On starting square jets and forced plumes

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2006

Ai, J. J. (2006). On starting square jets and forced plumes. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/5298>

<https://doi.org/10.32657/10356/5298>

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On Starting Square Jets and Forced Plumes

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A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

2006

SUMMARY

The present research focuses on the studies of the transitional stage of jets and plumes. It can be generally divided into two parts, namely, starting jets and starting forced plumes.

The first part attempts to improve the understanding of the vortex dynamics in starting jets and to examine the differences between that of the circular jets and square jets. Based on the qualitative (via PLIF) and quantitative results obtained (via PIV), it was found that streamwise vortices generated at the corners of the square nozzle interacted with the primary vortices. The interactions had altered the downstream flow characteristics tremendously. Vortex pinch-off, leapfrogging, coalescence and axis-switching were observed in starting square jets and the mechanisms were analyzed.

For starting forced plume, Boussinesq and non-Boussinesq cases had been investigated analytically and experimentally. An analytical model for the starting forced plume covering the entire range of jet-like, transitional and plume-like had been developed based on the understanding Wang & Law (2002) for forced plumes. The velocity ratio between the head vortex and the trailing buoyant jet was derived to vary from the jet-like to plume-like phases. The transitional changes for this ratio had also been quantified. Experimental results verified the analytical model. The quantitative PIV results showed disorder behaviors in the near-field region for the non-Boussinesq cases ($\Delta > 15\%$). Virtual origin correction approach was utilized to compensate the non-Boussinesq effects.

SUMMARY

All in all, the present work on starting jets and plumes provides basis for further study on starting jets and plumes phenomenon. The concept of optimal vortex formation can be generalized to enable the discovery of underlying design principles simulating the animals' (e.g. jellyfish, squid) or human's effective swimming and physical behaviors, e.g. aquatic locomotion, fluid dynamic energy conversion, and cardiac flows. The enhanced mixing in square jets provides fundamental knowledge for the fuel ejecting nozzle design for the propulsive combustors. In addition, the penetration model for the starting forced plume may offer assessment for the hazard area of the releases of hot ash during volcanic eruptions and the emission of chemical plumes by accidental or intentional releases in an indoor environment.

ACKNOWLEDGEMENTS

The author wishes to thank many people who have contributed to what might be considered worthwhile in this thesis. First of all, I would like to express my deepest appreciation and gratitude to my supervisors, A/P Simon, Yu Ching Man and A/P Adrian, Law Wing-Keung, for their constant guidance, inspiration and encouragements throughout this project. From them, I learned a lot not only in academic perspective but also in the aspect of philosophy of human life. Their non-stop source of ideas, inspiration and support are the motivation of this project.

Appreciation is also due to A/P Chua Leok Poh, for his generous financial support of the experimental rig manufacture and maintenance. I would also like to thank Mr. Eric Yap, Mr. Yuan Kee Hock in Thermal and Fluids Lab and Mr. Foo Shiang Kim in Hydraulic Lab, for their technical support. Financial support to this project from the Nanyang Technological University is greatly acknowledged.

Finally, thanks also to all the friends in Thermal and Fluids Research Lab and Hydraulic Lab who have brightened the tough but exciting time we spent together in the lab. A pair of helping hand, valuable advices are being offered during the period of experimental work. It is a kind of warm stream flowing through the heart with them around. Discussion with fellow students has provided much needed support. All the fun we had together will be unforgettable.

Last but not least, my family deserves particular thanks for their moral support during this period of study.

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LIST OF SYMBOLS

Symbol	Variable
A	Local area
A_b, A_c, A_t	Velocity ratio of trailing jet to the head vortex at the base, center and tip of the head vortex
B	Local cross-sectional buoyancy flux
B_0	Initial buoyancy flux
B_v	Buoyancy force of the head vortex
b_w, b_c	Velocity and concentration 1/e-widths, respectively
C	Dye concentration
C_1, C_2	Proportionality constants
C_e	Extinction coefficient for dye tracer
C_{ur}	Local curvature of tube
C_Z, C'_Z, C''_Z	Penetration coefficients for jet tip
C_t	Penetration coefficient for head vortex
D	Diameter of the round nozzle
D_{AB}	Diffusion coefficient
D_s	Width of the Square nozzle
D_e	Equivalent diameter of square nozzle
d	Estimated displacement for PIV data processing
E	Kinetic energy
E_{bias}, E_{rms}	Error for mean bias and turbulence in PIV measurements
E_{nd}	Non-dimensional energy, $= \frac{E}{I^{1/2} \Gamma^{3/2}}$
E_N	Non-dimensional energy for Norbury vortices
Fr	Froude number
g	Gravitational acceleration
h	Ring height
I	Impulse
I_{ll}	Function of light intensity

LIST OF SYMBOLS

I_N	Impulse for Norbury vortices
K, K', K''	Constants for dye tracer characteristics
$K_1, K_2, K_3, b_1, b_2,$	Control parameters in the function of nozzle contour
$k_1, k_2, k_3, k_4, \eta_1, \eta_2, \eta_3,$	Constants fitted by Wang and Law (2002), Eq. (6.11b)
k_M	The ratio of the total momentum flux to the mean momentum flux
k_P	Validation factor for cross-correlation process
k_{pc}	Decay rate in potential core region
k_{pw}, k_{jw}	Centerline velocity decay rates for jets and plumes
k_{wg}	Gaussian shape constants of the cross-sectional velocity profiles
l	Optical length, $= 1/(C_{avg} \sigma)$
L	Piston stroke
L_m	Morton length scale, $= M_0^{3/4} / B_0^{1/2}$
L_q	Length scale, $= Q_0 / M_0^{1/2}$
M	Momentum
M_0	Initial momentum flux
$Mix(X, Z)$	Function of mixedness
M_v	Momentum of the head vortex
N	Number of samples
N_e, N_i	Total number of photons emitted and photons per unit sheet area per unit time
Q	Local mass flux
Q_0	Initial mass flux
R	Characteristic ring radius, Eq. (6.2)
R_v	Vortex ring radius
Re	Reynolds number, $= W_0 D_e / \nu$
Ri	Richardson number with Ri_p for plumes
r	Radial coordinate
S	Growth rate for the vortex ring, $= X_w / Z_w$

LIST OF SYMBOLS

S_c	Schmidt number
t	Time
t_v	Travelling time of the head vortex
$\bar{t}_v$	Dimensionless time for thermals, $= t_v B_0 / M_0$
$\bar{t}'$	Dimensionless time scale, $= (D^4 / B_0)^{1/3}$
$\bar{t}_v$	Dimensionless time for puffs, $= t_v W_0 / D$
t^*	Dimensionless time for starting jets $= t W_0 / D_e$
U, V, W	Instantaneous velocity
$\vec{U}$	Vector Field
V_0	Initial volume flux
u_b, u_c, u_t	Velocity of the head vortex at the base, center and tip
W	Local axial velocity
W_0	Initial velocity
W_c	Instantaneous axial velocity at the centerline of the axisymmetric jet
W_{cmax}	Maximum value of the centerline velocity
W_{cb}, W_{cc}, W_{ct}	Centerline velocity of the trailing jet at the center, base and tip of the head vortex
W_{jet}	Trailing jet velocity W_c
W_N	Translational velocity of Norbury vortices
W_p	Piston velocity
W_{tr}	Translational velocity of vortices
X, Y, Z	Cartesian coordinates for streamwise, lateral and spanwise directions
X_w	Distance from vortex ring center to nozzle wall
Z_t	Jet tip length
Z_w	Distance from vortex ring center to exit plane
Z_0	Virtual origin
Z_b, Z_c, Z_t	Vertical distance from the source for the base, center and tip of the head vortex
$\bar{Z}_b, \bar{Z}_t$	Dimensionless distance, $= Z_b / L_m, Z_t / L_m$
$\bar{\bar{Z}}_b, \bar{\bar{Z}}_t$	Dimensionless distance, $= Z_b / D, Z_t / D$

LIST OF SYMBOLS

$\bar{Z}_t'$	Dimensionless distance, $= Z_t / (M_0 / \rho)^{1/4}$
Greek Letters	
α	Radial entrainment coefficient with α_j for jets and α_p for plumes
α'	Semi-spread angle of the visible cap edge
γ	$= (1 - \Lambda)^{1/5}$, modification factor for virtual origin correction
Γ	Ring circulation
Γ_N	Ring circulation for Norbury vortices
$\Gamma_{nd}, \Gamma_{nd}'$	Non-dimensional circulation
δ_s	Ring core diameter
Δ	local effective gravitational acceleration, $= \frac{\Delta \rho}{\rho_a} g$
Δ_0	Initial effective gravitational acceleration, $= \frac{\Delta \rho_0}{\rho_a} g$
Δ_c	Effective gravitational acceleration at the centreline
$\Delta_{\rho_0}, \Delta_\rho$	Initial and local density difference, respectively
ε	Nondimensional mean core radius for Norbury vortices
η_{wg}	Dimensionless velocity 1/e-width
λ	Ratio of concentration to velocity width, Eq. (6.17)
λ_L	Laser wavelength
Λ	$= \frac{5Q_0^2 B_0}{4\alpha M_0^{5/2}}$, Source parameter
ν	Kinematic viscosity
ξ	$= \log\left(\frac{Z}{L_m}\right)$, with $\xi_b = \log\left(\frac{Z_b}{L_m}\right)$, $\xi_c = \log\left(\frac{Z_c}{L_m}\right)$ and $\xi_t = \log\left(\frac{Z_t}{L_m}\right)$
ρ	Local density
ρ_v	Density of the head vortex
ρ_∞	Density of the ambient fluid
σ	Photons absorbed per unit length

LIST OF SYMBOLS

τ	Vortex pinch-off time, life time of photons
ϕ	Photons emitted per photon absorbed
ψ	Stream function
Ω	Vorticity density, $= \omega / r$
$\omega_x, \omega_y, \omega_z$	Cartesian components of vorticity,
ℓ_0	Dimensionless length scale
$\wp_0$	Dimensionless mass flux
$\hat{\lambda}_0$	Dimensionless momentum flux

Abbreviations

AR	Aspect ratio of non-circular nozzles
CS	Coherent structures
MW	Molecular weight
PLIF	Planar Laser Induced Fluorescence
PIV	Particle Image Velocimetry
rms	Root mean square
PDF	Period of Developed Flow
PFD	Period of Flow Development
ZEF	Zone of Established Flow
ZFE	Zone of Flow Establishment

Subscription

0	Initial value at the exit
<i>max</i>	Maximum value
<i>nd</i>	Non-dimensional
<i>over</i>	Over-shoot
<i>p</i>	Piston
<i>sat</i>	Saturation
<i>t</i>	Tip
<i>V1</i>	Leading vortex
<i>V2</i>	The first trailing vortex

LIST OF SYMBOLS

Superscription

— Time-average mean value or normalized value

Vector operators

$$\nabla = \frac{\partial}{\partial X} + \frac{\partial}{\partial Y} + \frac{\partial}{\partial Z}$$

Chapter 1 INTRODUCTION

1.1 General introduction

The study of starting jets and plumes is motivated by practical applications related to the unconfined and unsteady turbulent flows from the initiation of steady and interrupted gas, such as propulsive combustors, where the control of mixing is crucial for increasing combustion efficiency and liquid releases caused by process upsets, explosions, unwanted fires and volcanic eruptions. Another motivation is from the perspective of combination of passive and active flow control (Johari *et al.*, 1997). The general notion is that by forcing any flow at an appropriate frequency and at relatively low amplitudes, the flow characteristics, such as instability and mixing, can be significantly modified. On the other hand, geometrical modifications of the jet nozzle may also alter the flow development downstream relative to a conventional circular nozzle, and thereby enhance the mixing efficiency.

Jets and plumes are of great interest due to their effects on mixing and entrainment. A so-called forced plume is a kind of flow driven by both buoyancy and momentum by discharging fluids into an environment with different densities. In most practical situations, the plumes are initially driven by momentum. Hereafter, unless otherwise stated, ‘plume’ often implies a forced plume in the present report. A jet is the discharge of fluids from an orifice or slot into a large body of the same or similar fluid. A plume, on the other hand, is caused by a potential energy source that provides the fluid with positive or negative buoyancy relative to its surroundings (Fischer *et al.* 1979).

Compared to round jets, non-circular starting jets have received relatively little attention. In particular, starting square jets have not been examined in the laboratory previously. On the other hand, as far as the author can aware, starting forced plumes are also scarcely investigated in the previous literatures. The present study attempts to improve the understanding of the starting jets and plumes, spanning the whole range of their development. A synthesis of the literatures shows that there are two distinct stages in the plume development that can be termed as Period of Flow Development (PFD) and Period of Developed Flow (PDF), analogous to the steady buoyant jet behavior. PFD refers to the near source region whereby the penetration rate is governed by the complex vortex dynamics initiated by the exit conditions that can include vortex coalescence, vortex leapfrogging, pinching off of the head vortex from the trailing stem and eventually reconnection. The pinch-off and reconnection leads to an over-shoot of the plume front which is a common observation within this period. The penetration rate in PDF is more predictable and depends solely on the continuous feeding of buoyancy and momentum into the head vortex by the trailing stem.

1.2 Jets and plumes

Many discharges into the environment are classified as buoyant jets, which are derived from sources of both momentum and buoyancy. The initial flow is often driven mostly by the momentum of the fluid exiting from an orifice, but the effluent may be less or more dense than its surroundings and the resulting jet is thus acted upon by buoyancy forces. However, given enough flow distance, all buoyant jets eventually act like plumes.

Near the source, the flow in a jet or plume is usually controlled entirely by the initial conditions, which include the geometry of the jet, the mean jet exit velocity, the initial density difference

between the discharge and the ambient fluid, as well as the turbulence intensity and velocity distribution in the supply pipe. A general and useful description of the factors of prime importance to jet dynamics can be defined as below.

- (i) The mass flux, ρQ , which is the mass of fluid passing a jet/plume cross section per unit time. The mass flux is give by

$$\rho Q = \int_A \rho W dA, \quad (1.1)$$

in which A is the cross-sectional area and W the local velocity in the axial direction. Q is called the specific mass flux, or volume flux, of the jet.

- (ii) The momentum flux, which is the amount of streamwise momentum passing a jet across section per unit time. It is given by

$$\rho M = \int_A \rho W^2 dA, \quad (1.2)$$

M is called the specific momentum flux and differs from the momentum flux only by the density factor.

- (iii) The buoyancy flux, the buoyant or submerged weight of the fluid passing through a cross section per unit time, is defined by

$$\rho B = \int_A g \Delta_\rho W dA, \quad (1.3)$$

where Δ_ρ is the difference in density between the surrounding fluid and the fluid in the jet/plume. B is the specific buoyancy flux is related to the flux of the tracer that causes the density variation. It is convenient to define

$$\Delta = g\Delta_\rho / \rho_\infty, \quad (1.4)$$

as the effective gravitational acceleration.

Throughout this thesis, Q_0 , M_0 and B_0 refer to the initial values of volume, momentum and buoyancy flux, respectively. For a round jet, Q_0 , M_0 and B_0 are given by

$$Q_0 = \frac{1}{4}\pi D^2 W_0, \quad (1.5)$$

$$M_0 = \frac{1}{4}\pi D^2 W_0^2, \quad (1.6)$$

$$B_0 = \frac{1}{4}\pi D^2 W_0 \frac{\rho_0 - \rho_\infty}{\rho_\infty} g = \frac{\Delta_{\rho_0}}{\rho_\infty} g Q_0 = \Delta_0 Q_0. \quad (1.7)$$

A pure jet is a discharge of effluent without any density difference. Figure 1.1 depicts the development of the jet from the exit. In common definitions (Fischer *et al.* 1979), a steady jet may be classified as two regions, i.e. Zone of Flow Establishment (ZFE) and Zone of Established Flow (ZEF). ZFE comprises a potential core, where the velocity can be considered to be constant. The previous literature reveals that the length of the potential core is about $6D$ (Fischer *et al.* 1979). Beyond the potential core, the pure jet undergoes a transitional region and then enters into the asymptotic area, i.e. ZEF.

A pure plume is a discharge of effluent with buoyancy and no initial momentum. In practice, any form of discharge carries some momentum. Here, the initial momentum for the pure plume is assumed to be negligibly small compared to the action of buoyancy. Figure 1.2 depicts the development of a plume.

In the far field of pure jets and plumes, it is assumed that the velocity and concentration follow the Gaussian distribution:

$$\frac{W}{W_c} = \exp \left[-\left(\frac{r}{b_w} \right)^2 \right], \quad (1.8)$$

$$\Delta = g(\rho_\infty - \rho) / \rho_\infty = \Delta_c \exp \left[-\left(\frac{r}{b_c} \right)^2 \right] \quad (1.9)$$

A buoyant jet, or sometimes referred as forced plume, is a discharge with both buoyancy and momentum. Typically, the buoyant jet behaves like a jet (jet-like) near the port dominated by the discharge momentum, and then far away from the port becomes a plume (plume-like) affected primarily by the buoyancy (Fischer *et al.*, 1979, Wang & Law, 2002).

Two characteristic length scales, L_q and L_m , are commonly used. They are defined as follows:

$$L_q = \frac{Q_0}{M_0^{1/2}}, \quad (1.10)$$

$$L_m = \frac{M_0^{3/4}}{B_0^{1/2}} \quad (1.11)$$

L_m describes the relative importance of momentum and buoyancy. For $Z \gg L_m$ the flow behaves like a plume and for $Z \ll L_m$ it behaves like a jet. The dominant parameter of a plume-like buoyant jet (buoyancy driven flow) is its initial buoyancy flux B_0 whereas the initial volume and momentum fluxes, Q_0 and M_0 , are the controlling factors of a jet-like buoyant jet (momentum driven flow). On the other hand, if $Z \gg L_q$ the flow is fully developed (the region is known as Zone of Established Flow (ZEF)), and if $Z \sim O(L_q)$, then the flow is still controlled by the jet exit geometry (the region is known as Zone of Flow Establishment (ZFE)).

Thus if L_m and L_q are of the same order then the flow will be very similar to a plume from the outset.

The ratio of L_q / L_m is called the jet Richardson number Ri , which is defined by:

$$Ri = \frac{L_q}{L_m} = \frac{Q_0 B_0^{1/2}}{M_0^{5/4}} = \left(\frac{\pi}{4}\right)^{1/4} \left(\frac{\Delta_0 D}{W_0^2}\right)^{1/2} = \left(\frac{\pi}{4}\right)^{1/4} \frac{1}{Fr_0}, \quad (1.12)$$

where Fr_0 is commonly called the jet initial densimetric Froude number, defined as:

$$Fr_0 = \frac{W_0}{\sqrt{\Delta_0 D}} = \frac{W_0}{\sqrt{\frac{\Delta_{\rho_0}}{\rho_\infty} g D}}, \quad (1.13)$$

Fr_0 describes the ratio of the inertial to the buoyancy forces. In the limiting case of a pure jet,

$Fr_0 = \infty$, whereas in the limiting case of a pure plume, $Fr_0 = 0$.

1.3 Steady versus starting jets/plumes

Among the nature's omnipresent phenomena are fluid dynamic jets/plumes. Some are obvious to even the most casual observer — like the jets that exit from one's mouth when exhaling on a cold morning. In its simplest manifestation, a jet is generated in a quiescent environment by a source of momentum alone, and spreads into that environment free from the influence of external forces. The consequence is that the momentum crossing any downstream plane is nearly the same as that added at the source. Both experimental evidences and theories argue that even turbulent jets asymptotically could “remember” how they were generated (Ricou & Spalding 1961).

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The dynamics of a jet or plume are entirely controlled by vorticity. The generation of vorticity at the source, and its convection and diffusion into only a narrow portion of the total space available, gives a jet its defining appearance. However, for real jets that are viscous in nature, the vorticity will undergo deformation and diffusion due to instabilities. To understand the development of a jet, it is crucial that the local nature of the jet instabilities and their global nonlinear development in both space and time be recognized and understood. Especially important is the rate at which fluids from the jet and from its surroundings become entangled and mixed, as they meet at the mixing layers. The information is given by the “entrainment rate”, which is defined as the rate of propagation of the interface between the rotational and irrotational fluid. The viscous diffusion of vorticity, acting primarily at the smallest scales of motion where the gradients are the largest, acts to propagate vorticity into the irrotational fluid. However, the production of this small-scale vorticity is largely controlled by the strain provided by the much larger scales of motion. Thus, the entrainment rate is controlled by the interface and contortions of the largest scale vortices moving into the surrounding fluid, as reported by Gutmark *et al.* (1990). These controlling large-scale vortices tend to be coherent and easily recognizable, and hence their name – coherent structures (Brown & Roshko, 1974).

A single pulse of fluid issuing from a nozzle into a quiescent fluid is often referred to as a starting jet. If the discharge is initiated with buoyancy, it is called a starting plume. Both have long been associated with vortex ring formation (Krueger 2001). In the present report, the temporal evolution of a starting jet/plume with continuous supply of fluids from the nozzle will be studied. Comparing to the steady jet, a starting jet is a transient/unsteady jet, i.e. the starting process of a steady jet. This is a process that is of concern in many engineering applications, such as scramjet combustors, gas turbine combustion chambers, reciprocating internal

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combustion engines and steady-state industrial –type burners. Other examples of gaseous jets of great practical interest are found in vertical/short takeoff and landing aircraft, the cooling of turbine blades, and chimney stacks exhausting smoke into a wind.

Unsteady injection is imposed in many situations, by the periodic nature of the governing process, for example, in reciprocating engines where direct injection of natural gas into the cylinder of a diesel engine is an attractive solution because of its potential to reduce pollutant emissions (Rubas *et al.* 1998). Early studies of unsteady injection (Hill & Greene 1977, Bremhorst & Watson, 1981 and Bremhorst & Hollis, 1990) found that mixing is greatly enhanced by the use of unsteady or fully pulsed incompressible air jets, when compare with a steady jet. Pulsed injection of gaseous jets has been recently proposed as a way to increase the penetration of the injection in supersonic flows such as those of scramjet combustors (Randolph *et al.*, 1994).

A starting forced plume is driven by both initial buoyancy and momentum in a manner similar to a steady buoyant jet. However, the behavior of a starting forced plume is substantially more complex due to the dynamic interaction between the head vortex and the trailing stem of force plume. Two distinct periods in the transient plume penetration were identified, namely the Period of Flow Development (PFD) and Period of Developed Flow (PDF). PFD represents the initial phase after ejection, whereby the exit conditions generate longitudinal and azimuthal vortices and their interactions lead to complex vortical dynamics such as the occurrence of vortex coalescence and leapfrogging (will be further described in Chapter 4). In PFD, the starting jet would undergo head vortex pinch-off and reconnection. When the circulation of the head vortex reaches its maximum, the head vortex would then detach from the trailing plume

stem. The detachment is termed as the pinch-off phenomenon (Gharib *et al.* 1998). The detached head vortex and the trailing stem would however reconnect after some distance, at which point the PFD ends and the starting forced plume is fully developed independent of the exit conditions. Noticeably, because of the pinch-off and reconnection, an over-shoot in penetration is typically observed in PFD (refer to Chapters 4 and 5). The plume characteristics in PDF are relatively simpler and depend solely on the feeding of momentum and buoyancy into the head vortex by the trailing stem. More details on the PDF and PFD will be discussed in Chapter 6.

The original and pioneering conceptualization of a model for a starting plume, which consists of a slightly flattened spherical vortex at the head of a quasi-steady plume, should be attributed to Turner (1962). In Turner's original model, it was assumed that the trailing plume feeds mass, momentum and circulation into the spherical head vortex, and that the velocity of the head vortex, u_b at the base of the vortex is a constant fraction, A_b , of the mean velocity of the trailing plume at the base of the vortex, W_{cb} , i.e.

$$A_b = u_b / W_{cb} = \text{constant} . \quad (1.14)$$

The present analytical and experimental results confirm the Turner's conceptual approach. Furthermore, the analytical model shows that A_b is changeable in the transition region of the starting forced plume.

From the above description, it can be noted that the flow in the jet-like and plume-like regions can be analysed by dimensional similarity considerations. For steady buoyant jets, analytical integral model covering the three regions can be found in Morton *et al.* (1959), So & Aksoy (1993) and Wang & Law (2002). However, for starting forced plume, such an analysis is not

presently available in the transitional region. The present studies aim to propose an analytical model for the starting forced plume based on the present knowledge of vortex rings and steady forced plumes.

Hereafter, unless otherwise stated, “pure jets or plumes” often implies a “pure steady jets or plumes” in the present report. The usage of the term “starting” is necessary to avoid the confusion with a steady jet/plume which normally studies the mean characteristics.

1.4 Vortex ring pinch-off

When investigating the vortex ring formation from a piston-cylinder mechanism for large stroke ratio ($L/D > 4$), Gharib *et al.* (1995) discovered that after the non-dimensional, time-dependent piston displacement L/D passed a certain value, no additional energy or circulation entered the forming vortex ring and the remaining fluid in the pulse was simply ejected as a “trailing jet”. After this point, the vortex ring was said to have “pinched off” from the generating jet.

Gharib *et al.* (1998) defined the value of L/D at which pinch-off occurred as the “formation number”, F . The value of F was determined for several piston velocities and nozzle geometries by finding the value of L/D at which the circulation in the pinched off vortex ring equaled the total circulation ejected from the nozzle. For all of the cases tested by Gharib *et al.* (1998), F was found to be between 3.6 and 4.5. The narrow range of F observed led them to conclude that the formation number had a relatively universal value for vortex rings.

Subsequent work has revealed that F can be significantly manipulated by adjusting at least three properties of the jet used to generate a vortex ring. The first property that affects the formation number was discovered by Rosenfeld *et al.* (1998) through a series of numerical experiments simulating a piston-cylinder mechanism. In attempting to numerically replicate the results of Gharib *et al.* (1998), Rosenfeld *et al.* (1998) studied the effect of the velocity profiles of the jet at the nozzle exit. Varying the jet profile away from a nearly uniform profile was found to reduce the formation number. When the jet velocity profile was blunted to the limiting case of parabolic, F was reduced to 1.0. Rosenfeld *et al.* explained by noting that the normalized vortex ring circulation, $\Gamma_{ring} / W_{max} D$, (where W_{max} is the maximum piston velocity during the pulse) appears to universally fall between 1.8 and 2.6, while the vorticity flux from the nozzle for the parabolic profile increases by approximately a factor of the four over the uniform profile. Thus, the pinch-off should occur at a value of L/D four times smaller than for the case of uniform profile.

The second jet property affecting the formation number is the velocity history (i.e., the piston velocity as a function of time) used to generate the pulse. Rosenfeld *et al.* (1998) showed that the formation number could be increased to 5.22 for a linearly increasing velocity history (with a nearly uniform velocity profile). The suggestion that increasing the velocity during a pulse ejection can increase the formation number was also confirmed numerically by Mohseni *et al.* (2001), who simulated the compressible Navier-Stokes equations with a time-varying non-conservative force as the vortex generation mechanism. They reasoned that increasing the jet velocity during a pulse increases the formation number because it forces the shear layer of the trailing jet to keep up with the leading vortex (which continues to gain velocity as its circulation increases), thereby allowing vorticity to be added to the ring for a longer period of time.

The final property known to affect the formation number is the time dependent behavior of its diameter. Mohseni *et al.* (2001) numerically simulated a case analogous to a piston-cylinder mechanism with a time varying diameter. By effectively increasing the diameter during the generation of the vortex ring, they were able to achieve a leading vortex very near to the limiting case of Hill's spherical vortex (i.e., large formation number). The explanation was that increasing the diameter counteracts the tendency of the velocity field induced by the vortex ring to push the shear layer feeding the ring toward the symmetry axis where its strength is reduced.

The present results illustrated that in the vorticity flux from the nozzle for the starting jet was neither uniform nor asymptotic. The nondimensional pinch-off time was also found to vary with the Reynolds number.

1.5 Non-circular jets

Non-circular jets have been the topic of extensive research in the past fifteen years. These types of jet are identified as efficient passive flow control devices that allow significant improvements in mixing performance at a relatively low cost, because they rely solely on changes in the geometry of the nozzle (Gutmark & Grinstein 1999). The applications of noncircular jets include improved large- and small-scale mixing in low- and high-speed flows, and enhanced combustor performance, by improving combustion efficiency, reducing combustion instabilities and undesired emissions. Additional applications include noise suppression, heat transfer, and thrust vector control.

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The flow patterns associated with noncircular jets involve the mechanisms of vortex evolution and interaction, flow instabilities, and fine-scale turbulence augmentation (Gutmark & Grinstein 1999). Stability theory identified the effects of initial momentum thickness distribution, aspect ratio, and radius of curvature on the initial flow evolution. Experiments revealed complex vortex evolution and interaction related to self-induction and interaction between azimuthal and axial vortices, which lead to axis switching in the mean flow field. Numerical simulations described the details and clarified mechanisms of vorticity dynamics and effects of heat release and reaction on noncircular jet behavior.

The mixing process in jet shear layers involves bulk mixing driven by large-scale coherent structures, and smaller-scale mixing dominated by turbulent velocity fluctuations. Large-scale coherent structures (CS), characterized by organized vorticity distributions, are intrinsic features of the high Reynolds number (Re) mixing layer (Brown & Roshko 1974). The control of jet development in practical applications is strongly dependent on understanding the dynamics and topology of CS, in particular, how the jet properties can be affected by the control of the formation, interaction, merging, and breakdown of CS.

The formation of circular, azimuthally coherent vortex rings and their sequential merging dominate the shear-layer growth and entrainment in axisymmetric jets at moderately high Re (Crow & Champagne 1971). At a short distance downstream of the jet exit, three-dimensionality becomes the crucial feature of the jet structure, and the streamwise vorticity has the dominant role in entraining fluid from the surroundings (Liepmann & Gharib 1992). Self-induction, vortex stretching, and reconnection become the dominant fluid dynamical processes involved (Hussain 1986). Azimuthal non-uniformities at the jet exit add complexity to the

evolution of the jet shear layer; the three dimensional development of the jet is particularly sensitive to initial conditions determined, for example, by the nozzle geometry or by upstream disturbances. In noncircular jets, deformation dynamics of asymmetric vortices play an important role in the jet evolution. Asymmetric jets are naturally more unstable than their axisymmetric counterparts (Michalke 1971). Amplification of high-order instability modes rather than the axisymmetric mode that dominates the near-field region of a circular jet, brings about complex vortex topologies. Elongated jets, e.g. elliptic and rectangle tend to exhibit lateral flapping motion along their minor axes (Crighton 1973).

Substantial efforts have been devoted to investigate the properties of jets emerging from noncircular nozzles. Laboratory studies using elliptic nozzles (Ho & Gutmark 1987, Husain & Hussain 1983, Hussain & Husain 1989) and nozzles with corners, e.g. rectangular, triangular, star-shaped (Gutmark et al 1989a, Toyoda & Hussain 1989, Quinn 1992), have observed that as the jet spreads, its cross-section can regularly evolve through shapes similar to those of the jet nozzle but with axes successively rotated at angles characteristic of the jet geometry, denoted as the axis-switching phenomenon.

1.6 Mixing and its representation

Industrial fluid mixing is the motion and/or contacting of a single and multiphase process with a liquid continuous phase to achieve a desired process results. Two factors may affect the mixing efficiency, advection and diffusion. The advection is typically used to describe the movement, or transport, of scalar quantities due to the motion, or velocity field, of a fluid. Diffusion arises because fluid molecules are always in motion on a molecular level. These two types of motion relate to each other and thus affect the mixing at the same time. Putting advection and diffusion

together, the following equations is obtained as the general evolution/transport equation for conserved scalars,

$$\frac{\partial C}{\partial t} + \nabla \cdot (C\vec{U}) - D_{AB} \nabla^2 C = 0, \quad (1.15)$$

where C is the concentration, $\vec{U}$ represents the velocity field and D_{AB} stands for the diffusion coefficient of the species A and B.

Mixing occurs when the ambient fluid is engulfed into the core of the jet due to turbulent entrainment. The mixing process is characterized as a series of steps. The first step is entrainment of purely irrotational fluid into the mixing layer. The subsequent steps consist of the break down of eddies to successively smaller scales until the Kolmogorov micro-scale, the length scale at which viscous dissipation takes place, is reached. The diffusion length scale is proportional to the molecular diffusivity ($S_c^{-1/2}$), where S_c is the Schmidt number.

The measure for mixing is termed “mixedness”. Quantitatively, the mixedness at a point $Mix(x, z)$ can be expressed as:

$$Mix(X, Z) = \frac{C(X, Z)}{C_0}, \quad (1.16)$$

where $C(x, z)$ is the local dye concentration, and C_0 the initial dye concentration. Assuming weak excitation, i.e. no saturation and photo-bleaching effect, the fluorescence signal $I'_\parallel$ emitted from a small volume V_0 of the laser sheet corresponding to one pixel in the PLIF image can be modeled as (Guilbault 1973):

$$I'_\parallel(X, Z) = K\phi C_e I_\parallel(X, Z) C(X, Z) V_0, \quad (1.17)$$

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where ϕ is the quantum yield defined as the ratio between the emitted and the absorbed energy, C_e the extinction coefficient of the dye for the incident light, $I_{ll}(x, z)$ the local incident light intensity, $C(x, z)$ the local dye concentration, and K is a proportionality constant. V_0 and $I_{ll}(x, z)$ is constant when imaging window size, the camera spatial resolution, the laser sheet thickness and intensity are all fixed. In weak excitation region, both ϕ and C_e are constant. So the local concentration $C(x, z)$ can be expressed as:

$$C(X, Z) = K' I_{ll}'(X, Z), \quad (1.18)$$

where

$$K' = \frac{1}{K'' \phi C_e I_{ll}(X, Z) V_0}, \quad (1.19)$$

Substituting Eq. (1.19) into (1.16), the mixedness can be described by the light intensity.

$$Mix(X, Z) = K \frac{I_{ll}(X, Z)}{I_{ll0}}, \quad (1.20)$$

Therefore, by using the flow visualization images, the mean and fluctuating components of the concentration distribution from the starting jet can be obtained. In this case, the concentration distribution will define the mixing efficiency.

A well-established quantitative method of expressing “goodness of mixing” was reported in the early 1950’s (Danckwerts, 1952), in which, two quantities, the scale segregation and the intensity of segregation, were precisely defined to describe the degree of mixing. However, the first quantity is not applicable to the PLIF experimental results discussed here, because the spatial resolution in this study was not high enough to distinguish the dye molecules and water molecules. Nevertheless, the second quantity, the intensity of segregation, is well suited for the

study of mixing efficiency. For Planar Laser Induced Fluorescence (PLIF), the dilution rate of the dye during its own downstream traveling can be an indicator for the mixing performance.

1.7 Mixing control methods

Efficient mixing is essential for the operation of engineering devices, such as combustors, reactors, and lasers where chemical reaction is taking place. Rapid mixing is also critical for many applications in aircraft, such as thrust augmenting ejectors, commercial ground based ejectors, and waste water discharges.

Because of the practical importance, the mixing of jets has been widely studied for decades. It is now realized that improved jet mixing can be achieved through the manipulation of shear flow characteristics. The various mixing control methods developed to date can be broadly classified into two categories, namely, active and passive. Here, mixing control is interpreted in the fluid mechanical sense, i.e., to affect mixing via modifying the flow field by directing energy into selected turbulent scales, using mechanical, acoustic, hydrodynamic, thermal methods or combinations thereof.

Active control is defined as methodologies that affect the evolution properties of the shear flow characteristics by introducing external powered perturbation. Passive mixing-control strategies are based on geometrical modifications of the jet nozzle which alter the flow development downstream relative to a conventional circular nozzle. In recent years, there have been increasing efforts devoted to investigate the flow characteristics of non-circular jets, largely motivated by their abilities in enhancing the mixing relative to comparable circular jets, as shown in the works by Quinn (1992), Hertzberg & Ho (1995), Grinstein & DeVore (1996) and

Gutmark & Grinstein (1999). Among the non-circular jets, polygonal (for example square, rectangular or triangular) jets are the most popular ones since they combine the advantage of small-scale mixing at the corner regions with large-scale structures rolled-up from the flat surfaces (Gutmark *et al.* 1990).

