Dynamic spectrum allocation using game theoretical approaches

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NANYANG
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**DYNAMIC SPECTRUM ALLOCATION USING
GAME THEORETICAL APPROACHES**

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SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING
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Dynamic Spectrum Allocation using Game Theoretical Approaches

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Summary

The increasing demand for spectrum due to the growing wireless services causes spectrum shortage and thus motivates changes in the way the spectrum resources are managed. With the limited spectrum available, mobile users will have to compete for frequency channels to transmit data. Such problems can be formulated as games where the players involved interact with one another, each with the objective of maximizing its own utility function selfishly. This thesis aims to adopt game theory in devising efficient radio resource management (RRM) algorithms for dynamic spectrum allocation in wireless communication systems.

The RRM problem in this research is envisioned by using a two-tier framework. At the lower tier, the thesis considers games where players are the wireless devices, such as mobile users, access points, base stations, etc., which have data to transmit and need to access a common pool of the shared spectrum. For such systems, the underlying factor that may cause user performance degradation is co-channel interference (CCI). Consider a scenario with multiple transmitter-receiver pairs in an ad-hoc orthogonal frequency-division multiple access (OFDMA) network, where each pair of users needs to select OFDMA channels and a certain level of power to transmit in a distributed manner. In order to combat CCI, this problem is modeled as a strategic-form game with the utility function designed to optimize the signal-to-interference-and-noise (SINR) ratio. Iterative algorithms are devised to obtain the Nash equilibrium (NE), which is the stable operating point of the system. The iterative best-response techniques are frequently used in game theory but they might

not guarantee convergence, especially when pure-strategy NEs might not exist. However, they do exist for certain classes of games, such as potential games. By adopting an interference-minimizing utility function, a potential game for the OFDMA system under investigation can be formulated. The results are further extended to an infrastructure-based multi-cell OFDMA network where a player consists of a base station and multiple mobile users in the cell. As such, pure-strategy NEs exist and can always be reached by using the best/better-response dynamics. Other issues such as the fairness among the center-users and edge-users in cellular OFDMA systems or the optimality analysis of the algorithms will also be addressed in this research. These methods are able to satisfy the users' target SINR and system fairness in the numerical simulation studies.

At the upper tier, the thesis looks at a different scenario where the competition involves multiple licensed and unlicensed spectrum providers. The primary spectrum holders (PSHs) are the licensed spectrum users, e.g., operators in a geographical area, who are allowed to sell/lease portions of their available bandwidth to other unlicensed secondary service providers (SSPs) for monetary profits. Reallocating spectrum and encouraging sharing among the licensed and unlicensed service providers are new RRM methodologies proposed in future dynamic spectrum access systems, where micro-economic mechanisms could be employed to study the emerging spectrum market. This problem is viewed as an oligopolistic differential game between multiple PSHs, managed by a controller called the spectrum broker. By assuming the spectrum price is a dynamic continuous-time process governed by a differential equation, the proposed method seeks to find a Markov NE solution and determines the market equilibrium price. In addition, a discrete-time price adjustment which can be easily implemented at the spectrum broker is proposed. Extensive numerical studies are carried out to investigate the dynamic behaviors of the spectrum market.

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List of Abbreviations

3G	Third Generation
3GPP	Third-Generation Partnership Project
4G	Fourth Generation
ANSS	Autonomous number of subcarrier selection
AWGN	Additive white Gaussian noise
BER	Bit-error rate
BS	Base station
CCI	Co-channel interference
CDF	Cumulative distribution function
CDMA	Code-division multiple access
ComSoc	Communications Society
DSA	Dynamic spectrum access
DySPAN	Dynamic Spectrum Access Networks
FA	Fixed allocation scheme
FCC	Federal Communications Commission
FFT	Fast Fourier transform
GPS	Global positioning system
HJB	Hamilton-Jacobi-Bellman
IEEE	Institute of Electrical and Electronic Engineers
ISM	Industrial, Scientific and Medical

ITU-R	Radiocommunication Sector of the International Telecommunication Union
JSAC	Journal on Selected Areas in Communications
KKT	Karush-Kuhn-Tucker
LTE	Long term evolution
MAC	Medium access control
MC	Marginal cost
MIMO	Multiple-input multiple-output
MMS	Multimedia messaging service
MR	Marginal revenue
MS	Mobile station
NE	Nash equilibrium
ODE	Ordinary differential equation
OFDM	Orthogonal frequency-division multiplexing
OFDMA	Orthogonal frequency-division multiple access
OFDMA/WF	OFDMA iterative waterfilling scheme
PG-best	Potential Game with best-response scheme
PG-better	Potential Game with better-response scheme
PoA	Price of anarchy
PSH	Primary spectrum holder
QAM	Quadrature amplitude modulation
QoS	Quality of service
RRM	Radio resource management
SINR	Signal-to-interference-and-noise ratio
SM/PP	SINR maximization with power pricing scheme
SSP	Secondary service provider
UWB	Ultra-wideband

WF/PP	Waterfilling with power pricing scheme
WiMAX	Worldwide interoperability for microwave access
WLAN	Wireless local area network

Chapter 1

Introduction

1.1 Background

The impacts that wireless communications technologies have on our society are undeniable. We have seen a wide range of applications of these technologies in our daily life, from wireless remote controllers, radio broadcast to satellite TV antennas. Since the 1980s, mobile phones and wireless cellular networks have been available for commercial use and constantly flourished into an integrated part of the modern-day society. The recent emergence of wireless personal devices with high computing capability and connectivity, like smartphones and tablet computers, provides their users with not only traditional voice telephony, but also a huge variety of data services such as text messaging, multimedia messaging services (MMS), short-range data transfer using Bluetooth or infrared technologies, global positioning system (GPS) navigation, wireless Internet access and Internet-related applications such as multimedia streaming, gaming and social networking. It was estimated that more than 5.3 billion people used cellular phones in 2011 [1], which accounts for 70% of the global population; and furthermore, smartphones were owned by 18% of the population. Fourth generation (4G) wireless communications systems are currently being deployed in several regions in the world, including Singapore [2], which aim to support data rates of tens to hun-

dreds of megabits-per-second. The overwhelming demands for wireless services have been the key driving factor for research and development of wireless communications networks and technologies, which must be able to provide fast wireless access and high-speed data transmissions to support the ever-growing user traffics.

1.1.1 Resource Allocation in Wireless Communications

Nowadays, there are a large number of existing wireless communication systems and networks, which differ in various aspects, including their geographical coverage range, frequency bands, underlying technologies and standards, data rates, transmission power, etc. However, the design and implementation of these systems are all governed by some key principles and one of the most crucial and challenging tasks is the management and allocation of wireless resources, which is commonly referred to as radio resource management (RRM). The fundamental principle of RRM [3] is the management of wireless resources in a strategical, optimized manner in order for a wireless system in general or its individual users in particular to achieve the required network or user objectives, under the constraints imposed on the system. Fig. 1.1 demonstrates that RRM is a multifaceted discipline with various issues that one will have to consider.

As mentioned above, the main concern of a RRM problem is to fulfill certain objectives for both the system and its users. The network operators need to maintain a consistent, optimal network performance in the long run, which typically concerns the satisfaction of the individual Quality of Service (QoS). The supports of QoS are cross-layer design problems for wireless networks [15]. For example, if in the application layer, a video-streaming application is required to be of “high-definition”, this may translate to more specific measures in the physical layer such as bit rates, bit-error-rates (BER), and so on. As the data traffic is getting increasingly diversified, complex and application-dependent, each traffic type has its own QoS requirement and existing

Fig. 1.1: The various aspects of a RRM framework.

networks must be able to simultaneously accommodate all application services, which requires a flexible and customizable RRM framework capable of supporting of various QoS profiles.

However, this flexibility of satisfying QoS often incurs high design complexity. It generally requires some inherent tradeoffs between the system's and users' objectives, due to the fact that the system resources are often constrained. For example, a limited number of access points and radio channels in a certain coverage area may lead to traffic congestion, especially in peak periods. However, increasing the amount of system resources, such as installing more equipments, may otherwise result in under-utilization beyond the peak period, at the expense of additional infrastructure installation, maintenance, and operational costs. Another main difficulty is the unreliable time-varying wireless channels. Unlike wired or optical communications systems, the air medium in wireless communications is much less stable and the radio signals

are vulnerable to numerous irregularities such as attenuations, delays, multi-paths, etc., generally known as fadings, which cause degradations in transmission quality. Moreover, since multiple users access the medium simultaneously, they directly cause signal interferences on each other and this imposes severe limitations on the overall performance of the system. The tradeoff between throughput and fairness is one of the consequences of such constraints in wireless systems. For users at more geographically disadvantageous locations than others, e.g., behind some terrains where the signal suffers deep fades; or at the cell edge where interferences from neighboring cells are high, maintaining their QoS requirements under poor channel conditions while optimizing the system throughput is a very challenging task.

In a general RRM framework, different design criteria can be performed [9]. *Frequency allocation* is necessary in assigning frequencies to users for transmission so as to achieve the best system performance. Users have different preferences for which channels to be used due to different channel qualities, and such diversity can be exploited via a coordinated, adaptive scheme. *Power control* is another major consideration in multi-user systems vulnerable to signal interferences. Users will often have to adjust their transmission power because one's transmission has some negative impacts on the others' activities. Meanwhile, when there is a limitation on the number of users to be admitted to the system due to limited resources or interference constraints, *admission control* mechanism is required. On the other hand, the access of users to the wireless medium can be regulated by the *medium access control* (MAC) protocols. Design and analysis of such protocols to support different classes of service are important issues. Other tasks that may occur in RRM include *queueing* in packet-switching networks, *routing* in multi-hop networks, and so on.

The task of optimizing the allocation of resources requires a real-time, dynamic, computationally efficient process that can perform decision making by taking into account the current system state. Mathematical optimization as well as various mathematical branches in operational research provide rigorous analytical frameworks that

can aid researchers in achieving the optimization goals. There are a wide range of techniques and algorithms available. Convex programming, linear programming, integer programming and so on [22] are well-known techniques in optimization and can be used in a multitude of problems in wireless networks. Computational intelligence theory [23], including techniques such as machine learning, evolutionary algorithms and genetic algorithms can also be used in optimizing the system performance where the users have the learning ability in decision making. Optimal control techniques [24] may be employed when the network's dynamic behavior can be characterized over a time horizon via its state variables and one can have some degrees of control over this behavior. Besides, for distributed multi-user interactive scenarios, game theory [16, 17], which is the technique employed in this work, is the best tool to analyze the system's behaviors and outcomes.

1.1.2 Game-theoretical Approaches

Game theory [16–18] had long been exploited in microeconomics to deal with competition among selfish, intelligent decision makers. It is a useful tool for studying the interactive behaviors individually and collectively under conflict of interest. The motivations for communications researchers to adopt game theory come from the following main reasons:

- (i) *Rational decision making*: Intelligence and rationality are two fundamental assumptions for players in game theory. Nevertheless, “human decision making” hardly satisfies these assumptions perfectly and consistently. However, if one considers the “players” to be computerized machines, which are programmed to always aim at maximizing a predefined objective function, then the two assumptions can be expected to hold. Furthermore, with faster computing capability, machines have the computational power to determine the best actions. As such, the results in game theory hold better for computers than for people. In wireless

RRM problems, it is not unreasonable to view the transmitters and receivers, such as base stations (BS) and mobile stations (MS) as players in a “transmission game”, with their possible strategies being which frequency channels to use, for example. Then, a primitive form of a game can be formulated by simply assigning their utility functions to be the transmission rates that they must maximize. One can indeed see that this is the case of decision making under conflict: since the bandwidth is shared, some users may find themselves competing for the same channel, where one’s decision may hurt the other in terms of signal interferences.

- (ii) *Distributed optimization*: Traditionally, allocation of resources in wireless systems have been dealt with in a centralized manner, e.g., a radio network controller in cellular systems acting as a central authority who can monitor all the RRM tasks and can communicate with the BSs via a wired backbone network. However, such systems are becoming more and more complex as the number of users grows larger and larger. Additionally, some centralized algorithms such as exhaustively searching the solution set for the optimal point have an exponential or even factorial order of growth, while others are equivalent to known NP-hard problems such as the traveling salesman problem [104], which severely limits the system scalability. For the problem to be tractable, researchers may consider distributed optimization where the RRM tasks are performed on a per-user or per-BS basis. These can be performed either independently [37–39, 43–45], or in conjunction with a simpler centralized scheme to synchronize the system operations, in a multi-layer RRM architecture [46, 47]. Hence, the fact that game theory, particularly non-cooperative games, is set within a distributed context and studies selfish individuals who make decisions to maximize their own pay-offs, makes it an ideal tool for the wireless RRM context. It helps to lessen the computational burden; but maintaining the existence, stability, uniqueness, and

optimality of the solutions are still the key challenges to overcome.

(iii) *Analyzing risks and uncertainties:* The need to develop game theory is for analyzing the inherent risks and uncertainties involved in interactive decision making. Only in very few decision making situations can one exactly predict the outcomes, especially when the involved parties do not have complete and perfect information about the situations and the other parties. Random occurrences can also be present, which result in further risks in the outcome of an individual's decision. The same discussion applies very well when game theory is used in RRM for wireless systems. It is impossible for one user to have knowledge of the exact locations and channel state information of the neighboring users, and whether they have data to transmit or not. By acting individually, it is probable that they will cause unpredictable interferences to others due to the fact each selfish user has the tendency to greedily demand for more resources. This is one of the reasons why game theory can be employed in wireless communications to help minimize risks and uncertainties in making decisions.

(iv) *Solution concepts for studying interactive behaviors:* Last but not least, game theory provides a wide range of models and solution concepts which are helpful for a variety of scenarios. The Nash equilibrium (NE) is a well-celebrated concept which characterizes the stable outcome of a game. If the users act strictly by themselves, non-cooperative game model might be used. If long-term benefits are considered, repeated game model can be adopted, with mechanisms such as threats to enforce cooperation. Otherwise, if players can communicate with each other, they can play cooperative games which features solutions such as the Nash bargaining solutions. Groups of players can make joint decisions in a coalitional game model. In games with incomplete information where players are not aware of others' past and current moves and payoffs, one can employ Bayesian game model. There are even more branches of game theory, of which

new perspectives can be shed on traditional wireless RRM problems.

In Appendix A, the basics of game theory are presented. Applications of game theory in communications systems appeared early in the 1990s for problems such as flow control [25] and routing [26]. MacKenzie *et al.* [27] appeared to be the first to suggest some applications of game theory in wireless communications. Among the first few notable RRM game-theoretical approaches include results for code-division multiple access (CDMA) systems [28], orthogonal frequency-division multiple access (OFDMA) systems [37], broadband networks [29], medium access in ALOHA networks [30] and cognitive radio networks [58]. Since then, game-theoretical approaches to RRM have been extensively studied and become one of the prominent research topics in telecommunication, with several special issues dedicated to them by the IEEE Journal on Selected Areas in Communications (JSAC), as well as the IEEE Journal on Selected Topics in Signal Processing.

1.2 Objectives and Contributions

This thesis focuses on the game-theoretical approaches for RRM in wireless communications systems. As new wireless technologies and the next generation of communications networks are being researched and developed, the RRM aspect also faces new challenges. One of the main issues, widely recognized by the academia, industries as well as national and international policy makers, is the management of the frequency spectrum, the most crucial resource in any wireless networks. With the exponential growth in demands for wireless traffics, spectrum scarcity is unavoidable, which calls for better and more efficient methodologies to utilize the spectrum resource. To that end, research has been directed towards the realization of dynamic spectrum access (DSA) systems [8, 9]. In future DSA networks, users are equipped with computing capability and intelligent algorithms and are expected to improve the system performance as a whole. The design of such algorithms, especially RRM algorithms, must

rely on principles and results from various fields of sciences, among which game theory has recently been of primary interest. These RRM methodologies will also be the objective that the thesis strives to obtain.

DSA systems promote the mutual, efficient sharing of spectrum to overcome spectrum scarcity. This involves not only the performance-oriented frequency allocation among networks of wireless users, but must also involve reallocation of unused spectrum at a higher level among the spectrum holders [9]. However, in the literature, few works actually make a connection between these two types of players. With that purpose in mind, this thesis looks at the dynamic spectrum allocation problem as a two-tier allocation process, which addresses the spectrum access issues faced by both the spectrum end users and the spectrum service providers in a closely-related hierarchy. In the lower tier, players are direct users of spectrum, such as BS, MS, access points, relay nodes, etc., who have data to transmit and need to access a common pool of shared spectrum. Thus, game-theoretic mechanism design in this layer is concerned with issues such as throughput optimization, spectrum and energy efficiency, interference mitigation, etc. On the other hand, the upper tier involves the multiple spectrum providers and their issues are of the acquisition and ownership rights of spectrum. That is, the thesis investigates the issues arisen if there are both licensed and unlicensed spectrum providers; and they are allowed to sell and buy spectrum in exchange for monetary profits via market-driven mechanism. More details will be discussed in Chapter 2 of the thesis.

1.3 Organization of Thesis

The thesis is organized as follows.

- In Chapter 2, an overview of the traditional spectrum management as well as the key idea of future DSA systems are presented, together with an extensive review of the expanding literature on this topic. Chapter 2 also introduces the

RRM framework which the thesis envisions for the problems to be tackled in the later chapters. The RRM framework will feature separate RRM issues at both the user tier and the spectrum provider tier. Subsequently, Chapters 3 to 5 discuss game-theoretic schemes which are applicable to the user tier while Chapter 6 focuses on games for the spectrum provider tier.

- In Chapter 3, the thesis explores a resource allocation game in a distributed, ad-hoc scenario where multiple pairs of transmitters and corresponding receivers access the spectrum independently. Each pair of nodes will compete for a single frequency channel to transmit, with the objective of throughput/SINR maximization. This scenario resembles an open-spectrum-sharing model where “everyone can transmit”. Different power allocation schemes are examined in order to achieve either optimal user rates or QoS satisfaction plus fairness. In order to obtain the NE point of the game, iterative gameplay based on best-response dynamics is implemented. Simulation is performed to evaluate the effectiveness of the method.
- In Chapter 4, the thesis continues to investigate the distributed ad-hoc system, but under an OFDMA context. Here, the transmit-receive pairs with OFDM technologies enabled can compete for a subset of the available, reusable OFDMA subcarriers. In order to specifically tackle the interference mitigation issues, two novel methods are proposed. First, the use of the interference-sum utility function leads to the formulation of a potential game which possesses the property of pure-strategy NE existence and convergence. Secondly, by defining the interference radius, the players are able to identify their dominant interferers. This allows them to adaptively choose their exact number of subcarriers to transmit according to their QoS requirements, which in turn avoids the strategy domination issue of the original game.
- In Chapter 5, the thesis extends the potential game formulation of the preceding

chapter into the multi-cell OFDMA system. With a frequency reuse factor of 1, each cell is allowed to fully reuse any OFDMA subcarriers and each BS can independently assign subcarriers and power to the associated MSs in the cell in a decentralized manner. As such, a game-theoretical approach is proposed with each cell acting as a player, whose utility function is the generalized interference sum. Under a rigorous mathematical analysis, the cellular OFDMA game can be proven to be a potential game. Consequently, a stable NE can be obtained for such systems using best/better-response dynamics. Extensive numerical simulations are also done to investigate various performance issues of the proposed approach, including fairness for users at the cell edge and optimality bounds.

- Chapter 6 is dedicated to the dynamic spectrum allocation problem among multiple spectrum providers. Here, the thesis investigates an alternative market model to determine the spectrum price dynamics as well as the optimal bandwidth level that the participants of the market should offer to maximize their revenues in the long run. In the proposed model, each service provider is treated as an oligopolist. One of the key characteristics of the model is the presence of the spectrum broker. By assuming that the price follows a trajectory governed by a differential equation, which is controllable by each player, the problem will be formulated as a differential game. By using mathematical analysis and numerical simulation, the Markov NE solution can be established and this will uncover several interesting properties of the game.
- Finally, Chapter 7 concludes the thesis and recommends future directions.

Chapter 2

Overview of the Spectrum Allocation Problem

This chapter begins with an overview of the existing spectrum management methodologies and the proposed DSA systems. It also serves as a literature review on RRM schemes, especially those that adopted game-theoretic approaches. It is then followed by the discussion of the spectrum allocation framework that this work will focus on.

2.1 Issues in Spectrum Management

2.1.1 Traditional Spectrum Management

The radio spectrum is recognized as the most precious resource in wireless communications. It commonly refers to the useful electromagnetic wave medium (3 Hz - 300 GHz) that all wireless communications systems are built upon. The spectrum is divided into multiple frequency bands for use of different wireless applications and technologies. It plays a crucial role in not only technologies but also the economy, society and public welfare. Thus, regulating the radio spectrum should be based on careful and strategic decision makings from governmental institutions. Traditionally, large parts of the radio spectrum are licensed to operators or assigned to service

providers by the static assignment policies, where government agencies grant the usage of spectrum to licensed holders on a long term basis. However, only a very small part of the remaining spectrum, the unlicensed frequency bands, are more or less available for open access, for example, the Industrial, Scientific and Medical (ISM) bands. There are several ISM bands as defined by ITU-R (the Radiocommunication Sector of the International Telecommunication Union), among which the two most encountered ISM bands are the bands around frequencies 2.45 GHz and 5.8 GHz.

The existing regulatory mechanisms of the radio spectrum are best summarized in a report of the United States Federal Communications Commissions (FCC) Spectrum Policy Task Force [4], in which the following approaches were defined:

- *Exclusive-use model*: This model refers to licensing spectrum for exclusive usage, where the licensees (license holders) can use the spectrum exclusively subject to certain rules and are protected from interferences through regulations.
- *Command-and-control model*: This model refers to licensing spectrum for shared usage, restricted to a specific technology. For a particular band, regulators can specify certain technologies like TV or cellular phones, so that companies can compete for license to operate in the spectrum via methods such as spectrum auction. The command-and-control approach is the one currently employed by most of the regulators around the world.
- *Spectrum commons model* or *Open spectrum access*: This model refers to unlicensed spectrum, open for access by all wireless communication systems, conforming to technical standards and etiquettes; but users are not protected from interferences.

Up to the present, this approach has been serving well, as consumers have enjoyed benefits from the mobility and connectivity of wireless technologies and enormous revenues have been gained from the widespread and commercially successful wireless services. However, in the future, as new wireless technologies and applications

are emerging, the number of wireless devices will therefore substantially grow and tremendously increase the demands for more spectrum. As a result, new problems will arise, the most important being the scarcity of radio spectrum and the inefficiency of spectrum usage.

This scarcity of spectrum is caused by a seemingly paradoxical situation, i.e., spectrum under-utilization in the licensed bands, yet congestions in the unlicensed bands. Measurements [5] have shown that the licensed services only transmit in a few tens percent of the time. Berlemann *et al.* [8] also attributed the current inactivity of licensed spectrum at a given location and time to several social trends such as economic failures of wireless systems or the occasional and intermittent nature of various wireless services like public safety or military operations. On the other hand, the unlicensed bands are shared by more diversified services and applications and are likely to suffer from undesirably high level of interferences. For example, several important wireless applications with large consumer markets such as WiFi, IEEE 802.11a/b/g/n wireless local area network (WLAN), cordless phones and Bluetooth all coexist in the unlicensed ISM band. Despite being overused, there are no incentives for a unilateral technical improvement from all radio systems towards higher spectrum efficiency for the unlicensed band, due to an economic phenomenon known as “tragedy of the commons” [8]. Inefficient usage in the licensed bands, while the commercially potential unlicensed bands being over-crowded are indications for radical changes in the existing spectrum management towards more flexible spectrum access strategies.

The quest for new and novel spectrum management techniques has progressed in the last decades. We have witnessed successful wireless technologies in enhancing spectrum efficiency and capacity, such as OFDMA, Multiple Input Multiple Output (MIMO), and cognitive radio technology. *Dynamic spectrum access* (DSA) [6], aided by cognitive radio [11], is a recent attempt to directly confront the spectrum access problems. As the opposite of the current static spectrum management, DSA refers to the usage of spectrum in a time-varying, flexible manner, in accordance with regula-

tory and technical constraints [8]. Cognitive radios, originally proposed by J. Mitola III [10], can be thought of as autonomous radio systems with cognitive ability that can identify unused spectrum and use this spectrum in an intelligent and opportunistic manner. Future cognitive radio-based DSA networks will open a new spectrum management paradigm, in which users will self-regulate and coordinate among themselves and transmit over an agility bandwidth to avoid interfering each other and this will prevent the network from possible transmission outage.

2.1.2 Dynamic Spectrum Access

In the literature, DSA has been understood as any kind of techniques that aims to achieve the improved spectrum access objective. Such techniques are very diverse, as exemplified by the broad topics and works brought up at the annual IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN), which is dedicated to the realization of DSA. There are efforts to classify DSA into different categories, e.g., by Hossain *et al.* [9], Berlemann *et al.* [8] or Zhao *et al.* [7]. According to these authors, DSA can be divided into three different models which are counterparts to the three existing ones defined by FCC. They are, similarly, *exclusive-use* model, *shared-use* model (also referred to as hierarchical access model in [7]) and *spectrum commons* (or *open sharing*) model [7,9]. Each model can also be further divided into submodels. The precise categorization is summarized in Figure 2.1. Besides, in terms of the nature of the RRM schemes for spectrum sharing in DSA, one can loosely classify them into *centralized* vs. *distributed* spectrum sharing and *non-cooperative* vs. *cooperative* spectrum sharing [9].

2.1.2.1 Exclusive-use Model

In the DSA exclusive-use model, while the radio spectrum is still licensed to the service providers and network operators for exclusive usage as in the current policy,

Fig. 2.1: Models in DSA (adopted from [7, 9]).

certain degree of flexibility would be introduced in order to improve the utilization of spectrum. The license holders are allowed transferable rights of spectrum, so that when they do not fully use the spectrum at all times and locations, they can trade or lease parts of the spectrum to the unlicensed users to earn revenues. Coase [68] is often cited as the first in promoting spectrum ownership and property rights. Depending on the agreement, the spectrum owner may or may not need to specify the type of wireless technologies that the unlicensed users should use. There are two different approaches under this model, i.e., long-term exclusive-use vs. dynamic exclusive-use [9].

The long-term exclusive-use and dynamic exclusive-use models differ in scale. For the long-term exclusive-use model, the spectrum owner reallocates the unused spectrum exclusively to the unlicensed users for a longer period of time and often also specifies the services and specifications that the users must conform to when accessing the spectrum. For dynamic exclusive-use, there is more flexibility as the unlicensed users can be granted access to only small fractions of the spectrum, and for a relatively shorter interval. Moreover, multiple unlicensed operators with heterogeneous services may be allowed in the dynamic exclusive-use model [9].

For the exclusive-use strategies, current research efforts concerning dynamic spectrum allocation are focusing on issues known as spectrum market or spectrum trading [70, 87], which models the reallocation of transferable spectrum between the li-

censed and uncensed spectrum providers using economic approaches. Market demands, supplies, utility functions, equilibrium behaviors and pricing mechanisms will be carefully considered. Game theory and several micro-economic tools such as market analysis, auction mechanism design, etc. can serve as useful frameworks to investigate such problems. A number of applications of such models can be found in the DSA literature, e.g., spectrum pricing in [69,72,91,92], microeconomic approach in [71,72,93] and auction theory in [74,79,89,90].

2.1.2.2 Shared-use Model

Unlike the exclusive-use model where the licensed users have exclusive rights to the spectrum, the shared-use model promotes the simultaneous sharing of spectrum between the licensed and uncensed radio users. Spectrum sharing is the key point in DSA as a solution to inefficient utilization problems. In the context of the shared-use model, the co-existing licensed and uncensed users are also known as primary radios and secondary radios [9], which also suggests a hierarchical access structure [7]. The primary users have a higher priority and therefore should be protected from external interferences. The secondary users are permitted to access the spectrum as long as they do not interfere with the primary users' transmission. Two submodels are available in the shared-use model, i.e., spectrum underlay and spectrum overlay [6].

The spectrum underlay submodel permits the secondary users to transmit concurrently with the primary users temporally and spatially, but under strict interference constraints. The model assumes that the primary radios transmit all the time, so that the transmitted power of the secondary users must remain below a certain level (the interference temperature limit) so as to be treated as noise by the primary users. Sophisticated spread spectrum techniques are required to achieve this, such as ultra-wideband (UWB).

In contrast, the spectrum overlay submodel does not impose heavy interference constraints, but is based on opportunistic spectrum access [6]. Essentially, it means

that at an arbitrary time and location, if particular frequency bands are not used by the primary radios, they can be occupied by the secondary users in an ‘opportunistic’ manner. Such unused spectrum bands are often referred to as “spectrum hole”, “spectrum opportunity” or “white space”. In order to utilize such spectrum opportunities, the secondary users need to have cognitive radio technology enabled, in order to make use of the spectrum sensing functions and detect the spectrum holes opportunistically.

For the shared-use model, a variety of wireless networks and architectures, including OFDMA, CDMA, MIMO and cognitive radio networks can be considered; and multiple RRM tasks described previously such as frequency assignment, power allocation, etc. can be applied. Similarly, several optimization techniques can be employed, and game theory is one of the most important methodologies. Examples of spectrum underlay access problems can be found in [55, 76, 78]. Spectrum overlay access has also been considered by many authors in the literature, e.g., [54, 64, 82] and references therein.

2.1.2.3 Spectrum Commons or Open Sharing Model

The spectrum commons model is analogous to the FCC-defined counterpart, which advocates open sharing of spectrum among users. This model includes what have already been implemented for the 2.4 GHz ISM band and gains support due to the commercial success of various existing wireless applications operating under this band [7]. Open sharing can be applied in either a centralized network or a distributed, ad-hoc network. The licensed primary users are absent and the unlicensed users can share (or often compete for) the spectrum among themselves.

Because the unlicensed users do not have the rights of protection against interferences, and due to the lack of control on spectrum access, the radios may not function properly if the number of wireless devices at a given location becomes so dense that it results in excessive and severe interference power. Therefore, a major RRM challenge

in this model is to design reliable, feasible and scalable spectrum access mechanisms to avoid those problems. In certain scenarios, some protocols, rules and etiquettes should be defined so that radio users must follow. The main optimization objectives such as improving total network spectrum efficiency, meanwhile maintaining fairness among users, as well as minimizing interference, should be taken into account when performing frequency and power allocation. Lehr *et al.* [86] mentioned other important considerations for designing such spectrum access schemes. For example, users must still conform to a maximum power limit, and must be equipped with the ability to detect the presence of neighboring users, as well as the ability to exchange information and coordinate among themselves.

Similar to the shared-use model, several optimization approaches and techniques can be employed in the open sharing model. The topic has also been extensively studied; and various problem formulations exist including [53, 56, 77, 78, 80] and references therein.

2.1.2.4 Centralized, Distributed, Non-cooperative and Cooperative Spectrum Sharing

Centralized and distributed spectrum sharing refer to different types of architectures of the underlying networks that employ DSA. On the other hand, the network users may adopt different types of access strategies, such as non-cooperative and cooperative approaches. For the centralized case, there exists a central network controller (e.g., a BS in uplink cellular scenarios) that coordinates spectrum access of the individual users and makes decisions on the allocation of spectrum resource. The controller can gather all the necessary information of the users such as transmission parameters and QoS requirements, and compute the optimal solution that maximizes certain system performance criteria like total capacity and power. However, heavy computational overhead is expected at the central controller. On the other hand, in a distributed network, the individual users autonomously make decisions using only the locally

available information. The computational burden can be eased, but resulting in less efficient solutions. Conflict and competition may ensue in such scenarios where one has resource allocation games. Several approaches in the literature, such as those considered in [48, 55] and part of [64], can be seen as centralized, while those studied in [53, 54, 78] are distributed. There are also hybrid systems where centralized and distributed schemes are implemented in conjunction, such as [46] and [47].

For cooperative spectrum sharing mechanisms, communications, agreements and information exchange may exist among the users, in order for them to better utilize the spectrum and avoid interferences. When such cooperations are absent and the users try to access spectrum in a selfish manner, the system is considered non-cooperative. Cooperative mechanisms are employed in [56, 76, 84] and so on; and non-cooperative techniques can be found in [54], [85], etc. Centralized systems can be treated as cooperative as complete information can be obtained at the controller (e.g., in [64]). However, distributed systems can also be made cooperative to some extent if certain degrees of self-awareness and cooperation are introduced in the RRM mechanisms (e.g., in [53, 79]).

2.1.2.5 Game-theoretical Approaches to DSA in the Literature

Despite the late appearance of DSA in the 2000s, there has been a dramatic increase in not only the number of works, but also the variety in methodologies concerning game-theoretical spectrum sharing for DSA in recent years. Conventional non-cooperative and cooperative game approaches have been employed on a more frequent basis. For example, non-cooperative game for DSA was studied by Subramani *et al.* [85] and cooperative game by Suris *et al.* [84], and references therein. However, the existing methods are no longer restricted to these basic models, but instead cover a much larger multitude of more advanced game-theoretical concepts, sometimes even a mixture of different techniques, depending on the nature of the problems. A few significant formulations will be reviewed in this survey, but a lot more can be added to the

incomplete list.

Several approaches aimed at improving the NE solutions. Z. Han *et al.* [82] proposed the usage of *correlated equilibrium*, a refinement of the traditional NE, in distributed opportunistic DSA networks in order to improve the solution over the inefficient non-cooperative NE. On the other hand, *repeated game* was used by Etkin *et al.* in [56] for the problem of power allocation among coexisting cognitive radio users in the unlicensed bands. It was revealed that while the NE of a one-shot game was unlikely Pareto optimal, a repeated game with incentives for players to deviate toward higher rates, as well as punishment mechanisms, can achieve more optimal *subgame perfect equilibria*.

Potential games were often employed due to their desirable equilibrium properties. Neel *et al.* [58,59] suggested some models for signal-to-interference-and-noise (SINR)-based and power-based potential games in cognitive radio networks. Similarly, power-control potential games as well as best-response dynamics in distributed networks was also studied by Heikkinen [67]. Alternatively, Nie *et al.* [53] defined a potential game with an interference-minimizing utility function for the channel allocation problem among distributed players. To a lesser extent, Xing *et al.* [55] also formed another potential game based on users' classes and interference temperature constraint.

