is the round-trip phase shift for the size-untuned resonator $\mathbf{A}$.

$m = 1$ is shown in Fig. 4.7(a) for a CROW with 7 unit cells. Increasing κ^2 enhances the mutual interaction between the resonators and results in broader $2N$ -fold splitting of the resonance. The case of $\kappa_{intra}^2 \neq \kappa_{inter}^2$ is shown in Fig. 4.7(b) for $\kappa_{intra}^2 > \kappa_{inter}^2$. It can be observed that as $|\kappa_{inter}^2 - \kappa_{intra}^2| = |\Delta\kappa|$ increases, the primary passband is split into 2 smaller passbands separated by a mini-PBG. The mini-gap can be widened by simply increasing the intra-resonator coupling relative to the inter-unit cell coupling. The origin of this mini-PBG can be understood as follows: By increasing the difference between κ_{inter}^2 and κ_{intra}^2 , the CROW is no longer a chain of identical resonators but becomes a chain of unit cells, each containing a two-coupled-resonator system, and as the coupling between the resonators increases, the characteristic feature of a strongly-coupled two-resonator system, which is a split resonance with increased splitting, becomes more and more dominant. This additional feature will arise even for the case $\gamma = 2, 3, \dots$ (integer values). A similar effect can be achieved by decreasing κ_{inter}^2 while keeping κ_{intra}^2 fixed. In this case, decreasing κ_{inter}^2 reduces the linewidth, and thus lowers and broadens the transmission dip between the symmetric and anti-symmetric modes. With a suitable

choice of $\Delta\kappa$ for a unit cell so that a stopband forms, a PBG can thus be formed *at resonance* by directly coupling N unit cells together in a CROW. This is illustrated in Fig. 4.8. The transmission for a unit cell is shown in Fig. 4.8(a). For the transmission of a CROW shown in Fig. 4.8(b), N unit cells are coupled together resulting in two N -split Lorentzians within $\delta = 2\pi m$ that are separated from each other by a stopband. As the number of unit cells increases, the stopband deepens and broadens such that an approximate PBG results at resonance. Thus, in summary, *coupling tuning* may be useful, along with the earlier discussed *size tuning*, to introduce more flexibility in generating multi-passband transmission as well as PBG engineering at both on and off resonance.

Fig. 4.8. The origin of the mini-PBG at resonance: (a) The effect of increasing κ_{intra}^2 while keeping κ_{inter}^2 fixed at 0.3 for a single unit cell with 2 identical resonators ($\gamma = 1$), and (b) the effect of increasing the number of unit cells N of a CROW that has the coupling scheme of $\kappa_{inter}^2 = 0.3$ and $\kappa_{intra}^2 = 0.9$. Note that δ is the round-trip phase shift for the size-untuned resonator **A**.

(c) Unit Cell with other Configurations

For this work, we have focused on PBG engineering of CROW based on the unit cell of (**A B**) for simplicity. In principle, it is also possible to change the spectral and PBG properties by using any arbitrary combinations of resonators of different size and number in each unit cell. To illustrate this, we have shown in Fig. 4.9 the dispersion diagram of an infinite CROW of a different unit cell design of (**ABB**), where $\gamma = L_B/L_A = 2$ in Fig 4.9(a) and $\gamma = 3$ in Fig 4.9(b). The corresponding transmission for a finite CROW with around $7\frac{1}{2}$ unit cells ($(\text{ABB})^7 \text{A}$) with $\kappa_{intra}^2 = \kappa_{inter}^2 = \kappa^2 = 0.7$ and $r_{intra}^2 = r_{inter}^2 = r^2 = 0.3$ are shown at the top of each dispersion diagram. In contrast to the perfect CROW of (**AAA...A**), a CROW with unit cell of (**ABB**) produces 3-fold splitting in the passband and p -fold splitting in the stopband where p depends on γ . For $\gamma = 2$, $p = 3$ while for $\gamma = 3$, $p = 5$. This splitting into p mini-PBG occurs for two reasons. First is the splitting similar to that in a CROW with unit cell

(AB) as shown earlier in Fig. 4.5. A further splitting occurs in the secondary passbands within the stopband because there are actually two coupled **B** resonators in the unit cell (**ABB**). The transmission of the finite CROW matches well with the dispersion diagram at the defect region around non-integer m , except with just $7\frac{1}{2}$ unit cells and the 3-fold splitting, the mini-PBGs in the primary passbands around integer m are not fully formed.

Fig. 4.9. The transmission of a CROW with $7\frac{1}{2}$ unit cells of configuration $((\mathbf{ABB})^7\mathbf{A})$ with tuning factor: (a) $\gamma = 2$ and (b) $\gamma = 3$. For each respective γ , the dispersion diagram of infinite CROW is shown below the corresponding transmission graph. The PBGs are marked by the shaded regions while the photonic bands are represented by solid curves. Note that δ is the round-trip phase shift for the size-untuned resonator **A**.

(d) Effects of Addition of New Defects

Up till now, we have only looked into the effects of periodic defects, all of similar size tuning. It is also possible to introduce a new defect, which we denote as **G**, into the periodic CROW to generate additional localized states. We have shown in Fig. 4.10 the PBG at $\delta = \pi(2m+1)$ for a periodic CROW of $6\frac{1}{2}$ unit cells before (shown in dotted lines) and after (shown in solid lines) the defect **G** is added at the centre of the CROW. The new defect **G** is thus embedded between two equal arrays of unit cell of dual asymmetrical resonators so that the CROW arrangement becomes: $\{0 \mathbf{D} 0 \mathbf{D} 0 \mathbf{D} \mathbf{G} \mathbf{D} 0 \mathbf{D} 0 \mathbf{D} 0\}$, resulting in *disorder* in the periodic CROW. We set the cavity size **D** and **G** to be twice and thrice that of the size-untuned resonator 0, respectively. To excite the defect mode of **G**, we use $\kappa_{intra}^2 = \kappa_{inter}^2 = \kappa^2 = 0.7$ and $r_{intra}^2 = r_{inter}^2 = r^2 = 0.3$. This results in a new localized state within each of the original mini-PBGs that are generated by the cascade of $\{0 \mathbf{D}\}$. This shows the possibility of PBG

Fig. 4.10. The PBG structure *before* (in solid lines) and *after* (in dotted lines) a new defect **G** is added at the center of the periodic CROW. The coupling coefficients are set at $\kappa_{intra}^2 = \kappa_{inter}^2 = \kappa^2 = 0.7$ and $r_{intra}^2 = r_{inter}^2 = r^2 = 0.3$. Addition of a new defect **G** changes the CROW sequence from $\{0 \mathbf{D} 0 \mathbf{D} 0 \mathbf{D} 0 \mathbf{D} 0 \mathbf{D} 0 \mathbf{D} 0\}$ to $\{0 \mathbf{D} 0 \mathbf{D} 0 \mathbf{D} \mathbf{G} \mathbf{D} 0 \mathbf{D} 0 \mathbf{D} 0\}$ and generates a new localized state (marked as ‘G’) in each of the mini-PBGs. Note that δ is the round-trip phase shift for the size-untuned resonator **A**.

engineering within these mini-PBGs by implementing *disorder* in the original periodic-ordered system. As **D** is twice the size of resonator 0, there is 1 defect band and 2 mini-PBGs within the wide PBG around $\delta = \pi(2m+1)$, in accordance with our discussion earlier. The defect band has 6-fold splitting corresponding to the total number of defect **D**. The new localized “defect mode” of **G** appears as a sharp resonance line in the middle of each mini-PBG in this case. More generally, the position of the localized state of **G** can be tuned by its size relative to the regular resonator 0. As mentioned earlier, the passbands in between the PBGs occur where some or all of the resonators resonate. The localized state within each mini-PBG can thus only be observed if the defect mode of **G** is coupled to the propagating mode of the resonator array. This can only take place if the resonator array is finite and when the reflectivity $r_{intra}^2 = r_{inter}^2 = r^2$ is small enough to excite this new defect mode of **G**.

4.3. CROWs with Quasi-Periodic Order

4.3.1. Theory and Coupling Matrix Formulation

So far, we have only considered CROW with periodic size tuning where all unit cells are identical. In this section, we consider a deterministic aperiodic array where each “unit cell” is different, containing a different number of resonators determined according to a certain design rule. Specifically, we consider the case where the sequence of resonators in the array is given by the

Fibonacci number sequence $F_n = (1, 1, 2, 3, 5, 8, 13, \dots)$, where the n^{th} member of the sequence is defined by the recursive relation: $F_n = F_{n-1} + F_{n-2}$, where we define $F_0 = 0$ and $F_1 = 1$. Using two resonators labeled as **A** and **B**, the CROW analog of the Fibonacci number sequence can be generated by the above-mentioned recursive rule. Starting with $C_0 = \{\mathbf{A}\}$ and $C_1 = \{\mathbf{B}\}$, subsequent sequences of the Fibonacci CROW can be generated as follows:

$$\begin{aligned}
 C_2 &= \{A, B\}, \\
 C_3 &= \{B, A, B\}, \\
 C_4 &= \{A, B, B, A, B\}, \\
 C_5 &= \{B, A, B, A, B, B, A, B\}, \\
 C_6 &= \{C_4, C_5\}, \\
 C_7 &= \{C_5, C_6\}, \\
 &\vdots \\
 C_{N+1} &= \{C_{N-1}, C_N\}.
 \end{aligned} \tag{4.10}$$

In this way, we will get resonator CROW arrangement that is perfectly ordered but is not periodic. Thus, such CROWs fall in between periodic ordered and disordered systems: Their sequences are aperiodic but they are generated by a known ordering rule, thus forming a *deterministic aperiodic order* in the resonator arrangement. The deterministic aperiodic order generated by the rules of the Fibonacci sequence is generally known as *quasi-periodic* [23]. Fig. 4.11 shows a 5-member quasi-periodic CROW based on the Fibonacci sequence of order 4, or C_4 .

Fig. 4.11. A traveling wave microresonator-based quasi-periodic CROW of 5 resonators, following the Fibonacci sequence $C_4 = \{\mathbf{A}, \mathbf{B}, \mathbf{B}, \mathbf{A}, \mathbf{B}\}$.

Using the convention in Fig. 4.11 and following the previous matrix formulation in Eq. (4.3), the field amplitudes (a_{N+1}, b_{N+1}) in the N^{th} lattice of the quasi-periodic CROW can be defined as $[a_{N+1} \ b_{N+1}]^T = [H_N] [a_1 \ b_1]^T$, where $[H_N]$ is the transfer matrix for the Fibonacci sequence C_N of order N , which satisfies the same recursive relation: $[H_N] = [H_{N-1}] [H_{N-2}]$. Hence, for C_4 : $[H_4] = [H_3] [H_2] = [H_3] [A] [B] = [A] [B] [B] [A] [B]$, where $[A]$ and $[B]$ are the transfer matrices of unit cell (**A**) and (**B**), respectively. The transmissivity $T_D = C_{N+1}/a_1$ of the finite quasi-periodic CROW of order N can be derived by substituting the matrix elements H_{Nlq} for G_{lq} in Eq. (4.8), and we will use it to analyze

the transmission spectra of the Fibonacci class of CROWs. For simplicity, all the resonators have identical coupling coefficients of r and κ . Note that the subscripts l ($= 1$ or 2) and q ($= 1$ or 2) are used to represent the rows and columns of each matrix, respectively.

4.3.2. Results and Discussions

It can be seen in Fig. 4.12 that even for such quasi-periodic arrays, PBGs are still formed around $\delta = \pi(2m+1)$ and state localizations near these PBGs are still possible. The general trend is that with increasing order of the Fibonacci sequence, which leads to more resonators in the CROW, the resonance spectrum at $\delta = \pi(2m+1)$ in between the mini-PBGs develops from a single high Q localized resonant peak to a mini-band. The spectral shape of the mini-band will evolve to take on more complicated and interesting appearance with a higher order of the Fibonacci sequence. At the same time, the resonance bands at $\delta = 2\pi m$ become significantly broadened with more transmission dips and splittings. It is generally more difficult to excite the localized states centered at $\delta = \pi(2m+1)$

Fig. 4.12. Transmission spectra of a quasi-periodic CROW following the Fibonacci sequence of (a) C_3 , (b) C_4 , (c) C_5 and (d) C_6 , all with $\gamma = 2$ and using coupling coefficients of $\kappa^2 = 0.1$ and $r^2 = 0.9$.

when the number of resonators in the quasi-periodic CROW is large as more light is being reflected. Thus, r^2 is set to 0.3 for the Fibonacci sequences C_5 and C_6 , while for C_3 and C_4 , r^2 can take higher value of 0.9 for the resonance centered at $\delta = \pi(2m+1)$ to be noticeable. Note that δ is the round-trip phase shift for the size-untuned resonator **A**. As a general rule, as the number of resonators increases for higher order of the Fibonacci sequence, more transmission dips and ripples develop at both the primary and secondary passbands, which are also widened, as shown in Fig 4.12. This trend as well as the presence of mini-bands and mini-PBGs around $\delta = \pi(2m + 1)$ bear close resemblances to the PBG structures of periodic CROW discussed earlier in section 4.2, thus relating quasi-periodic ordered

resonator systems with periodic ones. It can be suggested that the presence of mini-PBGs and mini-bands in the PBG structures of quasi-periodic Fibonacci TWMR-based CROWs is a direct result of the *strong periodic correlation* in the Fibonacci sequence, which is also highlighted in [19]. For quasi-periodic CROW systems like the Fibonacci CROWs, there exist short-range periodic orders among the constituent sequences of the resonator cascade while the earlier discussed periodic resonator systems in section 4.2 has long-range periodic orders. In their PBG structures, therefore, quasi-periodic systems will exhibit several spectral properties of their periodic counterparts while at the same time have distinctive features, which are the signatures that characterize their quasi-periodic nature.

The results of Fig. 4.12 are for the cavity size ratio of $\gamma = 2$. In particular, C_3 and C_4 each exhibit a single high- Q resonant peak, which makes them suitable for ultra-narrow passband applications. The number of such ultra-narrow defect modes around $\delta = \pi(2m+1)$ for the Fibonacci CROW of C_4 is equal to $(\gamma - 1)$ for integer γ , as illustrated in Fig. 4.13. For order $N > 4$, the defect bands will generally have more complex shapes, which depend also on γ if other values of γ are used.

Fig. 4.13. Transmissions of quasi-periodic CROWs following the Fibonacci sequence of C_4 at $\kappa^2 = 0.1$ and $r^2 = 0.9$ for three different size tuning ratios: $\gamma = 2$ (leftmost plot), 3 (central plot) and 4 (rightmost plot). There are $(\gamma - 1)$ localized states within each wide PBG centered at half-integer resonance order.

4.4. Effects of Losses

Thus far, we have assumed all the resonators to be lossless. In this section, we model the effect of losses for a periodic CROW with $3\frac{1}{2}$ unit cells: $\{(\mathbf{AB})(\mathbf{AB})(\mathbf{AB})\mathbf{A}\}$, and an quasi-periodic CROW of C_4 as discussed in section 4.3. The resonator losses of a TWMR is defined using the round-trip attenuation factor τ (for a lossless resonator, $\tau = 1$), which is denoted as $\tau_A = \tau$ for the intra-resonator $\mathbf{A}$ and $\tau_B = \tau^\gamma$ for the intra-resonator $\mathbf{B}$, and is mainly due to sidewall roughness induced-scattering

losses and bending losses. In the presence of losses, the PBG is not strictly formed or defined as the propagation vector will be complex within these entire bands.

As shown in Fig. 4.14, it is found that even a small loss has a detrimental effect on the amplitude response. For satisfactory amplitude response, the round-trip attenuation factor τ_A must be larger than 0.99 for the periodic CROW with $3\frac{1}{2}$ unit cells. With increasing number of resonators, the amplitude response also suffers greater attenuation.

Fig. 4.14. The effects of losses on transmissivity: (a) Periodic CROW structure with $3\frac{1}{2}$ unit cells of sequence $\{A, B, A, B, A, B, A\}$, with $\gamma = 2$ at $r^2 = 0.3$, and (b) quasi-periodic CROW based on the Fibonacci sequence $C_4 = \{A, B, B, A, B\}$ with $\gamma = 2$ and $r^2 = 0.9$.

In conclusion, all CROWs are suitable for most practical applications only if all the resonators have very low losses, $\tau > 0.99$. This requirement might severely restrict the practical implementation of our proposed CROW designs. However, it has already been experimentally demonstrated [9, 28] that very large arrays of high- Q (low losses) microrings resonators with small bend radii for filter applications are possible. This implies that it is possible to achieve a round-trip amplitude transmission factor of $\tau > 0.99$. Similarly, the ultra-narrow high- Q resonance peak of the Fibonacci CROW as shown in Fig. 4.14(b) is very sensitive to and quickly degraded by any slight loss. However, as long as the losses can be kept to a minimal, such deterministic aperiodic structure will be suitable as a compact sensor due to the presence of sharp, high- Q resonance peaks (cf. Fig. 4.14(b) for $\tau = 1$ at $\delta/2\pi = 1.5$) in the transmission spectrum.

4.5. Comparison with FDTD Simulations

The transfer matrix model used so far is only approximate as it neglects several factors which will become apparent when compared with the benchmark Finite Difference Time Domain (FDTD)

method [29]. Below we present this comparison for the Fibonacci sequence C_4 using traveling wave microring resonators. In the FDTD simulation, a Gaussian pulse is launched into the input bus of the quasi-periodic CROW. In order to reduce computational resources and time, the CROW is designed on high-index contrast silicon-on-insulator photonic wire, which allows each ring to have a relatively small footprint of bend radius $2\ \mu\text{m}$ and $1\ \mu\text{m}$, respectively, for rings **A** and **B**. To obtain high- Q resonances, the waveguide-ring and ring-ring separations are set to be relatively large such that all the rings are weakly coupled ($r^2 = \sim 0.9$). The transmission at the output port, normalized to the input power, is shown in Fig. 4.15.

Fig. 4.15. FDTD simulated normalized transmission at the output port of a quasi-periodic CROW with the Fibonacci sequence $C_4 = \{\mathbf{A}, \mathbf{B}, \mathbf{B}, \mathbf{A}, \mathbf{B}\}$ and of $\gamma = 2$, using SOI microring resonators. The FDTD simulated results are consistent with the general trends of the analytical modeling of C_4 in Figs. 4.12(b) and Fig. 4.14(b).

In general, both the FDTD and transfer matrix method give similar trends in the transmission spectrum, with localized states generated within a PBG sandwiched between two primary passbands with resonance splitting. However, this agreement stops here and there are significant differences, in that the output is comparatively lower, and the primary passbands are rather asymmetrical, in the FDTD case. These are due mainly to the effect of bend losses. The transmission spectrum is expected to improve if larger ring radius (which will give smaller bend losses) and smaller grid size are used. This will, however, increase the computational time. We note that the transmission of the localized mode is generally lower at lower wavelengths and this is due to the wavelength dependence of the coupling strengths and losses, which is not accounted for in our transfer matrix model.

Also, there is a slight shift of the localized mode away from the centre of the PBG. This is most likely due to mesh discretization in the FDTD [4] as well as the effect of self-coupling that will cause a frequency shift but is not taken into consideration in the transfer matrix model. Finally, we note that FDTD can be used to analyze the effect of small deviations in γ from integer values. In general, the results are expected to be more complicated and irregular due to the presence of the Vernier effect.

4.6. Summary and Significant Contributions of this Research Work

In summary, we have studied, to our knowledge, for the first time, the effects on the PBG structures of TWMR-based CROWs in the presence of multiple periodic and quasi-periodic “defects”, all of similar sizes. Due to space limitation, we have presented only the most basic configurations with more intuitive results in this work. In this section, we discuss their potential applications and also possible variations and extension for our future work.

For the periodic case, the interaction of the multiple defects results in splitting of the eigenmodes corresponding to distinct localized states, leading to the formation of defects-induced mini-passbands within the broader PBG. The number and position of such narrow defect passbands can be tuned by adjusting the relative size of the intra-resonators. If the intra and inter unit cell coupling strengths are asymmetrical, new PBG forms at resonance. Other possible applications include multi-passband filters after the ripples or N -fold splittings in the transmission spectra are flattened by apodization techniques highlighted in [30]. To borrow further from the electronic analogy, we can consider a random distribution of defects, which is the normal state of defects, instead of the periodic distribution, and also “cluster” defects such as {AAAABBBAAAAABBBBAAAAA...}.

Somewhat more orderly than the random distribution is the quasi-periodic system exemplified by the Fibonacci sequence studied in Section 4.3, where we have shown that a sharp localized resonant state still exists within the PBG that slowly transforms to a broad passband as the order of the Fibonacci sequence is increased. If the total losses are sufficiently low, the ultra-narrow passband of the quasi-periodic CROW based on a low-order Fibonacci sequence, such as C_4 , may be highly useful as a compact sensor for bio-photonic and chemical applications. Deterministic aperiodic CROWs based on other sequences, such as the Thue–Morse [19, 20] and Cantor [23] sequences, and also hybrid ordered system whereby there is a mix of both periodic and quasi-periodic orders are also possible and are expected to show interesting spectral properties. As a general rule, systems with a perfect order or a complete disorder are simple, and it is in the middle where there is a mix of order and disorder that complexity arises that may produce novel properties and applications. Finally, we may also consider different types of arrays such as SCISSOR [11] for both the periodic and quasi-periodic designs considered in this work.

Note that the focus of this work is on how the *PBG structures* are affected by periodic and quasi-periodic orders, using *multiple defects*. As such, several other optical behaviors of our proposed devices are not covered in this work. One of these is the dispersion of the devices, which besides losses, is the primary restriction to the application of these resonator arrays. With suitable choice of coupling strengths, the dispersion can generally be reduced. For other applications, the dispersion can be turned into an advantage, and hence is an interesting subject for further study.

Finally, we mention that this research work has been published in the following journal:

- **Thomas Y. L. Ang** and Mee-Koy Chin, “The effects of periodic and quasi-periodic orders on the photonic bandgap structures of coupled microring resonators optical waveguides,” *Optics Express* 17, 5176-5192 (January 2009).

In a recent publication [31] dated May 2010, Chremmos et al. used periodic order consisting of alternating defects, which is identical to the one in this chapter (cf. Fig. 4.1), to improve the slow light behaviours in CROWs that is based on travelling wave microresonators. Though their work did not cite our above published paper dated January 2009, this, nevertheless illustrates the significance of our work in this chapter and also verified our above prediction (cf. seventh line of this page, which is also published in our above paper) that the dispersion of CROWs with periodic or quasi-order can be turned into an advantage. However, we have limited our study of CROWs in this thesis merely to the area of photonic bandgap engineering and not look further into their uses in slow and fast light generation. This is because the ripples in the transmission bands of the CROWs will limit the delay-bandwidth of the device [32]. Moreover, owing to the substantial length of the CROWs, valuable estate space is consumed in the photonic circuit. As such, in part two of this thesis, we look into slow and fast light generation in a single TWMR and in twin-coupled TWMRs, which will bring about greater simplicity in the design and yet with similar or better (slow and fast light) performances as compared to conventional CROWs and the CROWs with periodic and quasi-periodic orders in this chapter.

4.7. References for Chapter

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Chapter 5

Raman Scattering Processes in Submicron Silicon-on-Insulator Waveguides: Harnessing Temperature Effects

5.1. Introduction: Research Motivations and Objectives

In the last few chapters, we have focused our attention on the use of the silicon-on-insulator (SOI) (or in short, silicon) platform for passive waveguide (WG) applications at the submicron scale. For silicon-based photonic integrated circuits to have widespread commercial applications, both passive and active WG devices would be required. However, it is difficult to create active photonic WG devices using silicon due chiefly to its unfavourable physical properties, such as the near-absence of the Pockel's effect caused by the centrosymmetric crystal structure and the lack of efficient optical transitions due to the indirect band structure [1, 2]. To circumvent this, Raman scattering (RS) was proposed in 2002 [3]. This approach is effective due to: (1) The large Raman gain coefficient of silicon ($\sim 10^4$ times larger than the silica fiber glass), which, when combined with the small modal area in a silicon WG (~ 100 times smaller than the silica fiber glass), results in a large increase in optical intensity in the WG core; and (2) the large first-order RS frequency shift of silicon (± 15.6 THz at $T = 298$ K), which enables applications in the communication frequency windows [1, 2]. Consequently, a variety of RS processes [4-6] in silicon WGs was demonstrated in 2002 to 2004. Since then, RS has been extensively studied for optical communications functions, such as lasing [7-9], optical limiting [10], amplification [10-12], attenuation [13, 14] and parametric wavelength conversion [15, 16] in silicon photonic WG devices. We will focus on the last three optical functions in this chapter.

In order to simplify the analysis, existing studies on RS-based silicon WG devices have generally assumed a fixed operating temperature of $T = 298$ K (room temperature). Investigations on the effects of temperature variation on RS in silicon have been chiefly limited to bulk silicon [17-19]. Recently, such studies are extended to silicon nanowires [20-22] and micrometer-scale silicon WGs [9, 23], with the emphasis placed on either lasing or the general characteristics of RS. In contrast to these works, here we study, for the first time, to the best of our knowledge, the temperature dependence of RS in submicrometer-scale silicon WG for $100 \text{ K} \leq T \leq 500 \text{ K}$ with regard to three

specific types of RS-based functionalities in the continuous-wave regime: Raman amplification and attenuation near the C-band wavelength of $1.55\ \mu\text{m}$ and parametric Raman wavelength conversion (PRWC) between the C-band wavelength of $1.55\ \mu\text{m}$ and O-band wavelength of $1.31\ \mu\text{m}$.

Our motivation for this work stems from our belief that instead of viewing temperature change in submicron silicon WGs as deleterious [24], it could instead be used to tune the Raman performances of the WGs. It will be shown that by suitably detuning the operating temperature away from the room temperature and with the appropriate phase-matching condition, three types of Raman-based functionalities — Raman amplification, Raman attenuation and PRWC — can be achieved in a submicron silicon WG. To realize single-mode condition in the submicron WG, our earlier research results in chapter 2, which has been published in [25], will be utilized in this chapter. Submicron channel WGs (cf. Fig. 5.1) instead of rib WGs in [15] will be the main focus in this chapter as they have better modal power confinement, which will enhance the RS. In addition, we will deal exclusively with RS in the channel WGs that are fabricated parallel to the $[110]$ direction on a silicon $[001]$ surface (cf. Fig. 5.1) as such orientation is the preferred choice in commercial applications due to its favourable cleaving property in fabrication. A theoretical approach via the coupled mode formalism [7, 15] that considers the various key factors — pump depletion, two-photon absorption, free-carrier absorption, sidewall roughness, pump-to-signal-intensity ratio and phase matching conditions — that affect RS in submicron silicon WGs, is used to analyze Raman amplification/attenuation and PRWC at different temperatures in this work. Design rules for efficient Raman amplification/attenuation and PRWC in submicron silicon WGs when harnessing the effects of temperature variation are thereafter established.

Fig. 5.1. (a) 3-D view of a submicron silicon-on-insulator (SOI) channel waveguide. The upper cladding of silicon dioxide of thickness l_1 that covers the silicon core is not shown here. (b) Computed TE_{00} and TM_{00} modal profiles at $\lambda = 1.55\ \mu\text{m}$ and $T = 298\ \text{K}$ for the single-mode SOI channel waveguide with $w = 0.445\ \mu\text{m}$, $h = 0.22\ \mu\text{m}$, $l_1 = l_2 = 3\ \mu\text{m}$ and $L = 2\ \mu\text{m}$.

This chapter is organized as follows. In section 5.2, we review the basic theory of Raman scattering (RS). In section 5.3, we first analyze the temperature dependence of RS in silicon at the microscopic level, followed by a theoretical characterization of Raman amplification/attenuation and parametric Raman wavelength conversion (PRWC) in submicron silicon channel WGs at different net operating temperatures. In section 5.4, we look into practical design issues pertaining to the tuning of the temperature and bandwidth of the proposed Raman WG model. Finally, we conclude and highlight the important contributions of this research work in section 5.5.

5.2. Theoretical Background

The lowest-order nonlinear effects in silicon stems chiefly from its third-order nonlinear susceptibility $\chi^{(3)}$ (as silicon is centrosymmetric) [2], which has two main contributions [3, 26]:

$$\chi_{iplq}^{(3)} = \chi_{iplq}^{NR} + \chi_{iplq}^R, \quad (5.1)$$

where χ in Eq. (5.1) are all fourth-order tensors, the indices (i, p, l, q) are permutations of the crystallographic axes (x, y, z) , χ^{NR} is the electronic-non-resonant susceptibility while χ^R is the Raman-resonant susceptibility. In general, nonlinear effects (such as self-phase modulation (SPM), cross-phase modulation (XPM) and four-wave mixing (FWM)) due to χ^{NR} are too weak for practical applications in submicron silicon WG devices. Fortunately, χ^R is much larger than χ^{NR} ($\chi_{iplq}^R \approx 44 \chi_{iplq}^{NR}$ [15]), making Raman scattering (RS) the most suitable mechanism for nonlinear and active functionalities in silicon integrated photonics.

Essentially, Raman scattering is the inelastic scattering of the incident light by optical phonons. This is elaborated as follows. When the incident photons, each with energy $2\pi\hbar f_p$, frequency f_p and wavelength λ_p ($\hbar$ is the reduced Planck's constant), propagate through an optical material, they excite vibrational transition of the molecules. Consequently, each incident photon exits the optical material at a slightly different frequency (f_s or f_a) or wavelength (λ_s or λ_a) so as to satisfy energy conservation with each excited molecule. In other words,

$$f_s = f_p - f_v(T) \Rightarrow \lambda_s = c(f_p - f_v(T))^{-1}, \quad (5.2a) \quad f_a = f_p + f_v(T) \Rightarrow \lambda_a = c(f_p + f_v(T))^{-1}, \quad (5.2b)$$

Equation (5.2a) applies if the molecule is initially in the ground state (cf. Fig. 5.2(a)). In this case, energy is absorbed from the photon to form a phonon (with vibrational energy $2\pi\hbar f_v$ and Raman resonance frequency f_v), causing the output photon to exit at a lower frequency f_s (or higher wavelength λ_s). This is known as Raman Stokes scattering (RSS) and the output photon (wave) is termed as a Stokes photon (wave). On the other hand, if the molecule is already in an excited vibrational state (cf. Fig. 5.2(b)), a phonon is annihilated. Then the photon exits at a higher frequency f_a (or lower wavelength λ_a), which is described by (5.2b). This is called Raman anti-Stokes scattering (RASS) and the output photon (wave) is referred to as anti-Stokes photon (wave). In general, RSS is

dominant over RASS as the Stokes to anti-Stokes output intensity ratio, which we term as I_c , is described as

$$I_c = I_s/I_a = (1 + Z(f_v))/Z(f_v), \quad (5.3a) \quad Z(f_v) = \left\{ \exp(2\pi\hbar f_v(T)/[K_B T]) - 1 \right\}^{-1}, \quad (5.3b)$$

where I_s and I_a are the intensities of the Stokes and anti-Stokes waves, respectively, K_B is the Boltzmann's constant and T is the temperature. Note that all parameters in this work are in SI units, unless otherwise stated.

Fig. 5.2. Energy diagrams illustrating (a) Spontaneous Raman Stokes scattering, (b) Spontaneous Raman anti-Stokes scattering, (c) stimulated Raman Stokes scattering and (d) coherent anti-Stokes Raman scattering. Here, we use the angular frequency ω instead of the frequency f that is employed in the text.

The above-mentioned processes of RSS and RASS are usually weak and spontaneous. With an intense input laser pump, the stimulated Raman scattering (SRS) can occur. Through this, either Raman amplification/attenuation (of the Stokes wave) or parametric Raman wavelength conversion (PRWC) (between the Stokes and anti-Stokes wave) can be achieved, depending on the phase matching condition $\Delta\beta = 2\beta_p - \beta_s - \beta_a$, where $\beta_q = 2\pi N_{eff(q)}/\lambda_q$ and $N_{eff(q)}$ are, respectively, the propagation constant and effective index of each type of wave, with the subscripts $q = p$, $q = s$ and $q = a$ denoting, respectively, the pump, Stokes and anti-Stokes waves. At large $|\Delta\beta|$, Raman amplification (cf. Fig 5.2(d)) or Raman attenuation of the Stokes wave occurs. However, as $|\Delta\beta|$ tends to zero, Raman amplification/attenuation is suppressed and the dominant stimulated Raman scattering process is coherent anti-Stokes Raman scattering (CARS) (cf. Fig 5.2(d)), which involves nonlinear four wave mixing (FWM): $f_a = 2f_p - f_s$. In this case, the Stokes, anti-Stokes and pump waves undergo parametric coupling through the mediation of coherently generated phonons, leading to the coherent transfer of energy and information between the Stokes and anti-Stokes waves. Such a process is termed as CARS-assisted parametric Raman wavelength conversion or in short, parametric Raman wavelength conversion (PRWC).

5.3. Temperature Effects on Raman Performances in Submicron Silicon Waveguides

5.3.1. Microscopic Analysis

It will be instructive to first analyze the role of temperature in RS in silicon at the microscopic level. This can be achieved by noting that the Raman susceptibility tensor $\chi_{iplq}^R(\Omega_d, T) = \text{Re}[\chi_{iplq}^R(\Omega_d, T)] + j \text{Im}[\chi_{iplq}^R(\Omega_d, T)] = \chi_{iplq}^{R'}(\Omega_d, T) + j \chi_{iplq}^{R''}(\Omega_d, T)$ in Eq. (5.1) can be written as [2, 17-22, 26, 27]

$$\chi_{iplq}^R(\Omega_d, T) = 2\Gamma(T)f_v(T) \frac{2cn_{si}(f_d, T)g_{eff}(f_d, T)}{f_d(\mu_o/\epsilon_o)^{1/2}} \frac{\sum_{g=1,2,3} (R_{ip})_g (R_{lq})_g}{(f_v^2(T) - \Omega_d^2 - j2\Gamma(T)\Omega_d)}, \quad (5.4)$$

where

$$\Omega_d = f_p - f_d, \quad g_{eff}(f_d, T) = \left[\frac{c^2}{\hbar\pi f_d^3} \right] \left[\frac{S}{n_{si}^2(\lambda, T)(N_B(f_v(T), T) + 1)2\Gamma(T)} \right] \left[\frac{1}{1 + \Delta f_p/2\Gamma(T)} \right], \quad (5.5)$$

$$N_B(f_v(T), T) = \left\{ \exp(\hbar 2\pi f_v(T)/[K_B T]) - 1 \right\}^{-1}, \quad (5.6)$$

$$[R]_{g=1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad [R]_{g=2} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad [R]_{g=3} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (5.7)$$

$$f_v(T) = f_i + \Delta f_v(T), \quad (5.8)$$

$$\Delta f_v(T) = B_1 \left(1 + \frac{2}{\exp(\hbar\pi f_0/K_B T) - 1} \right) + B_2 \left(1 + \frac{3}{\exp(\hbar 2\pi f_0/3K_B T) - 1} + \frac{3}{(\exp(\hbar 2\pi f_0/3K_B T) - 1)^2} \right), \quad (5.9)$$

$$2\Gamma(T) = B_3 \left(1 + \frac{2}{\exp(\hbar 2\pi f_0/K_B T) - 1} \right) + B_4 \left(1 + \frac{3}{\exp(\hbar 2\pi f_0/3K_B T) - 1} + \frac{3}{(\exp(\hbar 2\pi f_0/3K_B T) - 1)^2} \right), \quad (5.10)$$

$$n_{si}(\lambda, T) = 3.41696 + \frac{0.138497}{\left((\lambda \times 10^6)^2 - 0.028 \right)} + \frac{0.013924}{\left((\lambda \times 10^{-6})^2 - 0.028 \right)^2} - 2.09 \times 10^{-5} (\lambda \times 10^6)^2 + 1.48 \times 10^{-7} (\lambda \times 10^6)^4 + 1.5 \times 10^{-4} \times (T - 298). \quad (5.11)$$

In the above equations, $\text{Re}[\chi_{iplq}^R(\Omega_d, T)] = \chi_{iplq}^{R'}(\Omega_d, T)$ and $\text{Im}[\chi_{iplq}^R(\Omega_d, T)] = \chi_{iplq}^{R''}(\Omega_d, T)$ are the real and imaginary parts of $\chi_{iplq}^R(\Omega_d, T)$, $j^2 = -1$, $2\Gamma(T)$ is the full-width at half maximum (FWHM) of the optical phonon spectrum, $n_{si}(\lambda, T)$ is the refractive index of silicon, $g_{eff}(f_d, T)$ is the Raman gain coefficient, $(R_{ip})_g$ and $(R_{lq})_g$ in Eq. (5.4) are the ip -th and lq -th components of the g -th Raman tensor (i.e., $[R]_g$ in Eq. (5.7), with $g = 1, 2$ or 3), μ_0 and ϵ_0 are the permeability and permittivity of free space, $\Omega_d = f_p - f_d$, f_p is the input pump frequency, f_d is the detuning frequency, S is the spontaneous scattering efficiency, $N_B(f_v(T), T)$ is the Bose occupancy factor, Δf_p is the FWHM of the pump beam spectrum, K_B is the Boltzmann's constant, $f_0 = 528c$ (in Hz), $\hbar$ is the reduced Planck's constant, $f_v(T)$ is the resonance frequency of the optical phonon, $\Delta f_v(T)$ is the change in $f_v(T)$, while $B_1 = -399.6c$, $B_2 = -23.5c$, $B_3 = 168.3c$ and $B_4 = 13.26c$ are anharmonic constants. The symbol c represents the speed of light in vacuum. All parameters are in SI units, unless otherwise stated. For convenience, we will drop $iplq$ in $\chi_{iplq}^R(\Omega_d, T)$. Note that the Raman WG may either operate at on-Raman-resonance (i.e., $|\text{Im}[\chi^R(\Omega_d, T)]| = \text{maximum}$) if $f_v(T) = \Omega_d$ or at off-Raman-resonance (i.e., $|\text{Im}[\chi^R(\Omega_d, T)]| \neq \text{maximum}$) if $f_v(T) \neq \Omega_d$. It can be seen that, while $[R]_g$ are temperature independent, other parameters are not, which render $\chi^R(\Omega_d, T)$ and any RS processes to be sensitive to temperature variations.

The temperature T dependence of $f_v(T)$, $2\Gamma(T)$, $n_{si}(\lambda, T)$ and $g_{eff}(f_d, T)$ are shown in Fig. 5.3. To ensure the accuracy of our modeling, experimental settings in [3] are used in this work: $\lambda_p = 1.427 \mu\text{m}$,

Fig. 5.3. Temperature dependence of the parameters that affect Raman scattering: (a) $f_v(T)$, (b) $2\Gamma(T)$, (c) $n_{si}(\lambda, T)$ for $\lambda = 1.3285 \mu\text{m}$ and $\lambda = 1.5413 \mu\text{m}$ and (d) $g_{eff}(\lambda, T)$ for $\lambda = 1.3285 \mu\text{m}$ and $\lambda = 1.5413 \mu\text{m}$. Note that $\lambda = 1.3285 \mu\text{m}$ and $\lambda = 1.5413 \mu\text{m}$ are, respectively, the anti-Stokes and Stokes wavelengths at $T = 298 \text{ K}$ for $\lambda_p = 1.427 \mu\text{m}$.

$S = (4.1 \pm 2.5) \text{ cm}^{-1}\text{Sr}^{-1}$, $\Delta f_p = 70 \text{ GHz/W}$. Also, λ_p , λ_s and λ_a are, respectively, in TE_{00} , TM_{00} and TM_{00} modes. Some general observations with regard to Fig. 5.3 are summarized below.

- At the room temperature ($T = 298 \text{ K}$), $f_v(T)$, $2I(T)$ and $g_{\text{eff}}(f_d = c/1.5413 \text{ } \mu\text{m}, T)$ are, respectively, $\sim 15.6 \text{ THz}$, 110 GHz and 20 cm/GW , which are consistent with Ref. [3].
- The resonance frequency $f_v(T)$ of the optical phonon decreases with T as seen in Fig. 5.3(a). Based on Eq. (5.2), this implies that, with a fixed f_p , f_s increases (decreases) when operating above (below) the room temperature, while the converse is true for f_a .
- The T dependence of the above-mentioned $f_v(T)$ and $2I(T)$ are related to the higher-order terms of the anharmonic oscillator model of the optical phonon in Eqs. (5.8)-(5.10). Details can be found in [17-22]. Essentially, an increase (decrease) in T results in a larger (smaller) interatomic spacing and lower (higher) lifetime of the optical phonon. These in turn decrease (increase) $f_v(T)$ and broaden (narrow) $2I(T)$.
- On the other hand, $2I(T)$ of the optical phonon increases with T in Fig. 5.3(b). The change in $2I(T)$ is much larger than the change in $f_v(T)$ for the same increment in T . For example, when reducing T from 298 K to 200 K , $2I(T)$ increases from 15.6 THz to 15.67 THz while $f_v(T)$ decreases from 110 GHz to 70 GHz . The former is only a $\sim 0.4 \%$ change while the latter is a much larger change of $\sim 36 \%$.
- The difference in the Raman gain coefficient $g_{\text{eff}}(f_d = c/\lambda, T)$ for $\lambda = 1.3285 \text{ } \mu\text{m}$ and $\lambda = 1.5413 \text{ } \mu\text{m}$ in Fig. 5.3(d) is due to their dispersion in $n_{\text{si}}(\lambda, T)$ in Fig. 5.3(c) (Note that $\lambda = 1.3285 \text{ } \mu\text{m}$ and $\lambda = 1.5413 \text{ } \mu\text{m}$ are, respectively, the anti-Stokes and Stokes wavelengths at $T = 298 \text{ K}$ for the pump wavelength $\lambda_p = 1.427 \text{ } \mu\text{m}$). In general, $g_{\text{eff}}(f_d, T)$ decreases as T rises; operating at $T < 298 \text{ K}$ ($T > 298 \text{ K}$) gives a higher (lower) Raman gain.

As T varies, the efficiency of any RS processes depends on the interplay of the above discussed parameters with $\text{Re}[\chi^R(\Omega_d, T)]$ and $\text{Im}[\chi^R(\Omega_d, T)]$. We have plotted $\text{Re}[\chi^R(\Omega_d = f_p - f_d, T)]$ and $\text{Im}[\chi^R(\Omega_d = f_p - f_d, T)]$ as a function of f_d near the anti-Stokes wavelength in Fig. 5.4(a) and near the Stokes wavelength in Fig. 5.4(b) at different T and with f_p fixed at $c/(1.427 \text{ } \mu\text{m})$ for our channel WG design that is directed along the $[110]$ orientation (cf. Fig. 5.1). Note that such a WG orientation (in Fig. 5.1) is used throughout this work in the study of the silicon WG Raman performances as this is the preferred choice in silicon WGs fabrication. From Fig. 5.4, we can observe that:

- Operating below (above) the room temperature blue-shifts (red-shifts) the location of the Raman resonance (where $|\text{Im}[\chi^R(\Omega_d, T)]| = \text{maximum}$) for the anti-Stokes wavelength, while the converse is true for the Stokes wavelength. This is consistent with the relation of $f_s = f_p - f_v(T)$ and $f_a = f_p + f_v(T)$ in Eq. (5.2).

- The peaks of $|\text{Re}[\chi^R(\Omega_d, T)]|$ and $|\text{Im}[\chi^R(\Omega_d, T)]|$ increase with decreasing T . Thus, the strength of the Raman resonance increases (decreases) when tuning from the room temperature ($T = 298$ K) to $T < 298$ K ($T > 298$ K).

Clearly, temperature variation impacts the location and magnitude of the Raman-resonance. This will subsequently affect the Raman performances of the WG. Assuming that we are initially operating at the room temperature ($T = 298$ K) using $\lambda_p = 1.427$ μm . Then based on Eq. (5.2), $\lambda_s = 1.5413$ μm and $\lambda_a = 1.3285$ μm (indicated by vertical dashed black lines in Fig. 5.4) will allow the WG to work at on-Raman-resonance. As T is reduced or raised from the room temperature while still fixing λ_p, λ_a and λ_s

Fig. 5.4. Temperature dependence of the real part of the Raman susceptibility $\text{Re}[\chi^R(\Omega_d, T)]$ and the imaginary part of the Raman susceptibility $\text{Im}[\chi^R(\Omega_d, T)]$ as a function of the (a) anti-Stokes wavelength λ_a and (b) Stokes wavelength λ_s for the pump wavelength of $\lambda_p = 1.427$ μm . Raman resonance occurs at the peaks of $|\text{Im}[\chi^R(\Omega_d, T)]|$, which shift with changes in the temperature. Note that the dotted vertical lines in all the graphs mark $\lambda_a = 1.3285$ μm and $\lambda_s = 1.5413$ μm , which are, respectively, the initial anti-Stokes and Stokes wavelengths for the pump wavelength $\lambda_p = 1.427$ μm when T is set to 298 K.

at 1.427 μm , 1.3285 μm and 1.5413 μm , respectively, the WG will move into off-Raman-resonance state (compare $|\text{Im}[\chi^R(\Omega_d, T)]|$ at different T along the vertical lines in Fig. 5.4). However, if we tune λ_s and λ_a (while still fixing λ_p at 1.427 μm) as T changes such that λ_s and λ_a always coincide with the maximum of $|\text{Im}[\chi^R(\Omega_d, T)]|$ in Fig. 5.4, then the WG will always be operating at on-Raman-resonance, with the strength of the Raman resonance increasing with decreasing T . The above-mentioned two scenarios are termed as **case A** and **case B** throughout this work, in which the WG is initially at $T = 298$ K when λ_p, λ_a and λ_s propagate through the WG. Let us formally define each case before moving on:

Case A — λ_p, λ_a and λ_s are always fixed at 1.427 μm , 1.3285 μm and 1.5413 μm , respectively, for all T . Then only $T = 298$ K allows operation at on-Raman-resonance state. All other T values will result in the WG to work at off-Raman resonance state.

Case B — λ_p is always fixed at 1.427 μm but λ_s and λ_a are tuned as T changes from 298 K such that the WG is always at the on-Raman-resonance state for any net T .

In the next section, the Raman performances are determined for both case A and case B in terms of Raman amplification/attenuation at λ_s and PRWC between λ_s and λ_a .

