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Regional Feedback Control of Robot with Application to Optical Manipulation of Biological Cells

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in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

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Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Date

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LI Xiang

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Abstract

Task-space sensory feedback information is used in many modern robot control systems as it improves robustness to model uncertainty. However, existing task-space sensory feedback control methods of robot are only valid locally in a finite task space within a limited sensing zone where singularity of the Jacobian matrix is avoided. The global stability problem of task-space control system has not been systemically solved.

It is interesting to observe from human vision guided tasks that sensor information is not used for the entire movement, but only at end phases when our hand is near the target. We are able to move our hand from an initial position that is not within our field of view and transit smoothly and easily into visual feedback when the target is near. Moreover, we can reach and manipulate an object even when the vision is occluded. The exploration of a control method that mimics such human behavior is an important step toward understanding dexterous movement of robot.

In this research, a novel regional feedback control methodology is proposed for robots. Each feedback information is employed in a local region, and the combination of regional information ensures the convergence of robot motion. Instead of designing multiple controllers in different regions and switching between them, the regional feedback method integrates the use of dual task-space information in a single controller. The transition from one feedback information to another is em-

bedded in the controllers without using any hard or discontinuous switching. It will be shown that the proposed control method is a unified formulation to address several open issues in task-space robot control systems, such as the singularity of the Jacobian matrix, the limited field of view of cameras, and the vision occlusion.

Using the proposed regional feedback, a new task-space robot controller is proposed, which consists of a reaching task variable that drives the end effector of the manipulator from one task space to another and a desired task variable to move the end effector to the desired position at the ending stage. It enables the end effector to start from any initial position outside sensing zone and in the vicinity of singular configurations, and reach for a desired trajectory in the end. The dynamic stability of the closed-loop systems is analyzed by using Lyapunov-like method. Numerous experimental results are presented to illustrate the performance of the proposed control methods.

The concept of regional feedback is also extended to the optical manipulation of biological cells with robot-assisted tweezers. For the optical tweezers system, the optical trapping works only when the cell is located in a small region around the center of the focused laser beam. To solve the problem, a unified robotic manipulation technique for optical tweezers is proposed to integrate trapping and manipulation of biological cells into a single control method. It allows the laser beam to start from an initial position that is far away from the cell and automatically trap then manipulate the cell, and it also works when the cell escapes from the optical trap during the course of manipulation. The dynamics of robotic manipulator is introduced into optical tweezers system so that a closed-loop manipulator control problem can be formulated and solved. The proposed formulation provides a theoretical framework that bridges the gap between traditional robotic manipulation techniques and optical manipulation techniques of cells.

Symbols

$\mathbf{x} \in \mathbb{R}^{\sum_{i=1}^m n_i}$	desired task feedback variable
$\mathbf{q} \in \mathbb{R}^n$	joint angles
$\mathbf{p} \in \mathbb{R}^{\sum_{i=1}^m n_{pi}}$	position and orientation of end effector in Cartesian space
$\mathbf{x}_I \in \mathbb{R}^{2m}$	position of image features
$\mathbf{J}_x(\mathbf{q}) \in \mathbb{R}^{\sum_{i=1}^m n_i \times n}$	Jacobian matrix from joint space to coordinates of $\mathbf{x}$
$\mathbf{J}_m(\mathbf{q}) \in \mathbb{R}^{\sum_{i=1}^m n_{pi} \times n}$	Jacobian matrix of manipulator
$\mathbf{J}_I(\mathbf{p}) \in \mathbb{R}^{2m \times \sum_{i=1}^m n_{pi}}$	image Jacobian matrix
ϕ, θ, ψ	Euler angles
$\mathbf{R}_{\phi, \theta, \psi} \in \mathbb{R}^{3 \times 3}$	rotation matrix
$\mathbf{T} \in \mathbb{R}^{4 \times 4}$	arm matrix
$\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n \times n}$	inertia matrix
$\mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{n \times n}$	skew-symmetric matrix
$\mathbf{g}(\mathbf{q}) \in \mathbb{R}^n$	gravitational force
$\boldsymbol{\tau} \in \mathbb{R}^n$	control inputs
$\mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \in \mathbb{R}^{n \times n_d}$	regressor matrix for dynamics of robot
$\mathbf{x}_C \in \mathbb{R}^2$	position of cell
$\mathbf{q}_L \in \mathbb{R}^2$	position of laser beam
k_1, k_2	laser parameters
$\mathbf{M}_C \in \mathbb{R}^{2 \times 2}$	inertial matrix of cell

$\mathbf{B}_C \in \Re^{2 \times 2}$	damping matrix of cell
$\mathbf{Y}_C(\dot{\mathbf{x}}_C, \ddot{\mathbf{x}}_C) \in \Re^{2 \times n_C}$	regressor matrix for dynamics of cell
$\mathbf{M}_L \in \Re^{2 \times 2}$	inertial matrix of manipulator of laser source
$\mathbf{B}_L \in \Re^{2 \times 2}$	damping matrix of manipulator of laser source
$\mathbf{Y}_L(\dot{\mathbf{q}}_L, \ddot{\mathbf{q}}_L) \in \Re^{2 \times n_L}$	regressor matrix for dynamics of manipulator of laser source

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Chapter 1

Introduction

Over the past decades, robotics technologies have been demonstrated as key drivers in manufacturing automation. Robotic manipulators are widely used in factories to perform repetitive operations, so as to increase efficiency of manufacturing and decrease cost of production. In recent years, the rapid advances in computing and sensor technologies also lead to the development of robot systems that are used in less structured environments, such as the space and undersea explorations, chemical and hazardous waste cleanup. In parallel, increasing demands for both accuracy and efficiency in biological engineering highlight the requirement for robotics and automation at micro and nano scales and the integration of robotics and biomedical technologies.

Many new initiatives in robot control systems require the sensory feedback such as vision to improve accuracy and robustness to modeling and calibration errors, but the sensory feedback control usually suffers from the limited sensing range. In addition, since the singularity issue impedes the robot from performing its task globally and produces large inputs that may damage the objects around the robot, some new initiatives such as robot surgery and human robot interaction require that

the singularity is avoided so that the safety of robot tasks can be guaranteed.

The emerging applications and new initiatives of robot systems open up new challenges in robot task-space control. Due to the limited sensing range and the singularity issue, more comprehensive and general control strategies are required to improve the performance of robot systems.

1.1 Literature Survey

1.1.1 Robot Control

Various control schemes have been proposed for robotic manipulators. As early as in the 1960s, Whitney [99] proposed the resolved motion rate control method which computed the desired velocity in joint space from the desired velocity in Cartesian space. However, it gave rise to an accumulation of tracking errors since the desired joint angles were obtained by integrating the desired joint velocity.

The dynamics of robot systems are highly nonlinear with strong couplings existing between joint. To solve the problem, a class of control schemes known as computed torque method have been proposed to directly cancel out the nonlinearity in robot dynamics [9, 43, 63, 87, 91]. By employing the feedback linearization, the control torques for a given desired joint trajectory are generated in real time. However, it is difficult to obtain exact dynamic parameters such as link inertias and evaluating frictional forces in practice. In addition, the cancellation of all the nonlinear terms does not provide any physical insight into the robot dynamics and control problem.

In joint-space robot control, the desired joint angles that correspond to the desired position of end effector in task space must be specified. The process of computing joint angles from a given position of end effector in task space is called inverse kine-

matics. Solving the inverse kinematics problem is necessary in joint-space control. However, as the number of degrees of freedom of robot increases, the inverse kinematics problem becomes complicated. In the presence of kinematic uncertainty, it is impossible to derive the desired joint angles from the desired end effector's path [16].

To describe a task for a robotic manipulator, it is convenient to specify the position of end effector in task space such as Cartesian space or visual space. Task-space control is introduced to eliminate the requirement of solving the inverse kinematics.

Various task-space controllers which either formulate the control problem in Cartesian space or sensory task space have been proposed for the robotic manipulator. Takegaki and Arimoto [88] proposed the first Cartesian-space regulator, and it was shown using Lyapunov method that the PD control plus gravity compensation was effective for setpoint control despite the nonlinearity and uncertainty of the robot dynamics. Inspired by the original work, much progress has been achieved in understanding the task-space regulation problems [4, 51]. To deal with tracking control, Slotine and Li [82], Niemeyer and Slotine [73] proposed Cartesian-space adaptive controllers, which contained a PD feedback and a full dynamics compensation.

However, in Cartesian-space control, the position of end effector is usually computed from forward kinematic equations, and the exact model of the Jacobian matrix is also required in the control law. Therefore, the Cartesian-space control methods are extremely sensitive to modeling and calibration errors. Small errors in robot kinematic model could impact the positioning accuracy of the entire system.

Recent advances in sensing technology have led to the research and development of sensory feedback control of robot systems. Among various sensing information, vision is frequently and commonly utilized in task-space control of robot. Vision system is a convenient and effective sensor that provides useful information of the

external environment. If cameras are used to monitor the position of robot, the task coordinates are defined in image space. Using visual feedback, the robot control systems are robust to modeling and calibration errors.

Most task-space control schemes are based on the assumption that the Jacobian matrix from joint space to task space is exactly known. Unfortunately, it is difficult to derive the Jacobian matrix accurately especially when the manipulator grasps several unknown tools, because it is difficult to obtain the exact lengths and grasping angles of the tools. Although much effort has been devoted to developing control schemes for robots with dynamic uncertainty and much progress has been achieved in understanding how the robot can deal with the dynamic uncertainty [53, 73, 74, 82], most works have assumed that the kinematics and the Jacobian matrix of robot are known exactly.

To overcome the problem of uncertain kinematics, Cheah *et al.* [16, 18] proposed approximate Jacobian control methods for setpoint control of robots. The proposed controllers do not require the exact knowledge of kinematics and Jacobian matrix, and sufficient conditions for the bound of the estimated Jacobian matrix are presented to guarantee the stability of the system. Dixon [31] developed an amplitude-limited regulator for robotic manipulators with uncertain kinematics and dynamics. Ozawa and Oobayashi [75] proposed an adaptive setpoint controller which employed sensory information only for kinematics adaptation. The aforementioned controllers are focusing on setpoint control problem. In some applications, it is necessary to specify a desired trajectory rather than simply stating the desired final position. To deal with tracking problem with uncertain kinematics, an adaptive Jacobian controller was proposed in [22]. It was shown that the robot was able to follow the desired trajectory with estimated Jacobian matrix, and novel update laws were designed to estimate the uncertain dynamic and kinematic parameters concur-

rently. The results of [22] were extended in [23] to include actuator parameters and redundant robots. In [94], an adaptive controller was proposed for free-floating space manipulator with uncertainties in both kinematics and dynamics. To solve the problem of representation singularities, an adaptive tracking controller using the unit quaternion representation was developed for robotic manipulators with uncertainty in the kinematic and dynamic models [10]. In adaptive task-space control, the convergence of task-space errors can be guaranteed without the convergence of estimated parameters of robot kinematics and dynamics.

For visual task-space control, the velocity of robot in image space and Cartesian space are related through an image Jacobian matrix [33, 98]. Due to the calibration errors of camera's intrinsic parameters and the uncertainty existing in the depth information, it is also difficult to obtain the exact image Jacobian matrix. In addition, in the image Jacobian matrix, the depth information is inversely proportional to the velocity of image features and hence the overall Jacobian matrix is not linearly parameterizable. Therefore, the estimated depth must be adapted with independent update laws. It is pointed out in [66] that if the depth information was coarsely calibrated, the image based control system might give degraded performance or even become unstable. In the recent past, only a few results have been obtained for the stability analysis of visual task-space control systems in the presence of the uncertainty in depth information. Liu *et al.* and Wang *et al.* [59, 92, 93] proposed the use of a depth-independent interaction matrix in vision based control. The depth-independent interaction matrix was obtained by eliminating the depth information in the image Jacobian matrix. However, the parameters of the depth information are not updated upon. In [24], an adaptive tracking controller was proposed for robot manipulators with uncertain kinematics, dynamics and depth information. It was shown that the end effector converged to the desired trajectory with the uncertain depth, kinematic and dynamic parameters updated online.

One common assumption in task-space control methods is that the Jacobian matrix is non-singular. To overcome the problem of singularity in joint-space control, Nakamura and Hanafusa [70] introduced a damped least-square inverse Jacobian matrix which provided an approximate motion when the robot was in the neighborhood of the singular positions. Sciavicco and Siciliano [81] proposed to use the Jacobian transpose instead of inverse Jacobian matrix in kinematic control, and hence the typical numerical instabilities were avoided. In addition, a large number of research works have been carried out on the singularity avoidance using redundant robots. In the presence of robot redundancy, inverse kinematic solutions can be exploited to increase the manipulability [102] and thus avoid singularity. Nakumura [71] proposed a redundancy control method based on task-priority strategy, while the tracking task was given a higher priority and the singularity avoidance was assigned with a lower priority. Chiaverini [28] proposed a task-priority strategy which can move the robots near or even through the singular positions. However, in those methods, the robot must possess more degrees of freedom than required to execute the task, which is thus not feasible for the control of non-redundant robots.

In task-space regulation control, the Jacobian transpose can be used in the control law to map the task-space error into joint torque, but the convergence of position errors can be ensured only when the Jacobian matrix is full rank [14]. In task-space tracking control, the null space approach is commonly used to avoid singularity [73]. However, it is only feasible for redundant robots, and the robot cannot start from singular points. In principle, the damped inverse Jacobian or the Jacobian transpose may be used in the control law to avoid singularity, but the stability of closed-loop system cannot be guaranteed. Therefore, in the stability analysis of task-space control, it is commonly assumed that the robot is operating in a finite task space such that singularity problem is avoided. Some work has been performed to analyze the stability with the damped inverse Jacobian [11, 72]. However, unlike

task-space control, the stability analysis of these methods is limited to kinematic control, without considering the effects of the robot dynamics. As a result, the global stability of the closed-loop dynamic system cannot be ensured.

1.1.2 Cell Manipulation

The research and development of robotics and automation technology in past few decades has completely revolutionized the modern manufacturing industries. Recent advances in biological sciences and nanotechnology have led to the requirement of robotics and automation at micro and nano scales, thus opening up new challenges to understanding robotic manipulation of cells or nanoparticles.

Many research groups have developed micromanipulation techniques and systems for various tasks. B. Nelson *et al.* [40, 77, 85] established several micropositioning and microinjection stations. They introduced the switching mechanism into the control framework, so that the system could switch from a camera with large field of view to a high-resolution microscope, or switch from position feedback to visual feedback to move the micro end effector from outside to inside the field of view of the microscope. Y. Sun *et al.* [60, 61, 62, 95, 96, 103] developed several micromanipulation systems. In [95], a purely vision-based contact detection method in vertical position was proposed. In [96], an automated system was developed for zebrafish embryo injection. In [62], a data synchronization mechanism between the cell deformations and applied pressure was introduced in micropipette aspiration. In [60], a vision-based cellular force sensing approach was proposed. In [103], a MEMS microgripper with a controllable plunging structure was employed to overcome adhesion forces. In [61], a cell orientation control system operated under inverted microscopes was established. The controllers implemented on the aforementioned systems were based on the standard PID control [61] or look-then-move strategy [96]. The look-then-

move strategy is an open-loop strategy and not robust to modeling uncertainty and disturbance, and neither the look-then-move strategy nor the PID controller is continuous when the micro end effector moves from outside to inside the field of view of the microscope. In addition, human-assisted operations are required for those micromanipulation systems, such as cell structure recognition or image coordinates selection [61].

D. Sun *et al.* [47, 48, 101] established several automatic microinjection systems. By considering the dynamics of robotic manipulator, they introduced the impedance control methods into the microinjection. In [48], a visual impedance force controller was proposed by exploring the relationship between the injection force and the displacement of the cell membrane. In [47], a position and force control method was introduced to control the injection pipette, which was mainly consisted of a feedback linearization in the $X - Y$ horizontal plane and a force impedance control in the Z direction. In [101], a force-control-based cell injection approach was developed to regulate the penetration force during the process of microinjection.

Among the diverse micromanipulation techniques, optical tweezers [7] are one of the most common and useful tools in non-contact cell manipulation because of the capability of manipulating tiny particles precisely without causing damage to the particles. By using a highly focused laser beam, the optical tweezers are able to trap particles as diverse as atoms, molecules, bacteria, viruses and live cells [6], and hence the optical tweezers have been successfully applied in biological sciences and nanotechnology for realization of various manipulation tasks, such as the cell separation [68], evaluation of nonsticky substrate coatings by moving live dissociated neurons [76], the analysis of membrane elasticity of human red blood cells [89], study of mechanical and structural properties of DNA [29, 79], etc.

Due to the laborious work of cell manipulation, the manual operation with optical

tweezers tends to induce the operator fatigue and thus the reduction of success rate. In addition, it also requires lengthy training and lacks reproducibility. Several automatic optical tweezers systems and control methods have developed to improve the efficiency [1, 2, 3, 5, 8, 12, 25, 26, 32, 41, 45, 78, 90, 97, 100]. In [8], the optical micromanipulation was modeled as an infinite-horizon partially observable Markov decision process, and a stochastic programming method was introduced for the real-time path planning of cell motion. A modified A-star path planning algorithm was proposed to transport cell in [100], and the force applied on the trapped cell was also analyzed. An automatic cell sorting system based on dual-beam trap was introduced in [41], and an image-processing system using thresholding, background subtraction and edge-enhancement algorithms was developed for identification of single cells. In [45], a PID closed-loop feedback controller and a synchronization control technology was proposed for cell transportation, based on a simplified dynamics model of the trapped cell. With the multiple trapping technology based on the computer-generated holographic optical-tweezers arrays [32], Arai *et al.* [3] developed an automatic system to flock micro-scale particles. In [2], a simple feedback controller was proposed for the positioning of a microscopic particle. In [78], the performance of proportional control, LQG control and nonlinear control in particle positioning was compared, and the dynamics of trapped particle was modeled as a first-order system by ignoring the particle mass. In [25], a region reaching control method [17] was used to flock multiple micro particles towards a static region, and the result was extended in [26] to include saturated velocities for trapped cells. In [90], an automated optical trapping technique was developed based on computer vision and multiple-force optical clamps. In [5], a flexible system framework on optical tweezers for nanoassembly was reported. In [12], an automated manipulation of multiple microbeads was performed using optical tweezers to simulate live cell transportation. In [97], optical tweezers were employed together with microwell ar-

rays to isolate then deposit cells. In [1], a robust control method was proposed to improve the bandwidth of force estimation in optically trapped bead systems.

One common assumption for the existing optical tweezers control systems is that optical trapping is maintained throughout the entire manipulation process. Therefore, current control techniques for optical tweezers are only valid locally when the cell is in a small neighborhood of the laser beam. The trapping fails when the laser beam starts from an initial position far away from the cell, or when the laser moves too fast to maintain the trapping. The hybrid control method such as [45] can be employed to switch from one controller to another, but the hard-switching mechanism results in chattering and vibration which is not desirable for micromanipulation.

In addition, in aforementioned optical tweezers systems, the cell dynamics is either simplified [2, 25, 45, 78, 100] or ignored [3, 8, 32, 41, 90], and an open-loop controller is employed for laser source without the consideration of manipulator dynamics. Investigating the interaction between robotic manipulator and trapped cell could gain understanding into the dynamic manipulation problem using optical tweezers.

1.2 Motivation

In conventional task-space control of robots, a single task-space feedback is used throughout the movement. However, it is well known that the task-space feedback information may not cover the entire robot workspace. Therefore, current task-space control methods are only valid locally in a finite task space and the stability of the closed-loop system cannot be ensured globally. In this section, some long standing problems in robot task-space control are first reviewed, and the motivation of research is then stated.

Singularity issue has been a long standing problem in task-space control of robot.

Singularity occurs when the Jacobian matrix is not full rank and it is commonly assumed in the theoretical analysis of task-space control system that the robot is operating in a finite task space such that singularity problem can be avoided. This limits the potential workspace of the robot when task-space control is employed. Fig. 1.1 shows the feasible workspace of a 3 DOF (degree of freedom) manipulator where singularities are avoided. By exploring the robot redundancy, several methods [28, 71, 69] have been proposed to overcome the problem of singularity. However, the robot must possess more degrees of freedom than required to execute the task. In addition, these are mainly avoidance techniques and hence the robot cannot start from or pass through singular points.

Figure 1.1: Singularity problem: the singular regions are not reachable.

For vision based control, since camera has limited field of view as illustrated in Fig. 1.2, the end effector of the robot must start within the field of view and cannot leave the field of view during the course of movement. A variety of approaches have been proposed in the literature of visual servoing [49] to keep the image features within the field of view [27, 30, 37, 38, 39, 65, 67], but these methods assume that the image features are observable at the beginning stage, and most methods do not

consider image occlusion which may cause the end effector to get stuck at a local minima. In addition, unlike task-space control, most visual servoing techniques are formulated as kinematic control methods without the consideration of the effects of robot dynamics. Therefore, existing visual feedback controllers are only valid locally within a limited field of view. The stability problem of visual task-space control that allows the end effector to start from an initial position outside the field of view and leave the field of view during the course of movement, has not been systematically solved.

Figure 1.2: Problem of limited field of view (FOV): the robot cannot leave the field of view during the course of movement.

Mobile robots are able to integrate information from various sensors for navigation and motion control. However, the sensory feedback control techniques for the mobile robot also suffer from the problem of limited sensing zone. For example, the use of GPS allows the navigation of mobile robot with high precision and fast response, but the signal is not feasible in tunnels or underwater environment, as illustrated in Fig. 1.3.

In micromanipulation tasks as illustrated in Fig. 1.4, there is always an inherent trade-off between resolution and field of view. A high-resolution microscope is required to improve end point accuracy of the manipulation tasks but it has very

limited field of view. A coarse-to-fine mechanism using mixed cameras is introduced in [77] by using a wide-angle camera to move the end effector inside the field of view of microscope. However, the stability of the closed-loop system is not considered. In addition, the feedback transition among different sensors should be smooth to avoid chattering that is not desirable for micromanipulation.

Figure 1.3: GPS is invalid in underwater environment.

The optical tweezers also have very limited field of view. Besides, to maintain an optical trap, the laser beam is required to move towards the cell to trap it first, and then transport the trapped cell to track the desired trajectory. Current control techniques for optical tweezers are only valid locally when the cell is in a small neighborhood of the laser beam [2, 3, 5, 8, 12, 25, 41, 78, 90, 100]. The trapping fails when the laser beam starts from an initial position far away from the cell, or when the laser moves too fast to maintain the trapping. An optical trap is illustrated in Fig. 1.5.

Figure 1.4: Since the microscope has limited field of view, another camera is required for the motion control of micro end effector initially.

Figure 1.5: An optical trap. The trapping works only when the cell is located in a small neighborhood of the centroid of the focused laser beam.

While sensory task-space feedback control improves robustness to modeling and calibration errors, it suffers from the limited sensing zone and is not feasible when the robot starts from an initial position at or near the singular configurations, and the control schemes also fail when the robot enters the vision occluded areas or the singular regions during the course of movement.

Therefore, existing task-space controllers assume that robots are operated in a finite task space within a limited sensing zone where the singularity of the Jacobian matrix is avoided, and the global dynamic stability problem has not been systematically solved. This thesis is devoted to the development of a novel regional feedback

method to address the open issues in robot task-space control.

1.3 Contributions

In the thesis, a novel regional feedback method is proposed for task-space robot control, which integrates different feedback information smoothly into one controller. Each feedback information is employed in a local region, and the combination of regional information ensures the global convergence of robot motion. A comprehensive framework based on the regional feedback is proposed and shown to be a unified formulation that allows various open issues in robot control systems to be addressed. The main contributions of this thesis are listed as follows:

- (i) This is the first result that solves the open issues on the dynamic stability of task-space robot control system with consideration of singularity of Jacobian matrix and limited sensing zone. Based on the proposed regional feedback, a global task-space controller is developed for the robotic manipulator. Instead of designing multiple controllers in different regions and switching between them, the proposed control strategy integrates the use of dual task-space information in a single controller. The transition from one feedback information to another is embedded in the controllers without using any hard or discontinuous switching.
- (ii) The results of the regional feedback control method are extended to a multiple regional feedback controller. By using the regional feedback from joint space, Cartesian space, and image space, the proposed multiple task-space controller allows the robot to start from any initial position outside the field of view and in the vicinity of singular configurations.
- (iii) The regional feedback control concept is also extended to the robot-assisted optical tweezers system. By using the regional feedback, a unified robotic manipulation

technique for optical tweezers that integrates automatic trapping and manipulation of biological cells into a single method is presented. Unlike the existing methods that assume the optical trapping is maintained throughout the manipulation process, the proposed method allows the laser to start from a large initial position far away from the cell and it also works when the cell escapes from the trap during the course of manipulation. The proposed method provides a theoretical framework that bridges the gap between traditional robotic manipulation techniques and optical manipulation techniques of biological cells.

1.4 Organization

The remaining chapters of the thesis is organized as follows.

Chapter 2 presents some background knowledge on robot dynamics, kinematics, visual servoing, and optical tweezers system. Several examples are given to illustrate the properties of robot kinematics and dynamics.

Chapter 3 introduces the concept of regional feedback for robot task space control. Several examples of regional feedback are given, and a general and systematic formulation of region functions and region errors is presented. By using the regional feedback, a global task-space control method is developed. The global stability of the closed-loop system is shown by using Lyapunov-like analysis. Experimental results are presented to illustrate the performance of the proposed controllers.

Chapter 4 presents a multiple regional feedback controller. While the global task-space controller is to integrate dual task-space information into a single control method, the multiple regional feedback controller is to extend the results to three categories of regional feedback from joint space, Cartesian space and image space respectively.

Chapter 5 extends the concept of the regional feedback to the robot-assisted optical tweezers system, so as to formulate a unified robotic manipulation technique which integrates trapping and manipulation of biological cells into a single method.

Chapter 6 summarizes the contributions of the thesis, and provides the outlines scope for future work.

Chapter 2

Robotic Manipulator and Optical Tweezers

The robotic manipulator is a mechanical system consisted of links connected by joints. To control the manipulator, mathematical models based on analysis of robot structure is required. Two main aspects of the robot models include kinematics and dynamics. The robot kinematics gives the relationship between the dimensions and connectivity of kinematic chains and the position, velocity of each links, while the robot dynamics provides the relationship between the motion and the related forces and torques. In this chapter, the background knowledge on robot kinematics and dynamics are introduced which constitute the basis of the proposed control schemes in this thesis.

This chapter also presents the basic principle of optical trap and the dynamics of optical tweezers system, to provide an understanding on how the control schemes for the robotic manipulator can be extended to the optical manipulation of biological cells.

2.1 Robotic Manipulator

2.1.1 Forward Kinematics and Jacobian Matrix

The forward kinematics provides the relationship between joint configurations of the robotic manipulator and positions and orientations of the tool or end effector. The mechanical structure of a manipulator is characterized by a number of degrees of freedom (DOF) which determine its configuration. Each degree of freedom is typically associated with a joint and constitutes a variable. The space denoted by the vector of joint variables is defined as joint space as:

$$\mathbf{q} = [q_1, \dots, q_n]^T \in \mathbb{R}^n, \quad (2.1)$$

where n represents the number of manipulator joints and q_i is the i^{th} joint angle.

Based on the joint configuration of the manipulator, the forward kinematic equation can be written in the following form [4, 51, 54]:

$$\mathbf{x} = \mathbf{h}(\mathbf{q}), \quad (2.2)$$

where $\mathbf{x} = [\mathbf{x}_1^T, \dots, \mathbf{x}_m^T]^T \in \mathbb{R}^{\sum_{i=1}^m n_i}$ is the position of robot features specified in task space, where $\mathbf{x}_i = [x_{i_1}, \dots, x_{i_{n_i}}]^T \in \mathbb{R}^{n_i}$ denote feature points, $i = 1, 2, \dots, m$, and m is the total number of features, and $\mathbf{h}(\bullet) \in \mathbb{R}^n \rightarrow \mathbb{R}^{\sum_{i=1}^m n_i}$ represents a nonlinear function which computes the task space variables from the joint space variables. In this thesis, the task space variable $\mathbf{x}$ can be specified in different task spaces such as Cartesian space and image space.

When $\mathbf{x}$ is specified in Cartesian space, it is denoted as:

$$\mathbf{x} = \mathbf{p} = [\mathbf{p}_1^T, \dots, \mathbf{p}_m^T]^T \in \mathbb{R}^{\sum_{i=1}^m n_{pi}}, \quad (2.3)$$

where $\mathbf{p}_i \in \mathbb{R}^{n_{pi}}$ correspond to features in Cartesian space, m is the number of features. Both the position and the orientation of the end effector are usually chosen as the features for most robot tasks. In that case, $\mathbf{p} = [\mathbf{p}_1^T, \mathbf{p}_2^T]^T$, where $\mathbf{p}_1 = \mathbf{p}_p$ denotes the position of end effector in Cartesian space, and $\mathbf{p}_2 = \mathbf{p}_o$ denotes the orientation of end effector. Multiple feature points [15, 50] are also required in Cartesian-space control for some robot tasks. For example, the position of wrist can be chosen as an additional feature to control the tool configuration and a point on the arm such as elbow can be used to narrow down the many possibilities of joint configurations in Cartesian-space control.

In Cartesian-space control, the velocity of the feature point $\dot{\mathbf{p}}_i$ is related to the joint-space velocity $\dot{\mathbf{q}}$ as:

$$\dot{\mathbf{p}}_i = \mathbf{J}_{mi}(\mathbf{q})\dot{\mathbf{q}}, \quad (2.4)$$

where $\mathbf{J}_{mi}(\mathbf{q}) \in \mathbb{R}^{n_{pi} \times n}$ is the manipulator Jacobian matrix. Next, the relationship between velocities of the multiple feature points and the joint-space velocity is represented by:

$$\dot{\mathbf{p}} = \mathbf{J}_m(\mathbf{q})\dot{\mathbf{q}}, \quad (2.5)$$

where

$$\mathbf{J}_m(\mathbf{q}) = [\mathbf{J}_{m1}^T(\mathbf{q}), \mathbf{J}_{m2}^T(\mathbf{q}), \dots, \mathbf{J}_{mm}^T(\mathbf{q})]^T \in \mathbb{R}^{\sum_{i=1}^m n_{pi} \times n} \quad (2.6)$$

is the Jacobian matrix for the multiple feature points.

In robot task-space control, the Jacobian matrix is used to investigate singularities, analyze redundancy, and represent the mapping from forces applied to the end effector to resulting torques at the joints.

The orientation of end effector in Cartesian space can be described by different representations, such as the tool configuration vector [80], Euler angles [4, 51, 69] or unit quaternion [10]. When the Euler angles are used to represent the orientation, the orientation vector $\mathbf{p}_o$ is specified as:

$$\mathbf{p}_o = [\phi, \theta, \psi]^T \in \mathbb{R}^3, \quad (2.7)$$

where ϕ , θ and ψ are the angles in each axis. The rotation matrix of the end effector is then expressed in terms of ϕ , θ , ψ as:

$$\mathbf{R}_{\phi, \theta, \psi} = \begin{bmatrix} c\phi \ c\psi - s\phi \ c\theta \ s\psi & -c\phi \ s\psi - s\phi \ c\theta \ c\psi & s\phi \ s\theta \\ s\phi \ c\psi + c\phi \ c\theta \ s\psi & -s\phi \ s\psi + c\phi \ c\theta \ c\psi & -c\phi \ s\theta \\ s\theta \ s\psi & s\theta \ c\psi & c\theta \end{bmatrix} \in \mathbb{R}^{3 \times 3}, \quad (2.8)$$

where $c\phi = \cos(\phi)$, $s\phi = \sin(\phi)$, $c\theta = \cos(\theta)$, $s\theta = \sin(\theta)$, $c\psi = \cos(\psi)$, $s\psi = \sin(\psi)$.

The Euler angles are functions of joint configurations, and the relationship can be derived by investigating the arm matrix. The arm matrix is to describe the position and orientation of the end effector with respect to the base coordinate frame, which

is expressed as [36]:

$$\mathbf{T} = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{p}_p \\ \mathbf{0} & 1 \end{bmatrix} \in \Re^{4 \times 4}, \quad (2.9)$$

where the vector $\mathbf{p}_p$ describes the position of the end effector, and the upper left 3×3 submatrix $\mathbf{R}$ describes the orientation of the end effector.

The values of ϕ , θ , ψ are thus obtained by equating the elements of $\mathbf{R}$ and $\mathbf{R}_{\phi,\theta,\psi}$, as:

$$\begin{aligned} \phi &= \cos^{-1}\left(-\frac{a_y}{\sin(\theta)}\right), \\ \theta &= \cos^{-1}(a_z), \\ \psi &= \cos^{-1}\left(\frac{s_z}{\sin(\theta)}\right). \end{aligned} \quad (2.10)$$

In equation (2.10), the variables of a_y , s_z , a_z are dependent on the specific robot structure.

Example 2.1 Consider a 6 DOF PUMA manipulator as illustrated in Fig. 2.1, a_y , s_z , a_z are given as [36]:

$$\begin{aligned} a_y &= s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5, \\ s_z &= s_{23}(c_4c_5s_6 + s_4c_6) + c_{23}s_5s_6, \\ a_z &= -s_{23}c_4s_5 + c_{23}c_5, \end{aligned} \quad (2.11)$$

where $s_1 = \sin(q_1)$, $c_1 = \cos(q_1)$, $s_{23} = \sin(q_2 + q_3)$, $c_{23} = \cos(q_2 + q_3)$, $s_4 = \sin(q_4)$, $c_4 = \cos(q_4)$, $s_5 = \sin(q_5)$, $c_5 = \cos(q_5)$, $s_6 = \sin(q_6)$ and $c_6 = \cos(q_6)$, and q_1 , q_2 , q_3 ,

q_4, q_5, q_6 are joint angles. From equations (2.7) and (2.11), the orientation vector is obtained as:

$$\mathbf{p}_o = \begin{bmatrix} \cos^{-1} \left\{ -\frac{s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5}{\sin[\cos^{-1}(-s_{23}c_4s_5 + c_{23}c_5)]} \right\} \\ \cos^{-1}(-s_{23}c_4s_5 + c_{23}c_5) \\ \cos^{-1} \left\{ \frac{s_{23}(c_4c_5s_6 + s_4c_6) + c_{23}s_5s_6}{\sin[\cos^{-1}(-s_{23}c_4s_5 + c_{23}c_5)]} \right\} \end{bmatrix}. \quad (2.12)$$

Figure 2.1: A 6 DOF PUMA manipulator.

△△△

2.1.2 Singularity of Jacobian Matrix

Singularities occur when the Jacobian matrix $\mathbf{J}_m(\mathbf{q})$ is not full rank. The singularity issue limits the workspace of robot and impedes the robot from coping with its task. It is therefore necessary to handle the singularity problem carefully so that the robot can be operated properly and stably within the entire workspace. The singular configurations of the manipulator can be obtained by investigating the Jacobian matrix, and a singularity occurs whenever the determinant of the Jacobian matrix is zero.

In general, the manipulator singularities can be divided into external singularity and internal singularity. The external singularity occurs at the external workspace boundary, and the internal singularity includes the internal boundary singularity and interior singularity [20].

Example 2.2 Consider a 2 DOF planar manipulator in Fig. 2.2, the forward kinematic equation is given as:

$$\begin{bmatrix} p_{p1} \\ p_{p2} \end{bmatrix} = \begin{bmatrix} l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) \\ l_1 \sin(q_1) + l_2 \sin(q_1 + q_2) \end{bmatrix} \in \mathbb{R}^2, \quad (2.13)$$

where $\mathbf{p}_p = [p_{p1}, p_{p2}]^T$ denotes the position of end effector, and l_1 and l_2 are the lengths of the first and the second link respectively, and q_1 and q_2 denotes the first and the second joint angles.

Figure 2.2: A 2 DOF planar manipulator.

The Jacobian matrix of the manipulator is specified as:

$$\mathbf{J}_m(\mathbf{q}) = \begin{bmatrix} -l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) \\ l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix} \in \mathbb{R}^{2 \times 2}. \quad (2.14)$$

The Jacobian matrix is singular when:

$$\det[\mathbf{J}_m(\mathbf{q})] = l_1 l_2 \sin(q_2) = 0. \quad (2.15)$$

Therefore, the singularities occur when $q_2 = 0$ (external boundary singularity) and $q_2 = \pi$ (internal boundary singularity), as illustrated in Fig. 2.3. $\triangle\triangle\triangle$

Figure 2.3: The singularities occur when (a) the manipulator is stretched out, or when (b) the manipulator is folded back.

Example 2.3 Consider the 3 DOF manipulator illustrated in Fig. 1.1, the forward kinematic equation is given as:

$$\begin{bmatrix} p_{p1} \\ p_{p2} \\ p_{p3} \end{bmatrix} = \begin{bmatrix} \cos(q_1)[l_2\cos(q_2) + l_3\cos(q_2 + q_3)] \\ \sin(q_1)[l_2\cos(q_2) + l_3\cos(q_2 + q_3)] \\ l_1 - l_2\sin(q_2) - l_3\sin(q_2 + q_3) \end{bmatrix} \in \mathbb{R}^3, \quad (2.16)$$

where $\mathbf{p}_p = [p_{p1}, p_{p2}, p_{p3}]^T$ denotes the position of end effector, and l_3 is the length of the third link, and q_3 denotes the third joint angle. Therefore, the Jacobian matrix is obtained as:

$$\mathbf{J}_m(\mathbf{q}) = \begin{bmatrix} -s_1(l_2c_2 + l_3c_{23}) & -s_2c_1l_2 - s_{23}l_3c_1 & -s_{23}l_3c_1 \\ -c_1(l_2c_2 + l_3c_{23}) & -s_2s_1l_2 - s_{23}l_3s_1 & -s_{23}l_3s_1 \\ 0 & -c_2l_2 - c_{23}l_3 & -c_{23}l_3 \end{bmatrix} \in \mathbb{R}^{3 \times 3}, \quad (2.17)$$

where $s_2 = \sin(q_2)$, $c_2 = \cos(q_2)$, $s_{23} = \sin(q_2 + q_3)$, $c_{23} = \cos(q_2 + q_3)$, and hence the Jacobian matrix of manipulator is singular when:

$$\det[\mathbf{J}_m(\mathbf{q})] = [l_2 \cos(q_2) + l_3 \cos(q_2 + q_3)] l_3 l_2 \sin(q_3) = 0. \quad (2.18)$$

Therefore, the singularities occur at the joint configurations where $q_3 = 0$ (external boundary singularity), $q_3 = \pi$ (internal boundary singularity) and $[l_2 \cos(q_2) + l_3 \cos(q_2 + q_3)] = 0$ (interior singularity). $\triangle\triangle\triangle$

Example 2.4 For the 6 DOF PUMA robotic manipulator illustrated in Fig. 2.1, the singular configurations can also be obtained by investigating $\det[\mathbf{J}_m(\mathbf{q})] = 0$, that is, $q_3 = 0$ (external boundary singularity), $q_3 = \pi$ (internal boundary singularity), $[l_2 \cos(q_2) + l_3 \cos(q_2 + q_3)] = 0$ (interior singularity), and $q_5 = 0$, $q_5 = \pi$ (wrist singularities). In addition, when Euler angles are used to represent the end effector's posture, the representation singularities occur at the joint configurations: $-\sin(q_2 + q_3) \cos(q_4) \sin(q_5) + \cos(q_2 + q_3) \cos(q_5) = \pm 1$ from equation (2.12). $\triangle\triangle\triangle$

2.1.3 Image Jacobian Matrix

When cameras are employed to measure the position of the end effector or tool, the task-space variable is specified in image space as:

$$\mathbf{x} = \mathbf{x}_I = [\mathbf{x}_{I1}^T, \dots, \mathbf{x}_{Im}^T]^T \in \Re^{2m}, \quad (2.19)$$

where m is the number of image features, and $\mathbf{x}_{Ii} = [x_{i_h}, x_{i_v}]^T \in \Re^2$ denotes an image feature point, and x_{i_h} represents the horizontal coordinate, and x_{i_v} is the vertical coordinate. Using the vision information from camera to control the movement of robotic manipulator is referred as visual servoing [49]. In visual servoing, the relationship between the robot frame and the camera is to be derived. In general,

the relationship is defined by a projection between the robot workspace and the camera image plane via the imaging geometry of the camera.

The pinhole camera model [52] is widely used to represent the mapping from Cartesian space to image space, and hence the projection of a point onto the image plane is modeled as a central projection through the center of the lens [59], which is illustrated in Fig. 2.4. In Fig. 2.4, note that the vision cannot cover the whole robot workspace due to the limited field of view.

Figure 2.4: The projection for a pinhole camera model.

Based on the pinhole camera model, the velocity of the image feature is related to the velocity of the feature point in Cartesian space by using the image Jacobian matrix [33, 98]:

$$\dot{\mathbf{x}}_{Ii} = \mathbf{J}_{Ii}(\mathbf{p}_i) \dot{\mathbf{p}}_i = \begin{bmatrix} \frac{f}{z_i} & 0 & -\frac{x_{ih}}{z_i} & -\frac{x_{ih}x_{iv}}{f} & \frac{f^2+x_{ih}^2}{f} & -x_{iv} \\ 0 & \frac{f}{z_i} & -\frac{x_{iv}}{z_i} & -\frac{f^2+x_{iv}^2}{f} & \frac{x_{ih}x_{iv}}{f} & x_{ih} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{p}}_{pi} \\ \dot{\mathbf{p}}_{oi} \end{bmatrix}, \quad (2.20)$$

where $\mathbf{J}_{Ii}(\mathbf{p}_i) \in \mathbb{R}^{2 \times 6}$ is the image Jacobian matrix, and z_i is the depth information of the i^{th} feature point with respect to the camera image frame, and f is the focal length of the camera. The dimensions of the image Jacobian matrix $\mathbf{J}_{Ii}(\mathbf{p}_i)$ may vary according to the degrees of freedom of robotic manipulator and the specific robot tasks. If the orientation of end effector is not con-

sidered, $\mathbf{J}_{Ii}(\mathbf{p}_i) = \begin{bmatrix} \frac{f}{z_i} & 0 & -\frac{x_{ih}}{z_i} \\ 0 & \frac{f}{z_i} & -\frac{x_{iv}}{z_i} \end{bmatrix} \in \mathbb{R}^{2 \times 3}$. If the end effector evolves in a 2-D plane while the camera is perpendicular to the plane as shown in Fig. 2.5, $\mathbf{J}_{Ii}(\mathbf{p}_i) = \begin{bmatrix} \frac{f}{z_i} & 0 \\ 0 & \frac{f}{z_i} \end{bmatrix} \in \mathbb{R}^{2 \times 2}$.

