Simulated modeling and image processing for intra-cardiac interventional diagnosis and therapy

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**SIMULATED MODELING AND IMAGE
PROCESSING FOR INTRA-CARDIAC
INTERVENTIONAL DIAGNOSIS AND THERAPY**

SUBMITTED

BY

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ABSTRACT

Coronary angioplasty and intra-coronary stent implantation are minimally invasive procedures to treat coronary heart disease through revascularization. A percutaneous intra-cardiac intervention offers an alternative treatment to restore myocardial function in ischemic regions through stem cell injection. This procedure allows for diagnosis and therapy in the same surgical setting where electromechanical mapping of the heart can detect ischemic regions and assess their severity for treatment.

Virtual reality simulators for vascular intervention have long been embraced for surgical training. In this research, we investigate technologies for developing an intra-cardiac intervention simulator to fulfill surgical training needs and improve the efficacy of the heart mapping procedure. Exploring from modeling of catheter-heart interaction, heart surface reconstruction to dynamic heart modeling, a real-time and patient specific intra-cardiac simulator is progressively being defined.

As a low complexity option for training, a novel geometrical approach is employed to model a catheter and its deformation when in contact with the moving heart wall. Through empirical measurements, we identify the non-slip and slip contacts for modeling. Non-slip contact is modeled by a memory recursive deformation between insertion and tip to minimize lateral and axial strains of the catheter. Slip contact is modeled by a progressive deformation from insertion to tip to constraint the catheter within the heart enclosure. The geometrical and subjective requirements are satisfied through positive user feedback and comparison with experimental work.

As electromechanical properties are increasingly acquired, they are mapped to a progressively reconstructed heart surface. An accurate surface representation aids in the constructive interaction between user and device. An algorithm is developed for an accurate and fast reconstruction from these sparse and non-uniform data through iterations of Laplacian deformation, local subdivision and edge swapping. The proposed algorithm has the best fit among the deformable mesh techniques.

To enable realistic heart mapping, dynamic heart motion is modeled from patient-specific tMRI scans. A B-Spline approach to phase unwrapping is proposed for motion tracking in tMRI. A bicubic B-Spline surface is used to model the absolute phase by minimizing the spatial gradient differences of unwrapped and wrapped phases and the residual differences of re-wrapped and wrapped phases. Compared to existing techniques, the approach overcomes the half-tag displacement limitation with the best motion tracking performance. Through

extensions of the deformable mesh in surface reconstruction, myocardial deformations are tracked spatio-temporally to reconstruct a dynamic heart model. The differences between tracked and projected displacements from orthogonal planes are minimized in a single optimization with constraints in surface curvature between deformations. Despite its reduced complexity, the measured strains and torsion from the proposed method are consistent with literature.

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LIST OF ABBREVIATIONS

DICOM	Digital Imaging and Communications in Medicine
FEM	Finite Element Method
GPU	Graphical Processing Unit
HARP	Harmonic Phase Imaging
HLA	Horizontal Long Axis
LA	Long Axis
LV	Left Ventricle
NURBS	Non-Uniform Rational B-Spline
MRI	Magnetic Resonance Imaging
RV	Right Ventricle
RF	Radiofrequency
SA	Short Axis
SPAMM	Spatial Modulation of Magnetism
tMRI	Tagged MRI
VLA	Vertical Long Axis
VR	Virtual Reality

Chapter 1 INTRODUCTION

1.1 Background

Cardiovascular disease is a major cause of death worldwide. Many of these cardiovascular related deaths are not caused by acute myocardial infarction but are due to a progressive decline in cardiac function from conditions such as ischemia and hypertension. Intra-cardiac interventional procedures are increasingly being performed for diagnostic and therapeutic applications in cardiac diseases. Electromechanical mapping of the heart [1][2] can detect the location and assess the severity of ischemic regions or viable myocardium regions in-situ. The early assessment heart wall damage is critical to the survival of the heart patient as early treatment lead to higher chances of recovery. Following the diagnostic procedure, subsequent therapeutic strategy focuses on the angiogenesis and cardiomyogenesis through cellular and gene injections to the mapped region of diseased but viable myocardium.

A cardiac navigation system known as NOGA® XP [3] is developed by Johnson and Johnson for intra-cardiac intervention. The system has unique features including fast and precise electromechanical mapping with an intra-cardiac mapping catheter, non-fluoroscopic real-time 3D display of electrical viability and wall motion data, accurate characterization of scar tissue and viable myocardium and precise therapeutic transendocardial delivery of biological agents through an intra-cardiac injection catheter.

The heart mapping procedure using the percutaneous intra-cardiac device is illustrated in Fig 1. A patient is first prepared for the procedure under local anesthesia and a catheter is inserted via the femoral artery and navigated to the aortic valve under the guidance of fluoroscopy. Three to four landmark points are acquired with the help of a pre-procedural echocardiograph for the initial reconstruction of the heart chamber. Fluoroscopy is then switched off to minimize radiation exposure for the patient. The catheter is maneuvered within the heart chamber through external twisting, bending, insertion and retraction by an interventional cardiologist. Contacts are acquired when the catheter tip is in stable contact with the heart wall. Cardiac measurements such as intra-cardiac electrocardiogram and catheter tip motion within a cardiac cycle are also captured simultaneously for each contact and post-processed for the reconstruction of an endocardial surface and the composite imaging of surface electrical and contractile activities. The contact acquisition process is repeated under visual aid of the progressively reconstructed surface until a mapping criterion is satisfied. The mapping criterion refers to the fair distribution and the sufficient density of contacts in the heart chamber. The composite image allows for the delineation of local

ischemic areas, marking the end of the diagnostic procedure. Ischemic areas are defined by regions of abnormal wall motion and higher sampling density is desired at their boundaries where stem cell treatment can be administered.

Fig. 1: Heart mapping procedure in intra-cardiac intervention.

1.2 Motivation

Despite the diagnostic and therapeutic potential of this intra-cardiac interventional device, the electromechanical mapping procedure remains complex. Interventional cardiologists have to attend instructional courses and intensive training of the mapping procedure on large animals before using the intra-cardiac intervention system on humans. The procedure requires the navigation of a catheter for local surface reconstruction under minimal visual aid and the securing of stable contacts of a catheter with the heart wall for acquisition of local properties. Rapid hand-eye coordination plays a critical role for the success of this heart mapping procedure as the intra-cardiac interventional system displays only the partially reconstructed surface and the catheter tip. Navigating the catheter in the heart lumen to acquire optimal stable contacts in a beating heart requires the learning of precise control to bend, twist and move the catheter tip to the intended location and to position the tip at a suitable angle to heart wall and apply the correct amount of force for mapping or injection.

A simulator system is being designed to help new interventional cardiologists gain sufficient experience before treating patients. The development of the simulator system requires research across various disciplines including mechanical, electrical, computer and medicine.

A successful simulator system lies in its ability to simulate real life heart conditions from patient specific data and to mimic the interventional procedure accurately in real-time. Simulated modeling and image processing are very important aspects to translate patient specific data and surgical setting to training rooms or simulated operation theatres. The simulator provides a platform for surgical skill training, as well as an opportunity for continuous improvement of the interventional procedure.

1.3 Objectives

This project focuses on the development of the simulator system for cardiac interventional diagnosis and therapy in three key areas: catheter-heart interaction, heart surface reconstruction and patient-specific heart model. In catheter-heart interaction, we model the catheter navigation, the contact of the catheter with the heart wall and the resultant catheter deformation to simulate the heart mapping and stable contact acquisition. In heart surface reconstruction, we provide real-time dynamic reconstructions of the heart surface as contacts are acquired progressively in the procedure. In patient-specific heart model, we track the planar cardiac motion and spatio-temporal deformations of myocardium from tMRI scans for reconstructing a patient-specific dynamic heart model for the simulator.

1.4 Scope

The research covers the simulation of electromechanical mapping of the heart for training purposes. It models the local mechanical properties of the heart for the manipulation of an intra-cardiac interventional device but not the electrical properties. There is no intention in this research to model the micro-architecture of the heart structure or its delicate biological, chemical or mechanical processes. The methods being developed in this project are for generic interventions and are not restricted to cardiac applications. This thesis documents original research work done by the author. The work on empirical measurements is done in collaboration with another project member. The virtual catheter unit by a third project member is reported in this thesis for completeness of the intra-cardiac simulator. Readers are encouraged to refer to their theses for more details.

1.5 Highlights

The thesis has three key highlights of original work by the author. First, a novel geometric-based method is proposed to model dynamic interaction between a mapping catheter and the heart wall, specifically the catheter deformation for two types of contacts, slip and non-slip, identified through force-displacement measurements of pushing a catheter against mounted heart tissue. Second, a linear variational mesh technique is proposed for an accurate and fast progressive surface reconstruction from a sparse and non-uniform data set through iterations of Laplacian deformation, local subdivision and edge swapping. Third, a novel B-Spline approach to phase unwrapping is proposed for motion tracking in tMRI scans and complements as a phase tracking method on long axis slices, where motion tracking is poor with prior art. The spatial-temporal myocardial motion is reconstructed from the planar tracked motion through a novel linear variational surface with projection optimization.

1.6 Organization of Dissertation

The thesis is organized into nine chapters:

- Chapter 1 gives an introduction, motivation and focus of the research work.
- Chapter 2 provides a literature review of existing arts in the areas of virtual reality (VR) simulators for minimally invasive procedures, device-tissue interaction for cardiovascular interventions, surface reconstruction techniques, motion tracking in tMRI and spatio-temporal tracking of myocardial deformation.
- Chapter 3 describes a general overview of the simulator for intra-cardiac intervention, including catheter model, heart surface voxelization, catheter-heart interaction, surface reconstructions, VR catheter unit and user evaluation study. The VR catheter unit is designed by Yu [4].
- Chapter 4 provides empirical measurements of contacts between catheter and heart wall. This work is done in collaboration with Ei [5].
- Chapter 5 presents a geometric modeling of dynamic catheter-heart interaction for heart mapping and acquisition of contacts.
- Chapter 6 describes a progressive surface reconstruction method for a sparse and random dataset.
- Chapter 7 proposes a B-Spline approach to phase unwrapping for motion tracking in tMRI.

- Chapter 8 presents a differential surface and projected displacement optimization for spatio-temporal tracking of left ventricle (LV) deformation.
- Chapter 9 concludes on the research work.

Chapter 2 LITERATURE REVIEW

2.1 VR Simulator

VR simulators have been developed for minimally invasive surgery [6] including needle biopsy [7], laparoscopic surgery [8] and interventional radiology [9]. Of specific interest to us are VR simulators for cardiology and vascular surgery that have rapidly been embraced for surgical training [10] and pre-treatment planning. Aiming to augment but not replace clinical experience, VR simulation can provide consistent training curriculum including detail assessment of the operators' skills and procedural errors. Learning curves with improved performance for carotid angiography has been demonstrated in commercial VR simulators for cardiovascular intervention [11]. In fact, VR simulation training has been listed as one of a mandatory requirement by Food and Drugs Administration for the high risk carotid stenting procedure. Of the many efforts made to simulate interventional cardiology, some simulators have given rise to commercialized catheterization systems like Cathi from Siemens, VISTTM from Mentice and ANGIO Mentor from Simbionix. VR technology has been used to simulate realism to enhance hand-eye coordination for training of vascular intervention [12] and coronary stent implantation. Despite the various advances in simulator technology for interventional cardiology, simulation work for myocardial mapping and intra-cardiac injection procedures, to the best of our knowledge, have yet to be reported.

Thus, we compare our work to that of simulators for vascular intervention [9]. While similar building blocks like rendering, haptic device, catheter and guidewire, physical and geometrical modeling are required, the technical framework for an intra-cardiac intervention differs from that of a vascular intervention in several important aspects. Instead of displaying fluoroscopic images, we need to progressively reconstruct and display the 3D heart surface from increasingly acquired mapping points and their properties. The catheter used in heart mapping procedure has a bending degree control to be modeled. A detailed heart anatomical structure replaces that of the vascular structure. The modeling focuses on interaction between the catheter and the heart wall in an enclosed volume rather than between the catheter and vessel wall in a tubular structure. There are additional requirements of the intra-cardiac intervention simulator for the modeling of dynamic contraction of the heart and electrical signal propagation.

2.2 Device-Tissue Interaction for Cardiovascular Intervention

During the intra-cardiac intervention procedure, the interventional cardiologist navigates the catheter through insertion, retraction, rotation and bending to acquire stable contacts with the heart wall. To simulate the acquisition of such contacts, we require the modeling of the interaction between the catheter and the heart wall. Given an input set of catheter movement and the dynamic contraction of the heart, we seek to determine the deformation of the catheter and if the tip has a stable contact with the heart wall.

Previous studies have focused on modeling the interaction between the catheter and the blood vessel for vascular intervention rather than the interaction of the catheter and the heart wall within a chamber enclosure. For example, a multi-body system is used to simulate the interaction of the catheter with the blood vessel [9]. The position and dynamics of the catheter are determined based on input force applied by a user at the proximal end of the catheter, contact forces due to the collision of catheter length along the blood vessels as well as forces from contrast dye injection. Cai *et al.* [13] employ a potential field representation of the vascular structure to support FEM based modeling for the catheter-vessel interaction. Lenoir *et al.* [14] model a catheter and a guidewire as a set of beam elements in FEM with static non-linear characteristics. The combination of translation and rotation imposes a set of constraints on the composite catheter-guidewire FEM model for deformation. An optimization strategy [16] based on substructure decomposition is used for faster computation. Collision detection is aided by using a connectivity graph that links a set of connected segments in the vasculature model. The collision response of the catheter with the blood vessel is iteratively found by applying a local set of concave and convex constraints within the detected segment and solving the system with the Gauss-Seidel method. The accuracy of collision detection depends on the resolution. Wang *et al.* [15] propose to simulate the elastic behavior of a catheter and a guidewire with a physics thread model represented by $(k+1)$ nodes connected by linear and angular springs. Each node has an assigned mass and polar momentum of inertia and is described by force and torsion dynamics. Bounding volume hierarchy is used for collision detection in which the vascular model is represented by axis-aligned bounding boxes [17] and the catheter and guidewire by spheres to balance collision detection accuracy with efficiency. At each movement, forces and torsion of a group of control nodes are computed. The deformation and torsion for non-control nodes are then solved by Euler integration. However, the method only models the elastic behavior of the catheter and guidewire. Another physics-based approach by Alderliesten *et al.* [18] models the propagation of guidewire by minimizing the sum of internal energy of guidewire bending and torsion and the external vessel wall deformation on

interaction with guidewire while considering the friction between guidewire and vessel wall. A semi-analytical approximation [19] is selected to minimize the energy of each joint from the tip to the bottom of guidewire with several iterations for global convergence.

In all of the above methods, the modeling considers the movement of the catheter along the vessel wall when inserted or retracted whereas the intra-cardiac intervention introduces another modeling scenario where the catheter tip can be fixed at a position on the heart wall when the catheter is inserted, retracted, rotated or bent. The deformation of the catheter is also limited in space due to the small volume enclosed by the vessel segment relative to the heart chamber in intra-cardiac intervention. There are typically a large number of contacts between a catheter and the vessel wall but there can be only few contacts in the heart surface. More importantly, the modeling of the vascular intervention considers only the movement of the catheter but not the vessel wall. The intra-cardiac intervention, however, requires the modeling of interaction between two moving bodies: the catheter and the heart wall. Therefore, these methods cannot be directly used in intra-cardiac intervention. The catheter kinematics involved in intra-cardiac navigation, as described by Ganji *et al.* [20], predict the catheter tip position based on external manipulation. However, they do not address catheter interaction with the heart wall within the heart chamber.

We propose a geometric-based approach to model catheter-heart interaction that can provide plausible results for simple VR training. The approach covers the catheter deformation for the use cases where the catheter tip remains at the contact position or it moves along the heart wall when the catheter is inserted or retracted or the heart contracts. The advantage of this approach is its low complexity that allows the system to be deployed with normal computing resources.

2.3 Surface Reconstruction

The quality of reconstruction of the 3D surface is an important aspect of the simulator. The surface reconstruction helps the interventional cardiologist to visualize the progress of the mapped surface and constructively influences his next decision. To the end, it affects the diagnosis of the cardiac disease. Rather than building a coarse mesh from which each vertex has to be acquired like in the existing intra-cardiac intervention system [3], we seek to deform an initial mesh and update its surface smoothly to fit a set of sparse and random contacts that is accumulated interactively during the mapping procedure.

The adaptive deformation of a triangular mesh model to approximate a 3D surface is common in computer graphics. Early work of deformable models combines geometry, physics and approximation theory for biomedical imaging [21]. McInerney *et al.* [22] use quintic triangular finite elements and bicubic rectangular finite elements to deform a planar surface through Lagrangian dynamics to fit 3D data where model-data distances are used as point forces. They extend the method to use a spherical thin plate [23] and global subdivision to fit dynamic 3D data. The computation for the global damping and stiffness matrix makes the FEM less suitable for minimal latency reconstruction.

With the non physics-based approach, Miller *et al.* [24] deform a closed geometric model to fit a volume dataset by optimizing deformation potential with feature voxel and topological constraints. However, their algorithm is applied to dense volume data rather than sparse data and it requires information on an object boundary unknown in our problem. Hoppe *et al.* [25] develop an algorithm based on the zero set of an estimated piecewise signed distance function for the reconstruction of a surface from a set of unorganized points. Similarly, the algorithm requires a dataset with dense samples and it cannot recover features where insufficient sampling occurs. Tognola *et al.* [26][27] reconstruct anatomical structures including the human heart model from a large unorganized set of range data. An icosahedron geometric model first undergoes a series of expansion and subdivision to fit the dataset. Re-sampling is then performed in regions of rich details with high local errors. Global smoothing without global shrinkages is achieved with several iterations of Taubin filtering [28]. Open surfaces are reconstructed from a planar surface with a number of vertices corresponding to 25% of the range data [26]. The projected distances between the points on the planar surface and the range data are minimized. As the targeted surface has a large set of range data with high level of details, the work cannot be applied directly in surface reconstruction with a few contacts. Similar in nature but with an input set of 2D images from 3D echocardiography, Legget *et al.* [29] propose a piecewise smooth subdivision to fit a set of 3D points sampled from manually traced borders of a LV. A control mesh model, specifically designed for the to-be-reconstructed structure, is rigidly aligned to the traced data through several iterations of translations, rotations and scaling. A subdivision surface representation is generated by a control mesh, where the topology and general shape, as well as local surface features are preserved. The optimal surface representation minimizes the residual energy between the dataset and anatomical features in the mesh model while maximizing the surface smoothness. The conjugate gradient method [30] is used in the optimization. Although the method reports good accuracies in the LV and RV [31]

approximations, manual indication of features and high computational time of 5 to 10 minutes are important deterrents for a heart mapping application.

For interactive applications, Botsch *et al.* [32] provide an excellent survey of linear variational mesh deformation techniques. They review the main approaches of multi-resolution editing and differential surface representations for various editing scenarios. By linearizing the deformation energies and details preservation objectives, the global optimization reduces to solving a sparse linear system of equations, making surface deformation particularly suitable for interaction. Recent work by Zhang *et al.* [33] applies the differential surface deformation for the reconstruction of LV from tMRI. They use a global deformation for translation, rotation and isotropic scaling, followed by Laplacian surface optimization (LSO) with a set of evenly selected landmark points for uniform smoothing. Laplacian surface deformation then adapts the model to a set of image landmarks by a local transformation and displacement of selected vertices. An additional LSO iteration is applied to improve the triangle quality. As an extension to surface motion reconstruction, Wang *et al.* [34] propose the use of meshless deformable models for the LV motion analysis from tMRI. Global deformation such as contraction and twisting is obtained by integrating a local parametric function over the entire cardiac volume while the local deformation is recovered by an intrinsic Laplacian representation. The geometric details of the surface are tracked by minimizing both the transform invariant Laplacian and the distances from the landmark points. The method relies on the tracking of a set uniformly distributed tag intersections and identification of landmarks. Most works assumes a one-off and not a progressive 3D reconstruction with an assumed dataset size and uniform distribution that is not appropriate for a heart mapping application. The prior art reconstruction may have large differences with the targeted geometry under conditions where the sparse contacts are concentrated in one region.

Rather than a surface-based approach that both computation and robustness increase linearly with complexity of surface tessellation, the volumetric space deformation technique is independent of the complexity and quality of surface representation. Botsch *et al.* [35] present an efficient incremental least square technique for the use of triharmonic radial basis function (RBF) for real-time freeform editing. Although triharmonic RBF can produce high quality deformation of fairness, the low density of control points in the regions of interpolation and the lack of C^2 constraints from support boundary in a heart mapping application made the RBF method unsuitable. By mapping the mesh vertices that are paired and unpaired with the contacts to the handle and free vertices in the algorithm, large undesirable changes in the reconstructed shape are observed between consecutive updates

depending on the random distribution of contacts. In the work by Reisfeld [36], a three dimensional reconstruction of intra-body organs is proposed through radial-distance-based adjustments. The mesh is first deformed as a weighted inverse distance of the mesh vertices from the contacts. Then the contacts are associated with nearest mesh vertices and all the mesh vertices are finely adjusted as a weighted inverse distance from the associated pair. A Gaussian kernel smoothens the adjusted mesh but results in a shrinkage that has to be compensated by affine transformation. As the deformation may not have an exact match of the contacts with the reconstructed surface, a matching process is added. The advantage of this method is its fast computation. The disadvantages are the high distortion for small number of contacts and the non-convergence of the smoothing process. Further addition of the exact matching process after smoothing may produce discontinuity and large shape discrepancies between successive contact acquisitions.

Sharf *et al.* [37] present yet another rough to fine approach to reconstruct a surface from a point cloud using multiple competing evolving fronts. The fronts adapt to the local feature in a coarse-to-fine manner by varying the weights between Laplacian and an attraction factor and then performing remeshing in areas of low triangle density. Topology control can be done through some user interaction. The final deformation is achieved via a moving least square projection. The reconstruction guarantees water-tightness and adaptive handling of sparse and non-uniform sampling. Since the fronts evolve from the initial mesh that is embedded within the object to be reconstructed, it limits the new contact acquisition to be outside the reconstructed surface during the progressive acquisition process. The reconstruction may not perform well under a few contacts without additional pseudo contacts to guide the evolution of the mesh.

Compared to the techniques mentioned above, our surface reconstruction method generates reasonable quality 3D surface with very sparse (minimally 2) and non-uniform dataset using a generic mesh of approximate shape and yet satisfies real-time requirement. Unlike most reconstruction from point cloud, it can progressively update a surface as new points are acquired.

2.4 Motion Tracking in Cardiac Tagged MRI

For realistic simulation of the catheter-heart interaction, a model describing the dynamic contraction of the heart is mandatory. To build our own model, two propositions are identified: raw position data captured during the intra-cardiac simulation and tMRI scans. Since we have not been able to access the data in the former due to data confidentiality,

tMRI scan is the only option. Comparing with raw position data, tMRI captures the heart motion in images and requires motion tracking algorithms. As a first step, we seek to recover dense LV planar motion from a set of patient-specific MRI data.

tMRI scan can provide important clinical information about the contractile abnormality of the heart wall for the diagnosis of cardiac diseases. Accurate motion tracking of the LV in the entire cardiac cycle can be performed with the tMRI for mapping of ventricular strain and reconstruction of a moving 3D LV. Among the literature for 2D motion tracking in tMRI, phase tracking is widely used compared to conventional tag line tracking, optical flow based tracking or frequency domain tracking. The Extrema Temporal Chaining method [38] is a recent line tracking method that uses temporal correlation in addition to spatial correlation for robust detection of local minimums of 1D signal along the direction perpendicular to tag lines. Local maximums are similarly detected and 2D displacement is estimated by interpolation with Laplacian equation after iterated retrieval of true motion. Tag line tracking [38][39][40][41] methods have a disadvantage of sparse motion tracking while the robustness of optical flow methods [42][43] is subjected to tag line decay and lack of differentiable features.

Phase tracking is a continuous motion tracking method that utilizes the phase of the tag pattern and tracks the motion assuming that phase is an invariant property of the material. The harmonic phase imaging (HARP) [44][45] is a phase tracking method widely used to measure ventricular strain and motion, and has been validated with *in-vivo* heart data [46]. The combination of HARP with an active contour method is developed to improve the robustness of motion tracking [47] but contour initialization is required for motion tracking. There have also been developments to extend the HARP method to 3D motion tracking [48][49]. Despite its successes, the HARP method is unable to detect phase displacement greater than the period of the magnetization field especially for long axis (LA) slices where the motion is larger compared to short axis (SA) slices. Although it is argued that imaging parameters can be adjusted to satisfy the condition but the MRI scan is limited by the characteristics of heart motion and the specification of the MRI machine. There is a need to balance among inter-tag distance, wall thickness, spatial resolution and temporal resolution. When the inter-tag distance is large, there can be frequency interference and motion artifacts.

The more recently developed frequency domain motion tracking method, SinMod [50], extracts motion from tMRI with local sine wave modeling where displacement is estimated as the phase shift from the demodulated tMRI image. It uses the auto and cross correlation of low and high bandpass filtered spectrum to describe the local pixel in a local frequency and

phase deviation. To enable comparable computational costs, SinMod performs frequency analysis in lower spatial resolution and then interpolates the estimated displacement. The method, however, estimates displacement in 1D prior to deriving 2D displacement and may not be presently as suitable as HARP for 2D grids. Further, it suffers from the half-tag displacement limitation.

The half-tag displacement limitation is attributed to motion tracking on wrapped phase. By unwrapping the phase before motion tracking, this limitation can be overcome. For successful motion tracking, both the spatial and temporal phases must be consistent. Phase unwrapping techniques are well-studied for interferometric synthetic aperture radar and sonar (InSAR/InSAS) and optical interferometry applications. Similar techniques have been applied for MRI. Phase unwrapping techniques can be classified into path following [51], minimum L^p norm [52][53], Bayesian [54][55][56][57][58] and parametric modeling [59]. A quality guided phase path following (QGPF) method has been applied for estimating ventricular strain in tMRI [60]. The quality is measured by partial derivatives variances of local unwrapped phase where lower variances from larger inter-tag distances are preferred. The method requires user intervention to specify branch cuts from identified phase inconsistencies for each image. It limits the majority of regions to have displacement smaller than half-tag spacing to ensure inter-frame temporal phase consistency. Similar quality guided unwrapping method [61] has been first applied in the spatial and temporal domain for high spatial resolution Cine DENSE (Displacement ENcoded imaging using Stimulated Echoes) imaging where myocardial displacement is encoded as wrapped phase during the image acquisition. However, path following methods are sensitive to the presence of discontinuities, resulting in inaccurate motion tracking.

From a statistical perspective, the Bayesian approach that provides a maximum *a-posteriori* probability (MAP) estimate to absolute phase has been applied for unwrapping MR images [57] by modeling *a-priori* information using a Markov random field. Dynamic programming with iterated conditional modes is used to reduce the complexity of optimizing MAP with smoothness constraints. The parametric algorithm [59] offers a more efficient fitting by modeling absolute phase with low-order polynomials and minimizing least-squares difference between spatial derivatives of absolute and wrapped phases. Parametric modeling imposes the smoothness constraint naturally but it can be a disadvantage where discontinuity is desired, and low-order polynomials may be insufficient to model phase accurately in practical MRI applications. These two methods, however, have mainly been applied for two dimensional images without temporal consistency.

We propose a B-Spline based unwrapped phase tracking approach for robust phase unwrapping in tMRI motion tracking. We seek to overcome the half-tag displacement limitation in HARP, to improve the robustness of tracking, and to minimize the user intervention required in the prior art for unwrapped phase tracking while having sufficient accuracy in modeling compared to polynomials, lower complexity than MAP optimization, and consistent temporal phases.

2.5 Spatial-Temporal Tracking of Myocardial Deformation

To reconstruct the beating heart, a spatial-temporal tracking of myocardial deformation is required where spatial refers to 3D space and temporal refers to frames in the cardiac cycle. The motion tracking in the tMRI can recover only 2D motion within the imaged slices. Through the integration of the 2D motion tracking from different orthogonal planes into a 3D model, we can recover the 3D motion of the heart.