5.3.2. Coupled Mode Analysis

Fig. 5.5 shows the longitudinal views of the WG, functioning as a Raman amplifier/attenuator (if $|\Delta\beta| \gg 0$) or Raman wavelength converter (if $|\Delta\beta| \approx 0$), where $\Delta\beta$ can be tuned by stress engineering [28-31]. A cross-sectional view of the WG is shown earlier in Fig. 5.1. In our WG design, we used width $w = 0.445$ μm , height $h = 0.22$ μm and length $L = 2$ cm, with cladding thickness of $l_1 = l_2 = 3$ μm , so that the device is single-mode at $\lambda = 1.31$ μm and 1.55 μm for $T = 298$ K [24]. The

Fig. 5.5. Longitudinal view of the submicron silicon WG of length L , functioning as (a) Raman amplifier/attenuator if $|\Delta\beta| \gg 0$ and (b) Raman wavelength converter if $|\Delta\beta| \approx 0$.

design procedures to obtain this set of WG dimensions for single-mode operation are mentioned in section 2.4. Letting $E_q^- (E_q^+)$ to represent the complex amplitude of the electric field of the backward (forward) propagating wave, where $q = p, q = s$ and $q = a$ denote, respectively, the pump, Stokes and anti-Stokes waves, the evolution of the different waves along the propagation direction z is described as [7, 15]

$$\begin{aligned} \pm \frac{dE_p^\pm}{dz} = & \left\{ -\frac{\alpha_p}{2} - \frac{\Upsilon}{2} \lambda_p^2 e_c - \frac{b}{2A_{wg}} (E_p^{\pm 2} + 2E_s^{\pm 2} + 2E_a^{\pm 2} + 2E_p^{\mp 2} + 2E_s^{\mp 2} + 2E_a^{\mp 2}) \right. \\ & \left. - \frac{\kappa_{ss}}{2A_{wg}} \frac{\lambda_s}{\lambda_p} (E_s^{+2} + E_s^{-2}) + \frac{\kappa_{aa}}{2A_{wg}} \frac{\lambda_a}{\lambda_p} (E_a^{+2} + E_a^{-2}) \right\} E_p^\pm, \end{aligned} \quad (5.12)$$

$$\begin{aligned} \pm \frac{dE_s^\pm}{dz} = & \left\{ -\frac{\alpha_s}{2} - \frac{\Upsilon}{2} \lambda_s^2 e_c - \frac{b}{2A_{\text{wg}}} (E_s^{\pm 2} + 2E_p^{\pm 2} + 2E_p^{\mp 2} + 2E_a^{\pm 2} + 2E_a^{\mp 2} + 2E_s^{\mp 2}) \right. \\ & \left. + \frac{\kappa_{ss}}{2A_{\text{wg}}} (E_p^{\pm 2} + E_p^{\mp 2}) \right\} E_s^\pm + \frac{\kappa_{sa}}{2A_{\text{wg}}} (E_p^{\pm 2}) (E_a^\pm)^* \exp(\pm j\Delta\beta z), \end{aligned} \quad (5.13)$$

$$\begin{aligned} \pm \frac{dE_a^\pm}{dz} = & \left\{ -\frac{\alpha_a}{2} - \frac{\Upsilon}{2} \lambda_a^2 e_c - \frac{b}{2A_{\text{wg}}} (E_a^{\pm 2} + 2E_p^{\pm 2} + 2E_p^{\mp 2} + 2E_s^{\pm 2} + 2E_s^{\mp 2} + 2E_a^{\mp 2}) \right. \\ & \left. - \frac{\kappa_{aa}}{2A_{\text{wg}}} (E_p^{\pm 2} + E_p^{\mp 2}) \right\} E_a^\pm + \frac{\kappa_{as}}{2A_{\text{wg}}} (E_p^{\pm 2}) (E_s^\pm)^* \exp(\pm j\Delta\beta z), \end{aligned} \quad (5.14)$$

where κ is the coupling coefficient between the propagating waves that can be expressed as

$$\kappa_{ss}(\Omega_d, T) = -j \frac{2\pi f_s^2}{c^2 k_s} (2\chi_{ijlq}^{NR}(\Omega_d, T) + \chi_{ijlq}^R(\Omega_d, T)), \quad (5.15)$$

$$\kappa_{sa}(\Omega_d, T) = -j \frac{2\pi f_s^2}{c^2 k_s} (\chi_{ijlq}^{NR}(\Omega_d, T) + \chi_{ijlq}^R(\Omega_d, T)), \quad (5.16)$$

$$\kappa_{aa}(\Omega_d, T) = j \frac{2\pi f_a^2}{c^2 k_a} (2\chi_{ijlq}^{NR}(\Omega_d, T) + \chi_{ijlq}^R(\Omega_d, T)), \quad (5.17)$$

$$\kappa_{as}(\Omega_d, T) = j \frac{2\pi f_a^2}{c^2 k_a} (\chi_{ijlq}^{NR}(\Omega_d, T) + \chi_{ijlq}^R(\Omega_d, T)), \quad (5.18)$$

while α_q ($q = p, s$ or a) is the linear propagation losses due mainly to fabrication-induced sidewall roughness, Υ quantifies the efficiency of the free-carrier absorption, e_c is the effective charge density, b is the two-photon absorption coefficient and A_{wg} is the modal effective area of our proposed submicron channel WG. We can express A_{wg} and e_c as [7]

$$A_{\text{wg}} = \left[\iint I_f(x, y) dx dy \right]^2 / \left[\iint |I_f(x, y)|^2 dx dy \right], \quad (5.19)$$

$$\begin{aligned} e_c = B_5 \{ & E_p^{+4} + E_p^{-4} + E_s^{+4} + E_s^{-4} + E_a^{+4} + E_a^{-4} + 4(E_p^{+2} E_s^{-2} + E_p^{+2} E_a^{-2} + E_p^{+2} E_s^{+2} + E_p^{+2} E_a^{+2} + E_p^{-2} E_s^{-2} \\ & + 4(E_s^{+2} E_p^{-2} + E_s^{+2} E_a^{-2} + E_s^{-2} E_p^{-2} + E_s^{-2} E_a^{-2} + E_s^{+2} E_s^{-2}) \\ & + 4(E_a^{+2} E_p^{-2} + E_a^{+2} E_s^{-2} + E_a^{-2} E_p^{-2} + E_a^{-2} E_s^{-2} + E_a^{+2} E_a^{-2}) \}, \end{aligned} \quad (5.20)$$

in which $B_5 = bv/(4\pi\hbar f_p A_{\text{wg}}^2)$, v is the free-carrier lifetime and $I_f(x, y)$ is the transverse intensity profile of the fundamental mode of the WG. In this work, we set $b \approx 0.44$ cm/GW, $\Upsilon \approx 6.0 \times 10^{-10}$ [7], $v_c \approx 0.77$ ns, $\alpha_q \approx 3.0$ dB/cm [11] while $A_{\text{wg}} \approx 0.06 \mu\text{m}^2$. Two important figures of merit that quantify the Raman amplification/attenuation and PRWC of the WG are the conversion efficiency (CE) and amplification efficiency (AE):

$$CE = I_a(L)/I_s(0), \quad (5.21a)$$

$$AE = I_s(L)/I_s(0), \quad (5.21b)$$

where $I_s(0)$ is the intensity of the input signal (or Stokes) wave, which is always fixed at 0.1 mW, while $I_a(L)$ and $I_s(L)$ are, respectively, the intensities of the anti-Stokes and Stokes waves that exit the WG. To analyze the CE and AE of the silicon WG, Eqs. (5.12) to (5.18) are numerically solved until steady-state solutions are obtained after a sufficiently large number of iterations.

5.3.3. Results and Discussions

(a) Effects of Temperature on the Phase Matching Condition

The WG Raman performances, i.e., the CE and AE, depend largely on the phase matching condition $\Delta\beta$. The characteristics of the CE and AE in relation to $\Delta\beta$ of the silicon WG under varying pump input intensity I_p (i.e., changing the pump-to-signal-intensity ratio) at different T are illustrated in Fig. 5.6 for case A and Fig. 5.7 for case B. For comparison, we have included the CE and AE curves at $T = 298$ K (room temperature) in both case A and case B (though the CE and AE curves at $T = 298$ K are the same for both cases). Let us now first look at the CE and AE at the room temperature ($T = 298$ K). In general, at $T = 298$ K, each CE curve obeys a Sinc^2 dependence. Thus, there is more than one solution of $\Delta\beta$ to each CE value. The number of solutions of $\Delta\beta$ for each CE value increases (decreases) as the magnitude of CE decreases (increases) with I_p . This is due to the fact that the ripples of the CE curves become increasingly flat (pronounced) with increasing (decreasing) I_p . If I_p is low ($I_p < 50$ mW), the maximum CE is at $\Delta\beta = 0$. Increasing I_p ($I_p > 50$ mW) forms a notch in each CE curve at $\Delta\beta = 0$ (consistent with [16]). This shifts the maximum CE to $|\Delta\beta| > 0$. On the other hand, the AE curves for $T = 298$ K are Lorentzian-like. Maximum values of AE are at $|\Delta\beta| \gg 0$ while the minimum is at $\Delta\beta = 0$. Increasing I_p changes the magnitude of these extrema of the AE curves but not their locations (unlike that of the CE curves). At a sufficiently large I_p , the AE at $\Delta\beta = 0$ rapidly decreases. Having analyzed the trends of the CE and AE curves in relation to the $\Delta\beta$ of the silicon WG at $T = 298$ K, we now look into the effects on these curves when the temperature deviates from the room temperature (i.e., $T \neq 298$ K) for case A and case B.

(ai) Case A

The effects of operating at $T < 298$ K and $T > 298$ K for case A on the properties of CE in relation to $\Delta\beta$ are shown in Fig. 5.6(ai) and 5.6(aiii). It can be seen that the CE curves are still marked by ripples, similar to that at $T = 298$ K. However, note three main changes: (i) Each CE curve is no longer centered at $\Delta\beta = 0$ but is translated towards $\Delta\beta < 0$ ($\Delta\beta > 0$) for $T < 298$ K ($T > 298$ K); (ii) no notch appears at the maxima of the CE curves at high I_p ; and (iii) there is a deepening of the ripples at high I_p . Consequently, the maximum CE for $T < 298$ K ($T > 298$ K) in case A always occur at $\Delta\beta < 0$ ($\Delta\beta > 0$) for all I_p , unlike at $T = 298$ K, where the maximum CE occurs at $\Delta\beta = 0$ ($\Delta\beta \neq 0$) for low (high) I_p .

Fig. 5.6. The effects of varying the pump intensity I_p on the relationship of the propagation constant mismatch $\Delta\beta$ with the (a) conversion efficiency (CE) and (b) amplification efficiency (AE) of the submicron silicon waveguide at different temperatures T for Case A (cf. section 5.3.1 for definition of case A). Note that I_p increases in the direction of the arrows.

Fig. 5.7. The effects of varying the pump intensity I_p on the relationship of the propagation constant mismatch $\Delta\beta$ with the (a) conversion efficiency (CE) and (b) amplification efficiency (AE) of the submicron silicon waveguide at different temperatures T for case B (cf. section 5.3.1 for definition of case B). Note that I_p increases in the direction of the arrows.

The properties of AE in relation to $\Delta\beta$ are also affected when $T \neq 298$ K. Comparing Figs. 5.6(bii) with Figs. 5.6(bi) and 5.6(biii), it can be seen that as T departs from 298 K, the AE curves lose their Lorentzian-like shapes and become asymmetric. Consequentially, for each I_p input, the maximum AE occurs at one single value of $\Delta\beta$ at $\Delta\beta < 0$ ($\Delta\beta > 0$) for $T < 298$ K ($T > 298$ K). Also, the magnitude of the AE for $T \neq 298$ K at all $\Delta\beta$ is lower than that at $T = 298$ K, with the AE at $T < 298$ K being larger than that at $T > 298$ K. This also applies to the CE except for high I_p , in which the appearance of a notch at $|\Delta\beta| = 0$ at $T = 298$ K results in the CE around $|\Delta\beta| = 0$ to be smaller than that at $T < 298$ K.

(aii) Case B

For case B, the CE curves are always centered at $\Delta\beta = 0$ at all T , as shown in Figs. 5.7(i) and 5.7(ii). Also, note that a notch at the maximum of each AE curve (which is previously observed at $T = 298$ K for high I_p), also occurs for case B when $T \neq 298$ K, provided that I_p is high. These are in contrast to the CE curves of case A when $T \neq 298$ K (in which the CE curves are no longer centered at $\Delta\beta = 0$ and no notch at the maximum or peak can be found in the CE curves). However, there are two main differences in the nature of such a notch of case B as compared to that at $T = 298$ K. First, the notch in the CE curves for case B is deeper as compared to that at $T = 298$ K. For example, at $I_p = 90$ mW, the CE at $\Delta\beta = 0$ is -1 dB for $T = 298$ K, which falls to -7 dB when T changes to 50 K at the same I_p and $\Delta\beta$. Second, for $T > 298$ K, the notch at $\Delta\beta = 0$ for case B occurs at a much higher I_p as compared to those at $T = 298$ K. As such, in Fig. 5.7(aiii), no notch is observed to occur at $T = 500$ K because for this T setting, the notch at $\Delta\beta = 0$ occurs at $I_p = \sim 300$ mW, which is outside the range of $I_p \leq 90$ mW used in the graph.

Likewise, for the AE curves in Fig. 5.7(b), the general shapes (Lorentzian-dips) stay the same at all T , with only change to the AE, particularly that at $\Delta\beta = 0$. It can be seen that as I_p increases under the condition of $T < 298$ K ($T > 298$ K), the notch at $\Delta\beta = 0$ deepens at a faster (slower) rate as compared to that at $T = 298$ K.

(b) Harnessing Temperature Effects for Improved Raman Performances

In practical applications, λ_p , λ_s and λ_a are typically set in quasi-TE₀₀, quasi-TM₀₀ and quasi-TM₀₀ modes, respectively, so as to maximize the Raman gain (as mentioned earlier). To tune the phase matching condition $\Delta\beta = 2\beta_p - \beta_s - \beta_a$ (where $\beta_q = 2\pi N_{eff(q)}/\lambda_q$ and $N_{eff(q)}$ are, respectively, the propagation constant and effective index of each type of wave, with the subscripts $q = p$, $q = s$ and $q = a$ denoting, respectively, the pump, Stokes and anti-Stokes waves) of the three waves to the desired value, static stress engineering [28, 29] and/or dynamically tunable stress engineering [30, 31] can be used to adjust the WG birefringence $\Delta\beta_B$. This is because $\Delta\beta$ can be approximated as [29]: $\Delta\beta \approx \Delta\beta_B + \Delta\beta_W + \Delta\beta_M$, where $\Delta\beta_B$ is the phase mismatch due to WG birefringence, $\Delta\beta_W$ is the phase mismatch

due to WG dispersion and $\Delta\beta_M$ is the phase mismatch due to material dispersion. Thereafter, to have active tunability in the Raman performances, all-optical tuning can be employed, in which the CE and AE are tuned by adjusting the input pump intensity I_p . We now demonstrate that such tuning of CE and AE can be improved and with enhanced magnitude (in the CE and AE) by exploiting the effects of T variation that we have uncovered in subsection 5.3.3(a), as compared to the performances of conventional Raman devices that operate at a fixed T of 298 K. For both Case A and Case B, we set $\Delta\beta = 0$ ($\Delta\beta = 2000 \text{ m}^{-1}$) for the WG to function as a Raman wavelength convertor (Raman amplifier/attenuator) while varying I_p at different T . Important results for each case are highlighted below.

(bi) Case A

The characteristics of the CE in Fig. 5.8(a) and AE in Fig. 5.8(b) at different T are dependent on I_p . In the regime of low I_p ($I_p < 20 \text{ mW}$), as shown in Fig. 5.8(ai), the CE at each T increases with I_p . Note that in this regime, given a fixed I_p , the CE degrades if $T \neq 298 \text{ K}$ as the maximum CE occurs at $T = 298 \text{ K}$. For example, at $I_p = 5 \text{ mW}$, the CE falls to -17 dB (-25 dB) when the T drops (rises) to $T = 100 \text{ K}$ (500 K) from $T = 298 \text{ K}$, which produces a CE of -13 dB . However, in regime of high I_p ($I_p > 20 \text{ mW}$), the slopes of the CE curves start to decrease as I_p increases, which is shown in Fig. 5.8(aii). In particular, the CE curves corresponding to temperatures at and near $T = 298 \text{ K}$ (in Fig. 5.8(aii)) become notch-like. This results in the CE at and near $T = 298 \text{ K}$ to drop rapidly and degrade as compared to those at $T \neq 298 \text{ K}$, which is in sharp contrast to the earlier trend in the regime of low I_p in Fig. 5.8(ai). Thus, for case A, operating in the regime of low (high) I_p would degrade (enhance) the CE when compared to the CE at the room temperature.

Similar to the CE, the AE, which is shown in Fig. 5.8(b), generally increases sharply with I_p in the regime of low I_p ($I_p < 100 \text{ mW}$) for all T . In the regime of high I_p ($I_p > 100 \text{ mW}$), the AE starts to decrease with increasing I_p for $T \leq 298 \text{ K}$ while for $T \geq 298 \text{ K}$, the AE is fairly constant. However, unlike the CE curves in Fig. 5.8(a), the maximum AE is always at $T = 298 \text{ K}$ regardless of the strength of I_p . In general, the further the T deviates from $T = 298 \text{ K}$, the lower its corresponding AE will be. Thus, for case A, the AE degrades at all values of I_p once $T \neq 298 \text{ K}$. Nevertheless, practical applications can still be obtained from the temperature effects of the AE. It can be seen from Fig. 5.8(b) that variable attenuation can be realized for $T \leq 150 \text{ K}$ while for $T > 150 \text{ K}$, both variable attenuation and amplification are possible. One can also vary T to switch the role of the WG between an attenuator and an amplifier at a fixed I_p . For example, at $I_p = 50 \text{ mW}$, the WG is an amplifier with $\text{AE} = 3 \text{ dB}$ for $T = 200 \text{ K}$. Lowering T to 100 K switches the WG to an attenuator with $\text{AE} = -3 \text{ dB}$.

Fig. 5.8. Demonstration of harnessing the effects of temperature T variation to enhance and improve the flexibility to tune the (a) conversion efficiency (at $|\Delta\beta| = 0$) and (b) amplification efficiency (at $|\Delta\beta| = 2000 \text{ m}^{-1}$) of the submicron silicon WG for case A (cf. section 5.3.1 for definition of case A).

(bii) Case B

For the regime of low I_p ($I_p < 25$ mW), as seen in Fig. 5.9(ai), the CE also increases with I_p . However, unlike case A, the maximum CE is no longer at $T = 298$ K but at $T = 100$ K as, in general, the CE increases (decreases) as T changes from $T = 298$ K to $T < 298$ K ($T > 298$ K) for a given I_p when working in the regime of low I_p . In the regime of high I_p (at $I_p > 25$ mW), which is shown in Fig. 5.9(aii), notches form in the CE curves at all T , resulting in the CE to fall rapidly. This is in contrast to case A, whereby notches only form in the CE curves for T close to or at $T = 298$ K. Though notch forms at all T for case B once I_p is high, we can still find a range of I_p at each T setting in which the AE in Fig. 5.9(aii) is higher or enhanced as compared to that at $T = 298$ K. Such a range of I_p is, in general, wider for the AE curves corresponding to $T > 298$ K. In conclusion, for case B, the effects of temperature variation can be used to enhance the CE at all strengths of I_p .

Similar trends of the CE curves in Fig. 5.9(ai) can also be observed for the AE curves in the regime of low I_p ($I_p < 60$ mW), which is shown in Fig. 5.9(bi). In the regime of high I_p ($I_p > 60$ mW), as shown in Fig. 5.9(bii), the AE for $T \geq 298$ K decreases with increasing I_p while the AE for $T \leq 298$ K stays rather constant for all I_p . This property is directly opposite to that of the AE of case A in Fig. 5.8(b). Also, for case B, the WG is able to function both as a variable attenuator and variable amplifier at all T , in contrast to case A earlier in Fig. 5.8(b), in which only $T > 150$ K allows the WG to fulfill both functions of optical amplification and attenuation.

Finally, we mention that, for both case A and case B, the effects of temperature variation can be used to provide another degree of freedom (in addition to the input pump I_p) to tune the CE and AE. For example, when the WG functions as a Raman wavelength convertor (amplifier) in case B (cf. Fig. 5.9(a) and 5.9(b)), the CE (AE) can be tuned in the range of -23 dB $\leq$ CE $\leq$ -1 dB (4 dB $\leq$ AE $\leq$ 20 dB) for the temperature range of 100 K $\leq T \leq 500$ K when $I_p = 180$ mW (40 mW) is used. Also, note that all the mentioned trends in the CE and AE of Figs. 5.8 and 5.9 in this subsection as well as the trends of $\Delta\beta$ in subsection 5.3.3(a) are due to the temperature dependence of the location and magnitude of the Raman-resonance in the frequency response of the Raman susceptibility tensor (cf. Fig. 5.4), which is different between case A and case B (as explained in subsection 5.3.1).

5.4. Practical Implementation Issues

In order to vary the net temperature of the WG to above the room temperature (i.e., $T > 298$ K), microheaters [32, 33] can be used. Such microheaters can be deposited on the upper cladding layer of the WG [32] and/or oxide buffer layer [33] during fabrication and can be used to adjust the net temperature of the WG to 298 K $\leq T \leq 500$ K by applying an external actuation-or-control signal with a driving power P_c in the range 1 mW $\leq P_c \leq 6$ mW, with an average response time of ~ 6 ms, as demonstrated in Ref. [32]. For greater tunability in the Raman performances of the WG, it is highly desirable that the net temperature of the WG can be adjusted to below the room temperature (i.e., $T <$

Fig. 5.9. Demonstration of harnessing the effects of temperature T variation to enhance and improve the flexibility to tune the (a) conversion efficiency (at $|\Delta\beta| = 0$) and (b) amplification efficiency (at $|\Delta\beta| = 2000 \text{ m}^{-1}$) of the submicron silicon WG for case B (cf. section 5.3.1 for definition of case B).

298 K), apart from adjusting it to $T > 298$, as just discussed. To do so, the microheaters can be used in conjunction with heat sinks [34, 35] and/or thermoelectric microcoolers [23, 36, 37]. Such devices, which can be deposited on the upper cladding and oxide buffer layers during fabrication, have the capability to cool the net temperature of the WG to $100 \text{ K} \leq T \leq 298 \text{ K}$. Alternatively, micro-fluidic channels [38-40] that are filled with liquid nitrogen [23] can be employed for such cooling purpose. Essentially, for this scheme, micro-channels are fabricated along each longitudinal side of the silicon WG, similar to that in Ref. [38]. The liquid nitrogen in the micro-channels will then cool the silicon WG to 77 K [23]. Thereafter, microheaters can be used to tune or fine tune the net temperature of WG to within $100 \text{ K} \leq T \leq 298 \text{ K}$ or above the room temperature. Therefore, it is feasible that temperature variation in the range of $100 \text{ K} \leq T \leq 500 \text{ K}$ can be achieved.

Finally, we mention that, though we have limited our modeling to only one particular set of WG dimensions and communication wavelengths, the results in this work can also be used to predict the general trends of the temperature effects of Raman amplification/attenuation and PRWC in submicron silicon WGs of other dimensions and at other wavelengths. Also, it may seem that our Raman waveguide might be limited in bandwidth capability as for Case A only one input signal wavelength can be amplified/attenuated and one wavelength can be converted. For Case B, the input signal wavelength and converted wavelength each has a tuning range of $\sim 2 \text{ nm}$. However, do note that in all our cases, the pump wavelength λ_p is fixed for convenience. For any Raman device, one simply needs to adjust λ_p in order to process different input signal wavelengths λ_s , or convert different wavelengths λ_a (cf. Eq. (5.2)). Static stress engineering [28, 29] and/or dynamically tunable stress engineering [30, 31] can be used to adjust the WG birefringence so as to achieve the desired phase-matching condition $\Delta\beta$, between the different wavelengths, where $\Delta\beta = 2(2\pi N_{\text{eff}(p)}/\lambda_p) - 2\pi N_{\text{eff}(a)}/\lambda_a - 2\pi N_{\text{eff}(s)}/\lambda_s$. This will mean that it is possible to increase the number of wavelengths that can be amplified/attenuated or converted by our Raman WG model presented in this work. For example, if λ_p is tuned in the range of $1.417 \mu\text{m} \leq \lambda_p \leq 1.427 \mu\text{m}$ (i.e., tuning range of $\sim 10 \text{ nm}$), then based on Eqs. (5.2), (5.8) and (5.9), λ_s and λ_a can, respectively, have a tuning range of $\sim 12 \text{ nm}$ and $\sim 9 \text{ nm}$ at all temperatures for our proposed scheme. In other words, our proposed Raman WG model is also suitable for communications applications around the C-band wavelength of $1.55 \mu\text{m}$ and O-band wavelength of $1.31 \mu\text{m}$ in dense wavelength division multiplexing (DWDM) and ultra-dense wavelength division multiplexing (UDWDM) networks, where the typical channel spacings are, respectively, $\sim 0.8 \text{ nm}$ (or 100 GHz) and $\sim 0.048 \text{ nm}$ (or 6.25 GHz) in the $1.55 \mu\text{m}$ window [41].

5.5. Summary and Significant Contributions of this Research Work

We have investigated for the first time, to the best of our knowledge, the temperature T dependence of three types of most commonly used Raman scattering processes – Raman amplification, Raman attenuation and parametric Raman wavelength conversion (PRWC) – in a

submicron silicon waveguide (WG) at the communication wavelengths. It has been shown that the effects of temperature variation ($100 \text{ K} \leq T \leq 500 \text{ K}$) can be harnessed to improve the Raman performances in the submicron silicon WG. To clearly show this, we have summarized in Table 5.1 the unique features of the proposed Raman WG model. In particular, the use of temperature variation provides an additional degree of freedom to tune the conversion efficiency (CE) or amplification efficiency (AE) of the WG, thereby allowing a more dynamical control of the Raman performances of the WG, as compared to those working at a fixed room temperature, which is the conventional practice. For example, using an input pump intensity of $I_p = 10 \text{ mW}$ in case A (cf. Fig. 5.8a(i)), the CE can be tuned in the range of $-19 \text{ dB} \leq \text{CE} \leq -7 \text{ dB}$ for $100 \text{ K} \leq T \leq 500 \text{ K}$. Also, the CE or AE can be enhanced as compared to that at the room temperature. For instance, at $I_p = 500 \text{ mW}$ in case A (cf. Fig. 5.8a(ii)), $\text{CE} = -12 \text{ dB}$ if $T = 298 \text{ K}$; however at $T = 200 \text{ K}$, the CE is increased to 8 dB . These research findings can be used as general design rules when engineering submicron silicon WGs for efficient Raman amplification, Raman attenuation and PRWC at different temperatures. Also, they are significant contributions to the field of Raman-based silicon photonics, which has largely overlooked the temperature effects of Raman scattering mechanisms in submicron waveguides. Our proposed Raman waveguide model has clearly highlighted the numerous advantages of harnessing the temperature effects of Raman scattering. This could have potentiality in revolutionizing the way in which Raman scattering is being utilized for active functionalities in silicon waveguide devices in the photonics community and industry.

Table 5.1. Raman performances of a submicron silicon waveguide[†] via temperature (T) variation

	Case A[*]		Case B[*]	
	Conversion Efficiency (CE) (at $\Delta\beta = 0$)	Amplification/ Attenuation Efficiency (AE) (at $\Delta\beta = 2000 \text{ m}^{-1}$)	Conversion Efficiency (CE) (at $\Delta\beta = 0$)	Amplification/ Attenuation Efficiency (AE) (at $\Delta\beta = 2000 \text{ m}^{-1}$)
General unique features	An additional degree of freedom to tune the conversion or amplification efficiency, besides using the input pump intensity I_p to do so.			
	Temperature variation can also be used to give an enhanced CE and AE as compared to the CE and AE at a fixed room temperature in the regime of high I_p .	For $T > 150 \text{ K}$, both variable attenuation and amplification are possible for the WG by tuning I_p .	Temperature variation can also be used to give an enhanced CE and AE as compared to the CE and AE at a fixed room temperature for all strength of I_p .	
		For $T \leq 150 \text{ K}$, only variable attenuation is possible for the WG.		For all T , the silicon WG can function both as a variable attenuator and a variable amplifier by tuning I_p .

[†]Based on a submicron silicon channel WG with design specifications stated in this work. ^{*} Refer to section 5.3.1 for definition of **Case A** and **Case B**.

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PART TWO

Generating Fast and Slow Light Effects in Photonic Integrated Circuits Using Traveling Wave Microresonators

Chapter 6

Harnessing Coupler-Induced Localized Backscattering for Enhanced Fast and Slow Light Effects in a Traveling Wave Microresonator

6.1. Introduction: Research Motivations and Objectives

We have ended the last chapter of part one of this thesis by studying the photonic bandgap engineering of coupled traveling wave microresonators (TWMRs). For part two of this thesis, we will focus exclusively on the fast light (FL) and slow light (SL) performances in TWMRs due to the numerous promising applications of FL and SL [1-5] and the current strong interest among the photonics community to employ semiconductor TWMR-based devices [6-13] for FL and SL research. In particular, FL and SL are envisioned to have profound implications for quantum information processing [14] and optical communications [15, 16], whereby the abilities to advance, delay and to coherently stop and store data trains are needed in order to enable far more flexible optical networks. Though atomic media [5, 17-19] are capable of generating larger pulse advancement or delay than semiconductor waveguide (WG) based schemes [6-13, 20-25], it is the ability of semiconductor schemes to operate at convenient communication wavelengths, and their compact size and ease of integration with other electronic and photonic components on a single chip that make them currently more attractive for communications applications. Among the various semiconductor schemes [6-13, 20-24], TWMRs [25], such as microrings, microspheres and microtoroids, have received particular emphasis in FL and SL research in recent years [6-13] owing to the fact that TWMRs are highly simple but yet effective photonic structures that are relatively easy to fabricate. Most importantly, TWMRs provide high dispersion per unit volume [26], which will make them highly suitable for FL and SL applications in integrated photonics.

An azimuthally symmetric TWMR inherently possesses Kramers degeneracy [27, 28]: The two possible directions of propagation in the cavity, clockwise (CW) and counterclockwise (CCW), have identical resonance frequency (i.e., degenerate resonance frequency). Lifting of this degeneracy will

occur in the presence of backscattering, which results in the mutual coupling of the CW and CCW modes, consequently altering the dispersive [13] and transmission [27-33] responses of the TWMR. Traditionally, studies pertaining to backscattering in TWMR have focused on distributed backscattering within the cavity [13, 27-30], ever since backscattering in TWMRs was first experimentally observed in [27]. Such distributed backscattering is due to surface perturbations, which either results from the intentional addition of surface microstructures, such as gratings [13] or surface imperfections [27-30], that are incurred during the fabrication process. Recently, degeneracy lifting and mode splitting originating from localized backscattering at the coupling junction when a TWMR is coupled to an external port WG has been analytically predicted in [31] and experimentally demonstrated in [32, 33]. We term such backscattering as coupler-induced localized backscattering (CILB). To the best of our knowledge, a detailed analytical study on the impacts of CILB on the nature of FL and SL in a TWMR is currently lacking in the literature, which will be the main objective of this work. In particular, it will be shown that weak CILB improves the FL and SL performances of a TWMR when the cavity possesses a small net optical gain.

This chapter is organized as follows. In section 6.2, we first review the basic theory of fast light (FL) and slow light (SL). Then in section 6.3, the FL and SL natures of a conventional TWMR, in which CILB effects are negligible, are analytically characterized so as to set the stage for section 6.4. In section 6.4, we demonstrate how CILB can be harnessed to improve the FL and SL performances of a TWMR. Finally, we conclude in section 6.5.

6.2. Basic Theories of Fast and Slow Light

6.2.1. Concepts of Pulse Propagation, Group Velocity, Group Index and Group Delay

A pulse, denoted as $U(z, t)$, with an angular spectral bandwidth $\Delta\omega$ around a central angular frequency ω_0 is formed by the superposition of different monochromatic waves of different angular frequencies ω , which are in the interval $\omega = \omega_0 \pm \Delta\omega$. This pulse can be mathematically described as

$$U(z, t) = A_0(z, t) \exp[j(\omega_0 t - \beta_0 z)] = \int_{\Delta\omega} E(z, t, \omega) d\omega, \quad (6.1)$$

where A_0 is the amplitude function of the pulse, β_0 is the propagation constant at ω_0 , z is the spatial coordinate (directed along the net propagation direction of the incident light in the optical medium), $j = \sqrt{-1}$ and t denotes time. Each monochromatic wave E in Eq. (6.1) can be described as

$$E(z, t, \omega) = a(\omega) \exp[j\{\omega t - \beta(\omega)z\}] = a(\omega) \exp[j\{\omega t - \beta'(\omega)z\}] \exp[-\alpha(\omega)z], \quad (6.2)$$

where $a(\omega)$ and $\beta(\omega)$ are, respectively, the amplitude and propagation constant at ω . Note that $\beta(\omega)$ can be expressed as $\beta(\omega) = \text{Re}[\beta(\omega)] + j\text{Im}[\beta(\omega)] = \beta'(\omega) + j\alpha(\omega) = N_{\text{eff}}\omega/c = (\text{Re}[N_{\text{eff}}] + j\text{Im}[N_{\text{eff}}])\omega/c$, in which $\text{Re}[\beta(\omega)] = \beta'(\omega)$ and $\text{Im}[\beta(\omega)] = \alpha(\omega)$ are, respectively, the real and imaginary part of β , N_{eff} is the effective index while $\text{Re}[N_{\text{eff}}]$ and $\text{Im}[N_{\text{eff}}]$ are, respectively, the real and imaginary part of

N_{eff} . The phase response $\delta(\omega)$ of the pulse is described by $\beta'(\omega)$ and $\text{Re}[N_{eff}(\omega)]$ while the amplitude response is due to $\alpha(\omega)$ and $\text{Im}[N_{eff}(\omega)]$. As implied in *Kelvin's method of stationary phase* [34], pulse propagation is essentially a wave interference phenomenon. When the various Fourier components are exactly in phase, constructive interference will occur and a peak appears at the pulse's envelope. On the other hand, if all the Fourier components of the pulse are exactly out of phase, destructive interference will occur and points of minima forms in the pulse [1-3]. For a typical Gaussian pulse, this corresponds to the wings of the Gaussian curve. Likewise, other points in between the peak and minimum of the pulse are formed by the interference of the various Fourier components. In particular, at the peak of the pulse, which is typically located at $\omega = \omega_0$, the rate of the effective phase shift $\Phi = \delta = \omega t - \beta'z$, which is imparted by the optical medium to the propagating pulse is zero, i.e., $d\Phi(\omega_0)/d\omega = 0$. This can be expanded as

$$\left. \frac{d(\beta'z - \omega t)}{d\omega} \right|_{\omega_0} = 0 \Rightarrow \left(z \frac{d\beta'}{d\omega} \right) - t \Big|_{\omega_0} = 0 \Rightarrow \frac{z}{t} = \frac{d\omega}{d\beta'} \Big|_{\omega_0} = v_g(\omega_0), \quad (6.3)$$

where the term $z/t = d\omega/d\beta'(\omega_0) = v_g(\omega_0)$ in Eq. (6.2) gives the speed of the pulse peak. We now proceed to show that if $\Delta\omega$ is small, the entire pulse will travel at a speed of $v_g(\omega_0)$. If $\Delta\omega$ is small, $\beta(\omega)$ will vary slowly near the centre frequency ω_0 of the pulse. Then $\beta'(\omega)$ can be approximated by the first two terms of the Taylor series: $\beta'(\omega) \approx \beta'(\omega_0) \pm \Delta\omega d\beta'(\omega_0)/d\omega = \omega_0/v(\omega_0) \pm \Delta\omega/v_g(\omega_0)$. Substituting this into $E(z, t, \omega)$ in Eq. (6.2) allows us to express the Fourier components of the pulse as

$$E(z, t, \omega) = a(\Delta\omega) \exp[j\Delta\omega(t - z/v_g(\omega_0))] \exp[j(\omega_0 t - \beta_0 z)], \quad (6.4)$$

where $v_g(\omega_0)$ is the group velocity of the light in the medium at the central frequency ω_0 . Making use of Eq. (6.3), the pulse envelope in Eq. 6.1 can be recasted as

$$U(z, t) = A_0(z, t - z/v_g(\omega_0)) \exp[j(\omega_0 t - \beta_0 z)]. \quad (6.5)$$

It is evident from Eqs. (6.4) and (6.5) that all the Fourier components of the pulse, i.e., the entire pulse envelope travels at a speed of $v_g(\omega_0)$ when $\Delta\omega$ is small. In this context, the Fourier components can be regarded as travelling as a *group* at a speed similar to that at the pulse peak $v_g(\omega_0)$. Thus, v_g is called the *group velocity*, which can also be expressed as $v_g(\omega_0) = c/\text{Re}[n_g]$, where $\text{Re}[n_g]$ is the real part of the group effective index (or simply the group index) n_g that can be easily derived from β' :

$$\text{Re}[n_g] = \frac{c}{v_g} = c \frac{d(\beta' = \text{Re}[N_{eff}]\omega/c)}{d\omega} = \text{Re}[N_{eff}] + \omega \frac{d \text{Re}[N_{eff}]}{d\omega}, \quad (6.6)$$

where c is the speed of light in vacuum and N_{eff} is the effective index of the medium. Note that the imaginary part of the group index is responsible for the loss of the pulse and is associated with $\text{Im}[\beta(\omega)] = \alpha(\omega)$. The significance of the pulse peak is that it contains intense energy that is used for

the actual detection of the pulse's arrival. For these reasons, the definition of the propagation velocity that is commonly used when quantifying the speed of the propagating pulse in any type of medium (be it an ordinary optical medium, such as a straight dielectric waveguide, or FL and SL resonant media in the next section) is $v_g(\omega_0)$. Alternatively, $\text{Re}[n_g(\omega_0)]$ or the group delay $t_g(\omega_0) = z/v_g(\omega_0)$ can also be used (note that from here onwards, we refer to $\text{Re}[n_g]$ simply as n_g . Also, all parameters of n_g , v_g and t_g are evaluated at $\omega = \omega_0$). If higher-order terms of the Taylor series are considered in $\beta'(\omega)$ in Eq. (6.2), it can be seen that there is group velocity dispersion (GVD), which together with the frequency dependence of α , might distort the pulse. Generally, v_g loses its original meaning if there is substantial pulse distortion [34, 35]. However, there is no clear benchmark on the extent of distortion that renders v_g , n_g or t_g to be unsuitable as an optical metric for optical systems; this issue is circumvented by minimizing GVD through dispersion compensation [36] or simply by limiting the pulse bandwidth to region of linear dispersion within the resonance spectrum.

6.2.2. Linear Pulse Propagation in Resonant Medium: Generation of Fast and Slow Light

To analyze the fast and slow light characteristics of any optical resonant medium, it is much more convenient to express Eq. (6.1) as

$$S_p(\ell, \omega) = H_T(\ell, \omega) S_p(0, \omega), \quad (6.7)$$

where $S_p(z, \omega)$ is the pulse profile in the spectral domain, while $H_T(z, \omega)$ is the steady-state transfer function of the through port, with the spatial coordinate $z = 0$ and $z = \ell$ denoting, respectively, the entry and exit of the linear resonant medium (cf. inset in Fig. 6.1). The transfer function H , which is a complex electric field transmission can be written as

$$H_T(\ell, \omega) = |H_T(\ell, \omega)| \angle \Phi(\omega) = \exp[j\beta_{\text{eff}}'(\omega)\ell] = \exp[-\alpha_{\text{eff}}(\omega)\ell] \exp[j\beta_{\text{eff}}'(\omega)\ell]. \quad (6.8)$$

The quantity $\Phi = \beta_{\text{eff}}'(\omega)\ell$ is the effective phase shift of the pulse after propagating through a linear resonant medium and gives all the information on the FL or SL. Note that we will use β_{eff}' and α_{eff} in place of β' and α (which are used earlier in section 6.2.1 for ordinary optical media) so as to quantify for the resonance effect. It can be seen in Eq. (6.8) that the resonant medium or system imparts a net amplitude change of $\exp(-\alpha_{\text{eff}}\ell)$ and effective phase shift of $\Phi(\omega) = \beta_{\text{eff}}'\ell$ to the input pulse. Using Φ , the phase delay t_p , group delay t_g and pulse front delay t_f and the corresponding phase velocity v_p , group velocity v_g and pulse front velocity v_f of the output pulse can be determined:

$$t_p = \Phi/\omega \Rightarrow v_p = \omega/\beta_{\text{eff}}', \quad t_g = d\Phi/d\omega \Rightarrow v_g = d\omega/d\beta_{\text{eff}}', \quad t_f = \lim_{\omega \rightarrow \infty} \Phi/\omega \Rightarrow v_f = \lim_{\omega \rightarrow \infty} \omega/\beta_{\text{eff}}' = c. \quad (6.9)$$

As mentioned, pulse propagation is a wave interference phenomenon. The nature of interference changes as the pulse propagates through a resonant medium. This modifies the phase relations among the Fourier components, resulting in the reshaping of the pulse that temporally shifts the point of

constructive interference and thus the pulse peak [1-3]. At the output port, the pulse peak has a temporal shift of $t_g = \ell / v_g$ with respect to the input pulse peak as the pulse envelope $A_0(t)$ becomes $\exp(-\alpha_{eff}\ell)A_0(t - \ell / v_g)$ after propagating through the optical medium of length ℓ (mentioned earlier). Whether a pulse advance ($t_g < 0$) or pulse delay ($t_g > 0$) is generated depends on the dispersion of the resonant medium. If the medium is anomalously dispersive, there are two possibilities: (i) $n_g < 0$ and $v_g < 0$, which produces a pulse advance ($t_g < 0$) as shown in Fig. 6.1(a), whereby the peak of the pulse emerges at the output port even before the peak of the input pulse has entered the medium or (ii) $0 < n_g < 1$ and $0 < c < v_g$, resulting in a pulse delay t_g that is less than that of vacuum, i.e., $t_g < t_{g(vac)} = \ell / c$, which is shown in Fig. 6.1(b). Both (i) and (ii) are known as fast light (FL), which does not violate Einstein's theory of relativity as the peaks of the input and output pulse are *not* causally related because the pulse undergoes continuous reshaping (through interference of the various Fourier components of the pulse) as it propagates through the optical medium [1-3]. Relativistic causality is

Fig. 6.1. In (a), (b) and (c), the dotted black lines represent the input Gaussian pulse centred at normalized time = 0, the thickened red lines represent the output Gaussian pulse after propagating through a linear resonant medium (shown in top inset) of length ℓ while the thin blue lines represent the pulse after propagating through vacuum of equal length as the resonant medium. The group delay of the pulse traveling through the linear resonant medium is denoted as t_g while the group delay of the pulse traveling through vacuum of similar distance ℓ is denoted as $t_{g(vac)}$. Depending on its dispersion, a resonant medium of length ℓ can generate output pulse with (a) fast light with negative t_g , (b) fast light with positive t_g , in which $t_g < t_{g(vac)}$ or (c) slow light in which $t_g > t_{g(vac)}$. For simplicity, the effects of GVD and waveguide losses or gain are not shown.

associated only with the wave (or signal) front, which carries the information and always has a speed of c (i.e., $v_f = c$ in Eq. (6.9)). On the other hand, if there is normal dispersion, a pulse delay of $0 < t_g < t_{g(vac)}$ is generated. This phenomenon is known as slow light (SL), as illustrated in Fig. 6.1(c).

Finally, it must be mentioned that FL and SL effects, which are usually associated with the v_g of the pulse peak, do not change the rate of information transfer v_i , which always has a speed of c , as mentioned above. Note that v_i is associated with the wavefront (but not pulse peak) as the information is impressed on each waveform only at the wave front. It is the sharp discontinuous wave front of the incoming pulse that carries the information, not the pulse peak. Thus, $v_i \neq v_g$. What FL and SL merely do is adjusting the detection latency between the time when the information (at the wave front) is first available at the detector and when it is actually detected via the pulse peak [37, 38].

6.3. Fast and Slow Light Performances in a Conventional Traveling Wave Microresonator

6.3.1. Theoretical Formulation

The schematic of the TWMR circuit, which is being studied in this chapter, is illustrated in Fig. 6.2. We define a *conventional* TWMR as one in which the cavity is degenerate (i.e., the direction of the light propagation in the resonator, be it clockwise or counterclockwise, does not affect the resonance frequency and thus the resonance frequency is also termed as the degenerate resonance

Fig. 6.2. Schematic of a traveling wave microresonator coupled to two output ports.

frequency), has negligible intracavity backscattering and is excited only by one input. For our work, we assume that a single input is launched at the left-hand side of the resonator circuit in Fig. 6.2. This

means $a_1' = b_1' = c_1' = d_1' = a_2' = b_2' = c_2' = d_2' = a_2 = 0$ in Fig. 6.2. Then the electric fields at the coupling junctions (CJs) in Fig. 6.2 can be described as [10, 36]:

$$\text{CJ 1: } \begin{pmatrix} b_1 \\ d_1 \end{pmatrix} = \begin{pmatrix} r_1 & j\kappa_1 \\ j\kappa_1 & r_1 \end{pmatrix} \begin{pmatrix} a_1 \\ c_1 \end{pmatrix}, \quad \text{CJ 2: } \begin{pmatrix} b_2 \\ d_2 \end{pmatrix} = \begin{pmatrix} r_2 & j\kappa_2 \\ j\kappa_2 & r_2 \end{pmatrix} \begin{pmatrix} a_2 \\ c_2 \end{pmatrix}, \quad (6.10)$$

where κ_i and r_i are, respectively, the forward “cross” amplitude coupling coefficient and forward “through” amplitude coupling coefficient of the coupler at the coupling junction i ($i = 1$ or 2). Also, $r_1^2 + \kappa_1^2 = r_2^2 + \kappa_2^2 = 1$ as we have assumed a lossless coupler for simplicity. Noting that $d_2 = d_1 r_2 \tau^{1/2} \exp(j\delta/2)$, the transfer function H_T (H_D) of the through (drop) port of a conventional TWMR can be written as

$$H_T = \frac{b_1}{a_1} = |H_T| \angle \Phi_T = \frac{r_2 \tau \exp(j\delta) - r_1}{r_1 r_2 \tau \exp(j\delta) - 1}, \quad (6.11)$$

$$H_D = \frac{b_2}{a_1} = |H_D| \angle \Phi_D = \frac{k_1 k_2 \sqrt{\tau} \exp(j\delta/2)}{r_1 r_2 \tau \exp(j\delta) - 1}, \quad (6.12)$$

where τ is the round-trip amplitude attenuation constant, Φ is the effective phase shift of the resonator, $\delta = 2\pi m + \Delta\omega/\text{FSR} = (n_g \omega/c) L_{rt}$ is the round-trip phase shift, $\Delta\omega$ is the detuning angular frequency, $m = \pm 1, \pm 2, \dots$ is the resonance order, FSR is the free spectral range, n_g is the group effective index and L_{rt} is the round-trip length of the cavity. In filter terminology, the two-port TWMR is known as a minimum phase filter (MPF). For MPFs, the amplitude and phase responses are uniquely related by the Hilbert transform [36], from which it can be easily deduced that a large enhancement in the group delay t_g and the transitions between FL to SL (or via versa) occurs near points where there is a large change in the transmission response. It is thus useful to locate these points so that the FL and SL of the TWMR can be analyzed. Generally, a large and sharp change in the transmission of any TWMR will occur near the critical coupling and oscillation conditions. Note that oscillation (critical coupling) refers to the phenomenon in which the transmission is infinite (zero) at the resonant frequency and is associated with the poles (zeros) of the transfer function. Applying the z -transform operation [36] ($z^{-1} = \exp(j\delta)$ is used) to Eqs. (6.11) and (6.12), the zero $z_{z,T}$ ($z_{z,D}$) and pole $z_{p,T}$ ($z_{p,D}$) of the through (drop) port of the conventional TWMR that is coupled to two ports can be expressed as

$$z_{z,T} = \tau r_2 / r_1, \quad z_{p,T} = \tau r_1 r_2, \quad (6.13)$$

$$z_{z,D} = 0, \quad z_{p,D} = \tau r_1 r_2. \quad (6.14)$$

Note that oscillation (critical coupling) occurs when the pole (zero) lies on the unit circle of the pole-zero diagram, i.e., $|z_z| = |z_p| = 1$. Then it is easy to see from Eqs. (6.13) and (6.14) that, for a conventional TWMR, it is possible for critical coupling to take place at the through port, while the drop port cannot produce critical coupling as $z_{z,D} = 0$. On the other hand, oscillation is possible at both

ports as $z_{p, T} = z_{p, D} \neq 0$. We shall analyze in more details in later sections on how the critical oscillation and oscillation points relate to the FL and SL natures of the TWMR. From Eqs. (6.11) and (6.12), the group delay $t_{g, T}(t_{g, D})$ at the through (drop) port can be derived, using $t_{g, T} = t_{rt} \partial \Phi_T / \partial \delta$ and $t_{g, D} = t_{rt} \partial \Phi_D / \partial \delta$, respectively:

$$t_{g, T} = \frac{\tau r_2 (r_1^2 - 1)(\tau^2 r_1^2 r_2^2 \cos \delta + r_1 \cos \delta - \tau r_1^2 r_2 - \tau r_2)}{(r_1^2 - 2\tau r_1 r_2 \cos \delta + \tau^2 r_2^2)(1 - 2\tau r_1 r_2 \cos \delta + \tau^2 r_1^2 r_2^2)} t_{rt} \quad (6.15)$$

$$t_{g, D} = \frac{1 - \tau^2 r_1^2 r_2^2}{2(1 - 2\tau r_1 r_2 \cos \delta + \tau^2 r_1^2 r_2^2)} t_{rt} \quad (6.16)$$

where t_{rt} is the round-trip time of the cavity. Thus, the resonator is capable of group delay or advance enhancement that is a multiplier of t_{rt} . Using t_g , three different types of light propagation velocity can be classified: (i) FL with negative v_g if $t_g < 0$, (ii) FL with positive v_g if $0 < t_g < 1$, and (iii) SL if $t_g > 1$, where v_g is the group velocity. We will use this classification to analyze t_g at the degenerate resonance frequency ω_0 , which has $\delta = 2\pi m$, in this work. Do note that t_g is expressed in terms of t_{rt} (i.e., t_g is normalized with t_{rt}) for the above classification of light propagation regimes. This convention will be used throughout this chapter, unless otherwise stated.

