Transduction matrix of AC motor driven mechanical systems : system modelling and fault diagnosis

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2012

Lian, K. F. (2012). Transduction matrix of AC motor driven mechanical systems : system modelling and fault diagnosis. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/51104>

<https://doi.org/10.32657/10356/51104>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**TRANSDUCTION MATRIX OF AC MOTOR DRIVEN
MECHANICAL SYSTEMS: SYSTEM MODELLING AND
FAULT DIAGNOSIS**

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SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING
2012**

TRANSDUCTION MATRIX OF AC MOTOR DRIVEN MECHANICAL SYSTEMS: SYSTEM MODELLING AND FAULT DIAGNOSIS

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A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2012

ABSTRACT

In this dissertation, a modelling technique for motor driven system based on transduction matrix is introduced. Transduction matrix describes the relationship between input and output of a system in frequency domain, thus it provides better understanding of the system's condition. For a motor driven system running at constant frequency, the transduction matrices for induction motor, power transmission system and mechanical loading can be obtained from respective governing equations. Since transduction matrix represents the system's properties, condition monitoring can be attained by studying these transduction functions. In addition, transduction matrix facilitates the analysis on both impedance propagation and power flow in the motor driven system. Consequently, the electrical input impedance that consists of transduction function is utilized in condition monitoring of induction motor. Studying the frequency components of electrical input impedance allows characteristic frequencies of motor faults to be identified. Therefore, the proposed monitoring signature is used to monitor the change of signature due to broken rotor bar, abnormal air-gap eccentricity and bearing faults. By comparing with the results from conventional monitoring technique, impedance signature proved to have better sensitivity as fault signature is amplified. Furthermore, wavelet packet transform is utilized in order to carry out fault detection in time-frequency analysis, where the signal to noise ratio is improved and make the fault detection easier.

ACKNOWLEDGEMENT

The author would like to express profound gratitude to his supervisor, Professor Ling Shih-Fu for his invaluable support, encouragement, useful suggestions and supervision he has given to the author throughout the research work. His moral support and continuous guidance enabled the author to complete work successfully. It has been a great honour and pleasure to work with him.

The author would also like to extend his appreciation to Assoc Prof Choo Fook Hoong for his precious suggestions in modelling of induction motor. The author also highly appreciates the kindness of Mr James Ee, for his valuable information of experimental setup and motor faults throughout the research.

In addition, the author also likes to extend his thanks to Mr Wong Yoke Rong, for his assistances on the programming of signal processing programme. His patience and enthusiasm to help are gratefully acknowledged. The author also likes to extend his appreciation to Mr Ivan Tanra, Ms Felicia Rezanda, Mr Pang Xin and Mr Ju Feng for many fruitful discussions and suggestions. Their suggestion and patience are praiseworthy.

The author would like to thank Mr Poon Weng Wai of Mechanic of Machines lab, Mr Cheo Hock Leong and Mrs Halimatun Bte Ma'Arof of Centre of Mechanics of Micro-Systems lab for their technical assistance.

Lastly, the author would like to express his appreciation to his family and friends for their understanding and support in his research journey.

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NOMENCLATURE

<i>A</i>	Area (m^2)
<i>a</i>	Scale Factor
<i>B</i>	Magnetic Field (T)
<i>BD</i>	Ball Diameter (m)
<i>C</i>	Damping Coefficient (Ns/m), Coefficient
<i>E</i>	Electric Field, Voltage Difference (V)
<i>F</i>	Force (N)
<i>f</i>	Frequency (Hz)
<i>g</i>	Air-Gap Length (m), Gravitational Acceleration (m/s^2)
<i>gr</i>	Gear Ratio
<i>I</i>	Current (A)
<i>J</i>	Mass Moment of Inertia (kgm^2)
<i>K</i>	Magnetic Constant
<i>K_s</i>	Spring Constant
<i>k</i>	Integer, Factor
<i>L</i>	Inductance (H), Length (m)
<i>l</i>	Axial Length of Rotor (m)
<i>m</i>	Integer
<i>N</i>	Shaft Rotation Speed (rpm), Number of Turn, Normal Force
<i>n</i>	Number of Bearing's Ball, Integer
<i>n_d</i>	Eccentricity Order
<i>n_ω</i>	Stator Time Harmonic
<i>P</i>	Power (W)
<i>PD</i>	Pitch Diameter (m)
<i>p</i>	Number of Pole
<i>q</i>	Electric Charge (C)
<i>R</i>	Resistance (Ohm), Rotor Slot Number
<i>r</i>	Residue, Radius (m)
<i>S</i>	Sum of Square
<i>s</i>	Motor Slip
<i>T</i>	Torque (Nm)
<i>t</i>	Time (s), Transduction Matrix Function
<i>V</i>	Voltage (V)
<i>v</i>	Velocity (m/s)
<i>W</i>	Weight (N)
<i>ω</i>	Angular Velocity (rad/s)
<i>X</i>	Reactance (Ohm)

Z	Impedance (Ohm)
β	Contact Angle (°)
Δ	Sampling Period (s)
ε	Electromotive Force, EMF (V)
η	Efficiency
θ	Phase Angle (°)
μ	Friction Coefficient
μ_0	Permeability of Free Space
ρ	Density (kg/m ³)
Φ	Flux (W/m ²), Phase
φ	Phase
Ψ	Analytic Signal

Subscripts

a	Phase ‘a’, Actual
B	Magnetic Field
c	Centre
D	Drag
E	Voltage Difference (V)
En	Engine
e	Electrical
ecc	Eccentricity
emf	Electromotive Force
FD	Final Drive Ratio
f	Fuel
h	Harmonic
i	Inner Race
L	Line-to-Line, Load
LSB	Lower Sideband
M	Magnetizing, Mass (kg)
m	Mechanical
max	Maximum
min	Minimum
mr	Magnetizing Component and Rotor
o	Outer Race
p	Peak
ph	Phase
r	Rotor, Rotational
rr	Rolling Resistance
S	Source

<i>ST</i>	Standard Transmission Ratio
<i>s</i>	Stator, Synchronous, Supply
<i>sf</i>	Stator Field
<i>T</i>	Torque
<i>t</i>	Total
<i>USB</i>	Upper Sideband
<i>w</i>	Wheel
<i>φ</i>	Line-to-Phase, Phase
<i>0</i>	Fundamental Supply
<i>1</i>	Port 1, Input
<i>2</i>	Port 2, Output
<i>3</i>	Port 3

CHAPTER 1

INTRODUCTION

In this chapter, brief backgrounds on induction motor and motor driven system are presented. Objectives and scopes of this research are indicated after recognizing the importance of having proper motor modelling technique. Finally, the outline of this dissertation is stated.

1.1 Background

The history of electric motor can be traced back to 1821, where British scientist Michael Faraday demonstrated the principle of conversion of electrical energy to mechanical energy by electromagnetic means. Later in 1828, Hungarian physicist Ányos Jedlik demonstrated the first device which contains stator, rotor and commutator. Subsequently, he built a model electric motor-propelled vehicle in 1855. In 1873, Zénobe Gramme invented the first modern DC motor when he used his dynamo and driving it as a motor. Nikola Tesla invented first AC motor with poly-phase power transmission system in 1888.

The invention of electric motor has revolutionized industry and enabled the development of motor driven system, as power transmission using shaft, belt or hydraulic pressure can be achieved by using electric motor. Thus improving power transmission efficiency and providing easy control for application. Electric motor has been applied to other fields as well, for example it is used in agriculture to eliminate human muscle power or in household appliances to improve on standard of convenience and safety.

Today, electric motors consume more than half of all electric energy produced. In fact, motor driven systems use 75% of the energy consumed by industries. Induction motor drives are the most widely used electrical drives system and

generally consume 40% to 50% of the total generating capacity of an industrialized nation. In the US, the total generating capacity is approximately 800,000 MW [1], [2]. As a result, motor driven systems are major assets for an industrialized nation. Due to vital role of motor driven machineries, a comprehensive electric motor modelling technique can provide better understanding in motor performance and contribute in optimizing motor design.

In modelling of electric motor, various modelling techniques have been developed. Equivalent circuit model is one of the most commonly used techniques. Much researches had been carried out to study the electric motor's performance by using equivalent circuit model. By studying a model which is fully characterized the motor condition provides better understanding on the motor's performance and its health condition. A proper modelling is the foundation of reliable condition monitoring method in analysis mechanical behaviour of an electric motor. The dynamic behaviour of induction motor's parameters under various motor's fault level may be different. Also, motor parameters may vary due to different loading conditions. With appropriate modelling on induction motor system, the relationship of motor's parameters with motor and loading condition can be studied.

It is reasonable that electric motor received much attention as electric motor is the major component in a motor driven system, where electrical energy is converted into mechanical energy to exert physical work. However, power transmission system and mechanical loading also contribute significantly in the analysis of performance of a motor driven system. Therefore, it would be desirable to extend the modelling to other components in motor driven system for better understanding of power flow within the system.

Modelling of electric motor enables the developments of new motor with improved performance. However, in order to improve the performance of in-service electric motor, condition monitoring and fault diagnosis techniques can

contribute significantly in this situation. Nowadays, many monitoring techniques are developed for motor driven system monitoring and fault diagnosis. The manufacturers are very keen to integrate monitoring and diagnosis system into electrical machines in order to improve efficiency and stability. Motor's input current is the most popular online monitoring signature for electrical motor, which is known as Motor Current Signature Analysis (MCSA). Other monitoring signatures such as speed, torque, noise and vibration are also explored. Most of these techniques are to discover the nature and the degree of fault. Fault diagnosis system is gradually changed to automated system from slow human involvement. For instance, development of automated tools such as Artificial Neural Networks (ANNs), Fuzzy-logic and etc help to increase efficiency of fault diagnosis process.

The major faults of electrical motors can generally be categorized into several groups. Some studies have been done to statistically classify motor faults and researchers show that bearing faults is one of the most common faults. Another two common faults are stator and rotor failure [1]. Monitoring and identifying these motor faults have been developed for years. Researchers are focusing in improving the monitoring technique for a better on-line monitoring of machines' condition. MCSA is able to monitor electrical machines' condition but the results are affected by the variation of input voltage. Also MCSA requires accurate slip information in order to compute motor faults frequencies and only able to analyse stationary data due to Fourier Transform. Therefore, another monitoring technique has been introduced in this project based on the model of induction motor with the intention of improving motor monitoring technique.

In this research, a transduction matrix model of induction motor is introduced, which is based on the idea of frequency response function. With the information from both electrical inputs and mechanical outputs as shown in *Figure 1-1*, the motor can be modelled into four frequency response functions which characterise the condition of the motor. In addition, transduction matrix

modelling can be applied in motor driven systems for better understanding and condition monitoring of motor loading, thus impedance and power flow within the motor driven system can be studied.

Figure 1-1: Schematic Diagram for Induction Motor System

Moreover, transduction matrix modelling techniques could be useful in fault diagnosis as well as quality inspection in motor production. In fact, transduction matrix modelling allows impedance flow within the motor driven system to be obtained, which describes dynamic properties of the system as well as its loading condition. Therefore, impedance would be suitable in performance analysis and fault diagnosis of a motor driven system. For instance, mechanical impedance Z_m from mechanical torque T and rotational speed ω allows the loading condition to be recognized. On the other hand, electrical impedance Z_e from voltage E and current I describes both motor condition and loading condition. Thus signals from electrical input port are able to provide enough information in order to monitor motor condition and diagnose motor faults. Consequently, electrical input impedance is introduced as an online monitoring signature which is more sensitive and accurate than input current in fault diagnosis of induction motor. In addition, electrical input impedance is rather easy and cost effective to be collected compare to mechanical outputs such as torque and speed.

1.2 Objectives

This research aims to study the capability of using transduction matrix for modelling a motor driven system. Firstly, the induction motor is modelled by

transduction matrix from governing equations of its equivalent circuit. The properties of transduction matrix of motor are studied since it describes the condition of motor. The four transduction functions are determined and its relationship with motor condition is studied. In addition, relationship between transduction functions with input impedance and efficiency are discussed. The theoretical expression of transduction matrix is constructed and compared with experimental results.

The transduction matrix modelling also extended to motor driven systems such as power transmission mechanism and mechanical loading. Power transmission mechanism such as gear trains or mechanical linkage is modelled using transduction matrix. Subsequently, power transfer and impedance change along the motor driven systems are studied based on its transduction matrix.

This research also aims to develop a motor monitoring technique which utilizes input impedance as a new monitoring signature. Experiments are set up in order to study the capability of motor fault detection by using input impedance. Hence the pros and cons of using input impedance signature are elaborated by comparing the results with existing technique, Motor Current Signature Analysis. Furthermore, advanced signal processing such as wavelet transform is utilized to improve the motor fault detection.

1.3 Scope

This research will focus on transduction matrix modelling on three phase squirrel cage induction motor and simple motor driven mechanisms such as gear and four bar linkage. Condition monitoring of alternating current three phase squirrel cage induction motor is focused because it is the most commonly used induction motor in industry. Impedance spectrum of various faults such as bearing fault, stator fault and rotor fault are analyzed and compared. MCSA is carried out in motor fault detection for reference and

comparison, because MCSA is currently the most widely used condition monitoring technique.

1.4 Overview of Report

Chapter 1 illustrates the background of this research on transduction matrix modelling and impedance monitoring on induction motor. The objectives and scope of this research is indicated. In the following chapter, basic motor theory and theoretical background are discussed to provide better understanding on squirrel cage induction motor. The literatures on existing modelling techniques and monitoring technologies are also presented in this chapter.

Modelling of induction motor by transduction matrix and the properties of transduction matrix are revealed in Chapter 3. Transduction matrix obtained from experiment is also presented in this chapter. Transduction matrix modelling on power transmissions mechanism and mechanical loading are illustrated in Chapter 4. Chapter 5 utilizes the transduction matrices developed previously to study the impedance change and power flow in a motor driven system.

In Chapter 6, electrical input impedance is used as the monitoring signature for induction motor fault detection. Additionally, discussions and comparison with existing monitoring techniques MCSA are included in this chapter. Chapter 7 discusses the time-frequency analysis for analytic signal of impedance signals by using wavelet packet transform.

Chapter 8 consists of the conclusions from all the results and discussions have been made. It also includes recommendations for the future works which includes development of software coding for pattern recognition, and power and energy management of motor system.

CHAPTER 2

INTRODUCTION TO INDUCTION MOTOR SYSTEM

This research focuses on the study of modelling and condition monitoring of a motor driven system, which consists of induction motor, power transmission system and mechanical loading. Therefore brief introduction on basic motor theory is presented in this chapter. Three phase circuits and rotating magnetic field in induction motor is studied for utilization in modelling of induction motor. Some commonly used motor modelling techniques are introduced such as equivalent electrical circuit modelling. These techniques aim to model a physical machine into simple mathematical models. Subsequently, existing monitoring and fault diagnosis techniques have been reviewed. Since motor monitoring is always related to possible faults on induction motor, some common motor faults will be elaborated.

2.1 Theoretical Background of Electric Machine

This section is to introduce and discuss some of the principles underlying the performance of electric machinery. These principles are applicable to both dc and ac machines. In electrical machines, electrical energy converts into mechanical energy such as force or torque. Most rotating machines consist of windings or groups of coils which rotate due to rotating magnetic field. The stationary part in rotating machines, which is known as stator consists of a group of individual electro-magnets arranged to form a hollow cylinder. The stator windings are arranged in a way that a rotating magnetic field will be created when current flows through the circuit. Rotor is the rotating electrical component, and consists of a group of electro-magnets facing towards the stator poles. When the polarity of stator poles is changed progressively to produce a rotating magnetic field, the rotor will follow and rotate with the magnetic field of the stator. *Figure 2-1* below is a simple representation of stator and rotor.

Figure 2-1: Basic Electrical Components of an Induction Motor

2.1.1 Electrodynamics Principles

In an electric machine, the motor converts electrical energy to mechanical energy, whereas generator is the reverse process which converts mechanical energy into electrical energy. However, the basic principles still remained the same as most of electric machines work by electromagnetism. For instance, Lorentz Force Law stated the electromagnetic force on a test charge at a given point and time is a certain function of its charge and velocity. Based on this principle, the Electromagnetic force acting on a conducting body with currents flowing through can be expressed by Eq. (2.1). The electromagnetic force is the cross product between current flowing over a length of conducting body and magnetic field. This is the basic of electromechanical energy conversion process, rotating machines or linear motion transducers work in the same way. [3]

$$F = IL \times B \quad (2.1)$$

2.1.2 Types of Electric Machine

Generally, electric machines can be categorized into two main types, namely direct current machines (DC) and alternating current (AC) machines. In a DC machine, a set of coils which is known as armature windings is mounted on the rotor. Field winding which carries DC current and is used to produce main magnetic flux is found in stator. Permanent magnets are also able to produce dc magnetic flux and they are used in field winding in some machines. The most common types of DC motor are brushed and brushless types, which use internal and external commutation respectively to create an oscillating current from the DC source. [4]

AC machines are the most common type of machines used in industrial as well as in home appliances. High power output, less expensive, durability and direct connection to AC power source are the main advantages of AC machines. As mentioned in the previous section, AC motor consists of two basic components. An outer stationary stator coil is supplied with AC current in order to produce rotating magnetic field, and an inner rotor attached to the output shaft that is given a torque by the rotating magnetic field. One of the most popular rotor types is squirrel cage rotor, its rotor bars are connected together mechanically and electrically by the use of rings. Almost 90% of induction motors have squirrel cage rotors, this is because squirrel cage rotor has a simple and rugged structure. Therefore a squirrel cage induction motor is studied in this research. [5]

2.1.3 Synchronous Speed and Slip

In stator, a rotating magnetic field is created by AC current supplied through groups of coils. This magnetic field rotates in a synchronous speed, N_s , and it is determined primarily by the frequency of AC supply and the number of pole pairs in the stator winding.

$$N_s = 60 \times \frac{f}{p} \quad (2.2)$$

Where f is AC supply frequency in Hz and p is the number of pole pairs in the stator. In a squirrel cage induction motor, the rotor always rotates in the same direction as that of the stator and tries to follow the rotating flux. In practice, the rotor runs slower than the speed of stator field, this speed is known as rotational speed or base speed, N_r . Hence the actual rotational speed of an AC induction motor will be less than calculated synchronous speed by an amount known as slip.

$$s = \frac{N_s - N_r}{N_s} \quad (2.3)$$

The slip is increased with torque produced. When there is no load, the slip will be small and the rotational speed will be very close to synchronous speed. When the load is increased, the rotor slowed down and increases the slip. A standard induction motor has 2% to 3% of slip. The motor slip describes the loading condition of an induction motor and it is important in identification of motor fault's characteristic frequencies.

2.1.4 Rotating Magnetic Field

In AC induction motor, one of the common methods to produce rotating electromagnetic field is to utilize a three phase power supply. At any instant, the electromagnetic field generated by single phase depends on current flows through that phase. Each of the phases is 120° out of phase, this three electromagnetic fields will combine to produce one rotating field which is acting on rotor. Through electromagnetic induction, the rotating magnetic field induces a current in the conductors in the rotor, which sets up a counterbalancing magnetic field that cause the rotor to turn in the direction of the field is rotating. More discussions on rotating magnetic field can be found in [6]. Because three phase circuit is used in induction motor, the voltage and current in the circuit will be discussed thoroughly in the following section.

2.1.5 Three Phase Circuit

Generally, a three phase circuit consists of three individual voltage sources which may each be connected to its own circuit. Each of the individual circuit can be considered as a single phase system. There are two possible arrangements of three phase windings, which are shown in *Figure 2-2*: (a) is Δ -Connection and (b) is Y-Connection. The three phase voltages supplied to the circuits are equal and displaced in phase by 120° , which is a general characteristic of a balanced three phase system. An unbalanced three phase system may be caused by unbalance source voltages or unbalance in impedance, either in magnitude or phase. Most of the practical analyses are carried out by assumption of balanced system. However, for a three phase induction motor with motor fault such as broken rotor bar or stator winding

fault, the impedance may not be equal in all the three phase circuits. This leads to unbalance loading and affects resulting current supplied to each single phase.

(a) Balanced Δ load, $Z_A = Z_B = Z_C = Z_\Delta$ (b) Balanced Y load, $Z_A = Z_B = Z_C = Z_Y$

Figure 2-2: Three Phase Connection: (a) Δ -Connection and (b) Y-Connection

The equations of voltage and current for both circuit connections are shown as follows. In the equations, V_L and I_L are line-to-line voltage and current, whereas V_ϕ and I_ϕ are line-to-neutral voltage and current.

Balanced Δ -Connection Circuit:

$$V_L = V_\phi \quad (2.4)$$

$$I_L = \sqrt{3}I_\phi \quad (2.5)$$

Balanced Y-Connection Circuit:

$$V_L = \sqrt{3}V_\phi \quad (2.6)$$

$$I_L = I_\phi \quad (2.7)$$

In term of power, the power of each phase is equal to each other and the total power for a balanced three phase system is three times of the average power per phase.

Power per Phase:

$$P_{\phi} = V_{\phi} I_{\phi} \cos \theta \quad (2.8)$$

Total Power:

$$P_t = 3V_{\phi} I_{\phi} \cos \theta \quad (2.9)$$

$$P_t = \sqrt{3}V_L I_L \cos \theta \quad (2.10)$$

Total power of balanced three phase system can be described in term of line-to-neutral voltage and current or in terms of line-to-line voltage and current. ‘ $\cos \theta$ ’ is referred to power factor which is the phase angle between voltage and current. The total instantaneous power of three phase system is constant and does not vary with time, which is an advantage of polyphase system. The understanding of three phase circuit helps in modelling induction motor when total power is concerned. In addition, it is important to identify the types of three phase connection before measuring the input voltage and current.

2.2 Motor Driven System

A motor driven system consists of electric motor, mechanical transmission system and mechanical load. The electric motor converts electric power to mechanical power in order to drive mechanical loads at its output, which has been discussed in the previous section. The mechanical transmission system plays a role in transferring mechanical power to loading. Generally, mechanical power can be transmitted directly by engineering mechanisms such as driveshaft, transmission gear, mechanical linkage and etc. The amount of torque and speed transmitted can be adjusted by different setup of the transmission system. Besides, hydraulic and pneumatic systems are also used for power transmission, they are using liquid under pressure and compressed air respectively. In the following section, some commonly used engineering mechanisms are introduced.

2.2.1 Engineering Mechanisms

A mechanism is a machine which consists of two or more components arranged together in order that the motion of one compels the motion of the others [7]. The purpose of mechanism is to transfer motion and mechanical work. Gears and gear trains, belt and chain drives, cam and follower and linkages are some of the examples of engineering mechanism. Gears are used to transmit mechanical power from one end to another, especially when the velocity ratio is constant. By using gears, the magnitude, speed and direction of mechanical power can be changed so that mechanical advantage can be produced at output end. In recent literatures, gear trains received substantial attention in wind turbine, where dynamic behaviour of the gear trains are focused [8], [9]. More examples on gear trains such as helical and worm gears can be found in [10].

Mechanical linkage comprises of several bodies connected together for the purpose of transforming a given input force and movement into a desired output force and movement. A linkage shares the same benefit of gear in providing mechanical advantage at the output, however mechanical linkage is always used for the situation where velocity ratio is not constant. For example, a crank-slider linkage is able to transfer a rotational motion into linear motion. Crank-slider linkage is one of the example of a four bar linkage, other examples of four bar linkage are crank-rocker, double-rocker, drag link, etc.

Figure 2-3: Four Bar Linkages in Micro-Positioning XY Stage

The applications of linkage can be found in robot designing. For instance, four bar linkages were used to control the position and stiffness of a robot joint [11]. Four bar linkage is also useful in micro-positioning, as a piezo driven XY stage was designed in [12], which is shown in *Figure 2-3*.

2.3 Modelling of Motor Driven System

Modelling of motor driven system provides a way to understand elements inside the system easily which is divided into a simpler form. Hence, a motor driven system with improved performance and better design can be achieved with accurate models. In addition, a proper modelling is the foundation of reliable fault diagnosis method in analysis mechanical behaviour of a faulty motor driven system. Some existing modelling techniques on motor driven system and its ability in monitoring the system's condition are discussed in this section.

2.3.1 Electric Motor

The electric motor is an essential element in motor driven system, thus modelling of electric motor especially induction motor deserved more attention in this research. Various modelling techniques have been applied on three phase dynamic system in induction motor, such as mathematical models which utilizes transformation of coordinate system to describe the three phase system. There are also finite element analyses on magnetic flux at air-gap. Modelling of induction motor not only provides information about the system's behaviour, but is also useful in understanding the correlation between the motor faults with fault signatures obtained. For instance, prediction of motor faults can be carried out based on simulated magnetic flux distribution or from the torque information obtained from parameters estimation. However, modelling of induction motor becomes complicated due to three phase circuit as well as dynamic behaviour of magnetic flux. The following section introduces some models used in three phase induction machines.

2.3.1.1 Electric Circuit Model

In induction motor, three phase power supply generates rotating magnetic field by groups of winding circuits. Hence the rotating magnetic field drives the rotor in order to provide torque and speed. In this case, the induction motor can be modelled in equivalent circuit which consists of resistance for electrical loss, inductance for magnetic field and voltage source. Indeed there are many researches are carried out using equivalent circuit model, especially in parameter estimation for induction motor.

Figure 2-4: Equivalent Circuit of a Squirrel Cage Induction Machine

The equivalent circuit shown above is used by E.B.S. Filho et al. [13] in the estimation of the parameter of squirrel cage induction machine. By relating the circuit parameters with loading condition, the estimation of parameters is achieved by non-linear least squares minimization. The researches were able to correlate estimated power slip curves with experimental results. Similar modelling has been carried out by K. M.-Kaminska et al. [14] where electromagnetic torque is expressed in term of circuit parameters. Hence the parameters are estimated by iterative process using Gauss-Newton Method. In general, these researches utilized equivalent circuit for induction motor modelling but different technique in parameters estimation. From parameter estimation, the induction motor's torque is computed and hence its performance can be simulated.

Another mathematical modelling is the well known d - q model of induction machines. It is a transformation of coordinate system from three phase a - b - c frame system to two phase d - q frame system. A generalized equivalent circuit

equation on an arbitrarily rotating frame is used to describe relationship between stator and rotor in two phase frame. Hence the three phase system can be solved in two phase system by the transformation. The following shows the equivalent circuit equations and transformation of the coordinate system [15]-[18], where L_s , L_r , and L_m are stator, rotor and mutual inductance respectively. ω_c is the frequency of arbitrary reference frame and ω_r is the rotor frequency:

Figure 2-5: Two and Three Phase Coordinate System

Equivalent Circuit Equation:

$$\begin{bmatrix} v_{qs}^c \\ v_{ds}^c \\ v_{qr}^c \\ v_{dr}^c \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_c L_s & L_m p & \omega_c L_m \\ -\omega_c L_s & R_s + L_s p & -\omega_c L_m & L_m p \\ L_m p & (\omega_c - \omega_r) L_m & R_r + L_r p & (\omega_c - \omega_r) L_r \\ -(\omega_c - \omega_r) L_m & L_m p & -(\omega_c - \omega_r) L_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs}^c \\ i_{ds}^c \\ i_{qr}^c \\ i_{dr}^c \end{bmatrix} \quad (2.11)$$

Transformation matrix:

$$[T_{abc}^s] = \frac{2}{3} \begin{bmatrix} \cos \theta_c & \cos \left(\theta_c - \frac{2\pi}{3} \right) & \cos \left(\theta_c + \frac{2\pi}{3} \right) \\ \sin \theta_c & \sin \left(\theta_c - \frac{2\pi}{3} \right) & \sin \left(\theta_c + \frac{2\pi}{3} \right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (2.12)$$

This sophisticated but precise mathematical model is formed based on several general assumptions, such as:

- Uniform air gap eccentricity
- Balance rotor and stator windings with sinusoidal distributed magnetomotive force
- Inductance and rotor position is sinusoidally related
- Negligible saturation and parameter changes
- Negligible inter-bar current

Several researches had been carried out in analysis of induction motor's fault by using this d - q modelling. For instance, A. R. Munoz et al. [19] utilized the d - q model which coupled between stator and rotor circuits, together with complex space vector to solve for rotor bar currents. Subsequently, the electromagnetic torque can be expressed in terms of rotor-stator mutual inductance matrix as well as stator and rotor current. The researchers proved his modelling required less computation time than full matrix model.

There are more studies performed by using such mathematical model in induction motor analysis [20]-[23]. However, these models have some drawbacks due to limitations in assumptions made. In d - q model, air-gap eccentricity is assumed uniform and hence this limits the analysis from abnormal air-gap eccentricity. In addition, core saturation and inter bar current have considerable effect on fault diagnosis in induction motor which have been neglected in assumptions. Improved modelling techniques such as winding function have been introduced and utilized for broken rotor bars and eccentricity faults [24], [25].

2.3.1.2 Finite Element Model

Development of powerful computational ability of modern computer has encouraged the introduction of numerical modelling in induction motor. The finite element modelling, which is well established for analyse dynamic

behaviour of induction motor. This tedious method is able to provide accurate evaluation of the magnetic field distribution inside the motor. Most of the motor faults will lead to perturbation in magnetic flux distribution, thus enable evaluation of the fault presence.

J. Cai et al. [26] studied transient response of radial force and torque for bearing-less wound-rotor induction motor by transient finite element method. The rotating machine is modelled by two dimensions electromagnetic field based on magnetic vector potential approach. The researchers extended the research with comparing the transient response of wound-rotor and squirrel cage rotor. FEM analysis can be used together with state space concept, the d - q model which was introduced previously [27]. The researchers utilized this combined model to predict the no load, blocked rotor, and load operating performance of induction motor.

In fault diagnosis, FEM has been developed to predict the perturbation of magnetic flux due to motor faults [28], [29]. J. Cusido et al. [28] simulated the magnetic field by magnetic vector potential and analysed by using CAD software. The researchers studied the effect of unbalance rotor and broken rotor bars, as illustrated in *Figure 2-6* below.

Figure 2-6: Flux Distribution for Rotor Unbalance (Left) and Current Distribution of Three Broken Rotor Bar (Right)

2.3.1.3 Transfer Matrix Model

Transfer matrix modelling relates response and excitation of a system, and it describes the dynamic properties of the system. Transfer matrix is well established in modelling complicated electric circuit, and progresses into mechanical system such as vibration and acoustic analysis. However, this modelling technique is not so popular in induction motor compared to the equivalent circuit and FEM modelling. Y. L. Kuo et al. [30] developed a transduction matrix based on equivalent circuit and mechanical governing equations of a DC motor as shown below. With the use of transduction matrix on DC motor, the researchers were able to analyse the output of the DC motor and used it to evaluate mechanical impedance of different road surface.

$$[t] = \begin{bmatrix} -\frac{Ls + R}{K_T} & K_E \\ \frac{1}{K_T} & 0 \end{bmatrix} \quad (2.13)$$

For AC induction motor wise, C. B. Jacobina et al. [31] tried to describe three phase circuit by transfer matrix which relates between current and voltage at each phase. Consequently, the researchers studied the surge distribution of three phase circuit due to power supply surge. Furthermore, the d - q model is applied and extended in transfer matrix modelling. Based on the coordinate transformation and complex transfer function which model a balanced three phase impedances system, frequency response and phase shift of the model can be predicted. This is particularly useful in study control system such as current controller for induction motor [32], [33]. Equations below show the impedance of a balanced resistance-induction circuit of induction motor and open loop transfer matrix of the model [35].

$$Z(s) = (R + sL)I + J\omega_1 L = \begin{bmatrix} R + sL & -\omega_1 L \\ \omega_1 L & R + sL \end{bmatrix} \quad (2.14)$$

$$G_k(s) = \frac{K(s + 1/T_i)e^{-sT_d}}{s(s^2 + \omega_1^2)L_d L_q} \begin{bmatrix} sL_q & \omega_1 L_q \\ -\omega_1 L_d & sL_d \end{bmatrix} \quad (2.15)$$

Modelling of induction motor has been developed extensively and most of the techniques involve complicated computation. Among those modelling techniques, transduction matrix model by Y. L. Kuo [30] is simple and comprehensive to describe dynamic properties of motor. However that is because DC motor does not consist of troublesome three phase circuit. In later stage of this project, a squirrel cage induction motor will be modelled by transduction matrix where the dynamic properties of motor would be described by four transduction functions. Hence the condition and performance of the induction motor can be analyzed from the evaluation of these functions.

2.3.2 Power Transmission System

Modelling of power transmission system can be carried out in various methods, depends on the focus of analysis. Some modelling techniques excel in force and stress analysis such as FEM which is useful in the design of transmission system. Other modelling techniques like bond graph modelling do well in power flow analysis. Some literatures on modelling of mechanical transmission system will be discussed in the following section.

Figure 2-7: A Lumped Mass Dynamic Model for Two Stage Spur Gear System

In mechanical transmission system such as gear trains, most of the modelling techniques are performed on the overall transmission system comprised of

several gear trains. For instance, a lumped mass dynamic model as shown in *Figure 2-7* is used to investigate the dynamic behaviour of a two stage gear system in a wind turbine [8]. Based on the model, the gear teeth deflection and fluctuation of gear ratio are studied under various kinds of wind excitation, which are important factors to be considered in the design of gear box system. Other similar research based on the distributed lumped parameter model can be found in [34], where the dynamic performance of driveshaft in power transmission system is investigated.

Finite element model (FEM) also applicable in analyzing power transmission system. For instance, a wind turbine gear box is studied by the combination of flexible multibody models and finite element model in [35], to gain sufficient insight in the dynamic of the gear trains. The FEM analysis is mainly used to compute the components stiffness values in the gear box model thus allows analyses such as shaft torsional and bending deformation to be carried afterwards. Application of FEM in mechanical linkage analysis can be found in [36]. In the analysis, FEM is used in non-linear problem in displacement analysis of a four bar linkage with flexible-link mechanism. However, most of the FEM analyses focus on the structure of transmission system rather than the performance or energy flow in the system.

Bond graph is another popular analysis used in transmission system, especially useful for comprehensive power flow analysis. In [37], a planetary gear train is modelled by multi-bond graph diagrams. The researchers make use of the dynamic model from bond graph, together with kinematic and kinetic equations in order to conduct power flow analysis. Besides of gear trains, bond graph model is also applicable in modelling mechanical linkage. A crank-slider mechanism is expressed in bond graph representation as shown in *Figure 2-8*, thus the dynamic equations for the system are derived with the help of Lagrange equations [38]. More examples of bond graph modelling can be found in [39]-[42]. One of the advantages of using bond graph modelling is the systematic analysis manner, which is suitable to be used in computer

simulation. In addition, the flow of energy information in bond graph makes it suitable to analyze the power and energy path in power transmission system.

Figure 2-8: Crank-Slider Mechanism and its Bond Graph Representation

From the literature reviews, various modelling techniques for induction motor and power transmission system are introduced. Most of the modelling techniques are able to describe the condition of modelled system. In fact, some models are utilized for condition monitoring. Condition monitoring of induction motor will be focused in latter section, because it is an essential component in motor driven system. Before that, some common induction motor faults are studied first.

2.4 Common Motor Faults

This section discusses about the common faults found in three phase induction motor, as it is the focus in this research. Due to mechanical, thermal and electrical stress, motor failures are unavoidable. Unexpected motor failures and unscheduled downtimes lead to expensive maintenance cost and lost of production cost. Study from researchers [1] shown percentage of different types of motor failure shown as follows:

Failure Mode	Percentage
<u>Bearing Related</u>	<u>Total – 41%</u>
Sleeve Bearings	16 %
Anti-Friction Bearings	8%
Seals	6%
Thrust Bearings	3%
Oil Leakage	3%
Other	3%
<u>Stator Related</u>	<u>Total – 37%</u>
Ground Insulation	23 %
Turn Insulation	4%
Bracing	3%
Wedges	1%
Frame	1%
Core	1%
Other	1%
<u>Rotor Related</u>	<u>Total – 10%</u>
Cage	5%
Shaft	2%
Core	1%
Other	2%
All Other	<u>Total – 12%</u>

Table 2-1: Percentage of Failure Modes as given by Major Components

2.4.1 Bearing Faults

Bearing failure is the most common fault of induction motor. Improper installations such as misalignment or deflection of shaft, as well as improper tilt of inner and outer race are the main reasons for physical bearing failure. For a bearing fault in a motor, it will allow the shaft to vibrate radically. The mechanical displacement resulting from damaged bearing causes the machine air gap geometry to be slightly disturbed and leading to a modulation of the current. This abnormal harmonic frequency of a bearing can be derived as follows [43]-[45]:

$$f_{bearing} = f_0 \pm mf_{i,o} \quad (2.16)$$

where f_0 is fundamental supply frequency, $m = 1, 2, 3$, and $f_{i,o}$ is characteristic vibration frequencies which are based on bearing dimension as follow:

$$f_{i,o} = \frac{n}{2} f_r \left[1 \pm \frac{BD}{PD} \cos \beta \right] \quad (2.17)$$

where n is number of bearing balls, f_r is mechanical rotor speed in hertz, BD is ball diameter, PD is bearing pitch diameter and β is the contact angle of the balls on the races.

