Analyses, simulation and control of a wheeled-vehicle with AWIS (Auxiliary Wheels of Involute-Shape)

Hendra Purnawali

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ANALYSES, SIMULATION AND CONTROL OF A WHEELED-VEHICLE WITH AWIS (AUXILIARY WHEELS OF INVOLUTE-SHAPE)

HENDRA PURNAWALI

**SCHOOL OF MECHANICAL & AEROSPACE ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY**

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A WHEELED-VEHICLE WITH AWIS
(AUXILIARY WHEELS OF INVOLUTE-SHAPE)**

HENDRA PURNAWALI

SCHOOL OF MECHANICAL & AEROSPACE ENGINEERING

A thesis submitted to the Nanyang Technological University
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Master of Engineering

2006

ABSTRACT

The environment where human beings live in has naturally uneven surfaces. Legs owned by most living creatures enable them to move easily and traverse a wide range of obstacles on the land. However, the efficiency of the legs is relatively low; therefore, means of transportation, especially wheeled-vehicles are developed to assist humans due to wheels' efficiency. Nonetheless, wheeled-vehicles have limited mobility while moving on rough terrain. This triggers worldwide researchers to develop land-vehicles based on others locomotion mechanisms, such as legged-vehicles, tracked-vehicles, or combination of the existing modes.

This project focuses on the development of the wheeled-vehicles by introducing AWIS, which stands for Auxiliary Wheels of Involute-Shape. The introduction of the AWIS to the wheeled-vehicle makes significant contribution towards the development of land-vehicles that have high negotiating movement capabilities on rough terrain while not imposing complexity in design and control. In-depth kinematics, statics, and dynamics analyses are performed on the AWIS. Subsequently, the dynamics simulations and controls of the wheeled-vehicle with AWIS on a flat surface, a block of obstacle, and a flight of stair case, are demonstrated, and the results are evaluated.

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TABLE OF CONTENTS

	Page
ABSTRACT	i
ACKNOWLEDGEMENTS	ii
TABLE OF CONTENTS	iii
LIST OF FIGURES	vi
LIST OF TABLES	xii
 CHAPTER 1: INTRODUCTION	
1.1 Background	1
1.2 Objective and Scope	4
 CHAPTER 2: LITERATURE REVIEW	
2.1 Wheeled-Vehicles	5
2.1.1 Purely-Wheeled-Vehicles	6
2.1.2 Articulated-Wheeled-Vehicles	6
1. Passive Articulated-Wheeled-Vehicles	6
2. Active Articulated-Wheeled-Vehicles	7
3. Hybrid Articulated-Wheeled-Vehicles	8
(a) SHRIMP	9
(b) WORKPARTNER	11
(c) OCTOPUS	13
(d) MARSOKHOD	15
2.2 Legged-Vehicles	17
2.2.1 RHEX	17
2.3 Tracked-Vehicles	20
2.3.1 URBAN	20
2.4 Wheeled-Legged-Vehicles	22
2.4.1 WHEELLEG	22

2.5	Wheeled-Tracked-Vehicles	25
2.5.1	HELIOS-VI	25
2.6	Wheeled-Legged-Tracked-Vehicles	27
2.6.1	AZIMUT	27
2.7	Summary and Remarks	30
CHAPTER 3: DESIGN AND KINEMATICS ANALYSIS OF AWIS		
3.1	Design of AWIS	32
3.2	Kinematics Analysis of AWIS	33
3.2.1	Finding the circumference of the involute-curve	37
3.2.2	Locations of Point C and Point P	
	on the Rotating AWIS	40
	1. Rotation Matrix	40
	2. The AWIS is at its initial position	41
	3. The AWIS rotates on the base circle	42
	4. Both base circle and involute-curve	
	of the AWIS touch ground	43
	5. The AWIS rotates on the involute-curve	46
	6. The AWIS hits an obstacle	48
3.2.3	Finding the activation distance of the AWIS	49
	1. Plotting the angular position of the AWIS	51
3.2.4	Finding the maximum height of	
	traversable obstacle of the AWIS	53
CHAPTER 4: KINETICS ANALYSIS OF AWIS		
4.1	Normal Wheel	63
4.1.1	Statics Analysis	63
4.1.2	Mass Moment of Inertia Calculation	64
4.1.3	Dynamics Analysis	65
4.2	AWIS	66
4.2.1	Statics Analysis	66
4.2.2	Dynamics Analysis	68

Table Of Contents

4.2.3	Mass Moment of Inertia Calculation	71
1.	Center of Mass of an AWIS	72
2.	Mass moment of inertia of an AWIS	77
CHAPTER 5: DYNAMICS SIMULATIONS AND CONTROLS OF A WHEELED-VEHICLE WITH AWIS		
5.1	A wheeled-vehicle with four normal wheels	79
5.2	A wheeled-vehicle with four AWIS	81
5.2.1	Moving on a flat ground without controller	82
5.3	A wheeled-vehicle with four AWIS and controllers	84
5.3.1	PID Controller	85
5.3.2	Moving on a flat ground	86
5.3.3	Traversing an obstacle	89
5.3.4	Traversing a staircase	94
5.4	A wheeled-vehicle with eight AWIS and controllers	99
5.4.1	Traversing an obstacle	100
CHAPTER 6: CONCLUSION		
6.1	Concluding Remarks	109
6.2	Limitations, Recommendations, and Future Works	111
6.3	Benefits	115
REFERENCES		117
APPENDIX A: MATLAB PROGRAMS		
APPENDIX B: SIMULATION RESULTS		

LIST OF FIGURES

	Page
 CHAPTER 2: LITERATURE REVIEW	
2.1 Passive articulated-wheeled-vehicles:	
(a) SOJOURNER, (b) MICRO5, (c) ROCKY7	7
2.2 Active articulated-wheeled-vehicles:	
(a) DANTE, (b) NANOROVER, (c) SPACECAT	8
2.3 The SHRIMP	9
2.4 The SHRIMP on a staircase	10
2.5 The WORKPARTNER	11
2.6 The HYBTOR Platform	12
2.7 The WORKPARTNER climbs a step	13
2.8 The OCTOPUS	14
2.9 The OCTOPUS climbs a step	15
2.10 The MARSOKHOD	16
2.11 The MARSOKHOD traverses an obstacle	16
2.12 The RHEX	18
2.13 The capabilities of the RHEX	19
2.14 The URBAN	20
2.15 The capabilities of the URBAN	21
2.16 The WHEELLEG	23
2.17 A sequence of the WHEELLEG	
while traversing an obstacle	24
2.18 The HELIOS-VI	26
2.19 The HELIOS-VI ascends a stair	26
2.20 The AZIMUT	28
2.21 The capabilities of the AZIMUT	29
 CHAPTER 3: DESIGN AND KINEMATICS ANALYSIS OF AWIS	
3.1 The AWIS	33

3.2	Point P on the involute-curve	34
3.3	Plot of Alpha (deg) vs. Theta (deg) (Program 3)	35
3.4	Plot of Alpha (deg) vs. Theta (deg) (Program 4)	36
3.5	Plot of Radius vs. Alpha (deg) and Radius vs. Theta (deg) with $r_b = 0.25$ m (Program 6)	37
3.6	Finding the length of arc AB	37
3.7	Plot of Arc Length (m) vs. Alpha (deg) with $r_b = 0.25$ m (Program 7)	39
3.8	Involute-curve circumference with $r_b = 0.25$ m (Program 8)	39
3.9	Rotation Matrix	40
3.10	The AWIS at its initial position	41
3.11	The AWIS rotates on the base circle	42
3.12	Both base circle and involute-curve of the AWIS touch ground	43
3.13	The schematic diagram of the AWIS at this instance	44
3.14	The AWIS rotates on the involute-curve	46
3.15	The end of the involute-curve touches ground	47
3.16	The AWIS hits an obstacle	48
3.17	Finding the activation distance of the AWIS	50
3.18	Graphical user interface to calculate the activation distance	51
3.19	The AWIS with $\beta_H = 0^\circ, 100^\circ, 200^\circ, 257.4528^\circ$	52
3.20	The AWIS are not able to traverse the obstacles	53
3.21	x-distances from center point C to points along the involute-curve for $\beta_H = 100^\circ, 200^\circ$	54
3.22	The maximum heights for the AWIS with $\beta_H = 0^\circ, 100^\circ, 200^\circ, 257.4528^\circ$	55
3.23	Plot of the maximum obstacle's height traversable for β_H ranging from 0 to 4.4934 radians	56
3.24	The maximum obstacle's height traversable for β_H ranging from 1 to 2.9226 radians	57

3.25	Plots of equations 3-17, 3-18	57
3.26	Proof of equation (3-18) geometrically	58
3.27	The highest value of $h_{\max}$ when the AWIS rotates on the involute-curve	59
3.28	The AWIS rotates about the end-point of the involute-curve	60
3.29	The highest value of $h_{\max}$ when the AWIS rotates about the end-point of the involute-curve	61
3.30	The ideal position of the AWIS while traversing an obstacle	62
 CHAPTER 4: KINETICS ANALYSIS OF AWIS		
4.1	Free-body-diagram of a normal wheel at rest	64
4.2	Mass moment of inertia of a cylinder	64
4.3	Free-body-diagram of a rotating normal wheel	65
4.4	Free-body-diagram of an AWIS at rest	67
4.5	Resultant reaction force's direction at contact point, P_B	68
4.6	Free-body-diagram of a rotating AWIS	68
4.7	Components of an AWIS, i.e. (a) involute, (b) half-circle	71
4.8	Area of a region bounded by a curve	72
4.9	Area of the involute part	73
4.10	Center of mass of the involute part	75
4.11	Center of mass of the half-circle part	76
4.12	Center of mass of an AWIS	77
 CHAPTER 5: DYNAMICS SIMULATIONS AND CONTROLS OF A WHEELED-VEHICLE WITH AWIS		
5.1	A virtual model of a wheeled-vehicle with four normal wheels: (a) Filled, (b) Wire frame	79
5.2	The torque profile exerted by the motor to the normal wheel	80
5.3	Plots of the simulation results of the normal wheel	81

5.4	A virtual model of a wheeled-vehicle with four AWIS: (a) Filled, (b) Wire frame	82
5.5	Plots of the simulation results of the vehicle with four AWIS when the maximum torque is 1800 Nm	82
5.6	The vehicle flies in the air	83
5.7	Plots of the simulation results of the vehicle with four AWIS when the maximum torque is 3000 Nm	84
5.8	Top-view of the wire frame model of the wheeled-vehicle with four AWIS and controllers	84
5.9	Angular velocity control mode	85
5.10	Moving on a flat ground at (a) $t=0$ s, (b) $t = 10$ s	86
5.11	Plots of the angular velocity's errors for the four AWIS	87
5.12	Plots of the simulation results of the AWIS	88
5.13	Traversing an obstacle	89
5.14	A sequence of traversing an obstacle	90
5.15	Plots of the angular velocity's errors of the four AWIS while traversing an obstacle	91
5.16	Plots of the simulation results of the four AWIS while traversing an obstacle	92
5.17	Zooming in the simulation results ($t= 0$ to 250 s)	92
5.18	The slippage between the rear AWIS and the ground	93
5.19	Final angular positions of the front and the rear AWIS	93
5.20	Traversing a staircase	94
5.21	The dimensions of the staircase	94
5.22	A sequence of traversing a staircase	95
5.23	Plots of the angular velocity's errors of the four AWIS while traversing a staircase	96
5.24	Plots of the simulation results of the four AWIS while traversing a staircase	97
5.25	The vehicle flies while ascending the staircase	97
5.26	The vehicle flies while descending the staircase	98

5.27 A wheeled-vehicle with eight AWIS:	
(a) Filled, (b) Wire frame	99
5.28 Top-view of the wire frame model of the wheeled-vehicle with eight AWIS	100
5.29 The horizontal distance between the vehicle's axles and the obstacle	100
5.30 Traversing an obstacle with eight AWIS	101
5.31 Plot of the desired angular velocity of the normal wheel	102
5.32 Plot of the desired angular velocity of the AWIS	103
5.33 A sequence of traversing an obstacle with eight AWIS	104
5.34 The final position of the vehicle at $t = 87$ s	105
5.35 Plots of the angular velocity's errors of the front and the rear AWIS	106
5.36 Plots of the simulation results of the front AWIS	107
5.37 Plots of the simulation results of the rear AWIS	108

APPENDIX B: SIMULATION RESULTS

B.1 Zooming in Figure 5.11:	
Angular velocity's errors graphs	B-4
B.2 Zooming in Figure 5.12: Motor torque	B-5
B.3 Zooming in Figure 5.12: Angular acceleration	B-6
B.4 Zooming in Figure 5.12: Angular velocity	B-7
B.5 Zooming in Figure 5.12: Translational acceleration	B-7
B.6 Zooming in Figure 5.15:	
Angular velocity's errors graphs	B-9
B.7 Zooming in Figure 5.16: Motor torque	B-11
B.8 Zooming in Figure 5.16: Angular velocity	B-12
B.9 Zooming in Figure 5.16: Translational velocity	B-13
B.10 Zooming in Figure 5.23:	
Angular velocity's errors graphs	B-14
B.11 Zooming in Figure 5.24: Motor torque	B-15

B.12 Zooming in Figure 5.24: Angular acceleration, angular velocity, and translational acceleration	B-16
B.13 Zooming in Figure 5.35: Angular velocity's errors graphs	B-18
B.14 Zooming in Figures 5.36 & 5.37: Motor torque	B-20
B.15 Zooming in Figures 5.36 & 5.37: Angular velocity	B-22

LIST OF TABLES

	Page
CHAPTER 2: LITERATURE REVIEW	
2.1 Comparison of the performance of each type of the land-vehicles	30
CHAPTER 3: DESIGN AND KINEMATICS ANALYSIS OF AWIS	
3.1 The maximum heights of traversable obstacle for the AWIS	55

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Vehicles are created and developed to assist in transporting people and goods from one place to another place. There are three basic substances that form our environment, i.e. solid, liquid, and gas and consequently, the vehicles developed have to interact with those substances.

Generally, based on the mode of the interactions between the vehicles and the environment, vehicles are categorized as land-vehicles, underwater-vehicles, and aerial-vehicles. The vehicles traveled on the soil, sandy, or rocky terrain are grouped into the land-vehicles. Moreover, some vehicles, able to travel across land, sea, and air, are being developed. However, the development of these vehicles is still in the early stage due to its complexity.

The underwater and aerial vehicles are mostly used for long journeys. Among all three groups of the vehicles, the land-vehicles are the most common means of transportation used by people around the world. This is because most people live and perform most of their activities on land. In addition to the purpose of transporting people and goods, the land-vehicles are deployed as well in hazardous environments, such as nuclear industry or in unreachable environments, e.g. forest. Furthermore, they are also used for military purposes, e.g. reconnaissance, search and rescue missions in buildings and cities. The land-vehicles have also been used for planetary explorations.

It is a challenge to develop a land-vehicle not only able to move on a flat surface, but also to traverse the uneven terrain or to avoid the obstacles, because generally, the terrain traversed by a land-vehicle is not always perfectly flat or regular, but

sometimes it is rugged, i.e. rough and uneven, or it has obstacles, like rocks, etc. Moreover, if the obstacles are spreaded over the terrain, then it is unlikely for a vehicle to avoid them, instead the vehicle has to overcome the obstacles in order to reach the destination. Otherwise, it lengthens the traveling time by finding the alternative routes to reach the destination. In overcoming the obstacles, new challenges are introduced such as handling the changes in orientation and acceleration, wheels slippage, position tracking, etc. In fact, there are still a lot of unexplored research areas available in developing a land-vehicle targeted for a special application. Therefore, many researches are focusing on the development of the land-vehicles.

In developing a land-vehicle, there are two main aspects to be considered. The first aspect is the locomotion mechanism of the vehicle itself, and secondly, the autonomous navigation of the vehicle. The second aspect involves the use of advanced technologies, for example in obstacle avoidance, whereas the first aspect deals with the design of the vehicle.

The locomotion mechanism of the land-vehicle determines the mobility of the land-vehicle in a rugged terrain. Currently, there are three options of locomotion mechanisms used for land-vehicles, i.e. wheels, legs, and tracks. Based on the locomotion mechanisms, the land-vehicles are classified into wheeled-vehicles, legged-vehicles, and tracked-vehicles.

The simplest locomotion mechanism is the wheel. The wheeled-vehicle has perfect performance in terms of energy efficient, speed, and stability, while traveling on a flat surface and slightly uneven terrain. The only drawback is that the wheeled-vehicle encounters difficulties while traversing a very rugged surface. On the other hand, the legged-vehicle is very suitable for traversing rugged terrain, but it involves a complex planning and control system for maintaining its stability during the motion. It is relatively slow compared to the wheeled-vehicle while traveling on a flat surface. Each locomotion mechanism used by the land-vehicle has its own advantages and disadvantages, as discussed in the literature review chapter.

Therefore, at the current stage, a land-vehicle which combines the available locomotion mechanisms has also been developed according to its intended applications. The combination of the locomotion mechanisms compromises the limitations of each locomotion mechanism. As the number of the locomotion mechanisms in the land-vehicle increases, consequently, the complexity in the design of the land-vehicle increases inevitably.

This project provides a solution that enables a wheeled-vehicle to extend its capabilities not only to travel on a flat surface, but also on a very rugged surface, like a staircase. The solution does not combine the available locomotion mechanisms, but only make use of the wheeled-vehicle, as the simplest type among the three categories.

The wheel's shape used in this project follows an involute-curve, instead of a circle. The involute-curve has continuous increasing radius, allowing a smooth motion for the wheeled-vehicle in the sagittal plane. AWIS is a term given to such wheel. It stands for Auxiliary Wheel of Involute-Shape. It is named auxiliary because the AWIS is used to assist the normal wheel in traversing the obstacles. On a flat surface or slightly uneven terrain, the wheeled-vehicle with normal wheels is able to maneuver without difficulties, but when the vehicle encounters more difficult terrain, the AWIS attached to the normal wheels are activated.

The AWIS offers the simplest solution to the wheeled-vehicle with the capability of traversing obstacles. It is simple in terms of design, and control. The AWIS is attached to the normal wheel through a clutch, without altering the original design of the wheeled-vehicle. For controlling the clutch, a simple on-off controller is used. Consequently, the lightweight design and high energy efficient vehicle can be achieved. Moreover, the size of the AWIS is customized according to the size of the normal wheel of the wheeled-vehicle.

1.2 OBJECTIVE AND SCOPE

The main objective of this project is to develop, analyze, model, and simulate a wheeled-vehicle with AWIS (Auxiliary Wheel of Involute-Shape).

The scope of this research project is as follows:

- Carry out the literature review on the conference papers, and journal articles related to the land-vehicle.
- Perform detailed Kinematics Analysis of the AWIS to find the maximum obstacle's height traversable, and the activation distance.
- Perform detailed Kinetics Analysis of the normal wheel and the AWIS, consisting of Statics and Dynamics Analysis to find the required motor torque.
- Model, and simulate the performance of the wheeled-vehicle with AWIS in three types of environment, i.e. flat surface, surface with a block of obstacle, and surface with a staircase, including the setting of PID controllers' gains, and the coordination among the AWIS.

This thesis is organized as follows:

- Chapter 2 presents literature review, which elaborates and classifies the available land-vehicles into six groups, and some well-known examples of the groups are discussed.
- Chapter 3 provides a brief description of the design of the AWIS and a detailed kinematics analysis of the AWIS.
- Chapter 4 covers the statics and dynamics analysis of the normal wheel and the AWIS, including the calculation of mass moment of inertia of the AWIS.
- Chapter 5 presents the dynamics simulation and control of the wheeled-vehicle with AWIS.
- Chapter 6 concludes this project with the summary of each chapter, and the benefits of the AWIS to the land-vehicle.
- Appendix A provides all the programs written in the Matlab software.
- Appendix B provides detailed descriptions of the simulation results presented in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

This Chapter discusses the existing land-vehicles, which can be classified into six groups based on their locomotion mechanisms, i.e. (1) wheeled-vehicles, (2) legged-vehicles, (3) tracked-vehicles, (4) wheeled-legged-vehicles, (5) wheeled-tracked-vehicles, and (6) wheeled-legged-tracked-vehicles. Each of the six groups is described, followed by the related examples. At the end of this chapter, the summary and remarks are provided, and finally a new solution offered by this project is briefly discussed.

2.1 WHEELED-VEHICLES

A wheeled-vehicle is defined as a land-vehicle, which uses wheels as locomotion mechanism, in order to move from one place to another place. Compared to other locomotion mechanisms, the advantages of the wheeled-vehicle are as follows. First, it has simple mechanisms' design, allowing a lightweight platform and accordingly, high energy efficient. Secondly, it can perform much faster and efficient movement on a flat or slightly rough terrain. Lastly, it is easy to control, which is a crucial factor in motion planning. However, the only drawback of the wheeled-vehicles is that they are difficult to traverse rugged terrain.

Based on the structure of the wheels' support, the wheeled-vehicles can be further categorized into two groups: purely-wheeled-vehicles and articulated-wheeled-vehicles.

