Development of an electrokinetic actuator and its applications in compressors

Alison Subiantoro

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DEVELOPMENT OF AN ELECTROKINETIC ACTUATOR AND ITS APPLICATIONS IN COMPRESSORS

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2006

Development of an Electrokinetic Actuator and Its Applications in Compressors

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ABSTRACT

Electroosmosis flow has been exploited by many for various applications. However none has ever published the exploitation of this working principle for its application in air compression system. Therefore, the aim of this project is to use electroosmotic flow principle to drive a compressor.

In this project, the theory and governing equations in accordance with the electroosmotic working principle were first derived. By using these equations, computer simulations on the evolution of the air compression models, starting from a simple compression system to an electrokinetic compressor, were carried out. The results obtained showed that the conceptual electrokinetic actuator can perform like a common reciprocating compressor. A discharge pressure of 600 kPa were obtained, however higher discharge pressures are clearly still achievable. The simulations also show that the loss can be minimized by using easier to move and bigger valves, while the mass of the piston does not affect the behavior of the piston too much.

Experiments were carried out to verify the theoretical predictions. The effects of three parameters, namely the applied electric field, the solution concentration and the hydraulic diameter were investigated. The results showed that in general, the theoretical predictions are in good agreement with the actual results. However, due to the ignoring of the temperature effects and the electrolysis effects, coupled with the independent parameter assumption adopted in the theoretical predictions, some discrepancies arose.

Chapter 1:

INTRODUCTION

1.1. Background

In the last two decades, the major advancement in precision technology has given way to the development of micron-sized channels. These channels are usually called microchannels. Microchannels can be fabricated in many materials ranging from glass, polymers, silicon and metals. These microchannels are now extensively used in many areas like electronic cooling or lab-on-a-chip in biotechnology industry.

Nominally, microchannels can be defined as channels with dimensions less than 1 mm and greater than 1 μm . These channels are unique for they have small volumes and high surface to volume ratio. The flow in these microchannels exhibits unique effects like electrokinetic effects, capillary effects and surface tension effects [1]. The aim of this project is to specifically observe and exploit the electrokinetic effects.

Electrokinetic effects occur when an electrolyte is in contact with a solid surface. This contact causes electric charge to be acquired by the surface and this charge will affect the flow. Generally, electrokinetic phenomenon can be categorized into: [2]

1. Electroosmosis, when a liquid moves in response to an applied electric field.
2. Electrophoresis, when particles suspended are induced to move by applying an electric field across the system.
3. Streaming potential, when an electrical potential difference is developed due to a pressure driven liquid flow.

4. Sedimentation potential, when an electrical potential difference is developed due to movement of charged colloidal particles.

More discussions about electrokinetic phenomenon will be presented in Chapter 2.

Some applications of electrokinetic phenomena are lab-on-a-chip (Figure 1.1), electrokinetic pump (Figure 1.2), protein separation, etc.

Figure 1.1 A lab-on-a-chip [3]

Figure 1.2 A planar electrokinetic pump [4]

In the application of electroosmosis pumps, a very interesting characteristic of electrokinetic phenomenon is the ability to pump without the need of a special mechanism. Because of this special characteristic, the pump has many special and advantageous characteristics [5]. More important of all, the pump is able to generate very high pressure, as high as high as 8000 psi or 55.16MPa [6].

With this reported high pressure generated, an idea to develop an electrokinetic actuator was developed upon [7]. In this project, this actuator was used to drive a compressor. It is expected that the actuator will have a lot of advantages like:

- no moving part needed to drive the actuator,
- wide range of size is possible, and
- fewer parts, hence easier maintenance.

1.2. Scope of this project

The scope of this project includes three major parts.

First, the numerical simulation of an electrokinetic actuator and an electrokinetic compressor will be developed. This numerical simulation was carried out using Fortran programming language. The objective of the simulation is to predict the behaviors of the conceptual electrokinetic actuator and compressor.

The second part of this project is to carry out experiments to characterize the fully-closed electroosmotic system which is very important for the development of the actuator. Together with the simulation results, these experimental results will be the basis to fabricate the prototype of an electrokinetic actuator.

The last part of this project gives recommendations for future research and development of the electrokinetic actuator and the electrokinetic compressor.

1.3. Objectives of the project

- To formulate the simulation model for an electrokinetic actuator,
- to formulate the simulation model for an electrokinetic compressor,
- to carry out experiments on a fully-closed electroosmotic system, and
- to lay the foundations to fabricate an electrokinetic compressor.

1.4. Organization of this thesis

Chapter 1 of this thesis presents the background of the project, the scope, the organization of the thesis and the objectives of the project. In Chapter 2, basic theories on electrokinetic effects and fluid motion are reviewed; and a comprehensive literature review of literature related to this work is presented in Chapter 3. Chapter 4 presents the derivations of the equations used to formulate the mathematical model. The procedures, results and discussions of the simulations carried out are presented in Chapter 5. After that, the design of experiments, the procedures to conduct the experiments and the results obtained are presented in Chapter 6. Lastly, this thesis ends with the conclusion of the project and plans for future works in Chapter 7.

Chapter 2:

BASIC THEORY

In this chapter, basic theories of general electrokinetic effects will be presented. After that, basic theories behind the electroosmotic flow follow and lastly, basic theories of potential distribution and equation of motion are presented.

2.1. Electrical Double Layer (EDL)

The theory of electrical double layer (EDL) is well established and has been well summarized by Hunter [2]. The basic concept behind it is the electrostatic concept. At the surface of any phase, there is a separation of positive and negative charge components. This separation creates a region of varying electrical potential which extends over a distance of the order of one or more molecular diameters. If the surfaces are liquid and solid, this potential difference will cause the ions near the surface to be arranged in a unique manner, where the co-ions (ions with the same charge as the solid's charge) are repelled by the solid surface, while the counter-ions (ions with opposite charge as the solid's charge) are attracted by the solid surface. The arrangement of charges that occurs is called the electrical double layer (EDL).

Gouy and Chapman as quoted by Hunter [2] proposed a model that assumes the potential distribution of the solid surface to be uniform over a plate and the plate is immersed in an electrolyte solution. The compensating ions are regarded as point charges immersed in a continuous dielectric medium while the fundamental

electrostatic equation is the Poisson's equation. This model is known as the Gouy-Chapman model and is the basis of the more advanced models developed later.

Stern, as quoted by Hunter [2], made the modification to the Gouy-Chapman model. He incorporated the finite size of the ions by dividing the space charge in the solution into two regions namely the compact or inner region and the diffuse region. The compact region exists right next to the solid surface. Here, the counter-ions are strongly attracted to the solid surface and are immobile. This region is usually several Angstroms thick and the potential distribution pattern is linear. The next region, which is the diffuse region, exists between the compact layer and the bulk liquid. Counter-ions are affected less strongly here and so they are mobile. The counter-ions concentration in this region gradually reduces to that of bulk liquid in a non linear pattern. Interestingly, accumulation of counter-ions in this diffuse region causes a non-zero net charge density. Between the compact region and the diffuse region, there is a plane that separates the two regions called Stern plane.

Debye and Huckel [8] introduced a parameter to estimate the thickness of EDL called the Debye-Huckel parameter:

$$\kappa = \left(\frac{e^2 \sum n_i z_i^2}{\epsilon k_b T} \right)^{1/2}$$

where: e is elementary charge

n_i is bulk concentration of ion

z_i is valence

ϵ is dielectric constant of the liquid

k_b is Boltzmann constant

T is absolute temperature of liquid

The inverse of Debye-Huckel parameter is called the Debye length, and the region of varying potential usually extends to a distance of about three times this Debye length before the potential has decayed to about 2% of its value at the surface [2].

Very close to the Stern plane, lies another plane called surface of shear. This surface is characterized as the plane at which the mobile portion of the diffuse layer can slip or flow past the charged surface. The shear plane is where the no slip boundary condition is assumed to apply. The surface potential at this surface is known to be zeta potential (ζ) which is a very important parameter to understand the electroosmotic flow. However, it is a common practice to take the potential at the Stern plane to be the zeta potential.

An illustration of the model is shown in Figure 2.1:

Figure 2.1 An illustration of Electrical Double Layer [9]

2.2. Electroosmotic Flow

As has been mentioned, the diffuse region has a non-zero net charge density. This causes the counter-ions to move when electric field is applied tangentially along the charged surface. This movement of ions will pull the surrounding fluid with them, resulting in a unique flow called electroosmotic flow [2] as depicted in Figure 2.2.

Figure 2.2 An illustration of an electroosmotic flow

According to von Helmholtz and Smoluchowski [2,10], the electroosmotic mobility can be calculated using the famous Helmholtz-Smoluchowski equation:

$$u = -\frac{\epsilon_r \epsilon_0 \zeta}{\eta} \cdot \frac{dE}{dx}$$

where: u is electroosmotic / electrophoretic velocity

ϵ_r is relative dielectric permittivity of the liquid

ϵ_0 is permittivity of a vacuum

ζ is zeta potential

η is viscosity of the liquid

$\frac{dE}{dx}$ is applied electric field gradient

More discussions on electroosmotic flow will be presented in Chapter 4.

2.3. Electrical potential distribution

The basic equation to describe the potential distribution in the electrolyte is the theory of electrostatics [11-18], which is also known as the Poisson's equation:

$$\nabla^2 \psi = \frac{-\rho}{\epsilon_r \epsilon_0}$$

where: ψ is electric potential

ρ is electric charge density

2.4. The equation of motion

The basic equation to describe the motion of this incompressible flow is the Navier-Stokes equation [19]:

$$\rho_f \frac{D\vec{V}}{Dt} = -\nabla P + F + \mu_f \nabla^2 \vec{V}$$

where: ρ_f is density of fluid

$\vec{V}$ is velocity vector

t is time

P is pressure

μ_f is viscosity of fluid

F is the body force experienced by the fluid, for electroosmotic flow in horizontal direction, this is carried out by the action of applied electrical field only. The direction of the electric field is parallel to the wall.

Chapter 3:

LITERATURE REVIEW

In this chapter, a review of what other researchers have done related to this project will be presented.

3.1. Effects of electrical double layer to fluid flow

The existence of electrical double layer (EDL) causes the unique ion arrangement of electrolyte near the channel wall. This unique characteristic results in many special effects to the flow and the liquid. Some of these effects are discussed below.

Sharp et al [1] and Hunter [2] stated that the mechanism for the spontaneous charging of surface layers is either the differential adsorption of ions from an electrolyte onto solid surfaces, the differential solution of ions from the surface to the electrolyte or the deprotonation / ionization of surface groups. The most common is the deprotonation of surface groups on the surface of materials such as silica, glass, acrylic and polyester. In the case of glass and silica, the deprotonation of surface silanol groups (SiOH) determines the generated surface charge density. The equilibrium reaction associated with this deprotonation can be represented as:

Sharp et al [1] published an equation to estimate the pressure generated for the case of zero flow rate electroosmotic flow which can be utilized for many applications:

$$\Delta p = -\frac{8\epsilon\zeta\Delta V}{a^2}$$

where a is the capillary radius.

In another paper, Mala et al [12] presented theoretical analysis on the influence of EDL on liquid flows in rectangular microchannels by solving 2D non-linear Poisson-Boltzmann equation. They found that when zeta potentials are larger than 25 mV, the linear approximations give observable error.

Mala et al [17] also investigated the effects of the EDL on liquid flow and heat transfer of a liquid flow through a microchannel between two parallel plates at constant and equal temperature. The fluid flow in a microchannel will produce the streaming potential and this will cause deviations from the normal flow. From their analysis, they found that the thinner the EDL the higher the velocity is, which means EDL affects pressure drop and heat transfer in the system. They also found that increasing zeta potential or increasing the Debye parameter results in higher friction constant. This means both the channel size and the fluid properties affect the friction constant. Lastly, they also found that heat transfer increases as zeta potential decreases, κ increases or Re increases.

Yang and Li [18] presented their work on analysis of electrokinetic effects on liquid flow in rectangular microchannels. From the derivations, they found that in microchannel flows, there exists streaming potential. This potential, in turn, will produce a liquid flow (electroosmotic flow) in the direction opposite to the pressure-driven flow causing a reduction in flow rate. The liquid thus appears to exhibit an enhanced viscosity. They noted some interesting findings such as the existence of corner effect and the effect of EDL and zeta potential to the flow which causes the reduction of flow rate and increase in viscosity due to the existence of streaming potential, especially in smaller capillaries.

Mala et al [20] also found that in a pressure driven flow, EDL does reduce the flow rate but only in flows with low ionic solutions the EDL effect is significant.

Yang et al [21] investigated the effects of EDL and the flow induced electrokinetic field on the pressure-driven flow and heat transfer through a rectangular microchannel. They found the results to be in agreement with Yang [18] and Mala [20] that EDL reduces flow rate and increases viscosity. This, in turn, increases the temperature in the microchannel. They also found that the axial thermal conduction in flow direction and the viscous dissipation have little impact on microchannel heat transfer and therefore can be safely neglected. Lastly, they noted that the Boltzmann distribution for the electrical potential can be safely applied only for a microchannel flow in a fully-developed hydrodynamic state.

Wan [22] observed how electroosmotic flow affects the electrical conductivity of a packed capillary column. He found that relative conductivity first increases with electrokinetic radius before finally dropping after a certain value. Relative conductivity also increases with higher zeta potential, smaller particle diameter or lower electrolyte concentration.

In another paper, Kasicka et al. [23] modified the capillaries in order to suppress the electroosmotic flow. They did this by coating the inner side of the capillary with low conductivity polymer. Other known methods to suppress electroosmotic flow include changing the electric field strength and modifying the electrolyte. Another method [24-26] is to apply additional electric field on the outer surface of the capillary to control the zeta potential.

3.2. Zeta potential

Zeta potential is a very important concept in understanding electrokinetic phenomenon. However, it is sometimes tedious to get the zeta potential in experiments. Some methods used are presented below. Most of the methods are not

applicable for more than one application since they are introduced only for specific applications.

One of the simplest techniques to measure zeta potential on a parallel plate microchannel or in a capillary is mentioned by Hunter [27] which is based on Smoluchowski's electrophoresis theory. It is done by experimentally measuring the electrophoresis mobility and then estimating the zeta potential using Smoluchowski's equation. Another technique is by using electroosmotic process, instead of electrophoresis [27-28]. This is done also by experimentally obtaining the electroosmosis velocity and then applying the Smoluchowski equation to estimate the zeta potential. Other than those two, streaming potential method has also been used to measure zeta potential on a parallel plate microchannel or in a capillary [29-30]. The basic principle is similar to the electrophoresis and electroosmosis methods.

Other methods to measure zeta potential of a specific system are available in literatures, such as [31-37].

3.3. Open Channel Electroosmotic Flow

Having discussed the basics of electrokinetic phenomenon by discussing the effects of electrical double layer to fluid flow and the existence of zeta potential, the subsequent discussion will focus on review of the characteristics of one kind of electrokinetic phenomenon known as electroosmotic flow (EOF), the basic theory of which is presented in section 2.2.

Marcos et al [11] and Kang et al [15,16] presented their theoretical analysis on dynamic aspects of electroosmotic flow in a rectangular and a cylindrical capillary respectively. They found that the velocity field generated by DC electric field in a simple electroosmotic flow exhibits a profile resembling a plug flow. In the case of

rectangular microchannel, there is a “corner effect” due to the overlapping of EDL at the corners. Experimentally, the analysis was carried out by Sinton et al [38,39]. They investigated the velocity profiles of electroosmotic flows in circular and rectangular microchannels. This is done by using a caged-dye based technique with a high degree of resolution. They found that the actual velocity profiles agree well with the theoretical predictions, which are plug-like in shape and that the velocity is indeed a linear function of electric field.

When an AC electric field is applied, the velocity profile is different and is dependent on the frequency of the field [11,15,16,40]. It was found that due to inertia, the velocity in the bulk region decreases as the frequency increases while in the near wall region, the velocity is always in phase with the driving electric field.

It is possible that the diameter of the channel is so small that the EDL overlaps each other. Wan [41] analyzed this special case and he found that the double-layer overlap reduces the electroosmotic flow velocity and the more closely packed the particles the lower the electroosmotic flow velocity is. The velocity profile is no longer plug-like as in the simple electroosmotic flow case but becomes parabolic in shape.

Gleeson [42] presented his analysis on the case of electroosmotic flow with random zeta potential. It was found that while the mean axial velocity is plug-like, the actual velocity actually varies and the variance of the axial velocity depends on radius.

It is possible that electroosmosis is created by surface defects and this case is analyzed by Long et al [43] by developing numerical modeling of the system. It was found that in any bounded configuration the defect produces a recirculating disturbance motion. Another effect of the defect is to create a step increase in the pressure.

Fu et al [44] presented an analysis of electroosmotic flow with a step change in zeta potential. It was found that due to this sudden change in zeta potential, the velocity

profile is not flat but becomes either parabolic or concave depends on the zeta potential of that region. One case demonstrated was a sudden jump from 0 to -75 mV of the zeta potential. It was reported that at the zero zeta potential region, the velocity profile is parabolic in shape and after the jump it becomes concave. This finding is in good agreement with experimental results presented by Herr et al [45].

Santiago [46] analyzed the effects of inertial and pressure forces on electroosmotic flow. It was found that the effects are negligible at the near-wall region but changes the velocity profile in the bulk region. It was also found that the streamline of electroosmotic flow follows the electric field lines closely. This last finding is also supported by results found by Kim et al [47] and Devasenathipathy and Santiago [48]. Although the theory of electroosmotic flow in capillaries is already established as has been presented, some papers published reported some challenging and controversial results. Yang et al [49] analyzed the electroosmotic flow through microchannels between two parallel plates and a 90 degree bend numerically. They found that there is a very small induced pressure drop in the tube and the most interesting result is that by including the convection term, the velocity profile becomes concave which is different from what other researchers have reported. In other paper, Yang et al [50] analyzed the entrance region on the electroosmotic flow. They reported that at the entry point, the velocity profile is parabolic in shape and eventually it grows to plug-like shape. This entry length is longer when Reynolds number is higher. Very interestingly it was reported that the velocity profile is concave when Reynolds number is high.

Electroosmotic flows in porous media have different characteristics from those in simple capillaries. Liapis and Grimes [51] reported that high velocity of electroosmotic flow can be obtained if zeta potential of the particles is very negative,

viscosity of the electrolyte is low, dielectric constant of electrolyte is high and if the EDL is thin.

Meyers and Liapis [52,53] investigated how the pore connectivity affects the electroosmotic flow velocity in the porous media. They found that as pore connectivity value increases, the intraparticle interstitial velocity and the pore diffusivity increase. They also found that intraparticle interstitial velocity is many times larger than the diffusion velocity, and as pore connectivity increases, this ratio increases significantly.

Coelho et al [54] investigated numerically electrokinetic effects in porous media with reasonably thick double layer. It was found that when EDL is thick, the flow is no longer inviscid but viscous due to overlapping EDL just like the case in capillaries with very small diameter.

In general, there are two models that are known to be used to analyze electrokinetic in porous media [55], namely the Overbeek's model and Dukhin's model. All of the above analysis uses Overbeek's model. The later model introduced by Dukhin and his group [56] provided alternative model known as electroosmosis of second kind which gives deeper understanding on how electroosmosis happens in porous media since a comprehensive theoretical basis for electroosmotic flow in porous media has not yet been really established. A publication on an experiment on electroosmotic flow in packed capillaries [57] supports the theoretical results presented. It was also found that the velocity profile of flow in porous media is not rectangular [58].

A case of electroosmotic flow in a T-shaped channel was analyzed by Hu et al [59]. It was found that the existence of a side channel changes the shape of the streamlines due to leakage to the side channel. This leakage is less when the length of the main channel is longer, the side channel is close to the outlet of the main channel, the side

channel is shorter, or if an appropriate potential is applied to provide enough electric force to balance the leakage. Pittman et al [60] supports these findings.

The effects of Joule heating process to electroosmotic flow is analyzed theoretically by Tang et al [61]. They found that the existence of Joule heating increases the velocity of the flow and that Joule heating effect can be neglected when the electrolyte is dilute (e.g. 10^{-4} M) or the applied electric field is low (e.g. 100 V/cm).

3.4. Closed Channel Electroosmotic Flow

Closed channel electroosmotic flow is one special case of electroosmotic flow when the net flow is zero. Not many publications focus on this special and very interesting case yet. Marcos et al [11-13] give a very comprehensive analysis on the theory of closed channel electroosmotic flow in a rectangular microchannel. Equations to predict the velocity, induced pressure gradient and electrical potential distribution were derived and used to analyze the case. It was found that the zero net flow condition is caused by a reverse flow to counter balance the electroosmotic flow. The velocity profile of the flow is parabolic due to the presence of the induced backpressure. When AC electric field is applied, the frequency of the field will affect the velocity profile and the induced pressure gradient. These findings are in good agreement with theoretical results reported by Reppert and Morgan [46].

In another paper, Marcos et al [62] presented a theoretical work on electroosmotic flow in a closed-end cylindrical microchannel. It was found that the behavior is very similar to the one predicted for a rectangular microchannel. Experimental analysis on closed channel electroosmotic flow in a capillary were carried out by Locke et al [58] and Manz et al [63] and it was found that the velocity profile is indeed as predicted by the theoretical model, which is parabolic in shape due to the induced backpressure.

Experiments on closed channel electroosmotic flow in porous media were done by Tallarek et al [57] and Locke et al [58], and it was found that there is reverse flow as can be seen from the velocity profile obtained.

3.5. Applications of Electroosmotic Flow

Having discussed the many aspects of electroosmotic flow, some applications of electroosmotic flow will be presented. The most common applications of electroosmotic flow include chemical separation processes, chromatography and other biochemistry and bio-MEMS applications [12,64,65]. However, there are some other applications of electroosmotic flow, like the micromixers developed by Oddy et al [66]. These micromixers are specially designed to rapidly mix fluids at very low Reynolds numbers as in the case of many bioanalytical, microfluidic devices.

Ren et al [67] use electroosmotic flow to displace one solution with another solution in a capillary. It was found that the velocity profile is no longer plug-like due to the existence of induced pressure gradient along the channel.

One of the most interesting applications of electroosmosis is the electroosmotic pump. This application is particularly important to this project because of the electroosmosis pressure generated for this pressure is the driving force for the actuator built in this project. By observing and understanding the many kinds of electroosmotic pumps built and how the pressure varies, a more concrete foundation on how electroosmotic pressure can be used can be obtained.

