Methodologies development and analysis for obtaining boundary conditions of heat transfer in a linear motor driven stage

Chow, Kelvin Joon Ho

2013

Chow, K. J. H. (2013). Methodologies development and analysis for obtaining boundary conditions of heat transfer in a linear motor driven stage. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/51777>

<https://doi.org/10.32657/10356/51777>

Methodologies Development and Analysis for
obtaining boundary conditions of Heat transfer
in a linear motor driven stage

KELVIN CHOW JOON HO

SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

2013

Methodologies Development and Analysis for obtaining boundary conditions of Heat transfer in a linear motor driven stage

KELVIN CHOW JOON HO

School of Mechanical and Aerospace Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2013

Acknowledgement

Firstly, I would like to express my deepest gratitude to Assoc Professor Zhong Zhaowei for his time and effort in supervising me on this thesis and paper writing. This thesis will not have been successful if not for his encouragement and guidance. As PhD studies are tedious to me, he has directed me step-by-step to reach several seemingly unreachable milestones. He has helped to break down big and difficult problems to numerous moderately achievable tasks with his advice. I am grateful for the amount of time he spent on guiding me.

Then, I would like to thank Professor Khoo Li Pheng for his advice and critique. His advice has allowed me to think more critically and decisively select the meaningful tasks to do. His advice has also strengthened the direction of my research focus and the methodology.

Lastly, I would like to thank Dr Lin Wei for the opportunity to undertake this thermal study. It is his vast experience and far-sightedness that determine thermal studies to be an important field from then onwards. He also introduces many important types of equipment that are essential for all the experiment setups.

Table of Contents

	<u>Page</u>
Acknowledgement	i
Table of Contents	ii
Summary	vii
List of Figures	viii
List of Tables	xv
List of Publications	xvi
 Chapter 1	
Introduction	1
1.1	Background
1.2	Motivation
1.3	Objective and Scope
1.4	Organisation of the Thesis
Chapter 2	
Literature Survey	8
2.1	Machine Positioning Errors Due to Heat Effects
2.2	Conventional Linear Stages Equipped with Ball-screw Type Actuation and Contact Roller Guides
2.3	Modern Linear Stages Equipped with Linear Motor Actuation and Aerostatic Bearing Guides
2.4	Research Areas in the Field of Linear Motor
2.5	Numerous Thermal Models Research in the Past Few Decades

2.5.1	Totally Enclosed Fan Cooled Induction Motor (TEFC)	16
2.5.2	Computer Numerical Control Machines (CNC)	18
2.5.3	Vertical Machining Centre (VMC)	19
2.5.4	Coordinate Measuring Machine (CMM)	21
2.6	Thermal Related Research with Application in Stages Equipped with Linear Motors	21
2.7	Conjugate Heat Transfer	24
2.7.1	Background	25
2.7.2	Numerical Methods	28
2.7.3	Analysis of the Summarized Method for Conjugate Heat Transfer	29
2.8	Summary of Chapter 2	32
Chapter 3	Research Motivation and Methodology	33
3.1	Limitations and Disadvantages of Existing Thermal Models	33
3.2	Thermal Modelling Approach	37
3.3	Lumped-Parameter Methodology for the Overall Modelling System	45
3.4	Heat Source Sub-Model for Heat Flux Generation from the Motor Coil	47
3.5	Contact Resistance Sub-Model for Heat Flow between Materials	48
3.6	The Approach for Solving Conjugate Heat Transfer	49
3.7	Summary of Chapter 3	50

Chapter 4	Evaluation of the Need to Analyse the Boundary Conditions	51
4.1	Thermal Induced Errors in Precision Machines	51
4.2	A Brief Description of the Thermal Induced Errors	51
4.3	Simulation of the Thermal Induced Errors	53
4.4	Heat Transfer and Heat Loss	57
4.5	Experimentation of the Thermal Induced Errors with Uncertain Boundary Conditions	60
4.6	Evaluation on the Effect of Radiation Heat Loss	66
4.7	Summary of Chapter 4	67
Chapter 5	Analysis of the Heat Source	68
5.1	Introduction	68
5.2	The Joule Heating Phenomenon	72
5.3	The Numerical Model for Motor Coil	83
5.3.1	Heat Conduction Equation in Solids	83
5.3.2	Lump-parameter Formulation	84
5.3.3	Convective Heat Loss Coefficients	89
5.4	Experimental Set-up	92
5.5	Results and Discussion	93
5.5.1	Steady State	93
5.5.2	Transient State	100
5.6	Summary of Chapter 5	110

Chapter 6	Analysis of the Effective Convection Heat Loss	111
6.1	Introduction	111
6.2	The Analytical Models for Thermal Analysis	116
6.2.1	Theoretical modeling basis	116
6.2.2	Heat Transfer and Fluid Flow Governing Equations	118
6.2.3	Numerical Lump-parameter Model Formulation	121
6.3	Verification of the Lumped-Parameter Model	123
6.4	Experimental Set-up	128
6.5	Results & Discussion	131
6.6	Summary of Chapter 6	139
Chapter 7	Analysis of the Contact Conductance of Machine Joints	140
7.1	Introduction	140
7.2	The Analytical Models for Thermal Analysis	144
7.2.1	Theoretical modeling basis	144
7.2.2	Numerical Lump-parameter Model Formulation	146
7.3	Experimental Set-up	149
7.4	Contact conductance acquisition and analysis	151
7.5	Results & Discussion	162
7.6	Summary of Chapter 7	165
Chapter 8	Evaluation of the Analysed Boundary Conditions	166
8.1	Introduction	166
8.2	The lumped parameter thermal model	166

8.3	Experimental Set-up	173
8.4	Results & Discussion	176
8.5	Summary of Chapter 8	177
Chapter 9	Conclusion & Recommendation	178
9.1	Conclusion	178
9.2	Recommendation	180
	References	182
	Appendix A	200
	Appendix B	214

Summary

Permanent magnet linear motors have been increasingly used in many machine applications that need precise and high speed motions since the last decade. A major drawback which is common with all other motors is high heat generation. The heat trap in the coil cannot be dissipated to the environment fast enough, and much of the heat is transferred to the carriage, which is the part of the structure that is fixed to the motor coil and a linear motion guide. The carriage is usually a platform that rigidly holds the specimen to be processed. The thermally induced deformations and stresses of the carriage, due to high heat influx from the coil, will distort large specimen, especially 300 mm wafers. Measurements of the thermally induced deformation errors were carried out and compared with the simulation results. The comparison shows discrepancies due to unknown boundary conditions. Further simulation studies show the importance of accurate boundary conditions for a reliable set of simulated results. A modified lumped-parameter method is adopted here to model the temperature and heat flux from the joining top of the coil to the carriage. The model has several essential elements such as a heat source sub-model, heat loss convective coefficients for complex geometry, and contact conductance values between structure parts. The methods of using the temperature sensors, to be placed strategically on the machine structure, to obtain the respective temperatures have been developed. The sub-models are combined and applied on the linear stage's carriage using modified lumped-parameter method and the deformations can be predicted reasonably well.

List of Figures

	Page
Fig. 2.1. Breakdown of thermal errors contribution	9
Fig. 2.2. Schematic diagram of a ball-screw linear stage with feedback control	11
Fig. 2.3. Assembled linear motor with air bearing guide	13
Fig. 2.4. Block diagram of a close-loop control with position and velocity feedbacks	15
Fig. 2.5. Power flow of a squirrel-cage induction machine	16
Fig. 2.6. Real-time error compensation with a vertical machine centre	20
Fig. 2.7. Typical single axis stage using linear motors	22
Fig. 2.8. Thermal resistance model of LVR motor for steady state heat transfer analysis	24
Fig. 2.9. Energy balance equation formulation	31
Fig. 3.1. Thermal path in motion system	38
Fig. 3.2. Existing models for heat transfer in machine structures	40
Fig. 3.3. Proposed framework for heat transfer modelling in machine structures	41
Fig. 3.4. Thermal error compensation for specific machine and specific processes	42
Fig. 3.5. Thermal error compensation for specific machine and general processes	44
Fig. 3.6. A proposed lumped-parameter method	46
Fig. 4.1. 3D CAD model of single axis stage equipped with linear motor.	52

Fig. 4.2.	Linearity errors maps that shows the increase of the errors as displacement is increased from the centre line	52
Fig 4.3a.	Simulation of a simple carriage with only the heat source and natural convection with structural boundary conditions at the heat source.	54
Fig 4.3b.	Simulation of a simple carriage with the heat source and natural convection with structural boundary conditions at the two long impractical guides.	55
Fig 4.3c.	Simulation of a simple carriage with the heat source and natural convection with structural boundary conditions at the four normal guides.	55
Fig. 4.4.	Simulation for thermal induced deformation (linearity deformation).	56
Fig. 4.5.	Simulation for thermal induced deformation (horizontal straightness deformation).	57
Fig. 4.6.	One dimensional lumped-parameter model	58
Fig. 4.7.	Experimental and lumped-parameter model results	59
Fig. 4.8.	Experimental setup of with a HP 5529A laser interferometer to measure the linearity, horizontal straightness and vertical straightness errors	61
Fig. 4.9.	Assembled single axis stage with linear motor and roller bearings for set-ups with straightness measurements.	62
Fig. 4.10.	Simulation with different values for convective coefficients and contact conductances.	65
Fig. 5.1.	Power flow chart of a typical motor.	73
Fig. 5.2.	Experimental set-up of the heater block supported by clamping on its thick cartridges wires.	74

Fig. 5.3.	Boundary conditions for FEA simulation and lumped-parameter modeling.	77
Fig. 5.4.	FEA temperature results of the heater block under steady state simulation.	78
Fig. 5.5.	Experimental set-up for evaluating the heating phenomenon of the motor coil.	80
Fig. 5.6.	Schematic of experimental set-up where the motor coil is actuating a load against gravity.	80
Fig. 5.7.	Boundary conditions for steady state temperature simulation of the motor coil.	81
Fig. 5.8.	Steady state temperature simulation of the motor coil with parameters from Fig. 5.7 and the calculated power from Table 5.6.	82
Fig. 5.9.	Schematic of the 3-D lumped parameter formulation.	85
Fig. 5.10.	Resistance calculation between two adjacent lumps.	85
Fig. 5.11.	The motor coil which will be used throughout the experiment portion.	87
Fig. 5.12.	The X-ray vision of the three pairs of coils inside the epoxy housing.	87
Fig. 5.13.	Illustrations of the copper coils in the linear motor coil model in CAD.	88
Fig. 5.14.	a) The line of symmetry shown to reduce the number of elements. b) The structure of the elements for the reduced model of the motor coil.	89
Fig. 5.15.	The boundary conditions allocated for the lumped-parameter method.	90
Fig. 5.16.	Position of the three temperature sensors on the top surface of the motor coil.	91
Fig. 5.17.	Transient temperatures of temperature sensor two and the corresponding lumped parameter surface node.	94

Fig. 5.18. Correlation between weight loading and the power constant for the lumped-parameter model.	95
Fig. 5.19. Correlation between current input and the power constant for the lumped-parameter model.	96
Fig. 5.20. Correlation between current input and the power constant for the lumped-parameter model.	97
Fig. 5.21. Heat flow in the various components of the linear motor coil.	99
Fig. 5.22. The amplification factor is plotted against time, after using least squares estimation.	101
Fig. 5.23. Amplification factor as plotted against the temperature of the coils.	102
Fig. 5.24a. The relationship of the transition temperature, in the transient amplification factor of Fig. 5.23, with respect to each load's current input.	103
Fig. 5.24b. The relationship of the gradient, in the transient amplification factor of Fig. 5.23, with respect to each load's current input.	103
Fig. 5.24c. The relationship of the Y-Axis intercept, in the transient amplification factor of Fig. 5.23, with respect to each load's current input.	104
Fig. 5.24d. The relationship of the steady state temperature, in the transient amplification factor of Fig. 5.23, with respect to each load's current input.	104
Fig. 5.24e. The relationship of the steady state amplification, in the transient amplification factor of Fig. 5.23, with respect to each load's current input.	105

Fig 5.25a. The verification of correlation equation (5.26) used in the lumped-parameter with a 1.5kg load.	108
Fig 5.25b. The verification of correlation equation (5.26) used in the lumped-parameter with a 2.5kg load.	109
Fig 6.1. Heat transfer analysis of a fin.	117
Fig 6.2. Internal temperature profile of a block with different value of convection coefficient values on its surfaces.	118
Fig 6.3. The aluminum grade 6061T6 block of size 100 by 100 by 75 mm used for the experiments.	122
Fig 6.4. The discretisation of the aluminum block.	122
Fig 6.5. The block and data acquisition chassis is mounted on the carriage of the linear motion stage.	129
Fig 6.6. The back view of the set-up.	130
Fig 6.7. Mounting of surface thermocouples for relating with convection coefficient values.	131
Fig 6.8. Temperature of one internal node under various speed profile of the aluminum block.	133
Fig 6.9. The evaluated convection coefficients from the lumped-parameter model.	134
Fig 6.10. The temperature difference of one of the two pairs of thermocouples on the surface.	135
Fig 6.11. Transient temperature of an internal node of the block.	137
Fig 6.12. Closed up view to show the temperature changes with carriage motion speed.	138

Fig 7.1	Temperature measurement points on the same side of material and across the interface.	145
Fig 7.2.	Thermal contact conductance simulation of two blocks.	146
Fig 7.3	The two blocks used for the present study. The left is a whole solid block, while the right is the block which has been separated into two pieces.	147
Fig 7.4.	The discretisation of the aluminum block.	148
Fig 7.5a.	The experimental set-up for the solid block.	149
Fig 7.5b.	The experimental set-up for the two separate pieces of solid block.	150
Fig 7.6.	The schematic diagram of the experimental set-up.	150
Fig 7.7a.	Temperature of two internal nodes, adjacent to each other across the interface under various power inputs.	152
Fig 7.7b.	Temperature of the three surface thermocouples placed as shown in Fig. 7.4.	153
Fig 7.8.	Schematic of lumped nodes 4 and 12 with a contact interface in between.	154
Fig 7.9.	The evaluated contact conductance from the lumped-parameter model.	158
Fig 7.10.	Temperature difference between the pair of thermocouples, P1 and P2, on the surface.	159
Fig 7.11.	Normalised temperature difference of the surface sensors.	160
Fig 7.12.	The correlation of the thermal contact conductance with normalised surface temperature difference.	161
Fig 7.13.	The correlation of the thermal contact conductance with node 4 temperature.	163

Fig 7.14.	Transient temperature of an internal node 12 of the block.	164
Fig 8.1.	The carriage assembly of the linear motor stage.	167
Fig 8.2.	The carriage of the linear motor stage.	168
Fig 8.3.	The lumped parameter discretisation of the carriage.	169
Fig 8.4.	The boundary conditions for the lumped parameter model.	171
Fig 8.5.	The schematic diagram of the thermal deformation experimental setup.	172
Fig 8.6a.	Experiment set-up for the thermally induced deformation measurement.	173
Fig 8.6b.	The experimental set-up for strain gauge mounting.	173
Fig 8.7.	Temperature measurements and prediction of the carriage and motor coil under cyclic loaded motion.	174
Fig 8.8.	Thermal deformation of the carriage under cyclic loaded motion.	175

List of Tables

	<u>Page</u>
Table 3.1 Limitations and Disadvantages of Existing Thermal Models	34
Table 4.1 Comparison between simulations and experimental results	63
Table 4.2 Deformation results after boundary condition values have been changed	63
Table 4.3. Convective coefficient and contact conductance values	63
Table 4.4 Air Properties at 40 °C	67
Table 5.1 Air properties at 60 °C	75
Table 5.2 Additional parameters for evaluating the dimensionless numbers	76
Table 5.3 Evaluated dimensionless numbers	76
Table 5.4 Properties of aluminium	77
Table 5.5 Temperature results	79
Table 5.6 Motor coil experimental results	79
Table 5.7 Energy stored in spring due to a 2 kg load.	79
Table 5.8 Convective heat transfer coefficients	91
Table 5.9 Temperature measurement at 7000s	92
Table 5.10 Power constant α and weight correlation verification	93
Table 6.1 Air properties at 40 °C	125
Table 6.2 Other parameters needed to calculate the dimensionless numbers	126
Table 6.3 Evaluated dimensionless numbers and convection coefficients	126
Table 6.4 Internal temperatures of the block	127

List of Publications

Journal papers

- 1) J.H. Chow, Z.W. Zhong, W. Lin, L.P. Khoo, A study of thermal deformation in the carriage of a permanent magnet direct drive linear motor stage, *Applied Thermal Engineering*, Vol. 48, 15 December 2012, pp89-96. (Published)
- 2) J.H. Chow, Z.W. Zhong, W. Lin, L.P. Khoo, A lumped-parameter thermal model of a three-phase coreless linear motor coil as a heat source, *International Journal of Thermal Science*. (Submitted and under review)
- 3) J.H. Chow, Z.W. Zhong, W. Lin, L.P. Khoo, An investigation to obtain the convection coefficient under sakaidis type flow of a linear motor stage's carriage, *Experimental Thermal and Fluid Science*. (Submitted and under review)
- 4) J.H. Chow, Z.W. Zhong, W. Lin, L.P. Khoo, An investigation to obtain the thermal contact conductance using lumped parameter method, *International Journal of Thermal Science*, Vol. 60, October 2012, pp 114-121. (Published)

Conference paper

- 1) J.H. Chow, Z.W. Zhong, W. Lin, L.P. Khoo, W.J. Lin, G.L. Yang, Investigation of thermal effect in permanent magnet linear motor stage, 11th International Conference of Control and Automation, Robotics and Vision, Singapore, 5-8th December 2010, pp. 258-262.

Chapter 1 Introduction

1.1 Background

In all kinds of machines with a purpose to do some useful work, such as drilling holes, cutting materials and inspecting product quality, positioning errors need to be addressed. These position errors may come from many sources, which include both static and dynamic errors as structural geometrical accuracy [1], material instability [1], vibration issues [2], and assembly errors [1]. Much research has been conducted on each of these areas of concern. As time passes, machine accuracy requirements have been increasing gradually. Thermally induced deformation errors have been identified as one obstacle that impedes further improvement in machine accuracy [3]. This once-neglected source of errors, in the past, due to lower accuracy requirements has come to a point where it no longer can be ignored under the high accuracy requirements. Thermal errors of machines are influenced by three key factors, namely processes, work specimen and time. Basically, thermally induced error is a dynamic error which is hard to predict compared to the previously mentioned static errors, which seldom change in a machine.

Many studies on thermal related issues on cutting and positioning machines have been carried out to predict and compensate the errors. These studies can be broadly classified into two categories namely analytical based and correlation based methods

[4]. Both methods have their merits and shortcomings, and can be further categorized into real-time and non-real-time strategies. Compared to correlation based methods, analytical based methods have far fewer published works and are usually analyzed with simple geometry, which may not be applicable to practical situations.

Thermally induced deformations on several types of precision machines have long been investigated by many researchers in the past decades. These include the study of temperature distributions and thermal characteristics of hydraulic motors [5], and rotary motor ball-screw driven systems with linear ball rollers as guides [6-8]. Thermal modelling and compensation of machining structures have been widely studied by using advanced simulation software. For example, predictions of thermal patterns have been done on coordinate measuring machines (CMM). Compensation of CMM's errors needs to be done within a very tight tolerance and is typically more precise as compared to other precision-based machine. A voice coil linear motor driven X-Y stage with an air bearing guide can attain 2nm repeatability without directly having thermal-based error compensation. It uses position control to attain the ultra-precise positioning [9].

As the requirement for positioning accuracy is getting more stringent, there is a gradual switch from coupled gearing with rotary actuator systems to contact-less direct drive systems. Permanent magnet linear motors, for example, are one of the commonly used contact-less direct drive actuators in many high precision machines. Basically, contact-less air bearing guide enables the linear motor to perform a near frictionless motion with very high accuracy. The absence of much of the friction also

allows high speed applications where high acceleration is possible. However, almost all current usage of linear motors is still coupled with roller bearing guides with very low friction. This is essentially due to the very high cost of air bearing components and the system that is included to support the air bearing pads. Another factor is that a high stiffness guide is usually needed in practical applications and air bearing guides are not able to provide that.

Using any electric motor, there is high heat generation in the rotor coil wires. A coupled gear mechanism with a rotary motor has the advantage of isolating its heat source as the gear transmission allows the rotary motor to be placed far from the important support structures and working areas. This would efficiently reduce or eliminate the effect of thermally induced deformation on the accuracy of machines.

A linear motor stator, with magnets, is usually fixed to the non-movable base structure of a machine while the rotor (motor coil) is fastened to the application stage. The motor coil is placed in the stator's U-shape channel and supported by linear guides. The compact design of the stage with linear motors and linear guides helps reduce offset errors such as the Abbe error [11]. It has the advantages of providing good static accuracy and better space utilization. On the other hand, the heat generated from the motor coil is more difficult to be dissipated to the ambient air. Thus, the unwanted heat is conducted to the stage, resulting in thermally induced deformations.

There were suggestions to use insulation materials, such as ceramics, between the motor coil and the stage so as to isolate the heat flow. However, a certain thickness of the insulation material is needed to effectively stop most of the heat flow. This is likely to result in a higher Abbe error and, at the same time, reduce the compactness. Moreover, the insulation material will retain the heat in it. This, in turn, will cause more distortion and result in thermal stresses within and between materials.

Some machines use coolant to counter the thermal heat flow. Coolant can be sprayed onto the work-piece in an enclosed machine or flowing in cooling tubes in the machine structure. As the work-piece generates very high heat during machining operations, coolant is able to dissipate most of the heat away and reduces the amount of distortions on the work-piece. Cooling of the entire precision machine structure is rare as it requires costly and frequent maintenance. Moreover, cooling the entire machine structure may not reduce the thermal errors effectively. As thermal errors are dynamic, the machine structure may be under-cooled or over-cooled at times and may result in more complex thermal distortions and create thermal stability issues.

1.2 Motivation

Many studies have been performed on thermally induced deformation errors compensation using different correlations methods. These studies mainly adopt neural networks to correlate the temperature inputs of machine structure with the displacement errors measurement. Neural network based method is machine and process specific. When there is a need to alter the task or process sequence on the

machine, the correlations have to be established again. This process can be time consuming and costly as learning requires substantial amount of data to be collected. Traditionally, neural network based method best suits low-mix-high-volume processes [1].

In recent years, in order to compete in the market place, a high-mix-low-volume process is often adopted. The aforementioned neural network method may not be suitable as its learning process can be very involved. Accordingly, a process-based compensation mechanism appears to be attractive. Moreover, such a mechanism may not require as many temperature sensors as the correlation-based methods do. However, to realise a process-based compensation mechanism, the knowledge about machine's structure and thermal effects need to be acquired and well understood.

As already mentioned, thus far, there are very few studies on the analytical method. Moreover, these studies mostly concern heat generation due to the interaction of tool and work-piece during machining [5]. A few of them look into modelling of friction of contact guides [6-8]. None of them attempts to deal with error compensation.

Further investigation into the possibility of applying analytical methods to deal with error compensation of complex machine structures also reveals that efforts to model thermal characteristics of materials has been lacking. There are numerous textbooks or online information about the conductivity of materials and thermal resistance. However, some important issues such as heat generation from actuators, thermal contact conductance between different materials at joints and natural convection heat

transfer coefficients of surfaces at different orientations, have not been thoroughly investigated. These issues, in the author's opinion, are critically needed for an analytical thermal model to work.

1.3 Objective and Scope

This work models the thermally induced deformation of linear motor driven stages. The relationship between process induced power input and process speed with temperature induced thermal deformation is examined and modelled.

The work here involves modelling of three different elements for heat transfer of a machine. They are namely heat generated by actuators, thermal resistance between different materials at joints, and natural convective heat transfer coefficients at surfaces of different orientations. The first model is used to predict the amount of heat energy generated by a linear motor coil and the transmission of heat to the motor carriage. The second model is developed to predict the convective heat loss coefficients on the surfaces of different orientations at different motion speeds. The third model is used to estimate the contact conductance across two surfaces with different contact pressures and temperature inputs.

A mathematical model will be established to combine the three different elements of heat transfer. It will be employed to estimate the thermal deformation on a machine carriage and use it for positioning error compensation.

1.4 Organisation of the Thesis

The thesis is organised as follows:

Chapter 2 presents a literature review on past works related to this thesis. It begins with a discussion on linear motors and ball-screw mechanisms, and highlights the advantages and disadvantages of using linear motors. Essentially, linear motors have an inherent high heat generation problem. Such a problem would deform the carriage and cause positioning errors. Techniques for the modelling of heat transfer are reviewed. A detailed study on conjugate heat transfer, an important mode of heat loss, is performed. Chapter 3 outlines the limitations of existing methods and proposes a methodology to overcome them. In Chapter 4, an investigation into the positioning errors resulted from the heat generated from the motor coil is carried out. It includes a discussion on the accuracy of the boundary conditions and the thermal sub-models. Chapter 5 presents the proposed method for the modelling of heat source boundary conditions. Proposal of the modelling and treatment of conjugate heat loss are presented in Chapter 6. Chapter 7 presents the proposal of the method to handle contact conductance between two solid materials. Chapter 8 presents the temperature and deformation prediction using the lumped parameter method developed in this work and the sub-models. Chapter 9 summarizes the main conclusions reached in this work and discusses the recommendations for future works.

Chapter 2 Literature Survey

2.1 Machine Positioning Errors due to Heat Effects

All machines have positioning errors that can be categorized into static and dynamic errors. Static errors are mainly due to manufacturing and assembly errors which can be compensated by simple and less costly methods such as using software programs. Dynamic errors are the results of manufacturing a work piece using more than one type of manufacturing processes or operating on one or more machines. They change with time and, frequently, require a mathematical model for error characterization and compensation. Vibration and thermal errors are two good examples of time-changing errors. As the requirement for accuracy is getting more and more stringent, thermal errors have contributed up to 70% of total machining errors [1]. Fig. 2.1 illustrates an analysis of various types of thermal errors.

This chapter aims at providing a thorough review on thermal effects of linear motors. In order to achieve better accuracy, many precision machine designs adopt linear motors over traditional ball-screw mechanisms [9]. The incorporation of air bearings further enhances the ability of linear motors in achieving higher accuracy as well as operation speed [9].

Fig. 2.1 Breakdown of thermal errors contribution [1]

Although vast amount of works had been done to improve the performance of linear motors [9], issues pertaining to thermal effects on linear motors have not been thoroughly examined. The analytical models developed for the study of thermal effects on electronics and rotary motor driven equipment cannot be straightforwardly extended to understand machines equipped with linear motors due to different working mechanisms.

2.2 Conventional Linear Stages Equipped with Ball-Screw Type Actuation and Contact Roller Guides

Conventional linear motion machines use ball-screw mechanisms actuated by rotary motors and guided with contact roller bearings [6-8] as shown in Fig. 2.2. Ball-screw mechanisms are still widely used in machines, even after the introduction of linear motors, because of lower cost and less stringent requirement on process accuracy. Much heat is primarily generated due to friction when operating at high speed. The secondary heat source is the roller guides. Many studies have been carried out to tackle the issue of thermally induced errors caused by the roller guides. New designs have been devised to minimize thermal errors by using different materials for machine structures [10]. Lee et. al [6] observed changes of frictional forces in roller guides. Such a phenomenon is due to the deformation of the carriage that is caused by the heat generated by the ball-screw mechanism [6].

However, the limiting factor on accuracy for this type of machines is not the thermal errors, but friction in the roller guides instead. Typically, their accuracy is in the micron range. Many studies have been carried out to characterize the friction so as to compensate thermal errors [8]. A so-called disturbance observer based model for control is able to achieve accuracy of several hundred nanometres. However, in reality, it is only accurate in the micron range and is unstable with slow response in the sub-micron range [7]. An improved model which separates the static and dynamic frictions is able to achieve accuracy of tens of nanometres. The response, however, is slow within the nanometre range and is unstable for long range applications [8].

Fig. 2.2. Schematic diagram of a ball-screw linear stage with feedback control [8].

Tool wear is a major issue in machining, which if reduced, can increase machining accuracy and reduce tool replacement cost and machining time significantly. Gordon

and Hillery [11] developed a high speed CNC machine using linear motors. The machine is able to achieve better accuracy and surface finishing as well as reduced tool-wear.

2.3 Modern Linear Stages Equipped with Linear Motor Actuation and Aerostatic Bearings Guides

In recent years, many modern machines are equipped with linear motors and aerostatic bearing guides (Fig. 2.3). These machines are able to achieve better accuracy compared to conventional machines fitted with ball-screw-rotary motors. They have found applications that require high precision, such as wafer alignment [12], CMM [13], inspection processes [14] and laser beam writers [15].

Aerostatic bearings require preloading. The application of linear motors with a single-sided magnet track [16] is adopted in a miniature stage for sub-micron machining [17] and in a slanted version for two-direction preloading [18]. The usual method of preloading requires aerostatic bearings to be mounted around a guide beam. This is replaced by single sided linear motors in order to reduce the number of expensive aerostatic bearings. Apart from high positioning accuracy, straightness and rotational errors can also be examined by using a single axis linear motor guided by aerostatic bearings [19]. As previously mentioned, the purpose of choosing aerostatic bearings over ball bearings is to tackle frictions. However, in reality, minute friction

still exists [20, 21]. Using linear motors and aerostatic bearings, it is possible to realize a design with nanometre resolution and high stability [22].

Fig. 2.3. Assembled linear motor (left) with air bearing guide (right).

Apart from high accuracy, the ability to achieve a high operation speed is also an important factor to be considered. The use of a ball-screw drive with rotary aerostatic bearings shows high accuracy [23] but its speed is slower than that of the linear motor with aerostatic bearings [24]. Conventional design of a two axes machine is usually achieved by stacking one axis onto the other. A new XY-axes stage, which adopts a decoupled planar design with aerostatic bearings, is able to realize high positioning and rotational accuracy operating at high speeds [9, 25, 26].

2.4 Research Areas in the Field of Linear Motors

For machines dealing with cutting processes, there are times when cutting processes are in operation and times when machines are idling, viz. non-processing time. The non-processing time is usually used to transport work piece to and from the work area. Linear motors are able to attain higher velocity and acceleration without much vibration compared to conventional machines driven by ball-screw mechanisms [27]. The ability to reduce processing time and achieve a higher accuracy prompts many to replace ball-screw mechanisms with linear motors.