2.1.1 PURELY-WHEELED-VEHICLES

A purely-wheeled-vehicle is a commonly used land-vehicle with normal wheels. The normal wheels are attached to the rigid structure of the vehicle's axles. For example, a car is an example of the purely-wheeled-vehicle with four normal wheels.

2.1.2 ARTICULATED-WHEELED-VEHICLES

This group of vehicles has a wheel placed at the end of every leg. The leg has a minimum of one degree-of-freedom. On a flat surface, an articulated-wheeled vehicle acts as a purely-wheeled-vehicle as it is not necessary to drive the legs when there is no obstacle encountered.

The articulated-wheeled-vehicles can be further classified into three categories, as follows:

1. Passive articulated-wheeled-vehicles [1]

This group of vehicles has no sensors or additional actuators attached to the legs to guarantee stable movement. This leads to the limited mobility, as it highly depends on the type of the environment and the typical size of encountered obstacles. Although a vehicle with passive locomotion is less mobile, however, it matches the required criteria of the vehicle for the planetary mission in the power consumption, complexity, and reliability aspects. The vehicles belong to this category can typically overcome obstacle of their wheel size, if friction is high enough. The examples of this group of vehicles are SHRIMP [2, 3], SOJOURNER [4, 5], ROCKY 7 [6-8], and MICRO5 [9-13] (Refer to Figure 2.1).

Figure 2.1 *Passive articulated-wheeled-vehicles:*

(a) SOJOURNER, (b) MICRO5, (c) ROCKY 7

2. Active articulated-wheeled-vehicles

This group of vehicles uses motorized degrees-of-freedom for their legs and implements closed-loop control systems for the movement and coordination of the legs. The active locomotion extends the mobility of the vehicle, but complexity in control, and power consumption increases as well. The examples of such vehicles are WORKPARTNER [14-17], walking machines [18-21], NANOROVER [22, 23], and SPACECAT [24, 25] (Refer to Figure 2.2).

Figure 2.2 Active articulated-wheeled-vehicles:
(a) DANTE, (b) NANOROVER, (c) SPACECAT

3. Hybrid articulated-wheeled-vehicles

This group of vehicles has both active and passive articulated-wheels, e.g. OCTOPUS [26-28], MARSOKHOD [29-32], NANOKHOD [33-35], and HYBTOR.

Now, the examples of these three categories are described consecutively. First, is the SHRIMP, followed by the WORKPARTNER, OCTOPUS, and MARSOKHOD.

a) SHRIMP

The SHRIMP is developed based on the passive locomotion. Its prototype is measured 60 cm in length, and 20 cm in height (Refer to Figure 2.3). Its total weight is 3.1 kg, including battery of 600 g. It has a total of six wheels, i.e. front and rear steering wheels, and two wheels arranged on a bogie on each side. Each wheel is actuated by a 1.75 W DC motor and the DC motor is controlled individually using open-loop control system.

Figure 2.3 *THE SHRIMP*

The front wheel has a spring suspension to ensure optimal ground contact of all wheels at any time. The use of parallel articulations enables the front wheel to elevate and move forward simultaneously if an obstacle is encountered.

The two wheels mounted on the bogie rotate freely at a central pivot between the two wheel axles, similar to a train suspension. The parallel architecture of the bogies enables to set a virtual center of rotation at the level of or below the wheel axes while keeping the bogies at maximum ground clearances. This will ensure maximum stability and climbing ability even if the friction coefficients between the wheels and the ground are very low. Furthermore, it provides a very smooth slope of the movement of the center of gravity even when the obstacle has a vertical slope, e.g. a stair.

The SHRIMP is designed to keep all its six motorized wheels in contact with a convex ground up to a minimal radius of 30 cm or a concave ground of up to a minimal radius of 35 cm.

The SHRIMP demonstrates the capabilities to:

- ⇒ move on a very rough terrain with minimal motor power even if the friction coefficients between the wheels and the ground are relatively low
- ⇒ passively overcome obstacles of up to two times its wheel diameter (22 cm) with maximum gripping capacity and stability
- ⇒ climb a stair with height of each step exceeds 20 cm, even when the SHRIMP does not approach the stair perpendicularly
- ⇒ maneuver precisely and turn on the spot with minimum slippage

The steering of the SHRIMP is realized by synchronizing the steering of the front and rear wheels and the speed difference of the bogies' wheels.

The SHRIMP was tested to move on structured environment, i.e. step climbing, stair climbing, and unstructured environment (Refer to Figure 2.4). The SHRIMP was not able to climb the step with all its wheels covered by tape.

Figure 2.4 *The SHRIMP on a staircase*

The SHRIMP can be deployed for planetary exploration or terrestrial applications in the field of mining, construction, agriculture, post-earthquake assistance or demining.

b) WORKPARTNER

WORKPARTNER is a lightweight vehicle designed to work interactively with humans in outdoor environment. It is being developed at Intelligent Machine and Special Robotics Institute, Helsinki University of Technology, Finland. Powered by a hybrid of electrical system and 3 kW gas engine generators, the WORKPARTNER is able to work for a long period of time.

Figure 2.5 *The WORKPARTNER*

The WORKPARTNER has two manipulators, like human arms for handling tools (Refer to Figure 2.5). It has an active body joint and four 3-DoF legs. Each leg has three actively controlled joints and an actively controlled wheel. Its total weight is about 250 kg, whereas its payload is about 40 kg. The maximum speed of 7 km per hour can be achieved while moving on the flat ground.

Figure 2.6 *The HYBTOR Platform*

The manipulators are built on the platform called HYBTOR, as shown in Figure 2.6. It can be operated in three modes, i.e. legs, wheels, hybrid legs and wheels. The walking mode uses legs to move like any four-legged machines. The maximum stride distance when walking is about 0.7 m. The wheeled-driving mode uses wheels like any wheeled-vehicles. The maximum speed attainable in wheeled-driving mode is 7 km/hour on a hard ground. Hybrid locomotion means combining the walking and wheeled-driving modes so that the propulsive force is generated by the legs and the wheels' joints simultaneously. Hybrid locomotion could also be called “rolking” (rolling-walking). The purpose of having the hybrid locomotion system is to provide a rough terrain capability and a wider speed range for the vehicle at the same time.

The WORKPARTNER is targeted for light outdoor applications, e.g. property maintenance, gardening, and light forestry tasks. Figure 2.7 shows the images captured from the simulation of the WORKPARTNER while climbing a step.

Figure 2.7 *The WORKPARTNER climbs a step*

c) OCTOPUS

OCTOPUS is an innovative off-road wheeled-vehicle able to deal autonomously with obstacles in rough terrain without getting stuck (Refer to Figure 2.8). It is developed at Autonomous Systems Laboratory, Swiss Federal Institute of Technology (EPFL).

The OCTOPUS is measured 43 cm in length, 42 cm in width, and 23 cm in height. The mass of the vehicle without payload is about 10 kg, and a 5 kg payload can be mounted on the central payload support. A set of batteries is mounted on the central payload support for autonomous operation. There are in total 8 motorized wheels and 15 degrees-of-freedom (with 14 of them are motorized).

Figure 2.8 *The OCTOPUS*

The OCTOPUS is constructed of a payload support, two bodies on each side, two arms for each body, one forearm for each arm, as shown in Figure 2.8. The payload support is linked to the two bodies in a passive differential configuration. The two arms are linked to the body in a motorized parallelogram configuration. The two forearms are linked to the arm by a motorized joint. Each forearm has two tactile, motorized wheels.

The pertinent combination of the mechanical structure with tactile wheels and tilt sensors insures very high autonomous climbing skills to the robot, as depicted in Figure 2.9.

Figure 2.9 *The OCTOPUS climbs a step*

d) MARSOKHOD

MARSOKHOD is a Russian term for a Mars rover. It weighs about 70 to 75 kg. It has totally six wheels. Its wheel's shape is unique with wide profile and it tapers from one side to another side, like a cone, as shown in Figure 2.10. The tapered wheel reduces the possibility of the wheel to be stuck with the obstacle and it prevents the vehicle from overturning. Another interesting feature of the design is a portion of scientific apparatus, service systems, and power sources are arranged as

individual module placed inside the wheel. Hence, it lowers the center of mass of the vehicle, which is crucial in improving the stability of the vehicle during motion.

Figure 2.10 *The MARSOKHOD*

The MARSHOKHOD is deployed in the Mars exploration mission (Refer to Figure 2.11), e.g. to take samples of outcrops of the rocks, or other particles from the ground for further investigation.

Figure 2.11 *The MARSOKHOD traverses an obstacle*

2.2 LEGGED-VEHICLES

A legged-vehicle is defined as a vehicle that uses legs to move. The examples of vehicles belong to this category are RHEX [36-39], SCOUT [40], etc. Compared to the vehicle which adopts others locomotion mechanisms, a legged-vehicle has better performance in rugged terrain because the contact points between the tips of the legs and the ground can be arbitrarily moved to achieve the overall stability of the vehicle.

On the other hand, the drawbacks associated with the use of the legs for the land-vehicle are as follows:

- It involves more complex design mechanically, hence heavier and consumes more power.
- It requires active control algorithms for controlling the dynamics of the vehicle, e.g. to keep the balance while moving.
- It moves much slower than the wheeled-vehicle on a flat surface due to posture stability control.

2.2.1 RHEX

Several legged-vehicles have successfully performed the stair climbing action, like HONDA biped, SCOUT I, and II. However, to date, there is only HONDA biped that is able to climb full-scale stairs. The RHEX is claimed to be the smallest and simplest legged-vehicle, capable of climbing a range of human-scale stairs in a reliable manner. The design and control of RHEX was inspired by recent research in biology, particularly cockroach locomotion.

The RHEX is a hexapod with six compliant legs and six actuated degrees-of-freedom (Refer to Figure 2.12). A single actuator is located at the hip of each leg. The prototype is measured 51 cm in length, 20 cm in width, and 12.7 cm in height. The length of the leg is 16 cm. It weighs about 8 kg.

Figure 2.12 *The RHEX*

The RHEX's legs had evolved from compass legs, four-bar legs, to half-circle legs. The development of the legs is intended for increasing compliance, improving ruggedness, and stair climbing performance. The compliant leg permits dynamic gaits by embedding similar mass-spring dynamics, as found in most legs of the legged-animals during running. In order to adapt to varying stair inclinations, each leg should have two actuated degrees-of-freedom, and consequently, this would double the number of actuators required, and the design objectives are difficult to achieve, i.e. to design the simplest possible robot's locomotion mechanisms and use the minimum number of actuators. Therefore, this scheme is not implemented in the RHEX's legs. Nonetheless, the half-circle legs eliminate the disadvantages associated with two degrees-of-freedom legs in terms of weight, reliability, and power consumption.

The RHEX demonstrates capabilities to:

- ⇒ traverse highly fractured and unstable terrain
- ⇒ ascend and descend a wide range of obstacles, and human-size stairs without operators' input during the actions, despite its small size, simple mechanical design, and simple pre-programmed legs' trajectories (Refer to Figure 2.13). It is enabled by using an open-loop controller.

Figure 2.13 *The capabilities of the RHEX*

The RHEX was tested on five different stairs' geometries and materials i.e. smooth concrete, rough concrete, heavy outdoor carpet, smooth stone, and metal grate. It is concluded that the best predictor of success was the height of each step, not the length, average slope, or surface finish. However, there are stairs that RHEX cannot climb yet, such as circular stairs, and stairs with very round edges.

The RHEX can be deployed in fire and rescue applications, land mine and bomb disposal, planetary exploration, and military and law enforcement activities.

2.3 TRACKED-VEHICLES

A tracked-vehicle is defined as a land-vehicle which uses tracks' mechanisms on its both sides to move from one place to another place. The most popular example of the tracked-vehicles is URBAN [41-45] made by iRobot. The tracked vehicle has good off-road capabilities due to its robustness, stability, and good friction coefficient during motion. On the other hand, it generally has large dimensions and heavy due to the complexity of the mechanisms. Furthermore, it consumes more power due to large friction loss between the tracks and the ground, especially when the vehicle turns.

2.3.1 URBAN

The URBAN has two main side-tracks of 68.6 cm in length each and two articulated tracks in the front that can perform continuous rotation of 360 degrees, as shown in Figure 2.14. The main tracks are used for driving and steering, whereas the two articulated tracks are used to mount a stair. When the articulated tracks are stretched out, the URBAN is measured 88 cm in length, 40 cm in width, and 18 cm in height. It weighs 20 kg, with 3 kg of batteries.

Figure 2.14 *The URBAN*

The URBAN has three degrees-of-freedom. Two motors controlled independently are used to turn the main tracks as well as the articulated tracks. One motor is used to turn the articulated arms about pivot points at the front axle.

The URBAN demonstrates the capabilities to:

- ⇒ cross curbs, climb stairs, and scramble over rubble (Refer to Figure 2.15)
- ⇒ flip over with the articulated tracks if it is inverted

The maximum velocity achieved by the URBAN is 80 cm per second on a flat surface. However, the latest version of the URBAN, named PACKBOT has the maximum velocity of 220 to 370 cm per second.

Figure 2.15 *The capabilities of the URBAN*

The Urban II chassis has been developed for the purpose of reconnaissance, search and rescue mission in buildings and cities.

2.4 WHEELED-LEGGED-VEHICLES

The vehicles categorized into this group have separate wheels and legs, which act together to provide locomotion. Some examples of wheeled-legged-vehicles are WHEELEG [46-50], CHARIOT II [51], ROBOTRAC, HYBRID WHEELCHAIR, and ALDURO.

2.4.1 WHEELEG

The WHEELEG is designed and built at the DEES Robotic Laboratory of the University of Catania. The dimensions of the WHEELEG prototype are 66 cm in width, 111 cm in length, and 40 cm in height. Its total weight is 25 kg. It has two front cylindrical-type legs and two rear wheels, as shown in Figure 2.16. The front legs are actuated pneumatically, and each leg is a serial 3-dof manipulator with three revolute joints and three prismatic joints. Each foot is equipped with a touching sensor, based on four optical switches, which provides information on which side of the foot is touching the surface. Each joint of the leg has a linear potentiometer used to feedback the joint's position to the pneumatic control board.

The rear wheels are independently actuated by two separate standard brush DC motors of 16W with 130:1 gear reducers. Each gear reducer is connected to the wheel using a chain transmission system, which gives further 2:1 speed reduction ratio. The rear wheels are designed to carry most of the vehicle's weight, whereas the front legs are used to improve gripping performance on the surface, which enable the vehicle to climb and overcome obstacles.

Figure 2.16 *The WHEELLEG*

The WHEELLEG was tested on four different kinds of surfaces, i.e. flat sandy surface, sandy surface with slope of up to 20° , flat surface with rocks, and inclined surface with rocks. The average travel speed is about 12 cm per second for flat surface, 7.5 cm per second for inclined surface. Figure 2.17 shows the sequence of the WHEELLEG while traversing an obstacle. The results obtained are entirely satisfactory. It is concluded that the role of the legs in climbing the obstacles is important. Furthermore, the cooperation between the legs and the wheels is crucial to prevent the WHEELLEG from skidding.

Compared to the wheeled-vehicles, the WHEELLEG has better traction capability on rough terrain, on the other hand if compared to the legged-vehicles, the WHEELLEG can move faster, and it is more stable. However, a more complex control system which allows the cooperation of the rear wheels and the front legs is required, especially when there is lack of traction, due to insufficient pressure while moving on rough terrain.

Figure 2.17 A sequence of the WHEELLEG while traversing an obstacle

The possible applications of the WHEELLEG are the exploration of unstructured environments, such as volcanoes, etc., post-disaster applications, humanitarian demining, and planetary exploration.

2.5 WHEELED-TRACKED-VEHICLES

A wheeled-tracked-vehicle combines wheels and tracks as its locomotion mechanisms. One example of the vehicles belongs to this group is HELIOS-VI [52-54].

2.5.1 HELIOS-VI

The HELIOS series land-vehicles are designed and developed in Japan to ascend and descend stairs. At this moment, the latest series developed is HELIOS-VI.

The HELIOS-VI prototype is measured 105.5 cm in length, 80 cm in width, and 40 cm in height with total weight of 85 kg. It is propelled by two tracks on both sides of the platform. It has a continuously variable transmission on each side to alter the velocity of each track and consequently change the orientation of the vehicle. The prototype also has two active arms attached to the axis of the one drive pulley of the active track. One of the arms has two passive tires installed at its tips, and the arm's angular position is movable through ± 90 degrees. The tires improve the vehicle's adaptability on rough terrain and act as footholds while ascending and descending the stair. The other arm is used to carry payload and adjust payload posture (Refer to Figure 2.18).

There are totally six actuators being used. Four DC motors (150 W) are for right track, left track, carrier, and front wheel arm, respectively. Two DC motors (11 W) are used for driving continuously variable transmission.

Figure 2.18 The HELIOS-VI

Figure 2.19 The HELIOS-VI ascends a stair

The HELIOS-VI demonstrates good capabilities in:

- ⇒ producing smooth, energy-efficient, and high-speed motion on a flat surface with a maximum velocity of 86.7 cm per second
- ⇒ traversing stairs or bumps inclined at angles from 30 to 40 degrees (Refer to Figure 2.19), craggy surface (e.g. rocky stretches), soft surface (e.g. marshland)
- ⇒ steering and turning that enables it to circle at a point on a flat surface without damaging the surface
- ⇒ carrying load of about 100 kg on the carrier

In comparison to the previously built HELIOS series, the HELIOS-VI possesses a simpler, lighter mechanical system, and improved rigidity.

The HELIOS series prototypes are deployed for the development of powered wheelchairs for the elderly and the disabled as well as carrier vehicles, e.g. to assist workers in carrying heavy loads up and down stairs.

2.6 WHEELED-LEGGED-TRACKED-VEHICLES

In this section, a vehicle which uses all three locomotion mechanisms is discussed. An example of this vehicle is AZIMUT [55, 56].

2.6.1 AZIMUT

The first prototype of the AZIMUT was completed in December 2002. The AZIMUT is designed symmetrically. It has four independent articulations attached to the corners of a square frame, and each articulation is made of two wheels, a leg, and a track (Refer to Figure 2.20). Modularity is a key specification for its design. There are totally twelve degrees-of-freedom and consequently, twelve motors are being used. The AZIMUT implements PID controllers for all of the motors. Each leg can

rotate 360 degrees around the y-axis, and 180 degrees around the z-axis. Once an articulation is placed at the right position, it is locked mechanically.

Figure 2.20 *The AZIMUT*

The first prototype of the AZIMUT demonstrates the capabilities of (Refer to Figure 2.21):

- ⇒ Changing the orientation of its articulations for omni-directional movements
- ⇒ Going up and down stairs and inclined surfaces

The first prototype of the AZIMUT is incapable of lifting itself up due to its heavy platform. However, this limitation can be overcome by using composite materials in building the platform, or reducing friction on the rotational joint of the articulation. Although the AZIMUT is much heavier than the URBAN, the AZIMUT provides more diverse applications. For example, the AZIMUT has minimal difficulties in climbing a circular staircase.

The second prototype is going to be built in the near future, opening up new research issues in the areas of distributed control of the articulations, four-wheel-steering control modes, and perception in 3D environment for navigation and obstacle avoidance.

Figure 2.21 *The capabilities of the AZIMUT*

2.7 SUMMARY AND REMARKS

In this chapter, all six groups of land-vehicles are described together with the well-known examples. Generally, the performance of a land-vehicle while traversing a rugged terrain can be assessed by evaluating the performance based on four major categories. Table 2.1 shows the performance of each type of the land-vehicles based on the four major categories.

Table 2.1 Comparison of the performance of each type of the land-vehicles

Locomotion Modes	Terrain Adaptability	Energy Efficiency	Locomotion Speed	Simplicity in Control & Mechanism
Wheeled-vehicle	Poor	Good	Good	Good
Legged-vehicle	Good	Poor	Poor	Poor
Tracked-vehicle	Average	Average	Average	Good
Wheeled-legged-vehicle	Good	Good	Good	Average
Wheeled-tracked-vehicle	Good	Average	Good	Average
Legged-tracked-vehicle	Good	Average	Average	Average
Wheeled-legged-tracked-vehicle	Good	Average	Good	Poor

The first three locomotion modes are the most basic categories. To compromise the drawbacks of each mode of the locomotion, more and more land-vehicles are designed and developed by using the combination of the available locomotion mechanisms, which have been discussed earlier in this chapter. Consequently, it involves more and more complex aspects, for example, in the design of the mechanisms, and control.

It can be seen from the table that the wheeled-vehicle satisfies the three categories, except the terrain adaptability. To compensate the drawbacks of the wheeled-vehicle, many wheeled-vehicles are built by incorporating additional legs, such that the vehicle is more adaptable to uneven terrain. However, there is another way to make the wheeled-vehicles more adaptable to any terrains, i.e. to add modified wheels to the existing wheels. This project emphasizes on the development of the wheeled-vehicle to enhance the vehicle's adaptability on rugged terrain by using these modified wheels.

