Design and optimization of ultrasonic array transducers for medical imaging applications

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Design and Optimization of Ultrasonic Array Transducers for Medical Imaging Applications

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Abstract

Currently array transducers are being studied with narrowband theory, however, it raises problems when the theory is used in medical imaging applications where broadband signals are used. This thesis investigates the properties of ultrasonic array transducers based on broadband analysis, in particular their radiation fields. And furthermore, the methods to design and optimize broadband sparse arrays are developed for both 1D and 2D arrays, which can be used to satisfy different application requirements.

The radiation fields from the broadband arrays as well as the narrowband arrays are first analyzed and compared starting from the same mathematical model. Despite the simplification of the model, broadband fields show much more complicated behaviours than the narrowband ones, due to the coupling of spatial dimension and temporal dimension in broadband fields. This means that narrowband theory is not adequate to be used in broadband analysis. The efficient algorithms to calculate the broadband radiation field are then developed. Although the rigorous model for broadband transducers is available, it is not suitable for array calculation because of its low efficiency. The developed algorithms dramatically reduce the complexity of calculation procedure while the accuracy is still comparable with the rigorous model.

Finite element method is used as alternative methodology to study arrays performance. The interaction between transducer and loading material, and cross coupling effect can be fully considered. The results show that when the cross coupling level is low, the radiation field matches well with the results from previously developed model. However, under high cross coupling level, the side lobe level dramatically increases, while the main lobe

keeps intact. On the other hand, for medical arrays, the FEM results show that the conventional quarter wavelength matching theory cannot achieve the best matching effect. The optimization of matching layer design is necessary.

Sparse array transducers as the possible solution to reduce array system complexity are the focus of this research work. Radiation field pattern is used as the criteria to evaluate performance of arrays. Current methods to design sparse array are mainly based on narrow band theory, which are not accurate for broadband arrays. Approaches to design and optimize broadband sparse array are developed and presented in this thesis, based on Genetic Algorithms (GAs). Traditional GAs shows poor performance in array optimization, and some novel GAs operators are developed. These operators are satisfied with some special requirements in this application, and the performance of GAs is also improved. The developed method can be used in various design strategies, and also possible to design arrays with trade-off among number of elements, maximum side lobe level, and beam width, with or without apodization weightings, for both 1D and 2D arrays. The optimization results show that the nonperiodic arrays can provide more flexibility and better performance than the periodic arrays.

Finally, experimental analysis is provided to validate the above analysis. A cost efficient synthesis measuring system is developed to fulfill the measurements, together with a 3×3 2D array prototype transducer. Both 1D and 2D arrays have been studied, and the experimental results show consistency with the simulation results for both dense arrays and sparse arrays optimized by the GAs approach. Furthermore, the measured data can be used to synthesize the field response of any sparse arrays that have same element characteristics.

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Table of Contents

Abstract	i
Acknowledgements	iii
Table of contents	iv
List of figures	vi
List of tables	xii
Nomenclatures	xiii
Chapter 1 Introduction	1
1.1 Background	1
1.2 Objectives	3
1.3 Scope	3
1.4 Outline of the thesis	4
Chapter 2 Literature Review	5
2.1 Introduction – ultrasonic imaging systems for medical diagnosis	5
2.2 Ultrasonic transducers used in imaging systems	10
2.3 Sparse array transducers design and optimization	12
2.4 Ultrasonic transducer modeling	14
2.5 Field response simulation	16
2.6 Summary	20
Chapter 3 Radiation Theory for Broadband Arrays	24
3.1 Introduction	24
3.2 Arrays one-way radiation fields	25
3.3 Simplifying of broadband radiation fields	31
3.4 Characteristics of broadband radiation fields	33
3.5 Conclusions	39
Chapter 4 Radiation Field Calculation for Array Transducers	47
4.1 Introduction	47
4.2 Calculation of radiation field based on spatial impulse response	48
4.3 Approximate methods for radiation field calculation	52
4.4 2D arrays radiation field	60
4.5 Pulse echo radiation field	65
4.6 Conclusions	67
Chapter 5 Element Cross Coupling and Its Influence on Radiation Field	84
5.1 Introduction	84
5.2 Finite element formulation for transient responses	85
5.3 Simulation of single array element	89

5.4	Cross coupling in 1D array transducers	92
5.5	Simulation of array transducers radiation field	96
5.6	Conclusions	101
Chapter 6	Sparse Array Transducers Design and Optimization	119
6.1	Introduction	119
6.2	Periodic and non-periodic sparse arrays	121
6.3	Introduction of Genetic Algorithms	125
6.4	GAs operators and parameters	127
6.5	Sparse array optimization	140
6.6	Conclusions	149
Chapter 7	Experimental Validation	165
7.1	Synthetic method for array radiation field measurement	165
7.2	Experimental set up and procedure	168
7.3	Experimental results – 1D arrays	170
7.4	Experimental results – 2D arrays	172
7.5	Conclusions	174
Chapter 8	Conclusions and Recommendations	189
8.1	Conclusions	189
8.2	Recommendations	193
References		195
Appendix A	Spatial Impulse Response of Rectangular Aperture	203
Appendix B	Material Properties of the 3x3 Prototype Array Transducer	208

List of Figures

- Fig. 2.1 Block diagram of B-mode ultrasound imaging system
- Fig. 2.2 Working principles of 1-D phased array
- Fig. 2.3 Various medical array transducers
- Fig. 3.1 Coordinate system for array radiation field calculation
- Fig. 3.2 Narrowband beam pattern of 64 elements, $\lambda/2$ pitch array
- Fig. 3.3 3.5 MHz 65% relative bandwidth waveform and its spectrum
- Fig. 3.4 Broadband beam pattern of 64 elements, $\lambda/2$ pitch array
- Fig. 3.5 Main lobes of field response
- Fig. 3.6 Main lobes of field response for different time durations of $p_0(t)$
- Fig. 3.7 Interference results of three different waveforms in main lobe region
- Fig. 3.8 Main lobes for three different waveforms $p_0(t)$ shown in Fig. 3.7
- Fig. 3.9 Beam patterns with different f numbers
- Fig. 3.10 Narrowband beam pattern of 32 elements, λ pitch array
- Fig. 3.11 Broadband beam pattern of 32 elements, λ pitch array
- Fig. 3.12 Interference of pulses in interference region and non-interference region
- Fig. 4.1 Coordinate system for 1D array radiation field calculation
- Fig. 4.2 Impulse response of single array element
- Fig. 4.3 Impulse response from a single array element, at $r = 25mm$ for different angles
- Fig. 4.4 Pressure waveform from a single array element, at $r = 25mm$ for different angles
- Fig. 4.5 Impulse response from a single array element, at $\theta = 30^\circ$ for different distances
- Fig. 4.6 Pressure waveform from a single array element, at $\theta = 30^\circ$ for different distance

- Fig. 4.7 The element directivity predicted by narrowband theory (dashed line) and impulse response method (solid line)
- Fig. 4.8 Variation of pressure amplitude along wave propagation direction at $\theta = 30^\circ$ (solid line) and $\theta = 60^\circ$ (dashed line)
- Fig. 4.9 Pressure waveforms at field point $r = 25mm$, $\theta = 0^\circ$ from 1D array focuses at $\theta = 45^\circ$
- Fig. 4.10 Pressure waveforms at field point $r = 50mm$, $\theta = 0^\circ$ from 1D array focuses at $\theta = 45^\circ$
- Fig. 4.11 Pressure waveforms at field point $r = 100mm$, $\theta = 0^\circ$ from 1D array focuses at $\theta = 45^\circ$
- Fig. 4.12 Beam pattern of 1D array focuses at $r = 25mm$, $\theta = 0^\circ$ calculated by four different methods
- Fig. 4.13 Beam pattern of 1D array focuses at $r = 25mm$, $\theta = 30^\circ$ calculated by four different methods
- Fig. 4.14 Coordinate system for 2D array radiation field calculation
- Fig. 4.15 Normalized waveform received at field point $(3mm, 0, 3mm)$ from single 2D array element (solid line), compared with $p_0(t)$ (dashed line)
- Fig. 4.16 Normalized waveform received at field point $(3mm, 3mm, 3mm)$ from single 2D array element (solid line), compared with $p_0(t)$ (dashed line)
- Fig. 4.17 Variation of pressure amplitude along wave propagation direction from a single 2D array element, calculated by exact impulse response method
- Fig. 4.18 Viewing angles $\theta_{i,j}$, $\phi_{i,j}$ for single 2D array element
- Fig. 4.19 Beam patterns of 2D array focuses at $r = 40mm$, $\theta, \phi = 0^\circ$ calculated by (a) *Field II* (b) Method developed in this chapter
- Fig. 4.20 Beam patterns of 2D array focuses at $r = 40mm$, $\theta, \phi = 30^\circ$ calculated by (a) *Field II* (b) Method developed in this chapter
- Fig. 4.21 Linear model of pulse-echo imaging system
- Fig. 5.1 Multilayer structure of 1D array elements
- Fig. 5.2 Plain strain FEM model of multilayer array transducer element
- Fig. 5.3 Pressure waveform predicted by 1D array element FEM model

- Fig. 5.4 Spectrum of waveform predicted by 1D array element FEM model
- Fig. 5.5 Element directivity calculated by FEM models (dashed line and dotted line) and impulse response method (solid line)
- Fig. 5.6 Pressure amplitude with different matching layer thickness
- Fig. 5.7 FEM model for 1-D array transducer
- Fig. 5.8 FEM models for evaluation of inter element cross coupling level
- Fig. 5.9 Coupled voltages of elements from No.2 to No.9 of different models
- Fig. 5.10 FEM model without transducer element (FEM1). Pressure is applied directly
- Fig. 5.11 FEM model of array transducer and water (FEM2)
- Fig. 5.12 Radiation pattern of results from FEM1 (solid line: $10\mu m$; dotted line: $25\mu m$) and impulse response method (dashed line) for 17 elements 1D array
- Fig. 5.13 Radiation pattern of model (c), predicted by three methods
- Fig. 5.14 Radiation pattern of model (d), predicted by three methods
- Fig. 5.15 Velocity profiles on element surface of low coupling level model (c)
- Fig. 5.16 Velocity profiles on element surface of high coupling level model (d)
- Fig. 5.17 Pressure wave transmitted from each element of low coupling level model (c)
- Fig. 5.18 Pressure wave transmitted from each element of high coupling level model (d)
- Fig. 5.19 Transient pressure waveform at field point for low coupling model (a) side lobe region $\theta = -45^\circ$ (b) focus point $\theta = \theta_F = 30^\circ$
- Fig. 5.20 Transient pressure waveform at field point for high coupling model (a) side lobe region $\theta = -45^\circ$ (b) focus point $\theta = \theta_F = 30^\circ$
- Fig. 5.21 Elemental directivity in the high cross coupling case (solid line), compared with the results predicted by impulse response method (dotted line)
- Fig. 6.1 One-way narrowband radiation pattern of 1D array focuses at $\theta_s = 45^\circ$, element pitch is $3\lambda/2$
- Fig. 6.2 Two-way narrowband radiation pattern of 1D Vernier array focuses at $\theta_s = 45^\circ$, element pitch is $3\lambda/2$ for transmitting and λ for receiving, respectively

- Fig. 6.3 One-way broadband radiation pattern of 1D array focuses at $\theta_s = 45^\circ$, element pitch is $3\lambda/2$ (solid line) and λ (dashed line)
- Fig. 6.4 Two-way broadband radiation pattern of 1D Vernier array focuses at $\theta_s = 45^\circ$, element pitch is $3\lambda/2$ for transmitting and λ for receiving, respectively
- Fig. 6.5 Two-way broadband radiation pattern of 2D Vernier array focuses at $\phi_s = 30^\circ, \theta_s = 0^\circ$. The maximum secondary lobe level is -31.8 dB
- Fig. 6.6 Two-way broadband radiation pattern of 2D random array focuses at $\phi_s = 30^\circ, \theta_s = 0^\circ$. The maximum secondary lobe level is -36.9 dB
- Fig. 6.7 Traditional crossover may change number of '1' before and after the operation
- Fig. 6.8 X2 crossover. The bits to perform crossover are equally selected from two groups, thus number of '1' will keep constant during crossover
- Fig. 6.9 Testing results for X1 (dashed line) and X2 (dash-dot line), compared with random search results (solid line)
- Fig. 6.10 $BLX - \alpha$ crossover for real number GAs
- Fig. 6.11 Testing results for common GAs (dashed line) and the modified one in this chapter (dash-dot line), compared with random search results (solid line)
- Fig. 6.12 GAs results for different population size, comparing with totally random search results
- Fig. 6.13 GAs results for different mutation rate, $mu = 0, 0.05, 0.1, 0.2$, population size is set to 100
- Fig. 6.14 GAs results for larger population size without mutation (dashed line) and smaller population size with mutation (solid line)
- Fig. 6.15 One-way beam patterns of GAs optimized 1D arrays, compared with dense array and periodic sparse array
- Fig. 6.16 Pulse-echo beam patterns of GAs optimized 1D arrays
- Fig. 6.17 The element layout of GAs optimized 2D sparse array. The number of active elements is 100
- Fig. 6.18 Beam pattern of the sparse array shown in Fig. 6.17. The maximum secondary lobe level is -43.6 dB

- Fig. 6.19 Radially sparse periodic design proposed by Austeng and Holm, 2002 (a,b). The transmitting aperture is (a), and receiving is (b)
- Fig. 6.20 Beam pattern of the array shown in Fig. 6.19. The maximum secondary lobe level is -39.8 dB
- Fig. 6.21 Beam pattern of random array with uniform weighting (dotted line), GAs optimized weighting (solid line) and cosine weighting (dashed line). The maximum SLL is -34.5 dB , -38.6 dB , -29.8 dB , respectively
- Fig. 6.22 Beam pattern of non-periodic array which is optimized by GAs first, with uniform weighting (dotted line) and GAs optimized weighting (solid line). The maximum SLL is -40.7 dB and -41.7 dB , respectively
- Fig. 6.23 Beam pattern of non-periodic array, which is optimized by GAs global synthesis. The maximum SLL is -40.9 dB
- Fig. 7.1 Synthetic method for arrays radiation field validation
- Fig. 7.2 Block diagram of the experimental system
- Fig. 7.3 The experimental system
- Fig. 7.4 The hydrophone holder allows hydrophone to rotate in two perpendicular planes $x-y$ and $x-z$
- Fig. 7.5 An example of the measured waveform from single array element
- Fig. 7.6 Spectrum of the measured signal shown in Fig. 7.5
- Fig. 7.7 Measured element directivity in $x-y$ plane (*), compared with theoretical results (solid line)
- Fig. 7.8 Element layouts of 1D array, black blocks represent active elements
- Fig. 7.9 Simulation results of 1D sparse arrays one-way radiation field. GAs optimized layout (Solid line), periodic layout (dashed line) and random layout (dotted line)
- Fig. 7.10 Experimental results of 1D sparse arrays one-way radiation field. GAs optimized layout (Solid line), periodic layout (dashed line) and random layout (dotted line)
- Fig. 7.11 Simulation results of 1D sparse arrays pulse-echo radiation field. GAs optimized layout (Solid line), Vernier layout (dashed line) and random layout (dotted line)

- Fig. 7.12 Experimental results of 1D sparse arrays pulse-echo radiation field. GAs optimized layout (Solid line), Vernier layout (dashed line) and random layout (dotted line)
- Fig. 7.13 Measured elemental directivity of 2D array element
- Fig. 7.14 Theoretical elemental directivity of 2D array element
- Fig. 7.15 Simulation results of 2D dense array pulse-echo radiation field. The maximum *SLL* is $-28.9dB$
- Fig. 7.16 Experimental results of 2D dense array pulse-echo radiation field. The maximum *SLL* is $-26.7dB$
- Fig. 7.17 Elemental layouts of 2D arrays
- Fig. 7.18 Simulation results of 2D Vernier array pulse-echo radiation field. The maximum *SLL* is $-15.9dB$
- Fig. 7.19 Experimental results of 2D Vernier array pulse-echo radiation field. The maximum *SLL* is $-15.1dB$
- Fig. 7.20 Simulation results of GAs optimized 2D sparse array pulse-echo radiation field. The maximum *SLL* is $-27.7dB$
- Fig. 7.21 Experimental results of GAs optimized 2D sparse array pulse-echo radiation field. The maximum *SLL* is $-22.1dB$
- Fig. 7.22 Simulation results of 2D random sparse array pulse-echo radiation field. The maximum *SLL* is $-14.1dB$
- Fig. 7.23 Experimental results of 2D random sparse array pulse-echo radiation field. The maximum *SLL* is $-12.7dB$
- Fig. A.1 Calculation of $\Omega(t)$ for 1D array elements. (a) Region I (b) Region II (c) Region III
- Fig. A.2 Calculation of $\Omega(t)$ for 2D array elements
- Fig. B.1 The diagram of the prototype 3x3 array.

List of Tables

Table 4.1 Time used to calculate single field point pressure response by different methods

Table 4.2 Amplitude errors (%) relative to the exact method

Table 5.1 Material properties of PZT-5H (Kunkel *et al.*, 1990)

Table 5.2 Material properties of Material properties of passive materials (Powell *et al.*, 1997)

Table 6.1 Maximum *SLL* and main lobe beam width for optimized 1D arrays

Table 6.2 Optimized results for 2D arrays, compared with other designs

Table 7.1 The maximum *SLL* of three 2D array designs and the dense array

Table B.1 Material properties of the prototype 3x3 array

Nomenclature

Symbol	Description
c	sound velocity in water
d	elemental pitch
D_i	electric displacement
E	Young's modulus
E_i	electric field
f	frequency
G	shear modulus
h	spatial impulse response
k	wave number
m	mass
p	pressure
$\vec{r}$	space vector
S_{ij}	strain
t	time
T_{ij}	stress
u	displacement
U	electric potential
v	particle velocity
V	sound velocity
Z	acoustic impedance
ε	electric permittivity
τ	time delay
θ, ϕ	angle
λ	wavelength
ρ	mass density
ω	angular frequency
∇^2	Laplace operator
$*$	Convolution
$\delta()$	Dirac Delta function
$FFT()$	Fast Fourier transform
$FFT^{-1}()$	Inverse Fast Fourier transform

Chapter 1

Introduction

1.1 Background

Ultrasonic imaging is one of the four modalities used nowadays for medical diagnosis imaging. It has various advantages over conventional X-ray, computed tomography (CT) and magnetic resonance imaging (MRI), because it does not utilize harmful electromagnetic radiation, but provides better soft tissue differentiation than X-ray. Furthermore, it is easy to perform, cost effective, capable of providing images in real time, and is also portable. Ultrasound usually operates in the range 1–10MHz for non-invasively abdominal, obstetrical, cardiological studies. Recently, frequency up to 30MHz has been used in intra-cardiac and intra-arterial imaging with ultrasound catheters.

Most of ultrasonic imaging system operates in the pulse echo mode, and piezoelectric transducer is the key part of the system. It works based on piezoelectricity phenomenon, converts electric energy to ultrasonic wave and directs the wave to propagate in human tissues, then receives the wave backscattered by human tissues and converts it to electric signal. It plays an important role in determining the quality of imaging, such as spatial resolution, dynamic range, depth of field and so on.

However, the performance of transducer is far from ideal due to the complexity of the problems. To design and fabricate ultrasonic transducer is the processes with broad interdisciplinary nature. It requires knowledge from a variety of disciplines such as

acoustics, mechanical vibration, electrical engineering, material science, signal processing, medical imaging, array theory, etc. It is of no wonder that till today the design of transducers is still mostly empirical, generally involving a trial-and-error approach. Comprehensive and accurate understanding of related physical problems is prerequisite requirement to designers.

During the last forty to fifty years, many studies have been done in this area from both academic and industry areas. In early stage, researchers focused on single element transducers, and the goal is to achieve broadband, thickness vibration transducer with high signal to noise ratio. From 1980's, transducers have developed from one element to multi element array, and from one-dimensional array (1D), which is widely used nowadays, to two-dimensional array (2D). With these array transducers, ultrasound can be electrically steered and focused dynamically, which improved the imaging quality and efficiency dramatically.

2D array transducers make it possible for emerging three-dimensional real time imaging, which has received extensive research interest in recent years. Three-dimensional ultrasound imaging is much more reliable, accurate and faster, and it provides more information to system user than 2D images. However, ultrasonic transducer is one of the bottlenecks of 3D imaging. To fabricate and use a dense 2D transducer array in medical ultrasound imaging system is still very difficult and costly in the foreseeing future, because it requires a huge amount of transducer elements and electrical channels (It can be several thousands to more than ten thousand) to be integrated in small aperture size. Sparse array is one of the potential solutions for this problem. But how to reduce and optimize the number of elements (to less than one thousand) to make a sparse array with

acceptable performance compared to dense array, is still a challenge to today's transducer researchers and designers, and hence the motivation of the current project.

1.2 Objectives

This research work aims to study the design and optimisation of ultrasonic array transducers used in medical systems. It includes,

- Study beam pattern for broadband arrays and develop an efficient method to calculate radiation field of broadband arrays.
- Finite element modeling of array transducers and elements, including study of dynamic characteristics of the array and its effect on the radiation field of the array.
- Develop an effective method to design and optimize broadband sparse array transducers to meet various design requirements and different applications.
- Experimental validation of the proposed design methods, in terms of field characteristics such as main lobe width, side lobe, grating lobe level, etc.

1.3 Scope

The scope of the study is limited to piezoelectric array transducers used in medical diagnosis imaging systems, including both 1D and 2D array transducers with operating frequency ranging from 1MHz to 10MHz. The methodologies include analytical modeling, finite element modeling, simulation, genetic algorithms optimization, and experimental measurement, etc. The general approach and models developed should also be applicable to high frequency imaging system, and high intensity focused ultrasound (HIFU) systems for medical therapeutic purpose.

1.4 Outline of the thesis

The thesis is organised as follows. After a brief introduction of the background and objectives of the thesis, a literature review is given in Chapter 2. It includes a general description of development of medical ultrasonic imaging system, array transducers, their working principle, and current status, which highlights the background information and motivation for the current research. In Chapter 3, array theory is analysed, with emphasis placed on the difference between broadband arrays and narrow band arrays. Chapter 4 describes an efficient method to calculate the beam pattern of broadband arrays, which is based on spatial impulse response calculations. These methods are integrated as an operator for optimization of sparse arrays. In Chapter 5, finite element method is used to study 1D array transducers, including the radiation field modeling. Chapter 6 presents the methodology developed based on GAs to design and optimize broadband sparse array. Chapter 7 utilizes the synthesis experimental method to study and validate the developed methods for array transducers in previous chapters. Finally, conclusions are drawn and future works are recommended in Chapter 8.

Chapter 2

Literature Review

Ultrasound imaging is one of the most important medical diagnostic tools in use today. This chapter reviews the past achievements and current developments of medical ultrasonic imaging system, and emphasises are placed on the ultrasonic array transducers, including modeling, design and optimization.

2.1 Introduction – ultrasonic imaging systems for medical diagnosis

Ultrasonic detection was first used in underwater applications, known as SONAR system, which was developed from the beginning of last century, after the famous Titanic sank. Another parallel and equally important development in ultrasonics that had started in the 1930's was the construction of pulse-echo ultrasonic metal flaw detectors for use in Non Destructive Test (NDT). In comparison, the history of ultrasonic medical imaging is much more recent. Following the pioneering work of Wild and Reid in the 1950's (Wild and Reid, 1952), the medical use of ultrasound began to progress slowly (Fenster and Downey, 1996).

The ultrasonic medical diagnosis imaging system detects differences in the behaviours of ultrasound wave in the human tissues and uses this information for medical diagnosis. These ultrasound diagnostic systems can be classified into the following three groups in accordance with their ultrasound detection methods (Fukumoto, 1982): Pulse echo method, Doppler method and Holography method. Systems combining pulse echo and Doppler method also can be found recently. Most of the ultrasound diagnostic

equipments currently adopted by the medical profession utilizes the pulse echo method, thus the study in this thesis is limited to this method only.

In early stage, with the limitation of electronics technology and display equipment, A-scan imaging is used which works like an oscilloscope. It displays the received echo directly, with horizontal axis as time of flight or depth of body (time multiplied by sound speed), and vertical axis as amplitude of echo wave. B-scan theory was established in 1960's when the scan converter was invented. After grey scale equipments with wide dynamic range had become widely available commercially by 1975, B-scan became the dominant method used in medical imaging system. With advancement in computer electronics, the analogy scan converter was soon replaced with digital scan converters (DSC) in the early 1980's. Subsequent developments in pulse echo imaging and scan conversion was based on the recognition that ultrasonic echoes originate not only from major interfaces but also from the smallest structures of the human body. The advancements were therefore directed towards the detection of small echoes in the presence of noise, and to display the subsequent information of spatial details with smaller spot size and wider range of brightness levels. A block diagram of B-mode system is shown in Fig.2.1.

What completely changed the practice of ultrasound scanning was the advent of the Real-time scanners. The first real-time scanner, better known as fast B-scanners at that time was developed and manufactured by Siemens Medical Systems of Germany in 1965. It used 3 rotating transducers housed in front of a parabolic mirror in a water coupling system and produced 15 images per second. The use of large face transducers with fixed focus produced a narrow beam to ensure good resolutions and image. Human organs and

motions could clearly be demonstrated. Griffith and Herny (1974) produced a mechanical oscillating real-time scanning apparatus in 1973, which was capable of producing clear 30° sector real-time images of good resolution. The transducer was employed extensively in cardiac scanners and the design was hailed as one of the significant milestones in the development of echocardiography. Although these systems used relatively heavy mechanically moved transducers, they produced outstanding real time resolution in the near and far field.

Ultrasound imaging technology continued to develop and improve in the 1980s. Real-time scanners had rather standard appearance, sizes and fabrication. Prior to the 1990s, B-scan ultrasound images made steady progress in resolution and quality, but the improvements were not dramatic and except for a few really top-end brands. Image quality saw real improvements in the early 1990s. It is interesting to note that the availability of new and effective technologies to ultrasound scanners had also progressively stemmed from advances in technology in other areas of science such as radar navigation, telecommunications and microelectronics. The very high-speed digital electronics required for ultrasound application had become available at an affordable cost. Multi element arrays with electronically steering and dynamic focusing replace single element mechanically steering transducers, though the theory was well established in 1960's.

Three-dimensional imaging system is being developed during the past 20 years. One disadvantage of 2-D imaging relates to the subjectivity of the conventional exam, which results from the dependence on the experience and knowledge of the diagnostician to manipulate the ultrasonic transducer, mentally transform the 2-D images into a 3-D tissue

structure, and make the diagnosis or perform an interventional procedure. Three-dimensional imaging is a way to overcome these limitations. Baba and Jurkovic (1992) first reported on a 3D ultrasound system in 1984 and succeeded in obtaining 3D ultrasound fetal images by using a mini-computer in 1986. Following that, many systems were reported. Most of these systems utilize transducers mounted on position-sensing articulated arm to achieve a series of 2-D images, then use some computer algorithms to generate 3-D images from these raw 2-D images together with the position information off-line (Fenster and Downey, 1996).