Next, the relationship between velocities of the multiple image features and robot end effector in Cartesian space is represented by:

$$\dot{\mathbf{x}}_I = \mathbf{J}_I(\mathbf{p})\dot{\mathbf{p}}, \quad (2.21)$$

where

$$\mathbf{J}_I(\mathbf{p}) = \begin{bmatrix} \mathbf{J}_{I1}(\mathbf{p}_1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{I2}(\mathbf{p}_2) & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{J}_{Im}(\mathbf{p}_m) \end{bmatrix} \in \mathbb{R}^{2m \times \sum_{i=1}^m n_{pi}} \quad (2.22)$$

is the image Jacobian matrix for the multiple image features. From equations (2.5) and (2.21), the velocities of multiple image features are related to the joint velocity as:

$$\dot{\mathbf{x}}_I = \mathbf{J}_I(\mathbf{p})\mathbf{J}_m(\mathbf{q})\dot{\mathbf{q}}. \quad (2.23)$$

Example 2.5 For a 2 DOF planar manipulator with a fixed camera configuration illustrated in Fig. 2.5, since the camera is placed perpendicular to the plane where the end effector evolves, the image Jacobian matrix is specified as:

$$\mathbf{J}_I = \begin{bmatrix} \frac{f}{z} & 0 \\ 0 & \frac{f}{z} \end{bmatrix} \in \mathbb{R}^{2 \times 2}. \quad (2.24)$$

Figure 2.5: A 2 DOF planar manipulator with a fixed camera configuration.

From equations (2.14) and (2.24), the composite Jacobian matrix $\mathbf{J}_I(\mathbf{p})\mathbf{J}_m(\mathbf{q})$ from joint space to image space is given as:

$$\mathbf{J}_I(\mathbf{p})\mathbf{J}_m(\mathbf{q}) = \begin{bmatrix} -\frac{f}{z}(l_1s_1 + l_2s_{12}) & -\frac{f}{z}l_2s_{12} \\ \frac{f}{z}(l_1c_1 + l_2c_{12}) & \frac{f}{z}l_2c_{12} \end{bmatrix} \in \Re^{2 \times 2}. \quad (2.25)$$

△△△

2.1.4 Robot Dynamics

The dynamic model of robotic manipulator can be derived from the Lagrange formulation. The Lagrange's equation of motion for a conservative system is given by [4]:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\mathbf{q}}} - \frac{\partial L}{\partial \mathbf{q}} = \boldsymbol{\tau}, \quad (2.26)$$

where L represents the difference between the kinetic and potential energy, and $\boldsymbol{\tau} \in \Re^n$ denotes a vector of control inputs.

Using the definition of L , the dynamic equation of the robotic manipulator is given

as: [4, 56]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + [\tfrac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}, \quad (2.27)$$

where $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n \times n}$ is an inertia matrix, and $\tfrac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})$ represents the Coriolis/centripetal vector, and $\mathbf{g}(\mathbf{q}) \in \mathbb{R}^n$ denotes a vector of gravitational force. Some important properties of the robot dynamics described by equation (2.27) are summarized as follows [4]:

Property 2.1 The inertia matrix $\mathbf{M}(\mathbf{q})$ is symmetric and positive definite. $\triangle\triangle\triangle$

Property 2.2 The matrix $\mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{n \times n}$ is skew-symmetric and satisfies:

$$\mathbf{y}^T \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) \mathbf{y} = 0 \quad \forall \quad \mathbf{q}, \dot{\mathbf{q}}, \mathbf{y} \in \mathbb{R}^n. \quad (2.28)$$

$\triangle\triangle\triangle$

Property 2.3 The dynamic model as described by equation (2.27) is linear in a set of physical parameters $\boldsymbol{\theta}_d = [\theta_{d1}, \dots, \theta_{dn_d}]^T \in \mathbb{R}^{n_d}$ as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + [\tfrac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\theta}_d, \quad (2.29)$$

where $\mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \in \mathbb{R}^{n \times n_d}$ is a dynamic regressor matrix. $\triangle\triangle\triangle$

Example 2.6 Consider the 2 DOF manipulator in Fig. 2.2 again, the dynamic equation is determined as [83]:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -C\dot{q}_2 & -C(\dot{q}_1 + \dot{q}_2) \\ C\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad (2.30)$$

where

$$\begin{aligned}
M_{11} &= m_1 l_{c1}^2 + I_1 + m_2 [l_1^2 + l_{c2}^2 + 2\cos(q_2)l_1 l_{c2}] + I_2, \\
M_{22} &= m_2 l_{c2}^2 + I_2, \\
M_{12} &= M_{21} = m_2 \cos(q_2)l_1 l_{c2} + m_2 l_{c2}^2 + I_2, \\
C &= m_2 \sin(q_2)l_1 l_{c2}, \\
g_1 &= m_1 g \cos(q_1)l_{c1} + m_2 g [\cos(q_1 + q_2)l_{c2} + \cos(q_1)l_1], \\
g_2 &= m_2 g \cos(q_1 + q_2)l_{c2},
\end{aligned} \tag{2.31}$$

and m_1 , m_2 are the masses of the first and second links, and I_1 , I_2 are the inertias, and l_{c1} , l_{c2} are the positions of gravity center, and g denotes the gravitational acceleration. Therefore, the inertia matrix $\mathbf{M}(\mathbf{q})$ is obtained as:

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \in \mathbb{R}^{2 \times 2}, \tag{2.32}$$

which is symmetric and positive definite, and the matrix $\mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})$ is specified as:

$$\mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} 0 & -C(\dot{q}_1 + \frac{1}{2}\dot{q}_2) \\ C(\dot{q}_1 + \frac{1}{2}\dot{q}_2) & 0 \end{bmatrix} \in \mathbb{R}^{2 \times 2}, \tag{2.33}$$

which is skew-symmetric, and $\mathbf{g}(\mathbf{q}) = [g_1, g_2]^T \in \mathbb{R}^2$ denotes the gravitational force, and $\boldsymbol{\tau} = [\tau_1, \tau_2]^T \in \mathbb{R}^2$ represents the control inputs. In addition, the dynamic model can be written into the product of a known regressor and a vector of estimated

physical parameters as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + [\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\theta}_d =$$

$$\begin{bmatrix} \ddot{q}_1 & \ddot{q}_1 + \ddot{q}_2 & 2c_2\ddot{q}_1 + c_2\ddot{q}_2 - s_2\dot{q}_1\dot{q}_2 - s_2(\dot{q}_1\dot{q}_2 + \dot{q}_2^2) & c_1 & c_{12} \\ 0 & \ddot{q}_1 + \ddot{q}_2 & c_2\ddot{q}_1 + s_2\dot{q}_1^2 & 0 & c_{12} \end{bmatrix} \begin{bmatrix} m_1l_{c1}^2 + m_2l_1^2 + I_1 \\ m_2l_{c2}^2 + I_2 \\ m_2l_1l_{c2} \\ m_1gl_{c1} + m_2gl_1 \\ m_2gl_{c2} \end{bmatrix}. \quad (2.34)$$

△△△

2.2 Optical Tweezers

2.2.1 Optical Trap

The basic principle of optical trap is based on the transfer of momentum from photons to microscopic objects, when a focused light travels through the object that is immersed in a medium. The refraction of the photons at the boundary between the object and the medium, results in a stable trap of the object. The force exerted on the object points to the center of the laser beam, and it consists of two components: a scattering force that acts in the direction of propagation of the light and a gradient force that acts in the direction of increasing intensity of the light. The trapping force is approximately linear to the distance between the laser center and the center of microscopic object when it is located in a small neighborhood of the laser beam, which is usually modeled as a Hookeian spring, as illustrated in Fig. 2.6.

Optical tweezers are the scientific instruments based on the optical trap, which

can manipulate the microscopic objects without physical contact. A typical optical tweezers system is shown in Fig. 2.7. The laser beam is expanded using a beam expander, reflected on a Dichroic mirror, and introduced into the inverted microscope. The offset between the laser beam and the object can be varied by directly adjusting the position of the laser beam with beam steering techniques or acousto-optic deflectors (AOD). It can also be varied by moving the stage with motor control while fixing the laser beam.

Figure 2.6: The optical trap behaves like a Hookeian spring when the microscopic object is very near the laser beam, and $\mathbf{x}_C$ represents the object position, and $\mathbf{q}_L$ represents the laser position.

2.2.2 Cell Dynamics in Optical Tweezers

In this thesis, the optical tweezers are employed to manipulate the biological cells. Both the position of the cell and the position of the laser beam are specified in the coordinate of camera frame Σ_C , and the cell dynamics in the optical tweezers is described by the following equation [2, 57]:

$$\mathbf{M}_C \ddot{\mathbf{x}}_C + \mathbf{B}_C \dot{\mathbf{x}}_C + k_1(\mathbf{x}_C - \mathbf{q}_L)e^{-k_2\|\mathbf{x}_C - \mathbf{q}_L\|^2} = \mathbf{0}, \quad (2.35)$$

Figure 2.7: A typical optical tweezers system.

where $\mathbf{M}_C \in \mathbb{R}^{2 \times 2}$ denotes the inertia matrix, and $\mathbf{B}_C \in \mathbb{R}^{2 \times 2}$ denotes the damping matrix, and k_1 and k_2 represent the parameters of laser beam, and $\mathbf{x}_C = [x_{C1}, x_{C2}]^T \in \mathbb{R}^2$ is the position of the cell, and $\mathbf{q}_L = [q_{L1}, q_{L2}]^T \in \mathbb{R}^2$ is the position of the laser. Both $\mathbf{M}_C$ and $\mathbf{B}_C$ are positive definite, and k_1 and k_2 are positive constants. The parameter k_1 is related to the laser intensity, and k_2 is related to the waist of the beam dimensions, and the values of k_1 and k_2 increase as the laser beam becomes thinner with higher intensity, and vice versa.

From equation (2.35), the behavior of the cell is regulated by the Gaussian potential field (see Fig. 2.6), which is described by the term $k_1(\mathbf{x}_C - \mathbf{q}_L)e^{-k_2\|\mathbf{x}_C - \mathbf{q}_L\|^2}$ in equation (2.35). If the cell is far away from the laser beam, $e^{-k_2\|\mathbf{x}_C - \mathbf{q}_L\|^2} \rightarrow 0$, thus there is no interaction between the cell and the laser beam. When the cell is very near the laser beam, $\mathbf{x}_C - \mathbf{q}_L \rightarrow \mathbf{0}$, $e^{-k_2\|\mathbf{x}_C - \mathbf{q}_L\|^2} \rightarrow 1$, and equation (2.35) is

simplified as:

$$\mathbf{M}_C \ddot{\mathbf{x}}_C + \mathbf{B}_C \dot{\mathbf{x}}_C + k_1(\mathbf{x}_C - \mathbf{q}_L) = \mathbf{0}, \quad (2.36)$$

thus the cell can be trapped by the laser beam.

Property 2.4 The terms $\mathbf{M}_C \ddot{\mathbf{x}}_C + \mathbf{B}_C \dot{\mathbf{x}}_C$ in equation (2.35) is linear in a set of physical parameters $\boldsymbol{\theta}_C = [\theta_{C1}, \dots, \theta_{Cn_C}]^T \in \mathbb{R}^{n_C}$ as:

$$\mathbf{M}_C \ddot{\mathbf{x}}_C + \mathbf{B}_C \dot{\mathbf{x}}_C = \mathbf{Y}_C(\dot{\mathbf{x}}_C, \ddot{\mathbf{x}}_C) \boldsymbol{\theta}_C, \quad (2.37)$$

where $\mathbf{Y}_C(\dot{\mathbf{x}}_C, \ddot{\mathbf{x}}_C) \in \mathbb{R}^{2 \times n_C}$ is a dynamic regressor matrix. △△△

2.2.3 Manipulator Dynamics in Optical Tweezers

From the cell dynamics described by equation (2.35), the optical trapping works only when the cell is located in a small neighborhood of the centroid of the focused laser beam. Therefore, the laser should be moved towards the cell to trap it, and then the trapped cell is manipulated by the laser beam towards a desired position.

Existing manipulation techniques treat the position of the laser beam $\mathbf{q}_L$ as the control input and open-loop controller without the feedback of $\mathbf{q}_L$ is designed to move the laser source. Without the feedback of $\mathbf{q}_L$, the accuracy of positioning may not be guaranteed, and the trapping also fails when the laser starts from a large initial position or when the cell escapes the trap during the course of manipulation, as illustrated in Fig. 2.8.

In this thesis, the variable $\mathbf{q}_L$ is set as the position of the laser beam with respect to the motorized stage, and it is varied by moving the motorized stage which thus acts as a robotic manipulator. Therefore, the position of laser beam $\mathbf{q}_L$ is con-

trolled by closed-loop robotic manipulation techniques, and the dynamic model of the manipulator of the laser source is described as:

$$\mathbf{M}_L \ddot{\mathbf{q}}_L + \mathbf{B}_L \dot{\mathbf{q}}_L = \mathbf{u}, \quad (2.38)$$

where $\mathbf{M}_L \in \mathbb{R}^{2 \times 2}$ denotes the inertial matrix and $\mathbf{B}_L \in \mathbb{R}^{2 \times 2}$ represents the damping matrix, and $\mathbf{u} \in \mathbb{R}^2$ is the control input for the manipulator. Both $\mathbf{M}_L$ and $\mathbf{B}_L$ are positive definite.

Figure 2.8: The optical trap fails when (a) the laser starts from a large initial position, or when (b) the cell escapes the trap during the movement.

Property 2.5 Equation (2.38) is linear in a set of physical parameters as:

$$\mathbf{M}_L \ddot{\mathbf{q}}_L + \mathbf{B}_L \dot{\mathbf{q}}_L = \mathbf{Y}_L(\dot{\mathbf{q}}_L, \ddot{\mathbf{q}}_L) \boldsymbol{\theta}_L, \quad (2.39)$$

where $\mathbf{Y}_L(\dot{\mathbf{q}}_L, \ddot{\mathbf{q}}_L) \in \mathbb{R}^{2 \times n_L}$ is a dynamics regressor matrix, and $\boldsymbol{\theta}_L = [\theta_{L1}, \dots, \theta_{Ln_L}]^T \in \mathbb{R}^{n_L}$ are a set of physical parameters. △△△

Chapter 3

Regional Feedback Control of Robot

Existing task-space sensory feedback robot control methods are only valid locally in a finite task space within a limited sensing zone where singularity of the Jacobian matrix is avoided. In this chapter, a regional feedback method is proposed for robot task-space control. Based on the concept of regional feedback, a global task-space controller is developed for the robotic manipulator, which enables the end effector to start from any initial position outside sensing zone and in the vicinity of singularity points, and reach for a desired trajectory. The proposed controller also allows the robot to enter vision occluded areas or singular regions during the course of movement.

3.1 Regional Feedback

As discussed in section 2.1.2 and section 2.1.3, the Cartesian-space feedback is not feasible when the robot is at or near the singular configurations, and the vision feed-

back is not feasible outside the field of view or inside the occluded areas. Therefore, the task-space feedback information is not accessible in the entire robot workspace. The stability analysis of current task-space feedback control system is only valid locally in a finite workspace, and the global dynamic stability problem of task-space control system has not been systematically solved.

Given a desired trajectory $\mathbf{x}_d$, the objective is to formulate and solve a regional feedback control problem such that the stability of closed-loop system can be guaranteed. In this thesis, the task-space feedback $\mathbf{x}$ is considered as a desired task variable in which the desired task is specified. To achieve the global stability, a reaching task variable $\mathbf{r}$ which specifies the position of the robot in a different coordinate space with available feedback information is introduced. The variable $\mathbf{r}$ is used to drive the robot into the working range of $\mathbf{x}$. That is, in the regions where $\mathbf{x}$ is not feasible, the reaching task variable $\mathbf{r}$ is adopted to drive the robot to leave those regions, as illustrated in Fig. 3.1.

Figure 3.1: An illustration of external and internal task-space regions. The feedback $\mathbf{x}$ is not feasible in these regions (dark regions), and the feedback $\mathbf{r}$ is activated to move the robot outside these regions.

By using these two task variables, a novel regional feedback method is proposed for task-space robot control in this thesis, which integrates different feedback information smoothly into one controller. Each feedback information is adopted in a corresponding local region, and the combination of regional information ensures global convergence of the robot motion. The regional feedback information consists of the reaching task variable $\mathbf{r}$ which drives the end effector to approach the desired task region, and the desired task variable $\mathbf{x}$ which converges to the desired trajectory.

Note that the regional feedback method is not to avoid the regions where the singularities or occlusion may occur, but to expand the feasible operating range of task-space control schemes to the entire workspace of robot.

The whole robot workspace is divided into external regions and internal regions as illustrated in Fig. 3.1. In these regions, the desired task variable $\mathbf{x}$ is not available due to the limited field of view, occlusion, singularities, and hence the reaching task variable $\mathbf{r}$ is employed. By specifying the reaching task variable $\mathbf{r}$ and the desired task variable $\mathbf{x}$ in different task coordinates according to specific robot tasks, it is shown that the proposed regional feedback method is a unified formulation to address various open issues in robot task-space control.

3.1.1 Singularity Problem

In Cartesian-space control [4, 51, 82, 88], the desired task variable $\mathbf{x}$ is specified in Cartesian space as:

$$\mathbf{x} = \mathbf{p}. \quad (3.1)$$

Figure 3.2: For the 2 DOF manipulator in Example 2.2, the Cartesian-space feedback is not feasible in the dark regions due to singularity issues, and it is feasible in the white regions away from singular configurations.

As discussed in section 2.1.2, the desired task feedback $\mathbf{x}$ is not feasible near the singular positions where the inverse Jacobian matrix becomes singular. Fig. 3.2 illustrates the regions for a 2 DOF planar manipulator where the Cartesian-space feedback is feasible. To solve the problem, the joint position is specified as the reaching task variable $\mathbf{r}$ and employed near or at the singular positions. The variable $\mathbf{r}$ is thus specified in joint space as:

$$\mathbf{r} = \mathbf{q}. \quad (3.2)$$

Consider the 2 DOF planar manipulator in Example 2.2, the singularities occur where $q_2 = 0$ (external boundary singularity) and $q_2 = \pi$ (internal boundary singularity). Based on the singular configurations, two regions can be specified as:

$$\begin{aligned} h_{E_1}(\mathbf{q}) &= b_{E_1}^2 - (q_2)^2 \geq 0, \\ h_{I_{11}}(\mathbf{q}) &= (q_2 - \pi)^2 - b_{I_{11}}^2 \leq 0, \end{aligned} \quad (3.3)$$

where $h_{E_1}(\mathbf{q}) \geq 0$ is the external region which covers the external boundary singularity, and $h_{I_{11}}(\mathbf{q}) \leq 0$ is the internal region which covers the internal boundary

singularity, and b_{E_1} and $b_{I_{11}}$ are positive constants. The constants b_{E_1} and $b_{I_{11}}$ represent the sizes of singular regions, which should be set to ensure that the manipulator is sufficiently far away singular configurations when the end effector leaves the singular regions. The joint information is used where $h_{E_1}(\mathbf{q}) \geq 0$ or $h_{I_{11}}(\mathbf{q}) \leq 0$ to drive the end effector away the singular configurations. After the end effector leaves the singular regions, the Cartesian-space feedback is activated so as to move the end effector towards the desired position.

For the 3 DOF manipulator illustrated in Example 2.3, the singularities occur when the end effector is near the positions where $q_3 = 0$, $q_3 = \pi$ and $[l_2\cos(q_2) + l_3\cos(q_2 + q_3)] = 0$. Similarly, three regions can be specified in joint space as:

$$\begin{aligned} h_{E_1}(\mathbf{q}) &= b_{E_1}^2 - (q_3)^2 \geq 0, \\ h_{I_{11}}(\mathbf{q}) &= (q_3 - \pi)^2 - b_{I_{11}}^2 \leq 0, \\ h_{I_{12}}(\mathbf{q}) &= [l_2\cos(q_2) + l_3\cos(q_2 + q_3)]^2 - b_{I_{12}}^2 \leq 0, \end{aligned} \tag{3.4}$$

where $h_{E_1}(\mathbf{q}) \geq 0$ is the external region which covers the external boundary singularity, and $h_{I_{11}}(\mathbf{q}) \leq 0$ is the internal region which covers the internal boundary singularity, and $h_{I_{12}}(\mathbf{q}) \leq 0$ is another internal region which covers the interior singularity, and $b_{I_{12}}$ is a positive constant. The joint information is used where $h_{E_1}(\mathbf{q}) \geq 0$, $h_{I_{11}}(\mathbf{q}) \leq 0$ or $h_{I_{12}}(\mathbf{q}) \leq 0$ to drive the end effector away the singular configurations.

For the PUMA manipulator in Example 2.4, when Euler angles are used to represent the end effector's posture, only a local description of the end effector orientation can be obtained for task-space robot control because of the representation singularity. This problem can be solved by introducing internal regions to enclose the joint configurations which lead to the representation singularity. Inside the internal regions,

only the reaching task variable $\mathbf{r}$ is employed to move the robot outside the internal regions.

From equation (2.10), the representation singularity occurs when $\sin(\theta) = 0$, and the joint configurations $-\sin(q_2 + q_3)\cos(q_4)\sin(q_5) + \cos(q_2 + q_3)\cos(q_5) = \pm 1$ correspond to $\sin(\theta) = 0$ that result in the representation singularity. To cover the singular joint configurations, internal regions can be introduced in joint space as:

$$\begin{aligned} h_{I_{11}}(\mathbf{q}) &= (-s_{23}c_4s_5 + c_{23}c_5 - 1)^2 - b_{I_{11}}^2 \leq 0, \\ h_{I_{12}}(\mathbf{q}) &= (-s_{23}c_4s_5 + c_{23}c_5 + 1)^2 - b_{I_{12}}^2 \leq 0. \end{aligned} \quad (3.5)$$

The joint information is used where $h_{I_{11}}(\mathbf{q}) \leq 0$, $h_{I_{12}}(\mathbf{q}) \leq 0$, to drive the end effector away from the singular orientation.

The representation singularity can also be eliminated if the orientation of end effector is represented by the unit quaternion. The corresponding region functions with the unit quaternion can be formulated by using the unit quaternion tracking error [10].

3.1.2 Limited Field of View

In vision-based control, the desired task variable $\mathbf{x}$ is specified in image space as:

$$\mathbf{x} = \mathbf{x}_I. \quad (3.6)$$

Due to the limited field of view of camera or image occlusion, the visual feedback $\mathbf{x}$ cannot cover the entire robot workspace as illustrated in Fig. 3.3. To solve the problem, the reaching task variable $\mathbf{r}$ is specified in Cartesian space as:

$$\mathbf{r} = \mathbf{p}. \quad (3.7)$$

Figure 3.3: For the vision based control system in Fig. 1.2, the vision feedback is not feasible in the dark regions due to limited field of view and vision occlusion, and it is feasible in the white regions.

Using the reaching task feedback $\mathbf{r}$, the end effector can start from an initial position outside the field of view or leave the field of view during the course of movement, and it can also move through the occluded area in the presence of image occlusion.

Consider a standard camera with a rectangular field of view, an external region can be specified in Cartesian space to cover those regions that are outside the field of view as [21]:

$$\mathbf{h}_{E_1}(\mathbf{p}) = \begin{bmatrix} \frac{(p_{p1_1} - p_{r1_1})^2}{(p_{b1_1} - p_{r1_1})^2} - 1 \\ \frac{(p_{p1_2} - p_{r1_2})^2}{(p_{b1_2} - p_{r1_2})^2} - 1 \\ \frac{(p_{p1_3} - p_{r1_3})^2}{(p_{b1_3} - p_{r1_3})^2} - 1 \end{bmatrix} \geq \mathbf{0}, \quad (3.8)$$

where the vector $\mathbf{p}_{b1} = [p_{b1_1}, p_{b1_2}, p_{b1_3}]^T$ represents a set of boundary positions, and the vector $\mathbf{p}_{r1} = [p_{r1_1}, p_{r1_2}, p_{r1_3}]^T$ denotes static reference positions. The function $\mathbf{h}_{E_1}(\mathbf{p})$ corresponds to a rectangular block which can match the field of view, and the Cartesian-space feedback is employed where $\mathbf{h}_{E_1}(\mathbf{p}) \geq \mathbf{0}$ to drive the end effector towards the field of view. Since the objective is to bring the end effector into the field of view, only the position information of the end effector is usually sufficient.

3.1.3 Micromanipulation and Mixed Cameras

In the micromanipulation system, the desired task variable $\mathbf{x}$ represents the position of end effector measured by the microscope, for the realization of the manipulation task. Therefore, the variable $\mathbf{x}$ is specified in image space of the microscope as:

$$\mathbf{x} = \mathbf{x}_z = [\mathbf{x}_{z1}^T, \dots, \mathbf{x}_{zm}^T]^T, \quad (3.9)$$

where $\mathbf{x}_{zi} = [x_{zi_h}, x_{zi_v}]^T \in \mathbb{R}^2$ denotes an image feature point, and m is the number of image features.

Figure 3.4: A combination of the wide-angle camera and the zoom camera. The wide-angle camera has a large field of view while the zoom camera can provide more details.

Another camera with wide angle can be used to control the movement of the end effector at the beginning stage, so as to drive the end effector towards the field of view of the microscope. Therefore, the reaching task variable $\mathbf{r}$ is defined as the vision information obtained from the wide-angle camera as:

$$\mathbf{r} = \mathbf{x}_w = [\mathbf{x}_{w1}^T, \dots, \mathbf{x}_{wm}^T]^T. \quad (3.10)$$

where $\mathbf{x}_{wi} = [x_{wi_h}, x_{wi_v}]^T \in \mathbb{R}^2$ denotes a feature point in the image space of the wide-angle camera, and m is the number of image features.

Using regional feedback, the transition from the feedback $\mathbf{r}$ to the feedback $\mathbf{x}$ is smooth, which prevents vibration during the course of micromanipulation. The proposed method can also be used in other mixed-camera control with the wide-angle lens such as the fish-eye camera, as illustrated in Fig. 3.4. Similarly, the wide-angle lens provides a large field of view for the robot's movement in the initial stage, while the zoom camera ensures the accuracy of robot movement by providing more details in the ending stage.

An external region function in the image space of the wide-angle lens is specified to match the shape of the FOV of the zoom camera as:

$$h_{E1}(\mathbf{x}_{w1}) = \frac{(x_{w1_h} - x_{E1_h})^{20}}{a_1^{20}} + \frac{(x_{w1_v} - x_{E1_v})^{20}}{a_2^{20}} - 1 \geq 0, \quad (3.11)$$

where $\mathbf{x}_{w1} = [x_{w1_h}, x_{w1_v}]^T$ is the position of the end effector in the image space of the wide-angle lens, the order is set as 20 to form a rectangular region with rounded corners, and the constants a_1, a_2 determine the size of the rectangle which are set to ensure that $h_{E1}(\mathbf{x}_{w1}) \leq 0$ is inside the field of view, and the reference position $\mathbf{x}_{E1} = [x_{E1_h}, x_{E1_v}]^T$ is set as the center of the field of view. $\triangle\triangle\triangle$

Remark 3.1 When the proposed method is employed for vision based robot control using mixed cameras, the vision feedback from either the zoom camera or the wide-angle lens is specified as the relative position between the feature point and the desired point. Therefore, the image based controller is robust with respect to calibration errors and radial distortion [13, 64]. To deal with the uncertain Jacobian matrix, adaptive control methods [22, 23, 24] can be used.

Figure 3.5: Unified projection model.

In the thesis, the image space of either the wide-angle lens or the zoom camera is related to the Cartesian space of the robot by using the simple perspective projection model. The image space of the wide-angle lens in the mixed cameras system can also be related to the Cartesian space of the robot with the unified projection model illustrated in Fig. 3.5. The image Jacobian matrix using the unified projection model is given in [34, 42], and several visual servoing techniques for wide-angle lens have been proposed by using various image features such as image moments [86] and three points [35]. △△△

3.2 Region Function

3.2.1 Region Function for Reaching Task

As discussed in the previous section, there exist some regions in the robot workspace, where the desired task feedback $\mathbf{x}$ is not feasible due to singularities, limited sensing zone or image occlusion. Therefore, several functions are formulated in the coordi-

nates of $\mathbf{r}$ to cover those regions, and the reaching task feedback $\mathbf{r}$ is used to drive the end effector out of these regions.

The regions formulated in the coordinates of $\mathbf{r}$ are classified as two categories: external regions and internal regions.

The external regions are specified to cover the positions that are beyond the external boundaries of the desired task variable $\mathbf{x}$, such as the external boundary singularity and the field of view. In general, the external region functions are specified in the coordinates of $\mathbf{r}$ as:

$$\mathbf{h}_E(\mathbf{r}) = [h_{E_1}(\mathbf{r}), h_{E_2}(\mathbf{r}), \dots, h_{E_m}(\mathbf{r})]^T \geq \mathbf{0}, \quad (3.12)$$

where m is the total number of external regions. The robot employs the feedback $\mathbf{r}$ inside the external regions where $\mathbf{h}_E(\mathbf{r}) \geq \mathbf{0}$. The vector inequality $\mathbf{h}_E(\mathbf{r}) \geq \mathbf{0}$ implies that $h_{E_i}(\mathbf{r}) \geq 0$ for all $i = 1, \dots, m$.

The specific forms of $\mathbf{h}_{E_i}(\mathbf{r})$ depend on the robot tasks. For example, consider the singularity problem of the 2 DOF manipulator in section 3.1.1, the reaching task variable $\mathbf{r}$ is specified in joint space (i.e. $\mathbf{r} = \mathbf{q}$), and the external boundary singularity occurs when the end effector is near the position $q_2 = 0$. Therefore, the external region is specified as: $h_{E_1}(\mathbf{q}) = b_{E_1}^2 - (q_2)^2 \geq 0$ to cover the external singular positions. The constant b_{E_1} represents the size of external region and it can be set to ensure that the manipulator is sufficiently far away the singular configuration when the end effector leaves the external region.

Similarly, the internal regions are specified to cover the positions that cannot be reached by the desired task variable $\mathbf{x}$ within the external boundaries, such as the internal boundary singularity, the interior singularity and the image occluded area. Each external region $h_{E_i}(\mathbf{r})$ may be accompanied with several internal regions

$h_{I_{i1}}(\mathbf{r}), h_{I_{i2}}(\mathbf{r}), \dots, h_{I_{iN_i}}(\mathbf{r})$. For example, in Fig. 1.1, there are two internal singular regions inside a single robot workspace. In Fig. 1.2, there are two occluded regions within a field of view. Therefore, the internal region functions are specified in general as:

$$\mathbf{h}_I(\mathbf{r}) = [\underbrace{h_{I_{11}}(\mathbf{r}), \dots, h_{I_{1N_1}}(\mathbf{r})}_{N_1}, \dots, \underbrace{h_{I_{i1}}(\mathbf{r}), \dots, h_{I_{iN_i}}(\mathbf{r})}_{N_i}, \dots, \underbrace{h_{I_{m1}}(\mathbf{r}), \dots, h_{I_{mN_m}}(\mathbf{r})}_{N_m}]^T \leq \mathbf{0}, \quad (3.13)$$

where N_i is the number of internal regions corresponding to the i^{th} external region $h_{E_i}(\mathbf{r})$. The robot employs the feedback $\mathbf{r}$ where $\mathbf{h}_I(\mathbf{r}) \leq \mathbf{0}$. The vector inequality $\mathbf{h}_I(\mathbf{r}) \leq \mathbf{0}$ implies that $h_{I_{ij}}(\mathbf{r}) \leq 0$ for all $i = 1, \dots, m$.

The specific forms of $h_{I_{ij}}(\mathbf{r})$ also vary according to different robot tasks. Consider again the singularity problem of 2 DOF manipulator, since the internal boundary singularity occurs when the end effector is near the position $q_2 = \pi$, the internal region is specified as: $h_{I_{11}}(\mathbf{q}) = (q_2 - \pi)^2 - b_{I_{11}}^2 \leq 0$ to cover the internal singular positions. The constant $b_{I_{11}}$ represents the size of internal region, and it is also set to ensure that the manipulator is sufficiently far away the singular configuration when the end effector leaves the internal region.

The feedback information $\mathbf{r}$ can be specified in different coordinates such as Cartesian space, image space or joint space, and it is employed inside the external regions $\mathbf{h}_E(\mathbf{r}) \geq \mathbf{0}$ or the internal regions $\mathbf{h}_I(\mathbf{r}) \leq \mathbf{0}$.

3.2.2 Region Function for Desired Task

The end effector is driven by the desired task variable $\mathbf{x}$ after it leaves the external and internal regions where $\mathbf{h}_E(\mathbf{r}) < \mathbf{0}$ and $\mathbf{h}_I(\mathbf{r}) > \mathbf{0}$ in the coordinates of $\mathbf{r}$. To

ensure a smooth transition from the feedback $\mathbf{r}$ to the feedback $\mathbf{x}$, another sets of external and internal regions are formulated in the coordinates of $\mathbf{x}$, to cover the remaining workspace where the desired task feedback $\mathbf{x}$ is feasible.

The external regions in the coordinates of $\mathbf{x}$ are formulated to include the regions where the desired task feedback is feasible, and the region functions are specified as:

$$\mathbf{f}_E(\mathbf{x}) = [f_{E_1}(\mathbf{x}_1), f_{E_2}(\mathbf{x}_2), \dots, f_{E_m}(\mathbf{x}_m)]^T \leq \mathbf{0}. \quad (3.14)$$

The robot employs the feedback $\mathbf{x}$ where $\mathbf{f}_E(\mathbf{x}) \leq \mathbf{0}$. The vector inequality $\mathbf{f}_E(\mathbf{x}) \leq \mathbf{0}$ implies that $f_{E_i}(\mathbf{x}_i) \leq 0$ for all $i = 1, \dots, m$.

Similarly, the internal regions in the coordinates of $\mathbf{x}$ are formulated to exclude the regions where the desired task feedback is not feasible, and the region functions are specified as:

$$\mathbf{f}_I(\mathbf{x}) = [\overbrace{f_{I_{11}}(\mathbf{x}_1), \dots, f_{I_{1N_1}}(\mathbf{x}_1)}^{N_1}, \dots, \overbrace{f_{I_{m1}}(\mathbf{x}_m), \dots, f_{I_{mN_m}}(\mathbf{x}_m)}^{N_m}]^T \geq \mathbf{0}. \quad (3.15)$$

The robot employs the feedback $\mathbf{x}$ where $\mathbf{f}_I(\mathbf{x}) \geq \mathbf{0}$. The vector inequality $\mathbf{f}_I(\mathbf{x}) \geq \mathbf{0}$ implies that $f_{I_{ij}}(\mathbf{x}_i) \geq 0$ for all $i = 1, \dots, m$.

The functions of $f_{E_i}(\mathbf{x}_i)$ in equation (3.14) can be specified as follows:

$$f_{E_i}(\mathbf{x}_i) = \frac{(x_{i1} - x_{Ei1})^{n_{Ei}}}{a_{i1}^{n_{Ei}}} + \dots + \frac{(x_{in_i} - x_{Ein_i})^{n_{Ei}}}{a_{in_i}^{n_{Ei}}} - 1 \leq 0, \quad (3.16)$$

where $\mathbf{x}_{Ei} = [x_{Ei1}, \dots, x_{Ein_i}]^T \in \mathbb{R}^{n_i}$ represent a set of reference positions, and $a_{i1}, \dots, a_{in_i}$ are positive constants and n_{Ei} are the orders of region functions which are even integers.

The functions of $f_{I_{ij}}(\mathbf{x}_i)$ in equation (3.15) can be specified as follows:

$$f_{I_{ij}}(\mathbf{x}_i) = \frac{(x_{i1} - x_{I_{ij}1})^{n_{I_{ij}}}}{b_{ij1}^{n_{I_{ij}}}} + \cdots + \frac{(x_{in_i} - x_{I_{ij}n_i})^{n_{I_{ij}}}}{b_{ijn_i}^{n_{I_{ij}}}} - 1 \geq 0, \quad (3.17)$$

where $\mathbf{x}_{I_{ij}} = [x_{I_{ij}1}, \dots, x_{I_{ij}n_i}]^T \in \mathfrak{R}^{n_i}$ are a set of reference positions and $b_{ij1}, \dots, b_{ijn_i}$ are positive constants, and $n_{I_{ij}}$ are the orders of region functions which are also even integers.

Both the functions $f_{E_i}(\mathbf{x}_i)$ and $f_{I_{ij}}(\mathbf{x}_i)$ denote superellipses, and the position and shape of superellipse can be varied by adjusting the reference positions $\mathbf{x}_{E_i}$, $\mathbf{x}_{I_{ij}}$ and the orders n_{E_i} , $n_{I_{ij}}$ respectively, as illustrated in Fig. 3.6. The reference positions and orders of $f_{E_i}(\mathbf{x}_i)$ and $f_{I_{ij}}(\mathbf{x}_i)$ should be chosen so that the external regions $f_{E_i}(\mathbf{x}_i) \leq 0$ include the task space where $\mathbf{x}$ is available and the internal regions $f_{I_{ij}}(\mathbf{x}_i) \geq 0$ exclude the task space where $\mathbf{x}$ is not feasible.

Figure 3.6: An illustration of superellipse $f(x) = \frac{x_1^{n_s}}{a_1^{n_s}} + \frac{x_2^{n_s}}{a_2^{n_s}} - 1$ in 2-D space. The shape of superellipse can be varied by setting the order n_s , and the dimensions in each axis a_1 and a_2 : (i) circle; (ii) oval; (iii) rectangle with rounded corners.

Therefore, the region functions $f_{E_i}(\mathbf{x}_i)$ and $f_{I_{ij}}(\mathbf{x}_i)$ denote the external boundary and internal boundary for the feedback $\mathbf{x}$ respectively. It is also possible that there is no internal boundary for $\mathbf{x}$ in some cases, such as no vision occlusion within the field of view. In that case, it is not required to specify the internal region functions $f_{I_{ij}}(\mathbf{x}_i)$.

The external regions $f_{E_i}(\mathbf{x}_i) \leq 0$ and $h_{E_i}(\mathbf{r}) \geq 0$ should match each other, to allow the robot employs the feedback $\mathbf{x}$ within the external boundaries of $\mathbf{x}$ and uses the feedback $\mathbf{r}$ outside the external boundaries. Similarly, the two internal regions $f_{I_{ij}}(\mathbf{x}_i) \geq 0$ and $h_{I_{ij}}(\mathbf{r}) \leq 0$ should also match each other, to allow the robot uses the feedback $\mathbf{r}$ in the regions where the feedback $\mathbf{x}$ is not feasible and uses the feedback $\mathbf{x}$ after it leaves the regions. The combination of the regions in different coordinates covers the entire workspace and ensures the global movement of robot.

Example 3.1 Consider the planar 2 DOF manipulator in Fig. 3.2, the desired task feedback $\mathbf{x}$ is specified in Cartesian space (i.e. $\mathbf{x} = \mathbf{p}$), and it is activated after the end effector leaves the singular regions. The singular joint configuration $q_2 = 0$ corresponds to a circle in Cartesian space where the manipulator is fully stretched out, and the boundaries of the external region $h_{E_1}(\mathbf{q}) = b_{E_1}^2 - (q_2)^2 \geq 0$ which covers $q_2 = 0$ also corresponds to a circle in Cartesian space. To match $h_{E_1}(\mathbf{q}) \geq 0$, the external region function $f_{E_1}(\mathbf{p}_1)$ in Cartesian space is specified as:

$$f_{E_1}(\mathbf{p}_1) = \frac{(p_{p1_1} - p_{E1_1})^2}{a^2} + \frac{(p_{p1_2} - p_{E1_2})^2}{a^2} - 1 \leq 0, \quad (3.18)$$

where $a_{1_1} = a_{1_2} = a$, and the order n_{E_1} is set as $n_{E_1} = 2$ so that the superellipse denoted by $f_{E_1}(\mathbf{p}_1)$ is specified as a circle, and the reference position $\mathbf{p}_{E_1} = [p_{E1_1}, p_{E1_2}]^T$ is the center of the circle which is set as the origin of the Cartesian coordinates in this case. The constant a denotes the radius and it is set to ensure a slight overlap between $h_{E_1}(\mathbf{q}) \geq 0$ and $f_{E_1}(\mathbf{p}_1) \leq 0$.

Similarly, the joint configuration $q_2 = \pi$ also corresponds to a inner circle in Cartesian space where the manipulator is fully folded back, and the boundaries of the internal region $h_{I_{11}}(\mathbf{q}) = (q_2 - \pi)^2 - b_{I_{11}}^2 \leq 0$ which covers $q_2 = \pi$ corresponds to a circle in Cartesian space as well. To match $h_{I_{11}}(\mathbf{q}) \leq 0$, the internal region function

Figure 3.7: External region $f_{E_1}(\mathbf{p}_1)$ and internal region $f_{I_{11}}(\mathbf{p}_1)$ for the 2 DOF planar manipulator.

$f_{I_{11}}(\mathbf{p}_1)$ in Cartesian space is specified as:

$$f_{I_{11}}(\mathbf{p}_1) = \frac{(p_{p1_1} - p_{I_{11}1_1})^2}{b^2} + \frac{(p_{p1_2} - p_{I_{11}2_1})^2}{b^2} - 1 \geq 0, \quad (3.19)$$

where the order $n_{I_{11}}$, reference position $\mathbf{p}_{I_{11}}$ and radius b are set in a similar argument. The region functions $f_{E_1}(\mathbf{p}_1)$ and $f_{I_{11}}(\mathbf{p}_1)$ are illustrated in Fig. 3.7. $\triangle\triangle\triangle$

Example 3.2 For the 6 DOF PUMA robot in section 3.1.1, two internal regions $h_{I_{11}}(\mathbf{q})$ and $h_{I_{12}}(\mathbf{q})$ described by equation (3.5) are introduced in joint space to cover the regions where the representation singularity occurs. The joint information is used where $h_{I_{11}}(\mathbf{q}) \leq 0$, $h_{I_{12}}(\mathbf{q}) \leq 0$, to drive the end effector away from the singular orientation.