CHAPTER 3

DESIGN AND KINEMATICS ANALYSIS OF AWIS

This chapter discusses firstly the design of AWIS (Auxiliary Wheel of Involute-Shape), followed by the kinematics analysis of AWIS. A generalized equation to represent the location of a point on the AWIS is derived. Finally, the maximum height of traversable obstacle is calculated.

3.1 DESIGN OF AWIS

AWIS stands for Auxiliary Wheel of Involute-Shape. It is intended for providing the wheeled-vehicle with the ability to traverse the obstacles. Its outer shape takes the form of involute-curve (Refer to Figure 3.1). Although other curves are possible, such as polynomial curves, however, in this project, the involute curve is chosen. Furthermore, it is also used in the design of teeth for an involute-gear.

In this case, the AWIS is attached to the normal wheel through a clutch. When the clutch is activated, the shaft of the AWIS is engaged to the shaft of the normal wheel, and consequently, the AWIS rotates. In the following, the involute-curve is briefly described.

- Involute-curve

Involute-curve is defined as a group of end points of a string resulted as the string is unwrapped around the circumference of a base circle with radius, r_b . For example, the string end point, P_C is obtained as the string is unwrapped from point D_0 to point D_C , which is measured as θ_{PC} (Refer to Figure 3.1). Line $D_C P_C$

is tangent to the base circle at point D_C . The length of line $D_C P_C$ is equal to the length of arc $D_0 D_C$, which is $r_b \cdot \theta_{PC}$. α_{PC} measures the angle between x-axis and line CP_C at center point, C, represented in local coordinate system $x_1 y_1$.

Figure 3.1 The AWIS

- Auxiliary Wheel of Involute-shape (AWIS)

The outer-shape of the AWIS consists of two parts, i.e. an involute-curve with α_{PC} ranging from 0 to 180 degrees, and a half circle of radius, r_b with α_{PC} ranging from 180 to 360 degrees.

3.2 KINEMATICS ANALYSIS OF AWIS

Kinematics analysis is performed to determine the kinematics parameters, e.g. position, velocity, or acceleration of a point of interest in an object. In this case, we will determine the kinematics parameters of a point, P on the involute-curve (Refer to Figure 3.2).

Figure 3.2 Point P on the involute-curve

The location of point P with respect to local coordinate system x_1y_1 , is derived to be:

$$x_P = r_b \cdot (\cos \theta + \theta \cdot \sin \theta) \quad (3-1a)$$

$$y_P = r_b \cdot (\sin \theta - \theta \cdot \cos \theta) \quad (3-1b)$$

In analyzing the AWIS, it is more convenient to express the equations in terms of α instead of using θ . The reason is that α directly shows the angular position of point P measured from x-axis of the local coordinate system. Hence, at this moment, the relationship between α and θ , is to be found. From equation (3-1),

$$\tan \alpha = \frac{y_p}{x_p} = \frac{r_b \cdot (\sin \theta - \theta \cdot \cos \theta)}{r_b \cdot (\cos \theta + \theta \cdot \sin \theta)} \quad (3-2a)$$

$$\alpha = a \tan \left(\frac{\tan \theta - \theta}{1 + \theta \cdot \tan \theta} \right) \quad (3-2b)$$

Equations (3-1) are expressed in terms of θ . To substitute θ for α , the inverse of equation (3-2b) is required. Using Matlab [57] (Program 1) (All programs are provided in Appendix A), the inverse was expressed in imaginary form, which indicated no inverse could be obtained. However, by differentiating and integrating equation (3-2b) respectively (Program 2), the simplified equation was obtained to be:

$$\alpha = \theta - \text{atan } \theta \quad (3-3)$$

Figure 3.3 Plot of Alpha (deg) vs. Theta (deg) (Program 3)

Figure 3.3 validates equation (3-3) as a simplified form of equation (3-2b). In fact, Figure 3.4 shows that equation (3-3) is more representative than equation (3-2b), as the function is continuous.

Figure 3.4 Plot of Alpha (deg) vs. Theta (deg) (*Program 4*)

It is obvious that α values are always less than θ values. When α is 180 degrees (π rad), the corresponding θ value is approximated to be 257.4535 degrees (4.4934 radians). This value is obtained from a table of theta vs. alpha in degrees (*Program 5*). The θ value is important because it represents the upper limit of the involute-curve of the AWIS.

The distance from center point C to point P is (Refer to Figure 3.2):

$$r_p = r_b \cdot \sqrt{1 + \theta^2} \quad (3-4)$$

As θ increases, the r_p value increases as well (Refer to Figure 3.5). The radius of the base circle of the AWIS, r_b , is 0.25 m, as it is used in the simulation.

Figure 3.5 Plot of Radius vs. Alpha (deg) and Radius vs. Theta (deg)
with $r_b = 0.25 \text{ m}$ (Program 6)

3.2.1 FINDING THE CIRCUMFERENCE OF THE INVOLUTE-CURVE

In order to calculate the distance traveled by the AWIS, as the AWIS rotates, the involute-curve's circumference must be worked out as follows:

Figure 3.6 Finding the length of arc AB

Arc AB is a segment of a curve with function $r_p = f(\alpha)$, starting from $\alpha=\alpha_1$ to $\alpha=\alpha_2$ (Refer to Figure 3.6). For polar coordinate system, the equation used to find the length of arc AB is:

$$\int_{\alpha_1}^{\alpha_2} \sqrt{r_p^2 + \left(\frac{dr_p}{d\alpha}\right)^2} d\alpha \quad (3-5)$$

By substituting equations (3-3) and (3-4) into equation (3-5), and using partial differentiation, the circumference of the involute-curve with function $r_p = f(\alpha)$, starting from $\alpha=\alpha_1$ to $\alpha=\alpha_2$ is:

$$\int_{\alpha_1}^{\alpha_2} r_b \cdot \theta d\theta \quad (3-6)$$

Note that because the integrand is integrated with respect to θ , hence θ_1 and θ_2 are used instead of α_1 and α_2 , as the lower and upper limits of the integration, respectively. The corresponding θ_1 and θ_2 are calculated from α_1 and α_2 by using equation (3-3).

Figure 3.7 shows the plot of arc length (m) vs. alpha (deg) when the radius of the base circle, r_b is 0.25 m. The arc length value read from this plot at alpha = x deg is the involute-curve circumference measured from alpha = 0 deg to x deg. The curve with concave-shape is obtained as the radius of the involute-curve increases with alpha. The involute-curve circumference with α ranging from 0 to 180 degrees is 2.5238 m with $r_b = 0.25$ m, and it is denoted by thick blue colour on the graph in Figure 3.8.

Figure 3.7 Plot of Arc Length (m) vs. Alpha (deg) with $r_b = 0.25$ m. (Program 7)

Figure 3.8 Involute-curve circumference with $r_b = 0.25$ m. (Program 8)

3.2.2 LOCATIONS OF POINT C AND POINT P ON THE ROTATING AWIS

In this section, the locations of point P and center point C on the rotating AWIS represented in local and global coordinate systems are derived in sequence. Firstly, the rotation matrix is derived as follows:

1. Rotation matrix

x_1y_1 is a local coordinate system which is attached to the center point C of the rotating AWIS, whereas x_0y_0 is a global coordinate system. The location of point P on the involute-curve can be expressed in either local or global coordinate systems. This requires the rotation matrix, which enables the conversion from one coordinate system to another coordinate system.

Figure 3.9 Rotation Matrix

$$\begin{bmatrix} x_{P0} \\ y_{P0} \end{bmatrix} = \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} x_{P1} \\ y_{P1} \end{bmatrix} \quad (3-7)$$

where β is the rotation angle measured from initial to final position, x_{P0} is the x-location of point P represented in x_0y_0 coordinate system, and x_{P1} is the x-location of point P represented in x_1y_1 coordinate system.

2. The AWIS is at its initial position (Contact Point is S_A and $\beta = 0$ rad)

This is the starting position when the AWIS is activated by engaging clutch (Refer to Figure 3.10).

Figure 3.10 The AWIS at its initial position ($\beta = 0$ rad)

Center Point C :

$$\begin{bmatrix} x_{C1} \\ y_{C1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (3-8a)$$

$$\begin{bmatrix} x_{C0} \\ y_{C0} \end{bmatrix} = \begin{bmatrix} x_C \\ r_b \end{bmatrix} \quad (3-8b)$$

where x_C is an initial distance between center point, C, and origin of the global coordinate system. Note that the x and y locations of point C in local coordinate system is always (0, 0), hence it will not be rewritten for the discussion onwards.

Point P :

$$\begin{bmatrix} x_{P1} \\ y_{P1} \end{bmatrix} = r_b \begin{bmatrix} \theta & 1 \\ 1 & -\theta \end{bmatrix} \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} \quad (3-8c)$$

$$\begin{bmatrix} x_{P0} \\ y_{P0} \end{bmatrix} = r_b \begin{bmatrix} \theta & 1 \\ 1 & -\theta \end{bmatrix} \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} + \begin{bmatrix} x_C \\ r_b \end{bmatrix} \quad (3-8d)$$

3. The AWIS rotates on the base circle of radius, r_b (Contact Point is S_B and $\beta > 0$ rad)

Now, consider when the AWIS rotates on the base circle of radius, r_b .

Figure 3.11 *The AWIS rotates on the base circle*

Center Point C :

$$\begin{bmatrix} x_{c0} \\ y_{c0} \end{bmatrix} = \begin{bmatrix} x_c \\ r_b \end{bmatrix} + \begin{bmatrix} r_b \cdot \beta \\ 0 \end{bmatrix} \quad (3-9a)$$

Point P :

$$\begin{bmatrix} x_{p1} \\ y_{p1} \end{bmatrix} = r_b \cdot \begin{bmatrix} \theta & 1 \\ 1 & -\theta \end{bmatrix} \cdot \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} \quad (3-9b)$$

$$\begin{bmatrix} x_{p0} \\ y_{p0} \end{bmatrix} = \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} x_{p1} \\ y_{p1} \end{bmatrix} + \begin{bmatrix} x_{c0} \\ y_{c0} \end{bmatrix} \quad (3-9c)$$

4. Both base circle and involute-curve of the AWIS touch ground (Contact Points are S_C and P_A , and $\beta = \beta_1$ rad)

When the AWIS rotates on the base circle, it eventually reaches a condition where the involute-curve touches ground as well. This is crucial, as at this time, the AWIS starts rotating on the involute-curve and moving in both vertical and horizontal directions, as depicted in Figure 3.12.

Figure 3.12 Both base circle and involute-curve of the AWIS touch ground

The angular position, β_1 where both base circle and involute-curve touch ground at points S_C and P_A , respectively, is determined as follows:

Contact Point P_A :

$$x_{P_A1} = r_b \cdot (\cos \theta_{P_A} + \theta_{P_A} \cdot \sin \theta_{P_A}) \quad (3-10a)$$

$$y_{P_A1} = r_b \cdot (\sin \theta_{P_A} - \theta_{P_A} \cdot \cos \theta_{P_A}) \quad (3-10b)$$

Then, the gradient of tangent line at point P_A is derived as follows:

$$\frac{dy_{P_A}}{d\theta_{P_A}} = r_b \cdot \theta_{P_A} \cdot \sin \theta_{P_A} \quad (3-11a)$$

$$\frac{dx_{P_A}}{d\theta_{P_A}} = r_b \cdot \theta_{P_A} \cdot \cos \theta_{P_A} \quad (3-11b)$$

$$\frac{dy_{P_A}}{dx_{P_A}} = \tan \theta_{P_A} \quad (3-11c)$$

Gradient of line CD_A is:

$$m_{CD_A} = \tan \theta_{P_A} \quad (3-11d)$$

It can be deduced that the tangent line at point P_A is parallel to line CD_A . Note that point P_A is a point on the involute-curve, which is formed by point D_A , and line D_AP_A is tangent to line CD_A according to the involute-curve's property. A schematic diagram of the angles is constructed as shown in Figure 3.13.

Figure 3.13 The schematic diagram of the AWIS at this instance

The θ_{PA} value at which length of line D_AP_A is equal to r_b , is 1 radian. To reach this position, the local coordinate system x_1y_1 , must rotate through 1 radian. Hence, β , which is the rotation angle of the AWIS, in this case, β_1 , is equal to 1 radian as well.

For any contact points P, the following relationship holds:

$$\beta = \theta_p \quad (3-12a)$$

In this position, $\beta_1 = \theta_{p_A} = 1 \text{ rad} = 57.32 \text{ deg}$. From Figure 3.13, the relationship between θ_{PA} and α_{PA} for this position, is found to be:

$$\theta_{p_A} - \alpha_{p_A} = \frac{PI}{4} \quad (3-12b)$$

α_{PA} is calculated to be $(1 - \frac{PI}{4}) \text{ rad} \approx 0.2146 \text{ rad} \approx 12.2958 \text{ deg}$.

Hence, the location of point C is:

$$(x_{C0}, y_{C0}) = (x_C + r_b, r_b)$$

By substituting θ_{PA} and β_1 values, which are 1 radian, to equations (3-10) and (3-9c), respectively, the location of point P_A is:

$$(x_{p_{A1}}, y_{p_{A1}}) = (1.3818 * r_b, 0.3012 * r_b)$$

$$(x_{p_{A0}}, y_{p_{A0}}) = ((x_C + 2 * r_b), 0)$$

Note that the location of contact point P_A represented in global coordinate system can be obtained directly by observing the distance of point P_A from the center point, C, if the location of point C in global coordinate system, is known in advance.

5. The AWIS rotates on the involute-curve (Contact Point is P_B and $\beta > 1$ rad)

Figure 3.14 The AWIS rotates on the involute-curve

It can be seen from Figure 3.14 that $y_{C0} = CP_B \cdot \sin(\theta_{P_B} - \alpha_{P_B})$ and by substituting $\alpha_{P_B} = \theta_{P_B} - a \tan \theta_{P_B}$ into the equation, the location of center point C is:

Center Point C :

$$\begin{bmatrix} x_{C0} \\ y_{C0} \end{bmatrix} = \begin{bmatrix} x_C \\ 0 \end{bmatrix} + \begin{bmatrix} r_b \cdot \beta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} \int_{\alpha_{P_A}}^{\alpha_{P_B}} r_b \cdot \theta_{P_B} \cdot d\theta \\ r_b \cdot \sqrt{1 + \theta_{P_B}^2} \cdot \sin(a \tan \theta_{P_B}) \end{bmatrix} \quad (3-13a)$$

Contact Point P_B :

$$\begin{bmatrix} x_{P_B1} \\ y_{P_B1} \end{bmatrix} = r_b \cdot \begin{bmatrix} \theta_{P_B} & 1 \\ 1 & -\theta_{P_B} \end{bmatrix} \cdot \begin{bmatrix} \sin \theta_{P_B} \\ \cos \theta_{P_B} \end{bmatrix} \quad (3-13b)$$

Likewise, by observing the location of contact point P_B with respect to center point C , the location of point P_B represented in global coordinate system is:

$$\begin{bmatrix} x_{P_B 0} \\ y_{P_B 0} \end{bmatrix} = \begin{bmatrix} x_{C0} + r_b \\ 0 \end{bmatrix} \quad (3-13c)$$

As the AWIS rotates, the contact point P_B moves along the involute-curve and eventually reaches the end of the involute-curve, as illustrated in Figure 3.15.

Figure 3.15 The end of the involute-curve touches the ground

In this case, α_{PB} is 180 degrees, and as calculated previously, the corresponding θ_{PB} is 257.4534 degrees or 4.4934 radians. From Figure 3.15, ϕ value is calculated to be 12.5466 degrees or 0.219 radians.

Substituting $\beta_1 = 1$ rad, $\alpha_{PA} = 0.2146$ rad, $\alpha_{PB} = \pi$ rad into equations (3-3), (3-13a), and (3-13b), the locations of center point C and contact point P_B are:

$$(x_{C0}, y_{C0}) = ((x_C + 10.5953 * r_b), 4.4934 * r_b)$$

$$(x_{P_B1}, y_{P_B1}) = (-21.1906 * r_b, 0)$$

The location of point P_B in global coordinate system follows the same rule as in equation (3-13c).

Note that, the AWIS rotates on the involute-curve starting from $\theta_{PA} = 1$ rad to the end point of the involute-curve, with $\theta_{PB} = 4.4934$ rad (Refer to Figures 3.12 and 3.15). Therefore, the equations mentioned in this sub-section are valid only for $1 \leq \beta \leq 4.4934$ radians.

Thus far, the contact point P_B moves as the AWIS rotates. After the contact point P_B reaches the end of the involute-curve (Refer to Figure 3.15), it will remain at the end point of the involute-curve and it serves as center of rotation, until the base circle or the line CP_B hits ground or obstacle.

6. The AWIS hits an obstacle at point P_C on the involute-curve

Figure 3.16 shows the angular position of the AWIS when it hits an obstacle at point P_C . The regular-shape obstacle is chosen, in this case a rectangular block is used since it has the steepest slope, which is 90 degrees, and hence it is the most challenging obstacle to be overcome.

Figure 3.16 *The AWIS hits an obstacle*

At this position, the location of center point C is the same as equation (3-13a). The location of contact point P_B is the same as equations (3-13b) and (3-13c).

The location of point P_C is:

$$\begin{bmatrix} x_{P_C1} \\ y_{P_C1} \end{bmatrix} = r_b \cdot \begin{bmatrix} \theta_{P_C} & 1 \\ 1 & -\theta_{P_C} \end{bmatrix} \cdot \begin{bmatrix} \sin \theta_{P_C} \\ \cos \theta_{P_C} \end{bmatrix} \quad (3-14a)$$

$$\begin{bmatrix} x_{P_C0} \\ y_{P_C0} \end{bmatrix} = \begin{bmatrix} \cos \theta_{P_B} & \sin \theta_{P_B} \\ -\sin \theta_{P_B} & \cos \theta_{P_B} \end{bmatrix} \cdot \begin{bmatrix} x_{P_C1} \\ y_{P_C1} \end{bmatrix} + \begin{bmatrix} x_{C0} \\ y_{C0} \end{bmatrix} \quad (3-14b)$$

Note that the value of θ_{P_B} is equal to β . Then, a generalized equation to represent the location of a point on the involute-curve, e.g. point P_C, represented in global coordinate system is derived to be:

$$\begin{aligned} \begin{bmatrix} x_{P_C0} \\ y_{P_C0} \end{bmatrix} &= \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix} \cdot r_b \cdot \begin{bmatrix} \theta_{P_C} & 1 \\ 1 & -\theta_{P_C} \end{bmatrix} \cdot \begin{bmatrix} \sin \theta_{P_C} \\ \cos \theta_{P_C} \end{bmatrix} \\ &+ \begin{bmatrix} x_C + r_b \cdot \beta_1 + \int_{\alpha_{P_A}}^{\alpha_{P_B}} r_b \cdot \theta_{P_B} \cdot d\theta \\ r_b \cdot \sqrt{1 + \theta_{P_B}^2} \cdot \sin(a \tan \theta_{P_B}) \end{bmatrix} \end{aligned} \quad (3-15)$$

3.2.3 FINDING THE ACTIVATION DISTANCE (x_d) OF THE AWIS

Activation distance, x_d is defined as the distance at which the AWIS is activated, such that the AWIS is able to traverse the obstacle that is lying ahead. The distance is measured horizontally from the center point C of the AWIS to point U_A on the obstacle. Assume that the initial angular position of the AWIS when it is activated, is at $\beta = 0$ deg, as shown in Figure 3.17.

Figure 3.17 Finding the activation distance of the AWIS

Point U_A is a point on the obstacle (Refer to Figure 3.17). As the AWIS rotates, it will eventually hit the obstacle. Point P_C is a point on the involute-curve that is in contact with point U_A when the AWIS hits the obstacle. Hence, it can be inferred that the locations of both points U_A and P_C in global coordinate system, are the same.

$$x_{P_C0} = x_{U_A0} = x_C + x_d \quad (3-16)$$

There are four variables involved in solving y-equation in equation (3-15), i.e. (1) y_{P_C0} whose value is the same as the height of the obstacle, y_{U_A0} , (2) β , angular position of the AWIS, (3) θ_{P_C} , and (4) θ_{P_B} , whose value is the same as β . In this case, a value of $\beta = \beta_H$, is specified. β_H represents the angular position of the AWIS when it hits the obstacle. Substituting these values into the y-equation, to calculate the value of θ_{P_C} .

Next, the θ_{P_C} value and equation (3-16) are substituted into x-equation of equation (3-15). The activation distance, x_d , is then calculated.

A graphical user interface has been created for the user to calculate the activation distance as shown in Figure 3.18 (Programs 9 & 10). In this case, the user is prompted to enter values into three boxes representing the base radius, r_b , obstacle height, y_{P_C0} , and angular rotation, β_H , respectively. After that, the user must click the

“Calculate” button, then the system will manipulate the input data, and the result, which is activation distance, is produced. Note that the AWIS rotates on the involute-curve when $1 \leq \beta \leq 4.4934$ rad. Consequently, the specified angular rotation, β_H , must fall within these lower and upper limits.

Figure 3.18 Graphical user interface to calculate the activation distance

Additionally, in order to help user to better visualize the angular position of the AWIS when it hits the obstacle, a program has been developed to produce the plot of the angular position of the AWIS (Program 8).

