Magneto-rheological fluid rotary brake used in wheeled vehicles

Alireza Farjoud

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Magneto-Rheological Fluid Rotary Brake used in Wheeled Vehicles

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NANYANG
TECHNOLOGICAL
UNIVERSITY

Agency for
Science, Technology
and Research

School of Mechanical and Aerospace Engineering

Magneto-Rheological Fluid Rotary Brake used in Wheeled Vehicles

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Abstract

MR fluids are currently attracting lots of interest because of their interesting rheological behavior. The viscosity of MR fluids changes rapidly in mili-seconds and this change is also reversible. MR fluids can be considered as a direct interface between mechanical and electrical systems.

This research is concerned with one important application of MR fluids which is clutch and brake systems. The main focus of the research was to fully understand the behavior of MR fluids in rotary devices, especially MR rotary brakes.

Generally, MR clutches and brakes are associated with high velocities; as a result the corresponding shear rate would be too high. However; currently most of the constitutive equations are based upon parameters which are obtained at low shear rates. The most widely used constitutive equation is the Bingham constitutive model. One of the deficiencies of this model is its inability to predict the shear thinning behavior shown by MR fluid at relatively high shear rates. To overcome this issue, the Herschel-Bulkley model is proposed in which the viscosity is a function of shear rate. In this research, the main focus is on the Herschel-Bulkley constitutive equation and the drum-type MR brake is mathematically modeled using Herschel-Bulkley constitutive equation. However; the Bingham model is still useful because of its simplicity. So; in some parts we also consider the Bingham model in addition to the Herschel-Bulkley model. For the disk-type MR brake, a three dimensional, tensorial constitutive equation is used.

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List of Symbols

$\vec{B}$	Magnetic induction
$\underline{\underline{B}}$	Strain tensor
Bn	Bingham number
c_1	Constant of integration
D	Half-width of the brake plate (chapter 5)
D_1, D_2	Diameter of strain guage wires before and after application of force (appendix A)
$\underline{D}^*$	Non – dimensional rate of strain vector
$\underline{\underline{D}}$	Rate of strain tensor
E	Magnetic potential energy
F	Magneto – motive force (chapter 2)
F	Load cell force (chapter 5)
G	Elasticity modulus (chapter 4)
GF	Guage factor
$\vec{g}$	Gravity field
G'	Storage shear modulus
G''	Loss shear modulus
G^*	Dynamic shear modulus
$\vec{H}$	Magnetic field
h	Half depth of the rectangular duct (section 4.1)
i	Electric current
j	Current density
k	Consistency index for the Herschel – Bulkley model
L	Length of strain guage wires
$\vec{M}$	Magnetization
$\vec{m}$	Magnetic dipole moment
n	Power law index for the Herschel – Bulkley model
P'	Pressure gradient
Q	Volumetric flow rate

R	Reluctance (chapter 2)
R	Electrical resistance (appendix A)
$R_1, R_2, h = R_2 - R_1$	Dimensions of MR brake
r, θ, z	Components of cylindrical coordinates
r^*	Non – dimensional radius
r_p	Plug flow radius
T	Torque
T_f	Field torque
T_η	Viscous torque
T^*	Non – dimensional torque
$\vec{t}_n$	Shear stress vector
u	Velocity of MR fluid in duct
V_0	Output voltage of the Wheatstone bridge
v^*	Non – dimensional velocity
$\vec{v}$	Velocity field
z^*	Non – dimensional coordinate in z direction
$\dot{\gamma}$	Shear rate
δ	Loss factor (chapter 2)
δ	Regularization parameter for the Bercovier and Engelman model (chapter 4)
δ^*	Non – dimensional regularization parameter
$\varepsilon_A, \varepsilon_L$	Axial and lateral strains
ζ	Non – dimensional radius (sec tion 4.3.5)
η	Viscosity
θ	Component of cylindrical coordinates
μ	Permeability
μ_0	Permeability of free space
μ_r	Relative permeability
ν	Poisson 's ratio
ρ	Density
τ_y	Yield stress of MR fluid
τ', τ''	Components of dynamic response stress

τ^*	Dynamic response stress
$\underline{\tau}^*$	Non – dimensional shear stress vector
$\underline{\underline{\tau}} = \tau_{ij}$	Stress tensor
ϕ	Magnetic flux (chapter 2)
ϕ	Dimensionless shear stress (section 4.1)
$\phi(\zeta)$	Non – dimensional velocity (sec tion 4.3.5)
χ	Susceptibility
ψ	Dimensionless volumetric flow rate (section 4.1)
ψ	Non-dimensional constant of integration (section 4.3.5)
ω	Rotational speed

Chapter 1

Introduction

MR fluids (MRF) are suspensions of micron-sized, magnetizable particles dispersed in a non-magnetic carrier fluid. MR fluids can respond to an applied magnetic field with a change in rheological behavior from free-flowing fluid in the absence of a magnetic field to a semi-solid in the presence of a magnetic field. The transformation is almost instantaneous and reversible. MR fluids have attracted considerable interest in recent years because of their potential as simple, quiet, and rapid-response interface between electronic controls and mechanical systems. Many applications have been studied, such as shock absorbers, engine mounts, brakes and clutches.

MR fluid was first discovered by Jacob Rabinow in the early 1940s (Rabinow 1948). Recently there seems to be a renewed interest in MR fluids and many countries are investing in the field of MR fluids and MR based devices. This is due to the huge application prospects in modern industrial areas including power transmission devices, aerospace systems, active and semi-active control of structural vibration, and robot control systems.

Although much research has been conducted on MR fluids, but there are very few commercial MR devices available in the market. This is possibly due to the unavailability of effective fluids and cost. For practical applications, the ideal MR fluid should have low zero-field viscosity, high yield stress, no toxicity, non-settling and quick response.

So far, a comprehensive understanding of MR rheology, whether experimental or in modeling is still lacking. In the present study, it is tried to fully understand the behavior of MR fluids, and MR fluid flow for different valve geometries and conditions and derive the velocity profile for each case. The mathematical models presented in this report are obtained by implementing appropriate MR fluid constitutive equation. We have used either the Bingham or the Herschel-Bulkley models. For the case of disk type brakes, we have used a modified version of the Bingham model proposed by Bercovier and Engelman (Bercovier and Engelman 1980).

1.1-Scope of the Research

This research aims at the application of Magnetorheological Fluids in rotary devices. These devices mostly use the MR effect to transmit rotary motion from one shaft to another. MR rotary brakes and clutches are examples of rotary devices. MR rotary brake is simply a clutch with the output velocity of zero. The present research focuses mainly on MR rotary brakes. The aim of the project was to fully understand the flow behavior of MR fluids in rotary devices. The main two types of MR rotary brakes are mathematically modeled using the Bingham and the Herschel-Bulkley constitutive models. The Bingham constitutive equation is mostly used to derive the design equations of the MR brakes.

Although the Bingham model gives satisfactory results for design purposes, but its main deficiency which is its inability to predict the shear thinning, raises the tendency to use the more precise model of Herschel-Bulkley. The Herschel-Bulkley model is capable of predicting the shear thinning phenomenon at high shear rates which is the case for most of the rotary brakes and clutches. There are not so many researches done on the behavior of MR fluids at high shear rates. This kind of experiments seems to be of great importance because most of the MR rotary devices operate at relatively high shear strain rates. The problem associated with the Herschel-Bulkley model is the relative complexity of the equations and the scarcity of the experimental data.

Mainly, the research aims are:

- To fully understand the mathematical models and constitutive equations used to model MR fluid flows.
- Mathematical modeling of Drum-type MR rotary brakes.
- Mathematical modeling of Disk-type MR rotary brakes.
- Designing and setting up the test rig for testing the disk type brake.

Due to time and financial constraints, the test rig is not built for testing.

1.2-Thesis Outline

Chapter 2

This chapter is the literature review. The literature review starts by an introduction to the general properties of field responsive fluids, then a review on magnetism and magnetic

materials is done. The most necessary topics in the field of magnetism needed for a good understanding of the operation on MR fluid devices is presented in this section. This chapter is continued by presenting other topics about MR fluid properties and MR technology. At the end of this chapter we have highlighted the limitations of MR technology.

Chapter 3

Chapter 3 discusses MR clutches and brakes. The main goal of this research project is to model the flow of MR fluids in rotary devices and specially brakes. In chapter 3 we have presented the main and most important concepts about MR clutches and brakes.

Chapter 4

This chapter is the main part of this report which considers the mathematical modeling of MR fluid flow for different geometries. We start with simple one dimensional flow in a rectangular duct and then move on to the case of MR fluid flow in an annular duct. Then the mathematical modeling of a drum type (or cylindrical type) brake is discussed. Finally the MR fluid flow in a disk type brake is analyzed where a numerical approach in conjunction with a computer code in MATLAB is used to solve the obtained governing equations and the results are presented.

Chapter 5

The main design procedures, considerations, goals and drawings are presented in chapter 5. Detailed drawings are presented in appendix B.

Chapter 6

This chapter is the conclusion. In this chapter, the main results of the research are highlighted.

Appendix A

Appendix A is included to give introductory knowledge on load cells and specifically strain gauges and the operation basics of strain gauge load cells.

Appendix B

Appendix B contains the detailed drawings of the designed test rig, the dimensions of individual parts, and the part list.

Appendix C

There have been three papers written on the mathematical modeling of MR rotary brakes. Two are journal papers and one is a conference paper. These three papers are addressed in appendix C.

Appendix D

The Matlab code itself, is presented in appendix D. section D.1 is the code and section D.2 is the PDE function which is called in the mail program.

Appendix E

A brief introduction to the Lambert W Function is given in appendix E. The Lambert W Function is used in mathematical modeling of drum-type MR rotary brakes.

Chapter 2

Literature Review

MR devices involve the following disciplines:

- 1) Magnetism
- 2) MR Fluid
- 3) Device

Since the MR fluid itself is the main medium of MR devices and also the main topic of this research, first we give a brief and general introduction to field responsive fluids including MR fluids.

One of the most important design aspects of MR devices is the design of the magnetic circuit. MR fluids do not show any change in their rheological properties without any magnetic field; so, it is very important to have a deep understanding about magnetism and magnetic properties of materials in order to design a good MR device. Thus, one part of the report is devoted to give a brief and useful introduction about fundamentals of magnetism.

After getting enough knowledge about magnetism and magnetic materials we go back to MR fluids and discuss the technical issues regarding MR fluids.

2.1-Field Responsive Fluids

2.1.1-Magnetorheological (MR) fluids

Magnetorheology is a branch of rheology that deals with the flow and deformation of the materials under an applied magnetic field. The discovery of MR fluids is credited to Jacob Rabinow (Rabinow 1948; Rabinow 1951). Magnetorheological (MR) fluids are suspensions of non-colloidal (~ 0.05 - $10\ \mu\text{m}$), multi-domain, and magnetically soft particles in organic or aqueous liquids. Many different ceramic metals and alloys have been described and can be used to prepare MR fluids as long as the particles are magnetically multi-domain and exhibit low levels of magnetic coercivity. Particle size, shape, density, particle size distribution, saturation magnetization and coercive field are important characteristics of the magnetically active dispersed phase. Other than magnetic particles, the base fluids, surfactants, anticorrosion additives are important factors that affect the rheological properties, stability and redispersibility of the MR fluid.

In the “off” state, in terms of their consistency, MR fluids appear similar to liquid paints and exhibit comparable levels of apparent viscosity (0.1 to 1 Pa.s at low shear rates) (Ashour, Rogers et al. 1996). Their apparent viscosity changes significantly (105 - 106 times) within a few milliseconds when the magnetic field is applied. The change in the viscosity is completely reversible when a magnetic field is removed. Once the magnetic field is applied, it induces a dipole in each of the magnetic particles.

The inert-particle forces originating from the magnetic interactions lead to a material with higher apparent viscosity. This dipolar interaction is responsible for the chain like formation of the particles in the direction of the field (Figure 2. 1).

Figure 2. 1 Schematic of the formation of chain-like formation of magnetic particles in MR fluids in the direction of an applied magnetic field (Genc 2002).

It is also believed that in addition to magnetic interactions between two particles, the formation of the chains contribute to a certain level to the increase in the apparent viscosity. Particles held together by magnetic field and the chains of the particles resist a certain level of shear stress without breaking which make them behave like a solid. When this shear stress exceeds a critical value, the structure breaks and the material starts to flow. MR fluid effect is often characterized by Bingham Plastic model which will be discussed in the proceeding sections. The critical value of the shear stress necessary to break the structure is the “yield stress” of the material.

The response time of MR fluids to an applied magnetic field is on the order of a millisecond. Note this is the response of the material itself to a magnetic field.

This is not a device response time where the effects of the magnetic circuits are included.

In MR fluids, materials with lowest coercivity and highest saturation magnetization (see section 2.5) are preferred, because as soon as the field is taken off, the MR fluid should come to its demagnetized state in milliseconds. Due to its low coercivity and high

saturation magnetization, high purity carbonyl iron powder appears to be the main magnetic phase of most practical MR fluid compositions.

2.1.2-Electrorheological (ER) Fluids

ER fluids are suspensions of electrically polarizable particles dispersed in electrically insulating oil (Havelka and W. 1996). The ER fluid is typically composed of 0.5 to 100 μm particles of cornstarch, silica, barium titanate or semiconductors (Havelka and W. 1996). For particles such as silica, polyelectrolytes need to be added to cause the adsorption of water onto the particulate material to enhance the ER effect, thus increasing the electrostatic force of attraction between the particles. The water also creates a conductive layer on the surface of the particles in which the ions in the water can drift in response to an electric field. These materials are called extrinsically polarizable materials in which the ER effect results from interfacial polarization. The ER effect decreases as the amount of water absorbed decreases. Therefore, at temperatures of $\sim 50^\circ\text{C}$, the ER activity decreases significantly and thus the temperature instability limits the potential use of the ER fluids. Materials such as ferroelectrics, inorganics, semiconductor polymers, metals, coated conductors, and liquid crystals has also been reported as producing water-free ER suspensions and these materials are called intrinsically polarizable materials and they function by bulk polarization or interfacial polarization. They have lower thermal coefficient of yield stress which may help expanding the temperature range of ER activity.

Similar to MR fluids, upon the application of electric field, particles become polarized and the local electric field is distorted. The polarizability of the particles is increased by the migration of the mobile charges to areas with greatest field concentration. This gives rise to larger dipole moments that attract one another and cause the particles to form chain in the direction of the field. ER fluid is characterized by the Bingham Plastic Model in which the change in viscosity from particle chain interactions under shear corresponds to the yield stress. Many of the models, developed for ER fluids can be adopted for MR fluids in low magnetic fields. However, at high magnetic fields, due to the non-linearity and magnetic saturation of the particles, the linear models used to treat ER fluids are no longer valid for MR fluids.

ER fluids were mostly developed for valves, mounts, clutches, brakes, and dampers. However, not much progress has been made in their commercialization.

2.1.3-Ferrofluids

Ferrofluids also known as magnetic liquids, are colloidal suspensions of ultra-fine (typically 5-10 nm), single domain magnetic particles such as iron oxides (Fe_2O_3 , Fe_3O_4), Mn-Zn ferrites (Auzans, Zins et al. 1999), Fe and Co in either aqueous or non-aqueous liquids. Since the particle size of the magnetic phase is very small, under ordinary field strengths, thermal agitation gives rise to Brownian forces that can overcome the alignment of the dipoles. Thus, MR fluids are based on ferromagnetic particles and ferrofluids are based on superparamagnetic materials. Instead, ferrofluids experience a body force on the entire material that is proportional to the magnetic field gradient. Ferro-fluids exhibit field dependent viscosity but they exhibit no yield stress

($\tau_y = 0$) under magnetic fields. Ferro-fluids are used in rotary seals, magnetic bearings, and motor dampers (Raj and Moskowitz 1990; Berkovsky, Medvedev et al. 1993).

2.2-Comparison of Field Responsive Fluids

More recently MR fluids have gained considerably more attention than their electric analogue electrorheological (ER) fluids.

One of the advantages of MR fluids is the higher yield stress value than ER fluids. The reason for having higher yield stress for MR fluids is the higher magnetostatic energy density, $\mu_0 H^2$ of MR fluids compared to electrostatic energy density, $\epsilon_0 E^2$ of ER fluids.

Where μ_0 , is the permeability of free space, H is the magnetic field and E is the electric field. Low voltage power supplies for MR fluids and relative temperature stability between -40 and $+150^\circ\text{C}$ make them more attractive materials than ER fluids. It is important to note that ER fluids can suffer from material instabilities such as dielectric break down under high fields, but MR fluids have high material stabilities. High material stability means such factors as extreme temperatures, impurities or excessive magnetic fields have a negligible effect on the material rheology. This high material stability is also referred to as robustness. Because of this robustness, industrial applications of these materials have been successful without the need for extensive basic research.

The ultimate magnetic property is "ductile". This ductile Behavior is associated with Bingham behavior. This means as the magnetic field is increased, the magnetization of the particles saturates and any increase in magnetic field will produce a diminishing

increase in the magnetorheological effect. The ultimate electric property is contrary to electrorheological fluids in that ER fluids fail electrically in a "brittle" manner. When the applied electric field exceeds a maximum value, the ER material breaks down via an electro-static discharge and the electrorheological effect is momentarily interrupted.

A long time scale is often necessary to develop or dissipate a strong magnetic field. Controllable devices that use MR fluids require electro-magnetic circuits to generate the variable magnetic fields. These electro-magnetic circuits require approximately $\frac{L}{R}$ seconds to develop their full magnetic fields, where L , is the inductance of the electro-magnetic circuit and R is the resistance. For large systems this development time can be on the order of 0.25 seconds. This time scale restricts the control implementation to low frequency or static applications. This time scale is also in contrast to those of ER fluid devices. These devices use electrodes whose electric field development time is equal to RC and has a time scale response on the order of micro seconds. One of the reasons for extensive research in ER fluids is to take advantage of the very short time scale.

Physically heavy apparatus are necessary for MR fluids. The iron used in the magnetic particles of the fluid, the iron of the electro-magnetic circuit can become heavy and cumbersome for large systems. ER devices on the other hand are relatively light since the electric fields can be generated by thin sheets of metal acting as electrodes.

Ferrofluids do not exhibit yield stress, but show an increase in the viscosity. The viscosity under an applied magnetic field increases almost twice as much as the viscosity when

there is no magnetic field applied. Since ferrofluids are synthesized by colloidal magnetic particles, these fluids are more stable than MR fluids which are based on non-colloidal magnetic particles. The comparison of MR, ER fluids, and ferrofluids is summarized in Table 2. 1.

	MR Fluids	ER Fluids	Ferrofluids
Particulate Material	Iron, ferrites, etc	Zeolites, Polymers, SiO ₂ , BaTiO ₃	Ceramics, Ferrites, iron, cobalt, etc.
Particle size	0.1-10 μ m	0.1-10 μ m	2-10 nm
Suspending fluid	Nonpolar oils, polar liquids, water and other	Oils	Oils, Water
Density (g/cc)	3-5	1-2	1-2
Off viscosity (mPa-s)	100-1000	50-1000	2-500
Required field	~ 3 kOe	3kV/mm	~1 kOe
Field Induced changes	$\tau_y(B) \sim 100 \text{ kPa}$	$\tau_y(E) \sim 10 \text{ kPa}$	$\frac{\Delta\eta(B)}{\eta(0)} \approx 2$
Device excitation	Electromagnets or permanent magnets	High voltage	Permanent magnet

Table 2. 1 Comparison of field responsive fluids (Genc 2002)

2.3-History of Magnetorheological Effect

The magnetorheological (MR) effect was first observed in the 1940's by Jacob Rabinow, an engineer who was working at the US National Bureau of Standards. He observed that the presence of a magnetic field would cause some fluids to thicken and become solid. This discovery led to the subsequent patent of Rabinow's "Magnetic Fluid Force and Torque Transmitting Device" (Rabinow 1951).

This discovery occurred almost concurrently with Willis Winslow's discovery of electrorheological (ER) fluids (Winslow 1947; Winslow 1953). These fluids were found to solidify when subjected to high electric fields. Although these two fluids were discovered at approximately the same time, most of the research and development of controllable fluids had been concentrated on the ER effect.

The primary feature of MR fluids is their controllability and reaction time. MR fluids react in milli-seconds to an applied magnetic field. To fully utilize these reaction time characteristics, one must have an electro-magnetic system that reacts more quickly than the MR fluid itself. Because the technology of the era did not offer practical control systems with sufficient reaction times to make use of the fast MR material reaction time, use of MR materials as a variable control device was not explored. This contributed to dormancy of MR material research, which lasted until the 1970's.

Research in MR fluids slowed down in 1980's, but did not go dormant. This slow down may be attributed to the rise in interest in electrorheological materials, as they are the

counterpart of MR fluids. The research on electrorheological materials showed a high degree of sensitivity of the material to contaminants and in the 1990's the focus switched back to the more robust MR fluids.

The robustness of MR fluids has been investigated and confirmed through research and device development. Some of the investigations in to the robust behavior of MR. fluids concentrated on such issues as temperature (Weiss and Duclos 1993), magnitude of applied magnetic field vs power supply (Tang and Conrad 1996), history dependency (repeatability of cycling the applied magnetic field on and off)(Ashour, Rogers et al. 1996), stability of the carrier medium and MR material reaction time. These investigations revealed that all of these issues favor the use of MR fluids in practical applications. A temperature range between -40°C and 150°C has minimal effect on the yield stress(Ginder, Davis et al. 1996; Ginder, Elie et al. 1996). A maximum value of the yield stress is obtained when magnetic saturation of the fluid is reached. Any further increase in magnetic field has a diminishing effect on the rheological properties of the MR fluid. The yield limit is associated with the magnetic saturation limit and is caused by limits on the magnetic flux density ($\vec{B}$) in the MR fluid.

The response of the MR fluid is reproducible when the magnetic field is cycled. There is negligible magnetic memory (Ginder, Davis et al. 1996; Ginder, Elie et al. 1996) that would cause a magnetic history dependent effect. Rosensweig (Rosenweig 1985) refers to particle size as the reason for little or no magnetic memory. Ferrofluids (particle sizes on the order of nano-meters) do have magnetic memory. The magnetic memory can be viewed as the ability of the particles to retain their dipole alignment after the magnetic

field has been removed. MR fluid particles are large enough so that gravity will cause the particles to move when no magnetic field is applied. This movement promotes rotation and displacement of the dipoles so they are no longer aligned. No dipole alignment means no magnetization. Thus MR fluids have a negligible magnetic memory. Ferrofluid particles are of such a size that the Brownian forces are greater than gravity forces. These Brownian forces keep the particles in place after the magnetic field has been removed. Thus the alignment of the particles remain and magnetic memory exists in ferrofluids.

MR fluids are currently being utilized by industry and some commercial devices are made using the magnetorheological effect, like MR linear dampers and rotary clutches and brakes. Research on MR material is being performed simultaneously by industry and academia. Industry research is focus on optimization of the MR effect by developing new MR fluids. Academic research is focusing on developing analytical descriptions of the fluid behavior.

2.4-Composition of MR Fluids

The compositions of MR fluid typically consist of three components: the ferromagnetic particles, a carrying fluid, and a stabilizer.

a) Ferromagnetic particles

Iron and iron-base alloys are the principal magnetosoft materials in use today (Carlson and Weiss 1995; Ashour, Rogers et al. 1996; Kordonsky and Demchuk 1996). The ferromagnetic particles are typically made from carbonyl iron and the magnetosoft

particles are usually spherical in shape. A particle size range of $1 \sim 10\mu m$ is preferred although broader range of $0.1\mu m \sim 100\mu m$ is also acceptable. In addition, the magnetosoft particles should have no residue magnetization and can randomly distribute in fluids. The traditional powders are iron particles, such as carbonyl particles and reduced iron particles. Iron alloy particles including iron-cobalt alloys and iron-nickel alloys are found to produce better performance than the traditional ones (Carlson and Weiss 1995). The drawback, however, is the high cost of such alloys in comparison to carbonyl iron particles.

b) Carrier fluid

The carrier fluid serves as a continuous isolating medium. Preferably, the carrier liquid should have a viscosity value in the range of $0.01 \sim 1.0 PaS$ at $40^\circ C$ (Ashour, Rogers et al. 1996). In general an ideal carrying liquid should have a low initial viscosity, good anti-corrosion properties, low toxicity, and a wide range of temperature stability. Some commonly used carrier fluids are silicon oil, kerosene and synethic oil (Ashour, Rogers et al. 1996). Water and glycol also may be used as the carrier fluid (Guangqiang 2001).

c) Stabilizer

The stabilizer serves to provide added stability to the fluid composition. Stability of MR fluids is one of the most important properties of the fluid. It determines to a large extent the lifetime and durability of the fluid. There are three distinct types of stability associated with MR fluids: agglomerative stability, sedimental stability, and thermal stability (Ashour, Rogers et al. 1996). Agglomerative stability refers to the resistance of the particles to the formation of aggregates. Sedimental or settling stability ensures that

the particles do not settle down with time. Thermal stability refers to the ability of the fluid to function at reduced and elevated temperatures, and its resistance to degradation when held at elevated temperatures over long periods of time. Most of the work on stability of MR fluids, in general has been on the first two kinds of stability (Ashour, Rogers et al. 1996). The stabilizer particles can be spherical in shape and preferably be non-porous. The stabilizer particles are approximately $0.005 \sim 0.015 \mu m$ in diameter. The relatively small diameter of the stabilizer particles results in a relatively large collective surface area. Stabilizer particles having a surface area of about $350 \text{ to } 400 m^2 / g$ is adopted. Typical stabilizers include inorganic silicon compounds. Table 2. 2 shows the general properties of MR fluids.

Specification	Remark
High Dynamic Yield Strength	Instantaneous generation of from 0 to 90 kPa yield stress for precise real-time control
Wide Operating Temperature Range	-40° to 150° with less than 10% variation in maximum dynamic yield stress
Viscosity (No field)	$0.08\text{-}0.25 \text{ Pa}\cdot\text{s}$ @ 25° C
Millisecond Response Time	Ideal for use in applications requiring continuously variable, high-precision control
Non-Abrasiveness	Additives in MR fluids reduce abrasiveness, enabling devices to achieve required life expectancies
Low Sedimentation Level	The formulation of MR fluids solves sedimentation and stratification problems found in other MR fluids
Current Density	Can energize with permanent magnets
Specific Gravity	2-4
Ancillary Materials	Iron/Steel
Color	Black/Gray

Table 2. 2 General properties of MR fluids (Weihua 2001)

The use of dispersant is not always sufficient for the provision of stability. The problem of stability over time had been dealt with in a number of ways.