Arulanandam and Li [68] investigated the electroosmotic pumping in rectangular microchannels numerically. They found that the volumetric flow rate increases with approximately the squared of hydraulic diameter. It also increases with the potential but is limited by the Joule heating effect.

Zeng et al [5] fabricated electroosmotic micropump that is able to generate high pressures by using a porous structure. This pump managed to produce a pressure as high as 23.5 atm and a flow rate as high as 4.8 $\mu\text{l}/\text{min}$. They concluded that the pressure is dependent primarily on the inversed square of the particle radius and the flow rate is independent of the particle radius as long as the porosity and tortuosity values remain the same.

Yao et al [69] presents an electroosmotic pump fabricated by using a commercially available sintered glass frit as a porous medium. The pump was designed with a large cross-sectional area and a short length to achieve the large flow rate. As a result, a pressure as high as 2.5 atm and a flow rate as high as 7 ml/min were obtained. Jiang et al [70] succeeded in utilizing this pump for cooling mechanism of small circuits.

Zeng et al [71] fabricated electroosmotic pumps with polymer frits which are able to generate a pressure of 2 atm and a flow rate of 0.8 ml/min. The low pressure generated is because of the resistance imposed by the frits. By reducing the thickness of the frit, the pressure is increased. Higher flow rate can be achieved by reducing the thickness of frits to minimize their flow and electrical current resistances, increasing the cross-sectional area of the pumps or using better solution. It is presented that when there is zero net flow, the maximum pressure is generated and when there is zero net back pressure, maximum flow rate is obtained.

One of the pumps that produces the highest pressure reported in publications is the one fabricated by Paul et al [6] which was reported to produce pressures in excess of 8000 psi (= 544 atm). It was found that the pressure should be independent of viscosity, proportional to the applied voltage and varies inversely with the square of the bead diameter. It was also found that there is no systematic dependence on the length or diameter of the packed capillaries to the pressure generated. The flow

velocity is found to be independent of bead diameter and that there is a linear relation between the measured pressures and flow rate with the applied voltage.

Another type of electroosmotic pump was built by Chen and Santiago [4], called planar electroosmotic micropump. The pump managed to generate 0.33 atm of pressure or 15 $\mu\text{l}/\text{min}$ of flow rate. However, after some time, the performance of the pump drops due to structural collapse.

Yao et al [72-74] built a porous glass electroosmotic pump using fabricated sintered-glass. This pump is equipped with a catalytic recombination device to remove bubbles generated during the process. It is reported that a pressure of 1.3 atm and a flow rate of 33 ml/min were generated. This pump can remove heat as high as 130 W.

Tripp et al [75] built an electroosmotic pump using a porous polymer monolith. A pressure of 0.38 MPa and a flow rate of up to 0.41 ml/min were obtained. It was reported that one main advantage of using porous polymer monolith is the ease of control of the properties, and hence ease of control of the pump performance.

Lazar and Karger [76] built an electroosmotic pump by bundling shallow microcapillaries in parallel arrangement. By having many capillaries, high flow rates can be obtained, and by using shallow capillaries, high pressures are also obtainable. It is reported that the highest pressure obtained was 80 psi (= 5.5 atm) and the maximum flow rate was 400 nl/min.

Morf et al [77] presented their work on the analysis of the behavior of partial electroosmotic and hydrodynamic flow in complex capillary system by linking six rectangular microchannels in parallel. The configuration gave 0.8648 $\mu\text{l}/\text{min}$ of flow rate or 321.5 Pa of pressure when all six channels were active.

Chen et al [78] developed another electroosmotic pump to be used in capillary column chromatography. The pump shows that pure water is the liquid that can generate the

highest pressure unless some buffer solution is used. Pressure as high as 18 MPa and flow rate as high as 50 $\mu\text{l}/\text{min}$ were reported.

Some other electroosmotic pumps developed include the one developed by McKnight et al [79] where the electrodes are integrated on the device and the one by Liu et al [80] where the electric field is decoupled from the fluidic conduit.

All the electroosmotic pumps reviewed above is summarized in the following table:

Ref	P (atm)	F.R (ml/min)	Electrolyte	Pump Size	Material	Field (V/cm)
4	0.33	15	DI water	38mm long, 1mm wide	Channel 0.9 μm	2100
5	23.5	4.8	DI water	0.53mm diam, 5.4cm long	Silica 3.5 μm	370.37
6	544	N/A	1mM borate	N/A	Silica 0.5 μm	N/A
69	2.5	7	1mM borate	3mm thick	Frit 1 μm pore	666.67
71	2	0.8	DI water	10mm diam, 15mm long	Silica 3 μm	666.67
72-74	1.3	33	1mM Na borate	40mm diam, 5mm thick	Glass frit 2 μm	200
75	3.75	0.41	DO water	13mm diam, 10mm long	Polymer monolith	50
76	5.5	0.4	Peptide mixture	30mm long	Cyl Cap 20 μm id	1000
77	0.0032	0.8648	10mM borax	83mm long	Rect Cap 300 μm x 50 μm	120.48
78	180	0.05	Pure water	150mm long	Silica 2 μm	800

Table 3.1 A summary of the published electroosmotic pumps

It has been mentioned briefly that pressures and flow rates vary with the size of channel, the size of particles, the existence of buffer, etc. The following papers will review the effects of each factor in more detail.

Chen et al [81] built an electroosmotic pump using packed column to perform observations on the effects of varying applied voltage, column internal diameter, solvent material, number of columns, and column length. The pump managed to generate a pressure of 15 MPa and 3.8 $\mu\text{l}/\text{min}$ of flow rate. From, the observation, it was found that the pressure is a linear function of the applied voltage, it is higher with narrower and longer column, and is a function of the properties of the solvent but independent of the pump area. The flow rate is also a linear function of the applied voltage. It is also a function of the properties of the solvent and can be doubled by doubling the area of the pump. It was also reported that Joule heating is a limitation for the design of an electroosmotic micropump.

Reichmuth et al [82] reported their success in increasing pumping efficiency and generated pressure. This is done by using TMAPS as the additive, which is found to be very effective. However it must be noted that there is an optimum point of the increase of performance caused by the addition of the additive. Variation in fabrication can lead to changes in the absolute performance of individual pump but these variations do not affect the relative impact of solute additives on performance. The reason for the increase of performance by the addition of additives is the increase in permittivity of the solution which is needed to enhance the performance of a pump. Zhao and Liao [87] presented an investigation on the thermal effects on electroosmotic pumping of liquid in microchannels. It should be noted that viscosity, specific electric conductivity, zeta potential and the dimensionless dielectric constant are influenced significantly by the temperature. Moreover, a non-uniform temperature distribution with the EOF flow channel also affected the charge density distribution and, thus, affected the electrokinetic force in the capillaries. It was found that with a decrease in Re , the temperature at the channel centerline becomes higher. With the decrease in backpressures, the flow rate increases with increasing Joule heating. On the other hand, with the increase in backpressures, an increase in Joule heating leads to a fluid viscosity gradient, with the fluid viscosity decreasing toward the channel centerline which then leads to a decrease in the cross-sectional mean velocity. It was then concluded that the isothermal assumption and the assumption of constant properties in the conventional theory leads to a substantial over prediction of the electroosmotic pumping performance when Joule heating is significant.

In another paper, Chen et al [84] found that the pressure generated can be increased without affecting the flow rate by having a multi-stage pump.

3.6. Basic reciprocating compressor model

As stated in Chapter 1, the final goal of the project is to build a conceptual electrokinetic compressor. Now, having reviewed the characteristics of the electrokinetic force and how other researchers have been utilizing and tapping them, it is clear that the compressor will be very similar to a positive displacement compressor, more specifically, the reciprocating compressor. This is because of the nature of the electroosmotic force, which is uni-directional. It is therefore necessary to review the basic principles of a reciprocating compressor.

Figure 3.1 A simple reciprocating compressor model [85]

A reciprocating compressor uses a piston to compress the fluid, which is normally in gaseous phase, in the chamber. As the piston compresses the fluid, the pressure increases, while as the piston retracts, the pressure drops. The suction valve is open when the pressure is sufficiently low, while the discharge valve is open when the pressure is sufficiently high. The complete process can be understood more clearly through a P-V diagram in Figure 3.2.

Figure 3.2 An ideal P-V diagram of a reciprocating compressor [85]

From point (a) to (b), the piston moves to compress the air but the valves are still closed because the air pressure is still less than the discharge pressure. The pressure at point (b) is the discharge pressure, so the discharge opens at this point. As the piston moves from point (b) to (c), air is discharged through the discharge valve. At point (c) the piston retracts back and the discharge valve closes. From point (c) to (d), the air expands but the mass is maintained since all the valves are closed. At point (d), the pressure is low enough for the suction valve to open, so from (d) to (a) air is sucked in. At point (a) the piston stops retracting and starts to compress the air again while the valves are closed again.

However, in reality, the P-V diagram will be slightly different from the ideal case because of losses caused by inertia of valves, temperature change, etc. The P-V diagram will be more like shown in Figure 3.3.

Figure 3.3 An actual P-V diagram of a reciprocating compressor [85]

The proposed compressor design is discussed in Chapter 5.

Chapter 4:

GOVERNING EQUATIONS

After discussing the basic theories of electrokinetic effects and fluid motion in Chapter 2 and then continued with literature review on electrokinetic effects with special emphasis on electroosmosis and its applications in Chapter 3, the discussion shall now continue with the presentation on the derivations of the governing equations used to formulate the computer model in this project.

4.1. Electroosmotic flow equations

The derivation for electroosmotic flow equations formulated is for rectangular microchannel with dimensions as shown in Figure 4.1. The derived equations can however be used for any shape of microchannel by making some modifications to the equations.

Figure 4.1 Geometry of the channel

4.1.1. Dynamics of electroosmotic flow

As any other fluid flows, electroosmotic flow can also be described by the famous Navier-Stokes equation [11-13] as presented in Chapter 2:

$$\rho_f \frac{D\vec{V}}{Dt} = -\nabla P + F + \mu_f \nabla^2 \vec{V} \quad (4.1)$$

The assumptions adopted to simplify the equation are:

1. The flow is laminar.
2. The flow is fully-developed ($\frac{D\vec{V}}{Dt} \rightarrow \frac{\partial \vec{V}}{\partial t}$).
3. Velocity ($\vec{V}$) can be represented by $u(x, y, t)$.
4. The only body force is the application of external electric field ($F = E(t)\rho_e(x, y)$).
5. The fluid is incompressible (ρ_f is constant), Newtonian (with constant μ_f).
6. The fluid is a symmetric electrolyte ($\rho_e(x, y) = -2z_v e n_0 \sinh\left(\frac{z_v e}{k_b T} \psi(x, y)\right)$).
7. Pressure gradient is only along the axial direction ($\nabla P \rightarrow \frac{dP(t)}{dz}$).

Inserting all the assumptions above to the general Navier-Stokes equation gives the simplified Navier-Stokes equation:

$$\rho_f \frac{\partial u(x, y, t)}{\partial t} = -\frac{dP(t)}{dz} + E(t)\rho_e(x, y) + \mu_f \left(\frac{\partial^2 u(x, y, t)}{\partial x^2} + \frac{\partial^2 u(x, y, t)}{\partial y^2} \right) \quad (4.2)$$

where:

$$E(t) = E_0 e^{i\alpha t}$$

$$P(t) = P_0 e^{i(\alpha t + \phi)}$$

Introducing the following dimensionless parameters:

$$\begin{aligned} \nu &= \frac{\mu_f}{\rho_f} & \text{Re}_0 &= \frac{D_h U}{\nu} & D_h &= \frac{4ab}{a+b} \\ \bar{u} &= \frac{u}{U} & \bar{t} &= \frac{\mu}{\rho_f D_h^2} t \\ X &= \frac{x}{D_h} & Y &= \frac{y}{D_h} & Z &= \frac{z}{D_h \text{Re}_0} \\ \bar{P}_0 &= \frac{P_0}{\rho_f U^2} & \bar{E}_0 &= \frac{E_0 D_h \text{Re}_0}{\zeta} & \Psi &= \frac{z_v e \psi}{k_b T} & \bar{G} &= \frac{2z_v e n_0 \zeta}{\rho U^2} \end{aligned}$$

where:

- ν = kinematics fluid viscosity
- μ_f = dynamic fluid viscosity
- ρ_f = fluid density
- Re_0 = reference Reynolds number
- D_h = hydraulic diameter of the channel
- U = reference velocity, taken to be 1 mm/s
- u = velocity of fluid
- t = time
- P_0 = magnitude of pressure
- E_0 = applied electric field strength
- ζ = zeta potential
- z_v = valence
- e = elementary charge
- ψ = local electrical double layer potential
- k_b = Boltzmann constant

T = absolute temperature

n_0 = bulk concentration of the ion

ω = angular frequency

ϕ = phase lag

i = unit of imaginary number

Substituting the dimensionless parameters to eq. (4.2) gives:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial X^2} + \frac{\partial^2 \bar{u}}{\partial Y^2} - \frac{d\bar{P}_0}{dZ} e^{i\left(\frac{\omega D_h^2 \bar{t}}{\nu} + \phi\right)} - \overline{GE_0} e^{\frac{i\omega D_h^2 \bar{t}}{\nu}} \sinh \Psi(X, Y) \quad (4.3)$$

The boundary conditions are:

$$\begin{aligned} \left. \frac{\partial \bar{u}}{\partial X} \right|_{X=0} &= 0 & \left. \frac{\partial \bar{u}}{\partial Y} \right|_{Y=0} &= 0 \\ \left. \bar{u} \right|_{X=b/D_h} &= 0 & \left. \bar{u} \right|_{Y=a/D_h} &= 0 \end{aligned}$$

The initial condition is:

$$\left. \bar{u} \right|_{\bar{t}=0} = 0$$

Solving eq. (4.3) with its boundary and initial conditions by using Green's function method gives the solution:

$$\bar{u}(X, Y, \bar{t}) = - \int_{\tau=0}^{\bar{t}} d\tau \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} G(X, Y, \bar{t} | X', Y', \tau) \left(\frac{d\bar{P}_0}{dZ} e^{i\left(\frac{\omega D_h^2 \tau}{\nu} + \phi\right)} + \overline{GE_0} e^{\frac{i\omega D_h^2 \tau}{\nu}} \sinh \Psi(X', Y') \right) dX' dY' \quad (4.4)$$

where $G(X, Y, \bar{t} | X', Y', \tau)$ is the Green's function.

$G(X, Y, \bar{t} | X', Y', \tau)$ can be obtained by solving the corresponding homogeneous equation of eq. (4.3.) as follow:

$$\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} = \frac{\partial \phi}{\partial \bar{t}} \quad (4.5)$$

The boundary conditions are:

$$\begin{aligned} \left. \frac{\partial \phi}{\partial X} \right|_{X=0} &= 0 & \left. \frac{\partial \phi}{\partial Y} \right|_{Y=0} &= 0 \\ \phi|_{X=b/D_h} &= 0 & \phi|_{Y=a/D_h} &= 0 \end{aligned}$$

The initial condition is:

$$\phi(X', Y', \bar{t}) \Big|_{\bar{t}=0} = R(X', Y')$$

Applying the separation of variable method, the solution of eq. (4.5) is:

$$\phi = A(X)B(Y)C(\bar{t})$$

Substituting this solution to eq. (4.5) gives:

$$\begin{aligned} BC \frac{d^2 A}{dX^2} + AC \frac{d^2 B}{dY^2} &= AB \frac{dC}{d\bar{t}} \\ \rightarrow \frac{1}{A} \frac{d^2 A}{dX^2} + \frac{1}{B} \frac{d^2 B}{dY^2} &= \frac{1}{C} \frac{dC}{d\bar{t}} \\ \rightarrow \frac{A''}{A} + \frac{B''}{B} &= \frac{C'}{C} \end{aligned} \quad (4.6)$$

Eq. (4.6) can be satisfied if $\frac{A''}{A} = -\alpha^2$, $\frac{B''}{B} = -\beta^2$, $\frac{C'}{C} = -\alpha^2 - \beta^2$ where both α and

β are constants, and hence the solutions are:

$$\begin{aligned} A'' + \alpha^2 A &= 0 \\ \rightarrow A &= C_1 \cos \alpha X + C_2 \sin \alpha X \end{aligned}$$

Using the boundary conditions:

$$\begin{aligned} BC \frac{dA}{dX} \Big|_{X=0} &= 0 \rightarrow C_2 = 0 \\ BCA \Big|_{X=b/D_h} &= 0 \rightarrow C_1 \cos \alpha \left(\frac{b}{D_h} \right) = 0 \rightarrow \alpha_m = \left(\frac{\pi D_h}{2b} \right) (2m-1) \end{aligned}$$

So, the solution for A(X) is:

$$A(X) = \sum_{m=1}^{\infty} C_m \cos \left(\frac{2m-1}{2} \frac{\pi D_h}{b} X \right) \quad (4.7)$$

In the same manner, the solution for B(Y) is found as follow:

$$B'' + \beta^2 B = 0$$

$$\rightarrow B = C_3 \cos \beta Y + C_4 \sin \beta Y$$

Using the boundary conditions:

$$AC \frac{dB}{dY} \Big|_{Y=0} = 0 \rightarrow C_4 = 0$$

$$ACB \Big|_{Y=a/D_h} = 0 \rightarrow C_3 \cos \beta \left(\frac{a}{D_h} \right) = 0 \rightarrow \beta_n = \left(\frac{\pi D_h}{2a} \right) (2n-1)$$

So, the solution for B(Y) is:

$$B(Y) = \sum_{n=1}^{\infty} C_n \cos \left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y \right) \quad (4.8)$$

And lastly, the solution for $C(\bar{t})$ is:

$$\frac{dC}{d\bar{t}} = -(\alpha^2 + \beta^2)C \rightarrow \int \frac{dC}{C} = \int -(\alpha^2 + \beta^2) d\bar{t}$$

$$\rightarrow \ln C = -(\alpha^2 + \beta^2)\bar{t}$$

$$C(\bar{t}) = e^{-(\alpha^2 + \beta^2)\bar{t}} \quad (4.9)$$

Combining (4.7), (4.8) and (4.9) gives:

$$\phi(X, Y, \bar{t}) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} e^{-(\alpha_m^2 + \beta_n^2)\bar{t}} \left[\cos \left(\frac{2m-1}{2} \frac{\pi D_h}{b} X \right) \right] \left[\cos \left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y \right) \right] \quad (4.10)$$

Using the initial condition:

$$\phi(X', Y', 0) = R(X', Y') = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \left[\cos \left(\frac{2m-1}{2} \frac{\pi D_h}{b} X' \right) \right] \left[\cos \left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y' \right) \right]$$

Introducing $Q_m = \sum_{n=1}^{\infty} C_{mn} \left[\cos \left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y' \right) \right]$ gives:

$$R(X', Y') = \sum_{m=1}^{\infty} Q_m \cos \left(\frac{2m-1}{2} \frac{\pi D_h}{b} X' \right)$$

which is a Fourier series and can be solved using Fourier series method:

$$Q_m = \frac{2D_h}{b} \int_0^{b/D_h} R(X', Y') \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) dX' \quad (4.11)$$

But, $Q_m = \sum_{n=1}^{\infty} C_{mn} \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) \right]$ is also a Fourier series, and hence:

$$C_{mn} = \frac{2D_h}{a} \int_0^{a/D_h} Q_m \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) dY' \quad (4.12)$$

Combining (4.11) and (4.12) gives:

$$C_{mn} = \frac{4D_h^2}{ab} \int_0^{a/D_h} \int_0^{b/D_h} R(X', Y') \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) dX' dY'$$

Finally, substituting C_{mn} to eq. (4.10), the solution is obtained:

$$\phi(X, Y, \bar{t}) = \frac{4D_h^2}{ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\int_0^{a/D_h} \int_0^{b/D_h} R(X', Y') \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) dX' dY' \right) \times e^{-(\alpha_m^2 + \beta_n^2) \bar{t}} \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \quad (4.13)$$

Using eq. (4.13) in Green's formulation gives:

$$\phi(X, Y, \bar{t}) = \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} G(X, Y, \bar{t} | X', Y', \tau) \Big|_{\tau=0} R(X', Y') dX' dY' \quad (4.14)$$

Comparing eq. (4.13) and eq. (4.14), Green's function is obtained:

$$G(X, Y, \bar{t} | X', Y', \tau) = \frac{4D_h^2}{ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(e^{-\left(\frac{D_h^2 \pi^2}{4} \left(\frac{(2m-1)^2}{b^2} + \frac{(2n-1)^2}{a^2} \right) (\bar{t} - \tau) \right)} \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) \right) \times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \quad (4.15)$$

Finally, substituting eq. (4.15) to eq. (4.4):

$$\begin{aligned} \bar{u}(X, Y, \bar{t}) = & - \int_{\tau=0}^{\bar{t}} d\tau \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} \frac{4D_h^2}{ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} e^{-\left(\frac{D_h^2 \pi^2}{4} \left(\frac{(2m-1)^2}{b^2} + \frac{(2n-1)^2}{a^2} \right) (\bar{t}-\tau) \right)} \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) \\ & \times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \times \left(\frac{d\bar{P}_0}{dZ} e^{i\left(\frac{\omega D_h^2 \tau}{\nu} + \phi\right)} + \overline{GE_0} e^{\frac{i\omega D_h^2 \tau}{\nu}} \sinh \Psi(X', Y') \right) dX' dY' \end{aligned} \quad (4.16)$$

Eq. (4.16) consists of two terms, namely the pressure and the electrical terms and they should be treated separately in order to solve the equation.

