Advances in 3D optical profilometry

Huang, Lei

2013

Huang, L. (2013). Advances in 3D optical profilometry. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/52124>

<https://doi.org/10.32657/10356/52124>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

Advances in 3D Optical Profilometry

HUANG LEI

SCHOOL OF MECHANICAL AND AEROSPACE

ENGINEERING

2013

Advances in 3D Optical Profilometry

HUANG LEI

HUANG LEI

School of Mechanical and Aerospace Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2013

Acknowledgements

I want to appreciate those people who helped me for this dissertation.

Primarily, I would like to show my gratitude to my supervisor, Prof. Anand Asundi. Thank to you for giving me the opportunity to join your research group. Thank you to let me work on my favorite domain as I want, and lead me to the right direction during my study. I always remember those precious moments that you patiently analyzed and thought for me when I turned to you with my tough issues. Appreciations are given to you for your comprehension, encouragement, and support when I am facing troubles in study and life. I have learned how to handle problems from the working experience and personal contact with my dear professor.

I would also like to extend my great thanks to Prof. Qian Kemaο (respecting your preferred order of name in Chinese tradition). It is always a pleasant time to discuss with Prof. Qian. I usually feel the time is so limited while chatting with you about the research in common. Your professional suggestion and conscientious attitude help me a lot in my own research.

Thank to Prof. Xianyu Su's 3D metrology group in Sichuan University, China. Thank Prof. Xianyu Su, Prof. Qican Zhang, and Prof. Yuankun Liu for your patience when discussing the research issues and replying my questions.

I want to thank Prof. Bing Pan and Prof. Liansheng Sui for your sharing your professional knowledge. I would also like to thank Mr. Koh for your assistance in my experiments. I am so happy to have research colleges of Ravi Kumar, Hao Yan, Chi Seng Ng, Kapil Dev, Hui Zhu, and Ming Zhao. Thank you all for your help during these years.

Special thanks are given to the Optics and Photonics Society of Singapore (OPSS), the international society for optics and photonics (SPIE), and School of Mechanical and Aerospace Engineering (MAE) in Nanyang Technological University (NTU) for their generous supports for my conference attendances.

Table of contents

Summary	1
List of symbols	5
List of tables	7
List of figures	9
CHAPTER 1 Introduction	19
1.1 Background	19
1.2 Motivation	20
1.3 Existing techniques and systems for dimensional metrology	20
1.3.1 Contact methods vs. non-contact methods	20
1.3.2 Mechanical scanning vs. full-field optical imaging	20
1.3.3 Full-field structured-light optical imaging: interference fringe vs. digital fringe	21
1.4 Challenges to digital fringe based optical shape measurement	22
1.5 Objective and scopes	25
1.5.1 Objective	25
1.5.2 Scopes	25
1.6 Organization	27
CHAPTER 2 Literature review	29

2.1	Introduction.....	29
2.2	Fringe analysis technique.....	30
2.2.1	Wrapped phase retrieval.....	31
2.2.2	Phase unwrapping	33
2.3	Multi-type surface profiling.....	37
2.3.1	Diffuse surface profilometry	37
2.3.2	Specular surface profiling	40
2.3.3	Profiling transparent surface	42
2.4	Conclusion	44
CHAPTER 3	Advances in fringe analysis.....	47
3.1	Introduction.....	47
3.2	Phase error analysis and compensation for non-sinusoidal waveforms in phase shifting technique.....	48
3.2.1	Phase errors from non-sinusoidal waveform.....	49
3.2.2	Analysis and compensation for phase error from non-sinusoidal waveform.....	49
3.2.3	Experiment and discussion.....	54
3.3	A practical phase invalidity identification framework with temporal phase unwrapping approach.....	56
3.3.1	Principle	57

3.3.2	Method: phase retrieval and phase error identification framework	59
3.3.3	Experiment.....	60
3.3.4	Discussion	63
3.4	Comparison of methods for wrapped phase retrieval from single fringe pattern.....	63
3.4.1	Principles of these transform based methods in fringe pattern processing	64
3.4.2	Annotations of algorithms and phase compensation.....	68
3.4.3	Comparison by simulation	71
3.4.4	Experimental comparison	75
3.4.5	Discussion	81
3.5	Quality-guided phase unwrapping technique: a comparison of the quality maps and the guiding strategies	81
3.5.1	Principle and problems of spatial phase unwrapping and QGPU	82
3.5.2	Comparison of quality maps	85
3.5.3	Guiding strategies	93
3.6	Conclusion	100
CHAPTER 4	Three-dimensional shape measurement for different types of surfaces....	101
4.1	Introduction.....	101
4.2	Diffuse surface profilometry with fringe projection method based on improved least squares calibration.....	102

4.2.1	Principle and implementation	103
4.2.2	Simulation	108
4.2.3	Experiment and discussion.....	114
4.3	An improved iterative least squares integration method for shape reconstruction from gradient data.....	118
4.3.1	Principle	120
4.3.2	Simulation	122
4.3.3	Discussion	126
4.4	Fast FRD and its application in dynamic 3D sensing of specular surfaces.....	128
4.4.1	Principle and method for the fast full-field out-of-plane deformation method with fringe reflection technique.....	130
4.4.2	System calibration.....	133
4.4.3	Experiments	134
4.4.4	Discussion	139
4.5	Partially diffuse and specular surface profilometry with combined fringe projection and reflection technique.....	141
4.5.1	Principles with difference	141
4.5.2	Experiment.....	142
4.5.3	Simulation	147

4.5.4	Discussion	149
4.6	Transparent surfaces profilometry with double-side fringe reflection technique	151
4.6.1	Methodology for phase retrieval from double-side reflective fringe patterns.....	152
4.6.2	Simulation	155
4.6.3	Discussion	160
4.7	Conclusion	160
CHAPTER 5	Conclusions and recommendations for future work.....	163
5.1	Conclusions.....	163
5.2	Recommendations for future work.....	166
	Publications.....	169
	References.....	173

Summary

The increased significance and need for three-dimensional (3D) shape measurement in various industries for 3D inspection, quality control, and reverse engineering, has promoted the interest in 3D metrology. Among the 3D shape measurement techniques, optical methods show great advantages by offering non-contact, fast and flexible measurement systems compared to tactile tools such as the ubiquitous Coordinate Measuring Machine (CMM). However, various challenges still need to be addressed as the needs for accuracy, robustness, speed, and reliability increase.

Basically 3D optical metrology has the following components – a pattern (fringe) illumination module, interaction of the fringe pattern with the object under test, recording of the fringe pattern using a digital camera (imaging module), and finally processing of the recorded results followed by display of the results. There are three types of surfaces – diffuse, specular, and transparent surfaces – typically encountered in engineering objects. The illumination, sample and imaging modules are treated as an entire system and combined effects are considered in the processing and data reduction stages. However, the phase which is obtained from the fringe processing is related to the profile in different ways for each of the methods.

Fringe processing is one of the key aspects in optical 3D metrology. In the derivation of phase shift equations, a sinusoidal profile is presumed. Practically this is not always true and a novel scheme to analyze and compensate phase error from non-sinusoidal waveforms is proposed. Phase error is compensated through an iterative process based on analysis of the theoretical phase error. Experimental results demonstrate the feasibility of the proposed compensation method in removal of the periodical phase error due to the non-sinusoidal waveform. In addition, a practical phase retrieval framework is proposed for multi-frequency phase shifting method to accurately calculate the absolute phase and automatically identify invalid phase points via a thresholding and criterion checking process. For dynamic application, only a single frame of data is available to retrieve the wrapped phase and then unwrap it. A comparison of

the transform-based algorithms for single-frame phase retrieval is conducted in the presence of noise and non-sinusoidal waveform conditions with simulation and experiment. The methods of 2D Windowed Fourier Ridges and 2D Wavelet Transform are the two superior phase retrieval algorithms in fringe projection profilometry. Quality-guided phase unwrapping technique is reviewed with a comparison on quality definitions and guiding strategies. When unwrapping noisy phase, transform-based methods perform better but with increased computation time. But none of the quality maps always succeeds in handling phase discontinuities. For guiding strategies, the classical guiding strategy with a data structure of indexed interwoven linked list is fastest to achieve the most reasonable unwrapping results. If a small unwrapping error is tolerated, stack-chain guiding strategy can work even faster.

Next, some specific aspects related to methods for different surface properties (reflectance) are studied. First, with a consideration of camera lens distortion, an advanced least squares calibration method is proposed for fringe projection profilometry. The proposed method involves the distortion of camera lens to estimate the system parameters. It is more accurate than the existing least squares calibration method which is verified with both simulation and experiment. Second, an improved least squares integration method is proposed to enhance the accuracy of surface reconstruction from gradient which is obtained from the phase in a reflection arrangement. Significant improvement in accuracy is verified by comparing with the traditional least squares integration method. The merits of the proposed method are accurate, fast, and able to handle large datasets. A fast fringe reflection method with a single shot of a two-directional fringe pattern is then proposed for measuring dynamic scene of a specular surface. The influence of surface reflective property is next discussed through a study on 3D shape measurement of partially diffuse and specular surfaces using both the fringe projection technique and fringe reflection technique. Fringe reflection technique is found more sensitive on the partially diffuse and specular surface in the experiment. The combination of both fringe projection and reflection techniques is a practical solution as well. At last, a method is proposed to analyze the fringe patterns of double-side reflection from the front and rear

interfaces of a transparent surface to retrieve the fringe phase from either surface for further dimensional reconstruction.

List of symbols

x, y, z	The world coordinates.
x_c, y_c, z_c	The camera coordinates.
x_n, y_n	The normalized camera coordinates.
x_p, y_p	The pixel coordinates.
I, f	The intensity of fringe pattern.
ϕ^w	The wrapped phase.
ϕ^u	The unwrapped phase.
δ	The phase shifting amount.
A	The background intensity.
M	The intensity modulation.
N	The total number of phase shifts.
$RMSE$	The root mean square error.
thr	The threshold.
j	The imaginary unit.
Q	The quality map.
S	The root mean square.

N	The surface normal.
p	The slope in x direction
q	The slope in y direction

List of tables

Table 3.1. Speed comparison is carried out for different data structures in classical quality guiding by using the language of C++.	96
Table 3.2. Comparison among thresholds is investigated in two-section guiding. Sample: Box (960×1,280) in Fig. 1. Language: C++.	96
Table 3.3. Comparison is carried out with Box (960×1,280) in Fig. 3.20.	98
Table 3.4. Comparison is carried out with Noisy Peaks (256×256) in Fig. 3.22.	98
Table 3.5. Comparison is carried out with Coffee Cover (400×400) in Fig. 3.27.	98
Table 4.1. Coefficients are determined with existing and proposed method.	111

List of figures

Fig. 1.1. Three-dimensional shape metrology is needed in different scales. The abbreviations are nm – nanometer, μm – micrometer, mm – millimeter, m – meter, km – kilometer, au – astronomical unit, and ly – light year. (Some of the pictures are downloaded from internet for illustration purpose only.)	19
Fig. 1.2. Schematic of digital fringe projection techniques generally include three steps.	22
Fig. 1.3. Reflection or refraction on diffuse, specular, and transparent surfaces are different conditions to be considered in optical 3D shape measurement.	23
Fig. 2.1. Information in the fringe pattern can be revealed through fringe analysis.	31
Fig. 2.2. Phase unwrapping breaks the phase measuring limitation of 2π . (a) Wrapped phase and unwrapped phase, (b) 3D view of phase unwrapping and (c) one line data of wrapped phase (blue circle) and unwrapped phase (green dot).	34
Fig. 2.3. Schematic diagram of FPP system mainly consists of a projector, a camera, and a processor.	38
Fig. 2.4. Schematic diagram of FRD systems mainly have three similar configurations. (a) Monoscopic FRD system without screen shifting, (b) monoscopic FRD system with screen shifting, and (c) stereoscopic FRD system.	40
Fig. 2.5. Some experimental results of transparent object profiling with digital fringe methods are found in literatures. (a) Diamond-shaped glass object in Kutulakos and Steger's work [113], (b) lens in Yamazaki's work [114], and (c) a piece of automotive glass in Xu's work [115, 116].	43
Fig. 3.1. The intensity of actual fringe images is expressed as a function of ideal sinusoidal fringes.	50
Fig. 3.2. Fringe pattern is projected onto a reference plane (a) and a test plaster model (b). (c) The plot shows frequency spectrum of the 512 th column of (a).	51

Fig. 3.3. Non-sinusoidal waveforms lead to periodic phase errors. (a) Phase error image, (b) phase error for 101 column points, the red line shows the average phase error of the 101 column points.....	54
Fig. 3.4. Phase error reduces after compensation. (a) Compensated phase error, and (b) compensated phase error for 101 column points, the red line shows the average phase error of the 101 column points.....	55
Fig. 3.5. Phase distributions of a sculpture are shown: (a) before compensation and (b) after compensation (unit in radian).	55
Fig. 3.6. A strategy of phase measurement is proposed with phase shifting plus temporal phase unwrapping method.	57
Fig. 3.7. Flowchart of phase invalidity identification framework shows the major procedures to retrieve the valid phase values.	60
Fig. 3.8. A prototype of the compact 3D profilometer is set up to demonstrate the proposed framework. (a) Schematic diagram and (b) photograph of the compact 3D profilometer system.....	61
Fig. 3.9. Experiments are carried out to test the validation of the proposed strategy. (a) Fringe patterns with different number of fringes, (b) wrapped phase maps corresponding to the fringe patterns in (a) and (c) unwrapped phase map without identification of invalid pixels.....	61
Fig. 3.10. Identification of invalid measuring points follows the flowchart of the proposed method. (a) Modulation map, (b) unreliable pixels from reliability identification step, (c) 3D result after reliability identification step, (d) least squares RMSE map in temporal phase unwrapping, (e) invalid pixels from RMSE identification step. (f) 3D result after RMSE identification step, (g) absolute phase map after RMSE identification step, (h) inharmonic pixels by checking neighboring phase relationship, and (i) final 3D result after the entire framework.	62

Fig. 3.11. Phase compensation is necessary for ridge-based algorithms. (a) Phase error by 1D WFR before compensation, (b) phase error by 1D WFR after compensation, (c) phase error by 1D WT before compensation, (d) phase error by 1D WT after compensation, (e) phase error by 2D WFR before compensation, (f) phase error by 2D WFR after compensation, (g) phase error by 2D WT before compensation, and (h) phase error by 2D WT after compensation.	70
Fig. 3.12. True phase and deformed fringe pattern are used in our simulation: (a) Phase using in simulation: $3.5 \times \text{peaks}(x, y)$, and (b) simulated deformed fringe patterns.	71
Fig. 3.13. Comparison is carried out under noisy condition. (a) Fringe pattern with noise of STD = 3, (b) fringe pattern with noise of STD = 10, and (c) standard deviation of phase error vs. standard deviation of additive noise.	73
Fig. 3.14. Comparison is carried out when the ratio of a_2 / a_1 is varying.	74
Fig. 3.15. Simulated true phase and phase errors are investigated for the usage in comparison as a benchmark. (a) True phase in simulation, (b) simulated reference and deformed fringe patterns, (c) phase distribution of the solid red line in (b) by using ‘N-step Phase Shifting + 2D WFF’ approach, (d) phase error by using phase shifting, and (e) phase error after 2D WFF.	76
Fig. 3.16. Mean values and $3 \times \text{STD}$ values of phase errors are calculated by using the true phase or the result of ‘N-step phase shifting + 2D WFF’ as the phase benchmark.	77
Fig. 3.17. Experiment is carried out and the benchmark can be obtained through noise reduction. (a) Reference and deformed fringe patterns under two sorts of conditions, (b) raw phase data from 16-step phase shifting method, (c) phase result after noise reduction by using 2D WFF, and (d) reduced noise in central area.	78
Fig. 3.18. Mean values and $3 \times \text{STD}$ values of phase errors are calculated under conditions of fringe pattern with no observable harmonics and with some harmonics.	79
Fig. 3.19. Phase and phase error are calculated with 2D WFF method under conditions of fringe pattern with no observable harmonics and with some harmonics. (a) Phase by 2D	

WFF with no observable harmonics, (b) phase error by 2D WFF with no observable harmonics, (c) phase by 2D WFF with some harmonics, and (d) phase error by 2D WFF with some harmonics.	80
Fig. 3.20. The QGPU unwraps the phase through a path according to a quality map. (a) Wrapped phase map, (b) quality map, (c) unwrapping path map (pixels in light yellow are unwrapped first and those in dark red are unwrapped last), and (d) unwrapped phase map.	82
Fig. 3.21. Spatial phase unwrapping method unwraps the phase of current pixel according to the information of its spatially neighboring pixel.	83
Fig. 3.22. Simulated noisy condition for phase unwrapping is investigated. (a) Four-step phase-shifted fringe patterns with speckle and random noise, (b) true wrapped phase, (c) true modulation, (d) retrieved noisy wrapped phase, (e) retrieved unwrapped phase (with phase noise).	87
Fig. 3.23. Comparison of phase unwrapping is carried out under noisy condition in case A. The quality maps used are (a) Q_{MOD} , (b) Q_{REL} , (c) Q_{FT} , (d) Q_{WFF} , (e) Q_{WFR} , and (f) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps. (In unwrapping error maps, correctly unwrapped pixels are shown with gray levels in proportion to their phase values, and wrongly unwrapped pixels are presented in colors of red or blue. Red and blue indicate that the unwrapped phase value is larger and smaller than its true value, respectively. Darker colors stand for larger errors.)	88
Fig. 3.24. Comparison of phase unwrapping is carried out under noisy condition in case B. The quality maps used are (a) Q_{PCC} , (b) Q_{PDV} , (c) Q_{FPD} , (d) Q_{SPD} , (e) Q_{FT} , (f) Q_{WFF} , (g) Q_{WFR} , and (h) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps.	89
Fig. 3.25. Additional phase unwrapping simulation with a wide spectrum and no carrier wave is carried out for transform-based methods. (a) Phase-shifted fringe patterns, (b)	

wrapped phase without noise, (c) modulation distribution, and (d) retrieved wrapped phase with phase noise.....	89
Fig. 3.26. Comparison of unwrapping results from transform-based methods in case A is investigated. The quality maps used are (a) Q_{FT} , (b) Q_{WFF} , (c) Q_{WFR} , and (d) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps.	90
Fig. 3.27. Experimental discontinuity condition for phase unwrapping is investigated. (a) eight-step phase-shifted fringe patterns with fringe projection technique, (b) retrieved wrapped phase, (c) retrieved modulation map, (d) unwrapped phase by using temporal phase unwrapping technique.	91
Fig. 3.28. Comparison of phase unwrapping is carried out under discontinuity condition in case A. The quality map used are (a) Q_{MOD} , (b) Q_{REL} , (c) Q_{FT} , (d) Q_{WFF} , (e) Q_{WFR} , and (f) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps.	91
Fig. 3.29. Comparison of phase unwrapping is carried out under discontinuity condition in case B. The quality map used are (a) Q_{PCC} , (b) Q_{PDV} , (c) Q_{FPD} , (d) Q_{SPD} , (e) Q_{FT} , (f) Q_{WFF} , (g) Q_{WFR} , and (h) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps.	92
Fig. 3.30. A failure example of QGPU is investigated. (a) Wrapped phase and unwrapped phase with temporal phase unwrapping, and unwrapping result under case A (b)-(g) and case B (h)-(o). The quality map used are (b) Q_{MOD} , (c) Q_{REL} , (d) Q_{FT} , (e) Q_{WFF} , (f) Q_{WFR} , (g) Q_{WT} , (h) Q_{PCC} , (i) Q_{PDV} , (j) Q_{FPD} , (k) Q_{SPD} , (l) Q_{FT} , (m) Q_{WFF} , (n) Q_{WFR} , and (o) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps.	93
Fig. 3.31. Data structures are considered for the sorting in quality guiding. (a) Array [140], (b) linked list [140], (c) indexed linked list, and (d) indexed interwoven linked list.....	95

Fig. 4.1. Schematic diagram of FPP system shows the target is placed in the common field of view of camera and projector.....	102
Fig. 4.2. Calibration gauge blocks are used in simulation. (a) Three-dimensional height distribution of calibration gauge blocks, (b) a representative simulated fringe pattern and (c) its unwrapped phase map.	108
Fig. 4.3. A target is used in simulation. (a) Three-dimensional true height distribution of target, and (b) a representative simulated fringe pattern and (c) its unwrapped phase map.	109
Fig. 4.4. Simulation is carried under no distortion condition. (a) Reconstructed result by the existing method, (b) reconstructed result by the proposed method, (c) reconstruction error from fringe patterns with no noise by the existing method, (d) reconstruction error from fringe patterns with no noise by the proposed method, (e) reconstruction error from fringe patterns with noise by the existing method and (f) reconstruction error from fringe patterns with noise by the proposed method.	110
Fig. 4.5. Simulation is carried out under camera lens distortion condition. (a) Reconstructed result by the existing method, (b) reconstructed result by the proposed method, (c) reconstruction error from fringe patterns with noise by the existing method and (d) reconstruction error from fringe patterns with noise by the proposed method.	112
Fig. 4.6. Simulation is carried under both camera lens and projector lens distortion condition. (a) Reconstructed result by the existing method, (b) reconstructed result by the proposed method, (c) reconstruction error from fringe patterns with noise by the existing method, and (d) reconstruction error from fringe patterns with noise by the proposed method.....	113
Fig. 4.7. Calibration experiment is carried out. (a) Representative fringe patterns in three separate measurements, and (b) their unwrapped phase data used for least-squares calibration.	114

Fig. 4.8. The calibration reference plane is reconstructed. (a) Reconstructed reference plane with the existing method, (b) reconstructed reference plane with the proposed method, and (c) histogram of the measured height results.	115
Fig. 4.9. Testing measurement is carried out. (a) A representative fringe pattern and (b) unwrapped phase map.	116
Fig. 4.10. Measurement results are investigated. (a) Point clouds with the existing method, (b) and (c) measured height distribution on the target block with the existing method, (d) point clouds with the proposed method, (e) and (f) measured height distribution on the target block with the proposed method.	117
Fig. 4.11. Error distribution shows the proposed method is very effective. (a) Error distribution with the existing method and (b) error distribution with the proposed method.	118
Fig. 4.12. Southwell grid model is used in least squares integration.	120
Fig. 4.13. True shape and gradient data are used in simulation. (a) True shape, (b) height map, and true slopes in x -direction $p = dz/dx$ (c) and in y -direction $q = dz/dy$ (d).	123
Fig. 4.14. Integration results are investigated. (a) Height error with no compensation, (b) after the 1st compensation, (c) after the 8th compensation, (d) standard deviations of height error after compensations, and (e) reconstructed result after the 8 th compensation.	124
Fig. 4.15. Integrate gradient data with noise. (a) True shape to be reconstructed, (b) slope $p = dz/dx$, (c) slope $q = dz/dy$	125
Fig. 4.16. Integration results show the error is effectively reduced through the proposed compensation. (a) Height error before compensation, (b) height error after iterative compensation, and (c) height error in one line ($y=0$).	125
Fig. 4.17. Integration comparison is carried out with small dataset. (a) True shape, and (b-c) height error with the RBFs based integration method (b) and the proposed integration method (c).	126

Fig. 4.18. Comparison of the RBFs method with the proposed method is carried out with two aspects. (a) Accuracy, and (b) speed.....	127
Fig. 4.19. Specular surface measurement with fringe reflectometry requires a camera, a pattern displayer, and a processor.	130
Fig. 4.20. Principle of surface normal calculation can be conducted.....	131
Fig. 4.21. Two-directional fringe pattern is used. (a) Displayed image, (b) captured image, and (c) Fourier spectrum of the captured fringe pattern (b).....	132
Fig. 4.22. Calibration is successfully carried out for a monoscopic fringe reflectometric system.	134
Fig. 4.23. Experiment is carried out and studied on fringe reflectometry. (a) Experimental setup, (b) captured two-directional fringe pattern, (c) and (d) retrieved wrapped phase maps in x and y directions, (e) and (f) unwrapped phase maps in x and y directions, (g) and (h) measured slope maps in x and y directions.....	135
Fig. 4.24. Results from measurement show the feasibility of the proposed method. (a) Three-dimensional results, and (b) single line profiles.....	136
Fig. 4.25. Small out-of-plane deformation is measured out. (a) Three dimensional surface difference, (b) single line profile of the height difference with 100 microns loading as their reference.	136
Fig. 4.26. A typical experimental result at $t = 0.05s$ is investigated. (a) Captured fringe pattern, (b) amplitude of Fourier spectrum shown with its natural logarithm value, (c) and (d) retrieved wrapped phase maps in x - and y -directions with the 2D WFR method (with boundary region removed).....	137
Fig. 4.27. Investigation is carried out for reflectometric result at $t = 0.05s$. (a) Slope in x -direction $p(x, y, z)$, (b) slope in y -direction $q(x, y, z)$, and (c) height distribution $z(x, y)$ from integration.	138

Fig. 4.28. Some 3D shapes of dynamic water wave at different moments are reconstructed. (a) $t = 0.00$ s, (b) $t = 0.05$ s, (c) $t = 0.10$ s, (d) $t = 0.15$ s, (e) $t = 0.20$ s, (f) $t = 0.25$ s, (g) $t = 0.30$ s, (h) $t = 0.35$ s, and (i) $t = 0.40$ s.	139
Fig. 4.29. Schematic diagrams of FPP (a) and stereoscopic FRD (b) show their difference in configuration.	142
Fig. 4.30. Experimental setup is composed of two cameras, a projector and an LCD screen.	143
Fig. 4.31. System calibration is carried out for both systems. (a) FPP system, and (b) FRD system.	144
Fig. 4.32. Test sample is a metal plate and its fringe patterns with FPP and FRD are captured. (a) A photo of tested target, (b) a captured fringe pattern with FPP, (c) a captured vertical fringe pattern with FRD, and (d) a captured horizontal fringe pattern with FRD... ..	144
Fig. 4.33. Absolute phase is retrieved from fringe patterns with phase shifting technique and temporal phase unwrapping. (a) FPP, and (b-c) FRD.	145
Fig. 4.34. Measuring results with FPP and FRD are compared. (a) Slope in y - direction, (b) slope in x - direction, (c) height directly from FPP, (d) height integrated with gradient data from FRD. (e) three-dimensional result from FPP, and (f) three-dimensional result from FRD.	146
Fig. 4.35. Comparison of one-line profiles in Fig. 4.34(c-d) is carried out. (a) Line a , and (b) line b	147
Fig. 4.36. Simulation is carried out with FPP under different phase noise conditions. STD = 0.0100 radians: reconstructed surface (a) and height error (b). STD = 0.0390 radians: reconstructed surface (c) and height error (d).	148
Fig. 4.37. Simulation is carried out with FRD under different phase noise conditions. STD = 0.0100 radians: reconstructed surface (a) and height error (b). STD = 0.5667 radians: reconstructed surface (c) and height error (d).	149
Fig. 4.38. The 3D shape is reconstructed with combined method.	150

Fig. 4.39. A typical double-side reflective fringe pattern is complex to analyze.....	152
Fig. 4.40. Images are simulated for investigation of proposed method. (a) Hann window, (b) true phase map of the front reflection ϕ_f , and (c) true phase map of the rear reflection ϕ_r . Phase maps are wrapped within $(-\pi, \pi]$ for visualization.....	156
Fig. 4.41. Double-side reflective fringe patterns are simulated with additive noise of $\sigma=3$. (a) I , and (b) I' with $k=1.1$	156
Fig. 4.42. An incorrect wrapped phase is retrieved with traditional phase shifting method...	157
Fig. 4.43. Wrapped phase and modulation are retrieved from double-side reflective fringe patterns. (a) Front phase ϕ_f , (b) front modulation B_f , (c) rear phase ϕ_r and (d) rear modulation B_r	157
Fig. 4.44. Phase is unwrapped with showing its corresponding unwrapping path maps. (a) Front unwrapped phase ϕ_f , (b) front unwrapping path map, (c) rear unwrapped phase ϕ_r and (d) rear unwrapping path map.	158
Fig. 4.45. Phase errors are compared. (a) Front phase error when noise Std. $\sigma=3$, (b) rear phase error when noise Std. $\sigma=3$, and (c) phase error Std. increasing with fringe noise Std. compared with conventional phase shifting methods.	159
Fig. 5.1. Compact 3D profilometers are shown with the user interface of software.	167
Fig. 5.2. Digital fringe reflectometer (a) and measuring result (b) are shown: local height variations of a measured Chinese magic mirror. Unit on color bar: nm.	167

CHAPTER 1 Introduction

1.1 Background

Human beings are able to analyze and make use of a variety of objects based on their cognition of the world. In object purpose, three-dimensional (3D) shape of object is one of the most helpful attributes. This most intuitive information can assist in clearer understanding and comprehension of the state and function of the object. For this reason, 3D shape measurement has been studied for a long time in many research and industrial domains [1].

At the same time with the improvement on 3D shape metrology, some related research and application fields have been impelled with a rapid development, such as MicroElectroMechanical Systems (MEMS), nanotechnology, biochemistry, and precision manufacturing industry.

Fig. 1.1. Three-dimensional shape metrology is needed in different scales. The abbreviations are nm – nanometer, μm – micrometer, mm – millimeter, m – meter, km – kilometer, au – astronomical unit, and ly – light year. (Some of the pictures are downloaded from internet for illustration purpose only.)

To meet the diverse requirements in these areas, different dimensional measurement techniques have been proposed and studied with measuring abilities varying from nanostructures by using Atomic Force Microscope (AFM) [2] and Scanning Electron Microscope (SEM) [3], to machine parts with Coordinate Measuring Machine (CMMs) [4-8], to the terrain on our planet by Interferometric Synthetic Aperture Radar (InSAR) [9-11], and even to galaxies with astronomical methods [12]. The dimensional metrology of different size scales are shown in Fig. 1.1.

1.2 Motivation

In engineering applications objects size range scales from nanometer to meters. Considering its large demand in this scale range from nowadays industry, this work will thus focus on this range of dimensional measurement as highlighted with a red bar in Fig. 1.1. A number of metrology techniques are proposed with many measurement equipments invented in order to satisfy the huge requirements for profiling objects in these scales. With the study is going further, the surface property of the sample to measure is also included in research.

1.3 Existing techniques and systems for dimensional metrology

1.3.1 Contact methods vs. non-contact methods

Dimensional metrology can be classed as contact methods and non-contact techniques. In contact techniques, the probe touches the object to make the measurement. To profile a surface, contact methods are usually a point-wise method and thus the probe or sample has to be scanned to provide the full 3D information. Contact probes are not suitable for certain applications, such as measuring soft or liquid target. Non-contact methods are gaining prominence due to their fast and non-scanning approach to shape measurement. Optical methods lead the way in this and with the advent of compact projection and high resolution imaging devices these methods are becoming increasingly prominent in industry and research. Various principles have been used for 3D measurement, such as optical triangulation [13], confocal technique [14], time-of-flight technique [15], electronic speckle pattern interferometry [16], white light interferometry [17], digital holography [18-20], and so on. Each of these methods target a specific object type, have specific sensitivity and range of measurement.

1.3.2 Mechanical scanning vs. full-field optical imaging

Optical methods evolved much the way of contact shape measurement systems. They were point-probes usually using a laser spot whose size could be adjusted to vary spatial resolution

[13, 21]. Of course for a full 2D scan, mechanical or other scanning was needed and depending on the resolution desired the speed could be quite slow. The next evolution was the line scanning system. The ability of optics to manipulate and shape the light beam enabled lines of light to be readily generated. This thus reduced scanning to one direction and was found to be an ideal system for monitoring objects on conveyor belts. Indeed, the photocopy and scanner utilize this approach although for 2D imaging. Finally with the advent of high resolution cameras and projectors -full field 3D imaging with no-scanning became a reality and has seen great advances in the current years [22, 23].

1.3.3 Full-field structured-light optical imaging: interference fringe vs. digital fringe

Full field optical 3D profiling methods use the projection of dark and bright lines called fringes on to the surface of the object to be measured. It was also known as structured or patterned light illumination. Generation of these fringes was readily available through the phenomenon of interference of light. The spacing between the dark and bright lines could be adjusted by changing the angle between two interfering beams. The field of optical interferometry is quite diverse and been used for testing of lenses using phase shift interferometry [24, 25], measurement of diffuse objects using electronic speckle pattern interferometry [16] and more recently techniques such as white light interferometry [17] and digital holography [18-20] for profiling of 3D objects. Interferometric methods offered very high sensitivity but were limited to small size of objects and needed a stable environment for measurement.