It has been proposed (Kordonsky and Demchuk 1996) that the addition of a magneto-solid powder as a dispersant helps to form a spatial structure within the carrier fluid due to its intrinsic magnetic moment in the absence of an externally applied magnetic field. This spatial structure not only provides the necessary temporal stability, but in the presence of an external magnetic field these magnetic particles play an active role and help to enhance the MR effect. The formation of this structure which occurs over a period of time and is quickly destroyed under shear flow conditions is referred to as a thixotropic network.

Thixotropy is defined (Harris 1977) as a decrease of the apparent viscosity under shear stress, followed by gradual recovery with time when the stress is removed. In order to prevent settling of the dispersed phase, it is desirable that a thixotropic network be formed (Kordonsky, Shulman et al. 1990; Margida, Weiss et al. 1996). A thixotropic network is defined as a suspension of particles that at low or zero shear rates form a loose network or structure over a period of time. This structure is often referred to as “clusters” or “floculates”. This loosely knit three-dimensional structure reduces particle settling. When a shearing force is applied to the fluid, this structure is quickly dispersed and the degree of rigidity obtained from the thixotropic formation vanishes.

Thixotropic agents which have been found to be effective are low molecular weight hydrogen bonding agents or polymer modified metal oxides. Various colloids can be

added in order to enhance this effect. Silica gel is also a preferred thixotropic agent and its porous time –induced structure not only provides stability against particle settling but its hygroscopic quality allows for the removal of any undesirable moisture.

A thixotropic agent which has been found to not only provide stability against settling but also to greatly enhance the MR effect is the addition of a porous carbon fiber powder (Shtarkman 1991). The intertwined fibers have been found to mechanically suspend the particulate phase. When added to a carrier fluid, these fibers tend to gel the fluid and prevent settling. However when exposed to shear, this structure breaks down and displays excellent flow properties.

Until now we have discussed the general properties of field responsive fluids including MR fluids. Some other properties and specifications of field responsive fluids require a deep understanding of magnetism. In the following section of the report, we discuss the most important topics in magnetism which are necessary to design a good MR device.

2.5-Magnetism and Magnetic Properties of Materials

A magnetic field is produced whenever there is electrical charge in motion. This can be due to an electrical current flowing in a conductor for example, as first discovered by Orested in 1819. A magnetic field is also produced by a permanent magnet. In this case there is no conventional electric current, but there are the orbital motions and spins of electrons (the so called “Amperian currents”) within the permanent magnet material which lead to a magnetization within the material and a magnetic field outside. The

magnetic field exerts a force on both current carrying conductors and permanent magnets. The direction of the magnetic field is such that it circulates the conductor obeying the right hand rule.

As a measure of strength of the magnetic field we define the magnetic field strength H . The unit of the magnetic field strength is amperes per meter. A current of 1 ampere passing through a straight 1 meter length of a conductor generates a tangential field strength of $\frac{1}{4\pi}$ amperes per meter at a radial distance of 1 meter.

The law which enables us to calculate the magnetic field H generated by an electrical current is the Biot-Savart law. It is a statement of experimental observation rather than a theoretical prediction. In its usual form the law gives the field contribution generated by a current flowing in an elementary length of conductor,

$$\delta \vec{H} = \frac{1}{4\pi r^2} i \delta \vec{l} \times \vec{u} \quad (2.1)$$

where i is the current flowing in an elemental length $\delta \vec{l}$ of a conductor, r is the radial distance, $\vec{u}$ is a unit vector along the radial direction and $\delta \vec{H}$ is the contribution to the magnetic field at r due to the current element $i \delta \vec{l}$.

Figure 2. 2 i passing through a length element of $d\vec{l}$ and its magnetic field.

From Equation (2.1) we can observe that the magnetic field strength H is solely determined by the size and distribution of currents producing it and is independent of the material medium. For steady currents the above equation is equivalent to the Ampere's law. According to Ampere the magnetic field generated by an electrical circuit depends on the shape of the circuit (i.e. the conduction path) and the current carried. By assuming that each circuit is made up of an infinite number of current elements each contributing to the field, and by summing or integrating these contributions at a point to determine the field, Ampere arrived at the result

$$\int_s \vec{j} \cdot d\vec{s} = \oint_c \vec{H} \cdot d\vec{l} \quad (2.2)$$

where j is the current density ($\text{amperes}/\text{m}^2$), $d\vec{l}$ is simply a line vector and $d\vec{s}$ is an area vector for the surface $\vec{s}$ as shown in Figure 2. 3. It should be noted that this equation is only true for steady currents.

Figure 2. 3 Closed path c and current density j passing through it.

When a magnetic field $\vec{H}$ is generated in a medium by a current, in according with Ampere's law, the response of the medium is its magnetic induction $\vec{B}$, also sometimes called the flux density. All media will respond with some induction and, the relation between magnetic induction and magnetic field is a property called the permeability of the medium.

Whenever a magnetic field is present in free space there will be a magnetic flux Φ . This magnetic flux is measured in units of webers and its rate of change can be measured since it generates an e.m.f. in a closed circuit of conductor through which the flux passes. One

weber is the amount of magnetic flux which when reduced uniformly to zero in one second produces an e.m.f. of one volt in a one-turn coil of conductor through which the flux passes.

The flux density in *webers/meter²* is also known as the magnetic induction $\vec{B}$ and consequently a flux density of one weber per square meter is identical to a magnetic induction of one tesla. The relation between magnetic induction and magnetic field is a property called the permeability of the medium.

There is often some confusion between the concept of magnetic field $\vec{H}$ and the magnetic induction $\vec{B}$. Here we discuss some important issues about the differences between the concepts of $\vec{H}$ and $\vec{B}$. In many media $\vec{B}$ is a linear function of $\vec{H}$. In particular in free space we can write

$$\vec{B} = \mu_0 \vec{H} \quad (2.3)$$

where μ_0 is the permeability of free space which is a universal constant. $\vec{H}$ is measured in *amps/metre* and $\vec{B}$ is measured in tesla ($=Vs/m^2$), the units of μ_0 are therefore *(volt sec)/(amp metre)*, also known as *henries/metre*, and its value is

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}.$$

However in other media, particularly ferromagnets and ferrimagnets, $\vec{B}$ is no longer a linear function of $\vec{H}$, nor is it even a single-valued function of $\vec{H}$, but $\vec{H}$ and $\vec{B}$ are still related by permeability of the medium μ through the equation,

$$\vec{B} = \mu \vec{H} \quad (2.4)$$

but now of course μ is not necessarily a constant. In paramagnets and diamagnets μ is constant over a considerable range of values of $\vec{H}$. However in ferromagnets μ varies rapidly with $\vec{H}$.

2.5.1-Magnetic dipole moment

As shown above in Ampere's theorem a current in an electrical circuit generates a field. A loop of a conductor carrying an electric current is the simplest circuit which can generate a magnetic field (Figure 2. 4). We can describe this current carrying coil with a single vector $\vec{m}$, its magnetic dipole moment. We take the direction of $\vec{m}$ to be that of the normal vector to the plane of the coil. The magnitude of the dipole moment is

$$m = NiA \quad (2.5)$$

in which N is the number of turns in the coil, i is the current through the coil and A is the area enclosed by each turn of coil.

Figure 2. 4 The concept of magnetic moment.

The torque on a magnetic dipole moment m in an external magnetic induction $\vec{B}$ is then simply

$$\vec{\tau} = \vec{m} \times \vec{B} \quad (2.6)$$

A magnetic dipole in an external magnetic field has a magnetic potential energy that depends on the dipole's orientation in the field.

$$E = -\vec{m} \cdot \vec{B} \quad (2.7)$$

2.5.2-Magnetization

When a magnetic field $\vec{B}$ is applied to a magnetic material, the material becomes magnetically polarized, i.e., it develops a magnetic dipole moment per unit volume $\vec{M}$, which is called the magnetization. Magnetization is a density- the density of magnetic moment. The dimensions of $\vec{M}$ are A/m . In some materials $\vec{M}$ points in the direction of the applied field $\vec{B}$, in which case the material is called paramagnetic; in others $\vec{M}$

points opposite to the direction of $\vec{B}$, in which case the material is called diamagnetic. In ferromagnetic materials $\vec{M}$ is parallel to $\vec{B}$ and very large.

The magnetic induction $\vec{B}$ consists of two contributions: one from the magnetic field, the other from the magnetization of the material. The magnetic induction on free space is $\mu_0 \vec{H}$ and the contribution to the induction from the magnetization of a material is $\mu_0 \vec{M}$. The magnetic induction is then simply the vector sum of these,

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) \quad (2.8)$$

where $\vec{B}$ is in tesla, and $\vec{H}$ and $\vec{M}$ are in apms per meter. The above equation which relates these three basic magnetic quantities is true under all circumstances.

The magnetic field $\vec{H}$ is generated by electrical currents outside the material either from a solenoid or electromagnet, or from a permanent magnet. The magnetization is generated by the resultant spin and orbital angular momentum of electrons within the solid.

2.5.3-Permeability and susceptibility

We can now make a general statement for the permeability μ and also introduce the susceptibility of magnetic materials. The permeability is defined as

$$\mu = \left| \frac{\vec{B}}{\vec{H}} \right| \quad (2.9)$$

and the susceptibility is defined as

$$\chi = \left| \frac{\vec{M}}{\vec{H}} \right| \quad (2.10)$$

Since $\vec{B}$ and $\vec{M}$ may or may not be linear functions of $\vec{H}$, depending on the type of material or medium, it should be noted that permeability and susceptibility may or may not be constant.

The relative permeability of a medium, denoted by μ_r , is given by

$$\mu_r = \frac{\mu}{\mu_0} \quad (2.11)$$

the relative permeability of free space is 1. The relative permeability is closely related to the susceptibility and the following equation is always true

$$\mu_r = \chi + 1 \quad (2.12)$$

2.5.4-Diamagnets, paramagnets and ferromagnets

In diamagnetism, weak magnetic dipole moments are produced in the atoms of the material when the material is placed in an external magnetic field $\vec{B}$; the combination of all those induced dipole moments gives the material as a whole only a feeble net magnetic field. The dipole moments and thus their net field disappear when the external

field is removed. The susceptibility value for diamagnetic materials is small and negative $\chi \approx -10^{-5}$. Examples of diamagnets are copper, silver, gold, bismuth and beryllium. Superconductors form another group of diamagnets for which $\chi = -1$.

Paramagnetism is exhibited by materials containing transition elements, rare earth elements, and actinide elements. Each atom of such a material has a permanent resultant magnetic dipole moment, but the moments are randomly oriented in the material and the material as a whole lacks a net magnetic field. However an external magnetic field $\vec{B}$ can partially align the atomic magnetic dipole moments to give the material a net magnetic field. The alignment and thus its field disappear when $\vec{B}$ is removed. For paramagnets the value of χ is small and positive and typically $\chi \approx 10^{-3}$ to 10^{-5} . The magnetization of paramagnets is weak but aligned parallel with the direction of magnetic field. Examples of paramagnets are aluminum, platinum and manganese.

Ferromagnetism is a property of iron, cobalt, nickel, and certain other elements (and of compounds and alloys of these elements). Some of the electrons in these materials have their resultant magnetic dipole moments aligned, which produces regions with strong magnetic dipole moments. An external field $\vec{B}$ can then align the magnetic moments of such regions, producing a strong magnetic field for a sample of the material; the field partially persists when $\vec{B}$ is removed. We usually use the term ferromagnetic material, and even the common term magnetic material, to refer to materials that exhibit primarily ferromagnetism. For ferromagnetic materials the susceptibility is positive, much greater than 1, and typically can have values $\chi \approx 50$ to 10000 .

In ferromagnetics neither χ nor μ_r have a constant value. Both permeability and susceptibility in ferromagnetics are strongly affected by the prevailing magnetic field $\vec{H}$ and the previous history of the material (Figure 2.5).

Figure 2.5 Permeability is not constant in ferromagnetics (Fitzgerald 2003)

2.5.5-Hard and soft magnetic materials

We can make a simple classification of ferromagnetic materials on the basis of their coercivity. Coercivity is a structure sensitive magnetic property, which is to say that it can be altered by subjecting the specimen to different thermal and mechanical treatments, in a way that for example saturation magnetization cannot. The meaning of coercivity becomes more clear as we discuss hysteresis in section 2.5.6.

It has been noticed in the past that iron and steel specimens that were mechanically hard also had high coercivity, while those that were soft had low coercivity. Therefore the

term ‘hard’ and ‘soft’ magnetic materials were used to distinguish ferromagnets on the basis of their coercivity. Broadly ‘hard’ magnetic materials were those with coercivities above 10 KA/m while soft magnetic materials were those with coercivities of below 1KA/m.

There are some other types of magnetic materials apart from the three classes of diamagnets, paramagnets and ferromagnets given above. These other materials are all very closely related to ferromagnets because they are magnetically ordered. They are ferrimagnets, antiferromagnets, helimagnets and superparamagnets. All were discovered many years after the three classical groups of magnetic materials discussed above. From bulk magnetic measurements the ferrimagnets are indistinguishable from ferromagnets, while the antiferromagnets and helimagnets were for many years mistaken for paramagnets.

2.5.6-Hysteresis

As discussed earlier when an external magnetization force is applied to a ferromagnetic material, the domain magnetic moments tend to align with the applied magnetic field. As a result the domain magnetic moments add to the applied field, producing a much greater value of flux density than would exist due to the magnetization force alone. Thus the effective permeability μ , equal to the ratio of the total magnetic flux density to the applied magnetic field intensity, is large compared with the permeability of free space μ_0 . As the magnetization force is increased, this behavior continues until all the magnetic

moments are aligned with the applied field; at this point they can no longer contribute to increasing the magnetic flux density, and the material is said to be fully saturated.

In the absence of an externally applied magnetization force, the domain magnetic moments naturally align along certain directions associated with the crystal structure of the domain, known as axes of easy magnetization. Thus if the applied magnetization force is reduced, the domain magnetic moments relax to the direction of easy magnetism nearest to that of the applied field. As a result, when the applied field is reduced to zero, although they will tend to relax towards their initial orientation, the magnetic dipole moments will no longer be totally random in their orientation; they will retain a net magnetization component along the applied field direction. It is this effect which is responsible for the phenomenon known as magnetic hysteresis.

Due to this hysteresis effect, the relation between $\vec{B}$ and $\vec{H}$ for a ferromagnetic material is both nonlinear and multivalued. In general, the characteristics of the material cannot be described analytically. They are commonly presented in graphical form as a set of empirically determined curves based on test samples of the material.

The most common curve used to describe a magnetic material is the $B-H$ curve or hysteresis loop. The first and second quadrants (corresponding to $B \geq 0$) of a set of hysteresis loop are shown in Figure 2. 6 for M-5 steel, a typical grain-oriented electrical steel used in electrical equipment. These loops show the relationship between the magnetic flux density $\vec{B}$ and the magnetization force $\vec{H}$. Each curve is obtained while cyclically varying the applied magnetizing force between equal negative and positive

values of fixed magnitude. Hysteresis causes these curves to be multivalued. After several cycles the $B-H$ curves form closed loop as shown. The arrows show the paths followed by $\vec{B}$ with increasing and decreasing $\vec{H}$. Notice that with increasing magnitude of $\vec{H}$ the curves begin to flatten out as the material tends toward saturation.

As $\vec{H}$ is decreased from its maximum value to zero, the flux density decreases but not to zero. This is the result of the relaxation of the orientation of the magnetic moments of the domains as described above. The result is that there remains a remanant magnetization $\vec{M}_R$ when $\vec{H}$ is zero.

The magnetic induction can be reduced to zero by applying a reverse magnetic field of strength $\vec{H}_C$. This field strength is known as the coercivity. It is strongly dependent on the condition of the sample, being affected by such factors as heat treatment and deformation.

Figure 2. 6 Hysteresis loop (Fitzgerald 2003)

Fortunately, for many engineering applications, it is sufficient to describe the material by a single-valued curve obtained by plotting the locus of the maximum values of $\vec{B}$ and $\vec{H}$ at the tips of the hysteresis loops; this is known as the *dc* or normal magnetization curve. A dc magnetization curve for M-5 grain oriented electrical steel is shown in Figure 2. 7.

Figure 2. 7 a) B-H loops for M-5 grain-oriented electric steel 0.012 in thick. Only the top halves of the loops are shown here. b) DC magnetization curve for M-5 grain-oriented electric steel 0.012 in thick (Fitzgerald 2003)

2.5.7-Magnetic circuits and reluctance

A magnetic circuit consists of a structure composed for the most part of high permeability magnetic material. The presence of high permeability magnetic material tends to cause magnetic flux to be confined to the paths defined by the structure, much as currents are

confined to the conductors of an electric circuit. A simple example of magnetic circuit is shown in Figure 2. 8. The core is assumed to be composed of magnetic materials whose permeability is much greater than that of the surrounding air ($\mu \gg \mu_0$). The core is of uniform cross section and is excited by a winding of N turns carrying a current of i amperes. This winding produces a magnetic field in the core, as shown in the Figure 2.8.

Figure 2. 8 Simple magnetic circuit (Fitzgerald 2003)

Because of high permeability of the magnetic core, an exact solution would show that the magnetic flux is confined almost entirely to the core, the field lines follow the path defined by the core, and the flux density is essentially uniform over the cross section because the cross sectional area is uniform. The magnetic field can be visualized in terms of flux lines which form closed loops interlinked with the winding.

The magnetic flux ϕ crossing a surface S is the surface integral of the normal component of $\vec{B}$; thus

$$\phi = \iint_S \vec{B} \cdot d\vec{A} \quad (2.13)$$

The above equation states that the net magnetic flux entering or leaving a closed surface is zero. This is equivalent to saying that all the flux which enters the surface enclosing a volume must leave that volume over some other portion of that surface because magnetic flux lines form closed loops.

These facts can be used to justify the assumption that the magnetic flux density is uniform across the cross section of a magnetic circuit such as the core of Figure 2. 8. So

$$\phi_c = B_c A_c \quad (2.14)$$

where, ϕ_c is the flux, B_c is the flux density and A_c is the cross-sectional area of the core.

From the ampere's law we also have

$$F = Ni = \oint \vec{H} \cdot d\vec{l} \quad (2.15)$$

The core dimensions are such that the path length of any flux line is close to the mean core length l_c . So we may write the simple following equation

$$F = Ni = H_c l \quad (2.16)$$

where H_c is the average magnitude of $\vec{H}$ in the core. The direction of H_c in the core can be found from the right hand rule.

In many devices there is an air gap in the magnetic circuit. In MR devices the MR fluid is the medium in the magnetic circuit. A magnetic circuit with an air gap is shown in Figure

2.9. When the air gap length g is much smaller than the dimensions of the adjacent core faces, the magnetic flux ϕ will follow the path defined by the core and the air gap. If the air gap length becomes excessively large, the flux will be observed to “leak out” of the air gap and the techniques of magnetic circuit analysis will no longer be applicable.

Figure 2.9 Magnetic circuit with air gap (Fitzgerald 2003)

Thus, provided the air gap length g is sufficiently small, the configuration of Figure 2.9 can be analyzed as a magnetic circuit with two series components: a magnetic core of permeability μ , cross-sectional area A_c , and mean length l_c , and an air gap of permeability μ_0 , cross-sectional area A_g , and length g . In the core the flux density can be assumed uniform; thus

$$B_c = \frac{\phi}{A_c} \quad (2.17)$$

and in the air gap

$$B_g = \frac{\phi}{A_g} \quad (2.18)$$

By applying the Ampere's law to this circuit we have

$$F = H_c l_c + H_g g \quad (2.19)$$

and assuming a linear relationship between B and H

$$F = \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g \quad (2.20)$$

Here the $F = Ni$ is the M.M.F applied to the magnetic circuit. A portion of mmf, $F_c = H_c l_c$, is required to produce magnetic field in the core while the reminder, $F_g = H_g g$, produces magnetic field in the air gap.

By using the equations developed so far we can write the mmf in terms of the magnetic flux

$$F = \phi \left(\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g} \right) \quad (2.21)$$

the terms that multiply the flux in this equation are known as the “reluctance $\mathcal{R}$ ” of the core and the air gap respectively,

$$R_c = \frac{l_c}{\mu A_c} \quad (2.22)$$

$$R_g = \frac{g}{\mu_0 A_g} \quad (2.23)$$

and thus

$$F = \phi (R_c + R_g) \quad (2.24)$$

finally, Equation (2.24) can be used to solve for the flux

$$\phi = \frac{F}{R_c + R_g} \quad (2.25)$$

In general, for any magnetic circuit of total reluctance R_{tot} , the flux can be found as

$$\phi = \frac{F}{R_{tot}} \quad (2.26)$$

Note that Equations (2.24) and (2.25) are analogous to the relationships between the current and voltage in an electric circuit. This analogy is illustrated in Figure 2. 10. Figure 2. 10a shows an electric circuit in which a voltage V drives a current I through resistors R_1 and R_2 . Figure 2. 10b shows the schematic equivalent representation of the magnetic circuit of Figure 2.9. Here we see that the mmf F (analogous to voltage in electric circuit) drives a flux ϕ (analogous to the current in the electric circuit) through the combination of the reluctances of the core and the air gap.

Figure 2. 10 Analogy between electric and magnetic circuits. a) Electric circuit, b) Magnetic circuit

(Fitzgerald 2003)

Having enough information about magnetism and magnetic materials we are now able to move on to discuss the magnetic properties and specifications of MR fluids and also the B-H curves used for MR fluids. These curves, in conjunction with $\tau - H$ curves are very crucial for designing MR devices. Other important issues regarding the flow models, flow modes, rheometry and stability of MR fluids are discussed after.

2.6-Magnetic Properties of MR Fluids

Besides the magnetic properties of isolated particles, the static magnetic properties of MR fluids characterized by B-H and M-H hysteresis are also important. Theoretical models for fluids and devices need magnetization as an input. These magnetic properties will also be helpful in predicting the dependence of the MR response on the applied current in the device.

At very low magnetic fields, the magnetic dipole moment induced in particles within MR fluid is given by (Gans 2000)

$$m = 4\pi\mu_0 R^3 \beta H \quad (2.27)$$

$$\beta = \frac{\mu_p - \mu_f}{\mu_p + 2\mu_f} \quad (2.28)$$

where R is the particle radius and μ_p and μ_f are the relative permeabilities of the particle and the fluid, respectively.

At higher fields where the magnetization of the particles reach saturation, the magnitude of the moments becomes independent of the field and it is given by (Lemaire, Bossis et al. 1993)

$$m = \frac{4}{3} \pi \mu_0 R^3 M_s \quad (2.29)$$

where $\mu_0 M_s$ is the saturation magnetization.

The saturation magnetization of the fluid, $\mu_0 M_f$ is related to the saturation magnetization of the bulk magnetic solid, $\mu_0 M_s$ through the volume fraction ϕ of the solid present

$$\mu_0 M_f = \phi \mu_0 M_s \quad (2.30)$$

2.7- Commercial MR Fluids

The LORD Corporation is the leader in the field of commercial MR fluids and MR devices and currently there are different types of MR fluids and devices manufactured by LORD, such as dampers, brakes, and various types of MR fluids.

Table 2. 3 presents the main properties of different types of MR fluids. Furthermore, the characteristics of MRFs such plastic viscosity, magnetic induction, and yield stress of LORD MRF-132DG oil-based MR fluid are shown in Figure 2. 11, Figure 2. 12, and Figure 2. 13.

	MRF-122-2ED	MRF-132DG	MRF-140CG	MRF-241ES	MRF-336AG
BASE FLUID	Hydrocarbon	Hydrocarbon	Hydrocarbon	Water	Silicone
COMPATIBILITY					
Buna N (Nitrile)	Good	Good	Good	Good	Good
Butyl	Poor	Poor	Poor	Good	Good
EPDM/EPR	Poor	Poor	Poor	Good	Good
Fluoroelastomer	Good	Good	Good	Good	Good
Natural Rubber*	Poor	Poor	Poor	Poor	Good
Neoprene	Good	Good	Good	Good	Good
Silicone	Fair	Fair	Fair	Fair	Poor
Iron	Good	Good	Good	Good	Good
Stainless Steel	Good	Good	Good	Good	Good
Aluminum	Good	Good	Good	Fair	Good
Polyurethane	Good	Good	Good	Poor	Fair
OPEN/CLOSED SYSTEM	Open or Closed	Open or Closed	Open or Closed	Closed	Closed
OPERATING TEMPERATURE, °C(°F)	-40 to +130 (-40 to +266)	-40 to +130 (-40 to +266)	-40 to +130 (-40 to +266)	-10 to +70 (14 to 158)	-40 to +150 (-40 to +302)
VISCOSITY					
Pa @40°C (104°F) calculated as slope 500-800 sec ⁻¹	0.07 ± 0.02	0.092 ± 0.015	0.225 ± 0.325	—	—
Pa at 10 sec ⁻¹	—	—	—	10.8 ± 1.5	8.5
Pa at 50 sec ⁻¹	—	—	—	2.2 ± 0.4	—

Table 2. 3 magnetorheological fluids (LORD Corp.)

Figure 2.11 Yield stress vs. magnetic field strength (LORD Corp.)

Figure 2.12 Shear stress as a function of shear rate with no magnetic field applied at 40°C (LORD Corp.)

Figure 2.13 Typical magnetic properties of MRF-132DG (LORD Corp.)

The biggest challenge of MR fluid is to have high yield stress, temperature stability, redispersibility, and durability. The second biggest challenge of MR fluids is the materials science oriented studies such as surface chemistry, polymer physics, in synthesizing stable and redispersible MR fluids.

To our knowledge there are not many systematic published studies on this aspect of MR fluid technology. Making durable MR fluids is also another challenge. There have been temperature studies in determining the yield stress of MR fluids at different temperatures and these studies revealed the temperature stability of MR fluids. However, there are not many studies conducted by exposing the MR fluid to high shear rates and high shearing stresses.

Although the “off-state” viscosity of MR fluids is a crucial parameter for having a high MR effect, it has not been investigated in detail. The rheology of MR fluids in their off state is also worth studying.

2.8-MR Rheology

Generally there are two distinct rheological domains over which MR fluids operate: the pre-yield and the post-yield regions. The pre-yield region is characterized by a strong viscoelastic nature while the post-yield region shows a dominant viscous behavior. The two rheological domains are separated by the yield point, which varies as a function of the applied magnetic field (Shulman, Kordonsky et al. 1986; Ginder, Davis et al. 1996).

In describing the flow behavior of MR fluid, shear strain rate, temperature, volume fraction, and field strength are often considered as important variables.

In the pre-yield regime the fluid acts like a solid and shows viscoelastic behavior.

Mathematically we can describe the pre-yield regime as

$$\tau = G\gamma, \quad \dot{\gamma} = 0 \quad \tau < \tau_y$$

Figure 2. 14 MRF behavior in pre-yield regime (Genc 2002)

For $\tau \geq \tau_y$, i.e. the post-yield regime, the material starts to flow. A typical shear stress versus shear strain rate behavior observed for a MR fluid in the presence of a magnetic field variation is illustrated in Figure 2. 15 (Yalcintas 1995). In order to describe MR rheology, the classical Newtonian definition of the apparent viscosity as the linear relationship between the shear stress and the shear strain rate can be used. But a Newtonian type of approximation is only valid for a MR fluid in the absence of a magnetic field.