6.3.2. Group Delay Classification at the Frequency ω_0

We first analyze the group delay at the through port. At ω_0 , we can simplify Eq. (6.15) to:

$$t_{g, T}(\omega_0) = \frac{\tau r_2 (1 - r_1^2)}{(\tau r_2 - r_1)(1 - \tau r_1 r_2)} t_{rt} = \frac{(z_{p, T} - z_{z, T})}{(z_{z, T} - 1)(z_{p, T} - 1)} t_{rt} \quad (6.17)$$

Stable light circulation in the cavity means $|z_{p, T}| < 1$ [36]. Imposing this on Eq. (6.17) gives:

$$\tau < r_1/r_2 \Rightarrow t_{g, T} < 0, \quad \tau > r_1/r_2 \Rightarrow t_{g, T} > 0 \quad (6.18)$$

The inequality $\tau < r_1/r_2$ ($\tau > r_1/r_2$) in Eq. (6.18) is termed as under coupling (over coupling) condition, which gives FL (SL). To better illustrate Eq. (6.18), we have plotted Eq. (6.17) in Fig. 6.3(a). It can be seen that the transmitted light at the through port transforms from “fast” to “slow” as the coupling condition changes from under-coupling ($\tau < r_1/r_2$) to over-coupling ($\tau > r_1/r_2$), with the reversal from FL to SL occurring at the first asymptote (at $\tau = r_1/r_2$), which is the critical-coupling point. A second reversal of dispersion occurs at the second asymptote (at $\tau = 1/(r_1 r_2)$), which is the oscillation point. This, however, leads to unstable FL. Note that at the critical coupling and oscillation points, $\partial \Phi / \partial \delta$ and t_g diverges and thus t_g (as well as n_g and v_g) loses its physical significance. The existence of the critical coupling and oscillation points at the through port is consistent with the presence of the pole and zero in the transfer function of the through port. The influence of the pole and zero on $t_{g, T}(\omega_0)$ can be better illustrated by the pole-zero diagram in Fig. 6.3(b). As seen in Fig. 6.3(b), when $\tau < r_1/r_2$, the zero always lies inside the unit circle, giving $t_{g, T} < 0$

Fig. 6.3. (a) Group delay t_g (in t_n) of the transmitted light at the degenerate resonance frequency ω_0 for the through port of a traveling wave microresonator with $\kappa_1 = \kappa_2 = 0.2$ as an example. Critical coupling (oscillation) occurs at $\tau = 1$ ($\tau = 1.0417$). (b) The pole-zero dynamics on the unit circle as τ is varied. The cross (circle) denotes the pole (zero) of the transfer function.

that corresponds with the FL regime in Fig. 6.3(a). Subsequent increase in τ moves the zero towards the unit circle and produces a rapidly decreasing function of $t_{g,T}$ that is bounded by the asymptote of the critical coupling point in Fig. 6.3(a). On the other hand, for $\tau > r_1/r_2$, the zero is always outside the unit circle in Fig. 6.3(b). This gives $t_{g,T} > 0$, corresponding to the SL regime in Fig. 6.3(a). Initial increment in τ moves the zero away from the unit circle, which leads to a sharp drop in t_g in Fig. 6.3(a). As τ is increased further, there is a sharp rise in t_g in the SL regime in Fig. 6.3(a) that is bounded by the asymptote of the oscillation point. Subsequent increase in τ moves the pole out of the unit circle, which corresponds to the unstable FL region in Fig. 6.3(a).

Next, we analyze the group delay at the drop port. At ω_0 , Eq. (6.16) is simplified to:

$$t_{g,D}(\omega_0) = \frac{1 + \tau r_1 r_2}{2(1 - \tau r_1 r_2)} t_n = \frac{1 + z_{p,D}}{2(1 - z_{p,D})} t_n. \quad (6.19)$$

Imposing the stability condition of $|z_{p,D}| < 1$ on Eq. (6.19) would mean:

$$\tau < 1/(r_1 r_2) \Rightarrow t_{g,T} > 0. \quad (6.20)$$

To show the implication of Eq. (6.20), we have plotted Eq. (6.19) in Fig. 6.4(a). It can be seen that at the drop port, only SL exists in the region of stable operation at $\tau < 1/(r_1 r_2)$. Similar to the through port case in Fig. 6.3(a), the reversal from stable SL to unstable FL occurs at the asymptote that is located at $\tau = 1/(r_1 r_2)$, which is also the oscillation point. However, the asymptote of the critical coupling, which is present for the through port case in Fig. 6.3(a), is absent for the drop port case in

Fig. 6.4(a). This occurs as only the pole, but not the zero, which produces critical coupling, is present in $t_{g,D}(\omega_0)$, as seen in Eq. (6.19). The zero is absent in Eq. (6.19) as the zero = 0 for the transfer function of the drop port in Eq. (6.14), which can also be seen in the pole-zero diagram of Fig. 6.4(b). The trends of the pole in regards to the FL and SL regimes of the drop port case are similar to those of the through port case, which have already been analyzed above and thus will not be repeated.

We have stated in section 6.2 that FL and SL are wave interference phenomena. Whether the transmitted light $b_1(t)$ at the through port is “fast” or “slow” depends on the interference $b_1(t) = r_1 a_1(t) + j\kappa_1 c_1(t)$ (based on Eq. (6.10)) at the coupling junction 1 in Fig. 6.2, where $a_1' = b_1' = c_1' = d_1' = a_2' = b_2' = c_2' = d_2' = a_2 = 0$ for a conventional TWMR. Note that the ballistic light $r_1 a_1(t)$ is π out of phase and time delayed with respect to the circulated light $j\kappa_1 c_1(t)$ at resonance. In the under (over) coupling

Fig. 6.4. (a) Group delay t_g (in t_r) of the transmitted light at the degenerate resonance frequency ω_0 for the drop port of a traveling wave microresonator with $\kappa_1 = \kappa_2 = 0.2$. Oscillation occurs at $\tau = 1.0417$. (b) The pole-zero dynamics on the unit circle as τ is varied. The cross (circle) denotes the pole (zero) of the transfer function.

regime, the circulated light is weaker (stronger) than the ballistic light. Thus, when $r_1 a_1(t)$ and $j\kappa_1 c_1(t)$ interfere, the trailing (leading) part of the ballistic (circulated) light vanishes through destructive interference, giving fast (slow) light. More details can be found in [8]. On the other hand, at the drop port of a conventional TWMR, the output light $b_2(t)$ only depends on the time delay incurred by the intracavity light circulation as $b_2(t) = j\kappa_2 c_2(t)$. As such, the transmission at the drop port is always “slow” in nature.

From this section, we can conclude that: (i) The group delay $t_g(\omega_0)$ of a TWMR depends on the coupling conditions, with a large enhancement in $t_g(\omega_0)$ near the critical coupling or oscillation point; (ii) the presence of pole (zero) in the transfer function, which is responsible for oscillation (critical

coupling), implies the possibility of stable SL (stable SL and FL); and (iii) the transition from stable FL to stable SL or vice versa occurs at the critical coupling point while the transition from stable SL to unstable FL or vice versa occurs at the oscillation point. These properties are later used to explain the effects of coupler-induced localized backscattering (CILB) on the traveling wave microresonator. Finally, note that only cases with zero z_z that is not equal to 0 are taken into consideration in our discussions as a zero z_z that is equal to 0 cannot produce the critical coupling, which is associated with SL, when a waveguide is coupled to a resonator. Here, the zero z_z refers to the zeros of the transfer function after a z -transform operation (see p. 144 and [36] for details). Critical coupling occurs when the zero z_z lies on the unit circle of the pole-zero diagram, i.e., $|z_z| = 1$. Nevertheless, for completeness, we have earlier included a zero that is $= 0$ in Eq. (6.14).

6.3.3. Limitations of Fast and Slow Light Performances

To realize a practical and versatile FL and SL device, one would require: (i) Continuously tunable group delay, with the ability to generate all three types of light propagation velocity (i.e., FL with negative v_g , FL with positive v_g and SL) in the stable operation regime, (ii) minimal insertion losses for both FL and SL, and (iii) a constant transmission and group delay over a broad bandwidth. We will focus only on (i) and (ii) in this chapter as fulfilling (iii) generally depends on expanding the delay-bandwidth product, which will be looked into in this thesis through the use of a twin-coupled resonators scheme in chapter 9. The transmission at ω_0 is plotted in Fig. 6.5, which is superimposed

Fig. 6.5. The relation between the transmission and the group delay at the degenerate resonance frequency ω_0 for the output light at the (a) through port and (b) drop port of a conventional traveling wave microresonator with $\kappa_1 = \kappa_2 = 0.2$ as τ is varied.

with Figs. 6.3 and 6.4. From Fig. 6.5, we can notice the limitations of the conventional TWMR in terms of its transmission. In particular, it is evident that at both ports, FL always suffers from substantial attenuation, while SL can be generated with high transmission. In fact, we can observe from Fig. 6.5 that each port has one specific value of τ that gives a transmission of 0-dB (i.e., lossless) for SL propagation. In addition, several limitations in the group delay performances of a conventional TWMR can be identified based on our earlier discussions in section 6.3.2. All these observations of the limitations of the conventional TWMR, i.e., with no coupler-induced localized backscattering (CILB), have been summarized in Table 6.1. Using Table 6.1, it can be concluded that a conventional TWMR does not meet (i) and (ii). However, when weak CILB is being harnessed, the FL and SL performances are improved, which are also shown in Table 6.1. These allow points (i) and (ii) to be met. In the next section, we will elaborate on how the use of weak CILB brings about such improved FL and SL performances in the TWMR.

Table 6.1. Summary of fast light (FL) and slow light (SL) performances at the degenerate resonance frequency ω_0 in the stable operation regime of a traveling wave microresonator

	Group Delay		Transmission	
	Without CILB	With Weak CILB	Without CILB	With Weak CILB
Through Port	Only two types of propagation velocity (FL with $v_g < 0$ and SL) can be produced.	All three types of propagation velocity (FL with $v_g < 0$, FL with $v_g > 0$ and SL) can be produced.	Possible to have SL with high transmission.	Possible to have SL with high transmission.
	Net loss region ($\tau < 1$) can produce only FL while the net gain region ($\tau > 1$) can produce both FL and SL.	Net loss ($\tau < 1$) and net gain regions ($\tau > 1$) can produce both FL and SL.	Fast light is always associated with low transmission.	Fast light is always associated with low transmission.
	Limited degree of freedom to tune the group delay.	Improved degree of freedom to tune the group delay		
Drop Port	Only one type of propagation velocity (SL) can be produced.	All three types of propagation velocity can be produced.	Possible to have SL with high transmission.	Possible to have SL with high transmission.
	Limited degree of freedom to tune the group delay.	Improved degree of freedom to tune the group delay.	FL is always associated with low transmission.	FL with amplified transmission can be generated.

6.4. Fast and Slow Light Performances in a Traveling Wave Microresonator with Coupler-Induced Localized Backscattering (CILB)

6.4.1. Theoretical Formulation

In the presence of CILB, both CW and CCW propagating modes will be excited in the resonator and the degeneracy is consequently lifted. As such, we will term the TWMR with CILB as a non-degenerate TWMR for the rest of this work. The coupling between the electric fields of the CW and CCW cavity modes in Fig. 6.2 in the presence of CILB can be described as

$$\begin{pmatrix} b_i' & b_i & d_i' & d_i \end{pmatrix}^T = \begin{pmatrix} [P_i] & [U_i] \\ [U_i] & [P_i] \end{pmatrix} \begin{pmatrix} a_i & a_i' & c_i & c_i' \end{pmatrix}^T = G_i \begin{pmatrix} a_i & a_i' & c_i & c_i' \end{pmatrix}^T, \quad (6.21)$$

$$\text{where } [P_i] = \begin{pmatrix} -jr_i' & r_i \\ r_i & -jr_i' \end{pmatrix}, \quad [U_i] = \begin{pmatrix} \kappa_i' & j\kappa_i \\ j\kappa_i & \kappa_i' \end{pmatrix}, \quad i = 1 \text{ or } 2.$$

In Eq. (6.21), κ_i' and r_i' are, respectively, known as the backward “cross” coupling coefficient and backward “through” coupling coefficient of the coupler at the coupling junction i ($i = 1$ or 2), while κ_i and r_i are the forward “cross” coupling coefficient and forward “through” coupling coefficient that are used earlier for the conventional TWMR. Note that here, the coupling coefficients are used to describe the coupling of the electric field amplitudes. For a TWMR with lossless couplers, G_i in Eq (6.21) satisfies the unitary condition $G_i^\dagger G_i = I$ (where the dagger † stands for the Hermitian conjugate and I is the identity matrix), which can be used to derive important relations between r_i , r_i' , κ_i and κ_i' :

$$|r_i|^2 + |r_i'|^2 + |\kappa_i|^2 + |\kappa_i'|^2 = \eta_i^2 + |r_i|^2 + |\kappa_i|^2 = 1, \quad (6.22)$$

$$\kappa_i r_i' = \kappa_i' r_i, \quad (6.23)$$

$$\eta_i^2 = |r_i'|^2 + |\kappa_i'|^2, \quad (6.24)$$

$$r_i = \sqrt{1 - \eta_i^2 - |\kappa_i|^2}, \quad (6.25)$$

$$\kappa_i = \sqrt{1 - \eta_i^2 - |r_i|^2}, \quad (6.26)$$

$$r_i' = \eta_i r_i / \sqrt{1 - \eta_i^2}, \quad (6.27)$$

$$\kappa_i' = \eta_i \sqrt{1 - \eta_i^2 - |r_i|^2} / \sqrt{1 - \eta_i^2}, \quad (6.28)$$

where $\eta = (|r_i|^2 + |\kappa_i|^2)^{1/2}$ has been introduced in the above equations to quantify for the CILB at each coupling junction. For the conventional TWMR in section 6.3, there is no CILB and thus $\eta = 0$. In this section, CILB is present in the TWMR. Then $\eta \neq 0$. The electric fields in the TWMR can be described as

$$c_1 = \sqrt{\tau} d_2 \exp(j\delta/2), \quad c_2 = \sqrt{\tau} d_1 \exp(j\delta/2), \quad (6.29)$$

$$c_1' = \sqrt{\tau} d_2' \exp(j\delta/2), \quad c_2' = \sqrt{\tau} d_1' \exp(j\delta/2). \quad (6.30)$$

Applying the boundary conditions $a_1' = a_2' = a_2 = 0$ and with the aid of Eqs. (6.22) to Eq. (6.30), the transfer functions for a TWMR with CILB can be derived from Eq. (6.21):

$$H_T = \frac{b_1}{a_1} = \frac{\tau^2 B_1 \exp(j2\delta) - \tau B_2 \exp(j\delta) + r_1}{\tau^2 B_3 \exp(j2\delta) + 2\tau B_4 \exp(j\delta) + 1}, \quad (6.31)$$

$$H_D = \frac{b_2}{a_1} = \frac{\tau^{3/2} B_5 \exp(j3\delta/2) + \sqrt{\tau} B_6 \exp(j\delta/2)}{\tau^2 B_3 \exp(j2\delta) + 2\tau B_4 \exp(j\delta) + 1}, \quad (6.32)$$

where

$$\begin{aligned} B_1 &= r_1 r_2^2 / \psi_2, \quad B_2 = \eta_1 r_2 (\eta_1 r_1^2 \psi_1^{-1/2} - 2\eta_2 \psi_1^{1/2}) / \sqrt{\psi_2} + r_2 (r_1^2 - 2\eta_1^2 + 1), \\ B_3 &= (r_1 r_2)^2 / (\psi_1 \psi_2), \quad B_4 = r_1 (\eta_1 \eta_2 r_2 / \sqrt{\psi_1 \psi_2} - r_2), \\ B_5 &= r_1 r_2 \sqrt{(\psi_1 - r_1^2)(\psi_2 - r_2^2)} / (\psi_1 \psi_2), \quad B_6 = (\eta_1 \eta_2 - \sqrt{\psi_1 \psi_2}) \sqrt{(\psi_1 - r_1^2)(\psi_2 - r_2^2)} / \sqrt{\psi_1 \psi_2}, \end{aligned}$$

in which $\psi_1 = (1 - \eta_1^2)$ and $\psi_2 = (1 - \eta_2^2)$. Using the z -transform, the zeros $z_{z,T}$ ($z_{z,D}$) and poles $z_{p,T}$ ($z_{p,D}$) of the through (drop) port can be obtained from Eqs. (6.31) and (6.32):

$$z_{z,T} = \left(B_2 \pm \sqrt{B_2^2 - 4B_1 r_1} \right) / (2r_1 \tau^{-1}), \quad z_{p,T} = \tau \left[(r_1 r_2 - r_1' r_2') \pm j(r_1 r_2' + r_1' r_2) \right], \quad (6.33)$$

$$z_{z,D} = -\tau B_5 / B_6, \quad z_{p,D} = \tau \left[(r_1 r_2 - r_1' r_2') \pm j(r_1 r_2' + r_1' r_2) \right]. \quad (6.34)$$

When $\eta = 0$, Eqs. (6.21), (6.31), (6.32), (6.33) and (6.34) will simplify, respectively, to Eqs. (6.10), (6.11), (6.12), (6.13) and (6.14), which are the expressions for the TWMR with no CILB (i.e., $\eta = 0$). This shows the validity of our modeling. It should also be noted that when CILB is present in the TWMR, the transmission spectrum at each output port is either a Lorentzian (with degenerate resonance frequency ω_0 at the peak/dip of the Lorentzian) or a pair of two-split Lorentzian (with the split resonance frequencies $\omega_{0,1}$ and $\omega_{0,2}$ at the peaks/dips of the two-split Lorentzian). Whether the former or latter case occurs depends on the nature of the backscattering (which in this case is CILB) in relation to the gain/loss and coupling of the cavity [29]. This is better illustrated later in Fig. 6.8 of section 6.4.4. Thus, the poles and zeros in Eqs. (6.33) and (6.34) can either correspond to the the degenerate resonance frequency ω_0 (at the the Lorentzian peak/dip) or the split resonance frequencies $\omega_{0,1}$ and $\omega_{0,2}$ (at the peaks/dips of the two-split Lorentzians).

6.4.2. Deducing the Impacts of CILB via the Poles and Zeros

Comparing with Eqs. (6.13) and (6.14), it is evident from Eqs. (6.33) and (6.34) that the presence of CILB in the cavity changes the nature as well as the number of poles and zeros of the

system. Utilizing the poles and zeros, the impacts of CILB can be deduced. This is highlighted below.

As seen from Eqs. (6.33) and (6.34), the through and drop ports of the non-degenerate TWMR (i.e., with CILB) have poles of the same values. This behavior also applies to the conventional degenerate ring (i.e., without CILB) as shown earlier in Eqs. (6.13) and (6.14). However, there are two poles in the non-degenerate TWMR (cf. Eqs. (6.33) and (6.34)) in contrast to only one pole for the degenerate TWMR (cf. Eqs. (6.13) and (6.14)) at both the through and drop ports. Using Eq. (6.33), it can be deduced that the two poles of the through port of the non-degenerate case are always symmetrical to each other, i.e., $a_{p1} = a_{p2}$ and $\phi_{p1} = -\phi_{p2} \neq 0$, where a_{p1} and a_{p2} denote the magnitudes of the two poles, while ϕ_{p1} and ϕ_{p2} denote the angles of the poles. This trend also applies to the poles of the drop port in Eq. (6.34). The presence of a pair of symmetrical poles indicates the existence of resonance splitting. In other words, when CILB is present, the poles will correspond to the split resonance frequencies $\omega_{0,1}$ and $\omega_{0,2}$ but not ω_0 . Consequently, oscillation takes place at $\omega_{0,1}$ and $\omega_{0,2}$ rather than at ω_0 .

As seen from Eqs. (6.33) and (6.34), the through and drop ports of the non-degenerate TWMR (i.e., with CILB) have zeros that are of different values. This property can also be seen in the conventional degenerate ring (i.e., without CILB) in Eqs. (6.13) and (6.14). However, we can see from Eq. (6.33) that the through port of the TWMR with CILB has two zeros. This is in contrast to the conventional TWMR in which there is only one zero as shown in Eq. (6.13). When η is small such that $B^2 \geq 4B_1r_1$ in Eq. (6.33), both the zeros are real and distinct, having different amplitudes, i.e., $a_{z1} \neq a_{z2}$ and $\phi_{z1} = \phi_{z2} = 0$, where a_{z1} and a_{z2} denote the magnitudes of the two zeros while ϕ_{z1} and ϕ_{z2} denote the angles of the two zeros. The zeros can only correspond to the split resonance frequencies $\omega_{0,1}$ and $\omega_{0,2}$ if only they are symmetrical to each other (i.e., $a_{z1} = a_{z2}$ and $\phi_{z1} = -\phi_{z2} \neq 0$). As such, these zeros in Eq. (6.33) correspond to ω_0 . Given fixed η and coupling coefficients, each zero will cross the unit circle on the pole-zero diagram at two different values of τ . This means that ω_0 has two critical coupling points at the through port for small η , in contrast to the degenerate case or conventional TWMR, where ω_0 has only one critical coupling point. At sufficiently large η such that $B^2 < 4B_1r_1$, the zeros change such that they are complex and are symmetrical to each other. Then the two zeros correspond to the split resonance frequencies $\omega_{0,1}$ and $\omega_{0,2}$. In this case, critical coupling occurs at the split resonances instead at ω_0 . On the other hand, only one zero is found at the drop port of the TWMR when CILB is present (cf. Eq. (6.34)). This property is similar to the case of the conventional TWMR (cf. Eq. (6.14)). However, it should be noted that the zero of the drop port is $= 0$ for the case of the conventional TWMR (cf. Eq. (6.14)). This is in contrast to the TWMR with CILB that has a zero that is $\neq 0$ at the drop port (cf. Eq. (6.34)). A zero that is $= 0$ ($\neq 0$) for the drop port means that critical coupling is impossible (possible) at the drop port when the waveguide is coupled to the resonator. Thus, CILB can be used to bring about critical coupling at the drop port.

In this work, the emphasis is on the fast light (FL) and slow light (SL) natures at the degenerate resonance frequency ω_0 as we have found that the group delay enhancement is generally larger at ω_0 as compared to that at the split resonance frequencies $\omega_{0,1}$ and $\omega_{0,2}$. To facilitate this, the number of poles and zeros that correspond to the degenerate resonance frequency ω_0 for a conventional TWMR (without CILB) and a non-degenerate TWMR (with weak CILB) are summarized in Table 6.2. These data in Table 6.2 are based on Eqs. (6.13), (6.14), (6.33) and (6.34) and the above-mentioned trends of the poles and zeros of the TWMR for the degenerate (i.e., no CILB) and non-degenerate (CILB is present) cases at ω_0 . From Table 6.2, it is clear that at ω_0 , CILB alters the number of poles and zeros (which are $\neq 0$). In particular, for both ports, the pole vanishes and is replaced by a zero (which is $\neq 0$) when moving from the regime of no CILB to regime of weak CILB. As mentioned in section 6.3.2, the pole (zero) is associated with oscillation (critical coupling) and the generation of stable SL (stable FL and SL). Then it can be deduced that for both ports, the oscillation point at $\tau > 1$ of the through

Table 6.2. [†]Number of poles and zeros that correspond to the degenerate resonance frequency ω_0 of a traveling wave microresonator.

	Number of Poles		Number of Zeros (which are $\neq 0$)	
	Without CILB	With Weak CILB	Without CILB	With Weak CILB
Through Port	One Pole	No Pole	One Zero	Two Zeros
Drop Port	One Pole	No Pole	No Zero (which is $\neq 0$)	One Zero

[†]For both ports, when moving from the regime of no coupler-induced localized backscattering (CILB) to the regime of weak CILB, the number of pole (zero) that corresponds to the degenerate resonance frequency ω_0 decreases (increases) by one. In other words, the pole of ω_0 vanishes and is replaced by a zero. It should be noted that only zero that is $\neq 0$ is taken into consideration in this table and in our discussion as a zero that is $= 0$ cannot produce critical coupling, which is associated with slow light properties, when a waveguide is coupled to a resonator.

port of a conventional TWMR will be replaced by a critical coupling point in the presence of weak CILB. Correspondingly, the stable SL and unstable FL near the oscillation point at $\tau > 1$ (shown earlier in Figs. 6.3(a) and 6.4(a)) of a conventional TWMR will transform, respectively, into stable FL and stable SL when there is weak CILB. This will be verified in the next section. It should be noted that only zero that is $\neq 0$ is taken into consideration in our discussion as a zero that is $= 0$ cannot produce critical coupling, which is closely linked to the SL properties, when a waveguide is coupled to a resonator.

6.4.3. Group Delay Classification at the Frequency ω_0

The group delay $t_g(\omega_0)$ at the through and drop ports can be derived from Eqs. (6.31) and (6.32):

$$t_{g,T}(\omega_0) = \left[\frac{2\tau^2 B_7 (|r_2|^2 + |r_2'|^2) - \tau B_8}{\tau^2 B_7 (|r_2|^2 + |r_2'|^2) - \tau B_8 + r_1} - \frac{2\tau^2 (|r_1|^2 + |r_1'|^2)(|r_2|^2 + |r_2'|^2) - \tau(2r_1 r_2 - 2r_1' r_2')}{\tau^2 (|r_1|^2 + |r_1'|^2)(|r_2|^2 + |r_2'|^2) + \tau(2r_1' r_2' - 2r_1 r_2) + 1} \right] t_{rt}, \quad (6.35)$$

where $B_7 = k_1^2 r_1 + r_1 |r_1'|^2 + |r_1|^3 - |k_1'|^2 r_1 + 2k_1 k_1' r_1'$, $B_8 = |k_1|^2 r_2 + 2r_1^2 r_2 - |k_1'|^2 r_2 - 2k_1 k_1' r_2' - 2r_1 r_1' r_2'$.

$$t_{g,D}(\omega_0) = 2 \left[\frac{\tau(3\tau B_9 + \kappa_1' \kappa_2' - \kappa_1 \kappa_2)}{4(\tau B_9 + \kappa_1' \kappa_2' - \kappa_1 \kappa_2)} + \frac{r_1 r_2 (1 - \tau r_1 r_2) - r_1' r_2' (1 + \tau r_1' r_2') - \tau(|r_1|^2 |r_2|^2 + |r_1'|^2 |r_2'|^2)}{\tau^{-1} (1 + \tau r_1' r_2')^2 + \tau |r_1|^2 (|r_2|^2 + |r_2'|^2) + r_2 (\tau r_2 |r_1'|^2 - 2r_1)} \right] t_{rt}, \quad (6.36)$$

where $B_9 = (r_1 r_2 - r_1' r_2')(\kappa_1 \kappa_2 - \kappa_1' \kappa_2') + (\kappa_1 \kappa_2' + \kappa_1' \kappa_2)(r_1 r_2' + r_1' r_2)$.

Using Eqs. (6.35) and (6.36), we have plotted $t_g(\omega_0)$ under different η for the through and drop ports in Figs. 6.6 and 6.7, respectively. For comparison, the $t_g(\omega_0)$ curves of a conventional TWMR with no CILB (i.e. $\eta = 0$), as shown earlier in Figs. 6.3 and 6.4, are also shown in Figs. 6.6 and 6.7. The effects of small increment in η on the $t_g(\omega_0)$ curve of a conventional TWMR at the through port is shown in the inset in Fig. 6.6(a), in which it can be seen that the $t_g(\omega_0)$ curve transforms from a rapidly increasing function (cf. $\eta = 0$) to a rapidly decreasing function (cf. $\eta = 0.0006$ or $\eta = 0.0009$) in the net gain region (i.e., $\tau > 1$). Changes in $t_g(\omega_0)$ in the net loss (i.e., $\tau < 1$) region, however, are minimal. Consequently, the stable SL (unstable FL) regime to the left (right) of the asymptote (at $\tau \approx 1.04$) in the net gain region switches to a stable FL (stable SL) regime when η changes from 0 to 0.0006 or 0.0009, as observed in the inset in Fig. 6.6(a). Such change in $t_g(\omega_0)$ in the net gain region when $\eta \neq 0$ is consistent with the deduction made using the poles and zeros of the system in section 6.4.2. Subsequent increase in η simply translates the hyperbolic curves in the direction of the arrows, as shown in Fig. 6.6(b). This will result in the FL and SL regions bounded by the two asymptotes to reduce in width. Eventually, at large η , the hyperbolic-like feature of $t_g(\omega_0)$ vanishes, as seen in Fig. 6.6(c). This degrades the group delay enhancement.

Similarly, for the case of the drop port, a small change in η will result in the $t_g(\omega_0)$ curve of a conventional TWMR to transform from an increasing function (cf. $\eta = 0$) to a decreasing function (cf. $\eta = 0.0006$ or $\eta = 0.0009$) in the net gain region, as shown in the inset in Fig. 6.7(a), which then correspondingly changes the regime of light propagation velocity. A subsequent increase in η in Fig. 6.7(b) smoothen the t_g curves and widens the FL and SL regions. At larger η , the t_g curves shift in the direction of the arrows in Fig. 6.7(c). Then the width of the FL (SL) region decreases (increases) as η increases. Note that unlike the through port case in Fig. 6.6(c), a large η does not destroy the sharp hyperbolic features of the $t_g(\omega_0)$ curves of the drop port, as seen in Fig. 6.7(c). In other words, working in the strong CILB regime does not degrade the sharp $t_g(\omega_0)$ enhancement at the drop port, while the converse is true for the through port case.

Fig. 6.6. Group delay at the degenerate resonance frequency ω_0 for the through port of a traveling wave microresonator with $\kappa_1 = \kappa_2 = 0.2$ as a function of τ at different η . (a) In the inset, the group delay curve changes from an increasing function to a decreasing function when η is tuned from 0 to 0.0006 or 0.0009. (b) Further increment in η shifts the asymptotes in the direction of the arrows. (c) If η becomes too large, the group delay enhancement is degraded.

Fig. 6.7. Group delay at the degenerate resonance frequency ω_0 for the drop port of a traveling wave microresonator with $\kappa_1 = \kappa_2 = 0.2$ as a function of τ at different η . (a) As shown in the inset, the group delay curve changes from an increasing function to a decreasing function when η is tuned from 0 to 0.0006 or 0.0009. Further increment in η smoothens the group delay curves. (c) At large η , the group delay curves shift in the direction of the arrows, without degrading the group delay enhancement, unlike the case at the through port in Fig. 6.6(c).

The above-mentioned changes in the nature of the FL and SL (as well as their associated poles and zeros in Table 6.2) as compared to those of the conventional TWMR in section 6.3 are due to the modification of the nature of interference at the output ports by the presence of CILB. For the through port, the interference process of the TWMR with CILB becomes $b_1(t) = r_1 a_1(t) + j\kappa_1 c_1(t) + \kappa_1' c_1'(t)$, in contrast to the conventional TWMR case of $b_1(t) = r_1 a_1(t) + j\kappa_1 c_1(t)$. With an additional net wave of $\kappa_1' c_1'(t)$ to interact with the ballistic wave $r_1 a_1(t)$, the behaviours of FL and SL at the through port are consequently altered. For the drop port, the formation of the transmitted wave of the non-degenerate case is described by $b_2(t) = j\kappa_2 c_2(t) + \kappa_2 c_2(t)$. Then both fast and slow light effects can be achieved at the drop port due to the presence of an additional wave $\kappa_2 c_2(t)$ to interfere with $j\kappa_2 c_2(t)$, unlike the case of the conventional TWMR, in which $b_2(t) = j\kappa_2 c_2(t)$, and thus the output light is always “slowed” by $c_2(t)$ that circulates in the cavity. These deductions are based on Eqs. (6.10) and (6.21).

6.4.4. Spectral Characteristics

In this section, we show the effects of CILB on the spectral responses in order to better explain the group delay trends at ω_0 in Figs. 6.6 and 6.7. The spectral responses for the through and drop ports are shown in Fig. 6.8(a) for the net loss regime and in Fig. 6.8(b) for the net gain regime at various η .

Fig. 6.8. Evolution in the transmission and group delay spectra around the degenerate resonance frequency ω_0 of a traveling wave microresonator with $\kappa_1 = \kappa_2 = 0.2$ for (a) the net loss regime ($\tau = 0.99$) and (b) net gain regime ($\tau = 1.035$) as η is varied.

To explain the trends of the spectral responses in Fig. 6.8, first note that two eigenmodes are formed due to the CILB that breaks the rotational symmetry of the cavity. These eigenmodes are clearly resolved in the transmission spectrum in the form of a two-split Lorentzian only if the mode

splitting or frequency separation between the eigenmodes (controlled by η) is much larger than the resonance linewidth (controlled by τ and r). Otherwise, the transmission spectrum has the shape of a single-Lorentzian near ω_0 . This is somewhat analogous to the Rayleigh criterion in diffraction optics. The above-mentioned occurrence of either a single sharp Lorentzian or a pair of two-split Lorentzian in the transmission spectrum has also been mentioned earlier in section 6.4.2.

For the net loss ($\tau < 1$) regime in Fig. 6.8(a), the presence of small CILB in the range of $\sim \eta \leq 0.01$ does not significantly affect the spectral responses; the spectral responses of $\eta = 0.006$ are nearly similar to that of $\eta = 0$. As a result, one can see that increasing η from 0 to 0.006 has a negligible effect for $\tau < 1$ for the group delay at ω_0 in Figs. 6.6(a) and 6.7(a). However, at larger η , the mutual coupling between the CW and CCW modes in the TWMR will be strong enough to induce mode splitting effects in the transmission spectrum. This can be seen in Fig. 6.8(aii) and Fig. 6.8(aiv), respectively, for the through and drop ports at $\eta = 0.06$. The occurrence of mode splitting in the transmission will then result in the group delay enhancement at the degenerate resonance frequency to be considerably reduced at the through port in Fig. 6.8(ai), and this is consistent with the trends in Fig. 6.6(c) earlier. However, for the drop port, such mode splitting effects will only cause the SL to switch to FL when η changes from 0 to 0.06 in Fig. 6.8(aiii), with no degradation to the group delay (or advance) enhancement. This matches with the trends in Figs. 6.7(a) to 6.7(c).

The presence of net optical gain ($\tau > 1$) in the cavity, which is the case in Fig. 6.8(b), will translate into amplification of the intracavity standing waves. This then results in observable mode splitting in the transmission spectra even at small η , as observed in Figs. 6.8(bii) and 6.8(biv). Such phenomenon has been experimentally observed in [32], which thus verifies the validity of our research methodology and results. The mode splitting effect in Figs. 6.8(bii) and 6.8(biv) will subsequently cause the original normal dispersion (i.e., SL) when $\eta = 0$ to be reversed to anomalous dispersion (i.e., FL) when $\eta = 0.006$, in Figs. 6.8(bi) and 6.8(biii). This accounts for the switch from SL (FL) to FL (SL) earlier in the dotted boxes of Figs. 6.6(a) and 6.7(a) when η changes from 0 to 0.0006 or 0.0009 for $\tau > 1$. Similar to the case of the net loss regime in Fig. 6.8(a), there will be large mode splitting at large η in the net gain regime, as seen in Figs. 6.8(bii) and 6.8(biv) at $\eta = 0.06$. This results in the group delay enhancement at ω_0 to be reduced considerably at the through port in Fig. 6.8(bi). This is consistent with the trends in Fig. 6.6(c), where the sharp hyperbolic feature vanishes and the $t_g(\omega_0)$ curves are degraded. However, for the drop port, a large η do not affect the group delay enhancement at ω_0 as seen in Fig. 6.8(biii), which is congruent with the trends in Fig. 6.7(c).

6.4.5. Improved Fast and Slow Light Performances

Earlier, we have shown in Table 6.1 on the improved FL and SL performances using weak CILB. We now elaborate on these based on the results in sections 6.4.2 to 6.4.4.

With weak CILB, the stable SL (unstable FL) regime to the left (right) of the asymptote at $\tau > 1$ in the $t_g(\omega_0, \tau)$ curve switches to a stable FL (stable SL) regime. This can be seen in the insets in Figs. 6.6(a) and 6.7(a) when η changes from 0 to 0.0006 or 0.0009. Thus, weak CILB gives an additional flexibility to generate stable FL. Also, weak CILB makes it possible to observe all three regimes of light propagations: FL with negative v_g , FL with positive v_g and SL at both the through and drop ports for both the net loss ($\tau < 1$) and net gain ($\tau > 1$) regions, which can also be seen in Figs. 6.6 and 6.7. This increases the versatility of the TWMR as a FL and SL device. Also, CILB offers an extra degree of freedom in addition to the parameters of κ and τ of the cavity to control the group delay as any change in η will modulate the group delay.

Finally, weak CILB also improves the transmission of the TWMR. To illustrate this, we have plotted the transmission at ω_0 for a TWMR with $\eta = 0.006$ in Fig. 6.9, which is superimposed with the group delay graphs of Figs. 6.6 and 6.7. Comparing Fig. 6.9 with the transmission of a conventional TWMR in Fig. 6.5, we can notice that with CILB, FL with amplified transmission can be generated at the drop port when there is a small net gain in the cavity. The lower and upper limits of the regions of FL with amplified transmission are marked by vertical dotted lines in Fig. 6.9(b). This is in contrast to the case of a conventional TWMR, whereby stable FL, which is generated at the through port, always has substantial attenuation, as seen in Fig. 6.5(a). Note that even by shifting the critical coupling point through adjusting the coupling coefficients (as critical coupling occurs at $\tau = r_1/r_2$) so that part of the FL regime fall into the net gain region (i.e., $\tau > 1$) in Fig. 6.5(a), substantial attenuation will still be observed in the transmission response. This is a fundamental constraint of conventional TWMR system. However, the constraint of fast light with low output power in a conventional degenerate

Fig. 6.9. The relation between the transmission and the group delay at the degenerate resonance frequency ω_0 for the output light at the (a) through port and (b) drop port of a non-degenerate traveling wave microresonator with $\kappa_1 = \kappa_2 = 0.2$ as τ is varied for $\eta = 0.006$.

TWMR can be mitigated by harnessing the CILB effects, in conjunction with a small amount of gain in the cavity. For example, at $\tau = 1.037$ and $\eta = 0.006$, a normalized group delay of $t_g = -90t_{rt}$ with an amplified transmission of 3 dB can be obtained at the drop port, as seen in Fig. 6.9(b).

To harness the CILB to realize a tunable FL and SL system, two approaches can be taken: (i) The round-trip amplitude attenuation constant τ of the cavity can be actively tuned by electrically [39] or optically [40] pumping the active material in the waveguide core or cladding, assuming that the coupling coefficients are fixed after fabrication in the presence of an optimal η ; and/or (ii) incorporating reflective optical elements [41, 42] at or near the coupling junctions with value of η . To actively tune η after fabrication, integrated micro-electro-mechanical systems (MEMS) can be employed at the coupling junctions [43]. This will give an extra degree of freedom to adjust t_g . For example, for a TWMR with $\kappa_1 = \kappa_2 = 0.2$ and $\tau = 0.995$, tuning η from 0.003 to 0.009 changes t_g from $-148t_{rt}$ to $-52t_{rt}$ at the through port, where t_{rt} is the round trip time of the resonator.

Note that the mode splitting effects as observed in this work are also consistent with that produced by distributed backscattering due to surface perturbations in the cavity [13, 27-30], which has been assumed to be of negligible effect in this work. This is a valid assumption for unless deliberate attempts are made to create perturbing micro-structures, such as gratings ridges [13], on the surface of the cavity, distributed backscattering in the cavity are generally minimal as advancement in the fabrication technology has made it possible to greatly reduce the sidewall roughness [44, 45], which is the dominant source of the distributed backscattering in the cavity. Lastly, we emphasize that our results in this work apply strictly to a *circular* TWMR. If a TWMR with other configurations, particularly a *racetrack* configuration, such as a racetrack microring resonator, is used, there will be additional backscattering within the cavity due to modal mismatch between the straight waveguide mode and the bend waveguide mode. This generally leads to Fresnel reflections at the junctions between the straight and bend waveguides. Such an effect on the dispersion of the TWMR can be looked into in a future work.

6.5. Evolution of the Output Pulse of the Traveling Wave Microresonator

For completeness, we give a brief discussion on how the shape of the output pulse evolves as τ is tuned to change the time delay or advancement of the pulse propagation. In this example, we use a conventional TWMR with no CILB, i.e., $\eta = 0$. In Fig. 6.10, the evolution of the shape of the pulse at the through port as the operation of the TWMR moves from the FL to the SL regime is shown. In the FL regime, in which the resonator is under-coupled, increasing τ brings the TWMR towards the critical coupling state. This increases the time advancement and gradually leads to pulse splitting. It is seen that at close to critical coupling (which occurs at $\tau = r_1/r_2 = 1$ for this example) in Fig. 6.10(c), pulse splitting starts to take place. The input pulse splits into two output pulses, one with time advance, which has a larger intensity, and the another with time delay, which has a lower intensity

and is magnified in the inset in Fig. 6.10(c). Moving the resonator closer to the critical coupling condition increases (decreases) the intensity of the output pulse with the time delay (advancement) such that at the critical coupling condition, the two output pulses have equal intensity. The occurrence of pulse splitting at critical coupling clearly validates our concluding paragraph in section 6.3.2 that the transition from FL to SL or vice versa occurs at the critical coupling point. As the resonator moves into the SL regime, in which the resonator is over-coupled, the intensity of the pulse with the time

Fig. 6.10. Evolution of the output pulse at the through port for a conventional traveling wave microresonator with $r_1 = r_2 = 0.9$ and $\eta = 0$ (as an example) as τ is being adjusted. For convenience, the input Gaussian pulse is only shown in Fig. 6.10(a). Note that critical coupling at the resonance frequency occurs at $\tau = r_1/r_2$ while oscillation at the resonance frequency occurs at $\tau = 1/(r_1 r_2)$. For this example, $\tau = 1$ gives critical coupling, which will split the input pulse into two output pulses (Fig. 6.10(e)), while $\tau = 1.2346$ gives oscillation, which will transform the input pulse into an optical step function ((in Fig. 6.10(j)).

advance (delay) will decrease so that at sufficiently large τ , only the output pulse with time delay remains. These are shown in Figs. 6.10(f) to 6.10(g). Thereafter, as the resonator moves towards the oscillation condition in the over coupling regime, the time delayed output pulse will progressively transform into an optical step function. This transformation is shown from Figs. 6.10 (g) to 6.10(j). The above-mentioned phenomenoma of the splitting of the input pulse into two output pulses (i.e., Fig. 6.10(e)) and the formation of an optical step function (i.e., Fig. 6.10(j)), respectively, at the critical coupling and oscillation conditions are consistent with the divergence of $\partial\Phi/\partial\delta$ and n_g at the resonance frequency (as mentioned in section 6.3.2). Such splitting of the input pulse has been harnessed in optical differentiation [46] while the formation of the optical step function has been used in optical integration [47]. Thus, by tuning τ , as shown in Fig. 6.10, a single two-port TWMR device can provide dual functions of optical differentiation and integration. Finally, note that the general

trend as shown from Figs. 6.10(g) to 6.10(j) can also be applied to the output pulse at the drop port, which only produces oscillation but not critical coupling. In addition, we mention that the trends in the evolution of the pulse shape in Fig. 6.10 as the coupling state of the TWMR approaches the critical coupling and oscillation conditions are also valid for the case of the TWMR with CILB, i.e., $\eta \neq 0$. Changing η from $\eta = 0$ to $\eta \neq 0$ only changes the value of τ at which critical coupling or oscillation take place, but not the nature of the evolution of the output pulse.

6.6. Summary and Significant Research Contributions

With the growing importance of traveling wave microresonators (TWMRs) in fast and slow light applications, it is crucial for an evaluation of the impact of coupler-induced localized backscattering (CILB) on the dispersion of the TWMRs, which has been achieved in this work. In particular, we have demonstrated that weak CILB aided by a small net optical gain in the cavity, will enhance the fast light (FL) and slow light (SL) performances of a TWMR in the following ways. First, all three types of light propagation velocity can be generated at both the through and drop ports at the degenerate resonance frequency ω_0 . This is in contrast to the conventional TWMR (where CILB is negligible), in which at ω_0 , only FL with negative v_g and SL are present at the through port, while for the drop port, only SL is produced. Second, the possibility to generate stable FL with amplified transmission can be achieved, unlike the case of a conventional TWMR, where stable FL always has substantial attenuation. Third, there is an extra degree of freedom in addition to the parameters of coupling coefficient κ and round-trip attenuation constant τ of the cavity to control the group delay as any change in the strength of the CILB will modulate the group delay. However, do note that in the regime of strong CILB, there will be some deleterious effects: (i) The group delay enhancement at the through port is degraded; and (ii) the width (i.e., range of τ , given fixed κ and magnitude of CILB) of the FL regime at the drop port is reduced. These research results are novel contributions to the field of FL and SL in TWMR devices and demonstrate that CILB can be harnessed to improve the FL and SL effects. The research results are significant contributions to the fields of FL and SL in TWMRs as CILB has generally been overlooked in the study of TWMR in the literature.

Finally, note that part of this research work has been published in:

- **Thomas Y.L. Ang** and Nam Quoc Ngo, “Harnessing coupler-induced localized backscattering for enhanced fast and slow light performances in a traveling wave microresonator,” *J. Opt. Soc. Am. B* **27**, 2639–2647 (December 2010).

On a positive note, the above-mentioned paper has been selected for the January 2011 issue of *The Virtual Journal of Ultrafast Science*, which can be found in [48]. This attests strongly to the significance and novelty of our research work in this chapter. Our findings in this paper would thus be of relevance and importance to the advancement in the science and technology of fast and slow light.

6.7. References for Chapter

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Chapter 7

Tunable Fast and Slow Light via Interaction of Intracavity Backscattering with Dual Contra-propagating Inputs

7.1. Introduction: Research Motivations and Objectives

In the previous chapter, we have shown that coupler-induced localized backscattering (CILB) can be exploited to improve the fast light (FL) and slow light (SL) performances of a traveling wave microresonator (TWMR). Besides CILB, another type of backscattering in TWMRs is the distributed backscattering within the cavity that is induced by surface imperfections. We will term this as intracavity backscattering. As mentioned in the previous chapter, intracavity backscattering has been the main emphasis in the study of backscattering in TWMRs ever since it was first observed in [1]. Here, in this chapter, we will look into the use of intracavity backscattering to tune and enhance the FL and SL performances of a TWMR by interacting it with dual contra-propagating inputs. Before going into the details, let us first look into the motivations behind this research proposal.

One of the reasons for the popularity of TWMR based devices is that they are capable of possessing whispering gallery modes (WGMs) with high-quality Q factor and small modal volume V , i.e., high Q/V ratio. In fact, the highest Q/V ratio to date has been achieved using TWMRs [2]. However, this high Q/V ratio of TWMRs also means that extremely small surface perturbations can easily lead to significant intracavity backscattering, which will result in: (i) The formation of spectral doublets [3, 4] in place of a single resonance peak/dip, which is often used in filtering [5]; (ii) the degradation of the Q -factor [6]; and (iii) the destruction of the directionality of the light propagation [7]. Due to this, the subject of intracavity backscattering in TWMRs has been studied thoroughly, primarily motivated by the goal of *reducing* surface perturbations due to fabrication-induced surface imperfections so as to minimize the above effects of intracavity backscattering in the TWMR devices [4, 6, 7]. On the other hand, intracavity backscattering can be put into practical uses. For example, in [8], the doublet resonance formed due to the fabrication-induced surface imperfections is used as a source of resonance optical feedback for narrowing the linewidth of the semiconductor laser. To further exploit the intracavity backscattering effect for more interesting applications, surface

perturbations in the TWMRs are sometimes *intentionally* induced either by deliberately introducing surface microstructures on the cavity during fabrication [9, 10, 11, 12] or the incorporation of nanoscale particles [13, 14] in the cavity after fabrication so as to enhance the intracavity backscattering effects. Thus far, applications that utilize such enhanced intracavity backscattering effects in TWMRs include lasing [11], higher-order filtering [12], sensing [13, 14] as well as fast and slow light effects [9, 10]. In particular, it is shown in [9, 10] that the enhanced intracavity backscattering, due to the addition of grating ridges on a microring resonator, can produce mode splitting that is accompanied by the generation of either FL or SL, depending on the coupling conditions. This demonstrates the possibility of harnessing the enhanced intracavity effects in a TWMR to generate fast light and slow light.