Just like the history of the 2-D system, 3-D real time imaging system is the next development goal. 3-D real time imaging makes an attractive and promising future of ultrasound diagnosis system. Real time is significant for cardiovascular applications. However, there are some difficulties to develop such kind of systems with today's technology. Firstly, there is a limited image frame rate due to the limited propagation speed of ultrasound in tissue. Conventional mechanical beam steering cannot meet the system requirements, and two dimensional multi-elements array transducers with electronically beam steering, which can manipulate ultrasound beam in full three dimensional space, is therefore highly necessary. Secondly, electronic beamformer with enough channels is another challenge. Other difficulties may come from signal processing techniques, fabrication technology, material science, image display, system cost-effective considerations and so on.

First report about 3-D real time system came from the research group in Duke University, USA (Ramm and Smith, 1990; Smith *et al.*, 1991; Ramm *et al.*, 1991). The system was commercially available in 1997. However, it is still costly to produce and poses problems

in manufacturing and in image quality due to the large amount of transducer elements and interconnections.

It is worth noting here, much improvement is seen with tissue harmonic imaging in recent years. This technology exploits the generation of harmonic frequencies as ultrasound waves propagate through tissue. The generation of harmonic frequencies is related to wave distortion due to slight nonlinearities in sound propagation that gradually deform the shape of the wave and result in development of harmonic frequencies that were not present in the transmitted wave. The significance of nonlinear propagation in medical ultrasonics was noted by Muir and Carstensen (1980). The observation of harmonics generated by contrast agents as described by Burns (1996) has focused research on acoustic harmonics. Contrast agent harmonics are generated by reflections from the injected contrast agent and not from reflections from tissue (Voormolen, *et al.*, 2004). Harmonic imaging considerably improves ultrasound images in terms of signal to noise ratio, spatial resolution and visualization of deep structures (Tranquart *et al.*, 1999; Taxt and Jirik, 2004), and have been used in various clinical applications (Kasprzak *et al.*, 1999; Spencer *et al.*, 1998). In more recent years, research on superharmonic and subharmonic are reported (Bouakaz and de Jong, 2003; Goetz *et al.*, 2005), and high frequency applications are also studied (Goertz *et al.*, 2005).

Furthermore, CMUT transducers developed in more recent years (Oralkan, *et al.*, 2002; Oralkan, *et al.*, 2003) provide new potentials to improve ultrasonic imaging quality. High frequency imaging application is another branch of current development (Lockwood, *et al.*, 1996(a); Ritter, *et al.*, 2002; Cannata, *et al.*, 2003). However, discussion of the above topics is beyond the scope of this thesis.

2.2 Ultrasonic transducers used in imaging systems

Ultrasonic transducer is a crucial part in an imaging system (Shung and Zipparo, 1996). It performs the electro-mechanical energy conversion, projects the strong acoustic pulse into tissue, and gathers weak echoes. It can be seen from the previous section that almost every progress in imaging system is always accompanied with breakthroughs in transducer technology.

Most of the ultrasonic transducers use piezoelectric effect, which was discovered by Curie brother in 1880. In 1954, Jaffe discovered the new piezoelectric ceramics lead titanate-zirconate (PZT). It soon replaced Barium Titanate and opened up the field of medical ultrasonics (Gooberman, 1968).

In the early stage, single element transducers were used. These transducers always consist of a piezoelectric ceramic disk with electrodes on both sides. On backside, a backing provides mechanical support and damps ultrasound energy propagating to this side. On the front side, one or two matching layers are used to match the difference of acoustic impedance between transducer and human tissue (Kossoff, 1966). In general there is an acoustic lens to focus ultrasound on-axis to a certain distance from the transducer surface and a housing to separate the transducer from the patient for safety. This kind of transducers has been used in both A-mode and B-mode system, moved by hand or mechanically during the scanning procedure.

Linear and phased multi elements array transducers replaced single element transducers about 20 year ago, and they improved imaging quality of systems vastly. Buschmann

(1965) and Somer (1968) first described the concepts of linear array and phased array, respectively. These arrays use multi-elements assembled in a line or concave or convex curve. In a linear array, groups of elements are fired in succession. After one group of elements finish the procedure of transmitting and receiving, multiplexing electronics will fire the next group, then the beam will move from one side of transducer to another side to finish one frame of image. In a phased array, all the elements of the array are used for each interrogation pulse and variable time delays are introduced between the various elements. Using different pre-determined time delays, steering and focusing are realized. Fig. 2.2 demonstrates the basic principle of array transducers in operation, and Fig. 2.3 shows various types of array transducers, which are commercially available.

Array transducers replaced mechanical scanning with electronically scanning, which dramatically improved the performance of real-time system. Due to the enhanced semiconductor fabrication techniques and higher levels of circuit integration in the past 20 years, today's linear array transducer can have up to 512 elements, and phased array consists of 64 to 256 elements.

However, commercially available arrays nowadays are mostly 1D arrays, in which elements are distributed in a line or curve in azimuth plane. Thus only one dimension (azimuth direction) of imaging field is under control. In elevation direction (perpendicular to azimuth direction), one still has to move the transducer manually or mechanically. Sometimes acoustic lenses placed before transducers are used to help focus in elevation direction, but their functions are very limited. It is desirable to develop 2D arrays, in which elements are distributed in a plane, in order to obtain full control throughout a three dimensional volume.

Theories of beam steering and dynamic focusing using 2D arrays were well established about 10 years ago (Smith *et al.*, 1991; Turnbull and Foster, 1992). However, fabrication and utilization of such transducers in system is still difficult. The major problem facing such development is the complexity due to very large number of elements to be fabricated and connected. For example, if a 2D array is extended from a 1D array with 128 elements, the 2D array will consist of total 16384 ($128 \times 128 = 16384$) elements. To use so many elements together with supporting electronics in a system is still impractical. One of possible solutions to this problem is to use sparse arrays, which means to remove some elements or only use fewer active elements selected from the original dense array. The problem then remained is how to find the optimal sparse layout with acceptable performance.

2.3 Sparse array transducers design and optimization

Array theory was established in 1960s, which is mainly accompanied with the development of Radar antenna system in military applications. Ultrasonic array transducers originate from this theory, more or less. Even some terms are borrowed from it, such as side lobes, phased array, etc, though their meanings are somewhat different in ultrasonic systems. However, Radar arrays and ultrasonic arrays are working under quite different principles. In general, Radar systems utilize continuous wave (CW) and are narrowband systems, while arrays used in ultrasonic imaging systems generate short pulses (PW) to detect human organs. The length of each pulse is smaller than the aperture dimension. In this case, the system should be treated as broadband system. Theory developed for narrowband system is only partially valid for broadband systems.

Sparse arrays as efficient ways to reduce system complexity and cost were first used in Radar system (Steinberg, 1976). Various ways to design and optimize sparse arrays have been developed (Trees, 2002). As the criteria of design and optimization, the radiation field from arrays can be modeled with theory based on Fourier transform. Thus the whole procedure can be performed in frequency domain. However, for broadband arrays, their radiation fields are both space and time dependent. Theories developed based on narrowband systems cannot be used directly.

Several different broadband ultrasonic sparse array designs have been reported in the past 10 years. Just like the narrowband arrays, they can also be divided into two categories, nonperiodic and periodic arrays. Davidsen *et al.* (1994) analysed several random distribution arrays, including uniformly random and Gaussian weighted random distribution. Nonperiodic sparse array can reduce grating lobes, but have a characteristic sidelobe pedestal. On the other hand, since the random array can only be described statistically, to find the optimized layout from so many elements is always difficult. For broadband arrays, time-consuming procedure to calculate radiation field is another difficulty. Weber *et al.* (1999) used genetic algorithm to optimize nonperiodic sparse arrays, but their results were not ideal. The main reason of the failure is due to the traditional genetic algorithm used. More recently, Trucco (2002), Cardone *et al.* (2002) utilized simulated annealing (SA) to design and optimize broadband nonperiodic arrays, respectively. However, only 1D arrays were studied in their works.

On the other hand, Lockwood and Foster (1996), Brunke and Lockwood (1997) have proposed periodic sparse or named Vernier array. In this design, active elements are spread evenly throughout the array by a certain distance. For transmit aperture and

receive aperture, the separate distances are different. With this method, grating lobes can be partially avoided. If element amplitude weighting or known as apodization is used, side lobes can also be reduced. However, due to the high impedance low sensitivity array elements, apodization is undesirable for 2D arrays (Holm *et al.*, 1997). Further development of these periodic arrays can be found in a series papers by Austeng and Holm (2002a, 2002b). Vernier arrays have some other disadvantages, for example, the transmitting and the receiving aperture are partially overlapped, which means some elements are used both in transmitting and receiving procedures. This will make system electronics more complicated. Another disadvantage is that the strict geometry fixes the number of elements (Austeng and Holm, 1999). Finally, it is not so effective for broadband arrays to reduce grating lobes, because it is based on narrowband theory.

Other designs are almost all derived from the above two categories, with more or less variations (Sumanaweera *et al.*, 1999; Hendricks, 1991; Schwarts and Steinberg, 1998; Yen *et al.* 2000; Yen and Smith, 2002).

Since 2-D arrays have difficulties to be in practice, some other arrays so called 1.25-D, 1.5-D, and 1.75-D have been developed at the same time to achieve partial control of the ultrasound beam in elevation direction (Zhang *et al.*, 1996; Gururaja and Panda, 1998). These arrays can be treated a trade-off between 1-D and 2-D arrays.

2.4 Ultrasonic transducer modeling

There are extensive investigations on the modeling of mechanical and electrical properties of piezoelectric transducers. Generally, these models can be divided into three categories.

The conventional method is the analytical method. Early models studied circular disk transducers, assumed that transducers vibrate like a piston in the thickness mode, and simplified the three dimensional problems into one-dimensional ones (Tiersten, 1963; Meeker, 1972). However one-dimensional models are too approximate, especially to some transducers whose shape cannot be treated as circular disk. For example, in 1-D array transducers, the transducer element is rectangular parallelepiped, using two-dimensional models is more reasonable. Bugdayci and Bogy (1981 a, b) established two-dimensional theory for piezoelectric layers. Lamberti *et al.* (1990; 1995; 2001) established a two-dimensional model of a multi element piezoelectric array transducer for imaging applications in a series of papers. Three-dimensional models are too complicated to be used, none of the reports can be found.

Equivalent Circuits modeling are also widely used (Ballato, 2001). These models are based on one-dimensional analytical models. Three most important models used in medical transducers are Mason's model (Berlincourt *et al.*, 1964), Redwood's model (Redwood, 1961), and KLM model (Krimholtz *et al.*, 1970). Those models are used in different situations. Using theories borrowed from electrical network, equivalent circuits models are helpful in studying ultrasonic transducers. Furthermore, for imaging system, transducers are always attached to electronics. With the equivalent circuit of transducers, the whole application systems can be designed and optimized more conveniently.

The most commonly used method nowadays is numerical methods, in particular the finite element method (FEM). FEM was first developed in early 1960's. It was initially developed on a physical basis for the analysis of problems in structural mechanics. Due

to its capability of handling complex geometry and boundary conditions, finite element methods were first formulated for piezoelectric materials by Allik and Hughes (1970). Since then the method has been expanded and is now widely used in the analysis of many piezoelectric devices. Many software based on different algorithms are commercially available.

Lerch (1990) first used FEM to analyze array transducers used in medical imaging system. Simulation results were used to optimize transducer design with respect to efficiency, bandwidth, angle of acceptance, inter-element cross coupling, etc. Gachagan *et al.* (1994) used FEM to study the matching layers and damping effects on the performance of transducers. FEM is also used widely in composite material transducers (Hossack and Hayward, 1991). However, it is the adoption of explicit wave propagation algorithms that made realistic transducer simulations practical (Wojcik *et al.*, 1993). With demonstrated speed/size advantage factors of 100 over conventional implicit algorithms, broadband imaging transducer models became tractable on desktop computers. Recently, Mckeighen (2001) used FEM software based on the above algorithms to simulate 2-D arrays with acceptable accuracy and simulation speed. In this thesis, FEM analysis is used to study the characteristics of the array, and their effect on the radiation field.

2.5 Field response simulation

The transducer pressure field response, or called beam pattern is most important in determining the imaging quality (Angelsen, 2000). The spatial resolution is dependent on the width of main lobe. The dynamic range or contrast resolution is related to the relative levels of side lobe and grating lobes. To compute the field response and find its relationship with transducer parameters including element size, element distribution, inter

element distance, array aperture shape and size, etc. is very important in the design of array transducer. The characteristics of the radiation pattern such as main lobe width, side lobe level always work as criteria to optimize array designs.

In general, the radiation pattern can be determined by transducer geometry and excitation signals. According to different types of excitation signals, the simulation methods can be divided into two categories. For continuous wave case, the well-known diffraction theory (Goodman, 1968) can be used to calculate the response in frequency domain, while for pulse wave transducers, impulse response method is more suitable.

For narrowband transducer, the temporal and spatial responses are completely separable. Furthermore, in far field, the relation between element distribution, normalized by wavelength (aperture function), and angular pressure field distribution (radiation pattern) can be approximated by spatial Fourier transform, if disregarding the amplitude scaling with distance from transducer surface (Steinberg, 1976). This important result makes the analysis of the far field straightforward and intuitive. For example, transmit-receive two-way radiation pattern can be obtained by product of transmit and receive radiation pattern, or by Fourier transform of the convolution of the transmit and receive aperture functions. Another important result is that if an array consists of identical elements, the radiation pattern of the array can be computed by the product of element factor and array factor, where the element factor is the radiation pattern of single element, and array factor is radiation pattern of an array, which replaces the elements in original array with point sources (Steinberg, 1976). This result decomposes the array properties into those associated with the excitation of the element and those resulting from the geometric positioning of the elements.

More recently, a semi-analytical method named distributed point source method (DPSM) was developed by Placko and Kundu (2001, 2003). By discretizing the transducer aperture into multiple point sources, it can deal with the aperture with complicated shapes, and also can be used to calculate the nonhomogeneous field and scattered field, as well as the interactions between pairs of phased arrays (Ahmad *et al.*, 2005).

The CW model is a very useful model in the design of array system, and it can also be used with pulse wave (PW) ultrasonic transducers, but only applicable in far field and near focusing region, where most of waves from each element interact with each other. With the above method, Lockwood *et al.* (1996 b) defined a new concept named effective aperture, which is the convolution of transmit and receive aperture functions, and proposed some sparse periodic arrays designs called vernier array, both for 1D and 2D arrays. They used different transmit and receive sparse aperture to achieve effective dense aperture. Pompei and Wooh (2002) optimized element shapes to obtain desired element factor in order to theoretically suppress grating lobes in phased array.

However, the CW model in general is not suitable for PW transducers in computer simulations. PW transducers used in medical imaging systems have broad bandwidths, and their transient pressure fields consist of a superposition of a wide spectrum of frequencies. This means computing using above methods must be integrated over the bandwidth of the transducers, which will be very time consuming. On the other hand, in near field the above method is also not applicable due to its far field assumption. Finally, PW transducers generate and receive short pulse, the steady-state response does not exist

at all, and thus the method based on time domain to analyze the transient response is preferred.

The impulse response method can be used to calculate the broadband radiation field rigorously in time domain, with the assumption imposed by Rayleigh integral (Stepanishen, 1971), which removes the constraints of far field. It first computes transducer spatial impulse response, which is the field response of transducer excited by an infinitely short impulse excitation. For the aperture with regular shape, the closed-form expression of the impulse response can be derived (Ullate and Emeterio, 1992; Xue, *et al.*, 1996). Two-way impulse response is the convolution of transmit and receive aperture impulse response. The field response can be achieved by the convolutions of the exciting signal with the electromechanical response of the transducer and the calculated spatial impulse response. The electromechanical response of the transducer can be computed by FEM or obtained from experimental results, where the velocity pattern of the transducer surface varying with time can be obtained.

However, this method is also time-consuming because of the very high sampling frequency needed in computer simulation. Turnbull and Foster (1991), and Jensen and Svendsen (1992) have simplified this method by assuming that all impulse responses are in trapezoid shape in far field. By dividing the transducer into small rectangular or triangular elements, the far field condition can be satisfied. And at the same time it can also deal with arbitrary shape of apertures (Jensen and Svendsen, 1992). The trapezoid assumption is not adequate in near axis region. Turnbull and Foster (1991) still adopt rigorous impulse response method for on axis calculations. For the above reasons, these two approaches can only increase the calculation speed to some extent.

The impulse response methods treat transducers as piston signal source, place the emphasis on array designs and ignore the interaction between transducer and loading material and the dynamic characteristics of transducers. The effect of cross coupling between array elements to radiation field is not considered either. If these issues are of interest, the approach combining FEM and BEM could be one possible solution (Lerch *et al.*, 1992).

2.6 Summary

From the literature review it can be seen that the piezoelectric ultrasonic transducer and its supporting electronics are the most important parts in a medical ultrasound imaging system. The imaging quality of the system depends on the performance of transducer. Array transducers are more powerful than single element transducers since they provide electronic steering and dynamic focusing, which is the prerequisite of real time imaging systems. For three-dimensional imaging system to be used in practice, 2D array transducer is indispensable, especially for three-dimensional real time imaging system.

However, with the limitations of current fabrication and electronic technology, 2D dense array transducer will not be practical in the near future. Transducers become one of the bottlenecks currently in the development of 3-D ultrasound imaging system. Using sparse arrays is one of the possible solutions to reduce the system complexity and cost. Despite extensive studies in this area, great potential and challenge still exists in design and optimization of the sparse arrays due to a huge number of possible element layouts.

In the following chapters, broadband arrays are studied and analyzed, compared with narrowband arrays. It shows that narrowband theory is not adequate to describe behaviors of broadband arrays. An efficient method to compute radiation field from broadband arrays is then developed. Finite element method is used as alternative methodology to study array performance where the characteristics of the arrays multilayer structure and radiation field are predicted. The relation between cross coupling and radiation field is demonstrated.

The remaining part of the thesis develops an efficient optimization method to design and optimize 1D and 2D sparse arrays based on Genetic Algorithms. It provides great flexibility to design sparse arrays. It is possible to make trade-off among sparse order, beam width, side lobe level, apodization, etc during design procedure with this method. Final part of this thesis describes and analyzes experimental results and future recommendations.

Fig. 2.1 Block diagram of B-mode ultrasound imaging system

Fig. 2.2 Working principles of 1-D phased array

Fig. 2.3 Various medical array transducers
(From website <http://www.ob-ultrasound.net/probes.html>)

Chapter 3

Radiation Theory for Broadband Arrays

The radiation field of broadband arrays and narrowband arrays are described and compared in this chapter. The results show that the two types of radiation fields have quite different characteristics. The broadband field is much more complicated because it is fully coupled with time dimension. Using narrowband theory is insufficient to study the broadband arrays.

3.1 Introduction

Array theory was well established around 40 to 50 years ago, however, it was mainly developed for narrowband systems. For medical ultrasound system, broadband array transducers are widely used. This chapter compares both types of arrays radiation field mathematically, and shows fundamental difference between these two systems. The analysis indicates that the broadband arrays are much more complicated than the narrowband ones. Some phenomena that exist in the narrowband system also appear in the broadband system, but their properties are quite different. That's the reason why when applying narrowband theory to design ultrasonic array transducers, the results are often not ideal and satisfactory. Better understanding of broadband array systems is crucial for further studies in design and optimization of the arrays.

In this chapter, Section 3.2 derives the one-way radiation field for broadband as well as narrowband arrays. The Rayleigh integral is applied to both cases. The similarities and differences between these two types are analyzed theoretically. Section 3.3 simplifies the

broadband response to the form similar to the narrowband case to facilitate further analysis. Section 3.4 analyzes the arrays radiation pattern, including grating lobes, side lobes, beam width, etc. which are directly related to the imaging quality. Emphasis is placed on the differences between narrowband and broadband. Finally, conclusions are drawn in Section 3.5.

In order to simplify the derivation and facilitate the comparison, elements of arrays are assumed to be omnidirectional point sources in this chapter. The influence of this assumption to the results of radiation field will be analyzed in Chapter 4.

3.2 Arrays one-way radiation fields

Narrow band arrays emit long time duration pulses, and the pulses always can be described by a transient part and followed by a steady-state part. Commonly, only the steady state of narrowband system is concerned, thus narrowband field is only spatial dependent for systems with given frequency, and time dimension can be erased during analysis. Different from narrowband system, broadband arrays generate pulses with very short time duration, only transient response exists. The field is fully temporal and spatial dependent. Steady state makes no sense for broadband systems. The coupling of time dimension and spatial dimension makes broadband field a more complicated one.

3.2.1 Narrowband array fields – near field

Although there are dramatic differences between the two types of arrays, the impulse response method can be applied to both arrays to derive and analyze their field response. To facilitate the description, a 1D array is illustrated in this chapter. Extension of the

theory from 1D to 2D arrays is straightforward. Coordinate system for the theory is shown in Fig. 3.1. A 1D array is placed along the x axis, with the centre as the origin of the coordinate system. The length of the array is L , consisting of N elements. The direction of wave propagation is defined as the z axis. θ_F is the steering angle, $\vec{r}$ is the field position vector, $\vec{r}_F$ is the focus point vector and $\vec{r}_i$ is the aperture position vector.

It is well known that the acoustic pressure field generated by a baffled transducer in a homogeneous, loseless liquid can be rigorously depicted by the Rayleigh integral (Stepanishen, 1971). If the boundary condition is the infinite rigid baffle, the field pressure,

$$p(\vec{r}, t) = \frac{\rho}{2\pi} \frac{\partial}{\partial t} \int_L \frac{v(t - \frac{|\vec{r} - \vec{r}_i|}{c})}{|\vec{r} - \vec{r}_i|} dl \quad (3.1)$$

where ρ is the mass density of propagation medium, c is the sound speed in this medium, v is the transducer surface velocity. For a soft baffle, an obliquity factor is necessary to be included, and the influence of this factor to the radiation field can be derived (Selfridge *et al.*, 1980). However, in this chapter, the following descriptions are all based on the rigid baffle.

Assuming the array elements perform piston-like vibration, the velocity v consists of only the normal component. Since the velocity of each element has the same shape of waveforms, only their amplitudes may be different due to the apodization amplitude weighting. Let $v_0(t)$ and A_i denotes the velocity waveform, the amplitude of the velocity for the i th element, respectively. The time delay to fulfil beam steering and focusing of

this element is defines as τ_i . Furthermore, if using Dirac delta function $\delta(t)$ and the time convolution symbol $*$, Equation (3.1) for a 1D array transducer can be in the form

$$p(\vec{r}, t) = \frac{\rho}{2\pi} \frac{\partial v_0(t)}{\partial t} * \sum_{i=1}^N \frac{A_i \delta(t - \frac{|\vec{r} - \vec{r}_i|}{c} - \tau_i)}{|\vec{r} - \vec{r}_i|} \quad (3.2)$$

In the above equation, each element is treated as an omnidirectional point source. Thus the integration is replaced by the summation of the waves from all elements.

If the system is narrowband, working on a single frequency of f_0 ,

$$v_0(t) = e^{-j2\pi f_0 t} \quad (3.3)$$

Substitution in Equation 3.2 yields

$$p(\vec{r}, t) = -j\rho f_0 \cdot e^{-j2\pi f_0 t} \cdot \sum_{i=1}^N \frac{A_i e^{-j2\pi f_0 (\frac{|\vec{r} - \vec{r}_i|}{c} - \tau_i)}}{|\vec{r} - \vec{r}_i|} \quad (3.4)$$

where the time dimension and the spatial dimension is separated. The time dependent term $e^{-j2\pi f_0 t}$ is a pure sine signal and can be ignored. The remaining part is only related to the space vector. Let's define

$$p(\vec{r}) = -j\rho f_0 \sum_{i=1}^N \frac{A_i e^{-j2\pi f_0 (\frac{|\vec{r} - \vec{r}_i|}{c} - \tau_i)}}{|\vec{r} - \vec{r}_i|} \quad (3.5)$$

3.2.2 Narrowband array field – far field

Furthermore, expanding term $|\vec{r} - \vec{r}_i|$ in a binomial series (Kreyszig, 1993) and ignoring the higher order terms gives,

$$|\vec{r} - \vec{r}_i| \cong r - x_i \sin \theta + \frac{x_i^2}{2r} \quad (3.6)$$

where x_i is the coordinate of the i th element, as shown in Fig. 3.1.

If the well known far field condition

$$\frac{L^2}{4r} \ll \lambda \text{ or } r \gg \frac{L^2}{4\lambda} \quad (3.7)$$

is satisfied, Equation (3.5) can be simplified further. The denominator of Equation (3.5) is replaced by the first term of Equation (3.6), since r is much greater than the array dimension in far field. For the exponent term, $|\vec{r} - \vec{r}_i|$ is necessary to be replaced by the first two terms of Equation (3.6). Move the element independent terms out of the summation symbol, one can obtain,

$$p(\vec{r}) = \frac{-j\rho f_0}{r} e^{-j2\pi f_0(\frac{r}{c})} \sum_{i=1}^N A_i e^{-j2\pi f_0(-\frac{x_i \sin \theta}{c} - \tau_i)} \quad (3.8)$$

Similarly, the time delay τ_i to focus and steer in far field is

$$\tau_i = T - \frac{|\vec{r}_F - \vec{r}_i|}{c} \cong T - \frac{(r_F - x_i \sin \theta_F)}{c} \quad (3.9)$$

where T is a constant to ensure all τ_i are positive values, $\vec{r}_F$ and θ_F are the focus vector and the steering angle, respectively, as shown in Fig. 3.1. Substituting Equation (3.9) into Equation (3.8) and change its form one can obtain,

$$p(\vec{r}) = \frac{-j\rho f_0}{r} e^{-j2\pi f_0(\frac{r}{c} - T)} \cdot \sum_{i=1}^N A_i e^{j2\pi f_0 \frac{x_i(\sin \theta - \sin \theta_F)}{c}} \quad (3.10)$$

Equation (3.10) means that in far field, narrowband radiation field can be further separated to a distance dependent term and an angle dependent term. The first term is not related to arrays configurations. Only the second term reveals the relationship between the array parameters and the radiation response. In narrowband theory, this term is always defined as arrays beam pattern, i.e.

$$p(\theta) = \sum_{i=1}^N A_i e^{j2\pi f_0 \frac{x_i (\sin \theta - \sin \theta_F)}{c}} \quad (3.11)$$

Furthermore, let

$$u = \sin \theta - \sin \theta_F \quad (3.12)$$

then

$$p(u) = \sum_{i=1}^N A_i e^{j2\pi f_0 \frac{x_i}{\lambda} u} \quad (3.13)$$

It's obviously that $p(u)$ and $A(x_i / \lambda)$ is a Fourier transform pair, and the Fourier variables are x_i / λ and u . With this result, the well-developed Fourier transform theory can be fully applied to analyze narrowband arrays. The performance of arrays in terms of field response can be predicted analytically. For example, the main lobe beam width is inversely proportional to the frequency and the aperture length; the existing and the positions of grating lobes depend on the frequency and the aperture pitch, which is defined as the distance between two neighbouring elements; weighting function can be designed analytically to suppress side lobes, etc.

