Energy absorption of thin-walled empty and foam-filled tubes

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**ENERGY ABSORPTION OF THIN-WALLED EMPTY
AND FOAM-FILLED TUBES**

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SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

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ENERGY ABSORPTION OF THIN-WALLED EMPTY AND FOAM-FILLED TUBES

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To my family

ABSTRACT

Recently, more attention has been paid to the energy absorption capability of novel structures. As one of the most versatile components in impact or blast protection, thin-walled tubes have been specifically explored by many researchers. Cellular materials, such as metallic foams, are used as impact energy absorbers in crash and blast protection because of their unique mechanical behavior. With this promising new material, the present project aims to develop energy-absorbing devices incorporating both thin-walled tubes and metallic foams.

One of the methods for improving crashworthiness performance of the thin-walled tubes under axial crushing is to vary the cross-sectional shape with convex polygons. It is evident that severe deformation occurs near the corner of tubes and most of the number of corners in the cross-section affects the efficiency of energy absorption, to a large extent. As an alternative, it is also necessary to develop tubes with concave polygon sections by introducing the extra non-convex corners. Four types of geometries were studied experimentally and numerically, including hexagon, octagon, 12-sided and 16-sided star. The increase in the number of inward corners demonstrates a promising improvement in energy absorption, but to a certain extent. In addition, for a regular triangular tube, which is a polygon with an acute angle and odd number of sides, a new type of basic plastic collapse folding element is proposed based on the Finite Element Analysis (FEA) and cardboards. The theoretical collapse modes are validated against corresponding experiments.

Furthermore, in order to explore the deformation behavior of the metallic foams, quantitative information is extracted on the state of stresses during the axisymmetric

expansion of a cylindrical hole located at the center of an infinite block of metallic foam. A macroscopic phenomenological constitutive model of crushable foam is employed, considering the initial and subsequent yielding surfaces in the space of the effective stress and hydrostatic stress. Simulations are performed using finite element analysis and the results demonstrate a good agreement with the analytical solutions newly obtained. The evolutions of plastic zone during the expansion are discussed and a map is further obtained exhibiting the evolution of three deforming zones, i.e. elastic, plastic and densification of the foam.

Quasi-static and dynamic lateral crushing behavior of cylindrical tubes filled with metallic foams was also examined. A total of 30 foam-filled tubes with different geometric configurations were crushed under Instron machine for the quasi-static tests. In the dynamic tests, unlike the conventional impact method, the specimen was placed on the bottom platen of an Instron VHS8000 machine with a constant upwards velocity and then the tube was collided with the fixed upper rigid platen. Three distinctive collapse types were observed from both the quasi-static and dynamic experiments. Detailed information about the energy absorption characteristics was given, including the load-deflection curves, absorbed energy as well as specific energy absorption (*SEA*). Both the experiments and corresponding FEA demonstrated that foam-filled tubes with thicker metallic foam cores are more energy efficient for both quasi-static and dynamic loading cases.

Keywords: Thin-walled tube; Foam-filled tubes; Axial and lateral crushing;
Deformation mode; Energy absorption

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LIST OF SYMBOLS

Chapter 2

α	angle between sides of the collapse tube and tube axis
b	radius of toroidal shell
β	angle between travelling hinge and tube axis
c	length of side of a rectangular tube, or called as flat width
C_{avg}	strengthening coefficient of foam filling in Eq. (2.20)
δ	crushing displacement
E_I, E_{II}	total energy dissipation of type I and II basic folding element, respectively
E_η	energy dissipation efficiency
$E_p, H(\varepsilon_{pl})$	hardening modulus the tube and foam material
$\varepsilon_a, \varepsilon_D, \dot{\varepsilon}$	plastic strain, densification strain and strain rate of the aluminium foam
$\varepsilon_{pl}, \varepsilon_{eng}$	plastic strain
$2\psi_0$	initial angle between two adjacent plates as viewed along the tube axis
γ	angle between sides of the collapse tube and travelling hinge
H	half-length of the fold
I_1, I_3	integrals in Eqns (2.4) and (2.5)
$M_0 = \frac{1}{4} \sigma_0 t^2$	fully plastic bending moment of tube wall per unit length
P_0, P_{0s}, P_{0h}	initial collapse load for single tube, square and hexagonally packed arrays
P, F_{av}	average crush force
P_{ae}, P_{af}	average crushing load of empty tube and foam-filled tube
R, D, t	radius, diameter and thickness of the empty tube, respectively
ρ^*, ρ, ρ_s	relative density of the foam, density of foam and solid material of the cell
σ_y, σ_0	yield stress and flow stress of the tube material

$\sigma_{pl}, \sigma_{ys}, \sigma_D$	plateau strength of foam core, yield stress of solid metal of which the foam is made, and dynamic crushing stress of foam, respectively
σ_y^0	initial yield stress and yield stress of foam under uniaxial compression
σ_m, σ_e	mean stress (or hydrostatic stress) and effective stress or von Mises stress
v	impact velocity

Chapter 3

α	angle between sides of the collapse tube and tube axis
A	cross-sectional area
b	radius of toroidal shell (approximately taken as $2.0 \sim 2.1t$)
β	angle between travelling hinge and tube axis
c	length of side of a polygonal tube, or called as flat width
d	flat width of a square tube
D	diameter of the equivalent circular tube
δ, l	crushing displacement and stroke
E, ν	Young's modulus and Poisson's ratio of material
E_1, E_2, E_3	internal energy absorption in Eqns. (3.19), (3.21) and (3.22), respectively
$E_I, E_{II}, E_{III}, E_{III}'$	total energy dissipation of type I, II and III basic folding element, respectively
F_{av}	average crush force
F_{max}	peak force
ψ	energy-absorbing effectiveness factor
$2\psi_0$	initial angle between two adjacent plates as viewed along the tube axis
γ	angle between sides of the collapse tube and travelling hinge
γ_1	angle of γ in type III folding element
γ_{min}	final inclination value of angle γ in a complete collapse of a single

	fold
$\gamma_{1\min}$	final inclination value of angle γ_1 in a complete collapse of a single fold
$2H$	fold-length, i.e. initial distance between plastic hinges at top and bottom of a basic folding element
$I_1, I_3, \dot{I}_1, \dot{I}_3$	integrals in Eqns (2.4) and (3.25)
η	structural effectiveness
L	initial length of a tube
L_1	length of travelling hinges
λ	c/H
M_0	fully plastic bending moment per unit length ($M_0 = \sigma_0 t^2 / 4$)
ρ	density of material
S	outer perimeter or section length
SEA	specific energy absorption
$\sigma_y, \sigma_u, \sigma_0$	yield stress, ultimate stress and flow stress of tube material, respectively
t	tube wall thickness

Chapter 4

a	initial radius of the hole in the infinite block of foam
α	aspect ratio of the yield surface, or the shape factor of the ellipse
b	outer radius of the foam block
β	shape of the flow potential ellipse in the effective-hydrostatic stress space
$d\lambda$	scalar rate variable
E, E_1, E_D	Young's modulus, linear hardening modulus and densification modulus of material, respectively.
ε_D	locking strain or densification strain
ε_z	strain in the axial direction
$\varepsilon_{pl}, \varepsilon_{eng}$	plastic strain and equivalent strain
g	flow potential

$H(\varepsilon_{pl})$	hardening modulus
k	compression yield stress ratio
p	internal pressure applied on the boundary of the hole
p_d	internal pressure at the locking position
p_e	maximum pressure before initial yielding
p_y^0	initial yield stress of foam under hydrostatic compression
r	radius for a generic layer
r_y	radius of plastic zone
r_y^0	radius of the first region
σ_y	absolute value of yield stress under uniaxial compression
σ_y^0	initial yield stress of foam under uniaxial compression
σ_m, σ_e	mean stress (or hydrostatic stress) and effective stress or von Mises stress
$\sigma_r, \sigma_\theta, \sigma_z$	radial, circumferential and axial stress, respectively
ν, ν_p	elastic and plastic Poisson's ratio
w	axial displacement on both the top and bottom surfaces
W	width of plastic zone

Chapter 5

δ	crush displacement
D_o, D_i	outer diameter of outer and inner tube
E, E_f	Young's modulus of tube and foam material, respectively
F	crush strength
L	length of specimens
m	mass of the specimens
ρ^*	relative density of foam
σ_y	yield stress of aluminium tube material
t_o, t_i	thickness of outer and inner tube

Chapter 6

A_0	initial cross-section area
C_p^0, C_p	longitudinal and flexural plastic stress wave velocity, respectively
Δt	time interval between the peak values in Fig. 6.12b
D, t	outer diameter and thickness of empty tube
D_o, D_i	outer diameter of outer and inner tube in foam-filled double tubes
δ	crush displacement
E_D, E_S	energy absorbed under dynamic and quasi-static crushing
E_p	strain-hardening modulus of the tube material
$E_{Proximal}, E_{Distal}$	internal energy of quadrants at the proximal and distal end
$\dot{\epsilon}$	strain rate of the aluminium foam
F	crush strength
I, K	moment of inertia of the beam cross section with respect to the neutral axis, in Eq. (6.5) and curvature, respectively
L	length of specimens
Λ	flexural wave length
m	mass of the specimens
m_p	fully plastic bending moment per unit width ($m_p = \sigma_y t^2 / 4$)
R_0, R_1, R_2	radius of curvature of the stripe BC, AB, CD in Fig. 6.14
ρ^*	relative density of foam
ρ_s	density of tube
S	outer perimeter or section length
$s = S / \pi D$	non-dimensional circumferential position
σ_y	yield stress of AA6061-T5 material
σ_{pl}, σ_{ys}	plateau strength of foam core and yield stress of solid metal of which the foam is made, respectively
t_o, t_i	thickness of outer and inner tube
v	compressive velocity
v_{cr}	critical compressive velocity

CHAPTER 1

INTRODUCTION

With increasing demand for energy and concern for environment, more research attention has been paid to developing and assessing light-weight materials and structures [1]. Thin-walled tubes and metallic foams are being broadly used as energy absorbers or cladding structures in many fields, which are vital to the protective structures. Thus, taking advantages of both components, tubular structures filled with metallic foams have attracted much attention in automotive and aerospace fields due to their higher strength-to-weight ratio, better impact resistance and associated energy absorption capacity [1-3].

1.1 Energy-absorbing Devices

Energy-absorbing devices or energy absorbers have been widely used to dissipate impact energy in many fields, such as civil, mechanical, marine and aeronautical engineering [1]. As shown in Fig. 1.1, they are usually attached on the outer side of the non-sacrificial main structures. The structural design of impact crashworthiness is a complex issue. Unlike traditional structures which undergo linear elastic deformation, they have to sustain intense impact or even blast loads. The geometrical nonlinearity, strain-hardening effect, strain-rate effect and various interactions effects are involved in the corresponding plastic deformation and failure history [1, 4]. Thus, a permanent deformation of the structure may be undesirable for repeatable and economical use under such circumstances. It is therefore possible to design a sacrificial energy-absorbing device subjected to direct loading that may undergo large plastic deformation and hence absorb impact energy.

Fig. 1.1. Blast resistant structures design.

1.2 Background and Motivations

In recent years, plenty of research interests have been directed to the protective structures against impact and blast loadings due to increased chance of terrorist attacks and accidents. Severe structural damage may be caused by the high pressure and loading rate produced by dynamic explosions, such as structural failure, progressive collapse and associated ballistic penetration [5], as shown in Fig. 1.2.

Fig. 1.2. Typical structural damage caused by explosions [5].
(Photo reproduced courtesy of SAFE Special Services Group)

In an attempt to protect the vital resources, equipment and people from those disasters, protective constructions as energy absorbers are involved. The plastic response and the energy-absorbing performance of structures, particularly under dynamic loadings, are of current interest in the research community. As typical energy absorbers, thin-walled tubes are usually crushed plastically under several loading conditions, such as axial crushing, lateral crushing, axial inversion and folding [4, 6]. The collapse modes and energy absorptions on axial crushing of thin-walled circular tubes in such systems have been developed by Ezra and Fay [7], and Yu [8]. Other shapes of regular or convex polygonal tubes subjected to axial crushing have been investigated [9-12]. However, little attention has been paid to the concave or star-shaped tubes. They may have potential in better energy absorption capacity due to the inwards and outwards folding around the corners. Thus, further research is needed to identify such superiority in the star-shaped design under axial loading. Furthermore, since triangular tube is the only type of regular polygon with acute angles and odd number of sides, its collapse mode needs to be investigated.

Following the above-mentioned work in which successful results were reported in the axial direction, lateral crushing takes place frequently in thin-walled structures. It will be useful to maximize the energy absorption, since structures under lateral crushing deform with a large volume of material reaching plasticity during collapse [4, 6, 13-16]. Therefore, thin-walled circular tubular systems are widely applicable when subjected to lateral crushing. Typically, the problem of crushing of those structures involves plastic deformation, which may be idealized as plastic hinges to model the lateral collapse of tubes. Because of the strain localization around the

plastic hinges, it may not be structurally efficient to dissipate energy under this condition. Therefore, to further improve the energy dissipating performance under lateral crushing, composite structure with other components sandwiched could be an alternative way, due to the additional interaction effects.

As typical core materials in the composite structures, cellular materials such as metallic foams have recently emerged as a novel material in energy-absorbing devices owing to their high specific stiffness and specific strength compared with traditional metals such as steels [17, 18]. So far, much attention has been paid by researchers to their mechanical properties and design [19-21]. However, since the constitutive relationship for metal foams is quite different from that of traditional metals [22], the plastic material model [23, 24] and plastic behavior of metallic foams under large deformation such as internal pressure still require further understanding.

Since the significant energy absorption capacity in the foam core material, foam-filled structures are now widely being considered for crashworthy structures. Researchers have studied the axial crushing behavior of foam-filled tubes [2, 4], which have the combined advantages of both the thin-walled tube and metallic foams. It is found that by combining both, the energy absorption capacity is improved under axial crushing, though not necessarily weight-effective [25, 26]. Nevertheless, work on foam-filled tube has proven that the foam filling advantage still exists under the quasi-static lateral loading condition [3]. It is also evident that the interaction effects between tube and foam core are significant in the load-carrying capacity [2, 3]. Hence, structures made up of double tubes with relatively

flexible metallic foam sandwiched are studied herein to explore the energy absorption under lateral crushing, due to the presence of the inner tube, which is in general more mass efficient than the outer one.

Despite the wide engineering applications of foam-filled structures [1, 27, 28], the behavior of foam-filled tubes under quasi-static and dynamic lateral crushing has not received much attention and is less reported in the literature. Previous work has focused on the axial and bending responses of foam-filled tubes [29-31]. However, an equally important consideration for plastic collapse is a thorough understanding of the structural response of foam-filled tubes when subjected to lateral crushing. Recent research on foam-filled structures subjected to impact or blast loading has underscored and enhanced their potential in a wider range of crashworthiness applications than ever before [32-34]. Compared with quasi-static loading case where relatively low velocity is involved, both the strain-rate and inertia effects are significant under dynamic test conditions [35, 36].

The motivation of current research is to design novel collapsing structures made of metallic foams and thin-walled structures which are subjected to collision and impact. First, systematic investigation on the crushing behavior of star tubes and new type of deformation patterns of triangular tubes would help us to have a comprehensive understanding of the deformation mechanism of axial crushing on thin-walled polygonal tubes with different cross-sectional shapes, whether they are convex or concave tubes. Second, the study on axisymmetric expansion of a cylindrical hole in a block of closed-cell metallic foam of infinite size will provide us a deeper understanding of the behavior of metallic foam in sandwiched

protective structures. Furthermore, systematic investigation into the lateral crushing response of foam-filled tubes may not only help obtain a deeper insight into the deformation mechanism of these structures, but may also offer them with significantly enhanced energy absorption and impact resistance performance in various configurations.

1.3 Research Objectives

The objective of the present thesis is, therefore, dedicated to explore the plastic structural response of foam-filled structures when subjected to quasi-static and dynamic lateral crushing, as well as to obtain a deep understanding of deformation mechanism of each component. For that purpose, efforts were made to better understand the deformation mechanism and energy absorption characteristics. Various double cylindrical tubes sandwiched with metallic foams were subjected to different load cases: quasi-static and impact loadings. A large number of quasi-static and dynamic experiments were carried out. Following the experimental work, a series of numerical studies were developed to verify and predict energy absorption characteristics with the help of increased computational power of modern computers. The purpose of this project can be briefly outlined, as follows,

1. To investigate the deformation modes of several structures made of thin-walled empty tubes with different cross-sectional shapes, including convex and concave shapes, as well as triangular tube with odd number of sides.
2. To better understand the mechanical behavior and material properties of metallic foam cores.
3. To investigate the performance of foam-filled double tubes with metallic foam core under quasi-static and impact loadings.

4. To determine an optimum configuration of thin-walled polygonal tube and foam-filled structures suitable for a given loading.

1.4 Scope of Research

The main research focus of this thesis is devoted almost exclusively to the plastic behavior of thin-walled empty and foam-filled tubes. First of all, quasi-static axial crushing experiments were conducted on thin-walled polygonal specimens, which are convex (hexagon and octagon) and concave (12-sided star and 16-sided star shaped) mild steel tube. The test results are further compared with those from the corresponding Finite Element Analysis (FEA) using ABAQUS. A further focus of the analytical work is to develop folding mechanism in a new type of basic plastic collapse folding element in the equilateral triangular tubes. Specifically, the theoretical collapse modes are based on the Super Folding Element theory and the idealised material models are assumed to a *rigid, perfectly plastic* model [1].

Additionally, a case concerning with metallic foam is being studied, where a block of closed-cell foam with a cylindrical hole in the center is subjected to internal pressure. Quasi-static responses on metallic foam are examined by numerical simulations and preliminary analytical treatment. A constitutive model of metal foam is introduced, considering the initial and subsequent yielding surfaces in the space of the effective stress and hydrostatic stress. The isotropic hardening model is incorporated into the material property of crushable foam. Quantitative information is revealed on the deformation status and evolution of plastic zone during expansion. Nevertheless, current analysis results are the preliminary studies for the foam-filled tubes subjected to internal pressure.

Furthermore, a series of quasi-static and dynamic experiments were performed to investigate the lateral crushing response of composite structure made up of two monolithic thin-walled aluminium alloy tubes and aluminium foam core. Lateral impact tests of short aluminium alloy empty tubes and double tubes with an annular region filled with aluminium foam were conducted via an Instron VHS8800 machine. Apart from the conventional impact method and to avoid the elastic bouncing of the tube, the specimen was placed on the bottom platen of the machine, which then gained a constant upwards velocity, followed by the impact between the tube and the upper rigid platen. Experiments were conducted with initial impact velocities up to 10 m/s. The structural response beyond the experimental range for impact velocities up to 100 m/s is numerically implemented by FEA, in which the strain rate effect of metallic foam is considered.

In this study, any material failure, imperfections and crack criteria are not incorporated throughout the entire analytical analysis and FEA.

1.5 Outline of the Thesis

The thesis consists of seven chapters and a reference list. The remainder of this thesis is organized as follows.

Chapter 2 overviews the literature related to the research fields in energy absorption characteristic of tubular systems filled with metallic foam. It summarizes up-to-date research about the structural responses and energy absorption characteristics on thin-walled tubes subjected to both the axial and lateral crushing.

It highlights the mechanical properties and the advantages of metallic foam. It also covers the currently available methodologies of composite tubular structural responses under quasi-static and dynamic loadings. **Chapter 3** gives detailed investigations on the quasi-static axial crushing performances of thin-walled tube with different cross-sections, including convex/regular polygons (hexagon and octagon) and concave (12-sided and 16-sided star) shapes. Some efforts towards developing deformation modes and collapse mechanism of regular triangular tubes with only acute angles are also made in this chapter. **Chapter 4** studies the behavior of expanding a hole in isotropic metal foam. It includes some observations from FEA and analytical results on the evolution of plastic zone during the expansion. The quasi-static responses of foam-filled double tubes subjected to lateral crushing by means of experiments and FEA are described in **Chapter 5**. The optimum configurations of foam-filled structures under quasi-static loading are determined. **Chapter 6** presents the dynamic responses of empty and foam-filled double tubes under dynamic lateral crushing, including experimental and numerical discussions about the energy absorption characteristics for both the empty tubes and foam-filled tubes. The effect of velocity on the deformation mechanism is studied as well. **Chapter 7** concludes the present thesis, summarizes major results and suggests future research topics.

CHAPTER 2

LITERATURE REVIEW

In this chapter an overview is presented about the experimental, theoretical and numerical studies on the energy absorption characteristics of tubular energy absorbers subjected to quasi-static and dynamic loadings. Typical energy absorbers made of single empty tube or double tubes with metallic foam cores sandwiched are highlighted.

2.1 Thin-Walled Empty Tubes

As one of the most common energy absorption devices, thin-walled metallic tubes are more versatile and efficient compared to other conduit materials [4]. The application of thin-walled tubes in energy absorption systems has attracted a considerable amount of attention [7, 8]. Generally, tubular structures are crushed plastically under several loading conditions, such as axial crushing, lateral crushing, internal expansion, axial inversion and bending [4, 6]. Since axial and lateral compressions are essential and widely utilized in engineering, current studies are focused on these two representative loading conditions.

2.1.1 Axial Crushing of Thin-Walled Tubes

Axial compression of tubular systems could support large load, and the deformation modes are relatively easy to distinguish [1]. A large number of experiments, numerical simulations and analytical models are reviewed in the following sections for thin-walled tubes with different cross-section shapes subjected to axial crushing.

2.1.1.1 Circular Tubes

Over the past decades, studies have been concentrated on the mechanical behavior and deformation modes of circular extrusions subjected to the axial crushing since they offer better energy absorption capability [9, 37-42] and are convenient to fabricate. As illustrated in Fig. 2.1, during the axial crushing of a thin-walled tube, it collapses either axisymmetrically or non-symmetrically, or a mixed mode of both, depending primarily on the diameter-to-thickness ratio (D/t) [37, 40, 43, 44]. The axisymmetrical mode in Fig. 2.1a is also called the ring mode [42], concertina mode [37] or extensible mode [43], while the non-symmetrical mode in Fig. 2.1b is referred to as the diamond mode [37] or inextensional mode [43]. The mixed mode occurs at certain value of D/t , which is a combination of both the axisymmetrical and non-symmetrical modes, see Fig. 2.1c.

Fig. 2.1. Examples of various axial collapse modes for thin-walled circular tubes: (a) axisymmetric mode; (b) non-symmetric mode; and (c) mixed mode [40].

The classifications of deformation modes have been given by Guillow et al. [40] and Andrew et al. [45] based on large number of experiments. For those deformation modes, the corresponding load-deformation curves differ a lot due to the distinctive plastic deformation mechanisms in the formation of successive lobes. Recently, Tabiei and Nilakantan [46] studied the concertina mode of deformation under axial crushing, which is an axisymmetric local buckling mode and usually

occurred in tubes that have a diameter to thickness ratio (D/t) less than around 80–90. The results confirmed that the concertina (symmetric) mode of deformation results in a higher specific energy absorption (SEA) than the diamond (non-symmetric) mode.

To further understand the progressive collapse mechanism in the symmetric collapse mode, the efforts towards the analytical solution to the axial crushing of circular tubes has been made by Abramowicz and Jones [41, 44]. A theoretical model was first proposed by Alexander [42] and further refined by considering the effective crush length to predict the average crushing force and energy absorption of cylindrical tubes subjected to axial loading [41, 47-49]. As shown in Fig. 2.2, the average crushing force F_{av} and the half-length of the fold H is simply given by

$$\begin{cases} H = 0.95\sqrt{Dt} \\ F_{av} = 6\sigma_0 t^2 \sqrt{D/t} \end{cases} \quad (2.1)$$

where D and t are the diameter and the thickness of the tube, respectively, and σ_0 is the flow stress in a rigid, perfectly plastic model. Theoretical models for the non-symmetric modes are less successful than those for the symmetric ones.

Fig. 2.2. A simple theoretical model for axisymmetric collapse [42].

Furthermore, concentrating on the quasi-static axial compression of circular tubes, an empirical formula between the average force and geometrical parameters for particular material was established by Guillo et al. [40], expressed as

$$F_{av}/M_0 = 72.3(D/t)^{0.32} \quad (2.2)$$

where $M_0 = \sigma_0 t^2 / 4$ is the fully plastic bending moment of tube wall per unit length. It is applicable for axisymmetric, non-symmetric and mixed modes of collapse. However, for circular tubes, a rigorous theoretical explanation of the functional dependence of the relationship for all modes of collapse, as observed in their experiments, needs to be developed.

2.1.1.2 Polygonal Tubes

Previous studies on the axial crushing of thin-walled circular tubes have been reported [9, 37-40]. Other geometrical shapes of thin-walled extrusion have been extensively studied [9-12], especially on the polygonal shapes. However, the cross-sectional shapes studied were restricted to square [41, 44, 50], hexagon [51], octagon [43, 52] or multi-corner convex column [53, 54]. Although the collapse modes of square tubes are quite different from those in circular tubes [41], the general characteristics of F - δ curves are similar [32] due to progressive collapse. In order to further understand the axial crushing behavior of square columns, a theory named Super Folding Element theory was proposed by Abramowicz [41, 44]. By adopting a rigid-plastic material model and condition of kinematic continuity on the boundaries between rigid and deformable zones, the collapse modes of column consist of two basic collapse elements, i.e. inextensible type I and extensible type II basic folding elements, as illustrated in Fig. 2.3. In type I, the hinge line needs to propagate during crushing to ensure a kinematically admissible model, while it is

stationary in type II mode. Wall deformation occurs by bending along the straight hinge lines only in inextensible Type I mode, but both bending and stretching take place during the collapse in extensible Type II mode. In theoretical analysis, inextensible Type I mode consists of four trapezoidal elements, a section of two horizontal cylindrical surfaces, two travelling conical surface and a toroidal surface [55], while extensional mode is simple and composed of four rectangular plates [41].

Fig. 2.3. Basic collapse elements: (a) inextensible Type I; and (b) extensible Type II [44, 56].