Some general studies on the performance of linear motors have been carried out. For example, a detailed review can be found in the work of McKean [28]. Chen et. al [29] employed finite element methods to analyze the performance of a linear permanent magnet motor. Different shapes and dimensions were used in the investigation. Abdou and Tereshkovich [30] studied the output force, velocity and acceleration of the end effector of a CNC vertical milling machine that was driven by linear motors using finite element method and compared the results with those obtained experimentally. A two dimensional planar motion of a linear motor was characterized. The study aimed at reducing the errors of machining of a milling machine [31].

Other studies were more specific and aimed at addressing certain aspects of linear motors. These include magnetic flux density [32, 33], thrust force [34, 35], relationship between magnetic flux and force [36, 37], force ripples [38, 39], control

techniques [40, 41], design optimization [42, 43] and power efficiency [44, 45]. A typical control technique using close-loop control of position and velocity is shown in Fig. 2.4. The outer loop is for positioning error feedback while the inner loop is for velocity control with disturbance included.

Fig. 2.4 Block diagram of a close-loop position and velocity feedback control [22].

2.5 Numerous Machine Thermal Models

Many thermal models have been established and used to study the thermal effects on rotary induction machines, computer-numerical-control (CNC) machines, vertical machining centres and coordinate measuring machines.

2.5.1 Totally Enclosed Fan Cooled Induction Motor (TEFC)

Rotary motors can be found in many types of machines that require linear motions. Ball-screws or lead-screws are used to transform rotary motion to linear motion. As rotary motors are fixed and protected by an outer casing, they can be easily heated up by electrical power during operation. Consequently, heat is trapped inside the casing (Fig. 2.5). This leads to high heat storage retained by the rotor and the stator, which may result in degrading of performance and failure of motor. Even with a strong cooling fan installed, it may not be able to cool the motor down effectively.

Fig 2.5. Power losses of a squirrel-cage induction machine [56].

Many studies have been performed to understand the thermal behaviour and effects. As the mathematical models of thermal effects have not been well established, for more than a decade, many researchers have relied on finite element methods to analyze heat transfer of motors and the environment [46-52]. Some of the research

efforts include electromagnetic and temperature distribution on a two-dimensional case [46], thermal network with losses [47], field-circuit coupling methods [48], simulation of an electrical and thermal coupling using air gaps approximated with equivalent conductive values [49], simulations of eddy current heat sources [50], temperature effects on eddy current generation [51], simulation and temperature profile of motors [52] and computational fluid dynamics and thermal simulation of motors [53, 54]. Dajaku and Gerking [55] developed an improved lumped parameter thermal model for electrical machines. Hwang et. al [56] established a so-called distribution loss model to study the temperature distribution in the stator of an induction motor. Bertin et. al [57] combined finite element analysis with transient time-history to realize a reduced model approach to examine the thermal behaviour of an electric motor. In the work of Rajjagopal et. al [58], Galerkin's weighted residual technique was used to improve the accuracy in temperature prediction.

Other than the finite element method, some researchers employed approximated models for real time simulations. These include a simple resistant network thermal model that ignore thermal capacitance and impedance for steady state simulation [59], models with approximated air gap equivalent values [49], model for sensitivity analysis [60] and circuit based models [61] that embedded Ohm's law of electrical conduction and Fourier's law of heat conduction to predict temperature increase of motors.

2.5.2 Computer Numerical Control (CNC) Machines

Regression methods that link temperature measurements by sensors and deformation errors have been widely studied. Thermocouples are used as temperature sensors while deformation errors are measured by using a one-dimensional ball array [62], non-contact displacement gauge [63], laser interferometers [64], or a capacitance sensor [65]. Simple auto-regression analysis between spindle speed, temperature and deformation errors [62], and multiple regressions based on statistics [63] with more parameters have been studied. In other studies, orthogonal regression modelling [64] as well as linear regression analysis [65] is used in assessing thermal errors of CNC machines. Artificial neural networks have also been employed to map deformation errors with temperature measurements so as to realize dynamic thermal error compensation [66] and active compensation of machining centres [67].

Finite element methods have also been used to evaluate convective heat transfer of a complex structure. These include the works of Zhao et. al [68] which is based on flat plates, and Wu and Kung [69], which estimates the strength of the heat source using experimental data. Yang et. al [70] employed a kinematic model embedded with a homogeneous coordinate transformation matrix to compensate thermal errors, while Yuan and Ni [71] adopted an inverse kinematic method coupled with temperature measurements to perform real-time error compensation. Srivastava et. al [72] used Denavit-Hartenberg transformation with small angle approximations to determine thermal errors in a five-axis industrial machine tool [72].

Hao et. al [73] developed a thermal model to compensate thermal errors in a CNC turning centre. In their work, thermal mode shape analysis that was coupled with a genetic algorithm-based back propagation neural network was used to obtain real-time error compensation. In a similar work, Yang et. al [74] carried out thermal error mode analysis and robust modelling to compensate thermal errors in a CNC turning centre. In another study, a modified lumped-parameter method and a finite element method were, respectively, used to deal with thermal errors resulted from the ball screw mechanism and the linear guides [75]. Other studies on thermally induced deformation errors include comparison of composites sandwich structures and steel structures [76], validity of free cutting models [77], finite element method based approximation model to reduce calculation time of heat generated [78], characterization of thermally induced deformation errors for three linear axes errors [79] and three rotation axes errors [14].

2.5.3 Vertical Machining Centre (VMC)

As thermal models of complex structures are difficult to established, many researchers reported the results of their investigations based on experiments, characterizations and correlations. These include bending of a machine structure due to thermal load [80], a flexible thermal error measurement technique for a four-axis dual-spindle turning centre (Fig. 2.6) [81]. The CNC controller is mounted on the machine structure with signal coming from the computer with compute the required position to move to. The computer computes the compensation base on the feedback from the temperature devices mounted on the machine structure.

Fig. 2.6. Real-time error compensation with a vertical machine centre [81].

Besides characterization and correlation, more robust compensation methods for thermal errors have also been developed. These cover the works by Okator and Ertekin [82] on kinematic error modelling of a three-axis vertical machining centre and Ramesh et. al [83, 84] on thermal error measurements and modelling of a three-axis vertical machine centre.

2.5.4 Coordinate Measuring Machine (CMM)

As for coordinate measuring machines, Lingard et. al [85] carried out a thermal response characterization. This was achieved by introducing multiple heat sources which generated temperature gradients in the machine structure and measuring the probe displacement by a ball-bar. Kruth et. al [86] developed a parametric approach and used it to describe the relationship between transient temperature distributions and deformation errors of the CMM. Attia and Fraser [87] proposed a generalized model for real-time compensation of thermal deformation of CMM. The generalized model was reported to be able to achieve better accuracy than those derived from finite element method.

2.6 Thermal Related Research with Applications in Stages Equipped with Linear Motors

Eun [88] compared the thermal behaviours of asynchronous and synchronous single sided linear motors. Two motors with exactly the same dimensions were chosen. These motors were able to fit onto the same single axis linear stage fitted with roller profile bearings. Two main heat sources, namely electrical power loss in the motor coil and frictional loss along the linear guides, were considered. Power loss through Ohmic resistance and frictional loss of motors were compared by using different loadings and speed ratings, respectively. Only temperature measurements of the carriage, motor coils, motor base structures and motor stators were taken.

Thermal behaviour of a machine tool equipped with linear motors in its X, Y and Z axes were studied by Kim et. al [89]. Thermocouples were positioned on the motors' coils and stators, linear scale and linear guide ways for the measurement of temperature. The positioning error of spindle ends was measured by using a laser interferometer, two gap sensors and a photo sensor. Measurements were made using 14 different process conditions. The results (with and without coolant) gathered were compared with those obtained by simulation with unspecified thermal heat fluxes were applied on the motors' coils and linear guides.

Fig. 2.7. Typical single axis stage using a linear motor [90].

Jang et. al [90] looked into temperature modelling of a single axis linear system equipped with an open-type linear motor (Fig. 2.7). In their work, simulation and experimental results were compared. Basically, temperature readings were taken at motor coils, carriages, stators and the base structures. Computational fluid dynamic was included in the simulation. This enabled natural convection coefficient for the heat loss on different surfaces to be determined. The thermal resistances between

contacts were estimated experimentally or by simulations. Their work concluded that the highly nonlinear thermal behaviour could not be accurately predicted.

Chayopitak and Taylor [91] performed thermal analysis of a linear variable reluctance motor by simulation. A two-dimensional thermal resistance model (Fig. 2.8) was developed. Each of the three columns depicts each of the three motor windings. The resistance symbols are thermal resistance referring to each component, of the three windings, specified on the right side. The current source symbols are the heat source resulting from joule heating effect of the wirings. As experiments to validate the model had not been done, its reliability could not be ascertained.

Huang et. al [162] investigate the temperature of the carriage of a single-axis linear stage driven by linear motor. Simulation using FEA with boundary conditions, such as convection coefficient and heat generation from motor coil, calculated from existing correlation are compared to experimental results. From the plotted graphs, there is a obvious 5°C to 10°C difference of the simulation and experimental results for most of the initial transient temperature profile of within 6000 s from the beginning. The plots stops at 12000 s, but it can be seen that the simulation temperature is still on the rise despite that the experimental temperature had already reached steady state by then. This does not mean the simulations and the experiments were done wrongly. The problem was the available existing boundary conditions that the author used. They were not wrong, and the correlations used were the closest and most suitable at that point of time. However, the problem is in the boundary conditions, which are the convection coefficient and heat generated. They are not

constants and cannot be calculated from traditional methods, as it quite obviously shown in the plotted graphs and later in Chapter 4.

It is apparent from the above review that thermal modelling of machine structures equipped with linear motors has not been well developed and still remains a research area in its infancy. In addition, in order to establish a reliable thermal model to describe temperature distribution, experimental works to estimate the convection coefficient and thermal contact resistance become necessary.

Fig. 2.8. Thermal resistance model for steady state heat transfer analysis [91].

2.7 Conjugate Heat Transfer

The convective heat transfer coefficient is very cumbersome to estimate as it needs the flow behaviour of air which is derived from highly non-linear equations with air velocity as input. Due to the complexity of calculating fluid flow coefficients of

Reynolds, Grashof, Prandtl and Nusselt, only flow and velocity boundaries are determined. These boundaries are subsequently employed to obtain the heat conduction boundaries.

Pure analytical methods have been developed to describe the convective heat transfer coefficient. These include the analytical cuboid shape [92] and Wilson plot [93]. Bousbaine et. al [94] proposed using an in-situ experimental method to determine thermal coefficients for electrical machines. In general, it is relatively easy to obtain the convective heat transfer coefficients for rotary motors' outer casing, as they are stationary. On the other hand, the convective heat transfer coefficients for linear motors are largely dependent upon the travelling speed of the carriage as it is exposed to air and is mobile.

2.7.1 Background

Conjugate heat transfer, which covers both conduction and convection, has been extensively studied in past decades [95, 96]. Many analytical and numerical techniques have been proposed to model fluid flow and heat transfer for conjugate thermal fluid problems. The heat transfer between fluid and a solid body, which is termed conjugate heat transfer, uses a set of coupled governing equations to describe the phenomenon. Conservation of mass, momentum and energy are used to describe heat transfer in the fluid region. These result in the derivation of the elliptical Navier-Stokes equation or the parabolic boundary layer equation. On the other hand, heat transfer in a solid body is governed by the elliptical Laplace equation or by the

parabolic differential equation, which is essentially the Fourier's law of conduction. Dorfman et al [96] gave a comprehensive review of the conjugate heat transfer. A total of 190 research works were cited. Seventy-nine of them were on processes where the materials undergo phase change or chemical reaction. Thirty-four of them were on forced convection of flow in conduits and heat transfer in heat exchangers. Sixty-three of them were on areas such as natural convection in enclosures, fins, rods and plates. The rest of the works were on mixed convection of external flat plates, internal conduits and external different object shapes. Out of these, only a few are on moving solid surface in a stationary fluid environment.

There are two types of conjugate heat transfer namely Blasius and Sakiadis [95]. The Blasius type of problems concern a static solid in flowing fluid while the Sakiadis type of problems consider a moving solid in quiescent fluid. The idea of studying a moving solid in quiescent fluid was first initiated by Sakiadis [97] in 1961. Sakiadis modeled a 2-D boundary layer of quiescent fluid with a moving infinite plate to understand the flow which was different from a Blasius type of problems. Subsequently, Tsou et. al [98] extended Sakiadis' study and showed that there would be a 25% error in estimation if a Sakiadis type of problem was modeled and solved according to a Blasius type of problems. The experiments conducted by Tou et. al agreed favorably with velocity boundary layer flow but not with temperature boundary flow.

Choudhury and Jaluria [99] proposed an analytical model to deal with a moving cylinder and a flat plate under quiescent air conditions. Their results show that the

analytical model was accurate only in the initial transient region while the existing numerical methods that they were comparing with were accurate after the initial transient region. Furthermore, there was a serious draw-back for the proposed model as it was very difficult to implement compared to those of the existing numerical methods. Several coefficients, heat transfer coefficient in particular, used in the analytical model required numerical methods to estimate. Thus, they found that a complete conjugate problem could not be solved without the application of numerical methods, unless major assumptions that would adversely affect the accuracy of prediction were made.

Talat [100] studied the skin friction coefficient for both Blasius and Sakiadis types of problems and solved the full time-dependent Navier-Stokes formulation numerically with the finite difference method. The estimated skin friction coefficients were 30% higher in the Sakiadis type of problems than those of the Blasius type of problems. Basically, the skin friction coefficient directly affects the drag force and heat transfer coefficient through the shear stress equations and the Nusselt number respectively. The above mentioned research works on conjugate heat transfer were done more than fifteen years ago. Only in the past three years, there were some newer research works in the same area. These research works are summarized as follows.

Helge and Aarseth [101] studied the effect of different fluid properties on the boundary layer formation in the Sakiadis type of problems. They showed that huge errors occurred when constant fluid properties were used in the model instead of temperature dependent properties. Pantokratoras [102] found that the momentum

boundary layer was largely affected by the viscosity and Prandtl number, and modeled a parabolic system which could predict both the Blasius and the Sakiadis types of problems. The parabolic system was able to predict the temperature boundary layer, which agreed favorably with the work of Pop [103]. However, both Pop's and Pantokratoras' works were seriously criticized for using a constant Prandtl number which resulted in erroneous predictions [101].

Al-Sanea [104] studied mixed convection heat transfer of a moving continuous flat plate with Blasius' equations, and compared the results obtained with previous research works on two different types of problems: pure natural convection [96] and pure forced convection situations [95]. The work showed good agreement in the forced convection region but not the natural convection's. The analytical and approximate methods are summarized in the appendix.

2.7.2 Numerical Methods

Numerical methods for solving conjugate heat transfer problem have gained much usage since the vast improvement in computing power. The differential equations are solved in each individual subdomains of the entire domain, whereas analytical and approximate analytical methods solve the entire domain all together. There are three basic groups of numerical methods, finite-difference method (FDM), finite-element method (FEM), and boundary element method (BEM). The main differences in these methods are in the way the domain is discretized. FDM uses regular grid system while both FEM and BEM use non-uniform grid system. The modern modification of

these methods use the control volume, which derive their nodal equation by integrating the entire control volume instead of the usual derivative equation. There were difficulties in using numerical methods. One difficulty was the computation of pressure gradient in the fluid flow. One way to overcome it was to totally ignore the pressure gradient, and the other way is to implement the stream function and cross-differentiating. The method of using Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) was developed to solve the velocity and pressure fields. However, an initial guess is needed. The numerical method using the central difference method will at times lead to unrealistic results and false diffusion. The upwind scheme implemented in place of the central difference method could solve the mentioned problems. The convergence of the numerical method to its solution is also a problem in many situations. The poor convergence leads to much time needed to converge or errors appearing in the solution. The method of changing to a finer grid and then back to the original grid is one way to correct the poor convergence. The other way is to use radial basis functions with meshless methods on the domain to improve the convergence.

2.7.3 Analysis of the Summarized Method for Conjugate Heat Transfer

In the first two sections, analytical and approximate methods were used to solve for the velocity flow of the fluid assuming that the temperature profile of the surface is known. The convection heat transfer coefficient is separately calculated based on isothermal surface temperature. Using the given surface temperature profile and constant convection coefficient, the velocity and thermal boundary are calculated.

The heat flux is then obtained. The velocity profile is derived with constants in its solution which needs a graph or look-up table. It is also stated in this review that the temperature of the solid surface is not known. The assumption of obtaining the constant convection coefficient from an isothermal surface for use on non-isothermal is invalid if the degree of nonisothermicity of the surface is high.

The third section approximates the conjugate heat transfer to an equivalent heat conduction problem. The application is more for internal flows of two fluids with a wall partition in it. The assumption is that the wall must be thin so that there is no temperature gradient in the thin wall and so there is no conduction along its length. Another problem is its singularity problem of its temperature at the surface.

All the analytic and approximate analytical methods attempt to solve the conjugate heat transfer with known surface temperature and coefficients correlations of the velocity profile of fluid. There is no attempt to calculate the convection coefficient which results from the fluid velocity profile which is created by the unknown temperature surface. The most important parameter of conjugate heat transfer is the convection coefficient, without which the heat transfer cannot be quantified accurately. However, it is possible to obtain the fluid velocity profile and surface temperature with measurement with particle image velocimetry (PIV) and thermal camera respectively. Using these equipments to measure sakiadis flow type convection is challenging as the equipment have to be in motion with the solid surface. Disadvantages are the special skills required to operate these equipment with correct technique to obtain accurate results and the cost to obtain such equipments.

Using the numerical method to solve the entire system of the conjugate heat transfer is possible but is very computational slow and tedious. It has to solve iteratively the fluid region and solid region. However, two crucial parameters are needed for the calculation and they are the friction coefficient and convection coefficient. These are usually not known for a typical fluid and surface roughness.

The numerical method and analytical method are derived from the same fundamental principles of heat transfer. Analytical methods have few exact solutions and especially for simple problem and so are often used as verification for more complex problems solved with numerical methods. To use analytical methods, one have to understand very deeply the knowledge of heat transfer while to use a numerical method, a common knowledge, together with moderate mathematical and programming skills, is enough. However, numerical methods have some disadvantages. The iteration can often met with convergence problem resulting in long computing time needed and poor accuracy. Using intuitive grid transformation between course and fine during computation can eliminated the convergence, but heavy on computing resources and time.

To eliminate the limitations of both analytical and numerical methods, a proposed semi-empirical and numerical method is proposed. It does not need to approximate the velocity profile, assume a wall temperature profile, nor the friction coefficient. The method will consist of a series of experiments to empirically obtain the convection coefficient with respect to the wall temperature. The convection coefficient values were empirically calculated with numerical method of heat

conduction and the obtained wall temperature. Similar approach will be used for the heat source and contact conductance modeling.

2.8 Summary of Chapter 2

In the recent decade, there is a sharp increase in the research for linear motors in terms of performance and characterisation. One of the limitations of linear motors is its high heat generation. It is a widely known problem especially for precision machines which generate positioning errors due to thermally induced deformation. To the best of the author's knowledge, there were no thermal models to predict the positioning errors in machine tools without using neural networks or error mapping. The boundary conditions for existing thermal models were lacking. Without the boundary conditions, the thermal models were unable to predict the temperatures well. In the recent half century, there were much more correlation studies for thermal models than analytical methods. In this thesis, correlation methods were adopted to predict the boundary conditions such as the heat source, heat loss convection coefficient and contact conductance. Each boundary condition will be further reviewed in their respective chapters. With the essential boundary conditions, a lumped-parameter method is used to predict the temperature and positioning errors of a precision stage's motion carriage.

Chapter 3 Research Methodology

3.1 Limitations and Disadvantages of Existing Thermal Models

A summary of the limitations and disadvantages of existing thermal models is presented in Table 3.1. Analytical heat transfer theories have been well established since the early twentieth century. The theories range from pure conduction and pure convection flow to coupled conjugate heat transfer problems. The pure conduction analytical solutions are based on one dimensional model with very simple geometry and using simplified assumptions without which the equations cannot be solved mathematically.

The most studied simple geometry is a flat plate with an infinite length, an infinite long cylinder, and a sphere. One of the assumptions made concerns the application of boundary conditions of the first kind, Dirichlet boundary condition [105]. The Dirichlet boundary condition postulates a fixed value temperature for a surface. Such a postulation would seriously over estimate the heat transfer and produce results with a much higher heat flux with a much lower temperature value input. This over-estimation affects the temperature predictions more in transient analysis than in steady state analysis.

Table 3.1 Limitations and disadvantages of existing thermal models

-
- ✧ Analytical solutions are based on 1-D models with simple geometry and simplified assumptions.
 - ✧ Dirichlet and Neumann boundary conditions over and under estimate the temperature distributions.
 - ✧ Extremely high proportion of research work is on empirical and correlation studies.
 - ✧ A complete thermal model needs a heat source, a transfer, and a heat loss sub-model.
 - ✧ Each sub-model is governed by different physical sciences which are heavily researched but unable to be connected to form a complete thermal model.
 - ✧ Neural network models have been the most used type of thermal model.
 - ✧ Neural network models are only suitable for high-volume-low-mix production.
 - ✧ The present low-volume-high-mix production needs a more analytical thermal model.
-

On the other hand, using the second kind of boundary condition, which is the Neumann boundary condition, would introduce a fixed heat flux boundary value for a surface [105]. As time passes, temperature difference reduces. Consequently, the actual heat flux value becomes smaller as heat flux is proportional to temperature difference in both the heat conduction and convection equations. Using a fixed heat

flux, the flow of heat would be over estimated. This would lead to lower temperature on the hotter side and higher temperature on the cooler side.

There is a third boundary condition which combines the first two boundary conditions. It is known as mixed boundary condition which specifies fixed values of both temperature and heat flux [105]. There is no analytical solution for the mixed boundary condition even for a one dimensional problem unless more treatments are applied to the governing equations with more simplified assumptions. Furthermore, the three types of boundary conditions mentioned above are constant. In reality, the actual boundary conditions vary with time and are modelled in the proposed research work.

Thus, it can be deduced that existing equations may not be effective in handling heat transfer problems of industrial machines. Accordingly, a three dimensional thermal model becomes necessary. Many empirical studies, experimental works and numerical computations have been carried out to address heat transfer problems as well as to understand the validity of three dimensional thermal models and mixed boundary conditions. Considerable amounts of experimental and numerical studies have been published in the later half of the twentieth century. With the advent in regression analysis and neural networks, 'black box' models or techniques for temperature prediction have been widely used at the turn of the century. The learning process for neural networks and regression analysis may take time to complete, but their temperature predictions are accurate to a degree or two. Their accuracies are better than that of analytical model [131].

Basically, a comprehensive thermal model should have at least three important sub-models. They are namely heat source sub-model, heat transfer sub-model and heat loss sub-model. The heat source sub-model deals with joule heating phenomena and friction from ball-screw, contact guides or cutting interactions. The heat transfer sub-model, which is a well researched area, handles the theoretical aspects of the governing equations on conduction, convection and radiation of a body. The heat loss sub-model tackles draining of the heat by natural and force convection.

The theoretical basis of each of these sub-models is built upon several scientific fundamentals. The heat source sub-module may take into consideration joule heating, electromagnetism and frictional theory. The heat transfer sub-model may cover pure conduction, pure convection, pure radiation, or conjugate heat transfer. The heat loss sub-model may involve natural or forced convection of the viscous flow of Newtonian fluid in Blasius or Sakiadis flow types. Each fundamental science has been individually well studied. However, there is no known knowledge of the fundamentals combined to realize a comprehensive thermal model for an industrial machine. There were researchers who shared the same view and they gave the reason that the existing techniques were very hard to realize in a comprehensive thermal model [106].

The objective of this research is to address this problem by formulating a comprehensive thermal model to predict the temperature profile of machine structures and heat deformations equipped with linear motors. The proposed research comprises four parts. The first part of this research is to develop a heat source

sub-model to describe the joule heating phenomenon in the motor coil. The second part is to formulate a heat loss sub-model on natural convection in conjugate heat transfer especially in the Sakiadis flow type. The third part is to develop a sub-model for contact conductance in machine joints, which is found to be lacking in the heat transfer area. The final part is to realize with a modified lumped parameter method which includes the three sub-models and estimate the temperature profile and deformation of machine structures.

In summary, all the sub-models are combined into a main thermal model using lumped parameter method for a machine structure. This would allow real-time prediction of position errors and provide compensation for a precision machine engaging in different operations or processes.

3.2 Thermal Modelling Approach

In order to predict the deformation of a machine structure during the design stage or compensate the thermal errors on a 2-dimensional surface, a thermal model that incorporates elements of control theory is proposed (Fig. 3.1). In this work, the main heat source is from the forcer coil. As for error compensation, this requires the ability to perform real-time error predictions.

As mentioned earlier in Section 3.1, analytical method is able to deal with heat transfer of parts with simple geometry and may not be suitable to handle practical

Fig. 3.1 Thermal path in motion system

problems. Neural networks require long learning time and data collection. Finite element simulation is accurate but it needs experimental data to specify the boundary conditions for computation.

The proposed research suggests using a three-dimensional lumped-parameter method to predict the deformation of surface on the carriage in order to compensate thermally induced deformation errors. The three-dimensional thermal model using lumped-parameter method needs a heat source sub-model that is able to characterize the heat dissipated from the motor coil by using time history current input and the current temperature by measuring the resistance value. It also requires contact heat resistance and convective coefficients to be established for the determination of the temperature profile of the carriage.

Some existing thermal models are shown in Fig. 3.2. A framework of the proposed research is depicted in Fig. 3.3. As shown, the output box on the right hand side is the ultimate goal which is to determine the deformation of machine structure affected by severe thermal heat transferred. The boundary condition box on the left hand side contains the sub-models, namely heat source, contact conductance and convective heat transfer, of the proposed thermal model using three-dimensional lumped-parameter method to be studied individually. The above-mentioned three-dimensional lumped-parameter method is used to combine these sub-models so as to predict the errors due to deformation.

Fig. 3.2. Existing models for heat transfer in machine structures.

Fig. 3.3 Proposed framework for heat transfer modelling in machine structures.

As discussed in Chapter 2, many researchers have investigated thermal models and errors compensation of machine structures. These research efforts mainly concentrated on rotary motors and ball-screw mechanisms. The final error predictions are very often on a single point of concern rather than the error profile of an area. The working principles of rotary motors and ball-screw mechanisms are very different from those of linear motors. Furthermore, the thermal models of linear motors have not been thoroughly studied.

As discussed in Section 2.6, existing thermal models for linear motors are analyzed using finite element methods so as to obtain the deformations of machine structures. This helps to reduce some errors due to the uneven thermal conduction of machine structures. However, the loading of heat source and boundary conditions have not been clearly defined. Their values vary with time due to two-way-coupling between thermal and structural changes, as well as process variations.

Fig. 3.4. Thermal error compensation for specific machine and specific processes

Traditionally, the thermal model of a process can be used to determine and, subsequently, compensate thermal errors for low-mixed and high-volume production. More recently, a process-specific thermal model alone (Fig. 3.4) is sufficient to correct thermal errors. However, such a model, as the name implied, is process specific. Hence, different processes using the same machine would require different process-specific thermal models to be developed. Fig 3.4 shows a typical process for cutting and positioning of substrates. A controller, which can be an industrial computer, is used to execute the planned motions and actions for the process. During machining process, heat is generated from two main sources: the heat generated from the cutting tool which is constantly removing materials from the substrate and the heat is transferred along the tool and substrate holder; and the heat generated from the coils of the motors, which are used to rotate the tool and position the substrate. The total heat generated is transferred to the rest of the machine structure nonlinearly, thereby creating a temperature profile on the machine structure. The temperature profile of the machine structure is captured by the temperature sensors mounted on essential points on the machine structure. As a result, the machine structure, influenced by the temperature profile, deforms non-linearly. However, as previously mentioned, neural networks can be employed to relate the amount of deformation of the structure to the temperature profile. Hence, the amount of the position errors can be estimated and compensated through the controller. When there is a change in the substrate dimensions or process, the above mentioned process has to be repeated.

A thermal model, that is capable to compensate thermally induced deformation errors for high-mixed and low-volume products, has to be machine specific as shown in Fig. 3.5. Regardless of any parameter changes, the model is able to predict the errors and compensate them with any type of processes. This is the basis on which the present research will work on. The only difference is that the heat source will be from the motor coil in the present research instead of process interaction.

Fig. 3.5. Thermal compensation for specific machine and general processes

With the intended process work to be done, commands are updated in the program and the controller will automatically execute the work on the substrate. In the process of doing work on the substrate, heat is produced from the interaction between the tool and the substrate. Power is supplied to the computer to execute the program and motors to actuate the required actions. The significant amount of heat generated is from the tool-substrate interaction and the motor coils. The heat generated will be

transferred to the rest of the machine structure and will cause an inaccurate cutting process. After work has been completed, measurements can be done, which is not possible during process interaction, on the work-piece to quantify the errors. A typical error is the straightness deviation of a straight line that is cut along the substrate. Together with the temperature and power input parameters, a mathematical model can be determined through the error analysis to quantify the amount of straightness deviation errors. With the characterization of the machine structure with a mathematical model, a change in process or substrate dimensions does not need any remodeling for compensation of the thermally induced deformation errors.

3.3 Lumped-Parameter Method for the Overall Modelling System

The lumped-parameter method has been used to represent thermally enclosed fan cooled rotary motors [60, 106] and verified with simulation results. Experimental data are needed in order to define the parameters used in the model [107, 108]. Some of its variations include a one-dimensional model for equivalent electrical networks [109, 110] and modified boundary nodes for improvement of accuracy [111]. It is postulated that the lumped-parameter method is able to predict real-time temperature profiles for error compensation.

Figure 3.6 A proposed lumped-parameter method.

A machine-specific thermal model using lumped-parameter method is proposed and developed in this work for the compensation of thermally induced deformation errors (Fig. 3.6). The method consists of a heat source sub-model, a component to handle convective heat transfer for complex geometry, and a component to deal with the contact conductance between structure parts or surfaces. In the proposed research, a mathematical model of the heat source is first established. The convective heat transfer coefficient for complex geometry and the contact conductance between two surfaces are the next to be tackled to realize a comprehensive thermal model using lumped-parameter method. It is anticipated that such a model is able to estimate the deformation caused by the heat dissipated from the motor coil in real time. A feedback control algorithm is developed for errors compensation in order to achieve better accuracy.