A large group of works consider spectrum access and sharing using economic-related techniques. Niyato *et al.* [73] examined an overlay scenario where multiple secondary users and a primary user played a non-cooperative *dynamic game* subject to oligopolistic market rules such as profit maximization. The same authors also analyzed the issue of pricing in spectrum sharing in [72] using a *Bertrand game* model among multiple primary service providers who offered spectrum to the secondary users. In their paper, repeated games and *collusion* mechanisms were considered alongside in order to enhance the performance of inefficient NEs. Another classical economic model, the *Stackelberg leader-follower game*, was adopted by Wang *et al.* in [83] for distributed cooperative relay networks. The source node played as a buyer/follower;

and the relay nodes competed among themselves in a seller/leader game, setting optimal price in order to gain profits. Auction-based spectrum sharing [79] is another notable approach where it was assumed that in a CDMA underlay system, power was similar to divisible goods which could be charged upon allocation via *auction*. Thus, auction mechanisms could be set up to obtain the socially optimal outcome. Halldorsson *et al.* [80] employed a different approach called spectrum sharing games with *bargaining* where players might bargain with others, used in conjunction with a graph-theoretic model. These models allowed for analysis of the game's *price of anarchy* (i.e., the difference between the social Pareto optimal outcome and that of a NE point of the game - see also Appendix A.1.7).

There are yet other approaches which engage other game-theoretical concepts than those mentioned earlier. For example, Huang *et al.* [78] related their distributed power control game to best-response myopic updates in a *fictitious play* as well as a strategy space transformation in order to characterize the game as a *supermodular game*. Sagduyu *et al.* [81] based their studies of relay channel on a *stochastic game* whose solution is a *perfect Markov equilibrium*. The utility of a player could be calculated based on the steady-state probability of a Markov chain. Additionally, Niyato *et al.* [75] modeled the dynamic behaviors of the secondary users in a multi-seller multi-buyer spectrum market using *evolutionary game* and *evolutionary equilibrium* solution. Two approaches, i.e., the deterministic model with replicators dynamics and the stochastic model based on Markov chain, were proposed to represent the evolutionary process.

The trends in the literature show that more and more game theoretical formulations will be appearing. The abundance of such approaches reaffirms the usefulness of game theory in DSA as well as RRM for wireless systems.

2.2 The Two-tier Spectrum Allocation Framework

As discussed earlier, the approaches to spectrum allocation in DSA are indeed diversified. Moreover, it can be highlighted that as opposed to models where mobile users and systems directly access the spectrum and performance issues are to be assessed, there exists concurrently another problem of a different nature, i.e., the reallocation of spectrum through transferable usage rights. This can be viewed as an *indirect* form of spectrum access, and is unlikely to take place at the user level. This problem is more often among the service providers and operators, specifically at the business planning levels, who perform transactions and thus can participate in the spectrum market. However, such distinction in who the key players are is often ambiguous in the literature on DSA and spectrum market. For example, Niyato *et al.* [71, 72] considered spectrum trading among primary and secondary users who could be both service providers or mobile users. In fact, the two different types of players as pointed out earlier imply a hierarchical structure, in which each level represents a separate spectrum allocation issue. In view of that, this thesis introduces a two-tier spectrum allocation framework which, as its name suggests, accommodates the two tiers of players in DSA systems, namely, the end-users in the lower tier and the service providers in the upper tier. Fig. 2.2 gives a graphical illustration of the two tiers and their relationships to DSA. It is implied that the spectrum acquired by the service providers through the mechanism in the upper tier will be available for allocation to the end users in the lower tier. Another view on this framework is that the spectrum allocation in DSA can be a two-step process, starting from the spectrum acquisition on the upper tier to the actual, physical allocation on the lower tier. Besides, from Fig. 2.2, one can see how the existing DSA models such as exclusive-use, shared-use and open-sharing models play their different roles in the framework. To the best of the author's knowledge, even though hierarchical models have appeared in the literature on DSA, such a framework has not been examined to the extent mentioned in

Fig. 2.2: The two-tier spectrum allocation framework.

this thesis. For example, Berry *et al.* [87] and Jia *et al.* [93] discussed multi-leveled market structure for service providers but did not incorporate the user-level of physical resource allocation. As such, the author believes that the proposed framework represents an alternative view and may provide possible future directions for research on the RRM issues for DSA systems.

Next, the two tiers are described in more details.

2.2.1 The Lower Tier: Spectrum Allocation among Users and Systems

The lower tier has previously been defined to be the distribution of spectrum resources among the users within the system, or among various users in coexisting networks. As such, it encompasses a wide range of traditional RRM problems in wireless networks as well as DSA. Fig. 2.3 gives a very general picture of what happens in this tier. Assuming that several wireless service providers have acquired the rights to use the spectrum through the procedures in the upper tier and begun to serve their mobile

Fig. 2.3: Spectrum access for mobile users and systems in the lower tier.

subscribers within their graphical coverage areas. In Fig. 2.3, two of such systems, SP1 and SP2, together with some of their mobile users, are depicted. SP1 and SP2 can either exclusively utilize their spectrum, in which case the RRM issues are restricted to allocating channels or transmission power within the system to optimize its performance criteria, such as throughputs and QoS-related objectives; or if they employ spatial reuse of frequency spectrum, mitigation of inter-system interferences should be addressed. The underlying wireless technologies used by SP1 and SP2 also give rise to separate RRM problems, such as resource allocation for OFDMA, CDMA, MIMO systems, etc. and among heterogeneous systems.

Alongside SP1 and SP2 who have the spectrum access rights, there also exist unlicensed systems. In Fig. 2.3, the shared-use spectrum access system coexists in the same locations and spectrum bands as SP1 or SP2. As described by the shared-use DSA model, users in this system can adopt an underlay strategy such as UWB systems, or an overlay approach in which they are cognitive radios trying

to explore spectrum holes opportunistically while not interfering with the existing systems. The recently emerged femtocell networks [12] also follow similar principles in coexisting with macrocell systems. To improve the spectrum efficiency in such a scenario, interference avoidance is more and more crucial.

Apart from SP1, SP2 and the shared-use system, there can also be an open spectrum access system, i.e., where the users transmit in the free, unlicensed spectrum such as WLAN in the ISM bands. Here, such a system is independent from the existing SP1 and SP2 as they utilize different spectrum bands. However, its users are not protected from interferences among themselves and RRM schemes need to address this challenging issue.

In this thesis, the spectrum allocation in the lower tier will be tackled by looking at several scenarios, which spans from Chapter 3 to Chapter 5. In Chapters 3 and 4, the systems under investigation consists of distributed pairs of transmitter-receiver in a wireless ad-hoc network. These scenarios can be categorized under the open spectrum access model in Fig. 2.3 where “everyone can transmit”. Chapter 5 studies the multi-cell OFDMA system. Here, the scenario is akin to the problem where a service provider like either SP1 or SP2 in Fig. 2.3 has acquired the spectrum and wants to allocate it to their subscribers by deploying a cellular network using OFDMA techniques.

RRM for the shared-use systems is not within the scope of this work and will not be covered in this thesis, but would be an interesting extension for future research.

2.2.2 The Upper Tier: Spectrum Competition among Service Providers

The spectrum access issues that take place in the upper tier are centered around the wireless operators and service providers and the transfer/competition of spectrum usage rights among them. The acquisition of spectrum might be done as a prior step

Fig. 2.4: Spectrum market model in the upper tier.

before the allocation of this spectrum to the wireless users. Moreover, it occurs over a relatively much longer time frame than the short-term, dynamic allocation of the resource to the users in the lower tier. In this sense, the upper tier problem can be treated independently from the lower tier problem. Recall that the FCC policy [4] did allow the licensees of spectrum flexible, transferable rights of spectrum subject to technical and governmental regulations. As such, DSA's exclusive-use model tries to exploit this aspect by promoting spectrum market [87] in order to improve the utilization of spectrum. In a similar manner, this thesis assigns such market-based spectrum allocation to the upper-tier of the DSA framework. Fig. 2.4 describes the mechanism of the upper tier problems.

Essentially, there are primary holders of spectrum, e.g., existing wireless network operators in a country or region, who own or are licensed to use the spectrum. Assuming that they do not fully make use of their entire bandwidth at certain areas or have long deprivation periods when no wireless traffic occurs, they are encouraged to share their spectrum with other unlicensed service providers. The incentives for sharing may come in the form of monetary revenues. Such spectrum reallocation mechanism

is known as secondary spectrum market [87]. In this regard, the spectrum is treated as divisible goods which can be traded for profits. Spectrum leasing is considered to be a more appropriate mechanism than permanent trading as a leasing contract can be shorter and provide more flexibility. Subsequently, there is a need to properly price the spectrum based on the demand and supply processes, while appropriately taking the various wireless communications parameters into account. Dynamic spectrum market is a relatively new approach to DSA systems but it has received growing attention from the academia, industries and policy makers. For example, the 2011 IEEE Dynamic Spectrum Access Networks (DySPAN) conference [101] as well as IEEE Communications Society (ComSoc) [102] have offered tutorials on this topic; while the IEEE JSAC features the special issue on economics of communications systems [103] which publishes the latest works on market-based approaches to DSA.

As mentioned earlier in Section 1.3, dynamic spectrum allocation schemes in the upper tier using game theory will be discussed in Chapter 6 of the thesis.

Chapter 3

Non-cooperative Game Approach to Open Access Ad-hoc Scenarios

3.1 Overview

A wireless ad-hoc network¹ [13] consists of independent, distributed mobile devices and nodes (such as laptops, tablet computers and smartphones) that are able to self-organize to form a wireless network without the need for any preexisting infrastructure like access points or BSs, as illustrated by Fig. 3.1. The nodes are able to form connections with one or many other surrounding nodes in multi-hop topologies and handle certain communication tasks such as transmitting, receiving, routing or forwarding data, via the use of distributed protocols and algorithms. The decentralized nature of ad-hoc networks makes them flexible and scalable compared to centralized ones and suitable for many applications, especially in the IEEE 802.11 WLAN systems that use the unlicensed 2.45 GHz ISM band.

In this chapter, this thesis looks at a game-theoretic approach to spectrum allocation in an ad-hoc, distributed scenario. Suppose that the nodes in the networks are configured so that every two mobile devices form a pair of transmitter and re-

¹*Ad hoc* is a Latin phrase that typically denotes a makeshift, situation-specific solution.

Fig. 3.1: A wireless ad-hoc network. Each node in the network communicates with neighboring nodes without a central coordinator. If two nodes are out of range, an intermediate node can forward their messages.

ceiver. Each pair has data to transmit and will try to gain access to a proportion of a common bandwidth for transmission. A frequency channel can be reusable by two or more pairs if they are at a sufficient distance away so as not to cause excessive co-channel interferences (CCI). Therefore, each pair of users have to distributively determine which channel to transmit and an appropriate power level in order to optimize their transmission throughput. Conflict of interest often arises among selfish nodes individually trying to fulfill their own objectives. For example, when a node suffers loss in its SINR, it often tries to increase its transmission power. As a result, other nodes will also tend to transmit at higher power to combat the increasing CCI imposed by the previous node's increased power, causing the entire system to end up in an inefficient state. Thus, the designs of efficient RRM algorithms in such a scenario are often challenging tasks. As previously mentioned in Section 2.2.1, the problem discussed here represents the RRM for an open-spectrum-access system of

unlicensed users in the lower tier (see Fig. 2.3).

In the literature, such a spectrum sharing scenario in the unlicensed bands were also considered by several authors. Raman *et al.* [32] suggested a centralized server to coordinate the scheduling of separate transmit-receive pairs to maintain maximum rates and fairness. Moreover, Xing *et al.* [55] also proposed a centralized scheme where the transmission and CCI levels of the unlicensed pairs of users were centrally monitored and regulated. These techniques are often not computationally efficient in this distributed context. On the other hand, a number of distributed approaches also exist for these problems. Larcher *et al.* [31] devised a decentralized scheduling algorithm for the unlicensed users based on time slots in order to optimize their delay and packet loss rate performance. In addition, Nie *et al.* [53] proposed two methods, namely, potential game and no-regret learning, in order to assign channels to the ad hoc transmit-receive pairs, while assuming constant power across all users. Etkin *et al.* [56] used repeated game for allocating power to all coexisting users in the unlicensed bands. However, the authors did not consider the problem of channel assignment. The problem of joint adaptive channel and power allocation will be tackled in this chapter.

The distributed approaches mentioned above adopted game theory and searched for the NE point of the system. Most of these works and a few other early game-theoretical formulations of spectrum allocation [37–39, 61] have the players maximize either their respective SINR or capacity. In these studies, radios which experience a significant amount of CCI would still prefer to transmit since they receive positive utilities, despite the fact that persistent transmission might bring down the overall network performance. In fact, the users make decisions in a selfish, distributed manner, without full knowledge of other users' choice of channel and transmission probability and often end up causing too much CCI on the other users. As such, these selfish decisions often contradict with the overall benefit of the network and lead to a situation when the final NE outcome is far from the Pareto optimal point (see Appendix A.1.3 for the discussion of Pareto optimality). Thus, it might be necessary for some users

to cease or delay their transmission if the channel conditions become too unfavorable, in order to maintain the link quality and improve the system performance as a whole.

In view of that, this thesis proposes the *no-transmission* strategy so that players with negative utilities will be blocked from transmission. With such a self-regulated method, the system will be able to achieve more efficient scheduling and performance, without the aid of a centralized controller like in [32]. At the same time, the utility function is designed to incorporate not only the player's "reward" in terms of the actual SINR or spectrum efficiency, but also the "cost" in terms of transmission power, which helps to neutralize the selfish aspect of distributed users. With a proper choice of utility function, iterative gameplays such as best-response dynamics might be used to seek the NE point of the game.

In the next section, the distributed ad-hoc system model will be introduced.

3.2 System Model

The system model considers a cell of radius r in which there are N wireless users, each consisting of a transmit-receive pair randomly located around the area. Fig. 3.2 gives a simple illustration where there are 6 pairs of nodes within the area. In a game-theoretical context, these users will form the set of N players. Henceforth, the terms *players*, *users* and *pairs* can be used interchangeably. Each transmit-receive pair needs to select one frequency channel in order to transmit, out of the K available channels. These channels are non-overlapping in the sense that transmission on a channel does not interfere with its adjacent channels. In order to avoid trivial situations, it is further assumed that $K < N$. As previously mentioned, a channel which has been used by one pair can be reused by another pair as long as the resulting CCI is negligible or does not severely affect the users' QoS. For the example, in Fig. 3.2, there are $K = 3$ channels, and certain players may access the same channel such as player 3 and player 6 on channel 1. The nodes' locations and channel conditions are assumed

Fig. 3.2: Six transmit-receive pairs accessing the spectrum.

to be static or changing slowly during the duration of the game. Moreover, the average communication distance of every pair is assumed to be generally short-ranged in order to allow the distant pairs to reuse the spectrum. Let us denote $\mathbf{D} \in \mathbb{R}^{N \times N}$ the distance matrix, whose element d_{ij} is the distance between the i^{th} transmitter and the j^{th} receiver. Then, it can be seen from Fig. 3.2 that the communication distance of pair 3, d_{33} is in fact significantly less than the distance from the 3^{rd} transmitter to the 6^{th} receiver, d_{36} .

Formally, the $N \times K$ channel assignment matrix is defined as $\mathbf{A} \in \{0, 1\}^{N \times K}$, whose element a_{ik} takes value 1 if the i^{th} pair transmits using the k^{th} channel; and 0 otherwise. As a matter of fact, it is possible that for some particular pairs, the entire spectral band is not suitable for transmission, i.e., these users cannot find an appropriate channel to meet their QoS requirement without disrupting the activities of the neighboring users. Therefore, an additional action is proposed, i.e., the no-transmission strategy, which can apply well to non-real-time users that constitutes

a large portion of wireless traffics. Such users in no-transmission mode may not transmit or simply delay it for the time being and retry after a certain period of time. However, this work considers only the instantaneous behaviors and not the exact retrying mechanism.

As such, the constraint on the channel assignment can be expressed as

$$\sum_{k=1}^K a_{i,k} \in \{0, 1\} \quad \forall i = 1, 2, \dots, N. \quad (3.1)$$

Next, the $N \times K$ channel gain matrix $\mathbf{G} \in \mathbb{R}^{N \times N \times K}$ is also defined, where g_{ij}^k gives the channel gain between transmitter i and receiver j through channel k . Note that in general $g_{ij}^k \neq g_{ji}^k$. In Fig. 3.2, player 3's own channel gain g_{33}^1 and the channel gain of a possible CCI path, g_{36}^1 , are illustrated. All the channel gains are subject to independent path loss and Rayleigh fadings. The i^{th} pair is assumed to have knowledge of its own channel gains g_{ii}^k for all k .

In the same manner, the $N \times 1$ power allocation vector is given by $\mathbf{p}^T \in \mathbb{R}^N$, whose element $p_i \geq 0$ is the transmission power used by the i^{th} pair. Note that transmission power for a vector can be dependent on the channel assignment parameters, i.e., $\mathbf{p}^T = \mathbf{p}^T(\mathbf{A})$. Consequently, for the i^{th} pair with channel k , its SINR is expressed as

$$\gamma_{ik} = \frac{p_i g_{ii}^k}{\sum_{j=1, j \neq i}^N a_{jk} p_j g_{ji}^k + \sigma^2} \quad (3.2)$$

where σ^2 is the power of the receiver additive white Gaussian noise (AWGN) and is assumed to be identical for any links.

The transmission capacity or spectrum efficiency of the i^{th} pair who transmits via channel k is then evaluated by

$$R_{ik} = \log_2 \left(1 + \frac{p_i g_{ii}^k}{\sum_{j=1, j \neq i}^N a_{jk} p_j g_{ji}^k + \sigma^2} \right). \quad (3.3)$$

In this study, the user's key criterion for channel selection is characterized by either

the SINR (3.2) or the transmission capacity (3.3). Let us assume that the i^{th} pair has its own QoS to achieve which is represented by a target SINR γ_i^* .

3.3 Game Formulation

In this section, the game-theoretical formulation of the channel assignment problem and the player's utility function will be defined.

3.3.1 Strategic-form Game

The design objective is how to allocate the K available channels to the N players such that each transmit-receive pair obtains their desirable transmission capacity or their SINR. Mathematically, this channel allocation problem can be formulated as a strategic-form non-cooperative game (see Appendix A.1 for a formal definition of strategic-form non-cooperative games).

First, let us denote this non-cooperative resource allocation game $\mathcal{G}$ as

$$\mathcal{G} = [\mathcal{N}, \mathbb{S}, \{U_i\}_{i \in \mathcal{N}}]. \quad (3.4)$$

Here, $\mathcal{N} = \{1, 2, \dots, N\}$ is the set of players which consists of the N transmit-receive pairs. On the other hand, $\mathbb{S} = \mathbf{S}_1 \times \dots \times \mathbf{S}_N$ is the strategy space. $\mathbf{S}_i$ is the strategy set associated with the i^{th} pair. Note also that the individual strategy set is identical for all players. Each strategy of $\mathbf{S}_i$ represents the i^{th} channel that a player can select to transmit. Let us symbolize the action of choosing channel k by c_k . Moreover, c_0 represents the no-transmission strategy. Thus,

$$\mathbf{S}_i = \{c_1, c_2, \dots, c_K\} \cup \{c_0\} \quad \forall i \in \mathcal{N}. \quad (3.5)$$

Equivalently, each action c_k of player i corresponds to a unique combination of

values for the i^{th} row of channel assignment matrix $\mathbf{A}$ where $a_{ik} = 1$ and $a_{ij} = 0$, $\forall j \neq k$. c_0 is the special case where the entire row is 0. For convenience, denote $\mathbf{a}_i^T$ the i^{th} row of $\mathbf{A}$.

Alternatively, $\mathcal{G}$ represents a distributed optimization problem applied to all individual player $i \in \mathcal{N}$, i.e.,

$$\begin{aligned} & \max_{\mathbf{a}_i^T} U_i, \forall i \in \mathcal{N} \\ & \text{subject to (s.t.)} \begin{cases} \mathbf{a}_i^T \in \{0, 1\}^K, & \sum_{k=1}^K a_{i,k} \in \{0, 1\} \\ \mathbf{p}^T(\mathbf{A}) \geq 0 \end{cases} . \end{aligned} \quad (3.6)$$

The actual utility function U_i is going to be defined in the next section.

3.3.2 Utility Functions with Power Pricing

The basic and natural optimizing objectives in wireless RRM are SINR/rate maximization and power minimization. Thus, the utility functions often take the form of SINR or capacity/transmission rate as in (3.2) and (3.3), or a negative sum of transmission power (equivalent to minimizing total power). In the literature, these approach has been adopted in various works as in [33–37] .

An improvement from these straightforward utility functions is the *net utility* concept, adopted in [29, 38, 60] and other works. It basically consists of a “reward” term minus a “cost” term. In non-cooperative games, the standard utility functions aforementioned normally represent the preference of a selfish user, who wants to maximize its own reward. A player in a RRM game may somehow modify its parameters in such a way that increases not only its own reward (SINR or capacity), but also the CCI caused to others. This produces a negative overall effect as a player deliberately neglects the harm it does to others. Hence, there may be a need to introduce some costs into the utility function to neutralize these negative impacts. Such an approach

is also known as *pricing*, as it resembles the pricing mechanism in micro-economic game theory [16]. It will be adopted in this game in order to enforce a certain level of cooperation and self-awareness among players.

Therefore, let us consider the two following alternatives as utility functions according to the net utility approach. These two candidate utility functions $U1_i$ and $U2_i$ for player $i \in \mathcal{N}$ are given by

$$U1_i(c_k, S_{-i}) = \begin{cases} \ln(\gamma_{ik}) - \ln(\gamma_i^*) - \alpha p_i, & \text{If } a_{ik} = 1, a_{im} = 0 \ \forall m \neq k, \\ 0, & \text{If } c_k = c_0, \end{cases} \quad (3.7)$$

and

$$U2_i(c_k, S_{-i}) = \begin{cases} R_{ik} - \log_2(1 + \gamma_i^*) - \beta p_i, & \text{If } a_{ik} = 1, a_{im} = 0 \ \forall m \neq k, \\ 0, & \text{If } c_k = c_0. \end{cases} \quad (3.8)$$

Here, γ_{ik} and R_{ik} are already defined in (3.2) and (3.3), respectively. The constants α and β are known as the cost or pricing coefficients.

The first utility function $U1_i$ emphasizes that SINR is the maximization criterion as well as the reward of the players, while that of the second function $U2_i$ is the capacity or spectrum efficiency of the players. It is ensured that the utility for no-transmission is equal to zero. Otherwise, a player i who transmits obtains the utility value indicated by the expression in the upper part of $U1_i$ or $U2_i$. Unlike the common net utility functions [29, 38], the target SINR γ_i^* is also present as part of the pricing in such a way that the utility value becomes negative if the SINR drops too close to the target value. Whenever this happens, the node has no incentives to transmit as it receives a negative utility and will prefer no-transmission instead. This mechanism has the advantage of strictly maintaining the QoS requirements for the users.

In (3.7) and (3.8), the two alternative pricing functions are implicitly defined as

linear functions of transmission power, i.e.,

$$L1_i(p_i) = \ln(\gamma_i^*) + \alpha p_i, \quad (3.9a)$$

$$L2_i(p_i) = \log_2(1 + \gamma_i^*) + \beta p_i. \quad (3.9b)$$

3.4 Power Allocation Mechanisms

In this section, two power allocation schemes are proposed for the players in order to estimate their transmission power. They are simply called Scheme 1 and Scheme 2. Moreover, Scheme 1 is used exclusively for the utility function $U1$ and Scheme 2 for $U2$.

3.4.1 Scheme 1

Scheme 1 tries to achieve the SINR target requirements of all players by using an interference margin to compensate for the channel loss and CCI.

Consider the i^{th} pair of transmitter and receiver. The receiver always senses the environment and is aware of the noise level and the average amount of interferences present in every frequency channel. Based on its own SINR target γ_i^* and using an interference margin μ_i , a smart player will then be able to compute its own receiver sensitivity or minimum allowable received power. Denote this quantity $p_i^{\min}$, then

$$p_i^{\min}(\text{dBm}) = \sigma^2(\text{dBm}) + \mu_i(\text{dB}) + \gamma_i^*(\text{dB}) \quad \forall i \in \mathcal{N}. \quad (3.10)$$

Subsequently, for player i , its estimated transmitted power for channel k , which can be expressed as a function of k , will be given by

$$p_{i,est}(k)(\text{dBm}) = p_i^{\min}(\text{dBm}) - g_{ii}^k(\text{dB}) \quad \forall i \in \mathcal{N}. \quad (3.11)$$

3.4.2 Scheme 2

In Scheme 2, the players are to seek the optimal power level, as the utility function $U2_i$ satisfies the mathematical property of concavity with respect to p_i .

Lemma 3.1. *For any i , the expression $\Lambda_i = R_{ik} - \log_2(1 + \gamma_i^*) - \beta p_i$ is a concave function with respect to p_i , $\forall p_i \geq 0$.*

Proof: One may prove this lemma using the second derivative test.

Since $\Lambda_i = \log_2 \left(1 + \frac{p_i g_{ii}^k}{\sum_{j=1, j \neq i}^N a_{jk} p_j g_{ji}^k + \sigma^2} \right) - \log_2(1 + \gamma_i^*) - \beta p_i$, we then have

$$\frac{\partial \Lambda_i}{\partial p_i} = \frac{1}{\ln 2} \cdot \frac{1}{p_i + \frac{\sum_{j=1, j \neq i}^N a_{jk} p_j g_{ji}^k + \sigma^2}{g_{ii}^k}} - \beta. \quad (3.12)$$

Thus, by differentiating with respect to p_i again,

$$\frac{\partial^2 \Lambda_i}{\partial p_i^2} = -\frac{1}{\ln 2} \cdot \frac{1}{\left(p_i + \frac{\sum_{j=1, j \neq i}^N a_{jk} p_j g_{ji}^k + \sigma^2}{g_{ii}^k} \right)^2}. \quad (3.13)$$

Clearly, $\frac{\partial^2 \Lambda_i}{\partial p_i^2} < 0 \forall p_i \geq 0$ which proves the lemma. ■

Lemma 3.1 implies that when player i does not choose the no-transmission strategy, its utility function $U2_i$ becomes Λ_i and is concave with respect to p_i . Now suppose that the i^{th} player is trying to allocate its power after it has selected its channel k , where the CCI and noise level has been measured and the only unknown parameter in $U2_i$ is p_i . It follows directly that the optimum power for a rational player i on this channel k is the one that maximizes its utility function, which is the solution to $\frac{\partial \Lambda_i}{\partial p_i} = 0$. The following corollary follows directly from (3.12), coupled with the constraint $p_i \geq 0$.

Corollary 3.1. *The optimal power can be estimated as*

$$p_{i,est}(k) = \max \left(\frac{1}{\beta \ln 2} - \frac{\sum_{j=1, j \neq i}^N a_{jk} p_j g_{ji}^k + \sigma^2}{g_{ii}^k}, 0 \right). \quad (3.14)$$

3.5 Iterative Algorithm

As previously mentioned and also discussed in Appendix A.1.5, an important consideration in RRM games is NE convergence via iterative gameplay. In the proposed game, each player determines its action (i.e., which channel to transmit) to maximize its own payoff, based on the information available locally, hoping to achieve a stable and desirable state (i.e., a NE point). The best-response dynamic (see Appendix A.1.6) can be employed to achieve the NE by letting the players play the game in a sequential or round-robin manner, such that players take turns in a predetermined order to find their respective best responses given the observation of opponents' actions. In each turn, a player only uses the game's current state information, the mechanism of which is known as myopic play [58]. The players still have to agree upon the order of play. One possible method to establish this order is to use the order of network entry prior to the gameplay.

An iterative best-response method for the non-cooperative allocation game is proposed in Algorithm 3.1. The algorithm serves as a distributed protocol that controls the spectrum access and power allocation of the transmit-receive pairs, which effectively engages them in the frequency assignment game.

At the beginning, without any prior interactions, players would simply randomize their channel selections while transmitting at a fixed power P_{init} set by the algorithm. Thereafter, a player i in its turn can adapt its strategy accordingly using the information it has observed. It can follow either Scheme 1 or Scheme 2 to estimate its power; and then it decides the new strategy in a best-response manner, defined by

Algorithm 3.1 Iterative algorithm for the frequency assignment game.

```

1: I. INITIALIZATION STEP:
2: Upon entry into the network, each player detects the other players and establishes
   its own order in the sequence of gameplay.
3: for player  $i = 1 \rightarrow N$  do
4:   Start with a random strategy  $c_k$ 
5:   if  $k \neq 0$  then  $p_i \leftarrow P_{init}$  ▷ Transmit at a predetermined power
6:   else  $p_i \leftarrow 0$  ▷ No-transmission
7:   end if
8: end for
9: II. ITERATION STEP:
10: while NE has not been reached do
11:   for player  $i$  in a predetermined sequence do
12:     for channel  $k = 1 \rightarrow K$  do
13:       The receiver measures the noise and CCI, then feeds those values back
14:       to the transmitter.
15:       The transmitter determines  $p_{i,est}(k)$  based on either (3.11) or (3.14).
16:       Estimate  $U1_i(c_k, S_{-i})$  or  $U2_i(c_k, S_{-i})$  from (3.7) and (3.8), respectively.
17:     end for
18:     Decide the new strategy  $c_m$  with best-response dynamics (A.8) or (3.15).
19:     if  $m \neq 0$  then  $p_i \leftarrow p_{i,est}(m)$ ,  $a_{im} \leftarrow 1$  and  $a_{ij} \leftarrow 0$ ,  $\forall j \neq m$ 
20:     else  $p_i \leftarrow 0$ ,  $a_{ij} \leftarrow 0$ ,  $\forall j$ 
21:     end if
22:   end for
23: end while

```

(A.8). Specifically in this case, the new strategy c_m is given by

$$c_m = \arg \max_{c_k \in \mathbf{S}_i} U_i(c_k, S_{-i}), \quad (3.15)$$

where U_i can be either $U1_i$ or $U2_i$.

After c_m is decided, the player updates the i^{th} row $\mathbf{a}_i^T$ of $\mathbf{A}$ and p_i accordingly. Player i 's turn terminates and the next player will make its move in the exact same manner. After all players have selected their strategies in sequence, one round is said to be completed. At the end of one round, NE convergence will be checked. If the equilibrium has not been achieved, certain players will be tempted to switch strategies and the game proceeds to the next iteration or another game round. This will repeat until no player intends to change its transmission strategy and the system converges.

At the point of convergence, the strategies chosen by the players will be stable, which is an indication that everyone achieves its desirable operating point. By definition, this corresponds to one of the possible NEs of the game. In general, there can exist multiple NEs when the dimensions of the strategy space are large.

As a final remark, in each player's turn, the computation is dominated by the best response, which searches a set of $K + 1$ elements (inclusive of the no-transmission strategy) for the maximum value. Thus, it is an $O(K)$ -operation. As such, the complexity of the entire algorithm is $O(INK)$, where I refers to the total number of rounds before convergence. Determining I is theoretically difficult but numerical simulations can be done to estimate the average number of rounds given a particular setting.

3.6 Simulation Results

Simulations were performed to confirm the NE convergence of the proposed channel allocation algorithm for both Scheme 1 and Scheme 2. The simulation was done using MATLAB. In this section, numerical results are documented.

Table 3.1 shows the simulation settings. In the initial state, all the pairs started with an initial power P_{init} that was conveniently set at 13 dBm (20 mW). The SINR target is $\gamma_i^* = 15$ dB for all players. Furthermore, in Scheme 1, all players estimated their transmission power with an identical interference margin $\mu_i = 5$ dB. Both Scheme 1 and Scheme 2 were simulated and compared based on the same nodes' locations and channel conditions. As an example, Fig. 3.3 illustrates an instance of the nodes' distribution during a one-shot of the game, where the locations of the nodes are shown. The transmitters and receivers were uniformly distributed over the area. The channels were assumed to undergo path loss and Rayleigh fadings.

Table 3.1: Simulation settings for the frequency assignment game.

System parameters	Values
Number of players, N	20
Number of channels, K	4
Cell radius, r	1 km
Path loss exponent, λ	3.7
Noise power level, σ^2	-130 dBm
Initial power level per user, P_{init}	13 dBm
Target SINRs per sub-carrier, γ_i^* , $\forall i$	15 dB
Interference margins, μ_i , $\forall i$	5 dB
Cost coefficient, α	40 W ⁻¹
Cost coefficient, β	200 bps/Hz/W

Fig. 3.3: Locations of nodes in one simulation.

3.6.1 Convergence of the Game

Fig. 3.4 shows the evolution of players' strategies over time during this particular one-shot until a NE was reached. A list of players who selected a particular strategy at NE are indicated next to the strategy graphs. Strategy 0 indicates no-transmission while strategies 1 to 4 refer to the channel index that players selected. Each iteration as seen on the horizontal axis is a complete game round as explained before.

Fig. 3.4: Convergence of the game for Scheme 1 and 2.

Both Scheme 1 and Scheme 2 are shown to quickly converge to a stable point. On average, they could converge within less than 10 rounds, which verifies the computational efficiency of the proposed algorithm. Moreover, it can be noticed that the two schemes converged to two almost similar NEs with 17 players out of 20 having the same channels, except for players 3, 4 and 11. This is because in both schemes, actions were taken in order by the same sequence of players; thus, the evolution of the allocation points is likely to follow the same path. However, the two final NEs may slightly differ due to the difference in strategies in terms of power estimation. On the other hand, a closer investigation shows that the allocations are reasonable. For instance, it is seen from Fig. 3.4 that both schemes assigned non-overlapping frequency channels to players 14, 15, 16 and 17: channel 2 to player 14, 1 to 15, 4 to 16 and 3 to 17. This is because these pairs were located in a close cluster (see Fig. 3.3). Such an assignment is thus able to avoid to potentially high CCI levels that would occur if the players were to select the same channels.

3.6.2 Power and SINR Performance

Next, Fig. 3.5 compares the transmission power (in W) of the 20 pairs of users and Fig. 3.6 compares their SINR levels after NE against the initial point using both Scheme 1 and Scheme 2. These results show that Scheme 1 seemed to maintain a minimal but fair level of SINR above the target value among all the players; while Scheme 2 exhibited apparently greater variations in terms of achieved SINR. This is due to the nature of the power allocation schemes: Scheme 1 (eq. (3.11)) emphasizes on fulfilling the SINR target whereas Scheme 2 (eq. (3.14)) allowed individuals to optimize their transmission rates. Moreover, in Scheme 2, players set their power reasonably higher and more evenly distributed than in Scheme 1 (although this is in part due to the value of the pricing coefficients α and β), because players were free to increase power to maximize their capacities. In particular, player 1 whose transmission distance is very close can achieve the same SINR level as others at a much lower power level in Scheme 1, or a much higher SINR in Scheme 2 than others with similar power. Nevertheless, in both schemes, the average power consumption had been reduced considerably from the initial power level of 20 mW, which indicates a more efficient usage of resources.

