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Neonatal Seizure Monitoring and Representation

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Abstract

Approximately 1 – 3/1000 newborn babies in the US experience some form of seizure activities. These seizure activities, in general, indicate that some form of brain abnormalities are present in the babies. Prolonged seizures can result in brain damage or even death. Therefore, it is important to detect and control seizure activities promptly to avoid permanent cerebral damage.

Clinical signs of seizures in adults are well defined and easily understood. They are usually accompanied by uncontrollable body jerking, tonic flexion of the muscles, repetitive winking or fluttering of eyelids. In newborns, however, the clinical signs are not obvious and can easily be missed out, resulting in undetected seizures. In this case, it is necessary to maintain constant and careful watch over the baby in order to detect seizures and administer the correct treatment and therapy. For this, monitoring brain activity through the electroencephalogram (EEG) plays an important role in the detection of seizure in a newborn.

However, the EEG is difficult to interpret, often requiring an expert in neonatal EEG to detect the presence of seizure. The long duration EEG recordings that are required to monitor the medical conditions of the neonates are usually corrupted by various artifacts. In addition, the accurate acquisition of EEG requires a level of expertise not always available in the Neonatal Intensive Care Unit (NICU). Since this is the primary method neurophysiologists use in diagnosing neonatal seizures, our research on neonatal seizure monitoring and representation has focused entirely on the EEG.

In a typical EEG monitoring system, the EEG clinician closely monitors the EEG recording for abnormalities such as seizures and takes the necessary actions. The recordings will be stored in a digitised form on a harddisk. Finally, the neurophysiologists annotate all activities such as artifacts and seizures in the entire recording. In order to mimic the process, our neonatal seizure monitoring and representation system is designed to comprise three automatic modules, namely the modelling, acquisition and compression of the EEG signals, the removal of artifacts and finally the detection of seizures.

In this thesis, we want to establish a new EEG signal model which departs from the traditional EEG models and set the groundwork for expanding the finite rate of innovation sampling theory into the field of EEG signal processing. We also envisage the potential of identification and formulation of new EEG signal processing techniques such as artifact removal based on the FRI framework. The EEG signals are modeled as signals with finite rate of innovation (FRI) and are effectively represented by a small set of Fourier coefficients corresponding to the signals' rate of innovation. Using methods developed in the FRI sampling theory, we reconstruct the original signals with low reconstruction errors. By modeling and acquiring the EEG signals at the rate of innovation, we inherently achieve signal compression. Hence our signal acquisition method allows us to represent the databases with a small set of data samples while retaining the morphologies of the EEG signals, unlike traditional compression methods which acquires many data samples and later discards redundant ones. It yielded promising results such as good compression ratio and low distortions at a low computational cost.

Artifacts are commonly found in EEG signals. Very often, correct diagnoses of abnormalities in EEG signals are not achievable because their patterns are corrupted by these artifacts. As such, reliable artifact removal methods are crucial to enable proper diagnoses of neurological conditions. In the second module of this thesis, we propose a novel EEG artifact removal method that models the EEG and artifact signals as FRI signals.

Three types of artifacts, namely electrostatic discharges, electrooculographic and electromyographic artifacts are studied and it is shown that EEG signals corrupted with a combination of these three artifacts can be recovered at a very low percentage root difference.

Lastly, a new time-frequency seizure detection method based on the local-energy variance measure of the Stockwell transform is proposed. The algorithm's performance is evaluated on neonatal EEG seizure signals and compared to some commonly cited neonatal seizure detection methods. It is shown that the local-energy variance measure of the Stockwell transform is a simple and yet effective feature to detect seizures with both high specificity and sensitivity. Our results indicate that slow wave and sharp spike seizures are detected with a sensitivity of 90.6%, a specificity of 92.2% and a false detection rate of 7.76%. Furthermore, our system achieved a positive predictive value of 92.6% and a negative predictive value of 90.2%. In the event when the dataset is corrupted by artifacts, we also showed that our system achieved a sensitivity of 87.65%, a specificity of 70.42% and a false detection rate of 29.58%. Our seizure detection algorithm performed very similarly on both original and FRI modelled datasets. The minor differences in the variances of the local-energies of the Stockwell spectra did not contribute to wrong or false detections on FRI modelled datasets. It is also demonstrated that the specificity, false detection rate, positive and negative predictive values of our seizure detection method improved upon applying our artifact removal scheme onto the Cork dataset. Hence we can conclude that our seizure detection method is robust and our FRI EEG model preserved the morphologies and frequency contents of the original signals.

In summary, the novel contribution of this thesis to the field of neonatal EEG signal processing is the successful development and application of sophisticated algorithms based on the finite rate of innovation theory and the Stockwell transform to the solutions of EEG signal compression, artifact removal and seizure detection.

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List of Abbreviations

Abbreviations

AR	Auto-Regressive
ARMA	Auto-Regressive, Moving Average
BSS	Blind Source Separation
BD	B-Distribution
BJD	Born-Jordan Distribution
CC	Cross Correlation
CTFS	Continuous Time Fourier Series
CWT	Continuous Wavelet Transform
CW	Choi-Williams Distribution
DFT	Discrete Fourier Transform
DFSV	Distribution Function of the Singular Variables
EOG	Electrooculogram
ECG	Electrocardiogram
EEG	Electroencephalogram
EMG	Electromyogram
FDR	False Detection Rate
FFT	Fast Fourier Transform
FN	False Negatives
FP	False Positives
FRI	Finite Rate of Innovation
GA	Gestational Age
GIF	Generalised Instantaneous Frequency
HSSI	Histogram of Successive Spike Intervals
HFC	High Frequency Coefficients
IC	Independent component
IF	Instantaneous Frequency
LFC	Low Frequency Coefficients
MAXERR	Maximum Error
MPA	Markov Process Amplitude
NICU	Neonatal Intensive Care Unit

NMSE	Normalised Mean Square Error
NPV	Negative Predictive Value
PARE	Peak Amplitude Related Error
PRD	Percent Root Difference
PPV	Positive Predictive Value
QTFD	Quadratic Time-Frequency Distribution
RID	Reduced Interference Distribution
ROC	Receiver Operating Characteristics
RMSE	Root Mean Square Error
SENS	Sensitivity
SPEC	Specificity
ST	Stockwell Transform
STFT	Short Time Fourier Transform
SVD	Singular Value Decomposition
TF	Time-Frequency
TFD	Time-Frequency Distributions
TFSA	Time-Frequency signal Analysis
TN	True Negatives
TP	True Positives
WVD	Wigner-Ville Distribution
ZAM	Zhao-Atlas-Mark Distribution

List Symbols and Notations

Symbols and notations

t	time variable
f	frequency variable
$*$	convolution operator
$x(t)$	1-dimensional time signal
ρ	rate of innovation of a signal
τ	period of a signal
f_m	maximum non-zero frequency of a bandlimited signal
f_s	sampling frequency of a signal
t_k	time locations of a stream of Dirac pulses
c_k	weights of a stream of Dirac pulses
$h_B(t)$	sampling kernel
B	bandwidth of sampling kernel
$x_D(t)$	a stream of Dirac pulses
$\delta(t)$	delta function
N	number of uniform sampling locations
T	sampling interval
$y(t)$	convolution of $x(t)$ and the sampling kernel
y_n	sampled values of $y(t)$
$X_D[m]$	continuous-time Fourier series of $x_D(t)$
$A[m]$	coefficients of the annihilating filter
L	bandlimit of a signal
$x_{BL}(t)$	bandlimited signal
$x_{NUS}(t)$	a nonuniform spline signal
R	degree of the nonuniform spline signal
$S(t, f)$	Stockwell transform with time and frequency variables
$w(t, f)$	analysis window of the Stockwell transform
$Hx(t)$	Hilbert transform of $x(t)$
$z(t)$	analytic signal of $x(t)$
$I_z(t, \nu)$	instantaneous autocorrelation function
$W_z(t, f)$	Wigner-Ville distribution

$R_z(t, f)$	Reduced Interference Distributions
$G(t, \nu)$	Choi-Williams and B distribution time-lag kernel
ς	parameter for CW time-lag kernel
β	parameter for BD time-lag kernel
$n(t)$	white noise component
$n_p(t)$	$1/f$ noise component
$\Gamma_w(f)$	power spectral density of white noise
$\Gamma_p(f)$	power spectral density of $1/f$ noise
$x_{FRI}(t)$	FRI signal
$h(t)$	linear time-invariant lowpass filter
$H(f)$	frequency amplitude response of $h(t)$
$\gamma_p(l)$	autocorrelation of the $1/f$ noise
σ_w^2	variance of the white noise signal
D	Toeplitz matrix created from spectral values of $x(t)$
E	Energy of a signal
$S(t_1, f)$	local spectrum for a particular slice of the Stockwell spectra at time t_1
$E_L(t_1, f)$	local-energy of the local spectrum for a particular time slice in the Stockwell spectra
$\mu_{E_L}(t_1)$	mean of the local-energy
$\sigma_{E_L}^2(t_1)$	variance of the local-energy
$\sigma(f)$	modification parameter to the Stockwell transform

Chapter 1

Introduction

Neonatal electroencephalogram (EEG) analysis is a growing topic of research as medical and engineering advances continue to improve the survival rate of infants. However, the infants still remain in a higher risk group for neurological problems later in life. Therefore, researchers continue to investigate into more accurate and efficient methods such as signal processing methods to detect brain abnormalities like seizures as early as possible to reduce this risk. This chapter outlines the motivation, the scope of the research and the structure of this thesis.