The general framework includes the setup of an appropriate model, the tracking of component motions and the modeling of 4D myocardial deformation. The models described in existing arts are of a varied representation including volume, surface, surface-bounded volume or point set having or missing spatial or temporal continuity. The input data to such algorithms can be tag points on tag lines, extracted tag lines, intersection of tag lines and contours. One important consideration of the algorithms is the conversion of the apparent 2D motion detected from these inputs to 3D motion through constraints, integration or component additions. The results are typically evaluated by motion profile on in-vivo MRI dataset and their error performance on simulation with kinematics model like Arts *et al.* [62], Waks *et al.* [63] and Waldmann *et al.* [64].

In early work by O'Dell *et al.* [65], a truncated power series is used to fit three orthogonal sets of tagged 1D data. The data are initially aligned to a first order power series in Cartesian co-ordinates and then the residual displacements are fitted to a power series in prolate spheroid coordinates. The method provides the ability to represent the 3D deformation in a small number of parameters but the mathematical model may not be sufficient to represent local deformations.

Using a physics-based approach, Park *et al.* [66] model the LV deformation with Lagrangian dynamics and FEM. Each volumetric prism is deformed with 3D nodal displacements due to external forces exerted by SPAMM data points and boundary displacements. The LV is further described by a deformable model with parametric functions that capture global

deformations like twisting and radial and longitudinal contraction intuitively for clinical evaluation. Haber *et al.* [67] extend the work of Park *et al.* to the RV. The volumetric finite element mesh is a piecewise continuous representation of the bi-ventricular geometry derived from epicardial and endocardial contours. Rather than using the tracked tag points in the slices directly, the tag surfaces are reconstructed from tag lines and then used to track the displacement of the material points. An internal force represented by material stiffness is added to the implementation of Lagrange's equation of motion. The Euler integration is improved with a step size adapted to the integration error for faster convergence. However, volumetric finite elements are vulnerable to degeneration and may require tedious remeshing for optimal representation.

Using similar Lagrangian dynamics, Wang *et al.* [34][68] propose a meshless model where each particle is grouped in phyxel with kernel functions instead of an array volumetric elements described with FEM. Local detected tag displacements and moving least squares mesh strains are used as external and internal forces in the motion equation. The global deformation is computed by integrating the local parameters of each phyxel over the myocardium. However, the iterative framework requires large iterations for a sparse control set and the model assumes isotropic physical characteristics.

From the perspective of geometry, the 3D myocardium deformation relates to a 4D fitting problem of B-Spline and Non-Uniform Rational B-Spline (NURBS) models. Luo *et al.* [69] improvise the work of Park *et al.* by representing the deformable model in terms of 4D B-Spline. A prolate spheroid model, rather than the super ellipsoid, is used to generalize the LV shape and to represent the global deformation. The initial prolate geometry is fitted with extracted contours from tMRI. The backward displacements are modeled with a B-Spline solid where each component displacement is fitted independently with a fast algorithm that resolves the controls for B-Spline solid in a hierarchical manner. The motion parameters are estimated with 3D B-Splines in sequential order by minimization of a regularization term and the sum errors between estimated motion from backward displacement and true motion. Ozturk *et al.* [70] model backward displacements with 2D Splines by performing a first fitting from displacements of tagged points and a second fitting from interpolated displacements of orthogonal slices. The forward displacements are further modeled with a 3D B-Spline tensor combining results from slices. Finally a 4D B-Spline tensor describes the spatial-temporal displacement. Validation is performed between two sets of image data from the same volunteer [71], taken with a z-offset and radial offset in the SA and LA slices. Huang *et al.* [72] models the myocardial deformation instead of displacements with 4D B-Spline. Each knot plane is first fitted to each tagged plane by minimizing the Chamfer

distance potential. Then the planes are combined to give a B-Spline solid in one time instant and finally the solids are combined to give the 4D B-Spline. The B-Splines provide a continuous representation but the grid-like control is not optimally distributed for an enclosure with relatively large lumen. To match the shape of the LV better, Li *et al.* [73] defines the B-Spline deformation model in prolate spheroid instead of Cartesian coordinates. As an extension to B-Spline, Tustison *et al.* [74] model biventricular deformations with a cylindrical parameterized Cartesian NURBS with 4x6x6(LV) and 3x4x4 (RV) control nodes. Liang *et al.* [75] apply the NURBS methodology to a set of LV human data for 3D motion reconstruction and strain analysis but the control nodes are sparse.

Rather than translating the apparent 2D motion to constraints in 3D deformation, Kerwin *et al.* [76] tracks the true motion of the intersections between orthogonal set of tag planes within the segmented myocardium. The deformation of each tag plane is modeled with a thin plate Spline. The intersections between tagged planes are determined by an iterative alternating projections algorithm. This algorithm however requires the alignment of the tagged planes to coincide at the initial tag application and much iteration for convergence. Pan *et al.* [48] propose a 3D HARP analysis on 2D tagged SA and ID tagged LA MRI scans. A half elliptical spheroid is initialized with 3D phase values and deformed with in-plane motion. The deformation is subsequently corrected through iterations with 2D motion tracking until the time-invariant phase property is satisfied at the intersection of the 3D model with the scan slices. However, the mesh comprises only 6x12 quad elements and the convergence of the iterations needs to be further accessed.

Instead of working on an algorithm to overcome the limitation of through-plane motion, Sampath *et al.* [49] improve the method of MRI imaging to measure true motions. The modified imaging technique SF-CSPAMM (Slice Following Complementary SPAtial Modulation of Magnetization) technique images a thick slice to cover the through-plane motion of the thin tagged slice. The 3D motion is derived directly by tracking the material points on intersections between orthogonal SA and LA slices by applying 2D HARP method on each slice. However this method is not available on commercial MRI machines.

Our proposal models the myocardial deformation with a surface representation. The representation is more suited for the simulation of the intra-cardiac intervention to assess the wall surface contractile properties. Unlike independent component displacement constraints in B-Splines or NURBS, we derive an optimization function that minimizes the local differential characteristics and the differences between the projected and tracked in-plane displacements and reduce this optimization to solving a sparse linear system.

Chapter 3 A VR SIMULATOR FOR INTRA-CARDIAC INTERVENTIONAL PROCEDURE: CONCEPT, DESIGN AND IMPLEMENTATION

The learning process for diagnostic and minimally invasive therapeutic procedures of heart diseases is challenging. A substantial amount of time is needed for training due to disease complexity to acquire the dexterity skill required to manipulate surgical devices, especially in percutaneous intra-cardiac cases. We aim to develop a novel simulator to create a realistic and low-cost training environment for an intra-cardiac technique used to determine the mechanical and electrical activity. This chapter focuses on the concept, design and implementation of a VR simulator for intra-cardiac intervention. Instead of using physics-based approaches, a new geometric-based method is proposed to model the interaction between an intra-cardiac catheter and the heart wall. A boundary-enhanced voxelization technique is developed to accelerate the process for detecting catheter-heart interactions. A tactile interface is designed incorporating a VR catheter unit to track the catheter movement within the virtual heart. A progressive approach is implemented to reconstruct the heart chamber for the application of heart mapping.

3.1 Design of a VR Simulator for Intra-cardiac Intervention

The VR simulator design (Fig. 2) for intra-cardiac intervention comprises of a foundation layer (software and hardware) and a simulation core engine (visualization, dynamic interaction and modeling) and a user interface (graphical and tactile). The user interface is the top layer communication for receiving user interaction of the VR catheter unit and graphical user interface, as well as for presenting visual and tactile feedback. Although intra-cardiac intervention is a computational and data intensive process, the geometrical modeling developed in this work enables the simulator to be supported on a normal PC with a mid range GPU card that takes advantage of parallel processing for visualization and computation. A VR catheter unit with linear, rotational and bending control is integrated to the system via a USB interface. The simulation core engine has a heart surface voxel representation pre-processed from a high resolution heart model, a catheter modeled as a series of rigid links connected by nodes and a surface reconstruction process for the heart mapping procedure. The simulator can be enriched with the modeling the heart dynamics by extracting patient specific heart motion data from MRI. In the system, a dynamic interaction

module, comprising of collision detection between catheter and heart wall, slip and non-slip deformation of catheter and stability assessment of contact, is responsible for the tracking of catheter inside the beating heart and the acquisition of stable contacts with the heart wall. Both the visualization of the reconstructed heart surface with composite display of electromechanical properties and the visualization of catheter navigation are parts of procedural requirement. The rendering of the beating heart is optionally included as part of the hand-eye co-ordination learning process.

Fig. 2: VR simulator design.

3.2 Heart Model

3.2.1 High Resolution Tetrahedral Heart Model

The simulator employs a high resolution anatomical structure dataset with 148,516 vertices and 728,321 tetrahedrons [77]. The dataset is extracted using top-down octree subdivision coupled with a dual contouring method from volumetric CT scan data. It has labels for vertices describing atria, ventricles, valves, veins and arteries. From the dataset, we derive an index referencing between the tetrahedron, vertices and triangle such that back referencing from an element type to the next is available without the need for explicit search during the simulation. From the tetrahedron array, a search is performed to locate adjoining triangular faces and generate an array of independent triangles. The tetrahedron array is updated with corresponding triangle indices while the triangle array is updated with corresponding vertex

indices and the vertex array is updated with adjoining triangle indices. This allows for fast identification of nearest triangle touched by the catheter during the heart mapping process.

3.2.2 Boundary-enhanced Voxelization of Heart Surface for Fast Collision Detection

Fast collision detection is often enabled by bounding volume hierarchy (BVH). Axis aligned bounding boxes (AABB) are used to model tissue surface boundaries [78] while devices are modeled with oriented bounded boxes or spheres [15] for tighter fit. However, such collision detection algorithm trades off false collision with inefficient update of BVH.

To have a real-time solution for fast collision detection and deformation problem, we re-encode the polygonal heart model as a cubic voxel representation. In geometrical modeling, voxel representation divides a defined 3D space with offset (x_0, y_0, z_0) and dimensions (width, height, depth) into regular sub-spaces to form $L \times M \times N$ cuboids. The cuboids allow for parallel processing by graphical units for volume rendering and complexity reduction. Collision detection becomes trivial because the test to determine if the catheter is inside or outside the ventricle lumen can be simply achieved by converting model (x, y, z) to voxel coordinates (u, v, w) to retrieve respective voxel information:

$$(u, v, w) = ((x - x_0) / q_v, (y - y_0) / q_v, (z - z_0) / q_v) \quad (1)$$

where q_v is the uniform quantization step used in voxelization.

The boundary-enhanced voxel representation of the heart comprises of the voxel type (heart, non-heart, heart-surface, non-heart surface), normalized distance from the nearest heart surface, and associated surface triangle index. The precision of the representation affects the interaction modeling accuracy and the rendering aspects. It is recommended that the unit voxel edge is at least half the smallest edge distance of the heart model. The method of voxelization involves three steps: firstly the binary heart vs. non-heart classification, then the heart surface identification, followed by the nearest surface and minimum distance search.

In the binary classification, we seek to achieve a volumetric representation using a tetrahedron test. For each tetrahedron in the heart model H comprising of n_H tetrahedrons, we define a bounding box to enclose the tetrahedron with a user-defined margin. We establish the plane equations for the four facets of the tetrahedron in a consistent manner with the plane normal pointing away from the tetrahedron. Then, we determine for each

voxel in bounding box if it is enclosed by the tetrahedron using the voxel-planes distances. The voxel is classified as heart voxel if it is enclosed by any tetrahedron in the model. The classification is defined by:

$$d(x, y, z) = \min_{j \in [0, n_H - 1]} \left\{ \max_{i \in [0, 3]} \{ d_{ij}(x, y, z) \} \right\} \quad (2)$$

$$voxeltype = \begin{cases} non-heart & d(x, y, z) > 0 \\ heart & d(x, y, z) \leq 0 \end{cases} \quad (3)$$

where $d_{ij}(x, y, z)$ is the distance of voxel at position (x, y, z) in 3D space from the i -th surface of the j -th tetrahedron. Due to voxel discretization, some vertices may be assigned to a *non-heart* voxel. A loop of the vertices in the model re-labels the voxels containing the vertices to voxel type *heart*. The accuracy of surface identification is critical to the establishment of the surface normal. We scan the volume of binary classification with a nearest neighborhood operating window. The current voxel is labeled as a *surface* voxel if any voxel within the window has both *non-heart* and *heart* voxel type. The triangular facet with all three vertices classified as *surface* voxels is labeled as potential heart surface candidate. We further refine the surface facets as those that generate voxel-surface distances with signs consistent with voxel classification (*heart* and *non-heart* voxels have negative and positive distances). Fig. 3a shows the classified voxels in 3 orthogonal slices with surface voxel represented in a darker color. Lastly for each voxel, we search to find the global minimum distance and associated nearest surface among all heart surface facets that contains the projection of the voxel. The voxelization is performed for all phases in the cardiac cycle. Fig. 3b shows the signed minimum absolute distance from the nearest heart surface where blue and red colors represent the negative and positive distance. Fig. 4 illustrates the volume rendering, transparency rendering and surface rendering of the same heart model.

Fig. 3: Multi-slice visualization: (a) Voxel type (non-heart-black, heart-colored, surface-darker colored); (b) Distance from heart surface (positive-red, negative-blue).

Fig. 4: Heart model visualization: (a) Volume rendering of heart model with surface representation using voxelization; A quadrant of the heart is removed to expose the different heart chambers (colored separately) and an embedded catheter in the LV; (b) Transparency rendering of heart model with an embedded catheter; (c) Surface rendering of heart model; A frontal plane is sliced to expose the internal heart surfaces.

3.2.3 Patient Specific Heart Motion

To develop a practical simulator for surgical training, the ability to incorporate patient specific data is an important consideration. A dynamic rather than a static heart model is required for simulating the intra-cardiac intervention procedure. The acquisition of human

heart data can be achieved with either minimally invasive intra-cardiac intervention or non-invasive functional imaging. The former requires the collaboration with the manufacturer and hospital for the release of confidential patient data. Although the data may be used directly, its spatial distribution is dependent on the contact acquisition. The latter has better flexibility and more regularity in the acquisition process but requires the post processing of image data. We develop a methodology to incorporate heart motion data from patient specific tMRI.

The dynamic motion of the LV is extracted from tMRI data. tMRI provides the motion correlation of the heart tissue between different time phases through imaging the time series of an irradiated tag pattern. The tMRI is captured on three orthogonal planes from a volunteer; one on the SA plane with a 2D tag grid and the other two on the LA plane with a 1D tag. Some markers lining the mid ventricular wall are inserted on the initial frame of all slices. The planar motion of these markers is tracked with a hybrid HARP B-Spline algorithm. A B-Spline approach to phase unwrapping in tMRI is developed to overcome the limitation of HARP [45] tracking algorithm in LA slices (Chapter 7). A prolate spheroid model is initially fitted to the markers lining the LV by a surface reconstruction method. The differential surface representation of LV is then linearly deformed spatial-temporally by minimizing the differences between projected and planar tracked motion from orthogonal planes (Chapter 8). The generic heart model is then registered to the LV model and deformed to fit the specific LV geometry and inter-correlated motion for 26 phases per cardiac cycle using free-form deformation techniques.

To incorporate the motion data into the simulator, the deformed heart model is re-encoded to surface voxel representation. The octree representation [79] is explored to determine the lossless storage efficiency for animation data in the context of heart surface voxelization. Given an octcell structure to store the location, level, parent, children and 8 bytes data per voxel, we require 16 bytes per intermediate node and 40 bytes per leaf node for a tree depth of less than 10. It is found that the octree structure suffers from significant overhead when the bounded volume of interest, in this case of a LV, is not a cube. For the same voxel resolution of $95 \times 110 \times 113$ units³ representing the LV, the octree representation has a negative savings of 32% with almost 43.7% of redundant cells compared to a linear array representation. Thus, the surface voxelization of the deformed LV model in all frames is stored in a simple 4D array.

3.3 Mapping Catheter Model

The electromechanical mapping catheter used in the intra-cardiac simulation comprises of 3 parts: the handle with control knobs, the shaft and the deflectable tip for bending control in Fig. 5. We model the catheter as a series of N rigid links with $(N+1)$ nodes in 3D space. For intra-cardiac intervention, we are particularly interested in the deflectable tip described by K nodes and a small distal length of the shaft that is enclosed within the heart chamber, where P_N refers to the catheter tip, P_{N-K} is the pivot node between the shaft and deflectable tip and k is the node index on deflectable tip away from the pivot node ($k = 0$) to the tip ($k = K$). When the catheter is inserted in the ventricle via the aortic orifice, the point of insertion is recorded between P_l and P_{l-1} while the direction of inserted shaft is denoted as principal axis v_{axis} . Besides typical linear translation τ along the axis and rotation r about the axis from user control, the electromechanical mapping catheter have an additional degree of freedom for user-controlled uni-/bi-directional bending in a 2D plane. This user-manipulated bending is not found in typical catheterization catheter that may have a particular memory shape. We define $h \in [0,1]$ as the normalized bending angle the catheter makes with its straight length at the pivot. The catheter nodes, prior to deformation within the heart chamber, are computed first using local coordinates. From the design of NaviStar Catheter F (J&J) and referring to Fig. 6a, we first determine *a-priori* inter-link angles $\{\theta_k(1)\}_{k=0}^K$ of various k nodes along catheter body at maximum specified bending $h = 1$. The inter-link angle $\theta_k(h)$ of the k -th node at h bending is easily interpolated as $\theta_k(h) = h\theta_k(1)$. The 2D position of P_k in XY plane for $k \in [0, K]$ is obtained through a rotation of P_{k-1} using matrix $R_z(h)$ about z axis for an inter-link angle $\theta_k(h)$:

$$P_k(x, y, h) = P_{k-1}(x, y) + R_z(h)(P_{k-1}(x, y) - P_{k-2}(x, y)) \quad (4)$$

The local coordinates of the k -th node at h bending are then determined through rotation r and translation τ of catheter about the local x-axis:

$$P_{k,local}(\tau, r, h) = R_x(r)[P_k(x, y, h), 0]^T + [\tau, 0, 0]^T \quad (5)$$

These local coordinates are converted into the model coordinates through affine transformations with principal axis v_{axis} and distal insertion node P_l . Fig. 6b illustrates various planar bi-directional bending units of the catheter model.

Fig. 5: Electromechanical mapping catheter with knob to control deflectable tip

Fig. 6: (a) Catheter model at $h=0, 0.5, 1$; (b) Simulated catheter tip at various h .

3.4 Dynamic Catheter-Heart Interaction

The modeling of the interaction between the catheter and the beating heart is to determine the complex relationship between the moving catheter and the changing volume enclosed by the heart chamber. The collision detection between these two moving bodies is accelerated by the boundary enhanced voxelization of the heart surface. In event of an interaction defined as at least one catheter linkage crossing the concave surface due to either catheter movement or heart contraction, the catheter has to deform such that the catheter nodes are within the described volume. The type of deformation the catheter undergoes is dependent on the collision point and the stability of the contact. The contact of the catheter with the heart wall is stable if the tip touches at least the upper volume boundary during diastole and the contact is maintained during the cardiac cycle where the catheter deforms during diastole. However, if the contact slips due to instability, the catheter will bend and move towards a new position along the heart wall. An experiment is designed to characterize the physical interaction of catheter with the porcine heart tissue in the LV (Chapter 4). From the experiment, slip and the non-slip interaction classes and the conditions under which they occur are identified (Chapter 4).

Consider a snapshot view where the catheter make contact with the heart wall either through catheter navigation within the heart lumen or heart contraction. The local heart wall deformation from the interaction is relatively less significant because the catheter is in soft contact with the muscular heart wall and this visualization is not required during the mapping process. The modeling work focuses on the catheter deformation in non-slip and slip interaction classes for quasi-static phase of either the catheter or the heart. Modeling in quasi-static state reduces the complexity from two to one moving object per interaction. A geometric approach with considerations of physical properties is proposed to enable fast catheter deformation (Chapter 5) that is similar to the deformation observed in an experimental setup (Chapter 4). Compared to vascular intervention, FEM may not suitable for a larger deformation range of intra-cardiac intervention and it may require a much larger number of iterations for convergence. In the case of non-slip deformation, the catheter deforms between insertion and tip to minimize the deformation energies contributed by the lateral and axial strains on the catheter. The deformation is also dependent on the previous catheter deformation to ensure shape continuity during deformation. In the case of slip deformation, the catheter is deformed progressively from the insertion to tip and the distal portion glides along the heart surface while changes to the catheter curvature are minimized. The contact stability of the catheter tip is assessed based on the deformation energies and the

catheter-tissue angle. The contact stability determines the transition from initial contact to deformation and between non-slip and slip deformations.

3.5 VR Catheter Unit Design

A VR catheter unit is designed by Yu [4] and integrated into the simulator to control the catheter movement for catheter-heart interaction. The VR catheter unit uses rotary-based displacement sensors for motion tracking with high accuracy and real-time response. The design provides linear and rotation motion tracking and bending control corresponding to the insertion, retraction, twist and bend control of the VR catheter unit. In addition, the design has a foot-switch for contact acquisition. The motion tracking part comprises an aluminum frame of dimension $300 \times 72 \times 101 \text{ mm}^3$ with a gear engaging shaft to integrate linear and rotational motion tracking. The linear motion tracking unit (Fig. 7a) uses a rack and pinion design where linear motion is converted to rotational motion and further encoded by an optical rotary encoder (HEDS-5500) operating at 100Hz and CPR (counts per revolution) of 500. The linear motion is tracked for 113.5mm with a step resolution of 0.12 mm. The rotational motion tracking unit (Fig. 7b) comprises a set of gears with a gear ratio of 3 where the smaller gear is attached to an optical rotary encoder (HEDS-5500) operating at 100Hz and CPR of 500, and the larger gear is connected to the driving end. The rotational motion is tracked for 360° with a step resolution of 0.24° . The catheter body (Fig. 7c) is made up of a shaft with 30cm long wire and a miniature optical kit encoder E4P, operating at 60Hz and CPR of 100 for bending control. The bending supports the full bidirectional deflection of the catheter with up to 50 steps per direction. The encoding signals of displacement sensors are transmitted via the USB4 Encoder Data Acquisition USB device to the computer. The data acquisition device supports real-time tracking of the incremental encoders up to 5MHz and digital samples of 44 kHz on analog channel used by the acquisition switch. Fig. 8 shows the VR catheter unit of the intra-cardiac simulator used to manipulate the virtual catheter.

On the software integration, the VR catheter unit is polled with a timer interrupt to detect if there is change in catheter status. The polling interval is tuned to 100 ms to balance between CPU load and smooth catheter motion tracking. Fig. 9 illustrates the flow chart for integration of the VR catheter unit. When the haptic device is activated, the timer is started and the device is initialized. The virtual catheter is synchronized with the device position and precision specified by CPR. When there is a timer callback, the registers for encoders are clocked and the readings are processed to determine the delta motion of the catheter. If there is motion detected in any DOF, the virtual catheter is moved correspondingly and its

interaction with the heart is checked. The digitized signal (analog source) from the foot switch is simultaneously checked to detect the edge pattern. If a falling edge is detected and the contact is stable, a contact acquisition is raised for surface reconstruction.

Fig. 7: Mechanical design of (a) linear motion tracking, (b) rotational motion tracking and (c) bending control; Figures extracted from [4].

Fig. 8: VR catheter unit that controls catheter-heart interaction; Figure extracted from [4].

Fig. 9: Flow chart for integration of VR catheter unit.

3.6 Progressive 3D Heart Surface Reconstruction for Heart Mapping

The 3D heart surface reconstruction is a unique feature of the intra-cardiac intervention. In the heart mapping procedure, the 3D heart surface, typically the LV, is progressively reconstructed with the acquisition of contacts. As the interventional cardiologist or radiologist is guided only by the reconstructed surface, a constructive interference between the contact acquisition and reconstructed surface is desired. High quality reconstruction can help to improve catheter navigation to accurately map the heart geometry and uncover the diseased regions (with the electromechanical data) for better diagnosis and cell-injection

therapy. Unlike typical surface reconstruction technique from large and fixed dataset, the reconstruction is based on very small dataset of typically 100 and below with sparse and random distribution. The reconstruction is performed progressively with each data point acquisition in real-time, instead of an offline one time reconstruction.

Reisfeld [36] proposes a fast 3D reconstruction of intra-body organs through fitting of generic mesh model to acquired contacts. Denoting p_j as the j -th vertex of the mesh and s_i as the i -th acquired contact, a rough adjustment f_R of the mesh is first performed as a weighted displacement $v_{ij} = s_i - p_j$ of the mesh vertices towards the contacts. The weights are dependent on the inverse distance between the j -th vertex and i -th contacts and the inverse density of the i -th contact $\rho_i = 1 / \sum_h \frac{1}{|s_h - s_i| + 1}$. However, this method cannot

differentiate between the cases where there are dense contacts around the mesh and far away from mesh because the weight is normalized across all contact-to-vertex distance. To reduce shape distortions, we add a factor $\hat{\lambda}_j$ representing the normalized local contact density around the j -th vertex. If $\lambda_j = \frac{1}{(|s_i - p_j| + 1)}$ is the local contact density for the j -th vertex and λ_0 is the average local contact density among the set of mesh vertices with the top 50%

density values, the normalized local contact density $\hat{\lambda}_j = \frac{\lambda_0 + 2\lambda_j}{2\lambda_0 + \lambda_j}$ ranges from 0 to 2. The

fine adjustment f_F makes a pairing of contacts with nearest mesh vertices to form set E_{GS} . While paired vertices are moved a fraction of distance towards the contacts, non-paired vertices are moved a weighted paired vertex-to-contact distance. The weight $\omega_{ij} = 1 / (|p_j - v_i|^k + \alpha)$ is the inverse distance of non-paired vertex from the contacts. The rough f_R and fine f_F adjustments are defined by:

$$f_R(p_j) = c_R \hat{\lambda}_j \sum_i \frac{\rho_i v_{ij}}{|v_{ij}|^k + \varepsilon} / \sum_i \frac{\rho_i}{|v_{ij}|^k + \varepsilon} \quad (6)$$

$$f_F(p_j) = \begin{cases} c_F \cdot (s_j - p_j) & G_j \in E_{GS} \\ c_F \sum_i \omega_{ij} (s_i - p_i) / \sum_i \omega_{ij} & G \notin E_{GS} \end{cases} \quad (7)$$

where c_R , k , ε , α and c_F are constants determined heuristically ($c_R = 0.75, k = 3, \varepsilon = 0.001, \alpha = 1, c_F = 0.66$).

A prolate spheroid surface is used and meshed into polygonal representation. The affine configuration of the surface is initialized by the location of the aortic valve, the mitral valve and the ventricular apex with the origin located at 2/3 base-to-apex distance from the apex [80]. Fig. 10 compares the effect of local contact density in progressively deforming the mesh to fit the acquired data points. Local contact density can prevent the collapse of part of the mesh where there are no contact but local mesh distortions remains in the final reconstruction. A linear variational surface deformation technique is developed to progressively reconstruct heart surface more efficiently from a small dataset with a better geometry fit and implicit inter-acquisition smoothness (Chapter 6).

Fig. 10: Prolate spheroid surface fitted to LV with increasing number $N(10,20,30)$ of contacts for Reisfeld method (top) and improved Reisfeld method (bottom) without smoothing.

3.7 User Evaluation of Intra-Cardiac Intervention Simulator

The objective of the simulator system is to train interventional cardiologists seeking to gain certification and experience in the intra-cardiac intervention procedure. In view of this

aspect, a preliminary user study (Fig. 11) is conducted to assess the efficacy of simulator and the areas for improvement. A static heart model is used at the time of the study. All participants of the study are doctors with an average 2.4 years of general clinical experience. The study protocol comprises of an introduction to intra-cardiac intervention and simulator, followed by 2 simulated procedures to map the LV surface. The participants work in pair where one manipulates the VR catheter unit within a virtual lumen to reconstruct the LV surface and the other to navigate the graphical view and acquire contacts on request. The participant decides the sampling density and pattern for mapping the closest representation of the LV. After each mapping, the roles of the participants are exchanged until each one completes 2 mapping assignments. On average, the second assignment took 5 min less time. Each group took about 1.5 to 2 hours to familiarize with the procedure and complete the 4 assignments. The participants then assess the simulator through ranking their responses on a scale from 1 to 5 for several questions per module. The list of questions and the average score of highlighted assessments are given in Table 1 and Table 2.