To match $h_{I_{11}}(\mathbf{q}) \leq 0$ and $h_{I_{12}}(\mathbf{q}) \leq 0$, another set of internal regions are formulated in Cartesian space, to ensure a smooth transition from the joint-space feedback to the Cartesian-space feedback. Since the singular joint configurations $-\sin(q_2 + q_3)\cos(q_4)\sin(q_5) + \cos(q_2 + q_3)\cos(q_5) = \pm 1$ correspond to the angles $\theta = 0$ and

$\theta = \pi$, two Cartesian-space region functions are specified as:

$$\begin{aligned} f_{I_{11}}(\theta) &= \frac{(\theta)^2}{b_{11}^2} - 1 \geq 0, \\ f_{I_{12}}(\theta) &= \frac{(\theta - \pi)^2}{b_{12}^2} - 1 \geq 0, \end{aligned} \quad (3.20)$$

where b_{11} and b_{12} are positive constants. The Cartesian-space feedback is activated where $f_{I_{11}}(\theta) \geq 0$, $f_{I_{12}}(\theta) \geq 0$ so as to move the end effector towards the desired orientation. △△△

Example 3.3 For the problem of limited field of view illustrated in section 3.1.2, the image-space regions are also specified to match the Cartesian-space regions, so that the end effector can transit smoothly from one region to another. For a rectangular field of view of a standard camera, the external region function in equation (3.16) is specified as:

$$f_{E_1}(\mathbf{x}_{I1}) = \frac{(x_{1h} - x_{E1h})^{20}}{a_1^{20}} + \frac{(x_{1v} - x_{E1v})^{20}}{a_2^{20}} - 1 \leq 0, \quad (3.21)$$

where the order n_{E1} is set high $n_{E1} = 20$ to form a rectangular region with rounded corners, and the constants a_1 , a_2 determine the size of the rectangle which are set to ensure that $f_{E_1}(\mathbf{x}_{I1}) \leq 0$ is inside the field of view, and the reference position $\mathbf{x}_{E1} = [x_{E1h}, x_{E1v}]^T$ is set as the center of the field of view.

Suppose that an oval occluded area exists within the field of view, the internal region function in equation (3.17) is specified as:

$$f_{I_{11}}(\mathbf{x}_{I1}) = \frac{(x_{1h} - x_{I11h})^2}{b_1^2} + \frac{(x_{1v} - x_{I11v})^2}{b_2^2} - 1 \geq 0, \quad (3.22)$$

where the order n_{I11} , reference position $\mathbf{x}_{I11}$ and constants b_1 , b_2 are set so that the internal region excludes the occluded area. The region functions $f_{E_1}(\mathbf{x}_{I1})$ and

$f_{I1}(\mathbf{x}_{I1})$ are illustrated in Fig. 3.8.

Figure 3.8: External region $f_{E1}(\mathbf{x}_{I1})$ and internal region $f_{I1}(\mathbf{x}_{I1})$ for the rectangular field of view of a standard camera.

△△△

Next, a set of task-oriented regions are introduced within the external regions so as to move the end effector towards the desired trajectory:

$$\mathbf{f}_T(\mathbf{x}) = [f_{T1}(\mathbf{x}_1), f_{T2}(\mathbf{x}_2), \dots, f_{Tm}(\mathbf{x}_m)]^T \leq \mathbf{0}. \quad (3.23)$$

Let $\mathbf{x}_d = [\mathbf{x}_{d1}^T, \dots, \mathbf{x}_{dm}^T]^T$ be the desired trajectory, where $\mathbf{x}_{di} = [x_{di1}, \dots, x_{din_i}]^T \in \mathfrak{R}^{n_i}$ is the desired trajectory for the i^{th} feature point. The task-oriented regions are defined to enclose the desired trajectory as:

$$f_{Ti}(\mathbf{x}_i) = \frac{(x_{i1} - x_{di1})^2}{(x_{bi1} - x_{di1})^2} + \dots + \frac{(x_{in_i} - x_{din_i})^2}{(x_{bin_i} - x_{din_i})^2} - 1 \leq 0, \quad (3.24)$$

where $\mathbf{x}_{bi} = [x_{bi1}, \dots, x_{bin_i}]^T \in \mathfrak{R}^{n_i}$ denote the boundary positions. The boundary positions are constituted by several parts, and the desired position $\mathbf{x}_{di}(t)$ is time-varying and not necessary the geometric center.

Remark 3.2 All the regions specified in the coordinates of $\mathbf{r}$ are static. The singular regions of a robotic manipulator are known and can be obtained by exploring the Jacobian matrix, while the size of the FOV can be determined by placing the markers that represent the boundaries since the camera is fixed. Therefore, it is not necessary

to specify task-space regions online, and the construction of region functions is a one-time setup that does not vary with the desired motion. $\triangle\triangle\triangle$

Remark 3.3 The regions specified in the coordinates of $\mathbf{r}$ are slightly overlapped with the regions in the coordinates of $\mathbf{x}$, so that the end effector does not get stuck when it transits from the feedback $\mathbf{r}$ to $\mathbf{x}$. The singular regions are overlapped with the Cartesian-space regions in section 3.1.1, and the overlap can be ensured by setting the joint configurations that correspond to the boundaries of singular regions to be within the regions in Cartesian space. The Cartesian-space region is overlapped with the image-space region in section 3.1.2, and the overlap can be ensured by placing a set of land markers that represent the boundaries of the regions in Cartesian space to be within the regions in image space. Similarly, the wide-angle lens and the zoom camera in section 3.1.3 are set to ensure that the robot is already inside the FOV of the zoom camera after it leaves the external region in the image space of the wide-angle lens. $\triangle\triangle\triangle$

3.3 Potential Energy

In this section, the potential energy functions in coordinates of $\mathbf{x}$ and $\mathbf{r}$ are constructed. The potential energy in coordinates of $\mathbf{r}$ will be used to drive the end effector to leave the regions where the feedback $\mathbf{x}$ is not feasible, and the potential energy in coordinates of $\mathbf{x}$ will be used to drive the end effector to move towards the desired position.

3.3.1 Potential Energy for Reaching Task

Using the external regions in equation (3.12), potential energy functions are introduced as:

$$P_{E_i}(\mathbf{r}) = \frac{k_{rE_i}}{N} [\max(0, h_{E_i}(\mathbf{r}))]^N, \quad (3.25)$$

where k_{rE_i} are positive constants, N is an even integer so that the potential energy $\in C^2$. Similarly, the potential energy functions $P_{I_{ij}}(\mathbf{r})$ for the internal regions in equation (3.13) are introduced as:

$$P_{I_{ij}}(\mathbf{r}) = \frac{k_{rI_{ij}}}{N} [\min(0, h_{I_{ij}}(\mathbf{r}))]^N, \quad (3.26)$$

where $k_{rI_{ij}}$ are positive constants.

Note that $P_{E_i}(\mathbf{r})$ and $P_{I_{ij}}(\mathbf{r})$ are given as:

$$P_{E_i}(\mathbf{r}) = \begin{cases} 0, & h_{E_i}(\mathbf{r}) \leq 0, \\ \frac{k_{rE_i}}{N} [h_{E_i}(\mathbf{r})]^N, & h_{E_i}(\mathbf{r}) > 0, \end{cases} \quad (3.27)$$

$$P_{I_{ij}}(\mathbf{r}) = \begin{cases} 0, & h_{I_{ij}}(\mathbf{r}) \geq 0, \\ \frac{k_{rI_{ij}}}{N} [h_{I_{ij}}(\mathbf{r})]^N, & h_{I_{ij}}(\mathbf{r}) < 0. \end{cases} \quad (3.28)$$

Therefore, the potential energy is smooth and lower bounded by zero. It is zero when the position of the end effector is inside the internal and external regions.

The overall potential energy with the reaching task variable $\mathbf{r}$ is the summation of $P_{E_i}(\mathbf{r})$ and $P_{I_{ij}}(\mathbf{r})$ as:

$$P_R(\mathbf{r}) = k_p \alpha_r \sum_{i=1}^m [P_{E_i}(\mathbf{r}) + \sum_{j=1}^{N_i} P_{I_{ij}}(\mathbf{r})], \quad (3.29)$$

where k_p and α_r are positive constants. Partial differentiating $P_R(\mathbf{r})$ with respect to $\mathbf{r}$ yields:

$$\begin{aligned} \left(\frac{\partial P_R(\mathbf{r})}{\partial \mathbf{r}}\right)^T &= k_p \alpha_r \sum_{i=1}^m \{k_{r_{E_i}} [max(0, h_{E_i}(\mathbf{r}))]^{N-1} \left(\frac{\partial h_{E_i}(\mathbf{r})}{\partial \mathbf{r}}\right)^T \\ &+ \sum_{j=1}^{N_i} k_{r_{I_{ij}}} [min(0, h_{I_{ij}}(\mathbf{r}))]^{N-1} \left(\frac{\partial h_{I_{ij}}(\mathbf{r})}{\partial \mathbf{r}}\right)^T\} \triangleq k_p \alpha_r \Delta \boldsymbol{\varepsilon}_r, \end{aligned} \quad (3.30)$$

where $\Delta \boldsymbol{\varepsilon}_r$ denotes the reaching regional feedback error which drives the end effector to leave the external and internal regions. From equation (3.30), note that $\Delta \boldsymbol{\varepsilon}_r = 0$ when $h_{E_i}(\mathbf{r}) \leq 0$ and $h_{I_{ij}}(\mathbf{r}) \geq 0$, which indicates that the regional feedback naturally reduces to zero after the end effector leaves those regions.

3.3.2 Potential Energy for Desired Task

For the construction of the potential energy in the coordinates of $\mathbf{x}$, two reference regions are formulated for the external region functions $f_{E_i}(\mathbf{x}_i)$ and the internal region functions $f_{I_{ij}}(\mathbf{x}_i)$ respectively.

First, a set of reference regions inside $f_{E_i}(\mathbf{x}_i) \leq 0$ are introduced as:

$$f_{Er_i}(\mathbf{x}_i) = \frac{(x_{i_1} - x_{Ei_1})^{n_{Ei}}}{(\kappa_{E_i} a_{i_1})^{n_{Ei}}} + \dots + \frac{(x_{i_{n_i}} - x_{Ei_{n_i}})^{n_{Ei}}}{(\kappa_{E_i} a_{i_{n_i}})^{n_{Ei}}} - 1 \leq 0, \quad (3.31)$$

where κ_{E_i} are positive constants and $\kappa_{E_i} < 1$.

By using $f_{E_i}(\mathbf{x}_i)$ in equation (3.16) and $f_{Er_i}(\mathbf{x}_i)$ in equation (3.31), the potential energy functions $P_{E_i}(\mathbf{x}_i)$ are introduced as:

$$P_{E_i}(\mathbf{x}_i) = \frac{k_{x_{E_i}}}{N^2} \{min[0, [min(0, f_{E_i}(\mathbf{x}_i))]^N - (\kappa_{E_i}^{n_{E_i}} - 1)^N]\}^N, \quad (3.32)$$

which can also be written as:

$$P_{E_i}(\mathbf{x}_i) = \begin{cases} 0, & f_{Er_i}(\mathbf{x}_i) \leq 0, \\ \frac{k_{xE_i}}{N^2} \{ [f_{E_i}(\mathbf{x}_i)]^N - (\kappa_{E_i}^{n_{E_i}} - 1)^N \}^N, & f_{E_i}(\mathbf{x}_i) < 0, f_{Er_i}(\mathbf{x}_i) > 0, \\ \frac{k_{xE_i}}{N^2} (\kappa_{E_i}^{n_{E_i}} - 1)^{N^2}, & f_{E_i}(\mathbf{x}_i) \geq 0, \end{cases} \quad (3.33)$$

where k_{xE_i} are positive constants. An illustration of $P_{E_i}(\mathbf{x}_i)$ in a 2-D space is shown in Fig. 3.9. From equation (3.33), it is seen that $P_{E_i}(\mathbf{x}_i)$ are smooth and lower bounded by zero. The potential energy $P_{E_i}(\mathbf{x}_i)$ is defined to drive the end effector from $f_{E_i}(\mathbf{x}_i) \leq 0$ towards $f_{Er_i}(\mathbf{x}_i) \leq 0$ and hence $f_{T_i}(\mathbf{x}_i) \leq 0$.

Similarly, to introduce the potential energy functions for $f_{I_{ij}}(\mathbf{x}_i)$, another set of reference regions enclosing $f_{I_{ij}}(\mathbf{x}_i) \geq 0$ are proposed as:

$$f_{I_{ij}}(\mathbf{x}_i) = \frac{(x_{i1} - x_{I_{ij}1})^{n_{I_{ij}}}}{(\kappa_{I_{ij}} b_{ij1})^{n_{I_{ij}}}} + \dots + \frac{(x_{in_i} - x_{I_{ij}n_i})^{n_{I_{ij}}}}{(\kappa_{I_{ij}} b_{ijn_i})^{n_{I_{ij}}}} - 1 \geq 0, \quad (3.34)$$

where $\kappa_{I_{ij}} > 1$ are constants.

Figure 3.9: Example of potential energy of $P_{E_i}(\mathbf{x}_i)$ with $n_{E_i} = 2$ in 2-D space. The top contour corresponds to $f_{E_i}(\mathbf{x}_i)$ while the bottom contour corresponds to $f_{Er_i}(\mathbf{x}_i)$.

Using the internal region functions and the reference region functions in equations

(3.17) and (3.34), the potential energy functions $P_{I_{ij}}(\mathbf{x}_i)$ are proposed as:

$$P_{I_{ij}}(\mathbf{x}_i) = \frac{k_{xI_{ij}}}{N^2} \{ \min[0, [\min(0, f_{Ir_{ij}}(\mathbf{x}_i))]^N - (\frac{1}{\kappa_{I_{ij}}} - 1)^N] \}^N. \quad (3.35)$$

The above equation can be written as:

$$P_{I_{ij}}(\mathbf{x}_i) = \begin{cases} 0, & f_{I_{ij}}(\mathbf{x}_i) \leq 0, \\ \frac{k_{xI_{ij}}}{N^2} \{ [f_{Ir_{ij}}(\mathbf{x}_i)]^N - (\frac{1}{\kappa_{I_{ij}}} - 1)^N \}^N, & f_{Ir_{ij}}(\mathbf{x}_i) < 0, f_{I_{ij}}(\mathbf{x}_i) > 0, \\ \frac{k_{xI_{ij}}}{N^2} (\frac{1}{\kappa_{I_{ij}}} - 1)^{N^2}, & f_{Ir_{ij}}(\mathbf{x}_i) \geq 0, \end{cases} \quad (3.36)$$

where $k_{xI_{ij}}$ are positive constants. An illustration of the potential energy in a 2-D space is shown in Fig. 3.10. From equation (3.36), it is seen that $P_{I_{ij}}(\mathbf{x}_i)$ are also smooth and lower bounded by zero. The potential energy $P_{I_{ij}}(\mathbf{x}_i)$ will be used to construct an energy function to avoid the use of the feedback $\mathbf{x}$ when it is not feasible.

Figure 3.10: Example of potential energy $P_{I_{ij}}(\mathbf{x}_i)$ with $n_{I_{ij}} = 8$ in 2-D space. The top contour corresponds to $f_{Ir_{ij}}(\mathbf{x}_i)$ while the bottom contour corresponds to $f_{I_{ij}}(\mathbf{x}_i)$.

By using the task-oriented regions in equation (3.24), the corresponding potential

Figure 3.11: Example of potential energy $P_{T_i}(\mathbf{x}_i)$ in 2-D space. The top contour corresponds to $f_{T_i}(\mathbf{x}_i)$ while the bottom point is the desired position.

energy functions $P_{T_i}(\mathbf{x}_i)$ are specified as follows:

$$P_{T_i}(\mathbf{x}_i) = \frac{k_{T_i}}{N} \{1 - [\min(0, f_{T_i}(\mathbf{x}_i))]^N\}, \quad (3.37)$$

where k_{T_i} are positive constants. An illustration of $P_{T_i}(\mathbf{x}_i)$ in a 2-D space is shown in Fig. 3.11, where the bottom of $P_{T_i}(\mathbf{x}_i)$ corresponds to the desired position, and the potential field ensures the convergence after the end effector enters the task-oriented regions.

The overall potential energy $P_D(\mathbf{x})$ with the desired task variable $\mathbf{x}$ is specified as the combination of $P_{E_i}(\mathbf{x}_i)$, $P_{I_{ij}}(\mathbf{x}_i)$ and $P_{T_i}(\mathbf{x}_i)$ as follows:

$$P_D(\mathbf{x}) = k_p \alpha_x \sum_{i=1}^m \{P_i(\mathbf{x}_i) \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) + \sum_{j=1}^{N_i} k_{c_{ij}} \left[\frac{k_{x_{I_{ij}}}}{N^2} \left(\frac{1}{\kappa_{I_{ij}}} - 1 \right)^{N^2} - P_{I_{ij}}(\mathbf{x}_i) \right]\}, \quad (3.38)$$

where α_x is a positive constant and $k_{c_{ij}}$ are positive constants, and $P_i(\mathbf{x}_i) = P_{T_i}(\mathbf{x}_i) + P_{E_i}(\mathbf{x}_i)$. An illustration of the combination is shown in Fig. 3.12, and the reasons for the combination of potential energy functions in equation (3.38) are summarized

as follows:

Figure 3.12: An illustration of the combination of potential energy functions in 2-D space.

(i) The top contour of potential energy $P_{E_i}(\mathbf{x}_i)$ (see Fig. 3.12(a)) is fixed to match $h_{E_i}(\mathbf{r})$, but its bottom is flat. Whereas the potential energy $P_{T_i}(\mathbf{x}_i)$ (see Fig. 3.12(b)) drives the end effector towards the desired position but its top contour cannot match $h_{E_i}(\mathbf{r})$. To combine the advantages, $P_i(\mathbf{x}_i)$ (see Fig. 3.12(c)) is defined as the summation of $P_{E_i}(\mathbf{x}_i)$ and $P_{T_i}(\mathbf{x}_i)$. Therefore, as $\mathbf{x}_{di}$ varies, the bottom of potential energy $P_i(\mathbf{x}_i)$ changes while the top contour remains the same. Since the top contour of $P_i(\mathbf{x}_i)$ corresponds to the external region functions $f_{E_i}(\mathbf{x}_i)$ and its bottom is the desired position, the potential field of $P_i(\mathbf{x}_i)$ enables the end effector

Figure 3.13: Example of overall potential energy in 2-D space. The robot only employs feedback $\mathbf{x}$ in the white area in the top view. Note that the robot can also start from or pass through the internal regions.

to move towards the desired position after it enters the external regions.

(ii) The potential energy $P_i(\mathbf{x}_i)$ is further multiplied with the potential energy $P_{I_{ij}}(\mathbf{x}_i)$ (see Fig. 3.12(d)). The multiplied potential energy is flattened at $f_{I_{ij}}(\mathbf{x}_i) < 0$ where the gradient reduces to zero. It avoids the use of feedback $\mathbf{x}$ where $f_{I_{ij}}(\mathbf{x}_i) < 0$, and the reaching task feedback $\mathbf{r}$ is employed in those regions.

(iii) After that, the potential energy obtained in step (ii) is offset inside the regions $f_{I_{ij}}(\mathbf{x}_i) < 0$ by adding the term $\sum_{j=1}^{N_i} k_{c_{ij}} \left[\frac{k_{x_{I_{ij}}}}{N^2} \left(\frac{1}{\kappa_{I_{ij}}} - 1 \right)^{N^2} - P_{I_{ij}}(\mathbf{x}_i) \right]$ (see Fig. 3.12(e)). Note that the potential energy function $P_{I_{ij}}(\mathbf{x}_i)$ is zero when $f_{I_{ij}}(\mathbf{x}_i) \leq 0$ (see Fig. 3.12(d)). The offset term is therefore introduced to raise the potential energy so as to enable the end effector to move away from the internal regions after leaving them. By adjusting the value of $k_{c_{ij}}$ in the offset term, the energy level can be varied to allow the end effector to pass through the internal regions or avoid them.

The aim of this combination is to move the end effector towards the desired position and keep it away the regions where the desired feedback $\mathbf{x}$ is not feasible. An example of the overall potential energy $P_D(\mathbf{x})$ is shown in Fig. 3.13.

Partial differentiating $P_D(\mathbf{x})$ with respect to $\mathbf{x}$ yields:

$$\begin{aligned} \left(\frac{\partial P_D(\mathbf{x})}{\partial \mathbf{x}}\right)^T &= k_p \alpha_x \sum_{i=1}^m \left\{ \left(\frac{\partial P_i(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) + \right. \\ &\left. P_i(\mathbf{x}_i) \sum_{j=1}^{N_i} \left[\left(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T \prod_{k \neq j}^{N_i} P_{I_{ik}}(\mathbf{x}_i) \right] - \sum_{j=1}^{N_i} k_{c_{ij}} \left(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T \right\} \triangleq k_p \alpha_x \Delta \boldsymbol{\varepsilon}_x, \end{aligned} \quad (3.39)$$

where $\Delta \boldsymbol{\varepsilon}_x$ is the desired regional feedback error which drives the end effector toward the desired trajectory, and it is activated automatically after the end effector is inside the external and internal regions in the coordinates of $\mathbf{x}$.

From equation (3.36), $\left(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T$ in equation (3.39) is given as:

$$\left(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T = \begin{cases} 0, & f_{I_{ij}}(\mathbf{x}_i) \leq 0, \\ k_{x_{I_{ij}}} \left\{ [f_{I_{r_{ij}}}(\mathbf{x}_i)]^N - \left(\frac{1}{\kappa_{I_{ij}}} - 1\right)^N \right\}^{N-1} f_{I_{r_{ij}}}(\mathbf{x}_i)^{N-1} \left(\frac{\partial f_{I_{r_{ij}}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T, & f_{I_{ij}}(\mathbf{x}_i) > 0, f_{I_{r_{ij}}}(\mathbf{x}_i) < 0, \\ 0, & f_{I_{r_{ij}}}(\mathbf{x}_i) \geq 0. \end{cases} \quad (3.40)$$

which can also be written as:

$$\begin{aligned} \left(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T &= k_{x_{I_{ij}}} \min\{0, [\min(0, f_{I_{r_{ij}}}(\mathbf{x}_i))]^N - \\ &\left(\frac{1}{\kappa_{I_{ij}}} - 1\right)^N\}^{N-1} [\min(0, f_{I_{r_{ij}}}(\mathbf{x}_i))]^{N-1} \left(\frac{\partial f_{I_{r_{ij}}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T. \end{aligned} \quad (3.41)$$

In addition, $\left(\frac{\partial P_i(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T$ in equation (3.39) is given as:

$$\left(\frac{\partial P_i(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T = \left(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T + \left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T, \quad (3.42)$$

where

$$\left(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T = \begin{cases} 0, & f_{E_i}(\mathbf{x}_i) \leq 0, \\ k_{x_{E_i}} \{ [f_{E_i}(\mathbf{x}_i)]^N - (\kappa_{E_i}^{n_{E_i}} - 1)^N \}^{N-1} f_{E_i}(\mathbf{x}_i)^{N-1} \left(\frac{\partial f_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T, & f_{E_i}(\mathbf{x}_i) > 0, f_{E_i}(\mathbf{x}_i) < 0, \\ 0, & f_{E_i}(\mathbf{x}_i) \geq 0. \end{cases} \quad (3.43)$$

which can also be written as:

$$\begin{aligned} \left(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T &= k_{x_{E_i}} \{ \min\{0, [\min(0, f_{E_i}(\mathbf{x}_i))]\}^N - \\ &(\kappa_{E_i}^{n_{E_i}} - 1)^N \}^{N-1} [\min(0, f_{E_i}(\mathbf{x}_i))]^{N-1} \left(\frac{\partial f_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T. \end{aligned} \quad (3.44)$$

and

$$\left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T = -k_{T_i} [\min(0, f_{T_i}(\mathbf{x}_i))]^{N-1} \left(\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T, \quad (3.45)$$

where the partial derivative $\left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T$ is the gradient of potential energy $P_{T_i}(\mathbf{x}_i)$ for the task-oriented regions. From equation (3.45), if the end effector is outside the task-oriented regions where $f_{T_i}(\mathbf{x}_i) > 0$, the gradient of $P_{T_i}(\mathbf{x}_i)$ reduces to zero. After the end effector enters the task-oriented regions, $f_{T_i}(\mathbf{x}_i) \leq 0$, and the gradient of $P_{T_i}(\mathbf{x}_i)$ becomes:

$$\left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T = -k_{T_i} [f_{T_i}(\mathbf{x}_i)]^{N-1} \left(\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T,$$

where $\left(\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T = [0, \dots, \frac{2(x_{i1} - x_{di1})}{(x_{bi1} - x_{di1})^2}, \dots, \frac{2(x_{in_i} - x_{din_i})}{(x_{bin_i} - x_{din_i})^2}, \dots, 0]^T \in \mathbb{R}^{\sum_{i=1}^m n_i}$, so it is nonzero until the end effector reaches the desired position.

Remark 3.4 In the presence of additional feature points $\mathbf{x}_{ai}$ that do not correspond to the tracking task (e.g. the position of wrist), the corresponding external and

internal region functions $f_{E_{ai}}(\mathbf{x}_{ai})$ and $f_{I_{aij}}(\mathbf{x}_{ai})$ can be similarly formulated to drive the additional features to pass and leave the singular or occluded positions. In that case, the potential energy in equation (3.38) is extended as:

$$\begin{aligned}
 P_D(\mathbf{x}) = & k_p \alpha_x \sum_{i=1}^m \left\{ P_i(\mathbf{x}_i) \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) + \sum_{j=1}^{N_i} k_{c_{ij}} \left[\frac{k_{x_{I_{ij}}}}{N^2} \left(\frac{1}{\kappa_{I_{ij}}} - 1 \right)^{N^2} - P_{I_{ij}}(\mathbf{x}_i) \right] \right\} \\
 & + k_p \alpha_x \sum_{i=1}^{m_a} \left\{ P_{E_{ai}}(\mathbf{x}_{ai}) \prod_{j=1}^{N_{ai}} P_{I_{aij}}(\mathbf{x}_{ai}) + \right. \\
 & \left. \sum_{j=1}^{N_{ai}} k_{c_{aij}} \left[\frac{k_{x_{I_{aij}}}}{N^2} \left(\frac{1}{\kappa_{I_{aij}}} - 1 \right)^{N^2} - P_{I_{aij}}(\mathbf{x}_{ai}) \right] \right\}, \quad (3.46)
 \end{aligned}$$

where m_a is the number of additional features, and N_{ai} is the number of internal regions for the i^{th} additional feature, and $P_{E_{ai}}(\mathbf{x}_{ai})$ are the potential energy functions for the external region functions $f_{E_{ai}}(\mathbf{x}_{ai})$, and $P_{I_{aij}}(\mathbf{x}_{ai})$ are the potential energy functions for the internal region functions $f_{I_{aij}}(\mathbf{x}_{ai})$. $\triangle\triangle\triangle$

Remark 3.5 The external regions $f_{E_i}(\mathbf{x}_i) \leq 0$ and the internal regions $f_{I_{ij}}(\mathbf{x}_i) \geq 0$ in the coordinates of $\mathbf{x}$ are defined to cover the task space where the desired task feedback $\mathbf{x}$ is feasible. The external region functions $f_{E_i}(\mathbf{x}_i)$ denote the external boundary of the feedback $\mathbf{x}$. Within the external boundary, there may be several regions excluded by $f_{I_{ij}}(\mathbf{x}_i) \geq 0$ where $\mathbf{x}$ is not feasible. This is a general formulation and it is also possible that there is no internal region where $\mathbf{x}$ is not feasible inside the external task-space boundary. For example, there may be no occlusion within the field of view or no internal singularity inside the external task-space boundary. In those cases, the potential energy $P_D(\mathbf{x})$ in equation (3.38) can be simplified as [21]:

$$P_D(\mathbf{x}) = k_p \alpha_x \sum_{i=1}^m P_i(\mathbf{x}_i) = k_p \alpha_x \sum_{i=1}^m [P_{E_i}(\mathbf{x}_i) + P_{T_i}(\mathbf{x}_i)]. \quad (3.47)$$

$\triangle\triangle\triangle$

Remark 3.6 The reference regions of the external and internal regions are used to introduce differences in the energy levels and note that the internal regions are within each external region. The potential energy $P_{E_i}(\mathbf{x}_i)$ is expressed in terms of the external region $f_{E_i}(\mathbf{x}_i)$ so that $f_{E_i}(\mathbf{x}_i)$ corresponds to the higher energy level of $P_{E_i}(\mathbf{x}_i)$ as illustrated in Fig. 3.12(a). The potential energy $P_{I_{ij}}(\mathbf{x}_i)$ is expressed in terms of the reference region $f_{I_{ij}}(\mathbf{x}_i)$ so that $f_{I_{ij}}(\mathbf{x}_i)$ corresponds to the lower energy level of the offset energy term $\sum_{j=1}^{N_i} k_{c_{ij}} \left[\frac{k_{x_{I_{ij}}}}{N^2} \left(\frac{1}{\kappa_{I_{ij}}} - 1 \right)^{N^2} - P_{I_{ij}}(\mathbf{x}_i) \right]$ as illustrated in Fig. 3.12(e). In both cases, the regions and their reference regions correspond to the higher levels and the lower levels of the energy functions respectively. $\triangle\triangle\triangle$

3.4 Task-Space Controller with Regional Feedback

The velocity of the end effector in the task-space coordinate of $\mathbf{x}$ is related to the joint velocity as:

$$\dot{\mathbf{x}} = \mathbf{J}_x(\mathbf{q})\dot{\mathbf{q}}, \quad (3.48)$$

where $\mathbf{J}_x(\mathbf{q}) \in \mathbb{R}^{\sum_{i=1}^m n_i \times n}$ is a Jacobian matrix from joint space to task space. When the desired task variable $\mathbf{x}$ is specified in Cartesian space, $\mathbf{J}_x(\mathbf{q}) = \mathbf{J}_m(\mathbf{q})$ represents the mapping from joint space to Cartesian space as described by equation (2.5). When $\mathbf{x}$ is specified in image space, $\mathbf{J}_x(\mathbf{q}) = \mathbf{J}_I(\mathbf{p})\mathbf{J}_m(\mathbf{q})$ represents the mapping from joint space to image space described by equation (2.23). Similarly, the velocity of the end effector in the different coordinate space of $\mathbf{r}$ is related to the joint velocity as:

$$\dot{\mathbf{r}} = \mathbf{J}_r(\mathbf{q})\dot{\mathbf{q}}, \quad (3.49)$$

where $\mathbf{J}_r(\mathbf{q}) \in \mathbb{R}^{n_r \times n}$ is the Jacobian matrix from joint space to the corresponding task space.

After the feedback variables $\mathbf{r}$ and $\mathbf{x}$ are specified in particular coordinates, the reaching region error $\Delta\boldsymbol{\varepsilon}_r$ and the desired region error $\Delta\boldsymbol{\varepsilon}_x$ in equations (3.30) and (3.39) are obtained accordingly. Using the region errors, the regional feedback controller for the robotic manipulator is proposed.

Note that the desired position is only specified in the region functions $f_{T_i}(\mathbf{x}_i)$, the desired velocity is specified as:

$$\dot{\mathbf{x}}_{di} = \mathbf{0}, \quad \text{if } f_{Er_i}(\mathbf{x}_i) \geq 0 \text{ or } f_{Ir_{ij}}(\mathbf{x}_i) \leq 0. \quad (3.50)$$

That is, the desired velocity is zero if the end effector is outside the external and internal reference regions in the coordinates of $\mathbf{x}$.

Equation (3.50) can be satisfied by using the weight factors [20] in the desired trajectory. In general, the desired trajectory $\mathbf{x}_{di}$ is consist of a time-varying part and a constant part, which is denoted as:

$$\mathbf{x}_{di}(t) = \mathbf{x}_{ci} + \mathbf{x}_{vi}(t), \quad (3.51)$$

where $\mathbf{x}_{ci} = [x_{ci_1}, \dots, x_{ci_{n_i}}]^T$ is the constant part, and $\mathbf{x}_{vi}(t) = [x_{vi_1}, \dots, x_{vi_{n_i}}]^T$ is the time-varying part. By using the weight factors, $\mathbf{x}_{di}$ is revised as:

$$\mathbf{x}_{di}(t) = \mathbf{x}_{ci} + w_i(\mathbf{x}_i)\mathbf{x}_{vi}(t), \quad (3.52)$$

where $w_i(\mathbf{x}_i)$ are weight factors which are defined so that $w_i(\mathbf{x}_i) = 0$ when $f_{Er_i}(\mathbf{x}_i) \geq 0$ or $f_{Ir_{ij}}(\mathbf{x}_i) \leq 0$, and $w_i(\mathbf{x}_i)$ smoothly transits to 1 when $f_{Er_i}(\mathbf{x}_i) < 0$ and $f_{Ir_{ij}}(\mathbf{x}_i) > 0$. The details are given in Appendix. Therefore, when the end ef-

fector is outside the external and internal reference regions, $w_i(\mathbf{x}_i) = 0$ and thus $\mathbf{x}_{di}(t) = \mathbf{x}_{ci}$ and $\dot{\mathbf{x}}_{di} = \mathbf{0}$, then equation (3.50) is satisfied. When the end effector enters the external and internal reference regions, $w_i(\mathbf{x}_i)$ smoothly increase to 1, and hence $\mathbf{x}_{di}(t) = \mathbf{x}_{ci} + \mathbf{x}_{vi}(t)$ which is the actual trajectory.

Next, a reference vector $\dot{\mathbf{x}}_a = [\dot{\mathbf{x}}_{a1}^T, \dots, \dot{\mathbf{x}}_{am}^T]^T$ is introduced, where $\dot{\mathbf{x}}_{ai}$ are specified as:

$$\dot{\mathbf{x}}_{ai} = [\dot{x}_{di1} - \dot{x}_{di1} \frac{x_{i1} - x_{di1}}{x_{bi1} - x_{di1}}, \dots, \dot{x}_{din_i} - \dot{x}_{din_i} \frac{x_{in_i} - x_{din_i}}{x_{bin_i} - x_{din_i}}]^T. \quad (3.53)$$

Using $\dot{\mathbf{x}}_a$ in equation (3.53), a sliding vector is proposed as:

$$\mathbf{s}_q = \dot{\mathbf{q}} - \mathbf{J}_x^+(\mathbf{q})\dot{\mathbf{x}}_a + \alpha_x \mathbf{J}_x^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_r, \quad (3.54)$$

where $\mathbf{J}_x^+(\mathbf{q})$ is the pseudo-inverse matrix of $\mathbf{J}_x(\mathbf{q})$, and $\Delta\boldsymbol{\varepsilon}_x$ and $\Delta\boldsymbol{\varepsilon}_r$ denote the desired region error and the reaching region error respectively.

From equation (3.52), note that the reference vectors $\dot{\mathbf{x}}_{ai}$ described by equation (3.53) can be expressed as:

$$\dot{\mathbf{x}}_{ai} = \mathbf{A}_i \dot{\mathbf{x}}_i + \dot{\mathbf{x}}_{fi}, \quad (3.55)$$

where

$$\mathbf{A}_i = \begin{bmatrix} \frac{\partial w_i(\mathbf{x}_i)}{\partial x_{i1}}(x_{vi1} - x_{vi1} \frac{x_{i1} - x_{di1}}{x_{bi1} - x_{di1}}) & \dots & \frac{\partial w_i(\mathbf{x}_i)}{\partial x_{in_i}}(x_{vin_i} - x_{vin_i} \frac{x_{i1} - x_{di1}}{x_{bi1} - x_{di1}}) \\ \vdots & \ddots & \vdots \\ \frac{\partial w_i(\mathbf{x}_i)}{\partial x_{i1}}(x_{vin_i} - x_{vin_i} \frac{x_{in_i} - x_{din_i}}{x_{bin_i} - x_{din_i}}) & \dots & \frac{\partial w_i(\mathbf{x}_i)}{\partial x_{in_i}}(x_{vin_i} - x_{vin_i} \frac{x_{in_i} - x_{din_i}}{x_{bin_i} - x_{din_i}}) \end{bmatrix}, \quad (3.56)$$

and

$$\dot{\mathbf{x}}_{fi} = [w_i(\mathbf{x}_i)\dot{x}_{vi_1} - w_i(\mathbf{x}_i)\dot{x}_{vi_1} \frac{x_{i_1} - x_{di_1}}{x_{bi_1} - x_{di_1}}, \dots, w_i(\mathbf{x}_i)\dot{x}_{vi_{n_i}} - w_i(\mathbf{x}_i)\dot{x}_{vi_{n_i}} \frac{x_{i_{n_i}} - x_{di_{n_i}}}{x_{bi_{n_i}} - x_{di_{n_i}}}]^T. \quad (3.57)$$

Next, a compound matrix is introduced as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_m \end{bmatrix}, \quad (3.58)$$

where the matrix $\mathbf{A}$ is introduced to avoid the use of joint acceleration in the proposed controller.

Using equation (3.58), the sliding vector $\mathbf{s}_q$ in equation (3.54) can be rewritten as:

$$\mathbf{s}_q = [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]\dot{\mathbf{q}} - \mathbf{J}_x^+(\mathbf{q})\dot{\mathbf{x}}_f + \alpha_x \mathbf{J}_x^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_r, \quad (3.59)$$

where $\mathbf{I}_n \in \Re^{n \times n}$ is an identity matrix, and $\dot{\mathbf{x}}_f = [\dot{\mathbf{x}}_{f1}^T, \dots, \dot{\mathbf{x}}_{fm}^T]^T$.

Next, a new sliding vector is defined as:

$$\begin{aligned} \mathbf{s} &= \dot{\mathbf{q}} - \dot{\mathbf{q}}_r = \dot{\mathbf{q}} - [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]^{-1} \times \\ &\quad [\mathbf{J}_x^+(\mathbf{q})\dot{\mathbf{x}}_f - \alpha_x \mathbf{J}_x^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_x - \alpha_r \mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_r], \end{aligned} \quad (3.60)$$

where $\dot{\mathbf{q}}_r = [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]^{-1} [\mathbf{J}_x^+(\mathbf{q})\dot{\mathbf{x}}_f - \alpha_x \mathbf{J}_x^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_x - \alpha_r \mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_r]$ and $\mathbf{s}_q = [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]\mathbf{s}$.

Using the region errors and the sliding vector, the regional feedback controller for

the robotic manipulator is proposed as:

$$\begin{aligned} \boldsymbol{\tau} = & -[\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]^T[k_p\alpha_x\mathbf{J}_x^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_x + \\ & k_p\alpha_r\mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_r] - \mathbf{K}_s\mathbf{s} + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\hat{\boldsymbol{\theta}}_d, \end{aligned} \quad (3.61)$$

where $\mathbf{K}_s$ is a positive definite matrix. When the end effector reaches the regions where the feedback $\mathbf{x}$ is not feasible due to singularities, occlusion or limited sensing zone, $\dot{\mathbf{x}}_a$ and $\Delta\boldsymbol{\varepsilon}_x$ reduce to zero and the reaching regional feedback $\Delta\boldsymbol{\varepsilon}_r$ enables the end effector to pass through those regions. After the end effector leaves the regions, $\Delta\boldsymbol{\varepsilon}_r$ reduces to zero, and the desired regional feedback $\Delta\boldsymbol{\varepsilon}_x$ is activated to drive the end effector to the desired trajectory. Therefore, the region errors $\Delta\boldsymbol{\varepsilon}_x$ and $\Delta\boldsymbol{\varepsilon}_r$ work within the corresponding regions, and the proposed controller is continuous since the region errors are continuous.

The estimated parameters $\hat{\boldsymbol{\theta}}_d$ are updated by the following update law:

$$\dot{\hat{\boldsymbol{\theta}}}_d = -\mathbf{L}_d\mathbf{Y}_d^T(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\mathbf{s}, \quad (3.62)$$

where $\mathbf{L}_d$ is a positive definite matrix.

From equation (2.27), the robot dynamic equation is stated as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + [\tfrac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}, \quad (3.63)$$

Using equation (3.60) and Property 2.3, equation (3.63) is written as:

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{s}} + [\tfrac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\mathbf{s} + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\boldsymbol{\theta}_d = \boldsymbol{\tau}. \quad (3.64)$$

The closed-loop equation is obtained by substituting equation (3.61) into equation

(3.64) to give:

$$\begin{aligned} & \mathbf{M}(\mathbf{q})\dot{\mathbf{s}} + [\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\mathbf{s} + [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]^T \\ & \times [k_p\alpha_x\mathbf{J}_x^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_x + k_p\alpha_r\mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_r] + \mathbf{K}_s\mathbf{s} + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \ddot{\mathbf{q}}_r)\Delta\boldsymbol{\theta}_d = \mathbf{0}, \end{aligned} \quad (3.65)$$

where $\Delta\boldsymbol{\theta}_d = \boldsymbol{\theta}_d - \hat{\boldsymbol{\theta}}_d$.