3.4 Heat Source Sub-Model for Generation of Heat Flux from the Motor Coil

An integral part of the thermal model is the heat source sub-model. The phenomenon of joule heating [112] had been investigated using a linear motor thermal model [91]. However, the measurement process has not been adequately reported. As coil resistance is known to change with temperature, through direct measurement of voltage, an estimation of the resistance was performed to estimate the temperature of the motor coil for failure warning [113, 114, 115]. A more accurate estimation was achieved by measuring both current and voltage of the motor coils [116, 117]. Small

DC signal was injected into the coil assembly. In doing so, the temperature profile of the motor coil was estimated by determining the resultant resistance [118, 119] and impedance [120].

As heat source is characterized by its heat flux and not temperature, in the proposed research, heat flux in transient state instead of temperature profile in steady state is estimated by measuring the resistance of motor copper coils and the current flowing through them. To the best of the author's knowledge, this has never been attempted by other researchers. Furthermore, the resistance of the motor coil is employed to monitor the temperature and used as a feedback to the heat source sub-model.

3.5 Contact Conductance Sub-Model for Heat Flow between Materials

Thermal conductance data of materials are available in books or databases. However, the contact conductance between two surfaces is not readily available as there are too many combinations of materials to be documented. In practice, many factors affect the thermal contact conductance between two materials [121]. An eigen-function [122] was used in conjunction with an analytical method to describe an ideal three--dimensional heat transfer between two rectangular slabs. Modified elements [123] were also used to characterize the heat transfer between two surfaces. To the best of the author's knowledge, the method to determine the contact conductance of machine structures has been lacking.

In the proposed research, a device, which consists of a heater and three temperature sensors, is designed and fabricated. This device is placed on the parting line between two contacting parts or surfaces. The heater is operated for different power levels and the temperature attained is measured and plotted. The relationship between the heat input and the temperature recorded by sensors are analysed and then used to compute the thermal contact conductance.

3.6 The Approach for Solving Conjugate Heat Transfer

A carriage carrying a motor coil moves along a linear guide constitutes a Sakiadis type of problem. Sakiadis flow refers to a situation where there is a moving convective surface in a stationary convective fluid medium. The principles of conservation of mass, momentum and energy are treated using similarity transformation and dimensionless parameter transformation. The model includes variable Prandtl number in the momentum boundary layer and variable fluid properties due to temperature change. The existing research works deal mainly with forced convection type fluid, but for the present situation, a mixed convection is more appropriate, although the major type of convection is natural. The fluid to be analyzed is a Newtonian fluid, air. The entire system of equations will be solved numerically using the lumped-parameter method, to calculate the heat transfer coefficient.

Experiments will be performed on the existing single axis positioning stage, to study the natural and forced convective coefficients with respect to the surface speed, surface orientation and temperature rise. Theoretically, these are the main parameters that will have significant effects on the values of the coefficients. More parameters will be added when they are proven to cause significant effects on the values of the convective coefficients.

3.7 Summary of Chapter 3

A review of the existing thermal models on machine structures had been conducted. The thermal model for machine structure were mainly based on black box methods such as neural networks and error mapping, especially suitable for process specific situations. A robust thermal model is needed for all process in machine specific situations. The modified lumped-parameter method is adopted to model the thermal characteristic of the linear motor stage. The method is robust and suitable for real-time control. The thermal model needs sub-models for heat source, heat loss convection, as well as radiation, and contact conductance. The respective sub-models will be addressed by subsequent chapters.

Chapter 4 Evaluation of the Need to Analyse the Boundary Conditions

4.1 Thermally Induced Errors in Precision Machines

As thermal errors account for 70% of total errors found in precision machines, there is a need to address the thermal issue. This chapter discusses what has been derived, experimented and simulated to establish the amount of thermal errors in a single axis stage equipped with a permanent magnet linear motor. It begins with how thermal errors are formed and why it is so difficult to model accurately.

4.2 A Brief Description of the Thermally Induced Errors

Fig. 4.1 shows a single axis linear stage equipped with a permanent magnet linear motor and a pair of roller bearings. It is a typical stage for wafer inspection or pick-and-place applications. The wafer or substrate is placed on the carriage (Fig. 4.1 insert), which is secured onto the linear motor coil. The motor coil is where the joule heating effects takes place when the linear motor is powered to move to a position in

Fig. 4.1. 3D CAD model of single axis stage equipped with a linear motor.

Fig. 4.2. Linearity errors along the Z-direction that show the increase of the errors as displacement is increased from the centre line.

order to perform a process. When this happens, large amount of heat flux is transmitted to the carriage, developing a temperature profile within the carriage, and causes non-linear deformations throughout the carriage. Fig. 4.2 shows the top view of the carriage. For a linear motion system, there is only one positional feedback sensor to track the position of the motion carriage. The thermal deformation errors occur on the entire carriage. Only along the line in Fig. 4.2, the process is undisturbed by the thermally induced deformation errors in the linearity direction (Z-axis). The linearity errors will increase non-linearly as displacement is increased away from the line.

4.3 Simulation of the Thermally Induced Errors

Simulation set-up needs several inputs. They are the structure geometry, material properties, boundary conditions and loadings. The speed and the accuracy of a meaningful simulation result depend very much on the inputs. Structure geometry can be simplified for faster computation but not at the expense of accuracy tolerance. Materials properties are very much constant and can be taken from books or websites. The boundary conditions for thermal simulations are specifically the surface convective coefficient and contact conductance. These are discussed in the later chapters. The loading in thermal context is the heat source and is discussed in Chapter 5. The following simulations are performed with the latest finite elements analysis package ANSYS of version 11.

Since it is a coupled simulation between structural and thermal physics, there is another boundary condition belonging to the structural domain. The use of support for structural domain is important to get the results for the thermal deformation profile. It is evident in the following simulation and will be discussed in Section 4.5. Fig. 4.3a shows a simple rectangular block with only the heat source, convective heat transfer, and fixed support at the heat source.

Fig 4.3a. Simulation of a simple carriage with only the heat source and natural convection with structural boundary conditions at the heat source.

The result shows a highly non-linear thermal deformation contour with respect to the temperature profile. Fig. 4.3b shows the inclusion of the structural boundary condition with fixed support as linear guides, but simplified in two rows, which is not usually used in industrial applications. Fig. 4.3c shows the structural boundary condition for four actual guides attached to the carriage block. It can be observed that

Fig 4.3b. Simulation of a simple carriage with the heat source and natural convection with structural boundary conditions at the two long impractical guides.

Fig 4.3c. Simulation of a simple carriage with the heat source and natural convection with structural boundary conditions at the four normal guides

the thermal deformation gets more complicated and less easy to predict by analytical methods when the simulation conditions are nearer to real conditions.

All three simulations have the same loading at a constant temperature of 100°C at the heat source surface, a convective heat transfer coefficient of $h = 10\text{W/m}^2\text{C}$ for all other surfaces, and material properties of AL6061-06 aluminium. The value of h for natural convection is taken to be between 0 and 20 Wm^{-2} from many books [114]. The meshing elements are a tetrahedral type with 10 mm mesh on regular shape and 1 mm fine mesh on curves, holes and heat source vicinity. Most researchers use 10 Wm^{-2} as the norm for their simulations where the coefficient is not a major concern. The only changes between the three simulations are the placement of the structural boundary conditions of fixed supports.

Fig. 4.4. Simulation for thermal induced deformation (linearity deformation).

Fig. 4.5. Simulation for thermal induced deformation (horizontal straightness deformation).

The actual geometry of the carriage is modelled in 3D CAD using Solidworks08 and simulated for linearity and horizontal straightness as shown in Fig. 4.4 and Fig. 4.5 respectively. The input parameters for this simulation are similar to the previous three simulations. Measurements of four points are taken for comparison with experimental data. Point A is on the extreme left, and Points B, C and D are along in the counter-clockwise direction respectively in Fig. 4.5.

4.4 Heat Transfer and Heat Loss

A one-dimension lumped-parameter method is adopted here to model the heat transfer from the motor coil to the carriage. This is to show that the lump-parameter

method can be used with satisfactory prediction of temperature in later chapters. Assuming constant thermo-physical properties of the aluminium material throughout the lump and temperature range, the lump is small enough to have a constant temperature throughout the solid. Taking the contact surface of the carriage with the linear motor coil as a high value convection heat transfer h_{in} , the energy balance equation [113],

$$h_{in}A(T_{Coil} - T) - h_{out}A(T_{Air} - T) = mc_p \frac{dT}{dt} \quad (4.1)$$

where A is the cross-sectional area of the rectangular lump element, m is mass, c_p is the specific heat capacity for aluminum, T_{Coil} is the temperature of the heated motor coil, T_{Air} is the temperature of ambient air, h_{out} is the convection heat transfer coefficient of ambient air, and $m = \rho V$, where ρ is the density and V is the volume.

Fig. 4.6. One dimensional element of the lumped-parameter method.

Solving for temperature $T(t)$,

$$T(t) = T_1(t) - T_2(t) \quad (4.2)$$

Where $T_1(t) = Ae^{-at} + T_{Coil}$, $A = T_i - T_{Coil}$

And $T_2(t) = T_{Air} - Be^{-bt}$, $B = T_{Air} - T_i$

The results of the individual and total temperature are shown in Fig. 4.7 with the experimental results. The pink curve shows the temperature of the solid if there is no heat loss. The yellow shows the heat loss of the solid when its surface is exposed to convection heat loss at 60°C.

Fig. 4.7. Experimental and lumped-parameter model results.

4.5 Experimentation of the Thermal Induced Errors with Uncertain Boundary Conditions

This section will investigate the effect on the thermal deformation prediction with different boundary conditions used. The single axis linear stage has been designed and fabricated to be used in this investigation as shown in Fig. 4.8. The magnetic ‘U’ channel stator is fixed onto the structure base. The motor coil is fixed to the underside of the carriage. With wiring assembled and linked to a computer, the linear stage is programmed to move within 0.5 μ m accuracy. An experimental setup is shown in Fig. 4.9, involving a HP 5529A laser interferometer which is on the extreme left in Fig. 4.8 but not shown. It is to measure the linearity, horizontal straightness and vertical straightness errors. The points of measurement of these errors are shown in Fig. 4.2, which are the same as those chosen in the simulation positions.

The carriage is programmed to run at 20 mm/s and a cyclic distance of 760 mm with 1 s dwell at each end. The time taken for one round is approximately 40 s. A 6.8 kg load is applied on the carriage which coincides with the operation of the motor coil at rated current after several tests performed previously. One of the errors, say linearity,

Fig. 4.8. Assembled single axis stage with a linear motor and roller bearings for straightness measurements.

is measured at one of the points at one time. The time taken for one experiment is between two to three hours as the temperature reaches a steady state with tolerance of less than a degree change with every five minutes. Another two hours is needed for cooling down when the machine shuts down. Temperature sensors, using resistance temperature detectors (RTDs), are placed on two locations; one is at the interface between the motor coil and carriage, while the other is at the top surface of the carriage.

Fig. 4.9 Experimental setup with a HP 5529A laser interferometer to measure the linearity, horizontal straightness and vertical straightness errors

Table 4.1. Deformation results of both simulated and experimental data

Points	A			B			C			D		
Tests	Horizontal Straightness (X)	Vertical Straightness (Y)	Linearity Error (Z)	Horizontal Straightness (X)	Vertical Straightness (Y)	Linearity Error (Z)	Horizontal Straightness (X)	Vertical Straightness (Y)	Linearity Error (Z)	Horizontal Straightness (X)	Vertical Straightness (Y)	Linearity Error (Z)
Simulation Result(°C)	-43.2	61.0	54.2	-42.0	57.7	-39.8	39.2	52.5	53.7	38.4	52.2	-37.5
Experiment Reading (°C)	-24.1	17.9	20.1	-22.5	22.3	-25.2	16.9	16.4	18.6	19.4	20.5	-23.1
% Error	79.3	240.8	169.7	86.7	158.7	57.9	132	220.1	188.7	97.9	154.6	62.3

Table 4.2. Deformation results after the boundary condition values have been changed.

Points	A			B			C			D		
Tests	Horizontal Straightness (X)	Vertical Straightness (Y)	Linearity Error (Z)	Horizontal Straightness (X)	Vertical Straightness (Y)	Linearity Error (Z)	Horizontal Straightness (X)	Vertical Straightness (Y)	Linearity Error (Z)	Horizontal Straightness (X)	Vertical Straightness (Y)	Linearity Error (Z)
Simulation Result(°C)	-20.0	24.9	25.9	-20.3	25.4	-19.4	17.7	22.4	25.4	17.7	23.3	-17.9
Experiment Reading (°C)	-24.1	17.9	20.1	-22.5	22.3	-25.2	16.9	16.4	18.6	19.4	20.5	-23.1
% Error	20.5	28.1	22.4	10.8	12.2	29.9	4.5	26.8	26.8	9.6	12.0	29.0

Table 4.3. Convective coefficient and contact conductance values

Simulation Case Study	1	2	3	4	5	6	7	8
Parameters								
Contact Conductance (W/mm ² °C)	PC	500	250	PC	500	250	250	250
Convective Coefficient (W/mm ² °C)	5	5	5	10	10	10	4.32 12.05	6.1 13.8
Steady State Temperature (°C)	59.32	52.05	47.47	58.74	49.34	42.55	46.64	44.43
Deformation in Z-axis (µm)	57.2	41.5	32.8	54.3	35.1	22.4	31.5	25.9

The interferometer has been factory calibrated and is accurate up to 1 nm. However, when used in the class 1000 clean-room, with temperature control of 20 ± 0.5 °C, the interferometer accuracy is at ± 100 nm. The temperature sensors (RTD) are calibrated and are accurate to 0.1 °C.

As for the simulation shown in Fig. 4.10, a constant temperature of 100°C, taken from the present experimental value, is applied at the bottom of the carriage where the motor is attached. The results shown in Table 4.1 compare the experimental readings with simulated results using the convective coefficients and contact conductance values shown in the third column of Table 4.3. These values are typically used in simulation of machine deformation in numerical analysis [88-92]. The errors in Table 4.1 are huge, due to the difference of the convective coefficients and contact conductance values between used values in simulation and the real values. A point on top and centre of the carriage was taken, and the experimental temperature is 44.43°C. The convective coefficients and contact conductance are manipulated in the simulation so that the temperature at this point in the simulation matches the temperature measured in the experiment. The fourth column in Table 4.3 show the values used for Table 4.1 simulation results. The two values for the convective coefficients shown in seventh column of Table 4.3 are calculative from flat plate correlation for flow in parallel and perpendicular to the surface respectively. Column eight of Table 4.3 shows the closest temperature compared for the chosen point. The results with the changed input values are shown in Table 4.2. There is huge drop of the difference between the simulated and measured deformation values.

This shows that the boundary conditions are very important to the thermal model for a meaningful result in terms of temperature profile of end-effector positioning error.

Fig. 4.10. Simulation with different values for convective coefficients and contact conductances.

The purpose of the experiment here shows the importance of specifying the boundary conditions for any simulation which is to be used for comparison with the experiments conducted. There is also this limitation where the simulation packages offer very simple yet impractical boundary conditions inputs. The inputs allow only temperature or heat flux while in an actual situation the boundary condition is both a varying temperature and heat flux situation due to conjugate heat transfer. The proposed research also attempts to use a lump-parameter method to incorporate the practical boundary conditions for a more meaningful simulation result to be compared with an experimental result.

4.6 Evaluation on the Effect of Radiation Heat Loss

The complexity and negligible amount of radiation heat transfer have prompted many researchers to omit it from their calculation [1, 161]. In this section, the evaluation of whether radiation heat transfer is significant in the research is discussed. The area of the carriage top is 260 by 155 mm. The linear motor was operating at the normal rating; the peak temperature of the carriage top was obtained by experiment and was 45 °C. The Nusselt number evaluation for free convection is evaluated with (B.4) to (B.6) to be 41.32. The conductivity of air taken at temperature of 40 °C, from Table 4.4, is $0.0271 \text{ Wm}^{-1}\text{k}$. With the length evaluated at 0.26 m, the free convection coefficient is $4.31 \text{ Wm}^{-2}\text{k}^{-1}$. The forced convection ranges from 25 to $500 \text{ Wm}^{-2}\text{k}^{-1}$ [125] so the value of $25 \text{ Wm}^{-2}\text{k}^{-1}$ is taken for minimum forced convection coefficient value. The amount of heat loss from convection is computed to be 3.47 W and 20.15 W for free and forced convection respectively. The emissivity of the grey surface of an aluminum block has an emissivity of 0.09 [162]. The Stefan-Boltzmann has a value of $5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$. With the temperature of the carriage surface and ambient evaluated at 318 K and 298 K, the heat loss from radiation is 0.48 W. The heat loss from radiation amounts to about 13.8 % and 2.39 % of the free and forced convection heat loss. There is no need to separately account for the radiation heat transfer in this case and would be lumped to the convective heat loss as the effective convection coefficient.

Table 4.4 Air properties at 40 °C

Air Properties @ 40 °C	Value	Unit
Dynamic Viscosity	1.91×10^{-5}	Ns/m ²
Kinematic Viscosity	1.7×10^{-5}	m ² /s
Conductivity	0.0271	w/mk
Specific Heat	1.005	KJ/kgK
Density	1.127	kg/m ³
Volumetric thermal expansion coefficient	0.0032	1/K
Prandtl Number, Pr	0.709255201	-

4.7 Summary of Chapter 4

The simulations showed that thermally induced deformation in machine structures is substantial and verified with experimentation. The simulations also showed that an accurate input of boundary conditions and loadings is needed for a thermal model to predict a more accurate resulting temperature profile. However, the boundary condition values can be manipulated so that the experimental and simulation results matched. This is due to the absence of knowledge of these boundary condition values. The modified lump-parameter method can be used in the thermal model to predict the transient temperature variation. To be able to predict the transient temperature is crucial so as to predict the deformation errors for either compensation or error budgeting. The following chapters will discuss more about the methods of obtaining these respective loadings and boundary conditions, in terms of the heat source, heat loss convective coefficients and contact conductance.

Chapter 5 Analysis of the Heat source

5.1 Introduction

For a better performance in precision machines, there is an increasing use of linear motors over ball-screw mechanisms. Basically, linear motors are able to provide higher speed and accuracy over ball-screw mechanisms, but they have the draw-back of producing more heat which easily heats up the connected carriage. Thermal errors can reach as much as 70% of the total machining errors [75]. The heated carriage deforms non-linearly according to the temperature profile which is caused mainly by convective heat loss [3]. Thermal analysis on precision stages equipped with linear motors was not conducted extensively. There were some finite element analysis (FEA) studies for carriage deformation and one-dimensional lumped models on carriage temperature profiles.

The research in thermal effects for linear motors is relatively fewer compared with ball-screw stages. Eun [88] compared the thermal behavior between asynchronous and synchronous single sided linear motors. The two main heat sources namely electrical power loss in the motor coil and frictional loss in linear guides were identified. Power loss through Ohmic resistance loss was also evaluated using different loadings. Kim and Jeong [89] investigated the thermal behavior of a machine tool equipped with linear motors for its X, Y and Z linear motions. Thermocouples were placed at the linear motors' coils and stators, linear scales and linear guide ways for temperature measurements. The positioning error at the spindle end was measured by a laser

interferometer, two gap sensors and a photo sensor. Fourteen (14) experiments were also conducted and their results were compared with the simulation results for both with and without coolants.

Temperature modeling was conducted on a single axis linear system equipped with an open-type linear motor [90]. Simulation based on a simple one-dimensional thermal resistance model and experimental results were compared using the temperatures of the motor coil, carriage, stator and the base. The computational fluid dynamic was employed in the simulation to find the natural convection coefficient for the heat loss at different surfaces. The thermal resistances between contacts were estimated from experiments and simulations. The comparison of the thermal resistance model and experimentation of processes with and without coolers showed that the highly nonlinear thermal behavior could not be accurately predicted. Thermal analysis simulations of a linear variable reluctance motor were conducted using a thermal resistance model [91]. A simple 2-dimensional thermal resistance model comprised an approximated heat source model of the motor coil, convective heat loss calculated by fluid flow over surfaces, and readily available heat resistance of individual materials. A positioning program was developed according to the speed requirement for the carriage to test the thermal resistance model and compared the temperatures with the simulation results. The accuracy of the model compared to experimental data was not known.

An estimation of surface heat flux by conjugate gradient method of a three-dimensional conduction problem on a flat plate was conducted [163]. An average of 2 % error estimation was found when compared with classical numerical

simulation. Temperature distribution of one surface of a flat plate was estimated, with temperature measurement on the others side, using modified one-dimensional correction method [164]. There is a 5% to 12 % error in the estimation of the surface temperatures. For a three dimensional simple shape, estimation of heat flux on one surface was done with an adaptive input estimator [165]. A one dimensional conduction model was used and up to 2 % error was found when compared with experimental values. The estimation of internal heat flux of a three-dimensional housing of high speed motor was conducted with steepest descent method [166]. Errors of about 5 % were found when compared to simulation with various values of connective heat loss coefficients. With the experimental temperature measurements of the machine structure, FEM is used to obtain the eigenvalues of the thermal equation for estimation the temperatures of the heat sources [167]. However the verification shows over 5 °C difference between the estimated and experimental temperature of the machine structure even at points near to the heat sources. The model for flat plates estimated the surface temperature well; however, it is not suitable for non-flat plate solids. The non-flat plate solid models are always performing well when compared with simulations. However the performance is bad when compared with actual experimental values.

From the above, it appears that the model used to study the thermal distribution on machine structures equipped with linear motors has not been well developed. Some analytical heat transfer models had been used to solve some industrial problems. A discrepancy of up to 400% was reported when compared with experimental data [124].

An integral part of the thermal model is the heat source sub-model as mentioned in Section 3.4. The phenomenon of joule heating [112] had been studied using a thermal model for a linear motor [91]. Motor coil's resistance is known to change with temperature. Frequently, the temperature of a motor coil is estimated indirectly through the measurement of coil resistance or voltage [113].

As already mentioned in Section 2.5.1, the lumped-parameter thermal models had been employed to study the thermal behaviors of thermally enclosed fan cooled rotary motors [60] and their effectiveness had also been verified with simulation results. However, the parameters used in the models were determined experimentally [107]. In the proposed research, the lumped-parameters thermal models are studied and adapted to predict real-time temperature profiles and compensate for thermal errors.

In this chapter, the joule heating phenomenon, which is based on the principle of conservation of energy, for heaters and motor coil is presented. The three-dimensional (3D) lumped-parameter method is used to model the heat source, which is the motor coil. As previously mentioned, the heat generated is transferred to the machine structure via the carriage that mounts the motor. Correlations are then made between the current input and power ratio constant. This enables the temperature of the top surface of the motor coil to be estimated based on the input current. Subsequently, heat flux is deduced from the estimated temperature. In doing so, the amount of heat transferred to the carriage can be obtained.

5.2 The joule heating phenomenon

The joule heating phenomenon happens as long as electrical power is used. Heat energy from joule heating can be quantified as [125],

$$P \propto I^2 R \quad (5.1)$$

where I is the current input, and R is the resistance of the conductor.

There are many products or equipment such as electric kettles and heaters that apply this principle. In other words, electric kettles and heaters are designed such that ideally all the electric power supplied to them viz. $P = VI$ or $P = I^2 R$, is converted to heat energy. However, there are also products or equipment that are driven by electrical power but generated unwanted heat as a by-product. Electric motor and light bulbs are good examples of this phenomenon. During operation, considerable amount of electrical energy is converted into heat energy because of joule heating. This is wasteful and undesirable. Thus, one of the considerations when designing them is to reduce wastage of energy that is due to joule heating. The ratio of useful work done to total input energy is known as efficiency. Fig. 5.1 shows a typical electric motor power flow chart. The output power or energy of the motor is a fraction of the total input energy.

Fig. 5. 1. Power flow chart of a typical motor.

Fig. 5.2. Experimental set-up of the heater block supported by clamping on its thick cartridges wires.

This section summarizes the investigation on the joule heating phenomenon of heater cartridges and motor coils. A numerical model and finite element analysis (FEA) are performed on the heater cartridges and verified with experimental data. The experimental set-up of the heater block, with the three heater cartridges inserted is shown in Fig. 5.2. The heater block and heater cartridges are supported by fastening the thick cartridges' wires to a clamp. The purpose of clamping the entire set-up by the wire is to create free convection boundary for all the sides of the heater block. The three heater cartridges have a resistance of 345.6, 347.5 and 345.5 ohms respectively. A power supply of 0.296 A and 33.86 V yields a power input of 10 W approximately. From the experimental data, a steady state surface temperature of $T_{surf} = 92.8^{\circ}\text{C}$ had been reached from the above mentioned power supply (with input power of about 10 W). Thus, the film temperature for air properties to be used in the numerical model and FEA, is evaluated from the following equation,

$$T_{film} = \frac{T_{surf} + T_{amb}}{2} \quad (5.2)$$

with the ambient air temperature at $T_{amb} = 22.5^\circ\text{C}$, and the film temperature at 57.65°C . The next nearest temperature for air properties evaluation is 60°C . The air properties at 60°C are shown in Table 5.1. The Prandtl number, Pr, is a dimensionless number which is a ratio of the gaseous kinematic viscosity to thermal diffusivity can be expressed as,

$$\text{Pr} = \frac{C_p \mu}{k} \quad (5.3)$$

where C_p is the heat capacity, μ is the dynamic viscosity of the fluid, and k is the thermal conductivity of the fluid.

Table 5.1. Air properties at 60°C

Air Properties @ 60°C	Value	Unit
Dynamic Viscosity	2.02×10^{-5}	Ns/m^2
Kinematic Viscosity	1.89×10^{-5}	m^2/s
Conductivity	0.0285	W/mK
Specific Heat	1.009	KJ/kgK
Density	1.067	kg/m^3
Volumetric thermal expansion coefficient	0.003	$1/\text{K}$
Prandtl Number, Pr	0.713	-

For the numerical model and FEA analysis, the convection heat loss coefficients have to be calculated. There are several dimensionless numbers that are needed to be evaluated. They are namely the Grashof number, the Rayleigh number and the Nusselt number shown in Appendix B.

Table 5.2. Additional parameters for evaluating the dimensionless numbers

Parameter	Value	Units
Gravitational Accel, g	9.81	m ² /s
Temperature Diff, ($T_{\text{surf}} - T_{\infty}$)	60	K
Block Dimension LxWxH	0.05x0.05x0.02	m
Characteristic Length	0.05	m

Table 5.3. Evaluated dimensionless numbers

Dimensionless Number	Evaluated Value
Grashof Number, Gr	6.18×10^5
Rayleigh Number, Ra	4.41×10^5
Nusselt Number, Nu (Top)	13.92
Nusselt Number, Nu (Bottom)	6.96
Nusselt Number, Nu (Sides)	13.94
Convection Coefficient, h (Top) Wm ⁻² K ⁻¹	7.93
Convection Coefficient, h (Bottom) Wm ⁻² K ⁻¹	3.97
Convection Coefficient, h (Sides) Wm ⁻² K ⁻¹	19.87 (27.41)

The additional parameters that are needed for the estimation of these dimensionless numbers are listed in Table 5.2. The values of the dimensionless numbers and the convection coefficients are shown in Table 5.3. The convection coefficient values for the side of the block are approximated to be 27.41 W/m²K after taking into account the extra surfaces of the fin-like structure. The convection coefficients are used in the numerical model and FEA analysis.

Table 5.4. Properties of Aluminium

Properties of Aluminium	Value	Unit
Heat Capacity	1300	J/kgK
Conductivity	170	W/mK
Density	2700	kg/m ³

Fig. 5.3. Boundary conditions for FEA simulation and lumped-parameter modeling.

In the FEA analysis, the properties of aluminum are shown in Table 5.4 and the boundary conditions for the simulation are shown in Fig. 5.3. The geometric heater block model was meshed with 2,144 tetrahedral elements with 4,510 nodes. Fig. 5.4 shows the simulation results of the steady state thermal analysis. The temperature of the heater block is 92.84 °C. It is taken from an internal point of the block where the actual thermocouple is placed for the experiment.

Fig. 5.4. FEA temperature results of the heater block under steady state simulation.

As mentioned previously, a lumped-parameter method is used to model the heater block in order to obtain the steady state temperature. It is based on the principle of conservation of energy. The energy supplied (Q_{in}) to the block is either stored (Q_{stored}) or lost (Q_{loss}) to the environment. The equation can be written as,

$$Q_{in} = Q_{stored} + Q_{loss} \quad (5.4)$$

where $Q_{in} = P_{in}$ is the electrical power input, $Q_{stored} = C_p \rho V \dot{T}$ is the amount of thermal energy store in the heater block material, and $Q_{loss} = h_{effective} A (T - T_{amb})$.

Substituting the individual energy in (5.4), we get

$$P_{in} = C_p \rho V \dot{T} + h A (T - T_{amb}) \quad (5.5)$$

At steady state, $\dot{T} = 0$. After rearranging,

$$T = R_e P_{in} + T_{amb} \quad (5.6)$$

where $R_e = \frac{1}{hA}$ is the sum of the total heat loss coefficient over all the surfaces of

the heater block from Table 5.3.

Table 5.5. Temperature results

Method	Temperature (°C)
Lumped-Parameter	92.02
Finite-Element Analysis	92.84
Experimental	92.88

Table 5.6. Motor coil experimental results

Phase 1 Current (A)	Phase 2 Current (A)	Phase 3 Current (A)	Phase Resistance (ohm)	Power Input (W)	Temperture (°C)
1.1889	0.0467	1.2304	32	38.67	25.67

With $P_{in} = 10$ W, the average temperature of the heater block is 92.02 °C (Table 5.5).

The results obtained from finite element analysis showed that if all the power input was converted to heat and after reaching equilibrium with heat loss to ambient air, a steady state temperature was reached. This steady state temperature agreed favorably with that obtained experimentally. This clearly shows that the lumped-parameter method is able to predict steady state temperature of a heater block. Next, motor coil is treated as a heat source and the problem is similarly solved using the lumped-parameter method.

Table 5.7. Energy stored in spring due to a 2 kg load.

Weight (kg)	Gravitational Acceleration (m/s ²)	Force (N)	Spring Constant (N/m)	Spring Displacement (m)	Stored Energy (J)
2	9.81	19.62	6	3.27	32.08

Fig. 5.5. Experimental set-up for evaluating the heating phenomenon of the motor coil

Fig. 5.6. Schematic of experimental set-up where the motor coil is actuating a load against gravity.