1.1 Motivation

Seizures are the most common sign of neurological dysfunction in neonates. They are the result of excessive, synchronised electrical discharges from a group of neurons. Usually, the causes of seizures are due to brain abnormalities such as hypoxic ischaemic encephalopathy, metabolic disturbances, cerebrovascular lesions and infections [1]. There are differing views over the exact prevalence of neonatal seizures. This is due to differing clinical views about what constitutes a neonatal seizure [3]. Furthermore, a lack of consensus on the minimum time duration of the EEG seizure events [4] poses a challenge to identify seizures objectively. Neonates with confirmed seizures usually fare extremely poorly in survival rates as described by Legido et al. [5]. They found that death or neurological impairment follow in 70% of cases after EEG confirmed seizures. Thus the scale and complexity of

neonatal neurological dysfunction diagnosis and management worldwide cannot be underestimated. A significant economic cost associated with neonatal care in the US alone has been estimated to be \$10.2 billion annually [6].

The estimated rate of seizures in fullterm neonates has not changed considerably in the past twenty years and is approximately 1 – 3/1,000 live births [7]. Seizures in the neonatal period often reflect serious underlying brain injury, and they are usually the first sign of neurologic dysfunction in the neonate. Seizures are also one of the few neurological conditions in the neonate that require immediate medical attention. This is because untreated seizures can result in long term neurological damage. Evidence that neonatal seizures have an adverse effect on neurodevelopmental outcome of the neonate in its later life is documented in [3]. Therefore, undetected seizure activities greatly increase the potential for brain damage [8]. Newborns with confirmed seizures was reported to have poor health outcomes (morbidity in 50% of survivors) and a high probability of death (30% of survivors) [2]. As a result, early detection and treatment of seizures is crucial to the improvement of prognosis.

The signs of neonatal seizure can be divided into two categories, namely clinical and electrical manifestations. Clinical seizures are recognised by certain physical manifestations while electrical seizures are identified by particular abnormal patterns observable in the EEG. These seizure signs may occur simultaneously or separately. An EEG seizure without clinical manifestations is called sub-clinical. Seizures where both seizure signs occur simultaneously are called electroclinical seizures. Seizures and non-epileptic seizure like events are typically differentiated on clinical grounds. However, such clinical signs are often very subtle, making them difficult to diagnose. They may be clinically silent, especially after antiepileptic drug treatment, making diagnosis by clinical observation alone very unreliable [9, 10]. Some subtle clinical signs of seizures in neonates include:

- Blinking, fixed stare

- Eye deviation
- Repetitive mouth and tongue movements such as sucking
- Pedalling and tonic posturing of limbs.

Unfortunately, these movements are often present in normal newborns. Thus it is difficult to diagnose and can be easily confused with normal infant movements, leading to misdiagnosis and unnecessary treatment with anti-epileptic medication [11]. Furthermore, in [12], Bye and Flanagan estimated that 85% of neonatal seizures are sub-clinical, this means that 85% of neonatal seizures have no clinical manifestations, having only an electrographic (EEG) seizure pattern and, consequently can only be detected with the EEG. In addition, visible effects of neonatal seizures are more subtle than adult epileptic seizures and require a considerable amount of medical attention for diagnosis. As a result clinical diagnosis becomes far more difficult than in adult patients.

The EEG is the primary tool for the clinical diagnosis and management of neonatal seizures. Abnormalities in the EEG have a good predictive value for a poor neurodevelopmental outcome in the newborn infant [2]. Detection of seizures in the neonatal EEG is complicated by a number of factors:

- Neonatal EEG is highly age and patient dependent
- Abnormal EEG patterns in older infants and children can constitute normal patterns in the neonatal EEG
- Electrographic signs are erratic and fragmented
- Patterns and morphologies highly variable
- Neonatal EEG requires expert interpretation, which is not available on a 24 hour basis

EEG activity that is normally associated with seizures in the adult brain such as spikes and abnormal waves occur spontaneously in neonates and are the result of extra electrical activity in the immature brain. This is because during the neonatal period, rapid maturational changes of the central nervous system take place. The electrographic changes reflect this rapid change. As such, the significance of certain EEG features varies greatly according to the conceptional age and the state of the newborn, which cannot be determined without the help of biomedical signal acquisition and knowledge of behavioural patterns [13]. The EEG is the only reliable method available to detect all neonatal seizures but requires special expertise for interpretation. This expertise is not readily available in most neonatal intensive care units on a 24 hour basis. Furthermore, visual EEG interpretation is an extremely laborious and time consuming task. Thus there is an urgent need for a system that can automatically detect, or even predict, the presence of seizures in newborn babies, allowing timely medical intervention.

1.2 Objectives

The problem which is addressed in this thesis will be formally stated below as:

To develop a robust automated neonatal seizure monitoring and representation system.

Contrary to existing neonatal EEG acquisition and monitoring devices that primarily acquire, store and display EEG tracings, we envision that our system encompasses a few objectives, namely

1. the intelligence to acquire EEG recordings at a low sampling rate and recovering them with high fidelity (finite rate of innovation framework)
2. the ability to automatically remove common artifacts in the EEG recordings (finite rate of innovation framework)

3. the capability to reliably locate and detect neonatal seizure activities (time-frequency framework).

We present our neonatal seizure monitoring and representation system in Figure 1.1.

Figure 1.1: The neonatal EEG seizure monitoring and representation system.

We intend to meet the objectives by focussing on two aspects, firstly by exploiting the theory of sampling signals with finite rate of innovation. This theory has, to date, not been investigated on biomedical signals such as EEG signals. Through sampling the EEG signals at their rates of innovation, we not only achieve signal compression inherently, but also allow the investigation of new signal analysis and processing techniques to EEG signals. In this case, we make use of the signal models to remove artifacts commonly found in such signals.

Secondly, we study how time-frequency features of EEG signals can be extracted for seizure detection. Using the features extracted, a superior seizure detection algorithm is developed.

1.3 Major contributions of the Thesis

The main contributions of the work covered in this thesis is the advancement of seizure detection in neonatal EEG signals. The following lists the detailed individual efforts

Chapter 1 Introduction

6

exposed in this thesis:

Chapter 2

- Conducts and presents a literature survey of existing EEG models, compression, artifact removal and seizure detection methods;

Chapter 3

- Investigates the suitability of modelling neonatal and adult EEG signals as Finite Rate of Innovation (FRI) signals [79];
- Establishes a new EEG signal model based on the FRI sampling theory. This sets the groundwork of expanding the FRI sampling theory into the acquisition and analysis of EEG signals;
- Presents a system to acquire EEG signals at their rate of innovation, thereby entailing a low-cost acquisition device that does not waste computational resources;
- Presents an EEG compression method based on the FRI framework. Although the method is lossy, it maintains the morphologies of the EEG signals;

Chapter 4

- Investigates the feasibility of modelling artifact signals such as electrostatic discharges, electrooculographic (EOG) and electromyographic (EMG) as FRI signals;
- Establishes an FRI model of EEG and artifacts signals;
- Presents a novel artifact removal algorithm using the FRI framework;

Chapter 5

- Analyses the clinical method of neonatal EEG interpretation and translated the observations into simple decision rules;

- Investigates the results of time-frequency distributions into different types of neonatal EEG signals;
- Presents a simple and effective seizure detection method based on an empirically derived feature, named the variance of the local-energy of the Stockwell transform;
- Detects two types of seizures, namely slow wave and sharp spike seizures together with their onset and duration;
- Validates the FRI signal models, namely the EEG and artifact models, by applying the seizure detection method on FRI compressed and artifact removed data.

1.4 Organisation of the Thesis

The scope of this research includes investigating the use of signal processing techniques in the task of analysing neonatal EEG processing specifically for features correlated with seizure onset and assist in clinical research. Artifact removal is also studied to improve the detection process. Seizure prediction, although a popular field in EEG signal processing, is not within the scope of the project.

The thesis is organised as follows. Firstly, Chapter 2 presents the characteristics of neonatal EEG signals and the literature reviews on EEG modelling and compression methods, artifacts removal and seizure detection. A review on sampling signals with finite rate of innovation and time-frequency analysis of signals will also be covered in Chapter 2. Next the thesis is divided into two main focus, namely methods based on FRI sampling theory in Chapters 3 and 4 and methods based on time-frequency theory in Chapter 5. Chapter 3 presents and discusses a new EEG model based on the FRI framework. The focus of this part of our research is to establish the validity of the FRI model and to apply it effectively. This model serves two functions in this thesis, namely as a new signal acquisition and compression function and more importantly, as a basis to utilise the FRI

sampling theory for EEG analysis. We make use of this model as a foundation to establish the artifact removal method in Chapter 4 (*Answers Objective 1*).

In Chapter 4, artifacts such as electrostatic discharge, EOG, EMG (and morphologically similar artifacts) and the background signals will be modelled as FRI signals, and their removal from the background EEG signal will be presented (*Answers Objective 2*).

Lastly, based on the Stockwell transform, a simple and effective seizure detection method is proposed in Chapter 5. The effectiveness of our neonatal seizure detection method will be demonstrated by comparing our results with those obtained from existing methods. As a test to validate our FRI models, results of the seizure detection method applied to FRI modelled, compressed and artifact removed datasets are also presented (*Answers Objective 3*).

Chapter 6 concludes and discusses future directions of this research topic.

Chapter 2

Background and literature review

This chapter aims to present a background on neonatal EEG signals and the signal processing methods investigated in this thesis. First, we review the methods of recording the EEG signals, the characteristics of the newborn EEG seizure and non seizure signals. This is followed by a literature review on EEG modelling, compression, artifact removal and seizure detection methods. Lastly, a brief review on sampling and reconstructing signals with finite rate of innovation and time-frequency analyses methods will be covered.