Figure 2. 15 Post-yield shear behavior of MRF (Weihua 2001)

In Figure 2. 15, τ_{yd} and τ_{ys} are respectively the dynamic and static yield stresses of the MR fluid. The dynamic yield stress τ_{yd} reflects the flow characteristics of MR fluids whereas the static yield stress τ_{ys} is defined as the stress necessary to initiate flow within the MR material. The dynamic yield stress is governed by the behavior of the MR fluid in the post yield flow region and is obtained by extrapolation of the shear stress versus shear strain rate measurements as shown in Figure 2. 15. It is seen that the dynamic yield stress is smaller than the static yield stress which is true for both ER and MR fluids (Ginder and Davis 1994; Lemaire, Bossis et al. 1996; Bossis, Lemaire et al. 1997; Volkova, Bossis et al. 2000). The static yield stress is due to the transient fracture of the particle chains which is highly dependent upon particle size, particle shape, and flow history of the material (Volkova, Bossis et al. 2000). The post yield behavior is used to model the properties of MR fluids for practical and industrial purposes.

The importance of the “off-state” viscosity of MR fluids comes from the Figure of merit for MR fluids which is given by the “turn up” ratio defined as the ratio of “on-state” yield stress to the “off-state” viscosity. “On-state” refers to the state of the MR fluid under an applied magnetic field and the on-state yield stress behavior depends on the magnetic properties and the volume fraction of the magnetic phase (Ginder, Davis et al. 1996). The off-state viscosity, which is a function of carrier liquid, additives, surfactants (Potanin, Shrauti et al. 1998), particle loading and particle size distribution (PSD) (Foister 1997), is the value when no magnetic field is applied. Due to the addition of additives and surfactants and changes in magnetic particle microstructure during shear, most MR fluids exhibit thixotropic behavior and shear thinning (Ginder 1998). The break up of weak agglomerates or bonds in the shear field is a major cause of a shear thinning behavior of MR fluids.

The structure of MR fluids is anisotropic (Figure 2. 16) due to the formation of the particles along the field lines of the applied magnetic field. Thus, the yield stress will depend on the orientation of the magnetic field. The direction of three fundamental orientations between magnetic field and shear direction where only shear in the x direction is considered, is given in Figure 2. 16. Most MR fluid applications such as clutches, brakes and dampers use the situation in which the magnetic field is applied along the gradient direction (Case B). However, for optical polishing application, Case C is preferred (Shorey, Kordonski et al. 1999). Shorey and co-workers compared the anisotropy of MR fluids by measuring the shear stress for case B and C where both cases have a field orthogonal to the direction of the flow. They found that in general the yield stresses were similar in magnitude but slightly higher for Case C (Shorey, Kordonski et

al. 1999). They concluded that the slight difference in the yield stress may be due to the internal structure formation which would need to be investigated further.

Figure 2. 16 Anisotropy of MR fluids. The value of the yield stress depends on the direction of the applied magnetic field and the shear direction (Genc 2002)

2.9-Rheometry

The design and realization of an actuator with MR fluid requires exact description of the rheological and magnetic properties of the MR fluids. Basic classes of rheometry are considered as stress driven and strain rate driven. Rotational rheometers such as concentric cylinder rheometer, cone and plate rheometer, parallel plate rheometer and double Couette rheometer are basic types of rheometers used in rheological measurements (Janocha and Rech 1994; Laun, Kormann et al. 1996; Macosko 1996)

Figure 2. 17. The shear stress, shear rate, strain and viscosity relations for different geometries are given in Table 2. 4.

Figure 2. 17 Types of rheometer geometries: a) double concentric cylinder, b) cone and plate, c) Parallel plate, and d) concentric cylinder (Genc 2002)

	Shear stress	Shear rate	Strain	Viscosity
Concentric Cylinder	$\frac{M}{(2\pi R_{ave}^2 h)}$	$\frac{2\Omega R_1^2 R_2^2}{r^2 (R_2^2 - R_1^2)}$	$\frac{\theta R_{ave}}{R_2 - R_1}$	$\frac{M(R_2^2 - R_1^2)}{4\pi h \Omega R_1^2 R_2^2}$
Parallel Plate	$\frac{M}{2\pi R^3}$	$\frac{\Omega R}{h}$	$\frac{R\theta}{h}$	$\frac{\pi \Omega M R^4}{2h}$
Cone and Plate	$\frac{3M}{2\pi R^3}$	Ω/α	$\frac{\theta}{\alpha}$	$\frac{3\alpha M}{2\pi R^3 \Omega}$
Double Concentric	$\frac{M}{2\pi h(R_1^2 + R_4^2)}$	$\frac{\Omega R_4^2}{(R_4^2 - R_3^2)} + \frac{\Omega R_1^2}{(R_2^2 - R_1^2)}$	$\frac{\theta R_4^2}{(R_4^2 - R_3^2)} + \frac{\theta R_1^2}{(R_2^2 - R_1^2)}$	$\frac{M(R_4^2 - R_3^2)(R_2^2 - R_1^2)}{2\pi h}$

Table 2. 4 Equations of rheological properties for different geometries. In these equations M is the torque, h is the height, R, is the radius (Figure 2.17), Ω is the angular velocity, θ is the angular displacement and α is the cone angle (Ferry 1961; Ferry 1980; Macosko 1996; Genc 2002)

The concentric cylinder rheometers are best for lower viscosity systems and high shear rates. The gravitational settling of suspensions has less effect than cone and plate and parallel plate. The shear rate across the gap is not uniform whereas in cone and plate rheometers, the shear rate is constant. Cone and plate rheometers are good for high and low viscosity systems. However, this geometry is not good as high shear rates. Rotational parallel plate rheometer is very useful for obtaining viscosity and normal stresses at high shear rates. It allows changing the gap height very easily which can be very useful in terms of determining the wall slip at two different gaps (Yoshimura and Prud'homme 1988).

A rotational parallel-plate geometry inserted into a coil was used by Lemaire and Bossis (Lemaire and Bossis 1991) whereas Laun et al. (Laun, Kormann et al. 1996) presented measurements performed with a concentric rheometer. A cone and plate type rheometer was utilized by Gans and co-workers in order to measure rheological behavior of the inverse ferrofluids. By choosing a small cone angle, they minimized the sample variations in the radial direction (de Gans, Blom et al. 1998). Shorey and co-workers developed a magnetometry for understanding properties of MR fluids for optical polishing (Shorey, Kordonski et al. 1999).

2.10-Micro-Structure

As stated, the effect of a magnetic field on MR fluids causes the particles to form chain-like fibrils and creates a micro-structure within the MR fluid. These chain-like fibrils align themselves parallel to the applied magnetic field. At small length scales, the fibril phenomenon is due to magnetic dipole interaction. The fibril phenomenon has been modeled considering the magnetic dipole interaction of adjacent particles along a single fibril at a constant magnetic saturation level (Ginder, Davis et al. 1996). The adjacent particle interaction effects were averaged over the entire fluid in order to model the effect at longer length scales. The fibril chain is assumed to consist of perfect spherical magnetic particles aligned in single columns (Figure 2. 18) along with the following assumptions and simplifications.

- The magnetic field $\vec{H}$ is constant in magnitude and orientation through out the fluid.
- The magnetic saturation of the fluid particles is uniform.

- Linear magneto-static principles are appropriate.
- The particles are perfect spheres of radius r .
- The particles are aligned with perfect symmetry.
- Only adjacent particles interaction (top & bottom) are calculated.
- No multi-dipole interaction or adjacent fibril interaction (side to side) is considered.
- The particle fibril volume remains constant as the fibril is sheared.
- For a single particle the fibril volume is $2h\pi w^2$, where h is the half height of a particle fibril volume and w is the radius of the particle fibril volume.

Figure 2. 18 Magnetic dipole fibril model (Baltimore 1998)

Tang et al. (1995) determined the force along axis z between two adjacent particles to be

$$F = 24\pi\mu_0\mu_f r^2 (\beta H)^2 \left(\frac{r}{2h} \right)^4 \quad (2.31)$$

The role of the induced magnetic flux ($\vec{B}$), the applied magnetic field ($\vec{H}$), and the magnetization ($\vec{M}$) are some of the most important variables in the response of MR fluids due to their non-linear relationship.

2.11-MR Flow Modes

All devices that use MR fluids can be classified as operating in: (a) a valve mode (fixed poles), (b) a direct shear mode (relatively movable poles), (c) a squeeze mode, or a combination of these modes (Carlson and Spencer 1996). Diagrams of these basic modes of operation are shown in Figure 2. 19. Examples of valve mode devices include servo-valves, dampers, shock absorbers and actuators. Shear mode devices include clutches, brakes, chucking and locking devices, dampers and structural composites.

The pressure drop developed in a device based on pressure driven flow mode can be divided into a field independent viscous component Δp_η and an applied field dependent induced yield stress component Δp_τ . These pressures may be approximated by (Baba, Yabushita et al. 1998):

$$\Delta p_\eta = \frac{12\eta QL}{g^3 w} \quad (2.32)$$

$$\Delta p_\tau(H) = \frac{c\tau_y L}{g} \quad (2.33)$$

where Q is the volumetric flow rate, η is the viscosity with no applied field and τ_y is the yield stress developed in response to an applied magnetic field H . The parameter c has a

value that ranges from a minimum value of 2 (for $\frac{\Delta p_\tau}{\Delta p_\eta}$ less than ~ 1) to a maximum value of 3 (for $\frac{\Delta p_\tau}{\Delta p_\eta}$ greater than ~ 100). The total pressure drop in a pressure driven flow mode device is approximately equal to the sum of Δp_τ and Δp_η .

In a like manner, the force developed by a direct-shear device can be divided into viscous F_η and field induced yield stress F_τ components:

$$F_\eta = \frac{\eta SA}{g} \quad (2.34)$$

$$F_\tau(H) = \tau_y(H)A \quad (2.35)$$

where the total force developed by a direct-shear device is the sum of F_η and F_τ .

While less well-understood than the other modes, the squeeze mode has been used in some small-amplitude vibration dampers.

Figure 2. 19 Basic operating modes for controllable fluid devices (Weihua 2001)

To date, several MR fluid devices have been developed for commercial use by the LORD Corporation (Carlson, Catanzarite et al. 1996; Jolly, Bender et al. 1999). Linear MR fluid

dampers have been designed for use as secondary suspension elements in vehicles. MR fluid rotary brakes are smooth-acting, proportional brakes which are more compact and require substantially less power than competing systems. MR fluid vibration dampers for real-time, active-control of damping have been used in numerous industrial applications.

In civil engineering applications, the expected damping forces and displacements are rather large in magnitude. Therefore, MR dampers primarily operating under direct shear mode or squeeze mode might be impractical. Usually valve mode or its combination with direct shear mode is employed. These dampers are capable of meeting real-world requirements and are presently either in commercial production or in production prototype trials.

2.12-MR Fluid constitutive Models

a) Yield stress: The yield stress of the MR fluids mainly depends on the saturation magnetization and volume fraction of the magnetic particles. In the analytical models developed by Ginder and co-workers, the yield stress increases linearly with increasing volume fraction (Ginder and Davis 1994; Ginder, Davis et al. 1996). However, at high volume fractions, the exponential increase of yield stress with increasing volume fraction was reported by Volkova and Chin (Chin, Park et al. 2001). This can be attributed to the higher packing of particles where the affine deformation can be restricted leading to higher stresses (Chin, Park et al. 2001).

b) Viscosity: The viscosity of a fluid can be defined as the constant of proportionality between the shear stress of a fluid and its velocity gradient or shear rate:

$$\tau = \eta \dot{\gamma} \quad (2.36)$$

For Newtonian fluids this relationship is linear and η is constant. However, for some fluids, including MR and ER fluids, this relationship is non-linear and a different means of relating the shear stress and shear rate is necessary.

Figure 2. 20 shows several types of rheological behavior. Curve E represents a Newtonian fluid with constant viscosity. Curve D represents a shear-thinning (pseudoplastic) fluid while curve F represents a shear-thickening (dilatant) fluid. Curves A, B and C represent fluids possessing a yield stress.

Often the flow behavior of a fluid is described by its “apparent viscosity”. This term represents the slope of the line connecting the origin to a specific point on the shear stress shear rate plot. The “differential viscosity” represents the derivative of the shear stress with respect to the shear rate. For fluids which possess a yield stress, the apparent viscosity is sometimes used to describe their rheological behavior; however, this one parameter model does not always provide sufficient insight into a fluid’s true behavior.

Figure 2. 20 Viscous types of rheological behavior (Gupta 1996)

Bingham model:

A simple Bingham visco-plasticity model (Phillips 1969), as shown in Figure 2. 21, is effective at describing the essential field-dependent fluid characteristics. In this model, the total shear stress τ is given by

$$\tau = \tau_y(H) \text{sgn}(\dot{\gamma}) + \eta \dot{\gamma} \quad (2.37)$$

where τ_y is yield stress caused by the applied field, H is magnitude of the applied magnetic field, $\dot{\gamma}$ is shear strain rate and η is field-independent plastic viscosity, defined as the slope of the measured post-yield shear stress versus shear strain rate.

Figure 2. 21 Idealized Bingham plastic behavior of MR fluids (Weihua 2001)

Herschel-Bulkley model:

Note that the fluid post-yield viscosity is assumed to be a constant in the Bingham model. But in many situations it is observed that the viscosity decreases as the strain (shear) rate increases. This phenomenon is called “shear thinning” which is shown in Figure 2. 22 and Figure 2. 23. Because MR fluids exhibit shear thinning effect the Herschel-Bulkley visco-plasticity model (Herschel and Bulkley 1926) can be employed to accommodate this effect. In this model, the constant post-yield plastic viscosity in the Bingham model is replaced with a power law model dependent on shear strain rate. Therefore,

$$\tau = \left[\tau_y(H) + K |\dot{\gamma}|^n \right] \text{sgn}(\dot{\gamma}) \quad (2.38)$$

where n and K are positive fluid parameters. K is called the consistency index and n is the power law index of the Herschel-Bulkley model. It can be seen that the equivalent plastic viscosity of the Herschel-Bulkley model is

$$\eta_e = K |\dot{\gamma}|^{(n-1)} \quad (2.39)$$

Figure 2.22 Decrease of apparent viscosity with shear rate (Weihua 2001)

Figure 2.23 Shear thinning in MRF (Guangqiang 2001)

Equation (2.39) indicates that the equivalent plastic viscosity η_e decreases as the shear strain rate $\dot{\gamma}$ increases when $n < 1$ (shear thinning). Furthermore, this model can also be used to describe the fluid shear thickening effect when $n > 1$. The Herschel-Bulkley model reduces to the Bingham model when $n = 1$, therefore $\eta = K$.

Figure 2. 24 shows a typical comparison between the Bingham model and the Herschel-Bulkley model. As shown the Herschel-Bulkley model is capable of predicting the shear thinning behavior of MR fluids while the Bingham model is not. This advantage of the Herschel-Bulkley model raises the interest of using the Herschel-Bulkley model in analyzing the flow of MR fluids at high shear rates where the effects of shear thinning is of great concern.

In Figure 2. 24 the data are based on the experiment done by Weihua, L.(Weihua 2001) with the parameters of Table 2. 5.

Model	Model Equation	Identified Parameters
Bingham model	$\tau = \tau_{yd} + \eta\dot{\gamma}$	$\tau_{yd} = 12910$ $\eta = 44$
Herschel-Bulkley model	$\tau = \tau_{yd} + K\dot{\gamma}^n$	$\tau_{yd} = 10200$ $K = 1058$ $n = 0.41$

Table 2. 5 Parameters for the Bingham and the Herschel-Bulkey model (Weihua 2001).

Figure 2. 24 Comparison between the Bingham model and the Herschel-Bulkley model

2.13-Applications of MR Fluids

As stated earlier the main advantages of the MR effect are the reversibility of properties, the fast response, low power requirements, and wide temperature stability (Carlson 1994; Carlson, Catanzarite et al. 1996). Hence, MR fluids have tremendous potential in providing simple and effective interfaces between electronic controls and mechanical systems. MR fluids are expected to play a significant role in modern industrial areas including power transmission systems, aerospace and aeronautic servo systems, active control of structure vibration, MEMS, adaptive structure, optical grinding, and robot systems (Carlson and Chrzan 1994; Carlson, Chrzan et al. 1994; Carlson and Spencer

1996; Kordonski and Jacobs 1996; Bansbach 1998; Carlson 1999). Most of MR applications to date have been on the development of dampers, clutches, valves, engine mounts, and brakes (Carlson 1994; Carlson and Chrzan 1994; Carlson, Catanzarite et al. 1996; Carlson and Spencer 1996; Bansbach 1998; Ginder 1998; Phule 1998; Jolly, Bender et al. 1999).

Optical polishing, which was first initiated by Kordonski and co-workers, is another promising application of MR fluids (Kordonski and Jacobs 1996; Jacobs, Arrasmith et al. 1999). MR fluid contains a nonmagnetic polishing abrasive. Under high shear the flow of non-magnetic abrasive particles cause material removal. The most commonly used polishing abrasive/carrier liquid combination for optical polishing for all optical glasses and crystals is cerium oxide/water combination. Abrasives like alumina and diamond are used for materials other than glasses.

The Lord Corporation has currently manufactured MR fluid devices for commercial applications including heavy-duty vehicle seat suspensions, rotary brakes that provide a tunable resistance for exercise equipment and vibration dampers for various industrial applications such as dampers for washing machines (www.lord.com). The damper that contains MR fluids for commercial applications such as seat suspensions is presented in Figure 2. 25.

Figure 2. 25 MR fluid damper, 1) Plastic shaft, 2) Sponge saturated with MR fluid, 3) Coil, 4) Steel tube, 5) Wire supplying current.

Figure 2. 26 shows the schematic of a damper used for seismic applications. A rotary brake is also shown in Figure 2. 27. Figure 2. 28 shows a three-dimensional view of a MR clutch.

Figure 2. 26 Schematic of a prototype damper for seismic applications (Jolly, Bender et al. 1999)

Figure 2. 27 MR rotary brake (LORD Corp.).

Figure 2. 28 A three dimensional view of a prototype MR brake (Kavlicoglu, Gordaninejad et al. 2002)

It has been found that the application of a magnetic field produces a significant change in heat transfer properties (Kordonsky, Shulman et al. 1990). When the direction of both

heat and magnetic flux are aligned, heat transfer properties are significantly enhanced while a mutually perpendicular orientation of magnetic field and direction of heat flux significantly decreases heat transfer.

2.14-Stability of MR Fluids

The stability and redispersibility of MR fluids have been one of the most important issues of these materials. Stable MR fluids are considered to exhibit no or very little amount of particle settling. For dilute systems, the dependence of the sedimentation velocity of a spherical particle can be obtained from Stoke's law as follows (Hiemenz 1986):

$$v = \frac{2}{9} \frac{R_s^2 (\Delta\rho) g}{\eta} \quad (2.40)$$

R_s is the particle radius, $\Delta\rho$ is the difference in density of the magnetic phase and carrier liquid, η is the viscosity of the carrier liquid and g is the gravitational acceleration ($9.8m/s^2$). Since, less viscous liquids will aggravate the settling of the particles in an MR fluid, Rankin and co-workers formulated a suspension with viscoplastic continuous phase (e.g., grease) to prevent sedimentation (Rankin, Horvath et al. 1999). When the yield stress of the viscoplastic medium is bigger than the critical yield stress that was defined for each particulate material and particle radius, the particles are suspended. Although, for most of the applications the Figure of merit for MR fluids is to keep the off state viscosity as small as possible, for applications such as control of seismic vibrations,

paste-like MR fluids can be more appropriate since the gravitational settling over an extended period of time can be prevented.

If the settled magnetic particles in the MR fluid can be easily dispersed by applying very little mechanical energy, i.e. stirring or shaking, then the fluid is said to be *redispersible*. Some fluids exhibit poor redispersibility and therefore “cake-like” formation is observed at the bottom of the fluid which would degrade the MR response.

Another problem associated with MR fluids was In-Use-Thickening (IUT), in an early version of Lord Corporation's MotionMaster RD-1005 truck seat damper, the off-state force (which would ideally remain constant at the initial low value over the life of the product) increased by a factor of 2.5 after 600000 on-state cycles, rendering it ineffective (Toy, Scriven et al. 1991). This increase in the off-state force is due to an increase in the off-state viscosity of the fluid, a phenomenon known as "In-Use Thickening" (IUT) and believed to be caused by spalling of the surface layer of the particles in the MR compound. After some years of fluid development, this problem was overcome and presently MR fluids can endure long on-state periods without deterioration.

The exact MR fluid life depends on a variety of factors, including the shear rate, temperature and duration. An estimate of the usable life of an MR fluid can be obtained from the following relation (Toy, Scriven et al. 1991):

$$LDE = \frac{1}{V} \int_0^{life} P dt \quad (2.41)$$

where V is the fluid volume (since LDE is generally given in J/cm^3 , this should be expressed in cm^3), and P is the dissipated power. Hence, LDE is the energy dissipated per unit volume of the fluid throughout its operation and can be compared to the permissible value obtained from experimental data.

2.15-Wall Slip Effects

Slip occurs in the flow of two or multi phase suspensions because of the displacement of the dispersed phases away from the walls of the geometries used in the rheometer leaving a layer of particles next to the wall more dilute than in the bulk dispersion. For suspensions, gravity can enhance the slip effect especially in parallel plate or cone and plate geometries. During flow, shear induced particle migration can occur due to the shear rate gradient (Acrivos 1995) and this results in a low viscosity layer adjacent to the wall that leads to an “apparent wall slip”. The conditions that lead to large and significant slip effects can be summarized as: large particles as the dispersed phase, smooth wall, small flow dimensions, low speeds and flow rates, walls and particles carrying electrostatic charges which would cause the walls of the platens to repel adjacent particles (Barnes 1995). The slipping at the walls can be prevented by roughening the surface of the walls as to increase the friction between the walls and the suspension. In rough surfaces, the particles become trapped whereas in the smooth surfaces, they can roll over small bumps more easily.

It is proposed by some researchers (de Vicente, Lopez-Lopez et al. 2004) that the yield of the MR fluid is not generally due to rupture of the aggregates but rather to their slip on

the plates of rheometer. This slipping occurs when the applied stress overcomes the solid friction of the particles on the plates. This yield stress is commonly known as frictional yield stress and is obtained by extrapolating the stress toward zero shear rates in a log-log scale of the plot of shear stress vs. shear rate.

2.16-Dynamic Investigation of MR Fluids

Many MR devices operate in dynamic modes where the fluid is subjected to oscillatory shear (Jolly, Bender et al. 1999). Though development of MR fluids and devices requires knowledge of their response under such condition, the MR rheology under such conditions has not been studied extensively and is not well understood. (Weiss, Carlson et al. 1994).

The linear viscoelastic properties of ER and MR suspensions for sufficiently small strain amplitudes have been investigated extensively by many researchers (Brooks, Goodwin et al. 1985; Shulman, Korobko et al. 1989; Korobko and Shul'man 1990; McLeish, Jordan et al. 1991; Weiss, Carlson et al. 1994). However the non-linear viscoelastic properties of ER and MR fluids under large amplitude oscillatory shear still needs further study. Although there is a unifying theory that describes linear viscoelasticity, there is no unifying constitutive theory for the non-linear case.

MR fluids, as well as other viscoelastic materials, such as polymers, emulsions, or ER fluids, exhibit linear viscoelastic properties at sufficiently small strain rang ($\gamma \leq \gamma_{lin}$).

Here, γ_{lin} is a critical strain, which is used to separate linear viscoelastic region from non-linear viscoelastic region.

Assuming MR suspension is subjected to an oscillatory strain with sufficiently small amplitude defined by:

$$\gamma = \gamma_0 e^{i\omega t} \quad (2.42)$$

where $\gamma_0 \ll 1$ is the strain amplitude and ω is the angular frequency in rad/sec . The shear rate is the time derivation of γ , which is in the form

$$\dot{\gamma} = \gamma_0 \omega e^{i(\omega t + \pi/2)} \quad (2.43)$$

In this case, the shear stress response (τ) to oscillatory shear would be

$$\tau = \tau_0 e^{i(\omega t + \delta)} \quad (2.44)$$

where δ is called the mechanical loss angle, whose range is $0^\circ < \delta < 90^\circ$ as shown in

Figure 2. 30. For fluids the constitutive equation is $\tau = \eta \frac{\partial v}{\partial y} = \eta \dot{\gamma}$, where η is the

viscosity and $\frac{\partial v}{\partial y}$ is the velocity gradient. If the fluid is subjected to an oscillatory strain

$\gamma = \gamma_0 e^{i\omega t}$, then the response to this oscillatory strain is a shear stress with 90° difference

in phase $\tau = \eta \gamma_0 e^{i(\omega t + \frac{\pi}{2})}$. If this oscillatory excitation is applied to an elastic solid with the

constitutive equation of $\tau = G\gamma$, the response would be $\tau = G\gamma_0 e^{i\omega t}$ which is in phase

with the strain. So the mechanical loss angle of $\delta = 0^\circ$ corresponds to perfect elastic

materials and $\delta = 90^\circ$ for purely viscous materials. The relationship between stress and strain under sinusoidal deformation for elastic, viscous and viscoelastic material is given in Figure 2. 29.

Figure 2. 29 Stress and strain relationship in oscillatory measurement

For a linear viscoelastic material we can divide the response stress τ into two components, an elastic stress (τ') in phase with strain, and a viscous stress (τ'') in phase with strain rate:

$$\tau^* = \tau' + i\tau'' \quad (2.45)$$

Figure 2. 30 Strain and stress in dynamic measurement and their corresponding components.

The dynamic shear modulus of the viscoelastic material is defined by dividing the complex shear stress by the applied strain amplitude

$$G^* = \frac{\tau^*}{\gamma_0} = \frac{\tau'}{\gamma_0} + i \frac{\tau''}{\gamma_0} = G' + iG'' \quad (2.46)$$

where G' is the storage modulus and G'' is the loss modulus presented by

$$G' = |G^*| \cos(\delta) \quad (2.47)$$

$$G'' = |G^*| \sin(\delta) \quad (2.48)$$

The storage modulus, G' , represents the ability of the viscoelastic material to store the energy of deformation, which contributes to the material to the store the energy of deformation, which contributes to the material or device stiffness. The loss modulus, G'' ,

represents the ability of the material to dissipate the energy of deformation as heat. The loss angle, δ , is an indication of whether the material is more solid-like, or liquid-like.