An investigation on the heat generated by a linear motor coil is carried out. Figure 5.5 shows an experiment set up. A weight of 2 kg has been used to act as a force to counteract the motor coil. The method of testing is shown in Fig. 5.6. With the linear motor having to withstand a 2 kg weight in stationary against gravity, the phase current of each of the three coils and the calculated total power input as $P = I^2 R$ are shown in Table 5.6. The temperature of the top of the coil is recorded as 25.67 °C after reaching steady state. With the input power of 38.67 W from Table 5.6 and the

heat loss convection coefficients from Table 5.3, an analysis is conducted using the FEA. The geometric motor coil model was meshed with 3,382 tetrahedral elements with 8,914 nodes. The solid model with its boundary conditions is shown in Fig. 5.7 with its corresponding values listed in Table 5.3, while the simulated result is shown in Fig. 5.8. The temperature of ‘top of the coil’ in the simulation is 108.26 °C. However, if an alternative power level is used in the simulation such that the simulated temperature of ‘top of the coil’ matches the experimental. The temperature of ‘top of the coil’ is 25.67 °C in the experiment, and to reach the same temperature in the simulation, a power of 2.53 W is applied.

Fig. 5.7. Boundary conditions for steady state temperature simulation of the motor coil.

This shows that from (5.1), the power used in the motor coil for heating is

$$P = a * I^2 R, \quad 0 < a \leq 1 \quad (5.7)$$

where a is the power constant.

Fig. 5.8. Steady state temperature simulation of the motor coil with parameters from Fig. 5.7 and the calculated power from Table 5.6.

From the simulation and experimental results, $a = 0.065$.

As most of the energy is used to hold the 2 kg mass under gravity, it can be estimated from the potential energy stored in a spring due to the load as follows,

$$E = \frac{1}{2} kx^2 \quad (5.8)$$

where k is the spring constant, and x is the displacement of the spring end due to a load.

The energy stored in the spring can be calculated with the parameters' values shown in Table 5.7. It is apparent that most of the energy from the power source is used to do work against gravity for the 2 kg load. There is some discrepancy in the amount of total energy as the total power input is more than the total energy loss. The energy loss is used to heat the motor coil and hold the 2 kg load against gravity. There are uncertainties in the estimation of coefficient of friction of the pulley and the boundary conditions used in the simulation. The boundary conditions refer to the contact conductance between the steel adaptor of the motor coils and aluminum carriage, and the convective heat loss coefficients on the surfaces of the motor coil. These uncertainties will result in a difference in the estimation of energy values.

5.3 The numerical model for motor coil

5.3.1 Heat conduction equations in solids

The governing equation for three-dimensional transient heat transfer with temperature dependent thermal conductivity k_x, k_y, k_z , in a homogeneous stationary medium can be expressed as,

$$c\rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial T}{\partial z} \right) + f \quad (5.9)$$

where c is the specific heat capacity, ρ is the density of the medium, and f is the internally generated heat source.

For applications with small operation range of temperature, the thermal conductivity can be safely assumed as a constant (k). The governing equation can be written in a simpler form as follows,

$$\frac{1}{\alpha} \frac{\partial T}{\partial t} = \Delta T + \frac{f}{k} \quad (5.10)$$

where $\alpha = \frac{k}{c\rho}$, is the thermal diffusivity.

5.3.2 Lump-parameter formulation

Taking into consideration of heat transfer in all six directions from a point, which is located in the centre of the block-shaped element, the six thermal resistances with seven nodes can be modeled as shown in Fig. 5.9. For node 1, the heat flow with the adjacent nodes can be expressed as,

$$C_1 \frac{dT_1}{dt} = \frac{T_1 - T_2}{R_{12}} + \frac{T_1 - T_3}{R_{13}} + \frac{T_1 - T_4}{R_{14}} + \frac{T_1 - T_5}{R_{15}} + \frac{T_1 - T_6}{R_{16}} + \frac{T_1 - T_7}{R_{17}} + aP_{in} \quad (5.11)$$

where dT_1 is the temperature difference of element one with respect to time, C_1 is the heat capacitance of element one, $R_{12}, R_{13}, R_{14}, R_{15}, R_{16}$ and R_{17} are the thermal resistance between indicated nodes number respectively, and P_{in} is the power input according to (5.7). T_7 is the thin layer element's node, which has a surface interacting with air, is governed by the convective boundary condition.

Fig. 5.9. Schematic of the 3-D lumped parameter formulation

Fig. 5.10. Resistance calculation between two adjacent lumps.

The thermal resistance between any two nodes, shown in Fig. 5.10, can be expressed as,

$$R_{li} = \frac{1}{\frac{k_l A}{L_1} + \frac{k_i A}{L_i}}, (i = 1, 2, 3, \dots, 6) \quad (5.12)$$

where k is the thermal conductivity, A is the elements interface area, and L is the length between the two elements.

While the thin layer element's node which has a surface interacting with air can be expressed as,

$$R_{7-air} = \frac{1}{hA} \quad (5.13)$$

where h is the convective heat transfer coefficient on the element surface area A .

A thermal model with n nodes can be written with (5.11) and the n coupled algebraic equations can be assembled in a state-space domain form shown as,

$$[\dot{x}] = A[x] + B[u] \quad (5.14)$$

$$[Y] = C[x] + D[u] \quad (5.15)$$

where $[x]$ is the state vector, $[u]$ is the input vector, $[Y]$ is the output vector, and A , B , C , and D are the state coefficient matrix, input matrix, output matrix and feedforward matrix respectively.

Fig. 5.11. The motor coil which will be used throughout the experiment portion.

Fig. 5.12. The X-ray image of the three pairs of coils inside the epoxy housing.

The linear motor coil which is used for this study is shown in Fig. 5.11. The motor coil consists of the epoxy part where it houses the copper coils, the aluminum top for mounting onto a carriage, and the Hall Effect sensor for detecting the motor initial position with respect to its stator magnets. The X-ray vision of the motor coil has been

taken as shown in Fig. 5.12 as it is necessary to know the internal structure and how the copper coils are placed inside. From the X-ray scan, 3 sets of coils labeled sets A, B and C are visible. The electronics of the Hall Effect sensor is also shown on the X-ray image on the far right of Fig. 5.12. With the help of the X-ray image, the internal structure of the copper coils for the motor is known. The top left hand side of Figure 5.13 shows a CAD model with a pair of single copper coils showing one of the phases of the motor, whereas, the top right hand side shows the positions of three pairs of copper coils banding together. This arrangement is similar to those appear on the X-ray image (Fig. 5.12). The bottom left hand side of Figure 5.13 shows the positions of the sets of copper coils in the motor. For greater strength, more than one set of copper coils may be used. However, the motor coil used in this study consists of only one set of copper coils. The bottom right hand side of the figure shows that the motor coil is always not visible and the actual motor coil is shown in Fig. 5.11.

Fig. 5.13. Illustrations of the copper coils in the linear motor coil model in CAD

Fig. 5.14. a) The line of symmetry shown to reduce the number of elements. b) The structure of the elements for the reduced model of the motor coil.

As mentioned, the lumped-parameter method is used in this study. A motor coil's CAD model is first constructed. There is symmetry in the motor coil structure. Accordingly, in Fig. 5.14a, the cross-sectional view of the motor coil with the two axis of symmetry line is drawn. Figure 5.14b shows the discretized elements after reducing the model structure by the two symmetry lines. The lumped-parameter thermal model has a very thin layer of elements on top of the coil for air convection. Each of the elements on the thin layer has its centre node near to the surface. This helps in improving the accuracy of the estimation of heat loss, as temperature measurements are done on the surface of the actual motor coil.

5.3.3 Convective heat loss coefficients

The convection heat loss coefficients are computed based on the surface orientation and speed of motion of the motor coil. The surfaces of the motor coil inside the magnet stator have heat transfer similar to steady Couette flow between plates [25].

These surfaces are labeled B in Fig. 5.15. The Nusselt number on the moving surface can then be written as,

$$Nu = \frac{3 + 9 \sum_{n=1}^{\infty} E_n \exp\left(-\frac{9}{4} \alpha_n^2 \xi\right)}{1 - 2 \sum_{n=1}^{\infty} F_n \exp\left(-\frac{9}{4} \alpha_n^2 \xi\right)} \quad (5.16)$$

The constants E_n and F_n in (5.22) are

$$E_n = \frac{1}{\left[\Gamma\left(\frac{1}{3}\right) \left(\frac{\alpha_n}{2}\right)^{\frac{2}{3}} J_{\frac{2}{3}}(\alpha_n) \right]} \quad \text{and} \quad F_n = \frac{1 - \left(\frac{\alpha_n}{2}\right)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}\right) J_{\frac{2}{3}}(\alpha_n)}{\Gamma^2\left(\frac{1}{3}\right) \left(\frac{\alpha_n}{2}\right)^{\frac{10}{3}} J_{\frac{2}{3}}^2(\alpha_n)}$$

where ξ is the dimensionless longitudinal distance, Γ is the gamma function, J_v is the Bessel function of the first kind, v is the order, and α_n is the eigenvalue.

Fig. 5.15. The boundary conditions allocated for the lumped-parameter method.

The average Nusselt number for the surfaces of the motor coil above the stator is given as [125],

$$Nu = 0.664 Re^{\frac{1}{2}} Pr^{\frac{1}{3}} \quad \text{for} \quad Pr \geq 0.6 \quad (5.17)$$

Table 5.8. Convective heat transfer coefficients

Surfaces	A	B
Convective coefficients (W/m^2K^{-1})	2.89	9.03

Fig. 5.16. Position of the three temperature sensors on the top surface of the motor coil.

The front and back surfaces of the motor coil can be approximated using (5.16) and (5.17) for surfaces within and above the motor stator respectively. These surfaces are labeled A in Fig. 5.15. The convective coefficient values for the respective surfaces shown in Fig. 5.15 are shown in Table 5.8.

5.4 Experimental Set-up

The experimental set-up is shown in Fig. 5.5. Temperature readings were taken with three mounted resistance temperature detectors (RTD) on the surface of the carriage. The temperature logging was done with a National Instrument DAQ-9174 acquisition chassis with an NI-9219 card for interfacing with the RTDs. The position of the three temperature sensors on the top of the motor coil are shown in Fig. 5.16. The measurements were used to validate the results obtained by simulation. The linear stage was built for the range of the particular type of a coreless three-phase linear motor. It consists of a three-phase linear motor, double sided magnet stator, a pair of

re-circulating ball-rollers guide and an optical scale encoder. Its actuation portion comprises motor power cables, signal transmission wires, Hall Effect sensor wires, driver, and the interface wires between the controller, the driver and the PC. The experiments were conducted with weights of 0.5, 1, 2 and 3 kg attached on one end with a pulley system as shown in Fig. 5.6. The carriage was programmed to run at 20 mm/s and a cyclic distance of 760 mm with 1 s dwell at each travel end. The time taken for one round is approximately 40 s.

Table 5.9. Temperature measurement at 7000s

Temperature Sensor	T1	T2	T3
Experimental	28.43 °C	28.72 °C	28.38 °C
Lumped-Model	28.51 °C	28.69 °C	28.51 °C

Table 5.10. Power constant α and weight correlation verification

Weight (kg)	1.5	2.5
Experimental (α)	0.175	0.189
Trend Line (α)	0.1731	0.1874
% Error	1.09	0.85

5.5 Results & Discussion

5.5.1 Steady State

Figure. 5.17 shows the temperatures obtained by the lumped-parameter method and the experimental data for a cyclic load of 0.5 kg. The results corresponded to those obtained using the T2 temperature sensor and a position-equivalent surface lumped-element using the lumped-parameter method. The power constant value of $\alpha = 0.158$ is used for the heat input power. The initial huge deviation would likely be due to the constant convective heat transfer used. The constant convective heat transfer value might have been over estimated in the initial stage. When the system reached 5000s, which was near to its steady state, the agreement between the predicted and experimental temperatures was much better. Table 5.9 shows the temperatures of the three measurement points in both experiment and the lumped parameter thermal model at 7000 s after starting the experiment and simulation. This shows that the spatial temperature difference between experimental and modeling results is quite small. The lumped-parameter method is able to predict the temperature variation of the top side of the motor coil with good accuracy. With the temperature profile of the top side of the motor coil established, the amount of heat transferred from the motor coil to the carriage can be readily estimated.

Fig. 5.17. Transient temperatures registered by temperature sensor two and the corresponding lumped parameter surface node.

Fig. 5.18. Correlation between weight loading and the power constant for the lumped-parameter model.

Fig. 5.19. Correlation between current input and the power constant for the lumped-parameter model.

Fig. 5.20. Correlation between current input and the power constant for the lumped-parameter model.

The experiment was conducted using loads of 0.5, 1, 2, and 3 kg. Power constants were calculated for each of the loads (Fig. 5.18). A linear relationship between the power constants and weights used was apparent from the plot. A trend line was drawn along the curve and an equation describing the trend line was obtained (Fig. 5.18). In order to ascertain the accuracy of the trend line drawn, loads of 1.5 and 2.5 kg were used to conduct the test and compared with the values derived from the trend line equation. As shown in Table 5.10, the discrepancies were very small.

The input to the model developed based on the lumped-parameter method for predicting the temperature of the motor coil is expressed in terms of current instead of weight. Fig. 5.19 shows the correlation between the power constant and the average current supplied to the motor coil. The verification points at load 1.5 and 2.5 kg which correspond to 1.7 A and 2.3 A are included in this plot. As shown in the figure, the curve plotted was quite straight and a trend line could be drawn using the data points. The equation for the trend line is included in the figure. It should be noted that same convection coefficients were used for the various loads, regardless of the steady state temperature reached by the linear motor. Fig. 5.20 shows the relationship between steady state temperatures and input current. It has a non-linear relationship and can be approximated with an exponential function. It can be deduced that the actual convective coefficients vary with the temperature of the motor surfaces. This may have an effect on the gradient of the power constant with respect to the input current. It can be seen that the initial transient portion of the temperature profiles was not predicted well. This may be due to the effect of the relationships between the heat generated, the current input and resistance of the copper coil. The heat generated depends on the current input and copper coil resistance as shown in Figure 5.21.

Fig. 5.21. Heat flow in the various components of the linear motor coil.

5.5.2 Transient state

It appears that using only the constant value amplification for the entire range of temperature, the temperature prediction for the transient portion of profile is not good. This may be due to the change of the resistance of the copper while heating up. For steady state analysis, the resistance R is assumed to be constant in (5.24). The input power at steady state, $P_{in,ss}$, is

$$P_{in,ss} = aI^2R \quad (5.18)$$

For the transient mode, both variables, a and R are grouped together to form an amplification factor function, $F(I,T)$, which depends on the current, I and temperature, T . The input power at transient mode, $P_{in,ts}$, is

$$P_{in,ts} = I^2F(I,T) \quad (5.19)$$

Using least square estimation to minimize the errors between the experimental and numerical results, the amplification factor is shown in Fig. 5.22. However, for predictive modeling, the amplification factor would be plotted against the temperature of the coils as shown in Fig. 5.23.

Figure. 5.22. The amplification factor is plotted against time, after using least squares estimation.

Figure. 5.23. Amplification factor as plotted against the temperature of the coils.

Figure. 5.24a. The relationship of the transition temperature, in the transient amplification factor of Fig. 5.23, with respect to each load's current input.

Figure. 5.24b. The relationship of the gradient, in the transient amplification factor of Fig. 5.23, with respect to each load's current input.

Figure. 5.24c. The relationship of the Y-Axis intercept, in the transient amplification factor of Fig. 5.23, with respect to each load's current input.

Figure. 5.24d. The relationship of the steady state temperature, in the transient amplification factor of Fig. 5.23, with respect to each load's current input.

Figure. 5.24e. The relationship of the steady state amplification, in the transient amplification factor of Fig. 5.23, with respect to each load's current input.

From Fig. 5.23, there is a close relationship between different loads and its amplification factor. Each load starts off from a constant value of 0.54 for its amplification. The relationships for the transition temperature, gradient, Y-intercept and the steady state shown in Fig. 5.23 are plotted in Fig. 5.24 with respect to the current input. Using these individual correlations, the final correlation of the amplification factor can be derived. The correlation of the amplification factor with coil temperature and current is,

$$F(I, T) = 0.54 \Big|_{T_0}^{T_{Trans}=F_1(I)} + \{ [F_2(I)]T + F_3(I) \} \Big|_{T_{Trans}=F_1(I)}^{T_{ss}=F_4(I)} + F_5(I) \Big|_{T_{ss}=F_4(I)}^{T_{\infty}} \quad (5.20)$$

where the functions are,

$$F_1(I) = 19.296I^2 - 45.564I + 56.808$$

$$F_2(I) = -0.0587I^2 + 0.2872I - 0.3667$$

$$F_3(I) = 1.5235I^2 - 7.3463I + 10.321$$

$$F_4(I) = 26.659I^2 - 56.32I + 62.478$$

$$F_5(I) = 0.0048I^2 + 0.0067I + 0.1433$$

and T_0, T_{Trans}, T_{SS} , and T_∞ , are the initial temperature, transition temperature, steady state temperature and temperature for time towards infinity respectively.

With (5.20), verification is conducted with loads of 1.5kg and 2.5 kg. The temperature of the top of the coil using the lumped-parameter model with correlation (5.20) is shown in Fig. 5.25. The temperature prediction agrees well with the experimental values with some minute errors.

By using this correlation, the heat generated from the motor copper coils can be predicted with a high accuracy, so that the surface temperature and heat flux on the surface of the motor coil can be calculated. The total heat transfer, Q , from the top of the coil can be calculated by the integral of the surface heat flux profile under the area concern,

$$Q = \int_A q \cdot n dA \quad (5.21)$$

where q is the heat flux profile on the top surface of the motor coil, n is the directional vector normal to the surface and A is the total area.

For the lumped-parameter method, it can be calculated by,

$$Q = \int_{i=n}^{i=n+N} \bar{q}_i d\xi \quad (5.22)$$

where $\bar{q}$ is the average heat flux of the elemental area, n is the beginning surface node number, N is the total number of surface elements on that surface, and ξ is a arbitrary variable for different surface elements.

The heat flux by conduction can be calculated by,

$$\bar{q}_i = \frac{kA}{L} (T_{i,surf} - T_{int}) \quad (5.23)$$

where $\bar{q}_i$ is the average heat flux evaluated from surface node i , k is the thermal conductivity, L is the distance between two nodes, $T_{i,surf}$ is the node temperature of the surface element, and T_{int} is the internal node of the element right underneath the corresponding surface element.

By using the correlation developed, the amount of the heat transferred to the carriage from the linear motor coil, as a heat source, can be quantified. This quantified heat can replace the previous method of using either constant temperature or heat flux on any thermal model or simulations. This will give a more meaningful thermal modeling technique with high accuracy temperature results. The basis for the prediction model is from the measured input current pumped into the motor coil.

Figure 5.25a. The verification of correlation equation (5.20) used in the lumped-parameter with a 1.5kg load.

Figure 5.25b. The verification of correlation equation (5.20) used in the lumped-parameter with a 2.5kg load.

Therefore, regardless of the process speed, duration and force, the input current will reflect the amount of energy needed by the linear motor coil. Therefore, with the input current value, the amount of the heat generated and transferred to the carriage can be determined. However, the convection coefficients used in the modeling were constants. This may results in the minute errors found. Further works need to be conducted to investigate how the convection coefficient varies with surface temperature under Sakiadis flow heat loss. The Sakiadis flow refers to a type of force convection when the solid moves in a stationary fluid medium.

5.6 Summary of Chapter 5

The joule heating phenomenon on heaters and motor coil has been discussed in detail, based on the conservation of energy. Initial investigations showed that almost all the input energy to pure heaters is converted to heat energy, with negligible losses. For motors, such as linear motor coil in this case, a large portion of the energy is used to do manual work. Thus, it is not possible to take the total input energy as the total heat generated from motor copper coils. The modified 3D lumped-parameter method has been successfully implemented to model the motor coil as the heat source for the machine structure, such as the carriage. Correlations have been made between the current input and power amplification factor. The temperature of the top surface of the motor coil can be accurately predicted with the developed correlation and numerical model, based on the current input values. The heat flux can be calculated from the temperatures to quantify the amount of heat transfer to the carriage. The agreement is good for the range of input current studied.

Chapter 6 Analysis of the Effective Convection Heat Loss

6.1 Introduction

As mentioned in Chapter 2, thermally induced errors in precision stages account for 70% of the total positioning errors [75]. Thermal analysis on precision stages equipped with linear motors was conducted in some works. Eun [88] compared thermal behavior between asynchronous and synchronous single sided linear motors. Thermal behavior and positional errors of spindle end were investigated on a machine tool equipped with linear motors for its X, Y and Z linear motions [89]. Temperature modeling was conducted on a single axis linear system equipped with an open-type linear motor [90]. Thermal analysis simulations of a linear variable reluctance motor were conducted using a developed thermal resistance model [91]. Machine structures are mostly made up of complex shapes and geometries. A variety of thermal models classified between principle-based and empirical-based, were reviewed [4].

The heat loss through the surface of a solid involves conduction within the solid's surface and convection outside the solid's surface, and it is termed conjugate heat transfer. Conjugate heat transfer involving both conduction and convection had been studied extensively in the past five decades [96]. Many analytical and numerical schemes have been developed to model the fluid flow and heat transfer for conjugate problems. The heat transfer between fluids flow and solid body, which is termed

conjugate heat transfer, has a set of coupled governing equations to describe the phenomenon. The conservation models of mass, momentum and energy describe the heat transfer in the fluid region which results in the elliptical Navier-Stokes equation or the parabolic boundary layer equation. The heat transfer in the solid body is governed by the elliptical Laplace equation or by the parabolic differential equation, which is the Fourier's law of conduction. Dorfman et al [96] gave a comprehensive review of the conjugate problems in the convective heat transfer.

There are two types of conjugate heat transfer problem. One is the more heavily researched Blasius type of problem and the other is the Sakiadis type of problem. The Blasius type of flow refers to a static solid in a medium of flowing fluid while the Sakiadis type of flow refers to a moving solid in a quiescent fluid. The problem of investigating the moving solid in a quiescent fluid was first initiated by Sakiadis [97] in 1961. Sakiadis modeled a 2-D boundary layer of the initially quiescent fluid with a moving infinite plate to study the type of flow which is different from a Blasius type of problem. Tsou et al [98] extended the study of the Sakiadis type of problem. They showed that there is a 25% error if a Sakiadis type of problem is modeled with a Blasius type of formulation. The experiments agree well on the velocity boundary layer flow but not as well on the temperature boundary flow. Choudhury and Jaluria [99] derived an analytical solution for a moving cylinder and a flat plate under quiescent air conditions. Results show that the analytical model is accurate only in the initial transient region while the existing numerical methods that they compared with are accurate after the initial transient region.

Talat [100] studied the skin friction coefficient for both Blasius type and Sakiadis type

of problems. He solved the full time-dependent Navier-Stokes formulation numerically with the finite difference method. The results show that the skin friction coefficient is 30% higher in the Sakiadis type of problems than in the Blasius type of problems. The skin friction coefficient directly affects the drag force and heat transfer coefficient through the shear stress equations and the Nusselt number respectively.

The above mentioned research works were done more than fifteen years ago. Only in the past three years, there are some research works in the same area. Helge and Aarseth [101] studied the effect of different fluid properties on the boundary layer formation in the Sakiadis type of problems. They demonstrated that huge errors are involved when using constant fluid properties throughout instead of temperature dependent properties in the model. Pantokratoras [102] stated that the momentum boundary layer is largely affected by the viscosity and Prandtl number. He modeled a parabolic system which could predict both the Blasius and the Sakiadis type of problems. The model was able to predict the temperature boundary layer which agreed well with research work from Pop [103]. However, both Pop and Pantokratoras have their work seriously criticized for using a constant Prandtl number which will result in erroneous predictions [101]. Al-Sanea [104] studied mixed convection heat transfer of a moving continuous flat plate with Blasius type of problem equations and compared the results with the previous research works on two separate problems, which were the pure natural convection and the pure forced convection situations. The agreement for the force convection region was excellent but the agreement for the natural convection region was not.

Blasius type and Sakiadis type of problem with flows over a plate were studied with

various fluid properties [126]. Velocity profiles were very different when compared to constant fluid properties, and also between flow by Blasius and Sakiadis types of problem. The effect of buoyancy forces on flows over continuous moving surface was studied. It was found to affect the heat transfer rate [127]. The conjugate heat transfer was investigated on stationary horizontal [128] and vertical [129] plates with internal heat generation. The thin wall and thick wall assumption was studied with different Prandtl numbers. Similar studies was conducted on moving vertical flat plates using similarity solutions and found that internal heat would hinder flow of heat within surface unless a critical Grashof number is reached [130].

A numerical method using finite difference and SIMPLE algorithm was used to study on the natural convection of a horizontal flat plate [131]. The temperature was not well predicted quantitatively. Numerical method alone is difficult to predict the temperature because the heat transfer characteristics are not accurately known. Correlation to predict the heat transfer characteristics of a rotating heat pipe was studied [132]. The correlations obtained were used to optimize the heat transfer with different parameters of the system. The heat transfer characteristic for conjugate boundary conditions was studied by inverse method using boundary element method [133]. An inverse heat transfer using Bayesian for the parameter estimation in the forward model was implemented in the Markov Chain Monte Carlo algorithm for sampling the probability distribution [168]. Experimentally measured temperatures were used to obtain the solid material conductivity and fluid convective coefficients. However, there was no verification of the obtained coefficients with further experimentations. Radziemska et al. attempt to analytically derive the coefficients to the previously experimentally established correlations of Nusselt-Rayleigh relations

for convective heat transfer of cuboid [169]. The derived correlations have a limited range where a short range of the Rayleigh number gives an error of below 10% of the experimental results. Further investigation of the Nusselt-Rayleigh relations was used on bigger dimensions of cuboids [170]. Results shows that there is 10% error when using the correlations parameter of smaller cuboids on bigger cuboids and a 20% errors when using flat plate correlations [171] on big cuboids. Popiel et al. compared the correlation model from McAdams and flat plate model from Churchill [172] with experimental and showed that flat plate model is not suitable for estimation of cuboid's convective heat transfer coefficient [173]. The Nusselt number for cuboids was derived using three existing models namely, the equivalent spheroid model, the cuboid model, and the equivalent flat plate model and compared with results from FEM simulation using FLOTHERM [174]. The spheroid model gives an opposite trend as compared to the numerical simulation and so was not recommended to be used. The flat plate model have a range of Nusselt number value which range between the lower and upper bound limit of the flow length. The numerical results fall from below the lower limit to half way between the limits for a large range of Reynold number. The flow length difference of 25% with respect to the lower limit gives a 15 % difference in the Nusselt number. The cuboid model gives the closest result to numerical simulation with both blending parameter and boundary layer parameter optimized with respect to the simulation results. Without the optimization, for the range of the parameters, the Nusselt number can differ by over 10%.

With various numerical models of heat transfer, to accurately predict the temperature profile, the most important are the boundary condition inputs. The heat source boundary condition was obtained from inverse method under steady state condition

[134]. The heat loss boundary condition, which was mainly the convection coefficient was obtained by estimation from fundamental solutions [135] and inverse methods [136]. Numerical methods such as lumped-parameter method are used widely in one dimensional conduction [137] and transient conduction with convective boundary conditions.

In this chapter, free and mixed convective heat losses are investigated based on conjugate heat transfer. The three-dimensional lumped parameter thermal model is used to model the solid with convective heat loss on all its surfaces. Correlations are made between solid surface speed with respect to quiescent air, surface temperature and the convective heat transfer coefficients. With the correlations, the heat loss can be quantified, so that the temperature profile of the solid can be accurately predicted.

6.2 The analytical models for thermal analysis

6.2.1 Theoretical modeling basis

Figure 6.1 shows the heat transfer analysis for a fin [125]. The conservation of energy can be written as,

$$q_{(in)} - q_{(out)} = 0 \quad (6.1)$$

The conservation of energy for the fin element is,

$$q(x) - [q(x + \Delta x) + q_{(conv)}] = 0 \quad (6.2)$$

where conduction within the fin, $q(x) = -kA_c(x) \frac{dT}{dx}$

convective heat loss on surface of the fins, $q_{(conv)} = -hp(x)\Delta x(T - T_{\infty})$

and infinitesimal heat transfer of the fin, $q(x + \Delta x) = q(x) + \frac{dq}{dx} \Delta x$

Figure 6.1 Heat transfer analysis of a fin [125]

Sub into (6.2), we have,

$$\frac{d}{dx} \left[A_c(x) \frac{dT}{dx} \right] - \frac{hP(x)}{k} (T - T_{\infty}) = 0 \quad (6.3)$$

Integrating,
$$A_c(x) \frac{dT}{dx} - \frac{h \int P(x)}{k} (T - T_{\infty}) = 0 \quad (6.4)$$

Bring over,
$$\frac{dT}{dx} = \frac{h \int P(x)}{k A_c(x)} (T - T_{\infty}) \quad (6.5)$$

Making h the subject,
$$h = \frac{k A_c(x)}{(T - T_{\infty}) \int P(x) dx} \frac{dT}{dx} \quad (6.6)$$

Eqn. (6.6) shows that the convective heat transfer coefficient can be deduced from the temperature difference measurement of the surface. This can be more easily seen with simulation shown in Fig 6.2. The three dimensional block is simulated with a heat input of 10 watts on the top, and a cross sectional view depicting the internal two dimensional temperature profile. The simulations had its surfaces input of $10 \text{ Wm}^{-1}\text{K}^{-1}$

and $50 \text{ Wm}^{-1}\text{K}^{-1}$ of convective heat loss for the results shown on the left and right respectively. Observing on the top of both surfaces, it can be seen that the right figure, with a higher convection coefficient, has a larger temperature difference for the same distance as compare with the left figure with a lower convection coefficient value.

Figure 6.2 Internal temperature profile of a block with different value of convection coefficient values on its surfaces

6.2.2 Heat transfer and fluid flow governing equations

The governing equation for three-dimensional transient heat transfer by conduction with temperature dependent thermal conductivity k , in a homogeneous stationary medium can be expressed as,

$$c_s \rho_s \frac{\partial T_s}{\partial t} = \frac{\partial}{\partial x} \left(k_s \frac{\partial T_s}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_s \frac{\partial T_s}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_s \frac{\partial T_s}{\partial z} \right) + f \quad (6.7)$$

where c_s is the specific heat capacity, ρ_s is the density of the medium, and f is

the internally generated heat source.

For a small range of temperature applications, the thermal conductivity can be safely assumed as a constant k . The governing equation can be written in a simpler form as follows

$$\frac{1}{\alpha_s} \frac{\partial T_s}{\partial t} = \Delta T_s + \frac{f}{k_s} \quad (6.8)$$

where $\alpha_s = \frac{k_s}{c_s \rho_s}$ is the thermal diffusivity.