From *Eq. (2.17)*, the specification of bearing construction is required in order to compute the exact characteristic frequencies. However, these characteristic frequencies can be simplified for approximation of bearings with 6 to 12 balls [44].

$$f_o = 0.4nf_r \quad (2.18)$$

$$f_i = 0.6nf_r \quad (2.19)$$

where n is integer. Generally, abnormal harmonic frequencies of bearing fault are in higher frequency band than those of rotor and stator faults. J.-H Jung and his colleges [45] identified abnormal harmonic frequencies of cracked bearing balls by using motor current signature analysis (MCSA)

2.4.2 Stator Faults

Stator winding failure is another major motor failure which comprises of 37% of the total. These faults are generally related to insulation failure which will lead to turn-to-turn fault (Turn Insulation) or phase-to-phase fault (Ground Insulation). One of the major reasons of stator winding's insulation degradation is thermal overloading [46]. In order to extend insulation life, precaution on thermal overloading conditions has to be taken into consideration. S. Nandi [43] has broadened the causes of stator windings failure to loose bracing for end windings, contamination, short circuit, electrical discharge and etc.

Stator winding faults such as turn fault will cause asymmetry between the three phases and leading to negative sequence current. Consequently, this results in negative magneto-motive force (MMF) which reduces the net MMF of the motor phase. Distortion of MMF changes the waveform of air-gap flux

and induces harmonic frequencies in stator winding current. The induced harmonic frequencies can be computed as follows, where $n = 1, 2, 3, \dots$, and $k = 1, 3, 5, \dots$, respectively:

$$f_{stator} = \left\{ \frac{n}{p}(1-s) \pm k \right\} f_0 \quad (2.20)$$

G. B. Kliman [47] and J.-H. Jung [45] utilized the negative sequence current for detection of stator faults. However, detection of stator faults required high resolution of stator current spectrum. In general, the abnormal harmonic frequencies of stator faults occur under frequency range under 1 kHz.

2.4.3 Rotor Faults

Almost 10% of total motor failures are related to rotor faults. Broken rotor bar and end ring breakage are common in rotor faults. The reasons behind these breakages are mainly due to thermal, magnetic, dynamic, mechanical and environmental stresses on the motor. A rotor bars failure does not initially cause a motor to fail, however it can lead to serious secondary effects. For example, the broken parts of the bar may hit the stator winding at high velocity. This can cause a serious mechanical damage to the insulation and consequential winding fault may follow, hence resulting in costly maintenance and lost in production. Rotor bars defects are detected by monitoring the motor current spectral components produced by the magnetic field irregularity of the broken bars. When there is a rotor fault, the flux distribution in the air gap is disturbed and resulting in a quite predictable set of frequencies in the line current. The broken rotor bar frequencies are given by [43], [45], [47]:

$$f_{rotor} = (1 \pm 2ks)f_0 \quad (2.21)$$

where $k = 1, 2, 3, \dots$. Lower sideband of the rotor frequencies is particularly related to broken rotor whereby upper sideband is due to consequent speed oscillation. However, more than one researcher has discovered other harmonic frequencies due to rotor faults which are given by [43], [44], [47], [48]:

$$f_{rotor} = \left\{ \frac{k}{p}(1-s) \pm s \right\} f_0 \quad (2.22)$$

These rotor frequency sidebands are further away from fundamental supply frequency and they are relatively easier to be identified compared to rotor frequencies from Eq. (2.21). Nevertheless, these rotor frequency sidebands are easily confused with abnormal harmonic frequency due to dynamic eccentricity of rotor [47].

2.4.4 Abnormal Air-Gap Eccentricity

Abnormal air-gap eccentricity is a motor fault where irregular air-gap happens between stator and rotor. Subsequently these lead to unbalance radial force and unbalance magnetic flux distribution which can damage stator and rotor. There are two types of abnormal air-gap eccentricity, namely static eccentricity and dynamic eccentricity. Static eccentricity is constant but uneven radial air-gap which fixed in space. It appears when stator core is in irregular shape or improper installation of rotor. For dynamic eccentricity wise, the radial air-gap changes with rotational of rotor. Dynamic eccentricity happens when the centre of rotor is not coinciding with centre of rotation, this might be due to bending of rotor shaft or misalignment.

The presences of static and dynamic eccentricity give rise to abnormal harmonic frequencies in stator current. There are two methods to detect the air gap eccentricity, which are monitoring the sidebands of the slot frequencies or monitoring the behaviour of the current at the fundamental sidebands of the supply frequency. The frequency components for the sidebands of slot frequencies can be determined as follows [43], [44]:

$$f_{slot,ecc} = f_0 \left[\left(kR \pm n_d \right) \frac{(1-s)}{p} \pm n_\omega \right] \quad (2.23)$$

where k is integer, R is the number of rotor slots, n_d is known as eccentricity order whereby $n_d = 0$ for static eccentricity and $n_d = 1, 2, 3, \dots$, for dynamic

eccentricity, and $n_\omega = \pm 1, \pm 3, \pm 5, \dots$, is the order of stator time harmonics. Sidebands of slot frequencies are separated from spectral components which are caused by broken rotor bars hence they are easier to be identified. However, the disadvantage is motor design parameters such as rotor slot number is required in the calculation.

On the other hand, abnormal air-gap eccentricity will be reflected on fundamental sidebands of supply frequency as follow:

$$f_{ecc} = f_0 \left[1 \pm m \frac{(1-s)}{p} \right] \quad (2.24)$$

where $m = 1, 2, 3, \dots$. These low frequency components do not required detailed information of motor design but it can be confused with rotor frequencies.

2.4.5 Other Faults

There are some other motor faults such as electrical faults, loading, bad coupling, weak mounting and etc. However, most of the researches that have been done so far do not consider these faults. One of the main reasons is these faults only contribute to a small percentage in total motor faults. Another reason is these faults are considered as minor motor faults which may not lead to critical motor failure.

2.5 Condition Monitoring of Induction Motor

Condition monitoring of induction motor became one of the popular research topics among motor manufacturers. For instance, leading manufacturers in Singapore are moving ahead to service and testing hub instead of traditional motor production. Since monitoring and identifying motor faults have been developed for years, literature reviews on condition monitoring techniques for induction motor are presented. Generally, practical monitoring techniques for induction motor are comprised of mechanical and electrical monitoring

techniques. Sensors have been used to monitoring health condition of motor but they are more often found in expensive or load-critical machine where the cost of continuous monitoring is worthy. Electrical monitoring techniques are concentrating on the use of spectral analysis of stator current which is widely known as Motor Current Signature Analysis (MCSA). Below shows some examples on motor monitoring technique which may involve various types of technology fields [43], [49]:

- Electromagnetic Field Monitoring
- Temperature Measurements
- Infrared Recognition
- Radio Frequency Emission Monitoring
- Noise and Vibration Monitoring
- Chemical Analysis
- Acoustic Noise Measurements
- Motor Current Signature Analysis (MCSA)
- Model, Artificial Intelligence and Neural Network Techniques

Current trend of motor monitoring technique is moving towards sensor-less and online methods. This means difficult installation of mechanical sensors in motor can be avoided and kept away from high-priced sensors. For example, vibration sensor such as proximity probes which is delicate but expensive has become less and less popular in induction motor monitoring system. Next section discusses about motor monitoring technique in detail.

2.5.1 Motor Current Signature Analysis (MCSA)

Motor Current Signal Analysis (MCSA) is one of the popular methods of online motor diagnosis for detecting motor faults. Using MCSA has advantages on simplicity of current sensors and their installations. After acquired input current signal, advanced signal processing techniques such as Fast Fourier Transform (FFT) or Short Time Fourier Transform (STFT) are used in spectral analysis for extracting useful information of motor faults. Diagnosis of motor faults becomes difficult with MCSA because the equations

of harmonic frequencies have unspecified harmonic numbers. Generally, abnormal magnitudes of spectra appear at dominant harmonic frequencies. If the dominant harmonic numbers are not detected prior to diagnosis process, it is unable to diagnose the motor faults. Dominant harmonic numbers of a motor are not the same in other motors. However, it can be determined with comparison of current spectral between healthy motor and known faulty motor.

G. B. Kliman and J. Stein [47] discussed about the techniques for monitoring and diagnosing motor faults by using motor current. They were able to identify rotor faults and distinguished broken rotor bar from rotor asymmetry by comparing higher harmonic sidebands. This paper also proposed a technique to detect stator windings faults by the presence of negative sequence currents. In addition, this paper presented application of MCSA on other rotating machinery such as motor-operated valves and compressor. However, issues such as A/D conversion and other digital signal processing techniques were not included in the research.

Similar research had been carried out by T. G. Habetler et al. [46] where they presented on the current based condition monitoring on stator, rotor, bearings and overload. They went one further step by implement neural network for learning and monitoring the component of stator current. *Figure 2-9* below is the example of rotor faults and bearing faults which the researchers identified. Current spectrums on left hand side shows radial misalignment of rotor will cause an increase in sidebands of rotor frequency. On the right hand side, a bearing with inner race slot will lead to a rise in higher frequency band.

Figure 2-9: Comparison of Current Spectrum between Radial Misalignment (Left) and Bearing Faults (Right)

W. T. Thomson and R. J. Gilmore [2] utilized MCSA to diagnosis motor faults on high power industrial motor which are 2900 kW and up to 3.6 MW squirrel cage induction motors. The researchers indicated frequency and magnitude of the sidebands of broken rotor bar in *Eq. (2.21)* will be affected by different rotor design, wide range of power rating, different loading condition, mechanical load characteristic and driven mechanical components. The research also showed that a serious broken rotor bar problem increases magnitude of rotor sidebands by 20 dB. This paper covers in depth analysis in rotor fault and air-gap eccentricity fault with comprehensive industrial case studies.

S. Nandi et al [43] has reviewed on different types of motor faults and listed out the causes of those motor faults. The researchers were able to demonstrate static and dynamic eccentricities will lead to a rise in low frequency components in *Eq. (2.24)* near the fundamental frequency. However, these low frequency components are only obvious for certain type of machines. *Figure 2-10* below shows the current spectra for a machine with rotor number of 44 and pole pair of 4, which has 50% of static eccentricity. Significant rise in

both low frequency components and high frequency components can be observed if compared with a healthy machine.

Figure 2-10: Current Spectra of Healthy (Above) and Static Eccentricity Fault (Bottom) Machines

Similar researches have been carried out by J.-H. Jung et al [45] and in addition to that, the researches presented an optimal slip estimation algorithm. Since MCSA required high precision of slip frequency information, computation accurate motor slip frequency becomes important to guarantee reliability of diagnosis results. Rotor slot harmonics are used to obtain the information of motor slip frequency, consequently avoids the use of another speed sensor. C. Bruzzese et al [50] carried out MCSA test on rotor fault of induction motor and they showed the magnitude of rotor sidebands frequencies increase while the loading of the motor is increased. In addition, the magnitude of rotor sidebands frequencies change with the number of broken rotor bar. H. Ma et al [51] analyzed current spectrum using zoom FFT (Fast Fourier Transform) and proper window function in order to improve the frequency resolution effectively.

According to all the researches carried out by researchers worldwide, Motor Current Signature Analysis (MCSA) is one of the most powerful online monitoring techniques for motor faults. About 20 years of developments, this technique is well established and has been utilized by induction motor industry. More researches are carrying out to improve the efficiency of MCSA with advanced signal processing algorithm and fault diagnosis algorithm. In this project, MCSA will be used as a reference for motor faults diagnosis results and compared with the proposed monitoring method. Despite of its popularity, there are some deficiencies in its accuracy of motor diagnosis because MCSA requires high precision of slip-frequency information to guarantee the reliability of diagnosis results. Also the stator current should be extracted after motor speed has achieved steady state condition. MCSA also ignored the change of voltage which might contribute to deficit in accuracy.

2.5.2 Other Monitoring Techniques

Plenty of techniques have been developed for condition monitoring of induction motor. For instance, flux sensors have been used to measure the change of magnetic flux at the motor's air-gap. Fault detection is done by diagnosis algorithm on peak ratio of certain frequency components [52], [53]. Noise and vibration monitoring is another monitoring techniques as most of motor faults lead to vibration of the machines. Accelerometer is able to pick up vibration of the machines and hence fault detection is done by spectrum analysis [54], [55]. However vibration monitoring reduces in popularity as installation of sensors are troublesome and not cost effective as well.

Hafezi, H. et al [56] installed digital temperature sensors on the rotor and stator windings on induction motor. The data from thermal sensors is acquired and monitoring temperature rise in rotor as temperature rise is one of the major cause of motor failures. High resolution optical sensing fiber has been used and installed at stator winding for long term temperature monitoring. Duncan, R.G. et al [57] utilized it for monitoring heat load and electrical condition of an industrial motor. Induced voltage [49], [58] and negative sequence

impedance [59]-[61] have been used by researchers as new monitoring signature. However, induced voltage has not yet proved to be a useful parameter for continuous monitoring due to difficulty in obtaining reliable measurements.

2.5.3 Pattern Recognition of Signal Features

In modern industrial, the demand for automated condition monitoring is increasing. In order to achieve automated and reliable condition monitoring, motor faults are required to be identified in early stage. A complete and successful fault diagnosis and monitoring is composed of several algorithms such as frequency search algorithm, fault detection algorithm, etc. Artificial Neural Networks (ANNs) and Fuzzy Logic are some of the common diagnosis algorithm. They are particularly suitable for induction machine where relationship between motor current and speed is nonlinear.

Artificial neural network is a mathematic model based on biological neural network. It consists of a group of interconnected artificial neurons and process information using a connectionist approach to computation. It is normally used to model complex relationship between input and output or to find patterns in data. Artificial neural networks can be trained to perform motor fault detection by learning experts' knowledge using a representative set of data. The difference between the correct decision made by the expert and neural network generates an error quantity. This error quantity is used to adjust the neural network internal parameters in order to yield a better output that is close to correct decision. The neural network can be trained to learn the fault detection by input-output examples. To utilize the neural network, appropriate neural network architecture is selected. Decide on proper initial network weights and training algorithm that allow the neural network to learn. Among those neural network architectures, Feedforward and Backpropagation training algorithm is one of the most popular one.

Based on these properties, a neural network can be trained to cope with all possible operating conditions and used to classify incoming data. Any spectral signature that occurs outside trained clusters is considered as potential sign of motor faults. Postprocessor sends an alarm to user when the fault signatures are observed persistently. A flow chart of ANN based fault diagnosis system is shown below [43]:

Figure 2-11: ANN based Fault Diagnosis

Fuzzy logic is a form of multi-value logic derived from fuzzy set theory to deal with reasoning. With preset fuzzy rules and membership functions, it is able to make a decision with degree of truth ranging from 0 (false) to 1 (truth) [62], [63]. Fuzzy logic provides a simple method to heuristically implement fault detection principles and to heuristically interpret and analyze their results. However fuzzy logic does not provide exact solution to the problems, it is approximation rather than precise. With hybrid neuralfuzzy systems can be used to solve motor fault detection problems with great accuracy and provide heuristic explanation for the fault detection process. Fuzzy logic has two major components, membership functions and fuzzy rules. Membership function is a generalization of the indicator function in classical sets which it represents the degree of truth. Fuzzy rules such as OR and the AND allow evaluation of membership functions. With this evaluation, conclusion may be drawn.

P. V. J. Rodriguez et al. [48] and L. Cristaldi [62] developed a fuzzy logic algorithm for motor fault diagnosis. They are able to use fuzzy logic to analyse the data and diagnosis broken rotor faults accurately.

In summary, modelling of motor driven system is important in understanding its condition and performance. Among those modelling techniques, equivalent circuit modelling of induction motor is very useful and it will be utilized in the transduction matrix modelling in the following chapter. Hence the transduction matrix that describes the induction motor's condition can be used in condition monitoring. From the studies of existing condition monitoring techniques, MCSA provides a convenient way for motor fault detection which avoids difficulty in installation of mechanical sensors. However, this method has some drawbacks such as the effect of input voltage's variation is ignored. Therefore input impedance that representing both the system properties and loading may improve the fault detection of induction motor.

CHAPTER 3

TRANSDUCTION MATRIX OF INDUCTION MOTOR

A transducer is a device which transforms energy from one type to another. For instance, induction motor is an electromechanical transducer that converts electrical energy to mechanical energy. A matrix that is used to describe the transducer's functions of transforming energy is named as transduction matrix. In this chapter, transduction matrix of a squirrel cage induction motor is studied. In general, transduction matrix describes the relationship between the inputs and outputs of an electromechanical transducer in frequency domain. Studying transduction matrix allows better understanding of the system's behaviour, which could be useful in fault diagnosis and quality inspection through the transfer of impedance quantified by the matrix. In addition, transduction matrix relates with power transmission between input energy domain and output energy domain, hence it can be useful for studying the energy flow of a motor system starting from power supply to loading. Since the elements of transduction matrix characterize the motor, determination of these parameters are presented and its relationship with input impedance have been studied.

3.1 Theoretical Background of Transduction Matrix

In an ideal dynamic system which has linear relationship between its response and excitation, its dynamic properties can be described by a frequency response function (FRF). If the system is time-variant, its FRF will be a function of both frequency and time. When this concept is applied to a system with m input variables and n output variables, its dynamic properties can then be illustrated by $m \times n$ of FRFs. These FRFs can be arranged in a $m \times n$ matrix. If this concept is applied to a transducer, the matrix obtained in this way is named as transduction matrix (TM), and those FRFs are known as transduction functions.

3.1.1 Frequency Response Function

Frequency response function is a frequency function of amplitude and phase between the response spectrum and excitation spectrum of a system. In an ideal single input/single output system with $m = 1$ and $n = 1$, where $x(t)$ is input of the system and $y(t)$ is the output, the dynamic properties of physical system are described in terms of linear transformation of the response function $h(\tau)$. A Fourier transformation producing a direct frequency domain description of the system properties as shown below [64], [65]:

Figure 3-1: Ideal Single Input/Single Output System

$$H(f) = \int_0^{\infty} h(\tau) e^{-j2\pi f\tau} d\tau \quad (3.1)$$

$H(f)$ is known as frequency response function. The FRF is generally a complex function with real and imaginary parts. Alternatively, frequency response function contains magnitude $|H(f)|$ and phase $\Phi(f)$ which are commonly referred as gain factor and phase factor respectively. There is few ways to identify FRF from $x(t)$ and $y(t)$, below is a direct way which uses finite Fourier transforms of the original data records. The finite Fourier transforms over a data of period T representing each process are given by:

$$X(f, T) = \int_0^T x(t) e^{-j2\pi ft} dt \quad (3.2)$$

$$Y(f, T) = \int_0^T y(t) e^{-j2\pi ft} dt \quad (3.3)$$

For single input/single output system, it can be described as below:

$$Y(f, T) = H(f) X(f, T) \quad (3.4)$$

$$H(f) = \frac{Y(f,T)}{X(f,T)} \quad (3.5)$$

It is important to note that the frequency response function $H(f)$ of a linear and time invariant system is a function of frequency only. If the system is nonlinear, $H(f)$ would also be a function of the applied input. Furthermore, $H(f)$ would also be a function of time if the system is time variant.

3.1.2 Four-Pole Model

A linear dynamic system with one input and one output terminal where a pair of variables exists will naturally have a four-pole model to describe the input-output relationship. The pair of variables in input and output terminal normally consists of an effort variable (Voltage, Force and Torque) and a flow variable (Current, Velocity and Rotational Speed). Product of effort variable and flow variable leads to power and division among the variables gives impedance at the port. Because four-pole model illustrates the input-output relationship of a system, it can be expressed by four frequency response functions between those input and output variables.

Since this study focuses on transducer such as electromechanical system, a four-pole model is used to represent the electromechanical system as shown in *Figure 3-2*, where E and current I as electrical inputs, force F and velocity V as mechanical outputs. Subsequently, the frequency response functions are arranged in matrix notation as transduction matrix in *Eq. (3.6)*. Other examples of four-pole model can be found in electric circuit [66], acoustics [67], [68] and mechanical system [69]-[72] as well.

Figure 3-2: Four-Pole Parameters for an Electromechanical System

Transduction Matrix:

$$\begin{bmatrix} E_1(f) \\ I_1(f) \end{bmatrix}_{IN} = \begin{bmatrix} t_{11}(f) & t_{12}(f) \\ t_{21}(f) & t_{22}(f) \end{bmatrix} \begin{bmatrix} F_2(f) \\ V_2(f) \end{bmatrix}_{OUT} \quad (3.6)$$

Transduction Functions:

$$t_{11}(f) = \left. \frac{E_1(f)}{F_2(f)} \right|_{V_2=0} \quad t_{12}(f) = \left. \frac{E_1(f)}{V_2(f)} \right|_{F_2=0} \quad (3.7 \text{ and } 3.8)$$

$$t_{21}(f) = \left. \frac{I_1(f)}{F_2(f)} \right|_{V_2=0} \quad t_{22}(f) = \left. \frac{I_1(f)}{V_2(f)} \right|_{F_2=0} \quad (3.9 \text{ and } 3.10)$$

All the transduction functions shown above are the frequency response function between the input and output variables. The frequency response function can be theoretically computed at condition where the other output variable is zero. Note that all the transduction functions are in frequency domain, hence inverse Fourier transform are required if time domain results are interested. According to the transduction matrix in *Eq. (3.6)*, the input variables can be expressed as:

$$E_1(f) = t_{11}(f)F_2(f) + t_{12}(f)V_2(f) \quad (3.11)$$

$$I_1(f) = t_{21}(f)F_2(f) + t_{22}(f)V_2(f) \quad (3.12)$$

Equations above show the input variables depend on both output variables and respective transduction functions. In general, the transduction functions for a linear system are function of frequency only. On the other hand, the transduction functions become function of time and frequency for linear time-variant system, as the system's parameters involved are time-varying. However in some special cases where the system is time invariant and running at single frequency, those complex transduction functions can be simplified to complex numbers at particular frequency. In this case, if time domain data of transduction functions are interested, it can be obtained from direct computation without using inverse Fourier transform, as all the functions in

transduction matrix are in the same frequency. Hence for an electromechanical system running at single frequency, the transduction matrix in Eq. (3.6) can be written as follows.

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \quad (3.13)$$

In addition, transduction matrix can be utilized to illustrate a system composed of cascaded sub-systems, providing all the sub-systems are running at constant frequency. In those systems, the overall transduction matrix is expressed as multiplication of transduction matrix of each sub-system, as illustrated in the mechanical system below.

Figure 3-3: Cascaded Sub-Systems of a Mechanical System

Transduction matrix of each sub system:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}_1 \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}_2 \begin{bmatrix} F_3 \\ V_3 \end{bmatrix} \quad (3.14)$$

Overall transfer matrix:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}_1 \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}_2 \begin{bmatrix} F_3 \\ V_3 \end{bmatrix} \quad (3.15)$$

3.1.3 Graphical Expression of Transduction Matrix

According to Eq. (3.11) and Eq. (3.12), the input variable of a four-pole system is shown depending on both output variables. In geometry, the equations describe a plane in a three dimensions space, where all the points in the plane fulfill the equation. For instance, Eq. (3.11) can be represented by general equation of a plane as shown below, with E , F and V represent voltage,

force and velocity respectively. The corresponding graphical representation is shown in *Figure 3-4*.

$$E = aF + bV \quad (3.16)$$

Figure 3-4: Graphical Representation of General Plane Equation

As shown in the figure, the plane lies on three dimension space with axes of F , V and E . In the graph, constants a and b represent transduction functions. However, they are assumed to be constant and unity in the following graph, for the ease of illustration. This graphical expression of plane applies on both equations in *Eq. (3.11)* and *Eq. (3.12)* for a particular frequency. Different planes will be used for transduction matrix with different frequencies. For a system with a constant operating frequency, the transduction functions are invariant, which means all the operating points of the system are placed within the plane. If any operating point lies outside the plane, it means the system's

properties have changed or the system is running at different frequency. When the plane intersects with E -plane at $F = 0$, the intersection line represents $E = bV$, for which the value of constant b is the slope of the line. Similarly, the value of constant a can be obtained with intersection line on E -plane at $V = 0$.

3.2 Theoretical Transduction Matrix of Squirrel Cage Induction Motor based on Cascaded Matrices

An electro-mechanical system such as induction motor is a transducer which transforms electrical energy to mechanical energy. Its transduction matrix describes system functions between electrical input and mechanical output. *Eq. (3.17)* below is the transduction matrix of an induction motor, t_{ij} is used to represent transduction functions, E_1 and I_1 represent voltage and current in electrical port, T_2 and ω_2 correspond to torque and rotational speed in mechanical port. In this section, the induction motor is assumed to be linear system, and is running at constant frequency under steady state operating condition. Therefore the transduction functions can be simplified as complex numbers at that frequency.

$$\text{Transduction Matrix, } \begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} T_2 \\ \omega_2 \end{bmatrix} \quad (3.17)$$

Transduction functions' value can be obtained via theoretical expression or experimental data. Theoretical expression is formed based on governing equations of the system. On the other hand, those transduction functions' value can be acquired experimentally by applying different loading condition to the system.

3.2.1 Equivalent Circuit of Stator and Rotor

Conventionally, an electric motor is modelled in an equivalent circuit which consists of resistance for electrical loss, inductance for magnetic field and voltage source. Indeed, there are many researches carried out analysis using equivalent circuit, especially in parameter estimation for electric motor.

Equivalent circuit of induction motor consists of stator and rotor components, as well as core loss component and magnetizing component. The rotor is to be replaced by an equivalent rotor with poly-phase winding with the same number of phases and turn as stator but still producing the same magnetic motive force and air gap flux as the actual rotor. This assumption is especially useful in modelling squirrel cage induction motor as the physical rotor phase winding is not obvious. For poly-phase induction motor, it is always assumed that the motor has symmetric poly-phase windings and driven by balance poly-phase power supply [4]. Also, the induction motor is assumed to be perfect with no energy loss, as efficiency is not included in the modelling.

In the following modelling, a steady state single phase equivalent circuit for squirrel cage induction motor (SCIM) shown in *Figure 3-5* is used. From the equivalent circuit, R_s , X_s , R_r and X_r are effective resistance and leakage reactance for stator and rotor respectively. A magnetizing reactance X_M is connected across E_{1s} , and generally assumed to remain constant for small variation of E_{1s} under normal operation condition of motor. T_{1e} and T_{2e} are the number of turns for stator and rotor windings, normally they are assumed to be equal for most of the induction motor. Core loss resistance and the associated core loss effect in electromechanical power have been omitted in this simplified equivalent circuit. This is due to the presence of air gap between stator and rotor leads to relatively higher leakage reactance. Hence core loss resistance is negligible compared to high leakage reactance.

Figure 3-5: Single Phase Equivalent Circuit of Induction Motor

In induction motor, the relative motion between stator and rotor has to be considered by taking slip frequency, s into account. In equivalent stationary rotor as view from stator, the leakage impedance of referred rotor with the consideration of slip frequency is shown below:

$$Z_r = \frac{E_{2r}}{I_r} = \frac{R_r}{s} + jX_r \quad (3.18)$$

In Eq. (3.18), the reflected resistance, R_r/s represents the combine effect of shaft load and rotor resistance. Hence the power delivered to this reflected resistance is equal to the total power transferred across the air gap from the stator. Total rotor loss has to be taken into account for useful power delivered by the motor. Thus, the electromagnetic power, P_{mech} developed by the motor can now be determined by subtracting the rotor dissipation loss from air-gap power.

$$P_{mech} = P_{gap} - P_{rotor} = \eta_{ph} I_r^2 \left(\frac{R_r}{s} \right) - \eta_{ph} I_r^2 R_r \quad (3.19)$$

where η_{ph} is number of phase, and Eq. (3.19) can be written as:

$$P_{mech} = P_{gap} - P_{rotor} = \eta_{ph} I_r^2 R_r \left(\frac{1-s}{s} \right) \quad (3.20)$$

With the expression of electromagnetic power, equivalent circuit of induction motor can be rearranged such that electromechanical power per stator phase is equal to the power delivered to resistance $R_r(1-s)/s$.

Applying Kirchhoff's Voltage and Current Law to the equivalent circuit of induction motor allows voltage and current equations of stator and rotor to be obtained. These equations will be used to form the transduction matrix of an induction motor.

Voltage and Current Equations of Stator:

$$E_s = I_s (R_s + jX_s) + E_{1s} \quad (3.21)$$

$$I_s = \frac{E_{1s}}{jX_M} + I_{1s} \quad (3.22)$$

Voltage and Current Equations of Air-Gap:

$$E_{1s} = E_{2r} \quad (3.23)$$

$$I_{1s} = I_r \quad (3.24)$$

Voltage and Current Equations of Rotor:

$$E_{2r} = I_r (R_r + jX_r) + I_r R_r \left(\frac{1-s}{s} \right) \quad (3.25)$$

$$I_r = I_r \quad (3.26)$$

3.2.2 Induced Electromotive Force and Mechanical Torque

In order to model an induction motor in transduction matrix that describes the relationship between electrical input and mechanical output, induced electromagnetic force (emf) is used to represent mechanical load in equivalent circuit. This is because induced emf is proportional to rotational speed of rotor. Any changes of rotational speed in squirrel cage induction motor are related to mechanical torque due to motor slip. Consequently, induced emf in rotor is able to describe mechanical load for an induction motor. The power across resistance $R_r(1-s)/s$ represents electromechanical power delivered from stator to produce torque. As a result, induced emf can be used to replace the resistance for the purpose of representing mechanical load in the equivalent circuit as shown in *Figure 3-6*. The voltage equation for the equivalent circuit of rotor is then modified as shown in *Eq. (3.27)* since E_{emf} is used to represent mechanical load. Therefore, an expression for E_{emf} in terms of mechanical parameters is required in order to compute its value.

Figure 3-6: Modified Equivalent Circuit of Induction Motor

Voltage and Current Equations of Modified Equivalent Circuit of Rotor:

$$E_{2r} = I_r (R_r + jX_r) + E_{emf} \quad (3.27)$$

$$I_r = I_r \quad (3.28)$$

Electromagnetic energy conversion occurs when flux linkage is changed due to mechanical motion. In rotating machine, voltages are generated in windings by rotating these windings mechanically through a magnetic field or vice versa. Voltages generated in this way are named as induced electromagnetic force. For induction machine, alternating currents are applied directly on stator windings to create rotating magnetic field. Rotor currents are then generated on rotor conductor bars by induction. In turn this induced rotor currents react with the magnetic field to produce tangential force, and resulting torque in the shaft. In order to compute induced emfs in rotor, the rotating magnetic field of squirrel cage induction motor has to be studied.

In squirrel cage induction motor, the air gap between stator and rotor is relatively small. Therefore under the assumption of small air gap, the stator windings can be assumed to produce radial space-fundamental air gap flux of constant peak magnetic flux density B_{peak} . If the air gap is uniform, B_{peak} can be found from Eq. (3.29), where μ_0 is permeability of free space, g is air gap length, k_s is stator winding factor, N_s is total series turns in the stator winding per phase and I_{sf} is stator field current. Since induction motor has multiple poles, the air gap flux per pole Φ_p can be found as shown in Eq. (3.30), where

l is axial length of the stator/rotor iron and r is radius to air gap. As the rotor rotating at constant angular velocity ω_m , the flux linkage per phase is shown in *Eq. (3.31)* where k_r and N_r are winding factor and number of turn per phase in rotor [4].

$$B_{peak} = \frac{4\mu_0}{\pi g} \left(\frac{k_s N_s}{p} \right) I_{sf} \quad (3.29)$$

$$\phi_p = \left(\frac{2}{p} \right) 2B_{peak} l r \quad (3.30)$$

$$\lambda_a = k_r N_r \phi_p \cos \left(\left(\frac{p}{2} \right) \omega_m t \right) \quad (3.31)$$

As stated by Faraday's law, the induced emf in any closed circuit is equal to the time rate of change of magnetic flux through the circuit. Hence, the emf induced in single phase windings can be found by differentiates the flux linkage:

$$E_{emf,a} = \frac{d\lambda_a}{dt} = - \left(\frac{p}{2} \right) \omega_m k_r N_r \phi_p \sin \left(\left(\frac{p}{2} \right) \omega_m t \right) \quad (3.32)$$

This equation describes general situation where Φ_p is the net air gap flux per pole produced by current on both rotor and stator, although *Eq. (3.32)* is derived based on assumption that only stator winding is producing the air gap flux. In addition, air gap flux is assumed to be constant as normal steady state operation condition is used. As a result, derivative due to changing air gap flux is zero. The polarity appears in this emf is such that induced emf would cause a current to flow in the direction that would oppose any change in the flux linkage of stator windings. However, only magnitude is concerned in following analysis. By substitute *Eq. (3.30)* into *Eq. (3.32)*, the maximum induced emf per phase can be written as:

$$E_{emf,a} = - \frac{8\mu_0 l r}{\pi g} \left(\frac{k_r N_r k_s N_s}{p} \right) I_{sf} \omega_m \quad (3.33)$$

A squirrel cage induction motor normally has three phase windings, thus consideration of three phase magnetic flux is necessary. In three phase machine, the windings are displaced from each other by 120 electrical degrees in space around the air gap. The total magnetomotive force (mmf) of three phase windings is the sum of contributions from each of three phases. The resultant positive travelling mmf waves reinforce to give 1.5 times the amplitude created by single phase whereas negative travelling mmf waves would sum to zero. In addition, the total induced emf from three phase winding would be 3 times of that in single phase. Consequently, the total induced emf is:

$$E_{emf,3\phi} = -3 \times \frac{12\mu_0 l r}{\pi g} \left(\frac{k_r N_r k_s N_s}{p} \right) I_{sf} \omega_m \quad (3.34)$$

The induced emf shown above is used to represent mechanical rotational speed from the output of a motor. An expression of torque generated by this induced emf can be obtained from power equivalent between mechanical energy and electrical energy at that point.

$$T \omega_m = E_{emf} I_r \quad (3.35)$$

$$T = 3 \times \frac{12\mu_0 l r}{\pi g} \left(\frac{k_r N_r k_s N_s}{p} \right) I_{sf} I_r \quad (3.36)$$

From Eq. (3.36), the mechanical torque is depended on stator field current and rotor current. However, under steady state operating condition, stator field current I_{sf} is constant and hence the generated magnetic flux can be assumed to be constant as well. In that case, the induced emf and total torque produced depends on ω_m and I_r respectively. By substituting Eq. (3.34) and Eq. (3.36) into rotor equation as in Eq. (3.27), a voltage equation in term of mechanical parameters, such as torque T_{total} and rotation speed ω_m can be obtained. Consequently, the voltage and current equations can be rearranged to form the transduction matrix.

3.2.3 Overall Transduction Matrix based on Cascaded Matrices

Transduction matrix of squirrel cage induction motor can be obtained from combination of all the transduction matrices of stator, air-gap, rotor and mechanical component. Transduction matrices of each component can be computed from respective governing equations such as Eq. (3.21) to Eq. (3.28). For transduction matrix of mechanical component, Eq. (3.34) and Eq. (3.36) are used. The following section shows the transduction matrices of stator, air-gap, rotor and mechanical component.

Transduction Matrix of Stator:

$$\begin{bmatrix} E_s \\ I_s \end{bmatrix} = \begin{bmatrix} \frac{R_s + jX_s}{jX_M} + 1 & R_s + jX_s \\ \frac{1}{jX_M} & 1 \end{bmatrix} \begin{bmatrix} E_{1s} \\ I_{1s} \end{bmatrix} \quad (3.37)$$

Transduction Matrix of Air-Gap:

$$\begin{bmatrix} E_{1s} \\ I_{1s} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} E_{2r} \\ I_r \end{bmatrix} \quad (3.38)$$

Transduction Matrix of Rotor:

$$\begin{bmatrix} E_{2r} \\ I_r \end{bmatrix} = \begin{bmatrix} 1 & R_r + jX_r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} E_{emf} \\ I_r \end{bmatrix} \quad (3.39)$$

Transduction Matrix of Mechanical Component under 3 Phase Circuit:

$$\begin{bmatrix} E_{emf} \\ I_r \end{bmatrix} = \begin{bmatrix} 0 & 3 \times \frac{12\mu_0 l r}{\pi g} \left(\frac{k_r N_r k_s N_s}{p} \right) I_{sf} \\ 1 & 0 \\ 3 \times \frac{12\mu_0 l r}{\pi g} \left(\frac{k_r N_r k_s N_s}{p} \right) I_{sf} & 0 \end{bmatrix} \begin{bmatrix} T_m \\ \omega_m \end{bmatrix} \quad (3.40)$$

Figure 3-7: Equivalent Circuit of Each Component in Induction Motor

Overall transduction matrix of a squirrel cage induction motor can be achieved by multiplying transduction matrices of each component in sequence as shown in Eq. (3.41) below.

$$\begin{bmatrix} E_s \\ I_s \end{bmatrix} = \begin{bmatrix} \frac{R_s + jX_s}{jX_M} + 1 & R_s + jX_s \\ \frac{1}{jX_M} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & R_r + jX_r \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 3K \\ \frac{1}{3K} & 0 \end{bmatrix} \begin{bmatrix} T_m \\ \omega_m \end{bmatrix} \quad (3.41)$$

$$\begin{bmatrix} E_s \\ I_s \end{bmatrix} = \begin{bmatrix} \left(\frac{R_s + jX_s}{jX_M} + 1 \right) \left(\frac{R_r + jX_r}{3K} \right) + \frac{R_s + jX_s}{3K} & \left(\frac{R_s + jX_s}{jX_M} + 1 \right) 3K \\ \left(\frac{R_r + jX_r}{jX_M} + 1 \right) \frac{1}{3K} & \frac{3K}{jX_M} \end{bmatrix} \begin{bmatrix} T_m \\ \omega_m \end{bmatrix} \quad (3.42)$$

$$\text{where } K = \frac{12\mu_0 l r}{\pi g} \left(\frac{k_r N_r k_s N_s}{p} \right) I_{sf} \quad (3.43)$$

Eq. (3.42) represents the overall transduction matrix of a squirrel cage induction motor and relates its electrical input with mechanical output. According to the transduction matrix developed, t_{11} , t_{12} and t_{21} consist of motor resistance, reactance as well as motor parameters. This shows that they may reflect motor condition as some of those motor faults would change the resistance of the motor. Loading condition of motor would be reflected on

those transduction parameters as well since the reactance of motor depends on its loading. All the transduction parameters are involving magnetic constant K that is used to compute induced emf. In K , motor parameters such as dimensions of stator and rotor, air-gap size, number of poles, etc are involved. These parameters will change with motor specifications, for instance different motor power rating or size would have different value of motor's parameters, and hence different transduction parameters.