As shown in Fig. 2.3a, for a square tube with inextensible collapse mode, the energy absorption of type I basic folding element [55] is

$$E_1 = 16M_0 I_1(\psi_0) Hb/t + 2M_0 \pi c + 4M_0 I_3(\psi_0) H^2/b \quad (2.3)$$

in which the first term represents the stretching energy in a sheet passing over a toroidal shell, the second term represents the bending energy in the fixed horizontal hinges, and the third term is the bending energy dissipated in the travelling hinges, respectively. b denotes the radius of the toroidal shell in the meridian direction. $2H$ is the fold length. c and t are the side length and thickness. $2\psi_0$ is the initial angle between two adjacent plates as viewed along tube's axial direction. Particularly,

$$\begin{cases} I_1(\psi_0) = \frac{\pi}{(\pi - 2\psi_0) \tan \psi_0} \int_0^{\pi/2} \cos \alpha \left[\cos \psi_0 - \cos \left(\psi_0 + \frac{\pi - 2\psi_0}{\pi} \beta \right) \right] d\alpha \\ I_3(\psi_0) = \frac{1}{\tan \psi_0} \int_0^{\pi/2} \frac{\cos \alpha}{\sin \gamma} d\alpha \end{cases} \quad (2.4)$$

where α is the angle of rotation of the side panels, β is the angle between travelling hinge and tube axis and γ is the angle between sides of the collapse tube and travelling hinge, respectively. The relationship among these angles is given by

$$\begin{cases} \tan \gamma = \tan \psi_0 / \sin \alpha \\ \tan \beta = \tan \alpha / \sin \psi_0 \end{cases} \quad (2.5)$$

Similarly, for a square tube with extensible collapse mode, the energy dissipation of type II basic folding element [41, 44] is, as shown in Fig. 2.3b, with an improved kinematic mechanism

$$E_{II} = 8M_0 H^2 \psi_0 / t + 2M_0 \pi c + 2M_0 H (\pi - 2\psi_0) \quad (2.6)$$

The first term in the above equation is the stretching energy dissipated during the extension, the second term is the bending energy along the stationary hinge lines, and the last term denotes the energy dissipated in rebending of conical surface to nearly circle for two adjacent plates, respectively. The crushing load could be predicted [41, 44, 50] from the dissipated plastic work. The Super Folding Element theory is also applicable for the non-convex multi-corner thin-walled column which demonstrates to have higher energy absorption capacity [57].

Several comparisons have already been made by Yamashita et al and Mamalis et al. [9, 52] among the thin-walled tubes with various convex polygonal cross-sections. The crush strength was found to increase as the number of side of the convex polygon increases up to 11 [9]. However, less work has been performed on concave polygons or star shapes, which might have potential in higher energy absorption capabilities due to their inwards and outwards folding during the plastic crushing processes. Thus, it is necessary to conduct comparative experiments between the convex and concave polygonal shaped tubes.

For the preparation of polygonal shape specimens, several fabrication methods were studied to minimize the material failure issue. The traditional one was by welding [52]. The thin-walled octagonal tube was formed by bending the low carbon steel plate into two halves. The two ends were then welded together in the middle of the two opposite faces of the octagonal shape. It is found that stress concentration would be likely to occur at the non-uniform welding seam to cause crack or material failure. To avoid this, the other method, wire Electrical Discharge Machining (EDM) technique is developed [58]. It gives high geometrical flexibility in cutting into tubes of various cross-sections by using metallic wire with a pre-programmed contour. Since the cutting mechanism of wire-cut EDM is applying the wire to cut through the work piece, the work piece must be predrilled with a hole in order to create the internal profile for a certain cross-sectional shape.

2.1.1.3 Tubes with Odd Number of Sides

Most of the research interests have been focused on the tubes with even number of sides. Meanwhile, little attention has been paid to polygonal tubes with odd number of sides. A typical example is the equilateral triangular tube, which is widely used in such structures as bridges, cranes and steel building, etc. [59]. Early analytical elastic buckling modes of triangular tubes were addressed by Merchant and Chattopadhyay et al. [60, 61] and modeled in FE analysis. The plastic mechanism of triangular tubes under pure bending was also investigated by Kotelko and Marian [62]. Because of the particular geometrical shape, it is important to explore deeper about the collapse modes of triangular tubes under large axial plastic deformation.

The theoretical solutions in previous Section 2.1.1.2 are applicable to the polygonal

tubes with the even number of sides, where all the walls reach instability at the same load in the lowest energy mode. However, when it comes to the triangular tubes, each side would be affected by the other two adjacent sides in the close tube and the alternative deformation mode needs to be developed. The average crushing force and associated energy absorption characteristics still remain unknown, which attract our interests. Recent investigations confirmed that the triangular tubes with the lowest number of edges have the lowest energy absorption capacity [12, 63, 64], but their collapse modes are quite different from those fully resolved inward–outward–inward–outward folding lobes in square or hexagonal sections. Due to an inherent incompatibility of folding mode of adjacent corner elements, previous deformation modes cannot be formed as there is always an inward–inward or outward–outward element in a triangular folding lobe [64], as shown in Fig. 2.4. Mixed deformation mode in a triangular tube with a trigger [12] is shown in Fig. 2.5. In the design of regular triangular structures, it is very important to determine the structure collapse modes as well as the load-carrying capacity.

Fig. 2.4. Triangular tube before (left), during (middle) and after (right) crushing [64].

Fig. 2.5. Deformation mode of a triangular tube (with an inward trigger). The solid-line box and dash-line box denote inextensible mode and extensible mode [12].

2.1.2 Lateral Crushing of Thin-Walled Tubes

Research on lateral crushing of polygonal tubes confirms that among various cross-sectional polygonal tubes, circular tube is the one with the best energy absorption capacity [31, 65, 66]. Meanwhile, the star-shaped specimens subjected to lateral compression are much more vulnerable to the fracture along the longitudinal lines, as clearly illustrated in Fig. 2.6. Generally, single circular tubes have better energy absorbing performance than single square tubes [67, 68]. Therefore, in the following sections concerning lateral crushing, only circular tubes are examined.

Fig. 2.6. Lateral compression of tubes with (a) star shape [65]; and (b) square [67].

The lateral crushing behavior of circular tubes has been well investigated by many researchers [4, 6, 13-16]. The methodologies on single tubes are well established by Reid and Reddy et al. [4, 16, 69, 70]. The mechanical characteristics on the lateral compression of tubes between rigid platens were studied by them for the first time and a simple theory was developed to obtain the material properties. The initial collapse load for single tube (radius R) under two-point loads is determined by

$$P_0 = 4M_0(1 + \sqrt{2})/R \quad (2.7)$$

Furthermore, based on the elastic/perfectly plastic material assumption, the lateral crushing load P of a tube under two flat plates at a deflection δ is given [4, 6]

$$P = \frac{P_0}{\sqrt{1 - \left(\frac{\delta}{2R}\right)^2}} \quad (2.8)$$

Considering the strain-hardening effect, the above equation becomes [71]

$$\frac{P}{P_0} = \frac{1}{\sqrt{1 - \left(\frac{\delta}{2R}\right)^2}} \left[1 + \frac{E_p}{15\sigma_y} \sin^{-1} \left(\frac{\delta}{2R} \right) \right] \quad (2.9)$$

where E_p and σ_y are the hardening modulus and yield stress of the tube material, respectively. In order to maximize the energy absorption, tubes under lateral crushing deform with a large volume of material reaching plasticity during collapse. To introduce more plastic hinges, tubes with side constraints were then investigated by Reddy and Reddy [72], Shim and Stronge [73]. It is evident that to form a collapse mechanism, more plastic hinges in the side constraint tubes are involved than free ones. However, due to the strain-rate effect and inertia effect, there seems to be little study on the deformation mechanisms of single tubes subjected to dynamic loading. Detailed explanation based on mechanics is required to elaborate the importance of the inertia effect, which is of importance to the design of energy-absorbing structures.

Tubes can be arranged into various tubular systems in order to introduce more constraints as well as desirable energy-absorption performance [74, 75]. As the basic unit of the arrays, quasi-static lateral collapse behavior in individual tube has been evaluated. Consequently, typical ring-system structures (see Fig. 2.7) were studied by Reid and Reddy et al. [74, 76, 77] on the shock wave theory of dynamic responses to demonstrate higher energy absorption efficiency.

Fig. 2.7. Typical ring-systems under impact [74].

The arrays of 2D/3D tubular systems were verified as well. Research on the lateral compression of thin-walled circular tubes has been presented experimentally and theoretically by Shim and Stronge [78]. The square and hexagonally packed arrays were presented, which could be clearly seen from Fig. 2.8. For square packed arrays, asymmetrical collapse modes were observed. Theoretical solutions were developed afterwards. The initial collapse load for square packed arrays is

$$P_{0s} = 4 \frac{M_0}{R} (1 + \sqrt{2}) \approx 2.414 P_0 \quad (2.10)$$

For hexagonally packed arrays, each tube is subjected to six equal and evenly spaced forces. Considering the forces acting on a ring (Fig. 2.8c) and using the equivalent structure technique developed by Reddy et al. [70], the initial collapse load is

$$P_{0h} = \frac{4M_0}{R} (3 + 2\sqrt{3}) \approx 2.678 P_{0s} \quad (2.11)$$

In addition, compared with experiments, asymmetrical modes of tube deformation predominate because the asymmetrical modes are more compliant than the alternative symmetrical modes of deformation. Both results indicated that when the

tube is a component of a column in the tightly-packed tubes, the capabilities of energy absorptions are enhanced sharply.

Fig. 2.8. Schematics of lateral crushing in tightly packed arrays of thin-walled metal tubes: (a) square-packed; (b) hexagonally-packed; and (c) forces acting on a one quadrant tube within a hexagonally packed system [78].

For other stacking arrays, Shim et al. [79] again analyzed the elastic stress transmissions in a closely packed array of thin-walled rings. In their studies, hexagonally packed arrays were further developed into another two types: transverse close-packed and vertical close-packed arrangement, as shown in Fig. 2.9. The influence of the size of the connection segment between two-neighboring cells is studied by Karagiozova and Yu [80], which indicated that the shape varies significantly depending on the connection. Composite tubular structures filled with foam core have been observed to affect the mode of collapse [40], and the capabilities of such structure will be improved significantly either under quasi-static, or under dynamic loadings.

Fig. 2.9. Ring packing arrangements: (a) square-packed; (b) transverse closed-packed; and (c) vertical closed-packed arrangement [79].

2.1.3 Comparisons between Axial and Lateral Crushing

The requirements for impact energy absorbers have become a necessity through last few decades. It is well known that the thin-walled metal tubes are very important components in the energy absorbers devices. Most tubes are loaded either axially or laterally. It has been found by Abdewi et al. [65] that structure shape for axial crushing affects energy absorption significantly. However, such phenomena are not detected in the case of lateral crushing [65]. Thus, circular tubes are the research objectives concerning with the lateral crushing. Under both loading profiles in the circular tubes, the typical load-displacement curves are shown in Fig. 2.10. In the axial compression, the corresponding energy absorbing capacities are significantly higher [2, 40, 46], as illustrated in Fig. 2.10a. Nevertheless, the axial buckling and fracture modes give rise to more fluctuations in loading and large amplitude of peak force [2]. In the design of crashworthiness structures for energy absorption, the peak load is a major consideration. An extremely high peak stress will cause damage to goods in their packaging, or severe injury to passengers in vehicles, for example [1]. Energy absorbers for crush and blast protection are chosen so that the plateau stress is just below the stress that will cause damage to the protective object; the best choice is then the one which has the longest plateau, and therefore absorbs the most energy. The global failure (Euler buckling) is more likely to occur in the axial crushing as well.

On the contrary, tubes under lateral compression can overcome these disadvantages. As shown in Fig. 2.10b, the load-deformation curve is much smoother with higher efficiency because of the extra plateau stress [4, 16], which is more desirable for energy absorption. Meanwhile, the strain-rate effect [81] and strain-hardening rate

[82] should also be considered in the lateral dynamic crushing. Although the operating loads of laterally compressed tubes are far lower than those of axially compressed, energy absorption capability of lateral compression could be improved by encouraging tubes to deform in alternative modes with more plastic hinges [72]. All the experimental investigations indicated that the crushing processes of those tubes are maintained by forming plastic hinges [6, 16]. Plastic hinges are adopted to analyze the lateral collapse of those tubes with different collapse mechanisms. Hence structures deform with a large volume of material reaching plasticity, which is exactly proposed for the array arrangements of foam-filled tubes.

Fig. 2.10. Crush force–displacement curves: (a) axially [2]; and (b) laterally [4].

2.2 Metallic Foam: Aluminum Foam

Metallic foam is a typical type of cellular material. It has higher specific stiffness, higher specific strength, and better thermal insulation properties compared with other engineering materials. The characteristics of foam are usually described by its relative density, $\rho^* = \rho/\rho_s$ (ρ and ρ_s are the densities of foam and solid material of the cell wall, respectively), and stating whether it is open or closed cell, as reported by Gibson and Ashby [17]. For the open-cell foam, the cell walls are broken and air fills all the spaces in the material (Fig. 2.11a). Thus, this type of

foam is soft and weak. While in the closed-cell foam (Fig. 2.11b), most of the cell walls are not broken and they are resembled together in a compact configuration in order to make them strong enough to undergo large pressure.

Fig. 2.11. (a) Open-cell foam; and (b) closed-cell foam [17].

2.2.1 Advantages of Metallic Foam

The most common type for the metallic foam is the stabilized aluminium foam, which is closed-cell foam. Aluminium foam is manufactured by introducing gas bubbles into a melt of metal matrix composite (MMC). The hard ceramic particles in the aluminum alloy stabilize the bubbles and allow the foam to be formed into its final shape. The geometry of a typical foam block is illustrated in Fig. 2.12 [83]. The relative density ρ^* is usually within 5% ~ 15%, provided by the suppliers (such as CYMAT[®] and Alporas[®]). The advantages of aluminium foams are listed below [17, 18],

- High strength and stiffness-to-weight ratio.
- Notch insensitive (holes do not affect material strength).
- Constant properties over time, temperature and moisture range.
- High mechanical energy absorption in all directions.
- Not flammable or susceptible to environmental degradation.
- Electromagnetic insulation properties.

- Acoustic and thermal insulation properties, and recyclable.

Due to these properties, aluminum foam is the first choice for the applications in automotive, transport, military and civil defense fields.

Fig. 2.12. Typical foam block and its corresponding coordinate system [83].

Note: Here, x denotes the longitudinal direction, whereas the transverse y direction is aligned with the width of the foam block; z is in the through-the-thickness, main expansion-influenced direction (first foaming direction).

2.2.2 Mechanical Property

Due to the production method, aluminum foam sheets have a natural directionality.

The typical compressive stress-strain curves [17, 18] are plotted in Fig. 2.13.

Fig. 2.13. Typical compressive stress-strain curves in (a) thickness direction; and (b) the other two directions [17, 18].

The common mechanical properties are listed as follows,

- Elastic Modulus E (compression): there are two elastic moduli that can be measured for aluminum foam in compression: loading and unloading. The slope

of the initial loading portion of the curve, which is the loading modulus, is much lower than that of the unloading curve (corresponding to the unloading modulus) in Fig. 2.13a. Surface strain measurements indicate that there is localized plastic yielding in foam at stresses well below the compressive strength of the foam, reducing the slope of the loading curve [18, 84]. As a result, measurements of Young's modulus should be made from the slope of the unloading curve.

- Upper Yield Point: when foam is compressed in the thickness direction, it exhibits an initial peak in the constitutive relationship, which is exactly the upper yield point (Fig. 2.13a).
- Lower Yield Point: the first trough in the compressive stress-strain curve in the thickness direction (Fig. 2.13a).
- Plateau Strength σ_{pl} : in the thickness direction, the plateau stress is obtained using an energy efficiency method [85]. In the other two directions, there is no obvious upper and lower yield point, in which stress-strain curve exhibits a flat plateau (Fig. 2.13b).
- Densification Strain ε_D : the strain at which the foam is completely crushed and there is a sharp increase in the slope of stress-strain curve (Fig. 2.13), determined by the energy efficiency method [85, 86], as illustrated below.

The plateau stress and densification strain are simplified given by [18]

$$\sigma_{pl} = 0.3\sigma_{ys} \cdot (\rho^*)^{1.6}, \quad \varepsilon_D = 1 - 1.5\rho^* \quad (2.12)$$

where σ_{ys} is the yield stress of solid metal of which the foam is made. To be more accurate, the energy dissipation efficiency (E_η) at strain (ε_a) [85, 86], is defined as

$$E_\eta(\varepsilon_a) = \frac{\int_0^{\varepsilon_a} \sigma(\varepsilon) d\varepsilon}{\sigma_a}, \quad 0 \leq \varepsilon_a \leq 1 \quad (2.13)$$

where σ_a is the stress at ε_a . The densification strain is defined as the maximum

value of ε_i which satisfies the condition of maximum efficiency

$$\left. \frac{dE_\eta(\varepsilon_a)}{d\varepsilon} \right|_{\varepsilon_a=\varepsilon_i} = 0, \quad 0 \leq \varepsilon_i \leq 1 \quad (2.14)$$

Then according to the obtained densification strain, the plateau stress is given by

$$\sigma_{pl} = \frac{\int_0^{\varepsilon_a} \sigma(\varepsilon) d\varepsilon}{\varepsilon_D} \quad (2.15)$$

In the impact case, the plateau stress is somewhat different due to the strain-rate effect, and given by [18, 86]

$$\frac{\sigma_{pl}}{\sigma_{ys}} = 0.59 \left(1 + 0.14 \dot{\varepsilon}^{0.17} \right) \left(\rho^* \right)^{1.7} \quad (2.16)$$

By adopting the rigid, perfectly-plastic, locking (RPPL) model for foam core, the dynamic crushing stress of the foam (σ_D) at impact velocity v is obtained by Tan et al. [87]

$$\sigma_D = \sigma_{pl} + \rho v^2 / \varepsilon_D \quad (2.17)$$

2.2.3 Numerical Modeling of Metallic Foam

Since experiments are expensive and time-consuming, numerical simulations can be used in order to minimize laboratory time and duration of research. Several material models in commercial software can be used to predict the behavior of aluminum foam with different structures. The yield criteria for these models are a set of six criteria where each stress component of the stress tensor has a corresponding yield value. These models assume uniaxial loading in any principal direction while insignificant deformation in the other directions. The most widely used model was developed by Deshpande and Fleck [23]. The yield surface depends on the effective stress and the hydrostatic stress. Besides, the plastic Poisson's ratio is not assumed to be zero and the load conditions in principal directions are inter-dependent. A general overview is given here.

According to the conventional Prandtl–Reuss J2 flow theory [88], the yield surface is given by

$$\sigma_e^2 + \alpha^2 \sigma_m^2 = \left[1 + \left(\frac{\alpha}{3} \right)^2 \right] \sigma_y^2 \quad (2.18)$$

where σ_y is the absolute value of yield stress under uniaxial compression, and the effective stress σ_e is a scalar measure of the deviatoric stress, also called von Mises stress. σ_m is the mean stress (or hydrostatic stress) and α is the aspect ratio of the yield surface, also called as the shape factor of the ellipse. The hardening law of the material is written in the incremental form

$$\sigma_y = \sigma_y^0 + H(\varepsilon_{pl}) \quad (2.19)$$

where σ_y^0 is the initial yield stress under uniaxial compression and $H(\varepsilon_{pl})$ is the hardening modulus. For the plastic part of the behavior, the yield surface is a von Mises circle in the deviatoric stress plane and an ellipse in the effective-hydrostatic stress $(\sigma_m - \sigma_e)$ space (see Fig. 2.14).

Two hardening models are available: volumetric and isotropic hardening model. For the former hardening model, the point on the yield ellipse in the effective-hydrostatic stress plane that represents hydrostatic tension loading is fixed and the evolution of the yield surface is driven by the volumetric compacting plastic strain. For the latter hardening model, the yield ellipse is centered at the origin in the $(\sigma_m - \sigma_e)$ stress plane and evolves in a geometrically self-similar manner. These phenomenological models were originally developed for metallic foams by Deshpande-Fleck [23], which has been incorporated into ABAQUS [89] as crushable foam model. It is currently being implemented into LS-DYNA [90]. For

other FE software this model could be programmed as a user-defined material model by a highly experienced user. One of the advantages of this model is that it requires only one uniaxial compression or hydrostatic test to calibrate it.

Fig. 2.14. Yield curve for hypothetical foam in deviatoric–hydrostatic stress components space [83].

Note: Differently from usual, the hydrostatic axis positive direction represents compression.

2.2.4 Foam Expansions

Traditionally, the solid expandable tubular is an emerging and promising technology in petroleum industry [91, 92]. Much more effective and efficient protective structures against impact and blast loadings are required due to enhanced chance of terroristic attacks and natural disasters. Thus, structures filled with metallic foams have attracted research interests [2, 3] for designing protective structures against impact, including those subjected to internal explosion [93] resulting from heated gas or terrorist attack. In order to assess the energy dissipation characteristics of foam-filled structure under dynamic loading, an understanding of the quasi-static response of such structure is essential. On one hand, the mechanical behavior and energy absorption characteristics of each component are crucial, including the deformation and interaction effect among each component in the foam-filled tubes design. The plastic behavior of monolithic expandable tubes

subjected to large plastic deformation has been studied in detail [92, 94, 95]. It is concerned with increasing the diameter of a tube by hydraulically pushing or mechanically pulling a conical mandrel through the inner tube.

On the other hand, due to the wide applications of metallic foams in engineering, much attention has been paid to the closed-cell metallic foams' mechanical properties and design [18, 19]. However, since the yielding, hardening and the associated plastic flow rule are different from the conventional isotropic elastic-plastic metal [22], the plastic behavior of metallic foams under large deformation still requires further understanding [24]. Two typical macroscopic material models have recently been developed for aluminum foams. One continuum plastic model was proposed by Miller [96] based on the Drucker-Prager [97] yield criterion for soil. Due to the difficulties in distinguishing elastic and plastic stages as well as the inhomogeneities of metallic foam, the initial yielding function and the shape of the subsequent yield surface may not be reliable. The other model is the crushable foam model [23] based on the proportional loading experiments, as mentioned in Section 2.2.3. This isotropic constitutive model has an elliptical yield surface defined in terms of the effective stress and hydrostatic stress, with the assumption of associated flow. It could be used to evaluate the response of foams under general loading with reasonable results.

2.3 Foam-filled Tubes

Composite structures have gained much attention since the last three decades. They are now widely used in the industry. In addition to their excellent performance with high specific strength and specific stiffness, they possess good energy absorption

behavior [98-102]. This section summarizes the energy absorption characteristics of tubes filled with metallic foam cores. As illustrated in Section 2.1.3, the load-deformation curves under lateral compression, are desirable for energy absorption [4]. Hence, the structure responses on lateral crushing of foam-filled tubes need to be developed. In order to provide a better understanding of the energy absorption characteristics of foam-filled tubular structures subjected to the impact or even blast loading, the mechanical behavior under quasi-static crushing is necessary.

2.3.1 Quasi-static Crushing Behavior

2.3.1.1 Foam-filled Empty Tube

After a thorough understanding of deformation modes of empty tube subjected to the axial loading, researchers began to study the axial crushing behavior of tube filled with metallic foam, which is found to significantly enhance the energy absorption capacity of the whole structure [25, 26, 32, 103-105]. The quasi-static compression experiments were also conducted by Güden and Kavi [2] on a single foam-filled tube (see Fig. 2.15). It was found that the deformation modes of single empty tube and multi-tubes with foam filling were transferred from diamond to concertina [2, 106]. Some possible causes, include folding parameter [107], load eccentricities [45], cut-off and grooving on the tube wall [108, 109], foam-filling and any disturbances in the periodicity of folding were studied.

Fig. 2.15. Top view of Al foam-filled tube [2, 106].

A theoretical formula on the average crushing strength of fully foam-filled column (P_{af}) was given by Hannsel et al. [104, 110],

$$P_{af} = P_{ae} + \sigma_{pl} \frac{\pi}{4} (D - 2t)^2 + C_{avg} \sqrt{\sigma_{pl} \sigma_y} \cdot (D - t) \cdot t \quad (2.20)$$

where P_{ae} , σ_y , D and t are the average crushing load, yield stress, diameter and thickness of the empty tube, respectively. σ_{pl} and C_{avg} are the plateau stress and the strengthening coefficient of foam filling. The interaction effect is included in the third item in the above equation. Such interaction between in-filled foam and tube walls can be understood as follows [1]. The foam provides constraint when a tube wall buckles inwardly. In the bonding case, depending upon the level of plateau stress, the tube collapse mode may be affected by the constraint from foam [40]. Also, the plastic fold length decreases with the presence of foam [32]. Especially, when the foams are compressed to densification, the stress increases rapidly with the strain and the fold length reduces by the locking strain, leading to a higher average force. Mainly due to this effect, the crushing loads of foam-filled tube are higher than the sum of crushing loads of individual component [104].

Then two different arrays of constraint multi-tubes, hexagonal and square-packed, with foam-filled were also discussed [2], as shown in Fig. 2.16. Moreover, in the multi-tubes design, the constraint interaction effects and friction among the tubes maybe the main factors to prevent deformation and increase the average crush forces than those of empty tubes [2]. It also validated that foam-filled tubes induce a higher strengthening coefficient in multi-tubes. Although foam filling increases the energy absorption ability in both designs, it is not weight effective in increasing the specific absorption energy (*SEA*) than that of the empty tube. However, a higher interaction effect in multi-tube is proved in the research in view that the multi-tube

design is found to be energetically more effective than empty tubes. Hence, only the multi-tube-packed tube design shows higher *SEA* values in the presence of the frictional forces and interaction effect, comparing with the foam-filled single and bi-tubular configurations [2, 106].

Fig. 2.16. Schematics of hexagonal and square packed multi-tubes: empty and foam-filled [2].

A number of experimental and numerical studies have been performed to investigate the axial crushing of foam-filled single tube [26, 32, 103, 104, 111]. However, comprehensive understandings of the lateral crushing of foam-filled tube have been less reported. The lateral crushing of aluminium fully foam-filled tube (shown in Fig. 2.17) was first studied by Hall et al. [3], indicating that the foam-filling effect has a vital influence on both the crushing strength and Specific Energy Absorption (*SEA*). Similar enhancement on energy absorption might be expected for foam-filled double tubes. However, the detailed deformation modes as well as the energy absorption need to be further clarified.