3.2.3 Broadband array fields

The analysis in this section show for broadband systems the situations are quite different. Starting from Equation (3.2), since broadband signal $v_0(t)$ is of limited time duration, its spectrum consists of a wide range of frequency components, and the nominal frequency is only the central frequency in its spectrum. So broadband systems are more suitable to be analysed in the time domain. If define

$$p_0(t) = \frac{\rho}{2\pi} \frac{\partial v_0(t)}{\partial t} \quad (3.14)$$

Equation (3.2) becomes

$$p(\vec{r}, t) = \sum_{i=1}^N \frac{A_i p_0(t - \frac{|\vec{r} - \vec{r}_i|}{c} - \tau_i)}{|\vec{r} - \vec{r}_i|} \quad (3.15)$$

$p_0(t)$ can be treated as the shape of pressure signal emitted by each element. Equation (3.15) means at any field point, the pressure equals to the summation of the delayed signal from each element, weighted by their amplitude and the spherical propagation decays. The results are totally coupled with time and space. This phenomenon still exists even in the region of far field defined by Equation (3.7). Ignoring the third term in Equation (3.6), substituting it and Equation (3.9) into Equation (3.15) and making some simple change, one can obtain,

$$p(\vec{r}, t) = \frac{1}{r} \cdot \sum_{i=1}^N A_i p_0[t - T - \frac{r - r_F}{c} - \frac{x_i(\sin \theta - \sin \theta_F)}{c}] \quad (3.16)$$

This equation means, in far field only the spherical propagation decays is element independent. For any further simplification it is necessary to analyse the specific $p_0(t)$ of the given systems. For example, Murino *et al.* (1997) assume a rectangular shape of envelope of the pulse $p_0(t)$ and provide a quasi-closed form of the broadband beam pattern. Due to the assumption of signal envelope their approach cannot be generally used for broadband arrays. Furthermore, the far field condition is not always satisfied for medical imaging applications. For example, for a 3 MHz arrays with the aperture length of 10mm, the far field condition calculated by Equation (3.7) is $r \gg 50\text{mm}$. The equivalent f number, which is defined by the ratio of the focus length and the aperture length will be much greater than 5. However, in medical imaging arrays, f number falls in a wide range from 1 to 10 for different applications (Angelsen, 2000). Thus it is necessary to study arrays performance both in near field and far field.

The above analysis and comparison indicate that the radiation field from broadband systems is much more complicated than the narrowband case. Thus most of narrowband theories do not hold for broadband arrays. Despite the term ‘phased array’ is still used for broadband arrays, the meaning is quite different from that in the narrowband arrays. The signal can never be described by a single frequency, thus the dominant factor to fulfil steering and focusing is the time delay rather than the signal phase.

3.3 Simplifying of broadband radiation fields

The radiation field generated by transducers is directly related to image quality. The main lobe beam width determines the lateral spatial resolution of the system. The level of side lobe floor is related to the dynamic range of image. The existence of the grating lobes may produce ambiguity in image due to their high amplitudes. The relationship between the narrowband array parameters and the radiation field can be fully studied by Fourier theory, but for broadband case, things are more difficult. As discussed in previous section, the broadband field response is dependent on not only space (r, θ) but also time, thus the response is four-dimensional data $p(r, \theta, t)$. Furthermore, if considering 2D arrays, the full three-dimensional space should be calculated, the response will be much more complicated. In order to facilitate the analyzing procedure, some simplifications are necessary.

Firstly, although the response is distance dependent, if the field points to be calculated are selected with the same distance to the array centre as the focus distance and only the angle changes from -90° to 90° , the distance dimension can be eliminated from the results. This is especially useful when field points fall in far field (Schwartz, 1996). If let

$$t' = t - T - \frac{r - r_F}{c} \quad (3.17)$$

and ignore the spherical propagation decay term $1/r$, Equation (3.16) becomes

$$p(\theta, t') = \sum_{i=1}^N A_i p_0 \left[t' - \frac{x_i (\sin \theta - \sin \theta_F)}{c} \right] \quad (3.18)$$

The field pattern shown above is only time and angle dependent. In near field, this method may also be used. This is because the above equation describes the variation of pressure versus angle. It can be used to analyze the system performance in lateral dimension. For the medical imaging applications that provide sector-scanning format images, this performance is always the first concern.

Furthermore, in order to reduce time dimension, generally two methods can be selected (Holm, 1996; Cardone *et al.*, 2002), and they are the maximum projection method and the integrated energy method. The first method reduces the time dimension by taking the maximum value of the response at each field point, i.e.

$$p(\theta) = \max_t \left| \sum_{i=1}^N \frac{A_i p_0 (t - \frac{|\vec{r} - \vec{r}_i|}{c} - \tau_i)}{|\vec{r} - \vec{r}_i|} \right| \quad (r = \text{constant}) \quad (3.19a)$$

$$p(\theta) = \max_{t'} \left| \sum_{i=1}^N A_i p_0 \left[t' - \frac{x_i (\sin \theta - \sin \theta_F)}{c} \right] \right| \quad (3.19b)$$

for near field and far field, respectively. With this method, the information of the temporal shape of pulse at each point is lost, but the maximum pressure is preserved.

The alternative method is to compute the beam pattern as the integrated pressure energy over time,

$$p(\theta) = \int_0^\infty \left| \sum_{i=1}^N \frac{A_i p_0 \left(t - \frac{|\vec{r} - \vec{r}_i|}{c} - \tau_i \right)}{|\vec{r} - \vec{r}_i|} \right|^2 dt \quad (r = \text{constant}) \quad (3.20a)$$

$$p(\theta) = \int_0^\infty \left| \sum_{i=1}^N A_i p_0 \left[t' - \frac{x_i (\sin \theta - \sin \theta_F)}{c} \right] \right|^2 dt' \quad (3.20b)$$

for near field and far field, respectively. This equation preserves the overall transmitted energy and takes the time duration of the pulse into account.

In many literatures (Smith *et al.*, 1991; Lockwood *et al.*, 1996(b); Schwartz and Steinberg, 1998; Trucco, 2002; Austeng and Holm, 2002(a, b)) the first method was used. It may be due to two reasons. First, this method is simple and requires less computation burden than the integration method. Second, for the narrowband field, this method is equivalent to the Fourier transform method. Since narrowband signals behave sinusoidally and continuously and only their amplitudes change at different points, Fourier transform also keeps the amplitude information only. In this thesis, the maximum projection method is adopted.

With the above simplifications, the time dimension and the distance dimension are hence removed from the broadband field response. The results are reduced to two-dimensional one for 1D arrays, which are the same as the narrowband case.

3.4 Characteristics of broadband radiation fields

A 1D phased array, which consists of 64 elements, working at 3.5 MHz, is used to illustrate the discussion in this section. The elements distribute in a line, the elemental pitch of this array is half wavelength. So the aperture length is around 32λ or 13.5mm.

The dimension of each element is ignored, and the elements work like point source. Assume all elements are uniformly weighted, for narrowband case, the far field beam pattern can be calculated with Equation (3.13). Fig. 3.2 shows the calculated pressure via spatial angle, where the pressure is normalized to the maximum value, which happens at focus angle. The focus angle of this beam pattern is 0° .

For broadband case, suppose the pulse $p_0(t)$ transmitted by each element is a 3 periods sine wave, weighted by Hanning window. The central frequency of this signal is 3.5 MHz , and its -6 dB relative bandwidth is 65% , as shown in Fig. 3.3. The beam pattern of the array focuses on $r_F = 100\text{ mm}$ and $\theta_F = 0^\circ$ can be calculated with Equation (3.15). This equation has no far field assumption, thus can be used for both near field and far field calculations. The field points to be calculated are distributed along a half circle with $r = 100\text{ mm}$, which is same as the focus distance. The time dimension is removed by Equation (3.19). Fig. 3.4 shows the beam pattern results.

Comparing Fig. 3.4 with Fig. 3.2, some differences can be found between the narrowband and the broadband response from the same array. Firstly, the well-known sidelobes in the narrowband response disappear in the broadband response, which is replaced by a sidelobe floor. This can be easily explained from the narrowband theory. The positions of the sidelobes in the narrowband response are determined by the frequency and the array configurations. Broadband signals consist of a wide range of frequency components, thus the sidelobes produced by each component appear at the different positions, and the summation of all the sidelobes from different frequency components results in a smoothed floor. Secondly, the main lobes above -10 dB of the two responses are almost the same. This is because the positions of the main lobes are only determined by the pre-

determined phase delays (for narrowband) or time delays (for broadband). Furthermore, the smoothing effect extends the main lobe region of broadband case on both sides of the focusing angle, while in the narrowband pattern, the near-in sidelobes start to appear below $-10dB$.

The beam width of the main lobe is an important parameter because it determines the lateral resolution of the transducer. So it needs further analysis. In the narrowband theory, the $-3dB$ beam width is always used to describe main lobe width, which corresponds to $\sqrt{1/2}$ amplitude decrease. The main lobe beam width is proportional to the wavelength and inversely proportional to the aperture length. However, for broadband case, the main lobe falls in a wider region, using $-3dB$ width is not always enough. For example, comparing the two responses shown in Fig. 3.5, which $p_0(t)$ are a 3 periods sine wave with and without Hanning weighting, the main lobe of the responses show different shapes but above $-3dB$ they are identical. Fig 3.6 shows another three responses, where $p_0(t)$ are 1, 2, 3 periods sine wave weighted by Hanning window, respectively.

The above comparisons indicate that not only the pulse length but also the pulse shape determines the main lobe width. This phenomenon can be explained in the time domain. Considering three extreme but simple cases, where $p_0(t)$ is a square wave, a triangular wave and a half circle wave, respectively. All of the above waves have the same time duration T and the amplitude A , as shown in Fig. 3.7. If the maximum time delay satisfies the condition $\tau_{\max} < T$ (Note $\tau_{\max}$ equals to zero at the focus point. As the field point deviates from the focus point, $\tau_{\max}$ increases), all the pulses from the elements will interfere. For the square wave, although the result wave shape may vary for different τ ,

the maximum amplitude always equals to the one at the focus point, which is $N \times A$. As $\tau_{\max}$ increases to be greater than T , only parts of the waves interfere, and then the maximum amplitude decreases from $N \times A$.

For the triangular and the half circle wave, even all the waves interfere, the amplitude of the resultant wave is always smaller than $N \times A$. This is because that the summation is performed on different part of the waves, where the signal values are different. The amplitude drops from $N \times A$ at the focus point to smaller value as field point becomes farther from the focus point and τ increases. The amplitude decrease rate is determined by the amplitude decrease rate of the original wave from the single element. Thus the main lobe shapes are similar as the original wave shapes, which can be observed from Fig. 3.8. The main lobe beam width also varies with the pulse shape.

The above analysis is based on very simple wave shapes. For more realistic broadband pulses, the conclusion still holds. This is because ultrasonic pulses are generated by the vibration ring down process of the transducer. There are always one to a few cycles of pulses has higher amplitude than other cycles. Like the pulse in Fig. 3.3, the middle cycle is the dominant cycle. The dominant cycle determines the characteristics of the main lobe. No doubt, as the time duration of the pulse increases, or the bandwidth decreases, more cycles of the pulse will affect the main lobe, and the main lobe coincides more with the narrowband case. From Fig. 3.6 this phenomenon can be seen.

The third difference between narrowband and broadband response is that the sidelobe levels are different. As mentioned in Section 3.2, the broadband field response is distance dependent. It is necessary to analyse the broadband patterns at different r values. Fig. 3.9

shows a series of responses when the array focus distances are 13.5mm , 27mm , 54mm and 108mm , respectively. If described by f number, they can be represented by $f1$, $f2$, $f4$ and $f8$. The responses have almost the same main lobes, but the sidelobes have different levels. As the focus distance decreases, the sidelobe level increases. This effect is obvious in near field. When the focus region falls in far field, the influence of the distance become very small. This can be seen by comparing the response of $f4$ and $f8$. This result is consistence with Equation (3.18).

The grating lobe does not appear in the above analysis. This is natural because the element pitch is not greater than half wavelength, and the Nyquist sampling theorem is satisfied. Thus in visible region, no grating lobes can be found. In order to analyse the grating lobes, the element pitch of the array is increased to λ in the following analysis. The off-axis beam patterns that focus on 45° are calculated and shown in Fig. 3.10 and Fig. 3.11 for narrowband and broadband respectively. For narrowband response, a grating lobe with same amplitude as main lobe can be found in Fig. 3.10. Its position agrees with the one predicted by the narrowband theory (Steinberg, 1976). However, for broadband case, it can be seen that the grating lobe does appear in the response, but its amplitude is -25dB lower than the main lobe (Fig. 3.11). The reason for the above phenomenon is similar as the sidelobes. The position of the grating lobes depends on the frequency and the element pitch, thus for the broadband arrays, the grating lobes spread in a wide range of angles rather than a single position and the amplitude also decreases.

Another interesting thing shown in Fig. 3.11 is that there is an almost flat sidelobe region between -35° and -90° . Note the amplitude of this part approximately equals $-20\log_{10}(1/N) = -30.1\text{dB}$, where $N = 32$. It means the amplitude of the signal almost

equals to the amplitude of the waveform generated by the single element. This is because in this region, the time delays are long enough that the dominant cycles of the waves from each element are well separated, and the interference will not increase the amplitude of the resultant waves. The resultant waves have long time durations, and obviously the energy of the signal from each element totally spreads in the time dimension at these field points. This phenomenon is demonstrated in Fig. 3.12. Schwartz and Steinberg, 1998, defined this field region as non-interference region. The pulse length and the shape generated by each element, and also the time delays determine the range of this non-interference region. The existence of the non-interference region leads to a type of array named *ultrasparse array* (Schwartz and Steinberg, 1998).

The non-interference region also explains why in near field the beam pattern has higher sidelobe at smaller focus distance. Note in Fig. 3.8, the influence of the distance to the sidelobe level becomes dramatic as the angle increases, especially at the region where $\theta > 60^\circ$. From the previous analysis, it is known that due to the short time duration pulse, in this region, the interfered wave spreads in time dimension. The maximum amplitude almost equals to the amplitude of the wave emitted from each element, weighted by the spherical propagation decays term $1/|\vec{r} - \vec{r}_i|$. In far field, this term can be approximated by $1/r$. It means the waves from each element have almost the same amplitude when they arrive in far field. Thus the side lobe level is independent of the distance r and the angle θ as long as it is in the non-interference region. However, in near field, Equation (3.6) is more suitable to replace the term $|\vec{r} - \vec{r}_i|$. The second term of the equation $x_i \sin \theta$ plays more and more important role as θ increases from 0° to 90° and r decreases. Thus $1/|\vec{r} - \vec{r}_i|$ may vary in a wide range in near field, especially as θ approaches to 90° .

The waves coming from each element will have different amplitude, which is weighted by $1/|\vec{r} - \vec{r}_i|$. As a result, the side lobe level is determined by the nearest element to the field point, which has the least decays and maximum amplitude. This amplitude is both r and θ dependent. So even in non-interference region, the side lobe level also varies with θ . The extreme case is that $\theta = 90^\circ$, the sidelobe reaches the maximum value, as shown in Fig. 3.9.

3.5 Conclusions

The analysis in this chapter show that the radiation field of the broadband arrays performs quite differently from the narrowband case. The reason is due to the short time duration of the pulses generated by array elements. The radiation field is fully spatial and time dependent. The pulse shape as well as the time duration of the wave generated by each element determines the beam pattern. Thus the sidelobe, grating lobe and main lobe, which determine the imaging quality, have different properties from the narrowband field response. The sidelobe floor replaces the sidelobe due to the wide range of frequency components. The grating lobes spread in a wide range and have lower amplitude than the main lobe for the same reason. The main lobe beam width is related to more factors than that of the narrowband case. Thus using narrowband theory to analyze broadband arrays is not sufficient, though based on a few assumptions the broadband field response can be simplified to the form same as that in the narrowband cases.

Figure 3.1 Coordinate system for array radiation field calculation

Fig 3.2 Narrowband beam pattern of 64 elements, $\lambda/2$ pitch array

Fig 3.3 3.5 MHz 65% relative bandwidth waveform and its spectrum

Fig 3.4 Broadband beam pattern of 64 elements, $\lambda/2$ pitch array

Fig. 3.5 Main lobes of field response. For solid line, $p_0(t)$ is Hanning weighted 3 periods sine wave. Dashed line is without weighting. Dotted line is narrowband response

Fig. 3.6 Main lobes of field response for different time durations of $p_0(t)$. Dotted line is narrowband response.

Fig. 3.7 Interference results of three different waveforms in main lobe region

Fig. 3.8 Main lobes for three different waveforms $p_0(t)$ shown in Fig. 3.7

Fig. 3.9 Beam patterns with different f numbers

Fig. 3.10 Narrowband beam pattern of 32 elements, λ pitch array Fig. 3.11 Broadband beam pattern of 32 elements, λ pitch array

Fig. 3.12 Interference of pulses in interference region and non-interference region (a) interference region (b, c) non-interference region

Chapter 4

Radiation Field Calculation for Array Transducers

The radiation field of broadband arrays needs to be calculated in time domain. The rigorous method is complicated and time-consuming, and is not suitable for array design and optimization. This chapter develops approximate methods that increase the efficiency and at the same time the accuracy is still acceptable.

4.1 Introduction

The radiation field directly determines systems imaging quality, as mentioned in previous chapter, and has always been one of the criteria to design and optimize array transducers. Thus an accurate and fast method to calculate this response is important. In Chapter 3, assuming array elements to be point sources reduces the complexity of the calculation procedure dramatically. However, results in Chapter 3 show that this assumption is too approximate, though it is enough to show the fundamental difference between broadband and narrowband fields. On the other hand, the method based on the closed form spatial impulse response to calculate the transducer field is rigorous but time consuming. This is because the discontinuity of impulse response requires high sampling frequency during numerical computing.

This chapter presents a fast method that simplifies the closed form method of calculating field response for array transducers. It is suitable for imaging array transducers in which the element size is in the order of half wavelength. Considering the element is small, this method avoids calculating the impulse response rigorously. Thus low sampling frequency

can be adopted and the calculation speed is increased dramatically. Comparing with the closed form results, the accuracy is still acceptable. Section 4.2 briefly introduces the closed form method. In Section 4.3, the simplified methods are described and compared with the rigorous solutions.

The methods are also extended to 2D array radiation field calculations in Section 4.4. The calculation for pulse echo two ways radiation field is discussed in Section 4.5. This is based on the linear model of imaging system and the acoustic reciprocal theorem. Section 4.6 is the conclusions for this chapter. The developed method will be used for sparse array optimization in Chapter 6.

4.2 Calculation of radiation field based on spatial impulse response

This section derives the calculation procedure of the radiation field of 1D arrays rigorously, which is based on the spatial impulse response method. This method was well developed (Stepanishen, 1971), thus the derivation is only introduced briefly for the convenience of further discussion and comparison. Closed form analytical solutions of transducers with regular geometries can also be found in the literature. For example, Xue, *et al.* (1996) gives the solutions for circular transducers. Ullate and Emeterio (1992) provide the solutions for rectangular pistons, which will be used for 1D arrays in this chapter.

The coordinate system is shown in Fig. 4.1, where a 1D array consisting of N elements is aligned along the x axis. In this chapter, both the dimension of the array in elevation direction (y axis) and in azimuth direction (x axis) are considered to improve the

accuracy from the results of Chapter 3, where the dimension of the array in elevation direction is ignored. All the field points to be calculated are all limited in $x-z$ plane, because the beam can only be manipulated in this plane by 1D arrays. To calculate the field response, applying the Rayleigh integral over the whole transducer surface, one can obtain

$$p(\vec{r}, t) = \frac{\rho}{2\pi} \frac{\partial}{\partial t} \iint_S \frac{v(t - \frac{|\vec{r} - \vec{r}_s|}{c})}{|\vec{r} - \vec{r}_s|} dS \quad (4.1)$$

Compared with Equation (3.1), it can be found that the line integral in Equation (3.1) is replaced by the surface integral, and S is the area of the transducer surface, $\vec{r}_s$ is the position vector in the transducer surface. Similar to the derivation in Chapter 3, assuming all array elements perform piston like vibration and the vibration profiles are same, only the amplitudes may be different because of the apodization. The time delay for steering and focusing of each element is defined as τ_i for the i th element, using Dirac delta function and the convolution operator $*$, Equation (4.1) changes to

$$p(\vec{r}, t) = \frac{\rho}{2\pi} \frac{\partial v_0(t)}{\partial t} * \sum_{i=1}^N A_i \iint_{S_i} \frac{\delta(t - \frac{|\vec{r} - \vec{r}_s|}{c} - \tau_i)}{|\vec{r} - \vec{r}_s|} dS \quad (4.2)$$

or

$$p(\vec{r}, t) = p_0(t) * h(\vec{r}, t) \quad (4.3)$$

If using Equation (3.14) and defining,

$$h(\vec{r}, t) = \sum_{i=1}^N A_i \iint_{S_i} \frac{\delta(t - \frac{|\vec{r} - \vec{r}_s|}{c} - \tau_i)}{2\pi |\vec{r} - \vec{r}_s|} dS \quad (4.4)$$

The physical meaning of Equation (4.4) can be explained as following, if the transducer is excited by a unit impulse, the field response equals the superposition of all impulses

propagate to the given spatial point $\vec{r}$ at given instant t from each point in the transducer surface. $h(\vec{r}, t)$ is defined as the spatial impulse response, which is a function of space and time, and depends on the configuration of the array only. This expression is also another form of Huygen's principle (Steinberg, 1976). With this definition, the field response is successfully separated to two parts, where $p_0(t)$ is related to the external excitation only, and $h(\vec{r}, t)$ is the inherent properties of the array. Once the impulse response is determined, the field response with any excitation signals can be easily calculated with Equation (4.3).

So far the key point of the problem is converted to the evaluation of $h(\vec{r}, t)$, which is defined by Equation (4.4). For array transducers, since each element has same geometric shape and dimension, the evaluation procedure can be simplified further. For a single element, one can define

$$h_i(\vec{r}, t) = \iint_{S_i} \frac{\delta(t - \frac{|\vec{r} - \vec{r}_s|}{c})}{2\pi|\vec{r} - \vec{r}_s|} dS \quad (4.5)$$

as the single element spatial impulse response. Note the fact that whatever the element shape, at any given field point P and time t , the impulse response is only related to the aperture points that lie in the intersection of the aperture surface with a sphere whose centre is at P and whose radius is $|\vec{r} - \vec{r}_s| = ct$. For a planar aperture, this intersection is one piece of arc or a few segments of arcs, as illustrated in Fig. 4.2. Thus the surface integral can be easily evaluated, and it is proportional to the arc angle or the summation of arc angles of each segment of arc (Stepanishen, 1971).

$$h_i(\vec{r}, t) = \frac{c}{2\pi} \cdot \Omega(\vec{r}, t) \quad (4.6)$$

where $\Omega(\vec{r}, t)$ is the arc angle, see Fig. 4.2.

The element of 1D arrays has a narrow rectangular shape. Ullate and Emeterio (1992) provide the closed form analytical expression of its impulse response without any paraxial or far-field approximation. The detailed equations can be found in Appendix A of this thesis. Some examples of the impulse responses at certain field points ($r = 25mm$) are shown in Fig. 4.3. From this figure and the expressions for impulse response calculations, it can be observed that the response at any points is a time-limited, amplitude bounded and piecewise continuous function. The discontinuities lie in the time instant when the arc is tangential to the aperture boundary or reaches the vertices of the aperture.

With the element impulse response, the 1D array impulse response and radiation field due to any excitation $p_0(t)$ can be calculated rigorously with the following equation

$$p(\bar{r}, t) = p_0(t) * \sum_{i=1}^N A_i \cdot h_i(\bar{r}, t - \tau_i) \quad (4.7)$$

The above method to calculate field response is exact for both near field and far field within the restriction imposed by the Rayleigh integral. Unfortunately, the discontinuities of impulse response require high sampling frequency for numerical calculation. The selection of sampling frequency highly depends on the position of field points and the aperture size. The paraxial points and small aperture size always need much higher frequency to fully distinguish the time intervals among the instants of discontinuities. For example, for the array working at the frequency $3.5 MHz$, the half wavelength element width is around $0.2mm$. To calculate the impulse response of the point located at $x = 0.15mm$ and $z = 25mm$, the time intervals between the control time instants τ_A and

τ_B calculated with Equation (A.1) is $5.7 \times 10^{-9} s$. This means a few *GHz* sampling frequency is necessary to ensure the accuracy. Such high sampling frequency always leads to a time consuming procedure to calculate the array response rigorously.

4.3 Approximate methods for radiation field calculation

The analysis in Section 4.2 shows the exact method is rigorous and accurate but with very low efficiency for array transducers. However, it may not be suitable for array design and optimization, particularly for 2D array design. The main reason is that the half wavelength constraint on the element dimensions makes the response from each element has very short time duration. In order to ensure the accuracy, the sampling frequency must be high enough. To increase the computation speed, this section considers two approximate methods, which avoid computing the element impulse response directly. Thus computational complexity is reduced and at the same time lower sampling frequency can be used.

A 1D array is used as the example in this section to illustrate the derivation. This array consists of 64 elements, working at $3.5 MHz$. The element pitch is 0.5λ , and the element width is 0.4λ . The element height, which is the dimension in elevation direction of the array, is $10mm$. The initial pressure waveform $p_0(t)$ is still Hanning weighted three periods sine wave, same as the waveform shown in Fig. 3.3.

4.3.1 Method 1 - modified point source model

For 1D arrays, the element width is much smaller than the height. One natural assumption is to ignore the element height. The whole calculation can be reduced to $x-z$ plane.

Furthermore, in this plane, the impulse response from each element can be replaced by a single impulse without time duration approximately because of the much smaller element width compared with the propagation distance. Thus the procedure to calculate the impulse response is simplified dramatically. The time instant of the impulse can be calculated directly from the distance between the element and the field point. The amplitude of the impulse is related to two factors, the first is the element directivity, i.e. the variation of the amplitude with the angle, which can be described by the element beam patterns. Another is the spherical propagation decay.

Since the element width is 0.4λ , which is much smaller than the pulse length of $p_0(t)$, which is 3λ , from the analysis of the previous chapter, the single element can be treated as the narrowband source. This is because the time delays of the pulses from different part of the source are much smaller relative to the pulse length. At any field point, all of the pulses will interfere with each other. The resultant pulse will have similar waveform as $p_0(t)$, but different amplitude. Thus the beam pattern of the single element can be predicted by the narrowband theory. For the single element, this pattern is only angle dependent in far field (For the single element, the far field condition is easily satisfied), and can be calculated by the Fourier transform. For a line source, the Fourier transform results in a simple **sinc** function (Steinberg, 1976),

$$A(\theta_i) = \text{sinc}\left[\frac{a}{\lambda} \sin(\theta_i)\right] \quad (4.8)$$

where a is the element width, and θ_i is the viewing angle as shown in Fig. 4.2.

On the other hand, in the far field, the amplitude of the pulse at any given point is inversely proportional to the propagation distance. So for the second factor, the amplitude

due to the spherical propagation decay can be simplified to a factor $1/|\vec{r} - \vec{r}_i|$. Based on the above assumption and simplifications, array impulse response in Equation (4.4) can be written as

$$h(\vec{r}, t) = \sum_{i=1}^N \frac{A_i \cdot \text{sinc}\left[\frac{a}{\lambda} \sin(\theta_i)\right] \cdot \delta\left(t - \frac{|\vec{r} - \vec{r}_i|}{c} - \tau_i\right)}{2\pi |\vec{r} - \vec{r}_i|} \quad (4.9)$$

Using Equation (4.3) to calculate pressure one can get

$$p(\vec{r}, t) = p_0(t) * \sum_{i=1}^N \frac{A_i \cdot \text{sinc}\left[\frac{a}{\lambda} \sin(\theta_i)\right] \cdot \delta\left(t - \frac{|\vec{r} - \vec{r}_i|}{c} - \tau_i\right)}{2\pi \cdot |\vec{r} - \vec{r}_i|} \quad (4.10)$$

Comparing Equation (4.10) with Equation (3.15), it can be found the only difference between these two equations is a correcting factor $A(\theta_i)$, which is defined by Equation (4.8). So Method 1 is in fact a modification of point source model, which considers the element directivity to improve the accuracy of simulation results.