The fluid flowing over the surfaces of the solid block can be considered with two-dimensional boundary layer flow. The velocity and temperature variations inside the momentum and thermal boundary layers are governed by the conservation equations of mass, momentum and thermal energy.

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad (6.9)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad (6.10)$$

$$u \frac{\partial T_f}{\partial x} + v \frac{\partial T_f}{\partial y} = \alpha_f \frac{\partial^2 T_f}{\partial y^2} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (6.11)$$

with boundary conditions,

$$\begin{aligned} u &= U_{solid}, v = 0, \quad \text{at } y = 0, \\ u &\rightarrow 0, \quad T \rightarrow T_{amb} \text{ as } y \rightarrow \infty \end{aligned} \quad (6.12)$$

In the present model, the solid block will be subjected to Dirichlet input boundary condition [138] from a heater cartridge. The value of the temperature with time is obtained from experiment. It specifically states a known temperature on a boundary surface for a three-dimensional model, which spans a space of Ω . The stated

temperature can be a constant or varies with time or space according to a known function. The boundary condition can be written as,

$$T(x, y, z, t) = \bar{T}(x, y, z, t) \quad \text{where } x, y, z \in \Omega \quad (6.13)$$

where $\bar{T}$ is the prescribed area temperature profile in terms of spatial coordinates and temporal period.

The surfaces of the block will be under the mixed boundary condition [10]. It is the combination of the Dirichlet and Neumann boundary conditions. The purpose of this boundary condition is to cater for the heat transfer of its surface interacting with a fluid, which is called convection heat transfer. The boundary condition can be expressed as,

$$k \frac{\partial T_s}{\partial n} + h[T_{s,surf}(x, y, t) - T_{Amb}(t)] = 0 \quad \text{where } x, y, z \in \Omega \quad (6.14)$$

where T_{Amb} is the temperature of the ambient air and $T_{s,surf}$ is the temperature profile of the solid surface.

The initial condition of the entire domain of interest is important, especially for transient thermal calculation. It specifically states the temperature profile of the domain and the ambient temperature prior to starting a heat transfer calculation. It is given by,

$$T(x, y, z, t) = T_0(x, y, z, 0) \quad \text{where } x, y, z \in \Omega \quad (6.15)$$

where T_0 is the initial temperature profile of the solid in terms of spatial coordinates at $t = 0$.

For the fluid portion,

$$u = v = 0 \quad \text{and} \quad T(x, y, 0) = T_{amb} \quad (6.16)$$

In analytical methods, (6.7) to (6.12) were solved together to obtain the solid and fluid temperature profile and convective heat transfer in (6.13) and (6.14) respectively. In numerical methods, (6.7) to (6.12) were solved iteratively to obtain the solid and fluid temperature profile and convective heat transfer in (6.13) and (6.14) respectively. For the proposed method, it is planned to be simple so as to reduce computational time and resources dramatically. Therefore, only (6.8) is solved with experimentally obtained effective convective heat transfer coefficients, to give the temperature profile of the solid block.

6.2.3 *Numerical lumped-parameter model formulation*

The numerical lumped-parameter formulation is similarly shown in Section 5.3.2. The aluminum grade 6061T6 block of size 100 by 100 by 75 mm is shown in Fig. 6.3. The aluminum block has its surfaces clear anodized and several holes drilled to insert the thermocouples. Each thermocouple is a state in the state-space lumped-parameter thermal model. The discretised block in state space is shown in Fig. 6.4. Fig. 6.4a shows the top view of the block while Fig. 6.4b provides the side view. The internal elements were surrounded by long and thin surface elements. By observation, there are a total of 24 lumped elements. However, the input to the state space model is not in terms of electrical power. It is by the Dirichlet boundary condition. The heater cartridge is inserted from the top of the block into the top layer's centre element shown in the figure of Fig. 6.4. A thermocouple is placed on the side of this centre element to record the temperature which is to be used as the Dirichlet boundary condition. The Dirichlet boundary condition is applied to the surface of the five elements connected directly to this centre element. As a result, there are 23 lumped

Figure 6.3. The aluminum grade 6061T6 block of size 100 by 100 by 75 mm used for the experiments

Figure 6.4. The discretisation of the aluminum block

elements.

The main heat loss for the precision machine is on the surfaces exposed to air. The heat lost phenomena consists mainly of free or forced convection and a little on radiation. Therefore, the combined effective convection heat loss, which is described

by Newton's law of cooling,

$$Q = h_{effective} (T_{Surf} - T_{Amb}) \quad (6.17)$$

where $h_{effective}$ is the effective convective coefficient for free or force convection,

and T_{Surf} is the temperature of the particular surface.

6.3 Verification of the lumped-parameter model

In this section, the lumped-parameter formulation which is solved numerically by finite difference method is verified. From Section 6.1, the Blasius flow forced convection coefficients developed for flat plate are not recommended for block surfaces, especially in Sakaidis flow. Therefore, free convection coefficients developed were used to verify the model, before the model is used to obtain the convection coefficients in Sakaidis flow situation. In the experiment, a steady state temperature was reached with a power input of a 6 Volts signal out of a 10 Volts range. The surface temperatures of the block are able to reach a steady state temperature after 6 hours. The film temperature for air properties to be used in the numerical model and FEA, is obtained from,

$$T_{film} = \frac{T_{surf} + T_{amb}}{2} \quad (6.18)$$

with the ambient air temperature $T_{amb} = 20.5^\circ\text{C}$ and the surface temperature

$T_{surf} = 63\sim 65^\circ\text{C}$, the film temperature $T_{film} = 39\sim 40^\circ\text{C}$.

The next nearest temperature for air properties evaluation is 40°C . Table 6.2 lists the air properties at 40°C . The Prandtl number, Pr, a dimensionless number which is the

ratio of the gaseous kinematic viscosity to thermal diffusivity, is expressed as,

$$\text{Pr} = \frac{C_p \mu}{k} \quad (6.19)$$

where C_p is the heat capacity, μ is the dynamic viscosity of the fluid, and k is the thermal conductivity of the fluid.

For the numerical lumped-parameter thermal model and FEA analysis, the convection heat loss coefficients have to be calculated. As mentioned in Section 2.7.2, there are several dimensionless numbers that are needed to be calculated, and they are the Grashof number, the Rayleigh number and the Nusselt number. The Grashof number is a ratio of the buoyancy to viscous force acting on a fluid and is expressed as,

$$\text{Gr} = \frac{g \beta (T_{surf} - T_{amb}) L^3}{\nu^2} \quad (6.20)$$

where g is the gravitational acceleration, β is the volumetric thermal expansion coefficient, L is the characteristic length, and ν is the kinematic viscosity.

The Rayleigh number of a fluid describes the buoyancy driven flow and is expressed as,

$$\text{Ra} = \text{Gr} * \text{Pr} = \frac{g \beta (T_{surf} - T_{amb}) L^3}{\nu \alpha} \quad (6.21)$$

where α is the thermal diffusivity of the fluid.

Table 6.1 Air properties at 40 °C

Air Properties @ 40 °C	Value	Unit
Dynamic Viscosity	1.91×10^{-5}	Ns/m ²
Kinematic Viscosity	1.7×10^{-5}	m ² /s
Conductivity	0.0271	w/mk
Specific Heat	1.005	KJ/kgK
Density	1.127	kg/m ³
Volumetric thermal expansion coefficient	0.0032	1/K
Prandtl Number, Pr	0.709255201	-

The Nusselt number is the fluid's ratio of free convective to conductive heat transfer of a surface interaction with a solid, from Churchill and Chu, and can be expressed for a heated surface facing direction, expressed as [125],

$$\text{Block surface facing up: } Nu_u = 0.54Ra^{1/4} \quad 10^4 \leq Ra \leq 10^7 \quad (6.22)$$

$$\text{Block surface facing down: } Nu_d = 0.27Ra^{1/4} \quad 10^4 \leq Ra \leq 10^7 \quad (6.23)$$

$$\text{Block surface facing side: } Nu_s = 0.68 + \frac{0.67Ra^{1/4}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9}} \quad Ra \leq 10^9 \quad (6.24)$$

The convective heat transfer can be evaluated from the expression,

$$h = \frac{Nu * k}{\Delta x} \quad (6.25)$$

where Δx is the evaluated length.

Table 6.2 Other parameters needed to calculate the dimensionless numbers

Parameter	Value	Units
Gravitational Accel, g	9.81	m ² /s
Temperature Diff, (T _{surf} -T _∞)	36	K
Block Dimension LxWxH	0.1x0.1x0.075	m
Characteristic Length	0.1	m

Table 6.3 Evaluated dimensionless numbers and convection coefficients

Dimensionless Number	Evaluated Value
Grashof Number, Gr	4.36x10 ⁶
Rayleigh Number, Ra	3.1x10 ⁶
Nusselt Number, Nu (Top)	22.65
Nusselt Number, Nu (Bottom)	11.33
Nusselt Number, Nu (Sides)	15.57
Convection Coefficient, h (Top)	6.14
Convection Coefficient, h (Bottom)	3.07
Convection Coefficient, h (Sides)	6.92

For the remaining parameters that require the determination of the dimensionless numbers, the values are listed in Table 6.3. The values of the dimensionless number and convection coefficients from (6.20) to (6.25) are shown in Table 6.4. The convection coefficients for all the surfaces on the block are inserted into the lumped-parameter thermal model. The internal node temperatures obtained by both experiment and lumped-parameter thermal model are tabulated in Table 6.5. They are in good agreement. The only significant errors are the surface temperatures. This might be due to the thermocouple measures not only the surface temperature, but also part of the air temperature. As a result, it has a much lower value than that obtained by the lumped model. Conversely, this also shows that the lumped-parameter model is able to predict the internal temperature well.

Table 6.4 Internal temperatures of the block

Sensor	Experimental	Lumped	Error	% Error
TC0	59.64	59.81	0.17	0.29
TC1	60.23	59.98	0.25	0.42
TC2	59.58	59.81	0.23	0.39
TC3	59.72	59.99	0.27	0.45
TC4	59.76	59.99	0.23	0.38
TC5	60.11	59.87	0.24	0.40
TC6	59.76	59.96	0.20	0.33
TC7	59.57	59.86	0.29	0.49
TC8	59.89	59.72	0.17	0.28
TC9	59.37	59.72	0.35	0.59
TC10	59.94	59.83	0.11	0.18
TC11	59.52	59.99	0.47	0.79
TC12	59.44	59.83	0.39	0.66
TC13	59.93	59.76	0.17	0.28
TC14	59.46	59.83	0.37	0.62
TC15	59.37	59.76	0.39	0.66
STC1	59.22	59.92	0.70	1.18
STC2	58.59	59.75	1.16	1.98
STC3	58.34	59.71	1.37	2.35
STC4	58.46	59.71	1.25	2.14
STC5	58.42	59.71	1.29	2.21
STC6	58.39	59.72	1.33	2.28

6.4 Experimental Set-up

The experimental set-up is shown in Fig. 6.5. Temperature readings were measured by thermocouples mounted on the surface and internal points of the block. Temperature logging was done with a National Instrument DAQ-9174 acquisition chassis with an NI-9219 card for thermocouple interfacing.

The experiment set up in Fig. 6.5 consists of the twenty-three J-type thermocouples and a 50 watts heater cartridge. The block is rested on four supports with insulation tapes pasted at the area of contact to prevent heat from transferring from the block to the supports (Fig. 6.6). The linear stage was built for a coreless three-phase linear motor and has a travel range of 300 mm. The stage consists of a three-phase linear motor, double sided magnet stator, a pair of ball-rollers guide and optical scale encoder. The actuation portion for the linear stage includes motor power cable, signal transmission wires, Hall Effect sensor wires, driver, and the interface wires between the controller, the driver and the PC.

The second set-up, which is for relating the temperature differences of the surfaces of the block to the convective coefficients, is shown in Fig. 6.7.

Figure 6.5. The block and data acquisition chassis is mounted on the carriage of the linear motion stage.

Figure 6.6. The back view of the set-up

Figure 6.7. Mounting of surface thermocouples for relating with convection coefficient values.

As shown in Fig 6.7, the heater is placed in the middle of the top surface of the block with four thermocouples, two on each side, to measure the temperature drop as the distance gets further away from the heater. As the convection coefficient increases, the temperature difference of each pair becomes greater. There are two pairs of thermocouples to make sure the values are consistent. The wooden sticks are used to press the thermocouple tip on the surface on the block without drawing much heat away.

6.5 Results and Discussion

Figure 6.8 shows the transient temperature profile of an internal node inside the block. The chosen node is right underneath the heater cartridge. The motion speeds are 0, 50, 100, 150, 200, 250, 300, 350, and 400 mm/s. The data shows that as the carriage

motion speed increases, the convection coefficient increases which results in an overall temperature decrease of the block.

Figure 6.9. shows that the evaluated convection coefficients have a linear relationship with respect to the motion speed of the carriage. With the alternative set-up shown in Fig. 6.7, the temperature difference of one of the two pairs of thermocouples is shown in Fig. 6.10. The temperature difference has a linear relationship with the carriage motion speed. The correlation in Fig 6.9 generates a linear equation of,

$$h = m_1 v + c_1 \quad (6.26)$$

where h is the convection coefficient, $m_1 = 4.6917$ is the gradient of the line, v is the speed of the carriage motion, and $c_1 = 3.0972$ is the y-intercept of the line. The correlation in Fig 6.10 generates a linear equation of,

$$\Delta T = m_2 v + c_2 \quad (6.27)$$

where ΔT is the temperature difference, $m_2 = 0.0299$ is the gradient of the line, and $c_2 = 0.3545$ is the y-intercept of the line.

Figure 6.8. Temperature of one internal node under various speed profile of the aluminum block.

Figure 6.9. The evaluated convection coefficients from the lumped-parameter model.

Figure 6.10. The temperature difference of one of the two pairs of thermocouples on the surface.

From (6.27), making v the subject,

$$v = \frac{\Delta T - c_2}{m_2} \quad (6.28)$$

Substituting (6.28) into (6.26), we get

$$h = m_1 \frac{\Delta T - c_2}{m_2} + c_1$$

Then,

$$h = \frac{m_1}{m_2} \Delta T + \left(c_1 - \frac{c_2}{m_2} \right) \quad (6.29)$$

After the values have been substituted accordingly, the correlation can be expressed as,

$$h = 156.91 * \Delta T - 8.76 \quad (6.30)$$

With the convection coefficients known at different motion speeds, a few series of runs of test with several different carriage motion speeds were carried out. The same internal node is used for verification between the predicted temperature using the lumped parameter model and the experiment shown in Fig 6.11. Fig 6.12 shows a closed up view of the temperature change of the node as the carriage motion speed changes with respect to the time.

The heater cartridge is powered with 30 watts throughout and during the first four hours, the carriage is stationary so that the block reaches a steady state temperature under free convection heat loss. After four hours, the speed of the carriage motion is increased every two hours with speed starting with 50 mm/s till 400 mm/s at 50 mm/s interval shown in Fig 6.12. In Fig 6.12, the purple line is the experimental data while the red line is the predicted temperature from the lumped parameter model. The predicted temperature agrees well with the experimental data.

Figure 6.11 . Transient temperature of an internal node of the block.

Figure 6.12. Closed up view to show the temperature changes with carriage motion speed.

By using the correlation developed, the amount of heat lost from the surface of the block, can be quantified. This can be applied to the surface of the carriage of the linear motor stage. This developed correlation can replace the previous method of using constant convection coefficient on any thermal model or simulations. This will give a more meaningful thermal modeling technique with better accuracy of temperature results. The basis for the prediction model is from the velocity input to the linear stage by the controller.

6.6 Summary for Chapter 6

In this work, the change of convection coefficients under Sakiadis type of problem of mixed convection when the carriage is in motion has been investigated. As the machines are assembled and aligned, it is not advisable to dismantle it to measure its surface roughness of all its exposed surface area. The method to predict the approximate convection coefficient was conducted using surface temperature differences. The lumped-parameter method was used to evaluate the convection coefficients with experimental data of temperature of the internal nodes. The method of deriving the mathematical equation was developed to predict the convection coefficient under different speed. The equation is different for different surface roughness, surface orientation and coatings. The predicted temperature agrees well with the experimental data for other speeds different from those for derivation. Thus, the convection coefficient can be obtained using this method with sufficient confidence on any surface orientation, roughness and coatings.

Chapter 7 Analysis of the Contact

Conductance of Machine Joints

7.1 Introduction

It is widely accepted that when two surfaces are in contact. The contact conductance plays an important role in the heat flow across the interface. In most joints, contact is made only at a few discrete areas while other areas are separated by a thin layer of air. In many practical situations, joints of two surfaces are usually filled with environment air. Heat transfer across contact surfaces is by conduction when solids are in contact, by conduction through the fluid in between solids and by radiation in the gaps between surfaces. As mentioned in Section 4.4, heat transfer by radiation is insignificant at moderately low temperature [139] as in this study. Radiation will not be considered here. Ideal surfaces refer to two completely flat surfaces with 100% solid-to-solid contact without any gap. However, engineering surfaces are never smooth and surface irregularities are observable when viewed under a microscope. As a result, only a few areas are in contact while the rest are with a layer of fluids or air in between when two solid surfaces are pressed together. The decrease in surface area for actual conduction between two surfaces is the cause of an extra of resistance on heat flow due to the existents of gaps.

Most of the research works before 1970 were based on the assumption that the distribution of the contact spots was known and the predictions were generally

over-estimated [140]. After that, there were many research works on prediction of the distribution of contact spots by the use of statistics. However, statistics does not predict the distribution well. For example, Williams [141] found that his theoretical values were over estimated as compared to the experimental data obtained. In a separate study, Veziroglu [142] found that the experimentally statistical nature of the contact conductance to have a $\pm 60\%$ variation with 15 pairs of similarly finished surfaces. Research works to experimentally obtain the contact spot distributions has also been carried out. For instance, the contact distribution of two contact surfaces was studied under the microscope or by the use of a profiler. The pressure gel which changes color under different pressures was used to map out the actual contact locations [142].

A comprehensive review of thermal models on contact conductance of metals was conducted and found that the theoretical models for flat surfaces did not agree with most non-flat surfaces [143]. Many empirical and semi-empirical correlations for non-flat surfaces suffer from limited applications. The few developed theoretical models are computationally intensive and cannot be readily applied to real applications. Shai and Santo [144] investigated the theoretical formulation for thermal contact conductance as most of the works done thus far were experimental. Their study consisted of three parts, surface profile analysis, deformation analysis and thermal analysis. The results obtained were compared with a few models developed previously and agrees well.

Numerical analysis using finite difference method was studied by some researchers [145-148]. Conjugate gradient method was used to calculate the thermal contact

conductance from the generated temperatures and verified with direct method by using the finite difference method [145]. Two-dimensional simulation was conducted with two different thermo-elastic materials, one being inside the other [147]. The simulated results showed the deformation, pressure on interfaces, gaps at interfaces, thermal contact conductance, and the temperature distribution. Simulation was also performed between two conical shaped cylinders [148]. The effect of cone radius ratio, cone angle and ambient vacuum on thermal contact conductance was examined. Wahid [146] studied changes of the thermal contact conductance between two cylinder's cone angles. Tang et. al [149] carried out finite element analysis on a tube-fin heat exchanger and validated the results experimentally. The relationship between thermal contact conductance with pressure and contact area was calculated with existing correlations. An inverse technique using boundary element method was used to predict the thermal contact conductance with input temperatures in a two-dimensional domain, and the sensitivity to temperature noise was investigated [150].

Many studies on thermal contact conductance were performed experimentally. The effects of surface topography and mean interfacial temperature on the thermal contact conductance were studied on cylindrical specimens [151]. Correlations of thermal contact resistance with different surface finishing and interstitial gases in gap were developed from experimental results [152]. There are 85% of the data points which fall within $\pm 4\%$ of the correlation. A predictive model for thermal contact conductance had been developed and a good agreement was found with experimental data [153]. The surface topography was obtained and analyzed with Hertz theory describing elastic contact under pressure and the resultant known contact area was simulated for

thermal analysis with CFD for the gas in between the gaps.

There were research works done experimentally to enhance the thermal contact conductance for better heat flow efficiency. The thermal contact conductance for honeycomb structures had been experimentally studied with different honeycomb pattern, joining pattern, interfacial contact pressure, and heat input for optimal heat transfer [154]. The thermal contact conductance on finned tube heat exchanger was optimized with different plating materials in the interface and different interstitial gases used for ambient [155]. The softness of the plating material has a higher effect on enhancing the thermal contact conductance than its conductivity. A cost effective way of enhancing the thermal contact conductance, without improving the surface flatness finishing or applying plating material at junction of two surfaces, was using cycling loading [156]. The interface surfaces were flattened under cyclic overloading to increase the contact area which enhanced the contact conductance. As the interface temperatures were difficult to measure, the interface temperatures were usually interpolated from internal situated temperature sensors which have uncertainties. A new method of experimentally measuring accurately the thermal contact conductance was developed by making use of stacked washers [157]. The multiple interfacing surfaces could reduce the uncertainty in the evaluated thermal contact conductance. The heat lost to ambient also played a part in temperature measurement accuracy and the resultant evaluated thermal contact conductance. A shield and vacuum environment had been shown to reduce heat loss to environment by radiation and convection respectively, and produced a more accurate thermal contact conductance value [158].

Most of the joints of machine structures were bolted for longer reliability and with adequate pressure for high stiffness. An experimental study was conducted on bolted joints for the thermal contact conductance values with different interfacing materials used and different pressures [159]. Another experimental study was conducted on thermal contact conductance with pairs of aluminum alloys with different numbers of bolts, blot patterns, pressures and thicknesses of silicon [160].

However, in precision machine tools, the machine structure is delivered after being assembled and aligned for precision. It is not advisable to dismantle the structure joints to study the joint surfaces as the alignment will not be there when the machine is assembled again. Thus, it is extremely difficult to know the profile of the two surfaces of the contact joint and thus, the distribution of the contact spot. The objective of the work described in this chapter is to predict the joint contact conductance without a complete knowledge of the surface profile and joint pressure. By using a few temperature sensors, it is possible to estimate the contact conductance of the contact joint.

7.2 The analytical models for thermal analysis

7.2.1 Theoretical modeling basis

Figure 7.1 shows the three temperature points on and across the interface of two mating material. Taking conduction along the x-axis,

$$q_{(x)} = -kA_c(x) \frac{dT}{dx} \quad (7.1)$$

and conduction along the y-axis,

$$q_{(y)} = -k_{cc} A_c(y) \frac{dT}{dy} \quad (7.2)$$

where cross sectional area $A_c(x) = A_c(y)$, heat input $q_{(x)} = q_{(x)} = q_{(x)}$, material

conductivity $k = \frac{1}{R}$, and contact conductivity $k_{cc} = \frac{1}{R_{cc}}$.

Figure 7.1 Temperature measurement points on the same side of material and across the interface.

Neglecting natural convective heat loss was neglected as they would be cancelled out between them. Dividing (7.1) by (7.2),

$$1 = \frac{k}{k_{cc}} \frac{dT/dx}{dT/dy} \quad (7.3)$$

$$\text{Substituting the conductivities, } 1 = \frac{R + R_{cc}}{R} \frac{dT/dx}{dT/dy} \quad (7.4)$$

$$\text{Rearranging, } R_{cc} = R \left[\frac{dT/dy}{dT/dx} - 1 \right], \text{ where } \frac{dT/dy}{dT/dx} \geq 1 \quad (7.5)$$

It can be shown that the contact conductance can be obtained if the temperatures for one side and across the interface are known. This can be more easily observed with Fig. 7.2. Two blocks of material are stack together and a heat input of 10 watts at the top block's square indentation with heat convection coefficient of $10 \text{ Wm}^{-1}\text{K}^{-1}$ and program controlled contact conductance. It can be observed that the temperature difference across the interface is more than the temperature difference on the top block, for the same distance measured.

Figure 7.2 Thermal contact conductance simulation of two blocks.

7.2.2 Numerical lumped-parameter model formulation

The formulation for the lump-parameter numerical model can be found in Section 5.3. The aluminum grade 6061T6 block of size 100 by 100 by 75 mm is shown in Fig. 7.3. It has its surfaces clear anodized and several holes drilled to insert the thermocouples. Each thermocouple is a state in the state-space lumped-parameter thermal model. The discretised block in state space is shown in Fig. 7.4. Fig. 7.4a shows the top view of

the block while Fig. 7.4b shows the side view. The internal elements were surrounded by long and thin surface elements. From observation, there were a total of 24 lumped elements. However, the input to the state space model is not in terms of electrical power. It will be Dirichlet boundary condition as explained in Section 6.2.2. The heater cartridge is inserted from the top surface into the top layer's centre element shown in the figure of Fig. 7.4. A thermocouple is situated on the side of the centre element. The Dirichlet boundary condition is applied to the surface of the five elements connected directly to this centre element. As a result, there are 23 lumped elements.

Figure 7.3 The two blocks used for the present study. The left is a whole solid block, while the right is the block which has been separated into two pieces.

The main heat loss for the precision machine is on the surfaces exposed to air. The heat lost phenomena consist of effective convection which can be described by Newton's law of cooling,

$$Q = h_{effective} (T_{Surf} - T_{Amb}) \quad (7.6)$$

where $h_{effective}$ is the convective coefficient for free or force convection, and

T_{Surf} is the temperature of the particular surface.

Figure 7.4. The discretisation of the aluminum block.

The convective heat transfer coefficient can be deduced from the following relation [125],

$$h = Nu \frac{k}{x} \quad (7.7)$$

where Nu is the Nusselt number, and the values can be obtain from (6.22) –

(6.24)

The verification of the lumped-parameter thermal model had been done and shown in Chapter 6.3.

7.3 Experimental set-up

The experimental setup used in this study is shown in Fig. 7.5. It consists of a specimen holder with four supports with insulator tape pasted on the supporting surface, a 50 watts cartridge heater, and 23 J-type thermocouples. The cartridge heater is powered by the amplifier which is controlled by the output of the NI data acquisition system. The 23 J-type thermocouples are also connected to the NI data acquisition system to record the temperature changes. The test blocks consist of a full block in Fig. 7.5a and a pair of half-blocks stacked together in Fig. 7.5b. The purpose of the full block in Fig 7.5a is to validate the lumped-parameter thermal model. The pair of half-blocks in Fig. 7.5b is used to model the contact conductance between them.

Figure 7.5a. The experimental set-up for the solid block.

Figure 7.5b. The experimental set-up for the two separate pieces of solid block.

The test block dimension is 100 by 100 by 75 mm while the two half-blocks are 100 by 100 by 37.5 mm. The material used is aluminum alloy 6061-T6 and the contact surfaces were machined to a smooth finish with surface roughness, R_a , in the range of 0.5-1 μm . In Fig 7.5b the two half-blocks were fastened by steel bolts and steel hexagonal nuts. The placements of the 4 bolts were at the corner of the square. For the two experimental set-ups, the schematic is shown in Fig. 7.6. The only difference is the aluminum block used.

Figure 7.6. The schematic diagram of the experimental set-up.

For both test setups, there are 17 thermocouples measuring internal temperatures of the blocks. There are 6 surface thermocouples with one on each side of the cuboid block in Fig 7.5b while there are 3 surface thermocouples on one side of the stacked half-blocks. Two surface thermocouples are on the top half while one thermocouple is on the bottom half. The spacing between P1 and P2, P2 and P3, is 20 mm apart. The three thermocouples are used to model and predict the thermal contact conductance by comparing the temperature difference between each pair. In the experiment, the power of the heater was set from 5 Watts to 30 Watts in steps of 5 Watts. The temperature logging was obtained with a National Instrument DAQ-9174 acquisition chassis with an NI-9219 card for thermocouple interfacing. The experimental data were taken until the temperature profile of each test achieved a steady state condition, which was assumed to have reached when none of the measured temperatures varied by more than 0.05°C over a period of sixty minutes. Besides the bare aluminum contact with bolted joint, this study also investigates the influence of the conducting paste, which is commonly used, on the bolt joint contact conductance.

7.4 Contact conductance acquisition and analysis

The experiments conducted are separated into four different situations. In the first two situations, the two pieces of the solid half blocks were assembled together without any conducting paste in between them. The first situation had four M5 bolts tightened to a torque of 1N, while the second situation each bolt was tightened to 3N. The last two situations consist of assembly of the two half blocks with ample conducting paste in between. The third and fourth situation differ by tightening of bolts to 1N and 3N respectively.

Figure 7.7a. Temperature of two internal nodes, adjacent to each other across the interface under various power inputs.

Figure 7.7b. Temperature of the three surface thermocouples placed as shown in Fig. 7.4.

In Fig. 7.7a and Fig. 7.7b illustrate the temperatures captured by the sensors for the second situation, i.e. no conducting paste in between and the bolts were tightened to 3N. Fig. 7.7a shows the temperature of the two adjection nodes separated by the interface of the two blocks shown in Fig 7.8 as T_4 and T_{12} . Fig 7.7b shows the corresponding temperatures captured by the three surface sensors, shown in Fig 7.8, in the same experiment. In each situations, a input power varied from 5 Watts, to 30 Watts in steps of 5 Watts.

Figure 7.8. Schematic of lumped nodes 4 and 12 with a contact interface in between.

The temperatures taken from all the four situations are used in the lumped parameter thermal model, to evaluate the contact conductance. The schematic of a pair of lumped nodes separated by a contact interface is shown in Fig. 7.8. The material resistance k_m is from the node centre to the side of the lump while k_{cc} is the contact resistance. When using an entire block of aluminum shown in Fig 7.5a, k_{cc} is zero and the resistance between the two nodes is $2k_m$.

For the evaluation of the contact resistance value, the experimental temperature input for node 4 is entered into the lumped model. The total resistance for the adjacent pair of nodes, $2k_m + k_{cc}$, is then obtained by searching for the total resistance value such that the lumped temperature for node 12 matches the experimental value. The total resistance is evaluated for all the contact pair of nodes for the entire interface between the two blocks. The contact resistance value is then obtained after subtracting the material resistance value. The thermal contact conductance is given by the inverse of the contact resistance.

The surface mounted thermocouples are shown in Fig. 7.8 as P_1 , P_2 , and P_3 . The temperature difference between P_2 and P_3 are due to heat transfer within the same solid while the temperature difference between P_1 and P_2 are due to the heat transfer with the contact interface. The heat transfer also includes convection and radiation which will cancel each other out since the distance apart is the same and the surface temperature does not differ much.