Non-circular nozzles typically entrain more ambient fluid than corresponding round nozzles (Ho and Gutmark 1987). The sharp corners of non-circular nozzles lead to the appearance of azimuthal concentrations of small-scale motions within the ensuing large-scale vortical structures, with considerable enhancement of mixing (Gutmark *et al.* 1989). In comparison, with active control, the formation and coalescence process of vortices can be manipulated. Hence, hybrid passive and active control methods have also been proposed in order to combine the virtues of both methods. For example, acoustic excitation (frequency modulation) had been applied to indeterminate-origin nozzles. The resulting highly three-dimensional instability-wave patterns generated complex interaction systems. It was concluded that active control applied to such nozzles is an effective means of imposing a high level of order on coherent turbulent scales in shear layers, and therefore is a technique that can be profitably employed for shaping flowfield development (Kibens and Wlezien 1985). More recently, strong instabilities of a jet column were observed when a square jet was forced using azimuthal modes excitation effected via amplitude modulation, although detailed analysis about the effects of nozzle geometry on the jet dynamics was not provided (Wiltse and Glezer 1993). In later work, spatial mode excitation had been implemented on triangular jets and elliptic jets with various initial velocity profiles (Vandsburger and Ding 1995; Ding 1995). It was shown that the initial velocity profiles have significant influence on the effects of such combined excitations in terms of the flow characteristics associated with nozzle geometry.

Hence starting square jets should possess the characteristics as composite active and passive controlled jets. The global flow pattern in a starting jet is related to the vortex dynamics of the coherent structures in the shear layers (Hertzberg and Ho 1995). The formation and evolution of an isolated vortex ring in this geometry hence provides clues to the behaviour of the starting jet. Vortex ring formation and evolution play an important role in the starting jet (Krueger 2001). Krueger also pointed out that a vortex ring, which pinches off in starting jets, represents a maximization principle for propulsion. This potential significance of vortex ring pinch-off was first conjectured by Gharib *et al.* (1998) for pulsatile propulsion. More generally, it is important to uncover the interaction between the leading vortex ring and the trailing jet in order to comprehend the dynamic behavior of a starting jet. Therefore, the mixing control becomes vortex generating control in the starting jets.

1.8 Objectives of present study

Most of the previous experimental mixing studies relied on the traditional point-based measurement techniques, such as Hot Wire Anemometry and Laser Doppler Anemometry for velocities as well as thermometry and LIF for passive scalars. With the rapid advances in the laser related techniques, planar measurement such as Particle Image Velocimetry (PIV) and Planar Laser Induced Fluorescence (PLIF) are becoming achievable in the laboratories. These techniques are more effective and efficient than the point-based techniques in revealing the mean and turbulent structures in complex flows. In the present study, these advanced laser imaging techniques are employed as the experimental tools to investigate the characteristics of starting jets and plumes.

Chapter 1 INTRODUCTION

The present work aims to investigate the vortex evolution in starting circular and square jets and plumes. Although much has been studied about circular and non-circular jets, most studies were focused on the steady cases. In particular there is a lack of study on the starting non-circular jets. Other than the published work by Auerbach (1987a and b), Hertzberg and Ho (1995) and Grinstein and DeVore (1996), no other investigation on the vortex dynamics of non-circular unsteady jets can be found by the author.

Square shape is selected in this study owing to two considerations. First, square jets belong to the family of non-circular jets, which have been the topic of extensive research in the past fifteen years (e.g. Quinn & Militzer 1988; Quinn 1992; Grinstein *et al.* 1995, 1996; Chua *et al.* 2001). The interest was motivated mainly by their increased entrainment and enhanced mixing properties relative to those of comparable circular jets. Second, in the absence of aspect ratio (AR) effect, square can be considered as the most regular non-circular shape and the easiest one to be compared with the circular counterpart.

It is also noted that the experimental studies of starting forced plumes are scarce. Most of the previous investigations focused on pure starting jets or plumes. Starting forced plume's behavior possesses the combined characteristics of the pure starting jets and plumes. However it is substantially more complex due to the co-action of the momentum and buoyancy. Moreover, few experiments have been devoted to the unsteady transition from jet-like to plume-like region. The transitional characteristics of both the mean and turbulent quantities are of interest because they allow the mathematical models to predict a smooth change from a starting jet to plume.

To summarize, the objectives for this thesis are:

1. To examine the physical behavior of circular and square starting jets under different initial conditions with Planar Laser Induced Fluorescence (PLIF). The focus will be on the observation of the vortex evolution and comparison between the steady and starting jets. Comparison will also be made between the square and circular jets. Streamwise and spanwise vortex evolution and their interactions in square jets which lead to axis-switching and sequent enhanced mixing, will be examined thoroughly within the range of $Z/D \leq 10$. At the same time, a nozzle and an orifice will be studied, respectively in order to investigate the effect due to the exit geometry;
2. Particle Image Velocimetry (PIV) will be applied to derive the flow quantities for the starting circular and square jets. The quantities of the vector field, vorticity distribution and energy of the generated vortex will be examined. The dynamics of the head vortex pinch-off process in starting jets and the effect of the nozzle geometry will be further quantitatively investigated;
3. Based on the PIV and PLIF results, a penetration model on the starting forced plumes, which is a synthesis of both starting jets and plumes, will be constructed. The experiments for the forced plume will cover the Boussinesq and non-Boussinesq range. In this project, the investigation will mainly focus in the range of $Z/D > 10$.

1.9 Outline of the thesis

This report is organized as below:

Chapter 1 INTRODUCTION

Chapter 1 gives a brief introduction on the starting jets and plumes and some important concept related to them. The significance of mixing in practical applications and its control methods are also briefly depicted;

Following Chapter 1, a brief review on the previous investigations is presented in Chapter 2;

Chapter 3 describes the experimental facilities, experimental techniques, as well as associated methods used for data acquisition and post-processing. The accuracy of the measurements will also be quantified;

Chapter 4 provides the qualitative results obtained by PLIF. The penetration rate for the starting jets will be examined. Some classic phenomena on the vortex interaction, i.e. axis-switching, leapfrogging and pinch-off and their influence will be examined;

Chapter 5 addresses the quantitative results of the flow field characteristics under the same conditions as depicted in Chapter 4. The phenomena discovered in the previous chapter will be further discussed based on the PIV results;

A penetration model on the starting forced plume, which combined the behavior of starting jets and plumes, will be developed in Chapter 6;

Finally, main concluding remarks will be drawn in the last chapter and some considerations and recommendations for future work will be proposed.

Fig. 1.1 Schematic representation of a pure jet (from Fischer *et al.* 1979).

Fig. 1.2 Schematic representation of a pure plume (from Fischer *et al.* 1979).

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2.1 Previous work on square jets

Square jets have been studied extensively in the past fifteen years. Early investigations of square nozzles concentrated on the mean flow field (Sforza *et al.* 1966, Trentacoste & Sforza 1967). Later studies of jets emerging from noncircular nozzles with corners showed that the introduction of sharp corners in the nozzle can increase significantly the fine-scale turbulence at the corners relative to the flat segments of the nozzle (Schadow *et al.* 1988, Toyoda & Hussain 1989) and enhance mass entrainment (Vandsburger & Ding 1995). In some cases, the small-scale turbulence was produced by three-dimensional flows, which were formed at the corner regions inside the nozzle (Su & Friedrich 1994). The evolution of square jets is characterized by differential shear-layer growth rates at the flat and vertex sections. This difference in spreading rate may lead to axis switching, or rotation of the jet geometry in the downstream location. The production of small-scale turbulence is also different at the various circumferential location. This behavior depends on the initial conditions at the jet exit that subsequently affect the vortex dynamics. Grinstein *et al.* (1995) discussed the role of initial conditions and other parameters that affect the near field of a subsonic square jet. They examined effects of initial conditions such as ratio of initial-momentum-thickness to circular-equivalent diameter, turbulence level, azimuthal momentum thickness distribution, and Reynolds number effect on the jet shear-layer growth pattern. Axis switching was shown to be related to vortex deformation dynamics, which in turn was enhanced at a low turbulence level and thin initial shear layer. The dependence on

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Reynolds number and jet Mach number (M) was very weak. High azimuthal turbulence level and small shear-layer thickness to jet diameter ratio were crucial for the initiation of strong vortex deformation and self-induction process. When these conditions were not satisfied, as in the case of a pipe square jet, axis switching was not observed, and lower entrainment rates were obtained. Vortex deformation in the square jet is initiated at the corners by downstream bending of the corner sections due to the small local radius of curvature. Stretching of the distorted vortex by the shear stresses produces small-scale streamwise vortices at the corners. This process, which is unique to jets flowing from nozzles with corners, results in the initial axis switching and amplification of small-scale turbulence at the vortices. The ensuing complex vortex interaction between azimuthal and streamwise structures limits the role of vortex ring deformation dynamics and the subsequent occurrence of axis switching (discussed further in Chapters 4 & 5).

Due to the azimuthal non-uniformities inherent in non-circular configuration, polygonal (triangular or square, for example) jet is characterized by large-scale coherent structures along the flat sides, and small-scale structures at the corner regions (Gutmark & Grinstein 1999). Combustion systems with these types of jets can benefit from the combination of large- and small-scale mixing: the large-scale vortices promote fast bulk mixing between fuel and air, while small-scale turbulence accelerates the combustion rate by enhancing mixing at the molecular level. Square jets with the same hydraulic diameter as a circular jet have been found to have a faster spreading rate, as shown by Quinn & Militzer (1988) and Quinn (1992). A further study by Grinstein *et al.* (1995) on square jets showed significant entrainment augmentation would be achieved in the jet near-field ($2 < Z/D_e < 4$) when using

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square geometry. Quinn (1992) has also observed off-center velocity peaks for a square slot orifice exit, which was attributed to the “vena contracta” effects.

In circular jets, the main mechanism leading to shear layer growth and mixing is the vortex ring interaction and merging process (Ho & Huerre 1984). In the case of non-circular jets, however, the dynamics of vortex self-induction turns out to be more important than the vortex merging process. Several non-circular configurations have been investigated to date: rectangular (Sfeir 1975, Husain & Hussain 1983, Quinn 1989, Austin 1992), square (Quinn 1992, Grinstein *et al.* 1995, Vandsburger & Ding 1995), triangular (Schadow *et al.* 1990, Mi *et al.* 2000), lobed (Wang *et al.* 2001, Zaman 1994, 1997) and elliptic (Dhanak & Debernardinis 1981, Ho & Gutmark 1987).

Koshigoe *et al.* (1989) conducted linear stability analysis on different initial conditions that might promote the non-uniform growth behavior underlying axis switching. Three conditions were determined to be necessary in order to initiate the deformation of non-circular vortex ring. First, the eigenfunctions of the instability mode should be localized within the jet circumference without excessive overlapping. The non-circular shape of the nozzle, with circumferentially varying curvature, causes separation of the different instability modes and localized amplification at specific segments along the circumference (e.g. one mode is dominant at the major axis, and another mode is dominant at the minor axis). Second, the amplification rates of the corresponding eigenmodes should be comparable. This requirement can be fulfilled if the azimuthal non-uniformities of the initial shear layer are relatively small, so that the growth rates of the velocity fluctuations associated with the different modes are comparable. Third, sufficient phase-speed differences must exist

between modes, so that the streamwise location where the vortex roll-up is completed relative to the edge of the nozzle is dependent on the azimuthal shedding location. The fulfillment of these conditions depends on the initial azimuthal distribution of the momentum thickness and on a particular geometry of the nozzle. As a result, when the initial shear layers grow at the similar rate, or when the initial momentum thickness at major axis is greater than that in minor axis, the instability modes would not be amplified sufficiently, and axis switching is hence suppressed. A typical example is in a fully-developed pipe jet with corners examined by Koshigoe *et al.* (1989).

2.2 Previous work on buoyant and nonbuoyant jets

Steady-state positively buoyant jets issuing into unbounded homogeneous fluids have been studied and modeled in greater details than negatively buoyant jets (Papanicolaou & List, 1988 and Peterson & Bayazitoglu, 1992). However, very little information is available regarding the transient behavior of positively buoyant jets. For neutrally buoyant jets, it can be shown that the transient jet penetration is proportional to $t^{1/2}$, where t is time (Turner 1962), but experimental results are limited. Presumably, positively buoyant jets would have similar dependence upon time, at least in the momentum-dominated region of the jet.

Instrumentation and techniques for the laboratory investigations of buoyant jets vary for different investigators and have become more advanced every year. Albertson *et al.* (1950) used pitot-tube to measure the mean velocity of an air jet up to 250 diameters downstream. Rouse *et al.* (1952) used thermocouple and vane anemometer in order to measure temperatures and velocities above a heated air plume. Forstall and Gaylord (1955) measured velocities and concentrations in a liquid jet using transversing impact tube and conductivity

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cell with 1% salt added as a tracer into the jet water. Lassiter (1957) measured the axial velocity component at the initial jet region using Hot Wire Anemometry (HWA). Becker *et al.* (1967) used light scattering technique to measure concentrations in an air jet traced by oil smoke. Abbiss, Bradbury and Wright (1975) measured the turbulence characteristics of an air jet up to 25 jet diameters using Laser Doppler Anemometry (LDA). George *et al.* (1977) measured velocities and temperatures simultaneously using a two-wire probe. Chevray and Tutu (1978) examined the turbulence parameters and the radial and axial turbulent mass transport of heat in a heated air jet using hot wire probes. Kotsovinos (1977) used LDA combined with microbead thermistor to observe the turbulence of a plane turbulent buoyant jet. Papanicolaou and List (1988) investigated axisymmetric turbulent buoyant jets in a stationary ambient using combined LDA and Laser Induced Fluorescence (LIF). Van Cruyningen *et al.* (1990) demonstrated the ability of Planar Laser Induced Fluorescence (PLIF) technique to investigate the mixing of turbulent air jet. Simoens and Ayrault (1994) combined the photographic-film-based Particle Image Velocimetry (PIV) and PLIF to investigate a turbulent jet. Weisgraber and Liepmann (1998) measured the turbulent structure of a round jet during transitional to self-similar region with digital PIV (DPIV). More recently, Webster *et al.* (1999) combined the DPIV-guided Digital Particle Tracking Velocimetry (DPTV) with PLIF in a turbulent jet investigation.

Extensive experimental database on the turbulent velocity measurements exists for the subject of a pure jet discharging into a stagnant ambient. The mean velocity measurements with pitot tubes were well summarised in Hinze (1959). Most of the turbulent quantities in the fully developed jet can be found in Corrsin (1943), Corrsin and Uberoi (1949, 1951), and Corrsin and Kistler (1955). The failure of the turbulence intensities to attain self-preservation

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in these early works motivated the efforts of Wygnanski and Fielder (1969) using linearised hot-wires. They performed a comprehensive study on the characteristics of a self-preserving turbulent jet, including mean velocity, turbulent stresses, intermittency, skewness and flatness factors, correlations, micro-scales and integral scales, low-frequency spectra and convection velocity. Their work became a standard reference. Rodi (1975) reported measurements of mean velocity and turbulent intensities and showed that the mean velocity decay along the axis of the jet conformed to the x^{-1} decay expected from similarity analysis, unlike the measurements of Wygnanski and Fiedler (1969). Baker (1980) discovered that the far-field data of Wygnanski and Fiedler (1969) failed to satisfy the constraint of the integrated axial momentum equation. Seif (1981) further analysed the data of both Wygnanski and Fiedler (1969) and Rodi (1975) and also found a significant shortfall in the momentum flux across the jet. These problems prompted the experimental study by Capp (1983) using LDA technique to investigate the source of the discrepancy. He found that the error was facility-related. Significant differences can exist between a jet in a confined or semi-confined enclosure and one in an infinite environment because of the recirculating flow entrained by the jet. Detailed discussions on this issue can also be found in Hussein et al (1994).

Measurements of a turbulent jet with traditional stationary hot-wire are often criticised because of the high local turbulence intensity, which is about 30% at the jet centreline and increases to greater than 100% past the velocity half-width (Antonia *et al.* 1980). The effects of the high turbulence intensity on hot-wire have been discussed by a number of authors including Tutu and Chevray (1975) and Beuther *et al.* (1987). The principal limitations are due to cross-flow, rectification and drop-out, all of which increase with the turbulence

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intensity. Hussein *et al.* (1994) investigated a turbulent jet in a large enclosure using burst-mode LDA, flying (FW) and stationary hot-wire (SHW). The velocity moments to third order obtained through these measurement techniques were compared. The results of LDA and FW differed substantially from the SHW, especially the higher moments and away from the flow centreline, raising doubt on the validity of the application of SHW technique on turbulent jets.

Panchapakesan and Lumley (1993) performed careful examination on a round air jet using x-wire and hot-wire probes mounted on a moving shuttle. The shuttle arrangement was to eliminate errors due to flow reversals in the jet's intermittent region. The turbulent characteristics of the round jet were analysed to the second and third moments and reported in great details.

Recently DPIV was employed by Weisgraber & Liepmann (1998) to measure the developing region ($17 < Z/D < 27$) of a turbulent jet transiting into the zone of established flow. They found that while the mean velocity profiles were nearly self-similar, the second-order and third-order turbulent correlations were still developing and increasing in magnitude in the downstream direction. PIV was shown to be an effective tool in quantifying the turbulent properties of this flow.

Experimental studies of plumes reported in literature are relatively scarce. George *et al.* (1977) measured velocities and temperatures of plumes simultaneously using a two-wire probe. The relative turbulent intensities were found to be 0.4 and 0.25 at the plume axis for the temperature and axial velocity, respectively. They found that about 15% of the total mass

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flux was contributed by turbulence. The velocity width was measured to be wider than the temperature width, contrary to the jet case and also opposite to the findings of Rouse *et al.* (1952). All measurements were made at a short distance from the nozzle of 8, 12 and 16 diameters and therefore the assumed self-similarity may not be correct.

Nakagome and Hirata (1976) performed plume experiments above a heated disc measuring velocities and temperatures simultaneously using hot wires. The recorded turbulence intensity of velocity and temperature are lower than those measured by George *et al.* (1977). They also found the concentration-to-velocity width ratio less than 1 in qualitative agreement with George *et al.* (1977). Their measurements were conducted at less than 11 disc diameters downstream, thus again whether the flow was fully developed is of concern.

Shabbir and George (1994) reported a comprehensive set of hot-wire measurements of a round plume which was generated by forcing a jet of hot air vertically up into a quiescent environment. This kind of experiment is very delicate because small convection currents usually exist in the ambient air. They made great efforts to ensure that the experiments were conducted in a non-stratified and stationary environment during the experimental period of up to 8.5 hours, and concluded that while the effect of buoyancy on the mean velocity field is evident, its influence on the velocity fluctuations is indirect. They also found that about 15%~18% of the total mass flux were carried by turbulence, in agreement with George *et al.* (1977). Their measurements was performed at a short distance of $Z/D < 28$ raising again the question that the flow may not be fully developed.

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Using temperature or conductivity as the scalar indicator makes the scalar quantity and buoyancy correlated. This imposes severe limitations in selecting the experimental parameters. As pointed by Papanicolaou (1984), with these methods, it is hard to obtain the required precision beyond a distance of 40 times of the jet nozzle diameter in the case of round buoyant jets. This is due to the fast decay of the initial concentration along the jet axis. Furthermore, temperature-based plume experiments normally require very high initial density difference. Together with the fact that the measurements can only be taken in the near field of the plume, whether Boussinesq's approximation can be applied in the analysis is another concern. For this reason, Chen and Rodi (1980) incorporated the effect of high initial density difference in their plume analysis.

Papanicolaou and List (1988) circumvented the above problems by adopting LIF technique instead of thermometry in the scalar measurements. The initial density difference was created by salt while the scalar transport was represented by the fluorescent tracer. They performed a comprehensive experimental study on buoyant jets from jet to fully developed plume regime with combined LDA and LIF. They obtained a velocity width in plumes similar to Rouse *et al.* (1952) but much thinner than George *et al.* (1977), Nakagoma & Hirata (1976) and Shabbir and George (1994). Consequently, the concentration-to-velocity width ratio was greater than 1. However, their LIF measurements overestimated the mean concentration by up to 20%, which may be caused by the combination of the two techniques.

Steady buoyant jets have three distinctive regions in ZEF, i.e. jet-like, transitional and plume-like. Experiments were conducted and empirical solutions were obtained on plane (Harris, 1967 and Kotsovinos and List 1977) and round (Ogino et al. 1980) buoyant jets.

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Utilizing the entrainment assumption, integral model (Morton *et al.* 1956, Fischer *et al.* 1979) had been proposed for jet-like and plume-like respectively. However, they are not appropriate for analysing the entire region of the buoyant jet until 1993, So and Sksoy suggested the velocity and temperature/density decay laws valid for all the three regions in plane or circular buoyant jets by introducing the eddy viscosity.

Recently, Wang and Law (2002) reported a second-order integral model for a round turbulent buoyant jet based on new experimental data on turbulent mass and momentum transport. The mean and turbulent characteristics of a round vertical buoyant jet covering the full range from jets to plumes were investigated using a recently developed combined digital particle image velocimetry (DPIV) and planar laser-induced fluorescence (PLIF) system. The second-order model conserved the mass and momentum fluxes introduced at the source accurately with closure errors of less than 5%. They suggested that although the axial turbulent mass and momentum fluxes are of second-order compared with the axial advective fluxes, they are not negligibly small and should be incorporated for better predictions.

2.3 Previous work on starting jets and plumes

As stated in the introduction, the study of starting jets is motivated by the practical importance and prospects in active flow control. Starting jets can be considered as the limiting case of a fully pulsed jet at zero pulsing frequency, or in another word, a pulsed jet with low pulsing frequency is a periodic series of starting jets (Krueger 2001). Hence starting square jets should possess the characteristics as composite active and passive controlled jets.

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Past experimental, theoretical and computational studies in starting flows cover the range of round turbulent nonbuoyant starting jets and puffs, buoyant starting plumes, and thermals. Turner (1962) studied a starting plume in stagnant surrounding. He proposed that the flows could be considered to be comprised of two simple parts with a quasi-steady-state jet region headed by a traveling vortex. He pictured the head vortex as spherical having modest rotational momentum, while being subjected to viscous and inertial retarding forces but not entraining ambient fluid to any significant extent. The vortex ball is continuously supplied with mass and momentum by the jet region which displays steady-state behavior. This conceptual model has been used successfully in estimating the development of transient jets. Abramovich and Solan (1973) applied this concept to calculate the penetration rate of non-buoyant transient laminar jets. They found that, in the far field, the jet tip velocity was approximately one-half of a steady jet with the same exit velocity at the same location. Witze (1980) extended the application to turbulent jets, using momentum integral methods coupled with steady-state jet growth rate information. Cantwell (1986) studied the starting process of a round jet using an axisymmetric Stokes flow approximation and obtained the particle paths at Reynolds number up to 30. The starting vortex becomes self-similar when the coordinates are scaled by the square root of time. Kuo *et al.* (1986) investigated a laminar starting jet by numerical methods and found a Reynolds number scaling that was different from that of Abramovich and Solan (1973). The laminar and turbulent starting jet models are both based on Turner's (1962) model for a starting plume.

Witze's (1980) model revealed that the time scale equal to the ratio of nozzle diameter to the jet velocity is an important parameter that can correlate the results. The jet tip celerity in the turbulent case was also proportional to the centerline velocity of a steady jet (with the same

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exit velocity) at the same location. Using an inert tracer and image processing techniques, Lahbabi *et al.* (1993) verified this proportionality in turbulent starting jets. They also showed that the starting jet entrainment, in a Lagrangian sense, was approximately 30% greater than the steady jet. Johari *et al.* (1997) studied the structure and mixing of impulsively started jets in a water tank utilizing an acid-base reaction. Experiments and modeling showed that the starting vortex followed the same scaling (i.e. mean velocity with axial position) as that of the steady jet and that its penetration scales with the square root of time (Witze, 1980, Lahbabi *et al.* 1993, Johari *et al.* 1997, Hill and Ouellette 1999). Cossali *et al.* (2001) investigated the near field entrainment in a starting gas jet. They found that the velocity profiles in near nozzle region is not self-similar, unlike those far downstream, and the unsteady jet head structure plays a significant role in the early jet development. They also showed that the jet tip grows linearly with time and the entrainment is larger than that of the steady jet in the near field.

Based on the concept of Turner (1962), Chu and Baines proposed an analytical penetration model for a confined two dimensional starting jet (plane jet). This model solved the spreading rate and penetration rate of the head vortex based on the experimental results of the plane jet. They also found that the spread rate in the starting plane jet is more than double the spreading rate of the steady jet behind the front due to the feeding of mass and momentum to the front which advances at a slower velocity than the velocity of the steady jet.

For the starting plume, most of the past works focus on the penetration rate. Little work looked into the evolution of the head vortex ring and the trailing jet. Actually, the

development of the leading vortex and its interaction with the trailing buoyant jet plays an important role on the starting buoyant jets. Furthermore the quasi-steady-trailing jet needs some time to attain its steady state and its behavior is different from that of the steady state. Middleton (1975) suggested that the penetration has a larger time exponent of $-1/4$ based on Turner's (1962) model. The $1/4$ time behavior for starting plume is also confirmed experimentally by Diez *et al.* (2003). On the other hand, Delichatsios (1979) pointed that the velocity ratio between the head vortex and the trailing jet at the base of the previous one has a maximum value of 0.42, which is larger than the value of 0.38 suggested by Turner in 1962.

Compared to round jets, non-circular starting jets have received relatively little attention. Other than the published work by Auerbach (1987a and b), Hertzberg and Ho (1995) and Grinstein and DeVore (1996), no other investigation on the vortex dynamics of non-circular unsteady jets can be found as far as the author is aware. In particular, starting square jets have not been examined in the laboratory previously.

2.4 Previous work on vortex rings

The transient turbulent jet can be considered to be comprised of a quasi-steady-state jet region headed by a traveling vortex. The leading vortex ring formation and its pinch-off play an important role in the starting jets (Krueger 2001). The interaction between the leading vortex and trailing jet also has a significant effect on the jet dynamic quantities. It determines the pulse frequency to obtain the greatest entrainment ratio.

The recent reviews of Shariff & Leonard (1992) and Lim & Nickels (1995) describe much of the current understanding of vortex ring phenomena as well as some unresolved issues.

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There are two main theoretical models that try to predict the ring structure based on generation parameters. The first is based on the self-similar rollup of vortex sheet. It accounts for the vortex sheet evolution during ejection and gives some insight into the core structure. Essentially the method solves the problem of a turning flow past a two-dimensional wedge. The vortex sheet evolution equation then becomes an integral equation, which can be computed numerically.

When a piston of diameter D moves through a length L ejecting fluid into an ambient fluid of similar density, a starting jet is created with a characteristic stroke ratio L/D . The vorticity generated that situates principally at the boundary layer at the inner surface of the nozzle then rolls up into an initially axisymmetric vortex ring. The evolution of the size, position, and circulation of these vortex rings have been studied experimentally by many researchers (Maxworthy 1977, Didden 1979 and Glezer 1988). Models of the roll-up process have also been considered analytically by Saffman (1975), using planar similarity theory. Nitsche (1996) was able to show numerically that the planar similarity theory correctly predicts the initial motion of a rolling-up vortex sheet issuing from a circular tube. At later times, however, the results from planar similarity theory do not agree with the investigations of vortex ring formation (Didden 1979, Nitsche and Krasny 1994).

Most of the work in vortex ring formation ignores the issue of the roll-up of a vortex sheet but rather focuses on the amount of the dynamic quantities (circulation, impulse, and energy) ejected by the piston-cylinder mechanism generating the ring. The final bulk properties can then be determined by assuming whatever is ejected from the cylinder is eventually entrained by the forming vortex ring. In almost every case, the amount of dynamic quantity generated

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is modeled using a fluid “slug” model where the ejected fluid is viewed as having a uniform velocity equal to the piston velocity, and a pressure equal to the ambient pressure. This model is most commonly used to estimate the circulation of the resulting vortex ring by considering the vorticity flux due to this slug flow. Glezer (1981 and 1988) also used this model to determine the vortex ring impulse. Despite the obvious oversimplifications by this model and the fact that its deficiencies in determining the vortex ring circulation have been demonstrated by Didden (1979), its simplicity is appealing and the model is still widely used in vortex ring research.

The apparently reasonable assumption that all (or nearly all) the circulation impulse, and energy ejected by a piston-cylinder mechanism eventually ends up in the vortex ring, independent of the stroke ratio, is actually not trivial. Since most vortex ring research focuses on $L/D < 3$, this assumption was not fully addressed until the recent work of Gharib *et al.* (1998). They considered large stroke ratios and introduced the important concept of vortex ring pinch-off.

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3.1 Experimental apparatus for the starting jets

The test tunnel shown in Fig. 3.1 was used for the PLIF and PIV experiments employed in Chapters 4 and 5. It consisted of an inlet, head tank, converging duct, test section, flowmeter, control valve and drainage pipe. The test section was made of transparent plexiglass in order to ensure easy optical access. Water was delivered to the test tank through the inlet, and a constant water level was maintained in the head tank during the experiments. To maintain a constant level in the head tank, a drainpipe was fixed on the tank (see Fig 3.1). Screen and honeycomb were installed between the tank and the converging duct in order to reduce the turbulence level of the flow. The water driven by gravity was accelerated through the converging nozzle before discharging into the test tank with a cross-section area of $400 \text{ mm} \times 400 \text{ mm}$ and 1000 mm in length. The initial turbulence intensity measured at the nozzle exit was about $0.5 \sim 0.6\%$ via a laser Doppler anemometer (Wang 2002). The flow rate was monitored by a flowmeter and adjusted by a control valve that was fixed at the end of the rig. For PLIF measurement, a mirror was posted at the far downstream position at 45° to the flow direction to reflect the illuminated cross-sectional images of the jet. The solenoid valve had an opening time of about 20 ms (manufacturers specification), ensuring that the opening procedure can be considered as an impulse.

It should be noted that the test channel shown in Fig. 3.1 is a close tank with the inlet and outlet at the two ends. The in and out flowrate is equivalent. The opening and closing of the

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flow are controlled by the solenoid valve located at the outlet. Another needle valve is employed to regulate the flow rate. In the present tests, the operating frequency of the solenoid valve is 0, i.e. only one operation occurred from the start to the end of the test.

Coordinates are specified from the centerline of the jet in the plane of the nozzle exit, with X , Y and Z standing for the lateral, spanwise and streamwise direction, respectively.

The fluorescent dye (2,7-Dichlorofluorescein) prepared in a solution of alcohol has the property of fluorescing green when illuminated by laser light. Illumination was provided by a light sheet (about 3 mm thick) produced by passing the beam from a 5.0W argon-ion continuous (Spectra-Physics) laser through a cylinder lens. In the experiments, the water from the source tank was seeded with dye, and thus the corresponding jet could be visualized when the laser sheet was shine through. The pictures were recorded by a COOKE Pixelfly (SVGA) 12bit high-resolution (1280×1024) digital camera. The shutter speed was set at 1/125 s, which was quick enough to capture the instantaneous features of jet as it evolved in time. The frame rate was 12.5 frames per second. The surrounding fluid contained no dye and appeared black in the captured images. Hence the green-black dye interfaces could be used to mark the instability, subsequent rollup, and mutual interaction of vorticity formed between the jets and surrounding fluid.

In the present studies, a commercial PIV system (include the laser, cameras and software), Flowmap which is integrated by Dantec, was applied for the velocity and turbulence measurement. The light source employed a dual-cavity pulsed mini Nd:YAG laser (Newwave) with a maximum repetition rate of 15 HZ for each cavity. The energy level was 50 mJ per

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pulse and the pulse duration was about 7 ns. The emitted laser light was green with a wavelength of 532 nm. The light sheet had a typical thickness of 3mm and a divergence angle of 32°. Double-frame 8-bit digital CCD camera (Kodak Megaplug ES1.0) fitted with Nikon 60mm lens was configured for PIV measurement. The spatial resolution of the cameras was 1008×1016 pixels. The real investigation area was about $30\text{cm} \times 30\text{cm}$.

FlowManager software for Dantec's FlowMap PIV systems is a general purpose tool for Particle Image Velocimetry investigations of fluid flows. It take advantage of the experience built into some of the best software packages in the world and combine them to enhance the analysis and visualisation of flows you record with the FlowMap systems. FlowManager software has a range of features for use in FlowMap PIV systems, including: (1) robust and reliable vector processing algorithms, (2) integrated measurement-user interface for control of camera and illumination system, and (3) comprehensive data analysis tools, e.g. FFT cross-correlation, validation filter and statistics. Most of the data acquiring and analyzing were accomplished by this software.

3.1.1 Nozzle configurations

3.1.1.1 Circular nozzle

The nozzle contour is shown in Fig. 3.2a. To avoid the formation of Görtler vortices in the contraction section, the nozzle were machined to follow a numerically function $F(x)$ proposed by Cohen and Ritchie (1962),

$$F(x) = 180(K_3 + b_1 \tan(K_1 Z) + b_2 e^{-K_2 Z^2}), \quad (3.1)$$

where $K_3 = 8.5$, $b_1 = 7.12$, $K_1 = 0.01$, $b_2 = 1.875$, $K_2 = 0.0001$. This function was selected because it is not a series function, and the adjustment of constants can be easily done.

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K_3 and b_1 are determined from the conditions at infinity, i.e. from the known velocities at inlet and outlet. K_1 is a parameter that to a large extent controls the slenderness of the contraction and K_2 should be selected sufficiently small, otherwise, the wall velocity distribution shows an unacceptable hump in the region near $Z = 0$ for certain values of b_2 . The validity of this function used for contracted nozzle has been verified experimentally by Vieira and Aparecido (1999).

The method to determine the constants was to fix the two end points and enable the tangent line at the exit and inlet to be horizontal. The nozzle size is shown as Fig. 3.2c. The nozzle contracted from a round section with diameter of 260 mm gradually to 36 mm with a wall thickness of 1mm at the exit. The contraction ratio was 52:1. A plane was fixed at the exit (identified by an arrow in Fig. 3.2b) for the experiments with an orifice.

3.1.1.2 Square nozzle

The detailed dimensions of the square nozzle are shown in Fig. 3.2d. The Equation (3.1) employed for the circular nozzle also applicable to the square nozzle. A transitional sector was used to convert the nozzle from circular to square cross-section. Here is should be mentioned that flow separation at the cusped joints which might be expected to produce some unsteadiness. However, such an effect should be small (Zaman 1996).

In order to achieve the same momentum flux at the same initial velocity, the exit area of the square nozzle should be equal to the circular nozzle. Hence, the equivalent diameter of the square nozzle D_e can be calculated by the following equation:

$$\frac{\pi D^2}{4} = \frac{\pi D_e^2}{4} = D_s^2 \Rightarrow D = D_e = \sqrt{\frac{4}{\pi}} D_s, \quad (3.2)$$

where D_s is the width of the square nozzle, D exit diameter of the circular nozzle. So the inner width of the nozzle was 32 mm with an equivalent diameter $D_e = 36$ mm (as shown in Fig. 3.2d). The other parameters were the same as the circular one.

3.2 Experimental apparatus for the starting forced plumes

The experiments for the starting forced plume were conducted in another tank. In order to examine the far field of the starting flow and reduce the effect of the return flow, the tests were carried out in a larger tank, with dimensions of $0.55\text{m} \times 0.55\text{m} \times 1.2\text{m}$ (height). A schematic diagram of the experimental set-up is shown in Fig. 3.3. The tank was constructed with glass panels and a stainless steel frame. The source solution was discharged from a round nozzle seated at the bottom of the tank. The nozzle was 100 mm in length and had a diameter of 7.5mm. The source fluid was supplied from a constant head tank located 2.8m above the test tank. The volumetric flow rate was controlled by a needle valve and monitored by a digital flow meter (model Endress+Hauser Promag 33). A solenoid valve with an opening time of about 20 ms was used to ensure that the starting process can be considered to be near instantaneous.

The Flowmap PIV system also employed for the experiments of the starting forced plume. However, PIV and PLIF were conducted simultaneously via a combined PIV/PLIF approach (Law & Wang 2000). Two double-frame 8-bit digital CCD cameras (Kodak Megaplug ES1.0) were configured, one for PIV and the other for PLIF. The PLIF camera was set to single-frame mode while the PIV camera worked in double-frame mode. Thus, the PLIF image was

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double-exposed by the laser pulse pair, the interval of which was determined by the cross-correlation PIV requirement. The two cameras were pointed to nearly the same area with a typical size of $30\text{cm} \times 30\text{cm}$.