1. Plotting the angular position of the AWIS

The program 8 mentioned above requires two parameters to be specified by the user, i.e. (1) base radius, r_b , and (2) angular rotation, β_H . Figure 3.19 illustrates the AWIS with four different values of angular rotations.

Figure 3.19 The AWIS with $\beta_H = (a) 0^\circ$, $(b) 100^\circ$, $(c) 200^\circ$, $(d) 257.4528^\circ$

3.2.4 FINDING THE MAXIMUM HEIGHT OF TRAVERSABLE OBSTACLE ($h_{\max}$) OF THE AWIS

Note that each angular position of the AWIS has its corresponding maximum obstacle's height traversable. This is due to the unique shape of the involute-curve itself. In certain cases, the AWIS is able to traverse the obstacle with height higher than the maximum allowable height. This is possible if the obstacle has irregular-shape such that the AWIS is supported by the obstacle at more than one contact points. The larger the area of contact, the more frictional effects resulted between the AWIS and the obstacle.

Figure 3.20 *The AWIS are not able to traverse the obstacles*

Figure 3.20 illustrates some examples when the AWIS are not able to traverse the obstacles. This is due to the obstacles' heights exceed the maximum height traversable for each angular position of the AWIS. As a result, the maximum allowable height for each angular position of the AWIS is to be calculated in this section ([Program 11](#)). Note that the maximum allowable heights are important for the user while entering the values of the obstacle's height and the angular rotation in calculating the activation distance.

Program 11 produces some results as follows. Firstly, for each angular position of the AWIS, β_H , specified by the user, a plot of x-values of the points on the involute-curve in global coordinate system is produced. Figure 3.21 shows the plot of x-values for $r_b = 0.25$ m, $\beta_H = 100$ deg and 200 deg, respectively.

Figure 3.21 *x*-distances from center point *C* to points along the involute-curve for $\beta_H = (a) 100^\circ$, $(b) 200^\circ$

Secondly, for each angular position of the AWIS, β_H , the maximum value of *x*, $x_{\max}$, the corresponding α_{P_C} , and the maximum height, $h_{\max}$ are calculated (Refer to Table 3.1). Two equations are used to calculate the $h_{\max}$, i.e.:

For $0 \leq \beta_H \leq 1$ rad (when the AWIS rotates on the base circle)

$$h_{\max} = -(\sin \beta) * \left[r_b \cdot \sqrt{1 + \theta_{P_C}^2} \cdot \cos \alpha_{P_C} \right] + (\cos \beta) * \left[r_b \cdot \sqrt{1 + \theta_{P_C}^2} \cdot \sin \alpha_{P_C} \right] + r_b \quad (3-17a)$$

For $1 < \beta_H \leq 4.4934$ rad (when the AWIS rotates on the involute-curve)

$$h_{\max} = -(\sin \beta) * \left[r_b \cdot \sqrt{1 + \theta_{P_C}^2} \cdot \cos \alpha_{P_C} \right] + (\cos \beta) * \left[r_b \cdot \sqrt{1 + \theta_{P_C}^2} \cdot \sin \alpha_{P_C} \right] + r_b \cdot \sqrt{1 + \theta_{P_B}^2} \cdot \sin(a \tan \theta_{P_B}) \quad (3-17b)$$

Table 3.1 The maximum heights of traversable obstacles for the AWIS

β_H (degrees)	α_{P_C} (degrees)	$x_{\max}$ (m)	$h_{\max}$ (m)
(a) 0	32.4819	0.3927	0.5000
(b) 100	116.7809	0.8290	0.6863
(c) 200	179.9995	1.0814	0.4790
(d) 257.4528	179.9995	0.25	0.0000

Note that table 3.1 is constructed based on the base circle of radius, r_b , of 0.25 m. Figure 3.22 shows the maximum heights for each angular position specified in table 3.1.

Figure 3.22 The maximum heights for the AWIS with $\beta_H = (a) 0^\circ, (b) 100^\circ, (c) 200^\circ, (d) 257.4528^\circ$

The maximum obstacle's height traversable, $h_{\max}$ is the y-distance measured from the ground to a point on the involute-curve represented in global coordinate system. The point has the furthest x-distance from center point, C (Refer to Figure 3.22).

Figure 3.23 shows the plot of the maximum obstacle's height traversable for angular position of the AWIS, ranging from 0 to 257.4528 degrees. Note that the radius of base circle, r_b , is 0.25 m.

Figure 3.23 Plot of the maximum obstacle's height traversable for β_H ranging from 0 to 257.4528 degrees

Now, we would like to find out the angular position, β_H , of the AWIS when it has the highest value among all the maximum obstacles' heights traversable. It is simply the maximum point on the graph plotted in Figure 3.23, i.e. when β_H is 2.9226 radians.

It is interesting to discover that the maximum obstacle's height traversable for angular position, β_H of the AWIS, ranging from 1 to 2.9226 radians can be calculated by using the following equation (Refer to Figure 3.24):

$$\begin{aligned} h_{\max} &= y_{C0} + r_b \\ &= r_b \cdot [\beta_H + 1] \end{aligned} \quad (3-18)$$

Note that $\beta_H = \theta_{P_B}$

Figure 3.24 The maximum obstacle's height traversable for β_H ranging from 1 to 2.9226 radians

Figure 3.25-b shows the plot of equation (3-18) with r_b equal to 0.25 m and β_H ranges from 1 to 2.9226 rad. Furthermore, if Figure 3.25-b is compared to Figure 3.25-a, then it is obvious that equation (3-18) is valid.

Figure 3.25 Plots of equations (a) 3-17, (b) 3-18

Additionally, to prove that equation (3-18) is valid, point D_C in Figure 3.26 is shown to have the same height as point P_C , which is the maximum obstacle's height traversable.

Figure 3.26 *Proof of equation (3-18) geometrically*

Suppose that point P_C is the farthest point on the involute-curve, measured from center point C in x -direction. Consequently, it corresponds to the maximum obstacle's height traversable for that angular position of the AWIS. A tangent line at point P_C means that this line does not intersect the involute-curve, except at point P_C . In other words, point P_C is either a maximum or minimum point. Point P_C is a point on the involute-curve, and it is originated from point D_C . Therefore, the gradient of the tangent line, i.e. line y_2 , at point P_C , represented in local coordinate system, x_1y_1 , is derived as follows:

$$\frac{dy_{P_C}}{d\theta_{P_C}} = r_b \cdot \theta_{P_C} \cdot \sin \theta_{P_C}$$

$$\frac{dx_{P_C}}{d\theta_{P_C}} = r_b \cdot \theta_{P_C} \cdot \cos \theta_{P_C}$$

$$\frac{dy_{P_c}}{dx_{P_c}} = \tan \theta_{P_c} \quad (3-19)$$

The gradient of line y_1 is:

$$m_{y_1} = \tan \theta_{P_C}$$

It can be concluded that lines y_1 and y_2 are parallel to each other. Point P_C has the same height as point D_C in global coordinate system, x_0y_0 .

The peak point in Figure 3.23 occurred if and only if both points D_C and P_C have the same vertical heights and the point P_C located at the end point of the involute-curve has the farthest x-distance from the center point C , as shown in Figure 3.27. Note also that as the AWIS rotates on the involute-curve, point P_C (as the farthest point measured horizontally from the center point C) moves along the involute-curve till it reaches the end point of the involute-curve.

Figure 3.27 The highest value of h_{max} when the AWIS rotates on the involute-curve

The value of β_H at the position shown in Figure 3.27 is calculated by substituting $\theta_{P_C} = 4.4934$ radians, and $r_b = 0.25$ m into equations (3-17b) and (3-18). Note that $\theta_{P_C} = \beta_H$ and $\alpha_P = \theta_P - a \tan \theta_P$. β_H value is obtained to be 2.9226 rad, which is the same as the value read from the graph in Figure 3.23.

It can be seen from Figure 3.23, when the angular position of the AWIS, β_H , ranges from 0 to 1 radian, the maximum obstacle's height traversable, $h_{\max}$, is at a constant value of 0.5 m. Subsequently, $h_{\max}$ increases from 0.5 m to 0.98065 m, as the AWIS rotates with β_H ranging from 1 to 2.9226 radians. Then, $h_{\max}$ decreases to 0 m, as the AWIS rotates with β_H ranging from 2.9226 to 4.4934 radians.

The maximum heights of traversable obstacles vary when the AWIS rotates on the involute-curve with β_H ranging from 0 to 4.4934 radians. From the range of the maximum obstacles' heights traversable, there is a highest value, i.e. 0.98065 m. It happens when the AWIS is at $\beta_H = 2.9226$ radians.

Now, consider when the AWIS rotates about a point at the end of the involute-curve, P_C , as illustrated in Figure 3.28. The locations of center point C and contact point P_C are determined as follows:

Figure 3.28 The AWIS rotates about the end-point of the involute-curve

Center Point C:

Substitute $\theta_{P_B} = 4.4934$ rad into equation (3-4)

$$(x_{C0}, y_{C0}) = ((x_C + 11.5953 * r_b), 4.6033 * r_b)$$

Contact Point P_C :

$$(x_{P_C1}, y_{P_C1}) = (-4.6033 * r_b, 0)$$

$$(x_{P_C0}, y_{P_C0}) = ((x_C + 11.5953 * r_b), 0)$$

Figure 3.29 The highest value of h_{max} when the AWIS rotates about the end-point of the involute-curve

When the AWIS rotates about the end-point of the involute-curve, i.e. for $\beta_H \geq 4.4934$ rad, the highest value of h_{max} is found to be the distance from point P_C to point C, as depicted in Figure 3.29.

$$h_{max} = 4.6033 * r_b \quad (3-20)$$

If the radius of the base circle, r_b , is 0.25 m, then the highest value of h_{max} when the AWIS rotates about the end-point of the involute-curve is 1.15083 m.

Figure 3.30 shows the position of the AWIS in traversing an obstacle with minimum slippage. In this case, the maximum height of traversable obstacle is the distance measured from point P_C to point U , which is calculated to be 0.90083 m.

Figure 3.30 The ideal position of the AWIS while traversing an obstacle

In conclusion, when the AWIS rotates on the involute-curve, the highest value of the $h_{\max}$ is 0.98065 m, whereas when the AWIS rotates about the end point of the involute-curve, the highest value of $h_{\max}$ is 1.15083 m.

CHAPTER 4

KINETICS ANALYSIS OF AWIS

In this chapter, a normal wheel and an AWIS are analyzed kinetically. It encompasses statics and dynamics analysis. It is ultimately aimed at deriving the motor torque required in order for the AWIS to perform the required motion. Firstly, the analysis is performed on the normal wheel, followed by the AWIS, as the vehicle rotates first using normal wheel when the AWIS is not activated. Finally, mass moment of inertia of the AWIS is derived to solve the dynamics equations for the rotating AWIS.

4.1 NORMAL WHEEL

In this section, the statics analysis is performed first, followed by the calculation of the mass moment of inertia of the normal wheel. Next, the dynamics analysis is performed and the required motor torque is derived.

4.1.1 STATICS ANALYSIS

Assume a normal wheel has a cylindrical-shape with uniform thickness, t . It has its own weight, w_n , which is assumed to act at the center, C , and at the same time, it bears part of total weight of a vehicle, w_p at the center of the normal wheel, C (Refer to Figure 4.1). Therefore, the total weight borne by a normal wheel, w_t , is:

$$w_t = w_n + w_p \quad (4-1)$$

Figure 4.1 Free-body-diagram of a normal wheel at rest

The normal wheel is at rest. Therefore, the resultant forces act on the normal wheel must be zero.

$$\sum F_y = 0 \rightarrow N = w_t \quad (4-2)$$

4.1.2 MASS MOMENT OF INERTIA CALCULATION

For a cylinder with radius r_n , and thickness t , the mass moment of inertia of the cylinder about z-axis at point C (Refer to Figure 4.2) is:

Figure 4.2 Mass moment of inertia of a cylinder

$$\begin{aligned} I_{zz} &= m.r_n^2 + m.(0.5*t)^2 \\ \rightarrow I_n &= \rho.\pi.r_n^2.t.(r_n^2 + 0.25*t^2) \end{aligned} \quad (4-3)$$

where I_n is the inertia of the normal wheel, and ρ is the material density of the normal wheel.

4.1.3 DYNAMICS ANALYSIS

Now, suppose a motor exerts torque, T_m , at the center of the normal wheel, C, and it drives the normal wheel. The frictional force created, i.e. F_{21} , between the normal wheel and the ground causes the normal wheel to rotate in clockwise direction as shown in Figure 4.3. Indexes 1 and 2 represent an acting body, i.e. normal wheel, and a reacting body, i.e. ground, respectively. Therefore, F_{21} means force from the reacting body acts on the acting body.

Figure 4.3 Free-body-diagram of a rotating normal wheel

The magnitude of the friction force is:

$$F_{21} = \mu \cdot N \quad (4-3)$$

where μ is coefficient of friction, it can be μ_s (static coefficient of friction) if the normal wheel is at rest or is going to rotate, or μ_d (dynamic coefficient of friction) if the normal wheel rotates.

The minimum torque required to drive the normal wheel is:

$$T_{m(\min)} = F_{21} \cdot r_n \quad (4-4)$$

Then, take moment about the center point C. The motor torque required to drive the normal wheel with specified α , is derived as follows:

$$\begin{aligned} \sum M_{-z(C)} &= I_n \cdot \alpha \\ \rightarrow T_m &= T_{m(\min)} + I_n \cdot \alpha \end{aligned} \quad (4-5)$$

where α is the angular acceleration of the normal wheel.

The acceleration of the center point C in x-direction is derived as follows:

$$\begin{aligned}\sum F_x &= m_t \cdot a_x \\ \rightarrow a_x &= \frac{F_{21} \cdot g}{w_t}\end{aligned}\tag{4-6}$$

where g is the gravitational force.

4.2 AWIS

An AWIS has uniform thickness of t_i and weight of w_i . The AWIS must also bear part of the total vehicle's weight, w_p , at the center C. Actually, the center of mass of the AWIS is not located at the center point C, as derived in section 4.2.3. However, the weight of the AWIS itself is much smaller compared to the part of the total vehicle's weight. Therefore, it is assumed that the total weight borne by the AWIS, w_t , acts at the center C, i.e.:

$$w_t = w_i + w_p\tag{4-7}$$

4.2.1 STATICS ANALYSIS

For a stationary AWIS, there is only total weight acts at the center C. Hence, the resultant forces act on the AWIS must be zero (Refer to Figure 4.4).

$$\sum F_y = 0 \rightarrow N = w_t\tag{4-8}$$

Figure 4.4 Free-body-diagram of an AWIS at rest

However, the line of action between the weight and the normal forces are not co-linear, hence, it creates a moment that tends to rotate the AWIS in opposite direction. The friction force resists the AWIS from rotating backwards, but it is still not sufficient to counter the moment created by the weight borne by the AWIS.

The magnitude of the friction force, F_s is:

$$F_s = \mu_s \cdot N \quad (4-9)$$

Therefore, to maintain static equilibrium of the AWIS, a certain amount of minimum torque must be exerted at the center C. Take moment about the center C, the minimum torque, $T_{m(\min)}$ is obtained to be:

$$\begin{aligned} \sum M_{-z(C)} &= 0 \\ T_{m(\min)} + F_s \cdot (D_B P_B) - N \cdot r_b &= 0 \\ T_{m(\min)} &= N \cdot r_b - F_s \cdot (r_b \cdot \theta_{P_B}) \\ \rightarrow T_{m(\min)} &= w_t \cdot r_b \cdot (1 - \mu_s \cdot \beta_H) \end{aligned} \quad (4-10)$$

Note that equation (4-10) is valid only for $1 \leq \beta_H \leq 4.4934$ rad. As the AWIS rotates, the required minimum torque decreases.

4.2.2 DYNAMICS ANALYSIS

Prior to deriving dynamics equations, it is necessary to determine the direction of the resultant reaction force. In this case, F_R is the resultant reaction force acts at the contact point P_B . The direction of F_R is always perpendicular to the line connecting the center of rotation, C and the contact point, P_B . (Refer to Figure 4.5)

Figure 4.5 Resultant reaction force's direction at contact point, P_B

Suppose a motor torque, T_m , is applied at the center C , such that the AWIS rotates with certain angular velocity, ω . Figure 4.6 shows all the forces that act on the AWIS.

Figure 4.6 Free-body-diagram of a rotating AWIS

The following relationships are derived geometrically (Refer to Figure 4.6):

$$\sin(\theta_{P_B} - \alpha_{P_B}) = \frac{D_B P_B}{C P_B} = \frac{\theta_{P_B}}{\sqrt{1 + \theta_{P_B}^2}} \quad (4-11a)$$

$$\cos(\theta_{P_B} - \alpha_{P_B}) = \frac{C D_B}{C P_B} = \frac{1}{\sqrt{1 + \theta_{P_B}^2}} \quad (4-11b)$$

$$F_{21X} = F_{21} \cdot \sin(\theta_{P_B} - \alpha_{P_B}) \quad (4-11c)$$

$$F_{21Y} = F_{21} \cdot \cos(\theta_{P_B} - \alpha_{P_B}) \quad (4-11d)$$

Divide equations (4-11c) by (4-11d),

$$F_{21X} = F_{21Y} \cdot \theta_{P_B} \quad (4-11e)$$

Note that $\theta_{P_B} = \beta_H$

Friction force at the contact point, P_B is:

$$F_{21X} = \mu_d \cdot (N + F_{21Y}) \quad (4-12)$$

Now, consider when the AWIS is about to rotate, then the minimum motor torque required to maintain this condition is:

$$\begin{aligned} \sum M_{-z(C)} &= 0 \\ \rightarrow T_{m(\min)} &= N \cdot r_b + F_{21} \cdot (C P_B) \end{aligned} \quad (4-13a)$$

The required motor torque when AWIS rotates with angular acceleration of α is:

$$\begin{aligned} \sum M_{-z(C)} &= I_A \cdot \alpha \\ \rightarrow T_m &= T_{m(\min)} + I_A \cdot \alpha \end{aligned} \quad (4-13b)$$

where I_A is the mass moment of inertia of the AWIS.

The resultant forces in x and y-directions cause the AWIS to accelerate in both x and y-directions, respectively.

$$\begin{aligned}\sum F_x &= m_t \cdot a_x \\ \rightarrow F_{21x} &= \frac{w_t \cdot a_x}{g}\end{aligned}\quad (4-14a)$$

$$\begin{aligned}\sum F_y &= m_t \cdot a_y \\ \rightarrow F_{21y} + N - w_t &= \frac{w_t \cdot a_y}{g}\end{aligned}\quad (4-14b)$$

Substitute equations (4-11e) and (4-14a) into equation (4-12), the normal reaction force, N is:

$$N = \frac{w_t \cdot a_x}{g} \cdot \left[\frac{1}{\mu_d} - \frac{1}{\beta_H} \right] \quad (4-15)$$

Using equations (4-11c) and (4-14a), the resultant reaction force, F_{21} is obtained to be:

$$F_{21} = \frac{w_t \cdot a_x}{g \cdot \beta_H} \cdot \sqrt{1 + \beta_H^2} \quad (4-16)$$

The x-distance of the center point, C, represented in global coordinate system, according to equation (3-13a) is:

$$x_{C0} = x_C + r_b \cdot \beta_1 + \int_{\alpha_{P_A}}^{\alpha_{P_B}} r_b \cdot \theta_{P_B} \cdot d\theta$$

By double differentiating the above equation, the acceleration in x-direction is derived to be:

$$a_x = \ddot{x}_{C0} = r_b \cdot (\theta_{P_B} \cdot \alpha + \omega^2) \quad (4-17)$$

Divide equations (4-14a) by (4-14b):

$$\frac{a_X}{a_Y} = \frac{F_{21X}}{F_{21Y} + N - w_t} \quad (4-18)$$

Equations (4-11e) and (4-18) show that the dynamics in the horizontal plane is coupled to the dynamics in the vertical plane.

Substituting equations (4-13a), (4-15), (4-16), and (4-17) into (4-13b), the required motor torque is:

$$T_m = \frac{w_t \cdot r_b^2}{g} \cdot (\beta_H \cdot \alpha + \omega^2) \left(\beta_H + \frac{1}{\mu_d} \right) + I_A \cdot \alpha \quad (4-19)$$

It can be observed that the required motor torque increases as the AWIS rotates. At this moment, there is still one variable that is unknown, i.e. I_A , the mass moment of inertia of the AWIS. In the following section, the mass moment of inertia of the AWIS is derived.

4.2.3 MASS MOMENT OF INERTIA CALCULATION

The AWIS is composed of two components, i.e. (1) involute part, and (2) half-circle part, as depicted in Figure 4.7.

Figure 4.7 Components of an AWIS, i.e. (a) involute, (b) half-circle

Firstly, the center of mass of the AWIS is derived, followed by the mass moment of inertia of the AWIS. The actual location of the center of mass of the AWIS is important in solving the dynamics system of the vehicle, to which the AWIS is attached.