Another equation widely used for viscoelastic materials is

$$\tan \delta = \frac{G''}{G'} \quad (2.49)$$

$\tan \delta$ is called the loss factor, which is the ratio of the two complex components. The two quantities, G'' and $\tan \delta$, are of interest for vibration damping. If they are small for a given vibration frequency and magnetic field, then the damping will be small. For large values of G'' and $\tan \delta$, the damping will be augmented.

2.17-Measures of Controllability

The controllability of MR fluids is most often assessed using either the shear ratio or the excess shear.

a) Shear ratio

The shear ratio of an MR fluid can be described as the ratio of the shear stress under an applied magnetic field divided by the shear stress of the fluid in the absence of a magnetic field.

$$shear\ ratio = \frac{\tau_H}{\tau_0} \quad (2.50)$$

This parameter allows one to judge the controllability of the fluid at a certain shear rate. The shear ratio varies with shear rate and is typically found to decrease with increasing shear rate which is due to the shear-thinning of the MR fluid by increasing the shear rate.

b) Excess shear

Excess shear in an MR fluid can be described as the difference between the observed shear stress under an applied magnetic field and the Newtonian shear stress under no applied magnetic field.

$$\tau_e = \tau_H - \tau_0 \quad (2.51)$$

Some other measures of controllability are defined in the literature which are useful for specific applications. For example Huang (Huang, Zhang et al. 2002) used the control torque ratio (ratio of the field induced torque to the viscous torque) as a parameter to measure the controllability of the rotary brake for design purposes.

2.18-Physical Phenomena behind the MR Effect

When subjected to a magnetic field, the magnetic particles within an MR fluid orient themselves along the direction of magnetic force and form a stable structure. It is this field induced structure which is responsible for the rheological behavior of magnetic suspensions. Bossis (Bossis, Clercx et al. 1993) states that “*the chaining of particles and the phase separation which are due to attractive forces between the dipoles induced of each solid particle by the external field is at the heart of the ER or MR effect.*” It has also

been stated that the equilibrium between the breaking and reforming of chains results in the MR fluid's field induced yield stress (Lemaire, Bossis et al. 1996).

Several researchers have found that the application of a magnetic field will result in the formation of separate, densely packed, elongated, practically non-interacting aggregates which possess a very small (close to point) contact area with the wall (Kordonsky and Gorodkin 1996). Bossis (Bossis, Clercx et al. 1993) has also stated that the experimentally observed structure does not consist of equally spaced chains of particles but of thick columns.

When considering the resistance to flow of the fluid, the MR effect is dependent on the relative orientations of the magnetic field and the fluid flow direction. This resistance to flow has been found to be most pronounced at a mutually perpendicular orientation of the magnetic field and direction of fluid flow (Kordonsky, Shulman et al. 1990). There is not much research done on other modes of operation of MR fluids such as parallel field/flow orientation and investigation into other modes of operation, and thus possible applications, for magnetorheological materials is hampered by the relative lack of data and models (Baltimore 1998). Figure 2. 31 and Figure 2. 32 show the difference between the chain formation in the perpendicular field/flow orientation and parallel field/flow orientation.

Figure 2. 31 The MR effect as the rupture of particle chains (fibrils) in perpendicular field/flow orientation (Baltimore 1998)

Figure 2. 32 The MR effect viewed as rupture between particle chains in parallel field/flow orientation (Baltimore 1998)

2.19-Super Strong MR Fluids

Since the discovery of MR fluids, there has been some expectation that MR fluids may revolutionize certain industrial sectors. This possibility will become reality only if we can enhance the strength of MR fluids. The current strength of MR fluids, for example, is not sufficient for automobile clutches and manufacturing flexible fixtures. Stronger MR fluids are needed for these applications. In addition, it is especially desirable if these strong MR fluids only require a moderate magnetic field. MR applications need electromagnets inside the MR devices. If the required magnetic field is strong, the electromagnets will be heavy and bulky, making the MR device bulky. The

electromagnets are also well known for their delay time since the magnetic coils are a RL circuit. To produce a strong magnetic field, the magnetic coils must have a low dc resistance R and a high inductance L . The stronger the magnetic field is, the longer is the delay time. For a large electromagnet, the delay time L/R can be as long as several seconds. While MR fluids themselves respond to an external field rapidly, the delay time of the electromagnets lengthens the response time of MR devices. Only if the required magnetic field is moderate will the size and delay time of electromagnets not become an issue and MR devices will remain agile. This, however, is not an easy task. Efforts in searching for new MR materials in the past decades achieved limited results. The results of current research on the material properties of MR fluid imply that improving the physical properties of magnetic particles alone is unlikely to bring the strength of MR fluids to a required level (Ginder and Davis 1994; Tao 2001). To improve the microstructure of the MR fluid is the key. Current research in this area is concentrated on employing a “rapid compression-assisted aggregation process” to force MR fluids to form a microstructure that is much stronger than the single chain structure (Gulley and Tao 1993).

Theoretical simulations and experimental results confirm that the formation of a conventional MR microstructure has two developmental stages (Tao and Sun 1991; Tao 2001). The first stage is the chain formation process, which only takes several milliseconds to complete after the magnetic field is applied. The second stage is a relatively slow process, involving the aggregation of chains into thick columns. In the simulation this process takes several seconds. When the MR system is large, this process may take a couple of minutes.

In formation of MR chains, there are two forces which are playing important roles. One is the dipolar interaction and the other is the Brownian forces. In comparison with the dipolar interaction, the Brownian force is very weak. For most MR fluids, the ratio of the interaction energy of two magnetic dipoles to the thermal energy is quite large. Therefore, at the first stage of chain formation, the Brownian force is negligible in comparison with the interaction between two dipoles. However, once the chains are formed, the dipolar interaction is screened. The interaction between two straight chains is weak and short-ranged. Then the Brownian force plays an important role and easily makes the chains bent or randomly fluctuating. This phenomenon is associated with the instability of a one dimensional solid. The attracting force between curved chains is stronger than that of straight chains (Halsey and Martin 1993). Therefore, the chains' fluctuations under Brownian motion help their aggregation together. However, once several chains get together to form a column, the 3D column is not easy to bend or fluctuate. The Brownian force has little effect on columns. Thus the column-column aggregation is difficult in the unassisted aggregation process and the developed columns are of limited thickness.

The rheological properties and strength of MR fluids are related to the field-induced microstructure. For example, a thick column structure is stronger than a single-chain structure (Gulley and Tao 1993). However, most agile MR applications require quick response and cannot wait long enough for the columns to form. Moreover, the unassisted aggregation process only produces columns consisting of 10–15 chains (Chen, Zitter et al. 1992; Chen, Zhang et al. 1993).

2.19.1-The Weak Points of MR Microstructure

To develop strong MR fluids, we need to know the rheological properties of the induced MR structures. In particular we want to identify the weak points of these structures and find a way to strengthen them. To begin with, we examine the single-chain structure, which is the basis of all other MR structures.

Consider applying an external shear force on the chain. Under the force, the chain deforms, becomes slanted, and then breaks into two broken chains if the shear force exceeds a threshold. The breaking point should be the chain's weak point under the shear. The conventional wisdom assumes that the breaking point is in the middle of a chain (Gulley and Tao 1993; Halsey and Martin 1993). The system is symmetric around the middle between the two poles; based on this symmetry, it is natural to guess that the chain could break at the middle. However, this is not the case. After a magnetic field is applied, the space is no longer isotropic. Assume that the shear process is quasi-static such that the chain relaxes into the lowest energy state at each shearing step. Under such a deformation, the configuration with the minimum energy is no longer symmetric. The symmetric configuration becomes unstable. In particular, during the initial deformation, a gap emerges at one end. The gap widens as the strain increases. When the strain exceeds a critical value, the chain breaks. The breaking point is at either end, but not in the middle. In another words, the weak points of a MR chain under shear are at its two ends (Figure 2. 33 and Figure 2. 34).

Figure 2.33 (a) A single chain without shear. (b) A slanted chain under a small shear strain. The gap is between the first and second particles. (c) A broken chain when the shear strain exceeds the critical value (Tao 2001).

Figure 2.34 (a) Under a shear force, the ER chain becomes slanted. (b) The chain breaks between the first and second particles when the shear strain exceeds the critical value (Tao 2001).

2.19.2-Super Strong MR Fluids

After understanding the microstructure of MR fluids and identifying its weak points, we are in a position to seek a new agile approach to producing super-strong MR fluids. Let us emphasize that the strength of MR fluids comes solely from the induced microstructure. Different microstructures deliver different strengths. Unfortunately, the unassisted-aggregation process, as discussed before, is slow, produces columns of a limited thickness, and is thus not very useful. A technology that can rapidly produce thick columns with strong and robust ends is therefore needed, i.e. thick columns with no weak points at their ends. This technology is a compression-assisted-aggregation process (Tang, Zhang et al. 2000). Immediately after a magnetic field is applied, we compress the MR fluid along the field direction before a shear force is applied. The magnetic field produces chains in milliseconds. The compression pushes these chains to form thick columns with strong and robust ends. In addition, each column consists of at least a couple of hundred chains. Once the weak points of MR microstructure are strengthened and the columns are very thick, the MR fluids become super-strong.

This process produces quite strong MR fluids (Tang, Zhang et al. 2000; Tang, Zhang et al. 2001; Tao 2001). The shear stress versus shear strain for a typical MR fluid with and without compression-assisted aggregation is shown in Figure 2. 35.

Figure 2. 35 a Shear stress versus shear strain curves with and without compression. The magnetic field for all curves is 372 kA m^{-1} .

This compression-assisted-aggregation process can be illustrated in Figure 2. 36. When a magnetic field is applied, magnetic particles quickly form chains. As the MR fluid is compressed, chains get shorter and bent. When the chains are bent, the attraction between the chains becomes stronger and pulls these chains quickly together (Tao and Jiang 1994). Meanwhile, as many particles are pushed by the plates, the ends of the columns are much thicker than their middle. These are the desirable robust structures. The columns produced by our compression process are much thicker than the product of natural aggregation.

Figure 2. 36 Formation of the robust MR microstructure during compression-assisted aggregation. (a) Chains before the compression. (b) The compression forces the chains to aggregate into thick columns with robust ends.

Super-strong MR fluids are expected to have many industrial applications. For example, the strength of super-strong MR fluids exceeds the requirement for manufacturing flexible fixtures and automobile clutches. The current approach is also applicable to ER fluids. If the structure-enhanced yield stress of ER fluids can also be 10 times higher than the yield stress without compression-assisted-aggregation, this method will enable ER fluids to have a yield stress strong enough for many industrial applications.

2.20-Limitations of MR Technology

While MR fluids have attracted considerable interests whether in academic or industrial fields, very few MR devices have been commercialized. This is mainly due to the limitations of MR fluids, such as particle settling, field saturation, wall effect and

response time (Shulman, Kordonsky et al. 1986; Ginder, Davis et al. 1996; Tang and Conrad 1996).

Because of the density mismatch between the solid particles and the carrying fluid, the particle settling is inevitable even though a certain stabilizer is added to the suspension to delay or reduce the settling (Ashour, Rogers et al. 1996).

As discussed earlier the maximum yield stress of MR fluids is dependant on their saturation magnetization, so it cannot be increased infinitely by just increasing the external magnetic field.

Wall effect is used to test the stress transmission efficiency, which depends on the electrode surface (Tang and Conrad 1996). The experimental results show that the rough surface has higher transmission efficiency than the smooth one (Tang and Conrad 1996). Therefore it is recommended to use rough surface with high permeability as electrodes (Lemaire, Bossis et al. 1996).

In applying MR fluids their response time also should be taken into account. It is known that MR fluids have two time constants τ_1 and τ_2 (Ashour, Rogers et al. 1996). τ_1 is called the chain formation time, which is within one milli-second. τ_2 is the system response, including field set up which is around 10 milli-seconds.

Chapter 3

MR Clutches and Brakes

MR fluids can be used in the magnetically controlled devices such as clutches or brakes (Rabinow 1948; Lee, Kim et al. 1999; Huang, Zhang et al. 2002). A MR clutch is a device that transmits torque, by the shear stress of the MR fluids, from the input shaft to the output shaft (see Figure 3. 2 and Figure 3. 3). A MR clutch has the property that its transmitting torque changes quickly in response to an external magnetic field strength.

3.1- MR Clutches

MRF-based clutches are garnering significant research attention in different areas, especially in the automotive industry. MR clutches are being examined as an alternative to torque converters and hydraulic starting clutches for the following reasons: the torque converter is not controllable during vehicle launch and it has very low efficiency at higher speeds of operation. While the hydraulic starting clutch can avoid the former problem, it suffers from low robustness of the hydraulic system and uncertainty in the stroke of the piston in the hydraulic system. These drawbacks may be overcome by using a MRF-based starting clutch (Figure 3. 1).

Table 3. 1 provides a comparison of the different systems that may be used to connect the engine to the transmission. The reason behind using a controllable torque transmitting device between the engine and the transmission is to achieve a smooth vehicle launch. During the launch of a vehicle, a sudden and improper engagement of the clutch could lead to undesirable torsional oscillations in the driveline of the powertrain. This is because during this improper engagement, one loads the engine abruptly with the whole inertia of the vehicle.

Figure 3. 1 Possible vehicle configuration with automatic transmission (Neelakantan and Washington 2005)

Torque converter	<ul style="list-style-type: none"> • No launch control • Low efficiency at higher speeds
Hydraulic starting clutch	<ul style="list-style-type: none"> • Controllable action • Improves efficiency • Uncertainty in stroking • Low robustness
MRF starting clutch	<ul style="list-style-type: none"> • Controllable action • Improves efficiency • Simpler construction • Direct interface with electronics

Table 3. 1 A comparison of various torque transmitting devices (Neelakantan and Washington 2005)

3.1.1- MR Clutch Types

In the absence of an applied magnetic field, the suspended particles of the MR fluid cannot restrict the relative motion between the two parts of the clutch. However, in the course of operation, a magnetic flux path is formed when electric current is put through the solenoid coil. As a result, the particles are gathered to form chain-like structures, in the direction of the magnetic flux path. These chain-like structures restrict the motion of the MR fluid, thereby increasing the shear stress of the fluid. Torque transmission or brake can be achieved by utilizing the shear force of the MR fluid. The transmitting (or braking) torque values can be adjusted continuously by changing the external magnetic field strength.

There are mainly two types of MR clutches being investigated by researchers, the disk type and the drum type (or cylindrical type). In the disk type, the activation region of the MR fluid is the space between two disks which are placed face to face and in the drum type the MR fluid is activated between two concentric cylinders.

Figure 3. 2 shows the operational principle drawing of a disk type (circular plate) MR clutch. This clutch uses the shear stress of the MR fluid acting on the surface of the plates connected to the rotating axis perpendicularly. The driving-plate and the driven-plate are connected to the driving-shaft and the driven-shaft, respectively. The shafts are rotatably supported by bearings. The MR fluid is contained within the operational gap between two circular plates. When the electric current is put through the coil assembly embedded in

the housing, the magnetic flux path is formed, perpendicular to the gap direction. The suspended particles within the MR fluid are magnetized to form the chain-like structures, with the direction of the magnetic flux path. These magnetic particles chain-like structures restrict the movement of the fluid within the gap, thereby increases the viscous characteristics of the suspension, which results in the connection between the driving-plate and the driven-plate. The power is transmitted by the shear stress of the fluid from the driving-shaft to the driven-shaft. The torque developed by the MR clutch goes up rapidly as the applied magnetic field increases. In the absence of the applied field, the particles are dispersed and the torque developed by the viscous force of the fluid cannot lift the load of driven-shaft.

Figure 3. 2 Disk type MR fluid clutch.

Figure 3. 3 Drum type (Cylindrical type) MR fluid clutch.

The drum type clutch has a configuration similar to the one shown in Figure 3. 3. Here, the input shaft is connected to the outer cylinder. The output shaft is connected to a rotary piston. The MRF is held in a small annular gap between the outer cylinder (input) and the rotary piston (output). The coil is housed in the piston. As the MRF is excited using the electromagnetic coil, the torque transmitted across the clutch increases. Compared to the disc-shaped device, the cylindrical clutch is preferred due to the complexities in the practical design of the disc-shaped clutch and also due to the exaggerated effect of centrifuging in disc-shaped clutches compared to their cylindrical counterparts.

3.1.2- Centrifuging in MR Clutches

The major drawback of MRF clutches is that the micron-sized ferrous particles centrifuge at high rotational speeds due to very high centrifugal forces. Since the particles have a higher density relative to the fluid, they move outward easily and the MRF loses its homogeneity. This leads to a change in the behavior of the MRF clutch from the initially

estimated theoretical behavior. To analyze the behavior in such situations, one needs to account for the distribution of the MRF particles along the critical volume (the volume in which the MRF is activated).

3.1.3- MR Sponge Clutches

The usual MRF devices have some disadvantages like the possibility of fluid leakage, etc. To avoid fluid leakage, one has to properly design fluid seals and components, which increases complexity and tightens tolerances in the machinable parts of the device. This can be avoided by using a MRF sponge as a replacement to the MRF itself. The result would make the whole equipment much simpler and also would reduce the cost significantly. Saturating highly absorbent foam material with MRF was first employed by Carlson et al. in devising simple low-cost–low-force vibration dampers to control vibrations in washing machines. The MRF sponge consists of an open-cell polyurethane foam, with about 95% open volume and high wicking capability, which is soaked and saturated with MRF. This foam when wound around the piston of the damper takes the place of the pure MRF itself without any loss of controllability and response time. (Neelakantan and Washington 2005)

Figure 3. 4 MR sponge clutch (Neelakantan and Washington 2005)

A similar concept can now be used to devise an MRF sponge-based transmission clutch. A simple prototype schematic of the MRF sponge clutch is shown in Figure 3. 4. The clutch consists of a rotary piston, which has a MR sponge wound around it. As the current through the electromagnetic coil is varied, one can change the yield stress of the MRF in the sponge and hence control the torque that is transmitted across the MR sponge clutch. By using an MRF sponge clutch one can reduce the effect of centrifuging significantly.

3.2- MR Brakes

In the above sections of this chapter we discussed the main characteristics of MR clutches and mentioned that there are mainly two types of MR clutches, Drum type and Disk type. One of the main applications of MR clutches in automotive industry was also discussed and its relative advantages in comparison with other types of clutches were also mentioned (Table 3. 1). MR brakes are another type of MR devices which are very similar to MR clutches. The work principles are exactly the same. As in MR clutches, applying current through the coils embedded in the housing of MR brake creates the magnetic field and as a result the viscosity of the MR fluid in the Brake changes rapidly. Hence a restraining torque is experienced by the output shaft. In other words, we can say that a MR brake is typically a clutch with the output velocity of zero.

3.2.1- Constitutive Equations Used for MR brakes

To design an MR fluid brake for a given specification, one must establish the relationship between the torque developed by MR fluids and the parameters of the structure and the magnetic field strength. The Bingham constitutive equation is mostly used to derive the design equations of the MR brakes. Although the Bingham model gives satisfactory results for design purposes, but its main deficiency which is its inability to predict the shear thinning, raises the tendency to use the more precise model of Herschel-Bulkley. The Herschel-Bulkley model is capable of predicting the shear thinning phenomenon at high shear rates which is the case for most of the rotary brakes and clutches. There are not so many researches done on the behavior of MR fluids at high shear rates. This kind

of experiments seems to be of great importance because most of the MR devices operate at relatively high shear strain rates. The problem associated with the Herschel-Bulkley model is the relative complexity of the equations and the scarcity of the experimental data.

In the following sections the Herschel-Bulkley model is used to model the flow of MR fluids in MR devices especially in rotary brakes as is the main goal of this research. The flow of MR fluid in circular and rectangular ducts is investigated first and the equations of motion are solved for these simple geometries. The equation of motion is then solved for two main types of MR brake which are disk type and drum type MR brakes.

Because there are not enough experimental data to evaluate the models developed here, as the second part of the research, a test rig is to be designed and built in order to investigate the validity of the equations and also to obtain the parameters associated with the Herschel-Bulkley model.

Chapter 4

Analysis of MR Fluid Flow

In previous chapters, the literature review was presented and an introduction to MR clutches and brakes was given. This chapter will deal with mathematical modeling of MR brakes. As stated earlier, there are two main types of MR brakes, the drum type and the disk type. In this chapter both types are mathematically modeled. But, in order to start the flow analysis in MR brakes, simpler cases are examined first. These simple cases are flow between two concentric cylinders and flow in an annular duct. Although some mathematical modeling for these two simple cases were available in literature, but very little is done on the mathematical modeling using the Herschel-Bulkley model (Wang and Gordaninejad 2000). Most of researchers have used the Bingham model to obtain simple design equations.(Baltimore 1998; Wang and Gordaninejad 2000)

4.1-Flow of MR Fluid in a Rectangular Duct

In this section, the flow of a Herschel-Bulkley fluid in a rectangular duct is considered. The geometry is a long, slender rectangular duct. The duct is long (end effects are negligible and steady flow is fully developed) and thin ($2h \ll b$, where b is the width of the duct). Referring to Figure 4. 1, an expression for maximum fluid velocity in terms of

the pressure gradient (P'), yield stress (τ_y) and the consistency index (k) can be developed. The development is performed by combining force equilibrium, the Herschel-Bulkley constitutive relationship and matching the boundary conditions.

The force equilibrium equations are derived from the differential element.

Figure 4. 1 Flow of MR fluid in a rectangular duct- differential element.

$$pdy - \left(p + \frac{\partial p}{\partial x} dx \right) dy + \tau dx - \left(\tau + \frac{\partial \tau}{\partial y} dy \right) dx = 0 \quad (4.1)$$

$$\frac{\partial \tau}{\partial y} = - \frac{\partial p}{\partial x} \quad (4.2)$$

$$p' \equiv - \frac{\partial p}{\partial x} \quad (4.3)$$

$$d\tau = p' dy \quad (4.4)$$

where τ is the shear stress in the fluid and p is the fluid pressure.(Baltimore 1998)

By applying the boundary conditions, the above differential equation can be solved.

$$\int_{\tau_w}^{\tau} d\tau = \int_h^y p' dy \quad (4.5)$$

$$\tau = \tau_w + p'(y - h) \quad (4.6)$$

where τ_w is the shear stress at the wall, i.e. $y = h$.

As can be seen from the above equation, the shear stress decreases as y decreases; so, the maximum shear stress is the wall stress τ_w . Shear stress decreases to the yield stress τ_y at $y = y_1$ where the fluid behaves like a solid and the shear rate becomes zero and so the velocity becomes constant.

$$\begin{aligned} \text{at } y = y_1 : \tau &= \tau_y \\ \tau_y &= \tau_w + p'(y - h) \end{aligned} \quad (4.7)$$

substituting Equation (4.7) in Equation (4.6) gives:

$$\tau = \tau_y + p'(y - y_1) \quad (4.8)$$

By using the Herschel-Bulkley model and Equation (4.8) and noting that at both bottom and upper part the shear rate is positive (if the direction of the flow is to the right as in Figure 4. 1), the differential equation for the velocity function is derived. Because of symmetry we solve this differential equation for the upper half of the duct ($y_1 < y < h$).

$$\tau = \tau_y \operatorname{sgn}(\dot{\gamma}) + k|\dot{\gamma}|^n \operatorname{sgn}(\dot{\gamma}) \quad (4.9)$$

$$\dot{\gamma} = -\frac{\partial u}{\partial y} > 0 \Rightarrow \tau = \tau_y + k\dot{\gamma}^n = \tau_y + k\left(-\frac{\partial u}{\partial y}\right)^n \quad (4.10)$$

Substituting Equation (4.10) into Equation (4.8) gives:

$$\frac{\partial u}{\partial y} = -\left[\frac{-p'(y_1 - y)}{k}\right]^{(1/n)} \quad (4.11)$$

The velocity is zero at the wall. By using this boundary condition the velocity profile is obtained.

$$\begin{aligned}
 -\int_0^u du &= \int_h^y \left[\frac{p'(y-y_1)}{k} \right]^{(1/n)} dy \\
 u &= -\frac{n}{n+1} \left(\frac{p'}{k} \right)^{(1/n)} \left[(y-y_1)^{(1+1/n)} - (h-y_1)^{(1+1/n)} \right]
 \end{aligned} \tag{4.12}$$

The maximum velocity occurs at the region $0 \leq y \leq y_1$ and its value is:

$$u_{\max} = \frac{n}{n+1} \left(\frac{p'}{k} \right)^{(1/n)} (h-y_1)^{(1+1/n)} \tag{4.13}$$

So the equation of velocity profile is:

$$u = \begin{cases} -\frac{n}{n+1} \left(\frac{p'}{k} \right)^{(1/n)} \left[(y-y_1)^{(1+1/n)} - (h-y_1)^{(1+1/n)} \right] & y_1 < y < h \\ u_{\max} & 0 < y < y_1 \end{cases} \tag{4.14}$$

The Herschel-Bulkley model reduces to the simple Bingham model if $n = 1$.

$$u = -\frac{1}{2} \left(\frac{p'}{k} \right) \left[(y-y_1)^2 - (h-y_1)^2 \right] \tag{4.15}$$

$$u_{\max} = \frac{1}{2} \left(\frac{p'}{k} \right) (h-y_1)^2 \tag{4.16}$$

here it should be noted the if $p' > 0$, then $u > 0$, which means if the pressure gradient is negative ($\frac{\partial p}{\partial x}$), i.e. pressure decreases as we go down stream of the duct, the velocity would be positive i.e. the fluid moves in the positive x direction. Similarly if $p' < 0$, $u < 0$.

Because of symmetry the shear stress must be zero at the centerline of the duct:

$$0 = \tau_y - p'(y_1)$$

$$y_1 = \frac{\tau_y}{p'} \quad (4.17)$$

The region $-y_1 < y < y_1$ where the shear rate is zero and hence the fluid acts as an elastic solid is called “the plug flow region”.

Variation of shear rate, shear stress, and velocity profile is illustrated schematically in Figure 4. 2.

Figure 4. 2 Velocity profile, shear stress diagram and shear rate diagram of a Herschel-Bulkley MR fluid.

The volume flow rate can be determined by integrating the velocity profile over the cross section of the duct.

$$\begin{aligned}
 Q &= 2 \int_0^h u dy \times b = 2b \left[\int_0^{y_1} u dy + \int_{y_1}^h u dy \right] \\
 &= 2b \left[u_{\max} y_1 - \int_{y_1}^h \frac{n}{n+1} \left(\frac{p'}{k} \right)^{1/n} \left[(y - y_1)^{(1+1/n)} - (h - y_1)^{(1+1/n)} \right] dy \right] \\
 &= 2b \left[u_{\max} y_1 + \left(\frac{p'}{k} \right)^{1/n} (h - y_1)^{(2+1/n)} \right]
 \end{aligned} \tag{4.18}$$

where Q is the volumetric flow rate and b is the width of the duct. By substituting Equation (4.13) into Equation (4.18) and simplifying we have:

$$Q = 2b \left(\frac{p'}{k} \right)^{1/n} (h - y_1)^{(1+1/n)} \left[\frac{n}{n+1} y_1 + h - y_1 \right] \tag{4.19}$$

by letting $n = 1$ the volumetric flow rate for a simple Bingham fluid is obtained.

$$Q = 2b \left(\frac{p'}{k} \right) (h - y_1)^2 \left(h - \frac{y_1}{2} \right) \tag{4.20}$$

The non-dimensional form of the volume flow rate can be derived by considering the following non-dimensional variables.

$$\psi = \frac{Q^n k}{2 p' b^n h^{2n+1}} \tag{4.21}$$

$$\phi = \frac{\tau_y}{p' h} \tag{4.22}$$

Where ψ is the non-dimensional flow rate, and ϕ is the non-dimensional yield stress.