Rhodamine B was selected as the fluorescent dye tracer for PLIF. The absorption spectrum of Rhodamine B is from 460 to 590 nm with a peak at 540 nm, while its emission spectrum ranges from 550nm to 680nm with a peak at 590 nm. An optical 532nm band-pass filter was placed in front of the PIV camera, allowing only a narrow band of wavelengths around 532nm scattered by the seeding particles to pass through. Another optical 590nm low-pass filter was attached to the PLIF camera, allowing through only the fluorescent light emitted from the dye tracer.

3.3 Flow visualization techniques

3.3.1 Basic principles for PLIF

For PLIF, the flow is illuminated by a laser sheet whose wavelength is tuned to excite a particular transition of a molecular tracer, which can be a species naturally occurring in the flow, such as CH, OH, NO, O₂, or seeded into the flow, such as biacetyl, acetone, I₂, NO, or dyes in liquid flows. A portion of the molecules in the lower energy level absorb the incident light and are raised to a higher energy state. Some of the excited molecules may dissociate. The undissociated molecules return to the equilibrium state either via non-radiative decay processes, such as collisional quenching, intersystem crossing, or internal conversion, or by emission of some or all of the excess energy in the form of photons. The emission from the illuminated slice of the flow can be imaged by a camera to acquire a 2-D concentration map of the marked fluid. Assuming a single absorbing and emitting species illuminated by a laser

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sheet propagating in the x direction without reaching saturation efficiency for this transition, the total number of photons emitted per unit sheet area per unit time, N_e , is given by

$$N_e(X, Z, t, \lambda) = \int \left\{ \begin{array}{l} \text{Photons incident} \\ \text{per unit area} \end{array} \right\} \left\{ \begin{array}{l} \text{Photons absorbed} \\ \text{per unit length} \end{array} \right\} \left\{ \begin{array}{l} \text{Photons emitted} \\ \text{per photon absorbed} \end{array} \right\} dY, \quad (3.3)$$

$$= \int \left\{ N_i(Y, Z, t) e^{-\sigma(\lambda_L) \int c(X, Y, Z) dX} \right\} \left\{ \sigma(\lambda) c(X, Y, Z) \right\} \left\{ \phi(\lambda_L) \right\} dZ$$

where λ is the wavelength of the laser beam; Y is the coordinate transverse to the laser sheet; $N_i(X, Y, t)$ is the number of photons incident on the flow per unit area per unit time; $(\sigma(\lambda_L))$ is the molecular absorption cross section of the tracer in cm^2 ; $C(X, Y, Z)$ is the concentration of the absorbing species in the lower energy level in cm^{-3} ; and $\phi(\lambda_L)$ is the species quantum, or Stern-Volmer factor, efficiency for this transition. Rayleigh scattering has been neglected since its cross section is normally much less than σ . The effect of intensity on the absorption has also been neglected due to a low excitation limit, and the ground state concentration has been considered constant with time.

The first term in the integral accounts for the attenuation of the incident laser beam due to the absorption by the fluid elements, which comes directly from Beer's law. If substituting $C(X, Y, Z)$ by a mean value C_{avg} , the integral can be approximated as

$$\sigma(\lambda_L) \int C(X, Y, Z) dX = \sigma(\lambda_L) C_{\text{avg}} X = X/l, \quad (3.4)$$

where $l = 1/(\sigma C_{\text{avg}})$ is known as the optical depth of the seeded fluid. The fluid is considered optically thin if $X \ll l$, where light absorption by the fluid is neglected and the exponential term can be approximated to 1.

The emission quantum efficient $\phi(\lambda_L)$, depends on wavelength, fluid temperature, pressure, bulk velocity, and gas composition. However, for some polyatomic tracers, such as acetone,

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the fluorescence efficiency is largely independent of temperature and pressure. It is also independent of the mixture composition for a large variety of gases. Thus, if the absorbing medium is optically thin, the fluorescence intensity from a point in the flow is directly proportional to the concentration $C(X,Y,Z)$ which is desired to measure. By collecting the fluorescence from a slice of a flow using a camera, a planar map of concentration of the seeded fluid is obtained. Absorption can be saturated at high laser intensities. Assuming a constant molecular absorption cross-section, a zero initial excited level population for homogeneous saturation, and an optically thin absorbing medium,

$$N_e = \frac{N_i \sigma C}{1 + \frac{N_i}{N_{sat}}} \phi, \quad (3.5)$$

where N_i is the flux of incident photons, and N_{sat} is the saturation photon flux. N_{sat} can in turn be expressed as

$$N_{sat} = 1 / \sigma \tau, \quad (3.6)$$

where τ is the life time of the photons in the emitting state.

It can be seen that the number of emitted photons depends linearly on the incident flux only when $N_i \ll N_{sat}$. It is usually desirable to work under this limit, known as low intensity limit, to simplify the data reduction procedure.

Briefly, a laser source is used to excite an electronic absorption transition of a species present in the flow of interest. Part of the laser light (energy) is absorbed and in turn, part of the absorbed energy is spontaneously re-emitted (fluorescence). Collisional quenching and other phenomena compete with fluorescence in the process of relaxation from the electronically excited state. The fluorescence occurs at a range of wavelength which is longer than the laser's

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excitation line. Fluorescence is a diffuse process, so the emission is equally probable over the 4π solid angle space.

3.3.2 PLIF Tracer selection and its photophysics

As mentioned above, PLIF has become a powerful technique used extensively to measure fluid-flow scalars such as species concentrations or temperature, or observing the turbulence structure. A major part in implementing PLIF is the choice of fluorescent tracer (dye) and of the laser used for excitation. The present project aims to study the turbulence structure and mixing efficiency using PLIF in an aqueous-phase flow. The flow of interest is a plane shear layer, where the flow is uniformly seeded with a fluorescent tracer, and the ambient tank filled with pure water.

For aqueous-phase experiments, the choice of tracer is dictated by the following factors (Karasso & Mungal 1997):

- (1) The dye must be water-soluble;
- (2) The dye must be stable under the laser excitation power needed to provide signal levels adequate for mixing measurements;
- (3) The dye must have absorption and emission spectra separated enough to avoid absorption of the emitted fluorescence wavelengths within the flow facility (optical trapping).

Following these requirements, 2,7-Dichlorofluorescein ($C_{20}H_{10}Cl_2O_5$, MW = 401.21) is chosen as the dye tracer in the experiments. Dichlorofluorescein is the most widely used class of fluorescent dye. The main concern was that this dye is quoted as not being water-soluble.

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To overcome this problem, the dye was first dissolved in some ethanol, which was subsequently diluted in water. Because the Dichlorofluorescein quantum yield for fluorescein (photons in/photons out) is near 0.9 in water, it is one of the brightest fluorescent dyes available (Fluorescein Derivatives). Several other features of the fluorescein series contribute to their popularity:

- (1) Dichlorofluorescein has a peak excitation near 504 nm, one of the wavelength for the argon-ion laser wavelength. The laser used in present experiment was a 5 W argon-ion laser (at $\lambda_L = 514.5$ nm). Its absorption and emission spectra are shown in Fig. 3.4. From the figure, the peak wavelength of emission is 529 nm. Hence the emitted light can be easily separated from the source light.
- (2) Protection of the hydroxyl groups of the xanthene (three ring) portion forms a non-fluorescent lactone. Cleavage of the protecting groups reveals the fluorophore and gives a huge increase in emission intensity. Because the protected dye has no fluorescence emission, there is no background signal (Fluorescein Derivatives).
- (3) Its chemical stability had been proven by Karasso and Mungal (1997). They exposed a still (non-stirred) dye solution to the laser light for 1 min continuously, and any measurable signal deterioration was not detected in images acquired during this time. This test implied that photobleaching or thermal blooming would not be of concern in the flowing system.

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Due to the change of the wave-length for the laser, the dye used for combined PLIF and PIV measurement for the starting forced plume was Rhodamine B. It has a high quantum yield and a remarkable chemical stability when exposed to laser. The absorption and emission spectra of this dye are shown in Fig. 3.5. The energy absorption spectra is found to be at wavelengths between 460 and 590 nm, with a peak around 540 nm. This characteristic of the dye suited well with the Nd:Yag laser which has a wave length of 532 nm.

The emission spectra that is between 550 and 670 nm with a peak at 590 nm, and does not conflict too much with the absorption spectrum. With a 570 nm low pass filter, the laser light can be blocked and only the fluorescence light can reach the camera.

3.3.3 Calibration for PLIF

In order to keep a uniform spatial resolution over the image when taking the front view image, a right angle between the laser sheet and the CCD camera must be kept. A graph paper was used for this purpose. The paper was positioned in the same plane as the laser sheet. And then the camera was adjusted till the squares on the image had no deformation (see Fig. 3.6).

In order to extract the vortex velocity, the spatial resolution should be calculated. A ruler was put on the center line, hence the actual length can be read from the ruler and the pixels number take from the image. Thus the spatial resolution is:

$$\text{Spatial resolution} = \text{Length} / \text{Pixels} = 185 / 970 = 0.19 \text{ mm} / \text{pixel} \quad (3.7)$$

The same calibration was done on the cross section planes, locating at $Z / D_e = 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0, 2.5, 3.0, 4.0, 5.0$. Table 3.1 illustrates the results of calibration on different horizontal planes.

3.4 PIV techniques

3.4.1 Theory of PIV

Particle Image Velocimetry (PIV) is a measurement technique for obtaining the velocity vectors based on the movement of small particles, or seeding. The technique is based on the well known and simple equation of speed equal to distance traveled divided by time taken. In PIV, the parameter that is actually measured is the distance traveled by a group of particles in the flow within a known time interval. A laser is used to generate a vertical light-sheet shining through the area of interest. A CCD camera is positioned at right angles to the light-sheet generated and is used to capture the scattered light by the illuminated seeding particles (Fig. 3.7).

During the test, the laser will generate two subsequent pulses at a very short time interval in between them. With the help of optics, the laser pulses spread into a flat sheet with a thickness around 3mm that strike into the area of interest. The scattered lights of the seeding particles in the flow are captured by the CCD camera. With current available laser technology, the camera is able to record the two images separately. A standard procedure called cross-correlation is used to correlate the second image to the first one to obtain the displacement vectors.

The primary objective of cross-correlation is to correlate a group of particles in two images. The principle can be illustrated in a one-dimensional case. For example, a curve $f(x)$ moves in x -direction and becomes $u(x)$ in Fig. 3.8. To determine the displacement of curve $f(x)$, a person will move it along the x -axis and find the best fit. It is obvious that the summation of the products of each point on the two curves will be maximized when $f(x)$ match $u(x)$.

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Therefore, the matching process is transferred to computer coding by finding the maximum summation of the product of two curves, $f(x-m)$ and $u(x)$, for different values of m along the x -axis. The calculated m value is the displacement of the curve $f(x)$.

In two-dimensional cases, the cross-correlation does the same thing. The camera images are first divided into rectangular regions called interrogation areas (IA) or interrogation regions. For each of these interrogation areas, the program will move the first-pulse-image and calculate the sum of products of every pixel with the second-pulse-image to obtain the average particles displacement vector. Doing this for all the interrogation regions produce a displacement vector map. The displacements are divided by the time interval between the two captured images to obtain velocity vectors. These are so-called raw velocity vectors.

In order to speed up the process of correlations, Fast-Fourier-Transform (FFT) algorithm is used. The captured particle images may be considered as two-dimensional signal field, analogous to a one-dimensional time series. The Fast Fourier Transformations of the two particle images are first obtained. They are then multiplied by each other. The results are inversely transformed and the peaks detected. The displacement vectors can then be obtained.

The cross-correlation will always produce results no matter what the initial input images are. Therefore, it is necessary to follow some rules of thumb to get meaningful results. The minimum number of particles within each interrogation area is five. The magnitudes of displacements are recommended to be kept below a quarter of the side of the interrogation area. The maximum velocities and the time interval between pulses are calculated based on

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this rule. The size of the particles should occupy five pixels in the area. Sometimes, the PIV camera is deliberately slightly de-focused to obtain the blurred particle images for this purpose.

The vector maps produced after cross-correlation are termed raw vector maps. Some vectors are incorrect due to the noise signal, which are also termed outliers. Therefore, a few techniques provided in FlowManager are used to validate the vectors on the raw vector maps. These include Peak-height validation, velocity-range validation, and moving-average validation.

The outliers or incorrect vectors occur in almost any PIV measurement. It is important to note that there will always be an outcome whether the input is meaningful or not. The Peak-height validation verifies or rejects individual vectors based on the values of the peak heights in the correlation plane where the vector displacement was measured. In the correlation plane, there will always be the highest peak, which is hopefully to be the signal peak. This highest peak should be greater than the second-highest peak by a certain factor if it is valid:

$$\frac{\text{Highest\_peak}}{\text{Second\_highest\_peak}} = k_p \quad (3.8)$$

This is also known as delectability criterion. It is recommended to use 1.2 for the value of k in Eq. (3.8) for validation. In the case of insufficient seeding particles in the images, it might be necessary to use a lower k value.

Velocity-range validation rejects vectors based on the maximum possible velocity vectors in the flow field provided by the user. The range can be calculated based on past experience, formulas provided by other researchers, the upstream conditions, criteria of displacement less

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than one-quarter of IA side, or any other methods. In the plume and jet experiments described here, the maximum velocity was first determined based on input from other researchers. In this experiment, the IA was assuming to be 32×32 pixels. The time interval between pulses was determined based on the maximum velocity calculated from previous experiments. About 80% to 90% of tolerance was allowed and the time interval between pulses was reduced to the nearest hundred micro-second. The maximum velocity was then back calculated from the time interval.

Moving-average validation is based on an implicit assumption of the continuity behavior of the flow field. It means that the velocity field is slowly changing so that there is not too much variation from one vector to its neighborhood vector. It rejects vectors by comparing them among the neighborhood vectors. Thus, if a vector deviates too much from its neighborhood vectors, it must be an error vector.

The choice of the seeding particle depends on a number of parameters. Primarily the seeding material should be chosen considering the flow that is to be measured, and the illumination system available. In generally seeding particles should be chosen as large as possible in order to scatter the most light, but the particle size is limited, since too large particles will not track the flow properly. In general the maximum allowable particle size decrease with increasing flow velocity, turbulence and velocity gradients. One should be aware also, that the camera images of seeding particles should have a diameter of at least 2 pixels, preferably 3 pixels or more. This will allow the system to estimate particle positions and displacements to subpixel accuracy, effectively increasing the resolution of the technique.

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Ideally the seeding material should also be chosen, so the seeding particles are neutrally buoyant in the carrying fluid, but in many flows this is a secondary consideration.

Finally it should be considered how the flow is seeded. They should be able to follow the flow streamlines without excessive slip. Water flows are often implemented using water in a closed circuit, and here commercial seeding particles such as latex beads or pine pollen can be used. Air flows on the other hand are usually not recirculated, and thus require the seeding to be generated at the inlet and disposed of at the outlet.

Another criterion for PIV is that more than 5 particles should be possessed in one IA, which determines the seeding density. According to the criteria of the PIV measurements mentioned above, Polyamide Seeding Particles (PSP), which have a mean particle diameter of 20 μm and a density of 1.03 g/cm^3 , were selected as the tracers for the PIV measurements, which has a seeding density of 10-20 particles in a 32 by 32 pixel sampling window.

3.4.2 Experimental procedures for PIV

The experimental procedures for taking velocity measurements by PIV can be summarized into four different stages. They were (1) preparation, (2) laser alignment and calibration, (3) focusing of cameras, and (4) selecting PIV measurement parameters.

3.4.2.1 Preparation

The first step was to prepare the ambient and jet fluids. For starting jet experiments, ambient and jet fluids are both fresh tap water, while for starting forced plume, saline water or calcium chloride solution was used as the ambient fluid. Fresh tap water or ethanol solution was

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employed as the jet fluid. Before the experiments, the solutions were prepared according to the expected density deficit and refraction indices. Density meter was used to measure the density and temperature of the solutions. The test tank was drained, cleaned thoroughly and then filled up with ambient fluids. Seeding particles were subsequently added into the ambient fluid to enhance the velocity measurements in the vicinity of the flow. The water in the test tank was then left to settle for a certain time to assure that the water in the test tank was sufficiently still. At the same time, make sure all the equipment run well and verify all the connections. Turn all the switch and valve to the specified position.

3.4.2.2 Laser Alignment and Focusing of Cameras

A string tied with a weight was hung along the centreline of the nozzle at the middle of the tank. The laser was then adjusted to align with the string to obtain a perfect verticality. With the laser sheet being properly aligned, the verticality of the nozzle can be checked again with laser flushing at its centreline. The distance between the laser and the area of measurement was kept to a minimum so as to obtain the maximum light intensity but yet wide enough to illuminate the whole imaging area evenly. During the actual run of the experiments, the lights were switched off to minimize any possible impacts from the ambient light.

Next was to adjust the focusing of the camera. This step was necessary to make sure that the camera axis was perpendicular to the laser light and thus geometric distortion could be avoided. This step was assured by placing a graph paper at the area of measurement. The grid paper was attached to a perspex sheet so as to ensure that it was hanging vertically downwards. The position of the camera was adjusted so that the camera was able to capture the desired

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area of measurement and the images captured were free from geometrical distortions. Fig. 3.9 shows an image of the grid paper with no geometrical distortion.

Once the position of the camera had been fixed, it was necessary to determine the object to image scale factor. The scale factor was required by the PIV system to scale the velocity vectors. Fig. 3.10 shows the Co-ordinate system and Image-/object –plane.

For PIV measurements, the camera was defocused so that the camera image of a seeding particle covered at least 3 to 5 pixels. By doing so, the PIV system was able to estimate the particles' position and displacement to sub-pixel accuracy. This would effectively increase the resolution of the technique. An example of the images of the particles after defocusing of the camera can be found in Fig. 3.10.

3.4.2.3 Selection of PIV Measurement Parameters

To obtain high quality PIV measurements, it was very important that the correct parameters were selected. These included the dimensions of the measurement area, the size of the interrogation areas, and the thickness of the light sheet and the optimization of the time interval between the pulse pairs.

In order to get the best combination of the experimental parameters, several trial runs with experimental conditions similar to that of the actual tests were conducted. The camera images are divided into rectangular regions called the interrogation areas (IAs). For the present study,

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each IA used was made of 32 x 32 pixels, with 50% or 75% overlapping. This allowed us to extract 62 or 124 cross-section profiles.

Once the size of the IAs had been determined, optimization of the time interval between pulse pairs should be determined. Typically, the larger the flow field displacements, the more accurate the measurements obtained. However, in PIV, the maximum displacement of the particles is recommended to be kept below a quarter of the side of the IA so as to reduce the effects of velocity biasing and signal drop out. This implied that the recommended maximum displacement on the camera should be 8 pixels (for further information refer to Section 3.5).

PIV requires a certain thickness of the laser sheet to prevent seeding particles from moving out of plane during the pulse pair interval that may lead to significant bias in the velocity determination. In this study, the thickness of the laser sheet was 3mm.

Another important parameter influencing the quality of the results is the number of seeding particles within each interrogation area. With a large number of seeding particles, there will be many true correlations ensuring a high signal to noise ratio, and in this case, an average displacement exceeding one quarter of the side of the IA may be acceptable. Generally a minimum of 5 particles per interrogation area is recommended to achieve reasonable results using correlation techniques.

Normally 600 hundred double-images were taken for one run. The first hundred is utilized to analyze the starting process. The last 400 images were employed to compute the statistics, i.e.

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average velocity and turbulence. A summary of the PIV parameters adopted can be found in Tables 5.1 and 6.1.

3.5 Uncertainty analysis

3.5.1 Error for PLIF measurements

The data from the PLIF flow visualization were obtained directly from the video images. The axial position of the front and the trajectory of the vortex ring core were measured as functions of time by analyzing the recorded images one frame at a time. Image preprocessing comprised background noise subtraction and contrast enhancement. The sampling rate was 12.5Hz, and the integration time for each frame was 8ms. Hence the exposure time was much smaller than the time associated with the large-scale motions of the jet and turbulence and molecule diffusion. The jet tip was clearly discernible in the PLIF images because the interface between the jet and ambient fluids was distinct. This interface was highly contorted due to the small scale eddies and K-H roll-ups. Even though the interface appearance varied among different runs, the axial location of the jet tip interface was quite repeatable. The uncertainty in the axial location of the jet tip stemmed primarily from the inaccuracies associated with visually measuring the interface location from the video monitor. Uncertainty of the jet tip position and vortex core position was estimated to be within 2 pixels. Its real value was about $2 \times 0.19 = 0.38$ mm, about 0.2% related to the whole investigate area. The jet tip position errors might be slightly larger at 4 pixels due to the ambient fluid pre-contaminant (dye diffused in the ambient fluid before the jet motion). In the far field, the accuracy of the measurement was estimated to be better than 2%. In the near field, the relative accuracy is slightly larger, but with smaller development time and thus smaller dye diffusion area. Thus, it is estimated that the accuracy of the measurement is lesser than 5%.

The frame rate 12.5 frame per second provided sufficient resolution for present low-speed flows and resulted in a temporal uncertainty of $1/25$ (0.04) second. The error for the time interval between the frames is minor and can be neglected. Therefore, the error in the velocity magnitude can be estimated to be less than 2%.

3.5.2 Error for PIV measurements

PIV measurement contain errors arising from several sources: (1) Random error due to noise in the recorded images; (2) Bias error arising from the process of computing the signal peak location to sub-pixel accuracy; (3) Gradient error resulting from rotation and deformation of the flow within an interrogation spot leading to loss of correction; (4) Tracking error resulting from the inability of a particle to follow the flow without slip; and (5) Acceleration error caused by approximating the local Eulerian velocity from the Lagrangian motion of tracer particles.

Certain errors can be minimized by careful selection of experimental conditions (e.g., tracking error). However, other sources are inherent to the nature of the correlation in PIV and cannot be eliminated. For example, even if the recorded images are free from noise, the location of the correlation peak can be influenced by random correlations between particle images not belonging to the same pair (such random correlations are visible as small peaks). In addition, bias error result from a phenomenon called pixel-locking in which the signal peak location is biased towards the nearest pixel while using a curve-fit or centroiding schemes to locate the discretized signal with sub-pixel accuracy. Similarly, gradient errors will occur in turbulent flow. Finally, acceleration error cannot be eliminated because of the very principle of PIV

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which uses the Lagrangian motion of particles to approximate the instantaneous Eulerian flow velocity.

In typical flows, particle streamlines are not straight, but curved. Consequently, an error is incurred when the Eulerian velocity is approximated using Lagrangian particle displacements. It is easy to understand that if the pulse separation Δt is greater, then is ΔX , and greater is the deviation of the measured velocity from the true velocity. This suggests that one must reduce Δt . On the other hand, if one were to reduce Δt too much, the measurement of ΔX becomes difficult; at very small values of Δt , it may become impossible to distinguish ΔX from random error. Obviously, simple error considerations suggest that the fractional random error can be reduced only by increasing ΔX , and corresponding Δt . So it is very important to properly select the value of Δt .

Theoretical analysis and extensive numerical simulations (Keane & Adrian 1990) show that the performance of the double-pulsed cross-correlation technique is optimized by the following approximate criteria:

$$N_I \geq 5, \quad (3.11a)$$

$$\sqrt{W^2 + U^2} \Delta t < \frac{1}{4} d_I \quad (3.11b)$$

$$|V| \Delta t < \frac{1}{4} \Delta Y_0 \quad (3.11c)$$

$$d_\tau = 2 \sim 4 \quad \text{pixels} \quad (3.11d)$$

where N_I is the seeding concentration, i.e. the number of particles in an interrogation area, W , U are the velocity components in the plane of the light sheet and V is the out-of-plane

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velocity component, Δt is the time interval between the two exposures, ΔY_0 is the thickness of the light sheet, d_l is the width of the interrogation area, and d_p is the particle image diameter.

Errors associated with PIV algorithms often are decomposed into two components: mean bias error and RMS error (Huang *et al.* 1997). Given the actual displacement, d_a , and the estimated PIV displacements, d_i ($i = 1, 2, 3, \dots, N$), the mean bias error can be defined as

$$E_{mean} = d_m - d_a, \quad (3.9)$$

where d_m represents the spatial mean of the N estimated displacements. Mean bias errors arise when the actual correlation peak is not well represented by the peak-fitting curve or procedure (Huang *et al.* 1997). This is often the case when out-of-pattern effects are prominent, or when strong gradients within the subimage act to broaden and/or distort the symmetry of the correlation peak (McKenna & McGillis 2002).

Bias errors arise during the process of calculating the particle displacement to sub-pixel accuracy. Essentially the correlation field is available on a discretized grid (typically 32×32). The location of the maximum value in the correlation field will correspond to the particle displacement, but obviously such a displacement will be an integer pixel value. In order to reduce measurement error, one attempts to locate the peak with sub-pixel accuracy using either a curve-fitting method or a centroiding scheme.

The term bias error was coined to describe the phenomenon of pixel-locking, i.e. during the process of determining the displacement to sub-pixel accuracy, the resulting value is always biased toward the nearest integer-valued pixel (Prasad *et al.* 1992). Bias error is zero if the

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particle is displaced exactly n pixels or $n+0.5$ pixels, the measured displacement $n < \Delta X < n+0.5$ pixels, the measured displacement is biased towards n , and for $n+0.5 < \Delta X < n+1$ pixels, the measured displacement is biased towards $n+1$. The true and measured values coincide at integer and at half-pixel values.

The bias error in PIV occurs primarily at the time of calculating the centroid or curve-fit of the correlation signal peak (Prasad *et al.* 1992). Uncertainty analysis of PIV measurements usually requires Monte Carlo simulation (Willert and Gharib, 1991). The software used, FlowManager employs the advanced adaptive-cross-correlation technique and the cross-correlation peak can be resolved with a sub-pixel accuracy of approximately 0.01 pixel (Willert & Gharib, 1991).

By satisfying the constraints mentioned above, it is possible to make valid measurements in over 90~95% of the interrogation spots with small bias errors. McKenna and McGillis (2002) reported that the maximum uncertainty based on FFT/correlation techniques for the local velocity and vorticity magnitude is $\pm 1\%$ and $\pm 3\%$, respectively. In present studies, using a 32×32 interrogation area, the dynamic range of PIV is $-8 \sim +8$ pixels. Through the careful alignment and configuration, the resolution of PIV depends on the sub-pixel interpolation accuracy and is estimated to be within $0.1 \sim 0.3$ pixel, which results in the relative accuracy of $0.3/8 = 3.75\%$ for the velocity.

Previous work shown that random errors in PIV usually scale with the particle image diameter (Adrian 1991), i.e.

$$E_{rms} = C_e d_\tau. \quad (3.10)$$

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where d_τ is the effective particle image diameter, and C_e is a constant whose value is between 0.05 and 0.10 depending upon the following experimental conditions (Westerweel *et al.* 1997):

(1) choice of interrogation method: auto-correlation or cross-correlation; (2) Choice of peak location technique: centroiding or curve-fitting; and (3) Irregularity of particle images, film gain, pixel readout noise, and other practical limitations on obtaining ideal images. Therefore,

taking $C_e = 0.1$ and $d_\tau = 3$, and $d_I = 300$, the estimated E_{rms} read

$$E_{rms} = 0.1 \times 3 = 0.3 \text{ pixel}, \quad (3.11)$$

which is the same relative accuracy of 3.75% as the bias error.

Therefore, the overall uncertainty of the measured velocity components can be calculated

$$E_{total} = E_{mean} + E_{rms} = 3.75\% + 3.75\% = 7.5\% . \quad (3.12)$$

As demonstrated by Prasad *et al.* (1992), the variation of the variation of the DPIV bias error with respect to the actual displacement is sine-wave-like with a wavelength of one pixel. When measuring turbulent flows, the velocity fluctuations cause both the negative and positive biasing to occur alternatively or intermittently. As such, the contribution of the bias errors to the measured mean velocity should be generally suppressed due to the neutralization of the negative and positive biasing.

3.5.3 Effects of the confinement of the test channel

Ideally, the jet should be introduced into an infinite environment. But in almost every laboratory experiment, the jet is confined by a finite enclosure that would unavoidably induce a co-flow as shown in the Fig. 3.11. Assuming the co-flow is uniform, the velocity of the co-flow, W_∞ , reads

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$$W_{\infty} = \frac{\overline{W}_0 A_0 - \pi W b^2}{A_{total} - \pi b^2}, \quad (3.13)$$

where A_{total} and A_0 are the cross-sectional area of the test channel and nozzle, respectively.

Because $A_{total} \gg \pi b^2$, the velocity ratio between the co-flow and jet flow can be written as,

$$\frac{W_{\infty}}{\overline{W}_0} < \frac{A_0}{A_{total}} = \frac{36 \times 36}{400 \times 400} = 0.008. \quad (3.14)$$

From the PIV results of the cross-sectional distribution for mean velocity, one can note that the co-flow around the jet flow is actually small (refer ahead to Figs. 5.25-32) and varies around 0. Thus, the co-flow in the present experiments can be neglected.

On the other hand, the mirror installed at the bottom of the tank could disturb the jet flow and co-flow. In order to ease the adverse effect of the mirror, a porous plate was fixed at the bottom of the test channel. The porous plate could ensure the co-flow to be uniform and ease the disturbance of the mirror. Furthermore, it was fixed at the bottom of the test channel and far from the test area. The test area was located within the top area of 324 mm, while the mirror is about 1 m away from the top, which is relatively far away from the nozzle. In addition, the relative areas between the nozzle and tank, A_0/A_{total} , is 0.008, as mentioned in Eq.3.14. Thus the influence of the mirror on the experimental results is minor.

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Table 3.1 Spatial resolution in different horizontal planes

Location Z/D_e	Resolution (<i>pixel/mm</i>)
0.25	11.63
0.5	11.78
0.75	11.86
1.0	11.99
1.25	12.03
1.5	12.15
1.75	12.26
2.0	12.38
2.5	12.64
3.0	12.82
4.0	13.42
5.0	13.84

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Fig. 3.1 Schematic diagram of PLIF and PIV experimental arrangements and the coordinate system used. Both cross-sectional (i.e. X - Y plane) and streamwise-sectional (i.e. X - Z plane) images were obtained.

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Fig. 3.2 Schematic diagram of the square and circular nozzle (all in mm).
 (a) Front view of the Nozzle; (b) Front view of the Orifice;
 (c) Circular end view; (d) Square end view;

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Fig. 3.3 Experimental set-up for the starting forced plumes.

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Fig. 3.4 Absorption and emission spectra of 2,7-Dichlorofluorescein (taken from Molecular probes Inc.).

Fig. 3.5 Absorption and emission spectra of Rhodamine B (taken from Dantec flowmap installation and user's guide, 1997).

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Fig. 3.6 Graph paper image for calibration avoiding distortion.

Fig. 3.7 Basic arrangement of PIV system.

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Fig. 3.8 Illustration of cross-correlation of a curve.

Fig. 3.9 Image of the graph paper.

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Fig. 3.10 Co-ordinate system and Image-/object-plane

Fig. 3.11 Schematic showing co-flow in the present test rig.

Chapter 4 QUALITATIVE RESULTS FOR THE STARTING CIRCULAR AND SQUARE JETS

4.1 Introduction

This chapter presents the results of an experimental study on starting square jets at three Reynolds numbers ($Re = W_0 D_e / \nu = 2358, 3528, 4716$) utilizing Planar Laser Induced Fluorescence (PLIF). Utilizing the qualitative flow visualization results, the present chapter globally describes the vortex evolution in starting jets. More details (i.e. the quantitative variables of energy, circulation, velocity) of the square and round jets will be further described in Chapter 5.

Starting circular jets under the same initial Re were also investigated as a basis for comparison. The present study on starting jets is motivated by practical applications related to the unconfined and unsteady turbulent flows from the initiation of steady and interrupted gases such as propulsive combustors. The control of mixing is necessary to increase combustion efficiency and to avoid characteristic instabilities that may lead to liquid releases by process upsets, explosions and unwanted fires. The global flow pattern in a starting jet is related to the vortex dynamics of the coherent structures in the shear layers (Hertzberg and Ho 1995). A starting jet can be considered as a pulsed jet with a low pulsing frequency, or in the limiting case of a fully pulsed jet at zero pulsing frequency.

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In order to evaluate the hazard area of unexpected fire or explosion and mixing efficiency of the fuel jet in combustors, the most important parameter is the penetration rate in starting jets. How the leading vortex and the trailing vortices, secondary vortices interact each other and thus affect the penetration rate and entrainment rate has become one of the most popular topics in the last decade.

In this chapter, the vortex structures generated in starting circular and square jets will be presented through the PLIF images. Some classic phenomena on the vortices interaction in the starting jets, such as vortices leapfrogging, coalescence, pinch-off, were observed via the flow visualization images. Axis-switching was also observed in the starting square jets. The data for the penetration rate of the starting circular and square jets were extracted from the images. The differences between the circular and square jets will also be discussed in the following sections. Some of the results in this chapter were reported in Ai *et al.* (2005a).

4.2 Experimental conditions

The square jet was issued through a converging nozzle attached to a constant head tank elevated above the test tank shown in Fig. 3.1. The water level at the head tank was maintained at about 800mm above the nozzle axis to provide a steady head. The water temperature was kept at about 20°C. The volumetric flow rate was controlled by a needle valve and monitored by a digital flowmeter (model Endress+Hauser Promag 33). The velocity at the jet exit, W_o , was calculated based on the cross-sectional nozzle area. The Reynolds number is defined as:

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$$Re = W_0 D_e / \nu \quad (4.1)$$

where D_e is the equivalent diameter of the square jet, W_0 the initial velocity and ν the kinematic viscosity. A solenoid valve with an opening time of about 20 ms was used to ensure that the opening procedure can be considered to be near instantaneous.

The two different types of nozzles used in this study are shown in Fig. 3.2. The contraction ratio for the round nozzle (A_{in} / A_{out}) was 52:1. As mentioned in Section 3.1.1.1, the contour of the nozzles was numerically machined to follow a family of high-order polynomial shapes designed to avoid the formation of Görtler vortices in the contraction section (Zaman, 1996). An orifice plate was subsequently fixed at the exit to form an orifice slot (Fig. 3. 2b).

Planar Laser Induced Fluorescence (PLIF) technique was used to monitor the evolution of the square jets, and 7-Dichlorofluorescein ($C_{20}H_{10}Cl_2O_5$, MW = 401.21) was chosen as the dye tracer. A 5W argon-ion laser through a cylinder lens with a 3mm thickness light sheet acted as the source of illumination. Images were captured by a 12 bit high-resolution ($1k \times 1k$) digital camera (COOKE Pixelfly). The shutter speed of the camera was set at 1/125 s, which should be quick enough to capture the instantaneous distribution of the jet. The frame rate of the camera was 12.5 frames per second.

The laser sheet and the CCD camera were posited perpendicular to each other in order to keep a uniform spatial resolution over the image when taking the side views. To achieve that, a graph paper was positioned in the same plane as the laser sheet. The position of the camera was then adjusted so that the camera was able to capture the image of the graph paper without any geometrical distortions.

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To determine the spatial resolution, an image of the ruler which was put at the laser sheet plane was taken so as to relate the actual length of the image to the image pixels. The spatial resolution of the front view was calculated to be typically of 0.19 mm/pixel. A total of 12 tests were performed on the starting circular and square jets. The initial parameters are listed in Table 4.1. The results of the tests are discussed in the following.

4.3 Results and discussion

4.3.1 Jet penetration

In the near field within approximately four exit diameters from the exit plane, the penetration rate of the jet head is nearly constant for both circular and square jets as shown in Fig. 4.2. After that, it changes to an asymptotic behavior that is proportional to the square root of time, as shown in Fig. 4.3. The transitional point from linear to asymptotic behavior varies with the Reynolds number. Within the range of Re in the present experiments, the transitional point was located at around $4D_e$ (see Fig. 4.3). Cossali *et al.* (2001) reported the turning point at a location around $6D_e$ downstream for a gas jet with a Re of 12,600. On the other hand, Lahbabi *et al.* (1993) suggested that the asymptotic rule holds true for downstream distances greater than $10D_e$.