Fig. 2.17. (a) Section through a laterally foam-filled tube; and (b) empty tube [3].

2.3.1.2 Foam-filled Double Tubes

Through previous analysis, it is found that the interaction effects play significant roles in improving the energy absorption capability. Hence, the deformation mechanism and energy absorption characteristics on the foam-filled double tubes have attracted research interests due to an extra component and corresponding interaction effect [29, 31, 112]. Double-cell profiles made of circular tubes have higher energy absorption efficiencies in axial crushing than those made of square tubes [31], hexagonal and octagonal tubes [29], due to the more mass-efficient inner tube, as shown in Fig. 2.18. The mean force level can be considerably improved by filling with aluminium foam.

Fig. 2.18. Axial crushing of empty, foam-filled, sandwich square [29] and sandwich octagonal tubes [31].

For the bending behavior of double tubes filled with aluminium foam core, recently, three-point quasi-static tests was conducted by Guo and Yu [30], as shown in Fig. 2.19. It transpires that under bending loading, foam-filled double tubes are the optimum design, in terms of higher specific energy absorption (*SEA*) than those of empty tubes and foam-filled tubes. Such enhancement in specific energy of double tubes filled with foams over the foam-filled single tubes subjected to oblique loading are also validated by Li et al. [113]. So far, little attention has been paid to the energy absorption characteristics of foam-filled double tubes subjected to lateral loading, whether through experimental or numerical investigations.

Fig. 2.19. (a) Empty tube; (b) foam-filled single tube; and (c) foam-filled double tubes [30].

2.3.2 Dynamic Crushing Behavior

Previous work has been more directed into structures under quasi-static loadings or the low-speed impact. Recent research on foam-filled structures subjected to impact or blast loading has enhanced their potential in a wider range of crashworthiness applications than ever [32-34]. Most work is concentrated on axial crushing on the single empty [114] or with foam-filled tube [32, 114-116], sandwich panels [99, 102, 117-125], layered structures [126, 127], frame structures [128] and cell micro-topology [129]. Studies on dynamic bending response demonstrate that with a proper design, the load carrying capacity can be achieved in foam-filled tubes [130]. Preliminary research on the quasi-static lateral responses has already identified this improvement in the crush strength and energy absorption capability in the foam-filled double tubes [131]. However, little attention has been paid to the lateral deformation modes and mechanism of foam-filled double tubes or even multi-tubes with tightly array arrangement under high-speed impact or blast loadings. Moreover, metallic foams can be strain-rate sensitive [18, 86, 132] and have a higher plateau stress and a higher energy dissipation capacity under impact or blast loadings, compared with quasi-static loading case. The literatures concerned with the energy absorption of foam-filled shell structures are summarized in the following subsections, experimentally, analytically and numerically.

2.3.2.1 Experimental Investigation

Different types of equipment are applied according to the strain rate. To investigate the structural responses of foam-filled tubes, some experimental facilities are illustrated in Fig. 2.20. There are standard Universal Testing Machine ($10^{-4} \sim 10^0 \text{ s}^{-1}$), drop weight machine ($10^1 \sim 10^2 \text{ s}^{-1}$), high velocity hydraulic machine ($10^1 \sim 10^3 \text{ s}^{-1}$), and Split Hopkinson Pressure Bar (SHPB, $10^2 \sim 10^4 \text{ s}^{-1}$) [133, 134].

Fig. 2.20. Experimental facilities under different loadings: (a) UTM machine; (b) drop weight machine; (c) high rate Instron machine and (d) SHPB [134].

Other facilities are introduced by Meyers [133], such as gas gun ($10^2 \sim 10^3 \text{ s}^{-1}$), Taylor anvil tests ($10^2 \sim 10^5 \text{ s}^{-1}$), etc. The Split Hopkinson pressure bar (SHPB) technique [135] has been used widely to measure the dynamic strength enhancement of materials at the strain-rate within $10^2 \sim 10^4 \text{ s}^{-1}$. But the weakness of SHPB lies in the generation of stress-strain curves for these samples from SHPB data. For the blast experiments with higher strain rate (up to 10^7 s^{-1}), a ballistic pendulum system can be used to measure the impulse imparted to various shock mitigation materials subjected to air blast explosion. A two-cable pendulum, which can be applied to measure the TNT impulse, was used by Hanssen et al. [136]. The four-cable pendulums presented by Nurick and Martin [137] for small explosive loading has been used for a number of years. These two types of pendulums are

shown in Fig. 2.21, as widely applied to the investigation on the blast mitigation of sandwich panels.

Fig. 2.21. Two types of ballistic pendulums: (a) a two-cable pendulum [136]; and (b) a four-cable pendulum [137].

Pioneer researchers Reid et al. [32] examined the dynamic mechanical responses of empty and foam-filled square sheet empty metal tubes experimentally via the drop hammer. For the double cylindrical tubes filled with closed-cell aluminium foam core, drop weight tests ($100\text{--}200\text{ s}^{-1}$) on the bending response of foam-filled double tubes are found to be with the highest specific energy absorption than empty tubes and fully foam-filled tubes [130]. Subsequent impact tests on transverse crushing of foam-filled tubes were conducted by Hall et al. [3] via SHPB (650 s^{-1}). It should be noted that unlike axial loading, lateral loading leads to a moderate lateral displacement of the foam. Those results are not strain-rate dependent in the lateral crushing of foam-filled tubes. However, further experiments are still necessary to reveal the energy absorption capacity of the novel foam-filled double tubes under dynamic lateral loading in the near future.

2.3.2.2 Analytical Modeling

Theoretical or analytical impulsive loaded models provide valuable information for locating damage and establishing criteria for acceptance and/or repair of structural

components. Analytical solutions that can describe deformation/damage would enable us to recognize impact parameters. Parametric studies can then show how the failure of structures varies with impact parameters. Furthermore, analytical solutions provide benchmark solutions for more refined finite element analysis. On one hand, for the simple protective structure, the theories of buckling and post collapse of plastic deformation are mature [38, 39, 46, 66, 138-153]. The problem in the dynamic deformation of the tubes involves both the material and geometric nonlinearities. The dynamic plastic behavior of different structures under impulsive loading were analyzed by Lee [152], Lellep and Torn [39]. A simplified analytical model for the rectangular metallic sandwich plates was developed by Hutchinson and Xue [154] for the effective design, based on the model proposed by Fleck and Deshpande et al. [123, 155]. However, the material property in the analysis was R-P-P-L (rigid-perfectly plastic-locking) model, which was an idealized model.

For the cylindrical shells filled with a compliant core, the elastic and buckling solution of such structures under uniform external pressure have been achieved [156-158]. However, for an effective energy absorber, the plastic deformation behavior is of importance. The analytical attempts on the quasi-static lateral crushing of foam-filled shells were made by Shen [134] and well validated against corresponding experiments, which were based on the equivalent structure technique [70]. Analytical solutions of the complex mechanics behavior under blast loadings are rare, especially when introducing the metallic foam core. No more theoretical analysis regarding the large deflections and associated energy absorbing performance in foam-filled double tube is available, due to the more complex nature of the problem, like inertia effect [36], strain-rate effect of the materials [18], etc.

2.3.2.3 Numerical Simulation

Most researchers concentrated on the theoretical and experimental results. Nowadays, since Finite Element Analysis (FEA) has capabilities in showing the initiation and propagation of the crushing and the effect of various parameters on the energy absorption characteristics. Some numerical solutions on foam-filled tubes were achieved by Aktay et al. [159] and Hanssen et al [160]. In addition, recent advancements in FE software and computational technologies allow researchers to successfully simulate the crushing of tubular structures in an acceptable period of time. Moreover, once FE models are well-validated, i.e. successfully capture experimental data or reach good agreements with theoretical solutions, and then parametric studies can be performed at a fraction of the cost of experimental studies. Above all, FE analysis could reveal some characteristics such as the distributions of plastic zone, element strain energy as well as better visualizations of structures under severe deformation, which are far beyond the capabilities of experiments.

Explicit approaches are generally recommended for dynamic loadings and usually more stable owing to the method of integration used in solving the equilibrium equations. As widely-used commercial nonlinear-dynamic explicit packages, LS-DYNA [90] and ABAQUS [89], numerical results of the dynamic behavior of tubular structures can be obtained using optimized algorithms. Most topics aimed at validating the corresponding analytical models [23, 46, 83, 88]. FE analysis on the Al circular tubes under an axial explosive load were conducted by Karagiozova et al. [38]. The blast modeling and failure criteria were further presented and discussed

by Gupta and Nagesh [143]. Few attempts of FE analysis on the blast loading of foam-filled circular multi-tubes have been made.

When structures and materials are subjected to dynamic loadings, dynamic effects such as strain rate and inertia effects are the principal factors to the yield stress and ultimate stress of materials. For traditional metals, two kinds of widely used representative material constitutive model are the Cowper-Symonds and Johnson-Cook models [1] considering those dynamic effects. In the commercial FE software [117], blast loads may be modeled as pressure-time history, initial velocity field over the front face [129], or with the blast pressure functions. In the recent simulations on sandwich panels involved with metallic foam [98, 100], the material property of charge is described with a high explosive material model using ALE (Arbitrary-Lagrangian-Eulerian) formulation. The interaction between the explosion and the target is an important issue from the viewpoint of explosion mechanics.

No structural damage (e.g. tearing or rupture) has been considered in the above studies. In other words, the failure criteria of material are not taken into account, whereas the material failure occurs frequently in experiments. New modeling techniques with better computational efficiency are needed to explore such complex problems. It is expected that, with controllable energy absorption and mass distribution, the energy absorbers made up of metallic foam and thin-walled structures will provide an excellent impact protection to main structures. The effectiveness of such foam-filled tubes for dynamic protection is primarily dominated by some factors. There are structural configuration, material behavior of metallic foams including the strain-rate effect, wave propagation in metallic foams,

interaction among each component and mechanism of energy absorption. These will be the main topics of our proposed research and they also represent the fundamental challenges of the problems.

2.4 Summary

Increasingly more and more attention is being directed toward the issues associated with the energy absorption capability of novel energy absorbers in the retrofit of impact or blast resistant structure. As the most versatile component, the axial crushing behavior of thin-walled tubes with different cross-sectional shape is investigated to explore the deformation mechanism and energy absorption capacity. To obtain longer plateau stress and stroke in the energy absorbers, thin-walled tubes subjected to lateral crushing are later examined. Cellular materials, such as metallic foams are intriguing materials with extraordinary properties and are used in many fields related with protective structures for their excellent energy absorption abilities. With this promising new material, energy absorbers incorporating metallic foams and thin-walled bi-tubes are naturally developed as good energy absorbers. Most research concentrate on the quasi-static axial crushing behavior of foam-filled structures. Few literature on the lateral crushing behavior on such structures were reported. Systematic investigations and more details about the deformation mechanism and crushing patterns of foam-filled double tubes would be made. This project attempts to resolve these issues, which are crucial to the future optimal structural design when subjected to quasi-static and dynamic loadings, especially impact and blast loadings.

CHAPTER 3

QUASI-STATIC AXIAL COMPRESSION OF THIN-WALLED TUBES

In this chapter, in order to assess the energy absorption capabilities, quasi-static axial crushing tests were performed on thin-walled low-carbon steel tubes with various polygonal cross-sections, namely hexagon, octagon, 12-sided star and 16-sided star, respectively. By using the commercial nonlinear finite element analysis software, ABAQUS/Explicit [89], the deformation behavior and energy absorption characteristics of structures were obtained. The results were compared with experimental data and more details about crush progressive collapses were found as well. Comparative studies were performed to further demonstrate the optimum design for different cross-sectional shapes. Furthermore, based on the numerical deformation modes and the use of cardboards, the progressive collapse mechanisms on the triangular tubes were theoretically analyzed and validated against corresponding experiments.

3.1 Experiments

A series of tests were conducted on the thin-walled polygonal tubes. There were four types of specimens in total, namely, hexagon (S6), octagon (S8), 12-sided star (S12) and 16-sided star (S16). Their shapes are shown in Fig. 3.1 and the related dimensions are listed in Table 3.1.

Fig. 3.1. Cross-section geometry of specimens (all dimension are in mm): (a) S6, Hexagon; (b) S8, Octagon; (c) S12, 12-sided star and (d) S16, 16-sided star. All the sections have the same perimeter of 199.2 mm and the same thickness of 1.5 mm.

Table 3.1 Test series of *ASTM* A36 (annealed)

Test ID.	Type of Geometry	Number of Side	Flat Width c (mm)	c/t	L/c
S6	Hexagon	6	33.2 ± 0.2	22.1	3.0
S8	Octagon	8	24.9 ± 0.2	16.6	4.0
S12	Star	12	16.6 ± 0.2	11.1	6.0
S16	Star	16	12.5 ± 0.2	8.3	8.0
SC*	Circular	Circle	$D = 63.5$	$D/t = 42.3$	$L/D = 1.6$

* SC is only examined in the FE model, not available in experiments.

3.1.1 Specimens

Instead of using conventional welding fabrication [10], mild steel tubes (*ASTM* A36) were machined directly from a steel block using wire cutting method on an Excetek® V850 Wire Cut EDM (Submerged) machine with a precision of 0.15 mm (minimum wire diameter). This method could eliminate the necessity of welding joints, which were vulnerable to crack [5]. In our work, a mild steel block (dimensioned $250 \times 250 \times 150$ mm) was firstly machined at the both ends to remove the uneven surface. In addition, five holes were drilled for wire penetration during cutting. To minimize the distortion, the inner material was removed in advance, followed by the outer material. These specimens (refer to Fig. 3.2) were prepared such that their outer perimeter or section length S remained constant ($S = 199.2$ mm), which was equivalent to that of a circular tube with an outer diameter of $D =$

63.5 mm. All the specimens were of a constant length ($L = 100 \pm 0.2$ mm) and thickness ($t = 1.5 \pm 0.1$ mm). The half-length of the buckle fold for a circular tube [42] is expressed as $H = 0.95\sqrt{Dt}$, which gives $2H = 18.54$ mm. The length of the tube should be at least three times the fold length in order to minimize the end effect. However, if it is too long, Euler buckling may occur. Hence, $L = 100$ mm was chosen.

Fig. 3.2. Photograph of the specimens fabricated by wire-cut EDM.

3.1.2 Material Property

In the present work, the material of the tubes was low-carbon steel (*ASTM A36*). In order to obtain its engineering stress-strain curve, which is the material property used in FE analysis, tensile tests of three specimens were conducted through an Instron 5569 testing machine at a controlled displacement rate of 5 mm/min. The average stress-strain curve for the annealed steel is shown in Fig. 3.3. The yield stress obtained was approximately 265 MPa. All the test specimens were annealed before test to remove any possible residual stress. Due to the surface oxidation, all specimens appeared to be darkened, as shown in Fig. 3.2. Formation of excessive oxidation scale would reduce the wall thickness, hence reducing the crushing capacity. Visual inspection was made to ensure that the oxidation scale was insignificant.

Fig. 3.3. Typical engineering stress-strain curves for annealed *ASTM A36*.

3.1.3 Experimental Set-up

The compression tests were performed using a universal testing machine, Instron 4400 under quasi-static loading circumstance. In order to study the effect of the number of sides and the cross-sectional shape on deformation behavior, all the test specimens were crushed at a constant crosshead speed of 5 mm/min. During the tests, the real time loading and compressive displacement were collected automatically. In general, all specimens failed in local buckling. Different deformation modes were observed, as discussed later in Section 3.3.

3.2 Numerical Simulation

For the numerical simulation of quasi-static collapse of the thin-walled tubes, ABAQUS/Explicit was used. Full structures are modeled despite the symmetry of the cross-sections. The thin-walled tube is rested on a fixed bottom base, while a total vertical displacement of 80 mm is gradually applied to the top platen, as illustrated in Fig. 3.4.

Fig. 3.4. FE model of specimen S16.

3.2.1 Material Model

The material for the tube is mild steel, which is assumed to be homogeneous, isotropic and elastic with power law hardening. It has the elastic property of Young's modulus $E = 210$ GPa and Poisson's ratio $\nu = 0.3$. The density of the tube is 7800 kg/m^3 . The constitutive relationship is the engineering stress-strain curve obtained from previous tensile tests (as shown in Fig. 3.3). The initial yield stress is 265 MPa. The strain-rate effect and material failure are not taken into account in the finite element analysis.

3.2.2 Finite Element Model

The thin-walled tubes are modeled using 4-node shell elements *S4R* with five-integration points through thickness, which is formulated by Belytschko et al. [161]. Two platens are both constrained as rigid bodies. Each platen is modeled by an 8-node linear brick element with no discretization. The bottom platen is fixed. Displacement of 80 mm is then applied gradually on the top platen to crush the polygonal tubes into several folds. The default value for the time period is 1.0 time unit to ensure quasi-static loading in an explicit code. General contact algorithm is

used to simulate the contact interaction between the thin-walled tube and two platens. The interaction properties are modeled using a contact surface with a penalty friction forces in tangential direction, as provided in ABAQUS. The tangent force is considered by setting the friction coefficient as 0.1. In the normal direction, the contact property is hard contact, which means that when surfaces are in contact, any contact pressure can be transmitted between them and the surfaces separate if the contact pressure reduces to zero. ABAQUS 6.9 is used for FE analysis on a HPC server platform (Intel Xeon 2.53 GHz), and the average CPU time is about 4 hours.

The mesh density of the FE model is determined by mesh sensitivity studies in Fig. 3.5. The coarser mesh results in a larger crush force and further refinement reduces the difference between results with different mesh. The moderate mesh density is selected when further refinement does not improve the accuracy of the simulation too much. The mesh size of each model is kept the same, with 3840 elements.

Fig. 3.5. Verification of mesh sizes of polygonal tubes

3.3 Experimental and Numerical Results

In this section, a number of deformation modes and recorded data from the tests are compared with the corresponding numerical results.

3.3.1 Deformation Modes and Load-Displacement Curves

Fig. 3.6 shows final deformation modes of the four specimens from both experiments and FEA. The experimental side view of compressed specimens S12 and S16 in Fig. 3.6a are quite different from those in FEA, due to the difference in the deformation modes, as shown in Figs.3.7-3.10. When analyzing the outward to inward fold length ratio in Fig. 3.6b, both the polygons S6 and S8 had almost equal amount ratio (ratio ≈ 1) and centered in the mid-line of the tube's thin wall section. However, greater outward fold length was found (ratio > 1) for both S12 and S16. From the comparison of the progressive collapse in Figs.3.7-3.10, it may be seen that numerical solutions have a fairly good agreement with the experimental results. The polygonal tubes displayed different local or global deformation properties during crushing process from the star shaped sections.

Fig. 3.6. Final deformation modes at displacement $\delta = 65$ mm (a) top: side view from experiment; bottom: side view from FEA and (b) top: top view from experiment; bottom: top view from FEA.

Fig. 3.7. Collapse mode for S6 (Hexagon) (top: experiment; bottom: FEA).

Fig. 3.8. Collapse mode for S8 (Octagon) (top: experiment; bottom: FEA).

(Continued on next page)

(Continued from previous page)

Fig. 3.9. Collapse mode for S12 (12-sided star) (top: experiment; bottom: FEA).

Fig. 3.10. Collapse mode for S16 (16-sided star) (top: experiment; bottom: FEA).

As shown in these figures, both the convex polygonal shapes (S6 and S8) underwent nearly axisymmetrical deformation, which was the ring mode [1]. Meanwhile mixed deformation modes existed in star shapes (S12 and S16). In terms of the number of buckle folds, S6 and S8 had three folds while S12 and S16 only had two. In addition, the buckle folds of star shapes were found irregularly spaced and tilted at an angle in comparison to the polygonal ones, which were uniformly distributed. It was also observed [57] that in these cases, the increase in corner elements in the star shape is equivalent to some sort of constraint on tube wall, which will dramatically improve the local buckling capacity. The specimens with

star shape will deform unstably compared to the traditional square box column. Numerical simulations showed that for a circular tube (SC) with $D/t = 42.33 (< 50)$ and $L/D = 1.58 (< 2)$ which is also shown in Table 3.1, axisymmetric mode occurred, as also elucidated by Guillo et al. [40] and Reid [69].

These four figures capture the progressive collapse of specimens from S6 to S16 in sequence. There is no deterministic criterion for prediction of the starting place of folds [42]. In Figs. 3.6 and 3.7, the first fold started at one end of the tube, either at the top or bottom. These were then followed by successive folds stacking onto the first fold. Such kind of deformation was favorable to the global collapse as it produced higher effective stroke and helped maintain the straightness of the tube during the axial crushing. In contrary, from both the experiments and FE results, the first folds in specimens S12 and S16 were initiated in the middle of the tube, as shown in Figs. 3.8 and 3.9. This might weaken the structure as global buckling may occur. The top portion of S12 was slightly shifted as compression progressed, causing irregular folds. The collapse patterns for S12 and S16 in the numerical simulation were almost the same, probably because the local buckling was prone at the middle of the tubes. For the detailed plastic deformation zone during the crushing process for all these specimens, the section views obtained from the numerical results are shown in Fig. 3.11. It gives a clear view of the interior features of the specimens which are hard to see from actual experiments. The buckle fold lengths ($2H$) of all specimens are revealed as well. It is shown that for the convex shape, increasing the number of side might shorten the fold length with more buckling lobes involved. Fracture was observed at the corner regions due to severe localized deformation (as illustrated in Fig. 3.12). In addition, since more cracks

were revealed in S16, it may be one of the major causes for the difference between the experiment results and the corresponding FE results.

Fig. 3.11. Section view along the center line for each specimen in FEA.

Note: the buckle fold length for each specimen is 25 mm, 22.5 mm, 35 mm, and 37.5 mm, respectively.

Crack/local rupture

Fig. 3.12. Local rupture in specimen S12 and S16.

The force-displacement curves of FE results are plotted in Fig. 3.13. Overall, these curves agree well with corresponding experimental results, validating the numerical models. However, the forces in FEA are lower than those from experiments, especially for specimen S8 and S12. The end effect of tubes may account for the deviations between FEA and experiments, which were probably owing to the limitation of the moderate length of specimens in the EDM machine. The results are also summarized in Table 3.2, in terms of the peak force, average force, etc. Here, the average force was obtained by dividing the total energy absorbed up to a displacement of 65 mm by 65 mm. From both sets of the results, it is evident that the peak force increases as the sides of polygon increases. On one hand, S12 has the

highest peak force of 115.4 kN as shown in Fig. 3.14a. However, its crushing pattern was random with several local peaks which had no relation to the number of folds formed. Only one fold was formed and further deformation appeared to displace the inward hinge to fold downward and eventually bottom out on the end of the tube (see Fig. 3.10). The ‘notch’ was found at compressive displacement $\delta = 42$ mm, which was due to the local rupture. The local rupture was located at the boundary where the circular fold interfaced with the corner edge of the star shape.

Fig. 3.13. Load-displacement curves from the experiments and FEA: (a) S6; (b) S8; (c) S12 and (d) S16.

The octagon (S8) had the second highest peak force. In comparison with the star shape, S8 had a much more regular curve pattern with its onset peak increased as displacement continued. The number of peak was found to be consistent with the number of observed folds. On the other hand, 16-sided star (S16) had the highest

peak force of 92.7 kN in Fig. 3.14b, which may be due to the largest side number of the star. Nevertheless, from Table 3.2, the increase in peak force with the side number is insignificant, most of which is approximately 10% higher than the hexagonal one.

Fig. 3.14. Load-displacement curves: (a) for test series S6 to S16, experimental; and (b) for test series S6 to SC, numerical.

Note: the horizontal dash-dot line represents the average force calculated by Eq. (3.1) for the circular tube (SC).

3.3.2 Energy Absorption Characteristics

Generally, there are many key variables to analyzing the energy absorption behavior

of a structure [1], such as the peak force ($F_{\max}$), average crush force (F_{av}), energy absorbed (EA) and specific energy absorption (SEA), as listed in Table 3.2.

Table 3.2 Summary of results for tubes of different cross-section (stroke: 65 mm), having flow strength $\sigma_0 = 332.5$ MPa and cross-sectional area $A = 298.8 \text{ mm}^2$

Test ID.	Peak Force		Average Crush Force		Specific Energy Absorption		Structural Effectiveness		Energy-absorbing Effectiveness Factor	
	$F_{\max}$ (kN)		F_{av} (kN)		SEA (J/g)		η		ψ	
	Exp.	FEA	Exp.	FEA	Exp.	FEA	Exp.	FEA	Exp.	FEA
S6	95.7	85.9	46.6	44.6	20	19.1	0.46	0.44	0.54	0.52
S8	110.2	90	58.8 (26.2%)	49.2 (10.3%)	25.2 (26.0%)	21.1 (10.5%)	0.59 (26.2%)	0.49 (10.3%)	0.68 (26.2%)	0.57 (10.3%)
S12	115.4	92.2	69.2 (48.5%)	52.7 (18.2%)	29.7 (48.5%)	22.6 (18.3%)	0.69 (48.5%)	0.53 (18.2%)	0.80 (48.5%)	0.61 (18.2%)
S16	96.4	92.7	47.2 (1.3%)	48.7 (9.3%)	20.3 (1.5%)	20.9 (9.4%)	0.47 (1.3%)	0.48 (9.1%)	0.55 (1.3%)	0.57 (10.0%)
SC	NA	81.0	NA	44.8 (0.4%)	NA	19.2 (0.5%)	NA	0.45 (0.4%)	NA	0.52 (0.4%)

Note: the percentage in the bracket is the improvement compared with those of the hexagonal tube (specimen S6).

In terms of the average crushing force, defined as the average value of the force history, increasing the number of corners leads to higher crush strength. Compared with the hexagon, 12-sided star shape (S12) shows an increase of 48.5% experimentally and 18.2% in FEA, while for the 16-sided star (S16), the experimental value shows only 1.3% improvement but 9.3% increase for FEA. Although the 16-sided star (S12) produced no improvement in contrast to S12, it indicates a significant improvement for displacement from 35 mm to 55 mm. With regard to the circular tube (SC), according to the empirical relation for axisymmetric folds [40], the average crush force is given by

$$F_{av} = 72.3(D/t)^{0.32} \cdot \sigma_0 t^2 / 4 \quad (3.1)$$

In the present case, the average flow stress $\sigma_0 = (\sigma_y + \sigma_u) / 2 = 332.5$ MPa. Then substituting the values into Eq. (3.1), $F_{av} = 44.8$ kN, which happens to equal to the

FE result obtained from the fifth curve in Fig. 3.14b.