To prove the stability, a Lyapunov-like candidate is proposed as:

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{M}(\mathbf{q})\mathbf{s} + P_D(\mathbf{x}) + P_R(\mathbf{r}) + \frac{1}{2}\Delta\boldsymbol{\theta}_d^T\mathbf{L}_d^{-1}\Delta\boldsymbol{\theta}_d. \quad (3.66)$$

Next, note that the time derivative of $P_D(\mathbf{x})$ in equation (3.38) is given as:

$$\begin{aligned} \dot{P}_D(\mathbf{x}) = & k_p\alpha_x \sum_{i=1}^m \{ \dot{P}_i(\mathbf{x}_i) \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) + \\ & P_i(\mathbf{x}_i) \sum_{j=1}^{N_i} [\dot{P}_{I_{ij}}(\mathbf{x}_i) \prod_{k \neq j}^{N_i} P_{I_{ik}}(\mathbf{x}_i)] - \sum_{j=1}^{N_i} k_{c_{ij}} \dot{P}_{I_{ij}}(\mathbf{x}_i) \}, \end{aligned} \quad (3.67)$$

where $\dot{P}_i(\mathbf{x}_i)$ in equation (3.67) is given as:

$$\dot{P}_i(\mathbf{x}_i) = \dot{P}_E(\mathbf{x}_i) + \dot{P}_T(\mathbf{x}_i). \quad (3.68)$$

From equation (3.37), the time derivative of $P_{T_i}(\mathbf{x}_i)$ is given as:

$$\begin{aligned} \dot{P}_{T_i}(\mathbf{x}_i) = & -k_{T_i}[\min(0, f_{T_i}(\mathbf{x}_i))]^{N-1} \dot{f}_{T_i}(\mathbf{x}_i) = -k_{T_i}[\min(0, f_{T_i}(\mathbf{x}_i))]^{N-1} \times \\ & \left(\frac{2(x_{i_1} - x_{di_1})(\dot{x}_{i_1} - \dot{x}_{di_1})}{(x_{bi_1} - x_{di_1})^2} + \frac{2(x_{i_1} - x_{di_1})^2 \dot{x}_{di_1}}{(x_{bi_1} - x_{di_1})^3} + \dots + \frac{2(x_{i_l} - x_{din_i})(\dot{x}_{in_i} - \dot{x}_{din_i})}{(x_{bin_i} - x_{din_i})^2} + \frac{2(x_{in_i} - x_{din_i})^2 \dot{x}_{din_i}}{(x_{bin_i} - x_{din_i})^3} \right) \\ & = [\dot{x}_{i_1} - \dot{x}_{di_1} + \frac{\dot{x}_{di_1}(x_{i_1} - x_{di_1})}{(x_{bi_1} - x_{di_1})}, \dots, \dot{x}_{in_i} - \dot{x}_{din_i} + \frac{\dot{x}_{din_i}(x_{in_i} - x_{din_i})}{(x_{bin_i} - x_{din_i})}] \times \\ & \left[-\frac{2k_{T_i}[\min(0, f_{T_i}(\mathbf{x}_i))]^{N-1}(x_{i_1} - x_{di_1})}{(x_{bi_1} - x_{di_1})^2}, \dots, -\frac{2k_{T_i}[\min(0, f_{T_i}(\mathbf{x}_i))]^{N-1}(x_{in_i} - x_{din_i})}{(x_{bin_i} - x_{din_i})^2} \right]^T \\ & = (\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_{a_i})^T \left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}_i} \right)^T = (\dot{\mathbf{x}} - \dot{\mathbf{x}}_a)^T \left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T, \end{aligned} \quad (3.69)$$

where the corresponding terms in $\dot{\mathbf{x}} - \dot{\mathbf{x}}_a$ are multiplied by zero in $(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T$.

Substituting equations (3.68) and (3.69) into equation (3.67) yields:

$$\begin{aligned} \dot{P}_D(\mathbf{x}) &= k_p \alpha_x \sum_{i=1}^m \left\{ [(\dot{\mathbf{x}} - \dot{\mathbf{x}}_a)^T (\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T + \right. \\ &\quad \left. \dot{\mathbf{x}}^T (\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T] \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) - \sum_{j=1}^{N_i} [k_{c_{ij}} \dot{\mathbf{x}}^T (\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}})^T] \right. \\ &\quad \left. + [P_{T_i}(\mathbf{x}_i) + P_{E_i}(\mathbf{x}_i)] \sum_{j=1}^{N_i} [\dot{\mathbf{x}}^T (\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{k \neq j}^{N_i} P_{I_{ik}}(\mathbf{x}_i)] \right\} \\ &= k_p \alpha_x \{ \dot{\mathbf{x}}^T \Delta \boldsymbol{\varepsilon}_x - \sum_{i=1}^m [\dot{\mathbf{x}}_a^T (\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i)] \}. \end{aligned} \quad (3.70)$$

Differentiating equation (3.66) with respect to time and substituting equation (3.70) into it, it is obtained:

$$\begin{aligned} \dot{V} &= \mathbf{s}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{s}} + \frac{1}{2} \mathbf{s}^T \dot{\mathbf{M}}(\mathbf{q}) \mathbf{s} + k_p \alpha_x \dot{\mathbf{x}}^T \Delta \boldsymbol{\varepsilon}_x + k_p \alpha_r \dot{\mathbf{r}}^T \Delta \boldsymbol{\varepsilon}_r \\ &\quad - k_p \alpha_x \sum_{i=1}^m [\dot{\mathbf{x}}_a^T (\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i)] - \dot{\boldsymbol{\theta}}_d^T \mathbf{L}_d^{-1} \Delta \boldsymbol{\theta}_d. \end{aligned} \quad (3.71)$$

Substituting equations (3.62) and (3.65) into equation (3.71) and using Property 2.2 and $\mathbf{s}_q = [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q}) \mathbf{A} \mathbf{J}_x(\mathbf{q})] \mathbf{s}$, it is obtained:

$$\begin{aligned} \dot{V} &= -\mathbf{s}^T \mathbf{K}_s \mathbf{s} - \mathbf{s}^T [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q}) \mathbf{A} \mathbf{J}_x(\mathbf{q})]^T [k_p \alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + k_p \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r] \\ &\quad - \mathbf{s}^T \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r) \Delta \boldsymbol{\theta}_d + k_p \alpha_r \dot{\mathbf{r}}^T \Delta \boldsymbol{\varepsilon}_r - \dot{\boldsymbol{\theta}}_d^T \mathbf{L}_d^{-1} \Delta \boldsymbol{\theta}_d \\ &= -\mathbf{s}^T \mathbf{K}_s \mathbf{s} - \mathbf{s}_q^T [k_p \alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + k_p \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r] \\ &\quad - \mathbf{s}^T \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r) \Delta \boldsymbol{\theta}_d + k_p \alpha_r \dot{\mathbf{r}}^T \Delta \boldsymbol{\varepsilon}_r - \dot{\boldsymbol{\theta}}_d^T \mathbf{L}_d^{-1} \Delta \boldsymbol{\theta}_d \\ &\quad + k_p \alpha_x \{ \dot{\mathbf{x}}^T \Delta \boldsymbol{\varepsilon}_x - \sum_{i=1}^m [\dot{\mathbf{x}}_a^T (\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i)] \} \\ &= -\mathbf{s}^T \mathbf{K}_s \mathbf{s} + \mathbf{s}_q^T [-k_p \alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x - k_p \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r] + k_p \alpha_r \dot{\mathbf{r}}^T \Delta \boldsymbol{\varepsilon}_r \\ &\quad + k_p \alpha_x \{ \dot{\mathbf{x}}^T \Delta \boldsymbol{\varepsilon}_x - \sum_{i=1}^m [\dot{\mathbf{x}}_a^T (\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i)] \}. \end{aligned} \quad (3.72)$$

Then substituting equation (3.54) into equation (3.72), yields:

$$\begin{aligned}
\dot{V} = & -\mathbf{s}^T \mathbf{K}_s \mathbf{s} - k_p [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r]^T [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r] \\
& + [\dot{\mathbf{q}} - \mathbf{J}_x^+(\mathbf{q}) \dot{\mathbf{x}}_a]^T [-k_p \alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x - k_p \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r] \\
& + k_p \alpha_x \{ \dot{\mathbf{x}}^T \Delta \boldsymbol{\varepsilon}_x - \sum_{i=1}^m [\dot{\mathbf{x}}_a^T (\frac{\partial P_{T_i}(\mathbf{x})}{\partial \mathbf{x}})^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i)] \} + k_p \alpha_r \dot{\mathbf{r}}^T \Delta \boldsymbol{\varepsilon}_r \\
= & -\mathbf{s}^T \mathbf{K}_s \mathbf{s} - k_p [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r]^T [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r] \\
& + k_p \alpha_r \dot{\mathbf{x}}_a^T \mathbf{J}_x^{+T}(\mathbf{q}) \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r + k_p \alpha_x \dot{\mathbf{x}}_a^T \Delta \boldsymbol{\varepsilon}_x \\
& - k_p \alpha_x \sum_{i=1}^m [\dot{\mathbf{x}}_a^T (\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i)], \tag{3.73}
\end{aligned}$$

where the term $\dot{\mathbf{x}}_a^T \Delta \boldsymbol{\varepsilon}_x$ is given as:

$$\begin{aligned}
\dot{\mathbf{x}}_a^T \Delta \boldsymbol{\varepsilon}_x = & \sum_{i=1}^m \{ \dot{\mathbf{x}}_a^T [(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T + (\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T] \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) + \\
& [P_{T_i}(\mathbf{x}_i) + P_{E_i}(\mathbf{x}_i)] \sum_{j=1}^{N_i} [\dot{\mathbf{x}}_a^T (\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{k \neq j}^{N_i} P_{I_{ik}}(\mathbf{x}_i)] - \sum_{j=1}^{N_i} [k_{c_{ij}} \dot{\mathbf{x}}_a^T (\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}})^T] \}. \tag{3.74}
\end{aligned}$$

Substituting equation (3.74) into equation (3.73), we have:

$$\begin{aligned}
\dot{V} = & -\mathbf{s}^T \mathbf{K}_s \mathbf{s} - k_p [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r]^T \times \\
& [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r] + k_p \alpha_r \dot{\mathbf{x}}_a^T \mathbf{J}_x^{+T}(\mathbf{q}) \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r + \\
& k_p \alpha_x \sum_{i=1}^m \{ \dot{\mathbf{x}}_a^T (\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) - \sum_{j=1}^{N_i} [k_{c_{ij}} \dot{\mathbf{x}}_a^T (\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}})^T] \\
& + [P_{T_i}(\mathbf{x}_i) + P_{E_i}(\mathbf{x}_i)] \sum_{j=1}^{N_i} [\dot{\mathbf{x}}_a^T (\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{k \neq j}^{N_i} P_{I_{ik}}(\mathbf{x}_i)] \}. \tag{3.75}
\end{aligned}$$

From equations (3.50) and (3.53), we have:

$$\dot{\mathbf{x}}_{ai} = \mathbf{0}, \quad \text{if } f_{Er_i}(\mathbf{x}_i) \geq 0 \text{ or } f_{Ir_{ij}}(\mathbf{x}_i) \leq 0. \tag{3.76}$$

The condition indicates that $f_{Er_i}(\mathbf{x}_i) \geq 0$ or $f_{Ir_{ij}}(\mathbf{x}_i) \leq 0$ includes that $h_{E_i}(\mathbf{r}) \geq 0$ or $h_{I_{ij}}(\mathbf{r}) \leq 0$ where the reaching regional feedback $\Delta \boldsymbol{\varepsilon}_r$ is nonzero, thus $\mathbf{x}_{ai}$ and

$\Delta\boldsymbol{\varepsilon}_r$ cannot be nonzero at the same time.

Equation (3.76) also indicates that $\dot{\boldsymbol{x}}_a$ is nonzero where $f_{E_i}(\boldsymbol{x}_i) < 0$ and $f_{I_{ij}}(\boldsymbol{x}_i) > 0$. The gradient of potential energy $P_{E_i}(\boldsymbol{x}_i)$ reduces to zero where $f_{E_i}(\boldsymbol{x}_i) < 0$, while the gradient of potential energy $P_{I_{ij}}(\boldsymbol{x}_i)$ reduces to zero where $f_{I_{ij}}(\boldsymbol{x}_i) > 0$. Thus, $\boldsymbol{x}_{ai}$ and $(\frac{\partial P_{E_i}(\boldsymbol{x}_i)}{\partial \boldsymbol{x}})^T$, $\boldsymbol{x}_{ai}$ and $(\frac{\partial P_{I_{ij}}(\boldsymbol{x}_i)}{\partial \boldsymbol{x}})^T$ cannot be nonzero at the same time.

Therefore, the last two terms in above equation (3.75) are always zero so that:

$$\begin{aligned} \dot{V} = & -\boldsymbol{s}^T \boldsymbol{K}_s \boldsymbol{s} - k_p [\alpha_x \boldsymbol{J}_x^T(\boldsymbol{q}) \Delta\boldsymbol{\varepsilon}_x + \alpha_r \boldsymbol{J}_r^T(\boldsymbol{q}) \Delta\boldsymbol{\varepsilon}_r]^T \\ & \times [\alpha_x \boldsymbol{J}_x^T(\boldsymbol{q}) \Delta\boldsymbol{\varepsilon}_x + \alpha_r \boldsymbol{J}_r^T(\boldsymbol{q}) \Delta\boldsymbol{\varepsilon}_r]. \end{aligned} \quad (3.77)$$

The following theorem is now stated:

Theorem 3.1: *The regional feedback control law (3.61), the update law (3.62) for the robot system (3.63) guarantee the convergence of the tracking errors. That is $\boldsymbol{x} \rightarrow \boldsymbol{x}_d$, $\dot{\boldsymbol{x}} \rightarrow \dot{\boldsymbol{x}}_d$ as $t \rightarrow \infty$.*

Proof: Since $V > 0$ and $\dot{V} \leq 0$, V is bounded. Hence, $\boldsymbol{s}$, $\Delta\boldsymbol{\theta}_d$, $P_R(\boldsymbol{r})$ and $P_D(\boldsymbol{x})$ are bounded. Since $P_R(\boldsymbol{r})$ is bounded, $P_{E_i}(\boldsymbol{r})$ and $P_{I_{ij}}(\boldsymbol{r})$ are bounded. The boundedness of $P_{E_i}(\boldsymbol{r})$ and $P_{I_{ij}}(\boldsymbol{r})$ ensures the boundedness of $h_{E_i}(\boldsymbol{r})$ and $h_{I_{ij}}(\boldsymbol{r})$. In addition, $P_{E_i}(\boldsymbol{x}_i)$, $P_{I_{ij}}(\boldsymbol{x}_i)$ and $P_{T_i}(\boldsymbol{x}_i)$ are also bounded since $P_D(\boldsymbol{x})$ is bounded. The boundedness of $P_{E_i}(\boldsymbol{x}_i)$, $P_{I_{ij}}(\boldsymbol{x}_i)$ and $P_{T_i}(\boldsymbol{x}_i)$ ensures the boundedness of $f_{E_i}(\boldsymbol{x}_i)$, $f_{I_{ij}}(\boldsymbol{x}_i)$ and $f_{T_i}(\boldsymbol{x}_i)$. Since the region functions are bounded, the reaching task variable $\boldsymbol{r}$ and the desired task variable $\boldsymbol{x}$ are bounded. Hence, $\frac{\partial h_{E_i}(\boldsymbol{r})}{\partial \boldsymbol{r}}$, $\frac{\partial h_{I_{ij}}(\boldsymbol{r})}{\partial \boldsymbol{r}}$, $\frac{\partial f_{E_i}(\boldsymbol{x}_i)}{\partial \boldsymbol{x}}$, $\frac{\partial f_{I_{ij}}(\boldsymbol{x}_i)}{\partial \boldsymbol{x}}$ and $\frac{\partial f_{T_i}(\boldsymbol{x}_i)}{\partial \boldsymbol{x}}$ are also bounded. Therefore, $\Delta\boldsymbol{\varepsilon}_r$ and $\Delta\boldsymbol{\varepsilon}_x$ are bounded. Since $\boldsymbol{x}$ is bounded, $\dot{\boldsymbol{x}}_a$ is bounded if $\dot{\boldsymbol{x}}_d$ is bounded. Hence, $\dot{\boldsymbol{q}}_r$ is bounded. From equation (3.60), $\dot{\boldsymbol{q}}$ is bounded because $\boldsymbol{s}$ is bounded. The boundedness of $\dot{\boldsymbol{q}}$ guarantees the boundedness of $\dot{\boldsymbol{x}}$ and $\dot{\boldsymbol{r}}$ since both $\boldsymbol{J}_x(\boldsymbol{q})$ and $\boldsymbol{J}_r(\boldsymbol{q})$ are trigonometric functions of $\boldsymbol{q}$ or constant. Therefore, $\Delta\dot{\boldsymbol{\varepsilon}}_r$ and $\Delta\dot{\boldsymbol{\varepsilon}}_x$ are bounded. Then $\ddot{\boldsymbol{q}}_r$ is bounded if $\ddot{\boldsymbol{x}}_d$ is

bounded. From the closed-loop equation (3.65), it is concluded that $\dot{\mathbf{s}}$ is bounded. Thus, $\ddot{V}$ is bounded since $\dot{\mathbf{s}}$, $\mathbf{s}$, $\Delta\dot{\boldsymbol{\varepsilon}}_r$, $\Delta\boldsymbol{\varepsilon}_r$, $\Delta\dot{\boldsymbol{\varepsilon}}_x$, $\Delta\boldsymbol{\varepsilon}_x$ are bounded. Therefore, $\dot{V}$ is uniformly continuous. Applying Barbalat's lemma [83], it is obtained that $\dot{V} \rightarrow 0$ which also indicates:

$$\begin{aligned} \alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta\boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta\boldsymbol{\varepsilon}_r &\rightarrow \mathbf{0}, \\ \mathbf{s} &\rightarrow \mathbf{0}. \end{aligned} \quad (3.78)$$

If the end effector is located where $h_{E_i}(\mathbf{r}) \geq 0$ or $h_{I_{ij}}(\mathbf{r}) \leq 0$, $\Delta\boldsymbol{\varepsilon}_x = \mathbf{0}$, $\Delta\boldsymbol{\varepsilon}_r \neq \mathbf{0}$, which contracts with equation (3.78) since $\mathbf{J}_r(\mathbf{q})$ is non-singular. Therefore, the end effector can only settle down where $f_{E_i}(\mathbf{x}_i) \leq 0$ and $f_{I_{ij}}(\mathbf{x}_i) \geq 0$, and hence $\Delta\boldsymbol{\varepsilon}_r = \mathbf{0}$. Since $\mathbf{J}_x(\mathbf{q})$ is also non-singular, from equation (3.78) $\Delta\boldsymbol{\varepsilon}_x = \mathbf{0}$. From equation (3.39), $\Delta\boldsymbol{\varepsilon}_x = \mathbf{0}$ can only be satisfied where $f_{T_i}(\mathbf{x}_i) \leq 0$, $f_{Er_i}(\mathbf{x}_i) \leq 0$ and $f_{Ir_{ij}}(\mathbf{x}_i) \geq 0$. Hence $\Delta\boldsymbol{\varepsilon}_x = \mathbf{0}$ means that $\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} = \mathbf{0}$. That is, $\mathbf{x} \rightarrow \mathbf{x}_d$ as $t \rightarrow \infty$. From the definition of $\dot{\mathbf{x}}_a$ in equation (3.53), $\mathbf{x} \rightarrow \mathbf{x}_d$ indicate that $\dot{\mathbf{x}}_a \rightarrow \dot{\mathbf{x}}_d$. Then from the definition of $\mathbf{s}$ in equation (3.60), $\dot{\mathbf{x}}_a \rightarrow \dot{\mathbf{x}}_d$, $\Delta\boldsymbol{\varepsilon}_r \rightarrow \mathbf{0}$, $\Delta\boldsymbol{\varepsilon}_x \rightarrow \mathbf{0}$ and $\mathbf{s} \rightarrow \mathbf{0}$ implies $\dot{\mathbf{x}} \rightarrow \dot{\mathbf{x}}_d$ as $t \rightarrow \infty$. △△△

Remark 3.7 The combination of potential energies does not result in local minimum for the tracking control task. This can be shown by analyzing the partial derivatives of the potential energy in equations (3.30) and (3.39) and the proposed controller in equation (3.61).

(i) If the end effector is located where $f_{E_i}(\mathbf{x}_i) \leq 0$, $f_{Er_i}(\mathbf{x}_i) > 0$ and $f_{Ir_{ij}}(\mathbf{x}_i) \geq 0$, the gradient of $P_i(\mathbf{x}_i)$ is nonzero, and the gradient of $P_{I_{ij}}(\mathbf{x}_i)$ reduces to zero. Therefore, $\Delta\boldsymbol{\varepsilon}_x = \sum_{i=1}^m [(\frac{\partial P_i(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i)]$, which is nonzero until the end effector is inside the external reference regions such that $f_{Er_i}(\mathbf{x}_i) \leq 0$.

(ii) After the end effector enters the external reference regions, it is inside the task-

oriented regions such that $f_{T_i}(\mathbf{x}_i) \leq 0$. If the end effector is located where $f_{T_i}(\mathbf{x}_i) \leq 0$ and $f_{I_{r_{ij}}}(\mathbf{x}_i) \geq 0$, then both the gradient of $P_{E_i}(\mathbf{x}_i)$ and $P_{I_{ij}}(\mathbf{x}_i)$ are zero, and $P_i(\mathbf{x}_i) = P_{T_i}(\mathbf{x}_i)$. Therefore, $\Delta\boldsymbol{\varepsilon}_x = \sum_{i=1}^m [(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i)]$, which is nonzero until the end effector reaches the desired position.

(iii) When the end effector starts outside the external or internal regions such that $f_{E_i}(\mathbf{x}_i) > 0$ or $f_{I_{ij}}(\mathbf{x}_i) < 0$, the reaching task feedback $\mathbf{r}$ is activated to enable the end effector to leave those regions.

(iv) However, there is one saddle point between the internal regions and the internal reference regions where $f_{I_{ij}}(\mathbf{x}_i) \geq 0$ and $f_{I_{r_{ij}}}(\mathbf{x}_i) < 0$, and the gradient of potential energy $(\frac{\partial P_D(\mathbf{x})}{\partial \mathbf{x}})^T$ at the saddle point is zero. The saddle point exists in the presence of internal regions. This is because that the potential energy $P_i(\mathbf{x}_i)$ is firstly multiplied by $P_{I_{ij}}(\mathbf{x}_i)$ and then added with the offset term. If the end effector reaches the saddle position, both $\Delta\boldsymbol{\varepsilon}_x$ and $\Delta\boldsymbol{\varepsilon}_r$ reduce to zero. However, both $\mathbf{s} = \dot{\mathbf{q}} - [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]^{-1}[\mathbf{J}_x^+(\mathbf{q})\dot{\mathbf{x}}_f - \alpha_x \mathbf{J}_x^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_x - \alpha_r \mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_r]$ and $\boldsymbol{\tau}$ are nonzero because the velocity control term $\dot{\mathbf{q}} - [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]^{-1}\mathbf{J}_x^+(\mathbf{q})\dot{\mathbf{x}}_f$ is not zero. When the system starts from rest at the saddle point with $\dot{\mathbf{x}}_d = \mathbf{0}$, the initial estimates of the uncertain parameters $\hat{\boldsymbol{\theta}}_d$ can be set as nonzero values so that $\mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\hat{\boldsymbol{\theta}}_d$ is nonzero and thus result in a non-zero initial control input. This enables the end effector to leave the saddle point. If all elements in the regressor matrix reduce to zero, the initial control input may reduce to zero, and an over-parameterization of dynamic model is needed to move the end effector out of the saddle point. $\triangle\triangle\triangle$

Remark 3.8 In the control law described by equation (3.61), note that the pseudo-inverse Jacobian $\mathbf{J}_x^+(\mathbf{q})$ can be replaced by $\mathbf{J}_x^{-1}(\mathbf{q})$ in the case of non-redundant robots. Hence the proposed controller can be applied to both non-redundant and redundant robots. $\triangle\triangle\triangle$

Remark 3.9 The end effector does not avoid the regions where the singularities or

occlusion may occur, and it can start from those regions. Once the end effector is outside the external and internal regions in the coordinates of $\mathbf{x}$ such that $f_{E_i}(\mathbf{x}_i) \geq 0$ or $f_{I_{ij}}(\mathbf{x}_i) \leq 0$, only the reaching task feedback $\mathbf{r}$ is employed. From equation (3.39), the desired regional feedback error $\Delta\boldsymbol{\varepsilon}_x = \mathbf{0}$. From the definition of the weight factors in Appendix, $w_i(\mathbf{x}_i) = 0$, and hence equation (3.50) is satisfied, i.e. $\dot{\mathbf{x}}_{di} = \mathbf{0}$. From equations (3.56), (3.57) and (3.58), $\dot{\mathbf{x}}_f = \mathbf{0}$, and $\mathbf{A}$ reduces to a zero matrix. Therefore, $[\mathbf{I}_n - \mathbf{J}^+(\mathbf{q})\mathbf{A}\mathbf{J}(\mathbf{q})]^{-1} = \mathbf{I}_n$, and the controller in equation (3.61) becomes

$$\boldsymbol{\tau} = -k_p\alpha_r\mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_r - \mathbf{K}_s\mathbf{s} + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\hat{\boldsymbol{\theta}}_d, \quad (3.79)$$

where $\mathbf{s} = \dot{\mathbf{q}} - \alpha_r\mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_r$, which drives the end effector to leave the regions where the singularities or occlusion may occur, without the pseudo inverse matrix $\mathbf{J}_x^+(\mathbf{q})$.

After the end effector leaves the regions such that $h_{E_i}(\mathbf{r}) < 0$ and $h_{I_{ij}}(\mathbf{r}) > 0$, from equation (3.30) the reaching regional feedback error $\Delta\boldsymbol{\varepsilon}_r$ naturally reduces to zero, and the desired regional feedback error $\Delta\boldsymbol{\varepsilon}_x$ is activated. In addition, the weight factors w_i smoothly increase to 1. In the case that $w_i(\mathbf{x}_i) = 1$, $\dot{\mathbf{x}}_{di} \neq \mathbf{0}$. $\triangle\triangle\triangle$

Remark 3.10 The results of regional feedback control can be extended to an adaptive regional feedback controller [58]. The adaptive controller with regional feedback is able to drive the robot from outside to inside the field of view and track the desired trajectory in the presence of uncertain robot kinematics and depth information. The main idea is to partition the robot kinematics into a known internal portion (manipulator kinematic parameters) and an unknown external portion (tool kinematic parameters), while the known portion is specified in Cartesian space and the unknown portion is specified in image space. The Cartesian-space region error $\Delta\boldsymbol{\varepsilon}_r$ is used for the region reaching control of known internal portion, and the image-space region error $\Delta\boldsymbol{\varepsilon}_x$ is used for the adaptive tracking control of unknown external

portion, as illustrated in Fig. 3.14.

Figure 3.14: A manipulator with a grasped tool, and $\hat{J}_t(q)$ is the approximate tool Jacobian matrix.

The adaptive regional feedback controller is proposed as:

$$\begin{aligned} \tau_a = & -[I_n - \hat{J}^+(q, \hat{\theta}_k) \hat{Z}(q, \hat{\theta}_z) A \hat{Z}^{-1}(q, \hat{\theta}_z) \hat{J}(q, \hat{\theta}_k)]^T [k_p \alpha_r J_r^T(q) \Delta \epsilon_r \\ & + k_p \alpha_x \hat{J}^T(q, \hat{\theta}_k) \hat{Z}^{-1}(q, \hat{\theta}_z) \Delta \epsilon_x] - K_s \hat{s}_a + Y_d(q, \dot{q}, \dot{\hat{q}}_{ar}, \ddot{\hat{q}}_{ar}) \hat{\theta}_d, \end{aligned} \quad (3.80)$$

where the estimated parameters $\hat{\theta}_d$, $\hat{\theta}_k$ and $\hat{\theta}_z$ are updated using the following update laws:

$$\begin{aligned} \dot{\hat{\theta}}_d &= -L_d Y_d^T(q, \dot{q}, \dot{\hat{q}}_{ar}, \ddot{\hat{q}}_{ar}) \hat{s}_a, \\ \dot{\hat{\theta}}_k &= -k_p \alpha_x L_k Y_k^T(q, \dot{q}) \hat{Z}^{-1}(q, \hat{\theta}_z) \Delta \epsilon_x, \\ \dot{\hat{\theta}}_z &= k_p \alpha_x L_z Y_z^T(q, \dot{x}) \hat{Z}^{-1}(q, \hat{\theta}_z) \Delta \epsilon_x, \end{aligned} \quad (3.81)$$

where L_k and L_z are symmetric positive definite matrices, and $Y_k(q, \dot{q})$ is the kinematic regressor, and $Y_z(q, \dot{x})$ is the depth regressor, and $\hat{Z}(q, \hat{\theta}_z)$ is the approximate depth matrix, and $\hat{J}(q, \hat{\theta}_k)$ denotes an approximated Jacobian matrix, and $\hat{s}_a$ is a

sliding vector defined as:

$$\begin{aligned} \hat{s}_a = \dot{\mathbf{q}} - \dot{\mathbf{q}}_{ar} = \dot{\mathbf{q}} - [\mathbf{I}_n - \hat{\mathbf{J}}^+(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \mathbf{A} \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \hat{\mathbf{J}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_k)]^{-1} \\ \times [\hat{\mathbf{J}}^+(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \dot{\mathbf{x}}_f - \alpha_x \hat{\mathbf{J}}^T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \boldsymbol{\varepsilon}_x - \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r]. \end{aligned} \quad (3.82)$$

The stability analysis can be found in [58] and is omitted here as it follows a similar argument as in the proof of Theorem 3.1. △△△

3.5 Experiment

The experimental setup consists of a Sony SCARA robot and a PSD camera (C5949) manufactured by Hamamatsu as shown in Fig. 3.15. The proposed controllers were implemented on the first two links of the robot. The lengths of the first and the second links are $l_1 = 0.35 \text{ m}$, $l_2 = 0.37 \text{ m}$ respectively. The Jacobian matrix of the robot is specified in equation (2.14).

The joint motors of the robot are driven by servo amplifiers. The amplifiers are connected to a Servo To Go I/O card (model II). The servo I/O card is an ISA-bus based general purpose data acquisition card. The optical incremental encoders in the robot monitors the joint positions with a resolution of 500 lines. Joint velocities are obtained from differentiation of the joint angles. The 24-bit counters of the card are read by a computer serving as the controller in which one Pentium III 450 MHz processor and 128 MB DRAM are installed. The control signals are fed through the digital-to-analogue converters of the servo I/O card to the amplifiers. The digital-to-analog converters have a 13-bit resolution and the output voltage has a -10 V to $+10 \text{ V}$ range.

Figure 3.15: Experimental setup

In the experiments, the regional feedback is specified in different task-space coordinates to address various task-space control problems.

3.5.1 Singularity Problem

In the first experiment, the desired task variable $\mathbf{x}$ is specified in Cartesian space, and the Jacobian matrix $\mathbf{J}_x(\mathbf{q}) = \mathbf{J}_m(\mathbf{q})$ is from joint space to Cartesian space and hence its determinant is $\det[\mathbf{J}_m(\mathbf{q})] = l_1 l_2 \sin(q_2)$. Therefore, the external singularity occurs where $q_2 = 0$, and the internal singularity occurs where $q_2 = \pi$. To solve the singularity problem, the reaching task variable $\mathbf{r}$ is specified in joint space, and the Jacobian matrix $\mathbf{J}_r(\mathbf{q}) = \mathbf{I}_2$ is an identity matrix which is not singular.

From equation (3.3), the external and internal singular regions are set in joint space

as:

$$\begin{aligned} h_{E_1}(\mathbf{q}) &= \pi/15^2 - q_2^2 \geq 0, \\ h_{I_{11}}(\mathbf{q}) &= (q_2 - \pi)^2 - \pi/10^2 \leq 0. \end{aligned} \quad (3.83)$$

Both the external and internal singularity corresponds to circular path in Cartesian space. Therefore, for the purpose of matching, another external region $f_{E_1}(\mathbf{p}_1) \leq 0$ and internal region $\mathbf{f}_{I_{11}}(\mathbf{p}_1) \geq 0$ are formulated in Cartesian space. From the general expression of external and internal region described by equations (3.16) and (3.17), the region functions are specified as:

$$\begin{aligned} f_{E_1}(\mathbf{p}_1) &= \frac{p_{p1_1}^2}{0.718^2} + \frac{p_{p1_2}^2}{0.718^2} - 1 \leq 0, \\ f_{I_{11}}(\mathbf{p}_1) &= \frac{p_{p1_1}^2}{0.112^2} + \frac{p_{p1_2}^2}{0.112^2} - 1 \geq 0, \end{aligned} \quad (3.84)$$

and the reference regions $f_{Er_1}(\mathbf{p}_1)$ and $\mathbf{f}_{Ir_{11}}(\mathbf{p}_1)$ are set with the parameters $\kappa_{E_1} = 0.65$ and $\kappa_{I_{11}} = 1.5$. From the general expression of task-oriented region described by equation (3.24), the function of the task-oriented region is specified as:

$$f_{T_1}(\mathbf{p}_1) = \frac{(p_{p1_1} - p_{d1_1})^2}{(p_{b1_1} - p_{d1_1})^2} + \frac{(p_{p1_2} - p_{d1_2})^2}{(p_{b1_2} - p_{d1_2})^2} - 1 \leq 0, \quad (3.85)$$

where $p_{b1_1} = -0.51 \text{ m}$ if $p_{p1_1} \leq p_{d1_1}$, else $p_{b1_1} = 0.51 \text{ m}$, and $p_{b1_2} = -0.51 \text{ m}$ if $p_{p1_2} \leq p_{d1_2}$, else $p_{b1_2} = 0.51 \text{ m}$.

The end effector is required to start from an initial position at $(-0.05 \text{ m}, 0.72 \text{ m})$ where q_2 is near the singular position $q_2 = 0$, and track a time-varying trajectory

specified in Cartesian space as:

$$\begin{cases} p_{d1_1} = -0.35 + 0.1\cos(0.4t - \frac{\pi}{2})w(\mathbf{p}_1), \\ p_{d1_2} = 0.13 + 0.1\sin(0.4t - \frac{\pi}{2})w(\mathbf{p}_1), \end{cases}$$

where the $w(\mathbf{p}_1)$ is the weight factor. The order of potential energy in equations (3.25), (3.26), (3.32), (3.35), (3.37) is $N = 4$, and the control parameters in equation (3.61) and (3.62) were set as: $k_{rE_1} = 200$, $k_{T_1} = 0.3$, $k_{xE_1} = 6000$, $k_{xI_{11}} = 200000$, $\mathbf{K}_s = \text{diag}\{0.0005, 0.0005\}$, $k_p = 1$, $\alpha_x = 1$, $\alpha_r = 1$, $\mathbf{L}_d = \text{diag}\{0.001, 0.001\}$. The experimental results are shown in Fig. 3.16(a)-3.16(c).

Next, the end effector starts from another initial position at $(-0.19 \text{ m}, 0.02 \text{ m})$ where q_2 is near the singular position $q_2 = \pi$. The control parameters remain the same, and the experimental results are shown in Fig. 3.17(a)-3.17(c).

From Fig. 3.16 and Fig. 3.17, it is seen that the end effector can start from the singular configurations and transit smoothly from joint space to Cartesian space. After it enters the task-oriented region $f_{T_1}(\mathbf{p}_1) \leq 0$, the position of end effector converges to the desired trajectory.

(a) Path of end effector in Cartesian space

(b) Tracking errors

(c) Control input

Figure 3.16: The end effector starts from an initial position at $(-0.05 \text{ m}, 0.72 \text{ m})$ near the external singular configuration.

(a) Path of end effector in Cartesian space

(b) Tracking errors

(c) Control input

Figure 3.17: The end effector starts from an initial position at $(-0.19 \text{ m}, 0.02 \text{ m})$ near the internal singular configuration.

3.5.2 Limited Field of View

In the second experiment, the PSD camera is used to measure the position of end effector and outputs the coordinate data in the unit of voltage [104] within $-5 \sim 5$ V. The size of image plane is $0.01 \text{ m} \times 0.01 \text{ m}$ while the focal length of the camera $f = 0.025 \text{ m}$.

The reaching task variable $\mathbf{r}$ is specified in Cartesian space, and the desired task variable $\mathbf{x}$ is specified in image space. Therefore, the Jacobian matrix $\mathbf{J}_r(\mathbf{q})$ is from joint space to Cartesian space as: $\mathbf{J}_r(\mathbf{q}) = \mathbf{J}_m(\mathbf{q})$. The Jacobian matrix $\mathbf{J}_x(\mathbf{q})$ from joint space to image space is described by equation (2.25), where the depth from the lens of camera to the end effector evolving plane is $z = 1.16$.

The external region $f_{E_1}(\mathbf{x}_{I1}) \leq 0$ in image space is used to specify the size of field of view. From the general expression of external region described by equation (3.16), the region function was specified to match the rectangular field of view as:

$$f_{E_1}(\mathbf{x}_{I1}) = \frac{(x_{1h}+0.22)^{10}}{1.85^{10}} + \frac{(x_{1v}-0.55)^{10}}{1.85^{10}} - 1 \leq 0, \quad (3.86)$$

where the reference region $f_{E_1}(\mathbf{x}_{I1})$ was set with the parameter $\kappa_{E_1} = 0.81$.

The external region $h_{E_1}(\mathbf{p}) \geq 0$ in Cartesian space enables the robot to reach the field of view. From equation (3.8), the region function was specified as:

$$\begin{aligned} h_{E_1}(\mathbf{p}) &= \frac{(p_{p1_1} + 0.42)^2}{(p_{b1_1} + 0.42)^2} - 1 \geq 0, \\ h_{E_2}(\mathbf{p}) &= \frac{(p_{p1_2} - 0.13)^2}{(p_{b1_2} - 0.13)^2} - 1 \geq 0, \end{aligned} \quad (3.87)$$

where $p_{b1_1} = -0.48 \text{ m}$ if $p_{p1_1} \leq p_{r1_1}$, else $p_{b1_1} = -0.36 \text{ m}$, and $p_{b1_2} = 0.06 \text{ m}$ if $p_{p1_2} \leq p_{r1_2}$, else $p_{b1_2} = 0.19 \text{ m}$, and $\mathbf{p}_{r1} = [-0.42, 0.13]^T$ represents the reference position.

From the general expression of task-oriented region in equation (3.24), the task-oriented region $f_{T_1}(\mathbf{x}_{I1})$ was specified as:

$$f_{T_1}(\mathbf{x}_{I1}) = \frac{(x_{1h} - x_{d1h})^2}{(x_{b1h} - x_{d1h})^2} + \frac{(x_{1v} - x_{d1v})^2}{(x_{b1v} - x_{d1v})^2} - 1 \leq 0, \quad (3.88)$$

where $x_{b1h} = -1.72$ V if $x_{1h} \leq x_{d1h}$, else $x_{b1h} = 1.78$ V, and $x_{b1v} = -1.45$ V if $x_{1v} \leq x_{d1v}$, else $x_{b1v} = 2.25$ V.

First, the end effector is controlled to start from an initial position at $(-0.01$ m, 0.61 m) which is outside the field of view, and track the desired trajectory in image space as:

$$\begin{cases} x_{d1h} = -0.22 + 0.7\cos(0.4t)w(\mathbf{x}_{I1}), \\ x_{d1v} = 0.55 + 0.7\sin(0.4t)w(\mathbf{x}_{I1}). \end{cases}$$

where the $w(\mathbf{x}_{I1})$ is the weight factor. The control parameters were set as: $k_{rE_1} = 3 \times 10^{-7}$, $k_{T_1} = 0.1$, $k_{xE_1} = 0.01$, $\mathbf{K}_s = \text{diag}\{0.0005, 0.0005\}$, $k_p = 1$, $\alpha_x = 1$, $\alpha_r = 1$, $\mathbf{L}_d = \text{diag}\{0.001, 0.001\}$. The experimental results are shown in Figure 3.18(a)-3.18(d).

Next, suppose that there is an occluded area locating near the position $[-0.22, -0.15]$ V within the field of view. To enclose the occluded area, an internal region $f_{I11}(\mathbf{x}_{I1}) \geq 0$ is introduced. From the general expression of internal region in equation (3.17), the region function was specified as:

$$f_{I11}(\mathbf{x}_{I1}) = \frac{(x_{1h} + 0.22)^2}{0.4^2} + \frac{(x_{1v} - 0.15)^2}{0.2^2} - 1 \leq 0, \quad (3.89)$$

and the reference region $f_{Ir11}(\mathbf{x}_{I1})$ was set with the parameter $\kappa_{I11} = 1.25$. The control parameters remain the same, and the experimental results are shown in Figure 3.19(a)-3.19(d).

(a) Path of end effector in Cartesian space

(b) Path of end effector in image space

(c) Tracking errors in image space

(d) Control input

Figure 3.18: The end effector starts from an initial position at $(-0.01 \text{ m}, 0.61 \text{ m})$ outside the field of view, and transits smoothly from outside to inside field of view to track the desired position in image space.

As seen from Figure 3.18(a)-3.18(d), the end effector can transit smoothly from outside to inside the field of view, and converge to the desired trajectory within the field of view. In the presence of image occlusion, the robot is able to pass through the occluded area with the Cartesian-space feedback and complete the tracking task after it leaves the occluded area, as seen from Figure 3.19(a)-3.19(d).

(a) Path of end effector in Cartesian space

(b) Path of end effector in image space

(c) Tracking errors in image space

(d) Control input

Figure 3.19: In the presence of image occlusion, the robot can pass through the occluded area with Cartesian-space feedback, and still converge to the desired trajectory in the end.

3.6 Summary

In this chapter, some examples of the feedback variables $\mathbf{x}$ and $\mathbf{r}$ and the corresponding task-space regions are first given, and a general and systematic formulation of region functions is then presented. By using the region functions, the corresponding potential energy functions are proposed, and the region errors are derived based on the gradient of the potential energy.

By using the regional feedback, a novel task-space control method is proposed for the robotic manipulator, where the reaching task variable drives the robot from one task space to another, and the desired task variable drives the robot to the desired position at the ending stage. The transition between the reaching task feedback and the desired task feedback is embedded in the controllers without using any hard or discontinuous switching. It is shown that the proposed regional feedback method is a unified formulation to address various open issues in task-space control problems such as singularity problem and limited sensing zone. The stability of the closed-loop system is shown by using Lyapunov-like analysis, and experimental results are presented to illustrate the performance of the proposed controllers.

Chapter 4

Multiple Regional Feedback Control

In chapter 3, the proposed regional feedback method integrates dual task-space variables into one controller which can solve the problems of singularity and limited field of view separately. In this chapter, the results are extended to a multiple regional feedback controller that is able to deal with the issues of singularity and limited field of view at the same time.

4.1 Multiple Regional Feedback

The multiple regional feedback method is consisted of two reaching task variables and one desired task variable. The joint-space information and the Cartesian-space feedback serve together as the reaching task feedback, while the vision is specified as the desired task feedback. Note that the Cartesian-space feedback is indispensable to the proposed multiple regional feedback method, which enables the robot to transit from joint space to image space without solving the inverse kinematics.

The concept of the multiple regional feedback can be illustrated by considering a 2 DOF manipulator system with a fixed camera configuration as shown in Fig. 4.1. In Fig. 4.1, the entire robot workspace is divided into three categories. The joint-space regions are defined around the singular positions, and the robot employs joint information inside the joint-space regions to leave singular configurations. The image-space regions are defined in the vicinity of the desired trajectory within the field of view of cameras, and the robot utilizes vision feedback only after the end effector enters the image-space region. Finally, the Cartesian-space regions cover the remaining workspace, to ensure a smooth transition from joint-space regions to image-space regions. Each feedback information is employed in a corresponding local region, and the combination of regional feedback guarantees the global stability of robot movement.

Figure 4.1: The end effector starts from an initial position near the external singular position, and tracks a trajectory within the field of view.