Table 4.2 and Fig. 4.2 show the experimental results of the average tip velocity $\overline{W}_t$. Comparing the $\overline{W}_t$ among all the cases (see Fig. 4.2), the tip velocity for a square jet was much lower due to vortex interaction and deformation (e.g. leapfrogging, axis-switching). For $Re = 3528$ and 4716, the average tip velocity was approximately 15% lower in square jet

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while the reduction is more significant, at 30% for the lower $Re = 2358$. Because of momentum conservation, the lower tip velocity indicates that more ambient fluids would be entrained resulting in higher mixing efficiency in square jets.

Hill and Ouellette (1999) reported a comprehensive analysis on transient gas jets. In the asymptotic range, they showed that a length scale $\bar{Z}_t'$ is defined by:

$$\bar{Z}_t' = Z_t / (M_0 / \rho)^{1/4} \quad (4.2)$$

where Z_t is the tip length, M_0 the momentum flux, ρ the density, and $\bar{Z}_t'$ varies linearly with $t^{1/2}$ i.e.,

$$\bar{Z}_t' = C_z' t^{1/2} \quad (4.3)$$

For the penetration rate C_z' , Cossali *et al.* (2001) and Sangras & Faeth (1999) suggested 3.4 and 2.76, respectively, and Johari *et al.* (1997) reported a value of 2.55 (see Figs. 4.4). Witze (1980) and Lahbabi *et al.* (1993) also followed Hill and Ouellette's (1999) format. In the present experiments, the results agreed with Eq. (4.3), with C_z' found to be 4.0 and 3.4 for the circular and square jets, respectively. The initial conditions of these studies are listed in Table 4.3 for a direct comparison. The scattering of C_z' values (with the range from 2.55 to 4) may be attributed to the differences in the experimental conditions, i.e. nozzle geometry, turbulence intensity and the history of the jet acceleration. In the present investigations, the nozzle geometry (circular vs square) significantly influenced the jet penetration rate. On the other hand, Wize (1983) attributed the difference in penetrate rates to the difference in turbulence levels, with larger entrainment from stronger turbulence causing the transient jets to

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penetrate at a slower rate. Gharib *et al.* (1998) also found that the history of the jet acceleration would affect the vortex formation and subsequently the head vortex penetration.

Hill and Ouellette's (1999) analysis implies that the nondimensional tip length, $\bar{\bar{Z}}_t = Z_t / D$, depends linearly on the square root of the normalized time $\bar{\bar{t}} = tW_0 / D$,

$$\bar{\bar{Z}}_t = C_z'' \sqrt{\bar{\bar{t}}} \quad (4.4)$$

This relationship was also reported by Lahbabi *et al.* (1993) and Sangras & Faeth (1999). Note that if the source jet velocity can be assumed to be uniform, then

$$M_0 = \rho W_0^2 \frac{\pi D^2}{4} \quad (4.5)$$

Substituting Eq. (4.5) into (4.3) yields

$$Z_t / D = C_z' \left(\frac{\pi}{4} \right)^{1/4} \sqrt{t \frac{W_0}{D}} \quad (4.6)$$

or

$$C_z' = C_z'' \left(\frac{\pi}{4} \right)^{1/4} \quad (4.7)$$

Hence, the two parameters, C_z' and C_z'' can be related to each other.

4.3.2 Vortex growth

Many researchers (Becker, 1968 and Beavers & Wilson, 1970) had investigated the vortex evolution in steady jets. However, very few studies had addressed the vortex growth in a starting jet. Turner (1962) suggested that a starting jet can be modeled as a head vortex and a quasi-steady jet. Hence the head vortex can be compared with an isolated vortex and the quasi-steady-jet can be related to a steady jet. The starting jet can also be considered as a

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sequential vortex ring generator (Liepmann & Gharib, 1992). In reality, the trailing jet may need some time/distance to develop due to the evolution of the vortex sheet before it becomes a quasi-steady jet. The interaction between the trailing jet and the head vortex can affect the vortex formation. However, the global flow pattern in a starting jet is linked to the vortex dynamics of the coherent structures in the shear layers. The formation and evolution of an isolated vortex ring may provide clues to the behaviour of the starting jet (Hertzberg & Ho, 1995).

Characteristically, the vortex ring diameter always over-shot its equilibrium level at the beginning. A typical trajectory of the vortex center is shown in Fig. 4.5 for a circular nozzle at $Re = 3528$. The growth rate S is also defined in the figure. After the over-shooting, the vortex size actually shrank slightly before increased steadily (see Fig. 4.6). With the influence of the streamwise vortices, the azimuthal vortex then grew suddenly at the normalized time of about 6. Finally, the starting jet would become completely turbulent. The over-shoot was observed for both circular and square starting jets (Figs. 4.6 and 7). Unlike a circular jet, the size of the vortex of a square jet oscillated as the jet traveled downstream, due to the three-dimensional vortex dynamics in the square jets. The over-shoot could also be observed in the evolution of the jet tip velocity (Figs. 4.8 and 9). This phenomenon was previously detected by Maxworthy (1977) and Didden (1979). The over-shoot happened shortly after the jet motion began. The time (t_{over}) and position (Z_{over}) of the over-shoot varied with Re . The larger the Re , the shorter the time of over-shoot would occur (see Table 4.4).

Table 4.4 also shows the average growth rate S of the head vortex. The growth rates for the present cases are much larger than those found by Maxworthy (1977) who did his experiments

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using a piston-driven vortex generator with a small stroke ratio L/D (less than 1.5). This can be explained as follows. The continuous injection from a starting jet creates streamwise vortices causing azimuthal motion. These azimuthal motions, which evolve and amplify in the braid region between primary vortical structures, are known to drastically alter the entrainment process in the near field and increase the rate at which fluid is entrained into the jet (Liepmann & Gharib, 1992). Gharib *et al.* (1998) found that the size of the leading vortex ring is approximately the same between a range of stroke ratio from 3.8 to 14.5 and smaller with a stroke ratio of 2.0. The present experimental results confirmed this trend (Figs. 4.6 and 4.7). From Table 4.4, it can also be noted that the growth rate and the size of the over-shoot vortex in the square jets are much larger than those of the circular jets. The larger growth rate implies that the streamwise vortices had enhanced the entrainment in square jets substantially.

The distance of vortex spiral center Z_w is shown in a log-log plot in Fig. 4.10. Saffman (1978) proposed the following similarity law for a starting jet:

$$\overline{Z}_w \sim \bar{t}^{2/3} \quad (4.8)$$

All quantities with an over-bar are rendered dimensionless using W_0 and D . On the contrary, Didden (1979) suggested that the best approximation of his experiment data for a starting circular jet was:

$$\overline{Z}_w = C_t \bar{t}^{3/2} \quad (4.9)$$

where $C_t = 2.8$. Didden (1979) suggested that the deviation revealed an essential difference between ring vortices and straight vortices formed at a flat plate. For ring vortices, a translational velocity in the X -direction is induced because of the curvature of the vortex axis, even if it is small. Since the induced velocity depends on the curvature $1/D$, a characteristic

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length would govern the flow from the beginning and thus the similarity arguments are not strictly valid. Auerbach (1987a) obtained the same time power of $3/2$ for a rectangular vortex generator with an aspect ratio of 2. As shown in Figs. 4.10 and 4.11, the present results agree well with the $3/2$ time power for both circular and square jets.

Comparing Figs. 4.10 and 4.11, C_t is much larger for the circular jets than the square jets at the same initial velocity. This is similar to the previous results on the jet tip penetration. Auerbach (1987a) also found that Didden's (1979) non-dimensionalized distance for the ring vortex was much larger than his rectangular vortex (both with $W_p = 46$ mm/s). This may be attributable to two factors. First, for the case of a ring vortex, the neighboring vortex element would induce an additional translational velocity from the very beginning of the motion. Second, the larger coefficient could be due to the effects of the azimuthal motion. In previous studies, almost all considered the center plane vortex pair as a two-dimensional flow. In reality, it could be significantly affected by the sidewall and corner vortices. Due to Biot-Savart self-induction, corner vortices would move faster downstream. Their interaction and deformation could be the main reason for the slow vortex axis motion especially with a small aspect ratio geometry. The three-dimensional effects also lead to the larger growth rate S (see Table 4.4) compared to the circular jet. The oscillating trajectory of the straight head vortex (see Fig. 4.7) also implies that there was significant azimuthal motion in square jets. It seems obviously that the asymmetry of geometry would also affect the self-similarity (Auerbach 1987a).

4.3.3 Head vortex pinch-off

Gharib *et al.* (1998) investigated the vortex ring formation from a piston-cylinder mechanism with a large stroke ratio ($L/D > 4$), They discovered that after the non-dimensional, time-

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dependent piston displacement (L/D) passed a certain value, no additional energy or circulation entered the forming vortex ring and the remaining fluid in the pulse was simply ejected as a “trailing jet”. After this point, the vortex ring was said to have “pinched-off” from the generating jet.

The vortex pinch-off phenomenon was detected in the present experiments as illustrated in Fig. 4.12a. The occurrence of vortex pinch-off is defined as the time when the head vortex was separated from the trailing jet which was observed in the experiments for both circular and square jets (Cases C1-3, C5-9, C11 and C12). In these cases, a spike can be clearly identified in the PLIF images (Fig. 4.12a). The spike was quite narrow and long at high Reynolds numbers. The head vortex was connected with the ensuing jet by the long spike until it broke down into turbulence.

The pinch-off time τ can be defined as the time when the starting vortex and the ensuing jet begin to be separated by pure ambient fluid. τ is highly dependent on the nozzle diameter and jet velocity (Johari *et al.*, 1997). The normalized vortex pinch-off time is plotted against D/W_0 in Fig. 4.12b. It varied from 6.1 to 7.3 among the cases when pinch-off was observed except for the Case CORE2358, when it was lowered at 4.4.

There are significant differences between the normalized starting vortex pinch-off time in the present study and those obtained by Kouros *et al.* (1993). Gharib *et al.* (1998) and Johari *et al.* (1997) (see Table 4.5). The differences probably stem from the different approaches in characterizing the pinch-off time. The pinch-off time from Gharib *et al.* (1998), which was referred to as the “formation number”, was based on vortex ring circulation measurements

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obtained by the particle image velocimetry technique. They defined that pinch-off occurs at the time of the maximum circulation of the vortex ring during its formation. With this definition, no vorticity is entrained into the vortex ring from the shear layer region of the trailing jet after pinch-off. At the same time, the spike pierces through the vortex ring. Some fluids from the trailing jet are entrained into the vortex, while some traverses through the vortex. Gharib *et al.* (1998) reported that the pinch-off process is not a sudden event and its completion might take up to two formation time units to complete. On the other hand, the present results were based on the visualization of pure ambient fluid between the jet tip and vortex ring. Since the separation of vorticity can occur earlier than the appearance of ambient fluid at the centerline, the present results on the pinch-off time can be expected to be longer than those of Gharib *et al.* (1998). In other words, although no pure ambient fluid can be seen between the head vortex and ensuing jet, the head vortex could have already attained maximum circulation (refer to Fig. 4.3 of Ref. 16).

Johari *et al.* (1997) also reported a much longer pinch-off time which may be explained as follows. The images of Johari *et al.* (1997) did not clearly reveal the spike-like stem clearly. Gharib *et al.* (1998) noticed that the circulation level increased with a step-like behavior in the large stroke-ratio ($L/D = 14.5$) case. They explained that the step jump is due to the leading vortex of the trailing jet eventually catching up with the pinched-off vortex. As a consequence, the circulation of the leading vortex increases impulsively. Johari *et al.* (1997) may have observed the second separation in the far field.

It is interesting to note that no vortex pinch-off was observed in SNRe2358 and SORe2358 and CNRe2358. Due to the slower penetrate rate of the head vortex (see Fig. 4.3), leapfrog

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occurred in these cases. In these cases, before the head vortex was pinched off, the ensuing vortex caught up with it (at $tW_0 / D \approx 5.5$), then leapfrogged through before broken into turbulence owing to the large strain. Eventually, the leading vortex of the trailing jet was entrained into the head vortex, coalescing into one. These phenomena, known as vortex leapfrogging and vortex coalescence, are discussed in more details in the following sections.

4.3.4 Coaxial vortices interaction: Leapfrogging and Coalescence

Two vortex rings at some distance apart will interact on a common axis of symmetry. The velocity field associated with the rear vortex ring has a radially outward component at the position of the front ring. As a result, the front ring gradually increases its radius and decreases its speed of travel. There is a corresponding increase in the speed of travel of the rear vortex, which ultimately passes through the upstream larger vortex and in turn becomes the front vortex. The maneuver is then repeated again (Batchelor, 1962). This phenomenon is known as vortex leapfrogging. Batchelor (1962) stated that it might be possible to demonstrate in the laboratory one or two such passages of one vortex through the other before both decay. Yamada and Matsui (1978) observed this phenomenon using air as working fluid. The present investigation appears to be the first recorded evidence of vortex leapfrogging in water.

The typical stages of leapfrogging are shown in Fig. 4.13 (from Case SNRe2358). The first image shows 2 vortices V1 and V2 being shed from the nozzle sequentially. As soon as V2 was formed, the interaction of the two vortices started (Fig. 4.13a). After about 0.64s, V2 was pulled down and pushed close to the axis by V1 (see also Fig. 4.14a). The size of V1 can be observed to be now considerably larger, while the size of V2 reduced (Figs. 13b and 14b). According to the Biot-Savart law,

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$$W_{tr} = \frac{\Gamma}{4\pi} \int_A \frac{\bar{r} \times ds}{r^3} \quad (4.10)$$

where Γ is the circulation, when the radius of the vortex r decreases, the translation velocity W_{tr} will sharply increase. So, V2 further accelerated while shrank at the same time. After another 0.48s, V2 caught up with V1, and penetrated it (Fig. 4.13c). Eventually, V2 broke down into turbulence and was integrated into V1 (Fig. 4.13d). This integration phenomenon can be called vortex coalescence.

Three factors, namely the core deformation, viscous diffusion and amplification of azimuthal waves, affect the process of vortex leapfrogging. The reasons are provided in the following. As stated previously, leapfrogging requires a transition stage when the leading vortex widens and travels more slowly while the trailing vortex shrinks and increases its speed. In the square cases, the acceleration time was longer than that of the circular jet (0.8s vs 0.5s). Hence, the leading vortex in square jet would slow down more due to the momentum distribution to the azimuthal motion and the larger entrainment, while the pursuing one picks up its velocity sharply and shrinks. At $t = 3$ s, when the vortices centers arrived at the same position in the streamwise direction where opposite extrema of the two cross-stream trajectories occurred, the leading vortex was penetrated (see Fig. 4.14). However, the pursuer was not strong enough to resist the straining during its own passage. Hence, it was broken into turbulence and was entrained into the leading one. Although leapfrogging was clearly observed in cases which $Re = 2358$, it did not occur in the other cases. Oshima and Asaka (1975) stated that leapfrogging only happens when the interval between their generations is of a certain duration and that the core radius might have to be taken into account. For the high Reynolds number cases, the time interval between vortex generations is not conclusive for the occurrence. In

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addition the observation of leapfrogging in SNRe2358 was only in the initial time. Further on, with the development of the shear layer, their strength became inadequate for leapfrogging.

4.3.5 Axis-switching in square jets

In non-circular jets, the deformation dynamics of asymmetric vortices play an important role in the jet evolution (Gutmark & Grinstein, 1999). In the early development of shear flows such as jets, the primary and secondary vortical structures determine the entrainment and growth. The streamwise vorticity may also be important in the actual mixing process as the jet evolves downstream (Liepmann & Gharib, 1992). One of the most peculiar aspects of noncircular jet evolution is axis-switching, which is interesting both from a fundamental scientific point of view and its potential application (Koshigoe *et al.*, 1989).

Two mechanisms are identified that govern the phenomenon. One is due to the difference in induced velocities for different segments of a roll-up azimuthal vortical structure. The other is due to the induced velocities of streamwise vortex pairs in the flow (Zaman, 1996). In the transient period, the jet performs like a vortex generator and the axis-switchings are mainly led by the vortex self-induction. The mechanism of square vortex deformation is illustrated in Fig. 4.15 (Grinstein & Devore, 1996). According to Batchelor (1962), the induced velocity of a vortex ring can be determined by:

$$W_{ir} \sim C_{ur} b \log(A^{-1}) \quad (4.11)$$

where C_{ur} is the local curvature of the tube, A is its local cross-section, and b is the binormal to the plane containing the tube. Due to high curvature in the neighborhood of the corners, the corner portions of a virtually flat and square vortex ring move ahead faster. In turn,

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as the flat portions that are left behind (e.g., at centers of initial flat sides) eventually develop higher curvature, they move ahead by the same mechanism, and the vortex ring becomes again approximately flat, with an axis rotated 45° relative to the previous planar configuration. After several axis rotations, the ring becomes quite distorted. These contortions can be attributed to the growth of azimuthal ring instabilities (Widnall, 1975).

Figure 4.16 illustrates the evolution of this axis-switching process in SNRe2358 as the head vortex ring was convected downstream. The trajectories in Fig. 4.16 represent the paths of the vortex-core centers in planes passing through the jet axis: (1) normal to the sides (“S-direction”) and (2) along the diagonals “D-direction”, (as denoted in Fig. 4.15). During the roll-up period ($t < D/W_0$), the cross-stream paths clearly show a faster spread of the jet shear layer at the centers of the flat sides, contrasting with a nearly constant spread along the diagonals (see Fig. 4.16a). At later times, the paths reflect the 45° -axis-switchings of the ring square section, with the ring undergoing a planar phase approximately at the locations where opposite extrema of the two cross-stream trajectories are aligned vertically (Fig. 4.16a), also roughly coinciding with the crossover of the streamwise paths (Fig. 4.16b). The behavior of the streamwise trajectories between crossovers in SNRe2358 hence reflects the basic features of self-induced vortex-ring deformation. The higher curvature “corner” portions of the ring first traveled downstream faster and then slowed down as the next crossover point is approached, when old “corners” become centers of “flat” sides in the new axis-rotated configuration. Note that the present results are different from Grinstein and DeVore’s (1996) numerical simulations as shown in Fig. 4.16. One can find that the square ring attempted to become circular and axis-switching would not occur after a few cycles which is contributed to the viscous dissipation. However, in the numerical experiments of Grinstein and DeVore

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(1996), the viscosity was neglected. On the other hand, the experimental results have higher occurring frequency for the axis-switching with the numerical results. These discrepancies might be due to the differences in the initial conditions and the interactions between the head vortices and the trailing jet.

Figure 4.17 shows the life-cycle of axis-switching in the starting square jet of SNRe2358 at $Z/D_e = 0.75$. The left images show a sequence of end views, while the middle and right images show the side views along the “S” and “D” (as denoted in Fig. 4.17a) directions, respectively. The images in the same row were taken at the same time after the jet was initiated. When the jet tip arrived, the front vortex appeared in a circular shape due to the high speed of the potential core. After the head vortex formed (see Fig. 4.17a-000), the square stem appeared with the corner portion moving faster, leaving behind the flat portion (see Fig 4.17a-001). At the same time, owing to the dominance of the primary vortical structures, the jet stretched out quickly and entrained more ambient fluid at the flat side. At the corner, the mushroom-shaped streamwise corner vortices emanated, counteracting the outward trend of the spanwise vortices, and remained stagnant. When the head vortex arrived in the observed position of $Z/D_e = 0.75$, the first axis switching occurred. Then, the head vortex traveled downstream according to the self-induced rule, and the second vortex formed 0.56 second later (see Fig. 4.17b-007). The axis-switching happened for a second time in Fig. 4.17b-007. All together axis-switching occurred four times in the initial image sequences of the starting square jet at this position of $Z/D_e = 0.75$. With the decay of nonuniformity and weakening of the streamwise and spanwise vortices, axis switching did not happen in latter time. Further downstream at $Z/D_e = 1.5$, continuous axis-switchings were observed for steady jets. In this case, the continuous jet axis-rotations are the result of strong interactions between the rings

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and braid vortices rather than being correlated with successive self-induced vortex ring deformations (Grinstein & DeVore, 1996).

4.3.6 Orifice plate effects

An orifice plate fixed at the exit of the nozzle produces a wall condition (see Fig. 3.2b). During the vortex formation, there is no motion in the streamwise direction. The orifice plate pushes the vortex downstream while holds back the vortex rolling-up due to the upstream velocity of the roll-up. Hence, there is lesser momentum transferred to the ambient fluid compared to a jet without an orifice plate.

Table 4.2 illustrates the orifice effect on the jet penetration rate. It can be seen that the wall plays an important role for low Reynolds number jets. The average velocity increased by 15% and 16%, respectively for circular and square jets at the Reynolds number of 2358. However, when the Reynolds number was higher at 3528 and 4716, the influence became negligible. This may be due to the fact that a longer formation time is required in the low Reynolds number cases. The lower the Reynolds number, the longer time the vortex maintains stagnation. Thus, at lower Reynolds number, the plate exerts large influence on the jet. With the increases in Reynolds number, this effect is weakened.

The difference between the nozzle and orifice jets is also shown in the two PLIF images in Fig. 4.18. The two images in the figure were taken at $t = 3s$ in CNRe2358 and CORE2358, respectively. Due to the negative vorticity created on the orifice plate, the circulation of the vortex ring generated in the orifice jet was somewhat smaller. The plate in fact prevented the ring rotation and spanwise movement. Hence the vortex sheet thickened, and the vortex

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traveled faster in the orifice jets, while the nozzle ring formed a well-organized spiral shape. Comparing these two images, it can be found that the orifice ring broke into turbulence much earlier due to the smaller momentum.

Gharib *et al.* (1998) tested the orifice wall effect for the vortex pinch-off time. They found that the formation number decreased from 3.8 to 3.6. Similar trend was noted in the present experiments. From Table 4.5, a slight reduction on pinch-off time for the orifice jet can be observed. However, a major shift was seen for the circular orifice jet with $Re = 2358$. Due to the significant wall effect in this case, the pinch-off occurred much in advance compared to the jet without an exit plane. It asserted the conclusion that the wall is effective at low Reynolds numbers. This deduction can be attributed to the opposite-sign vorticity generated in the secondary boundary layer on the outside wall and its entrainment by the vortex ring.

In the same manner, the orifice plate also affects the vortex evolution in starting jets. In the orifice starting jets, the growth rate was typically smaller (see Table 4.4) and the vortex convecting velocity was typically larger. Same conclusion can be drawn for both circular and square jets.

4.4 Summary

Utilizing Planar Laser Induced Fluorescence (PLIF), the characteristics of vortex evolution for starting square jets are examined and compared with starting circular jets with the same initial Reynolds number. It was found that for both geometries, the jet penetration propagates linearly in the near field within a distance of around four exit diameters, while obeying the

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similarity law ($Z_t \sim t^{1/2}$) further downstream. Compared to circular jets, square jets penetrate much slower due to the enhanced entrainment ratio and mixing efficiency.

The head vortex evolution in the starting jets is also investigated. The similarity law works well for the Z -direction trajectory of the vortex center for both square and circular jets. However, for starting square jets, the continuous axis-switchings and the vortex trajectory oscillating in the X -direction are phenomena related to each other, not necessary one leading to the other. The normalized head vortex pinch-off time based on the sequence of PLIF images was found to be approximately 7. Azimuthal non-uniformities also add complexity to the evolution of shear layer in starting square jets. These non-uniformities vortices leapfrogging and coalescence which were observed for the first time in starting jets experiments.

The orifice plate poses for a wall condition and has a drastic effect on the vortex ring formation when the Reynolds number is relatively low. With the orifice plate, the jet tip travels down faster and the vortex structure is significantly altered.

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Table 4.1 Cases and initial parameters for representative experiments

Cases	Name	Nozzle Geometry	Nozzle size D or D_s (mm)	Mass Flow rate (l/min)	Initial velocity $\bar{W}_0$ (mm/s)	$Re =$ $W_0 D_e / \nu$
C1	CNRe2358	Circular, Nozzle	36	4	65.5	2358
C2	CNRe3528	Circular, Nozzle	36	6	98.0	3528
C3	CNRe4716	Circular, Nozzle	36	8	131.0	4716
C4	SNRe2358	Square, Nozzle	32	4	65.5	2358
C5	SNRe3528	Square, Nozzle	32	6	98.0	3528
C6	SNRe4716	Square, Nozzle	32	8	131.0	4716
C7	CORE2358	Circular, Orifice	36	4	65.5	2358
C8	CORE3528	Circular, Orifice	36	6	98.0	3528
C9	CORE4716	Circular, Orifice	36	8	131.0	4716
C10	SORE2358	Square, Orifice	32	4	65.5	2358
C11	SORE3528	Square, Orifice	32	6	98.0	3528
C12	SORE4716	Square, Orifice	32	8	131.0	4716

Table 4.2 Average tip velocity

Cases	Initial Velocity W_0 (mm/s)	Tip Velocity ($\bar{W}_t$)	Cases	Initial Velocity W_0 (mm/s)	Tip Velocity ($\bar{W}_t$)
CNRe2358	65.5	57.6	SNRe2358	65.5	40.6
CNRe3528	98	84.2	SNRe3528	98	71.1
CNRe4716	131	113.2	SNRe4716	131	96.2
CORE2358	65.5	66.1	SORE2358	65.5	47
CORE3528	98	84.6	SORE3528	98	72.9
CORE4716	131	114.5	SORE4716	131	100.9

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Table 4.3 Test conditions of starting jet experiments in different studies.

Author	Instrument	Fluid medium	Nozzle Geometry	$W_0(m/s)$	D or D_e (mm)	Re	C'_z
Witze 1980	Hot Film	Air	Circular	22~103	1.2~1.8	2000~4660	3.0
Lahbabi <i>et al.</i> 1993	Image Processing	Water	Circular	0.51	5	2600	3.0
Johari <i>et al.</i> 1997	PLIF	Water	Circular	0.19~3.02	6.35~25.4	5000~20000	2.55
Sangras & Faeth 1999	Image Processing	Water	Circular	0.046~3.68	3.2~6.4	1450~11700	2.76
Cossali <i>et al.</i> 2001	Image Processing	Air	Circular	63.4~68.4	3	12600~13600	3.4
Present Square jet	PLIF	Water	Square	0.065~0.131	36	2358~4716	3.4
Present Circular jet	PLIF	Water	Circular	0.065~0.131	36	2358~4716	4.0

Table 4.4 Growth rate ε and parameters of over-shoot.

Cases	Stroke ratio	S	t_{over}	Norm t_{over}	Norm Z_{over}	Norm D_{over}
CNRe2358	∞	0.031	0.6	1.09	0.52	1.27
CNRe3528	∞	0.023	0.58	1.6	0.76	1.29
CNRe4716	∞	0.018	0.4	1.45	0.65	1.46
SNRe2358	∞	0.066	0.88	1.6	0.50	1.56
SNRe3528	∞	0.036	0.64	1.74	0.71	1.54
SNRe4716	∞	0.038	0.48	1.75	0.61	1.81
CORe2358	∞	0.014	0.5	0.91	0.33	1.29
CORe3528	∞	0.012	0.6	1.64	0.75	1.38
CORe4716	∞	0.014	0.62	2.26	0.98	1.47
SORe2358	∞	0.053	0.8	1.45	0.64	1.54
SORe3528	∞	0.028	0.48	1.30	0.53	1.70
SORe4716	∞	0.021	0.48	1.75	0.93	1.73
Maxworthy (1977)	0.66~1.51	0.005~0.014	0.1	0.45	0.45	1.21
Didden (1979)	3.0	-	-	-	0.35	1.39

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Table 4.5 Experimental results of normalized pinch-off time for representative cases

Nozzle		Orifice	
Exp.	Normalized pinch-off time $\tau W_0 / D_0$	Exp.	Normalized pinch-off time $\tau W_0 / D_0$
CNRe2358	-	CORe2358	4.4
CNRe3528	7.3	CORe3528	7.1
CNRe4716	7.3	CORe4716	7.1
SNRe3528	7.0	SORe3528	6.8
SNRe4716	7.0	SORe4716	6.1
Johari <i>et al.</i>	6.1~47.7		
Gharib <i>et al.</i>	3.6~4.5		
Kouros <i>et al.</i>	32.3		

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Fig. 4.1 Representation of a transient turbulent jet by the vortex-quasi-steady-jet model of Turner (from Case CNRe4716).

Fig. 4.2 Penetration of head vortex with time.

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Fig. 4.3 Normalized penetration vs square root of normalized time.

Fig. 4.4 Normalized penetration vs square root of time.

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Fig. 4.5 Trajectory of vortex center for the Case of CNRe3528.

Fig. 4.6 Growth in ring diameter for circular jets.

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Fig. 4.7 Growth in ring diameter for square jets.

Fig. 4.8 Temporal evolution of tip velocity for circular jets.

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Fig. 4.9 Temporal evolution of tip velocity for square jets.

Fig. 4.10 Distance Z_w of the vortex axis from the nozzle edge vs time.

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Fig. 4.11 Distance Z_w of the vortex axis from the nozzle edge vs time

Note: Auerbach's (1987a) experiments were performed with piston velocities at 120 and 284 mm/s.

Fig. 4.12 Head vortex pinch-off characteristics.

- (a) Representation of pinch-off in the Case of CORE2358
- (b) Normalized pinch-off time vs nozzle time

Note: CN circular nozzle; SN square nozzle

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Fig. 4.13 Vortex leapfrogging and coalescence in Expt. SNRe2358.

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Fig. 4.14 Evolution of the head and secondary vortices in Expt. SNRe2358.

Fig. 4.15 Topology of coherent structures in square jet (taken from Grinstein & DeVore 1996).

Note: D-direction: Radii along diagonal direction, S-direction: Radii normal to the side

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(a)

(b)

Fig. 4.16 Trajectories of head vortex core centers in the Case SNRe3528.
 (a) Cross-stream; (b) Streamwise

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Fig. 4.17 Sequential images with time for the Case SNRe3528.

- (a) Cross-sectional view at $Z/D_e=0.75$ (b) Side view in S-direction
(c) Side view in D-direction

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Fig. 4.17 Continued

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Fig. 4.18 Comparison with and without orifice plate, taken at $t=3s$, for the two cases of CNRe2358 and CORE2358.

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5.1 Introduction

Quantitative PIV measurements were conducted with the same initial conditions as the previous PLIF experiments shown in Chapter 4. A 2-D PIV system was employed as the key experimental tool to quantify the entrainment and mixing characteristics. The detailed velocities, vorticity and circulation evolution in the starting square and circular jets are presented and discussed in the present chapter.

A starting jet may be considered as a vortex generator (Liepmann & Gharib, 1992). Immediately after the initiation of the starting jet, the separation of boundary layers on either side of the nozzle wall would cause the formation of a head vortex. An important issue in vortex ring generation and evolution is to determine how much circulation and energy can be delivered to the vortex ring by the apparatus, and the time for its subsequent separation from the trailing jet. The speed of the vortex, which determines the penetration rate, is also an important point to be investigated. Gharib *et al.* 1998, had shown that for large piston stroke ($L/D = 14.5$) and starting jets, the generated flow field consists of a leading vortex ring followed by a trailing jet. Following the head vortex (also referred to as V1), a series of secondary vortices would subsequently be formed due to the Kelvin-Helmholtz instability (see Fig. 4.1).

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The generation, formation, evolution, and interactions of the vortex rings govern the subsequent development of a starting jet. The understanding in these vortex ring characteristics can shed important light to the behaviour of the starting jet, especially for the square jets which is more complex than the circular one. In this chapter, the flow field of the starting jets will be investigated quantitatively by PIV. Comparison will be made with the qualitative PLIF results shown in the previous chapter.

The existence of a universal time scale for vortex ring formation generated by impulsively starting jets was investigated in a recent study by Gharib *et al.* (1998). The jets were produced by a piston discharging a fluid column of length L through an orifice of diameter D . The study focused on vortex rings produced for large stroke ratios, L/D , between 2 and 14.5. The experimental findings revealed that the formation of the axisymmetric head vortex ring could be characterized by a single time scale. This time scale, called the formation number, is defined as the non-dimensional time factor $W_p t / D$, where W_p is the average piston velocity. The vortex ring was observed to detach from the trailing jet around a formation number of 4, over a range of different flow conditions. At this formation number, the vortex ring had reportedly reached its maximum circulation.

An analytical model, based on the Kelvin-Benjamin variational principle, was used by Gharib *et al.* (1998) to explain the existence of the formation number. A non-dimensional kinetic energy parameter was defined as:

$$E_{nd} = \frac{E}{I^{1/2} \Gamma^{3/2}}, \quad (5.1)$$

where E , I and Γ are the kinetic energy, impulse and circulation, respectively, and are given in Gharib *et al.* by the following relations:

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$$E = \pi \iint_A (U^2 + W^2) X dX dZ = \pi \iint_A \omega \psi dA, \quad (5.2a)$$

$$I = \pi \iint_A \omega X^2 dA, \quad (5.2b)$$

$$\Gamma = \iint_A \omega dA, \quad (5.2c)$$

where ψ is the streamfunction and A is the integral area for the vortex. According to the model, a single steady vortex ring is no longer possible to be delivered energy by the trailing jet, i.e. $E_{nd\_piston}(t) < E_{nd\_vortex}(t)$. Based on the measured kinetic energy, impulse and circulation of the vortex rings, the Kelvin-Benjamin principle correctly predicted the range of formation numbers observed in the experiments and obtained the limit value of $E_{nd\_vortex}(\lim) = 0.33$. It should be noted that Gharib *et al.*'s analytical model is based on a so-called slug-model, which assumes a uniform velocity profile at the exit plane. However, this assumption will not be valid for large stroke ratios and starting jets where the velocity profile in the pipe would show acceleration in the central region caused by the growth of the boundary layer (Didden 1979).

The model based on the Kelvin-Benjamin variational principle gives little insight into the physical mechanism leading to vortex ring pinch-off. This realization led Shusser *et al.* (1999) to consider a kinematic model of the formation process based on the assumption that pinch-off occurs when the ever-increasing velocity of the ring becomes equal to the velocity of the jet immediately behind the ring, causing the ring to physically separate from the generating jet. Shusser and Gharib (2000) were able to show that this model also predicts the limiting value of the non-dimensional energy based on the Kelvin-Benjamin variational principle. In a more recent paper, Shusser *et al.* (2002) extended the model to cases of time varying piston velocity

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programs and clearly demonstrated the mechanism that increases the formation number when the piston velocity increases with time, i.e. $W_p = \text{constant}$.

Another model for vortex ring pinch-off was proposed by Linden and Turner (2001). Here, the slug mode and Norbury family of vortex rings were employed as in Mohseni and Gharib (1998), but the equations were closed by imposing the additional constraint that the volume of fluid emitted by the jet and that in the pinched-off vortex ring are the same. This leads to a formation number of 3.5 for a piston with uniform velocity. However, as shown by Shusser and Gharib (2000), the model incorrectly predicts the formation number for time varying velocity.

Numerical predictions of the total and vortex ring circulation were recently published by Rosenfeld *et al.* (1998) and Mohseni *et al.* (2001). Rosenfeld *et al.* (1998) basically confirmed the experimental and analytical findings of Gharib *et al.* (1998). In addition, they showed that the vortex ring circulation is relatively insensitive to the details of the formation process, such as the velocity programme, velocity profile, vortex generator geometry and the Reynolds number. For thin shear layers, the formation number was confirmed to be approximately equal to four for the wide range of cases differing in the discharge time (piston stroke ratio), velocity programme, generator configuration and Reynolds number. They also extended the numerical experimental studies to thick shear layers, which in contrast, the total circulation does depend on the velocity profile and to some extent on the velocity programme. Consequently, the formation number (that is determined by the intersection of the vortex circulation with the total circulation curve) depends on the discharge velocity profile and the velocity programme. Non-uniform velocity profiles tend to decrease the formation number, while non-impulsive velocity

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programmers increase the formation number. They concluded that the relative change in the formation number was found to be as large as 400% (e.g. between the specified uniform and parabolic discharge velocity cases). The corresponding relative change of the scaled vortex circulation was found to vary less than 40%. Mohseni *et al.* (2001) obtained similar results by increasing the thickness of the shear layer.