There is a general agreement that the simulator is useful for LV mapping and is a cost-effective solution for multi-site training compared to training the procedure in animals. The participants also agreed that the simulator is a novel solution for training hand-eye coordination of the mapping procedure. The evaluation shows that the catheter interaction and the surface reconstruction are real-time. The simulation of catheter deformation matches the user expectation but there is room for improvement particularly in decreasing the sensitivity of the catheter in slip deformation. The sensitivity is attributed to the surface voxelization and we have since addressed this issue by using a moving average of the surface normals in the catheter-heart interaction modeling (Section 5.2.2). We have also improved on the non-slip deformation by having a closer correlation of the simulated deformation energy to physical characteristics (Section 5.2.4). Although the VR catheter unit manipulation matches the expectation and experience of the users, the difficulty experienced in rotation control can be improved by using a less malleable catheter wire. The bending control can also be made more sturdy and smooth by tightening the turn and providing a knob instead wedge-like dial (Fig. 8) to the user. The participants find the overall reconstruction of the heart surface quite realistic and smooth with improvement over the initial geometry as more contacts are acquired. They generally agreed that the surface reconstruction helps them to navigate the next contact. The study generated mapping data ranging from 13 to 78 contacts per reconstructed surface. Data analysis reveals an improvement in reconstructed geometry when comparing the first and second mapping in the learning curves of Fig. 12 with smaller square

distance error (mm²) between the vertices of the final reconstructed surface to corresponding LV model. An effective comparison requires larger sample size and more exercises.

Fig. 11: Medically trained users manipulating VR catheter unit to reconstruct an LV surface.

Table 1: List of questions in user study assessment.

<p>Module 1: Clinical Relevance & Personal Info</p> <p>1.1 How many years of clinical experience you have? Please state your area of experience.</p> <p>1.2 What is your medical training? Please indicate all training.</p> <p>Module 2: Learning Objectives & Technological Features</p> <p>2.1 The simulator is a cost-effective solution for training of cardiac disease diagnosis & treatment.</p> <p>2.2 The simulator provides useful training for mapping of the left ventricle.</p> <p>2.3 The simulator offers a novel solution of training for hand-eye coordination in intra-cardiac interventional procedure.</p> <p>Module 3: Learning Outcomes</p> <p>3.1 How fast are you able to master the mapping procedure with the simulator?</p> <p>3.2 How many points are you able to do satisfactory heart mapping with?</p> <p>3.3 In how many percent of the moves you made with the VR catheter does the response match well with your expectation?</p> <p>Module 4: Catheter Interaction</p> <p>4.1 The simulation of catheter in slip condition matches your expectation and experience.</p> <p>4.2 The simulation of catheter in non-slip condition matches your expectation and experience.</p> <p>4.3 The navigation of catheter using VR catheter unit in a virtual heart lumen is natural & realistic.</p>	<p>Module 5: VR Catheter Unit</p> <p>5.1 The catheter insertion and retraction in a virtual heart lumen matches well with your expectation and experience.</p> <p>5.2 The catheter rotation in a virtual heart lumen matches well with your expectation and experience.</p> <p>5.3 The catheter bending in a virtual heart lumen matches well with your expectation and experience.</p> <p>Module 6: Surface Reconstruction</p> <p>6.1 The overall geometry of the reconstructed heart surface is realistic.</p> <p>6.2 The initial geometry of the reconstructed heart surface during the acquisition of the first 10 contacts is realistic.</p> <p>6.3 The reconstructed surface becomes more realistic with increasing contact acquisitions.</p> <p>6.4 The update of the heart surface between successive acquisitions of the contacts is smooth and realistic.</p> <p>6.5 The reconstructed surface helps in the navigation of the next contact.</p> <p>6.6 The surface reconstruction has a real-time response.</p> <p>Suggestions for improvement</p>
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Table 2: Assessment of intra-cardiac simulator from 7 participants in user study. Average score in rank of 1 to 5 representing strongly agree, agree, acceptable, disagree and strongly disagree responses respectively.

Areas of assessment	Questions	Average Score	Standard Deviation	Percentile (%) Agreement
Cost-effective solution for training	2.1	1.43	0.53	89
Useful training for LV mapping	2.2	1.71	0.48	82
Novel training for hand-eye co-ordination	2.3	1.71	0.48	82
Simulation of catheter deformation matches expectation	4.1-4.3	2.57	0.87	61
▪ Slip condition	4.1	2.85	1.06	53
▪ Non-slip condition	4.2	2.29	0.95	68
▪ Catheter navigation	4.3	2.57	0.53	61
Real-time response of catheter interaction	4.4	1.57	0.53	86
VR catheter manipulation in line with experience	5.1-5.3	2.24	0.77	69
▪ Insertion and retraction	5.1	2.00	0.81	75
▪ Rotation	5.2	2.57	0.53	61
▪ Bending	5.3	2.14	0.69	72
Smooth and realistic reconstruction of LV	6.1-6.4	1.75	0.80	81
▪ Overall geometry	6.1	1.86	0.90	79
▪ Initial geometry	6.2	2.00	0.82	75
▪ Improvement over increasing acquisition	6.3	1.29	0.49	93
▪ Transition smoothness	6.4	1.86	0.90	79
Surface reconstruction aids navigation	6.5	1.71	0.49	83
Real-time surface reconstruction	6.6	1.71	0.49	83

Fig. 12: Average learning curves for mapping left ventricle in the first and second exercise.

3.8 Discussion

We have presented the various technologies behind an intra-cardiac simulator that shall assist in training interventional cardiologists and radiologists for the heart mapping procedure. Like coronary stent implantation, the simulation of intra-cardiac intervention may be seen as an extended module of interventional cardiology simulator. Compared to other minimally invasive cardiac-related interventions, the absence of fluoroscopy, bending control of the catheter, the stability assessment of the catheter tip with the endocardium and the visualization of the reconstructed heart surface are several aspects specific to the heart mapping procedure that we have addressed uniquely in our simulator system.

Starting with a tetrahedral heart model from CT scan data, we have created a boundary-enhanced heart surface voxelization for fast detection of catheter-heart contact compared to AABB method. Careful selection of the voxel precision can improve interaction accuracy and visualization at little expense of memory usage. This heart model is customized and enhanced with dynamic beating from patient-specific tMRI data. The motion of the cardiac wall is tracked from the deformed tag grids using a phase-tracking method and subsequently reconstructed with a linear variational mesh deformation technique. The heart model is deformed in accordance to the reconstructed motion using free-form deformation technique.

For the intra-cardiac intervention, we model only the part of the catheter inserted the heart lumen as a straight proximal segment and a distal deflectable tip. By representing the catheter as a series of rigid links connected by nodes, we derive nodal positions for various bending, twisting and inserting/retracting of the catheter using a-prior deflection characteristics. The part of the catheter in the vascular network can be considered for a more complete catheter modeling when integrating with the catheterization procedure.

We propose a geometric-based modeling of catheter-heart interaction as a low complexity and plausible option to physics-based modeling for training purposes. Our approach satisfies the geometrical and subjective requirements through positive feedback from the user study, similar force-displacement graphs and initial comparison of catheter deformation between the experiment and simulation (Section 5.3.3). It is anticipated that physics-based approaches may be designed in the future and that validation may be performed through carefully designed animal experiments with complex tracking equipment. In our approach, the non-slip interaction is more relevant to intra-cardiac intervention rather than vascular intervention to assess stability for contact acquisitions. The key advantage of the geometrical approach is that the catheter deformation can be achieved in a single step rather than several quasi-static steps and thus real-time interaction is possible.

Our system includes a VR catheter unit with a rotary-based motion tracking that the user can manipulate externally to control the bending, rotation and insertion/retraction of the catheter within the heart chamber and acquire contacts for mapping of the chamber geometry. However the catheter has yet to incorporate haptical feedback from the contact of catheter with the heart. The haptical feedback component can be realized with design of an air compressor with a balloon ring controlled by simulated energy functions. The strength of feedback is calibrated by tuning the simulated data to match the experiment.

The 3D surface reconstruction is an important part of the equipment display used during intra-cardiac intervention to visually guide the interventional cardiologists and radiologists for their next move to refine the area of interest. With each contact acquisition, the 3D heart surface is progressively updated. It should be noted that only a static 3D surface is reconstructed for a specific phase of the cardiac cycle during the actual procedure with local properties are mapped onto the reconstructed surface. We replicate this reconstruction capability in our simulator system but the quality of reconstruction is much better in terms of geometry compared to the coarse meshing of the set of contacts in the actual system. Currently, the simulator is enriched with motion data (linear local shortening) extracted from tMRI. Complementary electrical data (bi-/uni-polar voltages, local activation times) obtained either from procedural measurements of the actual system or modeling of electrical wave propagation from electrocardiogram may be added later.

3.9 Summary

In this chapter, the concept, design and first implementation of a cardiac simulator are presented, comprising the core units of visualization, interaction and modeling specific for the cardiac intervention. In particular, a novel geometric-based interaction modeling technique is proposed for cost-effective and realistic intra-cardiac interventional simulation. With the aid of boundary-enhanced voxel-based representation, non-slip and slip catheter-heart interaction is simulated in real-time (Section 5.3.1) for catheter contact and deformation. The navigation and visualization aspects of the simulator are addressed with the design of VR catheter unit using rotary motion tracking and the improvement of an existing surface reconstruction technique. However, the simulator has room for improvement in the areas of quantitative validation, haptical feedback and simulated medical conditions.

Chapter 4 EMPIRICAL MEASUREMENTS OF CONTACT BETWEEN CATHETER AND HEART TISSUE

The electromechanical mapping of the heart is a minimally invasive intra-cardiac intervention procedure for the diagnosis of regional cardiac wall damage. An understanding of how the catheter interacts with the heart wall is critical to the simulation of the heart mapping procedure. This chapter presents an experiment to investigate the contact between catheter and heart tissue. We identify two types of contact: non-slip and slip contacts. A non-slip contact is established when catheter first touches the heart tissue. It finally becomes a slip contact when more force is exerted.

4.1 Outline

In developing VR simulators for medical procedures, virtual tissue-force models must be able to display realistic displacement and force response. To develop these models, there are works to measure and characterize the tissue and their interaction ex-vivo in the laboratories. For example, force measurements for needle insertion procedure [81] and contact and friction between catheter and blood vessel for vascular catheterization [82]. In our work, we design an experiment investigate the contact between catheter and heart tissue. Our objective is to provide quantitative measurements for haptic devices in the context of an intra-cardiac intervention simulator. Force-displacement characteristics are investigated for different detection angles between the catheter and the heart tissue.

4.2 Material and Methods

A biomechanical apparatus (Fig. 13) specially designed for use in measuring small amounts of force against displacement in tissue is used for the experiment. The apparatus consists of a mechanical 3D positioning system, an optoelectronic laser distance sensor (optoNCDT 4021), a bipolar stepper motor (CTP21), a force sensor, a water tank with valves, a circulator and a computer system. The stepper motor runs at 200 steps per revolution with $\pm 3\%$ step accuracy. The distance sensor measures a range between 60 and 260 mm with linearity of 360 μm and resolution of 100 μm . The force sensor measures a range of ± 1500 g with a maximum sensitivity of 1.43×10^{-3} g. The circulator controls the water height and maintains the temperature at 37 °C in the water tank. The 3D positioning system and the recording of

sensor data are controlled using the LabView software. An electrophysiology mapping catheter (Fig. 5) from St. Judes Medical is used. The catheter has a deflectable tip controlled by rotating the top part of the catheter shaft. A holder for mounting the catheter to the force sensor is designed. The design requirements are that the holder is screwed to the sensor on the top surface, the catheter is secured to the holder at a fixed position to expose a short length from the bottom surface and that the catheter should maintain full length to preserve deflection control. The holder is fabricated with polymethyl-methacrylate (Fig. 14) and weighs 3.98 g excluding the screws. Fresh porcine hearts [83] are used in the experiment.

Fig. 13: Biomechanical apparatus for cardiac intervention measurement.

Fig. 14: Catheter holder: (a) Design; (b) Prototype.

The experiment is designed to characterize the interaction between the catheter and heart tissue, to determine the occurrence of slip condition quantitatively and to investigate the effect of catheter-tissue angle on interaction. In the first setup (Fig. 15a), the catheter-tissue angle is varied by adjusting an inclined wooden platform held in place by clamp with a ball and socket joint. In the second setup (Fig. 15b), the catheter-tissue angle is varied by adjusting the bending states of the catheter on a horizontally mounted tissue. Two subsets of measurements are taken: one subset in atmospheric air and the other subset by immersing both the heart tissue and catheter in a water bath. The water bath is maintained at 37°C with circulator to mimic closer physiological conditions (temperature and hydration) of the heart in-vivo.

The procedure included the following steps once the setup is in place. Tissue samples from the septal walls are incised from the LV of the fresh porcine hearts. In the first setup, the tissue is mounted onto the inclined platform with microclips. The angle of the inclined platform is confirmed with an inclinometer. In the second setup, the tissue is mounted on an elevated platform with nylon thread. Care is taken to ensure that the mounting is parallel to the platform. The system is first calibrated with force-displacement measurements for the sensor only and the sensor and catheter-holder only. The catheter is inserted into the catheter holder and held securely by a pair of set screws. The catheter tip is extended 95 mm beyond the holder. The force-displacement measurements of the catheter without contact are taken for a range of 40 mm. The tip of the catheter is then moved precisely 3 mm above the desired position of contact. It is necessary to maneuver the positioning system to ensure the catheter tip touches the heart surface at the same position to minimize the variation in experimental conditions. The catheter is further programmed to move a vertical distance of 40 mm towards the tissue at a speed of 2.8 mm/s. The corresponding force and vertical displacement measurements are recorded. In the first setup, the angle of the inclined platform is changed and calibrated by an inclinometer. In the second setup, the catheter deflection is changed using a calibrated knob on the catheter shaft. The steps from positioning the tip, moving it to recording the data are repeated for catheter-tissue angles ranging from 15° to 90° at 15° intervals.

Fig. 15: (a) Inclined platform setup and (b) Water tank setup.

4.3 Results and Discussion

The results and discussion for the inclined plane setup are detailed in [5] while those for the water-tank setup are described herein. It is found that catheter-heart interaction is highly dependent on the local tissue characteristics. Continuous slip occurs as early as 60° in smooth and mounted areas while it can occur late at 40° in highly trabecular region. The force-displacement graphs for the water-tank setup are shown in Fig. 16 and Fig. 17. The data has been compensated by calibration measurements.

Referring to the 90° graph, we can see four phases of catheter-heart interaction. During phase A, the catheter moves towards the heart tissue with a constant average force. When the catheter touches the tissue at a relative displacement level of 3 mm, an initial contact is established with a slight depression in the heart wall. Further displacement causes the catheter to deform in phase B. As the heart wall resists catheter insertion, the catheter deformation exerts upward force on the sensor, causing a decrease in the measured amount of force. The measured amount of force decreases linearly with the amount of displacement at a rate of 0.1 Nmm⁻¹. At a point near relative displacement of 30~33 mm, referred to as the sudden slip point, the contact becomes unstable, and the catheter enters phase C. The contact

between the catheter and tissue is lost, and the catheter slips suddenly in the direction of least obstruction, releasing the stored energy. The force detected increases sharply as the resistance is removed suddenly. Either the catheter tip hits another contact or the catheter wire glides along the heart wall, creating constant force displacement in phase D. The 75° graph similarly characterizes the catheter-heart interaction. However, the 60° graph depicts 2 small sudden slips. For angles smaller than 60°, the catheter slips directly along the heart surface and exhibits no initial firm contact. The force increases non-linearly upwards to a small degree as the displacement increases due to non-horizontal mounting (Fig. 19a). The mounting also affects early slip in larger catheter-tissue angle compared to experiment in water bath.

Similar characterizations of the interaction between catheter and heart tissue are observed in the water bath experiment in Fig. 17. In particular, a force-displacement trend is identified from angles 90° down to 45°. As the catheter-tissue angle decreases, the minimum force detected increases and the slip point occurs earlier in the relative displacement. The slope of force-displacement due to catheter deformation in phase B remains similar despite different catheter-tissue angle. It can be inferred that the catheter deformation is smaller with decreasing angles and the contact between the catheter and tissue is less stable. At catheter-tissue angle of 30°, the catheter tip slips gently on contact with tissue. The slight variation may be due to undulating surface of LV wall. The occurrence of continuous slip of catheter along heart surface is from 45° to 30° in the water bath experiment. The closeness of the 90° and 75° graphs is due to inaccurate bending angle of a worn out catheter.

Further comparison between the catheter-heart interaction with and without water bath is shown in Fig. 18. The slope of force-displacement due to catheter deformation in phase B has a larger magnitude in setup without the water bath compared to with water bath and the point of sudden slip occurs earlier without water bath. However, the minimum force detected is quite similar in both setups. The initial contact can be inferred to be more stable in the water bath and is contrary to the hypothesis that water can act as a lubricating agent for higher tendency of slip. The buoyancy effect of water may have reduced the resistive forces of deformation transmitted to the catheter holder, thereby preventing early slip. The same heart tissue and mounting is used for two experimental setups to minimize variation and facilitate comparison.

Fig. 16: Force-displacement graph for water-tank experiment without water-bath.

Fig. 17: Force-displacement graph for water-tank experiment with 37° water-bath

Fig. 18: Force-displacement comparison for water-tank experiment with and without water bath (air).

Fig. 19 shows two snapshots of the catheter-heart interaction experiment. The catheter with a pre-set bending and tip displacement of 3 mm above the mounted sepal tissue moved vertically downwards a distance of 40 mm. The catheter tip come into contact with the tissue and is deformed initially (in the non-slip condition), with the catheter tip fixed at a contact with the heart wall (Fig. 19a). Further vertical movement resulted in the sudden deflection of the catheter tip away from the initial contact. The catheter is subsequently deformed (in the slip condition), with the catheter tip gliding along the surface of the heart tissue and making multiple contacts along its distal length (Fig. 19b).

Fig. 19: Catheter deformation in non-slip contact (a) and slip contact (b) with the septal wall heart tissue.

The experiment designed to characterize the interaction between the catheter and heart tissue is performed in a controlled ex-vivo environment and may be insufficient to represent the specificities of an in-vivo catheter-heart interaction. First, the catheter-heart interaction investigated is for a static condition and not a dynamic condition with wall contraction and hemodynamics. Second, porcine heart tissue is excised from sacrificed animals. The characteristics of the tissue specimen including its contractile and elastic properties may differ from the living specimen although it is generally accepted for porcine hearts to be used in cardiovascular research [83]. Third, the setup mounts the tissue horizontally on a surface, exposing the catheter on an open top instead of inside an enclosure. Any slippage of the catheter has less obstruction than the practical environment. The catheter is also secured at a fixed position along the catheter length rather than an insertion point with relative motion between catheter and insertion point. The direction of the catheter against the heart wall does not cover all practical conditions. The measured force is at the distal position rather than proximal position of catheter manipulation. The measurements taken from the experiment can be a reference but the final values have to be calibrated by means of an experienced cardiologist.

4.4 Summary

Two types of contact: non-slip and slip contacts between catheter and heart tissue are identified. At smaller catheter-tissue angles, we have the slip contact where the catheter slips continuously against the tissue and multiple contacts are formed between the distal end of the catheter and the tissue. Early slippage starts at an initial catheter-tissue angle of 60° and full slippage occurs between 30° to 45° . The path of the slipping is dependent on local tissue structure. At a larger catheter-tissue angle, we have the non-slip contact where the catheter bends increasingly between the insertion and tip with force building at an average rate of 0.01Nmm^{-1} . Further pushing results in a transition from a non-slip to a slip contact. The force displacement measurements in the water tank setup provide a range of feedback forces under 1N for haptic devices in our simulator. Our proposed interaction modeling (Chapter 5) for catheter and heart wall is consistent with the data obtained from this experiment.

Chapter 5 A GEOMETRIC APPROACH TO THE MODELING OF CATHETER-HEART INTERACTION FOR VR SIMULATION OF INTRA-CARDIAC INTERVENTION

Cardiac intervention is a minimally invasive diagnostic and therapeutic procedure used to treat cardiac diseases. The mapping of heart geometry with minimal visual aid presents a technical challenge for interventional cardiologists attempting catheter navigation. This chapter presents a geometric approach to modeling the catheter-heart interaction for VR simulations of catheter navigation within a heart chamber. Three types of modeling are used to model the interaction between the catheter and the heart wall: non-slip, pseudo-slip and slip modeling. A two-step shape memory process that minimizes the lateral and axial strains on the catheter is designed for catheter deformation for non-slip and pseudo-slip contact, and a progressive group linkage bending process that constrains the catheter curvature and position within the volume enclosure is designed for catheter deformation for slip contact. The proposed model is consistent with the observations made during the experiment. The model is able to deform the catheter within the volume enclosure and is independent of local motion increment. Thus, it presents advantages in terms of complexity and real-time requirements.

5.1 Outline

The modeling of the interaction between the catheter and the heart-wall is a fundamental block of the intra-cardiac intervention simulator. Of the different modeling of device-tissue interaction for example needle-tissue [7] and spatula-brain [78], the modeling of catheter-vessel interaction [14][15][18] for vascular intervention simulators presents technical issues of similar nature. Like in vascular intervention, tissue deformation plays a less significant role in intra-cardiac intervention due to the flexibility of the catheter and rigidity of the tissue relative to the applied force. In both types of simulators, we are more interested in tracking the navigation of the catheter within the vasculature structure or the heart lumen. The catheter is a long flexible device with a memory shaped tip that can be modeled by a series of rigid links (Section 3.2.2). The catheter bends, deforms and creates multiple contacts with the vessel wall as it is navigated up the vascular structure through external manipulation like twisting, bending, insertion and retraction. Similarly, the catheter bends, deforms and glides along the heart wall, making multiple contacts with the wall along its distal portion. Besides

this type of deformation along the heart wall, the intra-cardiac intervention introduces another interaction where the catheter approaches the heart wall and establishes a firm contact at the tip while inserted catheter length deforms in the free space of the heart lumen. This deformation is important for the contact acquisition in intra-cardiac intervention. Compared to the narrow vasculature structure, the heart lumen allows a larger space for catheter deformation and thus there are fewer contacts between the catheter and heart wall.

Unlike most types of device-tissue interaction where the soft tissue deforms due to interaction with the device, the heart wall tissue deforms due to the heart contraction. This heart wall deformation comprises of global non-rigid motion and is relatively much larger than the local tissue deformation from the catheter contact. Besides the motion of the catheter, the catheter-heart interaction modeling needs to consider the motion of the heart wall for collision detection and response between moving bodies. The problem is simplified by simulating quasi-static phases of interaction where either the motion of the heart wall or motion of the catheter is considered per phase. The time interleaving of the catheter and heart phases allows for realistic moving bodies interaction.

By modeling the catheter contact and deformation within the moving heart chamber, the heart mapping procedure can be simulated. This chapter discusses the modeling of the catheter for a VR simulation of cardiac intervention and focuses on the geometric deformation of the catheter when it interacts with the heart wall under dynamic conditions. Although our approach is geometrically based and heuristic, the modeling is based on the characterization of the interaction between the catheter and heart tissue from a workbench experiment (Chapter 4). The experiment and user study (Section 3.7) indicate that our method can achieve real-time interaction and performance consistent with user experience and expectations. To the best of our knowledge, this is the first study that has proposed a means of simulating the catheter-heart interaction in the context of heart mapping.

5.2 Methods

In the heart mapping procedure, the catheter is navigated within the heart chamber by external means such as twisting, bending, insertion and retraction. The point of insertion of the catheter into the heart chamber is assumed to be fixed and held into position by the aortic valves and aortic arch, whereas the catheter tip is initially free. A collision occurs when the catheter touches or crosses the heart wall. When a collision occurs, the catheter is deformed. The contact of the catheter tip with the heart wall is deemed to be stable if the tip touches at least the upper volume boundary during diastole and if the contact is maintained during the

cardiac cycle with the catheter becoming deformed during diastole. If the contact slips due to instability, the catheter will bend and move to a new position along the heart wall. Therefore, we propose three types of catheter-heart interaction responses when a collision is detected between the heart wall and the catheter due to the input movement of the catheter: (a) non-slip contact, in which the catheter tip may be affixed to the heart wall at the initial contact while the catheter body undergoes deformation between the tip and the aortic insertion point; (b) pseudo-slip contact, in which the catheter tip slides a small distance on heart surface and the contact persists with a different tip position; and (c) slip contact, in which the initial contact is completely lost and the distal length catheter is glides along the heart wall.

5.2.1 Catheter-Heart Interaction Scheme

Based on the above analysis, we model the catheter-heart interaction as follows (Fig. 20). The catheter can be in either an idle or a touch or a touch-slip state depending on whether there is contact with the heart wall and whether the contact is at the tip. The input movement of the catheter or the heart can trigger a transition from an idle to a touch state or maintain the idle state through a series of processes. Upon catheter or heart motion, collision detection determines if the catheter has collided with the heart tissue. If no collision has occurred, the catheter returns to the idle state. If a collision has occurred, the contact position of the catheter with the heart wall is determined. Based on the first collision, the contact stability is assessed. If the contact is stable, the catheter undergoes non-slip deformation. If the contact is partially stable, a pseudo-slip occurs and a new contact will be found and the deformation is similar to non-slip deformation. If the contact is unstable, a slip occurs and the catheter will undergo slip deformation. The contact stability is re-assessed after each instance of deformation if the tip is in contact with the heart wall. The catheter transits to the touch state if the catheter tip is in contact with the heart wall, or it goes to touch-slip state if the contact remains, else it returns to the idle state if contact is lost.

Fig. 20: Scheme for modeling the catheter-heart interaction.

5.2.2 Collision Detection

Before collision detection, each catheter motion step is first checked to determine if it is obstructed by the insertion point especially during retraction and bending away from the principal axis of the catheter. If the full motion step cannot be realized, the partial motion step is computed. For each heart motion step, the position of the catheter is compensated by the shift in the insertion point. The position of the insertion point is defined as the center of aortic valve and can be re-computed with every heart motion.

Collision detection can be easily determined by converting the catheter node position to voxel coordinates to retrieve respective voxel information. Due to the discretization of the voxel representation, the voxel distance may result in a missed detection. When the catheter node is near the heart surface, the distance from surface is computed using voxel-based triangle index. In event that the tip of the catheter touches the heart wall, the contact position and its associated triangle are returned. The heart surface is not touched if all the catheter nodes are within non-heart voxel region or they have positive distance. A contact is detected if any catheter nodes belong to heart voxel class or if any node has zero or negative distance.

Due to the presence of degenerate elements and missing facets in the heart model [77], the voxel representation may have inaccurate surface information. To reduce the sensitivity of the catheter deformation to voxel defects, we compute the moving average of surface normal for a continuous surface curvature description. Let p_k be the position (x, y, z) of k -th catheter node, we derive the voxel co-ordinates (u, v, w) and a list of surface triangles $\mathbf{T}$ that are within a bounded region of the voxel. Let $\hat{n}_i$ and c_i represent the unit normal and

centroid of the i -th surface triangle, the moving average surface unit normal of the k -th catheter node is defined by:

$$\hat{n}(p_k) = \frac{\sum_{i \in T} \frac{\hat{n}_i}{|p_k - c_i| + \varepsilon}}{\sum_{i \in T} \frac{1}{|p_k - c_i| + \varepsilon}} \quad (8)$$

The moving average centroid is computed in a similar way to derive the interpolated surface equation for collision response where ε is a small constant ($\varepsilon = 0.01$).

Collision Detection From Catheter Motion

As collision detection is performed during a quasi-static phase in response to a catheter motion step or heart motion step, the point of collision or the initial contact is determined for an appropriate response. For collision detection due to catheter motion, we find the initial contact by back-tracing the path of the catheter from the previous to the current phase using the current catheter motion type. The intersection of the catheter path with the heart surface can be determined numerically by Newton Raphson method. If the initial contact is not located at the catheter tip, the catheter undergoes slip deformation. Otherwise, an initial contact of the catheter tip with the heart wall is established.

Collision Detection From Heart Motion

For collision detection due to heart motion, the search for the initial contact is more complicated because the intersection between the discrete representation of the moving heart surface and catheter tip is non-trivial. The possibility that the catheter position other than the tip touches the heart surface is first eliminated. The distances of the catheter nodes from their nearest surfaces are computed. If the catheter tip is not the node furthest into the surface, the initial contact is unlikely to be on the catheter tip and slip deformation is activated. Otherwise, we search for the most likely intersection surface. Let d_1 and d_2 be the distances of the catheter tip from the surface in the previous phase t_1 and current phase t_2 . The time of contact is estimated by $t_{\text{contact}} = d_1(t_2 - t_1)/(d_2 - d_1) + t_1$. Then a list of candidate surface triangles is compiled within a bounded region of the voxels representing the catheter tip in previous and current phases. Here we assume that the catheter in the current phase has already been compensated by the shift in the insertion point. All the listed triangles are interpolated bi-linearly to generate the intermediate surface at the estimated time of contact. The surface triangle nearest to the tip and its distance are recorded. A search is performed around the estimated time of contact to locate most likely initial contact. This contact is

represented by the minimum distance of the tip from the nearest surface among all time instances between previous and current phases.