The evaluated contact conductance values with respect to the steady state temperatures reached for different power rating are shown in Fig. 7.9. The contact conductance increases with increased temperature due to increased power input. This trend is consistent in all the four situations. The situation without paste, and having bolts tightened to 1 Nm, has the lowest contact conductance curve. The situation with paste, and having bolts tightened to 3 Nm, has the highest contact conductance curve. The situation without paste, and having bolts tightened to 3 Nm, has a higher contact conductance curve than the situation with paste, and having bolts tightened to 1 Nm. Only the steady state temperatures of each situation's power rating were taken for the evaluation of the thermal contact conductance.

The corresponding temperature differences of the surface sensors are shown in Fig. 7.10. The temperature difference increased with increased temperature due to increased power input. This trend is consistent in all the four situations. The situation without paste, and having bolts tightened to 1 Nm, has the lowest temperature difference curve. The situation with paste, and having bolts tightened to 3 Nm, has the highest temperature difference curve. The situation without paste, and having bolts tightened to 3 Nm, has a higher temperature difference curve than the situation with paste, and having bolts tightened to 1 Nm. This shows that the plain interface at higher pressure has a lower contact resistance than the interface at lower pressure with paste in between.

Directly using the graphs of the contact conductance with temperature differences of the surface alone does not have a single correlation to describe the contact conductance for all the situations. The normalized surface temperature difference is

calculated and shown in Fig. 7.11 according to the equation,

$$|\Delta T| = \frac{T_{P2} - T_{P1}}{T_{P2} - T_{P3}} \quad (7.8)$$

With the normalized temperature difference, a curve is plotted in Fig. 7.12 to represent the four over-lapping contact conductance curve. The trend line was created from this curve and the correlation is developed between the thermal contact conductance and the normalized temperature difference shown in Fig. 7.12. The correlation is,

$$K_{Contact} = 8.667|\Delta T|^4 - 26|\Delta T|^3 + 27.833|\Delta T|^2 + 3.5|\Delta T| + 116 \quad Wm^{-2}K^{-1} \quad (7.9)$$

For prediction of the thermal contact conductance with respect to temperature at node 4, the correlation is shown in Fig. 7.13, and the equation is

$$K_{Contact} = 0.0034T_4^4 - 0.5572T_4^3 + 33.667T_4^2 - 862.13T_4 + 12706 \quad Wm^{-2}K^{-1} \quad (7.10)$$

Figure 7.9. The evaluated contact conductance from the lumped-parameter model.

Figure 7.10. Temperature difference between the pair of thermocouples, P1 and P2, on the surface.

Figure 7.11 Normalised temperature difference of the surface sensors.

Figure 7.12 The correlation of the thermal contact conductance with normalised surface temperature difference.

7.5 Results and Discussion

With the correlation for the thermal contact conductance known, a verification test was conducted with a new thinner coat of conducting paste and the bolts were tightened with a torque of 2N. For simplicity, only nodes 4 and 12 of the lumped parameter thermal model were used for results presentation and discussion. Similar results can be observed with other pairs of nodes along the contact interface. Using the lumped parameter thermal model, the input from the experimental data was the temperature of node 4. The correlation developed from (7.10) was used to estimate for the thermal contact conductance, T_{CC} , with temperature data from P_1, P_2 and P_3 . The same internal node 12, which is on the opposite side of the contact interface from the heater cartridge, was used for verification.

The predicted temperature from the lumped parameter method and the experimental temperature reading of node 12 are shown in Fig 7.14. The predicted and experimental results agree well. The predicted temperature fits very well with the experimental data especially at the 5 and 30 W input powers. For more than 90% of the temperature range, the temperature difference between the predicted and experimental is less than 1 °C. The predicted temperature deviated the most when the input power was 15 W. There was a maximum of 1.8 °C difference between the results at less than 5% of the total range of temperature measured. The result shows that the lumped parameter thermal method to evaluate the thermal contact conductance is verified as the predicted temperature agrees well with the experimental measurements.

Figure 7.13. The correlation of the thermal contact conductance with node 4 temperature.

Figure 7.14. Transient temperature of an internal node 12 of the block.

By using the correlation developed, the amount of the heat transferred between joints of any materials, can be quantified. This quantified heat can replace the previous method of using constant contact conductance on any thermal model or simulations. This will give a more meaningful thermal modeling technique with high accuracy temperature results. The basis for the prediction model is from the measured temperature difference between the joint at a given temperature range. The dimensions used in the contact conductance modeling were for symmetric purpose so that the verification and consistency can be checked on the measurement values from the numerous temperature sensors.

7.6 Summary of Chapter 7

In this work, investigation was conducted on the change of contact conductance between two solid interfaces under different heat loading, pressure loading and interstitial material. As the machine joints are assembled and aligned, it is not advisable to dismantle it to measure its surface roughness to know the contact area for the interfaces. The method of using surface temperature difference to predict the approximate contact conductance was conducted. The lumped-parameter method was used to evaluate the contact conductance with experimental data of temperature of the internal nodes. The method of deriving the mathematical equation had been developed for predicting the contact conductance under different pressure and interstitial material. The equation is different for different pair of solid interfaces. The predicted temperature agrees well with the experimental data for other loadings different from those for derivation. Thus, the contact conductance can be obtained by this method with sufficient confidence for any pressure contact and interstitial material used.

Chapter 8 Evaluation of the Analysed Boundary Conditions

8.1 Introduction

The thermally induced deformation causes precision tools to consistently produce errors in their processes. A thermal model has been proposed and developed using the three-dimensional lumped parameter method. The unknown boundary conditions such as the heat source, heat loss convection and contact conductance have been modeled. With the input of the boundary conditions, the thermal model can then be used to predict the deformation errors.

8.2 The lumped parameter thermal model

The formulation for the lump-parameter numerical model can be found in Section 5.3. The boundary conditions used in the lumped parameter method are the heat source sub-model, convective heat loss sub-model and heat transfer contact conductance sub-model. From (5.26), the heat source sub-model correlation is given by,

$$F(I, T) = 0.54 \Big|_{T_0}^{T=F_1(I)} + \{[F_2(I)]T + F_3(I)\} \Big|_{T=F_1(I)}^{T_{ss}=F_4(I)} + F_5(I) \Big|_{T_{ss}=F_4(I)}^{T_{\infty}} \quad (8.1)$$

where, $F_1(I) = 19.296I^2 - 45.564I + 56.808$

$$F_2(I) = -0.0587I^2 + 0.2872I - 0.3667$$

$$F_3(I) = 1.5235I^2 - 7.3463I + 10.321$$

$$F_4(I) = 26.659I^2 - 56.32I + 62.478$$

$$F_5(I) = 0.0048I^2 + 0.0067I + 0.1433$$

The resulting amplification factor, $F(I, T)$, is used in the heat source sub-model. The heat source sub-model has input values as the current and the output as the heat flux of the top of the motor coil, where the carriage is fastened to. The output heat flux on the top of the motor coil is the heat flux input to the carriage.

From (6.32), the effective convective heat loss correlation is rewritten here as,

$$h_{effective} = 156.91 * \Delta T - 8.76 \quad Wm^{-2}K^{-1} \quad (8.2)$$

All the exposed surfaces of the carriage undergo convective heat loss. The surface convective heat loss coefficients for Sakaidis type of flow are derived from (8.2) for different orientations. The convective heat loss coefficient, h , is used in the thermal model for surface heat loss to ambient air.

Figure 8.1. The carriage assembly of the linear motor stage.

Figure 8.2. The carriage of the linear motor stage.

Figure 8.3. The lumped parameter discretisation of the carriage.

From (7.16), the contact conductance sub-model is rewritten here as,

$$K_{Contact} = 0.0034T_4^4 - 0.5572T_4^3 + 33.667T_4^2 - 862.13T_4 + 12706 \quad Wm^{-2}K^{-1} \quad (8.3)$$

The resulting contact conductance value is used in the thermal model for contact interfaces between any two solids.

The deformation is given by,

$$\delta = \alpha L(T_i - T_0) \quad (8.4)$$

where δ is the deformation, α is the linear coefficient of thermal expansion, L is the length of material, T_i is the temperature at which the deformation is to be evaluated, and T_0 is the ambient temperature.

The CAD model of the carriage assembly in the linear motor stage is shown in Fig 8.1. Figure 8.2 illustrates an aluminum grade 6061T6 carriage of size 260 by 155 by 55 mm fabricated and used in this research. The discretized lumped parameter thermal model of the carriage is shown in Fig. 8.3. A total of 156 lumped elements were used in the lumped numerical model. The boundary conditions described by (8.1)-(8.3) are applied to the lumped numerical method (Fig. 8.4). The inputs to the sub-model are shown respectively in Fig. 8.4.

Figure 8.4. The boundary conditions for the lumped parameter model

A thermal model with n nodes and the n coupled algebraic equations can be assembled in a state-space domain form as follows,

$$[\dot{x}] = A[x] + B[u] \quad (8.5)$$

$$[Y] = C[x] + D[u] \quad (8.6)$$

where $[x]$ is the state vector, $[u]$ is the input vector, $[Y]$ is the output vector, and A, B, C and D are the state coefficient matrix, input matrix, output matrix and feed-forward matrix respectively.

8.3 Experimental Set-up

A schematic diagram of the experimental set-up is shown in Fig. 8.5. Deformation data were collected with two strain gauges mounted on the surface of the carriage (Fig 8.4 and Fig. 8.6b). The strain gauges were placed symmetrically on both side of the carriage's centre line to ensure that the readings were consistent. Two thermocouples were also used: one on the surface of the carriage and near to a strain gage (Fig. 8.6b), and the other on the top of the motor coil (Fig. 8.4). Deformation and temperature logging were carried out using a National Instrument DAQ-9174 acquisition chassis with an NI-9219 card for strain gauge interfacing (Fig. 8.6a). The strain gauges are amounted as shown in Fig. 8.6b. The blue strain gauges are of type 23, 8 mm in length, and base material of polyester with copper-nickel alloy foil. It has a temperature usage range or -30°C to 80°C , with thermal variation of within $\pm 2\mu\epsilon/^{\circ}\text{C}$, and gauge factor variation of within $\pm 0.015\%/^{\circ}\text{C}$. The digital instrument can take in readings with resolution of $0.01\ \mu\text{m}$ strain, but with noise, the resolution becomes $0.5\ \mu\text{m} \pm 0.2\ \mu\text{m}$.

Figure 8.5. The schematic diagram of the thermal deformation experimental setup.

Figure 8.6a Experiment set-up for the thermally induced deformation measurement

Figure 8.6b. The experimental set-up for strain gauge mounting.

Figure 8.7. Temperature measurements and prediction of the carriage and motor coil under cyclic loaded motion.

Figure 8.8. Thermal deformation of the carriage under cyclic loaded motion.

8.4 Results & Discussion

The carriage is programmed to run in cyclic motion at 20 mm/s with a load of 2.3kg. The inputs to the lumped-parameter model are provided by the sub-models described in Chapters 5, 6 and 7 (Fig. 8.4). The sub-models are the heat source sub-model, convective heat loss sub-model and contact conductance sub-model. The results (i.e. temperature readings) obtained from the experiments and the lumped parameter thermal model are shown in Fig. 8.7. An error of 3.79% higher for the temperature registered on the top surface of the motor coil at steady state was observed. This could be due to the fact that the top surface of the motor coil was not exposed to the ambient air of 20°C. In the deformation experiment the solid carriage was mounted on top of the coil instead. There was a greater heat loss as heat was carried away from the motor coil. This results in over estimation of the temperature on top of the motor coil. There was a 4.19% error for the temperature registered on the top of the carriage surface at steady state. This could be due to the aspect ratio of the surface used in the contact conductance modelling. The aspect ratio refers to the interfacial surface length with respect to the thickness. The contact conductance modelling was conducted on aspect ratio 2.66, while the carriage interfacial has an aspect ratio of 17.33. The contact conductance further inside the interfacial could be better than expected which resulted in an overall higher average contact conductance value. The higher contact conductance values would provide better heat transfer and resulted in a higher temperature for the carriage. This explains why the predicted temperature of top surface of the carriage was lower. As the carriage was warmer than expected, the predicted deformation of the carriage was smaller than the experimental value by 2.62 μm (Fig. 8.8), which seemed reasonable.

The modeling and experiments were for a single axis stage. It can be applied to multiple axis stages. In a precision machine, for multiple axis motion, one axis is usually mount onto another axis. There will heat transfer between any two axis through the mounting and contact conductance in between the surface involved. The contact conductance modeling can be adopted here for joints between any two motion axis. The thermally induced deformation, which translate to positioning error in the end effector is culmulative for multiple axis motion stage. In other words, each axis will contribute to the positioning error of the end effector.

8.5 Summary of Chapter 8

In this work, verification was carried out using the modified lumped-parameter thermal model, with the boundary condition sub-models, can predict the temperature profile of the machine structure. The sub-models are the heat source sub-model, heat loss convection sub-model and contact conductance sub-model. The inputs for the thermal model are the currents and carriage speed. The current values are needed in the heat source sub-model to estimate the amount of heat generated. The carriage speed value is needed for the heat loss convection model to estimate the convection coefficients. The contact conductance values are varied according to the temperature of the internal node at the interface. With the temperature profile, the thermal deformation can be predicted with sufficient accuracy.

Chapter 9 Conclusion & Recommendation

9.1 Conclusion

The thermally induced deformation causes precision tools to consistently produce errors in their processes. The uncertainty of any thermal models is mainly due to the several boundary conditions which are unable to be stated accurately. This research work attempts to model the boundary conditions specifically for precision machine structures. The primary heat source is the heating of the motor coil. Heat source is the first boundary condition which needs to be modeled to evaluate how much thermal energy is transferred to the machine structure. The motor coil heats up due to joule heating effects during operation. The I^2R term for heating is different between a pure heater element and a copper coil motor. Almost all the electrical energy supplied to a pure heater is used to produce heat, while a motor have its electrical energy supply to spend on doing mechanical work and copper coil heating. The heating up process is a function of the current and temperature of the coil. A correlation is derived to predict the amount of thermal energy transferred to the machine structure from the power input of the amplifier. The method of obtaining the transient power constant, which include the experimental set-up of the temperature sensors and using the modified lumped-parameter method, can be used on any linear motors. The linear motors can have different size, force rating and copper coil arrangement depending on the design needs.

The second boundary condition is the heat loss convection from the surface of the machine structure. The stationary surface heat loss convective coefficient has well established correlation equations. The Blasius type of flow, which is the situation where the fluid moves on a stationary solid surface, is well studied. The Sakiadis type of flow, moving solid surface in a stationary fluid, has not been fully investigated. A method of measuring the surface temperature, and using it to correlate and predict the heat loss convective coefficients, has been developed. The derived correlation can predict the convective coefficients with sufficient accuracy. The method of obtaining the convection coefficients, which include the experimental set-up of the temperature sensors and using the modified lumped-parameter method, can be used on any machine structure's surfaces. The surfaces can have different types of coating or paint and whether it's moving or stationary.

The third boundary condition is the contact conductance, which is common in all machine structures. This is the result of imperfect contact of two solid surfaces, which gives higher resistance to heat transfer than ideal surface contact. A method is developed using the surface temperatures to predict the thermal contact conductance of two contact solid surfaces with different conditions. The derived correlation can predict the conductance with reasonable accuracy. The method of obtaining the contact conductance, which include the experimental set-up of the temperature sensors and using the modified lumped-parameter method, can be used on any machine structure's interfaces or joints. The interfaces or joints can have different types of pressure and interstitial material.

Finally, using the three sub-models, the modified lumped-parameter thermal model is

used to predict the deformation of the carriage at certain points. The experimental and predicted data have an average error of less than 3 μm at the steady state with one of the many test runs shown. This verifies that the boundary conditions obtained for the linear motor stage are accurate and reliable. The boundary condition values obtained from a particular machine structure can be used in simulation packages to study the heat transfer of the same machine structure. Another application for the boundary condition values is for positioning error compensation. With a real-time thermal model using lumped-parameter method, having inputs of the heat source sub-model and convective sub-model and contact conductance sub-model, the positioning errors can be compensated in real-time.

9.2 Recommendation

1. The heat source sub-model has been used, so far, only on one type of linear motor. It is important to verify that the correlation method can be used on other types of linear motors.
2. The convective heat transfer sub-model has been used so far on only aluminum surfaces with the same surface finish. It is good to have the correlation developed for other surface types, material types and different coatings to verify that the method is indeed a general one.
3. The contact conductance sub-model has been used only for limited contact situations. More conditions like, different materials of the two solids, larger range of interfacial pressure loadings, and different conducting fluids in between

contact surfaces, can be introduced to verify that the method can be generalized.

4. The single axis thermal model can be used in a multiple axis motion stage. This has to be investigated and verified, to study the interaction between each motion axis on the other. It is important to verify if the total positioning error of the end-effector is cumulative of the individual single axis thermal model.

References

- [1] R. Ramesh, M. A. Mannan, and A. N. Poo, Error compensation in machine tools- a review part I: geometric, cutting-force induced and fixture-dependent errors, *International Journal of Machine Tools and Manufacture*, Vol. 40, No. 9, 2000, pp.1235-1256.
- [3] S. K. Choudhury, N. N. Goudimenko, and V. A. Kudinov, On-line control of machine tool vibration in turning, *International Journal of Machine Tools and Manufacturer*, Vol. 37, No. 6, 1997, pp. 801-811.
- [3] R. Ramesh, M. A. Mannan, and A. N. Poo, Error compensation in machine tools- a review part II: thermal errors, *International Journal of Machine Tools and Manufacture*, Vol. 40, No. 9, 2000, pp.1257-1284.
- [4] J. W. Li, W. J. Zhang, G. S. Yang, S.D. Tu and X.B. Chen, Thermal-error modeling for a complex physical systems: the-state-of-arts review, *International Journal of Machine Tools and Manufacture*, Vol. 42, No. 1, 2009, pp.168-179.
- [5] R. L. Murty, Thermal deformation of a semi-automatic machine: a case study, *Journal of Precision Engineering*, Vol. 2, No. 1, 1980, pp.17-21.
- [6] S. K. Lee, J. H. Yoo, M. S. Yang, Effect of thermal deformation on machine tool slide guide motion, *Tribology International*, Vol. 36, 2003, pp. 41-47.
- [7] P. I. Ro, W. B. Shim, S. H. Jeong, Robust friction compensation for submicrometer positioning and tracking for a ball-screw-driven slide system, *Precision Engineering*, Vol. 24, 2000, pp. 160-173.
- [8] C. L. Chen, M. J. Jang, K. C. Lin, Modeling and high-precision control of a ball-screw-driven stage, *Precision engineering*, Vol. 28, 2004, pp. 483-495.
- [9] H. Shinno, H. Yoshioka, K. Taniguchi, A newly developed linear motor-driven aerostatic X-Y planar motion table system for nano-machining, *CIRP Annals – Manufacturing Technology*, Vol. 56, No. 1, 2007, pp. 369-372.
- [10] A. H. Slocum, Design to limit thermal effects on linear motion bearing performance, *International Journal of Machine Tools and Manufacture*, Vol. 27, No. 2, 1987, pp.239-245.

- [11] S. Gordon, M. T. Hillery, Development of a high-speed CNC cutting machine using linear motors, *Journal of Materials Processing Technology*, Vol. 166 , 2005, pp. 321-329.
- [12] C. W. Lee and S. W. Kim, An ultraprecision stage for alignment of wafers in advanced microlithography, *Precision Engineering*, Vol. 21, 1997, pp.113-122.
- [13] M. M. P. A. Vermeulen, P. C. J. N. Rosielle, and P. H. J. Schellekens, Design of a high-precision 3D-coordinate measuring machine, *Annals of the CIRP*, Vol. 47, No. 1, 1998, pp. 447-450.
- [14] A. C. Okafor, Y. M. Ertekin, Vertical machining center accuracy characterization using laser interferometer Part 2. Angular errors, *Journal of Materials Processing Technology*, Vol. 105, 2000, pp. 407-420.
- [15] H. Onda, Development of a unique high precision linear motor integrated air slide table and its application to laser beam writers, *Optical Review*, Vol. 6, No. 1, 1999, pp. 88-92.
- [16] S. K. Ro, S. Y. Kim, Y. K. Kwak, and C. H. Park, Development of magnetically preloaded air bearing for a linear slide: active compensation of three degrees of freedom motion errors, *Review of Scientific Instruments*, Vol. 79, No. 3, 2008, pp.104-107.
- [17] S. K. Ro, and J. K. Park, Development of a miniature air-bearing stage with a moving-magnet linear motor, *International Journal of Precision Engineering and Manufacturing*, Vol. 9, No.1, 1987, pp.19-24.
- [18] A. Slocum, M. Basaran, R. Cortesi, and A. J. Hart, Linear motion carriage with aerostatic bearings preloaded by inclined iron core linear electric motor, *Precision Engineering*, Vol. 27, 2003, pp. 382-394.
- [19] W. Gao, Y. Arai, A. Shibuya, S. Kiyono, C. H. Park, Measurement of multi-degree-of-freedom error motions of a precision linear air-bearing stage, *Precision Engineering*, Vol. 30, 2006, pp. 96-103.
- [20] Y. Fujii, Frictional characteristic of an aerostatic linear bearing, *Tribology International*, Vol. 39, 2006, pp. 888-896.

- [21] Y. Fujii and H. Fujimoto, Measurements of frictional characteristics of a pneumatic linear bearing, *Measurement Science Technology*, Vol. 10, 1999, pp. 362-366.
- [22] K. J. Stout, S. M. Barrans, The design of aerostatic bearings for application to nanometer resolution manufacturing machine systems, *Tribology International*, Vol. 33, 2000, pp. 803-809.
- [23] J. H. Mao, H. Tachikawa, and A. Shimokohbe, Precision positioning of a DC-motor-driven aerostatic slide system, *Precision Engineering*, Vol. 27, 2003, pp. 32-41.
- [24] H. Shinno, H. Hashizume, High speed nanometer positioning using a hybrid linear motor, *CIRP Annals – Manufacturing Technology*, Vol. 50, No. 1, 2001, pp. 246-246.
- [25] W. Gao, S. Dejima, H. Yanai, K. Katakura, S. Kiyono, Y. Tomita, A surface motor-driven planar motion stage integrated with XYθ_z surface encoder for precision positioning, *Precision Engineering*, Vol. 28, 2004, pp. 329-337.
- [26] Y. Tomita, Y. Koyanagawa, and F. Satoh, A surface motor-driven precise positioning system, *Journal of Dynamic Systems, Measurement and Control*, Vol. 117, No. 3, 1995, pp.311-319.
- [27] G. Pritschow, A comparison of linear and conventional electromechanical drives, *Annals of the CIRP*, Vol. 47, No. 2, 1998, pp. 541-548.
- [28] G. W. Mclean, Review of recent progress in linear motors, *IEE Proceedings*, Vol. 135 Part B, No. 6, 1988, pp. 380-416.
- [29] Y. M. Chen, S. Y. Fan, and W. S. Lu, Performance analysis of linear permanent-magnet motors with finite element analysis, *IEEE Transactions on Magnetics*, Vol. 44, No. 3, 2008, pp. 337-385.
- [30] G. Abdou and W. Tereshkovich, Experimental comparison of dynamic FEA results of a permanent magnet brushless DC linear motor, *International Journal of Computer Intergrated Manufacturing*, Vol. 13, No. 3, 2000, pp. 214-224.
- [31] M. A. Soltz, Y. L. Yao, and J. I.-Shalom, Investigation of a 2-D planar motor based machine tool motion system, *International Journal of Machine Tools & Manufacture*, Vol. 39, 1999, pp. 1157-1169.

- [32] D. K. Kim and B. I. Kwon, A novel equivalent circuit model of linear induction motor based on finite element analysis and its coupling with external circuits, *IEEE Transactions on Magnetics*, Vol. 42, No. 10, 2006, pp. 3407-3409.
- [33] S. M. Jang, J. H. Park, J. Y. Choi, and H. W. Cho, Analytical prediction and measurements for inductance profile of linear switched reluctance motor, *IEEE Transactions on Magnetics*, Vol. 42, No. 10, 2006, pp. 3428-3430.
- [34] H. Wakiwaka, N. Misron, A. Kamiya, H. Yajima, K. Tamura, N. Fujiwara, M. Hosono, and S. Takada, *International Journal of Applied Electromagnetics and Mechanics*, Vol. 15, 2001, pp. 129-133.
- [35] E. Byckling and K. Perkio, Dynamic properties of moving magnet transducers, *IEEE Transactions on Magnetics*, Vol. 16, No. 2, 1980, pp. 468-473.
- [36] A. Basak and F. J. Anayi, A DC linear motor with a square armature, *IEEE Transactions on Energy Conversion*, Vol. 10, No. 3, 1995, pp.462-469.
- [37] G. Y. Xiong and S. A. Nasar, Analysis of fields and forces in a permanent magnet linear synchronous machine based on the concept of magnetic charge, *IEEE Transactions on Magnetics*, Vol. 25, No. 3, 1989, pp.2713-2719.
- [38] T. S. Hwang, and J. K. Seok, Observer-Based ripple force compensation for linear hybrid stepping motor drives, *IEEE Transactions on Industrial Electronics*, Vol. 54, No. 5, 2007, pp.2417-2424.
- [39] Y. W. Zhu and Y. H. Cho, Thrust ripples suppression of permanent magnet linear synchronous motor, *IEEE Transactions on Magnetics*, Vol. 43, No. 6, 2007, pp.2537-2539.
- [40] S. L. Chen, and T. H. Hsieh, Repetitive control design and implementation for linear motor machine tool, *International Journal of Machine Tools and Manufacture*, Vol. 47, 2007, pp. 1807-1816.
- [41] M. F. Hsieh, C. J. Tung, W. S. Yao, M. C. Wu, Y. S. Liao, Servo design of a vertical axis drive using dual linear motors for high speed electric discharge machining, *International Journal of Machine Tools and Manufacture*, Vol. 47, 2007, pp. 546-554.

- [42] M. G. Lee, Y. M. Choi, S. Q. Lee, D. C. Lim, D. G. Gweon, Design of high precision linear stage with double-sided multi-segmented trapezoidal magnet array and its compensation for force ripples, *Mechatronics*, Vol. 16, 2006, pp. 331-340.
- [43] H. P. Chun, J. O. Yoon, H. H. Joo and W. L. Deug, Development of an ultra precision hydrostatic guideway driven by a coreless linear motor, *International Journal of Precision Engineering and Manufacturing*, Vol. 6, No. 2, 2005, pp.55-60.
- [44] A. H. Isfahani, B. M. Ebrahimi, and H. Lesani, Design optimization of a low-speed single-sided linear induction motor for improved efficiency and power factor, *IEEE Transactions on Magnetics*, Vol. 44, No. 2, 2008, pp. 266-272.
- [45] S. Y. Jung and H. K. Jung, Running characteristics of permanent magnet linear synchronous motor energized by partially excited primary current, *International Journal of Applied Electromagnetics and Mechanics*, Vol. 15, 2001, pp. 138-187.
- [46] S. Mezani, N. Takorabet, and B. Laporte, A combined electromagnetic and thermal analysis of induction motors, *IEEE Transaction on Magnetics*, Vol. 41, No 5, 2005, pp. 1572-1575.
- [47] M. Rizzo, Electromagnetic-thermal analysis of a hybrid switched-reluctance motor, *International Journal of Applied Electromagnetics and Mechanics*, Vol. 15, 2001, pp. 103-110.
- [48] S. Laissaoui, M. R. Mekideche, D. Sedira and A. Ladjimi, Dynamic modeling of induction motor taking into account thermal stresses, *The International Journal for Computation and Mathematics in Electrical and Electronics Engineering*, Vol. 26, No. 1, 2007, pp36-47.
- [49] J. P. Bastos, M. F. R. R. Cabreira, N. Sadowski, S. R. Arruda, A thermal analysis of induction motors using weak coupling modeling, *IEEE Transaction on Magnetics*, Vol. 33, No. 2, 1997, pp. 1714-1717.
- [50] Y. S. Lee, H. B. Lee, and S. Y. Hahn, Temperature analysis of induction motor with distributed heat sources by finite element method, *IEEE Transaction on Magnetics*, Vol. 33, No. 2, 1997, pp. 1718-1721.

- [51] J. A. D. Pinto, C. F. L. Antunes, and A. P. B. Coimbra, Transient heating and cooling analysis in an electromagnetics device, *IEEE Transaction on Magnetics*, Vol. 30, No. 5, 1994, pp. 3339-3342.
- [52] X. M. L. Fernandez, A. P. Coimbra, J. A. D. Pinto, C. L. Antunes, M. P. Donsion, Thermal analysis of an induction motor fed by unbalanced power supply using a combined finite element – symmetrical components formulation, 1998 International Conference on Power System Technology, *IEEE Proceedings. POWERCON 98*, Vol. 1, No. 18, 1998, pp. 620-624.
- [53] K. N. Srinivas, and R. Arumugam, A novel thermal characterization of switched reluctance motors involving computational fluid dynamics, *Electric Power Components and Systems*, Vol. 32, 2004, pp. 855-867.
- [54] K. N. Srinivas, and R. Arumugam, Analysis and characterization of switched reluctance motors: part 2 – flow, thermal, and vibration analysis, *IEEE Transaction on Magnetics*, Vol. 41, No. 4, 2005, pp. 1321-1332.
- [55] G. Dajaku, and D. Gerling, An improved lumped parameter thermal model for electrical machines, *European Conference on Power Electronics and Applications*, 2005, Vol 10, pp. P.10. Institute for Electrical Drives, University of Federal Defence Munich.
- [56] C. C. Hwang, S. S. Wu, and Y. H. Jiang, Novel approach to the solution of temperature distribution in the stator of an induction motor, *IEEE Transactions on Energy Conversion*, Vol. 15, No. 4, 2000, pp. 401-406.
- [57] Y. Bertin, E. Videcoq, S. Thieblin, and D. Petit, Thermal behaviour of an electrical motor through a reduced model, *IEEE Transactions on Energy Conversion*, Vol. 15, No. 2, 2000, pp. 129-134.
- [58] M. S. Rajagopal, K. N. Seetharamu, and P. A. A. Ashwathnarayana, Transient thermal analysis of induction motors, *IEEE Transactions on Energy Conversion*, Vol. 13, No. 1, 1998, pp. 62-69.
- [59] A. Boglietti, A. Cavagnino, M. Lazzari, and M. Pastorelli, A simplified thermal model for variable-speed self-cooled industrial induction motor, *IEEE Transactions on Industry Applications*, Vol. 39, No. 4, 2003, pp. 945-952.