$$\bar{u}(X, Y, \bar{t}) = \bar{u}_P(X, Y, \bar{t}) + \bar{u}_E(X, Y, \bar{t})$$

where:

$$\begin{aligned} \bar{u}_P(X, Y, \bar{t}) = & - \int_{\tau=0}^{\bar{t}} d\tau \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} \frac{4D_h^2}{ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} e^{-\left(\frac{D_h^2 \pi^2}{4} \left(\frac{(2m-1)^2}{b^2} + \frac{(2n-1)^2}{a^2} \right) (\bar{t}-\tau) \right)} \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) \\ & \times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \times \frac{d\bar{P}_0}{dZ} e^{i\left(\frac{\omega D_h^2 \tau}{\nu} + \phi\right)} dX' dY' \\ \bar{u}_E(X, Y, \bar{t}) = & - \int_{\tau=0}^{\bar{t}} d\tau \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} \frac{4D_h^2}{ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} e^{-\left(\frac{D_h^2 \pi^2}{4} \left(\frac{(2m-1)^2}{b^2} + \frac{(2n-1)^2}{a^2} \right) (\bar{t}-\tau) \right)} \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) \\ & \times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \times \overline{GE_0} e^{\frac{i\omega D_h^2 \tau}{\nu}} \sinh \Psi(X', Y') dX' dY' \end{aligned}$$

By introducing $R_{mn} = \pi^2 \left(\frac{(2m-1)^2}{b^2} + \frac{(2n-1)^2}{a^2} \right)$ the time integration inside

$\bar{u}_P(X, Y, \bar{t})$ can be solved as follow:

$$\begin{aligned} \int_{\tau=0}^{\bar{t}} e^{-\left(\frac{D_h^2}{4} R_{mn} (\bar{t}-\tau)\right)} \times \frac{d\bar{P}_0}{dZ} e^{i\left(\frac{\omega D_h^2 \tau}{\nu} + \phi\right)} d\tau &= \frac{d\bar{P}_0}{dZ} e^{-\left(\frac{D_h^2}{4} R_{mn} \bar{t} + i\phi\right)} \int_{\tau=0}^{\bar{t}} e^{\left(\frac{D_h^2}{4} R_{mn} \tau + i\frac{\omega D_h^2 \tau}{\nu}\right)} d\tau \\ &= \frac{d\bar{P}_0}{dZ} e^{-\left(\frac{D_h^2}{4} R_{mn} \bar{t} + i\phi\right)} \frac{e^{\left(\frac{D_h^2}{4} R_{mn} \bar{t} + i\frac{\omega D_h^2 \bar{t}}{\nu}\right)} - 1}{\frac{D_h^2}{4} R_{mn} + i\frac{\omega D_h^2}{\nu}} \\ &= \frac{d\bar{P}_0}{dZ} \frac{e^{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})} - e^{-\left(\frac{D_h^2}{4} R_{mn} \bar{t} + i\phi\right)}}{\frac{D_h^2}{4} R_{mn} + i\frac{\omega D_h^2}{\nu}} = \frac{d\bar{P}_0}{dZ} \frac{4}{D_h^2} \frac{e^{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})} - e^{-\left(\frac{D_h^2}{4} R_{mn} \bar{t} + i\phi\right)}}{R_{mn} + i\frac{4\omega}{\nu}} \end{aligned}$$

And the double integration inside $\bar{u}_P(X, Y, \bar{t})$ can be solved as follow:

$$\begin{aligned}
 & \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) dX' dY' \\
 &= \int_{X'=0}^{b/D_h} \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \left[\frac{\sin\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right)}{\frac{2n-1}{2} \frac{\pi D_h}{a}} \right] \Bigg|_0^{a/D_h} dX' \\
 &= \left(\frac{\sin\left(\frac{2n-1}{2} \pi\right)}{\frac{2n-1}{2} \frac{\pi D_h}{a}} \right) \left(\frac{\sin\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right)}{\frac{2m-1}{2} \frac{\pi D_h}{b}} \right) \Bigg|_0^{b/D_h} \\
 &= \left(\frac{\sin\left(\frac{2n-1}{2} \pi\right)}{\frac{2n-1}{2} \frac{\pi D_h}{a}} \right) \left(\frac{\sin\left(\frac{2m-1}{2} \pi\right)}{\frac{2m-1}{2} \frac{\pi D_h}{b}} \right) \\
 &= \frac{4ab}{\pi^2 D_h^2 (2m-1)(2n-1)} (-1)^n (-1)^m
 \end{aligned}$$

Hence,

$$\begin{aligned}
 \bar{u}_P(X, Y, \bar{t}) &= -\frac{64}{\pi^2 D_h^2} \frac{d\bar{P}_0}{dZ} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{e^{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})} - e^{(-\frac{D_h^2}{4} R_{mn} \bar{t} + i\phi)}}{R_{mn} + i \frac{4\omega}{\nu}} \frac{(-1)^{m+n}}{(2m-1)(2n-1)} \\
 &\times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right]
 \end{aligned}$$

And the time integration in $\bar{u}_E(X, Y, \bar{t})$ can be solved as follow:

$$\begin{aligned}
 & \int_{\tau=0}^{\bar{t}} e^{-\left(\frac{D_h^2}{4} R_{mn} (\bar{t}-\tau)\right)} \times \overline{GE_0} e^{i \frac{\omega D_h^2 \tau}{\nu}} d\tau = \overline{GE_0} e^{(-\frac{D_h^2}{4} R_{mn} \bar{t})} \int_{\tau=0}^{\bar{t}} e^{\left(\frac{D_h^2}{4} R_{mn} \tau + i \frac{\omega D_h^2 \tau}{\nu}\right)} d\tau \\
 &= \overline{GE_0} e^{(-\frac{D_h^2}{4} R_{mn} \bar{t})} \frac{e^{\left(\frac{D_h^2}{4} R_{mn} \bar{t} + i \frac{\omega D_h^2 \bar{t}}{\nu}\right)}}{\frac{D_h^2}{4} R_{mn} + i \frac{\omega D_h^2}{\nu}} \Bigg|_0^{\bar{t}} = \overline{GE_0} \frac{e^{(-\frac{D_h^2}{4} R_{mn} \bar{t})}}{\frac{D_h^2}{4} R_{mn} + i \frac{\omega D_h^2}{\nu}} \left(e^{\left(\frac{D_h^2}{4} R_{mn} \bar{t} + i \frac{\omega D_h^2 \bar{t}}{\nu}\right)} - 1 \right) \\
 &= \overline{GE_0} \frac{e^{i \frac{\omega D_h^2 \bar{t}}{\nu}} - e^{(-\frac{D_h^2}{4} R_{mn} \bar{t})}}{\frac{D_h^2}{4} R_{mn} + i \frac{\omega D_h^2}{\nu}} = \frac{4\overline{GE_0}}{D_h^2} \frac{e^{i \frac{\omega D_h^2 \bar{t}}{\nu}} - e^{(-\frac{D_h^2}{4} R_{mn} \bar{t})}}{R_{mn} + i \frac{4\omega}{\nu}}
 \end{aligned}$$

The double integration in $\bar{u}_E(X, Y, \bar{t})$ can not be solved yet because of the unknown variable $\Psi(X', Y')$, so it is better to leave it unsolved for the time being.

Therefore:

$$\bar{u}_E(X, Y, \bar{t}) = - \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} \frac{16}{ab} \bar{G} E_0 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{e^{\left(\frac{i\omega D_h^2 \bar{t}}{v}\right)} - e^{\left(\frac{-D_h^2}{4} R_{mn} \bar{t}\right)}}{R_{mn} + i \frac{4\omega}{v}} \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) \\ \times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \sinh \Psi(X', Y') dX' dY'$$

So finally, $\bar{u}(X, Y, \bar{t})$ is:

$$\bar{u}(X, Y, \bar{t}) = - \frac{64}{\pi^2 D_h^2} \frac{d\bar{P}_0}{dZ} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{e^{\left(\frac{i(\phi + \frac{\omega D_h^2 \bar{t}}{v})}{v}\right)} - e^{\left(\frac{-D_h^2}{4} R_{mn} \bar{t} + i\phi\right)}}{R_{mn} + i \frac{4\omega}{v}} \frac{(-1)^{m+n}}{(2m-1)(2n-1)} \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \\ - \frac{16}{ab} \bar{G} E_0 \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{e^{\left(\frac{i\omega D_h^2 \bar{t}}{v}\right)} - e^{\left(\frac{-D_h^2}{4} R_{mn} \bar{t}\right)}}{R_{mn} + i \frac{4\omega}{v}} \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) \\ \times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \sinh \Psi(X', Y') dX' dY' \quad (4.17)$$

4.1.2. Electrical potential distribution

The EDL potential distribution can be described by the Poisson-Boltzmann equation:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{2z_v e n_0}{\epsilon_r \epsilon_0} \sinh\left(\frac{z_v e \psi}{k_b T}\right) \quad (4.18)$$

With the following boundary conditions:

$$\left. \frac{\partial \psi}{\partial x} \right|_{x=0} = 0 \quad \left. \frac{\partial \psi}{\partial y} \right|_{y=0} = 0 \\ \psi|_{x=b} = \zeta \quad \psi|_{y=a} = \zeta$$

Substituting the dimensionless parameters introduced in 4.1.1 gives the dimensionless

form of eq. (4.18):

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = \frac{2z_v^2 e^2 n_0}{\epsilon_r \epsilon_0 k_b T} D_h^2 \sinh \Psi \quad (4.19)$$

But $\left(\frac{e^2 \sum n_i z_i^2}{\epsilon k_b T} \right)^{1/2}$ is the Debye-Huckel parameter, κ , and for symmetric electrolyte

$$\kappa = \frac{e^2 2 z_v^2 n_0}{\epsilon_r \epsilon_0 k_b T}. \text{ Introducing } K = \kappa D_h \text{ for further simplification and substituting to eq.}$$

(4.19) gives:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = K^2 \sinh \Psi \quad (4.20)$$

When zeta potential is small, Debye-Huckel linear approximation ($\sinh \Psi \approx \Psi$) can be adopted so that eq. (4.20) can be modified to:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = K^2 \Psi \quad (4.21)$$

And by introducing dimensionless zeta potential $\bar{\zeta} = \frac{z_v e \zeta}{k_b T}$, the boundary conditions

for eq. (4.21) are:

$$\begin{aligned} \left. \frac{\partial \Psi}{\partial X} \right|_{X=0} &= 0 & \left. \frac{\partial \Psi}{\partial Y} \right|_{Y=0} &= 0 \\ \Psi|_{X=b/D_h} &= \bar{\zeta} & \Psi|_{Y=a/D_h} &= \bar{\zeta} \end{aligned}$$

Eq. (4.21) can be solved by first modifying the boundary conditions to have only one non-homogeneous term. This is done by setting $\Psi = \Psi_1 + \Psi_2$ with the following boundary conditions:

$$\begin{aligned} \left. \frac{\partial \Psi_1}{\partial X} \right|_{X=0} &= 0 & \left. \frac{\partial \Psi_2}{\partial X} \right|_{X=0} &= 0 \\ \Psi_1|_{X=b/D_h} &= 0 & \Psi_2|_{X=b/D_h} &= \bar{\zeta} \\ \left. \frac{\partial \Psi_1}{\partial Y} \right|_{Y=0} &= 0 & \left. \frac{\partial \Psi_2}{\partial Y} \right|_{Y=0} &= 0 \\ \Psi_1|_{Y=a/D_h} &= \bar{\zeta} & \Psi_2|_{Y=a/D_h} &= 0 \end{aligned}$$

Ψ_1 and Ψ_2 can be solved separately by using the separation of variable method.

To solve Ψ_1 , let $\Psi_1 = F(X)G(Y)$, and from eq. (4.21):

$$\frac{\partial^2 \Psi_1}{\partial X^2} + \frac{\partial^2 \Psi_1}{\partial Y^2} = K^2 \Psi_1$$

$$F''G + G''F = K^2 FG$$

$$\frac{F''}{F} + \frac{G''}{G} = K^2$$

K^2 is a constant and hence $\frac{F''}{F}$ and $\frac{G''}{G}$ are also constants. Letting $\frac{F''}{F} = -\gamma^2$ and

$$\frac{G''}{G} = K^2 + \gamma^2 \text{ results in:}$$

$$F'' + \gamma^2 F = 0$$

$$\rightarrow F = D_1 \cos \gamma X + D_2 \sin \gamma X$$

$$G'' - (K^2 + \gamma^2)G = 0$$

$$\rightarrow G = D_3 \cosh(K^2 + \gamma^2)^{1/2} Y + D_4 \sinh(K^2 + \gamma^2)^{1/2} Y$$

$$\Psi_1 = (D_1 \cos \gamma X + D_2 \sin \gamma X) \left(D_3 \cosh(K^2 + \gamma^2)^{1/2} Y + D_4 \sinh(K^2 + \gamma^2)^{1/2} Y \right)$$

Applying the boundary conditions gives:

$$\left. \frac{\partial \Psi_1}{\partial X} \right|_{X=0} = (D_2 \gamma) \left(D_3 \cosh(K^2 + \gamma^2)^{1/2} Y + D_4 \sinh(K^2 + \gamma^2)^{1/2} Y \right) = 0$$

$$\rightarrow D_2 = 0$$

$$\Psi_1|_{X=b/D_h} = \left(D_1 \cos \gamma \frac{b}{D_h} \right) \left(D_3 \cosh(K^2 + \gamma^2)^{1/2} Y + D_4 \sinh(K^2 + \gamma^2)^{1/2} Y \right) = 0$$

$$\rightarrow \gamma_m \frac{b}{D_h} = (2m-1) \frac{\pi}{2} \rightarrow \gamma_m = \frac{(2m-1) \pi D_h}{2b}$$

$$\Psi_1|_{X=b/D_h} = \left(D_1 \cos \gamma \frac{b}{D_h} \right) \left(D_3 \cosh(K^2 + \gamma^2)^{1/2} Y + D_4 \sinh(K^2 + \gamma^2)^{1/2} Y \right) = 0$$

$$\rightarrow \gamma_m \frac{b}{D_h} = (2m-1) \frac{\pi}{2} \rightarrow \gamma_m = \frac{(2m-1) \pi D_h}{2b}$$

$$\left. \frac{\partial \Psi_1}{\partial Y} \right|_{Y=0} = \sum_{m=1}^{\infty} D_m \cos \left(\frac{(2m-1) \pi D_h}{2b} X \right) \left(D_4 (K^2 + \left(\frac{(2m-1) \pi D_h}{2b} \right)^2)^{1/2} \right) = 0$$

$$\rightarrow D_4 = 0$$

$$\begin{aligned}
 \Psi_1|_{Y=a/D_h} &= \sum_{m=1}^{\infty} D_m \cos\left(\frac{(2m-1)\pi D_h}{2b} X\right) \cosh\left(\left(K^2 + \left(\frac{(2m-1)\pi D_h}{2b}\right)^2\right)^{1/2} \frac{a}{D_h}\right) = \bar{\zeta} \\
 &\rightarrow D_m \cosh\left(\left(K^2 + \left(\frac{(2m-1)\pi D_h}{2b}\right)^2\right)^{1/2} \frac{a}{D_h}\right) = \frac{2D_h}{b} \int_0^{b/D_h} \bar{\zeta} \cos\left(\frac{(2m-1)\pi D_h}{2b} X\right) dX \\
 &\rightarrow D_m = \frac{2D_h \bar{\zeta} \left(\frac{2b}{(2m-1)\pi D_h}\right) \sin\left(\frac{(2m-1)\pi D_h}{2b} X\right) \Big|_0^{b/D_h}}{b \cosh\left(\left(K^2 + \left(\frac{(2m-1)\pi D_h}{2b}\right)^2\right)^{1/2} \frac{a}{D_h}\right)} \\
 &\rightarrow D_m = \frac{4\bar{\zeta}}{(2m-1)\pi} \frac{(-1)^{m+1}}{\cosh\left(\left(K^2 + \left(\frac{(2m-1)\pi D_h}{2b}\right)^2\right)^{1/2} \frac{a}{D_h}\right)}
 \end{aligned}$$

And finally:

$$\Psi_1 = \frac{4\bar{\zeta}}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)} \frac{\cos\left(\frac{(2m-1)\pi D_h}{2b} X\right) \cosh\left(\left(K^2 + \left(\frac{(2m-1)\pi D_h}{2b}\right)^2\right)^{1/2} Y\right)}{\cosh\left(\left(K^2 + \left(\frac{(2m-1)\pi D_h}{2b}\right)^2\right)^{1/2} \frac{a}{D_h}\right)}$$

To solve Ψ_2 , let $\Psi_2 = H(X)I(Y)$, and from eq. (4.21):

$$\begin{aligned}
 \frac{\partial^2 \Psi_2}{\partial X^2} + \frac{\partial^2 \Psi_2}{\partial Y^2} &= K^2 \Psi_2 \\
 H''I + I''H &= K^2 HI \Rightarrow \frac{H''}{H} + \frac{I''}{I} = K^2
 \end{aligned}$$

K^2 is a constant and hence $\frac{H''}{H}$ and $\frac{I''}{I}$ are also constants.

Letting $\frac{I''}{I} = -\lambda^2$ and $\frac{H''}{H} = K^2 + \lambda^2$ results in:

$$\begin{aligned}
 I'' + \lambda^2 I &= 0 \Rightarrow I = E_1 \cos \lambda Y + E_2 \sin \lambda Y \\
 H'' - (K^2 + \lambda^2)H &= 0 \Rightarrow H = E_3 \cosh(K^2 + \lambda^2)^{1/2} X + E_4 \sinh(K^2 + \lambda^2)^{1/2} X \\
 \Psi_2 &= (E_1 \cos \lambda Y + E_2 \sin \lambda Y) \left(E_3 \cosh(K^2 + \lambda^2)^{1/2} X + E_4 \sinh(K^2 + \lambda^2)^{1/2} X \right)
 \end{aligned}$$

Applying the boundary conditions:

$$\left. \frac{\partial \Psi_2}{\partial Y} \right|_{Y=0} = (E_2 \lambda) \left(E_3 \cosh(K^2 + \lambda^2)^{1/2} X + E_4 \sinh(K^2 + \lambda^2)^{1/2} X \right) = 0$$

$$\rightarrow E_2 = 0$$

$$\Psi_2|_{Y=a/D_h} = \left(E_1 \cos \lambda \frac{a}{D_h} \right) \left(E_3 \cosh(K^2 + \lambda^2)^{1/2} X + E_4 \sinh(K^2 + \lambda^2)^{1/2} X \right) = 0$$

$$\rightarrow \lambda_n \frac{a}{D_h} = (2n-1) \frac{\pi}{2} \rightarrow \lambda_n = \frac{(2n-1) \pi D_h}{2a}$$

$$\left. \frac{\partial \Psi_2}{\partial X} \right|_{X=0} = \sum_{n=1}^{\infty} E_n \cos \left(\frac{(2n-1) \pi D_h}{2a} Y \right) \left(E_4 (K^2 + \left(\frac{(2n-1) \pi D_h}{2a} \right)^2)^{1/2} \right) = 0$$

$$\rightarrow E_4 = 0$$

$$\Psi_2|_{X=b/D_h} = \sum_{n=1}^{\infty} E_n \cos \left(\frac{(2n-1) \pi D_h}{2a} Y \right) \cosh \left((K^2 + \left(\frac{(2n-1) \pi D_h}{2a} \right)^2)^{1/2} \frac{b}{D_h} \right) = \bar{\zeta}$$

$$\rightarrow E_n \cosh \left((K^2 + \left(\frac{(2n-1) \pi D_h}{2a} \right)^2)^{1/2} \frac{b}{D_h} \right) = \frac{2D_h}{a} \int_0^{a/D_h} \bar{\zeta} \cos \left(\frac{(2n-1) \pi D_h}{2a} Y \right) dY$$

$$\rightarrow E_n = \frac{2D_h \bar{\zeta} \left(\frac{2a}{(2n-1)\pi D_h} \right) \sin \left(\frac{(2n-1) \pi D_h}{2a} X \right) \Big|_0^{b/D_h}}{a \cosh \left((K^2 + \left(\frac{(2n-1) \pi D_h}{2a} \right)^2)^{1/2} \frac{b}{D_h} \right)}$$

$$\rightarrow E_n = \frac{4\bar{\zeta}}{(2n-1)\pi} \frac{(-1)^{n+1}}{\cosh \left((K^2 + \left(\frac{(2n-1) \pi D_h}{2a} \right)^2)^{1/2} \frac{b}{D_h} \right)}$$

And finally:

$$\Psi_2 = \frac{4\bar{\zeta}}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)} \frac{\cos \left(\frac{(2n-1) \pi D_h}{2a} Y \right) \cosh \left((K^2 + \left(\frac{(2n-1) \pi D_h}{2a} \right)^2)^{1/2} X \right)}{\cosh \left((K^2 + \left(\frac{(2n-1) \pi D_h}{2a} \right)^2)^{1/2} \frac{b}{D_h} \right)}$$

The closed equation for electrical potential Ψ is:

$$\begin{aligned} \Psi = & \frac{4\bar{\zeta}}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)} \frac{\cos\left(\frac{(2m-1)\pi D_h}{2b} X\right) \cosh\left(K^2 + \left(\frac{(2m-1)\pi D_h}{2b}\right)^2 Y\right)}{\cosh\left(K^2 + \left(\frac{(2m-1)\pi D_h}{2b}\right)^2 \frac{a}{D_h}\right)} + \\ & \frac{4\bar{\zeta}}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)} \frac{\cos\left(\frac{(2n-1)\pi D_h}{2a} Y\right) \cosh\left(K^2 + \left(\frac{(2n-1)\pi D_h}{2a}\right)^2 X\right)}{\cosh\left(K^2 + \left(\frac{(2n-1)\pi D_h}{2a}\right)^2 \frac{b}{D_h}\right)} \end{aligned} \quad (4.22)$$

4.1.3. Velocity equation of electroosmotic flow

When EDL is thin, eq. (4.17) can be simplified to be:

$$\begin{aligned} \bar{u}(X, Y, \bar{t}) = & -\frac{64}{\pi^2 D_h^2} \frac{d\bar{P}_0}{dZ} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{e^{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})} - e^{(-\frac{D_h^2}{4} R_{mn} \bar{t} + i\phi)}}{R_{mn} + i \frac{4\omega}{\nu}} \frac{(-1)^{m+n}}{(2m-1)(2n-1)} \\ & \times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \\ & - \frac{16}{ab} \bar{G} E_0 \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{e^{i(\frac{\omega D_h^2 \bar{t}}{\nu})} - e^{(-\frac{D_h^2}{4} R_{mn} \bar{t})}}{R_{mn} + i \frac{4\omega}{\nu}} \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) \\ & \times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \Psi(X', Y') dX' dY' \end{aligned} \quad (4.23)$$