In Fig. 3.6, it is observed that Scheme 1 guaranteed all players a channel to transmit and minimally fulfilled their SINR target without much variations among their SINRs, which implies a certain degree of fairness. On the other hand, Scheme 2 gave players better SINR (and subsequently higher rates) as seen from Fig. 3.6. The average SINR of Scheme 2 was 27.10 dB, while that of Scheme 1 was 18.90 dB. This is again because of the different natures of the two schemes. However, Scheme 2 might result in more players blocked from transmitting (e.g., player 4 in this one-shot). This is not unreasonable, as this player might have been in very bad channel conditions and already had to spend a relatively high power level in order to meet the target SINR as suggested by Scheme 1. Hence, in Scheme 2, its “optimal” power actually

Fig. 3.5: Transmission power at NE for Scheme 1 and 2.

Fig. 3.6: Players' SINR at the initial point and NE for Scheme 1 and 2.

resulted in a negative utility function and instead switched into the no-transmission strategy.

Additionally, in Scheme 2, it is remarked that the difference in transmission power levels among the players is small. This is because their power levels were set optimally according to (3.14). It is likely that the term $\frac{1}{\beta \ln 2}$ was dominant compared to the other one. In this case, an appropriate value of β can be set in order to adjust the final power levels.

3.6.3 Performance in the Long-run

In this simulation study, the long-run performance of the system is statistically evaluated by replicating 1000 independent one-shot games. The parameter settings of Table 3.1 were maintained, except the locations of the pairs and their channel conditions which were independently varied. Both Scheme 1 and Scheme 2 were simulated; and as a performance benchmark, the proposed game was compared with the random allocation (RA) scheme, where all transmit-receive pairs randomly select a frequency channel and a fixed power level to transmit, without any subsequent dynamic adaptation. As a result, the empirical cumulative distribution functions (CDF) for the SINRs of the players (omitting those that selected no-transmission) were obtained and plotted in Fig. 3.7. Once again the figure shows the contrast between Scheme 1 and Scheme 2. The SINR values of Scheme 1 are shown to minimally achieve the target SINR of 15 dB while those of Scheme 2 were notably better. However, the steep curve of Scheme 1's CDF also indicates a highly uniform range of values with low variance, i.e., a high degree of fairness among the players was achieved via Scheme 1. Meanwhile, the RA scheme shows a much wider range of SINR with more than 35% of the players unable to reach the target SINR of 15 dB. In terms of SINR (and as a result, system throughput), it is also inferior to Scheme 2, as indicated by the significant gap between its CDF and that of Scheme 2. The reason for the performance

Fig. 3.7: Empirical CDFs of SINR for Scheme 1, Scheme 2 and RA.

improvement is mainly due to the introduction of gameplay through best-response dynamics which allows the players to adapt their strategies to better utilize the frequency resources and avoid collisions with other players, unlike the stationary case in the RA scheme.

Finally, the effectiveness of the no-transmission strategy is assessed. An important criterion is how often no-transmission may occur as the number of users grows. Thus, 1000 one-shot games were simulated for each number of players N from 10 to 20 in order to study this behavior. For each one-shots, again the parameters in Table 3.1 were maintained except for the channel conditions and the nodes' locations. The percentage of occurrence of no-transmission for both Scheme 1 and Scheme 2 were recorded and plotted, as shown in Fig. 3.8. First of all, the frequency of no-transmission is seen to grow with the increasing system loads in both schemes. This is natural since resources get scarcer and scarcer as the number of competing users increases, hence more players opted for no-transmission. However, in Scheme 2, no-transmission oc-

Fig. 3.8: Percentage of no-transmission for Scheme 1 and 2.

curred significantly more frequent than in Scheme 1, as previously predicted. The reason is that for Scheme 2, since the individual optimization aspect of selfish players is emphasized, it may be better in terms of the overall throughput but worse in fairness than Scheme 1, as it blocks more players from accessing the spectrum resource.

3.7 Concluding Remarks

In this chapter, a new RRM scheme employing game theory for a distributed ad-hoc open-access network has been proposed. The problem of adaptively assigning the shared frequency channels to a large group of transmit-receive pairs is modeled as a non-cooperative game, in which players try to maximize either their SINRs or their spectrum efficiencies. The no-transmission strategy is introduced which provides further flexibility as users can decide whether to transmit or delay access if conditions are unfavorable, which applies to non-real-time traffics. Two alternatives are considered

for the utility function, both of which employ a pricing mechanism which takes into account not only the consumed power, but also the users' target SINR, so as to ensure that QoS requirements are strictly fulfilled. Furthermore, an iterative sequential algorithm using best-response dynamics is proposed in order to obtain the final NE solution. In addition, numerical results show that convergence can be obtained very quickly with acceptable SINR and power performance. The tradeoff aspect between Scheme 1 and Scheme 2 in terms of throughput and fairness is also explored in close details.

In the author's opinion, a few important indications from this preliminary work are:

- The potentials for efficient and fair dynamic spectrum sharing in a distributed, ad-hoc system have been unveiled and exploited.
- The existence of stable solutions for wireless RRM problems which can be obtained via the game-theoretical approach.
- In non-cooperative games, some degrees of "cooperation" and "self-awareness" can be enforced by proper designs of the utility functions to neutralize the selfish reward maximization aspect of selfish players.

There are still a few issues which have not been fully addressed. Although there is strong evidence of the existence of stable operating points as NEs have been obtained consistently in the simulations, such existence is not fully proven via a rigorous mathematical analysis. In the literature, only certain types of games, such as potential games, can be guaranteed with the existence of NE points. Furthermore, the game may not have a unique NE and the number of multiple NEs could well be large in some cases. How to ensure that the solution converges to the best achievable Nash equilibrium point remains an open research problem. Nevertheless, obtaining a performance bound for the achieved NE can be made possible using the price of anarchy (PoA -

see Appendix A.1.7). In the next chapter, the thesis will continue to develop better spectrum access solutions for distributed, ad-hoc systems by formulating games that possess the desirable NE existence and convergence properties, as well as provable PoA bounds.

Chapter 4

Potential Game Approach to Distributed OFDMA-based Systems

4.1 Overview

Orthogonal frequency division multiplexing (OFDM) [14] is a digital multi-carrier modulation technique, in which a large number of closely-spaced orthogonal subcarriers are used to transmit signals simultaneously, as depicted by Fig. 4.1. In OFDM systems, high-rate data which might otherwise suffer from severe frequency-selective fading, is first converted into a number of parallel orthogonal data streams occupying less than the coherence bandwidth before transmitting. The advantage of OFDM lies mainly in its robustness to multipath fadings and inter-symbol interferences due to the prolonged symbol duration on each subcarrier and the use of a cyclic prefix. Furthermore, its transceiver design and implementation can be simplified by the use of fast Fourier transform (FFT) and inverse FFT blocks, as shown in Fig. 4.2.

OFDMA [15] is the multiple-access version of OFDM. It has been adopted in various existing standards like IEEE 802.11 WLAN, IEEE 802.16e worldwide inter-

Fig. 4.1: The frequency spectrum of an OFDM signal. The subcarriers are orthogonal as at each subcarrier center frequency, contributions from adjacent subcarriers are canceled.

Fig. 4.2: Block diagram of a general OFDM transmitter.

operability for microwave access (WiMAX), Third-Generation Partnership Project (3GPP)'s Long Term Evolution (LTE) standards and will also be a crucial part of future wireless networks, such as 4G systems and the ongoing IEEE 802.22 standards. Multiple access in OFDMA can be achieved by partitioning separate subsets of subcarriers to users. OFDMA not only inherits all the advantages of OFDM but also provides multi-user diversity gain, because if a subcarrier is unfavorable to a user, it may still be favorable to another user and thus can be reassigned appropriately. Moreover, different number of subcarriers can be allocated for users with different QoS classes, which offers another degree of flexibility for RRM in OFDMA systems.

In multi-user OFDMA networks, a key RRM issue is how to dynamically allocating the subcarriers and power to users in order to combat CCI. Several subproblems exist

for OFDMA resource allocation, such as downlink [36] vs. uplink [43], single-cell [39] vs. multi-cell [37], or distributed [38] vs. centralized [48] approaches. Pioneering works on RRM for centralized OFDMA systems were done by Yong *et al.* [33], who was among the first few to treat OFDMA resource allocation as mathematical optimization problems. Since then, enormous research efforts have been contributed, providing a wide variety of approaches and solutions to the problem. At the same time, early game-theoretical approaches for OFDMA were proposed by [37–40] and references therein, which has been well-surveyed in a book chapter by the author [97]. These early works often focused on the traditional cellular systems involving uplink/downlink communications between the BS and several MSs. A common drawback of these approaches, as pointed out in [97], is that they do not always guarantee a stable solution, especially if no pure-strategy NE exists.

On the other hand, for the distributed, ad-hoc OFDMA system, Bazerque *et al.* [42] was one of the important works, where the authors relied on heavy information exchange in a fully connected network in order to obtain the decentralized algorithm. However, at the time of this research, there were still very few works directed towards the distributed OFDMA scenario. Nevertheless, it can be noticed that in the special case where each player only competes for a single frequency channel, the situation bears resemblance to the well-known distributed ad-hoc spectrum sharing games [31, 53, 56, 96], as well as cognitive radio spectrum access games [54, 58, 64]. Notably, Neel *et al.* [58] suggested the use of potential games for cognitive radio networks; and Nie *et al.* [53] formulated a potential game for the distributed, ad-hoc scenario. For such games, the existence of deterministic pure-strategy NEs is always guaranteed when assignment of a single channel is considered, which suggests the possibility of extending the same analytical platform to a multi-channel scenario. In summary, the lack of works in the literature on distributed, ad-hoc OFDMA systems and the stability issues of traditional OFDMA game-theoretical approaches provide the motivations to exploit the use of potential games to realize desirable solutions to

resource allocation for OFDMA.

In this chapter, the thesis extends its investigation into the distributed, ad-hoc networks of multiple transmit-receive pairs as similar to Chapter 3. However, the system has an OFDMA interface with the bandwidth being divided into different orthogonal subcarriers, which has been a more practical approach for wireless networks recently. Each player has an OFDM front-end and can measure the channels in order to select a subset of the available subcarriers to transmit. As such, more complicated spatial reuse of spectrum occurs over the cell's area; and one will have to pay closer attention to the issue of CCI mitigation. By adopting the interference-sum minimization objective in the players' utility function, a potential game can be formulated similar to [53]. When this game is played, due to domination in the strategy, it is best for the players to be allocated with a given number of subcarriers in advance. The number of subcarriers for each player can be determined prior to the gameplay via a simple proposed method. This situation notwithstanding, the universal properties of potential games are not lost. Hence, NE convergence can always be obtained through sequential dynamics. Besides, the optimality of the NEs obtained through this method can also be characterized with the PoA. Furthermore, extensive simulations are carried out to evaluate the proposed scheme as well as to compare the system and individual performance in terms of energy efficiency, fairness and optimality.

4.2 System Model

This section defines the mathematical model for the system and describes a framework for interference mitigation.

4.2.1 System Parameters

Similar to Chapter 3, the model considers a circular cell of radius r , where there are N players, each being a transmit-receive pair randomly distributed within the area.

Once again, the terms *players* and *pairs* can be used interchangeably. Each pair individually competes with others for different combinations of OFDMA subcarriers among the shared K subcarriers available. It is assumed that $N > K$ so as to avoid trivial solutions, as well as the nodes' locations and channel conditions being static or changing slowly during the game. Moreover, the assumption of short-range distance between the transmitter and receiver of a pair still holds. Recall that the distance matrix is $\mathbf{D} \in \mathbb{R}^{N \times N}$, whose element d_{ij} is the distance between the i^{th} transmitter and the j^{th} receiver. Then, in general, it is assumed that $d_{ii} \ll d_{ji}$ for $j \neq i$.

The other parameters can be defined as follows. Let $\mathbf{A}$ denote the subcarrier assignment matrix, where $\mathbf{A} \in \{0, 1\}^{N \times K}$. Each of the elements in $\mathbf{A}$, $a_{ik} \in \{0, 1\}$, implies whether player i occupies subcarrier k for transmission; and $a_{ik} = 1$ if this is true and 0 otherwise. Hence, the choice of subcarriers for player i is reflected by $\mathbf{a}_i^T$, the $1 \times K$ i^{th} row vector of $\mathbf{A}$. As the no-transmission strategy is not considered in this case, it can be understood that the all-zero vector $\mathbf{0}$ is not a valid choice.

Next, let us define the channel gain matrix $\mathbf{G} \in \mathbb{R}^{N \times N \times K}$, where g_{ij}^k gives the channel gain between transmitter i and receiver j through subcarrier k . Note that, in general, $g_{ij}^k \neq g_{ji}^k$. Similarly, the transmission power matrix is denoted by $\mathbf{P} \in \mathbb{R}^{N \times K}$, whose element p_{ik} is the transmitted power of player i over subcarrier k . The $1 \times K$ i^{th} row vector of $\mathbf{P}$, denoted by $\mathbf{p}_i^T$, must be element-wise non-negative and the sum of all elements should be less than $P_{\max}$. That is,

$$\mathbf{p}_i^T(\mathbf{A}) \succeq \mathbf{0}, \quad \mathbf{p}_i^T \mathbf{1} \leq P_{\max}, \quad \forall i \quad (4.1)$$

where $\mathbf{1}$ is the $K \times 1$ all-one column vector.

Then, for player i with subcarrier k , its SINR is expressed as

$$\gamma_{ik} = \frac{a_{ik} p_{ik} g_{ii}^k}{\sum_{j=1, j \neq i}^N a_{jk} p_{jk} g_{ji}^k + \sigma^2} \quad (4.2)$$

where σ^2 is the power of the receiver AWGN and is assumed to be identical between any two players.

Furthermore, the achievable rate for player i with subcarrier k is given by

$$R_{ik} = B \log_2 \left(1 + \frac{\gamma_{ik}}{\Gamma} \right) \quad (4.3)$$

where B is the bandwidth per subcarrier, $\Gamma = -\ln(5P_e)/1.5$ is a function of the required BER P_e , often known as the SINR gap [65]. Notice that there are some subtle changes from (3.2) to (4.2) and (3.3) to (4.3) due to the OFDMA context.

The objective of any player i is to optimize its performance individually, which is reflected by a utility function U_i . Mathematically, the distributed optimization problem is stated as

$$\begin{aligned} & \max_{\mathbf{a}_i^T} U_i, \forall i \\ & \text{s.t.} \quad \begin{cases} \mathbf{a}_i^T \in \{0, 1\}^K \setminus \{\mathbf{0}\}, \\ \mathbf{p}_i^T(\mathbf{A}) \succeq 0, \quad \mathbf{p}_i^T \mathbf{1} \leq P_{\max}, \end{cases} \end{aligned} \quad (4.4)$$

where the utility function $U_i(S)$ will be defined in Section 4.3.

4.2.2 An Interference Mitigation Framework

In this section, a framework for mitigation of CCI and the relevant parameters are discussed. Since the subcarriers are shared among the players, it is best that a subcarrier should not be reused by players which are within a certain vicinity of another player which is currently using it for transmission. As such, it is proposed that an area surrounding a player i with radius r_i be defined as the *circle of interference*, within which any device transmitting on the same subcarrier will cause significant CCI to the i^{th} player. r_i is called the radius of interference. A similar concept was briefly mentioned in [64]. However, it is developed differently here due to the fact that the

CCI affects mainly the receiver, so it makes sense to center the circle of interference at the receiver.

Using the previously defined distance matrix $\mathbf{D}$, two rules governing the reuse of subcarriers are generalized:

- (i) **Rule 1:** For player i to successfully transmit using subcarrier k , it is required that $(\forall j \neq i, d_{ji} < r_i): a_{jk} = 0$, i.e., all transmitters within the radius of interference should not use the same subcarrier as player i .
- (ii) **Rule 2:** Two players i and j can reuse any subcarrier k if $d_{ij} > r_j$ and $d_{ji} > r_i$, i.e., the transmitter of one player falls outside the radius of interference of the other, and vice versa. A *conflict* is said to occur between players i and j if rule 2 is violated.

The definition of conflict allows us to construct the so-called *conflict matrix* $\mathbf{C} \in \{0, 1\}^{N \times N}$, which maps the occurrence of conflicts in the entire network. Specifically, its elements are

$$C_{ij} = \begin{cases} 0 & i = j, \text{ or no conflict between } i \text{ and } j, \\ 1 & \text{otherwise.} \end{cases} \quad (4.5)$$

A simple example with $N = 4$ pairs of nodes is shown in Fig. 4.3. It is observed that player 2 does not have a conflict with any other player. On the other hand, player 4's transmitter is within both players 1 and 3's interference radii so it cannot use the subcarriers occupied by players 1 and 3. However, player 1 will not clash with player 3. Thus, the conflict matrix is given by

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}. \quad (4.6)$$

Fig. 4.3: Four transmit-receive pairs and their interference radii.

The symmetric conflict matrix is useful from a centralized point of view. In practice, an entry “1” in a row represents a one-hop neighbor of a particular player, and some available distributed MAC protocols can be used to obtain these one-hop neighbors. Hence, in the distributed context, it is reasonable to assume that each player knows only its corresponding row of the matrix. As will be seen in later parts of this chapter, this information can be useful for the players to autonomously compete for resources.

It has not been mentioned how the radius of interference is determined. This is a complex problem, as the value of r_i should depend on player i 's as well as its neighbors' transmitted power and locations, its own SINR requirement, and channel gains. To be more accurate, for different subcarriers, player i may have different radii as a result of frequency-selective fading. However, a simple way to estimate r_i with only the path loss taken into consideration will be proposed, so that r_i will be identical for all subcarriers.

Assuming that for pair i , its neighbor j who is using the same subcarrier k is the

most significant interferer, and the effects from the remaining pairs are negligible. Furthermore, let us assume that they use roughly the same power $p_{ik} \approx p_{jk} = p$, and consider only the effect of path loss.

Proposition 4.1. *Under the above assumptions, to maintain its SINR above a target level γ_i^* , player i might set*

$$r_i = d_{ii}(\gamma_i^*)^{1/\lambda} \quad (4.7)$$

where λ is the path loss exponent.

Proof: Following the assumptions, the received power levels at receiver i from the i^{th} and j^{th} transmitters are p/d_{ii}^λ and p/d_{ji}^λ , respectively. By neglecting the background noise, the SINR perceived by the i^{th} pair is roughly $(d_{ji}/d_{ii})^\lambda$. If we want this to be no less than γ_i^* , then $d_{ji} \geq d_{ii}(\gamma_i^*)^{1/\lambda}$. Thus, r_i can be estimated by $d_{ii}(\gamma_i^*)^{1/\lambda}$ according to rule 2 previously. ■

In practice, if more than one interferers need to be accounted for, a safeguard margin (i.e., a multiplier greater than 1) might be introduced to this computed value. In the next section, the utility function is introduced in order to analyze the resulting game.

4.3 Analysis of Potential Game

4.3.1 Preliminaries

As previously mentioned, potential games have the attractive properties of pure-strategy NE existence and convergence. Within the context of wireless RRM games, they have been considered by a few authors, e.g., [53], [58] and [67]. The definitions of such games, especially exact potential games, are given in details in Appendix A.2 and restated here as follows. A game $\mathcal{G}$ is an exact potential game if and only if an

exact potential function $F(S) : \mathbb{S} \mapsto \mathbb{R}$ exists, such that

$$U_i(T_i, S_{-i}) - U_i(S_i, S_{-i}) = F(T_i, S_{-i}) - F(S_i, S_{-i}). \quad (4.8)$$

For the distributed system of transmit-receive pairs where each player selects only a single frequency channel similar to Chapter 3, Nie *et al.* [53] formulated a potential game by adopting the following interference-minimizing utility function:

$$U_i = -(\text{Total CCI generated and experienced by } i). \quad (4.9)$$

The above game will be labeled $\mathcal{G}_1$. Next, a generalization of $\mathcal{G}_1$ will be formulated which applies to the distributed OFDMA system under investigation.

4.3.2 Potential Game Formulation

Let us denote the distributed OFDMA subcarrier allocation game $\mathcal{G}_2$, where

$$\mathcal{G}_2 = [\mathcal{N}, \mathbb{S}, \{U_i\}_{i \in \mathcal{N}}]. \quad (4.10)$$

The set of players is given by $\mathcal{N} = \{1, 2, \dots, N\}$. For any player i , its strategies S_i are the subcarriers. Thus, effectively $S_i = \mathbf{a}_i^T \in \mathbf{S}_i$, where $\mathbf{S}_i = \{0, 1\}^K \setminus \{\mathbf{0}\}$ is the strategy set of player i . For each S_i , S_{-i} is the joint strategy by opponents of player i . Hence, $S = (S_i, S_{-i})$ is the joint strategy of all players, also known as a strategy profile. All strategy profiles belong to the strategy space, here defined by $\mathbb{S} = \mathbf{S}_1 \times \dots \times \mathbf{S}_N$.

Every utility function $U_i(S)$ is then a mapping from $\mathbb{S}$ to $\mathbb{R}$, which is defined as a negative interference sum (4.9) over all the utilized subcarriers of player i , i.e., $U_i = \sum_{\{k|a_{ik}=1\}} U_{i,k}$. As the assignment of subcarriers $\{k|a_{ik} = 1\}$ is yet to be determined,

another set of variables δ_{ij}^k needs to be introduced, such that

$$\delta_{ij}^k = \begin{cases} 1 & \text{Players } i, j \text{ (} i \neq j \text{) both transmit via subcarrier } k, \\ 0 & \text{otherwise.} \end{cases} \quad (4.11)$$

Thus, δ_{ij}^k can be viewed as an indicator of whether two players i and j interfere each other through a common subcarrier k ; and $\delta_{ii}^k = 0$ because a player does not interfere with itself. It is not difficult to verify that $\delta_{ij}^k = \delta_{ji}^k = a_{ik}a_{jk}$ ($i \neq j$). With all these notations introduced, the utility function for player i can be expressed as

$$U_i(S) = - \sum_{k=1}^K \left(\sum_{j=1}^N \delta_{ji}^k p_{jk} g_{ji}^k + \sum_{j=1}^N \delta_{ij}^k p_{ik} g_{ij}^k \right). \quad (4.12)$$

Theorem 4.1. *Eq. (4.8) will hold if the following exact potential function is considered:*

$$F(S) = \frac{1}{2} \sum_{i=1}^N U_i. \quad (4.13)$$

Proof: To simplify, let us define $\omega_{ij}^k = -\delta_{ij}^k p_{ik} g_{ij}^k$. Then, U_i in (4.12) can be written as

$$U_i(S_i, S_{-i}) = \sum_{k=1}^K \left(\sum_{j=1}^N \omega_{ij}^k + \sum_{j=1}^N \omega_{ji}^k \right) = \sum_{k=1}^K U_{i,k}(S_i, S_{-i}) \quad (4.14)$$

where $U_{i,k}(S_i, S_{-i}) \triangleq \sum_{j=1}^N \omega_{ij}^k + \sum_{j=1}^N \omega_{ji}^k$.

The potential function (4.13) can be simplified to

$$\begin{aligned} F(S_i, S_{-i}) &= \frac{1}{2} \sum_{i=1}^N \left(\sum_{k=1}^K \left(\sum_{j=1}^N \omega_{ij}^k + \sum_{j=1}^N \omega_{ji}^k \right) \right) \\ &= \sum_{k=1}^K \left(\sum_{i=1}^N \sum_{j=1}^N \omega_{ij}^k \right) \\ &= \sum_{k=1}^K F_k(S_i, S_{-i}) \end{aligned} \quad (4.15)$$

where $F_k(S_i, S_{-i}) \triangleq \sum_{i=1}^N \sum_{j=1}^N \omega_{ij}^k$.

Suppose that player i switches its strategy from $S_i = \mathbf{a}_i^T = [a_{i1} \ a_{i2} \ \dots \ a_{iK}]$ to $T_i = \mathbf{a}'_i^T = [a'_{i1} \ a'_{i2} \ \dots \ a'_{iK}]$. Its power vector changes to $\mathbf{p}'_i^T = [p'_{i1} \ p'_{i2} \ \dots \ p'_{iK}]$. By comparing (4.14) and (4.15), it is seen that (4.8) holds if the following k separate identities hold simultaneously:

$$U_{i,k}(T_i, S_{-i}) - U_{i,k}(S_i, S_{-i}) = F_k(T_i, S_{-i}) - F_k(S_i, S_{-i}), \quad \forall k. \quad (4.16)$$

Noting that $\omega_{ii}^k = 0$, $F_k(S)$ can be rewritten into

$$\begin{aligned} F_k(S_i, S_{-i}) &= \sum_{l=1}^N \omega_{il}^k + \sum_{j=1}^N \omega_{ji}^k + \sum_{j=1, j \neq i}^N \sum_{l=1, l \neq i}^N \omega_{jl}^k \\ &= U_{i,k}(S_i, S_{-i}) + Q_k(S_{-i}) \end{aligned} \quad (4.17)$$

where $Q_k(S_{-i}) = \sum_{j=1, j \neq i}^N \sum_{l=1, l \neq i}^N \omega_{jl}^k$ is a constant value and independent of S_i , i.e., when player i changes its strategy to T_i , $Q_k(S_{-i})$ remains unchanged. Therefore,

$$\begin{aligned} F_k(T_i, S_{-i}) &= \sum_{l=1}^N \omega'_{il}^k + \sum_{j=1}^N \omega'_{ji}^k + \sum_{j=1, j \neq i}^N \sum_{l=1, l \neq i}^N \omega_{jl}^k \\ &= U_{i,k}(T_i, S_{-i}) + Q_k(S_{-i}). \end{aligned} \quad (4.18)$$

Subtracting (4.17) from (4.18) implies (4.16), and hence, (4.8) follows. $\blacksquare$

As the proof made no assumption of the strategy space $\mathbb{S}$, eq. (4.8) must be valid for any (S_i, S_{-i}) and $(T_i, S_{-i}) \in \mathbb{S}' \subset \mathbb{S}$. At the same time, the constraint in power plays no part in the proof and should not affect the analysis. The following corollaries are direct consequences, according to Theorem A.3 (see Appendix A.2).

Corollary 4.1. *The game $\mathcal{G}_2$ and every similar game whose strategy space is a subset of $\mathbb{S}$, including $\mathcal{G}_1$, are exact potential games.*

Corollary 4.2. *For such games, pure-strategy NE exists and can be obtained through*

a sequential best/better-response dynamic.

4.3.3 Domination in the Strategy Set

A careful examination reveals that the proposed game also exhibits dominated strategies, i.e., a strategy that always yields less utility than another for a player regardless of what its opponents' strategies are, as defined in Appendix A.1.1. This leads to the following theorem.

Theorem 4.2. *Given a strategy S_i of player i . Derive a new strategy T_i by replacing an element with value 1 in S_i by 0 (assuming T_i is valid). Provided that all other parameters remain the same, S_i is always (at least weakly) dominated by T_i for every player i , i.e.,*

$$U_i(T_i, S_{-i}) \geq U_i(S_i, S_{-i}), \quad \forall i \in \mathcal{N}. \quad (4.19)$$

Proof: Assume that $a_{im} = 1$ is replaced by $a'_{im} = 0$. Clearly, U_i can be rewritten into

$$\begin{aligned} U_i(S_i, S_{-i}) &= \left(\sum_{j=1}^N \omega_{ji}^m + \sum_{j=1}^N \omega_{ij}^m \right) + \sum_{k=1, k \neq m}^K \left(\sum_{j=1}^N \omega_{ji}^k + \sum_{j=1}^N \omega_{ij}^k \right) \\ &= U_{i,m}(S_i, S_{-i}) + W \end{aligned} \quad (4.20)$$

where $W = \sum_{k=1, k \neq m}^K \left(\sum_{j=1}^N \omega_{ji}^k + \sum_{j=1}^N \omega_{ij}^k \right)$ is a constant and $U_{i,m}(S_i, S_{-i}) \leq 0$.

With $a'_{im} = 0$, $\omega_{ij}^m = \omega_{ji}^m = 0$ for all i and j . Thus, $U_i(T_i, S_{-i}) = W$. Clearly, $U_i(S_i, S_{-i}) \leq U_i(T_i, S_{-i})$ and the theorem is proven. ■

The weakly dominated case corresponds to when player i is the only one using the subcarrier. Under this circumstance, when it stops using the subcarrier, it will not affect the rest of the players and their utility functions remain unaltered.

However, for most of the time, the domination will be strict. Basically, it means that players will favor a strategy which yields less CCI to obtain a less negative utility value. That is, on subcarrier k , there are other co-channel players besides player i

and by not transmitting, player i receives and generates less CCI. Following the same argument, players thus have a tendency to remove subcarriers. They will keep on doing so until each of them uses only one subcarrier, at which point the game $\mathcal{G}_2$ is reduced to $\mathcal{G}_1$. In that case, the following corollary is a direct implication.

Corollary 4.3. *In the strategy set of player i , the subset $\Phi_i = \{\mathbf{a}_i^T | \mathbf{a}_i^T \mathbf{1} = 1\}$ consists of dominant strategies. That is, transmitting on a single subcarrier is a dominant strategy.*

In defining the subset of dominant strategies above, it is understood that corresponding to each strategy S_i in Φ_i are a group of strategies dominated by S_i in the complement $\mathbf{S}_i \setminus \Phi_i$, but not all of them. For example, if $S_i = [1 \ 0 \ 0 \ 0]$, then $[1 \ 0 \ 1 \ 0]$, $[1 \ 1 \ 0 \ 1]$ and so on are dominated by S_i , but $[0 \ 1 \ 1 \ 0]$ is not dominated by S_i in the sense of Theorem 4.2. The domination in strategies is against the original objective for each player to have a sufficient number of subcarriers to transmit, so as to improve the overall spectrum usage. Hence, it is proposed that the domination in strategy for the game should be canceled by allowing a player to request in advance how many subcarriers for which it is going to compete. The exact number of subcarriers for each player will be determined via a proposed method in Section 4.4. Under this condition, the original game can be modified accordingly.

4.3.4 The Modified Game

In the modified game, the previous parameters remain the same except that each player requests for a fixed number of subcarriers K_i , where $1 \leq K_i \leq K$. For player i , an eligible strategy is S_i , drawn from $\mathbf{S}'_i = \{\mathbf{a}_i^T | \mathbf{a}_i^T \mathbf{1} = K_i\}$, which is a subset of $\{0, 1\}^K \setminus \{\mathbf{0}\}$. The overall strategy space is thus $\mathbf{S}' = \mathbf{S}'_1 \times \dots \times \mathbf{S}'_N$. The modified game is denoted as

$$\mathcal{G}'_2 = [\mathcal{N}, \mathbf{S}', \{U_i\}_{i \in \mathcal{N}}]. \quad (4.21)$$

Mathematically, the previous optimization problem can also be restated as

$$\begin{aligned} & \max_{\mathbf{a}_i^T} U_i, \quad \forall i \in \mathcal{N} \\ & \text{s.t.} \quad \begin{cases} \mathbf{a}_i^T \in \{0, 1\}^K \setminus \{\mathbf{0}\}, & \mathbf{a}_i^T \mathbf{1} = K_i, \\ \mathbf{p}_i^T(\mathbf{A}) \geq 0, & \mathbf{p}_i^T \mathbf{1} \leq P_{\max}. \end{cases} \end{aligned} \quad (4.22)$$

Remark 4.1. *The following observations can be made immediately from the formulation of $\mathcal{G}'_2$:*

- (i) *Since $\mathbb{S}' \subset \mathbb{S}$, by Corollaries 4.1 and 4.2, $\mathcal{G}'_2$ is also an exact potential game which has the desirable equilibrium properties.*
- (ii) *Moreover, in $\mathcal{G}'_2$, strategy domination in the sense of Theorem 4.2 does not exist.*

4.4 Allocation of the Number of Subcarriers

From the previous analysis, an important implication is that every player i will compete with a fixed number of subcarriers K_i to avoid the unnecessary strategy elimination. This fact gives rise to the need for a method to decide the number of subcarriers for the players before the actual gameplay. Thus, a distributed scheme is proposed where a player can use both its own QoS requirement, in terms of an expected data rate, and the locally available information about its neighbors in the network to autonomously decide the number of subcarriers. The method is referred to as the “Autonomous Number of Subcarrier Selection” (ANSS) scheme, which is described by Algorithm 4.1.

The ANSS scheme essentially defines a network etiquette by which at the beginning, players entering the network should go through some interactions prior to the actual gameplay. This allows them to locally explore the network, acquire and exchange information, thus enforcing certain degrees of “self-awareness” among the

Algorithm 4.1 The ANSS scheme.

-
- 1: I. ASSUMPTION:
 - 2: Player i determines its interference radius r_i according to (4.7).
 - 3: Player i 's requested number of subcarriers is $\lceil R_{i,req}/R_{sub} \rceil$.
 - 4: II. SELECTION STEP:
 - 5: **for** player $i = 1 \rightarrow N$ **do**
 - 6: Upon entering the network, broadcast a signal of identical power on the control
 - 7: channel.
 - 8: The receiver listens and detects the number of transmitters within its radius
 - 9: of interference.
 - 10: Identify the number of conflicting neighbors $\tau_i = \sum_{j=1}^N C_{ij}$.
 - 11: Thus, determine the number of subcarriers according to (4.23).
 - 12: **end for**
-

players. This approach can also be easily utilized to monitor new players who are entering the networks. If the listening process lasts for an ample period of time, a player will gather sufficient information to determine how many conflicting neighbors within its radius of interference, which for player i is given by the sum of elements of the i^{th} row in the conflict matrix $\mathbf{C}$, i.e., $\tau_i = \sum_{j=1}^N C_{ij}$.

The assumptions of ANSS and the selection rule given by eq. (4.23) below are further explained as follows. It is assumed that every player i may request a certain data rate $R_{i,req}$ bps. Although failing to satisfy this requested rate is undesirable, the type of data service does not require the players to strictly fulfill the rate requirement. Depending on the OFDM system design parameters, one can assume that each subcarrier can support a rate of R_{sub} bps. Hence, it is natural that player i should request a total of $\lceil R_{i,req}/R_{sub} \rceil$ subcarriers. Hence, it is proposed that a player i can determine its number of subcarriers as follows:

$$K_i = \max \left[1, \min \left(\left\lfloor \frac{K}{1 + \tau_i} \right\rfloor, \left\lceil \frac{R_{i,req}}{R_{sub}} \right\rceil \right) \right]. \quad (4.23)$$

Eq. (4.23) considers the subcarrier allocation problem from both the player and the network perspective. On the one hand, based on the environment information obtained, it is likely that player i should be sharing the K available subcarriers with

τ_i other conflicting neighbors. Apparently, among these τ_i neighbors there may exist non-conflicting pairs of players. However, due to incomplete information (player i not aware of the complete conflict matrix except its own row), and taking into account the fairness issue, the best action of that player is to take an equal proportion of the spectrum resource, i.e., $\lfloor K/(1 + \tau_i) \rfloor$. This way, to some extent, it also helps minimizing its own CCI imposed on other players by not making excessive demand for subcarriers. On the one hand, it gives a player at least one subcarrier in case the pair is in a very crowded neighborhood, thus maintaining fairness. Furthermore, if the pair is in a very favorable situation with no significant co-channel interferers, it gives the player only a sufficient number of subcarriers rather than all the available subcarriers (i.e., when $\tau_i = 0$, although $\lfloor K/(1 + \tau_i) \rfloor = K$, only $\lceil R_{i,req}/R_{sub} \rceil$ subcarriers will be allocated). This provides some flexibility for the player to select the good subcarriers and discard those with bad channel conditions, and hence achieve better energy efficiency while keeping the spectrum utilization high.