4.3.2 Method 2 – based on single element characteristics

Based on the idea of Method 1, if analyzing the radiation field results calculated by the rigorous impulse response method, it can be shown that the point source model can be improved further.

Firstly, consider the angular variation of pulse from the single element. Fig. 4.3 and 4.4 show the impulse response and radiation pressure waveform from the single array element at the points where $r = 25\text{mm}$ and $\theta = 0^\circ, 30^\circ, 60^\circ$ and 90° , respectively. It can be seen from Fig 4.4 that the waveforms at different angles vary little, in terms of their shapes and time durations. Only the amplitude changes with angle. This phenomenon can

be explained by their impulse responses as shown in Fig. 4.3. The amplitudes of these responses drop as the angle increases from 0° to 90° . But their shapes share one feature which is a high amplitude spike followed by amplitude-decreasing part. Thus the convolution to calculate the pressure just results in a distorted and scaled replica of the original pressure wave $p_0(t)$. Comparing Fig. 4.3 and Fig. 3.3, one can find the waveforms are similar but not completely the same. The amplitude-decreasing part is the reason of distortion. The existence of this part is due to the element height.

Furthermore, look at the variation of amplitude of pulse in the angular direction. Fig. 4.7 shows the directivity of the single element, which is calculated by the impulse response method. Comparing with the result by the narrowband theory shown in the same graph, it can be found that the two curves are coherent very well, and only small deviation can be observed when the curves are magnified. If the element directivity is defined as $A(\theta)$, the pressure waves at different angles from the single element can be described approximately by the following equation,

$$p(\theta, t) = p_r(t) \cdot A(\theta) \quad (4.11)$$

where $p_r(t)$ is the normalized waveform at given distance r and calculated at arbitrary angle θ .

Secondly, consider the variation of the pulse along the wave propagation direction. Fig. 4.6 shows a series waveforms at the same angle $\theta = 30^\circ$ but the different distance r . The corresponding impulse response is shown in Fig. 4.5. The influence of the element height to the pulse variation can be observed in two aspects. The first one is that the distortion effect to the waveform becomes weak as r increases. This is because the time duration of the impulse response decreases as r increases. The amplitude-decreasing parts affect less

to the pressure waveform. When $r > 40mm$, the waveform is almost resembled to $p_0(t)$, as shown in Fig.3.3. The second effect is that although the amplitude of the pulses still decreases along r , the spherical decay of amplitude in the propagation direction, which is observed in Chapter 3, does not hold now. To further study this effect, the normalized variation of the pressure amplitude along the wave propagation direction r at $\theta = 30^\circ$ and $\theta = 60^\circ$ are fully calculated and displayed in Fig. 4.8. r ranges between $1mm$ and $250mm$. This figure indicates that only when $r > 60mm$ the amplitude is roughly inversely proportional to r . The variation in the region that $r < 60mm$ is not easy to be described by simple equations.

On the other hand, it also can be seen from Fig. 4.8 that the normalized variations of amplitude at $\theta = 30^\circ$ and $\theta = 60^\circ$ are almost the same. This is because the element height is much greater than the element width. It has almost the same influence on the pulses at different θ . This phenomenon can also be explained by the element directivity. Since the directivity keeps intact at different r , the pressure amplitude along the propagation direction must vary with same trend at different θ theoretically. Thus, similar as the element directivity, the relation between the amplitude and the distance can also be defined as $A_r(r)$, which is totally angle independent.

Based on the above analysis on the characteristics of a single element, another approximate method to calculate radiation field from array transducers can be derived. Firstly, the elemental impulse response defined by Equation (4.5) can be simplified to

$$h_i(\vec{r}, t) = A_i \cdot A(\theta_i) \cdot A_r(r_i) \cdot \delta\left(t - \frac{|\vec{r} - \vec{r}_i|}{c} - \tau_i\right) \quad (4.12)$$

And array impulse response and radiation field are

$$h(\vec{r}, t) = \sum_{i=1}^N h_i(\vec{r}, t) \quad (4.13)$$

$$p(\vec{r}, t) = \sum_{i=1}^N p_{r_i}(t) * h(\vec{r}, t) \quad (4.14)$$

respectively.

It also can be found from Fig 4.6 than the waveform $p_r(t)$ does not change very much as r increases, especially the dominant cycles of $p_r(t)$ almost keep intact. So Equation (4.14) can be further simplified to

$$p(\vec{r}, t) = p_r(t) * \sum_{i=1}^N h(\vec{r}, t) \quad (4.15)$$

i.e., using the waveform from the central element to represent the waveforms from all elements.

Equation (4.15) dramatically reduces the computation time of the array field response due to two reasons. The first is that the calculation procedure is much simplified. The exact calculation of the impulse response is avoided, replaced by the summation of a series impulses from elements weighted by A_i , $A(\theta_i)$ and $A_r(r_i)$. The last two weighting coefficients can be obtained from the single element calculation. This is especially useful when Equation (3.19) is used to calculate the arrays beam pattern at the given r value. The second reason is that a lower sampling frequency can be used compared with that in the rigorous method. The minimum value of time delay τ_i becomes the control factor to select the sampling frequency in this method. Thus a few hundreds MHz sampling frequency is sufficient.

4.3.3 Comparison of approximate methods and accurate method

To compare the accuracy and computation time of Method 1, Method 2 and the exact impulse response method (Imp), the radiation field of the 1D array is calculated by all of the above three methods. The point source model (PS) is also considered. All simulations are performed with a *DELL* Precision workstation 620. The sampling frequency is 400MHz for Method1, Method2 and PS, and 4GHz for the exact method.

The pressure waveforms for the on axis points where $\theta = 0^\circ$ and $r = 25mm$, $50mm$ and $100mm$ are calculated by all four methods first to examine the transient pressure waveforms at the field points. The array focuses at $\theta = 45^\circ$, and all results are normalized to the amplitude at focus point calculated by each method, respectively. Results are shown in Fig. 4.9 to Fig. 4.11, and corresponding computing time used are listed in Table 4.1 for comparison. The relative errors of the maximum amplitude of each result are listed in Table 4.2, which are calculated by the following equation,

$$\text{error\%} = \frac{|\max|p_i(t)| - \max|p_{\text{Imp}}(t)||}{\max|p_{\text{Imp}}(t)|} \% \quad (4.16)$$

where $p_{\text{imp}}(t)$ is the result by the exact method, $p_i(t)$ is by other methods.

It can be seen from Table 4.1 that the point source model and Method1 are the fastest. The computation time is at the order of tens of microseconds for each point. Method 2 and the exact method are much slower, and they all need tens of seconds for each point. No doubt for single field point calculation, Method 2 has only limited advantage on efficiency when compared with the exact method. However, it should be noted that the time used by Method 2 is almost constant for different points. This is because almost all

of the time is spent on the calculation of $A(\theta)$ and $A_r(r)$ of the single element using the exact method. If $A(\theta)$ and $A_r(r)$ are pre-calculated, the time used by this method is also less than ten microseconds, which is the same as that with the point source model and Method 1, as shown in Table 4.1. The exact method has the lowest efficiency because of the complicated calculation procedure and the high sampling frequency.

The three approximate methods show different accuracy as shown in Table 4.2. Method 1 has poor accuracy, either in terms of the amplitude or the shape of pulses, especially when r is small. This is mainly due to the pulses that reach the field points from each element has much different shapes from $p_0(t)$ (comparing Fig. 4.4 with Fig. 3.3) when the field points are close to the transducer surface. Thus the superposition results also deviate from the result of the exact method. Its shape is the same as the one calculated by point source model, because they use the same approximation of $p_0(t)$. It can be observed from Fig. 4.9, (a) and (b) have same waveforms, which are different from the waveform by the exact method, which is shown in (d). Nevertheless, the amplitude accuracy is dramatically improved from the point source model, as indicated in Table 4.2. It means the correcting factor defined by the element directivity in Equation (4.8) is effective. As r increases, the shape of pulse from the single element becomes close to $p_0(t)$. The accuracy is improved, as demonstrated in Fig 4.11.

It is evident that the result from Method 2 is a good approximation to the exact impulse response method. Even in the region close to the transducer, the accuracy is still good, with the amplitude error around 2–3%. The pulse shapes are also accurate if compared with the exact method, both in near field and far field. This method is much more suitable to calculate arrays beam patterns, in which the field points are all located with the same

distance to the transducer centre. In this case the element parameters described by $A(\theta_i)$ and $A_r(r_i)$ can be pre-calculated. Then for each field point calculation, the parameters can be read in directly and the computing speed will be same as the point source model. In this way, the calculation time for one beam pattern ranges between -90° and 90° is only about one minute. To calculate beam pattern in the same range with the exact method, it commonly needs one to two hours. Obviously, Method 2 greatly improves the efficiency.

Fig. 4.12 and Fig. 4.13 show the beam patterns calculated by the above four methods, and the array focuses at $\theta_F = 0^\circ$ and $\theta_F = 30^\circ$, respectively. The radiation field is simplified to $p(\theta)$ using the methods introduced in Chapter 3. It can be seen that all of the methods are very accurate in the main lobe region. However, as the angle increases, the results by the point source model and Method1 start to deviate from the exact results. Because of their simplicity, these two methods may still be used in the situation when the accuracy requirements are not very strict. If the field points fall in far field, Method 1 also has enough accuracy and can be used to replace the exact method for its high efficiency. It can also be seen from Fig. 4.12 and Fig. 4.13 that the results by Method 2 agrees very well with the exact results in the full field region. When the performance of single element is determined, it has both very high efficiency and accuracy. This method is thus suitable for array design and optimization.

4.4 2D arrays radiation field

The algorithms derived in the above sections can be extended to 2D arrays. In 2D array transducers, array elements are distributed in two dimensions in the transducer plane.

Depending on the different fabrication techniques, the whole transducer apertures could be in various shapes (Mendelsohn and Wiener-Avnear, 2002). Using the traditional dice-and-fill method, 2D square arrays can be made, in which the elements are located on a 2D grid, with the same distance intervals that are defined as pitch. The shape of each element is usually square with same dimensions on both sides in order to achieve the same performance in both azimuth and elevation direction. Thus both element height and width are all limited by the half wavelength constraint in order to avoid grating lobes in the 3D radiation field. Theoretically, the beam from a 2D array transducer can be fully controlled in semi-infinite 3D region. 2D arrays are one of the potential solutions for 3D real time imaging systems. This is the most attractive advantage for 2D arrays. This section discusses the 2D square arrays only. Due to the small array elements, approximate method to calculate the radiation field also can be adopted.

The coordinate system for 2D array calculations is shown in Fig. 4.14. The array is located in $x - y$ plane. The steering angle is defined as θ in azimuth plane ($x - z$ plane) and ϕ in elevation plane ($y - z$ plane). Different from the 1D case, the beam can be controlled in both directions, and the field points are no longer limited within the azimuth plane. Thus the radiation fields of 2D arrays are five dimensional data $p(r, \theta, \phi, t)$. Using the methods described in Chapter 3, r can be removed by properly selecting the field point positions and t can be removed by the maximum projection. So after simplification, a three-dimensional curve surface can be used to demonstrate the 2D arrays beam patterns $p(\theta, \phi)$.

Although the exact spatial impulse response method can be used directly for 2D arrays, the procedure is much more complicated than the 1D case even for a single element. This

is because there are more possible relative positions between the field points and the elements than that of 1D case. On the other hand, the number of elements in 2D arrays is the square of the number of elements in the equivalent 1D arrays, and it also increases the simulation time dramatically. From the results of Section 4.3, it can be seen that the computing time for 2D arrays by the exact method will be much longer and is difficult to be used practically.

However, since the element size is small, the following study shows the calculation of the exact impulse response is not necessary for 2D arrays. Consider two special relative positions between the field point and the element as the examples, where the exact impulse responses can be calculated easily (Fig. A.2). The expressions of the impulse response at these two positions are shown in Appendix A. With this expression, the transient pressure pulse at these points can be calculated accurately. Suppose the array element height and width are $\lambda/2$ and initial $p_0(t)$ is the same as the one shown in Fig. 3.3. The normalized waveforms arrival at the field points $(3mm, 0, 3mm)$ and $(3mm, 3mm, 3mm)$ are shown in Fig. 4.15 and 4.16, compared with the normalized $p_0(t)$. To ensure the accuracy, the sampling frequency used is $10GHz$. It can be observed that even when the field points are very close to the element, the waveforms keep almost same shape as $p_0(t)$. No doubt, this is because the small element size generates little distortion on the waveforms. At the same time, the far field condition for the single element is easily satisfied, so the amplitude variation along the wave propagation direction is inversely proportional to the distance at the given angles. This can be observed from Fig. 4.17, which shows the pressure amplitude variations along the wave propagation direction.

On the other hand, for the same reason as 1D arrays, the elemental directivity can be predicted by the narrowband theory (Angelsen, 2000),

$$A(\theta, \phi) = \text{sinc}\left[\frac{a}{\lambda} \cdot \sin(\theta)\right] \cdot \text{sinc}\left[\frac{a}{\lambda} \cdot \sin(\phi)\right] \quad (4.17)$$

where a is the element dimension.

Based on the above analysis, the spatial impulse response and the radiation field from 2D square arrays, which consist of $N \times N$ elements, can be approximately calculated by the following equations,

$$h(\vec{r}, t) = \sum_{i=1}^N \sum_{j=1}^N \frac{A_{i,j} \cdot A(\theta_{i,j}, \phi_{i,j})}{2\pi |\vec{r} - \vec{r}_{i,j}|} \cdot \delta\left(t - \frac{|\vec{r} - \vec{r}_{i,j}|}{c} - \tau_{i,j}\right) \quad (4.18)$$

$$p(\vec{r}, t) = p_0(t) * \sum_{i=1}^N \sum_{j=1}^N \frac{A_{i,j} \cdot A(\theta_{i,j}, \phi_{i,j})}{2\pi |\vec{r} - \vec{r}_{i,j}|} \cdot \delta\left(t - \frac{|\vec{r} - \vec{r}_{i,j}|}{c} - \tau_{i,j}\right) \quad (4.19)$$

where the subscript i, j represent the element lies in the i th row and j th column. $A_{i,j}$ is the apodization coefficient for this element. $\vec{r}_{i,j}$ is element vector and $\theta_{i,j}, \phi_{i,j}$ is viewing angles from the field point to the given element, as shown in Fig. 4.18. Obviously, the above two equations is the extension of Method 1 to 2D arrays. However, the accuracy can be ensured for 2D arrays because the elements have small size in both two dimensions. While for 1D arrays, the element height leads to errors in Method 1.

Since to calculate the beam patterns using the exact impulse response method for 2D arrays is very time-consuming, the results by the above method are compared with another widely used software *Field II* (Jensen and Svendsen, 1991). This software is also an approximate algorithm based on the impulse response method. It uses a shape of trapezoid to replace the exact impulse response solution of the rectangular sources in far

field. Then only four control points values of the trapezoid need to be calculated. The transducer surfaces with different shapes or areas are divided into many small rectangular elements first. The far field condition is satisfied for each divided element. And further, by using the integrated spatial impulse response as an intermediate step in the calculations, the necessary sampling frequency is also reduced to the order of hundred MHz .

The 2D array used in the simulation consists of 20×20 square elements, and the elemental dimension is 0.4λ . The kerf between the neighbouring elements is 0.1λ , thus the element pitch is half wavelength. $p_0(t)$ is still Hanning weighted three periods sine wave, as shown in Fig. 3.3. The sampling frequency used in *Field II* is $150MHz$, somewhat higher than the recommended value. Each array element is divided into 4×4 elements during simulation. For the method developed in this chapter the sampling frequency is $400MHz$. A few cases have been calculated and compared, and the normalized mean square errors between the results of these two methods are within 2%. Although the developed method uses higher sampling frequency than that of *Field II*, the computing time used is only about one tenth of that used by *Field II* on average. The results of the two example cases are shown in this chapter. For the first case, the results are shown in Fig. 4.19, where the array focuses on axis. The results that the array focuses off axis, where $\theta, \phi = 30^\circ$ are shown in Fig. 4.20. All the field points calculated have the same distance to the array centre, in which $r = 40mm$, and the steering angles range between $\pm 45^\circ$. The results are all normalized to the pressure values at the focus points. From the figures it can be seen the results by the two methods match very well. Some numerical noise can be observed from the *Field II* results in Fig. 4.20. Further increasing the sampling frequency can reduce the noise, but at the price of longer simulation time. In

this case, the calculation time for each beam pattern is 8s to 10s , while *Field II* needs more than one minute.

4.5 Pulse echo radiation field

In medical imaging applications, the ultrasonic transducer always works in pulse echo mode. It generates and transmits ultrasonic waves, then receives the scattered waves from the loading materials. The received signals contain the information of imaging objects. In order to fully study the performance of transducers, the pulse echo two-way radiation field calculation is necessary. Based on the previous discussions in this chapter and the linear model of imaging system, the two-way field can be easily derived.

A pulse echo imaging system can be modelled as a linear system, as shown in Fig. 4.21 (Schwartz and Steinberg, 1998). With this model, the two-way radiation pressure can be described by the following equation,

$$p_{TR}(\vec{r}, t) = p_0(t) * h_T(\vec{r}, t) * s(\vec{r}, t) * h_R(\vec{r}, t) \quad (4.20)$$

where the subscript T, R represent transmitting and receiving aperture, $s(\vec{r}, t)$ is the scatterers in the field. The impulse response of transmitting aperture $h_T(\vec{r}, t)$ and receiving aperture $h_R(\vec{r}, t)$ are not necessary the same due to the possible different aperture designs or the different apodization functions. The widely used dynamic focusing technique in receiving procedure is also a possible reason.

Since the emphasis of the research work is transducers, the scatterers in the field is assumed to be a rigid point in this thesis, and it is time independent. Further, this point only reflects the incident waves and does not perturb the field distribution, and the

reflected strength from this point is angularly independent from the angle of the incident waves. In this way, the term $s(\vec{r}, t)$ is removed from Equation (4.20), and the pulse echo radiation field can be simplified to

$$p_{TR}(\vec{r}, t) = p_0(t) * h_T(\vec{r}, t) * h_R(\vec{r}, t) \quad (4.21)$$

In the literatures, Equation (4.21) is also called point spread function (PSF). It fully describes the pulse-echo properties of transducers. It should be noted here, in this equation, the electromechanical effect of the transducer element is not considered. If the electromechanical response of the transducer elements $h_{em}(t)$ is known, the more realistic pulse-echo response can be calculated,

$$v_{TR}(\vec{r}, t) = p_{TR}(\vec{r}, t) * h_{em}(t) \quad (4.22)$$

where $v_{TR}(\vec{r}, t)$ is the voltage signal generated by the transducers due to the received pressure signal $p_{TR}(\vec{r}, t)$.

Equation (4.21) and (4.22) can be used for both 1D and 2D arrays based on the linear assumptions. It is clear that the key procedure of calculation is still the calculation of spatial impulse response of arrays. The transmitting impulse response has been discussed in detail in previous sections. For the receiving impulse response, it is possible to derive the calculation directly like the transmitting case. However, if considering acoustic reciprocal theorem (Kinsler and Coppens, 1982), the transmitter and receiver can be replaced with each other, then this response can be calculated with the equivalent transmitting case. Thus all the developed methods in Section 4.2 to Section 4.4 can be used directly.

Obviously, the broadband pulse-echo field responses are also both spatial and temporal dependent. With the same methods introduced in Section 4.2, the responses can be reduced to simpler angular patterns. The patterns work as the criteria to design and optimise array transducers used in imaging systems. From it, the two way main lobe beam width, the secondary lobe level can be calculated, which directly determine pulse-echo imaging qualities. The broadband responses are also different from the narrowband ones. It is well known that the narrowband two-way patterns in far field are simply the multiplication of transmitting pattern and receiving pattern (Steinberg, 1976). However it does not hold for broadband arrays. It means many developed techniques for narrowband arrays such as the effective aperture methods (Lockwood *et al.* 1996 a, b) will not be valid for broadband arrays.

4.6 Conclusions

This chapter discusses and compares four different algorithms to calculate the radiation field from array transducers. The rigorous method is accurate but very time-consuming, especially for 2D arrays calculations. However, considering that the array elements are smaller than one wavelength, it is possible to use approximate methods to make a trade-off between the accuracy and the efficiency.

The point source model avoids calculation of the exact impulse response, thus has great efficiency. However for 1D arrays, due to the effect of the element height, the point source model has unacceptable errors, even after correcting the factor of element directivity (Method 1). The method developed in this chapter (Method 2) increases the accuracy by using the characteristics of the single element. The waveform at a given distance, the element directivity and the variation of the pressure amplitude along the

wave propagation direction can be first calculated from the single element. The results can then be used in array calculations. The calculation of the exact solution of the array impulse response is therefore avoided. If the single element results are pre-calculated, this method can have the same efficiency as that using the point source model. The accuracy is shown to be good even in the near field compared with the rigorous method.

For 2D arrays, since both sides of the array element are small, the developed method can be further simplified. Only the element directivity is necessary to be used as the correction factor. Compared with the results of widely used simulator *Field II*, the accuracy is within 2% . However, the calculating speed is dramatically increased by one order of magnitude on average. This makes the optimization of 2D array transducer possible, as it is shown in Chapter 6.

Table 4.1 Time used to calculate single field point pressure response by different methods (time unit: s)

	$r = 25mm$	$r = 50mm$	$r = 100mm$
point source model	0.04	0.07	0.09
Method 1	0.04	0.07	0.09
Method 2	21	22	22
Method 2*	0.05	0.07	0.09
exact method	32	45	56

* Not including the time for calculation of first single element

Table 4.2 Amplitude errors (%) relative to the exact method

	$r = 25mm$	$r = 50mm$	$r = 100mm$
point source model	17.9	16.1	7.0
Method 1	8.4	5.8	4.1
Method 2	3.5	3.5	2.7

Fig. 4.1 Coordinate system for 1D array radiation field calculation

Fig. 4.2 Impulse response of single array element

Fig. 4.3 Impulse response from a single array element, at $r = 25\text{mm}$ for different angles, (a) $\theta = 0^\circ$ (b) $\theta = 30^\circ$ (c) $\theta = 60^\circ$ (d) $\theta = 90^\circ$

Fig. 4.4 Pressure waveform from a single array element, at $r = 25\text{mm}$ for different angles, (a) $\theta = 0^\circ$ (b) $\theta = 30^\circ$ (c) $\theta = 60^\circ$ (d) $\theta = 90^\circ$

Fig. 4.5 Impulse response from a single array element, at $\theta = 30^\circ$ for different distances, (a) $r = 25\text{mm}$ (b) $r = 40\text{mm}$ (c) $r = 60\text{mm}$ (d) $r = 100\text{mm}$

Fig. 4.6 Pressure waveform from a single array element, at $\theta = 30^\circ$ for different distances, (a) $r = 25\text{mm}$ (b) $r = 40\text{mm}$ (c) $r = 60\text{mm}$ (d) $r = 100\text{mm}$

Fig. 4.7 The element directivity predicted by narrowband theory (dashed line) and impulse response method (solid line)

Fig. 4.8 Variation of pressure amplitude along wave propagation direction at $\theta = 30^\circ$ (solid line) and $\theta = 60^\circ$ (dashed line)

Fig. 4.9 Pressure waveforms at field point $r = 25\text{mm}$, $\theta = 0^\circ$ from 1D array focuses at $\theta = 45^\circ$, calculated by (a) point source model (b) Method 1 (c) Method 2 (d) exact method

Fig. 4.10 Pressure waveforms at field point $r = 50\text{mm}$, $\theta = 0^\circ$ from 1D array focuses at $\theta = 45^\circ$, calculated by (a) point source model (b) Method 1 (c) Method 2 (d) exact method

Fig. 4.11 Pressure waveforms at field point $r = 100\text{mm}$, $\theta = 0^\circ$ from 1D array focuses at $\theta = 45^\circ$, calculated by (a) point source model (b) Method 1 (c) Method 2 (d) exact method

Fig. 4.12 Beam pattern of 1D array focuses at $r = 25\text{mm}$, $\theta = 0^\circ$ calculated by four different methods

Fig. 4.13 Beam pattern of 1D array focuses at $r = 25\text{mm}$, $\theta = 30^\circ$ calculated by four different methods

Fig. 4.14 Coordinate system for 2D array radiation field calculation, $x-z$ plane is defined as azimuth plane and the steering angle in this plane is θ . $y-z$ plane is elevation plane, steering angle is ϕ

Fig. 4.15 Normalized waveform received at field point (3mm,0,3mm) from single 2D array element (solid line), compared with $p_0(t)$ (dashed line).

Fig. 4.16 Normalized waveform received at field point (3mm,3mm,3mm) from single 2D array element (solid line), compared with $p_0(t)$ (dashed line).

Fig. 4.17 Variation of pressure amplitude along wave propagation direction from a single 2D array element, calculated by exact impulse response method.

Fig. 4.18 Viewing angles $\theta_{i,j}$, $\phi_{i,j}$ for single 2D array element

Fig 4.19 Beam patterns of 2D array focuses at $r = 40\text{mm}$, $\theta, \phi = 0^\circ$ calculated by (a) *Field II* (b) Method developed in this chapter

Fig 4.20 Beam patterns of 2D array focuses at $r = 40\text{mm}$, $\theta, \phi = 30^\circ$ calculated by (a) *Field II* (b) Method developed in this chapter

Fig. 4.21 Linear model of pulse-echo imaging system

Chapter 5

Element Cross Coupling and Its Influence on Radiation Field

In this chapter, finite element method is used to study some characteristics of the array transducers, in particular, the cross coupling of the array elements and their effects on the radiation field of the array transducers.

5.1 Introduction

For medical piezoelectric array transducers, the array elements always have multilayer structure to achieve broadband performance. In-depth knowledge of the behaviour of the single element is important in array analysis and design. Furthermore, inter element cross coupling is another important phenomena for array transducers. High level cross coupling will seriously affect arrays radiation field. With FEM, the cross coupling can be fully studied. If the whole array transducers and loading materials are modeled simultaneously, the radiation field considering cross coupling effect can be simulated. This provides a more realistic prediction of arrays radiation field. And it also provides a tool to analyze the relationship between cross coupling and arrays radiation field.

The mechanism of the cross coupling in array transducers is quite complicated and it is difficult to study it analytically. Experiment and FEM are the two methods commonly adopted for this study (Adachi and Funakubo, 2000). However, the analysis of cross coupling and its influence on radiation field are always conducted in two separate steps. The first step focuses on the dynamic characteristics of array transducers, and then the

obtained results are used as the input to some extrapolation algorithms to calculate the radiation field (Assaad and Bruneel, 1997; Gori *et al.*, 2001). In this chapter, a hybrid algorithm based FEM code is used to simulate the transducer and the field in a single model, thus the cross coupling can be analyzed directly.