4.1.1 Joint-Space Feedback

As discussed in section 2.1.2, the singularities of the robotic manipulator can be divided into external singularity and internal singularity, where the former is denoted as the external boundary singularity and the latter includes the internal boundary singularity and the interior singularity. For the multiple task-space control, the joint-space region functions are formulated as in section 3.2.1 to cover the singular positions.

Since the joint-space feedback serves as the reaching task variable, the potential energy for the joint-space regions is introduced as in equation (3.29) as:

$$P_J(\mathbf{q}) = k_p \alpha_q \sum_{i=1}^m [P_{J_{E_i}}(\mathbf{q}) + \sum_{j=1}^{N_i} P_{J_{I_{ij}}}(\mathbf{q})] = k_p \alpha_q \times \sum_{i=1}^m \left\{ \frac{k_{qE_i}}{N} [\max(0, h_{J_{E_i}}(\mathbf{q}))]^N + \sum_{j=1}^{N_i} \frac{k_{qI_{ij}}}{N} [\min(0, h_{J_{I_{ij}}}(\mathbf{q}))]^N \right\}, \quad (4.1)$$

where k_{qE_i} , $k_{qI_{ij}}$ and α_q are positive constants, and $h_{J_{E_i}}(\mathbf{q})$ and $h_{J_{I_{ij}}}(\mathbf{q})$ represent the external and internal singular regions which are described in equations (3.12) and (3.13). The robot employs only the joint information within the singular regions so that the inverse Jacobian matrix is not required. In equation (4.1), $P_{J_{E_i}}(\mathbf{q})$ and $P_{J_{I_{ij}}}(\mathbf{q})$ are the potential energy functions for the external and internal singular regions respectively. Note that the potential energy in joint space is lower bounded by zero, and it smoothly reduces to zero where $h_{J_{E_i}}(\mathbf{q}) < 0$ and $h_{J_{I_{ij}}}(\mathbf{q}) > 0$. Therefore, the potential energy $P_J(\mathbf{q})$ will be used to move the end effector out of the singular regions.

Example 4.1 Consider the manipulator system in Fig. 4.1, the singularities occur when $q_2 = 0$ where the manipulator is fully stretched out, or $q_2 = \pi$ where the manipulator is fully folded back. Therefore, two joint-space regions are specified to

cover the singular positions $q_2 = 0$ and $q_2 = \pi$ respectively as:

$$\begin{aligned} h_{J_{E_1}}(\mathbf{q}) &= b_{E_1}^2 - (q_2)^2 \geq 0, \\ h_{J_{I_{11}}}(\mathbf{q}) &= (q_2 - \pi)^2 - b_{I_{11}}^2 \leq 0, \end{aligned} \quad (4.2)$$

where b_{E_1} and $b_{I_{11}}$ are positive constants that represent the sizes of singular regions. The singular regions $h_{J_{E_1}}(\mathbf{q})$ and $h_{J_{I_{11}}}(\mathbf{q})$ correspond to circles in Cartesian space, as illustrated in Fig. 4.2. △△△

Figure 4.2: The singular regions $h_{J_{E_1}}(\mathbf{q})$ and $h_{J_{I_{11}}}(\mathbf{q})$ correspond to circular path in Cartesian space.

4.1.2 Image-Space Feedback

Since the vision feedback serves as the desired task variable, the potential energy for the image-space regions is introduced as in equation (3.38) as:

$$P_D(\mathbf{x}_I) = k_p \alpha_x \sum_{i=1}^m \left\{ P_i(\mathbf{x}_{Ii}) \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_{Ii}) + \sum_{j=1}^{N_i} k_{c_{ij}} \left[\frac{k_{x_{I_{ij}}}}{N^2} \left(\frac{1}{\kappa_{I_{ij}}} - 1 \right)^{N^2} - P_{I_{ij}}(\mathbf{x}_{Ii}) \right] \right\}, \quad (4.3)$$

where m is the number of feature points, and $P_i(\mathbf{x}_{Ii}) = P_{E_i}(\mathbf{x}_{Ii}) + P_{T_i}(\mathbf{x}_{Ii})$, and $P_{E_i}(\mathbf{x}_{Ii})$, $P_{I_{ij}}(\mathbf{x}_{Ii})$ and $P_{T_i}(\mathbf{x}_{Ii})$ are the potential energy functions for the external regions, internal regions and task-oriented regions in image space respectively. The image-space regions are specified based on the general expressions in equations (3.16), (3.17) and (3.24). The potential energy in image space $P_D(\mathbf{x}_I)$ is constant when the end effector is outside the image-space region, and the bottom of potential energy corresponds to the desired position so that the convergence of tracking errors is guaranteed.

Example 4.2 Consider the manipulator system in Fig. 4.1 again, the external region in image space $f_{E_1}(\mathbf{x}_{I1})$ described by equation (3.16) is specified to match the rectangular field of view as:

$$f_{E_1}(\mathbf{x}_{I1}) = \frac{(x_{1h} - x_{E1h})^{20}}{a_1^{20}} + \frac{(x_{1v} - x_{E1v})^{20}}{a_2^{20}} - 1 \leq 0, \quad (4.4)$$

where a_1 , a_2 are positive constants which are set to ensure that $f_{E_1}(\mathbf{x}_{I1}) \leq 0$ is inside the field of view, and the reference position $\mathbf{x}_{E1} = [x_{E1h}, x_{E1v}]^T$ is set as the center of the field of view. In addition, the task-oriented region in equation (3.24) is introduced to ensure the convergence of tracking errors as:

$$f_{T_1}(\mathbf{x}_{I1}) = \frac{(x_{1h} - x_{d1h})^2}{(x_{b1h} - x_{d1h})^2} + \frac{(x_{1v} - x_{d1v})^2}{(x_{b1v} - x_{d1v})^2} - 1 \leq 0, \quad (4.5)$$

where $\mathbf{x}_{b1} = [x_{b1h}, x_{b1v}]^T$ is the boundary position. The image-space region is illustrated in Fig. 4.3. △△△

Figure 4.3: The external image-space region $f_{E_1}(\mathbf{x}_{I1})$ is introduced to match the field of view.

4.1.3 Cartesian-space Feedback

Besides the joint-space regions and image-space regions, the Cartesian-space regions are specified to cover the remaining robot workspace. Therefore, the Cartesian-space feedback serves as another reaching task variable to guarantee a smooth transition from joint-space regions to image-space regions. The Cartesian-space feedback is used for the proposed multiple regional feedback method, since solving the inverse kinematics is required if the joint information is employed as the reaching task variable to drive the end effector to enter the field of view of each camera.

Since the joint-space regions and image-space regions correspond to different shapes in Cartesian space, the Cartesian-space regions are specified as a set of sub regions to match the joint-space regions and image-space regions respectively.

Sub regions $h_{CE_i}(\mathbf{p}_i)$ to match the external singular regions

To match the external singular region $h_{JE_i}(\mathbf{q})$, Cartesian-space sub regions are specified as:

$$h_{CE_i}(\mathbf{p}_i) = \frac{(p_{i1} - p_{Ei1})^{n_{CEi}}}{a_{Ci1}^{n_{CEi}}} + \dots + \frac{(p_{in_i} - p_{Ein_i})^{n_{CEi}}}{a_{Cin_i}^{n_{CEi}}} - 1 \leq 0, \quad (4.6)$$

where n_{CEi} denote the orders of region functions which are also even integers, and $\mathbf{p}_{Ei} = [p_{Ei1}, \dots, p_{Ein_i}]^T$ are reference positions, and $a_{Ci1}, \dots, a_{Cin_i}$ are a set of positive constants. The functions $h_{CE_i}(\mathbf{p}_i)$ are specified to exclude the external singularity in Cartesian space, thus $h_{CE_i}(\mathbf{p}_i)$ represent the outer boundaries of Cartesian-space feedback. Therefore, the robot employs joint information where $h_{CE_i}(\mathbf{p}_i) > 0$ and Cartesian-space feedback where $h_{CE_i}(\mathbf{p}_i) \leq 0$.

To construct the potential energy, reference regions $h_{CE_i}(\mathbf{p}_i)$ within $h_{CE_i}(\mathbf{p}_i)$ are introduced as:

$$h_{CE_i}(\mathbf{p}_i) = \frac{(p_{i1} - p_{Ei1})^{n_{CEi}}}{(\kappa_{CE_i} a_{Ci1})^{n_{CEi}}} + \dots + \frac{(p_{in_i} - p_{Ein_i})^{n_{CEi}}}{(\kappa_{CE_i} a_{Cin_i})^{n_{CEi}}} - 1 \leq 0, \quad (4.7)$$

where κ_{CE_i} are positive constants and $\kappa_{CE_i} < 1$.

By using $h_{CE_i}(\mathbf{p}_i)$ and $h_{CE_i}(\mathbf{p}_i)$, the potential energy functions are introduced as:

$$P_{CE_i}(\mathbf{p}_i) = \frac{k_{CE_i}}{N^2} \{ \min[0, [\min(0, h_{CE_i}(\mathbf{p}_i))]^N - (\kappa_{CE_i}^{n_{CE_i}} - 1)^N] \}^N, \quad (4.8)$$

which can also be written as:

$$P_{CE_i}(\mathbf{p}_i) = \begin{cases} 0, & h_{CE_i}(\mathbf{p}_i) \leq 0, \\ \frac{k_{CE_i}}{N^2} \{ [h_{CE_i}(\mathbf{p}_i)]^N - (\kappa_{CE_i}^{n_{CE_i}} - 1)^N \}^N, & h_{CE_i}(\mathbf{p}_i) < 0, h_{CE_i}(\mathbf{p}_i) > 0, \\ \frac{k_{CE_i}}{N^2} (\kappa_{CE_i}^{n_{CE_i}} - 1)^N, & h_{CE_i}(\mathbf{p}_i) \geq 0, \end{cases} \quad (4.9)$$

where k_{CE_i} are positive constants. From equation (4.8), it can be seen that $P_{CE_i}(\mathbf{p}_i)$ are smooth and lower bounded by zero.

Sub regions $h_{CI_{ij}}(\mathbf{p}_i)$ to match the internal singular regions

Similarly, to match the internal singular regions $h_{JI_{ij}}(\mathbf{q})$, a set of sub Cartesian-space regions are specified as:

$$h_{CI_{ij}}(\mathbf{p}_i) = \frac{(p_{i1} - p_{Iij1})^{n_{CI_{ij}}}}{b_{CI_{ij}1}^{n_{CI_{ij}}}} + \dots + \frac{(p_{in_i} - p_{Iijn_i})^{n_{CI_{ij}}}}{b_{CI_{ijn_i}}^{n_{CI_{ij}}}} - 1 \geq 0, \quad (4.10)$$

and $\mathbf{p}_{Iij} = [p_{Iij1}, \dots, p_{Iijn_i}]^T$ represents reference points within $h_{CI_{ij}}(\mathbf{p}_i)$, and $b_{CI_{ij}1}, \dots, b_{CI_{ijn_i}}$ are positive constants, and $n_{CI_{ij}}$ denote the orders of region function which are also even integers. Different from sub regions $h_{CE_i}(\mathbf{p}_i)$, $h_{CI_{ij}}(\mathbf{p}_i)$ enclose singular regions so that $h_{CI_{ij}}(\mathbf{p}_i)$ represent the inner boundaries of Cartesian-space feedback., and the robot employs joint information where $h_{CI_{ij}}(\mathbf{p}_i) < 0$ and Cartesian-space feedback where $h_{CI_{ij}}(\mathbf{p}_i) \geq 0$. The orders $n_{CI_{ij}}$ in equation (4.10) can be varied to match different types of internal singularity in Cartesian space.

Next, reference regions which enclose $h_{CI_{ij}}(\mathbf{p}_i)$ are specified as:

$$h_{CI_{r_{ij}}}(\mathbf{p}_i) = \frac{(p_{i1} - p_{Iij1})^{n_{CI_{ij}}}}{(\kappa_{CI_{ij}} b_{CI_{ij}1})^{n_{CI_{ij}}}} + \dots + \frac{(p_{in_i} - p_{Iijn_i})^{n_{CI_{ij}}}}{(\kappa_{CI_{ij}} b_{CI_{ijn_i}})^{n_{CI_{ij}}}} - 1 \geq 0, \quad (4.11)$$

where $\kappa_{CI_{ij}} > 1$ are constants.

Using the sub regions $h_{CI_{ij}}(\mathbf{p}_i)$ and the corresponding reference regions described by equations (4.10) and (4.11), the potential energy $P_{CI_{ij}}(\mathbf{p}_i)$ are proposed as:

$$P_{CI_{ij}}(\mathbf{p}_i) = \frac{k_{CI_{ij}}}{N^2} \{ \min[0, [\min(0, h_{CI_{r_{ij}}}(\mathbf{p}_i))]^N - (\frac{1}{\kappa_{CI_{ij}}^{n_{CI_{ij}}}} - 1)^N] \}^N, \quad (4.12)$$

The above equation can be written as:

$$P_{C_{I_{ij}}}(\mathbf{p}_i) = \begin{cases} 0, & h_{C_{I_{ij}}}(\mathbf{p}_i) \leq 0, \\ \frac{k_{C_{I_{ij}}}}{N^2} \{ [h_{C_{I_{r_{ij}}}}(\mathbf{p}_i)]^N - (\frac{1}{\kappa_{C_{I_{ij}}}} - 1)^N \}^N, & h_{C_{I_{ij}}}(\mathbf{p}_i) > 0, h_{C_{I_{r_{ij}}}}(\mathbf{p}_i) < 0, \\ \frac{k_{C_{I_{ij}}}}{N^2} (\frac{1}{\kappa_{C_{I_{ij}}}} - 1)^{N^2}, & h_{C_{I_{r_{ij}}}}(\mathbf{p}_i) \geq 0, \end{cases} \quad (4.13)$$

where $k_{C_{I_{ij}}}$ are positive constants.

Example 4.3 For the robot system in Fig. 4.1, since the end effector evolves in a plane, the Cartesian-space regions are all specified in a 2-D space. The external singular region $h_{J_{E_1}}(\mathbf{q})$ in equation (4.2) corresponds to a circle in Cartesian space. To match the external singular region $h_{J_{E_1}}(\mathbf{q})$, the sub region in equation (4.6) is specified as a circle as:

$$h_{C_{E_1}}(\mathbf{p}_1) = \frac{(p_{11} - p_{E11})^2}{a^2} + \frac{(p_{12} - p_{E12})^2}{a^2} - 1 \leq 0, \quad (4.14)$$

where $\mathbf{p}_{E1} = [p_{E11}, p_{E12}]^T$ is the reference position that is set as the origin of Cartesian coordinates, and a denotes the radius. Similarly, to match the internal singular region $h_{J_{I1}}(\mathbf{q})$ in equation (4.2), the sub region in equation (4.10) is also specified as a circle as:

$$h_{C_{I1}}(\mathbf{p}_1) = \frac{(p_{11} - p_{I11})^2}{b^2} + \frac{(p_{12} - p_{I12})^2}{b^2} - 1 \geq 0, \quad (4.15)$$

where $\mathbf{p}_{I11} = [p_{I11}, p_{I12}]^T$ is the reference position that is also set as the origin of Cartesian coordinates, and b denotes the radius. The relationship of $h_{J_{E_1}}(\mathbf{q}) > 0$ and $h_{C_{E_1}}(\mathbf{p}_1) < 0$, and $h_{J_{I1}}(\mathbf{q}) < 0$ and $h_{C_{I1}}(\mathbf{p}_1) > 0$ are illustrated in Fig. 4.4.

△△△

Figure 4.4: The sub regions $h_{C_{E1}}(\mathbf{p}_1)$ and $h_{C_{I1}}(\mathbf{p}_1)$ are formulated in Cartesian space to match the singular regions.

Sub regions $h_{C_{V_i}}(\mathbf{p}_i)$ to match the image-space regions

To match the image-space region, another set of sub regions are formulated as:

$$h_{C_{V_i}}(\mathbf{p}_i) = \frac{(p_{i1} - p_{Vi1})^{n_{CV_i}}}{c_{C_{i1}}^{n_{CV_i}}} + \dots + \frac{(p_{in_i} - p_{Vin_i})^{n_{CV_i}}}{c_{C_{in_i}}^{n_{CV_i}}} - 1 \geq 0, \quad (4.16)$$

where $\mathbf{p}_{Vi} = [p_{Vi1}, \dots, p_{Vin_i}]^T$ are reference points inside the sub regions, and $c_{C_{i1}}, \dots, c_{C_{in_i}}$ are positive constants, and n_{CV_i} are the orders of region functions which are even integers. The robot employs vision feedback where $h_{C_{V_i}}(\mathbf{p}_i) < 0$ and Cartesian-space feedback where $h_{C_{V_i}}(\mathbf{p}_i) \geq 0$. In addition, the regions $h_{C_{V_i}}(\mathbf{p}_i) < 0$ are within the task-oriented image-space regions $f_{T_i}(\mathbf{x}_{I_i}) < 0$ so that the end effector does not get stuck during the transition from Cartesian space to image space.

A set of reference regions larger than $h_{C_{V_i}}(\mathbf{p}_i)$ are specified as:

$$h_{C_{V_i}}(\mathbf{p}_i) = \frac{(p_{i1} - p_{Vi1})^{n_{CV_i}}}{(\kappa_{CV_i} c_{C_{i1}})^{n_{CV_i}}} + \dots + \frac{(p_{in_i} - p_{Vin_i})^{n_{CV_i}}}{(\kappa_{CV_i} c_{C_{in_i}})^{n_{CV_i}}} - 1 \geq 0, \quad (4.17)$$

where $\kappa_{CV_i} > 1$ are constants.

Based on the region functions of $h_{C_{V_i}}(\mathbf{p}_i)$ and the corresponding reference regions, the potential energy $P_{C_{V_i}}(\mathbf{p}_i)$ for the sub region are proposed as:

$$P_{C_{V_i}}(\mathbf{p}_i) = \frac{k_{C_{V_i}}}{N^2} \{ \min[0, [\min(0, h_{C_{V_{r_i}}}(\mathbf{p}_i))]^N - (\frac{1}{\kappa_{C_{V_i}}^{n_{C_{V_i}}}} - 1)^N] \}^N. \quad (4.18)$$

The above equation can be written as:

$$P_{C_{V_i}}(\mathbf{p}_i) = \begin{cases} 0, & h_{C_{V_i}}(\mathbf{p}_i) \leq 0, \\ \frac{k_{C_{V_i}}}{N^2} \{ [h_{C_{V_{r_i}}}(\mathbf{p}_i)]^N - (\frac{1}{\kappa_{C_{V_i}}^{n_{C_{V_i}}}} - 1)^N \}^N, & h_{C_{V_i}}(\mathbf{p}_i) > 0, h_{C_{V_{r_i}}}(\mathbf{p}_i) < 0, \\ \frac{k_{C_{V_i}}}{N^2} (\frac{1}{\kappa_{C_{V_i}}^{n_{C_{V_i}}}} - 1)^{N^2}, & h_{C_{V_{r_i}}}(\mathbf{p}_i) \geq 0. \end{cases} \quad (4.19)$$

where $k_{C_{V_i}}$ are positive constants.

Example 4.4 Since the region function $f_{E_1}(\mathbf{x}_{I1})$ in equation (4.4) corresponds to a rectangle in image space, the sub region in equation (4.16) is specified to match $f_{E_1}(\mathbf{x}_{I1})$ as:

$$h_{C_{V_1}}(\mathbf{p}_1) = \frac{(p_{11} - p_{V1_1})^{20}}{c_1^{20}} + \frac{(p_{12} - p_{V1_2})^{20}}{c_2^{20}} - 1 \geq 0, \quad (4.20)$$

where $\mathbf{p}_{V1} = [p_{V1_1}, p_{V1_2}]^T$ is the reference position that is set as the center of the field of view, and c_1, c_2 are positive constants. △△△

Figure 4.5: The sub region $h_{C_{V_1}}(\mathbf{p}_1)$ is formulated in Cartesian space to match the image-space region.

Sub regions $h_{C_{EV_i}}(\mathbf{p}_i)$ to drive end effector from $h_{C_{E_i}}(\mathbf{p}_i) \leq 0$ to $h_{C_{V_i}}(\mathbf{p}_i) \leq 0$

Note that $P_{C_{E_i}}(\mathbf{p}_i)$ are zero when $h_{C_{E_i}}(\mathbf{p}_i) \leq 0$ and $P_{C_{V_i}}(\mathbf{p}_i)$ are constant when $h_{C_{V_i}}(\mathbf{p}_i) \geq 0$. Therefore, another set of sub regions $h_{C_{EV_i}}(\mathbf{p}_i)$ are introduced so that the end effector can be driven towards the image-space regions after it enters the Cartesian-space regions. The sub regions $h_{C_{EV_i}}(\mathbf{p}_i)$ are introduced as:

$$h_{C_{EV_i}}(\mathbf{p}_i) = \frac{(p_{i1} - p_{EV_{i1}})^2}{(p_{bi1} - p_{EV_{i1}})^2} + \dots + \frac{(p_{in_i} - p_{EV_{in_i}})^2}{(p_{bin_i} - p_{EV_{in_i}})^2} - 1 \leq 0, \quad (4.21)$$

where $\mathbf{p}_{bi} = [p_{bi1}, \dots, p_{bin_i}]^T$ represent the boundary positions and $\mathbf{p}_{EV_i} = [p_{EV_{i1}}, \dots, p_{EV_{in_i}}]^T$ are reference positions. The regions described by equation (4.21) are divided into several parts so that $\mathbf{p}_{EV_i}$ are not necessary the geometrical centers.

The potential energy functions for the sub regions $h_{C_{EV_i}}(\mathbf{p}_i)$ are proposed as:

$$P_{C_{EV_i}}(\mathbf{p}_i) = \frac{k_{C_{EV_i}}}{N} \{1 - [\min(0, h_{C_{EV_i}}(\mathbf{p}_i))]^N\}, \quad (4.22)$$

where $k_{C_{EV_i}}$ are positive constants. Note that the gradient of $P_{C_{EV_i}}(\mathbf{p}_i)$ is nonzero until the end effector converges to the reference position $\mathbf{p}_{EV_i}$.

Cartesian-space potential energy

The overall potential energy $P_C(\mathbf{p})$ in Cartesian space is formulated as a combination of potential energies $P_{C_{E_i}}(\mathbf{p}_i)$, $P_{C_{I_{ij}}}(\mathbf{p}_i)$, $P_{C_{V_i}}(\mathbf{p}_i)$ and $P_{C_{EV_i}}(\mathbf{p}_i)$ as:

$$\begin{aligned} P_C(\mathbf{p}) = & k_p \alpha_r \sum_{i=1}^m \{ [P_{C_{E_i}}(\mathbf{p}_i) + P_{C_{EV_i}}(\mathbf{p}_i)] P_{C_{V_i}}(\mathbf{p}_i) \prod_{j=1}^{N_i} P_{C_{I_{ij}}}(\mathbf{p}_i) \\ & + \sum_{j=1}^{N_i} k_{p_{ij}} \left[\frac{k_{CI_{ij}}}{N^2} \left(\frac{1}{\kappa_{CI_{ij}}} - 1 \right)^{N^2} - P_{C_{I_{ij}}}(\mathbf{p}_i) \right] \}, \end{aligned} \quad (4.23)$$

where $k_{p_{ij}}$ are positive constants. In equation (4.23), the term $P_{C_{E_i}}(\mathbf{p}_i) + P_{C_{EV_i}}(\mathbf{p}_i)$ enables the end effector to move towards the image-space region after it leaves the external singular positions. Then $[P_{C_{E_i}}(\mathbf{p}_i) + P_{C_{EV_i}}(\mathbf{p}_i)]P_{C_{V_i}}(\mathbf{p}_i)$ is flattened at $h_{C_{V_i}}(\mathbf{p}_i) < 0$ where the gradient reduces to zero. It avoids the use of Cartesian-space feedback inside the image-space region, and the vision feedback is employed there. The combination of $[P_{C_{E_i}}(\mathbf{p}_i) + P_{C_{EV_i}}(\mathbf{p}_i)]P_{C_{V_i}}(\mathbf{p}_i)$ is illustrated in Fig. 4.6.

Figure 4.6: An illustration of the combination of potential energy $[P_{C_{E_i}}(\mathbf{p}_i) + P_{C_{EV_i}}(\mathbf{p}_i)]P_{C_{V_i}}(\mathbf{p}_i)$ in 2-D space.

Figure 4.7: An illustration of the combination of potential energy $P_C(\mathbf{p})$ in 2-D space.

After that, the potential energy is multiplied with $\prod_{j=1}^{N_i} P_{CI_{ij}}(\mathbf{p}_i)$ so that it is flattened at $h_{CI_{ij}}(\mathbf{p}_i) < 0$ where the gradient reduces to zero. It avoids the use of Cartesian-space feedback inside the internal singular regions, and the joint information is employed in those regions. Next, the potential energy is offset inside the regions $h_{CI_{ij}}(\mathbf{p}_i) < 0$ by adding the term $\sum_{j=1}^{N_i} k_{p_{ij}} [\frac{k_{CI_{ij}}}{N^2} (\frac{1}{\kappa_{CI_{ij}}} - 1)^{N^2} - P_{CI_{ij}}(\mathbf{p}_i)]$. The offset term is introduced to raise the potential energy so as to enable the end effector to move away from the internal singular regions after leaving them. By adjusting the value of $k_{p_{ij}}$ in the offset term, the energy level can be varied to allow the end effector to pass through the internal singular regions or avoid them.

The aim of $P_C(\mathbf{p})$ is to move the end effector towards the image-space region while avoiding singularity during movement. An illustration of $P_C(\mathbf{p})$ is shown in Fig. 4.7.

4.2 Multiple Regional Feedback Controller

From equations (2.5) and (2.23), the velocities of end effector in Cartesian space and image space are related with the joint velocity as:

$$\begin{aligned}\dot{\mathbf{x}}_I &= \mathbf{J}_x(\mathbf{q})\dot{\mathbf{q}}, \\ \dot{\mathbf{p}} &= \mathbf{J}_r(\mathbf{q})\dot{\mathbf{q}},\end{aligned}\tag{4.24}$$

where $\mathbf{J}_r(\mathbf{q})$ is the Jacobian matrix from joint space to Cartesian space, and $\mathbf{J}_x(\mathbf{q})$ is the Jacobian matrix from joint space to image space.

The proposed multiple regional feedback controller is constructed by using the region errors obtained from the gradient of potential energy. Partial differentiating $P_J(\mathbf{q})$ described by equation (4.1) with respect to $\mathbf{q}$ yields:

$$\begin{aligned}\left(\frac{\partial P_J(\mathbf{q})}{\partial \mathbf{q}}\right)^T &= k_p \alpha_q \sum_{i=1}^m \{k_{qE_i} [\max(0, h_{J_{E_i}}(\mathbf{q}))]^{N-1} \left(\frac{\partial h_{J_{E_i}}(\mathbf{q})}{\partial \mathbf{q}}\right)^T \\ &+ \sum_{j=1}^{N_i} k_{qI_{ij}} [\min(0, h_{J_{I_{ij}}}(\mathbf{q}))]^{N-1} \left(\frac{\partial h_{J_{I_{ij}}}(\mathbf{q})}{\partial \mathbf{q}}\right)^T\} \triangleq k_p \alpha_q \Delta \boldsymbol{\varepsilon}_q,\end{aligned}\tag{4.25}$$

where $\Delta \boldsymbol{\varepsilon}_q$ denotes the joint-space region error which drives the end effector away from the singular regions. From equation (4.25), it is seen that $\Delta \boldsymbol{\varepsilon}_q$ naturally reduces to zero after the end effector leaves the singular positions where $h_{J_{E_i}}(\mathbf{q}) \leq 0$ and $h_{J_{I_{ij}}}(\mathbf{q}) \geq 0$.

Partial differentiating $P_D(\mathbf{x}_I)$ described by equation (4.3) with respect to $\mathbf{x}_I$ yields:

$$\begin{aligned} \left(\frac{\partial P_D(\mathbf{x}_I)}{\partial \mathbf{x}_I}\right)^T &= k_p \alpha_x \sum_{i=1}^m \left\{ \left(\frac{\partial P_i(\mathbf{x}_{Ii})}{\partial \mathbf{x}_I}\right)^T \prod_{j=1}^{N_i} P_{Iij}(\mathbf{x}_{Ii}) + \right. \\ &\left. P_i(\mathbf{x}_{Ii}) \sum_{j=1}^{N_i} \left[\left(\frac{\partial P_{Iij}(\mathbf{x}_{Ii})}{\partial \mathbf{x}_I}\right)^T \prod_{k \neq j}^{N_i} P_{Iik}(\mathbf{x}_{Ii}) \right] - \sum_{j=1}^{N_i} k_{c_{ij}} \left(\frac{\partial P_{Iij}(\mathbf{x}_{Ii})}{\partial \mathbf{x}_I}\right)^T \right\} \triangleq k_p \alpha_x \Delta \boldsymbol{\varepsilon}_x, \end{aligned} \quad (4.26)$$

where $\Delta \boldsymbol{\varepsilon}_x$ denotes the image-space region error moving the end effector to the desired trajectory, and it is only activated after the end effector enters the image-space region.

Partial differentiating $P_C(\mathbf{p})$ described by equation (4.23) with respect to $\mathbf{p}$ yields:

$$\begin{aligned} \left(\frac{\partial P_C(\mathbf{p})}{\partial \mathbf{p}}\right)^T &= k_p \alpha_r \sum_{i=1}^m \left\{ \left(\frac{\partial P_{C_{E_i}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T + \left(\frac{\partial P_{C_{EV_i}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T \right\} P_{C_{V_i}}(\mathbf{p}_i) \prod_{j=1}^{N_i} P_{C_{Iij}}(\mathbf{p}_i) \\ &\quad + [P_{C_{E_i}}(\mathbf{p}_i) + P_{C_{EV_i}}(\mathbf{p}_i)] \left(\frac{\partial P_{C_{V_i}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T \prod_{j=1}^{N_i} P_{C_{Iij}}(\mathbf{p}_i) \\ &\quad + [P_{C_{E_i}}(\mathbf{p}_i) + P_{C_{EV_i}}(\mathbf{p}_i)] P_{C_{V_i}}(\mathbf{p}_i) \sum_{j=1}^{N_i} \left[\left(\frac{\partial P_{C_{Iij}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T \prod_{k \neq j}^{N_i} P_{C_{Iik}}(\mathbf{p}_i) \right] \\ &\quad - \sum_{j=1}^{N_i} k_{p_{ij}} \left(\frac{\partial P_{C_{Iij}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T \right\} \triangleq k_p \alpha_r \Delta \boldsymbol{\varepsilon}_p. \end{aligned} \quad (4.27)$$

where $\left(\frac{\partial P_{C_{E_i}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T$, $\left(\frac{\partial P_{C_{Iij}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T$, $\left(\frac{\partial P_{C_{V_i}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T$ and $\left(\frac{\partial P_{C_{EV_i}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T$ are given as:

$$\begin{aligned} \left(\frac{\partial P_{C_{E_i}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T &= k_{C_{E_i}} \{ \min\{0, [\min(0, h_{C_{E_i}}(\mathbf{p}_i))]\}^N - \\ &(\kappa_{C_{E_i}}^{n_{C_{E_i}}} - 1)^N \} \}^{N-1} [\min(0, h_{C_{E_i}}(\mathbf{p}_i))]^{N-1} \left(\frac{\partial h_{C_{E_i}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T, \end{aligned} \quad (4.28)$$

$$\begin{aligned} \left(\frac{\partial P_{C_{Iij}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T &= k_{C_{Iij}} \min\{0, [\min(0, h_{C_{Iij}}(\mathbf{p}_i))]\}^N - \\ &\left(\frac{1}{\kappa_{C_{Iij}}^{n_{C_{Iij}}} - 1} - 1\right)^N \} \}^{N-1} [\min(0, h_{C_{Iij}}(\mathbf{p}_i))]^{N-1} \left(\frac{\partial h_{C_{Iij}}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T, \end{aligned} \quad (4.29)$$

$$\begin{aligned} \left(\frac{\partial P_{CV_i}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T &= k_{CV_i} \min\{0, [\min(0, h_{CV_i}(\mathbf{p}_i))]^N - \\ &\left(\frac{1}{\kappa_{CV_i}^n} - 1\right)^N\}^{N-1} [\min(0, h_{CV_i}(\mathbf{p}_i))]^{N-1} \left(\frac{\partial h_{CV_i}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T, \end{aligned} \quad (4.30)$$

$$\left(\frac{\partial P_{CEV_i}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T = -k_{CEV_i} [\min(0, h_{CEV_i}(\mathbf{p}_i))]^{N-1} \left(\frac{\partial h_{CEV_i}(\mathbf{p}_i)}{\partial \mathbf{p}}\right)^T. \quad (4.31)$$

The Cartesian-space region error $\Delta \boldsymbol{\varepsilon}_p$ defined in equation (4.27) drives the end effector from joint-space regions to image-space regions. From equation (4.27), $\Delta \boldsymbol{\varepsilon}_p$ reduces to zero when the end effector is inside the image-space region. Therefore, each region error acts in a corresponding local region, and the combination of region errors ensure the convergence of robot movement.

Similarly, since vision feedback is only used when the end effector is inside the image-space regions, the desired velocity is proposed as:

$$\dot{\mathbf{x}}_{di} = \mathbf{0}, \quad \text{if} \quad f_{Er_i}(\mathbf{x}_{Ii}) \geq 0 \text{ or } f_{Ir_{ij}}(\mathbf{x}_{Ii}) \leq 0. \quad (4.32)$$

Equation (4.32) can also be satisfied by introducing the weight factors w_i which are specified in Appendix.

Next, a new sliding vector is defined as:

$$\begin{aligned} \mathbf{s}_m &= \dot{\mathbf{q}} - \dot{\mathbf{q}}_{mr} = \dot{\mathbf{q}} - [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q}) \mathbf{A} \mathbf{J}_x(\mathbf{q})]^{-1} \times \\ &[\mathbf{J}_x^+(\mathbf{q}) \dot{\mathbf{x}}_f - \alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x - \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_p - \alpha_q \Delta \boldsymbol{\varepsilon}_q], \end{aligned} \quad (4.33)$$

where $\mathbf{A}$ is the matrix defined in equation (3.58), and $\dot{\mathbf{x}}_f$ is the vector defined in

equation (3.57), and

$$\begin{aligned}\dot{\mathbf{q}}_{mr} = & [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]^{-1}[\mathbf{J}_x^+(\mathbf{q})\dot{\mathbf{x}}_f - \alpha_x\mathbf{J}_x^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_x \\ & - \alpha_r\mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_p - \alpha_q\Delta\boldsymbol{\varepsilon}_q],\end{aligned}\quad (4.34)$$

is the reference vector. Note that the information of joint velocity $\dot{\mathbf{q}}$ is not required in the reference vector $\dot{\mathbf{q}}_{mr}$.

By using the region errors in equation (4.25) - (4.27) and the sliding vector in equation (4.33), a multiple regional feedback controller is proposed as:

$$\begin{aligned}\boldsymbol{\tau}_m = & -[\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]^T[k_p\alpha_x\mathbf{J}_x^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_x + k_p\alpha_r\mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_p \\ & + k_p\alpha_q\Delta\boldsymbol{\varepsilon}_q] - \mathbf{K}_s\mathbf{s}_m + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_{mr}, \ddot{\mathbf{q}}_{mr})\hat{\boldsymbol{\theta}}_d,\end{aligned}\quad (4.35)$$

When the end effector reaches the singular positions, $\Delta\boldsymbol{\varepsilon}_x$ and $\Delta\boldsymbol{\varepsilon}_p$ reduce to zero, and the joint-space region error $\Delta\boldsymbol{\varepsilon}_q$ pushes the end effector out of the singular positions. After the end effector leaves the singular regions, the Cartesian-space region error $\Delta\boldsymbol{\varepsilon}_p$ is activated to drive it towards the field of view. The image-space region error $\Delta\boldsymbol{\varepsilon}_x$ is only activated within the field of view to enable the end effector to converge to the desired trajectory. In addition, note that the multiple regional feedback controller is continuous since the region errors $\Delta\boldsymbol{\varepsilon}_x$, $\Delta\boldsymbol{\varepsilon}_p$ and $\Delta\boldsymbol{\varepsilon}_q$ are continuous.

The estimated parameters $\hat{\boldsymbol{\theta}}_d$ are updated using the following update law:

$$\dot{\hat{\boldsymbol{\theta}}}_d = -\mathbf{L}_d\mathbf{Y}_d^T(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_{mr}, \ddot{\mathbf{q}}_{mr})\mathbf{s}_m. \quad (4.36)$$

From equation (2.27), the robot dynamic equation is stated as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + [\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}_m, \quad (4.37)$$

Using the sliding vector $\mathbf{s}_m$ in equation (4.33) and Property 2.3, equation (4.37) is written as:

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{s}}_m + [\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\mathbf{s}_m + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_{mr}, \ddot{\mathbf{q}}_{mr})\boldsymbol{\theta}_d = \boldsymbol{\tau}_m. \quad (4.38)$$

The closed-loop equation of the system is obtained by substituting equation (4.35) into equation (4.38) to give:

$$\begin{aligned} & \mathbf{M}(\mathbf{q})\dot{\mathbf{s}}_m + [\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\mathbf{s}_m + [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]^T \\ & \times [k_p\alpha_x\mathbf{J}_x^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_x + k_p\alpha_r\mathbf{J}_r^T(\mathbf{q})\Delta\boldsymbol{\varepsilon}_p + k_p\alpha_q\Delta\boldsymbol{\varepsilon}_q] \\ & + \mathbf{K}_s\mathbf{s}_m + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_{mr}, \ddot{\mathbf{q}}_{mr})\Delta\boldsymbol{\theta}_d = \mathbf{0}. \end{aligned} \quad (4.39)$$

The following theorem is now stated:

Theorem 4.1: *The multiple regional feedback control law (4.35), the update law (4.36) for the robot system (4.37) guarantee the global convergence of the tracking errors. That is $\mathbf{x}_I \rightarrow \mathbf{x}_d$, $\dot{\mathbf{x}}_I \rightarrow \dot{\mathbf{x}}_d$ as $t \rightarrow \infty$.*

Proof: A Lyapunov-like candidate is proposed as:

$$V_m = \frac{1}{2}\mathbf{s}_m^T\mathbf{M}(\mathbf{q})\mathbf{s}_m + P_D(\mathbf{x}_I) + P_C(\mathbf{p}) + P_J(\mathbf{q}) + \frac{1}{2}\Delta\boldsymbol{\theta}_d^T\mathbf{L}_d^{-1}\Delta\boldsymbol{\theta}_d. \quad (4.40)$$

Next, note that the derivative of $P_C(\mathbf{p})$ in equation (4.23) is given as:

$$\begin{aligned}
\dot{P}_C(\mathbf{p}) = & k_p \alpha_r \left\{ [\dot{\mathbf{p}}^T \left(\frac{\partial P_{C_{E_i}}(\mathbf{p}_i)}{\partial \mathbf{p}} \right)^T + \dot{\mathbf{p}}^T \left(\frac{\partial P_{C_{EV_i}}(\mathbf{p}_i)}{\partial \mathbf{p}} \right)^T] P_{C_{V_i}}(\mathbf{p}_i) \prod_{j=1}^{N_i} P_{C_{I_{ij}}}(\mathbf{p}_i) \right. \\
& + [P_{C_{E_i}}(\mathbf{p}_i) + P_{C_{EV_i}}(\mathbf{p}_i)] \dot{\mathbf{p}}^T \left(\frac{\partial P_{C_{V_i}}(\mathbf{p}_i)}{\partial \mathbf{p}} \right)^T \prod_{j=1}^{N_i} P_{C_{I_{ij}}}(\mathbf{p}_i) \\
& + [P_{C_{E_i}}(\mathbf{p}_i) + P_{C_{EV_i}}(\mathbf{p}_i)] P_{C_{V_i}}(\mathbf{p}_i) \sum_{j=1}^{N_i} [\dot{\mathbf{p}}^T \left(\frac{\partial P_{C_{I_{ij}}}(\mathbf{p}_i)}{\partial \mathbf{p}} \right)^T \prod_{k \neq j}^{N_i} P_{C_{I_{ik}}}(\mathbf{p}_i)] \\
& \left. - \sum_{j=1}^{N_i} \dot{\mathbf{p}}^T \left(\frac{\partial P_{C_{I_{ij}}}(\mathbf{p}_i)}{\partial \mathbf{p}} \right)^T \right\} = k_p \alpha_r \dot{\mathbf{p}}^T \Delta \boldsymbol{\varepsilon}_p. \tag{4.41}
\end{aligned}$$

Differentiating equation (4.40) with respect to time, and substituting equation (4.41) into its time derivative yields:

$$\begin{aligned}
\dot{V}_m = & \mathbf{s}_m^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{s}}_m + \frac{1}{2} \mathbf{s}_m^T \dot{\mathbf{M}}(\mathbf{q}) \mathbf{s}_m + k_p \alpha_r \dot{\mathbf{p}}^T \Delta \boldsymbol{\varepsilon}_p + k_p \alpha_q \dot{\mathbf{q}}^T \Delta \boldsymbol{\varepsilon}_q \\
& + k_p \alpha_x \left\{ \dot{\mathbf{x}}_I^T \Delta \boldsymbol{\varepsilon}_x - \sum_{i=1}^m [\dot{\mathbf{x}}_a^T \left(\frac{\partial P_{T_i}(\mathbf{x}_{I_i})}{\partial \mathbf{x}_I} \right)^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_{I_i})] \right\} - \dot{\boldsymbol{\theta}}_d^T \mathbf{L}_d^{-1} \Delta \boldsymbol{\theta}_d. \tag{4.42}
\end{aligned}$$

Substituting equations (4.36) and (4.39) into equation (4.42) and using Property 2.2, it is obtained:

$$\begin{aligned}
\dot{V}_m = & -\mathbf{s}_m^T \mathbf{K}_s \mathbf{s}_m - k_p \mathbf{s}_m^T [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q}) \mathbf{A} \mathbf{J}_x(\mathbf{q})]^T \times \\
& [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_p + \alpha_q \Delta \boldsymbol{\varepsilon}_q] + k_p \alpha_r \dot{\mathbf{p}}^T \Delta \boldsymbol{\varepsilon}_p + k_p \alpha_q \dot{\mathbf{q}}^T \Delta \boldsymbol{\varepsilon}_q \\
& + k_p \alpha_x \left\{ \dot{\mathbf{x}}_I^T \Delta \boldsymbol{\varepsilon}_x - \sum_{i=1}^m [\dot{\mathbf{x}}_a^T \left(\frac{\partial P_{T_i}(\mathbf{x}_{I_i})}{\partial \mathbf{x}_I} \right)^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_{I_i})] \right\}. \tag{4.43}
\end{aligned}$$

Substituting equation (4.33) into equation (4.43), it is obtained:

$$\begin{aligned}
\dot{V}_m = & -\mathbf{s}_m^T \mathbf{K}_s \mathbf{s}_m - k_p [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_p + \alpha_q \Delta \boldsymbol{\varepsilon}_q]^T \\
& \times [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_p + \alpha_q \Delta \boldsymbol{\varepsilon}_q] \\
& + k_p \alpha_r \dot{\mathbf{x}}_a^T \mathbf{J}_x^{+T}(\mathbf{q}) \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_p + k_p \alpha_q \dot{\mathbf{x}}_a^T \mathbf{J}_x^{+T}(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_q \\
& + k_p \alpha_x \sum_{i=1}^m \left\{ \dot{\mathbf{x}}_a^T \left(\frac{\partial P_{E_i}(\mathbf{x}_{I_i})}{\partial \mathbf{x}_I} \right)^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_{I_i}) - \sum_{j=1}^{N_i} [k_{c_{ij}} \dot{\mathbf{x}}_a^T \left(\frac{\partial P_{I_{ij}}(\mathbf{x}_{I_i})}{\partial \mathbf{x}_I} \right)^T] \right. \\
& \left. + [P_{T_i}(\mathbf{x}_{I_i}) + P_{E_i}(\mathbf{x}_{I_i})] \sum_{j=1}^{N_i} \left[\dot{\mathbf{x}}_a^T \left(\frac{\partial P_{I_{ij}}(\mathbf{x}_{I_i})}{\partial \mathbf{x}_I} \right)^T \prod_{k \neq j}^{N_i} P_{I_{ik}}(\mathbf{x}_{I_i}) \right] \right\}. \quad (4.44)
\end{aligned}$$

From equations (4.32) and (3.53), it can be derived that:

$$\dot{\mathbf{x}}_{ai} = \mathbf{0}, \quad \text{if} \quad f_{Er_i}(\mathbf{x}_{I_i}) \geq 0 \text{ or } f_{Ir_{ij}}(\mathbf{x}_{I_i}) \leq 0. \quad (4.45)$$

The condition that $f_{Er_i}(\mathbf{x}_{I_i}) \geq 0$ or $f_{Ir_{ij}}(\mathbf{x}_{I_i}) \leq 0$ indicates that $h_{C_{E_i}}(\mathbf{p}_i) \leq 0$ and $h_{C_{I_{ij}}}(\mathbf{p}_i) \geq 0$, and $h_{J_{E_i}}(\mathbf{q}) \geq 0$ or $h_{J_{I_{ij}}}(\mathbf{q}) \leq 0$, where both the Cartesian-space region error $\Delta \boldsymbol{\varepsilon}_r$ and the joint-space region error $\Delta \boldsymbol{\varepsilon}_q$ are nonzero, thus $\dot{\mathbf{x}}_{ai}$ and $\Delta \boldsymbol{\varepsilon}_p$ and $\Delta \boldsymbol{\varepsilon}_q$ cannot be nonzero at the same time.