The alternate, so called incoherent approach to generate dark and bright lines - also called gratings evolved with the growth of the projector. In early days, gratings were written on slides which could be projected using a slide projector. With the development of the digital projector, gratings or fringes could now be generated via software and then projected. This afforded the flexibility of changing the period as well as incorporating phase shifting which enhanced the capabilities of these fringe projection methods for shape measurement. Use of

projectors enabled large objects to be measured and since no interference was required, the system could be used in harsh environments [26-28].

1.4 Challenges to digital fringe based optical shape measurement

Digital fringe projection techniques proved to be a versatile method for 3D shape measurement. As shown in Fig. 1.2, it involved various steps - first a pattern had to be generated and projected; second the pattern was distorted by the object to be measured; third the distorted fringe pattern was recorded using a digital camera; the next the pattern was processed to give the phase which had to be converted to the height through a calibration process. Various challenges in each of these steps need to be addressed as the needs for accuracy, robustness, speed and reliability increases.

Fig. 1.2. Schematic of digital fringe projection techniques generally include three steps.

Fringe processing methods use either transform-based methods [29-34] or phase shifting methods [23, 35-37]. Both these approaches require a sinusoidal pattern to be imaged. However, this is rarely achieved in practice and efforts have been done to address some aspects [27]. This is not just a matter of tweaking one or the other component but has to be looked at from a system level.

Secondly the interaction between the light and the object is important to understand how the object distorts the projected fringe pattern. For diffuse, reflecting and transparent objects as shown in Fig. 1.3, the relations are quite distinct.

Fig. 1.3. Reflection or refraction on diffuse, specular, and transparent surfaces are different conditions to be considered in optical 3D shape measurement.

Diffuse reflection happens when a light beam incidents upon a tiny surface and then reflects into various directions. The so called diffuse surface in this thesis stands for the surface on which the diffuse reflection makes the major contribution to the entire reflected amount. As a result, light reflected from a point on diffuse surface can be observed from many directions, ideally from all directions. By making use of this property, optical triangulation with digital fringe projection and full-field optical imaging technique is adopted to measure the 3D shape of diffuse surfaces, which is usually known as Fringe Projection Profilometry (FPP) [23, 29, 31, 35, 38]. Although FPP aiming for diffuse object is relatively well established, some measurement procedures are still necessary to be studied in detail to perfect measuring performance. For instance, some existing problems in FPP are phase measuring accuracy is significantly influenced by non-sinusoidal waveform of the fringe patterns, a flexible phase-height mapping method is still demanded for outdoor measurement in case a quick recalibration is required, a few error points might be “flying” within the resultant point cloud from multi-frequency phase unwrapping and pretty hard to remove them automatically, and the precision influence from surface quality and reflectivity.

On the other hand, specular reflection is distinct from diffuse reflection. The rays of incident light will only be reflected to a single outgoing direction if an ideal specular reflection happens. Polished smooth surface with certain materials, such as metals, will effectively reflect light by specular reflection. In order to measure this kind of specular surfaces with digital fringe full-field imaging method, Fringe Reflection Defectometry (FRD) [39, 40] becomes a preferred technique. From the direction of reflected light, 3D shapes as well as surface slopes can be obtained with fringe reflection technique or sometimes assisted by stereo vision techniques. In existing approaches for specular surface shape measurement with fringe reflection method, there are some practical issues to be solved to enhance the measuring ability. For example, the measuring speed is limited by multi-frequency phase shifted fringe capture and fast measurement for dynamic out-of-plane deformation is not able to carry out with existing FRD. Furthermore, the measuring accuracy of FRD highly depends on the numerical integration of gradient data instead of gradient measurement only, so it is eagerly required to develop an accurate, fast and flexible 2D integration algorithm, which can easily handle large dataset.

Instead of reflecting the light at the interface of two media, transparent object transmits most of the light energy through the interface. But enough information can still be obtained from its reflection, for example point-by-point measurement method can measure the profile of lens by distinguishing the signals from front reflection and rear one. Full-field non-contact measurement for transparent objects is a very tough issue, since it not only includes the height-slope ambiguity but also involves at least two times of refraction at the interfaces for each light ray. Furthermore, the refractive index of the material is also required to be known previously if the rear surface profile is also wanted to measure. Although there is some research work on this subject, the solutions are still far from satisfaction. One of the critical issues is how to successfully distinguish the front and rear surface reflections when digital fringe full-field imaging methods are utilized. Then hopefully some better solutions can be found to fill up the current blank in this area.

The next issue to deal with is to process the recorded fringe patterns. As mentioned two approaches are available - transform based methods and phase shifting. Various transform based methods have been proposed, but there is no systematic study as to the efficacy and reliability of these methods [29-34]. While phase shifting methods are well understood, there are still challenges in the presence of sharply changing object features which need to be addressed.

The result of this fringe is a phase which is wrapped between $-\pi$ to π . Unwrapping is a challenging task and no one approach seems to be able to handle all situations. Hence a need for a robust unwrapping scheme has to be explored.

Finally the need is to convert the phase map to dimensions. Once again depending on the type of object surface that is being profiled - the phase to height conversion might need additional steps such as integration. Of course calibration is a major part of this and also proper calibration can ensure best reliability and reproducibility of the data.

1.5 Objective and scopes

1.5.1 Objective

The objective of the work is to investigate digital fringe based full-field non-contact optical imaging 3D shape measurement methods for profiling not only diffuse objects, but also specular and transparent objects with varying height scales from submicron to meter.

1.5.2 Scopes

In order to achieve the objective mentioned above, several research scopes are specified for my study.

The first group is about fringe analysis technique which is a common technique in digital-fringe-based optical imaging methods. This technique retrieves intensity modulation,

fringe phase and other information from fringe patterns. So far some issues in this field, such as spatial phase unwrapping with discontinuities, are not well-studied yet.

- Although several methods were proposed for phase retrieval from a single fringe pattern, a comparison work is absolutely necessary since it is not clear which one is the best or relatively better under a specific condition.
- Another scope is the analysis and compensation of phase error from non-sinusoidal waveform. This kind of phase error should be carefully compensated or diminished by algorithms, as it significantly influences the final dimensional results. It seems a good strategy to study the phase shifting technique first, and then to deduce the phase error expression under non-sinusoidal waveform condition for its compensation.
- In addition, phase unwrapping algorithms including spatial phase unwrapping and temporal phase unwrapping should be studied for different applications. Some issues within phase unwrapping, such as “flying” points on phase result, should be carefully studied to improve the measuring accuracy.

Moreover, for specific techniques to deal with different type surfaces there are the other group of technical scopes to investigate.

- Firstly, for diffuse surfaces, FPP has been widely used to measure their 3D profiles. Some features need to be studied in detail to enhance its performance.
 - ✧ A flexible calibration is required in some outdoor applications to recalibrate system in a short time.
 - ✧ Additionally, surface quality and reflectivity will influence the intensity of reflected light and further the modulation and phase results will be impacted, which is a vital problem for some applications.

- Secondly, the following scopes are with respect to the technique of FRD which is to measure the 3D shape of specular surfaces.
 - ✧ However, the multi-frequency phase shifted fringe capture limits the measuring speed of FRD. A fast FRD method should be investigated to meet the requirement of dynamic specular surface out-of-plane deformation measurement.
 - ✧ Furthermore, the integration method for shape reconstruction from gradient data should be studied to enhance the accuracy, speed, and robustness of the 2D integration process.
- Thirdly, 3D shape measurement for transparent surfaces will be investigated by studying on how to distinguish the front and rear surface reflections when digital fringe full-field imaging methods are utilized.
- Finally, based on the advances in system calibration, fringe analysis, and data processing through my study - compact optical systems will be developed and implemented for high sensitive, robust 3D shape and profile measurement.

1.6 Organization

This thesis is organized as five chapters. 0 gives an introduction to multi-type surface 3D shape measurement with digital fringe based optical imaging techniques.

0 reviews the current optical 3D shape measurement methods. The 3D shape measurement methods based on digital fringe imaging techniques are especially presented in this chapter. The principles and applications of fringe projection technique and fringe reflection technique are introduced with discussing their key techniques in detail. The setups are different to measure different types of surfaces. Hence, there is a discussion on measuring setups and their relevant calibration methods.

0 demonstrates and discusses some advances in fringe analysis techniques. Fringe analysis techniques are separately studied with multi-frame fringe analysis and single-frame fringe analysis according to the required frame number for fringe processing. Multi-frame methods are applied to measure stationary surfaces and single-frame methods mainly aim for dynamic surface shape measurement.

0 discusses optical 3D shape metrology for multi-type surfaces with digital fringe imaging techniques. In this chapter, diffuse surfaces profilometry is first studied with least squares calibration with consideration of the camera lens distortion. Furthermore, the influence of the surface reflectivity is investigated with the measurement of partially diffuse and specular surfaces by using both fringe projection technique and fringe reflection technique. In addition, 3D metrology for specular surfaces is studied by proposing a fast FRD method for high speed or dynamic sensing of 3D shape variation of specular surfaces. Moreover, an accurate, fast, robust 2D integration method is proposed for shape reconstruction from gradient data.

0 concludes the work, discusses the contributions through my study on 3D optical profilometry, and gives some recommendations to the future work in full-field digital fringe imaging based 3D metrology.

CHAPTER 2 Literature review

Optical methods for 3D shape measurement and inspection have been investigated for a long time with many approaches invented by researchers all over the world. Some of them are well developed and widely applied, such as optical triangulation [29, 35], confocal technique [14, 21, 41, 42], time-of-flight technique [15], white light interferometry [43], digital holography [18]. Because the work is focused on full-field digital fringe imaging based optical 3D shape measurement, some literatures on optical 3D metrology with digital fringe imaging technique are reviewed in this chapter.

2.1 Introduction

Several approaches with digital fringe techniques to profile 3D shapes have been proposed in the last two decades. According to different surfaces types, 3D shape measurement techniques can be sorted as diffuse surface 3D metrology, specular surface 3D metrology, and transparent surface 3D metrology.

The diffuse surface 3D metrology with digital fringe technique is also well-known as fringe projection profilometry with an abbreviation of FPP mentioned above. This denomination is because fringe patterns shown on the diffuse surface are through a manner of light projection. Specifically, there are several different implementation approaches in fringe projection profilometry, such as phase measuring profilometry (PMP) [35], Fourier transform profilometry (FTP) [29, 31], and other transform-based approaches. Although these approaches use different phase retrieval method, they share a common measuring principle, optical triangulation.

For convenience in description and simplicity in differentiation, the specular surface 3D metrology with digital fringe technique is named as fringe reflection deflectometry, FRD for short. As what it is named, digital fringe patterns are displayed on an LCD screen and their

reflections on the tested specular surface are captured by digital cameras to measure the deflection of light for 3D shape reconstruction of the specular surface. In some research work, this technique is also called as phase measuring deflectometry [39], since the deflection of light is determined through fringe phase measurement.

The transparent surface 3D metrology with digital fringe technique is still little studied so far. Basically there is a lack of an effective digital fringe based method to nicely handle the 3D measurement for transparent surfaces. To some degree, it is the most challenging work in this study. Double-side reflection of the fringe pattern is the phenomena which should be investigated.

All these digital fringe based 3D shape measurement approaches have a technical background in common, fringe analysis technique for phase retrieval from fringe patterns. Since this report is focused on these digital fringe based measuring methods, following sections mainly review some fringe analysis techniques in common as well as their individual measuring methods in detail.

2.2 Fringe analysis technique

As the first data processing stage in fringe based techniques, fringe analysis on fringe patterns, as shown in Fig. 2.1, is actually to extract the intensity modulation, reliability, and most importantly phase information which is directly or indirectly related to dimensions of the target object. They provide fundamental information for following process in fringe based techniques and of great significance for accuracy and precision in 3D shape measurement.

Fig. 2.1. Information in the fringe pattern can be revealed through fringe analysis.

Phase retrieval from fringe patterns consequently becomes a hot research topic during the last two decades with many effective methods proposed to analyze fringe patterns in different situations. Generally speaking, the phase retrieval process can be divided into two steps, wrapped phase retrieval and phase unwrapping.

2.2.1 Wrapped phase retrieval

Raw data in fringe imaging techniques are fringe patterns, which are 8-bit light intensity values in a cosine waveform with phase modulation. The objective of this process is to demodulate the fringe intensity resulting in phase information wrapped within a range from $-\pi$ to π due to an arctangent operation in phase angle calculation. So far a few of fringe demodulation methods for wrapped phase retrieval have been proposed for various applications. According to the required frame number of fringe patterns, these wrapped phase retrieval methods can be sorted as two types: multi-frame methods and single-frame methods.

Multi-frame methods

Among various multi-frame methods, the traditional phase shifting method in PMP [35] is the most famous phase retrieval technique, which requires evenly phase-shifted fringe patterns. It is essentially based on least squares method. Because of its high accuracy, precision and robustness as well as pixel independence with parallel calculation, the phase shifting method becomes one of most widely used methods, especially for measuring surfaces with discontinuities. In order to achieve high speed phase shifting calculation, Huang and Zhang [44] proposed a fast three-step phase shifting algorithm. In addition, some advanced phase shifting algorithms are designed in order to reduce the sensitivity to inaccuracy of phase shift. Surrel [36, 45] studied the design of phase shifting algorithm and proposed a (N+1)-step algorithm to eliminate the phase errors caused by inaccuracy of phase shift. Zhang *et al.* [37] proposed a seven-step phase shifting algorithm insensitive to nonlinear phase-shift error and second-harmonic distortion and compared with other phase shifting algorithms. Novák *et al.* [46] analyzed and proposed a group of multi-step phase shifting algorithms with insensitivity of linear phase shifting error. Wang and Han [47, 48] proposed an advanced iterative algorithm for phase retrieval from randomly phase-shifted fringe patterns without knowing the phase shifting amount in advance. Nowadays, projectors with Digital Light Processing (DLP) and monitors with Liquid Crystal Display (LCD) are widely used in fringe based technique. The phase shift can be considered accurate by using a DLP or LCD to project or display computer generated fringe patterns. Therefore, although there are many discussions on the design of phase shifting algorithm which could tolerate inaccuracy in phase shifting, the traditional N-step phase shifting method seems accurate enough. But another issue, periodic phase errors from non-sinusoidal waveform due to the nonlinear response of digital photoelectric devices, becomes more evident. There are some discussions and solutions which are able to explain the phenomenon and reduce this sort of phase error. Guo *et al.* [27, 49] studied the influence of gamma effect in digital projectors and proposed two methods to correct the corresponding phase error. A practical compensation approach with a look-up table for phase errors caused by non-sinusoidal waveform is proposed by Zhang and Yau [50]. Baker *et al.* [51] analyzed the

phase error due to the γ nonlinear luminance of the projector and used a defocused method to reduce the phase error. Liu *et al.* [52] analyzed the mathematical model of gamma factor in PMP and proposed phase correction strategies under conditions with or without a gamma calibration. Li and Li [53] proposed an accurate gamma correction method for fast PMP.

Single-frame methods

For dynamic 3D metrology applications, it is more suitable to reconstruct the 3D shape by capturing only one frame fringe pattern during the dynamic deformation. In methodology of wrapped phase retrieval from single fringe pattern, there are mainly spatial domain methods and spectral domain methods. Spatial domain methods [54-60] are proposed by matching the fringes in spatial domain especially for fringe patterns with a low or even zero frequency carrier wave such as closed fringe patterns, which are not easy to handle in spectral domain due to the overlapping in spectrum. But in digital fringe methods, the carrier frequency is usually high enough, so spectral domain methods are widely studied and used. Spectral domain methods transfer the information in spatial domain into spectral domain, and retrieve the wrapped phase. These spectral domain methods include Fourier Transform (FT) method [29-31, 61-65], Windowed Fourier Transform (WFT) method [32, 66-70], Wavelet Transform (WT) method [33, 34, 71], and etc. These single-frame methods can calculate phase from one fringe pattern with great significance in dynamic sensing and measuring applications, although they have some limitations in phase retrieval accuracy comparing multi-frame methods when measuring surfaces with separations and discontinuities.

2.2.2 Phase unwrapping

After the wrapped phase is retrieved, the resultant phase values are lying in their principal value range $[-\pi, \pi)$, since the retrieved phase is usually calculated out through arctangent function with four quadrants. A procedure named as phase unwrapping is necessary to unwrap the wrapped phase from their principal values to relatively extended phase values as shown in Fig. 2.2. After

the phase unwrapping, the unwrapped phase values should directly have a certain relationship with the dimensions or gradient of the tested object.

Fig. 2.2. Phase unwrapping breaks the phase measuring limitation of 2π . (a) Wrapped phase and unwrapped phase, (b) 3D view of phase unwrapping and (c) one line data of wrapped phase (blue circle) and unwrapped phase (green dot).

The major issues in phase unwrapping are basically noise and discontinuities of the target profile. In order to remove the ‘wraps’ on phase, several phase unwrapping methods are proposed with considering how to reduce the influence of those two major issues in phase unwrapping. According to their implementation domains, phase unwrapping methods can be sorted into two types. They are spatial phase unwrapping methods and temporal phase unwrapping methods.

Spatial phase unwrapping

Spatial phase unwrapping is a natural and direct way to unwrap the wrapped phase retrieved from fringe pattern(s). Usually a single wrapped phase dataset is retrieved as a result and an unwrapping process is carried out on this frame of wrapped phase. These spatial phase unwrapping methods are designed based on the phase relationship between spatially neighboring pixels, so the whole unwrapping process is not pixel independent. With different considerations, a number of spatial phase unwrapping methods have been investigated in variety, which mainly include quality guided phase unwrapping or say reliability guided phase unwrapping methods by using the quality map or called reliability map as a guiding map to determine the unwrapping path [72, 73], mask cut method which utilizes the quality maps to

find branch cuts in phase images to block the integration through residues, multi-grid or weighted multi-grid method that calculates the unwrapped solution which minimizes the least-squared expression of the problem, Minimum L_p -norm method that minimizes the difference between the gradient of the wrapped phase and the gradient of the solution [74], phase unwrapping method via graph cuts [75, 76] and so on.

As mentioned above, noise and profile discontinuities are two major challenges in phase unwrapping. In spatial phase unwrapping, some phase denoising methods, such as the WFT method and the WT method, have been introduced into phase unwrapping and show their good performances [68, 77]. However, for discontinuities on profile, spatial phase unwrapping methods show their limitations. Since the spatial phase unwrapping methods are based on the phase information of spatial neighboring pixels, the judgment of phase jumps is according to the phase difference between neighboring pixels. In order to make the operation more insensitive to noise, any neighboring pixels with principal phase difference less than π will not be considered as a phase jump and their difference will be maintained in result. On the other hand, when the difference of principal phase values of the two neighboring pixels is bigger than π , only a phase jump of 2π will be confirmed. Hence, there is a serve limitation for spatial method to unwrap the phase image of an object with relatively large discontinuities and separations. Because if several fringes are unobservable at the discontinue regions and the neighboring pixels across a large discontinue edge, which gives large phase change, may have a difference of principal phase value less than π , incorrect judgment for phase jump will be made, or even if the judgment of phase jump is right, the phase adjustment with only 2π compensation still gives incorrect result, because only from a 2D phase data there is no idea at all how much the phase jump actually is. Under such conditions, in order to avoid the failure of spatial phase unwrapping in that region, coarser fringes are used with a sacrifice of losing the measuring sensitivity and the measuring precision later on.

Temporal phase unwrapping

Since the spatial phase unwrapping method has its limitation to unwrap the phase of a profile with large discontinuities and separations. Temporal phase unwrapping methods have been proposed with an idea of getting an unwrapped phase of coarser fringes or some fringe orders with black and white coded patterns, and using this unwrapped phase as a basic phase or these fringe orders to unwrap the other phase data pixel independently. In literatures, fringe patterns with two frequencies are suggested [78, 79], in which the spatial phase unwrapping is still used to unwrap the wrapped phase of coarser fringe pattern(s) and then the phase of finer fringe pattern(s) is pixel independently unwrapped based on the unwrapping phase of coarser one(s). Recently, the temporal phase unwrapping method has been developed into a completely space independent approach [80]. In this strategy, the fringe pattern with different fringe numbers are projected, and the coarsest fringe pattern has only one fringe in all, of which the phase without any ‘wraps’ is then used as fundamental information for further phase unwrapping because its phase values do not exceed the range of $[-\pi, \pi]$ in nature. The other phase maps are unwrapped based on their previous unwrapped phase maps one by one according to the relation of their frequencies or fringe numbers. This sort of temporal phase unwrapping method is also well known as multi-frequency phase unwrapping. Another big family of temporal phase unwrapping is gray-code temporal phase unwrapping [81-83]. Fringe orders are encoded with binary black and white patterns. The wrapped phase from the phase shifted sinusoidal fringe patterns is then unwrapped with those determined fringe orders from gray-code patterns. Sansoni *et al.* [81, 82] proposed a method for 3D imaging with gray-code light projection and combined it with phase shifting profilometry by using the gray codes to provide fringe orders for phase unwrapping. Zhang *et al.* [83] proposed an approach to solve the issue of binarization errors at black and white stripe boundaries by adding one more complementary gray-coded pattern.

Generally speaking, temporal phase unwrapping methods demand a very strong support from the fringe generator, as flexible fringe patterns with accurately determined fringe frequencies

are required to successfully realize the convenience and robustness of the later unwrapping algorithm. Fortunately, digital fringe techniques can offer these flexible fringe patterns very easily with computer generation and digital device illumination. Therefore, temporary phase unwrapping is widely used in digital fringe techniques. With an appropriate design of fringe patterns with different fringe numbers, the noises on fringe patterns will not make troubles in temporal phase unwrapping. On the other hand, the temporal phase unwrapping methods have solved the issue of unwrapping the phase with large discontinuities, with a sacrifice of projecting more fringe patterns onto the target. But considering the ephemeral projection or showing time by modern digital light projectors or LCD screens and high efficiency of the unwrapping process, temporal phase unwrapping is not time consuming. Because of its ability to carry out excellent phase unwrapping with dense fringe patterns that maintain high sensitivity, temporal phase unwrapping have become the most popular and robust phase unwrapping method in digital fringe pattern processing. With a further investigation on multi-frequency phase unwrapping [84, 85], there are some “flying” points on resultant dimensional points at the edge of discontinuities on target surfaces due to the wrongly unwrapped phase values.

2.3 Multi-type surface profiling

2.3.1 Diffuse surface profilometry

To measure the 3D shape of diffuse objects, the technique of FPP was proposed through a manner of fringe projection. Diffuse surfaces can clearly be observed from a large range of viewing angles due to the surface material and roughness. By studying the deformation of fringe pattern caused by the height distribution as well as projecting and observing directions, the surface height can be easily determined with system calibration parameters. Several techniques in literatures including PMP [35], FTP [29, 31], Windowed Fourier Transform Profilometry (WFTP) [32, 86], and Wavelet Transform Profilometry (WTP) [33] belong to the technique of FPP, since all of them use the same manner of projecting fringe patterns onto the target surface and then reconstruct the surface shape according to the fringe phase retrieved from captured fringe pattern(s) although their phase retrieval methods are different. As its

nomination, the technique of FPP is basically to project the fringe pattern(s) onto target surfaces and to capture the deformed fringe pattern(s) by using a digital camera. Thus, a basic setup of FPP contains a fringe projection device which is a digital projector nowadays instead of an actual grating projection setup, and a fringe image recording device, usually a digital camera, as shown in Fig. 2.3. To ensure the triangulation is sensitive with phase values, fringe patterns should be properly generated with a phase orientation parallel to the connection line between optical centers of projector and camera. Generally speaking, the systematic sensitivity will become higher if the angle between the two optical axes is larger.

Fig. 2.3. Schematic diagram of FPP system mainly consists of a projector, a camera, and a processor.

Taking a digital camera with a Charge Coupled Device (CCD) panel for an example, every pixel on its CCD panel can determine a tracing ray with the optical center of camera lens. If one point on the target surface is on that ray, there will be one and only one pixel location on the projector panel which can be determined by using the optical center of projector lens, once the projector can ‘see’ that point. In other words, if the optical centers of camera lens and projector lens, a pixel on CCD panel and its related pixel coordinate in projector are known, the observed point on the target surface can be uniquely determined.

In FPP including PMP, FTP, WFTP, and WTP, after the phase retrieval, a 2D phase dataset containing the 3D information of the objects can be obtained. However, the original aim for profilometry is actually to get the three dimensions of the objects. Hence, a conversion from 2D

phase to 3D coordinates is necessary and very significant for high measuring accuracy of the final dimensional results. Many different phase-coordinate conversion or say phase-height mapping methods have been proposed by researchers in the recent almost 30 years. The first calibration method relies on a certain measurement setup in which the geometric and other parameters must be accurately determined in advance [87, 88]. This is a very natural thought if the system geometric distances are known in advance, the 3D shape of the target can be calculated out according to the geometric relationship. Another sort of calibration approach is able to calibrate the system with several look-up tables by using a calibration plane, of which the height or depth should be precisely controlled during the calibration to ‘teach’ every pixel the relationship between its phase values and the corresponding dimensions in x -, y -, and z -directions [89, 90]. In addition, another kind of calibration method is based on least squares method to determine the phase-to-height relationship by using several standard gauge blocks [91, 92]. The least squares calibration method is more flexible in implementation, since it not only allows the system to be arbitrarily arranged, but also avoids the issues of accurate determination of geometric parameters and precise control of translation and alignment of a calibration plane. Furthermore, the gauge blocks are commonly used in metrology and easy to get with high accuracy. In the least squares calibration method, the out-of-plane phase-to-height mapping is carried out with least squares method by using several (at least two) gauge blocks to give the system different height or depth information. Once the out-of-reference-plane height is already known, the in-plane dimensions can be determined by the camera calibration parameters with a known height. Consequently, the out-of-plane phase-height mapping becomes quite critical in calibration, since in-plane dimensions are determined based on the out-of-reference-plane height or depth results and might be influenced by them. Moreover, a stereovision calibration method for FPP [93-95] is proposed to calibrate an FPP system as a stereovision system by considering the projector as the other camera. Using the phase value of the projected fringe pattern, the projector also can ‘see’ those feature points on a calibration pattern. The whole system can be treated as a stereovision system in calibration.

2.3.2 Specular surface profiling

For specular surfaces, specular reflection is the majority among reflection components and the light ray can only be captured within a small solid angle or even in only one direction with a theoretical solution. In this case, an LCD screen is adopted as the fringe display setup instead of a projector in order to make the camera see structure patterns on the illumination device. The technique of FRD or also called as phase measuring deflectometry (PMD) is specially proposed for measuring specular surfaces. A CCD camera can provide two points to determine the observation line just as how it works in FPP. However, an LCD screen does not have an imaging system as projector does in FPP. Thus only one certain point on the LCD screen can be provided by two fringe phase values. It is not enough to determine a line in 3D space. Consequently, the target point on the observation ray can hardly be determined. This situation is usually called as height-slope ambiguity, since the height and slope are related to each other and not able to be determined.

Fig. 2.4. Schematic diagram of FRD systems mainly have three similar configurations. (a) Monoscopic FRD system without screen shifting, (b) monoscopic FRD system with screen shifting, and (c) stereoscopic FRD system.

Bothe *et al.* [40] theoretically prove and experimentally demonstrate the ability, robustness and accuracy of the monoscopic fringe reflection technique for various applications. As shown in Fig. 2.4(a), this approach is especially suitable for almost flat specular surfaces with very tiny height variations. The phase change from height can be reasonably neglected, and the slope can

be determined with the phase values only. The second solution to solve the slope-height ambiguity is to use a translation stage to shift the LCD screen to two or more different positions shown in Fig. 2.4(b) [96-101]. Some specific applications by using this method is carried out, such as for measuring aspheric mirrors [98-100]. Another work by Guo *et al.* [101] keeps on how to use least squares method to track the light ray in measuring the shape of specular surfaces. A novel incident light tracking technique for measuring specular reflective surfaces is proposed and it successfully reconstructs a concave mirror without numerical integration. Moreover, the third solution for measuring specular surfaces is to utilize the stereovision system with an LCD screen [39, 97, 102]. As shown in Fig. 2.4(c) The stereovision system consists of two cameras, which will solve the height-slope ambiguity by searching points with the same height and slope. In a well-known work by Knauer *et al.* [39], physical limits of FRD are deduced by analyzing the limitation relations between the uncertainty of the slope measurement with the laterally resolvable distance. The basic idea of this so-called stereo deflectometry is to try to search a point along the observing ray from one pixel of the first camera, which can give an identical normal for both cameras. The merit of this method is no mechanical translation of the screen, and therefore it shortens the measuring time and does not require a costly mechanical translation device with high accuracy. Similar ideas are also proposed by Petz *et al.* [97] in another research work on absolute shape measurement of specular free-form surfaces.

Usually, the height distribution is obtained by integrating the determined slope data. There are some research publications on how to integrate the gradient values [103-107]. Evaluation methods for gradient measurement techniques have been investigated and demonstrated by Li *et al.* [106]. Several gradient evaluation methods including differentiation and integration as well as vector processing have been introduced and studied. Three gradient integration methods (path integration method, Fourier transform integration method, and full-field least squares integration method) have also been compared and discussed in different aspects. Ettl *et al.* [107] proposed another accurate shape reconstruction method from gradient data by dividing the whole area into small pieces with certain common regions, integrating the slope data piece by

piece based on radial basis functions (RBFs) with least squares sense, and then stitching all pieces together to reconstruct the entire surface.

2.3.3 Profiling transparent surface

It is not easy to reconstruct transparent surfaces by using optical methods. But there is an urgent requirement to measure 3D shape of transparent object with a non-contact method in industry, such as measuring the glasses and contact lens. Therefore, research on full-field non-contact 3D shape measurement method for transparent object is a significant and challenging study [108-112]. As mentioned, there are some issues to solve in measuring the 3D shape of transparent object. First, the reflection effect on the transparent object is weak. It is consequently not easy to directly measure the surface shape by using reflection effect on transparent object. Second, usually there are at least two times of refraction at the interfaces of transparent object. The ray tracing is more complicated than that in diffuse or specular surface measurement. Therefore the stereovision technique might be needed here in this more difficult situation. Third, the refractive index of the object is required to be a known parameter before the dimensional measurement. Sometimes it is not easy to know the refractive index in advance, so it might be a potential problem for transparent object measurement. Research work on how to measure the 3D shape of transparent object is a pretty new field which is not investigated too much.

Fig. 2.5. Some experimental results of transparent object profiling with digital fringe methods are found in literatures. (a) Diamond-shaped glass object in Kutulakos and Steger's work [113], (b) lens in Yamazaki's work [114], and (c) a piece of automotive glass in Xu's work [115, 116].

The theory for light ray tracing through multi-refraction or multi-reflection has been proposed by Kutulakos and Steger [113]. In their theory, in order to measure the transparent object with twice refraction, at least two positions for an LCD screen and at least three cameras are required to retrace the light beams. It shows the stereovision technique is a possible solution for measuring transparent object. Their experimental results shown in Fig. 2.5(a) demonstrate their method is feasible to measure some transparent, but the result is not so good and this technique still needs to be studied more in detail. A similar approach can be observed with showing some experimental results in Fig. 2.5(b) from Yamazaki's work [114]. Xu *et al.* [115, 116] use gray code and line shifting method proposed by Gühring [117] to solve the issue of double-side reflection in measuring automotive glass by setting a threshold to remove the reflected lines from the rear surface. This approach works when the lines reflected from front interface is obvious stronger than those from rear one in the whole field of view. If not, it is not easy to get rid of the rear lines by simply setting a threshold on image intensity. In practical measurement for transparent targets, the common way is to process the rear surface to weak the reflection on the rear interface. Obviously, this operation is not suitable in many applications, especially for high precision optical lens inspection. Another work by Miyazaki and Ikeuchi [118] uses inverse polarization ray tracing to reconstruct the transparent object. This method using iteration algorithm to find an optimized solution can give a result for some simple objects, usually with one surface being a flat plane.

2.4 Conclusion

According to the review on 3D shape measurement with digital fringe imaging techniques in literatures, many limitations of the current state-of-the-art techniques are disclosed, for instance, the periodic phase error due to non-sinusoidal waveform, the “flying” points with the current temporal phase unwrapping method, the integration error owing to the imperfection assumption in least squares integration method, etc. Many improvements can be made in various aspects within related technical procedures. The main effort will therefore be put on both processing algorithms and metrology techniques for samples with different surface characteristics.

For processing algorithms, the following aspects need to be investigated.

First, the non-sinusoidal fringe waveform is one of the major sources affecting the phase result. It is very necessary to analyze the influence of non-sinusoidal waveform in phase retrieval with both single-frame and multi-frame methods.

Second, temporal phase unwrapping is effective but with some “flying” points on the final result. It would be an enhancement if automatic identification and removal of those “flying” points can be done, especially for practical measurement.