The structure of this chapter is as follows: The FEM governing equation and a time domain explicit/implicit algorithm is briefly described in Section 5.2. The FEM code used in this chapter is based on this algorithm. Section 5.3 is about the single array element model. The multiplayer structure is studied. Cross coupling due to different array structures is analyzed in Section 5.4. The full models including the array transducers and the loading materials are studied in Section 5.5. The radiation field predicted by FEM is compared with the results in Chapter 4. Then the influence of cross coupling can be obtained. The final section gives the conclusions.

5.2 Finite element formulation for transient responses

Linear constitutive relations are commonly used to describe the mechanical and electrical behaviour of piezoelectric materials and structures (IEEE, 1987). The finite element formulation is well document in literature (Allik and Hughes, 1970; Lerch, 1990). It is only briefly introduced in this section, for the convenience of further discussion.

To establish the finite element equations, many methods can be used including virtual work, weighted residuals and variational method, etc. Lerch (1990) utilized Hamilton's variational principle to derive the global equation:

$$M_{uu}\ddot{u} + C_{uu}\dot{u} + K_{uu}u + K_{uU}U = F \quad (5.1)$$

$$K_{uU}^T u + K_{UU} U = Q \quad (5.2)$$

where M_{uu} , C_{uu} , K_{uu} are mass, damping and stiffness matrix, respectively, K_{uU} is piezoelectric stiffness matrix and K_{UU} is the dielectric matrix. F and Q are the force and the electric charge vector.

The solution of Equations (5.1) and (5.2) yields the mechanical displacements u and electrical potential U . The two equations are coupled by the matrix K_{uU} . For passive material (backing, matching layers and loading materials) in the multilayer element structure, $K_{uU}=0$. Equation (5.2) disappears and Equation (5.1) is reduced to a pure mechanical problem.

Many of early FEM implementations of piezoelectric materials are based on frequency domain (Jensen, 1988; Kunkel *et al.*, 1990; Guo *et al.*, 1992). If systems are excited by harmonic force or charge, the dynamic phenomenon is steady state. Time dependence can be eliminated from the problem and the system unknowns are converted to harmonic complex variables:

$$u(x, y, z, t) = \bar{u} e^{j\omega t} \quad (5.3)$$

$$U(x, y, z, t) = \bar{U}(x, y, z) e^{j\omega t} \quad (5.4)$$

$$\dot{u}(x, y, z, t) = j\omega \bar{u} e^{j\omega t} \quad (5.5)$$

$$\ddot{u}(x, y, z, t) = -\omega^2 \bar{u} e^{j\omega t} \quad (5.6)$$

The direct solutions such as Gaussian elimination or iterative solutions can be used to solve the problem. Eigen analysis is also always used to obtain the model's natural

frequencies and associated modal shapes. Algorithms related to the above solutions are well developed and can be found in many FEM textbooks (Bathe, 1996).

However, to study the medical piezoelectric transducers, transient response and broadband signals are of principle interest. In this case time domain analysis is desired. In time domain analysis, direct integration methods are often used, which compute the system unknowns by a step-by-step integration in time. In order to do so, Equation (5.1) needs further discretization in time. Finite difference expressions to approximate the accelerations ($\ddot{u}$) and velocities ($\dot{u}$) in terms of displacements are often used. For example, with the central difference method (Bathe, 1996)

$$\ddot{u}^t = \frac{1}{\Delta t^2} (u^{t-\Delta t} - 2u^t + u^{t+\Delta t}) \quad (5.7)$$

$$\dot{u}^t = \frac{1}{2\Delta t} (u^{t+\Delta t} - u^{t-\Delta t}) \quad (5.8)$$

After full discretization, the solution will only satisfy Equation (5.1) at discrete time intervals. Variations of the system unknowns within each time interval are assumed with different ways, which lead to different methods of determining the current solution at time $t + \Delta t$ from known solutions at the previous time step t . These methods can be divided into two categories: implicit methods and explicit methods.

Implicit methods compute current time $t + \Delta t$ system unknowns by satisfying Equation (5.1) at current time $t + \Delta t$. So the solution vector is coupled. At each time step, the global system of equations must be solved. These methods are unconditionally stable with respect to time. Explicit methods obtain solutions of current time $t + \Delta t$ by satisfying Equation (5.1) at time t . These methods decouple the current solution vectors, make the computing procedure element-by-element and need not assemble the global

matrix. This makes explicit methods much less demanding in computer resources than the implicit methods. The disadvantage of explicit methods is that they are only conditionally stable, i.e., there is a time step limit (CFL condition, Bathe, 1996). The central difference method is one of the explicit methods.

In elastic dynamics, the CFL condition is that the time step must be smaller than the shortest wave transit time across any element. This follows from the hyperbolic nature of the wave equations, i.e., during time step Δt the field at a point is only influenced by the field at neighbouring points within a sphere of radius $\Delta x = V_e \Delta t$ where V_e is the fastest local wave speed. In wave phenomena, the desired resolution and accuracy require a time step smaller than one-tenth the period of the highest frequency of interest, and there must be enough nodes per wavelength of the highest frequency (at least 10 nodes per wavelength or even more, depending on requirements of problems). These requirements make stricter conditions than CFL condition, and remove the principal advantage of implicit methods.

Thus, as long as the electromechanical problem maintains a wave propagation characteristic, a mixed explicit/implicit scheme is optimal. It means, with Equation (5.1), $u^{t+\Delta t}$ can be solved by explicit time integration methods using u^t and U^t achieved from the previous time step, and then solving quasi-static Equation (5.2) implicitly for $U^{t+\Delta t}$ using $u^{t+\Delta t}$.

In this thesis, a commercial code named *PZFlex* is used. This code is based on the above finite element method algorithms, specialized in piezoelectric problems. In piezoelectric

transducer simulations, it shows at least 10 times faster simulation speed than that of conventional implicit algorithms based FEM code (McKeighen, 2001).

5.3 Simulation of single array element

The structure of transducer element commonly consists of several layers (Kossoff, 1966). Fig. 5.1 shows a few elements separated for a 1D array. It can be seen that the element has a tall narrow shape. This is in order to achieve thickness vibration mode and push undesirable modes to higher frequency range to prevent them interfering with thickness mode. Commonly the ratio W/H is around 0.5 or smaller. Since the PZT thickness H is dependent on the application operating frequency, thus the element width W is also limited to small value. Note this is another constraint on element lateral dimension besides the half wavelength pitch requirement posed by array theory.

In addition to the active material layer that is made of piezoelectric materials and the electrode layers on both sides of it, there are one or more matching layers placed in front of active materials. The basic function of the matching layer is to match the great difference of acoustic impedance between PZT ceramics and loading materials. Classical acoustic theory provides guidelines to design matching layers. For example, the best matching effect can be achieved if the thickness of the matching layer equals quarter wavelength and the acoustic impedance satisfies the following equation (Wu and Cheng, 1988),

$$Z_i = Z_p^{(2n-2i+1)/2n} Z_L^{(2i-1)/2n} \quad (5.9)$$

where Z_p and Z_L are impedance of PZT and loading materials, respectively, n is the number of matching layers, i is from 1 to n . The more matching layers are used between

Z_p and Z_L , the better the matching results can be achieved. But practically, only one or two layers are adopted in medial arrays due to the fabrication difficulties.

On the backside of PZT, a backing material is always attached. Besides providing mechanical support for the whole structure, backing plays an important role to achieve broadband transducers. It damps the ultrasonic waves propagating backward from PZT. Then the internal reverberations of PZT layer are avoided, allowing a short pulse to be generated. Although the backing material with impedance close to PZT is the best on absorbing sound waves, it also will lead to poor sensitivity and serious cross coupling in array transducers. So low impedance but high damping coefficient material is a better choice for backing.

Another factor that need proper consideration is the downshift effect from passive layers to the resonant frequency of PZT (McKeighen, 1998). Because of the mass loading effect, the resonant frequency of the multilayer structure is lower than that of the single PZT layer. So it is necessary to design the PZT thickness with higher frequency initially to compensate this downshift effect.

Based on the above analysis, a FEM model of 1D array element is established, as shown in Fig. 5.2. Since the element thickness and width are much smaller than the element height, a 2D plain strain model is used. The material properties used in this model are listed in Tables 5.1 and 5.2. The transducer model is coupled with a 3mm thick water layer that works as the loading material. The boundary condition of this water layer is set to absorbing condition, which is provided by the code. This is to simulate a semi-infinite loading material and to avoid the waves reflecting from the boundary. The emphasis of

the analysis is placed on the simulation of transmitting field from the transducer model, so the results of impedance response and dynamic characters of the structure are not shown in this chapter. The model is excited with half period sine wave with a frequency of 3.5MHz . The pressure wave of the field point 2mm in front of the transducer surface is monitored, and the waveform and its spectrum are shown in Figs. 5.3 and 5.4, respectively. It can be seen that the central frequency of the pulse is around 3.5MHz , and its -6dB relative bandwidth is 65% .

The directivity of this single element model is also calculated and shown in Fig. 5.5. The results calculated by impulse response method are shown in this figure to make comparison. The boundary condition used here is the soft baffle to conform to the FEM model (refer to Section 3.2). Some deviation of the two results can be seen. A wider directivity is predicted by FEM model. The inconsistency comes from the piston-like vibration assumption used in the impulse response method. The FEM model shows that the front surface of the transducer element never keeps perfectly flat during operation even if it vibrates in the thickness mode. Thus the waves from different parts of transducer surface may have different amplitudes. In order to confirm this analysis, another model is simulated, in which the loading material is modeled independently. In order to calculate the pressure distribution, the pressure wave shown in Fig. 5.3 is applied to the boundary of the loading material directly, at the position same as the transducer element in the previous model. Other conditions are kept unchanged. At this time, the calculated directivity due to the applied pressure matches with the impulse response method very well, which is demonstrated in Fig. 5.5.

An interesting phenomenon is observed during this simulation. As mentioned earlier in this section, the thickness of matching layer is selected as a quarter wavelength, which is $245\mu m$ for the material used in this model. However, it is found that with this thickness, the matching effect is not the best. In order to fully study this problem, optimization of matching layer in the transducer model are performed. For each simulation, only the matching layer thickness changes. It varies from $120\mu m$ to $280\mu m$, with an interval of $20\mu m$. The maximum pressure amplitude of the same field point is recorded and plotted in Fig. 5.6. It can be seen that the maximum pressure doesn't occur at quarter wavelength thickness, but at $200\mu m$. It is about 18% less in comparison with the quarter wavelength thickness. With this thickness more than 6% of pressure can be transmitted to the loading material than using the quarter wavelength matching layer.

The reason of this effect is mainly due to the thin tall shape of the transducer element. The quarter wavelength principle is based on the plane waves transmitting from one material to another (Kino, 1987), and the geometries are assumed to be infinite in the lateral dimension. The shape of the array element does not satisfy this assumption. Further, the wave has more complicated behaviours than the plane waves. Secondly, the influence of the backing material cannot be ignored. However, theoretical analysis of this phenomenon is too complicated due to the multilayer structure. The results in this section show that numerical simulation is an efficient way to optimize the matching layers.

5.4 Cross coupling in 1D array transducers

The transducer element studied in the previous section is separated from a 1D array. In this section this array transducer is fully modeled. The emphasis is placed on the inter

element cross coupling and their reduction. Cross coupling among the elements of an array transducer can produce many undesirable effects. It directly affects elements ring down duration, increases the acoustic noise level, and limits aperture and steering angles. However, there are no general theoretical models for cross coupling due to its complexity. Most of research is based on experiments or numerical simulations.

Generally speaking, there are three main types of cross coupling. The first is the acoustic cross coupling. It is mainly caused by the lateral mode or the surface mode acoustic waves generated by transducer elements. The second is the electrical cross coupling due to stray capacitance or inductance in the cable or the terminal connection of electronics. The third is the electro-magnetic RF wave coupling in the wireless mode. In this section, finite element simulation is used to study the first type of cross coupling. The other two types are beyond the scope of this thesis.

The FEM model of a 1-D array transducer consisting of 17 elements is shown in Fig. 5.7. The level of cross coupling is evaluated as follow. The central element (No.1) is excited with a voltage signal. Then the induced voltages on the electrodes of other adjacent elements (No.2 to No.9) are computed and recorded. The ratio between the maximum value of the resulting voltage and the exciting voltage for a given element is used as the criteria of cross coupling level for this element. The higher the ratio is, the higher the cross coupling level is. If the ratios equal to zero, no cross coupling exists. Although the real arrays always consist of many more elements than this model, the coupling level is very low for the elements located far away from the excited element. So 7 elements are enough for the study of cross coupling, and the simulation results confirm this.

The materials used in these models are the same as those used in Section 5.4. The material properties are listed in Table 5.1 and Table 5.2. The epoxy resin is used as kerf filler, and its material properties are also listed in Table 5.2.

In order to evaluate the influence of backing, matching layers and kerf filler to the cross coupling level, a series of models are simulated, and shown in Fig. 5.8,

- (1) No diced kerf in matching layers and backing (Fig. 5.8(a)).
- (2) Only matching layers are diced (Fig. 5.8(b)).
- (3) The diced kerf is extended into backing (Fig. 5.8(c)).
- (4) The kerf is filled with epoxy resin (Fig. 5.8(d)).
- (5) Same as model c, but heavy backing is used.
- (6) Same as model c, but with loading material.

The calculated coupling levels of each element for the above models are shown in Fig. 5.9. It can be seen that the cross coupling depends strongly on whether the PZT elements are diced, the type of backing materials used and whether kerf is filled. The following can be observed:

- The paths of cross coupling include matching layer, backing and kerf filler. The influence from the loading materials can be ignored as compared with the above three factors.
- The cross coupling becomes weak from the most adjacent element to the far element.

- Diced matching layers are efficient to reduce the cross coupling level. Comparing the results of model b with model a, it can be found that the coupling level reduced more than $10dB$ on average.
- Diced backing is also useful. From results of model b and model c, it can be seen that coupling level is reduced slightly. It is because the diced thickness is only $50\mu m$. If diced thickness is increased to $150\mu m$, coupling level is reduced more than $5dB$ from model b. However, too deep dicing in backing will affect the strength of the transducer structure. A trade-off has to be made.
- When a heavy backing with high acoustic impedance is used, the cross coupling level is increased by more than $10dB$ as compared with the original model. The reason of increased coupling is because the acoustic impedance of the heavy backing is closer to the impedance of PZT, thus more acoustic energy will transmit to the backing material than that of the light backing case. This is another reason why light backing is preferred in array transducer design.
- Kerf filler is always used to enhance the structure strength. However, it will lead to strong cross coupling. From the above simulation results, kerf filler is the most important coupling path. It increases the coupling level for more than $20dB$ on average. So if kerf filler is necessary, the filler material must be selected with high damping to reduce the cross coupling effects.

The above analysis establishes the very general design guidelines for array transducers to avoid serious cross coupling. However, it also shows that the coupling cannot be eliminated completely. Another purpose of the above simulations is to make several examples that have different coupling levels, and then the radiation fields will be evaluated in the next section using FEM method to study how the coupling level will

affect the arrays radiation fields. The worst cross coupling is at $-25dB$ for model (d), while the model (c) has the least cross coupling at $-42dB$, their influence on radiation field will be discussed.

5.5 Simulation of array transducers radiation field

5.5.1 FEM models

The explicit algorithms based FEM code *PZFlex* makes it possible to model the whole loading material as well as the array transducers simultaneously with acceptable calculation speed. Thus the transient radiation field can be calculated directly from the model. Although it is much slower than the impulse response based methods to calculate field response, it allows to consider the effect of arrays dynamic performance and the cross coupling among array elements. It also can deal with complicated shape of transducers. The emphasis of this section is placed on the calculation of radiation field. It should be noted that the impulse response methods discussed in Chapter 4 ignore the elements interactions and assume piston-like vibrations. By comparison of the results of Chapter 4 and FEM, the effect of the above assumptions can be seen clearly.

It should be noted here, all discussions in Chapters 3 and 4 assume that transducers are surrounded by rigid baffle. This is in order to simplify the derivation. However, in the FEM model the array transducer is loaded with water, thus soft baffle is more suitable to be used in this chapter for the impulse response based methods. There exists only an obliquity factor exists between the two results of different baffles. For examples, for a 1D array element, Equation (4.8) describes its directivity for rigid baffle boundary. For soft baffle, the directivity becomes

$$A(\theta_i) = \text{sinc}\left[\frac{a}{\lambda} \sin(\theta_i)\right] \cdot \cos(\theta_i) \quad (5.10)$$

where θ_i is the viewing angle, as shown in Fig. 4.2. The radiation patterns calculated in this chapter are all within $\pm 45^\circ$. This is because beyond this region, the term $\cos(\theta_i)$ becomes the dominant factor, and the pressure drops quickly to zero.

The 1D array used in this section is same as the one in Section 5.4. It consists of 17 elements with the element pitch of half wavelength. The loading material is 10mm water, and absorbing boundary condition is adopted. Other conditions remain same.

The results of FEM and impulse response method are compared first. In order to eliminate the influence of the element behaviour and cross coupling to the radiation field and compare the two methods under the same conditions, the FEM model (FEM1) removes the transducer elements and simulate the ideal output that would be generated by the array. The pressure wave is applied to the boundary directly to replace the elements, at the 17 positions where the elements should be located. This is to simulate the ideal situation in which the cross coupling would not happen, and the element surface keeps flat during vibration duration. FEM1 considers wave propagations only, which is under the same conditions as the impulse response method. The model is shown in Fig. 5.10. The transient pressure waves at field points 7mm away from the array centre are calculated, which is corresponding to the f number of $f2$. To get the one-way beam pattern, the maximum amplitude of pressure at each point is recorded and plotted in Fig. 5.12, together with the result by impulse response method. The beam steering and focusing in FEM1 are realized by applying the pre-calculated time delays to the pressure boundary conditions on each elements. In this chapter, the arrays are all modeled to focus

off-line at $\theta_F = 30^\circ$, where the side lobe region can be shown clearly in the results of radiation field.

Agreement can be observed from the results though it shows some deviations beyond the main lobe region. The deviations are mainly caused by the spatial discretization of the finite element method. The discretization leads to calculation error at mesh grids. For wave propagation problems, these errors will accumulate as simulation proceeds in time domain. Another influence of discretization is that it directly creates errors on time delay calculations. Obviously, smaller mesh size yields more accurate results, but at the price of longer simulation time. The two results of FEM1 with different mesh size in Fig. 5.12 prove this. In this chapter, unless otherwise stated, the mesh size used in the following calculation is $10\mu m$. It is equivalent to around 40 elements per wavelength for the nominal $3.5MHz$ working frequency. Considering the bandwidth of signals, this is a reasonable value to make balance between accuracy and efficiency.

The full model that consists of transducer and loading material is referred as FEM2 in the following discussion. One example model is shown in Fig. 5.11. Two cases are studied. The array transducers used in these two configuration cases are model (c) and (d) as shown in Fig. 5.8, which are corresponding to models of the lowest and highest cross coupling level, respectively. The results predicted by three methods (Impulse response, FEM1, FEM2) are compared. For the first two methods, the initial pressure (Impulse response) or the applied pressure (FEM1) $p_0(t)$ is first calculated from the single element FEM model, which has the same structure and parameters as those of the array element in FEM2. Working in this way, the comparison of three results will show the influence of cross coupling to the radiation field.

Fig. 5.13 demonstrates the results of the radiation field for model (c). Clearly, the low level cross coupling (below -40dB) does not affect the beam pattern seriously. Only slightly increasing on side floor level of FEM2 result can be observed. However, for model (d), when the cross coupling is over -30dB , the pattern is distorted dramatically. The near-in side lobes appear as well as the side lobe floor increases more than 5dB as shown in Fig. 5.14 from the FEM2 results. However, in both cases, the main lobe is not affected by the elemental coupling.

5.5.2 Simulation results analysis

In order to explain the above phenomenon, it is necessary to study the behaviours of each element under coupling conditions. Since the projection of the focus point is out of the aperture range, the elements are excited one by one from one side of aperture to another during simulation procedure in FEM2. This is determined by the pre-calculated time delays. The interference of signals from all of the elements make it difficult to calculate the separated pressure signal generated by single element. However, the vibration velocity profile at the surface of each element can be calculated directly by FEM, this makes it possible to calculate the pressure signals transmitted by each element from the velocity results using Equation (3.14).

The results of velocity profiles are shown in Figs. 5.15 and 5.16 for model (c) and (d), respectively. The results for every other element from 17 elements are displayed, in the sequence that they are excited. It can be observed that the time delays of signals in the above figures increase from the first element to the last one. Comparing these two figures it can be seen that the direct effect of cross coupling is the extension of the element ring

down duration. High-level coupling extends the vibration duration dramatically. This is clearly shown in Fig. 5.16. Besides the first few periods of vibration excited by the voltage signal, all signals have ‘tails’ following the main signal. For the elements that are located in central part of aperture, this effect is more serious. This is because they are affected by the elements located on both sides of them. Even in low coupling model, this effect also can be observed from Fig. 5.15. Another effect can be observed in high coupling model is that the vibration amplitude of these seriously affected elements is lower than that of the elements at the boundary of aperture. Even the profiles of their ‘main’ vibration periods are also distorted somewhat from those of the less affected elements. For the low coupling model, this effect is not obvious.

The calculated pressure waves generated by each of the above elements are shown in Figs 5.17 and 5.18. Clearly, for low level coupling case, the signals from different elements are almost same in terms of their amplitude and the waveform. This uniform elemental performance is affected seriously when the coupling level is high. As shown in Fig. 5.18, the seriously affected elements generate weak signals with lower amplitude, and the waveforms also deviate from the ‘standard’ one. Furthermore, these signals have longer time durations than the desirable one. Besides the ‘main’ part of the signals, the ‘tail’s due to coupling have different amplitude and waveform. This is because the multiple coupling paths complicate further the coupling behaviours.

The reason for high relative side lobe due to the cross coupling can be explained as following. First, at the focal point, all pulses are in phase, but the signals from the seriously affected elements have lower amplitude, so the absolute pressure value of the superposition result at the focus point is lower than the case when the cross coupling

level is low. Note in the main lobe region, the ‘main’ part of each pulse is still the dominant factor for the superposition result. This is because the time intervals among pulses from all elements are small, compared with signal period. Thus cross coupling does not affect the main lobe shape. Secondly, in the side lobe region, the pulses are out of phase, the constructive and destructive effect should not lead to high amplitude signal, as analysed in Chapters 3 and 4. However, since the high amplitude ‘tail’s exist, the situation changes. The maximal amplitude at the field point in side lobe region is determined by the interference among these ‘tails’ and the ‘main’ periods. Due to the disorder of these ‘tail’s and high amplitude, the result amplitude is high. Figs. 5.19 and 5.20 show the transient pressure waveforms at side lobe region and focus point, which are calculated by FEM for model (c) and (d), respectively. It proves the above analysis.

Another important effect of cross coupling to the arrays performance is that the serious coupling greatly narrows the elemental directivity. Using model FEM2, this phenomenon can be demonstrated. For example, still using model (d), but only the central element is excited in order to calculate its directivity. The pressure wave is calculated in the region $-45^\circ < \theta < 45^\circ$. It is shown in Fig. 5.21, compared with the results of the impulse response method. The beam width is dramatically reduced under high coupling level. Clearly, the narrower elemental directivity will lead to lower signal intensity when array focuses off-axis than that of on-axis case.

5.6 Conclusions

FEM is adopted in this chapter to study the behaviours of single piezoelectric array element as well as 1D array transducers. Since the transient performance is of the most

concern, all simulations are performed in the time domain. An explicit/implicit algorithm based FEM code is utilized to fulfil the analysis. The algorithms are briefly introduced in this chapter.

If the cross coupling level in array transducers is low, the results by FEM match very well with the results calculated by the method developed in Chapter 4. This suggests that both methods are accurate. However, the impulse response based method has much higher efficiency. Any field point can be calculated independently. For FEM, it is not very suitable to calculate field response far from the transducer surface, since a large-scale of the propagating field has to be modeled. Simulation time is one factor, and the accumulated error is another prohibited factor.

However, the advantage of FEM is that it is not necessarily limited to field calculations only, it can deal with much more complicated problems such as optimization and design. This chapter analyses the cross coupling phenomenon in array transducer and its effect on the radiation field which can only be analysed by FEM. By comparing the FEM results with that of impulse response method, it is found that for phased array transducer, the cross coupling will generate high level side lobe or floors while the main lobe remains intact. The pulse from each element is extended to longer time duration and high amplitude ‘tail’s are the main reasons. Furthermore, the cross coupling will narrow the elemental directivity. This should provide general design guidelines to suppress the element cross-coupling effect.

Table 5.1 Material properties of PZT-5H (Kunkel *et al.*, 1990)

Elastic stiffness ($10^{10} \text{ N} / \text{m}^2$)	c_{11}^E	12.60
	c_{12}^E	7.95
	c_{13}^E	8.41
	c_{33}^E	11.70
	c_{44}^E	2.30
Piezoelectric constant (C / m^2)	e_{31}	-6.50
	e_{33}	23.30
	e_{15}	17.00
Relative permittivity	$\varepsilon_{11}^S / \varepsilon_0$	1700.00
	$\varepsilon_{33}^S / \varepsilon_0$	1470.00
Damping constant* (at 1 MHz)	Q_L	65
	Q_S	65
Mass density ($10^3 \text{ kg} / \text{m}^3$)	ρ	7.50

* (Wojcik *et al.*, 1996)

Table 5.2 Material properties of passive materials
(Powell *et al.*, 1997)

		Matching	Kerf Filler*	Backing 1	Backing 2*
Longitudinal sound velocity (m/s)	V_L	3950	2888.1	2180	3200
Shear sound velocity (m/s)	V_S	2170	1290.3	1115	1500
Mass density ($10^3 kg/m^3$)	ρ	1.901	1.150	0.987	2.6
Acoustic impedance ($MRals$)	Z	7.51	3.3	2.15	8.32
Longitudinal damping ($dB/cm @ 1MHz$)	α_L	3	2	9.5	$10(Q_L @ 1MHz)$
Shear damping ($dB/cm @ 1MHz$)	α_S	6	5	20	$10(Q_S @ 1MHz)$

* (PZFlex, 2000)

Fig. 5.1 Multilayer structure of 1D array elements

Fig. 5.2 Plain strain FEM model of multilayer array transducer element

Fig. 5.3 Pressure waveform predicted by 1D array element FEM model

Fig. 5.4 Spectrum of waveform predicted by 1D array element FEM model

Fig. 5.5 Element directivity calculated by FEM models (dashed line and dotted line) and impulse response method (solid line)

Fig. 5.6 Pressure amplitude with different matching layer thickness

Fig. 5.7 FEM model for 1-D array transducer

Fig. 5.8 FEM models for evaluation of inter element cross coupling level (a) Model a, only PZT is diced. (b) Model b, matching layer is diced. (c) Model c, partial backing is diced. (d) Model d, kerf filler is used.

Fig. 5.9 Coupled voltages of elements from No.2 to No.9 of different models

Fig. 5.10 FEM model without transducer element (FEM1). Pressure is applied directly.

Fig. 5.11 FEM model of array transducer and water (FEM2)

Fig. 5.12 Radiation pattern of results from FEM1 (solid line: $10\mu m$; dotted line: $25\mu m$) and impulse response method (dashed line) for 17 elements 1D array.