The transduction matrix is suitable to describe a motor which is running at single frequency, such as 50 Hz or 60 Hz. Different operating frequency only causes different value of transduction parameters, the determinant of the matrix still remains as unity. Other than looking at each of those transduction parameters, the determinant of the overall theoretical transduction matrix is equal to unity. This is always true when the determinants of all the transduction matrices of each component are unity as well. In latter discussion, the determinant of transduction matrix is shown to be related with energy conversion between the input and output of the matrix.

$$|Det(t)| = |t_{11}t_{22} - t_{12}t_{21}| = 1 \quad (3.44)$$

The value of each transduction function can be obtained by substituting all the motor's parameter into *Eq. (3.42)* and *Eq. (3.43)*. The equivalent circuit parameters can be obtained from no-load test, block rotor test and measurement of the DC resistance of stator windings. Some assumptions have been made in the process of determine those parameters. For instance, stator reactance X_s and rotor reactance X_r are assumed to be equal. Core loss resistance R_c also neglected in this analysis. Motor parameters such as winding factor, number of turns per phase and rotor dimension can be obtained from manufacturer. *Table 3-1* shows the value of all the motor parameters used for transduction matrix modelling on a 0.5 hp, three phase squirrel cage induction motor used in this study. Computational of transduction matrix is carried out under steady state operating condition with peak stator field current of 1.607A,

since a constant peak magnetic flux is assumed. Thus the theoretical transduction matrix of corresponding induction motor is shown in Eq. (3.45).

Parameter	Value	Parameter	Value
R_s	54.0 Ω	g	0.00035 m
X_s	28.3 Ω	l	0.0655 m
R_r	2.3 Ω	r	0.03525 m
X_r	28.3 Ω	K_s	0.933
R_c	0.0 Ω	N_s	255
X_m	464.8 Ω	K_r	1
μ_0	$4\pi \times 10^{-7}$ H/m	N_r	255

Table 3-1: Motor Parameters of 0.5 hp Squirrel Cage Induction Motor.

$$\begin{bmatrix} E_s \\ I_s \end{bmatrix} = \begin{bmatrix} 25.7811 + 25.0791j & 2.4561 - 0.2691j \\ 0.4582 - 0.0021j & 0 - 0.00498j \end{bmatrix} \begin{bmatrix} T_m \\ \omega_m \end{bmatrix} \quad (3.45)$$

$$|Det(t)| = |t_{11}t_{22} - t_{12}t_{21}| = |-0.9999 + 0.0001j| = 1 \quad (3.46)$$

According to the transduction matrix developed, t_{11} and t_{12} are much larger than t_{21} and t_{22} . This is because stator resistance is involved in both t_{11} and t_{12} and stator resistance is much larger than rotor resistance. Magnitude of t_{22} is closed to zero as value of X_m is large and $1/X_m$ would be approximated to zero. In addition, the magnitude of determinant for this transduction matrix is unity as stated earlier.

Figure 3-8: Graphical Expressions for Theoretical Transduction Matrix of Induction Motor based on Cascaded Matrices

The graphical expression of this theoretical transduction matrix can be plotted by taking only the magnitude of the matrix, as shown in the figure above. These surfaces contain all the operating points for the induction motor at full load condition.

3.3 Theoretical Transduction Matrix of Squirrel Cage Induction Motor based on Simplified Equivalent Circuit

Transduction matrix modelling on squirrel cage induction motor can be carried out by simplified equivalent circuit of induction motor which can be achieved by applying several simplifications and assumptions on existing equivalent circuit in *Figure 3-5*. This simplified equivalent circuit is commonly used in modelling induction motor, especially in parameters estimation. Therefore, a transduction matrix based on this simplified circuit is formulated. First, the rotor is assumed to be a poly-phase winding that has equal number of phase and turn as stator. Given that both stator and rotor windings have same number of phases and turns, the magnitude of voltage for rotor as seen from stator terminal would be unchanged. Subsequently, the circuit can be combined together as shown in *Figure 3-9*. Similarly, equivalent circuit above would be further simplified by neglecting core loss resistance R_c . Other than that, induced emf also used to represent mechanical loading in the simplified equivalent circuit shown in *Figure 3-10*.

Figure 3-9: Combined Equivalent Circuit of Induction Motor

Figure 3-10: Simplified Equivalent Circuit of Induction Motor

The relationship between I_s and I_r as shown in Eq. (3.47) can be obtained from applying Kirchhoff's current law at junction a . Hence the current equation can be expressed in term of mechanical parameters by combining torque equation in Eq. (3.36) and Eq. (3.47).

$$I_s = I_r \left[\frac{R_r + j(X_r + X_M)}{jX_M} \right] = I_r Z_{mr} \quad (3.47)$$

The simplified equivalent circuit shown in Figure 3-10 appears similar with equivalent circuit of direct current motor as in [30]. Therefore, this simplified equivalent circuit can be used to compute transduction matrix in the same way. Based on the simplified equivalent circuit developed above, voltage equation for this equivalent circuit is shown in Eq. (3.48). The induced emf in this voltage equation is the total induced emf generated from three phase circuit, which is shown previously in Eq. (3.34). The current equation is obtained from power equation based on Eq. (3.35) and Eq. (3.36). Transduction matrix for this simplified equivalent circuit of induction motor can be obtained with the following stator's voltage and current equations.

$$E_s = I_s (R_s + jX_s) + I_r (R_r + jX_r) + E_{emf, 3\phi} \quad (3.48)$$

$$I_s = \frac{Z_{mr}}{3 \times \frac{12\mu_0 l r}{\pi g} \left(\frac{k_r N_r k_s N_s}{p} \right)} T_m I_{sf} \quad (3.49)$$

$$\begin{bmatrix} E_s \\ I_s \end{bmatrix} = \begin{bmatrix} \frac{Z_{mr}(R_s + jX_s) + (R_r + jX_r)}{3 \times \frac{12\mu_0 l r}{\pi g} \left(\frac{k_r N_r k_s N_s}{p} \right) I_{sf}} & 3 \times \frac{12\mu_0 l r}{\pi g} \left(\frac{k_r N_r k_s N_s}{p} \right) I_{sf} \\ \frac{Z_{mr}}{3 \times \frac{12\mu_0 l r}{\pi g} \left(\frac{k_r N_r k_s N_s}{p} \right) I_{sf}} & 0 \end{bmatrix} \begin{bmatrix} T_m \\ \omega_m \end{bmatrix} \quad (3.50)$$

Determinant of this theoretical transduction matrix is equal to Z_{mr} , which is the equivalent impedance between rotor and magnetizing component. However, X_M is much larger than R_r and X_r in most of the induction motor. Consequently, the magnitude of Z_{mr} approximates to unity. Similarly, the transduction matrix above is used to model the 0.5 hp, three phase squirrel cage induction motor mentioned in *Table 3-1*. By substitute those motor parameters into *Eq. (3.50)*, and the motor is assumed to be running at steady state operating condition with peak stator field current of 1.607A. The theoretical transduction matrix for that motor is shown as follows:

$$\begin{bmatrix} E_s \\ I_s \end{bmatrix} = \begin{bmatrix} 25.784 + 25.080j & 2.315 \\ 0.458 - 0.002j & 0 \end{bmatrix} \begin{bmatrix} T_m \\ \omega_m \end{bmatrix} \quad (3.51)$$

$$|Det(t)| = |t_{11}t_{22} - t_{12}t_{21}| = |1.0603 - 0.005j| = 1.06 \quad (3.52)$$

From theoretical transduction matrix above, t_{11} is having highest value among the four parameters as it consists of both resistance and reactance from stator and rotor. Z_{mr} appears in both t_{11} and t_{21} , but the magnitude of Z_{mr} will be closed to unity. All the transduction functions consist of motor parameters such as air-gap distance, number of pole pairs and winding factor except t_{22} , which is equal to zero. Parameter t_{22} is equal to zero because according to *Eq. (3.49)*, the current is assumed to be independent from rotational speed.

Figure 3-11: Graphical Expressions for Theoretical Transduction Matrix of Induction Motor based on Simplified Equivalent Circuit

The corresponding graphical expression for this transduction matrix is shown in *Figure 3-11* above. Because this transduction matrix is similar with the transduction matrix obtained from cascaded matrix, therefore the graphical expressions of both transduction matrices are similar, only some slight different observed at current's surface plot.

3.3.1 Comparison of Transduction Matrices based on Cascaded Matrices and Simplified Equivalent Circuit

By comparing between the theoretical transduction matrix obtained from cascaded transduction matrix, *Eq. (3.42)* and simplified equivalent circuit, *Eq. (3.50)*, some differences are observed. For instance, in *Eq. (3.50)*, both t_{11} and t_{21} consists of stator and rotor impedance, t_{22} is zero, and determinant is equal to Z_{mr} . However, the values of each transduction functions from both methods are actually very similar. The reasons of those transduction parameters are similar is due to the fact that magnetizing reactance, X_m is much larger than impedance of stator and rotor. This is a common property in induction motor, and indeed it is always used as an assumption in parameter estimation of induction motor. Consequently, whenever $R_s + jX_s$ or $R_r + jX_r$ is divided by X_m , the value will be very small and approximated to zero. In addition, the

determinant of transduction matrix from simplified equivalent circuit, Z_{mr} is also approximated to unity.

<i>Parameter</i>	<i>Cascaded Method</i>	<i>Simplified Equivalent Circuit</i>
t_{11}	$\left(\left(\frac{R_s + jX_s}{jX_M} + 1 \right) (R_r + jX_r) + R_s + jX_s \right) \times \frac{1}{K} \approx \frac{(Z_{mr}(R_s + jX_s) + (R_r + jX_r))}{K}$	
t_{12}	$\left(\frac{R_s + jX_s}{jX_M} + 1 \right) K \approx K$	
t_{21}	$\left(\frac{R_r + jX_r}{jX_M} + 1 \right) \frac{1}{K} \approx \frac{Z_{mr}}{K}$	
t_{22}	$\left(\frac{1}{jX_M} \right) K \approx 0$	

Table 3-2: Comparison of Transduction Parameters between Cascaded Method and Simplified Equivalent Circuit

Although both theoretical transduction matrices are similar with each other, the matrix obtained from simplified equivalent circuit may have deficiency in modelling the induction motor. This is because it involves more assumptions in order to construct the matrix. For example, t_{22} is assumed to be zero in simplified equivalent circuit. In fact t_{22} is proven to be nonzero from experimental results in later section. Therefore, the theoretical transduction matrix based on cascaded method is preferred.

In summary, theoretical transduction matrix of a squirrel cage induction motor is carried out by cascaded method and simplified equivalent circuit in this section, which the former transduction matrix is preferred. Consequently, the theoretical transduction matrix is verified with experimental results.

3.4 Experimental Identification of Transduction Matrix for Squirrel Cage Induction Motor

Transduction functions can be obtained via theoretical expression or experimental data. Theoretical expression is formed based on governing equations of the system which has been discussed in previous section. On the other hand, the values of those transduction functions can be acquired experimentally by applying different loading condition to the system. In the following section, two experimental methods to obtain the transduction matrix of a mechanical system are introduced.

3.4.1 Direct Method

One of the methods to compute transduction matrix's parameters is based on information obtained from two extreme experimental conditions. The transduction functions of an induction motor which is running at constant frequency can be expressed as follows [70], [71]:

$$t_{11} = \left. \frac{E_1}{T_2} \right|_{\omega_2=0} \quad t_{12} = \left. \frac{E_1}{\omega_2} \right|_{T_2=0} \quad (3.53 \text{ and } 3.54)$$

$$t_{21} = \left. \frac{I_1}{T_2} \right|_{\omega_2=0} \quad t_{22} = \left. \frac{I_1}{\omega_2} \right|_{T_2=0} \quad (3.55 \text{ and } 3.56)$$

In experiment, value of t_{11} and t_{21} can be calculated by maintaining $\omega_2 = 0$, which is the clamp condition. Similarly t_{12} and t_{22} can be described by maintaining $T_2 = 0$, also know as free condition. However, clamp condition is difficult to be achieved in real experimental setup because relatively small vibration can still be observed at output terminal especially for large scale system [71]. For that reason, an improved method which is utilizing Reciprocity Theorem is used to avoid clamped condition in experimental setup. Reciprocity Theorem defined a system which input and output can be interchanged without altering the response of the system to a given excitation. This allows the inverse of transduction matrix and hence output of system can be expressed in term of input.

$$\begin{bmatrix} t_{22} & t_{12} \\ t_{21} & t_{11} \end{bmatrix} \begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} T_2 \\ \omega_2 \end{bmatrix} \quad (3.57)$$

In this case, t_{11} can be obtained from applying experiment condition, $E_1 = 0$:

$$t_{11} = \left. \frac{\omega_2}{I_1} \right|_{E_1=0} \quad (3.58)$$

The determinant of transduction matrix is unity due to characteristic of Reciprocity Theorem. Consequently, the last parameter, t_{21} can be computed as follow:

$$t_{21} = \frac{t_{11}t_{22} - 1}{t_{12}} \quad (3.59)$$

However, Reciprocity Theorem is only applicable to linear system with no energy loss or generation. Hence this method will have deficiency in accuracy especially for induction motor modelling where energy losses is significant.

3.4.2 Least Squares Method

Consequently, a method using least squares approximation is introduced. In this method, a set of readings of E , I , T and ω is taken with increasing loading and arranged in matrix form as shown below. Subsequently the matrix is solved by least squares method:

$$\begin{bmatrix} E_1 & E_2 & \dots & E_i \\ I_1 & I_2 & \dots & I_i \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} T_1 & T_2 & \dots & T_i \\ \omega_1 & \omega_2 & \dots & \omega_i \end{bmatrix}$$

Let $A = tB$

$$AB^t = tBB^t$$

$$\therefore t = AB^t(BB^t)^{-1} \quad (3.60)$$

The transduction matrix can be computed from the product of electrical input matrix, mechanical output matrix and their transpose. Above is the simplification from least squares approximation and the detail is shown as below:

$$\begin{aligned}
\text{Let} \quad & A_{ij} = t_j B_{ij} \\
\text{Residue,} \quad & r_i = A_{ij} - t_j B_{ij} \\
\text{Sum of Square,} \quad & S = \sum_{i=1}^{i=m} r_i^2 \\
\text{Least square,} \quad & \frac{\partial S}{\partial T} = 2 \sum_i r_i \frac{\partial r_i}{\partial T_j} = 0 \\
& -2 \sum_{i=1}^{i=m} \left(A_i - \sum_{j=1}^{j=n} t_j B_{ij} \right) B_{ij} = 0 \\
& \sum_{i=1}^{i=m} \sum_{k=1}^{k=n} t_k B_{ij} B_{ik} = \sum_{i=1}^{i=m} A_i B_{ij} \quad , j = 1, \dots, n \\
& t(BB^t) = AB^t \\
& \therefore t = AB^t (BB^t)^{-1}
\end{aligned}$$

This method avoids complicated experimental setup such as clamped condition. Only input and output data of a number of experiments under increasing loads or increasing stiffness are required. In addition, it avoids the assumption of Reciprocity Theorem used in conventional method. Accuracy of transduction matrix functions increases with the number of experimental data. Computation of these transduction matrix parameters can be carried out by matrix right division in MATLAB as well, which is based on least squares approximation. Nevertheless, the matrix computed in this method is merely an average result which describes motor's properties under varying loads. In this method, the motor is assumed running at constant operating frequency. In fact, transduction functions are frequency dependent functions and it should be obtained from frequency response function between input and output for better accuracy.

3.4.3 Experimental Results

A 0.5 HP (0.37 kW) three phase squirrel cage induction motor is used in experimental to obtain its transduction matrix. The experiment is set up as shown in schematic diagram of *Figure 3-12*. Basically, voltage probe and

current probe are used to acquire input voltage and current. In order to avoid unnecessary phase different, same line voltage and current is measured. Direct power supply is used to keep away from effect of induced harmonic due to frequency converter. With proper signal conditioning, input voltage and current is transferred to Dynamic Signal Analyzer (DSA) for signal processing. In DSA, input voltage and current are collected and transformed from time domain to frequency domain via Fast Fourier Transform (FFT). Thus the magnitude and phase of voltage and current at particular frequency are able to be collected. The torque and speed sensors are used to gather information at mechanical port. Torque and speed signals are sent to DSA for signal processing in order to obtain average torque and rotational speed of the induction motor. More details of experiment setup and equipments can be found in Appendix A.

A set of loadings ranging from 0.0 kg to 2.5 kg is applied on band brake as the loading applied to motor. Readings of E , I , T and ω are collected as shown in Table 3-3. The magnitude and phase of voltage and current at 50 Hz, and average torque and rotation speed are used in the calculation of transduction matrix for the corresponding induction motor.

Figure 3-12: Schematic Diagram for Experiment on Measuring E , I , T and ω of Induction Motor

Load (kg)	Voltage (V)				Current (A)				Torque (Nm)	Speed (rad/s)
	Mag	Phase	Real	Imag	Mag	Phase	Real	Imag		
0.0	560.534	15.923	539.027	153.780	1.206	87.757	0.047	1.205	0.019	158.477
0.5	558.366	17.771	531.723	170.421	1.222	100.641	-0.226	1.201	0.760	155.746
1.0	557.209	8.123	551.619	78.733	1.293	100.160	-0.228	1.273	1.419	153.187
1.5	557.286	21.755	517.595	206.552	1.369	119.976	-0.684	1.186	1.917	151.298
2.0	557.410	19.046	526.896	181.898	1.472	122.781	-0.797	1.238	2.415	149.089
2.5	556.369	15.680	535.664	150.367	1.646	126.179	-0.972	1.329	3.135	145.899

Table 3-3: Voltage, Current, Torque and Rotational Speed at Various Loads

Firstly, an experimental transduction matrix of the induction motor with peak value and phase of voltage and current is computed. Basically, complex form of voltage and current in Table 3-3 are substituted into matrix A , as well as torque and speed into matrix B as shown in Eq. (3.60). The data in matrix A and B is arranged in increasing trend and thus corresponding experimental transduction matrix is obtained from least squares approximation. In order to compare with experimental matrix based on peak value of voltage and current, theoretical transduction matrix in term of peak value of input voltage and current has been obtained as well. A factor of square root of 2 is used for transforming between peak value and rms value of input voltage and current. Therefore, the theoretical transduction matrix in term of peak value is shown in Eq. (3.61). Basically, each of the transduction matrix's parameters is multiply by square root of 2. The magnitude of determinant of this transduction matrix is equal to two due to the consideration of peak value.

Theoretical Transduction Matrix with Peak Value of Voltage and Current:

$$\begin{bmatrix} E_{s,p} \\ I_{s,p} \end{bmatrix} = \begin{bmatrix} \left(\left(\frac{R_s + jX_s}{jX_M} + 1 \right) (R_r + jX_r) + (R_s + jX_s) \right) \frac{\sqrt{2}}{3K} & \left(\frac{R_s + jX_s}{jX_M} + 1 \right) \frac{\sqrt{2}3K}{jX_M} \\ \left(\frac{R_r + jX_r}{jX_M} + 1 \right) \frac{\sqrt{2}}{3K} & \frac{\sqrt{2}3K}{jX_M} \end{bmatrix} \begin{bmatrix} T_m \\ \omega_m \end{bmatrix} \quad (3.61)$$

$$\begin{bmatrix} E_{s,p} \\ I_{s,p} \end{bmatrix} = \begin{bmatrix} 35.4560 + 35.4672j & 3.4734 - 0.3806j \\ 0.6480 - 0.0030j & 0 - 0.0070j \end{bmatrix} \begin{bmatrix} T_m \\ \omega_m \end{bmatrix} \quad (3.62)$$

$$|Det(t)| = |-2.0014 + 0.0089j| = 2 \quad (3.63)$$

Experimental Transduction Matrix with Peak Value of Voltage and Current:

$$\begin{bmatrix} E_{s,p} \\ I_{s,p} \end{bmatrix} = \begin{bmatrix} 10.870 - 9.069j & 3.390 - 0.935j \\ -0.340 - 0.062j & 0.001 - 0.008j \end{bmatrix} \begin{bmatrix} T_m \\ \omega_m \end{bmatrix} \quad (3.64)$$

$$|Det(t)| = |1.1457 + 0.1940j| = 1.162 \quad (3.65)$$

From the transduction matrix, generally t_{11} has the largest magnitude among the four functions, followed by t_{12} and t_{21} . Meanwhile t_{22} has relatively small value which is closed to zero. All the transduction functions are complex number as the system is assumed to have constant frequency. According to the results above, the input voltage is largely depended on torque compared with rotational speed. The input current is mainly depended on torque, and rotational speed does not have much effect on it.

By comparing between theoretical and experimental results, similar findings are observed where the magnitude of t_{11} and t_{21} are larger in theoretical transduction matrix. t_{12} and t_{22} are similar to theoretical results. However, the determinant for experimental results is different from theoretical determinant. The magnitude of determinants is 1.162 for matrix with peak value and phase information, which is about half of the theoretical transduction matrix's determinant. The difference is perhaps due to the values of t_{11} and t_{21} of experimental result are about half of that in theoretical result. The derivation of theoretical transduction matrix does not include efficiency and power lost. In addition, the induction motor's parameters in the calculation may not reflect the true condition of motor. As they are obtained from parameter estimation with a few assumptions are made. The theoretical transduction matrix is obtained based on information at full load condition, whereas the experimental transduction matrix is computed from the approximation of least squares method over a range of loading conditions. All these concerns might contribute to the differences between theoretical and experimental transduction matrices.

The experimental determinant is found to be slightly greater than unity. This is perhaps due to inefficiency of energy conversion inside the induction motor, which will be discussed in section 3.7 later. Nevertheless, the determinant is not too diverse from unity. This shows the energy loss for this induction motor is not very serious. In addition, determinant close to unity means the information from mechanical port transferred accordingly to electrical input port. It would be useful for load condition monitoring from electrical port. Furthermore, the determinant can be used as a quick indicator for induction motor health condition. If a motor faults exists, the efficiency and transduction parameters of the motor would change and reflect at its determinant value.

3.4.4 Graphical Expression for Experimental Transduction Matrix

The transduction matrix developed above can be demonstrated by two surface plots, one for electrical voltage and another one for current. These two surface plots contain all of the operating points for the induction motor at that frequency.

$$E_1 = t_{11}T_2 + t_{12}\omega_2 \quad (3.66)$$

$$I_1 = t_{21}T_2 + t_{22}\omega_2 \quad (3.67)$$

From the experimental data in *Table 3-3*, it is possible to plot all the operating points in a three dimension space and it is shown in *Figure 3-13*. Only magnitude of E , I , T and ω are considered in the following graphs. In the graphs, z -axis is representing voltage for *Figure 3-13* and current for *Figure 3-14*, x -axis is torque T_2 , and y -axis is rotational speed ω_2 . The solid coloured lines represent voltage and current values at different loading conditions. The grey colour surfaces are obtained from surface fitting of those experimental data.

The voltage equation in *Eq. (3.66)* describes the surface plotted in *Figure 3-13*. The transduction functions t_{11} and t_{12} are computed from least squares approximation from experimental data, according to the surface fitting. Hence the approximated values will be used to plot the surface. This is carried out

under assumption of the variation of transduction matrix of induction motor with loading condition is small. Therefore, an approximated transduction matrix is sufficient to describe the induction motor over different loading condition. Actually, for each of the loading condition, there will be a different value of transduction matrix, thus a different surface to describe the voltage.

In the surface plot, all the points in the surface would be able to fulfil the voltage equation, provided that torque and rotational speed are independent from each other. However, this is not the case for induction motor, because torque and rotational speed are related to each other, based on the provided power or applied mechanical impedance. As a result, the voltage equation turns into equation of line or curve as follows:

$$E_1 = t_{11}T_2 + t_{12} \frac{P}{T_2} \quad (3.68)$$

$$E_1 = t_{11}T_2 + t_{12} \frac{T_2}{Z} \quad (3.69)$$

Equations above imply that voltage of induction motor and output information can be described by a line as shown in the figure. Although the equation has changed into a line equation, the line is still placed within the plane obtained from the transduction matrix. This is the same for the current equation in Eq. (3.67).

From the results, the voltage and current fitted into respective plane rather nicely when the induction motor is operated in half load to full load condition. This shows that the transduction matrix can describe the induction motor precisely in those loading conditions. Whereas some voltage and current values at low loading condition are out of the plane, which means the transduction matrix may not be suitable to describe the induction motor at that loading condition. In order to have more precise modelling, transduction matrix at various loading condition should be considered.

Figure 3-13: Surface Plot for Voltage Equation

Figure 3-14: Surface Plot for Current Equation

3.5 Transduction Matrix at Various Operating Condition

In previous section, the theoretical transduction matrix of induction motor presented is based on steady state operating condition at full load. The experimental transduction matrix obtained representing average value over a range of loading condition. Indeed, for different loading condition, the transduction matrix of induction motor changes. As loading changes, the magnetic field generated in the stator of motor changes as well. Therefore, the transduction matrix will change according to magnetic field. In the following section, transduction matrices at various operating conditions are discussed.

3.5.1 Theoretical Transduction Matrix

Theoretical Transduction matrix developed in *Eq. (3.42)* is based on steady state full load operating condition. The constant K in *Eq. (3.70)* is related with magnetic field generated by stator field current. Since stator field current increases with mechanical torque produced by the induction motor, K will varies accordingly. Consequently, variation of transduction matrix at different loading conditions can be computed by substituting in the value of K at different loads.

$$\begin{bmatrix} E_s \\ I_s \end{bmatrix} = \begin{bmatrix} \left(\frac{R_s + jX_s}{jX_M} + 1 \right) \left(\frac{R_r + jX_r}{3K} \right) + \frac{R_s + jX_s}{3K} & \left(\frac{R_s + jX_s}{jX_M} + 1 \right) 3K \\ \left(\frac{R_r + jX_r}{jX_M} + 1 \right) \frac{1}{3K} & \frac{3K}{jX_M} \end{bmatrix} \begin{bmatrix} T_m \\ \omega_m \end{bmatrix} \quad (3.70)$$

$$\text{where } K = \frac{12\mu_0 l r}{\pi g} \left(\frac{k_r N_r k_s N_s}{p} \right) I_{sf} \quad (3.71)$$

Figure 3-15 shows the variation of transduction parameters due to various loading conditions. However, the resistance and reactance of stator along with rotor are assumed to have small changes that can be treated as constant over the load variation. Hence, the transduction matrix depends on magnetic field constant K only. The following analysis is based on the model of 0.5 hp squirrel cage induction motor presented previously.

Figure 3-15: Variation of TM based on Magnetic Field Constant K

From the results, loading is reflected by the torque produced from induction motor. When the torque is increasing, the rotational speed of motor decreased due to motor slip and in fact this is shown in all the transduction parameters. For t_{11} , its magnitude decreases from about 40 to 30 as torque is increased from zero to full load condition. Similarly, t_{21} shows the same trend as t_{11} , but value of t_{21} is near to zero and hence its increment is not obvious in this graph. The value of t_{12} is increased from about 2 to 2.8 as loading increased, also slight increments is observed for t_{22} . The trend of transduction parameters variation is mainly due to that K is increased proportionally with loading. The theoretical transduction matrix obtained in Eq. (3.45) only represents one point in the graph, at the full load condition.

Another analysis to study the variation of transduction matrix has been carried out. The transduction matrix can be solved simultaneously from the equations of transduction matrix shown in Eq. (3.72) and Eq. (3.75) below, together with several assumptions. First, the determinant of transduction matrix is assumed to be unity based on the finding that most of the determinants of transduction matrix found previously are unity. Secondly t_{22} is assumed to be zero, because

according to *Eq. (3.49)*, stator current depends on mechanical torque only. With the four equations developed below, the four unknowns in transduction matrix can be computed.

$$E_s = t_{11}T_m + t_{12}\omega_m \quad (3.72)$$

$$I_s = t_{21}T_m + t_{22}\omega_m \quad (3.73)$$

$$\text{Det}(t) = t_{11}t_{22} - t_{12}t_{21} = 1 \quad (3.74)$$

$$t_{22} = 0 \quad (3.75)$$

Figure 3-16: Variation of TM based on Solving Simultaneous Equations

Similar findings can be found in *Figure 3-16*, where t_{11} and t_{21} decrease while the loading increases. Besides, t_{12} increases with loading and t_{22} remains as zero. However, the variation of transduction matrix computed in this method is much larger than previous analysis. This method involves several assumptions in order to compute transduction matrix, therefore the results might have some deficiencies and therefore the range of computed data is so large. Based on this finding, the transduction matrices obtained from *Eq. (3.70)* are more suitable to describe the induction motor under various loading conditions.

3.5.2 Experimental Transduction Matrix

Experiments have been conducted under different loading conditions, so that experimental results for variation of transduction matrix are studied and compared with theoretical results. Least squares approximation still applicable in this analysis, but it is applied to different range of data. For instance, least squares approximation is applied to a few data near the vicinity of full load condition in order to obtain experimental transduction matrix at that loading. Similar method is employed to calculate transduction matrix at half load condition.

	t_{11}	t_{12}	t_{21}	t_{22}
<i>Overall TM</i>	4.3523 + 9.1032j	3.7140 – 0.8701j	0.2695 – 0.2080j	0.0064 + 0.0051j
<i>Full Load TM</i>	7.8608 + 30.2785j	3.5926 + 0.3753j	0.3140 + 0.3176j	0.0057 – 0.0064j
<i>Half Load TM</i>	10.4245 + 50.3691j	3.5739 + 0.0486j	0.3237 + 0.3386j	0.0055 – 0.0067j

Table 3-4: Experimental Transduction Matrix at Various Loading Condition

From the experimental results, the overall transduction matrix obtained here is quite similar with previous results in Eq. (3.64). This is because same induction motor is used in the test. For half load and full load transduction matrices, the magnitude of t_{11} and t_{21} decreases as loading increases. Slight increments can be observed at t_{12} , and t_{22} is almost constant. In short, the variation of transduction matrix obtained from experiments validates the theoretical findings in previous section.

Transduction matrix of induction motor is proven changing along with loading condition. Furthermore, transduction matrix does vary for different motor specifications. In order to verify this statement, experiments have been carried out on squirrel cage induction motors with different power rating, one of it is 0.5 hp (0.37 kW) and another motor is 0.25 hp (0.19 kW). The transduction matrices are compared as shown in Table 3-5 below:

	t_{11}	t_{12}	t_{21}	t_{22}	$Det(t)$	$ Det(t) $
<i>SCIM (0.25 hp)</i>	26.1980 - 11.0853j	5.1820 - 1.0653j	0.0318 + 0.2905j	-0.0066 + 0.0036j	-0.6066 - 1.3052j	1.4393
<i>SCIM (0.5 hp)</i>	10.8703 - 9.0687j	3.3900 - 0.9348j	-0.3395 - 0.0616j	0.0005 - 0.0075j	1.1457 - 0.1940j	1.1621

Table 3-5: Experimental Transduction Matrices of Squirrel Cage Induction Motor (SCIM) with Different Power Rating

By comparing the transduction matrices between the two induction motors of different power rating, some dissimilarities can be observed in both transduction functions and determinants of the matrix. For a 0.25 hp induction motor, it has larger value in both real and imaginary component of t_{11} and t_{12} , as well as larger magnitude of transduction matrix determinant. Perhaps these observations can be used to distinguish those induction motors with different power rating. From the point of view of theoretical transduction matrix, different motor specification will give rise to different value of magnetic field constant K . Since magnetic field constant involves in all transduction functions, changes are expected.

In summary, the transduction matrix of induction motor is proven to be varied with loading condition. Each transduction matrix represents a surface where voltage and current in response of associated load conditions are located in the surface. In order to represent the properties of motor accurately, different transduction matrices should be obtained for different loading conditions. However, for an induction motors that normally operates constantly around 70% to 80% of power rating, the induction motor can be assumed as linear and time invariant at the vicinity of that loading condition. Therefore, a transduction matrix obtained based on that loading condition would be sufficient to describe the induction motor precisely. In the following analysis, transduction matrix at full load condition is used because most of the analyses are carried out at that loading condition.

3.6 Validation of Transduction Matrix

In this section, verification of the theoretical transduction matrix is carried out. This is to ensure the transduction matrix is able to model the squirrel cage induction motor correctly and being able to illustrate mechanical output from a given electrical input. A measured electrical input signal based on constant full load condition is used as input for the theoretical transduction matrix and hence the mechanical output is computed from the model.

3.6.1 Sinusoidal Power Supply Signal

The power supply applied to the tested induction motor is three phase AC power supply at 400 V with 50 Hz frequency. Figure below shows the voltage and current supply over a period of time. The voltage supply is having peak value of 570 V, and the current's peak value are about 1.60 A.

Figure 3-17: Sinusoidal Voltage and Current Signals at 50 Hz

Voltage and current signals above are used as the input for the theoretical transduction matrix in *Eq. (3.62)*. Subsequently, the mechanical output can be obtained from matrix inverse as shown in below. The calculation is tedious as it is repeated for every point in the voltage and current signals. With the help of computer programme such as MATLAB, the calculation can be done easily.

$$\begin{bmatrix} T_m \\ \omega_m \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}^{-1} \begin{bmatrix} E_{s,p} \\ I_{s,p} \end{bmatrix} \quad (3.76)$$

Figure 3-18: Calculated Torque and Rotational Speed based on Sinusoidal Voltage and Current Signals

The calculated torque and rotational speed of induction motor are presented in *Figure 3-18* and compared with measured values from torque meter and speed sensor. From the comparison, the calculated torque is closed to the measured value of 2.45 Nm. However, the calculated torque has larger magnitude range from 1.8 Nm to 2.8 Nm. For rotational speed wise, the range of calculated rotational speed is even larger, ranging from 20 rad/s to 180 rad/s. This large range of calculated value is mainly due to alternating signal used at the input. The theoretical transduction matrix developed previously is based on either rms value or peak value. Whenever the alternating signal is changing from peak to peak value, output of the model changes accordingly. Therefore, variation of output data causes by alternating signal has to be solved, and it is discussed in following section.

3.6.2 Analytic Signals of Voltage and Current

In order to solve variation of data from alternating signal, analytic signals of voltage and current signals is introduced. Analytic signal has been progressively developed by researchers in various fields, such as electronics, radio and vibration signals. Analytic signal becomes more and more useful in signal processing because the attributes of analytic signal facilitates many mathematic manipulations.

Analytic signal is a complex signal where imaginary part is Hilbert transform of the real part as shown in *Eq. (3.78)*. Because of this, analytic signal is a Hermitian function where its complex conjugate is equal to the original function with variable changed in sign. As a result, the negative frequency components of the signal can be abandoned with no loss of information, thus enabling mathematical manipulation of complex signal.

The amplitude of analytic signal represents the envelope of the signal, which contains important information of the signal's energy [73]. Basically, amplitude envelope is the shape of variation of instantaneous amplitude for a

given signal. That means the signal and its envelope have common tangential points of contact, yet the signal do not cross the envelope. Consequently, amplitude envelope of an alternating signal is the waveform formed by the alternating signal's peak value along the time. This envelope would be a better representation of the alternating signal along the time domain compare to root mean square (RMS) value, as RMS value describes the signal's value averaged over a period of time. Therefore, by using this analytic signal, the oscillation in alternating signal will be removed and this signal is represented by direct signal expressing its envelope [74], [75].