1. Center of Mass of an AWIS

In this section, the centers of masses of the involute part and the half-circle part are calculated separately. Before deriving the center of mass of the AWIS, the area of the AWIS is derived as follows:

- Area of the involute part

Firstly, a general equation used to calculate the area of a region bounded by a curve is derived, followed by the calculation of the area of the involute part.

Figure 4.8 Area of a region bounded by a curve

The area of an infinitesimal region, dA is (Refer to Figure 4.8):

$$\begin{aligned}
 dA &= \text{Area} (\triangle CPQ - \triangle CMN) \\
 &= [0.5 * (r + dr)^2 . d\alpha] - [0.5 * r^2 . d\alpha] \\
 \rightarrow dA &\approx r . dr . d\alpha
 \end{aligned} \tag{4-20a}$$

The area of region CAB is:

$$A = \int dA = \iint_S r \cdot dr \cdot d\alpha$$

$$\rightarrow A = 0.5 * \int_{\alpha_1}^{\alpha_2} r^2 \cdot d\alpha \quad (4-20b)$$

Figure 4.9 Area of the involute part

Substitute equations (3-3) and (3-4) into (4-20b) and using partial differentiation with α ranging from 0 to π radians, the equation becomes:

$$A_i = 0.5 \cdot r_b^2 * \int_{\theta_1=0}^{\theta_2=4.4934} \theta^2 \cdot d\theta$$

Solving the above equation, the area of the involute part is calculated to be:

$$A_i = 15.120773 * r_b^2$$

- Mass of the involute part

Assume that region CAB has uniform thickness of t , then the total mass of the region, M is:

$$M = \int dm = \rho \cdot \int dV = \rho \cdot t \cdot \int dA \quad (4-20c)$$

where ρ is the material density of the mass.

The mass of the involute part is calculated to be:

$$M_i = 15.120773 * r_b^2 * \rho * t$$

- Moment of the involute part

The location of a point in the polar coordinate system is expressed as follows:

$$\begin{aligned} x &= r \cdot \cos \alpha \\ y &= r \cdot \sin \alpha \end{aligned} \quad (4-20d)$$

Substituting equations (4-20a), (4-20c), and (4-20d), the moment about x-axis is derived to be:

$$\begin{aligned} M_x &= \int y \cdot dm = \rho \cdot t \cdot \iint_S (r \cdot \sin \alpha) \cdot r \cdot dr \cdot d\alpha \\ \rightarrow M_x &= \frac{1}{3} * \rho \cdot t \cdot \int_{\alpha_1}^{\alpha_2} r^3 \cdot \sin \alpha \cdot d\alpha \end{aligned} \quad (4-20e)$$

Substitute equations (3-3), (3-4) into (4-20e), the equation becomes:

$$M_x = \frac{1}{3} * \rho \cdot t \cdot r_b^3 \cdot \int_{\theta_1=0}^{\theta_2=4.4934} \theta^2 \cdot \sqrt{1 + \theta^2} \cdot \sin(\theta - a \tan \theta) \cdot d\theta$$

Solving the above equation, the moment about x-axis for the involute part is:

$$M_{xi} = \frac{1}{3} * \rho \cdot t \cdot r_b^3 \cdot (61.2764)$$

Likewise, the moment about y-axis is:

$$\begin{aligned} M_y &= \int x \cdot dm = \rho \cdot t \cdot \iint_S (r \cdot \cos \alpha) \cdot r \cdot dr \cdot d\alpha \\ \rightarrow M_y &= \frac{1}{3} * \rho \cdot t \cdot \int_{\alpha_1}^{\alpha_2} r^3 \cdot \cos \alpha \cdot d\alpha \end{aligned} \quad (4-20f)$$

And again, repeat the same calculation as above, the moment about y-axis for the involute part is calculated to be:

$$M_{yi} = \frac{1}{3} * \rho * t * r_b^3 * (-59.1248)$$

- Center of Mass of the involute part

Figure 4.10 illustrates the location of center of mass of the involute part.

Figure 4.10 Center of mass of the involute part

The x and y locations of center of mass of the involute part are calculated as follows:

$$\bar{x}_{i_{cm}} = \frac{M_{yi}}{M_i} = -1.30339 * r_b$$

$$\bar{y}_{i_{cm}} = \frac{M_{xi}}{M_i} = 1.35082 * r_b$$

The center of mass is located at the depth of half of the involute part's thickness.

- Center of Mass of the half-circle part

In this case, the derivation is straightforward. The area of a half circle is:

$$A_h = 0.5 * \pi * r_b^2$$

Accordingly, the mass of the half-circle part is:

$$M_h = \rho \cdot t \cdot A_h = 1.57143 \cdot r_b^2 \cdot \rho \cdot t$$

Then, a constant radius of r_b , lower and upper limits of the integration of π and 2π , respectively, are substituted into equation (4-20e). The moment about x-axis is obtained to be:

$$M_{xh} = -\frac{1}{3} \cdot \rho \cdot t \cdot r_b^3 \cdot (2)$$

Likewise, the moment about y-axis is calculated to be:

$$M_{yh} = 0$$

Figure 4.11 Center of mass of the half-circle part

Figure 4.11 illustrates the location of center of mass of the half-circle part, which is calculated to be:

$$\bar{x}_{h_{cm}} = \frac{M_{yh}}{M_h} = 0$$

$$\bar{y}_{h_{cm}} = \frac{M_{xh}}{M_h} = -\frac{4}{3 \cdot \pi} \cdot r_b$$

- Center of Mass of the AWIS

Equations (4-21a) and (4-21b) are used to calculate the center of mass of a composite part, in this case, AWIS.

$$\bar{x}_{A_{cm}} = \frac{x_{h_{cm}} \cdot M_h + x_{i_{cm}} \cdot M_i}{M_h + M_i} \quad (4-21a)$$

$$\bar{y}_{A_{cm}} = \frac{y_{h_{cm}} \cdot M_h + y_{i_{cm}} \cdot M_i}{M_h + M_i} \quad (4-21b)$$

Substituting the previously calculated values into the above equations, the location of center of mass of the AWIS is calculated to be (Refer to Figure 4.12):

$$\bar{x}_{A_{cm}} = -1.180687 * r_b$$

$$\bar{y}_{A_{cm}} = 1.183713 * r_b$$

Figure 4.12 Center of mass of an AWIS

2. Mass moment of inertia of an AWIS

The mass moment of inertia of an AWIS is composed of mass moment of inertias of the involute part and the half-circle part.

- Mass moment of inertia of the involute part

The AWIS is assumed to rotate about z-axis. Therefore, the mass moment of inertia of the AWIS is derived with respect to z-axis. Generally, mass moment of inertia of a volume about z-axis, is defined as:

$$I_{zz} = \int r^2 \cdot dm \quad (4-22a)$$

Substitute equations (4-20a) and (4-20c) into equation (4-22a),

$$\rightarrow I_{zz} = \rho.t.\iint_S r^3 dr.d\alpha \quad (4-22b)$$

Substitute equations (3-3) and (3-4) into (4-22b) with the lower and upper limits of 0 and 4.4934 radians, respectively, the mass moment of inertia of the AWIS is obtained to be:

$$I_{izz} = 0.25 * \rho.t.r_b^4 \cdot \int_{\theta_1=0}^{\theta_2=4.4934} \theta^2 (1 + \theta^2) d\theta$$

$$\rightarrow I_{izz} = 99.149825 * \rho.t.r_b^4$$

- Mass moment of inertia of the half-circle part

Substitute radius of r_b , lower and upper limits of π and $2*\pi$, respectively, into equation (4-22b), the mass moment of inertia of the half-circle part is:

$$\rightarrow I_{hzz} = 0.25 * PI * \rho.t.r_b^4$$

- Mass moment of inertia of an AWIS

The mass moment of inertia of the AWIS is a summation of the mass moment of inertias of the involute part and the half-circle part.

$$I_{Azz} = I_{izz} + I_{hzz} \quad (4-23)$$

$$\rightarrow I_{Azz} = 99.93554 * \rho.t.r_b^4$$

Substitute the value of the mass moment of inertia of the AWIS above into equation (4-19) to obtain the required motor torque.

CHAPTER 5

DYNAMICS SIMULATIONS AND CONTROLS OF A WHEELED-VEHICLE WITH AWIS

This chapter presents the simulation results and control schemes for a wheeled-vehicle with AWIS. Firstly, a vehicle with four normal wheels is built. Then, four AWIS are built and connected to the four normal wheels through clutches. Next, PID controllers are implemented into the AWIS. Finally, four additional AWIS are added to the existing four AWIS such that the terrain adaptability of the vehicle is enhanced.

5.1 A WHEELED-VEHICLE WITH FOUR NORMAL WHEELS

Figure 5.1 shows the virtual model built using ADAMS/View Software [58].

Figure 5.1 A virtual model of a wheeled-vehicle with four normal wheels:

(a) Filled, (b) Wire frame

The masses of the vehicle's body and the normal wheel are 32.4 kg and 2.454 kg, respectively.

There are two types of motion sources that can be applied. The first is a velocity source. It means that the angular velocity of each normal wheel is specified regardless of the torque required. The second is a torque source, which is the actual one used. In this case, a motor is attached to each of the four normal wheels. The motor exerts torque with a value specified by the user. For example, in this simulation, a torque profile of a square wave form is specified (Refer to Figure 5.2) and it is implemented by defining a function as follows:

```
step(sin(2*pi*0.5*time),-0.01,-400,0.01,-1800)
```

The unit used is mks system. The torque profile resulted has a period of 2 seconds with minimum and maximum values of 400 and 1800 N.m., respectively. The minus sign of the torque value indicates that the motion resulted by the torque is in opposite direction to the coordinate system. The square wave function is intended to provide the deceleration in addition to the acceleration such that the vehicle moves in a more controlled manner.

Figure 5.2 The torque profile exerted by the motor to the normal wheel

The motor's torque profile specified above drives the normal wheel. The velocity and acceleration of the normal wheel are plotted in the graphs shown in Figure 5.3.

Figure 5.3 Plots of the simulation results of the normal wheel

It can be seen that the square wave input torque results in the square wave angular acceleration. Consequently, the angular velocity increases gradually as the angular acceleration is larger than zero at all times. By altering the torque profile, the kinematics parameters of the normal wheel can be adjusted such that the requirements are satisfied.

Note that in ADAMS/View Software, the contact between the wheel and the ground is modeled as a spring-mass system. The user must specify the values of the stiffness, the damping ratio, and the penetration depth of the spring. From the next section onwards, the AWIS are connected to the vehicle with normal wheels.

5.2 A WHEELED-VEHICLE WITH FOUR AWIS

In this section, four AWIS are created and attached to the four normal wheels, as shown in Figure 5.4.

Figure 5.4 A virtual model of a wheeled-vehicle with four AWIS:
(a) Filled, (b) Wire frame

The mass of each AWIS is 2.615 kg. In this case, the AWIS is directly attached to the normal wheel, such that the motor torque applied at the normal wheel is used to drive both normal wheel and connected AWIS.

5.2.1 MOVING ON A FLAT GROUND WITHOUT CONTROLLER

In this section, the wheeled-vehicle with four AWIS is simulated on a flat ground. If the motor uses the same torque profile as specified in Section 5.1, then the torque is not sufficient and the AWIS are not capable of lifting the vehicle up. The simulation results are plotted as shown in Figure 5.5.

Figure 5.5 Plots of the simulation results of the vehicle with four AWIS
when the maximum torque is 1800 N.m.

It can be seen from the translational displacement graph shown in Figure 5.5 that the vehicle moves forward and then moves back due to insufficient torque. However, if the maximum torque is increased to 3000 N.m instead of 1800 N.m, the vehicle exhibits uncontrolled behaviors, as shown in a captured image from the moving vehicle in Figure 5.6.

Figure 5.6 *The vehicle flies in the air.*

The plots of the simulation results are plotted in Figure 5.7. The angular velocity of the AWIS changes abruptly. It is explained as follows. The motor torque required to rotate the AWIS increases as the angular position of the AWIS increases (Refer to Chapter 4). As specified in the torque profile, the motor torque has a periodic constant value. This causes the angular velocity of the AWIS to decrease for that period of time. Once, the contact line between the AWIS and the ground reaches the end of the involute-curve of the AWIS, there is suddenly no contact between the AWIS and the ground, although the torque is still applied. As a result, the vehicle flies, then the vehicle falls and the AWIS touch the ground due to the weight of the vehicle. During the flying period, the motor torque required is only used to rotate the AWIS, whereas the motor torque available is far exceeding the requirement. This causes the AWIS to rotate with very high angular velocity and due to sudden contact between the AWIS and the ground; the angular velocity is suddenly decreased.

Figure 5.7 Plots of the simulation results of the vehicle with four AWIS when the maximum torque is 3000 N.m.

In order to avoid such abrupt changes of the angular velocity of the AWIS, a controller is added to each of the AWIS, as discussed in the following section.

5.3 A WHEELED-VEHICLE WITH FOUR AWIS AND CONTROLLERS

An independent wheel's control scheme is implemented, as illustrated in Figure 5.8.

Figure 5.8 Top-view of the wire frame model of the wheeled-vehicle with four AWIS and controllers
(L=left, R=right, F=front, B=back, NW=normal wheel)

In this section, the PID controllers and the control mode used in this simulation is firstly discussed. Overall, there are four controllers that are implemented in the wheeled-vehicle with four AWIS. Next, the wheeled-vehicle is simulated on a flat ground. Finally, the wheeled-vehicle is simulated to traverse an obstacle and a staircase.

5.3.1 PID CONTROLLER

In the simulation of the wheeled-vehicle with controllers, the angular velocity's profile of the AWIS is specified. The motor torque exerted on the AWIS is continuously adjusted such that the angular velocity of the AWIS follows the specified angular velocity's profile all the time. To achieve this objective, a PID controller is used to control the torque exerted by the motor. A schematic diagram showing the implementation of the PID controller is depicted in Figure 5.9.

Figure 5.9 Angular velocity control mode

The input to the PID controller is the error resulted from the difference between the desired angular velocity, ω_d and the actual angular velocity, ω_a of the AWIS. The PID controller outputs electrical signals to the motor such that the motor adjusts the output torque to the normal wheel. The normal wheel is connected to the AWIS through a clutch. For the time being, the clutch is assumed to be activated throughout the simulation. It means that the motor torque is transmitted directly to the AWIS.

The output of the PID controller is changed by adjusting the proportional, integral, and derivative gains.

In this simulation, a constant angular velocity, ω_d is specified. In addition, a closed-loop control system for the angular acceleration is also implemented. The desired angular acceleration value is zero, which minimizes the sudden changes in the actual angular velocity, ω_a .

5.3.2 MOVING ON A FLAT GROUND

In this section, the wheeled-vehicle with four AWIS and four PID controllers is simulated to move on a flat ground. Figure 5.10 shows the initial and final positions before and after the simulation.

Figure 5.10 Moving on a flat ground at (a) $t=0$ s, (b) $t=10$ s

In this simulation, the desired angular velocity is specified to be 50 degrees per second or 0.8727 radians per second. While the simulation runs with time starting from 0 to 10 seconds, the angular velocity's error for each AWIS is plotted, as shown in Figure 5.11. The expected angular position of the AWIS at $t = 10$ s is 500 degrees, as illustrated in Figure 5.10-b.

Figure 5.11 Plots of the angular velocity's errors for the four AWIS

The average error calculated for each AWIS is about 0.81% ($0.007068 / 0.872665$). The error is considered very small and it is achieved by fine-tuning the PID controller's gains, i.e. proportional gain to 10000, integrative gain to 200, and derivative gain to 100.

During the simulation, the vehicle moves smoothly with its AWIS rotate at the constant angular velocity. However, the translational velocity of the AWIS in x-direction is not constant, because of the profile of the involute-curve itself. It is constant for short period of time while AWIS rotates on the half-circle part, whereas afterwards, the velocity increases till the contact between the AWIS and the ground reaches the end of the involute-curve (Refer to Figure 5.12). When $5.4 < t < 7$ s the

velocity drops as the AWIS rotates about the end of the involute curve. When $t > 7$ s, the entire cycle repeats as now the AWIS has rotated for one full rotation.

Figure 5.12 Plots of the simulation results of the AWIS

From the plot of the motor torque (Refer to Figure 5.12), it can be seen that when the AWIS starts rotating, the required motor torque reaches its maximum value of up to 9000 N.m in order to overcome the friction and to accelerate the AWIS to the desired angular velocity. Subsequently, it decreases to less than 100 N.m. and varies from -100 N.m. to +100 N.m. throughout the simulation

5.3.3 TRAVERSING AN OBSTACLE

In this section, an obstacle is created to observe the traversing capability of the wheeled-vehicle with four AWIS and four PID controllers (Refer to Figure 5.13). The obstacle has dimensions of 4 m (l) x 0.5 m (h) x 1 m (d). The horizontal distance between the center of the front axle of the vehicle and the nearest side of the obstacle is 0.4 m.

Figure 5.13 *Traversing an obstacle*

When the activation distance is set to 0.4 m, the angular position of the AWIS is 94.85 degrees when it hits the obstacle, as shown in Figure 5.14-c. The motion of the vehicle is recorded, and a series of images is captured and shown in Figure 5.14. The figure shows the sequence of the vehicle while traversing the obstacle.

Note that the desired angular velocity is specified to be 1 degree per second or 0.017453 radians per second. The smooth motion of the vehicle is obtained by specifying the small angular velocity. Otherwise, the impact between the AWIS and the obstacle would be very high. Consequently, the simulation time has to be extended to 600 seconds for the vehicle to complete the action.

Figure 5.14 A sequence of traversing an obstacle

The same values of proportional, integrative, and derivative gains, i.e. 10000, 200, and 100, respectively, are being used, and the errors resulted from the difference between desired and actual angular velocity for each AWIS is plotted, as shown in Figure 5.15. The average of the angular velocity's error is obtained to be 0.00043348 radians per second or 2.48 %, which is considered small.

Figure 5.15 *Plots of the angular velocity's errors of the four AWIS while traversing an obstacle*

Figure 5.16 shows the plots of the simulation results of the front AWIS.

Figure 5.16 Plots of the simulation results of the AWIS while traversing an obstacle

Figure 5.17 Zooming in the simulation results ($t = 0$ to 250 s)

Figure 5.17 zooms in the simulation results plotted in Figure 5.17. It can be seen that when the front AWIS hits the obstacle, the angular velocity drops drastically, and the motor torque increases accordingly. Consequently, the angular velocity difference between the front and the rear AWIS causes slippage between the rear AWIS and the ground. Once the motor torque at the front AWIS is high enough to drive the AWIS, the motion is resumed. This interesting result was observed during the simulation, as captured in Figure 5.18.

Figure 5.18 *The slippage between the rear AWIS and the ground*

Moreover, the slippage between the rear AWIS and the ground causes the front AWIS for lagging behind the rear AWIS, as depicted in Figure 5.19.

Figure 5.19 *Final angular positions of the front and the rear AWIS ($t = 600$ s)*

5.3.4 TRAVERSING A STAIRCASE

In this section, the wheeled-vehicle with four AWIS and four PID controllers is simulated in an environment with a staircase (Refer to Figure 5.20). The vehicle is expected to ascend and descend the staircase steadily.

Figure 5.20 *Traversing a staircase*

In this model, a staircase with the dimensions shown in Figure 5.21 is built. The distance between the axles of the vehicle and the staircase is also shown.

Figure 5.21 *The dimensions of the staircase*

The desired angular velocity is specified to be 10 degrees per second or 0.174533 radians per second for all the AWIS. The proportional gain, integrative gain, and

derivative gain of all the PID controllers are set to 10000, 200, and 100, respectively. The end time of the simulation is set to 141 seconds, with step size of 0.1 seconds. A series of images showing the sequence of the vehicle traversing the staircase is depicted in Figure 5.22.

Figure 5.22 A sequence of traversing a staircase

First of all, all the AWIS have angular positions as shown in Figure 5.22-a. When the vehicle starts moving, all the AWIS are activated simultaneously. From the sequence, it can be seen that the vehicle shows the capability to traverse the staircase.

Figure 5.23 shows the plots of the angular velocity's errors for all the four AWIS. At the starting point, the four AWIS are at rest, such that the angular velocity's errors reach the value of 0.174533 radians per second. Afterwards, the errors decrease to nearly zero. The maximum average error is calculated to be 0.00093424 or 0.535 %, which is considered very small. Therefore, it can be concluded that the settings of the gains of the four PID controllers are satisfactory.

Figure 5.23 *Plots of the angular velocity's errors of the four AWIS while traversing a staircase*

Figure 5.24 shows the plots of the simulation results of the AWIS while traversing the staircase. It can be seen that the translational displacement and the translational velocity in x-direction, follow certain patterns. It is elaborated as follows: When the AWIS rotates with constant angular velocity, i.e. 0.174533 rad per second, the translational velocity of the center of the AWIS, is derived to be: $v = r_b \cdot \omega \cdot \beta$. As the angular rotation of the AWIS, β , increases, the velocity of the AWIS increases as well. After one full rotation, the translational velocity of the AWIS in x-direction repeats the same pattern, and so on. Accordingly, the translational displacement in x-direction follows a pattern which has the same period as the translational velocity has.