Fig. 5.13 Radiation pattern of model (c), predicted by three methods

Fig. 5.14 Radiation pattern of model (d), predicted by three methods

Fig. 5.15 Velocity profiles on element surface of low coupling level model (c). Top-left is the first excited element, bottom-right is the last.

Fig. 5.16 Velocity profiles on element surface of high coupling level model (d). Top-left is the first excited element, bottom-right is the last.

Fig. 5.17 Pressure wave transmitted from each element of low coupling level model (c). Top-left is the first excited element, bottom-right is the last.

Fig. 5.18 Pressure wave transmitted from each element of high coupling level model (d). Top-left is the first excited element, bottom-right is the last.

Fig. 5.19 Transient pressure waveform at field point for low coupling model (a) side lobe region $\theta = -45^\circ$ (b) focus point $\theta = \theta_F = 30^\circ$

Fig. 5.20 Transient pressure waveform at field point for high coupling model (a) side lobe region $\theta = -45^\circ$ (b) focus point $\theta = \theta_F = 30^\circ$

Fig. 5.21 Elemental directivity in the high cross coupling case (solid line), compared with the results predicted by impulse response method (dotted line).

Chapter 6

Sparse Array Transducers Design and Optimization

In this chapter, sparse array transducer design and optimization is presented to reduce the number of elements. Apart from periodic and random non-periodic methods, Genetic Algorithms is introduced and its application of this theory in broadband arrays is demonstrated.

6.1 Introduction

Array transducers provide versatile performance to imaging systems. However, array systems are always more complicated and thus costly than single element systems. The reason is obvious. To acquire the desirable spatial resolution, the aperture size cannot be small. But the number of elements cannot be reduced arbitrarily from array transducers due to the reasons introduced in previous chapters. Firstly, in order to generate longitudinal sound waves, the piezoelectric elements are made into thin tall shape. Their lateral dimensions should be kept small to achieve thickness vibration. Secondly, array theory requires the tiny array elements to be located very closely to avoid high-level secondary lobes. The distance between the two neighbouring elements is on the order of half wavelength. The above two constraints make the common arrays consist of hundreds (1D arrays) to thousands (2D arrays) of elements, which distribute very densely in the small aperture area. On the other hand, each element needs one electronic channel for excitation and receiving the scattered signals. Although with today's micro machining technique, it is possible to fabricate arrays with a few thousands of elements and connect them with electrical wires, enormous number of supporting electric channels is far from

practical and acceptable for commercial imaging systems. In fact, this is the main bottleneck for the applications of 2D arrays.

It has long been known that using sparse array is the potential solution to reduce system complexity. By using only part of elements from the original dense array, sparse arrays consist of fewer elements but at the price of deteriorated radiation field performance. The design and optimization of sparse array is to make trade-off between system complexity and performance. Sparse arrays are widely used in various narrowband applications nowadays. However, for broadband ultrasonic array transducers, the developed methods based on the narrowband theory are not effective. This is mainly because of the fundamental differences between these two types of systems, which are described in Chapter 3. To design and optimize broadband sparse arrays is more challenging than narrowband cases because of the complicated behaviours of their radiation fields, which is one of the most important criteria.

In this chapter, Section 6.2 discusses two commonly used methods for sparse array design. A few examples show the advantages and disadvantages for both methods. Since analytical solution for optimal broadband sparse array is not available, a stochastic optimization method called Genetic Algorithms (GAs) is briefly introduced in Section 6.3. Section 6.4 develops several GAs operators that are suitable for sparse array applications. The GAs parameters are also selected by testing. In Section 6.5, the developed GAs operators are applied to various sparse arrays designs based on the different requirements, both for 1D and 2D arrays. Section 6.6 is the conclusions for this chapter.

6.2 Periodic and non-periodic sparse arrays

Periodic and non-periodic arrays are two most widely used design methods of sparse arrays. Both are developed for narrow band arrays initially (Steinberg, 1976). For broadband arrays, these two methods are also adopted in the literature. However, they show different performance from the narrow band cases. The following part shows some examples to demonstrate their performance for broadband arrays. The performance is evaluated by their pulse-echo radiation field.

6.2.1 Periodic Vernier arrays

Vernier arrays (Lockwood *et al.* 1996b; Lockwood and Foster, 1996) are typical periodic sparse arrays. Two recent papers (Austeng and Holm, 2002 (a), (b)) developed this method further. This method is developed by narrowband theory. It is known that the positions of grating lobes are dependent on element pitch and signal frequency, i.e.

$$\theta_g = \sin^{-1}[\frac{i \cdot \lambda}{d} - \sin(\theta_s)] \quad (6.1)$$

where θ_g is the position of grating lobe, i is integer, θ_s is steering angle, λ is signal wavelength, d is elemental pitch. From this equation, it can be seen that in order to avoid grating lobes in full visible region $\pm 90^\circ$ during scanning procedure, d is necessary to be smaller than $\lambda/2$.

Vernier arrays select different element pitch d for transmitting aperture and receiving aperture, thus the grating lobes appear at different positions in transmitting pattern and receiving pattern. For narrowband arrays, the two-way radiation field are simply the

product of the above two patterns, so the grating lobes level may decrease due to the multiplication. For example, consider a 1D array, for which the aperture length is 32λ . If $d = \lambda$, it will consist of 32 element, which is half of the numbers of elements from dense arrays with half wavelength pitch. Its one-way radiation field is already shown in Fig. 3.10 when array focuses at $\theta_s = 45^\circ$. If $d = 3\lambda/2$, fewer elements will remain in the aperture, and as element pitch increases, two grating lobes can be seen in its one-way pattern (Fig. 6.1). If the above two apertures work as transmitting aperture and receiving aperture, respectively, its two-way radiation field can be calculated and shown in Fig. 6.2. Clearly, the grating lobes still exist, two of them come from transmitting aperture, as shown in Fig.6.1, and one comes from receiving aperture, as shown in Fig.3.10, but their levels drop to below $-25dB$.

For broadband case, the same apertures are considered. The initial signal $p_0(t)$ is still same as the one used in Chapter 3. The one-way patterns for the two apertures are shown in Fig. 6.3, using the methods presented in Chapter 4. The grating lobes appear at almost same positions of narrowband case, except the one near $\theta = 40^\circ$ which is replaced by the flat non-interference region, but the amplitudes are much lower than the main lobe. The pulse echo two-way pattern by the Vernier array is shown in Fig. 6.4. It can be seen that the grating lobe is also suppressed dramatically in the region between -40° and 20° . Comparing Figs. 6.4 and 6.2, the difference can be observed. For narrowband arrays, the suppressed grating lobes are still higher than the side lobes. But for the broadband arrays, these lobes are never the highest secondary lobes except the main lobe. From the viewpoint of optimization, if reduction of the secondary lobe level is used as an optimization criterion, it means the possibility still exists to utilize even fewer elements

until the grating lobes become almost of the same level as the other lobes. However, the design method of Vernier array does not provide this capability.

6.2.2 Non-periodic random arrays

Random arrays are the typical design of non-periodic arrays. In random sparse arrays, elements are selected by some random processes, and then they are removed from the dense array, so the resulting sparse array contains only a fraction of the original elements. The array is called sparse array of order n if the number of elements is reduced to $1/n$ of the original one. High level grating lobes can be avoided in random array by eliminating the uniform, periodic arrangement of elements. Another random process is the so-called binned arrays. Comparing with the totally random arrays, in binned array, the aperture is first divided into non-overlapping bins of equal area, and array elements are selected and distributed independently in each bin, according to some probability distribution functions. Hendircks (1991) has shown that binned arrays are efficient in suppressing near-in side lobes in narrowband transducers.

The beam patterns of random arrays show very different characteristics from the Vernier arrays. The grating lobes totally disappear in random arrays patterns. However, due to the non-periodic element layout, the sidelobe floor is high. In order to fully demonstrate the two different patterns, 2D arrays are calculated as an example. The 2D dense array consists of 20×20 elements with half wavelength pitch. For Vernier design, the transmitting aperture pitch is λ in both directions, consisting 10×10 elements. The receiving aperture pitch is $3\lambda/2$, consisting of 7×7 elements. The random array

consists of 120 elements, selected randomly from the dense array. The transmitting and receiving apertures are same.

Fig. 6.5 shows the pattern of Vernier array, and Fig. 6.6 is that of random array, for which the focus angles are set to $(0^\circ, 30^\circ)$. The two figures clearly show different features as mentioned above. It can be observed that the field pattern from the randomly selected element layout is better than that from the Vernier array design in terms of the maximum secondary lobe level. For Vernier array, this level is -31.8 dB , located on the grating lobe, which is about 5 dB higher than the random array case, as shown in Fig. 6.6. However, for random pattern, a floor level around -50 dB can be seen. On the other hand, for 2D arrays, the non-interference region can be hardly observed from their patterns. This is because the elements are distributed in two dimensions, and the conditions for non-interference analyzed in Chapter 3 are not satisfied by 2D arrays.

The above analysis shows that for broadband arrays, the two design methods of sparse arrays have their own advantages, respectively. Periodic designs are easy to implement. The beam patterns have no side lobe floor. However, this method faces some disadvantages such as high grating lobes, system complexity, and difficulties to select number of elements and design of optimization, etc. Non-periodic designs provide the flexibility to select elements. The radiation fields also have lower secondary lobe level. However, due to the method itself, no analytical solution can be derived to find the optimal pattern under given conditions. The free or constrained random procedure can only be described stochastically. The large number of possibilities of element layouts makes it far from practical to test all possible designs. These impose difficulties in design and optimizing non-periodic arrays.

6.3 Introduction of Genetic Algorithms

Since the non-periodic sparse array can only be described stochastically, it is suitable to use some stochastic optimization methods to study non-periodic arrays. Genetic Algorithms (GAs) is one of these methods developed in the past twenty to thirty years.

GAs is modeled on the principles and concepts of natural selection and evolution. In conventional GAs, values for all of the parameters to be optimized are represented by binary codes. Each encoded parameter is placed one by one to form a long binary string called a chromosome. Each chromosome maps to a solution of the original problem. GAs starts with a group of chromosomes called population, which are selected first by random process with or without constraints. Then the fitness value of each chromosome is calculated. Chromosomes are ordered by their fitness values. The one with better fitness values has more chance to be selected and to produce offspring by crossover and mutation operators, while the worse ones may die off. All offspring form new population transfer to next generation, and then all above procedures repeat, until the preset fitness value or number of generations are reached. The basic GAs operators include selection, crossover, mutation and fitness evaluation, etc.

The only link between GAs and objective problem is the fitness evaluation procedure. This procedure decodes each chromosome to problem parameters and evaluates its fitness value, then returns this value to GAs. The fitness value is directly related to the optimization objectives. “Better” chromosome has “better” fitness value. In this way, GAs works like a kind of “black box” method. The whole optimization procedure has no direct relations with how the fitness is calculated.

Genetic algorithms differ from most optimization methods. The advantages of GAs include that mathematic derivatives are not necessary during optimization; it works with the coding of the parameters to be optimized, not the parameters themselves, thus can deal with many parameters simultaneously; it also optimizes a population of potential solutions rather than a single one.

As stochastic optimization procedures, GAs is robust and simple to implement, and have been used for a variety of practical optimization problems. However, despite the powerful methods themselves, the theoretical basis for the GAs is not well developed so far, schema and building block theory developed by De Jong (1975) and Goldberg (1989) are probably the most widely quoted. However, these theories cannot provide sufficient guides to implement GAs to given applications. This is because that GAs' performance is believed to be highly application dependent. The theories explain only very general criteria. To obtain satisfactory results, operating routines and parameters for GAs should be selected, tested and improved carefully via numerical experiments.

The implementation of GAs to array design and optimization can be found in the literature (Haupt, 1994; Yan and Lu, 1997; Johnson and R-Samii, 1997; Marcano and Duran, 2000; Bray *et al.* 2002). However, they are all about narrowband arrays. This is because the beam patterns of narrowband arrays can be calculated by Fourier transform theory. The calculation speed is very fast if *FFT* algorithms are used. Thus for the iterative GAs procedure, the fitness evaluation procedure is not a problem. But for broadband arrays, Fourier theory fails to calculate patterns. The impulse response based methods become the main computing burden of optimization procedure. This is a

prohibitive factor to implement GAs to broadband arrays, especially for 2D arrays. However, the approximate methods to calculate broadband patterns are developed in Chapter 4, and these methods can ensure accuracy, and more importantly they are fast and suitable for iterative calculations. Thus, using GAs to optimize both 1D and 2D arrays are possible.

The conventional binary GAs is naturally suitable for sparse array layout optimization. To implement GAs, the sparse array transducer layout is coded by a binary string, the most straightforward way is using '0' to represent non-active element, and '1' to represent active element. For example, a string '0011001001000110' means a 1D 16-element sparse array transducer with 6 active elements. For 2D arrays, although there are some GAs utilize two-dimensional chromosome, they are too complicated to be implemented in array applications. In this thesis, the layouts of 2D arrays are first converted to 1D strings using certain routines. All the developed operators for 1D arrays can then be used directly. Only in fitness evaluation procedure, these 1D strings are converted back to 2D layouts for beam pattern calculations.

Real-code GAs is another branch of GAs optimization (Weight, 1991). It encodes problem parameters into chromosomes consisting of real numbers rather than binary code. It can avoid sampling errors that exist in binary GAs. Since the apodization weighting coefficients are continuous numbers, it is suitable to use real-code GAs for optimization.

6.4 GAs operators and parameters

In this section several GAs operators that are used for sparse array optimization are developed. The GAs parameters, such as population size, number of generation, mutation

rate, etc. are also tested and selected. The 1D array used in previous chapters is used as an example working with same configurations and working conditions. The sparse array has 16 active elements, totally randomly selected for first generation. The criterion of the optimization is to minimize the maximum secondary lobe level (*SLL*) in the pulse-echo radiation field of the array. The array focuses at angle $\theta_s = 30^\circ$. This model is referred as ‘Simple Model’ in the following part of this chapter.

It should be noted that considering *SLL* only would always result in beam patterns with wide main lobe beam width, which will reduce the resolution for imaging system in practical applications. However, at this time it is best to use a relative simple problem to facilitate the development of GAs. Once the development is accomplished, more practical problems will be considered in the next section. The reason of using $\theta_s = 30^\circ$ rather than broadside $\theta_s = 0^\circ$ is that the side lobe level is always the lowest when steering to broadside ($\theta_s = 0^\circ$) than that when steering to other angles. In other words, the worse case is considered. The maximum *SLL* is calculated from the pattern by finding the highest lobe besides the main lobe. To reduce the optimization time, the radiation pattern is calculated only in the region between $\theta = -45^\circ$ and 45° . This conforms to the common scanning range of current ultrasonic imaging systems. And another reason is that under soft baffle condition, the pressure beyond this region drops quickly to zero.

6.4.1 Fitness evaluation operator

The radiation pattern of array is calculated by the methods introduced in Chapter 4. Method 1 is used here as an example. However, the following discussion is not necessarily limited to Method 1. They are also suitable to other approximate methods of

field calculation including Method 2 and that for 2D arrays. In order to increase computing speed, a few approaches are adopted when implementing the fitness evaluation operator. Firstly, considering Equation (4.9) for field impulse response calculation, during the procedure to calculate the beam patterns for a group of sparse element layouts, some variables in this equation are computed repeatedly for each layout calculation, such as time delay τ_i , spatial vector $\vec{r}, \vec{r}_i$, etc. Thus, these values can be pre-calculated first and saved in computer hard disk. After that when the fitness evaluation is performed, these variables are first loaded into computer memory *RAM*. For a given sparse element layout, the program can read in the corresponding values needed from *RAM* and calculate the impulse response of the whole array. Working in this way, parts of the calculations are eliminated from optimization procedure. The whole optimization time can be reduced dramatically. Secondly, the time-consuming convolution is not calculated directly. It is well known that the fast Fourier transform and inverse Fourier transform can be used instead. After transforming signals to frequency domain, the convolution can be replaced by multiplication. This provides more efficient calculation than direct convolution. The procedure is expressed with the following equation,

$$\begin{aligned}
 p_{TR}(\vec{r}, t) &= p_0(t) * h_T(\vec{r}, t) * h_R(\vec{r}, t) \\
 &= FFT^{-1}[FFT(p_0(t)) \cdot FFT(h_T(\vec{r}, t)) \cdot FFT(h_R(\vec{r}, t))] \\
 &= FFT^{-1}[P_0(\omega) \cdot H_T(\vec{r}, \omega) \cdot H_R(\vec{r}, \omega)]
 \end{aligned} \tag{6.2}$$

where FFT and FFT^{-1} means fast Fourier transform and inverse Fourier transform, respectively.

After the beam pattern is calculated, the fitness value can be obtained from the pattern according to the optimization requirements. For example, if only the maximum *SLL* is considered, the value of maximum peak except the main lobe in the pattern can be assigned as fitness value. If more factors are considered, such as main lobe beam width,

the fitness value can be dealt with in two ways. One is to combine all the factors using weightings to calculate fitness value. For example, the fitness value can be expressed as,

$$\text{fitness value} = w_1 \cdot SLL_{\max} + w_2 \cdot \text{beamwidth} \quad (6.3)$$

where w_1 and w_2 are weightings, their values can be selected according to the specific requirements of given problems.

Another way to deal with the multiple requirements is to set part of them as threshold. The parameters for these requirements are calculated first. If the values are beyond the threshold, the chromosome is set with a ‘bad’ fitness value directly, without any further calculation. Using the above example, if the main lobe beam width is required to be smaller than a , the threshold can be set to a . The beam width is first calculated from the beam pattern, only if it is smaller than a , $SLL_{\max}$ is calculated. Otherwise, this chromosome is set with a very high $SLL_{\max}$ as its fitness value directly, and in the remaining of the optimization procedure this chromosome has very small possibility to be selected. The above method to calculate the fitness value can be expressed as,

$$\text{fitness} = \begin{cases} \text{'bad' value} & \text{beamwidth} \geq a \\ SLL_{\max} & \text{beamwidth} < a \end{cases} \quad (6.4)$$

In this chapter, the later one described with Equation (6.4) is preferred. This is because most of the optimization time is used for the beam pattern calculation. For all the chromosomes, the main lobe patterns can be calculated first, and the main lobe beam width can be obtained. For the chromosomes whose beam width is greater than the requirement, the calculation of the rest field patterns of these chromosomes, which are used to obtain $SLL_{\max}$ are no longer needed, thus the optimization time can be reduced further.

It should be noted here that the main lobe beam width can be controlled by other ways. For example, it is known that this width is determined directly by aperture length. For sparse arrays, to allocate the element in whole aperture position can ensure the same beam width as that of dense array. However, it is difficult to implement this constraint to GAs directly, since it's related to the structure of each chromosome. Testing results show that to apply the constraints in fitness evaluation procedure is much better than to interfere the GAs procedure directly.

6.4.2 Crossover operator

Several crossover operators are developed in this section. They are suitable to sparse array optimization under different requirements.

For binary GAs, the most widely used crossover methods include one-point crossover, two-point crossover (Goldberg, 1989) and uniform crossover (Syswerda, 1989). However, for sparse array optimizations, they may not be used directly in some applications. For example, if the sparse order needs to be kept constant, i.e. the solution layout should consist of given number of elements, these crossover operators could lead to problems. Using one-point crossover as example, it may change the number of elements before and after operation. Fig. 6.7 shows this phenomenon clearly. Note the two parents consist of four '1', respectively. After one-point crossover, the offspring have three '1' and five '1', respectively.

There are two ways to solve this problem. The first one is to leave GAs operators intact, and abandon the 'no proper' solution during selection procedure. The second one is to

modify the operators and avoid producing any illegal solutions. The later one is preferred in this thesis. This is also to avoid any unnecessary fitness evaluation calculations on undesirable solutions. It can not only save simulation time but also improve convergence speed. However, the risk to use modified operators is that their performance are difficult to be predicted by GAs theory directly. A lot of testing is necessary. One-point crossover is perhaps the most widely used operator in GAs. Firstly, constraints are applied to one-point crossover to ensure that this operator will not generate results out of solution space. This operator is named X1 in this chapter.

The obvious way to modify one-point crossover is to select the crossover point with conditions rather than totally random. To do so, one point is randomly selected first from parent chromosomes, then the number of '1' after the point for each parent is calculated and compared. Only when these two numbers are same, the crossover point is accepted. Otherwise it will be abandoned and selected again. Another constraint is that the parts of chromosomes after crossover point for each parent cannot be totally the same. This is to prevent the crossover from generating offspring which are totally same as their parents.

One problem for X1 is that there exists possibility that no crossover point can be found with the above constraints. For example, considering the following two strings, '11001' and '00111', crossover point doesn't exist at all based on the above constraints. This problem may fail the whole optimization procedure. Fortunately, for arrays consisting of more than 64 elements, this possibility is low. Once it happens, parents can be re-selected from the population to avoid crossover failure. This further reduces the failure possibility, if not completely avoided.

Empirical study (Syswerda, 1989) shows uniform crossover generally works better than both single-point and two-point crossover, though there are some exceptions. Uniform crossover is a kind of multi-point crossover. Based on this idea, the constrained multi-point crossover, X2 is developed. In order to keep the number of active elements constant before and after crossover, X2 is performed in 3 steps. Step one is to compare two parent chromosomes, and identify the different bit pairs. Only these different pairs are the candidates for crossover. For other bits, crossover does not make any sense. Step 2, the candidates are divided into two groups according to their values. Group one consists of all bits with value '0' in one parent and '1' in another parent, and group two is on the contrary. The size of two groups must be same, since two parent chromosomes have same number of bit '1' or '0'. Step 3, supposing the size of each group is K , the number of bits to be swapped from each group is forced to be same, and is randomly selected between 1 and $K-1$ from the above two groups. In this way, the number of '1' will be kept constant during crossover procedure. The number of swapped bits can not equal to K , otherwise all different bits will be swapped, and this will make the offspring be completely same as their parents. X2 is illustrated in Fig. 6.8.

The above two crossover methods are tested with 'Simple Model'. The results are shown in Fig. 6.9. Since GAs is basically stochastic procedures, in order to make more reasonable comparison, all results are average values from 20 independent programs running. The horizontal axis of the results is the number of evaluated solutions. Since the fitness evaluation operator takes almost all the program running time, the time for other operations can be ignored. Thus the horizontal axis is also an indicator of the used time. No mutation is used at this stage. Population size is pre-selected as 100. Selection by ranking (Goldberg, 1989) is used for all testing. In order to test crossover, a relative

simple selection method is used initially. In each generation, the chromosomes are sorted by their fitness values. The best 50 ones are candidatures to perform crossover. Each two of them are randomly selected as parents to generate 2 offspring by crossover. 50 offspring are generated. These 50 parents and offspring form the population of next generation. The results of totally random search are also shown in this figure. Similar as GAs results, it is also the average values of 20 independent running. The testing results show that X1 is almost the same as the random process. This may be due to the constraints which are applied to the one-point crossover deteriorate its performance greatly. However, the performance of X2 is much improved from X1. The convergence speed of X2 also increases.

For real code GAs, the crossover operator is developed based on the blend crossover ($BLX - \alpha$) (Eshelman and Schaffer, 1993). It uniformly picks values that lie between two points that contain the two parents, but may extend equally on either side determined by a user specified GA-parameter α , as shown in Fig. 6.10. p_1 and p_2 are two parents. $A_{\min}$ and $A_{\max}$ determine the range of legal solutions. The offspring can be generated with the following equation,

$$offspring = B_1 + u(B_2 - B_1) \quad (6.5)$$

where u is random selected from $[0,1]$, and,

$$\begin{aligned} B_1 &= p_1 - \alpha \cdot (p_2 - p_1) \\ B_2 &= p_2 + \alpha \cdot (p_2 - p_1) \end{aligned} \quad (6.6)$$

In order to ensure that the offspring are within the range $[A_{\min}, A_{\max}]$, B_1 and B_2 can be defined by,

$$\begin{aligned} B_1 &= \max(A_{\min}, B_1) \\ B_2 &= \min(A_{\max}, B_2) \end{aligned} \quad (6.7)$$

In this Chapter, α is selected as 0.5, if otherwise mentioned.

6.4.3 Mutation operator

Mutation is another important GAs operator. By mutating some genes in chromosome, it introduces new genes in population that cannot be produced by crossover operator. Thus the population diversity is ensured and pre-mature convergence can be avoided. The number of genes to be mutated is controlled by the parameter named mutation rate.

The mutation operator is relative easier to design than crossover. For binary GAs, the mutation is performed on pair of bits rather than a single bit. This pair of bits is randomly selected from the chromosome, consists of one '1' and one '0'. If the mutation is performed, the bit '1' flips to '0', and the bit '0' flips to '1' at the same time. Thus the total number of '1's keeps unchanged from the original chromosome. From the viewpoint of binary strings, mutation can create new combination of '1's and '0's to form new possible solutions.

For real code GAs, the mutation is performed by randomly selecting new gene values to replace the old ones. The testing results show that for real code GAs, the mutation plays less important roles than that in binary GAs. This is because that the $BLX - \alpha$ crossover can create almost all possible values for genes if α is not small.

6.4.4 Selection operator

In order to improve the performance of GAs, different selection operators are also tested. According to GAs theory (Miller and Goldberg, 1995), selection operators need to make

compromise between population diversity and convergence speed. Population diversity ensures enough good genes exist and transfer from current generation to next generation, and finally constitute the best solution. A selection strategy cannot simply select good chromosomes, which are measured by fitness value. Relative unfit chromosomes may also comprise good genes. There should be some chances that these genes can be transferred to next generation. Otherwise their genetic information may be lost. The optimization may converge to local optimal. In the above testing for crossover, the worst 50 chromosomes is simply abandoned, the convergence speed is very fast, but it can be seen from the later result in this chapter that the converged value is local minimum. On the other hand, if too many unfit chromosomes are selected and processed during optimization, the procedure to select good genes may be slow. The convergence speed will be reduced too.

There are various selection strategies. The most popular one may be the roulette-wheel selection (Goldberg, 1989). In this method, the possibility of each chromosome to be selected is proportional to its fitness value. The higher the fitness value is, the higher the possibility is. So even the worst chromosome has possibility to be selected, though sometimes it may be very low. The disadvantage of this selection strategy is that some “excellent” chromosomes, which have much better fitness values, may easily dominate the selection procedure. This is especially true when the fitness values vary in wide range. So this method is abandoned. For array optimizations, tournament selection (Miller and Goldberg, 1995) is a better choice. In this strategy, N chromosomes are pre-selected randomly from population to compete. The one with best fitness value becomes the selected one. Tournament selection selects chromosomes by their ranking rather than their originally fitness values. The possibility for each chromosome is proportional to

chromosomes ranking. Only the worst $N - 1$ one has no chance to be selected. Adjusting N could make the balance between population diversity and convergence speed for different applications and conditions. If $N = 2$, this operator is called binary tournament selection.

6.4.5 Other strategies

In order to improve the performance of GAs further, some new strategies different from the traditional GAs are used in arrays applications.

The first one is the duplicate-check in crossover and mutation operators. It prevents any chromosome being same as the one in current or previous populations by checking each new generated offspring with other chromosomes. If it is same as any existing one, then it is abandoned. The crossover or mutation will be performed again to create new one. This strategy avoids wasting calculation time in the duplicate solutions. It is important to reduce optimization time for array optimizations. On the other hand, it also prevents the population consisting of only the “excellent” member and many of its copies. Otherwise, the whole optimization procedure will repeat population by population and stop at local optimal.