2.1 Recording methods of EEG signals

Most recordings of scalp EEG involve the placement of electrodes over the head according to the International 10 – 20 standard. Twenty one electrode sites are defined under this standard [14] and they are determined by four standard positions on the head: the nasion, inion, and left and right preauricular points [15].

The EEG is a voltage measurement, which infers that a potential difference is recorded. The montage is the method by which the potential difference between electrode sites are selected as EEG channels. Two montages recommended by the American EEG society are the bipolar and the referential (monopolar) montages [14]. In the bipolar montage, an EEG channel is obtained by measuring the potential difference between two adjacent electrode sites. This method of recording removes common electrical activity between the two electrode sites such as unwanted artifacts, but may exclude some important EEG

Figure 2.1: The modified International 10 – 20 electrode placement for neonates.

information localized around either of the recording sites. In the referential montage, a common reference electrode to all EEG channels is used. Usually, it is placed away from the scalp on either the chin, nose or ear to minimize the possibility of including potentials from the brain [16]. This method of recording provides a localised representation of electrical activity but may be hindered by its susceptibility to noise and artifacts. As the size of the neonate’s head is smaller compared to an adult, a modified placement of the electrodes is adopted and shown in Figure 2.1. An example of a multi-channel neonatal EEG seizure is shown in Figure 2.2.

To interpret EEG recordings, clinicians generally examine 10 seconds of the tracing at a time, and group the channels in ways that emphasise features of interest. Instead of reading the 10-second windows in detail, most clinicians scan the traces where these 10-second windows are quickly scrolled through, to gain a general visual impression from the recordings. The clinician should then have a fair idea about the state of the patient at a general level. After the initial scanning, the clinician will then return to the EEG traces and go through the recording more thoroughly, spending more time examining any areas of interest in detail. The presence of artifacts is a problem for EEG, as muscle artifacts and electrical pulses from other areas (for example, ECG from the heart) can interfere with the EEG recording. Clinicians are trained to either ignore segments corrupted by

Figure 2.2: An example of a multi-channel EEG seizure signal. Seizure onset marked on the top left corner.

artifacts, or to look beyond the artifacts. For example, if a segment of the EEG recording is corrupted by high frequency muscle artifact, the clinician can still read the underlying low frequency activities. It is, however, difficult to determine whether some particular pattern is caused by artifact or has a cerebral origin. Usually, an educated guess is formed by viewing the video recording as well as comparing the channel displaying the patterns with other neighbouring channels.

2.2 The neonatal EEG

This section firstly describes the differences between neonatal EEG and adult EEG signals. Subsequently, a review on the characteristics of normal background neonatal EEG and neonatal seizure EEG is presented.

2.2.1 Comparison of neonatal EEG with adult EEG

The EEG signal is a complex waveform that appears like some form of noisy signal. For epileptic patients, the EEG can be divided into a background state and a seizure state. The background EEG activity is characterized by patterns that are relatively stable, and is generally without major temporal changes in frequency and voltage [17, 18]. This pattern is a normal behaviour for near term neonates and is defined by clinicians as a continuous EEG signal.

The frequency content of the background EEG has no clear upper and lower boundaries. Since current EEG measuring equipment generally concentrate on capturing the clinically relevant range between 0.1 Hz and 100 Hz [19], the frequency range is divided into a number of bands. These bands are defined as:

- Delta (δ) 0.1 – 3.5Hz
- Theta (θ) 4 – 7.5Hz
- Alpha (α) 8 – 13Hz
- Beta (β) 13 – 30Hz
- Gamma (γ) above 30Hz.

Delta activity, the predominant range in infants, is considered normal during deep sleep stages in the adult but abnormal in the EEG of awake adults. Theta activity, if focal or lateralized, with excessive amplitude is an indicator of possible localised cerebral pathology in the adult patient. However, it is often present in the EEG of normal children. The Alpha rhythm is observed during the resting-with-eyes-closed state of adult patient and is the classical EEG correlate of wakefulness [15, 19]. The Beta and Gamma activities are regarded as a higher frequency variation of the alpha activity. It has been observed that the EEG characteristics differ considerably from the adult and older child to the newborn,

particularly in frequency content. In the adult and older child, the predominant frequency range is between 8 – 30Hz (Alpha and Beta activity). Frequencies between 0.3 – 7Hz and above 30Hz are sparsely represented [19]. The neonate EEG, on the other hand, has most of its power in the low frequency range between 0.4 – 7.5Hz [20, 21].

The variations in neonate EEG are also more rapid with unstable stages of sleep-wake cycle and absent reaction to opening eyes. An interhemispheric asymmetry is also observable. Scher [22] reviewed that the unique neurophysiologic expressions of state transitions that occur in the newborn do not exist in adults. These differences include a shorter sleep cycle, prominent EEG delta rhythms, intra- and interhemispheric electrographic asynchrony, discrete neonatal waveform patterns (delta brush and theta bursts), a high percentage of periodic breathing, greater heterogeneity of rapid eye movements and unique motor patterns that reflect fetal movements. Sleep morphologies noted in older patients such as sleep spindles or vertex waves and K-complexes are not expressed during neonatal sleep [23]. Only term infants (i.e. 37 weeks gestational age or older) express an organised neonatal sleep cycle, while rudimentary states are expressed in preterm neonates. Applications of EEG-sleep analyses in the neonatal intensive care setting also remain a great challenge even for the experienced EEG technologist and neurophysiologist.

The amplitude of EEG in the adult is most commonly between 10 – 50V and is similar compared to the newborn EEG's amplitude [19],[23]. Observations show that the maximum amplitudes of various EEG rhythms, from the defined frequency bands, tend to be inversely related to their frequency [16]. Since Beta and Gamma bands occur in adult EEG, it is clear that the adult EEG has much more high frequency activity than the newborn and the amplitudes of the EEG signals are comparable in magnitude.

2.2.2 Characteristics of normal paroxysmal background neonatal EEG

The term paroxysmal refers to abnormalities in an EEG characterised by an abrupt onset and termination, which is significantly different from the background activity in frequency and amplitude [15]. Usually, abnormalities associated with some form of pathology occur paroxysmally [24]. Major forms of paroxysmal patterns in both adult and neonate EEG can be observed as

- spike wave pattern which is *"a transient, clearly distinguishable from the background activity with a pointed peak at conventional paper speed and a duration from 20 to under 70msec. Main component is generally negative to other areas. Amplitude is variable"* [25];
- sharp wave pattern which is *"a transient, clearly distinguishable from the background activity with a pointed peak at conventional paper speed and a duration from 70 to under 200msec. Main component is generally negative to other areas."* [25];
- low voltage traces. Usually patients exhibiting such patterns do not survive, or are left with severe neurological problems;
- burst suppression pattern which is characterised by a burst of high voltage lasting approximately 1 to 10 seconds. This is followed by a period of inactivity lasting 2 to 10 seconds [17].

However, it is more complicated to identify paroxysmal patterns in neonates because some patterns are normal at specific conceptual ages (CA).

2.2.3 Characteristics of the neonatal seizure EEG

Ictal or seizure patterns in the EEG are the most significant EEG abnormalities. Seizures are abnormal reactions of the brain and are the result of a number of diseases [60]. Seizures

may also be a result of craniocerebral trauma or brain damage.

The newborn EEG seizure event is generally characterized by a gradual, progressive build up of ictal activity. They are generally focal and spread across the brain in various ways [17]. However, newborn seizures which have highly rhythmical spiking are often multifocal with slow migration of the spike activity from one area to another [60].

The minimum duration of ictal patterns in the newborn EEG used to define a seizure event is a highly debated topic among neurologists. Some require that a seizure event display ictal patterns for no less than 10 seconds. However, others require a duration of at least 20 seconds [17]. Evidence from a study in the year 2000 has shown that even shorter durations than 10 seconds are clinically significant and provide prognostic value [61]. In another study of 487 neonatal electrographic seizures, the average seizure duration was 137 seconds, with a range of between 10 seconds to 46 minutes. An important result of this study was that 97% of all seizure events were less than 9 minutes [62].

The newborn EEG has been shown to be a nonstationary signal, particularly the ictal patterns. Therefore, representation of the rhythmic seizure patterns in the frequency domain does not clearly illustrate any of the important time-varying information. Due to this, time-frequency signal analysis (TFSA) has recently been applied to the newborn EEG [63, 64]. Many significant time-varying patterns, such as piecewise linear components, were found to exist in the EEG seizure. This clearly emphasizes the superiority of TFSA in analyzing nonstationary signals.

2.3 EEG signal modelling and compression

2.3.1 EEG signal modelling methods

EEG signal modeling is important to achieve a better understanding of the physical mechanisms generating these signals and to identify the causes of change in EEG signals.