4.5 Game Algorithm

In this section, the algorithm for allocating transmission power and achieving the NE point is described.

4.5.1 Power Mechanism

In OFDMA systems, waterfilling and its variations [37, 38] are often used for power allocation in order to optimize transmission capacity. However, when capacity-based power allocation is not the main concern and the key issue is CCI mitigation [53, 96], the common practice is to achieve a target transmission quality (e.g., target SINR) at minimal power. Similarly, in the proposed game where CCI minimization is the primary objective, a simpler power mechanism similar to Scheme 1 of the previous chapter can be employed which emphasizes more on fairness.

Consider player i and assume that the channel gain between its transmitter and receiver (g_{ii}^k) is known. During the pre-gameplay period (ANSS), the receiver has learned about its environment, measured the average amount of CCI from the surrounding players, and fed the information back to its transmitter. Player i who is aware of the expected amount of CCI previously would then set an interference margin μ_i to counter-measure this. The purpose of μ_i is to account for CCI. For player i to achieve a target SINR γ_i^* in an AWGN channel, it needs to transmit at a higher power to combat CCI in order to maintain the receiver performance. Real-time monitoring of CCI can be challenging and increases the implementation complexity. Therefore, the interference margin μ_i is used to account for the expected amount of CCI present at the receiver, the value of which can be estimated by the average amount of CCI seen at the receiver during the gameplay. The shortcoming of this is that the target SINR will not be strictly guaranteed after the gameplay, but it will be shown later via simulations that adequate SINR performance can still be obtained.

As such, if player i has a target SINR γ_i^* on each subcarrier, based on the channel gains (g_{ii}^k), the noise floor σ^2 and the margin μ_i it will be able to estimate the power level p_{ik} required to achieve this target. Furthermore, if the player uses multiple subcarriers, the same power budget will be distributed among K_i subcarriers and we can set maximum power per subcarrier at $P_{\max}/K_i$. The power level estimated in this way will be used during the gameplay. Such scheme reduces the complexity at the expense of slight derivation in SINR from the desired threshold. Specifically, assume player i uses subcarrier k then its transmitted power is estimated by

$$p_{ik(\text{dBm})} = \min \left\{ \sigma_{(\text{dBm})}^2 + \gamma_{i(\text{dB})}^* + \mu_{i(\text{dB})} - g_{ii(\text{dB})}^k, (P_{\max}/K_i)_{(\text{dBm})} \right\}. \quad (4.24)$$

Remark 4.2. *Eq. (4.24) ensures that there is a one-to-one correspondence from each subcarrier assignment matrix to a power matrix, i.e., $\mathbf{P} = \mathbf{P}(\mathbf{A})$. As such, subcarrier and power are two equivalent game strategies in this study.*

Algorithm 4.2 Best-response algorithm for the distributed OFDMA game.

```

1: I. INITIALIZATION STEP:
2: for player  $i = 1 \rightarrow N$  do
3:   Assign a random strategy  $S_i \in \{\mathbf{a}_i^T | \mathbf{a}_i^T \mathbf{1} = K_i\}$ .
4:   if  $a_{ik} = 1$  then set  $p_{ik}$  according to (4.24)
5:   else  $p_{ik} \leftarrow 0$ 
6:   end if
7: end for
8: II. ITERATION STEP:
9: while NE has not been reached do
10:  for player  $i$  in a predetermined sequence do
11:    for subcarrier  $k = 1 \rightarrow K$  do
12:      The receiver measures the noise and CCI, then feeds those values back
13:      to the transmitter.
14:    end for
15:    for subcarrier combination  $S_i \in \{\mathbf{a}_i^T | \mathbf{a}_i^T \mathbf{1} = K_i\}$  do
16:      Use the power mechanism (4.24) to estimate the utility function (4.12).
17:    end for
18:    Decide the best-response strategy  $T_i$  according to (4.25).
19:    Update the new subcarrier and power vectors  $\mathbf{a}_i^T$  and  $\mathbf{p}_i^T$ .
20:  end for
21: end while

```

4.5.2 Sequential Best-response Play

As stated in Corollary 4.2, NE convergence for a potential game is ensured and can be obtained through a sequential best-response dynamic among the players. Let T_i be the new strategy of player i obtained via the best-response dynamic, defined by eq. (A.8), which is restated here for the current game as follows

$$T_i = \arg \max_{\mathbf{a}_i^T \in \{\mathbf{a}_i^T | \mathbf{a}_i^T \mathbf{1} = K_i\}} U_i(T_i, S_{-i}). \quad (4.25)$$

Algorithm 4.2 summarizes the details of the sequential iterative best-response scheme. The game is initialized with a random subcarrier assignment for each player. This is likely to be a nonequilibrium state and the first player will take action to try to improve its utility value by looking for its best-response strategy after observing the opponents' actions. After one active player has made the decision, the power matrix and assignment matrix are updated. The next player will become active and

repeat the process to obtain its best response. The iteration continues until a NE is reached. Similar to Chapter 3, the sequential order by which players take turn can be determined by the order of network entry, during the ANSS stage.

The total search space for player i has a total of $\frac{K!}{K_i!(K-K_i)!}$ possibilities. The algorithm complexity is therefore $O(INL)$ where N is the number of players, I is the expected number of rounds until convergence (one round ends when all players have acted), and $L = \frac{1}{N} \sum_{i=1}^N \left(\frac{K!}{K_i!(K-K_i)!} \right)$ is the mean number of strategy combinations over all players.

4.6 Optimality Studies via Price of Anarchy

In this OFDMA potential game, a pure-strategy NE can always be obtained via best response, which is computationally tractable. However, a typical characteristic of games with large, discrete strategy spaces is that multiple NEs can exist and there can be vast differences between two different NEs in terms of performance. At the same time, it is well-known in game theory that NE is inefficient, compared to the social Pareto optimum point. This social optimum point can always be found using a centralized, exhaustive search method. However, it is known that such an approach is NP-hard. Therefore, it is important to assess the degree of optimality of the NE allocation point obtained via the best-response method with respect to the social optimum. The technique of PoA can provide us a tool to study this issue.

The concept of PoA is formally introduced in Appendix A.1.7. Briefly, a performance measure (i.e., a real number) is assigned to each outcome of the game, including the NEs as well as the social optimum. A common measure is the utilitarian welfare function $\Theta(S)$, which is defined to be the sum of all individual utility functions in (A.10). Denote $\hat{S}$ the social optimum point. Then, the PoA of strategy S is defined in (A.11) to be the ratio between $\Theta(S)$ and $\Theta(\hat{S})$ or vice versa, depending on whether the game maximizes or minimizes players' utility functions.

Based on the definition and convention for PoA, when applied to the proposed game, it is noticed that each player maximizes the negative interference sum in the utility functions, which is equivalent to minimization of a positive interference sum. Thus, mathematically, the PoA for strategy S will be given by

$$PoA(S) = \frac{\Theta(S)}{\Theta(\hat{S})}. \quad (4.26)$$

Moreover, since the exact potential function $F(S)$ in (4.13) is exactly half the sum of all utility functions, it can be used equivalently as a scaled version of the utilitarian welfare function of the game. As such, one can now formally define

$$\Theta(S) \triangleq F(S) = \frac{1}{2} \sum_{i=1}^N U_i(S). \quad (4.27)$$

As $\Theta(S)$ refers to a negative interference sum, $\Theta(\hat{S})$ will have the least absolute value, so $PoA(S) \geq 1, \forall S$. A quantity of interest is the worst-case PoA corresponding to the worst NE $S_{wc} = \arg \min_{S \in \mathbb{E}} \Theta(S)$. Here, $\mathbb{E} \subset \mathbb{S}$ denotes the set of NEs. S_{wc} gives the performance lower bound for $\mathcal{G}_2$. In the next lemma, the performance upper bound is discussed.

Lemma 4.1. *The social optimum point of $\Theta(S)$ for the proposed potential game coincides with the best NE, i.e., $\hat{S} = \arg \max_{S \in \mathbb{E}} \Theta(S)$. Thus, it is the upper bound for the achievable performance of the best-response algorithm.*

Proof: In a potential game, any NE corresponds exactly to a (local) maximum of the exact potential function $F(S)$ [57]. Therefore, the social optimum given by $\hat{S} = \arg \max_{S \in \mathbb{S}} \Theta(S)$, which is the global optimum of $\Theta(S)$ and $F(S)$, must be one NE (the best NE) of $\mathcal{G}_3$, i.e., $\hat{S} \in \mathbb{E}$. As such, $\hat{S}$ is reachable via the best-response dynamics and is the upper performance bound of this algorithm. ■

Table 4.1: Simulation settings for the distributed OFDMA system.

System parameters	Values
Number of players, N	20
Number of subcarriers, K	5
Cell radius, r	1 km
Path loss exponent, λ	3
Noise power level, σ^2	10^{-13} W
Bandwidth per subcarrier, B	10 kHz
Maximum power per user, $P_{\max}$	20 mW
Modulation scheme	16-QAM
Supported OFDM symbol rate	10 kSymbols/sec
BER requirement, P_e	10^{-5}
Target SINRs per subcarrier, $\gamma_i^*, \forall i$	13 dB
Interference margins, $\mu_i, \forall i$	3 dB

4.7 Simulation Results

Extensive computer simulation was carried out to study the system and the proposed approach using MATLAB. Firstly, the convergence property of the potential game will be verified. Secondly, in order to investigate the details of OFDMA resource allocation, the results of a particular one-shot are documented, as well as statistical results from long-run simulation in comparison with a few other schemes, which give more insights into the performance of the system. Numerical evaluation of PoA will also be carried out to estimate the optimality degree of the proposed algorithm. Furthermore, the effect of the number of allocated subcarrier K_i on the game's performance is also studied.

In the entire simulation, the parameters were set according to Table 4.1. In particular, the target SINR per subcarrier was $\gamma_i^* = 13$ dB for every player, which is necessary for 16-Quadrature amplitude modulation (QAM) to meet the BER requirement of 10^{-5} . In addition, all the links were assumed to undergo independently and identically distributed Rayleigh fading, as well as path loss. The OFDM subcarrier was designed to support a rate of 10 kSymbols/sec; thus, with 16-QAM, the bit rate supported is $R_{sub} = 40$ kbps. Moreover, the players' requested rates $R_{i,req}$

Fig. 4.4: Convergence of the game $\mathcal{G}'_2$ towards NE.

were assumed to be within 30 kbps and 150 kbps, following a truncated exponential distribution with mean at 60 kbps.

4.7.1 NE Convergence

To verify the convergence to a NE point of the best-response algorithm, a special case when all players occupied two subcarriers is demonstrated in Fig. 4.4. The following example is meant only for the purpose of graphical illustration. In an actual game, due to the ANSS scheme, the number of subcarriers varied from player to player, but NE would be obtained similarly and convergence properties would be maintained.

It is observed that the game quickly settled down to a NE solution. With $K_i = 2$, there were $L = 10$ possible valid strategies for each player to select from, each of which is labeled with a binary sequence on the vertical axis. At the same time, Fig. 4.5 shows the evolution of the exact potential function during the course of the example game. Each step on the horizontal axis refers to a decision made by a single player. Thus, $N (= 20)$ steps constitutes one round. As expected, a monotonically increasing

Fig. 4.5: Evolution of the exact potential function of $\mathcal{G}'_2$.

behavior until saturation can be noticed. Furthermore, in Fig. 4.4, it is seen that the number of rounds I was 4. It was observed that most of the actual simulations also quickly converged, where the expected value of the number of rounds I was only 4 or 5. Essentially, this illustrates three important facts: 1) existence of pure-strategy NE; 2) convergence of the sequential best-response dynamics; and 3) finite improvement paths with no improvement cycles. All of these exhibit the properties of a potential game.

4.7.2 Performance of a One-Shot Game

The performance of the system will be evaluated to assess the efficiency and fairness of the proposed game. The criteria for evaluation include power, SINR and capacity. Firstly, a one-shot is demonstrated. In the example, the nodes' distribution in the cell is depicted in Fig. 4.6. For viewing purpose, the players' indices are all labeled. Moreover, their radii of interferences which were determined according to (4.7) are also drawn. The players then followed the ANSS scheme and the best-response play

Fig. 4.6: Locations of the transmit-receive pairs for the one-shot.

Table 4.2: Allocation of subcarriers at equilibrium for the one-shot.

Subcarrier	Player(s)
1	1, 3, 9, 12, 15, 20
2	1, 2, 3, 5, 6, 12, 13, 14, 17
3	2, 5, 7, 10, 11, 12, 16, 20
4	2, 3, 5, 8, 10, 13, 14, 19
5	1, 4, 5, 11, 12, 13, 15, 16, 18

to successfully reach an NE point. Table 4.2 shows which players were occupying a certain subcarrier at NE.

Closer investigation shows that the algorithm yielded reasonable and desirable results. In Fig. 4.6, conflicts were observed among pairs 6, 7, 8, 9 and 11. They would likely cause heavy CCI if they transmitted on the same subcarrier. Nevertheless, the ANSS and iterative sequential play have allowed those players to intelligently avoid CCI by selecting non-overlapping subcarriers. This fact is evident in Table 4.2, which shows that none of the subcarriers was shared by more than one in the cluster. Notice that players 7 and 11 were an exception as they did not have conflict and could share subcarrier 3. For other conflicting clusters, heavy CCI was also avoided.

Fig. 4.7: Initial SINR seen from each subcarrier for the one-shot.

Next, in Fig. 4.7 and Fig. 4.8, both initial and final SINR values are displayed. The SINR values in dB are grouped according to the subcarrier, and plotted against the indices of the players occupying that subcarrier. It is clear that the final allocation point showed improvement in both spectrum efficiency and fairness. Spectrum efficiency was enhanced as most players achieved better SINR in all the subcarriers. Most significantly, in subcarrier 5, the average SINR was increased from 11.88 dB (see Fig. 4.7) to 15.50 dB (see Fig. 4.8). Furthermore, at NE, the variation in SINR of different players in the same subcarriers as well as the deviation from the SINR target of 13 dB had been decreased. Besides, no players had to suffer from severely low SINR (see Fig. 4.8).

The power and capacity performance obtained in Fig. 4.9 and Fig. 4.10 give more insights into the system performance. Compared to the initial power levels which were relatively high, the power consumption at NE was reduced considerably for several players and the whole system (see Fig. 4.9). Besides, no players had to use up their entire power budget $P_{\max}$. Hence, the higher efficiency in SINR performance

Fig. 4.8: Final SINR seen from each subcarrier for the one-shot.

could be achieved with lower power consumption, which reaffirms the efficiency of the algorithm. For the capacity performance shown in Fig. 4.10, the achievable capacities were compared with the desired (requested) capacities. For every player i , its achievable capacity is estimated by $\frac{1}{B} \sum_{k=1}^K a_{ik} R_{ik}$ with R_{ik} given in (4.3), while its requested capacity is $R_{i,req}/B$. It is observed that commonly the achievable capacity would satisfy the requested rate or was only below by a small margin. Exceptions occurred when the players suffered from excessive conflicts with other neighbors and could only use a smaller number of subcarriers than the requested one, as obtained by (4.23). Examples of these are players 8, 9, 19 and 20 which were in crowded neighborhoods (see Fig. 4.6) and thus had to give up their required rates. As CCI reduction is the primary objective, some players may need to sacrifice for the welfare of the entire network.

Fig. 4.9: Initial and final transmitted power for the one-shot.

Fig. 4.10: Players' achievable and desired capacities for the one-shot.

Fig. 4.11: Average power and capacity per user vs. number of subcarriers K_i .

4.7.3 Effects of the Number of Subcarriers

In this part of the simulation, the effects of K_i on the achievable capacity of the network were studied. To make the comparison compatible, it was assumed that every player had an identical number of subcarrier, i.e. $K_i = K_j, \forall i, j$ and the ANSS scheme was not in effect. For the same players' locations and channel conditions, the average achievable capacities per player of the game $\mathcal{G}'_2$ at NE were compared for the different number of subcarriers used by each player K_i . Note that if $K_i = 1$, the game $\mathcal{G}'_2$ reduces to $\mathcal{G}_1$. For fair comparison, the power mechanism was regulated so that for each K_i , all players consumed nearly the same amount of power. If one takes the single-subcarrier case as reference, then for K_i subcarriers, the expected SINR target per subcarrier should be reduced by K_i times. Specifically, $\gamma_{(dB)}^{*(K_i)} = \gamma_{(dB)}^* - 10 \log(K_i)$. All other parameters were the same as in Table 4.1. K_i took values from 1 to 5. Although only a small number of subcarriers is used in the graphical illustration, the algorithm is scalable to the size of network nodes and number of subcarriers.

Fig. 4.12: Value of $F(S)$ at NE vs. number of subcarriers K_i .

Fig. 4.11 plots the power usage as well as capacity per player at NE against K_i . First, it should be pointed out that with the previous regulation in transmission power, the variation in average power per player for different values of K_i was not significant. Despite that, the average capacity for per player in the system became better for higher K_i , implying that better efficiency can be achieved with more sharing of spectrum. The capacity curve became saturated since the higher K_i , the higher number of players occupying the same subcarrier and as a result, more CCI was introduced. This is also reflected in the change in the final values of the exact potential function at NE. Fig. 4.12 illustrates that the potential function F became more negative when K_i increased, since more interference terms were added to F . This is in accordance with Theorem 4.2, which predicts a lower potential value for more channel sharing. Nevertheless, in the proposed algorithm, the increase in CCI to a certain extent is still acceptable as long as improvement in average players' capacity can still be achieved.

Table 4.3: Comparison of distributed OFDMA schemes.

Schemes	Convergence	Fairness Index
Random Allocation	N.A.	0.693
Proposed Game	100%	0.840
SM/PP	94.06%	0.768
OFDMA/WF	0%	N.A.

4.7.4 Performance Evaluation in the Long Run

In addition to looking at one particular instance of the game, the system performance was also statistically evaluated over a long time horizon. This test case considered 5000 replications of the one-shot game, during which most of the global parameter settings were the same as before, except that the players' locations and channel conditions were randomly and independently generated for each replication. Furthermore, to benchmark the performance of the proposed algorithm, it was compared with a few existing RRM schemes, as shown in Table 4.3, including the *SINR Maximization with Power Pricing* (SM/PP) scheme [96], the *OFDMA Iterative Waterfilling* (OFDMA/WF) scheme (a commonly used method in OFDMA networks, partially adopted in [37] and [38]); as well as the *Random Allocation* scheme. In SM/PP [96], the utility function consists of a SINR reward and a power pricing term but does not possess the property of a potential game. In OFDMA/WF, the players also select subcarriers in a best-response manner and allocate power through the waterfilling algorithm to maximize their rates. Random allocation is an intuitive method where player i was randomly preassigned with K_i subcarriers to transmit regardless of the other players' actions.

The most important consideration for an allocation scheme is its ability to converge to a stable solution. From Table 4.3, the convergence probabilities for the schemes in comparison over the 5000 one-shot games can be seen. As expected, the proposed scheme converged for every one-shot game, as predicted by the property of potential games. The SM/PP scheme, despite not having 100% convergence, seemed to per-

form reasonably well as 94% of the one-shots converged. However, the OFDMA/WF scheme, which was employed in cellular OFDMA networks (e.g., [37] and [38]), did not stabilize in this distributed system. For the random allocation, testing for convergence is not necessary as the subcarrier assignment was preassigned.

Next, for the stable schemes (the proposed game, SM/PP and random allocation), the degree of fairness of the final spectrum allocation was compared. In order to obtain a measure of fairness, the test used the *fairness index* proposed by R. Jain *et al.* [66]. Considering an N -user system for which user i achieves a quantity of x_i under a particular resource allocation strategy, the fairness index for this allocation is defined as

$$\varphi = \frac{\left(\sum_{i=1}^N x_i\right)^2}{N \left(\sum_{i=1}^N x_i^2\right)}. \quad (4.28)$$

In the proposed game, this quantity is chosen to be the normalized rate/capacity achieved by the players. Normalized values are necessary as each player has a different requested rate $R_{i,req}$. At the final NE, the achievable rate of player i is given by $R_i = \sum_{k=1}^K R_{ik}$, and hence, the normalized rate is taken as $x_i = R_i/R_{i,req}$. Thus, for any one-shot game simulated, various fairness indices could be computed for different schemes. The mean values of the fairness indices over the 5000 replications are displayed in Table 4.3. It can now be seen that the proposed game obtained the highest degree of fairness over random allocation and SM/PP, with an index of 0.840. In practice, this implies that statistically, the proposed scheme let the players meet their requested rates better than the competing schemes. As the quantity x_i used in computing the Jain's index is a player's achievable rate normalized by its requested rate, if the index is closer to 1, then all players receive more even proportions of the resources and hence the values of x_i are closer to each other and to 1. Thus, players transmitting at higher than their requested rates can give up part of their capacities so that those transmitting at lower than their requested rates can improve their capacities. The reason for the proposed game's fairly high index despite a decentralized scheme can

Fig. 4.13: Empirical CDFs of SINR for multiple distributed OFDMA schemes.

be attributed to the utility function which allows for less selfish behaviors and more cooperation among competing players.

The empirical cumulative distribution functions (CDF) for the resulting players' SINR and capacities are also presented in Fig. 4.13 and Fig. 4.14 for the proposed, SM/PP and random allocation schemes. For SINR, the samples are the SINRs γ_{ik} for any player i over its used subcarrier k ; and for capacity, the samples are the individual total capacities $C_i = \sum_{k=1}^K C_{ik}$. The results show that the proposed game consistently maintained an advantage over the SM/PP scheme and outperformed the random allocation scheme in achieving both better SINR and capacities. From the CDF for SINR of the proposed game, it is noticed that $Prob[\gamma_{ik} < 13\text{dB}] \approx 0.3$. In other words, about 70% of the time, the achieved SINR for the players can satisfy the target SINR γ_i^* of 13 dB. In addition, based on this numerical studies, 70 - 75 % of the time, the requested rate can be adequately met by the achievable rate.

An interesting phenomenon could be noticed, which is the piecewise continuous

Fig. 4.14: Empirical CDFs of capacities for multiple distributed OFDMA schemes.

shapes of the CDFs for capacity. This is likely due to the differences in the number of subcarriers that the players were allocated, which is a discrete random variable. That is, if a player gets one subcarrier (i.e., its capacity consists of a single term C_{ik} only), then it is likely that its achievable capacity may fall into a certain region distinct from that of another player who gets two subcarriers (i.e., its capacity includes two terms $C_{jm} + C_{jn}$). In this sense, the first “segment” of the CDF curve corresponds to the capacities of those with one subcarrier, the second “segment” to those with two subcarriers, and so on. Therefore, the aggregate CDF might be seen as the sum of these individual curves.

4.7.5 PoA Evaluation

In order to examine the optimality degree of the proposed method, the PoA of the game was numerically computed during the simulations, which were done over 100 different randomized one-shot games using the global settings in Table 4.1. For each

Fig. 4.15: Comparison of PoA for the distributed OFDMA game.

one-shot game, the test searched for the set of NEs $\mathbb{E}$. The NE which gives the social optimum was then identified. On the other hand, an independent run of the proposed best-response sequential algorithm was performed, and the algorithm would converge to one of the NEs. For comparison purpose, a random allocation (as described in the previous section) of each one-shot game in question was also considered. Hence, for any one-shot game, different PoA metrics could be computed, including the PoA_{game} for the proposed game, PoA_{rand} for random allocation, and the PoA_{wc} which represents the price of anarchy for the *worst-case* NE. PoA_{wc} is an important performance metric which specifies a lower bound for the efficiency of a game-theoretical method.

In Fig. 4.15, the different PoAs and the social optimum PoA bound, which is always 1, are plotted for various one-shot games, sorted in an increasing order of PoA_{wc} to clearly indicate the performance bounds. As predicted by Lemma 4.1, the PoA_{game} graph is maximum bounded by the PoA_{wc} graph, and minimum bounded by 1. Moreover, it can also be observed that the optimality degree of the proposed

game was fairly acceptable as it was consistently close to the optimum bound, within the $10^0 - 10^1$ range, whereas the random allocation was generally worse, occasionally resulting in PoA of order 10^4 . In fact, numerical results showed that the mean value of PoA_{game} was about 1.65, and that of the worst case scenario was only 3.92, while on average the random allocation method had a mean PoA of around 10^3 . The results imply that the proposed game could reduce the interference power generated to the network by 10 - 20 dB over the random allocation and was only 2 - 4 dB behind the optimal solution in the worst case.

4.8 Concluding Remarks

This chapter studies the problem of designing game-theoretical approaches to RRM algorithms to achieve fair and efficient spectrum access for the distributed, ad-hoc OFDMA network of transmit-receive pairs with spatial frequency reuse. The problem applies well to the unlicensed spectrum access of the lower tier in the proposed spectrum access framework. The main contributions of this chapter include introducing a framework for distributed OFDMA spectrum sharing and CCI mitigation, formulating the problem into a potential game and analyzing its properties, implementing the iterative best-response algorithm to obtain NE solutions, and evaluating the system performance with extensive simulations. It is worth noting that the game exhibits strategy domination, which motivates the use of the ANSS scheme to deal with the allocation of number of subcarriers to players prior to the gameplay. Numerical results suggest that the scheme not only provides a stable solution to the OFDMA spectrum sharing system, but also improves the spectrum efficiency, and at the same time maintaining a reasonable degree of fairness and optimality. Compared to the non-cooperative formulation of the previous chapter, several issues have been overcome by the new approach.

- NE existence and convergence using best-response dynamics can always be guar-

anteed, which is a major advantage of the system when stable solutions are the main concern in any game-theoretical approaches.

- The use of PoA as an establishable performance bound provides a way to assess the efficiency of the NE point obtained by the best-response algorithm.
- The inclusion of the pre-play ANSS scheme could help users in the network to adapt themselves to the system environment (e.g., discovering conflicting neighbors) in order to successfully access the spectrum resource.

Although the game-theoretical solution proposed in this chapter is valid for a distributed OFDMA system, its methodology is general enough to be applied to several other systems, one of which is the infrastructure-based OFDMA networks, i.e., ones with existing infrastructures like BSs and access points. It is not difficult to visualize that if one BS serves only one MS, then this link is identical to a transmit-receive pair and the two problems are equivalent. However, for a BS serving multiple MSs, reformulation of the model in order to maintain the potential game properties is a complex, nontrivial problem. In the next chapter, the RRM issues for the multi-cell OFDMA system will be addressed.

Chapter 5

Potential Game Approach to Downlink Multi-cell OFDMA Networks

5.1 Overview

Unlike wireless ad-hoc networks, in an infrastructure-based wireless network, there exist infrastructures such as BSs, access points, switches, routers, etc. that provide wireless connectivities and spectrum access to mobile users. Over the past decades, cellular networks have emerged as a prominent type of infrastructure-based wireless network topologies where a large service area is divided into several smaller regions. Each region is called a cell, with a BS serving the MSs within the cell. OFDMA is one of the leading modulation and multiple access strategies for current and future cellular networks. The same set of frequency channels can be spatially reused by two cells if the resulting CCI levels are tolerable. In order to efficiently utilize the spectrum resources, the frequency channels and transmission power should be dynamically and adaptively allocated in such a way that minimizes CCI and fulfills the QoS performance of users in a single cell or in the entire system. A centralized RRM scheme for cellular OFDMA

(e.g., [36, 48–50]) must take into account the parameters of the entire system, where decisions are made at a central network controller, which is connected to each BS by wired, backhaul transmission. On the other hand, in distributed RRM schemes (e.g., [37, 38, 43–45, 52]), resources can be allocated on a per-BS basis, based on the performance of a particular cell. Distributed schemes might be less optimal compared to centralized schemes, but are less computationally complex. Centralized and distributed schemes can be also used together in multi-layered approaches [46, 47], where the network controller mitigates inter-cell interferences at a longer “super-frame” level while the BSs assist in decision-making by distributively assigning channels to MSs on a frame-by-frame basis. RRM schemes for cellular OFDMA networks have been studied to a considerable extent and comprise a rich body of works in the literature. Papers [33–36] were among the seminal works on OFDMA in the late 1990s and early 2000s, which formulated mathematical optimization problems in order to maximize the total rates or minimize the total power of the OFDMA system. Thereafter, a large number of research works have been proposed, employing analytical tools from various fields of sciences, including auction [49], graph-theoretical techniques [50], evolutionary algorithms [51], machine learning [52] and especially game-theoretical methods (e.g., [98, 99] and references therein). Comprehensive surveys of OFDMA RRM techniques for the interested readers can be found in [15] for earlier schemes, and [41] for more recent approaches.

Since the last decade, various approaches to cellular OFDMA resource allocation based on game theory have also been proposed by several authors. Some of the important works prior to 2010, which were extensively surveyed by the author in [97], can be loosely divided into *cooperative games* and *non-cooperative games*. For cooperative games, a notable work is [39], where the optimal rate allocation for multiple users was associated with the Nash bargaining solution; however, the drawback of the algorithm, which involves the Hungarian assignment method [106], was its high complexity degree. In Chee *et al.* [40], the Kalai-Smorodinsky bargaining solution was considered

and an algorithm of reduced complexity order was suggested. Ibing *et al.* [63] later made a comparison for four different fairness and bargaining schemes, i.e., the utilitarian, egalitarian, Nash and Kalai-Smorodinsky solutions. On the other hand, other authors also proposed non-cooperative game-theoretical approaches. For example, Z. Han *et al.* [37] devised a power-minimization game in which power was allocated via a dual waterfilling method, with further rate adaptation via a virtual referee game. Kwon *et al.* [38] considered rate maximization objectives with linear power pricing in their game, with a utility function comprised of weighted total throughputs and negative total power consumption. The same concept of power pricing was adopted by a few others, for example, by Wang *et al.* [61] for multi-cell OFDMA with a sigmoid-shaped reward function, or by Yu *et al.* [62] for the OFDMA-relay network. Capacity maximization was also studied by Liang *et al.* [45], where the authors introduced another dimension of integer bit-loading to the strategy set of the players.

One common strategy that the previous game-theoretical approaches for cellular OFDMA systems relied upon is the use of iterative, myopic gameplay in order to obtain convergence to a NE. As previously mentioned, such gameplay can often result in unstable, oscillatory behaviors [97], which is, at first glance, because of the non-existence of pure-strategy NEs. The underlying cause is due to the prevalence of CCI, which affects not only the stability, but also the fairness of the allocation, where the edge-users are likely to suffer more than the center-users. Meanwhile, in Chapter 4, a potential game formulation for the distributed, ad-hoc OFDMA scenario has been studied, which guarantees that stable NE solutions can be achieved via the use of best-response iterative dynamics. At the same time, the CCI minimization objectives exhibit a reasonable fairness index, implying the feasibility of such a direct approach.

Following up on the previous chapter, the current chapter further develops the potential game formulation for the multi-cell OFDMA systems, based on the earlier generalized interference-sum minimization objective (4.9). The resulting game will be labeled $\mathcal{G}_3$. Table 5.1 provides a description and comparison of the key potential

Table 5.1: Descriptions of $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_3$.

Game	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$
Player	Transmit-receive pairs (ad hoc).	Transmit-receive pairs (ad hoc).	Multiple BSs in downlink cellular OFDMA network.
Strategy	Each pair selects a single frequency channel.	Each pair selects a combination of subcarriers.	Each BS allocates to each of its MSs one subcarrier.
Utility Function	Interference sum (4.9).	Generalization of (4.9) for all used subcarriers.	Generalization of (4.9), to be formulated.
Potential Game	Yes.	Yes.	To be determined.

game candidates $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_3$. It will be proven in this chapter that $\mathcal{G}_3$ is indeed a potential game. The key contributions of this chapter are as follows.

- *Potential game formulation* for the downlink multi-cell OFDMA system. The mathematical analysis and proof of the potential game are provided.
- *Optimality analysis*. The performance bounds are again investigated using PoA.
- *Distributed algorithm to solve for NE*. Various aspects of the algorithm design are discussed, such as the classification of edge-users and center-users, the best-response and better-response dynamics, power mechanism and signaling issues.
- *Numerical results*. Simulations show that the proposed method could be a good alternative for cellular OFDMA systems due to its stability and fairness advantages.

In the following section, a system model for the multi-cell OFDMA networks under investigation will be described.

5.2 System Model

Consider the downlink of a multi-cell OFDMA network with N cells. Each cell has a BS at the center and several MSs randomly distributed around the cell area. For cell i , define Ω_i as the set of MSs belonging to this cell. The total number of MSs in the

Fig. 5.1: Example of a downlink cellular OFDMA network.

entire system is given by $M = \sum_{i=1}^N |\Omega_i|$, where $|\Omega_i|$ is the number of MSs of the set Ω_i . There are K available orthogonal subcarriers and each subcarrier has the same bandwidth, which is assumed smaller than the coherent bandwidth so that the links are subject to only flat fading. Interferences from adjacent subcarriers or adjacent symbols are assumed to be negligible. The channel conditions and locations of the MSs are further assumed to be static throughout the duration of a gameplay. Fig. 5.1 is an example of such an OFDMA network with $N = 3$, $M = 6$ and $K = 3$.

In the OFDMA downlink, each BS makes use of a subset of the available subcarrier pool and assigns one subcarrier to each of its MSs in a distributed manner. The cellular network has a reuse factor of 1. A subcarrier can be used by more than one BSs, if the resulting CCI is tolerable. Let us denote $\mathbf{A}$ the subcarrier assignment matrix, where $\mathbf{A} \in \{0, 1\}^{N \times M \times K}$, whose element a_{im}^k takes a value of 1 if BS i transmits to MS m ($m \in \Omega_i$) via subcarrier k , and 0 otherwise. Consequently, the following constraints apply to any BS i

$$\sum_{m \in \Omega_i} a_{im}^k \leq 1, \quad \forall k \quad (5.1a)$$

$$\sum_{k=1}^K a_{im}^k = \begin{cases} 1, & m \in \Omega_i, \\ 0, & m \notin \Omega_i. \end{cases} \quad (5.1b)$$

Eq. (5.1a) suggests that no subcarrier can be assigned to two MSs in the same cell, while (5.1b) means that each BS gives one subcarrier to a MS in its cell. From (5.1b), the overload of MSs in a cell occurs if $\exists i: |\Omega_i| > K$. In well-designed practical systems, this MS overload can be avoided by admission control mechanisms which limit the number of ongoing MSs in each cell. In this study, this specific scenario will be taken into account in Section 5.3.3. Otherwise, normally it is understood that $|\Omega_i| \leq K, \forall i$. As such, the system will always admit feasible solutions.