By some manipulations the non-dimensional form of flow rate equation is derived.

$$\psi = (1 - \phi)^{(n+1)} \left[1 - \frac{\phi}{n+1} \right]^n \quad (4.23)$$

For a Bingham fluid Equation (4.23) reduces to:

$$\psi = (1 - \phi)^2 \left(1 - \frac{\phi}{2} \right) \quad (4.24)$$

It should be noted that $\phi < 1$. As it is shown in Figure 4. 3, as the non-dimensional yield stress increases, there is more resistance to the flow and therefore, for a constant p' , there would be less volumetric flow going through the duct.

The plot of ψ versus ϕ is shown in Figure 4. 3 for different values of n from 0.2 to 1.

Figure 4.3 Plot of ψ versus dimensionless shear stress ϕ for different values of n .

4.2-Flow of MR Fluid in an Annular Duct

The procedure of solving for the flow field of MR fluid is the same, but the difference is that for the analysis of MRF in an annular duct we use the cylindrical coordinates and thus our equations of motion should be written in this coordinate system. Consider an annular duct with the inner radius of R_1 and an outer radius of R_2 , the inner wall is moving with a velocity of v_0 . As shown in the Figure 4. 4 the flow is symmetric in θ direction.

Figure 4. 4 Velocity profile of MRF flow in an annular duct

Writing the equation of motion in z direction :

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{1}{r} v_\theta \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} + \frac{1}{r} \tau_{rz} + f_z \quad (4.25)$$

assuming symmetry, no change of variables in z direction, no velocity in r direction and neglecting the gravitational effects, the above equation simplifies to:

$$\frac{\partial p}{\partial z} = \frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r} \tau_{rz} \quad (4.26)$$

we use τ instead of τ_{rz} for comfort. The solution to the above differential equation, assuming that the pressure gradient in z direction is constant, is:

$$\tau = \frac{1}{2} p' r + \frac{c_1}{r} \quad (4.27)$$

where $p' = \frac{dp}{dz}$ and c_1 is the constant of integration which should be identified.

To find the velocity profile we need to substitute the constitutive equation of MRF into the equation (4.27) above. Here we use the Herschel-Bulkley model. The equation of Herschel Bulkley model can be written as:

$$\tau = (\tau_y + k|\dot{\gamma}|^n) \text{sgn}(\dot{\gamma}) \quad (4.28)$$

It should be pointed out that Equation (4.28) is only applicable in the regions that the fluid has yielded (regions (I) and (II)). In the plug flow region the flow has constant velocity (region (III)).

Region (I) ($R_1 < r < r_1$)

In this region, as seen from Figure 4. 4, the shear rate is positive and substitution of Equation (4.28) in Equation (4.26) results in:

$$\begin{aligned} \dot{\gamma} = \frac{\partial v}{\partial r} > 0 &\Rightarrow \tau = \tau_y + k \left(\frac{\partial v}{\partial r} \right)^n \Rightarrow \tau_y + k \left(\frac{\partial v}{\partial r} \right)^n = \frac{1}{2} p' r + \frac{c_1}{r} \\ \Rightarrow \int_{v_0}^v dv &= \left(\frac{1}{k} \right)^m \int_{R_1}^r \left(\frac{1}{2} p' r + \frac{c_1}{r} - \tau_y \right)^m \\ v &= \left(\frac{1}{k} \right)^m \int_{R_1}^r \left(\frac{1}{2} p' r + \frac{c_1}{r} - \tau_y \right)^m + v_0 \end{aligned} \quad (4.29)$$

where $m = \frac{1}{n}$.

If $m=1$ then the Herschel-Bulkley model reduces to the Bingham model and:

$$v = \frac{p'r^2}{4k} + \frac{c_1 \ln r}{k} - \frac{\tau_y r}{k} - \left(\frac{p'R_1^2}{4k} + \frac{c_1 \ln R_1}{k} - \frac{\tau_y R_1}{k} \right) + v_0 \quad (4.30)$$

Region (II) ($r_2 < r < R_2$)

In this region the velocity gradient and so the shear rate is negative.

$$\begin{aligned} \dot{\gamma} &= \frac{\partial v}{\partial r} < 0 \\ \tau &= -\tau_y - k \left(-\frac{\partial v}{\partial r} \right)^n \Rightarrow \int_v^0 dv = - \int_r^{R_2} \left[-\frac{1}{k} \left(\frac{1}{2} p'r + \frac{c_1}{r} + \tau_y \right) \right]^m dr \\ v &= \int_r^{R_2} \left[-\frac{1}{k} \left(\frac{1}{2} p'r + \frac{c_1}{r} + \tau_y \right) \right]^m dr \end{aligned} \quad (4.31)$$

Similarly if $m = \frac{1}{n} = 1$, the result is the Bingham equation.

$$v = \frac{p'r^2}{4k} + \frac{c_1 \ln r}{k} + \frac{\tau_y}{k} r - \left(\frac{p'R_2^2}{4k} + \frac{c_1 \ln R_2}{k} + \frac{\tau_y}{k} R_2 \right) \quad (4.32)$$

Boundary conditions

To determine the boundary conditions, we must note the following points:

1- The magnitude of shear stress is equal to τ_y at r_1 and r_2 . According to the constitutive equation, the shear stress is of the same sign as the shear rate. So at r_1 , the shear stress is positive and at r_2 the shear stress is negative.

2- The velocity in region (III) is constant and is equal to the velocity at points $r = r_1$ and

$$r = r_2.$$

3- Another boundary condition is the equation of volumetric flow rate. This equation is used to determine the pressure gradient p' .

These boundary conditions are expressed mathematically as:

Region (I):

$$\begin{aligned} \dot{\gamma} > 0 &\Rightarrow \tau > 0 \\ \text{at } r = r_1 : \tau = \tau_y &\Rightarrow \tau_y = \frac{1}{2} p' r_1 + \frac{c_1}{r_1} \end{aligned} \quad (4.33)$$

Region (II):

$$\begin{aligned} \dot{\gamma} < 0 &\Rightarrow \tau < 0 \\ \text{at } r = r_2 : \tau = -\tau_y &\Rightarrow -\tau_y = \frac{1}{2} p' r_2 + \frac{c_1}{r_2} \end{aligned} \quad (4.34)$$

Combining Equations (4.33) and (4.34) we have:

$$\begin{aligned} \frac{1}{2} p' (r_1 + r_2) &= -c_1 \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \\ p' &= \frac{-2c_1}{r_1 r_2} \end{aligned} \quad (4.35)$$

$$r_1 = r_2 + \frac{2\tau_y}{p'} \quad (4.36)$$

Substituting Equation (4.35) into Equation (4.27) gives

$$\tau = \frac{p'}{2} \left(\frac{r^2 - r_1 r_2}{r} \right) \quad (4.37)$$

If the pressure gradient is negative, i.e. the fluid flows in the positive z-direction, τ is positive in region (I), and negative in region (II).

The velocity equals to the maximum velocity in region (III) and:

$$v(r_1) = v(r_2)$$

$$\left(\frac{1}{k}\right)^m \int_{R_1}^{r_1} \left(\frac{1}{2} p' r + \frac{c_1}{r} - \tau_y\right)^m dr + v_0 = \int_{r_2}^{R_2} \left[-\frac{1}{k} \left(\frac{1}{2} p' r + \frac{c_1}{r} + \tau_y\right)\right]^m dr \quad (4.38)$$

$$v_{\max} = \int_{r_2}^{R_2} \left[-\frac{1}{k} \left(\frac{1}{2} p' r + \frac{c_1}{r} + \tau_y\right)\right]^m dr \quad (4.39)$$

For the Bingham model ($m = 1$), Equation (4.38) simplifies to:

$$\begin{aligned} & \frac{p'}{4k} (r_1^2 - r_2^2 + R_2^2 - R_1^2) + \frac{\tau_y}{k} (R_1 - r_1 + R_2 - r_2) \\ & + \frac{c_1}{k} (\ln r_1 - \ln R_1 + \ln R_2 - \ln r_2) = v_0 \end{aligned} \quad (4.40)$$

The volumetric flow rate is obtained by integrating the velocity profile across the cross section of the duct.

$$Q = \int v dS = 2\pi \int v_I r dr + 2\pi \int v_{II} r dr + 2\pi \int v_{III} r dr \quad (4.41)$$

For the Herschel-Bulkley model, velocity equations (Equations (4.29) and (4.31)) must be replaced in Equation (4.41) and integrated numerically. For the Bingham model; however, we can derive the analytical formula as:

$$\begin{aligned}
 Q = 2\pi \int \left(\frac{p'r^2}{4k} + \frac{c_1 \ln r}{k} - \frac{\tau_y}{k} r - A \right) r dr \\
 + 2\pi \int \left(\frac{p'r^2}{4k} + \frac{c_1 \ln r}{k} + \frac{\tau_y}{k} r - B \right) r dr + \pi v_{\max} (r_2^2 - r_1^2)
 \end{aligned} \quad (4.42)$$

where

$$A = \left(\frac{p'R_1^2}{4k} + \frac{c_1 \ln R_1}{k} - \frac{\tau_y R_1}{k} \right), B = \left(\frac{p'R_2^2}{4k} + \frac{c_1 \ln R_2}{k} + \frac{\tau_y R_2}{k} \right),$$

So,

$$\begin{aligned}
 Q = 2\pi \left[\frac{p'r^4}{16k} + \frac{c_1 r^2 \ln r}{2k} - \frac{c_1 r^2}{4k} - \frac{\tau_y r^3}{3k} - \frac{1}{2} A r^2 \right]_{R_1}^{r_1} \\
 + 2\pi \left[\frac{p'r^4}{16k} + \frac{c_1 r^2 \ln r}{2k} - \frac{c_1 r^2}{4k} + \frac{\tau_y r^3}{3k} - \frac{1}{2} B r^2 \right]_{r_2}^{R_2} \\
 + \pi \left[\frac{p'r_2^2}{4k} + \frac{c_1 \ln r_2}{k} + \frac{\tau_y}{k} r_2 - B \right] (r_2^2 - r_1^2)
 \end{aligned} \quad (4.43)$$

So far we have 4 unknowns which are to be determined, they are: c_1, r_1, r_2, p' . These unknowns can be found by solving Equations (4.36), (4.37), (4.38) and (4.42); simultaneously.

Finally we can write the equation of velocity as:

$$v = \begin{cases} v = \left(\frac{1}{k}\right)^m \int_{R_1}^r \left(\frac{1}{2} p' r + \frac{c_1}{r} - \tau_y\right)^m + v_0 & R_1 < r < r_1 \\ v_{\max} = \int_{r_2}^{R_2} \left[-\frac{1}{k} \left(\frac{1}{2} p' r + \frac{c_1}{r} + \tau_y\right)\right]^m dr & r_1 < r < r_2 \\ v = \int_r^{R_2} \left[-\frac{1}{k} \left(\frac{1}{2} p' r + \frac{c_1}{r} + \tau_y\right)\right]^m dr & r_2 < r < R_2 \end{cases} \quad (4.44)$$

4.3- Flow of MR Fluid between Two Cylinders

4.3.1-Velocity profile

To obtain the governing equation of motion of MR fluid in a drum type brake, Newton equation of motion is applied for the case of an MR fluid confined between two concentric cylinders as shown in Figure 4. 5. The cylindrical coordinate system is used because of the geometry of the brake.

Figure 4. 5 Schematic of the cross section of the brake and the velocity profile

The equation of motion of a fluid particle is:

$$\rho \vec{g} - \vec{\nabla} p + \vec{\nabla} \cdot \vec{\tau}_{ij} = \rho \frac{d\vec{v}}{dt} \quad (4.45)$$

Where

$$\vec{g} = g_r \vec{e}_r + g_\theta \vec{e}_\theta + g_z \vec{e}_z \quad (4.46)$$

$$\vec{v} = v_r \vec{e}_r + v_\theta \vec{e}_\theta + v_z \vec{e}_z \quad (4.47)$$

$\vec{v}$ and $\vec{g}$ are the velocity and gravity fields; respectively, ρ is the fluid density, ∇p is the pressure gradient, and τ_{ij} is the ij th component of the stress tensor.

The equation of motion in θ direction results:

$$\begin{aligned} -\frac{1}{r} \frac{\partial p}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2}{r} \tau_{r\theta} + \rho g_\theta = \\ \rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{1}{r} v_\theta \frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} + \frac{1}{r} v_r v_\theta \right) \end{aligned} \quad (4.48)$$

Assuming steady state flow, symmetry, no velocity in the axial direction, and neglecting the gravitational effects, Equation (4.48) reduces to:

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r} \tau_{r\theta} = 0 \Rightarrow \frac{\partial \tau_{r\theta}}{\partial r} = -\frac{2}{r} \tau_{r\theta} \Rightarrow \frac{\partial \tau}{\tau} = -2 \frac{\partial r}{r} \quad (4.49)$$

where for simplicity τ is used instead of $\tau_{r\theta}$.

By solving the above differential equation, the equation of shear stress as a function of r is obtained as:

$$\tau = \frac{c_1}{r^2} \quad (4.50)$$

Where c_1 is a constant which is determined from the boundary conditions and has the dimension of force.

In this paper, we use the Herschel-Bulkley constitutive model for MR fluid to solve the above equation for the velocity profile within the gap.

$$\tau = (\tau_y + k|\dot{\gamma}|^n) \text{sgn}(\dot{\gamma}) \quad (4.51)$$

where τ_y is the yield stress of the MR fluid which is a function of the applied magnetic field, $\dot{\gamma}$ is the shear rate, n is the power law index, and k is the consistency index of the Herschel-Bulkley model.

The velocity decreases from $R_1\omega$ at $r = R_1$ to zero at $r = R_2$; so, the velocity gradient and shear rate would be negative.

$$\begin{aligned} \dot{\gamma} = \frac{dv}{dr} - \frac{v}{r} \leq 0 &\Rightarrow \tau = -\tau_y - k\left(\frac{v}{r} - \frac{dv}{dr}\right)^n = -\tau_y - k\left(-r\frac{d\omega}{dr}\right)^n \\ \Rightarrow \int_{\omega_0}^{\omega} d\omega &= \int_{R_1}^r \frac{-1}{r} \left(\frac{-c_1}{kr^2} - \frac{\tau_y}{k}\right)^{\left(\frac{1}{n}\right)} dr \end{aligned} \quad (4.52)$$

where Equation (4.50) is used to substitute for τ and at any radius r , the fluid rotational velocity is equal to $\omega = \frac{v}{r}$.

By integrating and applying the boundary conditions the velocity profile will be identified. To evaluate the above integral the third order Taylor expansion of the integrand around R_1 is written:

$$\begin{aligned}
 \frac{-1}{r} \left(\frac{-c_1}{kr^2} - \frac{\tau_y}{k} \right)^m = & \\
 & - \frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2} \right)^m}{R_1} + \left(\frac{2mc_1 \left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2} \right)^m}{R_1^2 (c_1 + \tau_y R_1^2)} + \frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2} \right)^m}{R_1^2} \right) (r - R_1) + \\
 & \left(\frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2} \right)^m \left(\frac{3mc_1}{R_1^2 (c_1 + \tau_y R_1^2)} + \frac{2m(m-1)c_1^2}{R_1^2 (c_1 + \tau_y R_1^2)^2} \right)}{R_1} - \frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2} \right)^m}{R_1^3} - \frac{2mc_1 \left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2} \right)^m}{R_1^3 (c_1 + \tau_y R_1^2)} \right) (r - R_1)^2 \\
 & + O((r - R_1)^3)
 \end{aligned} \tag{4.53}$$

Where $m = \frac{1}{n}$.

It should be noted that Equation (4.53) is the Taylor approximation of the integrand in Equation (4.52). This Taylor expansion gives the exact value at $r = R_1$ and as we move away from $r = R_1$, the Taylor approximation gives more erroneous results.

The integrand in Equation (4.52) is singular at $r=0$, so; the radius of convergence of the Taylor series expansion is R_1 , meaning that Equation (4.53) is approximating Equation

(4.52) in the interval $R_1 < r < 2R_1$. In MR rotary brakes the gap is quite small and is in the order of millimeters, so for MR rotary brakes Equation (4.53) is applicable.

The error term is of the order of $O\left((r - R_1)^3\right)$. Since the maximum value r can be is R_2 , the error term would be $O\left((R_2 - R_1)^3\right)$. But $R_2 - R_1 = h$ which is the gap, and for MR devices, the gap is in the order of 0.5 to 1mm. Assuming gap equal to 1mm ($10^{-3}m$), the maximum error term is in the order of 10^{-9} which gives the credit to use the Taylor expansion up to second order term.

By placing Equation (4.53) in Equation (4.52) and integrating, the velocity of Equation (4.54) is obtained:

$$\begin{aligned}
 \frac{v(r)}{r} = & \frac{1}{3} \left[-\frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m \left(\frac{3mc_1}{R_1^2(c_1 + \tau_y R_1^2)} + \frac{2m(m-1)c_1^2}{R_1^2(c_1 + \tau_y R_1^2)^2}\right)}{R_1} - \frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m}{R_1^3} - \frac{2mc_1 \left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m}{R_1^3(c_1 + \tau_y R_1^2)} \right] (r^3 - R_1^3) \\
 & + \frac{1}{2} \left[\frac{2mc_1 \left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m}{R_1^2(c_1 + \tau_y R_1^2)} + \frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m}{R_1^2} + 2 \left[\frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m \left(\frac{3mc_1}{R_1^2(c_1 + \tau_y R_1^2)} + \frac{2m(m-1)c_1^2}{R_1^2(c_1 + \tau_y R_1^2)^2}\right)}{R_1} + \frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m}{R_1^3} + \frac{2mc_1 \left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m}{R_1^3(c_1 + \tau_y R_1^2)} \right] R_1 \right] (r^2 - R_1^2) \\
 & - \left[\frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m}{R_1} + \frac{2mc_1 \left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m}{R_1^2(c_1 + \tau_y R_1^2)} + \frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m}{R_1^2} \right] R_1 (r - R_1) \\
 & + \left[-\frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m \left(\frac{3mc_1}{R_1^2(c_1 + \tau_y R_1^2)} + \frac{2m(m-1)c_1^2}{R_1^2(c_1 + \tau_y R_1^2)^2}\right)}{R_1} - \frac{\left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m}{R_1^3} - \frac{2mc_1 \left(-\frac{c_1 + \tau_y R_1^2}{k R_1^2}\right)^m}{R_1^3(c_1 + \tau_y R_1^2)} \right] R_1^2 (r - R_1) + \omega_0
 \end{aligned}
 \tag{4.54}$$

4.3.2-Plug Region

Depending on the thickness of the gap ($h = R_2 - R_1$), a plug flow region (Wang and Gordaninejad 2000) may exist. In the plug flow region (see Figure 4. 6), the fluid behaves as a solid and there would be no shear rate. In the rotary brake discussed here, if the gap is assumed to be large, then from a certain radius, say r_p , the fluid acts as a solid and the velocity would be zero. The value of the shear rate $\dot{\gamma}$ will be zero at the plug flow region.

Figure 4. 6 Plug flow region

The plug radius is calculated by imposing $\frac{d\omega}{dr} = 0$ at $r = r_p$, which results in:

$$r_p = \frac{R_1}{2 \left(3mc_1^2 + 5mc_1\tau_y R_1^2 + 2m^2c_1^2 + c_1^2 + 2c_1\tau_y R_1^2 + \tau_y^2 R_1^4 \right)} \quad (4.55)$$

$$\left(\begin{aligned} &4m^2c_1^2 + 3c_1^2 + 12mc_1\tau_y R_1^2 + 2\tau_y^2 R_1^4 + 8mc_1^2 + 6c_1\tau_y R_1^2 + \\ &\sqrt{-4m^2c_1^2\tau_y^2 R_1^4 - 8m^2c_1^3\tau_y R_1^2 - 32c_1^3m\tau_y R_1^2 - 16mc_1\tau_y^3 R_1^6 - 4m^2c_1^4} \\ &\sqrt{-8c_1^4m - 3\tau_y^4 R_1^8 - 18c_1^2\tau_y^2 R_1^4 - 12c_1^3\tau_y R_1^2 - 12\tau_y^3 R_1^6 c_1 - 3c_1^4 - 40mc_1^2\tau_y^2 R_1^4} \end{aligned} \right)$$

In order to evaluate the value of c_1 , the following velocity boundary condition is used:

$$v(r_p) = 0 \quad (4.56)$$

The values of c_1 and r_p can be found by solving Equations (4.55) and (4.56) simultaneously which can be done by a computer code. The equation of $v(r_p) = 0$ is in the form of m-th order non-linear equation for c_1 . If we consider m as a rational number of the form $m = \frac{p}{q}$, then $v(r_p) = 0$ would have q number of solutions. In general, these solutions are not necessarily real numbers. A real value of r_p in the interval $R_1 < r_p \leq R_2$ and the corresponding c_1 are acceptable. Otherwise there would be no plug flow region in the brake.

4.3.3-No Plug Region

If the thickness of the gap is small, then there would be no plug region any more and we have the following boundary condition:

$$v(R_2) = 0 \quad (4.57)$$

The value of c_1 can be determined by the above equation and thus the velocity profile would be identified.

4.3.4-Torque Equations

4.3.4.1-The Herschel-Bulkley Model

The torque developed by the brake is determined by integrating the shear stress on the inner cylinder.

$$\tau = \frac{c_1}{r^2} \quad (4.58)$$

$$dF = \tau dA = \tau (2\pi R_1 dz) \Rightarrow dT = R_1 \tau_{(R_1)} (2\pi R_1 dz) \Rightarrow T = 2\pi c_1 z$$

The values of c_1 and r_p are determined by solving Equations (4.55) and (4.56) if the plug region exists, or by solving Equation (4.57) if there is no plug region. One way to determine whether the plug region exists or not, is to solve Equations (4.55) and (4.56) for r_p . A value of r_p not in the interval $R_1 < r_p \leq R_2$ shows that there is no plug flow region.

4.3.4.2-The Bingham Model

If $n=1$ the velocity profile can be obtained analytically and there is no need to use the Taylor expansion. Integrating Equation (4.52) gives:

$$v = r \left[-\frac{c_1}{2kr^2} + \frac{\tau_y \ln r}{k} + c_2 \right] = r\omega(r) \quad (4.59)$$

Where $c_1 = -\frac{2R_1^2 R_2^2 \left(\tau_y \ln \left(\frac{R_2}{R_1} \right) + \omega_0 k \right)}{R_2^2 - R_1^2}$ and $c_2 = -\frac{\tau_y R_2^2 \ln R_2 - \tau_y R_1^2 \ln R_1 + \omega_0 k R_1^2}{k (R_2^2 - R_1^2)}$.

The shear stress equation then would be:

$$\tau(r) = -\frac{2R_1^2 R_2^2 \left(\tau_y \ln \left(\frac{R_2}{R_1} \right) + \omega_0 k \right)}{R_2^2 - R_1^2} \frac{1}{r^2} \quad (4.60)$$

To obtain the torque equation using the Bingham model, the shear stress on the surface of the inner cylinder is integrated which results in:

$$\begin{aligned} \tau(R_1) &= -\frac{2R_2^2 \left(\tau_y \ln \left(\frac{R_2}{R_1} \right) + \omega_0 k \right)}{R_2^2 - R_1^2} \\ dF &= \tau dA = \tau(R_1) \times 2\pi R_1 dz \\ dT &= R_1 dF \Rightarrow T = -4\pi R_1^2 R_2^2 \left[\frac{\tau_y \ln \left(\frac{R_2}{R_1} \right)}{R_2^2 - R_1^2} + \frac{\omega_0 k}{R_2^2 - R_1^2} \right] z. \end{aligned} \quad (4.61)$$

The minus sign is because the torque developed by MR fluid is a restraining torque and opposes the motion of the rotor; so, it is in the opposite direction of the rotation of the rotor.

In the Bingham model, k is the off-state viscosity of the fluid, often indicated by η . The first term on the right side of Equation (4.61) is the torque developed due to the field induced shear stress and the second term is due to the viscosity of the MR fluid.

For the case of Bingham model the plug radius can be calculated by letting $\frac{d\omega}{dr} = 0$ and using Equation (4.59) which results in:

$$r_p = \sqrt{\frac{-c_1}{\tau_y}}$$

$$c_1 = \tau_y R_1^2 W \left(-1, -\frac{\exp \left(-\frac{-2\tau_y \ln R_1 + \tau_y + 2\omega_0 k}{\tau_y} \right)}{R_1^2} \right) \quad (4.62)$$

where $W(a,x)$ is the Lambert W function (see appendix E). Lambert W function is the root of $W(x)e^{W(x)} = x$ (Corless, Gonnet et al. 1996; Caillol 2003). When $x > 0$, there is only one real solution to this equation. But there are two real solutions for W in the interval $-\frac{1}{e} < x < 0$. These two solutions can be referred to as $W(0,x)$ and $W(-1,x)$. In Equation (4.62), $W(-1,x)$ is chosen because it results in the physically meaningful range for the plug radius.

In Equation (4.62), letting $\tau_y = \infty$ results in $r_p = R_1$ and letting $\tau_y = 0$ gives $r_p = \infty$. So; the plug radius is in the range of $R_1 < r_p < R_2$. Values of r_p greater than R_2 show that there is no plug region in the flow and all of the fluid is moving in the gap.

Because experimental data on properties of MR fluids subjected to high shear rates are scarce (Maranville and Ginder 2003; Ahmadian and Norris 2004), the appropriate values of power law index (n) and consistency index (k) in the Herschel-Bulkley model are not available in order to model MR fluids at high shear rates.