Furthermore, pattern recognition and classification of EEG abnormalities can be achieved through the analysis of the estimated model parameters. Modeling can also be used for predicting the future neurological outcome and for data compression. Simulation based on EEG signal model can be used to better demonstrate the effectiveness of a certain quantitative analysis method or EEG feature extraction system. A popular signal model in EEG analysis is the linear prediction model. The EEG is modeled as the output of a linear system driven by stationary white noise. The parameters defining the system are estimated by fitting the linear model to the EEG signal using a suitable error criterion. The autoregressive, moving average (ARMA) model represents a general form of the linear model and a widely used model is the AR model [26, 27, 28, 29, 123]. Other models include the time-varying AR models and the multivariate AR models which takes care of the nonstationarity and spatial interactions between different EEG channels [30, 31, 32]. Although these models are widely used, they are inadequate in representing the EEG signals effectively [123]. More recently, different models for the EEG signal have been proposed. In [33], the authors proposed an adaptive EEG signal model based on Markov process amplitude (MPA) where some model parameters are determined adaptively using the least mean square algorithm. The model eliminates the stationarity constraints and the need to repeatedly and manually calculate the MPA model parameters. A non-stationary model of the neonatal EEG was described in [34]. The neonatal EEG background model is based on the short-time power spectrum with a timevarying power law. The relationship between the fractal dimension and the power law of a power spectrum is used for accurate estimation of the shorttime power law exponent. The neonatal EEG seizure model is based on a time-frequency signal model. This model addresses all significant time-frequency characteristics of newborn EEG seizure signal which includes multiple components or harmonics, piecewise linear instantaneous frequency laws and harmonic amplitude modulation. In [35], a Gaussian process modelling method was proposed. The number of tunable parameters for a Gaussian process model is

greatly reduced over its neural network counterpart. This model also provides an estimate of the variance of its predicted output, which can be interpreted as a level of confidence of the model. This measure of variance is a major advantage over neural network or fuzzy models as it gives an indication of when the model can be trusted.

2.3.2 EEG compression methods

Valuable information about the human brain conveyed by the EEG is used in various studies like the nervous system, sleep disorders, epilepsy and dementia [36]. These applications require acquisition, storage, and automatic processing of EEG during an extended period of time. For example, 24-hour monitoring of a multiple-channel EEG is needed for epilepsy patients. Traditionally, the EEG has been bandlimited to the frequency range between 0.1 and 100 Hz; thus a minimum Nyquist sampling rate of 200 Hz is needed. At the quantization level of 16 bit/sample, a 10-channel EEG for a 24-h period would amount to 346 megabytes. Hence, to efficiently store and transmit a huge amount of data, effective compression techniques are desired. While lossy techniques yield higher compression, because of reliability considerations, lossy data compression techniques are not used as the morphology of the signals are not always well retained. Excellent surveys of the performance of lossless and lossy EEG compression techniques can be found in [37] to [43]. Antoniol *et. al.* presented and discussed several classical lossless EEG signal compression methods such as Huffman coding, predictive compression and transform compression [37]. In [38], Memon *et al.* discussed lossless compression techniques ranging from simple dictionary searches to sophisticated context modeling. A long-term EEG compression method using features obtained from the signals' power spectral density was proposed in [39] while multi-channel EEG signals were compressed by exploiting the inter-correlation among the EEG channels through the Karhunen-Loève transform in [40]. Nielsen *et. al.* proposed a signal-dependent wavelet compression scheme that adapted optimal wavelets

to biomedical signals for compression [41]. A near-lossless compression method described in [42] compressed EEG signals using neural network predictors followed by nonuniform quantization. More recently, a new compression method based on the construction process of the classified signature and envelop vector sets of the EEG signals is described in [43].

The techniques presented above operated on EEG signals obtained at or above Nyquist rate. This acquisition process leads to a collection of huge amounts of irrelevant data, only to be discarded during the compression stage of the signals. Furthermore, transients, which are common in EEG signals, are not bandlimited, although we can treat the transients to be linearly superimposed on the otherwise bandlimited EEG signals. Hence, Shannon's sampling theory cannot be applied to sampling EEG signals. Over the last few years, advancements in signal processing and data acquisition introduced new sampling theories known as sampling signals with finite rate of innovation and compressed sensing. The fundamental difference between these two sampling theories is that the FRI sampling theory¹ compresses continuous and discrete time signals at their rate of innovation while compressed sensing compresses discrete-time signals by taking random projections of them [44]. Aviyente proposed a compressed sensing framework for EEG compression by exploiting the sparsity of EEG signal samples in a Gabor frame [45].

2.4 Artifact removal in EEG signals

The electrodes used to measure electroencephalographic (EEG) signals are sensitive to many interferences such as electrostatic discharges, eye and muscle movements. The electrostatic discharges occur in the presence of synthetic clothing materials and display themselves as spike artifacts in EEG signals [46]. Eye movements also produce electrical activities (electrooculographic artifacts or EOG) on the scalp and movements in the

¹Sampling signals with finite rate of innovation will be reviewed in Section 2.6.

head, jaw and cheeks (electromyographic artifacts or EMG) can also interfere with the EEG signals [47]. These interferences in the EEG signal can easily mask relevant features, such as seizures, in the EEG signal. Therefore, careful removal of all artifacts should be undertaken prior to any analysis.

In general, artifact rejection starts with the visual inspection of the EEG signals by a human expert. Identified artifact-contaminated epochs are subsequently removed, involving intensive human labour especially if the study involves a large number of subjects or a large amount of recorded data. A question of subjectivity of this process also poses major disadvantages. Furthermore the rejection of artifact-contaminated trials may lead to a substantial loss of data.

Linear filtering [48] removes artifacts located in certain frequency bands that do not overlap with those of the neurological phenomena of interest. Low-pass filtering can be used to remove EMG artifacts while high-pass filtering can be used to remove EOG artifacts. However, when the neurological phenomenon of interest and the artifacts overlap or lie in the same frequency band as the filters, linear filtering fails. As a result, a simple filtering approach cannot remove artifacts without removing a portion of the neurological phenomenon.

Linear combination of EOG contaminated EEG and referenced EOG signals is commonly used to remove EOG artifacts by subtracting the EOG signal from the contaminated EEG signal. However, a similar approach to remove EMG artifacts from the EEG signal is not as straight forward as the removal of ocular artifacts. Unlike the eye artifact, the EMG artifact does not come from a single source. The artifact caused by muscle has many sources distributed over the scalp and can be caused by small muscle activations producing muscle tension as well as larger muscle movements such as head movements [47]. Consequently, a good reference EMG signal cannot be obtained. Furthermore, reference signals are contaminated by the EEG signals as well.

Few studies have used blind source separation (BSS) to remove EMG artifacts, unlike

EOG artifact removal. Results in [49] show that EMG artifacts contaminate a much higher percentage of the independent components (ICs) than EOG artifacts. This is because EMG artifacts are composed of multiple sources. These sources tend to be isolated into individual ICs, which leaves fewer components for the signal of interest. Additionally, traces of muscle activity remain in most of the components, varying in their intensity, thus EMG and non-EMG components cannot be clearly separated.

2.5 Neonatal seizure detection methods

Automatic seizure detection methods were first investigated in the early 1970s. Ktonas et al. [50] first presented a digital computer method whereby abnormal EEG transients are magnified and processed to quantify detailed characteristics of the EEG transients based on six parameters. Prior et al. [51] developed a simple system aimed at recording the timing and frequency of seizure discharges. They were able to detect severe seizures by looking for large amplitude signals sustained over a period of time. Their system used a single channel of data recorded on paper running at 5 to 6 cm/hour. Babb et al. [52] developed electronic circuits for recording and detecting seizures. Their system used implanted electrodes to detect seizures which might otherwise go unreported. The system identified as seizure high-amplitude EEG signals of sufficient duration. They reported monitoring four patients and detecting 66 seizures, 20 of which were false detections. During this same period of time, the nurses detected 28 seizures, two of which were false detections. Therefore, the automated system detected 20 unreported seizures. Ives et al. [53] used implanted depth electrodes and a *PDP-12* computer to remotely monitor patients. The system was adjusted or calibrated for individual patients using their seizure patterns. If the EEG signals fell within an amplitude window for a certain period of time, the algorithm identified the event as a seizure and recorded eight channels of EEG data.

During two weeks of monitoring, thirteen seizures were detected and recorded by the computer. Only one seizure was reported by the nursing staff and only 3 were noted by the patient.

In [54, 55, 56] Gotman et al. introduced the concept of automatic detection of epileptic seizure events from the EEG. Their initial research in the late 1970s [54] focused on the detection of spike and sharp wave events on 16 channel EEG. An extension of this research in [55] led to decomposition of the EEG into elementary waves, allowing the detection of paroxysmal bursts of rhythmic activity. Simple procedures were introduced to measure the amplitude, duration and rhythmicity of waves relative to the background. Many of the concepts introduced continue to be pervasive in seizure detection research. In [56] Gotman improved on previously reported results with the introduction of a patient alarm button to automatically record an ictal events if the patient or a nurse became aware of a seizure onset. The results for this algorithm were evaluated on 293 recordings from 49 patients, for a total of 5303 hours of 16 channel EEG records. 76% of the 244 seizures recorded were detected automatically by the system, with a false detection rate of the order of 1 per hour of recording.

Lopes da Silva et al. [57] implemented an automatic EEG analysis method derived from an autoregressive filter model and based on inverse filtering. This method detected the occurrence of transient non-stationarities such as paroxysmal patterns of activity (spikes, spike-and-waves) characteristic of inter-ictal EEGs.

Whisler et al. [58] described in 1982 a largely analog device which detected irregular as well as classic spike and wave activity. This portable device processed the accelerated playback of 24 hour tape recorded as well as real-time EEG signals. It recognised spikes by their shape and waves by their frequency and made inter-channel comparisons to identify trains of bilateral synchronous spikes, generalized waves, and coincidence of spikes and waves. A limited description of each event in terms of these characteristics will be achieved. The device was tested against the judgment of 3 experienced and certified

electroencephalographers in 18 hours of EEG containing 769 bursts of spike-wave activity from 6 patients. It detected 96.5% of the consensus spike and wave activity identified by all 3 electroencephalographers. Only 0.56% of the machine's detections were false positives (i.e., activity identified by none of the electroencephalographers), though the false positive rate was higher in the presence of chewing artifact. It measured burst duration with an average error of 0.43 sec/burst. Subsequently, automatic seizure detection research expanded to neonatal EEG. In summary, EEG based neonatal seizure detection methods can be divided into three broad categories using the first author as the method's reference:

- Time based methods - Liu et al. [67], Celka et al. [68], Navakatikyan et al. [70]
- Frequency based methods - Gotman et al. [66]
- Time-frequency based methods - Hassanpour et al. [63, 74, 75], Rankine et al. [64]
- Other methods - Greene et al. [77], Deburchgraeve et al. [78]

2.5.1 Time based methods

The Liu et al. [67] method relies on periodicity as the dominant characteristic of neonatal seizures in the EEG. In this method the amount of periodicity in the autocorrelation of 30 second epochs of EEG data is scored and used to classify the epoch as a seizure or non-seizure epoch.