Substituting eq. (4.22) to eq. (4.23) gives:

$$\begin{aligned}
 \bar{u}(X, Y, \bar{t}) = & -\frac{64}{\pi^2 D_h^2} \frac{d\bar{P}_0}{dZ} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{e^{\frac{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})}{R_{mn} + i \frac{4\omega}{\nu}}} - e^{\frac{(-\frac{D_h^2}{4} R_{mn} \bar{t} + i\phi)}}{R_{mn} + i \frac{4\omega}{\nu}} \frac{(-1)^{m+n}}{(2m-1)(2n-1)} \\
 & \times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \\
 & - \frac{16}{ab} \bar{G} E_0 \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{e^{\frac{i(\frac{\omega D_h^2 \bar{t}}{\nu})}{R_{mn} + i \frac{4\omega}{\nu}}} - e^{\frac{(-\frac{D_h^2}{4} R_{mn} \bar{t})}}{R_{mn} + i \frac{4\omega}{\nu}} \cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X'\right) \cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y'\right) \\
 & \times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \\
 & \times \left(\frac{4\bar{\zeta}}{\pi} \sum_{p=1}^{\infty} \frac{(-1)^{p+1}}{(2p-1)} \frac{\cos\left(\frac{(2p-1)}{2} \frac{\pi D_h}{b} X'\right) \cosh\left(K^2 + \left(\frac{(2p-1)}{2} \frac{\pi D_h}{b}\right)^2 Y'\right)}{\cosh\left(K^2 + \left(\frac{(2p-1)}{2} \frac{\pi D_h}{b}\right)^2 \frac{a}{D_h}\right)} + \right. \\
 & \left. \frac{4\bar{\zeta}}{\pi} \sum_{q=1}^{\infty} \frac{(-1)^{q+1}}{(2q-1)} \frac{\cos\left(\frac{(2q-1)}{2} \frac{\pi D_h}{a} Y'\right) \cosh\left(K^2 + \left(\frac{(2q-1)}{2} \frac{\pi D_h}{a}\right)^2 X'\right)}{\cosh\left(K^2 + \left(\frac{(2q-1)}{2} \frac{\pi D_h}{a}\right)^2 \frac{b}{D_h}\right)} \right) dX' dY'
 \end{aligned} \tag{4.24}$$

The double integration in eq. (4.24) can be solved as follow:

Introducing some variables to simplify the equation:

$$\begin{aligned}
 \alpha_m &= \frac{2m-1}{2} \frac{\pi D_h}{b} & \alpha_p &= \frac{2p-1}{2} \frac{\pi D_h}{b} & \delta_p &= \left(K^2 + \left(\frac{(2p-1)}{2} \frac{\pi D_h}{b}\right)^2\right)^{1/2} \\
 \beta_n &= \frac{2n-1}{2} \frac{\pi D_h}{a} & \beta_q &= \frac{2q-1}{2} \frac{\pi D_h}{a} & \varepsilon_q &= \left(K^2 + \left(\frac{(2q-1)}{2} \frac{\pi D_h}{a}\right)^2\right)^{1/2}
 \end{aligned}$$

$$\begin{aligned}
 \bar{u}(X, Y, \bar{t}) = & -\frac{64}{\pi^2 D_h^2} \frac{d\bar{P}_0}{dZ} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{e^{\frac{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})}{R_{mn} + i \frac{4\omega}{\nu}}} - e^{\frac{(-\frac{D_h^2}{4} R_{mn} \bar{t} + i\phi)}}{R_{mn} + i \frac{4\omega}{\nu}} \frac{(-1)^{m+n}}{(2m-1)(2n-1)} \\
 & \times \left[\cos\left(\frac{2m-1}{2} \frac{\pi D_h}{b} X\right) \right] \left[\cos\left(\frac{2n-1}{2} \frac{\pi D_h}{a} Y\right) \right] \\
 & - \frac{16}{ab} \frac{b/D_h}{G E_0} \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{e^{\frac{i(\omega D_h^2 \bar{t})}{R_{mn} + i \frac{4\omega}{\nu}}} - e^{\frac{(-\frac{D_h^2}{4} R_{mn} \bar{t})}}{R_{mn} + i \frac{4\omega}{\nu}} \cos(\alpha_m X') \cos(\beta_n Y') [\cos(\alpha_m X)] [\cos(\beta_n Y)] \\
 & \times \left(\frac{4\bar{\zeta}}{\pi} \sum_{p=1}^{\infty} \frac{(-1)^{p+1}}{(2p-1)} \frac{\cos(\alpha_p X') \cosh(\delta_p Y')}{\cosh\left(\delta_p \frac{a}{D_h}\right)} + \frac{4\bar{\zeta}}{\pi} \sum_{q=1}^{\infty} \frac{(-1)^{q+1}}{(2q-1)} \frac{\cos(\beta_q Y') \cosh(\varepsilon_q X')}{\cosh\left(\varepsilon_q \frac{b}{D_h}\right)} \right) dX' dY'
 \end{aligned} \tag{4.25}$$

The double integration in eq. (4.25) consists of two terms which are solved separately:

$$\begin{aligned}
 Q1 = & \frac{4\bar{\zeta}}{\pi} \sum_{p=1}^{\infty} \frac{(-1)^{p+1}}{(2p-1) \cosh\left(\delta_p \frac{a}{D_h}\right)} \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} \cos(\alpha_m X') \cos(\alpha_p X') \cos(\beta_n Y') \cosh(\delta_p Y') dX' dY' \\
 Q2 = & \frac{4\bar{\zeta}}{\pi} \sum_{q=1}^{\infty} \frac{(-1)^{q+1}}{(2q-1) \cosh\left(\varepsilon_q \frac{b}{D_h}\right)} \int_{X'=0}^{b/D_h} \int_{Y'=0}^{a/D_h} \cos(\alpha_m X') \cosh(\varepsilon_q X') \cos(\beta_n Y') \cos(\beta_q Y') dX' dY'
 \end{aligned}$$

For $Q1$, remembering the orthogonality of trigonometric functions:

$$\int_{X'=0}^{b/D_h} \cos(\alpha_m X') \cos(\alpha_p X') dX' = \begin{cases} \alpha_m \neq \alpha_p \rightarrow 0 \\ \alpha_m = \alpha_p \rightarrow \frac{b}{2D_h} \end{cases}$$

The integration with respect to Y' can be solved using integration by parts method:

$$\begin{aligned}
 \int_{Y'=0}^{a/D_h} \cos(\beta_n Y') \cosh(\delta_p Y') dY' &= \cos(\beta_n Y') \frac{\sinh(\delta_p Y')}{\delta_p} \Big|_0^{a/D_h} + \beta_n \int_{Y'=0}^{a/D_h} \frac{\sinh(\delta_p Y')}{\delta_p} \sin(\beta_n Y') dY' \\
 &= 0 + \frac{\beta_n}{\delta_p} \sin(\beta_n Y') \frac{\cosh(\delta_p Y')}{\delta_p} \Big|_0^{a/D_h} - \frac{\beta_n^2}{\delta_p^2} \int_{Y'=0}^{a/D_h} \cos(\beta_n Y') \cosh(\delta_p Y') dY' \\
 &\rightarrow \left(1 + \frac{\beta_n^2}{\delta_p^2} \right) \int_{Y'=0}^{a/D_h} \cos(\beta_n Y') \cosh(\delta_p Y') dY' = \frac{\beta_n}{\delta_p^2} \sin\left(\frac{2n-1}{2} \pi\right) \cosh\left(\delta_p \frac{a}{D_h}\right)
 \end{aligned}$$

$$\rightarrow \int_{Y'=0}^{a/D_h} \cos(\beta_n Y') \cosh(\delta_p Y') dY' = \frac{\frac{\beta_n}{\delta_p^2} (-1)^{n+1} \cosh\left(\frac{\delta_p a}{D_h}\right)}{\left(1 + \frac{\beta_n^2}{\delta_p^2}\right)} = \frac{(-1)^{n+1} \cosh\left(\frac{\delta_p a}{D_h}\right)}{\frac{\delta_p^2}{\beta_n} + \beta_n}$$

So, the solution for $Q1$ is:

$$Q1 = \frac{4\bar{\zeta}}{\pi} \frac{(-1)^{m+1}}{(2m-1) \cosh\left(\delta_m \frac{a}{D_h}\right)} \frac{b}{2D_h} \frac{(-1)^{n+1} \cosh\left(\frac{\delta_m a}{D_h}\right)}{\frac{\delta_m^2}{\beta_n} + \beta_n} = \frac{2\bar{\zeta}b}{\pi D_h} \frac{(-1)^{m+n}}{(2m-1) \left(\frac{\delta_m^2}{\beta_n} + \beta_n\right)}$$

For $Q2$, again by remembering the orthogonality of trigonometric functions:

$$\int_{Y'=0}^{a/D_h} \cos(\beta_n Y') \cos(\beta_q Y') dY' = \begin{cases} \beta_n \neq \beta_q \rightarrow 0 \\ \beta_n = \beta_q \rightarrow \frac{a}{2D_h} \end{cases}$$

The integration with respect to X' can be solved using integration by parts method:

$$\begin{aligned} \int_{X'=0}^{b/D_h} \cos(\alpha_m X') \cosh(\varepsilon_q X') dX' &= \cos(\alpha_m X') \frac{\sinh(\varepsilon_q X')}{\varepsilon_q} \Big|_0^{b/D_h} + \alpha_m \int_{X'=0}^{b/D_h} \frac{\sinh(\varepsilon_q X')}{\varepsilon_q} \sin(\alpha_m X') dX' \\ &= 0 + \frac{\alpha_m}{\varepsilon_q} \sin(\alpha_m X') \frac{\cosh(\varepsilon_q X')}{\varepsilon_q} \Big|_0^{b/D_h} - \frac{\alpha_m^2}{\varepsilon_q^2} \int_{X'=0}^{b/D_h} \cos(\alpha_m X') \cosh(\varepsilon_q X') dX' \\ &\rightarrow \left(1 + \frac{\alpha_m^2}{\varepsilon_q^2}\right) \int_{X'=0}^{b/D_h} \cos(\alpha_m X') \cosh(\varepsilon_q X') dX' = \frac{\alpha_m}{\varepsilon_q^2} \sin\left(\frac{2m-1}{2} \pi\right) \cosh\left(\frac{\varepsilon_q b}{D_h}\right) \\ &\rightarrow \int_{X'=0}^{b/D_h} \cos(\alpha_m X') \cosh(\varepsilon_q X') dX' = \frac{\frac{\alpha_m}{\varepsilon_q^2} (-1)^{m+1} \cosh\left(\frac{\varepsilon_q a}{D_h}\right)}{\left(1 + \frac{\alpha_m^2}{\varepsilon_q^2}\right)} = \frac{(-1)^{m+1} \cosh\left(\frac{\varepsilon_q b}{D_h}\right)}{\frac{\varepsilon_q^2}{\alpha_m} + \alpha_m} \end{aligned}$$

So, the solution for $Q2$ is:

$$Q2 = \frac{4\bar{\zeta}}{\pi} \frac{(-1)^{n+1}}{(2n-1) \cosh\left(\varepsilon_n \frac{b}{D_h}\right)} \frac{a}{2D_h} \frac{(-1)^{m+1} \cosh\left(\frac{\varepsilon_n b}{D_h}\right)}{\frac{\varepsilon_n^2}{\alpha_m} + \alpha_m} = \frac{2\bar{\zeta}a}{\pi D_h} \frac{(-1)^{m+n}}{(2n-1) \left(\frac{\varepsilon_n^2}{\alpha_m} + \alpha_m\right)}$$

Substituting back $Q1$ and $Q2$ to eq. (4.25) gives the final solution of $\bar{u}(X, Y, \bar{t})$:

$$\begin{aligned} \bar{u}(X, Y, \bar{t}) = & -\frac{64}{\pi^2 D_h^2} \frac{d\bar{P}_0}{dZ} e^{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(1 - e^{\left(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu} \right)} \right) \frac{(-1)^{m+n}}{(2m-1)(2n-1)} \frac{[\cos(\alpha_m X)][\cos(\beta_n Y)]}{R_{mn} + i \frac{4\omega}{\nu}} \\ & - \frac{16}{ab} \overline{GE_0} e^{i(\frac{\omega D_h^2 \bar{t}}{\nu})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1 - e^{\left(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu} \right)}}{R_{mn} + i \frac{4\omega}{\nu}} [\cos(\alpha_m X)][\cos(\beta_n Y)] \\ & \times \left(\frac{2\bar{\zeta}b}{\pi D_h} \frac{(-1)^{m+n}}{(2m-1) \left(\frac{\delta_m^2}{\beta_n} + \beta_n \right)} + \frac{2\bar{\zeta}a}{\pi D_h} \frac{(-1)^{m+n}}{(2n-1) \left(\frac{\varepsilon_n^2}{\alpha_m} + \alpha_m \right)} \right) \end{aligned} \quad (4.26)$$

4.1.4. Induced pressure gradient

When the net flow in the channel is zero, there will be pressure gradient generated.

Substituting eq. (4.26) to this condition gives:

$$\begin{aligned} & \int_{X=-b/D_h}^{b/D_h} \int_{Y=-a/D_h}^{a/D_h} \bar{u}(X, Y, \bar{t}) dXdY = 0 \\ & \int_{X=-b/D_h}^{b/D_h} \int_{Y=-a/D_h}^{a/D_h} \left(-\frac{64}{\pi^2 D_h^2} \frac{d\bar{P}_0}{dZ} e^{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} [\cos(\alpha_m X)][\cos(\beta_n Y)] \left(\frac{(-1)^{m+n}}{(2m-1)(2n-1)} \right) \frac{1 - e^{\left(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu} \right)}}{R_{mn} + i \frac{4\omega}{\nu}} \right. \\ & \quad \left. + \frac{16}{ab} \overline{GE_0} e^{i(\frac{\omega D_h^2 \bar{t}}{\nu})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} [\cos(\alpha_m X)][\cos(\beta_n Y)] \frac{1 - e^{\left(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu} \right)}}{R_{mn} + i \frac{4\omega}{\nu}} \right. \\ & \quad \left. \times \left(\frac{2\bar{\zeta}b}{\pi D_h} \frac{(-1)^{m+n}}{(2m-1) \left(\frac{\delta_m^2}{\beta_n} + \beta_n \right)} + \frac{2\bar{\zeta}a}{\pi D_h} \frac{(-1)^{m+n}}{(2n-1) \left(\frac{\varepsilon_n^2}{\alpha_m} + \alpha_m \right)} \right) \right) dXdY = 0 \end{aligned}$$

$$\begin{aligned}
& \frac{64}{\pi^2 D_h^2} \frac{d\overline{P}_0}{dZ} e^{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin(\alpha_m X)}{\alpha_m} \Big|_{-b/D_h}^{b/D_h} \frac{\sin(\beta_n Y)}{\beta_n} \Big|_{-a/D_h}^{a/D_h} \left(\frac{(-1)^{m+n}}{(2m-1)(2n-1)} \right) \frac{1 - e^{(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu})}}{R_{mn} + i \frac{4\omega}{\nu}} \\
& + \frac{32\bar{\zeta}}{ab D_h \pi} \overline{GE}_0 e^{(i \frac{\omega D_h^2 \bar{t}}{\nu})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin(\alpha_m X)}{\alpha_m} \Big|_{-b/D_h}^{b/D_h} \frac{\sin(\beta_n Y)}{\beta_n} \Big|_{-a/D_h}^{a/D_h} \frac{1 - e^{(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu})}}{R_{mn} + i \frac{4\omega}{\nu}} \\
& \times \left(\frac{b(-1)^{m+n}}{(2m-1) \left(\frac{\delta_m^2}{\beta_n} + \beta_n \right)} + \frac{a(-1)^{m+n}}{(2n-1) \left(\frac{\varepsilon_n^2}{\alpha_m} + \alpha_m \right)} \right) = 0 \\
& \frac{64}{\pi^2 D_h^2} \frac{d\overline{P}_0}{dZ} e^{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{2 \sin\left(\frac{2m-1}{2} \pi\right)}{\frac{2m-1}{2} \frac{\pi D_h}{b}} \frac{2 \sin\left(\frac{2n-1}{2} \pi\right)}{\frac{2n-1}{2} \frac{\pi D_h}{a}} \left(\frac{(-1)^{m+n}}{(2m-1)(2n-1)} \right) \frac{1 - e^{(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu})}}{R_{mn} + i \frac{4\omega}{\nu}} \\
& + \frac{32\bar{\zeta}}{ab D_h \pi} \overline{GE}_0 e^{(i \frac{\omega D_h^2 \bar{t}}{\nu})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{2 \sin\left(\frac{2m-1}{2} \pi\right)}{\frac{2m-1}{2} \frac{\pi D_h}{b}} \frac{2 \sin\left(\frac{2n-1}{2} \pi\right)}{\frac{2n-1}{2} \frac{\pi D_h}{a}} \frac{1 - e^{(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu})}}{R_{mn} + i \frac{4\omega}{\nu}} \\
& \times \left(\frac{b(-1)^{m+n}}{(2m-1) \left(\frac{\delta_m^2}{\beta_n} + \beta_n \right)} + \frac{a(-1)^{m+n}}{(2n-1) \left(\frac{\varepsilon_n^2}{\alpha_m} + \alpha_m \right)} \right) = 0 \\
& \frac{1024ab}{\pi^4 D_h^4} \frac{d\overline{P}_0}{dZ} e^{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin\left(\frac{2m-1}{2} \pi\right)}{2m-1} \frac{\sin\left(\frac{2n-1}{2} \pi\right)}{2n-1} \left(\frac{(-1)^{m+n}}{(2m-1)(2n-1)} \right) \frac{1 - e^{(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu})}}{R_{mn} + i \frac{4\omega}{\nu}} \\
& = -\frac{512\bar{\zeta}}{\pi^3 D_h^3} \overline{GE}_0 e^{(i \frac{\omega D_h^2 \bar{t}}{\nu})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin\left(\frac{2m-1}{2} \pi\right)}{2m-1} \frac{\sin\left(\frac{2n-1}{2} \pi\right)}{2n-1} \frac{1 - e^{(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu})}}{R_{mn} + i \frac{4\omega}{\nu}} \\
& \times \left(\frac{b(-1)^{m+n}}{(2m-1) \left(\beta_n + \delta_m^2 / \beta_n \right)} + \frac{a(-1)^{m+n}}{(2n-1) \left(\alpha_m + \varepsilon_n^2 / \alpha_m \right)} \right)
\end{aligned}$$

$$\begin{aligned}
 & \frac{d\bar{P}_0}{dZ} e^{i(\phi + \frac{\omega D_h^2 \bar{t}}{\nu})} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^{m+1}}{2m-1} \frac{(-1)^{n+1}}{2n-1} \left(\frac{(-1)^{m+n}}{(2m-1)(2n-1)} \right) \frac{1 - e^{\left(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu}\right)}}{R_{mn} + i \frac{4\omega}{\nu}} \\
 &= -\frac{\pi D_h \bar{\zeta} G E_0 e^{i(\frac{\omega D_h^2 \bar{t}}{\nu})}}{2ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^{m+1}}{2m-1} \frac{(-1)^{n+1}}{2n-1} \frac{1 - e^{\left(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu}\right)}}{R_{mn} + i \frac{4\omega}{\nu}} \\
 &\times \left(\frac{b(-1)^{m+n}}{(2m-1) \left(\frac{\delta_m^2}{\beta_n} + \beta_n \right)} + \frac{a(-1)^{m+n}}{(2n-1) \left(\frac{\varepsilon_n^2}{\alpha_m} + \alpha_m \right)} \right) \\
 &\frac{d\bar{P}}{dZ} = -\frac{\pi D_h \bar{\zeta} G E}{2ab} \frac{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\frac{(-1)^{m+n}}{(2m-1)(2n-1)} \frac{1 - e^{\left(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu}\right)}}{R_{mn} + i \frac{4\omega}{\nu}} \right.}{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1 - e^{\left(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu}\right)}}{(2m-1)^2 (2n-1)^2 (R_{mn} + i \frac{4\omega}{\nu})}} \left. \times \left(\frac{b(-1)^{m+n}}{(2m-1) \left(\frac{\delta_m^2}{\beta_n} + \beta_n \right)} + \frac{a(-1)^{m+n}}{(2n-1) \left(\frac{\varepsilon_n^2}{\alpha_m} + \alpha_m \right)} \right) \right] \quad (4.27)
 \end{aligned}$$

Eq. (4.27) is the general form of pressure gradient generated in a rectangular microchannel.

When time elapsed is very short, then $e^{\left(-\frac{D_h^2}{4} R_{mn} \bar{t} - i \frac{\omega D_h^2 \bar{t}}{\nu}\right)} \approx 1 - \left(\frac{D_h^2}{4} R_{mn} + i \frac{\omega D_h^2}{\nu}\right) \bar{t}$, and

eq. (4.27) can be modified to be:

$$\begin{aligned}
 & \frac{d\bar{P}}{dZ} = -\frac{\pi D_h \bar{\zeta} G E}{2ab} \frac{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\frac{(-1)^{m+n}}{(2m-1)(2n-1)} \left(\frac{b(-1)^{m+n}}{(2m-1) \left(\frac{\delta_m^2}{\beta_n} + \beta_n \right)} + \frac{a(-1)^{m+n}}{(2n-1) \left(\frac{\varepsilon_n^2}{\alpha_m} + \alpha_m \right)} \right) \right]}{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{(2m-1)^2 (2n-1)^2}} \quad (4.28)
 \end{aligned}$$

When the applied electric field is DC ($\omega = 0$), eq. (4.27) becomes:

$$\frac{d\bar{P}}{dZ} = -\frac{\pi D_h \bar{\zeta} G E}{2ab} \frac{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\frac{(-1)^{m+n}}{(2m-1)(2n-1)} \frac{1 - e^{\left(-\frac{D_h^2}{4} R_{mn} \bar{t}\right)}}{R_{mn}} \left(\frac{b(-1)^{m+n}}{(2m-1) \left(\frac{\delta_m^2}{\beta_n} + \beta_n \right)} + \frac{a(-1)^{m+n}}{(2n-1) \left(\frac{\varepsilon_n^2}{\alpha_m} + \alpha_m \right)} \right) \right]}{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1 - e^{\left(-\frac{D_h^2}{4} R_{mn} \bar{t}\right)}}{(2m-1)^2 (2n-1)^2 R_{mn}}} \quad (4.29)$$

And for DC case when time elapsed is very small, eq. (4.29) can be modified to take the form of eq. (4.28).