4.3.5-Non-Dimensional Forms of Equations

There are several different parameters involved in previous equations and it is more convenient to reduce the number of variables by non-dimensionalizing the equations developed so far by considering the following non-dimensional variables:

$$\psi = \frac{-c_1}{\tau_y R_1^2}, \phi(\zeta) = \left(\frac{v(r)}{r} - \omega_0 \right) \left(\frac{k}{\tau_y} \right)^m, \zeta = \frac{r}{R_1}$$

which results in:

$$\begin{aligned} \phi(\zeta) = & \frac{-1}{3}(\psi-1)^m \left(\frac{2m(m-1)\psi^2 + (\psi-1)^2 + 5m\psi(\psi-1)}{(\psi-1)^2} \right) (\zeta^3 - 1) \\ & + \frac{1}{2}(\psi-1)^m \left[\frac{4m(m-1)\psi^2 + 3(\psi-1)^2 + 10m\psi(\psi-1)}{(\psi-1)^2} \right] (\zeta^2 - 1) \\ & - (\psi-1)^m \left[\frac{2m(m-1)\psi^2 + 3(\psi-1)^2 + 3m\psi(\psi-1)}{(\psi-1)^2} \right] (\zeta - 1) \end{aligned} \quad (4.63)$$

and

$$T^* = -2\pi\psi \quad (4.64)$$

Where $T^* = \frac{T}{\tau_y R_1^2 z}$ is the non-dimensional torque.

These dimensionless equations can be used as the design equations for the drum-type brake. For a specific MR fluid which the parameters of the Herschel-Bulkley model at high shear rates are available and for a required torque T , one can find the value of dimensionless torque by choosing the values of R_1 and z . Using Equation (4.64), the value of ψ is determined. Knowing that at $r = R_2$ the velocity should be zero the value of ϕ is known by its definition. Substituting the values of ϕ , ψ and m into Equation (4.63) and solving for ζ results in the value of $\zeta_2 = \frac{R_2}{R_1}$ which can subsequently be used to obtain the value of R_2 .

4.3.6-The Difference between Two Models

As stated earlier there is not enough experimental data on high shear rates; so, we evaluate the mathematical model developed here by using some available data which are not obtained from high shear rate experiments; so, the torque calculated by the Herschel-Bulkley model can be compared to the value obtained from the Bingham plastic model.

According to experiment done by Chaudhuri (Chaudhuri, Wereley et al. 2005), the following parameters were found for the Herschel-Bulkley and the Bingham model for nano-iron based MR fluid.

Bingham :

$$\tau_y = 2.25 \text{ KPa}$$

$$\eta = 0.5 \text{ Pa.s}$$

Herschel – Bulkley :

$$\tau_y = 0.8 \text{ KPa}$$

$$k = 425 \text{ Pa.s}^n$$

$$n = 0.225$$

The geometric values are as follows:

$$R_1 = 0.05 \text{ m}$$

$$R_2 = 0.051 \text{ m}$$

$$h = R_2 - R_1 = 0.001 \text{ m} \text{ Which is common in most applications}$$

The speed of rotation is taken to vary between 0 rad/sec and 245 rad/sec . The lower and upper values of rotational speed, correspond to shear rate values of 0 1/sec and 12000 1/sec , respectively. As the shear rate increases, the difference between the Herschel-Bulkley model prediction and the Bingham model prediction becomes more significant as seen in Figure 4. 7. This phenomenon is due to shear thinning of MR fluid at higher shear rates. Figure 4. 7 shows that using the Bingham constitutive equation to model MR fluids at high shear rates can result in over estimations in torque. The values in Figure 4. 7 are calculated per 1cm depth of the brake ($z = 1 \text{ cm}$).

Figure 4. 7 Comparison of developed torque between two constitutive models used.

If the brake rotates at a speed of 200 rad/sec , Equations (4.55) and (4.56) do not give us any real value but a complex value for c_1 , so; it can be concluded that there is no plug flow in this case. Since there is no plug flow region, Equation (4.57) is used and c_1 is found to be $c_1 = -12.305$. Using Equation (4.58), the value of torque can be calculated as $T = -0.77 \text{ N.m}$. On the other hand, for the Bingham model, we use Equation (4.61) to calculate the torque. The torque obtained is $T = -1.17 \text{ N.m}$. The torque calculated by using the Bingham model is much larger the torque from the Herschel-Bulkley model. The value of shear rate at $r = R_1$ is $\dot{\gamma} = -9800 \text{ 1/sec}$. The schematic of velocity profile for the case of no-plug flow is shown in Figure 4. 8.

Figure 4. 8 Schematic of velocity profile for the case of no-plug flow.

4.3.7- Design Equations for an MR Drum-Type Rotary

Brake

Although the Bingham model is not so satisfactory at high shear rates, it can still be used to derive the design equations for the brake for preliminary design. The Bingham model has another advantage, which is its simplicity, but by using Herschel-Bulkley model at high shear rates one can obtain more accurate and reliable results.

In this section, the Bingham model is used to derive the design equations of the brake and finally a design example is represented. In previous sections, the torque equation was calculated as:

$$T = -4\pi R_1^2 R_2^2 \left[\frac{\tau_y \ln\left(\frac{R_2}{R_1}\right)}{R_2^2 - R_1^2} + \frac{\omega_0 k}{R_2^2 - R_1^2} \right] z \quad (4.65)$$

Let

$$T_f = \frac{4\pi\tau_y R_1^2 R_2^2 \ln\left(\frac{R_2}{R_1}\right)}{R_2^2 - R_1^2} z \quad (4.66)$$

$$T_\eta = \frac{4\pi R_1^2 R_2^2 \omega_0 k}{R_2^2 - R_1^2} z \quad (4.67)$$

Where, T_f is the field torque and T_η is the viscous torque.

The ratio $\frac{T_f}{T_\eta}$ is the control torque ratio (Huang, Zhang et al. 2002) and shows the controllability of the brake. If the field torque is much greater than the viscous torque the brake is more controllable, which means the viscous torque becomes less significant than the field torque and the field torque has more contribution in the total developed torque.

From Equations (22) and (23), we can find the control torque ratio is:

$$\frac{T_f}{T_\eta} = \frac{\tau_y \ln\left(\frac{R_2}{R_1}\right)}{\omega_0 k} \quad (4.68)$$

4.3.8- Sample Design Problem

Let's assume a brake with 10 N.M torque at 1000 rpm with the control torque ratio of 15 is to be designed. The MR fluid to be used has a yield stress of 40000 Pa and a viscosity of 0.33 Pa.s .

The torque developed by the brake is the summation of the field torque and viscous torque.

$$T = T_f + T_\eta \rightarrow T = T_\eta \left(1 + \frac{T_f}{T_\eta} \right) \quad (4.69)$$

By substituting the values for torque and the control torque ratio the viscous torque is found to be 0.625 N.M .

By choosing the minimum radius of the brake as $R_1 = 50\text{ mm}$ and using the Equations (4.66) and (4.67), we are able to calculate the fluid thickness and the width of the brake.

In this particular example $h = 0.6\text{ mm}$ and $z = 15\text{ mm}$.

4.4-MRF flow Between Two Disks

In this section, MR fluid flow in disk-type MR rotary brakes is modeled (Figure 4. 9). The results are used to find the yield surface and plug flow region in disk-type MR rotary brakes. Plug flow region is considered as the region where the MR fluid resembles a bulk of solid (Wang and Gordaninejad 2000; Farjoud, Vahdati et al. 2007). In the plug flow region, the fluid is in the pre-yield phase. Plug flow region plays an important role in design and analysis of MR devices. In MR suspensions, the damping coefficient is a function of the plug thickness (Wereley and Pang 1998). In MR valves, the plug thickness is used to control flow rate through, and pressure drop across, the MR valve (Yoo and Wereley 2002). A MR clutch is performing at highest efficiency when the whole gap is the plug region. For a MR rotary brake, the highest restraining torque is obtained when the whole gap is the plug region as far as there are no wall slip effects (de Vicente, Lopez-Lopez et al. 2004). In particular, for disk type MR rotary brakes, the torque due to viscosity is approximated as (Li and Du 2003):

$$T_{vis} = \frac{\pi\eta\omega}{2h} (R_2^4 - R_1^4) \quad (4.70)$$

where η is the viscosity of the MR fluid, ω is the speed of rotation, R_1 is the inner radius, and R_2 is the outer radius (Figure 4. 9). h is the real gap thickness where the MR fluid is flowing and the plug flow thickness should not be included in h . h is, and should be, a function of magnetic field, so h should decrease as the plug flow region increases. The value of viscous torque is dependant on the value of h . That is why determination of the plug flow region in MR rotary brakes is important.

In this section, the flow of magnetorheological fluids between two disks (the rotor and the stationary brake housing) is discussed. (Figure 4. 9)

The flow between two disks is a two dimensional problem. In this case the velocity v of the MRF is a function of both r and z , namely $v(r, z)$. Since the shear rate is imposed by the velocity gradient in two directions, the one dimensional constitutive equations (e.g. Bingham, Herschel-Bulkley, etc.) are no longer useful. According to literature, there are different constitutive models for MRF in three dimensions (Bercovier and Engelman 1980; Papanastasiou 1987; Burgos, Alexandrou et al. 1999; Zhu, Kim et al. 2005).

Figure 4. 9 Schematic of the disk type MR brake.

4.4.1- Constitutive Equation

The most common three-dimensional constitutive relation uses the von Mises criterion (Papanastasiou 1987):

$$\underline{\underline{\tau}} = \begin{cases} G\underline{\underline{B}} & |\underline{\underline{II}}|_{\underline{\underline{\tau}}} < \tau_y^2 \\ 2 \left(\eta + \frac{\tau_y}{2|\underline{\underline{II}}|_{\underline{\underline{D}}}^{1/2}} \right) \underline{\underline{D}} & |\underline{\underline{II}}|_{\underline{\underline{\tau}}} > \tau_y^2 \end{cases} \quad (4.71)$$

Here, $\underline{\underline{\tau}}$, $\underline{\underline{D}}$ and $\underline{\underline{B}}$ are the viscous stress, the rate of strain, and the strain tensors; respectively. G is the elasticity modulus for the un-yielded material, and η the viscosity of the yielded fluid. τ_y is the yield stress, and $II_{\underline{\underline{D}}}$ is the second invariant of $\underline{\underline{D}}$. The mentioned parameters will be described in more details later in this paper. The one dimensional analog of Equation (4.71) is:

$$\tau_{12} = \begin{cases} GB_{12} & |\tau_{12}| \leq \tau_y \\ \left(\eta + \frac{\tau_y}{|\dot{\gamma}|} \right) \dot{\gamma} & |\tau_{12}| > \tau_y \end{cases} \quad (4.72)$$

where $\dot{\gamma}$ is the shear rate. The above equation for $|\tau_{12}| > \tau_y$ is the famous Bingham constitutive equation.

As will be discussed in more details, solving the equation of motion for MR fluid flow between two disks results in a system of non-linear Partial Differential Equations (PDE) which cannot be solved analytically. So; numerical methods have to be used. However the constitutive equations used for one dimensional problems, (e.g. the Bingham or Herschel-Bulkley models) are not amenable to numerical analysis because in some applications, parts of the fluid flow, while the rest remain as solid (plug region). This

introduces difficulties in tracking the shape and the location of the MR-fluid yield surface and applying two different constitutive equations at either side of the yield surface. In addition; at vanishing shear rates ($\dot{\gamma} = 0$), the apparent viscosity ($\frac{\tau_y}{|\dot{\gamma}|}$) becomes infinite, which leads to discontinuity and numerical difficulties. To overcome this issue, Bercovier and Engelman (Bercovier and Engelman 1980) and Papanastasiou (Papanastasiou 1987) have proposed modified models of Bingham and Herschel-Bulkley model to approximate the rheological behavior of MR fluids.

Here in this paper, the Bercovier and Engelman model is used. The constitutive equation proposed by Bercovier and Engelman is (Bercovier and Engelman 1980):

$$\underline{\underline{\tau}} = \left[2\eta + \frac{\tau_y}{|\underline{\underline{II}}|_{\underline{\underline{D}}}^{1/2} + \delta} \right] \underline{\underline{D}} \quad (4.73)$$

δ is the regularization parameter, having the dimensions of strain rate. As shown in Figure 4. 10, for one dimensional case, as δ gets smaller the Bercovier model approaches the Bingham model.

Figure 4. 10 The Bercovier & Engelman model as δ varies

By using this regularization parameter, δ , in the Bercovier model the un-yielded solid-like material is approximated by a highly viscous fluid (Figure 4. 10). The value of δ in the Bercovier and Engelman model must be as small as possible. However this parameter cannot be changed arbitrarily because its value affects the numerical stability and convergence of the computer codes.

The Bercovier and Engelman model does not have a well defined yield surface ($v = 0, \tau = \tau_y$). One way to define the yield surface, is to find the locus of points where $\tau = \tau_y$ and then track this surface as the regularization parameter reduces to zero ($\delta \rightarrow 0$). As it was mentioned earlier, very small values cannot be assigned to the regularization parameter because of numerical difficulties associated with the convergence. So; the location of the yield surface can be found approximately.

4.4.2- Governing Equations

Equation of motion in θ direction for cylindrical coordinate is:

$$\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{1}{r} v_\theta \frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} + \frac{1}{r} v_\theta v_r \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{z\theta}}{\partial z} + \frac{2}{r} \tau_{r\theta} + f_\theta \quad (4.74)$$

Where ρ is the fluid density, v_θ is the tangential velocity, t is time, v_r is the radial velocity, f_θ is the body force exerted on the fluid in the direction of θ , and τ_{ij} 's are the stress tensor components.

For the disk-type MR rotary brake in Figure 4. 9, assuming steady state condition ($\frac{\partial}{\partial t} = 0$), symmetry ($\frac{\partial}{\partial \theta} = 0$), no radial and axial velocity ($v_r = v_z = 0$) but only tangential, no body force (neglecting the effects of gravity), and no extra stress, Equation (4.74) simplifies to:

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2}{r} \tau_{r\theta} = 0 \quad (4.75)$$

The general form of the rate of deformation tensor in cylindrical coordinates is,

$$\underline{\underline{D}} = \begin{pmatrix} \frac{\partial v_r}{\partial r} & \frac{1}{2} \left(\frac{\partial v_\theta}{\partial r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} - \frac{1}{r} v_\theta \right) & \frac{1}{2} \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) \\ \frac{1}{2} \left(\frac{\partial v_\theta}{\partial r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} - \frac{1}{r} v_\theta \right) & \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{1}{r} v_r & \frac{1}{2} \left(\frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \right) \\ \frac{1}{2} \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) & \frac{1}{2} \left(\frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \right) & \frac{\partial v_z}{\partial z} \end{pmatrix} \quad (4.76)$$

Simplifying Equation (4.76) by the stated assumptions,

$$\underline{\underline{D}} = \begin{pmatrix} 0 & \frac{1}{2} \left(\frac{\partial v_\theta}{\partial r} - \frac{1}{r} v_\theta \right) & 0 \\ \frac{1}{2} \left(\frac{\partial v_\theta}{\partial r} - \frac{1}{r} v_\theta \right) & 0 & \frac{1}{2} \left(\frac{\partial v_\theta}{\partial z} \right) \\ 0 & \frac{1}{2} \left(\frac{\partial v_\theta}{\partial z} \right) & 0 \end{pmatrix} \quad (4.77)$$

The second invariant of the rate of deformation tensor is:

$$|\underline{\underline{D}}| = \frac{1}{4} \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right)^2 + \frac{1}{4} \left(\frac{\partial v_\theta}{\partial z} \right)^2 \quad (4.78)$$

Considering Equations (4.77) and (4.78), the Bercovier and Engelman constitutive

Equation (4.73) simplifies to:

$$\underline{\underline{\tau}} = \left[\frac{2\eta + \frac{\tau_y}{\left(\frac{1}{2} \sqrt{\left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right)^2 + \left(\frac{\partial v_\theta}{\partial z} \right)^2} \right) + \delta}} \right] \times \begin{pmatrix} 0 & \frac{1}{2} \left(\frac{\partial v_\theta}{\partial r} - \frac{1}{r} v_\theta \right) & 0 \\ \frac{1}{2} \left(\frac{\partial v_\theta}{\partial r} - \frac{1}{r} v_\theta \right) & 0 & \frac{1}{2} \left(\frac{\partial v_\theta}{\partial z} \right) \\ 0 & \frac{1}{2} \left(\frac{\partial v_\theta}{\partial z} \right) & 0 \end{pmatrix} \quad (4.79)$$

where $\underline{\underline{\tau}} = \begin{pmatrix} 0 & \tau_{r\theta} & 0 \\ \tau_{r\theta} & 0 & \tau_{\theta z} \\ 0 & \tau_{\theta z} & 0 \end{pmatrix}$ is the shear stress tensor. As there are only two non-zero

components of rate of deformation tensor and stress tensor present (namely θz and $r\theta$ components), they can be written in the vector form as

$$\underline{\underline{\tau}} = \begin{bmatrix} \tau_{r\theta} \\ \tau_{\theta z} \end{bmatrix}, \quad \underline{\underline{D}} = \frac{1}{2} \begin{bmatrix} \frac{\partial v}{\partial r} - \frac{v}{r} \\ \frac{\partial v}{\partial z} \end{bmatrix} \quad (4.80)$$

Here the velocity notation v is used instead of v_θ for simplicity.

By considering Equations (4.75) and (4.79), the system of governing equations can be written as

$$\begin{aligned}
 \tau_{r\theta} &= \frac{1}{2} \left[2\eta + \frac{\tau_y}{\left(\frac{1}{2} \sqrt{\left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right)^2 + \left(\frac{\partial v_\theta}{\partial z} \right)^2} \right) + \delta} \right] \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right) \\
 \tau_{\theta z} &= \frac{1}{2} \left[2\eta + \frac{\tau_y}{\left(\frac{1}{2} \sqrt{\left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right)^2 + \left(\frac{\partial v_\theta}{\partial z} \right)^2} \right) + \delta} \right] \left(\frac{\partial v}{\partial z} \right) \\
 \frac{\partial \tau_{r\theta}}{\partial r} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2}{r} \tau_{r\theta} &= 0
 \end{aligned} \tag{4.81}$$

After substituting the first two Equations in (4.81) into the third equation we have a PDE. This partial differential equation is a nonlinear one and cannot be solved by analytical methods. It is simpler to nondimensionalize this PDE and then solve it as lots of parameters gather in one parameter and working with nondimensional parameters is much easier. The Bercovier constitutive equation and the equation of motion are non-dimensionalized by considering the following parameters:

$$\begin{aligned}
 \underline{\tau}^* &= \frac{\underline{\tau}}{\tau_y} \\
 \underline{D}^* &= \frac{\underline{DH}}{v_0} \\
 \delta^* &= \frac{H\delta}{v_0} \\
 Bn &= \frac{H\tau_y}{\eta v_0} \\
 v^* &= \frac{v}{v_0} \\
 r^* &= \frac{r}{H} \\
 z^* &= \frac{z}{H}
 \end{aligned} \tag{4.82}$$

where H and v_0 are the characteristic length and characteristic velocity; respectively. $\underline{\tau}^*$ and $\underline{D}^*$ are the non-dimensional stress and strain vectors; respectively, δ^* is the non-dimensional regularization parameter, Bn is the Bingham number, v^* is the non-dimensional velocity, and r^* and z^* are non-dimensional coordinates. The non-dimensional form of Equations (4.79) and (4.75) are:

$$\begin{bmatrix} \tau_{r\theta}^* \\ \tau_{\theta z}^* \end{bmatrix} = \frac{H}{2v_0} \left(\frac{2}{Bn} + \frac{1}{\frac{1}{2} \sqrt{\left(\frac{\partial v^*}{\partial r^*} - \frac{v^*}{r^*} \right)^2 + \left(\frac{\partial v^*}{\partial z^*} \right)^2} + \delta^*} \right) \begin{bmatrix} \frac{\partial v^*}{\partial r^*} - \frac{v^*}{r^*} \\ \frac{\partial v^*}{\partial z^*} \end{bmatrix} \tag{4.83}$$

$$\frac{\partial \tau_{r\theta}^*}{\partial r^*} + \frac{\partial \tau_{\theta z}^*}{\partial z^*} + \frac{2}{r^*} \tau_{r\theta}^* = 0 \tag{4.84}$$

4.4.3-Boundary Conditions

The solution domain is the rectangular cross section of the brake in r - z plane (Figure 4. 11).

Figure 4. 11 Boundary conditions for the MRF flow between the two disks.

It is assumed that the fluid in touch with the moving disk has the same velocity as the moving disk. Moving toward the stationary disk, the velocity of the MR fluid decreases and its value is zero at the stationary disk. It is assumed that there is no magnetic flux line at the upper ($r = R_2$) and lower ($r = R_1$) portion of the domain. So; the fluid behaves like a Newtonian fluid. By this assumption, the Newton viscosity law ($\tau = \eta \frac{\partial v}{\partial z}$) can be used.

Since the thickness of the gap between the two disks is small, linear velocity profile at the top and bottom portion of the domain is assumed (Figure 4. 11).

By considering these issues the boundary conditions can be written as:

$$\begin{aligned}
 v(r, 0) &= r\omega & R_1 < r < R_2 \\
 v(r, z_2) &= 0 & R_1 < r < R_2 \\
 v(R_1, z) &= \frac{R_1\omega}{z_2}(z_2 - z) & 0 < z < z_2 \\
 v(R_2, z) &= \frac{R_2\omega}{z_2}(z_2 - z) & 0 < z < z_2
 \end{aligned} \tag{4.85}$$

The dimensionless form of boundary conditions is:

$$\begin{aligned}
 v^*(r^*, 0) &= \frac{r\omega}{v_0} = \frac{r^*}{R_1^*} & R_1^* < r^* < R_2^* \\
 v^*(r^*, z_2^*) &= 0 & R_1^* < r^* < R_2^* \\
 v^*(R_1^*, z^*) &= 1 - \frac{z^*}{z_2^*} & 0 < z^* < z_2^* \\
 v^*(R_2^*, z^*) &= \frac{R_2^*}{R_1^*} \left(1 - \frac{z^*}{z_2^*} \right) & 0 < z^* < z_2^*
 \end{aligned} \tag{4.86}$$

where $v_0 = R_1\omega$, and ω is the angular velocity of the rotary disk. Note that we could have chosen any other velocity as v_0 . For example by choosing $v_0 = R_2\omega$ the boundary conditions will be:

$$\begin{aligned}
 v^*(r^*, 0) &= \frac{r\omega}{v_0} = \frac{r^*}{R_2^*} & R_1^* < r^* < R_2^* \\
 v^*(r^*, z_2^*) &= 0 & R_1^* < r^* < R_2^* \\
 v^*(R_1^*, z^*) &= \frac{R_1^*}{R_2^*} \left(1 - \frac{z^*}{z_2^*} \right) & 0 < z^* < z_2^* \\
 v^*(R_2^*, z^*) &= \left(1 - \frac{z^*}{z_2^*} \right) & 0 < z^* < z_2^*
 \end{aligned} \tag{4.87}$$

4.4.4-Finite Difference Method

As stated earlier, the PDE obtained, cannot be solved analytically and has to be dealt with by the aid of numerical methods. We have used the finite difference model to solve this PDE (Chapra and Canale 2006). The finite difference method can be used for steady state problems as well as transient problems. In this method the partial derivatives are substituted by finite difference formulas and the PDE reduces to an algebraic equation. The algebraic equation then can be solved by use of computer software like MATLAB.

First we have to divide the domain into discrete points called nodes. The value of velocity and other calculations is performed on these nodes and the other portions of the domain are not engaged in the calculations. We may increase the number of nodes in order to increase the accuracy. The domain in the present problem is a rectangular region in r-z plane. By solving the PDE we find the values of velocity at the nodes which can be used to evaluate other parameters such as shear stress and torque. Figure 4. 12 shows the solution domain. The notation used for identifying the nodes is used in such a way that makes the writing of the code as simple as possible.

Figure 4. 12 Solution domain and finite difference notation.

The calculations are done as following:

- 1- Analytically, $\frac{\partial \tau_{r\theta}^*}{\partial r^*}$, and $\frac{\partial \tau_{\theta z}^*}{\partial z^*}$ were found from Equation (4.83) using Maple

software. The mentioned partial derivatives are functions of velocity.

- 2- The velocity partial derivatives were substituted by finite difference formulas.

The partial derivatives of velocity can be written as:

$$\begin{aligned}
 \frac{\partial v^*}{\partial r^*} &= \frac{v^*(i-1, j) - v^*(i+1, j)}{2h_r} \\
 \frac{\partial v^*}{\partial z^*} &= \frac{v^*(i, j+1) - v^*(i, j-1)}{2h_z} \\
 \frac{\partial^2 v^*}{\partial r^{*2}} &= \frac{v^*(i-1, j) - 2v^*(i, j) + v^*(i+1, j)}{h_r^2} \\
 \frac{\partial^2 v^*}{\partial z^{*2}} &= \frac{v^*(i, j+1) - 2v^*(i, j) + v^*(i, j-1)}{h_z^2} \\
 \frac{\partial^2 v^*}{\partial r^* \partial z^*} &= \frac{v^*(i-1, j+1) - v^*(i-1, j-1) - v^*(i+1, j+1) + v^*(i+1, j-1)}{4h_r h_z}
 \end{aligned} \tag{4.88}$$

Where the notations of Figure 4. 12 are used, $v(i, j)$ is the velocity at node $\begin{matrix} i \\ j \end{matrix}$, h_r and h_z are the distance between two adjacent points in r and z direction, respectively.

- 3- The obtained partial derivatives were replaced in Equation (4.84) which resulted in an algebraic equation.
- 4- A MATLAB code is written to solve the algebraic equation in step (3). Velocity value at each node is calculated. The boundary conditions are given in Equation (4.87).
- 5- Knowing velocities at each node, the velocity partial derivatives at each node can be calculated. These partial derivatives are used to calculate the shear stress at each node by using Equation (4.83).
- 6- Having the shear stress at each node, the yield surface can be easily determined. The yield surface is the locus of points where the non-dimensional shear stress equals one ($\tau^* = 1$).

4.4.5-MATLAB Code

To find the solution, a program is written in MATLAB. As discussed earlier, the code is written based on a FDM (Finite Difference Method) numerical scheme. The procedure of solving equations and calculations are given in previous sections.

4.4.5.1-Inputs to the Code

For a specific geometry the inputs to the program are:

1- Number of divisions in r and z directions. We may choose more number of nodes in order to get more accurate results or less number of nodes for faster speed. The number of grid points are depicted as m and n in r and z directions, respectively (Figure 4. 12).

2- R_1^* . This is the nondimensional small radius of the disk which is $\frac{R_1}{H}$, where H is the characteristic length.

3- R_2^* . This is the nondimensional larger radius of the disk which is $\frac{R_2}{H}$.

4- Z_2^* . This is the nondimensional thickness of the fluid (or the gap between the two disks). $Z_2^* = \frac{Z_2}{H}$

5- Bn is the Bingham number defined by $Bn = \frac{H\tau_y}{\eta v_0}$. H is the characteristic length,

τ_y is the yield stress, η is the viscosity of MRF and v_0 is the characteristic velocity.