The Celka [68] method is a time domain method employing pre-processing. The pre-processing involves estimating an ARMA model of pre-recorded normal EEG for each record, the inverse of this is applied to the signal being analysed, leaving only seizure components and white Gaussian noise (WGN). A singular variable decomposition (SVD) algorithm is then employed to extract seizure features from noise. Navakatikyan et al. [70] present a neonatal seizure detection algorithm that detects seizures by measuring the regularity of EEG wave sequences. The algorithm makes use of each wave's timing, morphology and

amplitude as measures of its regularity in order to distinguish seizure waveforms from non-seizure waveforms.

Liu et al. report a sensitivity and a specificity² of 84% and 98% respectively for this method, while Faul et al. [65] report a sensitivity of 42.9% and a specificity of 90.2% for the Liu algorithm. The results reported by Liu et al. were for 117 thirty second seizure and nonseizure EEG epochs (0.975 hours of EEG). The seizure epochs used by Liu et al. were selected based on them being the common typical seizure waveforms and may have had the effect of biasing the results, producing higher sensitivities. Celka's method reports a sensitivity of 93% and a specificity of 96% for four newborns. This method was also validated by experiments using simulated data generated by a model of neonatal seizure EEG also introduced by the authors [72]. Faul et al. report a sensitivity of 66.1% and a specificity of 56.0% for their modified Celka method. In the results reported by Celka, channels known to contain seizures were chosen for processing. This does not bias the results on a per-channel basis. However, a real-time seizure detection system would require processing and polling of all channels in parallel as the spatial location of the seizure is a priori unknown. Furthermore the Celka method requires a pre-processing stage, which limits its potential for a real-time detection system. A further paper by Celka and Colditz [73] reported a new approach based on an adaptive autocovariance eigenspectrum along with Rissanen's Minimum Description length. It was used to examine five channels of seizure EEG from a single neonate. The analysis on this set of data revealed that during seizure, the EEG was more likely to be deterministic with a large number of degrees of freedom. The Navakatikyan algorithm was trained using two separate training sets consisting of six neonates each and tested on a third, larger data set containing multi-channel EEG from 17 neonates with seizures and 22 control neonates. Navakatikyan et al. reported that 87 of 97 seizure events were correctly detected by their algorithm leading to a good detection rate of 89.69% and 2.0 false detections per hour. The main drawback

²Sensitivity and specificity are defined in Section 5.3.2.

of this algorithm is its reliance on measures of EEG waveform morphology. Neonatal seizures have a large variety of waveform morphologies [71], as a result it is arguable that algorithms based on empirical measures of waveform morphology such as documented by Navakatikyan may require testing on more data.

2.5.2 Frequency based methods

The Gotman algorithm [66] is a frequency based method that assumes the dominant characteristic of neonatal seizures is periodicity, many algorithms are based on this premise [66, 67, 69, 70]. The Gotman method uses fast Fourier transform (FFT) based frequency spectrum analysis to detect periodic discharge in epochs of EEG. Based on a number of features such as the frequency and width of the dominant spectral peak as well as the relative power of that frequency band compared to the same band in the background, a seizure epoch is detected. An earlier paper by Gotman [54] shows a method for detection of multiple spikes. Spikes are generally of longer duration in newborns than in adults. An epoch contains a seizure if there are six or more spikes in each 10 second epoch. Each epoch is then classified as seizure or non-seizure using a set of empirically derived thresholds for each feature. Gotman reports a seizure sensitivity of 71% and two false detections per hour. These results were derived from 281.5 hours of EEG recordings from 43 neonates with seizures and 12 control neonates. All EEG records were included regardless of quality or duration. In contrast Faul et al. [65] reported a sensitivity of 62.5% and a specificity of 64.0% for the Gotman method. The results for the Gotman method were validated in a subsequent paper [59] by results for a separate data set containing 237 hours of EEG data from 54 patients. The average seizure detection rate for this set was 69% with an average of 2.3 false detections per hour. The Gotman method is based on a single channel of EEG. Furthermore, the threshold values used in this method were empirically derived as opposed to being derived from real training data. However it must be pointed out that

the Gotman algorithm is the most rigorously tested.

2.5.3 Time-frequency based methods

Hassanpour et al. [63] investigated time-frequency (TF) based neonatal seizure detection methods by performing a singular value decomposition (SVD) on the time-frequency domain representation of an EEG epoch. To discriminate between seizure and non-seizure activities in each EEG epoch, features are extracted through the histograms of the four singular values which contain time and frequency information of the signal. These features are organised into a feature vector and fed into a trained neural network classifier. Further research by Hassanpour et al. [74] investigated high frequency (70Hz) signatures of seizures combined with the low frequency signatures described above. In [74], the three published methods by Liu, Gotman and Celka are compared with the Histogram of Successive Spike Intervals (HSSI) [75] and the Distribution Function of the Singular Variables (DFSV) [76] methods on five neonatal subjects. Most literature discussing neonatal seizure EEG concentrates on activity in the low frequency bands (below 10Hz), while previous research from Hassanpour et al. looked at activity in bands above 70Hz for detecting seizure related spikes [75] (defined as short time broadband events with high instantaneous energy). In [75], the authors combined information from the two bands and analyzed the data in the time-frequency domain (TFD). Rankine et al. [64] suggested that seizures can be detected by detecting repetitive spikes that are harmonically related. The method makes use of an adaptive spectrogram with an optimal window scale function to detect these spikes. In order for a seizure event to be detected, at least 3 out of 5 successive epochs must contain multiple spikes.

The results cited in [63] for eight babies give an average sensitivity of 92.5% and a specificity of 96.3%. Further results given in [75] report a sensitivity of 92.4% an specificity of 95.1% for the HSSI method while the DFSV method gave a sensitivity of 92.6% and a

specificity of 96.2%. Results given by Faul et al. for the Liu, Gotman and Celka methods do not significantly disagree with those given by Hassanpour et al. One issue with these results is that the author selected an average of 10 minutes of recorded data to evaluate each method. As previously mentioned selecting of 'typical' seizure EEG has the effect of biasing results. The methods should be examined on several hours of continuous recordings wherever possible. Such short duration recordings cannot be considered in the same light as results presented for other methods using more typical recording lengths. The method by Rankine was tested on 8 sets of neonatal EEG data and showed an average good detection rate of 95.8% and an average false detection rate of 2.38%.

2.5.4 Other methods

Recently, a neonatal seizure detection method is described by Greene et al. which combines EEG and ECG features together to enhance detection accuracy [77]. A set of ECG and EEG features are extracted and combined into a feature vector in two ways, either by an early integration or a late integration of the features using pattern classifiers. Deburchgraeve et al. [78] capitalised on the characteristics of neonatal seizures used by observers to conclude whether a seizure is present. The neonatal seizures are divided into two types. For each type, a fully automated detection algorithm was developed based on the identified human observer characteristics. The first algorithm analyzes the correlation between high-energetic segments of the EEG while the second detects increases in low-frequency activity (< 8 Hz) with high autocorrelation.

In the method by Greene et al., for a patient-specific system, 97.52% of seizures are correctly detected with a false detection rate of 13.18% and for a patient-independent system, 81.44% of seizures are correctly detected with a false detection rate of 28.57%. The algorithm described by Deburchgraeve et al. was tested on multi-channel EEG recordings of 21 patients with, and 5 patients without, electrographic seizures, totaling 217 hours of

Table 2.1: A summary of the performances of neonatal seizure detection methods reviewed in this thesis.

Algorithm	Testing on segments / continuous EEG	Duration of data (Hours)	Results			
			GDR	FDR	Sensitivity (%)	Specificity (%)
Liu	segments	1.0	-	-	74	98
Celka	segments	-	-	-	93	96
Hassanpour	segments	5.8	-	-	92.5	96.3
Gotman	continuous	281.5	71.0	2.0	-	-
Navakatikyan	continuous	24	-	-	82.8	87
Greene	continuous	154	81.44	28.57	71.73	71.43
Deburchgraeve	continuous	217	88	0.66	-	-

EEG. The sensitivity of the combined algorithms was found to be 88% with a positive predictive value³ 75% and a false detection rate of 0.66 per hour. A summary of the various neonatal seizure detection methods reviewed in this section is presented in Table 2.1. As observed, the algorithms generally perform better in the case of segment based EEG in contrast to continuous EEG. This is due to the removal of artifacts and other difficult segments in the EEG which would otherwise have affected the performance of the algorithms.