Third, quality guided phase unwrapping method is an excellent spatial phase unwrapping approach, considering its relatively high success rate, high speed, and automatic process. But an improvement between speed and success rate as well as automation can be studied in detail to make this good method more effective in practical measurement.

For measurement techniques, the investigation into several aspects mentioned below is necessary for measuring samples with different surface characteristics.

Least squares calibration method is easy to handle and flexible to use. However, I found the camera lens distortion is not considered in the current system calibration model. This leads to a

large systematic error in measurement results, which is vital to a measurement system. It is necessary to have a study on how to improve the least squares calibration method with consideration of the camera lens distortion.

Furthermore, the current least squares integration method is not able to reconstruct free-form surfaces accurately due to its imperfect assumption. It becomes an unavoidable issue that how to keep the least squares integration method free from the error owing to its imperfect assumption, if an accurate reconstruction is required. With the consideration above, a study on compensate the error introduced by the imperfect assumption is badly needed to reconstruct the surface from gradient data in high accuracy.

For profiling specular surfaces with FRD, there is currently no effective approach to carry out a fast or even dynamic measurement. Therefore dynamic 3D shape measurement of a specular surface is a big challenge nowadays. It is consequently a very interesting topic to investigate the way to handle a fast or dynamic 3D shape measurement of specular surfaces.

In addition, there is no watershed between diffuse surfaces and specular surfaces. The influence of the reflective property is also significant to investigate for partially diffuse and specular surfaces profiling.

Currently there is no effective digital fringe imaging based method to measure the 3D shape of transparent surfaces. However, the demand on this field is huge no matter in optical industry or in other manufacture fields. This is a big challenge since none of the digital fringe imaging based methods can offer a satisfactory 3D result of freeform transparent surfaces. Consequently, instead of directly solve the whole problem, it is more reasonable to firstly concentrate on a critical issue that how to get rid of the influence of the reflections from both front and rear interfaces.

CHAPTER 3 Advances in fringe analysis

3.1 Introduction

Fringe imaging based metrology measures the deformation of physical quantity through the distortion of fringe patterns from reference stage and deformed stage. Observations of the distorted fringe patterns provide a qualitative view of the shape changes of the object. For quantitative analysis, the fringes have to be processed according to one of many fringe analysis techniques. The growth and research into fringe analysis techniques have progressed the fringe profiling methods to a whole new level by providing robust, reliable and repeatable results. These fringe analysis techniques can also be applied to a broader field of interferometric and other similar fringe patterns.

With digital fringe imaging based method, the fringe analysis methods can be more flexible because of the elasticity of fringe generation in digital devices. In order to perfect the performance of the 3D measurement systems for different types of targeted surfaces, such as diffused, specular or transparent surfaces, fringe process and analysis methods are investigated and some advances in fringe analysis technique are presented in this chapter.

According to the frame numbers required in fringe analysis, these advances in fringe analysis are divided into two aspects, multi-frame study and single-frame study. Multi-frame study is mainly for measuring stationary surfaces with higher accuracy and precision. Single-frame study is carried out to meet the requirement for dynamic shape measurement.

Multi-frame study is focused on the static object profiling with multiple fringe patterns capture and processing. There are two issues studied in this thesis about the aspect. One is the phase error analysis and compensation for non-sinusoidal waveforms in phase shifting technique. The other topic is phase invalidity identification in multi-frequency phase unwrapping.

Dynamic surface 3D shape measurement requires the high speed data recording, for which the single-frame fringe imaging method is more suitable. Therefore, how to analyze the single fringe pattern and successfully retrieve the fringe phase becomes a critical issue. Two studies are carried out by investigating the two major steps of fringe analysis, wrapped phase retrieval and phase unwrapping, but of course especially for single fringe pattern processing. First, so far there are a few fringe demodulation approaches in literatures but without knowing which one is superior at image processing in digital fringe 3D shape measurement. A comparison of wrapped phase retrieval methods from a single fringe pattern is consequently studied to investigate the performance of the existing fringe demodulation methods. Second, since only one frame fringe pattern is used for one scene in dynamic measurement, phase unwrapping of course inevitably needs to be done in spatial domain at least once. Within the spatial phase unwrapping approaches, quality guided phase unwrapping is an excellent method with relatively high accuracy and speed with automatic processing ability. A review of quality guided phase unwrapping technique is presented by comparing the quality maps and guiding strategies.

3.2 Phase error analysis and compensation for non-sinusoidal waveforms in phase shifting technique

The nonlinear intensity response of a digital FPP system causes the captured fringe patterns to be non-sinusoidal waveforms and leads to an additional phase measurement error for commonly used three-step and four-step phase shifting algorithms. Theoretical analysis of the phase error due to the non-sinusoidal waveforms is performed. Based on the derived theoretical model, a novel and simple iterative phase compensation algorithm is proposed to compensate the non-sinusoidal phase error. Experiments show that the proposed algorithm can be used for effective phase error compensation in practical phase shifting FPP.

3.2.1 Phase errors from non-sinusoidal waveform

FPP using phase shifting technique [29, 30, 34, 35, 38, 93, 119-121] has been extensively investigated and recognized as one of the most effective techniques for practical shape measurement. Due to its easy availability, low cost and highly flexibility, a digital video projector (DVP) has been commonly adopted in FPP for projecting fringe patterns. One significant advantage of digital FPP is that phase shifting error (or called phase shifting miscalibration) can be totally avoided because the digital fringes are generated using a computer. However, the commercial DVP as well as the CCD camera used in a FPP system are generally fabricated to be nonlinear devices for a better visual effect. The nonlinear characteristic of an FPP system inevitably leads to the deviation of the captured fringe pattern from ideal sinusoidal waveforms and introduces an additional phase error [27]. This phase error can be alleviated or eliminated by using a larger number of phase shift fringe patterns [36]. However, for commonly used three-step or four-step phase shifting FPP, the non-sinusoidal phase error is considered as the dominant error source. It should be noted that the photometric calibration technique is able to calibrate the overall response curve of the FPP system and eliminate the non-sinusoidal phase error effectively, but the whole calibration procedure is time consuming and laborious. A substantial theoretical analysis which can help us understand the phase error distribution in phase shifting FPP is still lacking. A simple yet effective technique to minimize or eliminate the non-sinusoidal phase error is also not available.

3.2.2 Analysis and compensation for phase error from non-sinusoidal waveform

In phase shifting FPP, a series of ideal sinusoidal fringe patterns are generated by a computer and projected to the test object surface using DVP. The camera captures the reflected intensity from test object surface for further processing. Due to the nonlinear intensity response of an FPP system including the overall influences of the DVP, object surface reflectivity, environment brightness and the CCD camera, the captured fringe patterns will deviate from its ideal sinusoidal waveform as shown in Fig. 3.1.

Fig. 3.1. The intensity of actual fringe images is expressed as a function of ideal sinusoidal fringes.

The models used in previous works [51] only considered the nonlinear effect of the DVP. In this work, we use a nonlinear function to depict the overall nonlinearity of a FPP system, therefore the intensity of the actual fringe images $I_i^c(x, y)$ is expressed as a nonlinear function of the fringe patterns obtained from an ideal FPP system as follows

$$I_i^c(x, y) = f(I_i(x, y)), \quad (3.2.1)$$

where $I_i(x, y)$ is the i th fringe pattern of a sequence of N ideal sinusoidal fringe patterns, which can be given as

$$I_i(x, y) = I_b(x, y) + I_m(x, y) \cos(\varphi(x, y) + \delta_i), \quad (3.2.2)$$

where $I_b(x, y)$ is the background intensity, $I_m(x, y)$ is the intensity modulation, $\varphi(x, y)$ is the desired phase information, and $\delta_i = 2\pi i/N$ is the shifted phase of each of the N fringe patterns.

The desired phase information can be solved by least-squares algorithm [35],

$$\varphi(x, y) = -\arctan \left[\frac{\sum_{i=0}^{N-1} I_i^c(x, y) \sin(\delta_i)}{\sum_{i=0}^{N-1} I_i^c(x, y) \cos(\delta_i)} \right]. \quad (3.2.3)$$

In the following derivation, the actual non-sinusoidal waveforms are approximated as an ideal sinusoidal function with high order harmonics. The Fourier series expansion of Eq. (3.2.1) to the fifth order harmonic yields

$$\begin{aligned}
 I_i^c(x, y) &= f(I_i(x, y)) = a_0 + \sum_{k=1}^{\infty} a_k \cos[k(\varphi(x, y) + \delta_i)] \\
 &\approx a_0 + \sum_{k=1}^5 a_k \cos[k(\varphi(x, y) + \delta_i)]
 \end{aligned}
 \quad (3.2.4)$$

where $a_0, a_1, \dots, a_5$ are constants. It is seldom observed that harmonic waves larger than five-order in FFP system as shown in Fig. 3.2(c). However, it should also be noted that the following derivation can be extended to higher harmonics easily.

Fig. 3.2. Fringe pattern is projected onto a reference plane (a) and a test plaster model (b). (c) The plot shows frequency spectrum of the 512th column of (a).

Submitting Eq. (3.2.4) into Eq. (3.2.2), the actual phase calculated with the traditional phase shifting algorithm can be written as

$$\varphi'(x, y) = -\arctan \left[\frac{\sum_{i=0}^{N-1} \left\{ a_0 + \sum_{k=1}^5 a_k \cos[k(\varphi(x, y) + \delta_i)] \right\} \sin(\delta_i)}{\sum_{i=0}^{N-1} \left\{ a_0 + \sum_{k=1}^5 a_k \cos[k(\varphi(x, y) + \delta_i)] \right\} \cos(\delta_i)} \right]. \quad (3.2.5)$$

For three-step phase shifting algorithm (*i.e.*, $N = 3$), the computed phase is given as

$$\varphi'(x, y) = \arctan \left[\frac{a_1 \sin(\varphi(x, y)) - a_2 \sin(2\varphi(x, y)) + a_4 \sin(4\varphi(x, y)) - a_5 \sin(5\varphi(x, y))}{a_1 \cos(\varphi(x, y)) + a_2 \cos(2\varphi(x, y)) + a_4 \cos(4\varphi(x, y)) + a_5 \cos(5\varphi(x, y))} \right]. \quad (3.2.6)$$

The measured phase can be considered as a sum of the actual phase and the phase error caused by nonlinearity of the FPP system. Thus, we have

$$\varphi'(x, y) = \varphi(x, y) + \Delta\varphi(x, y). \quad (3.2.7)$$

The residual phase error can be derived as the following form

$$\begin{aligned} \Delta\varphi(x, y) &= \arctan \left[\frac{-(a_2 - a_4) \sin(3\varphi(x, y)) - a_5 \sin(6\varphi(x, y))}{a_1 + (a_2 + a_4) \cos(3\varphi(x, y)) + a_5 \cos(6\varphi(x, y))} \right], \\ &\approx -c_1 \sin(3\varphi(x, y)) - c_2 \sin(6\varphi(x, y)) \end{aligned} \quad (3.2.8)$$

with c_1, c_2 are constants. Normally, c_1 is much larger than c_2 , thus Eq. (3.2.8) can be further approximated as $\Delta\varphi(x, y) \approx -c_1 \sin(3\varphi(x, y))$.

For four-step phase-shifting method (*i.e.*, $N = 4$), the computed phase can be expressed as:

$$\varphi'(x, y) = \arctan \left[\frac{a_1 \sin(\varphi(x, y)) - a_3 \sin(3\varphi(x, y)) + a_5 \sin(5\varphi(x, y))}{a_1 \cos(\varphi(x, y)) + a_3 \cos(3\varphi(x, y)) + a_5 \cos(5\varphi(x, y))} \right]. \quad (3.2.9)$$

The non-sinusoidal phase error caused can similarly derived as,

$$\Delta\varphi(x, y) = \arctan \left[\frac{-(a_3 - a_5) \sin(4\varphi(x, y))}{a_1 + (a_3 + a_5) \cos(4\varphi(x, y))} \right] \cong -c \sin(4\varphi(x, y)), \quad (3.2.10)$$

where c is a constant.

Similarly, for five-step phase-shifting method (*i.e.*, $N = 5$), the corresponding phase error can be given by

$$\Delta\varphi(x, y) = \arctan \left[\frac{-a_4 \sin(5\varphi(x, y))}{a_1 + a_4 \cos(5\varphi(x, y))} \right] \cong -c \sin(5\varphi(x, y)), \quad (3.2.11)$$

where c is a constant. Note that assumptions of $a_2, a_3, a_4, a_5 \ll a_1$ are used in Eqs. (3.2.8), (3.2.10), and (3.2.11).

Eqs. (3.2.8), (3.2.10), and (3.2.11) indicate that the phase error due to nonlinearity of the system can be approximated as periodic sinusoidal function with its period related to the phase-shifting steps and the pitch of the fringe pattern. For three-step phase shifting method, this periodicity is in good agreement with the results presented in Zhang's work [50] and in our study.

Based on Eqs. (3.2.7), (3.2.8), (3.2.10), and (3.2.11), we propose a simple iterative algorithm to reduce the phase error due to non-sinusoidal waveforms. For the four-step phase shifting method, the iteration algorithm can be written as

$$\varphi^{k+1}(x, y) = \varphi'(x, y) + c \sin(4\varphi^k(x, y)), \quad (3.2.12)$$

where φ^{k+1}, φ^k are the solutions of $(k+1)^{\text{th}}$ and k^{th} round iteration, c denotes the amplitude of the phase error, which can be estimated from the phase error distribution of the reference plane (see Fig. 3.3(b)). The initial guess of the iteration can be used as $\varphi^0(x, y) = \varphi'(x, y)$. The convergence condition is set as $\max |\varphi^{k+1}(x, y) - \varphi^k(x, y)| < 0.001$ rad. The algorithm converges after 3 to 5 iterations in all the following calculations. The above iterative algorithm can also be applied to the three-step and five-step phase shifting algorithms for phase error reduction.

3.2.3 Experiment and discussion

To verify the correctness of the theoretical derivation and the effectiveness of the compensation algorithm, three-step, four-step and five-step phase shifting experiments have been conducted. All experimental results agree well with theoretical predictions. Here, we demonstrate the four-step phase shifting experiments. In experiment, the computer generated ideal sinusoidal fringes with a pitch of 64 pixels were projected using a commercial projector (NEC, Model 1050). Fig. 3.2(a) and (b) show the typical fringe patterns projected onto the reference plane and the test plaster, respectively. Fig. 3.2(c) plots the Fourier transform of the 512th column of the fringe, in which harmonic to five orders can be clearly viewed.

Fig. 3.3. Non-sinusoidal waveforms lead to periodic phase errors. (a) Phase error image, (b) phase error for 101 column points, the red line shows the average phase error of the 101 column points.

The phase distribution of the reference plane is computed using Eq. (3.2.3). After subtracting the real linear phase (carrier frequency) for each pixel, the residual sinusoidal phase error distribution is shown in Fig. 3.3(a). The phase errors for the 101st column points (arbitrarily selected) within a fringe pitch located in the central area of the phase error map is plotted in Fig. 3.3(b). It is observed that the phase error distribution agrees well with the prediction of Eq. (3.2.10), or say the period of phase error is 1/4 times of that of the recorded fringe pattern. The amplitude of the phase error is estimated to be 0.08 radians from Fig. 3.3(b).

Fig. 3.4. Phase error reduces after compensation. (a) Compensated phase error, and (b) compensated phase error for 101 column points, the red line shows the average phase error of the 101 column points.

Using Eq. (3.2.12), the compensated phase error map and the phase errors for the same 101st column points are shown in Fig. 3.4. The average phase errors for the 101st column points before and after compensation are plotted as the red lines in Fig. 3.3(b) and Fig. 3.43H (b). The phase error is about 8 times smaller after compensation, with the maximum phase error decreases significantly from 0.08 to 0.01 radians. The results clearly prove the correctness of the theoretical model and the validity of the compensation algorithm.

Fig. 3.5. Phase distributions of a sculpture are shown: (a) before compensation and (b) after compensation (unit in radian).

As a practical application of the proposed phase error compensation technique, we measured a complex plaster model. Fig. 3.5(a) gives the phase distributions of the plaster model by directly subtraction of the reference plane from the phase of object. Fig. 3.5(b) shows the phase

distributions of the same subtraction procedure after phase error compensation applied to both the reference plane and the object. It is evident that the phase error compensation technique can effectively remove the periodical phase error due to the non-linearity intensity response characteristic of the FPP system and obtain a more accurate result with better visual effect.

3.3 A practical phase invalidity identification framework with temporal phase unwrapping approach

Nowadays, temporal phase unwrapping techniques – more specifically multi-wavelength or multi-frequency phase unwrapping techniques [38, 84, 121] – are very popular in dealing phase unwrapping issues especially for stationary measurement. Temporal phase unwrapping techniques not only solve the modulo 2π problem, but also provide absolute phase maps, which are preferred in FPP. Because of their pixel-independent unwrapping procedure and common phase reference for each unwrapped phase map, targets with large discontinuities can be easily measured.

Recently some phase unwrapping issues at discontinuous boundaries have been observed [84, 85]. Zhang [84] provides a solution that retrieves phase step integer numbers from the smoothed fringe functions, and removes “bad” phase points by comparing two neighboring smoothed phase values. A spatial Gaussian filter will, however, not only reduce phase noise but also smooth step height changes and details. Recently, Chen et al [85] have analyzed the phase calculation problem at the discontinuity boundaries, and proposed a method to identify invalid points by setting a threshold to the least squares fitting errors in temporal phase unwrapping. This method is easy to use and quite effective. Most of error points can be identified and removed. However, in the presence of noise, there may still be a few ‘bad’ points with relatively small least squares fitting errors.

Here, we present a practical phase invalidity identification framework with temporal phase unwrapping, which identifies and removes or fills the unavailable measuring points at the phase

retrieval stage. Consequently, more reliable 3D results with better visualization can be reconstructed. Experimental study shows the feasibility and robustness of our frame work.

3.3.1 Principle

The phase shifting method along with temporal phase unwrapping as shown in Fig. 3.6, has been adopted as the state-of-the-art strategy for stationary target profiling. It is well known that the phase shifting method is essentially a least squares method to fit measured phase-shifted values with a cosine function.

Fig. 3.6. A strategy of phase measurement is proposed with phase shifting plus temporal phase unwrapping method.

Wrapped phase, ϕ^w and intensity modulation, M can be calculated using the phase shifting equations, Eq. (3.3.1) and Eq. (3.3.2), where n is the phase shifting number, N is the total number of phase shifting steps, and I_n is the n th phase-shifted fringe pattern. Usually, the modulation M is selected as the reliability map R . Sometimes, the reliability map R can be formed by combining two or more quantities to optimize the judgment [72].

$$\phi^w = -\arctan \frac{\sum_{n=0}^{N-1} I_n \cdot \sin \frac{2n\pi}{N}}{\sum_{n=0}^{N-1} I_n \cdot \cos \frac{2n\pi}{N}}. \quad (3.3.1)$$

$$M = \frac{2}{N} \sqrt{\left[\sum_{n=0}^{N-1} I_n \cdot \sin \frac{2n\pi}{N} \right]^2 + \left[\sum_{n=0}^{N-1} I_n \cdot \cos \frac{2n\pi}{N} \right]^2}. \quad (3.3.2)$$

A least squares fitting of the unwrapped phase with different fringe frequencies from temporal phase unwrapping will improve the phase measuring precision [122], and at the same time it can also provide Root Mean Square Error (RMSE) as a judging quantity for identification of invalid phase points.

The Eq. (3.3.3) gives a common temporal phase unwrapping procedure [38],

$$\phi_k^u = \begin{cases} \phi_k^w, & k = 1 \\ \phi_k^w + \text{Round} \left(\frac{\phi_{k-1}^u \cdot m_k / m_{k-1} - \phi_k^w}{2\pi} \right) \cdot 2\pi, & k > 1 \end{cases}, \quad (3.3.3)$$

and RMSE can be calculated using Eq.(3.3.4),

$$RMSE = \sqrt{\frac{\sum_{k=1}^K (\phi_k^u - m_k \cdot X)^2}{K}} = \sqrt{\frac{\sum_{k=1}^K \left(\phi_k^u - m_k \cdot \frac{\sum_{k=1}^K \phi_k^u m_k}{\sum_{k=1}^K m_k^2} \right)^2}{K}}, \quad (3.3.4)$$

where the slope X in the relationship of $\phi_k^u = f(m_k) = X \cdot m_k$ can be determined as

$$X = \frac{\sum_{k=1}^K \phi_k^u m_k}{\sum_{k=1}^K m_k^2}, \quad (3.3.5)$$

while minimizing the least squares error

$$S = \sum_{k=1}^K (X \cdot m_k - \phi_k^u)^2 \rightarrow \min. \quad (3.3.6)$$

where ϕ_k^w and ϕ_k^u indicate wrapped and unwrapped phase of the k^{th} frequency fringe patterns, function Round() means round the input to its nearest integer, m_k is the fringe number of the k^{th} frequency fringe patterns, and K is the total frequency number.

The phase relationship between neighboring pixels is considered next. Suppose the phase values increase along the horizontal direction. Considering the phase noise, ‘good’ points should meet the criterion shown in Eq.(3.3.7), or it can be marked as inharmonic pixel and so identified.

$$\begin{aligned}
 &-\frac{\pi}{2} < \phi^u(x, y) - \phi^u(x-1, y) < 2\pi \\
 &\text{or} \\
 &-\frac{\pi}{2} < \phi^u(x+1, y) - \phi^u(x, y) < 2\pi
 \end{aligned} \tag{3.3.7}$$

3.3.2 Method: phase retrieval and phase error identification framework

Since data from fringe patterns may not always reliably reflect height information of the target surface, *e.g.* in regions of shadows, dark color, and step height boundaries, the task of phase retrieval is not only to calculate the phase values from those fringe patterns, but also to identify invalid pixels on the retrieved absolute phase map. The framework can thus be composed of the following stages:

- (I). Calculate the absolute phase map from fringe patterns by using phase shifting method and temporal phase unwrapping method.
- (II). Calculate intensity modulation M or other parameter maps from the set of fringe patterns with the highest frequency. Define a proper reliability map R . Set a threshold, ‘ thr_{ps} ’ to identify the invalid and unreliable points in phase-shifting procedure.
- (III). Calculate the least squares RMSE in temporal phase unwrapping $RMSE$. Identify the invalid points from phase unwrapping procedure by setting a threshold ‘ thr_{tpu} ’.
- (IV). Compare phase relationship between neighboring pixels. Mark those points, which do not satisfy Eq.(3.3.7), as invalid phase points.

For easy understanding, a flowchart for this framework is drawn in Fig. 3.7, which visually indicates the main steps of the proposed framework.

Fig. 3.7. Flowchart of phase invalidity identification framework shows the major procedures to retrieve the valid phase values.

3.3.3 Experiment

As a demonstration, a compact 3D profilometer shown in Fig. 3.8 is used to profile an optical mouse along with some computer cables.

Fig. 3.8. A prototype of the compact 3D profilometer is set up to demonstrate the proposed framework. (a) Schematic diagram and (b) photograph of the compact 3D profilometer system.

The total fringe numbers, m_k that are projected are 1, 4, 16, and 64. Some typical fringe patterns and their wrapped phase maps as well as the absolute phase map are shown in Fig. 3.9.

Fig. 3.9. Experiments are carried out to test the validation of the proposed strategy. (a) Fringe patterns with different number of fringes, (b) wrapped phase maps corresponding to the fringe patterns in (a) and (c) unwrapped phase map without identification of invalid pixels.

The intensity modulation, M of fringe patterns with $m = 64$ is selected as the reliability map R as shown in Fig. 3.10(a). The identified invalid pixels by modulation are shown (in color of red) in Fig. 3.10(b) with $thr_ps = 2$. It shows the three-dimensional result after reliability identification step in Fig. 3.10(c).

Fig. 3.10. Identification of invalid measuring points follows the flowchart of the proposed method. (a) Modulation map, (b) unreliable pixels from reliability identification step, (c) 3D result after reliability identification step, (d) least squares RMSE map in temporal phase unwrapping, (e) invalid pixels from RMSE identification step. (f) 3D result after RMSE identification step, (g) absolute phase map after RMSE identification step, (h) inharmonic pixels by checking neighboring phase relationship, and (i) final 3D result after the entire framework.

The following step is to identify the invalid step height boundary points by using least squares RMSE in temporal phase unwrapping. Images of Fig. 3.10(d), (e), and (f) show the RMSE map, newly identified invalid points by RMSE (in color of red) with $thr_tpu = 0.25$, and the three-dimensional result after the RMSE step. Finally, based on the absolute phase result after

the RMSE step shown in Fig. 3.10(g), the inharmonic phase points are indentified by checking neighboring pixel phase relationship as shown in Fig. 3.10(h) (in color of red and highlighted with red circles). It shows the final three-dimensional result in Fig. 3.10(i) after the whole framework.

3.3.4 Discussion

The proposed phase retrieval and invalidity identification framework is suitable for stationary target profile measurement with the following advantages.

- (I). It identifies invalid pixels for further processing (removal or interpolation depending on different applications), while valid points are maintained.
- (II). No smoothing or filtering process is involved.
- (III). The calculation process is simple.

Here, an invalid phase point identification framework for temporal phase unwrapping is proposed. With three invalidity identifying steps after phase calculation, invalid pixels can be successfully identified without losing too much valid information or smoothing raw phase values. The experimental results indicate the feasibility of the proposed framework. With phase invalidity identification, reliable three-dimensional dataset with better visualization can be reconstructed as a result.

3.4 Comparison of methods for wrapped phase retrieval from single fringe pattern

Phase extraction methods from a single fringe pattern using different transform methods are compared using both simulations and experiments. The principles of Fourier transform, windowed Fourier transform and wavelet transform methods for fringe pattern processing are introduced. Implementation of 1D and 2D algorithms and phase compensation are discussed. Noisy and non-sinusoidal waveforms are involved into this comparison. The merits and limitations of each of these processing methods are indicated.

The whole-field capability coupled with high speed and high accuracy of the fringe projection methods has resulted in varied research and industrial applications. Phase retrieval is generally a crucial process in FPP. The well-known PMP uses multiple fringe patterns along with the phase-shifting algorithm to obtain 3D profiles with high accuracy [35, 36]. However, for dynamic objects measurement, a series of single projected pattern is processed using transform methods, such as the FT method [29-31, 61-65, 123], WFT method [32, 67-69], and WT method [33, 34, 43, 71].

In this section, the FT, WFT, and WT are compared for different fringe patterns characteristics. Both the 1D FT [29] and 2D FT [61] methods are tested, so are the 1D WT and 2D WT algorithms. For WFT, the Windowed Fourier Filtering (WFF) and Windowed Fourier Ridges (WFR), in both 1D and 2D are examined. Since the phase extraction from one fringe pattern is usually influenced by noise and non-sinusoidal waveform, the comparisons of the different algorithms under noisy condition or non-sinusoidal waveform condition are compared.

3.4.1 Principles of these transform based methods in fringe pattern processing

At first, transform-based methods for fringe pattern analysis are introduced as a background and preparation for the comparison and discussion.

A fringe pattern, $f(x, y)$ captured by a CCD camera can be expressed as:

$$f(x, y) = A(x, y) + B(x, y) \cos[\varphi(x, y)], \quad (3.4.1)$$

where $A(x, y)$, $B(x, y)$, and $\varphi(x, y)$ are respectively the background intensity, the fringe modulation and the phase to be measured at the location of (x, y) in a fringe pattern.

The Fourier Transform (FT) method

The principle of FT is to extract the fundamental frequency component of the fringe pattern in the 1D or 2D frequency domain. An inverse transform of the filtered frequency domain signal

then provides the modulo 2π phase of the fringe pattern. For example for a 2D fringe pattern as described by Eq. (3.4.1), after the Fourier transform, filtering in the frequency domain, and inverse Fourier transform, the filtered exponential fringe pattern $\overline{f}(x, y)$ can be obtained as

$$\overline{f}(x, y) = \mathbf{F}^{-1} \left\{ \overline{\mathbf{F} \{ f(x, y) \}} \right\} = a(x, y) \exp(j \times \varphi(x, y)), \quad (3.4.2)$$

Where functions of $\mathbf{F}$ and $\mathbf{F}^{-1}$ represent forward and inverse Fourier transform, respectively. The over-bar denotes the filtering process, after which the fundamental component can be extracted, and $a(x, y)$ is the amplitude of the filtered exponential fringe pattern. From the exponential fringe pattern $\overline{f}(x, y)$, the modulo 2π phase $\varphi(x, y)$ can be retrieved as

$$\varphi(x, y) = \tan^{-1} \left[\frac{\text{Im } \overline{f}(x, y)}{\text{Re } \overline{f}(x, y)} \right], \quad (3.4.3)$$

The Windowed Fourier Transform (WFT) method

The 2D WFT and 2D inverse WFT can be expressed as [67]

$$Sf(u, v, \xi, \eta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) g(x - u, y - v) \exp(-j\xi x - j\eta y) dx dy, \quad (3.4.4)$$

$$f(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Sf(u, v, \xi, \eta) g(x - u, y - v) \exp(j\xi x + j\eta y) d\xi d\eta du dv. \quad (3.4.5)$$

where the imaginary unit $j = \sqrt{-1}$ and the window function $g(x, y)$ is a normalized Gaussian function given as

$$g(x, y) = \frac{1}{\sqrt{\pi\sigma_x\sigma_y}} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right), \quad (3.4.6)$$

where σ_x and σ_y are the standard deviations of the Gaussian function in x and y directions. In the Windowed Fourier Filtering (WFF) algorithm, the phase φ and filtered fringe pattern can be expressed as

$$\varphi(x, y) = \tan^{-1} \left[\frac{\text{Im} \bar{f}(x, y)}{\text{Re} \bar{f}(x, y)} \right], \quad (3.4.7)$$

$$\bar{f}(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{Sf}(u, v, \xi, \eta) g(x-u, y-v) \exp(j\xi x + j\eta y) d\xi d\eta du dv. \quad (3.4.8)$$

The fundamental frequency component is extracted from the filtered spectrum by adjusting ξ and η

$$\bar{Sf}(u, v, \xi, \eta) = \begin{cases} Sf(u, v, \xi, \eta) & \text{if } |Sf(u, v, \xi, \eta)| \geq thr, \\ 0 & \text{if } |Sf(u, v, \xi, \eta)| \leq thr. \end{cases} \quad (3.4.9)$$

where thr is a preset threshold to suppress noise.

For the Windowed Fourier Ridges (WFR) algorithm, local frequencies ω_x and ω_y , and phase φ can be expressed as

$$[\omega_x(u, v), \omega_y(u, v)] = \arg \max_{\xi, \eta} |Sf(u, v, \xi, \eta)|, \quad (3.4.10)$$

$$\varphi(u, v) = \tan^{-1} \left\{ \frac{\text{Im} Sf[u, v, \omega_x(u, v), \omega_y(u, v)]}{\text{Re} Sf[u, v, \omega_x(u, v), \omega_y(u, v)]} \right\} + \omega_x(u, v)u + \omega_y(u, v)v. \quad (3.4.11)$$

The Wavelet Transform (WT) method

As a non-constant signal processing method, WT method has been introduced into many signal processing fields, including phase retrieval from fringe pattern. In WT, the window size is changed according to the frequency. For 1D WT, the phase φ can be retrieved as [34]

$$\varphi(b) = \tan^{-1} \left[\frac{\text{Im} Wf(a_{r,b}, b)}{\text{Re} Wf(a_{r,b}, b)} \right], \quad (3.4.12)$$

where $a_{r,b}$ is the value of scale dilation parameter a relating to the ridge point at position b ; 1D wavelet transform of $f(x)$, $Wf(a, b)$, and 1D Morlet wavelet $\psi(x)$ can be expressed as [34]

$$Wf(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(x) \psi^* \left(\frac{x-b}{a} \right) dx, \quad (3.4.13)$$

$$\psi(x) = \frac{1}{\pi^{1/4}} \left(\frac{2\pi}{\gamma} \right)^{\frac{1}{2}} \exp \left[-\frac{(2\pi/\gamma)^2 x^2}{2} + j2\pi x \right], \text{ with } \gamma = \pi \sqrt{2/\ln 2}. \quad (3.4.14)$$

Furthermore, for 2D WT the phase φ can be calculated by [34]

$$\varphi(\mathbf{b}) = \tan^{-1} \left[\frac{\text{Im} Wf(\mathbf{b}, a_{r,\mathbf{b}}, \theta_{r,\mathbf{b}})}{\text{Re} Wf(\mathbf{b}, a_{r,\mathbf{b}}, \theta_{r,\mathbf{b}}) - f_0(\pi/m) \exp(-\pi^2/m)} \right], \quad (3.4.15)$$

where $a > 0$ is a scale dilation parameter corresponding to the width of the wavelet, θ is a rotation angle, $a_{r,\mathbf{b}}$ and $\theta_{r,\mathbf{b}}$ are the values of a and θ relating to the ridge point of Wf at position $\mathbf{b}$ while $\mathbf{b}$ is a 2D translation parameter corresponding to the position of the wavelet. The wavelet transform of $f(\mathbf{x})$ and the modified wavelet can be expressed as

$$Wf(\mathbf{b}, a, \theta) = a^{-n} \int_{-\infty}^{\infty} f(\mathbf{x}) \cdot \psi^* \left[a^{-1} \mathbf{r}_{-\theta}(\mathbf{x} - \mathbf{b}) \right] d^2 \mathbf{x}, \quad (3.4.16)$$

$$\psi(\mathbf{x}) = \exp \left(-m |\mathbf{x}|^2 + j2\pi x \right), \quad (3.4.17)$$

where $\mathbf{x} = (x, y)$ comprises the 2D coordinates of the individual pixel in the fringe pattern; $\mathbf{r}_{-\theta}$ is the conventional 2×2 rotation operator matrix and m is a window-modifying parameter.