However, for the fast and slow light scheme based on the above-mentioned enhanced intracavity backscattering effect to be viable for practical applications, the group delay must be *actively tunable* (i.e., the group delay is variable and responsive to an external actuation-or-control signal). We have summarized in Table 7.1 of the features of the various active tuning schemes in the literature, labeled as schemes (a)-(d), which can be used to continuously vary the group delay and transmission of a TWMR. Note that thus far, only scheme (d) has been implemented on TWMR with enhanced backscattering to realize tunable fast and slow light [10]. Generally speaking, the main drawback of schemes (a)-(c), which we term as conventional active tuning (CAT) schemes, is that additional procedures and controls are required in the fabrication process so as to incorporate the active tuning components, such as p-i-n diodes [15-17], metal oxide semi-conductor (MOS) [18, 19], electrodes [20, 21], microheaters [22-24], gain media (like semi conductor optical amplifiers (SOAs) [25] or Erbium ions [26]) and integrated micro-electro-mechanical systems (MEMS) [27, 28] in the cavity system, which would increase the overall complexity and cost in the production of the cavity for practical applications. The use of such active components generally also considerably increases the footprint of the cavity, particular for schemes (b) and (c). Moreover, as the active components of the CAT schemes are implanted in the cavity system, it would be difficult to reconfigure these active components or to replace them after fabrication. In the case in which a passive cavity has already been fabricated, it might not be feasible or possible in certain situations to do additional alternation to the device so as to render it active. To circumvent these issues, scheme (d), which is all-optical tuning (AOT) [10, 29], can be employed. However, such a scheme is highly dependent on the material nonlinearity of the cavity and as a result, it might require larger energy requirement as compared to the CAT schemes in some cases. For example, in [10], as the cavity is based on silicon, thermal nonlinearity primarily due to two-photon absorption is utilized for all-optical tuning, which consumes more energy in contrast to the CAT scheme based on the electro-optic effect in [16].

In this work, we will demonstrate the possibility to harness the intracavity backscattering in a TWMR to realize a fast and slow light system with continuous tunability in the group delay that does

Table 7.1. Comparisons between the commonly used active tuning schemes in TWMRs with our proposed dual-input scheme

Types of active tunable schemes to control group delay (GD)	Conventional Active Tuning (CAT) Schemes			(d) All-optical tuning (AOT) scheme	(e) Our proposed dual-input scheme
	(a) Modulation of group index n_g of cavity	(b) Modulation of bus waveguide-to-cavity coupling κ	(c) Modulation of amplitude attenuation constant τ of cavity		
Methods of implementation	Using thermo-optic [22] or electro-optic [15-20] effect for the cavity.	Using MEMS [27, 28] or electro-optic [21] / thermo-optic [24] effect to realize tunable couplers.	Electrically [25] or optically [26] pumping a cavity, which contains active media.	Two co-propagating beams (control and probe) of different frequency and power are inserted into the cavity [10, 29].	Two contra-propagating beams of the same frequency are inserted into a cavity with backscattering.
Operation principle	The resonance is shifted as n_g is modulated. This tunes the GD.	The linewidth of the resonance is changed as κ is modulated. This tunes the GD.	The linewidth of the resonance is changed as τ is modulated. This tunes the GD.	Adjusting the power of the control beam modulates n_g . This shifts the resonance of the probe beam and tunes the GD.	Phase and/or amplitude (θ and/or A) modulation between the dual inputs changes the shape of the resonance. This tunes the GD.
Compactness of cavity system	Fairly compact as active tuning components (such as p-i-n [15-17] and MOS [18, 19] diodes) occupy extra area in the cavity	Reduced compactness due to the large area of the tunable couplers.	Reduced compactness as the cavity must be large enough in order to have sufficient loop gain.	Fairly compact as the active tuning components (such as laser diode and phase/amplitude modulators) occupy extra area in the port WG.	
Fabrication complexity	High as additional procedures are required in the fabrication process to incorporate the active tuning components (such as microheaters [21], p-i-n [15-17], MOS [18, 19] diodes, tunable coupler [21, 24, 27, 28] and active medium like SOA [25] and Erbium ions [26]) in the cavity.			Low as the cavity is passive. Tuning components are attached to the cavity via the bus WG after fabrication.	
Reconfigurability of active tuning components	Difficult as the active tuning components (as listed above) are implanted within the cavity system. They cannot be easily reconfigured or replaced after the fabrication process.			Easy as the active tuning components are separated from the cavity and thus can be easily reconfigured or replaced.	
Device cost	High fabrication cost as additional procedures are needed to implant active tuning components in the cavity.			Low fabrication cost as only a passive cavity is needed. However, the use of dual inputs might increase the cost.	
Energy consumption	Fairly low except for MEMS based schemes, which require considerable energy			Higher than most CAT schemes (except MEMS scheme) due to use of dual inputs.	
Number of degree of freedom (DOE)	One DOE; adjusting n_g tunes the GD	One DOE; adjusting κ tunes the GD	One DOE; adjusting τ tunes the GD	One DOE; amplitude modulation of control beam tunes the GD	Two DOE; amplitude and/or phase modulation between dual inputs tunes the GD
Minima of GD function [†]	-69 ps	-2 ps	-4 ps	-69 ps	-223 ps (if $A = 0.4$) +24 ps (if $A = 1.0$)
Maxima of GD function [†]	+15 ps	+10 ps	+34 ps	+15 ps	+15 ps (if $A = 0.4$) +70 ps (if $A = 1.0$)
GD Tuning Range [†]	+84 ps	+12 ps	+38 ps	+84ps	+238 ps (if $A = 0.4$) +46 ps (if $A = 1.0$)

[†] Applicable to a probe beam (fixed at $\lambda = 1.55 \mu\text{m}$) emerging from the through port of a TWMR with $Q_{\text{int}} = 2.2 \times 10^4$ (i.e., surface-defects are intentionally introduced on the cavity) and initial Q_{int} and Q_{ext} values of 6.4×10^4 and 3.2×10^4 , respectively.

not stem from the use of the CAT or AOT scheme listed in Table 7.1. This is achieved by: (i) The intentional addition of surface perturbations to enhance and tune the intracavity backscattering; and (ii) by launching *dual* contra-propagating inputs of the same frequency into the TWMR to interact with the enhanced intracavity backscattering, as shown in Fig. 7.1. To set the stage, we first present a detailed analysis on the unique physical effects, in terms of the transmission and dispersion responses, that arise from the presence of dual inputs to interact with the enhanced intracavity backscattering in

Fig. 7.1. A traveling wave microresonator (TWMR) coupled to one input bus with dual contra-propagating inputs and outputs via a point coupler at the coupling junction (CJ). The intentionally addition of surface microstructures, such as grating ridges, on the cavity will lead to enhanced mutual energy coupling (with coefficient u_m) between the CW and CCW cavity modes. Each cavity mode has a net decay rate of $\Gamma_{ext} + \Gamma_{int}$, where Γ_{ext} is the decay rate due to the evanescent energy coupling (with coefficient u_0) between the bus waveguide mode and the cavity mode while Γ_{int} is the decay rate due to the intrinsic losses of the cavity.

the cavity. Subsequently, we will look into exploiting these physical effects for continuous tunable fast and slow light applications at the communication wavelength of $\lambda = 1.55 \mu\text{m}$. It will be shown that by controlling the phase and/or amplitude differences between the external dual inputs (laser diodes), which can be achieved through the use of external modulators [15, 30] and/or variable optical attenuators [31, 32], the structural dispersion of the TWMR can be tuned and switching between fast and slow light can easily be achieved. This will allow the possibility to achieve continuously tunable group delay in a *passive* and *linear* cavity, without the need to implant active components within the cavity system, thereby reducing the fabrication complexity and footprint of the cavity. The reconfigurability and serviceability of the proposed cavity system (in terms of its active tunability) are also improved as the external active tuning components (laser diodes and modulators) can be easily adjusted or replaced. More importantly, as elaborated later in this work, our proposed scheme enhances the tunable group delay performance over the commonly used active tuning schemes. These

improved features of our proposed scheme are also listed in Table 7.1. The results in this chapter can be applied to different types of TWMRs.

This work is organized as follows. In section 7.2, we present the mathematical modeling of our proposed scheme using the temporal coupled mode theory (CMT) [33]. This is followed by an analysis of the unique physical effects that result from the use of the dual-input scheme in section 7.3. In section 7.4, tunable fast and slow light applications at $\lambda = 1.55 \mu\text{m}$ are looked into. This is followed by a time-domain analysis and a short discussion on potential fast and slow light applications of our proposed scheme in section 7.5. In section 7.6, we compare the performances of our proposed scheme with other conventional tuning schemes in the literature. Finally, we conclude in section 7.7, highlighting the significance and contributions of this work.

7.2. Theoretical Formulation

The proposed TWMR circuit is shown schematically in Fig. 7.1, in which an azimuthally symmetric TWMR is evanescently coupled to a bus waveguide (WG) of length $L = L_1 + L_2$ via a coupling junction (CJ) that is taken to be infinitesimally short. The phase and/or amplitude differences between the dual inputs — S_{+1} at the reflection port and S_{+2} at the through port— can be controlled through the use of external optical phase/amplitude modulators [15, 30] and/or variable optical attenuators [31, 32] such that the dual inputs entering the coupling junction (CJ) are related to each other by $S'_{+2} = MS'_{+1} = [A \exp(j\theta)]S'_{+1}$, where M is the overall modulation factor while A and θ are, respectively, the amplitude and phase modulations between S'_{+1} and S'_{+2} . We will term S_{+1} (S_{+2}) as the signal (control) beam. Surface perturbations in the form of microstructures, such as grating ridges [9, 10], which are intentionally introduced on the cavity, will produce enhanced intracavity backscattering that results in mutual coupling between the clockwise (CW) and counterclockwise (CCW) cavity modes. In Fig. 7.1, the CW and CCW modes, respectively, have normalized field amplitudes of E_{ccw} and E_{cw} . We will assume that S_{+1} (S_{+2}) only directly generates the CCW (CW) mode; localized backscattering at the coupling junction that directly couples S_{+1} (S_{+2}) with the CW (CCW) cavity mode, which has already been analyzed in the previous chapter, is negligible in comparison to the intracavity distributed backscattering in this work. Assuming that all the inputs and cavity modes have $\exp(j\omega t)$ time dependences, i.e., $S_{+1} = \tilde{S}_{+1} \exp(j\omega t)$, $S_{+2} = \tilde{S}_{+2} \exp(j\omega t)$, $E_{ccw} = \tilde{E}_{ccw} \exp(j\omega t)$ and $E_{cw} = \tilde{E}_{cw} \exp(j\omega t)$, the interplay between the coherent coupling of the cavity modes and the losses out of the resonator can be described as [33]:

$$\begin{aligned} \frac{dE_{ccw}}{dt} &= (-j\Delta\omega - \Gamma_{net})E_{ccw} - ju_m E_{cw} - ju_0 S_{+1}, \\ \frac{dE_{cw}}{dt} &= (-j\Delta\omega - \Gamma_{net})E_{cw} - ju_m E_{ccw} - ju_0 S_{+2}, \end{aligned} \quad (7.1)$$

where $j = \sqrt{-1}$, $\Delta\omega = \omega - \omega_0$ is the (angular) frequency detuning and ω_0 is the degenerate (angular) resonance frequency of the cavity. The rate of energy coupling between the cavity and the bus waveguide (WG) is denoted as u_0 , which leads to an energy loss at a rate of $\Gamma_{ext} = u_0^2/2$ for each cavity mode, as shown in Fig. 7.1. Intrinsic losses in the cavity will give rise to additional energy loss at a rate of Γ_{int} . After taking account of this, the decay rate of each cavity mode becomes $\Gamma_s = \Gamma_{ext} + \Gamma_{int}$. In addition, the mutual coupling between the cavity modes is quantified by u_m . We then define four different types of quality or Q -factors for each cavity mode: (1) The intrinsic Q -factor $Q_{int} = \omega_0/(2\Gamma_{int})$; (2) the WG evanescent coupling Q -factor $Q_{ext} = \omega_0/(2\Gamma_{ext})$; (3) the mutual coupling Q -factor $Q_{mut} = \omega_0/(2u_m)$; and (4) the sum of the effects of the intrinsic and extrinsic Q -factors $Q_s = \omega_0/(2\Gamma_s) = (1/Q_{ext} + 1/Q_{int})^{-1}$. Note that the net Q -factor Q_{net} of each cavity mode is $Q_{net} = (1/Q_{int} + 1/Q_{ext} + 1/Q_{mut})^{-1}$. The above *energy* parameters u_0 , Γ_{ext} , and Γ_{int} are related to the *field* parameters κ , r and τ of the coupling matrix method in [34] by: $u_0 = \kappa/t_{rt}^{1/2}$, $\Gamma_{int} = (1 - \tau)/(t_{rt} \tau^{1/2})$ and $\Gamma_{ext} = (1 - r)/(t_{rt} r^{1/2})$, where $|\kappa|^2 + |r|^2 = 1$ (assuming lossless coupler), κ is the amplitude cross-coupling coefficient, r is the amplitude through-coupling coefficient, τ is the round-trip amplitude attenuation constant of the cavity, $t_{rt} = n_g L_c / c$ is the round-trip time of the cavity, c is the speed of light in vacuum, n_g is the group effective index and L_c is the length of the cavity. Note that we will focus our attention on light propagation in the net loss (i.e. $\Gamma_{int} > 0$ or $\tau < 1$) regime and for operating wavelengths near $\lambda = 1.55$ μm . Solving the coupled equations in Eq. (7.1), solutions to E_{ccw} and E_{cw} can be obtained:

$$\begin{aligned} E_{ccw} &= -\frac{(\Delta\omega - u_m M - j\Gamma_s)u_0}{(\Delta\omega - u_m - j\Gamma_s)(\Delta\omega + u_m - j\Gamma_s)} S_{+1}, \\ E_{cw} &= \frac{(M[j\Gamma_s - \Delta\omega] + k_m)k_0}{(\Delta\omega^2 - k_m^2) - 2j\Delta\omega\Gamma_s - \Gamma_s^2} S_{+1}, \end{aligned} \quad (7.2)$$

where $M = [A \exp(j\theta)]$ is the modulation factor. New cavity eigenmodes (or supermodes) will form in the TWMR from the symmetric and anti-symmetric superposition of the degenerate $\tilde{E}_{ccw}$ and $\tilde{E}_{cw}$, which are centered around ω_0 :

$$\begin{aligned} E_+ &= \frac{1}{\sqrt{2}}(E_{ccw} + E_{cw}) = -\frac{1}{\sqrt{2}} \frac{(1+M)u_0}{\Delta\omega - j\Gamma_s + u_m} S_{+1}, \\ E_- &= \frac{1}{\sqrt{2}}(E_{ccw} - E_{cw}) = -\frac{1}{\sqrt{2}} \frac{(1-M)u_0}{\Delta\omega - j\Gamma_s - u_m} S_{+1}, \end{aligned} \quad (7.3)$$

where E_+ (E_-) is the anti-symmetric (symmetric) eigenmode. For a linear system, the backward (forward) transmission in the bus WG, denoted as S_{-1} (S_{-2}), can be expressed in terms of the incident wave and the cavity modes using power conservation theorems [33]:

$$S_{-1} = e^{-j\beta_w L} (S_{+2} - ju_0 E_{cw}), \quad S_{-2} = e^{-j\beta_w L} (S_{+1} - ju_0 E_{ccw}), \quad (7.4)$$

where β_w is the propagation constant of the mode in the bus WG of length $L = L_1 + L_2$. Focusing only on the artificial dispersion due to the structural resonance of the TWMR, the steady-state forward transmission $T_T(\Delta\omega_{nor}) = |T_T| \exp(\Phi_T) = S_{-2}/S_{+1}$ at the through port and the steady-state backward transmission $T_R(\Delta\omega_{nor}) = |T_R| \exp(\Phi_R) = S_{-1}/S_{+1}$ at the reflection port can then be derived by using Eqs. (7.2) and (7.4) and simplified with the aid of the above-mentioned parameters of Q_{ext} , Q_{int} and Q_{mut} :

$$T_T(\Delta\omega_{nor}) = 1 - \frac{j \left[\Delta\omega_{nor} - \frac{M}{2Q_{mut}} \right] + \frac{1}{2Q_{ext}} + \frac{1}{2Q_{int}}}{Q_{ext} \left(j \left[\Delta\omega_{nor} + \frac{1}{2Q_{mut}} \right] + \frac{1}{2Q_{ext}} + \frac{1}{2Q_{int}} \right) \left(j \left[\Delta\omega_{nor} - \frac{1}{2Q_{mut}} \right] + \frac{1}{2Q_{ext}} + \frac{1}{2Q_{int}} \right)},$$

$$T_R(\Delta\omega_{nor}) = M - \frac{j \left[M\Delta\omega_{nor} - \frac{1}{2Q_{mut}} \right] + M \left[\frac{1}{2Q_{ext}} + \frac{1}{2Q_{int}} \right]}{Q_{ext} \left(j \left[\Delta\omega_{nor} + \frac{1}{2Q_{mut}} \right] + \frac{1}{2Q_{ext}} + \frac{1}{2Q_{int}} \right) \left(j \left[\Delta\omega_{nor} - \frac{1}{2Q_{mut}} \right] + \frac{1}{2Q_{ext}} + \frac{1}{2Q_{int}} \right)}, \quad (7.5)$$

where $\Delta\omega_{nor} = (\omega - \omega_0)/\omega_0$ is the normalized detuning. Using Eq. (7.5), we then derive the critical coupling condition Q_{cri} , which we define as the complex solution of Q_{mut} to $T_T = 0$ or $T_R = 0$:

$$Q_{cri,T} = \frac{-jMQ_{int} \pm \sqrt{M^2 Q_{int}^2 - Q_{ext}^2 + Q_{int}^2}}{(Q_{ext}^2 - Q_{int}^2)(Q_{ext} Q_{int})^{-1}}, \quad Q_{cri,R} = \frac{-jQ_{int} \pm \sqrt{M^2 (Q_{int}^2 - Q_{ext}^2) - Q_{int}^2}}{M(Q_{ext}^2 - Q_{int}^2)(Q_{ext} Q_{int})^{-1}}. \quad (7.6)$$

Finally, we derive the group delay t_g of the TWMR. To do so, the effective phase shifts Φ_T and Φ_R of the TWMR are first obtained from $\Phi_T = \tan^{-1}(\text{Im}(T_T)/\text{Re}(T_T))$ and $\Phi_R = \tan^{-1}(\text{Im}(T_R)/\text{Re}(T_R))$. Then using $t_g(\Delta\omega_{nor}) = \partial\Phi_{T,R}(\Delta\omega_{nor})/\partial(\Delta\omega_{nor})$, the normalized group delay for the through and reflection port, which are, respectively, denoted as $t_{g,T}(\Delta\omega_{nor})$ and $t_{g,R}(\Delta\omega_{nor})$, can be expressed as

$$t_{g,T}(\Delta\omega_{nor}) = \frac{B_1 \left[\frac{Q_{ext}}{Q_s} - 1 \right] - 2Q_{ext} \Delta\omega_{nor}^2 \left[\frac{Q_{ext}}{Q_s} + \frac{A \cos \theta}{2Q_{mut} \Delta\omega_{nor}} - 1 \right]}{B_1^2 + \Delta\omega_{nor} \left[\frac{Q_{ext}}{Q_s} + \frac{A \cos \theta}{2Q_{mut} \Delta\omega_{nor}} - 1 \right]^2} + \frac{\Delta\omega_{nor}^2 + \frac{1}{4Q_{mut}^2} + \frac{1}{4Q_s^2}}{Q_{net} \left(\left[\Delta\omega_{nor}^2 - \frac{1}{4Q_{mut}^2} - \frac{1}{4Q_s^2} \right]^2 + \left[\frac{\Delta\omega_{nor}}{Q_s} \right]^2 \right)},$$

$$t_{g,R}(\Delta\omega_{nor}) = A \frac{\sin\theta}{B_3} \left(\frac{A \left(\left[\frac{Q_{ext}}{Q_s} + 1 \right] \cot\theta - 2Q_{ext}\Delta\omega_{nor} \right) + \frac{B_2}{B_3} \left(\left[\frac{Q_{ext}}{Q_s} - 1 \right] + 2Q_{ext}\Delta\omega_{nor} \cot\theta \right)}{A^2 + \left[\frac{B_2}{B_3} \right]^2} \right) + \frac{1}{Q_s} \left(\frac{\Delta\omega_{nor}^2 + \frac{1}{4Q_{mut}^2} + \frac{1}{4Q_s^2}}{\left[\Delta\omega_{nor}^2 - \frac{1}{4Q_{mut}^2} - \frac{1}{4Q_s^2} \right]^2 + \frac{\Delta\omega_{nor}^2}{Q_s^2}} \right), \quad (7.7)$$

$$\text{where } B_1 = Q_{ext}\Delta\omega_{nor}^2 + \frac{1}{2} \left(\frac{1}{Q_s} + \frac{A\sin\theta}{Q_{mut}} \right) - \frac{Q_{ext}}{4} \left(\frac{1}{Q_{mut}^2} + \frac{1}{Q_s^2} \right),$$

$$B_2 = \left(\left[Q_{ext}\Delta\omega_{nor}^2 - \frac{Q_{ext}}{4Q_{mut}^2} - \frac{Q_{ext}}{4Q_s^2} + \frac{1}{2Q_s} \right] A\sin\theta - \left[\frac{Q_{ext}}{Q_s} + 1 \right] A\Delta\omega_{nor} \cos\theta - \frac{1}{2Q_{mut}} \right)^2,$$

$$B_3 = \left(1 - \frac{Q_{ext}}{Q_s} \right) \Delta\omega_{nor} \sin\theta + \left(\frac{1}{4Q_{mut}^2} + \frac{1}{4Q_s^2} - \frac{1}{2Q_{ext}Q_s} - \Delta\omega_{nor}^2 \right) Q_{ext} \cos\theta.$$

Note that the absolute group delay is $t_g(\omega) = t_g(\Delta\omega_{nor})/\omega_0$. From Eqs. (7.5), it can be seen that the control beam (i.e., $S'_{+2} = MS'_{+1} = [A\exp(j\theta)]S'_{+1}$) influences the transmission and dispersion by altering the numerator (but not the denominator) of the transfer functions of the TWMR. This is also evident by the presence of A and θ only in the first fractional term of the group delay functions of the through and drop ports in Eq. (7.7). In fact, we can also observe in Eq. (7.7) that the group delay functions of the through and drop port share the same second fractional term while the first fractional term is different for both ports. It must be mentioned that the group delay function in Eq. (7.7) is expressed as a sum of two fractional terms. We now use Eqs. (7.5) to (7.7) to analyze how such changes by the control beam would give unique physical effects.

7.3. Physical Effects of the Interaction of Intracavity Backscattering with Dual Inputs

7.3.1. Phase Modulation of the Control Beam

The effects of phase θ modulation are looked into for two types of coupling states: (i) The under-coupling state $Q_{int} < Q_{ext}$, and (ii) the over-coupling state $Q_{int} > Q_{ext}$. With no loss of generality, we set $Q_{int} = 2.1 \times 10^4$, $Q_{ext} = 5.1 \times 10^4$, $Q_{mut} = 1.5 \times 10^4$ for case (i) and $Q_{int} = 5.1 \times 10^4$, $Q_{ext} = 2.1 \times 10^4$, $Q_{mut} = 1.5 \times 10^4$ for case (ii), while θ is being varied, with $A = 1$. These arbitrary chosen physical parameters of Q_{int} , Q_{ext} and Q_{mut} (as well as other values near these chosen values) can be easily implemented by current nanofabrication technology, as shown in the literature. For example, it has been experimentally demonstrated in [9] that near an operating wavelength of $\lambda_0 = 1.55 \mu\text{m}$, $Q_{int} = 4 \times$

10^4 , $Q_{ext} = 3 \times 10^4$ and $Q_{mut} = 1.37 \times 10^4$ can be easily achieved by using a silicon circular microring resonator having a bend radius of $R = 5 \mu\text{m}$, waveguide-to-resonator air gap g of 120 nm and cavity surface defects in the form of periodic grating ridges (with each grating ridge having a width of ~ 20 nm and the separation distances between adjacent ridges range from ~ 50 nm to ~ 100 nm). These correspond to a round-trip (electric field) amplitude attenuation constant τ of ~ 0.992 and (electric field) amplitude cross coupling coefficient κ of ~ 0.146 . Note that the conversion from the *energy* parameters of Q_{ext} and Q_{mut} (or Γ_{ext} and Γ_{int}) to the *field* parameters κ , r and τ of the coupling matrix method can be easily done using the formulae mentioned earlier in section 7.2.

The transmission and dispersive responses of the TWMR at our above-mentioned chosen values of Q_{int} , Q_{ext} and Q_{mut} are plotted in Fig. 7.2 for the under-coupling state $Q_{int} < Q_{ext}$ and in Fig. 7.3 for the over-coupling state $Q_{int} > Q_{ext}$. We denote the degenerate resonance frequency as ω_0 , which has been set as $\omega_0/(2\pi) = 193.55$ THz (or $\lambda_0 = 1.55 \mu\text{m}$) in Figs. 7.2 and 7.3. For the split resonance, we label it as ω_{s+} and ω_{s-} , where ω_{s+} (ω_{s-}) is the anti-symmetric (symmetric) mode. Note that the anti-symmetric (symmetric) mode is located at $\omega > \omega_0$ ($\omega < \omega_0$) or $\lambda < \lambda_0$ ($\lambda > \lambda_0$). Also, we mention that all the transmission spectra in this work are presented in the wavelength domain near the degenerate resonance wavelength $\lambda_0 = 1.55 \mu\text{m}$, though the (angular) frequency domain are used earlier in the analytical expressions (i.e., Eqs. (7.1) to (7.7)) for consistency with the literature. Also, λ_0 and ω_0 will be used interchangeably throughout the discussion, where $\omega_0 = 2\pi f_0 = 2\pi(c/\lambda_0)$.

It can be observed from Figs. 7.2 and 7.3 that as θ is being modulated, the general trend is that the transmission spectra of both ports undergo either mode splitting or a shift in resonance from λ_0 . In addition, the trends in the evolution of the mode splitting as θ is being varied are periodic. This means: (i) For a given coupling state, the spectra of a particular port at θ are similar to those at $\theta + 2\pi$; and (ii) the spectra of the through port found within the interval $p\pi \leq \theta \leq (p+1)\pi$ is symmetrical to the spectra at the reflection port at $(p+1)\pi \leq \theta \leq (p+2)\pi$, where p is an integer. Note that (i) and (ii) apply to any TWMR with mode splitting in the presence of dual inputs, not just for our cases in Fig. 7.2 and Fig. 7.3; we can see later in Fig. 7.5 that changing the physical parameters will give similar trends described in (i) and (ii).

Let us now take a closer look at the nature of the mode splitting and the resonance shift as θ is being modulated. For comparison, we have included the transmission and dispersive responses for the case of $Q_{mut} \rightarrow \infty$, which is the case of a degenerate cavity, in Figs. 7.2 and 7.3. In general, for both coupling states of $Q_{int} > Q_{ext}$ and $Q_{int} < Q_{ext}$, mode splitting at $A = 1$ can be observed in the transmission spectra of the through and reflection ports only at certain non-integer values of θ/π (i.e. $p\pi < \theta < (p+1)\pi$). Otherwise, the resonance mode is simply shifted from λ_0 . To determine whether mode splitting or resonance shifting takes place for a non-integer θ/π , we utilize the parameter $|Q_{cri, T}|$ and $|Q_{cri, R}|$ introduced earlier in Eq. (7.6), which is plotted in Fig. 7.4. Mode splitting occurs at $p\pi < \theta < (p+1)\pi$ if the criterion of $|Q_{cri}| \gg Q_{mut}$ is met. To understand this, first note from Eq. (7.5) that the

Fig. 7.2. Evolution in the (i) transmission, (ii) effective phase shift and (iii) group delay responses for the output light at the (a)-(c) through port and (d)-(f) reflection port of a TWMR in the coupling state of $Q_{int} < Q_{ext}$ with fixed $Q_{int} = 2.1 \times 10^4$, $Q_{ext} = 5.1 \times 10^4$ and $Q_{mut} = 1.5 \times 10^4$ as the phase θ of the control beam $S_{+2} = A \exp(j\theta) S_{+1}$ (with $A = 1$) is varied.

Fig. 7.4. Variation of $|Q_{cri, T}|$ and $|Q_{cri, R}|$ with θ for $Q_{int} < Q_{ext}$ ($Q_{int} = 2.1 \times 10^4$, $Q_{ext} = 5.1 \times 10^4$) and $Q_{int} > Q_{ext}$ ($Q_{int} = 5.1 \times 10^4$, $Q_{ext} = 5.1 \times 10^4$) of a TWMR. Only $Q_{mut} = 1.5 \times 10^4$ and $A = 1$ apply to Figs. 7.2 and 7.3 while $Q_{mut} = 8 \times 10^3$ and $A = 0.1$ are used to show the effects of a different Q_{mut} or amplitude A of the control beam. Mode splitting occurs if $|Q_{cri}| \gg Q_{mut}$. This explains the mode splitting trends in Figs. 7.2 and 7.3.

degenerate mode ω_0 is split into two eigenmodes, which are $\omega_0 - \omega_0/(2Q_{mut})$ and $\omega_0 + \omega_0/(2Q_{mut})$, due to the backscattering that breaks the rotational symmetry of the cavity. These eigenmodes can only be clearly distinguishable in the resonance spectra in the form of a two-split Lorentzian only if the frequency separation between the eigenmodes (proportional to $1/Q_{mut}$) is much larger than the resonance linewidth (proportional to $1/|Q_{cri}|$), which translates to $|Q_{cri}| \gg Q_{mut}$. Otherwise, the superposition of the eigenmodes is no longer resolvable into two distinct eigenmodes and thus no distinguishable mode splitting will appear in the Lorentzian resonance spectra. This is somewhat analogous to the Rayleigh criterion in diffraction optics. As shown in Fig. 7.4, $|Q_{cri, T}|$ and $|Q_{cri, R}|$, respectively, have a $-\sin(X)$ and $+\sin(X)$ relation. Hence, given an arbitrary Q_{mut} , there are two possible scenarios whereby $|Q_{cri}| \gg Q_{mut}$: (i) p is an even (odd) integer for the reflection (through) port, which occurs if $Q_{mut} = 1.5 \times 10^4$ in Fig. 7.4 and (ii) p is any integer for both ports, which occurs if $Q_{mut} = 8 \times 10^3$ in Fig. 7.4. At a fixed θ/π , scenario (i) means that mode splitting occurs either at the through or reflection port but not simultaneously at both ports while scenario (ii) translates into simultaneous mode splitting for both ports. For Figs. 7.2 and 7.3, $Q_{mut} = 1.5 \times 10^4$ and thus scenario (i) can be seen: Mode splitting takes place at the through port for $\pi < \theta < 2\pi$ whereas at the reflection port, mode splitting can be observed for $0 < \theta < \pi$ and $2\pi < \theta < 3\pi$. If Q_{mut} is reduced, for example to 8×10^3 in Fig. 7.4, scenario (ii) is observed. This is shown in Fig. 7.5, in which mode splitting occurs simultaneously at both ports for all non-integer θ/π .

The mode splitting that occurs at any non-integer θ/π when $|Q_{cri}| \gg Q_{mut}$ follows a general evolution trend. We have enlarged Fig. 7.2(bi) in Fig. 7.6(ai) to show this. At $\theta = p\pi$, the resonance

mode is shifted from λ_0 . As θ is adjusted to a non-integer θ/π within the interval $p\pi < \theta < (p + 1/2)\pi$, the Lorentzian becomes an asymmetrical doublet: the split resonances ω_{s+} and ω_{s-} have different notch depths. At odd (even) p , ω_{s+} (ω_{s-}) will be the split resonance that has a shallower notch depth for the interval $p\pi < \theta < p\pi/2$ while at a later interval $p\pi/2 < \theta < (p + 1)\pi$, the trend reverses and ω_{s-} (ω_{s+}) becomes the resonance with a shallower notch depth. As the non-integer θ/π approaches $\theta = (p + 1/2)\pi$, there is a redistribution of energy between ω_{s+} and ω_{s-} , resulting in the difference between the notch depths of ω_{s+} and ω_{s-} to progressively reduce such that at $\theta = (p + 1/2)\pi$, a symmetrical doublet centred at ω_0 forms. Essentially, the spectra for $p\pi \leq \theta < p\pi/2$ is symmetrical to those for $p\pi/2 < \theta \leq (p + 1)\pi$ with respect to $\theta = (p + 1/2)\pi$. For example, in Fig. 7.6(ai), the spectrum of $\theta = 1.3\pi$ is symmetric to that of $\theta = 1.7\pi$ with respect to the dotted vertical line at $\theta = 1.5\pi$. This property also means that the trends in the evolution of the mode splitting process as $\theta = p\pi$ is increased to $\theta = (p + 1/2)\pi$ are symmetric to those as $\theta = (p + 1)\pi$ is decreased to $\theta = (p + 1/2)\pi$.

Fig. 7.5. Evolution in the transmission for the (a) through and (b) reflection ports of a TWMR in the coupling state of $Q_{int} < Q_{ext}$, with $Q_{int} = 2.1 \times 10^4$, $Q_{ext} = 5.1 \times 10^4$ and $Q_{mut} = 8 \times 10^3$ for $A = 1$ as the phase θ of the control beam is varied.

We now analyze the spectra with no mode splitting, which occurs at: (i) All integer values of θ/π , and (ii) at non-integer θ/π where $|Q_{cri}| \ll Q_{mut}$. For both cases, the resonance is shifted from λ_0 by about $\pm f_0 \lambda_0^2 / (2Q_{mut}c)$. In general, the resonance is red (blue) shifted from λ_0 if $p\pi \leq \theta < p\pi/2$ ($p\pi/2 < \theta \leq (p + 1)\pi$) for even integer of p . Conversely, if p is an odd integer, the resonance mode will be blue (red) shifted from λ_0 for $p\pi \leq \theta < p\pi/2$ ($p\pi/2 < \theta \leq (p + 1)\pi$). Also, the set of spectra for $p\pi \leq \theta < p\pi/2$ are symmetrical to those for $p\pi/2 < \theta \leq (p + 1)\pi$ with respect to $\theta = (p + 1/2)\pi$, similar to the above-mentioned mode splitting case. Finally, for non-integer θ/π , the resonance spectra are asymmetric, with the asymmetry of each resonance reducing as θ approaches either $\theta = p\pi/2$, at

which the resonance becomes a symmetric Lorentzian with a broadened notch, or at $\theta = p\pi$, where a sharp symmetric Lorentzian is present. Note that at $\theta = p\pi/2$, the resonance is located at ω_0 .

The phenomena of mode splitting and resonance shift are due to the interaction between the symmetric and anti-symmetric cavity eigenmodes, which are earlier denoted as E_+ and E_- in Eq. (7.3). This is shown in Fig. 7.6(aii) and Fig. 7.6(bii). Essentially, the mutual coupling between the CCW and CW modes results in the formation of new cavity eigenmodes E_+ and E_- . Whether E_+ and E_- can be resolved in the transmission spectrum depends if $|Q_{cri}| \gg Q_{mut}$. In Fig. 7.6(aii), $|Q_{cri}| \gg Q_{mut}$ and thus mode splitting occurs in Fig. 7.6(ai). For this case, modulation of θ adjusts the intensity of the eigenmodes in Fig. 7.6(aii), which in turn shapes the degree of symmetry of the transmission in Fig. 7.6(ai). Initially at $\theta = p\pi$, E_- (E_+) is the dominant mode when p is an odd (even) integer, resulting in a blue (red) shifted resonance mode in Fig. 7.6(ai). As θ moves towards $(p + 1/2)\pi$, E_+ and E_- change in intensity. Their difference in intensity decreases as θ gets nearer to $(p + 1/2)\pi$, with the mode splitting being asymmetrical as long as one of the eigenmodes is stronger. Eventually at $\theta = (p + 1/2)\pi$, both E_+ and E_- have similar strength, which results in symmetric mode splitting. On the other hand, in Fig. 7.6(bii), $|Q_{cri}| \ll Q_{mut}$ and thus no mode splitting occurs in Fig. 7.6(bi). Then the effect of $E_+ + E_-$ only results in the Lorentzian resonance to be shifted from ω_0 . For this case, whether the resonance is red or blue shifted from λ_0 depends if E_+ or E_- is the dominant mode; when E_+ and E_- are

Fig. 7.6. The transmission and intracavity intensity at the through port of a TWMR ($Q_{int} = 2.1 \times 10^4$, $Q_{ext} = 5.1 \times 10^4$, $Q_{mut} = 1.5 \times 10^4$) as the phase θ of the control beam is being varied. The dotted vertical line marks the degenerate resonance mode ω_0 . The mode splitting and resonance shift are shown, respectively, in (ai) and (bi), where the resonances located to the left (right) of ω_0 are denoted as ω_s (ω_{s+}). In (aai) and (bai), at each θ , there are two cavity eigenmodes, E_+ and E_- . The sum of E_+ and E_- gives the transmission in (ai) and (bi). If $|Q_{cri}(\theta)| \gg Q_{mut}$, mode splitting occurs. Otherwise, $E_+ + E_-$ results in a resonance shift from ω_0 . In this case, $|Q_{cri}(\theta)| \gg Q_{mut}$ only for (bi). Note that in (aai) and (bai), the peaks located to the left (right) of the vertical dotted line (at $\lambda = 1550$ nm) corresponds to E_+ (E_-).

of equal strength, which occurs at $\theta = (p + 1/2)\pi$, a Lorentzian with a slightly broadened notch forms.

The dispersion of the TWMR depends on the coupling state. For $Q_{int} < Q_{ext}$, anomalous dispersion is always present at both the through and reflection ports at all θ , regardless of whether there is mode splitting in the transmission. Then only fast light is generated at the output ports, as observed in Fig. 7.2(iii). On the other hand, for $Q_{int} > Q_{ext}$, either normal or anomalous dispersion is possible, depending if mode splitting can be observed. Normal dispersion, which produces slow light, is present at the output ports, when there is no mode splitting. This can be seen in Fig. 7.3 when $0 \leq \theta \leq \pi$ and $2\pi \leq \theta \leq 3\pi$ for the through port and when $\pi \leq \theta \leq 2\pi$ for the reflection port. On the contrary, the occurrence of mode splitting will result in a reversal of normal to anomalous dispersion at the output ports, thereby switching the subluminal propagation to superluminal propagation. This phenomenon can be clearly seen in Fig. 7.3 at the through port when $\pi \leq \theta \leq 2\pi$ and at the reflection port when $0 \leq \theta \leq \pi$ and $2\pi \leq \theta \leq 3\pi$. However, do note that such a change in dispersion response is observed at the split resonance only if symmetrical or close to symmetrical mode splitting is observed. This is evident in Fig. 7.3(b): though mode splitting can be observed at $\theta = 1.2\pi, 1.4\pi, 1.5\pi$ and 1.8π in Fig. 7.3(bi), fast light is only produced at $\theta = 1.4\pi$, in which the mode splitting is nearly symmetrical and at $\theta = 1.5\pi$, where the mode splitting is symmetrical, in Fig. 7.3(biii). Another important characteristic is that, for both coupling states, the absolute magnitude of the group delay at resonance progressively reduces as θ approaches $(p + 1/2)\pi$, as seen in Fig. 7.2 and 7.3.

7.3.2. Amplitude Modulation of the Control Beam

Thus far, all the discussions are for the case of $A = 1$. We will now look at the effects of modulating A . The characteristics in the transmission and dispersion are summarized in Table 7.2, where $A > 0$ collectively refers to the case of $A = 1$ in the earlier subsection and the present case of amplitude A modulation. Modulating A simply changes the absolute magnitude of $|Q_{cri}|$ and its relation to Q_{mut} , which will affect the mode splitting. For example, in Fig. 7.4, when A is decreased to 0.1, the locus of $|Q_{cri}|$ is compressed. As a result, for $Q_{mut} = 1.5 \times 10^4$, the mode splitting criterion $|Q_{cri}| \gg Q_{mut}$ is then satisfied for all p , resulting in mode splitting at every θ for both ports, in contrast to the case $A = 1$ in Figs. 7.2 and 7.3, where mode splitting occurs at only specific non-integer θ/π . Another main effect of A modulation is that at $\theta = p\pi$, either mode-splitting or an asymmetric Lorentzian resonance, which is shifted from ω_0 , will be produced, in contrast to the earlier case of $A = 1$, in which the transmission response at $\theta = p\pi$ is always a symmetric Lorentzian resonance that is shifted from ω_0 , with no mode splitting. This occurs because when A is modulated, the intensity of the eignemodes E_+ and E_- , will be adjusted such that both E_+ and E_- have influence on how the transmission spectrum is being shaped, in comparison to the case of $A = 1$, in which the intensity of one of the eigenmode is negligible at $\theta = p\pi$, as shown earlier in Fig. 7.6. As such, depending if $|Q_{cri}| \gg Q_{mut}$, either mode

Table 7.2. Transmission and dispersive responses at resonance for various coupling states for a TWMR with dual inputs

Coupling State	Relation of $ Q_{cri} $ to Q_{mut}	Mode splitting observed in transmission spectra?				Type of dispersive response i.e. fast light (FL) or slow light (SL)							
						Through Port				Reflection Port			
		$\theta = p\pi$		$p\pi < \theta < (p+1)\pi$		$\theta = p\pi$		$p\pi < \theta < (p+1)\pi$		$\theta = p\pi$		$p\pi < \theta < (p+1)\pi$	
		Value of A		Value of A		Value of A		Value of A		Value of A		Value of A	
		$A \rightarrow 0$	$A > 0$	$A \rightarrow 0$	$A > 0$	$A \rightarrow 0$	$A > 0$	$A \rightarrow 0$	$A > 0$	$A \rightarrow 0$	$A > 0$	$A \rightarrow 0$	$A > 0$
$Q_{ext} > Q_{int}$	$ Q_{cri} \ll Q_{mut}$	No	No	No	No	FL	FL	FL	FL	SL	FL	SL	FL
	$ Q_{cri} \gg Q_{mut}$	Yes	No ($A = 1$) Yes, ($A \neq 1$)	Yes	Yes	FL	FL	FL	FL	SL	FL	SL	FL
$Q_{ext} < Q_{int}$	$ Q_{cri} \ll Q_{mut}$	No	No	No	No	SL	SL	SL	SL	SL	SL	SL	SL
	$ Q_{cri} \gg Q_{mut}$	Yes	No ($A = 1$) Yes ($A \neq 1$)	Yes	Yes	FL	SL ($A = 1$) FL ($A \neq 1$)	FL	[†] SL/ FL	SL	SL ($A = 1$) FL ($A \neq 1$)	SL	[†] SL/ FL

[†]For $Q_{ext} < Q_{int}$, mode splitting for $p\pi < \theta < (p+1)\pi$ produces fast light only when θ is near or at $(p+1/2)\pi$. Further away from $(p+1/2)\pi$, the splitting becomes increasingly asymmetrical, which will generate slow light. Note that p is an integer.

splitting or a resonance shift from ω_0 occurs at $\theta = p\pi$ when A is modulated (i.e. $A \neq 1$). In the event of $A \rightarrow 0$, only normal dispersion and thus slow light will be generated at the reflection port for both coupling states, irrespective of whether there is mode splitting. Other than these changes, all the properties in the transmission and dispersive responses when A is tuned are in agreement with those mentioned in this section for $A = 1$. This can also be seen in Table 7.2.

7.4. Harnessing the Dual-Input Scheme for Continuously Tunable Fast and Slow Light

We now look into how we can harness the physical effects arising from the use of dual inputs in a TWMR with enhanced surface-perturbations induced backscattering for continuous fast and slow light applications. First of all, note that in a conventional TWMR, the surface perturbations due to fabrication-induced surface roughness can be made very small due to advances in fabrication technology [35, 36]. As such, the intracavity backscattering in most TWMRs is relatively weak, resulting in $Q_{mut} \rightarrow \infty$. Then we have $|Q_{cri}| \ll Q_{mut}$, which gives a single sharp Lorentzian resonance with no mode splitting as mode splitting occurs only if $|Q_{cri}| \gg Q_{mut}$. By intentionally introducing

surface perturbations on the TWMR, the intracavity backscattering is enhanced, giving rise to two possibilities in the shape of the resonance: (i) A single sharp Lorentzian if $|Q_{cri}| \ll Q_{mut}$ or (ii) a two-split Lorentzian (that can be either symmetrical or asymmetrical), i.e., a doublet, if $|Q_{cri}| \gg Q_{mut}$. In the presence of dual inputs, $|Q_{cri}|$ can be adjusted (from $|Q_{cri}| \ll Q_{mut}$ to $|Q_{cri}| \gg Q_{mut}$ or vice versa) by modulating the phase and/or amplitude differences between the dual inputs. Consequently, the shapes of the transmission and group delay resonance spectra will gradually change from a single Lorentzian to a two-split Lorentzian and vice versa, which has been shown in Figs. 7.2 and 7.3 for the different coupling regimes. By exploiting such a change in the spectral shape and the corresponding dispersion response as $|Q_{cri}|$ is adjusted, continuous tunability in the group delay and transmission of the output light at a fixed λ can consequently be achieved. Switching between FL and SL can also be achieved when the change in spectral shape is accompanied with a change in the nature of the structural dispersion from anomalous dispersion to normal dispersions or vice versa. We now analyze the properties of such continuous tunability at $\lambda = \lambda_0 = 1.55 \mu\text{m}$.

The effects of modulating θ on the group delay and transmission for $\lambda_0 = 1.55 \mu\text{m}$ are illustrated in Fig. 7.7, where the parameters used are similar to Figs. 7.2 and 7.3. To better describe the tunability in the group delay and transmission, several figures-of-merit (FOM) are introduced to quantify for the $t_g(\theta)$ responses: (i) $t_{g(max)}$ — maximum of the group delay vs θ response, (ii) $t_{g(min)}$ — minimum of the group delay vs θ response, (iii) Δt_g — tunable range of the group delay vs θ response, which is

Fig. 7.7. Demonstration of the continuous tunability of the transmission and group delay at $\lambda = \lambda_0 = 1.55 \mu\text{m}$ for a TWMR with (a) $Q_{int} < Q_{ext}$ ($Q_{int} = 2.1 \times 10^4$, $Q_{ext} = 5.1 \times 10^4$) and (b) $Q_{int} > Q_{ext}$ ($Q_{int} = 5.1 \times 10^4$, $Q_{ext} = 2.1 \times 10^4$) as the phase θ of the control beam $S_{+2} = A \exp(j\theta) S_{+1}$ (with $A = 1$). The graphs with dotted lines have $Q_{mut} = 8 \times 10^3$ while the graphs with solid lines have $Q_{mut} = 1.5 \times 10^4$. All the blue (red) graphs represent the transmission (group delay) on the left (right) axis.

defined as $|t_{g(max)} - t_{g(min)}|$, (iv) $T_{(max)}$ — maximum of the transmission vs θ response, (v) $T_{(min)}$ — minimum of the transmission vs θ response and (vi) ΔT — tunable range of the transmission vs θ response, which is defined as $T_{(max)} - T_{(min)}$. For the through port, $t_{g(max)}$, $t_{g(min)}$, Δt_g , $T_{(max)}$, $T_{(min)}$ and ΔT are, respectively, denoted as $t_{g, T(max)}$, $t_{g, T(min)}$, $\Delta t_{g, T}$, $T_{T(max)}$, $T_{T(min)}$ and ΔT_T . Likewise, for the reflection port, they are labeled as $t_{g, R(max)}$, $t_{g, R(min)}$, $\Delta t_{g, R}$, $T_{R(max)}$, $T_{R(min)}$ and ΔT_R . As an example, we have illustrated the parameters of $t_{g, T(max)}$, $t_{g, T(min)}$, $\Delta t_{g, T}$, $T_{T(max)}$, $T_{T(min)}$ and ΔT_T for the through port case in Fig. 7.7. In addition, the transmission and group delay responses as a function of θ at a fixed wavelength of $\lambda = \lambda_0$ will be labeled, respectively, as $T_T(\theta, \lambda_0)$ and $t_{g, T}(\theta, \lambda_0)$ for the through port and as $T_R(\theta, \lambda_0)$ and $t_{g, R}(\theta, \lambda_0)$ for the reflection port or collectively as $T(\theta, \lambda_0)$ and $t_g(\theta, \lambda_0)$.

It is evident from Fig. 7.7 that continuous tunability of both the group delay and transmission can be elegantly achieved. Switching between subluminal and superluminal propagation at each port can be simply achieved by modulating the control beam. The main features in the tunability of the transmission $T(\theta, \lambda_0)$ and group delay $t_g(\theta, \lambda_0)$ are highlighted below.