Hilbert Transform of $y(t)$:

$$\hat{y}(t) = \frac{1}{\pi} \times y(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{y(\tau)}{t - \tau} d\tau \quad (3.77)$$

Analytic Signal of $y(t)$:

$$\Psi_y(t) = y(t) + j\hat{y}(t) \quad (3.78)$$

Envelope and Instantaneous Phase of Analytic Signal of $y(t)$:

$$|\Psi_y(t)| = \sqrt{y^2(t) + \hat{y}^2(t)} \quad (3.79)$$

$$\phi_y(t) = \tan^{-1} \left(\frac{\hat{y}(t)}{y(t)} \right) \quad (3.80)$$

The reason that magnitude of analytic signal is able to represent an alternating signal as a direct signal, can be shown as follow. Basically, a real time alternating voltage signal is computed and transformed into analytic signal here. Throughout the transformation, the voltage components at its main frequency will remains and become a constant value.

Assuming a time record of voltage signal $E_{max} \sin(\omega_0 t)$, and its harmonics $E_{har} \sin(k\omega_0 t)$ with supply frequency of ω_0 , 50 Hz for example.

$$E(t) = E_{max} \sin \omega_0 t + E_{har} \sin k\omega_0 t \quad (3.81)$$

Hence the analytic signal of $E(t)$:

$$\begin{aligned}\Psi_E(t) &= E_{\max} \sin \omega_0 t + E_{\text{har}} \sin k\omega_0 t + j(-E_{\max} \cos \omega_0 t + E_{\text{har}} \cos k\omega_0 t) \\ \Psi_E(t) &= E_{\max} (\sin \omega_0 t - j \cos \omega_0 t) + E_{\text{har}} (\sin k\omega_0 t - j \cos k\omega_0 t)\end{aligned}\quad (3.82)$$

Analytic signal above can be rewritten together with Euler's formula:

$$\text{Euler's Formula, } e^{j\omega t} = \cos \omega t + j \sin \omega t \quad (3.83)$$

$$\Psi_E(t) = -jE_{\max} e^{j\omega_0 t} - jE_{\text{har}} e^{jk\omega_0 t} \quad (3.84)$$

The magnitude of analytic signal would be:

$$|\Psi_E(t)| = |-je^{j\omega_0 t}| |E_{\max} + E_{\text{har}} e^{jk}| \quad (3.85)$$

$$|\Psi_E(t)| = |E_{\max} + E_{\text{har}} e^{jk}| \quad (3.86)$$

Eq. (3.86) shows that magnitude of analytic signal comprises of a constant value of $E_{\max}$ and $E_{\text{har}} e^{jk}$, where $E_{\text{har}} e^{jk}$ oscillates around the value of $E_{\max}$. If the voltage supply is maintained at maximum voltage with 50 Hz, then the corresponding analytic signal will be a constant value along the time. When the voltage supply changes in magnitude, the analytic signal will give an envelope that describes the change. With this technique, the alternating signal can be represented by a direct signal that shows its maximum value along the time.

Accordingly, the alternating signals of voltage and current can be converted into analytic signal in following figures. Envelope of voltage's analytic signal is having voltage peak value of about 570 V and current's analytic signal is at 1.5 A.

Figure 3-19: Envelope of Analytic Signal of Voltage and Current

Similarly, analytic signals of voltage and current above are used as the input for transduction matrix and hence the mechanical output is computed. The calculated mechanical torque and rotational speed are shown in *Figure 3-20*.

Figure 3-20: Calculated Torque and Rotational Speed based on Analytic Signal of Voltage and Current

Now, the calculated output has more reasonable value range where torque is computed for around 3.0 Nm and calculated rotational speed is about 155 rad/s. Comparing the calculated mechanical output with measured value at 2.45 Nm and 142 rad/s, the calculated torque and rotational speed are higher by 122% and 109%. Hence there are some discrepancies in the theoretical transduction matrix of induction motor. Perhaps the theoretical transduction motor does not include some losses such as energy and mechanical losses, therefore the theoretical torque and rotational speed is slightly higher than the

measured values. In addition, the motor parameters such as rotor resistance, rotor reactance, magnetizing reactance are obtained from parameters estimation. Therefore, these values might be different under actual operating condition. The accuracy of transduction matrix based on these motor parameters will be affected as well.

However, the output obtained from transduction matrix is at least able to describe mechanical torque and rotational speed. Of course, the accuracy of theoretical transduction matrix of induction motor can be further improved by considering energy efficiency or losses issues, as well as using more precise motor parameters in the calculation. In summary, the theoretical transduction matrix is able to model a squirrel cage induction motor and can be used to obtained mechanical outputs based on signals collected at electrical input.

3.6.3 Validation of Transduction Matrix obtained from Least Squares Method

In this section, the experimental transduction matrix obtained from least squares method is validated and its accuracy in describing information at one port based on another port is studied. From the experimental transduction matrix in *Eq. (3.64)*, the electrical input information at different loadings can be computed from measured torque and rotational speed. The results are presented in *Table 3-6*, the accuracies of calculated voltage and current are within 6% of measured values.

<i>Load</i> (kg)	<i>E (V)</i> <i>Measured</i>	<i>I (A)</i> <i>Measured</i>	<i>T (Nm)</i> <i>Measured</i>	ω (rad/s) <i>Measured</i>	<i>E (V)</i> <i>Calculated</i>	<i>Accuracy (%)</i>	<i>I (A)</i> <i>Calculated</i>	<i>Accuracy (%)</i>
0.0	560.534	1.206	0.019	158.477	557.542	99.5	1.278	106.0
0.5	558.366	1.222	0.760	155.746	557.507	99.8	1.297	106.1
1.0	557.209	1.293	1.419	153.187	557.047	99.9	1.354	104.7
1.5	557.286	1.369	1.917	151.298	556.875	99.9	1.420	103.7
2.0	557.410	1.472	2.415	149.089	555.592	99.7	1.501	102.0
2.5	556.369	1.646	3.135	145.899	553.780	99.5	1.643	98.8

Table 3-6: Calculated E and I from Experimental Transduction Matrix.

To validate the accuracy of experimental transduction matrix in computing torque and rotational speed, least squares method is carried out to obtained

transduction matrix as shown in *Eq. (3.87)*. Subsequently, the torque and rotational speed can be calculated from the matrix and compared with measured voltage and current, the results are shown in *Table 3-7*. According to the results, the calculated torque and rotational speed are generally closed to measured values, except for the calculated torque at free load. This is because measurement at low torque is difficult and small difference will leads to large discrepancy in accuracy.

$$\begin{bmatrix} T_m \\ \omega_m \end{bmatrix} = \begin{bmatrix} 0.0015 + 0.0063j & -3.0027 - 0.3061j \\ 0.2776 - 0.1008j & 10.5660 + 6.0414j \end{bmatrix} \begin{bmatrix} E_{s,p} \\ I_{s,p} \end{bmatrix} \quad (3.87)$$

<i>Load</i>	<i>E (V)</i>	<i>I (A)</i>	<i>T (Nm)</i>	<i>ω (rad/s)</i>	<i>T (Nm)</i>		<i>ω (rad/s)</i>	
<i>(kg)</i>	<i>Measured</i>	<i>Measured</i>	<i>Measured</i>	<i>Measured</i>	<i>Calculated</i>	<i>Accuracy (%)</i>	<i>Calculated</i>	<i>Accuracy (%)</i>
0.0	560.534	1.206	0.019	158.477	0.057	300.0	158.363	99.9
0.5	558.366	1.222	0.760	155.746	0.752	98.9	155.228	99.7
1.0	557.209	1.293	1.419	153.187	1.394	98.2	152.519	99.6
1.5	557.286	1.369	1.917	151.298	1.884	98.3	150.729	99.6
2.0	557.410	1.472	2.415	149.089	2.399	99.3	148.814	99.8
2.5	556.369	1.646	3.135	145.899	3.164	100.9	145.620	99.8

Table 3-7: Calculated T and ω from Experimental Transduction Matrix.

The reason of using *Eq. (3.87)* obtained from least squares method is because inverse of *Eq. (3.64)* incurred more computational error from the approximation of least squares method and inverse of matrix. Therefore, calculated torque and rotational speed deviate away from the measured values.

3.7 Properties of Transduction Matrix

According to the analysis carried out so far, the transduction matrix represents a system's properties, where each of the transduction functions relates with motor condition. Beside of transduction functions, there are other properties from transduction matrix which could be useful in induction motor analysis. For instance, determinant of transduction matrix which has been mentioned several times in previous section would be related with energy conversion. In addition, other properties such as electrical and mechanical impedance, as well as power and efficiency in term of transduction matrix will be focused in this section.

3.7.1 Determinant of Transduction Matrix

Determinant is a value associated with square matrix and it provides important information for a system with linear equations. In geometry, determinant represents the scalar factor for area described by a 2 by 2 matrix. Therefore it can be presumed as the magnitude of a matrix. In the theoretical transduction matrix of induction motor, its determinant is unity. Hence the magnitude of input matrix is equal to the magnitude of output matrix. For a given operating condition, the area under the electrical input's graph should be matched with the area under the mechanical output's graph as shown in *Figure 3-21*. This might imply that the energy or power from input is equal with that from output.

Figure 3-21: Geometry Illustration of Transduction Matrix's Determinant

Figure 3-22: Comparison of Power at Electrical Input and Mechanical Output Ports

Comparison of powers at electrical input port and mechanical output port is presented in *Figure 3-22*. The electrical input power is not similar with mechanical output power, perhaps this is due to nonlinearity of induction motor. As transduction matrix of induction motor varies significantly when loading increased. More power is lost when the loading is small, but the electrical power and mechanical power become comparable at higher loading. The power at input and output port, which is related with the area under curve in *Figure 3-21* would be much similar in a linear system, such as piezoelectric transducer.

A real matrix with unity determinant is also known as unimodular matrix which is invertible. The presence of determinant indicates that transduction matrix has an inverse operation. By inversion of transduction matrix, the output functions can be expressed in term of input functions and inverse of transduction matrix. Unity determinant means the magnitude of inverse matrix does not change. Because of these properties, the information at hard to reach

output port can be obtained based on information at input port and transduction matrix without much loss of information.

According to experiment, the determinant of transduction matrix for an induction motor is always greater than 1. This indicates the energy conversion between input and output is not perfect due to nonlinearity and inefficiency of power consumption in induction motor. The inequality is shown as follows with only their magnitudes are considered, where E_I , I_I are electrical input and F_2 , V_2 are mechanical outputs.

$$t_{11}t_{22} - t_{12}t_{21} \geq 1 \quad (3.88)$$

$$\frac{t_{11}t_{22}}{t_{21}} - t_{12} \geq \frac{1}{t_{21}} \quad (3.89)$$

$$Z_{12} \geq Z_{21} \quad (3.90)$$

$$\frac{E_1}{V_2} \geq \frac{F_2}{I_1} \quad (3.91)$$

$$E_1 I_1 \geq F_2 V_2 \quad (3.92)$$

Eq. (3.89) can be expressed in term of impedance parameters, according to the relationship of transduction matrix and impedance matrix.

$$Z_{12} = \frac{t_{11}t_{22}}{t_{21}} - t_{12} \quad Z_{21} = \frac{1}{t_{21}} \quad (3.93 \text{ and } 3.94)$$

Where the impedance parameters are defined as:

$$Z_{12} = \left. \frac{E_1}{V_2} \right|_{I_1=0} \quad Z_{21} = \left. \frac{F_2}{I_1} \right|_{V_2=0} \quad (3.95 \text{ and } 3.96)$$

Equations above show that the inequality of transduction matrix's determinant is caused by inefficiency of energy conversion from electrical energy to mechanical energy. However, conversion from Eq. (3.91) to Eq. (3.92) may not be theoretically correct. Because by definition, E_I and V_2 occurs when $I_I = 0$, whereas F_2 and I_I occurs when $V_2 = 0$. They are in two different boundary conditions, thus Eq. (3.92) may not be proper as:

$$E_1|_{I_1=0} I_1|_{V_2=0} \neq F_2|_{V_2=0} V_2|_{I_1=0} \quad (3.97)$$

Nonetheless, the derivation of transduction matrix's determinant from *Eq. (3.88)* to *Eq. (3.92)* could be a good reference for energy conversion between input port and output port. Determinant is larger than unity means the magnitude of transduction matrix is more than one. This implies that the magnitude of input matrix is larger than the magnitude of output matrix, since input matrix is the product of transduction matrix and output matrix. Although the determinant of transduction matrix may not represent energy conversion between input and output exactly, it is still useful to indicate the energy at input and output. Perhaps in some special case of transduction matrix, the condition in *Eq. (3.92)* can be fulfilled, but further study might be needed in order to prove this statement.

Unity determinant also leads to another property which is the Reciprocity theorem, because unity determinant is normally used to describe a reciprocal system [69], [70], [76]. The Reciprocity theorem stated that the ratio of response to excitation is invariant to an interchange of the positions of the excitation and response in a single source network. In short, the theorem stated a physical system which input and output can be interchanged without altering the response of system to a given excitation. Given a two-port network with electrical input E_1 , I_1 and electrical output E_j , I_j , the expression for the Reciprocity theorem is shown as below [66]:

$$\frac{E_j}{I_1} = \frac{E_1}{I_j} \quad (3.98)$$

$$E_1 I_1 = E_j I_j \quad (3.99)$$

In electromechanical system, reciprocity means if a torque and rotational speed is applied at mechanical port, the voltage and current obtained at electrical port will be same as the voltage and current applied in order to generate same amount of torque and rotational speed. However, this may not be true in induction motor. This is because applied torque and rotational speed

cannot generate equivalent voltage and current due to absent of magnetic field in the windings. Therefore, the theoretical unity determinant of induction motor's transduction matrix does not imply that induction motor is reciprocal. This is because the induction motor is assumed to be linear in the theoretical derivation, for which it is nonlinear in practical. On the other hand, permanent magnet DC motor or piezoelectric transducer with unity determinant could be reciprocal because they have better linearity.

3.7.2 Mechanical and Electrical Impedance Relationship

In a transduction matrix which describes the system's properties between electrical input and mechanical output, its mechanical impedance can be written in term of electrical impedance and transduction functions, and vice versa.

From transduction matrix of electromechanical system, the input voltage and current can be expressed as follow:

$$E_1 = t_{11}F_2 + t_{12}v_2 \quad (3.100)$$

$$I_1 = t_{21}F_2 + t_{22}v_2 \quad (3.101)$$

According to the definition of electrical impedance, Z_e is the ratio between voltage and current.

$$Z_e = \frac{E_1}{I_1} = \frac{t_{11}F_2 + t_{12}v_2}{t_{21}F_2 + t_{22}v_2} \quad (3.102)$$

Equation above can be rearranged in term of mechanical impedance:

$$Z_e = \frac{Z_m t_{11} + t_{12}}{Z_m t_{21} + t_{22}} \quad (3.103)$$

Similarly, mechanical impedance can be expressed in term of electrical impedance:

$$Z_m = \frac{Z_e t_{22} - t_{12}}{t_{11} - Z_e t_{21}} \quad (3.104)$$

These expressions are particularly useful in obtaining impedance information at a port where measurement is difficult to be achieved. In most of the case, force and velocity are more difficult to be measured and more costly comparing to voltage and current. *Eq. (3.104)* allows mechanical impedance to be identified by monitoring the signatures of electrical input. Thus the loading condition and other information of mechanical port can be analysed. Using experimental transduction matrix is able to compute Z_e with more than 90% of accuracy compared to measured results, as shown in *Table 3-8*. Comparing between Z_e and Z_m , Z_e is normally having larger value than Z_m and this leads to easier signature identification.

<i>Load (kg)</i>	<i>$Z_m (\Omega)$</i>	<i>$Z_e (\Omega)$</i>	<i>$Z_e (\Omega)$</i>	
	<i>Measured</i>	<i>Measured</i>	<i>Calculated</i>	<i>Accuracy (%)</i>
0.0	0.0001	464.788	436.261	93.86
0.5	0.0049	456.928	429.843	94.07
1.0	0.0093	430.943	411.408	95.47
1.5	0.0127	407.075	392.165	96.34
2.0	0.0162	378.675	370.148	97.75
2.5	0.0215	338.013	337.054	99.72

Table 3-8: Calculated Z_e from Experimental Transduction Matrix.

In fact, *Eq. (3.104)* allows the transducer operates as an actuator as well as a sensor to describe the loading condition. Some researchers used this transduction matrix property for sensing purposes by PZT actuator [77], [78], monitoring drill pit's condition [79], welding [80] and other mechanical system [30].

3.7.3 Power and Efficiency

Efficiency of electric motor can be expressed in term of its transduction functions. If the relationship between electric motor conditions with each transduction functions is acknowledged, the efficiency or condition of the motor can be recognized.

Since electrical power is the product of input voltage and current, mechanical power is the product of output force and velocity, the efficiency of electric motor can be written as follows:

$$P_e = E \times I \quad (3.105)$$

$$P_e = t_{11}t_{21}F_2^2 + t_{12}t_{22}v_2^2 + (t_{11}t_{22} + t_{12}t_{21})F_2v_2 \quad (3.106)$$

$$P_e = t_{11}t_{21}Z_m P_m + t_{12}t_{22} \frac{P_m}{Z_m} + (t_{11}t_{22} + t_{12}t_{21})P_m \quad (3.107)$$

Efficiency can be obtained from the ratio of mechanical power and electrical power:

$$\frac{P_e}{P_m} = t_{11}t_{21}Z_m + t_{12}t_{22} \frac{1}{Z_m} + (t_{11}t_{22} + t_{12}t_{21}) \quad (3.108)$$

$$\eta = \frac{P_m}{P_e} = \left| \frac{1}{t_{11}t_{21}Z_m + t_{12}t_{22} \frac{1}{Z_m} + (t_{11}t_{22} + t_{12}t_{21})} \right| \quad (3.109)$$

Eq. (3.109) shows that efficiency of induction motor is related with the magnitude of power and impedance at mechanical output port, as well as transduction functions. This also suggests that electrical power supplied to induction motor not only used to generate mechanical power, but also used in other power consumption inside the induction motor. For instance, t_{12} is related with magnetic field generated by stator windings and it is involved in the power equation, which means some electrical power has been drawn to form the magnetic field. Consequently, the electrical power supplied for the induction motor is always greater than the mechanical power it can produce.

The expression of power in Eq. (3.108) also indicate that determinant of transduction matrix is not enough to describe power conversion between input and output. According to the power equation, the only condition that electrical power is equal to mechanical power while having unity determinant, is a transduction matrix with:

$$t_{12} = t_{21} = 0 \quad (3.110)$$

$$t_{11} = \frac{1}{t_{22}} \quad (3.111)$$

Under these conditions, the determinant of transduction matrix can be simplified as:

$$|Det(t)| = |t_{11}t_{22}| = 1 \quad (3.112)$$

Subsequently, the power equation can be written as:

$$P_e = (t_{11}t_{22})P_m \quad (3.113)$$

$$P_e = Det(t)P_m \quad (3.114)$$

As a result, the determinant of transduction matrix can accurately represent the power transfer between electric input and mechanical power only if the two conditions in *Eq. (3.110)* and *Eq. (3.111)* are fulfilled. In this case, unity determinant indicates that power conversion between input and output is perfect.

Eq. (3.109) also shows that efficiency of an induction motor is related to transduction functions and mechanical impedance. It has been shown in previous section, the transduction functions which are representing the system properties, associated with efficiency of energy conversion from electrical energy to mechanical energy. For mechanical impedance wise, impedance matching of loading impedance becomes another factor in efficiency of a transducer. Perhaps the equations developed in this section would be helpful to study the performance of induction motor in term of efficiency and maximum power transfer. Maximum power transfer occurs when output impedance of the source is equal to the input impedance of the load. However, maximum power transfer does not result in maximum efficiency.

In alternating power supply, it is important to consider the real power transferred to the load instead of the apparent power, which is the product of

voltage and current in the circuit. The electrical power computed from *Eq. (3.107)* consists of real and imaginary components which are corresponding to real and reactive power. Real power should be considered in the computational of efficiency, therefore *Eq. (3.109)* can be modified as follow:

$$\eta = \frac{P_m}{P_e} = \left| \frac{1}{\operatorname{Re} \left(t_{11}t_{21}Z_m + t_{12}t_{22} \frac{1}{Z_m} + (t_{11}t_{22} + t_{12}t_{21}) \right)} \right| \quad (3.115)$$

In conclusion, a transduction matrix based modelling technique on a squirrel cage induction motor is introduced. The transduction matrix represents the induction motor's system properties, hence motor monitoring and fault diagnosis can be attained by monitoring these parameters or determinant of the matrix. Furthermore, the efficiency of motor depends on transduction functions thus performance evaluation of motor is achievable. Mechanical loading condition can be monitored without torque and speed sensors since they can be estimated from electrical inputs and transduction matrix

Based on the idea of transduction matrix developed in this chapter, this modelling technique is extended to motor driven system. This is to provide better understanding on power flow within a motor driven system from the view point of transduction matrix. In addition, this modelling technique can be used to study how impedance changes in mechanical process for a given motor system. This motor driven system and its transduction matrix will be further discussed in following chapter.

CHAPTER 4

TRANSDUCTION MATRIX OF MOTOR DRIVEN SYSTEM

Many work exertion processes employed by human to improve productivity are in the form of a dynamic system containing a driver, a power transmission and a load in series. In the last few decades, motors play greater role as drivers of such systems. Transduction matrix of induction motor has been studied in the previous chapter. This chapter focuses on transduction matrices of power transmission mechanism and mechanical loading. Transduction matrix of power transmission mechanisms such as gears and mechanical linkage is modelled from governing equations of respective system. Similarly, transduction matrix of mechanical loading is studied in this chapter. Lastly, an overall transduction matrix which describes a motor driven system is obtained from combining all the matrices from cascaded transduction matrices.

4.1 Introduction

Typical motor driven systems utilized in industry include compressed air system, pump system, conveyor belts, and other machines that are driven by an electrical motor. Many researches have been focused in improving the performance, applications, as well as energy efficiency of the motor driven system [81].

Figure 4-1: A Typical Motor Driven System

Figure 4-1 shows a typical motor driven system, which consists of electric motor, power transmission mechanism and mechanical loading. Electric motor converts electrical energy to mechanical energy. In industry, the most widely used electric motor is induction motor due to its reliability and low cost. Power transmission is another essential sub-system as it plays the role of transferring mechanical power from motor to exert work on loading in the system. Power transmission mechanism such as mechanical linkage not only transferring power but also provide mechanical motion. Finally, the mechanical power will be transferred to drive a mechanical load as the mechanical energy is converted into another energy form.

In order to have better understanding on motor driven systems and its energy flow from electrical source to mechanical loading, a motor driven system is modelled and analyzed by using transduction matrix. In previous chapter, transduction matrix of induction motor has already been covered. Basically, transduction matrix describes a system's properties and enables obtaining output information based on input information, or vice versa. In this chapter, transduction matrix of power transmission mechanism and mechanical loading will be focused. Transduction matrices of power transmission mechanism such as gear system and mechanical linkage can be obtained from governing equations of respective system.

4.2 Transduction Matrix of Gears

Gear is a power transmission mechanism with teeth or spurs that meshes with one and another in order to transmit mechanical power from one shaft to another shaft, usually with a constant speed ratio. Using gears system can change the speed, torque and direction of a given mechanical power, thus mechanical advantages can be achieved at output. Mechanical advantage is a measure of the force amplification achieved by using a mechanical device. In addition, some power transmissions offer multiple gear ratios, such as bicycle and cars transmission system. This setup allows selection of different gear

ratios in order to attain various mechanical advantages. Gear system is so common in power transmission system, and it has high efficiency in power transfer as well as accuracy in motion.

4.2.1 Governing Equation

In gear system, there are many types of gear such as helical gear, worm gear, rack and pinion, which is to accommodate with various applications. However, spur gears are used in following analysis due to its simplicity.

Figure 4-2: Spur Gears System

A pair of spur gear is presented in *Figure 4-2*. The larger gear is often named as gear or wheel, the smaller one is normally called pinion. Gear 1 is driving by torque and rotational speed of T_1 and ω_1 . This torque and rotational speed will then be transferred to Gear 2. Equations below show the relationship of torque and rotational speed from both gears and its gear ratio [10], [82].

$$v_p = r_1 \omega_1 = r_2 \omega_2 \quad (4.1)$$

$$gr = \frac{r_2}{r_1} = \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} \quad (4.2)$$

Based on the gear ratio in *Eq. (4.2)*, the governing equations of the spur gear system can be obtained as follows:

$$T_1 = \frac{1}{gr} T_2 \quad (4.3)$$

$$\omega_1 = gr \omega_2 \quad (4.4)$$

Equations above describe the change of torque and rotational speed from Gear 1 to Gear 2. Hence they can be used in transduction matrix which also describes the relationship between input and output.

4.2.2 Transduction Matrix of Gears

Given a gear set which is applied with input torque T_1 and rotational speed ω_1 , the output torque and rotational speed would be T_2 and ω_2 , as shown in *Figure 4-2*. Transduction matrix of the simple spurs gear system can be obtained from *Eq. (4.3)* and *Eq. (4.4)*:

$$\begin{bmatrix} T_1 \\ \omega_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{gr} & 0 \\ 0 & gr \end{bmatrix} \begin{bmatrix} T_2 \\ \omega_2 \end{bmatrix} \quad (4.5)$$

This transduction matrix represents the spurs gear system, where gr is corresponding to gear ratio. From the transduction matrix, t_{11} is equivalent to the gear ratio and t_{22} is reciprocal of that. Both t_{12} and t_{21} are zero in this case. This shows that for a given gear system with constant gear ratio, its transduction matrix is constant as well.

$$|Det(t)| = \left| \frac{1}{gr} gr - 0 \right| = 1 \quad (4.6)$$

The determinant of gears' transduction matrix is unity as shown in *Eq. (4.6)*. From previous analysis in section 3.7, determinant of transduction matrix is related with power transfer from input to output. The power of this gear system can be obtained from the product of torque and rotational speed as shown in *Eq. (4.7)* below. The input power to the gear system is indeed equal to its output power.

$$T_1 \omega_1 = \frac{1}{gr} T_2 \times gr \omega_2 \quad (4.7)$$

$$T_1 \omega_1 = T_2 \omega_2 \quad (4.8)$$

$$P_1 = P_2 \quad (4.9)$$

In the transduction matrix of gear system, input power is equal to output power while having unity determinant. This is because it fulfilled the two conditions below:

$$t_{12} = t_{21} = 0 \quad (4.10)$$

$$t_{11} = \frac{1}{t_{22}} \quad (4.11)$$

Consequently, *Eq. (4.7)* can be rearranged in term of power and transduction matrix's determinant as:

$$P_1 = \left(\frac{1}{gr} \times gr \right) P_2 = \text{Det}(t) P_2 \quad (4.12)$$

This indicates that determinant of gear system's transduction matrix represents power transfer between input and output of gear system. Unity determinant means power transfer from input to output is ideal and there is no loss of energy. The gear system used in this analysis is assumed to have perfect efficiency, thus its determinant is unity.

Another purpose of using gear system is to achieve mechanical advantage at the output. This can be reflected by studying the mechanical impedance of gear system. Mechanical impedance can be computed from division of torque by rotational speed.

$$\frac{T_1}{\omega_1} = \frac{\frac{1}{gr} T_2}{gr \omega_2} \quad (4.13)$$

$$\frac{T_1}{\omega_1} = \frac{1}{gr^2} \frac{T_2}{\omega_2} \quad (4.14)$$

$$Z_1 = \frac{1}{gr^2} Z_2 \quad (4.15)$$

From *Eq. (4.15)*, mechanical input impedance for gear is equal to the square of gear ratio multiply with mechanical output impedance. For instance, a gear

system with gear ratio of 2 is having output mechanical impedance four times larger than its mechanical input impedance. The increment of mechanical output impedance corresponds to the mechanical advantage that system can achieve. Higher mechanical output impedance means larger force can be produced at output, thus the gear system is able to drive a loading that required large force.

As a result, a gear system is able to transmit mechanical power and change the system's mechanical impedance in order to match with loading requirements. The transduction matrix of gear system in *Eq. (4.15)* can be used to represent gear system, especially in the analysis of motor driven system later.

4.3 Transduction Matrix of Four Bar Linkage

A mechanical linkage is one type of mechanical mechanism which comprises of several bodies together in order to handle force and motion. In mechanical linkage, the predominant purpose of using this mechanism is to achieve a desired motion, despite it can transmit mechanical power or force as well. Because of this, mechanical linkage is always used in the situation where velocity ratio is not constant, unlike the gear system.

4.3.1 Crank-Rocker Four Bar Linkage

The most commonly used mechanical linkage is four bar linkage that consists of four rigid links and connected in a loop by four joints. In four bar linkage, achieving mechanical advantage is not its main purpose. It is mainly used in transmitting motion from input to output [7], [10], [82]. In fact, four bar linkage is by far the most widely used linkage for irregular motion generation. For instance, a crank rocker linkage as shown in *Figure 4-3(a)* is able to generate an irregular circular shape by taking the path of point *P*. Besides that, some special linkages are able to generate straight line with a high degree of accuracy, which is very useful in machinery for converting rotational motion into linear motion. Watt's straight line mechanism in *Figure 4-3(b)* is a four

bar linkage which is able to approximate a straight line with point P of its coupler link. This linkage can be found in some automobile suspensions, to restrain lateral motion of the axle of a vehicle.

Figure 4-3: Motion Generation by Four Bar Linkage

Generally, force and motion transmission of all types of four bar linkage can be modelled by transduction matrix. Since transduction matrix describes the relationship between input and output for its force and velocity, thus understanding of the four bar linkage's force and velocity at each position is necessary. Kinematic analysis that covers four bar linkage's position, velocity and acceleration is carried out in latter section. In addition, force analysis will be carried out to identify forces acting on each links, especially the input and output links. In the following analysis, a crank-rocker four bar linkage as shown in *Figure 4-4* is used. In the following crank-rocker linkage, point A and D is fixed, hence r_1 is the ground link. The input link is r_2 , which is the shortest. Link r_3 is the coupler and r_4 is the output link respectively. As a result, link r_2 is able to make complete revolution hence it is also named as crank. Link r_4 is known as rocker as it can only rotate between motion limits. The dimensions of each link are given in *Table 4-1*, and this linkage indeed satisfies both linkage criterion and Grashof's law.

Figure 4-4: A Crank-Rocker Four Bar Linkage

Link	Length (cm)
r_1	40
r_2	10
r_3	50
r_4	30

Table 4-1: Dimensions of Each Link in Crank-Rocker Linkage

Consequently, kinematic analysis is performed on the crank-rocker shown above. There are several methods to carry out kinematic analysis, such as graphical, analytical, and numerical methods. Analytical analysis of four bar linkage is presented in Appendix B.

4.3.2 Transduction Matrix of 4-Bar Linkage

The kinematic and static force analysis of four bar linkage has been conducted. From the analytic equations developed, the transduction matrix of a four bar linkage can be constructed by the torque and velocity equations from Eq. (B.27) and Eq. (B.16).

$$T_2 = \frac{r_2 \sin \phi}{r_4 \sin \eta} T_4 \quad (4.16)$$

$$\omega_2 = \frac{r_4 \sin \eta}{r_2 \sin \phi} \omega_4 \quad (4.17)$$

Hence the transduction matrix:

$$\begin{bmatrix} T_2 \\ \omega_2 \end{bmatrix} = \begin{bmatrix} \frac{r_2 \sin \phi}{r_4 \sin \eta} & 0 \\ 0 & \frac{r_4 \sin \eta}{r_2 \sin \phi} \end{bmatrix} \begin{bmatrix} T_4 \\ \omega_4 \end{bmatrix} \quad (4.18)$$

Transduction matrix above describes the relationship between input port and output port of a four bar linkage. This transduction matrix represents system properties of linkage that do not affected by external force. Indeed the transduction parameters are depends on the length of input and output link, as well as the angle ϕ and η . These parameters for a given four bar linkage are varying with respect to position but they will be the same for every cycle it turns, this is illustrated in *Figure 4-5*. In the graph, t_{12} and t_{21} are zero as stated. Parameter t_{11} is changing in between 0.38 and -0.53 in sinusoidal curve, and t_{22} is indeed the reciprocal of t_{11} .

Figure 4-5: Transduction Matrix of Crank-Rocker Linkage with respect to Position of Input Link

The transduction matrix of four bar linkage shares great similarity with transduction matrix of gear. For instance, t_{11} is a constant ratio between input and output link, whereas t_{22} is reciprocal of t_{11} . Both t_{12} and t_{21} are zero as well. Hence this transduction matrix fulfilled the conditions in *Eq. (4.10)* and *Eq. (4.11)*. As a result, the unity determinant of this transduction matrix explained that the power transfer between input and output is equal. Perhaps these are the general properties of transduction matrix for a mechanical mechanism that aims to transmit power and provides mechanical advantages.

$$|Det(t)| = \left| \frac{r_2 \sin \phi}{r_4 \sin \eta} \times \frac{r_4 \sin \eta}{r_2 \sin \phi} - 0 \right| = 1 \quad (4.19)$$

The input power is equal to output power as follows:

$$T_1 \omega_1 = \frac{r_2 \sin \phi}{r_4 \sin \eta} T_2 \times \frac{r_4 \sin \eta}{r_2 \sin \phi} \omega_2 = T_2 \omega_2 \quad (4.20)$$

$$P_1 = P_2 \quad (4.21)$$

Similarly, the mechanical impedance from input to output has changed. Normally the mechanical impedance is increased as mechanical advantage is achieved.

$$\frac{T_1}{\omega_1} = \frac{r_2 \sin \phi}{r_4 \sin \eta} T_2 \left/ \frac{r_4 \sin \eta}{r_2 \sin \phi} \omega_2 \right. \quad (4.22)$$

$$\frac{T_1}{\omega_1} = \left(\frac{r_2 \sin \phi}{r_4 \sin \eta} \right)^2 \frac{T_2}{\omega_2} \quad (4.23)$$

$$Z_1 = \left(\frac{r_2 \sin \phi}{r_4 \sin \eta} \right)^2 Z_2 \quad (4.24)$$

In summary, the transduction matrix of a four bar linkage is developed in this section and it is similar with gear's transduction matrix. This transduction matrix can be used to model a four bar linkage in a motor driven system.

4.4 Transduction Matrix of Loading

Mechanical loading is one of the components in motor driven system, and it is normally the final components in the whole mechanical system, as the mechanical energy is converted into another form of energy. Some examples of literature on the load modelling for power flow and dynamic analysis can be found in [84]-[88]. In this section, transduction matrix will be used to model mechanical load instead. Basically, a mass-spring-damper system is studied because it represents a typical mechanical system. Subsequently, the study is extended to the modelling of band brake used in the experiments by transduction matrix.

4.4.1 Mass-Spring-Damper System

Mass-spring-damper model is a simple, powerful and widely used model to describe the function of a dynamic component in our pursued system. For example, it could be the load of a motor driven system to characterize a sample of material or to identify a machinery structure. An ideal mass-spring-damper system consists of a mass M to represent the inertia of system, a spring constant K_s to describe the system's stiffness, and finally a damper with damping coefficient C to illustrate the system's viscosity or resistance. Because of its versatility, a mass-spring-damper system is used to represent a general mechanical load. Thus the transduction matrix of a general mass-spring-damper system is derived for utilization later. In literature, similar derivation appeared in [69], [70].

Figure 4-6: Mass-Spring-Damper Model

In the mass-spring-damper system above, F_1 and V_1 are representing the input force and velocity to the mechanical system, whereas F_2 and V_2 are the output. In mechanical system, mass M is always associated with inertia force of rigid body. Stiffness K_s is usually related to the position of mass and the energy stored inside the system. Damping coefficient C is correlated with dissipated energy in the system, which includes friction force or force used to drive a load. The governing equations relate the force and velocity of input and output of the mass-spring-damper system are shown as follow [70], [89], [90]:

$$F_1 = M \frac{dV_1}{dt} + F_2 \quad (4.25)$$

$$F_2 = C(V_1 - V_2) + K_s \int (V_1 - V_2) dt \quad (4.26)$$

$$\text{Let, } F = F(\cos \omega t + i \sin \omega t) = Fe^{i\omega t} \quad (4.27)$$

$$V = V(\cos \omega t + i \sin \omega t) = Ve^{i\omega t} \quad (4.28)$$

$$\frac{d}{dt} V = i\omega Ve^{i\omega t} \quad (4.29)$$

$$\int V dt = \frac{1}{i\omega} Ve^{i\omega t} \quad (4.30)$$

Hence the Eq. (4.25) and Eq. (4.26) can be rewritten as follow by eliminating $e^{i\omega t}$ at both sides.

$$F_1 = M \omega i V_1 + F_2 \quad (4.31)$$

$$F_2 = C(V_1 - V_2) + \frac{K_s}{\omega i} (V_1 - V_2) \quad (4.32)$$

Rearranging Eq. (4.32) to express V_1 in term of F_2 and V_2 gives:

$$V_1 = \frac{1}{\left(C + \frac{K_s}{\omega i}\right)} F_2 + V_2 \quad (4.33)$$

Substitute Eq. (4.33) back into Eq. (4.31) yields the expression of F_1 in term of F_2 and V_2 .

$$F_1 = \left(\frac{M \omega i}{C + \frac{K_s}{\omega i}} + 1 \right) F_2 + M \omega i V_2 \quad (4.34)$$

Finally, the transduction matrix of a mass-spring-damper system can be obtained with *Eq. (4.33)* and *Eq. (4.34)*.