5.2.3 Contact Stability Assessment

The stability of the contact is dependent on the deformation of the catheter and the characteristics of the surface. The validation experiment shows that both the surface inclination and the contact force strength affect the contact stability and risk of slippage. Thus we define metrics to measure the energy $M_{lateral}$ and M_{axial} from lateral and axial loading of the catheter:

$$M_{lateral}(t) = \sum_{i=I(t)}^{N-1} |\theta_i(t-1) - \phi_i(t)|^2 \bigg/ \sum_{i=I(t)}^{N-1} l_i^2 \quad (9)$$

$$M_{axial}(t) = \sum_{i=I(t)}^{N-1} |\lambda_i(t-1) - l_i|^2 \bigg/ \sum_{i=I(t)}^{N-1} l_i^2 \quad (10)$$

where $I(t)$ denotes the I -th catheter node at time t where the point of insertion lies between the I -th and $(I+1)$ -th catheter nodes, $\theta_i(t-1)$ and $\phi_i(t)$ refers to the slope of the i -th catheter link relative to the $(i-1)$ -th catheter link of the deformed catheter at phase $t-1$ and deformed catheter at phase t while $\lambda_i(t-1)$ and l_i are the catheter link lengths of the deformed catheter at phase $t-1$ and undeformed catheter. The inclination between the catheter and tissue ψ is computed as the angle between the current deformed catheter state and the normal of the nearest surface plane. The non-slip interaction is assigned for stable contacts while the slip interaction is assigned for unstable contacts with higher energy metrics and larger inclination angle. The decision regions for are shown in Fig. 21 and pseudo-slip is the transition between slip and non-slip interactions. The thresholds ($T_{1a} = 0.04$; $T_{1b} = 0.08$; $T_{2a} = 0.03$; $T_{2b} = 0.06$; $T_{3a} = 0.16$; $T_{3b} = 0.22$) are determined heuristically. For the contact acquisition in the heart mapping procedure, the catheter tip is required to maintain a stable contact with the heart wall over a few cardiac cycles.

Fig. 21: Decision of non-slip, pseudo-slip and slip deformation.

5.2.4 Non-slip Deformation

In this section, we use $R_i(t)$ and $P_i(t)$ to denote the i -th node on the undeformed catheter and deformed catheter at time t , respectively. $R_i(t)$ is obtained using the method described in Section 3.3. Typical stable contact may be described as catheter deformation with two fixed ends: the catheter point nearest to the point of insertion and the tip that makes contact with the heart wall. The point of insertion lies between the I -th and $(I+1)$ -th catheter nodes while the tip is represented by the N -th catheter node. The deformation is modeled to ensure shape continuity during deformation and to minimize the strain and bending of the catheter with respect to its initial shape. For this purpose, we propose a two-step heuristic shape memory algorithm for deformation.

First, the initial shape deformation is computed as the sum of weighted force displacement at insertion $f_{I,i}(t)$, weighted compensation displacement $c_{I,i}(t)$ at insertion and weighted force displacement at tip contact $f_{N,i}(t)$ with boundary constraints. $f_{I,i}(t)$ represents force displacement due to catheter motion from external maneuver or heart motion at insertion point while $c_{I,i}(t)$ refers to the component from reaction forces at the contact between catheter and heart wall. $f_{N,i}(t)$ represents force displacement due to heart motion at catheter tip. We let $P_i^0(t)$ denote the nodes on the initial deformed catheter at time t :

$$P_i^0(t) = P_i(t-1) + w_{a,i}f_{I,i}(t) + w_{b,i}c_{I,i}(t) + w_{c,i}f_{N,i}(t) ; i = I+1, \dots, N-1 \quad (11)$$

where $w_{a,i}$, $w_{b,i}$ and $w_{c,i}$ are weight distributions representing the effect of respective force or compensation displacements along the catheter length at the i -th catheter node. The linear combination of the first two force displacement terms constraints the displacement at the insertion point to be tangential to the principal axis of insertion. The application of the initial shape deformation adapts to the scenarios of catheter motion and heart motion.

Next, the initial shape deformation obtained is refined by adjusting the nodes towards or away from the straight line connecting the fixed catheter node near the point of insertion and the catheter tip. We let Q_i denote the projection of P_i^0 onto the straight line $P_I^0 P_N^0$. Then, the adjustment is made using $P_i = Q_i + \mu(P_i^0 - Q_i)$ where μ is a global coefficient. The global coefficient is computed by minimizing the following energy function:

$$E = \alpha \sum_{i=1}^N |\theta_i - \phi_i| / 2\pi \sum_{i=1}^{N-1} l_i + (1 - \alpha) \sum_{i=1}^{N-1} |\lambda_i - l_i| / \sum_{i=1}^{N-1} l_i \quad (12)$$

The first term in the above equation represents the energy from lateral loading and is estimated by catheter curvature θ_i deviation from undeformed curvature ϕ_i while the second term represents the energy from axial loading representing catheter length λ_i deviation from original length $l_i = |R_{i+1} - R_i|$. θ_i and ϕ_i are computed from the angle that the i -th catheter link makes with the $(i - 1)$ -th catheter link in deformed and undeformed states while catheter length is defined by $\lambda_i = |P_{i+1} - P_i|$. Both terms are normalized by inserted catheter length. α balances the catheter deformation between the curvature and length deviation. Since physical quantities are not known, α ($\alpha = 0.5$) can be tuned to match force displacement curves with visual deformation. Fig. 22 shows the two-step process through which non-slip catheter deformation occurs.

Fig. 22: Catheter deformation during non-slip interaction.

Non-slip Deformation From Catheter Motion

For the scenario of catheter motion, the boundary constraints are $P_N^0(t) = P_N(t-1)$, which keeps the catheter tip P_N stationary and $P_I^0(t) = R_I(t)$, which maintains the location of $P_{I-1}P_I$ (which is the link containing the point of insertion at time t). The force displacement at the insertion point is attributed to the insertion, retraction, bending and twisting of catheter. It is represented by the vector difference between the undeformed catheter states at times t and $t-1$:

$$f_{I,i}(t) = R_i(t) - R_i(t-1) \quad (13)$$

The compensation displacement is a component complementary to the force displacement by the catheter. It is a continuity constraint at the stationary insertion point that keeps the displacement tangential to the catheter length:

$$c_{I,i}(t) = R_i(t-1) - P_i(t-1) \quad (14)$$

$w_{a,i}$ distributes the force displacement with a maximum value near the point of insertion and a smaller value near the catheter tip while $w_{b,i}$ distributes the compensation displacement

across the proximal part of the catheter length. We set $w_{a,i} = 1 - \left(\frac{i-l}{N-l}\right)^3$ and $w_{b,i} = \text{clip}\left(\left(\frac{2i-2l}{N-l}\right)^2, 0, 1\right)$.

Non-slip Deformation From Heart Motion

However for heart motion, the boundary displacement conditions are reversed. When the heart contracts, the insertion point is held in position with the moving aortic valve orifice by applied external force while the tip moves with the tip contact on the contracting surface. The force displacement at the insertion is attributed to the shift in the insertion point due to heart contraction:

$$f_{I,i}(t) = P_I(t) - P_I(t-1) \quad (15)$$

We can easily determine the new insertion position $P_I(t)$ as the center position of the aortic valves. Since the insertion point shifted with the same catheter link $P_{I-1}P_I$ in tandem, no compensation displacement is required.

The displacement at the tip contact however requires the consideration of the underlying discrete voxel representation to ensure consistency in interaction:

$$f_{N,i}(t) = f_{N,N}(t) = P_N(t) - P_N(t-1) \quad (16)$$

As the heart moves from time $t-1$ to t , the catheter tip $P_N(t-1)$ moves to $P_N(t)$ while remaining in contact with a surface triangle T_A at local co-ordinates (u_a, u_b) . If the surface triangle has nodes V_1, V_2 and V_3 and Q_a and Q_b are the projections of V_1 and V_2 on edges V_3V_2 and V_3V_1 , we can express P_N as the intersection between Q_aV_1 and Q_bV_2 (Fig. 23). However, changes in voxel representation with time may result in change in the surface triangle from T_A to T_B for the contact position $P_N(t)$ due to discretization. The change in surface triangle affects the interaction process. Two searches are performed to check if a better candidate contact $P_N(t)$ with zero surface distance can be found. The first search returns a candidate with the same surface triangle. The search is performed in the neighborhood of voxel (u, v, w) to find voxels with similar surface triangle and their projections of the voxel center to the triangle facet are compared (Fig. 24). The projection nearest to $P_N(t)$ that returns the same surface triangle with its voxel data, replaces $P_N(t)$. The second search returns a candidate with an immediately adjacent surface triangle. The projections of $P_N(t)$ to each edge of surface triangle T_A are checked. If any projection

returns a surface triangle immediately adjacent to T_A with its voxel data, the one with the nearest distance to $P_N(t)$ replaces $P_N(t)$.

Fig. 23: Local triangle co-ordinates.

Fig. 24: Contact position and shift in nearest triangle.

$w_{c,i}$ distributes the force displacement $f_{N,i}(t)$ with a maximum value near the tip contact and a minimum value at the insertion while $w_{a,i}$ fades the force displacement $f_{I,i}(t)$ in the reverse direction. We set $w_{c,i} = 1 - \left(\frac{N-i}{N-1}\right)^3$ and $w_{a,i} = \left(\frac{N-i}{N-1}\right)^3$.

5.2.5 Pseudo-slip deformation

Pseudo-slip contact represents the transition between stable and non-stable contact in which the catheter transits from the non-slip to the slip condition. As the heart surface has a network of trabeculae, the catheter tip will meet obstacles that slow down the slipping. Pseudo-slip contact is modeled as a sub-class of the non-slip contact interaction in which the

catheter tip is allowed to deviate slightly from the original contact when the external force that is applied to the catheter is insufficiently counteracted by the frictional wall forces.

Pseudo-slip Deformation From Catheter Motion

The initial slip vector S^0 of the catheter tip is computed as the component of force displacement $f_{I,N}(t)$ along the projection vector $p_{\tan}$ of the catheter onto the tangent plane of the heart surface at $P_N(t-1)$:

$$S^0 = [f_{I,N}(t) \cdot p_{\tan}] p_{\tan} / \|p_{\tan}\|^2 \quad (17)$$

If the unit normal vector of the surface at $P_N(t-1)$ is $\hat{n}$, then $p_{\tan} = [(P_{N-1}(t) - P_N(t)) \cdot \hat{n}] \hat{n} - P_{N-1}(t) + P_N(t)$. The new catheter tip position is obtained by moving $P_N(t-1) + S^0$ perpendicularly towards the heart surface:

$$P_N(t) = P_N(t-1) + S^0 + \tau \hat{n} \quad (18)$$

for some τ such that $P_N(t)$ is on the heart surface, and therefore, the final slip vector becomes $S = P_N(t) - P_N(t-1)$.

Pseudo-slip Deformation From Heart Motion

For the case of heart motion, pseudo-slip contact occurs when the frictional forces on catheter wall are insufficient to maintain the contact on the moving heart wall. Unlike the former case where the slip vector follows the direction of catheter motion, the slip vector is in opposite direction to the movement of the heart wall. From the perspective of the contact on the heart wall, the catheter is exerting a force displacement in opposite direction (Fig. 25).

The initial slip vector S^0 is defined similarly tangential to the heart surface at $P_N(t-1)$ but with displacement $f_{N,N}(t)$ at the tip contact and tangential plane vector $p_{\tan} = [(P_{I+1}(t) - P_I(t)) \cdot \hat{n}] \hat{n} - P_{N-1}(t) + P_N(t)$:

$$S^0 = -[f_{N,N}(t) \cdot p_{\tan}] p_{\tan} / \|p_{\tan}\|^2 \quad (19)$$

The new catheter tip position is derived in similar way using $P_N(t-1) + f_{N,N}(t)$ instead of $P_N(t-1)$ as the starting point:

$$P_N(t) = P_N(t-1) + f_{N,N}(t) + S^0 + \tau \hat{n} \quad (20)$$

and the final slip vector becomes $S = P_N(t) - P_N(t-1) - f_{N,N}(t)$.

The initial shape deformation is modified to compensate for the displacement of the contact due to the slip vector. The compensation displacement $c_{N,i}(t)$ is a component orthogonal to the final displacement of the contact. The modified shape deformation is defined as follows:

$$P_i^0(t) = P_i(t-1) + w'_{a,i} f_{1,i}(t) + w_{b,i} c_{1,i}(t) + w'_{c,i} f_{N,i}(t) + w_{d,i} c_{N,i}(t) \quad (21)$$

The weight distributions are also modified by scaling factors s_1 and s_2 to ensure that the boundary conditions $P_i^0(t) = P_N(t-1) + S$ and $P_i^0(t) = P_N(t-1) + f_{N,N}(t) + S$ are met for both the catheter and heart motion cases. In the case of catheter motion, we set scaling factor

$$s_1 = \left(1 - \frac{\|S\|^2}{f_{1,N}(t) \cdot S}\right) \quad \text{and} \quad \text{assign weight distributions} \quad w'_{a,i} = 1 - s_1 \left(\frac{i-1}{N-1}\right)^3 \quad \text{and}$$

$w_{d,i} = (1 - s_1) \left(\frac{2i-N-1}{N-1}\right)^2$. $w_{b,i}$ is not modified because the compensation displacement does not affect the catheter tip position. In the case of heart motion, we set scaling factor

$$s_2 = \left(\frac{\|f_{N,N}(t) + S\|^2}{f_{N,N}(t) \cdot (f_{N,N}(t) + S)}\right) \quad \text{and weight distributions} \quad w'_{a,i} = s_2 \left(\frac{N-i}{N-1}\right)^3, \quad w'_{c,i} = 1 - s_2 \left(\frac{N-i}{N-1}\right)^3 \quad \text{and}$$

$w_{d,i} = s_2 \left(\frac{2i-N-1}{N-1}\right)^2$. Finally, the refinement is computed based on the non-slip contact.

Fig. 25 illustrates the initial shape deformation weights $w_{a,i}$ and $w'_{a,i}$ for force displacement from the catheter movement and weights $w_{c,i}$ and $w'_{c,i}$ for the heart wall contraction under non-slip and slip conditions. The deformation decreases from insertion to tip for the catheter movement where catheter tip position remains unchanged while it decreases from tip to insertion for the moving heart wall. When a pseudo-slip condition occurs, an offset is added at the catheter tip to compensate for the slip vector. Due to the inverse direction of the slip vector, the final displacement vector of the contact is the same as the slip vector for the catheter motion case but not for the heart motion case.

Fig. 25: Initial deformation weights for pseudo-slip and non-slip due to force displacement from (a) catheter motion and (b) heart motion (excluding compensation displacement at insertion point)

5.2.6 Slip Deformation

Non-stable contact occurs when there is catheter deformation involving a fixed end (the catheter node nearest the point of insertion) and a moving end (the catheter tip). The non-stable contact occurs when the catheter deformation is large and the frictional forces of the contact are unable to hold the catheter tip in position.

For this mode of interaction, our approach is to adjust the catheter nodes iteratively, progressing from the fixed catheter node near the aortic valve to the moving catheter tip until all catheter nodes outside the lumen fit within the volume enclosure. First, the intersection between the heart wall and the catheter is computed. Then, the new position of the intersecting catheter link is determined. Using the proximal node as reference, the distal node is manipulated so that it touches the tangent plane of the surface. To simulate a more realistic deformation and to model catheter stiffness, a group of catheter links are bent collectively instead of as an individual link. Next, the bending is extended from the intersecting link to the catheter tip to preserve the curvature of the catheter. This interaction process is repeated for the next catheter link. During this iterative adjustment process, the catheter bends and deforms from the insertion to the moving tip.

Beginning with the fixed end of the catheter, each catheter node is assessed to determine if it has penetrated the heart wall. If a catheter node R_{cur} has moved within the heart wall, the new position of R_{cur} must be determined, which should be on the heart wall in that region. Suppose that the previous catheter node proximal to the current catheter node R_{cur} is R_{ref} ; then, link $R_{ref}R_{cur}$ intersects the heart wall surface at some point. We denote the normal of the heart surface at the intersection point as n_l . Based on the principle of minimal energy for deformation, the new contact position must lie within the plane, denoted by π , that contains $R_{ref}R_{cur}$ and has a normal vector orthogonal to n_l . Thus, the normal of plane π is computed using $n_b = (R_{cur} - R_{ref}) \times n_l$. In addition, the length of the catheter link $R_{ref}R_{cur}$ should remain constant. The problem of finding the new position of R_{cur} can be described geometrically as the intersection of a sphere with center R_{ref} and radius $\|R_{cur} - R_{ref}\|$, plane π and the heart wall surface. Note that the intersection of the sphere and plane π is a circle (Fig. 26). If we let $T = n_b \times (R_{cur} - R_{ref}) / \|n_b\|$, then the circle in 3D has the following parametric representation:

$$P(\alpha) = R_{ref} + \cos \alpha (R_{cur} - R_{ref}) + T \sin \alpha, \alpha > 0 \quad (22)$$

The new position of R_{cur} can be obtained by computing the intersection of this parametric curve and the heart wall surface.

Fig. 26: Interaction process between a catheter link and the heart wall surface.

To simulate catheter stiffness and resistance to bending, the above interaction process is used with a group linkage bending scheme. Instead of exhibiting a local bend at a link, the catheter starts to bend M links proximal to the interaction to ensure smooth bending across a larger region. Referring to Fig. 27, let catheter node R_m be the node moved within the heart

wall; the group linkage, comprising nodes $R_m, R_{m-1}, \dots, R_{m-M}$ is bent collectively based on the following constraints:

1. $R_m'' \in \text{heart} - \text{chamber}$, where R_m'' is the new position at the m -th link after the interaction process. This process guarantees that the new position stays within the heart chamber.
2. $\angle R_{m-M} R_m R_{m-1} \leq \Theta_1$, where Θ_1 is an angle threshold controlling the deviation of the group linkage from the local linkage. Segment $R_{m-M} R_m$ is analogous to a piecewise approximation of the length of catheter $\{R_{m-M}, R_{m-M-1}, \dots, R_{m-1} R_m\}$ for group linkage bending. The group linkage should provide an acceptable approximation of the local linkage. It should not include sudden bends or turns that require multiple piecewise approximations. A large deviation is more likely to incorporate such bends. Θ_1 is set to 60° in this work to lower the risk of poor approximation.
3. $\Theta_2 < \angle R_m R_{m-M} R_m'' < \Theta_3$ is a group linkage bending angle and where Θ_2 and Θ_3 are the minimum and maximum angles corresponding to the bending of the group linkage. The minimum angle Θ_2 is required for effective bending and precision. If the group linkage angle is close to zero, the near-zero rotation may have large precision errors. The maximum angle Θ_3 is dependent on the length of the group linkage (M links), and together, they determine the flexural rigidity of the beam. Θ_3 must be selected such that it does not result in a sharp bend issue with local linkage bending. Too small a Θ_3 may result in larger number of iterations. In this study, Θ_2 and Θ_3 are set at 3° and 45° respectively.
4. $\theta_j \leq \theta'_j$ and $\angle R_{m-M} R_m R_{m-1} \leq \theta'_j$, where θ_j and θ'_j refer to the angle between vector $R_m R_{m-M}$ and normal n_j and the angle between vector $R_m R_{m-1}$ and n_j , respectively. Normal n_j is the normal of the j -th surface nearest R_m . This constraint ensures that the local and group linkages both make an acute angle with the surface on the same side of the normal to pre-empt the group linkage from bending in the direction opposite the intended local linkage.

Fig. 27: Catheter deformation during slip interaction: (a) Local deformation ($M=1$); (b) Group deformation.

The above four constraints are used to determine the length of the group linkage and the regional bending behavior. Given an input M linkage associated with catheter inertia, the group linkage is tested to determine that it satisfies all constraints in a process referred to as the group linkage search. If the conditions are not satisfied, M is decremented, and the process is repeated until the conditions are satisfied. The corresponding group linkage bending angle $R_m R_{m-M} R_m''$ is computed using the interaction process described earlier in Fig. 26 with $R_m = R_{cur}$ and $R_{m-M} = R_{ref}$.

The group linkage bending angle indicates the collective deviation of links from the current catheter shape. The group bending angle is effected between the $(m-M-1)$ -th and $(m-M)$ -th link by rotating R_m to approximate R_m' on the plane formed by R_{m-M} , R_m and R_{m-1} . After each adjustment, the remaining length of the catheter comprised of the nodes beyond the adjusted node towards the free end is readjusted. The chain adjustment is performed with respect to the curvature (or the 2nd order derivative) of the undeformed catheter:

$$R'_j = 2R'_{j-1} - R'_{j-2} + (R_j - 2R_{j-1} + R_{j-2}) \quad (23)$$

Because the group linkage bending involves the collective adjustment of individual links, there may be some rotation that results in collision of some intermediate nodes with the heart wall. Hence, multiple attempts may be required to ensure that the catheter nodes reside inside the heart lumen.

5.3 Results and Discussion

The modeling of the catheter-heart interaction is performed on an Intel Xeon system with dual 2.67GHz cores, a NVIDIA GeForce GTX285 and 4G RAM. The heart surface of the LV is pre-computed from a cardiac dataset [77] comprised of 148,516 vertices and 728,321 tetrahedra. The LV is represented with dimensions 106x126x152x25 unit-voxel³ and is digitalized at a resolution of 0.8x0.8x0.8 mm³. The deflectable portion of the catheter is represented by 16 data points with 4 units per link and up-sampled (x8) via linear interpolation.

5.3.1 Real-time Performance

The real-time catheter-heart interaction is mainly attributed to voxel representation of the heart and geometrical approach for catheter deformation. Without these optimizations, a 3D volume intersection problem has to be solved and a deformation solution has to be iterated with every catheter or heart movement. Table 3 summarizes the time and contact type statistics taken for various catheter and heart motion with a minimum of 1000 non-slip contacts. The catheter is first initialized to a common position close to the surface of the LV, and then with the either catheter is moved with equal probability of the different catheter motion types or the phase of the heart cycle is incremented with equal probability to the catheter motion. The step motions of the catheter are 15° per rotation, 8 mm per translation (8% of LV length dimension) and 4 units per bending (0.5% maximum deflection). From the data, it is observed that interaction analysis and associated deformation for each movement can be computed within 6 ms. Each of the null, non-slip and slip contacts consumes an average of 0.33, 6.62 and 1.76 ms. Given a 30% interaction task load on the system, it is possible to sample up to 150 catheter movements per second using the VR catheter unit or to have 150 cardiac phases per second up to a heart rate of 360 beats per min for 25 phases per cardiac cycle. In fact, the real-time movement of the catheter is validated by the virtual device whose movement is only sampled every 100 ms. Under normal circumstances, the

catheter is carefully navigated within the heart chamber, and high acceleration or large catheter movements are unlikely. Any hard contact of the catheter with the heart wall should be minimized during the cardiac intervention procedure to prevent defibrillation or injury to the heart wall. The catheter movement used in the statistics tests is intended to provide a worst case simulation. For contact acquisitions, specific skills are to be trained as non-slip interaction occurs intermittently during random motion.

Table 3 : Time taken (ms) and contact profile for equal probability of heart and catheter motion; Each dataset is taken with a start point where the catheter tip first is slightly above the heart wall.

No	Contact						Catheter Motion			Heart Motion	Time per motion
	Null	Time	Non-Slip	Time	Slip	Time	Translate	Bend	Rotate		
1	1,608	525	1,000	6,468	254,485	445,873	42,893	42,700	42,805	128,694	1.761
2	1,595	475	1,000	6,766	220,694	383,987	37,061	37,227	37,347	111,654	1.752
3	1,781	643	1,000	6,618	315,942	553,400	53,501	52,986	53,151	159,085	1.759
4	2,876	906	1,000	5,563	481,866	847,551	80,803	81,104	80,548	243,287	1.758
5	2,290	822	1,000	6,423	431,160	770,219	72,591	72,288	72,169	217,402	1.790
6	1,763	582	1,000	6,584	317,499	568,580	53,337	53,416	53,416	160,083	1.798
7	1,731	632	1,000	6,466	288,623	515,724	48,519	48,500	48,519	145,816	1.794
8	824	376	1,000	8,107	426,329	763,094	71,487	71,487	71,650	213,915	1.802
9	1,881	591	1,000	6,734	1,387,260	2,394,726	232,136	232,136	231,976	694,028	1.728
10	2,004	652	1,000	6,511	1,250,235	2,198,783	209,852	202,852	209,127	626,067	1.760

5.3.2 Visual Effects

Fig. 28 illustrates various catheter-heart interactions with the catheter bent (a), rotated (b), and inserted (c) from the initial catheter position shown in the left column. The LV is represented by the mesh with the aortic and mitral valves. The catheter is represented by the blue-striped rod in its undeformed state and by the red-striped rod in its deformed state. The shape and position of the catheter are determined initially by the insertion point at the aortic valve and tilt angles. Subsequently, the deformation of the catheter via a set of bending, rotating, and pull-push motions is determined using catheter-heart interaction modeling. Progressive incremental catheter motion may displace part of the catheter outside the heart lumen. If the catheter tip has penetrated the LV wall (indicated by blue stripe), the catheter will be deformed using the interaction algorithm (indicated by red stripe). In non-slip contact (Fig. 28a), the catheter is deformed with the catheter tip fixed at initial contact position. In

slip contact (Fig. 28b and Fig. 28c), the catheter is progressively deformed from the fixed catheter point near the insertion point and ending with the catheter tip.

Fig. 28: Catheter-heart interaction with various maneuvering mode

Fig. 29 shows the effect of global linkage bending on the catheter-heart interaction. Without group linkage bending, a kink may result from the simple interaction process. If care is taken

in controlling catheter stiffness via group linkage bending, the deformation of the catheter is much smoother, and the results of the deformation are much more acceptable.

Fig. 29: Effect of group linkage on catheter deformation: (a) Sharp bend occurs without group linkage; (b) Smooth deformation occurs with group linkage.

Fig. 30 shows an example of dynamic catheter-heart interaction where a catheter touches the lateral wall with a stable contact in a beating heart. The undeformed catheter (blue-striped) remains static while the catheter (red-striped) deforms and the heart contracts and expands. From top left to right, then bottom, the catheter undergoes various stages of non-slip deformation while the LV contracts and then expands through the cardiac cycle. It can be observed that the catheter deformation gets more pronounced with increased bending from the first to the sixth picture as the LV contracts towards systole. Then after, the catheter deformation decreases as the LV expands and enters diastole. The point of contact between the catheter and the heart wall remains in the entire process. The pictures also illustrate the shift in the insertion point as the difference between the deformed and undeformed catheter changes near the aortic valve (top left).

Fig. 30: Dynamic catheter-heart interaction. (phase 3,4,5,6,9,12,15,18,22 out of 26 phases per cardiac cycle from left, right, top, bottom)

5.3.3 Qualitative Validation

The suitability of the geometrical model for VR training is assessed from a qualitative rather than a quantitative perspective. The energy metrics from lateral and axial loads are computed from equations (9) and (10). Fig. 31a corresponds to an experiment where a mounted catheter is programmed to move towards the heart tissue mounted on an inclined plane setup for a distance of 60 mm. Note that the force displacement graph is inverted to represent positive haptical feedback force. Fig. 31b corresponds to a simulation of continuously

inserting a catheter into the heart lumen for 50 mm. The tip of the catheter touches the inferior lateral wall. Comparing the curves in Fig. 31, both the simulation and experiment have two distinct sections of non-slip and slip interaction and the trend of the curves are similar. After the first contact (5 mm), the lateral load begins with a slow increase, subsequent accelerated increase, then decelerated increase in energy while the axial load has a steady increase in energy during non-slip deformation. There is a sharp decrease in both energies when the slip condition occurs (33 vs. 52 mm).

Fig. 31: Force-displacement characteristic of a catheter moving against LV tissue: (a) Experiment; (b) Simulation; Energy metrics from axial (9) and lateral (10) loading.