As for *SEA*, which is the energy absorbed per unit mass of crushed structure and has the unit of J/g, it is given as

$$SEA = E_{energy} / w = \int F dl / (\rho \cdot t \cdot S \cdot l) \quad (3.2)$$

where density ρ , thickness t , section length S and stroke l (65mm) are the same for each specimen. *SEA* values can be easily deduced from the total energy. FE results were also worked out. The 12-sided star (S12) has an increase of 18.3% compared with those of S6. Besides, for the 16-sided star (S16), energy absorption ability was better than that of the hexagon (S6), with 1.5% increase experimentally and 9.4% increase numerically. Both show that the 12-sided star is the optimum design. This means that considerable weight saving can be achieved by using the star design as it can absorb a larger amount of plastic strain energy compared with regular polygon.

Two more key factors to assess the energy dissipation are also investigated in current study, which are the structural effectiveness η [37] and energy-absorbing effectiveness factor ψ proposed by Jones [162], respectively. The former dimensionless parameter η , defined as $\eta = F_{av} / (A\sigma_0)$, allows comparisons to be made of the effectiveness of various geometrical shapes and of energy absorbers made from different materials. The latter is defined as $\psi = 3F_{av} / (4A\sigma_0\epsilon_r)$, where ϵ_r is the rupture strain and A is the net cross-sectional area, respectively. Both η and ψ are proportional since all specimens have the same dimension, material property and weight. Table 3.2 also shows the improvement of other shapes in percentage (%) relative to the hexagon shape, experimentally and numerically. It is found that

the 12-sided star shape dissipates the most energy from the viewpoints of crush strength, *SEA*, structural effectiveness, and energy-absorbing effectiveness factor.

3.4 Discussions on the Polygonal Tubes

The study has shown that the energy absorption ability varies with the different number of sides due to local buckling of the specimens, which are calculated from the area under the curves in Fig. 3.13. While for the polygon, global buckling is more prone to occur during crushing. The experimental values are larger than the numerical results, probably due to the strain hardening and frictional effect which may not be suitably captured.

In the experiments, specimens of only one thickness were used, due to the cost consideration. However, to overcome this shortcoming, further finite element analysis was conducted for specimens with different thicknesses, and so that the value of D/t ranged from 20 to 200. For four sets of specimens, the average force non-dimensionalized with respect to the fully plastic bending moment $M_0 (= \sigma_o t^2 / 4)$ is plotted in Fig. 3.15a. In addition, considering the different weights of specimens with different D/t ratio, the structural effectiveness η is also plotted in Fig. 3.15b. It remains valid that the octagon (S8) is better than the hexagon (S6) in energy absorption for convex shapes. However, the average crushing force and structural effectiveness of 16-sided star are the largest, which are different from the previous results. It may be said that if the ratio of D/t is less than 50, the optimum design is still S12; while for the ratio beyond 50, S16 instead of S12 may be the optimum. It is also evident that the structure is more effective in terms of the structural effectiveness η at the smaller D/t ratio.

Fig. 3.15. FE analysis results: (a) F_{av}/M_0 vs. D/t ratios; and (b) structural effectiveness vs. D/t ratios.

A general remark may be made herein. When selecting tubes as energy absorbing devices, it is important that tubes of high values of SEA are chosen, in order to maximize the energy absorption while keeping the structure light. During an impact loading, the tube must be able to absorb the specified kinetic energy, and hence the value of SEA will be useful in the initial design. However, impact loading can cause difference in the tube behavior and energy absorption. For small velocity impact, the deformation mode should be similar to the quasi-static case and the SEA from the quasi-static case can be a good guide. For high velocity impact, inertia may play a significant role and the deformation mode changes; then the SEA under quasi-

static cannot be used at all. Strain rate effect of the material tends to increase the yield stress of the material and hence, for a given energy to be absorbed, the deflection in the dynamic case will be smaller than that in the quasi-static case.

3.5 FE and Analytical Model for Equilateral Triangular Tubes

The above analysis is applicable to the tubes with even number of sides. However, when it comes to an equilateral triangular tube having an odd number of sides, the alternating collapse mode needs to be developed due to the distinctive folding modes of adjacent corner elements. The theoretical analysis used for the predictions of the mechanics of the triangular tube is based on the super folding element method developed for tubes with rectangular cross-sections [50, 55, 163], but modified to account for the odd number of sides. The material failure or crack, Euler buckling and change of the thickness in tube wall are not considered in the current analytical model. The material is assumed to be rigid, perfectly plastic with a constant flow stress $\sigma_0 (= 332.5\text{MPa})$. Furthermore, there is no interaction between bending and stretching in the yielding criterion, and hence the material yields either by bending only or stretching only. The top and bottom edges of the tube remain plane throughout the progressive collapse. It is further assumed that the tube is collapsed by inextensional folding. Similar to Section 3.2, preliminary FEA is conducted via ABAQUS/Explicit and further validated against corresponding experiments. During the formation of the folds, two types of collapse modes are identified and investigated in details.

3.5.1 Diamond Mode

The diamond mode consists of one entirely outward and two inwards lobes, or one

entirely inward and two outwards lobes.

3.5.1.1 A New Basic Folding Element

The assumed collapse mode is demonstrated in a series of cardboards and successive deformation events in FEA shown in Fig. 3.16. In the formation of a single fold, an outward lobe is formed on one plate and two inwards lobes are formed on the rest plates. It consists of a layer with a total initial height $2H$ and three basic folding elements at each corner. As collapse processes, a series of such collapse modes occurs one after another for a longer tube, maybe occasionally the individual lobes change position. Apart from those two basic folding elements [44] illustrated in Fig. 2.3, an alternative basic folding element (called as Type III) is observed, in which the travelling hinges only exist at one plate.

Fig. 3.16. Triangular tubes ($L/c = 1.5$ and $c/t = 44.3$) with diamond mode showing various stages of the deformation process corresponding to a series of cardboards.

3.5.1.2 Idealized Collapse Mechanism

Generally, two sorts of plastic hinges, fixed horizontal hinges and inclined travelling (also known as rolling) hinges are involved. In the Type III basic collapse element reported herein, at one inward plate at the corner, there are only stationary horizontal plastic hinges but no travelling hinges, which means only bending energy contributes to the collapse of this tube section. However, at the other inward plate, both travelling and fixed horizontal plastic hinges exist, as shown in Fig. 3.17 and

Fig. 3.18.

Fig. 3.17. Stationary hinges (horizontal lines) and travelling regions (shadowed regions, includes dotted and netted region) on the developed tube surface of diamond mode.

Note: In the shaded region, the dotted region (such as surface UCO) is the area in which the inclined hinges are bent and unbent travelling through the whole area to leave the region eventually flat [50]. The surface BJV in the netted region will be fully contacted and overlapped with the rigid trapezoidal plate $ABJI$ during collapse.

Fig. 3.17 shows a developed view of the final form of diamond mode. Especially in type III basic folding element, three travelling hinges are involved during the deformation, i.e. inclined hinges BV , FV , and horizontal travelling hinge JV , with the remaining hinges, such as BU , BS , JI , VL , FQ and FR , being stationary. The horizontal hinges form around the circumference of the tube at the mid-length of the fold [50]. The length of the stationary hinges does not change except for VL , which decreases since it moves inward. In Fig. 3.17, the dotted regions are the areas in which the inclined hinges are bend and unbent travelling through the whole areas to leave the region eventually flat. In Type III basic folding element, the surface BJV in the netted region will be fully contacted with the rigid trapezoidal plate $SBJI$ during collapse. Fig. 3.18 represents an intermediate stage of collapse, in which the travelling hinge originates from the vertical corner (BF) and the inclination angles increase as point B moves downwards. In particular, the travelling hinge JV coincides with the horizontal hinge IJ and moves inward increasing the length. In an idealized model, the angle of rotation of the side plates $\alpha (= \angle BFJ)$, and the angle

between travelling hinge and tube axis $\beta(=\angle BFV)$ are shown. Both angles increase progressively from zero to $\pi/2$.

Fig. 3.18. Close-up view of the diamond mode with a type III basic fold element: (a) FE model; (b) cardboards; and (c) theoretical model.

Note that the surface BJV will be fully contacted and overlapped with the rigid trapezoidal plate $SBJI$. The point V is on the hinge line IJ .

Some important angles are examined to determine the final orientation of the inclined hinge. They are γ and γ_1 as shown clearly in Fig. 3.18c, which are the angle between the sides of the collapse tube and travelling hinge in type I folding element and the angle between the sides of the collapse tube and travelling hinge in type III folding element, respectively. To determine the relationship between both angles, the collapse mode viewed from the terminal collapsed state of plastic deformation is shown in Fig. 3.19. For a complete collapse of a single fold, as the tube is inextensible, the length of NO (or PK) and IJ at mid-length are constant, which are just the length of the side(c). These angles must be determined since

they affect the final status of collapse. The initial angle between two adjacent plates is $2\psi_0 (= 2\pi/3)$.

Fig. 3.19. Collapse mode of a complete collapse of a triangular tube with diamond mode (plan view): (a) FE model; (b) cardboards; and (c) theoretical model.

Note: the letters in bracket are the nodes at the bottom of complete collapsed fold.

In Fig. 3.19c, we have $\gamma_1 = \angle BVJ = \angle VBS$ and $BJ = FJ = H$. The initial values of these two angles are the same, i.e. $\pi/2$. Moreover, $2\gamma_1 < \angle JBS = \pi/2$, see Fig. 3.19c.

In $\triangle NTV'$, we have, $NT = c/2, NV' < BS < c$ and hence

$$\sin \gamma_1 = NT/NV' > 0.5 \Rightarrow \pi/6 < \gamma_1 < \pi/4 \quad (3.3)$$

and in $\triangle MXS$, $\angle SXM = \angle MSX = \gamma$, gives

$$\pi/2 - \gamma_1 + 2\gamma = \pi \Rightarrow 2\gamma = \gamma_1 + \pi/2 \quad (3.4)$$

From the above two equations, the limits of the angle between two adjacent plates

are $(2\psi_0 =) 2\pi/3 < 2\gamma < 3\pi/4$. It is different from the traditional basic folding element, where the angle is equal to $2\psi_0$. Thus, in the current collapse mode, the angle between two adjacent plates as viewed along the tube axis is not kept as constant.

Also from Fig. 3.19c, in $\triangle BYS$

$$\left. \begin{aligned} \sin \gamma_1 &= YS/BS \\ YS &= YA + AS = (c + H \cot \gamma)/2 \\ BS &= c - XI = c - H \cot \gamma/2 \end{aligned} \right\} \Rightarrow 2 \sin \gamma_1 = (c + H \cot \gamma)/(c - H \cot \gamma/2) \quad (3.5)$$

Substituting the above equation back into Eq. (3.4), the values of final inclination angles at the complete collapse status are

$$\left\{ \begin{aligned} \gamma_{\min} &= \arccos \left\{ \sqrt{[2c^2 + H^2 - \sqrt{H^2(3c^2 + H^2)}]/(8c^2 + 2H^2)} \right\} \\ \gamma_{1\min} &= 2\gamma_{\min} - \pi/2 \end{aligned} \right\} \quad (3.6)$$

For the extreme case, the gap between two plates (MM') is zero, expressed as

$$MM' = c - H \cot \gamma - 2H \tan \gamma_1 = c - H \cot \gamma + 2H \cot 2\gamma = 0 \quad (3.7)$$

Combining Eqns (3.4) and (3.5) results in

$$-2 \cos 2\gamma = (c + H \cot \gamma)/(c - H \cot \gamma/2) \quad (3.8)$$

Solving Eqns (3.7) and (3.8), the minimum value of c/H and γ is obtained

$$(c/H)_{\min} = \sqrt{5} = 2.24 \text{ and } \gamma_{\min} = 1.15 \quad (3.9)$$

We may assume that $c = \lambda \cdot H$ to estimate the final orientation angles, where

$\lambda (\geq \sqrt{5})$ is the coefficient. The final inclination angles in Eq. (3.6) reduces to

$$\left\{ \begin{aligned} \gamma_{\min} &= \arccos \left[\sqrt{(1 + 2\lambda^2 - \sqrt{1 + 3\lambda^2})/(2 + 8\lambda^2)} \right] \\ \gamma_{1\min} &= 2\gamma_{\min} - \pi/2 \end{aligned} \right\} \quad (3.10)$$

For a given value of λ , the explicit solution of both angles is listed in Table 3.3. It is evident that the angle $\gamma_{\min}$ doesn't change too much, up to 0.1 ($= 5.7^\circ$). For convenience, we may assume that the angle between the two adjacent plates ($2\gamma_{\min}$) remains constant.

Table 3.3. $\gamma_{\min}$ and $\gamma_{1\min}$ for a given value of λ

λ	$\gamma_{\min}$ (radian)	$\gamma_{1\min}$ (radian)
∞	1.047	0.524
100	1.050	0.529
10	1.072	0.573
5	1.096	0.620
4	1.107	0.644
3	1.126	0.681
$\sqrt{5}$	1.150	0.730

To evaluate the energy dissipation of type III basic folding element, the relationships of those angles at different deformation status are examined first.

From Fig. 3.18c, in ΔBJF

$$BF = 2 \cdot BJ \cdot \cos \alpha = 2H \cos \alpha \quad (3.11)$$

In ΔBJV

$$BV = BJ / \sin \gamma_1 = H / \sin \gamma_1 \quad (3.12)$$

In ΔBVF

$$\cos \beta = (BF/2) / BV \quad (3.13)$$

Substituting Eqns (3.11) and (3.12) into Eq. (3.13) yields

$$\sin \gamma_1 = \cos \beta / \cos \alpha \quad (3.14)$$

It is evident that the projection of ΔBVF on plane BJF is just ΔBVF , which is also parallel to the tube axis. Consequently,

$$\cos(2\gamma_{\min}) = \cos\left(\frac{\pi}{2} - \gamma_{1\min}\right) = \frac{S_{\Delta BJF}}{S_{\Delta BVF}} = \frac{H^2 \sin 2\alpha / 2}{(H^2 / \sin^2 \gamma_1) \cdot \sin 2\beta / 2} \quad (3.15)$$

Substituting Eq. (3.14) into (3.15), yields

$$\tan \beta = \tan \alpha / \sin \gamma_{1\min} \quad (3.16)$$

Furthermore, α and γ are determined by the angle between two adjacent plates $\gamma_{\min}$, as expressed by Abramowicz and Jones [41], through

$$\tan \gamma = \tan \gamma_{\min} / \sin \alpha \quad (3.17)$$

3.5.1.3 Average Crushing Force

When analyzing the energy absorption behavior of a structure, the average crushing force (indicated as F_{av}) is usually concerned because it is the most significant parameter in evaluating the energy absorption capacity [1]. For a complete collapse of a single fold, the plastic energy includes bending and stretching energy. In ΔBJV

$$JV = BJ/\tan \gamma_1 = H \cot \gamma_1 \quad (3.18)$$

The energy contribution along the stationary hinge lines (SB , XJ , QF , BU , FR and VL) is

$$\begin{aligned} E_1 &= 2\pi M_0 c + M_0 \int_{\frac{\pi}{2}}^{\gamma_{1\min}} H \cot \gamma_1 d\gamma_1 = 2\pi M_0 c + M_0 H \cdot \ln(\sin \gamma_{1\min}) \\ &= 2\pi M_0 c + M_0 H \cdot \ln(-\cos 2\gamma_{\min}) \end{aligned} \quad (3.19)$$

where $2\gamma_{\min} > 2\pi/3$. From Eqns (3.12) and (3.18), the total length of travelling hinges BV , FV and JV is expressed as

$$L_1 = 2H/\sin \gamma_1 + H \cot \gamma_1 \quad (3.20)$$

The bending energy in the travelling hinges BV , FV and JV is then given by

$$E_2 = 4M_0 I_3'(\gamma_{\min}) H^2/b \quad (3.21)$$

where b is the radius of the toroidal shell in the meridian direction [55]. The stretching energy in a sheet passing over a toroidal surface [3] around plane $BJVF$ is

$$E_3 = 16M_0 I_1'(\gamma_{\min}) Hb/t \quad (3.22)$$

where $\pi/3 < \gamma_{\min} \leq 1.15$, $I_1'(\gamma_{\min})$ and $I_3'(\gamma_{\min})$ are defined by

$$\begin{cases} I_1'(\gamma_{\min}) = \frac{\pi}{(\pi - 2\gamma_{\min}) \tan \gamma_{\min}} \int_0^{\frac{\pi}{2}} \cos \alpha \left[\cos \gamma_{\min} - \cos \left(\gamma_{\min} + \frac{\pi - 2\gamma_{\min}}{\pi} \beta \right) \right] d\alpha \\ I_3'(\gamma_{\min}) = \frac{1}{\tan \gamma_{\min}} \int_0^{\frac{\pi}{2}} \left(\frac{\cos \alpha}{\sin \gamma_1} + \frac{\cos \alpha}{2 \tan \gamma_1} \right) d\alpha \end{cases} \quad (3.23)$$

where the relationships between those angles are determined by means of Eqns (3.14), (3.16) and (3.17). Hence, Eq. (3.23) is solved for a given value of $\gamma_{\min}$, as listed in Table 3.4. The integrals of $I_1(\gamma)$ and $I_3(\gamma)$ in basic folding element type I

are calculated by Eqns (2.4) and (2.5) as well.

Table 3.4. Values of integrals for a given value of $\gamma_{\min}$

$\gamma_{\min}$	$I_1(\gamma_{\min})$	$I_3(\gamma_{\min})$	$I_1'(\gamma_{\min})$	$I_3'(\gamma_{\min})$
1.150	0.255	0.462	0.303	0.654
1.10	0.291	0.530	0.362	0.815
$\pi/3=1.047$	0.330	0.608	0.436	1.048

Thus, the total energy dissipation in basic collapse element type III is

$$E_{\text{III}} = E_1 + E_2 + E_3 \quad (3.24)$$

For the diamond mode, which consists of a layer with a total initial height $2H$ and two Type I and one Type III basic folding elements, the internal energy absorption is $2E_I + E_{\text{III}}$. From Eq. (2.3) in Section 2.1.1.2 and energy balance method, we have

$$2E_I + E_{\text{III}} = F_{av} \cdot 2H \quad (3.25)$$

We assumed that the radii of torodial shell (b) in Type I and Type III basic folding elements are the same. A numerical scheme shows that the relative error for assuming that $b = 2.1t$ is less than 1% for $5 \leq c/t \leq 100$ [41, 44]. It would, therefore, seem reasonable for such assumption in the rest of the theoretical prediction. Hence, the dimensionless mean force equals

$$\begin{aligned} F_{av}/M_0 = & 33.6I_1(\gamma_{\min}) + 0.5\ln[-\cos(2\gamma_{\min})] + 16.8I_1'(\gamma_{\min}) \\ & + 3\pi c/H + 1.9I_3(\gamma_{\min})H/t + 1.0I_3'(\gamma_{\min})H/t \end{aligned} \quad (3.26)$$

which when minimized with respect to H gives

$$H = \sqrt{ct / \left(0.2I_3(\gamma_{\min}) + 0.1I_3'(\gamma_{\min}) \right)} \quad (3.27)$$

From Table 3.4, for different values of $\gamma_{\min}$, Eq. (3.27) becomes

$$H_{\min} = 2.09\sqrt{ct}; H_{av} = 2.30\sqrt{ct}; H_{\max} = 2.43\sqrt{ct} \quad (3.28)$$

Since those values do not vary too much, the average values are approximately regarded as the ideal solution. This gives a mean crushing load

$$(F_{av}/M_0)_{av} = 15.56 + 15.05\sqrt{c/t} \quad (3.29)$$

It is interesting to note that the average crushing load for a square tube and circular tube with ring mode is $F_{av}/M_0 = 2\pi + 8\pi\sqrt{d/t}$ [41] and $F_{av}/M_0 = 24\sqrt{D/t}$ [42], with d and D being the side length of a square tube and diameter of a circular tube, respectively. For the triangular, square and circular tubes with the same outer perimeter, the triangular tube absorbs the lowest energy in the range of $c/t > 20$. As shown in Fig. 3.16, diamond modes are observed in the tubes with $c/t \leq 55.3$, $l/c \leq 2.3$. The above theoretical predictions are further compared with the FE results, as listed in Table 3.5. The values of the half-fold length (H) and the average force (F_{av}) are calculated from the load-displacement curve in numerical simulations. It is found that the average force increases as the thickness increase and the theoretical average forces are in well agree with FE ones. Some discrepancies may be due to an inaccuracy on approximating the friction coefficient in FEA.

Table 3.5. Comparisons among theoretical predictions, FE and experimental results

Mode	c (mm)	t (mm)	c/t	H (mm)		F_{av} (kN)		
				FE	Theory	FE	Theory	Exp.
Diamond	66.4	2.0	33.2	26.3	26.5	34.9	37.3	39.0
		1.5	44.3	23.3	22.9	22.0	21.6	22.3
Rotational symmetry	66.4	1.0	66.4	9.8	9.0	14.5	12.6	10.0
		0.5	132.8	9.0	6.3	4.4	4.4	2.1
Diamond	44.3	2.2	20.1	24.0	24.2	39.3	41.0	68.2
		2.0	22.1	22.5	21.6	34.1	28.7	39.8
		1.5	29.5	19.0	18.7	20.5	18.2	22.9
		1.2	36.9	17.3	16.7	13.2	12.8	13.4
		1.0	44.3	16.3	15.3	9.8	9.6	9.7
		0.8	55.3	13.8	13.7	7.8	6.8	7.2

Note that all tubes are of the same length, $L = 100$ mm, flow stress $\sigma_0 = 332.5$ MPa.

3.5.2 Rotational Symmetrical Mode

From the FE results, when the equilateral triangular tube is considerable thin (e.g., $c/t > 60$), rotational symmetrical mode may occur; in which three identical type III

basic folding elements are involved. A series of photographs of cardboards and the corresponding collapse stages of such mode in FE are shown in Fig. 3.20. At all stages of the deformation process, agreement is seen to be good. Similar to the approach concerning with the diamond mode in previous section, the developed view of final form of deformation mode and the final collapsed view are included from Fig. 3.21 to Fig. 3.23. It is concluded that in this case, the final inclination angles $\gamma_{\min} = \pi/3$, $\gamma_{1\min} = \pi/6$, and the angle between two adjacent plates is kept as constant, $2\psi_0 = 2\pi/3$, due to the rotational symmetry in this mode.

Fig. 3.20. Triangular tubes ($L/c = 1.5$ and $c/t = 66.4$) with rotational symmetrical mode showing various stages of the deformation process corresponding to a series of cardboards.

Fig. 3.21. Stationary plastic hinges (horizontal lines) and travelling region (area with netted shadow) on the developed tube surface for rotational symmetrical mode. Note that the surface such as BJO in netted region will be fully contacted with the rigid trapezoidal plate $ABJN$ during collapse.

Fig. 3.22. Close-up view of the rotational symmetrical mode with three basic fold element type III: (a) FE model; (b) cardboards; and (c) theoretical model.

Fig. 3.23. Top view of the completely crushed triangular tube with rotational symmetrical mode: (a) FE model; (b) cardboards and (c) theoretical model.

Hence, from Eq. (3. 24), the total energy dissipated (E_{III}') in this mode leads to

$$E_{III}' = 2\pi M_0 c + 13.96 M_0 H + 2.00 M_0 H^2 / t \quad (3. 30)$$

After looking into more details about this mode from FEA, as a matter of fact, there always accompanies with another layer of folds with three extensional type II basic

elements (see Fig. 2.3b in Section 2.1.1.2), as illustrated in Fig. 3.24. Hence, the mixed collapse mode consists of two layers with a total initial height $4H$ and three identical type II and three identical type III basic folding elements. The energy dissipation of Type II basic collapse element (E_{II}) is given by Eq. (2.6).

Fig. 3.24. Rotational symmetrical mode of triangle tubes.

Thus, equating the internal energy absorption to the external work gives

$$3E_{II} + 3E_{III}' = F_{av} \cdot 4H \quad (3.31)$$

which when minimized F_{av} with respect to H predicts

$$\begin{cases} H = 1.10\sqrt{ct} \\ F_{av}/M_0 = 12.04 + 17.13\sqrt{c/t} \end{cases} \quad (3.32)$$

From FE analysis, it is found that some of the triangular tubes ($c/t > 60$) developed the rotational symmetric mode. The above theoretical solutions are compared with FEA and good agreements are reached, as listed in Table 3.5.

3.6 Experimental Validations of Triangular Tubes

In order to validate the approximate theoretical predication, experimental investigations were performed. Same as previous polygons, a series of triangular tubes with various thicknesses (t), ranging from 0.8 mm to 2.2 mm, were crushed under the same experimental setup in Section 3.1. The length ($L = 100$ mm) was the same for all specimens, while the side length or flat width (c) was 66.4 mm and 44.3 mm, respectively. Details on specimens' geometry are given in Table 3.5. All

specimens were made of low-carbon steel (*ASTM A36*) and fabricated by EDM wire cutting method as well. All the specimens were annealed before tests to remove any possible residual stress caused in the pre-fabrication processes. Due to the geometrical characteristic of triangular tubes, two cameras were placed to capture the whole description of real deformation history. One shot the front view and the other shot the back. The tests stopped at the onset of visible tearing and large crack lines in the specimens. Load and displacement were recorded by an automatic data acquisition system.

For some typical specimens, satisfactory correlations are found between FE predictions and corresponding experimental results in Fig. 3.25, in which the tubes have the same wall thickness ($t = 1.5$ mm) but different flat width. The experimental forces are somewhat larger than those of the numerical ones, probably due to the strain hardening effect and actual friction effect is not involved in FE analysis and the deformation modes in experiments were not easy to differentiate, as discussed later. The load-displacement curves of all specimens are plotted in Fig. 3.26, from which the average forces are calculated and presented together with the theoretical values in Table 3.5. It is found that the experimental results are slightly higher than those of the theoretical prediction for most cases. It may also be noted that the theoretical average force were based on a complete collapse of the fold. Nevertheless, the average forces for a given deformation in the experiments are found as the absorbed energy divided by deformed distance from the load-displacement curve [164]. The discrepancy between the theoretical and experimental data indicates a shortcoming in the theoretical analysis, which are given minimum values of average force on the assumption of a constant radius of

toroidal shell (b) [50]. Imperfections and frictions during the interactions are also ignored in the theoretical analysis, which maybe another possible error source.