Recently, Zhao *et al.* (2000) studied the interaction of the trailing jet instability (i.e. secondary roll-up of vorticity in the trailing jet) with the leading vortex ring. They concluded that this interaction accelerated the pinch-off process and was the cause of up to 20% variation in vortex ring circulation, when non-dimensionalized with the orifice diameter and maximum piston velocity.

Shusser and Gharib (2000) and Pottebaum and Gharib (2004) studied the vortex ring formation in starting buoyant plumes experimentally and analytically. The analytical model and experimental results showed that the non-dimensional time varies from 4.4 to 4.9, a little larger than that of the vortex ring without buoyancy.

In the present chapter, the evolution of the head vortex is explored. At the same time, the interactions between the trailing jet and the head vortex are also examined. The pinch-off time and transitional velocity of the leading vortex in starting jets will be firstly investigated.

5.2 Experimental conditions

The test rig and initial parameters were the same as these described in the PLIF experiments (cf. Figs. 4.1 and 4.2). Four additional cases with $Re = 7056$ and 9432 (i.e. CNRe7056,

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CNRe9432, SNRe7056 and SNRe9432) were investigated. The initial velocity was 0.196 and 0.262 m/s, respectively. Only the mean and turbulent velocity characteristics were examined for these four cases. Particle Image Velocimetry (PIV) was employed to measure the flow field of the starting jets. The flow was seeded with neutrally buoyant Polyamide Seeding Particles (average diameter of 50 μm) and illuminated by a sheet of laser light with a thickness of approximately 2~3 mm. Similar to the PLIF experiments, the video camera was positioned normal to the measurement plane and recorded an image sequence of the particle fields with a spatial resolution of 1008×1016 pixels. The time interval between two successive exposures was set properly in order to limit the maximum displacement of particles. The interval time for all the cases is shown in Table 5.1.

The measurements were performed in the region of $0 < Z/D < 9$. Using a window size of 32×32 pixels and a step size for the moving average of 8×8 pixels (75% window overlap), the processing resulted in a field measurement of 123×124 velocity vectors with a temporal resolution of 6.25 or 5.0 Hz. With a typical field of view of $\approx 31.9 \text{ cm} \times 32.1 \text{ cm}$, the spatial resolution was $1.01 \times 1.01 \text{ cm}$.

In square jets, 2-D PIV experiments were conducted in two planes (S-S and D-D plane, as shown in Fig. 5.1) in order to investigate the three-dimensional effects. The measurements were fulfilled via a 45° rotation of the nozzle. In the following section, unless otherwise mentioned, the results are mainly for the S-S plane. The results acquired on the D-D plane are only presented in Section 5.4.10.

The vorticity was calculated by the vector field as,

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$$\omega_y = \frac{\partial U}{\partial Z} - \frac{\partial W}{\partial X}. \quad (5.3)$$

From the vorticity distributions, one can obtain the total discharged circulation by integrating the vorticity contained within the lowest detectable contours for either positive or negative senses, e.g. Eq. (5.2c). This contour level was determined to be at 2 s^{-1} . Attempt had been made to select 1 s^{-1} as the boundary of the vortex ring, but the total circulation level was found to be insensitive to the choice of the cut-off level (the difference was within 5%). However, when selecting 2 s^{-1} , a clear separation between the vorticity contours of the head vortex from those of the trailing region can be identified and thus the head vortex circulation can be computed. Moreover, most of the noises can be eliminated by selecting the level of 2 s^{-1} . In this analysis, the increment for the vorticity contours is also chosen to be 2 s^{-1} for all the cases.

Following Gharib *et al.* (1998) and Rosenfeld *et al.* (1998), the non-dimensionalized time t^* is defined by

$$t^* = t \bar{W}_0 / D_e, \quad (5.4a)$$

where $\bar{W}_0$ is the average velocity at the nozzle plane which reads

$$\bar{W}_0 = \frac{\int W_0 dA}{A} \quad (5.4b)$$

and D_e is the hydraulic diameter of the nozzle exit plane which is also employed as the length scale

$$\bar{Z} = Z / D_e. \quad (5.4c)$$

And the circulation is normalized as

$$\Gamma_{nd} = \Gamma / (\bar{W}_0 D_e). \quad (5.4d)$$

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Note that there are other ways of normalization. For example, Mohseni and Gharib (1998) suggested

$$\Gamma'_{nd} = \frac{\Gamma}{I^{1/3} \overline{W}_0}. \quad (5.4e)$$

5.3 Results and discussion

5.3.1 Case CNRe2358

The temporal evolution for the vortex dynamics (velocity field, vorticity field and PLIF images) for this case is shown in Figs. 5.2a to 5.2i. At the earliest time, $t^* = 1.16$ (Fig. 5.2a), the head vortex (hereafter denoted as V1) was shed and separated from the nozzle. The PLIF image and vorticity distribution plots show that the head vortex core was initially circular in shape. As soon as the vortex core separated from the nozzle, it elongated in the direction of the ring propagation into an ellipse due to the self-induced strain field (Fig. 5.2b). From Fig. 5.3, it is interesting to note that V1 had attained a maximum circulation level at $\Gamma^* = 1.36$ where $Z/D \approx 0.4$ in such a short time ($t^* = 1.16$) comparing to Gharib et al. (1998) experimental results ($t^* = 4$). At later time, $t^* = 2.03$, another pair of vortex (henceforth denoted as V2) started to emerge from the nozzle. At the same time, V1 did not entrain any more fluids into the core and started to shed excessive vorticity away to the trailing jet to form hair-pin vorticity (as shown by the arrow in the Fig. 5.2c (iii)). Therefore, the energy and circulation were kept constant (Kelvin-Benjamin variational principle).

At about $t^* = 2.62$, V2 was formed and detached from the nozzle exit plane (e.g. Fig. 5.2d). It was noted that the streamwise distance between V1 and V2 grew further apart (see Fig. 5.4) immediately after the formation of V2. Because of the thinner boundary layer, it can be seen

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that the circulation of V2 was less than that of V1 from Fig. 5.3 (about 25%). From the onset of V2 formation, its diameter was already observed to be smaller than that of V1. As soon as V2 was formed at the nozzle exit, the interaction of the two vortices started. Until $t^* = 3.20$, the streamwise distance between V1 and V2 attained its maximum value ($Z/D = 1.1$). From then on, due to the push of the trailing jet, V2 traveled faster than the first one and the streamwise distance began to diminish (see Figs 5.4). At the same time, another trailing vortex (V3) started to emerge from the nozzle. The induced velocity of the head vortex (V1), caused V2 to contract and accelerate (see Figs. 5.2f and 5.4). In contrast, the velocity field of V2 caused V1 to expand in diameter and slow down, which can be explained by the Biot-Savart law (already discussed in Chapter 4). By $t^* \approx 6.7$ (see Fig. 5.2g), V2 caught up with V1, and finally slip through its center and emerge ahead of the front ring. Due to the viscosity diffusion, the circulation declined slightly during its traveling (i.e. Fig. 5.3).

It is also interesting to note that when the two vortices connected each other at $t^* = 4.95$ (see Fig. 5.2f) and started to undergo the leapfrogging process, the strength of the vortices declined sharply due to their interaction (see Fig. 5.3). The PLIF images show clearly that the interaction between these V1 and V2 hastened the process of their breaking-down into turbulence. It can also be seen in Fig. 5.2g that after the leapfrogging, V1 and V2 was no longer coaxial with the symmetric axis. Instead, they were tilted while they were undergoing the leapfrogging process. The PLIF images illustrated that the tilted rings were more likely to lead to an early transition to turbulence (see also Broze & Hussain, 1996 and Lim, 1997b) because of the induced azimuthal bending waves. The vortex filaments become entangled, causing the flow to become three-dimensional and fine-scale structures to be generated in the vortex ring.

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It is also interesting to note that, resembling to V1 and V2, the third (V3) and fourth (V4) vortices were undergoing the same procedure (Fig. 5.2h). It should be mentioned that the total circulation of V1 and V2 fell to the level of V1 alone before the interaction (Fig. 5.3). Fig. 5.2i shows the reverse role and the attempt of another passage (V1 slipping though V2). The subsequent trailing vortices could no longer perform leapfrogging, because they broke down interacting each other. Finally, the velocity of the combined vortex (V1 and V2) decayed and broke into turbulence. At the same time, the trailing jet caught up with the head vortex.

A final note is that the total circulation began to level off at time $t^* \approx 6$, (as shown in Fig. 5.3). Two factors may attribute to this phenomenon. First is the interaction among the head vortex and the trailing vortices which lead to the loss of circulation. Viscous diffusion and the thinning of the boundary layer may be another cause for the leveling off of the total circulation.

Lim (1997a) pointed out that apart from the effect of core distortion as noted by Shariff *et al.* (1989), the time interval between the generation of the two rings (T) also plays an important role in the leapfrogging process. He further pointed that the time scale of the rolling up of the vortex sheet increases with the decrease of the Reynolds number. This implies that T needed to produce successful leapfrogging at low Reynolds numbers must therefore be greater than that at higher Reynolds numbers. He successfully observed this phenomenon with $T^* = W_0 T / D \approx 0.055$ and 0.175 at $Re = 2077$ and 478 , respectively.

In starting jets, the time interval and the time scale of the rolling up for the vortex ring cannot be controlled manually. As mentioned in the previous chapter, the leapfrogging behavior was

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only observed at a particular Reynolds number case ($R_e = 2358$). In this case, one can see the rings underwent as many as two slip-throughs (or passages) before they eventually paired off to form a single ring at further downstream station. At higher Reynolds numbers, the head vortex over-shot and pinched off from the main stem at the onset of the jet development. Because the induced velocity for V2 cannot be caused by V1, the classical leapfrogging behavior failed to occur in the higher Reynolds number cases. It is also of interest to note that V3 and V4 merged into one vortex when undergoing the leapfrogging in the present case. The failure to leapfrog can be partly attributed to the effect of core distortion, since the vortex cores are thicker and therefore more susceptible to distortion during the leapfrogging (Shariff *et al.*, 1989).

5.3.2 Case CNRe3528

The vortex evolution for the Case CNRe3528 is shown from Fig. 5.5a to Fig 5.5g. The obvious difference between this case and the previous one is that the distance between V1 and V2 was larger although the time interval of the generation for the two vortices is similar. The streamwise distance between V1 and V2 was almost $1.64D$ comparing to about $1.2D$ in the previous case. The leading vortex also seemed to attain the maximum circulation strength as early as $t^* = 1.74$ (see Fig. 5.3). Comparing to the previous case, the leading vortex sheet was thinner with more circulation due to higher speed rotation. The nondimensional circulation was about 33% larger, 1.8 comparing with 1.36. From $t^* = 3.05$ onward, the vortex core of V1 became elliptic (elongate in the streamwise direction) and began to shed excessive fluids entrained from the jet stem (Fig. 5.5b). At this instant, V2 was rolling up in the trailing shear layer. In contrast to V1, the vortex sheet for V2 was thicker and lower strength of circulation was achieved.

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At $t^* = 5.23$, three distinct vortices (V1, V2 and V3) were formed (Fig. 5.5c). It is interesting to note that the distance between any two of the vortices was nearly the same. Comparing Figs. 5.2e with 5.5c, it can be found that the leading vortex entrained almost all the trailing fluid in Case CNRe3528, while a strong stem was left at the center of the vortex pair in the Case CNRe2358. The streamwise distance between V1 and V2 started to diminish at $t^* = 6.96$ due to the shrinking of V2 (see Figs. 5.5d and 5.6). With the severe self-deformation and shrinking, the vortex pairs came into contact and interacted with each other at time $t^* = 10.01$ (e.g. Fig. 5.5e). The concentration of large vorticity near the axis of symmetry increased the destruction of vorticity by the vorticity cancellation mechanism. At $t^* = 10.89$ (i.e. Fig. 5.5f), V2 broke into turbulence completely (e.g. Fig. 5.5f). Leapfrogging failed to occur between V1 and V2. However, the circulation history of V1 (as shown in Fig. 5.3) indicated that the trailing jet caught up with V1 at $t^* = 9.15$, although V2 broke down at this instant. On the contrary, V3 and V4 were also attempting to perform leapfrogging which is similar to the previous case. At $t^* = 13.50$, V3 and V4 coalesced together and were overtaken by the trailing jet.

5.3.3 Case CNRe4716

The sequential images of the flow for this case are shown in Figs. 5.7a to 5.7g. At this Reynolds number, the streamwise distance difference between the formation of the leading vortex (V1) and the first trailing vortex (V2) lengthened further ($1.3D$, $1.7D$ and $2.5D$ for cases CNRe2358, CNRe3528 and CNRe4716, respectively). The leading vortices achieved the maximum circulation also within a short time (at $t^* \approx 2.33$, e.g. Fig. 5.3). At $t^* \approx 3.49$ (Fig. 5.7b), the first trailing vortex (V2) began to roll up. At the same time, a small vortex rolled up in the middle of V1 and V2 (as shown in Fig. 5.7b). Due to the higher flux rate from

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the nozzle, the jet stem fluids were not being injected fast enough into the head vortex. The surplus vorticity left in the trailing shear layer was strong enough to form another vortex. Under the pressure of the high-speed trailing vortex, kinks appeared which could be due to the Kelvin-Helmholtz-instability. At $t^* = 4.66$, the three vortices (V1, V2 and the kinks) attained their maximum energy (Fig. 5.7(c)). Some braid vortices were also found behind the leading vortex. Figure 5.7d illustrated that V1 tilted at time $t^* = 5.82$ due to the induced azimuthal instability (Lim 1997b). The misalignment of V1 seemed to hasten its breaking down (as shown in Fig. 5.7e). At $t^* = 7.28$, V2 was attempting to entrain the kinks (see Fig. 5.7e). Owing to the interaction between V2 and the kinks, the transitional velocity of V2 slowed down. In contrast, V1 accelerated at this instant (as shown in Fig. 5.8). Fig. 5.7e also illustrates that the head vortex pinched off from the trailing stem completely and left no wakes behind it. From Fig. 5.3, it can be seen that the circulation of V2 fell sharply during the process of the interaction between V2 and the kinks. However, it is interesting to note that the circulation of V1 underwent some increase at this moment (see Fig. 5.3). Finally, after shedding three main vortices, the shear layer formed and the trailing stem achieved its steady state. The following weaker trailing vortices rolled up in the shear layer with higher shedding frequency and shorter wave length (Fig. 5.7f).

5.3.4 Case SNRe2358

Sequential images of the flow for this case are shown from Fig. 5.9a to Fig. 5.9k. At $t^* = 1.16$ (Fig. 5.9a), the separation of boundary layers on either side of the nozzle wall caused the formation of vortices. Noticing that the regions within the two vortices contained both jet and ambient fluids, this indicates that the entrainment of the ambient fluids had already started. Fig. 5.10 shows that V1 had attained its maximum circulation at this instant.

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At $t^* = 2.18$, another pair of vortices emerged from the nozzle (Fig. 5.9b). Entrainment of the ambient fluids continued to take place. At the same time, it was noted that the streamwise distance between the two head vortices (V1 and V2) grew further apart (see Fig. 5.11). The increase in distance for the leading vortex was due to the fact that it had reached the maximum circulation (or strength) and, as a consequence, they were not able to entrain more fluids from the jet stem. Since the continuous supply of fluids from the nozzle cannot be entrained into the leading vortices anymore, this led to their accumulation at the tip area which in turn pushed the two leading vortices further away from one another in the Z direction (see Fig. 5.11). Fig. 5.12a clearly shows that the velocity at the location of the head vortex core center steadily increased from $t^* \approx 1.16$ to 5.24. Moreover, the translational velocity of V1 was found to be less than that of the trailing jet until the leapfrogging process completed (ref. Fig. 5.12a). At about $t^* = 2.91$, the vortices appeared to have reached their maximum circulation (see Fig. 5.10). The distance between V1 and V2 also achieved a maximum value (as shown in Fig. 5.11). This is clearly shown by the thickening of the outer boundary of each vortex (Fig. 5.9c). The vortices started to shed excessive fluids away (as shown by the arrow in Fig. 5.9d). During this period (from $t^* \approx 2 \sim 3$), the diameter of V1 decreased while that of V2 increased (see Fig. 5.13).

The diameter for V1 started to expand at $Z/D_e \approx 1.4$ (i.e. Fig. 5.13). However, the diameter for V2 began to decline. At $t^* = 4.37$ (Fig. 5.9d), the shedding of excessive fluids became very apparent for the trailing vortices (V2) and the streamwise distance between the two trailing vortices diminished further. It is obvious from the figures that, between $t^* = 2.91$ to 5.24 (Figs 5.11d and e), the decrease in distance was accompanied by a corresponding increase in the translational velocity. The size of the trailing vortex (V2) also decreased by

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about 75% (from $R/D_e = 0.6$ to 0.15), an obvious indication of vortex stretching. By $t^* = 5.46$ (Fig. 5.11f), the trailing vortices had surged forward and penetrated into the tip of the jet.

The reduction in ring diameter was consistent with an increase in ring velocity if momentum was to be conserved, as it would also induce a higher translational velocity (i.e. Figs. 5.11 and 12). Flow visualization had also been conducted on the cross-sectional plane (see Fig. 5.14). It was noted that streamwise vortices were formed at the corners of the square jet. The presence of the vortices had in turn stretched the vortex tube connecting any two corners. Due to the conservation of angular momentum, when the vortex tube was stretched, the corresponding circulation strength increased. This in turn enhanced the translational velocity via the Biot-Savart Law, and explained the sudden surge forward of the vortices shown at $t^* = 5.46$ (Fig. 5.9f). As shown in Fig. 5.13, the obvious difference shown by the trajectory of V1 was that the size of V1 kept increasing in Case CNRe2358 while oscillated in Case SNRe2358 when undergoing the leapfrogging process. In addition, the location of the pinch-off was far downstream in the circular case (see Fig. 5.13), and was doubled for Case CNRe2358 ($Z_{pinch-off}/D_e = 5$ vs. 2.5). It is also interesting to note that the total circulation of V1 and V2 decreased quicker in the circular case after the leapfrogging (compare Fig. 5.3 and Fig. 5.10). These may also be due to the axis-switching, an important phenomenon already discussed in Chapter 4. Owing to the axis-switching in the square jets, V1 stretched in the traverse direction when V2 leapfrogged V1. Therefore, V2 slipped through smoothly and the strength of the two vortices were not significantly lost during the leapfrogging process.

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At $t^* = 6.19$ (Fig. 5.9g), the trailing vortices were residing at the tip of the jet. It should be noted that from $t^* = 2.91$ (Fig. 5.9c) onward as the trailing vortices were being stretched, intensified and surged forward, the trailing vortices also entrained some of the jet fluids to the tip area. Figure 5.12b showed that the tip velocity was very high when V2 stayed there. This can be seen clearly via the thinning of the jet fluids around the stem region (ref. Fig. 5.9e). Although the strength of V2 decayed, it did not seem to break down, as shown at $t^* = 6.19$ (ref. Fig. 5.9g).

It is interesting to note that at this instant, the leading vortex still maintained its strength and continued to shed excessive fluids away (at $t^* = 6.19$ and Fig. 5.9g). At the same time, the third trailing vortices (V4) had also reached its maximum strength in circulation. At $t^* = 7.64$ (i.e. Fig. 5.9h), V2 was amalgamated into the leading vortices when it attempted to repeat the leapfrogging process. One can note that the combined vortex of V1 and V2 traveled down faster than the pursuing jet and detached from it (ref. 5.12b).

At the same time, V4 started to shrink in diameter as a result of the formation of corner streamwise vortices. Although they were also surging forward as expected from the increase in translational velocities, they seemed to have broken down by the time they caught up with V3, see $t^* = 9.09$ (Fig. 5.9j). It should be noted that the vortex ring was stretched and tilted at an angle between the major axis of the ellipse and the axis of symmetry of the ring in the measured plane. According to Cater (2002), this behavior was an artifact of a wavy azimuthal distribution in the measured plane. This can be seen clearly at $t^* = 9.82$, the marked region had become turbulence and no clear presence of vortices could be found (Fig. 5.9i). Although no three-dimensional flow visualization was performed in the present investigation, it was

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believed that “kinks” may have occurred in the azimuthal vortex filament. The breakdown of azimuthal waves was often seen immediately prior to transition (Windnall & Tsai, 1977). The jet flow had almost become time-independent after $t^* = 13.83$ (Fig. 5.9k). In short, the transition to turbulence was facilitated by the axis switching mechanism between the third and the fourth vortex rings. This agree well with the findings of Lim (1997b) that the misalignment of the vortex rings is instrumental in producing fine-scale structures in the flow thus causing vortex ring to become turbulent.

Figures 5.11 and 5.10 plot the distance between the leading vortices and the first trailing vortices, the corresponding circulation strength and peak circulation (evaluated from the PIV). The above results offered some interesting observation. First, the separating distance between the two leading vortex started to decrease after the circulation had reached their maximum, i.e., at about $t^* \approx 3$. Second, the first trailing vortex (V2) started to surge forward after the formation of corner streamwise vortices.

Figure 5.10 illustrates that the circulation did not vary much after certain distance from the nozzle exit and the peak was first achieved at about $t^* = 1.16$. This result differs from Gharib *et al.*'s results by 70 %. Rosenfeld *et al.* (1998) studied the effect of the velocity profile of the jet at the nozzle exit. Varying the jet profile away from a nearly uniform profile was found to reduce the formation number. For the case of parabolic profile, a factor of four over the uniform profile was found. Based on the present investigation, it is obvious that axis switching mechanism also played an important role in determining the formation number.

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A final note is the occurrence of vortex stretching. From the results, it appeared only at the first trailing vortices and the third vortices. It is not clear why it did not appear at the leading vortex ring, based on the cross-sectional images obtained for this case at $Z/D_e = 0.5$ (i.e. Fig. 5.14). The leading vortices along the diagonal axes were moving forward at a different speed as those at the centre-plane. The curvature effects did not seem to have many effects on and the leading vortices. The trailing vortices showed differences which lead eventually to the stretching of vortices.

5.3.5 Case SNRe3528

The sequential images of the flow for this case are shown in Figs. 5.15a to 5.15f. The obvious difference between this case and that of the previous one is that the leading vortex sheet was thicker and entrained more fluid from the stem. Although it did not take a longer time for the first trailing vortices to appear after the formation of the leading vortex, the distance between the leading vortices (V1) and the first trailing vortex (V2) were almost $2D_e$ comparing to about $0.83D_e$ in the previous case. This may be due to a higher velocity and thinner boundary layer regions at higher Reynolds numbers. The leading vortex also attained the maximum circulation strength earlier at $t^* = 2.18$ (see Fig. 5.10). From $t^* = 3.26$ (Fig. 5.15b) onward the leading vortex started to shed excessive fluids being entrained from the jet stem and thus maintained the same vortex core size as the previous case. It should be noted that from $t^* = 2.18$ to 3.26 (Fig. 5.15a to Fig. 5.15b), the thickness of the stem shrank by almost 50% and by $t^* = 5.44$ (Fig. 5.15c), some hair-pin vortices formed by the excess fluids, were left behind of V1. Until $t^* = 8.71$, the main jet stem and the hair-pin vortices had almost disappeared. At the same time, the first trailing vortices (V2) started to intensify and were moved closer towards the jet centre axis. It is interesting to note that some ambient fluids were

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entrained to the centre of the jet stem, at around the region between the two trailing vortices (Fig. 5.15d). This is probably the result of entrainment from the two vortices on the plane orthogonal to the plane of visualization, due to the formation of the streamwise vortices at the corner. Contrary to the Case SNRe2358, the first trailing vortices did not accelerate forward although they continued to intensify (as shown in the shedding of excessive fluids from the jet stem). If the definition of vorticity disconnection is being used to identify pitch-off, then the leading vortices pinched off at about $t^* = 2.18$ (Fig. 5.16). Figure 5.17 shows that from the formation of V2 to its break-down, V1 and V2 traveled down parallel with the same traverse distance. It is also interesting to note that $W_{tr\_V1} = W_{jet}$ at time $t^* = 10.89$ where W_{jet} is defined as the minimum centerline axial velocity behind V1 (Fig. 5.16). This indicated that V1 began to detach from the trailing jet. However, Fig. 5.10 showed that the circulation for the first trailing vortex began to increase due to the feeding by the trailing stem at $t^* = 10.89$ (Fig. 5.15f). It also confirmed that the trailing stem caught up with the head vortex, i.e. and began to engulf the broken-down V2 (see Fig. 5.16). From the vorticity contour (Fig. 5.15f), it appears that the leading vortex was still strong. Unfortunately, the leading vortex was beyond the investigation area in the flow visualization image. However, the stem was seen to achieve the steady state and broke down into turbulence. Until $t^* = 14.70$ (Fig. 5.15f), the head vortex was still entraining the trailing fluids. However, it was severely deformed and began to break down.

It is interesting to note that for both cases (SNRe2358 and SNRe2358), axis switching occurred (Figs. 4.17 and 5.14) but the intensification for the first trailing vortices (V2) for Case SNRe3528 did not surge forward rapidly as in the Case SNRe2358. It can be deduced that the distance between V1 and V2 (or rather formation time for respective vortices) were crucial for

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leapfrogging to occur. The significant self-deformation and strong strength led to the breaking down of V2 before it can leapfrog V1. It can be noted that $W_{tr\_V1} = W_{tr\_V2}$ from Fig. 5.17. At $t^* = 10.89$, $W_{tr\_V1} = W_{jet}$ and V1 attempted to pinch-off from the trailing jet. However, it was caught up by V2 immediately after this instant and failed to perform the pinch-off process. Therefore, it can be concluded that $Re = 3528$ is the critical Reynolds number for the head vortex pinch-off to occur. In the next section, it can be noted that $W_{tr\_V1} > W_{jet}$ at $Re = 4716$. Another interesting observation was that for Case SNRe3528, the trailing vortices attained the maximum circulation within a short time. As shown in Fig. 5.10, maximum circulation was almost constant after $Z/D_e = 0.8$, which provided a value of about 2.18 for the dimensionless pinch-off time, $\overline{tW_0}/D_e$ (or formation number as defined by Gharib *et al.*).

5.3.6 Case SNRe4716

At this Reynolds number, the time difference between the formation of the leading vortices and the first trailing vortices lengthened further. The leading vortices achieved the maximum circulation ($\Gamma^* \approx 2.1$) also within a shorter time comparing to the last two cases ($t^* = 1.75$, i.e. Fig. 5.18a). It is interesting to note that as the leading vortices continued to entrain fluids from both the jet stem and ambient, kinks (as shown in Fig. 5.18b) appeared at the jet stem at about the location between the leading vortices and first trailing vortices. It is not clear why these kinks appeared but it is possible that the jet stem fluids were not being entrained fast enough to the leading vortices while at the same time the first trailing vortices emerged out from the nozzle. Figure 5.19 clearly displayed that the translational velocity of V1 was larger than that of the trailing jet (i.e. $W_{tr\_V1} > W_{jet}$) from $t^* = 2.91$. Alternatively, when examining the traverse velocity profiles at around the same region but prior to the formation of

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the kink. It appeared that the streamwise velocity profiles showed some inflexion points. This may explain in part the subsequently formation of the vortices.

At $t^* = 2.91$, the jet stem further stretched and the kinks kept growing (Fig. 5.18c). The leading vortex (V1) continued to travel fast and by $t^* = 6.40$ (Fig. 5.18d), had detached completely from the jet (pinched off had occurred). From $t^* = 7.59$ to $t^* = 11.06$, the first trailing vortices underwent intensification. Due to the decline of the velocity of V2, the distance between the two vortices further increased (Fig. 5.20), as may have been expected from the results of axis switching. The vortices subsequently broke down to turbulence and emerged into the trailing stem (see Fig. 5.18e) at about $t^* = 11.06$. From this time onward, the jet had almost achieved a steady state. As shown in Fig. 5.10, the circulation of V1 began to grow at $t^* = 11.06$. At the same time, the distance between V1 and the trailing jet declined (as shown in Fig. 5.18f). This indicates that V1 slowed down due to the viscous diffusion and the trailing jet was attempting to reconnect with it.

5.3.7 Mean characteristics for the steady trailing jets

In order to compare with the starting process and validate the PIV system, experiments for the steady trailing jets were conducted. The data obtained in this section were averaged by 400 recordings which were recorded about 30s after the jets initiated. From the PIV and PLIF results, one can note that 30s was enough for a jet to attain its steady state. Some important characteristics for the steady trailing jets within the near field ($0 < Z/D_e < 9$), such as mean velocity distribution, turbulence intensity, will be presented in the following sections. The mean velocity was the averaged value of the 400 records, while the turbulence intensity was the results of the standard deviation divided by the local centerline velocity.

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5.3.7.1 Mean Characteristics of velocity

Figures 5.21 and 5.22 show the mean axial velocity ratio ($\overline{W}_c / W_{c\max}$) decay along the centerline of the jets, where $\overline{W}_c$ and $W_{c\max}$ are the local centerline mean velocity and maximum of $\overline{W}_c$, respectively. Here, for the convenience of description, D is replaced with D_e for the circular cases. At the region very close to the exit ($Z_0 / D_e < 3$), the centerline velocity decayed linearly. This region is the potential core area of the jet. The decay rate was found to be 0.027 in the higher Reynolds number cases and 22% higher in the lower Reynolds number cases (about 0.033). The formula for the mean centerline evolution can be written approximately as

$$\frac{\overline{W}_c}{\overline{W}_{\max}} = -k_{pc} \frac{Z}{D_e} + 1. \quad (5.5)$$

The decay rate k_{pc} can be approximated to be 0.03. The cross-sectional profile for the mean axial velocity shows a top-hat shape in this potential core region (i.e. 5.23). The average length of the jet potential core (L_{pc}) varied between 2.5 and 3.0, which was shorter than the generally recognized value of 6.0 proposed by Fischer *et al.* (1979). This difference may be due to the difference in Re and nozzle size. The present data were obtained at $Re < 10000$. However, most of the previous works were conducted at higher Reynolds numbers. Wang (2000) also reported that the larger the port, the shorter the dimensionless length of the ZFE.

However, one can note that the mean axial velocity along the centerline line kept constant in the potential core region from the previous reports (Fischer *et al.*, 1979 and Crow & Champagne, 1971). In non-circular jets, the centerline velocity was found to accelerate before

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it began to decay due to the vena contracta (Hussain & Husain, 1989, Quinn, 1992 and Mi *et al.* 2000). The difference between the present and previous results might be caused by large-scale coherent structures rolling-up near the nozzle exit. The previous sections (i.e. sections 5.4.4, 5.4.5 and 5.4.6), illustrate that the centerline velocity was significantly affected by the rolling-up vortices. The centerline velocity showed a high peak at the location of the vortex core center, and then followed by a trough. This confirms that the mixing layer extended to the centerline even in the potential core region. On the other hand, the vortex structures was relatively smaller and not strong enough to affect the centerline characteristics in fully turbulent jets. The smaller decay rate in the higher Reynolds number cases as shown in Figs. 5.21 and 5.22 verified the conjecture. The acceleration of the mean axial velocity can contribute to the vena contracta effect triggered by the sharp-edged exit of the slot (Quinn & Militzer, 1988).

Beyond the potential core, the decay rate reaches an asymptotic behavior that is inversely proportional to $(Z / D_e)^{-1}$, i.e.

$$\frac{\overline{W}_c}{\overline{W}_{\max}} = k_{jw} \left(\frac{Z}{D_e} - \frac{Z_0}{D_e} \right)^{-1}. \quad (5.6)$$

A best fit-line to the data points of the present study yields a decay constant of 6.03 and 6.49 for the lower and higher Reynolds number cases, respectively and the virtual origin is about 3.86. For higher Reynolds number cases, the present results (obtained 6.49) are consistent with Wang and Law (2002), who obtained 6.477, and compare well with the value 6.22 proposed by Fischer *et al.* (1979) and Chen and Rodi (1980) for k_{jw} . Due to the faster decay rates in the potential core region which affect the virtual origin for the following similar area, the decay constant k_{jw} lowers to 6.03 for the lower Reynolds number cases.

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Next to be noted is the deviation for the cases CNRe2358 and CNRe3528. The mean axial velocity along the symmetric axis attenuated very fast in these two cases. The phenomena can be explained by the PLIF images (as shown in Fig. 5.24). Comparing Figs. 5.24a and 5.24b, the stem was smooth near the exit plane and no coherent structures rolled up in the circular jet, while in square jet, the pursuing vortices formed much earlier due to the thick shear layer (as shown in Fig 5.23). With the accumulation of the vorticity in the shear layer, strong K-H instabilities appeared and significantly altered the evolution of the jet stem. Because of the strong roll-up, the K-H vortices entrained almost all the fluid fed by the stem. With the breaking down of the vortices, the stem disappeared and merged into the ambient fluids. However, the stem maintained for a longer time in the square jet despite the presence of more K-H vortices formed. This was because the pursuing vortices were not strong enough to affect the stem.

A final note is that the hot wire results of Mi *et al.* (2000) in square jets showed a slightly higher value comparing with the present data. This deviation can be attributed to the different behaviors in the potential core. Fig. 5.22 demonstrates that the maximum centerline velocity occurred at $Z_0 / D_e \approx 2$ for the Mi *et al.*'s (2000) results, but the present data showed steady drop in the potential core region. The deviation may also be due to the different Reynolds numbers as Malmström *et al.* (1997) pointed at that the decay rate of $\overline{W}_c$ decreases as Re increases until a critical value is reached.

The cross-sectional variation of the mean axial velocity near the exit (as shown in Fig. 5.23) indicates that the velocity in the potential core was higher for the square cases. This may be

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attributed to the thicker shear layer at the corner of the square jets. It is interesting to note that the width for the Case SNRe2358 was larger than the other cases. The larger width is believed to be due to the axis-switching phenomenon depicted in previous chapters.

The figures from Figs. 5.25 to 5.32 illustrate the cross-sectional variation of the mean axial velocity of the jets at different locations. The cross-sectional velocity demonstrated top-hat profiles within the potential core ($Z_0 / D_e < 3$). Beyond that, the cross-sectional distribution showed a Gaussian profile, with the Gaussian width increasing as the jet traveled downstream.

Previous reports (Fischer *et al.* 1979 and Wang and Law 2002) proposed that the Gaussian-fit equation can be written as

$$\frac{\overline{W}}{\overline{W}_{\max}} = \exp \left[-k_{jwg} \left(\frac{R}{Z} \right)^2 \right]. \quad (5.7)$$

where k_{jwg} is the constant for Gaussian shape. Hence,

$$\sqrt{\frac{1}{k_{jwg}}} = b_w \quad (5.8)$$

is the dimensionless velocity 1/e- width defined as the radius where the mean axial velocity is $\overline{W}_{\max} / e$. Wang and Law (2002) suggested $k_{jwg} \approx 91$. To enable a comparison with them, the normalized data are plotted in Figs 5.26, 28, 30 and 32. The figures indicated that within the measurement area, the velocity was not fully developed and had not attained its self-similarity. The value of the width, b_w , for the Gaussian profile is larger than the value suggested by Wang and Law (2002). That implies that the jet did not attain its asymptotic state in the present investigate area. On the other hand, the Gaussian profile was narrower in square jets due to the initial narrower width (32 vs. 36 mm) in square jets.

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In summary, the coherent structures formed in the shear layer can significantly influence the evolution of the jet. The Reynolds number can also alter the behavior of the potential core region and thus change the virtual origin of the asymptotic region.

5.3.7.2 Turbulence Characteristics of Velocity

The evolution of the turbulence intensity can also explain why the mean axial velocity along the centerline decayed faster in cases CNRe2358 and CNRe3528. Comparing Figs. 5.33-36, the axial and radial turbulent intensity for the cases CNRe2358 and CNRe3528 were both larger than the other cases, especially for the Case CNRe2358 which the axial turbulent intensity kept increasing from about 0.1 to 0.55 throughout the measurement area. The turbulence intensity of the radial velocity for the two cases was also among the largest. The larger turbulence intensity was believed to be the results of the strong coherent structures.