Equation (4.45) also indicates that $\dot{\mathbf{x}}_a$ is nonzero where $f_{Er_i}(\mathbf{x}_{I_i}) < 0$ and $f_{Ir_{ij}}(\mathbf{x}_{I_i}) > 0$. The gradient of potential energy $P_{E_i}(\mathbf{x}_{I_i})$ reduces to zero where $f_{Er_i}(\mathbf{x}_{I_i}) < 0$, while the gradient of potential energy $P_{I_{ij}}(\mathbf{x}_{I_i})$ reduces to zero where $f_{Ir_{ij}}(\mathbf{x}_{I_i}) > 0$. Thus, $\mathbf{x}_{ai}$ and $(\frac{\partial P_{E_i}(\mathbf{x}_{I_i})}{\partial \mathbf{x}_I})^T$, $\mathbf{x}_{ai}$ and $(\frac{\partial P_{I_{ij}}(\mathbf{x}_{I_i})}{\partial \mathbf{x}_I})^T$ cannot be nonzero at the same time.

Therefore, the last three terms in equation (4.44) are zero and hence:

$$\begin{aligned}
\dot{V}_m = & -\mathbf{s}_m^T \mathbf{K}_s \mathbf{s}_m - k_p [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_p + \alpha_q \Delta \boldsymbol{\varepsilon}_q]^T \\
& \times [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_p + \alpha_q \Delta \boldsymbol{\varepsilon}_q]. \quad (4.46)
\end{aligned}$$

Since $V_m > 0$ and $\dot{V}_m \leq 0$, V_m is bounded. Hence, $\mathbf{s}_m$, $\Delta \boldsymbol{\theta}_d$, $P_J(\mathbf{q})$, $P_D(\mathbf{x}_I)$ and

$P_C(\mathbf{p})$ are bounded. Hence, the region functions are bounded. The boundedness of region functions ensures the boundedness of $\mathbf{q}$, $\mathbf{x}_I$ and $\mathbf{p}$. Therefore, $\Delta\boldsymbol{\varepsilon}_q$, $\Delta\boldsymbol{\varepsilon}_x$ and $\Delta\boldsymbol{\varepsilon}_p$ are bounded. Since $\mathbf{x}_I$ is bounded, $\dot{\mathbf{x}}_a$ is bounded if $\dot{\mathbf{x}}_d$ is bounded. Hence, $\dot{\mathbf{q}}_{mr}$ is bounded. From equation (4.33), $\dot{\mathbf{q}}$ is bounded because $\mathbf{s}_m$ is bounded. The boundedness of $\dot{\mathbf{q}}$ guarantees the boundedness of $\dot{\mathbf{x}}_I$ and $\dot{\mathbf{p}}$ since $\mathbf{J}_x(\mathbf{q})$ and $\mathbf{J}_r(\mathbf{q})$ are bounded. Therefore, $\Delta\dot{\boldsymbol{\varepsilon}}_q$, $\Delta\dot{\boldsymbol{\varepsilon}}_p$ and $\Delta\dot{\boldsymbol{\varepsilon}}_x$ are bounded. Then $\ddot{\mathbf{q}}_{mr}$ is bounded if $\ddot{\mathbf{x}}_d$ is bounded. From the closed-loop equation (4.39), it is concluded that $\dot{\mathbf{s}}_m$ is bounded. Thus, $\ddot{V}_m$ is bounded since $\dot{\mathbf{s}}_m$, $\mathbf{s}_m$, $\Delta\dot{\boldsymbol{\varepsilon}}_q$, $\Delta\boldsymbol{\varepsilon}_q$, $\Delta\dot{\boldsymbol{\varepsilon}}_x$, $\Delta\boldsymbol{\varepsilon}_x$, $\Delta\dot{\boldsymbol{\varepsilon}}_p$, $\Delta\boldsymbol{\varepsilon}_p$ are bounded. Therefore, $\dot{V}_m$ is uniformly continuous. Applying Barbalat's lemma [83], it is obtained that $\dot{V}_m \rightarrow 0$ which also indicates:

$$\begin{aligned} \alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta\boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta\boldsymbol{\varepsilon}_p + \alpha_q \Delta\boldsymbol{\varepsilon}_q &\rightarrow \mathbf{0} \\ \mathbf{s}_m &\rightarrow \mathbf{0}. \end{aligned} \quad (4.47)$$

If the end effector is located at the singular positions, $\Delta\boldsymbol{\varepsilon}_q \neq \mathbf{0}$, $\Delta\boldsymbol{\varepsilon}_p = \mathbf{0}$, $\Delta\boldsymbol{\varepsilon}_x = \mathbf{0}$, which contracts with equation (4.47). If the end effector is located at the Cartesian-space region, $\Delta\boldsymbol{\varepsilon}_q = \mathbf{0}$, $\Delta\boldsymbol{\varepsilon}_p \neq \mathbf{0}$, $\Delta\boldsymbol{\varepsilon}_x = \mathbf{0}$, which also contracts with equation (4.47). Therefore, the end effector can only settle down within the image-space region. Then $\Delta\boldsymbol{\varepsilon}_q = \mathbf{0}$ and $\Delta\boldsymbol{\varepsilon}_p = \mathbf{0}$, and from equation (4.47), $\Delta\boldsymbol{\varepsilon}_x = \mathbf{0}$, which means $\mathbf{x}_I \rightarrow \mathbf{x}_d$ as $t \rightarrow \infty$. From the definition of $\dot{\mathbf{x}}_a$ in equation (3.53), $\mathbf{x}_I \rightarrow \mathbf{x}_d$ indicate that $\dot{\mathbf{x}}_a \rightarrow \dot{\mathbf{x}}_d$. From the definition of $\mathbf{s}_m$ in equation (4.33), $\mathbf{s}_m \rightarrow \mathbf{0}$, $\dot{\mathbf{x}}_a \rightarrow \dot{\mathbf{x}}_d$, $\Delta\boldsymbol{\varepsilon}_x \rightarrow \mathbf{0}$, $\Delta\boldsymbol{\varepsilon}_q \rightarrow \mathbf{0}$, $\Delta\boldsymbol{\varepsilon}_p \rightarrow \mathbf{0}$ implies $\dot{\mathbf{x}}_I \rightarrow \dot{\mathbf{x}}_d$ as $t \rightarrow \infty$. $\triangle\triangle\triangle$

Remark 4.1 Unlike position-based visual servoing, the proposed regional feedback method for robot task-space control does not require the translation from Cartesian space to image space. The tracking errors specified in task spaces such as Cartesian space or image space are directly transformed into the torque input of the robot in

joint space by using the Jacobian matrices $\mathbf{J}_x(\mathbf{q})$ and $\mathbf{J}_r(\mathbf{q})$, as given in equation (4.35). △△△

Remark 4.2 The results in this chapter can also be extended to an adaptive controller with multiple regional feedback, which is able to deal with the uncertainties in robot kinematics and depth information. The controller can be similarly formulated as in Remark 3.10 of chapter 3. △△△

4.3 Experiment

The proposed multiple task-space controller in equation (4.35) was also implemented in the first two links of the SCARA manipulator system. The experimental setup is reproduced in Fig. 4.8. The PSD camera is used to measure the position of end effector, and it is placed perpendicular to the end effector's evolving plane so that the image plane is parallel to the 2-D Cartesian space, and hence the relation between the position of end effector in image space and that in Cartesian space is specified as:

$$\begin{bmatrix} x_{1_h} \\ x_{1_v} \end{bmatrix} = \frac{f}{z} \begin{bmatrix} p_{p1_1} \\ p_{p1_2} \end{bmatrix}. \quad (4.48)$$

The end effector is controlled to start from an initial position near the external singular configuration, and track a time-varying trajectory specified in image space.

Since the singularities occur where $q_2 = 0$ and $q_2 = \pi$, the external and internal singular regions in equation (4.2) were specified to cover the singular configurations

as:

$$\begin{aligned} h_{J_{E_1}}(\mathbf{q}) &= \pi/15^2 - q_2^2 \geq 0, \\ h_{J_{I_{11}}}(\mathbf{q}) &= (q_2 - \pi)^2 - \pi/10^2 \leq 0. \end{aligned} \quad (4.49)$$

Figure 4.8: Experimental setup

Both the external and internal singularity correspond to circular path in Cartesian space. To exclude the external singular position and enclose the internal singular position, the sub Cartesian-space regions $h_{C_{E_1}}(\mathbf{p}_1)$ in equation (4.6) and $h_{C_{I_{11}}}(\mathbf{p}_1)$ in equation (4.10) were specified as:

$$\begin{aligned} h_{C_{E_1}}(\mathbf{p}_1) &= \frac{p_{1_1}^2}{0.72^2} + \frac{p_{1_2}^2}{0.72^2} - 1 \leq 0, \\ h_{C_{I_{11}}}(\mathbf{p}_1) &= \frac{p_{1_1}^2}{0.12^2} + \frac{p_{1_2}^2}{0.12^2} - 1 \geq 0, \end{aligned} \quad (4.50)$$

where the reference positions $\mathbf{p}_{E1} = \mathbf{p}_{I11} = [0, 0]^T$ were the origin of Cartesian coordinates, and the reference regions $h_{C_{Er1}}(\mathbf{p}_1)$ and $h_{C_{Ir11}}(\mathbf{p}_1)$ were set with the parameters $\kappa_{CE1} = 0.65$ and $\kappa_{CI11} = 1.5$ respectively.

The external image-space region $f_{E1}(\mathbf{x}_{I1})$ in equation (4.4) was specified to match the rectangular field of view as:

$$f_{E1}(\mathbf{x}_{I1}) = \frac{(x_{1h}+0.22)^{20}}{1.85^{20}} + \frac{(x_{1v}-0.55)^{20}}{1.85^{20}} - 1 \leq 0, \quad (4.51)$$

where the order n_{E1} was set as $n_{E1} = 20$ so that $f_{E1}(\mathbf{x}_{I1})$ was specified as a rectangle with rounded corners. The reference region $f_{Er1}(\mathbf{x}_{I1})$ was set with the parameter $\kappa_{E1} = 0.81$. The task-oriented image-space region was specified as:

$$f_T(\mathbf{x}_{I1}) = \frac{(x_{1h}-x_{d1h})^2}{(x_{b1h}-x_{d1h})^2} + \frac{(x_{1v}-x_{d1v})^2}{(x_{b1v}-x_{d1v})^2} - 1 \leq 0, \quad (4.52)$$

where $x_{b1h} = -1.72 \text{ V}$ if $x_{1h} \leq x_{d1h}$, else $x_{b1h} = 1.78 \text{ V}$, and $x_{b1v} = -1.45 \text{ V}$ if $x_{1v} \leq x_{d1v}$, else $x_{b1v} = 2.25 \text{ V}$.

To match the image-space regions, the sub region $h_{CV1}(\mathbf{p}_1)$ in equation (4.16) was specified as:

$$h_{CV1}(\mathbf{p}_1) = \frac{(p_{11}+0.37)^{20}}{0.1^{20}} + \frac{(p_{12}+0.13)^{20}}{0.1^{20}} - 1 \leq 0, \quad (4.53)$$

and the reference regions $h_{CVr1}(\mathbf{p}_1)$ was set with the parameter $\kappa_{CV1} = 1.2$.

Finally, the sub region $h_{CEV1}(\mathbf{p}_1)$ in equation (4.21) was specified to drive the end effector to move from Cartesian-space region to image-space region as:

$$h_{CEV1}(\mathbf{p}_1) = \frac{(p_{11}+0.37)^2}{(p_{b11}+0.37)^2} + \frac{(p_{12}-0.13)^2}{(p_{b12}-0.13)^2} - 1 \leq 0, \quad (4.54)$$

where the boundary positions were set as: $p_{b1_1} = -0.51 \text{ m}$ if $p_{p1_1} \leq -0.37 \text{ m}$, else $p_{b1_1} = 0.51 \text{ m}$, and $p_{b1_2} = -0.51 \text{ m}$ if $p_{p1_2} \leq 0.13 \text{ m}$, else $p_{b1_2} = 0.51 \text{ m}$.

The initial position is set as $(-0.23, 0.68)m$ where q_2 is near the singular position $q_2 = 0$, and the desired trajectory is specified in image space as:

$$\begin{cases} x_{d1_h} = 0.83 + 0.7\cos(0.4t)w(\mathbf{x}_{I1}), \\ x_{d1_v} = 0.34 + 0.7\sin(0.4t)w(\mathbf{x}_{I1}), \end{cases} \quad (4.55)$$

where the w is the weight factor. The order of potential energy in equations (4.1), (4.3), (4.8), (4.12), (4.18) and (4.22) was set as $N = 4$, and the control parameters in equation (4.35) and (4.36) were set as: $k_{qE_1} = 5000$, $k_{CE_1} = 0.0001$, $k_{CI_{11}} = 0.7$, $k_{CV_1} = 0.4$, $k_{CEV_1} = 0.0005$, $\mathbf{K}_s = \text{diag}\{0.0005, 0.0005\}$, $k_p = 1$, $\alpha_x = 0.002$, $\alpha_q = 1$, $\alpha_r = 1$, $\mathbf{L}_d = \text{diag}\{0.01, 0.01\}$. The experimental results are shown in Fig. 4.9(a)-4.9(d).

As seen from Fig. 4.9, the end effector started from an initial point near the external singular position, and transited from the Cartesian-space region towards the image space region. After it entered the image space region, the position of end effector converged to the desired trajectory.

(a) Path of end effector in Cartesian space

(b) Path of end effector in image space

(c) Tracking errors in image space

(d) Control input

Figure 4.9: The end effector started from an initial position at $(-0.23 \text{ m}, 0.68 \text{ m})$ near the external singular position, and transited from the Cartesian-space region toward the image-space region, and eventually converged to the desired trajectory.

4.4 Summary

In this chapter, the results of regional feedback control method are extended to a multiple regional feedback controller which integrates the feedback from joint space, Cartesian space and image space into one controller. The joint-space feedback is employed to drive the end effector away from the singular configurations. The image-space feedback is introduced to ensure the convergence of tracking errors after the end effector enters the image-space region. The Cartesian-space feedback is employed to ensure a smooth transition from joint-space regions to image-space regions. Each feedback information is employed in a corresponding local region, and the combination of regional feedback guarantees the global stability of robot movement. Therefore, the proposed controller enables the robot to start from any initial position outside the field of view and in the vicinity of singularity points, and eventually track the desired trajectory in image space. The stability of the closed-loop system is shown by using Lyapunov-like analysis, and the experimental results have been presented to illustrate the performance of the proposed multiple regional feedback controller.

Chapter 5

Dynamic Trapping and Manipulation of Biological Cells

Rapid advances in biological sciences and nanotechnology have led to the requirement of robotics and automation at micro and nano scales, thus opening up new challenges to understanding robotic manipulation of cells or nanoparticles. Current optical manipulation techniques ignore the effect of Gaussian field, and the dynamics of manipulator of the laser source is not considered in the formulations of the controllers. Therefore, existing control methods treat the position of the laser beam as the control input and open-loop controller is designed to move the laser source. Without the feedback of the laser position, the accuracy of positioning may not be guaranteed, and the trapping also fails when the laser starts from a large initial position or when the cell escapes the trap during the course of manipulation.

In this chapter, the concept of regional feedback control method is extended to the robot-assisted optical tweezers system. A unified robotic manipulation technique for optical tweezers that integrates automatic trapping and manipulation of biological cells into a single method is proposed. Instead of using open-loop control of the

position of laser source as assumed in the literature, the dynamics of robotic manipulator is introduced into optical tweezers system so that a closed-loop manipulator control problem is formulated and solved.

5.1 Dynamic Trapping and Manipulation

As discussed in section 2.2, an optical trap works only when the cell is located in a small neighborhood of the centroid of the focused laser beam. A trapping region is introduced in this chapter to monitor the offset between the laser beam and the cell. When the cell is outside the region, it cannot be trapped by the laser beam. When the cell is inside the region, it is attracted towards the centroid of the laser beam which results in a stable trap.

Figure 5.1: (i) The laser beam moves towards the cell to trap it; (ii) The trapped cell is manipulated to follow the desired trajectory.

Therefore, the optical manipulation is consisted of two phases as illustrated in Fig. 5.1: (i) *trapping phase* - the laser beam is controlled to move towards the cell so as to trap it; (ii) *manipulation phase* - the trapped cell stays within the trapping region and is manipulated to follow the desired trajectory. The control schemes for the optical tweezers should be able to smoothly transit between the trapping phase

and the manipulation phase to avoid any chattering and vibration which are not desirable for micromanipulation. Since the position of trapping region varies with the moving laser beam, the control problems in optical tweezers systems are significantly different from the robot regional feedback control in the previous chapters.

5.1.1 Trapping Region

To monitor the distance between the laser and the cell, a trapping region is introduced near the position of the cell as:

$$f(\mathbf{x}_C, \mathbf{q}_L) = \|\mathbf{x}_C - \mathbf{q}_L\|^2 - b_T^2 \leq 0, \quad (5.1)$$

where $\mathbf{x}_C$ represents the cell position, and $\mathbf{q}_L$ denotes the laser position, and b_T is a positive constant which is set so that the region $f(\mathbf{x}_C, \mathbf{q}_L) \leq 0$ is inside the Gaussian potential field. If the position of the cell is outside the trapping region, $f(\mathbf{x}_C, \mathbf{q}_L) > 0$, and the laser should be moved towards the cell for trapping. If the cell is inside the trapping region, $f(\mathbf{x}_C, \mathbf{q}_L) \leq 0$, the cell is trapped by the laser, and the trapped cell can be transported along the desired trajectory. Therefore, the position of the laser is controlled to ensure that $f(\mathbf{x}_C, \mathbf{q}_L) \leq 0$ to maintain trapping.

Next, a reference region smaller than $f(\mathbf{x}_C, \mathbf{q}_L)$ is introduced as:

$$f_r(\mathbf{x}_C, \mathbf{q}_L) = \|\mathbf{x}_C - \mathbf{q}_L\|^2 - (\kappa_T b_T)^2 \leq 0, \quad (5.2)$$

where $\kappa_T < 1$ is a positive constant. By using $f(\mathbf{x}_C, \mathbf{q}_L)$ and $f_r(\mathbf{x}_C, \mathbf{q}_L)$, a weight

factor $w_T(\mathbf{x}_C, \mathbf{q}_L)$ is introduced as:

$$w_T(\mathbf{x}_c, \mathbf{q}_L) = \begin{cases} 1, & f_r(\mathbf{x}_C, \mathbf{q}_L) \leq 0, \\ 1 - \frac{\{[f(\mathbf{x}_C, \mathbf{q}_L)]^{N_T} - [(\kappa_T b_T)^2 - b_T^2]^{N_T}\}^{N_T}}{[(\kappa_T b_T)^2 - b_T^2]^{N_T^2}}, & f(\mathbf{x}_C, \mathbf{q}_L) < 0, f_r(\mathbf{x}_C, \mathbf{q}_L) > 0, \\ 0, & f(\mathbf{x}_C, \mathbf{q}_L) \geq 0, \end{cases} \quad (5.3)$$

Figure 5.2: An illustration of weight factor with $\mathbf{q}_L = [0, 0]^T$, $b_T = 3 \mu m$, $\kappa_T = 0.7$, and $N_T = 6$.

where $N_T \geq 6$ is an even integer, so that $w_T(\mathbf{x}_C, \mathbf{q}_L) \in C^3$. An illustration of $w_T(\mathbf{x}_C, \mathbf{q}_L)$ is show in Fig. 5.2. From equation (5.3) and Fig. 5.2, the weight factor $w_T(\mathbf{x}_C, \mathbf{q}_L)$ smoothly increases from 0 to 1 when the cell moves from outside to inside the trapping region, and vice versa. The weight factor $w_T(\mathbf{x}_C, \mathbf{q}_L)$ will be used to enable the controller to transit between the trapping operation and the manipulation operation, and he gradient of the weight factor can be varied by adjusting the parameter κ_T .

5.1.2 Desired Position Input of Laser Beam

Using the trapping region and the weight factor, a unified dynamic control method for automatic trapping and manipulation is proposed. Firstly, based on the cell dynamics (2.35), a desired position input of laser beam $\mathbf{q}_{Ld}$ is developed to ensure the convergence of the tracking error. Then based on the dynamics of manipulator of

the laser source (2.38), a backstepping procedure is used to derive a control input $\mathbf{u}$ for the robotic manipulator of the laser source to guarantee that the actual position input $\mathbf{q}_L$ tracks the desired position input $\mathbf{q}_{Ld}$.

Since the desired position is only required after the cell is trapped, the desired trajectory is specified as:

$$\mathbf{x}_{Cd}(t) = \mathbf{x}_{Cc} + w_T(\mathbf{x}_C, \mathbf{q}_L)\mathbf{x}_{Cv}(t), \quad (5.4)$$

where $\mathbf{x}_{Cd}(t)$ is the desired position for the trapped cell, and $\mathbf{x}_{Cc}$ is the constant part, and $\mathbf{x}_{Cv}(t)$ is the time-varying part of the desired trajectory. When the cell is outside the trapping region, $w_T(\mathbf{x}_C, \mathbf{q}_L) = 0$ and $\dot{\mathbf{x}}_{Cd} = \mathbf{0}$. After the cell is trapped by the laser, $w_T(\mathbf{x}_C, \mathbf{q}_L)$ smoothly increases to 1, and hence $\mathbf{x}_{Cd}(t) = \mathbf{x}_{Cc} + \mathbf{x}_{Cv}(t)$ which is the actual trajectory.

First, a sliding vector is introduced as:

$$\begin{aligned} \mathbf{s}_x &= \dot{\mathbf{x}}_C - \dot{\mathbf{x}}_{Cr} = \frac{d}{dt}(\mathbf{x}_C - \mathbf{x}_{Cd}) + \alpha_{Cx} \frac{w_T(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta} (\mathbf{x}_C - \mathbf{x}_{Cd}) \\ &= \Delta \dot{\mathbf{x}}_C + \alpha_{Cx} \frac{w_T(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta} \Delta \mathbf{x}_C, \end{aligned} \quad (5.5)$$

where $\Delta \mathbf{x}_C = \mathbf{x}_C - \mathbf{x}_{Cd}$, and $\Delta \dot{\mathbf{x}}_C = \dot{\mathbf{x}}_C - \dot{\mathbf{x}}_{Cd}$, and $\dot{\mathbf{x}}_{Cr}$ is a reference vector defined as:

$$\dot{\mathbf{x}}_{Cr} = \dot{\mathbf{x}}_{Cd} - \alpha_{Cx} \frac{w_T(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta} \Delta \mathbf{x}_C, \quad (5.6)$$

and α_{Cx} is a positive constant, and δ is a very small positive constant. The role of δ is to ensure that the term $\frac{w_T(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta}$ is not ill-defined when $w_T(\mathbf{x}_C, \mathbf{q}_L) = 0$.

Differentiating equation (5.5) with respect to time yields:

$$\dot{\mathbf{s}}_x = \ddot{\mathbf{x}}_C - \ddot{\mathbf{x}}_{Cr}. \quad (5.7)$$

From equation (2.35), the cell dynamic equation is stated as:

$$\mathbf{M}_C \ddot{\mathbf{x}}_C + \mathbf{B}_C \dot{\mathbf{x}}_C + k_1(\mathbf{x}_C - \mathbf{q}_L) e^{-k_2 \|\mathbf{x}_C - \mathbf{q}_L\|^2} = \mathbf{0}, \quad (5.8)$$

By using the sliding vector $\mathbf{s}_x$ and Property 2.4, the cell dynamics in equation (5.8) can be represented as:

$$\mathbf{M}_C \dot{\mathbf{s}}_x + \mathbf{B}_C \mathbf{s}_x + \mathbf{Y}_C(\dot{\mathbf{x}}_{Cr}, \ddot{\mathbf{x}}_{Cr}) \boldsymbol{\theta}_C + k_3(\mathbf{x}_C, \mathbf{q}_L)(\mathbf{x}_C - \mathbf{q}_L) = \mathbf{0}, \quad (5.9)$$

where we note that $k_3(\mathbf{x}_C, \mathbf{q}_L) = k_1 e^{-k_2 \|\mathbf{x}_C - \mathbf{q}_L\|^2}$ is a positive time-varying gain.

Supposing the desired position input for the trapped cell is denoted as $\mathbf{q}_{Ld}$, then equation (5.9) can be written as:

$$\begin{aligned} \mathbf{M}_C \dot{\mathbf{s}}_x + \mathbf{B}_C \mathbf{s}_x + \mathbf{Y}_C(\dot{\mathbf{x}}_{Cr}, \ddot{\mathbf{x}}_{Cr}) \boldsymbol{\theta}_C + k_3(\mathbf{x}_C, \mathbf{q}_L) \mathbf{x}_C \\ = k_3(\mathbf{x}_C, \mathbf{q}_L) \Delta \mathbf{q}_L + k_3(\mathbf{x}_C, \mathbf{q}_L) \mathbf{q}_{Ld}, \end{aligned} \quad (5.10)$$

where $\mathbf{Y}_C(\dot{\mathbf{x}}_C, \ddot{\mathbf{x}}_C) \in \mathbb{R}^{2 \times n_C}$ is the dynamic regressor matrix, and $\Delta \mathbf{q}_L = \mathbf{q}_L - \mathbf{q}_{Ld}$ represents an input perturbation to the cell dynamics. The system in equation (5.10) can be viewed as being controlled by the input $k_3(\mathbf{x}_C, \mathbf{q}_L) \mathbf{q}_{Ld}$ with the perturbation $k_3(\mathbf{x}_C, \mathbf{q}_L) \Delta \mathbf{q}_L$.

The desired position input is proposed as:

$$\begin{aligned} \mathbf{q}_{Ld} = & \mathbf{x}_C - k_3^{-1}(\mathbf{x}_C, \mathbf{q}_L)w_T(\mathbf{x}_C, \mathbf{q}_L)\mathbf{K}_p\Delta\mathbf{x}_C \\ & - k_3^{-1}(\mathbf{x}_C, \mathbf{q}_L)w_T(\mathbf{x}_C, \mathbf{q}_L)\mathbf{K}_d\mathbf{s}_x + k_3^{-1}(\mathbf{x}_C, \mathbf{q}_L)\mathbf{Y}_C(\dot{\mathbf{x}}_{Cr}, \ddot{\mathbf{x}}_{Cr})\hat{\boldsymbol{\theta}}_C, \end{aligned} \quad (5.11)$$

where $\mathbf{K}_p$ and $\mathbf{K}_d$ are positive definite, and $\hat{\boldsymbol{\theta}}_C$ is the uncertain dynamic parameters of cell. An update law is proposed as follows:

$$\dot{\hat{\boldsymbol{\theta}}}_C = -\mathbf{L}_C\mathbf{Y}_C^T(\dot{\mathbf{x}}_{Cr}, \ddot{\mathbf{x}}_{Cr})\mathbf{s}_x, \quad (5.12)$$

where $\mathbf{L}_C \in \Re^{n_C \times n_C}$ is a positive definite matrix.

From the definition of trapping region $f(\mathbf{x}_C, \mathbf{q}_L)$ described by (5.1) and the definition of weight factor $w_T(\mathbf{x}_C, \mathbf{q}_L)$ described by (5.3), it is clear that if the cell is outside the trapping region, $f(\mathbf{x}_C, \mathbf{q}_L) \geq 0$, $w_T(\mathbf{x}_C, \mathbf{q}_L) = 0$, and hence $\dot{\mathbf{x}}_{Cd} = \mathbf{0}$. In addition, $\dot{\mathbf{x}}_{Cr}$ and $\ddot{\mathbf{x}}_{Cr}$ are also equal to zero since $\dot{\mathbf{x}}_{Cr} = \dot{\mathbf{x}}_{Cd} - \alpha_C x \frac{w_T(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta} \Delta\mathbf{x}_C$. Therefore, the regressor matrix $\mathbf{Y}_C(\dot{\mathbf{x}}_{Cr}, \ddot{\mathbf{x}}_{Cr})$ reduces to zero. From equation (5.11), the desired position input is specified as:

$$\mathbf{q}_{Ld} = \mathbf{x}_C. \quad (5.13)$$

That is, when the desired position input is the position of the cell, the laser is moved towards the cell. The desired position input (5.13) is in *trapping phase*.

After the cell enters the trapping region, $w_T(\mathbf{x}_C, \mathbf{q}_L) = 1$, and the desired position input in equation (5.11) becomes:

$$\begin{aligned} \mathbf{q}_{Ld} = & \mathbf{x}_C - k_3^{-1}(\mathbf{x}_C, \mathbf{q}_L)\mathbf{K}_p\Delta\mathbf{x}_C - k_3^{-1}(\mathbf{x}_C, \mathbf{q}_L)\mathbf{K}_d\mathbf{s}_x \\ & + k_3^{-1}(\mathbf{x}_C, \mathbf{q}_L)\mathbf{Y}_C(\dot{\mathbf{x}}_{Cr}, \ddot{\mathbf{x}}_{Cr})\hat{\boldsymbol{\theta}}_C, \end{aligned} \quad (5.14)$$

Figure 5.3: Two phases of optical manipulation.

and it drives the trapped cell towards the desired trajectory. The desired position input (5.14) is in *manipulation phase*. In the case that the cell escapes from the trapping region, the weight factor reduces to zero and the *trapping phase* is activated again. The proposed trapping and manipulation mechanism is illustrated in Fig. 5.3.

Substituting the desired position input for the trapped cell (5.11) into equation (5.10), it is obtained:

$$\begin{aligned} \mathbf{M}_C \dot{\mathbf{s}}_x + \mathbf{B}_C \mathbf{s}_x + w_T(\mathbf{x}_C, \mathbf{q}_L) \mathbf{K}_p \Delta \mathbf{x}_C + w_T(\mathbf{x}_C, \mathbf{q}_L) \mathbf{K}_d \mathbf{s}_x \\ + \mathbf{Y}_C(\dot{\mathbf{x}}_{Cr}, \ddot{\mathbf{x}}_{Cr}) \Delta \boldsymbol{\theta}_C = k_3(\mathbf{x}_C, \mathbf{q}_L) \Delta \mathbf{q}_L. \end{aligned} \quad (5.15)$$

To analyze the stability of the trapped cell system, a Lyapunov-like candidate V_x is proposed as:

$$V_x = \frac{1}{2} \mathbf{s}_x^T \mathbf{M}_C \mathbf{s}_x + \frac{w_T(\mathbf{x}_C, \mathbf{q}_L)}{2} \Delta \mathbf{x}_C^T \mathbf{K}_p \Delta \mathbf{x}_C + \frac{1}{2} \Delta \boldsymbol{\theta}_C^T \mathbf{L}_C^{-1} \Delta \boldsymbol{\theta}_C. \quad (5.16)$$

Differentiating V_x with respect to time yields:

$$\begin{aligned}\dot{V}_x = & \mathbf{s}_x^T \mathbf{M}_C \dot{\mathbf{s}}_x - \dot{\hat{\boldsymbol{\theta}}}_C^T \mathbf{L}_C^{-1} \Delta \boldsymbol{\theta}_C + \frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2} \Delta \mathbf{x}_C^T \mathbf{K}_p \Delta \mathbf{x}_C \\ & + w_T(\mathbf{x}_C, \mathbf{q}_L) (\dot{\mathbf{x}}_C - \dot{\mathbf{x}}_{Cd})^T \mathbf{K}_p \Delta \mathbf{x}_C,\end{aligned}\quad (5.17)$$

and substituting equation (5.12) and equation (5.15) into equation (5.17), it is obtained:

$$\begin{aligned}\dot{V}_x = & -\mathbf{s}_x^T (\mathbf{B}_C + w_T(\mathbf{x}_C, \mathbf{q}_L) \mathbf{K}_d) \mathbf{s}_x - w_T(\mathbf{x}_C, \mathbf{q}_L) \mathbf{s}_x^T \mathbf{K}_p \Delta \mathbf{x}_C + k_3(\mathbf{x}_C, \mathbf{q}_L) \mathbf{s}_x^T \Delta \mathbf{q}_L \\ & + \frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2} \Delta \mathbf{x}_C^T \mathbf{K}_p \Delta \mathbf{x}_C + w_T(\mathbf{x}_C, \mathbf{q}_L) (\dot{\mathbf{x}}_C - \dot{\mathbf{x}}_{Cd})^T \mathbf{K}_p \Delta \mathbf{x}_C.\end{aligned}\quad (5.18)$$

Then substituting equation (5.5) into equation (5.18) to give:

$$\begin{aligned}\dot{V}_x = & -\mathbf{s}_x^T (\mathbf{B}_C + w_T(\mathbf{x}_C, \mathbf{q}_L) \mathbf{K}_d) \mathbf{s}_x + k_3(\mathbf{x}_C, \mathbf{q}_L) \mathbf{s}_x^T \Delta \mathbf{q}_L \\ & - w_T(\mathbf{x}_C, \mathbf{q}_L) (\dot{\mathbf{x}}_C - \dot{\mathbf{x}}_{Cd} + \alpha_{Cx} \frac{w_T(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta} \Delta \mathbf{x}_C)^T \mathbf{K}_p \Delta \mathbf{x}_C \\ & + \frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2} \Delta \mathbf{x}_C^T \mathbf{K}_p \Delta \mathbf{x}_C + w_T(\mathbf{x}_C, \mathbf{q}_L) (\dot{\mathbf{x}}_C - \dot{\mathbf{x}}_{Cd})^T \mathbf{K}_p \Delta \mathbf{x}_C \\ & = -\mathbf{s}_x^T (\mathbf{B}_C + w_T(\mathbf{x}_C, \mathbf{q}_L) \mathbf{K}_d) \mathbf{s}_x + k_3(\mathbf{x}_C, \mathbf{q}_L) \mathbf{s}_x^T \Delta \mathbf{q}_L \\ & + \frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2} \Delta \mathbf{x}_C^T \mathbf{K}_p \Delta \mathbf{x}_C - \alpha_{Cx} \frac{w_T^2(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta} \Delta \mathbf{x}_C^T \mathbf{K}_p \Delta \mathbf{x}_C \\ & = -\mathbf{s}_x^T (\mathbf{B}_C + w_T(\mathbf{x}_C, \mathbf{q}_L) \mathbf{K}_d) \mathbf{s}_x + k_3(\mathbf{x}_C, \mathbf{q}_L) \mathbf{s}_x^T \Delta \mathbf{q}_L \\ & \quad - \Delta \mathbf{x}_C^T (\alpha_{Cx} \frac{w_T^2(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta} \mathbf{K}_p - \frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2} \mathbf{K}_p) \Delta \mathbf{x}_C.\end{aligned}\quad (5.19)$$

Since δ is very small, the bound of $\frac{\delta}{w_T^2(\mathbf{x}_C, \mathbf{q}_L)}$ exists when $w_T(\mathbf{x}_C, \mathbf{q}_L) \neq 0$. In addition, since the derivative $\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)$ is continuous and $\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L) = 0$ where $f(\mathbf{x}_C, \mathbf{q}_L) \geq 0$ or $f_r(\mathbf{x}_C, \mathbf{q}_L) \leq 0$, $\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)$ is bounded.

From equation (5.5), note that α_{Cx} is only employed when $w_T(\mathbf{x}_C, \mathbf{q}_L) \neq 0$. There-

fore, the control parameter α_{Cx} can be chosen large enough so that

$$\alpha_{Cx} > \mathbf{b}_{max}\left(\frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)(w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta)}{2w_T^2(\mathbf{x}_C, \mathbf{q}_L)}\right) = \mathbf{b}_{max}\left(\frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2}\left(1 + \frac{\delta}{w_T^2(\mathbf{x}_C, \mathbf{q}_L)}\right)\right), \quad (5.20)$$

where $\mathbf{b}_{max}\left(\frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2}\left(1 + \frac{\delta}{w_T^2(\mathbf{x}_C, \mathbf{q}_L)}\right)\right)$ denotes the upper bound of $\frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2}\left(1 + \frac{\delta}{w_T^2(\mathbf{x}_C, \mathbf{q}_L)}\right)$. If the condition (5.20) is satisfied, $\alpha_{Cx} \frac{w_T^2(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta} - \frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2}$ in equation (5.19) is positive.

In the following lemma, the case when $\Delta \mathbf{q}_L = \mathbf{0}$ is considered to show the convergence of the tracking errors first, and an input of manipulator of the laser source is proposed to ensure the convergence of $\Delta \mathbf{q}_L \rightarrow \mathbf{0}$ in the next section.

Lemma 5.1: *The desired position input of the laser beam (5.11) and the update law (5.12) for the closed-loop equation of the cell (5.15) guarantee the convergence of $\mathbf{x}_C \rightarrow \mathbf{x}_{Cd}$ and $\dot{\mathbf{x}}_C \rightarrow \dot{\mathbf{x}}_{Cd}$ as $t \rightarrow \infty$ if $\Delta \mathbf{q}_L = \mathbf{0}$, and the control parameter α_{Cx} is chosen to satisfy equation (5.20).*

Proof: Since $\Delta \mathbf{q}_L = \mathbf{0}$, it is derived that $V_x > 0$ and $\dot{V}_x \leq 0$. Therefore, V_x is bounded, and $\mathbf{s}_x$, $\Delta \boldsymbol{\theta}_C$ and $\Delta \mathbf{x}_C$ are bounded. The boundedness of $\mathbf{s}_x$ and $\Delta \mathbf{x}_C$ ensures the boundedness of $\Delta \dot{\mathbf{x}}_C$ from equation (5.5). Therefore, $\Delta \mathbf{x}_C$ is uniformly continuous. Since $\Delta \mathbf{x}_C$ is bounded, the reference vector $\dot{\mathbf{x}}_{Cr}$ (5.6) is bounded. Since $\Delta \dot{\mathbf{x}}_C$ is bounded, $\ddot{\mathbf{x}}_{Cr}$ is bounded. From equation (5.15), since $\Delta \mathbf{q}_L = \mathbf{0}$, $\Delta \mathbf{x}_C$, $\mathbf{s}_x$, $\dot{\mathbf{x}}_{Cr}$, $\ddot{\mathbf{x}}_{Cr}$ and $\Delta \boldsymbol{\theta}_C$ are all bounded, it is concluded that $\dot{\mathbf{s}}_x$ is bounded. Therefore, $\mathbf{s}_x$ is uniformly continuous. Moreover, from equation (5.19), it is easy to verify that $\Delta \mathbf{x}_C$, $\mathbf{s}_x \in L_2(0, +\infty)$. Then it follows [4, 84] that $\Delta \mathbf{x}_C \rightarrow \mathbf{0}$ and $\mathbf{s}_x \rightarrow \mathbf{0}$.