Fig. 3.25. Comparisons of force-displacement curves between experiments and FEA: (a) $c = 66.4$ mm, $t = 1.5$ mm; and (b) $c = 44.3$ mm, $t = 1.5$ mm.

Fig. 3.26. Experimental load-displacement curves of the triangular tubes: (a), (b) $c = 66.4$ mm; and (c), (d) $c = 44.3$ mm.

The progressive collapse histories for typical specimens are shown from Fig. 3.27 to Fig. 3.31. In fact, the diamond mode governed the actual quasi-static progressive behavior of triangular tubes, approximately over the entire experimental range of c/t (< 130). No visual evidence was shown for the rotational symmetrical mode.

Furthermore, unlike the previous FE studies, the folds were not formed sequentially in the experiments. Two layers of folds existed immediately at the initial crushing process. No obvious boundary of each individual fold was observed during crushing. The other main source of discrepancy between the theoretical prediction and experiments is the fact that the theoretical analysis assumes inextensional deformation whereas in the actual experiments there were somewhat extensions, as clearly illustrated in those figures. Furthermore, cracks were observed during the crushing and marked in the corresponding positions. The initiations of compressive cracking in the specimens were observed to be sensitive to the thickness. There appears to be more potential of crack for the thicker tubes. Experimental evidences in Fig. 3.32 show that less fracture occurs when the tube is sufficiently thinner.

Fig. 3.27. Collapse mode for a triangular tube ($c = 66.4 \text{ mm}$, $t = 1.0 \text{ mm}$). Top: front view; bottom: back view.

$\delta = 10 \text{ mm}$ $\delta = 20 \text{ mm}$ $\delta = 30 \text{ mm}$ $\delta = 40 \text{ mm}$ $\delta = 50 \text{ mm}$ $\delta = 60 \text{ mm}$

Fig. 3.28. Collapse mode for a triangular tube ($c = 66.4 \text{ mm}$, $t = 1.5 \text{ mm}$). Top: front view; bottom: back view.

$\delta = 5 \text{ mm}$ $\delta = 15 \text{ mm}$ $\delta = 25 \text{ mm}$ $\delta = 35 \text{ mm}$ $\delta = 45 \text{ mm}$ $\delta = 55 \text{ mm}$

Fig. 3.29. Collapse mode for a triangular tube ($c = 44.3 \text{ mm}$, $t = 0.8 \text{ mm}$). Top: front view; bottom: back view.

Fig. 3.30. Collapse mode for a triangular tube ($c = 44.3 \text{ mm}$, $t = 1.5 \text{ mm}$). Top: front view; bottom: back view.

Fig. 3.31. Collapse mode for a triangular tube ($c = 44.3 \text{ mm}$, $t = 2.0 \text{ mm}$). Top: front view; bottom: back view.

Fig. 3.32. Typical specimens (top: before test; bottom: after test).

3.7 Conclusions

A total of four quasi-static axial compression tests have been conducted on mild steel tube (*ASTM A36*) in order to check if the corners within the tubes could indeed absorb much energy and hence increase the values of *SEA*, structural effectiveness (η) and energy-absorbing effectiveness (ψ). Four types of geometries, i.e. hexagon, octagon, 12-sided and 16-sided star were tested, respectively, with $D/t = 42.3$. As the number of corners varies the value of peak force changes accordingly. To further understand the deformation/failure mechanism of the tubes, corresponding numerical simulations were performed and reasonably good agreements were obtained. It was found that increasing the number of corners may help improve the energy absorption but limited to a certain extent. The 12-sided star shape had the best energy absorption where it produced about 48.5% (experiment) and 18.3% (FEA) improvement in terms of *SEA*, compared with the hexagon of the same cross-sectional area. However, further increasing the number of corners showed a reduction in the energy absorption performance, as demonstrated for the 16-sided star shape. It must be noted that for other values of D/t larger than 50, S16 may be the optimum shape for the thin-walled section. Clearly further more comprehensive studies are needed to investigate this problem.

An approximate analysis was presented for predicting the large plastic deformation of equilateral triangular tubes when they were subjected to axial crushing. The analysis used a modified super folding element method which was developed previously for square tubes. Two kinds of inextensible collapse modes were identified, which are diamond and rotational symmetrical modes. The progressive collapse mechanism for each mode is successfully developed by the energy balance method. Reasonably good agreements have been found between the models and corresponding experiments, which shows the feasibility of using the simplified analytical method to analyze the energy absorption of regular polygonal tubes with odd number of sides. However, for the rotational symmetric mode, further investigations are needed and triggers may be introduced to achieve stable deformation modes [12].

CHAPTER 4

AXISYMMETRIC PLASTIC EXPANSION OF A CYLINDRICAL HOLE IN ISOTROPIC FOAM

A system of two concentric monolithic metal tubes with metal foam sandwiched may be used as a protective structure to contain, for example, explosives inside. The response of such structures subjected to internal pressure should be understood and hence the resistance of the metal foam as well as each of the two monolithic tubes needs to be assessed. As a first step, a numerical analysis using ABAQUS/Explicit [89] is conducted to study the expansion of a cylindrical hole in an infinite block of closed-cell metallic foam, as shown in Fig. 4.1. Detailed deformation history and stress history at the inner boundary are explored in this chapter. The evolution and the size of plastic zone are discussed during the expansion. Based on the isotropic constitutive models with inelastic, linear hardening with densification, the evolution of plastic zone is then compared with the simplified analytical solution.

Fig. 4.1. Schematic representation of the hole expansion under internal pressure p (the initial radius of the hole is $a = 16$ mm at the inner boundary. r is the radius for a generic layer and r_y is the radius of the plastic zone. σ_r and σ_θ are the stress components).

4.1 Finite Element Analysis

A finite element model of the pressure-loaded foam is given below, where the foam is simplified as a continuum of an inelastic, linear hardening with densification, represented by a uniaxial compressive constitutive relationship. Previous studies [84] have found that the plastic deformation occurs locally even before the stresses reach the plastic collapse strength, and so the elastic stage of crushing metallic foam would be very short. Therefore, the main research focus of our study will be on the plastic response in the expansion. Due to the axisymmetry of the problem, including the circular hole and the loading, an axisymmetrical model is employed. The initial radius of the hole in the infinite block of foam is assumed to be $a = 16$ mm. Using the convention for foams, a compressive stress is regarded as positive while tensile stress is negative.

4.1.1 Foam Constitutive Relationship

The crushable foam model assumes uniaxial loading in any principal direction while insignificant deformation occurs in the other directions. Besides, Poisson's ratio in plastic regime is assumed to be non-zero and the load conditions in principal directions are interdependent [23]. As stated in the literature review, typical yield surface of elliptical shape in the $(\sigma_m - \sigma_e)$ space [83] is shown in Fig. 4.2. Isotropic hardening model is adopted for the metallic foam in the current study, where the yield ellipse is centered at the origin in the stress space.

Fig. 4.2. Yield surface for metallic foam in effective–hydrostatic stress space.

Note: Different from conventional notion, the hydrostatic axis positive direction represents compression.

4.1.2 Material Property

Typical properties for commercial ALPORAS[®] aluminium foam of the relative density (ρ^*) of 8% are used in the FE analysis. The elastic properties of the foam are given as Young's modulus $E = 1.1$ GPa and Poisson's ratio $\nu = 0.3$. The uniaxial compressive constitutive relation is assumed to govern the quasi-static response of the foam under internal pressure, which is idealized in Fig. 4.3 with initial yield stress $\sigma_y^0 = 1.2$ MPa, the linear hardening modulus $E_1 = 1.2$ MPa and, after the locking strain $\varepsilon_D = 0.65$ the densification modulus $E_D = 120$ MPa. In addition, to determine the shape factor of the yield ellipse that defines the relative magnitudes of the axes, the value of compression yield stress ratio k should be defined first. According to the guideline in ABAQUS [89], for a valid yield surface the choice of yield stress ratio must be such that $0 \leq k < 3$. For many low-density foams, the initial yield surface is close to a circle (corresponding to $\alpha = 1$) in the stress plane. From experimental data by Shen [134], Doyoyo and Wierzbicki [165], it is taken as 1.21. The shape factor is then 1.32 by using

$$\alpha = \frac{3k}{\sqrt{9-k^2}} \quad \text{with} \quad k = \frac{\sigma_y^0}{p_c^0} \quad (4.1)$$

where p_c^0 is the initial yield stress under hydrostatic compression.

Fig. 4.3. Idealized engineering stress-strain curve of ALPORAS[®] aluminium foam with Young's modulus $E = 1.1$ GPa, initial yield stress $\sigma_y^0 = 1.2$ MPa, linear hardening modulus $E_1 = 1.2$ MPa, and after densification strain $\varepsilon_D = 0.65$ the densification modulus $E_D = 120$ MPa.

Note that in drawing this curve, the elastic region is ignored. In ABAQUS, the engineering strain is converted to the corresponding logarithmic strain.

4.1.3 Finite Element Model

This study treats the present hole expansion problem as quasi-static. To explore the plastic deformation behavior of the foam and provide some basic information for comparison with the analytical consideration, which will be given in the next section, ABAQUS/Explicit finite element program is employed. The material is modeled as crushable foam and the hardening curve is represented by the true (Cauchy) stress as a function of the corresponding logarithmic strain. Radial displacement of up to 4 mm is gradually applied along the hole boundary of the foam (i.e. initially at $r = 16$ mm) to simulate the expansion process. The infinite foam beyond 100 mm is represented by using one 4-node, axisymmetric, infinite element with the element type of *CINAX4*. No axial displacement is allowed at both the top and the bottom surfaces of the foam, and so it is a plane strain problem.

To obtain reasonably accurate results, the mesh density of the foam is also determined through mesh sensitivity studies shown in Fig. 4.4. More refined meshes would improve the smoothness of the pressure–deflection curves. The current results are adequate to show the nature of the response. A mesh size of 0.25 mm in the region within 40 mm from the hole boundary has therefore been selected. The outer radius of the foam is 100 mm. The mesh size is 1 mm between the radius of 56 mm and 100 mm. Detailed results including the stress state, strain state and the evolution of the yield zone will be presented and discussed in Section 4.3.

Fig. 4.4. Verification of mesh sizes in foams.

4.2 Analytical Consideration

The analytical consideration starts by following the theoretical analysis of plastic deformation in a thick-walled cylinder [166]. In the present case, the axial displacement on both the top and bottom surfaces is given as $w = 0$. This corresponds to a plane strain problem, i.e. $\varepsilon_z = 0$.

4.2.1 Initial Yielding Pressure

The yield criterion we have adopted for the foam is as given in Eq. (2.18), and the effective stress and hydrostatic stress are expressed in terms of the stress

components:

$$\begin{cases} \sigma_e = \sqrt{\left[(\sigma_r - \sigma_\theta)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2\right]}/2 \\ \sigma_m = (\sigma_r + \sigma_\theta + \sigma_z)/3 \end{cases} \quad (4.2)$$

where σ_r , σ_θ and σ_z are the radial, circumferential and axial stresses, respectively.

For an elastic-plastic tube under internal pressure only, the elastic stress at radius r is given by

$$\sigma_r = -p \left(\frac{b^2}{r^2} - 1 \right) / \left(\frac{b^2}{a^2} - 1 \right) = -\frac{a^2}{r^2} p \quad \text{for } b \rightarrow \infty \quad (4.3a)$$

$$\sigma_\theta = p \left(\frac{b^2}{r^2} - 1 \right) / \left(\frac{b^2}{a^2} - 1 \right) = \frac{a^2}{r^2} p \quad \text{for } b \rightarrow \infty \quad (4.3b)$$

$$\sigma_z = pa^2 / (b^2 - a^2) = 0 \quad \text{for } b \rightarrow \infty \quad (4.3c)$$

where p is the internal pressure applied on the boundary of the hole and b is the outer radius of the block, the value of which approaches infinity here. Substituting the above equations into the yielding criterion Eq. (2.18), the maximum pressure before initial yielding, P_e , is

$$p_e / \sigma_y^0 = \sqrt{\left[1 + (\alpha/3)^2\right]}/3 = 0.634 \quad (4.4)$$

Note that from the finite element results shown in Fig. 4.5, the value of the radial stress or the maximum pressure at the initial yielding is 0.76 MPa or $p_e / \sigma_y^0 = 0.633$.

This value of the initial yielding pressure agrees well with that of the analytical solution in Eq. (4.4). It is interesting to note that $p_e / \sigma_y^0 = 1/2 (=0.5)$ and $p_e / \sigma_y^0 = 1/\sqrt{3} (=0.577)$ for metals satisfying Tresca and von Mises criteria, respectively. This pressure for initial yielding does not depend on the radius, which may be easily seen from the dimensional analysis for this problem: radius is the only parameter with length dimension and there would be “no room” for it to come in.

Fig. 4.5. Results of FE analysis: (a) non-dimensional internal pressure vs. displacement curve; (b) comparison between the non-dimensional radial stress and circumferential stress at the hole boundary ($r = a = 16$ mm).

4.2.2 Plastic Deformation after Initial Yielding

During deformation the stresses in the foam must always satisfy the equilibrium equation

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (4.5)$$

Note that in this equation, σ_r and σ_θ are positive when they are tensile. Substituting the value of the foam yield stress $\sigma_y^0 (= 1.2 \text{ MPa})$ into Eq. (2.18), we obtain the initial yield surface as

$$\sigma_r^2 + \sigma_\theta^2 = 1.44 + 0.512\sigma_r\sigma_\theta \quad (4.6)$$

which defines a domain in the space of these two stresses. Thus to ensure real roots of σ_θ existing, we must have

$$\Delta = (0.512\sigma_r)^2 - 4(\sigma_r^2 - 1.44) = 5.76 - 3.74\sigma_r^2 \geq 0 \quad (4.7)$$

The above equation indicates that during the initial yield state the value of radial stress should not be beyond 1.242 MPa ($= 1.035\sigma_y^0$). Eliminating σ_θ from Eqns (4.5) and (4.6) leads to

$$\frac{d\sigma_r}{\sqrt{5.76 - 3.74\sigma_r^2 - 1.488\sigma_r}} = \frac{1}{2} \frac{dr}{r} \quad (4.8)$$

Because of the discontinuities in the integrant, an explicit solution is not available for indefinite integral in this case. Letting the internal pressure p be large enough to cause plastic yield of the material within radius r_y , then integrating Eq. (4.8) gives

$$\int_{-p_e}^{-p} \frac{d\sigma_r}{\sqrt{5.76 - 3.74\sigma_r^2 - 1.488\sigma_r}} = \int_a^{r_y} \frac{1}{2} \frac{dr}{r} = \frac{1}{2} \ln \frac{r_y}{a} \quad (4.9)$$

Hence for a given value of pressure, the value of r_y defining the plastic zone can be obtained numerically from the above equation. By numerical integration, an empirical relationship of the dimensionless radius of plastic zone versus dimensionless pressure is given by

$$\frac{r_y}{a} = 0.8117 \left(\frac{p}{\sigma_y^0} \right)^2 - 0.3451 \frac{p}{\sigma_y^0} + 0.8969, \quad \text{for } \frac{p_e}{\sigma_y^0} \leq \frac{p}{\sigma_y^0} \leq 1.035 \quad (4.10)$$

At the initial yielding, the above equation gives $r_y/a = 1$, as expected. When the non-dimensional internal pressure p/σ_y^0 is less than p_e/σ_y^0 , no plastic deformation occurs and the radius of plastic zone r_y does not exist. When the internal pressure exceeds the above definition domain, the associated flow rule of metallic foam is not well defined in the previous research of yielding behavior of metal foams. Nevertheless, using the same approach, it is possible to develop a solution for the radius of plastic zone when the internal pressure exceeds 1.242 MPa ($=1.035\sigma_y^0$). From the constitutive relationship, the yield stress σ_y is 1.98 MPa when the effective strain reaches the value of locking strain $\varepsilon_D = 0.65$. Thus the corresponding yield surface in Eq. (4.6) is rewritten as

$$\sigma_r^2 + \sigma_\theta^2 = 3.92 + 0.512\sigma_r\sigma_\theta \quad (4.11)$$

From the above equation, the internal pressure P increases to P_d at the locking position, where the foam starts to densify and P_d is 1.623 MPa ($=1.3525\sigma_y^0$). The plastic deformation zone is then divided into two separate regions, depending on

whether the radial stress is less than 1.242 MPa in Eq. (4.10). In the first region, the radial stress along the radial direction is in the range from $p_d (=1.623 \text{ MPa})$ to $1.035\sigma_y^0 (=1.242 \text{ MPa})$. In the second region, the radial stress ranges from 1.242 MPa to $p_e (=0.76 \text{ MPa})$. Beyond the second region, there is no plastic deformation. The radius of the first region is r_y^0 and that of the second region is the radius of plastic deformation zone, r_y . It is assumed that within each region the yield surface does not change considerably because of the low value of hardening modulus. Therefore, the yield surface for each individual region is expressed as Eqns (4.11) and (4.6). For the former region, Eq. (4.9) becomes

$$\int_{-1.242}^{-p_d} \frac{d\sigma_r}{\sqrt{15.6816 - 3.74\sigma_r^2 - 1.488\sigma_r}} = \int_a^{r_y^0} \frac{1}{2} \frac{dr}{r} = \frac{1}{2} \ln \frac{r_y^0}{a} \quad (4.12)$$

The calculated radius for the former region is $r_y^0 = 18.6653 \text{ mm}$. With regard to the latter region of the plastic zone, the relationship between the pressure and the radius can be obtained by numerical integration of the following equation:

$$\int_{-p_e}^{-1.242} \frac{d\sigma_r}{\sqrt{5.76 - 3.74\sigma_r^2 - 1.488\sigma_r}} = \int_{r_y^0}^{r_y} \frac{1}{2} \frac{dr}{r} \quad (4.13)$$

Solving the above integration, we obtain the radius of the total plastic deformation zone $r_y = 26.3789 \text{ mm}$ at the locking strain. Similarly, when the internal pressure p is less than the critical densified pressure p_d , the radius of the first region can be solved by replacing the pressure p_d with pressure p from Eq. (4.12). Upon substituting each calculated radius of the first region r_y^0 into Eq. (4.13) and using the quadratic-polynomial fitting for the numerical results, we have the radius of plastic zone in terms of the applied internal pressure, namely,

$$\frac{r_y}{a} = -0.1339 \left(\frac{p}{\sigma_y^0} \right)^2 + 1.0602 \frac{p}{\sigma_y^0} + 0.4572, \quad \text{for } 1.035 \leq \frac{p}{\sigma_y^0} \leq 1.3525 \quad (4.14)$$

The non-dimensional relationship between the internal pressure and the radius of the plastic zone is plotted in Fig. 4.6. It is shown that after the early yielding stage, the expanding rate of the plastic zone is slower than that in the previous plastic deformation stage. For the internal pressure beyond the initial yield stress, a distinctive disagreement between the analytical solution and the numerical result is found in this stage. This is probably because under the non-proportional loading in the expansion, the deformation theory of plasticity adopted here may result in errors and not be suitable for the current loading case. More appropriately, a theoretical model based on the incremental theory of plasticity should be formulated in future to predict the quasi-static response.

Fig. 4.6. Analytical non-dimensional radius of plastic zone vs. internal pressure.

4.3 Results and Discussions

4.3.1 Stress Distribution

From the finite element analysis, the stresses, strain and size of the plastic zone at different magnitudes of displacement of the internal hole are summarized in Table 4.1. They are also plotted in Fig. 4.7. On one hand, it is evident that the radial stress at the hole boundary increases during the whole process of expansion, rapidly before the initial yield but much more smoothly with the evolution of yielding zone.

On the other hand, the magnitude of circumferential stress at the hole boundary is increasing during the initial elastic stage up to point *A*, but decreasing afterwards. After this point, the effective stress or Mises stress decreases for a while, and the mean stress increases in the early stage of yielding zone's evolution. The tensile circumferential stress decreases at point *D*, where the hydrostatic stress is equal to 0.4 MPa or $\sigma_y^0/3$. After this critical point, the nodal circumferential stress at the hole boundary increases and remains tensile afterwards. Overall as the expansion region is broadened the plastic zone of the metallic foam evolves further, which can be seen from both the values of the radius and the width of the plastic zone. The magnitudes of radial stresses are larger than those of the circumferential stress due to the fact that during the expansion of such foam the radial compression dominates.

Also from Fig. 4.7, two stages of stress state are revealed from the FE results, i.e. initial yield stage I and further hardening-densification stage II, respectively. In the former stage, the radial stress is smaller than the initial yield stress of the foam under uniaxial compression, σ_y^0 , and in the latter stage the radial stress is greater than σ_y^0 . Yield point *D* is the transition point between the two stages. Elastic deformation only takes place at the initial period in stage I and plastic deformation occurs in the major range of expansion. In the stress stage I, the radial stress and circumferential stress are both linear up to the elastic limit at initial yield at point *A*. Beyond the elastic limit, they become nonlinear and, in particular, the circumferential stress drops to almost zero while the radial stress still increases with the displacement. It should be noted that in stage II both stresses are gradually increasing, but not at the same rate.

Table 4.1 Stress distribution and the related plastic zone property obtained from FEA

Point	Displacement δ (mm)	Radial stress $\sigma_r = p$ (MPa)	Circumferential stress σ_θ (MPa)	Effective stress σ_e (MPa)	Mean stress σ_m (MPa)	Outer radius of plastic zone r_y (mm)	Width of plastic zone W (mm)	Equivalent strain ε_{eng}
<i>A</i>	0.016	0.76	-0.76	1.31	0.00	16.00	0.000	0.1%
<i>B</i>	0.024	1.02	-0.43	1.28	0.23	19.25	3.245	0.2%
<i>C</i>	0.032	1.16	-0.17	1.24	0.34	21.25	5.239	0.5%
<i>D</i>	0.040	1.22	-0.01	1.22	0.40	22.00	5.982	1.6%
<i>E</i>	0.120	1.29	-0.10	1.34	0.40	23.75	7.653	10.9%
<i>F</i>	0.216	1.34	-0.14	1.41	0.40	24.75	8.558	17.0%
<i>G</i>	0.480	1.42	-0.22	1.54	0.40	26.5	10.046	28.5%
<i>H</i>	0.631	1.46	-0.25	1.60	0.40	27.25	10.646	33.8%
<i>I</i>	0.807	1.50	-0.27	1.65	0.41	28.00	11.221	39.4%
<i>J</i>	0.959	1.52	-0.30	1.69	0.41	28.75	11.819	43.9%
<i>K</i>	1.129	1.55	-0.32	1.73	0.41	29.50	12.400	48.5%
<i>L</i>	1.331	1.57	-0.34	1.77	0.41	30.25	12.949	53.7%
<i>M</i>	1.610	1.60	-0.38	1.82	0.41	31.25	13.671	60.3%
<i>N</i>	2.092	1.65	-0.43	1.90	0.41	32.75	14.690	65.0%
<i>O</i>	3.987	1.77	-0.62	2.15	0.38	38.25	18.301	104.2%

Note: Point *A* means the onset of initial yield; point *D* indicates when the hydrostatic stress remains approximately constant; and point *N* indicates densification or locking strain reached.

Fig. 4.7. Stress versus displacement curves at $r = a = 16$ mm from FE analysis: (a) stage I of expansion; and (b) stress state at stage II.

4.3.2 Evolution of the Plastic Zone

In comparison with the yield surface under uniaxial compression, the non-dimensional radial stress versus circumferential stress obtained from the finite element analysis is plotted in Fig. 4.8. Before point A, it is elastic deformation and the two stresses are of the same magnitude but opposite sign. Afterwards, they

closely satisfy the yield condition defined by Eq. (4.6). However, later at point *D*, both the stresses increase in magnitude again, similar to those in the early elastic deformation. This could be attributed to the hardening effect of crushable foam, with enlarged yield surface in the principal stress space. Considerable hardening occurs over an initial strain increment of 1.62%, since the constant hydrostatic stress component has less contribution towards the evolution of the yield surface. The slope of line is approximately equal to that of a line from the origin to the initial yielding point *A*, which is -1. It is seen that point *D* separates stages I and II as well.

Fig. 4.8. σ_θ and σ_r in non-dimensional principal stress space at $r = a = 16$ mm (compressive stress is defined positive).

Note that the solid line is for FEA and the dashed line represents the analytical initial yield surface in Eq. (4.6).

In order to further examine the evolution of yield surface during foam's expansion, the locus of yield points is plotted together with the initial elliptical yield surface in terms of σ_m and σ_e , again non-dimensionalized by the initial yield stress of the

foam material σ_y^0 , as shown in Fig. 4.9. After the onset of plastic strain at yield point A, the plastic yield surface is enlarged compared with the initial yield surface ellipse and in a geometrically self-similar manner. Again the similar slightly hardening effect could be observed until yield point D in stage I, where the effective stress reaches the critical value of σ_y^0 , which confirms the isotropic hardening in the crushable foam as well. The value of σ_e decreases initially with the hydrostatic stress component before the yield point D. But subsequently in the stress stage II the stress path given by the simulation shows no change in the hydrostatic stress component. It is shown that when the yield surface and the loading path for uniaxial compression intersect at point D, the shape of the subsequent yield surface changes dramatically. From the yield criterion of Eq. (2.18), the effective stress increases in the expansion process since the yield stress increases. Oscillations can be observed in the expansion of plastic zone during the evolution of the yield surface in the compressible elements.

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Fig. 4.9. Stress state in non-dimensional effective-hydrostatic stress space: (a) stage I; (b) all the expansion stages (at yielding point D , $\sigma_m/\sigma_y^0 = 1/3$).

Note that the solid line represents the values from FEA, the dashed line denotes the analytical initial yield surface and the dotted line is the loading path for the uniaxial compression.