2.6 Sampling signals with finite rate of innovation (FRI)

In the year 2002, Vetterli, Marziliano, and Blu [79] showed that it is possible to sample and perfectly reconstruct a large class of signals that are not bandlimited. However, these non bandlimited signals are finite complexity parametric signals. A common feature of such signals is that they are characterised by a finite number of degrees of freedom per unit of time. The number of degrees of freedom per unit of time is defined as rate of innovation, and therefore, such signals are termed as signals with finite rate of innovation [79]. Vetterli et al. showed that it is possible to sample and perfectly reconstruct 1-D FRI signals such as streams of Dirac pulses, nonuniform splines, and piecewise polynomials from a finite

³Positive predictive value is defined in Section 5.3.2.

number of uniform samples of a filtered version of the signals. The reconstruction is based on the annihilating filter also found in spectral estimation [80] and error-correction coding [81]. The results of [79] provide theorems for sampling and perfect reconstruction of 1-D FRI signals in the presence of noise [82]. The results of [79] are also extended further for 2-D FRI signals such as sets of 2-D Dirac pulses and polygons using sinc and Gaussian kernels by Maravic and Vetterli in [83, 84]. A detailed description on 1-D and 2-D FRI sampling is given in the PhD theses of Marziliano [85] and Maravic [86] respectively. Further research into different types of kernels, namely polynomial reproducing, exponential reproducing and rational kernels can be found in [87]. This theory has recently been investigated for a compression technique for electrocardiogram (ECG) signals [88] and neonatal EEG seizure signals [89, 90] as well as for EEG seizure source localisation [91].

2.6.1 Definition of FRI signals

Consider a $1 - D$ signal whose representation is

$$x(t) = \sum_{n \in \mathbb{Z}} \sum_{r=0}^R c_{nr} \varphi_r\left(\frac{t - t_n}{T}\right), \quad (2.1)$$

where the set of functions $\varphi_r(t), r = 0, 1, \dots, R$ is known. Notice that the degrees of freedom of $x(t)$ are the time instants t_n and coefficients c_{nr} . By introducing a counting function $C_x(t_a, t_b)$ that counts the number of free parameters of $x(t)$ over an interval $[t_a, t_b]$, the rate of innovation of $x(t)$ is then defined as

$$\rho = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} C_x\left(-\frac{\tau}{2}, \frac{\tau}{2}\right). \quad (2.2)$$

Definition 1 (Vetterli, Marziliano, and Blu, [79]). A signal with a finite rate of innovation is a signal that is characterised by Eq. (2.1) and with a finite ρ as given in Eq. (2.2).

Shift-invariant signals, including bandlimited signals, also fall under Definition 1. For

example, if f_m is the maximum non-zero frequency in a bandlimited real signal, then the rate of innovation $\rho = 1/T = f_s = 2f_m$, where T is a sampling interval and f_s is the sampling frequency. One possible interpretation is that it is possible to sample bandlimited signals because they have a finite rate of innovation (rather than because they are bandlimited) [87]. It is shown in [79] that although these signals are not bandlimited, they can be sampled uniformly at (or above) the rate of innovation using an appropriate kernel, and then perfectly reconstructed by finding the roots of the z-transform of the annihilating filter and by solving systems of linear equations. Examples of signals with finite rate of innovation include streams of Dirac pulses, nonuniform splines, and piecewise polynomials and sinusoids.

2.6.2 Sampling and reconstruction schemes for FRI signals

Consider the sampling system in Fig. 2.3. We will next give a step by step description of the reconstruction scheme illustrated in Fig. 2.4 for the cases when $x(t)$ is a periodic stream of Dirac pulses, a periodic nonuniform spline and a periodic bandlimited signal + a stream of Dirac pulses, respectively.

Figure 2.3: Sampling signals with finite rate of innovation.

Figure 2.4: The FRI reconstruction block diagram.

Stream of Dirac pulses

Consider a stream of K Dirac pulses with a period τ ,

$$x_D(t) = \sum_{n \in \mathbb{Z}} c_n \delta(t - t_n)$$

where $t_{n+K} = t_n + \tau$ and $c_{n+K} = c_n, \forall n \in \mathbb{Z}$. This signal has $2K$ degrees of freedom per period, thus the rate of innovation is

$$\rho = \frac{2K}{\tau}. \quad (2.3)$$

By taking a continuous-time periodic sinc sampling kernel $h_B(t) = B \text{sinc}(Bt)$ with bandwidth B such that B is greater or equal to the rate of innovation ρ given by Eq.(2.3), and sampling $y(t) = (h_B * x_D)(t)$ at N uniform locations $t = nT$ with $n = 0, \dots, N-1$, where $N \geq 2M+1, M = \lfloor \frac{B\tau}{2} \rfloor$ and $M \geq K$, then the samples defined by

$$y_n = \langle h_B(t - nT), x_D(t) \rangle, n = 0, 1, \dots, N-1$$

sufficiently represent $x_D(t)$ [79].

The algorithm for sampling and reconstruction of a periodic stream of Diracs is summarised as follows:

Step 1 Sampling of $x_D(t)$ at the rate of innovation.

Let $B = \rho = 2K/\tau$ and consider the sampling kernel $h_B(t) = B \text{sinc}(Bt)$, then the sample values are

$$y_n = \langle h_B(t - nT), x_D(t) \rangle, n = 0, 1, \dots, N-1. \quad (2.4)$$

Step 2 Find $2K+1$ contiguous spectral values of $x_D(t)$ from N samples where $\tau/T = N$

$$y_n = \sum_{m=-K}^K X_D[m] e^{i \frac{2\pi mn}{N}}. \quad (2.5)$$

Step 3 Determine the locations of the Dirac pulses.

Consider a filter $A[m]$ whose z -transform has K zeros at $u_k = e^{-j\frac{2\pi t_k}{\tau}}$, that is,

$$A(z) = \prod_{k=0}^{K-1} (1 - u_k z^{-1}).$$

Since the continuous time Fourier series (CTFS) of the signal $x_D(t)$ is a linear combination of K complex exponentials u_k , $A[m]$ is an annihilating filter if it satisfies the following condition

$$A[m] * X_D[m] = 0. \quad (2.6)$$

The coefficients of the annihilating filter are found solving Eq. (2.6) which is equivalent to the following Toeplitz linear system of equations

$$\sum_{k=0}^K A[k] X_D[m - k] = 0, m = -K, \dots, K. \quad (2.7)$$

Let $A[0] = 1$ then Eq. (2.7) can be expressed in matrix form by

$$\begin{pmatrix} X_D[0] & X_D[-1] & \cdots & X_D[-K+1] \\ X_D[1] & X_D[0] & \cdots & X_D[-K] \\ \vdots & \vdots & \ddots & \vdots \\ X_D[K-1] & X_D[K-2] & \cdots & X_D[0] \end{pmatrix} \cdot \begin{pmatrix} A[1] \\ A[2] \\ \vdots \\ A[K] \end{pmatrix} = - \begin{pmatrix} X_D[1] \\ X_D[2] \\ \vdots \\ X_D[K] \end{pmatrix}. \quad (2.8)$$

Thus the locations $\{t_k\}_{k=0}^{K-1}$ of the Dirac pulses are given by the roots of $A(z)$.

Step 4 Determine the weights of the Diracs.

The weights $\{c_k\}_{k=0}^{K-1}$ of the Dirac pulses are given by solving the Vandermonde system of equations given by

$$X_D[m] = \frac{1}{\tau} \sum_{k=0}^{K-1} c_k e^{\frac{-j2\pi m t_k}{\tau}}, m = 0, \dots, K-1. \quad (2.9)$$

With the t_k 's and c_k 's, the original periodic stream of Dirac pulses can be perfectly reconstructed.

Nonuniform splines

A signal $x_{NUS}(t)$ is a nonuniform spline of degree R with knots at $\{t_k\}_{k=0}^{K-1}$ if and only if its $(R+1)$ th derivative is a stream of K weighted Dirac pulses in one period τ [92]

$$x_D^{(R+1)}(t) = \sum_{k=0}^{K-1} c_k \delta(t - t_k).$$

The rate of innovation of the nonuniform spline is $\rho = 2K/\tau$.

Consider a continuous-time periodic nonuniform linear spline $x_{NUS}(t)$ with period τ , containing K pieces of maximum degree $R = 1$. By following the sampling method described in Section 2.6.2, $x_{NUS}(t)$ is uniquely defined by [79]

$$y_n = \langle h_B(t - nT), x_{NUS}(t) \rangle, n = 0, 1, \dots, N - 1.$$

The sampling and reconstruction of nonuniform linear splines is similar to that described in Section 2.6.2 with the addition of Step 2a after Step 2 and Step 4a after Step 4:

Step 2a Determine the spectral values of the stream of Dirac pulses from the spectral values of the nonuniform splines:

$$X_{NUS}^{(R+1)}[m] = \left(\frac{j2\pi m}{\tau}\right)^{R+1} X_{NUS}[m], m \in [-K, K].$$

Step 4a Integrate $(R+1)$ times the stream of Dirac pulses to get the original nonuniform spline.

Bandlimited signal + stream of Dirac pulses

Consider an L bandlimited signal⁴ with period τ added to a stream of K weighted Dirac pulses

$$x(t) = x_{BL}(t) + x_D(t).$$

⁴ L is an integer which corresponds to the index of the maximum frequency of the bandlimited signal.