Figure 5.24 Plots of the simulation results of the AWIS while traversing a staircase

However, on several occasions, the vehicle seems to loss its stability, for example the vehicle flies, as illustrated in Figure 5.25.

Figure 5.25 The vehicle flies while ascending the staircase

The instability is explained as follows: When the vehicle is at the position shown in Figure 5.25-a, both of the front AWIS touch the edge of the highest step, whereas both of the rear AWIS touch the edge of the second step of the staircase. The front AWIS resist the rear AWIS to rotate backwards and the inertia of the front part of the vehicle assists the rear AWIS to lift the vehicle up. Once, the rear AWIS overcomes the edge of the step, the inertia of the vehicle causes a sudden increase in the angular velocity of the front AWIS. Consequently, it causes the loss of stability for the rear AWIS. Likewise, while descending the stair, the vehicle undergoes the same problem, as shown in Figure 5.26.

Figure 5.26 *The vehicle flies while descending the staircase*

Furthermore, the vehicle undergoes a sudden fall as well, as illustrated in Figure 5.22-g, h. Therefore, in the next section, a new solution to solve the problem of instability is introduced, i.e. by adding four AWIS to the existing four AWIS. Apart from the purpose of solving the instability problem, the new solution is expected to make the vehicle more adaptable to a wide range of obstacles. In one sense, the combination of the AWIS allows the flexible adjustment of the clearance between the vehicle's body and the ground. Moreover, the weight of the vehicle is shared among the eight AWIS, instead of only four AWIS, such that the slippage possibility of each AWIS is reduced.

5.4 A WHEELED-VEHICLE WITH EIGHT AWIS AND CONTROLLERS

In the previous section, the wheeled-vehicle is equipped only with four AWIS. The simulation results show the capability of the AWIS to assist the wheeled-vehicle in traversing obstacles, from a single block to a flight of stairs. However, the performance of the wheeled-vehicle can be further enhanced, for example in the stability aspect, such that the vehicle is able to traverse a wide range of obstacles.

In this section, four additional AWIS are attached to the wheeled-vehicle to improve the performance of the wheeled-vehicle (Refer to Figure 5.27). The wheeled-vehicle with eight AWIS and twelve PID controllers is simulated to traverse an obstacle.

Figure 5.27 A wheeled-vehicle with eight AWIS: (a) Filled, (b) Wire frame

The four additional AWIS complements the existing four AWIS attached to the wheeled-vehicle. The four additional AWIS smooth the vehicle's motion during ascending or descending the obstacles.

Figure 5.28 shows the top-view of the vehicle. The red color components represent the four additional AWIS, whereas the cyan color components represent the four existing AWIS.

Figure 5.28 Top-view of the wire frame model of the wheeled-vehicle with eight AWIS

5.4.1 TRAVERSING AN OBSTACLE

Figure 5.29 shows the horizontal distance between the vehicle's axles and the obstacle. The heights of the vehicle's axles and point U are 0.25 m and 0.5 m, respectively. Both heights are measured vertically from the ground. In the simulation, the x-location of point U with respect to the origin in the global coordinate system is -3 m.

Figure 5.29 The horizontal distance between the vehicle's axles and the obstacle

Figure 5.30 illustrates the initial position of the vehicle. In the actual implementation, the first set of the four AWIS (cyan color) is linked to a set of four normal wheels (black color) through four clutches. Likewise, the second set of the four additional AWIS (red color) is linked to the first set of the four AWIS through four clutches. Therefore, a complete system of the vehicle requires only one actuator to be attached to each of the normal wheel. When a certain clutch is activated, the corresponding shaft of the driven AWIS is engaged to the respective actuator, and hence the torque is transmitted to the driven AWIS.

However, in this simulation, each wheel is driven by one actuator. Overall, the vehicle is driven by twelve actuators. Consequently, the vehicle uses twelve PID controllers. The objective of this simulation is to show the improved performance of the wheeled-vehicle to traverse the obstacle by using eight AWIS.

Figure 5.30 *Traversing an obstacle with eight AWIS*

The desired input velocity of all the wheels is set to be 10 degrees per second or 0.174533 radians per second. Nevertheless, to simulate the function of the clutch in driving the respective AWIS in a certain period of time during the simulation, a certain function which specifies the desired velocity profile is written for each AWIS. In ADAMS/View software, the functions are written as inputs to the controllers, as follows:

- For normal wheels:

The angular velocity of 0.174533 radians per second is written. As a result, the constant angular velocity is produced during the simulation, as shown in Figure 5.31.

Figure 5.31 Plot of the desired angular velocity of the normal wheel

- For RF AWIS and LF AWIS:

STEP(DX(44,69,69), -1.6, 0, -1.4, 10*PI/180) +
STEP(DX(44,69,69), 4.9, 0, 5.1, -10*PI/180)

This function analyzes the distance between point C_F and point U (Refer to Figure 5.29). If the distance between the points (location of point C_F - location of point U) is -1.6 m, then the respective AWIS is activated, or else if the distance is +4.9 m, then it is de-activated. The same definitions apply to the following programs for the others AWIS.

- For RB AWIS and LB AWIS:

STEP(DX(39,69,69), -3.7, 0, -3.5, 10*PI/180) +
STEP(DX(39,69,69), 2.9, 0, 3.1, -10*PI/180)

- For RF AWIS 2 and LF AWIS 2:

STEP(DX(80,69,69), -1.1, 0, -0.9, 10*PI/180) +
STEP(DX(80,69,69), 3.5, 0, 3.8, -10*PI/180)

- For RB AWIS 2 and LB AWIS 2:
 $\text{STEP}(\text{DX}(85,69,69), -2.2, 0, -2.0, 10 \cdot \text{PI}/180) +$
 $\text{STEP}(\text{DX}(85,69,69), 2.6, 0, 2.9, -10 \cdot \text{PI}/180)$

The desired angular velocity for each of the AWIS is plotted, as shown in Figure 5.32. Note that Figure 5.32 shows only the plots of the desired angular velocity for one side of the vehicle, as another side is exactly the same. This arrangement of the angular velocity's profile is adequate for this simulation, as the vehicle traverses a regular-shape obstacle. To cater for irregular-shape obstacles, the functions are defined differently for all the AWIS.

Figure 5.32 Plots of the desired angular velocity of the AWIS
 (“1” denotes the existing AWIS, “2” denotes the additional AWIS)

It can be seen that both existing AWIS at the front rotate first, followed by both existing AWIS at the rear and both additional AWIS at the front. Eventually, both additional AWIS at the rear rotate.

Next, a series of images are captured during the simulation, as shown in Figure 5.33.

To produce a smooth motion, the simulation is set with step size of 0.05 s, and as a result, the vehicle takes a longer time to complete the action. In this case, a total time of 87 seconds is needed for the vehicle to traverse the obstacle.

Figure 5.33 A sequence of traversing an obstacle with eight AWIS

During the first few seconds (Figures 5.33-a, b), the vehicle moves using the four normal wheel. Next, both existing AWIS at the front start rotating (c-d), followed by the others (e-h). The vehicle's body is supported by the additional AWIS while landing on the ground (i-l). The final position of the vehicle is captured, as depicted in Figure 5.34.

Figure 5.34 The final position of the vehicle at $t = 87$ s

In the previous simulations in sections 5.2 and 5.3, the four existing AWIS are activated concurrently to traverse the obstacle and the staircase. In this section, the motion profile of each of the four AWIS on the left side of the vehicle follows exactly the respective four AWIS on the right side. However, each of the four AWIS on each side of the vehicle has different activation period, as defined in the above programs. The coordination among all the eight AWIS is very important in determining the capability of the vehicle to traverse different types of obstacles.

Figure 5.35 plots the differences between the desired and the actual angular velocity for each AWIS at one side of the vehicle. Using ADAMS/PostProcessor, the average of the angular velocity's errors for the existing AWIS at the front and back, the additional AWIS at the front and back are 0.0042, 0.0016, 0.0001, 0.00048 radians per second, respectively. The maximum angular velocity's error is obtained to be 2.4 %. Hence, it can be concluded that the PID controllers used for the existing and the additional AWIS, with proportional gain, integrative gain, and derivative gain values of 10000, 5, 1, respectively, are suitable for this application.

Figure 5.35 Plots of the angular velocity's errors of the front and the rear AWIS

Figure 5.36 shows the plots of the simulation results of the existing and the additional AWIS at the front of the vehicle. It can be seen that the horizontal distance (x-direction) between the front axle and the obstacle does not form a straight line. It can be explained as follows: Before the front axle passes the obstacle, the AWIS have to lift the vehicle up, and the angular velocity of the AWIS drops, especially when the AWIS hits the obstacle. Accordingly, the respective motor has to increase its output torque to drive the AWIS with the specified angular velocity, i.e. 0.174533

rad per second. It is illustrated in Figure 5.36 that there are some ripples on the angular velocity graphs, and the motor torque graphs. On the other hand, once the front axle passes the obstacle, the gravitation effect works on the vehicle, causing the vehicle to tumble down fast. This results in a sudden increase in the angular velocity and consequently, the motor exerts torque in the reverse direction to reduce the angular velocity to the specified value.

Figure 5.36 Plots of the simulation results of the front AWIS

The minus signs of the angular velocity and the motor torque indicate that the motion's directions are opposite to the positive local coordinate system of the wheels. It can be observed from the graph that the angular velocity of the normal wheel is always constant at 0.1746 radians per second, as the AWIS perform most of the time while traversing the obstacle.

Figure 5.37 shows the plots of the simulation results of the existing and the additional AWIS at the rear of the vehicle. The same arguments hold as of the front AWIS.

Figure 5.37 Plots of the simulation results of the rear AWIS

CHAPTER 6

CONCLUSION

The result of this project is summarized in this chapter. Subsequently, the limitations of the proposed solution are discussed, followed by the recommendations such that the performance of the vehicle is fully optimized. Next, future works are proposed as well on certain topics to explore all possible aspects in realizing sophisticated wheeled-vehicles. Lastly, the benefits and advantages of using the AWIS for a land-vehicle are concluded.

6.1 CONCLUDING REMARKS

Chapter 2 provides the literature review that summarizes all the groups of the land-vehicles developed around the world. Overall, six categories are classified and the examples of each category are described. The advantages and drawbacks of every group are discussed as well.

In Chapter 3, the geometrical derivation of the involute-curve is briefly discussed. It is followed by the discussion on the general shape of the AWIS. Next, the kinematics analysis is performed. Firstly, the location of the center point, the point of contact between the AWIS and the ground, and a point on the outer surface of the AWIS, are derived in both local and global coordinate systems. The calculation of the circumference of the AWIS is also included. Then, a generalized equation of the location of the hitting point between the AWIS and the obstacle is derived. Following that, the activation distance of the AWIS is defined. The activation distance is measured between the center point of the AWIS to the nearest point on the obstacle, and it indicates the distance at which the AWIS is activated such that the AWIS can traverse the obstacle. A graphical user interface is also created for the user. After inputting the parameters, it automatically calculates the activation

distance. Subsequently, a graph showing the maximum height of traversable obstacle for each angular position of the AWIS is plotted. From the calculation, the AWIS with the base radius of r_b has the capability to traverse the obstacle with maximum height of $4.6033 \cdot r_b$.

Chapter 4 discusses the kinetics analysis of the normal wheel and the AWIS. Firstly, the statics analysis, the moment of inertia calculation, and the dynamics analysis are performed on the normal wheel, and the required motor torque is derived. After which, the statics and dynamics analyses are carried out for the AWIS to find the required motor torque. Likewise, the center of mass and the mass moment of inertia calculations are calculated.

Chapter 5 presents the simulation results of the vehicle with AWIS. The vehicle with four normal wheels is firstly modeled with each wheel is driven by an actuator or a motor. Then, four AWIS are added to the four normal wheels. The vehicle with four AWIS is simulated on a flat ground without controller. Same torque profiles for all motors are given as inputs to the AWIS. The simulation results show that the angular velocity of each AWIS changes abruptly, and the vehicle flies in uncontrolled manners. Next, four PID controllers are added, and the desired velocity profile is specified instead of the torque profile. This scheme is applied to the vehicle and as a result, the vehicle is capable of moving smoothly on the flat ground. Now, a block of obstacle is created and the images showing the sequence of the vehicle traversing the obstacle are provided. The vehicle moves smoothly during the simulation, and the plots of errors, displacement, velocity, acceleration, and motor torque are presented.

A flight of staircase is then modeled in the environment. Again, the previous scheme for the vehicle traversing the obstacle is applied to this scenario. The images showing the motion of the vehicle are presented together with the plots of errors, displacement, velocity, acceleration, and motor torque. In this simulation, the vehicle seemed to lack stability while ascending and descending the stair. Therefore, a new solution is proposed to improve the stability performance of the vehicle while traversing any obstacles. Additional four AWIS are attached to the existing four

AWIS, making the total number of AWIS used by the vehicle amounted to eight. Consequently, there are four more PID controllers being used. In this simulation, the AWIS are activated at different period of time to provide a smooth and stable motion to the vehicle. Again, the images showing the steps of the vehicle in traversing a block are given and graphs of the desired input velocity, distance, displacement, velocity, and acceleration, and motor torque for each AWIS are plotted.

6.2 LIMITATIONS, RECOMMENDATIONS AND FUTURE WORK

The followings are some key issues related to the design, analysis, and simulation of the wheeled-vehicles with AWIS, including the limitations and drawbacks of using AWIS in this project, the recommendations to overcome these limitations, and future works:

In this project, a wheel with involute-shape was used to assist a wheeled-vehicle in traversing obstacles. The involute-shape wheel was chosen because it is derived from a base circle with a certain radius and its radii change accordingly once the base radius is changed. However, it is not only limited to this shape, there are still many studies can be done to explore the possibility of adopting other shapes. This opens up the possibility of further improving the performance of the wheeled-vehicle with these types of auxiliary wheels.

The involute-curve of the AWIS provides the wheel with smoothly increasing radius, and it is conceptually and mathematically-proven. The AWIS consists of a half circle and an involute-curve joined by a straight line. It can be seen that there are two transitions do exist. The first is the transition from half base circle to the involute-curve, and the second is from the involute-curve to the half circle. Although it is theoretically sound from the conceptual point-of-view, there are some concerns in realizing the AWIS. First, the transitions cause the discontinuities to the AWIS geometrically, and they involve complexity from the manufacturing point-of-view.

The discontinuity can be seen clearly as a sharp corner at the end of the involute-curve. Hence, the AWIS must be modified at those transitions before building the real prototype. For example, some fillets (radius corners) must be designed to replace the sharp edges. Additionally, the straight line can be arranged such that the line starts at the end of the involute-curve, and it ends at a point tangent to the base circle.

The efficiency of the wheeled-vehicle with AWIS would also be reduced while traversing small obstacles with sizes smaller than the AWIS' radius and larger than normal wheel's radius. In this case, the wheeled-vehicle with normal wheels is not able to traverse the obstacles. On the other hand, while the wheeled-vehicle with AWIS can easily traverse the obstacle, the wheeled-vehicle is also being lifted up to a height as biggest as the radius of the AWIS, which is not necessary for the obstacle with radius smaller than the radius of the AWIS. Moreover, if the length of the obstacle is small, then instead of just stepping on the obstacle, as done with the legged-vehicle, the vehicle with AWIS needs to rotate at least one full rotation of its AWIS before lowering to its initial position.

In the simulation of the wheeled-vehicle with four AWIS, the input desired angular velocity to all AWIS was the same and is specified to be constant all the time. The PID controllers then detect the errors and send the signals to adjust the output motor torque such that the desired angular velocity is achieved. However, this same and constant angular velocity causes the instability problem, as mentioned in Section 5.3.4. It is a good idea to alter the angular velocity profile so that the profile is different for each AWIS. Furthermore, the profile is adjusted to be time varying in order to enhance the performance of the vehicle. This type of angular velocity profile involves a dynamic velocity control system. Note also that due to the varying radius of the AWIS, the constant angular velocity results in different translational velocity at the contact point for different angular position of the AWIS. Consequently, if all the AWIS have different angular positions, the slippage is inevitably occurred. In addition, the coordination among the AWIS has to be considered in order to obtain the most optimal performance for the vehicle in traversing various sizes of the obstacles.

There is constraint associated with the wheeled-vehicle with eight AWIS. Each additional AWIS is attached side-by-side to the existing AWIS and the normal wheel by using clutches. Due to this arrangement, the existing AWIS has to be activated first before the additional AWIS can be activated. For instance, when the existing AWIS stops rotating in order to maintain the vehicle's clearance, the additional AWIS could not be activated as well although the additional AWIS is supposed to be activated in order to drive the vehicle. Therefore, it poses difficulties in coordinating the AWIS while traversing an irregular shaped obstacle. However, this issue can be solved mechanically, for example, by designing the gears and clutches system such that the engagement of the existing and additional AWIS can be easily controlled.

Throughout this project, the wheeled-vehicle with AWIS was analyzed and simulated to move in forward direction only. It limits the capability of the vehicle to perform a complex motion in a confined working space. Furthermore, the shape of the AWIS itself causes the tendency to drive the AWIS in only one direction. It requires high torque to drive the AWIS in another direction as there is discontinuity between the base circle and the involute-curve. However, for this case, the torque is not necessarily being supplied most of the time as the radius keeps decreasing after the point of discontinuity, hence torque is created. In this case, brakes would be used in order to prevent the AWIS from rotating too fast, which means that the energy loss would be compensated. One way to solve this issue is to come up with a wheel's design that has smooth radius changes such that the wheel can be driven forwards or backwards.

In chapter 3, the kinematics and dynamics analyses had been done on one AWIS. However, the wheeled-vehicle discussed previously requires at least four AWIS. It is obvious that the kinematics and dynamics parameters of the whole vehicle as one system and one AWIS are different. For example, the entire vehicle's ground clearance and one AWIS' clearance are obviously different due to different angular positions between one AWIS and another. Therefore, it affects the maximum obstacle height traversable by the vehicle. The investigation of the system's center of

gravity and the maximum speed achievable are significant topics because it affects the stability of the vehicle. Future analyses should be focused on these issues.

In chapter 5, the wheeled-vehicle with four AWIS was simulated to traverse regular shaped-obstacle like staircase. The vehicle demonstrates a good capability in overcoming the regular terrain. However, in natural environment, irregular and complex-shaped obstacles are mostly found like rubble, etc, and the terrain is hardly found to be flat surfaces. At times, recovery after flip over capability is an added advantage. Therefore, further study must be carried out to further improve the performance of the wheeled-vehicle with AWIS. For instance, various shapes of obstacles can be created in the environment of the simulating software and the performance of the vehicle traversing them is observed. Moreover, the motion of the AWIS is monitored and coordinated such that the vehicle is able to move smoothly.

Another aspect that would be interesting to probe into is the steering of the wheeled-vehicle with AWIS. By applying the differential speed algorithm between left and right hand-sides AWIS, the steering of the vehicle can be achieved. However, the height difference between left and right hand-sides of the vehicle would pose the stability issue that need to be considered. The clearance between the vehicle and the ground during this action would limit the range of obstacles traversable.

The capability of a wheeled-vehicle with AWIS in traversing a huge obstacle depends on many factors, mainly the vehicle's weight and its distribution among the wheels, the condition of the contact surface between the AWIS and the obstacle, the AWIS' material which determines the frictional effects between the AWIS and the obstacle. The larger the frictional coefficient, the larger the traction, and it prevents the wheels from sliding, hence increase energy efficiency. Apart from those simulations performed, it would be very useful in the future to actually build a real prototype in order to test it in various environments and to implement the sensing, planning, control, and coordination for complex maneuvers and steering. With the real prototype, it is easier to validate the parameters modeled in the simulated environment.

After considering the above-mentioned issues, it is worth exploring the idea to further the study on designing a time-varying radius wheel. In this case, the radius of the wheel at every angular position keeps changing according to the oncoming number of obstacles and their sizes and the distance between the wheel and the obstacle. This idea enables the wheeled-vehicle to move forwards or backwards without difficulties. Furthermore, it gives more flexibility to the extent of the involute-curve being used in the AWIS. For example, the involute-curve of the AWIS discussed previously starts from angle 0° and ends at angle 180° . With time-varying radius wheel, the involute-curve can be designed such that it can start and end at any angular positions. Certainly, the vehicle with this type of wheel is able to easily overcome difficult terrains, for example, while crossing the river. It allows the vehicle to be optimally balanced in almost every situation. This solution will provide a wider range of obstacles traversable and provide solutions to improve the performance of the wheeled-vehicles.

6.3 BENEFIT

This project introduces the AWIS as innovative auxiliary devices attached to the wheeled-vehicle. Wheeled-vehicles equipped with the AWIS are superior to those land-vehicles using other locomotion mechanisms (e.g. legged-vehicles, or tracked-vehicles) in negotiating rugged terrain.