The force-displacement graph from the simulation is similar to the one from the experiment and we seek to validate the correlation between the simulation and experimental data. The lateral and axial strain energies of the catheter simulated from external manipulation can be tuned to the relative force from the experiment to indicate the scalable resistive force as

haptical feedback in the simulator. We define the resistive force $F(t)$ as the weighted square sum of the lateral and axial strain energies:

$$F(t) = k_l M_l(t) + k_a M_a(t) \tag{24}$$

where k_l and k_a are constants both set to 16.5 to match the force range of the experiment.

Fig. 32: Sequence of a moving catheter against the LV wall from (a) experiment, (b) simulation in volume rendering and (c) simulation in transparency rendering; Catheter length above insertion segment (blue) is not deformed in simulation.

The time sequences of a catheter pushing against the LV wall are compared in Fig. 32. The first set of pictures is taken from the experimental setup with porcine heart tissue and catheter. A mechanical positioning system lowers the catheter onto the heart tissue for a pre-set distance. The second and third sets of pictures are captured from the simulator. To replicate the condition of the experiment, the insertion is positioned at a distance above the LV wall and the catheter is then pushed towards the LV wall. Due to enclosure of the LV, a segment of the wall is cut to show the catheter-heart interaction. The pictures are rendered in

perspective view. The catheter deforms between two fixed points from pictures 1 to 4 under the non-slip condition. The slip transition occurs between pictures 4 and 5. The catheter tip glides along the wall from pictures 5 to 10 under the slip condition. From the comparison of the pictures, we see the simulated catheter slips in a similar manner as the experiment. However, it should be noted that the conditions of experiment differ from the simulation in that the catheter slips in open space above the mounted tissue instead LV enclosure and that the insertion point is fixed along a catheter length instead a 3D position (blue).

5.4 Summary

This chapter has presented a geometric approach to addressing the catheter-heart interaction in a VR simulation of intra-cardiac intervention. We represent the catheter using a series of rigid links and the heart using a hybrid representation consisting of mesh and voxels. The interaction between the catheter and the heart wall is classified as non-slip, pseudo-slip or slip deformation under external twisting, bending, insertion and retraction of the catheter. Catheter deformation under non-slip or pseudo-slip contact is achieved via recursive shape deformation that minimizes the lateral and axial strains of the catheter. Catheter deformation under slip contact is achieved using a progressive geometric deformation that constrains the catheter curvature and position within the volume enclosure. The key advantage of the proposed approach is that it can be used to deform the catheter into any free-state shape within the volume enclosure and is independent of local motion increment. The deformation can be achieved in a single motion step, thus ensuring real-time interaction. Each interaction from a catheter or heart motion can be achieved within 1.8 ms on a Xeon 2.67GHz system and it is possible to process 150 interactions per second for a typical heart rate of 60-100 beats per min. Although the modeling is consistent with the observations from our experimental procedure, the validation is mainly via qualitative observation and subjective user opinion. For the virtual simulator to be accepted by the medical community, more rigorous quantitative validation by means of carefully designed experiments with dynamic interaction with positional tracking should be done. In addition, an implementation with a physics-based method like mass-spring model would be better received and can serve as a comparison with the geometric-based method. It should be pointed out that this is the first attempt to model dynamic catheter-heart interaction within a VR simulation.

Chapter 6 PROGRESSIVE SURFACE RECONSTRUCTION FOR HEART MAPPING PROCEDURE

The composite imaging of measured cardiac properties like electrical activation and contractile properties on a reconstructed endocardial surface allows for the diagnosis of cardiac arrhythmia and myocardial infarct. However, it is difficult for an interventional cardiologist to acquire heart surface contacts by navigating a catheter to the desired region of interest under minimal visual aid. This chapter discusses a new method for the progressive reconstruction of an endocardial surface during a heart mapping procedure. A generic mesh is first aligned with a set of anchor points to obtain a first approximation of the surface. Subsequent deformations are constrained by the preservation of local surface characteristics and the fidelity of new contacts. The mesh is refined by local subdivision and its geometrical shape is further improved by edge swapping. Compared to prior art, the new method can reconstruct a realistic surface from a set of sparse and random data. It can advantageously provide a smooth reconstruction at initial acquisition and ensure a geometrical consistency between consecutive reconstructions. The accurate reconstruction of a heart chamber provides important visual cues for an interventional cardiologist to decide on the next mapping site, thus constructively influencing the final diagnosis.

6.1 Outline

In computer graphics (CG) and CG industrial applications, a surface can be reconstructed from a set of point clouds or different sets of image slices that are produced by a 3D scanner. The object of interest is measured from its outer surface either through non-contact laser scanners that operate from 10 to 500kHz or contact probes that move slowly up to a few hundred Hz. Compared with the heart mapping procedure that acquires each contact only when the catheter tip forms a stable contact with heart wall for a few cardiac cycles, the data acquisition of 3D scanners are much denser. Typical heart mapping procedures can acquire about a hundred contacts in an hour. Further, the procedure requires a contact with the inner surface of the heart by navigating a catheter device skillfully through a small opening located proximal to the surface whereas both the non-contact scanning, or contact probing obtain contacts from the outer surface. The entire or partial heart surface is to be reconstructed with these acquired contacts. The heart surface may be represented by a discrete polygonal mesh. The reconstructed mesh has to match the geometry of the sampled heart chamber and to fit

the acquired contacts on its surface. The degree of fit may be measured by the Hausdorff distance [84], vertex-to-vertex distance and curvature error-fit metrics.

The major difficulty of the surface reconstruction lies in the progressive acquisition of contacts. The surface has to be refined with each contact acquisition as the reconstructed surface is a visual aid for an interventional cardiologist to decide on the location of the next contact. The next contact affects the subsequent surface reconstruction. Thus an accurate representation of the heart geometry allows for the constructive interactions between the interventional cardiologist and his catheter device. During initial acquisition, any reconstruction algorithm must be able to reconstruct a reasonable surface with a few contacts such that the interventional cardiologist can match the surface with his mental 3D image of heart chamber hinted by the echocardiograph and previous fluoroscopic guidance. As each contact is updated, the deformation of the surface must progress towards the true surface with acceptable transitional shapes. There must be little discrepancies between the reconstructed surfaces at consecutive point acquisitions.

The random acquisition of the procedure also imposes sparse sample point constraints on the surface reconstruction. The distribution of the contacts need not be uniformly spaced. There may be some regions where the catheter tip cannot reach and hence no contact can be acquired. On the other hand, there may be particular regions of interest often pathologically related where higher density of sampling is desired. In this case, the mesh model must be sufficiently fine to represent such details. However, finer meshing requires more computational time and has a smaller contact-to-mesh-vertices ratio. As an online diagnostic tool for the intra-cardiac interventional procedure, the surface reconstruction also has to meet real-time requirements. It must be efficient enough to reconstruct the surface with a minimal latency for a sampling speed of at least several sample points per minute.

6.2 Methods

The description of the progressive surface reconstruction is organized into following subsections of generic mesh model, alignment of mesh model to heart chamber, first approximation, progressive deformation, local subdivision and edge swapping. Other than the selected mesh model and its alignment that is specific to LV, the other sections can be applied to other enclosures with progressive acquisition of contacts.

6.2.1 Generic Mesh Model

The choice of a generic mesh model is associated with the geometry of the surface to be reconstructed. Mesh regularities, non-occurrence of high degree, absence of sharp corners are typical properties desired for reconstructing enclosures like heart chambers. In line with literature [80], a prolate spheroid model is selected to approximate the shape of LV shown in Fig. 33. The equations of the prolate spheroid model are defined as follows in an elliptical co-ordinate system where ϕ represents the circumferential angular co-ordinate, η represents the longitudinal angular co-ordinate, ξ represents the radial co-ordinate and a represents the focal length:

$$\begin{aligned} x &= a \sinh \xi \sin \eta \cos \phi \\ y &= a \sinh \xi \sin \eta \sin \phi \\ z &= a \cosh \xi \cos \eta \end{aligned} \quad (25)$$

The parameter set is given by $0 \leq \phi \leq 2\pi$, $\frac{\pi}{3} \leq \eta \leq \pi$, $\xi = 0.8$ and $a = 40$.

Fig. 33: Prolate spheroid model.

The range of η provides a geometric model opened on the superior surface. The prolate spheroid model does not describe the base of LV, including the mitral valve and aortic valve as the apex and mid walls undergo larger changes during contraction than the base wall. ξ is heuristically assigned to match the geometry of sampled LV while a ensures that the model encloses the LV. The prolate spheroid model is sampled digitally by varying $\{\eta, \phi\}$. Higher resolution decreases the speed of computation while lower resolution has insufficient smoothness. A resolution of 32x32 was set to keep the latency of reconstruction within 1s for interactive applications while maintaining vertex-to-vertex distance and curvature error-fit

within 6 mm^2 and 0.25° respectively with acceptable subjective results. After the triangulation process, the generic mesh model has 1025 vertices, 2016 triangles and 3040 edges. Alternatively, a LV model derived from a CT scan may be used for better approximation.

6.2.2 Alignment

The generic mesh model is globally aligned to the heart chamber for specific rotation, scaling and transformation in Fig. 34. In the case of LV, the initial contacts, namely, apex, mitral valve center and aortic valve center are acquired under the guidance of fluoroscopy. The spheroid co-ordinates are rotated to align the Y-axis on base-apex axis, formed by the line joining the mitral valve center and the apex. The base center is estimated as the projection of the aortic valve center onto the base-apex axis. The spheroid co-ordinates are translated to align its origin to $2/3$ the distance from the apex to base. The scale factor is determined by the ratio of apex-to-base distance to $a \cosh \xi$.

Fig. 34: Aligning LV to prolate spheroid co-ordinates.

6.2.3 First Approximation

The first approximation is important to the progressive reconstruction process as a more precise approximation leads to a better shape in subsequent recursive updates. The surface characteristics of the target LV are unknown at initialization and the reconstructed surface can be refined with the addition of new contacts to better approximate the target surface. The global shape approximation of LV is determined by two landmark points: the apex and aortic valve center. The approximation in our work is adapted from Reisfeld method [36] where we

added a normalized local contact density $\tilde{\lambda}_i$ for the i -th vertex to the mesh adjustment process (Section 3.6).

6.2.4 Progressive Deformation

The process of progressive deformation needs to satisfy the requirements of a smooth surface and a non-abrupt transition between consecutive contact acquisitions. The preservation of local mesh curvature is desired. The Laplacian differential from traditional mesh editing is applied for soft non-rigid deformation [85]. The technique generates a globally continuous and smooth surface while preserving local Laplacian characteristics that is closely related to the mean curvature and normal of the mesh. The set of new deformed vertices minimizes the mean square Laplacian difference from previous deformation while fitting the mesh to the current array of contacts. That is, after the $(n+1)$ -th contact is acquired, the set of mesh vertices $\mathbf{P}_{n+1}$ is the solution to the following minimization problem:

$$\begin{aligned} \min & \|\mathbf{L}\mathbf{P}_{n+1} - \mathbf{T}_{n+1}\mathbf{L}_n\|^2 \\ \text{s.t. } & \mathbf{P}_{n+1}^* = \mathbf{S} \end{aligned} \quad (26)$$

where $\mathbf{L}$ is the Laplacian matrix that depends on the connectivity of the reconstructed mesh with vertices $\mathbf{P}_n$ at the n -th contact acquisition; $\mathbf{L}_n$ is the Laplacian values of the mesh with vertices $\mathbf{P}_n$; $\mathbf{P}_{n+1}^*$ consists of those mesh vertices in $\mathbf{P}_{n+1}$ that correspond to the vertices in $\mathbf{P}_n$, having the smallest Euclidean distance to each of the contacts in $\mathbf{S}$; and $\mathbf{T}_{n+1}$ is a diagonal matrix with each diagonal element corresponding to a local transformation matrix applied to a vertex on the reconstructed mesh at the n -th acquisition. The local transformation matrix is introduced to compensate for the sensitivity of differential coordinates to rotation and scaling at the $(n+1)$ -th acquisition. The minimization problem (26) can be converted to the following problem using the method of Lagrange multipliers, which can be solved with a linear solver package:

$$\min \|\mathbf{L}\mathbf{P}_{n+1} - \mathbf{T}_{n+1}\mathbf{L}_n\|^2 + \alpha \|\mathbf{P}_{n+1}^* - \mathbf{S}\| \quad (27)$$

where the weight α tradeoffs the smooth term and the fidelity term. α is set to 1.

The Laplacian vector at vertex p_i is a discrete differential vector defined as $\delta_i = \sum_{j \in N(i)} \omega_{ij} (p_j - p_i)$ where δ_i approximates the local mean curvature. $N(i)$ refers to the

first connected neighborhood of vertex p_i and ω_{ij} is the weight corresponding to each edge formed by p_j and p_i . The set of all differential vectors can be expressed in matrix form. E is the set of edges in the mesh. L_{ij} is the (i,j) -th element of Laplacian matrix L .

$$\mathbf{L}_n = \mathbf{L}\mathbf{P}_n$$

$$L_{ij} = \begin{cases} -\sum_{j \in N(i)} \omega_{ij}, & i = j \\ \omega_{ij}, & (i, j) \in E \\ 0, & \text{otherwise} \end{cases} \quad (28)$$

Several Laplacian weighting schemes have been evaluated for remeshing and smoothing effects and they include uniform weighting $\omega_{ij} = 1$, inverse length weighting $\omega_{ij} = \frac{1}{e_{ij}}$ and cotangent weighting $\omega_{ij} = \frac{1}{2}(\cot \alpha_{ij} + \cot \beta_{ij})$. α_{ij} and β_{ij} are the opposite angles of edge e_{ij} . It is observed that the cotangent weighting scheme [86] gives the smallest fitting error of contacts and allows for better fit with minimal distortion. With each update of new contact, the new positions of the set of vertices are solved by the constrained linear system. The Laplacian deformation values are computed and used for the next update. The Laplacian technique provides a smooth surface. While the recursive process contributes to the smooth transition of shapes, it may lead to distortion because of error propagation.

The local transformation for each vertex p_i can be estimated from a naive solution of minimization problem (26) [13] by setting $T_i = I$ where I is the identity matrix. The local transformation matrix T_i can be decomposed into a scaling matrix S_i and a rotation matrix R_i such that $T_i = S_i R_i$. The rotation matrix can be computed from the changes in the edge vectors [87] from the n -th acquisition and naive solution of the $(n+1)$ -th acquisition. Denoting the edge $e_{ij}(n)$ joining vertices p_i and p_j at the n -th acquisition, the covariance matrix for p_i is defined by:

$$c_i = \sum_{j \in N(i)} w_{ij} e_{ij}(n) e_{ij}(n+1)^T \quad (29)$$

If we derive the singular value decomposition of $c_i = U_i \Sigma_i V_i^T$ where U_i and V_i are the eigenvectors and Σ_i is the diagonal matrix of eigenvalues, we obtain $R_i = V_i U_i^T$. The scaling matrix S_i can be derived from the minimization of errors in edge transformation:

$$E(S_i) = \sum_{j \in N(i)} w_{ij} \|e_{ij}(n+1) - S_i R_i e_{ij}(n)\| \quad (30)$$

The scaling transformation can be simplified to isotropic scaling $S_i = \varsigma_i I$ such that scaling factor is derived by $\varsigma_i = \sum_{j \in N(i)} w_{ij} e_{ij}(n+1)^T R_i e_{ij}(n) / \sum_{j \in N(i)} w_{ij} e_{ij}(n)^T R_i^T R_i e_{ij}(n)$. The estimated local transformations are used in global optimization (26) to solve for the n -th acquisition.

6.2.5 Local Subdivision

A large mesh requires a longer computation time. To satisfy the real-time requirement of mesh deformation, we can employ a smaller mesh and perform adaptive local refinement where necessary. Local subdivision is used to refine the local connectivity and topology of the mesh in order to provide more degrees of freedom to represent the local curvature. Compared to global subdivision that doubles the number of vertices, local subdivision brings the advantage of a lower computation time for solving the minimization problem (26). For each contact s_j , a search is performed to locate the mesh vertex $p_{idx(j)}$ with the smallest Euclidean distance criterion.

$$idx(j) = \arg \min_{p_i \in \mathbf{P}} \|p_i - s_j\|^2 \quad (31)$$

A subdivision is applied locally around the nearest mesh vertex $p_{idx(j)}$. Among the methods of subdivision, the Loop subdivision [88] is selected for its simplicity and effectiveness. Referring to Fig. 35a and Fig. 35b before and after Loop subdivision, a new main vertex p_9 is created by subdividing an edge into two and additional four new side vertices are generated by further subdividing each of its two adjacent triangles into four triangles. Rather than performing the complete global subdivision, each of the edges adjacent to $p_{idx(i)}$ is subdivided to generate candidate vertices. Each contact is searched among these candidate vertices to locate the nearest vertex to the contact. If the nearest candidate vertex has a smaller Euclidean distance to s_i than $p_{idx(i)}$ to s_i , it is added to $\mathbf{P}$ and the mesh near the candidate vertex is refined according to local subdivision scheme.

The local subdivision scheme is illustrated in Fig. 35c. Instead of subdividing every triangle to 4 triangles to create vertices with valence of six like global subdivision in Fig. 35b, the four side vertices of local subdivision form T-like vertices with valence of five. T-like

vertices constrains the subdivision to be within second ring of triangles sharing adjacent edges. Due to the change in valence from six to five, the Loop subdivision rule should be modified to handle these newly generated side vertices p_A, p_B, p_C, p_D . The new side vertices are moved away from the centre of local subdivision p_9 , towards the adjacent outlier vertices p_5, p_6, p_7, p_8 to compensate for the asymmetry in Fig. 36c. Similar to the Loop subdivision (Fig. 36a), the local subdivision assigns a 3:1 weight ratio for the adjacent to non-adjacent vertices. The exceptions are the outlier vertices that are assigned larger weights than non-adjacent vertices to move the new side vertices towards the outlier and yet smaller weights than adjacent vertices to moderate the asymmetry. The weights $[\frac{1}{3} \ \frac{1}{3} \ \frac{2}{9} \ \frac{1}{9}]^T$ used in our work are determined heuristically.

Fig. 35: (a) Mesh before subdivision; (b) Mesh after Loop subdivision; (c) Mesh after local subdivision; Current and new vertices are indicated in black and white respectively; Current and new edges are indicated in black and grey respectively.

Referring to the Fig. 36d, the current vertex in the middle (black vertex with weight α_0) is moved away from the edges that are subdivided. Unlike Loop subdivision where all adjacent edges are subdivided (Fig. 36b), local subdivision introduces a mix of intact and subdivided edges. In Loop subdivision, the neighboring n vertices contribute equally to the new position of the current vertex with weights defined as $[1 - n\alpha \ \alpha \ \dots \ \alpha \ \dots]^T$. However, local subdivision does not affect all vertices or edges. Suppose m neighboring current edges are affected by local subdivision, the weights for the neighboring vertices may be modified to $[1 - (n - m)\alpha - m\alpha' \ \alpha \ \dots \ \alpha' \ \dots]^T$ where α and α' represent the weights for the adjacent and non-adjacent vertices. Higher weights are applied to adjacent vertices than non-adjacent vertices i.e. $\alpha > \alpha'$ as they affect directly the local normal and curvature of the mesh. $\frac{\alpha}{\alpha'} = 2$ is used in our work. Fig. 36 shows the modification of Loop subdivision computation for the new (Fig. 36c) and current vertices (Fig. 36d). Current vertices in the

neighborhood of the newly generated main vertex are re-adjusted. To control oversampling in a specific region, local subdivision is limited to generating a new edge with a minimum edge length equivalent to a fraction of typical edge length in the initial mesh. This fraction is set to $\frac{1}{4}$ in our work. With the local subdivided mesh and edge swapping process, the local connectivity and topology of mesh is improved. The Laplacian constraints of the new vertices are added while the Laplacian constraints of modified current vertices are updated. The Laplacian matrix weights are then re-computed for the entire mesh and the vertex positions for the $(n+1)$ -th acquisition are solved with the global optimization (26).

Fig. 36: Subdivision weights. Top: Loop subdivision for (a) new and (b) current vertices; Bottom: local subdivision for (c) new and (d) current vertices.

6.2.6 Edge Swapping

Edge swapping is used to improve the geometry of the mesh by changing the connectivity of the mesh. It is useful as a post-processing tool after local subdivision. With reference to Fig. 37, the edge e_{12} (comprising vertices p_1 and p_2) is flipped within the quadrilateral to form edge e_{03} (comprising vertices p_0 and p_3) if the swap criterion is satisfied. Several edge swap criteria are defined. A reasonably good criterion for this application is the minimization of surface area [89]. Defining A_i as the area of triangle T_i , the edge e_{ij} (comprising vertices

p_i and p_j) is flipped if and only if area sum of swap configuration is smaller than existing configuration i.e. $(A_0 + A_3) < (A_1 + A_2)$. Another good criterion for improving the triangulation is the minimization of local mean curvature [90]. The criterion suppresses the formation of local depression or protrusion and enables global smoothness. For each edge e_{ij} , the curvature is computed as the angle θ_{ij} between the normals of 2 adjacent triangles T_i and T_j . For simplicity, the dot product of the normals, $\cos \theta_{ij}$, is computed and an edge swap is performed if the swapped edge is locally Delaunay and the sum of the local inverse curvatures $\cos \theta_{ij}$ of edges associated with T_i and T_j is larger for swap configuration i.e. $(\cos \theta_{03} + \cos \theta_{43} + \cos \theta_{50} + \cos \theta_{63} + \cos \theta_{70}) > (\cos \theta_{12} + \cos \theta_{41} + \cos \theta_{51} + \cos \theta_{62} + \cos \theta_{72})$. An edge e_{ij} is locally Delaunay if the sum of the opposite angles α and β is not greater than π or the Laplacian cotangent weight $\frac{1}{2}(\cot \alpha + \cot \beta)$ associated with the edge is non-negative [91]. The Delaunay condition maximizes the minimal angle and pre-empts the formation of elongated triangles. The edge swapping process is simple and fast and terminates in a finite number of iterations to provide a better geometry.

Fig. 37: Edge swapping configurations.

The procedure for progressive surface reconstruction may be summarized by first generating a generic mesh model for approximating the heart chamber, aligning the mesh to heart chamber when the landmark points are acquired and then creating a first approximation. With any subsequent point acquisition, a search is performed to match the contact to the nearest mesh vertex and local subdivision is performed. This is followed by edge swapping with the area or mean curvature criterion and then computing the weighted Laplacian deformation for the entire mesh. The new positions for the set of mesh vertices are found by solving the constrained least square Laplacian difference. The process is repeated with every new contact acquisition and the heart surface is progressively reconstructed. With a smaller initial mesh, the computation requirements are satisfied while local subdivision and edge swapping are applied for fine tuning of the mesh.

6.3 Results and Discussion

The progressive surface reconstruction algorithm is implemented in C++ on a Intel Xeon system with dual 2.67GHz cores, 4GB RAM and a NVIDIA GeForce GTX 285 graphics card. The LAPACK [92] linear equation solver is preferred over iterative methods mainly due to speed reasons. Dynamic updates in iterative methods may be slower than linear solver and may not converge with an open top. A GPU accelerated solver (CULATM) is used in our work. A heart model [77] is first converted from a polygonal mesh into a voxel-based surface representation for collision detection (Section 3.2.2) and catheter deformation in an intra-cardiac intervention simulator (Chapter 5). To emulate the heart mapping procedure, contacts are sampled progressively on the endocardial surface of the LV in a random fashion. There is higher density of contacts near the apex and lower density near the base. Without prior knowledge of the LV geometry, the acquired contacts are used to reconstruct the surface progressively in accordance to prior art and the proposed algorithm.

Fig. 38: Progressive LV surface reconstruction comprising the rough and fine adjustment steps in Reisfeld method (excluding surface smoothing and affine transformation); Contacts are indicated in yellow; Vertices matched to contacts are indicated in red; Light green and light blue vertices are first and second ring neighborhood surrounding the matched vertices; Different snapshots are captured as N contacts are acquired.

Fig. 38 shows the progressive surface reconstruction of the LV using the method of rough and fine adjustment in the Reisfeld method. The green finely dotted outline in the background is the point representation of the dense LV mesh from which the contacts are sampled. It should be noted that the LV shape is not available during the mapping procedure and is shown here for comparison purposes. The mesh vertices matched to the contacts are indicated in red. Although the smoothing and affine transformation processes are excluded due to the lack of implementation details, the method shows large shape distortion with reference to the LV and local collapse in the region without the support of contacts.

Fig. 39 shows an improvement to the surface reconstruction in Reisfeld method by including the normalized local contact density for i -th vertex (Section 3.6). Although the global shape deformation is much improved, some visible distortion is observed near the apex for the fitting of the larger number of contacts. As the reconstruction is not recursive, consecutive addition of contacts results in a ripple transition effect.

Fig. 39: Progressive LV surface reconstruction with improved Reisfeld method for multiple contacts.

The new algorithm for progressive surface reconstruction strikes a balance in global and local smoothing. It generates a more visually pleasing shape that fits to the set of contacts and ensures a continuous transition between consecutive contact acquisitions with recursive shape deformation. From Fig. 40, the cotangent weighting scheme for Laplacian deformation

provides the minimum surface fitting error for the set of contacts among the uniform and inverse length weighting schemes on a 64x64 mesh. It can also be observed that the top part of reconstructed surface for uniform and inverse length weighting appears distended compared to mid and bottom parts where more contacts are concentrated. The performances of the edge swap area or curvature criterion are also compared on 16x16 initial meshes for clarity. The shape deformation appears smoother for mean curvature criterion than area criterion in Fig. 41. There are some local protrusions on the mesh with area swap condition but the overall shape for the mesh with mean curvature swap condition is smoother.

Fig. 40: Different weighting schemes for Laplacian deformation with a set of 65 contacts.

Fig. 41: Area and curvature minimization swap criteria on a 16x16 mesh.

Fig. 42 shows the progressively reconstructed LV using the new algorithm. As the contacts increase, the reconstructed surface approximates the LV better. Local adaptation is observed with each new contact with minimal change to the previous contacts. The global shape of the reconstructed surface remains largely consistent throughout the contact acquisition process. Fig 12 illustrates the step from Laplacian smoothing, local subdivision to edge swapping.

Initially, the mesh has a minor indentation with Laplacian smoothing shown in Fig. 43a. The mesh is then locally refined near the contacts (indicated in yellow) in Fig. 43b and the edges of the mesh are re-arranged for a more optimal shape representation in Fig. 43c.

Fig. 42: Progressive LV surface reconstruction with proposed algorithm for multiple contacts; Local regions with higher density of vertices are added by subdivision.

Fig. 43: Steps for progressive surface reconstruction with 65 contacts: Laplacian smoothing with cotangent weighting, Laplacian smoothing and local subdivision, Laplacian smoothing, local subdivision and edge swapping.

To measure the improvement, a distance error-fit metric is defined as the Euclidean distance square per reconstructed surface vertex from the corresponding nearest LV vertex. Let $\mathbf{Q}$ represent the set of n_q vertices of the dense LV mesh $\{q_i\}_{i=0}^{i=n_q}$ representing the surface where contacts are acquired. The distance error-fit metric is described by

$$E_{dist} = \sum_{i=0}^{n_p} \|p_i - q_{idx(i)}\|^2 / n_p \text{ where } idx(i) = \arg \min_{q_k \in \mathbf{Q}} \|p_i - q_k\|^2.$$
 Fig. 44 provides the distance error fit plot for a range of 4 to 65 contacts. The Reisfeld method has a square distance error above 20 mm² for less than 24 contacts whereas the improved Reisfeld method has a stabilized error of close to 8 mm² from the 25-th contact onwards. The co-tangent weighted Laplacian deformation method without local transformation generates steadily decreasing square distance error from 6 to 4.8 mm². The proposed algorithm has the lowest error from the fifth contact and achieves a minimum error of 3.2 mm² after 65 contacts. The square error distances of 3 to 6 mm² represent an error range of 2.2 to 3.2% along the shortest length of a mapped ventricle with dimensions 77x92x96 mm³.