The dimensional pressure gradient is:

$$\frac{dP}{dz} = \frac{d\bar{P}}{dZ} \times \left(\frac{\rho_f U^2}{D_h \text{Re}_0} \right) = \frac{d\bar{P}}{dZ} \times \left(\frac{\mu_f U}{D_h^2} \right) \quad (4.30)$$

More in-depth observation to eq. (4.27) and (4.30) shows that while the induced pressure gradient is a linear function of the applied electric field strength, E , and the zeta potential, ζ , it is a complex function of the other parameters. By assuming that each parameter is independent, some of the effects of the parameters to the pressure difference generated between the two ends of the capillary are shown in Figure 4.2 - 4.6. However, it is important to remember that in reality, many of these parameters are actually inter-dependent.

Figure 4.2 Effects of applied electric field strength only to induced pressure gradient

Figure 4.3 Effects of zeta potential only to induced pressure gradient

Figure 4.4 Effects of hydraulic diameter only to induced pressure gradient

Figure 4.5 Effects of electrolyte concentration and hydraulic diameter to induced pressure gradient

Figure 4.6 Effects of valence, electric field strength applied and electrolyte concentration to induced pressure gradient

4.2. Compression system equations

4.2.1. Compressor equations

The basic equation to describe the thermodynamic processes in a compressor is the 1st law of thermodynamics in a control volume system:

Figure 4.7 An illustration of a control volume system

$$\dot{Q} - \dot{W} + \Sigma \frac{dm_i}{dt} h_i - \Sigma \frac{dm_e}{dt} h_e = u \frac{dm}{dt} + m \frac{du}{dt} \quad (4.31)$$

where:

$$W = \int_{V_1}^{V_2} p dV \rightarrow \frac{dW}{dt} = p \frac{dV}{dt}$$

$$h = C_p T$$

$$u = C_v T \rightarrow \frac{du}{dt} = C_v \frac{dT}{dt}$$

$$\frac{dm}{dt} = \Sigma \frac{dm_i}{dt} - \Sigma \frac{dm_e}{dt}$$

And so, eq. (4.31) can be modified to be:

$$\frac{dT}{dt} = \frac{1}{m C_v} \left[\dot{Q} - p \frac{dV}{dt} + \Sigma \frac{dm_i}{dt} C_p T_i - \Sigma \frac{dm_e}{dt} C_p T_e - C_v T \left(\Sigma \frac{dm_i}{dt} - \Sigma \frac{dm_e}{dt} \right) \right] \quad (4.32)$$

According to Longo & Gasparella [86], $\dot{Q}$ can be estimated with the following equation:

$$\dot{Q} = 2.1 \frac{k}{D} \left(\frac{\rho u_p D}{\mu} \right)^{0.7} \times A \times (T_w - T) \quad (4.33)$$

where:

k = thermal conductivity of the compressible fluid

D = diameter of the compressor

ρ = density of the compressible fluid

u_p = mean piston speed

μ = dynamic compressible fluid viscosity

A = heat transfer area

T_w = temperature of the compressor wall (assumed to be constant)

T = temperature of the compressible fluid

The compressible fluid is assumed to be ideal gas:

$$pV = mRT$$

$$\Rightarrow \frac{dp}{dt} = \frac{1}{V} \left[mR \frac{dT}{dt} + RT \frac{dm}{dt} - p \frac{dV}{dt} \right] \quad (4.34)$$

4.2.2. Compressed air related equations

The suction and discharge processes through the valves are assumed to be isentropic:

Figure 4.8 An isentropic flow through an orifice

$$\frac{p_1}{\rho_1^k} = \frac{p_2}{\rho_2^k} \rightarrow \frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2} \right)^k \quad (4.35)$$

Modifying the ideal gas equation gives the following relation:

$$\frac{p_1}{p_2} = \frac{\rho_1 RT_1}{\rho_2 RT_2} \rightarrow \frac{p_1}{p_2} = \frac{\rho_1 T_2}{\rho_2 T_1} \quad (4.36)$$

Combining eq. (4.35) and eq. (4.36) gives:

$$\frac{p_1}{p_2} = \left(\frac{p_1 T_2}{p_2 T_1} \right)^k \rightarrow T_2 = T_1 \left(\frac{p_1}{p_2} \right)^{\frac{1-k}{k}} \quad (4.37)$$

Energy equation of the flow gives an estimation of the velocity of the flow:

$$v_2^2 = 2(h_1 - h_2) = 2C_p(T_1 - T_2) \rightarrow v_2 = \sqrt{2C_p(T_1 - T_2)} \quad (4.38)$$

While the mass flow rate through the orifice can be calculated by:

$$\dot{m} = \frac{dm}{dt} = C_d \rho_2 A_2 v_2 \quad (4.39)$$

Combining eq. (4.38) and eq. (4.39) gives:

$$\frac{dm}{dt} = C_d \frac{p_2}{RT_2} A_2 \sqrt{2C_p(T_1 - T_2)} \quad (4.40)$$

Knowing the velocity of the piston u_p , eq. (4.32), (4.34) and (4.40) can be used to estimate the temperature, pressure and mass of the compressible fluid in the compression system during the process.

4.2.3. Piston related equations

The movement of the pistons are governed by the force balance equation at the two-sides of the pistons:

Figure 4.9 Force balance on a piston

$$p_1 A_p - p_2 A_p - F_p = m_p \frac{d^2 x_p}{dt^2} \quad (4.41)$$

where:

p_1 = pressure at the 1st side of the piston

x_p = displacement of the piston

A_p = cross-sectional area of the piston

p_{air} = pressure at the 2nd side of the piston

F_p = friction force at the piston

m_p = mass of the piston

The change in the volume of air in the compression as the piston moves follows:

$$\frac{dV_{air}}{dt} = -A_p \frac{dx_p}{dt} \quad (4.42)$$

4.2.4. Valves related equations

The movements of the suction and discharge valves in a compressor can be described with the following equation:

Figure 4.10 A spring valve mechanism

$$(P_1 - P_2)A_v = k \cdot x_v + R \cdot \frac{dx_v}{dt} + m_v \cdot \frac{d^2 x_v}{dt^2} \quad (4.43)$$

where:

A_v = cross-sectional area of the opening

x_v = spring displacement

k = spring stiffness

R = damping constant

m_v = mass of the valve

The change in the pressure and the density of a liquid as its volume changes follow these equations:

$$E_v \frac{dV}{dt} = -V_0 \frac{dP_{liq}}{dt} \quad (4.44)$$

$$\rho_{liq} = \rho_0 \frac{V_0}{V_0 + dV} \quad (4.45)$$

where:

E_v = bulk modulus of elasticity of the liquid

V_0 = initial volume of the liquid

ρ_0 = initial density of the liquid

4.2.5. Solving the compression system equations

When simulating a compression system, most of the equations in eq. (4.31)-(4.45) need to be solved simultaneously. The well known 4th order Runge-Kutta Method [87] can be utilized to perform the task.

Chapter 5:

SIMULATIONS:

RESULTS AND DISCUSSIONS

After deriving all the governing equations in Chapter 4, the simulations carried out will be discussed in this chapter. The descriptions, the basic working principles, the results and the discussions of each model will be presented. The presentation begins with the basic model of a simple compression system, continues with the model where the actuator is employed to actuate a compressor and ends with the improvements implemented to the compressor.

5.1. Model #1

5.1.1. Descriptions

Figures 5.1 and 5.2 show the schematic drawing of model #1 and its dimensions respectively:

Figure 5.1 A schematic drawing of model #1

Figure 5.2 Basic dimensions of model #1

Model #		1
Applied electric field strength per unit length		100 V/cm DC
Chamber 1	Fluid	Electrolyte
	Diameter	10 cm
	Length	12 cm
Insert A	Length	10 cm
	Hydraulic Diameter	100 μm
Electrolyte	Concentration	0.001 M
Piston 1	Diameter	10 cm
	Thickness	0.2 cm
	Mass	0.1 kg
	Static Friction Coefficient	0.1
	Dynamic Friction Coefficient	0.05
Gap between Insert A and Piston 1		2 cm
Chamber 2	Fluid	Air
	Diameter	10 cm
	Length	1 cm

Table 5.1 Table of dimensions of model #1

Model #1 consists of two chambers and a piston encapsulated in a 10 cm diameter, 13.2 cm long cylinder. The cylinder wall is made of a solid and electrically non-conducting material.

Chamber 1 is 12 cm long with an insert, called Insert A which is a 10 cm long porous material or a bundle of capillaries with a 100 μm hydraulic diameter. Electrodes are attached to the ends of Insert A to provide the electric field (1000 V DC). All the space in Chamber 1, including all the pores and holes in Insert A, is filled with an electrolyte which is a 0.001 M salt solution ($z_v = 1$). An application of electric field will generate the electroosmotic pressure gradient. The direction of the pressure depends on the polarity of the DC power source, and it was set such that the pressure

generated is in the direction of Polarity 1 as shown in Figure 5.1. Pressure 1 is the pressure acting on the base wall, which is the electrolyte pressure and Pressure 2 is the pressure acting on the piston which is the summation of Pressure 1 and the electroosmotic pressure gradient generated. Chamber 1 is also called the Electroosmotic Chamber because of this generation of electroosmotic pressure gradient in this chamber.

Chamber 2 has the same diameter as Chamber 1 and is 1 cm long. It is filled with air with Pressure 3 denoting the air pressure. This chamber is where the compression happens and therefore it is called the Compression Chamber.

Piston 1 has a mass of 1 kg, a diameter that is the same as the chambers and a thickness of 2 mm. It is located in between the two chambers and it is free to move according to the force balance acting on it caused by Pressure 2 and Pressure 3. A proper sealing is needed at the contact surface between the piston and the inner cylinder wall to prevent the leaking of fluid from Chamber 1 to Chamber 2 or vice versa. A static friction coefficient of 0.1 and a dynamic friction coefficient of 0.05 were assumed at the contact surface between the piston and the slider surface of the wall. A 2 cm gap is provided between Insert A and Piston 1 to give sufficient space for the piston to move.

5.1.2. Basic working principles

In the beginning, the pressures are assumed to be uniform in both Chambers 1 and 2 which are taken to be the atmospheric pressure, for simplicity. Once the DC power source is switched on, a pressure gradient builds up in Chamber 1. The electric field is supplied throughout the whole operation of the system and hence the pressure gradient always exists. The polarity of the power source is arranged such that the

pressure gradient is in the direction of Polarity 1 as shown in Figure 5.1. If the unbalanced force caused by Pressure 2 and Pressure 3 is big enough to overcome the static friction of Piston 1, the piston will move to compress the air according to eq. (5.1), which is an adaptation of eq. (4.41):

$$\left(p_{liq} + L \frac{dP}{dz} \right) A_p - p_{air} A_p - F_p = m_p \frac{d^2 x_p}{dt^2} \quad (5.1)$$

where:

p_{liq}	= pressure of the electrolyte
L	= length of Insert A
x_p	= displacement of the piston
$\frac{dP}{dz}$	= electroosmotic pressure gradient generated by Insert A
A_p	= cross-sectional area of the piston
p_{air}	= air pressure, also called as Pressure 3
F_p	= friction force at the piston
m_p	= mass of the piston
t	= time

If the unbalanced force acting on Piston 1 caused by Pressure 2 and Pressure 3 is too small to overcome the static friction of Piston 1, then the piston will remain stationary. During the operation, when Piston 1 has moved far enough such that the Pressure 3 is higher than Pressure 2, it decelerates and slows down. Then, it will finally move in the reverse direction; compressing the liquid and expanding the air. The changes in the air pressure, air temperature and air volume are estimated with eq. (4.32), (4.34), and (4.42) respectively while the changes of the electrolyte pressure and electrolyte density are estimated by eq (4.44) and (4.45) respectively. The air compression

process was assumed to be an adiabatic process while the temperature of the water was assumed constant throughout the whole process (isothermal condition).

During the process, it is possible that the solution pressure (Pressure 1) drops below the saturation pressure and so flashing occurs. This is an unwanted situation since flashing generates gas phase that may disturb and even stop the electroosmosis process by isolating the electrodes from the electrolyte. Therefore, it is necessary to keep Pressure 1 above the saturation point.

Some universal constants used in the simulations include [88-89]:

Atmospheric pressure, $P_{atm} = 101325 Pa$

Boltzmann constant, $k_b = 1.3807 \times 10^{-23} J / K$

Elementary charge, $e = 1.6022 \times 10^{-19} C$

The properties of the electrolyte were assumed to be very close to the water properties at the specified temperature (303 K) [90-91]:

Bulk modulus of elasticity, $E_v = 2.25 \times 10^9 N / m^2$

Dynamic fluid viscosity, $\mu = 0.798 \times 10^{-3} Ns / m^2$

Density, $\rho_0 = 995.7 kg / m^3$

Vacuum permittivity, $\epsilon_0 = 8.8542 \times 10^{-12} F / m$

Relative permittivity, $\epsilon_r = 80$

Saturation pressure, $P_{sat} = 4246 Pa$

The zeta potential is assumed to be a constant value of $\zeta = -50 mV$.

The properties of the air were assumed to be [92]:

Specific heat at constant pressure, $C_p = 1005 J / kg.K$

Specific heat at constant volume, $C_v = 718 J / kg.K$

Specific heat ratio, $k = 1.4$

Gas constant, $R = 287 J / kg.K$

5.1.3. Results and discussions

The results obtained from model #1 are shown in Figures 5.3-5.4. An examination on equations (4.27) and (4.30) in Chapter 4 as shown in Figures 4.2-4.6 shows that varying the applied electric field strength or the hydraulic diameter are the two most practical and simplest way to control the generated electroosmotic pressure gradient hence the effects of the two are investigated further here.

Figure 5.3 Voltage effects to model #1

Figure 5.4 Hydraulic diameter effects to model #1

Figure 5.3 shows that higher applied electric strengths give larger piston displacements and higher air pressures. This is expected as higher applied electric field strength gives higher electroosmotic pressure gradient. The relations are found to be linear as expected. Further investigations suggest that there is a limit to the allowable increase of applied electric strength before the electrolyte pressure (Pressure 1) drops to below the saturation point which will cause flashing. This limit is called the flashing limit. It was found to be around 9.5 kV/cm which is a value that is almost of no practical value for electroosmosis processes due to the rapid electrolysis and Joule heating processes involved with such enormous electric strength.

Figure 5.4 shows that another way to get larger piston displacements and higher air pressures is by using materials with smaller hydraulic diameters. This is also expected as smaller hydraulic diameter generates higher electroosmotic pressure gradient. It was also found from further investigations that the hydraulic diameter flashing limit is 4 μm .

One important thing to observe from all the results presented is the very small piston displacements and very small air pressure increase obtained with model #1. The largest piston displacement shown is less than 1 μm and the biggest air pressure rise is less than 15 Pa. These values are definitely too small to be utilized and hence a better design is needed to improve the outcomes.

5.2. Model #2

5.2.1. Introduction

Model #1 showed its limitations in producing large piston displacements and high air pressures due to the undesirable pressure drop in the electrolyte as the piston moves to

compress the air. Model #2 is designed to overcome this limitation by controlling the drop in the solution pressure. This is done by introducing a movable base at Chamber 1 to reduce the change in the electrolyte volume and hence reduces the change in the electrolyte pressure (Pressure 1), thus avoiding the flashing limit of the electrolyte.

5.2.2. Descriptions

Figures 5.5 and 5.6 show the schematic drawing and the dimensions of model #2:

Figure 5.5 A schematic drawing of model #2

Figure 5.6 Basic dimensions of model #2

Model #		1	2
Applied electric field strength per unit length		100 V/cm DC	100 V/cm DC
Chamber 1	Fluid	Electrolyte	Electrolyte
	Diameter	10 cm	10 cm
	Length	12 cm	14 cm
Insert A	Length	10 cm	10 cm
	Hydraulic Diameter	100 μm	100 μm
Electrolyte	Concentration	0.001 M	0.001 M
Piston 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.1 kg	0.1 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Piston 1		2 cm	2 cm

Chamber 2	Fluid	Air	Air
	Diameter	10 cm	10 cm
	Length	1 cm	1 cm
Base 1	Diameter	N/A	10 cm
	Thickness	N/A	0.2 cm
	Mass	N/A	0.1 kg
	Static Friction Coefficient	N/A	0.1
	Dynamic Friction Coefficient	N/A	0.05
Gap between Insert A and Base 1		N/A	2 cm

Table 5.2 Table of comparisons between model #1 and #2

The design of model #2 is very similar to model #1 but the former has a movable base, called Base 1. Base 1 was designed to have exactly the same mass and friction coefficients as Piston 1. A 2 cm gap between Base 1 and Insert A is also added to provide a sufficient space for the base to move as shown in Figure 5.6.

5.2.3. Basic working principles

The basic working principles behind model #2 are very similar with model #1 since other than the movable base in model #2, they have identical designs.

As electric field is applied, electroosmotic pressure gradient is generated by Insert A inside Chamber 1. If the unbalanced force acting on Piston 1 caused by Pressure 2 and Pressure 3 is big enough to overcome the static friction between Piston 1 and the inner cylinder wall of Chamber 2, the piston will move to compress the air while at the same time expanding the electrolyte. Due to this expansion, Pressure 1 drops to be less than the surrounding pressure and hence there is an unbalanced force at Base 1 as well. If this unbalanced force is big enough to overcome the static friction between Base 1 and the inner cylinder wall of Chamber 1, the base will move to compress the electrolyte and hence the pressure of the electrolyte (Pressure 1) increases. This continues until the point where Piston 1 starts to retract when the net force direction on the piston reverses. As the piston move back to expand the air and so compressing the electrolyte, the electrolyte pressure (Pressure 1) increases. When this electrolyte

pressure is higher than the surrounding pressure, the base will be pushed back and hence preventing the electrolyte pressure to rise further. Just like the piston, in the actual case, the sealing on the contacting surface between the base and the inner cylinder wall of Chamber 1 requires special attention to prevent the electrolyte in Chamber 1 leak out to the surrounding or vice versa.

The movement of the base follows eq. (5.2) which is an adaptation of eq. (4.41):

$$p_{atm}A_b - p_{liq}A_b - F_b = m_b \frac{d^2x_b}{dt^2} \quad (5.2)$$

where:

- p_{atm} = pressure of the surrounding
- A_b = cross-sectional area of the base
- F_b = friction force at the base
- m_b = mass of the base
- x_b = displacement of the base

5.2.4. Results and discussions

The results obtained with model #2 are shown in Figure 5.7:

Figure 5.7 Results obtained from model #2

The results obtained with model #2 showed that the piston displacements and the increase in air pressures are much better compared to the ones obtained with model #1. However, it can be seen that flashing eventually stopped the process as can be seen from the low electrolyte pressure at the end of the process. The reason behind the flashing is the inability of Base 1 to follow Piston 1 properly. In fact, it can be observed that the amplitude of the electrolyte pressure becomes larger with time, implying the less concurrent movements of the base (Base 1) and the piston (Piston 1). The reason behind this inability of Base 1 to follow the movement of Piston 1 properly is the different characteristics of the unbalanced forces acting on the two. The net force causing the movement of Base 1 is dependent on the pressure difference across the base, that is between the electrolyte pressure (Pressure 1) and the surrounding pressure. Pressure 1 can rise to a very high value when the electrolyte is compressed, but it cannot drop below the saturation pressure, which is 4246 Pa in this model, while the surrounding pressure has a constant value of one atmospheric pressure (101325 Pa). Meanwhile, the net force causing the movement of Piston 1 is dependent on the pressure difference across the piston, which is between Pressure 2 and Pressure 3. Pressure 2 is the summation of Pressure 1 and the electroosmotic pressure gradient which the latter has a steady-state value of around 100 Pa with this design. The lowest value of Pressure 2 is therefore around 4346 Pa, which is the value when Pressure 1 is at the saturation pressure value. On the other hand, as Pressure 1 can rise very high when the electrolyte is compressed, Pressure 2 can be of a very high value too. The air pressure (Pressure 3) is dependent only on the piston displacement and hence can be very high depending on the design. With these characteristics of Pressure 2 and Pressure 3, it is obvious that the pressure difference across Piston 1 can be very big unlike the pressure difference across Base 1 which is

limited by the constant value of the surrounding pressure. With this, the piston can move more rapidly than the base. Therefore the base needs to be designed to be able to match the movement of the piston. This can be done by adjusting the masses of Base 1 and Piston 1. A combination of 0.1 kg base and 0.5 kg piston was chosen arbitrarily to be tested further. Hence, the proposed dimensions of model #2.2 are shown in Figure 5.8.

Figure 5.8 Basic dimensions of model #2.2

Model #		2	2.2
Applied electric field strength per unit length		100 V/cm DC	100 V/cm DC
Chamber 1	Fluid	Electrolyte	Electrolyte
	Diameter	10 cm	10 cm
	Length	14 cm	14 cm
Insert A	Length	10 cm	10 cm
	Hydraulic Diameter	100 μm	100 μm
Electrolyte	Concentration	0.001 M	0.001 M
Piston 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.1 kg	0.5 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Piston 1		2 cm	2 cm
Chamber 2	Fluid	Air	Air
	Diameter	10 cm	10 cm
	Length	1 cm	1 cm
Base 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.1 kg	0.1 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Base 1		2 cm	2 cm

Table 5.3 Table of dimensions of model #2 and #2.2

The results obtained with model #2.2 are shown in Figures 5.9 -5.10.

Figure 5.9 Voltage effects to model #2.2

Figure 5.10 Hydraulic diameter effects to model #2.2

In general, the results in Figures 5.9 and 5.10 show the same trends as model #1, which is higher applied electric strength and smaller hydraulic diameter give more piston displacements and higher air pressures. However, with the exact same

parameters, the amplitudes obtained with model #2.2 are significantly bigger than in model #1. A piston displacement of more than 120 micron and air pressure increase of more than 2 kPa were observed with this model.

However, the results also show that flashing is still a major problem with model #2.2. Another major problem is the damping issue. The results shown in Figures 5.9(f), 5.10(e) and 5.10(f) show that Base 1 still cannot follow Piston 1 properly as can be seen from the trend of the electrolyte pressure diagram which shows bigger amplitude as time passes when damping is not very significant. In the cases when damping is significant, like in the cases shown in Figures 5.9(d), 5.9(e) and 5.10(d), both the base and the piston ceases to move too fast.