Several observations can be made. First, for both axial and radial velocities, the initial turbulence intensity varied from 0.05 to 0.1 for all the cases. However, the axial turbulence intensity remained steady relatively in the potential core region. Beyond that, it increased quickly to a constant level of about 0.25 at $Z / D_e \approx 7$. This level is consistent well with the previous results report in the literature for steady jets, i.e. Papanicolaou & List (1988) which performed the experiments with LDA and Wang & Law with PIV (2002), but a little higher than Panchapakesan and Lumley (1993) with hot-wire.

Next, the turbulence intensity increased with the Reynolds number. However, when the Reynolds number was high enough, the turbulence intensity became a constant in the far field.

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The turbulence intensity for the axial velocity maintained constant in the region of potential core and fully developed area, but increased in the transitional zone. The turbulent intensity in the radial component along the centerline jet axis was recorded to range from 0.12 to 0.2 with an average value 0.16 in the similarity region. The measured values are in the same range as what were measured by Papanicolaou (1984) who employed LDA as the experiment tools and Panchapakesan and Lumley (1993) who used hot-wire. The consistency of the mean and turbulent characteristics confirmed the present experimental arrangements were properly set and the other results are reliable.

Finally, it is also interesting to note that the turbulence intensity for the radial velocity kept growing in square jets while it remained steady in the circular ones, which further confirmed the existence of the streamwise vortices in the square jets.

5.3.8 Dynamics of the pinch-off process

The present section deals with the so called “pinch-off” process, where the leading vortex ring separates from the trailing jet after a short duration of jet fluids shedding. One important problem in vortex ring generation is to determine how much circulation and energy can be delivered to the vortex ring by the apparatus. The experiments of Gharib *et al.* (1998) showed that for large piston stroke vs. diameter ratios (L/D), the generated flow field consists of a leading vortex ring followed by a trailing jet. As mentioned above, they observed that for a set of flow parameters, there is a limiting stroke length that imposes an upper bound on the circulation that a vortex ring can acquire. The vorticity field of the formed leading vortex would be disconnected from the trailing jet at a critical value of L/D (dubbed the “formation number”), at which time the vortex ring attains a maximum circulation. Furthermore they

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observed that the maximum circulation is achieved for a narrow range of the formation number L/D , between 3.6 and 4.5, for a variety of flow parameter and nozzle geometries.

In a cylinder-piston mechanism, the so called slug-model assumes that the exit velocity is uniform and equals to the piston velocity. However, in starting jets, the mean velocity at the exit plane is shown to have a top-hat profile (as shown in Fig. 5.23). In the present report, the average velocity $\overline{W}_0$, which is defined as

$$\overline{W}_0 = \frac{Q_0}{A}, \quad (5.9)$$

is used to replace the piston velocity W_p . Hence, the formation number becomes $t^* = t\overline{W}_0 / D_e$, i.e. the nondimensional time.

The leading vortex ring is the outcome of the instability and roll-up of the cylindrical shear layers generated by shedding fluids. Unlike the slug fluid, the exit velocity in starting jets is not uniform (e.g. Fig. 5.23). The shear layer is intrinsically unstable and will roll up into a spiral structure. The resulting vortex ring is fed by the shear layer at a specific rate (velocity), which is the velocity of the shear layer. The rate at which the invariants of motion (in an inviscid case, E , Γ , and I) are delivered is essential in determining the properties of the resulting leading vortex ring. This rate is in fact the main characteristic of the vortex generator and varies among different vortex generation tools.

Two mechanisms were proposed for the pinch-off process by Mohensi *et al.* (2001). The first one considers the phenomenon as the relaxation process of the leading ring to its final

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equilibrium state. The second approach describes the process as a dynamical problem, which focuses on the transitional dynamics of the leading vortex ring and its following shear layer.

5.3.8.1 Relaxation approach

The relaxation approach is based on the Kelvin-Benjamin variational principle, which states that steadily translating vortex rings have a maximum energy with respect to isovortical perturbations that preserve the impulse. In simple terms, the idea is as follows. The constant water head delivers energy to the flow at a certain rate. At earlier time, all the energy is delivered to the leading vortex ring. As the circulation of this vortex ring grows, the ring eventually obtains the maximum Kelvin energy. At this point, the shear layer cannot deliver more energy to the leading vortex ring, and therefore, the vortex ring separates from the trailing jet and translates down-stream faster than the following shear layer. Thus, the ring pinches off at the point in time when it obtains its maximum Kelvin energy.

Based on the so-called “slug-model” and Norbury vortices assumptions, Mohseni and Gharib (1998) proposed a model for the universal time scale of vortex ring formation. The derivation can be briefly described as follows.

The kinetic energy E , circulation Γ and impulse I for a slug of fluid with diameter D , Length L , and uniform velocity W_p are given by

$$E = \frac{1}{8} \pi D^2 L W_p^2 = (W_p / 2) I, \quad (5.10a)$$

$$\Gamma = \frac{1}{2} L W_p, \quad (5.10b)$$

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$$I = \frac{1}{4} \pi D^2 L W_p = \frac{1}{2} \pi D^2 \Gamma . \quad (5.10c)$$

Incorporating the above equations, the energy can be obtained as

$$E = \Pi / L , \quad (5.10d)$$

Hence for a fixed impulse and circulation, the minimum stroke length L which results in a Norbury vortex would maximize the energy.

The corresponding invariant quantities for the Norbury vortices are non-dimensionalized as follows:

$$E = (\Omega R_v \varepsilon)^2 R_v^3 E_N , \quad (5.11a)$$

$$\Gamma = (\Omega R_v \varepsilon) R_v \Gamma_N , \quad (5.11b)$$

$$I = (\Omega R_v \varepsilon) R_v^3 I_N , \quad (5.11c)$$

where the subscript N refers to the corresponding nondimensional quantity, for the Norbury vortices, $\Omega = \omega / r$ the vorticity density, R_v the vortex radius and ε is the nondimensional mean core radius (see Norbury, 1973). The dimensional translation velocity W_{tr} and the nondimensional translation velocity W_N are related by

$$W_{tr} = (\Omega R_v \varepsilon) W_N , \quad (5.11d)$$

An estimate for W_{tr} may be obtained from the following relation (Roberts, 1972)

$$\left. \frac{\partial E}{\partial I} \right|_{\text{fixed } \Gamma \text{ \& vol.}} = W_{tr} , \quad (5.12)$$

which for the slug model (Eqs. 5.10) results in

$$W_{tr} = W_p / 2 , \quad (5.13)$$

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Equating the kinetic energy, the total circulation and the impulse for the slug model (Eqs. 5.10) with the corresponding quantities in the Norbury vortices (Eqs. 5.11a, b, c and d) and using (5.13) we obtain,

$$L/D = \sqrt{\frac{\pi}{2}} \frac{I_N^{1/2} \Gamma_N^{3/2}}{E_N} = f(\varepsilon), \quad (5.14a)$$

$$L/D = \sqrt{\frac{\pi}{2}} \frac{I_N^{1/2} \Gamma_N^{3/2}}{I_N^{1/2} W_N} = g(\varepsilon), \quad (5.14b)$$

According to Norbury (1973), the right-hand side of Eqs. (5.14a and 14b) are only a function of ε . The nondimensional stroke length L/D from the Eqs. (5.14a and 14b), as a function of ε , are shown in Fig. 5.37. Their intersection yields the parameter ε of the resulting Norbury vortex. The corresponding stroke length L/D for the resulting Norbury vortex is approximately equal to 3.

By varying W_{tr} between $0.5W_p$ and $0.6W_p$ Mohseni and Gharib (1998) reported that $3.0 \leq L/D \leq 4.5$, $0.27 \leq E_{nd} \leq 0.4$, and $1.77 \leq \Gamma_{nd} \leq 2.07$. At the same time, experimental results (Gharib *et al.* 1998) showed that $E_{nd} = 0.33$ and $L/D \approx 4.0$ for a cylinder-piston mechanism.

The invariant quantities (kinetic energy and circulation) obtained from the experiments for the starting circular and square jets are shown in Table. 5.2 and Figs. 5.38 and 39. The circulation was measured from the initiation of the jets (see 5.3 and 5.10). It is interesting to note that the circulation of the head vortex underwent an over-shoot at higher Reynolds number cases and followed by a sudden decrease. Furthermore, the total circulation ceased to grow during the period of over-shooting. It should be noted that the head vortex attained its maximum

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circulation in a relatively short time (about 0.6~0.64s) for all cases. However, the normalized time was not as uniform as previously reported (Gharib *et al.* 1998, Rosenfeld *et al.* 1998). The nonuniformity for the formation number may partly be contributed to the modification of the discharge velocity profiles by the change in the Reynolds number. The average characteristics depicted in the previous section showed that the higher the Reynolds number, the smaller the displacement thickness will be. Therefore, the higher the Reynolds number, the more uniform the exit velocity profile would be, which would lead to the extension of the formation time. The temporal evolution for actual discharge velocity (Fig. 5.40) shows that the exit velocity profile was neither uniform, nor consistent. Instead, it varied with time.

Figures 5.3 and 5.10 illustrate that during the evolution, the circulation of the head vortex slightly decreased due to the viscous dissipation. Finally, the head vortex was caught up by the trailing stem which reconnected with it. Subsequently, the circulation began to rise. The only difference was that in the lower Reynolds number cases, the first trailing vortex leapfrogged the leading vortex while in the higher Reynolds number cases it broke down before exceeding the leading one. Thus, the circulation of V1 showed a step-like increase (see Figs. 5.3 and 5.10). It is also interesting to note that the circulation of the head vortex only related to the initial velocity (see Table 5.2). Gharib *et al.* obtained similar results in their experiments. However, the velocity scale did not seem to make the normalized circulation converge to a universal value. Another scale ($\Gamma'_{nd\_V1} = \Gamma / (I^{1/2} \overline{W}_0)$), as shown in Eq. 5.4e) also can not achieve that (see Table 5.1). In summary, these two scales are probably not the best choice for non-uniform or non-impulsive discharge velocity profiles.

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The circulation of the head vortex decayed faster in square jets than that in circular cases. This might be due to the interaction between the streamwise and spanwise vortices which led to the axis-switching and hastened the breaking-down of the vortex, as described in Chapter 4. This further confirms that the streamwise vortices play a significant role in the evolution of the starting jets.

As shown in Figs. 5.38 and 39 for the circular and square cases, respectively, the nondimensional energy fell to about 0.2 for all the cases. Comparing with the results of Gharib *et al.* (1998), the present nondimensional energy was much smaller either for total or the head vortex (40%). The formation number varied between 1.16 and 2.33, which was also less than Gharib *et al.*'s results (4.0). The difference again might be due to the different exit velocity profiles in the starting jets. Rosenfeld *et al.* (1998) reported that the exit velocity profile has a significant influence on the formation number. In particular, for a parabolic profile, the formation number is reduced to unity from 4 for a uniform profile. Here in starting jets, the average axial exit velocity profile is trapezoidal (see Fig.5.41). In real flow cases, the axial velocity profile is neither uniform nor parabolic; rather it develops with time (i.e. Fig. 5.40).

It should be noted that the velocity shown in Fig. 5.40 was not measured on the exact exit plane but at the location of $Z/D = 0.36$, because of the limitation of the PIV system. However, it is believed that the measured velocity can closely represent the development of the exit velocity. At the start of the flow discharge ($t^* = 0.44$), the velocity profile showed a Gaussian profile, and the magnitude grew from zero. At $t^* = 0.87$ the vortex rolled up very close to the exit. The centerline velocity was much larger than the average velocity with two strong peaks (27.6%) at either two sides of the symmetric axis, owing to the over-shoot and

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large entrainment of the head vortex. The flow meter also revealed an over-shoot during the initiation of the jets that can be related to the larger pressure gradient at the beginning. With the releasing of the jet, the pressure gradient decreased and thus the volume flow rate lowered. The over-shoot directly speeded up the pinch-off process. The peaks indicate that the leading vortex was passing the measurement location. At $t^* = 1.31$, the velocity lowed and the peaks disappeared. At a later time $t^* = 1.74$, the axial velocity declined further, while the leading vortex traveled down-stream. At this instant, the leading vortex pinched off from the main stem. It should be noted that the velocity was much lower than the average velocity while the width was larger. Because the continuous discharge of vorticity cannot be entrained into the head vortex, it accumulated in the tail that was significantly longer and strong enough to form a secondary vortex ring. Hence, at $t^* = 3.05$ (Fig. 5.5b), the first trailing vortex formed with a lower intensity of vorticity than the leading one. Then, the velocity lowered when V2 traveled downward and pinched off from the stem.

Comparing to the previous experimental and numerical simulations with cylinder-piston mechanism (ref. to Fig. 8 of Didden (1979) and Fig. 13 of Rosenfeld *et al.* (1998)), the peaks shown in the axial velocity profile were higher for the present results. The higher peaks denoted that the over-shoot vortex was stronger in starting jets.

An important implication of these findings is that the variations in the velocity profile affect the formation of the head vortex. The thick shear layer in the jet nozzle can accelerate the formation of the leading vortex. Another indication from the quantitative results is that the gravity-driven jet mechanism is quite different from the cylinder-piston vortex generator. The large over-shoot in starting jets also increased the amplitude of the circulation and hastened the

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evolution of the leading vortex. Furthermore, the velocity programme can be considered as an impulse one in starting jets. The acceleration time was very short ($<0.2s$), including the opening time of the solenoid valve.

5.3.8.2 Dynamical approach

Although the Kelvin-Benjamin principle offers an elegant theory for the mere existence of a limiting process, it fails to suggest a process for the dynamics of pinch-off. In the following, the author will try to describe the kinematic model suggested by Shusser *et al.* (1999), which is reported as the physical manifestation of the Kelvin-Benjamin principle.

The dynamical criterion for the pinch-off is that the maximum axial velocity in the rings equals the maximum axial velocity of the following shear layer, i.e.

$$W_{tr} = W_{jet} . \quad (5.15)$$

Utilizing the thin ring approximation, the translation velocity W_{tr} can be expressed as

$$W_{tr} = \frac{E}{2I} + \frac{3}{8} \sqrt{\frac{\Gamma^3}{\pi I}} . \quad (5.16)$$

In the thin ring approximation, the ring radius of the ring $R_v = D / \sqrt{2}$. Based on the mass conservation law, the jet velocity can be obtained, i.e. $W_{jet} = W_p / 2$. Equating W_{jet} with (5.16), the formation number can be obtained,

$$\frac{L}{D} = \frac{2\pi\sqrt{2}}{3} \approx 2.96 . \quad (5.17)$$

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Several velocity programs, i.e. impulse, linear, trapezoidal, fast and slow, were considered by Shusser *et al.* (1998) and the results showed a possible range (2.96~7.9) of formation number values.

Case CNRe2358 was chosen to explain the head vortex pinch-off process from the dynamical approach. Figure 5.41 illustrates the temporal evolution of the axial velocity profile in the plane through the vortex core center of V1. It can be found that the head vortex core size, circulation, translational velocity and shear layer thickness maintained at a constant level. The amplitude and shape of the cross-sectional velocity profile at the plane through the center of the V1 at different instances (as shown in Fig. 5.41) were similar, which suggested that the head vortex ring was very stable. The velocity profiles along the streamwise direction correspond to Fig. 5.2d (at $t^* = 2.62$), are shown in Fig. 5.42. The profiles illustrate that the maximum velocity located at the plane the vortex core center. Moreover, V1 was shown to be stronger than V2.

The centerline axial velocity profile also showed a peak at the location of V1 core center at the instant of the leading vortex formation ($t^* = 1.16$) (see Fig. 5.43). Because of the powerful rolling up, the velocity close to the vortex core center was much larger than the average exit one. Figure 5.43 also shows that the maximum velocity along the centerline occurred at the center plane of the head vortex ring. A notch also disappeared in the centerline velocity plot between the two vortices.

As mentioned before, at the time when $W_{tr} = W_{jet}$, the vortex will pinch-off from the trailing jet. The jet velocity, W_{jet} , was measured just on the top of V1 (i.e. the minimum centerline

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velocity between V1 and V2). From the results of the present Case CNRe2358, one can observe that the normalized translational velocity for V1 was $W_{tr\_V1} / \overline{W}_0 = 0.68$, while $W_{jet} / \overline{W}_0$ was equal to 0.94 (as shown in Fig. 5.43). Furthermore, from $t^* = 1.16$ to 2.62, W_{jet} was always larger than W_{tr_V1} . However, V1 attained its maximum circulation and energy as earlier as $t^* = 1.16$. Therefore, the criterion $W_{tr} = W_{jet}$ for vortex pinch-off was not identical with that of the leading vortex attaining its maximum energy.

The PLIF images (shown in Figs. 5.2a to 5.2i) may help to explain why the vortex ring pinch-off might not occur at the time the leading vortex attaining its maximum energy. At the very beginning of the starting jet, a head vortex with a very strong stem was formed. With the entrainment of V1, the stem became thin. At the same time, V1 underwent elliptic deformation while the trailing fluids were engulfed by V1. However, V1 began to discharge the excess fluid at a later time in order to maintain its characteristics (energy and circulation) at a constant level. At the same time, some hair-pin vortices formed behind the head vortex. The PLIF images for the Case CNRe2358 never show that V1 distinctly separated from the trailing stem. But Fig. 5.3 reveals that V1 attained its maximum circulation shortly after the jet beginning. However, it is interesting to note that the amalgamative vortex pinched-off due to the acceleration of V2 during the leapfrogging process (as shown in Fig. 5.2g and 5.43).

Another interesting phenomenon to be noted is the W'_{tr_V1} , which is the theoretical velocity of a single vortex, was less than the measured one. According to the second-order formulae for Norbury vortices, the velocity and the energy of a vortex ring are given by (Fraenkel 1972)

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$$W'_{tr} = \frac{\Gamma}{4\pi R_v} g(\varepsilon), \quad (5.18)$$

Here $\varepsilon = a / R_v$, where a is the core radius, R_v the radius of the ring and

$$g(\varepsilon) = \frac{\ln \frac{8}{\varepsilon} - \frac{1}{4} + \frac{\varepsilon^2}{64} \left(37 \ln \frac{8}{\varepsilon} - \frac{223}{4} \right)}{1 + \frac{21}{64} \varepsilon^2}. \quad (5.19)$$

Taking the value of 0.43 for ε (from Table 5.2), $W'_{tr-V1} / \overline{W}_0 = 0.43$, which is much less than the measured value of 0.68 for Case CNRe2358. This confirms that the stem pushed the leading vortex forward when it traveled downstream.

Table 5.2 reveals the transitional velocity in the higher Reynolds number cases CNRe3528 and CNRe4716 ($W'_{tr} / \overline{W}_0 = 0.70$ and 0.87, respectively). This value is larger than the lowest trailing stem velocity behind the leading vortex as shown in Figs. 5.44 and 5.45 ($W_{jet} / \overline{W}_0 \approx 0.60$ and 0.85, respectively). Thus in these cases, the leading vortices indeed “pinched-off” from the trailing jet. From the PLIF images (i.e. Figs. 5.5 and 5.7), the head vortex was thoroughly detached from the trailing stem without any ambient fluids appearing between leading vortex and the trailing jet. As mentioned in Section 5.4.5, $R_e = 3528$ is the critical value for the pinch-off occurring in square jets. However, owing to the larger translational velocity, the critical Reynolds number for the pinch-off occurring should be smaller in circular jets.

In summary, the relaxation criterion for the pinch-off occurring at the time when the leading vortex attains the maximum circulation or energy is not proper for the actual starting jets. Alternatively, the pinch-off should occur when the translational velocity of V1 equals to that

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of the trailing jet, i.e. $W_{tr} = W_{jet}$. After the leading vortex attains its maximum energy, the translation velocity may still be lower than the trailing jet. At the same time, the trailing jet still attempts to feed the leading vortex. The head vortex will eject the surplus fluids to maintain its steady state and form hair-pin vortices behind it. In other words, pinch-off may not occur in starting jets. Instead, leapfrogging may take place if the conditions, as described in section 5.4.1, is suitable.

5.3.9 Orifice plate effects

In previous chapters, the PLIF results illustrated that the orifice plate fixed at the exit of the nozzle produces a wall condition. The wall condition speeds up the head vortex and hastens its pinch-off process. Due to the disturbance of the negative vorticity created on the orifice plate, the breaking down process is also accelerated. In this section, some quantitative results obtained from the orifice cases are further presented to investigate the orifice plate effect in starting jets.

James & Madnia (1996) presented a numerical study of vortex ring formation which showed that the impulse in the flow field of the ring is approximately the same for the nozzles with and without a vertical wall at the nozzle exit plane. Gharib *et al.* (1998) reported that the maximum level of circulation for the ring decreased slightly resulting in lower value of the formation number (≈ 3.6).

Figures 5.46 and 5.47 display the total and head circulation evolution for the orifice cases. As shown in Table 5.2, the steady state value of the circulation was similar to those cases without the orifice plate. Furthermore, the nondimensional energy, E_{nd} , was nearly the same (≈ 0.2).

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However, comparing with Figs. 5.38, 39, 48 and 49, E_{nd} decayed faster for all the orifice cases. The faster attenuation in the orifice case might be due to the disturbance of the negative vortices produced on the orifice plate. As shown in Table 5.2, the mean maximum velocity $\overline{W}_{\max}$ along the centerline was also unaffected by the orifice plate. After the jet attained the steady state, the pressure gradient near the orifice plate disappeared. The K-H vortices rolled up at the location far from the nozzle exit and no negative vortices were generated on the outside of the orifice plate. Therefore, the radial pressure gradient created was negligible. Hence, the effect on the fully-developed steady jets was minor.

Figures 5.50a to 5.50j present the evolution of the negative vortices. At the very beginning, owing to the pressure gradient created by the orifice plate, the head vortex was distorted while it was shed from the nozzle (as shown in Fig. 5.50a). Comparing with Fig. 5.2a, the diameter of the head vortex ring was actually smaller in the orifice cases. Due to the counter-rotation effect of the orifice plate, opposite-sign vortices appeared on the outside of the vortex ring at time $t^* = 0.87$ (see fig. 5.50b). At later time, the negative vortices further developed, pushing the primary vortex V1 ahead and making it flatten. At time $t^* = 1.31$, the negative vortices seemed to have broken down, and V2 began to be shed from the jet exit. Finally, the negative vortices crashed and mixed with the ambient fluids. When V2 formed, similar process was occurred (see Fig. 5.50j). The unchanged circulation indicates that the negative vortices broke down completely before they were entrained. The axial velocity profile (as shown in Fig. 5.51) in plane through the vortex core center also reveals that the leading vortex traveled down at a higher speed and with a smaller diameter comparing with the Case CNRe2358 (see Fig. 5.43).

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As shown in Fig. 5.52, the trajectory of the head vortex core was significantly affected by the orifice plate. In the cases CORE3528 and CORE4716, the leading vortex kept shrinking during its travel (see Fig. 5.52b). However, in the circular nozzle cases, the diameter of the leading vortex kept growing. For both square nozzle and square orifice jets, the size of the head vortex increased with fluctuations. This difference may be related to the axis-switching in the square jets.

As shown in Table 5.2, the pinch-off time was shortened with the orifice plate. This might be due to two factors. First is the high pressure gradient generated on the outside of the orifice plate. The high initial pressure gradient speeded up the vortex ring and hastened its detachment from the trailing jet. The transitional velocity of the head vortex was found to be greatly enhanced also (ref. to Table 5.2). The normalized transitional velocity $W_{tr\_v1} / \overline{W}_0$ rose to 0.85 and 0.89 for the cases CORE2358 and SORe2358, respectively, and about 1 for the other orifice cases. The interactions between the opposite-sign vortices and the primary vortex might be the other factor. The counter-rotating effect of the orifice plate prevented the head vortex to eject the excess fluids and causing elliptic deformation. Therefore, the fluids fed by the trailing jet can be accumulated quickly and the pinch-off occurred in a shorter time.

The mean characteristics in the orifice cases were similar to the nozzle cases (see Fig. 5.53 to 55). As shown in Fig 5.53, the trend for the mean centerline velocity was consistent with the fitted line for the nozzle cases. The decay rate and the turbulence intensity for the axial and radial velocity at the lower Reynolds number of 2358 were also greater than the other cases. It is interesting to note that the turbulence intensity for the radial velocity kept rising in the

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square jets, while it remained steady in the circular jets. This trend implies that the stream-wise and span-wise vortices kept interacting to increase the turbulence intensity.

5.3.10 Three-dimensional effects in square jets

As mentioned in previous chapter, the asymmetry of square jets resulted in slower penetration rate and axis-switching which enhance mixing efficiency. In this section, the three-dimensional effects in square jets are discussed and quantitative results obtained in D-D plane are presented. Through the comparison of the results obtained in D-D plane and S-S plane, the differences between the circular and square jets will be further explored.

As shown in Fig. 5.56, the circulation obtained in D-D plane for the head vortex was consistent with that obtained in the S-S plane. The values was 30, 62 and 100 cm^2/s for the three cases SNRe2358, SNRe3528 and SNRe4716, respectively. However, the nondimensional energy was obtained at about 0.18, slightly smaller than 0.2 in the S-S plane (see fig. 5.57). The decrease in the nondimensional energy might be attributed to the thicker shear layer in the diagonal direction.

Figures 5.58a to 5.58f show the vortex evolution in the D-D plane for the Case SNRe3528. In the previous chapter, the head vortex trajectory for this case was discussed. From the trajectory, the phenomenon of axis-switching was explained. Fig. 5.58a shows an extremely thick shear layer shedding from the nozzle. Because of the strong vorticity pre-generated in the nozzle, a large part of the original shear layer was left behind and not entrained by the leading vortex. The thicker shear layer speeded up the leading vortex and hastened the pinch-off process (see Fig. 5.59). The PLIF images also illustrated that the vortex sheet was thicker in the D-D plane

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(Ref. Fig. 4.17). At the same time, the faster transition speed at the corner portion led to the axis-switching.

Comparing the instantaneous vorticity field plot taken at $t^* = 2.18$ (Fig. 5.15a vs. Fig. 5.58c), there was strong trailing shear layer behind the leading vortex in Fig. 5.58c. However, the leading vortex engulfed most of the vorticity in Fig. 5.15a and only left a small part of the original shear layer. Until $t^* = 3.48$, the trailing shear layer broken down into small pieces and the leading vortex separated completely from the trailing jet. V2 also left a strong trailing shear layer as shown in Figure 5.58f.

The cross-sectional profile of the mean axial velocity in the D-D plane (i.e. Figs 5.60 and 5.61) also displays a thicker shear layer near the exit. Instead of a top-hat profile near the exit, a U-shape profile is illustrated. As shown in Figs. 5.62 and 5.63, the fitted Gaussian profile implies that the velocity width, b_w , was still larger in D-D plane, but Figs. 5.62 and 5.63 illustrate a fast decay rate. With this trend, b_w should eventually become similar both for D-D and S-S planes.

5.3.11 Contribution of the over-pressure

The commonly assumed slug model ignores the over-pressure at the nozzle exit (Krueger 2001). However, recent literature reported that the over-pressure can play a significant role in the head vortex formation process (Krueger 2001), which further confirmed that the over-pressure might only be ignored during the ejection phase of a trailing jet. In the present section, the contribution of the over-pressure to the vortex formation will be explored.

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An isolated, axis-touching vortex ring bubble comprises the ejected fluids, entrained fluids and added mass (refer to Fig.10 of Krueger 2001). The added mass was the ambient fluids set into motion as the ring forms and moved downstream. The fluids ahead or behind the leading vortex should be both accelerated or brought in once the jet was initiated. In starting jets, the trailing jet occupied the part place of the added mass behind the vortex ring. Furthermore, as mentioned above, the head vortex of a starting jet may also contain the irrotational fluids, causing the head vortex to become non-axis-touching in a gravity-driven-starting jet (as shown in Fig. 5.64).

The rolling up process is sensitive to the history of the exit velocity (Didden 1979). The twin-peak exit velocity profile resulted in larger mass fluxes as the vortex rolled up (as shown in Fig 5.40). The over-mass-fluxes, which are due to the over-pressure resulted larger average exit velocity, $\overline{W}_e(t)$, cannot all be entrained by the vortex ring. Therefore, the fluids at the ring center are left to be irrotational, but travel with the head vortex downstream (as shown in Fig. 5.64). As a result, the head vortex cannot attain axis-touching state. It should be pointed out that in the present studies, the computation of the characteristics of the vortex ring, including the energy, circulation and impulse, are limited within an iso-vorticity contour of 2 s^{-1} . Thus, the added mass and the irrotational fluids were not included in the integration.

For a starting jet, the total energy of a starting jet, E_{total} , account for the propulsion required to push the flow ahead which is produced by the gravity and over-pressure, i.e.

$$E_{total} = E_p + E_g, \quad (5.20)$$

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where E_p and E_g represent energy generated by the over-pressure and gravity, respectively.

E_{total} can be computed from the PIV results as shown in Eq. (5.2a) while E_g can be expressed as

$$E_g = \frac{1}{2} m W_0^2 = \frac{1}{2} \frac{\rho Q_0^3}{A^2} t. \quad (5.21)$$

By substituting Eq. (5.21) into (5.20), E_p can then be obtained,

$$E_p = E_{total} - E_g = \int_{\Omega} (U^2 + W^2) dV - \frac{1}{2} \frac{\rho Q_0^3}{A^2} t. \quad (5.22)$$

Figures 5.65-67 illustrate the vortex ring energies of E_{V1} , E_{V2} , E_{V3} , and E_{total} , E_g , E_p .

Several observations can be drawn from the three figures. First, the energy produced by the over-pressure increased with the rise of the Reynolds number (as shown in Table 2). Secondly, Figs. 5.66 and 67 (Cases CNRe3528 and CNRe4716) show that the energy of the head vortex, E_{V1} was larger than E_g during the formation process of V1, which confirmed that E_p contributed to the rolling up of V1. However, for Case CNRe2358, $E_{V1} < E_g$ and the overshoot was not as strong as the other two cases. This implied that the overpressure was only delivered to the irrotational fluids and the added mass, which further confirmed that the overshoot was induced by the over-pressure. Another conclusion is that the over-pressure only took effect during the period of the head vortex formation as proven by the constant offset between E_{total} and E_g after V1 was formed. Third, the proportion of the E_p to E_{total} , E_p/E_{total} , was smaller in Case C1. The maximum value of E_p/E_{total} was around 0.43, while about 0.65 for the other two cases (see Fig. 5.68). It may be one of the other factors causing the head vortex not to be able to pinch-off from the trailing stem in case C1. After the overshoot, the value of E_p/E_{total} decreased linearly with the diminishing of the overpressure.

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A final note is that total energy underwent a sudden increase when a new vortex ring was shed from the nozzle. The increase may be contributed to the negative pressure gradient created during the vortex rolling up and entrainment process. However, it is smaller than the over-pressure at the beginning of the gravity-driven starting jet. The energy created by the pressure gradient may determine whether pinch-off will occur. Since E_p may be delivered to the rotational ring core, irrotational flow and the trailing jet, the amount of energy obtained by the vortex ring will directly determine whether the vortex ring can pinch off from the trailing stem. It can be conjectured that if the maximum energy of the vortex ring exceeds the E_g during the period of the vortex ring formation, pinch-off will happen. This criterion may be expressed as

$$\frac{dE_g}{dt} < \frac{dE_v}{dt}, \text{ where } E_v \text{ denotes the energy of the vortex ring. In other words, only then } E_p$$

conveys enough energy, can the vortex ring separate from the main stem. Since E_g relates to the energy of the trailing jet stem, this criterion is essentially equivalent to the dynamical criterion of Eq. (5.15). Figures 5.65-67 provide same supports to this conjecture.

In particular, as shown in Fig. 5.66 (Case CNRe3528), although the energy strength for V2 and V3 are within 10% of each other, V2 pinched off from the trailing jet while V3 did not. It appears from Fig. 5.66 that the energy of the vortex must reach a certain level before the pinch-off phenomenon to occur. It is likely that the effect of pressure impulse may also have some residual effects on the first trailing vortex, as shown in the twin-peak velocity profiles in Fig. 5.40. The twin-peak velocity profiles did not appear before the formation of V3 ($t^* = 5.23$).

5.4 Summary

Quantitative PIV results, corresponding to the PLIF results depicted in Chapter 4, are presented in this chapter. Temporal evolution of the velocity field and vorticity distribution is documented. The measured mean and turbulent characteristics for the steady trailing jet are also discussed.

The development of the vortex dynamics in the starting jets reveals that the head vortex attained the maximum circulation and energy shortly after it was shed from the nozzle ($\approx 0.6s$). The normalized formation time for the head vortex was shorter in starting jets than that generated by the cylinder-piston mechanism, which can be approximated by the slug-mode. Furthermore, owing to the variation of the exit velocity profile, the nondimensional formation time changed with the Reynolds number.

The analysis of the dynamics of the head vortex pinch-off process confirms the Kelvin-Benjamin variational principle. The non-dimensional energy E_{nd} shows a consistent value of around 0.2 for all cases, which was smaller than that of the vortex created by cylinder-piston mechanism. On the other hand, the dynamical analysis illustrated that the Kelvin-Benjamin variational principle could not be applied for the head vortex pinch-off process in starting jets. The time when the head vortex attains the maximum energy, was not consistent with the time of pinch-off. Whether the leading vortex generated by starting jets can pinch-off from the trailing jets is determined by the Reynolds number. Critical Reynolds number was found to be 3528 in the starting square jets. It should be smaller in the circular jets.

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The mean characteristics for the steady trailing jet illustrate that the axial velocity along the centerline declined linearly in the potential core region. Beyond that, the decay rate reached an asymptotic behavior that is inversely proportional to $(Z / D_e)^{-1}$. At the same time, the strong coherent structures significantly affected the decay rate of the centerline axial velocity when the Reynolds number was relatively lower.

The influence of the strong coherent structures was also revealed in the turbulent characteristics. The turbulence intensity for both mean axial and radial velocity were found to be enhanced. Due to the fine scale vortices produced in the square jets, the turbulence intensity for the radial velocity kept growing in square jets, while maintaining constant in circular jets.

The orifice plate fixed at the exit of the nozzle was found to produce high initial pressure gradient that speeded up the head vortex traveling down and thus hastened the pinch-off process. The interaction between the leading vortex and the opposite-sign vortices created by the orifice plate accelerated the breaking down of the previous one. With the development of the jet, the pressure gradient diminished and the influence of the orifice plate on the steady trailing jets can be neglected.

The exit velocity profile displayed a thicker shear layer in the D-D plane that accelerated the corner portion of the leading vortex. The different velocities between the corner and flat part in a square vortex ring resulted in a 45° axis-switching. The Gaussian profile for the mean axial velocity in D-D plane attempt to become consistent with that in the S-S plane.

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Finally, a general note from the vortex evolution in starting jets is that the interaction between the vortices (i.e. leapfrogging) and the self deformation can lead to a sharp decay of the circulation and subsequently hasten the breaking down of the vortices.

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Table 5.1 Initial parameters and configurations for the PIV experiments.

Expts.	Initial velocity $\overline{W}_0$	X range of image (mm)	Z range of image (mm)	Interrogation area (Pixel)	Overlap (%)	Re	Time interval (ms)	Sampling rate (Hz)	Number of Recording
CNRe2358	65.5	31.9	32.1	32×32	75	2358	10	6.25	600
CORE2358	65.5	31.9	32.1	32×32	75	2358	10	6.25	600
CNRe3528	98.0	31.9	32.1	32×32	75	3528	8	6.25	600
CORE3528	98.0	31.9	32.1	32×32	75	3528	8	6.25	600
CNRe4716	131.0	31.9	32.1	32×32	75	4716	5	6.25	600
CORE4716	131.0	31.9	32.1	32×32	75	4716	5	6.25	600
CNRe7056	196.0	31.9	32.1	32×32	75	7056	4	6.25	600
CNRe9432	262.0	31.9	32.1	32×32	75	9432	4	6.25	600
SNRe2358	65.5	31.9	32.1	32×32	75	2358	10	5.00	300
SORE2358	65.5	31.9	32.1	32×32	75	2358	10	6.25	600
SNRe3528	98.0	31.9	32.1	32×32	75	3528	8	5.00	300
SORE3528	98.0	31.9	32.1	32×32	75	3528	8	6.25	600
SNRe4716	131.0	31.9	32.1	32×32	75	4716	5	6.25	600
SORE4716	131.0	31.9	32.1	32×32	75	4716	5	6.25	600
SNRe7056	196.0	31.9	32.1	32×32	75	7056	4	6.25	600
SNRe9432	262.0	31.9	32.1	32×32	75	9432	4	6.25	600

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Table 5.2 Nondimensional quantities for the starting jets.