- [60] N. Benamrouche, M. Bouheraoua, and S. Haddad, A thermal model for a TEFC induction motor – development and sensitivity analysis, *Electric Power Components and Systems*, Vol. 34, 2006, pp. 259-269.
- [61] R. L. Kotnik, An equivalent thermal circuit for nonventilated induction motors, *Transactions of the American Institute of Electrical Engineers, Power Apparatus and Systems, Part 3*, Vol. 73, No. 2 Part3-B, 1954, pp. 1604-1609.
- [62] S. H. Li, Y. Q. Zhang, and G. X. Zhang, A study of pre-compensation for thermal errors of NC machine tools, *International Journal of Machine Tools and Manufacturing*, Vol. 37, No. 12, 1997, pp. 1715-1719.
- [63] S. C. Huang, Analysis of a model to forecast thermal deformation of ball screw feed drive systems, *International Journal of Machine Tools and Manufacturing*, Vol. 35, No. 8, 1995, pp. 1099-1104.
- [64] Z. C. Du, J. G. Yang, Z. Q. Yao, B. Y. Xue, Modeling approach of regression orthogonal experiment design for the thermal error compensation of a CNC turning center, *Journal of Materials Processing Technology*, Vol. 129, 2002, pp. 619-623.
- [65] J. H. Lee, S. H. Yang, Statistical optimization and assessment of a thermal error model for CNC machine tools, *International Journal of Machine Tools and Manufacturing*, Vol. 42, 2002, pp. 147-155.
- [66] J. S. Chen, Computer-aided accuracy enhancement for multi-axis CNC machine tool, *International Journal of Machine Tools and Manufacturing*, Vol. 35, No. 4, 1995, pp. 593-605.
- [67] M. Mitsuishi, S. Warisawa, and R. Hanayama, Development of an intelligent high-speed machining center, *CIPR Annals – Manufacturing Technology*, Vol. 50, No. 1, 2001, pp. 275-280.
- [68] H. T. Zhao, J. G. Yang, and J. H. Shen, Simulation of thermal behaviour of a CNC machine tool spindle, *International Journal of Machine Tools and Manufacturing*, Vol. 47, 2007, pp. 1003-1010.
- [69] C. H. Wu, and Y. T. Kung, Thermal analysis for the feed drive system of a CNC machine, *International Journal of Machine Tools and Manufacturing*, Vol. 43, 2003, pp. 1521-1528.

- [70] J. G. Yang, Y. Q. Ren, and Z. C. Du, An application of real-time error compensation on an NC twin-spindle lathe, *Journal of Materials Processing Technology*, Vol. 129, 2002, pp. 474-479.
- [71] J. X. Yuan, and J. Ni, The real-time error compensation technique for CNC machining systems, *Mechatronics*, Vol. 8, 1998, pp. 359-380.
- [72] A. K. Srivastava, S. C. Veldhuis, and M. A. Elbestawit, Modelling geometric and thermal errors in a five-axis CNC machine tool, *International Journal of Machine Tools and Manufacturing*, Vol. 35, No. 9, 1995, pp.1321-1337.
- [73] W. Hao, H. T. Zhang, Q. J. Guo, X. S. Wang, and J. G. Yang, Thermal error optimization modeling and real-time compensation on a CNC turning center, *Journal of Materials Processing Technology*, Vol. 207, No. 1, 2008, pp. 172-179.
- [74] J. G. Yang, J. X. Yuan, and J. Ni, Thermal error mode analysis and robust modeling for error compensation on a CNC turning center, *International Journal of Machine Tools and Manufacturing*, Vol. 39, 1999, pp. 1367-1381.
- [75] W. S. Yun, S. K. Kim, and D. W. Cho, Thermal error analysis for a CNC lathe feed drive system, *International Journal of Machine Tools and Manufacturing*, Vol. 39, 1999, pp. 1087-1101.
- [76] J. D. Suh, and D. G. Lee, Thermal characteristic of composite sandwich structures for machine tool moving body applications, *Composite Structures*, Vol. 66, 2004, pp. 429-438.
- [77] J. S. Chen, A study of thermally induced machine tool errors in real cutting conditions, *International Journal of Machine Tools and Manufacturing*, Vol. 36, No. 12, 1996, pp. 1401-1411.
- [78] Y. Takeuchi, M. Sakamoto, and T. Sata, Improvement in the working accuracy of an lathe by compensating for thermal expansion, *Precision Engineering*, Vol. 4 No.1, 1982, pp. 19-24.
- [79] A. C. Okafor, Y. M. Ertekin, Vertical machining center accuracy characterization using laser interferometer Part 1. Linear positional errors, *Journal of Materials Processing Technology*, Vol.105, 2000, pp. 394-406.

- [80] T. J. Ko, T. W. Gim, and J. Y. Ha, Particular behavior of spindle thermal deformation by thermal bending, *International Journal of Machine Tools and Manufacturing*, Vol. 43, 2003, pp. 17-23.
- [81] C. H. Lo, J. X. Yuan and J. Ni, An application of real-time error compensation on a turning center, *International Journal of Machine Tools and Manufacturing*, Vol. 35, No. 12, 1995, pp. 1669-1682.
- [82] A. C. Okator, and Y. M. Ertekin, Derivation of machine tool error models and error compensation procedure for three axes vertical machining center using rigid body kinematics, *International Journal of Machine Tools and Manufacturing*, Vol. 40, 2000, pp. 1199-1213.
- [83] R. Ramesh, M. A. Mannan, and A. N. Poo, Thermal error measurement and modeling in machine tools. Part I. Influence of varying operating conditions, *International Journal of Machine Tools and Manufacturing*, Vol. 43, 2003, pp. 391-404.
- [84] R. Ramesh, M. A. Mannan, A. N. Poo, and S. S. Keerthi, Thermal error measurement and modeling in machine tools. Part II. Hybrid Bayesian network: support vector machine model, *International Journal of Machine Tools and Manufacturing*, Vol. 43, 2003, pp. 405-419.
- [85] P. S. Lingard, M. E. Pursst, C. M. Sona, E. G. Thwaite, and G. H. Mariasson, Temperature perturbation effects in a high precision CMM, *Precision Engineering*, Vol. 13, No.1, 1991, pp.41-51.
- [86] J. P. Kruth, P. Vanherck, and C. Van den Bergh, Compensation of static and transient thermal errors on CMMs, *CIRP Annals – Manufacturing Technology*, Vol. 50, No. 1, 2001, pp. 377-380.
- [87] M. H. Attia, and S. Fraser, A generalized modelling methodology for optimized real-time compensation of thermal deformation of machine tools and CMM structures, *International Journal of Machine Tools & Manufacture*, Vol. 39, 1999, pp. 1001-1016.
- [88] I. U. Eun. Comparison between asynchronous and synchronous linear motors as to thermal behavior. *International Journal of the Korean Society of Precision Engineering*, Vol. 2 No. 3, 2001, pp. 61-68.

- [89] J. J. Kim, Y. H. Jeong, D. W. Cho. Thermal behaviour of a machine tool equipped with linear motors. *International Journal of machine tools and structures*, Vol. 44, 2004, pp. 749-758.
- [90] C. S. Jang, J. Y. Kim, Y. J. Kim, J. O. Kim. Heat transfer analysis and simplified thermal resistance modeling of linear motor driven stages for SMT applications, *IEEE Transaction on Components and Packaging Technologies*, Vol. 26, No. 3, 2003, pp. 532-540.
- [91] N. Chayopitak, D. G. Taylor. Thermal Analysis of Linear Variable Reluctance Motor for Manufacturing automation applications. *IEEE Conference on Electric Machines and Drives*, 2005, p866-873. Georgia Institute of Technology, Atlanta, GA 30332-0250 USA.
- [92] J. R. Culham, M. M. Yovanovich, P. Teertstra, C. S. Wang, Simplified analytical models for forced convection heat transfer from cuboids of arbitrary shape, *Advances in Electronic Packaging*, Vol. 123, No. 3, 1999, pp. 105-114.
- [93] J. Fernandez-Seara, F. J. Uhia, J. Sieres, A. Campo, A general review of the Wilson plot method and its modifications to determine convection coefficients in heat exchange devices, *Applied Thermal Engineering*, Vol. 27, 2007, pp. 2745-2757.
- [94] A. Bousbaine, M. McCormick, W. F. Low, In-situ determination of thermal coefficients for electrical machines, *IEEE Transactions on Energy Conversion*, Vol. 10, No. 3, 1995, pp. 385-391.
- [95] Y. Yener, S. Kakac, *Heat Conduction* 4th edition, 2008, Taylor & Francis Group, New York
- [96] A. Dorfman, Z. Renner, Conjugate problems in convective heat transfer: review, *Mathematical Problems in Engineering*, Vol. 2009, 2009, pp. 1-27.
- [97] B.C. Sakiadis, Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equation for two-dimensional and axisymmetric flow, *AIChE Journal*, Vol. 7, 1961, pp. 26-28.
- [98] F.K. Tsou, E.M. Sparrow and R.J. Goldstein, Flow and Heat Transfer in the boundary Layer on a Continuous Moving Surface, *International Journal of Heat and Mass Transfer*, Vol. 10, 1967, pp. 219-235.

- [99] S. Roy Choudhury and Y. Jaluria, Analytical solution for the transient temperature distribution in a moving rod or plate of finite length with surface heat transfer, *International Journal of Heat and Mass Transfer*, Vol. 37, No. 8, 1994, pp. 1193-1205.
- [100] Talat A. Abdelhafez, Skin friction and heat transfer on a continuous flat surface in a parallel free stream, *International Journal of Heat and Mass Transfer*, Vol. 28, No. 6, 1985, pp.1234-1237.
- [101] Hedge I. Andersson, Jan B. Aarseth, Sakiadis flow with variable fluid properties revisited, *International Journal of Engineering Science*, Vol. 45, 2007, pp. 554-561.
- [102] A. Pantokratoras, Furthur results on the variable viscosity on flow and heat transfer to a continuous moving flat plate, *International Journal of Engineering Science*, Vol. 42, 2004, pp. 1891-1896.
- [103] I. Pop, R.S.R. Gorla, M. Rashi, The effect of variable viscosity on the flow and heat transfer to a continuous moving flat plate, *International Journal of Engineering Science*, Vol. 30, 1992, pp. 1-6.
- [104] Sami A. Al-Sanea, Mixed convection heat transfer along a continuously moving heated vertical plate with suction or injection, *International Journal of Heat and Mass Transfer*, Vol. 47, 2004, pp. 1445-1465.
- [105] M. Massoud, *Engineernig Thermofluids*, 2005, Springer, Berlin.
- [106] G. Dajaku, D. Gerling, An improved lumped parameter thermal model for electrical machines, *European Conference on Power Electronics and Applications*, 2005, Vol 10, pp. P.10. Institute for Electrical Drives, University of Federal Defense Munich.
- [107] H. Rouhani, J. Faiz, C. Lucas, Lumped thermal model for switched reluctance motor applied to mechanical design optimization, *Mathematical and Computer Modelling*, Vol. 45, 2007, pp. 625-638.
- [108] J. M. Ortiz-Rodriguez, A. R. Hefner, D. Berning, M. Velez-Reyes, M. Hernandez-Mora and J. Gonzalez, Lumped-parameter thermal modeling of an IPED using thermal components models, *IEEE Workshop Proceedings on Computers in Power Electronics*, 2004, pp. 43-48, Urbana.

- [109] C. Kral, T. G. Habetler, R. G. Harley, F. Pirker, G. Pascoli, H. Oberguggenberger and C. J. M. Fenz, Rotor temperature estimation of squirrel-cage induction motors by means of a combined scheme of parameter estimation and a thermal equivalent model, *IEEE Transactions on Industry Applications*, Vol. 40, No. 4, 2004, pp. 1049-1057.
- [110] P. H. Mellor, D. Roberts, D. R. Turner, Lumped parameter thermal model for electrical machines of TEFC design, *IEE Proceedings-B*, Vol. 138, No. 5, 1991, pp. 205-218.
- [111] D. Gerling, G. Dajaku, Novel lumped-parameter thermal model for electrical systems, Institute for Electrical Drives, 2005 European Conference on Power Electronics and Applications, 2005, pp. 1-10, Dresden.
- [112] C. Hager, Y. Tronel and W. Fichter, Electro-thermo-mechanical simulations of aluminum bond wires in IGBT-Packages, *International Conference on Modeling and Simulation of Microsystems 1998*, pp. 163-167, California.
- [113] S. B. Lee, T. G. Habetler and R. G. Harley, An evaluation of model-based stator resistance estimation for induction motor stator winding temperature monitoring, *IEEE Transactions on Energy Conversion*, Vol. 17, No. 1, 2002, pp. 217-222.
- [114] P. Gnacinski, Effect of unbalanced voltage on winding temperature, operational life and load carrying capacity of induction machine, *Energy Conversion and Management*, Vol. 49, 2008, pp. 761-770.
- [115] P. Gnacinski, Prediction of windings temperature rise induction motors supplied with distorted voltage, *Energy Conversion and Management*, Vol. 49, 2008, pp. 707-717.
- [116] R. M. Tallam, S. B. Lee, G. C. Stone, G. B. Kliman, J. Y. Yoo, T. G. Habetler and R. G. Harley, A survey of methods for detection of stator-related faults in induction machines, *IEEE Transactions on Industry Applications*, Vol. 43, No. 4, 2007, pp. 920-933.
- [117] K. R. Cho, J. H. Lang, S. D. Umans, Detection of broken rotor bars in induction motors using state and parameter estimation, *IEEE Transactions on Industry Applications*, Vol. 28, No. 3, 1992, pp. 702-709.

- [118] Y. Wu, H. W. Gao, Induction-motor stator and rotor winding temperature estimation using signal injection method, *IEEE Transactions on Industry Applications*, Vol. 42, No. 4, 2006, pp. 1038-1044.
- [119] D. A. Paice, Motor thermal protection by continuous monitoring of winding resistance, *IEEE Transactions on Industrial Electronics and Control Instrumentation*, Vol. IECI-27, No. 3, 1980, pp. 137-141.
- [120] F. Briz, M. W. Degner, J. M. Guerrero, and A. B. Diez, Temperature estimation in inverter-fed machines using high-frequency carrier signal injection, *IEEE Transactions on Industry Applications*, Vol. 44, No. 3, 2008, pp. 799-808.
- [121] M. G. Cooper, B. B. Mikic and M. M. Yovanovich, Thermal Contact conductance, *International Journal of Heat Mass Transfer*, Vol. 12, 1969, pp. 279-300.
- [122] J. Driesen, R. Belmans and K. Hameyer, Finite element modeling of thermal contact resistances and insulation layers in electrical machines, *IEEE Transactions on Industry Applications*, Vol. 37, No. 1, 2001, pp. 15-20.
- [123] A. HajI-Sheikh, J. V. Beck, D. Agonafer, Steady-state heat conduction in multi-layer bodies, *International Journal of Heat and Mass Transfer*, Vol. 46, 2003, pp. 2363-2379.
- [124] Eugene F. Adiutori, *The new heat transfer*, 1989, Ventuno Press, US.
- [125] Frank P. Incropera, David P. Dewitt, Theodore L. Bergman, Adrienne S. Lavine, *Fundamentals of Heat and Mass Transfer*, 2006, John Wiley & Sons, US.
- [126] A. Pantokratoras, The Blasius and Sakiadis flow with variable fluid properties, *Heat Mass Transfer*, Vol. 44, 2008, pp. 1187-1198.
- [127] A.J. Chamkha, H.S. Takhar, G. Nath, Effect of buoyancy forces on the flow and heat transfer over a continuous moving vertical or inclined surface, *International Journal of Thermal Science* Vol. 40, 2001, pp. 825-833.
- [128] C. Trevino, Heat transfer in a thin facing up horizontal strip with internal heat generation, *Heat Mass Transfer* Vol. 35, 1999, pp. 243-249.
- [129] F. Mendez, C. Trevino, The conjugate conduction-natural convection heat transfer along a thin vertical plate with non-uniform internal heat generation,

International Journal of Heat and Mass Transfer, Vol. 43, 2000, pp. 2739-2748.

- [130] O.D. Makinde, Similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition, Thermal Science, Vol. 15, 2011, pp. S137-S143.
- [131] S. Pretot, B. Zeghmami, G. Le Palec, Theoretical and experimental study of natural convection on a horizontal plate, Applied Thermal Engineering, Vol. 20, 2000, pp. 873-891.
- [132] N. Waowaew, P. Terdtoon, S. Maezawa, P. Kamonpet, W. Klongpanich, Correlation to predict heat transfer characteristics of a radially rotating heat pipe at vertical position, Applied Thermal Engineering, Vol. 23, 2003, pp. 1019-1032.
- [133] T.J. Martin, G.S. Dulikravich, Inverse determination of boundary conditions and sources in steady heat conduction with heat generation, Journal of Heat Transfer, Vol. 118, Iss. 3, 1996, pp. 546-554.
- [134] M. Slodicka, D. Lesnic, T.T.M. Onyango, Determination of a time-dependent heat transfer coefficient in a nonlinear inverse heat conduction problem, Inverse Problems in Science and Engineering, Vol. 18, No. 1, 2010, pp. 65-81.
- [135] M.F. Valle, M.J. Colaco, F.S. Neto, Estimation of the heat transfer coefficient by means of the method of fundamental solutions, Inverse Problems in Science and Engineering, Vol. 16, No. 6, 2008, pp. 777-795.
- [136] H. Sadat, A general lumped model for transient conduction in one-dimensional geometries, Applied Thermal Engineering, Vol. 25, 2005, pp. 567-576.
- [137] H. Sadat, A second order model for transient heat conduction in a slab with convective boundary conditions, Applied Thermal Engineering, Vol. 26, 2006, pp. 962-965.
- [138] Cengel, Yunus A., Heat and mass transfer: fundamentals and applications, 2010, McGraw-Hill, Boston.
- [139] C.V. Madhusudana, Thermal Contact Conductance, Mechanical Engineering Series, Springer-Verlag, New York Inc, 1996, pp. 45-61.

- [140] C.V. Madhusudana, Contact Heat Transfer-The Last Decade, American Institute of Aeronautics and Astronautics Journal, Vol. 24, No. 3, pp. 510-521.
- [141] Williams A., Heat Flow Through Single Spots of Metallic Contacts of Simple Shapes, American Institute of Aeronautics and Astronautics Journal, July 1974, pp. 674-692.
- [142] Veziroglu T.N., Statistical Study of Thermal Contact Conductance, Univ of Miami, Coral Gables, FL, NASA Grant NG 10-007-010 Report, June 1972.
- [143] M.A. Lambert, L.S. Fletcher, Review of Models for Thermal Contact Conductance of Metals, Journal of Thermophysics and Heat Transfer, Vol. 11, No. 2, April-June 1997, pp.129-140.
- [144] I. Shai, M. Santo, Heat Transfer with Contact Resistance, International Journal of Heat Mass Transfer, Vol. 25, No. 4, pp. 465-470, 1982.
- [145] Y.C. Yang, Simultaneously Estimating the Contact Heat and Mass Transfer Coefficients in a Double-layer Hollow Cylinder with Interface Resistance, Applied Thermal Engineering, 27 (2007), pp. 501-508.
- [146] Syed M.S. Wahid, Numerical Analysis of Heat Flow in Contact Heat Transfer, International Journal of Heat and Mass Transfer, Vol. 46, 2003, pp. 4751-4754.
- [147] S.L. Lee, C.R. Ou, Gap Formation and Interfacial Heat Transfer Between Thermoelastic Bodies in Imperfect Contact, Journal of Heat Transfer, Vol. 123 , April 2001, pp. 205-212.
- [148] C.V. Madhusudana, Heat Flow Through Conical Constrictions, American Institute of Aeronautics and Astronautics Journal, Vol. 18, No. 10, Oct 1980, pp. 1261-1262.
- [149] D. Tang, D. Li, Y. Peng, Z. Du, A New Approach in Evaluation of Thermal Contact Conductance of Tube-Fin Heat Exchange, Applied Thermal Engineering, Vol. 30, 2010, pp. 1991-1996.
- [150] J. Gill, E. Divo, A.J. Kassab, Estimating Thermal Contact Resistance Using Sensitivity Analysis and Regularization, Engineering Analysis with Boundary Elements, Vol. 33, 2009, pp. 54-62.

- [151] S.M.S. Wahid, C.V. Madhusudana, E. Leonardi, Solid Spot Conductance at Low Contact Pressure, *Experimental Thermal and Fluid Sciences*, Vol. 28, 2004, pp.489-494.
- [152] Syed M.S. Wahid, C.V. Madhusudana, Gap Conductance in Contact Heat Transfer, *International Journal of Heat and Mass Transfer*, Vol. 43, 2000, pp. 4483-4487.
- [153] V. Singhal, P.J. Litke, A.F. Black, S.V. Garimella, An Experimentally Validated Thermo-mechanical Model for the Prediction of Thermal Contact Conductance, *International Journal of Heat and Mass Transfer*, Vol. 48, 2005, pp.5546-5459.
- [154] C.L. Yeh, Y.F. Chen, C.Y. Wen, K.T. Li, Measurement of Thermal Contact Resistance of Aluminum Honeycombs, *Experimental Thermal and Fluid Sciences*, Vol. 27, 2003, pp. 271-281.
- [155] W.W. Cheng, C.V. Madhusudana, Effect of Electroplating on the Thermal Conductance of Fin-Tube Interface, *Applied Thermal Engineering*, Vol. 26, 2006, pp. 2119-2131.
- [156] Y.Z. Li, C.V. Madhusudana, E. Leonardi, On the Enhancement of the Thermal Contact Conductance: Effect of Loading History, *Journal of Heat Transfer*, Vol. 22, Feb 2000, pp. 46-49.
- [157] S. Woodland, A.D. Crocombe, J.W. Chew, S.J. Mills, A New Method for Measuring Thermal Contact Conductance- Experimental Technique and Results, *Journal of Engineering for Gas and Turbines and Power*, Vol. 133, Jul 2011, pp. 071601-1-8.
- [158] C.V. Madhusudana, Accuracy in Thermal Contact Conductance Experiments- The Effect of Heat Losses to the Surroundings, *International Communication of Heat Mass Transfer*, 2000, Vol. 27, No. 6, pp. 877-891.
- [159] C.L. Yeh, C.T. Lin, Thermal Contact Resistance Correlation For Metals Across Bolted Joints, *International Communication of Heat Mass Transfer*, 2001, Vol. 30, No. 7, pp. 987-996.
- [160] C.L. Yeh C.Y. Wen, Y.F. Chen, S.H. Yeh, C.H. Wu, An Experimental Investigation of Thermal Contact Conductance Across Bolted Joints, *Experimental Thermal and Fluid Science* Vol. 25, 2001, pp. 349-357.

- [161] D.S. Huang, J.S. Shih, H.C. Hsia, M.T. Lin, Three phase linear motor heat transfer analysis using finite-element method, *Heat Transfer Engineering*, 31(7), 2010, pp. 617-624.
- [162] W.H. McAdams, *Heat Transmissions*. Third Edition, McGraw-Hill, New York, 1954.
- [163] C.H. Huang, S.P. Wang, A three-dimensional inverse heat conduction problem in estimating surface heat flux by conjugate gradient method, , *International Journal of Heat and Mass Transfer* 42 (1999), pp. 3387-3403.
- [164] C.L. Fan, F.R. Sun, L. Yang, A simple method for inverse estimation of surface temperature distribution on a flat plate, *Inverse Problems in Science and Engineering* Vol. 17, No 7, October 2009, pp. 885-899.
- [165] K.Y. Kim, B.S. Kim, H.C. Kim, M.C. Kim, Inverse estimation of time-dependent boundary heat flux with an adaptive input estimator, *International Communication of Heat Mass Transfer*, Vol. 30 (2003), No. 4, pp. 475-484.
- [166] C.H. Huang, H.C. Lo, A three-dimensional inverse problem in estimating the internal heat flux of housing for high speed motors, *Applied Thermal Engineering* 26 (2006) pp. 1515-1529.
- [167] M. Matsuo, T. Yasui, T. Inamura, M. Matsumura, High-speed test of thermal effects for a machine-tool structure based on modal analysis, *Precision Engineering*, Vol. 8, Iss 2, April 1986, pp. 72-78.
- [168] N. Gnanasekaran, C. Balaji, A Bayesian approach for the simultaneous estimation of surface heat transfer coefficient and thermal conductivity from steady state experiments on fins, *International Journal of Heat and Mass Transfer* 54 (2011) pp. 3060-3068.
- [169] E. Radziemska, W. Lewandowski, Experimental verification of natural convective heat transfer phenomenon from isothermal cuboids, *Experimental Thermal and Fluid Science* 32 (2008), pp.1034-1038.
- [170] E. Radziemska, W. Lewandowski, Natural convective heat transfer from isothermal cuboids, *International Journal of Heat and Mass Transfer* 46 (2003) pp. 2169-2178.

- [171] S.W. Churchill, Free convection around immersed bodies, Heat Exchanger Design Handbook, Hemisphere, Washington DC, 1983.
- [172] S.W. Churchill, H.H.S. Chu Correlating equations for laminar and turbulent free convection from a vertical plate, International Journal of Heat and Mass Transfer 18 (1975) pp. 1323-1329.
- [173] C.O. Popiel, J. Wojtkowiak, Experiments on free convection heat transfer from side walls of a vertical square cylinder in air, Experimental Thermal and Fluid Science 29 (2004), pp. 1-8.
- [174] J.R. Culham, M.M. Yovanovich, P. Teertstra, C.-S. Wang, G. Refai-Ahmed, Ra-Min Tain, Simplified analytical models for forced convection heat transfer from cuboids of arbitrary shape, Journal of Electronic Packaging ASME, Vol. 23, September 2001, pp. 182-188.
- [175] A.S. Dorfman, Conjugate problems in convective heat transfer, CRC Press, Boca Raton FL, 2010.

Appendix A

A.1.1 Review of Conjugate Heat Transfer Formulation and Solution

A detail summary of the methods of formulating and solving of the conjugate heat transfer problem from [175], is reviewed here. This section is split into the earlier approximate analytical methods with and without time factor, exact formulations of laminar boundary flow, modern analytical methods, modern numerical methods and the conjugate criterion.

A.1.2 Earlier Approximate Analytical Methods

In many earlier applications, there flow problems in heat transfer are often having high Reynolds (Re) and Peclet (Pe) numbers. This causes the viscosity and conductivity to have a heavier effect only in thinner boundary layers. The effect causes the Navier-Stokes and energy equations to be simplified. For laminar steady-state flow of an incompressible fluid with constant thermophysical properties, they are the [106];

$$\text{Continuity equation:} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (\text{A.1})$$

$$\text{Momentum equation:} \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - U \frac{dU}{dx} - \nu \frac{\partial^2 u}{\partial y^2} = 0, \quad (\text{A.2})$$

$$\text{Energy equation:} \quad u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} - \alpha \frac{\partial^2 T}{\partial y^2} - \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 = 0 \quad (\text{A.3})$$

with the boundary conditions,

At the solid-fluid interface with $y = 0$,

$$\text{we have;} \quad u = v = 0 \text{ and } T = T_w(x) \quad (\text{A.4})$$

Far away from the solid-fluid interface $y \rightarrow \infty$,

$$\text{we have; } u \rightarrow U \text{ and } T \rightarrow T_{\infty} \quad (\text{A.5})$$

using variable x , stream function ψ , and $Z = U^2 - u^2$, the boundary layer equation can be simplified to about a conduction equation,

$$\text{Continuity equation: } u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (\text{A.6})$$

$$\text{Momentum equation: } \frac{\partial Z}{\partial x} - \nu u \frac{\partial^2 Z}{\partial \psi^2} = 0, \quad (\text{A.7})$$

$$\text{Energy equation: } \frac{\partial T}{\partial x} + \alpha \frac{\partial}{\partial \psi} \left(u \frac{\partial T}{\partial \psi} \right) - \frac{\nu}{c_p} u \left(\frac{\partial u}{\partial \psi} \right)^2 = 0 \quad (\text{A.8})$$

with corresponding boundary conditions,

$$\text{for } \psi = 0, \quad Z = U^2(x) \text{ and } T = T(x) \quad (\text{A.9})$$

$$\text{for } \psi \rightarrow \infty, \quad Z \rightarrow 0 \text{ and } T \rightarrow T_{\infty} \quad (\text{A.10})$$

the laminar velocity flow profile near the solid-fluid interface is a analytic function,

$$\text{Velocity profile: } u \sim c_1 y + c_2 y^2 + \dots \quad (\text{A.11})$$

$$\text{Stream function: } \psi = \int_0^y u dy \sim \frac{c_1}{2} y^2 + \frac{c_2}{3} y^3 + \dots \quad (\text{A.12})$$

$$\text{So when } y \sim \psi^{1/2}, \text{ we have } \frac{\partial y}{\partial \psi} \sim \psi^{-1/2} \quad (\text{A.13})$$

$$\text{and when, } \psi \rightarrow 0, \text{ we have } \frac{\partial y}{\partial \psi} \rightarrow \infty \quad (\text{A.14})$$

A.1.3 Self-Similar solution

When a solid have a wedge shape with opening $\pi\beta$, the fluid velocity profile, as well as the temperature profile, follows the power law distribution,

Velocity profile: $U(x) = Cx^m$, where $m = \beta/(2 - \beta)$ (A.15)

Temperature Profile: $T_w(x) - T_\infty = C_1 x^{m_1}$ (A.16)

In self-similar, the velocity and temperature profile are the same. So with the self-similar variables:

$$\frac{u}{U} = \varphi'(\eta), \quad \frac{T - T_w}{T_\infty - T_w} = \theta(\eta), \quad \eta = y \sqrt{\frac{(m+1)Cx^{m-1}}{2\nu}}, \quad \varphi = \psi \sqrt{\frac{m+1}{2\nu Cx^{m+1}}} \quad (\text{A.17})$$

Using (A.17), the partial differential equations (A.1) ~ (A.3) reduces to ordinary differential equations:

$$\varphi'''' + \varphi\varphi'' + \beta(1 - \varphi'^2) = 0, \quad \theta'' + \text{Pr} \varphi\theta' + \frac{2m_1}{m+1} \text{Pr} \varphi'(1 - \theta) = 0 \quad (\text{A.18})$$

the friction coefficient, Nusselt number and heat flux can be evaluated with numerically integrating of (A.18):

$$C_f = \frac{\sqrt{2(m+1)}\varphi''(0)}{\text{Re}_x^{1/2}}, \quad Nu_x = \sqrt{\frac{m+1}{2}}\theta'(0)\sqrt{\text{Re}_x}, \quad q_w = C_1 \lambda \sqrt{\frac{(m+1)}{2\nu}}\theta'(0)x^{m_1 + \frac{m-1}{2}} \quad (\text{A.19})$$

A.1.4 Power Series Solution

The self-similar solution is only usable in a few applications. In most cases, the velocity and temperature profile are described by different functions. Blasius [2] came out with the assumption that the velocity and temperature profile around a symmetrical solid can be described by power series:

$$U(x) = u_1 x + u_3 x^3 + u_5 x^5 + \dots, \quad \theta_w(x) = \theta_{w0} + \theta_{w2} x^2 + \theta_{w4} x^4 + \dots \quad (\text{A.20})$$

The boundary layer solutions are also in power series,

$$\begin{aligned}\psi(x, y) &= \psi_1(y)x + \psi_3(y)x^3 + \psi_5(y)x^5 + \dots \\ T - T_\infty &= \theta(x, y) = \theta_0(y) + \theta_2(y)x^2 + \theta_4(y)x^4 + \dots\end{aligned}\quad (\text{A.21})$$

For asymmetrical flow, Gortler [2], propose the power law variable as:

$$\Phi = \frac{1}{\nu} \int_0^x U(\zeta) d\zeta, \quad \eta = \frac{yU}{\nu\sqrt{2\Phi}}, \quad \varphi = \frac{\psi}{\nu\sqrt{2\Phi}}, \quad \theta = \frac{T - T_w}{T_\infty - T_w} \quad (\text{A.22})$$

With the variables (22), (1)~(3) becomes,

$$\frac{\partial^3 \varphi}{\partial \eta^3} + \varphi \frac{\partial^2 \varphi}{\partial \eta^2} + \beta(\Phi) \left[1 - \left(\frac{\partial \varphi}{\partial \eta} \right)^2 \right] = 2\Phi \left[\frac{\partial^2 \varphi}{\partial \Phi \partial \eta} \frac{\partial \varphi}{\partial \eta} - \frac{\partial \varphi}{\partial \Phi} \frac{\partial^2 \varphi}{\partial \eta^2} \right] \quad (\text{A.23})$$

$$\frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial \eta^2} + \varphi \frac{\partial \theta}{\partial \eta} + 2\Phi \beta_t(\Phi) \frac{\partial \varphi}{\partial \eta} (1 - \theta) = 2\Phi \left[\frac{\partial \varphi}{\partial \eta} \frac{\partial \theta}{\partial \Phi} - \frac{\partial \varphi}{\partial \Phi} \frac{\partial \theta}{\partial \eta} \right] \quad (\text{A.24})$$

$$\text{where } \beta(\Phi) = \frac{2}{U^2} \frac{dU}{dx} \int_0^x U(\zeta) d\zeta \quad \beta_t(\Phi) = \frac{2\nu}{U\theta_w} \frac{d\theta_w}{dx} \quad (\text{A.25})$$

When β and β_t are constants, the Gortler equations (A.23) and (A.24) becomes self-similar equations (18). These equations are used to study the free and forced convection heat transfer from nonisothermal surfaces. The Gortler solution converges faster than Blasius power series solution, but the convergence are not always good.