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \frac{M \omega i}{C + \frac{K_s}{\omega i}} + 1 & M \omega i \\ \frac{1}{C + \frac{K_s}{\omega i}} & 1 \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \quad (4.35)$$

From the transduction matrix, perhaps t_{11} and t_{21} can be useful in studying loading condition of a mechanical load, as damping coefficient C appears in these two transduction functions. The existence of damping coefficient C means the input mechanical power results in output mechanical power from loading, as well as dissipated energy such as heat lost to surrounding. If C is removed from the model, the output mechanical power is smaller than the input mechanical power. Since C is included in the model, the determinant of this transduction matrix is equal to unity, which is related to energy conversion between input and output.

$$|Det(t)| = \left| \left(\frac{M \omega i}{C + \frac{K_s}{\omega i}} + 1 \right) \times 1 - \frac{M \omega i}{C + \frac{K_s}{\omega i}} \right| = 1 \quad (4.36)$$

The analysis of mass-spring-damper system is suitable to describe general situation where there are no constraint at both end of the system. In the case of the mass-spring-damper system is fixed with a base, such that the output is under the clamped condition. The transduction matrix in this situation will be slightly different from previous one.

If the mass-spring-damper load is grounded, where $V_2 = 0$. Physically, a fixed material specimen or a clamped structure component satisfies this condition. Therefore the two equations in Eq. (4.33) and Eq. (4.34) will be expressed as:

$$F_1 = \left(\frac{M\omega i}{\left(C + \frac{K_s}{\omega i}\right)} + 1 \right) F_2 \quad (4.37)$$

$$V_1 = \frac{1}{\left(C + \frac{K_s}{\omega i}\right)} F_2 \quad (4.38)$$

The corresponding transduction matrix:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \frac{M\omega i}{C + \frac{K_s}{\omega i}} + 1 & 0 \\ \frac{1}{C + \frac{K_s}{\omega i}} & 0 \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \quad (4.39)$$

If the load moves freely with the output terminal of the power transmission mechanism, the load reduces to a pure mass and thus:

$$F_1 = M\omega i V_2 \quad (4.40)$$

$$V_1 = V_2 \quad (4.41)$$

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} 0 & M\omega i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \quad (4.42)$$

4.4.2 Band Brake

In testing of dynamic behaviors of a motor, a proper power sink is required. In the case where the power involved is not heavy, band brake as shown in Figure 4-7 is considered an appropriate power sink. Note that the friction which consumes the power supplied by the motor under testing is adjustable by changing the weight hung at the end of the belt.

Figure 4-7: Band Brake used in Experiments

A band brake can be modelled as a mass-spring-damper system in rotational motion. Firstly, the linear governing equations are changed to rotational motion and hence reorganize the rotational governing equations into transduction matrix.

Figure 4-8: Rotational Mass-Spring-Damper System

Let, $\omega = \omega(\cos \omega_0 t + i \sin \omega_0 t) = \omega e^{i\omega_0 t}$ (4.43)

Hence,
$$T_1 = \left(\frac{J\omega_0 i}{\left(C + \frac{K_s}{\omega_0 i} \right)} + 1 \right) T_2 + J\omega_0 i \omega_2$$
 (4.44)

$$\omega_1 = \frac{1}{\left(C + \frac{K_s}{\omega_0 i}\right)} F_2 + \omega_2 \quad (4.45)$$

In the rotational mass-spring-damper system above, the system is assumed to be running at frequency of ω_0 . T_1 and ω_1 represent input torque and rotational speed. On the output port, T_2 and ω_2 are representing output torque and rotational speed. Mass moment of inertia J is used to describe the inertia of mass in rotational motion. Similarly, K_s and C are the rotational system's stiffness and damping coefficient. Together with all the parameters described above, and the governing equations, transduction matrix of rotational mass-spring-damper system is shown as follow:

$$\begin{bmatrix} T_1 \\ \omega_1 \end{bmatrix} = \begin{bmatrix} \frac{J\omega_0 i}{C + \frac{K_s}{\omega_0 i}} + 1 & J\omega_0 i \\ \frac{1}{C + \frac{K_s}{\omega_0 i}} & 1 \end{bmatrix} \begin{bmatrix} T_2 \\ \omega_2 \end{bmatrix} \quad (4.46)$$

Equation above shows the general expression of the rotational mass-spring-damper system. However it can be further simplified according to the actual mechanical system. For example, the band brake used in experiments is always rotating at constant rotational speed. Therefore, $\omega_0 = 0$ and the inertia of rotating mass can be neglected here. In addition, the cylinder and shaft can be assumed to be rigid body, thus stiffness K_s can be discarded as well. Under these circumstances, the transduction matrix of band brake is simplified with only the damping coefficient that indicating friction force.

$$\begin{bmatrix} T_1 \\ \omega_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{C} & 1 \end{bmatrix} \begin{bmatrix} T_2 \\ \omega_2 \end{bmatrix} \quad (4.47)$$

Transduction matrix above illustrates the properties of band brake, and actually the value of the transduction matrix can be obtained from

experiments. The only variable in the matrix is damping coefficient, in order to compute its value, a set of experiment is carried out. The band brake is tested under increasing load by placing more and more masses at the load hanger. Meanwhile, the torque and rotational speed required to drive the cylinder against the load are measured. In band brake which is stationary, the ω_2 equals to zero. Hence the damping coefficient can be computed from Eq. (4.48) below. The experiments are conducted with two induction motors with different power rating and rotational speed. The results are shown in Figure 4-9.

$$C = \frac{T_2}{\omega_1} = \frac{T_1}{\omega_1} \quad (4.48)$$

Figure 4-9: Damping Coefficient of Band Brake

According to the result, the damping coefficient increases as the loading increased. For instance, a band brake with loading mass of 2.0 kg will required a torque of 2.4 Nm to drive the cylinder. Hence the damping coefficient at that loading is about 0.016 Ns/m, and this value does not vary much even if

induction motor with different rating is used. The variation of damping coefficient also proved that transduction matrix of band brake changing with loading as well. The damping coefficient obtained can be justified with friction coefficient between metal and belt. Firstly, the normal force acting on cylinder by belt, N_D based on free body diagram in *Figure 4-10* is computed.

$$N_D = 2 \times \frac{0.23}{0.1} \times Mg \quad (4.49)$$

Damping coefficient is related with friction coefficient by:

$$T = C\omega \quad (4.50)$$

$$\mu N_D r = C\omega \quad (4.51)$$

$$\mu = \frac{C\omega}{N_D r} \quad (4.52)$$

Figure 4-10: Free Body Diagram of Band Brake

Based on the calculations above, at damping coefficient of 0.016 Ns/m and rotational speed of 149.09 rad/s, together with loading mass of 2.0 kg. Consequently, the friction coefficient at that experiment setting is 0.54. In general, the static friction coefficient between surface of metal and leather is around 0.4 – 0.6 [91]. As results, the computed friction coefficient is acceptable as it falls within the theoretical range.

In this section, a transduction matrix of band brake is developed. Basically, the transduction matrix in Eq. (4.47) contains damping coefficient which can be found by a set of experiments. Since the damping coefficient is changing with loading condition thus variation of the transduction matrix is expected. However, the transduction matrix comprises of output torque caused by friction force from belt, which is rather difficult to be visualized. For that reason, it is better to express the transduction matrix of band brake with weight of loading mass and velocity as the output port. This can be done by dividing the band brake into several sections, and modelled each section with transduction matrix. Finally, combine all the transduction matrices with cascaded method in order to obtain the overall transduction matrix of band brake.

Figure 4-11: Segmentation of Band Brake

Each segment of band brake has a transduction matrix respectively, for example, the transduction matrix that describes friction force between rotating mass and belt is already presented in Eq. (4.47). After that, the transduction matrix of belt converts rotational motion to linear motion as follows, where r_D is the radius of cylinder.

$$T_D = r_D \times F_D \quad (4.53)$$

$$\omega_D = \frac{1}{r_D} \times V_D \quad (4.54)$$

$$\begin{bmatrix} T_D \\ \omega_D \end{bmatrix} = \begin{bmatrix} r_D & 0 \\ 0 & \frac{1}{r_D} \end{bmatrix} \begin{bmatrix} F_D \\ V_D \end{bmatrix} \quad (4.55)$$

Likewise, the transduction matrix for power transmitted from lever arm to weight can be calculated from governing equations below.

$$F_D = \frac{0.23}{0.10} \times \frac{1}{\sin 70^\circ} W_M \quad (4.56)$$

$$V_D = \frac{0.10}{0.23} \times \sin 70^\circ V_M \quad (4.57)$$

$$\begin{bmatrix} F_D \\ V_D \end{bmatrix} = \begin{bmatrix} \frac{0.23}{0.10} \times \frac{1}{\sin 70^\circ} & 0 \\ 0 & \frac{0.10}{0.23} \times \sin 70^\circ \end{bmatrix} \begin{bmatrix} W_M \\ V_M \end{bmatrix} \quad (4.58)$$

Consequently, the overall transduction matrix of band brake is obtained by multiplying matrix of each segment sequentially.

$$\begin{bmatrix} T_1 \\ \omega_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{C} & 1 \end{bmatrix} \times \begin{bmatrix} r_D & 0 \\ 0 & \frac{1}{r_D} \end{bmatrix} \times \begin{bmatrix} \frac{0.23}{0.10} \times \frac{1}{\sin 70^\circ} & 0 \\ 0 & \frac{0.10}{0.23} \times \sin 70^\circ \end{bmatrix} \begin{bmatrix} W_M \\ V_M \end{bmatrix} \quad (4.59)$$

$$\begin{bmatrix} T_1 \\ \omega_1 \end{bmatrix} = \begin{bmatrix} r_D \times \frac{0.23}{0.10} \times \frac{1}{\sin 70^\circ} & 0 \\ \frac{r_D}{C} \times \frac{0.23}{0.10} \times \frac{1}{\sin 70^\circ} & \frac{1}{r_D} \times \frac{0.10}{0.23} \times \sin 70^\circ \end{bmatrix} \begin{bmatrix} W_M \\ V_M \end{bmatrix} \quad (4.60)$$

Transduction matrix at above describes the band brake based on its input torque to the weight of loading mass. Among the parameters, only damping coefficient C is changing with loading, the rest are constant. Since the damping coefficient is found previously in *Figure 4-9*, thus the torque created by various loading mass can be computed from the transduction matrix. The determinant of this transduction matrix is equal to unity. This is because all the determinants of each sub-system's transduction matrix involved are unity as well.

$$Det(t) = \left(r_D \times \frac{0.23}{0.10} \times \frac{1}{\sin 70^\circ} \right) \times \left(\frac{1}{r_D} \times \frac{0.10}{0.23} \times \sin 70^\circ \right) = 1 \quad (4.61)$$

4.5 Overall Transduction Matrix of a Motor Driven System

In a motor driven system which consists of motor, power transmission mechanism and mechanical load, its overall transduction matrix can be obtained from multiplying each matrix in sequence. Representing the motor driven system with a overall transduction matrix allows the output of the system to be studied based on the information at input, and vice versa. In addition, the transduction matrix describes the system's condition thus any changes in the downstream of system can be acknowledged. On the other hand, the power flow and how the impedance change along the energy path of motor driven system can be investigated if the each of the system is analyzed with respective transduction matrix. The testing set-up employed in this research work to identify the transduction matrix of a motor is taken as an example here to demonstrate the validity of the method explained so far.

4.5.1 Motor Driven System

The motor driven system used in experiments is very simple, which only consists of induction motor and band brake. This example may not be able to fully describe a motor driven system as it lacks of transmission system. Therefore, an example of motor driven system is presented below for the purpose of illustrate its transduction matrices.

Figure 4-12: Segmentation of Motor Driven System

In this example, all the transduction matrices introduced previously have been included. Basically, a mechanical load which is modelled as rotational mass-spring-damper system is driven by induction motor. In between the motor and load, gear and four-bar linkage are used to transmit the mechanical power. E_e and I_e representing the electrical input for induction motor, and the following

T and ω indicating the torque and rotational speed produced by respective system .

4.5.2 Cascaded Transduction Matrix

The motor driven system shown in *Figure 4-12* can be expressed in transduction matrices as below. The first matrix is induction motor's transduction matrix that was developed in previous chapter. The second matrix is the gear's transduction matrix and follows with four bar linkage's matrix. The last matrix represents mechanical load.

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \left(\left(\frac{R_s + jX_s}{jX_M} + 1 \right) (R_r + jX_r) + (R_s + jX_s) \right) \frac{\sqrt{2}}{3K} & \left(\frac{R_s + jX_s}{jX_M} + 1 \right) \sqrt{2} 3K \\ \left(\frac{R_r + jX_r}{jX_M} + 1 \right) \frac{\sqrt{2}}{3K} & \frac{\sqrt{2} 3K}{jX_M} \end{bmatrix} \quad (4.62)$$

$$\times \begin{bmatrix} \frac{1}{gr} & 0 \\ 0 & gr \end{bmatrix} \times \begin{bmatrix} \frac{r_2 \sin \phi}{r_4 \sin \eta} & 0 \\ 0 & \frac{r_4 \sin \eta}{r_2 \sin \phi} \end{bmatrix} \times \begin{bmatrix} \frac{J\omega_0 i}{C + \frac{K_s}{\omega_0 i}} + 1 & J\omega_0 i \\ \frac{1}{C + \frac{K_s}{\omega_0 i}} & 1 \end{bmatrix} \begin{bmatrix} T_3 \\ \omega_3 \end{bmatrix}$$

Hence the overall transduction matrix:

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} T_3 \\ \omega_3 \end{bmatrix} \quad (4.63)$$

where,

$$\begin{aligned} t_{11} = & \left(\left(\left(\frac{R_s + jX_s}{jX_M} + 1 \right) (R_r + jX_r) + (R_s + jX_s) \right) \frac{\sqrt{2}}{3K} \right) \times \frac{1}{gr} \frac{r_2 \sin \phi}{r_4 \sin \eta} \times \left(\frac{J\omega_0 i}{C + \frac{K_s}{\omega_0 i}} + 1 \right) \\ & + \left(\frac{R_s + jX_s}{jX_M} + 1 \right) \sqrt{2} 3K \times gr \frac{r_4 \sin \eta}{r_2 \sin \phi} \times \frac{1}{C + \frac{K_s}{\omega_0 i}} \end{aligned} \quad (4.64)$$

$$t_{12} = \left(\left(\left(\frac{R_s + jX_s}{jX_M} + 1 \right) (R_r + jX_r) + (R_s + jX_s) \right) \frac{\sqrt{2}}{3K} \right) \times \frac{1}{gr} \frac{r_2 \sin \phi}{r_4 \sin \eta} \times J\omega_0 i + \left(\frac{R_s + jX_s}{jX_M} + 1 \right) \sqrt{2} 3K \times gr \frac{r_4 \sin \eta}{r_2 \sin \phi} \quad (4.65)$$

$$t_{21} = \left(\left(\frac{R_r + jX_r}{jX_M} + 1 \right) \frac{\sqrt{2}}{3K} \right) \times \frac{1}{gr} \frac{r_2 \sin \phi}{r_4 \sin \eta} \times \left(\frac{J\omega_0 i}{C + \frac{K_s}{\omega_0 i}} + 1 \right) + \frac{\sqrt{2} 3K}{jX_M} \times gr \frac{r_4 \sin \eta}{r_2 \sin \phi} \times \frac{1}{C + \frac{K_s}{\omega_0 i}} \quad (4.66)$$

$$t_{22} = \left(\left(\frac{R_r + jX_r}{jX_M} + 1 \right) \frac{\sqrt{2}}{3K} \right) \times \frac{1}{gr} \frac{r_2 \sin \phi}{r_4 \sin \eta} \times J\omega_0 i + \frac{\sqrt{2} 3K}{jX_M} \times gr \frac{r_4 \sin \eta}{r_2 \sin \phi} \quad (4.67)$$

Transduction matrix from Eq. (4.63) to Eq. (4.67) represents the overall transduction matrix for the motor driven system. According to the derivation, almost all parameters from each system are included in all of the four transduction parameters. Despite that, t_{11} and t_{12} might be useful for indicating the condition of stator winding in induction motor, as they consists of stator resistance and reactance. On the other hand, perhaps t_{21} and t_{22} are able to show the changes in rotor resistance and reactance. For mechanical loading wise, t_{12} and t_{22} may be able to describe the inertia from loading, whereas the loading condition and stiffness can be shown in t_{11} and t_{21} .

There are a few advantages to model the motor driven system in transduction matrix above. Firstly, the condition of each system's parameters can be studied by looking at the transduction parameters that describe it. However, identifying the transduction parameters which relates with system condition can be troublesome. Secondly, the power and impedance at each individual system can be identified when the motor driven system is illustrated in cascaded transduction matrix. Consequently, it can be used to study how the power flows within the system, as well as the role of impedance in matching up induction motor with mechanical loading.

In summary, the transduction matrix of motor driven system has been introduced in this chapter. The use of transduction matrix in motor driven system enables the study of power flow and impedance change within the system. In mechanical system, the power flows from induction motor to mechanical load in response of the impedance's level at loading. As a result, these issues on power and impedance will be focused in the following chapter.

CHAPTER 5

IMPEDANCE PROPAGATION AND POWER TRANSMISSION IN MOTOR DRIVEN SYSTEMS

In a motor driven system, the electrical power converts to mechanical power by electric motor. The mechanical power is then transmitted by means of various transmission systems to drive mechanical loading. During this process, the power transmitted to mechanical load is changed according to loading impedance. Therefore, the relationship between impedance and power transmission of a motor driven system will be investigated in this chapter. The transduction matrix developed previously facilitates the power transmission analysis on each of the components in motor driven system.

5.1 Introduction

When power transmission is mentioned, much attention has been given to electric power system such as electric transmission grid, electrical cable, power transformer and etc [92], [93]. They are effective and economical method for transferring power over a very long distance. They are also the main source of power supply in most industry in order to drive mechanical machines. In the infrastructure, more and more motor driven system plays essential role to transmit the electrical power into mechanical power that can carry out physical work.

Much effort has been put into the study of power transmission for the purpose of upgrading and control of power transmission. Research and development works are carried out based on analytic expression on the power flow of the system and improving it by optimizing the system's parameters. For instance, the power flow of a gear train system has been studied by looking at the expressions of efficiency and power ratio. Subsequently, improve the gear system by minimizing the power losses [94]-[96]. On the other hand, bond

graph is another popular and useful analysis in studying power flow. In bond graph, the elements in a system are linked together with “bonds”, and the power flow in each bond is illustrated by “effort” and “flow” as the power variables. Bond graph analysis has been used in mechatronic system such as transmission of a robot, which the information gathered are very valuable in its control system [97], [98]. Other examples of bond graph modelling can be found in [38], [42], [99].

Furthermore, some studies on power flow analysis on multi input / multi output (MIMO) system had been conducted by transmission matrix approach. This method allows each structure in MIMO system to be modelled as a group of transmission elements, thus the power flow is formulated analytically from force and velocity functions based on the transmission matrices. [100]. Power flow analysis using transmission matrix shares some conceptual similarities with the modelling of motor driven system by transduction matrix in this chapter. The fact is that impedance plays significant role in determining the amount of power flow between two bodies. Transduction matrix facilitated the analysis on both impedance propagation and power transmission.

5.2 Electrical and Mechanical Impedance

Electrical impedance is defined as the ratio of voltage to current in frequency domain, not only describing the relative amplitude of voltage and current but also the relative phase. In term of complex quantity, the electrical impedance in *Eq. (5.1)* is expressed in resistance as its real component and reactance as the imaginary component. Similarly, mechanical impedance is the ratio of applied force to resulting velocity at that point as shown in *Eq. (5.2)*. It is a function of frequency, as the applied force and resulting velocity can vary with frequency.

$$Z_e(f) = R_e + jX_e = \frac{E_e(f)}{I_e(f)} \quad (5.1)$$

$$Z_m(f) = R_m + jX_m = \frac{F_m(f)}{V_m(f)} \quad (5.2)$$

In a motor driven system, the electric power is converted into mechanical power and hence the electrical impedance at input is changed to mechanical impedance at output. For an induction motor, the electrical impedance at input is inversely proportional to the mechanical impedance at output. The mechanical impedance from motor is normally increased by transmission system, thus the increased mechanical impedance is able to match with loading impedance.

From a transduction matrix of mechanical system running at single frequency, the input impedance can be expressed in term of output impedance together with transduction functions, and vice versa. This has been discussed in Chapter 3.

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix}_{IN} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix}_{OUT} \quad (5.3)$$

$$Z_1 = \frac{Z_2 t_{11} + t_{12}}{Z_2 t_{21} + t_{22}} \quad (5.4)$$

$$Z_2 = \frac{Z_1 t_{22} - t_{12}}{t_{11} - Z_1 t_{21}} \quad (5.5)$$

This relationship allows the computational of impedance at another port if the transduction functions and impedance at the other end are known. Therefore, in a motor driven system that is modelled by transduction matrix as shown in *Figure 5-1*, the impedance at each port can be computed if the input power supply or the loading condition is known.

The transduction matrices of induction motor and transmission system have been identified in previous chapters. In order to drive a given load, the output impedance from transmission system must be able to match with loading impedance. Based on the output condition, the input impedance required for

the transmission system can be computed from its transduction matrix. Correspondingly, the input impedance for transmission system has to match with the output impedance for induction motor. Under a given voltage, the motor will draw in higher or lower current in order to match with the output. Thus the electrical input impedance for motor will change accordingly. Besides, the input power of induction motor is also changed in response to the loading impedance.

Figure 5-1: Simple Motor Driven System with Transduction Matrix

In short, the impedance at loading propagates backward via transmission system to induction motor. In response to the impedance, the motor produces required power correspondingly and the power is then transmitted downward to the load through transmission system again. Obviously, this power drawn from the electricity sources is limited by the motor properties specified by the machinery designer

Studying the impedance of a system enables the understanding of system properties and the power transfer of inter or intra-systems. Hence the system's power transfer can be optimized. This has been utilized in piezoelectric energy harvesting system, where the impedance of a vibration source is made to match with energy storage in electric circuit. As a result, the energy flows in the system is optimized and the power output of energy harvesting system improved as well [101]-[104].

In fact, the relationship of impedance and power transfer has been well described in the concept of impedance matching. Impedance matching was initially developed for electrical circuit design, where it is stated that the input impedance of an electrical load Z_L is designed in a way to match with the output impedance of the corresponding signal source Z_s , thus maximum power transfer to the load can be achieved. The concept of impedance matching for maximum power transfer in electromechanical system has been discussed comprehensively in [105]. The researcher gave an insight into how the impedance matching of different domain in electromechanical system affects the power transferred. It is desirable to apply the concept of impedance matching in the analysis of power flow in a motor driven system. In opinion, mechanical power transmission is an example for impedance matching, because transmission system changes the source's impedance and matches it with loading impedance for maximum power transfer.

5.3 Power Flow and Efficiency

In power system, power flow studies are the backbone of its analysis and design, it is especially important for systems that involve exchange of power between two energy domains. In addition, the demand of control techniques over electrical and mechanical system also gives rise to the development power flow analysis, as power flow analysis able to provide precise system's information as a feedback to the control techniques. In this section, the power flow of a motor driven system is expressed in term of transduction matrix.

5.3.1 Power Flow

For a transduction matrix describing input port 1 and output port 2 of a system as in Eq. (5.3), the input power at port 1 can be expressed as follow:

$$P_1 = t_{11}t_{21} \left(\frac{P_2}{V_2} \right)^2 + t_{12}t_{22} \left(\frac{P_2}{F_2} \right)^2 + (t_{11}t_{22} + t_{12}t_{21})P_2 \quad (5.6)$$

Power equation above can be rearranged in term of power and impedance at port 2, the output:

$$P_1 = t_{11}t_{21}Z_2P_2 + t_{12}t_{22}\frac{P_2}{Z_2} + (t_{11}t_{22} + t_{12}t_{21})P_2 \quad (5.7)$$

The equation above is a general expression of power transfer between input and output of a system. It is obvious that the power depends on transduction functions and output impedance which represents the properties of system and the loading respectively. Given a system and its transduction matrix is known, the input power required to drive a given loading could be computed. In the power flow analysis in this chapter, the above power equation will be used to calculate the power of each component in the system.

5.3.2 Efficiency

Efficiency is the ratio of output power to input power, and it is another important consideration in power transmission analysis. In mechanical transmission system, gear is generally considered as high efficiency transmission system. For instance, spur gear has power efficiency as high as 98% to 99%. Belt drive also has high efficiency, which is around 90% to 98%. In electromechanical system which involving conversion of energy of different domain, the efficiency is normally lower. For instance, efficiency of induction motor is around 70% to 90%. According to the transduction matrix, the efficiency of a system can be obtained from rearranging *Eq. (5.7)* and considering its magnitude only. This efficiency describes the power transfer between input and output of a system and enables computational of efficiency at each component in motor driven system. From the equation, it is a function of transduction functions and output impedance.

$$\eta = \frac{P_2}{P_1} = \left| \frac{1}{t_{11}t_{21}Z_2 + t_{12}t_{22}\frac{1}{Z_2} + (t_{11}t_{22} + t_{12}t_{21})} \right| \quad (5.8)$$

5.4 Impedance and Power

In the previous sections, the expressions of both impedance and power for a system have been introduced based on transduction matrix. This allows the following analysis on impedance propagation and power flow in a motor driven system.

5.4.1 Impedance Propagation and Power Flow in Motor Driven System

In order to illustrate the impedance propagation and power flow, a simple motor driven system as shown in *Figure 5-2* is used in the analysis. This simple motor driven system drives a given load from an induction motor via transmission gear.

Figure 5-2: Simple Motor Driven System with Induction Motor, Gear and Load

Assuming the loading requires input torque, T_{m2} of 7.2 Nm and running at rotational speed, ω_{m2} of 50 rad/s. The T_{m3} and ω_{m3} are zeros as the load is assumed as a power sink. Based on this loading condition, the loading impedance is 0.144 Ω , which is obtained from ratio of torque to rotational speed. The power required for the loading is 360 W, and it is obtained from the product of torque and rotational speed. In order to drive this load, the output from gear has to be able to match with the loading requirement. Therefore, the output torque T_{m2} and rotational speed ω_{m2} of gear have to be 7.2 Nm and 50 rad/s as well. Hence the output impedance of gear is 0.144 Ω . If the output from gear does not reach the required load condition, such that the output torque is lower than 7.2 Nm. The output impedance would be lower than loading impedance, thus it is not able to drive the load. Hence there is not

power flow from gear to the load. On the other hand, the loading impedance can be matched with lower value of torque and rotational speed. However, in this situation the power generated from corresponding torque and rotational speed is not sufficient to drive the load.

According to the analysis above, the output impedance of gear has to be 0.144Ω , as the reflection of loading impedance. In this example, the gear is assumed to be a set of gears with gear ratio of 3 and the corresponding transduction matrix would be:

$$\begin{bmatrix} T_{m1} \\ \omega_{m1} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} T_{m2} \\ \omega_{m2} \end{bmatrix} \quad (5.9)$$

Hence the input torque and rotational speed of the gear set can be computed from transduction matrix above. The calculated value of input torque is 2.4 Nm, and rotational speed is 150 rad/s. Subsequently, the input impedance of gear is 0.016Ω , which can also be obtained from dividing the square of gear ratio with output impedance as shown in *Eq. (4.15)*. Now the mechanical impedance from loading is changed to lower impedance due to transmission gear. This impedance will be the reference for output impedance from induction motor.

In order for the induction motor to be able to drive the gear, its output has to be 2.4 Nm and 150 rad/s as well, thus the output impedance of induction motor matches with the input impedance of gear set. Based on the output of induction motor calculated above, it is not possible to drive the loading of 7.2 Nm and 50 rad/s without using a gear set. This is because the output impedance of motor 0.016Ω does not match with loading impedance of 0.144Ω , no power flows between these two points. Gears are required to change the impedance in the backward propagation of impedance, so that the output impedance of motor is increased and matched with loading impedance.

According to the output impedance of motor, the input voltage and current can be obtained based on the transduction matrix of induction motor developed previously. In general, the voltage applied to the induction motor is fixed at constant value, for instance 400 V of RMS value or about 570 V of maximum voltage. However, direct computational of input voltage and current based on transduction matrix does not account for this voltage limitation, hence the results computed in this way will have deficiency in accuracy.

In order to obtain electrical input information under constant voltage supply, the following analysis is carried out based on impedance. Firstly, the output impedance of induction motor is deduced as the propagation of loading impedance, and this impedance will be reflected at electrical input impedance of motor. Hence the electrical input impedance can be obtained according to equation below, together with transduction functions of induction motor at the corresponding frequency.

$$Z_e = \frac{Z_{m1}t_{11} + t_{12}}{Z_{m1}t_{21} + t_{22}} \quad (5.10)$$

$$\begin{bmatrix} E_{e,p} \\ I_{e,p} \end{bmatrix} = \begin{bmatrix} 35.4560 + 35.4672j & 3.4734 - 0.3806j \\ 0.6480 - 0.0030j & 0 - 0.0070j \end{bmatrix} \begin{bmatrix} T_{m1} \\ \omega_{m1} \end{bmatrix} \quad (5.11)$$

According to Eq. (5.10), the electrical impedance is computed as below, with output impedance Z_{m2} of 0.016 Ω :

$$Z_e = 259.067 + 192.343j \Omega = 322.66 \Omega \quad (5.12)$$

Since the peak voltage $E_{e,p}$ is kept at 570 V, the input current $I_{e,p}$ can be computed as follow:

$$I_{e,p} = \frac{E_{e,p}}{Z_e} \quad (5.13)$$

$$I_{e,p} = 1.418 - 1.053j \text{ A} = 1.767 \text{ A} \quad (5.14)$$

Finally, the electrical input for induction motor is calculated. According to the magnitude of voltage and current, the electrical input power to the motor is:

$$P_e = \frac{1}{2} E_{e,p} I_{e,p} = 503.60 \text{ W} \quad (5.15)$$

When the output impedance of motor is changed due to the variation of loading condition, the electrical input impedance will change accordingly, so does the input current of motor. As a result, the electrical power drawn by motor varies according to loading condition. Subsequently, the electrical power will be converted into mechanical power as the output of motor. Then the mechanical power will be further transmitted downward via transmission system and finally used to drive the mechanical loading. The figure below gives a graphical illustration on feedback of impedance and power flow.

Figure 5-3: Impedance Change and Power Flow

The calculated electrical input power is higher than mechanical power at motor output. This might be due to power loss such as heat loss in the windings of induction motor. For instance, the electrical power in Eq. (5.15) can be written in term of complex numbers and it describes real and reactive power.

$$P_e = 404.13 - 300.11j \text{ W} \quad (5.16)$$

$$P_{loss} = 404.13 - 360 = 44.13 \text{ W} \quad (5.17)$$

Therefore, about 10% of real power is lost in the induction motor at the given loading. From the electrical impedance equation in Eq. (5.10), the electrical impedance not only depends on mechanical impedance, but also relates with

transduction functions. These transduction functions describe properties and operating condition of induction motor. Hence the electrical power drawn by motor in response of the electrical impedance is used to drive both the mechanical load and the motor. This explains the reason why calculated electrical power is higher than mechanical power.

The impedance and power analysis above only covered one loading condition, and it can be extended to analyze the impedance and power flow for different loading. *Figure 5-4* and *Figure 5-5* show the impedance and power of a simple motor drive system at different loading conditions, such as 3.6 Nm, 7.2 Nm and 14.4 Nm all at 50 rad/s, thus the loading impedances and powers are different in each case.

According to the results, mechanical impedance at Z_{m2} and Z_{m1} decrease as the given mechanical loading is getting smaller. The value of Z_{m1} is smaller and rather constant compared to Z_{m2} . The presence of gears increases Z_{m1} in order to match with Z_{m2} . For electrical impedance wise, Z_e is decreasing as loading condition increased. This is because electrical impedance is inversely proportional to mechanical impedance. In electromechanical system, normally voltage such as back emf is proportional to rotational speed, whereas current relates with generated torque.

$$E_{emf} = k\omega \quad (5.18)$$

$$Ik = T \quad (5.19)$$

Division between *Eq. (5.18)* and *Eq. (5.19)* gives the relationship of electrical impedance and mechanical impedance.

$$Z_e \propto \frac{1}{Z_m} \quad (5.20)$$

In the analysis of power flow, the power required is generally getting larger as loading condition is increasing. P_{m1} and P_{m2} is the same because the gear set is assumed to have perfect efficiency thus input power is equal with output

power. The reason of electrical power P_e is larger than mechanical power under same loading has already been discussed, which is due to power loss in the motor. This additional power becomes less and less significant in total power as loading condition is increased, since the different between electric power and mechanical power is diminishing with increasing load.

Figure 5-4: Impedance Change in Simple Motor Driven System

Figure 5-5: Power Flow in Simple Motor Driven System

5.4.2 Impedance Propagation and Power Flow in Induction Motor and Band Brake

In this section, motor testing equipment used in the previous experiments is modelled by transduction matrix, thus its impedance and power are analyzed. The motor testing equipment consists of a squirrel cage induction motor and a fiction belt as loading only, as shown in figure below. According to the setup, this motor driven system is modelled as transduction matrix of induction motor and transduction matrix of band brake. The torque meter as a sensor, does not affect power flow nor impedance propagation in the system. It is used for measuring mechanical torque and rotational speed.

Figure 5-6: Induction Motor and Band Brake as Motor Driven System

Figure 5-7: Schematic Diagram of Induction Motor and Band Brake

The transduction matrices for both systems have been developed in previous chapter as in Eq. (3.42) and Eq. (4.60). Combining the two transduction matrices yield:

$$\begin{bmatrix} E_{s,p} \\ I_{s,p} \end{bmatrix} = \begin{bmatrix} \left(\left(\frac{R_s + jX_s}{jX_M} + 1 \right) (R_r + jX_r) + (R_s + jX_s) \right) \frac{\sqrt{2}}{3K} & \left(\frac{R_s + jX_s}{jX_M} + 1 \right) \sqrt{2} 3K \\ \left(\frac{R_r + jX_r}{jX_M} + 1 \right) \frac{\sqrt{2}}{3K} & \frac{\sqrt{2} 3K}{jX_M} \end{bmatrix} \quad (5.21)$$

$$\times \begin{bmatrix} r_D \times \frac{0.23}{0.10} \times \frac{1}{\sin 70^\circ} & 0 \\ \frac{r_D}{C} \times \frac{0.23}{0.10} \times \frac{1}{\sin 70^\circ} & \frac{1}{r_D} \times \frac{0.10}{0.23} \times \sin 70^\circ \end{bmatrix} \begin{bmatrix} W_M \\ V_M \end{bmatrix}$$

At any given weight applied at hanger of band brake, the torque and rotational speed required to drive the band brake can be obtained from its transduction matrix. Subsequently, the torque and rotational speed will be used as reference to compute electrical input by using transduction matrix of induction motor. The impedance and power at each port can be easily obtained from division and multiplication of those effort and flow variables.

In this study, a set of increasing weights is applied to the band brake. Because the transduction matrix of band brake involves damping coefficient which depends on loading, therefore the transduction matrix is varying with loading as shown in *Table 5-1*. Transduction matrix of induction motor can be assumed as constant in this case because the changes are less significant compared with another matrix.

Weight (NM)	<i>t</i> 11	<i>t</i> 12	<i>t</i> 21	<i>t</i> 22
4.91	0.1224	0	25.0794	8.1712
9.81	0.1224	0	13.2115	8.1712
14.72	0.1224	0	9.6588	8.1712
19.62	0.1224	0	7.5551	8.1712
24.53	0.1224	0	5.6954	8.1712

Table 5-1: Transduction Matrix of Band Brake at Various Loadings

Based on the transduction matrix of band brake, together with transduction matrix of induction motor in *Eq. (3.45)*, the impedance and power at each port are presented in figures below:

Figure 5-8: Impedance Change in Induction Motor and Band Brake System

Figure 5-9: Power Flow in Induction Motor and Band Brake System

In the analysis of impedance change, Z_{mI} is the input impedance of band brake due to the applied weights. Z_{mI} is generally increasing as more and more weight is added. The measured Z_{mI} is obtained from measured torque and rotational speed according to torque meter, whereas calculated Z_{mI} is computed from transduction matrix. They are very similar because the damping coefficient in the transduction matrix is obtained from the measured torque and rotational speed. This Z_{mI} will then referred as output impedance of motor and is reflected to electrical input. The electrical input impedance Z_e is decreasing as loading is increased, because the induction motor takes in more and more current. Measured and calculated Z_e are comparable, in spite of some discrepancies which might be due to errors in calculation. Furthermore, measurements of voltage and current are not consistent due to voltage supply variation.

In the power flow analysis, P_e is the electrical input power applied to induction motor, and more P_e is required as loading increased. Both the measured and calculated P_e are similar. P_{mI} is the input power for band brake and increases together with the loading condition. The measured P_{mI} is slightly larger than calculated value, perhaps this is due to slight differences between measured and calculated torque. The difference between P_e and P_{mI} has been explained previously, which is due to additional power used for maintaining operation of motor.