3.4.2 Annotations of algorithms and phase compensation

Considering the flexibility of these methods in practical applications, some annotations for the implementation of the different algorithms and phase compensation of ridge-based methods are given to enhance the reproducibility of the results in this section.

Annotations for implementations of algorithms:

- The implementation of 2D FT is conventional [61] and 2D Hanning window is chosen as the frequency response of the band pass filter [123, 124]. Four algorithms of WFT (1D WFF, 1D WFR, 2D WFF, 2D WFR) are implemented according to the Kemao's method [32, 67] and the window size is set as $(6\sigma_x + 1) \times (6\sigma_y + 1)$ to achieve an unbiased estimation for phase distribution [69]. The 1D WT implemented here is the same as in reference [33, 71, 77] and uses a 1D complex Morlet wavelet, while the 2D WT is implemented according to approach proposed by Wang [34] with the parameter $m=2$.
- The DC component in a fringe pattern is suppressed by simply subtracting the average intensity of the fringe pattern to reduce the influence of the DC component.
- Reliability-guided phase unwrapping algorithm is used in the experimental comparison in following section. The reliability for WFF is the amplitude of the filtered fringe pattern $\bar{f}(x, y)$ in Eq.(3.4.8), while those for WFR and WT are the calculated ridge maps.
- The pixels at image boundaries are not involved in our comparison because of the boundary effect of these methods. The tested objects are usually suggested to be settled at the center of the image in practical measurements.

Phase compensation in ridge-based algorithms:

In these eight algorithms, there are four ridge-based algorithms: 1D WFR, 1D WT, 2D WFR, 2D WT. It should be noted that the extracted phase by using these ridge-based algorithms needs

phase compensation to reduce the theoretical phase error [69]. The phase results of WFR should be compensated by adding a compensatory phase φ_c , and φ_c can be expressed as

$$\varphi_c = -\frac{1}{2} \arctan\left(\sigma_x^2 \frac{\partial^2 \varphi}{\partial x^2}\right) - \frac{1}{2} \arctan\left(\sigma_y^2 \frac{\partial^2 \varphi}{\partial y^2}\right), \quad (3.4.18)$$

In the implementation of phase compensation for WFR, the same Gaussian window used in WFR in Eq. (3.4.6) is suggested to smooth the extracted local frequencies by spatial convolution, and then the second partial derivatives of the phase with respect to x and y can be easily calculated.

The phase compensation in WT uses a Gaussian window with an average size of the windows used in WT to smooth the extracted phase for 1D WT or local frequencies for 2D WT, respectively. Then central difference is applied twice to extracted phase for 1D WT and once to local frequencies for 2D WT to get the second partial derivatives of the phase. Finally, the compensatory phase φ_c in WT can be calculated by Eq. (3.4.18).

Fig. 3.11. Phase compensation is necessary for ridge-based algorithms. (a) Phase error by 1D WFR before compensation, (b) phase error by 1D WFR after compensation, (c) phase error by 1D WT before compensation, (d) phase error by 1D WT after compensation, (e) phase error by 2D WFR before compensation, (f) phase error by 2D WFR after compensation, (g) phase error by 2D WT before compensation, and (h) phase error by 2D WT after compensation.

Simulation examples are shown in Fig. 3.11 to indicate the need for phase compensation in ridge-based algorithms. The true phase in the simulation is given as $3.5 \times \text{peaks}(x, y)$ shown in Fig. 3.12. The *peaks* is a built-in MATLAB[®] function expressed as

$$\begin{aligned} \text{peaks}(x, y) = & 3 \times (1 - x)^2 \times \exp(-(x^2) - (y + 1)^2) \\ & - 10 \times (x / 5 - x^3 - y^5) \times \exp(-x^2 - y^2) - 1 / 3 \times \exp(-(x + 1)^2 - y^2) \end{aligned} \quad (3.4.19)$$

Fig. 3.12. True phase and deformed fringe pattern are used in our simulation: (a) Phase using in simulation: $3.5 \times \text{peaks}(x, y)$, and (b) simulated deformed fringe patterns.

The size of fringe patterns is 256 pixels $\times$ 256 pixels. This true phase will be used again in the next section for the simulation comparison. Throughout the section, all results from the ridge-based algorithms have been compensated.

3.4.3 Comparison by simulation

In this section, effect of noise and non-sinusoidal waveform will be simulated to compare the performances of the eight algorithms.

Noise

Noise in fringe patterns is unavoidable in practical measurements. In order to compare the sensitivity to noise of these eight algorithms, normally distributed noise with a mean value of

zero and different standard deviations (STD) is added onto fringe patterns. The reference and deformed fringe patterns can be expressed as

$$I_{ref}(x, y) = 255 \left[0.5 + 0.3 \cos \left(\frac{2\pi x}{p} \right) \right] + n(x, y), \quad (3.4.20)$$

$$I_{def}(x, y) = 255 \left\{ 0.5 + 0.3 \cos \left[\frac{2\pi x}{p} + 3.5 \times peaks(x, y) \right] \right\} + n(x, y), \quad (3.4.21)$$

where the standard deviation of $n(x, y)$ ranges from 0 to 10 with an interval of 1. The noisy fringe pattern with standard deviation of 3 and 10 are shown in Fig. 3.13(a) and (b), respectively. We also include four-step phase shifting (4-step PS) technique as a reference. The results are shown in Fig. 3.13(c). The comparison results in Fig. 3.13(c) indicate that the four-step phase-shifting method is sensitive to noise for it is a pixel-wise fringe pattern processing method, and all the 1D local methods (1D WFF, 1D WFR, and 1D WT) and the global methods (1D and 2D FT) are more sensitive to noise than 2D local methods (2D WFF, 2D WFR, and 2D WT). If parameters are appropriately selected, 2D WFF, 2D WFR and 2D WT can retrieve the phase from a more noisy fringe pattern with relatively small phase error. This is because the 1D algorithms calculate the phase distribution line by line and are thus more vulnerable to noise than 2D methods. The 2D FT extracts the fundamental component with a suitable window in frequency domain. Being a global method, the fundamental component usually has wide frequency band. Noise in this frequency band cannot be suppressed in the 2D FT method, which ultimately affects the phase precision. Therefore 2D FT could also be sensitive to noise. In addition, the 2D local algorithms retrieve the phase information from 2D local data with different noise reduction strategies: for a point in the spatial domain, the 2D WFF uses a threshold for the local spectrum to differentiate the noise from the signal, while 2D WFR and 2D WT retrieve the phase from the most reliable point (*i.e.*, the ridge) of the local spectrum. Thus the 2D local algorithms are usually more tolerant to noise than 1D local algorithms.

Fig. 3.13. Comparison is carried out under noisy condition. (a) Fringe pattern with noise of STD = 3, (b) fringe pattern with noise of STD = 10, and (c) standard deviation of phase error vs. standard deviation of additive noise.

Noise in practical structured-light-based 3D optical profilometric fringe patterns is usually not severe. Considering the speed, the 1D local algorithms are much faster than the 2D local algorithms. Also the 2D FT, which involves one forward and one inverse Fourier transforms, is much faster than the 2D local algorithms. Thus, the 2D local algorithms are not suitable for real-time processing, which is their major limitation and hence 2D FT, 1D methods, and phase-shifting method are still widely applied in practice.

Non-sinusoidal waveform condition

Digital light projectors have been widely used in optical profilometry nowadays. Non-sinusoidal waveform may be due to the gamma parameter [27] of the digital projector or the non-linear response function of the whole system. In addition, for transform-based methods, the DC component and the second harmonic hinder the extraction of the fundamental frequency component in the frequency domain and are thus considered in this simulation. The reference and deformed fringe patterns are hence simulated as

$$I_{ref}(x, y) = 255 \left[a_0 + a_1 \cos\left(\frac{2\pi x}{p}\right) + a_2 \cos\left[2 \cdot \left(\frac{2\pi x}{p}\right)\right] \right], \quad (3.4.22)$$

$$I_{def}(x, y) = 255 \left\{ a_0 + a_1 \cos\left(\frac{2\pi x}{p} + 3.5 \times peaks(x, y)\right) + a_2 \cos\left[2 \cdot \left(\frac{2\pi x}{p} + 3.5 \times peaks(x, y)\right)\right] \right\}, \quad (3.4.23)$$

where $a_0 = 0.5$, $a_1 = 0.4$, and the ratio of a_2/a_1 changes from 0 to 25% with a step size of 5%.

Fig. 3.14. Comparison is carried out when the ratio of a_2/a_1 is varying.

From the simulation results in Fig. 3.4, it can be seen that the ridge-based methods (1D WFR, 1D WT, 2D WFR and 2D WT) are less sensitive to the higher harmonics than methods based on forward and inverse transforms (1D FT, 1D WFF, 2D FT and 2D WFF). In the forward and inverse transforms methods, higher harmonics extend into the fundamental frequency component, thus affecting phase retrieval. In the ridge-based methods, the ridge will not be

severely affected by the higher harmonics since the ridge has the highest amplitude of the spectrum and is usually quite far away from the harmonics. Therefore, these ridge-based methods are less sensitive to the higher harmonic components.

3.4.4 Experimental comparison

The results of a scanning mechanical stylus could possibly be considered as the standard, although the mechanical stylus directly gives the height rather than phase. Therefore errors from the phase-height mapping and the calibration of camera will be involved. We propose an approach for constructing a phase standard for continuous phase distributions.

It is well known that in the phase-shifting technique, the phase error due to non-sinusoidal waveforms can be reduced by using a large number of phase-shifting steps, and in addition, the noise can also be partially suppressed [36]. Furthermore, the 2D WFF has been shown theoretically and experimentally to be very effective in removing noise in exponential fringe patterns with continuous phase maps with a SNR gain of around 20 [69]. It is thus reasonable to construct an experimental phase standard using the N -step phase shifting approach (N should be large enough to reduce the error from non-sinusoidal waveform), followed by the 2D WFF for denoising. To verify this ‘ N -step PS + 2D WFF’ approach, a pair of fringe patterns are simulated as shown in Fig. 3.15(b) with the phase, Fig. 3.15(a), similar to the experimental phase distribution in respect of the fringe period, noise level, and second harmonic wave. In simulation, the true phase $\varphi_t(x, y)$ over 256 pixels $\times$ 256 pixels is set as

$$\varphi_t(x, y) = -(x - 30)^2 / 1000. \quad (3.4.24)$$

The fringe patterns can be written as

$$I_{ref}(x, y) = 255 \left[a_0 + a_1 \cos\left(\frac{2\pi x}{p}\right) + a_2 \cos\left[2 \cdot \left(\frac{2\pi x}{p}\right)\right] \right] + n(x, y), \quad (3.4.25)$$

Fig. 3.15. Simulated true phase and phase errors are investigated for the usage in comparison as a benchmark. (a) True phase in simulation, (b) simulated reference and deformed fringe patterns, (c) phase distribution of the solid red line in (b) by using ‘N-step Phase Shifting + 2D WFF’ approach, (d) phase error by using phase shifting, and (e) phase error after 2D WFF.

$$I_{def}(x, y) = 255 \left\{ a_0 + a_1 \cos \left(\frac{2\pi x}{p} + \varphi_t(x, y) \right) + a_2 \cos \left[2 \cdot \left(\frac{2\pi x}{p} + \varphi_t(x, y) \right) \right] \right\} + n(x, y), \quad (3.4.26)$$

where $a_0 = 0.5$, $a_1 = 0.3$, $a_2 = 0.045$, $p = 8$ and the standard deviation of noise $n(x, y)$ is 2. A phase $\varphi_{ps}(x, y)$ can be extracted from N phase shifted reference and deformed fringe patterns by using conventional N -step phase shifting algorithm [35], and then converted to an exponential fringe pattern $f_{exp}(x, y)$ as

$$\varphi_a^u = \varphi_a^w + 2\pi \times \text{round} \left(\frac{\varphi_b^u - \varphi_a^w}{2\pi} \right). \quad (3.4.27)$$

Since in this simulation only the second harmonic is added in the fringe patterns, $N = 4$ phase shifts is sufficient [36]. The phase along the solid red line in Fig. 3.15(b) are shown in Fig. 3.15(c) to indicate efficiency of ‘ N -step PS + 2D WFF’ approach. Phase errors by using only the phase shifting technique and the ‘ N -step PS + 2D WFF’ approach are shown in Fig. 3.15(d) and (e) respectively, indicating that the 2D WFF performs well in filtering exponential fringe patterns. Furthermore, phase errors from the different algorithms are compared to either the true phase or the result of phase standard ‘ N -step PS + 2D WFF’. The results in Fig. 3.16 show the mean and $3 \times \text{Standard Deviation}$ of phase error for these algorithms.

Fig. 3.16. Mean values and $3 \times \text{STD}$ values of phase errors are calculated by using the true phase or the result of ‘ N -step phase shifting + 2D WFF’ as the phase benchmark.

Form Fig. 3.16, it is seen that for each algorithm, the error means based on these two different standards are quite close, so are the standard deviations of the errors. It suggests that the phase results from ‘ N -step PS + 2D WFF’ approach can be used as a phase standard for our experimental comparison.

Fig. 3.17. Experiment is carried out and the benchmark can be obtained through noise reduction. (a) Reference and deformed fringe patterns under two sorts of conditions, (b) raw phase data from 16-step phase shifting method, (c) phase result after noise reduction by using 2D WFF, and (d) reduced noise in central area.

For the experimental demonstration, a model is first analyzed using the 16-step phase shifting method (*i.e.*, $N = 16$). The reference and deformed fringe patterns in two experiments are shown in Fig. 3.17(a). Due to the boundary effect, only the central part, as shown inside the blue solid rectangle in Fig. 3.17(b), is processed. The phase calculated by a 16-step phase shifting algorithm and further denoised by the 2D WFF are shown in Fig. 3.17(b) and (c), respectively,

and the denoised phase is considered as the phase standard for experimental comparison. The reduced noise in central area is shown in Fig. 3.17(d).

In the first experiment, there are no observable higher harmonics in the fringe patterns, as shown in the left of Fig. 3.17(a), while in the second experiment, some harmonics, especially a strong second harmonic is seen (right of Fig. 3.17(a)). The phase errors of these algorithms under both conditions are shown in Fig. 3.18. The results in Fig. 3.19 show the retrieved phase by the 2D WFF under both conditions and their corresponding phase errors. For fringe patterns with no higher order harmonics, there is no overlap in the spectrum and the WFF performs well for phase retrieval. However, the non-sinusoidal waveform situation, there are obvious overlaps. Consequently the WFF results would be affected by the higher harmonics.

Fig. 3.18. Mean values and $3 \times \text{STD}$ values of phase errors are calculated under conditions of fringe pattern with no observable harmonics and with some harmonics.

The experimental results agree with the results from the simulation. First, 2D local methods are generally more tolerant to noise than those 1D methods. Second, local methods are usually less sensitive to noise than global methods. Third, ridge-based methods are more tolerant to higher order harmonics as seen with a non-sinusoidal waveform than methods based on forward and

inverse transforms. Finally, 2D WFR and 2D WT seem to be the most robust methods in these eight algorithms under continuous phase condition, but they are more time consuming than the other six algorithms. In addition, the essence of 2D WFR and 2D WT is almost the same. The only main difference between them is that 2D WFR uses a fixed window size while 2D WT changes the window size according to the frequencies. Nevertheless, when processing the fringe patterns with carrier fringe, the window size in the 2D WT usually changes within a certain range which generally covers the selected window size in the 2D WFR. Consequently the WFR and WT give very similar results.

Fig. 3.19. Phase and phase error are calculated with 2D WFF method under conditions of fringe pattern with no observable harmonics and with some harmonics. (a) Phase by 2D WFF with no observable harmonics, (b) phase error by 2D WFF with no observable harmonics, (c) phase by 2D WFF with some harmonics, end (d) phase error by 2D WFF with some harmonics.

3.4.5 Discussion

In FPP, several one-frame fringe pattern processing methods are evaluated and compared in this study. Effects of noise and the higher harmonics in a non-sinusoidal waveform are considered in the comparison using both simulation and actual experiments. Among the transform based methods, the 2D WFR and 2D WT seem to be the most robust algorithms, but they are time-consuming and not suitable for real-time processing. All transform-based algorithms perform poorly around discontinuities in a phase map, which needs to be further investigated. In summary, algorithms should be chosen properly according to the requirement of an experiment to foster strengths and circumvent weaknesses of these algorithms.

3.5 Quality-guided phase unwrapping technique: a comparison of the quality maps and the guiding strategies

Quality-guided phase unwrapping (QGPU) is a widely used technique [72, 125, 126]. Various quality definitions and guiding strategies are possible. A detailed comparison is necessary to select a good quality map and a good guiding strategy. For quality maps, under the noisy condition, transform-based methods perform satisfactory, while under discontinuity condition, phase unwrapping is still a challenging problem and so far cannot be solved by QGPU alone. For guiding strategies, classical, two-section, and stack-chain guiding strategies are compared with showing respective advantages and disadvantages. To achieve the most reasonable unwrapping results, the classical guiding strategy with a data structure of indexed interwoven linked list is fastest. If a small unwrapping error is tolerated, stack-chain guiding strategy can perform even faster.

As an effective, fast and automatic spatial phase unwrapping method, QGPU technique has been well studied in last two decades [68, 72, 77, 126-134]. As shown in Fig. 3.20, with a quality-guiding procedure, the wrapped phase (Fig. 3.20(a)) can be unwrapped with a result shown in Fig. 3.20(d) by following a certain unwrapping path (Fig. 3.20(c)) according to a quality map (Fig. 3.20(b)).

Fig. 3.20. The QGPU unwraps the phase through a path according to a quality map. (a) Wrapped phase map, (b) quality map, (c) unwrapping path map (pixels in light yellow are unwrapped first and those in dark red are unwrapped last), and (d) unwrapped phase map.

For unwrapping path map, the pixels in light yellow are unwrapped first and those in dark red are unwrapped last. In the QGPU technique, quality definition and guiding strategy are two major aspects affecting the success and speed of phase unwrapping. Many quality maps have been proposed, including the correlation coefficient map in InSAR [135], modulation map or reliability map from optical metrology [72, 128], pseudo correlation coefficient map [125], phase derivative variance map [136], maximum phase gradient map [125], first phase difference map [72], second phase difference map [129, 137, 138], amplitude map or ridge map from transform methods [68, 72, 77]. For guiding strategy, the most widely applied one seems to be the classical quality guiding strategy [72], which always unwraps the pixel with the highest quality. Other guiding strategies include two-section guiding strategy and stack-chain guiding strategy [127].

How to select a proper quality map and a guiding strategy is of interest and importance. This work provides a detailed comparison on different quality maps and guiding strategies.

3.5.1 Principle and problems of spatial phase unwrapping and QGPU

Basic principle of spatial phase unwrapping

In spatial phase unwrapping, a pixel a can be unwrapped as follows if it has an unwrapped neighboring pixel b ,

$$\varphi_a^u = \varphi_a^w + 2\pi \times \text{round}\left(\frac{\varphi_b^u - \varphi_a^w}{2\pi}\right), \quad (3.4.28)$$

where the function of $\text{round}()$ converts the input to its nearest integer; φ_b^u is the unwrapped phase of pixel b; φ_a^w and φ_a^u represent wrapped and unwrapped phase of pixel a , respectively. The underlying hypothesis of Eq. (3.4.28) is that the difference of the unwrapped phase between neighboring pixels is within $(-\pi, \pi)$. In Fig. 3.21, dark green pixels have already been unwrapped, based on which light pink pixels can be unwrapped using Eq. (3.4.28). Sometimes noise reduction is carried out for noisy phase maps, and the denoised wrapped phase maps are unwrapped. Based on the unwrapping results from denoised wrapped phase maps, the unwrapping results of original noisy wrapped phase maps can also be obtained through a phase congruence operation [125].

Fig. 3.21. Spatial phase unwrapping method unwraps the phase of current pixel according to the information of its spatially neighboring pixel.

Unwrapping failure

Although the phase unwrapping described in Eq. (3.4.28) seems to be an easy task, it becomes problematic in the following circumstances.

- Under-sampling causes the absolute phase difference between two neighboring pixels beyond the range of $(-\pi, \pi)$ and thus violates the hypothesis of Eq. (3.4.28). This situation is not considered herein.
- Noise can cause wrong unwrapping values in Eq. (3.4.28) and result in the failure of phase unwrapping. Suppressing noise before phase unwrapping becomes useful.

- Invalid areas do not reflect the actual physical quantity. The decorrelation in InSAR and the shadow in FPP are such examples. The unwrapping results within the invalid areas are not of interest, but they may propagate errors into other pixels, which is a major concern.
- Discontinuities on true phase cannot be recognized by the phase unwrapping algorithm and is treated as if they are continuous in Eq. (3.4.28). The unwrapping will often be wrong.
- If a pixel is wrongly unwrapped due to previous reasons, the subsequent unwrapping based on this pixel will also be wrong. This is the nature of all path-following phase unwrapping techniques.

Basic process of QGPU

Referring to Fig. 3.21 again, all light pink pixels can be immediately and simultaneously unwrapped based on their dark green neighbors using Eq. (3.4.28). Rather than unwrapping all light pink pixels, we push them into an adjoining list and only select and unwrap the pixel with the highest quality. Namely, at any time, only the most “trustworthy” pixel is unwrapped. Noisy pixels and invalid areas can be effectively postponed in quality-guide phase unwrapping if they are assigned low quality values. Problems of discontinuities can sometimes, although not always, be avoided by QGPU. The process can be described as follows.

Step 1: Find the seed pixel with the highest quality from the quality map and push it into the adjoining list;

Step 2: Find the pixel with the highest quality from the adjoining list, and unwrap it, and remove it from the adjoining list. Push the unprocessed neighbors into the adjoining list;

Step 3: Repeat step 2 until the adjoining list is empty.

3.5.2 Comparison of quality maps

Resources for quality calculation

There are two possible cases for quality calculation according to the data resources. In case A: a quality map is calculated from raw data of a measurement. A typical example is using phase shifting technique, from which a complex-valued fringe pattern f_c can be obtained as [67]

$$f_c = M \cdot \exp(j \cdot \varphi^w), \quad (3.4.29)$$

where j is the imaginary unit, φ^w is the wrapped phase, and M is the intensity modulation.

From f_c both φ^w and M can be extracted. In case B: quality map is calculated from a wrapped phase, or equivalently, a normalized complex-valued fringe pattern f_n

$$f_n = 1 \cdot \exp(j \cdot \varphi^w) = \frac{f_c}{|f_c|}. \quad (3.4.30)$$

Classification of quality maps

The first group of quality maps is defined only for case A. Typical quality maps considered include the correlation coefficient map in InSAR [125] and modulation map Q_{MOD} [72], and reliability map Q_{REL} [72, 128] from least squares fitting in phase shifting method.

The second group of quality maps is defined only for case B. Typical examples include the pseudo correlation coefficient map Q_{PCC} [125], the phase derivative variance map Q_{PDV} [125], the first phase difference map Q_{FPD} [72], and the second phase difference map Q_{SPD} [129]. Some of these quality maps are combined together as a new quality map [130, 139].

The third group of quality maps is calculated from filtering in spectrum with transform methods, including Fourier transform method [29-31], windowed Fourier filtering method [32, 67-69], windowed Fourier ridges method [32, 67-69], and wavelet transform method [34, 77]. For

QGPU, their related quality maps are amplitude map from Fourier transform method Q_{FT} [72], amplitude map from windowed Fourier filtering method Q_{WFF} [68], ridge map from windowed Fourier ridges method Q_{WFR} [68], and ridge map from wavelet transform method Q_{WT} [68], respectively. Although these methods can be used to retrieve phase from a fringe pattern, in this work, they are only responsible for noise removal and quality measure. This group suits both cases A and B since both f_c in Eq. (3.4.29) and f_n in Eq. (3.4.30) can be transformed and processed. For this group, it is worth noting that the filtered phase values are used for unwrapping, followed by a phase congruence operation.

In summary, quality maps for case A include Q_{MOD} , Q_{REL} , Q_{FT} , Q_{WFF} , Q_{WFR} , and Q_{WT} , while quality maps for case B include Q_{PCC} , Q_{PDV} , Q_{PFG} , Q_{SPD} , Q_{FT} , Q_{WFF} , Q_{WFR} , and Q_{WT} . These two cases will be compared separately, where noise and discontinuities are two major concerns.

Comparison of different quality maps for noisy phase maps

The noisy condition is implemented by computer simulations for easy controlling and benchmarking. Four-step phase-shifted fringe patterns (256×256 pixels) with both speckle noise and normally distributed random noise are simulated as shown in Fig. 3.22(a). The true phase map (shown in Fig. 3.22(b) as wrapped) is

$$\varphi(x, y) = \frac{2\pi}{8}(x - 128) + 3 \times peaks(x, y), \quad (3.4.31)$$

where φ denotes the true phase map and the function of $peaks()$ is a built-in MATLAB[®] function. The true modulation map M shown in Fig. 3.22(c) is

$$M(x, y) = \frac{255}{2} \times normalize(peaks(x, y)), \quad (3.4.32)$$

where the function of $normalize()$ linearly scales the input into [0, 1]. The wrapped phase is then retrieved using the least squares phase-shifting algorithm [35] and is shown in Fig. 3.22(d). The wrapped phase is then unwrapped using a congruence operation based on the simulated true

phase and is shown in Fig. 3.22(e). It needs to note that the signal-to-noise ratio is almost zero in the regions of low modulation, and the wrapped phase and unwrapped phase result in that region are not reliable. However, this is intentionally simulated to investigate the performance of the quality maps.

Fig. 3.22. Simulated noisy condition for phase unwrapping is investigated. (a) Four-step phase-shifted fringe patterns with speckle and random noise, (b) true wrapped phase, (c) true modulation, (d) retrieved noisy wrapped phase, (e) retrieved unwrapped phase (with phase noise).

Six quality maps, Q_{MOD} , Q_{REL} , Q_{FT} , Q_{WFF} , Q_{WFR} , and Q_{WT} , are calculated from raw fringe patterns and then their performances are compared. The results using these quality maps are shown in Fig. 3.23.

Fig. 3.23. Comparison of phase unwrapping is carried out under noisy condition in case A. The quality maps used are (a) Q_{MOD} , (b) Q_{REL} , (c) Q_{FT} , (d) Q_{WFF} , (e) Q_{WFR} , and (f) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps. (In unwrapping error maps, correctly unwrapped pixels are shown with gray levels in proportion to their phase values, and wrongly unwrapped pixels are presented in colors of red or blue. Red and blue indicate that the unwrapped phase value is larger and smaller than its true value, respectively. Darker colors stand for larger errors.)

In unwrapping error maps in Fig. 3.23, correctly unwrapped pixels are shown with gray levels in proportion to their phase values, and wrongly unwrapped pixels are presented in colors of red or blue. Red and blue indicate that the unwrapped phase value is larger and smaller than its true value, respectively. Darker colors stand for larger errors. The results show that, under the noisy condition, transform-based quality maps, Q_{FT} , Q_{WFF} , Q_{WFR} , and Q_{WT} , are much better than the others, Q_{MOD} and Q_{REL} .

Eight quality maps, Q_{PCC} , Q_{PDV} , Q_{FPD} , Q_{SPD} , Q_{FT} , Q_{WFF} , Q_{WFR} , and Q_{WT} , are calculated from the wrapped phase map and then their performances are compared. The results are given in Fig. 3.24, which show that, once again, transform-based methods perform better.

Fig. 3.24. Comparison of phase unwrapping is carried out under noisy condition in case B. The quality maps used are (a) Q_{PCC} , (b) Q_{PDV} , (c) Q_{FPD} , (d) Q_{SPD} , (e) Q_{FT} , (f) Q_{WFF} , (g) Q_{WFR} , and (h) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps.

In order to compare the performances within transform-based methods, a more complicated example is simulated, where the fringe patterns, true phase, true modulation, and retrieved wrapped phase are shown in Fig. 3.25(a-d), respectively. The phase is designed to have wide spectrum but no carrier wave.

Fig. 3.25. Additional phase unwrapping simulation with a wide spectrum and no carrier wave is carried out for transform-based methods. (a) Phase-shifted fringe patterns, (b) wrapped phase without noise, (c) modulation distribution, and (d) retrieved wrapped phase with phase noise.

Unwrapping results with transform-based methods in case A are given in Fig. 3.26. Fig. 3.26(a) shows that FT as a global method is not able to reduce noise in high frequency regions. Fig. 3.26(b) and (c) demonstrate good results by using two local methods, WFF and WFR, because of their good noise reduction abilities. Fig. 3.26(d) shows the unwrapping result by using the WT method. The WT method is not suitable to process very low frequency regions where the WT method requires a very large window. As shown, the phase unwrapping error in Fig. 3.26(d) is almost within the low frequency regions. Similar results are observed in case B.

Fig. 3.26. Comparison of unwrapping results from transform-based methods in case A is investigated. The quality maps used are (a) Q_{FT} , (b) Q_{WFF} , (c) Q_{WFR} , and (d) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps.

To summarize, under noisy condition, transform-based methods show better performances because they provide both denoised wrapped phases and reasonable quality maps.

Comparison of different quality maps for discontinuous phase maps

Another problem in phase unwrapping is that the true phase distribution is not continuous somewhere. It is required to avoid unwrapping through the discontinuous regions. For more actual comparison study under discontinuity condition, eight frames of experimental phase-shifted fringe patterns are captured with fringe projection technique as shown in Fig. 3.27(a). The images in Fig. 3.27(b) and (c) show the retrieved wrapped phase and modulation maps. The unwrapped phase map with temporal phase unwrapping technique is shown in Fig. 3.27(d) for benchmarking.

Fig. 3.27. Experimental discontinuity condition for phase unwrapping is investigated. (a) eight-step phase-shifted fringe patterns with fringe projection technique, (b) retrieved wrapped phase, (c) retrieved modulation map, (d) unwrapped phase by using temporal phase unwrapping technique.

Six quality maps are compared for discontinuous phase in case A. The results are shown in Fig. 3.28, which indicate that the phase unwrapping procedure for this discontinuous example is successful with assistance of quality maps of Q_{MOD} , Q_{REL} , Q_{FT} , and Q_{WFF} . The ridges based quality maps of Q_{WFR} and Q_{WT} fail in leading the unwrapping process by directly passing through discontinuity.

Fig. 3.28. Comparison of phase unwrapping is carried out under discontinuity condition in case A. The quality map used are (a) Q_{MOD} , (b) Q_{REL} , (c) Q_{FT} , (d) Q_{WFF} , (e) Q_{WFR} , and (f) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps.

Eight quality maps are compared for discontinuous phase in case B. The results are shown in Fig. 3.29, which indicate that quality maps of Q_{PDV} , Q_{FPD} , Q_{SPD} , Q_{FT} , and Q_{WFF} , and Q_{WFR}

successfully guide the phase unwrapping process in this experiment. Q_{PCC} and Q_{WT} , fail in leading the unwrapping process with directly passing through discontinuity.

Fig. 3.29. Comparison of phase unwrapping is carried out under discontinuity condition in case B. The quality map used are (a) Q_{PCC} , (b) Q_{PDV} , (c) Q_{FPD} , (d) Q_{SPD} , (e) Q_{FT} , (f) Q_{WFF} , (g) Q_{WFR} , and (h) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps.

However, it is necessary to notice that, under discontinuity condition, it is hard to say which quality map can successfully guide the phase unwrapping. The case in Fig. 3.30 shows an example in which all quality maps fail to guide the unwrapping process.