Its CTFS can be expressed as:

$$X[m] = \begin{cases} X_{BL}[m] + X_D[m] & \text{if } m \in [-L, L] \\ X_D[m] & \text{if } m \notin [-L, L] \end{cases} \quad (2.10)$$

and its rate of innovation $\rho = \frac{2L+2K+1}{\tau}$, where $2L + 1/\tau$ and $2K/\tau$ are the number of degrees of freedom of the bandlimited signal and the stream of K Dirac pulses respectively. The signal, $x(t)$ can be reconstructed from a contiguous set of CTFS coefficients $X[m]$, $m \in [-M, M]$ obtained from a uniform set of samples of the low pass approximation of $x(t)$. Since the CTFS coefficients of the bandlimited signal $X_{BL}[m]$ is zero outside the band $[-L, L]$, it follows that the CTFS coefficients of $x(t)$ outside this band is equal to the CTFS coefficients of the stream of K weighted Dirac pulses $X_D[m]$. As a contiguous set of $2K$ CTFS values outside the band $[-L, L]$, for example, in $[L + 1, L + 1 + 2K]$, is available, the stream of K weighted Dirac pulses can be recovered. The CTFS coefficients of the bandlimited signal, $X_{BL}[m]$ can then be obtained by subtracting $X_D[m]$ from $X[m]$ in the band $[-L, L]$. Hence, we can state the following:

Consider an L bandlimited signal with period τ added to a stream of K weighted Dirac pulses $x(t) = x_{BL}(t) + x_D(t)$ with a rate of innovation $\rho = \frac{2L+2K+1}{\tau}$. By taking a sinc sampling kernel $h_B(t) = B\text{sinc}(Bt)$ and B is greater or equal to the rate of innovation $\rho = \frac{2L+2K+1}{\tau}$, and sampling $y(t) = (h_B * x)(t)$ at N uniform locations $t = nT; n = 0, \dots, N-1$, $M = \lfloor \frac{BT}{2} \rfloor$, then the samples defined by $y_n = \langle h_B(t - nT), x(t) \rangle, n = 0, 1, \dots, N-1$ sufficiently represent $x(t)$ [93].

2.7 Time-frequency analysis

To analyse a non-stationary signal, the information about the time and frequency variations of the signal is crucial. This information can be obtained by representing the signal in the time-frequency domain. Time-frequency distributions (TFDs) are important tools

to fulfil this function.

Generally, TFDs can be classified into two main groups namely the linear and quadratic TFDs. Linear TFDs analyse the signal through a limited window localised in time. Hence the time and frequency resolutions of this class of TFDs depend on the size of the analysis window and cannot be improved simultaneously. Some examples of linear TFDs include the short time Fourier transform (STFT), the wavelet transform (WT) [95] and the Stockwell transform (ST). [96].

Quadratic TFDs estimate the instantaneous power spectrum of the signal using a bilinear operator. Their resolutions are superior to the linear TFDs. The Wigner-Ville distribution (WVD) [94] is the fundamental quadratic TFD and it is the optimal TFD to represent monocomponent linear frequency modulated signals. However, for multicomponent signals, crossterms create severe artifacts [94]. Hence a class of reduced interference distributions (RIDs) are introduced to reduce the spurious features of the WVD by filtering the TFD in time and frequency. Several RIDs were designed for different applications and examples of RIDs include Choi-Williams distribution (CWD) [97], the B-distribution (BD) [98], the Born-Jordan distribution (BJD) [99] and the Zhao-Atlas-Mark distribution (ZAM) [100]. In our discussion, we will focus on the ST, the CW and the BD. An excellent review of the advances of time-frequency analyses can be found in [101].

2.7.1 The Stockwell transform

The Stockwell transform [96] (ST), recently introduced in geophysics, combines the time-frequency (TF) representation of the short-time Fourier transform with the multi-scaling feature of the continuous wavelet transform. It provides a unique time-frequency representation of a signal by adapting the Fourier transform to analyze the localized signal, using frequency dependent time-scaling windows. The interpretation in a TF domain becomes much easier and the multiresolution analysis allows the Stockwell transform to

detect subtle Fourier spectral changes over time. In addition, the Fourier and Stockwell spectra are intimately related; they can be readily converted from one to the other. Thus, the properties of the Stockwell transform make it a potentially valuable processing tool for EEG signal analysis. In [102], we investigated the ability of the Stockwell transform to characterise both low frequency and high frequency signatures of neonatal EEG seizures. It was shown that the Stockwell transform can concurrently capture both low frequency EEG patterns and high frequency EEG spike patterns.

The ST is a linear time-frequency spectral localisation technique and the ST of a signal $x(t)$ is defined in [96] as

$$S(t, f) = \frac{|f|}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(\nu) e^{-\frac{(\nu-t)^2 f^2}{2}} e^{-j2\pi f\nu} d\nu = \int_{-\infty}^{\infty} x(\nu) w(\nu - t, f) e^{-j2\pi f\nu} d\nu \quad (2.11)$$

where t and f represent the time and frequency variables. The ST, in essence, combines the short time Fourier transform with the continuous wavelet transform and can be considered as a multi-scaled local Fourier transform. The analysis window $w(t, f) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2}}$ is Gaussian, hence $\int_{-\infty}^{\infty} w(t, f) dt = 1$. In contrast with the constant width of the short time Fourier transform, the localizing time window becomes $1/|f|$ in the ST, i.e., the window width is scaled according to the inverse of the temporal frequency. As a result, narrower time windows are used at higher frequencies and wider time windows at lower frequencies, similar to that of the wavelet transform. Thus, the ST provides better frequency resolution at low frequencies. This facilitates the identification of low frequency components that usually have a long duration and represent structural identities of signals such as slow wave seizures. Conversely, narrow time windows at high frequencies give good temporal resolution, which allows for accurate detection of the temporal locations of high frequency changes. High frequency events often have a short duration and contain distinct detailed features in signals such as sharp spikes seizures and spike epileptiforms. However, the Gaussian window has no parameter to allow its width in time or frequency to be adjusted. Hence Mc Fadden et al. [103] and later Pinnegar [104, 105]

introduced a generalized ST which has a greater control over the window function. Therefore, the multi-resolution characteristic of the ST enables a more precise assessment of signal changes. The TF representation of the ST also provides a direct and easily interpretable description of the signal behaviour in terms of the more familiar Fourier context, as opposed to using the time-scale representation of the wavelet transform. In fact, it provides a frequency dependent resolution while maintaining a direct relationship with the Fourier transform [96] as shown in Eq. (2.12)

$$\int_{-\infty}^{\infty} S(t, f) dt = \int_{-\infty}^{\infty} x(\nu) e^{-j2\pi f\nu} d\nu = X(f). \quad (2.12)$$

Thus the integration of the ST over time results in the Fourier transform of the signal $x(t)$. This allows reconstruction of the signal using the inverse Fourier transform. Furthermore, the ST maintains the absolute phase of every signal component.

Recently, a clear relation was formed between the ST and the continuous wavelet transform [106]. The ST can be rewritten as a wavelet multiplied by some data-independent phase and amplitude adjustments. In particular, the ST with a Gaussian window can be rewritten in terms of a Morlet wavelet transform [106]. With this relation, a new inverse ST that allows the use of logarithmic frequency scales is inferred from the wavelet reconstruction formula. The authors in [106] also proposed the usage of wavelet frames in combination with the relation between the ST and the wavelet transform as a method to overcome the efficiency and resolution limitations of the discrete ST inherent to an inappropriate sampling. Like the wavelet transform, the frequency resolution of the ST is logarithmic, its bandwidth being proportional to the frequency. Therefore, the mismatch between the linear frequency scale required by this inverse and the logarithmic frequency resolution of the window introduces a high frequency oversampling rate at high frequency bands and a small one at the lowest ones. This is what is known as inappropriate sampling. These characteristics represent an important drawback in the design of efficient algorithms because they impede the use of a sampling scale adapted to the

frequency resolution of the S transform and the reduction of the number of frequencies used. Additional explanation can be found in [106]. Furthermore, it has been shown that the ST is capable of providing visual representation emphasising the morphological differences between sounds with a sharper time-frequency resolution than the STFT and WT [107, 108]. More recently, at the time of writing this thesis, fast algorithms known as the fast Stockwell transform with the computational complexity of $O(N \log N)$ and $O(N)$ have been introduced by Wang et al. [109] and Bhandari et al. [110], respectively. This greatly enhanced the possibility of real time implementation of the ST. Another recent publication presents a technique that estimates the phase and time offsets between different channels in EEG recordings of seizure activity using the ST. The offset information is visually combined with amplitude information to emphasize the most significant signal features [111].

2.7.2 Differences between the Stockwell transform and continuous wavelet transform (CWT)

The main distinguishing factors of the ST from the many time-frequency representations available is that the ST uniquely combines three characteristics:

1. progressive resolution;
2. absolutely referenced phase information; and
3. frequency invariant amplitude response.

Daubechies has stated that progressive resolution gives a fundamentally more sound time-frequency representation [114]. A detailed discussion of these unique characteristics can be found in [112].

One can view the local spectral analysis as a generalization of the global Fourier spectrum. Since no record of observations is infinite, all discrete spectral analysis ever

performed on measured data have been restricted to the time of observation. Hence there must be a direct relationship between a local spectral representation and the global Fourier spectrum. Thus the fundamental principle of ST analysis can be expressed as: The time average of the local spectral representation should result identically in the complex-valued global Fourier spectrum, as stated in Eq. (2.12).

This leads to phase and amplitude values of the local spectrum that are obvious and meaningful. Consider a signal $x(t) = Ae^{(j2\pi f_o t + \phi)}$. The transform spectrum of this signal at the frequency f_o would return the amplitude of A and the phase constant ϕ . In order to carry this understanding of phase into the realm of time-frequency representations and local spectra, it would require a transform that would return a voice (function of time) for the frequency f_o that had a constant amplitude A and a constant phase ϕ . Such is the case with the ST (not the case with the CWT), and thus it can be described as a generalization of the Fourier transform to the case of nonstationary signals. In comparison with the CWT, there are a few distinct differences, namely:

Frequency sampling

The discrete ST has a very definite sampling of the frequency space in order to be complete and orthonormal. It also has the same sampling of the time domain as the original time series. Therefore there are N^2 points in the redundant ST representation. On the other hand, the CWT has a loosely defined sampling of the time-frequency domain. It is commonly employed with an octave scaling of the frequencies which provides an oversampled representation of the lowest frequencies, and undersamples (relative to the Fourier transform) the highest frequencies.