The flashing issue will be dealt first here by proposing a new design with a better combination of the masses of Base 1 and Piston 1. The proposed new design, called model #2.3, is shown in Figure 5.11. Other than the mass combination of the base and the piston, the applied electric field strength and the length of Chamber 2 were also modified to enhance the performance. A 0.1 kg base and a 0.8 kg piston were chosen in model #2.3. The applied electric field strength is doubled to 2000 V to increase the generated electroosmotic pressure gradient. The length of the Compression Chamber is changed to 5 cm to let the design be more practical.

Figure 5.11 Basic dimensions of model #2.3

Model #		2.2	2.3
Applied electric field strength per unit length		100 V/cm DC	200 V/cm DC
Chamber 1	Fluid	Electrolyte	Electrolyte
	Diameter	10 cm	10 cm
	Length	14 cm	14 cm
Insert A	Length	10 cm	10 cm
	Hydraulic Diameter	100 μm	100 μm
Electrolyte	Concentration	0.001 M	0.001 M
Piston 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.5 kg	0.8 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Piston 1		2 cm	2 cm
Chamber 2	Fluid	Air	Air
	Diameter	10 cm	10 cm
	Length	1 cm	5 cm
Base 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.1 kg	0.1 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Base 1		2 cm	2 cm

Table 5.4 Table of comparisons between model #2.2 and #2.3

The results obtained with model #2.3 are shown in Figure 5.12-5.13.

Figure 5.12 Voltage effects to model #2.3

Figure 5.13 Hydraulic diameter effects to model #2.3

The results show that like model #2 and model #2.2, model #2.3 failed to overcome the flashing problem. The trends of the electrolyte pressure diagrams still show that Base 1 cannot properly follow Piston 1.

However, in general the results obtained with this model show improvements in the compression ability. The increase in air pressure can reach more than 4 kPa, which is twice of the value obtained with model #2.2.

After reviewing the results obtained with models #2, #2.2 and #2.3, the concept of movable base introduced here has improved the compression processes. However, some major problems exist. These include the flashing of the electrolytes involved and the damping of the oscillations of the piston. Varying the combination of the

masses of Base 1 and Piston 1 alone cannot solve the flashing problem, and therefore a special mechanism must be introduced to the design to solve this problem.

5.3. Model #3

5.3.1. Introduction

Model #2, model #2.2 and model #2.3 have shown that electroosmotic pressure can be utilized to drive an actuator. However, flashing was a major problem and modifying the combination of the base mass and the piston mass alone cannot solve the problem. Therefore a special mechanism must be added to the design to solve this problem. The mechanism introduced in Model #3 is a spring mechanism which is used to control the movements of the piston and the base better.

5.3.2. Descriptions

The schematic drawing and the basic dimensions of model #3 are shown in Figures 5.14-5.15.

Figure 5.14 A schematic drawing of model #3

Figure 5.15 Basic dimensions of model #3

Model #		2.3	3
Applied electric field strength per unit length		200 V/cm DC	200 V/cm DC
Chamber 1	Fluid	Electrolyte	Electrolyte
	Diameter	10 cm	10 cm
	Length	14 cm	14 cm
Insert A	Length	10 cm	10 cm
	Hydraulic Diameter	100 μm	100 μm
Electrolyte	Concentration	0.001 M	0.001 M
Piston 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.8 kg	0.8 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Piston 1		2 cm	2 cm
Chamber 2	Fluid	Air	Air
	Diameter	10 cm	10 cm
	Length	5 cm	5 cm
Base 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.1 kg	0.1 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Base 1		2 cm	2 cm
Spring	Length	N/A	14 cm
	Spring constant	N/A	5*10 ⁹ kg/s ²

Table 5.5 Table of comparisons between model #2.3 and #3

Model #3 is basically the same as model #2.3 but it has a spring connecting Base 1 and Piston 1. This spring is 14 cm long and has a spring constant of 5*10⁹ kg/s². It will pull or push the piston and the base accordingly, depending on the relative movement between the two.

5.3.3. Basic working principles

The basic working principle of model #3 is identical to that of model #2. However, with the addition of the spring mechanism, the movements of Base 1 and Piston 1 are now restricted. When the distance between the base and the piston is less than 14 cm, the spring is compressed and will push both the piston and the base. However, when this distance is more than 14 cm, the spring is stretched and it pulls both the piston and the base closer. The spring needs to be sufficiently stiff for it to be effective. In this model a spring constant of $5 \times 10^9 \text{ kg/s}^2$ was chosen arbitrarily. The force of the spring and the equations of motion for Piston 1 and Base 1 in the direction to compress the air are given in eq. (5.3)-(5.5) respectively.

$$F_s = k * (x_p - x_b) \quad (5.3)$$

$$\left(p_{liq} + L \frac{dP}{dz} \right) A_p - p_{air} A_p - F_p - F_s = m_p \frac{d^2 x_p}{dt^2} \quad (5.4)$$

$$p_{atm} A_b - p_{liq} A_b - F_b + F_s = m_b \frac{d^2 x_b}{dt^2} \quad (5.5)$$

where:

F_s = pulling force of the spring

k = spring constant

5.3.4. Results and discussions

The results obtained with model #3 are shown in Figures 5.16-5.18.

Figure 5.16 Voltage effects to model #3

Figure 5.17 Hydraulic diameter effects to model #3

Figure 5.18 Spring stiffness effects to model #3

The results shown in Figures 5.16-5.18 show that model #3 is quite successful in controlling the movements of the base and the piston better than model #2.3. Although the amplitude of the electrolyte pressure still increases with time, it is now

comparably low. The results also show that having a very stiff spring is a very good method to reduce the increase in the electrolyte pressure amplitude without having to compromise the piston displacement and the air pressure generated. A special case when the spring constant is of a very big value is shown in Figure 5.19. The results obtained are shown in Figures 5.20.

Figure 5.19 Basic dimensions of model #3.2

Model #		3	3.2
Applied electric field strength per unit length		200 V/cm DC	200 V/cm DC
Chamber 1	Fluid	Electrolyte	Electrolyte
	Diameter	10 cm	10 cm
	Length	14 cm	14 cm
Insert A	Length	10 cm	10 cm
	Hydraulic Diameter	100 μm	100 μm
Electrolyte	Concentration	0.001 M	0.001 M
Piston 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.8 kg	0.8 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Piston 1		2 cm	2 cm
Chamber 2	Fluid	Air	Air
	Diameter	10 cm	10 cm
	Length	5 cm	5 cm
Base 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.1 kg	0.1 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Base 1		2 cm	2 cm
Spring	Length	14 cm	14 cm
	Spring constant	$5 \times 10^9 \text{ kg/s}^2$	∞

Table 5.6 Table of comparisons between model #3 and #3.2

Figure 5.20 Results obtained from model #3.2

Figure 5.20(c) shows that an infinitely stiff spring will be able to solve the flashing problem thoroughly since it will be able to keep the distance between Base 1 and Piston 1 constant throughout the process.

In conclusion, the introduction of the spring mechanism to prevent flashing in model #3 has been quite successful. However, the damping of the oscillation has not been addressed. With this damping, the piston will eventually stop moving and hence the process stops. Therefore it is necessary to improve model #3 furthermore to overcome this damping issue.

5.4. Model #4

5.4.1. Introduction

Model #3 has given very promising results. However, the issue of damping has not been overcome. Model #4 is developed to overcome this issue. This is done by switching the polarity of the DC power source.

5.4.2. Descriptions

The schematic drawings and dimensions of model #4 are shown in Figures 5.21-5.22.

Figure 5.21 A schematic drawing of model #4

Figure 5.22 Basic dimensions of model #4

Model #		3.2	4
Applied electric field strength per unit length		200 V/cm DC	200 V/cm DC
Chamber 1	Fluid	Electrolyte	Electrolyte
	Diameter	10 cm	10 cm
	Length	14 cm	14 cm
Insert A	Length	10 cm	10 cm
	Hydraulic Diameter	100 μm	2 μm
Electrolyte	Concentration	0.001 M	0.001 M
Piston 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.8 kg	0.8 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Piston 1		2 cm	2 cm
Chamber 2	Fluid	Air	Air
	Diameter	10 cm	10 cm
	Length	5 cm	5 cm
Base 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.1 kg	0.1 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05

Gap between Insert A and Base 1		2 cm	2 cm
Spring	Length	14 cm	14 cm
	Spring constant	∞	10^{12} kg/s^2
Piston 1 Displacement when switching to Polarity 2 (Sensor 1)		N/A	1 mm
Piston 1 Displacement when switching to Polarity 1 (Sensor 2)		N/A	0

Table 5.7 Table of comparisons between model #3.2 and #4

The design of model #4 is very similar to model #3 however some modifications have been done to the spring stiffness and the hydraulic diameter. The spring stiffness has been changed to 10^{12} kg/s^2 as investigations with model #3 and model #3.2 show that stiffer spring handles the flashing problem better. The hydraulic diameter has been reduced to $2 \mu\text{m}$ in order to get higher electroosmotic pressure gradient.

In model #3, the polarity of the DC power source is always in the direction of Polarity 1. This polarity generates electroosmotic pressure gradient in the direction of compressing the air. Therefore, when Piston 1 retracts, the electroosmotic pressure generated (Pressure 2) actually resists the movement. However, in model #4, the polarity of the power source is set to switch every time the piston changes direction to always help the piston movement.

5.4.3. Basic working principles

The basic working principles of model #4 are very similar to that of model #3 with one exception, which is the polarity of the DC power source.

In model #3, when the air pressure (Pressure 3) is high enough to push the piston (Piston 1) back, the piston will retract and since the polarity is always in the direction of Polarity 1, it has to resist the electroosmotic pressure generated since Pressure 2 is higher than Pressure 1. However, in model #4, as the piston starts to retract, the polarity of the DC power source is reversed to Polarity 2, such that the electroosmotic pressure is now pushing Base 1 instead of Piston 1. Pressure 1, which is now the summation of the electrolyte pressure (Pressure 2) and the generated electroosmotic

pressure gradient, is now higher than Pressure 2, which is the electrolyte pressure only. By doing this, the electroosmotic pressure gradient will assist the retraction of the piston. Another advantage of applying this mechanism is that the piston movement can now be fully controlled. It is no longer dependent on the air pressure (Pressure 3). It can be retracted and pushed at any point required. Another advantage is due to the characteristic of the electroosmotic pressure that is very high at the beginning before dropping to the steady state value. When the polarity is switched, the generation of the electroosmotic pressure is restarted and hence a very high pressure is generated every time the switching happens. However, this poses another problem since a high pressure pushes the piston or the base a lot faster so that flashing can easily happen here.

In this simulation, a sensor (Sensor 1) is put at the point where the piston displacement is 1 mm. This number was taken from the maximum piston displacement obtained with model #3. The sensor will switch the polarity to the direction of Polarity 2 when Piston 1 activates it. Another sensor (Sensor 2) is put at the initial position of Piston 1. This sensor switches the polarity back to direction of Polarity 1 when the piston activates it.

5.4.4. Results and discussions

The results obtained with model #4 are shown in Figures 5.23.

Figure 5.23 The results from model #4

The results obtained with model #4 show surprising results. The switching mechanism does not only resolve the damping issue, it has even helped the compression process as can be seen from the increasing piston displacement and increasing air pressure with time. The reason behind this behaviour is the inertia effect. As the piston oscillates longer, it gains more momentum due to the continuous push from the pressures involved. This gain of momentum causes the difficulties in stopping the piston movement. One caution with this result is the limited gap available between Piston 1 or Base 1 and Insert A. The displacements of both Piston 1 and Base 1 cannot exceed the gaps and hence they need to be controlled further. Figure 5.23(d) also show that flashing is no longer a problem with model #4.

5.5. Model #5

5.5.1. Introduction

The results obtained with model #4 have shown that the conceptual electrokinetic actuator can perform well. With these results, model #5 utilizes the actuator to drive a simple compressor.

5.5.2. Descriptions

The schematic drawings and dimensions of model #5 are shown in Figures 5.24-5.25.

Figure 5.24 A schematic drawing of model #5

Figure 5.25 Basic dimensions of model #5

Model #		4	5
Applied electric field strength per unit length		200 V/cm DC	200 V/cm DC
Chamber 1	Fluid	Electrolyte	Electrolyte
	Diameter	10 cm	10 cm
	Length	17 cm	17 cm
Insert A	Length	10 cm	10 cm
	Hydraulic Diameter	2 μm	2 μm
Electrolyte	Concentration	0.001 M	0.001 M

Piston 1	Diameter	10 cm	10 cm
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	Thickness	0.2 cm	0.2 cm
	Mass	0.8 kg	0.8 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Piston 1		2 cm	2 cm
Chamber 2	Fluid	Air	Air
	Diameter	10 cm	10 cm
	Length	1 cm	1 cm
Heat Transfer to/from Chamber 2		Ignored	Counted
Base 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.1 kg	0.1 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Base 1		2 cm	5 cm
Spring	Length	14 cm	17 cm
	Spring constant	10^{12} kg/s^2	10^{12} kg/s^2
Piston 1 Displacement when switching to Polarity 2 (Sensor 1)		1 mm	3 cm
Piston 1 Displacement when switching to Polarity 1 (Sensor 2)		0	0
Suction Valve	Mass	N/A	0.001 kg
	Damping constant	N/A	0.1 kg/s
	Spring constant	N/A	1 kg/s²
	Diameter	N/A	3 cm
Suction Pressure		N/A	101.325 kPa
Discharge Valve	Mass	N/A	0.001 kg
	Damping constant	N/A	0.1 kg/s
	Spring constant	N/A	1 kg/s²
	Diameter	N/A	2 cm
Discharge Pressure		N/A	300 kPa
Distance between Stopper 1 and Base 1 at initial position		N/A	0 cm
Distance between Stopper 2 and Piston 1 at initial position		N/A	0 cm
Distance between Stopper 3 and Piston 1 at initial position		N/A	4 cm

Table 5.8 Table of comparisons between model #4 and #5

Model #5 is model #4 with the addition of two valves at Chamber 2. Valve A is the Suction Valve which opens when the pressure in Chamber 2 is low enough and valve B is the Discharge Valve which opens when the pressure in Chamber 2 is high enough. The diameter of Valve A is 3 cm and the diameter of Valve B is 2 cm. The suction pressure is set to be 101.325 kPa while the discharge pressure is set to be 300 kPa. Three stoppers have been installed to prevent Piston 1 and Base 1 to move beyond the permissible boundaries. They are located at the initial positions of Piston 1 and Base 1 and at the point where the piston displacement is 4 cm. Two sensors to switch the

polarity of the power source are installed. Sensor 1 is located at the point where piston displacement is 3 cm and Sensor 2 is located at Stopper 2.

The adiabatic assumption in the air process has been dropped, and so there is heat interaction between the air in Chamber 2 and the surroundings.

The gap between Base 1 and Insert A has been lengthened to be 5 cm to allow more space for the base to move.

5.5.3. Basic working principles

The working principles behind model #5 are identical to model #4. However, since this model has two valves, the mass of the air is no longer constant.

As the DC power source is turned on with the polarity in the direction of Polarity 1, Piston 1 moves to compress the air. The air pressure (Pressure 3) will eventually rise to a level of the discharge pressure. This pressure will open the discharge valve (Valve B) and so air is discharged. The air will still be discharged as long as the pressure is above the discharge pressure. As Piston 1 move further, it will activate Sensor 1, the polarity is reversed to Polarity 2 and Piston 1 will retract. When the air pressure (Pressure 3) has dropped below the discharge pressure, the discharge valve (Valve B) closes and the discharging process stops. As Piston 1 retracts further, the air pressure (Pressure 3) will eventually drop to the suction pressure level. With this low air pressure, the suction valve (Valve A) will open and air is sucked in until the air until a point where the air pressure has risen back to above the suction pressure. Piston 1 will keep on retracting until it hits Stopper 2. When it hits this stopper, the piston will stop moving and Sensor 2 is activated so that the polarity is switched back to Polarity 1 to push the piston to compress the air again.

The change of mass of air in Chamber 2 follows eq. (4.40) while the heat transferred to or from Chamber 2 is governed by eq. (4.33). The movements of the valves follow the spring equation in eq. (4.43). The constants of both the valves are:

Valve spring constant, $k = 1kg / s^2$

Valve damping constant, $R = 0.1kg / s$

Valve mass, $m_v = 0.001kg$

5.5.4. Results and discussions

The results obtained with model #5 are shown in Figure 5.26-5.27. The electrolyte pressure will no longer be shown unless when flashing occurs during the process.

Figure 5.26 The results from model #5

Figure 5.27 The P-V diagram of model #5

The results in Figures 5.26-5.27 show that the proposed electrokinetic compressor of model #5 can perform like a common reciprocating compressor with a frequency of 100 Hz. The P-V diagram indicates that the losses are quite significant as can be seen from the big hump at the discharging process and the smaller hump at the suctioning process. Improvements to the model can be done to reduce these losses and to increase the frequency. The different lines observed in the P-V diagram during the compression of the air and the discharging processes are due to the condition of the system which is still in its unsteady state causing the different path from the 1st cycle taken during 2nd cycle.

5.6. Model #6

5.6.1. Introduction

The results obtained from model #5 show that the conceptual electrokinetic compressor can perform like a common reciprocating compressor. The design can be improved further by adding a displacement magnifier. This improvement is adopted in model #6

5.6.2. Descriptions

The schematic drawings and the dimensions of model #6 are shown in Figures 5.28-5.29.

Figure 5.28 A schematic drawing of model #6

Figure 5.29 Basic dimensions of model #6

Model #		5	6
Applied electric field strength per unit length		200 V/cm DC	200 V/cm DC
Chamber 1	Fluid	Electrolyte	Electrolyte
	Diameter	10 cm	10 cm
	Length	14 cm	14 cm
Insert A	Length	10 cm	10 cm
	Hydraulic Diameter	2 μm	2 μm
Electrolyte	Concentration	0.001 M	0.001 M
Piston 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.8 kg	0.8 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Piston 1		2 cm	2 cm

Chamber 2	Fluid	Air	Air
	Diameter	10 cm	10 cm
	Length	5 cm	5 cm
Heat Transfer to/from Chamber 2		Counted	Counted
Base 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.1 kg	0.1 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Base 1		5 cm	5 cm
Spring	Length	17 cm	17 cm
	Spring constant	10^{12} kg/s^2	10^{12} kg/s^2
Piston 1 Displacement when switching to Polarity 2 (Sensor 1)		3 cm	4 cm
Piston 1 Displacement when switching to Polarity 1 (Sensor 2)		0	0
Suction Valve	Mass	0.001 kg	0.001 kg
	Damping constant	0.1 kg/s	0.1 kg/s
	Spring constant	1 kg/s ²	1 kg/s ²
	Diameter	3 cm	3 cm
Suction Pressure		101.325 kPa	101.325 kPa
Discharge Valve	Mass	0.001 kg	0.001 kg
	Damping constant	0.1 kg/s	0.1 kg/s
	Spring constant	1 kg/s ²	1 kg/s ²
	Diameter	2 cm	2 cm
Discharge Pressure		300 kPa	300 kPa
Distance between Stopper 1 and Base 1 at initial position		0 cm	0 cm
Distance between Stopper 2 and Piston 1 at initial position		0 cm	0 cm
Distance between Stopper 3 and Piston 1 at initial position		4 cm	N/A
Distance between Stopper 3 and Piston 2 at initial position		N/A	4 cm
Piston 2	Diameter	N/A	5 cm
	Thickness	N/A	0.1 cm
	Mass	N/A	0.1 kg
	Static Friction Coefficient	N/A	0.1
	Dynamic Friction Coefficient	N/A	0.05
Chamber 3	Fluid	N/A	Oil
	Diameter	N/A	10 cm, 5 cm
	Length	N/A	3 cm, 3 cm

Table 5.9 Table of comparisons between model #5 and #6

Model #6 is model #5 but with a piston displacement magnification chamber (Chamber 3) between Chamber 1 and Chamber 2 and a piston in between Chamber 1 and Chamber 3 called Piston 2. Hence, model #6 has 3 chambers (Chamber 1, Chamber 2 and Chamber 3), 2 pistons (Piston 1 and Piston 2) and 1 movable base (Base 1). The dimensions of each of them are shown in Figure 5.29 and Table 5.9.

Chamber 3 is filled with incompressible fluid such as oil and it magnifies the displacement of Piston 1 by transferring the displacement of Piston 2. The

magnification ratio is 4 since the ratio of the cross-sectional areas of Piston 2 and Piston 1 is 1:4.

Like model #5, three stoppers are installed at the positions shown in Figure 5.29. One sensor to change the DC power source polarity to Polarity 2 (Sensor 1) is installed at Stopper 3, while another sensor to change the power source polarity to Polarity 1 (Sensor 2) is installed at Stopper 2.

5.6.3. Basic working principles

The general principles behind model #6 are similar to model #5. One major difference is the presence of Chamber 3 and Piston 2. As the DC power source is turned on, an electroosmotic pressure gradient in the direction of Polarity 1 is generated. This pressure will push Piston 2 to compress the incompressible fluid in Chamber 3. The fluid will therefore push Piston 1 to compress the air in Chamber 2. The incompressibility assumption means the volume of fluid displaced by the movement of Piston 2 will be the same as the volume of the fluid displaced to push Piston 1. However, since Piston 2 is bigger than Piston 1, the displacement of Piston 1 will be dependent on the ratio between the areas of Piston 2 and Piston 1. In this model #6, the ratio of the areas is 4:1 and so it is expected that the displacement of Piston 1 is 4 times of the displacement of Piston 2.