- $T_T(\theta, \lambda_0)$ and $t_{g, T}(\theta, \lambda_0)$ are, respectively, out of phase from $T_R(\theta, \lambda_0)$ and $t_{g, R}(\theta, \lambda_0)$ by $\theta = \pi$, i.e. $t_{g, R}(\theta, \lambda_0) = t_{g, T}(\theta + \pi, \lambda_0)$ and $T_R(\theta, \lambda_0) = T_T(\theta + \pi, \lambda_0)$, which is due to the periodic characteristic of the wavelength spectra in Fig. 7.2 and Fig. 7.3. We could thus adjust the contrast in the transmission and dispersion between the two ports simply by tuning θ . However, for fast and slow light applications which stringently require minimal power variation accompanying change in the group delay, the use of variable optical attenuator and/or gain elements (such as semiconductor optical amplifiers) at the output ports can be used to adjust the transmission.
- For the through port, $T_{T(min)}$ ($T_{T(max)}$) is located at $\theta = (p + 1/2)\pi$, where p is an even (odd) integer. As $T_T(\theta, \lambda_0) = T_R(\theta + \pi, \lambda_0)$, the reverse will apply to the reflection port: $T_{R(min)}$ ($T_{R(max)}$) is located at $\theta = (p + 1/2)\pi$, where p is an odd (even) integer.
- Likewise, the extrema of $t_g(\theta, \lambda_0)$ are located at $\theta = (p + 1/2)\pi$. However, the nature of p depends on the coupling state. For $Q_{int} < Q_{ext}$, p is always even (odd) for $t_{g, T(min)}$ ($t_{g, T(max)}$) and $t_{g, R(max)}$ ($t_{g, R(min)}$), which also means that $t_{g(min)}$ ($t_{g(max)}$) occurs at the same δ as $T_{(min)}$ ($T_{(max)}$). For $Q_{int} > Q_{ext}$, p swings between even and odd as the extrema of $t_g(\theta, \lambda_0)$ might switch between the minima and the maxima, as seen in Fig. 7.7(b). This is because for $Q_{int} > Q_{ext}$, any change to the mode splitting results in a reversal from normal to anomalous dispersion, as mentioned earlier. In this case, for Fig. 7.7(b), the change in the mode splitting when Q_{mut} switches between 1.5×10^4 and 8.0×10^3 can also be seen earlier in Fig. 7.4.

To better understand the propagating regime (i.e. fast or slow light) of the output light at the extrema of $t_g(\theta, \lambda_0)$ as well as the tunable range and the trade-off between the group delay, bandwidth and transmission, we have shown in Figs. 7.8 and 7.9 the plots of $t_{g(max)}$, $t_{g(min)}$, Δt_g , $T_{(max)}$, $T_{(min)}$ and ΔT for $\lambda = \lambda_0 = 1.55 \mu\text{m}$ as a function of Q_{int} for different Q_{mut} . With no loss in generality, Q_{ext} is fixed at 5.1×10^4 to facilitate this discussion. Using the trends in Figs. 7.8 and 7.9, the desired device

performances can then be achieved by a proper choice of the physical parameters (Q_{ext} , Q_{int} , Q_{mut}), which depends on the physical situation under consideration and the fabrication tolerance. Note that Q_{ext} is determined by the bus WG-to-cavity air gap distance g , Q_{int} depends mainly on the bend radius R while the surface defects on the cavity will determine Q_{mut} . For example, as mentioned earlier in section 7.3.1, using a silicon microring resonator with $R = 5 \mu\text{m}$, air gap g of 120 nm and with surface defects in the form of grating ridges, with each grating ridge having a width of ~ 20 nm and the separation distances between adjacent ridges range from ~ 50 nm to ~ 100 nm, Q_{ext} , Q_{int} and Q_{mut} are, respectively, 3×10^4 , 4×10^4 and 1.37×10^4 near $\lambda_0 = 1.55 \mu\text{m}$ [9].

Two different cases are analyzed: (i) Critical coupling is absent, which is shown in Fig. 7.8; and (ii) critical coupling occurs as Q_{mut} is increased, which is shown in Fig. 7.9. Note that both Fig. 7.8 and Fig. 7.9 apply to both the through and reflection ports as $t_{g(max)}$, $t_{g(min)}$, $T_{(max)}$ and $T_{(min)}$ are the same for both ports. The general trend in Figs. 7.8 and 7.9 is that increasing Q_{int} has the effect of increasing $|t_{g(min)}|$ and $|t_{g(max)}|$ up to a certain critical turning point, which we term as C_p , after which $|t_{g(min)}|$ and $|t_{g(max)}|$ start to decrease with any increment in Q_{int} . Consequently, the tunable range of the group delay Δt_g increases (decreases) with Q_{int} before (after) the critical turning point C_p . For small Q_{mut} such as that in Fig. 7.9(a), C_p is located in $Q_{int} < Q_{ext}$. As a result, for small Q_{mut} , both tunable fast and slow light are possible at a fixed λ for $Q_{int} < Q_{ext}$ as $t_{g(min)}$ and $t_{g(max)}$ can both be of the same or different sign while for $Q_{int} > Q_{ext}$, only tunable slow light is possible at a fixed λ as both $t_{g(min)}$ and $t_{g(max)}$ are positive. Increasing Q_{mut} will move C_p towards the regime of $Q_{int} > Q_{ext}$ such that at a sufficiently large

Fig. 7.8. (i) The FOMs of $t_{g(min)}$, $t_{g(max)}$, and Δt_g , and (ii) the FOM $T_{(min)}$, $T_{(max)}$ and ΔT as a function of Q_{int} for a TWMR with $Q_{ext} = 5.1 \times 10^4$ and $\lambda = \lambda_0 = 1.55 \mu\text{m}$, with no critical coupling. The value of Q_{mut} is 8.0×10^3 for (a), 1.5×10^4 for (b) and 2.0×10^4 for (c). The region to the left (right) of the dotted vertical line is the coupling state of $Q_{int} < Q_{ext}$ ($Q_{int} > Q_{ext}$).

Q_{mut} , $t_{g(min)}$ and $t_{g(max)}$ are in different light propagation regime at every Q_{int} , making it possible to have tunable fast and slow light for both $Q_{int} < Q_{ext}$ and $Q_{int} > Q_{ext}$. This can be seen in Fig. 7.8(c). Besides shifting C_p , increasing Q_{mut} will also increase $|t_{g(min)}|$ and $|t_{g(max)}|$ as well as Δt_g . However, such an increment is substantially more for $|t_{g(min)}|$ than for $|t_{g(max)}|$. This is beneficial if the main focus is to enhance the tunability of the pulse advancement, rather than the pulse delay.

As Q_{mut} is increased, critical coupling occurs. This is illustrated in Fig. 7.9, in which critical coupling occurs at around $Q_{int} = 10 \times 10^4$ for $Q_{mut} = 2.2 \times 10^4$. This has two main effects on the FOM of the group delay. First, there is an enhancement in the absolute magnitude of the loci of $t_{g(min)}$ and $t_{g(max)}$, which are bounded by the critical coupling asymptote while other portions of $t_{g(min)}$ and $t_{g(max)}$ that are not bounded by this asymptote are comparatively smaller in absolute magnitude. This happens as divergent dispersion occurs at the critical coupling point; moving towards this point leads to an increase in the strength of the dispersion. The second effect is that both $t_{g(min)}$ and $t_{g(max)}$ are in the same propagation regime of slow light after the critical coupling asymptote. This occurs in Fig. 7.9(a) for $Q_{int} > \sim 12 \times 10^4$. Then only tunable pulse delay is possible. As $Q_{mut} \rightarrow \infty$, which is shown in Fig. 7.9(b), the cavity will approach degeneracy and behaves like a conventional TWMR that has a typical notch-filter Lorentzian resonance response. For the FOM of the transmission, increasing Q_{mut} or Q_{int} results in the increment (decrement) of $T_{(max)}$ ($T_{(min)}$), which corresponding leads to a rise in ΔT . As Q_{int} is increased such that $Q_{int} > Q_{ext}$, $T_{(max)}$, $T_{(min)}$ and ΔT will gradually stabilize.

Fig. 7.9. (i) The FOMs of $t_{g, (min)}$, $t_{g, (max)}$, and Δt_g , and (ii) the FOM of $T_{(min)}$, $T_{(max)}$ and ΔT as a function of Q_{int} for a TWMR with $Q_{ext} = 5.1 \times 10^4$ and at $\lambda = \lambda_0 = 1.55 \mu\text{m}$ when critical coupling occurs. The value of Q_{mut} is set at 2.2×10^4 for (a) and $Q_{mut} \rightarrow \infty$ for (b). In (a)-(b), the region to the left (right) of the dotted vertical line is the coupling state of $Q_{int} < Q_{ext}$ ($Q_{int} > Q_{ext}$) while the asymptote (vertical solid line) marks the critical coupling (cc) point.

We now relate the trends of the FOM of the transmission with that of the group delay to better understand the trade-offs in the device performance. Notice that $|t_{g(max)}|$ generally increases with $T_{(max)}$, while increasing $|t_{g(min)}|$ reduces $T_{(min)}$. This has difficult implications for difficult coupling states. For the coupling state of $Q_{int} < Q_{ext}$, $t_{g(min)}$ and $t_{g(max)}$ always occur at the same θ as $T_{(min)}$ and $T_{(max)}$, respectively, as shown earlier in Fig. 7.7(a). This means that the output power can be increased with the group delay in the slow light regime. However, a trade-off would exist in the fast light regime: Increasing the absolute magnitude in the negative group delay gives a reduction in the output power. This is a fundamental constraint of any cavity based system, which is commonly mediated through the use of gain at the output ports. For the coupling state of $Q_{int} > Q_{ext}$, $t_{g(min)}$ and $t_{g(max)}$ also occur at the same θ as $T_{(min)}$ and $T_{(max)}$, provided that Q_{int} is before the critical coupling point, which is at $Q_{int} \sim 12 \times 10^4$ in Fig. 7.9(a). Then the above-mentioned properties of slow and fast light for the coupling state of $Q_{int} < Q_{ext}$ will also apply to the coupling state of $Q_{int} > Q_{ext}$. However, after the occurrence of critical coupling (which is at $Q_{int} > 12 \times 10^4$ for the case in Fig. 7.9(a)), the locations of $t_{g(min)}$ and $t_{g(max)}$ at each port will switch such that $t_{g(min)}$ and $t_{g(max)}$ occur at the same θ as $T_{(max)}$ and $T_{(min)}$, respectively. As $T_{(max)}$ is relatively large (and relatively constant as well) while $T_{(min)} \approx 0$, it would be better to operate around $t_{g(min)}$ though $t_{g(max)}$ has a much larger group delay.

Finally, we look at the relation between the bandwidth and group delay. To do so, several FOMs of the bandwidth are introduced: (i) $BW_{(min)}$ — the full-width at half maximum (FWHM) for the resonance spectrum centered at $\lambda = \lambda_0 = 1.55 \mu\text{m}$ when $t_g(\theta, \lambda_0) = t_{g(min)}$ and $T(\theta, \lambda_0) = T_{(min)}$, (ii) $BW_{(max)}$ — the FWHM for the resonance spectrum centered at $\lambda = \lambda_0 = 1.55 \mu\text{m}$ when $t_g(\theta, \lambda_0) = t_{g(max)}$ and $T(\theta, \lambda_0) = T_{(max)}$ and (iii) ΔBW — the tunable bandwidth, defined as $|BW_{(max)} - BW_{(min)}|$. In other words, $(BW_{(min)}, BW_{(max)}, \Delta BW)$ corresponds to $(t_{g, (min)}, t_{g, (max)}, \Delta t_g)$ and $(T_{(min)}, T_{(max)}, \Delta T)$. We have illustrated in Fig. 7.10 the FOMs of the bandwidth that correspond to the FOMs of the group delay

Fig. 7.10. The FOMs of $BW_{(min)}$, $BW_{(max)}$, and ΔBW as a function of Q_{int} for a TWMR with $Q_{ext} = 5.1 \times 10^4$, $Q_{mut} = 2.0 \times 10^4$ and $\lambda = \lambda_0 = 1.55 \mu\text{m}$. Note that $(BW_{(min)}, BW_{(max)}, \Delta BW)$ corresponds to $(t_{g, (min)}, t_{g, (max)}, \Delta t_g)$ in Fig. 7.8(cii) and $(T_{(min)}, T_{(max)}, \Delta T)$ in Fig. 7.8(cii).

and transmission in Fig. 7.8(c). Comparing Fig. 7.10 with Fig. 7.8(c), the general trend is that an increase in $|t_{g, (min)}|$ and $|t_{g, (max)}|$ leads to a decrease in $BW_{(min)}$ and $BW_{(max)}$, while the tunable bandwidth ΔBW remains relatively constant. This is the case because the delay-bandwidth product (DBP), which is defined as: $DBP = t_{g, R} \times BW$, where $t_{g, R}$ is the time delay at resonance, is approximately constant for any system due to causality, which implies that increasing the group delay results in the narrowing of the bandwidth. As both $|t_{g, (min)}|$ and $|t_{g, (max)}|$ generally increase with Q_{int} , $BW_{(min)}$ and $BW_{(max)}$ will thus decrease with Q_{int} . For single cavity systems, including our proposed scheme, the bandwidth is inherently small (< 1 nm for $t_{g, R} > 20$ ps), which will severely limit fast and slow light applications in transmission systems that utilize ultra-short pulses. To increase the DBP and thus the bandwidth, our scheme can be extended to coupled-cavity systems [37, 38]. Another method to mitigate the DBP constraint would be the use of dynamic tuning mechanism [39] in which the dispersion relation of the system is tuned in an adiabatic manner as a function of time after the pulse has entered the system.

From the above discussion, it is thus evident that a trade-off condition exists between the transmission ($T_{(max)}$, $T_{(min)}$, ΔT), bandwidth ($BW_{(min)}$, $BW_{(max)}$, ΔBW) and group delay ($t_{g, (max)}$, $t_{g, (min)}$, Δt_g) when utilizing the dual-input scheme. A compromise must therefore be reached depending on whether a high transmission, bandwidth or group delay is needed for the specific applications. The results in Figs. 7.8 to 7.10 can be used as a general design rule in trading off between the various device performances.

Finally, we mention that modulating the amplitude A of the control beam will further enhance the continuous tunability in the transmission and group delay. This is demonstrated in Fig. 7.11, in

Fig. 7.11. Demonstration of the effect of amplitude A modulation of the control beam on the continuous tunability of the transmission and group delay at a fixed wavelength of $\lambda = \lambda_0 = 1.55 \mu\text{m}$ for the through port of a TWMR with (a) $Q_{int} < Q_{ext}$ ($Q_{int} = 3.2 \times 10^4$, $Q_{ext} = 6.4 \times 10^4$) and (b) $Q_{int} > Q_{ext}$ ($Q_{int} = 6.4 \times 10^4$, $Q_{ext} = 3.2 \times 10^4$) and fixed Q_{mut} of 2.2×10^4 .

which it is shown that additional adjustment of the dispersion and transmission can be made at any fixed θ and λ by modulating A . Thus, amplitude modulation of the control beam can be used to provide another degree of freedom to manipulate the group delay and output power in the dual-input scheme.

7.5. Time Domain Analysis and Potential Fast and Slow Light Applications

The pulse propagation through the TWMR near the communication wavelength of $1.55 \mu\text{m}$ is shown in Fig. 7.12. At each input port, a Gaussian pulse $U(t) = \exp(-[t - t_0]^2/[0.72t_{pw}^2]) \cdot \exp(j\omega_c t)$ is launched into the TWMR, with physical parameters similar to those in Fig. 7.11(a). Here, t_0 is the temporal location of the peak of the input pulse, t_{pw} is the temporal full width at half maximum (FWHM) of the pulse and ω_c is the frequency of the carrier wave. We have used $\omega_c = \omega_0/(2\pi) = 193.55 \text{ THz}$ (or $\lambda_0 = 1.55 \mu\text{m}$), $t_0 = 0$ and $t_{pw} = 8t_{cav} \approx 281 \text{ ps}$ in our modeling, where t_{cav} is the photon life time of the cavity, which is defined as the average time that a photon will stay in the cavity before it is coupled out of the cavity. t_{pw} is set to be longer than t_{cav} so that the output pulse is in steady state.

Multiplying Eq. (7.5) with the Fourier transform of $U(t)$ will give the spectral profile $S_p(\omega)$ of the output pulse. The temporal expression of the output pulse follows from the inverse Fourier transform of $S_p(\omega)$. It can be seen in Fig. 7.12(a) that for slow (fast) light, the pulse experiences delay

Fig. 7.12. (a) Tunable pulse advancement or delay at the through port by modulating the amplitude and phase of the control beam for a TWMR with (ai) $Q_{int} < Q_{ext}$ ($Q_{int} = 3.2 \times 10^4$, $Q_{ext} = 6.4 \times 10^4$) and (aii) $Q_{int} > Q_{ext}$ ($Q_{int} = 6.4 \times 10^4$, $Q_{ext} = 3.2 \times 10^4$). The input pulse has a pulse width of $t_{pw} = 8t_{cav}$ and central wavelength $\lambda_0 = 1.55 \mu\text{m}$. The dotted vertical line for each pulse marks the location of the pulse peak. (b) The effects of adjusting the input pulse width t_{pw} on the output pulse at the through port of a TWMR with $Q_{int} = 6.4 \times 10^4$, $Q_{ext} = 3.2 \times 10^4$ and control beam setting of $A = 1.0$ and $\theta = 1.5\pi$. Note that $Q_{mut} = 2.2 \times 10^4$ for both (a) and (b).

(advancement), where the peak of the output pulse appears later (earlier) than the peak of the input pulse. The extent of the pulse advancement and delay can be adjusted by simply modulating the amplitude or phase of the control beam. Note that there is no violation of causality for the fast light as the leading part of the input pulse always precedes that of the output pulse. Consistent with what we have mentioned earlier, it can be seen in Fig. 7.12(a) that the pulse advancement is associated with a substantial attenuation whereas a high throughput is possible for the pulse delay.

Finally, we look at the impacts of the earlier mentioned DBP constraint on our proposed scheme regarding pulse propagation. Firstly, to maintain the pulse shape for any cavity based system, be it fast or slow light, the temporal pulse width must be larger than the photon life time of the cavity, i.e. $t_{pw} > t_{cav}$, where $t_{cav} = 1/\Delta\omega_{1/2}$ and $\Delta\omega_{1/2}$ is the FWHM (in Hz) of the resonance. Otherwise, higher-order dispersions will cause pulse distortion. As we have used $t_{pw} = 8t_{cav}$ in Fig. 7.12(a), the output pulse is thus relatively undistorted. However, it can be seen in Figs. 7.12(bi) to 7.12b(iii) that as t_{pw} is decreased, pulse distortion starts to occur. If t_{pw} becomes too small, the single input pulse will split into two output pulses, one with negative group delay and one with positive group delay, which can be observed in Fig. 7.12(biv). Hence, there is a limit to how small t_{pw} can be: the input pulse must be made adequately long in time so that its spectral bandwidth is smaller than that of the fast and slow light system, consistent with the DBP constraint.

The DBP constraint also implies a small fractional delay (or advancement) in the output pulse for single-cavity systems. Note that the fractional delay F_d is defined as $F_d = |t_g(\omega)|/t_{pw}$ [40]. For example, in Fig. 7.11(b), though using a TWMR with $Q_{int} = 6.4 \times 10^4$, $Q_{ext} = 3.2 \times 10^4$ and $Q_{mut} = 2.2 \times 10^4$ will result in a relatively large negative (positive) group delay of 225 ps (80 ps) when $A = 0.4$ and $\theta = 0.5\pi$ ($A = 1$ and $\theta = 0.5\pi$) for the control beam, the temporal pulse width t_{pw} must be set to around $8t_{cav}$ in Fig. 7.13(a) in order to avoid pulse distortion, which will correspondingly imply a rather small fractional advancement (delay) of $F_d = 0.801$ ($F_t = 0.285$) as compared to coupled-resonators systems [37, 38] where $F_d \gg 1$. However, it must be noted that even fractional variation in F_t is beneficial for certain applications. One such example is data stream regeneration [41, 42], which requires optical data pulses to be actively centred in their time windows. To perform this function well, the abilities to precisely delay and advance the pulse by a fractional amount of F_t is of utmost importance. Our dual-input scheme, with the ability to control the group delay by modulating the control beam will be suitable for such an operation. In addition, the proposed scheme can also be useful in optical-time-division-multiplexing (OTDM) [43, 44], where variable time delay is needed for the synchronization of optical signals or buffering of optical data.

7.6. Comparison of Our Proposed Scheme with Other Commonly Used Tuning Schemes

We end this work by comparing the performances in the group delay of our proposed dual-input scheme with other commonly used active tuning schemes (scheme (a)-(d) in Table 7.1). For a fair

comparative analysis, we will compare the group delay performances of these schemes with our proposed dual-input scheme by using a TWMR with the same physical parameters of $Q_{mut} = 2.2 \times 10^4$, $Q_{int} = 6.4 \times 10^4$ and $Q_{ext} = 3.2 \times 10^4$ for all these schemes. Using these physical parameters, the maximum achievable group delay $t_{g(max)}$, minimum achievable group delay $t_{g(min)}$ and tunable delay range $\Delta t_g = |t_{g(max)} - t_{g(min)}|$ for the probe beam at the fixed degenerate resonance wavelength of $1.55 \mu\text{m}$ for the schemes (a)-(d) are then calculated and summarized in Table 7.1. Note that for our proposed scheme, $t_{g(min)}$, $t_{g(max)}$ and Δt_g for the above chosen parameters of Q_{mut} , Q_{int} and Q_{ext} has been shown earlier in Fig. 7.11(bi) and they are also listed in Table 7.1 for the cases of $A = 0.4$ and $A = 1.0$.

It is evident from Table 7.1 that our proposed dual-input scheme has a better group delay performance as compared to other active tuning schemes. In particular, when the amplitude A modulation of the control beam of our proposed beam is fixed at 0.4, the tunable group delay range is ~ 239 ps and the minimum group delay is ~ -224 ps. These are major improvements as compared to schemes (a)-(d), which, respectively, has a tunable group delay range of around 84 ps, 9 ps, 9 ps and 84 ps and minimum group delay of around -69 ps, $+1$ ps, $+1$ ps and -69 ps. A unique feature of the dual-input scheme is that it has two degrees of freedom (amplitude A and phase modulation θ of the control beam) to tune the group delay. As a result, the maximum achievable group delay (or advancement) and the tunable group delay range can be varied as shown in Fig. 7.11 and Table 7.1. This is in contrast to conventional tuning schemes that have only one degree of freedom and thus their maximum achievable group delay (or advancement) and tunable group delay range are generally fixed by their physical parameters. This advantage of the dual-input scheme will allow the optimization of the device performance. For example, at $A = 0.4$, the maximum group delay that can be realized is only around $+15$ ps. This can be improved by tuning A to 1.0, which will increase the maximum group delay to around $+70$ ps, albeit the device performances in terms of the minimum group delay and the tunable group delay range are affected.

7.7. Summary and Significant Contributions of this Research Work

We have presented a detailed analysis on the transmission and dispersive behaviours of a TWMR with distributed intracavity backscattering in the presence of dual opposing inputs. It is found that, by controlling the phase and amplitude differences between the inputs, the transmission and dispersion of the TWMR can be controlled. Consequently, continuous tunability in the group delay and output spectrum can be achieved. If the physical parameters of the TWMR are properly chosen, a wide tunable range and large group delay can be achieved. For example, using a TWMR with $Q_{int} = 6.4 \times 10^4$, $Q_{ext} = 3.2 \times 10^4$ and $Q_{mut} = 2.2 \times 10^4$, a tunable range of 239 ps can be achieved at the communication wavelength of $1.55 \mu\text{m}$ at the through port, with a maximum pulse advancement (delay) of 225 ps (15 ps), when the phase difference between the dual inputs S_{+1} and S_{+2} is varied from 0.5π to 1.5π for $|S_{+2}|/|S_{+1}| = 0.4$, which is shown in Fig. 7.11(bi). Also, by modulating the amplitude

and phase differences between the dual inputs, the contrast in the group delay and transmission spectra at the different ports of the TWMR can be controlled and switching between fast and slow light can be attained. These demonstrate that the dual-input scheme allow us to better capitalize on the backscattering effects in a TWMR to generate fast and slow light, offering a more dynamic control of the speed of light, as compared to [9, 10], which uses only a single input. More importantly, the proposed dual-input scheme realizes the possibility to achieve tunable fast and slow light in a passive and linear cavity. This is in contrast to conventional TWMR-based slow and fast light systems [9, 10, 16, 17, 22, 27, 29], which require either the use of active tuning mechanisms or nonlinearity in the cavity so as to have tunable group delay.

Like the all-optical tuning (AOT) scheme (which is labeled as scheme (d) in Table 7.1), the only disadvantages of our proposed scheme as compared to the conventional active tuning (CAT) schemes (which are labeled as schemes (a)-(c) in Table 7.1), are higher implementation cost and energy requirement due to the use of dual inputs and external active tuning components like modulators and laser diodes to control the dual inputs. However, it can be seen in Table 7.1 that the advantages of the proposed scheme significantly outweigh these disadvantages. In particular, it offers unique features (which have been mentioned in Table 7.1, such as the ability to achieve active tuning by merely using a passive cavity that does not require additional fabrication processing) that are often required in practical applications, but are absent in the CAT schemes. Comparing with the all-optical tuning scheme, labeled as scheme (d) in Table 7.1, the main advantage of our scheme is that the tunability in the group delay is not generated via exploiting the nonlinearity of the cavity. This will circumvent the issue of weak material nonlinearity in the cavity in certain situations. Moreover, we have shown that the tunable group delay range and the maximum achievable pulse delay (or advancement) are significantly enhanced. For example in Table 7.1, at the degenerate resonance wavelength, the maximum pulse delay achieved is +10 ps when scheme (a), i.e., the CAT scheme, is used as compared to a maximum pulse delay of +70 ps offered by our scheme when $A = 1.0$, which is a sevenfold improvement. Thus, our proposed dual-input scheme will be highly attractive for practical fast and slow light applications.

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7.8. References for Chapter

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Chapter 8

Harnessing Intracavity Backscattering for Enhanced Tunable Coupled-Resonator- Induced Transparency and Optical Fano Resonance

8.1. Introduction: Research Motivations and Objectives

In recent years, coupled traveling wave microresonators (TWMRs) structures of various configurations [1-12] are used to realize the optical analogues of electromagnetically induced transparency (EIT) [13] and Fano resonance (FR) [14], which are known, respectively, as coupled-resonator-induced transparency (CRIT) and optical Fano resonance (OFR) in photonic systems. This ability to mimic such traditional atomic effects via TWMRs is highly useful as it is generally much easier to manipulate TWMRs as compared to atomic systems. Characterizing CRIT and OFR in TWMRs [1-12] or other integrated photonic devices [15, 16] has attracted tremendous attention as CRIT and OFR can be utilized in various applications, such as tunable delay lines [4-7], sensing [8, 9], optical switching [15] and nonlinear optics [16]. This is due to the high dispersive property, high transmission, large Q -factor and enhanced sensitivities of the CRIT and OFR resonance lineshapes. It should be noted that due to the strong dispersive property of the CRIT and OFR lineshapes, CRIT and OFR have also been categorized as fast and slow light effects in the literature [5, 12]. We have adopted this convention in this thesis and thus included the study of CRIT and OFR in part two of this thesis that deals with fast and slow light effects.

Essentially, EIT and FR are quantum interference phenomena. In EIT (cf. Fig. 8.1(a)) [13], the quantum interference between the transition amplitudes of the two different excitation pathways to the upper $|3\rangle$ state — the direct pathway ($|1\rangle - |3\rangle$) that is excited by the probe field ω_{prob} , and the indirect pathway ($|1\rangle - |3\rangle - |2\rangle - |3\rangle$) that is excited by the coupling field ω_{cou} — leads to the cancellation of the original absorption $|1\rangle - |3\rangle$ of the probe field. This results in the formation of a sharp Lorentzian peak of high transparency at the resonance frequency ω_R in between two resonance dips in the transmission T spectrum of the probe field (cf. Fig. 8.1(c)). The sharp peak at ω_R also has strong dispersion and

Fig. 8.1. The mechanisms of the different types of quantum interference phenomena in a three-level atom, explained using the multiple routes to excitation approach in (a) and (b) and their resultant transmission spectra, which are shown in (c) and (d). (a) Illustration of the mechanism of electromagnetic induced transparency (EIT). In EIT, the weaker probe field ω_{prob} can excite population by the direct pathway $|1\rangle - |3\rangle$. An alternative pathway within the atom for the population to reach $|3\rangle$ is by the indirect pathway $|1\rangle - |3\rangle - |2\rangle - |3\rangle$, in which the population is moved between $|2\rangle$ and $|3\rangle$ by the stronger coupling field ω_{cou} . Destructive interference between the direct and indirect pathways will cancel the original absorption $|1\rangle - |3\rangle$ of the weak probe field, leading to a sharp high transmission within a narrow frequency window, which is illustrated in (c). Note that transition between states $|1\rangle$ and $|2\rangle$ is forbidden. (b) Illustration of the mechanism of Fano resonance (FR). In FR, excitation takes place between the lower state $|1\rangle$ and a continuum ionising state $|2\rangle$. In addition, it can also occur between the lower state $|1\rangle$ and an autoionized state $|3\rangle$. Once in state $|3\rangle$, the atom will relax to the continuum ionising state $|2\rangle$. In other words, there are two routes to the final state $|2\rangle$. This, in general, will cause the transition probability amplitude to vanish on one side of the resonance, leading to a sharp asymmetric transmission spectrum in (d), known as FR. Note that FR spectra with different degree of asymmetry are shown in (d). The asymmetry can be tuned by the operating conditions.

large group delay enhancement. Turning off the coupling field ω_{cou} will result in the disappearance of the sharp Lorentzian peak at ω_R within the Lorentzian dip. It must be mentioned that the EIT scheme (and likewise for the CRIT scheme) is unique in that it simultaneously allows high transmission T and large enhancement in the group delay (positive and negative group delay). A somewhat similar situation occurs in FR (cf. Fig. 8.1(b)), in which the quantum interference between the transition amplitude of the resonance pathway ($|1\rangle - |2\rangle$) with that of the coherent background pathway ($|1\rangle - |3\rangle - |2\rangle$) will result in the atomic absorption to vanish at one side of the resonance frequency ω_R , leading to a sharp asymmetric resonance line shape (cf. Fig. 8.1(d)) in the transmission spectrum. Note that different types of asymmetric lineshapes (cf. Fig. 8.1(d)) can be generated by FR. In photonic systems, EIT- and FR-like effects (i.e., CRIT and OFR) can be realized in an analogous manner via the coherent interference of electric fields (rather than transition amplitudes in atoms) between the whispering gallery modes (WGMs) from different optical resonance pathways in the coupled-TWMRs structure. Different configurations of coupled-TWMRs structures [1-12] have been employed to generate CRIT and OFR, with the most common and simplest configuration being the twin coupled-TWMRs structure [5-8, 11, 12], in which two resonators are directly coupled to each another, with one of the resonators coupled to an external port waveguide. For such twin coupled-TWMRs structure (cf. Fig. 8.2), which is the focus of this work, it has been shown that the destructive interference between the decaying WGMs from the different resonators will result in an EIT-like Lorentzian peak in the output transmission spectrum (similar to Fig. 8.1(c)) when the two resonators are weakly coupled and provided that either the loss in the resonator farthest from the port waveguide is zero [5] or the net effective loss rate of the coupled-resonators system is zero [7]. By introducing gain to one of the resonators [11] or by using resonators of different sizes [12] in the twin-coupled TWMRs system, the EIT-like or CRIT symmetric Lorentzian peak in the transmission spectrum will be reshaped into a distinctly sharp asymmetric lineshape (similar to Fig. 8.1(d)), which is the optical equivalent of FR, known as optical Fano resonance (OFR). However, to the best of our knowledge, the study of CRIT and OFR in the above-mentioned twin coupled-TWMRs structure generally does not consider intracavity backscattering and dual inputs, which have recently been employed to realize tunable fast and slow light in a single TWMR system in our work in chapter 7 and [17].

In this chapter, we will explore the possibility of harnessing intracavity backscattering to generate enhanced CRIT and OFR in the twin-coupled TWMRs structure (cf. Fig. 8.2). To do so, surface perturbations in the form of microstructures, such as grating ridges [18], are intentionally introduced into the resonators. Also, either single or dual inputs can be used. We will term our TWMR structure as the modified twin-coupled TWMRs structure as it departs from the traditional twin-coupled TWMRs [5-8, 11, 12], which neither harnesses intracavity backscattering nor dual inputs. It will be shown that the intracavity backscattering modifies the critical coupling conditions and the mode-splitting features, which consequently lead to different and enhanced CRIT and OFR

Fig. 8.2. Schematic of the proposed modified twin-coupled TWMRs structure in which surface perturbations in the form of microstructures like grating ridges are intentionally introduced on the cavities. This induces mutual coupling between the CW and CCW modes in each cavity. Enhanced CRIT and OFR effects can be generated when the CW and CCW modes from resonator 1 interact with those from resonator 2 and the single or dual inputs.

effects in contrast to the traditional twin-coupled TWMRs. In the event of using dual inputs, the CRIT symmetric peak can be reshaped into a sharp asymmetric line shape of OFR simply by controlling the phase and/or amplitude differences between the inputs. We can thus switch between CRIT and OFR simply by using a second input to interact with the intracavity backscattering. This is in contrast to the traditional twin-coupled TWMRs, in which gain [11] or phase detuning [12] is typically employed to switch between CRIT and OFR. As mentioned in section 7.1, the main drawback of such gain or phase detuning schemes in the cavities, such as [11] and [12], is that additional procedures and controls are required in the fabrication process so as to incorporate the active tuning components. Moreover, as the active components are implanted in the cavity system, it would be difficult to reconfigure or to replace them after fabrication. In the case in which a passive cavity has already been fabricated, it might not be feasible or possible in certain situations to do additional alternation to the device so as to render it active. By using our proposed dual-input scheme, such constraints are eliminated as only external dual inputs (laser diodes) as well as phase modulators [19] and/or variable optical attenuators [20] are required to achieve tuning between CRIT and OFR. In other words, a passive and linear cavity can be used, without the need to implant active components within the cavity. This reduces the fabrication complexity and footprint of the cavity. The reconfigurability and

serviceability of the proposed cavity system (in terms of its active tunability) are also improved as the external active tuning components can be easily adjusted or replaced.

This chapter is organized as follows. In section 8.2, we present the theoretical modeling of the interplay between the intracavity backscattering and losses out of the resonator in our modified twin coupled-TWMR structure. In section 8.3, we show how the presence of intracavity backscattering leads to enhanced CRIT as well as modifying the coupling conditions. In section 8.4, the generation of sharp asymmetric OFR lineshapes by manipulating the intracavity backscattering is demonstrated. In section 8.5, we show that switching between CRIT and OFR can be achieved when dual inputs are employed to interact with the backscattering in the TWMRs. Finally, we conclude in section 8.6.

8.2. Theoretical Formulation

The proposed modified twin-coupled TWMR structure is shown schematically in Fig. 8.2, in which two azimuthally symmetric TWMRs are evanescently coupled to each other, with one of the resonators coupled to an excitation port waveguide of length $L = L_1 + L_2$ via a coupling junction (CJ) that is taken to be infinitesimally short. In this work, we will assume that all the waveguide based components only support the fundamental modes (either TE_0 or TM_0) and that any backscattering due to fabrication-induced sidewall roughness [21] in the bus waveguide or resonators is negligibly small due to advancement in fabrication technology [22, 23]. In order to effectively produce the intracavity backscattering that will result in mutual coupling between the clockwise (CW) and counterclockwise (CCW) cavity modes, surface microstructures [18] are intentionally introduced into the two resonators (but not the port waveguide). Also, either a single input or dual inputs can be employed to harness the intracavity backscattering effect. In the case of using dual inputs, the phase and/or amplitude differences between the dual inputs — S_{+1} at the reflection port and S_{+2} at the through port — can be controlled through the use of external optical phase modulators [19] and/or variable optical attenuators [20] such that the dual inputs entering the coupling junction (CJ) are related to each other by $S'_{+2} = MS'_{+1} = [A \exp(j\theta)]S'_{+1}$, where $j = \sqrt{-1}$, $M = A \exp(j\theta)$ is termed as the overall modulation factor, while A and θ are, respectively, the amplitude and phase modulations between the dual inputs S'_{+1} and S'_{+2} . We will term S_{+1} (S_{+2}) as the signal (control) beam. Do note that the amplitude A and phase θ can be varied, respectively, by external amplitude and phase modulators [19, 20]. If only one input is employed, $M = 0$ (i.e. only S_{+1} is present and $S_{+2} = 0$), which will be the case for our discussion in sections 8.3 and 8.4 while section 8.5 deals with the case of the dual-input scheme (i.e., $M \neq 0$). By treating each resonator as a lumped oscillator, with all the inputs and cavity modes having $\exp(j\omega t)$ time dependences, i.e., $S_{+1} = \tilde{S}_{+1} \exp(j\omega t)$, $S_{+2} = \tilde{S}_{+2} \exp(j\omega t)$, $E_{ccw} = \tilde{E}_{ccw} \exp(j\omega t)$ and $E_{cw} = \tilde{E}_{cw} \exp(j\omega t)$, the temporal evolution of the instantaneous intracavity electric fields in our modified twin-coupled TWMRs structure can be described using the following set of coupled mode equations [24]:

$$\begin{aligned}
\frac{dE_{ccw,1}}{dt} &= (-j\Delta\omega_1 - \Gamma_{s,1})E_{ccw,1} - ju_{m,1}E_{cw,1} - ju_0S_{+1}, \\
\frac{dE_{cw,1}}{dt} &= (-j\Delta\omega_1 - \Gamma_{s,1})E_{cw,1} - ju_{m,1}E_{ccw,1} - ju_0S_{+2}, \\
\frac{dE_{ccw,2}}{dt} &= (-j\Delta\omega_2 - \Gamma_{s,2})E_{ccw,2} - ju_{m,2}E_{cw,2} - ju_1E_{cw,1}, \\
\frac{dE_{cw,2}}{dt} &= (-j\Delta\omega_2 - \Gamma_{s,2})E_{cw,2} - ju_{m,2}E_{ccw,2} - ju_1E_{ccw,1},
\end{aligned} \tag{8.1}$$

where $E_{cw,n}$ ($E_{ccw,n}$) is the energy amplitude of the CW (CCW) intracavity mode, $\Delta\omega_n = \omega - \omega_n$ is the frequency detuning, ω_n is the uncoupled (or degenerate) resonance frequency of each resonator, $\Gamma_{net,n}$ is the net decay rate for each cavity mode, u_0 is the rate of energy coupling between resonator 1 and the bus waveguide, u_1 is the rate of energy coupling between the resonators, $u_{m,n}$ is the mutual coupling between the cavity modes in each resonator and the subscript $n = 1$ refers to the resonator nearest to the port waveguide while $n = 2$ refers to the resonator furthest away from the port waveguide. Considering both the intrinsic cavity losses and the evanescent coupling, each cavity mode in resonator 1 has a decay rate of $\Gamma_{s,1} = \Gamma_{int,1} + \Gamma_{ext}$, while the decay rate for each cavity mode in resonator 2 is simply $\Gamma_{s,2} = \Gamma_{int,2}$. Note that $\Gamma_{ext} = u_0^2/2$ is the decay rate due to the evanescent coupling between resonator 1 and the port waveguide and $\Gamma_{int,n}$ ($n = 1$ or 2) is the decay rate of each resonator that arises from the cavity intrinsic losses. We also define five different types of quality or Q -factors for each cavity mode: (1) The intrinsic Q -factor $Q_{int,n} = \omega_n/(2\Gamma_{int,n})$; (2) the WG evanescent coupling Q -factor $Q_{ext} = \omega_1/(2\Gamma_{ext})$; (3) the inter-cavity evanescent coupling Q -factor $Q_{cou} = \omega_2/(2u_1)$; (4) the mutual coupling Q -factor $Q_{mut,n} = \omega_n/(2u_{m,n})$; and (5) the sum of the effects of the intrinsic and extrinsic Q -factors $Q_{s,n} = \omega_n/(2\Gamma_{s,n}) = (1/Q_{ext} + 1/Q_{int,n})^{-1}$. Note that the net Q -factor Q_{net} of each cavity mode, which is coupled to the bus WG, is $Q_{net} = (1/Q_{int,1} + 1/Q_{mut,1} + 1/Q_{ext} + 1/Q_{cou})^{-1}$. For our system, the backward (forward) transmission in the bus WG, denoted as S_{-1} (S_{-2}), can be expressed in terms of the incident wave and the cavity modes using power conservation theorems [24]:

$$S_{-1} = e^{-j\beta_w L} (S_{+2} - ju_0 E_{cw}), \quad S_{-2} = e^{-j\beta_w L} (S_{+1} - ju_0 E_{ccw}), \tag{8.2}$$

where β_w is the propagation constant of the mode in the bus WG of length $L = L_1 + L_2$. Note that the artificial dispersion due to the structural resonance of the TWMR is much stronger than the material dispersion. As such, the $\exp(-j\beta_w L)$ term in Eqn 8.2 is considered as negligible because it is a pure linear phase term of the two sections of the straight waveguides with lengths L_1 and L_2 and thus it does not affect the structural dispersion characteristics of the microresonator circuit. The steady-state complex electric field transmittivities at the through and reflection ports are, respectively, defined as $\xi_T(\Delta\omega_n) = S'_{-2}/S'_{+1}$ and $\xi_R(\Delta\omega_n) = S'_{-1}/S'_{+1}$ for the case of two co-resonant coupled-resonators (i.e., $\omega_1 = \omega_2 = \omega_0$ and $\Delta\omega_1 = \Delta\omega_2 = \Delta\omega_0$, where ω_0 is the common degenerate resonance frequency for the two resonators). Both $\xi_T(\Delta\omega_n)$ and $\xi_R(\Delta\omega_n)$ can be solved by first, substituting the steady-state solutions

E_{cw} and E_{ccw} of Eqn. (8.1) into Eqn. (8.2) and subsequently arranging Eqn. (8.2) into the form $\xi_T(\Delta\omega_n) = S'_{-2}/S'_{+1}$ and $\xi_R(\Delta\omega_n) = S'_{-1}/S'_{+1}$. The final results $\xi_T(\Delta\omega_n)$ and $\xi_R(\Delta\omega_n)$ can be expressed as:

$$\begin{aligned}\xi_T(\Delta\omega_0) &= 1 - \frac{u_0^2}{B_1} \left\{ j\Delta\omega_0^3 - (\Gamma_{s,1} + 2\Gamma_{s,2} - jMu_{m,1})\Delta\omega_0^2 - [\Gamma_{s,2}(2j\Gamma_{s,1} + j\Gamma_{s,2} + 2Mu_{m,1}) \right. \\ &\quad \left. + ju_{m,2}^2 + ju_1^2] \Delta\omega_0 + [\Gamma_{s,2}(u_1^2 + \Gamma_{s,1}\Gamma_{s,2}) + \Gamma_1 u_{m,2}^2] + jM(u_1^2 u_{m,2} - u_{m,1} u_{m,2}^2 - u_{m,1} \Gamma_2^2) \right\}, \\ \xi_R(\Delta\omega_0) &= M - \frac{u_0^2}{B_1} \left\{ jM\Delta\omega_0^3 + (ju_{m,1} - M\Gamma_{s,1} - 2M\Gamma_{s,2})\Delta\omega_0^2 \right. \\ &\quad \left. + [jM(u_1^2 + u_{m,2}^2 + \Gamma_{s,2}^2 + 2\Gamma_{s,1}\Gamma_{s,2}) + 2\Gamma_{s,1}\Gamma_{s,2}] \Delta\omega_0 \right. \\ &\quad \left. - j(u_{m,1}\Gamma_{s,2} - u_{m,2}u_1^2 + u_{m,1}u_{m,2}^2) + M(\Gamma_{s,1}u_{m,2}^2 + \Gamma_{s,2}u_2^2 + \Gamma_{s,1}\Gamma_{s,2}^2) \right\},\end{aligned}\tag{8.3}$$

where $B_1 = [\Delta\omega_0^2 + j(\Gamma_1 + \Gamma_2 + ju_{m,1} + ju_{m,2})\Delta\omega_0 - j(u_{m,1}\Gamma_2 + u_{m,2}\Gamma_1) - u_1^2 - \Gamma_1\Gamma_2 + u_{m,1}u_{m,2}] \times$
 $[\Delta\omega_0^2 + j(\Gamma_1 + \Gamma_2 - ju_{m,1} - ju_{m,2})\Delta\omega_0 + j(u_{m,1}\Gamma_2 + u_{m,2}\Gamma_1) - u_1^2 - \Gamma_1\Gamma_2 + u_{m,1}u_{m,2}].$

Note that $M = [A \exp(j\theta)]$, which is the modulation factor between the dual inputs. Details can be found at the beginning of this section. The complex electric field transmittivities ξ_T and ξ_R in Eq. (8.3) can also be expressed as $\xi_T = |\xi_T| \exp(j\Phi_T)$ and $\xi_R = |\xi_R| \exp(j\Phi_R)$, respectively. Here, Φ_T and Φ_R are, respectively, the effective phase shifts for the through and reflection ports. They contain all the necessary information to determine the transmission and dispersive properties of the output lights. In particular, the transmission T_q and group delay $t_{g,q}$ are given by $T_q = |\xi_q|^2$ and $t_{g,q}(\Delta\omega_0) = \partial\Phi_q(\Delta\omega_0)/\partial(\Delta\omega_0)$, where the subscripts $q = T$ and $q = R$, respectively, denote the through and reflection ports. Using Eq. (8.3), we will show in the next section of how the intracavity backscattering alters the mode-splitting features, which will lead to different CRIT and OFR properties between our modified twin-coupled TWMRs structure and the traditional twin-coupled TWMRs (no backscattering) structure in the literature for the case of co-resonant resonators, i.e., $\omega_1 = \omega_2 = \omega_0$ and $\Delta\omega_1 = \Delta\omega_2 = \Delta\omega_0$, with $\omega_0/(2\pi) = 193.55$ THz (or $\lambda_0 = 1.55$ μm).

8.3. Enhanced CRIT Effects in Proposed Structure

8.3.1. General Transmission and Dispersive Properties

The mutual coupling between the cavity modes in each resonator is described by $u_{m,n}$, where the subscript $n = 1$ refers to the resonator nearest to the port waveguide, while $n = 2$ refers to the resonator furthest away from the port waveguide. The Q -factor describing the mutual coupling of the contra-propagating cavity modes is denoted as $Q_{mut,1}$ for resonator 1 and $Q_{mut,2}$ for resonator 2. To clearly illustrate how the presence of intracavity backscattering modifies and enhances the CRIT effects, we first look into the scenario in which both resonators in our proposed scheme in Fig. 8.2 have the same intracavity backscattering strengths. This means $u_{m,1} = u_{m,2} = u_m$, which also implies $Q_{mut,1} = Q_{mut,2} = Q_{mut}$. Also, only one single input is launched into the coupled-resonator system, i.e., $M = 0$ in Eq.

Fig. 8.3. Evolution in the transmission at the through port of the twin-coupled TWMRs with $Q_{int,1} = Q_{int,2} = 6e4$ and $Q_{ext} = 5e4$ as the resonator-to-resonator coupling (controlled by Q_{cou}) changes in the presence of different magnitude of intracavity backscattering (controlled by Q_{mut}). The two resonators have equal Q_{mut} , i.e., $Q_{mut} = Q_{mut,1} = Q_{mut,2}$. In each row, Q_{cou} is varied while Q_{mut} is fixed. Insets shown the blown-up of the regions marked by 'X'. These spectra can be obtained using silicon resonators, each with a radius of 5 μm .

Fig. 8.4. Evolution in the transmission at the reflection port of the twin-coupled TWMRs with $Q_{int,1} = Q_{int,2} = 6e4$ and $Q_{ext} = 5e4$ as the resonator-to-resonator coupling (controlled by Q_{cou}) changes in the presence of different magnitude of intracavity backscattering (controlled by Q_{mut}). The two resonators have equal Q_{mut} , i.e., $Q_{mut} = Q_{mut,1} = Q_{mut,2}$. In each row, Q_{cou} is varied while Q_{mut} is fixed. Insets show the blown-up of the regions marked by 'X'. All the spectra can be obtained using silicon resonators, each with a radius of 5 μm .