Some land-vehicles combine the available locomotion mechanisms. As a result, they involve much more complex designs than the solution that is proposed in this project. In Section 2.7 Summary and Remarks, the table showing the comparison of the performance of all the six groups of the vehicles based on four categories, is constructed. It can be seen that out of six categories, the wheeled-legged-vehicle has the best overall performance in all four categories, followed by the wheeled-vehicle. The wheeled-legged-vehicle has good performance in terrain adaptability, energy efficiency, and locomotion speed categories, but it requires more advanced control and mechanisms system. However, the wheeled-vehicle has good performance in

energy efficiency, locomotion speed, and simplicity in control and mechanism categories, but it is constrained to move only on flat or slightly uneven terrains. In this case, it is easier to extend the capability of the wheeled-vehicle in negotiating rough terrains, for example by using the AWIS, than to simplify the control and mechanism of the wheeled-legged-vehicle, which is quite complex because of the adoption of two locomotion modes.

The AWIS does not only increase the stability of the vehicle, but also extends the applications of the wheeled-vehicle from the flat surface or slightly uneven terrain to the most extreme surface, e.g. staircase. In addition, the AWIS enables the wheeled-vehicle to have a smooth dynamics in the sagittal plane. The most interesting fact is that the maximum height of traversable obstacle reaches almost five times the radius of the normal wheel. Furthermore, the AWIS can be used without going through major modifications to the original design of the vehicle with normal wheels. The wheeled-vehicle with eight AWIS has one potential application which outperforms the wheeled-vehicle with four AWIS. The former has the ability to maintain its clearance between the vehicle and the ground, which is especially important feature if the vehicle is used while crossing a river. The concept of wheeled-vehicle with AWIS has a promising future in sophisticated applications such as unmanned planetary exploration. In conclusion, AWIS offers the simplest solution to a land-vehicle with normal wheels, in terms of design, planning, control, coordination, and sensing.

The author realized that there is room for improvement; however, this project serves as a useful platform and valuable stepping stone for future development of the sophisticated land-vehicles.

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APPENDIX A

MATLAB PROGRAMS

- 1: INVERSE.M
- 2: DIFFERENTIATING_INTEGRATING.M
- 3: PLOTALPHA.M
- 4: PLOT_ALPHA_THETA_DEG.M
- 5: THETAALPHA.M
- 6: PLOT_RADIUS_ALPHA_THETA_DEG.M
- 7: PLOT_ARCLENGTH_ALPHA_DEG.M
- 8: CREATE_AWIS.M
- 9: ACTIVATION_DISTANCE.M
- 10: ACTIVATION_DISTANCE_GUI.M
- 11: MAXIMUM_OBSTACLE_HEIGHT.M

- **PROGRAM 1: inverse.m**

% This program finds the inverse of a function

% COMMAND WINDOW:

% >> inverse

function inverse

syms theta

alpha = (atan((tan(theta)-theta)./(1+theta*tan(theta))));

pretty(alpha)

theta = finverse(alpha);

pretty(theta)

- **PROGRAM 2: differentiating_integrating.m**

% This program differentiates and integrates a function

% COMMAND WINDOW:

% >> differentiating_integrating

function differentiating_integrating

syms theta

alpha = (atan((tan(theta)-theta)./(1+theta*tan(theta))));

pretty(alpha)

differentiating = diff(alpha);

differentiating_simple_form = simplify(differentiating)

integrating = int(differentiating)

- **PROGRAM 3: plotalpha.m**

% This program plots two functions in a same window

% COMMAND WINDOW:

% >> plotaplha

function plotalpha

alpha=[];

for a=0:360

 theta = deg2rad(a);

 temp1 = atan((tan(theta)-theta)./(1+theta*tan(theta)));

 temp1 = rad2deg(temp1);

 alpha=[alpha temp1];

end

plot(alpha,'k *--')

xlabel ('Theta (deg)')

ylabel ('Alpha (deg)')

hold on;

simplifiedalpha=[];

for a=0:360

 theta = deg2rad(a);

 temp2 = theta - atan(theta);

 temp2 = rad2deg(temp2);

 simplifiedalpha = [simplifiedalpha temp2];

end

plot(simplifiedalpha,'b')

- **PROGRAM 4: plot_alpha_theta_deg.m**

%COMMAND WINDOW: plot_alpha_theta_deg

%This function:

%1.Creates table of theta_deg vs. alpha_deg

%2.Plots alpha_deg (y) vs. theta_deg (x)

%Input is theta_deg from 0 to 360

%Output is alpha_deg

function output=plot_alpha_theta_deg

alpha_theta_deg=[];

for theta_deg = 0:360

 theta_rad = deg2rad(theta_deg);

 alpha_rad = theta_rad - atan(theta_rad);

 alpha_deg = rad2deg(alpha_rad);

 alpha_theta_deg = [alpha_theta_deg;[theta_deg alpha_deg]];

end

output=alpha_theta_deg;

plot(alpha_theta_deg(:,1),alpha_theta_deg(:,1),'k');

hold on

plot(alpha_theta_deg(:,1),alpha_theta_deg(:,2),'b*');

xlabel('Theta (deg)');

ylabel('Alpha (deg)');

- **PROGRAM 5: thetaalpha.m**

%COMMAND WINDOW: theta_alpha

%This function creates table of theta_deg vs. alpha_deg

function output=thetaalpha

thetasimplifiedalpha=[];

for theta = 257.4530:0.000001:257.4540

thetarad = deg2rad(theta);

simplifiedalpha = thetarad - atan(thetarad);

simplifiedalpha = rad2deg(simplifiedalpha);

thetasimplifiedalpha = [thetasimplifiedalpha ;[theta simplifiedalpha]];

end

output=thetasimplifiedalpha;

- **PROGRAM 6: plot_radius_alpha_theta_deg.m**

%COMMAND WINDOW: plot_radius_alpha_theta_deg

%This function:

%1.Creates table of theta_deg vs. alpha_deg vs. radius

%2.Plots radius (y) vs. alpha_deg (x) and radius (y) vs. theta_deg (x) in the same plot

%Input is theta_deg from 0 to 360

%Output is radius

function output=plot_radius_alpha_theta_deg

rb = 0.25;

radius_alpha_theta_deg=[];

for theta_deg = 0:360

 theta_rad = deg2rad(theta_deg);

 alpha_rad = theta_rad - atan(theta_rad);

 alpha_deg = rad2deg(alpha_rad);

 radius = rb * ((1+theta_rad^2)^0.5);

 radius_alpha_theta_deg = [radius_alpha_theta_deg;[theta_deg alpha_deg radius]];

end

output=radius_alpha_theta_deg;

plot(radius_alpha_theta_deg(:,2),radius_alpha_theta_deg(:,3),'k');

hold on

plot(radius_alpha_theta_deg(:,1),radius_alpha_theta_deg(:,3),'b');

• PROGRAM 7: plot_arclength_alpha_deg.m

% COMMAND WINDOW:

```
%[alpha_theta_deg,theta_initial_rad,theta_final_rad,arclength]=plot_arclength_alpha_deg(0.25,0,180)
```

% 0 is alpha initial in degree

% 180 is alpha final in degree

% This function creates a table of alpha_deg vs. theta_deg

% It calculates theta_initial_rad, and theta_final_rad from the table using interpolation

% It calculates arc_length

% It plots the arclength vs. alpha_deg from theta_rad = theta_initial_rad to theta_final_rad, based on the alpha_initial_deg and alpha_final_deg

function

```
[alpha_theta_deg,theta_initial_rad,theta_final_rad,arclength]=plot_arclength_alpha_deg(rb,alpha_initial_deg,alpha_final_deg)
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
alpha_theta_deg=[];
```

```
for theta_deg = 0:360
```

```
    theta_rad = deg2rad(theta_deg);
```

```
    alpha_rad = theta_rad - atan(theta_rad);
```

```
    alpha_deg = rad2deg(alpha_rad);
```

```
    alpha_theta_deg=[alpha_theta_deg;[alpha_deg theta_deg]];
```

```
end
```

```
theta_initial_deg=interp1(alpha_theta_deg(:,1),alpha_theta_deg(:,2),alpha_initial_deg,'linear');
```

```
theta_initial_rad=deg2rad(theta_initial_deg);
```

```
theta_final_deg=interp1(alpha_theta_deg(:,1),alpha_theta_deg(:,2),alpha_final_deg,'linear');
```

```
theta_final_rad=deg2rad(theta_final_deg);
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
syms theta_rad_2;
```

```
func_2 = rb*theta_rad_2;
```

```
arclength=double(int(func_2,theta_rad_2,theta_initial_rad,theta_final_rad));
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
syms theta_rad_3;
```

```
arclength_theta=[];
```

```
for theta_rad_4 = theta_initial_rad:0.0001*pi:theta_final_rad;
```

```
    func_3 = rb*theta_rad_3;
```

```
    arclength_3 = double(int(func_3,theta_rad_3,theta_initial_rad,theta_rad_4));
```

```
    alpha_rad_4 = theta_rad_4 - atan(theta_rad_4);
```

```
    alpha_deg_4 = rad2deg(alpha_rad_4);
```

```
    arclength_theta=[arclength_theta;[alpha_deg_4 arclength_3]];
```

```
end
```

```
plot(arclength_theta(:,1),arclength_theta(:,2),'b*');
```

```
xlabel('Alpha (deg)')
```

```
ylabel('Arc Length (m)')
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```



```

alpha_rad_1 = theta_rad_1 - atan(theta_rad_1);
alpha_deg_1 = rad2deg(alpha_rad_1);
alpha_rad_1_rotate = alpha_rad_1 - beta_rad;
alpha_deg_1_rotate = alpha_deg_1 - beta_deg;
table_alpha_vs_radius_1 = [table_alpha_vs_radius_1;[alpha_rad_1_rotate
alpha_deg_1_rotate radius_avis]];
end
table_alpha_vs_radius_1;
polar(table_alpha_vs_radius_1(:,1),table_alpha_vs_radius_1(:,3),'b*')

hold on

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% PLOT THE HALF CIRCLE FROM ALPHA = PI TO 2PI

table_alpha_vs_radius_2 = [];

for alpha_rad_2 = pi:0.001*pi:2*pi
    radius_2 = rb;
    alpha_deg_2 = rad2deg(alpha_rad_2);
    alpha_rad_2_rotate = alpha_rad_2 - beta_rad;
    alpha_deg_2_rotate = alpha_deg_2 - beta_deg;
    table_alpha_vs_radius_2 = [table_alpha_vs_radius_2;[alpha_rad_2_rotate
alpha_deg_2_rotate radius_2]];
end
table_alpha_vs_radius_2;
polar(table_alpha_vs_radius_2(:,1),table_alpha_vs_radius_2(:,3),'r')

hold on

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```


% DRAW LINE BETWEEN 2 POINTS, i.e. STARTING AND ENDING POINTS
OF INVOLUTE CURVE

alpha_rad_rotate = [(pi-beta_rad) (pi-beta_rad)]

start_end_points = [radius_awis radius_2]

polar(alpha_rad_rotate,start_end_points,'r')

%%
%%

else

%%

fprintf('the rotation angle exceeds the limit. The AWIS is rotating with the end point
of the involute curve\n')

fprintf(' as the center of rotation\n')

%%

end

%%
%%
%%


```

theta_pc_rad = solve (y_equation,theta_pc);
theta_pc_deg = double(rad2deg(theta_pc_rad))

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

alpha_pc_rad = theta_pc_rad - atan(theta_pc_rad);
alpha_pc_deg = double(rad2deg(alpha_pc_rad))

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% CALCULATE ARC LENGTH

alpha_pb_rad = (beta_rad-(0.25*pi));           % same as alpha_final_deg
alpha_pb_deg = rad2deg (alpha_pb_rad)

%-----

alpha_theta_deg=[];

for theta_deg = 0:360
    theta_rad = deg2rad(theta_deg);
    alpha_rad = theta_rad - atan(theta_rad);
    alpha_deg = rad2deg(alpha_rad);
    alpha_theta_deg=[alpha_theta_deg;[alpha_deg theta_deg]];
end

alpha_theta_deg

%-----

theta_pa_deg=interp1(alpha_theta_deg(:,1),alpha_theta_deg(:,2),alpha_pa_deg,'linear');

```

```
theta_pa_rad=deg2rad(theta_pa_deg)
```

```
theta_pb_deg=interp1(alpha_theta_deg(:,1),alpha_theta_deg(:,2),alpha_pb_deg,'linear');
theta_pb_rad=deg2rad(theta_pb_deg)
```

```
syms theta_rad_2
```

```
func_2 = rb*theta_rad_2;
```

```
arclength=double(int(func_2,theta_rad_2,theta_pa_rad,theta_pb_rad))
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
distance = (cos(beta_rad)) * ( rb * (( 1 + ((theta_pc_rad)^2) ) ^ 0.5) *
    ( cos(alpha_pc_rad) ) ) + ...
    (sin(beta_rad)) * ( rb * (( 1 + ((theta_pc_rad)^2) ) ^ 0.5) *
    ( sin(alpha_pc_rad) ) ) + ...
    rb * (1) + ...
    arclength
```

```
output=distance;
```

- **PROGRAM 10: activation_distance_gui.m**

% This program creates the graphical user interface for activation_distance.m

```
function varargout = activation_distance_gui(varargin)
```

```
gui_Singleton = 1;
```

```
gui_State = struct('gui_Name',      mfilename, ...  
                  'gui_Singleton', gui_Singleton, ...  
                  'gui_OpeningFcn', @activation_distance_gui_OpeningFcn, ...  
                  'gui_OutputFcn', @activation_distance_gui_OutputFcn, ...  
                  'gui_LayoutFcn', [] , ...  
                  'gui_Callback', []);
```

```
if nargin && ischar(varargin{1})
```

```
    gui_State.gui_Callback = str2func(varargin{1});
```

```
end
```

```
if nargin
```

```
    [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});
```

```
else
```

```
    gui_mainfcn(gui_State, varargin{:});
```

```
End
```

```
function activation_distance_gui_OpeningFcn(hObject, eventdata, handles, varargin)
```

```
handles.output = hObject;
```

```
guidata(hObject, handles);
```

```
function varargout = activation_distance_gui_OutputFcn(hObject, eventdata, handles)
```

```
varargout{1} = handles.output;
```

```
function calculate_button_Callback(hObject, eventdata, handles)
```

```
rb = str2double(get(handles.rb,'String'));
```

```
obst_height = str2double(get(handles.obst_height,'String'));  
beta_deg = str2double(get(handles.beta_deg,'String'));  
distance=activation_distance(rb,obst_height,beta_deg)  
set(handles.distance,'String', double(distance));
```

- **PROGRAM 11: maximum_obstacle_height.m**

```
% COMMAND WINDOW:
```

```
% clear
```

```
% maximum_obstacle_height(0.25,150)
```

```
% @ is function handle
```

```
% This function calculates the maximum obstacle height that is traversable for each  
angular position, beta of the awis.
```

```
function output= maximum_obstacle_height(rb,beta_deg)
```

```
beta_rad = deg2rad(beta_deg)
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% PLOT THE X_VALUES VS. ALPHA_PC FROM 0 TO PI RAD FOR CERTAIN  
BETA
```

```
table_x_vs_alpha_pc = [];
```

```
for theta_pc_rad = 0:0.01*pi:4.4934
```

```
    alpha_pc_rad = theta_pc_rad - atan (theta_pc_rad);
```

```
    alpha_pc_deg = rad2deg(alpha_pc_rad);
```

```
    x = (( cos(beta_rad)) ) * ( rb * (( 1 + ((theta_pc_rad)^2))^0.5 ) * ( cos(theta_pc_rad  
        - (atan(theta_pc_rad))) ) ) + ...
```

```
        (( sin(beta_rad)) ) * ( rb * (( 1 + ((theta_pc_rad)^2))^0.5 ) * ( sin(theta_pc_rad -  
        (atan(theta_pc_rad))) ) ) );
```

```
    table_x_vs_alpha_pc = [table_x_vs_alpha_pc;[theta_pc_rad alpha_pc_deg x]];
end
```

```

fprintf('theta_pc_rad   alpha_pc_deg   x')
table_x_vs_alpha_pc

plot(table_x_vs_alpha_pc(:,2),table_x_vs_alpha_pc(:,3))
xlabel('Alpha P_c (deg)')
ylabel('x (m)')
title('Plot of Alpha P_C vs x')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% FOR CERTAIN BETA:

% FIND THETA_PC_RAD AT WHICH X IS MAXIMUM
% CALCULATE THE CORRESPONDING X_MAX VALUE

% CALCULATE THE MAXIMUM OBSTACLE HEIGHT AT POINT WHERE X
IS MAXIMUM

syms theta_pc
x = @(theta_pc)(-1*((( cos(beta_rad)) ) * ( rb * (( 1 + ((theta_pc)^2))^0.5 ) *
                                     ( cos(theta_pc - (atan(theta_pc)))) ) ) + ...
               (( sin(beta_rad)) ) * ( rb * (( 1 + ((theta_pc)^2))^0.5 ) *
                                     ( sin(theta_pc - (atan(theta_pc)))) ) ) );

theta_pc_rad_max = fminbnd(x,0,4.4934);      % Lower and Upper boundaries are
the same as in theta_pc_rad
theta_pc_deg_max = rad2deg(theta_pc_rad_max);
alpha_pc_rad_max = theta_pc_rad_max - atan (theta_pc_rad_max);
alpha_pc_deg_max = rad2deg(alpha_pc_rad_max);

```

```

x_max = ((( cos(beta_rad)) ) * ( rb * (( 1 + ((theta_pc_rad_max)^2))^0.5 ) *
        ( cos(theta_pc_rad_max - (atan(theta_pc_rad_max)))) ) ) + ...
        (( sin(beta_rad)) ) * ( rb * (( 1 + ((theta_pc_rad_max)^2))^0.5 ) *
        ( sin(theta_pc_rad_max - (atan(theta_pc_rad_max)))) ) ) )

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

if ( (0 <= beta_rad) && (beta_rad <= 1) )

```

```

    max_obst_height_1 = -(sin(beta_rad)) * ( rb * (( 1 + ((theta_pc_rad_max)^2))^0.5 )
        * ( cos(theta_pc_rad_max - (atan(theta_pc_rad_max)))) ) ) + ...
        (cos(beta_rad)) * ( rb * (( 1 + ((theta_pc_rad_max)^2))^0.5 )
        * ( sin(theta_pc_rad_max - (atan(theta_pc_rad_max)))) ) ) + ...
        rb

```

```

elseif ( (1 <= beta_rad) && (beta_rad < 4.4934) )

```

```

    max_obst_height_2 = -(sin(beta_rad)) * ( rb * (( 1 + ((theta_pc_rad_max)^2))^0.5 )
        * ( cos(theta_pc_rad_max - (atan(theta_pc_rad_max)))) ) ) + ...
        (cos(beta_rad)) * ( rb * (( 1 + ((theta_pc_rad_max)^2))^0.5 )
        * ( sin(theta_pc_rad_max - (atan(theta_pc_rad_max)))) ) ) + ...
        rb * ( ( 1 + (beta_rad^2) )^0.5 ) * (sin(atan(beta_rad)))

```

```

end

```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% PLOT MAXIMUM OBSTACLE HEIGHT TRAVERSABLE VS. BETA
```

```
table_max_height_vs_beta = [];
```

```
for beta_rad_range = 0:0.01*pi:4.4934
```

```
    syms theta_pc_range
```

```
    x_range = @(theta_pc_range)(-1*((( cos(beta_rad_range)) ) *
        ( rb * (( 1 + ((theta_pc_range)^2))^0.5 ) *
        ( cos(theta_pc_range - (atan(theta_pc_range))) ) ) + ...
        (( sin(beta_rad_range)) ) *
        ( rb * (( 1 + ((theta_pc_range)^2))^0.5 ) *
        ( sin(theta_pc_range - (atan(theta_pc_range))) ) ) ));
```

```
    theta_pc_rad_range_max = fminbnd(x_range,0,4.4934);
```

```
    theta_pc_deg_range_max = rad2deg(theta_pc_rad_range_max);
```

```
    alpha_pc_rad_range_max = theta_pc_rad_range_max -
        atan (theta_pc_rad_range_max);
```

```
    alpha_pc_deg_range_max = rad2deg(alpha_pc_rad_range_max);
```

```
    x_range_max = ((( cos(beta_rad_range)) ) *
        ( rb * (( 1 + ((theta_pc_rad_range_max)^2))^0.5 ) *
        ( cos(theta_pc_rad_range_max - (atan(theta_pc_rad_range_max))) ) ) + ...
        (( sin(beta_rad_range)) ) *
        ( rb * (( 1 + ((theta_pc_rad_range_max)^2))^0.5 ) *
        ( sin(theta_pc_rad_range_max - (atan(theta_pc_rad_range_max))) ) ) );
```