To compare the local mesh characteristics of the reconstructed surface and dense LV mesh, a curvature error is defined as the angle between the normals of a reconstructed surface at a vertex and its corresponding nearest LV vertex. For a subset of vertices $\mathbf{P}_{50}$ with the top 50% curvature error, the curvature error-fit metric is described by

$$E_{curve} = \frac{2}{n_p} \sum_{p_i \in \mathbf{P}_{50}} \cos^{-1}(\hat{n}_{p_i} \cdot \hat{n}_{q_{idx(i)}})$$
 where $\hat{n}_{p_i}$ is the normal of the i -th vertex of the reconstructed surface and $\hat{n}_{q_{idx(i)}}$ is the normal of the LV vertex matching the i -th vertex. Fig.

45 shows the curvature error plot for the progressive reconstruction from 4 to 65 contacts. The proposed algorithm reconstructs the surface with a better curvature fit of 0.195, 0.040 and 0.036 ° to the dense LV mesh when comparing with Reisfeld, improved Reisfeld and Laplacian methods. Hence, the improvements for progressive surface reconstruction are evident both subjectively and objectively. The symmetric Hausdorff distances are computed between the dense LV mesh and final reconstructed surface using MESH routine [84] and the root mean square values are summarized in Table 4. The proposed algorithm presents the lowest symmetric Hausdorff distances among Reisfeld, improved Reisfeld, triharmonic radial basis function [35] and Laplacian methods. The symmetric distance is the maximum distance between distance from reference to reconstructed and from reconstructed to reference.

Fig. 44: Distance error fit of reconstructed LV surface for 4 to 65 contacts comparing Reisfeld method, improvised Reisfeld in section 3.3, Laplacian method and proposed algorithm.

Fig. 45: Curvature error fit of reconstructed LV surface for 4 to 65 contacts comparing Reisfeld method, improvised Reisfeld, Laplacian method and proposed algorithm.

Table 4: Root mean square Hausdorff surface distance [84] between the dense LV mesh and the reconstructed surface with the various methods; Sampling step = 0.5 and sampling frequency=14

Method	Reisfeld	Improved Reisfeld	Radial basis function[35]	Laplacian [85]	Proposed algorithm
Reference to reconstructed	7.341	4.927	4.291	4.557	4.354
Reconstructed to reference	3.487	2.741	7.638	2.840	2.255
Symmetric distance	7.341	4.927	7.638	4.557	4.354

Fig. 46 shows the results for 3D reconstruction of 2 different left ventricles. The leftmost column shows the LV surface from which the contacts are acquired. The proposed algorithm has the closest resemblance to the original LV with least distortion among the methods compared. Table 5 summarizes the total time taken for reconstructing the LV surface 64 and 80 times during the acquisition of 65 and 81 contacts. The time taken per reconstruction has an average of 221 ms and 235 ms, indicating that the reconstruction satisfy real-time requirements.

Fig. 46: Comparison of 2 left ventricles and corresponding 3D reconstructions with Reisfeld, improved Reisfeld and proposed algorithm. LV1 and LV2 have 65 and 81 acquired contacts.

Table 5: Performance data for progressive reconstruction of 2 left ventricles with 65 and 81 contacts; Data represents an average over 10 sets of reconstructions.

	LV1 (65 contacts)	LV2 (81 contacts)
Total time (ms)	14160	18814
Time per reconstruction (ms)	221	235
Final reconstructed mesh		
Triangles (no.)	1155	1265
Vertices (no.)	2276	2946

This method of surface reconstruction can be used to map the atria for electrophysiology study of arrhythmias. Besides the LV, it can be applied to the RV for similar ischemia diagnosis. The method can be used for offline 3D reconstruction of organs from imaging modalities. An example is the recovery of dynamic LV motion from tMRI. As the tag pattern in tMRI reduces the overall contrast of the image, the segmentation of cardiac tissue becomes more difficult. The clinical laboratory may only be able to identify limited tissue contours and representative point set that tracks LV motion. The proposed algorithm can reconstruct the 3D model from these sparse datasets and track the LV motion through the deformation of the reconstructed surface.

6.4 Summary

A progressive surface reconstruction algorithm comprising the components of mesh alignment and approximation, weighted Laplacian deformation, local subdivision and edge swapping has been presented for the heart mapping procedure. The algorithm is capable of generating a natural and smooth shape having a close fit to the set of contacts. Unlike prior art, it does not suffer from high distortion or regional collapse under conditions of few contacts and uneven distribution of contacts. Being recursive in nature, the reconstructed surface is progressively adapted to increasing contacts with a smooth variation and continuity. As an example, the chapter has described the mapping and reconstruction of LV. Some adjustments to the recursive linear system should be made to enable more responsive shape deformation to a contact acquired away from the area of high sampling. The contractile (or electrical) properties are composited on the reconstructed geometry to enhance the diagnostic capability of the system (Chapter 8).

Chapter 7 A B-SPLINE APPROACH TO PHASE UNWRAPPING IN CARDIAC TAGGED MRI FOR MOTION TRACKING

A novel B-Spline based approach to phase unwrapping in cardiac tMRI is proposed for motion tracking. A bicubic B-Spline surface is used to model the absolute phase. The phase unwrapping problem is formulated as a mixed integer optimization problem that minimizes the sum of the difference between the spatial gradients of absolute and wrapped phases and the difference between the re-wrapped and wrapped phases. In contrast to the existing techniques for motion tracking, the proposed approach can overcome the limitation of inter-frame half-tag displacement and increase the robustness of motion tracking. The chapter further presents a hybrid HARP B-Spline method to take the advantage of the HARP method for small motion and the efficiency of the B-Spline approach for large motion. The proposed approach has been successively applied to a full set of cardiac MRI scans in both long and short axis slices with superior performance when compared with the HARP and quality guided path following methods.

7.1 Outline

Standard non-invasive medical imaging techniques including MRI and computed tomography can identify cardiac landmarks and atrial and ventricular boundaries but they cannot be used to track tissue motion due to lack of transmural variation and differentiable features in the heart tissue. tMRI allows the motion of cardiac tissue to be tracked using the deformation of tagging patterns.

7.1.1 Tagged MRI

Tagging uses pulse sequences similar to standard MRI to induce longitudinal spatial variation in the magnetization of tissue. One of the more commonly used sequences in tagging is spatial modulation of magnetization (SPAMM). In SPAMM, a series of radiofrequency (RF) pulses first induces transverse magnetization. Then a gradient pulse produces periodic spatial modulation along direction of gradient. Finally, a crusher of RF pulses inverts the modulated transverse magnetization back to vertical. The SPAMM sequences produce a set of parallel tagging planes orthogonal to the image plane and the tagging planes intersect the image plane with parallel lines. Fig. 47 shows the initial and

deformed orthogonal tag planes with the image plane at the tag application and some time after. The initial and deformed tag planes forms straight tag lines and curves on the image plane respectively. As the tagging pattern is induced in the tissue once during the time of application, any visible distortions in tagging pattern during subsequent image acquisitions is a result of tissue motion. The tag pattern provides a means for tracking the correlated motion between different time slices within the same slice. However, the tag pattern reduces the contrast needed to identify tissue boundaries and causes blurring on untagged area. The tag pattern also fades rapidly with time as signal to noise ratio decreases with the time of acquisition in the later cardiac phases. Generally, tagging provides a good alternative for tracking myocardial motion as opposed to invasive radiopaque markers or sonomicrometers. tMRI has been used as a research tool for LV motion and strain analysis to better understand myocardial motion abnormalities in cardiovascular diseases.

Fig. 47: Tag plane deformation and apparent motion. Figure extracted from [44].

7.1.2 Planar Motion Tracking

To reconstruct a moving 3D heart model from patient specific tMRI data, we first need to compute the 2D motion accurately on tMRI slices from orthogonal imaging planes. Among the tracking algorithms for tMRI, tagged line tracking methods have a disadvantage of sparse motion tracking while the robustness of optical flow methods is affected by tag line decay and lack of differential features between tag lines. Phase tracking (HARP) [45] is more suitable for dense motion tracking in tMRI due to its phase invariant material property. However, phase tracking suffer from the theoretical limitation of half-tag displacement like the other tracking algorithms. It is unable to detect phase displacement greater than π or lesser than $-\pi$ due to the periodicity of the magnetization field. As the temporal resolution and tag distance is limited, we need a new algorithm to track the larger than half-tag distance

motion along the long axis. To overcome the half-tag displacement limitation, we perform the motion search on unwrapped phase instead than wrapped phase.

Compared to other phase unwrapping applications, tMRI may have better image quality with less noise and aliasing. However, phase unwrapping in tMRI is complex in nature. It has unique discontinuities arising from myocardial motion, non-tissue regions, myocardium boundaries and tag fading. The moving myocardium may be disjoint from immediate near-static tissue regions where the smoothness of phase unwrapping is disrupted. The myocardium may be surrounded by non-tissue regions that have high uncertainty in phase. There may be temporally short appearance of untagged blood vessels of high image amplitude due to through-plane motion that affect phase unwrapping process. Unlike typical phase unwrapping problems, tMRI motion tracking specifically requires phase consistencies between consecutive cardiac frames so that motion tracking can be performed reliably.

In this chapter, we propose a B-Spline based unwrapped phase tracking approach for robust phase unwrapping in tMRI motion tracking. Our approach overcome the half-tag displacement limitation in HARP and ensures spatial and temporal consistencies with minimal user intervention. We use a bicubic B-Spline surface for a continuous approximation of the absolute phase which is appropriate since the tag lines are continuous at initial application of the tagging. It is important to distinguish our approach from prior arts that use B-Splines to model the deformation of tag planes or tag lines [72] rather than to approximate the absolute phases in tMRI.

7.2 Methods

7.2.1 Imaging Protocol

The approach is validated on a set of human heart data (Fig. 49) from a healthy volunteer imaged with Siemens Magnetom TrioTim 3T scanner using a CINE breath-hold protocol. Continuous slices of the LV are imaged 8 mm apart in three orthogonal planes consisting of 13 short axis (SA), 9 horizontal LA (HLA) and 8 vertical LA (VLA) slices. Two orthogonal sets of tag planes at 45° and 135° to the horizontal were applied in the SA slice while one set was applied horizontally and vertically in the HLA and VLA slice. Each slice was acquired using segmented k-space during suspended respiration for a temporal resolution of 27 cardiac frames in 12 consecutive beats. The tagging signal was applied at end-diastole. Other imaging parameters were: field of view 249.6mm (frequency) and 195mm (phase),

acquisition-matrix 192x150, pixel size 1.3x1.3mm², inter-tag distance 8mm, pulse repetition time 37.85ms, echo time 3.52ms and flip angle 10°.

Prior to this imaging, we had a first tMRI scan with an inter-tag distance of 4 mm for denser tag lines. Table 6 captures the differences in imaging parameters of the scans. In the first scan, the tagging pattern fades rapidly during high motion and it affected the visibility and the motion tracking in early systole. The tag period barely meets the Nyquist sampling period and the phase variances are too coarse for motion search. The shorter inter-tag distance limits the phase tracking performance for larger motion. The current scan improves over the first scan in terms of image quality and tag line visibility beyond systole. The scan uses orthogonal planes rather than cardiac planes, providing a better reference for 3D reconstruction. Similar to the first scan, the image resolution remains low and tissue boundaries have low visibility. The only problem is that LV wall is thin with barely one tag period spanning the wall thickness. The cardiac scan layout in Fig. 48 illustrates the relationship between the imaging planes. Only the LV is fully covered by the scan.

Table 6: Cardiac tMRI scan imaging parameters and comparison.

Image quality	Fading during systole	Good with some fading
Slice Type	2C,4C,SA	SA,HLA,VLA
Manufacturer/Model	Siemens TrioTim	Siemens TrioTim
Software version	syngo MR B15	syngo MR B15
Scanning sequence	gradient recalled	gradient recalled
Sequence variant	segmented k-space, spoiled	segmented k-space, spoiled, oversampling
Scan options	G4/CT	G8/CT
MR acquisition type	2D	2D
Slice thickness	5 mm	8 mm
Repetition time	27.60 ms	37.85 ms
Echo time	2.71 ms	3.52 ms
No of averages	1	1
Imaging frequency	123.26 MHz	123.26 MHz
Imaged Nucleus	1H	1H
Echo numbers	1	1
Magnetic field strength	3T	3T
Number of phase encoding steps	160	153
Echo train length	1	1
Percent sampling	80%	100%
Percent phase field of view	78.13%	78.13%
Pixel bandwidth	455	230
Number of cardiac phases	26	27
Transmitting coil	Body	Body

Acquisition matrix	Frequency column: 250 Phase row: 160	Frequency row: 192 Phase column: 150
Phase encoding direction	ROW	COL
Flip angle	12 °	10 °
SAR	0.19 watts/kg	0.11 watts/kg
Patient position	head first supine	head first supine
Row	256	150
Columns	200	192
Pixel spacing	1.25/1.25 mm	1.302/1.302 mm
Bits allocated/stored	16 bits/12 bits	16 bits/12 bits
Nominal interval	779 ms	1131 ms
Heart rate	77 cycles/min	53 cycles/min
Inter-Tag distance	4 mm	8 mm
Inter-Slice distance	10 mm	8 mm

Fig. 48: Cardiac MRI layout for SA, HLA and VLA planes and their relationship; Numbering indicates the slice number in the scan.

7.2.2 Phase Tracking

Referring to Fig. 49, the images are first transformed from spatial domain to frequency domain using Fast Fourier Transform (FFT). A bandpass filter, designed to trade-off accuracy with artifacts, is used to isolate the l -th spectral peak centered at the lowest harmonics frequency ω_l . Each set of filtered data is inverse Fourier transformed to yield complex images in which amplitude and phase images are extracted. In our work, we propose to perform motion tracking on the absolute phase instead of the wrapped phase. A phase unwrapping process is introduced to generate unwrapped phase image (UWPI) instead of wrapped phase image (WPI) prior to motion search. The difference of our approach from wrapped phase based approaches (such as HARP) is shown in Fig. 50. The approach can overcome the half-tag displacement limitation while ensuring a more robust inter-frame consistency than the QGPF method [60]. In the following, we present our B-Spline approach for unwrapped phase tracking first and then present a hybrid method that combines our B-Spline approach with the HARP method.

Fig. 49: Fourier transformed (FFT), filtered FFT, magnitude and phase images from sample SA, HLA and VLA slices.

Fig. 50: Comparison of process flow for unwrapped and wrapped phase tracking methods.

7.2.3 B-Spline Unwrapped Phase Tracking

Let $\phi(x, y)$ and $\alpha(x, y)$ represent the absolute and wrapped phases of the signal at position (x, y) in an MRI image and $\gamma(x, y)$ represent the amplitude. $\phi(x, y)$ and $\alpha(x, y)$ are related by k integer multiples of 2π :

$$\phi(x, y) = 2\pi k(x, y) + \alpha(x, y) \quad (32)$$

The objective of phase unwrapping is to find $\phi(x, y)$ from $\alpha(x, y)$. It involves two important components: (i) a model for $\phi(x, y)$ and (ii) an algorithm for recovering $\phi(x, y)$ based on the model.

Here, a bicubic B-Spline surface is proposed to model the absolute phase. Bicubic B-Spline surfaces are piecewise polynomial and have C^2 continuity [93][94]. They have been widely used in geometric modelling and engineering computation. Compared to polynomial surfaces, B-Splines are more flexible since the number of the control coefficients (i.e., de-Boor points) is independent of the degree of the B-Splines. Thus with bicubic B-Splines we can have as many control coefficients as we need to model complicated behaviours of the phase within the tissue region while maintaining the smoothness of the phase. Another advantage of using B-Splines for phase is that the B-Spline model can be reconstructed from partial points in the tissue region and this is especially suitable for the case where the phase in some small regions is very inaccurate or missing.

First we identify a region of interest R of dimensions $w_R \times h_R$, sufficiently large to envelop myocardial motion. Without loss of generality, we assume that the upper-left corner of R has coordinates $(0, 0)$. Then two uniform knot vectors $X : \{x_i\}_0^{n+4} = \{-3, -2, \dots, n, n+1\} \cdot \delta_x$

and $Y: \{y_i\}_0^{m+4} = \{-3, -2, \dots, m, m+1\} \cdot \delta_y$ are generated in x and y directions at intervals $\delta_x = w_R/(n-2)$ and $\delta_y = h_R/(m-2)$ for given integers m and n . In our work, we use $m = n = 18$ for regions of size 56x60, 90x52 and 62x90 in cropped 128x128 images. We define the absolute phase ϕ as:

$$\phi(x, y) \equiv \sum_{j=0}^m \sum_{i=0}^n P_{ij} N_i^3(x) N_j^3(y), \quad x \in [x_3, x_{n+1}] , \quad y \in [y_3, y_{m+1}] \quad (33)$$

where P_{ij} are the de-Boor points to be determined and $N_i^3(x)$ and $N_j^3(y)$ are the cubic B-Spline basis functions over the uniform knot vectors X and Y . $N_i^3(x)$ is defined by:

$$N_i^3(x) = \begin{cases} \frac{x-x_i}{x_{i+3}-x_i} \cdot \frac{x-x_{i+1}}{x_{i+2}-x_{i+1}} \cdot \frac{x-x_{i+2}}{x_{i+3}-x_{i+2}} & x \in [x_i, x_{i+1}] \\ \frac{x-x_i}{x_{i+3}-x_i} \cdot \left(\frac{x-x_{i+1}}{x_{i+2}-x_{i+1}} \cdot \frac{x_{i+2}-x}{x_{i+3}-x_{i+2}} + \frac{x-x_{i+1}}{x_{i+2}-x_{i+1}} \cdot \frac{x_{i+3}-x}{x_{i+4}-x_{i+3}} \right) & x \in [x_{i+1}, x_{i+2}] \\ \frac{x-x_i}{x_{i+3}-x_i} \cdot \frac{x_{i+3}-x}{x_{i+4}-x_{i+3}} + \frac{x_{i+4}-x}{x_{i+5}-x_{i+4}} \cdot \left(\frac{x-x_{i+1}}{x_{i+2}-x_{i+1}} \cdot \frac{x_{i+2}-x}{x_{i+3}-x_{i+2}} + \frac{x-x_{i+1}}{x_{i+2}-x_{i+1}} \cdot \frac{x_{i+3}-x}{x_{i+4}-x_{i+3}} \right) & x \in [x_{i+2}, x_{i+3}] \\ \frac{x_{i+4}-x}{x_{i+5}-x_{i+4}} \cdot \frac{x_{i+5}-x}{x_{i+6}-x_{i+5}} & x \in [x_{i+3}, x_{i+4}] \\ 0 & x \in [-\infty, x_i] \cup [x_{i+4}, +\infty] \end{cases} \quad (34)$$

$N_j^3(y)$ is defined similarly.

Thus, the absolute phase ϕ is completely determined by de-Boor points P_{ij} . We now find P_{ij} and wrapped count $k(x, y)$ by minimizing the following objective function:

$$\begin{aligned} & \sum_{(x,y) \in R} \omega_1(x, y) \|\nabla \phi(x, y) - \nabla^* \alpha(x, y)\|^2 \\ & + \sum_{(x,y) \in R \setminus Anchor} \omega_2(x, y) \|(\phi(x, y) - 2\pi k(x, y)) - \alpha(x, y)\|^2 \\ & + \sum_{(x,y) \in Anchor} \omega_A(x, y) \|\phi(x, y) - \phi_0(x, y)\|^2 \end{aligned} \quad (35)$$

where *Anchor* is the set of anchor points, ϕ_0 is the true absolute phase of the anchor points, ∇ is a spatial gradient operator with respect to the position of the material point, and ∇^* is defined by:

$$\nabla^* \alpha(x, y) = \begin{cases} \nabla \alpha & \|\nabla \alpha\| < \|\nabla W(\alpha + \pi)\| \\ \nabla W(\alpha + \pi) & \text{otherwise} \end{cases} \quad (36)$$

with the wrapping function [44]:

$$W(\phi) = ((\phi + \pi) \% 2\pi) - \pi \quad (37)$$

The first term of objective function (35) measures the difference of the absolute and wrapped phase gradients over region R in a weighted L^2 norm. The wrapped phase gradient is derived from the wrapped phase images (7). The absolute phase gradient is:

$$\nabla \phi(x, y) = \left(\frac{\partial \phi(x, y)}{\partial x}, \frac{\partial \phi(x, y)}{\partial y} \right) \quad (38)$$

where the partial derivative of the absolute phase with respect to x is:

$$\frac{\partial \phi(x, y)}{\partial x} = \sum_{j=0}^m \sum_{i=0}^n P_{ij} N_j^3(y) \left(\frac{3}{x_{i+3}-x_i} N_i^2(x) + \frac{3}{x_{i+3+1}-x_{i+1}} N_{i+1}^2(x) \right) \quad (39)$$

with the quadratic B-Spline basis function:

$$N_i^2(x) = \begin{cases} \frac{x-x_i}{x_{i+2}-x_i} \cdot \frac{x-x_{i+1}}{x_{i+1}-x_i} & x \in [x_i, x_{i+1}] \\ \frac{x-x_i}{x_{i+2}-x_i} \cdot \frac{x_{i+2}-x}{x_{i+2}-x_{i+1}} + \frac{x-x_{i+1}}{x_{i+2}-x_{i+1}} \cdot \frac{x_{i+3}-x}{x_{i+3}-x_{i+1}} & x \in [x_{i+1}, x_{i+2}] \\ \frac{x_{i+3}-x}{x_{i+3}-x_{i+1}} \cdot \frac{x_{i+3}-x}{x_{i+3}-x_{i+2}} & x \in [x_{i+2}, x_{i+3}] \\ 0 & x \in [-\infty, x_i] \cup [x_{i+3}, +\infty] \end{cases} \quad (40)$$

and $\frac{\partial \phi(x, y)}{\partial y}$ can be derived similarly. The weighting function ω_1 is introduced to

emphasize the tissue regions with continuous phase and de-emphasize the non-tissue regions with discontinuity. ω_1 is dependent of the amplitude $\gamma(x, y)$ and corresponds to good quality tag lines on the myocardium. Unlike traditional amplitude weighting, we use a cumulative density function g that de-couples the amplitude from the limitations of tag fading. It has a similar effect of histogram contrast stretching in the spatial domain. The weighting function ω_1 is chosen as:

$$\omega_1(x, y) = \exp(h \times g(\gamma(x, y))) \quad (41)$$

where h is an integer (we choose $h = 8$ in our work). The amplitudes $\gamma_1(x, y), \gamma_2(x, y)$ of two grid directions are combined for consistent myocardium identification for tMRI with 2D grid:

$$\omega_1(x, y) = \exp(h \times g(\max(\gamma_1(x, y), \gamma_2(x, y)))) \quad (42)$$

The second term of (35) seeks to minimize the difference between the re-wrapped and wrapped phases to provide better approximation to the absolute phase. The re-wrapped phase refers to the residual component of the absolute phase mod 2π operation and lies in the principal range. Integer k is the unknown phase period or wrapped count. The weighting

function ω_2 is distributed to bias towards smaller range errors to minimize the gap between the true and estimated wrapped counts. To this end, ω_2 is chosen as:

$$\omega_2(x, y) = \exp(h \times (1 - |W(\phi(x, y)) - \alpha(x, y)| / \pi)) \quad (43)$$

As the first and second terms of (35) do not provide a unique solution, the third term that measures the difference of the true and estimated absolute phase at a few anchor points introduced as boundary conditions. The difference is scaled by a maximum weight $\omega_A(x, y)$. We choose $\omega_A(x, y) = \exp(h)$. The inclusion of the anchor points ensures inter-frame consistency and assists in the unwrapping process for cardiac frames after systole and boundary slices where the image quality is poor. The anchor points are selected once for each slice, having a displacement range of less than π in the entire cardiac cycle where the displacement refers to the temporal difference between the wrapped phases in two different frames. The phase of the anchor points has to stay within the principal range $[-\pi, \pi)$ so that the true absolute phase ϕ_0 of the anchor points at position (x_A, y_A) can be computed. The true absolute phase ϕ_0 is described by:

$$\phi_0(x_A, y_A, t) = 2\pi k(x_A, y_A) + \alpha(x_A, y_A, t) \quad (44)$$

Without causing ambiguity, we add the time parameter t explicitly in ϕ_0 and α to highlight time-dependent frames. Wrapped phase $\alpha(x_A, y_A, t)$ can be extracted from its position at different cardiac frames while wrapped count $k(x_A, y_A)$ is independent of the cardiac frame and is determined from its position in the initial undeformed cardiac frame. A simple parser can compute the temporal phase difference between consecutive frames and presents the candidate anchor points that have inter-frame displacements of less than π in a cardiac cycle. From the composite images shown in Fig. 51, one to four anchor points can be selected to be located on the moving myocardium, to minimize inter-frame displacement and to maximize its distribution.

Fig. 51: Region for reference point selection overlayed with original MRI image in (a,b) SA, (c) HLA and (d) VLA slices; Whiter parts indicate smaller displacement and candidates for anchor points.

The minimization of (35) is a *mixed integer* optimization problem where de-Boor points P_{ij} are floating points and wrapped count k is integer. Instead of resorting to sophisticated mixed integer minimization solvers, here we present a simple iterative approach, which consists of the following steps:

Step 1: ω_2 is initialized to be 0. The minimization of (35) is a typical least-squares problem whose solution gives an initial approximation to the unwrapped phase ϕ .

Step 2: Using the approximate ϕ , we estimate the wrapped count $k(x, y) = (\phi(x, y) - \alpha(x, y) \pm \pi) / 2\pi$ where $\pm\pi$ is a rounding value following the sign of the difference.

Step 3: Now we set ω_2 according to (8). Fixing $k(x, y)$, we solve the minimization of (35) for de-Boor points, which is also a least-squares problem. If the maximum iteration number is reached or a satisfactory solution is obtained, the process stops; Otherwise, it goes to step 2.

The motion search is performed on the two derived unwrapped phase images to determine the temporal correlation. Consider a material point p_m at frame t_m and its corresponding point p_{m+1} at frame t_{m+1} . We have $\phi(p_{m+1}, t_{m+1}) = \phi(p_m, t_m)$. p_{m+1} can be found by numerically solving $\phi(p, t_{m+1}) = \phi(p_m, t_m)$ using the Newton-Raphson approach [44]:

$$p^{(n+1)} = p^{(n)} - [\nabla \phi(p^{(n)}, t_{m+1})]^{-1} [\phi(p^{(n)}, t_{m+1}) - \phi(p_m, t_m)] \quad (45)$$

where the superscript η refers to the iteration number. The iteration terminates when the temporal difference $\phi(p^{(n+1)}, t_{m+1}) - \phi(p_m, t_m)$ is close to zero. Then the material point is

$p_{m+1} = p^{(n+1)}$ and the motion vector is $v_{m+1} = p_{m+1} - p_m$. Compared to the wrapped motion search in HARP, the true spatial gradient and temporal difference are used instead of their estimate.

7.2.4 Hybrid HARP B-Spline Phase Tracking

The performance of the B-Spline approach outperforms the HARP method in LA planes where the motion along the axis is large but it has less accurate performance where phase image is affected by frequency aliasing or has inconsistent phase with corresponding tag lines across the cardiac wall. In such cases, the wrapped motion search may produce more consistent results than the unwrapped motion search. Thus, we propose a hybrid phase tracking method that can combine the advantages of both the HARP method and the B-Spline phase unwrapping method for motion tracking by switching between the HARP and B-Spline methods at pixel level.

During the evaluation of the HARP method, we found two potential improvements. One concerns the reduction of tag jumping frequency by adding an upscaling process (Fig. 50b) before the Fourier Transform. The pre-FFT upscaling by finite response interpolation filters increases the spatial resolution while maintaining the same frequency resolution. Since the increased spatial resolution provides less discretized and smoother phase images with reduced phase inconsistencies, motion tracking is improved. A factor of 4 is chosen to balance between the performance gain and the computation complexity. In order for the B-Spline approach to match the spatial resolution for the improvised HARP method, the unwrapped phase image is upscaled by the B-Spline interpolation (Fig. 50a) with the de-Boor points derived from the phase unwrapping process.

The other aspect relates to the motion search strategy. Conventional search strategy performs motion search between consecutive cardiac frames. The cardiac wall motion is tracked from the initial frame onwards. As this strategy uses the motion tracking results from the previous search, any inaccuracies from the previous search is propagated to the next. Low image quality associated with tag line fading affects the search results negatively. For short temporal searches, the conventional search may be satisfactory but the results for tracking entire cardiac cycle are unacceptable. We revise the motion search strategy by using the cardiac frame at initial tag application as a reference to search with each new cardiac frame (Fig. 52). This reference-based search strategy reduces the effect of low image quality of reference images and removes the error propagation from the previous searches. At each

cardiac frame, forward motion search is performed on the current cardiac image to derive the motion vectors that match the pixel phase values in the initial reference image. Since the same reference frame is used, the coherence between the searches is guaranteed. The motion vectors from the previous search are used as initial seeds to the current search to ensure the search starts within the tracked phase period.