Fig. 3.30. A failure example of QGPU is investigated. (a) Wrapped phase and unwrapped phase with temporal phase unwrapping, and unwrapping result under case A (b)-(g) and case B (h)-(o). The quality map used are (b) Q_{MOD} , (c) Q_{REL} , (d) Q_{FT} , (e) Q_{WFF} , (f) Q_{WFR} , (g) Q_{WT} , (h) Q_{PCC} , (i) Q_{PDV} , (j) Q_{FPD} , (k) Q_{SPD} , (l) Q_{FT} , (m) Q_{WFF} , (n) Q_{WFR} , and (o) Q_{WT} , respectively. From left to right, each group shows quality maps, path maps, and unwrapping error maps.

To summarize, phase unwrapping under discontinuity condition is still a challenging problem and so far cannot be solved by QGPU alone. Besides, the parameter selection in transform-based methods reduces the intrinsic automaticity in QGPU.

3.5.3 Guiding strategies

A good quality map helps to achieve correct phase unwrapping, while a good guiding strategy helps to reduce the phase unwrapping time. In this section, different guiding strategies are introduced, followed by a comparison of their performances.

Classical quality guiding strategy

The simplest and classical guiding strategy has been illustrated above. However, it is the most time-consuming in processing the adjoining list. The pixels in the adjoining list can be unsorted

or sorted according to their quality values. If the pixels are unsorted, to maintain the adjoining list needs three operations, inserting a new pixel, searching the pixel with the highest quality value to unwrap, and removing the unwrapped pixel. On the other hand, if the pixels are sorted, there are also three operations, locating the inserting position, inserting a new pixel, and removing the pixel with the highest quality value. By using the sorted list, it only needs to visit a part of the adjoining list to find out the inserting position, while it needs to traverse the whole list to find the highest quality value if the unsorted list is used. Thus the sorted adjoining list is preferred and used in this work as default.

The adjoining list can be implemented by several data structures. The image in Fig. 3.31(a) shows the data structure of an array [140]. The array is stored in a continuous memory area, and it is inefficient to insert or remove pixels due to memory re-allocation. The image in Fig. 3.31(b) shows the data structure of a linked list (LL) [140]. A node of a linked list contains pixel properties and a pointer to the next node according to the order of quality values. Unlike an array, an LL is very efficient to insert or remove pixels by modifying the pointers. However, traversing the whole LL is still not efficient because the LL is usually very long.

One possible solution to improve the efficiency of LL is to cut the whole LL into several sections based on the quality values. Each section can be viewed as a “mini-LL”, while the whole structure forms array of pointers and each pointer points to a mini-LL. This data structure is called indexed linked list (ILL) and is shown in Fig. 3.31(c), where quality values decreases from an upper row to a lower row, and also decrease from left to right. The ILL is efficient in inserting and removing pixels, similar to an LL, but its traversal is much faster as the length of each section is much shorter than that of an LL.

For each mini-LL in an ILL, rather than dynamically allocating and releasing memory for each pixel, a sufficient memory of an array can be pre-allocated for all pixels to reduce time consumption. However, the size of a mini-LL cannot be pre-determined and in the worst cases it has the same size of the wrapped phase map. In other words, each mini-LL should be pre-allocated a big memory, which is often too demanding. Fortunately, these mini-LLs can

share one such array, as illustrated in Fig. 3.31(d). The mini-LLs are interwoven in the pre-allocated array. According to the pointers stored in the array, different mini-LLs can be easily recognized without ambiguity. The array is one-dimensional (1D). The 1D index of an element of the array can be converted to the 2D location of a pixel in the wrapped phase map. The data structure is named as Indexed Interwoven Linked List (I2L2) is shown in Fig. 3.31(d). From our practical experiences, section number in ILL and I2L2 is recommended to be the square root of pixel number of a phase map. To the best of our knowledge, the data structures ILL and I2L2 are proposed by the authors in this work, with the hint from the data structure for stack-chain guiding strategy which will be introduced later.

In addition, an algorithm called TRIM is discussed and implemented in reference [125], which splits the original list into two sections if it is too long. One section is still called the adjoining list and the other one is called the postponed list. Once the adjoining list is empty, higher quality pixels from the postponed list will be inserted into it.

Fig. 3.31. Data structures are considered for the sorting in quality guiding. (a) Array [140], (b) linked list [140], (c) indexed linked list, and (d) indexed interwoven linked list.

A comparison of the speed of different data structures is carried out and the result is shown in Table 3.1. The performances from the worst to the best are given by an array, an LL, an ILL and an I2L2, which is consistent to our previous analysis. TRIM performs better than an array and an LL, but is worse than an ILL and an I2L2.

Table 3.1. Speed comparison is carried out for different data structures in classical quality guiding by using the language of C++.

	Data Structures	Array	LL	ILL	I2L2	TRIM
	Box (960×1,280)	33.441	14.437	0.813	0.595	2.749
Time (s)	Noisy Peaks (256×256)	1.762	0.831	0.051	0.037	0.070
	Coffee Cover (400×400)	2.687	1.214	0.069	0.045	0.128

Two-section guiding strategy

In the classical quality guiding strategy, a pixel has to compare its quality value with others before it is unwrapped. However, pixels with high quality values always give correct results and do not need quality comparison. A two-section guiding strategy is thus proposed: the high quality pixels form a high-quality section without any comparison; the rest pixels still need the quality comparison and form a low-quality section which can adopt the data structure of I2L2. Because only partial pixels need to be sorted in the proposed guiding strategy, speedup is expected. A threshold is needed to separate these two sections, which is difficult to set if no prior experience is available. Experimental results with different thresholds are given in Table 3.2. The improvement is not significant: to guarantee a zero-error rate, an improvement of only 0.1s can be obtained, while to tolerate 1% error rate, about two times speedup can be achieved.

Table 3.2. Comparison among thresholds is investigated in two-section guiding.

Sample: Box (960×1,280) in Fig. 1. Language: C++.

Threshold (Percentage of the highest quality)	Time (s)	Error Rate (%)
10%	0.202	0.0994
20%	0.281	0.1085
30%	0.340	0.4278
40%	0.399	0.0139
50%	0.445	0.0008
60%	0.487	0
70%	0.527	0
80%	0.566	0
90%	0.593	0
100%	0.595	0

Stack-chain guiding strategy

Referring to Fig. 3.31(c) again where the data structure includes many sections and pixels in each section have similar quality values. Thus ordering pixels in each section can be avoided to reduce the time cost. The initial idea was proposed in [127, 141] and formed through personal communication [142]. In each section, since no ordering is needed, the concept of a stack with simple “first in, last out” data management can be adopted. Similarly to Fig. 3.31(d), many stacks can be interwoven and the technique is called stack-chain guiding strategy [142]. Again, it is empirically a good choice to set the section number as the square root of pixel number of a phase map. For the Box example used for Table 3.2, which has 960 rows and 1,280 columns, the section number is $(960 \times 1,280)^{1/2} \approx 1,109$. The time cost is 0.82s and the error rate is 0.0009%. Comparing to classical and two-section strategies, the time cost of stack-chain strategy is slightly lower in C++, but the error rate of stack-chain strategy is not zero although it is often very small.

Comparison and discussion

To evaluate the performance of all the strategies discussed above, three wrapped phase maps, Box in Fig. 3.20, Noisy Peaks in Fig. 3.22, and Coffee Cover in Fig. 3.27, are used. Four methods, TRIM, I2L2, two-section, and stack-chain, are implemented in two programming languages, MATLAB and C++. Three thresholds to separate two sections, 25%, 50% and 75% of the largest quality value, are used for the two-section strategy. The results are shown in Table 3.3, Table 3.4, and Table 3.5.

Table 3.3. Comparison is carried out with Box (960×1,280) in Fig. 3.20.

Box (960×1,280)				
		Time (s)		Error Rate (%)
		MATLAB	C++	
CQ	TRIM	65.226	2.749	0
	I2L2	19.406	0.595	0
Two-section	75%	18.323	0.548	0
	50%	16.462	0.445	0.0008
	25%	11.914	0.305	0.1152
Stack-chain		1.007	0.162	0.003

Table 3.4. Comparison is carried out with Noisy Peaks (256×256) in Fig. 3.22.

Noisy Peaks (256×256)				
		Time (s)		Error Rate (%)
		MATLAB	C++	
CQ	TRIM	2.335	0.070	0
	I2L2	0.705	0.037	0
Two-section	75%	0.692	0.037	0
	50%	0.660	0.035	0
	25%	0.540	0.030	0.4486
Stack-chain		0.056	0.010	0.0656

Table 3.5. Comparison is carried out with Coffee Cover (400×400) in Fig. 3.27.

Coffee Cover (400×400)				
		Time (s)		Error Rate (%)
		MATLAB	C++	
CQ	TRIM	4.783	0.128	0
	I2L2	1.936	0.045	0
Two-section	75%	1.821	0.042	0
	50%	1.492	0.031	0.0006
	25%	1.431	0.029	22.5375
Stack-chain		0.102	0.025	0.2338

From these results, it can be summarized that,

- I2L2 performs satisfactorily. The result is theoretically most reasonable and acceptable as all the pixels are sorted. The computing time is also short due to the effective data structure. TRIM gives same unwrapping results but it is much slower. I2L2 is highly recommended.
- Stack-chain strategy is fastest among all strategies. However, it is not error-free, although the error is often small. This algorithm is most useful when the speed is essential.
- Comparatively two-section strategy is less attractive regarding both error rate and speed. However, if a batch of phase maps with similar properties is to be unwrapped, a proper threshold can usually be pre-determined. In this situation, there is a possibility that two-section strategy is able to unwrap the phase maps with zero-error rate but faster than I2L2. Another situation is that the majority of pixels in a wrapped phase map are high, for which two-section can also quickly unwrap the phase without error. However, it can be noticed that the gain of speed is not much.

The QGPU method is discussed with two aspects, the quality map and the guiding strategy. A good quality map is important for successful phase unwrapping. For noisy phase maps, transform-based methods perform better with a cost of heavier computation. Besides, the parameter selection in transform-based methods reduces the intrinsic automaticity in QGPU. For the discontinuity case, QGPU alone is not always successful so far. Guiding strategy is the other important aspect. Three strategies, classical, two-section and stack-chain, are discussed. The classical quality guiding strategy is the most reasonable method and can be accelerated by using a proper data structure such as the interwoven index linked list. The two-section method runs faster, but its success depends on a threshold. The stack-chain method runs fastest but is not error-free.

3.6 Conclusion

This chapter presents and discusses our recent work on the advances in fringe analysis technique. Multi-frame fringe analysis methods are studied for stationary surface metrology with high accuracy and reliability, and presented in Section 3.2 and Section 3.3. Sections 3.4 and 3.5 discuss the wrapped phase retrieval methods from a single frame fringe pattern and quality-guided phase unwrapping technique which are especially studied for dynamic surface shape measurement with a single shot for each measurement.

Section 3.2 analyzes the phase error from nonsinusoidal waveforms in phase shifting technique, which is a multi-frame phase retrieval method. This work is significant to digital fringe projection profilometry with phase shifting method.

Section 3.3 describes a practical phase retrieval frame work to accurately calculate the absolute phase within reliable regions only, with which those unavailable phase points can be automatically identified with thresholds selection and criterion testing and then removed or interpolated according to applications.

Section 3.4 is a comparison work for single-frame phase retrieval methods. The methods of 1D and 2D FT, WFT, and WT have been compared in FPP. From the comparison, the 2D WFR and 2D WT methods performs better than the others to process the fringe pattern from the technique of FPP.

Section 3.5 reviews the quality-guided phase unwrapping technique with a detailed comparison on quality definitions and guiding strategies to select a good quality map and a good guiding strategy.

CHAPTER 4 Three-dimensional shape measurement for different types of surfaces

4.1 Introduction

After the previous discussion on fringe analysis, it is the major objective to measure the 3D shape of different types of surfaces (diffuse, specular, and transparent) by using the fringe analysis technique. As the light propagation is different for each type of surface, the measurement techniques are similar but different in principle. Their similarity is basically because they use structure light illumination with fringe patterns and the fringe analysis is the image processing technique in common. However, they are distinct in measuring principle and data processing procedures.

In this chapter, some developments in 3D shape measurement for different types of surfaces will be described. With fringe projection technique, it is well developed and called as FPP to profile the diffuse surfaces. Study on the topic is mainly to develop a fast and flexible calibration method, especially for outdoor applications if the system recalibration is required. The influence of surface reflective property is discussed through a study on 3D shape measurement of partially diffuse and specular surfaces with both FPP and FRD. A fast FRD method is proposed for measuring dynamic scene of a specular surface. In addition, an improved least squares method is proposed as well to enhance the integration accuracy in FRD when measuring specular surfaces. At last, a method is proposed to analyze the fringe patterns with double-side reflection at the front and rear interfaces of transparent surfaces.

4.2 Diffuse surface profilometry with fringe projection method based on improved least squares calibration

As a fast, accurate, and precise three-dimensional shape measurement method, FPP has been studied and used in many fields, such as product inspection, reverse engineering, and animation [1, 23, 123, 124]. The diagram of FPP system is shown in Fig. 4.1. The target is placed within the common field of view of the camera and projector. A series of fringe patterns are generated by computer and then projected by using a projector. After the projection of fringe patterns, the fringes are distorted due to the height variation of the target if a camera is set up from a height-sensitive viewing angle. Images of the deformed fringe patterns are captured by a digital camera. From the images with fringe patterns, the 2D phase map can be calculated with phase retrieval methods which have been well developed in the last two decades [30, 32, 34, 35, 45, 61].

Fig. 4.1. Schematic diagram of FPP system shows the target is placed in the common field of view of camera and projector.

Furthermore, one of the issues in FPP is how to calibrate the system to determine the relationship between the retrieved 2D phase map and the three dimensions. The calibration procedure usually includes out-of plane calibration and in-plane calibration. The out-of plane calibration is to convert a 2D phase map to height or depth, which is also called as phase-height

mapping [119]. The calibration procedure with phase-height mapping greatly influences the accuracy and precision of FPP system.

A good calibration method should be able to acquire the 3D measurement result with high accuracy, high precision and easy implementation. In experiment, it is noticed that the camera lens distortion does influence the measurement result. However, it is not involved in the existing least-squares calibration in FPP [92]. In order to achieve a high accuracy in measurement with the existing method, high quality lenses with little distortion are required, which necessarily increase the cost of the system. In this work, a modified least-squares expression is proposed to have the camera lens distortion considered in our least-squares calibration for FPP system.

4.2.1 Principle and implementation

Generally speaking, the purpose of calibration in FPP is to convert a 2D phase map into a 3D coordinates result. In the least-squares calibration method, the out-of-plane phase-height mapping is carried out with least-squares method by using several (at least two) gauge blocks to give the system different height or depth information. Once the out-of-reference-plane height is already known, the in-plane dimensions can be determined by the parameters from camera calibration with the known height. Consequently, the out-of-plane phase-height conversion becomes quite critical in the whole calibration procedure, since the in-plane dimensions might be influenced by the out-of-reference-plane height or depth results.

In the existing least-squares calibration method [92], the pinhole camera model is utilized, in which the lens distortion is not considered. The existing governing equation of the out-of-plane height determination [92] can be written as

$$z = \frac{1 + c_1\phi + (c_2 + c_3\phi)x_n + (c_4 + c_5\phi)y_n}{d_0 + d_1\phi + (d_2 + d_3\phi)x_n + (d_4 + d_5\phi)y_n}, \quad (4.2.1)$$

where z is the out-of-reference-plane height or depth at point (x, y, z) on the target, c_1 - c_5 and d_0 - d_5 are constant coefficients which need to be determined, ϕ is the fringe phase at the same

point, and x_n and y_n are the normalized coordinates of the pinhole image point of the same point, with the definition as $x_n = x_c/z_c$ and $y_n = y_c/z_c$. It should be noted that the camera coordinates (x_n, y_n) in Eq. (4.2.1) is different from those in reference [92], but the phase-height equation forms are the same, because the transformation from the pixel coordinates to our camera coordinates (x_n, y_n) does not change its mathematical description form. The camera lens distortion is discussed as follows.

Including the distortion of lens, the new normalized point coordinate (x_d, y_d) is defined as

$$x_d = \left[1 + k_{c1}r_n^2 + k_{c2}r_n^4 + k_{c5}r_n^6 \right] x_n + dx, \quad (4.2.2)$$

$$y_d = \left[1 + k_{c1}r_n^2 + k_{c2}r_n^4 + k_{c5}r_n^6 \right] y_n + dy, \quad (4.2.3)$$

and the tangential distortion dx and dy are defined as

$$dx = 2k_{c3}x_ny_n + k_{c4}(r_n^2 + 2x_n^2), \quad (4.2.4)$$

$$dy = k_{c3}(r_n^2 + 2y_n^2) + 2k_{c4}x_ny_n, \quad (4.2.5)$$

where $k_{c1}, k_{c2}, k_{c3}, k_{c4}$, and k_{c5} are constant coefficients of camera lens distortion, and r_n is defined as $r_n^2 = x_n^2 + y_n^2$. Since the symmetric radial distortion is usually much larger than tangential distortion in modern optics, k_{c3} and k_{c4} can be reasonably discarded. Moreover, it is often not recommended to expand the radial component of distortion model beyond the fourth order, i.e. $k_{c5} = 0$. Therefore, only k_{c1} and k_{c2} are finally included in our lens distortion model for simplicity of expression and efficiency of least-squares procedure. The simplified coordinates of x_d and y_d can be rewritten as

$$x_d = \left[1 + k_{c1}r_n^2 + k_{c2}r_n^4 \right] x_n, \quad (4.2.6)$$

$$y_d = \left[1 + k_{c1}r_n^2 + k_{c2}r_n^4 \right] y_n. \quad (4.2.7)$$

It is not so easy to directly derive an expression of $x_n = f(x_d)$ from Eq. (4.2.6). But usually the distortion amount ($x_d - x_n$) is much smaller than x_n itself, i.e. $k_{c1}r_n^2 + k_{c2}r_n^4 \ll 1$. Then, with Taylor expansion, Eq. (4.2.6) and Eq. (4.2.7) can be rewritten as

$$x_n = \frac{x_d}{1 + k_{c1}r_n^2 + k_{c2}r_n^4} \approx (1 - k_{c1}r_n^2 - k_{c2}r_n^4) x_d, \quad (4.2.8)$$

$$y_n = \frac{y_d}{1 + k_{c1}r_n^2 + k_{c2}r_n^4} \approx (1 - k_{c1}r_n^2 - k_{c2}r_n^4) y_d. \quad (4.2.9)$$

These equations can be considered as a reverse imaging process from (x_d, y_d) to (x_n, y_n) through another lens with inverse distortion.

Substituting x_n and y_n in Eq. (4.2.1) with Eq. (4.2.8) and Eq. (4.2.9), the expression for phase-height relation becomes

$$z = \frac{\left[1 + c_1\phi + (c_2 + c_3\phi)x_d + (c_4 + c_5\phi)y_d + (c_6 + c_7\phi)r_d^2x_d + (c_8 + c_9\phi)r_d^2y_d + (c_{10} + c_{11}\phi)r_d^4x_d + (c_{12} + c_{13}\phi)r_d^4y_d \right]}{\left[d_0 + d_1\phi + (d_2 + d_3\phi)x_d + (d_4 + d_5\phi)y_d + (d_6 + d_7\phi)r_d^2x_d + (d_8 + d_9\phi)r_d^2y_d + (d_{10} + d_{11}\phi)r_d^4x_d + (d_{12} + d_{13}\phi)r_d^4y_d \right]}, \quad (4.2.10)$$

where c_1 - c_{13} and d_0 - d_{13} are constant coefficients. According to the well-established camera calibration theory, the final pixel coordinates (x_p, y_p) on the image plane can be determined by

$$x_p = f_{c1}x_d + \alpha_c y_d + c_{c1}, \quad (4.2.11)$$

$$y_p = f_{c2}y_d + c_{c2}, \quad (4.2.12)$$

where f_{c1} and f_{c2} are the focal distance expressed in units of pixels, the coefficient α_c encodes the angle between x and y axes, and for modern camera, the axes of x and y are generally

perpendicular to each other, i.e. this coefficient can be assumed as zero ($\alpha_c = 0$), and the point of (c_{c1}, c_{c2}) is the image point of the principal point. These parameters all can be determined from camera calibration.

With the discussion above, two coordinates X and Y are defined, which are directly used in least-squares.

$$X = f_{c1}x_d = x_p - c_{c1}, \quad (4.2.13)$$

$$Y = f_{c1}y_d = y_p - c_{c2}. \quad (4.2.14)$$

And Eq. (4.2.10) can be finally expressed as

$$z = \frac{\left[1 + C_1\phi + (C_2 + C_3\phi)X + (C_4 + C_5\phi)Y \right. \\ \left. + (C_6 + C_7\phi)R^2X + (C_8 + C_9\phi)R^2Y + (C_{10} + C_{11}\phi)R^4X + (C_{12} + C_{13}\phi)R^4Y \right]}{\left[D_0 + D_1\phi + (D_2 + D_3\phi)X + (D_4 + D_5\phi)Y \right. \\ \left. + (D_6 + D_7\phi)R^2X + (D_8 + D_9\phi)R^2Y + (D_{10} + D_{11}\phi)R^4X + (D_{12} + D_{13}\phi)R^4Y \right]}, \quad (4.2.15)$$

where C_1 - C_{13} and D_0 - D_{13} are constant coefficients to be determined and R is defined as

$$R^2 = X^2 + Y^2. \quad (4.2.16)$$

In order to calculate the height or depth z by using Eq. (4.2.15), the coefficients C_1 - C_{13} and D_0 - D_{13} should be determined in advance with least-squares method based on the phase data on the reference plane and several (at least two) gauge blocks of which the heights are known. The coefficients here can be determined by using linear least-squares, and least-squares error S can be defined as

$$S = \sum_{n=1}^N \left\{ \begin{aligned} & \left[\begin{aligned} & 1 + C_1\phi_n + (C_2 + C_3\phi_n)X_n + (C_4 + C_5\phi_n)Y_n \\ & + (C_6 + C_7\phi_n)R_n^2X_n + (C_8 + C_9\phi_n)R_n^2Y_n \\ & + (C_{10} + C_{11}\phi_n)R_n^4X_n + (C_{12} + C_{13}\phi_n)R_n^4Y_n \end{aligned} \right] \\ & - \left[\begin{aligned} & D_0 + D_1\phi_n + (D_2 + D_3\phi_n)X_n + (D_4 + D_5\phi_n)Y_n \\ & + (D_6 + D_7\phi_n)R_n^2X_n + (D_8 + D_9\phi_n)R_n^2Y_n \\ & + (D_{10} + D_{11}\phi_n)R_n^4X_n + (D_{12} + D_{13}\phi_n)R_n^4Y_n \end{aligned} \right] z_n^g \end{aligned} \right\}^2, \quad (4.2.17)$$

where n is the ordinal number of each valid point, N is the total number of valid points on the reference plane and gauge blocks which are used in our calculation, z_n^g is the out-of-reference-plane height or depth of the n th valid point on calibration reference plane or gauge blocks, which should be known in advance as basic geometric information.

The least-squares criterion can be given as

$$\begin{cases} \frac{\partial S}{\partial C_i} = 0, & i = 1, 2, \dots, 13 \\ \frac{\partial S}{\partial D_j} = 0, & j = 0, 1, \dots, 13. \end{cases} \quad (4.2.18)$$

In implementation, usually the coefficients C_1 - C_{13} are firstly determined with the valid points on reference plane. Then the coefficients D_0 - D_{13} can be determined by using the valid points on gauge blocks and the known C_1 - C_{13} as well. With the determination of the coefficients C_1 - C_{13} and D_0 - D_{13} , the out-of-reference-plane height or depth of each valid point can be calculated. Moreover, if the extrinsic parameters from camera calibration and the out-of-reference-plane height or depth data from out-of-plane calibration are available, the in-plane dimensions of each point can be easily calculated as well. Once the FPP system is calibrated, the pattern plane for camera calibration, the reference plane, and gauge blocks are not needed anymore in actual 3D measurement, unless a new calibration is required.

4.2.2 Simulation

In order to check the feasibility and validity of the proposed calibration method without the influence from other factors in FPP system, simulations of the 3D measurement by using fringe projection are carried out. In the simulation, the heights of five calibration gauge blocks are 10 mm, 20 mm, 25 mm, 30 mm, and 40 mm, respectively. The true height distribution of calibration gauge blocks, a representative simulated fringe pattern, and its unwrapped phase map are shown in Fig. 4.2.

Fig. 4.2. Calibration gauge blocks are used in simulation. (a) Three-dimensional height distribution of calibration gauge blocks, (b) a representative simulated fringe pattern and (c) its unwrapped phase map.

The target to be measured in simulation is given as $z = 40 \times \text{membrane}$ with its height varying from -10 mm to 40 mm. The function of *membrane* is a build-in MATLAB[®] function. The true height distribution of the target, a representative simulated fringe pattern, and its unwrapped phase map are shown in Fig. 4.3.

Fig. 4.3. A target is used in simulation. (a) Three-dimensional true height distribution of target, and (b) a representative simulated fringe pattern and (c) its unwrapped phase map.

The phase data on reference plane and calibration gauge blocks are adopted to determine the calibration coefficients. Then the target is reconstructed to show the performance of the calibration method. Three conditions are discussed in this simulation: no distortion, distortion on camera lens only, and distortion on both camera lens and projector lens. Both the existing method and proposed method are implemented to show a comparative result. Comparing with the existing method, the performance of the proposed method under these different simulated conditions will give us better understandings.

Condition 1: no distortion on either camera lens or projector lens

If both camera lens and projector lens has no distortion, the existing Du and Wang's [92] least-squares expression, Eq. (4.2.1) should be accurate enough. The aim of the simulation under this condition is to check if the proposed calibration method can also be utilized under no distortion or little distortion conditions. The results of comparison are shown in Fig. 4.4.

Fig. 4.4. Simulation is carried under no distortion condition. (a) Reconstructed result by the existing method, (b) reconstructed result by the proposed method, (c) reconstruction error from fringe patterns with no noise by the existing method, (d) reconstruction error from fringe patterns with no noise by the proposed method, (e) reconstruction error from fringe patterns with noise by the existing method and (f) reconstruction error from fringe patterns with noise by the proposed method.

Simulation results with no distortion in Fig. 4.4(c) and (d) indicate that the proposed least-squares expression also can give an accurate phase-to-height conversion as Du and Wang's expression does [92]. The images in Fig. 4.4(e) and (f) indicate that the proposed method is not sensitive to the normally distributed noise on fringe patterns just as the existing method. The coefficients from the existing method and proposed method in simulation with noise on fringe patterns under no distortion condition are shown in Table 4.1.

Table 4.1. Coefficients are determined with existing and proposed method.

	Coefficients	Values
Existing method	c_1-c_5	[-0.8617, 0.0000, -0.0000, 0.7983, -0.0003]
	d_0-d_5	[-0.7975, -0.0003, 0.0000, 0.0000, 0.0005, -0.0000]
Proposed method	c_1-c_{13}	[-0.8617, 0.0000, -0.0000, 0.7983, -0.0003, 0.0000, -0.0000, -0.0000, 0.0000, -0.0000, -0.0000, 0.0000, -0.0000]
	d_0-d_{13}	[-0.7975, -0.0003, 0.0000, 0.0000, 0.0005, -0.0000, 0.0000, -0.0000, -0.0000, 0.0000, -0.0000, 0.0000, 0.0000]

It is noticed that the determined coefficients of the same term in these two expressions of Eq. (4.2.1) and Eq. (4.2.15) are quite close to each other. A coefficient of zero means there is no contribution of that component and the coefficients of the higher order terms in proposed expression are equal to zero, since no higher order components are involved by distortion under this no distortion condition. Therefore, the proposed calibration algorithm could also be used under this little distortion condition as a generalized algorithm of the existing least-squares calibration algorithm. The next step is to check if the proposed method is more accurate than the existing method with camera lens distortion.

Condition 2: distortion on the camera lens only

Here the comparison is pushed to the condition that only the camera lens has distortion while the projector lens has no distortion. In implementation, the distortion effect on camera lens is simulated with the distortion parameters as $k_{c1} = -0.2042$, $k_{c2} = 0.2102$, $k_{c3} = 0.0031$, $k_{c4} =$

0.0006 and $k_{c5} = 0$, and the distortion model as described from Eq. (4.2.2) to Eq. (4.2.5). The simulation results under this condition are shown in Fig. 4.5.

Fig. 4.5. Simulation is carried out under camera lens distortion condition. (a) Reconstructed result by the existing method, (b) reconstructed result by the proposed method, (c) reconstruction error from fringe patterns with noise by the existing method and (d) reconstruction error from fringe patterns with noise by the proposed method.

The error distributions in Fig. 4.5(c) and (d) show that under the camera lens distortion condition, the proposed method can involve the camera lens distortion into the least-squares to achieve an accurate phase-to-height conversion, while it is difficult for the existing least-squares calibration method to accurately reconstruct the target or even the reference plane, which has already been used in its calibration procedure.

Condition 3: distortion on both the camera lens and the projector lens

Usually, there are distortions on the camera lens as well as the projector lens in an actual measurement. It is more meaningful to simulate this kind of condition to get more actual understanding about how the existing and proposed least-squares phase-to-height conversion methods behave in real case. In simulation, the camera lens distortion parameters k_{c1} - k_{c5} are identical with that in previous section, and the projector lens distortion parameters are set as $k_{p1} = 0.1429$, $k_{p2} = -0.1471$, $k_{p3} = -0.0021$, $k_{p4} = -0.004$, $k_{p5} = 0$.

Fig. 4.6. Simulation is carried under both camera lens and projector lens distortion condition. (a) Reconstructed result by the existing method, (b) reconstructed result by the proposed method, (c) reconstruction error from fringe patterns with noise by the existing method, and (d) reconstruction error from fringe patterns with noise by the proposed method.

From the comparison shown in Fig. 4.6, the proposed modified method is able to get a phase-to-height conversion with fewer errors. The result suggests the proposed calibration method is suitable and easy to calibrate the FPP system with some distortions on its optical components.

4.2.3 Experiment and discussion

In order to demonstrate the performance of the proposed calibration method, some experiments are carried out. Since these experiments have very similar results, only one of them with calibration and testing is presented here for brevity. The existing method is also implemented with the same phase data to make a comparison. To keep from drawing a biased conclusion through an experiment with the contingency of available region selection, a separately measured phase map of the reference plane is completely adopted in least squares.

Fig. 4.7. Calibration experiment is carried out. (a) Representative fringe patterns in three separate measurements, and (b) their unwrapped phase data used for least-squares calibration.

As shown in Fig. 4.7, a phase map of the reference plane is separately measured by the fringe projection profilometric system in Fig. 3.8 and wholly used for the determination of the parameters of C_1 - C_{13} , and the phase maps of two gauge blocks with the heights of 20.00 mm and 30.00 mm, respectively, are measured and utilized to determine the rest parameters of D_0 - D_{13} . Each of these two calibration gauge blocks is placed at two different positions within two separate measurements.

Fig. 4.8. The calibration reference plane is reconstructed. (a) Reconstructed reference plane with the existing method, (b) reconstructed reference plane with the proposed method, and (c) histogram of the measured height results.

In order to study the performance of the existing and proposed methods, the calibration reference plane which is used in least-squares is reconstructed by the existing and proposed methods shown in Fig. 4.8. It indicates with Fig. 4.8(a) and (b) that the experimental results are similar to the simulation ones in Fig. 4.5(c) and (d) or Fig. 4.6(c) and (d). With the consideration of the camera lens distortion, the proposed expression is more close to the actual phase-to-height relation, and as a result shown in Fig. 4.8(c), the residual error is less by using the proposed calibration method. In order to make sure the undulation in Fig. 4.8(a) is not the real height distribution of the reference plane, it is measured once more after it is rotated on its normal through about 90 degrees to. A result with the existing method has a very similar undulation to that shown in Fig. 4.8(a).

After our calibration, a testing measurement is carried out as well. Another two gauge blocks with the same height of 25.00 mm are measured as the target. A representative fringe pattern and its unwrapped phase map are shown in Fig. 4.9.

Fig. 4.9. Testing measurement is carried out. (a) A representative fringe pattern and (b) unwrapped phase map.

The measurement results based on the existing calibration as well as the proposed calibration are shown in Fig. 4.10. The point clouds of these two results in Fig. 4.10(a) and (d) seem quite similar to each other, but they are actually different in details.

Comparing the results shown in Fig. 4.10(b) and (e), as well as (c) and (f), the height distribution from the proposed method is more uniform, while tilts are quite commonly observed in the result with the existing method.

Fig. 4.10. Measurement results are investigated. (a) Point clouds with the existing method, (b) and (c) measured height distribution on the target block with the existing method, (d) point clouds with the proposed method, (e) and (f) measured height distribution on the target block with the proposed method.