Results in this section justified previous analysis on impedance and power flow of motor driven system. Basically the induction motor draws in current in respond to the loading impedance. Thus electrical power is converted into mechanical power and able to flow towards loading.

5.4.3 Transmission Gear System in Vehicle

A transmission gear system in vehicle is used as an example to study the impedance and power flow in a transmission system. Transmission gear system normally consists of several gears and gear trains to provide multiple

gear ratios that can be switched manually or automatically. The present of transmission gear system enables the vehicle to run on different road surfaces by changing the torque and speed ratio from the output of engine. Hence it would be a good example to illustrate impedance change and power flow in mechanical system.

Figure 5-10: Schematic Diagram of Transmission System in Vehicle

Schematic diagram in Figure 5-10 shows an example of transmission system in vehicle and its components. Basically, the engine converts the chemical energy in fuels into useful mechanical energy. Therefore the input variables for engine are perhaps the pressure P_f and flow rate Q_f of fuel, and the corresponding output variables would be mechanical torque and rotational speed. This mechanical power will be transmitted to wheel via a series of transmission gears as shown in Figure 5-11.

Figure 5-11: Six Speed Manual Transmission

Gears with standard transmission ratio typically have 5 or 6 gear ratios so that the driver can switch it according to load condition. Final drive ratio is a final

gear set with constant gear ratio in transmission system. Lastly, the transmitted torque reaches wheel and turned into forward driving force as F_w and V_w .

According to transduction matrix developed previously, the transmission system as in *Figure 5-10* can be modelled as follows, where t_{ij} is transduction matrix of engine, gr_{ST} and gr_{FD} are standard transmission ratio and final drive ratio respectively. Lastly, R is the radius of wheel.

$$\begin{bmatrix} P_f \\ Q_f \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}_{engine} \times \begin{bmatrix} \frac{1}{gr_{ST}} & 0 \\ 0 & gr_{ST} \end{bmatrix} \times \begin{bmatrix} \frac{1}{gr_{FD}} & 0 \\ 0 & gr_{FD} \end{bmatrix} \times \begin{bmatrix} R & 0 \\ 0 & \frac{1}{R} \end{bmatrix} \begin{bmatrix} F_w \\ V_w \end{bmatrix} \quad (5.22)$$

In order to illustrate the impedance change and power flow in transmission system, an example of transmission system from BMW Z4 sDrive35is is used in following analysis. The analysis is carried out when the engine is running at maximum power of 225 kW at 5800 rpm. Based on this output power, the impedance and power according to different combinations of transmission gear ratio are analyzed. In this example, the transduction matrix of engine is unknown, but the mechanical output torque and rotational speed of engine can be obtained as follow:

$$T_{En} = \frac{P}{\omega} = 370.45 \text{ Nm} \quad (5.23)$$

$$\omega_{En} = 5800 \text{ rpm} = 607.37 \text{ rads}^{-1} \quad (5.24)$$

Hence the output impedance from engine is:

$$Z_{En} = \frac{T_{En}}{\omega_{En}} = 0.610 \Omega \quad (5.25)$$

As a result, the impedance and power flow analysis will start from mechanical output of engine. This is acceptable since the focus of this section is on the transmission system. Once all the gear ratios are known, the mechanical information at each port in the transmission system can be computed.

$$\begin{bmatrix} T_{En} \\ \omega_{En} \end{bmatrix} = \begin{bmatrix} \frac{1}{gr_{ST}} & 0 \\ 0 & gr_{ST} \end{bmatrix} \times \begin{bmatrix} \frac{1}{gr_{FD}} & 0 \\ 0 & gr_{FD} \end{bmatrix} \times \begin{bmatrix} R & 0 \\ 0 & \frac{1}{R} \end{bmatrix} \begin{bmatrix} F_w \\ V_w \end{bmatrix} \quad (5.26)$$

The transmission gear ratio of BMW Z4 sDrive35is can be found from its manufacturer's catalogue, and it is summarized in *Table 5-2* below. Also, the radius of wheel is 0.2159 m.

Gear	Standard Transmission Ratio	Final Drive Ratio
I	4.06	3.08
II	2.40	
III	1.58	
IV	1.19	
V	1.00	
VI	0.87	

Table 5-2: Transmission Gear Ratio of BMW Z4 sDrive35is

With all the information above, the force and velocity produced at the wheel of vehicle can be computed according to transduction matrix in *Eq. (5.26)*. The force, velocity and output impedance for various transmission gear ratios are presented in following figures.

Figure 5-12: Output Force and Velocity at Various Gear Ratios

Figure 5-13: Output Mechanical Impedance at Various Gear Ratios

From the results, the output force is decreasing when higher transmission gear is used. On the contrary, the velocity of vehicle is getting faster for higher transmission gear. Consequently, the output mechanical impedance is decreasing for higher transmission gear since lower force but higher velocity are produced. At low transmission gear, for instance first transmission gear is able to produce the largest mechanical impedance. Hence vehicle drivers always switch to lower transmission gear when they are driving on rough road surface, or driving up to a slope. This is because rough road surface and driving up a slope are corresponding to larger loading impedance, as more force are required to overcome the friction or gravity force. On the other hand, higher transmission gear provides lower mechanical impedance, thus it is not suitable for driving on rough road surface or up a slope. However, higher transmission gear is preferred at normal road surface where velocity is more concerned.

After understanding the impedance change with respect to various transmission gear ratios, the following analysis focuses on the impedance change along the transmission system starting from engine output.

Figure 5-14: Impedance Change in Vehicle Transmission System

Generally, the mechanical impedance is getting larger from output of engine to wheel. The output of engine is normally having low mechanical impedance as the rotational speed produced is very high, can easily reach a few thousand rpm. This mechanical impedance will be enhanced by transmission gear so that the mechanical impedance at the end of transmission gear is high enough to match with load impedance. At sixth transmission gear, the mechanical impedance is decreased because it is having gear ratio of 0.87, which further reduced the mechanical torque. However, the final mechanical impedance still increased in the overall transmission. This analysis also shows that various gear ratios lead to different mechanical impedances at output.

An example of vehicle driving up a slope is used to illustrate the loading condition. The larger of the degree of slope, the more mechanical impedance is required to overcome it. A free body diagram in *Figure 5-15* describes all the forces involving when a vehicle is going up a slope.

Figure 5-15: Free Body Diagram of Vehicle and Slope

In the figure, F_w is the driving force produced at wheel, Mg is the weight of the car, F_D is the drag force, F_{rr} is rolling resistance and θ is the degree of slope. If the vehicle is assumed to be driving at constant velocity, the force equilibrium equation for the situation above would be:

$$F_w = Mg \sin \theta + F_D + F_{rr} \quad (5.27)$$

where,
$$F_D = \frac{1}{2} \rho V^2 C_D A \quad (5.28)$$

$$F_{rr} = C_{rr} N \quad (5.29)$$

In the equations above, ρ is air density which is 1.225 kg/m^3 . V is velocity of vehicle which is the same as V_w , the velocity at wheel. C_D is the drag coefficient of this vehicle, and it is 0.35 according to its manufacturer. A is 1.96 m^2 , the reference area of vehicle. C_{rr} is rolling resistance coefficient and it is 0.010 for ordinary car tires on concrete. Lastly, N is the normal force of the vehicle applied on ground. According to all these equations, Eq. (5.27) can be rearranged so that the degree of slope is expressed in term of output force at wheel.

$$\theta = \sin^{-1} \left(\frac{F_w - F_D + F_{rf}}{Mg} \right) \quad (5.30)$$

From Eq. (5.30), the degree of slope that the vehicle can overcome can be found, with given mechanical torque and rotational speed. Figure below shows the results based on the output from different transmission gear ratio.

Figure 5-16: Degree of Slope against Various Transmission Gear Ratios

From the result, the slope that vehicle can overcome is becoming smaller and smaller when higher transmission gear is used. For instance, at fourth gear, the output force and velocity at wheel is about 6300 Nm with 35.8 m/s, with output impedance of 175.8 Ω . This output enables the vehicle to drive up a slope of 20.9°. However, fourth gear does not able to drive up to any slope that is greater than 20.9°, because the loading impedance is greater than output impedance produced at wheel. The driver has to switch to lower gear which gives higher output impedance in this case. For first gear, the output force is greater than the weight of the vehicle itself, therefore in theoretical it is capable to drive the vehicle up a slope with nearly 90°. However, this is not possible in practical.

In conclusion, the analysis of impedance and power can be carried out with the help of transduction matrix of the system. The role of transmission system not only helps to transmit power, but also increases the impedance from power

source in order to match with loading impedance. The electrical impedance as the reflection of loading impedance determines the amount of electrical power drawn by the induction motor. Subsequently, the electrical power is converted to mechanical power and then transmitted to loading.

According to the equation of impedance in *Eq. (5.4)*, the input impedance is the reflection of loading impedance and the system condition. Because of this, the input impedance can be used in condition monitoring of the system, as well as loading. At a given loading condition, any change at input impedance indicates change of system condition which might be due to system faults or abnormality. Actually, this idea leads to condition monitoring of induction motor by electrical input impedance in the following chapter. If the system and its transduction matrix remain constant, the output mechanical information can be expressed by input electrical information. Therefore, mechanical torque and speed can be monitored from voltage and current, thus difficulty of torque and speed sensors installation is avoided.

CHAPTER 6

FAULTS DIAGNOSIS BY FREQUENCY ANALYSIS OF MOTOR INPUT IMPEDANCE

In previous chapters, the concept and applications of transduction matrix of a motor-driven work-applying system was established and demonstrated. The impedance caused by the loading to the system propagates backwards to the driving motor and the power exerts provided by the motor transmits forwards to the loading for work exertion. Using transduction matrix the impedance at every interface can be quantified including the input electrical impedance at the input port of the motor. It is well known that input impedance of a dynamic system characterizes the dynamic properties of the system. In system dynamics, vibration, control theories, the input impedance have been used to explore, identify and investigate the system properties. If abnormality happens in the system, the input electrical impedance measures at the input port of the motor reflects the health condition of the whole system. In this chapter, the impedance-based monitoring and diagnosis of motor driven systems will be presented.

In case the health condition of subsystems after the motor in a motor driven system is normal, which can be indicated by the mechanical impedance at the output of the motor, abnormalities in the input electrical impedance reflects the health conditions the driving motor. In many applications, ensuring the health of the driving motor is the key reliability issue because the reliability of the mechanical subsystems is often better than that of the electro-mechanical motor. As indicated in Chapter 2, the Motor Current Signature Analysis (MCSA) utilizes the spectrum of input current of an induction motor for monitoring and diagnosing. It is a successful method. In this chapter, the advantages and disadvantages of replacing current by impedance for the same purpose will be explored and investigated. Therefore, Motor Impedance Signature Analysis (MISA) which utilizes electrical input impedance of

induction motor for monitoring is introduced. There are several numbers of common motor faults. In this chapter, broken rotor bar, asymmetric air-gap eccentricity and bearing fault will be focused.

6.1 Introduction

From literature reviews, MCSA is one of the popular methods for diagnosis of induction motor faults by utilizing spectrum of input current. It has advantages of using current probe which avoids difficulty in installation of torque and speed sensors. However, MCSA ignores the change of voltage which might contribute to deficiency in its accuracy. In practice, voltage input of electric motor is not constant most of the time. Operation of electric motor at voltage other than nominal value will affect its performance characteristic and life expectancy. Research has shown that performance of AC induction motor operated outside nominal voltage and frequency could reduce motor life significantly. AC induction motor is recommended to run within $\pm 10\%$ of rated voltage and $\pm 5\%$ of rated frequency [106]. Variation of voltage can be due to voltage sags and the use of additional frequency converter. Voltage sags are variation of magnitude and duration of voltage supply which could cause unbalances, non-sinusoidal waveforms and phase angle shift and affect motor performance. Figure below shows variation of voltage for a 50 Hp motor driving a pump [106], [107]:

Figure 6-1: Voltage Variation of AC Induction Motor

From the graph above, the voltage supplied to AC motor is not constant. Interference of other electric appliances which sharing the same source will definitely cause voltage variation. In addition, frequency converter or current controlled device which used to drive electrical motor will induce inter-harmonics or cross-modulation harmonics in power supply. The switching frequency of these control techniques may give rise to harmonic in voltage and current spectrum. Consequently, the task for fault detection on electrical motor becomes more difficult [108]-[110].

Variation of voltage should be considered in order to have better accuracy in fault detection. MCSA do not consider change of voltage in the analysis, hence there is deficiency in accuracy of analysis results. This is one of the main reasons why input impedance is chosen as monitoring signature in this project. Motor Impedance Signature Analysis (MISA) takes into consideration of voltage variation in the analysis and it is expected to have better accuracy in analyzing motor faults. Fault detection of induction motor by current signature is not enough to provide complete information without considering the variation of voltage.

In addition, the electrical input impedance is proved to be related with mechanical output impedance and transduction functions in the previous chapter. These transduction functions represent the induction motor's properties. Therefore, any changes in the motor condition due to bearing, rotor, stator or other faults will change the transduction functions, thus reflected at input impedance. In fact, experiments results in *Table 6-1* show the correlation between transduction functions and various motor faults. The results verified that input impedance would be a suitable monitoring signature for induction motor since it consists of the motor's transduction functions, and its frequency components would be good enough to illustrate characteristic frequencies of motor faults.

	<i>t11</i>	<i>t12</i>	<i>t21</i>	<i>t22</i>
<i>Good Motor</i> (0.5 hp)	10.8703 - 9.0687j	3.3900 - 0.9348j	-0.3395 - 0.0616j	0.0005 - 0.0075j
<i>One Broken</i> <i>Rotor Bar</i> (0.5hp)	1.7677 - 32.7984j	3.5037 - 0.7403j	-0.3811 - 0.0377j	0.0010 - 0.0081j
<i>Asymmetric Air-</i> <i>Gap Eccentricity</i> (0.5hp)	30.0706 + 37.9202j	3.1614 - 1.6662j	-0.2183 - 0.1116j	-0.0014 - 0.0072j

Table 6-1: Experimental Transduction Matrix of Induction Motor with Various Motor Faults

By definition, the input impedance is defined as quotient of excitation to response at the input port in frequency domain, and it fully characterizes the properties of the dynamic system. Therefore, the electrical input impedance used in MISA can be computed using frequency response function between voltage and current as shown in *Eq. (6.1)*.

$$Z_e(f) = R(f) + jX(f) = \frac{E(f)}{I(f)} \quad (6.1)$$

Similar to MCSA, MISA is a sensor-less and online monitoring technique which avoids installation of accelerometer. MISA only requires voltage and current probes at supplied input power hence this method is versatile for most kind of electric motor. Input impedance will be a suitable monitoring signature as it describes the system properties. Besides, the variation of input voltage has been considered, thus this error source of motor current monitoring is reduced and better accuracy can be expected by using input impedance. In addition, input current is in the denominator of impedance, therefore variation of input current is amplified in impedance. Hence MISA might be better in sensitivity.

6.1.1 Time Domain Analysis

By definition, impedance is a function of frequency. However, when there is only one frequency, the variation of the impedance at this frequency along time is observed for time domain analysis. For instance, the mean, variance and maximum value are some of the good indicators to illustrate the changes in impedance. In addition, analytic signal obtained by Hilbert Transform is able to show the envelope of signal. This is useful in monitoring alternating power supply where the mean of signal is not useful. If there is huge statistical history data of impedance for a good motor, it can be used to monitoring motor condition by comparing its statistical pattern. For instance, feature selection of time domain records of impedance can be carried out by Principal Components Analysis. Hence those features can be statistically compared by statistical pattern recognition methods. However, time domain analysis always have drawback in fault diagnosis as it contains too many frequency components, resulting in difficulty for monitoring particular motor fault frequency components. Therefore, time domain analysis will not be focused in this project as it is not suitable for motor faults identification and diagnosis.

6.1.2 Frequency Domain Analysis

Since impedance characterizes the whole system after the measured point in the power flow path and time history of impedance at the operating point is obtainable, the frequency contents of this time record brings in the time varying characteristics of impedance. These pieces of information are very useful in diagnosis of motor faults. Moreover, input impedance can be expressed in term of transduction functions, which are frequency functions that describe a system's properties. Therefore, frequency analysis on input impedance would be a good method to extract and study the system's condition.

Since most of the motor faults have its own characteristic frequency, by monitoring the amplitude of those frequencies would able to notify user about the severity of corresponding motor faults. Frequency analysis can be achieved

by applying Fourier Transform on the time record of signal. In this report, frequency analysis on impedance will be used extensively for motor fault diagnosis. However, the limitations of this method are having low time resolution and stationary data is required. Hence it hinders the monitoring process for transient condition.

6.1.3 Time-Frequency Analysis

Time-frequency analysis is the combination of both time and frequency analysis. Short Term Fourier Transform and Wavelet Transform are some of the time-frequency analysis methods. These analysis methods have decent resolution in both time and frequency domain, hence it can be used in examination of fault frequency under transient condition. It might be suitable in motor monitoring if continuous monitoring on motor faults in real time is preferred.

6.2 Measurement and Signal Processing to obtain Impedance

6.2.1 Frequency Response Function

As mentioned in previous section, input impedance of a motor can be obtained by computational of frequency response function between input voltage and current. Frequency response function (FRF) can be obtained directly from division of finite Fourier transforms of excitation and response's data records. Alternatively, the FRF can be computed from auto-correlation function and cross-correlation function of excitation and response's data records. In the computational of FRF, the data are assumed as stationary random processes with zero mean values. Stationary data is preferred in analysis in order to minimize statistical sampling errors. Stationary data is defined as a set of data with all average values of interest remain constant with respect to time [64], [65].

Two methods introduced above lead to frequency domain of response function and they are used to acquire impedance spectrum from input voltage and

current spectrum. As a consequence of fast computation of Fourier series and development of computational software, direct transformation using finite Fourier transform on original time history records has become dominant and relatively easier. For analysis below, LABVIEW has been utilized to obtain finite Fourier Transform of voltage and current and hence the frequency response function for impedance. In Appendix C, a LABVIEW programme which collects signals, converts the signals to frequency domain and computes frequency response function is shown.

6.2.2 Stationary Data

In the computational of Fourier Transform of voltage and current signals, the signals have to be collected under steady state condition. This is also known as stationary data and below is some studies on voltage and current under constant loading, in order to ensure the captured signals are stationary data.

A three phase induction motor, 3 poles, 0.18 kW, 920 rpm, 50 Hz is used for this experiment under 2 kg loading and the measured voltage, current, torque and rotational speed are presented in *Figure 6-2* to *Figure 6-5*. From the results, the mean value and standard deviation of all the data can be considered constant over time. The fluctuation of mean and standard deviation is below 5% of respective mean value. In conclusion, the input voltage, input current, torque and speed measured from the AC induction motor can be considered as stationary data under constant experimental conditions. This will ease the analysis of experimental results when frequency analysis is applied at latter stage.

Figure 6-2: Mean and Standard Deviation of Input Voltage over Time

Figure 6-3: Mean and Standard Deviation of Input Current over Time

Figure 6-4: Mean and Standard Deviation of Torque over Time

Figure 6-5: Mean and Standard Deviation of Speed over Time

6.2.3 Schematic Diagram for Experimental Setup

Figure 6-6: Schematic Diagram for MISA on AC Induction Motor

From the schematic diagram above, voltage and current probes are used to acquire input voltage and current. In order to avoid unnecessary phase difference, same line voltage and current are measured. Direct power supply is used to keep away from the effect of induced harmonic due to frequency converter. With proper signal conditioning, input voltage and current are transferred to Dynamic Signal Analyzer (DSA) for signal processing. In DSA, input voltage and current are collected and transformed from time domain to frequency domain via Fast Fourier Transform (FFT), hence the input impedance of AC induction motor can be computed by obtaining FRF between input voltage and current. The FRF is calculated with current as the response over voltage as the excitation. The computed FRF is known as admittance and hence inverse of it will be impedance. The computed impedance spectrum will be transferred to computer for fault diagnosis.

The torque and speed sensors gather information of mechanical port to ensure proper loading is applied to the ac induction motor. Torque and speed signals are sent to DSA for signal processing in order to determine mechanical impedance.

6.2.4 Experimental Setup and Equipment Lists

Figure below shows actual experimental setup for MISA on AC induction motor. In this experiment, a 0.5 hp (0.37 kW) three phase squirrel cage induction motor is used. The detail of equipments is listed in Appendix A:

Figure 6-7: MISA Experimental Setup on AC Induction Motor

- i. 0.5 HP, 4 Poles, Three Phase Squirrel Cage Induction Motor
- ii. High Voltage Differential Probe (Tektronix P5205) & TEKPROBE Power Supply (Tektronix 1103)
- iii. Current Probe (Tektronix A6303) & Current Probe Amplifier (Tektronix AM 503B)
- iv. Torque Detector (Ono Sokki SS 100) & Torque Converter (Ono Sokki TS-2600)
- v. Dynamic Signal Analyzer (Dynamic Signal Acquisition Module NI 4462)

6.2.5 Experiment Flowchart

Figure 6-8: Experiment Flowchart

Flowchart above shows the procedures for obtaining current and impedance spectrums and hence identifies those fault characteristic frequencies. Firstly, input data such as voltage and current are collected from high voltage differential probe and current probe respectively. Torque and rotational speed are collected from torque meter and tachometer for reference. A dynamic signal analyzer is used to record these data. In the experiments, a sampling rate of 1 kS/s is used with a sample size of 10,000 samples. FFT is carried out on time domain data of voltage and current. Based on the setting above, the maximum frequency of the spectrum is 500 Hz with frequency resolution of 0.1 Hz. RMS averaging is used in order to improve signal to noise ratio and reduces the effect of signal fluctuation. Hanning window is used to reduce the effect of leakage. Impedance is obtained from FRF between voltage and current. Consequently, the motor fault characteristic frequencies are identified and its amplitude is monitored.

Experiments have been carried with various loadings, from no load until full load of the tested induction motor. Also the experiment is repeated for induction motor with known fault and comparison is carried out between good and faulty motor. The differences between current and impedance spectrum are studied as well. In order to identify and monitor those motor faults, the

characteristic frequencies of commons motor faults are studied as discussed in the following sections.

6.3 Rotor Faults

Most of the rotor faults are related to broken rotor bar or breakage of end ring. Rotor bar failure does not initiate serious motor fault, it results in unbalances and causes vibration while motor is operating. These unbalances reduce motor efficiency and lead to serious mechanical damage to stator winding if the situation is getting worse. As mentioned in previous chapter, a broken rotor bar will result in air-gap magnetic field irregularity. Perturbation of magnetic field can be observed in current and impedance spectrum, the frequency which characterizes broken rotor bar can be obtained by *Eq. (2.21)*.

6.3.1 Characteristic Frequencies of Rotor

In the following, current power spectrum of good rotor is shown and the sidebands of broken rotor bar are located. The experiment is carried out on 0.5 hp, 4 poles, three phase squirrel cage induction motor with a loading of 2.4 Nm, about 98% of full load at 1410 rpm. Information of loading condition enables calculation of motor slip by *Eq. (2.3)*. Hence the sidebands of rotor frequency can be verified as follow:

Motor Slip:

$$s = \frac{N_s - N_r}{N_s} = \frac{1500 - 1410}{1500} \quad (6.2)$$

$$s = 0.06 \quad (6.3)$$

Sidebands of Rotor Frequency:

$$f_{rotor} = (1 \pm 2ks) f_0 = (1 \pm 2(1)(0.06))50 \quad (6.4)$$

$$f_{rotor} = 50 \pm 6 \text{ Hz}$$

$$f_{rotor,LSB} = 44 \text{ Hz} \quad @ \quad f_{rotor,USB} = 56 \text{ Hz} \quad (6.5)$$

Figure 6-9: Rotor Frequencies on Current Spectrum of Good Motor

From Figure 6-9, the experimental sidebands of rotor frequency are 44.0 Hz and 55.9 Hz respectively and they are similar to calculated sidebands. As sidebands of rotor frequency depend on motor slip, it will change according to loading condition. For instance, the sidebands of rotor frequency located nearer to supply frequency of 50 Hz under no load condition. In this case, observation on rotor sidebands becomes difficult as signal of supply frequency dominates signal of rotor sidebands. Beside of frequency, the magnitude of rotor sidebands increases proportionally with loading. These rotor sidebands will be monitored and used to identify broken rotor bar.

Sidebands of shaft frequency are related with rotational speed of rotor and it can be obtained as follow:

$$f_{shaft} = f_0 \pm k f_s \quad (6.6)$$

$$f_{shaft} = 50 \pm 1 \times 1410 \times \frac{2\pi}{60} \times \frac{1}{2\pi}$$

$$f_{shaft} = 50 \pm 23.5 \text{ Hz}$$

$$f_{shaft, LSB} = 26.5 \text{ Hz} \quad @ \quad f_{shaft} = 73.5 \text{ Hz} \quad (6.7)$$

Similarly, sidebands of supply frequency vary with loading condition. These sidebands shift towards 50 Hz while the loading is increased, and the magnitude of sidebands is increased as well. Rotor faults may affect the magnitude of these sidebands, however these sidebands are more useful in monitoring air-gap eccentricity, which will be discussed in more detail later.

There are other harmonic frequencies due to rotor faults which can be used to monitor rotor fault:

$$f_{rh} = \left\{ \frac{k}{p}(1-s) \pm s \right\} f_0 \quad (6.8)$$

$$f_{rh} = \left\{ \frac{2}{4}(1-0.06) \pm 0.06 \right\} 50$$

$$f_{rh} = 23.5 \pm 3 \text{ Hz}$$

$$f_{rh,LSB} = 20.5 \text{ Hz} \quad @ \quad f_{rh,USB} = 26.5 \text{ Hz} \quad (6.9)$$

Harmonic of rotor sidebands can be observed clearly in current spectrum as *Figure 6-10* below. Some researchers prefer to monitor rotor fault by using this harmonic of rotor sidebands because they are further away from main supply frequency. Hence these harmonic rotor sidebands would not affect much by main supply frequency, resulting in easier detection of signal.

Figure 6-10: Harmonic of Rotor Sidebands of Good Motor

In impedance power spectrum, similar sidebands of rotor frequency and shaft frequency can be observed. *Figure 6-11* below is an impedance power spectrum of similar test. From the graph, sidebands of rotor frequency and shaft frequency can be observed. The reason of these sidebands are facing towards negative value is because of the magnitude of current sidebands are greater than magnitude of voltage sidebands. Since impedance is FRF of voltage over current, the magnitude of sidebands shown in impedance power spectrum would be small in value. Therefore, decibel (dB) is a suitable scaling unit to display small signal in impedance power spectrum. Harmonic of rotor sidebands can be observed as well, located at few hertz away towards left hand side of shaft frequencies.

Figure 6-11: Rotor Frequencies on Impedance Spectrum of Good Motor

In impedance spectrum, the magnitude of main supply frequency at 50 Hz is limited around 50 dB. Consequently the effect of main supply frequency on rotor sidebands is reduced hence the sidebands can be observed clearly. The magnitudes of rotor and shaft frequency are rather constant after sample averaging of 50 samples.

Rotor frequencies and shaft frequencies depend on motor slip, hence they vary with loading. With increasing load on induction motor, rotor frequencies move further away from 50 Hz but shaft frequencies shift closer to 50 Hz. Apart from frequency, the magnitude of those sidebands changes with loading as well. As present in *Figure 6-12* below, the rotor frequency increased nonlinearly when the loading applied to induction motor is increased. As a result, loading effect should take into consideration in fault detection so that it does not confuse with rotor fault.

Figure 6-12: Variation of Rotor Frequencies against Loading

6.3.2 Comparison between Good Rotor and Broken Rotor Bar

In previous section, rotor frequencies have been identified in both current and impedance spectrums, thus they will be monitored for rotor fault detection. Experiments have been carried out on two rotors, one is good rotor and another has one broken rotor bar. These rotors would be tested on a same induction motor by replacing one and another. This is to minimize the uncertainty from experiment by maintaining constant experiment conditions.

Below is a picture on rotor with one broken rotor bar, however the fault can not be seen as it is sealed. The broken bar is specially made and created during manufacturing of rotor. A section of rotor mould is blocked in order to form discontinuity in between rotor bar when the rotor is under casting.

Figure 6-13: Rotor with Broken Rotor Bar

To the difference between good rotor and broken rotor, current spectrum is used as a reference and impedance spectrum is studied to ensure the difference in signature can be detected. Figures below are experimental results on two different rotors under same loading conditions, 98% loading (2.4 Nm) at 1410 rpm on a three phase squirrel cage induction motor. The spectrums are limited up to 100 Hz as characteristic frequencies of rotor occur around 50 Hz. Both rotor frequencies and shaft frequencies will be observed and compared.

Figure 6-14: Current and Impedance Spectrums of Good Rotor and One Broken Rotor Bar

In current spectrum, rotor frequencies are identified and compared between good rotor and one broken rotor bar. From the results, the lower sideband and upper sideband of rotor frequencies for broken rotor bar are increased in magnitude from -55.3 dB to -42.3 dB and -57.1 dB to -47.8 dB respectively. About 10 dB increment of sidebands' magnitude is due to abnormal magnetic flux distribution in air-gap by rotor fault. Comparing current spectrum of good rotor and broken rotor bar, more harmonics of rotor frequencies with smaller magnitude are observed. These harmonics are the results of rotor fault, but only rotor frequencies are focused in this section because these frequencies are the characteristic frequency that can be computed and identified easily. The magnitudes of shaft frequencies remain about the same for lower sideband and upper sideband respectively.

For impedance spectrum wise, the magnitude of rotor frequencies is decreased about 15 dB, from 22.4 dB to 6.6 dB and from 27.5 dB to 11.7 dB. The change in amplitude of rotor frequencies in impedance is slightly larger than that of current spectrum. Similar magnitude change is observed at both upper and lower sidebands of rotor frequencies. However, the magnitude of shaft frequencies observed in impedance spectrum is almost constant. The lower sideband of shaft frequency and the upper sideband experienced small increment in magnitude which is negligible.

In short, rotor fault causes larger magnitude change at rotor frequencies, and only minor magnitude change at shaft frequencies. One possible reason is that shaft frequencies are not directly related to rotor fault. Shaft frequencies are associated with rotational speed of the rotor, since motor speed is restricted as constant due to invariance loading for all tests, thus shaft frequencies can be expected to have no changes as well.

The experiments on good rotor and broken rotor have been carried out under different loadings. *Figure 6-15* shows the change of current and impedance

magnitudes for rotor frequencies under increasing loading. The results of good rotor and one broken rotor bar are compared as well.

Figure 6-15: Comparison between Good Rotor and One Broken Rotor Bar under Increasing Load

In the experiment, 0.5 hp (0.37 kW) induction motor is used and its full load torque is about 2.55 Nm. During no load test, the rotor frequencies can not be observed as the signature falls very close to 50 Hz and dominated by main supply frequency's signature. Therefore, the results on zero loads do not shown in graph. Generally, the magnitude of rotor frequencies in impedance decreases for increasing load. The trends are corresponding with that of current spectrum. In addition, the difference between magnitudes of good rotor and broken rotor is larger at higher loading. In short, detection of rotor fault is more obvious at higher loading especially at full load condition.

In conclusion, MISA is able to detect rotor faults same as MCSA. Rotor faults such as broken rotor bar will lead to decrease in magnitude of rotor signature in impedance spectrum. By computing the FRF between input voltage and current, larger variation in magnitude of rotor frequencies can be observed for broken rotor bar compared to current spectrum analysis. Using impedance analysis, the effect of power supply harmonic is eliminated hence sidebands of rotor and shaft frequencies are more obvious. This allows easier fault frequency identification in peak search process. Also these rotor signatures vary with the loading applied to induction motor. However, only slight

changes have been observed at magnitude of shaft frequencies for broken rotor bar. This could be the reason of shaft frequencies are independent with rotor faults, which can be used to distinguish between rotor fault and asymmetric eccentricity, since asymmetric eccentricity would affect shaft frequency.

6.4 Abnormal Air-Gap Eccentricity

Abnormal air-gap eccentricity is irregular air-gap occurs in between stator and rotor which normally due to improper installation of rotor or irregular shape of stator core. Abnormal air-gap eccentricity leads to unbalance magnetic flux distribution which gives rise to abnormal harmonic frequencies in stator current. These abnormal harmonics frequencies can be obtained by *Eq. (2.23)* and *Eq. (2.24)*. There are two types of abnormal air-gap eccentricity, namely static eccentricity and dynamic eccentricity. In the following experiments, static eccentricity of rotor will be focused.

6.4.1 Characteristic Frequencies of Abnormal Air-Gap Eccentricity

Firstly, the characteristic frequencies of abnormal air-gap eccentricity are located in current spectrum. The following current spectrum is obtained by applying 98% of loading to 0.5 hp, 4 poles three phase squirrel cage induction motor. The motor slip is about 0.06. As introduced in Chapter 2, characteristic frequencies of abnormal air-gap eccentricity can be observed at both sidebands of slot frequencies and supply frequency. With the information of motor slip and *Eq. (2.23)*, the sidebands of slot frequencies can be determined as follows. The rotor slot number, $R = 34$ and $n_d = 0$ for static eccentricity. Pole pair, $p = 4$. k is integer and n_ω is stator time harmonics.

$$f_{slot,ecc} = f_0 \left[(kR \pm n_d) \frac{(1-s)}{p} \pm n_\omega \right] \quad (6.10)$$

$$f_{slot,ecc} = 50 \left[(1 \times 34 \pm 0) \frac{(1-0.06)}{4} \pm 1 \right]$$

$$f_{slot,ecc} = 399.5 \pm 50 \text{ Hz}$$

$$f_{slot,ecc,LSB} = 349.5 \text{ Hz} \quad \& \quad f_{slot,ecc,USB} = 449.5 \text{ Hz} \quad (6.11)$$

Figure 6-16: Characteristic Frequencies of Abnormal Eccentricity on Current Spectrum

Figure 6-17: Characteristic Frequencies of Abnormal Eccentricity on Impedance Spectrum

Based on the calculated air-gap eccentricity frequencies, they are located near the harmonics of supply frequency, 350 Hz and 450 Hz. As the harmonics of supply frequency dominate the signature around those frequencies, the

observation of air-gap eccentricity frequencies becomes difficult. However, two peaks are detected at frequency components of 343 Hz and 443 Hz, which are possibly related with asymmetric eccentricity. Hence the magnitude of these frequency components will be monitored and compared for detection of asymmetric eccentricity. According to S. Nandi [43] and M. E. H. Benbouzid [44], abnormal air-gap eccentricity will be reflected on fundamental sidebands of supply frequency, also known as shaft frequencies in previous section. The computational of these low frequency components are shown below:

$$f_{ecc} = f_0 \left[1 \pm m \frac{(1-s)}{p} \right] \quad (6.12)$$

$$f_{ecc} = 50 \left[1 \pm 2 \times \frac{(1-0.06)}{4} \right]$$

$$f_{ecc} = 50 \pm 23.5 \text{ Hz}$$

$$f_{ecc,LSB} = 26.5 \text{ Hz} \quad \& \quad f_{ecc,USB} = 73.5 \text{ Hz} \quad (6.13)$$

These frequencies are easier to be identified and located at lower frequency range. However, their magnitudes are affected by loading condition, rotor faults as well as abnormal air-gap eccentricity. In order to ensure the change of magnitude at fundamental sidebands of supply frequency is solely depended on abnormal air-gap eccentricity, the test samples have to be isolated from other faults. From the calculation, motor slip information is essential to determine air-gap eccentricity frequencies. For sidebands of slot frequencies at high frequency range, slight changes on motor slip will result in a shift of few hertz on spectrum. Consequently, sidebands of supply frequency are preferred in some papers for abnormal air-gap eccentricity monitoring. In the following analysis, both sidebands of slot frequencies and supply frequencies will be studied and compared.

6.4.2 Comparison between Normal and Abnormal Eccentricity

In this section, the current spectrum and impedance spectrum of normal as well as abnormal air-gap eccentricity will be compared. In order to create static eccentricity between stator and rotor, a customized bracket with spigot

eccentricity offset by 0.2 mm is used. This bracket will be installed at one end of induction motor thus the rotor is offset at that end to provide the static eccentricity. Normal air-gap between stator and rotor for the testing induction motor is 0.35 mm. Figures below are the picture on customized bracket and a diagram to illustrate the setting.

Figure 6-18: Static Eccentricity created by Customized Bracket

Figure 6-19: Current and Impedance Spectrums of Normal and Abnormal Eccentricity below 100 Hz

Same 0.5 hp induction motor and loading conditions, full load at 1410 rpm are used in the experiments. Both current and impedance spectrums of good and abnormal eccentricity are compared as shown in *Figure 6-19*. Results above are based on averaging of 50 samples on both normal and abnormal eccentricity. At low frequency, the changes on sideband of supply frequency (shaft frequencies) from current spectrum are not significant, only a few dB of variations are observed. Magnitudes of rotor frequencies at 44.1 Hz and 55.8 Hz remain constant on both current and impedance spectrums. These findings show that monitoring sidebands of supply frequency may not be enough to indicate abnormal eccentricity.