6- δ^* . The nondimensional regularization parameter used in Bercovire and

Engelman model which is defined by $\delta^* = \frac{H\delta}{v_0}$. As seen from Figure 4. 13 the

value of $\delta^* \approx 10^{-3}$ gives a good approximation of the true model. In Figure 4. 13 the size of stress vector is plotted versus Bn (the Bingham number) and D (the size of rate of deformation vector) for different values of δ^* .

7- Desired error percentage is also required as the stopping criterion of the code.

The percentage error at each node is defined as

$$\%error = \left| \frac{New\ solution - Current\ value}{New\ solution} \right| * 100 \quad (4.89)$$

The velocity values at each point are stored in an m*n matrix and each time the code solves for a velocity of point a comparison is done and the result is stored in the error matrix. The maximum value of the error matrix is the maximum error and is displayed at each iteration. Once the maximum error is less than the value assigned by the user the main loop is terminated and the results are displayed.

Figure 4. 13 Approximation of true model by decreasing the value of δ^* .

4.4.5.2- Outlook of the MATLAB Code

All input parameters to the code are entered via an input box (Figure 4. 14). This input box is created by using the Matlab's GUI. The default values in the box correspond to a sample case which is used as an example in the next sections.

first iterat...

grid point numbers in r direction
26

grid point numbers in z direction
26

R1
3

R2
6

z2
0.1

Bingham number
130

delta
0.02

OK Cancel

Figure 4. 14 Input dialog box.

After entering the parameters to the code, the desired error percentage should be entered. The desired error percentage can be any number between 0 and 100 and the default error percentage is set to 50. (Figure 4. 15)

Desired err...

Enter the desired error percentage
50

OK Cancel

Figure 4. 15 Desired error percentage error dialog box.

The code is written in a manner that it allows increasing the accuracy by decreasing the desired error percentage without any need to start from the first iteration. It stores all the

parameter values into a data file (the name of the data file is “velocitymatrix.mat”) and if the user wants to decrease the desired error percentage to increase the accuracy of the solution, the code calls the data file and restores all the parameters into the Matlab workspace and continues from the last iteration. This enables the user to enter a large value for the desired error percentage in order to see the results of the first few iterations and then continue for more accurate solutions. So; when the code is executed a window appears asking the user whether to start from the first iteration or continue a current solution from a data file provided by the user. (Figure 4. 16)

Figure 4. 16 The first window appearing when the code is executed.

When the code is running, a window appears on the screen showing the number of iterations, current maximum error, and entered parameters. (Figure 4. 17)

Figure 4. 17 Program Status Window.

After the maximum errors is lower the desired error percentage the code stops and shows the results. The results are:

- 1- Velocity Distribution Profile.
- 2- Percentage Error Distribution Profile.
- 3- Plot of Maximum Error vs. Iteration Number.
- 4- Shear Stress Distribution profile.
- 5- Non-dimensional Torque Value.

The results are shown for a sample case in later sections.

4.4.6-Torque Equation

The value of the torque exerted to the disk is calculated by the integration of shear stress on the rotor surface. The component of shearing stress acting on the surface of the disk is $\tau_{\theta z}$, this can be shown by using the tensor form of shear and using the fact that the stress vector acting on a surface can be calculated by multiplying the matrix of the stress tensor by the normal vector to that surface.

As stated earlier the shear stress tensor in this case is:

$$\underline{\underline{\tau}} = \begin{pmatrix} 0 & \tau_{r\theta} & 0 \\ \tau_{r\theta} & 0 & \tau_{\theta z} \\ 0 & \tau_{\theta z} & 0 \end{pmatrix}.$$

Considering Figure 4. 18, we can find the shearing stress acting on the rotor.

Figure 4. 18 Determination of the shear component of the stress vector acting on the surface of the disk.

$$\vec{t}_n = \underline{\underline{\tau}} \vec{n} = \begin{pmatrix} 0 & \tau_{r\theta} & 0 \\ \tau_{r\theta} & 0 & \tau_{\theta z} \\ 0 & \tau_{\theta z} & 0 \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \tau_{\theta z} \\ 0 \end{bmatrix} \Rightarrow \text{the shear stress on the disk} = \tau_{\theta z} \hat{e}_\theta$$

where $\vec{t}_n$ is the shear stress vector acting on the surface of the disk with the normal vector $\vec{n}$. The first two components of $\vec{t}_n$ are the shear stresses and the third one is the normal stress. So the total shear stress acting on the disk is:

$$\tau_{shear} = \sqrt{(\vec{t}_n)_r^2 + (\vec{t}_n)_\theta^2} = \sqrt{0^2 + \tau_{\theta z}^2} = \tau_{\theta z} \quad (4.90)$$

To calculate the torque we integrate the shear stress ($\tau_{\theta z}$) on the surface of the disk from R_1 to R_2 .

$$T = \int r \tau dA = \int \tau (2\pi r^2 dr) = 2\pi \int \tau r^2 dr \quad (4.91)$$

here τ in the integral is $\tau_{\theta z}$ on the surface of the disk. The above integral has to be evaluated numerically.

$$T = 2\pi \sum_{i=2}^{m-1} \tau_i r_i^2 \Delta r = 2\pi \Delta r \left(\sum_{i=2}^{m-1} \tau_i r_i^2 \right) \quad (4.92)$$

where the summation is done on the shear stress on the surface of the disk from $i = 2$ to $i = m - 1$ (Figure 4. 12), and $\Delta r = h_r$.

The above equation is still not appropriate for use in conjunction with our earlier analysis because we have already used the nondimensional form of equations; so, Equation (4.92)

first must be nondimensionalized. This is done by considering the nondimensional torque as

$$T^* = \frac{T}{\tau_y H^3} \quad (4.93)$$

and,

$$T^* = 2\pi h_r \left(\sum_{i=2}^{m-1} \tau_i^* r_i^{*2} \right) \quad (4.94)$$

The nondimensional torque is computed by the code. To find the value of the torque, T^* must be multiplied by $\tau_y H^3$.

As we stated earlier, by applying this numerical approach to complex problems we can find the yield surface. Yield surface tracking can be done by solving the equations while decreasing the value of δ^* , the locus of the points with $\tau^* = 1$ ($\tau = \tau_y$) then indicates the yield surface.

A MR rotary brake delivers the highest torque when the whole gap is in pre-yield phase or plug flow region (neglecting wall slip).

4.4.7-Results

Here a sample problem is solved by the aid of the above concept and the result is compared to experimental data. The most important advantage of this approach is to detect the yield surface and thus improve the design.

Consider a disk-type MR rotary brake with the following dimensions (see Figure 4.11 and 4.12).

$$\begin{aligned}
 R_1 &= 30mm && \text{Inner radius of the brake} \\
 R_2 &= 60mm && \text{Outer radius of the brake} \\
 h = z_2 &= 1mm && \text{Gap thickness} \\
 \eta &= 0.94Pa.s && \text{MR fluid viscosity} \\
 \omega &= 82rad/sec && \text{Rotational speed of the left disk} \\
 \tau_y &= 60KPa && \text{Yield stress of MR fluid}
 \end{aligned} \tag{4.95}$$

The non-dimensional parameters can be calculated from the above parameters. The non-dimensional parameters are the inputs to the program and so are the grid point divisions and the desired relative error.

By letting the characteristic length as $H = 10mm$ and the characteristic velocity as $v_0 = R_2\omega = 60 \times 10^{-3} \times 82 = 4.92 \text{ m/s}$, the non-dimensional parameters are:

Number of divisions in r and z direction: 50

Desired percentage error: 2%

$$\begin{aligned}
 R_1^* &= 3 \\
 R_2^* &= 6 \\
 z_2^* &= .1 \\
 Bn &= \frac{H\tau_y}{\eta v_0} = \frac{10 \times 10^{-3} \times 60000}{0.94 \times 4.92} \approx 130 \\
 \delta^* &= 0.02
 \end{aligned} \tag{4.96}$$

By entering the above parameters to the code, the following results will be displayed.

Figure 4. 19 The results obtained for the sample case.

Figure 4. 19 shows the results shown by the computer code. The top left Figure is the velocity distribution in the domain, the top right Figure is the plot of maximum error as defined earlier vs. number of iterations done, the lower left Figure shows the shear stress distribution in the domain. Note that, because the θ_z component is responsible for the torque developed by the brake we have not plotted the other component ($r\theta$). Finally the lower right Figure shows the error distribution at the last iteration for the domain.

4.4.7.1-Velocity Distribution

The velocity distribution is shown in the Figure 4. 20. As seen from the Figure, the velocity increases in the radial direction and obtains a value of 1 at $r = R_2$ (because we have used the value of speed at R_2 as the characteristic value). The velocity is zero at the right hand side of the domain. Note the inclined nature of the constant velocity lines.

These lines are inclined to the right because as we move in the direction of r , the linear velocity of the disk increases; so, a velocity of the same magnitude as the one in the smaller r will occur at a greater distance from the moving disk.

Figure 4. 20 Velocity distribution.

4.4.7.2-Error Distribution

As seen in Figure 4. 21, as we move to the right of the domain the error becomes greater. This is because of the nature of the finite difference method used. this method starts from the upper left node and starts to march to the lower left portion of the domain; so, in this fashion the error associated with the calculations is accumulated and thus the error at the right is greater than the error at the left. But the error at each node is below the desired percentage error which in this case is 2%.

After the program solves the problem it also shows a window representing the maximum value of error.

Figure 4. 21 Error distribution.

4.4.7.3-Maximum Error Trend

The plot of the maximum error vs. iteration number is presented in Figure 4. 22. As it is obvious the error reduces as the number of iterations increases. There is a peak seen in the Figure where the error is increased and after that starts to decrease again.

Figure 4.22 Maximum error vs. iteration number.

4.4.7.4-Shear Stress Distribution

The shear stress distribution is shown in Figure 4. 23. This Figure can be used to detect the yield surface in the flow domain. Since this is the non-dimensional shear stress that is shown in the Figure, the yield stress corresponds to the locus of points where the value of non-dimensional shear stress is one ($\tau^* = 1$). This can be done either by looking at the Figure or by checking the shear stress matrix which is saved in the workspace of MATLAB by the code (by the name of tautetaz).

Figure 4.23 Shear stress distribution.

4.4.7.5-Developed Torque

One of the results given by the computer code is the value of non-dimensional torque T^* .

The value of T^* is displayed in a window. The value of developed torque can be obtained by multiplying T^* by $\tau_y H^3$. H is the characteristic length and τ_y is the yield stress.

$$\left. \begin{array}{l} T = T^* \times \tau_y H^3 \\ T^* = -4.193 \times 10^2 \\ \tau_y = 60 \text{ KPa} \\ H = 10 \text{ mm} \end{array} \right\} T = -4.193 \times 10^2 \times 60 \times 10^3 \times 10^{-6} = -25.16 \text{ N.m} \quad (4.97)$$

The value obtained here is comparable to the value of 23.54 N.m obtained by Huang (Huang, Huang et al. 2004). The slightly larger value obtained here is because of considering the yield stress as its maximum value of 60KPa throughout the region. There is always a variation in the yield stress along the radius of the brake because the magnetic flux lines are not homogeneous in the region and as we move farther from the magnetic coils the magnetic induction decreases.

4.4.7.6- Mesh Convergence Study

The Matlab code has been implemented for different mesh sizes to satisfy the mesh convergence. The results should be almost the same as the number of nodes exceeds a certain amount. Here in this section, the problem is solved for two mesh sizes and the results are as followed. The domain is divided into 25x25, and 50x50 cells, respectively and the results show the code is convergent in terms of mesh numbers. The velocity

distribution and shear stress distribution for 25x25, and 50x50 divisions are presented here.

Note that the gap is much smaller than the radial distance (R_1 to R_2), and the following figures are not plotted in scale just for illustrative purposes.

Figure 4. 24 Velocity distribution for 25x25 divisions.

Figure 4. 25 Shear stress distribution for 25x25 divisions.

Figure 4. 26 Velocity distribution for 50x50 divisions.

Figure 4. 27 Shear stress distribution for 50x50 divisions.

4.4.7.7- Summary of the Numerical Results

A sample numerical problem was solved in this section using the Finite Difference scheme developed in previous sections. The results were presented in subsections 4.4.7.1 to 4.4.7.5. The mesh convergence study was performed and the results of the mesh convergence study were given in subsection 4.4.7.6.

Figure 4.20 shows the velocity distribution. The left wall is the rotary disk which is rotating by an angular velocity of ω and the right wall is the stationary disk. The linear velocity is increased from $r=R_1$ to $r=R_2$ ($v = r\omega$). The non-dimensional velocity, as defined in Equation (4.82), increases from $v^* = \frac{R_1^*}{R_2^*}$ to $v^*=1$ on the rotary disk and is zero on the stationary disk. The inclined nature of the constant velocity contours is because of the increase in the linear velocity with the radius r . This can be seen clearly in Figure 4.20.

The shear stress distribution which is the most important result obtained from this study is shown in Figure 4.23. At places where the velocity (or equally, shear rate) is high the MR fluid will yield and the magnitude of the non-dimensional yield stress is larger than one. As the velocity is decreased near the stationary disk, the magnitude of the non-dimensional yield stress gets smaller. The locus of points where the value of non-dimensional yield stress is one is called the yield surface. Beyond the yield surface, the velocity is zero and the fluid behaves as a solid. This region is known as the plug flow

region. Determination of plug flow region is of interest in this study. The determination of the plug flow region is useful for design of MR rotary brakes. The reason to this issue is discussed in the beginning of section 4.4.

In Figure 4.23, at the plug flow region, although the magnitude of the non-dimensional shear stress is less than one, a very small value of velocity exists. This is because of the approximate method that is used in FD solution. As the value of δ in the Bercovier and Engelman model gets smaller, the FD solution gives more accurate results. However, due to numerical difficulties, the value of δ cannot be assigned smaller than a specific value.

The shear stress can be integrated numerically on the surface of either of the disks to calculate the torque. This is done in subsection 4.4.7.5. The value of the torque obtained from this calculation is compared with published experimental data with the same geometry. Good agreement is observed between the two. There is no way to experimentally observe the plug flow region in the literature. One way to validate the numerical results is to compare the torque with experimental data. However, this can be a good criterion for validation of the numerical model. The reason is that the value of the torque is directly related to the shear stress values and the plug flow region is directly obtained from the shear stress.

The mesh convergence study is also another way to assess the accuracy of the numerical solution. This study is performed and the results were presented in subsection 4.4.7.6.

Chapter 5

Test Rig Design

In this chapter, a MR test rig is designed to carefully analyze the MR effect in MR rotary brakes. The test rig is designed according to the following considerations:

- 1- Simple to manufacture
- 2- Easy to assembly
- 3- Adequate accuracy
- 4- Precise alignment
- 5- Ability to retain the test rig for future usage
- 6- Low in cost

The test rig has to be designed in order to hold the measuring instruments, motor and MR brake in place. The measuring instruments for the test parameters have to be carefully planned in the initial stage of the design process, to ensure that the best measurements are obtained. A torque sensor is one of the best instruments to measure the torque of the brake in the course of the experiment. However, it is not within the budget allocated and a load cell has to be designed to measure the torque.

It should be noted here that the design calculations are not included in this thesis. The stress analysis and size calculations are done as a FYP project (Hon 2007) under supervision of Associate Professor Nader Vahdati.

5.1- Design Considerations

The main goal in designing the test rig is to experimentally test the MR rotary brake characteristics especially the torque of the brake. The test rig should have the capability to mount different MR rotary brakes with minimum change in its structure. Currently there is only one MR rotary brake available for test. This brake is obtained from LORD Corporation and is a disk-type MR rotary brake.

After the tests have been carried out, the experimental data are compared with the theoretical simulations. Simulations are fully done for the two types of MR rotary brakes, namely the drum-type, and the disk-type (see Chapter 4). Because of time constraints, the experiments could not be carried out during this research and comparisons between the experimental data and mathematical models are not available. However, the simulations are compared to experiments carried out by other researchers. The comparisons show that the mathematical models developed here well agree with the experimental results. It is expected that later experiments on the test rig designed here also approve the mathematical models.

The test rig has to be capable of:

- 1- Measuring the torque of the brake.
- 2- Controlling the speed of rotation.
- 3- Temperature measurements are also needed in order not to exceed the safe temperature limit of the brake. This is done by using a thermocouple on the surface of the brake and comparing the value with the working temperature of the brake.
- 4- The brake current has also to be controlled by a current source.

5.1.1- Precision Alignment

The alignment of the motor to the brake is one of the most important considerations for the entire project. The aim is to accurately align the two shafts together precisely after the whole test rig is assembled so that there will be an efficient power transfer from the motor to the brake during operation.

In very broad terms, shaft misalignment occurs when the centerlines of rotation of two machinery shafts are not in line with each other (angular misalignment). This can be due to axial misalignment as illustrated in Figure 5. 1 below. Another reason that gives rise to shaft misalignment is due to the differences in height of the centerlines (linear misalignment). These misalignments will give rise to bending stress as well as possible vibration during operation. In order to reduce the shaft misalignment, a coupling is used to hold the two shafts together.

The purpose of a shaft coupling is to absorb the shaft restoring loads while maintaining some amount of rigidity in rotation and transmission of torque. There are two main types of couplings widely used, namely the flexible type coupling, and the rigid type coupling. The key difference between the two is the allowance for backlash. For purpose of the project, the rigid type coupling is selected as it provides greater efficiency in torque transmission and hence greater accuracy in measurement.

Figure 5. 1 Misalignment of motor and brake shaft

Apart from solving axial misalignment, the test rig should also align both shafts three-dimensionally. In view of this, the three different dimensions are carefully planned and design so that after assembly, the centerline for both shafts will coincide. Following are the designs used to address the problem of alignment.

The initial idea is to have a separate block for each the MR brake as well as the motor. This is to allow for troubleshooting in event that the shaft do not meet after assembly, where individual block for instance the block that holds the motor can be machined to offset the differences in height. This design also solves the alignment in the two other axis, namely front axis (into the page) and horizontal axis since the brake is detached from the motor and they are not constrained in mobility. However, one issue that arises from this design is the difficulty in holding the individual blocks down on a platform (Figure 5. 2).

Figure 5. 2 Separate block for MR brake and motor

Another design is to hold both the MR brake as well as the motor on a common base plate so that the components can be held stationary and firmly during operation (Figure 5. 3). However, even if the centerline of the shafts can be aligned accurately in terms of

height through careful calculations, the same cannot be ensured in the front and horizontal axis. Furthermore, if there are misalignments in either axis, it would be difficult to perform any machining to the base plate for offset. For this design to work, the tolerances of the machined parts are required to be relatively low. To summarize, this design requires meticulous calculations for high accuracy in dimensions and allow for only a low margin of error.

Figure 5. 3 MR brake and motor attached to one common base plate

To improve the design, slots can be included into both the base platform as well as the brake holder. The purposes of these slots are to allow room for adjustment after assembly in all 3 dimensions. This will also allow for a reduction in the requirement for tolerances and hence reduce the overall cost of the design. As locating a manufacturer with the tools for high tolerance machining is time-consuming, this improvement can also save time as a wider group of manufacturers can be considered. Often, even with great caution and a

reliable graphics software, there exist some degree of misalignment. With this design add-on, this problem can also be eliminated.

To allow for sufficient room for adjustment after assembly, the tolerances in the 3 axis are as follows:

Y – Axis (top-down): 43mm

X – Axis (left- right): 65mm

Z – Axis (into the page): 30mm

The slots designed for each component is apparent due to its size. For instance, the plate on which the motor rest on has slots created in the Z-axis due to its bulky size and weight. The MR brake holder has slots for it for it to translate in the x-axis and within the holder itself are slots create to allow the MR brake to move in the Y-axis. (See Figure 5. 4 and Figure 5. 5)

Figure 5. 4 Isometric view of the base plate.

Figure 5. 5 Isometric view of brake plate.

With the consideration of alignment as well as implementation of the coupling to reduce axial misalignment, the objective of shaft alignment is fulfilled.

5.1.2- Ease of Assembly

For this design, there is not much difficulty in installation since the MR brake and the motor can be easily attached using bolts and nuts. Bolts and nuts are decided to be the primary connecting mechanism for the entire design. Once this is decided, the challenge lies in the location where these connectors should be placed. Considerations for these connectors include, ease for user to fasten bolts without much obstacle, the ability to hold the components in place as well as the number of these connectors required. More details to be presented in the next sections.

5.1.3- Future Usage of the Test Rig

A considerable amount of resources are used to build the test rig and the ideal is for the test rig to be able to conduct further testing with different sizes of MR brake. Hence during the design of the MR holder, one of the considerations is to allow for different sizes of the MR brake to be able to be mounted. More details will be presented in next sections.

5.2- Components

The use of the test rig is to facilitate the experiment so that accurate measurements can be taken during operation. The first approach for this design process is to understand the key limiting factors and parameters needed for the experiment as all these factors will ultimately contribute to how the test rig is to be designed. The two major components to observe are namely the MR brake and the motor. Other components have also to be designed carefully to prevent any future problems and at the same time satisfying the design goals.

5.2.1- MR Rotary Brake

The brake available for this experiment is a disk-type MR rotary brake purchased from LORD Corporation. The actual MR brake will be modeled using design software Solidworks to show all critical dimensions. These dimensions will then be referenced during the design of the test rig (Figure 5. 6 and Figure 5. 7).

Figure 5. 6 Front view of MR brake

Figure 5. 7 Side view of MR brake

The working conditions of the MR brake will limit the operating range of the motor and limit the criteria for motor selection. The working conditions for the MR rotary brake are as follows:

Supply voltage :	24V
Range of input current :	0 – 1.5 Amperes
Coil resistance :	21 Ω Nominal
Typical torque range :	0.4 – 5 Nm
Operating speed :	120 RPM maximum
Operating temperature :	-35 °C – 60 °C

5.2.2- Motor

A motor is selected base on the requirements and is similarly modeled using solidworks. The motor should be able to rotate the MR brake easily without any unexpected oscillations. It has to be able of delivering at least 5N.m torque at the speed of 120rpm. It should be noted that other MR rotary brakes may be used for testing in the future, so; the motor should be chosen such that it is able to support other brakes with higher capacities. The motor should also be capable of rotating at different speeds and the speed control system should be accurate and easy to use

According to the above considerations and availability of the motor, a AC motor with an accompanying speed control system is chosen (Figure 5. 8 and Figure 5. 9).

Figure 5. 8 Motor used in the design

Figure 5. 9 The selected motor and speed controller.

The key specifications of the motor are:

1 hp, 3-phase, 50Hz induction motor. The dimensions of the motor are:

Height: 164 mm max

Length: 256 mm max

Width: 205 mm max

A platform shall be designed according to these dimensions.

5.2.3- Brake Plate

The initial design for the MR brake plate is to have longer slots for different sizes of MR brake to fit. Such a design will be cost efficient since the same plate can be used for any brake to fit. As shown in Figure 5. 10, the extended slot is a good design to cater for MR brake of different sizes.

Figure 5. 10 MR brake holder with expansion slots.

However, this design will not allow the MR brake to be adjusted in any direction. As a result, there might be a possibility that the MR brake might be misaligned as discussed earlier. Since the main aim of the design is to provide for accurate test readings, the brake plate will be designed to have slots for it to be adjusted instead, as shown in Figure 5. 11.

The key dimensions of the brake plate are decided as shown in the following figure.

Figure 5. 11 Front view of brake plate.

5.2.4- Base Plate

Similarly, the concept of extended slots will be used in the base plate so to allow for small adjustments to be made for alignment. Seen in Figure 5. 12 is the design of slots for the base plate, such that it enables the brake plate to be adjusted front and back. The motor can also be adjusted side to side to ensure alignment with the brake plate. This two

alignment slots enable the motor and the brake to have accurate alignment as far as possible.

Figure 5. 12 Plan view of the base plate used for holding motor and brake plate.

In the design of the base plate, another consideration is to have additional space for easy installation of the bolts and nuts. Hence, the bottom part of the base plate is made hollow for this purpose (Figure 5. 13).

Figure 5. 13 Base plate as seen from below showing hollow sections.

5.2.5- Load Cell

The load cell is designed to take force measurement during operation. A load cell acts like a transducer that converts force into output that can be interpreted by human. A variety of load cells are available and the most common type of load cell is the strain gauge load cell. An introduction to load cells and their operating basics is given in Appendix A.

The selected design for the load cell will be based on a full-bridge configuration (Appendix A). All four arms of the strain gauge will be attached to the each of the four

faces of the metal block (Figure 5. 14), thus forming a load cell. The advantage of a full bridge design is to increase the sensitivity of the load cell to ensure better accuracy in the observations. The load cell will be placed directly below the MR brake holder to capture any forces during operation.

Figure 5. 14 The Load Cell.

When the motor is turned on, it will transmit torque to the MR brake and the MR brake will in turn transfer this force to the MR brake plate and eventually to the load cell as shown in Figure 5. 15.

Figure 5. 15 transmission resolve into linear force components.

Due to the symmetrical arrangement of the load cell with the MR brake plate, the force on each of the load cell can be estimated to have the following expression:

$$F = \frac{T}{2D} \quad (5.1)$$

Where T is the total torque transmitted to the brake plate, F is the force transmitted to the load cell, and D is the horizontal distance from centre of MR brake to centre of load cell (Figure 5. 15). It should be noted that the force exerted to the load cells from the weight of the brake assembly is causing a static deflection in the load cells which can be

set to zero when calibrating the load cell. So, F is the force resulted from torque transfer only.

5.2.6- Other Components

In addition to the key components mentioned above, there are other minor components that need to be brought together in order to complete the design. These components are considered not newly designed items but serve as part of the assembly. These components include:

- 1- Cylindrical shaft cap: To be attached to the shaft of the MR brake so that it better fits into a coupling (Figure 5. 16).

Figure 5. 16 The Cylindrical Shaft Cap.

- 2- Coupling: Used for connecting shafts to minimize misalignment (Figure 5. 17).

Figure 5. 17 The Couplings.

3- Bolts and nuts: Used due to their good strength and easy installation.

5.3- Full Assembly

Figure 5. 18 and Figure 5. 19 show the full assembly of all the components mentioned.

Details drawings will be provided in appendix B.

Figure 5. 18 Full assembly.

Figure 5. 19 Exploded view of the test rig.

5.4- Some additional design considerations

In addition to considerations mentioned in previous sections, some additional considerations and technical descriptions are address in this subsection:

- It is expected that the temperature has effects on the torque capacity of the MR rotary brake. Additionally, in order not to exceed the allowed temperature of the rotary brake, it is recommended to measure the operating temperature.
- In order to increase the operating temperature range of the MR rotary brake, it is possible to design a cooling system (e.g. heat sinks or fins on the surface of the MR rotary brake, etc.) to keep the temperature of the brake in the safe range.
- One of the possible factors that may affect the measurements and performance of the test rig is the vibration issue. This issue may require additional design considerations. For example, including some vibration isolators would be useful in reducing the vibration of the test rig and improving the measurements.
- The mounting holes on both ends of the load cells should not go deep into the load cell. This ensures that the stress field in the load cell is homogeneous and the strain gauge readings are more accurate (See the load cell drawing in appendix B).
- Appendix A gives an introduction about the load cell configuration and strain gauge assembling. Refer to appendix A for more details.