The errors of reconstructed results with existing and proposed methods are shown in Fig. 4.11(a) and (b), respectively. The error with the existing method is large especially at the boundary region of the field of view, and its standard deviation and varying range are 0.066 mm and $-0.216 \text{ mm} \sim 0.337 \text{ mm}$, respectively. However, the proposed method is accurate in the whole field with a standard deviation of 0.017 mm and a varying range of $-0.073 \text{ mm} \sim 0.064 \text{ mm}$.

Fig. 4.11. Error distribution shows the proposed method is very effective. (a) Error distribution with the existing method and (b) error distribution with the proposed method.

In this work, a modified least-squares calibration method for FPP is proposed. On one hand, the proposed method is still very flexible in actual implementation just as the existing least-squares calibration method, since no extra experimental operation is required and the system can be arbitrarily arranged as well. On the other hand, with the consideration of the camera lens distortion, the proposed least squares calibration method gives a result with high accuracy and high precision even under the lens distortion condition.

4.3 An improved iterative least squares integration method for shape reconstruction from gradient data

There are a variety of optical techniques in three-dimensional (3D) shape metrology for industrial inspection applications [143]. Of these measurement techniques, some are referred to direct methods, since they get 3D results directly from the captured optical information, *e.g.* locations of laser spot or pattern images. Other techniques deduce the shape by integration of the surface gradient information. The fringe reflection technique [39, 40] and Shack-Hartmann methods [144] are two representatives of these indirect shape measurement methods. A process to integrate the shape from gradient data is consequently required in order to reconstruct the shape/profile of the surface/wave front.

This two-dimensional (2D) integration will influence the profile results, and hence research efforts to make this process more accurate and faster are being explored. Fried [103], and Hudgin [104, 105] studied the wavefront reconstruction from phase difference or slope using least squares fitting and evaluated the error propagation in reconstructions. Southwell [145] provided the wave front estimation from slope with least squares method in more details including the grid models, solutions, and errors of least squares integration method. Li *et al.* [106] reviewed several evaluation methods for gradient measuring techniques and a comparison of these integration methods shows the superiority of the least squares integration method in terms of accuracy, robustness and flexibility. Ettl *et al.* [107] proposed another accurate shape reconstruction method from gradient data by dividing the whole area into in small pieces with certain common regions, integrating the slope data piece-wise by radial basis functions (RBFs) with least squares sense, and then stitching all pieces together to reconstruct the entire surface.

The Southwell grid model widely used in least squares integration method, assumes that the surface can be expressed as a biquadratic function [145]. Obviously this assumption is not satisfied in many situations resulting in integration errors. Ettl's RBFs based method [107] is not able to directly handle large matrices without dividing and stitching process and it is slow even for one small piece of data with RBFs, although it performs with high accuracy and flexibility.

Least squares integration method with Southwell grid model [145] basically is good for handling large datasets, because sparse matrices can be utilized to reduce the memory usage during the least squares integration. This work consequently aims to improve the integration accuracy due to the imperfection of the assumption from Southwell grid model.

4.3.1 Principle

In some slope-measuring-based optical metrology methods, integration converts gradient data to surface shape using least squares fitting. Following Southwell grid model [145] as shown in Fig. 4.12, the relationship between slope and shape can be expressed as

$$\begin{cases} \frac{p_{i,j+1} + p_{i,j}}{2} = \frac{z_{i,j+1} - z_{i,j}}{x_{i,j+1} - x_{i,j}}, & i = 1, \dots, M, \quad j = 1, \dots, N-1 \\ \frac{q_{i+1,j} + q_{i,j}}{2} = \frac{z_{i+1,j} - z_{i,j}}{y_{i+1,j} - y_{i,j}}, & i = 1, \dots, M-1, \quad j = 1, \dots, N \end{cases}, \quad (4.3.1)$$

where x, y, z are the coordinates. p and q are the slopes in x - and y -directions, respectively, *i.e.* $p = dz/dx$ and $q = dz/dy$.

Fig. 4.12. Southwell grid model is used in least squares integration.

Hence, the height information can be given as

$$\begin{cases} z_{i,j+1} - z_{i,j} = \frac{1}{2}(p_{i,j+1} + p_{i,j})(x_{i,j+1} - x_{i,j}), & i = 1, \dots, M, \quad j = 1, \dots, N-1 \\ z_{i+1,j} - z_{i,j} = \frac{1}{2}(q_{i+1,j} + q_{i,j})(y_{i+1,j} - y_{i,j}), & i = 1, \dots, M-1, \quad j = 1, \dots, N \end{cases}. \quad (4.3.2)$$

This can be rewritten in terms of matrices as

$$\mathbf{DZ} = \mathbf{G}, \quad (4.3.3)$$

where

$$\mathbf{D} = \begin{bmatrix} -1 & 0 & \dots & 0 & 1 & 0 & \dots & \dots & 0 \\ 0 & -1 & 0 & \dots & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 & -1 & 0 & \dots & 0 & 1 \\ -1 & 1 & 0 & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & -1 & 1 & 0 & \dots & \dots & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & \dots & \dots & \dots & 0 & -1 & 1 \end{bmatrix}, \quad (4.3.4)$$

$$\mathbf{Z} = \begin{bmatrix} z_{1,1} \\ z_{2,1} \\ \vdots \\ z_{M,N} \end{bmatrix}, \quad (4.3.5)$$

$$\mathbf{G} = \frac{1}{2} \begin{bmatrix} (p_{1,2} + p_{1,1})(x_{1,2} - x_{1,1}) \\ (p_{1,3} + p_{1,2})(x_{1,3} - x_{1,2}) \\ \vdots \\ (p_{M,N} + p_{M,N-1})(x_{M,N} - x_{M,N-1}) \\ (q_{2,1} + q_{1,1})(y_{2,1} - y_{1,1}) \\ (q_{3,1} + q_{2,1})(y_{3,1} - y_{2,1}) \\ \vdots \\ (q_{M,N} + q_{M-1,N})(y_{M,N} - y_{M-1,N}) \end{bmatrix}. \quad (4.3.6)$$

Commonly Eq. (4.3.3) is over determined in practical situations. Hence, a least squares estimation can be carried out to provide the height distribution as an optimized solution.

The assumption implied in Eq. (4.3.1) is that the slope distribution is bilinear within each tiny quadrilateral, *i.e.* the variation of surface height is biquadratic. As we mentioned above, this assumption is not always satisfied, and hence is a main source of error. An iterative compensation procedure is thus proposed to improve the integration accuracy. Basically higher order terms are contained in these residual gradient data, which cannot be fit with the biquadratic surface. By integrating these residual gradient data it is possible to compensate errors due to the higher order terms. Following a few iterations, the final shape can be

accurately reconstructed with no influence of the imperfect assumption. The specific steps of the algorithm are described as follows.

Step 1: Integrate the gradient data with the traditional least squares integration method to get initial height $z_0(x, y)$. Set the current height $z(x, y) = z_0(x, y)$;

Step 2: Calculate the residual slopes in x - and y -directions $dp(x, y)$ and $dq(x, y)$ with current height distribution $z(x, y)$.

Step 3: Integrate the residual slopes $dp(x, y)$ and $dq(x, y)$ with the traditional least squares integration method to get $z_c(x, y)$ for compensation.

Step 4: Compensate the height $z(x, y) = z(x, y) + z_c(x, y)/n_k$, where the parameter n_k are empirical constants which can be determined through simulation and k stands for the number of compensation.

Step 5: Repeat the compensation loop from step 2 to step 4 till the compensating term $z_c(x, y)$ is less than the preset threshold z_{thr} . The current $z(x, y)$ is the final height.

Note: the empirical parameters n_k are determined by simulation with surfaces containing different higher order components. Form our calculation, the empirical parameters n_k are $n_1 \approx 3$, $n_2 \approx 4.0909$, $n_3 \approx 4.9476$, $n_4 \approx 5.6768$ when $k=1, 2, 3, 4$.

4.3.2 Simulation

In order to investigate the validity of the proposed iterative compensation, a series of simulation were conducted. The first example is to show the iterative compensation works efficiently to improve the integration accuracy when the assumption of biquadratic shape is not satisfied. The x and y coordinates range from -5mm to 5 mm with an interval of 0.02mm, *i.e.* the size of matrices is 500 by 500 which yields 500,000 x - and y -slope values in all. As shown in Fig. 4.13(a) and (b), the true out-of-plane dimension is predefined as

$$z = \cos(0.4x^2 + 2x)\cos(0.4y^2 + 2y), \quad (4.3.7)$$

and its corresponding slopes in x - and y -directions p and q shown in Fig. 4.13(c) and (d) can be derived as

$$p = -(0.8x + 2)\sin(0.4x^2 + 2x)\cos(0.4y^2 + 2y), \quad (4.3.8)$$

$$q = -(0.8y + 2)\cos(0.4x^2 + 2x)\sin(0.4y^2 + 2y). \quad (4.3.9)$$

In Fig. 4.13(c) and (d), the surface gradient varies slightly at the lower left corner and acutely at the upper right corner. The sample surface shape is specially chosen by purpose to test the iterative performance of the proposed method.

Fig. 4.13. True shape and gradient data are used in simulation. (a) True shape, (b) height map, and true slopes in x -direction $p = dz/dx$ (c) and in y -direction $q = dz/dy$ (d).

By applying the proposed integration method with iterative compensation, the out-of-plane height distribution can be accurately reconstructed as shown in Fig. 4.14. The result from traditional least squares integration method with no compensation is demonstrated in Fig. 4.14(a), which shows large integration errors as the true surface does not satisfy the biquadratic surface assumption.

Fig. 4.14. Integration results are investigated. (a) Height error with no compensation, (b) after the 1st compensation, (c) after the 8th compensation, (d) standard deviations of height error after compensations, and (e) reconstructed result after the 8th compensation.

As shown in Fig. 4.14(b-c), with the proposed compensation process, height errors significantly reduced to around 1% as compared to Fig. 4.14(a). The majority of compensated amount is completed after the first one or two compensations, which is also shown in Fig. 4.14(d). After eight compensations, the out-of-plane height is effectively and accurately reconstructed as shown in Fig. 4.14(e).

The second example demonstrates the ability of the proposed method to operate in the presence of noise. The true height shown in Fig. 4.15(a) is defined by

$$z = 3(1-x)^2 \cdot e^{-x^2-(y+1)^2} - 10\left(\frac{x}{5} - x^3 - y^5\right) \cdot e^{-x^2-y^2} - \frac{1}{3}e^{-(x+1)^2-y^2}, \quad (4.3.10)$$

where the in-plane dimensions x and y are limited within a range from -1 to 1 with sampling points of 400 by 400, and the corresponding out-of-plane height varies by more than 5 mm.

Fig. 4.15. Integrate gradient data with noise. (a) True shape to be reconstructed, (b) slope $p = dz/dx$, (c) slope $q = dz/dy$.

In practical measurement, noise is normally distributed on fringe phase, and furthermore can be approximately considered as normally distributed on surface normal directions. Here normally distributed angular noise with a standard deviation of 8 arcsec is added in the normal directions, which is a typical noise level in fringe reflection technique. The associated noisy slopes in x - and y -directions are shown in Fig. 4.15(b) and (c).

Fig. 4.16. Integration results show the error is effectively reduced through the proposed compensation. (a) Height error before compensation, (b) height error after iterative compensation, and (c) height error in one line ($y=0$).

The integration result from the traditional least squares method is shown in Fig. 4.16(a). Without any compensation, the height error is not only because of the influence from gradient

noise but also the mismatch between the assumed biquadratic surface and the actual surface to be reconstructed. However, most of the errors due to this assumption can be compensated through the iterative process with the height error shown in Fig. 4.16(b). A line profile ($y=0$) of errors before and after compensation is compared in Fig. 4.16(c), from which it can be easily found that the iterative process improves the integration accuracy with successful elimination of the waviness, and the residual error is mainly from the gradient noise. The result shows the iterative least squares integration method is able to reconstruct shapes from gradient data in the presence of noise.

4.3.3 Discussion

As noted in literature, Ettl's RBFs based method [107] is an accurate integration approach as well. A comparison of these two methods is also conducted to show the relative performances. As mentioned above, with no stitching process, the RBFs based integration method can only handle small piece of dataset because of its large memory consumption. Therefore the true height distribution is set as a small piece of spherical surface shown in Fig. 4.17(a) with

$$z = 90 - \sqrt{90^2 - x^2 - y^2}, \quad (4.3.11)$$

where the in-plane dimensions x and y vary from -4 to 4 with 40 by 40 sampling points such that the dataset can be handled by both methods. Normally distributed angular noise with a standard deviation of 8 arcsec is added onto the surface normal directions.

Fig. 4.17. Integration comparison is carried out with small dataset. (a) True shape, and (b-c) height error with the RBFs based integration method (b) and the proposed integration method (c).

Integration results of RBFs based method and the proposed method are shown with their height errors in Fig. 4.17(b) and (c), respectively. It is found that both methods are able to successfully reconstruct the shape from gradient data with a comparable accuracy. Although least squares estimation is involved in both methods, there are some differences between them, especially when handling large datasets. The RBFs based integration method accurately integrates every subset in a sequence and then stitch all subsets together to get the entire shape. That means the heavy work is spatially distributed and completed in sequence. On the other hand, the proposed method works on the whole region or at least few relatively large blocks if blocking is necessary for some huge datasets. The issue of large memory occupation is solved by implementing simple integration models with sparse matrices and reusing the same memory during iterative calculation in time axis.

The comparison on accuracy and speed of these two methods is shown in Fig. 4.18. First, the accuracy is compared by integrating the same noisy gradient dataset with different sampling rates. The plot in Fig. 4.18(a) shows the standard deviations of height errors with different methods. It is obvious that the proposed method is more accurate. Also the larger the size of subset the more insensitive to noise as well as the stitching process the integration result is.

Fig. 4.18. Comparison of the RBFs method with the proposed method is carried out with two aspects. (a) Accuracy, and (b) speed.

Considering the necessary overlaps for stitching, the subset number is actually larger than the size ratio between the whole surface and the subset. The total runtime for integration is thus composed of the integration time in sequence and stitching time. It is also important to notice that if the subsets are identical for each patch, the integration time of the RBFs based method can be reduced by calculating the common matrix beforehand. Thus the RBFs based method is very fast when identical subsets are used as shown in Fig. 4.18(b) and also smaller subsets yields shorter runtime. However, in practical situation, some sampling data might be missing due to boundaries, unavailable regions, or poor quality, thus subsets are not identical and need to be recalculated, which increases the runtime as shown in Fig. 4.18(b). The proposed method copes with the complete and incomplete dataset in the same way and almost the same speed. Generally speaking the proposed method is fast, especially when the dataset can be directly handled without stitching. the proposed method runs faster, comparing to the reported runtime in Ref. [107] as around 300 seconds when reconstructing 10^6 surface values, which is also marked in Fig. 4.18(b).

The limitation for this proposed method is also investigated. The same as the traditional least squares integration method, the proposed method can only handle gradient dataset with rectangular mesh grid. It is not able to integrate arbitrarily distributed dataset. However, very luckily, the data are generally in rectangular mesh grid in many practical optical measurements.

4.4 Fast FRD and its application in dynamic 3D sensing of specular surfaces

There are numerous applications in research and industry for metrology of specular objects [98-100, 146, 147]. Phase measuring reflectometry (or deflectometry) using computer aided gratings [148, 149] (or fringes) is a suitable and practical approach to measure specular surfaces [39, 40, 97]. From the phase of the grating, geometric information of target surface can be obtained using several different approaches. Petz and Tutsch [97] propose a deflectometric

system with fringe reflection technique, which mainly consists of one camera, one LCD screen, and a translation stage for moving the screen to different select positions. This method requires mechanical translation during both calibration and measurement procedure. Knauer et al. [39] propose a stereoscopic fringe reflectometric method with a gradient-based stereovision principle. Bothe *et al.* [40] suggest monoscopic fringe reflectometry without mechanical movement. This method is applicable for measuring almost flat specular surfaces with the height values roughly known a priori. Due to its fast measurement speed, the system has a potential to carry out dynamic shape measurement.

The phase measurement in grating reflectometry aims to determine the pattern positions on the LCD screen, then surface normal with system calibration. So far the phase shifting method or the multi-frequency phase shifting method with temporal phase unwrapping has been used to get the phase [40, 100]. With advancements in fringe processing, some novel methods with the capability of retrieving phase from single fringe pattern with high accuracy have been developed [31, 32, 34]. By using these single frame processing methods, the speed of the fringe reflectometric measurement are enhanced.

Dynamic 3D shape measurement as seen from current literature is mainly limited to measurement of diffuse objects. Su and Zhang review the development of dynamic 3D shape measurement in recent years [123, 124, 150-153]. Zhang [23, 28] proposed a real-time 3D shape measurement system with phase shifted digital fringe projection technique for diffuse objects, which utilizes a color wheel-removed digital light processing projector as the structured light source and accelerates the processing by a graphics processing unit. The fringe projection technique is more suitable for diffuse targets, while the fringe grating reflection technique is more suited for specular surfaces. My research concentrates on a fast full-field out-of-plane deformation measurement method for specular reflecting surfaces from a single frame fringe pattern and its application for dynamic shape measurement of specular reflective surfaces.

4.4.1 Principle and method for the fast full-field out-of-plane deformation method with fringe reflection technique

The basic setup of monoscopic fringe reflectometry is shown in Fig. 4.19. The system is composed of an LCD screen as a structured light source, a camera with imaging lens as optical sensing device, and a computer as a controlling and processing unit. The system is set up such that the camera observes the target and the reflection of the grating displayed on an LCD screen. The directions of incident and reflected light follow the reflection law such that the angle of reflection θ_r equals the angle of incidence θ_i . Hence, the surface normal needs to be determined, from the directions of the incident and reflected light.

Fig. 4.19. Specular surface measurement with fringe reflectometry requires a camera, a pattern displayer, and a processor.

For convenience, the camera sight ray is usually considered as the probe ray and the system sketch can be adjusted using the conventional model of camera calibration [154, 155]. As a result, the system sketch is shown in Fig. 4.20.

Fig. 4.20. Principle of surface normal calculation can be conducted.

In Fig. 4.20, a camera pixel x_p with its projection point $x_n = (X_x^C/X_z^C, X_y^C/X_z^C)^T$ in X_z^C -normalized image projection plane ($X_z^C = 1$) observes the point X on the target surface, represented by the vector, $X^C = (X_x^C, X_y^C, X_z^C)^T$ in the camera coordinate system. The point X reflects the incident light from a point S on the LCD screen, which is coded with two-dimensional fringe pattern. The incident light direction vector L can be expressed as

$$L = X - S. \quad (4.4.1)$$

Here, lowercase letters x , l , and etc. are used to denote the normalized vectors. That means

$$x := \frac{X}{\|X\|}, \text{ and } l := \frac{L}{\|L\|}. \quad (4.4.2)$$

Considering the reflection law as well as the geometric relationship among vectors of x , l and N , the surface normal N can be calculated by

$$N = -(x + l) =: \begin{pmatrix} N_x \\ N_y \\ N_z \end{pmatrix}. \quad (4.4.3)$$

Therefore, gradient data can be determined from the calculated normal vector by

$$\nabla z(x, y) = -\frac{1}{N_z} \begin{bmatrix} N_x \\ N_y \end{bmatrix} = \begin{bmatrix} p(x, y, z) \\ q(x, y, z) \end{bmatrix}, \quad (4.4.4)$$

where $p(x, y, z)$ and $q(x, y, z)$ are slope values at point $\mathbf{X} = (x, y, z)$ in x and y direction, respectively. Note the point $\mathbf{X} = (x, y, z)$ can be denoted both as $\mathbf{X}^W = (X_x^W, X_y^W, X_z^W)^T$ in real world coordinate and $\mathbf{X}^C = (X_x^C, X_y^C, X_z^C)^T$ in camera coordinate, i.e. the superscript denotes the coordinate system. The height distribution $z(x, y)$ is then calculated by integration of the gradient data [106, 145].

Fig. 4.21. Two-directional fringe pattern is used. (a) Displayed image, (b) captured image, and (c) Fourier spectrum of the captured fringe pattern (b).

Although the phase shifting method is commonly used for phase retrieval [38, 40, 78, 85, 100], reasonable faster way is to retrieve both horizontal and vertical phase maps from a single image. The single image can be designed as a two-directional fringe pattern shown in Fig. 4.21(a) and the captured fringe pattern is shown in Fig. 4.21(b) with its spectrum in Fig. 4.21(c). From the spectrum, higher order harmonics can be easily observed due to the systematic nonlinear response. The additive two-directional fringe pattern with an intensity value $f(S_x^S, S_y^S)$ at the pattern point $\mathbf{S}^S = (S_x^S, S_y^S, 0)$ can be expressed as

$$f(S_x^S, S_y^S) = 255 \times \left[\frac{1}{2} + \frac{1}{4} \cos \left(2\pi \frac{S_x^S}{p_x} \right) + \frac{1}{4} \cos \left(2\pi \frac{S_y^S}{p_y} \right) \right], \quad (4.4.5)$$

where superscript letter S denotes the screen coordinate system, p_x and p_y are fringe periods in x and y directions. Usually, periods in two directions are set as the same value, i.e. $p_x = p_y$.

There are some existing phase retrieval methods which can be used to calculate the phase map from a single fringe pattern, such as FT method, WFT method, and WT method [31-34, 61, 67, 69]. Among these methods, 2D WFR method is selected since it is least affected by noise and non-sinusoidal waveform, and has a well-established phase compensation approach. Phase unwrapping procedure is performed by using quality guided phase unwrapping approach with the ridge image as its quality map [68].

There are two sets of sinusoidal components in spectrum as shown in Fig. 4.21(c). Therefore, in order to retrieve both horizontal and vertical phase maps, the 2D WFR algorithm should be applied for two different frequency components.

4.4.2 System calibration

The system calibration process mainly consists of three steps. The first step is the calibration of the LCD screen to determine the relation between phase values and geometric dimensions in screen coordinate system. The second step is to calibrate the camera. It is suggested to use both printed calibration pattern and images reflected from a flat mirror for camera calibration. The toolbox for camera calibration in MATLAB [155] is used to determine the camera intrinsic and extrinsic parameters. The last calibration step is to establish the relationships among screen coordinate system, world coordinate system, and camera coordinate system. Although unwrapped phase is not absolute phase, it is still able to calibrate the system and measure the surface out-of-plane deformation if the non-deformed region is known or another calibration device is used to calibrate the system, which is commonly possible in many applications. The calibrated system in camera coordinate system is shown in Fig. 4.22.

Fig. 4.22. Calibration is successfully carried out for a monoscopic fringe reflectometric system.

4.4.3 Experiments

Experiment 1: out-of-plane deformation measurement of a piece of silicon wafer with Fast FRD method

In order to verify the feasibility and performance of the proposed out-of-plane deformation measurement method, a series of experiments are carried out. The photo in Fig. 4.23(a) shows the experimental setup, which is basically composed of an LCD screen (Dell® E193FP with a resolution of 1280×1024 pixels) and a CCD camera (The Imaging Source® DMK 41AU02 with a resolution of 1280×960 pixels) with its camera lens. The specimen is a polished wafer as shown in Fig. 4.23(a). It is clamped on all sides and a load is applied with a micrometer head (Mitutoyo® Digimatic Electronic Micrometer Head with a resolution of 0.001 mm) at the centre. The Fringe patterns are shown on the LCD screen and the camera captures the reflection of the fringe from the wafer.

Fig. 4.23. Experiment is carried out and studied on fringe reflectometry. (a) Experimental setup, (b) captured two-directional fringe pattern, (c) and (d) retrieved wrapped phase maps in x and y directions, (e) and (f) unwrapped phase maps in x and y directions, (g) and (h) measured slope maps in x and y directions.

A typical deformed fringe pattern is shown in Fig. 4.23(b). With fringe pattern analysis techniques (WFR method is used in this work), the wrapped phase maps in both directions can be retrieved as shown in Fig. 4.23(c) and (d). Images in Fig. 4.23(e) and (f) show the unwrapped phase maps respectively. According to the reflectometric principle, the gradient data can be determined and are shown in Fig. 4.23(g) and (h).

Fig. 4.24. Results from measurement show the feasibility of the proposed method. (a) Three-dimensional results, and (b) single line profiles.

In the experiment, the out-of-plane deformations in steps of 50 microns, 100 microns, 150 microns, and 200 microns, are applied. The experimental results from the calibrated system are shown in Fig. 4.24. After integration of the gradient data, three-dimensional results can be reconstructed as shown in Fig. 4.24(a) magnified for clarity. The line profile of the deformed wafer along a select line are shown in Fig. 4.24(b), which shows that the out-of-plane deformation is measured successfully and accurately.

Fig. 4.25. Small out-of-plane deformation is measured out. (a) Three dimensional surface difference, (b) single line profile of the height difference with 100 microns loading as their reference.

In order to study the system performance for small out-of-plane deformations, loading caused by relative movement of the micrometer head of ± 5 microns are measured as shown in Fig. 4.25. Three-dimensional surface changes are shown in Fig. 4.25(a), along with the line profiles in Fig.

4.25(b). From the measurement results, the whole surface can be successfully measured. Since the resolution of the loading device is 1 micron, the measured results have small deviations (within ± 1 micron) between the desired values.

Experiment 2: dynamic 3D sensing of water wave with fast FRD

A series of experiments are carried out to verify the feasibility and performance of the proposed dynamic 3D sensing approach. The experimental setup is basically composed of an LCD screen (PHILIPS® 190S with a resolution of $1,024 \times 1,280$ pixels) and a high speed camera (The Photron® FASTCAM 1024PCI with a resolution of 500×500 pixels at a frame rate of 60 frames per second). The fringe periods of the two-directional fringe pattern are $p_x = p_y = 20$ pixels. Similar to normal monoscopic fringe reflectometry, three calibration steps, screen calibration, camera calibration, and geometric calibration, are carried out to calibrate the measuring system. But it should be noted that the unwrapped phase used in calibration is based on the same seed pixel of the 3D phase unwrapping in measurement stage. Therefore the unwrapped phase maps at different time are based on the reference.

Fig. 4.26. A typical experimental result at $t = 0.05s$ is investigated. (a) Captured fringe pattern, (b) amplitude of Fourier spectrum shown with its natural logarithm value, (c) and (d) retrieved wrapped phase maps in x - and y -directions with the 2D WFR method (with boundary region removed).

Here the example scene is water wave spreading from center to surroundings. A typical fringe image captured from the high speed camera is shown in Fig. 4.26(a), and Fig. 4.26(b) shows its related Fourier spectrum. With the assistance of the 2D WFR method, the wrapped phase in both x and y directions can be retrieved as shown in Fig. 4.26(c) and (d). Note due to the windowed strategy, the boundary region is unreliable and then not used for further process.

Fig. 4.27. Investigation is carried out for reflectometric result at $t = 0.05s$. (a) Slope in x-direction $p(x, y, z)$, (b) slope in y-direction $q(x, y, z)$, and (c) height distribution $z(x, y)$ from integration.

A 3D (two spatial dimensions and one temporal dimension) phase unwrapping process with a common seed pixel for all frame phase maps is carried out. The pixel the with the highest ridge value from WFR method is selected as the seed pixel for the whole 3D phase dataset. The resultant unwrapped phase maps are used for the gradient calculation. According to the principle of fringe reflectometry in Section 2, slope data $p(x, y, z)$ and $q(x, y, z)$ in Eq. (4.4.4) can be calculated out as shown in Fig. 4.27(a) and (b), once unwrapped phase maps are obtained for determination of pattern points S . Least squares integration method is used to determine the height variation of water wave as shown in Fig. 4.27(c) [106, 145]. The height variation range in this experiment is about 20 microns and the field of view in this experiment is around $150mm \times 150mm$.

Fig. 4.28. Some 3D shapes of dynamic water wave at different moments are reconstructed. (a) $t = 0.00$ s, (b) $t = 0.05$ s, (c) $t = 0.10$ s, (d) $t = 0.15$ s, (e) $t = 0.20$ s, (f) $t = 0.25$ s, (g) $t = 0.30$ s, (h) $t = 0.35$ s, and (i) $t = 0.40$ s.

After frame-by-frame reconstruction, the dynamic scene can be digitalized. Here, nine frames of typical 3D shape results from 0.00s to 0.40s with an interval of 0.05s are shown in Fig. 4.28, from which the dynamic water wave spreading can be nicely demonstrated.

Since the monoscopic fringe reflection technique and the 2D WFR method are adopted [32, 67], this approach is more suitable to sense the dynamic shape of a continuous specular surface, which is almost flat during its shape variation.

4.4.4 Discussion

The experimental results have demonstrated that grating reflectometry can successfully measure out-of-plane deformation of specular surfaces. The major advantages of the proposed method are:

- Non-contact measurement. Usually the test specular object has requirements of surface roughness, hence contact methods are not suitable and a non-contact solution is commonly preferred.
- Accurate and precise measurement. The WFR method is an unbiased phase estimation approach with good performance of noise reduction [69], thus the measurement can be carried out with high accuracy and high precision, once the system is calibrated.
- Full-field measurement. As long as the fringe pattern can be observed, the whole field of view in camera can be measured without any mechanical scanning procedure.
- High resolution. Since the in-plane calculation process is pixel by pixel, the lateral resolution is the same as camera resolution and out-of-plane measuring resolution is determined by the system configuration but typically reflectometric systems have micron, or even nanometer resolution in out-of-plane direction [156].
- Fast data acquisition. The whole measurement only requires a single shot. Therefore, the proposed technique has ability to measure dynamic out-of-plane deformation.

This work presents a fast, high resolution, full-field, and non-contact method to measure out-of-plane deformation of specular surfaces. The principle and calibration of deformation measurement system is described. From our experimental studies, this method can successfully measure the out-of-plane deformation of the specular reflecting surfaces with a single shot of two-directional fringe pattern and the validity of this novel method to sense the dynamic 3D shape of a continuous specular surface.

4.5 Partially diffuse and specular surface profilometry with combined fringe projection and reflection technique

Three dimensional (3D) shape measurement techniques have been well researched and increasingly adopted in industry. Applications include but not limited to reverse engineering, quality control, animation, medical diagnosis, and so on. As members of the 3D shape metrology family, FPP [23, 35, 38] and FRD [39, 40] are specifically developed for diffuse and specular targets, respectively.

In some instances, the target to be measured is neither completely diffuse nor fully specular. For these objects both FPP and FRD can be used. Thus it would be necessary to see which approach and under what conditions would be the appropriate choice. This work focuses on comparing the performance of FPP and FRD with both experimental and simulation studies. Samples which can be used for both methods is chosen and tests are conducted with and without fringe processing to see if defects programmed in the sample can be highlighted. To justify this claim, simulations are done.

4.5.1 Principles with difference

The basic principle of FPP is optical triangulation. A digital projector is used to generate the patterns to be projected, which are then recorded using a digital camera, and processed using the ubiquitous phase shifting routines. As shown in Fig. 4.29(a), two points in the 3D space can be determined along both incident ray and observing ray in fringe projection profilometric system. Once the system is calibrated, the optical centers O_c and O_p of both camera and projector are uniquely determined. For each camera pixel C , its position in world coordinates can be determined through camera calibration parameters. Moreover, the world coordinates of the related point P from the projector side, which is ‘observing’ the same target point T , can be determined with both fringe phase and projector calibration parameters. Therefore, the two rays can be determined to measure 3D coordinate of the target point T .

Fig. 4.29. Schematic diagrams of FPP (a) and stereoscopic FRD (b) show their difference in configuration.

The FRD method is based on the law of reflection and uses an LCD screen to display the software generated fringe patterns. However, in a common monoscopic FRD system, only the observing ray can be determined as the requirement of two position-known points are satisfied, but the incident ray cannot be determined with only one position-known point S_1 (Fig. 4.29(b)). Several solutions have been suggested to overcome this issue. The first solution is to regularize this ill-posed problem [40] by assuming the height of the surface is approximately known. This approach is best suited for nearly flat surfaces. The second approach adds one or more additional pattern points along the incident ray by placing the LCD screen at different positions [97]. As shown in Fig. 4.29(b), the third solution uses a second camera to form a stereoscopic fringe reflectometric system [39]. In this study, the third solution, stereoscopic fringe reflectometry is adopted due to its larger measuring range and no mechanical movement.

4.5.2 Experiment

In order to compare the performance of these two techniques in measuring partially diffuse and specular surfaces condition, the measuring system is set up with both FPP and FRD as shown in Fig. 4.30. The projector used for FPP is 3MTM Pocket Projector MPro120 with a resolution of 768×1,024 pixels. Two USB CCD cameras (Imaging Source[®] DMK 41AU02 with a resolution of 960×1,280 pixels and DMK 41BU02 also with a resolution of 960×1,280 pixels) are utilized for stereoscopic fringe reflectometric system and shared with fringe projection system. In

addition, An LCD screen, PHILIPS® 190S with a resolution of 1,024×1,280 pixels, is used to display fringe patterns for FRD. The measuring volume is located in the common field of view of the two cameras and the projector. The fringe patterns can be generated by the computer and then shown on the LCD display or the projector. Hence the fringe patterns can be captured by the two cameras for further analysis with FPP method or FRD method.