Fig. 52: Motion search with the initial frame as a reference search frame; Motion vectors (MV) derived from each frame compensates the initial marker positions to generate an output frame.

Fig. 53: Hybrid HARP-B-Spline phase tracking method; Final motion is selected from wrapped and unwrapped motion search using a motion range detector and a phase alignment error detector; Wrapped phase image (WPI) and unwrapped phase image (UWPI) are inputs to the motion search.

In our proposed hybrid phase tracking method, the B-Spline phase unwrapping complements HARP in image regions where the performance of HARP is poor when the assumption of a half-tag displacement does not hold. The process is shown in Fig. 53, where unwrap motion

search and wrap motion search are performed by B-Spline and HARP methods and two pixel-level performance metrics are used to decide the output of the search.

A predictor MR for the performance of HARP at pixel level is derived from the motion range detector as:

$$MR = \begin{cases} \frac{\partial^2}{\partial t^2} \alpha, & \left\| \frac{\partial}{\partial t} \alpha \right\| < \left\| \frac{\partial}{\partial t} W(\alpha + \pi) \right\| \\ \frac{\partial^2}{\partial t^2} W(\alpha + \pi), & otherwise \end{cases} \quad (46)$$

When the half-tag displacement limitation is violated, there is an immediate reversal of temporal derivative. Thus higher detected motion range implies a higher risk of poor performance of HARP. The phase alignment error detector, on the other hand, provides an indicator PA of the performance of the B-Spline method. It computes the phase alignment errors of the unwrapped and wrapped phases using:

$$PA = (\phi - \alpha \pm \pi) \% 2\pi \quad (47)$$

A lower phase alignment error implies higher probability of good performance of the B-Spline method. A binary decision between HARP and B-Spline method is made at pixel level using the two performance indicators. The maximum of each indicator is taken over a 3x3 overlapping spatial processing window for increased robustness. A fading region between the motion range and phase alignment error detectors is designed to switch between the performances of the HARP (Wrap) and B-Spline (Unwrap) methods. T_1 and T_2 are the thresholds for higher confidence of good and poor performance of HARP (we heuristically choose $T_1 = 3\pi/8, T_2 = 5\pi/8$). With the hybrid phase tracking method, we take advantage of the maturity of the HARP method and the larger motion tracking capability of the B-Spline approach while not compromising the robustness of the tracking.

7.3 Results

The HARP process is implemented according to the details given in literature [44]. A circular bandpass filter [45], centered on the lowest harmonic peak with a normalized peak-to-peak radius of 0.4 and a deviation $\sigma=0.1$, is used to isolate the frequency spectral of the tag direction for phase image generation. The motion search for the LA slices is interpreted from the 2D motion search algorithm from HARP for the SA slices where the 2D spatial

gradient is projected along the axis perpendicular to the tag line and the search space is limited to 1D instead of 2D.

Fig. 54 demonstrates the forward tracking of markers at initial frame 3 by original HARP and the improvements by higher resolution tracking and reference-based motion search strategy. It should be noted that the motion tracking is dense and that the markers are purely for illustration purposes. All markers in original HARP method are shifted right downwards after systole (frame 12). Both higher resolution and reference-based search shows better tracking than HARP except for one marker deviation (frame 10) while their combination provides the best tracked markers. All the comparison of HARP that follows in subsequent description refers to HARP with reference-based search at 4x resolution.

Fig. 54: Motion tracking of markers (white) from frame 6 (left) to frame 14 (right) with (a) HARP, (b) HARP and upscaling, (c) HARP and upscaling, (d) HARP, upscaling and reference based search; Higher resolution search results are downsampled for comparison purposes; Arrows highlight shift in marker position relative to tagging grid.

Fig. 54: Motion tracking of markers (white) from frame 14 (6, 8, 10, 12, 14) with (a) HARP, (b) HARP and upscaling, (c) HARP and reference-based search (d) HARP, upscaling and reference based search; Higher resolution search results are downscaled for comparison purposes; Arrows highlight shift in marker position relative to tagging grid.

Fig. 55 highlights the tag line tracking performance difference by the conventional and reference-based motion search in higher resolution. The tag lines are first detected at $\text{phase}=\pi$ in initial frame 3 and used for motion tracking. Acceptable tracking is seen for shorter temporal tracking from end diastole to systole at frame 11 (Fig. 55b). Due to error propagation between successive frame tracking and poor references, the markers become increasingly fuzzy and the tracking results are unacceptable towards the end of cardiac cycle (Fig. 55c). However with reference-based motion search, there is little loss at systole (Fig. 55d) and the quality of tracked tag lines at end diastole (Fig. 55e) is better.

Fig. 55: Motion search comparison in continuous tracking of (a) detected tag lines at $\text{phase}=\pi$ from frame 3; (b-c) Conventional motion search in frames 11 and 27; (d-e) Reference-based motion search in frames 11 and 27.

Using the B-Spline approach, the wrapped phase images of cropped images are unwrapped and the reference-based motion search is performed. The unwrapping computation is accelerated with a GPU-based linear algebra solver package for example CULA. The HARP and B-Spline methods are compared for their motion tracking efficiency. A few clusters of markers, each covers a 5×5 pixel-square in $4 \times$ resolution, are initially inserted in frame 3 and their positions are tracked through the cardiac cycle. Fig. 56 shows some horizontal motion tracking results from the two methods in mid HLA slices for a small period around systole from frame 12 to 15. As evident in their relative tag lines, the HARP method (Fig. 56a)

shows some abnormal tracking of the markers. Tag jumping starts from frame 13 to 15 and is indicated by the solid head arrows pointing antagonistic to the motion direction. The corresponding motion plots of clusters 8 and 9 show strong rightward horizontal motion (28×0.325 mm) in the HARP method compared to other methods (5×0.325 mm). The split of the cluster 1 in frame 15 is highlighted by the dashed circle and large standard deviation. On the other hand, successful motion tracking of markers using the B-Spline approach can be observed in Fig. 56b. The corresponding plots of forward displacement vectors during systole where the LV contracts (from frame 12 to frame 13) then expands (from frame 14 to frame 15) are shown in Fig. 57. It can be observed that the tracking by HARP goes in the opposite direction (right) to LV motion (left) in frame 14 while and the tracking by B-Spline has most vectors in similar direction as LV motion.

Fig. 58 provides further tag jumping examples for vertical motion tracking results with the HARP method in mid VLA slices. In frame 14, there is downward movement of clusters 1 (35×0.325 mm) and 3 (20×0.325 mm) in the HARP method while the marker clusters in the B-Spline approach moves upwards with the heart wall expands upwards. From the corresponding plots in Fig. 59, collective downward displacements for HARP and upward displacements for B-Spline can be observed for the upward expansion of the LV in frame 14. For illustration purposes, the displacement vectors are drawn only for amplitude segmented region in center of the image to a scale of 0.25 of the actual displacement.

Fig. 56: Horizontal motion tracking of marker clusters (white) in mid HLA slices from frame 12 (left) to 15 (right) using (a) HARP, (b) B-Spline, (c) hybrid HARP B-Spline, and (d) QGPF methods; Right-pointed arrows highlight shift in marker position relative to tagging grid and in opposite direction to general leftward motion while dash circle highlights cluster splits; (e) Plots of horizontal motion (average and standard deviation in 0.325 mm units, rightward positive) of marker clusters (numbered from 1 in clockwise direction) with respect to initial position in frame 3.

Fig. 56: Horizontal motion tracking of marker clusters (white) in mid HLA slices from frame 12 (left) to 15 (right) using (a) HARP, (b) B-Spline, (c) hybrid HARP B-Spline, and (d) QGPF methods; Right-pointed arrows highlight shift in marker position relative to tagging grid and in opposite direction to general leftward motion while dash circle highlights cluster splits; (e) Plots of horizontal motion (average and standard deviation in 0.325 mm units, rightward positive) of marker clusters (numbered from 1 in clockwise direction) with respect to initial position in frame 3.

Fig. 56: Horizontal motion tracking of marker clusters (white) in mid HLA slices from frame 12 (left) to 15 (right) using (a) HARP, (b) B-Spline, (c) hybrid HARP B-Spline, and (d) QGPF methods; Right-pointed arrows highlight shift in marker position relative to tagging grid and in opposite direction to general leftward motion while dash circle highlights cluster splits; (e) Plots of horizontal motion (average and standard deviation in 0.325 mm units, rightward positive) of marker clusters (numbered from 1 in clockwise direction) with respect to initial position in frame 3.

Fig. 57: Horizontal forward displacement vectors (white lines from black dots) for mid HLA slices from frame 12 (left) to frame 14 (right) using (a) HARP and (b) B-Spline methods. Bottom left of image enlarged from box region (blue).

Fig. 58: Vertical motion tracking of marker clusters (white) in mid VLA slices from frame 13 (left) to 15 (right) using (a) HARP, (b) B-Spline, (c) hybrid HARP B-Spline, and (d) QGPF methods; Down-pointed arrows highlight shift in marker position relative to tagging grid and in opposite direction to general upward motion; (e) Plots of vertical motion (average and standard deviation in 0.325 mm units, downward positive) of marker clusters (numbered from 1 in clockwise direction) with respect to initial position in frame 3.

Fig. 58: Vertical motion tracking of marker clusters (white) in mid VLA slices from frame 13 (left) to 15 (right) using (a) HARP, (b) B-Spline, (c) hybrid HARP B-Spline, and (d) QGPF methods; Down-pointed arrows highlight shift in marker position relative to tagging grid and in opposite direction to general upward motion; (e) Plots of vertical motion (average and standard deviation in 0.325 mm units, downward positive) of marker clusters (numbered from 1 in clockwise direction) with respect to initial position in frame 3.

QGPF [60] is a method of motion tracking from unwrapped phase images that is implemented for comparison purposes. QGPF employs the technique of phase integration along specific paths guided by a quality metric to unwrap the local phase and obtain the absolute phase. The quality metric is described by the spatial derivative variances of the absolute phase. Smaller local variances indicate the priority of candidates for integration. Fig. 60a shows the composite display of the input image and corresponding pole residue. The poles are computed by summing the wrapped inter-phase differences of each 4x4 pixel-square (16). The left ventricle is segmented manually (black region in Fig. 60b) instead of semi-automatically for higher accuracy. Then phase inconsistencies are identified with both the phase image and the pole residue image and branch cuts (Fig. 60b) are specified for the segmented region by matching each pole to either the nearest opposite pole or outside the region. The phase gradient variances are computed from the phase images. Following the

flood-fill algorithm [61], a seed is initialized within the segmented region and the wrapped phase pixels are integrated along the path of lowest variances. As each pixel is unwrapped, its adjoining pixels in the segmented non-branch-cut region are added as unwrapping candidates. The unwrapping stops when there are no more pixels to unwrap. The unwrapped phase is globally adjusted by inter-frame temporal consistency. The motion of marker clusters is tracked with the reference-based search at 4x resolution. The performance of QGPF is better than that of HARP and comparable to that of the B-Spline method for frames near systole (Fig. 56d and Fig. 58d) except for a glitch on frame 13 in HLA slice (Fig. 56d). However unlike the B-Spline method (Fig. 60f) in frames after systole, tag jumping occurs for three clusters from frame 22 onwards (Fig. 60h). The split of cluster 6 is indicated by larger standard deviations (Fig. 60d). The corresponding wrapped count image (Fig. 60c) shows some horizontal left shift in tag lines in the lateral wall (lower) near the apex (indicated by dashed circle). Fig. 56, Fig. 58 and Fig. 60 also show the results using hybrid HARP B-Spline for comparison.

Fig. 59: Vertical forward displacement vectors (white line from black dot) for mid VLA slices from frame 12 (left) to frame 14 (right) using (a) HARP and (b) B-Spline methods. Top right of image enlarged from box region (blue).

Fig. 61 shows the performance of the HARP, B-Spline, hybrid HARP B-Spline and QGPF methods for SA slices near the base and apex. The 2D motions data is plotted as radial contraction or expansion (positive) with respect to center of the left ventricle for different frames and different methods. The B-Spline method (Fig. 61b) has some diagonal shift in tracking for frames 10-12 whereas HARP (Fig. 61a) has good tracking. The hybrid HARP-B-Spline method (Fig. 61c) has comparable performance as HARP but the QGPF method (Fig. 61d) shows very poor performance, where most clusters are wrongly positioned in frames 10 and 14 as observed by different radial distance trends (Fig 10e). Clusters 4-6 in frame 16 disperse with large standard deviations (Fig. 61e).

SinMod [50] is a frequency domain motion tracking method, where the images are modeled by local sine-wave travelling in direction orthogonal to the tagging line and displacement between two images is extracted from the phase shift of the local sine wave. By the use of a pair of bandpass filters skewed to low and high frequencies on a pair of temporally correlated images, both the local frequency and displacement are estimated from the high frequency and low frequency power and cross-power images. SinMod is implemented with successive image comparison at low resolution. The lowest harmonic frequency is filtered with a normalized radius of 0.9 for 2D tags and 1.0 for the 1D tags. The best performance is achieved with the original image size with no reduction in resolution for the inverse FFT transform. For the 2D grid, 2 frequency peaks are processed separately and the local sine wave is rotated at 45° and 135° before deriving the final motion vector. Fig. 62 shows the tracking of markers for HLA, VLA and SA slices on frame 15. Comparing to the initial markers at frame 3 and tracked markers at frame 15, the SinMod method has poorer performance than hybrid HARP B-Spline method. Some shifts in markers can be observed at the 6 o'clock position in the SA and HLA slices and at the 3 o'clock position in the VLA slice.

Fig. 60: QGPF phase unwrapping for HLA slices on frames 21 (top) and 22 (bottom): (a) Input image and pole residues; (b) Left ventricle region mask (black) and branch cut (white); (c) Wrapped count image with shifted period in circle; Comparison of motion tracking results: (e) HARP, (f) B-Spline, (f) Hybrid HARP B-Spline, and (f) QGPF methods; Solid head arrows highlight shift in marker position relative to tagging grid while outline arrows represent local motion of heart wall; (d) Plots of horizontal motion (average and standard deviation in 0.325 mm units) of marker clusters (numbered from 1 in clockwise direction) with respect to initial position in frame 3.

Fig. 60: QGPF phase unwrapping for HLA slices on frames 21 (top) and 22 (bottom): (a) Input image and pole residues; (b) Left ventricle region mask (black) and branch cut (white); (c) Wrapped count image with shifted period in circle; Comparison of motion tracking results: (e) HARP, (f) B-Spline, (g) Hybrid HARP B-Spline, and (h) QGPF methods; Solid head arrows highlight shift in marker position relative to tagging grid while outline arrows represent local motion of heart wall; (d) Plots of horizontal motion (average and standard deviation in 0.325 mm units) of marker clusters (numbered from 1 in clockwise direction) with respect to initial position in frame 3.

Fig. 61: 2D motion tracking of marker clusters in SA slices from frame 8 to 16 (8,10,12,14,16 row-wise) using (a) HARP, (b) B-Spline, (c) hybrid HARP-B-Spline and (d) QGPF methods; Arrows highlight shift in marker position relative to tagging grid while circle highlights split of clusters; (e) Plots of radial motion (average and standard deviation in 0.325 mm units, outwards positive) of marker clusters (numbered from 1 in clockwise direction) with respect to initial position in frame 3.

Fig. 61: 2D motion tracking of marker clusters in SA slices from frame 8 to 16 (8,10,12,14,16 row-wise) using (a) HARP, (b) B-Spline, (c) hybrid HARP-B-Spline and (d) QGPF methods; Arrows highlight shift in marker position relative to tagging grid while circle highlights split of clusters; (e) Plots of radial motion (average and standard deviation in 0.325 mm units, outwards positive) of marker clusters (numbered from 1 in clockwise direction) with respect to initial position in frame 3.

Fig. 61: 2D motion tracking of marker clusters in SA slices from frame 8 to 16 (8,10,12,14,16 row-wise) using (a) HARP, (b) B-Spline, (c) hybrid HARP-B-Spline and (d) QGPF methods; Arrows highlight shift in marker position relative to tagging grid while circle highlights split of clusters; (e) Plots of radial motion (average and standard deviation in 0.325 mm units, outwards positive) of marker clusters (numbered from 1 in clockwise direction) with respect to initial position in frame 3.

Fig. 62: Motion tracking of (a) markers at original resolution by (b) SinMod and (c) hybrid B-Spline methods.

The profiling data (Table 7) for the various methods to process one slice, was taken from an Intel Xeon 2.67 GHz processor system with 4GB RAM and an NVIDIA GeForce GTX285 graphics card. The profiling for QPGF method excludes the time for segmentation and manual branch cut specification while the profiling for Hybrid HARP-B-Spline method excludes the anchor point selection.

Table 7: Time taken for tracking the cardiac motion in a slice of 24 time frames using HARP, hybrid B-Spline, and QGPF methods.

Method/Slice	HARP (s)	Hybrid HARP-B-Spline (s)	QGPF (s)
Hla.46	77.11	360.98	130.20
Vla.57	87.96	428.16	133.81
Sa.20	94.46	518.07	153.10

7.4 Discussion

With the introduction of an increased spatial resolution and reference-based motion search, the HARP performance is good in mid ventricular SA slices. However, its performance is poor in LA slices with large motion. Left ventricular shortening causes a large displacement along the LA during systole. It is difficult to keep inter-frame displacement within the half-tag distance due to the physical constraint of temporal resolution. In this case, we have already acquired 27 frames for each cardiac cycle. In the LA slice, the 1D motion tracking is affected by through-line motion and is analogous to the through-plane motion of 2D tracking. The projected 1D motion may be displaced outside the tissue region during lateral movements parallel to the tag lines into non-tissue regions where the phase information may not be valid. Although the application of HARP processing is extended from 2D tags to 1D tags, the tracking of invariant phase in 1D is theoretically similar to 2D. Further, HARP processing is not expected to work well for 2D tags in LA slices because of the half-tag displacement limitation. It is further reported in literature [38] that HARP performance is not robust at myocardial boundaries that are noisier due to the tissue properties and through-plane motion of the heart.

The frequency-domain motion tracking method, SinMod, models local frequency and displacement with a local sine wave. The estimation of displacement as a phase shift with the aid of auto and cross correlation functions subjects the motion tracking to a limited range of displacement within its periodicity. As a result, SinMod displays tag jumping behaviour, similar to HARP. The hybrid HARP B-Spline method has a better performance on all image slices than frequency domain tracking SinMod method.

QGPF is a path-following algorithm in the early works of phase unwrapping for interferometric synthetic aperture radar and sonar (InSAR/InSAS) applications. One shortcoming of QGPF is that it is dependent on the path of integration where different specification of branch cuts leads to different results. The more recent approaches formulate phase unwrapping as a regularized inverse problem with path-independent optimization techniques. Although the QGPF method overcomes the half-tag displacement limitation, it requires tedious user intervention on every frame in every slice to identify branch cuts. Its motion tracking performance is comparable to B-Spline on frames before systole in mid LA slices and is worse on poorer quality images like boundary slices and cardiac frames after systole. The identification of branch cut is also highly dependent on user experience for good phase unwrapping results. Path-following is a classic phase unwrapping method that may

lead to inconsistent path-dependent results in the presence of discontinuities and noise. The regularization of unwrapping problem as a minimization of L^P norm provides a path-independent solution. The least-squares norm can be computed efficiently with fast cosine transforms (17) or Poisson equation with Neumann boundary conditions (18) but the fitting results are unsuitable for localized phase changes. More optimal $L1$ solutions coupled with smoothness constraints have non-convex optimization issues and higher computational loads. While graph cut approach (23) seems more attractive than network programming methods (21) in terms of complexity, more constraints remain to be resolved for specific application to tMRI scans. The direct use of low-order polynomials (24) or bipolynomials in modeling absolute phase reveals the inadequacies in representing ventricular contraction.

Although B-Spline has been applied to tMRI scans, they are only used to approximate the tagging planes [72] and to model the 4D displacements [70]. While there is no related work in B-Spline for phase unwrapping, the B-Spline method can be classified as a parametric modeling technique for phase unwrapping. A B-Spline is commonly used to approximate a curve as a piecewise polynomial function. Its excellent local support and continuity control makes it a better candidate than Poisson-Neumann or low order polynomials for modeling absolute phase. Compared to QGPF, it is a path-independent optimization technique and does not restrict phase unwrapping to the segmented regions. B-Spline modeling also offers more flexibility than polynomial modeling in representing the variations in absolute phase of a surface. Similar to QGPF, the B-Spline method overcomes the half-tag displacement limitation. However, it is able to correctly detect motion for all LA slices (6-HLA and 6-VLA) and mid SA slices, in particular it has much better performance than HARP in the LA slices and QGPF in the SA and LA slices. Generally, the B-Spline method is suitable for motion tracking in slices where a set of anchor points meeting the π inter-frame displacement criterion can be selected. The contraction of the LV wall typically satisfies the criterion. The B-Spline method also has better continuity near the tissue boundaries due to the C^2 continuity of B-Spline but may be affected by an immediately adjacent static tissue where phase discontinuity is preferred.

The performance of B-Spline deteriorates faster than HARP but slower than QPGA for SA slices nearer the base and apex. This is because frequency aliasing in highly deformed tag planes causes inconsistencies in the filtered amplitude and phase images. As phase unwrapping depends on the integration of wrapped phase, such inconsistencies can lead to the isolation of cardiac regions and subsequently undesired unwrapped images. Phase unwrapping is more vulnerable where the tagged lines are parallel to the cardiac surface. This disadvantage can be overcome with the hybrid HARP B-Spline method that maximizes

the motion tracking of the HARP method in SA slices and yet complements the large motion tracking with the B-Spline method in LA slices. HARP can assist in the neighborhood of frequency aliasing where phase unwrapping is affected but local phase is still intact. The motion range detector and phase alignment error detector estimate the confidence of HARP and B-Spline methods and make an appropriate decision between the two methods.

However, all the methods including proposed HARP B-Spline method have poor performance where there exist merged tag lines for example during contraction for boundary slices during systole. The B-Spline and QPGA phase unwrapping treat the merged tag line as one and unwrapped phase beyond the merged tag will have a phase error of 2π . HARP does not give good motion tracking because of the large regional motion. The occurrence of this condition should be minimized with careful selection of imaging parameters as merges lines represent an unrecoverable information loss.

The different methods are profiled for their computational time on LA and SA slices for the defined region subsets. The hybrid HARP-B-Spline method takes 4.78x and 5.48x longer than the HARP method at 4x resolution, requiring 394s and 518s to process 24 frames per LA and SA slice on an Intel Xeon 2.67GHz system with 4GB RAM and a NVIDIA GeForce GTX 285 graphics card. The QGPF method takes 1.60x and 1.62x longer than HARP on unwrapping and motion search. If hybrid HARP-B-Spline takes 60s to specify anchor points per slice, QGPF must not exceed 13.4s and 17.7s to segment and specify branch cuts per frame to be comparable.

7.5 Summary

We have presented a B-Spline approach for robust phase unwrapping in tMRI. A bicubic B-Spline surface is used to model the absolute phase. The phase unwrapping is a solution of a mixed integer optimization problem where the sum of the difference between re-wrapped phase and wrapped phase and the difference between the gradients of absolute and wrapped phases is minimized. Unlike existing model-based techniques, the B-Spline approach has sufficient flexibility to model phase based on initial continuity assumption and it provides a more regularized optimization formulation than the path-following methods. By performing motion tracking on unwrapped instead of wrapped phase, we can overcome the half-tag displacement constraint on successive image motion that frequently occurs in LA slices. We further propose a hybrid HARP B-Spline method for tMRI motion tracking, which maximizes the use of the HARP on small displacement and the B-Spline method on large displacement with low phase alignment errors. Also, we suggest an initial reference based

motion tracking strategy to ensure that small errors in previous motion tracking are not propagated and an increase in spatial resolution of images to improve smoothness in motion tracking. Comparing with the HARP and QGPF methods on a full set of cardiac MRI scan, our method gives the best motion tracking results and correlation. In addition, there is flexibility in our framework to replace the HARP method with other wrapped phase tracking methods. Our work is extended to reconstruct 3D heart motion using results from planar motion tracking (Chapter 8).

Chapter 8 SPATIO-TEMPORAL TRACKING OF MYOCARDIAL DEFORMATION FOR RECONSTRUCTION OF DYNAMIC HEART MODEL

A dynamic heart model incorporating patient-specific data can enhance the realism of a VR simulator for training in intra-cardiac intervention. In this chapter, we describe a methodology to reconstruct the dynamic heart model from tMRI scans. A linear variational surface representation is proposed for spatio-temporal tracking of myocardial deformation of the LV. The 4D tracking problem is formulated as a linear optimization function that minimizes the changes in local differential characteristics and differences between projected and measured in-plane displacements. Compared to existing arts that fit displacements to model in separate components, the proposed method resolves the apparent motion by global and local mesh deformation considerations. The method has successfully been applied to reconstruct the motion for the LV for a set of tMRI scans.

8.1 Outline

A dynamic heart model is critical to simulate the realistic interaction between the catheter and heart wall in a VR simulator for intra-cardiac intervention. The modeling of whole heart [95] comprises of construction of heart geometry, inclusion of special conduction network and anisotropic myofibers, electrophysiological considerations and propagation process. Traditional modeling simulates the heart wall contraction from electrical measurements (typically ECG) using electromechanical models at multi-scale level [96][97][98] that incorporate Purkinje network, detailed myofiber orientation and sheet structure. However, these complex models require validation and cannot be easily tuned for patient-specific modeling. Alternatively, patient specific motion data can be extracted directly from non-invasive imaging techniques like cardiac tMRI for reconstructing dynamic heart model. The spatio-temporal tracking of myocardial deformation from tMRI can provide regional and global LV functional measures such as strain and torsion.

tMRI applies temporary non-invasive tagging pattern to the myocardium at end diastole. The tagging pattern comes from a periodic planar RF variation that is orthogonal to the imaging plane. Tag planes are formed by points having phase of π in the volume. As the heart contracts, the tag plane deforms with the motion of myocardium and the tag pattern is captured in an imaging slice for a time series. The imaging plane is fixed for the capture but

the motion of myocardium may not be parallel to the imaging plane. For a point y in the myocardium at time t , the backward motion tracking locates a point $q(y,t)$ on the initial time frame (Fig. 47). This point y may have come from another position in the tag plane of q above or below the imaging plane. The backward displacement vector from y at time t to q at time 0 is known as the apparent motion. Hence, planar motion tracking on a slice cannot capture true motion due to through plane motion.

Generally, the spatial-temporal tracking of myocardial deformation integrates different sets of the apparent motion into the 4D domain to provide useful myocardial functional measures. The imaging planes may be orthogonal or inclined at an angle. The data input depends on the planar tracking method and may be dense or sparse from the tag lines. The methods for the reconstruction of 3D motion from tMRI can be briefly classified by the type and continuity of model representation. Volumetric model requires transmural input while continuous model may not be able to describe motion fully. Table 8 summarizes the features of various spatio-temporal tracking methods. We propose the use of a differential surface representation for spatio-temporal tracking in our work. Rather than working on constraints for different components of motion separately or adding the motion components, we integrate the apparent motion into our optimization equation to find a global solution. The surface representation has an advantage in measuring in surface-based contractile function. The discrete mesh has a lower complexity and a higher flexibility to representing deformation compared to continuous models or FEM models. Although our method have neither direct representation of clinical intuitive parameters nor strain analysis, the clinical parameters can be analyzed at a post-processing stage.