Expts.	Γ_{V1} (cm ² /s)	Γ_{nd_V1}	Γ'_{nd_v1}	E_{nd_V1}	ε_{V1}	$t_{pinch-off}$ (s)	$t^*_{pinch-off}$	$W_{tr\_V1} / \overline{W}_0$	$W'_{tr\_V1} / \overline{W}_0$	$W_{tr\_V2} / \overline{W}_0$	$\overline{W}_{max} / \overline{W}_0$	$\overline{W}_{max}$ cm/s
CNRe2358	31	1.3	1.4	0.20	0.43	-	-	0.68	0.43	0.86	7.1	
CNRe3528	65	1.8	1.7	0.20	0.48	0.64	1.74	0.70	0.51	0.71	10.6	
CNRe4716	100	2.1	1.75	0.20	0.50	0.64	2.33	0.87	0.54	0.58	14.2	
SNRe2358	31	1.3	1.4	0.20	0.46	-	-	0.56	0.40	0.94	7.5	
SNRe3528	65	1.8	1.7	0.20	0.46	0.6	1.63	0.65	0.45	0.65	11.3	
SNRe4716	103	2.2	1.75	0.20	0.5	0.64	2.33	0.81	0.49	0.68	15.2	
CORE2358	30	1.3	-	0.20	0.48	-	-	0.85	-	1.15	7.2	
CORE3528	60	1.7	-	0.20	-	0.56	1.52	0.98	-	0.91	10.5	
CORE4716	97	2.1	-	0.20	-	0.56	2.04	1.00	-	0.93	13.8	
SORE2358	33	1.4	-	0.20	-	-	-	0.89	-	1.35	7.8	
SORE3528	66	1.9	-	0.20	-	0.48	1.31	0.96	-	1.06	11.7	
SORE4716	103	2.2	-	0.20	-	0.48	1.75	1.02	-	1.09	15.1	
Gharib <i>et al.</i> (1998)	28 ($R_e = 1905$) 60 ($R_e = 3810$)	-	2.2~2.7	0.33	-	-	≈ 4	-	-	-	-	
Rosenfeld <i>et al.</i> (1998)	-	1.85~2.61 ($R_e = 7500$)	-	-	-	-	0.9~5.22	-	-	-	-	
Mohseni <i>et al.</i> (1998)	-	-	1.8~2.84	0.11~0.37	-	-	-	-	-	-	-	

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Fig. 5.1 Schematic of measurement planes in square jets.

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Fig. 5.2 (i) Velocity field; (ii) vorticity contour; (iii) PLIF image for Case CNRe2358 at different time.

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Fig. 5.2(c) $t^* = 2.03$.

Fig. 5.2(d) $t^* = 2.62$.

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(i)

(ii)

(iii)

Fig. 5.2(e) $t^* = 3.20$.

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(i)

(ii)

(iii)

Fig. 5.2(f) $t^* = 4.95$.

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(i)

(ii)

(iii)

Fig. 5.2(g) $t^* = 6.70$.

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(iii)
Fig. 5.2(h) $t^* = 7.86$.

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(i)

(ii)

(iii)

Fig. 5.2(i) $t^* = 9.61$.

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Fig. 5.3 Temporal evolution of the circulation for total, V1 and V2 .

Fig. 5.4 Streamwise evolution of the vortex core center for V1 and V2 in Case CNRe2358

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Fig. 5.5 (i) Velocity field; (ii) vorticity contour; (iii) PLIF image for Case CNRe3528 at different time.

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(i)

(ii)

(iii)

Fig. 5.5 (c) $t^* = 5.23$.

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(i)

(ii)

(iii)

Fig. 5.5(d) $t^* = 6.96$.

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Fig. 5.5 (e) $t^* = 10.01$.

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Fig. 5.5 (f) $t^* = 10.89$.

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Fig. 5.5 (g) $t^* = 13.50$.

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Fig. 5.6 Streamwise evolution of the vortex core center for V1 and V2 in Case CNRe3528.

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(a) $t^* = 2.33$
 Fig. 5.7 (i) Velocity field; (ii) vorticity contour; (iii) PLIF image for Case CNRe4716 at different time.

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Fig. 5.7 (b) $t^* = 3.49$.

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Fig. 5.7 (c) $t^* = 4.66$

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Fig. 5.7 (d) $t^* = 5.82$.

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(i)

(ii)

(iii)

Fig. 5.7 (e) $t^* = 7.28$.

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Fig. 5.7 (f) $t^* = 9.90$.

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Fig. 5.8 Streamwise evolution of the vortex core center for V1 and V2 in Case CNRe4716

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Fig. 5.9 (i) Velocity field; (ii) vorticity contour; (iii) PLIF image for Case SNRe2358 at different time.

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Fig. 5.9 (c) $t^* = 2.91$.

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Fig. 5.9 (d) $t^* = 4.37$.

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(i)

(ii)

(iii)

Fig. 5.9 (e) $t^* = 5.24$.

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(i)

(ii)

(iii)

Fig. 5.9 (f) $t^* = 5.46$.

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Fig. 5.9 (g) $t^* = 6.19$.

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Fig. 5.9 (h) $t^* = 7.64$.

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(iii)

(i) $t^* = 9.09$

Fig. 5.9 continued.

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Fig. 5.9 (j) $t^* = 9.82$.

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Fig. 5.9 (k) $t^* = 13.83$.

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Fig. 5.10 Temporal evolution of the circulation for total, V1 and V2.

Fig. 5.11 Streamwise evolution of the vortex core center for V1 and V2 in Case SNRe2358

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Fig. 5.12 Temporal evolution of the centerline axial velocity corresponding to Fig. 5.9.

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Fig. 5.13 Trajectory of V1 and V2 in Case SNRe2358 and CNRe2358

Fig. 5.14 Snap-shot of cross-sectional views in a plane parallel to the exit plane at $Z/D_e = 0.5$ for the Case of SNRe2358 (taken at $t^* = 2.58$).

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Fig. 5.15 (i) Velocity field; (ii) vorticity contour; (iii) PLIF image for Case SNRe3528 at different time.

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(iii)
Fig. 5.15 (c) $t^* = 5.44$.

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(i)

(ii)

(iii)

Fig. 5.15 (d) $t^* = 8.71$.

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Fig. 5.15 (e) $t^* = 10.89$.

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Fig. 5.15 (f) $t^* = 14.70$.

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Fig. 5.16 Temporal evolution of the centerline axial velocity corresponding to Fig. 5.15.

Fig. 5.17 Streamwise evolution of the vortex core center for V1 and V2 in Case SNRe3528

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Fig. 5.18 (i) Velocity field; (ii) vorticity contour; (iii) PLIF image for Case SNRe4716 at different time.

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Fig. 5.18 (c) $t^* = 4.08$.

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Fig. 5.18 (d) $t^* = 6.40$.

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Fig. 5.18 (e) $t^* = 7.59$.

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Fig. 5.18 (f) $t^* = 11.06$.

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Fig. 5.19 Temporal evolution of the centerline axial velocity corresponding to Fig. 5.18.

Fig. 5.20 Streamwise evolution of the vortex core center for V1 and V2 in Case SNRe4716.

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Fig. 5.21 Development of the normalized streamwise mean velocity along the jets centerline for Lower Reynolds number cases.

Fig. 5.22 Development of the normalized streamwise mean velocity along the jets centerline for Higher Reynolds number cases.

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Fig. 5.23 Normalized exit streamwise velocity profile (measured at $Z/D_e = 0.22$).

(a) Case CNRe2358

(b) SNRe2358

Fig. 5.24 Trailing jets (taken at $t = 20s$).

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Fig. 5.25 Cross-sectional variation of mean axial velocity for Case CNRe2358.

Fig. 5.26 Cross-sectional variation of mean axial velocity with Gaussian fit for Case CNRe2358.

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Fig. 5.27 Cross-sectional variation of mean axial velocity for Case CNRe4716.

Fig. 5.28 Cross-sectional variation of mean axial velocity with Gaussian fit for Case CNRe4716.

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Fig. 5.29 Cross-sectional variation of mean axial velocity for Case SNRe2358.

Fig. 5.30 Cross-sectional variation of mean axial velocity with Gaussian fit for Case SNRe2358

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Fig. 5.31 Cross-sectional variation of mean axial velocity for Case SNRe4716.

Fig. 5.32 Cross-sectional variation of mean axial velocity with Gaussian fit for Case SNRe4716.

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Fig. 5.33 Turbulence intensity of the axial velocity along jet's centerline (for circular cases).

Fig. 5.34 Turbulence intensity of the radial velocity along jet's centerline (for circular cases).

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Fig. 5.35 Turbulent intensity of the axial velocity along jet's centerline (for square cases).

Fig. 5.36 Turbulent intensity of the radial velocity along jet's centerline (for square cases).

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Fig. 5.37 Relation between the formation number (L/D) and the parameter of Norbury's family of vortices ε (taken from Mohseni and Gharib, 1998).

Fig. 5.38 Variation of nondimensional energy with time for circular cases

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Fig. 5.39 Variation of nondimensional energy with time for square cases.

Fig. 5.40 The evolution of the discharge velocity for the Case CNRe3528 (measured at $Z/D = 0.36$).

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Fig. 5.41 Temporal evolution of the axial velocity profile in the plane through the vortex core center of V1 for Case CNRe2358.

Fig. 5.42 Axial velocity profile at different cross section at $t^* = 2.62$ for Case CNRe2358.

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Fig. 5.43 Temporal evolution of the centerline axial velocity for Case CNRe2358.

Fig. 5.44 Temporal evolution of the centerline axial velocity for Case CNRe3528.

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Fig. 5.45 Temporal evolution of the centerline axial velocity for Case CNRe4716.

Fig. 5.46 Temporal evolution of the circulation for total and V1 in circular orifice cases.

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Fig. 5.47 Temporal evolution of the circulation for total and V1 in circular orifice cases.

Fig. 5.48 Variation of non-dimensional energy with time for circular orifice cases.

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Fig. 5.49 Variation of non-dimensional energy with time for square orifice cases.

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Fig. 5.50 Representation of the evolution of the negative vortices for Case COr2358.

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Fig. 5.51 Temporal evolution of the axial velocity profile in the plane through the vortex core center of V1, corresponding to Fig. 5.51.

Fig. 5.52 Head vortex core trajectory.

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Fig. 5.53 Development of the normalized streamwise mean velocity along the jets centerline.

Fig. 5.54 Turbulent intensity of the axial velocity along jet's centerline for orifice cases.

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Fig. 5.55 Turbulent intensity of the radial velocity along jet's centerline for orifice cases.

Fig. 5.56 Temporal evolution of the circulation in square jets (D-D plane).

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Fig. 5.57 Variation of nondimensional energy with time in square jets (D-D plane).

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(a) $t^* = 1.31$

(b) $t^* = 1.74$

(c) $t^* = 2.18$

(d) $t^* = 2.61$

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(e) $t^* = 3.05$

(f) $t^* = 3.48$

Fig. 5.58 Vortex evolution in D-D plane for the case SNRe3528.

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Fig. 5.59 Temporal evolution of the discharge velocity for the case SNRe3528 (D-D plane, measured at $Z/D_e = 0.15$).

Fig. 5.60 Cross-sectional variation of mean axial velocity for Case SNRe2358 (D-D Plane)

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Fig. 5.61 Cross-sectional variation of mean axial velocity for Case SNRe4716 (D-D Plane)

Fig. 5.62 Cross-sectional of mean axial velocity with Gaussian fir for Case SNRe2358 (D-D plane)

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Fig. 5.63 Cross-sectional of mean axial velocity with Gaussian fit for Case SNRe4716 (D-D plane)

Fig. 5.64 Illustration of the components of a vortex bubble in starting jets. The PLIF image was taken in Case C2 at $t^* = 3.32$.

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Fig. 5.65 Vortex rings energy, total energy, gravity energy and overpressure energy for Case C1.

Fig. 5.66 Vortex rings energy, total energy, gravity energy and overpressure energy for Case C2.

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Fig. 5.67 Vortex rings energy, total energy, gravity energy and overpressure energy for Case C3.

Fig. 5.68 Variation of E_p/E_{total} .

Chapter 6 BOUSSINESQ AND NON-BOUSSINESQ STARTING FORCED PLUMES

6.1 Introduction

Starting forced plumes are common phenomena in nature with a wide spectrum of physical scales from the releases of hot ash during volcanic eruptions to the ejection of chemical gas plumes at the sea bed. The knowledge is needed to analyze these natural phenomena as well as for analysis in situation such as interrupted gas and liquid releases caused by process upsets, explosions and unwanted fires (Sangras *et al.* 1999). Quantifying the mixing and entrainment of the starting forced plumes is thus of substantial engineering and environmental interests.

A forced plume, or buoyant jet, possesses both initial momentum and buoyancy fluxes. A function of the dimensionless source parameters,

$$\Lambda = \frac{5Q_0^2 B_0}{4\alpha M_0^{5/2}}, \quad (6.1)$$

where α represents the entrainment coefficient, can be used to categorize the buoyant plumes as suggested by Morton (1959). He defined the pure plumes as $\Lambda = 1$, the forced plumes to be those with the initial momentum flux greater than that of an equivalent pure plume of the same initial buoyancy flux (i.e. $0 < \Lambda < 1$), and lazy plumes as $\Lambda > 1$. The Λ values shown in Table 6.2 indicate that the experiments conducted in the present study were all with forced plumes.

A starting forced plume is driven by both initial buoyancy and momentum in a manner similar to a steady buoyant jet. However, the behavior of a starting forced plume is substantially more

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complex due to the dynamic interaction between the head vortex and the trailing buoyant jet stem. A synthesis of recent literatures (e.g. Gharib *et al.* 1998, Ai *et al.*, 2005a) shows that there are two distinct periods in the temporal development of a starting forced plume that can be termed the Period of Flow Development (PFD) and Period of Developed Flow (PDF), in analogous with the ZFE and ZEF that are well known for a steady buoyant jet (ref. Section 1.2). The schematic diagram for a starting forced plume is shown in Fig. 6.1. As shown in Fig. 6.1, PFD represents the initial phase after the fluids ejection, whereby the exit conditions generate longitudinal and azimuthal vortices and their interactions lead to complex vortical dynamics such as the occurrence of vortex coalescence and leapfrogging (Ai *et al.*, 2005a). When the circulation of the head vortex reaches its maximum, the head vortex would then detach from the trailing plume stem. The detachment is known as the pinch-off phenomenon (Gharib *et al.*, 1998). The detached head vortex and the trailing stem would however reconnect after some distance, at which point the PFD ends and the starting forced plume becomes fully-developed independent of the exit conditions. Noticeably, because of the pinch-off and reconnection, an over-shoot in penetration is typically observed in PFD (Ai *et al.*, 2005a). Figure 6.2 shows a sketch of the processes in PFD.

The plume characteristics in PDF are relatively simpler and depend primarily on the feeding of momentum and buoyancy into the head vortex by the trailing stem. Again, analogous to the interplays between initial and buoyancy-induced momentum in a steady buoyant jet, a fully developed forced plume would also experience three stages of behavior during its vertical rise: jet-like, transitional and plume-like. The three stages are depicted in Fig. 6.1. In the asymptotic jet-like and plume-like regions, the starting forced plume is dominated by initial momentum and buoyancy, respectively, and its behaviors are comparable with a starting pure jet and pure

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plume, respectively. In the transitional region, however, the forced plume is influenced by both the initial momentum and buoyancy. Note that, depending on the initial momentum, the developing phase of the starting forced plume can overwhelm the jet-like portion of the transitional region, such that a fully developed forced plume can directly enter into the transitional and plume-like behavior in PDF.

The objective of the present study is to extend the previous models developed, for example, by So & Aksoy (1993), Turner (1962) and Wang & Law (2002) to the penetration mode for starting forced plume. Subsequently, experiments were conducted to verify the analytical model utilizing combined PIV and PLIF with refractive index matching. The scope of the experiments included a density difference, Δ_0 , of up to 25% and thus also covered cases of non-Boussinesq forced plumes ($\Delta_0 > 15\%$) in addition to the Boussinesq forced plumes. Note that the cutoff of 15% was only set approximately as a guide for the validity of Boussinesq approximation based on small density differences. The results and observations of the non-Boussinesq cases will also be discussed, and it will be shown that the plume behaviors are distinctly different in these higher density difference cases.

In the following, the current understanding on the behavior of isolated vortex rings and buoyant jets will be firstly reviewed. Subsequently, their coupling dynamics for a starting forced plume is addressed. Some of the results in this chapter were reported in Ai *et al.* (2006).

6.2 Previous studies on the starting plume

Previous experimental, theoretical and computational studies focus mostly on either starting jets or starting plumes. They include Turner (1962), Delichatsios (1979), Sangras *et al.* (1999),

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Pantzlaff & Lueptow (1999) and Diez *et al.* (2003). For starting jets, it has been shown that the transient jet penetration velocity, u_t , is inversely proportional to the square root of time (Witze 1980, Johari *et al.* 1997, Hill & Ouellette 1999, Cossali *et al.* 2001 and Ai *et al.* 2005a). On the other hand, Middleton (1975) suggested that the starting plume penetration has a smaller time exponent of -1/4 based on Turner's (1962) model (i.e. $u_t \propto B_0^{1/4} t^{-1/4}$, where B_0 and t are initial buoyancy flux and time, respectively). The $1/4$ time behavior for starting plume is also supported by the recent analysis of Diez *et al.* (2003). Delichatsios (1979) further reported that the advancing velocity at the leading edge of the plume is less than the velocity in the plume interior with a maximum ratio of 0.42, although both bear the same temporal development.

Comparatively, there have been very few studies on starting forced plumes. Plantzlaff and Lueptow (1999) reported experimentally that positively buoyant jets would have similar dependence on time as the pure jet at least in the momentum-dominated region. Diez *et al.* (2003) proposed a time scale $\bar{t}' = (D^4 / B_0)^{1/3}$ (D is the diameter of the round exit port). They suggested that the penetration distance normalized by D is proportional to the time normalized by this time scale with a power of $3/4$ (cf. Section 6.4). However, it should be noted that these previous works used a virtual origin for a bulk correction of the non-self-preserving behaviors, and did not include a detailed description of the various developmental stages of the plume penetration as performed here.

As discussed previously, the dynamics of a starting forced plume in PDF relates directly to the coupling behavior between the head vortex and the trailing stem of buoyant jet. In terms of the individual behavior of the two phenomena, there has been substantial knowledge available in

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the recent literature. On the head vortex, much is now known on the characteristics of momentum puffs and thermals (without and with buoyancy, respectively) in a neutral surrounding (Turner 1957, Maxworthy 1977, Saffman 1975, 1978, Didden 1979, Pullin 1979, Glezer 1988 and Lundgren & Mansour 1991). The comprehensive reviews of Shariff & Leonard (1992) and Lim and Nickels (1995) discussed the current understanding of vortex ring phenomena as well as some unresolved issues. For the trailing buoyant jets, steady-state Boussinesq characteristics had been investigated in great details (e.g. Wyganski & Fiedler 1969, Morton *et al.* 1956, and Fischer *et al.* 1979) and quantitative relationships for the velocity and scalar behavior are now available with high accuracy (Papanicolaou & List 1988, So & Aksoy, 1993, Shabbir & George 1994 and Wang & Law 2002).

Here, it should be noted here that the research literature on non-Boussinesq plumes is very limited. Rooney and Linden (1996) derived the theoretical form of entrainment velocity across the plume edge which had been postulated from the experimental observations by Ricou & Spalding (1961). Subsequently, their model was applied to different fire experiments in Rooney and Linden (1997) and Woods (1997). Zhou *et al.* (2001) performed a detailed simulation on the non-Boussinesq plumes with two different large density ratios of 2.0 and 3.0.

6.3 Isolated vortex ring

Depending on the developmental stage, the cap of a starting forced plume can be considered as a vortex ring that is similar to a momentum puff (jet-like) or thermal (plume-like). According to Shariff & Leonard (1992), two main theoretical approaches exist that quantify the ring structure with the generation parameters. The first approach is based on the self-similar rollup of the vortex sheet. It attempts to account for the vortex sheet evolution during

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ejection and gives insight into the core structure. This approach is accurate but difficult to implement as detailed knowledge of the complex vortex evolution is required. The second approach is known as the slug-flow model. It estimates the circulation by adopting some empirical corrections, and is an easier choice for model simulation at present. the slug-flow model will be employed to study the “bulk” motion of the head vortex.

For a momentum puff, the turbulent vortex ring can be described as a compact turbulent region with a net linear momentum that is statistically invariant with respect to the rotation about a certain axis. The typical axis chosen is the azimuthal direction (Shariff and Leonard 1992). Helmholtz (1858) proposed a method for computing the translational speed of thin rings without knowing the exact ratio of a/R (where a and R are the characteristic ring core size and ring radius, respectively) which was also used by Kelvin (1880) to obtain the speed of a thin steady ring with uniform vorticity. The method is described as follows.

The impulse momentum of all the fluid moving with a vortex ring reads (Lamb 1932),

$$M_v = \pi \rho_v \Gamma R^2, \quad (6.2a)$$

where Γ is the circulation, ρ_v is the mean density of the thermal or vortex ring and R is the ‘mean ring radius’ defined by Lamb (1932) as (denoted in Fig. 6.1(iii)),

$$R = \left(\frac{\int_0^\infty \int_0^\infty r^2 \omega dr dh}{\int_0^\infty \int_0^\infty \omega dr dh} \right)^{1/2} \quad (6.3)$$

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where r and h are the radial and vertical coordinates, respectively, and ω is the local vorticity. Assuming that the distribution of vorticity in the cap remains similar at all heights, the velocity of the vortex centre, u_c , can be expressed as

$$u_c = C_1 \Gamma / R, \quad (6.4)$$

where C_1 is a constant. The value for C_1 should differ between a momentum puff and a thermal, although their order of magnitude is probably similar. It is convenient for the present purpose to eliminate Γ from (6.4) and (6.2a) to give,

$$M_v = (\pi \rho_v / C_1) R^3 u_c. \quad (6.2b)$$

Turner (1957) suggested that a thermal can be regarded as a special case of a buoyant vortex ring. For an isolated buoyant vortex ring, the increase in momentum is solely due to the total buoyancy force B_v . Hence, the time rate of change of momentum can be expressed as

$$\frac{dM_v}{dt} = \rho_\infty B_v. \quad (6.5)$$

where ρ_∞ is the density of the ambient fluid. The laboratory experiments of Sangras *et al.* (1999) showed that a thermal consists of a region of turbulent buoyant fluid with its radius expanding linearly with distance as the thermal mixes with its surroundings. The addition of outside fluid takes place partly by mixing over the thermal front and partly by drawing up fluid from behind. In this sense, it can be said that a thermal leaves no wake behind.

Overall, the head vortex can be consider as a buoyant ring vortex that is growing by the turbulent entrainment of the external ambient fluid and the trailing buoyant jet.

6.4 Trailing buoyant jet

The starting forced plume possesses a trailing stem that with time would develop into a steady buoyant jet. Similarity solutions for steady buoyant jets were first investigated by Morton *et al.* (1956), using conservation equations representing the mass, momentum and buoyancy. The common assumption is that the profiles of mean velocity and density differences are Gaussian with length scales b_w and b_c , and maximum centerline values W_c and Δ_c , respectively, all being functions of the height Z above a virtual origin. Thus, the cross-sectional variations of the mean axial velocity W and density deficiency Δ can be expressed as,

$$\frac{W}{W_c} = \exp\left[-\left(\frac{r}{b_w}\right)^2\right], \quad (6.6)$$

$$\Delta = g(\rho_\infty - \rho) / \rho_\infty = \Delta_c \exp\left[-\left(\frac{r}{b_c}\right)^2\right], \quad (6.7)$$

where ρ the local density of the buoyant jet, and g the gravitational acceleration.

The volume flux, Q , can be integrated using the Gaussian velocity profile (Eq. 6.6) and evaluated over a cross-section as $\pi b_w^2 W_c$. If this volume flux is considered to be distributed uniformly over a cross-sectional area of πb_w^2 , then W_c can be taken as the average value over the cross-section rather than the maximum value. In other words, W_c can also act as a velocity scale (Middleton 1975).

Morton *et al.* (1956) proposed the entrainment principle, namely, that the rate of increase of volume flux per unit height is equal to $2\pi b_w \alpha W_c$, where α is the entrainment coefficient.

The conservation equations over a particular cross-section can then be written as:

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$$d(b_w^2 W_c) / dZ = 2\alpha b_w W_c, \quad (6.8a)$$

$$d(b_w^2 W_c^2) / dZ = 2 b_c^2 \Delta_c, \quad (6.8b)$$

$$dB / dZ = 0. \quad (6.8c)$$

where the specific buoyancy flux, B , is

$$B = \frac{\pi b_w^2 b_c^2}{b_w^2 + b_c^2} W_c \Delta_c = B_0 \quad (6.9)$$

For a round port with a diameter of D , an exit velocity W_0 and an average initial density ρ_0 ,

B_0 can be defined as,

$$B_0 = \frac{1}{4} \pi D^2 W_0 \frac{\rho_0 - \rho_\infty}{\rho_\infty} g = \frac{\Delta \rho_0}{\rho_\infty} g Q_0 = \Delta_0 Q_0. \quad (6.10)$$

An important assumption in this approach is that the local variations in buoyancy are small, such that the profiles of mean vertical velocity and mean buoyancy are of similar form at all cross-sections; and that the Reynolds number similarity holds for the turbulent fluid within the plume (Turner 1969). The assumption is equivalent to the so-called Boussinesq assumption. The model provides serviceable results for a buoyant jet, but it cannot be applied to analyze the exit region near the origin of the forced plume where the profiles are developing and not self-similar.

A detailed experimental investigation on the quantitative characteristics of a vertical buoyant jet was performed by Wang and Law (2002) using combined PIV and PLIF. The investigation covered the three stages of a developed steady buoyant jet from jet-like, transitional to plume-like. Based on the experimental results, they reported that the decay of the centerline velocity over the developed range is as follow:

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$$\log\left(\frac{\sqrt{M_0}}{ZW_c}\right) = g(\xi), \quad (6.11a)$$

where M_0 is the initial momentum flux ($M_0 = \frac{1}{4}\pi D^2 W_0^2$ for a round port) and

$$\begin{aligned} g(\xi) = & -\frac{1}{3}[1 + \tanh(k_1\xi - \eta_1)]\xi + \log\left(\frac{1}{k_{pw}}\right) \\ & + \frac{1}{2}\left[\log\left(\frac{1}{k_{pw}}\right) - \log\left(\frac{\sqrt{\pi}}{2k_{jw}}\right)\right][\tanh(k_2\xi - \eta_2) - 1], \\ & - k_4 \exp[-k_3(\xi - \eta_3)^2] \end{aligned} \quad (6.11b)$$

$$\xi = \log\left(\frac{Z}{L_m}\right), \quad (6.11c)$$

where L_m is the Morton scale which is defined as (Morton 1959, List 1982):

$$L_m = \frac{M_0^{3/4}}{B_0^{1/2}}. \quad (6.12)$$

L_m is pertinent to buoyant flows and measures the distance needed for the buoyancy-induced momentum to dominate over the momentum of the source flow.

The constants in Eq. (6.11b) were determined by the experiments in Wang and Law (2002) as:

$$k_1 = 3.110, \quad k_2 = 2.815, \quad k_3 = 6.123, \quad k_4 = 0.0797, \quad \eta_1 = 1.831, \quad \eta_2 = 2.067,$$

$$\eta_3 = 0.4529, \quad k_{jw} = 6.477 \text{ and } k_{pw} = 4.13. \quad \text{These constants define quantitatively the self-}$$

similar behaviour of the centreline velocity for the entire range. Equation (6.11a)

automatically satisfies the two asymptotic cases of buoyant jets, i.e. pure jets and pure plumes,

as demonstrated in the following.

For jet-like behaviour, $\xi \rightarrow -\infty$, Eq. (6.11b) reduces to:

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$$g(\xi) = \log\left(\frac{\sqrt{\pi}}{2k_{jw}}\right), \quad (6.13)$$

which is equivalent to the well-known equation for pure jets (Fischer *et al.* 1979), i.e.

$$W_c = k_{jw} W_0 (Z / D)^{-1}, \quad (6.14)$$

For plume-like, $\xi \rightarrow +\infty$ and Eq. (6.11b) becomes:

$$g(\xi) = -\frac{2}{3}\xi + \log\left(\frac{1}{k_{pw}}\right), \quad (6.15)$$

which is identical to the asymptotic behaviour for the centerline velocity of a pure plume (Fischer *et al.* 1979), i.e.

$$W_c = k_{pw} B_0^{1/3} Z^{-1/3}. \quad (6.16)$$

Wang and Law (2002) further suggested the ratio of concentration to velocity width,

$\lambda = b_c / b_w$, is related to the Richardson number Ri as:

$$\lambda = b_c / b_w = \lambda_j - (\lambda_j - \lambda_p) \left(\frac{Ri}{Ri_p} \right)^{1.5}, \quad (6.17)$$

where the subscripts j and p denote the corresponding variables for jet and plume,

$Ri_p = 0.584$, $\lambda_j = 1.23$ and $\lambda_p = 1.04$. For a round buoyant jet, Ri can be defined as:

$$Ri = \frac{QB^{1/2}}{M^{5/4}} = \frac{\frac{\pi 10^{-g(\xi)} M_0^{1/2} Z}{k_{wg}} B_0^{1/2}}{\left(k_M \frac{\pi 10^{-2g(\xi)} M_0}{2k_{wg}} \right)^{5/4}} = 2^{5/4} \pi^{-1/4} k_M^{-5/4} k_{wg}^{1/4} 10^{3g(\xi)/2} \frac{Z}{L_m}, \quad (6.18)$$

where $k_M = 1.1$ and $k_{wg} \cong 90$. In other words, λ can be related directly with Z / L_m .

6.5 Penetration rate of a starting forced plume

In this section, a model for the penetration rate of a starting forced plume will be developed. The model spans the entire developed range by quantifying the linkage between the head vortex and the trailing buoyant jet stem.

As mentioned previously, the original and pioneering conceptualization should be attributed to Turner (1962) who proposed that a starting plume can be modeled as a slightly flattened spherical vortex at the head of a quasi-steady plume (see Fig. 6.1). In Turner's original model, it was assumed that the trailing plume feeds mass, momentum and circulation into the spherical head vortex, and that the velocity of the head vortex, u_b at the base of the vortex is a constant fraction, A_b , of the mean velocity of the trailing plume at the base of the vortex, W_{cb} , i.e.

$$A_b = u_b / W_{cb} = \text{constant} , \quad (6.19a)$$

With this conceptualization, the velocity at the center and front of the head vortex, u_c and u_t , respectively, will also be proportional to the plume velocity at the equivalent height, W_{cc} and W_{ct} . Using A_c and A_t to denote the ratios correspondingly,

$$A_c = u_c / W_{cc} , \quad (6.19b)$$

$$A_t = u_t / W_{ct} . \quad (6.19c)$$

These ratios, A_b , A_c and A_t were all assumed constant throughout the evolution of the starting plume.

In reality, as the starting forced plume enters into the different regimes as shown in Figure 6.1(b), the ratios will vary because of the change in the characteristics of the head vortex from

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puff to thermal and the trailing buoyant jet from jet-like to plume-like. The constant assumption can be relaxed and compute the ratios directly from the conservation equations.

The variations in A_b , A_c and A_t can be determined by considering the centerline velocity decay, Equation (6.11a), and the rate of growth of the cap radius. Taking $\alpha' = dr_v / dZ$ as the semi-spread angle of the visible cap edge, the relationships for the ratios can be converted to

$$A_b = A_c (1 - \alpha')^2 10^{-g(\xi_c) + g(\xi_b)}, \quad (6.20a)$$

$$A_t = A_c (1 + \alpha')^2 10^{-g(\xi_c) + g(\xi_t)}, \quad (6.20b)$$

where $\xi_c = \log(Z_c / L_m)$, $\xi_b = \log(Z_b / L_m)$.

For a pure jet or pure plume, Eqs., 6.20a and 6.20b can be simplified to

$$A_b = A_c (1 - \alpha')^2, A_t = A_c (1 + \alpha')^2 \text{ for trailing pure jet,} \quad (6.21a)$$

$$A_b = A_c (1 - \alpha')^{4/3}, A_t = A_c (1 + \alpha')^{4/3} \text{ for trailing pure plume.} \quad (6.21b)$$

The flux of buoyancy through the base of the head vortex can be computed as (see Fig. 6.1(iii))

$$dB_v / dt_v = B_0 (1 - A_b), \quad (6.22)$$

where t_v represents the time for the the head vortex to reach the location from its source.

The velocity of the trailing buoyant jet stem at the base of the head vortex is faster than the vortex by a factor of A_b at any particular time. Thus, if t represents the rise time for the trailing buoyant jet to reach from its source, we have

$$t = A_b t_v. \quad (6.23)$$

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The vortex length scale R bears a constant relationship with the Gaussian velocity width of the trailing buoyant jet (Turner 1962). This relationship can be expressed as

$$R = C_2 b_w. \quad (6.24)$$

For pure plumes, Turner (1962) suggested that $C_2 = 1.2$.

The rate of change of the head vortex momentum can be expressed as:

$$dM_v / dt_v = \rho_\infty B_v + \rho \frac{1}{2} \pi b_w^2 W_{cb}^2 (1 - A_b), \quad (6.25)$$

where ρ can be considered as the average density at the base of the head vortex. The first and second terms of the RHS of Eq. (6.25) represent the action of the buoyant force and the rate of injection of momentum from the trailing buoyant jet, respectively.

The buoyancy flux of the head vortex can be calculated by integrating Eq. 6.22, to give,

$$B_v = \int_0^{t_v} B_0 (1 - A_b) dt_v'. \quad (6.26)$$

Utilizing the behavior of steady buoyant jets (Eq. 6.11a) and replacing b_w with $\eta_{wg} Z$, Eq. (6.25) can be integrated to become

$$M_v = \rho_\infty B_0 \int_0^{t_v} \left(\int_0^{t_v'} (1 - A_b) dt_v' \right) dt_v'' + \frac{\pi}{2} \rho \eta_{wg}^2 M_0^{1/2} \int_0^{Z_b} (1 - A_b) / A_b 10^{-g(\xi_b)} Z_b' dZ_b'. \quad (6.27)$$

Incorporating Eq. (6.4) into (6.27) yields,

$$\begin{aligned} (\pi \rho_v / C_1) R^3 u_c = \rho_\infty B_0 \int_0^{t_v} \left(\int_0^{t_v'} (1 - A_b) dt_v' \right) dt_v'' \\ + \frac{\pi}{2} \rho \eta_{wg}^2 M_0^{1/2} \int_0^{Z_b} (1 - A_b) / A_b 10^{-g(\xi_b)} Z_b' dZ_b'. \end{aligned} \quad (6.28)$$

Thus, the relationship between Z_b and t_v can be computed as:

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$$t_v = \int_0^{Z_b} \frac{1}{u_b} dZ'_b = \int_0^{Z_b} \frac{dZ'_b}{A_b W_{cb}}. \quad (6.29)$$

Replacing t_v , R and u_c with Z_b , b_w and W_{cb} , respectively, Eq. (6.28) can be transferred to,

$$\left(\frac{\pi \rho_v}{C_1} \right) C_2^3 \eta_{wg}^3 \frac{A_b}{1-\alpha'} 10^{-g(\xi_b)} \bar{Z}_b^2 = \rho_\infty B_0 \int_0^{\bar{Z}_b} \left(\int_0^{\bar{Z}_b} (1-A_b) \frac{Z'_b dZ'_b}{A_b 10^{-g(\xi_b)}} \right) \frac{Z_b'' dZ_b''}{A_b 10^{-g(\xi_b)}} + \frac{\pi}{2} \rho \eta_{wg}^2 M_0^{1/2} \int_0^{\bar{Z}_b} (1-A_b) / A_b 10^{-g(\xi_b)} Z'_b dZ'_b, \quad (6.30)$$

where $\bar{Z}_b = Z_b / L_m$.