Fig. 4.30. Experimental setup is composed of two cameras, a projector and an LCD screen.

For FPP, there are a few calibration methods [87, 89, 91, 93, 94, 119]. Here, stereovision calibration method [94, 157, 158] is adopted with considering the projector as a camera, since its projected phase values can be used to determine the related pixel coordinates in projector. On the other hand, when calibrating FRD system, the calibration data will be obtained both from camera calibration patterns located within the measuring volume, which the tested sample will be placed in, and from the pattern points on screen reflection when the reference mirror is placed at several different tilted positions. The calibrated FPP system and the FRD system after system calibration are shown in Fig. 4.31.

Fig. 4.31. System calibration is carried out for both systems. (a) FPP system, and (b) FRD system.

A sample with metal texture and several dents on its surface is chosen as the measuring target as shown in Fig. 4.32(a). The fringe patterns captured with FPP are shown in Fig. 4.32(b), while those recorded using FRD are shown in Fig. 4.32 (c-d).

Fig. 4.32. Test sample is a metal plate and its fringe patterns with FPP and FRD are captured. (a) A photo of tested target, (b) a captured fringe pattern with FPP, (c) a captured vertical fringe pattern with FRD, and (d) a captured horizontal fringe pattern with FRD.

By visual observation of the fringe patterns in Fig. 4.32(b-d), the FRD seems to highlight the fringe distortions near the defect more clearly than the FPP. Phase shifting techniques was then adopted for both methods to see if this processing would improve the results for the FPP. With phase shifting plus temporal phase unwrapping technique, the absolute phase can be retrieved as shown in Fig. 4.33.

Fig. 4.33. Absolute phase is retrieved from fringe patterns with phase shifting technique and temporal phase unwrapping. (a) FPP, and (b-c) FRD.

Only the data in region of interest are used in the following processing steps. In FRD, the absolute phase values are used for slope calculation and rough height estimation as shown in Fig. 4.34(a-b), while the absolute phase can be directly converted to three dimensions in FPP shown in Fig. 4.34(c). Since the surface is neither completely specular not fully diffuse both methods show some noise. A least squares integration procedure [106, 145] is applied to integrate the slope data with FRD to reconstruct the 3D shape of the tested surface as shown in Fig. 4.34(d).

Fig. 4.34. Measuring results with FPP and FRD are compared. (a) Slope in y -direction, (b) slope in x -direction, (c) height directly from FPP, (d) height integrated with gradient data from FRD. (e) three-dimensional result from FPP, and (f) three-dimensional result from FRD.

After the in-plane calculation to determine x and y results, the final 3D results are shown in Fig. 4.34(e-f). Two lines in each height map (Fig. 4.34(c-d)) are plotted in Fig. 4.35 to show the difference of the reconstructed height values with these two techniques.

Fig. 4.35. Comparison of one-line profiles in Fig. 4.34(c-d) is carried out. (a) Line a , and (b) line b .

The processed result indicates that FPP can also detect the dents but with lower signal-to-noise ratio compared with FRD, which is able to obviously show height variations from the reconstructed 3D shape in Fig. 4.34(f) even if the gradient data obtained are noisy.

4.5.3 Simulation

In order to have a further study on the measurement precision of these two techniques under partially diffuse and specular surface condition, computer simulation is carried out with the same system calibration parameters. The simulated target expressed in Eq. (4.5.1) becomes the benchmark for accuracy and precision study.

$$z = 0.003 \times (x^2 + y^2). \quad (4.5.1)$$

The in-plane dimensions are within $[-10, 10] \text{ mm} \times [-10, 10] \text{ mm}$ with a sampling resolution of 512×512 pixels.

Firstly, the simulation is set under the preferred condition for each technique. Hence the phase noise with each technique is small. The phase noise standard deviation (STD) is set as 0.0100

radians in FPP, and 0.0100 radians in FRD, which are close to usual values in actual experiment under their preferred condition when 16-steps phase-shifting fringe patterns are used. The reconstructed 3D shapes are shown in Fig. 4.36(a) with FPP and Fig. 4.37(a) with FRD. The images in Fig. 4.36(b) and Fig. 4.37(b) show their related height errors.

Fig. 4.36. Simulation is carried out with FPP under different phase noise conditions. STD = 0.0100 radians: reconstructed surface (a) and height error (b). STD = 0.0390 radians: reconstructed surface (c) and height error (d).

Secondly, the phase noise is set as the same noise level of that in experiment, which can be calculated by using phase after windowed Fourier filtering as phase benchmark. In the above experiment, the STD of phase noise in FPP is calculated to be 0.0390 radians, and the STD of phase noise is 0.5667 radians in FRD. The same phase noise is simulated and put into reconstruction for precision study. Images in Fig. 4.36(c) and (d) show the reconstructed 3D shape and height error with FPP, while Fig. 4.37(c) and (d) show the reconstructed 3D shape and height error with FRD in simulation.

Fig. 4.37. Simulation is carried out with FRD under different phase noise conditions. STD = 0.0100 radians: reconstructed surface (a) and height error (b). STD = 0.5667 radians: reconstructed surface (c) and height error (d).

From the simulation with our configuration, height precisions with FPP are 0.0079 mm when phase noise STD = 0.0100 radians and 0.0307 mm when phase noise STD = 0.0390 radians. With FRD, height precisions are 1.3314×10^{-4} mm when phase noise STD = 0.0100 radians and 1.0991×10^{-4} mm when phase noise STD = 0.5667 radians. It is shown that the precision with FRD is much higher than that with FPP. In addition, under partially diffuse and specular surface condition when phase noise is obviously severer than under their preferred conditions, FRD is not so sensitive to phase noise compared with FPP.

4.5.4 Discussion

The accuracy of a fringe-based system does depend on the system calibration. The sensitivity to height with FPP depends on the system geometric configuration as well as the fringe frequency

used. With FRD these two factors do not directly determine the sensitivity to height, but the sensitivity to gradient. Thus once a fringe-based system (with FPP or FRD) is fixed, the system sensitivity is already determined. Thus the height measuring precision depends on the phase measuring precision for FPP. But for FRD, the phase measuring precision can only determine the slope measuring precision, and the height measuring precision will be influenced by the integration process from slope data. Hence, the height measuring precision with FRD is determined by the phase measuring precision and the applied integration method. Under the condition of our experiment and simulation, the sensitivity of FRD is higher than FPP.

For measuring partially diffuse and specular surfaces, the main problem for both methods is the phase measuring precision is not good as that under their preferred conditions. But actually there is a property or so-called ascendancy for these partially diffuse and specular surfaces that they can be measured by using both techniques. Thus a natural idea is to combine these two techniques. FPP can be used to estimate the height and used as an input to FRD for slope calculation, so the time-consuming iterative height estimation process can be avoided. The result from the combined method is shown in Fig. 4.38.

Fig. 4.38. The 3D shape is reconstructed with combined method.

Three-dimensional shape metrology using FPP and FRD is an effective way to reconstruct three-dimensional shape with different reflectance properties. FPP is used for measuring objects with diffuse surfaces and relies on the principle of triangulation, while FRD is used for specular specimens based on principle of reflection. While fringe projection technique directly provides the profile, fringe reflectometry measures the slope of the surface from which the profile is

integrated. For both methods, projected patterns are computer generated and phase shifting methods are applied.

For fringe projection technique, the sensitivity of three-dimensional measurement directly depends on system geometric configuration. The precision of dimensional measurement directly relies on the system sensitivity and the precision of phase measurement. For the fringe reflection technique, the sensitivity of dimensional measurement does not directly depend on geometry, which actually only directly determines the slope measuring sensitivity. Hence, phase measuring precision only directly yields the precision of slope measurement, which will influence the accuracy and precision of three-dimensional dataset through the following integration process. The performances of these two fringe-based techniques are analyzed with respect to their sensitivity and precision when measuring partially diffuse and specular reflective targets. The measuring precision of both techniques are compared and discussed in simulation and with actual experiment.

4.6 Transparent surfaces profilometry with double-side fringe reflection technique

Gratings have been widely used in metrology due to its convenience in generation, abundance in information, and practicality in analysis. Digital methods for grating analysis [149] provide qualitative information of deformation, shape, etc. by demodulating the phase from these patterns. Phase retrieval from fringe pattern(s) consequently is an interesting topic with much effort given by researchers around the world. Phase shifting methods [35, 36, 47] have been widely studied for grating analysis for about 30 years, and it is usually utilized for shape measurement. For dynamic measurement, fringe phase is usually retrieved from a single fringe pattern using transform-based methods [29-32, 34].

However, some more complicated patterns appear in practical measurement under special conditions. In the grating reflection technique [39, 40, 97, 148], for example, when measuring

transparent objects, each pattern is actually the superimposition of fringes reflected from both the front and rear surfaces. This type fringe pattern cannot be demodulated by using the conventional phase shifting technique. Xu et al. [115, 116] use gray code and line shifting method [117] to solve the issue of double-side reflection in measuring automotive glass by setting a threshold to remove the reflected lines from the rear surface. This approach works when the lines reflected from front interface is stronger than those from rear surface in the whole field of view. If not, it is not easy to get rid of the rear lines by simply setting a threshold on image intensity.

This study focuses retrieval of the phase from two surfaces patterns without using a thresholding approach.

4.6.1 Methodology for phase retrieval from double-side reflective fringe patterns

There is a phase retrieval issue when measuring transparent surface with fringe reflection technique. Since the both sides of the transparent object will reflect the fringes into the camera, the captured fringe pattern will be similar to that shown in Fig. 4.39.

Fig. 4.39. A typical double-side reflective fringe pattern is complex to analyze.

Theoretically, the n th step phase-shifted fringe intensity of the double-side reflective fringe pattern can be expressed as

$$I_n = A + B_f \cos(\phi_f + \delta_n) + B_r \cos(\phi_r + \delta_n) \quad (4.6.1)$$

where A is the background intensity, B_f and B_r are the fringe intensity modulation, ϕ_f and ϕ_r are fringe phase values from front and rear surfaces, respectively. The n th phase shifting amount $\delta_n = 2\pi n/N$, and N is the total number of phase shifting. Eq. (4.6.1) can be rewritten as

$$I_n = A + C \cos \delta_n + S \sin \delta_n \quad (4.6.2)$$

where

$$C = B_f \cos \phi_f + B_r \cos \phi_r \quad (4.6.3)$$

$$S = -B_f \sin \phi_f - B_r \sin \phi_r \quad (4.6.4)$$

According to the least squares principle, the parameters C and S can easily be determined as

$$C = \frac{2}{N} \sum_{n=0}^{N-1} I_n \cdot \cos \delta_n \quad (4.6.5)$$

$$S = \frac{2}{N} \sum_{n=0}^{N-1} I_n \cdot \sin \delta_n \quad (4.6.6)$$

Hence,

$$B_f \cos \phi_f + B_r \cos \phi_r = \frac{2}{N} \sum_{n=0}^{N-1} I_n \cdot \cos \delta_n \quad (4.6.7)$$

$$-B_f \sin \phi_f - B_r \sin \phi_r = \frac{2}{N} \sum_{n=0}^{N-1} I_n \cdot \sin \delta_n \quad (4.6.8)$$

The total intensity modulation is

$$M = \sqrt{C^2 + S^2} = \frac{2}{N} \sqrt{\left(\sum_{n=0}^{N-1} I_n \cdot \cos \delta_n \right)^2 + \left(\sum_{n=0}^{N-1} I_n \cdot \sin \delta_n \right)^2} \quad (4.6.9)$$

However, there are four unknowns and only two equations (4.6.7) and (4.6.8) although the right side of these equations can be determined through phase-shifted fringe intensity. This is why the conventional phase shifting method is unable to retrieve the phase in the presence of fringes from the rear surfaces. Normally the reflection from rear surface is reduced by diminishing the rear fringe modulation B_r in order to weaken the influence from the two terms of $B_r \cos \phi$ and $B_r \sin \phi$.

In the proposed method, another group of phase shifted fringe patterns with a frequency $f' = k \cdot f$ (k is close to but not equal to 1) is used in order to provide more information by capturing fringe patterns I'_n . Thus, two additional equations can be obtained as

$$B_f \cos(k \cdot \phi_f) + B_r \cos(k \cdot \phi_r) = \frac{2}{N} \sum_{n=0}^{N-1} I'_n \cdot \cos \delta_n \quad (4.6.10)$$

$$-B_f \sin(k \cdot \phi_f) - B_r \sin(k \cdot \phi_r) = \frac{2}{N} \sum_{n=0}^{N-1} I'_n \cdot \sin \delta_n \quad (4.6.11)$$

We thus have four unknowns with four equations (4.6.7), (4.6.8), (4.6.10), and (4.6.11). The variables, B_f , B_r , ϕ_f and ϕ_r can be solved numerically. A nonlinear least squares approach can be used with the following steps for numerical solution:

Step 1. Calculate the total intensity modulation M from Eq. (4.6.9). Mask the region of interest by setting a threshold to M . Pixels without fringe have very low modulation values and can be highlighted.

Step 2. Select a starting point for phase calculation. Using initial values of $[B_{f0}, B_{r0}, \phi_{f0}, \phi_{r0}] = [128, 128, 0, 0]$, four unknown variables B_f , B_r , ϕ_f and ϕ_r of this point can be determined with a nonlinear least squares approximation.

Step 3. Implement the flood-fill algorithm by using the previous resultant B_f , B_r , ϕ_f and ϕ_r as the initially approximated values to diminish the iterative calculation time.

Step 4. Generally the resultant phase is already unwrapped, but probably a few phase jumps may present. Unwrap the phase to get a continuous phase distribution, if necessary.

4.6.2 Simulation

In order to show the feasibility and effectiveness of the proposed method, a simulation study is carried out. In this simulation, double-side reflective fringe patterns are generated as

$$I_n = \frac{255}{2} + B_f \cos(\phi_f + \delta_n) + B_r \cos(\phi_r + \delta_n) + I_{noise} \quad (4.6.12)$$

$$I'_n = \frac{255}{2} + B_f \cos(k \cdot \phi_f + \delta_n) + B_r \cos(k \cdot \phi_r + \delta_n) + I_{noise} \quad (4.6.13)$$

where the simulated fringe modulation are not uniform in the field of view which can be expressed by $B_f = 42 + 18 \cdot \text{Hann}(\cdot)$, and $B_r = 24 + 8 \cdot \text{Hann}(\cdot)$. The function of $\text{Hann}(\cdot)$ is the 2D Hann window shown in Fig. 4.40(a). In simulation, the image size is 256 pixels by 256 pixels. The phase maps of front and rear sides are simulated by

$$\phi_f = \frac{2\pi(0.1x + 5 \times 10^{-4}x^2 + y)}{9} \quad (4.6.14)$$

$$\phi_r = \frac{2\pi y}{16} + 35 \quad (4.6.15)$$

Fig. 4.40. Images are simulated for investigation of proposed method. (a) Hann window, (b) true phase map of the front reflection ϕ_f , and (c) true phase map of the rear reflection ϕ_r . Phase maps are wrapped within $(-\pi, \pi]$ for visualization.

The simulated front and rear phase maps are shown in Fig. 4.40(b) and (c) wrapped from $(-\pi, \pi]$ for visualization. A four-step phase shifted fringe patterns are generated with phase shifts $\delta_n = 2n\pi/4$, $n = 0, 1, 2, 3$. The grating periods are 9 and 16 pixels for front and rear surfaces, respectively. Noise, I_{noise} , is simulated with normally distributed random noise with standard deviation (Std.) $\sigma = 0, 1, 2, 3, 4, 5, 6$ for different noisy conditions. The frequency multiple for the second grating is $k = 1.1$. The simulated fringe patterns with additive noise of Std. $\sigma = 3$ are shown in Fig. 4.41(a) and (b). This noise level is close to that found in practical measurement using the fringe reflection technique.

Fig. 4.41. Double-side reflective fringe patterns are simulated with additive noise of $\sigma = 3$. (a) I , and (b) I' with $k = 1.1$.

Phase results of fringe patterns in Fig. 4.41(a) with traditional phase shifting method are demonstrated in Fig. 4.42. Comparing with true wrapped phase distribution in Fig. 4.40, it is easy to find the obtained phase results are influenced by both front and rear phase distributions.

Fig. 4.42. An incorrect wrapped phase is retrieved with traditional phase shifting method.

The proposed method is then applied to retrieve the phase from the simulated noisy fringe patterns. As a result, two phase maps and two modulation maps can be simultaneously demodulated from the two sets of phase-shifted fringe patterns with different frequencies. The phase maps of the front reflection ϕ_f and the rear reflection ϕ_r can be retrieved with the modulation maps B_f and B_r as shown in Fig. 4.43. Again, the demodulated phase distribution is rewrapped for good visualization of the existing of noise on phase maps.

Fig. 4.43. Wrapped phase and modulation are retrieved from double-side reflective fringe patterns. (a)

Front phase ϕ_f , (b) front modulation B_f , (c) rear phase ϕ_r and (d) rear modulation B_r .

Sometimes the resultant phase values might jump with 2π during the optimization process. In order to get rid of the possible phase jumps in phase distribution, phase unwrapping could be implemented. Quality-guided phase unwrapping technique [68, 72, 77, 127] is utilized with adoption of the retrieved modulation maps as related quality maps for front and rear phase maps, respectively. The unwrapped phase maps shown in Fig. 4.44(a) and (c) can be retrieved with a certain corresponding unwrapping paths as shown in Fig. 4.44(b) and (d). In path maps, the brighter pixel means it is unwrapped earlier.

Fig. 4.44. Phase is unwrapped with showing its corresponding unwrapping path maps. (a) Front unwrapped phase ϕ_f , (b) front unwrapping path map, (c) rear unwrapped phase ϕ_r and (d) rear unwrapping path map.

The front and rear phase errors are presented in Fig. 4.45(a) and (b) which show the front phase error is smaller than rear one. It is because the fringe modulation of reflected fringe patterns from the rear surface B_r is lower than that of the front surface B_f in simulation settings.

Fig. 4.45. Phase errors are compared. (a) Front phase error when noise Std. $\sigma=3$, (b) rear phase error when noise Std. $\sigma=3$, and (c) phase error Std. increasing with fringe noise Std. compared with conventional phase shifting methods.

Furthermore, the noise influence is studied by adding noise with different standard deviation ranging from 0 to 6. The phase error is compared with the well-known conventional phase shifting algorithm [35]. Of course the conventional algorithm is not able to solve the double-side reflection issue, so the rear fringe modulation B_r is set as 0 in Eq. (4.6.12) for the conventional algorithm. The relation between standard deviation of phase error of each method and standard deviation of fringe noise is shown in Fig. 4.45(c). The proposed method uses twice the number of fringe patterns than the conventional N-step phase shifting method since frequency multiplication is utilized for more information to solve the double-side reflection issue. Statistically speaking, phase errors due to random fringe noise will be smaller if more repeatable data are provided. Hence it is found that the phase error of the proposed method with 4-step phase shifting by 2 frequencies is lower than that of the conventional 4-step phase shifting method. Compared to the 8-step phase-shifting method, the proposed method uses the same total image number to estimate two more parameters and therefore its phase error is higher.

4.6.3 Discussion

A novel method for dual surface fringe reflectometry is suggested with several merits which cannot be achieved by conventional phase shifting methods. The phase and modulation values of the fringe reflected from both sides can be retrieved simultaneously by using the novel method.

While the novel method shows its abilities, some limitations are also observed. When the phase difference between the front and rear fringes

$$\Delta\phi = 2m\pi \quad \text{and} \quad k \cdot \Delta\phi = 2m'\pi \quad (4.6.16)$$

where m and m' are integral numbers, equations (4.6.7), (4.6.8), (4.6.10), and (4.6.11) will be converted to conventional single side fringe intensity situation. The nonlinear least squares estimation of the four parameters B_f , B_r , ϕ_f and ϕ_r will be inaccurate in these areas. That means the phase measuring range is limited by the phase difference between front and rear fringes. Currently the algorithm is slower than the conventional phase shifting algorithm. Future work will focus on speeding up the algorithm to meet the requirement for more applications.

4.7 Conclusion

This chapter describes our recent research on the 3D shape measurement techniques for diffuse, specular, and transparent surfaces. According to the distinct developing stages of measurement techniques for different types of surfaces, different research aspects are investigated to push these techniques forward. For diffuse surface profiling, flexible and accurate calibration method and surface reflection property are our investigation points, since many approaches for this type of surface with fringe projection technique are already established. For specular surface, dynamic measurement and integration method are our interests. How to distinguish the front and rear reflection from the two interfaces of a transparent plate or lens is the major problem for transparent surface shape measurement.

Section 4.2 proposes a least squares calibration method for FPP with an improvement of considering camera lens calibration in the system model. Compared to the previous method, the proposed one is more accurate, and at the same time it will not increase the complexity of the calibration procedure.

Section 4.3 reports an improved least squares integration method to enhance the integration accuracy without losing the capability of handling large dataset.

Section 4.4 proposes a fast shape measurement method for specular surfaces and demonstrates its feasibility with measuring dynamic scenes.

Section 4.5 discusses the surface reflection property and its influence on the FPP and FRD by studying the partially diffuse and specular surface profiling.

Section 4.6 describes an approach to analyze the double-side reflection fringe patterns in transparent surface measurement. Both front and rear phase data can be retrieved from some phase shifted multi-frequency fringe patterns with this method. The phase retrieval is the main issue in transparent surface profiling, so the transparent surfaces are able to be measured as a result.

CHAPTER 5 Conclusions and recommendations for future work

Accomplishments and recommendations for future work are summarized in this chapter.

5.1 Conclusions

The following contributions are made through my research and described in this thesis.

- A phase error analysis and compensation method for non-sinusoidal waveforms in phase shifting technique is proposed.

The phenomena of non-sinusoidal waveforms are commonly observed in digital fringe imaging based methods due to the nonlinear response of digital elements. It influences the phase retrieval with phase shifting technique obviously especially when the phase shifting number is small. The proposed method can compensate this sort of phase error with an iterative calculation based on analysis of the phase error theoretically. Simulation and experimental results demonstrate the feasibility of the proposed compensation method.

- A framework to identify the invalid phase in multi-frequency phase unwrapping is developed.

Phase retrieval from fringe patterns is a primary procedure in fringe projection profilometry. Only accurate phase values result in three dimensions with certain accuracy. Phase shifting method plus temporal phase unwrapping approach provides not only the unwrapped absolute phase, but also the modulation map, background map, root mean square errors of least squares fitting, and phase relationship between two neighboring pixels, which can be used for the identification of phase invalidity. A practical phase retrieval frame work is presented to accurately calculate the absolute phase within reliable regions only, with which those unavailable phase points can be automatically identified with thresholds selection and criterion testing and then removed or interpolated according

to applications. Experimental results show practical feasibility of the proposed framework.

- A comparison of transform-based single-frame fringe analysis methods is conducted.

As a structure-light-based optical profilometry, FPP is widely used for 3D surface shape profiling. Phase extraction methods using a single fringe pattern in FPP are compared by both simulations and experiments. Principles of FT, WFT and WT methods in fringe pattern processing are introduced. Implementation of 1D and 2D algorithms and phase compensation are discussed. Noisy and non-sinusoidal waveform conditions are discussed and compared by both simulations and experiments. The merits and limitations are indicated for these fringe pattern processing methods.

- The quality guided phase unwrapping is reviewed and improved with comparisons on quality maps and guiding strategy.

Quality-guided phase unwrapping is a widely used technique. Various quality definitions and guiding strategies are possible. A detailed comparison is necessary to select a good quality map and a good guiding strategy. For quality maps, under the noisy condition, transform-based methods perform satisfactory, while under discontinuity condition, phase unwrapping is still a challenging problem and so far cannot be solved by quality-guided phase unwrapping alone. For guiding strategies, classical, two-section, and stack-chain guiding strategies are compared with showing respective advantages and disadvantages. To achieve the most reasonable unwrapping results, the classical guiding strategy with a data structure of indexed interwoven linked list is fastest. If a small unwrapping error is tolerated, stack-chain guiding strategy can perform even faster.

- An improved least squares calibration method for FPP system is proposed with a consideration of camera lens distortion in the system model.

By using the least squares fitting approach, the calibration procedure for FPP becomes more flexible and easier, since neither the measurement of system geometric parameters, nor precise control of plane moving is required. With a consideration of camera lens

distortion, a modified least-squares calibration method for FPP is proposed. In this method, camera lens distortion is involved into the mathematical description of the system for least squares fitting to reduce its influence. Both simulation and experimental results are shown to verify the validity and easiness of this modified calibration method.

- A novel fast FRD method is proposed for dynamic specular surface shape measurement. Full-field out-of-plane deformation measurement of specular surfaces can be implemented quickly and conveniently using fringe reflectometry. The system configuration is simple and processing is fast. With the assistance of an advanced fringe pattern processing technique, the windowed Fourier ridges method, only a single two-directional fringe pattern is necessary for determination of the deformed surface profile. Thus, the whole measurement only requires a single image. Dynamic full-field three-dimensional sensing of specular reflective surfaces can be conveniently implemented with this technique. Experimental results of water wave variations are shown to demonstrate the feasibility of the proposed approach.

- An improved least squares integration method is implemented with high speed, high accuracy, and large dataset processing ability.

The proposed method is investigated by several simulations. Significant improvement in accuracy is verified by comparing with the traditional least squares integration method. Moreover, a comparison with the RBFs based integration method is carried out, and the results show the accuracy of both methods are comparable. The merits of the proposed method are accurate, fast, and able to handle large datasets. In summary, this least squares integration with iterative compensation method is an effective and accurate 2D integration tool to handle shape from slope problems in some gradient measuring based optical inspection applications.

- A phase retrieval method is proposed for double-side reflection fringe patterns in measuring transparent surfaces.

Successful retrieved results indicate the feasibility of the proposed method and study on

noise influence in simulation is also presented to show its capability of handling fringe patterns from practical fringe reflection technique. Although there are some issues to be considered to improve the performance of this new method, it is a suggested solution for phase retrieval from double-side reflective fringe patterns.

5.2 Recommendations for future work

- In the calibration of a FPP system, the number of feature points for camera calibration should be larger than what are current being used in measurement, since the fringe projection technique itself has a large number of dataset. Every part of the system calibration should have the same level of data amount to ensure the measuring accuracy and precision.
- A flexible calibration method is necessary to develop for the FRD system. So far, the calibration method is complicated which is one of the issues to be solved for practical applications in the future.
- The issue of transparent surface profiling is still quite challenging and not completely solved, especially for actual measurements. The fringe phase retrieval of double-side reflection and the refraction process still need to be studied in order to get both front and rear surfaces.
- Several prototypes of measurement systems are developed for the research on 3D shape metrology purpose as shown below. Compact optical 3D shape measurement systems with fringe projection technique and fringe reflection technique are going to develop for flexible usage in various applications.

For profiling diffuse surfaces, several prototypes of fringe projection profilometric system are developed with a compact design. By using the novel digital light projectors, the

profilomter can be packaged into an even compact size. In addition, a software with a friendly user interface is developed as well as shown in Fig. 5.1.

Fig. 5.1. Compact 3D profilometers are shown with the user interface of software.

For measuring specular and transparent surfaces, a prototype of digital fringe reflectometric system is developed as shown in Fig. 5.2(a).

Fig. 5.2. Digital fringe reflectometer (a) and measuring result (b) are shown: local height variations of a measured Chinese magic mirror. Unit on color bar: nm.

The performance of the system is also tested by various specular samples. As shown in Fig. 5.2(b), the surface of a Chinese magic mirror is measured with showing the detailed structure with tiny local variations on surface height.

Based on the current technology, compact optical 3D shape measurement systems with fringe projection technique and fringe reflection technique are going to develop.

Publications

Journal paper:

[J-1] B. Pan, Q. Kemao, L. Huang, and A. Asundi, "Phase error analysis and compensation for nonsinusoidal waveforms in phase-shifting digital fringe projection profilometry," *Opt. Lett.* **34**, 416-418 (2009).

[J-2] L. Huang, P. S. K. Chua, and A. Asundi, "Least-squares calibration method for fringe projection profilometry considering camera lens distortion," *Appl. Opt.* **49**, 1539-1548 (2010).

[J-3] L. Huang, Q. Kemao, B. Pan, and A. K. Asundi, "Comparison of Fourier transform, windowed Fourier transform, and wavelet transform methods for phase extraction from a single fringe pattern in fringe projection profilometry," *Optics and Lasers in Engineering* **48**, 141-148 (2010).

[J-4] L. Huang, and A. K. Asundi, "Phase invalidity identification framework with the temporal phase unwrapping method," *Measurement Science and Technology* **22**, 035304 (2011).

[J-5] L. Huang, and A. K. Asundi, "Practical Framework for Phase Retrieval and Invalidity Identification with Temporal Phase Unwrapping Method in Fringe Projection Profilometry," *Applied Mechanics and Materials* **83**, 179-184 (2011).

[J-6] L. Huang, C. S. Ng, and A. K. Asundi, "Dynamic three-dimensional sensing for specular surface with monoscopic fringe reflectometry," *Opt. Express* **19**, 12809-12814 (2011).

- [J-7] M. Zhao, L. Huang, Q. Zhang, X. Su, A. Asundi, and Q. Kemao, "Quality-guided phase unwrapping technique: comparison of quality maps and guiding strategies," *Appl. Opt.* **50**, 6214-6224 (2011).
- [J-8] L. Huang, C. Seng Ng, and A. Krishna Asundi, "Fast full-field out-of-plane deformation measurement using fringe reflectometry," *Optics and Lasers in Engineering* **50**, 529-533 (2012).
- [J-9] L. Huang, and A. K. Asundi, "Phase retrieval from reflective fringe patterns of double-sided transparent objects," *Measurement Science and Technology* **23**, 085201 (2012).
- [J-10] L. Huang, and A. Asundi, "Improvement of least-squares integration method with iterative compensations in fringe reflectometry," *Appl. Opt.* **51**, 7459-7465 (2012)

Conference paper:

[C-1] L. Huang, S. S. Chng, C. P. Lee, P. S. K. Chua, and A. Asundi, "Compact fringe projection profilometer," C. Quan, K. Qian, A. K. Asundi, and F. S. Chau, eds. (SPIE, Singapore, Singapore, 2009), pp. 752256-752256.

[C-2] L. Huang, P. S. K. Chua, and A. Asundi, "Least-squares phase-height mapping for fringe projection profilometry considering camera lens distortion," C. Quan, K. Qian, A. K. Asundi, and F. S. Chau, eds. (SPIE, Singapore, Singapore, 2009), pp. 75220I-75228.

[C-3] L. Huang, K. Qian, and A. Asundi, "Comparison of two-dimensional phase retrieval methods from single interference fringe pattern," F. Song, S. Tao, F. T. S. Yu, S. Jutamulia, K. A. S. Immink, and K. Shono, eds. (SPIE, Beijing, China, 2010), pp. 785107-785110.

[C-4] L. Huang, and A. Asundi, "Study on three-dimensional shape measurement of partially diffuse and specular reflective surfaces with fringe projection technique and fringe reflection technique," K. G. Harding, P. S. Huang, and T. Yoshizawa, eds. (SPIE, San Diego, California, USA, 2011), pp. 813304-813307

[C-5] L. Huang, C. Ng, and A. Asundi, "Dynamic 3D Measurement for Specular Reflecting Surface with Monoscopic Fringe Reflectometry," in Computational Optical Sensing and Imaging, OSA Technical Digest (CD) (Optical Society of America, 2011), paper CWC3.