Next, let us denote the channel gain matrix $\mathbf{G} \in \mathbb{R}^{N \times M \times K}$, where g_{im}^k is the channel gain between BS i and MS m through subcarrier k . If $m \notin \Omega_i$, g_{im}^k is in fact the channel gain of the interference path from external BS i to MS m . On the other hand, the transmission power matrix is defined by $\mathbf{P} \in \mathbb{R}^{N \times K}$. Each element p_{ik} represents the transmitted power of BS i on subcarrier k . If a subcarrier is unused, the BS simply transmits 0. The $1 \times K$ i^{th} row vector $\mathbf{p}_i^T$ of $\mathbf{P}$ belongs entirely to BS i and has to satisfy the non-negative and the maximum constraints, given by

$$\mathbf{p}_i^T(\mathbf{A}) \succeq 0, \quad \mathbf{p}_i^T \mathbf{1} \leq P_{\max}, \quad \forall i \quad (5.2)$$

where $\mathbf{1}$ is the all-one $K \times 1$ column vector.

For the link between BS i and MS m on subcarrier k , its SINR is expressed as

$$\gamma_{im}^k = \frac{a_{im}^k p_{ik} g_{im}^k}{\sum_{j=1, j \neq i}^N a_{jm}^k p_{jk} g_{jm}^k + \sigma^2} \quad (5.3)$$

where σ^2 is the power of the receiver AWGN and is assumed to be identical for all

links. Consequently, the achievable capacity for this link in bps/Hz is given by

$$C_{im}^k = \log_2 \left(1 + \frac{\gamma_{im}^k}{\Gamma} \right). \quad (5.4)$$

Again, $\Gamma = -\ln(5P_e)/1.5$ is a function of the required BER P_e , often known as the SINR gap [65].

5.3 Potential Game Formulation and Analysis

5.3.1 Game Formulation

In the multi-cell OFDMA system, each of the N BSs will independently distribute subcarriers to its MSs in order to optimize the performance of the individual cell, which can be viewed as an N -player strategic-form game. One can denote the set of players $\mathcal{N} = \{1, 2, \dots, N\}$, which represents the N BSs. For BS i , its available strategy S_i is a feasible assignment of subcarriers to its MSs. The action of choosing subcarrier k can be symbolized by its index k , $1 \leq k \leq K$. Then, an eligible strategy S_i of BS i may be represented by a $1 \times |\Omega_i|$ vector

$$S_i = [k_1 \ k_2 \ \dots \ k_{|\Omega_i|}] \quad (5.5)$$

in which BS i has assigned subcarrier k_m to the m^{th} MS of cell i (not the m^{th} MS of the system). Referring to Fig. 5.1, the strategies taken by B1, B2 and B3 are $S_1 = [1 \ 2]$, $S_2 = [3 \ 2]$ and $S_3 = [1 \ 3]$, respectively. Let the strategy set $\mathbf{S}_i$ denote the set of all possible combinations that S_i can take. Moreover, a strategy profile S can be understood as the joint strategy of all players, i.e., $S = (S_i, S_{-i})$. The domain of S is called the strategy space, defined by $\mathbb{S} = \mathbf{S}_1 \times \dots \times \mathbf{S}_N$.

Now, every player's objective in the game is to maximize its own utility function $U_i(S) : \mathbb{S} \mapsto \mathbb{R}$. The original interference-sum utility function (4.9) for player i can

now be redefined, which incorporates the total CCI which BS i imposes on other MSs $m \notin \Omega_i$, as well as those which all MSs $m \in \Omega_i$ suffer as a result of external BSs. This sum can be written as

$$U_i(S) = - \left[\sum_{m \notin \Omega_i} I[i, m] + \sum_{m \in \Omega_i} \sum_{j=1, j \neq i}^N I[j, m] \right] \quad (5.6)$$

where $I[i, m]$ is the CCI caused by BS i on MS m . If the CCI is present, it will equal $p_{i\kappa(m)} g_{im}^{\kappa(m)}$, where $\kappa(m)$ is the subcarrier assigned to MS m . Since the assigned subcarrier is yet to be determined, the indicator variable δ_{im}^k is introduced such that

$$\delta_{im}^k = \begin{cases} 1 & \text{If BS } i \text{ interferes with MS } m \text{ } (m \notin \Omega_i) \text{ via subcarrier } k, \\ 0 & \text{Otherwise.} \end{cases} \quad (5.7)$$

Basically, δ_{im}^k indicates whether there is an interference path between BS i and MS m via subcarrier k . In particular, note that $\delta_{im}^k = 0$ if $m \in \Omega_i$ because the BS does not interfere its own MSs. The utility function can now be expressed as

$$U_i(S) = - \sum_{k=1}^K \left[\sum_{m \notin \Omega_i} \delta_{im}^k p_{ik} g_{im}^k + \sum_{m \in \Omega_i} \sum_{j=1}^N \delta_{jm}^k p_{jk} g_{jm}^k \right]. \quad (5.8)$$

The formulated game will therefore be denoted by

$$\mathcal{G}_3 = [\mathcal{N}, \mathbb{S}, \{U_i\}_{i \in \mathcal{N}}]. \quad (5.9)$$

Mathematically, the multi-cell OFDMA game can be described by the following distributed optimization problem applied to any player i , i.e.,

$$\max U_i \quad \text{s.t. (5.1a), (5.1b) and (5.2).} \quad (5.10)$$

In the next section, the proof that the multi-cell OFDMA game $\mathcal{G}_3$ given above is

an exact potential game will be derived.

5.3.2 Existence of the Exact Potential Function

To verify the potential game property of $\mathcal{G}_3$, the exact potential function $F(S)$ must be established such that the defining equation (4.8) for exact potential games is satisfied. In order to derive this function, a novel approach is employed where a fictional game $\mathcal{G}_4$ which is a dual to $\mathcal{G}_3$ is built, whose utility function will be defined so that it becomes a potential game. First, the concept of a “pseudo-player” is introduced as follows.

Definition 5.1. *In the multi-cell OFDMA system, a pseudo-player refers to any pairing of a BS with one of its MSs.*

In fact, a pseudo-player is a transmit-receive pair where the BS is the transmitter and the MS is the receiver. The number of pseudo-players is exactly M , the number of MSs. Hence, in Fig. 5.1, six pseudo-players are identified: B1-M1, B1-M2, B2-M3, B2-M4, B3-M5 and B3-M6.

The fictional game $\mathcal{G}_4$ is the one played by the M pseudo-players, i.e., the subcarrier allocation game among multiple transmit-receiver pairs. The set of players will be $\mathcal{M} = \{1, 2, \dots, M\}$. Note that even though a transmitter can be common to a few pseudo-players, they will be treated as different entities in this fictional game. The grouping of the MSs into N cells corresponds exactly to a disjoint partition of the player set $\mathcal{M} = \bigcup_{i=1}^N \Omega_i$. For each pseudo-player m , an eligible strategy is $S_m = \mathbf{a}_m^T$ which is a binary $1 \times K$ row vector serving as the subcarrier assignment indicator (when subcarrier k is used, its k^{th} element is 1). The strategy set of a pseudo-player is given by $\mathbf{S}'_m = \{0, 1\}^K \setminus \{\mathbf{0}\}$; and the strategy space is $\mathbb{S}' = \mathbf{S}'_1 \times \dots \times \mathbf{S}'_M$. The counterpart of the subcarrier assignment matrix in $\mathcal{G}_4$ is given by $\mathbf{A}' = [\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_M]^T$. The alternative channel gain matrix $\mathbf{H} \in \mathbb{R}^{M \times M \times K}$ and power matrix $\mathbf{Q} \in \mathbb{R}^{M \times K}$ can be defined in the same manner as $\mathbf{G}$ and $\mathbf{P}$ of $\mathcal{G}_3$. Furthermore, the cross-interference

indicators ϵ_{im}^k for pseudo-players are defined in the same manner as (5.7).

The utility function V_m of the pseudo-player m is modeled after the game $\mathcal{G}_2$ of Chapter 4, i.e.,

$$V_m = - \sum_{k=1}^K \left[\sum_{j=1}^M \epsilon_{mj}^k q_{mk} h_{mj}^k + \sum_{j=1}^M \epsilon_{jm}^k q_{jk} h_{jm}^k \right]. \quad (5.11)$$

Finally, the game $\mathcal{G}_4$ is fully defined as follows.

$$\mathcal{G}_4 = [\mathcal{M}, \mathbb{S}', \{V_m\}_{m \in \mathcal{M}}]. \quad (5.12)$$

Regarding the non-overlapping assignment of a single subcarrier to the MSs in the same cell, a joint strategy such that those two MSs get the same subcarriers, which is valid in $\mathcal{G}_4$ but not in $\mathcal{G}_3$, should be deemed infeasible for $\mathcal{G}_4$. Hence, the feasible strategy space $\tilde{\mathbb{S}}$ is defined as

$$\tilde{\mathbb{S}} = \{S \mid \forall i : \forall m, j \in \Omega_i, \mathbf{a}_m^T \text{ and } \mathbf{a}_j^T \text{ have a single 1 at different locations} \}. \quad (5.13)$$

Thus, $\mathcal{G}_4$ may be considered over the feasible strategy space $\tilde{\mathbb{S}}$.

Lemma 5.1. $\mathcal{G}_4$ is an exact potential game with the exact potential function $F(S) = \frac{1}{2} \sum_{m=1}^M V_m$.

Proof: $\mathcal{G}_4$ is the OFDMA game among transmit-receive pairs similar to $\mathcal{G}_2$ (See Table 5.1). Its strategy space, either $\mathbb{S}'$ or $\tilde{\mathbb{S}}$, is a nonempty subset of that of $\mathcal{G}_2$, so it is an exact potential game and admits the exact potential function $F(S)$, according to Corollary 4.1 from the previous chapter. ■

The next lemma features an important result which connects the two games $\mathcal{G}_3$ and $\mathcal{G}_4$.

Lemma 5.2. Within $\tilde{\mathbb{S}}$, the same subcarrier and power allocation profile for both $\mathcal{G}_3$ and $\mathcal{G}_4$ yields $\sum_{m \in \Omega_i} V_m = U_i$.

Proof: The two expressions will be equal as they refer to the same physical quantity, i.e., the total CCI. Mathematically, it can indeed be shown that they are equivalent.

In eq. (5.8), let $U_i = -\sum_{k=1}^K (X_k + Y_k)$ where $X_k = \sum_{j \notin \Omega_i} \delta_{ij}^k p_{ik} g_{ij}^k$ (index m has been changed to j without affecting the formula) and $Y_k = \sum_{m \in \Omega_i} \sum_{j=1}^N \delta_{jm}^k p_{jk} g_{jm}^k$.

For the term X_k , under the conditions stated in the lemma, for all k and j , it follows that $g_{ij}^k = h_{mj}^k$ and $p_{ik} = q_{mk}$, $\forall m \in \Omega_i$. Moreover, due to the restriction of non-overlapping subcarrier assignment in the same cell, the relationships between the δ -parameters and the ϵ -parameters are established as follows.

- If $\delta_{ij}^k = 1$, there exists only one $m \in \Omega_i$ that $\epsilon_{mj}^k = 1$.
- If $\delta_{ij}^k = 0$, then $\epsilon_{mj}^k = 0, \forall m \in \Omega_i$.

For any of the two cases above, the following identity holds

$$\delta_{ij}^k p_{ik} g_{ij}^k = \sum_{m \in \Omega_i} \epsilon_{mj}^k q_{mk} h_{mj}^k. \quad (5.14)$$

Thus,

$$X_k = \sum_{j \notin \Omega_i} \sum_{m \in \Omega_i} \epsilon_{mj}^k q_{mk} h_{mj}^k. \quad (5.15)$$

Besides, if both $m, j \in \Omega_i$, then $\epsilon_{mj}^k = 0, \forall k$. It allows us to add the dummy term $\sum_{j \in \Omega_i} \sum_{m \in \Omega_i} \epsilon_{mj}^k q_{mk} h_{mj}^k = 0$ to X_k , which yields

$$\begin{aligned} X_k &= \sum_{m \in \Omega_i} \sum_{j \in \Omega_i \cup (\mathcal{M} \setminus \Omega_i)} \epsilon_{mj}^k q_{mk} h_{mj}^k \\ &= \sum_{m \in \Omega_i} \sum_{j=1}^M \epsilon_{mj}^k q_{mk} h_{mj}^k. \end{aligned} \quad (5.16)$$

For the second term Y_k , similar to (5.14), it can be derived that

$$\delta_{jm}^k p_{jk} g_{jm}^k = \sum_{l \in \Omega_j} \epsilon_{lm}^k q_{lk} h_{lm}^k. \quad (5.17)$$

Hence,

$$\begin{aligned}
 Y_k &= \sum_{m \in \Omega_i} \sum_{j=1}^N \sum_{l \in \Omega_j} \epsilon_{lm}^k q_{lk} h_{lm}^k = \sum_{m \in \Omega_i} \sum_{l \in \bigcup_{j=1}^N \Omega_j} \epsilon_{lm}^k q_{lk} h_{lm}^k \\
 &= \sum_{m \in \Omega_i} \sum_{l=1}^M \epsilon_{lm}^k q_{lk} h_{lm}^k \\
 &= \sum_{m \in \Omega_i} \sum_{j=1}^M \epsilon_{jm}^k q_{jk} h_{jm}^k \quad (\text{index } j \text{ is used to replace } l). \tag{5.18}
 \end{aligned}$$

From (5.11), (5.16) and (5.18), it is clear that $U_i = \sum_{k=1}^K (X_k + Y_k) = \sum_{m \in \Omega_i} V_m$. ■

We are ready to prove the main result in the next theorem.

Theorem 5.1. $\mathcal{G}_3$ is an exact potential game with the exact potential function $F(S) = \frac{1}{2} \sum_{i=1}^N U_i$.

Proof: First, due to Lemma 5.2,

$$\frac{1}{2} \sum_{i=1}^N U_i = \frac{1}{2} \sum_{i=1}^N \sum_{m \in \Omega_i} V_m = \frac{1}{2} \sum_{m=1}^M V_m = F(S)$$

which is the same potential function given in Lemma 5.1. Now, suppose that an arbitrary BS i changes its strategy from $S_i = [k_1 \ k_2 \ \dots \ k_{|\Omega_i|}]$ to $T_i = [k'_1 \ k'_2 \ \dots \ k'_{|\Omega_i|}]$ where both (S_i, S_{-i}) and (T_i, S_{-i}) are in $\tilde{\mathcal{S}}$. One can now prove that (4.8) is true by introducing a strategy sequence $\{S_i^{(n)}\}$ as follows.

$$\left\{ \begin{array}{ll} S_i^{(0)} &= S_i, \\ S_i^{(1)} &= [k'_1 \ k_2 \ \dots \ k_{|\Omega_i|}], \\ &\vdots \\ S_i^{(n)} &= [k'_1 \ \dots \ k'_n \ k_{n+1} \ \dots \ k_{|\Omega_i|}], \\ &\vdots \\ S_i^{(|\Omega_i|)} &= T_i. \end{array} \right. \tag{5.19}$$

An intermediate strategy $S_i^{(n)}$ may correspond to an infeasible allocation in $\mathcal{G}_3$ where BS i assigns a subcarrier to more than one MSs (e.g., in multi-casting), as a new subcarrier index k'_j may overlap with an existing subcarrier index k_l . These are allowed as transitional strategies in the context of the proof but are not treated as the feasible actions in the game.

Nevertheless, $(S_i^{(n)}, S_{-i})$ corresponds to a valid strategy profile of the fictional game $\mathcal{G}_4$ in the general strategy space $\mathbb{S}'$. Each step from $S_i^{(n)}$ to $S_i^{(n+1)}$ corresponds to a change in strategy due to a *single* pseudo-player $m_n \in \Omega_i$, e.g., from (S_{m_n}, S_{-m_n}) to (T_{m_n}, S_{-m_n}) . As $\mathcal{G}_4$ is a potential game with potential function F , it follows that

$$\begin{aligned} F(S_i^{(n+1)}, S_{-i}) - F(S_i^{(n)}, S_{-i}) &= F(T_{m_n}, S_{-m_n}) - F(S_{m_n}, S_{-m_n}) \\ &= V'_{m_n} - V_{m_n} \end{aligned} \quad (5.20)$$

where V'_{m_n} is the utility function of pseudo-player m_n because of the strategy change. As a result,

$$\begin{aligned} F(T_i, S_{-i}) - F(S_i, S_{-i}) &= (F(T_i, S_{-i}) - F(S_i^{(|\Omega_i|-1)}, S_{-i})) + \dots + (F(S_i^{(2)}, S_{-i}) \\ &\quad - F(S_i^{(1)}, S_{-i})) + (F(S_i^{(1)}, S_{-i}) - F(S_i, S_{-i})) \\ &= (V'_{m_{|\Omega_i|}} - V_{m_{|\Omega_i|}}) + \dots + (V'_{m_2} - V_{m_2}) + (V'_{m_1} - V_{m_1}) \\ &= \sum_{m \in \Omega_i} V'_m - \sum_{m \in \Omega_i} V_m = U_i(T_i, S_{-i}) - U_i(S_i, S_{-i}). \end{aligned} \quad (5.21)$$

The final equality of (5.21) is a consequence of Lemma 5.2 and it requires only S_i and T_i to be feasible strategies. ■

5.3.3 A Special Case: Overload of MSs

The singular scenario where there can be more MSs than the total number of available subcarriers in one cell is addressed. Suppose for cell i , $|\Omega_i| > K$. It is then proposed that BS i continues to allocate one subcarrier to K MSs. For the other $|\Omega_i| - K$, the

BS employs the “no-transmission” strategy on them, leaving them unoccupied temporarily until the resource is released by other MSs. The no-transmission strategy has been introduced in Chapter 3, which allows the users to cease or delay transmissions if the conditions are unfavorable.

If no-transmission is represented by the index 0, then in (5.5), each k_j , $1 \leq j \leq |\Omega_i|$, can take values from 0 to K . This results in a new strategy space $\mathbb{T}$ and a modified game

$$\mathcal{G}'_3 = [\mathcal{N}, \mathbb{T}, \{U_i\}_{i \in \mathcal{N}}]. \quad (5.22)$$

Theorem 5.2. $\mathcal{G}'_3$ is also an exact potential game with the potential function $F(S)$.

Proof: It is observed that the only difference introduced by the addition of no-transmission is that the all-zero vector $\mathbf{a}_m^T = \mathbf{0}$ is now valid in the strategy set of a pseudo-player m in $\mathcal{G}_4$. However, such an inclusion does not affect the proof of Theorem 4.1 that $\mathcal{G}_2$ (and $\mathcal{G}_4$) are potential games, as the addition of terms of zero values does not affect the exact potential function. ■

5.4 Price of Anarchy Analysis

In this section, the PoA of the proposed game $\mathcal{G}_3$ is studied. Similar to the previous chapter, due to the interference minimization of the game and the definition of the exact potential function, the PoA for a particular strategy profile $PoA(S)$ can be defined according to (4.26), with the same utilitarian welfare function $\Theta(S)$ given by (4.27). The fact that the utilitarian welfare can be represented equivalently by the exact potential function $F(S)$ illustrates a convenient property of potential games, where each strategy profile has a latent “potential” equal to the value of the exact potential function taken at this profile, indicative of its relative strength.

Analogous to the previous chapter, similar performance bounds can be made regarding the PoA of the game $\mathcal{G}_3$.

Lemma 5.3. *The performance of the game $\mathcal{G}_3$ under sequential dynamics such as best/ better responses are upper bounded by the social optimum point of $\Theta(S)$ which is also the best NE, i.e., $\hat{S} = \arg \max_{S \in \mathbb{E}} \Theta(S)$; and lower bounded by the worst NE $S_{wc} = \arg \min_{S \in \mathbb{E}} \Theta(S)$, where $\mathbb{E}$ is the set of all pure-strategy NEs of $\mathcal{G}_3$.*

Proof: The proof is obtained by using the same argument as in Lemma 4.1. ■

The best/better-response dynamics can always converge to a NE for potential games, which is another convenient property of this class of games. In the next section, details of these NE convergence dynamics will be discussed, together with other design issues.

5.5 Distributed Algorithm for Nash Equilibrium

5.5.1 Edge-users versus Center-users

For cellular systems, the BS requires higher power to communicate with MSs at the cell edge than to those nearer to the center. Since all BSs tend to do so, CCI increases significantly and the throughputs of edge-MSs hardly improve, sometimes even worsen. A simple greedy throughput maximization approach will end up assigning all resources to the center-MSs, leaving very little to the edge-MSs, consequently resulting in a poor system fairness index. To overcome this issue, besides adopting interference avoidance, it is proposed that all the MSs be separated into two groups and subsequently allocated power based on such grouping, which has not been considered in some previous approaches in the literature (e.g., [37–39, 43]). Specifically, different weighing factors w_m are given to MSs belonging to different groups. Denote r and d_{im} the cell radius and the distance from MS m to its BS i , respectively. Then, a threshold distance $r_{th} < r$ can be defined, such that MS m is a center-user if $d_{im} \leq r_{th}$, and an edge-user if $r_{th} < d_{im} \leq R$.

5.5.2 Power Mechanism

In Chapter 4, when the main objective of the system is CCI mitigation, the previously used power mechanism which aims at achieving a target SINR at a minimal power consumption level and maintaining fairness has shown its feasibility without any significant loss in users' performance. Therefore, it will again be adapted in this current game with necessary adjustments to account for the weighing factors.

Consider the link between BS i and its own MS m . The BS computes the receiver sensitivity at the m^{th} MS based on a target SINR γ_m^* , an interference margin μ_m , and the weighing factors w_m , as follows

$$p_{m(dBm)}^{\min} = \sigma_{(dBm)}^2 + \mu_{m(dB)} + \gamma_{m(dB)}^* + w_{m(dB)}. \quad (5.23)$$

Then, the estimated power that BS i should use on subcarrier k assigned to MS m is the function of the MS index m , i.e.,

$$p_{ik}(m)_{(dBm)} = \max \{ p_{m(dBm)}^{\min} - g_{im(dB)}^k, P_{\max}/|\Omega_i|_{(dBm)} \} \quad (5.24)$$

where $P_{\max}/|\Omega_i|$ is present to ensure the power constraint. Note that in (5.24), in the special case when $|\Omega_i| > K$ for some i , $P_{\max}/|\Omega_i|$ is replaced by $P_{\max}/K$. Finally, eq. (5.24) ensures that there is a one-to-one correspondence from each strategy profile to a power matrix, i.e., $\mathbf{P} = \mathbf{P}(S)$. As such, subcarrier and power are two equivalent game strategies in this study.

5.5.3 Signaling Issues

It is worth mentioning that the feasibility of the utility function (5.8) requires complete channel state information. Therefore, the number of channel gains to be estimated, the amount of signaling traffic and information exchange are expected to be very high. It is also necessary to modify the existing protocol to support these estimation and

information exchange.

There is a need for each BS to know the amount of CCI it imposes on all the MSs in neighboring cells. For BS i and neighboring MS $m \in \Omega_j$, the MS needs to estimate the gains g_{im}^k . Assuming that channel conditions do not change significantly over the period of gameplay, the following method is suggested. That is, an out-of-band common pilot channel is employed using sufficiently high power to form a fully connected network to perform channel estimations and information exchange. Thus, a network protocol can be enforced such that, prior to the actual gameplay, BS i can send probing messages to external MSs in order for them to estimate their channel gains g_{im}^k to BS i . Each MS then takes turn to broadcast its estimate to BS i . Once BS i completes, it signals another BS and the process is repeated for all other BSs.

5.5.4 Iterative Convergence Dynamics

This section covers the iterative schemes that are used to obtain the NE. For potential games, there exists an improvement path of the potential function to reach a pure-strategy NE. In order to establish such a path, the game can be played sequentially, where each player employs actions such as best/better responses.

The best response is already employed for the games proposed in the previous chapters. For the current game $\mathcal{G}_3$, player i is to select its best-response strategy T_i if and only if

$$T_i = \arg \max_{S_i \in \mathbf{S}_i} U_i(S_i, S_{-i}). \quad (5.25)$$

The best-response dynamic requires an exhaustive search over the strategy set $\mathbf{S}_i$ for the best option. Since BS i serves $|\Omega_i|$ MSs, where each MS is allocated with one out of K subcarriers, the total number of possible strategies that belongs to player i amounts to ${}_K P_{|\Omega_i|} = \frac{K!}{(K-|\Omega_i|)!}$ options, which limits the scalability of the system. Thus, in this game, the algorithm also considers a method that required less searching, i.e., the better-response dynamic (A.9), whereby player i 's better-response strategy T_i is

Algorithm 5.1 Iterative gameplay for the multi-cell OFDMA game.

```

1: I. INITIALIZATION STEP:
2: for player  $i = 1 \rightarrow N$  do
3:   Select a random strategy  $S_i$  that satisfies (5.1a) and (5.1b).
4:   if  $a_{im}^k = 1$  then  $p_{ik} \leftarrow p_{ik}(m)$  according to (5.24).
5:   else if  $\sum_{m \in \Omega_i} a_{im}^k = 0$  then  $p_{ik} \leftarrow 0$ .
6:   end if
7: end for
8: II. ITERATION STEP:
9: while NE has not been reached do
10:  for player  $i$  in a predetermined sequence do
11:    for MS  $m = 1 \rightarrow |\Omega_i|$  do
12:      for subcarrier  $k = 1 \rightarrow K$  do
13:        Measure the noise and CCI, then feed those values back to the BS.
14:      end for
15:    end for
16:    for subcarrier combination  $S_i \in \mathbf{S}_i$  do
17:      Use the power mechanism (5.24) to estimate the utility function (5.8).
18:    end for
19:    Decide the new strategy  $T_i$  according to (5.25) or (5.26).
20:    Update the new subcarrier and power matrices  $\mathbf{A}$  and  $\mathbf{P}$ .
21:  end for
22: end while

```

any random strategy such that

$$U_i(T_i, S_{-i}) > U_i(S_i, S_{-i}). \quad (5.26)$$

Algorithm 5.1 shows the detailed steps. The multiple BSs should act in a sequential order, which can easily be determined at the system level during the network planning stage. This is different and easier to handle than in the ad-hoc context where players need to self-organize before each gameplay.

5.6 Simulation Results

Simulations were done to study the performance of the proposed solutions. The system parameters were specified in Table 5.2. The network consisted of $N = 7$ hexagonal cells with omnidirectional BSs and the layout is similar to Fig. 5.2. There were

Table 5.2: Simulation settings for the multi-cell OFDMA system.

System parameters	Values
Number of BSs, N	7
Total number of MSs, M	21
Number of subcarriers, K	5
Cell radius, r	100 m
Threshold radius, r_{th}	$2r/3$
Path loss exponent, λ	3
Noise power level, σ^2	10^{-13} W
Maximum power, P_{max}	40 mW
BER requirement, P_e	10^{-5}
Target SINRs per user per subcarrier, γ_m^* , $\forall m$	13 dB
Interference margins, μ_m , $\forall m$	3 dB
Weighing factor for edge-users, w_e	1
Weighing factor for center-users, w_c	2
Probability of a MS being an edge-user, P_{edge}	0.5

Fig. 5.2: The OFDMA cellular network in the one-shot.

Fig. 5.3: Convergence of the game $\mathcal{G}_3$ in (a) The potential function, and (b) Separate utility functions.

$M = 21$ MSs randomly distributed around the area competing for $K = 5$ subcarriers. On average, there were 3 MSs per cell. The threshold distance to classify MS in the cell edge and cell center was given by $r_{th} = 2r/3$. There was a 50% probability that a MS fell within the edge area. This study assigned the edge-MSs a smaller weight $w_e = 1$ than the center-MSs' $w_c = 2$ with the aim to limit CCI. Moreover, in the simulation, the system was not overloaded. The channels were subject to identically and independently distributed Rayleigh fading as well as path loss. The path loss exponent was 3.

5.6.1 Convergence of the Game

The convergence of the proposed algorithm was demonstrated via a one-shot simulation in Fig. 5.2. The indices of the BSs and MSs are indicated, with the edge-MSs depicted by stars and the center-MSs by circles. The better-response dynamic (5.26) was employed in the iterative algorithm. The game was played until a NE was found

as the convergent point of the dynamics. In each step which represents a player's decision, the values of the potential function as well as the values of the $N = 7$ utility functions were plotted in Fig. 5.3. As expected, the potential function increased monotonically and converged to a NE. Note that one's utility function improved within its turn by the same amount as the potential function according to the game's property (e.g., in Fig. 5.3(b), player 7 in steps 1, 8, and so on). It is also seen that for each player, only 5 rounds (i.e., 35 steps) were needed before convergence occurred.

5.6.2 Performance Evaluation of the One-Shot Game

The achieved SINR, power and capacity allocation for this particular one-shot were examined in this section. In Fig. 5.4, initial SINRs and final SINRs at NE in dB were displayed for the edge-MSs and center-MSs separately. Note that an improvement in SINR could be seen within both groups. In fact, the average SINR increased from 8.84 dB to 13.63 dB for the edge-MSs, and from 10.78 dB to 16.06 dB for the center-MSs. Furthermore, it is observed that at the NE, all the center-MSs and almost all the edge-MSs achieved SINR near their target SINR of 13 dB, except for example, MS 15 which was in a more crowded cell. The overall Jain fairness index of the system, as given by (4.28), with the quantity x_i represented by the individual SINR was estimated to be 0.937, which suggests a high degree of fairness in this approach.

In Fig. 5.5, the achievable power and capacity were compared from the BSs' perspective. Both showed a significant improvement at the NE over the initial point. Due to the interference minimization nature of the game, players showed the tendency to avoid overlapping subcarriers and preferred subcarriers that cost less power. Consequently, the total power and CCI were reduced, which led to improved throughputs. Statistically, the average power consumption of a BS was reduced from 22.9 mW initially to 14.0 mW at the NE. Moreover, the average capacity per BS (i.e., $\frac{1}{N} \sum_{i=1}^N C_i$ where $C_i = \sum_{m \in \Omega_i} C_{im}^{\kappa(m)}$) increased from 4.49 to 7.61 bps/Hz.

Fig. 5.4: Comparison of SINR for (a) Edge-MSs, and (b) Center-MSs.

Fig. 5.5: Comparison of (a) BS's total power, and (b) BS' total capacity.

Table 5.3: Comparison of multi-cell OFDMA schemes.

Schemes	Convergence	Iterations	Fairness	Energy Eff.
FA	N.A.	N.A.	0.7030	260.27
PG-better	100%	7.892	0.9072	583.63
PG-best	100%	3.604	0.9032	561.39
WF/PP	87.20%	27.89	0.2808	644.42

5.6.3 Performance Evaluation in the Long Run

In this section, the performance of the proposed game with the previous settings was studied over 500 independent one-shots, each with different MS locations and channel conditions. Four RRM methods were compared, namely, Fixed Allocation (FA), Potential Game with better response (PG-better), Potential Game with best response (PG-best), and Waterfilling with Power Pricing game [38] (WF/PP). In the FA scheme, subcarriers and power were randomly assigned to the MSs at the beginning without further adaptation. The power allocation given in (5.24) was also used for fair comparison. WF/PP was subject to the same power constraint as the PG schemes, and the subcarriers were assigned greedily to the MSs with highest throughputs and reallocated iteratively until convergence. Its utility function consists of the total transmission rate and a negative cost related to the power consumed [38]. The key performance indicators for comparison were shown in Table 5.3.

The convergence probability is a critical measure for any game theoretic approaches. Both PG schemes could guarantee pure-strategy NE convergence with a 100% convergence rate. WF/PP achieved a reasonably high convergence rate of 87.2%. However, the number of iterations (rounds) required for WF/PP before convergence was considerably higher than for PG. PG-best needed less rounds than PG-better; however, within a round, the searching required for PG-better was less due to the non-optimal selection of new strategy. Convergence and iteration counts did not apply to FA.

The other two indicators are the Jain fairness index (4.28), and the energy effi-

Fig. 5.6: Empirical CDFs of BS's achievable capacity.

ciency defined as the achieved capacity per unit power, i.e.,

$$\eta = \frac{\sum_{i=1}^N C_i}{\sum_{i=1}^N \mathbf{p}_i^T \mathbf{1}} \quad (\text{bps/Hz/W}). \quad (5.27)$$

In terms of fairness, PG-better and PG-best were both superior in the ranking, with fairness indices around 0.9. WF/PP returned a low fairness score, perhaps due to the greedy allocation which allowed some favorable MSs to hog the resources. Nevertheless, WF/PP obtained the best energy efficiency (644.42 bps/Hz/W), due to its throughput maximization nature. PG-better gave roughly the same energy efficiency as PG-best (583.63 vs. 561.39 bps/Hz/W); and both of them were significantly superior to FA (260.27 bps/Hz/W).

For this simulation studies, the empirical CDFs of the achievable capacity of BSs for the four schemes were also obtained. Fig. 5.6 shows that FA was consistently worst than the rest. WF/PP, however, obtained occasionally high capacity (over 15 bps/Hz, for about 6% of the time) for some BSs hogging the resources, while the rest

Fig. 5.7: Comparison of PoA for the multi-cell OFDMA game.

of the BSs had much lower throughputs (50% having capacity less than 5 bps/Hz) compared to PGs (only 15%). This partially explains its low fairness score. There was no significant performance difference in the long run between PG-best and PG-better.

5.6.4 Price of Anarchy Evaluation

In order to study the degree of optimality of the game, the PoA was numerically evaluated. In each of the 50 simulated one-shots, the set of NEs $\mathbb{E}$ was obtained. Independent runs of PG-best and PG-better will converge to one of the NEs. Hence, PoAs can be computed for the NEs achieved by both dynamics, i.e., PoA_{best} and PoA_{better} , for the worst-case NE, PoA_{wc} , and for the initial allocation point, PoA_{init} . The various PoAs (sorted in increasing order of PoA_{wc}) are plotted in Fig. 5.7. The social optimum bound (which is always 1) is also included. As explained before, PoA_{best} and PoA_{better} are bounded between PoA_{wc} and the optimum bound. Both PG-best and PG-better were able to reach the best NE once or twice, and none of

them was consistently better than the other. The mean values of PoA_{best} , PoA_{better} and PoA_{wc} were 1.385, 1.344 and 2.611, respectively. PoA_{init} was far behind at 30.898 on average. A relatively low PoA value suggests the proposed game has a fair degree of optimality.