If the cell is outside the trapping region, $f(\mathbf{x}_C, \mathbf{q}_L) > 0$, thus $w_T(\mathbf{x}_C, \mathbf{q}_L) = 0$, $\mathbf{q}_{Ld} = \mathbf{x}_C$, and the laser is moved towards the cell. When the cell enters the trapping region, $f(\mathbf{x}_C, \mathbf{q}_L) \leq 0$, and the Gaussian field is in effect. The Gaussian field drives the cell towards the centroid of the laser beam, and $w_T(\mathbf{x}_C, \mathbf{q}_L)$ increases from 0

to 1. Therefore, the cell can be successfully trapped by the laser beam in the end, and $\Delta \mathbf{x}_C \rightarrow \mathbf{0}$ indicates that the trapped cell converges to the desired trajectory. That is, $\mathbf{x}_C \rightarrow \mathbf{x}_{Cd}$. From equation (5.5), $\mathbf{x}_C \rightarrow \mathbf{x}_{Cd}$, and $\mathbf{s}_x \rightarrow \mathbf{0}$ indicate that $\dot{\mathbf{x}}_C \rightarrow \dot{\mathbf{x}}_{Cd}$. △△△

5.1.3 Control Input of Robotic Manipulator of Laser Beam

In the previous section, the desired position input for the trapped cell $\mathbf{q}_{Ld}$ is proposed. In this section, a robot input $\mathbf{u}$ which ensures the convergence of $\mathbf{q}_L \rightarrow \mathbf{q}_{Ld}$ is formulated.

First, another sliding vector by using the desired position input is proposed as:

$$\mathbf{s}_L = \dot{\mathbf{q}}_L - \dot{\mathbf{q}}_{Lr} = \dot{\mathbf{q}}_L - \dot{\mathbf{q}}_{Ld} + \alpha_{Lq} \Delta \mathbf{q}_L, \quad (5.21)$$

where α_{Lq} is a positive constant, and $\dot{\mathbf{q}}_{Lr}$ is a reference vector defined as:

$$\dot{\mathbf{q}}_{Lr} = \dot{\mathbf{q}}_{Ld} - \alpha_{Lq} \Delta \mathbf{q}_L. \quad (5.22)$$

Next, the control input for the robotic manipulator of laser beam is proposed as:

$$\mathbf{u} = -\mathbf{K}_{sq} \mathbf{s}_L - \mathbf{K}_q \Delta \mathbf{q}_L + \mathbf{Y}_L(\dot{\mathbf{q}}_{Lr}, \ddot{\mathbf{q}}_{Lr}) \hat{\boldsymbol{\theta}}_L, \quad (5.23)$$

where $\mathbf{K}_{sq}$ and $\mathbf{K}_q$ are positive definite matrices, and the estimated parameters $\hat{\boldsymbol{\theta}}_L$ are updated by the following update law:

$$\dot{\hat{\boldsymbol{\theta}}}_L = -\mathbf{L}_L \mathbf{Y}_L^T(\dot{\mathbf{q}}_{Lr}, \ddot{\mathbf{q}}_{Lr}) \mathbf{s}_L. \quad (5.24)$$

where $\mathbf{L}_L \in \Re^{n_L \times n_L}$ is a positive definite matrix. Note that the desired position input

$\mathbf{q}_{Ld}$ in equation (5.11) is continuous, and hence the control input $\mathbf{u}$ in equation (5.23) is also continuous without chattering that is not desirable for micromanipulation.

From equation (2.38), the dynamic equation for the manipulator of the laser beam is stated as:

$$\mathbf{M}_L \ddot{\mathbf{q}}_L + \mathbf{B}_L \dot{\mathbf{q}}_L = \mathbf{u}. \quad (5.25)$$

By using the sliding vector $\mathbf{s}_L$, the manipulator dynamics in equation (5.25) can be rewritten as:

$$\mathbf{M}_L \dot{\mathbf{s}}_L + \mathbf{B}_L \mathbf{s}_L + \mathbf{Y}_L(\dot{\mathbf{q}}_{Lr}, \ddot{\mathbf{q}}_{Lr}) \boldsymbol{\theta}_L = \mathbf{u}. \quad (5.26)$$

Substituting the control input (5.23) into equation (5.26), the closed-loop equation is given as:

$$\mathbf{M}_L \dot{\mathbf{s}}_L + (\mathbf{B}_L + \mathbf{K}_{sq}) \mathbf{s}_L + \mathbf{K}_q \Delta \mathbf{q}_L + \mathbf{Y}_L(\dot{\mathbf{q}}_{Lr}, \ddot{\mathbf{q}}_{Lr}) \Delta \boldsymbol{\theta}_L = \mathbf{0}. \quad (5.27)$$

To prove the stability of the overall system, a Lyapunov-like candidate is proposed as:

$$V_T = V_x + V_q = V_x + \frac{1}{2} \mathbf{s}_L^T \mathbf{M}_L \mathbf{s}_L + \frac{1}{2} \Delta \mathbf{q}_L^T \mathbf{K}_q \Delta \mathbf{q}_L + \frac{1}{2} \Delta \boldsymbol{\theta}_L^T \mathbf{L}_L^{-1} \Delta \boldsymbol{\theta}_L, \quad (5.28)$$

where V_x is defined in equation (5.16).

Differentiating V_T in equation (5.28) with respect to time, it is obtained:

$$\dot{V}_T = \dot{V}_x + \dot{V}_q = \dot{V}_x + \mathbf{s}_L^T \mathbf{M}_L \dot{\mathbf{s}}_L + \Delta \mathbf{q}_L^T \mathbf{K}_q \Delta \dot{\mathbf{q}}_L - \dot{\boldsymbol{\theta}}_L^T \mathbf{L}_L^{-1} \Delta \boldsymbol{\theta}_L. \quad (5.29)$$

Substituting equations (5.24) and (5.27) into equation (5.29), it is obtained:

$$\begin{aligned}
\dot{V}_T &= \dot{V}_x + \Delta \mathbf{q}_L^T \mathbf{K}_q \Delta \dot{\mathbf{q}}_L - \dot{\boldsymbol{\theta}}_q^T \mathbf{L}_q^{-1} \Delta \boldsymbol{\theta}_q \\
&\quad - \mathbf{s}_L^T [(\mathbf{B}_L + \mathbf{K}_{sq}) \mathbf{s}_L + \mathbf{K}_q \Delta \mathbf{q}_L + \mathbf{Y}_L(\dot{\mathbf{q}}_{Lr}, \ddot{\mathbf{q}}_{Lr}) \Delta \boldsymbol{\theta}_L] \\
&= \dot{V}_x + \Delta \mathbf{q}_L^T \mathbf{K}_q \Delta \dot{\mathbf{q}}_L - \mathbf{s}_L^T (\mathbf{B}_L + \mathbf{K}_{sq}) \mathbf{s}_L - \mathbf{s}_L^T \mathbf{K}_q \Delta \mathbf{q}_L.
\end{aligned} \tag{5.30}$$

Substituting equations (5.19) and (5.21) into equation (5.30) yields:

$$\begin{aligned}
\dot{V}_T &= \dot{V}_x + \Delta \mathbf{q}_L^T \mathbf{K}_q \Delta \dot{\mathbf{q}}_L - \mathbf{s}_L^T (\mathbf{B}_L + \mathbf{K}_{sq}) \mathbf{s}_L - (\Delta \dot{\mathbf{q}}_L + \alpha_{Lq} \Delta \mathbf{q}_L)^T \mathbf{K}_q \Delta \mathbf{q}_L \\
&= -\Delta \mathbf{x}_C^T (\alpha_{Cx} \frac{w_T^2(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta} \mathbf{K}_p - \frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2} \mathbf{K}_p) \Delta \mathbf{x}_C - \alpha_{Lq} \Delta \mathbf{q}_L^T \mathbf{K}_q \Delta \mathbf{q}_L \\
&\quad - \mathbf{s}_L^T (\mathbf{B}_L + \mathbf{K}_{sq}) \mathbf{s}_L - \mathbf{s}_x^T (\mathbf{B}_C + w_T(\mathbf{x}_C, \mathbf{q}_L) \mathbf{K}_d) \mathbf{s}_x + k_3(\mathbf{x}_C, \mathbf{q}_L) \mathbf{s}_x^T \Delta \mathbf{q}_L \\
&= -\Delta \mathbf{x}_C^T (\alpha_{Cx} \frac{w_T^2(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta} \mathbf{K}_p - \frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2} \mathbf{K}_p) \Delta \mathbf{x}_C - \mathbf{s}_L^T (\mathbf{B}_q + \mathbf{K}_{sq}) \mathbf{s}_L \\
&\quad - \begin{bmatrix} \mathbf{s}_x^T & \Delta \mathbf{q}_L^T \end{bmatrix} \mathbf{Q} \begin{bmatrix} \mathbf{s}_x^T & \Delta \mathbf{q}_L^T \end{bmatrix}^T,
\end{aligned} \tag{5.31}$$

where $\mathbf{I}_2 \in \mathbb{R}^{2 \times 2}$ is an identity matrix, and $\mathbf{Q} = \begin{bmatrix} \mathbf{B}_C + w_T(\mathbf{x}_C, \mathbf{q}_L) \mathbf{K}_d & -\frac{k_3(\mathbf{x}_C, \mathbf{q}_L)}{2} \mathbf{I}_2 \\ -\frac{k_3(\mathbf{x}_C, \mathbf{q}_L)}{2} \mathbf{I}_2 & \alpha_{Lq} \mathbf{K}_q \end{bmatrix}$.

Note that $k_3(\mathbf{x}_C, \mathbf{q}_L) = k_1 e^{-k_2 \|\mathbf{x}_C - \mathbf{q}_L\|^2} \leq k_1$. Therefore, if the controller parameters α_{Lq} and $\mathbf{K}_q$ are chosen sufficiently large so that

$$\alpha_{Lq} \lambda_{\min}[\mathbf{K}_q \mathbf{B}_C] > \frac{k_1^2}{4}, \tag{5.32}$$

then the matrix $\mathbf{Q}$ is positive definite. The notation $\lambda_{\min}[\bullet]$ denotes the minimum eigenvalue of the matrix.

The following theorem is stated:

Theorem 5.1: *If the manipulator control input $\mathbf{u}$ given by equations (5.23), (5.24), (5.11) and (5.12) is applied to the robot-assisted cell manipulation system (5.8),*

(5.25), then the closed-loop system gives rise to the convergence of $\mathbf{x}_C \rightarrow \mathbf{x}_{Cd}$, $\dot{\mathbf{x}}_C \rightarrow \dot{\mathbf{x}}_{Cd}$ and $\mathbf{q}_L \rightarrow \mathbf{q}_{Ld}$ as $t \rightarrow \infty$ when the control parameters α_{Cx} , α_{Lq} and $\mathbf{K}_q$ are chosen to satisfy conditions (5.20) and (5.32).

Proof: If equations (5.20) and (5.32) are satisfied, it is derived that $V_T > 0$ and $\dot{V}_T \leq 0$. Therefore, V_T is bounded, and $\mathbf{s}_x$, $\Delta\boldsymbol{\theta}_C$, $\Delta\mathbf{x}_C$, $\mathbf{s}_L$, $\Delta\mathbf{q}_L$ and $\Delta\boldsymbol{\theta}_L$ are all bounded. The boundedness of $\mathbf{s}_L$ and $\Delta\mathbf{q}_L$ ensures the boundedness of $\Delta\dot{\mathbf{q}}_L$ from equation (5.21). Therefore, $\Delta\mathbf{q}_L$ is uniformly continuous. From equation (5.31), it is easy to verify that $\Delta\mathbf{q}_L \in L_2(0, +\infty)$. Then it follows [4, 84] that $\Delta\mathbf{q}_L \rightarrow \mathbf{0}$ and thus $\mathbf{x}_C \rightarrow \mathbf{x}_{Cd}$ and $\dot{\mathbf{x}}_C \rightarrow \dot{\mathbf{x}}_{Cd}$. $\triangle\triangle\triangle$

Remark 5.1: The manipulator of the laser source can also be extended to a general manipulator, and the dynamic model of the manipulator is specified as follows [4]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + [\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}, \quad (5.33)$$

and the velocity of the robot end effector in image space is related to the joint velocity as follows [22, 24]:

$$\dot{\mathbf{q}}_L = \mathbf{J}_L(\mathbf{q})\dot{\mathbf{q}}, \quad (5.34)$$

where $\mathbf{J}_L(\mathbf{q}) \in \mathbb{R}^{2 \times n}$ is the Jacobian matrix. The end effector is controlled to manipulate the laser source.

In this case, the control input for the manipulator of the laser beam is proposed as:

$$\boldsymbol{\tau} = -\mathbf{K}_{sq}\mathbf{s}_c - \mathbf{J}_L^T(\mathbf{q})\mathbf{K}_q\Delta\mathbf{q}_L + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_{cr}, \ddot{\mathbf{q}}_{cr})\hat{\boldsymbol{\theta}}_d, \quad (5.35)$$

and $\mathbf{s}_c = \dot{\mathbf{q}} - \dot{\mathbf{q}}_{cr} = \dot{\mathbf{q}} - \mathbf{J}_L^+(\mathbf{q})\dot{\mathbf{q}}_{Ld} + \alpha_{Lq}\mathbf{J}_L^+(\mathbf{q})\Delta\mathbf{q}_L$ where $\mathbf{J}_L^+(\mathbf{q})$ is the pseudo-inverse of $\mathbf{J}_L(\mathbf{q})$, and $\hat{\boldsymbol{\theta}}_d$ are updated by $\dot{\hat{\boldsymbol{\theta}}}_d = -\mathbf{L}_d\mathbf{Y}_d^T(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_{cr}, \ddot{\mathbf{q}}_{cr})\mathbf{s}_c$. Substituting the

control input into the robot dynamic equation, the closed-loop equation is obtained as: $\mathbf{M}(\mathbf{q})\dot{\mathbf{s}}_c + [\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{K}_{sq}]\mathbf{s}_c + \mathbf{J}_L^T(\mathbf{q})\mathbf{K}_q\Delta\mathbf{q}_L + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_{cr}, \ddot{\mathbf{q}}_{cr})\Delta\boldsymbol{\theta}_d = \mathbf{0}$.

To prove the stability, a Lyapunov-like candidate is proposed as: $V_c = V_x + \frac{1}{2}\mathbf{s}_c^T \mathbf{M}(\mathbf{q})\mathbf{s}_c + \frac{1}{2}\Delta\mathbf{q}_L^T \mathbf{K}_q \Delta\mathbf{q}_L + \frac{1}{2}\Delta\boldsymbol{\theta}_d^T \mathbf{L}_d^{-1} \Delta\boldsymbol{\theta}_d$, where V_x is defined in equation (5.16). Differentiating V_c with respect to time and substituting the closed-loop equation and the update law into it, we have: $\dot{V}_c = -\Delta\mathbf{x}_C^T (\alpha_{Cx} \frac{w_T^2(\mathbf{x}_C, \mathbf{q}_L)}{w_T^2(\mathbf{x}_C, \mathbf{q}_L) + \delta} \mathbf{K}_p - \frac{\dot{w}_T(\mathbf{x}_C, \mathbf{q}_L)}{2} \mathbf{K}_p) \Delta\mathbf{x}_C - \mathbf{s}_c^T \mathbf{K}_{sq} \mathbf{s}_c - \begin{bmatrix} \mathbf{s}_x^T & \Delta\mathbf{q}_L^T \end{bmatrix} \mathbf{Q} \begin{bmatrix} \mathbf{s}_x^T & \Delta\mathbf{q}_L^T \end{bmatrix}^T$. Therefore, if the controller parameters α_{Cx} , α_{Lq} and $\mathbf{K}_q$ are chosen sufficiently large to satisfy equations (5.20) and (5.32), $\dot{V}_c \leq 0$. Since $V_c > 0$ and $\dot{V}_c \leq 0$, it can be proved similarly that the closed-loop system gives rise to the convergence of the tracking errors. $\triangle\triangle\triangle$

Remark 5.2: When the viscous friction force and the disturbance is taken into consideration, the robot dynamic model is specified as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + [\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{B}_q\dot{\mathbf{q}} = \boldsymbol{\tau} + \mathbf{d}, \quad (5.36)$$

where $\mathbf{B}_q\dot{\mathbf{q}}$ represents the viscous friction force, and $\mathbf{d}$ represents the disturbance force. The dynamic model in equation (5.36) can also be parameterized as: $\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + [\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{B}_q\dot{\mathbf{q}} = \mathbf{Y}_f(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\theta}_f$, where $\mathbf{Y}_f(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ is a known regressor matrix, and $\boldsymbol{\theta}_f$ represents a set of dynamic parameters.

In this case, the control input for the manipulator of the laser beam is proposed as:

$$\boldsymbol{\tau} = -\mathbf{K}_{sq}\mathbf{s}_c - \mathbf{J}_L^T(\mathbf{q})\mathbf{K}_q\Delta\mathbf{q}_L + \mathbf{Y}_f(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_{cr}, \ddot{\mathbf{q}}_{cr})\hat{\boldsymbol{\theta}}_f, \quad (5.37)$$

where $\hat{\boldsymbol{\theta}}_f$ are updated by $\dot{\hat{\boldsymbol{\theta}}}_f = -\mathbf{L}_f \mathbf{Y}_f^T(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_{cr}, \ddot{\mathbf{q}}_{cr})\mathbf{s}_c$ where $\mathbf{L}_f$ is a positive definite matrix. Substituting $\boldsymbol{\tau}$ into equation (5.36), the closed-loop equation is obtained as: $\mathbf{M}(\mathbf{q})\dot{\mathbf{s}}_c + [\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{K}_{sq}]\mathbf{s}_c + \mathbf{J}_L^T(\mathbf{q})\mathbf{K}_q\Delta\mathbf{q}_L + \mathbf{Y}_f(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_{cr}, \ddot{\mathbf{q}}_{cr})\Delta\boldsymbol{\theta}_f = \mathbf{d}$.

The disturbance force $\mathbf{d}$ in the closed-loop equation can be attenuated with the H_∞ tuning [19]. If $\mathbf{K}_{sq}$ is chosen sufficiently large so that $2\lambda_{\min}[\mathbf{K}_{sq}] - 1 > 0$, the following H_∞ tuning can be established for disturbance attenuation as:

$$\int_0^t \|\mathbf{s}_e(\varsigma)\|^2 d\varsigma \leq \gamma^2 \int_0^t \|\mathbf{d}(\varsigma)\|^2 d\varsigma + \gamma^2 \mu, \quad (5.38)$$

where μ is a positive constant depending on initial conditions of the state variables, and $\frac{1}{\gamma^2} \triangleq 2\lambda_{\min}[\mathbf{K}_{sq}] - 1$.

Thus viscous frictions can be compensated by expressing the dynamic model in the form of regressor, and bounded disturbances or uncompensated bounded frictions can be attenuated. It has been shown in [4] that Coulomb's frictions can also be compensated by expressing them in the form of regressor (see section 7.7), but the proof is more sophisticated. △△△

5.2 Adaptive Observers for Optical Tweezers

In the previous section, the concept of the dynamic trapping and manipulation is demonstrated. However, since the overall dynamics of the manipulator interacting with the cell described by equations (2.35) and (2.38) is a fourth-order dynamics, the acceleration information and its derivatives are required in the proposed control method in equations (5.11) and (5.23). In this section, a set of adaptive observers are introduced to avoid the use of the acceleration and its derivatives due to the fourth-order dynamics.

5.2.1 Desired Position Input of Laser Beam

To eliminate the requirement of the acceleration and its derivatives in the desired position input, observed signals $\hat{\mathbf{x}}_C$ and $\hat{\mathbf{q}}_L$ instead of the actual position information $\mathbf{x}_C$ and $\mathbf{q}_L$ are employed to construct the weight factor. First, the observer dynamics for $\hat{\mathbf{x}}_C$ is given as:

$$\begin{cases} \dot{\hat{\mathbf{x}}}_C = \boldsymbol{\eta}_x + \beta_x \mathbf{e}_x \\ \dot{\boldsymbol{\eta}}_x = \hat{\mathbf{M}}_o^{-1}[-k_3(\mathbf{x}_C, \mathbf{q}_L)(\mathbf{x}_C - \mathbf{q}_L) - \hat{\mathbf{B}}_o \boldsymbol{\eta}_x + \mathbf{K}_x \mathbf{e}_x], \end{cases} \quad (5.39)$$

where $\mathbf{e}_x = \mathbf{x}_C - \hat{\mathbf{x}}_C$ is the observation error, and $\boldsymbol{\eta}_x$ is an auxiliary variable, and β_x is a positive constant and $\mathbf{K}_x$ is a positive definite matrix. The matrices $\hat{\mathbf{M}}_o$ and $\hat{\mathbf{B}}_o$ are the approximate models for $\mathbf{M}_C$ and $\mathbf{B}_C$ respectively, which are updated through the update laws:

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}}_{M_o} &= -\mathbf{L}_{M_o} \mathbf{N}_{M_o}(\dot{\boldsymbol{\eta}}_x) \mathbf{z}_x, \\ \dot{\hat{\boldsymbol{\theta}}}_{B_o} &= -\mathbf{L}_{B_o} \mathbf{N}_{B_o}(\boldsymbol{\eta}_x) \mathbf{z}_x, \end{aligned} \quad (5.40)$$

where $\mathbf{z}_x = \dot{\mathbf{x}}_C - \boldsymbol{\eta}_x$, and $\hat{\boldsymbol{\theta}}_{M_o}$ and $\hat{\boldsymbol{\theta}}_{B_o}$ are the vectors of estimated parameters, and $\mathbf{L}_{M_o}$ and $\mathbf{L}_{B_o}$ are positive definite matrices, and the matrices $\mathbf{N}_{M_o}(\dot{\boldsymbol{\eta}}_x)$ and $\mathbf{N}_{B_o}(\boldsymbol{\eta}_x)$ are defined as:

$$\begin{aligned} \mathbf{N}_{M_o}(\dot{\boldsymbol{\eta}}_x) &= \begin{bmatrix} \dot{\eta}_{x1} & 0 \\ 0 & \dot{\eta}_{x2} \end{bmatrix}, \\ \mathbf{N}_{B_o}(\boldsymbol{\eta}_x) &= \begin{bmatrix} \eta_{x1} & 0 \\ 0 & \eta_{x2} \end{bmatrix}. \end{aligned} \quad (5.41)$$

Similarly, the observer dynamics for $\hat{\mathbf{q}}_L$ is given as:

$$\begin{cases} \dot{\hat{\mathbf{q}}}_L = \boldsymbol{\eta}_q + \beta_q \mathbf{e}_q \\ \dot{\boldsymbol{\eta}}_q = \hat{\mathbf{M}}_{qo}^{-1}(\mathbf{u} - \hat{\mathbf{B}}_{qo} \boldsymbol{\eta}_q + \mathbf{K}_z \mathbf{e}_q), \end{cases} \quad (5.42)$$

where $\mathbf{e}_q = \mathbf{q}_L - \hat{\mathbf{q}}_L$ is the observation error, and $\boldsymbol{\eta}_q$ is an auxiliary variable, and β_q is a positive constant, and $\mathbf{K}_z$ is a positive definite matrix. The matrices $\hat{\mathbf{M}}_{qo}$ and $\hat{\mathbf{B}}_{qo}$ are the approximate models for $\mathbf{M}_L$ and $\mathbf{B}_L$ respectively, which are updated as follows:

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}}_{M_{qo}} &= -\mathbf{L}_{M_{qo}} \mathbf{N}_{M_{qo}}(\dot{\boldsymbol{\eta}}_q) \mathbf{z}_q, \\ \dot{\hat{\boldsymbol{\theta}}}_{B_{qo}} &= -\mathbf{L}_{B_{qo}} \mathbf{N}_{B_{qo}}(\boldsymbol{\eta}_q) \mathbf{z}_q, \end{aligned} \quad (5.43)$$

where $\mathbf{z}_q = \dot{\mathbf{q}}_L - \boldsymbol{\eta}_q$, and $\hat{\boldsymbol{\theta}}_{M_{qo}}$ and $\hat{\boldsymbol{\theta}}_{B_{qo}}$ are the vectors of estimated parameters, and $\mathbf{L}_{M_{qo}}$ and $\mathbf{L}_{B_{qo}}$ are positive definite matrices, and the matrices $\mathbf{N}_{M_{qo}}(\dot{\boldsymbol{\eta}}_q)$ and $\mathbf{N}_{B_{qo}}(\boldsymbol{\eta}_q)$ are defined as:

$$\begin{aligned} \mathbf{N}_{M_{qo}}(\dot{\boldsymbol{\eta}}_q) &= \begin{bmatrix} \dot{\eta}_{q1} & 0 \\ 0 & \dot{\eta}_{q2} \end{bmatrix}, \\ \mathbf{N}_{B_{qo}}(\boldsymbol{\eta}_q) &= \begin{bmatrix} \eta_{q1} & 0 \\ 0 & \eta_{q2} \end{bmatrix}. \end{aligned} \quad (5.44)$$

Using the estimated positions of the cell and the laser beam $\hat{\mathbf{x}}_C$ and $\hat{\mathbf{q}}_L$, a new desired trajectory is defined as:

$$\hat{\mathbf{x}}_{Cd}(t) = \mathbf{x}_{Cc} + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) \mathbf{x}_{Cv}(t), \quad (5.45)$$

where $\hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)$ is the approximate weight factor of $w_T(\mathbf{x}_C, \mathbf{q}_L)$. The difference

is that the estimated cell position and $\hat{\mathbf{x}}_C$ and the estimated laser position $\hat{\mathbf{q}}_L$ is employed in $\hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)$ instead of the actual positions $\mathbf{x}_C$ and $\mathbf{q}_L$.

Next, a new reference vector $\dot{\hat{\mathbf{x}}}_{Cr}$ is introduced as:

$$\dot{\hat{\mathbf{x}}}_{Cr} = \dot{\hat{\mathbf{x}}}_{Cd} - \alpha_{Cx} \frac{\hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) + \delta} \Delta \hat{\mathbf{x}}_C, \quad (5.46)$$

where $\Delta \hat{\mathbf{x}}_C = \mathbf{x}_C - \hat{\mathbf{x}}_{Cd}$, and a new sliding vector $\hat{\mathbf{s}}_x$ is defined as:

$$\hat{\mathbf{s}}_x = \dot{\mathbf{x}}_C - \dot{\hat{\mathbf{x}}}_{Cr} = \dot{\mathbf{x}}_C - (\dot{\hat{\mathbf{x}}}_{Cd} - \alpha_{Cx} \frac{\hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) + \delta} \Delta \hat{\mathbf{x}}_C). \quad (5.47)$$

A new estimated desired position input for the laser beam by using $\dot{\hat{\mathbf{x}}}_C$ and $\dot{\hat{\mathbf{q}}}_L$ is proposed as:

$$\begin{aligned} \bar{\mathbf{q}}_{Ld} = & \mathbf{x}_C - k_3^{-1}(\mathbf{x}_C, \mathbf{q}_L) \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) \mathbf{K}_p \Delta \hat{\mathbf{x}}_C \\ & - k_3^{-1}(\mathbf{x}_C, \mathbf{q}_L) \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) \mathbf{K}_d \hat{\mathbf{s}}_x + k_3^{-1}(\mathbf{x}_C, \mathbf{q}_L) \mathbf{Y}_C(\dot{\hat{\mathbf{x}}}_{Cr}, \ddot{\hat{\mathbf{x}}}_{Cr}) \hat{\boldsymbol{\theta}}_C, \end{aligned} \quad (5.48)$$

where the uncertain parameters $\hat{\boldsymbol{\theta}}_C$ are updated as follows:

$$\dot{\hat{\boldsymbol{\theta}}}_C = -\mathbf{L}_C \mathbf{Y}_C^T(\dot{\hat{\mathbf{x}}}_{Cr}, \ddot{\hat{\mathbf{x}}}_{Cr}) \hat{\mathbf{s}}_x. \quad (5.49)$$

Substituting equation (5.48) into equation (5.8) and using the sliding vector $\hat{\mathbf{s}}_x$, it is obtained:

$$\begin{aligned} & \mathbf{M}_C \dot{\hat{\mathbf{s}}}_x + \mathbf{B}_C \hat{\mathbf{s}}_x + \mathbf{Y}_C(\dot{\hat{\mathbf{x}}}_{Cr}, \ddot{\hat{\mathbf{x}}}_{Cr}) \Delta \boldsymbol{\theta}_C + \\ & + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) \mathbf{K}_p \Delta \hat{\mathbf{x}}_C + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) \mathbf{K}_d \hat{\mathbf{s}}_x = k_3(\mathbf{x}_C, \mathbf{q}_L) \Delta \bar{\mathbf{q}}_L. \end{aligned} \quad (5.50)$$

where $\Delta \bar{\mathbf{q}}_L = \mathbf{q}_L - \bar{\mathbf{q}}_{Ld}$.

Next, multiplying both sides of equation (5.39) with $\hat{\mathbf{M}}_o$ and using equation (5.8), the closed-loop observer dynamics for $\hat{\mathbf{x}}_C$ is obtained as:

$$\begin{aligned} \mathbf{M}_C \dot{\mathbf{z}}_x + \mathbf{B}_C \mathbf{z}_x + \mathbf{K}_x \mathbf{e}_x &= (\hat{\mathbf{M}}_o - \mathbf{M}_C) \dot{\boldsymbol{\eta}}_x + (\hat{\mathbf{B}}_o - \mathbf{B}_C) \boldsymbol{\eta}_x \\ &= -\mathbf{N}_{M_o}(\dot{\boldsymbol{\eta}}_x) \Delta \boldsymbol{\theta}_{M_o} - \mathbf{N}_{B_o}(\boldsymbol{\eta}_x) \Delta \boldsymbol{\theta}_{B_o}, \end{aligned} \quad (5.51)$$

where $\Delta \boldsymbol{\theta}_{M_o} = \boldsymbol{\theta}_{M_o} - \hat{\boldsymbol{\theta}}_{M_o}$ and $\Delta \boldsymbol{\theta}_{B_o} = \boldsymbol{\theta}_{B_o} - \hat{\boldsymbol{\theta}}_{B_o}$. Similarly, multiplying both sides of equation (5.42) with $\hat{\mathbf{M}}_{qo}$ and substituting $\mathbf{u}$ with the right side of equation (2.38), the closed-loop observer dynamics for $\hat{\mathbf{q}}_L$ is obtained as:

$$\begin{aligned} \mathbf{M}_L \dot{\mathbf{z}}_q + \mathbf{B}_L \mathbf{z}_q + \mathbf{K}_z \mathbf{e}_q &= (\hat{\mathbf{M}}_{qo} - \mathbf{M}_L) \dot{\boldsymbol{\eta}}_q + (\hat{\mathbf{B}}_{qo} - \mathbf{B}_L) \boldsymbol{\eta}_q \\ &= -\mathbf{N}_{M_{qo}}(\dot{\boldsymbol{\eta}}_q) \Delta \boldsymbol{\theta}_{M_{qo}} - \mathbf{N}_{B_{qo}}(\boldsymbol{\eta}_q) \Delta \boldsymbol{\theta}_{B_{qo}}, \end{aligned} \quad (5.52)$$

where $\Delta \boldsymbol{\theta}_{M_{qo}} = \boldsymbol{\theta}_{M_{qo}} - \hat{\boldsymbol{\theta}}_{M_{qo}}$ and $\Delta \boldsymbol{\theta}_{B_{qo}} = \boldsymbol{\theta}_{B_{qo}} - \hat{\boldsymbol{\theta}}_{B_{qo}}$.

5.2.2 Control Input of Manipulator of Laser Beam

To avoid the use of the acceleration and its derivatives in the control input for the manipulator of the laser beam, another observed signal $\hat{\mathbf{q}}_{Ld}$ is employed instead of the actual desired position input $\mathbf{q}_{Ld}$. Therefore, equation (5.50) can be rewritten as:

$$\begin{aligned} \mathbf{M}_C \dot{\hat{\mathbf{s}}}_x + \mathbf{B}_C \hat{\mathbf{s}}_x + \mathbf{Y}_C(\dot{\hat{\mathbf{x}}}_{Cr}, \ddot{\hat{\mathbf{x}}}_{Cr}) \Delta \boldsymbol{\theta}_C + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) \mathbf{K}_p \Delta \hat{\mathbf{x}}_C + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) \mathbf{K}_d \hat{\mathbf{s}}_x \\ = k_3(\mathbf{x}_C, \mathbf{q}_L) [\mathbf{q}_L - \hat{\mathbf{q}}_{Ld} - (\bar{\mathbf{q}}_{Ld} - \hat{\mathbf{q}}_{Ld})] = k_3(\mathbf{x}_C, \mathbf{q}_L) (\Delta \hat{\mathbf{q}}_L - \mathbf{e}). \end{aligned} \quad (5.53)$$

where $\Delta \hat{\mathbf{q}}_L = \mathbf{q}_L - \hat{\mathbf{q}}_{Ld}$, and $\mathbf{e} = \bar{\mathbf{q}}_{Ld} - \hat{\mathbf{q}}_{Ld}$.

The estimated desired position input is updated by the following observer as:

$$\begin{cases} \dot{\hat{\mathbf{q}}}_{Ld} = \boldsymbol{\eta} + \beta \mathbf{e} \\ \dot{\boldsymbol{\eta}} = \hat{\mathbf{M}}_{do}^{-1}(\mathbf{u} - \hat{\mathbf{B}}_{do}\boldsymbol{\eta} + \mathbf{K}_e \mathbf{e}), \end{cases} \quad (5.54)$$

where $\boldsymbol{\eta}$ is an auxiliary variable, and β is a positive constant. The matrices $\hat{\mathbf{M}}_{do}$ and $\hat{\mathbf{B}}_{do}$ are the approximate models for $\mathbf{M}_L$ and $\mathbf{B}_L$ respectively, which are updated as follows:

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}}_{M_{do}} &= -\mathbf{L}_{M_{do}} \mathbf{N}_{M_{do}}(\dot{\boldsymbol{\eta}}) \mathbf{z}, \\ \dot{\hat{\boldsymbol{\theta}}}_{B_{do}} &= -\mathbf{L}_{B_{do}} \mathbf{N}_{B_{do}}(\boldsymbol{\eta}) \mathbf{z}, \end{aligned} \quad (5.55)$$

where $\mathbf{z} = \dot{\mathbf{q}}_L - \boldsymbol{\eta}$, and $\hat{\boldsymbol{\theta}}_{M_{do}}$ and $\hat{\boldsymbol{\theta}}_{B_{do}}$ are the vectors of estimated parameters, and $\mathbf{L}_{M_{do}}$ and $\mathbf{L}_{B_{do}}$ are positive definite matrices, and the matrices $\mathbf{N}_{M_{do}}(\dot{\boldsymbol{\eta}})$ and $\mathbf{N}_{B_{do}}(\boldsymbol{\eta})$ are defined as:

$$\begin{aligned} \mathbf{N}_{M_{do}}(\dot{\boldsymbol{\eta}}) &= \begin{bmatrix} \dot{\eta}_1 & 0 \\ 0 & \dot{\eta}_2 \end{bmatrix}, \\ \mathbf{N}_{B_{do}}(\boldsymbol{\eta}) &= \begin{bmatrix} \eta_1 & 0 \\ 0 & \eta_2 \end{bmatrix}. \end{aligned} \quad (5.56)$$

Next, a new reference vector $\dot{\hat{\mathbf{q}}}_{Lr}$ is defined as:

$$\dot{\hat{\mathbf{q}}}_{Lr} = \dot{\hat{\mathbf{q}}}_{Ld} - \alpha_{Lq}(\mathbf{q}_L - \hat{\mathbf{q}}_{Ld}), \quad (5.57)$$

and its derivative is obtained as:

$$\ddot{\hat{\mathbf{q}}}_{Lr} = \ddot{\hat{\mathbf{q}}}_{Ld} - \alpha_{Lq}(\dot{\mathbf{q}}_L - \dot{\hat{\mathbf{q}}}_{Ld}), \quad (5.58)$$

then a new sliding vector is obtained as:

$$\hat{\mathbf{s}}_L = \dot{\mathbf{q}}_L - \dot{\hat{\mathbf{q}}}_{Lr} = \dot{\mathbf{q}}_L - \dot{\hat{\mathbf{q}}}_{Ld} + \alpha_{Lq}(\mathbf{q}_L - \hat{\mathbf{q}}_{Ld}), \quad (5.59)$$

and the control input of robotic manipulator is proposed as:

$$\mathbf{u} = -\mathbf{K}_q \Delta \hat{\mathbf{q}}_L - \mathbf{K}_{sq} \hat{\mathbf{s}}_L + \mathbf{Y}_L(\dot{\hat{\mathbf{q}}}_{Lr}, \ddot{\hat{\mathbf{q}}}_{Lr}) \hat{\boldsymbol{\theta}}_L, \quad (5.60)$$

where the uncertain dynamic parameters $\hat{\boldsymbol{\theta}}_L$ are updated as follows:

$$\dot{\hat{\boldsymbol{\theta}}}_L = -\mathbf{L}_L \mathbf{Y}_L^T(\dot{\hat{\mathbf{q}}}_{Lr}, \ddot{\hat{\mathbf{q}}}_{Lr}) \hat{\mathbf{s}}_L. \quad (5.61)$$

Using the sliding vector $\hat{\mathbf{s}}_L$, equation (5.25) can be expressed as:

$$\mathbf{M}_L \dot{\hat{\mathbf{s}}}_L + \mathbf{B}_L \hat{\mathbf{s}}_L + \mathbf{Y}_L(\dot{\hat{\mathbf{q}}}_{Lr}, \ddot{\hat{\mathbf{q}}}_{Lr}) \boldsymbol{\theta}_L = \mathbf{u}. \quad (5.62)$$

Substituting equation (5.60) into equation (5.62), the closed-loop equation for the control input of the robotic manipulator is obtained as:

$$\mathbf{M}_L \dot{\hat{\mathbf{s}}}_L + \mathbf{B}_L \hat{\mathbf{s}}_L + \mathbf{K}_q \Delta \hat{\mathbf{q}}_L + \mathbf{K}_{sq} \hat{\mathbf{s}}_L + \mathbf{Y}_L(\dot{\hat{\mathbf{q}}}_{Lr}, \ddot{\hat{\mathbf{q}}}_{Lr}) \Delta \boldsymbol{\theta}_L = \mathbf{0}. \quad (5.63)$$

Next, multiplying both sides of equation (5.54) with $\hat{\mathbf{M}}_{do}$ and substituting $\mathbf{u}$ with the right side of equation (2.38), the closed-loop observer dynamics is obtained for $\dot{\hat{\mathbf{q}}}_d$ as:

$$\begin{aligned} \mathbf{M}_q \dot{\mathbf{z}} + \mathbf{B}_q \mathbf{z} + \mathbf{K}_e \mathbf{e} &= (\hat{\mathbf{M}}_{do} - \mathbf{M}_L) \dot{\boldsymbol{\eta}} + (\hat{\mathbf{B}}_{do} - \mathbf{B}_L) \boldsymbol{\eta} \\ &= -\mathbf{N}_{M_{do}}(\dot{\boldsymbol{\eta}}) \Delta \boldsymbol{\theta}_{M_{do}} - \mathbf{N}_{B_{do}}(\boldsymbol{\eta}) \Delta \boldsymbol{\theta}_{B_{do}}, \end{aligned} \quad (5.64)$$

where $\Delta\theta_{M_{do}} = \theta_{M_{do}} - \hat{\theta}_{M_{do}}$ and $\Delta\theta_{B_{do}} = \theta_{B_{do}} - \hat{\theta}_{B_{do}}$.