References

- [1] K. Harding, "Industrial metrology: Engineering precision," *Nat Photon*, vol. 2, pp. 667-669, 2008.
- [2] G. Binnig, C. F. Quate, and C. Gerber, "Atomic force microscope," *Physical Review Letters*, vol. 56, pp. 930-933, 1986.
- [3] N. D. Browning, M. F. Chisholm, and S. J. Pennycook, "Atomic-resolution chemical analysis using a scanning transmission electron microscope," *Nature*, vol. 366, pp. 143-146, 1993.
- [4] G. Zhang, R. Veale, T. Charlton, B. Borchardt, and R. Hocken, "Error Compensation of Coordinate Measuring Machines," *CIRP Annals - Manufacturing Technology*, vol. 34, pp. 445-448, 1985.
- [5] H. Kunzmann, E. Trapet, and F. Wäldele, "A Uniform Concept for Calibration, Acceptance Test, and Periodic Inspection of Coordinate Measuring Machines Using Reference Objects," *CIRP Annals - Manufacturing Technology*, vol. 39, pp. 561-564, 1990.
- [6] A. J. Spyridi and A. A. G. Requicha, "Accessibility analysis for the automatic inspection of mechanical parts by coordinate measuring machines," 1990, pp. 1284-1289.
- [7] W. E. Singhose, W. P. Seering, and N. C. Singer, "Improving repeatability of coordinate measuring machines with shaped command signals," *Precision Engineering*, vol. 18, pp. 138-146, 1996.
- [8] G. Lee, J. Mou, and Y. Shen, "Sampling strategy design for dimensional measurement of geometric features using coordinate measuring machine," *International Journal of Machine Tools and Manufacture*, vol. 37, pp. 917-934, 1997.
- [9] F. Amelung, D. L. Galloway, J. W. Bell, H. A. Zebker, and R. J. Lacznia, "Sensing the ups and downs of Las Vegas: InSAR reveals structural control of land subsidence and aquifer-system deformation," *Geology*, vol. 27, pp. 483-486, 1999.
- [10] A. Hooper, H. Zebker, P. Segall, and B. Kampes, "A new method for measuring deformation on volcanoes and other natural terrains using InSAR persistent scatterers," *Geophysical Research Letters*, vol. 31, pp. 1-5, 2004.

-
- [11] T. J. Wright, B. Parsons, P. C. England, and E. J. Fielding, "InSAR observations of low slip rates on the major faults of western Tibet," *Science*, vol. 305, pp. 236-239, 2004.
- [12] S. M. Hansen, T. A. McKay, R. H. Wechsler, J. Annis, E. S. Sheldon, and A. Kimball, "Measurement of galaxy cluster sizes, radial profiles, and luminosity functions from SDSS photometric data," *Astrophysical Journal*, vol. 633, pp. 122-137, 2005.
- [13] R. G. Dorsch, G. Häusler, and J. M. Herrmann, "Laser triangulation: fundamental uncertainty in distance measurement," *Appl. Opt.*, vol. 33, pp. 1306-1314, 1994.
- [14] A. Gruber, A. Dräbenstedt, C. Tietz, L. Fleury, J. Wrachtrup, and C. Von Borczyskowski, "Scanning confocal optical microscopy and magnetic resonance on single defect centers," *Science*, vol. 276, pp. 2012-2014, 1997.
- [15] S. May, D. Droschel, D. Holz, S. Fuchs, E. Malis, A. Nuchter, and J. Hertzberg, "Three-Dimensional Mapping with Time-of-Flight Cameras," *Journal of Field Robotics*, vol. 26, pp. 934-965, Nov-Dec 2009.
- [16] C. Wykes, "Use of Electronic Speckle Pattern Interferometry (ESPI) in The Measurement of Static and Dynamic Surface Displacement," *Optical Engineering*, vol. 21, pp. 400-406, 1982.
- [17] L. Deck and P. de Groot, "High-speed noncontact profiler based on scanning white-light interferometry," *Applied Optics*, vol. 33, pp. 7334-7338, 1994.
- [18] S. Ulf and P. O. J. Werner, "Digital recording and numerical reconstruction of holograms," *Measurement Science and Technology*, vol. 13, p. R85, 2002.
- [19] A. Asundi and V. R. Singh, "Amplitude and phase analysis in digital dynamic holography," *Opt. Lett.*, vol. 31, pp. 2420-2422, 2006.
- [20] D. Carl, M. Fratz, M. Pfeifer, D. M. Giel, and H. H. Müller, "Multiwavelength digital holography with autocalibration of phase shifts and artificial wavelengths," *Appl. Opt.*, vol. 48, pp. H1-H8, 2009.
- [21] R. K. K, V. J. Shen, A. Talukdar, and A. Asundi, "Adaptive confocal system for 3-D profiling," in *Ninth International Symposium on Laser Metrology*, 2008, pp. 71551W-7.
- [22] M. Vo, Z. Wang, B. Pan, and T. Pan, "Hyper-accurate flexible calibration technique for fringe-projection-based three-dimensional imaging," *Opt. Express*, vol. 20, pp. 16926-16941, 2012.

-
- [23] S. Zhang and P. S. Huang, "High-resolution, real-time three-dimensional shape measurement," *Optical Engineering*, vol. 45, pp. 123601-8, 2006.
- [24] L. L. Deck, "Suppressing phase errors from vibration in phase-shifting interferometry," *Appl. Opt.*, vol. 48, pp. 3948-3960, 2009.
- [25] J. Schwider, "Phase shifting interferometry: reference phase error reduction," *Appl. Opt.*, vol. 28, pp. 3889-3892, 1989.
- [26] R. Sitnik, M. Kujawinska, and J. Woznicki, "Digital fringe projection system for large-volume 360-deg shape measurement," *Optical Engineering*, vol. 41, pp. 443-449, 2002.
- [27] G. Hongwei, H. Haitao, and C. Mingyi, "Gamma correction for digital fringe projection profilometry," *Applied Optics*, vol. 43, pp. 2906-14, 2004.
- [28] S. Zhang, "Recent progresses on real-time 3D shape measurement using digital fringe projection techniques," *Optics and Lasers in Engineering*, vol. 48, pp. 149-158, 2010.
- [29] M. Takeda and K. Mutoh, "Fourier transform profilometry for the automatic measurement of 3-D object shapes," *Appl. Opt.*, vol. 22, pp. 3977-3982, 1983.
- [30] D. J. Bone, H. A. Bachor, and R. J. Sandeman, "Fringe-pattern analysis using a 2-D Fourier transform," *Appl. Opt.*, vol. 25, pp. 1653-1660, 1986.
- [31] X. Su and W. Chen, "Fourier transform profilometry: A review," *Optics and Lasers in Engineering*, vol. 35, pp. 263-284, 2001.
- [32] Q. Kemao, "Windowed Fourier transform for fringe pattern analysis," *Applied Optics*, vol. 43, pp. 2695-2702, 2004.
- [33] J. Zhong and J. Weng, "Spatial carrier-fringe pattern analysis by means of wavelet transform: Wavelet transform profilometry," *Applied Optics*, vol. 43, pp. 4993-4998, 2004.
- [34] Z. Wang and H. Ma, "Advanced continuous wavelet transform algorithm for digital interferogram analysis and processing," *Optical Engineering*, vol. 45, p. 045601, 2006.
- [35] V. Srinivasan, H. C. Liu, and M. Halioua, "Automated phase-measuring profilometry of 3-D diffuse objects," *Applied Optics*, vol. 23, pp. 3105-3108, 1984.
- [36] Y. Surrel, "Design of algorithms for phase measurements by the use of phase stepping," *Applied Optics*, vol. 35, pp. 51-60, 1996.

-
- [37] H. Zhang, M. J. Lalor, and D. R. Burton, "Robust, accurate seven-sample phase-shifting algorithm insensitive to nonlinear phase-shift error and second-harmonic distortion: A comparative study," *Optical Engineering*, vol. 38, pp. 1524-1533, 1999.
- [38] Z. Wang, H. Du, S. Park, and H. Xie, "Three-dimensional shape measurement with a fast and accurate approach," *Appl. Opt.*, vol. 48, pp. 1052-1061, 2009.
- [39] M. C. Knauer, J. Kaminski, and G. Häusler, "Phase measuring deflectometry: a new approach to measure specular free-form surfaces," in *Optical Metrology in Production Engineering*, Strasbourg, France, 2004, pp. 366-376.
- [40] T. Bothe, W. Li, C. von Kopylow, and W. P. O. Jüptner, "High-resolution 3D shape measurement on specular surfaces by fringe reflection," in *Optical Metrology in Production Engineering*, Strasbourg, France, 2004, pp. 411-422.
- [41] R. Kiesslich, J. Burg, M. Vieth, J. Gnaendiger, M. Enders, P. Delaney, A. Polglase, W. McLaren, D. Janell, S. Thomas, B. Nafe, P. R. Galle, and M. F. Neurath, "Confocal laser endoscopy for diagnosing intraepithelial neoplasias and colorectal cancer in vivo," *Gastroenterology*, vol. 127, pp. 706-713, 2004.
- [42] J. Korlach, P. Schwille, W. W. Webb, and G. W. Feigensohn, "Characterization of lipid bilayer phases by confocal microscopy and fluorescence correlation spectroscopy," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 96, pp. 8461-8466, 1999.
- [43] P. Sandoz, "Wavelet transform as a processing tool in white-light interferometry," *Optics Letters*, vol. 22, pp. 1065-1067, 1997.
- [44] P. S. Huang and S. Zhang, "Fast three-step phase-shifting algorithm," *Appl. Opt.*, vol. 45, pp. 5086-5091, 2006.
- [45] Y. Surrel, "Phase stepping: a new self-calibrating algorithm," *Applied Optics*, vol. 32, pp. 3598-3600, 1993.
- [46] J. Novák, P. Novák, and A. Mikš, "Multi-step phase-shifting algorithms insensitive to linear phase shift errors," *Optics Communications*, vol. 281, pp. 5302-5309, 2008.
- [47] Z. Wang and B. Han, "Advanced iterative algorithm for phase extraction of randomly phase-shifted interferograms," *Optics Letters*, vol. 29, pp. 1671-1673, 2004.
- [48] Z. Wang and B. Han, "Advanced iterative algorithm for randomly phase-shifted interferograms with intra- and inter-frame intensity variations," *Optics and Lasers in Engineering*, vol. 45, pp. 274-280, 2007.

-
- [49] H. Guo and Z. Zhao, "Nonlinearity correction in digital fringe projection profilometry by using histogram matching technique," in *Proceedings of SPIE - The International Society for Optical Engineering*, Munich, 2007.
- [50] S. Zhang and S.-T. Yau, "Generic nonsinusoidal phase error correction for three-dimensional shape measurement using a digital video projector," *Applied Optics*, vol. 46, pp. 36-43, 2007.
- [51] M. J. Baker, J. Xi, and J. F. Chicharo, "Elimination of γ non-linear luminance effects for digital video projection phase Measuring profilometers," in *Proceedings - 4th IEEE International Symposium on Electronic Design, Test and Applications, DELTA 2008*, 2008, pp. 496-501.
- [52] K. Liu, Y. Wang, D. L. Lau, Q. Hao, and L. G. Hassebrook, "Gamma model and its analysis for phase measuring profilometry," *J. Opt. Soc. Am. A*, vol. 27, pp. 553-562, 2010.
- [53] Z. Li and Y. Li, "Gamma-distorted fringe image modeling and accurate gamma correction for fast phase measuring profilometry," *Opt. Lett.*, vol. 36, pp. 154-156, 2011.
- [54] J. L. Marroquin, M. Servin, and R. Rodriguez-Vera, "Adaptive quadrature filters and the recovery of phase from fringe pattern images," *J. Opt. Soc. Am. A*, vol. 14, pp. 1742-1753, 1997.
- [55] K. G. Larkin, D. J. Bone, and M. A. Oldfield, "Natural demodulation of two-dimensional fringe patterns. I. General background of the spiral phase quadrature transform," *J. Opt. Soc. Am. A*, vol. 18, pp. 1862-1870, 2001.
- [56] J. C. Estrada, M. Servín, J. A. Quiroga, and J. L. Marroquín, "Path independent demodulation method for single image interferograms with closed fringes within the function space C^2 ," *Opt. Express*, vol. 14, pp. 9687-9698, 2006.
- [57] O. S. Dalmau-Cedeño, M. Rivera, and R. Legarda-Saenz, "Fast phase recovery from a single closed-fringe pattern," *J. Opt. Soc. Am. A*, vol. 25, pp. 1361-1370, 2008.
- [58] H. Wang and Q. Kemao, "Frequency guided methods for demodulation of a single fringe pattern," *Opt. Express*, vol. 17, pp. 15118-15127, 2009.
- [59] L. Kai and Q. Kemao, "Fast frequency-guided sequential demodulation of a single fringe pattern," *Opt. Lett.*, vol. 35, pp. 3718-3720, 2010.

-
- [60] H. Wang, K. Li, and Q. Kemao, "Frequency guided methods for demodulation of a single fringe pattern with quadratic phase matching," *Optics and Lasers in Engineering*, vol. 49, pp. 564-569, 2011.
- [61] J.-F. Lin and X. Su, "Two-dimensional Fourier transform profilometry for the automatic measurement of three-dimensional object shapes," *Optical Engineering*, vol. 34, pp. 3297-3302, 1995.
- [62] M. Takeda, Q. Gu, M. Kinoshita, H. Takai, and Y. Takahashi, "Frequency-multiplex Fourier-transform profilometry: a single-shot three-dimensional shape measurement of objects with large height discontinuities and/or surface isolations," *Appl. Opt.*, vol. 36, pp. 5347-5354, 1997.
- [63] H. Guo and P. S. Huang, "Absolute phase retrieval for 3D shape measurement by Fourier transform method," in *Two- and Three-Dimensional Methods for Inspection and Metrology V*, Boston, MA, USA, 2007, pp. 676204-10.
- [64] H. Guo and P. S. Huang, "3-D shape measurement by use of a modified Fourier transform method," in *Two- and Three-Dimensional Methods for Inspection and Metrology VI*, San Diego, CA, United States, 2008, pp. 70660E-8.
- [65] H. Guo and P. S. Huang, "Absolute phase technique for the Fourier transform method," *Optical Engineering*, vol. 48, pp. 043609-7, 2009.
- [66] K. Qian, H. S. Seah, and A. Asundi, "Fault detection by interferometric fringe pattern analysis using windowed Fourier transform," *Measurement Science and Technology*, vol. 16, pp. 1582-1587, 2005.
- [67] Q. Kemao, "Two-dimensional windowed Fourier transform for fringe pattern analysis: Principles, applications and implementations," *Optics and Lasers in Engineering*, vol. 45, pp. 304-317, 2007.
- [68] Q. Kemao, W. Gao, and H. Wang, "Windowed Fourier-filtered and quality-guided phase-unwrapping algorithm," *Appl. Opt.*, vol. 47, pp. 5420-5428, 2008.
- [69] Q. Kemao, H. Wang, and W. Gao, "Windowed Fourier transform for fringe pattern analysis: theoretical analyses," *Appl. Opt.*, vol. 47, pp. 5408-5419, 2008.
- [70] K. Li and B. Pan, "Frequency-guided windowed Fourier ridges technique for automatic demodulation of a single closed fringe pattern," *Appl. Opt.*, vol. 49, pp. 56-60, 2010.
- [71] J. Zhong and J. Weng, "Phase retrieval of optical fringe patterns from the ridge of a wavelet transform," *Optics Letters*, vol. 30, pp. 2560-2562, 2005.

- [72] X. Su and W. Chen, "Reliability-guided phase unwrapping algorithm: A review," *Optics and Lasers in Engineering*, vol. 42, pp. 245-261, 2004.
- [73] A. Asundi and Z. Wensen, "Fast phase-unwrapping algorithm based on a gray-scale mask and flood fill," *Applied Optics*, vol. 37, p. 5416, 1998.
- [74] E. Zappa and G. Busca, "Comparison of eight unwrapping algorithms applied to Fourier-transform profilometry," *Optics and Lasers in Engineering*, vol. 46, pp. 106-116, 2008.
- [75] J. M. Bioucas-Dias and G. Valadao, "Phase unwrapping via graph cuts," *IEEE Transactions on Image Processing*, vol. 16, pp. 698-709, 2007.
- [76] J. Bioucas-Dias and G. Valadao, "Phase unwrapping via diversity and graph cuts," Piscataway, NJ 08855-1331, United States, 2008, pp. 495-498.
- [77] S. Li, W. Chen, and X. Su, "Reliability-guided phase unwrapping in wavelet-transform profilometry," *Applied Optics*, vol. 47, pp. 3369-3377, 2008.
- [78] J. M. Huntley and H. Saldner, "Temporal phase-unwrapping algorithm for automated interferogram analysis," *Appl. Opt.*, vol. 32, pp. 3047-3052, 1993.
- [79] H. Zhao, W. Chen, and Y. Tan, "Phase-unwrapping algorithm for the measurement of three-dimensional object shapes," *Appl. Opt.*, vol. 33, pp. 4497-4500, 1994.
- [80] J. Tian, X. Peng, and X. Zhao, "A generalized temporal phase unwrapping algorithm for three-dimensional profilometry," *Optics and Lasers in Engineering*, vol. 46, pp. 336-342, 2008.
- [81] G. Sansoni, S. Corini, S. Lazzari, R. Rodella, and F. Docchio, "Three-dimensional imaging based on Gray-code light projection: characterization of the measuring algorithm and development of a measuring system for industrial applications," *Appl. Opt.*, vol. 36, pp. 4463-4472, 1997.
- [82] G. Sansoni, M. Carocci, and R. Rodella, "Three-Dimensional Vision Based on a Combination of Gray-Code and Phase-Shift Light Projection: Analysis and Compensation of the Systematic Errors," *Appl. Opt.*, vol. 38, pp. 6565-6573, 1999.
- [83] Q. Zhang, X. Su, L. Xiang, and X. Sun, "3-D shape measurement based on complementary Gray-code light," *Optics and Lasers in Engineering*, vol. 50, pp. 574-579, 2012.

-
- [84] S. Zhang, "Phase unwrapping error reduction framework for a multiple-wavelength phase-shifting algorithm," *Optical Engineering*, vol. 48, pp. 105601-8, 2009.
- [85] F. Chen, X. Su, and L. Xiang, "Analysis and identification of phase error in phase measuring profilometry," *Opt. Express*, vol. 18, pp. 11300-11307, 2010.
- [86] J. Zhong and H. Zeng, "Multiscale windowed Fourier transform for phase extraction of fringe patterns," *Applied Optics*, vol. 46, pp. 2670-2675, 2007.
- [87] Q. Hu, P. S. Huang, Q. Fu, and F.-P. Chiang, "Calibration of a three-dimensional shape measurement system," *Optical Engineering*, vol. 42, pp. 487-493, 2003.
- [88] A. Maurel, P. Cobelli, V. Pagneux, and P. Petitjeans, "Experimental and theoretical inspection of the phase-to-height relation in Fourier transform profilometry," *Appl. Opt.*, vol. 48, pp. 380-392, 2009.
- [89] H. Liu, W. H. Su, K. Reichard, and S. Yin, "Calibration-based phase-shifting projected fringe profilometry for accurate absolute 3D surface profile measurement," *Optics Communications*, vol. 216, pp. 65-80, 2003.
- [90] M. Fujigaki, A. Takagishi, T. Matui, and Y. Morimoto, "Development of real-time shape measurement system using whole-space tabulation method," in *Two- and Three-Dimensional Methods for Inspection and Metrology VI*, San Diego, CA, United States, 2008, pp. 706606-8.
- [91] H. Guo, H. He, Y. Yu, and M. Chen, "Least-squares calibration method for fringe projection profilometry," *Optical Engineering*, vol. 44, pp. 033603-9, 2005.
- [92] H. Du and Z. Wang, "Three-dimensional shape measurement with an arbitrarily arranged fringe projection profilometry system," *Optics Letters*, vol. 32, pp. 2438-2440, 2007.
- [93] R. Legarda-Saenz, T. Bothe, and W. P. Juptner, "Accurate procedure for the calibration of a structured light system," *Optical Engineering*, vol. 43, pp. 464-471, 2004.
- [94] S. Zhang and P. S. Huang, "Novel method for structured light system calibration," *Optical Engineering*, vol. 45, pp. 083601-8, 2006.
- [95] X. Zhang and L. Zhu, "Projector calibration from the camera image point of view," *Optical Engineering*, vol. 48, pp. 117208-13, 2009.

-
- [96] M. Petz and R. Tutsch, "Measurement of optically effective surfaces by imaging of gratings," in *Optical Measurement Systems for Industrial Inspection III*, Munich, Germany, 2003, pp. 288-294.
- [97] M. Petz and R. Tutsch, "Reflection grating photogrammetry: a technique for absolute shape measurement of specular free-form surfaces," in *Optical Manufacturing and Testing VI*, San Diego, CA, USA, 2005, pp. 58691D1-58691D12.
- [98] Y. Tang, X. Su, Y. Liu, and H. Jing, "3D shape measurement of the aspheric mirror by advanced phase measuring deflectometry," *Opt. Express*, vol. 16, pp. 15090-15096, 2008.
- [99] Y. Tang, X. Su, F. Wu, and Y. Liu, "A novel phase measuring deflectometry for aspheric mirror test," *Opt. Express*, vol. 17, pp. 19778-19784, 2009.
- [100] W. Zhao, X. Su, Y. Liu, and Q. Zhang, "Testing an aspheric mirror based on phase measuring deflectometry," *Optical Engineering*, vol. 48, pp. 103603-5, 2009.
- [101] H. Guo, P. Feng, and T. Tao, "Specular surface measurement by using least squares light tracking technique," *Optics and Lasers in Engineering*, vol. 48, pp. 166-171, 2010.
- [102] J. Balzer, S. Höfer, S. Werling, and J. Beyerer, "Optimization on Shape Curves with Application to Specular Stereo," in *Pattern Recognition*, vol. 6376, M. Goesele, S. Roth, A. Kuijper, B. Schiele, and K. Schindler, Eds., ed: Springer Berlin / Heidelberg, 2010, pp. 41-50.
- [103] D. L. Fried, "Least-square fitting a wave-front distortion estimate to an array of phase-difference measurements," *J. Opt. Soc. Am.*, vol. 67, pp. 370-375, 1977.
- [104] R. H. Hudgin, "Wave-front reconstruction for compensated imaging," *J. Opt. Soc. Am.*, vol. 67, pp. 375-378, 1977.
- [105] R. H. Hudgin, "Optimal wave-front estimation," *J. Opt. Soc. Am.*, vol. 67, pp. 378-382, 1977.
- [106] W. Li, T. Bothe, C. von Kopylow, and W. P. O. Jüptner, "Evaluation methods for gradient measurement techniques," in *Optical Metrology in Production Engineering*, Strasbourg, France, 2004, pp. 300-311.
- [107] S. Ettl, J. Kaminski, M. C. Knauer, and G. Häusler, "Shape reconstruction from gradient data," *Appl. Opt.*, vol. 47, pp. 2091-2097, 2008.

-
- [108] S. Hata, Y. Saitoh, S. Kumamura, and K. Kaida, "Shape extraction of transparent object using genetic algorithm," in *Pattern Recognition, 1996., Proceedings of the 13th International Conference on*, 1996, pp. 684-688 vol.4.
- [109] M. Ben-Ezra and S. K. Nayar, "What does motion reveal about transparency?," in *Computer Vision, 2003. Proceedings. Ninth IEEE International Conference on*, 2003, pp. 1025-1032 vol.2.
- [110] M. Ferraton, C. Stolz, and F. Meriaudeau, "Shape measurement of transparent objects with polarization imaging," in *Optical and Digital Image Processing*, Strasbourg, France, 2008, pp. 70001R-8.
- [111] R. Rantoson, C. Stolz, D. Fofi, and F. Meriaudeau, "3D reconstruction by polarimetric imaging method based on perspective model," in *Optical Measurement Systems for Industrial Inspection VI*, Munich, Germany, 2009, pp. 73890C-12.
- [112] L. Joannes, T. Hough, X. Hutsebaut, X. Dubois, R. Ligot, B. Saoul, P. Van Donink, and K. De Coninck, "The reproducibility of a new power mapping instrument based on the phase shifting schlieren method for the measurement of spherical and toric contact lenses," *Contact Lens and Anterior Eye*, vol. 33, pp. 3-8, 2010.
- [113] K. Kutulakos and E. Steger, "A Theory of Refractive and Specular 3D Shape by Light-Path Triangulation," *International Journal of Computer Vision*, vol. 76, pp. 13-29, 2008.
- [114] M. Yamazaki, S. Iwata, and G. Xu, "Dense 3D Reconstruction of Specular and Transparent Objects Using Stereo Cameras and Phase-Shift Method," in *Computer Vision – ACCV 2007*, ed, 2007, pp. 570-579.
- [115] J. Xu, N. Xi, C. Zhang, Q. Shi, and J. Gregory, "A Robot-Assisted Back-Imaging Measurement System for Transparent Glass," *Mechatronics, IEEE/ASME Transactions on*, vol. PP, pp. 1-10, 2011.
- [116] J. Xu, N. Xi, C. Zhang, Q. A. Shi, and J. Gregory, "A geometry and optical property inspection system for automotive glass based on fringe patterns," *Optica Applicata*, vol. 40, pp. 827-841, 2010.
- [117] J. Gühring, "Dense 3-D surface acquisition by structured light using off-the-shelf components," 2001, pp. 220-231.
- [118] D. Miyazaki and K. Ikeuchi, "Shape Estimation of Transparent Objects by Using Inverse Polarization Ray Tracing," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. 29, pp. 2018-2030, 2007.

-
- [119] X. Su, W. Song, Y. Cao, and L. Xiang, "Phase-height mapping and coordinate calibration simultaneously in phase-measuring profilometry," *Optical Engineering*, vol. 43, pp. 708-712, 2004.
- [120] Z. Wang, H. Du, and H. Bi, "Out-of-plane shape determination in generalized fringe projection profilometry," *Optics Express*, vol. 14, pp. 12122-12133, 2006.
- [121] Z. Wang, D. A. Nguyen, and J. C. Barnes, "Some practical considerations in fringe projection profilometry," *Optics and Lasers in Engineering*, vol. 48, pp. 218-225, 2010.
- [122] J. M. Huntley and H. O. Saldner, "Error-reduction methods for shape measurement by temporal phase unwrapping," *J. Opt. Soc. Am. A*, vol. 14, pp. 3188-3196, 1997.
- [123] Q. Zhang and X. Su, "High-speed optical measurement for the drumhead vibration," *Opt. Express*, vol. 13, pp. 3110-3116, 2005.
- [124] Q. Zhang, X. Su, Y. Cao, Y. Li, L. Xiang, and W. Chen, "Optical 3-D shape and deformation measurement of rotating blades using stroboscopic structured illumination," *Optical Engineering*, vol. 44, pp. 113601-7, 2005.
- [125] D. C. Ghiglia and M. D. Pritt, *Two-Dimensional Phase Unwrapping: Theory, Algorithms, and Software*. New York: John Wiley and Sons, 1998.
- [126] S. Zhang, X. Li, and S.-T. Yau, "Multilevel quality-guided phase unwrapping algorithm for real-time three-dimensional shape reconstruction," *Appl. Opt.*, vol. 46, pp. 50-57, 2007.
- [127] A. Asundi and Z. Wensen, "Fast Phase-Unwrapping Algorithm Based on a Gray-Scale Mask and Flood Fill," *Appl. Opt.*, vol. 37, pp. 5416-5420, 1998.
- [128] W.-S. Li and X.-Y. Su, "Phase unwrapping algorithm based on phase fitting reliability in structured light projection," *Optical Engineering*, vol. 41, pp. 1365-1372, 2002.
- [129] M. A. Herráez, D. R. Burton, M. J. Lalor, and M. A. Gdeisat, "Fast two-dimensional phase-unwrapping algorithm based on sorting by reliability following a noncontinuous path," *Appl. Opt.*, vol. 41, pp. 7437-7444, 2002.
- [130] Y. Lu, X. Wang, and G. He, "Phase unwrapping based on branch cut placing and reliability ordering," *Optical Engineering*, vol. 44, pp. 055601-9, 2005.
- [131] M. Ramji and K. Ramesh, "Adaptive Quality Guided Phase Unwrapping Algorithm for Whole-Field Digital Photoelastic Parameter Estimation of Complex Models," *Strain*, vol. 46, pp. 184-194, 2010.

- [132] Q. Kemao, W. Gao, and H. Wang, "Windowed Fourier filtered and quality guided phase unwrapping algorithm: on locally high-order polynomial phase," *Appl. Opt.*, vol. 49, pp. 1075-1079, 2010.
- [133] Z. Heping, T. Jinsong, Z. Sen, and C. Ming, "An Improved Quality-Guided Phase-Unwrapping Algorithm Based on Priority Queue," *Geoscience and Remote Sensing Letters, IEEE*, vol. 8, pp. 364-368, 2011.
- [134] W. Huifang and et al., "A three-dimensional quality-guided phase unwrapping method for MR elastography," *Physics in Medicine and Biology*, vol. 56, p. 3935, 2011.
- [135] C. V. Jakowatz, D. E. Wahl, P. H. Eichel, D. C. Ghiglia, and P. Thompson, *Spotlight-Mode Synthetic Aperture Radar: A Signal Processing Approach*. Boston: Kluwer Academic Publishers, 1996.
- [136] M. D. Pritt, "Phase unwrapping by means of multigrid techniques for interferometric SAR," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 34, pp. 728-738, 1996.
- [137] D. J. Bone, "Fourier fringe analysis: the two-dimensional phase unwrapping problem," *Appl. Opt.*, vol. 30, pp. 3627-3632, 1991.
- [138] J. A. Quiroga, A. González-Cano, and E. Bernabeu, "Phase-unwrapping algorithm based on an adaptive criterion," *Appl. Opt.*, vol. 34, pp. 2560-2563, 1995.
- [139] Y. Lu, X. Wang, X. Zhong, G. He, Y. Liu, and D. Zheng, "A new quality map for quality-guided phase unwrapping," *Chin. Opt. Lett.*, vol. 2, pp. 698-700, 2004.
- [140] R. Sedgewick, *Algorithms in C: Parts 1-4, Fundamentals, Data Structures, Sorting, and Searching*: Addison-Wesley Longman Publishing Co., Inc, 1997.
- [141] Y. Li and X.-Y. Su, "Fast algorithm for reliability-guided phase unwrapping," *Guangdian Gongcheng/Opto-Electronic Engineering*, vol. 32, pp. 76-79, 2005.
- [142] Y. Li and Z. Hou, "personal communications," 2011.
- [143] F. Chen, G. M. Brown, and M. Song, "Overview of three-dimensional shape measurement using optical methods," *Optical Engineering*, vol. 39, pp. 10-22, 2000.
- [144] B. C. Platt and R. Shack, "History and principles of Shack-Hartmann wavefront sensing," *Journal of Refractive Surgery*, vol. 17, pp. S573-S577, 2001.

-
- [145] W. H. Southwell, "Wave-front estimation from wave-front slope measurements," *J. Opt. Soc. Am.*, vol. 70, pp. 998-1006, 1980.
- [146] S. Werling, M. Mai, M. Heizmann, and J. Beyerer, "Inspection of specular and partially specular surfaces," *Metrology and Measurement Systems*, vol. 16, 2009.
- [147] J. Balzer and S. Werling, "Principles of Shape from Specular Reflection," *Measurement*, vol. 43, pp. 1305-1317, 2010.
- [148] A. Asundi, "Novel techniques in reflection moiré," *Experimental Mechanics*, vol. 34, pp. 230-242, 1994.
- [149] A. Asundi, "Computer Aided Moiré Methods," *Optics and Lasers in Engineering*, vol. 18, pp. 213-238, 1993.
- [150] X. Su and Q. Zhang, "Dynamic 3-D shape measurement method: A review," *Optics and Lasers in Engineering*, vol. 48, pp. 191-204, 2010.
- [151] Q.-C. Zhang and X.-Y. Su, "An optical measurement of vortex shape at a free surface," *Optics & Laser Technology*, vol. 34, pp. 107-113, 2002.
- [152] Y. Watanabe, T. Komuro, and M. Ishikawa, "955-fps Real-time shape measurement of a moving/deforming object using high-speed vision for numerous-point analysis," in *In Proceedings of ICRA*, 2007, pp. 3192-3197.
- [153] L.-C. Chen, Y.-T. Huang, X.-L. Nguyen, J.-L. Chen, and C.-C. Chang, "Dynamic out-of-plane profilometry for nano-scale full-field characterization of MEMS using stroboscopic interferometry with novel signal deconvolution algorithm," *Optics and Lasers in Engineering*, vol. 47, pp. 237-251, 2009.
- [154] Z. Zhang, "A flexible new technique for camera calibration," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 22, pp. 1330-1334, 2000.
- [155] J. Y. Bouguet, "Camera calibration toolbox for matlab, please see http://www.vision.caltech.edu/bouguetj/calib_doc/."
- [156] G. Häusler, C. Richter, K.-H. Leitz, and M. C. Knauer, "Microdeflectometry—a novel tool to acquire three-dimensional microtopography with nanometer height resolution," *Opt. Lett.*, vol. 33, pp. 396-398, 2008.
- [157] W. Gao, L. Wang, and Z. Hu, "Flexible method for structured light system calibration," *Optical Engineering*, vol. 47, pp. 083602-10, 2008.

- [158] Z. Li, Y. Shi, C. Wang, and Y. Wang, "Accurate calibration method for a structured light system," *Optical Engineering*, vol. 47, pp. 053604-9, 2008.