Chapter 6

Summary and Conclusion

6.1-Summary

This report started with an in-depth literature review of MR effect. It started by introducing field responsive fluids, and an overview of field responsive fluids (MR, ER, etc.) and their comparison was presented. A brief history of MR fluids was also given. The next section was devoted to description of the composition of MR fluids, and to commercially available MR fluids in the market and their specifications. The fundamental concepts of magnetism necessary for an indepth understanding of MR effect like magnetic field $\vec{H}$, magnetic induction $\vec{B}$, permeability μ , susceptibility, magnetic dipole, magnetization M , etc. were discussed. . Then a brief introduction to different types of magnetic materials (ferromagnets, paramagnets, etc.) was presented. Then the concept of magnetic circuits which is an important part in any MR device was discussed. MR rheology and rheometry, which are of great importance in designing MR devices were discussed and design equations for different types of rheometers were presented. MR flow modes were introduced and then the most commonly used constitutive equations for MR fluids were explained. There are many different types of constitutive

equations in the literature for MR fluids, but the most important constitutive equations are the Bingham and the Herschel-Bulkley models which are used for modeling of MR fluid flows in this report. For the case of MR fluid flow in the disk type MR brake, due to numerical problems associated with the Bingham model, the Bercovier and Engelman's model was used. This model is appropriate for numerical calculations where in, the singularity associated with the Bingham model is omitted by introducing a regularization parameter δ . Other important issues like applications of MR fluids, Stability, wall slip effects, dynamic characteristics, controllability and limitations of MR technology were also discussed in the literature review.

The next chapter was completely devoted to the MR brakes and clutches as the main goal of this research project was to develop more accurate mathematical models for MR brakes. In chapter 3, two main types of MR rotary brakes which are the disk type and the drum type (or cylindrical type) and their features were explained. A potential application of MR rotary brakes and clutches is their usage in automobile industry as either brake or clutch. This issue was also pointed out in this chapter.

The main part of this research project was the analytical modeling of MR fluid flow in different conditions. In chapter 4, the flow of MR fluids for different geometries was explained completely. We have started from the simplest geometry and moved to the more complex cases. As our first analysis, we have analyzed the MRF flow in a rectangular duct. The next modeling was done for the case of an annular duct. Then the mathematical modeling of the two types of MR rotary brakes was presented. For the case of drum type brakes, the equations can be written in such a way to reduce the problem to

a one dimensional problem by the aid of cylindrical coordinates, but for the case of disk type brakes, because of the nature of the problem, although we have used the cylindrical coordinates, the problem is two dimensional. By writing the governing equations for this case, a system of partial differential equations was obtained. The resulting equation was solved by the aid of numerical methods. A computer program was written in MATLAB to solve the equations. The results were presented and discussed. As stated earlier, the developed computer program can be used to track the yield surface in different designs of disk type MR rotary brakes. It also gives accurate results for the torque developed by the MR effect.

In chapter 5, a test rig was completely designed. The test rig was designed to experimentally test the MR rotary brakes. A disk-type MR rotary brake from LORD Corporation was purchased, and the test rig was designed based on this particular MR brake. But, the test rig was designed such that the changes and modifications are minimal for adoption of another MR brake. The design considerations and procedures were given in chapter 5 and the detailed drawings were presented as an appendix to the thesis. The design calculations were not included in this thesis. The stress analysis and size calculations were done as a FYP project (Hon 2007) under supervision of Associate Professor Nader Vahdati.

Since the A*STAR financial support for this project was only on manpower, no funding was available to build the test rig. The experimental testing of the MR-brakes will later take place when funding is available. So; the author's focus was mainly on analytical modeling and design of the MR-rotary brake test rig.

6.2-Conclusion

As discussed in details in chapter 4, the Bingham model is not able to predict the behavior of MR fluid in the Drum type rotary brakes when high shear rates are engaged. As the shear rate increases, the difference between the Herschel-Bulkley model prediction and the Bingham model prediction becomes more significant. This phenomenon is due to shear thinning of MR fluids at high shear rates. Using the Bingham constitutive equation to model MR fluids at high shear rates may result in over estimations in torque. So using the Bingham model to estimate the developed torque should be done with care, especially when dealing with high shear rates.

Design equations for Drum type MR brakes were also derived in chapter 4. The design equations were developed by using both the Bingham model and the Herschel-Bulkley model.

The Bercovier model was used to model the MR fluid flow in Disk type MR brakes. This model is presented by the aid of tensor notation which makes it useful for 2-Dimensional calculations. By writing the equations of motion, the resulting equation was not solvable by analytical methods; so a numerical method was implemented. In the Bercovier model the regularization parameter δ was set to a minimum. Less values of δ caused the solution to diverge and high values of δ resulted in inaccurate solutions. So; assigning the correct value to regularization parameter is very crucial. Solving the partial differential equation resulted in velocity distribution profile in the brake. Having the

velocity distribution profile, we were able to calculate the shear stress and also the developed torque. The results were in complete agreement with experimental results from other researchers.

In the current literature, no one has predicted the location of the plug flow region for disk type MR rotary brakes. Here in this study, we have introduced a way to predict the plug flow region, which can be very useful for design purposes. A MR rotary brake delivers the highest torque when the whole gap is in pre-yield phase or plug flow region (neglecting wall slip).

Figure 4. 23 shows the shear stress distribution in the flow field. As can be seen, the locus of points where the values of non-dimensional shear stress, τ^* , is equal to 1 is the yield surface. Locating the yield surface can be helpful in design of MR brakes and better estimation of the parameter h (the real gap thickness). Existence of yield surface in a brake means that there is a region behind the yield surface where the fluid velocity is zero and the fluid is stagnant (plug flow region). Plug flow region should be considered when designing MR rotary brakes.

Appendix A: Load Cells

A.1 Types of Load Cell

There are several types of load cell available and they serve different purposes. They can be classified into mechanical load cell, strain gauge load cell and other types of load cell.

A.1.1 Hydraulic Load Cell

Mechanical load cells consist of hydraulic and pneumatic load cells. The hydraulic load cell can take up to 10 million kg of weight with 0.25% accuracy. These devices work by measuring weight as a change in pressure of internal filling fluid. The pressure of the hydraulic fluid increases with an increase in force. As they are able to take high impact and are insensitive to temperature, they can be applied in tanks, bins and hoppers, as well as in hazardous areas. The weight of the tank has to be obtained by placing a load cell at each point of support. The disadvantage of hydraulic load cells is that they are expensive and complex.

A.1.2 Pneumatic Load Cells

Pneumatic load cells can withstand a wide range of weight with high accuracy. They operate using multiple dampener chambers to provide higher accuracy than hydraulic load cell. They are used in areas where small weights are measured and contamination is avoided, such as the food industry and also hazardous areas. The advantage is that it is safe and no fluid is used in the system. However, it has a slow response and requires clean and dry air.

A.1.3 Strain Gauge Load Cell

A strain gauge load cell is one that converts load into electrical signals. The gauge is bonded to a beam or a structural member that deforms when weight is applied. An electric circuit with resistors is incorporated into the load cell, so that the force changes will result in a change in resistance and thereby, producing a change in electrical signal. There are many types of strain gauge load cell available in the market, each catering to different applications. Some of the examples include S shape load cell, beam load cell, canister load cell etc. For this research, a commercial load cell that is professionally built is desirable. However, funds are limited and a load cell has to be built.

A.2 Working Principle of a Strain Gauge Load Cell

The strain gauges found on load cell are responsible for taking force measurements. A typical strain gauge is made up of a thin metal foil embeds a wire grid.

A.2.1 Strain Gauge

The strain gauge is made up of a tiny flat coil of conductive wire that is subjected to changes in resistance when it is bent. It should be placed on the beam of the load cell with special adhesive and the strain is directly related to the force applied to the beam.

Figure A. 1 Strain gauge patch.

Seen in Figure A. 1 is the strain gauge patch. When the patch is compressed in the direction shown in the figure, the resistance of the strain gauge decreases as the cross sectional area of the wire in the strain gauge increases.

The definition of strain is given by

$$\varepsilon_A = \frac{\Delta L}{L} \quad (\text{A.1})$$

where ΔL is the change in length and L is the original length.

When a member is subjected to uniaxial stress in the elastic range, lateral strains result with the following relation (Figure A. 2).

$$\nu = -\frac{\varepsilon_L}{\varepsilon_A} \quad (\text{A.2})$$

where ν is the Poisson's ratio, and ε_L is the lateral strain given by

$$\varepsilon_L = \frac{D_2 - D_1}{D_1} \quad (\text{A.3})$$

Figure A. 2 gauge wire before and after strain

The working principle of a strain gauge is to make use of the changes in resistance associated with the change in diameter to measure force.

A fundamental parameter of the strain gauge is its sensitivity to strain, expressed quantitatively as the gauge factor (GF). Gauge factor is defined as the ratio of fractional change in electrical resistance to the fractional change in length strain. Gauge factor are determined experimentally and provided by the manufacturers. The typical gauge factor for metallic gauge factor is 2 but it can also be calculated by:

$$GF = \frac{\Delta R}{R} \left(\frac{1}{\varepsilon} \right) \quad (\text{A.4})$$

Where R is the electrical resistance.

A.2.2 Wheatstone bridge circuit

The Wheatstone bridge is invented by Hunter Christie in 1833 and popularized by Charles Wheatstone. It is actually used to measure the difference in resistance and is commonly used in strain gauge. The circuit consisted of four resistors, a battery and a galvanometer. To measure such small changes in resistance, strain gauges are almost always used in a bridge configuration with a voltage excitation source. The general Wheatstone bridge, illustrated in Figure A. 3, consists of four resistive arms with an excitation voltage, V_{EX} , that is applied across the bridge.

Figure A. 3 Wheatstone bridge circuit.

The output voltage of the bridge, V_0 , will be equal to:

$$V_0 = \left[\left(\frac{R_3}{R_3 + R_4} \right) - \left(\frac{R_2}{R_1 + R_2} \right) \right] V_{EX} \quad (\text{A.5})$$

It can be shown that from this equation, when $\frac{R_1}{R_2} = \frac{R_4}{R_3}$, the voltage output V_0 will be zero. Under these conditions, the bridge is said to be balanced. Any change in resistance in any arm of the bridge will result in a nonzero output voltage.

Ideally, the resistance of the strain gauge should change only in response to applied strain. However, strain gauge material, as well as the specimen material to which the gauge is applied, will also respond to changes in temperature.

To minimize the effect of temperature two strain gauges instead of one, can be used in the bridge. Figure A. 4 illustrates a strain gauge configuration where one gauge is active ($R_G + \Delta R$), and a second gauge is placed transverse to the applied strain. Therefore, the strain has little effect on the second gauge, called the dummy gauge. However, any changes in temperature will affect both gauges in the same way. Because the temperature

changes are identical in the two gauges, the ratio of their resistance does not change, the voltage V_O does not change, and the effects of the temperature change are minimized.

Figure A. 4 Use of dummy gauge to eliminate temperature effects.

Making both gauges active in a half-bridge configuration can double the sensitivity of the bridge to strain. This half-bridge configuration, whose circuit diagram is illustrated in Figure A. 5, yields an output voltage that is linear and approximately doubles the output of the quarter-bridge circuit, where the latter only uses one strain gauge on the test specimen.

Figure A. 5 Half-Bridge Circuit.

Finally, we can further increase the sensitivity of the circuit by making all four of the arms of the bridge active strain gauges in a full-bridge configuration. The full-bridge circuit is shown in Figure A. 6.

Figure A. 6 Full-Bridge Circuit.

In practice, resistance tolerances and strain induced by gauge application will generate some initial offset voltage. This can be compensated by measuring the initial unstrained output of the circuit and compensating afterwards.

A.3 Strain Measurement Considerations

Strain gauge measurement involves sensing extremely small changes in resistance. To ensure accurate strain measurements, it is important to consider the following:

- Excitation
- Amplification
- Offset
- Shunt calibration

Excitation – It is important to have a constant voltage source to power the bridge. While there is no standard voltage level, excitation voltage levels of around 3 and 10 V are common.

Amplification – The output of strain gauges and bridges is relatively small. In practice, most strain gauge bridges and strain-based transducers will output less than 10 mV/V (10 mV of output per volt of excitation voltage). Therefore, amplifiers are necessary to boost the signal level to increase measurement resolution.

Offset – When a bridge is installed, it is very unlikely that the bridge will output exactly zero volts when no strain is applied. Hence, offset will be required to compensate for the difference.

Shunt Calibration – This involves simulating the input of strain by changing the resistance of an arm in the bridge by some known amount. This is accomplished by shunting, or connecting, with a large resistor of known value across one arm of the bridge, creating a known DR. The output of the bridge can then be measured and compared to the expected voltage value. The results can then be used to correct any errors in the entire measurement process

Appendix B: Detailed Drawings

B.1- Final Assembly and Part List of the Test Rig

REV

DESCRIPTION

DATE

DR

-

See Pg 1 SC

-

-

Typical Torque Range

Torque (Nm)

Current (A)

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B.3- Brake Plate

B.5- Load Cell

B.6- Cylindrical Cap

Appendix C: Publications

- 1- Farjoud, A., N. Vahdati and F. F. Yap (2007). "Mathematical Model of Drum-type MR Brakes using Herschel-Bulkley Shear Model." Journal of Intelligent Material Systems and Structures (**In Press**).
- 2- Farjoud, A., N. Vahdati and F. F. Yap (2007). "MR-fluid Yield Surface Determination in Disk-type MR Rotary Brakes." Journal of Intelligent Material Systems and Structures (**Under review**).
- 3- Vahdati, N., A. Farjoud and F. F. Yap (2007). "Drum-Type MR-Rotary Brakes under High Shear Rate Conditions." International Conference on Modeling, Simulation, and Applied Optimization (ICMSAO). March 24-27, 2007. Abu Dhabi, UAE. (**Accepted**)

Appendix D: The Matlab Code

D.1- The Main Program

```
% finite difference solution for MRF flow between 2 disks
% solving the Non-Dimensional form of equations
% for simplicity we avoid using the * superscript which we used to
denote a
% dimensionless variable
% the domain is a rectangle and the coordinates are r and z
% the characteristic velocity is v0=R2*omega
clc;
clear all;
warning off;
format long;
%k=input('press 1 if you want to start from the first iteration or
press 2 if you have a mat file and you want to continue from the data
on that file: ');
k=menu('','Start from the first iteration','I have a data file (MAT
file) already from previous executions of the code and I want to start
from its last iteration');
global dvdr curr vu vr vle vlo hr dvdz dvdrdz delta Bn hz error dvdr2
dvdz2
% m & n are the grid points in r and z direction respectively (known)
if k==1
    prompt = {'grid point numbers in r direction','grid point numbers
in z direction','R1','R2','z2','Bingham number','delta'};
    dlg_title = 'first iteration and applying boundary conditions: ';
    num_lines= 1;
    def      = {'26','26','3','6','0.1','130','0.02'};
    in1= inputdlg(prompt,dlg_title,num_lines,def);
    %disp('first iteration and applying boundary conditions: ');
    %m=input('enter the grid point numbers in r direction: ');
    %n=input('enter the number of grid points in z direction: ');
    %R1=input('enter R1: ');
    %R2=input('enter R2: ');
```

```

    %z2=input('enter z2: '); % this is dimensionless thickness of the
fluid (200306 page 20)
    %Bn=input('enter Bingham number: ');
    %delta=input('enter delta: '); % Bercovier and Engelman (1980)
    m=str2num(cell2mat(in1(1)));
    n=str2num(cell2mat(in1(2)));
    R1=str2num(cell2mat(in1(3)));
    R2=str2num(cell2mat(in1(4)));
    z2=str2num(cell2mat(in1(5)));
    Bn=str2num(cell2mat(in1(6)));
    delta=str2num(cell2mat(in1(7)));
    r=zeros(1,m);
    z=zeros(1,n);
    v=zeros(m,n);
    hr=(R2-R1)/(m-1);
    hz=(z2-0)/(n-1); % z varies from 0 to 1 because we have chosen
the charactrisic length as z2
    dvdr=zeros(m,n);
    dvdz=zeros(m,n); % we need to have the velocity gradient in each
point in order to find the shear stress at the domain so we define it
as a matrix
    taurteta=zeros(m,n);
    tautetaz=zeros(m,n);
    iter=0; % number of iterarions
    errorlog=100*ones(1,1);
    for i=1:m
        r(i)=R2-(i-1)*hr;
    end
    for j=1:n
        z(j)=(j-1)*hz;
    end
    %Boundary Conditions:
    % left part of the domain
    for i=1:m
        v(i,1)=r(i)/R2;
    end
    %lower part of domain
    for j=1:n

```

```

        v(m,j)=R1/R2*(1-z(j)/z2);
    end
    %right part of the domain
    for i=1:m
        v(i,n)=0;
    end
    %upper part of the domain
    for j=1:n
        v(1,j)=(1-z(j)/z2);
    end
    error=100*ones(m,n);
end
if k==2
    h = warndlg('please make sure that the file velocitymatrix.mat is
available and contains the proper data for this simulation','Warning');
    %disp('please make sure that the file velocitymatrix.mat is
available and contains the proper data for this simulation');
    %disp('press any key to continue');
    pause (5);
    load velocitymatrix
end
prompt = {'Enter the desired error percentage'};
dlg_title = 'Desired error PERCENTAGE: ';
num_lines= 1;
def      = {'50'};
in2= inputdlg(prompt,dlg_title,num_lines,def);
er=str2num(cell2mat(in2(1)));
%er=input('enter the desired error percentage: ') %desired percentage
error
while max(max(error(2:m-1,2:n-1)))>=er
    msgbox({'Number of iterations:',num2str(iter),'Maximum
error:',num2str(max(max(error(2:m-1,2:n-1))))},'','-----Entered
Parameters-----','','Desired Error Percentage:',num2str(er),'Non-
dimensional R1:',num2str(R1),'Non-dimensional R2:',num2str(R2),'Non-
dimensional Gap Thickness:',num2str(z2),'Bingham
Number:',num2str(Bn),'Regularization Parameter
(delta):',num2str(delta)},'parameters','replace');
    iter=iter+1;
end

```

```

    %msgbox({'Number of iterations:',num2str(iter)},'replace');
    %disp('number of iterations:');
    %disp(iter);
    errorlog(iter)=max(max(error(2:m-1,2:n-1)));
    for i=2:m-1
        for j=2:n-1
%            if error(i,j)>.5
                dvdr(i,j)=(v(i-1,j)-v(i+1,j))/(2*hr);
                dvdz(i,j)=(v(i,j+1)-v(i,j-1))/(2*hz);
                dvdrdz=(v(i-1,j+1)-v(i-1,j-1)-v(i+1,j+1)+v(i+1,j-
1))/(4*hr*hz);
                curr=r(i);
                vu=v(i-1,j);
                vle=v(i,j-1);
                vlo=v(i+1,j);
                vr=v(i,j+1);
                x0=v(i,j);
                dvdr2=dvdr(i,j);
                dvdz2=dvdz(i,j);
                s=fsolve(@myfunction2_modified_bercovier,x0);
                if s<0
                    beep
                end
                if s>0                                %at first few iterations the
program avoids accepting negative values
                    error(i,j)=100*abs((s-v(i,j))/s);
                    v(i,j)=s;
                end
                if iter>50 & s<0                        % after a certain number of
iterations if there still exists a negative number the program accepts
it and assigns it to the velocity matrix
                    error(i,j)=100*abs((s-v(i,j))/s);
                    v(i,j)=s;
                end
                    %disp(i);
                    %disp(j);
                    %disp('percentage error at (i,j):');
                    %disp(error(i,j));

```

```

                                %disp('value of velocity:');
                                %disp(v(i,j));
                                %disp('maximum error:');
                                %disp(max(max(error(2:m-1,2:n-1))));
                                %disp('-----');
%                                end
                                end
                                end
                                end
                                end
                                surface(flipud(v));
                                title('velocity distribution');
                                xlabel('Gap Thickness ( 0 to z2 )');
                                ylabel('Radial Distance ( R1 ro R2 )');
                                figure;
                                surface(flipud(error(2:m-1,2:n-1)));
                                title('error distribution(percentage error)');
                                xlabel('Gap Thickness ( 0 to z2 )');
                                ylabel('Radial Distance ( R1 ro R2 )');
                                disp('solution converged');
                                disp('maximum error:');
                                disp(max(max(error(2:m-1,2:n-1))));
                                figure;
                                plot([1:1:iter],errorlog);
                                title('maximum error vs iteration no. ');
                                xlabel('Iteration Number');
                                ylabel('Percentage Error');
                                % evaluating the value of shear stress in the domain
                                for i=2:m-1
                                    for j=2:n-1
                                        taurteta(i,j)=1/2*(2/Bn+1/(1/2*((dvdr(i,j))-
v(i,j)/r(i))^2+dvdz(i,j)^2)^(1/2)+delta))*(dvdr(i,j)-v(i,j)/r(i));
                                        tautetaz(i,j)=1/2*(2/Bn+1/(1/2*((dvdr(i,j))-
v(i,j)/r(i))^2+dvdz(i,j)^2)^(1/2)+delta))*(dvdz(i,j));
                                    end
                                end
                                end
                                save velocitymatrix.mat
                                figure;
                                surface(flipud(taurteta(2:m-1,2:n-1)));

```

```

title('shear stress distribution (rteta)');
xlabel('Gap Thickness ( 0 to z2 )');
ylabel('Radial Distance ( R1 ro R2 )');
figure;
surface(flipud(tautetaz(2:m-1,2:n-1)));
title('shear stress distribution (tetaz)');
xlabel('Gap Thickness ( 0 to z2 )');
ylabel('Radial Distance ( R1 ro R2 )');
%torque calculations
%as seen in 280306 tautetaz incorporates in generation of the shear
stress
%on the surface of the disk
% note that all the variables are non-diemnsional as calculated in
280306
sum=0;
for i=2:m-1
    sum=sum+tautetaz(i,2)*r(i)^2;
end
torque=2*pi*hr*sum;
msgbox({'Number of iterations:',num2str(iter),'Maximum
error:',num2str(max(max(error(2:m-1,2:n-1))))},'','-----Entered
Parameters-----','','Desired Error Percentage:',num2str(er),'Non-
dimensional R1:',num2str(R1),'Non-dimensional R2:',num2str(R2),'Non-
dimensional Gap Thickness:',num2str(z2),'Bingham
Number:',num2str(Bn),'Regularization Parameter
(delta):',num2str(delta),'','-----Results-----','','the
nondimensional torque value is:',num2str(torque)},'Problem
Solved','replace');
%disp('the nondimensional torque value is:');
%disp(torque);

```

D.2- The PDE Function

```
function F=myfunction2_modified_bercovier(x)
global dvdr curr vu vr vle vlo hr dvdz dvdrdz delta Bn hz error dvdr2
dvdz2
F=[-1/8*(2*(dvdr2-x/curr)*((vu-2*x+vlo)/hr^2-
dvdr2/curr+x/curr^2)+2*dvdz2*dvdrdz)*(dvdr2-x/curr)/((1/2*((dvdr2-
x/curr)^2+dvdz2^2)^(1/2)+delta)^2*((dvdr2-
x/curr)^2+dvdz2^2)^(1/2))+1/2*(2/Bn+1/(1/2*((dvdr2-
x/curr)^2+dvdz2^2)^(1/2)+delta))*((vu-2*x+vlo)/hr^2-
dvdr2/curr+x/curr^2)-1/8*(2*(dvdr2-x/curr)*(dvdrdz-
dvdz2/curr)+2*dvdz2*(vr-2*x+vle)/hz^2)*dvdz2/((1/2*((dvdr2-
x/curr)^2+dvdz2^2)^(1/2)+delta)^2*((dvdr2-
x/curr)^2+dvdz2^2)^(1/2))+1/2*(2/Bn+1/(1/2*((dvdr2-
x/curr)^2+dvdz2^2)^(1/2)+delta))*(vr-
2*x+vle)/hz^2+(2/Bn+1/(1/2*((dvdr2-
x/curr)^2+dvdz2^2)^(1/2)+delta))*(dvdr2-x/curr)/curr];
```

Appendix E: The Lambert W Function

In the equation of plug radius, the Lambert W function appears. The Lambert function is defined to be the multi-valued inverse of the function $W \mapsto W(x)e^{W(x)}$. It has many applications in pure and applied mathematics (Corless, Gonnet et al. 1996; Caillol 2003). Here, a brief introduction of this function is presented by which the range of plug radius is explained. A summery is also addressed in the text. Please also see: http://en.wikipedia.org/wiki/Lambert_W_function

For $W > 0$, We^W increases monotonically from 0 to ∞ (Figure E. 1). When $W < 0$, We^W is negative. Thus, for $x > 0$, $W(x)$ is positive and well defined and increases monotonically.

For $W < 0$, We^W reaches a minimum of $-\frac{1}{e}$ at $W=-1$. On $-1 < W < 0$, We^W increases monotonically from $-\frac{1}{e}$ to 0. On $-\infty < W < -1$, We^W decreases monotonically from 0 to $-\frac{1}{e}$. Thus, on $\left(-\frac{1}{e}, 0\right)$, $W(x)$ can have one of two values, one in $(-1, 0)$ and another in $(-\infty, -1)$. The value in $(-1, 0)$ is called the principal value and is usually shown by $W_0(x)$ or $W(0, x)$. The other value is shown by $W_{-1}(x)$ or $W(-1, x)$. A plot of the Lambert W function is shown in Figure E. 1.

Figure E. 1 Plot of the Lambert W function.

In our calculations the plug radius is

$$r_p = \sqrt{\frac{-c_1}{\tau_y}} = R_1 \sqrt{-W \left(-1, -\frac{e^{\frac{-2\tau_y \ln R_1 + \tau_y + 2\omega_0 k}{\tau_y}}}{R_1^2} \right)} \quad (\text{E.1})$$

Let $x = -\frac{e^{\frac{-2\tau_y \ln R_1 + \tau_y + 2\omega_0 k}{\tau_y}}}{R_1^2}$, then

$$\lim_{\tau_y \rightarrow 0} x = 0 \text{ and } \lim_{\tau_y \rightarrow \infty} x = -\frac{1}{e}.$$

So; x is in the interval $\left(-\frac{1}{e}, 0\right)$. As is seen in Figure E. 1, when $-\frac{1}{e} < x < 0$, W has two values. To have a reasonable value for the plug radius, the value in $(-\infty, -1)$ is accepted (this implies that $W(-1, x)$ is the answer which physically makes sense and corresponds to the characteristics of MR rotary brakes) which results in $R_1 < r_p < \infty$. So; the range of plug radius is (R_1, R_2) . The details are not included in the paper because the paper may get too long but a summary is presented.

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