(8.3). The situation of $u_{m,1} \neq u_{m,2}$ and $M \neq 0$ will be dealt with shortly in the next few sections. We have shown in Figs. 8.3 and 8.4 the evolution of the transmission responses of the twin-coupled TWMRs for different u_m . Note that all the transmission spectra in this work are presented in the wavelength domain near the degenerate resonance wavelength $\lambda_0 = 1.55 \mu\text{m}$, though the frequency domain is used in the analytical expressions and discussions for consistency with the literature. Also, λ_0 and ω_0 will be used interchangeably throughout the discussion.

Let us start from the very basic — the case of $Q_{mut} \rightarrow \infty$ or $u_m \rightarrow 0$. This refers to the traditional twin-coupled resonators in the literature that has no intracavity backscattering. For this case, we can see from Fig. 8.4a(i) to Fig. 8.4a(iii) that there are no output transmission at the reflection port (i.e., $T_R = 0$). On the other hand, for the through port, the transmission at $\lambda_0 = 1.55 \mu\text{m}$ transforms from a single Lorentzian dip (cf. Fig. 8.3a(i)) to a sharp CRIT peak with near complete transparency (cf. Fig. 8.3a(ii)) that is accompanied by strong normal dispersion and finally to a broadened Lorentzian (cf. Fig. 8.3a(iii)) as Q_{cou} decreases. To explain these phenomena, first note that the uncoupled resonance mode ω_0 splits into two eigenmodes — ω_{spc+} and ω_{spc-} — whenever two resonators are coupled together. These eigenmodes $\omega_{spc\pm}$ are clearly resolvable or distinguishable in the transmission spectrum in the form of a two-split Lorentzian only if the mode splitting or frequency separation between ω_{spc+} and ω_{spc-} (proportional to $1/Q_{cou}$) is much larger than the average resonance linewidth, i.e., when $\omega_{spc+} - \omega_{spc-} > \Gamma_{ave}$ or, in other words, $1/Q_{cou} > \sim \Gamma_{ave}$ (or $u_1 > \Gamma_{ave}$), where $\Gamma_{ave} = (\Gamma_{s,1} + \Gamma_{s,2})/2$. Otherwise, the transmission spectrum has the shape of a single-Lorentzian near ω_0 . This is somewhat analogous to the Rayleigh criterion in diffraction optics. In Fig. 8.3a(i), a single Lorentzian dip is formed at ω_0 as the split modes ω_{spc+} and ω_{spc-} are indistinguishable due to the large Q_{cou} (or weak u_1), which results in $1/Q_{cou} < \Gamma_{ave}$. Then the two coupled resonators behave like a single resonator system. However, for smaller Q_{cou} (or stronger u_1), we have $1/Q_{cou} > \Gamma_{ave}$, which causes $\omega_{spc\pm}$ (note that the two split modes ω_{spc+} and ω_{spc-} are termed collectively as $\omega_{spc\pm}$) to be clearly resolved into a two-split Lorentzian, which can be observed in Figs. 8.3a(ii) and 8.3a(iii). In this regime, if Q_{cou} is sufficiently small, CRIT can occur, which is the case in Fig. 8.3a(ii). The conditions for such CRIT with complete transparency (i.e., $T_T = 1$) at ω_0 for $u_m = 0$ can be derived by letting $u_{m,n} \rightarrow 0$ in Eq. (8.3):

$$\xi_T(\Delta\omega_0) = \frac{\Delta\omega_0^2 + j(\Gamma_{s,1} + \Gamma_{s,2} - u_0^2)\Delta\omega_0 - \Gamma_{s,2}(u_1^2/\Gamma_{s,2} + \Gamma_{int,1} - \Gamma_{ext})}{\Delta\omega_0^2 + j(\Gamma_{s,1} + \Gamma_{s,2} + u_0^2)\Delta\omega_0 - \Gamma_{s,2}(u_1^2/\Gamma_{s,2} + \Gamma_{int,1} + \Gamma_{ext})}. \quad (8.4)$$

It can be deduced from Eq. (8.4) that CRIT with complete transparency at $\Delta\omega_0 = 0$ (or $\lambda = \lambda_0$) occurs if either there is no net loss in resonator 2 (i.e., $\Gamma_{s,2} = 0$) or the net effective loss rate of the two resonators is zero (i.e., $u_1^2/\Gamma_{s,2} + \Gamma_{int,1} = 0$). Do note that gain can be used to achieve $u_1^2/\Gamma_{s,2} + \Gamma_{int,1} = 0$ (as $\Gamma_{s,n} < 0$ for cavity with net gain). For our case in Fig. 8.3a(ii), $\Gamma_{s,2} \approx 0$ and thus a CRIT peak is observed with $T_T \approx 1$ at $\Delta\omega_0 = 0$ (or $\lambda = \lambda_0$). This is the result of the destructive interference between

the optical resonance pathways in resonator 1 and 2, analogous to the EIT effect in atomic system. The CRIT peak is sandwiched in between two resonance dips, which are the split modes ω_{spc+} and ω_{spc-} that have $\text{Im}[\zeta_T] \approx 0$ (where $\text{Im}[\]$ denotes the imaginary component) and thus ω_{spc+} and ω_{spc-} can be expressed as

$$\omega_{spc\pm} \approx \omega_0 \pm \sqrt{u_1^2 - \Gamma_{s,2}^2}. \quad (8.5)$$

In the event that Q_{cou} is sufficiently small (or u_1 is sufficiently large) for the above-mentioned strong coupling regime of $1/Q_{cou} > \Gamma_{ave}$, there will be a strong mode splitting that pulls ω_{spc+} and ω_{spc-} far apart, resulting in a broadened Lorentzian at $\Delta\omega_0 = 0$ in between the two split Lorentzian, as seen in Fig. 8.3a(iii). Note that this is not CRIT even though there is transparency in the transmission at $\lambda = \lambda_0$ as such transparency (which is accompanied by a weak dispersive response) is due to the strong mode splitting rather than the destructive interference between the light from the different resonators.

We now move on to the case of our proposed modified twin-coupled resonators, in which there is intracavity backscattering (i.e., $Q_{mut} < \infty$ and $u_m \neq 0$). A unique difference between this case and that of $Q_{mut} \rightarrow \infty$ or $u_m = 0$ as described above and in Figs. 8.3a(i) to 8.3a(iii) is the presence of transmission at the reflection port (in contrast to $T_R = 0$ for $u_m = 0$), as seen in Fig. 8.4b and 8.4c. In addition, there are also changes in the mode splitting features of the transmission T_T spectrum at the through port in Fig. 8.3 for $u_m \neq 0$, as compared to those of $u_m = 0$. To explain these, first note that the presence of intracavity backscattering in the two resonators results in the resonance frequency ω_n ($n = 1$ or 2) in each resonator to split into eigenmodes ω_{sps+}^n and ω_{sps-}^n , where $\omega_{sps\pm}^n$ have frequencies of $\omega_{sps\pm}^n = \omega_n \pm u_{m,n} = \omega_n \pm \omega_n/(2Q_{mut,n})$ (cf. chapter 7 for more details). Note that these eigenmodes or split modes are totally different from the earlier mentioned split modes ω_{spc+} and ω_{spc-} , which results from the evanescent coupling between the two resonators, rather than intracavity backscattering. As both resonators 1 and 2 have the same backscattering strength (i.e., $u_{m,1} = u_{m,2} = u_m$) and size (i.e., $\omega_1 = \omega_2 = \omega_0$) in this section, the eigenmodes in these resonators are equal (i.e., $\omega_{sps\pm}^1 = \omega_{sps\pm}^2$), which will give rise to symmetric splitting in the resonance spectra in Fig. 8.3. Later, in sections 8.4 and 8.5, it will be shown that the splitting in the resonance spectra will be unbalanced when $u_{m,1} \neq u_{m,2}$. This will give rise to OFR.

For a sufficiently large Q_{cou} (i.e., weak evanescent coupling), the twin-coupled TWMRs behave like a single-resonator system, which is to say that the split modes $\omega_{sps\pm}^1$ in resonator 1 has weak interaction with the split modes $\omega_{sps\pm}^2$ in resonator 2. In such a scenario, the transmission spectrum of the twin-coupled TWMRs system at the output ports will be chiefly determined by $\omega_{sps\pm}^1$. Depending on the magnitude of the intracavity backscattering, either a single Lorentzian or a two split Lorentzian can form in the transmission spectrum, with both types of spectra appearing reproducibly at every free spectral range (FSR). This is in contrast to the earlier case for $u_m = 0$, where a Lorentzian always appears at every FSR in the transmission spectrum. For Figs. 8.3b(i) and 8.4b(i), the intracavity

backscattering is weak such that $Q_{mut} \gg Q_{cri}$, where Q_{cri} is the value of Q_{mut} that gives critical coupling. As mentioned earlier in chapter 7, mode-splitting due to intracavity backscattering can only be resolved in the transmission spectrum if $Q_{mut} \ll Q_{cri}$. Thus for this case, $\omega_{sps\pm}^1$ cannot be clearly distinguishable, thus giving a single-Lorentzian dip (peak) at $\omega = \omega_0$ in the transmission T_T (T_R) spectrum of the through (reflection) port in Fig. 8.3b(i) (Fig. 8.4b(i)). On the other hand, for Figs. 8.3c(i) and 8.4c(i), the intracavity backscattering is strong enough such that $Q_{mut} \ll Q_{cri}$. This results in $\omega_{sps\pm}^1$ to be clearly distinguishable in the form of a two-split Lorentzian dips (peaks) in the transmission T_T (T_R) spectrum of the through (reflection) port in Fig. 8.3c(i) (Fig. 8.4c(i)), with each dip (peak) or split mode located at $\omega = \omega_0 \pm u_m = \omega_0 \pm \omega_0/(2Q_{mut})$. Whether slow or fast light is produced at the resonances at $\omega = \omega_0$ and $\omega = \omega_0 \pm u_m$ for this case of weak evanescent coupling between the resonators is chiefly determined by the coupling state (i.e., relation of Q_{ext} to $Q_{int, 1}$) and the intracavity backscattering strength of resonator 1. This has been elaborated in chapter 7 and [12].

As Q_{cou} is decreased, the inter-resonator coupling strength becomes much stronger and thus $\omega_{sps\pm}^1$ and $\omega_{sps\pm}^2$ can interact with one another. Destructive interference between $\omega_{sps\pm}^1$ and $\omega_{sps\pm}^2$, which will give CRIT with $T \approx 1$, is also possible if $\Gamma_{int, 2} \approx 0$ for the pair of high-finesse resonators. This occurs in Fig. 8.3b(ii) and Fig. 8.3c(ii) when Q_{cou} is decreased to $1.5e5$. However, the strength of the intracavity backscattering will affect the features of the CRIT. For the case of $Q_{mut} = 4.1e8$ in Fig. 8.3b(ii), a sharp CRIT peak at $\omega = \omega_0$ (similar to that of $u_m = 0$ in Fig. 8.3a(ii)) can be seen in the transmission T_T spectrum of the through port, while a sharp Lorentzian peak with a considerably low power can be observed in the transmission T_R spectrum of the reflection port in Fig. 8.4b(ii). However, for the case of $Q_{mut} = 4.1e3$ in Fig. 8.3c(ii), we can observe a pair of CRIT peaks within the same frequency window in the transmission T_T spectrum. This is accompanied by a pair of Lorentzian dips or notches in the transmission T_R spectrum in Fig. 8.4c(ii). For the former case of $Q_{mut} = 4.1e8$, both the split modes $\omega_{sps\pm}^1$ and $\omega_{sps\pm}^2$ cannot be clearly resolved. In other words, $\omega_{sps\pm}^1$ and $\omega_{sps\pm}^2$ are both approximately equal to the uncoupled resonance modes ω_1 and ω_2 , respectively (which is $\omega_1 = \omega_2 = \omega_0$ in this section). Then the destructive interference between $\omega_{sps\pm}^1$ and $\omega_{sps\pm}^2$ will lead to a single CRIT peak at $\omega = \omega_0$ in the T_T spectrum. On the other hand, for the case of $Q_{mut} = 4.1e3$ in Fig. 8.3c(ii), both the split modes $\omega_{sps\pm}^1$ and $\omega_{sps\pm}^2$ are clearly distinguishable in each resonator. Then each split mode (ω_{sps+}^1 and ω_{sps-}^1) from resonator 1 will interfere destructively with one of the split modes (ω_{sps+}^2 or ω_{sps-}^2) from resonator 2, giving a total of two CRIT peaks at $\omega = \omega_0 \pm u_m = \omega_0 \pm \omega_0/(2Q_{mut})$ in the same frequency window in the T_T spectrum. Each such CRIT peak is sandwiched between two split modes that are due to inter-resonator coupling. We will label these split modes due to inter-resonator coupling in the case of inter-cavity backscattering as $\omega_{sps\pm 1}$ and $\omega_{sps\pm 2}$. They are located at

$$\omega_{sps\pm 1} \approx \omega_0 \pm u_m + \sqrt{u_1^2 - \Gamma_{s, 2}^2}, \quad \omega_{sps\pm 2} \approx \omega_0 \pm u_m - \sqrt{u_1^2 - \Gamma_{s, 2}^2}, \quad (8.6)$$

while the two split modes that sandwich the single CRIT peak for the case of weak intracavity backscattering in Fig. 8.3b(ii) are labeled as $\omega_{spc\pm}$ and are located at $\omega_{spc\pm} = \omega_0 \pm (u_1^2 - \Gamma_{s,2}^2)^{1/2}$, similar to the case of $u_m = 0$ in Fig. 8.3a(ii). As Q_{cou} becomes sufficiently small, the CRIT peaks and Lorentzian notches, respectively, in the T_T and T_R spectra disappear due to the strong mode splitting that pulls the split modes further apart, resulting in broad Lorentzians at $\omega = \omega_0$ in Fig. 8.3b(iii) and Fig. 8.4b(iii) and at $\omega = \omega_0 \pm u_m$ in Fig. 8.3c(iii) and Fig. 8.4c(iii).

From the above discussion, we can see that the presence of intracavity backscattering in the twin-coupled TWMRs makes it possible to either generate a single sharp CRIT peak or a pair of CRIT peaks in the transmission spectrum of the through port, depending on the magnitude of the backscattering, in contrast to the case of the traditional twin-coupled TWMRs (no intracavity backscattering) in the literature, in which only a single sharp CRIT peak can be realized within the same frequency window. The distance between the twin-CRIT peaks can be controlled by adjusting the intracavity backscattering magnitude u_m or Q_{mut} (as $Q_{mut} = \omega_0/(2u_m)$). This is shown clearly in Fig. 8.5a(i): The distance between the twin CRIT peaks decreases with increasing Q_{mut} (or decreasing u_m). As $u_m \rightarrow 0$ or $Q_{mut} \rightarrow \infty$, the two CRIT peaks will merge into one single CRIT peak at $\omega = \omega_0$ as the split modes $\omega_{spc\pm}$ (which has frequencies $\omega = \omega_0 \pm u_m$) cannot be clearly resolved in the transmission spectrum. This is consistent with the results of the conventional twin-coupled TWMRs in [5-8, 11, 12]. Note that in Fig. 8.5 (and in all other Figs), the wavelength $\lambda = 1550$ nm corresponds to $\omega = \omega_0$.

Fig. 8.5. The transmission, effective phase shift and group delay responses resulting from the CRIT effects of the twin-coupled TWMRs with $Q_{int,1} = Q_{int,2} = 6e4$, $Q_{ext} = 5e4$ and $Q_{cou} = 1.5e5$, using different magnitude of intracavity backscattering (quantified by Q_{mut}). Evidently, adjusting Q_{mut} tunes the separation distance between the CRIT peaks. In all the graphs, the spectra move in the direction of the arrows as Q_{mut} increases. Note that $Q_{mut} = Q_{mut,1} = Q_{mut,2}$. All the spectra can be obtained using silicon resonators, each with a radius of 5 μm .

In addition, another unique feature of the modified twin-coupled TMWRs system is that the CRIT peaks at the transmission spectrum of the through port is always accompanied by a notch filter transmission spectrum (i.e., Lorentzian dip) at the reflection port, which can be seen in Fig. 8.5b(i). This is unlike the traditional twin-coupled TMWRs system, which does not utilize intra-cavity backscattering and thus only has forward transmission but no backward transmission ($T_R = 0$). In such a circumstance, the CRIT peak and notch filter transmission spectrum cannot be simultaneously generated, unlike our proposed modified twin-coupled TMWRs system described above. Thus, the presence of intracavity backscattering enhances the CRIT features and improves the versatility of the twin-coupled TMWRs system. Note that the intracavity backscattering u_m can be easily controlled by intentionally introducing microstructures (such as grating ridges) on the surfaces of the cavities during fabrication. This has been demonstrated in [12], as mentioned earlier. Thus, our proposed modified twin-coupled TMWRs system is suitable for practical implementation in integrated optics platform.

Finally, note that the CRIT peaks at the through port, which are located at $\omega = \omega_0 \pm u_m$ are always accompanied by a strong normal dispersion (i.e., slow light), as shown in Fig. 8.5a, while anomalous dispersion (i.e., fast light) can be found around the vicinity of the CRIT peaks. In general, such anomalous dispersion increases in strength as we approach the resonance dips that sandwich the CRIT peaks. Note that these resonance dips are the split modes $\omega_{spc\pm1}$ and $\omega_{spc\pm2}$ (cf. Eq. (8.6)). Reversal from the normal to the anomalous dispersion or vice versa will occur as we move from the through port to the reflection port at the same wavelength or frequency. In particular, there is a strong anomalous dispersion (i.e., fast light) at the Lorentzian dips or notches, which is accompanied by the normal dispersion (i.e., slow light) in the vicinity of these Lorentzian dips. These are shown in Fig. 8.5b. The strength of such a normal dispersion is strongest at the split modes $\omega_{spc\pm1}$ and $\omega_{spc\pm2}$, which are the peaks that sandwich the Lorentzian dips, and decreases as we depart away from these split modes. This trend is similar to the case at the through port. Thus, it is evident from Fig. 8.5 that, by harnessing intracavity backscattering, it is possible to simultaneously produce both fast and slow light, one from each port, at the various resonant modes. For example, at $\omega = \omega_0 \pm u_m$, a CRIT peak, which is accompanied by slow light, can be generated at the through port while a notch filter (i.e., Lorentzian dip) response, which is accompanied by fast light, can be simultaneously generated at the reflection port of our proposed device. This is in contrast to the case of $u_m = 0$ (i.e., traditional twin-coupled resonators system), in which there is zero transmission at the reflection port and thus at any one time, only slow light can be produced at the through port if ω that corresponds to the CRIT peak is chosen. The ability to simultaneously generate fast and slow light will make our proposed scheme highly attractive for fast and slow light applications.

8.3.2. Coupling Regimes

In any resonator-based system, three types of coupling regimes are possible. They are critical coupling, which gives $T_T = 0$, and under-coupling and over-coupling, both of which give $0 < T_T < 1$. In this subsection, we will demonstrate that the presence of intracavity backscattering (i.e., $u_m \neq 0$) in the modified twin-coupled TWMRs system can alter the effective loss rate of the resonator system, which consequently changes the conditions to realize the various coupling regimes for the split modes $\omega_{sps\pm}$ that sandwich the CRIT peaks (for example in Figs. 8.3b(ii) and 8.3c(ii)) in comparison to those of the traditional twin-coupled TWMRs system (for example in Fig. 8.3a(ii)) where $u_m = 0$. For completeness, the conditions to realize the various coupling regimes at $\Delta\omega_0 = 0$ and the split modes $\omega_{sps\pm}$ at $\Delta\omega_0 = \pm u_m$ when CRIT is absent (for example in Figs. 8.3b(i) and 8.3c(i)) will also be shown.

Let us start with the case of $u_m = 0$. The condition for critical coupling at $\Delta\omega_0 = 0$ can be derived by letting $\zeta_T = 0$ in Eq. (8.3). It is found that critical coupling at $\Delta\omega_0 = 0$ occurs if

$$\Gamma_{ext} = u_1^2 / \Gamma_{int, 2} + \Gamma_{int, 1}, \quad (8.7)$$

in which $u_1^2 / \Gamma_{s, 2} + \Gamma_{int, 1}$ is the net effective loss rate of the twin-coupled TWMRs system with $u_m = 0$. Consequently, at $\Delta\omega_0 = 0$ and for $u_m = 0$, under-coupling will occur if $u_1^2 / \Gamma_{s, 2} + \Gamma_{int, 1} > \Gamma_{ext}$ while over-coupling will occur if $u_1^2 / \Gamma_{s, 2} + \Gamma_{int, 1} < \Gamma_{ext}$. On the other hand, the condition for critical coupling at the split modes $\omega_{sps\pm}$ for $u_m = 0$ can be obtained by letting $\Delta\omega_0 \approx \pm(u_1^2 - \Gamma_{s, 2}^2)^{1/2}$ and $\zeta_T = 0$ in Eq. (8.3), which gives

$$\Gamma_{ext} = \Gamma_{int, 1} + \Gamma_{int, 2}, \quad (8.8)$$

while under-coupling and over-coupling, respectively, occur at $\Gamma_{int, 1} + \Gamma_{int, 2} > \Gamma_{ext}$ and $\Gamma_{int, 1} + \Gamma_{int, 2} < \Gamma_{ext}$, in which the term $\Gamma_{int, 1} + \Gamma_{int, 2}$ is the total intrinsic losses of the two resonators. Earlier in Fig. 8.3(a), we have used $\Gamma_{int, 1} + \Gamma_{int, 2} = \Gamma_{ext}$ and thus CRIT with critical coupling (i.e., $T_T(\omega_{sps\pm}) = 0$) at the split modes $\omega_{sps\pm}$, which is due to evanescent coupling between the resonators, is achieved in Fig. 8.3(a(ii)). We will term CRIT with critical coupling at the split modes simply as critically coupled CRIT in this work.

In the presence of intracavity backscattering (i.e., $u_m \neq 0$) in the twin-coupled TWMRs system, new split modes $\omega_{sps\pm}$ are formed. By letting $\Delta\omega_0 = \pm u_m$ and $\zeta_T = 0$ in Eq. (8.3), it is found that critical coupling at $\omega_{sps\pm}$ is achieved only if

$$\Gamma_{ext} = \Gamma_{eff, sps} = \left\{ \frac{\Gamma_{int, 2} \left(u_1^2 / \Gamma_{int, 2} + \Gamma_{int, 1} \right) \left[u_1^2 / \Gamma_{int, 2} + \Gamma_{int, 1} - 2u_m (2u_m + j\Gamma_{int, 1} + j\Gamma_{int, 2}) / \Gamma_{int, 2} \right]}{(\Gamma_{int, 2} - 2ju_m)} \right\}^{1/2}, \quad (8.9)$$

while under-coupling and over-coupling, respectively, occur at $\Gamma_{eff, sps} > \Gamma_{ext}$ and $\Gamma_{eff, sps} < \Gamma_{ext}$, in which the term $\Gamma_{eff, sps}$ is the net effective loss rate of the twin-coupled TWMRs system at $\omega_{sps\pm}$ in the

presence of intracavity backscattering. For a sufficiently small u_m , the split mode $\omega_{sps\pm}$ cannot be resolved in the transmission spectrum. Then $\omega_{sps\pm} \approx \omega_0$, i.e., a single Lorentzian forms at ω_0 in the transmission spectrum. In such a situation, the critical coupling condition $\Gamma_{ext} = \Gamma_{eff, sps}$ reduces to $\Gamma_{ext} \approx u_m^2/\Gamma_{s, 2} + \Gamma_{int, 1}$, which is the critical coupling condition of the twin-coupled TWMRs system with $u_m = 0$ in Eq. (8.7). Finally, the critical-coupling condition at the split modes $\omega_{sps\pm 1}$ and $\omega_{sps\pm 2}$, which result from the splitting of $\omega_{sps\pm}$ due to evanescent coupling between the two resonators, can be derived by letting $\Delta\omega_0 = \pm u_m \pm (u_1^2 - \Gamma_{s, 2}^2)^{1/2}$ and $\xi_T = 0$ in Eq. (8.3):

$$\Gamma_{ext} = \Gamma_{eff, spc} = \left\{ \left(\Gamma_{int, 1} + \Gamma_{int, 2} \right) \left[u_1^2 + 2\Gamma_{int, 2} (ju_m - \Gamma_{int, 2}) + 2B_2 (j\Gamma_{int, 2} + u_m) \right] \times \right. \\ \left. \left[u_1^2 (\Gamma_{int, 1} + \Gamma_{int, 2} - 4ju_m) - 2(\Gamma_{int, 1} + \Gamma_{int, 2} - 2ju_m)(jB_2 - \Gamma_{int, 2})(ju_m - \Gamma_{int, 2}) \right] \times \right. \\ \left. \left[u_1^2 + 2\Gamma_{int, 2} (jB_2 - \Gamma_{int, 2}) + 2u_m (j\Gamma_{int, 2} + B_2) \right]^{-2} \right\}^{1/2}, \quad (8.10)$$

where $B_2 = (u_1^2 - \Gamma_{s, 2}^2)^{1/2}$, while $\Gamma_{eff, spc}$ is the net effective loss rate of the twin-coupled TWMRs system at $\omega_{sps\pm 1}$ and $\omega_{sps\pm 2}$ in the presence of intracavity backscattering. Under-coupling and over-coupling, respectively, occur at $\Gamma_{eff, spc} > \Gamma_{ext}$ and $\Gamma_{eff, spc} < \Gamma_{ext}$. For our example earlier in Fig. 8.3c(ii) in which there is intracavity backscattering, the critical coupling condition in Eq. (8.10) is not fulfilled at the split modes $\omega_{sps\pm}$ that sandwich each CRIT peak, resulting in $T_T(\omega_{sps\pm}) > 0$. Later on, we will show that a critically-coupled CRIT (i.e., $T_T(\omega_{sps\pm}) = 0$) can be generated by employing dual inputs to the twin-coupled TWMRs. As mentioned earlier, for a sufficiently small u_m , the intracavity backscattering-induced split modes $\omega_{sps\pm}$ cannot be clearly resolved in the transmission spectrum. This will change the frequencies of the inter-resonator coupling-induced split modes $\omega_{sps\pm 1}$ and $\omega_{sps\pm 2}$ from $\omega = \omega_0 \pm u_m \pm (u_1^2 - \Gamma_{s, 2}^2)^{1/2}$ to $\omega = \omega_0 \pm (u_1^2 - \Gamma_{s, 2}^2)^{1/2}$. Then the critical coupling condition $\Gamma_{ext} = \Gamma_{eff, spc}$ will reduce to $\Gamma_{ext} = \Gamma_{int, 1} + \Gamma_{int, 2}$, which is the critical coupling condition of the twin-coupled TWMRs system with $u_m = 0$ in Eq. (8.8). Clearly, it can be seen that the presence of intracavity backscattering alter the conditions to realize the various coupling regimes at the different split modes that sandwich the CRIT peaks as well as at $\Delta\omega_0 = 0$ or at the location $\Delta\omega_0 = \pm u_m$, in which CRIT peaks can be formed.

8.4. Generating Optical Fano Resonance (OFR) Using Proposed Scheme

Thus far, our discussions are based on resonators with equal intracavity backscattering strength (i.e., $u_{m, 1} = u_{m, 2} = u_m$) and with a single input. As seen in the last section, for such a configuration, if $\Gamma_{s, 2} \approx 0$ and Q_{cou} is sufficiently large, we can either have twin CRIT peaks (each, respectively, located at $\omega = \omega_0 + u_m$ and $\omega = \omega_0 - u_m$) when the backscattering magnitude is large ($Q_{mut} \ll Q_{cri}$), or a single CRIT peak at $\omega = \omega_0$ when the backscattering magnitude is small ($Q_{mut} \gg Q_{cri}$). In this section, we will show that if the resonators have unequal intracavity strength (i.e., $u_{m, 1} \neq u_{m, 2}$) and with a single input, the symmetric

CRIT sharp peaks in the last section will be reshaped into sharp, asymmetric lineshapes, which is known as optical Fano resonances (OFRs). Do note that in this section, we use resonators with smaller Q_{cou} and Q_{ext} than those in the previous sections so that the asymmetric OFR lineshapes can extend over a much wider transmission bandwidth.

We now explain the principle of using our modified twin-coupled TWMRs structure to generate asymmetric OFR lineshapes in the transmission spectrum when $u_{m,1} \neq u_{m,2}$, which resemble the Fano resonances (FRs) in atomic systems that involve quantum interference between a continuum and a discrete energy level [14]. We have earlier mentioned that the presence of intracavity backscattering results in the resonance mode ω_1 of resonator 1 to split into eignemodes ω_{sps+}^1 and ω_{sps-}^1 and the resonance mode ω_2 of resonator 2 to split into eignemodes ω_{sps+}^2 and ω_{sps-}^2 , where $\omega_1 = \omega_2 = \omega_0$ in this work and the split modes $\omega_{sps\pm}$ have frequencies $\omega = \omega_0 \pm u_{m,n}$. In this section, $u_{m,1} \neq u_{m,2}$. Then the split modes $\omega_{sps\pm}$ in the two resonators have different resonance frequencies: ω_{sps+}^1 and ω_{sps-}^1 have $\omega = \omega_0 \pm u_{m,1}$ while ω_{sps+}^2 and ω_{sps-}^2 have $\omega = \omega_0 \pm u_{m,2}$. In such a scenario, if the previous conditions used to achieve CRIT when $u_{m,1} = u_{m,2}$ are used, the Lorentzian resonance fields of the split modes $\omega_{sps\pm}^2$ from resonator 2 will interfere with the coherent background (but not with $\omega_{sps\pm}^1$, unlike the CRIT case) that is generated by resonator 2 to form sharp asymmetric OFR lineshapes. The shape and location of the OFR will depend on the absolute difference between $Q_{mut,1}$ and $Q_{mut,2}$ and also on whether $Q_{mut,1} < Q_{mut,2}$ or $Q_{mut,1} > Q_{mut,2}$. These are illustrated in Figs. 8.6 and 8.7.

In Fig. 8.6a(i), a twin CRIT peak is first observed within the given frequency window in the transmission T_T spectrum of the through port while in Fig. 8.6b(i), a twin Lorentzian dip is found at the transmission T_R spectrum of the reflection port. However, once we move into the regime of $u_{m,1} \neq u_{m,2}$ in Fig. 8.6a(ii) to Fig. 8.6a(iv), each CRIT peak of the through port is reshaped into two sharp, asymmetric lineshapes. This also applies to the Lorentzian dips of the reflection port in Fig. 8.6b(ii) to Fig. 8.6b(iv). The unique feature of such sharp and asymmetric lineshapes, which is known otherwise as OFR, is that the transmission falls abruptly over a narrow frequency range, with the frequency range being much narrower than the full width of the resonance itself. In particular, the OFR in the transmission T_T spectra allows tuning between near-zero to near unity transmission over a narrow frequency range. The shape of the OFR changes but the location of the asymmetric lineshapes in the transmission spectra is fixed as $Q_{mut,1}$ is increased progressively in Fig. 8.6a and 8.6b, while keeping $Q_{mut,2}$ fixed (note that $Q_{mut,n} = \omega_0/(2u_{m,n})$). On the other hand, if $Q_{mut,2}$ is increased progressively while keeping $Q_{mut,1}$ fixed, which occurs in Figs. 8.6(c) and 8.6(d), the change in the shapes of the OFRs is accompanied by a frequency or wavelength shift of the resonance spectra. To explain these, first note that the two asymmetric lineshapes correspond to the reshaped Lorentzian fields of the split modes $\omega_{sps\pm}^2$ (from resonator 2), which has resonance frequency $\omega = \omega_0 \pm u_{m,2}$. Thus, by adjusting $u_{m,2}$ or $Q_{mut,2}$, the location of each asymmetric lineshape or OFR in the transmission spectrum is also tuned. On the other hand, the broad Lorentzians that are found near the sharp and asymmetric OFR in the

Fig. 8.6. The reshaping of the symmetric CRIT peaks in the transmission T_T at the through port (first column) and the Lorentzian dips in the transmission T_R at the reflection port (second column) of the twin-coupled MRs with $Q_{int,1} = 5.0e4$, $Q_{int,2} = 6.0e8$, $Q_{ext} = 4.0e2$ and $Q_{cou} = 1.2e3$. In (a) and (b), $Q_{mut,1}$ is adjusted, while fixing $Q_{mut,2}$ at $1.0e2$. Different asymmetric Fano-like lineshapes are produced, with the spectral location of each Fano-like lineshape fixed in this case in (a) and (b). On the other hand, in (c) and (d), $Q_{mut,2}$ is adjusted, while fixing $Q_{mut,1}$ at $1.0e2$. Different asymmetric Fano-like lineshapes are produced like those in (a) and (b). However, the spectral location of each Fano-like lineshape changes with $Q_{mut,2}$ in (c) and (d).

transmission spectra corresponds to the split modes $\omega_{sps\pm}^1$ (from resonator 1), which has resonance frequency $\omega = \omega_0 \pm u_{m,1}$. Adjusting $u_{m,1}$ or $Q_{mut,1}$ determines the splitting between ω_{sps+}^1 and ω_{sps-}^1 . Too small a value in $Q_{mut,1}$ will result in ω_{sps+}^1 and ω_{sps-}^1 to be indistinguishable and thus only a single Lorentzian can be found in the transmission spectra (cf. Fig. 8.6a(iv) and Fig. 8.6b(iv)), while a sufficiently large $Q_{mut,1}$ clearly resolves ω_{sps+}^1 and ω_{sps-}^1 into two Lorentzians in the transmission spectra (cf. Fig. 8.6a(ii), Fig. 8.6a(iii), Fig. 8.6b(ii) and Fig. 8.6b(iii)). Such control of the nature of the mode splitting of $\omega_{sps\pm}^1$ via $Q_{mut,1}$, together with the change in the resonance frequency of $\omega_{sps\pm}^2$ via $Q_{mut,2}$, leads to a tuning of the spectral shapes and locations of the asymmetric lineshapes of the OFRs in Figs. 8.6(c) and 8.6(d).

In conclusion, our proposed scheme is also capable of generating OFR with asymmetric lineshapes. By using two co-resonant resonators with unequal intracavity backscattering strength ($u_{m,1} \neq u_{m,2}$), the symmetric CRIT sharp peaks discussed in the earlier section (i.e., section 8.3) can be reshaped into OFR lineshapes. This proposed method of generating OFR is an alternative to conventional methods in the literature, which typically involves the introduction of gain into the resonators [11] or the phase tuning of the resonators [12], when the traditional twin-coupled TWMRs (with $u_{m,1} = u_{m,2} = 0$) scheme is used to generate OFR. Our proposed modified twin-coupled TWMRs can thus also be utilized for optical switching applications as it is well known that resonances with asymmetric lineshapes will greatly enhance the switching sensitivity and output extinction ratio [9].

8.5. Optically Tunable CRIT and OFR using Proposed Scheme

We have seen that a twin-CRIT peak can be formed in the transmission T_T spectrum of the through port for the case of $u_{m,1} = u_{m,2}$ in section 8.3.1 and that the twin-CRIT peak can be reshaped into asymmetric OFR lineshapes in section 8.4 when $u_{m,1} \neq u_{m,2}$. We can thus choose between CRIT and OFR simply by adjusting the magnitude of the intracavity backscattering $u_{m,1}$ and $u_{m,2}$ during the fabrication process. However, limitation to control the CRIT and OFR will arise after fabrication when $u_{m,1}$ and $u_{m,2}$ are generally fixed. To overcome this, active tuning can be used to adjust the net gain or phase shift of the modes in the cavities. These have been conventionally employed to tune the CRIT and OFR features [11, 12] in the transmission spectrum. Though effective, the main disadvantage of such method is that gain or phase tuning elements like semiconductor optical amplifiers and micro-heaters must be incorporated into the resonators during fabrication, which would increase the complexity and cost of the fabrication process. A legitimate question then is: Is it possible to *actively* switch between OFR and CRIT and to adjust the shape of the lineshape, given a fixed $u_{m,1}$ and $u_{m,2}$ after fabrication and in the absence of gain and/or phase detuning elements in the resonators? Building upon our work in chapter 7, we will demonstrate in this section that by employing dual inputs in the proposed modified twin-coupled TWMRs system, it is possible to switch between OFR and CRIT simply by controlling the phase and/or amplitude differences between the dual inputs.

We shown in Fig. 8.7 the evolution of the transmission responses of the through and reflection ports as the phase θ difference between the dual inputs — S'_{+1} and $S'_{+2} = MS'_{+1} = [A \exp(j\theta)]S'_{+1}$ — is adjusted for our proposed modified twin-coupled TWMRs structure in Fig. 8.2. In order for the dual-input scheme to achieve switching between CRIT and OFR, the split modes $\omega_{sps\pm}$ and $\omega_{spc\pm}$ must be clearly distinguishable in the transmission spectrum, which requires $Q_{mut} \ll Q_{cri}$ and Q_{cou} to be sufficiently large, as mentioned earlier in section 8.2. To achieve this, we use $Q_{mut,1} = Q_{mut,2} = Q_{mut} = 4.1e3$ with $Q_{cou} = 400$, $Q_{ext} = 900$, $Q_{int,1} = 4000$ and $Q_{int,2} = 900$. Do note that the configuration with $Q_{mut,1} \neq Q_{mut,2}$ can also be employed for the dual-input scheme as long as $Q_{mut} \ll Q_{cri}$. It can be observed from Fig. 8.7 that as θ changes, the transmission spectra of the through and reflection ports transform from symmetric CRIT Lorentzian peaks to asymmetric FR-like lineshapes. Let us now take a closer look at such a transformation.

Initially, when there is only a single input ($A = 0$), a twin-CRIT peak is observed in the transmission T_T spectrum for the through in Fig. 8.8a(i), while a twin-Lorentzian peak is observed in the transmission T_R spectrum for the through in Fig. 8.8b(i). Upon the application of dual inputs with $A = 1$, $\theta = \pi$ in Fig. 8.7a(ii) and 8.7b(ii), there are three noticeable changes: (i) The right CRIT peak

Fig. 8.7. The evolution of the transmission T_T spectrum at the through port (first column) and the transmission T_R spectrum at the reflection port (second column) of the twin-coupled TWMRs with $Q_{int,1} = Q_{int,2} = 6e4$, $Q_{ext} = 5e4$, $Q_{cou} = 1.5e5$ and $Q_{mut,1} = Q_{mut,2} = Q_{mut} = 4.1e3$ as dual inputs are employed to interact with the intracavity backscattering. Tuning the phase difference θ between the dual inputs (while keeping the amplitude difference A fixed) allows active switching between CRIT peaks and sharp asymmetric lineshapes. All the spectra can be obtained using silicon resonators, each with a radius of $5 \mu\text{m}$.

at the longer wavelength region (i.e., at $\omega = \omega_0 - u_m$) in the T_T spectrum vanishes, resulting in only 1 CRIT peak found at the shorter wavelength region (i.e., at $\omega = \omega_0 + u_m$); (ii) the split modes ω_{sps+}^2 and

ω_{sps-}^2 that sandwich the surviving CRIT peak is now critically coupled ($T_T = 0$ for $\omega_{sps\pm}^2$) as compared to the case of a single input in Fig. 8.3c(ii) in which ω_{sps+}^2 and ω_{sps-}^2 are not critically coupled. However, the transmission of the CRIT peak is slightly reduced from $T_T = 1$ to $T_T = 0.98$; and (iii) the transmission T_R spectrum of the reflection port is now identical to that of the through port. The case of $\theta = \pi$ that we have just analyzed can be generalized as $\theta = p\pi$, where p is an odd integer. In other words, using dual inputs with $A = 1$, $\theta = 3\pi$, 5π or 7π will achieve similar effects as that of $\theta = \pi$. For even integers of p , such as $\theta = 0$, 2π or 4π , as shown in Fig. 8.7 a(vi) and Fig. 8.7b(vi), the above-mentioned features in points (i) to (iii) can also be found in the transmission spectra of both the through and reflection ports except that in these cases, it is the CRIT peak at the shorter wavelength region, instead of that at the longer wavelength region, that vanishes in the transmission spectrum, resulting in only one CRIT peak found at the longer wavelength region (i.e., at $\omega = \omega_0 - u_m$).

As the phase modulation between the dual inputs is adjusted (with $A = 1$) such that $\theta = s\pi$, where s is a non-integer, with $p < s < (p + 1)$, the original symmetric lineshape of the single CRIT peak (at $\omega = \omega_0 - u_m$ if p is even and at $\omega = \omega_0 + u_m$ if p is odd) is reshaped into an asymmetric lineshape for both the through and reflection ports. This can be seen in Fig. 8.7a(iii) and Fig. 8.7b(iii) for $\theta = 1.2\pi$, in which the reshaped CRIT peak is found at $\omega = \omega_0 + u_m$. In addition to this reshaping of the CRIT peak, an asymmetric lineshape appears within the frequency window (at $\omega = \omega_0 + u_m$ if p is even and at $\omega = \omega_0 - u_m$ if p is odd, which is originally the location of the vanished CRIT peak when $\theta = p\pi$) adjacent to the reshaped CRIT peak. This can be found at $\omega = \omega_0 - u_m$ in the transmission spectrum of the through and drop ports in Fig. 8.7a(iii) and 8.8b(iii) for $\theta = 1.2\pi$. The degree of asymmetry of each lineshape in the transmission spectra can be tuned by adjusting θ . Such tuning of the asymmetry of the lineshapes via θ will also result in the asymmetric lineshapes, which are originally dissimilar, to be transformed into two identical asymmetric lineshapes that are symmetrical about $\omega = \omega_0$ at $\theta = (p + 1/2)\pi$ or $s = p + 1/2$ (where p is an integer) in each transmission spectrum. This is illustrated in Fig. 8.7a(iv) and Fig. 8.7b(iv) for $\theta = 1.5\pi$. In addition, notice that for all cases of $\theta = s\pi$ in Fig. 8.8, where s is a non-integer, the two asymmetric lineshapes (centered at $\omega = \omega_0 \pm u_m$) found at the through port exhibit a mirror symmetry to that found at the reflection port.

Besides using θ to tune the shapes of the resonances in the transmission spectrum, one can also adjust A to achieve a similar effect, which is not illustrated here. Thus, amplitude modulation between the dual inputs can be used to provide an additional degree of freedom to tune the shapes of the resonances. Finally, we mention that to further tune the asymmetry or the modulation depth MD (i.e., $MD = |T_{max} - T_{min}|$, where T_{max} and T_{min} are, respectively, the maximum and minimum points of the transmission spectrum) of the line-widths of the OFR, the evanescent coupling strengths u_0 and u_1 can be adjusted, either during the fabrication process or by using tunable couplers.

In conclusion, for this section, we have shown that the resonance lineshapes in our proposed modified twin-coupled resonators can be tuned between CRIT and OFR simply by adjusting the phase

difference between the contra-propagating dual inputs of the resonator circuit. This also means that our proposed scheme can generate both CRIT and OFR on a single circuit if dual inputs are used. Note that for the case of the traditional twin-coupled resonators (i.e., intracavity backscattering is not utilized) in the literature, either active gain [11] or active phase tuning [12] is used to switch the lineshape of the resonance spectrum between CRIT and OFR. However, as mentioned in chapter 7, such use of active gain or phase tuning requires the incorporation of active elements in the cavities, which will often complicate the fabrication process. Also, as the active components of such conventional active tuning scheme are implanted in the cavity system, it would be difficult to reconfigure these active components or to replace them after fabrication. In the case in which a passive cavity has already been fabricated, it might not be feasible or possible in certain situations to do additional alternation to the device so as to render it active. Our proposed scheme thus offers a better alternative to switch between CRIT and OFR as only passive cavities with external dual inputs (which are typically laser diodes as well as phase modulators to tune the phase and/or amplitude differences of the inputs) are needed to realize such functionality. This will reduce the fabrication complexity of the cavity as well as improving the reconfigurability and serviceability of the cavity system as the external active tuning components (laser diodes and phase modulators) can be easily adjusted or replaced.

8.6. Summary and Significant Contributions of this Research Work

We have theoretically explored the feasibility of harnessing intracavity backscattering to generate enhanced tunable CRIT Lorentzian sharp peaks of high transparency and sharp asymmetric OFR lineshapes in a twin-coupled TWMRs structure. This is contrary to the popular assumption that backscattering usually brings about deleterious effects. To clearly illustrate this, we have summarized our research results in Table 8.1, highlighting the improved CRIT and OFR features of our proposed modified twin-coupled TWMRs scheme that exploits intracavity backscattering, as compared to the conventional twin-coupled TWMRs system that does not utilize intracavity backscattering. Our proposed scheme makes it possible to simultaneously produce fast and slow light effects (one at each output port) in a single device. For example, the CRIT peak, which is accompanied by slow light, can be generated at the through port while the notch filter (i.e., Lorentzian dip) response, which is accompanied by fast light, can be simultaneously generated at the reflection port of our proposed device. This is in contrast to the case of $u_m = 0$ (i.e., traditional twin-coupled resonators system), in which there is zero transmission at the reflection port and thus at any one time, only slow light can be produced at the through port at the CRIT peaks. Such ability to simultaneously generate fast and slow light from a single device will also make our proposed device highly attractive for fast and slow light applications. However, do note that the CRIT peaks, which have large group delay, have small usable resonance bandwidth. This will severely limit fast and slow light applications in transmission

systems that utilize ultra-short pulses. To increase the bandwidth, our proposed scheme can be implemented on a cascade of N -unit cells ($N = 1, 2, 3, 4, \dots$) [25], where each cell is formed by our proposed modified twin-coupled TWMRs structure in Fig. 8.2. Another method to mitigate the DBP constraint would be the use of dynamic tuning mechanism [26] in which the dispersion relation of the system is tuned in an adiabatic manner as a function of time after the pulse has entered the system.

It is also shown that by tailoring u_m in the two resonators in our proposed modified twin-coupled TWMRs structure, the CRIT peaks can be reshaped into sharp asymmetric OFR lineshapes. We can

Table 8.1. Comparisons of the CRIT and OFR features between the conventional twin-coupled traveling wave microresonators (TWMRs) in the literature and our proposed modified twin-coupled TWMRs

	Conventional twin-coupled TWMRs (intracavity backscattering is not utilized)	Our proposed modified twin-coupled TWMRs (intracavity backscattering is utilized)
Main features of Coupled- resonator- induced transparency (CRIT)	<ul style="list-style-type: none"> • Only one CRIT peak, which has high transmission and slow light, is possible at the through port for every FSR. • No output at the reflection port. • Either fast or slow light can be generated at any one time from the device. 	<ul style="list-style-type: none"> • Two CRIT peaks, both of which produce high transmission and slow light, are possible at the through port for every FSR. • Output spectra with a Lorentzian dip is produced at the reflection port. • Both slow and fast light can be simultaneously generated from the device, one at each output port, at the various resonant modes.
Main Feature of Optical Fano resonance (OFR)	Active gain or active phase tuning is needed to generate OFR.	Two resonators with different intracavity backscattering strengths can be used to generate OFR.
Main Feature of Active Switching between CRIT and OFR	Active gain or active phase tuning is needed to switch between CRIT and OFR.	Dual inputs with phase modulation can be used to switch between CRIT and OFR.

thus choose between CRIT and OFR by an appropriate choice of u_m during the fabrication process. This method of generating OFR is an alternative to the traditional twin-coupled TWMRs with no backscattering, in which either gain is introduced to the resonators [11] or by phase tuning [12] in

order to generate OFR. Our proposed modified twin-coupled TWMRs can thus also be utilized for optical switching applications in communication systems as it is well known that resonance with an asymmetric lineshape has enhanced switching sensitivity and output extinction ratio [9].

In addition, we have also shown that if dual inputs are employed to interact with the intracavity backscattering in the resonators, the CRIT symmetric peaks of our proposed modified twin-coupled TWMRs systems can be reshaped into asymmetric OFR lineshapes simply by controlling the phase and/or amplitude differences between the inputs. Active switching between CRIT and OFR can then be achieved simply by controlling the dual inputs. This is a highly attractive and useful scheme that can be deployed after the fabrication process when the physical parameters are fixed and in the absence of active gain and/or phase tuning elements in the resonators. Note that conventional twin-coupled TWMRs scheme uses either gain [11] or phase tuning [12] to switch between CRIT and OFR. As mentioned in section 7.1 and 8.1, the main drawback of such gain or phase detuning schemes in the cavities, such as [11] and [12], is that additional procedures and controls are required in the fabrication process so as to incorporate the active tuning components. Moreover, as the active components are implanted in the cavity system, it would be difficult to reconfigure or to replace them after fabrication. In the case in which a passive cavity has already been fabricated, it might not be feasible or possible in certain situations to do additional alternation to the device so as to render it active. By using our proposed dual-input scheme, such constraints are eliminated as only external dual inputs (laser diodes and phase modulators) are required to achieve tuning between CRIT and OFR. In other words, a passive and linear cavity can be used, without the need to implant active components within the cavity. This reduces the fabrication complexity and footprint of the cavity. The reconfigurability and serviceability of the proposed cavity system (in terms of its active tunability) are also improved as the external active tuning components can be easily adjusted or replaced.