5.6.5 Impacts of Weighing Factors

In this section, the effects of assigning different weights to the edge-MSs (w_e) and center-MSs (w_c) were examined. Three scenarios were considered: (a) Equal weights (w_e, w_c) = (1, 1), (b) Center-MSs had higher weights (w_e, w_c) = (1, 2) and (c) Center-MSs had lower weights (w_e, w_c) = (1, 0.5). Note that more power is consumed by the group with a larger weight. The performance was compared in terms of players' capacity and system fairness over 500 independent one-shots with the previous settings. In Fig. 5.8, the CDFs of BS achievable capacity for the three cases are shown. The obtained fairness indices for case (a), (b) and (c) were 0.9250, 0.9043 and 0.9531, respectively. As expected, there was a trade-off between fairness and throughput for the two opposite weighing schemes (b) and (c). The higher weight assigned to the center-MSs, the better the player's throughput, as shown by the gain in the CDF of case (b) over the other two. Case (c) resulted in the highest fairness index, although the differences were relatively small. Hence, different weight combinations can be flexibly assigned depending on whether the concerned objective is fairness or throughput.

5.6.6 System Performance with Increasing Loads

The impact of the number of MSs on the system performance was investigated for $N = 7$ cells and $K = 16$ subcarriers. A maximum power $P_{\max} = 0.1$ W was imposed on each BS, while the other parameters in Table 5.2 was maintained. For the PG scheme, best-response dynamic was used and the performance was compared with FA and WF/PP, in terms of the fairness index (Fig. 5.9(a)) and the average BS

Fig. 5.8: Comparison of various weighing factor combinations.

achievable capacity (Fig. 5.9(b)), for different loads. It was seen that the higher the system load was, the overall system fairness deteriorated for all schemes, probably due to the increasing CCI and MS density which further widened the performance gaps among different users. For PG, however, the decrease in fairness had a smaller impact as its score was consistently in the range of 0.9. WF/PP was still worst in terms of fairness due to reasons previously discussed, i.e., the performance gap between the few users hogging the subcarriers and the rest in WF/PP was emphasized and reduced its fairness index. However, because of the users with very high throughputs, WF/PP showed a gain in average BS achievable capacity over PG. The trade-off between fairness and throughput was expected of PG, whose throughput was still significantly better than FA.

Fig. 5.9: Impacts of the number of MSs on (a) Fairness and (b) Capacity.

5.7 Concluding Remarks

In this chapter, an OFDMA subcarrier assignment method is proposed using game theory. The derivation of the potential game is achieved via the concept of a fictional game involving the pseudo-players. This property also holds for an overloaded system if the no-transmission strategy is included for those MSs who are not able to get a free subcarrier. The system always admits a stable NE solution and can be played using best/better responses, both of which were studied and whose performances were compared. Simulation results indicate the validity of the interference minimization approach in combating CCI, achieving a convergent solution for the RRM problem and enhancing fairness among users, especially when the system load becomes high. Although throughput maximization is not the main objective, the proposed game is still able to maintain a fair degree of energy efficiency and optimality.

The method documented in this chapter applies well to a multi-cell OFDMA system, which corresponds to a scenario in the lower tier of the spectrum allocation

framework where a service provider who has gained the rights to exclusive spectrum usage is deploying a cellular network in order to serve its mobile subscribers. Due to the generality of the potential game formulation, although the proposed method is considered for a traditional cellular network, the results discussed in this chapter can also be valid for a much broader range of systems and for other different contexts. One of the important examples is the new femtocell network, which are small-range low-power BSs serving clusters of users. Nowadays, due to the growing wireless traffics, there can be increasingly many wireless hotspots in public areas like shopping centers, airports and train stations, within which there are many densely-placed femtocell configurations. Similarly, in the unlicensed bands, there are also situations where multiple clusters of users try to access the shared spectrum through their own network gateways; and the level of CCI needs to be controlled among these clusters. Each of the coexisting systems can be thought of as an independent player and the protocols described in the chapter can be used as a spectrum access mechanism.

Chapter 6

Dynamic Spectrum Allocation via Oligopolistic Market Competition

6.1 Overview

For the past few chapters, the capacity of game-theoretical methods has been exploited in order to devise spectrum access schemes from the users' point of view for given types of wireless systems. The key design aspect in these problems is to perform CCI mitigation in order to satisfy the QoS requirements of users. In the proposed spectrum access framework, this type of problems is related to the direct-access problems, as distinguished from the indirect type of spectrum access, i.e., *spectrum market*, which belongs to the upper tier of the proposed spectrum access framework. This problem deals with how the service or spectrum providers compete among themselves for the scarce spectrum resources available in the market for setting up their spectrum access networks. In this chapter, the thesis will focus on the design of schemes and analytical platforms for market-based spectrum allocation.

The motivations and ideas behind market-based spectrum allocation had been presented earlier in Chapter 2. Essentially, under the new DSA paradigm, the existing licensed primary spectrum holders (PSHs) are allowed to reallocate its excessive

spectrum by selling or leasing to the unlicensed secondary service providers (SSPs) in exchange for some forms of benefits, which at the same time helps to improve the overall spectrum efficiency. The benefits will most often be monetary gains, although there can be exceptions, like in [88] where the PSHs agreed to lease spectrum to the SSHs as long as these unlicensed operators were committed to cooperate with them in relaying their transmission. Nevertheless, the SSHs will most often be charged a price for the rights to use the licensed spectrum; and for such a spectrum access model, the establishment of a spectrum pricing model and market mechanism will be required. Therefore, micro-economic models, and especially game-theoretic techniques, can again be useful for the analysis of market-driven spectrum allocation.

A number of approaches to DSA have been proposed based on well-known economic models. Niyato *et al.* [73] examined a scenario where multiple SSPs and a PSH played a game subject to market rules under the Cournot model¹. The oligopoly market among multiple PSHs was studied via a dynamic Bertrand² game recently in [72] and [94]. Another classic economic model, the Stackelberg³ leader-follower game, was adopted by Wang *et al.* [83]. Auction-based spectrum sharing (e.g., [79, 89, 90] and references therein) is another notable economics-driven approach in which the resources are regarded as divisible goods which can be charged upon allocation via auction.

A crucial aspect of spectrum markets is how the spectrum can be priced by the competitive players. Three different pricing models, including market-equilibrium pricing, competitive pricing and cooperative pricing, were studied by Niyato *et al.* [71]. Ileri *et al.* [91] suggested that the players offered spectrum through demand-responsive pricing based on a user acceptance probability. Here, the authors also employed the

¹The Cournot competition is a classical economic model where firms compete to determine the quantities of products they should produce. It is named after French mathematician Antoine Augustin Cournot (1801-1877).

²The Bertrand competition is another classical economic model where firms compete to determine the prices for their products, as opposed to the Cournot model. It is named after French mathematician Joseph Louis François Bertrand (1822-1900).

³The Stackelberg leader-follower competition is a type of games in economics and game theory where one firm (the leader) first makes a move and another firm (the follower) subsequently makes the next move. It is named after German economist Heinrich Freiherr von Stackelberg (1905-1946).

concept of a spectrum server that mediates spectrum transactions. On the other hand, Xing *et al.* [92] investigated the price dynamics for two different buyer types, the quality-sensitive and the price-sensitive. Furthermore, Isiklar *et al.* [95] assumed that spectrum price and demand could be modeled based on the spatial distribution of the SSPs on a unit interval.

Most of the aforementioned schemes focused on the secondary markets among licensed and unlicensed users. In [87], Berry *et al.* discussed the possible emergence of a two-tier market structure, with the involvement of an upper-tier trading among spectrum owners to reallocate frequency resources. A relatively similar model was considered by Jia *et al.* [93], where the service providers would go through a spectrum acquisition process, prior to the actual duopoly competition to attract end users.

A particular issue with the aforementioned works is that they relied on discrete-time update rules to model the price dynamics. Although discrete-time processes can be easily implemented in real-time, they do not describe the exact time-varying behaviors of price change due to the dynamics of competition and interaction in the spectrum market. In order to characterize the dynamics in a deterministic manner, one may assume a continuous-time model where the market's movement is akin to a dynamical control system [24], with market price as the state variable, governed by a differential equation. The dynamic sticky-price oligopoly [109] in microeconomics is one of such models that could be useful in formulating the spectrum market and would be adopted in the thesis.

This chapter further investigates the spectrum allocation among licensed and unlicensed spectrum providers via market competition. The multiple PSHs are the players of the game and would like to reallocate portions of their excessive bandwidth to other SSPs by participating in the secondary market. Unlike other existing schemes where the players propose their own prices (e.g., [72,91,93,94]), an alternative market model is considered where the players compete non-cooperatively and simultaneously in terms of their output levels (i.e., offered bandwidth) under a common market price

moderated by the spectrum broker. Such a competition is believed to be a more suitable model for capacity-constrained players [107]. Moreover, the use of the dynamic sticky-price model helps to capture the deterministic exact continuous-time behaviors of the price dynamics, which is an advantage of such formulation compared to discrete-time counterparts. As such, the oligopolistic competition is an N -player differential game [20] and one can characterize the solution by the feedback Markov NE. In practice, such dynamics can also be approximated by a complementary discrete-time price adjustment scheme, which can be easily implemented at the spectrum broker.

In short, the key contributions of this chapter can be summarized as follows.

- The formulation of the market-based spectrum allocation as an oligopolistic differential game is presented. Moreover, the general outcomes for a two-player game are discussed and the Markov NE for the symmetric N -player scenario is derived. The equilibrium may lie in several regions and transitions may occur as time elapses, which will be thoroughly analyzed.
- Discrete-time price mechanism and protocol are proposed for the implementation of the game in practice. The conditions for the adjustment interval will be derived.
- Numerical studies are carried out to gain insight into the system behaviors. In particular, the impacts of parameters such as the number of players or maximum bandwidth are addressed.

6.2 Spectrum Market Formulation

First of all, let us look at the spectrum market model under investigation in this chapter, as depicted in Fig. 6.1. The system consists of N PSHs, which are currently deploying wireless services in the same geographical area, e.g., operators providing wireless access to a group of subscribers, which are referred to as primary spectrum

Fig. 6.1: Structure of the spectrum market.

usage as shown in Fig. 6.1. Since the entire frequency bands may not be fully utilized at a particular time and location, in order to increase spectrum utilization, the PSHs are encouraged to lease portions of their available bandwidth to other unlicensed SSPs. By entering this secondary spectrum market, the PSHs become oligopolists (i.e., sellers in a market dominated by a small number of firms) and compete among themselves to maximize monetary profits.

6.2.1 The Spectrum Broker

The system model assumes the presence of a spectrum broker, which has also been part of a few existing works, such as [87,95]. The advantages of having such an entity in the system are evident:

- The broker can act as a mediator between the PSHs and the SSPs, synchronizing

and coordinating the activities of the spectrum market.

- The broker can also announce the spectrum price to the players and adjust it over time such that the revenue can be maximized.
- Equivalently, it is a resource allocator which manages the spectrum assets via market-driven mechanism. Network protocols and algorithms can be executed at the spectrum broker.

The underlying principles of the Cournot oligopoly [107], i.e., the oligopolists producing homogeneous goods which are sold at a common market price, can be expected to hold for this model. This is due to the fact that the SSPs who provide their own wireless services might only be interested in the physical bandwidth, and the infrastructures of all the PSHs are assumed to be capable of meeting the SSPs' QoS requirements. Such equal preferences from the buyers (i.e., SSPs) among the spectrum offered by different PSHs have also been assumed by existing works, e.g., in [95]. Under these conditions, the SSPs will behave like price-takers who go along with the market price, and the market can always be cleared (i.e., selling out all offered bandwidth) by the spectrum broker. As such, the SSPs play a passive role in this model and can be collectively represented by the spectrum broker.

6.2.2 Dynamic Oligopoly

Next, the underlying mechanism for the model is further established. Under the previous conditions, one assumes that each PSH i has a limited amount of bandwidth B_i , from which a portion b_i can be offered to the secondary market, and $B_i - b_i$ units are allocated to primary services. A unit bandwidth is sold at market price p , which is monitored by the spectrum broker. Then, each oligopolist i competes in terms of its offered bandwidth or output b_i , which in turn controls the market price level, according to

$$p = f(b_1, b_2, \dots, b_N) \tag{6.1}$$

where the continuous, differentiable function f is called the *inverse demand function*. Player i is assumed to maximize the net profit $U_i = R_i - C_i$, where $R_i(p, b_i)$ and $C_i(p, b_i)$ are respectively its revenue and cost functions, dependent on both the market price as well as the player's own output, which will be defined later on.

The model described above is a simple static market mechanism. Static market model has its shortcomings as it does not address the dynamic process in which the market price changes, but only the outcomes before and after the changes. In reality, the price hardly changes abruptly, which necessitates the use of “stickiness” assumption in a dynamic oligopoly model [108]. In dynamic markets, the competition takes place over time. The system as a whole can be modeled as a *differential game* [20] where the time-dependent price $p(t)$ acts as a *state variable* and the time-and-state-dependent outputs $b_i(t, p)$ interact dynamically to control the movement of $p(t)$. In differential games, the general strategy $b_i(t, p)$ can be categorized into several types which may or may not be dependent of the current price or state variable (see Appendix A.3). From here onwards, we are only concerned with the *stationary Markov* strategy, given in Definition A.14, where b_i is solely a function of the current state, i.e., $b_i \equiv b_i(p(t))$.

At time t , the inverse demand equation (6.1) above leads to a desirable price level

$$\hat{p}(t) = f(b_1(p(t)), b_2(p(t)), \dots, b_N(p(t))). \quad (6.2)$$

However, this quantity is generally not equal to the current price $p(t)$. As a result, $p(t)$ adjusts towards $\hat{p}(t)$, not in an abrupt manner but gradually over time, in response to the difference $\Delta p(t) = \hat{p}(t) - p(t)$, following the *trajectory* equation

$$\dot{p}(t) = \frac{dp(t)}{dt} = \mathbf{G}(\Delta p(t)), \quad p(0) = p_0. \quad (6.3)$$

where $\mathbf{G}(\cdot)$ is a function of $\Delta p(t)$, which should be monotonically increasing and has

a zero at the origin [108], i.e., $\mathbf{G}(0) = 0$ and $d\mathbf{G}(u)/du > 0 \forall u$.

As such, a player's instantaneous utility function is $U_i(t, p(t), b_i(p(t)), b_{-i}(p(t)))$. However, its objective now is to maximize the accumulated payoff, discounted over time by a discount rate $r \geq 0$, i.e.,

$$J_i(p(t), b_i(p(t)), b_{-i}(p(t))) = \int_0^\infty e^{-rt} U_i(t, p(t), b_i(p(t)), b_{-i}(p(t))) dt. \quad (6.4)$$

6.2.3 Linear Constrained Sticky-price Oligopoly Model

A general framework for the dynamic oligopoly has been presented. Next, let us define the utility function and the actual price dynamic that will be used to study the dynamic spectrum market.

The utility function of each player should represent the satisfaction of a PSH in terms of monetary payoff, resulting from allocating its spectrum to both its primary services and the secondary market. As such, the following revenue and cost functions for player i are proposed, i.e.,

$$R_i = \underbrace{\pi_i(B_i - b_i(p(t)))}_{R_{i,1}} + \underbrace{p(t)b_i(p(t))}_{R_{i,2}} \quad (6.5a)$$

$$C_i = \alpha_i b_i(p(t)) + \beta_i b_i^2(p(t)). \quad (6.5b)$$

Here, the revenues of player i includes $R_{i,1}$ from the primary services and $R_{i,2}$ from the secondary market, since $B - b_i(p(t))$ and $b_i(p(t))$ are the amount of spectrum to be allocated to the primary services and the spectrum market, respectively. The existing primary subscribers pay a fixed fee, and one can assume that each PSH has a fixed number of subscribers, which translates into a constant return rate π_i dollars per unit spectrum allocated to the primary service. Note that $-\pi_i b_i(p(t))$ can also be regarded as the opportunity cost from not leasing the spectrum. On the other hand, the revenue from secondary market depends directly on the price $p(t)$.

Fig. 6.2: A quadratic profit function. Profit maximization occurs for output quantity at the saturation point, where marginal revenue (MR) equals marginal cost (MC).

The cost $\alpha_i b_i(p(t))$ accounts for the commission paid to the spectrum broker and miscellaneous cost, which are assumed to be linearly proportional to the amount of spectrum leased; while $\beta_i b_i^2(p(t))$ is proportional to the square of the primary service's bandwidth loss, which indicates that a larger penalty should be imposed to compensate for the performance degradation of the primary service. Here, α_i and β_i are positive constants. Quadratic cost has been adopted frequently in microeconomics, as well as in market-based DSA schemes for cognitive radio networks [71, 72], as one reason for adopting such cost in the utility function is because as a concave function, it can best represent the saturation of player's utility (see Fig. 6.2).

For the price dynamics, it is assumed that $\mathbf{G}(\cdot)$ in (6.3) is linear in $\Delta p(t)$, i.e.,

$$\mathbf{G}(\Delta p(t)) = k(\hat{p}(t) - p(t)) \quad (6.6)$$

where the constant $k > 0$ is called the price adjustment speed. The inverse demand function is a commonly adopted function in Cournot oligopoly, given by

$$\hat{p}(t) = f(b_1(p(t)), b_2(p(t)), \dots, b_N(p(t))) = a - \lambda \sum_{i=1}^N b_i(p(t)), \quad (6.7)$$

where constants $a, \lambda > 0$ are the intercept and slope of the inverse demand curve, respectively. Hence, the price trajectory is

$$\dot{p}(t) = k \left[a - \lambda \sum_{i=1}^N b_i(p(t)) - p(t) \right], \quad p(0) = p_0. \quad (6.8)$$

Note that the dynamic oligopoly is a variant of the sticky-price duopoly [108–110], which was used originally to study a market of two players. The model is also one of linear-quadratic control models [24], which are very important in control theory as well as economics. Its popularity is partially due to the tractability and uniqueness of analytical solutions.

Under this mathematical formulation, the PSHs are involved in a differential game. For each player i , the problem can be stated as the following optimization, i.e.,

$$\begin{aligned} & \max_{b_i} J_i(p(t), b_i(p(t)), b_{-i}(p(t))) \\ &= \int_0^\infty e^{-rt} [p(t)b_i(p(t)) + \pi_i(B_i - b_i(p(t))) - \alpha_i b_i(p(t)) - \beta_i b_i^2(p(t))] dt \\ & \text{s.t.} \begin{cases} \dot{p}(t) = k \left[a - \lambda \sum_{i=1}^N b_i(p(t)) - p(t) \right], & p(0) = p_0. \\ p(t) \geq 0, \quad 0 \leq b_i(p(t)) \leq B_i \quad \forall t, i. \end{cases} \end{aligned} \quad (6.9)$$

The maximum output constraints $b_i(p(t)) \in [0, B_i]$ are induced by the limited available bandwidth of the PSHs. Essentially, a differential game $\mathcal{G}$ has been formulated for the N PSHs over the infinite time horizon $[0, \infty)$. Differential games, which are introduced in details in Appendix A.3, are characterized by the existence of the state variables, which evolve dynamically and continuously over time following some differential equations. In $\mathcal{G}$, the state is the price $p(t)$ and the state trajectory is described by (6.8). The state space is the domain of $p(t)$, i.e., $\mathbb{X} = [0, \infty)$. The strategy space is given by the Cartesian product $\mathbb{S} = \times_{i=1}^N [0, B_i]$. Each player i maximizes its accumulated utility $J_i(p(t), b_i(p(t)), b_{-i}(p(t))) : \mathbb{X} \times \mathbb{S} \mapsto \mathbb{R}$.

For this differential game, the goal is to establish a Markov feedback NE, as defined in Definition A.15. In order to obtain this Markov NE of the game, one will need to solve the Hamilton-Jacobi-Bellman (HJB) equation (A.17) as given in Theorem A.4. In the subsequent sections, such solutions will be investigated in more details.

6.3 Outcomes of Dynamic Duopoly

First of all, let us briefly discuss the possible outcomes of the formulated constrained differential game in the simple case when there are only two players. When $N = 2$, the game becomes a dynamic duopoly. The dynamic sticky-price duopoly under constraints was first investigated by Simaan *et al.* [108], albeit for a different utility function and inverse demand function. As such, the Markov feedback NE can be obtained by following the same procedures in solving the HJB equation, the details of which can be omitted. Akin to [108], there exist nine different outcomes in the final NE solution, which is shown in Table 6.1. The table indicates the form of the player's strategy¹ $b_i(p)$ at the NE, the condition for each case and their implications on the market. Note that the constants X_j and Y_j , $j = 1, \dots, 9$ can be derived from the given parameters; and Φ_i , $i = 1, 2$ which occurs during the derivation, is given by

$$\Phi_i = \frac{\partial U_i}{\partial p} - k\lambda \frac{\partial V_i}{\partial p} = p - (\alpha_i + \pi_i) - 2\beta_i b_i(p) - k\lambda \frac{\partial V_i}{\partial p}, \quad i = 1, 2, \quad (6.10)$$

where V_i is the value function of Theorem A.4. The interpretation of the conditions are as follows.

- For player i , $\Phi_i < 0$ implies

$$p < (\alpha_i + \pi_i) + 2\beta_i b_i(p) + k\lambda \frac{\partial V_i}{\partial p}. \quad (6.11)$$

¹In dealing with the HJB equation and its subsequent analysis, it is understood that p is treated as a variable and the results should hold for all t . Thus, from here onwards, p , $b_i(p)$ and $V_i(p)$ can be used instead of $p(t)$, $b_i(p(t))$ and $V_i(p(t))$.

Table 6.1: Nine possible outcomes of the dynamic duopoly.

Case	$b_1(p)$	$b_2(p)$	Conditions	Player 1	Player 2
i	0	0	$\Phi_1 < 0, \Phi_2 < 0$	Out	Out
ii	0	$X_2p + Y_2$	$\Phi_1 < 0, \Phi_2 = 0$	Out	Monopolist
iii	$X_3p + Y_3$	0	$\Phi_1 = 0, \Phi_2 < 0$	Monopolist	Out
iv	0	B_2	$\Phi_1 < 0, \Phi_2 > 0$	Out	Saturation
v	B_1	0	$\Phi_1 > 0, \Phi_2 < 0$	Saturation	Out
vi	$X_6p + Y_6$	B_2	$\Phi_1 = 0, \Phi_2 > 0$	Monopolist	Saturation
vii	B_1	$X_7p + Y_7$	$\Phi_1 > 0, \Phi_2 = 0$	Saturation	Monopolist
viii	B_1	B_2	$\Phi_1 > 0, \Phi_2 > 0$	Saturation	Saturation
ix	$X_9p + Y_9$	$X'_9p + Y'_9$	$\Phi_1 = 0, \Phi_2 = 0$	Duopolist	Duopolist

Here, p can be seen as the MR (i.e., earnings from leasing one more unit of spectrum in the market). The other expression, $(\alpha_i + \pi_i) + 2\beta_i b_i(p) + k\lambda \frac{\partial V_i}{\partial p}$, is the MC [108] (i.e., additional cost of an additional unit spectrum). As $\text{MR} < \text{MC}$, the player gains no profits and will drop out of the market.

- Similarly, if $\Phi_i > 0$, then $\text{MR} > \text{MC}$. Hence, the revenue from the secondary market will be so high that player i will output maximum quantities in the market.
- When $\Phi_i = 0$, $\text{MR} = \text{MC}$. Hence, player i faces perfect competition and will offer an output that maximizes his profits. The optimal solution corresponds to the Markov NE strategy.

Therefore, each PSH may end up either: (a) allocating all the spectrum to its primary service (i.e., staying out of the market); (b) offering all the spectrum to the market (i.e., output saturation), or (c) balancing the primary and secondary spectrum allocation. Note that when the PSH does balance its spectrum allocation between its primary services and the secondary market, the exact amount of bandwidth that it should offer to the market is then determined as a linear feedback function $X_j p + Y_j$ of the current market price level p . This is due to the adopted linear quadratic model. The combination of these three possibilities for the two players results in the nine

cases as suggested by Table 6.1.

The final outcome depends on various parameters, such as the players' cost and profit factors ($\alpha_{i,j}$, $\beta_{i,j}$ and $\pi_{i,j}$), as well as the market inherent property (a , k and λ). From the system's point of view, possibility (a) for a certain player is not preferable as one would like to encourage mutual spectrum access; and so is possibility (b) since the primary service should be guaranteed priority access to spectrum. As such, the system manager is interested in designing the market so that the *true duopoly* case will occur, i.e., case (ix) in Table 6.1. Under such free competition, at the Markov NE, it is observed that both players adapt their strategies as a linear function of the price p .

6.4 Solutions to Symmetric Constrained Dynamic Oligopoly

The oligopolistic competition is a generalized scenario from the dynamic duopoly. For each player, each of the three possible outcomes (i.e., staying out, free competition and saturation) will also happen. As a result, the Markov NE of the game can branch out into 3^N different possibilities. Characterization of the NE involves solving a system of N simultaneous partial differential equations. Moreover, the game may start in one of these regions and cross into another as time progresses. Such strategy transition was not addressed adequately in existing dynamic duopoly models, such as [109]. As such, a complete analysis is much more complex than the duopoly case and may be intractable.

In this analysis, the thesis considers a special case which is the symmetric dynamic oligopoly, i.e., $\pi_i = \pi$, $\alpha_i = \alpha$, $\beta_i = \beta$ and $B_i = B$, $\forall i$. Also, it is assumed that the slope of the inverse demand function $\lambda = 1$ for simplicity. As such, it is possible to not only obtain the symmetric stationary Markov feedback NE for the game, but also

thoroughly investigate the transition of strategy across different regions.

A stationary Markov feedback NE $(b_i^*(p), b_{-i}^*(p))$ for the proposed game can be characterized by applying the HJB equation (refer to Theorem A.4). That is, for each player i , there exists a corresponding value function $V_i(p) : [0, \infty) \mapsto \mathbb{R}$, which is everywhere continuous and differentiable, such that

$$rV_i(p) = \max_{b_i} \left\{ (p b_i + \pi(B - b_i) - \alpha b_i - \beta b_i^2) + \frac{\partial V_i(p)}{\partial p} \cdot k \left[a - \sum_{i=1}^N b_i - p \right] \right\}. \quad (6.12)$$

As the expression under maximization in (6.12) is quadratic in b_i , carrying out the maximization yields

$$\tilde{b}_i = \frac{1}{2\beta} \left[p - (\alpha + \pi) - k \frac{\partial V_i(p)}{\partial p} \right], \quad \forall i. \quad (6.13)$$

Substituting (6.13) into (6.12) gives

$$\begin{aligned} 2\beta rV_i(p) = & (p - (\alpha + \pi)) \left(p - (\alpha + \pi) - k \frac{\partial V_i(p)}{\partial p} \right) - \frac{1}{2} \left(p - (\alpha + \pi) - k \frac{\partial V_i(p)}{\partial p} \right)^2 \\ & + k \frac{\partial V_i(p)}{\partial p} \left[2\beta a - (2\beta + N)p + N(\alpha + \pi) + k \sum_{j=1}^N \frac{\partial V_j(p)}{\partial p} \right], \quad \forall i. \end{aligned} \quad (6.14)$$

Eq. (6.14) is a system of N non-linear first-order ordinary differential equations (ODE) in $V_i(p)$. Similar to [109], one may conjecture that the solution $V_i(p)$ is such that $\partial V_i / \partial p = D_i p - E_i$ and subsequently, $V_i = \frac{1}{2} D_i p^2 - E_i p + G_i$. Then, solving (6.14) gives two possible solutions, the symmetric one where $D_i = D_j$, $E_i = E_j$, and $G_i = G_j$, $\forall i, j$; and the asymmetric one where $D_i \neq D_j$, $E_i \neq E_j$ and $G_i \neq G_j$. For the asymmetric one, following [109] it may be shown that the price will not be asymptotically stable (i.e., $\lim_{t \rightarrow \infty} p(t) = \infty$). This leaves us with the symmetric case. Subsequently, this leads to $V_i = V_j = V$, $\forall i, j$. Hence, the Markov NE will be symmetric where $b_i^*(p) = \tilde{b} = \frac{1}{2\beta} \left[p - (\alpha + \pi) - k \frac{\partial V(p)}{\partial p} \right]$, $\forall i$ if there are no bandwidth constraints. However, when coupled with the constraint $b(p) \in [0, B]$, three

possibilities can occur:

- (i) *Case A:* $\tilde{b} < 0$. The actual strategy taken by the players will be $b^*(p) = 0$ regardless of p .
- (ii) *Case B:* $0 \leq \tilde{b} < B$. The players follow the feedback strategy $b^*(p) = \tilde{b}$.
- (iii) *Case C:* $\tilde{b} \geq B$. Here, the strategy will saturate at $b^*(p) = B$ regardless of p .

6.4.1 Solution for Case B

Case B represents the true oligopoly scenario where all PSHs dynamically optimize their strategies in the market. At this point, one needs to solve the HJB equation which is given by

$$2\beta rV(p) = (p - (\alpha + \pi)) \left(p - (\alpha + \pi) - k \frac{\partial V(p)}{\partial p} \right) - \frac{1}{2} \left(p - (\alpha + \pi) - k \frac{\partial V(p)}{\partial p} \right)^2 + k \frac{\partial V(p)}{\partial p} \left[2\beta a - (2\beta + N)p + N(\alpha + \pi) + kN \frac{\partial V(p)}{\partial p} \right]. \quad (6.15)$$

The above equation is a non-linear first-order ODE in $V(p)$. Although there is no direct solving method, one may conjecture that a particular solution $V(p)$ is quadratic in p as follows.

Lemma 6.1. *The value function and its derivative that satisfy (6.15) are of the following forms*

$$\frac{\partial V(p)}{\partial p} = Dp - E, \quad (6.16a)$$

$$V(p) = \frac{1}{2}Dp^2 - Ep + G, \quad (6.16b)$$

where

$$D = \frac{(2\beta + N)k + \beta r - \sqrt{((2\beta + N)k + \beta r)^2 - (2N - 1)k^2}}{(2N - 1)k^2} < \frac{1}{k}, \quad (6.16c)$$

$$E = \frac{2\beta kDa + (NkD - 1)(\alpha + \pi)}{(2N - 1)k^2D - (2\beta + N)k - 2\beta r}, \quad (6.16d)$$

$$G = \frac{(\alpha + \pi)^2 + (2N - 1)k^2E - 2NkE(\alpha + \pi) - 4\beta kEa}{4\beta r} + \frac{\pi B}{r}. \quad (6.16e)$$

Proof: One can substitute $V(p)$ as in (6.16b) back to (6.15) and simplify, which yields a quadratic equation $A_1p^2 + A_2p + A_3 = 0$ in p . Since the original equation is an ODE in $V(p)$, it should hold for all values of p , which means that A_1 , A_2 and A_3 must be simultaneously 0. In particular,

$$A_1 = (2N - 1)k^2D^2 - 2((2\beta + N)k + \beta r)D + 1 = 0 \quad (6.17)$$

which is quadratic in D . By taking the smaller root, D can be obtained as in (6.16c). Additionally, it can be shown that $D < \frac{1}{k}$, as after some algebraic manipulation, this will be equivalent to $2k\beta(r + 2k)(2N - 1) > 0$ which is true. Choosing such D will ensure stability of the state variable, as will be shown later on.

Next, A_2 and A_3 are linear in E and G , respectively. Therefore, E and G as in (6.16d) and (6.16e) can be obtained readily, the details of which are neglected. ■

The analysis proceeds to find the stationary Markov feedback NE and the steady-state price level as follows.

Proposition 6.1. *The stationary Markov feedback NE strategy of players in Case B is*

$$b^*(p) = \frac{1}{2\beta} [(1 - kD)p + (kE - \alpha - \pi)]. \quad (6.18a)$$

Additionally, if Case B contains the trajectory's steady state, then this point is

$$\Gamma = \frac{2\beta a + N(\alpha + \pi - kE)}{2\beta + N(1 - kD)}. \quad (6.18b)$$

Proof: With $\frac{\partial V(p)}{\partial p}$ given in (6.16b), (6.18a) follows directly from (6.13). For (6.18b), one needs to solve the state differential equation (6.8) using b in (6.18a), which gives

$$\dot{p}(t) = k \left(a - \frac{N[(1-kD)p(t) + (kE - \alpha - \pi)]}{2\beta} - p(t) \right). \quad (6.19)$$

The general solution to this is

$$p(t) = \Gamma + C e^{-k \left(1 + \frac{N(1-kD)}{2\beta} \right) t}, \quad C = \text{const}, \quad (6.20)$$

with Γ in (6.18b). In (6.20), the constant C can be found with some initial condition. In particular, if Case B takes place at the starting point of the game $p(0) = p_0$ then $C = p_0 - \Gamma$.

Moreover, since $D < \frac{1}{k}$, it follows that the exponent $-k \left(1 + \frac{N(1-kD)}{2\beta} \right) t$ is negative. Hence, the exponential term vanishes as $t \rightarrow \infty$, and the price reaches the asymptotically stable steady state Γ , provided that Γ lies in the region for Case B. ■

The steady-state price Γ is the *market equilibrium*, where the supplies and demands are balanced. This is not to be confused with the other equilibrium concept, i.e., the Markov NE.

6.4.2 Solution for Case A

In Case A, all PSHs supply zero outputs to the market. It can be implied that the price level may drop too low for any of the players to gain positive profits, so all of them are dropped out of the competition. The solution to this case can be found via similar procedures.

Proposition 6.2. *In Case A, the players' NE strategy is $b^*(p) = 0$ for all p in the region. The market equilibrium in this case is given by $p = a$.*

Proof: It remains to compute the steady state price. With $b = 0$, eq. (6.8) becomes

$\dot{p}(t) = k(a - p(t))$, whose general solution is

$$p(t) = a - C e^{-kt}, \quad C = \text{const.} \quad (6.21)$$

This proves the proposition, as the exponential term is convergent. If the game begins in Case A, then one can apply initial condition $p(0) = p_0$ to get $C = p_0 - a$. ■

The value function $V(p)$ in Case A can be subsequently determined from the HJB equation $rV(p) = k(a - p)\frac{\partial V(p)}{\partial p} + \pi B$, whose solution is

$$V(p) = \frac{C}{(a - p)^{r/k}} + \frac{\pi B}{r}, \quad C = \text{const.} \quad (6.22)$$

6.4.3 Solution for Case C

Case C is the direct opposite of Case A, where the PSHs have no incentives to provide primary services and they are now leasing all their available bandwidth B . Therefore, by applying the same analysis, similar results can be obtained.

Proposition 6.3. *In Case C, the players' NE strategy is $b^*(p) = B$ for all p in the region. The market equilibrium in this case is given by $p = a - NB$.*

Proof: By solving the price trajectory $\dot{p}(t) = k(a - NB - p(t))$, one can obtain

$$p(t) = a - NB - C e^{-kt}, \quad C = \text{const}, \quad (6.23)$$

which converges to $a - NB$ if the point lies in the region for Case C. Also, if the game begins in Case C, then the constant $C = p_0 - (a - NB)$. ■

The HJB equation in Case C gives

$$k(p - a + NB)\frac{\partial V(p)}{\partial p} + rV(p) = Bp - \alpha B - \beta B^2, \quad (6.24)$$

whose solution is

$$V(p) = Rp + S + \frac{C}{(a - NB - p)^{r/k}}, \quad C = \text{const}, \quad (6.25)$$

where

$$R = \frac{B}{r + k}, \quad (6.26a)$$

$$S = \frac{1}{r} \left[\frac{kB}{r + k} (a - NB) - \alpha B - \beta B^2 \right]. \quad (6.26b)$$

6.4.4 Transition Between Different Market Scenarios

In the previous analysis, one has obtained the solutions for different possible market situations, i.e., cases A, B and C. However, it is still necessary to determine the required conditions and define the exact regions in which each of these cases holds. It is possible that the price trajectory may reach a boundary point and enter another region over time and a strategy transition is said to take place.