Since sidebands of supply frequency cannot be the evidence for abnormal eccentricity, sidebands of slot harmonics at higher frequency range are studied as follows.

Figure 6-20: Current and Impedance Spectrums of Normal and Abnormal Eccentricity below 500 Hz

From the comparison of spectrums between abnormal and normal eccentricity, abnormal eccentricity will give rise to higher magnitude of frequency components near 349.5 Hz and 449.5 Hz. However, these sidebands of slot harmonics could not be monitored as harmonics of supply frequency at 350 Hz and 450 Hz dominated the signature at those frequencies. No obvious change in current spectrum can be monitored between normal and abnormal eccentricity. On the other hand, some amplitude changes are detected at frequency of 343 Hz and 443 Hz in impedance spectrum. The amplitudes change from 45.0 dB to 38.1 dB and from 46.9 dB to 43.7 dB respectively. Beside of these two frequencies, around 6 dB of amplitude changes are observed at 338 Hz and 438 Hz. According to the comparison, impedance spectrum able to show some amplitude changes due to asymmetric eccentricity. This could be a good sign that impedance spectrum has slightly better sensitivity compared to current spectrum.

Figure 6-21: Current and Impedance Spectrums of Normal and Abnormal Eccentricity below 3000 Hz

At even higher frequency, some amplitude changes can be detected in impedance spectrum analysis. For instance, amplitude changes of about 10 dB

are detected at frequency components of 802 Hz and 903 Hz. These two frequencies approximate with the static eccentricity frequencies with $k = 2$. Besides that, about 6 dB of amplitude changes are observed at 1184 Hz, which is similar to calculated dynamic eccentricity frequencies with $k = 3$. This finding shows the eccentricity fault created for this experiment not only leads to static eccentricity but also causes small degree of dynamic eccentricity.

6.4.3 Comparison between Normal and Nonlinear Eccentricity

The effect of static eccentricity on spectrums has been studied in previous section. Another eccentricity fault will be introduced here, which is nonlinear eccentricity. Nonlinear eccentricity is due to uneven surface of rotor thus different air-gap distances occur along the rotor. Not many research works have been carried out on this type of faults. However the manufacturer of induction motor does encounter this problem during rotor manufacturing. Therefore, the effect of nonlinear eccentricity on spectrum has been studied in this project. Figure below is the picture of rotor with nonlinear eccentricity:

Figure 6-22: Rotor with Nonlinear Eccentricity

The experiments are carried out under full load condition using same induction motor. Current and impedance spectrums have been collected and analysed with results from averaging of 50 samples. The experimental results are presented in *Figure 6-23*. Comparing current and impedance spectrums for normal and nonlinear eccentricity, the sidebands of supply frequency do not change significantly, whereas the sidebands of rotor frequency decrease in current spectrum and increase in impedance spectrum for nonlinear eccentricity. Sidebands of rotor frequency characterize backward rotating

magnetic field created by rotor asymmetry. If backward rotating magnetic field is stronger, the magnitude of rotor frequency's sidebands becomes larger. In this case, decreases in rotor frequency's sidebands illustrate the backward rotating magnetic field is weaker. It is interesting to see this result, as backward rotating magnetic field can be considered as a loss in energy, The lower it is, the better efficiency that machine can have. This phenomenon perhaps can be explained by the uneven surface of rotor, which has reduced the magnitude of eddy current induced by forwards rotating magnetic field. Correspondingly the induced backward rotating magnetic field is reduced in magnitude.

Figure 6-23: Current and Impedance Spectrums of Normal and Nonlinear Eccentricity below 500 Hz

In conclusion, MISA is able to detect asymmetric eccentricity fault of induction motor by monitoring the sidebands of slot frequencies. However, asymmetric eccentricity fault will not be reflected at sidebands of supply frequency which have been used for monitoring eccentricity fault in many research papers. This is because sidebands of supply frequency describe the motor rotation speed and if the motor rotation speed is kept as constant, the sideband of supply frequency remains unchanged in impedance spectrum. In order to detect eccentricity fault, high frequency components are used in the analysis. Asymmetric eccentricity fault increases the magnitude of frequency components around 343 Hz and 443 Hz for the test motor. In addition, eccentricity fault is detected at higher harmonic of the slot frequency. At high frequency region, signal to noise ratio is improved and hence those frequencies can be identified easily.

Nonlinear eccentricity is studied and it reduces the magnitude of rotor frequency's sidebands. This finding shows the backward rotating magnetic field is reduced, which can be valuable in reducing the loss in efficiency of induction motor.

6.5 Bearing Faults

Bearing faults is one of the most common faults for an electric motor. Improper installation, misalignment and deflection of shaft are the main reasons behind the bearing failure. Mechanical vibration from damaged bearing causes perturbation in magnetic field in air-gap. This perturbation can be observed in spectrum analysis with abnormal harmonic frequency from *Eq. (2.16)* to *Eq. (2.17)*. From the equations, bearing faults such as inner race and outer race faults will be focused in the following analysis.

6.5.1 Characteristic Frequencies of Bearing Faults

In order to detect bearing fault in current spectrum, bearing inside the induction motor has been replaced by a faulty bearing. The faulty bearing has

unknown fault but jagged rotation can be felt when the bearing is rotating. For bearing faults, its faulty frequencies can normally be observed at high frequency range depending on the bearing size. The bearing used in the 0.5 hp induction motor is NTN 6202Z, single row deep groove ball bearing with 8 balls $n = 8$, ball diameter $BD = 6$ mm, pitch diameter $PD = 25$ mm and zero contact angle $\beta = 0^\circ$. The computational of inner race and outer race bearing faults frequencies in spectrum analysis are shown as below:

$$f_{i,0} = \frac{n}{2} f_r \left[1 \pm \frac{BD}{PD} \cos \beta \right] \quad (6.14)$$

$$f_{i,0} = \frac{n}{2} f_r \left[1 \pm \frac{6}{25} \cos 0 \right]$$

$$f_{i,0} = \frac{n}{2} f_r [1 \pm 0.24]$$

$$f_i = 0.62nf_r \quad \& \quad f_o = 0.38nf_r \quad (6.15)$$

From the equations above, characteristic frequencies of inner race (f_i) and outer race (f_o) are approximated to Eq. (2.18) and Eq. (2.19). These imply those equations are good approximation for the bearing used in this experiment, hence the equations will be used in the following analysis. Under full loading condition with motor slip of 0.06, the characteristic frequencies of bearing are shown as follows:

$$\begin{aligned} f_i &= 0.6 \times 8 \times 23.5 \quad \& \quad f_o = 0.4 \times 8 \times 23.5 \\ f_i &= 112.8 \text{ Hz} \quad \& \quad f_o = 75.2 \text{ Hz} \end{aligned} \quad (6.16)$$

Characteristic frequencies of bearing's inner and outer race will be reflected in current and impedance spectrums due to perturbation created by vibration resulted from bearing faults. The abnormal harmonic frequencies can be obtained by Eq. (2.16). Equations below show the abnormal harmonic frequencies of bearing faults up to third harmonic.

$$f_{bearing} = f_0 \pm mf_{i,0} \quad (6.17)$$

$$f_{b,i} = |50 \pm m(112.8)| \quad \& \quad f_{b,o} = |50 \pm m(75.2)| \quad (6.18)$$

$$f_{b,i} = \begin{Bmatrix} 62.8, & 175.6, & 288.4 \end{Bmatrix} \text{Hz}$$

$$f_{b,o} = \begin{Bmatrix} 25.2, & 100.4, & 175.6 \\ 125.2, & 200.4, & 275.6 \end{Bmatrix} \text{Hz}$$

6.5.2 Comparison between Good and Faulty Bearing

A faulty bearing use in the experiments consists of a crack near the inner race in order to produce vibration of shaft when the bearing rotates. The experiment is carried out with same 0.5 hp induction motor without any other motor faults except bearing fault. Current and impedance spectrums of the input of motor are analyzed and abnormal harmonic frequencies obtained previously are monitored. The results are shown in *Figure 6-24*.

Figure 6-24: Current and Impedance Spectrums of Good and Faulty Bearing below 300 Hz

By comparing current spectrum of faulty bearing and good bearing, the difference in amplitude is not obvious. Only slight amplitude changes can be observed in frequency components of 96.9 Hz and 199.9 Hz, which is comparable to the calculated second harmonic of characteristic frequencies for outer race bearing fault. In impedance spectrum, the change in amplitude of those frequency components is more noticeable. For instance, there are more than 10 dB of amplitude changes at 102.9 Hz and 273.1 Hz. These two frequencies are similar to harmonic frequencies of outer race bearing fault. More amplitude changes are observed at 126.2 Hz and 179.2 Hz. For 179.2 Hz, it is approximated to harmonic frequency of inner race bearing faults.

However, for all the abnormal harmonic frequencies discovered in impedance spectrum do not match with calculated frequencies exactly. This might be because of the loading applied to the motor has small degree of variations. The variation of loadings affects rotor rotational speed and this variation will be magnified in the process of identifying higher order harmonic frequencies. Consequently, the variation of loadings will cause few hertz of frequency variations. Another reason which characteristic frequencies of bearing fault are hard to be identified is because of low power induction motor used in the experiment. Higher power induction motor generally involves with higher value of induced voltage and current that reflects motor faults, thus motor faults signature can be detected with ease.

Generally, impedance spectrum contains more amplitude changes not only at those abnormal harmonic frequencies of bearing faults, but also at other frequency components. Compared to current spectrum whereby the amplitude change is not obvious, impedance spectrum may be more sensitive to detect abnormal harmonic frequencies at high frequency region.

6.6 Induced Harmonics by Frequency Converter

In spectrum analysis, the switching frequency of control techniques such as frequency converter may give rise to induce harmonic in voltage and current spectrums. These induced harmonics would increase the magnitude of fault signature and lead to confusion in fault detection. The effect of induced harmonics due to frequency converter has been studied and compared with direct power supply.

Figure 6-25: Induced Harmonics due to Frequency Converters

A three phase, 0.25 hp, 6 poles squirrel cage induction motor is used for the experiment and running at 80% of full load with 900 rpm. From the results, the effect of induced harmonics can be noticed clearly at voltage spectrum. Comparing the voltage spectrum of power supply from frequency converter and the voltage spectrum of direct power, shaft frequencies and its harmonics can be easily observed in the voltage spectrum of power supply from frequency converter. On the other hand, voltage spectrum from direct power

does not consist of high magnitude shaft frequencies signature. Beside from voltage spectrum, different in magnitude on shaft frequencies can be seen in current spectrum from both power supply. Current spectrum of power supply given by frequency converter has higher magnitude in shaft frequencies compared with that of direct power.

Induced harmonics from frequency converter resulted from switching process, as AC power supply from frequency converter is comprised of many DC components at high switching frequency. This effect of induced harmonic may cause confusion in fault detection as motor fault such as asymmetric eccentricity increases magnitude of shaft frequencies as well. Consequently, the task for fault detection becomes even more difficult. Generally, Total Harmonic Distortion (THD) is used to characterize the influence of induced harmonics. By installation of a harmonic filter, the induced harmonics can be mitigated.

In the previous experiments, direct power supply has been used to provide power to induction motor in order to avoid the effect of induced harmonics. However, impedance monitoring will be conducted under effect of frequency converter in order to examine its capability when voltage supply is varied.

6.6.1 Effects of Induced Harmonic on MISA

Frequency converter gives rise to induced harmonic on voltage and current spectrum. In order to study the effect of induced harmonic on impedance spectrum analysis, experiments have been conducted in such a way that comparison between current spectrum and impedance spectrum can be carried out. 0.5 hp, 3 phase squirrel cage induction motor with one broken rotor bar has been used in the experiments. The purpose of these experiments is to study the effect of induced harmonic on broken rotor bar's spectrum. The induction motor is operated under full load condition and the results are collected based on averaging of 50 samples.

Figure 6-26: Voltage, Current and Impedance Spectrums of Broken Rotor Bar under Direct and Frequency Converter Supply

Results above are the spectrum analyses of broken rotor bar. In the voltage spectrum, induced harmonic causes an increase of amplitude in the entire spectrum. With the power supply from frequency converter, the sidebands in voltage spectrum increase in amplitude for about 10 dB. Similarly, current spectrum experienced overall increment in amplitude. The rotor's sidebands in

current spectrum increase from -42.3 dB to -41.5 dB and from -48.5 dB to -47.0 dB for lower and upper sidebands respectively. About 1 dB of increment in amplitude is caused by induced harmonic. As a result, current spectrum is affected by induced harmonic but with a smaller level compared to voltage spectrum. For impedance spectrum wise, as impedance takes into consideration of both voltage and current, it does not have overall change in amplitude. However, the rotor's sidebands of impedance spectrum decrease in amplitude. This might be because of the change in voltage is greater than that of current. Therefore the FRF between current and voltage is having smaller amplitude. Since the amplitude of rotor's sidebands decrease, identification of rotor's sidebands becomes slightly difficult as those sidebands may not be obvious under the effect of induced harmonic. This could be one of the drawbacks in impedance analysis.

Figure 6-27: Current and Impedance Spectrums of Broken Rotor Bar and Good Rotor under Frequency Converter Supply

Figure above shows the comparison of broken rotor bar and good rotor's spectrum analysis under power supply from frequency converter. In current spectrum, broken rotor bar increases rotor sideband's amplitude by 9 dB. Whereas broken rotor bar reduces impedance amplitude by 7 dB for upper rotor sideband. At lower rotor sideband, impedance's amplitude does not change significantly. Generally, broken rotor bar causes an amplitude change at rotor sidebands of both current and impedance spectrums. Furthermore, harmonics of rotor sidebands increase in amplitude as well. The results above show both current and impedance spectrums are able to detect rotor fault even under the influence of induced harmonics. Nevertheless, the amount of amplitude change due to rotor fault under frequency converter is lesser than that of direct power supply.

According to the results, motor faults such as broken rotor bar can still be identified even under the effect of induced harmonic by frequency converter. The amount of amplitude change on rotor sidebands is lesser. In voltage and current spectrums, amplitude increases in general whereas impedance is independent from the effect of induced harmonics. However, the sidebands become less obvious and this may hinder the detection of rotor sidebands.

In conclusion, the impedance spectrum is shown to have better sensitivity compared to current spectrum as impedance has larger amplitude change due to motor faults. The elimination of main supply frequency component leads to easier fault frequency identification process. Furthermore, induced harmonics from controller have little effect on impedance spectrum. Despite of the advantages of impedance spectrum, it still has some limitations such as low time resolution due to the properties of FFT. In order to overcome this problem, analytic signal from Hilbert Transform is suggested to obtain the time domain data of electrical impedance which will be discussed in depth in the following chapter.

CHAPTER 7

FAULTS DIAGNOSIS BY TIME-FREQUENCY ANALYSIS OF MOTOR INPUT IMPEDANCE

Electrical input impedance of induction motor is proven containing rich information of the motor's health condition, hence it can be used as a monitoring signature for induction motor's fault. In previous chapter, the electrical input impedance is analyzed in frequency domain for fault detection and the results are convincing. However, frequency domain analysis has limitations such as low time resolution and stationary data are required. Therefore, this chapter focuses on time-frequency analysis. Time-frequency analysis has the ability to identify the time when transient occurred or the moment when motor fault is happened. This is difficult to achieve in frequency analysis by FFT. The following time-frequency analysis is carried out based on Hilbert transform and wavelet packet transform.

7.1 Introduction

Time-frequency analysis is a technique to study a signal in both time and frequency domains simultaneously. There are several different methods to formulate a time-frequency distribution function, one of the most basic techniques is Short Time Fourier Transform (STFT), and more sophisticated techniques such as Wavelet Transform. One of the main advantages of using time-frequency analysis is the ability to study transient signal, in which the signal's frequency characteristic varies with time. STFT is a Fourier transform which applied to local sections of a signal as it changes over time. By definition, the function to be transformed is multiply by a time window function, thus the Fourier transform is carried out as the window function is slid along the time axis. Therefore the resulting function would be in term of time and frequency. However, the drawback of STFT is that it has fixed resolution which is determined by the width of the window function. For

instance, a wide window function gives better frequency resolution but poor in time resolution. Conversely, a narrow window function gives good time resolution but poor in frequency resolution. In fact, this is one of the reasons that Wavelet transform is introduced.

The purpose of Wavelet transform is to decompose a signal into sub-signals with different resolution levels. Mathematically, the wavelet transform is a convolution of scaled and translated wavelet function with the original signal. Wavelets are localized waveform functions of small duration with average value of zero. The frequency and time information of a signal can be extracted by matching the scaled and translated wavelet with the signal, thus allowing time and frequency analysis simultaneously.

In this chapter, the electrical input impedance of induction motor will be analyzed by using wavelet transform. Analytic signal facilitates mathematic manipulation of signals, thus time record of impedance can be obtained from division of voltage and current's analytic signals. In the previous chapter, the electrical impedance of induction motor is proven to be related with transduction functions and mechanical impedance, which is corresponding to the motor and loading conditions respectively. Consequently, by monitoring the time record of electrical impedance, the motor and loading conditions can be acknowledged in time domain.

7.2 Time Domain Analysis

In this section, input electrical impedance as a monitoring signature in time domain is studied. Analytic signal based on Hilbert Transform is utilized to compute time records signal of electrical impedance. Hilbert Transform is defined in time domain as a convolution between $y(t)$ with the function $1/(\pi t)$. In signal processing, a real signal $y(t)$ can be converted by Hilbert Transform into a new analytical signal where the real part is the time domain signal $y(t)$ and the imaginary part is its Hilbert Transform $\hat{y}(t)$. The amplitude of analytic

signal expresses the envelope of signal and the phase represents the instantaneous angle of signal in polar notation that defines where vector is pointing. The mathematical expressions of analytic signal were shown in section 3.6.

The input voltage and current obtained from induction motor can be converted into analytic signals and hence impedance is obtained from division between the two analytic signals.

$$\text{Voltage:} \quad \Psi_E(t) = E(t) + j\hat{E}(t) \quad (7.1)$$

$$\text{Current:} \quad \Psi_I(t) = I(t) + j\hat{I}(t) \quad (7.2)$$

$$\text{Impedance:} \quad \Psi_Z(t) = \frac{E(t) + j\hat{E}(t)}{I(t) + j\hat{I}(t)} \quad (7.3)$$

Subsequently, the analytic signal of electrical impedance is used in following time-frequency analysis. A LABVIEW programme is written to compute the Hilbert Transform of input voltage and current, hence impedance is obtained from division between two signals. The programme is shown in Appendix C

7.2.1 Time Domain Analysis on Load Variation

In this section, the impedance change due to load variation is analysed in time domain by analytic signal based on Hilbert Transform. The 0.5 hp three phase squirrel cage induction motor used in previous experiments is monitored. Two cases of load variations are created for this experiment. First experiment is removal of load in the middle of test for about 10 seconds. Another experiment is based on random load variation given to the motor. The electrical input voltage and current are measured and computed into analytic signal by using LABVIEW programme. At the same time, torque and rotational speed are measured for verification purpose.

Figure 7-1: Time Analysis by Hilbert Transform on Constant Load Variation

Figure on the left shows the variation of torque at mechanical port when the loading is removed at 12th second, then the loading is restored at 23rd second. The motor's full load torque is about 2.5 Nm. From the impedance graph on the right, which is computed from analytic signal of input voltage and current, an increase in magnitude can be observed at 12th second and it decreases at 23rd second. As torque decreases, the current drawn by the motor is reduced. But voltage remains the same, hence impedance would increase since it is the ratio between voltage over current.

Figure 7-2: Time Analysis by Hilbert Transform on Random Load Variation

Figure above shows the variation of impedance's analytic signal due to random load given to the motor. By comparing the torque and impedance, the impedance is able to follow the changes of torque closely. Moreover impedance amplifies the changes in torque. For instance, the first 0.2 Nm torque change at about 8th second is hard to be observed at torque graph.

Whereas the corresponding change in impedance is obvious, and about 70 ohms of changes is detected. Results above show that impedance is a sensitive monitoring signature for load condition, which is suitable for monitoring of small load variation.

In conclusion, impedance obtained from analytic signal based on Hilbert Transform is able to describe motor loading condition precisely and it is sensitive to load variation. However, more studies on analytic signal have to be carried out if condition monitoring on motor fault is focused. Since most of motor faults are expressed in respective characteristic frequencies, therefore the analytic signal of electrical impedance is studied by frequency analysis.

7.3 Frequency Domain Analysis

In previous section, impedance can be computed in time domain by the use of analytic signal. This allows monitoring impedance in real time as analytic signal contains magnitude and phase information. However, the analytic signal of impedance contains all the characteristic frequencies of motor faults. In order to diagnosis motor faults, the corresponding motor faults frequencies have to be identified. Hence the analytic signal is analyzed in frequency domain by Fast Fourier Transform (FFT) as below. The understanding on frequency components of impedance's analytic signal will be helpful in time-frequency analysis in latter section.

7.3.1 Fast Fourier Transform on Impedance's Analytic Signal

Voltage and current signals of the squirrel cage induction motor under full load condition are computed in analytic signal and hence its impedance is calculated from division of voltage and current. Then the frequency components of the impedance's analytic signal are extracted by FFT.

Figure 7-3: FFT of Impedance's Analytic Signal of Good Motor

From the FFT of impedance's analytic signal, the frequency components of analytic signal are different from the frequency components obtain from FRF of voltage and current as shown in *Figure 6-11*. For instance, the rotor frequencies at 44.1 Hz and 55.8 Hz in that impedance spectrum are missing. Whereas in the spectrum of impedance's analytic signal, some low frequencies components such as 6.2 Hz, 23.6 Hz and 29.6 Hz are observed. In addition, the frequency components in impedance's analytic signal are facing towards positive side.

7.3.2 Characteristic Frequencies of Motor Faults on Current's Analytic Signal

In this section, the frequency components which have been found previously from the FFT of analytic signal are identified by carrying out the transformation of signal analytically. The derivation is quite similar to that in section 3.6, but the characteristic frequencies of motor fault are focused in this section. First of all, the frequency components in current signature are chosen to be analysed and figure below is the spectrum of original current signature.

Figure 7-4: FFT of Original Current Signature of Good Motor

At frequency below 100 Hz, the major frequency components can be categorized as main supply frequency (f_0), rotor frequencies (B_1 and B_2), shaft rotational frequencies (C_1 and C_2) and harmonics of rotor frequencies (D_1 to D_4). Subsequently, the current signature in time domain can be written as follow with the assumption that the signature only consists of main supply frequency and upper rotor frequency components. The following equations are expressed in term of angular frequency, where $\omega_0 = 2\pi f_0$.

$$I(t) = A \sin \omega_0 t + B \sin \omega_0 (1 + 2s)t \quad (7.4)$$

Hilbert transform of above equation:

$$\hat{I}(t) = -A \cos \omega_0 t - B \cos \omega_0 (1 + 2s)t \quad (7.5)$$

In analytic signal, real part is the original signal and imaginary part is Hilbert Transform of the signal:

$$\Psi_I(t) = I(t) + j\hat{I}(t) \quad (7.6)$$

$$\Psi_I(t) = A(\sin \omega_0 t - j \cos \omega_0 t) + B[\sin \omega_0 (1 + 2s)t - j \cos \omega_0 (1 + 2s)t] \quad (7.7)$$

Together with Euler Formula, analytic signal of current can be rearranged as:

$$\Psi_I(t) = -jAe^{j\omega_0 t} - jBe^{j\omega_0 t} e^{j2s\omega_0 t} \quad (7.8)$$

$$\Psi_I(t) = -je^{j\omega_0 t} [A + Be^{j2s\omega_0 t}] \quad (7.9)$$

Thus the magnitude of analytic signal:

$$|\Psi_I(t)| = |-je^{j\omega_0 t}| |A + Be^{j2s\omega_0 t}| \quad (7.10)$$

$$|\Psi_I(t)| = |A + Be^{j2s\omega_0 t}| \quad (7.11)$$

From Eq. (7.11), it can be concluded that main supply frequency components does not present in the magnitude of current's analytic signal, it has been simplified into constant A . The rotor frequency which originally appeared at $f_0(1+2s)$ will be relocated at $2sf_0$, so this frequency component can be used to monitor rotor faults in induction motor [111], [112]. The magnitude of current's analytic signal, and its spectrum are shown in Figure 7-5. In fact, there is a peak observed at the spectrum graph which is corresponding to $2sf_0$. That frequency is located at 6.5 Hz, as the motor is running at full load condition with motor slip of 0.06 and $f_0 = 50$ Hz.

Figure 7-5: Spectrum and Magnitude of Analytic Signal of Current

From the magnitude of current's analytic signal, A is the magnitude of signal's envelope and B is the magnitude of signal fluctuation at that value. Normalization can be carried out in order to further simplify the analytic signal, thus it only consists of rotor frequency component.

$$|\Psi_I(t)|_{Normalized} = |\Psi_I(t)| - |\Psi_I(t)|_{mean} \quad (7.12)$$

$$|\Psi_I(t)|_{Normalized} = |Be^{j2s\omega_0 t}| \quad (7.13)$$

However, in actual current signal, it consists of not only rotor frequency but also supply harmonics and other motor's characteristic frequencies. Other frequency components observed in FFT of analytic signal are the results of those harmonics components. The identification of other frequency components is shown in the following.

The characteristic frequencies of each frequency component are listed below:

$$\text{Rotor Frequency: } f_{rotor} = f_0(1 \pm 2s) \quad (7.14)$$

$$\text{Shaft Rotational Frequency: } f_{shaft} = f_0 \left[1 \pm \frac{1}{2}(1-s) \right] \quad (7.15)$$

$$\text{Rotor Frequency's Harmonic: } f_{rotor,h} = f_0 \left[1 \pm \frac{1}{2}(1-s) \pm 2s \right] \quad (7.16)$$

Thus the current signal in time domain is shown below:

$$\begin{aligned} I(t) = & A \sin \omega_0 t + B \sin \omega_0 (1 \pm 2s)t \\ & + C \sin \omega_0 \left[1 \pm \frac{1}{2}(1-s) \right] t + D \sin \omega_0 \left[1 \pm \frac{1}{2}(1-s) \pm 2s \right] t \end{aligned} \quad (7.17)$$

Hilbert Transform of the signal:

$$\begin{aligned} H[I(t)] = & -A \cos \omega_0 t - B \cos \omega_0 (1 \pm 2s)t \\ & - C \cos \omega_0 \left[1 \pm \frac{1}{2}(1-s) \right] t - D \cos \omega_0 \left[1 \pm \frac{1}{2}(1-s) \pm 2s \right] t \end{aligned} \quad (7.18)$$

Analytic Signal of the signal:

$$\Psi_I(t) = -je^{j\omega_0 t} \left[\begin{aligned} & A + \\ & B_1 e^{-j2s\omega_0 t} + B_2 e^{j2s\omega_0 t} + \\ & C_1 e^{-j\frac{1}{2}\omega_0 t} e^{j\frac{1}{2}s\omega_0 t} + C_2 e^{j\frac{1}{2}\omega_0 t} e^{-j\frac{1}{2}s\omega_0 t} + \\ & D_1 e^{-j\frac{1}{2}\omega_0 t} e^{-j\frac{3}{2}s\omega_0 t} + D_2 e^{-j\frac{1}{2}\omega_0 t} e^{j\frac{5}{2}s\omega_0 t} + D_3 e^{j\frac{1}{2}\omega_0 t} e^{-j\frac{5}{2}s\omega_0 t} + D_4 e^{j\frac{1}{2}\omega_0 t} e^{j\frac{3}{2}s\omega_0 t} \end{aligned} \right] \quad (7.19)$$

The frequency components of $|\Psi_I(t)|$ can be computed from magnitude of equation above. *Table 7-1* below shows the frequency components of both $I(t)$ and $|\Psi_I(t)|$. Frequency components of D_2 and D_4 are excluded in the analysis as there are not significant in current spectrum. The motor is tested under full load condition with motor slip of 0.06, and $f_0 = 50$ Hz.

		$I(t)$		$ \Psi_I(t) $	
Rotor Frequency	B1	$f_0(1-2s)$	44.5 Hz	$-2sf_0$	-6.5 Hz
	B2	$f_0(1+2s)$	56.5 Hz	$2sf_0$	6.5 Hz
Shaft Rotational Frequency	C1	$\frac{1}{2}f_0(1+s)$	26.5 Hz	$-\frac{1}{2}f_0(1-s)$	-23.5 Hz
	C2	$\frac{1}{2}f_0(3-s)$	73.5 Hz	$\frac{1}{2}f_0(1-s)$	23.5 Hz
Rotor Frequency's Harmonic	D1	$\frac{1}{2}f_0(1-3s)$	20.5 Hz	$-\frac{1}{2}f_0(1+3s)$	-29.5 Hz
	D3	$\frac{1}{2}f_0(3-5s)$	67.5 Hz	$\frac{1}{2}f_0(1-5s)$	17.5 Hz

Table 7-1: Comparison of Frequency Components in Original and Analytic Signal of Current

From all the frequency components shown in the table above, a 50 Hz of frequency shift is observed in between frequency components of $I(t)$ and $|\Psi_I(t)|$. Negative frequency components are obtained in analytic signal. However, because of one side spectrum properties of analytic signal's Fourier Transform, those negative frequency components will be reflected in positive frequency region respectively. One side spectrum properties is one of the properties of analytic signal and it is explained in the following section.

Given a Hilbert transformed signal $\hat{y}(t)$ from original signal $y(t)$, its Fourier transform is:

$$\text{Hilbert Transform of } y(t): \quad \hat{y}(t) = \frac{1}{\pi t} \cdot y(t) \quad (7.20)$$

$$\text{Fourier Transform of } \hat{y}(t): \quad \hat{Y}(f) = -j \operatorname{sgn}(f) Y(f) \quad (7.21)$$

Where signum function, $\operatorname{sgn}(f)$ is defined as below:

$$\text{sgn}(f) = \begin{cases} 1 & \text{for } f > 0 \\ 0 & \text{for } f = 0 \\ -1 & \text{for } f < 0 \end{cases} \quad (7.22)$$

In frequency domain of analytic signal,

$$\Psi(f) = U(f) + j[-j \text{sgn}(f)U(f)] \quad (7.23)$$

$$\Psi(f) = [1 + \text{sgn}(f)]U(f) \quad (7.24)$$

Hence the signum function is:

$$1 + \text{sgn}(f) = \begin{cases} 2 & \text{for } f > 0 \\ 1 & \text{for } f = 0 \\ 0 & \text{for } f < 0 \end{cases} \quad (7.25)$$

From Eq. (7.24) and Eq. (7.25), the Fourier transform of an analytic signal has one side spectrum at positive frequencies. Its frequency components are two times larger in magnitude, compared to Fourier transform of original signal [74]. This is perhaps one of the advantages for using analytic signal.

According to all the analysis above, the frequency components of analytic signal are identified. These findings are applicable to analytic signal of impedance as they share the same frequency components as follows.

$$\text{Voltage:} \quad \Psi_E(t) = -je^{j\omega_0 t} [E_1 + E_2 e^{j2s\omega_0 t}] \quad (7.26)$$

$$\text{Current:} \quad \Psi_I(t) = -je^{j\omega_0 t} [I_1 + I_2 e^{j2s\omega_0 t}] \quad (7.27)$$

$$\text{Impedance:} \quad \Psi_Z(t) = \frac{[E_1 + E_2 e^{j2s\omega_0 t}]}{[I_1 + I_2 e^{j2s\omega_0 t}]} \quad (7.28)$$

If normalization is applied to current and voltage analytic signals, the constants E_I and I_I are able to be eliminated from impedance's analytic signal. The frequency components in impedance's analytic signal only consist of rotor frequency at $2sf_0$ and other harmonics, no main supply frequency exists in the signal.

7.3.3 Comparison between Good and Broken Rotor Bar

After the motor fault's characteristic frequencies in $|\Psi_Z(t)|$ are identified, the capability of this signal in describing motor fault is studied. Comparison has been made between good rotor and broken rotor bar by using $|\Psi_Z(t)|$. The experiments are carried out under full load condition with motor slip at about 0.06. The $|\Psi_Z(t)|$ is computed and normalized in order to eliminate main supply frequency components.

From the comparison as shown in *Figure 7-6*, obvious amplitude change can be observed at rotor frequency, 6.2 Hz for a broken rotor bar. In addition, amplitude change can be observed at 17.4 Hz, which is corresponding to the harmonic of rotor frequency. For frequency components at 23.6 Hz and 47 Hz, they are shaft rotational frequency and its harmonics respectively. At 50 Hz, it is harmonic of supply frequency. From the result, a broken rotor bar leads to amplitude change in those frequency components, especially in rotor frequency and shaft rotational frequencies.

Figure 7-6: Fourier Transform of Impedance's Analytic Signal of Good Rotor and One Broken Rotor Bar

In short, impedance from alternating signal can be computed in time domain by using analytic signal. The analytic signal can be used to monitor load

changes over time via envelopes of the signal. Subsequently, the analytic signal is studied by frequency analysis in order to recognize its frequency components. In frequency domain, the main power supply frequency is eliminated which enables easier observation on other sidebands. In addition, rotor frequency is found to be located at low frequency region (<10 Hz), thus leads to easier rotor frequency identification. In the case where only rotor fault of motor is focused, only low frequency region is required to be analyzed. Consequently, the numbers of data to be processed is reduced and hence the computation process is faster. However, the use of FFT does not provide good time resolution for a real time monitoring. Therefore Wavelet Transform with decent frequency and time resolution is used in the following section in order to carry out time-frequency analysis.

7.4 Wavelet Transform

Wavelet transform is particularly useful in analyzing signals which are noisy, transient, intermittent, and so on. It is able to analyze signal simultaneously in both time and frequency domains in a different way from conventional short time Fourier transform (STFT). Wavelet transform is a convolution of scaled and translated mother wavelet with the original signal. Mother wavelet, $\psi(t)$ is a waveform of small duration that has average value of zero. Using scaling and translation operations on the mother wavelet, a family of wavelet functions is created with same shape but different in frequency and time location. The frequency contents and time locations of a given signal can be known by matching the scaled and translated wavelet to the given signal. The wavelet coefficient in *Eq. (7.29)* indicates the frequency content of a signal $x(t)$. High coefficient means the scaled and translated wavelet well matched with the section of original signal. Scale factor, a is related to frequency of wavelet and translation factor, b is related to shifting of wavelet in time record [113]-[116].

$$T(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi\left(\frac{t-b}{a}\right) dt \quad (7.29)$$

7.4.1 Continuous Wavelet Transform

Continuous Wavelet Transform (CWT) can be carried out by *Eq. (7.29)* with continuous functions of the scaled and translated wavelets in both time and frequency domains. This means the scale factor a and translation factor b in the wavelet are real continuous variables. In order to have better understanding on the capability of time-frequency analysis by CWT, it is applied to analytic signal of electrical input impedance obtained in the previous section.

In the following analysis, the squirrel cage induction motor is operated at full load condition, and its analytic signal of electrical impedance is obtained and analyzed by CWT. In order to carry out the CWT, Wavelet Toolbox in MATLAB programme is utilized. In the CWT analysis, Gaussian wavelet with order of 4 (gaus4) is chosen as the mother wavelet, because it is applicable in CWT and symmetry in shape. This wavelet has centre frequency f_c of 0.5 Hz. The result of CWT on magnitude of electrical impedance's analytic signal is shown in *Figure 7-7*.

From the figure, $|\Psi_Z(t)|$ shows the envelope of electrical impedance's analysis signal, which is about 360 Ω over a period of time. In the CWT plot, y-axis is the scale factor a , which represents the frequency component and x -axis is the time step in 0.001 second. High wavelet coefficient can be observed within the range of 50 to 160 of scale factor, which is corresponding to 10 Hz and 3.125 Hz in frequency. The matching between scale factor and frequency can be expressed in *Eq. (7.30)*, where f_a is the pseudo-frequency corresponding to the scale factor a in Hz, f_c is the centre frequency of wavelet, and Δ is the sampling period. The sampling period in this analytic signal is 0.001 s.

$$f_a = \frac{f_c}{a \times \Delta} \quad (7.30)$$

Figure 7-7: CWT on $|\Psi_Z(t)|$ of Induction Motor with Good Rotor

According to the frequency analysis of $|\Psi_Z(t)|$ in Figure 7-6, the rotor frequency is found at 6.2 Hz, which is located within the high wavelet coefficient region. In fact, the wavelet coefficient at scale factor of 81 represents the frequency components at 6.173 Hz, which is related to rotor frequency. Therefore, the wavelet coefficient at the rotor frequency can be extracted as shown in the figure, and the magnitude is about ± 10 for induction motor with good rotor.

Similar CWT analysis is performed on $|\Psi_Z(t)|$ from induction motor with one broken rotor bar. The comparison of wavelet coefficients at scale factor of 81 are presented as follows.

Figure 7-8: Comparison of Wavelet Coefficient between Induction Motor with Good Rotor and One Broken Rotor Bar

The comparison shows that wavelet coefficient of broken rotor bar increased by two times of the wavelet coefficient of good rotor. This implies that CWT is indeed able to detect motor fault such as broken rotor bar. In short, analysis from CWT is often easier to interpret because it demonstrates the change of frequency over time continuously in one chart. However, the drawback of using CWT is the huge computational load.