Although the initial and subsequent yield surfaces could describe the deformation history, the evolution of plastic zone may give us more detailed information about the deformation mechanism. Comparisons are made in Fig. 4.10a with the analytical solution developed in Section 4.2 for the size of the plastic zone. It is noted that the calculated radius of plastic zone versus the internal pressure matches the predicted solution in Eq. (4.10) before the critical point D and diverts a little as the pressure gradually increases. When the internal pressure is beyond σ_y^0 , the predicted results from Eq. (4.14) are much smaller than the calculated values obtained by finite element analysis, but closer to that given by polynomial curve fitting from Eq. (4.10).

Fig. 4.10. (a) Non-dimensional radius of plastic zone vs. internal pressure. Both solid lines are from FEA and the other two lines (dashed and dotted) are the corresponding results from analytical solution; and (b) non-dimensional radius of plastic zone vs. displacement at the hole boundary from FEA.

As shown in Fig. 4.10a, the width of the plastic zone is almost proportional to the internal pressure. In other words, when the internal pressure increases, the corresponding size of the plastic region extends. In addition, the dimensionless outer radius of plastic zone and the present position of the hole boundary during

expansion are also plotted against the corresponding displacement in Fig. 4.10b to describe the evolution in the subsequent plastic yield process. In fact, the present position of the hole boundary is the inner radius of the plastic zone during expansion. In stage I, the plastic zone spreads almost linearly with the displacement. However, when it approaches the transition to stage II, the ascending tendency of plastic zone evolution slows down and is even smooth during the large deformation period as more densification elements emerge.

To further investigate the characteristics at the transitional point D , which separates the two stress stages, the developments of plastic zone at different radial positions are examined. Since the outer radius of plastic zone at point D is 22 mm from Table 4.1, which means that the corresponding width of plastic zone W is 6 mm, the yield surface in the effective-hydrostatic stress space for different radial positions is examined. Several nodes located at radial positions $r = 19$ mm, $r = 20$ mm and $r = 22$ mm are selected. The related dimensionless yield surface from FE and the subsequent stress states are plotted in Fig. 4.11. Initially when the node at the hole boundary starts to experience the plastic deformation, other nodes are still under elastic deformation though for a short period (shown in Fig. 4.11a). When the yield state moves along the yield surface to the point B , the yield state of node located at 3 mm away from the hole boundary shifts directly from the elastic to the initial yield ellipse. However, instead of exactly following the original yield surface, the yielding state at this moment is not at the vertex but somewhat off it. After passing yield point D , since the nodes at the hole boundary positioned at $r = 16$ mm have already suffered the plastic strain, the plastic deformation rapidly propagates along the radial direction, similar to the nodes at the hole boundary.

Fig. 4.11. Stress states of different nodes: (a) at $r = 16$ mm and $r = 19$ mm. Note that solid circles indicate points A, B, C, and D during FEA. The corresponding points in the analytical initial yield surface are indicated using empty circles; and (b) at $r = 16$ mm, $r = 20$ mm and $r = 22$ mm in FEA. Solid circle indicates the transition point D at $r = 16$ mm, and the corresponding points D' and D'' at $r = 20$ mm and $r = 22$ mm are indicated using an empty circle and solid square.

The same phenomenon is shown Fig. 4.11b for two nodes positioned at $r = 20$ mm and $r = 22$ mm. At the outer radius of the plastic zone with the radial position of 22

mm, elastic deformation could still be revealed from Fig. 4.11b. At the critical point D , the dimensionless hydrostatic stresses for the node located at $r = 20$ mm and $r = 22$ mm are 0.113 and 0.006, respectively. Similarly, the initial yielding points on the yield surface for the nodes on each layer move closer to the pure shear line, which is on the vertical axis, where the von Mises stress dominates. The closer the node is to the hole boundary, the sooner the plastic strain appears. From the commencement of initial yielding point at each node on a different layer, clear progressive expansion could be observed in Fig. 4.11. It is noted that once the yield surface or the subsequent yield curve intersects with the uniaxial compressive stress path, the stress state will immediately change its evolution path to follow an approximately vertical line, parallel to the dimensionless effective stress axis.

The radial and circumferential strains at the node at the hole boundary are also shown in Fig. 4.12. From the initial yield state to the critical yield point D , the radial strain increases monotonically. The hardening effect plays an important role in the propagating behavior of plastic zone. Thus the radial strain increases until the densification starting at point N , where the equivalent strain is equal to the locking strain ε_D and the densified layer of foam appears at the hole boundary. The tensile circumferential strain increases linearly with the displacement, as expected from geometry. It is evident that the radial strain is larger than the circumferential one.

Fig. 4.13 plots the dimensionless pressure against the equivalent plastic strain at the hole boundary. It is seen that the critical point D is also the transition point, dividing the strain variation into two stages. Significant hardening occurs at the transition point where the equivalent strain is 1.62%.

Fig. 4.12. Strain vs. non-dimensional displacement at hole boundary ($r = 16$ mm) obtained from FEA: (a) radial strain; and (b) circumferential strain.

Note that point N indicates the instant when densified layer first occurs.

Fig. 4.13. Plot of non-dimensional internal pressure vs. equivalent plastic strain obtained from FE analysis.

4.3.3 Examination of the Plastic Flow Rule

For many types of low-density foam, the plastic flow rule in ABAQUS/Explicit simulation is non-associated with the yield surface, and so the plastic flow behavior has to be taken into account in our analysis on the expansion. Apart from the initial and subsequent yield surfaces, the flow potential for the isotropic hardening model is chosen as [13]

$$g = \sqrt{\sigma_e^2 + \beta^2 \sigma_m^2} = \sqrt{\frac{(\sigma_r - \sigma_\theta)^2 + \sigma_r^2 + \sigma_\theta^2}{2} + \frac{\beta^2}{9} (\sigma_r + \sigma_\theta)^2} \quad (4.15)$$

where $\beta = \frac{3}{\sqrt{2}} \sqrt{\frac{1-2\nu_p}{1+\nu_p}}$ represents the shape of the flow potential ellipse in the effective-hydrostatic stress space. It is related to the plastic Poisson's ratio, ν_p , defined by the ratio of the transverse to the longitudinal plastic strain under uniaxial compression. For the crushable foam in the numerical model, ν_p is nearly zero, which leads to a value of $\beta = 2.12$. According to the plastic flow rule [166], the ratios of the components of the plastic strain increments, $d\varepsilon_{ij}^p$, are determined by partially differentiating the flow potential function with respect to the stress components, σ_{ij} ; that is,

$$d\varepsilon_{ij}^p = d\lambda \cdot \frac{\partial g}{\partial \sigma_{ij}} \quad (4.16)$$

where $d\lambda$ is a scalar rate variable. At the initial yield point *A*, where the radial stress σ_r and the circumferential stress σ_θ are 0.76 MPa and -0.76 MPa, respectively, the direction of the outward normal on the flow potential surface is (1.299, -1.299), normalized to (1, -1). From the yield surface plotted in Fig. 4.8, the direction of the strain increment vector is (1, -1), which agrees with the normality of the plastic strain increment vector [166]. However, at the transition point *D*, after substituting the values of stresses into Eqns (4.15) and (4.16), the direction of plastic flow is then (0.999, -0.008), or about (1, 0). But the direction of the outward normal on the yield surface at this point in Fig. 4.8 is (1.093, -0.288), which is different from the direction of plastic flow. It is also noted that at this critical point *D*, the direction of plastic flow is exactly the same as the vector of the loading path for the uniaxial compression, which causes a sudden change in the shape of the yield surface. This apparent violation of the conventional normality rule may warrant further investigation.

4.3.4 Deformation Map

At a given pressure, foam at different zones (r/a) can be in an elastic, plastic or densified state. Fig. 4.14 plots those three regions into a map, where the three curves divide the deformation domain into three different deformation zones. The corresponding deformation distribution stages are the linear elastic area I, plastic plateau region II and densification zone III. It is shown that after the initial yield at pressure p_e , as the pressure increases, the whole plastic zone expands. Beyond the elastic limit, no densification layer exists until the pressure reaches a critical pressure for densification $p_d = 1.375\sigma_y^0$, when the first element on the hole boundary becomes densified. When the applied pressure exceeds p_d , more elements are densified. The relationship between the applied pressure and the radius of the plastic deformation zone is nonlinear. Compared with the plastic plateau region, the densified zone occurs much later. Fig. 4.14 also demonstrates the evolution of the three deformation zones with the increase of the applied pressure.

Fig. 4.14. Map of deformation obtained from FEA showing the non-dimensional plastic zone: plateau region and densification.

Note that the deformation distribution stage I is the linear elastic zone, II is the plastic plateau region and III is the densification zone. r is the radial coordinate.

4.4 Conclusions

The quasi-static plastic expanding process of a cylindrical hole in a block of closed-cell metallic foam subjected to an internal pressure is studied. A combined analytical treatment and numerical simulation is carried out to verify and validate the foam model. Two stress stages are revealed with regard to the stress levels: initial yield stage I and further hardening-densification stage II. For the idealized Alporas foam studied, when the pressure reaches $p_e/\sigma_y^0=0.633$ initial yielding occurred; when it reaches $p_d/\sigma_y^0=1.375$, densification starts to occur. At the effective plastic strain of approximately 1, the pressure is close to $1.5\sigma_y^0$.

Key variables that are used to characterize the property of metallic foams are given, and the effects of some of these parameters (mainly the effective stress and hydrostatic stress) on the evolution of yield surfaces of foams are also analyzed. The non-dimensional plastic deformation distribution along the radial position of the foam is investigated. It is found that the yield surfaces based on the finite element results are quadratic in the effective-hydrostatic stress space and the stress path almost follows a vertical line in this space after the critical point D , where the uniaxial compressive stress path intersects with the yield surface ellipse. The foam shows greater hardening effect during the hole expansion than under uniaxial compression.

The deformation and stress distribution of idealized Alporas foams under proportional axisymmetric compressive loading are established herein. Compared with the simplified analytical results, the shape of the initial yield surface and its subsequent evolution have been verified, including the dimensionless radius and the

width of the plastic zone, which provide an insight on the evolution mechanism of the plastic zone and the hole expansion. However, the results indicate that the flow potential significantly affects the shape of the yield surface at the transition point D , where the direction of flow potential is identical with the loading path for uniaxial compression. Since the plastic flow rule does not comply with the conventional normality rule after the transition point D , the plastic flow potential seems non-associated with the yield surface evolutions.

This preliminary study on the expansion of a cylindrical hole within a block of metallic foams could provide guidance on the future design of energy absorbers made of double tubes filled with metallic foams. Future experiments are also required to validate both the numerical and the analytical models. The effects of the parameters such as the relative density of the foam and the hardening modulus should be further discussed.

CHAPTER 5

QUASI-STATIC LATERAL CRUSHING ON FOAM-FILLED TUBES

This chapter presents systematic investigations into the thin-walled circular tubes with aluminium foam sandwiched under quasi-static lateral crushing. A total of 30 foam-filled double tubes with different diameter-to-thickness ratios were crushed under Instron machine at a constant crosshead speed. The effects on the key parameters, such as the average radius of curvature, the thickness ratio of both tubes and the thickness of foam core were evaluated. Bonding with foam core and tubes might display a noticeable effect in such structures [167]. Consequently, two bonding cases between the tubes and foam core, i.e. fully glued and packed cases were considered for each model. Three distinct crushing patterns were observed and classified with respect to the collapse patterns. The progressive collapse histories for typical specimens were successfully simulated by ABAQUS/Explicit and the corresponding load-displacement curves were validated against experimental data. More useful information on plastic energy absorption was further explored based on FEA to demonstrate the energy enhancement of the foam-filled tubular structures. The energy absorption characteristics, including energy partition and *SEA* were assessed throughout FE analysis.

5.1 Material and Testing

5.1.1 Specimens

Foam-filled double tubes were constructed by bonding two stiff, thin-walled tubes

with the foam core. The preparation of specimens and the experiments were conducted by the author's seniors and co-workers, Dr. Shen Jianhu [134] and Dr. Ruan Dong at Swinburne University of Technology, Melbourne, Australia. A photograph of all the 30 specimens is illustrated in Fig. 5.1 and the geometrical features of all specimens are listed in Table 5.1. All the specimens were of constant length ($L = 50$ mm). Two bonding cases among foam core and solid tubes, i.e. fully glued and packed, were considered as well. The thixotropic epoxy liquid adhesive (FORTIS AD825) was used to glue the solid tubes together with the foam core for fully glued case. Half of the specimens (marked as 'G') were glued and the others (marked as 'P') were just packing two solid tubes and foam core without any tie constraint. The foam-filled tubes were manufactured by cutting the inner, outer solid tubes and foam core separately. The solid aluminium tubes were cut from extruded AA6060-T5 sections with the yield stress σ_y of 150 MPa [168]. The aluminum foams were cut from an initial Alporas[®] aluminium foam block of 50 mm in thickness. The material properties of the foams were the same as those reported by Shen et al. [86], from which the relative density (ρ^*) of aluminium foam is 8%, and the Young's modulus E_f is 1.2 GPa, listed in Table 5.2 as well.

Fig. 5.1. Photograph of all specimens [134].

Table 5.1 Specimens list for foam-filled tubes

Specimens No.	D_o (mm)	t_o (mm)	D_i (mm)	t_i (mm)	m (g)	Collapse pattern	SEA (J/g)
Group 1. Effect of radius of curvature							
P01, G01	49.8	1.55	31.8	1.54	63.2	III	2.29
P02, G02	59.9	1.98	39.8	1.52	88.9	I	2.07
P03, G03	98.1	1.88	79.9	1.98	168.0	I	1.27
P04, G04	125.1	2.76	98.7	1.88	266.5	II	0.98
Group 2. Effect of thickness ratio of both tubes							
P05, G05	99.8	1.97	49.7	1.58	177.2	II	1.77
P06, G06	98.2	1.95	49.9	1.94	179.5	II	2.42
P07, G07	98.7	1.97	49.8	2.89	200.1	III	2.70
P08, G08	97.9	2.84	49.8	1.99	212.7	II	2.18
P09, G09	101.3	1.50	49.7	1.98	170.8	II	2.03
Group 3. Effect of thickness of foam core							
P10, G10	150.5	3.28	127.2	2.76	399.2	I	0.98
P11, G11	149.8	3.32	98.4	1.98	391.3	II	1.18
P12, G12	150.7	3.34	79.8	1.99	410.3	II	1.58
P13, G13	98.9	1.98	59.7	1.89	179.8	II	2.00
P14, G14	98.2	1.95	49.9	1.94	179.5	II	2.42
P15, G15	98.8	1.96	31.8	1.57	175.0	III	2.82

Note: (D_o , D_i) and (t_o , t_i) are the diameters and thicknesses of tubes. The subscript ‘ o ’ and ‘ i ’ denote the outer and inner tube, respectively. m is the mass of specimen. All the specimens are of the same length, $L = 50$ mm. ‘P’ refers to packed specimens and ‘G’ means glued specimens. The values of SEA are for the glued cases at the strain of 65%. The standard deviation for all the specimens is ± 0.2 mm.

Table 5.2 Material properties of ALPORAS[®] aluminium foam [86]

Composition (%)	Density (kg/m ³)	Young’s Modulus (GPa)	Shear Modulus (GPa)	Shear Strength (MPa)
Al + 1.5%Ca + 1.5% Ti	230 $\pm$ 20	1.1 $\pm$ 0.1	0.33 $\pm$ 0.02	1.2 $\pm$ 0.05
Tensile Strength (MPa)	Bending Strength (MPa)	Poisson’s Ratio	Compressive Peak Stress (MPa)	Average Pores Size (mm)
1.6 $\pm$ 0.2	2.8 $\pm$ 0.3	0.33	1.9 $\pm$ 0.3	2.88

5.1.2 Set-up of Experiments

A series of quasi-static lateral crushing tests were performed under Instron 5569 machine with the maximum capacity of 50 kN. During the tests, each specimen was positioned centrally between two rigid steel plates. The specimen was laterally

crushed individually with a fixed downward crosshead speed of 2 mm/min. The load cell was mounted on the top platen. The loading procedures were ended right after the occurrence of the self-contact within the inner tube to protect the apparatus. The load and corresponding displacement were recorded automatically through the inductive transducers on the load cell. The progressive collapse processes were manually captured by a camera with a regular interval of 30 seconds.

5.2 Finite Element Analysis

5.2.1 Finite Element Model

In order to investigate the typical quasi-static lateral crushing behaviors of the foam-filled double tubes and to further understand the mechanisms of plastic deformation, finite element analysis is developed in ABAQUS/Explicit. Any material failure is not taken into account.

The top and bottom steel plates are regarded as rigid bodies during the analysis. This assumption is reasonable because the mass of the platen is considerably larger than that of the specimen. The thin-walled tubes can be modeled as 3D solid elements. To prevent the penetrations of the protruding nodes of the rigid bodies (steel plates) into the slave surfaces (foams), the mesh on the deformable (slave) surface needs to be refining due to the interactive algorithm. It can result in high computational cost and it cannot be able to reveal the overall response of structure. To simplify the problem, the tubes are modeled as shell element (*S4R*) with five integration points through thickness. Second order accuracy method is employed since the shell element may undergo large bending and warping deformation.

The aluminium foam core is modeled as an 8-node three-dimensional solid element, *C3D8R*. Since the foam might experience extremely high local compression and sever distortion, element distortion controls are required for the foam elements to allow simulation not being terminated. Fine mesh should be made to prevent the penetration of the foam cores, which are the slave surfaces during the interaction. Crushable foam with volumetric hardening is selected for foam material model [23]. The advantage of such material model is that only one uniaxial compression test is sufficient to calibrate, in which the true stress vs. logarithmic strain or true strain curve is provided in Fig. 5.2a. As described in Section 4.1, the yield surface of crushable foam is an ellipse in the $\sigma_m - \sigma_e$ (hydrostatic stress-Mises equivalent stress) plane. From Eq. (4.1), the parameter α defines the shape of the yield surface, which is taken as 1.32 [134, 165].

For each tube filled with metallic foam, the interaction type general contact (Explicit) is used to prevent the self-penetration and mutual penetration of each component. The interaction algorithm is penalty contact algorithm, which allows for the treatment of contacts involving rigid bodies. For glued cases, both tubes are connected with the foam core by defining the tie constraint. A vertical downward displacement is gradually applied to the top plate while the bottom plate is fixed to provide the support. The simulation process stops when the self-contact of the inner surface emerges, where the stroke is about $0.8D$. The duration of simulation is 1.0 time unit to ensure quasi-static loading in an explicit code, corresponding to an equivalent loading strain rate of 0.8 s^{-1} . The typical run time of a computation model on HPC platform is approximately 8 hours.

5.2.2 Validation Material Model of Foam Core

Before conducting numerical analysis for all specimens, the validation of material model is necessary because of the non-homogeneity of metallic foam [18]. It might be a challenge to simulate the discrete microstructure of metallic foams with a macroscopic model by a continuum FE model. Some natural features are not possible to model by the FE code, such as the non-uniform distribution and imperfection during the casting process, which may lead to the local softening associated with the individual collapse of cell. Thus the global characteristics in terms of energy dissipation are of interest in the present study. The uniaxial compressive tests of a foam cylinder are executed and simulated to verify the foam material model. The input constitutive relationship in ABAQUS is the true stress-strain curve from the uniaxial tests, as plotted in Fig. 5.2a. The output nominal stress-strain curve (calculated from load-displacement curve) is also evaluated and plotted against the original data in Fig. 5.2b. It confirms that the material model captures the global characteristics of the metallic foam. It is observed with small discrepancy in the initial stage of deformation since during the uniaxial compressive tests, the localized nonlinear deformation is prior to the onset of plastic yielding. Similar phenomenon is detected in recent study on the foam material [84], where Young's modulus of the foam (E_f) is the unloading modulus, rather than the slope of the loading curve. Nevertheless, according to the tests on uniaxial compression, FE results are still reasonable to represent the deformation behavior of the foam core.

Fig. 5.2. (a) Constitutive relationship of crushable foam; and (b) verification of foam model.

5.3 Results and Discussions

Based on the experimental observations and FE analysis on lateral crushing of foam-filled double tubes, the deformation behavior and energy absorption characteristics are described in details.

5.3.1 Deformation Profiles

Fig. 5.3 shows the test results for non-dimensionalized crush force, $F/(\sigma_y D_0 L)$, plotted versus dimensionless displacement δ/D_0 . The deformation process appeared to have three common phases, namely elastic deformation phase, plastic collapse phase and densification/inner self-contact phase, respectively. From the experimental tests, non-symmetric deformation was detected in specimens G07, P07, G15, P15, etc. Generally, glued cases are more inclined to deform unevenly than the packed ones because of the extra interactions and cohesive constraints. The collapse loads of the glued cases are larger than those of the packed ones, as shown in Fig. 5.3, a linear relationship between the dimensionless crush force and displacement is observed in the initial elastic deformation phase. The elastic deformation of glued foam-filled double tubes is less than that of the simply packed

ones. The curves level off after the elastic phase and increase gradually in the following plastic collapse phase. These phenomena are possibly attributed to some factors, such as the hardening of the aluminium material at plastic hinges, geometric changes, and the hardening or densification of foam during crushing. Afterwards, the densification/inner self-contact phase starts at where the foam core either goes into densification stage or the upper and lower portions of the inner tube begins to contact. As a result, a rapid increase in the reaction force is observed from the curves. Detailed progressive collapse would be discussed next.

Fig. 5.3. Plots of non-dimensional experimental forces versus displacement for foam-filled tubes.

5.3.2 Collapse Patterns

The plastic deformation phase is of great importance in energy dissipation capacity of the foam-filled double tube. The crushing capacity depends on the collapse pattern. The progressive collapse might be further classified with respect to deformation features. Three types of collapse modes were observed in the current

tests.

Generally, for an effective foam-filled tubular structure, three components tend to be deformed simultaneously (crushing pattern I, DS pattern) in the plastic collapse phase. The foam core does not crush severely in this mode. Typical deformation histories are illustrated in Fig. 5.4 and Fig. 5.5, together with the load-displacement curves of specimens P10 and G10 as plotted in Fig. 5.6. Good correspondence of specimen P10 is achieved between the experiments and numerical predicted deformation, whereas the numerical results of specimen G10 appear to slightly underestimate the amount of crush strength during the tests. One possible reason for the deviation was that the tangential slippage occurred in the lower part of the foam, which made the foam core thicker and the micro-crack speared widely along the surrounding region, as highlighted in Fig. 5.6b. Compared with the packed one (P10), the bonding effect in specimen G10 shows an increase in crush strength of 1.62% experimentally and 10.48% in FE (around 65% strain).

Fig. 5.4. Progressive collapse of P10 (experiment and FEA).

Fig. 5.5. Progressive collapse of G10 (experiment and FEA).

Fig. 5.6. Load-displacement curves of (a) P10; and (b) G10.

In some cases, especially when the thick foam core was involved, tear fractures and cracks occurred in the plastic collapse phase (characterized as crush pattern II) due to severe deformation, as indicated in Fig. 5.7 and Fig. 5.8. Visible fractures were observed at the bottom of the foam core when the displacement reached 20 mm in specimens P13 and G13. It is interesting to note that similar phenomenon also existed in previous experiments on foam-filled tubes [3]. In Crushing Pattern II, all the components of the foam-filled tubes always deformed simultaneously, as shown in FE simulation in Fig. 5.7. Nevertheless, from the experimental observations, after the onset of distinctive foam fracture at $\delta = 40$ mm, nonsymmetrical deformation

occurred and considerable variation in the thickness of foam core is observed in specimen P13. Overall, the comparisons between experimental and numerical results are made and well agreement for the specimen P13 is shown in Fig. 5.9a. However, for the glued case (specimen G13), the average crushing load in FE is less than that of experimental data, as plotted in Fig. 5.9b. From both figures, the adhesive effects have 15.48% and 6.28% improvement in crushing strength, experimentally and numerically, respectively.

Fig. 5.7. Progressive collapse of P13 (experiment and FEA).

Fig. 5.8. Progressive collapse of G13 (experiment and FEA).

Fig. 5.9. Load-displacement curves of (a) P13; and (b) G13.

Considering the three components of the structure, namely outer tube, foam core and inner tube, both tubes always deformed simultaneously in the above-mentioned specimens. The foam core was crushed only after a certain displacement. However, in some other cases, the inner tube did not deform in the beginning while visible deformation took place only after the foam cores crushed for a certain amount, which were close to the compression platen. It indicated a Sequential Crushing Pattern (SCP, crushing pattern III) for such foam-filled tubes. Typical deformation processes are illustrated in Fig. 5.10 and Fig. 5.11, where thicker foam core or inner tube was involved. In the experiments, crush pattern III occurred in the specimens G07 and G15. Probably because of the non-uniformity of aluminium foam core, shear failure and non-symmetric deformation were observed for this type of crushing pattern in experiments.

As depicted in Fig. 5.10 and Fig. 5.11, the plastic collapse phase in this pattern might be further divided into two individual phases. The first one is called as OC crushing phase, where the outer tube deforms with the foam core. The second one is OCI crushing phase, where the three components deform synchronized. The

corresponding load-deflection curves of the specimen G15 are shown in Fig. 5.12. The discrepancies between FE and experimental curves were probably attributed to the non-uniform distribution of foam core and the initial cracks or imperfections of metallic foam. Especially in specimen G15, non-symmetric deformation of foam took place at the displacement of 40mm. As shown in Fig. 5.11, severe deformation is localized in the right portion of the inner tube, resulting in a rapid increase in force shown in Fig. 5.12b. In light of the limitations in simulating the foams' fractures, crack rupture and the adhesive failures, the numerical views herein were more likely to be symmetrical compared with experimental observations. With respect to the estimation of the bonding effects, in glued specimen G15 there was an increase of 11.37% experimentally and 4.10% numerically, respectively. It is evident that in the OC crushing phase, the inner tube experienced almost no plastic deformation and thus it was not responsible for any plastic strain energy dissipation. A plateau stage in the load-deflection curve appeared during this stage. In the subsequent OCI crushing phase, as the deformation developed and the inner tube was participated in the energy dissipation, a step increase in the crushing strength is revealed in Fig. 5.12.

Fig. 5.10. Progressive collapse of P15 (experiment and FEA).

Fig. 5.11. Progressive collapse of G15 (experiment and FEA).

Fig. 5.12. Load-displacement curves of (a) P15; and (b) G15.