The equations governing the movements of Piston 2, Piston 1 and the magnification processes are eq. (5.6)-(5.8) respectively.

$$\left(p_{liq} + L \frac{dP}{dz} \right) A_{inc} - p_{inc} A_{inc} - F_{inc} = m_{inc} \frac{d^2 x_{inc}}{dt^2} \quad (5.6)$$

$$p_{inc} A_{air} - p_{air} A_{air} - F_{air} = m_{air} \frac{d^2 x_{air}}{dt^2} \quad (5.7)$$

$$x_{inc} A_{inc} = x_{air} A_{air} \Rightarrow \frac{d^2 x_{inc}}{dt^2} A_{inc} = \frac{d^2 x_{air}}{dt^2} A_{air} \quad (5.8)$$

where:

A_{inc} = cross-sectional area of Piston 2

p_{inc} = pressure of the incompressible fluid in Chamber 3

F_{inc} = friction force at Piston 2

m_{inc} = mass of Piston 2

x_{inc} = displacement of Piston 2

A_{air} = cross-sectional area of Piston 1

p_{air} = air pressure (Pressure 3)

F_{air} = friction force at Piston 1

m_{air} = mass of Piston 1

x_{air} = displacement of Piston 1

Piston 1 will then move to compress the air in Chamber 2 until it activates Sensor 1 to switch the polarity of the power source to Polarity 2. Meanwhile the air pressure (Pressure 3) keeps increasing until it opens the discharge valve and therefore the air is discharged. With the polarity now in the direction of Polarity 2, Piston 2 retracts and again this movement is transferred and magnified to Piston 1. Piston 2 retracts all the way until it touches Stopper 2 and the polarity is switched back to the direction of Polarity 1.

5.6.4. Results and discussions

Figure 5.30 The results from model #6

Figure 5.31 The P-V Diagram of model #6

The results obtained show that the addition of a displacement magnification chamber (Chamber 3) increases the frequency of the system. While the frequency of model #5 is 100 Hz, the frequency of model #6 is 200 Hz. This is linearly dependent to ratio of the sizes of the electroosmotic chamber (Chamber 1) and the compression chamber

(Chamber 2). While in model #5, the ratio is 1:1, in model #6 it is 2:1 and hence the frequency is doubled.

The result in Figure 5.30(a) also shows that the displacements of Piston 1 are as expected, which is four times of Piston 2. As mentioned in Section 5.6.3, this is because of the ratio of the area of the pistons, where Piston 2 has twice the diameter of Piston 1, which is translated to a 4:1 area ratio.

Another important thing to be observed is the high loss generated as shown by the big humps in the suctioning and discharging processes in the P-V diagram in Figure 5.31.

These losses can be minimized by using valves that are easier to move. Figure 5.32 shows the P-V diagram obtained with valves #6.2 with properties as shown in Table 5.10.

Valves model #		6	6.2
Suction Valve	Mass	0.001 kg	0.0001 kg
	Damping constant	0.1 kg/s	0.1 kg/s
	Spring constant	1 kg/s ²	1 kg/s ²
	Diameter	3 cm	3 cm
Suction Pressure		101.325 kPa	101.325 kPa
Discharge Valve	Mass	0.001 kg	0.0001 kg
	Damping constant	0.1 kg/s	0.1 kg/s
	Spring constant	1 kg/s ²	1 kg/s ²
	Diameter	2 cm	2 cm
Discharge Pressure		300 kPa	300 kPa

Table 5.10 Table of comparisons between valves #6 and #6.2

Figure 5.32 The P-V Diagram of model #6 with valves #6.2

Figure 5.32 shows that with valves that are easier to move, the losses can be reduced.

Another way to reduce the losses is by using bigger valves. Figure 5.33 shows the effects of using smaller valves to the P-V diagram. This was done with valves #6.3 with the diameters of 2 cm for the suction valve and 1 cm for the discharge one as shown in Table 5.11.

Valves model #		6	6.3
Suction Valve	Mass	0.001 kg	0.001 kg
	Damping constant	0.1 kg/s	0.1 kg/s
	Spring constant	1 kg/s ²	1 kg/s ²
	Diameter	3 cm	2 cm
Suction Pressure		101.325 kPa	101.325 kPa
Discharge Valve	Mass	0.001 kg	0.001 kg
	Damping constant	0.1 kg/s	0.1 kg/s
	Spring constant	1 kg/s ²	1 kg/s ²
	Diameter	2 cm	1 cm
Discharge Pressure		300 kPa	300 kPa

Table 5.11 Table of comparisons between valves #6 and #6.3

Figure 5.33 The P-V Diagram of model #6 with valves #6.3

Figure 5.33 shows that with smaller valves, the losses are a lot bigger now and so it is always recommended to use bigger and lighter valves to reduce the losses.

The effects of the mass of Piston 1 to the displacement of Piston 1 and the air pressure are shown in Figure 5.34.

Figure 5.34 The mass of Piston 1 effects

The results shown in Figure 5.34 interestingly suggest that the mass of Piston 1 does not affect the compression process at all. This is caused by the characteristic of the equations of motion of the pistons which are heavily dependent on the pressure balance across them. Because of this characteristic, the only way to increase the maximum air pressure generated is by increasing Pressure 2, which means increasing the electroosmotic pressure gradient. Modifying the mass or the size of the pistons will not change the maximum air pressure attainable.

The perfectly incompressible fluid assumption in Chamber 3 also causes the movement of Piston 1 to be wholly dictated by Piston 2. Therefore, although mass of Piston 1 is modified, its movement is not affected. This is of course not really true in reality. Heavier Piston 1 has more inertia and causes more friction; therefore in reality the movements will be more difficult than the lighter ones.

From the results in Figures 5.30(a) and 5.30(b), it can also be seen that Piston 1 move further back than its initial position when retracting. It is also realized that there is nothing stopping the movement of Piston 2 when it compresses the incompressible fluid in Chamber 3. This situation is unwanted because the pistons may eventually move beyond limit. Therefore, more stoppers are needed to control the displacements of the pistons (Piston 1 and Piston 2) and the base (Base 1) better. In addition of the installed three stoppers installed in model #6, two additional stoppers are installed

Suction Valve	Mass	0.001 kg	0.001 kg
	Damping constant	0.1 kg/s	0.1 kg/s
	Spring constant	1 kg/s ²	1 kg/s ²
	Diameter	3 cm	3 cm
Suction Pressure		101.325 kPa	101.325 kPa
Discharge Valve	Mass	0.001 kg	0.001 kg
	Damping constant	0.1 kg/s	0.1 kg/s
	Spring constant	1 kg/s ²	1 kg/s ²
	Diameter	2 cm	2 cm
Discharge Pressure		300 kPa	300 kPa
Distance between Stopper 1 and Base 1 at initial position		0 cm	0 cm
Distance between Stopper 2 and Piston 1 at initial position		0 cm	0 cm
Distance between Stopper 3 and Piston 2 at initial position		4 cm	4 cm
Distance between Stopper 4 and Piston 2 at initial position		N/A	2 cm
Distance between Stopper 5 and Piston 1 at initial position		N/A	0 cm
Piston 2	Diameter	5 cm	5 cm
	Thickness	0.1 cm	0.1 cm
	Mass	0.1 kg	0.1 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Chamber 3	Fluid	Oil	Oil
	Diameter	10 cm, 5 cm	10 cm, 5 cm
	Length	3 cm, 3 cm	3 cm, 3 cm

Table 5.12 Table of comparisons between model #6 and model #6.4

The results are shown in Figures 5.36-5.37.

Figure 5.36 The results from model #6.4

Figure 5.37 The P-V Diagram of model #6.4

The results show that with the addition of the stoppers, the displacements of the pistons and the base are now fully controlled.

Observing the results obtained suggest that higher discharge pressures are also possible with the design. Therefore, in model #6.5, the discharge pressure is set to 600 kPa. The illustration of model #6.5 is shown in Figure 5.38.

Figure 5.38 Basic dimensions of model #6.5

Model #		6.4	6.5
Applied electric field strength per unit length		200 V/cm DC	200 V/cm DC
Chamber 1	Fluid	Electrolyte	Electrolyte
	Diameter	10 cm	10 cm
	Length	14 cm	14 cm
Insert A	Length	10 cm	10 cm
	Hydraulic Diameter	2 μ m	2 μ m
Electrolyte	Concentration	0.001 M	0.001 M

Piston 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.8 kg	0.8 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Piston 1		2 cm	2 cm
Chamber 2	Fluid	Air	Air
	Diameter	10 cm	10 cm
	Length	5 cm	5 cm
Heat Transfer to/from Chamber 2		Counted	Counted
Base 1	Diameter	10 cm	10 cm
	Thickness	0.2 cm	0.2 cm
	Mass	0.1 kg	0.1 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Gap between Insert A and Base 1		5 cm	5 cm
Spring	Length	17 cm	17 cm
	Spring constant	10^{12} kg/s ²	10^{12} kg/s ²
Piston 1 Displacement when switching to Polarity 2 (Sensor 1)		4 cm	4 cm
Piston 1 Displacement when switching to Polarity 1 (Sensor 2)		0	0
Suction Valve	Mass	0.001 kg	0.001 kg
	Damping constant	0.1 kg/s	0.1 kg/s
	Spring constant	1 kg/s ²	1 kg/s ²
	Diameter	3 cm	3 cm
Suction Pressure		101.325 kPa	101.325 kPa
Discharge Valve	Mass	0.001 kg	0.001 kg
	Damping constant	0.1 kg/s	0.1 kg/s
	Spring constant	1 kg/s ²	1 kg/s ²
	Diameter	2 cm	2 cm
Discharge Pressure		300 kPa	600 kPa
Distance between Stopper 1 and Base 1 at initial position		0 cm	0 cm
Distance between Stopper 2 and Piston 1 at initial position		0 cm	0 cm
Distance between Stopper 3 and Piston 2 at initial position		4 cm	4 cm
Distance between Stopper 4 and Piston 2 at initial position		2 cm	2 cm
Distance between Stopper 5 and Piston 1 at initial position		0 cm	0 cm
Piston 2	Diameter	5 cm	5 cm
	Thickness	0.1 cm	0.1 cm
	Mass	0.1 kg	0.1 kg
	Static Friction Coefficient	0.1	0.1
	Dynamic Friction Coefficient	0.05	0.05
Chamber 3	Fluid	Oil	Oil
	Diameter	10 cm, 5 cm	10 cm, 5 cm
	Length	3 cm, 3 cm	3 cm, 3 cm

Table 5.13 Table of comparisons between model #6.4 and model #6.5

The results are shown in Figures 5.39-5.40.

Figure 5.39 The results from model #6.5

Figure 5.40 The P-V Diagram of model #6.5

The results obtained with model #6.5 again suggested that the compressor design can perform well even with discharge pressure of 600 kPa. In fact, the results also indicated that the discharge pressure can be further increased. However, the P-V diagram shows that the losses are very big in this case and so a multi-stage compressor needs to be considered if a very high discharge pressure is needed.

From all the results obtained with model #6 and its variations, it can be concluded that the conceptual electrokinetic actuator can perform just like a common reciprocating compressor. More improvements to the design to increase the performance are still possible.

5.7. Summary

Six main models with their variations have been simulated and presented in this chapter. The summary on the evolution of the model designs is shown in Table 5.14.

Model No	Improvement from the previous design	Method
#1	The utilization of electroosmosis in an actuator	-
#2	An Increase in the piston displacement	Adding a movable base
#3	Overcoming the flashing problem	Adding a spring mechanism
#4	Overcoming the damping problem	Adding the switching mechanism
#5	The application of the actuator to drive a compressor	Adding valves
#6	The improvement of the compressor design	Adding a magnification chamber

Table 5.14 A summary on the evolution of the models

Chapter 6:

EXPERIMENTS:

RESULTS AND DISCUSSIONS

As has been mentioned in Chapter 1, the objectives of the experiments are to collect data on a fully-closed electroosmotic system and then to compare them with the theoretical results. The equipments used and the steps taken to carry out the experiments on a fully-closed channel electroosmotic flow are presented in this chapter.

6.1. List of equipments and apparatus

The experiments were conducted in Thermal and Fluids Research Lab 1, School of Mechanical and Aerospace Engineering, Nanyang Technological University. The equipments available can be grouped into:

1. Equipments to perform electroosmotic flow
2. Equipments to monitor electroosmotic flow
3. Equipments to collect experimental data

6.1.1. Equipments and apparatus to perform electroosmotic flow

As any other electroosmosis experiments, some basic equipments and apparatus are needed to perform the electroosmotic flow. These include a power source, electrodes, microchannels, electrolyte, and reservoirs if needed.

6.1.1.1. Power supply

Figure 6.1 The Spellman CZE1000R High DC Voltage Power Supply

To provide the high DC voltage needed to induce electroosmotic flow, a Spellman CZE1000R high DC voltage power source is employed. It can provide voltages up to 30 kV DC and allows both voltage and current output to be controlled. This apparatus is connected to a computer to allow an automatic control and easier data analysis.

6.1.1.2. Electrodes

Figure 6.2 The Aldrich Chem. Co. 100 μ m diameter platinum wire (in red circle)

Platinum wires from Aldrich Chemicals Co. with diameters of 100, 200 and 500 micron are used for the electrodes. In principle, any conductive material can be electrodes, however, when the voltage is high enough, electrolysis will occur for some materials. Platinum is one of the materials with less electrolysis even at a very high voltage, and that makes platinum wire to be one of the ideal electrodes.

6.1.1.3. Microchannels and cutter

Figure 6.3 The Polymicro Technologies 30 μm internal diameter microcapillary (gold colour)

Figure 6.4 The Agilent Technologies capillary cutter

Fused silica capillaries from Polymicro Technologies are used to be the microchannels in where the electroosmosis should happen. These capillaries are made of glass with polyimide coating to enhance its endurance and flexibility. This coating can be removed by using some chemicals or by burning. There are two types of these capillaries, they are circular and rectangular capillaries. A special cutter by Agilent Technologies is used to cut the capillaries smoothly to the length needed.

6.1.1.4. Electrolyte

Figure 6.5 The Aldrich Chem. Co. salt and salt solution

The electrolyte tested is prepared by mixing DI water produced by NTU Micromachines Centre with a proportion of salt from Aldrich Chemicals Co. The salts available include KCl, KOH, NaCl, etc.

6.1.1.5. Reservoir

Figure 6.6a The first example of reservoir

Figure 6.6b The second example of reservoir

There are many types of reservoirs available to be used. Two of them are shown in Figure 6.6a and Figure 6.6b. The first one consists of two blocks of reservoirs that are free to slide along the support. The support has electrode knobs for easy connection of electrodes to the reservoirs. Both the support and the blocks are made of Teflon material. The second example is made of two big glass tubes. The tubes have two holes at their sides for the electrodes and microchannels.

6.1.2. Equipments and apparatus to monitor electroosmotic flow

There are many methods to monitor the electroosmotic flow behaviors. One of the most common methods is the current method which is employed in this project. By employing this method, some special equipments and apparatus are needed.

6.1.2.1. I/O Card

Figure 6.7 The National Instrument BNC-2110 data acquisition card

A data acquisition card from National Instrument is employed to connect the high voltage DC power supply mentioned in section 6.1.1.1 to a computer. It is able to transmit current signal from the experimental setup to be monitored by the computer and to allow remote control of input voltage.

6.1.2.2. Computer

Figure 6.8 The IBM Pentium 3 desktop computer

The data acquisition card mentioned in section 6.1.2.1 is connected to an IBM Pentium 3 desktop computer from the high-voltage DC power supply. A computer program is installed in this computer in order to perform monitoring of current signal and remote control of input voltage from the power supply.

6.1.2.3. Monitoring software

Figure 6.9 The LabVIEW 6.1 interface

LabVIEW version 6.1 software from National Instrument is installed into the computer mentioned in section 6.1.2.2 to do the monitoring and data collection of the current and the controlling of the input voltage.

6.1.3. Equipments to collect experimental data

It has been mentioned that the goal of the experiments is to confirm that the theoretical model presented in Chapter 4 is correct. Therefore, some special equipment to collect relevant data from the rigs set is needed. These include a pressure transducer to detect the existence of pressure increase and a scope to record the readings. However, it is worthy to note that using pressure transducer is not the only method to measure pressure.

6.1.3.1. Pressure transducer

Figure 6.10 The Honeywell 24PC series pressure transducer

Figure 6.11 The detailed drawings of a 20PC series pressure transducer

Honeywell 24PC and 26PC series pressure transducers are employed to measure pressure generated by the electroosmotic flow. When using the very small range type, these transducers can measure very small pressure accurately. The output is in the form of voltage and each of the transducers requires 10 V of input voltage to operate.

6.1.3.2. DC power supply

Figure 6.12 The Abletech regulated DC power supply 3003AD4

An Abletech regulated DC power supply is employed to supply input power for the pressure transducers. This power supply can supply up to 60 V of DC electric potential.

6.1.3.3. Digital scope

Figure 6.13 The DL716 16-channel Yokogawa digital scope

The pressure transducers mentioned in section 6.1.4.1 are connected to a Yokogawa digital scope to provide monitoring and data collection. The scope has a 10.4 inch TFT colour LCD with an in-built printer which allows immediate screen print-out. Data can also be transferred to the computer by means of a floppy in the form of a text file.

6.2. Presentation and discussions on the experimental set-ups

There are many set-ups built and tested, however only a few gave promising results.

Due to the limitation of space, only some of the set-ups tested will be discussed here.

6.2.1. Set-up #1

Set-up #1 is made of four 3 cm long capillaries, a pressure transducer and a reservoir.

One side of the capillaries is in the transducer, while the other side is submerged in the reservoir. An anode is dipped into the reservoir, while a cathode is in the transducer.

Figure 6.14 A diagram of set-up #1

Figure 6.15 A picture of set-up #1

The space inside the transducer and in the capillaries must be fully filled with the electrolyte. This is to prevent disturbances from trapped gas. A syringe with a plastic tube is used to fill the capillaries, as shown in Figure 6.15.

Figure 6.16 The use of syringe to fill a capillary

It was hypothesized that as an electric field is applied, electroosmotic flow will be generated in the direction to the cathode. With this flow, pressure will build up in the transducer and so the transducer will read this pressure.

Some tests were carried out using set-up #1, however no result was obtained. There are two most plausible reasons. First reason is the very long time needed to reach the steady state pressure gradient value. The long time needed is due to the very small flowrate involved which is in the range of microliter per minute. This long time needed is longer than the time needed to produce bubbles by electrolysis process, and it is predicted that this is one of the most plausible reasons for the failure. The second reason is imperfect sealing that causes leaking. The shape and the scale of the items used in set-up #1 makes it very difficult to get perfect sealing. In addition, the very small flowrate generated makes even a very small leak fail the test. There are many varieties of sealants tried, they include silicones, super glue and epoxy as shown in Figure 6.16. Each sealant has its own unique characteristics and hence its own advantages and disadvantages. Silicone tends to be softer but is faster to dry as compared to epoxy that is the hardest but takes a long time to dry, while super glue is too fluid and is not recommended to be used for sealing purposes.

Figure 6.17 Various types of sealants used

6.2.2. Set-up #2

Set-up #2 is fabricated from a block of acrylic glass. Two concentric holes were drilled through the block, one is for the pressure transducer and the other one is for a plastic tube to insert a capillary to be connected directly to the transducer and an electrode. Two more holes were drilled for clamping purpose. The main idea behind set-up #2 is to prevent leaking in set-up #1 by modifying the sealing method. So unlike set-up #1, this set-up does not use any sealant to seal the transducer, instead it uses o-rings.

Figure 6.18 Detailed drawings of set-up #2

Figure 6.19 A picture of set-up #2

A fully filled capillary and a reservoir filled with the electrolyte are then prepared. After the tube and the transducers were inserted into their respective places, electrolyte was injected into the transducer and sealant was then applied to seal the gaps. After that, one side of the capillary is dipped in the reservoir while the other one is inserted through the tube to meet the transducer. The diagram of the complete set-up is as follow:

Figure 6.20 Set up for set-up #2

It was hypothesized that when DC electric field is applied, electrolyte flows from the reservoir to the transducer and causing pressure build up at the transducer side.

A lot of tests were carried out with set-up #2 with many parameters varied including the length and size of capillary, the type of electrolyte. However, no result was

obtained. The most possible explanation is again the very small flowrate involved. The number of the capillary used here, which is only one, makes it worse. Leakage through gaps in between the o-rings is also a possible explanation for the failure.

6.2.3. Set-up #3

While set-up #2 uses only one capillary, set-up #3 uses a bundle of capillaries. So instead of having a short tube to insert a capillary, a long glass column filled with many bare capillaries is used. This column is inserted to the transducer hole just like the capillary in set-up #2. The idea is to increase the flow rate so that the time needed for the establishment of pressure can be shortened.

The whole set up is as follow:

Figure 6.21 A diagram of set-up #3

Figure 6.22 A picture of set-up #3

By using this set-up, a promising sign of result was obtained. A reading was shown in the scope when the electric field was applied. This reading was, however, not repeatable. Some possible reasons of the unrepeatability are the failure to do the sealing well enough and the inability of the transducers to perform the task.

6.2.4. Set-up #4

Set-up #4 is designed after realizing that the pressure transducer available may not be suitable to perform the task assigned. Fortunately, using pressure transducer is not the only method available to measure pressure. There are some other methods that can be adopted to do the task, such as the manometer principle. This principle is the principle behind set-up #4. Two reservoirs are connected with a bundle of 3 cm long capillaries. Each reservoir has a 3 mm internal diameter, 35 cm long vertical column on it. As DC electric field is applied, electroosmotic flow should occur and hence displacing water from the anode side to the cathode side until a point where the pressure at the cathode side is high enough for the electroosmotic flow to overcome. This pressure is the closed-channel electroosmotic flow pressure that needs to be tapped.

Figure 6.23 A diagram of set-up #4

Figure 6.24 Detailed drawing of the reservoirs of set-up #4

Figure 6.25 A picture of set-up #4

A few tests were done using set-up #4 and some of them gave results which gave us optimism in continuing to use the design. However generation of bubbles around the electrodes that eventually isolated the electrodes from the electrolyte gives a lot of problems. The large columns used also caused longer time spent to get the results. So we decided to improve the design and set-up #5 came out.

6.2.5. Set-up #5

Generation of bubbles around the electrodes caused by electrolysis is unavoidable but it can be very disturbing like in set-up #4. Therefore it is important to find a way to keep the bubbles not to disturb the electroosmotic process. One way is by modifying the shape of the reservoir such that the bubbles are kept far away from the capillaries and also not to cover the electrode completely. This modification of reservoir is adopted in set-up #5. Another modification adopted is the smaller column, from 3 mm i.d. in set-up #4 to 1 mm i.d. in set-up #5 to speed up the process.

Figure 6.26 A diagram of set-up #5

Figure 6.27 Detailed drawing of set-up #5

Figure 6.28 A picture of set-up #5

The results from set-up #5 are shown in Figure 6.29-6.36.