For Boussinesq starting forced plumes, $\rho_v \approx \rho_\infty \approx \rho$, Eq. (6.30) can then be written as

$$\left(\frac{\pi}{C_1} \right) C_2^3 \eta_{wg}^3 \frac{A_b}{1-\alpha'} 10^{-g(\xi)} \bar{Z}_b^2 = B_0 \int_0^{\bar{Z}_b} \left(\int_0^{\bar{Z}_b} (1-A_b) \frac{Z'_b dZ'_b}{A_b 10^{-g(\xi_b)}} \right) \frac{Z_b'' dZ_b''}{A_b 10^{-g(\xi_b)}} + \frac{\pi}{2} \eta_{wg}^2 M_0^{1/2} \int_0^{\bar{Z}_b} (1-A_b) / A_b 10^{-g(\xi_b)} Z'_b dZ'_b. \quad (6.31)$$

Taking the values of $C_1 = 0.14$, $C_2 = 1.2$ from Turner (1962), and $\eta_{wg} = 0.105$ from Wang and Law (2002), and with $\alpha' = 0.16$ obtained in the present experiments (further discussion will be provided in Section 6.1.2), the value of A_b can be computed from Eq. (6.31) using iteration methods.

Figure 6.2 shows the results of the iterations. It can be seen that the value of A_b starts at 0.33 in the jet-like region, increases throughout the transition and then reaches a steady value of 0.418 when $Z / L_m > 30$. In other words, the head vortex is being injected with a progressively larger amount of mass and momentum by the trailing stem as the forced plume enters into the jet-like, transitional and plume-like regimes consecutively. It is important to note that Turner (1962) obtained a measured value of 0.38 for A_b which is approximately in

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the middle of the computed range (0.33~0.418). The value of 0.418 agrees excellently with the maximum theoretical value (0.42) derived by Delichatsios (1979) in his analytical work on starting plumes. His theoretical value was obtained by applying a shock (jump) condition at the base of the vortex with the value of buoyancy after the shock assumed to be 0. In reality, the shock would not be so abrupt and thus the value of A_b can only approach 0.42 asymptotically.

The computational results in Fig. 6.3 in the asymptotic regimes can be verified. The rate of change of momentum for the head vortex as a momentum puff (Eq. 6.25) can be simplified to

$$dM_v / dt_v = \frac{\pi}{2} \rho b_w^2 W_{cb}^2 (1 - A_b), \quad (6.32)$$

Substituting Eq. (6.14) into (6.32), we obtain

$$\int_0^{\bar{Z}_b} \frac{1 - A_b}{2A_b} Z_b' dZ_b' = \frac{C_2^3}{C_1} \frac{A_b}{1 - \alpha'} \eta_{wg} \left(\bar{Z}_b \right)^2, \quad (6.33)$$

From the above equation, A_b can be solved analytically as,

$$A_b = \frac{-1 + \sqrt{\frac{16\eta_{wg} C_2^3}{C_1(1 - \alpha')} + 1}}{\frac{8\eta_{wg} C_2^3}{C_1(1 - \alpha')}} = 0.33, \quad (6.34)$$

The value of 0.33 is identical to the value determined by the iteration methods shown in Fig. 6.3. With the same reasoning, replacing $g(\xi)$ in Eq. (6.31) with the results of a steady pure plume (Eq. 6.15), A_b can be obtained as a constant with the magnitude of 0.418.

With the quantification of A_b , the velocity at the base of the head vortex can be determined as:

$$u_b = A_b W_{cb} = A_b 10^{-g(\xi_b)} Z^{-1} M_0^{1/2}, \quad (6.35)$$

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Substituting the results of the buoyant jet velocity (Eq. 6.35) into Eq. (6.29), the penetration rate for a starting Boussinesq forced plume can finally be obtained as,

$$\bar{t}_v = \int_0^{\bar{Z}_b} \frac{Z'_b dZ'_b}{A_b 10^{-g(\xi_b)}} = \int_0^{\bar{Z}_t} \frac{Z'_t (1 - \alpha') / (1 + \alpha') d[Z'_t (1 - \alpha') / (1 + \alpha')]}{A_b 10^{-g(\xi_b)}}, \quad (6.36)$$

where $\bar{t}_v = t_v B_0 / M_0$ and $\bar{Z}_t = Z_t / L_m$ are normalized traveling time and distance for the front of the head vortex, respectively.

The penetration behavior described by Eq. (6.36) is plotted in Fig. 6.4. The penetration time derived in Eq. (6.36) covers the various regimes in PDF. For verification, the predictions of the penetration time to the measurements of starting jets and starting plumes reported in the literature will be compared now.

For jet-like behavior, Eq. (6.36) becomes,

$$\bar{t}_v = \int_0^{\bar{Z}_b} \frac{Z'_b dZ'_b}{A_b k_{jw}} = \frac{1}{2 A_b k_{jw}} \left[\bar{Z}_t (1 - \alpha') / (1 + \alpha') \right]^2, \quad (6.37)$$

where $\bar{t}_v = t_v W_0 / D$ and $\bar{Z}_t = Z_t / D$. Utilizing 0.33 and 6.477 to replace the constants A_b and k_{jw} , respectively, Eq. (6.37) gives

$$\bar{Z}_t = 2.853 (\bar{t}_v)^{1/2}. \quad (6.38)$$

Sangras *et al.* (2003) obtained a penetration constant of 2.6 in their experiments on starting jets which is approximately 10% less than the prediction in Eq. (6.38), while Lahbabi *et al.* (1993) measured a value of 2.9 and Witze (1980) from 2.5 to 3.2 which is nearly identical to the present analytical results. Note that Sangras *et al.* used a video-based technique for their

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laboratory measurements and thus it can be expected that the accuracy might be lower compared to the other more advanced imaging techniques such as PLIF.

For plume-like behavior, Eq. (6.36) simplifies to,

$$\bar{t}_v = \int_0^{\bar{Z}_b} \frac{(Z'_b)^{1/3} dZ'_b}{A_b k_{pw}} = \frac{3}{4} \frac{1}{A_b k_{pw}} [\bar{Z}_t (1 - \alpha') / (1 + \alpha')]^{4/3}. \quad (6.39)$$

Substituting the constants ($A_b = 0.418$ and $k_{pw} = 4.13$) into Eq. (6.39), yields,

$$\bar{Z}_t = 2.579(\bar{t}_v)^{3/4}. \quad (6.40)$$

The penetration coefficient is very close to Middleton's (1975) results on starting pure plumes. From analytical derivations, Middleton concluded that the velocity of the head vortex for a starting plume decays in the following manner

$$u_c = \frac{5^{1/2}}{2\alpha^{1/2}\pi^{1/4}} A_c B_0 (t_v)^{-1/4}. \quad (6.41)$$

Based on the Turner's experimental results, $\alpha = 0.09$ and $A_c = 0.49$, Eq. (6.41) can be converted to,

$$\bar{Z}_t = 2.536(\bar{t}_v)^{3/4}. \quad (6.42)$$

It can be seen that Eqs. (6.40) and (6.42) are nearly identical.

The unified penetration time given in Eq. (6.36) covers the three different regimes in PDF including jet-like, transitional and plume-like. Although laboratory measurements on starting jets and starting plumes have been reported previously, no information on the transitional phase can be found in the literature as far as the authors are aware. In the following,

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experimental investigations with PIV and PLIF will be reported, which will provide information on the full range of development of a starting forced plume.

6.6 Experimental conditions

A schematic diagram of the experimental set-up is shown in Fig. 3.3. The experiments were conducted in a glass test tank with dimensions of $0.55\text{m} \times 0.55\text{m} \times 1.2\text{m}$ (height). The tank was constructed with glass panels and a stainless steel frame. The source solution was discharged from a round nozzle seated at the bottom of the tank. The nozzle was 100mm in length and had a diameter of 7.5mm. The source fluid was supplied from a constant head tank located 2.8m above the test tank. The water temperature was kept constant at about 25°C to avoid the influence of temperature on dye fluorescence. The volumetric flow rate was controlled by a needle valve and monitored by a digital flow meter (model Endress+Hauser Promag 33). A solenoid valve with an opening time of about 20 ms was used to ensure that the starting process can be considered to be near instantaneous.

The velocity and concentration measurements were based on a combined PIV/PLIF approach (Law & Wang 2000). The light source employed a dual-cavity pulsed mini Nd:YAG laser with a maximum repetition rate of 15 Hz for each cavity. The energy level was 50 mJ per pulse and the pulse duration was about 7 ns. The emitted laser light was green with a wavelength of 532 nm. The light sheet had a typical thickness of 3mm and a divergence angle of 32° .

Two double-frame 8-bit digital CCD cameras (Kodak Megaplug ES1.0) were configured, one for PIV and the other for PLIF. The spatial resolution of the cameras was

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1008×1016 pixels. Both cameras were fitted with Nikon 60mm lens. The PLIF camera was set to single-frame mode while the PIV camera worked in double-frame mode. Thus, the PLIF image was double-exposed by the laser pulse pair, the interval of which was determined by the cross-correlation PIV requirement. The two cameras were pointed to nearly the same area with a typical size of 30cm×30cm.

Rhodamine B was selected as the fluorescent dye tracer for PLIF. The absorption spectrum of Rhodamine B is from 460 to 590 nm with a peak at 550 nm, while its emission spectrum ranges from 550nm to 680nm with a peak at 590 nm. An optical 532nm band-pass filter was placed in front of the PIV camera, allowing only a narrow band of wavelengths around 532nm scattered by the seeding particles to pass through. Another optical 590nm low-pass filter was attached to the PLIF camera, allowing through only the fluorescent light emitted from the dye tracer.

A total of 11 tests were performed and the experimental parameters are listed in Table 1. Experiments C1~C7 had a relatively small density difference of up to 15%. In these cases, saline water was employed as the ambient solution and fresh water as the plume fluid. This combination of fresh and saline water is commonly adopted for experimentation on buoyant jets within the Boussinesq range (e.g. Wang and Law 2002).

Four experiments (C8-C11) with larger density differences (20% and 25%) were also conducted to investigate the behavior of non-Boussinesq starting forced plumes. In order to include these higher density difference cases, a combination of light-weight ethanol solution and heavy calcium chloride solution was used instead. The two solutions were

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chosen because they can have the same refractive index while having vastly different densities depending on the concentrations of ethanol and calcium chloride, as shown in Fig. 6.5. The matching refraction indices were needed in order to avoid blurring of the PIV/PLIF images.

6.7 Experimental Results and discussions

The general observation of a typical starting forced plume in the experiments is shown in Fig. 6.6 (Case C3). The first image (000) was recorded at 0.2 second after the forced plume was initiated. The series of images had a time interval of 0.2 sec. In the beginning, the forced plume was pushed upward into the ambient fluid by the initial momentum, and a momentum puff was formed with the intrusion. The starting forced plume then began to initiate a complex sequence of vortex leapfrog, coalescence, pinch-off and reconnection (cf. Fig. 6.2 and Ai *et al.*, 2005a). During this developing phase, the penetration accelerated to a maximum value and then started to decelerate towards the temporal behavior in the developed phase. This phenomenon of initial acceleration is called over-shooting. In addition to the penetration rate, over-shoot was also commonly observed in the growth of the head vortex size (Maxworthy 1977, Didden 1979 and Ai *et al.* 2005a). In our experiments, over-shoot was typically recorded in the penetration results (cf. section 6.7.1.1).

From the images, a trailing jet was formed behind the head intrusion (see Fig. 6.6) immediately after the forced plume was started. After some time, a cap vortex became somewhat distinguishable (e.g. Fig. 6.6-009) with an obvious boundary (see Fig. 6.6). At the same time, the advancing head vortex was fed with fluid from the trailing buoyant jet which added buoyancy and momentum to the vortex. The flow visualization records showed that the

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cap vortex grew continuously in size, and then emanated into the ambient fluid at the time of the pinch-off (see Fig. 6.6-003). The detachment occurred momentarily and visibly (although in some cases the pinch-off could not be observed well because of the instant reconnection) and the head vortex was subsequently reconnected (see Fig. 6.6-004) to the stem which completed the developing phase in PFD. After the reconnection, a new head vortex was formed engulfing the initial vortex while leading the trailing buoyant jet.

The Boussinesq and non-Boussinesq cases will be addressed separately in the following.

6.7.1 Boussinesq starting forced plumes

6.7.1.1 Streamwise Penetration of head vortex

For Boussinesq starting forced plumes, Diez *et al.* (2003) proposed the penetration time as:

$$(Z_t - Z_0) / D = C_z (t / \bar{t}')^{3/4}, \quad (6.43)$$

where C_z and Z_0 are best-fit empirical parameters of self-preserving scaling relationships, $\bar{t}'$ is defined as:

$$\bar{t}' = (D^4 / B_0)^{1/3}. \quad (6.44)$$

Utilizing this time scale, the experimental results on the penetration rate are plotted in Fig. 6.7. The virtual origin was set as $Z_0 / D = 7.0$ in this figure following the suggestion by Diez *et al.* (2003). It can be seen that although the time exponent can be fitted to be 3/4, the coefficient C_z does not appear to be a constant. Figure 6.7 clearly shows that the penetration of the starting forced plume experienced three different stages: PFD, transitional and plume-like. Note that the hump at $t / \bar{t}' \approx 10$ signifies the developing phase of the forced plume. This

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hump can be related to the over-shoot phenomenon as mentioned above. In all experiments, the developing phase of the forced plume dominated the initial period such that the jet-like behavior was not observed.

Utilizing L_m as the length scale, Fig. 6.8 shows the penetration height as a function of time for the Boussinesq cases. A time scale, defined as M_0 / B_0 , can be used to identify the three developed regimes of the forced plume (Fischer *et al.*, 1979). The jet-like region occurs when $t \ll M_0 / B_0$, while the transitional region is dominated by both initial momentum and buoyancy when $t \sim M_0 / B_0$. When $t \gg M_0 / B_0$, the forced plume becomes similar to a starting pure plume. From the experimental results shown in Fig. 6.8, the buoyant jet penetration became self-similar when $tB_0 / M_0 = 1$. Thus M_0 / B_0 can also be used a time scale for the developing phase of a starting forced plume.

It can be observed that the analytical model in Eq. (6.36) gives creditable prediction on the developed behavior of the starting forced plume, including the transitional regime as well as the asymptotic plume regime. Note that no virtual origin correction is required given the detailed simulation of the different regimes in this model. As shown in Fig. 6.8, the experimental data in the present experiments agree excellently with Eq. (6.36) in the PDF, while in the PFD, the starting forced plume behaves differently as expected.

6.7.1.2 Radial penetration

The corresponding temporal variation of the maximum radial penetration distance can be expressed in terms of the streamwise penetration distance, as follows:

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$$(r_v - D/2)/Z_L = \alpha', \quad (6.45)$$

where α' is the half spreading angle of the trailing buoyant jet, and Z_L is the local height (rather than the distance to the vortex front). Most of previous work on starting and steady plumes assumes that α' is a constant. Figure 6.9 shows the values of α' to be approximately 0.16 ± 0.03 in PDF, in reasonable agreement with Diez *et al.* (2003) and Turner (1962) who obtained 0.16 and 0.18 for starting plumes, respectively. From Fig. 6.9, it can also be seen that due to the initial vortex dynamics, the spread angle of the head vortex in PFD was larger. Subsequently, as the pinch-off occurred and the feeding of buoyancy and circulation into the head vortex ceased, the spreading angle began to decrease because of the lower entrainment until the head vortex reconnected with the trailing stem. Throughout the development, the Gaussian velocity width showed a monotonic increase until it reached the steady state values (cf. section 6.6).

6.7.1.3 Boussinesq trailing buoyant jet

For the Boussinesq cases, the mean axial velocity decay along the centerline of the trailing buoyant jet is plotted in Fig. 6.10. In the figure, the solid line represents the results of Wang and Law (2002) for a steady buoyancy jet. They reported that for $Z/L_m < 0.6$ the flow is jet-like, and for $Z/L_m > 6$ the flow is plume-like. Near the forced plume exit, the experimental data slight deviated from the self-similar solution. As mentioned above, this is because the flow in this zone has not established self-similarity. Similar to the results of the penetration rate, the mean centerline velocity became self-similar at the location of $Z/L_m \approx 2$. Figure 6.10 shows that the trailing buoyant jet over-shot the jet-like area and entered the transitional regime in PDF directly. The figure further implies that the stronger the initial momentum, the shorter the dimensionless length of the PFD would be.

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Figure 6.11 shows the temporal evolution of the axial velocity with the plume penetration. The centerline velocity collapsed onto each other after achieving a self-preserving state, which is expected since the trailing buoyant jet should eventually develop into a steady buoyant jet with time.

6.7.2 Non-Boussinesq starting forced plumes

6.7.2.1 Streamwise penetration of head vortex

Figure 12a shows that the penetration rate for the non-Boussinesq starting forced plumes in comparison with the penetration model of Eq. (6.36). Comparing Figs. 6.8 and 6.13a, it can be observed that the deviation was larger in these higher density difference cases.

The differences between the Boussinesq and non-Boussinesq forced plumes occurred mainly in the region near the source. With continuous dilution, the non-Boussinesq effects diminished away from the source. Thus, the difference between the two cases can be corrected by the virtual origin method to account for the near-field effects.

Morton's (1959) theoretical description provides a practical solution for the determination of the virtual origin for a steady forced plume. Hunt and Kaye (2001) summarized that the solution can be described as a two-step procedure. The first step involves replacing the extended source (B_0, M_0, Q_0) with a virtual point source forced plume of modified strength $(B_0, \gamma M_0, 0)$ located at $Z = Z_v$, where $\gamma = (1 - \Lambda)^{1/5}$ and Λ is related to the source conditions as defined in Eq. 6.1.

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Morton (1959) categorized various types of buoyant plume in terms of Λ . He defined forced plumes as those for which the initial momentum flux is greater than that of an equivalent pure plume of the same initial buoyancy flux (i.e. $0 < \Lambda < 1$), pure plumes as $\Lambda = 1$ and lazy plumes as $\Lambda > 1$. The second step is to relate the flow above this modified source to a pure buoyancy source $(B_0, 0, 0)$ at a virtual origin $Z_0 = Z_v + Z_{avs}$.

For forced plumes, the momentum flux of the modified source is positive and the total magnitude of the two-step correction is

$$\frac{Z_0}{L_m} = \frac{Z_v + Z_{avs}}{L_m} = -10^{1/2} \int_{\gamma}^1 v^3 (v^5 - \gamma^5)^{-1/2} dv - 1.057 \gamma^{3/2}. \quad (6.46)$$

Based on the above described method for Boussinesq forced plumes, Carlotti and Hunt (2005) extended the virtual origin correction approach for non-Boussinesq cases. They replaced the actual non-Boussinesq area source located at $Z = 0$ with an idealized point source located at a virtual origin $Z = Z_0$, and proposed the asymptotic analytical expressions for the pure, lazy and forced plumes valid for large vertical distances above the non-Boussinesq source. A characteristic length scale, related to the buoyancy B_0 and gravity g , is proposed as:

$$\ell_0 = \frac{B_0^{2/5}}{(C_3 g)^{3/5}}, \quad (6.47)$$

where the constant $C_3 = 0.1154$, was employed to normalize the mass and momentum fluxes, i.e.

$$\phi_0 = \frac{g \frac{\rho_0}{\rho_\infty} Q_0}{(C_3 g)^{3/2} \ell_0^{5/2}} \quad (\text{dimensionless mass flux}), \quad (6.48a)$$

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$$\text{and } \lambda_0 = \frac{M_0}{C_3^{3/2} g \ell_0^{5/2}} \quad (\text{dimensionless momentum flux}). \quad (6.48b)$$

According to Carlotti and Hunt (2005), the virtual origin for the non-Boussinesq steady forced plumes can be expressed as,

$$\frac{Z_0}{\ell_0} = -\frac{3}{5} (\lambda_0^{5/2} - \phi_0^2)^{3/10} \int_0^{\phi_0} \frac{dx}{(\lambda_0^{5/2} - \phi_0^2)^{1/2}} + \frac{3}{5} C_4 (\lambda_0^{5/2} - \phi_0^2)^{3/10}, \quad (6.49)$$

where the constant $C_4 = -0.84$. Table 2 lists the values of Z_0/L_m for the above two methods.

The approach of for the non-Boussinesq shows better results than Morton's approach for the Boussinesq plume performed.

Table 2 lists the values of Λ and Z_0/L_m . The Λ values show that the experiments conducted in the present study were all with forced plumes.

The above virtual origin determination was developed for a steady forced plume. For starting forced plumes, it is uncertain whether similar approach can be adopted. Utilizing the correction results of Z_0/L_m in Table 2, the non-Boussinesq effects on the starting forced plume penetration rates are illustrated in Fig. 6.12b. Compared with Fig. 6.12a, the corrected penetration rates fitted significantly better with the model prediction for Boussinesq plumes (Eq. 6.36). Hence, the current experimental results show that the approach of virtual origin correction can also be applied for the penetration rate of non-Boussinesq starting forced plumes.

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6.7.2.2 Radial penetration

Fig. 6.9 shows that the radial penetration in terms of the spreading rate is the same ($\alpha' = 0.16 \pm 0.03$) for both Boussinesq and non-Boussinesq starting forced plumes in the PDF. It is however important to note that flapping, a critical phenomenon which is related to the large-scale vortices and can significantly increase the spreading rate in steady plumes (Papps and Wood, 1997), was also commonly observed in the present experiments on starting forced plumes. In the PFD where the non-Boussinesq effects were more significant due to the higher density differences, it can be noted an important observation that significantly more flapping was observed in cases C8-11 than in C1-7 (shown in Figs. 6.13a and b). However, although the magnitude of the flapping motion was stronger in non-Boussinesq cases, the spreading rate in PDF did not increase according to Fig. 6.9. The reason may be due to the fact that the strong buoyancy in the near field pushed the starting forced plume to move up faster, and thus counteracted the increased spreading by the flapping motion.

6.7.2.3 Non-Boussinesq trailing buoyant jets

The variation of centreline velocity decay with penetration distance for the non-Boussinesq cases is shown in Fig. 6.14. From $Z/L_m > 6$ onward, the data began to follow the asymptotic behaviour of Boussinesq pure plumes with a net displacement due to the virtual origin. As discussed previously, the asymptotic behaviour was due to the continuous dilution with the entrainment of ambient fluid and the diminishing of density differences between the forced plume and the surrounding. In the region near the source, however, the non-Boussinesq effects were prevalent and the strong buoyancy played a significant role in the evolution as evidenced by the deviations in the initial region in Fig.

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6.14. In the following, the non-Boussinesq effect will be examined more closely in this region.

Figure 6.15 shows the variation of the centreline velocity with the penetration distance. Comparing Figs. 6.11 and 6.15, the velocity variation was not so orderly for the non-Boussinesq cases (see Fig. 6.15). In the initial region of non-Boussinesq starting forced plume, the strong buoyancy overwhelmed the shear stress at the plume edge and led to an “increase” in the centerline velocity in the beginning. The velocity eventually reached a peak at $Z/L_m \approx 3.5$. With the entrainment and the corresponding dilution, the buoyancy began to weaken and then the velocity started to decrease with distance. For the Case 20%05, the centerline velocity maintained steady for some time suggesting that the buoyancy balanced the turbulence stresses during this period. For Case 25%05 the centerline velocity had a similar trend as 20%05 but decreased in a slower pace. Figs. 6.14 and 6.15 demonstrated that it would take a longer time for a strongly buoyant forced plume to develop into the asymptotic states.

6.7.2.4 Plume width and entrainment in non-Boussinesq cases

Table 3 shows the plume characteristics for the present experiments in the PDF. From the table, it can be observed that the velocity radius spreading rate, b_w/Z , varied from 0.098 to 0.105, and was near Fischer *et al.*'s (1979) suggested range, but slightly smaller than the value of 0.105 obtained by Wang and Law's (2002) for steady Boussinesq plumes. The results imply that the proportional constant for the Gaussian radius b_w is nearly the same throughout the three different regimes of jet-like, transitional and plume-like.

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Figures 6.16 and 6.17 demonstrate the Gaussian cross-sectional velocity profiles at different locations for cases 03%05 and 28%10. The expansion in radius was similar for both Boussinesq and non-Boussinesq cases, although the locations where the plumes became asymptotic were different. The stronger the initial buoyancy, the shorter the distance would be before the plume developed into a self-preserving state.

Figures 6.18 and 6.19 plot the variations of volume entrainment rate and entrainment coefficient along the penetration distance. The profiles of the entrainment coefficient and the volume entrainment rate were quite similar. For Boussinesq cases ($\Delta\rho_0 / \rho_\infty < 15\%$), their behavior were asymptotic (see Figs. 6.18 and 6.19) and nearly identical to previous observations in steady buoyant plumes (Wang & Law, 2002), with the entrainment coefficient increasing steadily from jet-like (0.0528) to plume-like (0.0874) during the penetration (Fig. 6.19). For non-Boussinesq starting forced plumes, the entrainment behavior was vastly different in the initial region. Corresponding to the centerline velocity profiles (Fig. 6.14), there were a sharp increase initially for the entrainment coefficient and volume entrainment for both cases 25%10 and 20%10. The entrainment coefficient jumped to a peak of 0.12 for Case 25%10 and 0.1 for Case 20%10. The plume fluid was rapidly diluted with the turbulent mixing with the surrounding fluids and the behavior finally achieved a self-preserving state as the Boussinesq cases (see Figs. 6.14 and 6.19).

6.8 Temporal Evolution of the velocity distribution

According to the penetration rate, the arrival time scale of the plume front should be

$t \sim \frac{Z}{L_m} \times \frac{M_0}{B_0}$. In the following, this time scale will be applied to estimate the transitional time

for the centerline evolution.

Figure 6.20 shows the variations of the dimensionless velocity versus the penetration time normalized with the above time scale. From the figure, it can be seen that the assumption is reasonable and the time scale agreed well with the present results. The normalized rising time T_r was around 0.5 (see Fig. 6.20). After reaching a local peak due to over-shoot, the velocity began a periodic pulsation. The pulsation was stronger during the transition period and was attenuated slightly after the trailing stem became near steady state. The figure also shows that the centerline velocity fluctuation in the axial direction was about 0.3 which was rather close to the previous results reported in Shabbir & George (1994) and Wang & Law (2002) for steady buoyant jets.

The typical Gaussian profile evolution with time is illustrated in Fig. 6.21. From the figure, it can be observed that when the plume front first arrived, the cross-sectional velocity profile was initially top-hatted. An over-shoot was recorded at $t = 2.8s$. Subsequently the centerline velocity oscillated around the mean value and the cross-sectional profile became Gaussian with the radius increasing monotonically towards the steady value. The Gaussian radius took much longer time to attain a steady value than the centerline velocity.

6.9 Summary

Two distinct periods, the Period of Flow Development and Period of Developed Flow, are identified in the penetration behavior of a starting forced plume. An analytical model is developed for the penetration rate in the Period of Developed Flow by coupling the dynamics of the head vortex and the trailing forced plume. The unified model predicts the penetration rates in the three different phases of jet-like, transitional and plume-like behavior in PDF, and computes directly the variations of the velocity ratios between the head vortex and the trailing forced plume.

Experiments were conducted on both Boussinesq and non-Boussinesq starting forced plumes using combined PIV and PLIF. For the Boussinesq cases, the measurements on the penetration rate agreed well with the model predictions. Other characteristics such as the radial penetration and entrainment rate also demonstrated reasonable agreement with previous results reported in the literature. For non-Boussinesq cases, flapping motion was observed to be more significant, and the temporal change in the centerline velocity had a strong overshoot in the initial region unlike the exponential decay observed in the Boussinesq cases. It was found that the non-Boussinesq effects near the source can be corrected by a virtual origin in the plume-like region similar to a non-Boussinesq steady forced plume.

A complete analytical model of the penetration of a starting forced plume requires the detailed simulation of the processes inside the PFD in addition to the PDF. Further research is obviously needed in this area and is being currently pursued.

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Table 6.1 Initial parameters of the experiments

Cases	Name	W_0 (m/s)	ρ_∞ (kg/l)	ρ_0 (kg/l)	Δ_0 (%)	Re	L_m (mm)	Fr_0	Time interval (ms)
C1	03%05	0.503	1.0258	0.9974	2.79	3773	78.1	9.80	4
C2	05%05	0.503	1.0552	0.9974	5.52	3773	55.4	6.95	4
C3	05%10	0.987	1.0552	0.9974	5.52	7500	110.8	13.91	2
C4	10%05	0.503	1.1007	0.9974	9.42	3750	42.4	5.32	4
C5	10%10	0.987	1.1007	0.9974	9.42	7403	84.8	10.64	2
C6	15%05	0.503	1.1518	0.9974	13.44	3773	35.5	4.46	4
C7	15%10	0.987	1.1518	0.9974	13.44	7403	71.0	8.91	2
C8	20%05	0.503	1.0997	0.8857	20.22	3773	29.0	3.63	4
C9	20%10	0.987	1.0997	0.8857	20.22	7403	57.9	7.27	2
C10	25%05	0.503	1.1171	0.8370	25.07	3773	26.1	3.26	4
C11	25%10	0.987	1.1171	0.8370	25.07	7403	52.2	6.53	2

Table 6.2 Virtual origin correction for starting forced plumes

Cases	Q_0 (cm ³ /s)	Δ_0 (cm/s ²)	B_0 (cm ⁴ /s ³)	M_0 (cm ⁴ /s ²)	Λ	Z_0/L_m	Z_{avs}/L_m	ℓ_0 (mm)	Z_0/L_m (Carloti & Hunt)	Z_0/L_m (Morton)
20%05	22.22	198.16	4403.5	1117.7	0.161	-0.050	-0.9654	16.86	-1.668	-1.053
20%10	43.61	198.16	8641.6	4304.9	0.040	-0.024	-0.9918	22.0	-0.390	-1.068
25%05	22.22	245.69	5459.7	1117.7	0.199	-0.056	-0.9566	18.3	-1.203	-1.662
25%10	43.61	245.69	10714.4	4304.9	0.050	-0.026	-0.9898	24.0	-0.946	-1.384

Table 6.3 Comparison of plume width

Cases	$\Delta\rho_0 / \rho_\infty$ (%)	b_w / Z
03%05	2.79	0.103
05%05	5.52	0.100
05%10	5.52	0.101
10%05	9.42	0.099
10%10	9.42	0.098
15%05	13.44	0.102
15%10	13.44	0.098
20%05	20.22	0.104
20%10	20.22	0.100
25%05	25.07	0.105
25%10	25.07	0.104
Wang and Law (2002)	<3%	0.105
Fischer <i>et al.</i> (1979)	-	0.100

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Fig. 6.1 Schematic diagram of the four different phases of the development of a starting plume.

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Fig. 6.2 A pictorial sketch of the vortical dynamics in PFD: (i) flow visualization by PLIF, and (ii) corresponding vector and vorticity map by PIV. The image was obtained in a starting jet experiments with $Re=4716$.

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Fig. 6.3 Variation of A_b with Z_b/L_m

Fig. 6.4 Penetration rate of a fully-developed starting forced plume (Eq. 6.36).

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Fig. 6.5 Refraction index versus densities of calcium chloride and ethanol.

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Fig. 6.6 Sequence of PLIF images through the central axis of symmetry showing the evolution of a starting forced plume for the Case of 05%10. The first image was 0.2s after the initial start of the forced plume. The time interval of the images was 0.2s.

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Fig. 6.7 Normalized streamwise penetration distance $(Z_t - Z_0)/D$ as a function of normalized time $t/\bar{t}$

Fig. 6.8 Normalized streamwise penetration distance Z_t/L_m as a function of normalized time tB_0/M_0 (Boussinesq cases).

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Fig. 6.9 Radial penetration vs tB_0 / M_0

Fig. 6.10 Centerline decay of mean axial velocity with Equation (6.11a) for Boussinesq cases.

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Fig. 6.11 Centerline velocity decay as a function of Z/L_m for Boussinesq cases

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Fig. 6.12 Normalized streamwise penetration distance Z_t / L_m as a function of normalized time tB_0 / M_0 for non-Boussinesq cases: (a) without virtual origin correction (b) with virtual origin correction

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Fig. 6.13 Flapping motion in starting forced plumes:(a) $t = 1.4s$ in Case 05%05 and (b) $t = 1.0s$ in Case 20%10

Fig. 6.14 Centerline decay of the mean axial velocity with Equation (6.11a) for non-Boussinesq cases

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Fig. 6.15 Centerline velocity decay as a function of Z/L_m

Fig. 6.16 Cross-sectional variation of the mean axial velocity in Case 03%05, showing that the profiles became self-similar when $Z/D > 25$

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Fig. 6.17 Cross-sectional variation of mean axial velocity in Case 25%10. The profiles became self-similar only after $Z/D > 15$

Fig. 6.18 Variation of volume entrainment rate with Z/L_m

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Fig. 6.19 Variation of entrainment coefficient with Z / L_m

Fig. 6.20 Time evolution of centerline velocity

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Fig. 6.21 Time evolution of cross-sectional velocity at $Z/D = 30$ for Case 03%05

Chapter 7 CONCLUSIONS

The present research focuses on the studies of the transitional stage of jets and plumes. It can be generally divided into two parts, namely, starting jets and starting forced plumes. The first part attempts to improve the understanding of the vortex dynamics in starting jets and to examine the differences between that of the circular jets and square jets. Based on the qualitative (via PLIF) and quantitative results obtained (via PIV), it was found that streamwise vortices generated at the corners of the square nozzle interacted with the primary vortices. The interactions had altered the downstream flow characteristics tremendously. For starting circular and square jets, the following concluding remarks can be extracted from the preceding chapters.

1. Vortices Leapfrogging and coalescence were observed at $Re=2358$ for both circular and square jets;
2. Pinch-off effect was detected for the first and second vortices at $Re=3528$ for both circular and square jets;
3. Single pinch-off for the leading vortex was found at $Re=4716$ for both circular and square jets;
4. Analyses utilizing the Dynamic approach showed that pinch-off process was found to be not equivalent to the formation process proposed by Gharib *et al.* (1998). Reynolds number at around 3528 was found to be the critical Reynolds number for the pinch-off to occur;
5. Axis-switching was observed and investigated in starting square jets. The interactions between the corner vortices and primary vortices were shown to reduce the jet tip

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penetration rate, but mixing with ambient fluids was enhanced for the square jets, as compared to the circular jets;

For starting forced plume, Boussinesq and non-Boussinesq cases had been investigated analytically and experimentally. The main results are summarized as below:

1. An analytical model for the starting forced plume covering the entire range of jet-like, transitional and plume-like had been developed based on the understanding of So & Aksoy (1993), Turner (1962) and Wang & Law (2002) for forced plumes;
2. The velocity ratio, A_b , between the head vortex and the trailing buoyant jet was derived to vary from the jet-like to plume-like phases. The transitional changes for this ratio had also been quantified;
3. Another contribution by the present study was the experimental results for non-Boussinesq starting forced plumes using advanced measurement tools of PIV and LIF. Larger entrainment coefficient and centerline velocity increase were found for the non-Boussinesq forced plumes;
4. PFD/PDF model for the starting forced plume was proposed for starting forced plume;
5. Experimental results showed disorder behaviors in the near-field region for the non-Boussinesq cases ($\Delta > 15\%$). Virtual origin correction approach was utilized to compensate the non-Boussinesq effects.

In closing, the present work on starting jets and plumes provides basis for further study on starting jets and plumes phenomenon. The concept of optimal vortex formation can be generalized to enable the discovery of underlying design principles simulating the animals'

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(e.g. jellyfish, squid) or human's effective swimming and physical behaviors, e.g. aquatic locomotion, fluid dynamic energy conversion, and cardiac flows. The enhanced mixing in square jets provides fundamental knowledge for the fuel ejecting nozzle design for the propulsive combustors. In addition, the penetration model for the starting forced plume may offer assessment for the hazard area of the releases of hot ash during volcanic eruptions and the emission of chemical plumes by accidental or intentional releases in an indoor environment.

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