The above-described three types of modes were identified with good accuracy through FE analysis. Comparisons between the packed and glued results were consistent with the test data, suggesting that the bonding effect might make an important contribution to the energy-absorption capability. The foam cores were assumed to be isotropic with the same strength in the three orthogonal directions during FE modeling. However, the anisotropic nature of foam cores exists due to the variations in the cell size and cell wall-thickness. No correction was made to the experimental results for this effect. In addition, some energy might be dissipated in the experimental apparatus, particularly the frictional effect in the bottom platen which held the specimen in position. This might explain why the numerical results in Fig. 5.6b, Fig. 5.9 and Fig. 5.12b are lower than corresponding experimental

results. It is therefore estimated from the areas underneath the load-displacement curves that the discrepancies in absorbed-energy between experiments and FEA varied from 6.1% to 12.8% for glued specimens (G10, G13 and G15), at around 65% strain.

5.3.3 Key Parameters related to Energy Absorption

To further investigate the key parameters that dominate the energy dissipation capacities of foam-filled double tubes, the specimens were preliminarily chosen with respect to some common parameters [134]. There were radius of curvature of the whole foam-filled tubes, thickness ratio of both tubes and thickness of the foam core. Hence, all specimens were divided into three groups with respect to those effects in Table 5.1.

The specimens in the first group were presented to test the effect of the radius of curvature of the whole structure. In this group, where specimens P01-P04 and G01-G04 were prefabricated, the foam cores of all specimens were of the same thicknesses but different diameters of the inner and outer tubes. The corresponding force-displacement curves for packed and glued cases are plotted in Fig. 5.13. It shows that the increase in the average radius of curvature results in an increase in the deflection but decrease in the crush strength. It is also evident that with the increase of the average radius of curvature, the energy dissipation capacity increases in the presence of the extra crushing stroke. Unexpectedly, softening behavior phenomena [169] were encountered in the specimens with the smallest diameter (i.e. G01 and P01) in the experiments, which were primarily resulted from the material failures or fractures along the plastic hinges of the outer tubes. In those specimens,

the average radius of curvature was less than the length/width of the tubes, where the foam core was relatively thick to enlarge the plastic region around the two horizontal plastic hinges in the tubes. The softening phase disappeared in other specimens in current experiments, where relatively larger diameters of the inner tubes were involved. Furthermore, the adhesive effect may not sharply affect the general trend of force-displacement curves but increase the plastic strain energy dissipation capacity.

Fig. 5.13. Effect of average radius of curvature of foam-filled tubes: (a) packed; and (b) glued.

In the second group, in which both tubes of the specimens P05-P09 and G05-G09 were of the same diameter, the influence of the thickness ratio in tubes was studied. The corresponding load-deflection curves are presented in Fig. 5.14. It appeared that varying the thickness ratio might change the original collapse pattern into a more efficient collapse mode, such as in specimens P07, P08 and G07, G08. The increase in the thickness ratios resulted in the decrease in crushing strength at the beginning of plastic collapse. However, it would dramatically improve the crushing strength and the corresponding strain energy in the coming large plastic deformation process. Even if the collapse pattern did not change, such as in specimens P05, P09 and G05, G09, an increase in the thickness ratios led to the

increase in both crushing strength and energy absorption capacity. The foregoing statements are applicable for both glued cases and packed ones.

Fig. 5.14. Effect of thickness ratio T_o/T_i : (a) packed; and (b) glued.

The last group, group 3 with specimens G10-G12, P10-P12 and G13-G15, P13-P15 was designed to study the effect of the thickness of the foam core. Hence, in this group, the outer tubes owned similar diameters and thicknesses. The load-displacement curves obtained from experiments are plotted in Fig. 5.15 and Fig. 5.16 for the specimens with different profiles of the outer tubes, where D_o were 150 mm and 100 mm, respectively. Distinctive thicknesses of foam core led to distinctive collapse pattern in specimens G13 and G15. For the foam-filled tubes under the same collapse pattern, a minor variation in thickness of the foam did not affect the crush strength and energy absorption. However, when the thickness of foam core was large enough to change the collapse pattern from II to III, the crushing strength was increased as well as the energy absorption ability. These results remain true for this group, whether for different outer tubes or under different bonding circumstances. It is noted that the glued effect, decreasing the average radius of curvature and thickening the foam core improve energy dissipation of foam-filled tubular structures.

Fig. 5.15. Effect of foam core thickness ($D_0 = 150$ mm): (a) packed; and (b) glued.

Fig. 5.16. Effect of foam core thickness ($D_0 = 100$ mm): (a) packed; and (b) glued.

5.3.4 Energy Absorption Characteristics

The plastic strain energy dissipation and specific energy absorption (*SEA*) are often worked out as indications of energy-absorption capacity for a particular structural design.

5.3.4.1 Energy Partition

To illustrate the responsibility of each component during the crushing, the corresponding plastic energy by the foam-filled tubes is partitioned, in terms of the plastic strain energy for each individual component, which are beyond the limits of the experiments and only available from FE analysis. The contribution of strain

energy shared by each component in those typical crushing patterns during crushing tests is shown in Fig. 5.17. Herein, typical specimens G03, G13 and G15 are selected due to their identical outer tubes. The dimensions of the specimens are also listed in Fig. 5.17 for reference. The specimen G15 has the largest value of diameter ratio, D_o/D_i , and thus the foam core is the thickest. It is evident from the observations that the transition in crushing pattern leads to a decrease in the contribution of plastic energy for both tubes. However, the plastic deformation of the foam core is primarily accounted for the plastic energy in the plastic collapse, which consumes up to 75% of the total energy dissipation at a compression strain around 80%. Particularly in a more efficient collapse pattern III, the foam core dominates the energy absorption due to the extensive plastic deformation of foam as well as the constraint interaction between the tube and the foam filler.

Fig. 5.17. Partition energy dissipation for each crushing pattern.

5.3.4.2 Specific Energy Absorption (SEA)

In axial crushing on foam-filled tube, it was found that foam-filling was not more weight effective than empty tubes [25]. In order to investigate the superiority of the

double tubular structure with metallic foam sandwiched subjected to lateral compression scenario, the experimental results were again studied in terms of *SEA*. The values of *SEA* for all glued cases are plotted in Fig. 5.18 and also listed in Table 5.1 at a stroke of $0.65D_o$.

The specimens are divided into three groups according to their geometrical features. For group 1 in Fig. 5.18a, the thicknesses of foam cores among all specimens are identical. Although the specimen G01 has the smallest diameter and lightest weight, it is the most weight efficient structure with the largest value of *SEA* up to 2.29 J/g at 65% strain, whereas the specimen G04 is less efficient even with the thickest outer tube. For specimens G02 and G03 with the same crushing pattern, the value of *SEA* is significantly greater in magnitude compared to the ones with larger average radius of curvature. Overall, it suggests that specimens deform in the crushing pattern III are favorable to energy absorbers. Likewise, for the second group in Fig. 5.18b, tubes have the same geometrical parameters except different thickness of inner tubes in specimens G05, G06 and G07. It is evident that the specimen G07 dissipates energy much more than those with smaller thicknesses of inner tubes. In addition, for the specimens G08 and G09, in which the only different geometrical parameter is the thickness of the outer tube, the one with thicker outer tube is much better in energy dissipation. This finding demonstrates that the foam-filled structures having thicker inner or outer tubes are better design for quasi-static loading for a fixed crushing pattern.

Fig. 5.18c plots the results of specimens with different foam core thicknesses, but similar diameters and thicknesses of outer tubes in the last group. As anticipated,

specimen G15 with the most foam core filling is the one that absorbed the most energy. It is concluded that depending upon the level of plateau stress, the foam core may change the original collapse mode into a more efficient one with higher energy efficiency. Besides, the interaction between the foam core and tube walls provides additional enhancement in energy dissipation. When foams are compressed, densification occurs after the plateau stage and the stress increases rapidly with the strain, which might lead to a higher stoke efficiency and a larger average force as well as better specific energy. A large amount of material sustains plastic deformation, which dissipates plastic energy remarkably.

Fig. 5.18. Plots of Specific Energy Absorption (SEA) versus non-dimensional deformation for glued specimens in (a) group 1; (b) group 2 and (c) group 3.

5.4 Conclusions

The quasi-static lateral crushing tests of composite structures filled with metallic foam core are presented to investigate the deformation behavior in this chapter. Thirty pieces of foam-filled double tubes with different specifications were crushed under an Instron machine. Detailed progressive collapses were obtained during the tests. This chapter aims to demonstrate the advantage of foam-filled tube in the lateral direction as well, which have already been identified when subjected to axial crushing. Three kinds of collapse patterns were observed from the experiments, which were simultaneous collapse pattern, simultaneous collapse pattern with fractures of the foam core and sequential collapse pattern, respectively. The average collapse forces and energy absorption capacities were dependent upon the corresponding collapse patterns. In addition, parametric studies were performed on some key factors which might affect the crushing strength in plastic collapse phase.

The quasi-static crushing processes were further modeled in ABAQUS/Explicit. Good agreement between both results was obtained. Energy absorption characteristics such as energy partitions and specific energy absorption were examined with regard to different collapse patterns. The initial foam crack, non-uniform distribution of foam, foam failures or crack rupture and failure of the adhesive joints were very difficult to evaluate and no failure criterion of the foam material were considered in FE analysis, which might cause the discrepancies. Based on a comparative study on the energy absorption behavior of foam-filled double tubes by experiments and corresponding FEA, foam-filled tubular structures with the thickest foam core and inner tube are the optimum design under quasi-static loading.

CHAPTER 6

DYNAMIC LATERAL CRUSHING ON FOAM-FILLED TUBES

In this chapter, the dynamic response of empty and foam-filled double tubes under lateral compression is investigated. The effect of velocity was examined experimentally and by using finite element analysis. A series of tests on empty tubes and foam-filled double tubes were performed by using an Instron High Rate Test System, which enabled a single shot test to be conducted at a constant crosshead speed, up to 10 m/s for compression. The specimens were placed on the bottom platen and then they moved upwards together, until they collided with the top platen, with very little change in the speed during the whole crushing process. The dynamic force and deformation histories were recorded simultaneously by a high-speed data acquisition package. Finite element code ABAQUS/Explicit is then employed to analyze the response of the tubes under higher velocities, up to approximately 100 m/s. Based on the deformation modes from FE analysis, a critical compression velocity is defined above which the tube deformation is localized around the impact end. A comparative study of the load-deflection curves under quasi-static and impact tests is described and the effect of velocity on the total energy absorption is studied.

6.1 Dynamic Compression Experiments

6.1.1 Material Properties of Specimens

Fig. 6.1 shows the specimens used in the tests. The foam-filled double tubes were manufactured by assembling together the individual components, i.e. the inner tube,

the outer tube as well as the aluminium foam core. The short aluminium tubes were made of AA6060-T5 and the length was fixed at 50 ± 0.2 mm for each specimen [93]. The stress-strain curves of the aluminium tube were obtained from the standard uniaxial tensile tests [168], from which the average yield stress σ_y is 150 MPa. The density of tube ρ_s is 2760 kg/m^3 . Hollow cylindrical ring of aluminium foam was cut from an initial ALPORAS[®] aluminium foam block ($400 \times 700 \times 2400 \text{ mm}$), with the nominal relative density of 9%, supplied by Gleich Ltd., Germany. The material properties of the foam core were the same as those reported by Shen et al. [86], in which the compressive plateau stress at strain rates in the range from 10^{-3} s^{-1} to $2.2 \times 10^2 \text{ s}^{-1}$ was investigated. The compressive plateau stress of ALPORAS[®] foam (σ_{pl}) at different strain rates ($\dot{\epsilon}$) is given by

$$\frac{\sigma_{pl}}{\sigma_{ys}} = 0.59 \left(1 + 0.14 \dot{\epsilon}^{0.17} \right) (\rho^*)^{1.7} \quad (6.1)$$

where σ_{ys} is the yield stress of the base material of the foam. For bonding cases, a thixotropic epoxy liquid adhesive was used to paste, separately, the two monolithic aluminium tubes with the aluminium foam core. Four empty tubes and seven foam-filled double tubes were fabricated and tested separately in the Instron VHS8800 machine and their details are listed in Table 6.1. The definitions of geometric parameters in the tests are shown in Fig. 6.2.

Fig. 6.1. Photograph of typical specimens for dynamic lateral crushing, including empty tubes and foam-filled double tubes.

Fig. 6.2. Schematic of a foam-filled tube under lateral impact: (a) impact method I: specimen and bottom platen gain a velocity and then it impacts to the top rigid platen; and (b) impact method II: bottom platen has a velocity and then impacts to the stationary specimen. Insert: illustrating of the outer diameters of inner and outer tubes, D_i and D_o , as well as the thicknesses of both tubes, t_i and t_o , respectively.

Table 6.1(a). Specimens for empty tubes

Test	Diameter D (mm)	Thickness t (mm)	Mass m (g)	EA (J)	SEA (J/g)	t/D
ET01	99.81±0.2	1.92±0.1	83.1	62.3	0.75	0.019
ET02	99.57±0.2	1.92±0.1	83.1	-	-	0.019
ET03	79.64±0.2	2.00±0.1	69.1	95.5	1.38	0.025
ET04	49.87±0.2	2.96±0.1	64.0	232	3.63	0.059

Table 6.1(b). Specimens for glued foam-filled tubes

Test	Diameter D_o (mm)	Thickness t_o (mm)	Diameter D_i (mm)	Thickness t_i (mm)	Mass m (g)	Velocity v (m/s)	Collapse pattern	EA (J)	SEA (J/g)
SWT01	99.85	1.87	50.34	1.90	180.0	10.0	II	431	2.39
SWT02	99.71	1.87	49.91	2.94	204.0	1.0	III	588	2.88
SWT03	99.62	1.89	79.81	1.97	172.0	10.0	I	260	1.51
SWT04	99.61	1.89	59.76	1.86	177.0	10.0	II	346	1.70
SWT05	99.59	1.91	49.98	2.95	201.0	10.0	III	796	3.96
SWT06	99.59	1.91	49.90	2.95	201.0	10.0	III	796	3.96
SWT07	99.91	1.93	49.95	1.90	182.0	1.0	II	434	2.12

6.1.2 Set-up of Dynamic Experiments

All the lateral crushing tests were performed at a room temperature of 25 °C. The dynamic tests were performed using a High Rate Test System (INSTRON VHS8800). A photograph of the experimental setup is shown in Fig. 6.3. The top

platen was fixed and the finishing position of the bottom platen was controllable. This system provided a feedback mechanism by its FastTrack™ VHS8800 controller package to adjust the drive profile of the hydraulic system according to the experimental condition. After some initial trials with several iterations, an almost constant loading rate could be achieved for the subsequent experiments.

Fig. 6.3. Experimental setup for dynamic tests.

In our tests, for the bottom platen, a constant velocity of 1.0 m/s with a sampling rate of 50 kHz was used for low rate compressions and a constant velocity of 10.0 m/s (which was the maximum possible) with a sampling rate of 500 kHz for high velocity compression tests. The load history was measured by Kistler® load cell (Type 9071A) without data filter. The displacement history was measured by a linear variable differential transformer (LVDT) with data filter with a cut-off frequency of 1000 Hz. During the initial tests, the empty tubular specimen was placed either on the bottom platen or top platen, in order to explore the difference of these two arrangements. Subsequently, for all the tests with empty and foam-filled

tubes the specimen was placed on the bottom platen and then moved upwards together with the base platen. Double-sided tapes were used to stick the specimen on the bottom platen during the movement. A high-speed camera was used to track the deformation profile of the specimen during crushing. The dynamic experiments were carried out at Swinburne University of Technology, Melbourne, Australia.

The tube's contact forces with the top and the bottom platens are quite different in the presence of a tube's inertia. To study the possible difference, two kinds of impact methods were explored for the empty tubes. Impact method I was to attach the specimen to the bottom platen and then move together at a velocity, as sketched in Fig. 6.2a. The other, impact method II, was to fix the specimen with the top stationary platen, as depicted in Fig. 6.2b. The load cell was mounted in the top platen and the displacement of the bottom platen was measured. The force-displacement curves of both the methods are plotted in Fig. 6.4 for specimens ET01 and ET02, respectively. The two methods resulted in different forces in the initial stage, but later the two forces were almost identical. For the second method, at the early crushing phase of small deflection, the magnitude of contact force was very small compared with that of the first one. This delay in experiencing the force by the load cell was due to the inertia of the tube. Also, a drop in the force after the initial small peak might indicate elastic bouncing of the tube from the top platen. As the displacement developed further, the tube wall started to interact with the top platen again, rendering a steeper increase of the force in the subsequent phase. This bouncing-back was also detected from the high speed photography. For all the tests reported here, method I was used.

Fig. 6.4. Comparison of Force-displacement curve from the two impact methods. Specimens ET01 and ET02 were tested using method I and II, respectively.

6.2 Experimental Results

6.2.1 Empty Tubes

Fig. 6.5 shows the load-deflection results of several empty tubes (ET01, ET03 and ET04) from the quasi-static and dynamic tests with a crushing velocity of 10 m/s. Apart from the initial difference at the early deformation phase, the magnitudes of the crushing strength are nearly the same under each loading case. From the deformation profile recorded by the camera, the deformation modes of the empty tubular specimens were almost the same when they were subjected to a constant velocity. The progressive collapse has three phases, i.e. initial tube wall collision, steady dynamic collapse and unloading phase. In the first phase, the contact force had large fluctuations probably due to the elastic effect during the collision of the tube wall. The fluctuations were damped by the plastic deformation of the tube, in a mode similar to that in quasi-static compression [131]. Following this initial phase, a slight increase in the compressive strength of the plastic collapse phase was found,

which commenced with a series of very small peaks and troughs. To avoid possible damage to the apparatus, the crushing process was stopped before the self-contact of inner surface of the tube and this caused an unloading phase in the load-deflection curve.

Fig. 6.5. Comparison of load-displacement curves of empty tubes between the quasi-static and dynamic crushing ($v = 10 \text{ m/s}$).

6.2.2 Foam-filled Tubes

To improve the energy dissipation of foam-filled tubes, it is important for the plastic deformation to occur extensively in the protective structure. From the analysis on the quasi-static lateral compressive responses of foam-filled tubes, the crushing strength depends on the collapse patterns [131]. Similar to the quasi-static case, three types of collapse patterns were observed in dynamic crushing. There were simultaneous collapse pattern I, plastic collapse pattern II with foam fracture and

sequential collapse pattern III, respectively. Some photographs of the deforming specimens showing those collapse patterns, together with finite element results to be discussed next, are illustrated in Figs. 6.6-6.8.

Generally, for the foam-filled tubular structure with collapse pattern I, both the inner and outer tubes deformed simultaneously in the plastic collapse. The foam core was less deformed, as illustrated in Fig. 6.6a. In some cases, especially when thick foam was involved, ductile fracture of the foam core occurred during the crushing stage, resulting in collapse pattern II. As shown in Fig. 6.7a, a distinct crack can be seen near the lower portion of foam core at the displacement of 20 mm. It is worth noting that a similar phenomenon was revealed in the previous experiments with foam-filled tube [3].

When the foam core thickness increased, the inner and outer tubes deformed at different displacement, unlike the previous patterns. In the beginning, the inner tube did not deform at all. Noticeable deformation in the inner tube took place only after a certain displacement, indicating a sequential collapse pattern III, as described in Fig. 6.8a. This collapse pattern occurred with the specimens of the thickest foam, in both quasi-static loading and dynamic impact (SWT02, SWT05 and SWT06, for instance). Because of the inherent large variation in the size, shape and distribution of foam cells, slightly non-symmetric deformation occurred in this collapse pattern in the experiments.

Fig. 6.6. Illustration of collapse pattern I of specimen (SWT03): (a) deformation history; and (b) comparison of force-displacement curves between quasi-static and dynamic crushing.

Note that the side-load protection devices partially blocked the full view of specimens.

Fig. 6.7. Illustration of collapse pattern II: (a) deformation history of SWT01; and (b) comparison of force-displacement curves between quasi-static and dynamic crushing.

Fig. 6.8. Illustration of collapse pattern III: (a) deformation history of SWT06; and (b) comparison of force-displacement curves between quasi-static and dynamic crushing.

6.2.3 Comparison between Quasi-static and Dynamic Crushing

Figs. 6.6 to 6.8 show a comparison of force-displacement curves between the dynamic and quasi-static compression for each deformation pattern. For specimens such as SWT03 with the collapse pattern (I) shown in Fig. 6.6a, the crush strength subjected to dynamic impact is lower than in quasi-static situation. A drop of dynamic collapse force was found near the displacement of 10 mm, which was due to the failure of the adhesive joint. The adhesive might be vulnerable to failure under dynamic situation because of the large intensive stress and heat. The force-displacement curve of dynamic case was very close to that of foam-filled tube without glue under quasi-static compression.

For specimens with collapse pattern II (SWT01, SWT04 and SWT07) as shown in Fig. 6.7a, the collapse forces under dynamic loading ($v = 10$ m/s) have the same magnitude as those fully bonded specimens under the quasi-static compression. However, the collapse force under dynamic compression at $v = 1.0$ m/s was lower than that in quasi-static loading. This phenomenon indicates that the failure of two adhered surfaces is sensitive to the impact at the beginning of the test, regardless of the compression velocity. The enhancement of crushing strength is more significant as the compression velocity increases.

For specimens with collapse pattern III as shown in Fig. 6.8a, foam-filled tubes exhibited a larger collapse strength under dynamic compression at $v = 10$ m/s than that under quasi-static loading. When the normalized displacement became larger than 0.2, small amount of foam fracture occurred, leading to the reduction of curve slope. Similar trend was observed for the collapse force under compression velocity

of 1.0 m/s. When the foam core became densification and the inner tube started to deform, a sharp increase in the force was found.

6.3 Finite Element Analysis

Experimentally the maximum velocity of dynamic compression is limited to 10 m/s for this machine. In order to investigate the behavior of tubes at higher velocity and to further understand the mechanics and mechanisms of plastic deformation, a finite element (FE) analysis is conducted. Fracture is not considered in the simulation.

6.3.1 Dynamic Response of Empty Tubes

6.3.1.1 Deformation Modes and Critical Velocity

The empty tube is modeled as a 4-node doubly curved shell element *S4R* [93, 145], with reduced integration. The Simpson thickness integration rule with five integration points along the thickness direction is adopted [161]. The actual stress-strain curve from tensile tests for the tube is used and no strain rate effect is considered, because the strain rate effect of aluminium alloy is of minor importance [170]. The tube and the bottom platen are given an initial velocity, which for the platen is maintained constant throughout each test.

Fig. 6.9 shows deformed profiles of an empty tube (Specimen ET01: $D = 99.81$ mm, $t = 1.92$ mm) at three values of deflection (25, 50 and 75 mm), subjected to a velocity of 10 m/s and 50 m/s, respectively. When the compression velocity is 10m/s (Fig. 6.9a), the deformation mode is very much the same as that for a typical quasi-static case [13]. But for $v = 50$ m/s, plastic deformation initially takes place around the impact region of the tube, with little bending deformation in the

remaining part. As the tube is further compressed by the bottom platen, plastic deformation is extended to the whole tube.

Fig. 6.9. Deformation profile of an empty tube (ET01: $D = 99.81\text{mm}$, $t = 1.92\text{ mm}$) under two different compression velocities: (a) $v = 10\text{ m/s}$; and (b) $v = 50\text{ m/s}$.

Fig. 6.10a shows the energy dissipation against deflection for an empty tube (ET01) compressed at various velocities. It shows that when the compression velocity is less than a certain value, say 20 m/s , energy dissipated by the structure is not affected by velocity significantly. This is because the deformation mode remains the same as that for the quasi-static case, with most plastic deformation localised around the four “plastic hinges”. For higher velocity, considerable increase in energy absorption is observed. This could be attributed to the fact the plastic deformation starts from the impact region and then gradually propagates to the whole tube, as explained in the next section.

To further quantify the possible existence of a critical velocity for the dynamic mode change as shown in Fig. 6.9, a tube is divided into four equal segments, one quadrant for the proximal (front) end, one for the distal (back) end, and one each for the two sides (Fig. 6.10b). The plastic energy for each quadrant is then evaluated

from FEA and this broadly indicates the energy distribution within the tube. The effect of compression velocity on the energy distribution for tubes with different values of D/t is then studied. The energy absorbed by the front quadrant and back quadrant, normalized by the total energy, is plotted in Fig. 6.10b. It is striking that, for a given value of D/t , there exists a narrow band of velocity, below which the front and back quadrants dissipate the same amount of energy. Above this velocity, the front dissipates relatively significant amount and the back dissipates less and less. This is due to the fact that at low velocity the deformation mode is very much the same as quasi-static ones, involving four stationary hinges. But for the high velocity impact, inertia plays a significant role and deformation starts from the impact face and then propagates in a plastic “double-moving hinge”, as explained later. This region for critical velocity seems to change with D/t .

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Fig. 6.10. (a) Plastic energy profile of an empty tube (ET01) under different compression speeds; and (b) comparison of plastic energy ratios between quadrants for the distal end (E_{Distal}) and proximal end ($E_{Proximal}$).

For each value of D/t , the value of critical velocity is defined by locating the intersection of two straight lines, one for low velocity and the other for high velocity region. To study the effect of material properties and geometry on the critical velocity (v_{cr}), two dimensionless groups are chosen, $\rho_s v_{cr}^2 / \sigma_y$ and t/D , where σ_y is the yield stress of the material. Their relationship is plotted in Fig. 6.11, on a double-logarithmic scale. From the best fitted straight line, we have,

$$\frac{\rho_s v_{cr}^2}{\sigma_y} = 0.41 \left(\frac{t}{D} \right)^{0.94} \quad (6.2)$$

Since the exponent of the above equation is very close to 1, it is reasonable to approximate it to 1. Hence, an empirical equation from the plots is obtained,

$$\frac{\rho_s v_{cr}^2}{\sigma_y} = 0.3 \frac{t}{D} \quad (6.3)$$

The above relationship demonstrates that the dimensionless critical velocity squared

is a constant for a given tube and it is linearly proportional to t/D . Hence, the critical velocity is rewritten as

$$v_{cr} = \sqrt{0.3 \cdot \frac{t}{D} \cdot \frac{\sigma_y}{\rho_s}} \quad (6.4)$$

Fig. 6.11. Double-logarithmic graphic plot of $\rho_s v_{cr}^2 / \sigma_y$ versus t/D .