```

if ( (0 <= beta_rad_range) && (beta_rad_range <= 1) )

    max_obst_height_range = -(sin(beta_rad_range)) *
        ( rb * (( 1 + ((theta_pc_rad_range_max)^2))^0.5 ) *
        ( cos(theta_pc_rad_range_max - (atan(theta_pc_rad_range_max))) ) ) + ...
        (cos(beta_rad_range)) *
        ( rb * (( 1 + ((theta_pc_rad_range_max)^2))^0.5 ) *
        ( sin(theta_pc_rad_range_max - (atan(theta_pc_rad_range_max))) ) ) + ...
        rb ;

elseif ( (1 <= beta_rad_range) && (beta_rad_range < 4.4934) )
    max_obst_height_range = -(sin(beta_rad_range)) *
        ( rb * (( 1 + ((theta_pc_rad_range_max)^2))^0.5 ) *
        ( cos(theta_pc_rad_range_max - (atan(theta_pc_rad_range_max))) ) ) + ...
        (cos(beta_rad_range)) *
        ( rb * (( 1 + ((theta_pc_rad_range_max)^2))^0.5 ) *
        ( sin(theta_pc_rad_range_max - (atan(theta_pc_rad_range_max))) ) ) + ...
        rb * ( ( 1 + (beta_rad_range^2) )^0.5 ) *
        (sin(atan(beta_rad_range)));
end

table_max_height_vs_beta = [table_max_height_vs_beta;
[beta_rad_range alpha_pc_deg_range_max x_range_max max_obst_height_range]];

end

fprintf('beta_rad_range          alpha_pc_deg_range_max          x_range_max\n'
max_obst_height_range')

table_max_height_vs_beta

```

```

figure
plot(table_max_height_vs_beta(:,1),table_max_height_vs_beta(:,4))
xlabel('beta rad range (rad)')
ylabel('max obst height range (m)')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% CREATE TABLE & PLOTTING OF X_AT_180_RANGE VS. BETA

theta_pc_max = 4.4934;

table_x_at_180_range_vs_beta = [];

for beta_rad_when_xmax_at_alphapc_180_range = 1:0.01*pi:4.4934
    x_at_180_range = ((( cos(beta_rad_when_xmax_at_alphapc_180_range)) ) *
        ( rb * (( 1 + ((theta_pc_max)^2))^0.5 ) *
        ( cos(theta_pc_max - (atan(theta_pc_max)))) ) ) + ...
        (( sin(beta_rad_when_xmax_at_alphapc_180_range)) ) *
        ( rb * (( 1 + ((theta_pc_max)^2))^0.5 ) *
        ( sin(theta_pc_max - (atan(theta_pc_max)))) ) ) );

    table_x_at_180_range_vs_beta = [table_x_at_180_range_vs_beta;
        [beta_rad_when_xmax_at_alphapc_180_range    x_at_180_range]];
end

figure
plot(table_x_at_180_range_vs_beta(:,1),table_x_at_180_range_vs_beta(:,2))
xlabel('beta (rad)')
ylabel('x p_c (m)')

```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% CALCULATE BETA SO THAT X_MAX AT ALPHA_PC = 180 DEG IS THE
% MAXIMUM FOR A RANGE OF BETA VALUES
```

```
% The result is the position where x_max is at 180 && x_max is the maximum
% at all beta values, i.e. when beta is 180 degrees
```

```
syms beta_rad_when_xmax_at_alphapc_180
```

```
x_at_180 = @(beta_rad_when_xmax_at_alphapc_180)
    (-1*((( cos(beta_rad_when_xmax_at_alphapc_180)) ) *
        ( rb * (( 1 +((theta_pc_max)^2))^0.5 ) *
        ( cos(theta_pc_max - (atan(theta_pc_max))) ) ) + ...
        (( sin(beta_rad_when_xmax_at_alphapc_180)) ) *
        ( rb * (( 1 + ((theta_pc_max)^2))^0.5 ) *
        ( sin(theta_pc_max - (atan(theta_pc_max))) ) ) ) ));
```

```
beta_rad_when_xmax_at_alphapc_180_answer = fminbnd(x_at_180,1,4.4934)
```

```
% this is when x max is the maximum value
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% Prove that  $h_{\max} = r_b + y_{c0}$  for beta between 1 and 2.9226 rad
```

```
table_h_max_vs_beta = [];
```

```
for beta = 1:0.01*pi:2.9226
```

```
    h_max = r_b + [ r_b * ( ( 1 + (beta^2) )^0.5 ) * (sin(atan(beta))) ];
```

```
    table_h_max_vs_beta = [table_h_max_vs_beta;[beta h_max]];
```

```
end
```

```
table_h_max_vs_beta;
```

```
figure
```

```
plot(table_h_max_vs_beta(:,1),table_h_max_vs_beta(:,2))
```

```
xlabel('beta')
```

```
ylabel('h max')
```

APPENDIX B

SIMULATION RESULTS

- 1: A WHEELED-VEHICLE WITH FOUR NORMAL WHEELS
- 2: A WHEELED-VEHICLE WITH FOUR AWIS
MOVING ON A FLAT GROUND WITHOUT CONTROLLER
- 3: A WHEELED-VEHICLE WITH FOUR AWIS AND
CONTROLLERS:
 - 3.1: MOVING ON A FLAT GROUND
 - 3.2: TRAVERSING AN OBSTACLE
 - 3.3: TRAVERSING A STAIRCASE
- 4: A WHEELED-VEHICLE WITH EIGHT AWIS AND
CONTROLLERS TRAVERSING AN OBSTACLE

1. A WHEELED-VEHICLE WITH FOUR NORMAL WHEELS

(REFER TO PAGE 80)

Torque

At $t=0$ s, the exerted torque is somewhere between 400 and 1800 Nm due to the specified torque function. It can be seen that there are some gradual transitions from 400 to 1800 or vice versa. This is due to the period given in the specified torque function, i.e. 0.02s, to allow sufficient time for the motor to gradually increase or decrease the torque. Moreover, the gradual transitions shown are not a smooth curve, depending on the sampling time of the simulation itself.

Angular acceleration

At $t=0$ s, the angular acceleration is not zero due to the nonzero value of the torque. Shortly thereafter, it overshoots, drops, and stabilizes to the constant value. It happened because of the slippage between the wheel and the ground. Although the plot is similar to the torque plot, it can be seen that there are ripples while the torque is constant. This phenomenon happened due to the slippage.

Angular velocity

The vehicle is at rest initially. The larger the angular acceleration, the steeper the angular velocity curve.

Translational acceleration

Initially, the translational acceleration jumps from 0 to almost 2 m/s^2 in a very short period, and stables down to 1 m/s^2 . At first, the torque supplied is much higher than required such that the wheel is accelerated from rest to a certain velocity. The supplied torque is always higher than the required torque to drive the normal wheel, hence always causing slippage between the wheel and the ground. As a result, the maximum allowable translational acceleration is limited to 1 m/s^2 , if the static and dynamic frictional coefficients are 0.3 and 0.1, respectively.

Translational velocity

It increases constantly and approximates a straight line with a certain positive gradient, due to the constant translational acceleration.

2. A WHEELED-VEHICLE WITH FOUR AWIS MOVING ON A FLAT GROUND WITHOUT CONTROLLER (REFER TO PAGE 82)

Angular velocity

At $t < 1\text{s}$, the AWIS rotates on the base circle, and the angular velocity keeps increasing due to the torque. At $t = 1\text{s}$, the involute face touches the ground when torque is 1800 Nm , causing the angular velocity to drop. At $t = 1.2\text{s}$, the angular velocity drops at a higher rate, as the torque is reduced to 400 Nm . At $t = 2.2\text{s}$, the angular velocity increases slightly due to the increased torque at 1800 Nm . At $t = 3\text{s}$, the angular velocity drops due to the decreased torque. Moreover, as the rotation angle of the AWIS increases, the required torque increases as well. Therefore, the supplied torque is now unable to drive the AWIS. As a result, the AWIS rotates in

the reverse direction. It is indicated in the graph when it passes through zero. At $t=4\text{s}$, the angular velocity increases, but the AWIS still rotates in backward direction.

Translational displacement

At $t=3.75\text{ s}$, the AWIS reaches its maximum displacement. It occurs when the angular velocity is zero.

REFER TO FIGURE 5.6 ON PAGE 83

The angular positions for the four AWIS are initially the same. However, it can be seen that after the simulation, the angular positions between the front and rear AWIS are different. This is due to the different load distribution between the front and rear AWIS. The rear AWIS have lower weight compared to the front AWIS, hence lower frictional force, such that for the same supplied torque, the rear AWIS undergo more slippages than the front AWIS.

REFER TO FIGURE 5.7 ON PAGE 84

The interpretation as in Figure 5.5 holds for this figure. Note that at $t>2.5\text{s}$, there are abrupt changes in the angular velocity. This happens many times after the vehicle flies and falls on the ground. Instead of moving on the ground, the vehicle bounces up and down on the ground. This is due to the spring-mass-damper model is applied to the contact between the AWIS and the ground.

3. A WHEELED-VEHICLE WITH FOUR AWIS AND CONTROLLERS

3.1 MOVING ON A FLAT GROUND (REFER TO PAGE 87)

Figure B.1 Zooming in Figure 5.11: Angular velocity's errors graphs

Angular velocity errors

The angular velocity errors graphs are the same for both left and right front AWIS, likewise for both left and right rear AWIS. However, the front AWIS have larger errors than the rear AWIS. It is due to the bigger vehicle's body at the front and it causes heavier load distributions, less slippages, hence more time is required for the front AWIS to achieve the desired angular velocity with the changing torque.

Note that at $5.4 < t < 7$ s, the errors are negative. It means that the actual angular velocity is larger than the desired angular velocity. It happens when the center C (axle point) of the AWIS is ahead of the contact point between the AWIS and the ground. In other words, it happens when the AWIS rotates about the end of the involute-curve. It resulted in the sudden increase of the angular velocity due to the inertial load of the vehicle borne by the AWIS.

Figure B.2 Zooming in Figure 5.12: Motor torque

Motor torque

The initial value of the torque reaches 9000 Nm. In reality, it is unreasonable to achieve such high value of the torque in order to meet the desired angular velocity requirement, which is 50 degrees per second. In the simulation, it can be achieved

easily in a short period. However, by setting a gradual increment of the desired angular velocity, this issue can be solved. Subsequently, the torque is reduced to a constant value, as it is required to merely maintain the angular velocity of the AWIS. At this time, the AWIS is rotating on the base circle.

When $t > 1.2$ s, the torque increases since the AWIS rotates on the involute-curve, hence required more torque. It can be seen that there are ripples in the torque generated as a result of the slippages occur between the AWIS and the ground. Furthermore, the torque is adjusted continuously by the controller in order to achieve the desired angular velocity and angular acceleration. At $5.4 < t < 7$ s, the opposite torque is generated and used to reduce the angular velocity of the AWIS as elaborated in the angular velocity errors section.

Figure B.3 Zooming in Figure 5.12: Angular acceleration

Angular acceleration

At $t=0$ s, the angular acceleration is zero. The value increases as the torque increases. At $t > 1.2$ s, it fluctuates as the AWIS rotates on the involute-curve according to the motor torque. Note that the desired angular acceleration is set to zero throughout the simulation to minimize sudden changes in the angular velocity.

Figure B.4 Zooming in Figure 5.12: Angular velocity

Angular velocity

Within $t < 0.3$ s, the angular velocity reaches the desired value, which is 0.8727 radians per second. At $t > 1.2$ s, the angular velocity drops and fluctuates as the AWIS rotates on the involute-curve. When $5.4 < t < 7$ s, the angular velocity increases (see angular velocity errors for the explanation). After $t = 7$ s, the angular velocity profile repeats the first cycle ($t < 7$ s), except now the initial value is not zero.

Figure B.5 Zooming in Figure 5.12: Translational acceleration

Translational acceleration

Note that the peak surge occurred at $t=5.4$ s as seen in the translational acceleration plot in Figure 5.12 on page 88, is omitted in this plot. The spike happened when the AWIS rotates about the end of the involute-curve and the vehicle is going to fly. This sudden loss contact with the ground causes discontinuity in the integrator calculation. This discontinuity is still within the error tolerance specified in the integrator, hence the spike disappears quickly. The integrator in this case is used to control the numerical integration of the equations of motions for analyzing dynamically the system in motion.

Initially, when $t < 0.3$ s, the translational acceleration surges to about 2 m/s^2 due to the large motor torque. At $0.3 < t < 1.2$ s, it is constant as the AWIS rotates on the base circle. At $1.2 < t < 5.4$ s, it slightly increases as the AWIS rotates on the involute-curve. When $5.4 < t < 7$ s, it drops, as the AWIS rotates about the end of the involute-curve, and the motor torque prevents the AWIS from accelerating further in order to satisfy its desired angular velocity. When $7 < t < 7.5$ s, the AWIS lands on the ground and touches the base circle. In this period, the spike appears again due to sudden contact with the ground. The same explanation holds as in the previous paragraph. After $t=7.5$ s, the profile repeats itself as the cycle repeats.

3.2 TRAVERSING AN OBSTACLE (REFER TO PAGE 91)

Figure B.6 Zooming in Figure 5.15: Angular velocity's errors graphs

Angular velocity errors

The angular velocity errors graphs share similarity in shapes for both left and right rear AWIS, likewise for both left and right front AWIS. Nonetheless, the values are slightly different. It can be caused by factors such as integrator errors, the weight balance between left and right sides of the vehicle during motion.

Moreover, the graphs for the front AWIS are different from those for the rear AWIS. The factors contributing to those differences include the weight distribution between front and rear AWIS, the time when the AWIS encounter obstacles. In this scenario, the front AWIS hit the obstacle and lift the vehicle up from the ground, whereas the rear AWIS only need to lift the vehicle with minimum effort, as most works are done by the front AWIS, resulting in higher errors at the front AWIS.

At the beginning of the simulation, the vehicle is at rest, such that the initial values of the angular velocity errors for all four AWIS are 0.017453 radians per second, which are the desired angular velocity. Afterwards, the errors lower to zero.

At $t=60$ s, the errors for the four AWIS surge as the AWIS start rotating at the involute-curves, hence lower angular velocity, and higher angular velocity errors. The front part of the vehicle is heavier than the rear part such that it contributes to higher friction forces between front AWIS and ground, hence less slippages and higher angular velocity errors than the rear AWIS.

At $t=120$ s, both front AWIS hit obstacle, causing the errors values surge to almost equal to the desired angular velocity. At the same time, slippages occur at the rear AWIS, causing the ripples to the errors graphs. Due to the slippage at the rear AWIS, at $t=260$ s, the rear AWIS reach and rotate about the end of involute-curve first while the front AWIS are still rotating on the involute-curve. During this period, the vehicle's weight accelerates the AWIS, causing the higher actual velocity compared to the desired angular velocity. This results in the negative values of the angular velocity errors. At $t=325$ s, the rear AWIS touch obstacle, whereas at $t=330$ s, the

front AWIS touch ground. These cause the angular velocity to drop drastically, hence positive angular velocity errors. It can be seen that there is no numerical integration error resulted because the front and rear AWIS have different angular positions due to the existence of the obstacle such that the vehicle is always in contact with either the obstacle or the ground. At $t=425$ s, all the four errors surge due to AWIS start rotating at the involute-curves. Note that different scales are used between the front and rear AWIS' plots.

Figure B.7 Zooming in Figure 5.16: Motor torque

Motor torque

It can be seen that the initial torque required is ± 160 Nm in order to achieve desired angular velocity of 1 degree per second. At $t=60$ s, more torque is required as the AWIS start rotating on the involute-curve. At $t=120$ s, both front AWIS hit obstacle. During this period, more torque is exerted at the front AWIS until the torque is sufficient to lift the vehicle up. When $225 < t < 275$ s, the front AWIS continue rotating on the involute-curve. Starting at $t=275$ s, the front AWIS rotate about end of involute-curve and it results in continuous increment in the angular velocity. Consequently, reverse motor torque is exerted to decelerate the motion until at $t=330$ s, when the front AWIS touch ground as explained in the preceding section. It is

reflected as well in the negative surge of the translational acceleration as a result of the opposite motor torque. After touching the ground, the torque is back to normal to drive the AWIS in the forward direction and so on.

Angular acceleration

The range of the magnitude of all angular acceleration values are in the exponential power of 10^{-4} , which is very small and approaching zero, as specified in the desired angular acceleration. Note that at $t=310$ s, the acceleration reaches its peak value and it is countered by the peak value of the torque exerted.

Figure B.8 Zooming in Figure 5.16: Angular velocity

Angular velocity

At $t=0$ s, the initial angular velocity is zero. In a short period, the angular velocity reaches its desired value which is 0.017453 radians per second. The rest of the explanations are provided in the angular velocity's errors section above.

Figure B.9 Zooming in Figure 5.16: Translational velocity

Translational velocity

At $t=0$ s, the initial velocity is zero. Soon afterwards, the velocity increases to 0.004192 m/s and remains till $t=60$ s when the AWIS rotates on the base circle. At $t=60$ s, the AWIS starts rotating on the involute-curve, hence the velocity drops a little bit. As the AWIS rotates, the velocity increases due to the increasing radius of the involute-curve while the angular velocity is slightly increasing. At $t=120$ s, the velocity drops to almost zero since the AWIS hits the obstacle. At $120 < t < 220$ s, the change in the contact point location of the AWIS from the ground to the obstacle leads to the high velocity. At $225 < t < 275$ s, as the torque increases, the AWIS is able to lift the vehicle up, the velocity continues increasing at a very low rate. At $275 < t < 300$ s, the AWIS rotates about the end of involute-curve, and the velocity increases. Subsequently, at $300 < t < 330$ s, the velocity decreases as the result of the reversed motor torque. At $t=330$ s, the front AWIS touch the ground, and the slippage occurs, causing the velocity to surge. These sequences are then repeated again afterwards.

3.3 TRAVERSING A STAIRCASE (REFER TO PAGE 96)

Figure B.10 Zooming in Figure 5.23: Angular velocity's errors graphs

Angular velocity errors

All the four plots can be interpreted in the same way as described in the previous sections. However, in this simulation, the graphs show more fluctuations in the errors values compared to those in the previous section (Refer to page 91). The main factor contributing to this fluctuation is the shape of the obstacle. The more complex the obstacle's shapes are, the more fluctuations the errors would be.

Figure B.11 Zooming in Figure 5.24: Motor torque

Motor torque

The above plot shows the abrupt changes of the motor torque during the motion. When the actual angular velocity is higher than the desired angular velocity, the reverse motor torque (in this case, positive torque) is exerted or the other way around.

Figure B.12 Zooming in Figure 5.24: Angular acceleration, angular velocity, and translational acceleration

Angular acceleration, angular velocity, and translational acceleration

A staircase is a combination of obstacles with different dimensions of the same profile. Therefore, the same reasoning from previous sections applies in interpreting the above graphs. Although a large number of spikes occurred in the angular acceleration graph, the mean value is approximately zero, which meets the desired angular acceleration, i.e. zero. For angular velocity profile, during ascending the staircase, the higher the step, the higher the angular velocity drops, hence higher motor torque exerted, whereas during descending the staircase, the higher the step, the higher the angular velocity surges, hence higher reverse motor torque.

**4. A WHEELED-VEHICLE WITH EIGHT AWIS AND
CONTROLLERS TRAVERSING AN OBSTACLE
(REFER TO PAGE 103)**

Appendix B – Simulation Results

Figure B.13 Zooming in Figure 5.35: Angular velocity's errors graphs

Angular velocity errors

In this section, only the graph for existing front AWIS (front_1) is elaborated, whereas other AWIS' graphs can be interpreted in the same way.

At $t < 20$ s, the angular velocity's error fluctuates. During this period, the normal wheels are rotating, whereas all AWIS are at rest. The existing AWIS are in contact with ground and their desired angular velocity is set to be zero, while the vehicle moves. It causes the existing AWIS to rotate, hence angular velocity's error. Subsequently, the motor exerts torque to drive the AWIS to come to a stop, satisfying the specified angular velocity. However, this is not the case for additional AWIS, since it has smaller base radius, hence there is no slippage. The center of gravity of the AWIS is located at the back with respect to the center of the base circle, causing the AWIS to rotate backward, hence positive angular velocity errors.

At $t = 20$ s, the existing front AWIS activates. Nonetheless, the error only increases to 0.003, instead of 0.1745, which is the desired angular velocity. It is because of the "STEP" function which generates a gradual change in achieving the desired angular velocity. At $t = 28$ s, the AWIS undergoes the transition from the base circle to the involute-curve, hence surging the error to 0.02. At $t = 35$ s, the AWIS hits obstacle, causing the error to increase further to 0.06. Afterwards, the errors keep fluctuating due to the angular positions of rotating AWIS. As the angular position of the AWIS increases, the radius increases, and the angular velocity reduces, hence error increases. The PID controller then adjusts the torque to minimize the errors. There are times when the graph shows negative errors. It means that the actual angular velocity is higher than the desired angular velocity. It occurs when the AWIS rotates about the end of the involute-curve and the axle point is ahead of the contact point between the AWIS and the ground, causing the AWIS to accelerate due to its inertia and accordingly, the motor torque is reversed.

Appendix B – Simulation Results

Figure B.14 Zooming in Figures 5.36 & 5.37: Motor torque

Motor torque

The PID controller adjusts the motor torque according to the actual angular velocity. When the actual angular velocity is higher than the desired angular velocity, the reverse motor torque (in this case, positive torque) is exerted or the other way around.

Figure B.15 Zooming in Figures 5.36 & 5.37: Angular velocity

Angular velocity

The above plots are interpreted in the same way as the plots in the previous sections.