First, let us look for these boundary values of the price p . Notice that in Case B, $b(p)$ is not only a linear function in p , but also bounded in $[0, B)$. Thus, $p(t)$ must also be bounded in $[p_1, p_2)$ which defines the boundaries for Case B. By solving $b(p) = 0$ and $b(p) = B$, respectively, with $b(p)$ in (6.18a), one can obtain

$$p_1 = \frac{\alpha + \pi - kE}{1 - kD}, \quad (6.27a)$$

$$p_2 = \frac{\alpha + \pi - kE + 2\beta B}{1 - kD}. \quad (6.27b)$$

From this result, it is clear that the region for Case A to occur is $p(t) \in [0, p_1)$ and for Case C $p(t) \in [p_2, \infty)$. Therefore, at any time instant t , the current market scenario and players' strategies depend on current state $p(t)$ and which interval it is in. Specifically, at the beginning $t = 0$, the value of p_0 will determine the starting scenario of the market.

Next, the transition mechanisms are examined. Suppose that at a certain time $t = \hat{t}$, $p(t)$ reaches a boundary value p_u (i.e., either p_1 or p_2) and is about to cross over to the neighboring region. Note that $\hat{t}$ can easily be calculated by solving $p(t) = p_u$ with $p(t)$ in either (6.20), (6.21) or (6.23). At $\hat{t}$, the strategy $b(t)$, the trajectory $p(t)$ and the value function $V(p)$ take on new shapes as a transition occurs. Here, according to Theorem A.4, it is mandatory that $V(p)$ is continuous and differentiable. Moreover, the “stickiness” assumption requires $p(t)$ to be continuous.

The continuity of $p(t)$ can be satisfied if the new initial condition $p(\hat{t}) = p_u$, $u = 1, 2$ is enforced when re-solving the state equation. For example, if a switch from Case A or Case C to Case B happens, solving (6.19) with the new initial condition yields a new shape of $p(t)$ as given by

$$p(t) = \Gamma + (p_i - \Gamma) e^{-k \left(1 + \frac{N(1-kD)}{2\beta}\right) (t - \hat{t})}, \quad \text{for } t \geq \hat{t}. \quad (6.28)$$

On the other hand, $V(p)$ is comprised of several piecewise smooth functions in $[0, p_1)$, $[p_1, p_2)$ and $[p_2, \infty)$. In order to ensure that it is continuous at p_1 , one can let $V_1 = \frac{1}{2}Dp_1^2 - Ep_1 + G$ and solve (6.22) with the boundary condition $V(p_1) = V_1$. Similarly, one can let $V_2 = \frac{1}{2}Dp_1^2 - Ep_1 + G$ and incorporate $V(p_2) = V_2$ in (6.25). Hence, the complete value function is given by

$$V(p) = \begin{cases} \left(V_1 - \frac{\pi B}{r}\right) \frac{(a - p_1)^{r/k}}{(a - p)^{r/k}}, & p < p_1 \\ \frac{1}{2}Dp^2 - Ep + G, & p_1 \leq p < p_2 \\ Rp + S + (V_2 - Rp_2 - S) \frac{(a - NB - p_2)^{r/k}}{(a - NB - p)^{r/k}}, & p \geq p_2. \end{cases} \quad (6.29)$$

Finally, it remains to verify that

$$\begin{cases} \lim_{p \rightarrow p_1^+} \frac{\partial V(p)}{\partial p} = \lim_{p \rightarrow p_1^-} \frac{\partial V(p)}{\partial p} \\ \lim_{p \rightarrow p_2^+} \frac{\partial V(p)}{\partial p} = \lim_{p \rightarrow p_2^-} \frac{\partial V(p)}{\partial p}. \end{cases} \quad (6.30)$$

The calculation is lengthy but it can be shown to be true. With that, the complete solution for the proposed differential game is fully characterized. It is summarized by the following theorem.

Theorem 6.1. *With the value function in (6.29), the symmetric differential game has a symmetric stationary Markov feedback NE in*

$$b^*(p) = \begin{cases} 0, & p < p_1 \\ \frac{1}{2\beta} [(1 - kD)p + (kE - \alpha - \pi)], & p_1 \leq p < p_2 \\ B, & p \geq p_2. \end{cases} \quad (6.31)$$

with p_1 and p_2 in (6.27). Moreover, it will result in an asymptotically stable market equilibrium at either $p = a$, $p = \Gamma$ defined in (6.18b) or $p = a - NB$. Additionally, the behaviors of the players' strategy depend on the region of the current state $p(t)$ and transition between regions can occur at boundary points in a manner similar to (6.28).

Proof: The proof is evident from the previous analysis. ■

6.4.5 A Visualization of the Market Regions

It has been shown that the symmetric oligopoly's Markov NE may occur in three regions. In order to gain a better understanding of these market scenarios and transitions, Fig. 6.3 (not to scale) is used to help illustrate the results. It depicts the different regions in the $p - \pi$ plane, which offers a look into the spectrum market in

 Fig. 6.3: Three regions of the spectrum market on the p - π plane.

terms of comparative market price. Naturally, π and p can be understood as the price of spectrum for the primary services and the secondary market, respectively. Thus, at a given time, the point (p, π) in the p - π plane determines which region the game currently is in (see Fig. 6.3). The three regions are separated by the two boundary lines $p = p_1(\pi)$ and $p = p_2(\pi)$, as p_1 and p_2 in (6.27) can be shown to be linear in terms of π ; and these two lines can be proven to be parallel. Consequently, if p is too low compared to π , the secondary market investment is worse than the primary investment, which leads to Case A. Meanwhile, an appropriate range of p for a given π can result in true oligopoly; and Case C occurs when p is much higher than π .

The dotted lines $p = a$, $p = \Gamma(\pi)$ and $p = a - NB$ in Fig. 6.3 are the steady-state prices for the three cases. Besides, the group of lines $p = a$, $p = \Gamma(\pi)$ and $p = p_2(\pi)$ are proven to be concurrent; and so are $p = a - NB$, $p = \Gamma(\pi)$ and $p = p_1(\pi)$, as depicted in the figure. As such, the actual steady-state price is represented by the segments of these three lines that lie within their corresponding regions, represented by arrows; and as π increases, the actual steady-state follows the continuous path

highlighted by these arrows in Fig. 6.3. Moreover, for a given π and initial price p_0 , a trajectory of $p(t)$ can be depicted as a vertical straight arrow pointing towards the steady-state path. For example, in Fig. 6.3, for the initial price p_{02} and given π_2 , they correspond to the trajectory that starts from the point (p_{02}, π_2) within Case C. This trajectory later crosses into Case B until it reaches the steady-state path, which is the arrow segment of $p = \Gamma(\pi)$ within Case B. It should be noted that Fig. 6.3 depicts the behaviors of the game as a whole, not the time-varying behaviors of a particular variable. Such behaviors will be numerically studied in Section 6.6.

6.5 Discrete-time Price Adjustment

Under the previous analysis, theoretically, the PSHs in the market adapt their offered bandwidth continuously as a function of the current market price, which is not feasible in practice since the process requires price feedback and players' decisions almost instantaneously. Therefore, to implement the dynamic spectrum market, a discrete-time price adjustment protocol will be proposed, in which the price is adjusted dynamically in a time-slotted manner. The protocol can be effectively implemented at the spectrum broker and should be agreed upon by all participating oligopolists in the spectrum market. The basic procedures are presented in Algorithm 6.1.

Algorithm 6.1 Discrete-time price adjustment method.

```

1:  $n \leftarrow 0$ 
2:  $p[t_n] \leftarrow p_0$ 
3: repeat
4:    $n \leftarrow n + 1$ 
5:    $p[t_n] \leftarrow p[t_{n+1}]$  ▷ Updated by the broker according to (6.32).
6:    $b[t_{n+1}] \leftarrow b^*(p[t_{n+1}])$  ▷ Updated by players with  $b^*(.)$  defined in (6.31).
7: until  $|p[t_{n+1}] - p[t_n]| < p_{tol}$  ▷  $p_{tol} > 0$  is a tolerance value.

```

In step 5 of Algorithm 6.1, the price adjustment equation is given by

$$p[t_{n+1}] = p[t_n] + h \times k(a - p[t_n] - \sum_{i=1}^N b_i[t_n]). \quad (6.32)$$

Eq. (6.32) is in fact the gradient update method in numerical analysis [105]. The parameter h is the stepsize or the adjustment interval. An appropriate choice for h is required to ensure its convergence, which is stated in the following lemma.

Lemma 6.2. *Given the game's parameters, in order for the update process to converge, the stepsize h should not exceed a maximum value $h_{\max}$, given by*

$$h_{\max} = \frac{2}{k(1 + \frac{N(1-kD)}{2\beta})}. \quad (6.33)$$

Proof: According to Heath [105], the stability of the numerical method (6.32) implies $|1 + hJ| < 1$. In other words, the Jacobian $J = \frac{d\mathbf{G}}{dp}$ of the price trajectory has eigenvalues of magnitudes less than 1. Recall that $\mathbf{G}(p(t))$ is the right-hand side of (6.8). Evaluating J yields

$$J = \begin{cases} k \left(1 + \frac{N(1-kD)}{2\beta} \right), & p \in [p_1, p_2], \\ k, & p \notin [p_1, p_2]. \end{cases} \quad (6.34)$$

Thus, the condition $|1 + hJ| < 1$ gives $h < h_{\max} = \min(\frac{2}{k}, \frac{2}{k(1 + \frac{N(1-kD)}{2\beta})})$. ■

6.6 Numerical Results

In this section, numerical examples and analysis are presented in order to further investigate the properties and behaviors of the proposed differential game for spectrum market. Furthermore, computer simulations using MATLAB are also carried out to study the convergence of the discrete-time price adjustment method.

6.6.1 A Numerical Example

First of all, let us observe the theoretical behaviors of some important functions in the game and the strategy transitions over the time horizon through a specific example. In

Table 6.2: Parameter settings for the spectrum market.

Parameters	Values
Number of PSHs, N	4
Total bandwidth of each PSH, B	3000 kHz
Intercept of inverse demand curve, a	11000 \$/kHz
Slope of inverse demand curve, λ	1 \$/(kHz) ²
Price adjustment speed, k	0.5
Discount rate, r	0.3
Income rate of primary services, π	800 \$/kHz
First-order cost coefficient, α	400 \$/kHz
Second-order cost coefficient, β	0.5 \$/(kHz) ²

this example, the various game parameters are conveniently chosen as given in Table 6.2, which gives a price trajectory equation $\dot{p}(t) = 0.5 [11000 - 4b(t) - p(t)]$ and an accumulated revenue of $\int_0^\infty e^{-0.3t} [p(t)b(t) + 800(3000 - b(t)) - (400b(t) + 0.5b^2(t))] dt$. Thereafter, for each of three different values of the initial spectrum price, $p_0 = 50, 2500$, and 7000 \$/kHz, the actual price $p(t)$, the Markov NE strategy $b(p(t))$, the value function $V(p(t))$ and its derivative $\partial V/\partial p$ are tracked following their derived theoretical values according to Theorem 6.1 and plotted against time t , as shown in Fig. 6.4.

As verified by the previous analysis, the price $p(t)$ is asymptotically stable and converges to the market equilibrium as $t \rightarrow \infty$; and so will $b(p(t))$ converge to its final value. This market equilibrium price $\Gamma \approx 3508.7$ \$/kHz lies in Case B, within $p_1 \approx 1425.3$ \$/kHz and $p_2 \approx 4762.6$ \$/kHz; so the players all balance their spectrum allocation to both primary services and the spectrum market. In this case, approximately $b^* \approx 1873$ kHz is to be offered to the secondary market. It is clear that the unique equilibrium point is independent of the initial state p_0 , which is due to the global stability of the system. $V(p)$ is shown to be continuous. Furthermore, the continuity of $\partial V/\partial p$ confirms the differentiability of $V(p(t))$.

The value of p_0 affects the starting region and the subsequent shapes of $p(t)$, $b(p(t))$ and $V(p(t))$ (see Fig. 6.4). For a very low value $p_0 = 50$ \$/kHz $< p_1$, there is at first

Fig. 6.4: An example of the oligopoly.

no incentive for the PSHs to allocate any spectrum to the SSPs due to negative profit, so they stay out of the market (i.e., Case A). At this point, the inverse demand law gives a desirable price $\hat{p}(0) = a > p(0)$, which implies that the demands exceed the supplies and will drive $p(t)$ up. As time t elapses, $p(t)$ increases and reaches p_1 at $\hat{t}_1 \approx 0.27$ time unit (this point is marked by an asterisk in Fig. 6.4), where a transition to Case B occurs and the players start to offer positive outputs to the market. The game then proceeds towards the NE. Similarly, for $p_0 = 7000$ $\$/\text{kHz} > p_2$, the game starts in Case C and switches midway to Case B at $\hat{t}_2 \approx 0.66$ time unit. Case B occurs from the beginning for $p_0 = 2500$ $\$/\text{kHz}$.

Fig. 6.5: Convergence of the discrete-time price adjustment method.

6.6.2 Convergence of the Discrete-Time Method

The proposed discrete-time price adjustment scheme was simulated for the game where the parameters were set similar to the previous example (see Table 6.2), except for the initial price $p_0 = 7500$ \$/kHz. As such, $p_0 > p_2 = 4762.6$ \$/kHz and one would expect $p(t)$ to start in Case C, then cross into Case B midway and converge to the market equilibrium $\Gamma \approx 3508.7$ \$/kHz. For this simulation, two different time adjustment intervals were selected, i.e., $h_1 = 0.5h_{\max}$ and $h_2 = 0.2h_{\max}$, where the maximum allowable interval is estimated to be $h_{\max} \approx 0.87$ according to Lemma 6.2. The tolerance threshold is $p_{tol} = 0.01$. The two discrete-time processes were plotted in Fig. 6.5, together with the theoretical $p(t)$.

Clearly, with a properly chosen interval below the maximum allowable value, the discrete-time dynamics of $p[t]$ and $b[t]$ converged to the theoretical equilibrium after a period of time, as shown in Fig. 6.5. However, the convergence speed depends on how small the stepsize is. Also, with h_2 , the discrete solution approximates the theoretical

solution with less errors than with h_1 . However, there is a tradeoff between accuracy and computational cost. A longer interval means less computations on the spectrum broker, but gives a larger error which translates into higher profit loss between the actual and the optimal value. Such profit loss can be on the PSHs (negative errors) or the SSPs (positive errors). Therefore, the realization of such market protocol should be agreed among the PSHs, SSPs and the spectrum broker so that the costs for all sides are within acceptable margins.

6.6.3 Impact of Number of Players and Maximum Bandwidth

Next, the steady-state behaviors of the game, particularly of the spectrum price $p(t)$ and the optimal bandwidth output $b(p(t))$ are studied when the number of players N and the total bandwidth B are varied. For the previous parameter settings as in Table 6.2, the steady-state values (i.e., limits as $t \rightarrow \infty$) of $p(t)$ and $b(p(t))$ are plotted against the number of players N for three different values of the maximum bandwidth $B = 1500, 2000$ and 3000 kHz in Fig. 6.6. Some observations can be made as follows.

Firstly, as N increases, the steady-state price $p(\infty)$ and offered bandwidth $b(p(\infty))$ tend to drop for the same B . This agrees with the economic principle that increasing supplies lead to reduction of the equilibrium market price, hence the decrease in $p(\infty)$. At the same time, the more competitors, the smaller share of the spectrum market a PSH will get, which explains the drop in $b(p(\infty))$.

Secondly, the decline in $p(\infty)$ is initially linear with respect to N , corresponding to Case C's equilibrium price $p = a - NB$. This is particularly evident in the shape of $p(t)$ in Fig. 6.6 for $B = 1500$ kHz where $N \leq 5$. However, when $N > 5$, it is implied that Case C's equilibrium no longer holds and the trajectory switches midway to Case B where its equilibrium price Γ given in (6.18b) becomes nonlinear in N . On the other hand, $b(p(\infty))$ is seen to remain at its maximum bandwidth level B in Case C for

 Fig. 6.6: Steady-state values of $p(t)$ and $b(p(t))$ versus N .

$N \leq 5$, and then also drop nonlinearly with respect to N in Case B for $N > 5$. Similar observations are also made for other values of B , where $p(\infty)$ is linear and $b(p(\infty))$ is constant for small N ; and they drop nonlinearly for sufficiently large N .

Interestingly, it can be easily verified that as $N \rightarrow \infty$, $\lim_{N \rightarrow \infty} \Gamma = \alpha + \pi$, $\lim_{N \rightarrow \infty} p_1 = \alpha + \pi$, $\lim_{N \rightarrow \infty} p_2 = \alpha + \pi + 2\beta B$ and $\lim_{N \rightarrow \infty} b(\Gamma) = 0$. That is, for an infinitely large number of players, the Case B steady-state output approaches 0 as the Case B equilibrium price Γ approaches the first-order cost factor.

Furthermore, it is seen that as B increases, for the same N , both $p(\infty)$ and $b(p(\infty))$ will converge to Γ and $b(\Gamma)$ in Case B if N is large. Note that Γ in (6.18b) is independent of B . Theoretically, one can show that if $B > b(p_0)$ with $b(p)$ in (6.18a), it no longer has any effects on $p(\infty)$ and $b(p(\infty))$. The implication is that once the PSHs have an abundant amount of unused bandwidth, they will ultimately benefit from participating in the secondary spectrum market and hence achieve better utilization of spectrum. However, due to the fixed demand from the SSHs, the optimal offered bandwidth to the market will stabilize even if the maximum available bandwidth gets

excessively large.

6.7 Number of Players under Fixed Total Bandwidth

The following discussion can be considered a case study with practical implications. Suppose that the regulatory authority wants to enact the spectrum market at a particular geographical location for a fixed frequency band $\mathcal{B}$, by dividing it equally for N licensed PSHs who in turn can lease spectrum to the SSPs (i.e., $NB = \mathcal{B} = \text{const}$). It is of particular interest to find out an appropriate number of players N so as to encourage free competition (i.e., true oligopoly), and at the same time ensure benefits for the SSPs by reducing the equilibrium market price as much as possible. In order to illustrate this objective, let us look at the behaviors of the dynamic oligopoly under this particular constraint when the number of participating players N vary through a numerical example. One can assume that the system parameters also follow Table 6.2 except for N and B , and additionally, $p_0 = 1500$ \$/kHz and $\mathcal{B} = 7200$ kHz. As such, Fig. 6.7 and Fig. 6.8 show the various trajectories of $p(t)$ and their corresponding outputs $b(p(t))$ over time t for different values of N . At first glance, it can be observed that the steady-state point falls within either Case B for small N , or Case C after N exceeds 4, as seen in Fig. 6.8. At the point when N reaches 4, the steady-state price lies in Case C which is equal to the constant $a - \mathcal{B}$; so it ceases to drop, as shown in Fig. 6.7. Thus, $N = 3$ is the critical point at which true oligopoly is still in effect and beyond which the equilibrium price cannot be decreased further. Therefore, ideally, three oligopolists should participate in the market if free competition and reduced equilibrium price are desired.

In general, the desirable value of N can be mathematically specified as follows.

Proposition 6.4. *The desired number of players N in this case study is the one that*

Fig. 6.7: Price trajectory $p(t)$ for different N when $NB = \mathcal{B} = \text{const.}$

Fig. 6.8: Optimal strategy $b(p(t))$ for different N when $NB = \mathcal{B} = \text{const.}$

satisfies $\Omega(N) \leq \mathcal{B} < \Omega(N+1)$, where

$$\Omega = \frac{a(1-kD) - (\alpha + \pi - kE)}{(1-kD) + (2\beta/N)}. \quad (6.35)$$

Proof: Since the steady state in Case B is desired, the condition requires that $\Gamma \in [p_1, p_2]$. $\Gamma > p_1$ can be ensured from the start by assuming $a > \alpha + \pi$, following the discussion in Dockner *et al.*, chapter 10 [20]. Hence, it remains to satisfy $\Gamma \leq p_2$, where both sides are dependent on N . By simplifying $\Gamma = p_2$ while noting that $B = \mathcal{B}/N$, one can obtain $\mathcal{B} = \frac{a(1-kD) - (\alpha + \pi - kE)}{(1-kD) + (2\beta/N)} = \Omega(N)$, which is a function of N . If N is relaxed to take real values in $[2, +\infty)$, then it can be shown that $\Omega(N)$ is monotonically increasing in N (details neglected), which is evident in Fig. 6.9. Since $\lim_{N \rightarrow \infty} D = \lim_{N \rightarrow \infty} E = 0$, this implies $\lim_{N \rightarrow \infty} \Omega(N) = a - \alpha - \pi$, i.e., $\Omega(N)$ is bounded above by $a - \alpha - \pi$. Therefore, if $\Omega(2) \leq \mathcal{B} \leq a - \alpha - \pi$, there exists a unique integer $N \geq 2$ such that $\Omega(N) \leq \mathcal{B} < \Omega(N+1)$ which is the desired N . ■

The proposition also reveals two extreme cases. That is, when $\mathcal{B} < \Omega(2)$, any number of players will always result in saturation (i.e., Case C), implying a shortage of total supplies. On the other hand, when $\mathcal{B} > a - \alpha - \pi$, Case B will occur for any number of players, implying that the authority may want to reduce the total bandwidth $\mathcal{B}$ down.

6.8 Concluding Remarks

New spectrum allocation schemes such as spectrum markets are currently being researched in order to improve the utilization of the scarce spectrum resources and to realize future DSA systems. In this chapter, one such market-driven model is studied where the licensed holders of spectrum allocate their total bandwidth to both their primary services and the spectrum market, managed by the spectrum broker. The thesis formulates this oligopolistic competition as a sticky-price differential game un-

Fig. 6.9: Plot of the function $\Omega(N)$, showing it is monotonically increasing and bounded.

der maximum bandwidth constraints. As such, the game is shown to have multiple outcomes even in the 2-player case. For the N -player scenario, the Markov feedback NE is derived when the game is symmetric and the strategy transition mechanism is properly addressed. An important design issue is to select proper parameters so that the costs incurred by all participants are minimized, while maintaining free competition (true oligopoly) so as to encourage mutual sharing of spectrum. Under both numerical and theoretical analyses, this research examines the impact of parameters such as the number of player and the total bandwidth, as well as their desirable values. Moreover, a discrete-time price adjustment scheme which is easily executed by the spectrum broker is also proposed. It is believed that the proposed model has addressed the following important questions in dynamic spectrum market:

- How much spectrum that a PSH should allocate to the secondary market and its own primary service in order to maximize its benefits, given the market parameters and its own revenue and cost functions.

- What the desired spectrum price levels and conditions would be in order for certain market outcomes to happen, such as no sharing, mutual sharing or saturation.
- Practical scenarios, e.g., given a fixed total amount of spectrum at a specific region, what the optimal number of players would be that the authority can allow in order to promote free competition.

Dynamic spectrum market has been gaining a lot of attention in the literature and there have been numerous research works and projects in recent years in order to fully realize it within the new DSA paradigm. The author believes that the presented model can serve as an alternative view on the dynamics and mechanisms of spectrum allocation taking place in the upper tier of the proposed framework and as a contribution to the growing body of works on this emerging research landscape.

Chapter 7

Conclusions and Recommendations

In this chapter, the conclusions of the thesis are drawn. At the same time, several problems which are open to future research and developments are discussed.

7.1 Conclusions

The traditional management and allocation of the radio frequency spectrum has seen a lot of difficulties in coping with the development of wireless communications technologies and systems, as well as the enormous demands for wireless traffics and spectrum access. Thus, a transition from the current spectrum management towards a future DSA era, believed to provide greater agility and more efficient utilization of the frequency resources, has been envisioned and extensively investigated in the literature. In order to successfully make such a transition, it is of paramount importance that better RRM techniques and methodologies are devised, so as not only to allow the smart wireless mobile devices and systems to adapt to the surrounding environment and optimize their performance, but also enable the wireless service providers at the system level to flexibly reallocate any under-utilized bandwidth to the unlicensed spectrum users, via a reasonable, market-driven mechanism that benefits both the spectrum “sellers” and “buyers”. At the same time, the mathematical beauty of game theory

and its appropriateness as a tool for computer and communications researchers have also provided a strong incentive for this thesis to adopt such methodologies within the context of wireless communications in the quest for better resource allocation strategies.

In tackling the dynamic spectrum allocation problems, firstly, the thesis has made a voluminous literature survey of the existing DSA techniques and corresponding game-theoretical schemes in Chapter 2, discussing the major models, trends and issues that require attention. Based on the literature study, this thesis has identified two main research areas in dynamic spectrum allocation and thereby proposed a two-tiered spectrum allocation framework, which specifically accommodates the end-user-oriented RRM issues within the lower tier, as well as the market-based “indirect” spectrum access problems under the upper tier. Henceforth, Chapter 3 to 5 are dedicated to game-theoretical techniques to deal with the allocation of frequency resources in the lower tier, where the key design objectives are users’ QoS requirements, system throughput and fairness, as well as CCI avoidance and mitigation. Subsequently, Chapter 6 turns the attention towards spectrum market games, where one is concerned with pricing of spectrum and equilibrium market behaviors.

Chapter 3 looks at the frequency assignment problem for distributed pairs of nodes in a wireless ad-hoc network. For non-real-time data applications, scheduling of user transmission can be made more effective by the introduction of the no-transmission strategy. That is, as evident from the simulation study, when the number of users increases, more and more users must temporarily delay transmission in order to guarantee the QoS requirements of the transmitting users. The utility functions with pricing are designed as an effort to correct the selfish behaviors of the individual players. The proposed scheme further explores the throughput and fairness tradeoff in the system by suggesting two alternative power allocation methods. Scheme 1 can guarantee fairness and satisfy the SINR target of 15 dB at a minimal power consumption, whereas Scheme 2 allows the users to attain their optimal rates at the cost of fairness

while blocking more users from accessing the spectrum. Simulations show that the game achieves convergence to NE consistently via best-response iterative gameplay.

In order to improve the spectral efficiency of the system while maintaining its stability of allocation, a potential game approach has been employed for distributed ad-hoc systems with an OFDMA interface in Chapter 4. The interference-sum objective function, which is chosen specifically to minimize the prevailing CCI in the system, is shown to be able to establish an exact potential game, which always admits pure-strategies NEs and their convergence through best-response dynamics. Another novel aspect of the proposed method is the ANSS scheme which is integrated in the pre-gameplay period for determining the number of subcarriers for each user to avoid interfering with neighboring users and to meet their QoS requirements. As such, the game is shown to achieve desirable SINR and capacity through simulations. That is, the SINR target of 13 dB is fulfilled for 70% of the time, while 75% of the users can obtain their requested rates. Furthermore, a reasonably low PoA level of about 1.5 to 3 indicates the ability of the algorithm to achieve good optimality.

The feasibility of the potential game formulation allows us to extend the concept to analyze a large-scale, infrastructure-based OFDMA system in Chapter 5, where a player is no longer a transmit-receive pair but a BS serving multiple MSs in a downlink multi-cell configuration. Establishing the exact potential game properties in this particular scenario is not a trivial task; however, it can be achieved by looking at the pseudo-player concept and the corresponding dual game. The convergence property of the game is the most important advantage of the potential game method compared to the earlier works on multi-cell OFDMA systems such as [38, 45]. As a result, one can rely on sequential dynamics such as best/better responses in order to play the game and achieve its NE points. Simulation results indicate that the system obtains superior fairness to existing OFDMA allocation methods such as [38], even under high system loads, with its fairness index consistently in the range of 0.9 to 0.95.

The efficiency of future DSA systems also depends on how the spectrum is allocated in the spectrum market. In Chapter 6, the thesis addresses this problem by looking at an alternative model for spectrum pricing which is based on two key principles. Those are, firstly, oligopolists offering homogeneous spectrum goods and competing on quantities based on a single market price set by a spectrum broker; and secondly, the spectrum price being sticky and changing as a dynamic, differential system. The problem is therefore solved as a differential game where its solutions exist in terms of Markov feedback NEs. Once the game's multiple outcome regions and strategy transitions are identified, the final market equilibrium price can be determined, both theoretically and numerically via the use of a discrete-time adjustment method. An important conclusion from the differential game analysis is that a market equilibrium price level can be uniquely obtained, at which point the PSH knows the optimal bandwidth it should allocate to its own primary services and the secondary market. Another interesting finding is that for a fixed total bandwidth and market condition, one can also identify the optimal number of PSHs to compete in the market to allow free competition and ensure the most benefits for the SSPs.

7.2 Future Research

As mentioned throughout the thesis, this research strives to identify new spectrum sharing and management methods that may help the future development and deployment of DSA, which has been and will continue as a fascinating and stimulating research area. There can be a number of ways whereby research efforts can be expanded.

- (i) *Practical implementation issues for potential games:* It is noted that in order to implement the proposed potential game in practice, especially for the distributed, ad-hoc systems, heavy signalings and information exchange need to be carry out in order to measure the channel gains and obtain the complete

channel state information. The signaling mechanism presented in the thesis is a simplified procedure, which highlights the key steps to be done in order to execute the game. However, in the long terms, one may need major modifications of the existing protocols in order to adapt not only this scheme, but also the majority of the existing RRM algorithms for DSA systems in the literature, including but not limited to game-theoretical approaches.

(ii) *Shared-use opportunistic spectrum access:* Opportunistic spectrum access for cognitive radio networks, or the spectrum overlay model in DSA, has been an important research area in the literature. This is also part of the RRM issues in the lower tier, albeit not investigated in this thesis. Thus, this research could naturally be expanded to cover this scenario, in order to obtain a complete RRM framework and methodologies for dynamic spectrum allocation. With the coexistence of the primary licensed users and secondary unlicensed users, the primary transmission needs to be protected from CCI and the unlicensed users will have to closely monitor the environment and compete among themselves under stricter interference constraints than in the previous studies. A primary user can also actively participate in the decision making process as a game player in order to improve the overall system performance. Hence, there could be a lot of open and challenging issues that deserve further investigation.

(iii) *Femtocell OFDMA resource allocation:* Femtocell networks with OFDMA techniques are a relatively new area but are recognized as a promising technique for future DSA networks; and research works focusing on femtocells have been widely proposed in recent years. In an ongoing work that the author is involved [100], game-theoretical algorithms have been proposed in order to mitigate CCI among femto-BSs, as well as cross-tier CCI between the femto-BS and macro-BS. At the same time, centralized algorithms are also considered which can help the femtocell users to optimize their power and ergodic capacity when

deployed near the macro-BS.

- (iv) *New perspectives on spectrum market:* Implementation of the spectrum market is still a very difficult task since a number of regulatory issues as well as economic considerations are involved. In the market model proposed in Chapter 6 of this thesis, there are a few assumptions and considerations that can be re-examined and improved. It is previously assumed that the demands for spectrum from the SSHs are not changing; therefore, an enhanced mechanism that allows for variable demands is of great interest. Another feasible model is one in which the primary spectrum demands are also dynamic as reflected in the time-varying income $\pi(t)$, where one would also need to define another state equation to capture this behavior. Finally, there are still many analytical tools and techniques offered by game theory in micro-economics, such as supermodular games [112], that one could adopt to characterize the spectrum market.

Despite a huge amount of research interest in this area, genuine DSA networks still require a lot of efforts in order to be successfully put in practice. In the lower tier, several challenges still need to be overcome which include signaling overheads, synchronization in spectrum sensing, user mobility, CCI mitigation and more importantly, the selfish behaviors of the individuals as an autonomous decision maker, which are often simplified or assumed to be ideal in the problem formulation stage. In the upper tier, the problem is far more complicated which involves not only the service providers but also policy makers, regulatory bodies and governments, in order to clearly define the market and successfully bring the spectrum to market. The directions of our research are hence in terms of designing RRM methods that efficiently deal with distributed selfish players and concurrently ensure network stability all the time.

Appendix A: Game Theory

Background

Appendix A covers the background on game theory. The presentation is limited to a minimal level that is sufficient to understand the techniques used in the thesis. Particularly, the discussion focuses on strategic non-cooperative games and other particular types of games of interest, including potential games and differential games. For thorough and in-depth treatments of game theory, it is suggested that interested readers refer to dedicated texts such as [16], [17] or [18] for the fundamentals of game theory; and [20], [21] for the more advanced topics of differential games.

A.1 Strategic Non-cooperative Games

Strategic non-cooperative games are often encountered in wireless resource allocation problems. They are referred to simply as *games* here for brevity. Such games can always be established if the three fundamental elements are clearly defined:

- *The set of players.*
- *The strategies associated with the players*, i.e., all the actions that a player can possibly select from.
- *The utilities (payoffs) for the players*, i.e., a function/rule that governs the payoff that a player will be awarded for taking a certain move given the other players'

moves.

A game refers to the scenario where every participating player interacts by choosing different strategies to determine the final outcome of the game. An important, implicit assumption in game theory is the rationality assumption: *Players are always rational*. A player is rational if he or she makes decisions which are consistent to the purpose of maximizing his/her own utility function. Consequently, a rational player always favors strategy A over strategy B if A gives him/her a better payoff than B.

Mathematically, let us denote $\mathcal{N} = \{1, 2, \dots, N\}$ the set of N participating players. For every i^{th} player in $\mathcal{N}$, his/her collection of all possible actions or strategies forms a *strategy set*, which can be informally expressed as

$$\mathbf{S}_i = \{S_i \mid S_i \text{ is a valid strategy for the } i^{th} \text{ player}\}. \quad (\text{A.1})$$

Definition A.1. *The strategy space $\mathbb{S}$ is defined as the Cartesian products of all individual strategy sets, i.e.,*

$$\mathbb{S} = \mathbf{S}_1 \times \dots \times \mathbf{S}_N. \quad (\text{A.2})$$

Each element $S = [S_1, S_2, \dots, S_n] \in \mathbb{S}$ is said to be a strategy profile.

Often, one is only interested in a single player (e.g., the i^{th} player), so S can be rewritten as $S = (S_i, S_{-i})$ where S_{-i} is the joint strategy adopted by player i 's opponents.

Definition A.2. *For each player i , his/her utility function U_i is a function that maps each strategy profile S to a real number, that is, $U_i(S) : \mathbb{S} \mapsto \mathbb{R}$, where $\mathbb{R}$ is the set of real number.*

The notations $U_i(S)$ and $U_i(S_i, S_{-i})$ are both frequently used in standard texts on game theory.