6.3.1.2 Dynamic Deformation Mechanism: Double-moving-hinges

Fig. 6.10a indicates that in the dynamic case when velocity was 100 m/s, the energy absorbed can be as high as five times that of the static case. It is therefore natural to explore the mechanisms of such high energy dissipation under high velocities. As the empty tube is relatively thin, two possible mechanisms exist: plastic bending and in-plane stretching/compression in the circumferential direction. At a given instant (and therefore displacement), the complete profile of the deforming tube is obtained from FE and from that the curvature around the circumference is computed, and so is the mid-plane circumferential strain. Fig. 6.12 and Fig. 6.13 plot the curvature distribution and in-plane strain distribution, respectively, at three values of displacements and for both $v = 10$ m/s and 50 m/s, while the deformed profiles

are shown in Fig. 6.14. Fig. 6.12a shows that for low velocity impact ($v = 10$ m/s), at the both proximal end (the non-dimensional circumferential position $s = S/(\pi D) = 0$) and distal end ($s = 0.5$), the curvature changes from the initial 20 m^{-1} to zero (becoming flat) and this flattening zone extends as deflection increases. At the mid-range of the tube ($s = 0.25$), the curvature increases monotonically, with the plastic deformation confines within the same size (or even getting smaller) of 4 to 6 times thicknesses.

For the case of $v = 50$ m/s, Fig. 6.12b shows a complete different mechanism with two significant features. First, at the proximal end the curvature finally becomes to zero and this “flattened” zone is increasing with deflection, which appears the same as that for the low velocity case. However, the material ahead of this zone has undergone a positive change of curvature first before it then changes back to zero curvature. Let us look at a small segment around position $s = 0.15$, for example. The initial curvature will be approximately 20 m^{-1} , corresponding to a radius of 99.8 mm. At displacement of 25 mm, the segment is bent further to a curvature of 46 m^{-1} . As displacement proceeds, the segment starts unbending with a reducing curvature, until it reaches almost zero at displacement of 75 mm. There is practically little deformation at the distal end ($s = 0.5$) and, for small displacement, even mid-segment ($s = 0.25$).

The second feature is that, as displacement proceeds, the position of peak curvature is propagating and the peak value increases. The maximum curvature of 52 m^{-1} occurs at $s = 0.18$ when $\delta = 25$ mm, but it is 72 m^{-1} at $s = 0.21$ when $\delta = 50$ mm. At this instant, the distal end starts to flatten. Between this two peak curvature points,

the time interval (Δt) is 0.5 ms and the flexural wave propagates with a speed of 20.70 m/s ($=0.03s/\Delta t$). When the displacement is 75 mm, the flattened distal end has extended considerably and the peak curvature is about 100 m^{-1} occurring at $s = 0.28$, which is five times the initial curvature. The plastic flexural wave at the same time interval propagates with a speed of 43.98 m/s ($=0.07s/\Delta t$). The propagation velocity of plastic flexural wave is not a constant. According to the theory of plastic wave propagation in beams for linear strain-hardening material [171, 172], the velocity of plastic flexural wave C_p is given by

$$C_p = \frac{2\pi\sqrt{I/A_0} \cdot C_p^0}{\Lambda} \quad (6.5)$$

where Λ is the flexural wave length, I is the moment of inertia of the beam cross section with respect to the neutral axis, and A_0 is the initial cross-sectional area, respectively. $C_p^0 = \sqrt{E_p/\rho_s}$ denotes the speed of the longitudinal plastic stress wave [1] and E_p is the strain-hardening modulus of the material. For the empty tube, E_p is about 314 MPa [168], and thus C_p^0 is about 340 m/s. Λ is about $0.3\pi D$ measured from Fig. 6.12b. Hence, from the above equation, the plastic bending wave velocity $C_p = 12.46 \text{ m/s}$. It is noted that the velocity of the bending wave in Fig. 6.12b lies between the plastic flexural and plastic longitudinal wave velocities.

Fig. 6.12. Curvature distribution along the mid-surface of an empty tube (ET01) at different values of displacement: (a) $v = 10$ m/s; and (b) $v = 50$ m/s.

Note that $s = S/(\pi D) = 0$ corresponds to the proximal end and $s = 0.5$ the distal end.

For the possible in-plane deformation, Fig. 6.13a shows that there is tension around the contact regions, which propagates with increasing deflection, but compression around the mid-segment ($s = 0.25$), for low velocity impact. The peak value is around 9×10^{-4} for compression and 6×10^{-4} for tension. For high velocity impact, there is a very small tension zone at proximal end, followed by a compression zone

of a peak strain 30×10^{-4} and then another tension zone and compression zone. The distribution and peak values do not seem to change with increasing displacement.

Fig. 6.13. Mid-surface circumferential strain distribution along the mid-surface of the empty tube at different values of displacement: (a) $v = 10$ m/s; and (b) $v = 50$ m/s. Note that tensile strain is positive.

In order to assess the contribution of energy from bending and stretching/compression, the magnitudes of strain should be compared. Using Kirchhoff's hypothesis [173], the mean strain corresponding to curvature κ would be $\kappa t/4$. This gives a value of 480×10^{-4} for a peak curvature of 100 m^{-1} . This is 16

times the peak compressive strain at the mid-surface. Hence the energy absorbed by plastic bending is significantly higher than that would be by in-plane stretching/compression and it should be the dominant energy dissipation mechanism.

The above described curvature evolution may also be qualitatively observed from the profiles shown in Fig. 6.14(a, b). In essence, the dynamic deformation mechanism at high velocity involves a travelling plastic deformation zone which invokes bending of a strip to a curvature and then unbending to another curvature. If we adopt the term “hinge” to indicate a point of change in curvature, where plastic bending is concentrated, then we can say that a mechanism with a pair of moving hinges is involved. This term of “double moving hinges” is used in order to emphasize the important feature of both bending at one hinge and unbending at the other, as opposed to the conventional one travelling hinge which could involve one-way bending only (and unbending also, though). The bending energy associated with such double-moving-hinges can be worked out by considering an arc section with two plastic hinges (B and C) in Fig. 6.14c. The segment hinge BC has a radius R_0 and the strip before bending and after final unbending has a radius of R_1 and R_2 , respectively. It is easy to understand this double-hinge moving process if one imagines that the material is flowing from AB to BC , and at hinge B the strip is bent to radius $R_0 (< R_1)$. When the strip moves to hinge C , unbending occurs and the strip's radius becomes $R_2 (> R_1)$. For a strip of unit length and unit width, the energy absorbed by bending the material at B and unbending at C are given, respectively, by,

$$W_B = m_p \cdot \left(\frac{1}{R_0} - \frac{1}{R_1} \right), \quad W_C = m_p \cdot \left(\frac{1}{R_0} - \frac{1}{R_2} \right) \quad (6.6)$$

where $m_p = \sigma_y t^2 / 4$ is the fully plastic bending moment per unit width. The total energy dissipated by plastic bending when the unit strip passes this hinge is equal to the sum of the two. For a strip which is flat both before and after bending, this total energy is $2m_p / R_0$. In the present case, $1/R_2$ is approximately zero. This equation shows that the total plastic energy is almost inversely proportional to the radius of the rolling hinge, R_0 . A small value of R_0 would need a large amount of energy for the hinge to roll forward. Indeed, an infinitely sharp crease (which has a zero radius) would not roll forward, as it would need an infinite amount of energy to do so. In the present case, low velocity impact involves stationary plastic deformation zones (or hinges). But the high velocity impact is associated with dynamic double moving hinges, which may not only have higher curvatures than their static counterparts, but also travel and spread over a larger area. It is highly plausible that the observed large difference in energy dissipation may be attributable to this fundamental difference in the deformation mechanisms between the static and dynamic cases. Nevertheless, further analytical models based on impact mechanics should be established to accurately explain the fundamental mechanics of the deformation modes observed.

Fig. 6.14. Deformation profiles for $v = 10$ m/s (a); and $v = 50$ m/s (b). (c) Illustration of double-moving-hinges. Note that in (a) and (b), the solid squares represent the position of maximum curvature, and the solid triangles denote the quarter positions at $s = 0.25$, respectively. Only a half of the tube section is shown.

6.3.2 Dynamic Crushing of Foam-filled Tubes

A crushable foam model was employed for the aluminium foam core in FE analysis for the lateral crushing of foam-filled tubes by two rigid platens. The representative FE model in ABAQUS/Explicit is shown in Fig. 6.15. The element type of both inner and outer tubes is *S4R*, which is the same as in the previous analysis for empty tubes. The aluminium foam core is modeled as a three dimensional 8-node linear

brick element *C3D8R*, with reduced integration. Distortion control of foam elements is introduced since large plastic deformation was observed in the tests. The inner surface of outer tube and the outer surface of inner tube are tied to the foam core to model perfect glue without debonding. Surface to surface contacts are applied for the cases without glue and the friction coefficient is set at 0.1. Both platens are modeled as rigid bodies. The top platen, which is at the proximal surface, is fixed during the whole crushing stages. A constant velocity is applied on the bottom platen. To replicate the impact method adopted in previous experiments, an initial velocity is applied to the foam-filled tube. The interactions among the rigid platen, outer tube and foam core are considered by adopting a penalty function, as provided in the software package.

Fig. 6.15. Schematic of FE model subjected to impact loading (specimen: SWT01).

Since the metallic foam core is involved in the dynamic analysis, it is necessary to validate the numerical model of foam first. Based on the uniaxial compressive tests and corresponding finite element analysis on the cylindrical foam extrusion, the crushable foam model [23] is confirmed to represent the macroscopic mechanical behavior of metallic foam [86]. Further numerical simulations are made for the foam-filled tubes. A good agreement between the prediction and the experimental

result is shown in Fig. 6.16. Thus, the FE modeling could be used to study the dynamic behavior of foam-filled tubes. More detailed information such as partition of the energy absorption for each component is discussed in the later section and might provide potential support for the analytical models.

Fig. 6.16. Validation of FE models against experimental results.

The broad agreement in Fig. 6.16d is also noted for the specimen with deformation pattern I. The discrepancy and sudden drop of the crush strength at a displacement of 24 mm were probably due to the failure of adhesive. This may be inferred also from the further comparison with the unglued case, where the load-displacement curve was much closer to that without glue. It implies that the adhesive was probably prone to fail at this point, which led to a reduced load.

6.3.2.1 Deformation Modes of Foam-filled Tubes

From the collapse modes observed in the experiments with foam-filled tubes under low and moderate velocity impact, symmetric behavior about both vertical and horizontal planes was reported. The deformation at both the lower and upper portion of such structures were symmetric, as depicted in Figs. 6.6 to 6.8 for both quasi-static [131] and low velocity crushing. However, similar to the case of a single empty tube, when the compression velocity increases beyond a certain value, the deformation near the proximal and distal surface become non-symmetric with respect to the horizontal plane during the high speed crushing process. Fig. 6.17 shows typical deformation patterns of two different specimens subjected to high velocity impact up to 100 m/s. It can clearly be seen that the severe deformation of tubes is localized at the upper portion of structure, which is near the proximal surface. The lower portion of foam-filled tube remains undeformed even when the stroke reaches a value equal to the radius of the outer tube. Fig. 6.17a is for a foam-filled tube (SWT03) with thin foam core, identified as collapse pattern I. Both the outer tube and foam core deform from the beginning of the crushing. Visible deformation of the upper portion of inner tube occurs when the foam core becomes densified. Nevertheless, the lower portion of the foam-filled tube remains undeformed. For a tube (SWT06) filled with thick foam core, sequential deformation is also shown in Fig. 6.17b, as collapse pattern III. In both the cases when foam-filled tubes subjected to high speed impact, crushing propagates from the contact surface into the undeformed layer, which is different from the observation in the tests with low velocity. The double-moving-hinges as a dynamic bending mechanism presented for empty tubes should be applicable here.

In FEA, foam fracture and failure of adhesive are not considered. A homogenized foam constitutive relation is used. Such a relation is validated for foam cores under quasi-static loading. However, it is unclear whether such a constitutive description is adequate for the high strain rate loading, since different deformation modes are observed in Fig. 6.17 and the foam core is more vulnerable to crack under high velocity impact. Validation of FEA under high velocity impact is required in future.

Fig. 6.17. Deformation history of foam-filled tubes under high velocity ($v = 100 \text{ m/s}$): (a) SWT03 with thin foam core; and (b) SWT06 with thick foam core.

6.3.2.2 Collapse Load under High Velocity Impact

To understand the effect of velocity on energy absorption, the foam-filled tubes are dynamically compressed at different velocities, ranging from 10 m/s to 100 m/s. Fig. 6.18a shows the variation of collapse load in specimen (SWT03) for different compression velocities. It is evident that when the compression speed is lower than 20 m/s, the average collapse strength of the foam-filled tube having thin foam is almost the same regardless the value of velocity. Small discrepancy appears only at the initial crushing process where extensive plastic deformation takes place at the upper part of the tubes. It may be noted that for high velocities, the force drops sharply after the initial peak, which resembles the characteristic of Type II structures [35]. For the tube filled with thick foam, noticeable difference in the load-deformation curves for each loading velocity exists, as shown in Fig. 6.18b. Based on the area underneath each of these curves, the energy absorbed is larger for the specimens with thick foam and small inner tube, compared with those with thin foam.

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Fig. 6.18. Load-deformation curves of foam-filled tubes under different compression velocities: (a) SWT03 with thin foam core; and (b) SWT06 with thick foam core.

6.3.3 Energy Absorption of Empty and Foam-filled Tubes

From the finite element results on empty tube (ET01) and foam-filled tube (SWT03) in Fig. 6.17 and Fig. 6.18a, the ratio of energy dissipation under dynamic impact and that of the quasi-static counterpart is plotted in Fig. 6.19a, corresponding to a displacement of 70 mm. It is interesting to note that the points seem to fall into a single curve, whether it is for the empty or foam-filled tubes. When the velocity is less than 20 m/s, the ratio of dynamic energy to quasi-static energy is around 1.1. Afterwards, the energy absorption capability under the dynamic crushing increases sharply with the compression speed, and especially it is over five times when the compression velocity reaches 100 m/s. For the empty tubes studied, the dynamic energy enhancement over its static one is obtained, empirically, as follows

$$E_D/E_S = 1 + 38.86 \left(v / \sqrt{\sigma_y / \rho} \right)^{2.5} \quad (6.7)$$

which is also plotted in Fig. 6.19a.

Fig. 6.19. (a) Plots of enhancement over its quasi-static counterpart of energy dissipation versus velocity; and (b) plot of SEA versus compressive velocity. Note that solid square points are the results of empty tube and other points are the results of foam-filled tubes. The equation in solid line is obtained via similar double logarithmic methodology to Fig. 6.11.

The specific energy absorption (SEA), which is the energy absorbed (EA) per unit mass of a crushed structure, is studied. The values of EA and SEA for specimens under dynamic experiments are also given in Table 6.1, for a displacement of $0.7D$. It is shown that foam-filled tubes with collapse pattern III is the most weight efficient, with a maximum SEA value of 3.96 J/g. For velocities up to 100 m/s, the values of SEA for typical specimens of comparable dimensions (ET01, SWT03 and SWT06) are plotted in Fig. 6.19b. The trend is similar to that shown in Fig. 6.19a, as expected. The dynamic effect would lead to an increase in the SEA for all specimens. It is also illustrated that for specimen SWT06 with the thickest foam and smallest and thickest inner tube, the value of SEA is almost four times that of the corresponding empty tube (ET01). This result demonstrates that tubes filled with a

thicker foam core and thicker inner tube have better weight efficiency.

To assess the energy dissipation characteristic of foam-filled tubes, the total energy absorbed by the foam-filled tubes is partitioned for each component, i.e. the outer and inner tubes as well as the foam core. Fig. 6.20a shows the strain energy of each component in a tube (SWT06) filled with thick foam core subjected to a velocity of 10 m/s. When the dynamic displacement is less than 20 mm, the inner tube does not contribute to energy dissipation process until the crushing layer becomes severely deformed and stiffened. The impact energy is almost equally dissipated by the outer tube and foam core. Afterwards, all the three components start to dissipate energy. It is noted that at the displacement of 70 mm, the foam core dissipates 56.2% of the total strain energy. For the outer tube and inner tube it is 28.5% and 15.3%, respectively. To further investigate the effect of velocity on energy absorption, a graph of energy dissipation for the tube under different compression speeds is plotted in Fig. 6.20b, in terms of the proportion of energy absorbed. The proportion for the foam core is over half of the whole internal strain energy, nearly 60%, and this percentage does not vary with the compression speed. However, for compression velocity up to 50 m/s, the outer tube absorbs more energy than the inner one. On the contrary, when the compression velocity is large, the inner tube is crushed and deformed severely with more strain energy. Overall, for the foam-filled tubes with collapse pattern III, the energy absorption is dominated by metallic foam core, up to 60%, which again confirms the foam-filling effect in the structural design. Future work is needed to elucidate the mechanics embedded.

Fig. 6.20. Energy characteristics for each component of SWT06 subjected to different velocities: (a) energy dissipation; and (b) partition of energy dissipation.

6.4 Concluding Remarks

This chapter presents both the experimental and finite element results on the dynamic responses of empty tubes and foam-filled tubes by two flat plates. The lateral crushing tests of several empty tubes and foam-filled tubes with different geometrical parameters have been carried out using an INSTRON machine. The deformation histories and load-deformation profiles have also been obtained. The energy is dissipated by plastic deformation starting at the section near the proximal surface, which is the impact region. A critical velocity has been identified for mode change and is related to t/D by means of dimensional analysis.

Furthermore, three types of collapse patterns in the quasi-static tests have also been observed in the dynamic crushing experiments on the foam-filled tubes subjected to compression velocity up to 10 m/s, resulting in different energy dissipation mechanisms. The dynamic behaviour of foam-filled tubes under different velocities has been discussed, in terms of deformation patterns and energy dissipation. Non-symmetric deformation pattern about the horizontal plane has been observed in the

case of high velocity impact. The energy absorption characteristics of foam-filled tubes under impact crushing are analysed. For both empty and foam-filled tubes, significant increase has been shown for the dynamic crushing load and energy absorption over its quasi-static counterpart, for which the inertia effect would be responsible. This can be explained by employing the concept of a double-moving-hinges mechanism to describe the propagation of plastic deformation from the proximal surface. Such moving hinges with a small radius in-between would dissipate a significant amount of energy, which is completely different from a conventional stationary plastic hinge. The inertia effect present in the dynamic crushing plays two roles. First, it invokes a deformation mechanism with the double-moving-hinges. Second, it increases the curvature of the segment in-between the two moving hinges. Future work is needed to establish analytical models in order to explain, by means of impact mechanics, how the impact velocity determines the radius of the moving hinge.

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

This thesis presents a systematic study on the energy-absorbing performance of thin-walled tubes and double tubes filled with metallic foam. A summary of current work is included and future work is suggested in this chapter.

7.1 Conclusions

7.1.1 Thin-Walled Tubes with Different Cross-sectional Shapes

The axial crushing behavior of thin-walled empty tubes of various cross-sections was studied by including both convex and concave sections, in terms of better *SEA*, structural effectiveness (η) and energy-absorbing effectiveness (ψ). A series of experiments were presented. The specimens were fabricated using wire-cut EDM technology to provide desirable results without the occurrence of crack or material failure at the non-uniform welding seam in the traditional method. Four types of geometries were tested. They were hexagon, octagon, 12-sided and 16-sided star, respectively. The quasi-static axial crushing experimental data were then compared with those predicted by FE analysis using ABAQUS/Explicit. Good agreement was achieved in both deformation modes and the load-displacement curves. An increase in corner elements in the star shape improves the local buckling capacity as well as the energy absorption capacity, but limited to a certain extent. It is found that the 12-sided star shape has the best energy absorption capability when diameter-to-thickness is less than 50. The 16-sided star shape performs poorly compared to other shapes studied.

Furthermore, collapse behavior of the triangular tube having only acute angles was studied. Based on data from Finite Element Analysis (FEA), theoretical inextensional collapse modes are proposed to describe the plastic progressive collapse of thin-walled equilateral triangular tubes. The progressive folding around the stationary horizontal hinges and inclined travelling hinges are involved to develop a new basic folding element. The corresponding fold length and mean crushing load are predicted from the Super Folding Element theory and validated against experiments. Two types of inextensible collapse modes are discovered, i.e. diamond mode and rotational symmetrical mode. The approximate quasi-static theoretical predictions for the mean crushing loads of triangular tubes are in reasonable agreement with corresponding experimental results in some areas.

7.1.2 Metallic Foams

By combining an FEA with an analytical treatment, quantitative information is provided on the stress, strain and deformation states induced during the axisymmetric expansion of a cylindrical hole of an initial radius, located at the center of a block of closed-cell metallic foam of infinite size. Uniformly distributed radial loading is applied on the surface of the hole. Crushable foam with isotropic hardening model is incorporated. Two deformation stages are revealed from the numerical simulation, i.e. the initial yielding and then subsequent expansion of the hole accompanied with hardening. Preliminary analytical formulation is performed with respect to the pressure at the initial yielding and the size of the subsequent plastic deformation zone. It is found that the initial yielding occurs at the pressure $p_e = 0.633 \sigma_y^0$, which is identical to that from FEA. The evolution of plastic zone during the expansion is discussed in terms of the results from the analytical and

finite element studies. When the internal pressure is increased to $p_d = 1.375 \sigma_y^0$, foam densification is observed. In addition, a deformation map is obtained showing the evolution of the three deforming zones, i.e. elastic, plastic and densification of the foam, when the applied pressure increases.

7.1.3 Foam-filled Tubes

Both the experimental and finite element results on the energy absorption of empty tubes and tubes filled with metallic foam compressed laterally by two flat plates were presented in this thesis. Firstly, the deformation characteristics of foam-filled double tubes subjected to quasi-static lateral crushing were studied based on extensive experimental observations. Different collapse patterns were exhibited and classified: simultaneous collapse (pattern I), simultaneous collapse with fracture of the foam core (pattern II) and sequential collapse (pattern III). It is evident that the bonding effect, decreasing the average radius of curvature and thickening the foam core improve energy dissipation of foam-filled tubular structures. Based on the detailed FE analysis on foam-filled tubes, energy absorption characteristics were clarified, in terms of energy partition and specific energy absorption. It is shown that composite tubes with the highest proportion of foam filling absorb the maximum amount of energy when subjected to quasi-static loadings.

Subsequently, the dynamic lateral crushing behavior of short empty and foam-filled circular tubes was examined. Unlike the conventional impact method and to avoid rebound phenomenon, the specimens and bottom platen were impactors. Specimens were compressed at a constant velocity. From the viewpoint of deformation modes and plastic strain energy absorbed by the quadrants of empty tube, the energy is

initially dissipated by plastic deformation at the section near the proximal impact surface. A critical impact velocity is determined which corresponds to a mode change in the empty tube. Through dimensional analysis, a relationship is found to exist between the critical velocity and the thickness-to-diameter ratio as well as the yield stress and the density of the material.

A simple deformation mechanism of the empty tube subjected to different impact velocities is proposed by combining experiments and FEA (ABAQUS), including the curvature and circumferential strain distribution along the mid-surface. A dynamic plastic double-moving-hinge is found to be the main mechanism of energy dissipation, as opposed to stationary hinges in the quasi-static mode. There are some circumferential compressive and tensile strains at the mid-surface. Nevertheless, the circumferential strain seems an order of magnitude lower than that due to bending and hence bending is still regarded as the dominant dynamic energy dissipating mechanism.

Three types of collapse patterns in the quasi-static tests were observed in the dynamic crushing of the foam-filled tubes. Similar to the observations in the quasi-static tests, the mode of dynamic collapse was bending; with the formation of plastic bending zones accompanied with core crushing. The corresponding FE models for ABAQUS/Explicit are developed and validated against the experimental observations. Detailed energy absorption characteristics during the crushing process are identified. For both empty and foam-filled double tubes, significant enhancement has been shown for the total internal plastic energy dissipation in dynamic loadings when the impact velocity is greater than 20 m/s, for which the

inertia effect would be responsible. It is evident that tubes filled with a thicker metallic foam core are more energy efficient, not only in quasi-static situation, but also in dynamic loading cases. The propagation of plastic bending in the form of double-moving-hinges is the main mechanism of energy dissipation, as opposed to the low velocity impact which involves stationary plastic deformation zones.

7.2 Future Work

This thesis investigates the energy absorption characteristics on foam-filled shell structures and their components. Several promising future work is recommended in this section.

To further validate the current FE results about various polygonal tubes as well as triangular tubes, experiments are still required for a wider range of D/t ratios. Furthermore, in the current analysis, FEA is quite time costly and advanced schemes are inevitable to reach the goals within an acceptable period of time. Further research might be able to resolve issues concerned with more polygonal tubes with odd number of sides, like the pentagonal tubes. Since aluminium foam fillers could noticeably improve the energy absorption capacity of a tube during crushing, studies on the effect of aluminium foam on star-shaped tube are still incomplete and more detailed work is to be conducted. In addition, expansion of foam-filled double tubes under internal pressure needs to be investigated in future. Theoretical model based on the incremental theory of foam plasticity should be employed in future to predict the quasi-static response in foam-filled tubes. Experiments are also required to validate both the numerical and analytical models during the expansion.

Besides, much of the current research focuses on the large plastic deflection without taking into account the structural failure in the foam-filled tubular structures, which would occur very often in the actual experiments, especially under impact and blast loadings. Analysis needs to be concerned to cover such failure modes. For numerical investigations, a more physical model of foam-filled tubes involving some failure criteria of each component should be formalized as well. Corresponding experiments will be conducted to explore the blast resistant capacity in order to accomplish the optimization design. Different foam densities may need to be determined since current researches are only concentrated on a certain density of foam, as well. If possible, different layouts or arrangements of foam-filled tubes arrays, hexagonal and squarer packed arrays, may be studied under quasi-static and dynamic loadings, by using experiments and corresponding finite element analysis.

Following the experiments and FE analysis, considerably more analytical work are to be conducted to capture the fundamental physics involved in the problem. More details about the collapse mechanisms may be conducted. The resulting equations will in turn serve as a basis for the optimum design of such energy-absorbing devices under a given loading.

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PUBLICATIONS ARISING FROM THIS RESEARCH

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