The above-mentioned points strongly demonstrate that intracavity backscattering and dual inputs can be harnessed for enhanced tunable CRIT and OFR effects in the twin-coupled TWMRs structure. Such usage of the intracavity backscattering and dual inputs to generate CRIT and OFR in the twin-coupled TWMRs structure has previously been overlooked. This work, thus, represents a significant contribution to the field of microresonators research regarding CRIT and OFR as well as slow and fast light (as CRIT and OFR can be regarded as slow and fast light phenomena). Specific CRIT and OFR applications, such as tunable delay line and optical switching, based on our proposed modified twin-coupled TWMRs will be the subject of our future work.

Finally, note that this research work has been submitted to the Journal of Optical Society B.

8.7. References for Chapter

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Chapter 9

Tunable Flat-Band Slow Light via Contra-Propagating Cavity Modes in Twin-Coupled Resonators

9.1. Introduction: Research Motivations and Objectives

Paradoxically, slow light (SL) holds the key to increase the speed of optical communications [1, 2] as SL offers the abilities in delaying and coherently stopping and storing data trains [3], which are needed for enhanced performances in optical networks. In general, the most simple and compact scheme to generate SL in traveling wave microresonators (TWMRs) can be accomplished by using a single unit cell of N' -TWMRs in one of two ways (where $N' = 1$ or 2): (1) Either by resonance enhancement — the group delay t_g is increased by forcing the light to circulate many times either in a single TWMR that is coupled to one bus waveguide (WG), i.e., 1-cavity-1-bus configuration [4, 5], or a single TWMR that is coupled to two bus WGs, i.e., 1-cavity-2-buses configuration [6, 7], which is also used earlier in chapter 7; or (2) by optically mimicking the atomic concept of electromagnetically induced transparency (EIT) [8, 9] — this is known as coupled-resonator-induced transparency (CRIT) and is typically realized by coupling two TWMRs [10-13] such that the coupled TWMRs behave like atoms with EIT settings, which then generate a large t_g with high transmission at the through port. Alternatively, one TWMR can also be used to achieve the same CRIT effects of a two-coupled TWMRs system simply by simultaneously exciting two modes in the same resonator [14]. We have earlier employed the scheme of resonance enhancement in chapters 6 and 7 while the scheme of CRIT was used in chapter 8. Both of these schemes were used in conjunction with intracavity backscattering and dual contra-propagating inputs, which have shown to improve the slow and fast light performances. However, a fundamental limitation in any SL schemes that uses a single unit cell is the inherent small usable resonance bandwidth Δf_u . To increase Δf_u as well as the group delay t_g for the above-mentioned schemes of resonance enhancement and CRIT in the literature as well as for our work in chapters 6 to 8, a cascade of N -unit cells ($N = 1, 2, 3, 4, \dots$) can be used, where each unit cell consists of N' -resonators ($N' = 1, 2, 3, 4, \dots$) [15-21]. However, this approach will consume additional estate space on the chip and also does not improve Δf_u in an effective manner, as we will show

subsequently. Improving Δf_u in a manner that does not degrade the system performance and without consuming much estate space on the photonic circuit will be the focus of this chapter.

A practical SL system should ideally have: (1) A large t_g enhancement; and (2) flat or minimal dispersion in the t_g and transmission spectra (preferably with transmission ≈ 1) over a large usable bandwidth Δf_u . One fundamental figure of merit to quantify criteria (1) and (2) is the delay-bandwidth product (DBP), defined as $\text{DBP} = t_{gm} \times \Delta f_u \approx \text{constant}$ [15], where t_{gm} is the maximum group delay at resonance. A SL system that fulfills the above criteria (1) and (2) would allow an input pulse to be delayed many times its temporal width, without significant distortion and over a large Δf_u or in other words, a large DBP. For the earlier mentioned TWMRs based SL schemes of resonance enhancement and CRIT in a single unit cell, the DBPs are inherently small – the DBPs of the resonance enhancement scheme are computed to be $\sim 2/\pi$ and $\sim 1/(2\pi)$ for the 1-cavity-1-bus and 1-cavity-2-buses configurations, respectively, while that of the CRIT scheme with a two coupled TWMRs is $\sim 1/\pi$. Thus, the 1-cavity-1-bus configuration has the largest DBP among the single unit cell SL systems based on TWMR. Note that these figures of DBP are computed using the full width at half maximum (FWHM) as Δf_u . In reality, to minimize higher-order dispersion, $\Delta f_u \ll \text{FWHM}$. This implies a much lower DBP than the above computed figures. Though cascading N -unit cells of N' -TWMRs together would increase t_g by $\sim (N \times N')$ -times, the DBP remains approximately the same as that of a single unit cell, which has a low DBP (as mentioned above), due to the formation of $(N \times N')$ -ripples in the resonance spectra. This reduces Δf_u by $1/(N \times N')$, besides decreasing the transmission. Note that N ($= 1, 2, 3, \dots$) is the number of unit cells, while N' ($= 1, 2, 3, \dots$) is the number of resonators in each unit cell. A low DBP means that a large t_g results in a small Δf_u . In other words, for large t_g , the input pulse must be made adequately long in time. This will inevitably restrict the use of the SL media in transmission systems with ultra-short pulses.

To mitigate the DBP constraint, one could flatten the above-mentioned ripples in the resonance spectra of the coupled TWMRs, either by apodisation of the coupling coefficients between the resonators [22] or by using the WG losses to smoothen the ripples [17]. However, it might be difficult to accurately control the coupling coefficients in actual fabrication for the former approach, while for the latter approach, there is a trade-off of reduced transmission and t_{gm} . Alternatively, the DBP (and thus Δf_u) can be expanded by using dynamic tuning [23], in which the dispersion relation of the system is tuned in an adiabatic manner as a function of time after the pulse has entered the system. However, it is generally much easier and cheaper to implement a SL system in a static setting, i.e., without using dynamic tuning, which will be the objective of this work.

In this chapter, we propose using another type of twin-coupled TWMRs structure, as shown in Fig. 9.1 (in which both the resonators are coupled to the bus waveguide, in contrast to chapter 8, whereby only one resonator is coupled to the bus waveguide) to realize a SL system that not only has large t_g and high transmission but the t_g and transmission are fairly constant over a wide bandwidth

Table 9.1. Comparison of the delay-bandwidth products (DBPs) between the conventional traveling wave microresonator (TWMR) based slow light systems and the proposed scheme in this work

Traveling wave microresonator (TWMR)-based slow light schemes	Conventional TWMR-based systems of one or N -unit cells in literature			This work	
	Resonance enhancement-based MR systems of one or N -unit cells		<i>System C:</i> Coupled-resonator-induced transparency (CRIT)-based MR configuration Example: [10-14, 19]	Output light at through Port	Output light at reflection Port
	<i>System A:</i> N -unit-cells-1-bus configuration* Example: [4, 5, 15, 16]	<i>System B:</i> N -unit-cells-2-buses configuration† Example: [6, 7, 17, 18, 20, 21]			
#Delay-bandwidth product (DBP)	$\ll 2/\pi$	$\ll 1/(2\pi)$	$\ll 1/\pi$	$\approx 2/\pi$ (or ≈ 0.637)	$\approx 4/\pi$ (or ≈ 1.273)
	Assuming that only 1/3 of the FWHM is the usable resonance bandwidth in order to avoid higher order dispersion			This is an improvement of 3 fold , 12 fold and 6 fold as compared, respectively, to the DBPs of systems A, B and C.	This is an improvement of 6 fold , 24 fold and 12 fold as compared, respectively, to the DBPs of systems A, B and C.
	≈ 0.212	≈ 0.0531	≈ 0.106		

#DBP = maximum group delay at resonance $\times$ usable resonance bandwidth, in which the usable resonance bandwidth is taken as the region where the slow light has high transmission and minimal dispersion in the group delay and transmission. * N -unit-cells-1-bus configuration: A single resonator or a chain of N unit cells of N' resonators coupled to one bus waveguide. This is also known as all-pass filter configuration if all the cavities are lossless. † N -unit-cells-2-buses configuration: A single or a chain of N unit cells of N' resonators coupled to two bus waveguides. This is also known as add-drop filter configuration. Note that N ($= 1, 2, 3, \dots$) is the number of unit cells, while N' ($= 1, 2, 3, \dots$) is the number of resonators in each unit cell.

(i.e., flat-band). As shown in Table 9.1 (which will be elaborated later), the proposed scheme improves the DBP by 3- to 24-fold as compared to the earlier mentioned conventional TWMR based SL systems (mentioned in paragraph one), which are either of one or N -unit cells. Though the proposed configuration has previously been studied in [11, 24, 25], the analyses in these works are chiefly limited to the filtering characteristics, without a detailed study of the SL performances. It must also be mentioned that our twin-coupled TWMRs is different from that in [26], which has only one resonator being directly coupled to the bus waveguide (WG), while both the resonators are directly coupled to the bus WG in this work.

This chapter is arranged as follows. In section 9.2, we present the theoretical modeling of the proposed twin-coupled TWMRs. In section 9.3, the group delay t_g and transmission T properties of the device are looked into, in which we illustrate how flat-band SL can be achieved for both forward and backward light propagations as well as quantifying the characteristics of the flat-band SL. In section 9.4, we analyze the sensitivity of the device to propagation losses and fabrication errors. Finally, in section 9.5, we conclude and highlight the significance of this research work.

Fig. 9.1. Schematic of the twin-coupled TWMRs structure in which each resonator is directly coupled to the bus waveguide as well as to the adjacent resonator. Such coupling arrangement inherently sets up contra-propagating modes in the resonators and bus waveguide.

9.2. Theoretical Formulation

The schematic of the twin-coupled TWMRs system is shown schematically in Fig. 9.1, whereby two resonators, which are mutually coupled to each other, are both directly coupled to the port waveguide (WG) as well. This excites both clockwise (CW) and counter-clockwise (CCW) propagating modes in each resonator as well as forward and backward propagations in the bus WG, even without the presence of surface perturbations within the cavity, which was employed in our earlier work in chapters 7 and 8, or the presence of coupling-induced localized backscattering (CILB) effects, which was employed in our earlier work in chapter 6. Thus, this configuration in Fig. 9.1 provides another alternative to generate and harness the contra-propagating cavity modes. In Fig. 9.1, the electric fields of the propagating waves in each resonator are represented by a_n^x , b_n^x , c_n^x and d_n^x , in which $x = -$ ($x = +$) represents the CW (CCW) mode, while $n = 1$ ($n = 2$) denotes the leftmost (rightmost) resonator. Likewise, the electric fields in the port WG are denoted as A_n^x , B_n^x , C_n^x and D_n^x , with $x = -$ ($x = +$) representing the backward (forward) propagating mode, while $n = 1$ ($n = 2$) denotes the side of the waveguide (WG) located below the leftmost (rightmost) resonator. Note that we have assumed that the effects of perturbations at the surface of the cavity and at the coupling junctions are negligible for the configuration in Fig. 9.1. This is in contrast to our earlier work in chapters 6 to 8, whereby perturbations at the surface of the cavity and coupling junctions need to be intentionally introduced so that their related effects can be of any significance. As such, the contra-propagating cavity modes in Fig. 9.1 in this work can only indirectly couple with each other via the evanescent coupling at the coupling junctions (CJs). Using the matrix formalism [17, 22], the evanescent coupling between the cavity modes and the bus WG at CJ 2 and CJ 3 can be described as

$$\begin{bmatrix} a_1^+ & b_1^- & b_1^+ & a_1^- \end{bmatrix}^T = S_2 \begin{bmatrix} A_1^- & B_1^- & B_1^+ & A_1^+ \end{bmatrix}^T, \quad (9.1)$$

$$\begin{bmatrix} A_2^+ & B_2^+ & B_2^- & A_2^- \end{bmatrix}^T = S_3 \begin{bmatrix} a_2^+ & b_2^+ & b_2^- & a_2^- \end{bmatrix}^T, \quad (9.2)$$

while the evanescent coupling between the resonators at CJ 1 can be written as

$$\begin{bmatrix} c_2^- & d_2^- & d_2^+ & c_2^+ \end{bmatrix}^T = S_1 \begin{bmatrix} c_1^+ & d_1^+ & d_1^- & c_1^- \end{bmatrix}^T, \quad (9.3)$$

in which S_i ($i = 1, 2$ or 3) is termed as the coupling matrix that is defined as

$$S_i = \begin{pmatrix} [U_i] & 0 \\ 0 & [U_i] \end{pmatrix}, \quad \text{with} \quad U_i = \begin{pmatrix} r_i/(j\kappa_i) & -1/(j\kappa_i) \\ (\kappa_i^2 + r_i^2 + \sigma_i^2)/(j\kappa_i) & -r_i/(j\kappa_i) \end{pmatrix}, \quad (9.4)$$

where $j = \sqrt{-1}$, r_i and κ_i are, respectively, the “through” and “cross” coupling coefficients while σ_i quantifies for the coupler loss. The subscript $i = 1, 2$ or 3 represents the position of the CJs in Fig. 9.1, i.e., $i = 1, 2$ and 3 corresponds to CJ 1, CJ 2 and CJ 3, respectively. Note that r_i , κ_i and σ_i are real values and $r_i^2 + \kappa_i^2 + \sigma_i^2 = 1$. For a lossless coupler, $\sigma_i = 0$. Within the resonators, the electric fields are described as

$$\begin{bmatrix} c_1^+ & d_1^+ & d_1^- & c_1^- \end{bmatrix}^T = P_1 \begin{bmatrix} a_1^+ & b_1^- & b_1^+ & a_1^- \end{bmatrix}^T, \quad (9.5)$$

$$\begin{bmatrix} a_2^+ & b_2^+ & b_2^- & a_2^- \end{bmatrix}^T = P_2 \begin{bmatrix} c_2^- & d_2^- & d_2^+ & c_2^+ \end{bmatrix}^T, \quad (9.6)$$

in which P_n ($n = 1$ or 2) is termed as the propagation matrix that is defined as

$$P_n = \begin{pmatrix} 0 & [X_n] \\ [X_n] & 0 \end{pmatrix}, \quad \text{with} \quad X_n = \begin{pmatrix} 0 & \tau_n^{1/4} \exp(j\delta_n/4) \\ \tau_n^{-3/4} \exp(-j3\delta_n/4) & 0 \end{pmatrix}, \quad (9.7)$$

where τ_n is the round-trip amplitude transmission coefficient (lossless resonator: $\tau_n = 1$) and $\delta_n = 2\pi R_n \beta_{R,n}$ is the round-trip phase shift, in which R_n and $\beta_{R,n}$ are, respectively, the bend radius and propagating constant of the resonator, with $n = 1$ ($n = 2$) denoting the leftmost (rightmost) resonator.

The electric fields in the bus WG can be described as

$$\begin{bmatrix} A_2^+ & A_2^- \end{bmatrix}^T = P_w \begin{bmatrix} A_1^+ & A_1^- \end{bmatrix}^T, \quad \text{with} \quad P_w = \begin{pmatrix} \exp(j\beta_w L) & 0 \\ 0 & \exp(-j\beta_w L) \end{pmatrix} \quad (9.8)$$

where β_w and L are, respectively, the propagating constant of the bus WG and the separation distance between the resonators. Note that in this chapter, we will only use one input, unlike chapters 7 and 8, in which dual inputs are employed, as we have found that a second input will complicate the conditions to achieve flat-band SL. Setting $B_2^- = 0$ and using Eqs. (9.1) to (9.8), the fields propagating in the bus WG can then be described as

$$\begin{bmatrix} A_1^+ \exp(-j\beta_w L) & B_2^+ & 0 & A_1^- \exp(j\beta_w L) \end{bmatrix}^T = Y \begin{bmatrix} A_1^- & B_1^- & B_1^+ & A_1^+ \end{bmatrix}^T, \quad \text{with } Y = S_3 P_2 S_1 P_1 S_2, \quad (9.9)$$

where Y is a 4×4 matrix. From Eq. (9.9), four simultaneous equations can be obtained:

$$\begin{aligned} -Y_{13}B_1^+ &= Y_{11}A_1^- + Y_{12}B_1^- + (Y_{14} - \exp(j\beta_w L))A_1^+, & -Y_{23}B_1^+ &= Y_{21}A_1^- + Y_{22}B_1^- - B_2^+ + Y_{24}A_1^+, \\ -Y_{33}B_1^+ &= Y_{31}A_1^- + Y_{32}B_1^- + Y_{34}A_1^+, & -Y_{43}B_1^+ &= (Y_{41} - \exp(-j\beta_w L))A_1^- + Y_{42}B_1^- + Y_{44}A_1^+. \end{aligned} \quad (9.10)$$

In particular, we are interested to solve for B_2^+/B_1^+ and B_1^-/B_1^+ , which are, respectively, the complex electric field transmittivities (i.e., the ratio of the output field to the input field) at the through and reflection ports. To solve for B_2^+/B_1^+ and B_1^-/B_1^+ , we simply recast Eq. (9.10) into a matrix form and thereafter, perform the necessary matrix inversion. The final form, which is consistent with the results in [24, 25], is shown below:

$$\begin{bmatrix} B_2^+/B_1^+ \\ B_1^-/B_1^+ \\ A_1^+/B_1^+ \\ A_1^-/B_1^+ \end{bmatrix} = \begin{bmatrix} Y_{41} - \exp(-j\beta_w L) & Y_{42} & Y_{44} & 0 \\ Y_{21} & Y_{22} & Y_{24} & -1 \\ Y_{31} & Y_{32} & Y_{34} & 0 \\ Y_{11} & Y_{12} & Y_{14} - \exp(j\beta_w L) & 0 \end{bmatrix}^{-1} \begin{bmatrix} -Y_{43} \\ -Y_{23} \\ -Y_{33} \\ -Y_{13} \end{bmatrix} \quad (9.11)$$

We will term B_2^+/B_1^+ and B_1^-/B_1^+ as ξ_T and ξ_R , respectively, or collectively as ξ_q ($q = T$ or R), where the subscript T (R) denotes the through (reflection) port. Note that ξ_q contains all the information needed to characterize the SL properties: $|\xi_q|^2$ gives the transmittance or transmission T_q , while $\Phi_q = \arg(\xi_q)$ is the resonator-induced effective phase shift of the output light that determines the group delay $t_{g,q}$. For convenience of our modeling in this chapter, we use the below relation to derive $t_{g,q}$, instead of $t_{g,q} = [\partial\Delta\Phi_q(\omega)/\partial(\omega)]$ or $t_{rr}\partial\Delta\Phi_q/\partial\delta$ that is used in the previous chapters:

$$t_{g,q} = t_{rr} \frac{\partial\Phi_q}{\partial\delta} = t_{rr} \frac{1}{|\xi_q|^2} \left(\text{Re}[\xi_q] \frac{\partial \text{Im}[\xi_q]}{\partial\delta} - \text{Im}[\xi_q] \frac{\partial \text{Re}[\xi_q]}{\partial\delta} \right), \quad (9.12)$$

where $t_{rr} = 1/\text{FSR}$ is the cavity round-trip time, $\text{FSR} = c/(n_g L_c)$ is the free spectral range (in Hz), n_g is the group effective index of the cavity mode, L_c is the length of the cavity, while $\text{Re}[\]$ and $\text{Im}[\]$ denote, respectively, the real and imaginary components. Eqs. (9.11) and (9.12) will form the theoretical basis of our discussions in subsequent sections.

9.3. Generating Flat-Band Slow Light via the Proposed Scheme

9.3.1. General Operating Mechanism

To understand the physical mechanism behind the proposed twin-coupled MRs, it is instructive to first analyze a single resonator system (i.e., only resonator 1 exists in Fig. 9.1) and a twin-coupled resonators system that has only one cavity coupled to the bus WG (i.e., $\kappa_3 = 0$ in Fig. 9.1). We will assume that the losses of the resonators and couplers are negligible (i.e., $\tau_n \approx 1$ and $\sigma_n \approx 0$ or $r_i^2 + \kappa_i^2$

≈ 1) throughout this section; the effects of insertion losses are studied in section 9.4. Also, for simplicity, we will focus on the case of co-resonant cavities (i.e., $\delta_1 = \delta_2 = \delta$) in this work. This will generate flat-band slow light centred at the resonance frequency $\delta = \delta_0 = 2\pi m$ (where $m = 1, 2, 3..$) if certain physical conditions (governed by Eq. (9.19), as derived below) are met. Although a full study of the case of $\delta_1 \neq \delta_2$ is outside the scope of this work, we have found that $\delta_1 \neq \delta_2$ or detuning δ_1 and δ_2 simply changes the physical conditions to realize flat-band SL due to the Vernier effects.

For the single resonator system, the complex transmittivity ζ_T has the expression $\zeta_T = (exp(j\delta) - r_2)/(r_2 exp(j\delta) - 1)$. Note that for any resonator system, the characteristic roots of the denominator of ζ_q ($q = T$ or R) correspond to the complex eigenmodes (or complex resonance frequencies $\omega_0 = \text{Re}[\omega_0] + j\text{Im}[\omega_0]$). Each such root or eigenmode can be written as $\delta' = \delta_0 + j\Gamma$, where δ_0 and Γ are, respectively, the real-valued normalized resonance frequency and the half-resonance linewidth, while n_g is the group effective index and c is the speed of light in vacuum. In the case of the single resonator, there is only one eigenmode of the form $\delta' = \delta_0 + j\ln(1/r_2)$, where $\delta_0 = 2\pi m$. Consequently, a sharp Lorentzian resonance lineshape is formed at $\delta = 2\pi m$, as shown in the transmission and group delay spectra in Figs. 9.2(ai) and 9.2(aiii) in solid line plots, with a rapid phase shift of 2π across the resonance in Fig. 9.2(aii). When a second resonator is coupled to the single resonator such that it is not directly coupled

Fig. 9.2. The transmission, effective phase shift and group delay spectra of different resonator systems — (ai) to (aiii): (in solid curves) A single resonator system, i.e., only resonator 1 exists in Fig. 9.1, with $\kappa_2 = 0.08$, $\tau_1 = 0.9999 \approx 1$, and (in dotted curves) a twin-coupled resonators system that has only one resonator coupled to the bus waveguide, i.e., $\kappa_3 = 0$ in Fig. 9.1, and with $\kappa_1 = 0.05$, $\kappa_2 = 0.08$, $\tau = \tau_1 = \tau_2 = 0.9999 \approx 1$; (bi) to (biii): The through port case of our proposed twin-coupled resonators with $\kappa_1 = 0.05$, $\kappa = \kappa_2 = \kappa_3 = 0.08$ and $\tau = \tau_1 = \tau_2 = 1$; and (ci) to (ciii): The reflection port case of the proposed twin-coupled resonators with $\kappa_1 = 0.05$, $\kappa = \kappa_2 = \kappa_3 = 0.08$ and $\tau = \tau_1 = \tau_2 = 1$. The cavity and coupler losses are assumed to be negligible in this section.

to the bus WG (i.e., $\kappa_3 = 0$ in Fig. 9.1), ξ_T becomes $\xi_T = (\xi_2 \exp(j\delta) - r_2)/(r_2 \exp(j\delta) - 1)$, where $\xi_2 = (\exp(j\delta) - r_1)/(r_1 \exp(j\delta) - 1)$ is the loading factor due to the addition of the second resonator. This gives two characteristic roots and thus two eigenmodes, which are denoted as δ'_1 and δ'_2 , where $\delta'_1 = \delta_0 + j\Gamma_1 = \delta_0 + j\ln\{[r_1 + r_1 r_2 - (r_2^2 r_1^2 + 2r_2 r_1^2 + r_1^2 - 4r_2)^{1/2}]/(2r_2)\}$ and $\delta'_2 = \delta_0 + j\Gamma_2 = \delta_0 + j\ln\{[r_1 + r_1 r_2 + (r_2^2 r_1^2 + 2r_2 r_1^2 + r_1^2 - 4r_2)^{1/2}]/(2r_2)\}$. In the limit of $\kappa_2^2 \ll 1$, $\text{Im}[\xi_T] \approx 0$ at δ'_1 and δ'_2 , assuming that the change in $\text{Re}[\xi_T]$ is small at these resonance frequencies. Then we have $\delta'_1 \approx \delta_0 + j\ln[r_1 - (r_2^2 - 1)^{1/2}]$ and $\delta'_2 \approx \delta_0 + j\ln[r_1 + (r_2^2 - 1)^{1/2}]$, which can be approximated as

$$\delta'_1 = \delta'_0 - \sin^{-1} \kappa_1, \quad \delta'_2 = \delta'_0 + \sin^{-1} \kappa_1. \quad (9.13)$$

In other words, the real-valued normalized resonance frequency δ'_0 of the single resonator is split into δ'_1 and δ'_2 , with the mode splitting being controlled by κ_1 . As a result, the resonance spectra have the shape of a two-split Lorentzian, as shown in Figs. 9.2(ai) and 9.2(aiii) in dotted line plots, with the effective phase shift being split into two 2π swings in Fig. 9.2(aii). Finally, if both resonators are coupled to the bus WG (i.e., $\kappa_3 \neq 0$) such that one has the proposed twin-coupled resonators in Fig. 9.1, contra-propagating modes are excited in the cavities. Consequently, both forward and backward transmissions, i.e., $T_T = |\xi_T|^2$ and $T_R = |\xi_R|^2$, are observed. Note that throughout this work, the coupling junctions 2 and 3 for our system in Fig. 9.1 will have the same evanescent coupling strengths, i.e., $r = r_2 = r_3$ and $\kappa = \kappa_2 = \kappa_3$, so as to simplify the design. Using this assumption and Eq. (9.11), the complex transmittivities ξ_T and ξ_R can be expressed as

$$\xi_T = \frac{4r^2 \cos^2(\delta) + (1+r^2)^2 r_1^2 - \left[(1-r^2)^2 (1-r_1^2) + (1+r^2) 2rr_1 \right] \exp(-j\delta) - (1+r^2) 2rr_1 \exp(j\delta)}{\left[r^2 \exp(j2\delta) - 2rr_1 \exp(j\delta) + 1 \right] \exp(-3j\delta)},$$

$$\xi_R = \frac{j(1-r^2) \kappa_1 \left\{ r \exp(-j3/2\delta) - [r^2 r_1 + r_1] \exp(-j/2\delta) + r \exp(j/2\delta) \right\} [1 + \exp(j\delta)]}{\left[r^2 \exp(j2\delta) - 2rr_1 \exp(j\delta) + 1 \right] \exp(-3j\delta)}. \quad (9.14)$$

There are four characteristic roots or eigenmodes for ξ_q ($q = T$ or R) in Eq. (9.14), which are denoted as $\delta'_{1,1}$, $\delta'_{1,2}$, $\delta'_{2,1}$ and $\delta'_{2,2}$. These eigenmodes can be written as: $\delta'_{1,1} = \delta'_{2,1} = \delta_0 + j\ln\{r/[r_1 + (r_2^2 - 1)^{1/2}]\}$ and $\delta'_{1,2} = \delta'_{2,2} = \delta_0 + j\ln\{r/[r_1 - (r_2^2 - 1)^{1/2}]\}$, where $\delta'_{n,m}$ ($n, m = 1$ or 2) is of the form $\delta'_{n,m} = \delta_0 + j\Gamma_{n,m}$. In other words, there are now two repeated roots and thus two pairs of degenerate (or identical) eigenmodes. When roots are repeated, an interesting new phenomenon emerges. This is illustrated in Figs. 9.2(bi) and 9.2(ci): There are now two (four) sharp peaks and four (two) sharp dips in the transmission spectra of the through (reflection) port due to the newly formed split modes $\delta'_{1,1}$, $\delta'_{1,2}$, $\delta'_{2,1}$ and $\delta'_{2,2}$, besides the earlier mentioned split modes δ'_1 and δ'_2 that results from the splitting of δ'_0 . These spectra of Figs. 9.2(bi) and 9.2(ci) are in contrast to the dotted curve of Fig. 9.2(ai), which has only two dips (δ'_1 and δ'_2) because $\kappa_3 = 0$ in Fig. 9.1 for this case. The newly formed split modes $\delta'_{1,1}$, $\delta'_{1,2}$, $\delta'_{2,1}$ and $\delta'_{2,2}$ in Figs. 9.2(bi) and 9.2(ci) can be mathematically described as

$$\delta'_{1,1} = \delta'_1 - \Gamma_{1,1}/2, \quad \delta'_{1,2} = \delta'_1 + \Gamma_{1,2}/2, \quad \delta'_{2,1} = \delta'_2 - \Gamma_{2,1}/2, \quad \delta'_{2,2} = \delta'_2 + \Gamma_{2,2}/2 \quad (9.15)$$

where $\Gamma_{n,m} = |\text{Im}[\delta'_{n,m}]|$ is the resonance half-linewidth of the eigenmode $\delta'_{n,m}$ ($n, m = 1$ or 2) of the complex transmittivity ξ_q , as defined above. In the limit of $\kappa^2 \ll 1$, we can approximate Eq. (9.15) as

$$\begin{aligned} \delta'_{1,1} &\approx \delta_1 - \kappa^2/2 = \delta_0 - \sin^{-1} \kappa_1 - \kappa^2/2, & \delta'_{1,2} &\approx \delta_1 + \kappa^2/2 = \delta_0 - \sin^{-1} \kappa_1 + \kappa^2/2, \\ \delta'_{2,1} &\approx \delta_2 - \kappa^2/2 = \delta_0 + \sin^{-1} \kappa_1 - \kappa^2/2, & \delta'_{2,2} &\approx \delta_2 + \kappa^2/2 = \delta_0 + \sin^{-1} \kappa_1 + \kappa^2/2, \end{aligned} \quad (9.16)$$

where Eq. (9.16) is derived using Eq. (9.13) and the fact that $\Gamma_{n,m} = \text{Im}[\delta'_{n,m}] \approx \kappa^2/2$, in which $\kappa = \kappa_2 = \kappa_3$. It is then evident from Eq. (9.16) that κ controls the mode splitting of δ'_1 and δ'_2 that forms the multi-peaks or dips of $\delta'_{1,1}$, $\delta'_{1,2}$, $\delta'_{2,1}$ and $\delta'_{2,2}$, which appear reproducibly at every FSR in the transmission T_q spectra of both the through and reflection ports. In particular, these peaks or dips have sharper transitions between $T_q = 0$ and $T_q = 1$, as compared to those of the single coupled resonator and twin-coupled resonators systems that has only 1 cavity coupled to the bus waveguide in Fig. 9.2(ai). Such sharp transitions in the transmission can be attributed to CRIT [10] for the case of the through port and Fano-interference [13] for the case of the reflection port as the cavity modes can decay either to the port WG or adjacent resonators for the proposed twin-coupled resonators scheme.

The mode splitting of δ'_1 and δ'_2 of the proposed twin-coupled resonators produces strong normal dispersion (i.e., large t_g) with high transmission T_q : The two (four) sharp peaks of $T_q \approx 1$ each has a rapid π swing for the case of the through (reflection) ports in Fig. 9.2(bii) (Fig. 9.2(cii)). The proposed structure can thus be employed in SL applications. However, for practical SL applications utilizing ultra-short pulses, the sharp multi-peaks in the resonance spectra of the proposed device must be flattened. In general, it is not possible to simultaneously flatten both the transmission and t_g spectra of any resonator system due to its Hilbert transform properties [15]. Nonetheless, the group delay dispersion (GDD) of our proposed device is found to be minimal, being much lower (by ~ 2 to 10 times) than those of conventional TWMR-based SL schemes for the same group delay, when the transmission spectrum is maximally flat. As such, for practical design considerations in this work, we define flat-band SL as one with: (1) Maximally flat transmission T_q ($q = T$ for through port, $q = R$ for reflection port) spectrum, (2) $T_q \approx 1$, and (3) minimal GDD (i.e., close-to flat-band t_g). To achieve such a flat-band SL at both the through and reflection ports for the proposed device, one only needs to control the mode splitting via Eq. (9.16) such that the multi-peaks of the various resonance frequencies become undistinguishable and merge into one single broaden peak that is maximally flat or close-to flat at around $\delta = \delta_0 = 2\pi m$ in the resonance spectra (where m is an integer), while at the same time producing $T_q \approx 1$. This is intuitively shown in Fig. 9.3: The multi-peaks of the transmission spectra move inwards (in the direction of the arrows), towards $\delta = \delta_0 = 2\pi m$ and merge into one single, flatten peak as the resonator-to-resonator coupling κ_1 is progressively decreased, while fixing κ (where $\kappa = \kappa_2 = \kappa_3$), with the initial effective phase shift of two (four) π swings, converging into one

Fig. 9.3. Evolution in the resonance spectra towards flat-band slow light (i.e., maximally flat transmission with $T \approx 1$ and minimal group delay dispersion) for the proposed device with $\kappa = \kappa_2 = \kappa_3 = 0.3$ as κ_1 is decreased progressively, in the direction of the arrows – (ai) to (aiii): resonance spectra at the *through port* for $\kappa_1 = 0.2$, $\kappa_1 = 0.1135$, $\kappa_1 = 0.07$ and $\kappa_1 = 0.0471$; and (bi) to (biii): resonance spectra at the *reflection port* for $\kappa_1 = 0.2$, $\kappa_1 = 0.1135$, $\kappa_1 = 0.0471$ and $\kappa_1 = 0.0195$. The cavity and coupler losses are assumed to be negligible in this section.

single 2π (4π) for the through (reflection) port when flat-band SL is achieved. We now proceed to analytically quantify for the conditions needed to achieve the above-mentioned flat-band SL.

9.3.2. Optimization for Flat-Band Slow Light

From Fig. 9.2(a), one can easily observe that, at the through port, there are two peaks at δ'_1 and δ'_2 , with $|\xi_T(\delta'_n) - \xi_T(\delta'_0)| > 0$ and $|\xi_T(\delta'_n) - \xi_T(\delta'_{n,m})| > 0$, while at the reflection port, there are four peaks at $\delta'_{1,1}$, $\delta'_{1,2}$, $\delta'_{2,1}$ and $\delta'_{2,2}$, with $|\xi_R(\delta'_n) - \xi_R(\delta'_0)| < 0$ and $|\xi_R(\delta'_n) - \xi_R(\delta'_{n,m})| < 0$, where δ'_n refers to δ'_1 or δ'_2 while $\delta'_{n,m}$ refers to $\delta'_{1,1}$, $\delta'_{1,2}$, $\delta'_{2,1}$ or $\delta'_{2,2}$. Flat-band SL for the proposed twin-coupled resonators can be generated at the through port if the mode splitting is adjusted such that the two peaks of δ'_1 and δ'_2 are converted into a single flatten peak at $\delta = \delta'_0 = 2\pi m$. This can be mathematically described as

$$\begin{aligned} \delta'_1 = \delta'_2 = \delta'_0 = 2\pi m &\Rightarrow \delta'_{1,1} = \delta'_{2,1} \approx 2\pi m - \kappa^2/2, \quad \delta'_{1,2} = \delta'_{2,2} \approx 2\pi m + \kappa^2/2, \\ \left| \xi_T(\delta'_n) \right| - \left| \xi_T(\delta'_0) \right| &= 0, \\ \text{with } \frac{\partial^2 |\xi_T(\delta'_0)|}{\partial \delta^2} &= 0, \quad \text{where } \frac{\partial |\xi_T(\delta'_0)|}{\partial \delta} = \frac{1}{|\xi_T(\delta'_0)|} \left[\xi_T(\delta'_0) \cdot \frac{\partial \xi_T(\delta'_0)}{\partial \delta} \right]. \end{aligned} \quad (9.17)$$

Likewise, for the reflection port, flat-band SL can be generated by adjusting the mode splitting such that the four peaks of $\delta'_{1,1}$, $\delta'_{1,2}$, $\delta'_{2,1}$ and $\delta'_{2,2}$ are converted into a single flatten peak at $\delta = \delta'_0 = 2\pi m$. Mathematically, this means

$$\delta'_1 = \delta'_2 = \delta'_{1,1} = \delta'_{1,2} = \delta'_{2,1} = \delta'_{2,2} = 2\pi m \Rightarrow \left| \xi_R(\delta'_n \text{ or } \delta'_{n,m}) \right| - \left| \xi_R(\delta'_0) \right| = 0, \quad (9.18)$$

$$\text{with } \frac{\partial^2 |\xi_R(\delta_0)|}{\partial \delta^2} = 0, \quad \text{where } \frac{\partial |\xi_R(\delta_0)|}{\partial \delta} = \frac{1}{|\xi_R(\delta_0)|} \left[\xi_R(\delta_0) \cdot \frac{\partial \xi_R(\delta_0)}{\partial \delta} \right].$$

Solving Eqs. (9.17) and (9.18) analytically gives the following solutions:

$$\kappa_{s,T} = \kappa_1 = \frac{\kappa^2}{2 - \kappa^2}, \quad \kappa_{s,R} = \kappa_1 = \frac{\kappa^2}{\sqrt{2} \left[2 - \kappa^2 + \sqrt{2(1 - \kappa^2)} \right]}, \quad (9.19)$$

where $\kappa_{s,T}$ and $\kappa_{s,R}$ gives the conditions need to achieve flat-band SL at the through port and reflection port, respectively. Specifically, Eq. (9.19) gives the strength of the inter-resonator evanescent coupling (i.e., values of κ_1) needed to realize flat-band slow light at each port when the evanescent coupling between resonator 1 and the bus waveguide is the same as the evanescent coupling between resonator 2 and the bus waveguide (i.e., $\kappa_2 = \kappa_3 = \kappa$). Using Eq. (9.19), we have plotted the different combinations of κ_1 and κ required to achieve flat-band SL in Fig. 9.4, from which

Fig. 9.4. Different combinations of inter-resonator coupling coefficient κ_1 and resonator-to-bus waveguide coupling coefficient κ needed for flat-band slow light at the through and reflection ports of the proposed device. The ratio $\Psi = \kappa_{s,T} / \kappa_{s,R}$ is also shown, where $\kappa_{s,T}$ and $\kappa_{s,R}$ are the values of κ_1 that gives flat-band slow light at the through port and reflection ports, respectively, for a given κ . Note that we have used $\kappa = \kappa_2 = \kappa_3$ for the cavity system. The cavity and coupler losses are assumed to be negligible in this section.

it can be observed that to achieve flat-band SL at either the through or reflection port, we must have $\kappa > \kappa_1$, i.e., the resonator-to-bus coupling must be larger than the resonator-to-resonator evanescent coupling, with $\kappa \gg \kappa_1$ when $\kappa \ll 1$. Note also that the inter-resonator coupling strength κ_1 needed for flat-band SL at the through port is generally larger than that of the reflection port, given the same κ , i.e., $\kappa_{s,T} > \kappa_{s,R}$, with $\kappa_{s,T} = \Psi \kappa_{s,R}$, where $\Psi = [2^{1/2} + 2r/(1+r^2)]$. This translates to $\kappa_{s,T} \approx 2.4\kappa_{s,R}$ for $\kappa < \sim 0.7$, converging to $\kappa_{s,T} \approx 1.4\kappa_{s,R}$ in the strong coupling limit, as also illustrated in Fig. 9.4.

Using different combinations of κ_1 and κ from Fig. 9.4, the flat T_q and the close-to flat group delay spectra are then plotted in Fig. 9.5(a) for the through port and in Fig. 9.5(b) for the reflection

Fig. 9.5. Demonstration of tunable flat-band slow light with maximally flat transmission T_q and minimal group delay dispersion for the proposed device for different κ_1 and κ . The resonance spectra at the *through port* are shown in (ai) to (aiii) while the resonance spectra at the *reflection port* are shown in (bi) to (biii), with $\kappa = 0.3$ (blue plots), $\kappa = 0.35$ (red plots), $\kappa = 0.4$ (green plots), $\kappa = 0.45$ (pink plots), $\kappa = 0.5$ (brown plots), $\kappa = 0.55$ (yellow plots) and $\kappa = 0.6$ (black plots). κ decreases in the direction of the arrow in the graphs. The corresponding values of κ_1 used for each port can be found from Fig. 9.4 or Eq. (9.19). Note that we have used $\kappa = \kappa_2 = \kappa_3$ for the cavity system. The cavity and coupler losses are assumed to be negligible in this section.

port. It is evident that maximally flat transmission T_q spectra, which are centered at $\delta_0 = 2\pi m$, can be achieved with $T_q \approx 1$ and minimal GDD by controlling the evanescent coupling coefficients κ_1 in relation to κ , with the bandwidth of the flat-band region increasing with κ , albeit a smaller group delay due to the DBP constraint. Such DBP constraint can also be verified in the effective phase shift responses in Figs. 9.5(aii) and 9.5(bii): For all combinations of κ_1 and κ , there is a fixed phase swing of $\sim 2\pi$ for the through port and a slightly higher fixed phase swing of $\sim 4\pi$ for the reflection port. Comparing these with the effective phase shift (of 2π across the FWHM in Fig. 9.2a(ii)) for the N -

cavities-1-bus system, which has a DBP of $2/\pi$ (cf. Table 9.1), we can then conclude that the DBP (in terms of the FWHM) of our proposed coupled resonator system for the through port case is $2/\pi$ and is thus similar to that of the N -cavities-1-bus system, while that of the reflection port is $4/\pi$, which is twice that of the N -cavities-1-bus system. However, as mentioned in section 9.1, the actual DBP of the N -cavities-1-bus system is much lower than the computed value of $2/\pi$ as it has a sharp Lorentzian response. Thus, the through port of our proposed device will also outperform the N -cavities-1-bus system in terms of the DBP performance. Likewise, the DBP of our proposed device will also outperform other TWMR-based SL systems mentioned in section 9.1 (cf. Table 9.1) since these systems have lower DBPs than the above-mentioned N -cavities-1-bus system.

9.4. Characterizing the Flat-Band Slow Light

Let us now take a closer look at the nature of the flat-band SL in terms of the maximum group delay at resonance $t_{gm, q}$, usable resonance bandwidth $\Delta f_{u, q}$ ($q = T$ or R) and the trade-off between $t_{gm, q}$ and $\Delta f_{u, q}$, where the subscript $T(R)$ denotes the through (reflection) port. Both $t_{gm, q}$ and $\Delta f_{u, q}$ are commonly normalized in the literature so as to facilitate fair comparative analyses of the performances between slow light devices (especially for cavity-based systems) of different material platforms and sizes. Following this approach, we define $t_{gm, q}$ and $\Delta f_{u, q}$ of the flat-band SL of our proposed device at around $\delta = 2\pi m$ as $t_{gm, q} = t'_{gm, q} \times t_{rt}$ and $\Delta f_{u, q} = \text{FSR} \times \Delta\delta_{u, q}/(2\pi)$, respectively, where $t'_{gm, q}$ is the normalized group delay, $t_{rt} = 1/\text{FSR}$ is the cavity round-trip time, $\text{FSR} = c/(n_g L_c)$ is the free spectral range (in Hz), n_g is the group effective index of the cavity mode, L_c is the length of the cavity and $\Delta\delta_{u, q}$ is the normalized usable bandwidth. Using Eqs. (9.11)-(9.19) and our earlier mentioned design specifications of $\kappa = \kappa_2 = \kappa_3$ and $r = r_2 = r_3$ for our twin-coupled TWMRs system, $t'_{gm, q}$ can be derived for the through and reflection ports:

$$t'_{gm, T} = \frac{2(1+r^2)}{(1-r^2)} t_{rt}, \quad t'_{gm, R} = \frac{4(r-1)^2 (r+1)^2 (B_1-1) (-2r+r^2 B_1^{1/2} + B_1^{1/2})^2 (-2+2r B_1^{1/2} - \kappa^2)}{(2-2r B_1^{1/2} - \kappa^2)^5} t_{rt}, \quad (9.20)$$

$$\text{where } B_1 = 1 - \frac{\kappa^4}{2(r^2 + r\sqrt{2} + 1)^2}.$$

Next, we proceed to derive the usable normalized resonance bandwidth $\Delta\delta_{u, q}$ ($q = T$ or R). Unlike most SL works, which express $\Delta\delta_{u, q}$ in terms of the full-width at half maximum (FWHM), here we define $\Delta\delta_{u, q}$ as the region of bandwidth in which the transmission T_q spectrum is maximally flat and there is minimal GDD, so as to minimize dispersion in both the transmission and group delay. Using Eqs. (9.11)-(9.19), $\Delta\delta_{u, q}$ for our device can be approximated as

$$\Delta\delta_{u,T} \approx \sin^{-1} \left[\frac{\kappa^2}{2} \sqrt{\frac{1}{r(2-\kappa^2)}} \right], \quad \Delta\delta_{u,R} \approx \frac{4}{3} \sin^{-1} \left[\frac{\kappa^2}{2} \sqrt{\frac{1}{r(2+r\sqrt{2}-\kappa^2)}} \right]. \quad (9.21)$$

Using Eqs. (9.20) and (9.21), we have plotted the maximum group delay t'_{gm} at resonance and the normalized usable resonance bandwidth $\Delta\delta_u$ of the flat-band SL of the propagating light emerging from the through and reflection ports in Fig. 9.6. The delay-bandwidth product (DBP) is shown in the inset in Fig. 9.6. Some general observations and pertinent points with regard to Fig. 9.6 are summarized below.

Fig. 9.6. The normalized group delay (left axis) and the normalized usable resonance bandwidth (right axis) of the region of flat-band slow light for the through port (dotted curves) and reflection port (solid curves) of the proposed device at different values of κ . Corresponding values of κ_1 can be found from Fig. 9.4 or Eq. (9.19). The inset shows the delay-bandwidth product (DBP). Note that we have used $\kappa = \kappa_2 = \kappa_3$ for the cavity system.

- For both the reflection and through ports, the t_{gm} and $\Delta\delta_u$ characteristics share an inverse relationship – t_{gm} increases with decreasing κ , while $\Delta\delta_u$ increases with increasing κ . This occurs as t_{gm} is proportional to the photon lifetime of the cavity t_{cav} , while $\Delta\delta_u$ is proportional to the cavity decay rate $1/t_{cav}$, with t_{cav} increasing with decreasing κ . Therefore, a trade-off condition exists between t_{gm} and $\Delta\delta_u$; one needs to decide whether a large t_{gm} or $\Delta\delta_u$ is critical for the intended SL applications as it is not possible to achieve both.
- A large κ in the range of $\kappa \geq 0.6$ results in a relatively small t_{gm} of $t_{gm} \leq 9t_{rt}$ for the through port and $t_{gm} \leq 16t_{rt}$ for the reflection port, where t_{rt} is the round-trip cavity time. However, even small group delay is beneficial for certain applications such as data stream regeneration [1].

- The inverse relationship between t_{gm} and $\Delta\delta_u$ (cf. point (i) above) results in the earlier mentioned delay-bandwidth product (DBP) constraint. As shown in the inset in Fig. 9.6, the DBP stays approximately constant as κ changes with the through port having a DBP of ~ 0.6 while that of the reflection port is twofold higher, at ~ 1.2 . These match closely with our earlier prediction of a DBP of $2/\pi$ (≈ 0.64) for the through port and a DBP of $4/\pi$ (≈ 1.27) for the reflection port in section 9.3 (using Fig. 9.5). This further verifies the results in Table 9.1, which shows that our device outperforms the conventional TWMR-based SL systems.
- It is also worth mentioning that t_{gm} and $\Delta\delta_u$ for the reflection port are larger than that of the through port as the output light at the reflection port spends more time in the cavities. The difference between $t_{gm, T}$ and $t_{gm, R}$ increases as $t_{gm, T}$ and $t_{gm, R}$ become larger and likewise for the difference between $\Delta\delta_{u, T}$ and $\Delta\delta_{u, R}$.

In summary, we have demonstrated in this section that the mechanisms of resonance splittings and broadenings of our proposed twin-coupled resonators structure can be used to generate flat-band SL, i.e., slow light with maximally flat transmission T_q ($q = T$ for through port, $q = R$ for reflection port) spectrum, $T_q \approx 1$ and close-to flat-band group delay t_g . The maximum group delay $t_{gm, q} = t'_{gm, q} \times t_{rt}$ and usable resonance bandwidth $\Delta f_{u, q} = \text{FSR} \times \Delta\delta_{u, q}/(2\pi)$ are tunable by controlling the coupling coefficients κ and κ_1 (cf. Fig. 9.4 and Fig. 9.6), where $\kappa = \kappa_2 = \kappa_3$, $t'_{gm, q}$ is the normalized group delay, $t_{rt} = 1/\text{FSR}$ is the cavity round-trip time, $\text{FSR} = c/(n_g L_c)$ is the free spectral range (in Hz), n_g is the group effective index of the cavity mode, L_c is the round-trip length of the cavity and $\Delta\delta_{u, q}$ is the normalized usable bandwidth. Design rules to realize the maximum achievable $t'_{gm, q}$ and $\Delta\delta_{u, q}$ given a set of coupling coefficients (κ, κ_1) , as well as the trade-off between $t'_{gm, q}$ and $\Delta\delta_{u, q}$ have also been clearly illustrated (cf. Fig. 9.6).