The steady-state GAs is already used in testing crossover operators. Steady-state GAs differs from common GAs, by transferring some of the best chromosomes from one generation to next generation. Working in this way can increase convergence speed. This is because due to the stochastic properties of GAs operators, if all of the chromosomes are generated by crossover and mutation, it is possible that the new created chromosomes

are not better than the previous generation, the best solution from previous generation is lost.

By adopting the above strategies, the performance is improved dramatically from the common GAs. Fig. 6.11 compares the results of the common GAs with X2 crossover operator and the results of modified GAs. The random result is also shown.

6.4.6 GAs parameters

The two most important parameters are population size and mutation rate. The GAs theory provides basic principle for these parameters. This section tests a series of values to select the suitable ones.

A small population can make optimization faster, but may converge to local optimal, because of lacking genetic diversity. However, large population always means longer optimization time. Population size is related to the length of each chromosome.

In order to select optimal population size, different values are tested using Single Array. Fig. 6.12 shows the results for population size 50, 100, 150 and 200, respectively. The effect of population size to GAs performance can be seen clearly. The smaller the size is, the faster the convergence speed is. The larger the size is, the better the final convergence value is. Further, comparing the results of size 50 with size 100, and size 100 with size 200, we can see that there is limitation to improve performance by simply increasing population size. The results suggest that population size around 150 is a good trade-off between convergence value and speed.

Mutation rate determines the possibility of how many genes mutate in each generation. Mutation is another way to maintain population diversity. However, since mutation is a random procedure, a very high mutation rate may make the optimization procedure to be more random, and reduce the convergence speed. The mutation rate 0, 0.05, 0.1 and 0.2 is tested. Results are shown with Fig. 6.13. The population size used in test is 100. It can be seen clearly the effect of mutation rate. For mutation rate higher than 0.1, the convergence rate is too low to be acceptable. Changing mutation rate from 0 to 0.05, the convergence rate is lowered, but the convergence value is also a little improved. From the above results, the mutation rate can be selected among the range between 0 and 0.05 to make the trade-off between convergence rate and optimal results. However, if comparing the results in Figs. 6.12 and 6.13, it can be seen that simply increasing population size will realize the same effect as that of using mutation operator. These results are compared in Fig. 6.14. It can be observed that using population size of 200 has almost same effect as population size of 100 with mutation rate of 0.05, in terms of convergence rate and convergence value. Thus it's possible to achieve same performance by abandoning mutation operator in sparse array optimization and simply increasing population size.

The number of generations for program to run is determined in the following way. Firstly, a pre-determined value is set considering the chromosome length, population size. When the program is running, it checks the best fitness value for each generation. If the value keeps unchanged in 15 to 20 generations, the convergence is reached. Or the program will stop at the pre-determined generation, and the number of generations is suggested to increase.

6.5 Sparse array optimization

In this section, the tailored GAs is used in several applications of sparse array optimization. It shows great flexibility in design arrays active elements number, field performance, weighting coefficients. It is also successfully used for 2D arrays design and optimization due to its fast fitness evaluation operator.

6.5.1 Optimization of sparse array with given sparse order

For many design applications, one of the common problems is that with the given hardware configurations or the limit of system cost, what is the best system performance that can be achieved. For sparse array applications, this means if the number of electronic channels is pre-determined, the number of active elements in the array is also given. With the given sparse order, the purpose of optimization is to find the element layout that can provide the best field performance. The solution should provide minimum secondary lobe level (*SLL*) in its radiation field, and with other constraints for given application, such as that beam width is not greater than a given value to achieve desirable resolution, etc. The combinations of elements at different positions in dense array form the solution space of this problem. It should be noted here that even for 1D arrays, this solution space is so large that exhaustive checking for all possible solution is almost impossible. For example, if a 1D sparse array to be designed consists of 16 active elements, which are chosen from a 64 elements dense array, the number of total possible element layouts in sparse arrays will be $64 \times 63 \times \cdots \times (64 - 16 + 1) / [16 \times 15 \times \cdots \times 1] \approx 4.9 \times 10^{14}$.

For this problem, the binary crossover operator X2 is used. The fitness value for each possible layout is determined by Equation (6.3). The GAs parameters for the optimization

of 1D sparse arrays are as following, the population size is 150, mutation rate is 0. The program runs for 200 generations or less if the convergence conditions are satisfied.

The sparse way is first optimized for its one-way radiation pattern. The example 1D array consists of 16 elements, selected from the half wavelength 64-element dense array. The $-6dB$ and $-15dB$ main lobe beam width constraints are set to the values of dense array. The beam pattern of the optimized sparse array is shown in Fig. 6.15, compared with the pattern of dense array and the periodic array. The elemental pitch for periodic one is 2λ . Their main lobes are same. The maximum *SLL* of optimized one is $-19.1dB$, $0.5dB$ lower than the periodic one, which is $-18.6dB$. If the constraint of main lobe beam width at $-15dB$ is released, and 4° constraint is set at $-6dB$, the optimized beam pattern will provide about $-1.5dB$ lower maximum *SLL*, as shown in Fig. 6.15, at the price of wider main lobe.

For the sparse arrays optimized with their two-way patterns, several cases are studied. In all these cases, the sparse order is 2, i.e. half of 64 elements are selected as active elements. For the first case, the main lobe beam width constraints are set at $-6dB$ and $-30dB$ down from the peak of main lobe. While the second and the third case are all at $-6dB$ and $-25dB$ but with different value. The details of constraints values are listed in Table 6.1. These three cases represent loose or strict constraints on beam patterns. 10 independent running of each case are performed. The results show that the convergence values vary in narrow range for each running, though the optimised layouts are not same. The maximum *SLL* values of the best results for each case are also listed in Table 6.1, together with the main lobe beam width.

The beam pattern of Vernier design is calculated and compared with the optimized results. The elemental pitch of the Vernier array is $3/2\lambda$ for transmitting and 2λ for receiving. It utilizes of 22 and 16 elements for transmitting and receiving, respectively. The whole active elements in Vernier array are 32 because some of the elements are commonly used in both transmitting and receiving. It has same sparse order as the GAs optimized ones.

Fig. 6.16 shows the beam patterns of optimized 1D arrays and Vernier array. These results demonstrate the robustness of tailored GAs. The shapes of main lobes clearly show the effect of applied beam width constraints. If there are looser constraints on main lobe width, more radiation energy is allocated to main lobe region and the maximum secondary lobe level can be reduced. Case 2 has the strictest beam width constraint. Its maximum *SLL* is highest among the three cases. Its field performance is almost same as Vernier array in terms of main lobe width and maximum *SLL*. For Case 3, the main lobe beam width becomes wider than those of other cases and Vernier array under -25dB due to no constraints applied. Its maximum *SLL* is also reduced more than 1.5dB . The similar phenomena can be observed in Case 1. This indicates that it is possible to use GAs to design the desirable main lobe shape, or make compromise between the main lobe width and the maximum *SLL* for different applications.

The 2D arrays considered in this section consist of 20×20 elements. The sparse arrays to be optimized utilize 100 active elements. The chromosomes in GAs thus are strings with 400 bits. The population size is set to 800 considering the much longer chromosomes than that used in 1D arrays. The main lobe constraints are set at -6dB and -20dB , with the values calculated from dense array pattern.

The optimized 2D array layout is shown in Fig. 6.17, and Fig. 6.18 is its field pattern. The pattern clearly shows the constraints of main lobe beam width at $-6dB$ and $-20dB$. The maximum SLL is $-43.6dB$. This pattern can be compared with Fig. 6.5, which is the field pattern of Vernier array that consists of 100 elements in transmitting aperture and 49 elements in receiving aperture. The whole number of active elements used in Vernier array is 133 if account for the commonly used elements. Although Vernier array uses almost more than one third of elements, it can be seen that the GAs result has much lower maximum SLL . Even compared with Fig. 6.6, which is the pattern of random array consisting of 120 elements, the optimized one also provide $6.7dB$ lower maximum SLL with less active elements.

Further, the optimized 2D array is compared with the proposed designs by Austeng and Holm, 2002 (a, b). Among their periodic designs, Rad4 has the highest sparse order except the regular Vernier sparse design. This method is applied to the 20×20 dense array to get sparse arrays. The transmitting and receiving apertures are shown in Fig. 6.19, which consist of 145 and 120 active elements, respectively. Considering the elements used in both apertures, the number of total used active elements in this design is 181. The calculated beam pattern is shown in Fig. 6.20. The maximum SLL in this pattern is $-39.8dB$, which is also higher than that of the optimized one by GAs. The above results are summarized in Table 6.2.

6.5.2 Minimizing number of elements for given field performance

Another commonly encountered question for design is that what the minimal system requirements to achieve the desirable performance. For sparse array design, this question

is to minimize the number of active elements (or maximum the sparse order) under the given field response requirements.

To implement GAs to this design problem, elements layouts are encoded in binary chromosomes first (both for 1D and 2D arrays). Initially, both number of elements and elements positions are random selected for each chromosomes. All these chromosomes form the population of first generation. The fitness evaluation procedure is same as the one discussed in Section 6.5.1. The main lobe beam pattern of each chromosome is calculated. Then the beam width can be obtained from the beam pattern. If the beam width is greater than the given constraints, the chromosome is assigned a “bad” fitness value. Otherwise, the whole beam pattern is calculated and the maximum *SLL* is found. In order to minimize the number of active elements, the fitness value for each chromosome is the combination of weighted maximum *SLL* and element number, i.e.

$$fitness = \begin{cases} 'bad' \text{ value} & beamwidth \geq a \\ w_1 \cdot SLL_{\max} + w_2 \cdot N & beamwidth < a \end{cases} \quad (6.8)$$

where N is number of elements for each chromosome. w_1 and w_2 are weight factors.

In this problem, since there is no requirement for the chromosome to keep the number of active elements constant during the optimization procedure, the regular multi-point crossover operator can be used directly. Even though, if the chromosomes consist of too few elements during optimization procedure, clearly they cannot meet the field response requirements. These ‘bad’ chromosomes are not necessary to perform fitness evaluation, and a ‘bad’ value is assigned to them directly to reduce optimization time. Other strategies used in Section 6.5.1 are all adopted.

The above GAs is applied to 1D sparse arrays first. The conditions for an example running are set as following: the main lobe beam width constraints are set to 2° at $-6dB$ and 4.5° at $-25dB$. The required maximum *SLL* is smaller than $-40dB$. GAs population size is 150, and runs for 150 generations. The program runs 10 times independently. The average value of minimized element number is 31.9. The best one is 31, while the worst one is 33. Note this results match well with the one in Section 6.5.1, where the array consists of 32 elements under the same beam width constraints and the maximum *SLL* is the criterion to be minimized (Fig. 6.16).

For 2D arrays, the population size it set to 800 and the program runs 250 generations. The main lobe beam width constraints are set to the values calculated from the beam pattern of dense array, and maximum *SLL* is set to be $-40dB$. The GAs optimized 2D layout consists of 94 elements, which satisfies the above requirements.

6.5.3 Optimization of apodization weighting coefficients

Apodization is a widely used method to control side lobe levels in arrays radiation field. It controls the amplitude of signal transmitted or received by each element and allocates part of energy from side lobe region to main lobe. Thus the side lobe level can be reduced, but the main lobe width will increase at the same time. For narrow band arrays, theories based on Fourier theory are well developed to design apodization functions (Trees, 2002) for different requirements of beam shapes or *SLL* level. For example, the well-known Dolph-Chebyshev array provides uniform height sidelobes (Steinberg, 1976). However, for broadband arrays, it is somewhat difficult to design the weighting functions due to the beam shape are both time and space dependent. Only the main lobe region is under the

control of Fourier theory. On the other hand, the sparse arrays result in undersampling of weighting functions, which strongly affects the desirable effect. This is especially true for non-periodic arrays. No analytical theories are currently available to describe this problem. In this section, real-code GAs is used to design the apodization coefficients for sparse arrays. The results show that for non-periodic arrays, it is also possible to use apodization to reduce side lobe level.

Real-code GAs is more suitable to design the weighting coefficients than traditional GAs because the parameters to be considered are continuous numbers rather than discrete numbers. The real-code GAs can encode problem parameters into chromosomes consist of real numbers directly, and avoid sampling errors that exist in binary GAs. Since using larger population size can ensure the population diversity, mutation is not used either in real-code GAs. The $BLX - \alpha$ crossover operator discussed in Section 6.4.2 is used in this Section.

A sparse array that consists of 32 randomly selected elements is shown as the first example. The weighting coefficients for each element form one chromosome with 32 real-number bits. The population size is 100 and the program runs 100 generations due to the relative shorter chromosome. The fitness value is calculated with Equation (6.3). The value for beam width constraints is calculated from the uniformly weighted pattern of this array. Fig. 6.21 shows the beam pattern under optimized apodization weighting, compared with the uniformly weighted and cosine weighted pattern. It can be seen that the optimized apodization reduces the maximum SLL more than $4dB$ while it keeps the main lobe almost intact within the beam width constraints, and an almost flat side lobe is

achieved. Compared with the optimized result, the cosine weighting fails, instead it increases the sidelobe as well as the main lobe beam width.

For the layouts whose elements positions have been optimized by GAs already, using apodization weighting can further reduce the maximum SLL , though the effect is not as dramatic as the random one. Fig. 6.22 shows the results with and without optimized apodization. It can be seen that the maximum SLL is further reduced by 1.4 dB, and flat side lobe region can also be observed. No doubt, if the main lobe constraints can be relaxed, it is also possible to use GAs to design apodization weightings to make trade-off between side lobe level and main lobe beam width.

To apply the above algorithms to 2D arrays is straightforward. However, practically weighting is not used with 2D arrays, because the low signal to noise ratio (SNR) is a serious problem for 2D arrays due to the tiny array elements. Apodization could worsen this problem. So 2D arrays are not considered in this section, though GAs can be used to optimize 2D weighting coefficients with same effect as 1D arrays.

6.5.4 Global synthesis

In this section, the developed GAs algorithms are used to deal with so-called global synthesis optimization problem, as mentioned by Trucco (2002). Here global synthesis means to optimize the element positions and the weighting coefficients, and, at the same time, minimize the number of elements. The resulting array layout can generate beam pattern that is satisfied with pre-determined requirements, such as main lobe beam width, maximum SLL , etc. Or for given number of elements, optimized the element layout, weighting coefficients, and minimize maximum SLL with main lobe constraints

simultaneously. The spatial aperture is not included in the parameters to be optimized in this thesis. This is because the acoustic windows of different medical imaging applications always determine this parameter.

Based on the algorithms discussed in Section 6.5.1 to 6.5.3, it's nature to combine these algorithms to fulfill global synthesis. Hybrid chromosomes are used to form population. Each chromosome comprises two parts. The first part is binary string, represents the element layouts. The second part is weighting coefficients for each element. It is real-code string. The length of both parts equals to the number of elements in dense array. The population for the first generation is random generated. The number of '1' and their positions are random selected for first part of chromosome. For second part of chromosome, values between the given weighting ranges are also generated and assigned to each bit. Each value corresponds to the weighting for each element in the first part of chromosome, if this element is active, i.e.,

$$chromosome = [a_1, a_2, a_3, \dots, a_N, w_1, w_2, w_3, \dots, w_N] \quad (6.9)$$

where N is number of elements in dense array, and

$$a_1, a_2, a_3, \dots, a_N = 1 \text{ or } 0 \quad (6.10)$$

$$w_1, w_2, w_3, \dots, w_N \in [weighting_{\min}, weighting_{\max}] \quad (6.11)$$

Array with weighting can be decoded from chromosome by following equation,

$$layout = [a_1 \cdot w_1, a_2 \cdot w_2, \dots, a_N \cdot w_N] \quad (6.12)$$

The GAs operators, including crossover and mutation for binary and real-code chromosomes are also applied to the two parts of the hybrid chromosome respectively. They can be used directly without any modification.

The hybrid GAs is used in the problem discussed in Section 6.5.1 but considering the apodization at the same time. The number of active elements in sparse array is given to be 32. The optimizing parameter is the element position, weighting coefficients, and minimizing the maximum *SLL* with main lobe width constraints same as that in Section 6.5.1. The population size used in this optimization is 200. GAs runs 250 generations and converges. The result layout with optimized weighting can generate beam pattern with maximum *SLL* in almost same level as the result shown in Fig. 6.22. This pattern is displayed in Fig. 6.23.

The above algorithms are also used to optimize the combined problem in Section 6.5.2 and 6.5.3. The desired beam pattern is controlled by the following parameters: the maximum *SLL* is -40dB , -6dB main lobe beam width constraint is 2° , and -20dB main lobe beam width constraint is 4.5° . The parameters to be optimized include the number of active elements, the element layout and apodization weighting coefficients for each active element. The population size in this GAs is 300. After running for 200 generations, the result converges. The program runs for 10 times independently. The result sparse array consists of 31.3 active elements on average. This result is equivalent to the results in the previous section. Thus the hybrid GAs is efficient in global synthesis problems.

6.6 Conclusions

This chapter studies the design and optimization of broadband sparse array transducers. The traditional methods, which are mainly developed with narrowband theories, are introduced first. When applying them to broadband arrays, the performance is not good.

For example, the Vernier array is efficient to reduce grating lobe level for narrowband arrays. But for broadband case, its performance is very limited, especially for 2D arrays.

The failure of the traditional methods to design and optimize broadband sparse arrays is due to the different mechanisms of broadband radiation field from the narrowband ones, as discussed in Chapter 3 and 4. The broadband radiation field is determined by not only element layouts, weightings, etc. but also the bandwidth and waveform of the transmitted signal from each element. When traditional methods are used to design sparse arrays, the later factors are ignored.

Genetic algorithms as stochastic optimization and search method is adopted in this chapter. Some GAs operators and parameters are tailored for array applications. With the modified GAs, and the approximate methods for field calculation, the broadband sparse arrays can be optimized, both for 1D and 2D arrays. Many parameters and related factor of broadband arrays can be considered simultaneously, which include element layout, sparse order, apodization weightings, etc. Especially, with the efficient method of field calculation that is developed in Chapter 4, 2D arrays can also be optimized, and relative large population size and number of generations can be adopted in GAs, which avoid that the optimized results converge to local optimal.

Several example applications are demonstrated in this chapter. Results show that the developed GAs is powerful in design and optimization of broadband arrays. It is possible to make trade-off among arrays main lobe shape, maximum *SLL*, element sparse order for different practical applications. It also provides the methods to design apodization weighting coefficients for non-periodic arrays. With the method, the design of array

transducers may be tailored to application specification in terms of layout, element sparse order, main lobe width and maximum SLL , etc.

Table 6.1 Maximum *SLL* and main lobe beam width for optimized 1D arrays

	Main lobe Constraints	Maximum <i>SLL</i> (dB)	Main lobe Beam width ($-6dB$)	Main lobe Beam width ($-25dB$)
Case 1	$-6dB : 2^\circ$ $-30dB : 8^\circ$	$-42.12dB$	1.99°	6.95°
Case 2	$-6dB : 2^\circ$ $-25dB : 4.5^\circ$	$-40.69dB$	2.0°	4.23°
Case 3	$-6dB : 2^\circ$ $-25dB : 6.5^\circ$	$-42.36dB$	1.89°	6.48°
Vernier	—	$-40.63dB$	1.82°	4.11°

Table 6.2 Optimized results for 2D arrays, compared with other designs

	No. of active elements	Maximum <i>SLL</i> (dB)	Sparse order
Dense array	400	—	—
Vernier	133	$-31.8dB$	3
Random	120	$-36.9dB$	3.3
Rad4*	181	$-39.8dB$	2.2
GAs	100	$-43.6dB$	4

* (Austeng and Holm, 2002)

Fig. 6.1 One-way narrowband radiation pattern of 1D array focuses at $\theta_s = 45^\circ$, element pitch is $3\lambda/2$. Two grating lobes can be observed.

Fig. 6.2 Two-way narrowband radiation pattern of 1D Vernier array focuses at $\theta_s = 45^\circ$, element pitch is $3\lambda/2$ for transmitting and λ for receiving, respectively.

Fig. 6.3 One-way broadband radiation pattern of 1D array focuses at $\theta_s = 45^\circ$, element pitch is $3\lambda/2$ (solid line) and λ (dashed line).

Fig. 6.4 Two-way broadband radiation pattern of 1D Vernier array focuses at $\theta_s = 45^\circ$, element pitch is $3\lambda/2$ for transmitting and λ for receiving, respectively.

Fig. 6.5 Two-way broadband radiation pattern of 2D Vernier array focuses at $\phi_s = 30^\circ, \theta_s = 0^\circ$. The maximum secondary lobe level is -31.8 dB .

Fig. 6.6 Two-way broadband radiation pattern of 2D random array focuses at $\phi_s = 30^\circ, \theta_s = 0^\circ$. The maximum secondary lobe level is -36.9 dB .

Fig. 6.7 Traditional crossover may change number of '1' before and after the operation

Fig. 6.8 X2 crossover. The bits to perform crossover are equally selected from two groups, thus number of '1' will keep constant during crossover

Fig. 6.9 Testing results for X1 (dashed line) and X2 (dash-dot line), compared with random search results (solid line).

Fig. 6.10 $BLX - \alpha$ crossover for real number GAs

Fig. 6.11 Testing results for common GAs (dashed line) and the modified one in this chapter (dash-dot line), compared with random search results (solid line).

Fig. 6.12 GAs results for different population size, comparing with totally random search results.

Fig. 6.13 GAs results for different mutation rate, $\mu = 0, 0.05, 0.1, 0.2$, population size is set to 100.

Fig. 6.14 GAs results for larger population size without mutation (dashed line) and smaller population size with mutation (solid line).

Fig. 6.15 One-way beam patterns of GAs optimized 1D arrays, compared with dense array and periodic sparse array.

Fig. 6.16 Pulse-echo beam patterns of GAs optimized 1D arrays

Fig. 6.17 The element layout of GAs optimized 2D sparse array. The number of active elements is 100.

Fig. 6.18 Beam pattern of the sparse array shown in Fig. 6.17. The maximum secondary lobe level is -43.6 dB .

Fig. 6.19 Radially sparse periodic design proposed by Austeng and Holm, 2002 (a,b). The transmitting aperture is (a), and receiving is (b).

Fig. 6.20 Beam pattern of the array shown in Fig. 6.19. The maximum secondary lobe level is -39.8 dB .

Fig. 6.21 Beam pattern of random array with uniform weighting (dotted line), GAS optimized weighting (solid line) and cosine weighting (dashed line). The maximum SLL is $-34.5dB$, $-38.6dB$, $-29.8dB$, respectively.

Fig. 6.22 Beam pattern of non-periodic array which is optimized by GAS first, with uniform weighting (dotted line) and GAS optimized weighting (solid line). The maximum SLL is $-40.7dB$ and $-41.7dB$, respectively.

Fig. 6.23 Beam pattern of non-periodic array, which is optimized by GAs global synthesis. The maximum SLL is $-40.9dB$.

Chapter 7

Experimental Validation

This Chapter presents a prototype 2D array transducer and a synthetic field measuring system, which are developed and used to validate the methods for field calculation and sparse array optimization, which are described in previous chapters.

7.1 Synthetic method for array radiation field measurement

Array transducers are always controlled by multiple channels beamformers. Each channel excites one array element with voltage as high as several hundreds volts and receives the returned echoes, which can be very small in amplitude. It also has the ability to provide time delays and apodization weightings. The received signals are processed further to form images. To develop such a system for experiment is expensive and time-consuming. On the other hand, to fabricate array transducers with tens to hundreds of elements is also difficult, especially for 2D arrays. In order to overcome the above difficulties, a synthesis method is used in the experiment to validate the simulation results.

The basic idea of this method is to perform the beam forming procedure using software rather than hardware. Fig. 7.1 demonstrates the method with an 1D array as an example. In order to avoid using array transducer and multiple channel beamformer, a single array element is used only. If the array to be studied consists of N elements, this single element will be moved to N positions, where each array element should be located. At each position, the element is excited to transmit the pulse. A needle-like hydrophone is used to receive the transmitted signal from the element. The hydrophone is located at the

positions where the field points to be measured. Thus for each field position, there will be N waveforms to be collected and recorded. These N groups of data represent the wave from each array element respectively. These data can be added by applying proper time delays that are calculated according to the array configuration, focus point position and the field point positions. Then the one-way radiation field can be achieved. i.e.

$$p_{\vec{r}}(t) = \sum_{i=1}^N p_{\vec{r},i}(t - \tau_i) \quad (7.1)$$

where $p_{\vec{r},i}(t)$ is the i th record at field point $\vec{r}$, τ_i is the time delay for this record.

On the other hand, if the element location is fixed, and the hydrophone is moved to different positions according to the array element positions, the measured and calculated result is the one-way receiving pattern. However, it must be equivalent to the radiation field due to the acoustic reciprocal theorem (Kinsler *et al.*, 1982).

To measure the pulse-echo two way field, it is possible to put a small metal ball in the field to work as the point scatterer. The hydrophone and the array element are located on the same side, which work as receiver and transmitter, respectively. However, there are two obstacles to do so. Firstly, the array aperture is so small that it is very difficult to arrange the array element and hydrophone so closely. Secondly, the signal to noise ratio is very poor because the scatter is small. For 2D array element, the transmitted signal from the transducer is weak, and the reflected signal can be totally immersed in noise. On the other hand, if the scatter volume is increased, the accuracy of measurement will be reduced.

Due to the above two reasons, the two way radiation field cannot be measured directly. However, it can be calculated with the measured one-way data. From Equation (4.21), it can be achieved

$$p_0(t) * p_{TR}(\bar{r}, t) = p_0(t) * h_T(\bar{r}, t) * p_0(t) * h_R(\bar{r}, t) \quad (7.2)$$

and

$$p_0(t) * p_{TR}(\bar{r}, t) = p_T(\bar{r}, t) * p_R(\bar{r}, t) = w(\bar{r}, t) \quad (7.3)$$

where $p_T(\bar{r}, t)$ and $p_R(\bar{r}, t)$ are one-way transmitting and receiving pressure field, respectively. They can be achieved by Equation (7.1) from the experimental results. Thus the two-way radiation field $p_{TR}(\bar{r}, t)$ can be obtained by a deconvolution process.

However, the deconvolution problem is highly ill conditioned and a simple deconvolution process will not produce any meaningful results. Several deconvolution techniques are available. A common approach is to apply one of the frequency domain Wiener filters. This filter is formulated based on the minimization of the least square error. In frequency domain, applying the Wiener filter to Equation (7.3) one can obtain (Jansson, 1997),

$$P_{TR}(\bar{r}, \omega) = \frac{W(\bar{r}, \omega) \cdot P_0^*(\omega)}{|P_0(\omega)|^2 + S_n(\omega) / S_{p_{TR}}(\omega)} \quad (7.4)$$

where the variables in capital letter represent the Fourier transforms of the corresponding variables in small letter in Equation (7.3). $P_0^*(\omega)$ is the complex conjugate of $P_0(\omega)$. $S_{p_{TR}}(\omega)$ and $S_n(\omega)$ are the power spectral densities of $p_{TR}(\bar{r}, t)$ and additive noise in signals. In practice, this term is usually set equal to a frequency-independent constant Q^2 to simplify the calculation procedure (Chen and Sin, 1992),

$$P_{TR}(\vec{r}, \omega) = \frac{S(\vec{r}, \omega) \cdot P_0^*(\omega)}{|P_0(\omega)|^2 + Q^2} \quad (7.5)$$

This constant is called the noise desensitising factor. One commonly recommended value for Q^2 is (Chen and Sin, 1992)

$$Q^2 = 0.01 \cdot |P_0(\omega)|_{\max}^2 \quad (7.6)$$

7.2 Experimental set up and procedure

The block diagram of the experimental system is shown in Fig. 7.2 and the system is shown in Fig. 7.3.