Figure 6.29 The pressures generated by a 5e-4M NaCl solution in a 100 μ m i.d., 5cm long rectangular capillary

Figure 6.30 The pressures generated by a 1e-2M NaCl solution in a 100 μ m i.d., 5cm long rectangular capillary

Figure 6.31 The pressures generated by a 5e-4M NaCl solution in a 75 μ m i.d., 5cm long rectangular capillary

Figure 6.32 The pressures generated by a 1e-2M NaCl solution in a 100 μ m i.d., 5cm long rectangular capillary

Figure 6.33 The concentration effects on the pressures generated by a 100 μ m i.d., 5cm long rectangular capillary

Figure 6.34 The concentration effects on the pressures generated by a 75 μ m i.d., 5cm long rectangular capillary

Figure 6.35 The hydraulic diameter effects on the pressures generated by a 5e-4M NaCl solution in 5cm long rectangular capillaries

Figure 6.36 The hydraulic diameter effects on the pressures generated by a 1e-2M NaCl solution in 5cm long rectangular capillaries

Figures 6.29-6.32 show the effects of applied electric field to the pressure generated.

It is unfortunate that due to the electric current limitations of the equipment, the applied electric field ranges have to be different in Figure 6.29 and 6.30. As it is known, as the concentration of the solution goes higher, the current also increases with the same potential difference. This is also the case here. When the concentration is 5e-4 M NaCl, the potential difference can go up to 3000 V before it reaches the current limit of the equipment. However, when the concentration is higher at 1e-2 M

NaCl, this current limit is already reached at a potential difference of around 1500 V. Therefore, the difference in voltage range is unavoidable here.

In general, Figures 6.29-6.32 show that the pressure varies linearly with the applied electric field. This is in good agreement with the theoretical predictions shown in Figure 4.2. However there are some obvious discrepancies at the lower voltage region between the experimental and the theoretical results in this region. The reason behind these discrepancies is mainly due to the inability of the power supply to supply accurate voltage at this region. Another possible factor is the wrong zeta potential values assumption for this region. It was assumed that zeta potential is independent of the applied electric field, and zeta potential values at the highest voltages were used to get the theoretical predictions in these experiments. However, it was found that actually zeta potential changes with the applied electric field and hence the discrepancies were present.

It can also be seen that the disparity of values is quite observable at some points. This is thought to be mainly because of the effects of the change in the solution's temperature and the electrolysis process. As electric field is applied, the temperature of the solution may change because of Joule heating. It was found that this change can be as high as 15 degrees. This change of temperature will change the properties of the solution which in turn will affect the pressure generated. The application of electric field will also cause electrolysis process in the solution. This process will affect the chemical composition of the solution. This change in composition will also change the properties of the solution and so the pressure will be affected.

Figures 6.33-6.34 show the effects of change in solution concentration in the electrolyte to the pressures generated. It was found that different from the theoretical predictions shown in Figure 4.5, the more dilute solution gave higher pressures in the

100 μ m capillary, while it gave slightly lower pressures in the 75 μ m one. The reason behind this discrepancy is the independent parameter assumption taken in the theoretical predictions. It was assumed that a change in the solution concentration does not affect any other parameter. However, this is not the case in reality since a change in the concentration will change the properties of the electrolyte and also the zeta potential value. Therefore, the real effects of a change in solution concentration are not as simple as what were predicted.

Figures 6.35-6.36 show the effects of hydraulic diameter to the pressure generated. It was found that smaller diameter capillaries give higher pressure, which is what is predicted by the theoretical prediction shown in Figure 4.4. However, due to the independent parameter assumption adopted in the theoretical prediction, the effects of change of zeta potential with the size of the channel were ignored. In fact, it was assumed that zeta potential is independent of the channel size. However, it was found that zeta potential values are actually dependent on the size of the channel. Because of this, the results obtained from the theoretical predictions are quite different from the experimental results.

Chapter 7:

CONCLUSIONS AND RECOMMENDATIONS

7.1. Conclusions

After going through all the chapters in this thesis and observing the results obtained, some conclusions can be drawn.

The simulations showed the evolution of the models from the simple electrokinetic actuator to the conceptual electrokinetic compressor. From the results obtained, it can be concluded that electroosmotic flow can be used to drive an actuator. This actuator can then be used to drive a compressor. The simulations showed that the conceptual electrokinetic compressor can perform like a common reciprocating compressor. The simulations also showed that the electrokinetic compressor has some different characteristics from the common reciprocating compressors. These include the heavy dependence of the piston movement to the pressure in the compression chamber and hence the unique movement behaviours of the piston were observed.

The experiments on the closed-channel electroosmotic flows faced three major problems:

1. Leakage, caused by the difficulties in having proper sealing due to the complexity in shape and the small size of the set-ups. Leakage is a big problem since the flow rates involved in electroosmotic flows are very small, so even a small leakage can be disastrous.

2. Electrolysis, which causes the change in chemical composition of solution and the generation of bubbles. Electrolysis is caused by the requirements to use high potential difference.
3. Evaporation, caused by the condition of the surrounding air and the possible increase in water temperature because of the applied electric field.

The effects of three parameters, namely the applied electric field, the solution concentration and the hydraulic diameter of the channel were investigated experimentally. The results showed some discrepancies between the theoretical values and the actual pressure values, mainly because of the independent parameter assumption adopted in the theoretical predictions. It was also found that the effects of change in solution's temperature and the electrolysis cannot be ignored either.

7.2. Recommendations

The simulations showed that the conceptual electrokinetic compressor can function like a normal reciprocating compressor. More in-depth analysis on the effects of each parameter to the overall compressor's performance needs to be carried out in the future to optimize the design. Possibilities to incorporate more accurate assumptions, such as the dependence of zeta potential to applied electric field, need to be observed as well. Better methods to overcome all the limitations mentioned also need to be investigated further. The possibility of using a multi-stage electrokinetic compressor to achieve higher pressure with better overall performance should also be investigated. Further experimental investigations on the effects of the parameters to the electroosmotic pressure generated need to be carried out in the future to better understand the behaviour. Methods to eliminate the three major problems mentioned in section 7.1 also need to be investigated. In addition, although the manometer

principle adopted for set-up #5 worked well, a more accurate method to measure the electroosmotic pressure also needs to be found in the future.

Finally, after collecting sufficient simulation and experimental results, the project should proceed to the fabrication of the prototype of an electrokinetic compressor.

LIST OF REFERENCES

1. Sharp, Kendra V., et al. (2002). Liquid Flows in Microchannels. The MEMS Handbook. USA: CRC Press.
2. Hunter, Robert J. (1988). Zeta Potential in Colloid Science – Principles and Applications. London: Academic Press Limited.
3. <http://www.ornl.gov/hgmis> (March 4, 2004).
4. Chen, Chuan-Hua & Santiago, Juan G. (2002). A planar electroosmotic micropump, Journal of Microelectromechanical System. 11, no. 6, pp. 672-683.
5. Zeng, Shulin, et al. (2001). Fabrication and characterization of electroosmotic micropumps, Sensors and Actuators B. 79, pp. 107-114.
6. Paul, Phillip H., Arnold, Don W. & Rakestraw, David J. Electrokinetic generation of high pressures using porous microstructures.
7. Mosier, B.P, et al. (2003). High-Pressure Microhydraulic Actuator, Proceedings of IMECE'03. IMECE2003-44401.
8. <http://chem.ch.huji.ac.il/~eugeniik/history/debye.html> (March 6, 2004).
9. Probstein, Ronald F. (1994). Physicochemical hydrodynamics an Introduction. 2nd ed. New York: John Wiley & Sons, Inc.
10. Lyklema, J. (2003). Electrokinetics after Smoluchowski. Colloids and Surfaces A: Physicochemical Engineering Aspects. 222, pp.5-14.
11. Marcos, et al. (2004). Dynamic aspects of electroosmotic flow in rectangular microchannels. International Journal of Engineering Science. 42, pp. 1459-1481.

12. Marcos, et al. (2004). Frequency dependent laminar electroosmotic flow in a closed-end rectangular microchannel. Journal of Colloid and Interface Science. 275, pp. 679-698.
13. Marcos. (2003). Dynamic aspects of electroosmotic flow in open and closed rectangular microchannels. School of NTU – MPE, FYP report. Singapore.
14. Mala, Gh. Mohiuddin, et al. (1997). Electrical double layer potential distribution in a rectangular microchannel. Colloids and Surfaces A: Physicochemical and Engineering Aspects. 135, pp. 109-116.
15. Kang, Yuejun, et al. (2002). Dynamic aspects of electroosmotic flow in a cylindrical microcapillary. International Journal of Engineering Science. 40, pp. 2203-2221.
16. Kang, Yuejun, et al. (2002). Electroosmotic flow in a capillary annulus with high zeta potentials. Journal of Colloid and Interface Science. 253, pp. 285-294.
17. Mala, G. Mohiuddin, et al. (1997). Heat transfer and fluid flow in microchannels. International Journal of Heat and Mass Transfer. 40, pp. 3079-3088.
18. Yang, Chun & Li, Dongqing. (1998). Analysis of electrokinetic effects on the liquid flow in rectangular microchannels. Colloids and Surfaces A: Physicochemical and Engineering Aspects. 143, pp. 339-353.
19. Potter, Merle C. & Wiggert, David C. (1997). Mechanics of Fluids. 2nd ed. USA: Prentice Hall, Inc.
20. Mala, G. Mohiuddin, et al. (1997). Flow characteristics of water through a microchannel between two parallel plates with electrokinetic effects. International Journal of Heat and Fluid Flow. 18, pp.489-496.

21. Yang, Chun, et al. (1998). Modeling forced liquid convection in rectangular microchannels with electrokinetic effects. International Journal of Heat and Mass Transfer. 41, pp. 4229-4249.
22. Wan, Qian-Hong. (1997). Effect of electroosmotic flow on the electrical conductivity of packed capillary columns. Journal of Physical Chemistry B. 101, pp. 4860-4862.
23. Kasicka, Vaclav, et al. (2000). External electric field control of electroosmotic flow in non-coated and coated fused-silica capillaries and its application for capillary electrophoretic separations of peptides. Journal of Chromatography B. 741, pp. 43-54.
24. Ghowsi, K. & Gale, R.J. (1991). Field effect electroosmosis. Journal of Chromatography A. 559, pp. 95-101.
25. Lee, C.S., et al. (1990). Analysis of separation efficiency in capillary electrophoresis with direct control of electroosmosis by using an external electric field. Journal of Chromatography A. 559, pp. 133-140.
26. Lee, C.S., et al. (1990). Direct control of the electroosmosis in capillary zone electrophoresis by using an external electric field. Analytical Chemistry. 62, pp. 1550-1552.
27. Hunter, R.J. (1993). Introduction to Modern Colloid Science. Oxford: Oxford University Press.
28. Sze, Alice, et al. (2003). Zeta potential measurement using the Smoluchowski equation and the slope of the current-time relationship in electroosmotic flow. Journal of Colloid and Interface Science. 261, pp. 402-410.
29. Werner, Carsten, et al. (1998). Extended electrokinetic characterization of flat solid surface. Journal of Colloid and Interface Science. 208, pp. 329-346.

30. Zimmermann, Ralf, et al. (2001). Electrokinetic measurements reveal interfacial charge at polymer films caused by simple electrolyte ions. Journal of Physical Chemistry. 105, pp. 8544-8549.
31. Fievet, P., et al. (2000). Determining the zeta potential of porous membranes using electrolyte conductivity inside pores. Journal of Colloid and Interface Science. 235, pp. 383-390.
32. Huisman, I.H. & Tragardh, Gun. (1999). Determining the zeta potential of ultrafiltration membranes using their salt retention. Colloids and Surfaces A: Physicochemical and Engineering Aspects. 157, pp. 261-268.
33. Huisman, I.H., et al. (1998). Determining the zeta-potential of ceramic microfiltration membranes using the electroviscous effect. Journal of Membrane Science. 147, pp. 187-194.
34. Yang, Chun, et al. (2001). Measurement of the zeta potential of gas bubbles in aqueous solutions by microelectrophoresis method. Journal of Colloid and Interface Science. 243, pp. 128-135.
35. Phianmongkhon, Aphirak & Varley, Julie. (2003). ζ potential measurement for air bubbles in protein solutions. Journal of Colloid and Interface Science. 260, pp. 332-338.
36. Hunter, R.J. (2001). Measuring zeta potential in concentrated industrial slurries. Colloids and Surfaces A: Physicochemical and Engineering Aspects. 195, pp. 205-214.
37. Moncho, A., et al. (2001). Comparative study of theories of conversion of electrophoretic mobility into ζ -potential. Colloids and Surfaces A: Physicochemical and Engineering Aspects. 192, pp. 215-226.

38. Sinton, David & Li, Dongqing. (2003). Electroosmotic velocity profiles in microchannels. Colloids and Surfaces A: Physicochemical Engineering Aspects. 222, pp. 273-283.
39. Sinton, David, et al. (2002). Direct and indirect electroosmotic flow velocity measurements in microchannels. Journal of colloid and interface science. 254, pp. 184-189.
40. Reppert, P.M. & Morgan, F.D. (2002). Frequency dependent electroosmosis. Journal of Colloid and Interface Science. 254, pp. 372-383.
41. Wan, Qian Hong. (1997). Effect of electrical double layer overlap on the electroosmotic flow in packed capillary columns. Analytical Chemistry. 69, pp. 361-363.
42. Gleeson, J.P. (2002). Electroosmotic flows with random zeta potential. Journal of Colloid and Interface Science. 249, pp. 217-226.
43. Long, D., et al. (1998). Electroosmotic flows created by surface defects in capillary electrophoresis. Journal of Colloid and Interface Science. 212, pp. 338-349.
44. Fu, L.M., et al. (2002). Analysis of electroosmotic flow with step change in zeta potential. Journal of Colloid and Interface Science. 258, pp. 266-275.
45. Herr, A.E., et al. (2000). Electroosmotic capillary flow with nonuniform zeta potential. Analytical Chemistry. 72, pp. 1053-1057.
46. Santiago, J.G. (2001). Electroosmotic flows in microchannels with finite inertial and pressure forces. Analytical Chemistry. 73, pp. 2353-2365.
47. Kim, M.J., et al. (2002). Electro-osmosis driven micro-channel flows: A comparative study of microscopic particle image velocimetry measurements and numerical simulations. Experiments in Fluids. 33, pp. 170-180.

48. Devasenathipathy, Shankar & Santiago, J.G. (2002). Particle tracking techniques for electrokinetic microchannel flows. Analytical Chemistry. 74, pp. 3704-3713.
49. Yang, R.J., et al. (2001). Electroosmotic flow in microchannels. Journal of Colloid and Interface Science. 239, pp. 98-105.
50. Yang, R.J., et al. (2001). Electroosmotic entry flow in microchannels. Journal of Colloid and Interface Science. 244, pp. 173-179.
51. Liapis, A.I. & Grimes, B.A. (2000). Modeling the velocity field of the electroosmotic flow in charged capillaries and in capillary columns packed with charged particles: interstitial and intraparticle velocities in capillary electrochromatography systems. Journal of Chromatography A. 877, pp. 181-215.
52. Meyers, J.J. & Liapis, A.I. (1998). Network modeling of the intraparticle convection and diffusion of molecules in porous particles packed in a chromatographic column. Journal of Chromatography A. 827, pp. 197-213.
53. Meyers, J.J. & Liapis, A.I. (1999). Network modeling of the convective flow and diffusion of molecules adsorbing in monoliths and in porous particles packed in a chromatographic column. Journal of Chromatography A. 852, pp. 3-23.
54. Coelho, D, et al. (1996). Electroosmotic phenomena in porous media. Journal of Colloid and Interface Science. 181, pp. 169-190.
55. Rathore, A.S. & Horvath, Cs. (1997). Capillary electrochromatography: theories on electroosmotic flow in porous media. Journal of Chromatography A. 781, pp. 185-195.

56. Dukhin, S.S. (1991). Electrokinetic phenomena of the second kind and their applications. Advances in Colloid and Interface Science. 35, pp. 173-196.
57. Tallarek, U., et al. (2002). Electroosmotic flow phenomena in packed capillaries: from the interstitial velocities to intraparticle and boundary layer mass transfer. Journal of Physical Chemistry B. 106, pp. 12709-12721.
58. Locke, B.R., et al. (2001). Electro-osmotic flow in porous media using magnetic resonance imaging. Langmuir. 17, pp. 6771-6781.
59. Hu, L, et al. (1999). Numerical model of electrokinetic flow for capillary electrophoresis. Journal of Colloid and Interface Science. 215, pp. 300-312.
60. Pittman, J.L., et al. (2003). Experimental studies of electroosmotic flow dynamics in microfabricated devices during current monitoring experiments. Analytical Chemistry. 75, pp. 361-370.
61. Tang, G.Y., et al. (2004). Joule heating effect on electroosmotic flow and mass species transport in a microcapillary. International Journal of Heat and Mass Transfer. 47, pp. 215-227.
62. Marcos, et al. (2004). Frequency dependent velocity and vorticity fields of electroosmotic flow in a closed-end cylindrical microchannel. Journal of Micromechanics and Microengineering. 15, pp. 301-312.
63. Manz, B, et al. (1995). NMR imaging of the time evolution of electroosmotic flow in a capillary. The Journal of Physical Chemistry. 99, pp. 11297-11301.
64. Manetto, G., et al. (2003). Application of capillaries with minimized electroosmotic flow to the electrokinetic study of acidic drug- β -oleoyl- γ -palmitoyl-L- α -phosphatidyl choline liposome interactions. Journal of Chromatography A. 990, pp. 205-214.

65. Zhu, Lingyan & Hian, Kee Lee. (2001). Field-amplified sample injection combined with water removal by electroosmotic flow pump in acidic buffer for analysis of phenoxy acid herbicides by capillary electrophoresis. Analytical Chemistry. 73, pp. 3065-3072.
66. Oddy, M.H., et al. (2001). Electrokinetic instability micromixing. Analytical Chemistry. 73, pp. 5822-5832.
67. Ren, Liqing, et al. (2001). Electroosmotic flow in microcapillary with one solution displacing another solution. Journal of Colloid and Interface Science. 242, pp. 264-271.
68. Arulanandam, Sarah & Li, Dongqing. (2000). Liquid transport in rectangular microchannels by electroosmotic pumping. Colloids and Surfaces A: Physicochemical and Engineering Aspects. 161, pp. 89-102.
69. Yao, Shuhuai, et al. (2001). A large flowrate electroosmotic pump with micron pores. Proceedings of IMECE 2001 ASME International Mechanical Engineering Congress and Exposition. New York.
70. Jiang, Linan, et al. (2002). Closed-loop electroosmotic microchannel cooling system for VLSI circuits. IEEE Transactions on Components and Packaging Technologies. 25, pp. 347-355.
71. Zeng, Shulin, et al. (2002). Electroosmotic flow pumps with polymer frits. Sensors and Actuators B. 82, pp. 209-212.
72. Yao, Shuhuai & Santiago, J.G. (2003). Porous glass electroosmotic pumps: theory. Journal of Colloid and Interface Science. 268, pp. 133-142.
73. Yao, Shuhuai, et al. (2003). Porous glass electroosmotic pumps: design and experiments. Journal of Colloid and Interface Science. 268, pp. 143-153.

74. http://www.electronics-cooling.com/html/2002_november_techbrief.html
(May 21, 2004)
75. Tripp, J.A., et al. (2004). High-pressure electroosmotic pumps based porous polymer monoliths. Sensors and Actuators B. 99, pp. 66-73.
76. Lazar, I.M. & Karger, B.L. (2002). Multiple open-channel electroosmotic pumping system for microfluidic sample handling. Analytical Chemistry. 74, pp. 6259-6268.
77. Morf, W.E., et al. (2001). Partial electroosmotic pumping in complex capillary systems – Part 1: Principles and general theoretical approach. Sensors and Actuators B. 72, pp. 266-272.
78. Chen, Lingxin, et al. (2004). Study of an electroosmotic pump for liquid delivery and its application in capillary column liquid chromatography. Journal of Chromatography A. 1028, pp. 219-226.
79. McKnight, T.E., et al. (2001). Electroosmotically induced hydraulic pumping with integrated electrodes on microfluidic devices. Analytical Chemistry. 73, pp. 4045-4049.
80. Liu, Shaorong, et al. (2003). Electric field-decoupled electroosmotic pump for microfluidic devices. Journal of Chromatography. 1013, pp. 57-64.
81. Chen, Lingxin, et al. (2003). Generating high-pressure sub-microliter flow rate in packed microchannel by electroosmotic force: potential application in microfluidic systems. Sensors and Actuators B. 88, pp. 260-265.
82. Reichmuth, D.S., et al. (2003). Increasing the performance of high-pressure, high-efficiency electrokinetic micropumps using zwitterionic solute additives. Sensors and Actuators B. 92, pp. 37-43.

83. Zhao, T.S. & Liao, Q. (2002). Thermal effects on electro-osmotic pumping of liquids in microchannels. Journal of Micromechanics and Microengineering. 12, pp. 962-970.
84. Chen, Lingxin, et al. (2005). Fabrication and characterization of a multi-stage electroosmotic pump for liquid delivery. Sensors and Actuators B. 104, pp.117-123.
85. Pierre, Pichot. (1986). Compressor Application Engineering Volume 1: Compression Equipment. Houston, USA: Gulf Publishing Company.
86. Longo, G.A. & Gasparella, A. (2003). Unsteady state analysis of the compression cycle of a hermetic reciprocating compressor. International Journal of Refrigeration. 26, pp. 681-689.
87. Kreyszig, Erwin. (1999). Advanced Engineering Mathematics. Singapore: John Wiley & Sons, Inc.
88. http://physics.nist.gov/cgi-bin/cuu/Value?k/search_for=boltzmann (June 11, 2004)
89. <http://scienceworld.wolfram.com/physics/ElectronCharge.html> (June 11, 2004)
90. Daugherty, R.L., et al. (1989). Fluid Mechanics with Engineering Applications. Singapore: McGraw-Hill Book Co.
91. <http://schools.matter.org.uk/SchoolsGlossary/permittivity.html> (June 11, 2004).
92. Cengel, Y.A. (1997). Introduction to Thermodynamics and Heat Transfer. USA: McGraw-Hill Book Co.