Of particular importance is the fact that a range of κ , with its corresponding values of κ_1 from Fig. 9.4 can be chosen to achieve flat-band SL with normalized values of $t_{gm, q}$ and $\Delta f_{u, q}$, i.e., $t'_{gm, q}$ and $\Delta\delta_{u, q}$, that are dictated by the design rules in Fig. 9.6. It should be noted that the absolute values of t_{gm} and Δf_u depend on the round-trip length L_c of the cavity and the chosen values of κ and κ_1 . To show this, we have plotted in Fig. 9.7 the absolute values of $t_{gm, q}$ and $\Delta f_{u, q}$ as a function of the bend radius of a circular microring resonator (MRR) that has a cavity length of $L_c = 2\pi R$ and is based on silicon-on-insulator (SOI) channel waveguide with width = 500 nm and height = 500 nm. MRRs are used here and, in later sections, due to their simplicity in design and ease of fabrication as compared to other types of TWMRs, such as the microspheres.

Two different set of coupling coefficients (κ, κ_1) are arbitrary chosen for the plots in Fig. 9.7: In Fig. 9.7(a), we use $(\kappa, \kappa_1) = (0.3, 0.0195)$, while in Fig. 9.7(b), we use $(\kappa, \kappa_1) = (0.5, 0.0594)$. Based on the design rules in Fig. 9.6, one can see that with $(\kappa, \kappa_1) = (0.3, 0.0195)$, flat-band SL with $t'_{gm} \approx 73$

Fig. 9.7. The absolute usable resonance bandwidth (left axis) and absolute group delay (right axis) of the region of flat-band slow light for a circular microring resonator as a function of bend radius R for fixed coupling coefficients of $(\kappa, \kappa_1) = (0.3, 0.0195)$ in (a) and $(\kappa, \kappa_1) = (0.5, 0.0594)$ in (b). These chosen coupling coefficients correspond to the design rules of flat-band slow light in Fig. 9.6. Solid (dotted) curves correspond to the performances of the through (reflection) port. Note that the microring resonators are based on silicon-on-insulator channel waveguide with core height = 500 nm and core width = 500 nm. Also, $\kappa = \kappa_2 = \kappa_3$ for the system of circular microring resonators.

and $\Delta\delta_u/2\pi \approx 0.0054$ can be generated at the reflection port, while using $(\kappa, \kappa_1) = (0.5, 0.0594)$ gives a lower t_{gm} of $\sim 25t_{rt}$ but with a larger $\Delta\delta_u/2\pi$ of ~ 0.017 for the transmitted light at the reflection port.

For a circular MRR that is based on SOI channel waveguide with width = 500 nm and height = 500 nm, using $(\kappa, \kappa_1) = (0.3, 0.0195)$ at a bend radius R of 6 μm in Fig. 9.7(a) would translate into flat-band SL with a group delay of ~ 46 ps and a bandwidth of ~ 10 GHz at the reflection port, while using the second set of $(\kappa, \kappa_1) = (0.5, 0.0594)$ in Fig. 9.7(b) will give SL with a group delay of ~ 16 ps and bandwidth of ~ 26 GHz at the reflection port for the same R of 6 μm . In this case, it is evident that the actual group delay (bandwidth) decreases (increases) with κ and κ_1 . On the other hand, if a smaller bend radius R of 2 μm for the above-mentioned circular MRR is used, the set of parameters of $(\kappa, \kappa_1) = (0.3, 0.0195)$ in Fig. 9.7(a) will give flat-band SL with a group delay of ~ 15 ps and bandwidth of ~ 26 GHz at the reflection port, while the set of parameters of $(\kappa, \kappa_1) = (0.5, 0.0594)$ in Fig. 9.7(b) will give flat-band SL with a group delay of ~ 6 ps and bandwidth of ~ 78 GHz at the reflection port. Comparing these quantities of group delay and bandwidth with those of the above-mentioned larger R of 6 μm , it is easy to see that a smaller bend radius R (or larger FSR) brings about a much broader usable bandwidth $\Delta f_{u,q}$ (as $\Delta f_{u,q}$ is proportional to the FSR) but at a trade-off of a smaller group delay $t_{gm,q}$ (as $t_{gm,q}$ is inversely proportional to the FSR) in actual applications. A compromise must therefore be reached depending on whether a large bandwidth or group delay is needed for the specific

applications. The results in Figs. 9.6 and 9.7 can be used as general design rules in trading off between the various device performances. Generally speaking, the actual R that can be used for circular MRRs depends on the available fabrication technology and the minimum allowable cavity losses that allow a high transmission T_q to be realized as the dominant component of the cavity losses is the bend loss, which increases with decreasing R . On the other hand, if a racetrack MRR is used instead of a circular MRR, the group delay will be larger albeit a smaller usable bandwidth. For this reason, we will focus on circular MRRs in the next section, which looks into the effects of fabrication errors and cavity losses on the slow light (SL) performances of our proposed scheme. Note that SOI waveguides are used for the MRRs (in this subsection as well as in the next section) due to their compatibility with the mature silicon integrated circuits (ICs) manufacturing technology, as mentioned in section 1.1.4.

9.5. Effects of Fabrication Errors and Cavity Losses of the Proposed Flat-Band Slow Light Scheme Based on Circular Microring Resonators

The actual performances of the proposed flat-band SL system will depend on two factors: (i) The fabrication tolerance (i.e. the sensitivity of the fabricated device to fabrication errors in physical parameters, such as the size of the cavities and coupling coefficients); and (ii) the cavity losses of the fabricated devices. To date, deep UV lithography and improved fabrication recipes have made it possible to control the waveguide dimensions to within $\pm 1\%$ [27] when fabricating MRRs based on silicon-on-insulator (SOI) waveguides. We have found that, at such fabrication tolerance of $\pm 1\%$, the flat-band SL of our proposed system is relatively unaffected. Moreover, for small arrays of MRRs, any fabrication errors in the size of the cavities can easily be remediated or offset by the use of phase-shifters [13, 28, 29], such as micro-heaters, in the cavities. The use of phase shifters also allows the resonance wavelength or central wavelength of the input pulse to be actively tuned to a desired value, which is widely adopted in the development of commercial devices. In other words, fabrication tolerance is not much of an issue for our proposed device if MRR based on SOI waveguides are used. On the other hand, the cavity losses of the fabricated MRRs based on SOI waveguides are expected to be the dominant sources of degradation to the SL performances. Such cavity losses consist of the propagation losses and the coupler losses. These are discussed below.

9.5.1. Propagation Losses

The chief components of the propagation losses of any MRRs are the fabrication-induced sidewall roughness losses and the bending losses. With optimization of the etching recipe and improvement in the fabrication technologies, the fabrication-induced sidewall roughness losses can generally be reduced to a negligible level [30, 31] for SOI-based WG devices. Then the bend losses will be the main source of the propagation losses in the SOI-based MRRs. Note that the propagation

losses of each cavity are characterized by the round-trip amplitude transmission coefficient τ_n , where the subscripts $n = 1$ and $n = 2$, respectively, denote the leftmost and rightmost resonators in Fig. 9.1. As the two MRRs are of equal size (i.e., $\delta_1 = \delta_2 = \delta$) in this work, we use $\tau = \tau_1 = \tau_2$, where $\tau < 1$ corresponds to net optical losses, $\tau = 1$ corresponds to a lossless cavity, which is the case in sections 9.2 to 9.4, while $\tau > 1$ corresponds to net optical gain. We will not be looking into the regime of $\tau > 1$ as net optical gain results in additional mode-splitting effects in the resonance spectra, as concluded earlier in section 6.4.4, thus rendering flat-band SL difficult to be realized.

Let us first look at the effect of varying τ (while first assuming no coupler loss, i.e., $\sigma_1 = \sigma_2 = 0$; coupler losses are looked into in the next section) on the flat-band SL for one particular example of $\kappa = 0.3$ for our proposed device (with the corresponding value of κ_1 that gives flat-band SL at each port from Fig. 9.4). This is shown in Fig. 9.8. For this case in Fig. 9.8, flat-band SL with high transmission of $0.8 \leq T_q \leq 1$ can still be realized at both the through and reflection ports in the presence of losses, provided that the losses are in the range of $0.999 \leq \tau \leq 1$. For such small losses of $0.999 \leq \tau \leq 1$, notice that the group delay around the flat-band region of $\delta = 2\pi m$ is rather independent of τ , although T_q fluctuates slightly. Increasing the losses to $\tau < 0.999$ results in the transmission T_q around $\delta = 2\pi m$ to rapidly decrease (cf. $\tau = 0.99$ and 0.96 for the through port and $\tau = 0.99$ for the reflection port) until the critical coupling point is reached (cf. $\tau \approx 0.93$ for the through port and $\tau \approx 0.96$ for the reflection port), in which $T_q = 0$. We will term the value of τ where $T_q = 0$ as τ_c . Further increasing the losses

Fig. 9.8. The effect of cavity losses (characterized by τ , with $\tau = 1$ for a lossless cavity) on the transmission T_q spectrum and group delay t_g spectrum of the flat-band slow light at the (ai)-(a(ii)) *through port* and the (bi)-(b(ii)) *reflection port* of the proposed device with $(\kappa, \kappa_1) = (0.3, 0.0471)$ for the through port and $(\kappa, \kappa_1) = (0.3, 0.0195)$ for the reflection port. The values of τ used are $\tau = 1$, $\tau = 0.9999$, $\tau = 0.999$, $\tau = 0.99$, $\tau = 0.96$, $\tau = 0.93$ and $\tau = 0.86$ as shown in the graphs. Note that we have used $\kappa = \kappa_2 = \kappa_3$ and $\tau = \tau_1 = \tau_2$ for the cavity system. Coupler losses are negligible in this case.

such that $\tau_c > \tau$ will increase the transmission (cf. $\tau = 0.86$ for the through port and $\tau = 0.93$ and 0.86 for the reflection port). Note that the MRRs are said to be operating in the under-coupled (over-coupled) regime when $\tau_c > \tau$ ($\tau_c < \tau$) and that the group delay around $\delta = 2\pi m$ is closely related to these coupling regimes; $\tau_c > \tau$ ($\tau_c < \tau$) gives fast (slow) light, with a large change in the group delay occurring when τ is approaching τ_c , at which the group delay is divergent. As a result, flat-band SL is observed in Fig. 9.8 only for $0.999 \leq \tau \leq 1$, in which the MRRs are in the over-coupled regime (i.e., $\tau_c < \tau$) and when τ is considerably far away from the critical coupling point τ_c . For values of τ close to τ_c , there will be a large change in the group delay, giving rise to sharp Lorentzian responses and low T_q (though T_q around $\delta = 2\pi m$ is rather flat), which fail our criteria of flat-band SL, as defined in section 9.3.

The above described phenomenon applies to any combinations of κ_1 and κ for our proposed device, not just for the specific case of $(\kappa, \kappa_1) = (0.3, 0.0471)$ in Fig. 9.8. However, different combinations of κ_1 and κ have different critical coupling points τ_c , which also implies different range of values of τ that gives flat-band SL. To show this, we have presented the transmission and group delay responses at the resonance frequency $\delta = 2\pi m$ of our proposed device as a function of τ for

Fig. 9.9. The effect of cavity losses (characterized by τ , with $\tau = 1$ for a lossless cavity) on the transmission T_q and group delay t_g at $\delta = 2\pi m$ (which is the centre of the region of flat-band slow light) for the (ai)-(aii) *through port* and the (bi)-(bii) *reflection port* of the proposed device for different κ_1 and κ , where $\kappa = \kappa_2 = \kappa_3$. The used values of κ are $\kappa = 0.3$ (blue plots), $\kappa = 0.4$ (red plots), $\kappa = 0.5$ (green plots) and $\kappa = 0.6$ (black plots), while the corresponding values of κ_1 used for each port can be found from Fig. 9.4 or Eq. (9.19). The insets in all the graphs show the enlarged region of $\tau \geq 0.958$. Note that flat-band slow light with $T_q \approx 1$ is only achieved when τ is in the region to the right of the critical coupling (cc) point (cc means $T = 0$ and group delay is divergent), where $\tau > \sim 0.99$. For $\tau < \sim 0.99$, the flat-band slow light has $T_q < 1$ and progressively transforms into fast or slow light with a sharp Lorentzian response. Note that $\tau = \tau_1 = \tau_2$ for the cavity system. Coupler losses are negligible in this case.

different combinations of κ_1 and κ (based on the data in Fig. 9.4 that gives flat-band SL) in Fig. 9.9. Evidently, τ_c (cf. points of asymptote and $T_q = 0$ in Fig. 9.9) decreases with increasing κ_1 and κ and thus the transmission T_q and group delay near the point corresponding to $T_q \approx 1$ become less sensitive to changes in τ at larger κ_1 and κ . In particular, it can be seen that the group delay is less sensitive to changes in τ than the transmission. This is consistent with the trends in Fig. 9.8. For both ports, the group delay is fairly constant for the range of $0.96 \leq \tau \leq 1$ for all values of κ in the interval $\kappa > 0.3$. On the other hand, for $0.1 \leq \kappa \leq 0.3$ (note that only the plots of $\kappa = 0.3$ are shown due to space constraint; the properties of the plots of $\kappa < 0.3$ can be inferred from those of $\kappa \geq 0.3$), the propagation losses must be such that $0.96 \leq \tau \leq 1$ ($0.98 \leq \tau \leq 1$) in order for the group delay to remain fairly constant for the through (reflection) port. However, to achieve flat-band SL with high transmission of $T_q \geq 0.8$ (cf. Fig. 9.9(ai) and 9.9(bi)) for the through (reflection) ports, the allowed range of τ would decrease to $0.995 \leq \tau \leq 1$ ($0.997 \leq \tau \leq 1$) for $\kappa > 0.3$ while that for $0.1 \leq \kappa \leq 0.3$ is $\sim 0.999 \leq \tau \leq 1$ for both ports. These imply that for our device to be feasible for the generation of flat-band SL, any propagating losses present in each cavity must be kept to a level such that τ is at least at $\sim 0.999 \leq \tau \leq 1$.

Theoretically speaking, it is possible to achieve τ in the range of $\sim 0.999 \leq \tau \leq 1$ for a passive microring resonator with bend radius of $R \geq 2 \mu\text{m}$ based on submicron silicon-on-insulator (SOI) wires with optimized WG dimensions [32]. Recently, it has been demonstrated that round-trip amplitude transmission coefficient as high as $\tau \approx 0.9886$ can be achieved for a passive microring resonator based on submicron SOI channel waveguide with an ultrasmall bend radius of $R = 1.0 \mu\text{m}$ by optimizing the aspect ratio of the WG [33]. This implies the technical possibility to realize the theoretical limit of $\sim 0.999 \leq \tau \leq 1$ for a microring resonator with $R \geq 2 \mu\text{m}$ as τ increases exponentially with R . As mentioned earlier, the usable resonance bandwidth Δf_u increases with decreasing R while the group delay t_g increases with increasing R in an actual device. Since MRRs with $R \geq 2 \mu\text{m}$ that give low round-trip loss of $\sim 0.999 \leq \tau \leq 1$ can be realized, our proposed scheme in Fig. 9.1 can thus be used to generate flat-band SL with widely variable Δf_u and t_g (based on the design rules earlier in Figs. 9.4 and 9.6) that are suitable for practical applications (cf. section 9.4 for some examples of actual figures of Δf_u and t_g), such as in wavelength division multiplexing systems [34]. For greater versatility, τ can be actively tuned by electrically [35] or optically [36] pumping the active material in the WG.

9.5.2. Coupler Losses at the Coupling Junctions

Besides the propagation losses, the coupler losses at the coupler junctions of the MRRs would also influence the flat-band SL. The coupler loss at each coupler junction is characterized by the parameter σ_i (wherein $r_i^2 + \kappa_i^2 + \sigma_i^2 = 1$), with $\sigma_i = 0$ corresponding to an ideal lossless coupler, which has been the case thus far. For simplicity, we assume equal coupler losses for all the three coupling

junctions for our proposed device, i.e., $\sigma = \sigma_1 = \sigma_2 = \sigma_3$. In general, coupler losses arise due to the coupling of the input light to higher-order modes as well as radiation modes at the coupling junctions. Details can be found in Ref. [32, 33, 37].

Earlier in section 9.5.1, we have concluded that, to realize flat-band SL with high transmission T_q for any given κ at the coupling junctions, any propagating losses present in each cavity must be kept to a level such that τ is at least ~ 0.999 . We thus set $\tau = 0.999$ to demonstrate the effects of varying σ on the flat-band SL of our proposed device in this section. This is shown in Fig. 9.10 for the case of $\kappa = 0.3$ (with corresponding values of κ_1 for each port from Fig. 9.4), in which it can be seen that for the through (reflection) port, a coupler loss of $\sigma \leq \sim 0.04$ ($\sigma \leq \sim 0.02$) at each coupling junction will keep the transmission at both ports to a high level of $T_q \leq 0.8$ at $\delta = 2\pi m$. The group

Fig. 9.10. The effects of varying coupler losses (characterized by σ , with $\sigma = 0$ for a lossless coupler) on the transmission T_q and group delay t_g of the flat-band slow light at the (ai)-(aai) *through port* and the (bi)-(bii) *reflection port* of the proposed device with $(\kappa, \kappa_1) = (0.3, 0.471)$ for the through port and $(\kappa, \kappa_1) = (0.3, 0.0195)$ for the reflection port. Propagating loss of each cavity is fixed at $\tau = 0.999$. Note that $\sigma = \sigma_1 = \sigma_2 = \sigma_3$, $\kappa = \kappa_2 = \kappa_3$ and $\tau = \tau_1 = \tau_2$ for the cavity system.

delay, on the other hand, is relatively independent of the coupler losses. This trend is somewhat similar to the sensitivity test regarding the propagation losses described earlier in Fig. 9.8.

The above-observed trends of reduced transmission and rather stable group delay when coupler losses are present apply to any values of κ and κ_1 , not just for the specific case in Fig. 9.10. To show this and also to better illustrate the effects of the coupler losses on the flat-band SL, we have presented the transmission and group delay responses at $\delta = 2\pi m$ of our proposed device as functions of the coupler loss σ for different combinations of κ and κ_1 (based on the data in Fig. 9.4 that gives flat-band SL) in Fig. 9.11. It can be seen from Figs. 9.11(ai) and 9.11(bi) that, for each distinct combinations of

κ and κ_1 , the transmission T_q decreases with increasing coupler loss σ until the critical coupling point $T_q = 0$ is reached. Such critical coupling point at $T_q = 0$ is reflected as an asymptote in the group delay function in Figs. 9.11(aii) and 9.11(bii), in which the group delay changes rapidly around the asymptote. These properties are similar to the trends earlier in Figs. 9.9(aii) and 9.9(bii), where τ instead of σ is varied. We will term the critical coupling points (points of asymptote and $T_q = 0$ in Fig. 9.11) that arise from adjusting the coupler loss σ as σ_c . In general, σ_c increases with κ_1 and κ and thus the transmission T_q and group delay near the point corresponding to $T_q \approx 1$ becomes less sensitive to

Fig. 9.11. The effect of coupler losses (characterized by σ , with $\sigma = 0$ for a lossless cavity) on the transmission T_q and group delay t_g of the region of flat-band slow light (centered at $\delta = 2\pi m$) for the (ai)-(aii) *through port* and the (bi)-(bii) *reflection port* of the proposed device for different κ_1 and κ . The used values of κ are $\kappa = 0.3$ (blue plots), $\kappa = 0.4$ (red plots), $\kappa = 0.5$ (green plots) and $\kappa = 0.6$ (black plots), while the corresponding values of κ_1 used for each port can be found from Fig. 9.4 or Eq. (9.19). Note that flat-band slow light with $T_q \approx 1$ is only achieved when σ is in the region to the left of the critical coupling (cc) point (cc means $T = 0$ and group delay is divergent), where $\sigma < \sim 0.02$. For $\sigma > \sim 0.02$, the flat-band slow light has $T_q < 1$ and progressively transforms into fast or slow light with a sharp Lorentzian response.

changes in σ at larger κ_1 and κ . This is also similar to the sensitivity test pertaining to varying τ earlier in Fig. 9.9. Also, as shown in Fig. 9.11, the group delay is less sensitive to changes in σ than the transmission. This is consistent with the trends in Fig. 9.10. For the through (reflection) port, the group delay are fairly constant for all coupling values κ in the range of $\kappa > 0.3$ provided that $\sigma \leq \sim 0.3$ ($\sigma \leq \sim 0.2$). On the other hand, for all coupling values κ in the range of $0.1 \leq \kappa \leq 0.3$, the group delay at the through (reflection) port can be kept fairly constant if $\sigma \leq \sim 0.2$ ($\sigma \leq \sim 0.1$). However, to achieve flat-band SL with high transmission of $T_q \geq 0.8$ (cf. Fig. 9.11(ai) and 9.11(bi)) for the through (reflection) ports, the allowed range of σ would considerably decrease to $\sigma \leq \sim 0.06$ ($\sigma \leq \sim 0.04$) for

all coupling values κ in the range of $\kappa > 0.3$, while that for $0.1 \leq \kappa \leq 0.3$ is $\sigma \leq \sim 0.04$ ($\sigma \leq \sim 0.02$) for the through (reflection port). In other words, the coupler loss σ at each coupling junction can at most be 0.02 in order for high transmission T_q of 0.8 to be observed at both ports when using our proposed device for generating flat-band SL in the presence of propagating losses in the cavity with $\tau = \sim 0.999$ (which is the maximum τ that renders the device performances rather insensitive to propagating losses, as concluded earlier). Note that σ , in general, increases exponentially with decreasing bend radius and the coupling coefficient κ_i [32, 33]. It has been found that for a circular microring resonator with bend radius R , the coupler loss σ at each coupling junction is $\sigma < \sim 0.01$ for $R \geq 1 \mu\text{m}$ [33]. This implies that, in actual fabricated devices, the coupler loss at each coupling junction is too low to have any severe degradation on the flat-band SL of our proposed system as the maximum allowable coupler loss of our proposed MRRs system is $\sigma = 0.02$ (as mentioned above), while the bend radius of each resonator is in the range of $R \geq 2 \mu\text{m}$ (cf. subsection 9.5.1). Based on the results in this subsection and those in the earlier subsection 9.5.1, we can conclude that our proposed twin-coupled TWMRs can be suitable for practical flat-band SL applications when circular microring resonator (MRR) based on the SOI platform is being used.

9.6. Summary and Significant Contributions of this Research Work

In summary, we have proposed a scheme of twin-coupled traveling-wave microresonators (TWMRs) to generate flat-band slow light (SL) at both the through and reflection ports. Such flat-band SL has the characteristics of (1) maximally flat transmission T_q spectrum, (2) high transmission of $T_q \approx 1$ and (3) minimal group delay dispersion. The bandwidth Δf_u and the group delay t_g (cf. Fig. 9.6 for magnitudes of Δf_u and t_g) of the region of flat-band SL can be tuned by adjusting the coupling coefficients κ_1 , κ_2 and κ_3 of the coupling junctions of the TWMRs such that the design rule that is governed by Eq. (9.19) and illustrated in Fig. 9.4 is fulfilled. The coupling coefficients can be actively tuned by the use of integrated micro-electromechanical system (MEMS) to adjust the resonator-to-resonator and waveguide-to-resonators air-gap separations. This has been experimentally been demonstrated in [38, 39]. Our proposed device can thus effectively mitigate the constraint of conventional TWMR-based SL system, in which large group delay is achieved at the expense of narrow usable bandwidth with low transmission T_q (i.e., flat-band SL with high T_q is not possible). A fundamental figure of merit to quantify the capability of the SL system to simultaneously fulfill the bandwidth and group delay enhancement requirements is the delay-bandwidth product (DBP) that is defined as $\text{DBP} = \text{maximum group delay at resonance} \times \text{usable resonance bandwidth}$, in which the slow light has high transmission and minimal dispersion in the group delay and transmission. We have demonstrated that the DBP (in terms of the FWHM) of our proposed coupled TWMR system for the through port case is $2/\pi$ while that of the reflection port is $4/\pi$, which are, respectively, 3- to 12-fold and 6- to 24-fold higher than conventional SL systems (cf. Table 9.1) that typically have DBPs of

$\leq \sim 0.21$, assuming that the usable bandwidth of each conventional TWMR-based SL system occupies only one-third of its FWHM due to the high dispersive nature of the Lorentzian resonance spectrum. This further illustrates that our proposed system is capable of better SL performances as compared to conventional TWMR-based SL systems [4-7, 10-21].

We have also found that the fabrication errors in the waveguide dimensions must be kept within $\pm 1\%$ and that the size of the bend radius R used should be $R \geq 2 \mu\text{m}$ so that the flat-band SL is not severely degraded by the deviation in the physical parameters (induced by the fabrication errors) and the cavity losses (due to propagation and coupler losses, which are dependent on R). These can easily be achieved due to advancement in fabrication technology, which has allowed a fabrication tolerance of $\leq \pm 1\%$ [27] and the fabrication of low-loss TWMRs with R as small as $1 \mu\text{m}$ [33]. Therefore, our proposed device will be highly suitable for practical SL applications. Our work is also a significant contribution to the field of SL as a large resonance bandwidth with a large delay-bandwidth product can be generated, without the need for either a complicated device design or a large device footprint (as only two resonators are needed in our proposal).

Finally, note that this research work has been submitted to the Journal of Optical Society B.

9.7. References for Chapter

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Chapter 10

Summary, Conclusions and Future Works

10.1. Summary and Conclusions

In summary, in this thesis, theoretical optimizations and new research proposals for efficient microphotonic components, with particular emphasis on communication applications, have been presented. These research works have been completed in two stages: (i) Stage one, which is part one of this thesis (chapters 2 to 5), focuses on basic functionalities of two fundamental microphotonic components, namely, the straight waveguide and the microring resonator (MRR); and (ii) stage two, which is part two of this thesis (chapters 6 to 9), focuses on more advanced functionalities, namely the fast and slow light performances of traveling wave microresonators (TWMRs). Significant research results and contributions have been achieved in these two stages of the research work, which are highlighted in the following subsections. Note that in chapters 2, 3 and 5, we have focused on the use of the silicon-on-insulator (SOI) platform for the optimization and design process of waveguides and one particular type of traveling wave microresonators (TWMRs) — the microring resonator. For all other chapters, we are concerned with generic ideas that can be applied to any type of TWMRs and material platforms, although the microring resonator based on SOI has been used in our case studies.

10.1.1. Research Works in Part One of Thesis

In part one of this thesis, the main focus is on the optimization and design of two fundamental microphotonic components — the straight waveguides and the microring resonator — for efficient communication applications while shrinking the device size into the submicron scale on the silicon-on-insulator (SOI) platform.

To build a good research foundation in this stage, we have first looked into the stringent conditions needed to realize both single-mode (SM) and polarization-independent (PI) operations at the submicron scale for three types of SOI waveguides, namely, the slab, the channel and rib waveguides at the communication wavelengths of 1.31 μm and 1.55 μm . Our research results, as presented in chapter 2, clearly distinguish the dimensions of SOI slab, channel and rib waveguides required to satisfy both SM and PI conditions simultaneously at the submicron scale for the communication wavelengths of 1.31 μm and 1.55 μm . These can be used as design rules when fabricating silicon waveguides for SM and PI operations. In fact, Ref. [1] and [2] have used our design rules to fabricate microring resonators. In addition, our work has also inspired the use of SOI

waveguides at the submicron scale in the design of several PI couplers [3-5], a PI demultiplexer [6], a slow light device [7], microbends [8] and hybrid plasmonic waveguide devices [9]. In many of these works, our design rules in chapter 2 have been used to optimize the device performances. Note that previously, SOI submicron rib waveguides are seldom used for photonic devices as the SM and PI conditions at the submicron scale are not very well understood. Our work has thus, in one way or another, led to the trend of using submicron SOI waveguides for SM and PI devices. This highlights the fact that our research work in this chapter is a significant contribution to the field of silicon photonics. Last, but not least, by using analytical modeling, we have clearly shown the modal characteristics in SOI slab waveguides and its relation to the SOI channel and rib waveguides. This has often been ignored in most of the literature. Also, we have demonstrated that the approximations provided by the analytical modeling for the submicron SOI waveguides matches well with the general trends of the results of the numerical modeling. Such use of both analytical and numerical modeling in this research work has provided a better physical insight of the stringent conditions needed for the realization of both SM and PI operations in the SOI waveguides at the submicron level.

With the above-mentioned research work as foundations, we have then designed a small SOI-based microring resonator that is capable of single-mode (SM) and polarization-independent (PI) operations at the input wavelength of $1.55\text{ }\mu\text{m}$ in chapter 3. To be specific, we are interested to find out how small can a SOI-based microring resonator (MRR) be, i.e., how small can we shrink both its cross-sectional dimensions and radius, and yet maintain the overall polarization-independence and single-mode conditions of the MRR, while ensuring low round-trip losses. Our design approach to achieve both SM and PI operations for compact resonators is modular and hierarchical. We have first shown how SM and close to birefringence-free conditions can be achieved for straight waveguides using deeply etched ribs, followed by how PI and low-loss bends as well as PI multi-mode interferometer (MMI) coupler can be designed as key components for a bus-coupled resonator. Note that PI also implies SM in our work. All the PI components are then integrated into a single-bus all-pass filter configuration and the transmittance spectra are simulated for both TE and TM polarizations. The results for a specific design where the core thickness is $0.4\text{ }\mu\text{m}$ have shown that a resonance mismatch of about 0.22 nm , representing only 11% of the FWHM linewidth of 2 nm , can be achieved. This small polarization dependence represents a large improvement over what can usually be achieved with the conventional PI resonators [10-13] that have much larger device size and very small FSR. Most importantly, we have demonstrated the feasibility of realizing MRRs of submicron cross-sectional dimensions with a small radius of a few microns that have substantially reduced polarization sensitivity while maintaining the essential SM and low-loss conditions. We have also provided an approximate but reasonably accurate theoretical lower size limit of the MRR for simultaneous SM and PI operations, at our considered upper oxide cladding thickness and using the more robust deeply etched rib waveguide, which is seldom used for MRR. Recently, in July 2011, this

research work in chapter 3 has been cited in [14], which looked into the design and fabrication of a TE-TM polarization convertor based on the SOI platform. This shows the significance of our work.

After analyzing the single-mode and polarization-independent behaviours in a single microring resonator, which is one particular class of traveling wave microresonators (TWMRs), we then move on in chapter 4 to look into some of the interesting phenomena that emerge when a chain of TWMRs are coupled together. In particular, we are interested in the photonic-bandgap (PBG) properties of coupled-TWMRs, which are also known as coupled-resonator optical waveguides (CROWs). For the first time, to the best of our knowledge, we have proposed the use of periodic and quasi-orders to achieve PBG engineering of the resonance spectra of the TWMR-based CROWs. Such periodic and quasi-periodic orders are implemented by embedding multiple periodic and quasi-periodic “defects” resonators among the regular resonators in the CROWs. For the periodic case, the interaction of the multiple defects results in the splitting of the eigenmodes corresponding to distinct localized states, leading to the formation of defects-induced mini-passbands within the broader PBG. The number and position of such narrow defect passbands can be tuned by adjusting the relative size of the intra-resonators in each unit cell. If the intra and inter unit cell coupling strengths are asymmetrical, new PBG forms at resonance. These features make the deployment of such schemes for practical PBG engineering very attractive in the absence of active elements to achieve tuning. Other possible applications include multi-passband filters after the ripples are flattened by apodization techniques as highlighted in [15]. For the aperiodic or quasi-periodic case, we have shown that a sharp localized resonant state still exists within the PBG that slowly transforms to a broad passband as the order of the sequence is increased. If the total losses of the constituent unit cells are sufficiently low, the ultra-narrow passband of the quasi-periodic CROW based on a low-order Fibonacci sequence may be highly useful as a compact sensor for bio-photonics and chemical applications. As a general rule, systems with a perfect order or a complete disorder are simple, and it is in the middle where there is a mix of order and disorder that complexity arises that may produce novel properties and applications.

Finally, in the last chapter of part one of this thesis, we have studied the temperature T dependence of Raman scattering in submicrometer-scale SOI waveguide for $100\text{ K} \leq T \leq 500\text{ K}$ in regard to three specific types of Raman scattering-based functionalities in the continuous-wave regime: Raman amplification and attenuation near the C-band wavelength of $1.55\text{ }\mu\text{m}$ and parametric Raman wavelength conversion (PRWC) between the C-band wavelength of $1.55\text{ }\mu\text{m}$ and O-band wavelength of $1.31\text{ }\mu\text{m}$. It has been demonstrated in chapter 5 that the effects of temperature variation ($100\text{ K} \leq T \leq 500\text{ K}$) can be harnessed to improve the Raman performances in the submicron SOI waveguides. In particular, the use of temperature variation provides an additional degree of freedom to tune the conversion efficiency (CE) or amplification efficiency (AE) of the SOI waveguide, thereby allowing a more dynamical control of the Raman performances of the SOI waveguide, as compared to those working at a fixed room temperature, which is the conventional practice. Furthermore, the CE or

AE can be enhanced as compared to that at the room temperature. The detailed research findings in chapter 5 can be used as general design rules when engineering submicron SOI waveguides for efficient Raman amplification, Raman attenuation and PRWC at different temperatures. Most importantly, our results have highlighted the numerous advantages of harnessing the temperature effects of Raman scattering, which has largely been overlooked in the literature. These could have potentiality in revolutionizing the way in which Raman scattering is being employed for active functionalities in silicon waveguide devices in the future and are thus important contribution to the field of silicon photonics.

10.1.2. Research Works in Part Two of Thesis

In part two of the thesis, the theme is on fast and slow light effects in either a single traveling wave microresonator (TWMR) or a system of twin-coupled TWMRs. The main research objective here is on the use of degeneracy lifting of the cavity via the excitation of contra-propagating cavity modes to improve the fast and slow light effects in the TWMR-based circuits. Four different schemes to generate contra-propagating cavity modes in the TWMRs — the coupler-induced localized backscattering (CILB) in chapter 6, the intracavity distributed backscattering that is assisted by the use of dual inputs in chapter 7, the intracavity distributed backscattering in a system of twin-coupled TWMRs that has only one resonator coupled to the bus waveguide in chapter 8, and finally the contra-propagating cavity modes that are established due to the evanescent coupling of both the resonators to the bus waveguide in the twin-coupled TWMRs system that has a negligible intracavity distributed backscattering in chapter 9 — have been looked into for the enhancement of fast and slow light effects. These are elaborated as follows.

We have started part two of the thesis by demonstrating in chapter 6 that weak coupler-induced localized backscattering (CILB) that is aided by a small net optical gain in the cavity will enhance the fast light (FL) and slow light (SL) performances of a TWMR in the following ways. First, all three types of light propagation velocity (which are FL with negative v_g , FL with positive v_g and SL) can be generated at both the through and drop ports at the degenerate resonance frequency ω_0 . This is in contrast to the conventional TWMR (where CILB is negligible), in which at the degenerate resonance frequency ω_0 , only FL with negative v_g and SL are present at the through port while, for the drop port, only SL is produced. Second, the possibility to generate stable FL with an amplified transmission can be achieved, unlike the case of a conventional TWMR, where stable FL always has substantial attenuation. Third, there is an extra degree of freedom in addition to the physical parameters of coupling coefficient κ and round-trip amplitude attenuation constant τ of the cavity to control the group delay as any change in the strength of the CILB will modulate the group delay. However, do note that in the regime of strong CILB, there will be some deleterious effects: (i) The group delay enhancement at the through port is degraded; and (ii) the width of the FL regime at the drop port is

reduced. On a positive note, this research work regarding CILB in chapter 6 has been selected for the January 2011 issue of *The Virtual Journal of Ultrafast Science* [16]. This attests strongly to the significance and novelty of this research work.

Besides CILB, another type of backscattering in TWMRs is the surface imperfections-induced distributed backscattering within the cavity, which we have termed as intracavity backscattering. Following the fast light (FL) and slow light (SL) work based on the CILB effect in chapter 6, we have subsequently proceed to look into the use of intracavity backscattering effects to generate enhanced tunable FL and SL in TWMRs. Research in this area is described in chapter 7, in which we have harnessed the physical effects that arise from the interaction of the intracavity backscattering with the dual contra-propagating inputs to achieve continuously tunable FL and SL from a single TWMR. To explain the rationale behind the use of such mechanism, first note that in a conventional TWMR, the surface perturbations due to fabrication-induced surface roughness can be made very small due to advances in fabrication technology [17, 18]. As such, the intracavity backscattering in most TWMRs is relatively weak, resulting in $Q_{mut} \rightarrow \infty$. Then we have $|Q_{cri}| \ll Q_{mut}$, which gives a single sharp Lorentzian resonance with no mode splitting; mode splitting only occurs if $|Q_{cri}| \gg Q_{mut}$, where Q_{mut} is the Q -factor that characterizes the intracavity backscattering, while Q_{cri} is the Q -factor that quantifies the critical coupling (which is defined as the phenomenon whereby the output transmission is zero). By intentionally introducing surface perturbations in the form of surface microstructures, such as grating ridges, on the TWMR in our work in chapter 7, the intracavity backscattering is enhanced, giving rise to two possibilities in the shape of the resonance: (i) A single sharp Lorentzian if $|Q_{cri}| \ll Q_{mut}$ or (ii) a two-split Lorentzian, i.e., a doublet, if $|Q_{cri}| \gg Q_{mut}$. In the presence of dual inputs, $|Q_{cri}|$ of the TWMR system can be adjusted (from $|Q_{cri}| \ll Q_{mut}$ to $|Q_{cri}| \gg Q_{mut}$ or vice versa) simply by modulating the phase and/or amplitude differences between the dual inputs. This will then allow the shape of the transmission and group delay resonance spectra to be switched between a Lorentzian and a doublet. By exploiting such change in the spectral shape as $|Q_{cri}|$ is adjusted, continuous tunability in the group delay and transmission of the output light at a fixed λ can consequently be achieved. This offers a more dynamic control of the speed of light, as compared to systems that use only a single input. More importantly, the proposed dual-input scheme realizes the possibility to achieve tunable fast and slow light effects in a passive and linear cavity. This is in contrast to conventional TWMR-based FL and SL systems (listed in Table 7.1 of chapter 7), which require either the use of active tuning mechanisms, such as p-i-n diodes, gain media or nonlinearity in the cavity, so as to have tunable group delay. The fact that the active tunability of the group delay stems from the use of external dual inputs also means that the reconfigurability and serviceability of the cavity system can be improved as the external dual inputs can be easily adjusted or replaced.

Having exploited the intracavity backscattering in a single TWMR for enhanced FL and SL applications in chapter 7, we have subsequently looked into the use of the intracavity backscattering in

a system of dual or twin-coupled TWMRs that has only one resonator coupled to the external bus waveguide in chapter 8. By intentionally introducing intracavity backscattering in the twin-coupled TWMRs, enhanced coupled-resonator-induced transparency (CRIT) and optical Fano resonance (OFR) effects can be generated as compared to conventional twin-coupled TWMRs systems that do not utilize the intracavity backscattering effects. In particular, the presence of intracavity backscattering in our modified twin-coupled TWMRs makes it possible to either generate a single sharp CRIT peak or a pair of CRIT peaks in the transmission spectrum of the through port within one free spectral range (FSR), depending on the magnitude of the backscattering, in contrast to the case of the traditional twin-coupled TWMRs structure (i.e., intracavity backscattering is not utilized), in which only a single sharp CRIT peak can be realized within one FSR. The distance between the twin-CRIT peaks can be controlled by adjusting the intracavity backscattering magnitude u_m . In addition, our proposed scheme makes it possible to simultaneously produce fast and slow light (one at each output port) in a single device. For example, the CRIT peak, which is accompanied by slow light, can be generated at the through port while the Lorentzian dip, which is accompanied by fast light, can be simultaneously generated at the reflection port of our proposed device. Such ability to simultaneously generate fast and slow light in a single device will make our proposed device highly attractive for fast and slow light applications. By tailoring u_m in our proposed modified twin-coupled TWMRs structure, the CRIT peaks can be reshaped into sharp asymmetric OFR lineshapes. We can thus switch between CRIT and OFR by proper choice of u_m during the fabrication process. More importantly, the ability to generate OFR means that our proposed modified twin-coupled TWMRs can also be utilized for optical switching applications in communication systems as it is well known that the sharp asymmetric lineshape of OFR is highly desirable for optical switching. In addition, we have demonstrated that if dual inputs are employed to interact with the intracavity backscattering in the resonators system, the CRIT symmetric peaks of our proposed modified twin-coupled TWMRs systems can also be reshaped into asymmetric OFR lineshapes simply by controlling the phase differences between the inputs. Active switching between CRIT and OFR can then be achieved simply by controlling the external dual inputs. This is a highly attractive and useful scheme that can be deployed after the fabrication process when the physical parameters are fixed and in the absence of active gain and/or phase tuning elements in the cavities. Note that conventional twin-coupled TWMRs scheme uses either gain [19] or phase tuning [20] to switch between CRIT and OFR. Our proposed scheme thus offer greater versatility as the external active tuning components can be easily adjusted or replaced, as compared to the gain or phase tuning elements, which are integrated into the cavities.

Finally, in the last chapter (i.e., chapter 9) of part two of this thesis, we have utilized the contra-propagating cavity modes that arise purely from the evanescent coupling (but not the surface perturbations-induced backscattering) of both the resonators to the bus waveguide in the twin-coupled TWMRs system to generate flat-band slow light (SL) at both the through and reflection ports. The

presence of such contra-propagating cavity modes in the twin-coupled TWMRs system will generate multi-peaks in the resonance spectra. Flat-band SL can be generated if such multi-peaks become undistinguishable and merge into one single broadened peak that is maximally flat or almost flat in the resonance spectra, which will only happen if the inter-resonator evanescent coupling strength is optimized relative to the resonators-to-bus-waveguide coupling strength, as shown in chapter 9. The generated flat-band SL from our proposed scheme has the characteristics of (1) maximal flat transmission T_q spectrum, (2) high transmission of $T_q \approx 1$, and (3) minimal group delay dispersion. The bandwidth Δf_u and the group delay t_g of the region of flat-band SL can be tuned by adjusting the coupling coefficients of the TWMRs. These mitigate the constraint of a conventional TWMR-based SL system, in which large group delay is achieved at the expense of a narrow usable bandwidth with low transmission. A fundamental figure of merit to quantify the capability of the SL system to simultaneously fulfill the large bandwidth and group delay enhancement requirements is the delay-bandwidth product (DBP), which is defined as $\text{DBP} = \text{maximum group delay at resonance} \times \text{usable resonance bandwidth}$. We have demonstrated that the DBP of our proposed twin-coupled TWMRs system for the output light at the through port case is $2/\pi$ while that of the reflection port is $4/\pi$. These DBPs of our device are 3 to 24-fold higher than those of the conventional SL systems based on TWMRs (cf. Table 9.1) that typically have DBP of $\leq \sim 0.21$, assuming that the usable bandwidth of each conventional TWMR-based SL system occupies only one-third of its full width at half maximum (FWHM) due to the high dispersive nature of the sharp or ripples-like Lorentzian resonance spectrum. This illustrates that our proposed system is capable of better SL performances as compared to the conventional TWMR-based SL systems. Fabrication tolerance and cavity losses analyses have also revealed that our proposed scheme is rather robust to the fabrication errors and limitations of current state-of-the-arts semiconductor processing technology.

10.2. Future Works

We will now elaborate on our future work, which are extensions of our current achievements in this thesis.

First and foremost, the design of the single-mode (SM) and polarization-independent (PI) SOI submicron rib in chapter 2 has only focused on waveguides with a fixed cladding thickness and at two specific communication wavelengths of $\lambda = 1.31 \mu\text{m}$ and $1.55 \mu\text{m}$ in order to expedite the simulation time and to save computational resources. For a complete understanding of the modal characteristics at the submicron scale, the waveguide analysis in chapter 2 can be extended to different cladding thickness and the entire communication wavelength spectrum. This extension is critical as: (i) Studies have shown that the cladding thickness can affect the modal characteristics [21], which is also illustrated in our earlier work in chapters 2 and 3; and (ii) moreover, if a pulse with a large spectral width (i.e., ultra-fast pulse) is used, the design rule that we have formulated specifically for $\lambda = 1.31$

μm and $1.55 \mu\text{m}$ would be insufficient to ensure that the transmission of the pulse can also be SM and PI. Also, waveguides with slant sidewalls [22] can also be used to achieve SM and PI conditions. The slant sidewalls angle will give an additional degree of freedom in the design process when used with other geometrical parameters, such as etch depth and width, which have been considered in our work in chapter 2 and 3. Likewise, the design of the SM and PI microring resonator (MRR) in chapter 3 can also be extended for different wavelengths and slant sidewalls angles, so as to improve the versatility of the MRR. In addition, our present SM and PI research work for both the submicron straight waveguides and MRR are based on passive media. The case of active media can also be considered for future work in the design of SM and PI submicron SOI waveguides and MRR.

For our work on photonic bandgap engineering in chapter 4, we may also consider different types of arrays such as SCISSOR [23] for both the periodic and quasi-periodic designs considered as we have limited our studies to only CROWs with a typical add-drop filter configuration (i.e., the chain of N -resonators is coupled to two bus waveguides, one at the first resonator and the other at the N^{th} resonator). Also, we have only focused on the Fibonacci CROWs in the section on the aperiodic order. Deterministic aperiodic CROWs based on other sequences, such as the Cantor [24] and Thue–Morse sequences [25], and hybrid ordered systems (with a mix of both periodic and quasi-periodic orders) are also possible and are expected to show interesting spectral properties. In addition, note that the focus of this work is on how the PBG structures are affected by periodic and quasi-periodic orders using multiple defects. As such, several other optical behaviors of our proposed devices have not been investigated. One of these is the dispersion of the device. In particular, the presence of multiple defects in both the periodic and quasi-periodic ordered ring systems would influence the slow wave characteristics of the device. This would in turn affect the non-linear phase sensitivity because of the relation: $d\Phi/dI = LS_s^2 \times (2\pi n_{\text{kerr}}/\lambda)$, where $d\Phi/dI$ is the non-linear phase sensitivity, L is the length of the device, S_s is the slowing ratio, n_{kerr} is the Kerr coefficient of the material and I is the input intensity. In addition, the presence of multiple coupled defects will impact the ring intensity build-up factor and this could enhance the bistable and multi-bistable performances of our proposed device. These mentioned issues can be analyzed in future works.

In chapter 5, our investigation of the temperature effects of the various Raman scattering-based processes of Raman amplification, Raman attenuation and parametric Raman wavelength conversion are limited to the continuous wave (CW) regime. Future work can be on the effects of temperature variation on different types of pulses in the Raman waveguide. In addition, the pump wavelength can also be varied in order to improve the tunability and versatility of the Raman waveguide. Also, instead of using a simple straight waveguide as a Raman medium, we could alternatively use a microring resonator or any type of resonant cavity. It would then be interesting to study how the resonance effects of the resonant cavity influence the different types of Raman scattering processes in the cavity at different operating temperatures.

In addition, we emphasize that our results in chapter 6 regarding enhanced fast and slow light effects based on the coupler-induced localized backscattering (CILB) apply strictly to a circular TWMR. If a TWMR with other configurations, particularly a racetrack configuration, such as a racetrack microring resonator, is used, there will be additional backscattering within the cavity due to the modal mismatch between the straight waveguide mode and the bend waveguide mode. This generally leads to Fresnel reflections at the junctions between the straight and bend waveguides. Such an effect on the fast and slow light behaviors of the TWMR can also be looked into in a future work.

Finally, for chapter 8, we have proposed the use of intracavity backscattering to enhance the CRIT and OFR effects in the twin-coupled TWMRs system. However, the analysis in chapter 8 regarding this proposed scheme is only limited to the general characteristics of the CRIT and OFR. Specific CRIT and OFR applications, such as tunable delay line and optical switching, based on our proposed modified twin-coupled TWMRs scheme can also be considered in our future work.

10.3. References for Chapter

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