7.4.2 Discrete Wavelet Transform

Discrete Wavelet Transform (DWT) utilizes scaled and translated wavelets with discrete value based on power of two. Normally, DWT is expressed as series expansion of both approximation and detail coefficients. The approximation coefficients are the high scale, low frequency components of the signal, whereas the detail coefficients contain the low scale, high frequency components. Consequently, the DWT is also known as multiresolution analysis which is computed from a series of high (detail) and low (approximation) pass filters.

In the following DWT analysis, impedance's analytic signals from previous section are studied. Similarly, the Wavelet Toolbox from MATLAB is used. The mother wavelet used in the DWT is Daubechies wavelet with order of 10 (db10). There are 8 levels of decomposition of signal, in order to ensure the rotor frequency can be extracted clearly. From the result in *Figure 7-9*, the decomposed signals are presented on the left, and the corresponding frequencies are shown on the right. Decomposed signal d_7 contains rotor frequency which is at 6.2 Hz, thus it is monitored for rotor fault. Comparison between d_7 of good rotor and one broken rotor bar is shown in *Figure 7-10*. The magnitude of d_7 for good rotor is around ± 0.5 and it is increased to about ± 2.0 for one broken rotor bar. The result shows that DWT is capable in detecting induction motor's rotor fault. This analysis method has lighter computational load compared to CWT.

Figure 7-9: DWT on $|\Psi_Z(t)|$ of Induction Motor with Good Rotor

Figure 7-10: Comparison of Decomposed Signal d_7 between Good Rotor and One Broken Rotor Bar

However, these two conventional wavelet transform methods have limitation on time-frequency resolution. Generally, high frequency resolution analysis is achieved on the expense of low time resolution or vice versa. This is good for monitoring rotor fault as its characteristic frequency occurs at low frequency region. However, difficulty rises when motor fault's characteristic frequencies at high frequency region are monitored. Identification of those characteristic frequencies could be hindered by low frequency resolution. Consequently, wavelet packet transform is preferred in these cases, as it allows finer and adjustable frequency resolution at higher frequency regions.

7.5 Wavelet Packet Transform

Wavelet Packet Transform (WPT) is a generalization of DWT that offers a richer range of possibilities for signal analysis. It is performed by further signal decomposition for some or all of the sub-signals of the preceding level. Therefore, finer and uniform time-frequency resolution can be achieved even at high frequency region. However, the drawback is the rise of computational complexity. There are many researches on fault diagnosis systems which utilize wavelet packet transform or wavelet transform to diagnosis induction machines' faults [117]-[119]. This shows that wavelet transform is feasible and getting popular in fault diagnosis sector.

WPT is particularly useful in bearing or asymmetric eccentricity fault detection, as those fault characteristic frequencies are located at high frequency region. In order to identify those motor faults accurately, fine and uniform frequency resolution is essential. Therefore, WPT will be used as time-frequency analysis method in following section. After the signal is decomposed by WPT, an array of wavelet packet coefficients with M levels each with N coefficients can be obtained. A total of N coefficients from the $M \times N$ array can be selected to represent the signal. For instance, if a signal is analyzed by WPT with decompose level of $M = 7$, there will be a total of 128 coefficients in that level. Part of the decomposition tree is shown in *Figure 7-11* below.

Figure 7-11: Part of Decomposition Tree from WPT with $M = 7$

Each node contains wavelet packet coefficients over a certain frequency range, depends on the composition level. In the following analysis, a number of wavelet coefficients from N are selected, which its frequency components consist of interested motor faults characteristic frequencies. Those selected wavelet coefficients will be extracted and reconstruct by inverse wavelet transform in order to represent it in a signal that only consists of corresponding frequency range. The node in the decomposition tree is arranged in natural order, from (7.0) to (7.127) in regular order. However, the frequency components of WPT do not arrange in this order. For the ease of analysis, it is better to arrange the WPT in frequency order as shown in *Figure 7-12*. This is mainly due to different decomposition path in WPT. Normally, left branch in decomposition tree is the combination of low pass filtering (0) and followed by high pass filtering (1). The combination at right branch is high pass filtering and followed by low pass filtering [120].

Figure 7-12: WPT arranged in Frequency Order

Experiments are carried out with the same setup to obtain $|\Psi_Z(t)|$ for various motor faults and then analyzed by WPT. The WPT is carried out with decomposed level of 7 and using db10 as mother wavelet. The signal is sampled at 1000 kS/s hence it has maximum frequency of 500 Hz. Based on the setting, each decomposed signal has frequency resolution of 3.91 Hz. The

following WPT analyses are carried out by making use of another computational programme, LABVIEW. The graphical programming for WPT can be found in Appendix C.

7.5.1 Rotor Fault

The $|\Psi_Z(t)|$ from induction motor with good rotor and one broken rotor bar are analyzed by WPT, and the comparison result is shown in *Figure 7-13*. The figure shows that decomposed signal at node (7.1) is increased from 0.6 to 2.5 due to the rotor fault, about 4 times of magnitude change is observed. Node (7.1) is chosen because it has frequency range from 3.91 Hz to 7.81 Hz, and so rotor frequency at 6.5 Hz is embedded in the node. In addition, increment of wavelet coefficient from 2.0 to 2.5 also observed at node (7.5), with frequency ranging from 23.44 Hz to 27.34 Hz. This frequency range contains shaft rotational frequency at 23.6 Hz, which will change in response of rotor fault as well.

Figure 7-13: WPT Analysis of Rotor Faults at Node (7.1) and (7.5)

7.5.2 Asymmetric Air-Gap Eccentricity

The WPT analysis is also carried out on $|\Psi_Z(t)|$ of induction motor with asymmetric air-gap eccentricity. In *Figure 7-14*, some increments from 2.5 to 5.0 are observed at node (7.42), as a consequence of asymmetric air-gap eccentricity. Node (7.42) corresponds to frequency range of 199.2 Hz to 203.13 Hz, which consists of asymmetric air-gap eccentricity's characteristic

frequency. This characteristic frequency, 199.5 Hz matches with the air-gap characteristic frequency of 245.5 Hz which is obtained from *Eq. (6.10)* with $n_{\omega} = 3$, after considered 50 Hz offset due to the properties of analytic signal. In addition, increment of magnitude is observed at node (7.58) which has frequency range from 171.88 Hz to 175.78 Hz, it is believed to be caused by harmonics of asymmetric eccentricity's characteristic frequency. Similarly for node (7.106) with frequency from 296.88 Hz to 300.78 Hz, the decomposed signal is having more waviness due to asymmetric eccentricity.

Figure 7-14: WPT Analysis of Asymmetric Air-Gap Eccentricity at Node (7.42), (7.58) and (7.106)

7.5.3 Bearing Fault

The results of WPT analysis on $|\Psi_Z(t)|$ of induction motor with bearing faults are shown in *Figure 7-15*. Amplitude changes from 0.03 to 0.08 due to bearing fault can be observed at node (7.48). The frequency range of node (7.48) is ranging from 125.00 Hz to 128.91 Hz, and it consists of bearing fault frequency of 125.6 Hz. This bearing frequency can be correlated with $f_{b,i}$ from *Eq. (6.18)* with $m = 2$, together with 50 Hz offset. Small increments of magnitude can be observed at other frequency ranges, such as node (7.10) with frequency range of 46.88 Hz to 50.78 Hz and node (7.85) with 398.44 Hz to 402.34 Hz. Node (7.10) consists of shaft frequency's harmonic. An increase in magnitude at this frequency shows bearing fault leads to vibration that will

perturb magnetic field at shaft rotational speed. Node (7.85) consists of inner race bearing characteristic frequency with $m = 4$. All the characteristic frequencies above are obtained under consideration of 50 Hz offset from using analytic signal.

Figure 7-15: WPT Analysis of Bearing Fault at Node (7.10), (7.48) and (7.85)

All the results above show that $|\Psi_Z(t)|$ is capable to detect various motor faults by WPT analysis. Although the computation of this method is slightly more complicated compared to spectrum analysis, but the fault signal is enhanced, which can be observed from the higher ratio of amplitude change. Perhaps this is able to improve sensitivity of fault detection and make the process of fault detection easier. Also the analysis is able to be carried out in time-frequency domain, which supports the analysis of transient signal.

7.6 Comparison of Various Analysis Techniques

The WPT analysis is extended to input current signal and its analytic signal, such that comparison can be made between these three monitoring signatures. In addition, different analysis techniques have been applied on these signatures and the results are shown in *Table 7-2*. Only the results of rotor faults detection are presented as the characteristic frequencies of rotor faults are the easiest and the most distinct to be identified.

In the table, amplitude of rotor frequency and wavelet coefficients of corresponding decomposed signals are collected for comparison. By comparing different analysis techniques, wavelet analysis such as DWT and WPT are having larger magnitude change due to rotor fault compared to FFT. Generally, wavelet analysis gives about 4 times of changes but FFT only gives about 3 times of changes. If the entropy or normalized energy of decomposed signal from WPT is considered, the increment ratios are even higher. It is believed due to multiplication factor within the formula in order to obtain entropy or normalized energy. This shows that wavelet analysis is a better and more sensitive analysis tool compared to FFT despite it is slightly more complicated.

By comparing between different monitoring signatures, $I(t)$ shows smaller changes compared to analytic signal of current and $\Psi_Z(t)$. Which means that pre-processing the monitoring signature into analytic signal will enhance the motor fault's signature. About the same increment can be observed in $|\Psi_I(t)|$ and $|\Psi_Z(t)|$. However, $|\Psi_Z(t)|$ is having larger signature amplitude compared to $|\Psi_I(t)|$. This shows that by using impedance is able to magnify the fault signal and improves the signal to noise ratio.

From the results above, WPT analysis on $|\Psi_Z(t)|$ gives the best result in motor fault detection, among the other tested techniques. Although the computation of this method is slightly more complicated comparing to conventional spectrum analysis, but the motor fault's signature is enhanced, which can be

observed from the higher ratio of amplitude change. Perhaps this is able to improve sensitivity of fault detection and make the process of fault detection easier.

Signature		FFT	DWT	WPT	Entropy	Normalized Energy
$ \Psi_Z(t) $	Good	-7.332 dB (0.4299)	0.6	0.6	-18165.81	0.226357
	Faulty	3.380 dB (1.4757)	2.5	2.5	-164836.5	2.58370
	$\Delta\%$	10.712 dB (343%)	416%	416%	907%	1141%
$ \Psi_I(t) $	Good	-55.435 dB (1.69×10^{-3})	2.5×10^{-3}	2.5×10^{-3}	0.362	3.48450×10^{-6}
	Faulty	-44.508 dB (6.15×10^{-3})	10.0×10^{-3}	10.0×10^{-3}	2.132	4.15255×10^{-5}
	$\Delta\%$	10.927 dB (364%)	400%	400%	589%	1191%
$I(t)$	Good	-50.951 dB (2.83×10^{-3})	-	3.0×10^{-3}	-611.94	9.16461×10^{-4}
	Faulty	-42.793 dB (7.25×10^{-3})	-	10.0×10^{-3}	-621.50	8.27353×10^{-4}
	$\Delta\%$	8.158 dB (256%)	-	333%	102%	90.2%

Table 7-2: Comparison of Rotor Fault Detection based on Various Analysis Techniques on Different Monitoring Signature

In conclusion, wavelet packet transform is applied on $|\Psi_Z(t)|$ for motor fault detection in time-frequency domains. The findings are encouraging where WPT on $|\Psi_Z(t)|$ gives higher ratio of amplitude change at motor fault characteristic frequencies. This illustrates that WPT on $|\Psi_Z(t)|$ has better sensitivity and signal to noise ratio compared to conventional spectrum analysis.

The process of motor fault diagnosis can be further improved by introducing some feature selection algorithm, which helps to identify the decomposed signal that contains motor fault's characteristic frequency. Moreover, some existing pattern recognition techniques such as artificial neural network and fuzzy logic can be utilized in order to achieve automated and reliable fault diagnosis.

CHAPTER 8

CONCLUSIONS & RECOMMENDATIONS

8.1 Conclusions

8.1.1 Transduction Matrix of Motor Driven System

In this research, the induction motor, power transmission system and mechanical loading in a motor driven system were thoroughly studied by using transduction matrix. The theoretical transduction matrix of a squirrel cage induction motor is obtained from its equivalent circuit, and compared with experimental transduction matrix computed from least squares approximation. The transduction matrix is a two by two matrix consisting of four frequency response functions between the electrical inputs and mechanical outputs. Because the induction motor is running at constant single frequency, the transduction functions can be simplified into four complex numbers at that frequency. According to both theoretical and experimental results, the transduction matrix varies with loading condition, where each transduction matrix represents the system properties which relates the input and output at the corresponding loading condition.

Other properties of transduction matrix also investigated, such as the determinant of transduction matrix which is related to the energy transfer between input and output. The determinant of experimental transduction matrix is found to be slightly greater than unity, which implies that energy conversion from electrical input to mechanical output is not perfect due to nonlinearity and power lost in induction motor. In addition, the electrical impedance was evaluated in term of transduction functions along with mechanical impedance, and vice versa.

The transduction matrix model was applied to power transmission mechanism and mechanical loading, where gear train, four bar linkage and band brake were studied. In power transmission mechanism, its transduction matrix shared

some common properties, such as transduction functions t_{12} and t_{21} are zero, and t_{11} is the reciprocal of t_{22} . Because of these properties, the determinant of the transduction matrix can be directly related with power transfer between input and output. Besides, the impedance changes from input to output, thus output impedance is able to match with loading impedance. The overall transduction matrix of motor driven system can be obtained from multiplying each cascaded matrix in sequence. By doing so, the condition of the motor driven system can be observed by studying its transduction functions. This also allows investigation on the power flow and impedance change in the motor driven system, once the transduction matrix of each sub-system is known.

8.1.2 Power Transmission Analysis

In order to illustrate power transmission by using transduction matrix, a simple motor driven system consists of motor, gear and mechanical loading is studied. When the motor driven system is applied with increasing loading, the mechanical impedance at loading increased as well and it will be reflected back to induction motor via transmission gears. The reflected impedance at induction motor determines how much current or power the motor required in order to drive the given loading. The transmission gears serve as a role to adjust the mechanical impedance between the input of loading and the output of induction motor, so that impedance matching is achieved and enables transmission of power towards the mechanical loading. Examples of motor driven system such as induction motor with band brake and power transmission system of an automobile are utilized to further explain power flow and impedance change in the motor driven system.

In short, modelling a motor driven system by transduction matrix allows the condition of the system and loading to be acknowledged. Any changes on the system condition will be reflected on its transduction functions. This leads to the study of fault detection on induction motor by using input impedance which comprises of the motor's transduction functions and loading impedance.

8.1.3 Fault Diagnosis by Electrical Input Impedance

Studying the transduction functions of an induction motor allows its condition to be acknowledged, but it requires huge computational process and information at both input and output which is not feasible in certain case. Therefore electrical input impedance that consists of transduction functions is utilized in condition monitoring of induction motor. Electrical input impedance is obtained from frequency response function between input voltage and current, studying frequency components of electrical input impedance allows characteristic frequencies of motor faults to be identified. In addition, voltage variation has been taken into consideration in impedance hence it leads to more accurate diagnosis results.

Experiments are done by introducing known faults to a three phase squirrel cage induction motor. Broken rotor bar, abnormal air-gap eccentricity and bearing fault are introduced and studied. For each of the faults, its corresponding characteristic fault frequencies are identified and monitored. For instance, rotor fault frequencies are observed at 50 ± 6 Hz at full load condition. For abnormal air-gap eccentricity and bearing wise, the fault characteristic frequencies are found located at higher frequency region, and not all of the harmonics of fault characteristic frequency can be observed.

Under varying loads applied to the induction motor, the fault characteristic frequencies change with loading. Few hertz of frequency shifts are detected while the loads changed from no load to full load. Moreover the magnitude of fault characteristic frequency changes with increasing loads. Thus in order to capture fault signatures clearly, >80% of full load condition is suggested to apply on the motor.

From the comparison between impedance and current spectrums, impedance spectrum has a few advantages:

- Impedance signature is capable of detect motor fault more sensitively. For instance, change of magnitude of fault characteristic frequencies in impedance spectrum is 1.5 times higher than that of current spectrum for rotor fault. Similarly, impedance has more significant change in magnitude at higher frequency region for abnormal eccentricity and bearing fault detection.
- Fault characteristic frequencies are more obvious and easier to be detected. Because the power supply harmonic is eliminated in impedance spectrum, fault characteristic frequency will not be concealed by supply harmonic frequency components. Peak search algorithm would be simpler and less computation loads.

Despite the advantages of impedance spectrum, there are some limitations such as low time resolution in the analysis. In order to further improve fault diagnosis by input impedance, time-frequency analysis such as wavelet transform is introduced. Before that, time record data of input impedance is obtained from analytic signal based on Hilbert transform. This is particularly useful for alternating signal where direct division between voltage and current signals is not applicable.

The frequency components of magnitude of impedance's analytic signal have been studied where a 50 Hz of frequency shift is observed. This leads to easier rotor faults detection as its characteristic frequency is found to be located at lower frequency region. In addition, main supply frequency is eliminated which enables easier observation on others characteristic frequencies. Consequently the magnitude of impedance's analytic signal is used in wavelet transform for time-frequency analysis.

Wavelet packet transform which has better time and frequency resolution is performed to analyze magnitude of impedance's analytic signal, and it is capable in detecting broken rotor bar, asymmetric air-gap and bearing faults. For instance, about 4 times of amplitude changes are observed for broken rotor

bar at the decomposed signal which contains rotor's characteristic frequency. High ratio of amplitude changes can be observed for asymmetric air-gap and bearing fault as well. In fact, from the comparison between various analysis techniques, WPT on magnitude of impedance's analytic signal is able to improve the signal to noise ratio and make the fault detection easier.

8.2 Recommendations

8.2.1 Transduction Matrix Modelling on Motor Driven System

From the studies carried out in this thesis, transduction matrix has been proved to be a useful modelling technique to describe the condition of a motor driven system, but there is room for improvements. For instance, the transduction matrix of induction motor can be improved by using more precise motor parameters. Furthermore, the modelling can be extended to other motor driven system such as motor driven system in air-conditioner in order to study its performance and efficiency. There are other possible applications of using transduction matrix, which could be worthwhile in future study:

- Motor quality inspection. A similar model of induction motor from a production line should have identical transduction matrix. Consequently those motors which do not meet the lowest margin of acceptable range for transduction parameters are rejected.
- Motor performance evaluation, since the power and efficiency of motor is related to transduction functions.
- Load monitoring. As mechanical impedance can be expressed in term of electrical impedance and transduction functions, information at mechanical port can be acquired without the use of torque meter and speed sensor.

According to the studies, the transduction matrix for induction motor changes with loading condition. Therefore, transduction matrices at different loading are required to describe induction motor with frequent load changing. To overcome the troublesome of transduction matrix identification at various

loading conditions, formulation of constitutive equations to obtain those transduction matrices are suggested. These equations can be obtained by further investigation on the theoretical formulation of transduction matrix.

8.2.2 Transduction Matrix Modelling on Energy Harvesting System

In piezoelectric energy harvesting system, the impedance and power flow in the system are one of the research topics of many researchers. As a properly designed energy harvesting system with the impedance of vibration source matches with the impedance of energy storage will optimize energy flow within the system. Therefore, transduction matrix which enables impedance and power analysis would be suitable in studying the performance of energy harvesting system. In addition, the transduction matrix modelling on piezoelectric actuator has already been successfully implemented in other research works.

8.2.3 Pattern Recognition for Signal Features

Impedance signature is proven to be capable in detecting various motor faults. However, there is a need for automated pattern recognition for identifying faulty motor signature. Artificial neural network and fuzzy logic are the most popular diagnosis tools in solving motor fault detection problem. Artificial neural network has learning ability to classify various operation conditions, whereas fuzzy logic is useful in transforming heuristic decisions into numerical values for complex machine computations.

Development of fault diagnosis programme could be one of the future works. Firstly, types of diagnosis tools have to be decided and the most suitable tools is identified particularly for diagnosis purpose in single motor or multiple motors. For example, combination of artificial neural network and fuzzy logic can be considered. Alternatively, statistical pattern recognition which utilizes historical data to diagnose motor faults can be considered as well.

On the other hand, a frequency search algorithm is essential in order to locate motor fault's characteristic frequencies. The frequency search algorithm has to be able to estimate motor slip at a given loading. Hence computes and locates the faulty frequencies according to the motor slip, and compared with reference data by diagnosis tools.

8.2.4 Fault Diagnosis on Motor Driven System

In this thesis, only the fault diagnosis on squirrel cage induction motor is focused. This analysis is carried out in case of the health condition of subsystems after the motor in a motor driven system is normal. Fault diagnosis on subsystems can be carried out if the fault's characteristic frequencies of the subsystems which reflected on electrical input of electrical machine are known. Hence the health condition of the subsystem can be acknowledged by monitoring the magnitude change of those fault's characteristic frequencies, provided the health condition of electric machine is normal. In the case of fault diagnosis on overall motor driven system, the use of pattern recognition techniques will be helpful in identifying the fault's characteristic frequencies of electric machine and its subsystems. Consequently, more studies are required to understand the faults of motor driven system and development of pattern recognition techniques are essential in identification of those faults.

8.2.5 Multi Motor Monitoring

In most of the industry or on a large motor driven system, normally it has many systems which operate simultaneously. As a result, there is a need to expand the motor monitoring technique for multi motor monitoring. In addition, electrical impedance can be used in monitoring the loading condition and power for each system. One of the concerns in multi motor monitoring is data transfer, since huge amount of data are required to be handled at once. Wireless network would be one good method in transferring data to computer for analysis and it avoids messy cable connections. Therefore the application of wireless network in motor monitoring would be studied.

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APPENDIX A

EQUIPMENT LISTS

i. Three Phase Squirrel Cage Induction Motor: *Figure A-1*

Model	: TECO, AEEDKB04R500FMX
Power	: 0.5 HP (0.37 kW)
Full Load Speed	: 1395 rpm
Supply Frequency	: 50 Hz
Circuit Connection	: Delta Connection
Voltage	: 400 V
Full Load Current	: 1.2 A
Number of Pole	: 4
% Efficiency	: 71.5 %
% Power Factor	: 70.5 %

ii. High Voltage Differential Probe (Tektronix P5205) & TEKPROBE Power Supply (Tektronix 1103): *Figure A-2*

Maximum Operating Voltage	: ± 1.3 kV (DC + peak AC)
Attenuation Range	: 50X, 500X
Rise Time	: 3.5 ns
Bandwidth Limit	: 5 MHz
AC Noise	: 50 Hz
Propagation Delay	: 17 ns

iii. Current Probe (Tektronix A6303) & Current Probe Amplifier (Tektronix AM 503B): *Figure A-3*

Maximum Operating Current	: 20 Amperes
Bandwidth	: DC to 50 MHz, -3 dB
Rise Time	: ≤ 7 ns
Gain Accuracy	: $\leq 3\%$
Signal Delay	: Approximately 30 ns

iv. Torque Detector (Ono Sokki SS 100) & Torque Converter (Ono Sokki TS-2600): *Figure A-4*

Torque Range	: 5 Nm / 1 kgf·m
Maximum Speed	: 8000 rpm
Time Constant	: 63 ms

v. Dynamic Signal Analyzer (Dynamic Signal Acquisition Module NI 4462): *Figure A-5*

Analog Input	: 4
Sampling Rate per Channel	: 204.8 kS/s
Dynamic Range	: 118 dB
Maximum Signal Bandwidth	: 92 kHz
Input Range	: ± 316 mV to 42.4 V

Figure A-1: Three Phase Squirrel Cage Induction Motor

Figure A-4: Torque Detector & Torque Converter

Figure A-2: High Voltage Differential Probe & Power Supply

Figure A-5: Dynamic Signal Analyzer

Figure A-3: Current Probe & Current Probe Amplifier

APPENDIX B

KINEMATIC AND FORCE ANALYSIS OF FOUR BAR LINKAGE

B.1 Position Analysis

In analyzing position and displacement of linkage, the most basic concept is identifying the position in term of reference coordinate system. For instance, if the position of linkage is defined upon the xyz coordinate system, the coordinate axes describe the direction along which measurements are to be made. Hence, the unit distance along any of the coordinate axes provides scalar quantity description of distance along that direction. Since the crank-rocker linkage in this analysis is planar, therefore xy coordinate system is used as reference.

Figure B-1: Crank-Rocker Linkage and xy Coordinate System

Position analysis for four bar linkage is very common and the solution can be found in many reference books, such as [10]. Basically the position of θ_3 and θ_4 in term of θ_2 can be obtained by solving Eq. (B.1) and Eq. (B.2).

$$\text{x-axis:} \quad r_2 \cos \theta_2 + r_3 \cos \theta_3 = r_1 \cos \theta_1 + r_4 \cos \theta_4 \quad (B.1)$$

$$\text{y-axis:} \quad r_2 \sin \theta_2 + r_3 \sin \theta_3 = r_1 \sin \theta_1 + r_4 \sin \theta_4 \quad (B.2)$$

According to the solution from position analysis, θ_3 and θ_4 are expressed as follow, where r_1 , r_2 , r_3 , and r_4 represent the length of respective links and θ_1 , θ_2 , θ_3 , and θ_4 describe the direction of each link.

$$\theta_4 = 2 \tan^{-1} \left(\frac{-B \pm \sqrt{B^2 - C^2 + A^2}}{C - A} \right) \quad (B.3)$$

$$\theta_3 = \tan^{-1} \left[\frac{r_1 \sin \theta_1 + r_4 \sin \theta_4 - r_2 \sin \theta_2}{r_1 \cos \theta_1 + r_4 \cos \theta_4 - r_2 \cos \theta_2} \right] \quad (B.4)$$

where, $A = 2r_1r_4 \cos \theta_1 - 2r_2r_4 \cos \theta_2$ (B.5)

$$B = 2r_1r_4 \sin \theta_1 - 2r_2r_4 \sin \theta_2 \quad (B.6)$$

$$C = r_1^2 + r_2^2 + r_4^2 - r_3^2 - 2r_1r_2 (\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2) \quad (B.7)$$

From the position analysis above, the position of links r_3 and r_4 can be found with respect to the driving link r_2 . *Figure B-2* shows the position of each link while the driving link is making one full revolution.

Figure B-2: Position Analysis of Crank-Rocker Linkage

Figure B-3: Crank-Rocker Linkage with its Motion Limits

From the results of position analysis above, θ_1 is zero since link r_1 is fixed in position and horizontal to x-axis. θ_2 is the angle for driving link hence it changes from 0° to 360° for one revolution. Meanwhile, θ_4 varies with respect to driving link, the position changes from 62.7° to 112.0° . These are the two motion limits of the output link. When r_4 reaches its minimum position of 62.7° , the position of driving link is at 26° . After the driving link further rotates until 224° , r_4 reaches its maximum position of 112.0° . The coupler r_3 also changing with position between 22.3° to 51.3° when r_2 reaches position from 72° to 283° . Once the position analysis of the crank-rocker is obtained, the following velocity and acceleration analysis become easier.

B.2 Velocity Analysis

The analytic form of velocity equation for crank-rocker can be developed by differentiating the position equation obtained previously. Noted that the ground link is fixed, hence r_1 and θ_1 are constant and will be eliminated in the differentiation.

$$x\text{-axis:} \quad r_2 \dot{\theta}_2 \sin \theta_2 + r_3 \dot{\theta}_3 \sin \theta_3 = r_4 \dot{\theta}_4 \sin \theta_4 \quad (B.8)$$

$$y\text{-axis:} \quad r_2 \dot{\theta}_2 \cos \theta_2 + r_3 \dot{\theta}_3 \cos \theta_3 = r_4 \dot{\theta}_4 \cos \theta_4 \quad (B.9)$$

In equations above, the position of each link is known from previous position analysis.

$\dot{\theta}_2$ is given since r_2 is the driving link and the remaining unknowns are $\dot{\theta}_3$ and $\dot{\theta}_4$.

The equations can be solved easily by elimination of variables or using matrix inverse.

The solutions for $\dot{\theta}_3$ and $\dot{\theta}_4$ are shown as follow:

$$\dot{\theta}_3 = \frac{r_2 (\cos \theta_4 \sin \theta_2 - \sin \theta_4 \cos \theta_2)}{r_3 (-\sin \theta_3 \cos \theta_4 + \cos \theta_3 \sin \theta_4)} \dot{\theta}_2 \quad (B.10)$$

$$\dot{\theta}_4 = \frac{r_2 (\cos \theta_3 \sin \theta_2 - \sin \theta_3 \cos \theta_2)}{r_4 (-\sin \theta_3 \cos \theta_4 + \cos \theta_3 \sin \theta_4)} \dot{\theta}_2 \quad (B.11)$$

In addition, $\dot{\theta}_4$ above can be expressed in term of transmission angle η and ϕ as shown in *Figure B-5*. The expressions of ϕ and η are shown below.

$$\text{Angle } \phi: \quad \phi = 180^\circ - \theta_2 + \theta_3 \quad (B.12)$$

$$\sin \phi = \sin(180^\circ - \theta_2 + \theta_3)$$

$$\sin \phi = -\sin(-\theta_2 + \theta_3)$$

$$\sin \phi = \sin \theta_2 \cos \theta_3 - \cos \theta_2 \sin \theta_3 \quad (B.13)$$

$$\text{Angle } \eta: \quad \eta = \theta_4 - \theta_3 \quad (B.14)$$

$$\sin \eta = \sin(\theta_4 - \theta_3)$$

$$\sin \eta = \sin \theta_4 \cos \theta_3 - \cos \theta_4 \sin \theta_3$$

$$\sin \eta = -\sin \theta_3 \cos \theta_4 + \cos \theta_3 \sin \theta_4 \quad (B.15)$$

Therefore *Eq. (B.11)* can be rewritten as:

$$\dot{\theta}_4 = \frac{r_2 \sin \phi}{r_4 \sin \eta} \dot{\theta}_2 \quad (B.16)$$

For example, the crank-rocker has a input angular velocity of 157.08 rad/s (1500 rpm), then the angular velocity of $\dot{\theta}_3$ and $\dot{\theta}_4$ can be computed from *Eq. (B.10)* and *Eq. (B.11)*. *Figure B-4* shows the velocity analysis of each link at the given input voltage.

Figure B-4: Velocity Analysis of Crank-Rocker Linkage under Constant Angular Velocity as Input

Figure B-5: Crank-Rocker Linkage with its Angular Velocity

Under input angular velocity of 157.08 rad/s, r_4 rotates with angular velocity ranging from 59.7 rad/s to -83.5 rad/s as input link undergoes one revolution. The velocity of r_4 becomes zero when it reaches at both motion limits, as it changes rotation direction. Although velocity of r_3 is not the main concern in this crank-rocker analysis, the velocity also varies from 33.2 rad/s to -55.6 rad/s

B.3 Acceleration Analysis

Acceleration analysis is carried in similar way as velocity analysis. The equations for acceleration can be developed by differentiating Eq. (B.8) and Eq. (B.9).

$$\begin{aligned} x\text{-axis:} \quad & r_2 \ddot{\theta}_2 \sin \theta_2 + r_2 \dot{\theta}_2^2 \cos \theta_2 + r_3 \ddot{\theta}_3 \sin \theta_3 + r_3 \dot{\theta}_3^2 \cos \theta_3 \\ & = r_4 \ddot{\theta}_4 \sin \theta_4 + r_4 \dot{\theta}_4^2 \cos \theta_4 \end{aligned} \quad (B.17)$$

$$\begin{aligned} y\text{-axis:} \quad & r_2 \ddot{\theta}_2 \cos \theta_2 - r_2 \dot{\theta}_2^2 \sin \theta_2 + r_3 \ddot{\theta}_3 \cos \theta_3 - r_3 \dot{\theta}_3^2 \sin \theta_3 \\ & = r_4 \ddot{\theta}_4 \cos \theta_4 - r_4 \dot{\theta}_4^2 \sin \theta_4 \end{aligned} \quad (B.18)$$

The angular acceleration of $\ddot{\theta}_2$ is normally given as input data and the unknowns are $\ddot{\theta}_3$ and $\ddot{\theta}_4$ since all other parameters can be found by previous analysis. Since this crank-rocker is running at constant velocity as its input, therefore the input acceleration $\ddot{\theta}_2$ is zero. However, this does not lead to zero acceleration at other links. In fact the angular acceleration $\ddot{\theta}_3$ and $\ddot{\theta}_4$ are computed and it is presented in Figure B-6.

Figure B-6: Acceleration Analysis of Crank-Rocker Linkage under Zero Angular Acceleration

Figure B-7: Crank-Rocker Linkage with its Angular Acceleration

The result of acceleration analysis shows that, at zero input angular acceleration, both angular acceleration of $\ddot{\theta}_3$ and $\ddot{\theta}_4$ are changing with respect to position of input link. For the output link r_4 , its angular acceleration becomes zero when the velocity is reaching maximum or minimum.

In summary, kinematic analysis for the crank-rocker linkage is carried out. Among those analyses, the velocity equation that describes output velocity in term of input velocity in Eq. (B.16) will be used to construct the transduction matrix of corresponding four bar linkage.

B.4 Force Analysis

Figure B-8: Crank-Rocker Linkage and Applied Torque

Figure B-9: Free Body Diagram of Crank-Rocker Linkage

In free body diagram above, r_2 is the input link where input torque is acting on this link. Because of force equilibrium, the force is then transmitted to the coupler, r_3 and to the output link r_4 . Finally, the force transmitted to link r_4 will create the output torque. The equations for force and moment equilibrium for each link are shown in below. For force equilibrium, the forces acting on each link is normally equal in magnitude but opposite in direction. For moment equilibrium wise, the torque applied on the link is always equal and opposite to the cross product of $r \times F$.

At link r_2 :

$$\sum F = 0: \quad F_{12} = F_{32} \quad (B.19)$$

$$\sum M_A = 0: \quad T_2 = r_{AB} \times F_{32} = r_2 F_{32} \sin \phi \quad (B.20)$$

At link r_3 :

$$\sum F = 0: \quad F_{32} = F_{23} = F_{43} = F_{34} \quad (B.21)$$

$$\sum M_B = 0: \quad r_{BC} \times F_{43} = r_3 F_{43} \sin 180 = 0 \quad (B.22)$$

At link r_4 :

$$\sum F = 0: \quad F_{34} = F_{14} \quad (B.23)$$

$$\sum M_D = 0: \quad T_4 = r_{DC} \times F_{34} = r_4 F_{34} \sin \eta \quad (B.24)$$

From the force analysis above, the relationship between input torque and output torque can be expressed with combining Eq. (B.20), Eq. (B.21) and Eq. (B.24).

$$F_{32} = F_{34} \quad (B.25)$$

$$\frac{T_2}{r_2 \sin \phi} = \frac{T_4}{r_4 \sin \eta} \quad (B.26)$$

$$T_2 = \frac{r_2 \sin \phi}{r_4 \sin \eta} T_4 \quad (B.27)$$

In Eq. (B.27), T_2 and T_4 are the torque acting on input and output links respectively. Parameters r_2 and r_4 are the length of corresponding links. Angle ϕ is the angle between r_2 and F_{32} , which is also the angle between r_2 and r_3 . Angle between r_3 and r_4 is the transmission angle η , which plays significant role in design of optimal linkages. Because the mechanical advantage of the four bar linkage is directly proportional to $\sin \eta$. In general, the transmission angle is held within 30° to 150° . As the transmission angle approaches 0° or 180° , high loads, excessive wear and locking of joints are likely to occur.

Consequently, the analysis is continued on a crank-rocker example with constant input torque of 2 Nm acting on link r_2 . The result of output torque with respect to position of driving link is presented in *Figure B-10*.

Figure B-10: Output Torque of Crank-Rocker Linkage based on Input Torque of 2 Nm

According to figure above, the output torque at link r_4 is justified varying with respect to position of linkage. From previous position analysis, link r_4 is reaching its motion limits at 62.7° and 112.0° when the input link r_4 rotates to 26° and 224° . Whenever link r_4 is approaching its motion limits, the torque increases tremendously and approximates to infinity. This is because there is a change of torque direction every time link r_4 reaches the limits. Generally the output torque produced at link r_4 is much larger than the input torque. This mechanical advantage enables the linkage to drive heavier load that requires larger torque.

APPENDIX C

LABVIEW PROGRAMME

Figure C-1: LABVIEW Programme for Frequency Response Function (Front Panel)

Figure C-2: LABVIEW Programme for Frequency Response Function (Block Diagram)

Figure C-3: LABVIEW Programme for DWT and WPT with Limit Test (Front Panel) - 1

Figure C-4: LABVIEW Programme for DWT and WPT with Limit Test (Front Panel) - 2

Figure C-5: LABVIEW Programme for DWT and WPT with Limit Test (Block Diagram) - I

Figure B-6: LABVIEW Programme for DWT and WPT with Limit Test (Block Diagram) - 2