A.1.5 Integral Methods

Integral methods are sometime chosen over differential equations due to its relative simplicity. However, the conservation laws are not abided in each point of flow, but only on the average for each cross section of the boundary layer. The equation (2.1) is rewritten in integral form as;

$$(1)*U: \quad \frac{\partial(Uu)}{\partial x} + \frac{\partial(Uv)}{\partial y} - u \frac{dU}{dx} = 0 \quad (\text{A.26})$$

$$\Sigma(1)^*u: \quad \frac{\partial(u^2)}{\partial x} + \frac{\partial(uv)}{\partial y} - U \frac{dU}{dx} - v \frac{\partial^2 u}{\partial y^2} = 0 \quad (A.27)$$

$$\int[(26)-(27)]: \quad \frac{d}{dx}(U\delta^{**}) + U \frac{dU}{dx} \delta^* = \frac{\tau_w}{\rho} \quad (A.28)$$

$$\int[(1)-(3)]: \quad \frac{d}{dx}(U\theta_w\delta_t^{**}) = \frac{q_w}{\rho c_p} \quad (A.29)$$

$$\text{where } \delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy, \quad \delta^{**} = \int_0^\infty \frac{u}{U} \left(1 - \frac{u}{U}\right) dy, \quad \delta_t^{**} = \int_0^\infty \frac{u}{U} \frac{T - T_\infty}{T_w - T_\infty} dy \quad (A.30)$$

The integral boundary layer equations satisfy the conservation on an average basis across the flow, so there is no information on the velocity and temperature distribution on the boundary layer.

A.1.6 Method of Superposition

The principle of superposition is applicable for the linear thermal boundary layer equation. The principle is generic for any nonisothermal wall. For nonuniform heat for wall, assuming an initial unheated length of $x = \xi$, and from then, a constant temperature of T_w is applied, the thermal boundary layer equation,

$$T - T_\infty = \int_0^x f(x, \xi) \frac{dT}{d\xi} d\xi + \sum_{k=1}^i f(x, \xi_k) \Delta T_{wk} \quad (A.31)$$

The Heat flux is obtained using the influence function $f(x, \xi)$, and differentiating (A.31) with respect to y to get,

$$q_w = h_s \left[\int_0^x f(x, \xi) \frac{dT}{d\xi} d\xi + \sum_{k=1}^i f(x, \xi_k) \Delta T_{wk} \right] \quad (A.32)$$

The difficult part is to obtain the influence function with affect the coefficient of heat transfer on the unheated part of the wall. h_s can be obtained by available methods.

A.1.7 Series of Shape Parameters Solution

The series of shape parameter of the first type,

$$f_k = \frac{x^k}{U} \frac{d^k U}{dx^k}, \quad f_{tk} = \frac{x^k}{\theta_w} \frac{d^k \theta_w}{dx^k} \quad (\text{A.33})$$

Equations (A.33) are used to obtain the solution of the velocity boundary layer in terms of stream function,

$$\frac{\psi}{\sqrt{\nu x U}} = \psi_0(\eta) + \psi_1(\eta)f_1 + \psi_2(\eta)f_2 + \psi_{11}(\eta)f_1^2 + \psi_3(\eta)f_3 + \psi_{12}(\eta)f_1f_2 + \dots \quad (\text{A.34})$$

and thermal boundary layer,

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} = T_0(\eta) + T_1^{(d)}(\eta)f_1 + T_1^{(t)}(\eta)f_{t1} + T_2^{(d)}(\eta)f_2 + T_2^{(t)}(\eta)f_{t2} + \dots \quad (\text{A.35})$$

To include the thickness characteristic of the boundary layer, the shape parameters of the second type are used,

$$f_k = \left(\frac{\delta^{**2}}{\nu} \right)^k U^{k-1} \frac{d^k U}{dx^k}, \quad f_{tk} = \left(\frac{\delta_t^{**2}}{\nu} \right) \frac{U^k}{\theta_w} \frac{d^k \theta_w}{dx^k} \quad (\text{A.36})$$

From (A.36), it can be seen that the thickness layer δ^{**} and δ_t^{**} have to be evaluated before the shape parameters can be calculated. The frequent problem with shape parameters is the convergence of the series.

A.1.8 Solutions of Boundary Layer with any Nonisothermal Surface

Conjugate heat transfer consist of thermal interaction between a solid body and a fluid and the there is a interface surface between them. This interaction will cause a temperature distribution on this interface. The following are theories to evaluate the flow velocity and

temperature profile assuming the interface temperature distribution is known. However, this interface temperature distribution is always an unknown.

A.1.9 The Exact Solution of the Thermal Boundary Layer Equation for an Arbitrary Surface Temperature Distribution

Using the Gortler variables Φ , and the boundary conditions are in their Prandtl-Mises form, the thermal boundary layer equation for a power law free stream velocity,

$$2\Phi \frac{\partial \theta}{\partial \Phi} - \varphi \frac{\partial \theta}{\partial \varphi} - \frac{1}{\text{Pr}} \frac{\partial}{\partial \varphi} \left(\frac{u}{U} \frac{\partial \theta}{\partial \varphi} \right) = \frac{U^2}{c_p} \frac{u}{U} \left[\frac{\partial}{\partial \varphi} \left(\frac{u}{U} \right) \right]^2 \quad (\text{A.37})$$

$$\text{For when } \varphi = 0, \theta = \theta_w(\Phi) \quad \text{and when } \varphi \rightarrow \infty, \theta = 0 \quad (\text{A.38})$$

$$\text{Where } \Phi = \frac{1}{v} \int_0^x U d\zeta, \quad \varphi = \frac{\psi}{v\sqrt{2\Phi}}, \quad \text{and} \quad \theta = T - T_\infty \quad (\text{A.39})$$

The solution is expected to be in the form,

$$\theta = \sum_{k=0}^{\infty} G_k(\varphi) \Phi^k \frac{d^k \theta_w}{d\Phi^k} + G_d(\varphi) \frac{U^2}{c_p} \quad (\text{A.40})$$

Substituting (A.37) into (A.34) will reduce the system to a set of ordinary equations,

$$(1/\text{Pr})[\omega(\varphi, \beta)G'_k] + \varphi G'_k - 2kG_k = 2G_{k-1} \quad (k = 0, 1, 2, \dots) \quad (\text{A.41})$$

$$\text{Where } G_k = \sum_{i=0}^{k-1} \frac{(-1)^{k+i}}{(k-i)!} F_i, \quad F_i = \frac{1}{2^i i! (2i-1)!} \exp(-\xi^2) \frac{d^{2i}}{d\xi^{2i}} \left[\exp(\xi^2) \left(1 - \frac{2}{\sqrt{\pi}} \int_0^\xi \exp(-\zeta^2) d\zeta \right) \right]$$

$$\text{And } \xi = \varphi \left(\frac{\text{Pr}}{2} \right)^{1/2}$$

The heat flux at the interface can then be evaluated,

$$q_w = h_* \left[\theta_w + \sum_{k=1}^{\infty} g_k \Phi^k \frac{d^k \theta_w}{d\Phi^k} - g_d \frac{U^2}{c_p} \right] \quad (\text{A.42})$$

A.1.10 Generalization for an Arbitrary Velocity Gradient in a Free Stream Flow

The solution is similar to (A.38) but the pressure gradient is ignored. The heat flux can be evaluated by

$$q_w = h_* \left[\theta_w + \sum_{k=1}^{\infty} g_k \Phi^k \frac{d^k \theta_w}{d\Phi^k} \right], \quad q_w = h_* \left[\theta_w + \sum_{k=1}^{\infty} g_k x^k \frac{d^k \theta_w}{dx^k} \right] \quad (\text{A.43})$$

Using self-similar solution $\omega(\varphi, \beta)$ for the velocity ratio u/U , integrating and subtracting between (A.43), we have the boundary layer free stream velocity profile,

$$\int_0^{\infty} \frac{\partial}{\partial \varphi} \left\{ \left[\frac{u}{U} - \omega(\varphi, \beta) \right] \frac{\partial \theta}{\partial \varphi} \right\} d\varphi = \left[\frac{u}{U} - \omega(\varphi, \beta) \right] \frac{\partial \theta}{\partial \varphi} \Big|_0^{\infty} = 0 \quad (\text{A.44})$$

A.1.11 General Form of the Influence Function of the Unheated Zone: Convergence of the Series

Using the superposition principle, the heat flux for a temperature head distribution without breaks for gradientless flow,

$$q_w = \left[\theta_w(0) + \int_0^x f(\xi/x) \frac{d\theta_w}{d\xi} d\xi \right] \quad (\text{A.45})$$

Using the influence function, $f(\xi/x) = [1 - (\xi/x)^{c_1}]^{c_2}$, and after repeated integration by parts,

$$q_w = h_* \left[\Phi \frac{d\theta_w}{d\Phi} f_1(1) - \Phi^2 \frac{d^2\theta_w}{d\Phi^2} f_2(1) + \dots (-1)^k \Phi^{k+1} \frac{d^k\theta_w}{d\Phi^k} f_k(1) + \dots + \theta_w(0) \right. \\ \left. + \frac{\Phi}{1!} \frac{d\theta_w}{d\Phi} \Big|_{\Phi=0} + \dots \frac{\Phi^k}{k!} \frac{d^k\theta_w}{d\Phi^k} \Big|_{\Phi=0} + (-1)^k \Phi^k \int_0^\Phi \frac{d^{k+1}\theta_w}{d\Phi^{k+1}} f_k(\xi/\Phi) d\xi \right] \quad (A.46)$$

$$\text{Where} \quad f_k(\xi) = \int_0^\xi d\zeta \int_0^\zeta d\zeta \dots \int_0^\zeta f(\zeta) d\zeta + \sum_{n=1}^{n=k} \frac{(-1)^n \xi^{k-n}}{n!(k-n)!} \quad (A.47)$$

And the error for the convergence can be calculated by,

$$J_k = \Phi^k \int_0^\Phi f_k \left(\frac{\xi}{\Phi} \right) \frac{d^{k+1}\theta_w}{d\xi^{k+1}} d\xi \quad (A.48)$$

A.1.12 The Exact Solution of the Thermal Boundary Layer Equation for an Arbitrary Surface Heat Flux Distribution

Equations (A.43) are solved for the heat flux to get,

$$\theta_w + \sum_{k=1}^{\infty} g_k \Phi^k \frac{d^k\theta_w}{d\Phi^k} = \frac{q_w}{h_*} = \theta_{w*}(\Phi) \quad (A.49)$$

The solution for the thermal boundary layer is,

$$\theta_w = \frac{C_1}{\Gamma(C_2)\Gamma(1-C_2)} \int_0^\Phi \left[1 - \left(\frac{\xi}{\Phi} \right)^{C_1} \right]^{C_2-1} \left(\frac{\xi}{\Phi} \right)^{C_1(1-C_2)} \frac{q_w(\xi)}{h_*(\xi)\xi} d\xi \quad (A.50)$$

A.1.13 The Exact Solution of an Unsteady Thermal Boundary Layer Equation for Arbitrary Surface Temperature Distribution

The unsteady boundary layer equation,

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (A.51)$$

With dimensionless variables (A.39) becomes,

$$\left[\frac{2\Phi v}{Ux} \left(\frac{U}{u} - z \right) + \frac{2\Phi v}{U^2} \frac{dU}{dx} z \right] \frac{\partial \theta}{\partial z} + 2\Phi \frac{\partial \theta}{\partial \Phi} - \phi \frac{\partial \theta}{\partial \phi} - \frac{1}{\text{Pr}} \frac{\partial}{\partial \phi} \left(\frac{u}{U} \frac{\partial \theta}{\partial \phi} \right) = 0 \quad (\text{A.52})$$

The boundary layer solution is,

$$\begin{aligned} \theta = \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} G_{ki}(z, \phi) \frac{x^{k+i}}{U^i} \frac{\partial^{k+i} \theta_w}{\partial x^k \partial t^i} = G_{00} \theta_w + G_{10} x \frac{\partial \theta_w}{\partial x} + G_{01} \frac{x}{U} \frac{\partial \theta_w}{\partial t} \\ + G_{20} x^2 \frac{\partial^2 \theta_w}{\partial x^2} + G_{02} \left(\frac{x}{U} \right)^2 \frac{\partial^2 \theta_w}{\partial x^2} + G_{11} \frac{x^2}{U} \frac{\partial^2 \theta_w}{\partial x \partial t} + \dots \end{aligned} \quad (\text{A.53})$$

A.1.14 The Exact Solution of a Thermal Boundary Layer Equation for a Surface with Arbitrary Temperature in a Compressible Flow

Using the Dorodnizin's and Gortler's variables,

$$\eta = \sqrt{\frac{U}{2v_{\infty} Cx}} \int_0^y \frac{\rho}{\rho_{\infty}} d\xi, \quad \phi = \frac{\psi}{\rho_{\infty} \sqrt{2Cv_{\infty} x U}} \quad (\text{A.54})$$

The thermal boundary equation,

$$2x \frac{\partial i}{\partial x} - \phi \frac{\partial i}{\partial \phi} - \frac{1}{\text{Pr}} \frac{\partial}{\partial \phi} \left(\frac{u}{U} \frac{\partial i}{\partial \phi} \right) - (k-1) M_{\infty}^2 \frac{u}{U} \left[\frac{\partial}{\partial \phi} \left(\frac{u}{U} \right) \right]^2 = 0 \quad (\text{A.55})$$

And the solution can be expressed as,

$$i = \sum_{k=0}^{\infty} G_k(\phi) x \frac{\partial^k i_w}{\partial x^k} + G_d(\phi) (k-1) M_{\infty}^2 \quad (\text{A.56})$$

A.1.15 The Exact Solution of a Thermal Boundary Layer Equation for a Moving Continuous Surface with Arbitrary Temperature Distribution

$$T_{\infty}^* = \frac{(\text{Re}/u_s^*) + \int_{u_s^*}^{u^{**}} (u_s^* - u^{**}) \Gamma^*(dy^*/du^{**}) du^{**}}{\int_0^{u_s^*} (u_s^* - u^{**}) (dy^*/du^{**}) du^{**}} \quad (\text{A.57})$$

The evaluated coefficient of friction and heat transfer for a moving plate is 34% and 20% higher than a stationary plate in Blasius flow.

A.2 Analytical Methods for Solving Conjugate Convective Heat Transfer Problems

A.2.1 A biot number as the Criterion of the Conjugate Heat Transfer Rate

The biot number, which is the ratio of thermal resistance of the solid body and moving fluid, is taken as a measurement of the intensity of conjugate heat transfer.

$$Bi_* = \frac{h_* \Delta_{fluid}}{\lambda_{solid}} \quad (A.58)$$

With the biot number, the effect of nonisothermicity of the solid surface temperature on the accuracy of the solution can be evaluate by,

$$g_1 \Phi \frac{d\theta_{solid}}{d\Phi} = \frac{1}{Bi_*} \frac{\partial T}{\partial (y/\Delta_{fluid})} \quad (A.59)$$

The general boundary condition of the solid wall of the third kind, which is in the form of a series of successive derivatives, causes errors to the final solution that are computable from the truncated derivatives.

A.2.2 Reduction of a Conjugate Convective Heat Transfer Problem to an Equivalent Heat Conduction Problem

The new approach to solve a conjugate problem is to solve conduction part of the solid body instead of the fluid flow. The derived equation for the conduction heat transfer with biot number and dimensionless variable is,

$$\theta_1 + \sum_{k=1}^{\infty} \bar{g}_{k1} Bi_i^k \frac{d^k \theta_1}{dBi_*^k} = \theta_2 + \sum_{k=1}^{\infty} \bar{g}_{k2} Bi_i^k \frac{d^k \theta_2}{dBi_*^k} \quad (A.60)$$

$$\text{And} \quad Bi_{*2}(1 - \theta_1) + Bi_{*1}(1 - \theta_2) = Bi_{*1} Bi_{*2} \left(\theta_1 + \sum_{k=1}^{\infty} \bar{g}_{k1} Bi_*^k \frac{d^k \theta_1}{dBi_*^k} \right) \quad (A.61)$$

$$\text{Where} \quad Bi_K = \frac{Bi_{*1}(1 - \theta_1) + Bi_{*2}(1 - \theta_2)}{Bi_{*1} + Bi_{*2}} \quad (A.62)$$

$$\text{And} \quad \bar{g}_k = \sum_{n=k}^{\infty} g_n \sum_{j=1}^k \frac{(-1)^{k-j}}{j!(k-j)!} \left(-\frac{1}{2} j \right) \left(-\frac{1}{2} j - 1 \right) \dots \left(-\frac{1}{2} j - n + 1 \right) \quad (A.63)$$

A.2.3 The Singularities of Temperature boundary at the Solid-Fluid Interface of Thermally thin Body

The wall temperature of a thermally thin plate at the origin is not an analytic function of distance from the origin. It is in the form of series in integer power of $x^{1/2}$. In general, the temperature of the surface is in terms of $x^{1/s}$, where s is derived from the isothermal heat transfer coefficient,

$$h_* = h_{*L} (x/L)^{-r/s} \quad (A.64)$$

For assuming a thermally thin body, the following equalities have to be adhered to,

$$\frac{1}{T_{fluid}} \left| \frac{Bi_1 \theta_{w1}}{3} - \frac{Bi_2 \theta_{w2}}{6} \right| \ll 1, \quad \frac{1}{T_{fluid}} \left| \frac{Bi_2 \theta_{w2}}{3} - \frac{Bi_1 \theta_{w1}}{6} \right| \ll 1, \quad \text{and} \quad \frac{Bi \theta_w}{6T_{fluid}} \ll 1 \quad (A.65)$$

The errors resulting from assuming a thermally thin body can be calculated from,

$$T/T_{fluid} = 1 - (\omega_1/3) + (\omega_2/6) + \omega_1(y/\Delta) - (\omega_1 + \omega_2)(y/\Delta)^2/2 \quad (A.66)$$

Where $\omega = q_w \Delta / \lambda_w T_{fluid} = Bi(\theta_w / T_{fluid})$ (A.67)

A.2.3 Analytical Methods for solving the Conjugate Problems in their Equivalent Heat Conduction Form

Using universal functions to solve the heat conduction form, the general solution in the form

$$\vartheta = C_1 x F(\alpha, \beta, \gamma, D_2 x^{2-r/s}) + C_2 F(\alpha - \gamma + 1, \beta - \gamma + 1, 2 - \gamma, D_2 x^{2-r/s}) + \sigma_{Bi} + \vartheta_q \quad (A.68)$$

Where $\alpha + \beta = \frac{D_1 + D_2}{D_2(2-r/s)}, \quad \alpha\beta = \frac{D_1 + D_0}{D_2(2-r/s)}, \quad \sigma_{Bi} = \frac{Bi_{*L2}}{Bi_{*L1} + Bi_{*L2}}$ (A.69)

Using integral transform method, the quasi-steady approach for the heat flux is,

$$q_w = \int_0^{\bar{x}} G(\bar{x} - \xi) \frac{\partial \theta}{\partial \xi}(\xi, \tau) d\xi \quad (A.70)$$

Where $\bar{x} = \alpha x / R^2 u_m$ and $\tau = t - x / u_m$

The kernel $G(\bar{x})$ found with the actual velocity profile is,

$$G(\bar{x}) = \frac{\lambda}{R} \sum_{n=0}^{\infty} G_n \exp\left(-\frac{\mu_n^2 \bar{x}}{8}\right) \quad (A.71)$$

Using Asymptotic series in eigenfunctions solutions with fourier transform, taking governing mathematical model for conjugate heat transfer,

$$cy \frac{\partial T}{\partial x} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad \frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} = 0 \quad (A.72)$$

Solving for wall temperature T_w , after fourier transform and using Airy functions,

$$\tilde{T}_w = \frac{\bar{q}(s)LAi}{\lambda(isLPe)^{1/3} \frac{Ai'[(iPe)^{-2/3}(sL)^{4/3}]}{Ai[(iPe)^{-2/3}(sL)^{4/3}]} - sL\lambda_w \tanh(s\Delta)} \quad (A.73)$$

There is no known method to inverse fourier transform (A.73), so an asymptotic solution for large x was investigated to give,

$$T_w(x) \cong \frac{3^{1/6} Q \Gamma(1/3)}{2\pi(x\lambda)^{2/3} (c\rho c_p)^{1/3}} - \frac{Q_{ml} \Gamma(1/3)}{3^{5/6} \pi \lambda^{2/3} x^{5/3} (c\rho c_p)^{1/3}} + O(x^{-2/3}) \quad (A.74)$$

Using the superposition method, the equivalent conjugate heat conduction equation with heat flux of nonisothermal wall temperature is given by,

$$-\int_0^{\bar{x}_1} f\left(\frac{\xi}{\bar{x}_1}\right) \frac{d\theta_1}{d\xi} d\xi = H \frac{\bar{x}_1^{1-m_1}}{\bar{x}_2^{1-m_2}} \int_0^{\bar{x}_2} f\left(\frac{\xi}{\bar{x}_2}\right) \frac{d\theta_2}{d\xi} d\xi \quad (A.75)$$

Equation (A.72) is solved by breaking to several parts to give,

$$-\sum_{k=1}^j s_{i,k} \int_{\bar{x}_{1,k-\Delta_k}}^{\bar{x}_{1,k}} f\left(\frac{\xi}{\bar{x}_{1,j+1}}\right) d\xi = H \frac{\bar{x}_{1,j}^{1-m_1}}{\bar{x}_{2,j+1}^{1-m_2}} \sum_{k=1}^{j+1} s_{2,k} \int_{\bar{x}_{2,k-\Delta_k}}^{\bar{x}_{2,k}} f\left(\frac{\xi}{\bar{x}_{2,j+1}}\right) d\xi \quad (A.76)$$

Using green's function to solve (A.72), the surface temperature can be written as,

$$T_w = \int_{-\infty}^{\infty} q_w(\xi) G(x-\xi) d\xi \quad (A.77)$$

Where $G(x-\xi)$ is the green's function. Using (2.77) give a set of algebraic equations,

$$\sum_{j=1}^m [\phi(x_1, x_j) + \phi_w(x_1, x_j)] h^+(x_j) = \sum_{j=1}^m \phi_w(x_1, x_j) f_i \quad (A.78)$$

which is solved numerically for its surface heat flux

Appendix B

Review of Heat Transfer Fundamentals and Assumptions

Conduction

The equation that describe one-dimensional conduction heat transfer in solid is [125],

$$q_x = \frac{kA}{L}(T_1 - T_2) \quad (\text{B.1})$$

where k is thermal conductivity in $Wm^{-1}K$, A is the cross-sectional area in m^2 , L is the length, between the two temperature points T_1 and T_2 in $^{\circ}C$, in m .

Convection

The equation that describe convection heat transfer over a surface is [125],

$$q = hA(T_s - T_{\infty}) \quad (\text{B.2})$$

where h is the convection coefficient in $Wm^{-2}K$, A is the cross-sectional area in m^2 , T_s and T_{∞} are the surface and ambient temperatures in $^{\circ}C$.

There are a number of available empirical convection correlations using Nusselt number in terms of Rayleigh (Ra) and Prandtl (Pr) numbers.

The Prandtl number, Pr, a dimensionless number which is the ratio of the gaseous kinematic viscosity to thermal diffusivity, is expressed as [125],

$$\text{Pr} = \frac{C_p \mu}{k} \quad (\text{B.3})$$

where C_p is the heat capacity, μ is the dynamic viscosity of the fluid, and k is the thermal conductivity of the fluid.

For the numerical lumped-parameter thermal model and FEA analysis, the convection heat loss coefficients have to be calculated. As mentioned in Section 2.7.2, there are several dimensionless numbers that are needed to be calculated, and they are the Grashof number, the Rayleigh number and the Nusselt number. The Grashof number is a ratio of the buoyancy to viscous force acting on a fluid and is expressed as [125],

$$\text{Gr} = \frac{g \beta (T_{\text{surf}} - T_{\text{amb}}) L^3}{\nu^2} \quad (\text{B.4})$$

where g is the gravitational acceleration, β is the volumetric thermal expansion coefficient, L is the characteristic length, and ν is the kinematic viscosity.

The Rayleigh number of a fluid describes the buoyancy driven flow and is expressed as [125],

$$\text{Ra} = \text{Gr} * \text{Pr} = \frac{g \beta (T_{\text{surf}} - T_{\text{amb}}) L^3}{\nu \alpha} \quad (\text{B.5})$$

where α is the thermal diffusivity of the fluid.

The convective heat transfer can be evaluated from the expression,

$$h = \frac{\text{Nu} * k}{\Delta x} \quad (\text{B.6})$$

where Δx is the evaluated length.

For free convection over a flat vertical plate, we have from Churchill and Chu, for the whole range of Rayleigh number [125],

$$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492 / Pr)^{9/16} \right]^{4/27}} \right\} \quad (B.7)$$

However, a better accuracy can be obtain for $Ra_L \leq 10^9$,

$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1 + (0.492 / Pr)^{9/16} \right]^{4/9}} \quad (B.8)$$

For horizontal flat plates and in the case of upper surface of a hot plate or lower surface of a cold plate, we have [125],

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} \quad (10^4 \leq Ra_L \leq 10^7) \quad (B.9)$$

$$\overline{Nu}_L = 0.15 Ra_L^{1/3} \quad (10^7 \leq Ra_L \leq 10^{11}) \quad (B.10)$$

For horizontal flat plate in the case of lower surface of hot plate or upper surface of cold plate,

$$\overline{Nu}_L = 0.27 Ra_L^{1/4} \quad (10^5 \leq Ra_L \leq 10^{10}) \quad (B.11)$$

For long horizontal cylinders,

$$\overline{Nu}_L = \left\{ 0.60 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.557 / Pr)^{9/16} \right]^{4/27}} \right\}^2 \quad (B.12)$$

For spheres with $Pr \geq 0.7$ and $Ra_D \leq 10^{11}$

$$\overline{Nu}_L = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + (0.469 / Pr)^{9/16}\right]^{4/9}} \quad (B.13)$$

For forced convection over flat plate with constant heat flux with laminar flow[125],

$$Nu_x = 0.453 Re_x^{1/2} Pr^{1/3} \quad Pr \geq 0.6 \quad (B.14)$$

While for turbulent flow,

$$Nu_x = 0.0308 Re_x^{4/5} Pr^{1/3} \quad 0.6 \leq Pr \leq 60 \quad (B.15)$$

For forced convection over flat plate with constant surface temperature with laminar flow[125],

$$\overline{Nu}_x = 0.664 Re_x^{1/2} Pr^{1/3} \quad Pr \geq 0.6 \quad (B.16)$$

And for turbulent flow,

$$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3} \quad 0.6 \leq Pr \leq 60 \quad (B.17)$$

Although the equations of this section are suitable for most engineering calculations, in practice they rarely provide exact values for the convection coefficients. Conditions vary according to free stream turbulence and surface roughness, and errors as large as 25% may be

incurred by using the expressions. For mixed convection, it is the combination of both free and forced convection and thus may be expressed as,

$$Nu^n = Nu_F^n \pm Nu_N^n \quad (B.18)$$

While Nu_F and Nu_N are determined from existing correlations for pure forced and natural convection respectively.

Radiation

The equation that describe radiative heat transfer over a surface is [125],

$$q = \varepsilon \sigma A (T_s^4 - T_{wall}^4) \quad (B.19)$$

where ε is the emissivity have a constant value, σ is the Stefan-Boltzmann constant with the value of $5.67 \times 10^{-8} W m^{-2} K^4$, A is the cross-sectional area in m^2 , T_s and T_{wall} are the material surface and room wall temperatures respectively in *Kelvin*.