Table 8: Comparison of prior arts versus current proposal in spatio-temporal tracking of myocardial motion

	Clinical Intuitive Params (Y/N)	Model (V-Volume, S-Surface, P-Point)	Continuity (S-Spatial, T-Temporal)	Input Data	Stain Analysis (Y/N)	Apparent Motion	Validation	Technique
O'Dell <i>et al.</i> (1995)	N	V	S	Tag points along tag line	Y	Component constraints	Monte Carlo simulation, In-vivo	Power Series
Park <i>et al.</i> (1996)	Y	V	S,T	2D tag grid intersection, boundary points	Y	Component constraints	In-vivo	FEM
Sampath <i>et al.</i> (2007)	N	P	N	3D Intersection of tagging planes	N	Component addition	Phantom, In-vivo	3D HARP, imaging
Tustison <i>et al.</i> (2004)	N	Surface Bounded Volume (SBV)	S,T	Contour, normal displacement, 3D intersection, contour-tag intersection	Y	Not specified.	Arts <i>et al.</i>	NURBS
Kerwin <i>et al.</i> (1998)	N	P	N	Tag points along tag line	N	Component integration	Arts <i>et al.</i> , Waks <i>et al.</i> , in-vivo	Thin spline plates
Liang <i>et al.</i> , (2007)	N	V, P	S,T	2x2D tag grid, tag plane intersection	Y	Component addition	In-vivo	NURBS
Guo <i>et al.</i> (2005)	Y	SBV, V	S,T	Contour, tag lines	Y	Component constraints	In-vivo	B-Spline
Pan <i>et al.</i> (2003)	N	S (low density)	N	image	N	Component integration	In-vivo	3D HARP
Ozturk <i>et al.</i> (1999, 2000)	N	T	S, T	Tag point along tag line	Y	Component integration	In-vivo	B-Spline
Huang <i>et al.</i> , (1999)	N	T	S,T	Tag lines	N	Component constraints	Arts <i>et al.</i> , in-vivo	B-Spline
Wang <i>et al.</i> (2008)	Y	P	N	2D tag intersections	Y	Component constraints	In-vivo	Meshless model
Haber <i>et al.</i> (2000)	N	V	S	Epi and endocardial contours, tag points along tag line	Y	Component addition	Waldmann <i>et al.</i> , In-vivo	FEM
Our proposal	N	S	N	Myocardial points on not restricted to tag line or contour	N	Component integration	In-vivo	Differential Surface

8.2 Methods

8.2.1 Motion Reconstruction Methodology from tMRI

The overview of our methodology in 3D motion reconstruction from patient specific tMRI is detailed in Fig. 63. From the cardiac tMRI DICOM files provided by a MRI scanner, the images are cropped and scaled for motion tracking. Some markers lining the ventricular lumen are inserted on the initial frame of slices in SA and LA planes. The boundaries of the LV are typically not suitable for tracking because of the poor visibility of the endocardial surface and the disappearing tagging pattern from through-plane motion. Thus, the markers are limited to a distance from the LV boundaries. The markers from orthogonal planes may give conflicting local indications of boundaries and affect the smoothness of the reconstructed LV. This can be avoided by considering the density of markers from orthogonal planes. With the time series of cardiac images in different slices, the inserted markers are tracked with the hybrid HARP B-Spline method (Chapter 7) throughout the cardiac cycle. The tracked markers are converted to 3D positions using respective slice orientation and origin extracted from DICOM headers. Cardiac landmarks of aortic valve, mitral valve and apex are identified in the MRI scans for mesh model alignment. A prolate spheroid mesh is first fitted to the initial frame using the surface reconstruction algorithm (Chapter 6) to give the LV model. The fitted LV model is subsequently deformed with the planar tracked motion data to reconstruct the 3D motion.

Fig. 63: Process for 4D LV reconstruction from patient specific tMRI.

8.2.2 Initial Fitting

Referring to Fig. 64, the initial geometry of the LV is first achieved by deforming a generic mesh to fit a set of data at time 0. In the surface reconstruction algorithm, the mesh is globally transformed through the processes of rotation, scaling and translation with an input set of cardiac landmarks. Then, the set of material points is paired to the mesh vertices using the closest Euclidean distance criterion (31) and the mesh is locally subdivided at the paired vertex for a better fit. The geometry of the mesh is improved by swapping the edges with a mean curvature criterion. The co-tangent weights of the mesh are computed for Laplacian optimization.

Fig. 64: Spatio-temporal tracking of myocardial deformation with initial geometry fitting.

Let $p_i(t)$ be the i -th vertex in a set of mesh vertices $\mathbf{P}$ at time t and $s_j(t)$ be the j -th material point in a dataset $\mathbf{S}$ from the planar tracked motion at time t ; $p_{idx(j)}(t)$ corresponds to the mesh vertex that is paired with the j -th material point; The Laplacian vector at vertex p_i is defined as $L(p_i) = \sum_{k \in N(i)} \omega_{ik} (p_i - p_k)$ where ω_{ik} refers to the co-tangent weight corresponding to the edge formed by vertex p_i and another vertex p_k in its

neighborhood; T_i is the local transformation matrix at vertex is p_i . We rewrite the Laplacian optimization problem in (26) as:

$$F(\mathbf{P}, t) = \sum_{p_i \in \mathbf{P}} \|L(p_i(t)) - T_i(t)L(p_i(t-1))\|^2 + \alpha \sum_{s_j \in \mathbf{S}} \|p_{idx(j)}(t) - s_j(t)\|^2 \quad (48)$$

where α weighs between the smooth term and the fidelity term. The custom mesh at $t = 0$ is a solution to the minimization of (48).

8.2.3 Spatial-temporal Deformation

After the initial geometry, the mesh is deformed for spatio-temporal tracking of material points. The correlation between the material point and mesh vertex is maintained as the material points from myocardial tissue are tracked from initial frame to end frame of the cardiac cycle. The edge swapping process is first performed to improve the mesh geometry from the previous deformation and the cotangent weights are updated for the new positions and connectivity of the vertices.

As the planar tracking provides only the apparent motion of the material points, we have to modify the fidelity term in (26). Here, we present a new deformation optimization function that integrates the apparent motion from orthogonal planes with differential surface characteristics. Let $\mathbf{S}^1$, $\mathbf{S}^2$ and $\mathbf{S}^3$ be exclusive subsets of dataset $\mathbf{S}$ from different imaging planes in SA, HLA and VLA slices where $\hat{n}_1$, $\hat{u}_2$ and $\hat{u}_3$ are respective imaging plane unit normal and tagging plane unit normals. In our work, we have three orthogonal sets of images, a 2D tagging grid on the SA slices and 1D tagging lines on both the HLA and VLA slices. The deformation of the mesh at time t is estimated by minimizing the following optimization function:

$$\begin{aligned} F(\mathbf{P}, t) = & \lambda_1 \sum_{p_i \in \mathbf{P}_t} \|L(p_i(t)) - T(t)L(p_i(t-1))\|^2 \\ & + \lambda_2 \sum_{s_j \in \mathbf{S}_t^1} \|(\hat{n}_1 \hat{n}_1^T - I)(p_{idx(j)}(t) - s_j(t))\|^2 \\ & + \lambda_3 \sum_{s_j \in \mathbf{S}_t^2} \|\hat{u}_2 \hat{u}_2^T (p_{idx(j)}(t) - s_j(t))\|^2 + \lambda_4 \sum_{s_j \in \mathbf{S}_t^3} \|\hat{u}_3 \hat{u}_3^T (p_{idx(j)}(t) - s_j(t))\|^2 \end{aligned} \quad (49)$$

where λ_1 , λ_2 , λ_3 and λ_4 are weights to balance the smoothing and fidelity terms. The first term preserves the local surface curvature by minimizing the square sum of Laplacian from the deformation at time $t - 1$. The remaining terms integrate the motion data by minimizing

the sum of differences between apparently displaced material points and the projections of their paired mesh vertices for various imaging planes. The second term is used to fit motion data from an image plane with a 2D tagging grid. A mesh vertex $p_{idx(j)}$ that is paired to a material point s_j , is projected onto the image plane with surface normal $\hat{n}_1$ to give $p'_{idx(j)} = (I - \hat{n}_1 \hat{n}_1^T)(p_{idx(j)} - s_j) + s_j$. Both the third and fourth terms are used for fitting motion data from image planes with 1D tags. A paired mesh vertex $p_{idx(j)}$ is projected to the tagging plane normal $\hat{u}_2$ to give $p'_{idx(j)} = \hat{u}_2 \hat{u}_2^T (p_{idx(j)} - s_j) + s_j$. In our work, we use the SA plane data for plane projection and the LA planes for line projections. This formulation can be applied to combine motion data from imaging planes that are not orthogonal to each other. By using projection data in a global optimization, we eliminate the need to model separately for displacements in each component. The solution to (49) generates a custom mesh approximating the surface deformation of the LV at each time t . We set $\lambda_1 = 1$, $\lambda_2 = 0.0317$ and $\lambda_3 = \lambda_4 = 0.0156$ heuristically. The weight λ_2 should be set larger than weights λ_3 and λ_4 , since the second term represents two components of displacement while the third and fourth terms represent only one component. A larger λ_1 will provide a solution of a closer fit to the initial geometry of the LV while larger λ_2 , λ_3 and λ_4 allows for larger deformation of the reconstructed moving LV. Visible distortion might result if the latter weights are too large.

8.2.4 Heart Model Deformation

The tetrahedron heart model [77] is deformed according to motion of the reconstructed LV model via 3D free-form deformation [99]. Let $h(x, y, z, t)$ represent a point in the Cartesian space of the heart model $\mathbf{H}$. Denoting the corners of deformation region that encloses the heart model by $(X_{\min}, Y_{\min}, Z_{\min})$ and $(X_{\max}, Y_{\max}, Z_{\max})$, we compute the local co-ordinates (u, v, w) of the vertices at time $t = 0$ in the deformation region ranging between 0 and 1:

$$\begin{aligned} u &= (x - X_{\min}) / (X_{\max} - X_{\min}) \\ v &= (y - Y_{\min}) / (Y_{\max} - Y_{\min}) \\ w &= (z - Z_{\min}) / (Z_{\max} - Z_{\min}) \end{aligned} \tag{50}$$

We estimate the deformation of heart model $\tilde{h}(u, v, w, t)$ through a trivariate tensor product Bernstein polynomial function:

$$\tilde{h}(u, v, w, t) = \sum_{k=0}^l \sum_{j=0}^m \sum_{i=0}^n P_{ijk}(t) B_i^n(u) B_j^m(v) B_k^l(w) \quad (51)$$

where $B_i^n(u)$, $B_j^m(v)$ and $B_k^l(w)$ are the Bernstein polynomials. In our work, we use Bézier solid of degree $n = m = l = 3$, giving a total of 4x4x4 control points. P_{ijk} is the (i, j, k) -th control point with initial position at $t = 0$ in a rectangular lattice:

$$P_{ijk}(0) = \left(X_{\min} + \frac{i}{n}(X_{\max} - X_{\min}), Y_{\min} + \frac{j}{m}(Y_{\max} - Y_{\min}), Z_{\min} + \frac{k}{l}(Z_{\max} - Z_{\min}) \right) \quad (52)$$

Let p_g be the g -th vertex in the reconstructed LV mesh $\mathbf{P}$ and $h_{idx(g)}$ be the $idx(g)$ -th vertex with local co-ordinates $(u_{idx(g)}, v_{idx(g)}, w_{idx(g)})$ in the heart model $\mathbf{H}$ corresponding to p_g through a prior pairing process (31). The control points $P_{ijk}(t)$ for the deformation at time t can be found by minimizing the following cost function:

$$F(\mathbf{H}, t) = \sum_{p_g \in \mathbf{P}_1} \left\| \sum_{k=0}^l \sum_{j=0}^m \sum_{i=0}^n P_{ijk}(t) B_i^n(u_{idx(g)}) B_j^m(v_{idx(g)}) B_k^l(w_{idx(g)}) - p_g(t) \right\|^2 + \alpha \sum_{k=0}^l \sum_{j=0}^m \sum_{i=0}^n \|P_{ijk}(t) - P_{ijk}(0)\|^2 \quad (53)$$

The first term in the cost function is the sum of error differences between the estimated deformation of the $idx(g)$ -th vertex $h_{idx(g)}$ in the heart model and true deformation of the g -th vertex p_g in the reconstructed LV mesh. The second term is a regularization to minimize the displacement of control points from their initial undeformed positions with a weight α , which is set to 1. By solving the control points, the deformed heart model at time t can be computed by (51). Thereafter the dynamic heart model is re-encoded as surface voxelization for dynamic catheter-heart interaction (Section 3.2.2).

8.3 Results and Discussion

The 3D motion of the LV is reconstructed from tMRI by solving the proposed deformation optimization function in section 8.2.3 and we present the results for LV strain and torsion. Rather than the Lagrangian strain tensor derived from a volumetric model [75] or the

circumferential strain from a latitude segment length change [48], the mechanical function of the LV and its local wall motion can be assessed by means of linear local shortening (LLS) [2][100]. LLS provides the diagnosis of the myocardial surface contractile function. The range of LLS when comparing end diastole and end systole states and their indication of myocardial cells is listed in Table 9 derived from [2].

Table 9: LLS range and contractile description

LLS Range	Contractile Description
$LLS \geq 12\%$	normal
$8\% < LLS < 12\%$	mild impairment
$4\% < LLS < 8\%$	moderate impairment
$0 < LLS < 4\%$	severe impairment (hypokinesis or akinesis)
$LLS \leq 0\%$	paradoxical increase (dyskinesis)

Denoting p_i as the i -th vertex of the reconstructed LV mesh and $d_{ij} = \|p_i - p_j\|$ as the Euclidean distance between the i -th and j -th vertices, we define the LLS of p_i at time t with respect to the initial frame at end diastole t_{ED} by:

$$lls(p_i, t) = \sum_{j \in N(i)} \omega_{ij} (1 - d_{ij}(t) / d_{ij}(t_{ED})) \tag{54}$$

where $N(i)$ is the neighborhood of the i -th vertex and ω_{ij} is the radial basis weight defined as $\omega_{ij} = e^{-k(d_{ij}-c)^2}$. We set $c = 8$, $k = 0.1$ and $k = 0.05$ when $d_{ij} < c$ and $d_{ij} \geq c$ heuristically. c affects the radial distance of influence while k controls the slope of the tapering weight. In all results presented thereon, we use an initial frame captured after end diastole at $t = 3$ because the qualities of preceding frames are poor. Fig. 65 illustrates the LLS profile of the reconstructed LV mesh with respect to the time in frame and the distance from apex. The data at x distance from apex refers to the averaging of the vertex data from a slice located at a normalized distance $[x - 0.1, x]$ from the apex. The LLS increases rapidly from frame 3 near end diastole. It peaks during systole at frames 10-12 before decreasing quickly back to zero at frame 17. Across the distance from apex to base, the LLS is higher near apical slices with a maximum of 0.2 at a normalized distance of 0.3 and then it gently decreases towards the base. The range of LLS and its general decrease along apex-base axis is consistent with the circumferential strain reported in [48].

(a) Surface plot

(b) Distance from apex series

Fig. 65: LV linear local shortening profile.

The torsion of the LV is measured by the axial twist. Defining the apex-to-base axis to be the axis on which twist is evaluated, we project each mesh vertex p_i to the apex-to-base axis at p'_i . The twist angle is computed as the rotation angle of vector $p_i p'_i$ from t to t_{ED} with respect to the axis less the global axial rotation of the LV. A positive twist indicates a counter clockwise rotation directed from apex to base while a negative twist indicates clockwise rotation directed from apex to base. The twist angles are further compensated by the alignment of the base-apex axis from t to t_{ED} . The first 1% of vertices on the apex-to-base axis is excluded as large angle errors are generated at the apex. Fig. 66 illustrates the axial twist profile varying with time and slice. The profile shows that the slices near the apex (0.1-0.4) undergo counter clockwise rotation while the slices near the base (0.6-1.0) undergo clockwise rotation. This anti-rotation along the base-apex axis produces the axial twist with its center migrating from 0.57 to 0.4 towards the apex as time increases. From the timeline, the twist angle near the apex increases as the apical slices rotate counter clockwise up to a maximum at frame 10 during systole, then it decreases as the apical slices rotate clockwise. On the other hand, the basal slices rotate clockwise to twist the LV during pre-systole and they rotate counter-clockwise to untwist the LV during post-systole. The frame series results are consistent with prior arts [66][69] while the surface plot is similar with meshless model described in [68]. However, the total twist angle deviation from base to apex is 10.3° and is smaller than the 14° reported in [66] and the 10.3° outer wall and 20° inner wall

measurements in [69]. It is found that the twist angle deviation can increase up to 13° without compensation from the base-apex axis alignment.

(a) Surface plot

(b) Distance from apex series

(c) Frame series (3-12)

(d) Frame series (11-20)

Fig. 66: LV axial twist profile.

To determine the global shape deformation, the radial and longitudinal compressions are measured. Fig. 67a illustrates the average radius profile for different slices and time. From the apex to the base, the radius increases gradually, peaks at distance 0.7, and then decreases. The average apical radius has a larger offset due to the exclusion of some apex vertices while the confidence of the basal radius is affected by the open LV mesh model. Denoting $r(t)$ as average radius of a slice with distance from the apex, we define the radial strain by:

$$\varepsilon_r(t) = r(t)/r(t_{ED}) - 1 \tag{55}$$

From Fig. 67b and Fig. 67c, the radial compression (negative strain) increases and then decreases across the apex-base axis and the time axis. The largest radial compression is observed at mid ventricular slice of distance 0.4 and at systole frames 10-12. The expansion for upper half of the LV from frame 17 to frame 26 is due to the initial frame being captured after end diastole. The maximum radial compression of 25% is between the outer wall radial strain of 15-20% in [66] and inner wall radial strain of 35% in [69]. The basal radial compression of 10% is similar to [66].

(a) Surface plot

(b) Distance from apex series

(c) Frame series (3-12)

Fig. 67: LV radius and associated radial strain profile.

From Fig. 68a, we observe a downward displacement of vertices along base-apex axis during pre-systole and an upward displacement during post-systole and the crossing of the zero line at frame 14. This displacement along base-apex is computed by tracking the center of base-apex axis and projecting it to the axis at end diastole. Let the length of the base-apex axis at time t be $l(t)$, the longitudinal strain is defined by:

$$\varepsilon_l(t) = l(t)/l(t_{ED}) - 1 \tag{56}$$

The profile for longitudinal strain in Fig. 68b shows a similar trend along the time axis as radial strain. There is an increase in longitudinal compression (negative strain) during pre-systole and a decrease during post-systole with an expansion from frame 18 for similar reason of the initial frame. However, the maximum range of longitudinal compression of 16% is below the 40% reported in [68] but greater than 12% in [66].

Fig. 68: LV base-apex displacement and associated longitudinal strain profile.

Fig. 69 shows the LLS distribution on the reconstructed LV in time series and different views. The maximum LLS is observed from frame 9 to frame 12 near the apex region. Fig. 70 shows the distribution of the axial twist on the reconstructed LV in time series and different views. The maximum twist is effected from frame 9 to frame 12. An oblique direction of twist center is observed. The anterior view on frame 9 shows clockwise rotation on basal septal region and anti-clockwise rotation on apical lateral region. This oblique twist

is due to the helical arrangement of the muscle fibers [102] and in this case the right handed helix of the subendocardium or the descending segment.

Fig. 69: Composite map of LLS on LV model with anterior, lateral, posterior, septal views for frames 3-18.

Fig. 70: Composite map of twist on LV model with posterior, septal, anterior and lateral views for frames 3-18.

The ventricular motion [101] is classified into 4 major phases in the cardiac cycle and these phases are described herein with comparison to observations from the 3D motion

reconstruction. In the isovolumic contraction phase, global counter-clockwise twisting (directed at base) occurs, resulting in narrowing, bidirectional transient lengthening and downward displacement of the ventricle. The initial global counter-clockwise and transient lengthening is missed due to the delay of the initial frame. In preparation for the ejection phase, contraction down the descending segment occurs with clockwise rotation at the base and counter-clockwise rotation at the apex and results in longitudinal shortening. Both the twisting and shortening are observed in Fig. 66b and Fig. 68b from frame 3 to frame 11. This is followed by the isovolumic relaxation phase, where contraction up the ascending segment occurs with untwisting of apex, resulting in brisk upward motion of base and lengthening. The untwisting is seen in Fig. 66b with upward displacement and lengthening in Fig. 68 from frame 12 to frame 18. There is subsequent widening of basal area, eventually building sufficient negative pressure to transit to diastolic filling phase, where rapid clockwise untwisting and widening occurs with rapid filling of ventricle. The basal widening is observed in Fig. 67b near frames 17-19.

8.4 Summary

The 3D motion of the LV is reconstructed from tMRI scans. A differential surface model is proposed to track spatial-temporal deformations. The proposed model minimizes the sum differences of Laplacian surface and the sum differences of projected and tracked planar displacements. In this way, the deformation is optimized between the apparent motions from orthogonal planes while maintaining the surface curvature of the model. The spatio-temporal tracking produces strains (surface, radial and longitudinal) and axial twists from the reconstructed LV that are consistent with literature and yet offers a lighter complexity than 4D B-Splines and FEM models. With the reconstructed 3D motion of LV, we integrate patient specific motion data into the heart model of the VR simulator through free-form deformation and surface voxelization.

Chapter 9 CONCLUSIONS

A VR simulator for intra-cardiac intervention has been designed and developed in our research. The system is complete with a dynamic heart model and catheter model and is capable of simulating the heart mapping procedure for assessment of local heart wall contractile function. By maneuvering a virtual catheter in a virtual beating heart model through a VR catheter unit, a contact and its associated LLS can be acquired when a stable contact is formed. The acquired contact updates the heart surface reconstruction with the LLS in real-time and influences the decision for the next contact. A positive evaluation of the simulator has been received from a user study.

9.1 Highlights

The research work has addressed three key aspects of the intra-cardiac intervention simulator: catheter-heart interaction, progressive surface reconstruction and patient-specific heart model.

Unlike physics-based approaches used in vascular intervention, a novel geometric-based modeling of dynamic catheter-heart interaction has been proposed. The modeling work has simplified the collision of two moving bodies into quasi-static phases where either the catheter moves in response to input forces or the heart surface deforms with the heart contraction. Collision detection between catheter and heart wall is accelerated by a boundary-enhanced voxel representation of the heart surface. Two types of contact: non-slip and slip contacts are identified in empirical measurements of pushing a catheter against mounted heart tissue. A non-slip contact is established when the catheter first touches the heart tissue where the catheter deforms between the insertion and the tip. It finally becomes a slip contact when more force is exerted or the angle of incidence is low where the distal length of catheter glides along the heart surface. Catheter deformation for the non-slip contact is modeled as a two-step memory recursive shape deformation that minimizes the lateral and axial strains of the catheter. The initial shape deformation is computed as the sum of weighted force displacements at insertion due to catheter and heart motion, weight force displacements at tip due to heart motion, weighted force compensations at both ends from slippage of the catheter. Catheter deformation for the slip contact is a progressive geometrical deformation from insertion to tip that bends a group of catheter links collectively within the volume enclosure to minimize catheter curvature deviation. The modeling

produces a simulated force-displacement with a close relationship to the empirical measurement and displays similar manner of slipping compared to the experiment.

From the heart mapping procedure, the heart surface is progressively reconstructed with continuous acquisition of surface contacts. Our reconstruction algorithm achieves a smooth progressive update of the heart surface having a close fit to acquired contacts despite having a sparse and non-uniform dataset. The novelty lies in the use of recursive iterations of Laplacian deformation for local smoothness, local sub-division for mesh refinement and edge swapping for geometry improvement. Among the prior arts of radial basis function and Laplacian deformation, our reconstruction generates the lowest Hausdorff symmetric distance, distance and curvature error-fit metrics.

To simulate the realistic interaction for heart mapping procedure, a patient-specific dynamic heart model reconstructed from tMRI measurements is integrated into the VR simulator. The reconstruction of 3D motion is achieved by first motion tracking in tMRI slices and then spatial temporal tracking using a 3D deformable model. A B-Spline approach to phase unwrapping is proposed for motion tracking in tMRI. Motion search is performed in unwrapped instead of wrapped phase in existing arts to overcome a half-tag displacement limitation. The absolute phase modeled by bicubic B-Spline surface is a solution to the minimization of the spatial gradients differences between absolute and wrapped phases and the differences between the re-wrapped and wrapped phases. The proposed approach has the best performance when compared with HARP, frequency domain tracking and quality guided path following methods. A differential surface model is then proposed to track spatial-temporal deformations by optimizing between these planar tracked motions from orthogonal planes and minimizing surface Laplacian differences between deformations. In contrast to existing arts, the apparent motion from orthogonal planes is integrated in a global optimization problem instead of separate components. The proposed methods have successfully been applied to reconstruct the motion for the LV from a set of tMRI scans and the reconstructed LV mesh produces strain and axial twists that are consistent with literature.

Fig. 71 summarizes the functional relationship between these research modules for building the intra-cardiac simulator. The thesis focuses on mapping the patient tMRI to the heart model of the simulator through the steps of planar motion tracking and spatio-temporal LV deformation, modeling of the dynamic interaction between the catheter controlled by the catheter unit and the patient-specific heart model and the reconstruction of mapped surface from the acquisition during the interventional procedure simulation. The experiment to investigate the force displacement of catheter and heart tissue for tuning purposes and the

haptic control to maneuver the virtual catheter for interaction with the heart model are essential building blocks of intra cardiac simulator where more research details can be found in the theses [4] and [5].

Fig. 71: Functional block diagram of the intra-cardiac simulator.

9.2 Limitations and Future Work

Although the key aspects of the simulator have been addressed to produce a reasonable simulation of the heart mapping procedure for training, more research work is required in the areas of quantitative validation for catheter-heart interaction, haptical feedback, motion reconstruction for the RV, reconstruction of normal and diseased heart model and simulation of electrical properties. The geometrical modeling of catheter-heart interaction has plausible results that are subjectively confirmed by user study, comparison of force-displacement graphs and time series of images. To have quantitative validation, we require the design of embedded sensors on the length of the intervention catheter to track the path in time and space and yet not affect the interaction process. Further experiments should be designed to determine the static and dynamic frictional coefficients of the contact between catheter and heart tissue [82]. The haptical force feedback from the VR catheter unit should be added for better user experience. The simulated forces, coupled with the experimental force range, shall be inputs in a force feedback design.

To extend the mapping functionality to the RV, the existing dynamic catheter-heart interaction can be applied while the proposed methods for planar cardiac motion tracking and spatio-temporal tracking of myocardial deformation should be adapted. The manipulation of the virtual catheter within the RV via pulmonary artery may be one difficulty to be overcome for RV mapping. The current MRI data set is focused on the LV and it lacks the third orthogonal plane of the RV. The B-Spline method is found to be less suitable for the tracking of the RV motion. It is mainly attributed to the very thin RV cardiac wall of the current dataset that affects the confidence of unwrapping, the shear between the thin wall and neighboring static tissue that disrupts continuity of B-Spline and the limited candidates anchor points with the larger vertical displacement of RV than LV. Other potential unwrapping methods like graph cut may be investigated for shear robustness. Like the LV, the linear variational mesh technique can be used to reconstruct the RV. However, there are additional challenges to select a suitable model to approximate the crescent shape of the RV and to describe the strains and rotation of the RV in a clinical intuitive way. There is room for improvement in the alignment of the mesh model and the matching of the tracked points to the mesh vertices. The Euclidian distance criterion alone is insufficient for the matching the crescent shape RV to the prolate spheroid model. The tangential surface normal approximation may be considered in tandem.

On the deformation of the heart model, the errors between the LV region and the reconstructed LV model need to be minimized and the non-LV regions are required to be extrapolated. The existing free-form deformation technique for heart model deformation is insufficient to represent accurately the LV deformation. Radial-based volumetric deformation or free-form deformation with B-Spline may be investigated for increased flexibility of representation. To integrate both the LV and RV models for heart deformation, there are more considerations for the alignment since patient-specific LV and RV models may have different correlation between cardiac landmarks of the heart model. The two geometric transformations used for alignment may be interpolated by means of exponential map or quaternion approach. Further data needs to be extracted from patients with heart disease and processed to represent the different variations of heart disease for a practical simulator.

The simulator can be enhanced with local electrical properties in addition to local mechanical properties for a more complete analysis of electromechanical function of the heart wall. The electrical properties can be simulated by embedding special conducting system in the voxel-based representation and propagating the electrical signal from electrocardiogram input [103].

Besides the heart mapping procedure, the simulator can be extended for the cellular and gene injection procedure in intra-cardiac intervention. An interesting area is the modeling of catheter-heart injection where we can classify the modeling into three stages [104]: free movement, surface contact and piercing and dynamic tissue penetration. Unlike prior arts of needle insertion under static condition [7], the modeling of catheter-heart injection should take into consideration of the normal heart contraction, the regional chaotic contraction of heart tissue (flutter) induced by needle penetration and the cellular or gene injection into the heart tissue.

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