To analyzed the stability, a Lyapunov-like candidate $\hat{V}_T$ is proposed as:

$$\begin{aligned}\hat{V}_T = & \frac{1}{2}\hat{\mathbf{s}}_x^T \mathbf{M}_C \hat{\mathbf{s}}_x + \frac{1}{2}\Delta\theta_C^T \mathbf{L}_C^{-1} \Delta\theta_C + \frac{\hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{2} \Delta\hat{\mathbf{x}}_C^T \mathbf{K}_p \Delta\hat{\mathbf{x}}_C + \frac{1}{2}\hat{\mathbf{s}}_L^T \mathbf{M}_L \hat{\mathbf{s}}_L \\ & + \frac{1}{2}\Delta\theta_L^T \mathbf{L}_L^{-1} \Delta\theta_L + \Delta\hat{\mathbf{q}}_L^T (\frac{1}{2}\mathbf{K}_q + \alpha_{Lq} \mathbf{K}_{sq}) \Delta\hat{\mathbf{q}}_L + \frac{1}{2}\mathbf{e}_x^T \mathbf{K}_x \mathbf{e}_x + \frac{1}{2}\mathbf{e}_q^T \mathbf{K}_z \mathbf{e}_q \\ & + \frac{1}{2}\mathbf{z}_x^T \mathbf{M}_C \mathbf{z}_x + \frac{1}{2}\mathbf{z}_q^T \mathbf{M}_L \mathbf{z}_q + \frac{1}{2}\mathbf{z}^T \mathbf{M}_L \mathbf{z} + \frac{1}{2}\Delta\theta_{M_o}^T \mathbf{L}_{M_o}^{-1} \Delta\theta_{M_o} + \frac{1}{2}\Delta\theta_{B_o}^T \mathbf{L}_{B_o}^{-1} \Delta\theta_{B_o} \\ & + \frac{1}{2}\Delta\theta_{M_{qo}}^T \mathbf{L}_{M_{qo}}^{-1} \Delta\theta_{M_{qo}} + \frac{1}{2}\Delta\theta_{B_{qo}}^T \mathbf{L}_{B_{qo}}^{-1} \Delta\theta_{B_{qo}} + \frac{1}{2}\Delta\theta_{M_{do}}^T \mathbf{L}_{M_{do}}^{-1} \Delta\theta_{M_{do}} + \frac{1}{2}\Delta\theta_{B_{do}}^T \mathbf{L}_{B_{do}}^{-1} \Delta\theta_{B_{do}}. \quad (5.65)\end{aligned}$$

Differentiating $\hat{V}_T$ with respect to time, it is obtained:

$$\begin{aligned}\dot{\hat{V}}_T = & \hat{\mathbf{s}}_x^T \mathbf{M}_C \dot{\hat{\mathbf{s}}}_x - \dot{\hat{\theta}}_C^T \mathbf{L}_C^{-1} \Delta\theta_C + \frac{\dot{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{2} \Delta\hat{\mathbf{x}}_C^T \mathbf{K}_p \Delta\hat{\mathbf{x}}_C + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) (\dot{\mathbf{x}}_C - \dot{\hat{\mathbf{x}}}_{Cd})^T \mathbf{K}_p \Delta\hat{\mathbf{x}}_C \\ & + \hat{\mathbf{s}}_L^T \mathbf{M}_L \dot{\hat{\mathbf{s}}}_L - \dot{\hat{\theta}}_L^T \mathbf{L}_L^{-1} \Delta\theta_L + \Delta\dot{\hat{\mathbf{q}}}_L^T (\mathbf{K}_q + 2\alpha_{Lq} \mathbf{K}_{sq}) \Delta\hat{\mathbf{q}}_L + \mathbf{e}_x^T \mathbf{K}_x \dot{\mathbf{e}}_x + \mathbf{e}_q^T \mathbf{K}_z \dot{\mathbf{e}}_q \\ & + \mathbf{z}_x^T \mathbf{M}_C \dot{\mathbf{z}}_x + \mathbf{z}_q^T \mathbf{M}_L \dot{\mathbf{z}}_q + \mathbf{z}^T \mathbf{M}_L \dot{\mathbf{z}} - \dot{\hat{\theta}}_{M_o}^T \mathbf{L}_{M_o}^{-1} \Delta\theta_{M_o} - \dot{\hat{\theta}}_{B_o}^T \mathbf{L}_{B_o}^{-1} \Delta\theta_{B_o} \\ & - \dot{\hat{\theta}}_{M_{qo}}^T \mathbf{L}_{M_{qo}}^{-1} \Delta\theta_{M_{qo}} - \dot{\hat{\theta}}_{B_{qo}}^T \mathbf{L}_{B_{qo}}^{-1} \Delta\theta_{B_{qo}} - \dot{\hat{\theta}}_{M_{do}}^T \mathbf{L}_{M_{do}}^{-1} \Delta\theta_{M_{do}} - \dot{\hat{\theta}}_{B_{do}}^T \mathbf{L}_{B_{do}}^{-1} \Delta\theta_{B_{do}}. \quad (5.66)\end{aligned}$$

Substituting equations (5.40), (5.43), (5.49), (5.51), (5.52), (5.53), (5.55), (5.61), (5.63), and (5.64) into equation (5.66) yields:

$$\begin{aligned}\dot{\hat{V}}_T = & -\hat{\mathbf{s}}_x^T (\mathbf{B}_C + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) \mathbf{K}_d) \hat{\mathbf{s}}_x + k_3(\mathbf{x}_C, \mathbf{q}_L) \hat{\mathbf{s}}_x^T \Delta\hat{\mathbf{q}}_L - k_3(\mathbf{x}_C, \mathbf{q}_L) \hat{\mathbf{s}}_x^T \mathbf{e} \\ & - \Delta\hat{\mathbf{x}}_C^T (\alpha_{Cx} \frac{\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) + \delta} \mathbf{K}_p - \frac{\dot{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{2} \mathbf{K}_p) \Delta\hat{\mathbf{x}}_C \\ & - \hat{\mathbf{s}}_L^T \mathbf{B}_L \hat{\mathbf{s}}_L - \Delta\dot{\hat{\mathbf{q}}}_L^T \mathbf{K}_{sq} \Delta\hat{\mathbf{q}}_L - \Delta\hat{\mathbf{q}}_L^T (\alpha_{Lq} \mathbf{K}_q + \alpha_{Lq}^2 \mathbf{K}_{sq}) \Delta\hat{\mathbf{q}}_L - \mathbf{z}_x^T \mathbf{B}_C \mathbf{z}_x - \mathbf{z}_q^T \mathbf{B}_L \mathbf{z}_q \\ & - \mathbf{z}^T \mathbf{B}_L \mathbf{z} - \mathbf{z}_x^T \mathbf{K}_x \mathbf{e}_x - \mathbf{z}_q^T \mathbf{K}_z \mathbf{e}_q - \mathbf{z}^T \mathbf{K}_e \mathbf{e} + \mathbf{e}_x^T \mathbf{K}_x \dot{\mathbf{e}}_x + \mathbf{e}_q^T \mathbf{K}_z \dot{\mathbf{e}}_q. \quad (5.67)\end{aligned}$$

Note that $\mathbf{z}_x = \dot{\mathbf{x}}_C - \boldsymbol{\eta}_x = \dot{\mathbf{e}}_x + \beta_x \mathbf{e}_x$, $\mathbf{z}_q = \dot{\mathbf{q}} - \boldsymbol{\eta}_q = \dot{\mathbf{e}}_q + \beta_q \mathbf{e}_q$ and $\mathbf{z} = \dot{\mathbf{q}} - \boldsymbol{\eta} = \Delta\dot{\hat{\mathbf{q}}} + \beta \mathbf{e}$.

Therefore, equation (5.67) can be rewritten as:

$$\begin{aligned}
\dot{V}_T = & -\hat{\mathbf{s}}_x^T (\mathbf{B}_C + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) \mathbf{K}_d) \hat{\mathbf{s}}_x + k_3(\mathbf{x}_C, \mathbf{q}_L) \hat{\mathbf{s}}_x^T \Delta \hat{\mathbf{q}}_L - k_3(\mathbf{x}_C, \mathbf{q}_L) \hat{\mathbf{s}}_x^T \mathbf{e} \\
& - \Delta \hat{\mathbf{x}}_C^T \left(\alpha_{Cx} \frac{\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) + \delta} \mathbf{K}_p - \frac{\dot{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{2} \mathbf{K}_p \right) \Delta \hat{\mathbf{x}}_C \\
& - \hat{\mathbf{s}}_L^T \mathbf{B}_L \hat{\mathbf{s}}_L - \Delta \hat{\mathbf{q}}_L^T \mathbf{K}_{sq} \Delta \hat{\mathbf{q}}_L - \Delta \hat{\mathbf{q}}_L^T (\alpha_{Lq} \mathbf{K}_q + \alpha_{Lq}^2 \mathbf{K}_{sq}) \Delta \hat{\mathbf{q}}_L - \mathbf{z}_x^T \mathbf{B}_C \mathbf{z}_x - \mathbf{z}_q^T \mathbf{B}_L \mathbf{z}_q \\
& - \mathbf{z}^T \mathbf{B}_L \mathbf{z} - \beta_x \mathbf{e}_x^T \mathbf{K}_x \mathbf{e}_x - \beta_q \mathbf{e}_q^T \mathbf{K}_z \mathbf{e}_q - \Delta \hat{\mathbf{q}}^T \mathbf{K}_e \mathbf{e} - \beta \mathbf{e}^T \mathbf{K}_e \mathbf{e} \\
= & - \Delta \hat{\mathbf{x}}_C^T \left(\alpha_{Cx} \frac{\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) + \delta} \mathbf{K}_p - \frac{\dot{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{2} \mathbf{K}_p \right) \Delta \hat{\mathbf{x}}_C - \hat{\mathbf{s}}_L^T \mathbf{B}_L \hat{\mathbf{s}}_L - \mathbf{z}_x^T \mathbf{B}_C \mathbf{z}_x \\
& - \mathbf{z}_q^T \mathbf{B}_L \mathbf{z}_q - \mathbf{z}^T \mathbf{B}_L \mathbf{z} - \beta_x \mathbf{e}_x^T \mathbf{K}_x \mathbf{e}_x - \beta_q \mathbf{e}_q^T \mathbf{K}_z \mathbf{e}_q \\
& - [\mathbf{s}_x^T \ \Delta \hat{\mathbf{q}}_L^T \ \Delta \hat{\mathbf{q}}_L^T \ \mathbf{e}^T] \mathbf{P} [\mathbf{s}_x^T \ \Delta \hat{\mathbf{q}}_L^T \ \Delta \hat{\mathbf{q}}_L^T \ \mathbf{e}^T]^T, \tag{5.68}
\end{aligned}$$

where $\mathbf{P} \in \mathbb{R}^{8 \times 8}$ is:

$$\mathbf{P} = \begin{bmatrix} \mathbf{B}_C + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) \mathbf{K}_d & -\frac{k_3(\mathbf{x}_C, \mathbf{q}_L)}{2} \mathbf{I}_2 & \mathbf{0} & \frac{k_3(\mathbf{x}_C, \mathbf{q}_L)}{2} \mathbf{I}_2 \\ -\frac{k_3(\mathbf{x}_C, \mathbf{q}_L)}{2} \mathbf{I}_2 & \alpha_{Lq} \mathbf{K}_q + \alpha_{Lq}^2 \mathbf{K}_{sq} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{sq} & \frac{1}{2} \mathbf{K}_e \\ \frac{k_3(\mathbf{x}_C, \mathbf{q}_L)}{2} \mathbf{I}_2 & \mathbf{0} & \frac{1}{2} \mathbf{K}_e & \beta \mathbf{K}_e \end{bmatrix}. \tag{5.69}$$

If the controller parameters α_{Cx} , α_{Lq} , $\mathbf{K}_q$, $\mathbf{K}_{sq}$, β and $\mathbf{K}_e$ are chosen so that

$$\begin{aligned}
\alpha_{Cx} & > \mathbf{b}_{max} \left(\frac{\dot{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) (\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) + \delta)}{2 \hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)} \right) = \mathbf{b}_{max} \left(\frac{\dot{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{2} \left(1 + \frac{\delta}{\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)} \right) \right), \\
\alpha_{Lq} \lambda_{min}[(\mathbf{K}_q + \alpha_{Lq} \mathbf{K}_{sq}) \mathbf{B}_C] & > \frac{k_1^2}{4}, \\
\beta \lambda_{min}[\mathbf{K}_{sq}] & > \frac{1}{4} \lambda_{max}[\mathbf{K}_e], \\
\lambda_{min}[\mathbf{K}_e] \{ \beta \lambda_{min}[\mathbf{K}_{sq}] - \frac{1}{4} \lambda_{max}[\mathbf{K}_e] \} \{ \alpha_{Lq} \lambda_{min}[(\mathbf{K}_q + \alpha_{Lq} \mathbf{K}_{sq}) \mathbf{B}_C] - \frac{k_1^2}{4} \} & > \\
\frac{k_1^2}{4} \alpha_{Lq} \lambda_{max}[(\mathbf{K}_q + \alpha_{Lq} \mathbf{K}_{sq}) \mathbf{K}_{sq}], \tag{5.70}
\end{aligned}$$

then $\mathbf{P}$ is positive definite, and $\alpha_{Cx} \frac{\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{\hat{w}_T^2(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) + \delta} - \frac{\dot{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)}{2} > 0$ and hence $\dot{V}_T \leq 0$.

The proof that $\mathbf{P}$ is positive definite if equation (5.70) is satisfied is given in the

Appendix.

The following theorem is stated:

Theorem 5.2: *The input of the robotic manipulator (5.60), and the update laws (5.40), (5.43), (5.55), and (5.61) ensure the convergence of the closed-loop system. That is, $\mathbf{x}_C \rightarrow \mathbf{x}_{Cd}$, $\dot{\mathbf{x}}_C \rightarrow \dot{\mathbf{x}}_{Cd}$ as $t \rightarrow \infty$ when the control parameters α_{Cx} , α_{Lq} , $\mathbf{K}_q$, $\mathbf{K}_{sq}$, β , and $\mathbf{K}_e$ are chosen to satisfy conditions (5.70).*

Proof: If equation (5.70) is satisfied, it is derived that $\hat{V}_T > 0$ and $\dot{\hat{V}}_T \leq 0$, and hence $\hat{V}_T$ is bounded. The boundedness of $\hat{V}_T$ ensures the boundedness of $\hat{\mathbf{s}}_x$, $\Delta\boldsymbol{\theta}_C$, $\Delta\hat{\mathbf{x}}_C$, $\hat{\mathbf{s}}_L$, $\Delta\boldsymbol{\theta}_L$, $\Delta\hat{\mathbf{q}}_L$, $\mathbf{e}_x$, $\mathbf{e}_q$, $\mathbf{z}_x$, $\mathbf{z}_q$, $\mathbf{z}$, $\Delta\boldsymbol{\theta}_{Mo}$, $\Delta\boldsymbol{\theta}_{Bo}$, $\Delta\boldsymbol{\theta}_{Mqo}$, $\Delta\boldsymbol{\theta}_{Bqo}$, $\Delta\boldsymbol{\theta}_{Mdo}$ and $\Delta\boldsymbol{\theta}_{Bdo}$. The boundedness of $\hat{\mathbf{s}}_x$ and $\Delta\hat{\mathbf{x}}_C$ ensures the boundedness of $\Delta\dot{\hat{\mathbf{x}}}_C$ from equation (5.47). Therefore, $\Delta\hat{\mathbf{x}}_C$ is uniformly continuous. The boundedness of $\Delta\hat{\mathbf{x}}_C$ ensures the boundedness of $\dot{\hat{\mathbf{x}}}_{Cr}$. Moreover, $\ddot{\hat{\mathbf{x}}}_{Cr}$ is bounded since $\Delta\dot{\hat{\mathbf{x}}}_C$ is bounded. In addition, since $\hat{\mathbf{s}}_L$ and $\Delta\hat{\mathbf{q}}_L$ are bounded, $\Delta\dot{\hat{\mathbf{q}}}_L$ is bounded. Since $\Delta\dot{\hat{\mathbf{q}}}_L$ and $\mathbf{z}$ are bounded, $\mathbf{e}$ is bounded. From the closed-loop equation (5.53), it is obtained that $\dot{\hat{\mathbf{s}}}_x$ is bounded. Therefore, $\hat{\mathbf{s}}_x$ is also uniformly continuous. In addition, the boundedness of $\mathbf{z}_x$ and $\mathbf{e}_x$ ensures the boundedness of $\dot{\mathbf{e}}_x$. Therefore, $\mathbf{e}_x$ is uniformly continuous. The boundedness of $\mathbf{z}_q$ and $\mathbf{e}_q$ ensures the boundedness of $\dot{\mathbf{e}}_q$. Therefore, $\mathbf{e}_q$ is also uniformly continuous. From equation (5.68), it is easy to verify that $\hat{\mathbf{s}}_x$, $\Delta\hat{\mathbf{x}}_C$, $\mathbf{e}_x$, $\mathbf{e}_q \in L_2(0, +\infty)$. Then it follows [4, 84] that $\hat{\mathbf{s}}_x \rightarrow \mathbf{0}$, $\Delta\hat{\mathbf{x}}_C \rightarrow \mathbf{0}$, $\mathbf{e}_x \rightarrow \mathbf{0}$ and $\mathbf{e}_q \rightarrow \mathbf{0}$. The convergence of $\mathbf{e}_x \rightarrow \mathbf{0}$ and $\mathbf{e}_q \rightarrow \mathbf{0}$ implies that $\hat{\mathbf{x}}_C \rightarrow \mathbf{x}_C$ and $\hat{\mathbf{q}}_L \rightarrow \mathbf{q}_L$, and hence $\hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L) \rightarrow w_T(\mathbf{x}_C, \mathbf{q}_L)$, $\hat{\mathbf{x}}_{Cd} \rightarrow \mathbf{x}_{Cd}$. Then $\hat{\mathbf{s}}_x \rightarrow \mathbf{0}$ and $\Delta\hat{\mathbf{x}}_C \rightarrow \mathbf{0}$ indicates that $\mathbf{x}_C \rightarrow \mathbf{x}_{Cd}$ and $\dot{\mathbf{x}}_C \rightarrow \dot{\mathbf{x}}_{Cd}$. $\triangle\triangle\triangle$

Remark 5.3 The desired position of the trapped cell can also be specified as a dynamic region to provides flexibility in the specifications of the cell manipulation tasks [57]. In an application where the high precision is required, it is possible to define the region arbitrarily small. When the precision is not critical, the region

could be scaled up so that less control effort is required. Therefore, the dynamic region is a generalization of conventional setpoint or trajectory. $\triangle\triangle\triangle$

5.3 Simulation

Simulation has been carried out to verify the performance of the proposed control methods. The optical tweezers system is illustrated in Fig. 5.4. In Fig. 5.4, the cell is placed on a motorized stage and the laser beam is fixed downwards, and the relative distance between the laser beam and the stage is dependent on the variation of the stage position. The motorized stage thus act as a robotic manipulator, and the relative distance is controlled between the laser beam and the cell. In the simulation, the variable $\mathbf{q}_L$ is set as the position of the laser beam with respect to the motorized stage.

The parameters of the cell dynamics in equation (5.8) were set as: $\mathbf{M}_C = \text{diag}\{10^{-9}, 10^{-9}\}kg$, $\mathbf{B}_C = \text{diag}\{2 \times 10^{-9}, 2 \times 10^{-9}\}kg/s$, $k_1 = 2 \times 10^{-5}$, $k_2 = 9.5 \times 10^9$. From the value of k_1 and k_2 , it is obtained that the Gaussian field is in effect when the cell is less than $18\mu m$ from the laser. The parameters of the manipulator dynamics in equation (5.25) were set as: $\mathbf{M}_L = \text{diag}\{0.02218, 0.011386\}kg$, $\mathbf{B}_L = \text{diag}\{0.04749, 0.04023\}kg/s$. The values of the dynamic parameters of biological cells and the manipulator of the laser beam are selected with reference to [2, 46, 55].

In the first simulation, the laser started from a large initial position at $(1.1, 0.7) \mu m$ and moved towards a cell located at $(126, -130.8) \mu m$. Therefore, the cell was outside the Gaussian field initially, which is shown in Fig. 5.6(a).

Figure 5.4: The optical tweezers system. The laser beam is fixed downwards, and the relative distance between the cell and the laser is adjusted by the robotic stage.

The trapping region was set as a circle as:

$$f(\mathbf{x}_C, \mathbf{q}_L) = (x_{C1} - q_{L1})^2 + (x_{C2} - q_{L2})^2 - b_T^2 \leq 0, \quad (5.71)$$

where $b_T = 10 \mu m$ is the radius, and the parameters of the reference region in equation (5.2) were set as: $\kappa_T = 0.99$. The cell was trapped by the laser beam after it entered the trapping region. Then the trapped cell was moved to follow the desired trajectory together. The desired trajectory was specified as a lemniscate of Bernoulli:

$$\begin{cases} x_{Cd1} = 200 + \frac{15\cos(0.35t)}{1+\sin^2(0.35t)} w_T(\mathbf{x}_C, \mathbf{q}_L) \mu m, \\ x_{Cd2} = -150 + \frac{15\sin(0.35t)\cos(0.35t)}{1+\sin^2(0.35t)} w_T(\mathbf{x}_C, \mathbf{q}_L) \mu m. \end{cases} \quad (5.72)$$

The parameters of the desired position input for the trapped cell in equations (5.11) and (5.12) were proposed as: $N_T = 6$, $\alpha_{Cx} = 1$, $\mathbf{K}_p = 3 \times 10^{-6} \mathbf{I}_2$, $\mathbf{K}_d = 3.5 \times 10^{-7} \mathbf{I}$, $\mathbf{L}_C = 10^{-10} \mathbf{I}_4$ where $\mathbf{I}_4 \in \mathbb{R}^{4 \times 4}$ is an identity matrix, and the parameters of the robot input in equations (5.23) and (5.24) were set as: $\alpha_{Lq} = 1$, $\mathbf{K}_q = \mathbf{I}$, $\mathbf{K}_{sq} = 1.5 \mathbf{I}_2$, $\mathbf{L}_L = 10^{-5} \mathbf{I}_4$.

Figure 5.5: Simulation 1: The tracking error.

The tracking errors are shown in Fig. 5.5, and the tracking errors converge to zero in less than 1s, which indicates the successful realization of the proposed controller. The path of the laser and the trapped cell is shown in Fig. 5.6, which consists of three stages: (1) the laser starts from a large initial position and moves towards the cell (Fig. 5.6(a)); (2) the cell enters the trapping region and is trapped by the laser (Fig. 5.6(b)); (3) the trapped cell is manipulated to follow the desired trajectory (Fig. 5.6(c)).

(a) $t=0$ s: initial positions (b) $t=0.15$ s: the cell is trapped by the laser beam (c) $t=24$ s: the cell position converges to the desired position

Figure 5.6: Simulation 1: The cell is trapped then transported to the desired position.

In the second simulation, the adaptive observers are introduced to eliminate the requirement of acceleration and its derivatives. The initial positions of the cell and the laser beam remained the same, and hence the cell was outside the Gaussian field as well. After the cell was trapped by the laser beam, it was moved to track the desired trajectory. The desired trajectory was the same as that specified in equation (5.72).

The parameters of the adaptive observer for $\hat{\mathbf{x}}_C$ in equation (5.39) are set as: $\beta_x = 3 \times 10^{-6}$, $\mathbf{K}_x = 10^{-10} \mathbf{I}_2$, $\mathbf{L}_{M_o} = 10^{-10} \mathbf{I}$, $\mathbf{L}_{B_o} = 10^{-10} \mathbf{I}_2$, while the initial estimates of the dynamic parameters of the cell were set as: $\hat{\mathbf{M}}_o = \text{diag}\{5 \times 10^{-10}, 5 \times 10^{-10}\} kg$ and $\hat{\mathbf{B}}_o = \text{diag}\{1.8 \times 10^{-9}, 1.8 \times 10^{-9}\} kg/s$.

The parameters of the adaptive observer for $\hat{\mathbf{q}}_L$ in equation (5.42) are set as: $\beta_q = 3$, $\mathbf{K}_z = 0.01 \mathbf{I}$, $\mathbf{L}_{M_{qo}} = 10^{-10} \mathbf{I}$, $\mathbf{L}_{B_{qo}} = 10^{-10} \mathbf{I}$, while the initial estimates of the dynamic parameters of the robotic manipulator were set as: $\hat{\mathbf{M}}_{qo} = \text{diag}\{0.0222, 0.0114\} kg$ and $\hat{\mathbf{B}}_{qo} = \text{diag}\{0.05, 0.04\} kg/s$.

The parameters of the adaptive observer for $\hat{\mathbf{q}}_{Ld}$ in equation (5.54) are set as: $\beta = 9$, $\mathbf{K}_e = 0.01 \mathbf{I}_2$, $\mathbf{L}_{M_{do}} = 10^{-10} \mathbf{I}_2$, $\mathbf{L}_{B_{do}} = 10^{-10} \mathbf{I}_2$, while the initial estimates of the dynamic parameters of the robotic manipulator were set as: $\hat{\mathbf{M}}_{do} = \text{diag}\{0.0222, 0.0114\} kg$ and $\hat{\mathbf{B}}_{do} = \text{diag}\{0.05, 0.04\} kg/s$.

The parameters of the trapping region remained the same, but now the estimated laser position $\hat{\mathbf{q}}_L$ and the estimated cell position $\hat{\mathbf{x}}_C$ instead of the actual positions $\mathbf{q}_L$ and $\mathbf{x}_C$ were used to construct the weight factor $\hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)$.

The parameters of the desired position input for the trapped cell in equations (5.48) and (5.49) were proposed as: $N_T = 8$, $\kappa_T = 0.99$, $b_T = 10 \mu m$, $\alpha_{Cx} = 1$, $\mathbf{K}_p = 10^{-6} \mathbf{I}_2$, $\mathbf{K}_d = 2 \times 10^{-7} \mathbf{I}_2$, $\mathbf{L}_C = 10^{-10} \mathbf{I}_4$, and the parameters of the robot input in equations (5.60) and (5.61) were set as: $\alpha_{Lq} = 1$, $\mathbf{K}_q = \mathbf{I}_2$, $\mathbf{K}_{sq} = 1.5 \mathbf{I}_2$, $\mathbf{L}_L =$

$10^{-5} \mathbf{I}_4$.

Figure 5.7: Simulation 2: The tracking error.

The tracking errors are shown in Fig. 5.7, and the tracking errors converge to zero in about 4s. The path of the laser and the trapped cell is shown in Fig. 5.8, which also consists of three stages: (1) the laser starts from a large initial position and moves towards the cell (Fig. 5.8(a)); (2) the cell enters the trapping region and is trapped by the laser (Fig. 5.8(b)); (3) the trapped cell is manipulated to follow the desired trajectory (Fig. 5.8(c)). From Fig. 5.7 and Fig. 5.8, it is seen that the cell is successfully trapped by the laser and manipulated to follow the desired trajectory.

(a) $t=0$ s: initial positions (b) $t=0.15$ s: the cell is trapped by the laser beam (c) $t=24$ s: the cell position converges to the desired position

Figure 5.8: Simulation 2: The cell is trapped then transported to the desired position.

5.4 Experiment

The proposed control method was also implemented in a robot-tweezer manipulation system in the City University of Hong Kong, as shown in Fig. 5.9. The system is constituted of three modules for sensing, control and execution [45]. The sensing module consists of a microscope and a CCD camera, and the cell positions can be obtained through image processing. The control module consists of a phase modulator and a stepping motor controller. The execution module consists of the holographic optical trapping and the motorized stage, and the offset between the laser beam and the cell can be varied by directly adjusting the position of the laser beam or by moving the stage with motor control while fixing the laser beam. All of the mechanical components are supported by an anti-vibration table in a clean room. The optical tweezers were controlled to manipulate the yeast cell, but due to the limited access to the software interface, the desired position of the laser source is set as the control input.

Figure 5.9: A robot-tweezer manipulation system.

Figure 5.10: Experiment: The tracking error.

Figure 5.11: Experiment: The path of the laser beam and the trapped cell.

In the experiment, the position of the laser beam is fixed, and the desired position input $\mathbf{q}_{Ld}$ in equation (5.11) is applied on the motorized stage to vary the relative distance between the laser beam and the cell. In the beginning, the trapped cell was initially located at $(0, 31.97)\mu m$, and the laser beam was at $(0, 0.08)\mu m$. Therefore, the initial distance between the laser and the cell was very large, and the cell was outside the range of Gaussian field. The laser was controlled to move towards the cell. After the cell was trapped, the laser transported the cell to follow the desired

trajectory which was specified as:

$$\begin{cases} x_{Cd_1} = -15 + 15\cos(0.01t)w_T(\mathbf{x}_C, \mathbf{q}_L) \mu m, \\ x_{Cd_2} = 32 + 15\sin(0.01t)w_T(\mathbf{x}_C, \mathbf{q}_L) \mu m. \end{cases} \quad (5.73)$$

Different desired trajectories for the trapped cell are used in the simulation and the experiment respectively, to illustrate that the proposed method is feasible for various trajectory tracking control problems.

The control parameters were set as: $N_T = 8$, $\kappa_T = 0.99$, $\mathbf{K}_p = 0.00006\mathbf{I}_2$, $\alpha_{Cx} = 1$, $\mathbf{K}_d = 0.2\mathbf{I}_2$, $\mathbf{L}_C = 10^{-7}\mathbf{I}_4$. The tracking error is shown in Fig. 5.10, and the path of the laser and the trapped cell is shown in Fig. 5.11. The pictures of the trapped cell at different time instants are shown in Fig. 5.12. From Fig. 5.10 to Fig. 5.12, it is seen that the cell was successfully trapped by the laser beam and transported to follow the desired trajectory.

Figure 5.12: Experiment : Positions of the trapped cell at various time instants.

5.5 Summary

In this chapter, the concept of the regional feedback is extended to the robot-assisted optical tweezers system. A unified control method is proposed for robot-assisted optical tweezers systems, which can integrate automatic trapping and manipulation of biological cells into a single method. The proposed control method is able to transit between the operation of trapping and manipulation without hard switching, and it allows the laser beam to start from an initial position that is far away from the cell and automatically trap then manipulate the cell, and it also works when the cell escapes from the optical trap during the course of manipulation.

Unlike the existing methods that assume open-loop control of the position of laser source, the dynamics of robotic manipulator is introduced into optical tweezers system so that a closed-loop manipulator control problem can be formulated and solved. The proposed formulation provides a theoretical framework that bridges the gap between traditional robotic manipulation techniques and optical manipulation techniques of cells.

The stability of the closed-loop system is analyzed by using Lyapunov-like analysis, with the consideration of the dynamics of both the cell and the manipulator of the laser source. Simulation and experimental results are presented to illustrate the performance of the proposed control methods.

Chapter 6

Conclusion and Recommendations

6.1 Conclusion

For control tasks defined in task space, traditional solutions in the literature employ single sensory feedback information for the entire task. However, it is well known that the task-space feedback information cannot cover the entire robot workspace, due to the singularity of the Jacobian matrix, the limited field of view of the camera and the vision occlusion. In principle, hybrid control theory can be applied to switch from one controller to another when the robot reaches the boundary of a task space, and hence it can enable the robot to move from an initial position outside the sensing zone to a desired position within the sensing zone. However, in hybrid control method, the controllers must be designed separately in different coordinates. Since most hybrid control methods are based on hard-switching strategy, the transition from one controller to another is not continuous. The discontinuity results in chattering and vibration of robot control inputs that may damage the objects around the robot. Therefore, the global stability problem of task-space control system has not been systemically solved.

This thesis is devoted to developing a comprehensive framework to address various open issues in robot task-space control. Towards this objective, a novel regional feedback method is proposed in this thesis and a set of regional feedback based control schemes are presented. Some conclusions of this thesis are summarized as follows:

- (i) A regional feedback method is presented for robot task-space control. Each feedback information is employed in a local region, and the combination of regional information ensures the convergence of robot motion. The transition from one feedback information to another is embedded in the controllers without using any hard or discontinuous switching. The regional feedback variables can be specified in different task-space coordinates to address various task-space control problems in a unified way.
- (ii) Based on the proposed regional feedback, a novel task-space controller is developed for the robotic manipulator. Instead of designing multiple controllers in different regions and switching between them, the proposed control strategy integrates the use of dual task-space information in a single controller. This is the first result in task-space control that ensures the dynamic stability with consideration of singularity issue and limited sensing zone.
- (iii) The results of the regional feedback method are further extended to a multiple task-space control method that is able to deal with the issues of singularity and limited field of view at the same time. The main idea is to divide the whole task space into three categories including joint-space region, Cartesian-space region, and image-space region. The joint-space feedback drives the end effector away from the singular configurations, and the Cartesian-space feedback enables the end effector to reach the field of view, and the image-space feedback is activated within the field of view to ensure the convergence of tracking task. The concept of multiple task-space

control can be easily extended to the cases that more than three kinds of feedback information are required.

(iv) The concept of regional feedback is also extended to the robot-assisted optical tweezers system. By using the regional feedback, a unified robotic manipulation technique for optical tweezers that integrates automatic trapping and manipulation of biological cells into a single method is proposed. Instead of using open-loop control of the position of laser source as assumed in the literature, the dynamics of robotic manipulator is introduced into optical tweezers system so that a closed-loop manipulator control problem is formulated and solved. The proposed formulation also provides a theoretical framework that bridges the gap between traditional robotic manipulation techniques and optical manipulation techniques of cells.

6.2 Recommendations for Further Research

Some future research works and possible extensions are identified as follows.

- (i) The results of regional feedback control cannot guarantee the transient performance of robot which is necessary for some new initiatives such as robot surgery. In the future, a new task-space control scheme would be developed to ensure the transient response of robot with a performance bound. The task-space control with the performance bound would required less energies compared with conventional setpoint or tracking control since it allows more flexibility in the tasks.
- (ii) The proposed regional feedback control method has been implemented in the first two links of a SCARA robot system with a PSD camera. The results will be extended to more complex robot systems with stereo cameras after the improvement of experimental setup in future.

(iii) The dynamic model of optical tweezers proposed in this thesis applies to spherical cell trapping. For non-spherical microobjects such as nanowires and nanotubes, the trapping force in the dynamic model is different and should be experimentally validated. In addition, when the optical tweezers manipulate nanoscale objects, viscous drag dominates inertia and thus the mass of trapping object can be ignored. Therefore, new dynamic models would be established for trapping of non-spherical nanoscale objects.

(iv) Orientation control is important when optical tweezers trap non-spherical microtools to manipulate cells or when cells are imaging for birefringent features. Future research efforts would be devoted to developing orientation control strategies by using optical tweezers.

(v) Existing control techniques for optical tweezers are based on the assumption that the optical trap can be modeled as a linear spring with exact stiffness when the cell is very near the laser beam. Adaptive control methods may be developed to deal with the uncertain stiffness due to the errors in parameter identification for the laser beam.

(vi) The results of dynamic trapping and manipulation in this thesis are limited to single cell. Future works would be devoted to develop an optical manipulation technique for multiple groups of cells. Decomposition of cells into smaller groups give capability and flexibility in tasks and remarkably increases environmental adjustability. Independent control of each group of cells gives capability in studying group interactions between different types of cells or between bacterial and cells.

(vii) The manipulation of multiple groups of cells using optical tweezers requires each laser beam to be controlled independently and simultaneously. To satisfy the requirement, new control strategies for the laser beam will be proposed based on beam steering techniques.

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Appendix

Weight Factor

To satisfy equations (3.50) and (4.32), weight factors are introduced as:

$$w_i(\mathbf{x}_i) = w_{E_i}(\mathbf{x}_i) \prod_{j=1}^{N_i} w_{I_{ij}}(\mathbf{x}_i),$$

where $w_{E_i}(\mathbf{x}_i)$ and $w_{I_{ij}}(\mathbf{x}_i)$ are the weight factors corresponding to the i^{th} external region and the ij^{th} internal region respectively. The weight factor $w_{E_i}(\mathbf{x}_i)$ smoothly changes from 0 to 1 when the end effector transits from $f_{Er_i}(\mathbf{x}_i) \geq 0$ to $f_{Er_i}(\mathbf{x}_i) < 0$. The weight factor $w_{I_{ij}}(\mathbf{x}_i)$ smoothly changes from 1 to 0 when the end effector transits from $f_{Ir_{ij}}(\mathbf{x}_i) \geq 0$ to $f_{Ir_{ij}}(\mathbf{x}_i) < 0$. Both $w_{E_i}(\mathbf{x}_i)$ and $w_{I_{ij}}(\mathbf{x}_i)$ must be continuous and smooth between 0 and 1. An illustrated is shown in the following Figure.

To construct $w_{E_i}(\mathbf{x}_i)$, two regions inside the $f_{Er_i}(\mathbf{x}_i)$ are defined as:

$$\begin{aligned} f_{w_{E_i}}(\mathbf{x}_i) &= \frac{(x_{i1} - x_{Ei1})^{n_{Ei}}}{c_{i1}^{n_{Ei}}} + \dots + \frac{(x_{in_i} - x_{Ein_i})^{n_{Ei}}}{c_{in_i}^{n_{Ei}}} - 1 \leq 0, \\ f_{w_{Er_i}}(\mathbf{x}_i) &= \frac{(x_{i1} - x_{Ei1})^{n_{Ei}}}{(\kappa_{w_i} c_{i1})^{n_{Ei}}} + \dots + \frac{(x_{in_i} - x_{Ein_i})^{n_{Ei}}}{(\kappa_{w_i} c_{in_i})^{n_{Ei}}} - 1 \leq 0, \end{aligned}$$

where $c_{i1}, \dots, c_{in_i}$ are positive constants, and $\kappa_{w_i} < 1$ are also positive constants.

Hence the size of $f_{w_{Er_i}}(\mathbf{x}_i) \leq 0$ is smaller than the size of $f_{w_{E_i}}(\mathbf{x}_i) \leq 0$ and the size of $f_{w_{E_i}}(\mathbf{x}_i) \leq 0$ is smaller than the size of $f_{Er_i}(\mathbf{x}_i) \leq 0$. Based on the $f_{w_{E_i}}(\mathbf{x}_i)$ and $f_{w_{Er_i}}(\mathbf{x}_i)$, the weight factor is proposed as:

$$w_{E_i}(\mathbf{x}_i) = \begin{cases} 0, & f_{w_{E_i}}(\mathbf{x}_i) \geq 0, \\ 1 - \frac{\{[f_{w_{E_i}}(\mathbf{x}_i)]^4 - (\kappa_{w_i}^{n_{E_i}} - 1)^4\}^4}{(\kappa_{w_i}^{n_{E_i}} - 1)^{16}}, & f_{w_{Er_i}}(\mathbf{x}_i) > 0, f_{w_{E_i}}(\mathbf{x}_i) < 0, \\ 1, & f_{w_{Er_i}}(\mathbf{x}_i) \leq 0. \end{cases}$$

Similarly, to construct $w_{I_{ij}}(\mathbf{x}_i)$, two regions are defined as:

$$f_{w_{I_{ij}}}(\mathbf{x}_i) = \frac{(x_{i1} - x_{I_{ij}1})^{n_{I_{ij}}}}{d_{ij1}^{n_{I_{ij}}}} + \dots + \frac{(x_{in_i} - x_{I_{ij}n_i})^{n_{I_{ij}}}}{d_{ijn_i}^{n_{I_{ij}}}} - 1 \leq 0,$$

$$f_{w_{Ir_{ij}}}(\mathbf{x}_i) = \frac{(x_{i1} - x_{I_{ij}1})^{n_{I_{ij}}}}{(\kappa_{w_{ij}} d_{ij1})^{n_{I_{ij}}}} + \dots + \frac{(x_{in_i} - x_{I_{ij}n_i})^{n_{I_{ij}}}}{(\kappa_{w_{ij}} d_{ijn_i})^{n_{I_{ij}}}} - 1 \leq 0,$$

where $d_{ij1}, \dots, d_{ijn_i}$ are positive constants, and $\kappa_{w_{ij}} < 1$ are also positive constants.

Based on the $f_{w_{I_{ij}}}(\mathbf{x}_i)$ and $f_{w_{Ir_{ij}}}(\mathbf{x}_i)$, the weight factor is proposed as:

$$w_{I_{ij}}(\mathbf{x}_i) = \begin{cases} 0, & f_{w_{Ir_{ij}}}(\mathbf{x}_i) \leq 0, \\ \frac{\{[f_{w_{I_{ij}}}(\mathbf{x}_i)]^4 - (\kappa_{w_{I_{ij}}}^{n_{I_{ij}}} - 1)^4\}^4}{(\kappa_{w_{I_{ij}}}^{n_{I_{ij}}} - 1)^{16}}, & f_{w_{Ir_{ij}}}(\mathbf{x}_i) > 0, f_{w_{I_{ij}}}(\mathbf{x}_i) < 0, \\ 1, & f_{w_{I_{ij}}}(\mathbf{x}_i) \geq 0. \end{cases}$$

Positive-Definite Matrix $\mathbf{P}$

According to Sylvester's Theorem [44], the symmetric matrix $\mathbf{P}$ in equation (5.69) is positive definite if all its leading principal minors ($A_1, \dots, A_8$) are strictly positive.

Since $\mathbf{B}_C = \text{diag}\{b_1, b_2\}$, $\mathbf{K}_d = \text{diag}\{k_{d1}, k_{d2}\}$, $\mathbf{K}_q = \text{diag}\{k_{q1}, k_{q2}\}$, $\mathbf{K}_{sq} = \text{diag}\{k_{s1}, k_{s2}\}$, and $\mathbf{K}_e = \text{diag}\{k_{e1}, k_{e2}\}$, the leading principal minors of $\mathbf{P}$ are computed as follows:

(i) $A_1 = b_1 + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)k_{d1}$;

(ii) $A_2 = (b_1 + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)k_{d1})(b_2 + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)k_{d2})$;

(iii) $A_3 = (b_2 + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)k_{d2})[(b_1 + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)k_{d1})(\alpha_{Lq}k_{q1} + \alpha_{Lq}^2k_{s1}) - \frac{k_3^2(\mathbf{x}_C, \mathbf{q}_L)}{4}]$;

(iv) $A_4 = [(b_1 + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)k_{d1})(\alpha_{Lq}k_{q1} + \alpha_{Lq}^2k_{s1}) - \frac{k_3^2(\mathbf{x}_C, \mathbf{q}_L)}{4}][(b_2 + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)k_{d2})(\alpha_{Lq}k_{q2} + \alpha_{Lq}^2k_{s2}) - \frac{k_3^2(\mathbf{x}_C, \mathbf{q}_L)}{4}]$;

(v) $A_5 = k_{s1}A_4$;

(vi) $A_6 = k_{s2}A_5$;

(vii) $A_7 = k_{e1}k_{s2}(\beta k_{s1} - \frac{k_{e1}}{4})[(b_1 + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)k_{d1})(\alpha_{Lq}k_{q1} + \alpha_{Lq}^2k_{s1}) - \frac{k_3^2(\mathbf{x}_C, \mathbf{q}_L)}{4}][(b_2 + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)k_{d2})(\alpha_{Lq}k_{q2} + \alpha_{Lq}^2k_{s2}) - \frac{k_3^2(\mathbf{x}_C, \mathbf{q}_L)}{4}]$;

(viii) $A_8 = \{k_{e1}(\beta k_{s1} - \frac{k_{e1}}{4})[(b_1 + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)k_{d1})(\alpha_{Lq}k_{q1} + \alpha_{Lq}^2k_{s1}) - \frac{k_3^2(\mathbf{x}_C, \mathbf{q}_L)}{4}] - k_{s1}(\alpha_{Lq}k_{q1} + \alpha_{Lq}^2k_{s1})\frac{k_3^2(\mathbf{x}_C, \mathbf{q}_L)}{4}\}\{k_{e2}(\beta k_{s2} - \frac{k_{e2}}{4})[(b_2 + \hat{w}_T(\hat{\mathbf{x}}_C, \hat{\mathbf{q}}_L)k_{d2})(\alpha_{Lq}k_{q2} + \alpha_{Lq}^2k_{s2}) - \frac{k_3^2(\mathbf{x}_C, \mathbf{q}_L)}{4}] - k_{s2}(\alpha_{Lq}k_{q2} + \alpha_{Lq}^2k_{s2})\frac{k_3^2(\mathbf{x}_C, \mathbf{q}_L)}{4}\}$.

Next, the conditions in equation (5.70) is reproduced as:

$$\alpha_{Lq}\lambda_{\min}[(\mathbf{K}_q + \alpha_{Lq}\mathbf{K}_{sq})\mathbf{B}_C] > \frac{k_1^2}{4}, \quad (C.1)$$

$$\beta\lambda_{\min}[\mathbf{K}_{sq}] > \frac{1}{4}\lambda_{\max}[\mathbf{K}_e], \quad (C.2)$$

$$\lambda_{\min}[\mathbf{K}_e]\{\beta\lambda_{\min}[\mathbf{K}_{sq}] - \frac{1}{4}\lambda_{\max}[\mathbf{K}_e]\}\{\alpha_q\lambda_{\min}[(\mathbf{K}_q + \alpha_{Lq}\mathbf{K}_{sq})\mathbf{B}_C] - \frac{k_1^2}{4}\} > \frac{k_1^2}{4}\alpha_{Lq}\lambda_{\max}[(\mathbf{K}_q + \alpha_{Lq}\mathbf{K}_{sq})\mathbf{K}_{sq}]. \quad (C.3)$$

Therefore, we have:

A_1 and A_2 are positive;

A_3 and A_4 are positive if condition (C.1) is satisfied;

A_5 is positive if A_4 is positive;

A_6 is positive if A_5 is positive;

A_7 is positive if conditions (C.1) and (C.2) are satisfied.

A_8 is positive if condition (C.3) is satisfied.

Therefore, the matrix $\mathbf{P}$ is positive definite if the conditions in equation (5.70) are satisfied.