A 3×3 2D array prototype transducer is designed, and fabricated by Duke University. The active material in this array is PZT-5H, which is bonded with ground foil, backing and matching layer by epoxy and cut in two directions with $50\mu m$ thick diamond saw to obtain each single element. The element pitch is $250\mu m$, or 0.6λ with nominal frequency of $3.5MHz$. Each element is terminated in a 10-pin header connector, and wired with $30cm$ of micro-coaxial cable. On the other end of each cable, BNC connector is used. In order to facilitate the experiment, the whole structure and cables are packaged with a $30cm$ of waterproof flexible plastic tube $3mm$ in diameter. Material properties of this array can be found in Appendix B.

One of the elements from the prototype transducer is used as the signal source. It is excited with spike voltage pulse at 200 volts, with repeat rate of $1KHz$, which is generated from the Pulser/Receiver (model FUI2100, Fallon Ultrasonic Inc). In the measuring system, the transducer is mounted on the transducer holder, which allows the

angular adjustment in order to align the transducer with the hydrophone. Furthermore, since the whole experiment may last a few weeks, in order to avoid immersing the transducer in water for too long, the holder can move in vertical direction manually, and the transducer can be removed from water after measurement each day and re-positioned to the same location with enough accuracy to continue the measurement without re-alignment.

A needle-like PVDF hydrophone (model HPM 05/3, Precision Acoustics) is used to measure the pressure wave transmitted from the transducer in water. The active material of the hydrophone is $9\mu\text{m}$ thick PVDF and 0.5mm in diameter. The bandwidth of its frequency response is wide enough for the signals to be measured. The hydrophone is connected with the submersible preamplifier (model HP1, Precision Acoustics) with nominal 8dB voltage gain at 3.5MHz . The signal is amplified further by the booster amplifier fabricated by the same company (model HA1, Precision Acoustics). It provides another 25dB gain in voltage. The hydrophone and the preamplifier are mounted in the hydrophone holder. The holder allows angular adjustment in two planes that are perpendicular with each other. Then the hydrophone can be aligned to the signal source at any position, as demonstrated in Fig. 7.4. This eliminates the influence to the measured results from the hydrophone directivity. The holder and the hydrophone can move in three directions by a computer controlled 3-axis actuator system (IAI Automation) to measure different field points, with the positioning accuracy of $\pm 0.025\text{mm}$.

Before the first measurement, the hydrophone is aligned along the x axis by the device in the system. Then the acoustic axis of the transducer is adjusted to be parallel with the x axis too by scanning the hydrophone back and forth between two points along this

direction and ensuring that at each point the hydrophone signal is a maximum in the $y - z$ plane (IEEE, 1989). Once the transducer is aligned, it is fixed and cannot change its angle during the whole measuring procedure.

The measured and amplified signal is digitized and recorded by a 12-bit data acquisition card (DAQ card, model DP306, Acqiris) at 100MHz sampling frequency. The high sampling frequency ensures the higher frequency components in the broadband signal can be recorded properly. The trigger signal to the DAQ card comes from the Pulser/Receiver. A digital oscilloscope (model 9450A, LeCroy) is also used for monitoring the signal for assisting in alignment and set up.

The whole system is integrated with *LABVIEW* and run on a PC computer. The movements of hydrophone and the data acquisition procedure can be controlled by the developed software directly. The coordinates of the field points and the corresponding waveforms are recorded at the same time in data files for post processing to synthesize the radiation field. The whole measuring procedure is fully automated and can be user-defined. In addition to field measurement in this project, the system can be used for measuring other characteristics of transducers.

7.3 Experimental results – 1D arrays

To increase the signal to noise ratio, all recorded data are the results of 10 averages, and noise-filtered by the 5th order Chebyshev filter over a bandwidth of approximately 6MHz with the central frequency 3.2MHz. One of the waveform example and its

spectrum is shown in Figs. 7.5 and 7.6. The $-6dB$ relative bandwidth of the waveform is around 36% , with the central frequency of $3.2MHz$.

The 1D arrays are studied first. The arrays consist of 64 elements, half wavelength pitch. For the 1D arrays, all measurements are performed in $x-y$ plane, where the array elements are located along y axis. The elemental directivity is first measured between the angular ranges $\pm 55^\circ$, and the result is shown in Fig. 7.7. Compared with the theoretical result, the measured one is narrowed dramatically. It is due to the cross coupling effect among the elements, and confirmed by the conclusions of the FEM studies in Chapter 5.

The waveforms at all possible relative positions that between the array elements and field points are measured and recorded. These data are used to calculate the radiation field with the synthetic methods discussed in Section 7.2. They are thus referred as the experimental results in the following discussions. The simulation results mentioned in the following part of this chapter is referred to the ones calculated by the methods developed in Chapter 4, where the waveform shown in Fig. 7.5 is used as the initial pressure waveform $p_0(t)$, and the measured element directivity shown in Fig. 7.7 replaces the theoretical one.

The arrays optimized with their one-way radiation field are calculated first. The optimized sparse array consists of 16 elements, which is shown in Fig. 7.8(a). Figs 7.8(b) and (c) show one example of random array and the periodic sparse array, respectively. The simulation results of the optimized array by GAs are shown in Fig. 7.9, and compared with the results of the other two designs. The corresponding experimental

results can be seen in Fig. 7.10. The optimized sparse array provides around $2-3\text{dB}$ lower maximum *SLL* than that of the periodic array, and the random array has the highest maximum *SLL*. This can be observed from both simulation and experimental results. Some deviations can be seen between the above two results. The experimental results have wider main lobe, and in particular higher floor noise of $2-3\text{dB}$. The deviations are due to two factors. Firstly, the measured waveforms are not uniform due to the influence of system noise. This is especially true at the field points with large angles, where the pressure signal is much weaker than those of on-axis points. Secondly, the errors in time delays also increase the *SLL*. Despite the above deviations, general agreement in the characteristics of radiation fields between the experiments results and simulation results can be observed.

The results for arrays optimized with pulse-echo field are demonstrated with Fig. 7.11 and 7.12, which are simulation results and experimental results, respectively. The optimized sparse arrays consist of 32 elements, and the Vernier array in comparison utilizes the same number of active elements. The pattern of one example of random array with same number of active elements is also shown. The element layouts of the above three arrays can be found in Figs. 7.8(c-f). The same phenomena as those of one-way case can be observed, and it is evident that the GAs optimized array offers the best characteristics in terms of the maximum *SLL*.

7.4 Experimental results – 2D arrays

The data for 2D arrays are also measured. The method used is same as that of the 1D arrays. The 2D dense array studied consists of 10×10 elements, with the pitch of half

wavelength in two directions. The transducer plane is $y-z$ plane, and the field points that measured are located in full 3D space. The measured elemental directivity is shown in Fig. 7.13, compared with the theoretical result shown in Fig. 7.14. The narrowed directivity can also be observed from the experiment result. The simulation and experimental results for dense array are shown in Figs. 7.15 and 7.16. A grating lobe can be observed from the results due to the higher frequency components in signals. Besides some deviations between the two patterns, the main features of field pattern such as the main lobe, sidelobe and grating lobe agree well. The experimental result has higher maximum *SLL* due to the same reasons as that of 1D array results.

The sparse arrays are then compared experimentally. The GAs optimized sparse arrays consist of 25 elements. The criteria of optimization are same as that discussed in Section 6.5.1, i.e. given the number of sparse order and constraints on main lobe beam width in pulse-echo radiation field, while the solution provides the lowest maximum *SLL*. Several random arrays with same sparse order are also tested, and the one with the lowest *SLL* is selected and demonstrated in this chapter. The corresponding 2D Vernier array consists of 37 active elements, considering the commonly used elements in both transmitting and receiving aperture. The element layouts of the above three designs are shown in Fig 7.17. The simulation results of pulse echo radiation field are shown in Figs. 18, 20, 22, while the experimental results are in Figs. 19, 21, 23, for Vernier array, GAs optimized array and random array, respectively. The maximum *SLL* achieved are also listed in Table 7.1 for all three designs and the dense array.

From the above results, it can be seen that though the Vernier array utilizes more active elements than the other two designs, the grating lobe cannot be avoided. The high level

grating lobes in transmitting and receiving apertures cannot be eliminated with each other effectively due to the broadband signals, which results high *SLL* in the two-way patterns. Random design does avoid grating lobes, however, the high level sidelobes appear in some regions, and furthermore, the main lobe beam width is also widen due to it cannot be controlled efficiently. The layout optimized by GAs provides the lowest *SLL* among the three designs, where the maximum *SLL* is almost at the same level as the results of dense array, though a sidelobe floor exists.

The above phenomenon can be observed from both simulation results and experimental results. Despite the existence of noise, the experimental results agree well with the corresponding simulation results. In both results, the GAs optimized layout shows the best performance among the three designs, as evidenced in Table 7.1.

7.5 Conclusions

In this chapter, experimental methods are used to validate the radiation field calculation methods and GAs optimization for sparse array design. A cost efficient synthesis measuring system is developed to fulfil the measurements, together with a 3×3 2D array transducer. The measured data can be used to synthesis the field response of any sparse array designs that have similar element characteristics in terms of the operating frequency, bandwidth and directivity, etc. Both 1D and 2D arrays have been studied experimentally, and the experimental results show consistence with the simulation results for both dense arrays and sparse arrays, which include the random design, Vernier design and the design by the GAs approach. All results show that the GAs optimized layout can provide the best performance among the three designs.

Table 7.1 The maximum *SLL* of three 2D array designs and the dense array

	Simulation results (dB)	Experimental results (dB)
Dense array	-28.9	-26.7
Vernier	-15.9	-15.1
GAs	-27.7	-22.1
Random	-14.1	-12.7

Fig. 7.1 Synthetic method for arrays radiation field validation.

Fig. 7.2 Block diagram of the experimental system

Fig. 7.3 The experimental system

Fig. 7.4 The hydrophone holder allows hydrophone to rotate in two perpendicular planes $x-y$ and $x-z$.

Fig. 7.5 An example of the measured waveform from single array element

Fig. 7.6 Spectrum of the measured signal shown in Fig. 7.5

Fig. 7.7 Measured element directivity in $x - y$ plane (*), compared with theoretical results (solid line).

Fig. 7.8 Element layouts of 1D array, black blocks represent active elements, (a) GAs optimized by one-way radiation field, 16 elements (b) Random array, 16 elements (c) Vernier array, transmitting aperture, 2λ pitch, 16 elements (d) Vernier array, receiving aperture, $3/2\lambda$ pitch, 22 elements (e) GAs optimized by two-way radiation field, 32 elements (f) Random array, 32 elements.

Fig. 7.9 Simulation results of 1D sparse arrays one-way radiation field. GAs optimized layout (Solid line), periodic layout (dashed line) and random layout (dotted line).

Fig. 7.10 Experimental results of 1D sparse arrays one-way radiation field. GAs optimized layout (Solid line), periodic layout (dashed line) and random layout (dotted line).

Fig. 7.11 Simulation results of 1D sparse arrays pulse-echo radiation field. GAs optimized layout (Solid line), Vernier layout (dashed line) and random layout (dotted line).

Fig. 7.12 Experimental results of 1D sparse arrays pulse-echo radiation field. GAs optimized layout (Solid line), Vernier layout (dashed line) and random layout (dotted line).

Fig. 7.13 Measured elemental directivity of 2D array element

Fig. 7.14 Theoretical elemental directivity of 2D array element

Fig. 7.15 Simulation results of 2D dense array pulse-echo radiation field. The maximum SLL is $-28.9dB$.

Fig. 7.16 Experimental results of 2D dense array pulse-echo radiation field. The maximum SLL is $-26.7dB$.

(a)

(b)

(c)

(d)

Fig. 7.17 Elemental layouts of 2D arrays (a) GAs optimized sparse array, 25 elements (b) Random array, 25 elements (c) Vernier array, transmitting aperture, λ pitch, 25 elements (d) Vernier array, receiving aperture, $3/2\lambda$ pitch, 16 elements.

Fig. 7.18 Simulation results of 2D Vernier array pulse-echo radiation field. The maximum SLL is $-15.9dB$.

Fig. 7.19 Experimental results of 2D Vernier array pulse-echo radiation field. The maximum SLL is $-15.1dB$.

Fig. 7.20 Simulation results of GAs optimized 2D sparse array pulse-echo radiation field. The maximum SLL is $-27.7dB$.

Fig. 7.21 Experimental results of GAs optimized 2D sparse array pulse-echo radiation field. The maximum SLL is $-22.1dB$.

Fig. 7.22 Simulation results of 2D random sparse array pulse-echo radiation field. The maximum SLL is $-14.1dB$.

Fig. 7.23 Experimental results of 2D random sparse array pulse-echo radiation field. The maximum SLL is $-12.7dB$.

Chapter 8

Conclusions and Recommendations

8.1 Conclusions

Ultrasonic imaging is one of the four modalities used nowadays in medical diagnosis imaging. As the signal transmitter and receiver, the transducer plays a very important role in imaging systems. The performance of the transducers is directly related to the image quality, including the spatial resolution, dynamic range, frame rate, etc. Array transducers provide more versatile function with the techniques named dynamic focusing and electronic scanning, and are widely used nowadays. However, 2D arrays, which are necessary for 3D real time imaging system, are still far from practical due to the large number of array elements and supporting electronic channels. This thesis presents a detailed study of array transducers, with focus placed on the design and optimization of sparse arrays. The arrays radiation field, which directly determines the imaging quality, is used as the criterion of optimization.

In Chapter 3, array theory is discussed. It is evident that the traditional array theory is mainly based on narrowband signals, while the ultrasonic waves used in imaging systems are signals with short time durations and broad bandwidth in order to increase the axial spatial resolution. The fundamental differences of the radiation fields between the two different arrays are analyzed. For broadband field, no steady-state response exists, and the field is fully spatial and time dependent. The field response is determined by not only the array configurations and operating frequency, but also the bandwidth and waveform of the signal transmitted from each element. This makes the problem a more complicated

one than the narrowband case. It also explains the reason why using traditional array theory is not adequate to describe broadband field, though after some simplifications, the broadband fields are in the similar form as the narrowband ones. However, the methods based on broadband field are necessary for studying the imaging system.

A more efficient method for broadband field calculation based on the spatial impulse response methods is developed in Chapter 4. Although the rigorous method is available to calculate the field response, it is too complicated and time consuming for array optimization. This is because the discontinuity in arrays impulse response requires sampling frequency as high as GHz in computer simulation. With this sampling frequency, it will take tens of minutes to calculate the full field response of 1D arrays. For 2D arrays, it may need several hours. Obviously, that is not suitable to be used for array designs. The methods developed in this chapter avoid calculating the impulse response directly. In realizing that the element dimension is at the order of half wavelength in one side (for 1D arrays) or both sides (for 2D arrays), the impulse response from each element can be assumed to be single impulse, and modified by some factors calculated from single element, such as the elemental directivity. With this assumption, the sampling frequency can be reduced by one order to a few hundreds KHz . The testing results show that the calculation time by the developed method can be reduced to within one second to a few seconds for both 1D and 2D arrays. The errors compared with the rigorous results are within $\pm 5\%$.

Finite element method is adopted as an alternative methodology in Chapter 5 to study the array transducers. With the FEM code based on fast hybrid explicit/implicit algorithms, it is possible to model both the array transducers and the radiation field. Thus the influence

of the element dynamic performance and the inter element cross coupling to the radiation field can be taken into account. A few models are simulated, and the results show that the cross coupling has significant effect on the arrays field. It shows the side lobe level increases at high coupling level, and the elemental directivity is greatly narrowed. This is confirmed by the subsequent experimental results in Chapter 8. When the coupling level is low, the FEM results match well with the theoretical results by the methods developed in Chapter 4.

The broadband field depends on more factors than that of the narrow band case, inevitably the methods based on narrowband theory for sparse array design do not perform well. This is demonstrated in Chapter 6. To design the broadband sparse array, the signal from each element must be taken into account, as well as the array configurations. This chapter develops the design and optimization approach for broadband sparse arrays. The approach is based on stochastic methods Genetic Algorithms. Some novel GAs operators and routines are developed for various array applications. The GAs parameters are also tested and selected. The developed approach shows great flexibility in design sparse arrays. For example, the trade-off between sparse order and performance can be realized. Apodization weighting coefficients can be selected for nonperiodic arrays. The design criteria can be determined simply by modifying fitness evaluation functions. Global synthesis is also possible. Due to the fast methods developed in Chapter 4 to calculate the field response, the approach can be applied to 2D arrays successfully. The results show that the GAs optimized nonperiodic arrays outperform the periodic designs such as the Vernier array in terms of maximum *SLL*. The maximum *SLL* can also be reduced further by GAs designed weighting coefficients, even for nonperiodic arrays.

In Chapter 7, a prototype 3×3 2D array transducer is used to validate the optimization methods developed in this thesis. The elemental directivity and signal transmitted from single element is measured and recorded. They are used in design and optimization of assumed arrays that consist of practical number of elements. The performance of the array element is different from the one used in Chapter 6, but the developed methods also generated satisfied results. On the other hand, a synthesis experimental system is developed to measure the arrays radiation field. The waveforms for all the possible relative positions between array elements and field points are measured and recorded. Thus the radiation field from any sparse arrays can be synthesized. The radiation field of the optimized arrays as well as the arrays designed by other methods is validated by the developed experimental method. The optimized arrays show lowest secondary lobe level, and the experimental results agree with the simulation results.

The major contributions of the research work can be summarised as follows:

- Approximate methods for broadband arrays radiation field calculations are developed. Compared with the rigorous theoretical method, the calculation time is dramatically reduced, with satisfactory accuracy.
- Finite element method is used to study arrays radiation field, while the transducer element dynamic response and cross coupling are taken into account. This provides more practical predictions of array transducers.
- Approach to design and optimize broadband sparse arrays is developed. The method can be used for 2D arrays as well as 1D arrays. It provides great flexibility for various sparse array designs. The trade-off among sparse order,

main lobe width, maximum SLL, with or without weighting coefficients can be realized.

- A synthesis experiment system is developed for arrays radiation field measurement. The data collected can be used to validate any sparse array designs.

8.2 Recommendations

- The optimization criterion for sparse array used in the thesis is maximum SLL with main lobe width constraints. For practical applications, this may not be the only criterion. The selection of criteria is mainly determined by the application requirements on the images. For example, in some literatures (Holm, 1996), the integrated sidelobe energy is also considered. These cases are not considered in the thesis. However, the developed approach does provide the capability to deal with the different criteria. Only the fitness evaluation operator needs modification according to the given criteria, other operator and strategies can be used directly.
- The GAs optimization starts with totally random sparse arrays. This may not be the best and most efficient choice. It is possible to apply some constraints with the known information to the first generation. For example, it is known the main lobe beam width is determined by the aperture dimension. For sparse array, if the elements are distributed in the aperture area as large as possible, a narrower main lobe can be achieved. The binned sparse array (Hendricks, 1991) is based on this idea. If these constraints can be applied to the initial population properly, the convergence of Gas can be accelerated. However, to do so, the trade-off between the gene diversity and convergence value must be considered. On the other hand,

some operators such as crossover need modification to ensure the offspring is within the possible solution range.

- As discussed in Chapter 4, for broadband field, the initial pressure waveform from each element $p_0(t)$ also affects the radiation field, both for main lobe width and SLL . Although it may be difficult to control this waveform in real applications, it is possible to design this waveform to reduce the maximum SLL or tailor the main lobe shape. In the optimization methods developed in this thesis, this signal is not considered. It is possible to develop the methods further to take $p_0(t)$ into consideration.

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Appendix A

Spatial Impulse Response of Rectangular Aperture

Closed form solutions of spatial impulse response from rectangular apertures can be calculated. As discussed in Chapter 4, this response is proportional to the angle $\Omega(\vec{r}, t)$ as shown in Fig. 4.2. This appendix lists the expressions to calculate $\Omega(\vec{r}, t)$ for various cases according to the different relative positions between field points and aperture (Ullate and Emeterio, 1992).

For 1D arrays, the aperture of single element has narrow rectangular shape. Suppose the aperture width and length are $2a$ and $2b$ and $b \gg a$. The projections of field points all lie in the y axis. Three possible relative positions are shown in Fig. A.1. The temporal discontinuities are produced when the arcs contain any of rectangle vertices or when they are tangential to a side of the aperture. For the field point $R(x, 0, z)$, its projection on $x-y$ plane is $P(x, 0)$, the vertex discontinuities are produced at the instants,

$$\begin{aligned}\tau_A(x) &= \tau_C(x) = ((x-a)^2 + b^2 + z^2)^{1/2} / c \\ \tau_B(x) &= \tau_D(x) = ((x+a)^2 + b^2 + z^2)^{1/2} / c\end{aligned}\tag{A.1}$$

The instants of the tangential discontinuities τ_1 , τ_2 and τ_3 when the arcs become tangential to the sides AC, AB and BD, respectively, are

$$\begin{aligned}\tau_1(x) &= ((x-a)^2 + z^2)^{1/2} / c \\ \tau_2(x) &= (b^2 + z^2)^{1/2} / c, \quad \text{for } x \leq a \\ \tau_3(x) &= ((x+a)^2 + z^2)^{1/2} / c\end{aligned}\tag{A.2}$$

In addition, for points with $x \leq a$, there is a discontinuity at

$$\tau_0(x) = z / c\tag{A.3}$$

Explicit analytical expressions of $\Omega(x,t)$ can be found directly in the time domain dividing the x axis in three regions: region I for $x \leq a$, region II for $a \leq x \leq b^2/4a$ and region III for $b^2/4a \leq x$. $x = b^2/4a$ corresponds to the field point where $\tau_3 = \tau_A$.

For region I,

$$\begin{aligned}
 \Omega(x,t) &= 2\pi; & \tau_0 \leq t \leq \tau_1 \\
 &= \pi - 2\alpha_1; & \tau_1 \leq t \leq \tau_3 \\
 &= 2\alpha_3 - 2\alpha_1; & \tau_3 \leq t \leq \tau_2 \\
 &= -2\pi - 2\alpha_1 + 2\alpha_3 + 4\alpha_2; & \tau_2 \leq t \leq \tau_A \\
 &= -\pi + 2\alpha_2 + 2\alpha_3; & \tau_A \leq t \leq \tau_B \\
 &= 0 & \text{otherwise}
 \end{aligned} \tag{A.4}$$

where α_i correspond to the angles shown in Fig. A.1 and are given by

$$\begin{aligned}
 \alpha_1(x,t) &= \sin^{-1}((x-a)/\sigma(t)) \\
 \alpha_2(x,t) &= \sin^{-1}(b/\sigma(t)) \\
 \alpha_3(x,t) &= \sin^{-1}((x+a)/\sigma(t))
 \end{aligned} \tag{A.5}$$

$\sigma(t)$ is defines as

$$\sigma(t) = (c^2 t^2 - z^2)^{1/2} \tag{A-6}$$

For region II,

$$\begin{aligned}
 \Omega(x,t) &= \pi - 2\alpha_1; & \tau_1 \leq t \leq \tau_3 \\
 &= 2\alpha_3 - 2\alpha_1; & \tau_3 \leq t \leq \tau_A \\
 &= -\pi + 2\alpha_2 + 2\alpha_3; & \tau_A \leq t \leq \tau_B \\
 &= 0 & \text{otherwise}
 \end{aligned} \tag{A.7}$$

For region III,

$$\begin{aligned}
 \Omega(x,t) &= \pi - 2\alpha_1; & \tau_1 \leq t \leq \tau_A \\
 &= 2\alpha_2; & \tau_A \leq t \leq \tau_3 \\
 &= -\pi + 2\alpha_2 + 2\alpha_3; & \tau_3 \leq t \leq \tau_B \\
 &= 0 & \text{otherwise}
 \end{aligned} \tag{A.8}$$

For 2D arrays, the element has square shape. The field points $R(x, y, z)$ are never limited within $x - z$ plane. Thus there are many more possible relative positions between the aperture and the projections of field points. The detailed expressions for general cases are not listed, and only two special and simple cases are discussed here as examples. These two cases are shown in Fig. A.2. For the first case, the expression of response is same as the Equation (A.8) for 1D element. For the second case, the instants of discontinuities are

$$\begin{aligned}\tau_A(x, y) &= ((x-a)^2 + (y-a)^2 + z^2)^{1/2} / c \\ \tau_B(x, y) &= \tau_C(x) = ((x-a)^2 + (y+a)^2 + z^2)^{1/2} / c \\ \tau_D(x, y) &= ((x+a)^2 + (y+a)^2 + z^2)^{1/2} / c\end{aligned}\tag{A.9}$$

The expression for $\Omega(x, y, t)$ is

$$\begin{aligned}\Omega(x, y, t) &= -\pi / 2 + 2 \cos^{-1} ((y-a) / \sigma(t)) & \tau_A \leq t \leq \tau_B \\ &= \pi / 2 - 2 \cos^{-1} ((y+a) / \sigma(t)) & \tau_B \leq t \leq \tau_D \\ &= 0 & \text{otherwise}\end{aligned}\tag{A.10}$$

Fig. A.1 Calculation of $\Omega(t)$ for 1D array elements. (a) Region I (b) Region II (c) Region III

Fig. A.2 Calculation of $\Omega(t)$ for 2D array elements

Appendix B

Material Properties of the 3x3 Prototype Array Transducer

The 3x3 2D array prototype transducer is designed and fabricated by Duke University. The active material in this array is PZT-5H, which is bonded with ground foil, backing and matching layer by epoxy and cut in two directions with $50\mu\text{m}$ thick diamond saw to obtain each single element. The matching layer is silver epoxy. The back is complex. The first layer is polyimide, which is used as a wire guide. The backing behind the wire guide is a filled epoxy. There is no kerf filler, only air in between the elements. The element pitch is $250\mu\text{m}$, or 0.6λ with nominal frequency of 3.5MHz . Each element is terminated in a 10-pin header connector, and wired with 30cm of micro-coaxial cable. On the other end of each cable, BNC connector is used. In order to facilitate the experiment, the whole structure and cables are packaged with a 30cm of waterproof flexible plastic tube 3mm in diameter.

The diagram of the array is shown in Fig. B.1 and the material properties are listed in Table B.1 (refer to Table 5.1 for material properties of PZT-5H).

The author would like to thank Duke University to fabricate the prototype array and provide the information about materials.

Table B.1 Material properties of the prototype 3x3 array

	Matching	Kerf Filler	Backing (Wire Guide)	Backing
Material	Silver Epoxy	Air	Polyimide	Epoxy
Longitudinal sound velocity (m/s)	1900	-	2200	1850
Mass density ($10^3 kg/m^3$)	2.5	-	1.5	2.01

Fig. B.1 The diagram of the prototype 3x3 array.