Definition A.3. *The game $\mathcal{G}$ that consists of the player set $\mathcal{N}$, the strategy space $\mathbb{S}$*

and the utility functions U_i , $\forall i \in \mathcal{N}$ will be denoted as

$$\mathcal{G} = [\mathcal{N}, \mathbb{S}, \{U_i\}_{i \in \mathcal{N}}]. \quad (\text{A.3})$$

The strategy set can contain discrete actions (e.g., mobiles can choose to transmit or not); or continuous actions (e.g., transmission power level can be chosen from a continuous interval). For the discrete case (also known as finite-strategy games), it may be necessary to distinguish between a *pure strategy* from a *mixed strategy*. A pure strategy requires a player to play a certain move with absolute certainty, i.e., with probability 1. Meanwhile, a mixed strategy is defined as a portfolio of pure strategies with a predetermined probability distribution assigned to each component. In the context of wireless resource allocation, only pure strategies are considered so far in this thesis.

A.1.1 Dominant and dominated strategies

In some games, there are situations in which a certain strategy always gives a player better utility than any other strategies, independent of what actions the rest of the players take. Such a strategy is called a dominant strategy, and it dominates some other strategies. The strategies dominated by a dominant strategy are known as dominated strategies.

Definition A.4. For player i , $S_i \in \mathbf{S}_i$ is strictly dominant if and only if

$$\forall T_i \in \mathbf{S}_i, T_i \neq S_i : \quad U_i(S_i, S_{-i}) > U_i(T_i, S_{-i}) \quad \forall S_{-i} \in \mathbb{S} \setminus \mathbf{S}_i. \quad (\text{A.4})$$

If one changes the “>” sign in the above by the “ $\geq$ ”, “<” and “ $\leq$ ” signs, the definitions refer to *weakly dominant*, *strictly dominated* and *weakly dominated* strategies, respectively.

A.1.2 Nash equilibrium

Nash equilibrium (NE) is a crucial concept in predicting a game's outcome. By definition, a NE is a strategy profile such that if the opponents' strategies remain unaltered, no player would be tempted to move away from his/her current strategy.

Definition A.5. *The strategy profile $\sigma \in \mathbb{S}$ is a NE if and only if*

$$U_i(\sigma_i, \sigma_{-i}) \geq U_i(\sigma'_i, \sigma_{-i}) \quad \forall i \in \mathcal{N}. \quad (\text{A.5})$$

NE is perhaps the most important concept in game theory. At NE, no player is able to gain by deviating from the current point. Thus, it is seen as a “stable operating point” from the system perspective. Obtaining NE is often the ultimate objective in a resource allocation game. As a celebrated result in game theory, the existence of NE in certain games is established in the following theorems [16].

Theorem A.1. *A game has a NE if for all $i \in \mathcal{N}$, the set $\mathbf{S}_i$ is a nonempty, convex and compact subset of a Euclidean space, and the utility function U_i is continuous and quasi-concave in each S_i .*

Proof: Refer to Fudenberg *et al.* [16], pp. 29-30. ■

Theorem A.2. *Every finite strategy game has a NE in mixed strategy.*

Proof: Refer to Fudenberg *et al.* [16], pp. 34. ■

A.1.3 Pareto optimal strategies

Another significant concept encountered in game theory is that of Pareto optimality. A strategy profile is Pareto optimal if improving the utilities of some players would lead to decreased utilities for some other players. No strategy deviation from the original point can *mutually* improve the payoffs of all players.

Definition A.6. *The strategy profile $\sigma \in \mathbb{S}$ is Pareto optimal if and only if*

$$\nexists S \in \mathbb{S} : \quad \forall i \in \mathcal{N}, \quad U_i(S) \geq U_i(\sigma) \quad (\text{A.6})$$

with strict inequality for at least one i .

A.1.4 Example: The prisoners' dilemma

The game of prisoners' dilemma is very often used as an introductory example to illustrate the basic concepts in game theory. Its name was first given by W. Poundstone in his book of the same title [111]. The description of the game is as follows.

“Suppose that two suspects are arrested for a crime, and questioned separately. If they both keep quiet (*cooperate* with each other), they will both go to prison for a year. If one suspect supplies evidence (*defects*) then he will be freed, and the other one who *cooperates* will be imprisoned for six years. If both *defect* then they will both be imprisoned for four years.”

Table A.1 depicts the game in its non-cooperative, strategic-form, one-shot (i.e., only played once) version. The game elements are $\mathcal{N} = \{1, 2\}$, $\mathbf{S}_1 = \mathbf{S}_2 = \{C, D\}$, $\mathbb{S} = \{(C, C), (C, D), (D, C), (D, D)\}$, $U_1 : \mathbb{S} \mapsto \{-1, -6, 0, -4\}$ and $U_2 : \mathbb{S} \mapsto \{-1, 0, -6, -4\}$. Note that the 4-tuple of values $(-6, -4, -1, 0)$ used in the tables can be changed, provided that the increasing order among them is preserved.

Table A.1: Payoff matrix for prisoners' dilemma.

		Player 2	
		<i>C</i>	<i>D</i>
Player 1	<i>C</i>	(-1,-1)	(-6,0)
	<i>D</i>	(0,-6)	(-4,-4)

Lemma A.1. *In the prisoners' dilemma above, D is the strictly dominant strategy for both players; and C is the strictly dominated one. (D, D) is the NE, representing the case where both non-cooperatively defect, which is however not Pareto optimal.*

The Pareto optimal strategy is (C, C) , where both prisoners cooperate.

Proof: The proof can be obtained directly via definitions. ■

In the following section, some game-theoretical mechanisms are discussed which are of interest in the context of wireless resource allocation and in the thesis.

A.1.5 Iterative gameplay vs. Repeated games

It is worth mentioning that iterative gameplay is not a formal concept in game theory, but instead a technique frequently adopted in most game-theoretical algorithms in wireless resource allocation. The basic idea is that for a one-shot game with incomplete information, NE may not be obtained in just one simultaneous round of play since a high degree of uncertainty is involved. However, the game may be iterated where players update their strategies based on their observations of other players' actions until a NE is reached. Based on the current game outcome, players may gradually improve their utilities towards a more stable and optimal solution by adopting a certain plan of how to adopt their strategies. Such iterations and strategy adaptations are called *game dynamics*.

The underlying principles of iterative play are somehow different from *repeated games*, a branch in game theory. In repeated games, an original one-shot game is also repeated in several stages, and the strategies adopted over time can also be learned from history. Rigorous mathematical treatment of repeated games often introduces a discount factor $0 < \alpha < 1$ into the original utility function in such a way that at a later t^{th} stage, the expected payoff for player i corresponding to a strategy profile S becomes $U_{i,t}(S) = \alpha^t U_i(S)$ where $U_i(S)$ is the payoff that player i would get in the one-shot version. Therefore, the accumulated payoff if the game is played from stage 0 to stage T is $U_i = \sum_{t=0}^T U_{i,t}$. In repeated games, one can define *subgame perfect NE* and invoke *Folk theorem* [16] to find the solutions.

As a matter of fact, most wireless resource allocation problems in the literature

do have the game repeated in multiple stages. However, they are different from repeated games as the discount factor and the accumulated utilities are not used; and the main objective is only to achieve *convergence* to NE. Hence, the scope of this discussion can be restricted to games where the game convergence dynamics are based only on knowledge of the current state of the game. Folk theorem does not hold for these *myopic games* [58]. As such, this discussion makes a point in distinguishing the concept of *iterative gameplay* in wireless resource allocation context against the *repeated games* traditionally known in game theory.

A.1.6 Best-response and better-response dynamics

Other key concepts encountered in the thesis are the best-response and better-response dynamics, which are two important mechanisms in iterative gameplay in order to seek NE convergence.

In game theory, the *best response* of a player given his opponents' strategies [16] is what he/she should play in order to maximize his/her utility.

Definition A.7. Consider a game $\mathcal{G}$ as in (A.3). For each player i , the best response for him/her is a set-valued mapping $B_i(S_{-i}) : (\mathbb{S} \setminus \mathbf{S}_i) \mapsto \mathbf{S}_i$ such that

$$B_i(S_{-i}) = \{S_i \in \mathbf{S}_i \mid U_i(S_i, S_{-i}) \geq U_i(T_i, S_{-i}) \quad \forall T_i \neq S_i\}. \quad (\text{A.7})$$

In the context of iterative gameplay, the best-response dynamic is defined as the following rule [58]:

Definition A.8. Player $i \in \mathcal{N}$ will select the best-response strategy T_i over the current strategy S_i if and only if

$$T_i = \arg \max_{T_i \in \mathbf{S}_i} U_i(T_i, S_{-i}). \quad (\text{A.8})$$

Best-response dynamic is often implemented in sequential games in which players take turns in a determined, round-robin order. It requires an exhaustive search over

the strategy set $\mathbf{S}_i$ for the best option, which may incur a high complexity order in games with large strategy sets. Alternatively, another mechanism which requires less searching is also considered, i.e., the better-response dynamic [58] which is defined as follows.

Definition A.9. *Player i 's better-response strategy is T_i if and only if T_i is any random strategy such that*

$$U_i(T_i, S_{-i}) > U_i(S_i, S_{-i}). \quad (\text{A.9})$$

A.1.7 Price of anarchy

Price of anarchy (PoA) is a concept in game theory, which literally means the amount of damage suffered by the members of a system (i.e., the players) due to the absence of a central authority. Specifically, it measures the discrepancy in efficiency of the game behaviors between the socially Pareto optimal point due to centralized behavior and the one due to players' selfish behaviors (which is the NE, or any other allocation point obtained by a distributed method) [19]. Often, a ratio between the performance measures of the two points is computed. In that sense, PoA is seen as a metric to evaluate how efficient an allocation is, compared to the social optimum.

The performance measure is also known in the literature as the *welfare function*. Assuming that to each strategy profile $S = (S_i, S_{-i}) \in \mathbb{S}$, one can assign a real number $\Theta(S)$ as a measure of efficiency, then the welfare function is mathematically a mapping $\Theta : \mathbb{S} \mapsto \mathbb{R}$. A common choice for the welfare function is the *utilitarian welfare*, defined as follows.

Definition A.10. *Given $S \in \mathbb{S}$, the utilitarian welfare function $\Theta : \mathbb{S} \mapsto \mathbb{R}$ is simply the sum of all utility functions, i.e.,*

$$\Theta(S) \triangleq \sum_{i=1}^N U_i(S). \quad (\text{A.10})$$

Profile S is preferred to T if $\Theta(S) > \Theta(T)$ for a maximization game, or $\Theta(S) < \Theta(T)$ for a minimization game.

Now, let $\hat{S} \in \mathbb{S}$ be the socially optimal point with respect to the welfare function Θ , i.e., $\Theta(\hat{S}) = \max_{S \in \mathbb{S}} \Theta(S)$ for maximization, or $\Theta(\hat{S}) = \min_{S \in \mathbb{S}} \Theta(S)$ for minimization. Also, let $S \in \mathbb{S}$ be a particular outcome of the game, e.g., a NE. Then, by convention, the PoA is defined separately for these two different scenarios as follows.

Definition A.11. Let $PoA_{\max}(S)$ and $PoA_{\min}(S)$ be the PoAs of strategy profile S for utility-maximization and utility-minimization games, respectively. Then,

$$PoA_{\max}(S) = \frac{\Theta(\hat{S})}{\Theta(S)}, \quad (\text{A.11a})$$

$$PoA_{\min}(S) = \frac{\Theta(S)}{\Theta(\hat{S})}. \quad (\text{A.11b})$$

A.2 Potential Games

Within the context of spectrum allocation games where convergence to NE is the primary consideration, potential games are very useful since they can ensure the existence of pure-strategy NEs and their convergence via some means. They were proposed originally by Monderer *et al.* [57]. Here, the concept and properties of potential games were reviewed.

One can distinguish two types of potential games, namely, *exact* (or *cardinal*) potential games and *ordinal* potential games. Consider a game $\mathcal{G}$ as in (A.3). Their definitions are given as follows.

Definition A.12. $\mathcal{G}$ is an exact potential game if and only if an exact potential function $F(S) : \mathbb{S} \mapsto \mathbb{R}$ exists such that for any player i ,

$$U_i(T_i, S_{-i}) - U_i(S_i, S_{-i}) = F(T_i, S_{-i}) - F(S_i, S_{-i}) \quad \forall S_i, T_i \in \mathbf{S}_i, \forall S_{-i} \in \mathbb{S} \setminus \mathbf{S}_i. \quad (\text{A.12})$$

Basically, the change in a single player's utility due to his/her own strategy devia-

tion only results in exactly the same amount of change in the exact potential function.

Definition A.13. $\mathcal{G}$ is an ordinal potential game if and only if an ordinal potential function $\Phi(S) : \mathbb{S} \mapsto \mathbb{R}$ exists such that for any player i ,

$$\text{sgn}[U_i(T_i, S_{-i}) - U_i(S_i, S_{-i})] = \text{sgn}[\Phi(T_i, S_{-i}) - \Phi(S_i, S_{-i})] \quad \forall S_i, T_i \in \mathbf{S}_i, \forall S_{-i} \in \mathbb{S} \setminus \mathbf{S}_i. \quad (\text{A.13})$$

In ordinal potential games, the change in the (ordinal) potential function only needs to be of the same sign as that of the individual utility function, unlike in exact potential games. Moreover, it implies that all exact potential games are ordinal potential games.

All types of potential games possess very attractive properties regarding their NEs, which were proven in [57] and restated here with some modifications.

Theorem A.3. *If $\mathcal{G}$ is a potential game with a finite strategy space then*

- *It admits a pure-strategy NE.*
- *All the NEs of $\mathcal{G}$ correspond to the maxima of the potential function F or Φ .*
- *Sequential dynamics such as best/better responses will converge to a pure-strategy NE (originally referred to as the existence of Finite Improvement Path).*

Proof: Refer to Monderer *et al.* [57]. ■

As a concluding note, let us look at one example of an exact potential game.

Proposition A.1. *The prisoners' dilemma given in Section A.1 is an exact potential game with the exact potential function $F : \mathbb{S} \mapsto \mathbb{R}$, where $F(C, C) = k - 3$, $F(C, D) = F(D, C) = k - 2$ and $F(D, D) = k$, for any arbitrary real number k .*

Proof: The proof is intuitive. One just has to list all possible strategy switches in the prisoners' dilemma and see if the utility change matches the change in the proposed potential function. For instance, for the switch (C, C) to (D, C) (here,

player 1 is considered), $U_1(C, C) - U_1(D, C) = (-1) - 0 = -1$. Correspondingly, $F(C, C) - F(D, C) = (k - 2) - (k - 3) = -1$. The rest can be verified similarly. ■

A.3 Differential Games

Differential games [20] are a special group of problems in game theory. Specifically, they can be classified as dynamic infinite-strategy continuous-time state-space games. They are *dynamic*, as opposed to *static* games, since the gameplay does not happen in a single moment but recurs over a time period, in which previous outcomes and history can be observed and used in the decision process. Moreover, such dynamic games can be played either in several stages, where they are called *discrete-time* or sequential games, or on the other hand, in *continuous time*. The finiteness of the strategy space, and subsequently the manner of evolution of the game's history and action paths, provide yet another way of classifying those games. Unlike dynamic games with *finite* strategy space which can be formulated in extensive form and hence represented by a tree structure, dynamic games with *infinite* (continuum) strategy space have their decision process characterized by some parameters known as *state variables*. The evolution of such variables is governed by either *difference* (discrete-time) or *differential equations* (continuous-time), hence the name 'differential games'. A close analogy with dynamical systems in control theory can be noticed, but here, there are N controls and objectives for N players instead of a single optimization problem. As a matter of fact, various techniques in optimal control [24] can be applied in differential games.

Let us consider an N -player game $\mathcal{G}$, with the set of player $\mathcal{N} = \{1, 2, \dots, N\}$. Furthermore, the game is assumed to take place over a time interval $[0, T]$. When $T \rightarrow \infty$, the game is said to be played over an infinite horizon. Let $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_M]^T$ be the (column) vector of M state variables and $\mathbb{X}$ be the domain of $\mathbf{x}$, called the state space, then $\mathbf{x}(t) \in \mathbb{X} \subseteq \mathbb{R}^M$, for each $t \in [0, T]$. Similar to dynamical systems, the

state variables follow a system of M differential equations, called the state trajectory, i.e.,

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}(t)}{dt} = \mathbf{f}(t, \mathbf{x}, b_1, b_2, \dots, b_N), \quad \mathbf{x}(0) = \mathbf{x}_0, \quad (\text{A.14})$$

where $\mathbf{f} = [f_1 \ f_2 \ \dots \ f_M]^T$ is a vector-valued function. b_i denotes the strategy (or control input) of player $i \in \mathcal{N}$. Each b_i is a function of time t over the game interval, and additionally can be dependent or independent of the state $\mathbf{x}$, whose various types will be explained shortly. For each $t \in [0, T]$, $b_i(t) \in S_i$ where S_i denotes the strategy set of player i . Thus, with b_{-i} denoting the strategies of player i 's opponents, a strategy profile (b_i, b_{-i}) belongs to $\mathbb{S} = S_1 \times S_2 \times \dots \times S_N$, the joint strategy space. Each S_i is a subset of $\mathbb{R}$, or $\mathbb{R}^{k_i}$ if b_i is a k_i -element vector. From (A.14), it can be seen that the movement of the state vector at any time depends not only on the current state but also the current actions (controls) of all the players.

In differential games, one needs to specify how a player adapts his/her strategy b_i over time. This issue is known as the classification of the player's information structure [20]. In state-space games, the history of the game is available via the state variables $\mathbf{x}$. A player can choose to base his/her strategy on the current value of $\mathbf{x}$, past values of $\mathbf{x}$, or without $\mathbf{x}$. A formal definition of various strategy types is then given as follows.

Definition A.14. *In differential games, the strategy b_i of player i can be classified as*

- (i) Open loop: $b_i \equiv b_i(t)$ is solely a function of time.
- (ii) Closed loop: $b_i \equiv b_i(t, \mathbf{x}(\tau))$ for $\tau \in [0, T]$. In other words, b_i is dependent on time as well as the past and current states.
- (iii) Closed-loop Memoryless (Markov): $b_i \equiv b_i(t, \mathbf{x}(t))$ is dependent only on time and the current state (having no memory of past states).
- (iv) Stationary Markov: $b_i \equiv b_i(\mathbf{x}(t))$ can be considered solely as a function of the current state.

Next, let us define the payoff or utility functions. It is assumed that at a particular instant t , the instantaneous payoff of player i is given by $U_i(t, \mathbf{x}, b_i, b_{-i})$. Mathematically, U_i is the mapping $[0, T] \times \mathbb{X} \times \mathbb{S} \mapsto \mathbb{R}$. Over $[0, T]$, player i seeks to maximize his/her total accumulated payoff $J_i(\mathbf{x}, b_i, b_{-i}) : \mathbb{X} \times \mathbb{S} \mapsto \mathbb{R}$, discounted at a rate $r_i \geq 0$, given by

$$\begin{aligned} \max_{b_i} J_i(\mathbf{x}, b_i, b_{-i}) &= \int_0^T U_i(t, \mathbf{x}, b_i, b_{-i}) e^{-r_i t} dt + q_i(\mathbf{x}(T)), \\ \text{subject to } \dot{\mathbf{x}} &= f(t, \mathbf{x}, b_1, b_2, \dots, b_N), \quad \mathbf{x}(0) = \mathbf{x}_0, \end{aligned} \quad (\text{A.15})$$

where the term $q_i(x(T))$ denotes the terminal payoff at $t = T$. When $T \rightarrow \infty$, it is assumed that $q_i(\mathbf{x}) = 0$ for all $\mathbf{x}$.

The Markov strategies are of particular concern in this thesis. If all players use Markov strategies, then (A.15) is in fact an optimal control problem [20]. Solutions of the optimal control problems correspond to the Markov NEs of the game. It is also known that stationary Markov feedback NEs are *subgame-perfect* [20]. The definition of such NEs is as follows.

Definition A.15. A stationary Markov strategy profile $(b_1^*, b_2^*, \dots, b_N^*) \in \mathbb{S}$, $b_i^* = b_i^*(\mathbf{x}(t))$ is a stationary Markov feedback NE if for any $\mathbf{x} \in \mathbb{X}$,

$$J_i(\mathbf{x}, b_i^*, b_{-i}^*) \geq J_i(\mathbf{x}, b_i', b_{-i}^*), \quad \forall b_i' \neq b_i^*, \forall i \in \mathcal{N}. \quad (\text{A.16})$$

Using techniques from optimal control and dynamic programming, a characterization of the stationary Markov feedback NEs can be obtained. In particular, for infinite-horizon games, the conditions for such NEs are stated in the following theorem [21].

Theorem A.4. The differential game discussed herein admits a stationary Markov strategy profile $(b_1^*, b_2^*, \dots, b_N^*)$, $b_i^* = b_i^*(\mathbf{x})$ as a stationary Markov feedback NE if for any i , there exists a continuously differentiable function $V_i(\mathbf{x}) : \mathbb{X} \mapsto \mathbb{R}$ (the “value

function”) that satisfies

$$\begin{aligned} rV_i(\mathbf{x}) &= \max_{b_i} \{U_i(\mathbf{x}, b_i, b_{-i}^*) + (\nabla V_i)^T \mathbf{f}(\mathbf{x}, b_i, b_{-i}^*)\} \\ &= U_i(\mathbf{x}, b_i^*, b_{-i}^*) + (\nabla V_i)^T \mathbf{f}(\mathbf{x}, b_i^*, b_{-i}^*). \end{aligned} \quad (\text{A.17})$$

Proof: Refer to Dockner *et al.* [21], pp. 34. ■

In (A.17), the gradient ∇V_i results when differentiating V_i with respect to each element of $\mathbf{x}$. Eq. (A.17) is known as the Hamilton-Jacobi-Bellman (HJB) equation.

Appendix B: Related RRM Schemes

In Appendix B, RRM schemes which have been used for comparison in this thesis are described. These include the SINR maximization with power pricing (SM/PP) scheme (Chapter 4), the OFDMA iterative waterfilling (OFDMA/WF) scheme (Chapter 4) and the waterfilling with power pricing (WF/PP) scheme (Chapter 5).

B.1 The SM/PP Scheme

In formulating the subcarrier assignment games $\mathcal{G}_2$ and $\mathcal{G}_2'$ for the distributed OFDMA system defined in Chapter 4, one can adopt a different utility function for the transmit-receive pair other than the interference-sum utility function (4.12). One such alternative is to look at the SINR maximization objective (e.g., [96]). In the SM/PP scheme, the utility function of a player is defined as a “net utility” which consists of two parts: the reward in terms of the player’s achieved SINR and the cost/pricing in terms of power consumption.

Consider the same ad-hoc OFDMA system model presented in Chapter 4 with N players and K subcarriers where player i selects K_i subcarriers, using the same set of notations such as $\mathbf{A}$, $\mathbf{G}$ and $\mathbf{P}$ as the assignment, channel gain and power matrices, respectively. Then, the alternative utility function, which is modified for this OFDMA

context, is given by

$$U_i = \sum_{k=1}^K (a_{ik} \ln \gamma_{ik}) - \alpha \sum_{k=1}^K (a_{ik} p_{ik}) \quad (\text{B.1})$$

where γ_{ik} is the SINR, given by (4.2); and α is a positive pricing coefficient.

Consequently, the same distributed optimization problem as in (4.22) applies in the SM/PP scheme, i.e.,

$$\begin{aligned} & \max_{\mathbf{a}_i^T} U_i, \quad \forall i \in \mathcal{N} \\ \text{s.t.} \quad & \begin{cases} \mathbf{a}_i^T \in \{0, 1\}^K \setminus \{\mathbf{0}\}, & \mathbf{a}_i^T \mathbf{1} = K_i, \\ \mathbf{p}_i^T(\mathbf{A}) \geq 0, & \mathbf{p}_i^T \mathbf{1} \leq P_{\max}. \end{cases} \end{aligned} \quad (\text{B.2})$$

To find the NE for this game, the same procedures for $\mathcal{G}'_2$ will be executed. By doing so, one obtains a fair comparison between this scheme and the proposed game.

- (i) First, the ANSS scheme (Algorithm 4.1) is run which lets players select their required number of subcarriers.
- (ii) Next, players follow the sequential best-response gameplay (Algorithm 4.2) in order to converge to one of the NEs.

In the simulation settings, the SM/PP scheme shares the parameters provided in Table 4.1. In addition, the cost α is set at 40 W^{-1} .

B.2 The OFDMA/WF Scheme

The OFDMA/WF method is based on the waterfilling algorithm and has been extensively used for allocating power and/or rate in OFDMA networks, such as in [37, 38]. The waterfilling problem is a problem in convex optimization (see [22], pp. 245), whose solution is analogous to pouring water into a container of uneven grounds, so that more water should occupy regions of greater depths. The same idea applies when

a user allocates a total power to multiple channels of different gains and CCI levels in order to maximize its transmission capacity. With multiple players, iterative waterfilling can be used where players sequentially take turn to apply the best-response subcarrier assignment strategy.

Here, the same set of notations in Chapter 4 is used. Player i will not use its utility function U_i in (4.12) but try to maximize its total rate $R_i = \sum_{k=1}^K R_{ik}$ with R_{ik} given in (4.3). A key different constraint for the OFDMA/WF method is the fixed total power constraint, i.e., $\mathbf{p}_i^T \mathbf{1} = P_{\max}$, $\forall i$. Thus, the OFDMA/WF scheme considers the following distributed optimization:

$$\begin{aligned} \max_{\mathbf{a}_i^T, \mathbf{p}_i^T} R_i &= B \sum_{k=1}^K \log_2 \left[1 + \frac{a_{ik} p_{ik} g_{ii}^k}{\Gamma(\sum_{j=1, j \neq i}^N a_{jk} p_{jk} g_{ji}^k + \sigma^2)} \right] \\ \text{s.t. } &\begin{cases} \mathbf{a}_i^T \in \{0, 1\}^K \setminus \{\mathbf{0}\}, & \mathbf{a}_i^T \mathbf{1} = K_i, \\ \mathbf{p}_i^T(\mathbf{A}) \geq 0, & \mathbf{p}_i^T \mathbf{1} = P_{\max}. \end{cases} \end{aligned} \quad (\text{B.3})$$

The same ANSS scheme (Algorithm 4.1) is used to select the required number of subcarriers K_i so as to provide a compatible and fair comparison with the proposed scheme. Thereafter, each player takes turn sequentially to solve its respective optimization problem, which will be repeated until convergence. For a player i , the OFDMA/WF method is carried out in two steps, namely, subcarrier assignment, followed by power allocation.

In the first step, player i selects K_i subcarriers, which can be chosen using a best-response manner as follows

$$\mathbf{a}_i^{*T} = \arg \max_{\mathbf{a}_i^T \in \{\mathbf{a}_i^T \mid \mathbf{a}_i^T \mathbf{1} = K_i\}} R_i. \quad (\text{B.4})$$

The best-response method may require advanced knowledge of the values of power allocated to calculate R_i . Since the player can only utilize the currently available knowledge which are the values of CCI on each subcarrier, a simpler strategy is to

select the subcarriers such that the total CCI is minimized, i.e.,

$$\mathbf{a}_i^{*T} = \arg \min_{\mathbf{a}_i^T \in \{\mathbf{a}_i^T | \mathbf{a}_i^T \mathbf{1} = K_i\}} \left(\sum_{j=1, j \neq i}^N a_{jk} p_{jk} g_{ji}^k \right). \quad (\text{B.5})$$

In the next step, once the K_i subcarriers are chosen, player i can carry out water-filling over these subcarriers. The original problem (B.3) is now reduced to

$$\begin{aligned} & \max_{\mathbf{p}_i^T} R_i \\ & \text{s.t. } \mathbf{p}_i^T(\mathbf{A}) \geq 0, \quad \mathbf{p}_i^T \mathbf{1} = P_{\max}. \end{aligned} \quad (\text{B.6})$$

Problem (B.6) is a standard concave optimization problem and can be solved via the Karush-Kuhn-Tucker (KKT) conditions [22]. The optimal power level is thus given by

$$p_{ik}^* = \begin{cases} 0 & \text{if } a_{ik} = 0 \\ \max \left[0, \mu_i - \frac{\Gamma(\sum_{j=1, j \neq i}^N a_{jk} p_{jk} g_{ji}^k + \sigma^2)}{g_{ii}^k} \right] & \text{if } a_{ik} = 1 \end{cases} \quad (\text{B.7})$$

where the Lagrangian multiplier μ_i , also known as the water level, is introduced when obtaining the KKT conditions.

In the simulation settings, the OFMDA/WF scheme shares the parameters provided in Table 4.1.

B.3 The WF/PP Scheme

The WF/PP method is a variation of the standard OFDMA waterfilling technique where the utility function of a player has an additional power pricing term. It was adopted in [38] to study the multi-cell OFDMA scenario. The major difference between the cellular system and the ad hoc system is that the BS's strategy involves

allocating non-overlapping subcarriers to different MSs. This is achieved via *greedy channel assignment*.

Assuming the same system model and parameter definitions as in Chapter 5, each player in the WF/PP scheme encounters the following problem

$$\begin{aligned} \max_{\mathbf{A}_i, \mathbf{p}_i^T} U_i &= B \sum_{k=1}^K \log_2 \left(1 + \frac{\gamma_{im}^k}{\Gamma} \right) - c \sum_{k=1}^K p_{ik} \\ \text{s.t.} \quad &\begin{cases} \sum_{m \in \Omega_i} a_{im}^k \leq 1 \quad \forall k, & \sum_{k=1}^K a_{im}^k \leq 1 \quad \forall m \in \Omega_i, \\ \mathbf{p}_i^T(\mathbf{A}) \geq 0, & \mathbf{p}_i^T \mathbf{1} \leq P_{\max}. \end{cases} \end{aligned} \quad (\text{B.8})$$

where B is the subcarrier bandwidth; $\mathbf{A}_i$ is the 2-dimensional submatrix of $\mathbf{A}$ that contains the assignment parameters under player i ; and c is the pricing coefficient.

Similarly, the WF/PP scheme can also be done in two steps. Firstly, by assuming that power is known, the BS will greedily assign the subcarriers, i.e., assigning subcarrier k to the MS that obtains the highest rate on that subcarrier. Note that at the beginning, the BS can assume equal power per subcarrier; and a subsequent iteration can use the power obtained from the last round. The greedy method can be stated as

$$a_{im}^k = \begin{cases} 1 & \text{if } k = \tilde{k} = \arg \max_k C_{im}^k, \\ 0 & \text{otherwise.} \end{cases} \quad (\text{B.9})$$

where C_{im}^k is given in (5.4).

Once the subcarriers are assigned to the MSs, the problem (B.8) is reduced to

$$\begin{aligned} \max_{\mathbf{p}_i^T} U_i \\ \text{s.t.} \quad \mathbf{p}_i^T(\mathbf{A}) \geq 0, \quad \mathbf{p}_i^T \mathbf{1} \leq P_{\max}. \end{aligned} \quad (\text{B.10})$$

The KKT conditions can also be introduced to find the optimal power allocation

in (B.10) since U_i is quasi-concave in p_i . As a result,

$$p_{ik} = \max \left[0, \frac{B}{(c + \mu_i) \ln 2} - \frac{\Gamma(\sum_{j=1, j \neq i}^N a_{jk} p_{jk} g_{ji}^k + \sigma^2)}{g_{ii}^k} \right] \quad (\text{B.11})$$

where μ_i denotes the Lagrangian multiplier such that $\mu_i \geq 0$ and $\mu_i(\mathbf{p}_i^T \mathbf{1} - P_{\max}) = 0$.

In the simulation settings, the WF/PP scheme shares the parameters provided in Table 5.2. In addition, the cost c is set at 10 Mbps/W.

Author's Publications

Journal Articles

- (i)* **Q. D. La**, Y. H. Chew and B. H. Soong, "An Interference-Minimization Potential Game for OFDMA-based Distributed Spectrum Sharing Systems," *IEEE Trans. Veh. Technol.*, Vol. 60, No. 7, pp. 3374-3385, Sep. 2011.
- (ii)* **Q. D. La**, Y. H. Chew and B. H. Soong, "Performance Analysis of Downlink Multi-cell OFDMA Systems based on Potential Game," *IEEE Trans. Wireless Commun.*, Vol. 11, No. 9, pp. 3358-3367, Sep. 2012.
- (iii)* **Q. D. La**, Y. H. Chew and B. H. Soong, "Dynamic Spectrum Allocation Game via Oligopolistic Market Competition under Spectrum Broker," *IEEE Trans. Wireless Commun.*, submitted for publication.
- (iv) S. Han, B. H. Soong and **Q. D. La**, "Power Control Based on Subcarrier Exclusion to Mitigate Downlink Cross-Tier Interference in OFDMA Tiered Networks," *IEEE Wireless Commun. Lett.*, Vol. 2, No. 2, pp. 179-182, Apr. 2013.

Book Chapter

- (i) **Q. D. La**, B. H. Soong, Y. H. Chew and W. H. Chin, "Chap. 4: Game Theory and OFDMA Resource Allocation," *Orthogonal Frequency Division Multiple Access: Fundamentals and Applications*, (Eds: T. Jiang, L. Song and Y. Zhang). Boca Raton: Auerbach, pp. 67-100, 2010.

Conference Proceedings

- (i)* **Q. D. La**, Y. H. Chew, W. H. Chin and B. H. Soong, "A Game Theoretic Distributed Dynamic Channel Allocation Scheme with Transmission Option," in *Proc. IEEE Milcom*, pp. 1-7, Nov. 2008.
- (ii)* **Q. D. La**, Y. H. Chew and B. H. Soong, "An Interference Minimization Game Theoretic Subcarrier Allocation Algorithm for OFDMA-based Distributed Systems," in *Proc. IEEE Globecom*, pp. 1-6, Nov. 2009.

* Papers that are related to this thesis.

- (iii)* **Q. D. La**, Y. H. Chew and B. H. Soong, "Subcarrier Assignment in Multi-cell OFDMA Systems via Interference Minimization Game," in *Proc. IEEE WCNC*, pp. 1321-1325, Apr. 2012.
- (iv) S. Han, B. H. Soong and **Q. D. La**, "Interference Mitigation in Resource Allocation for OFDMA-based Macro/Femtocell Two-tier Wireless Networks," in *Proc. IEEE International Symposium on Broadband Multimedia Systems and Broadcasting*, pp. 1-5, Jun. 2012.
- (v) S. Han, B. H. Soong and **Q. D. La**, "Subcarrier Allocation in Multi-Cell OFDMA Wireless Networks with Non-Coherent Base Station Cooperation and Controllable Fairness," in *Proc. IEEE PIMRC*, pp. 524-529, Sep. 2012.

* Papers that are related to this thesis.

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