Figure 2.5: The phase properties of the ST [96].

ST phase vs CWT phase

The ST approach provides an extension of Instantaneous Frequency to broadband signals. The voice for a particular frequency ν_1 can be written as

$$S(\tau, \nu_1) = A(\tau, \nu_1)e^{j\Phi(\tau, \nu_1)}. \quad (2.13)$$

One can use the phase in Eq. (2.13) to define the Generalised Instantaneous Frequency similar to that defined by Bracewell [113]

$$GIF(\tau, \nu_1) = \frac{1}{2\pi} \frac{\partial}{\partial t} 2\pi\nu_1\tau + \Phi(\tau, \nu_1). \quad (2.14)$$

Thus the absolutely referenced phase information leads to a generalization of the Instantaneous Frequency to broadband signals. A peak in the ST is a region of stationary phase as can be seen in Eq. (2.14).

The stationary phase of the ST can be seen in the phase voices of Figure 2.5. This figure shows a particular voice for a signal with a mid frequency component. In Figure 2.5a, the phase of the CWT for that voice is shown, and it is rapidly varying through the

full range of phase. In Figure 2.5b, the phase of the ST for the mid frequency component is shown. It is constant, and is equal to 0.4radians (which is the correct phase). In Figure 2.5c, the phase of local CWT spectrum is shown for time $t = 0$. The amplitude of the local CWT spectrum is overplotted as a dotted line. The phase at the peak is not equal to the phase of the signal. In Figure 2.5d, the phase of the ST local spectra is shown for time $t = 0$, and the amplitude local spectrum is overplotted as a dotted line. The phase varies quite a bit for very low frequencies, but phase is meaningless when the amplitude is negligible (as shown by the dotted line). For the region where the amplitude has a significant value, the phase of the ST local spectrum is constant and equal to 0.4radians which is the correct value.

ST amplitude vs CWT amplitude

The ST also has superior amplitude response. The normalization of the ST is a vital distinction from the wavelet approach. The unit area localizing function (the Gaussian window) preserves the amplitude response of the ST and ensures that the amplitude response of the ST is invariant to the frequency. This means that the amplitude of the ST is the same as the amplitude of the Fourier transform. In Figure 2.6, a chirp function with an increasing frequency, and two high frequency bursts are shown. The amplitude is constant for all three of the components, and is equal to 10 units. Figure 2.6(a) shows the ST amplitude for this signal. The ST returns the correct amplitude for all three components, showing the high frequency bursts to have the same amplitude as the chirp signal. Figure 2.6(b) shows the CWT (with Morlet wavelets) amplitude for this signal. The amplitude is large for the lower frequencies, and diminishes as the frequency of the chirp increases. This is due to the normalization of the wavelet transform. It inherently diminishes the higher frequency components. The high frequency bursts, which have the same amplitude as the chirp, are relatively weak in the CWT representation.

Figure 2.6: The amplitude response of the ST in comparison to that of the CWT [96].

Direct signal extraction

Since the characteristics of the ST allows one to read off the amplitude $A(t) = \text{abs}(S(t, f))$, the frequency f , and the phase $\phi(t) = \text{atan}(\text{real}(S(t, f)), \text{imaginary}(S(t, f)))$ for each time step from the ST, one can directly extract that signal by reproducing it in the time domain as follows:

$$x(t) = A(t) \cos(2\pi f(t) * t + \phi(t)). \quad (2.15)$$

Such a direct extraction is not possible with the CWT.

2.7.3 Quadratic time-frequency distributions - The Choi-Williams and B distributions

Quadratic time-frequency distributions (QTFDs) are special forms of the bilinear transform and has the basic definition in the form of the Wigner-Ville distribution (WVD). All other forms of QTFDs are a derivation of the WVD through operations such as averaging or smoothing [94].

The WVD of a signal $x(t)$ can be defined in terms of its analytic signal $z(t)$. The analytic signal of $x(t)$ can be obtained from the Hilbert transform shown in Eq. (2.16)

$$H\{x(t)\} = X_H(\kappa) = \frac{1}{\pi} \oint_{-\infty}^{\infty} \frac{x(t)}{t - \kappa} dt \quad (2.16)$$

such that $z(t) = x(t) + H\{x(t)\}$.

The instantaneous autocorrelation function of $z(t)$ can then be defined as

$$I_z(t, \nu) = z(t + \frac{\nu}{2})z^*(t - \frac{\nu}{2}) \quad (2.17)$$

and finally the WVD of $x(t)$ is given by the Fourier transform of $I_z(t, \nu)$

$$W_z(t, f) = \mathcal{F}\{I_z(t, \nu)\} = \int_{-\infty}^{\infty} z(t + \frac{\nu}{2})z^*(t - \frac{\nu}{2})e^{-j2\pi f\nu} d\nu. \quad (2.18)$$

Although the WVD has many desirable properties, it suffers from crossterm interferences, especially in the case when the signal of interest contains multiple components [94]. Consider a multicomponent signal defined in its analytic form as

$$z(t) = z_1(t) + z_2(t). \quad (2.19)$$

Its instantaneous autocorrelation function $I_z(t)$ is shown to be

$$I_z(t, \nu) = I_{z_1}(t, \nu) + I_{z_2}(t, \nu) + I_{z_1 z_2}(t, \nu) + I_{z_2 z_1}(t, \nu) \quad (2.20)$$

where $I_{z_1 z_2}(t, \nu) + I_{z_2 z_1}(t, \nu)$ are the instantaneous cross correlation functions between the two components. If we take the Fourier transform with respect to ν , we obtain the WVD such that

$$W_z(t, f) = W_{z_1}(t, f) + W_{z_2}(t, f) + 2\text{Re}\{W_{z_1 z_2}(t, f)\}. \quad (2.21)$$

Thus there exists an interference in terms of the cross-WVD of the two components and hence, such interferences must be reduced via other techniques, namely the reduced interference distributions (RIDs). By introducing a specially designed time-lag kernel $G(t, \nu)$, these crossterms can be effectively attenuated while maintaining the resolution provided by the WVD. In general, we can define the RIDs as follow

$$R_z(t, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(t - u, \nu) z(u + \frac{\nu}{2})z^*(u - \frac{\nu}{2})e^{-j2\pi f\nu} du d\nu. \quad (2.22)$$

The Choi-Wiliams distribution can be obtained by defining the time-lag kernel

$$G(t, \nu) = \frac{\sqrt{\pi} \varsigma}{|\nu|} e^{-\pi^2 \varsigma t^2 / \nu^2} \quad (2.23)$$

while the B distribution can be obtained by defining

$$G(t, \nu) = |\nu|^\beta \cosh^{-2\beta} t. \quad (2.24)$$

The parameters ς for CWD and β for BD are tunable and can be changed to suit various signals to improve the time-frequency representations obtained by these distributions where ς is a real parameter that can control the resolution and the crossterm reduction. The CWD has shown good performance in reducing crossterms while keeping high resolution, with a compromise between these two requirements determined by ς while β of the BD is a real parameter that controls the sharpness of cutoff of the 2-D filter in the ambiguity domain. The range of β is between zero and unity. It has been shown that both CWD and BD gives high resolution and minimal crossterm interferences and are suitable for EEG signal analysis [115, 75].

Chapter 3

Modelling EEG signals using the finite rate of innovation framework

In this chapter, we focus on modelling EEG signals as signals with finite rate of innovation (FRI). The purpose of modelling the EEG signals are twofold. Firstly we want to establish a new EEG signal model which departs from the traditional EEG models as discussed in Chapter 2.3.1 and set the groundwork to expanding the FRI sampling theory into the field of EEG signal processing. Secondly, we envisage the potential of identification and formulation of new EEG signal processing techniques based on the FRI framework. In Chapter 4, we make use of the FRI model as a preprocessing step to remove some types of artifacts commonly found in EEG signals. Our scheme will be presented by first modelling the EEG signals as signals with FRI embedded in $1/f$ noise and then sampling EEG signals at their rate of innovation. The signals can then be effectively represented by a small set of Fourier coefficients corresponding to the signals' rate of innovation. Using methods developed in the FRI sampling theory, we can reconstruct the original signals using this small set of Fourier coefficients with low reconstruction errors. As a result of modelling the EEG signals as signals with finite rate of innovation, we propose a signal acquisition scheme which compresses EEG signals during acquisition while maintaining the morphologies of these signals. Our results based on various distortion measures show that the new FRI model for EEG signals maintains the signals' morphologies when compared to the original signals.

3.1 The FRI model for EEG signals

We present and validate the FRI model for EEG signals in this section. We first consider three types of nonuniform spline models to ascertain which type fits the EEG signals best. Next we consider embedding the nonuniform spline models in two types of noise models and finally arrive at a suitable FRI model for EEG signals.

3.1.1 Spline-based FRI models with additive noise

In this section we model the EEG signals as

$$x(t) = x_{NUS}(t) + n(t), \quad (3.1)$$

where $x_{NUS}(t)$ is the nonuniform spline component, and $n(t)$ is the noise component. We consider the cases of nonuniform linear spline, nonuniform quadratic spline and lastly the nonuniform cubic spline (where $R = 1, 2, 3$ respectively in Section 2.6.2) with additive noise. We also compare the models with the original signals based on the performance metrics described in Section 3.4.2 and conclude with a suitable FRI model for EEG signals. Two noise models are considered in our model. The first type of noise signal considered is white noise, which is a zero-mean signal characterised by a flat power spectral density. The second type of noise signal is $1/f$ noise whose power spectral density is inversely proportional to its frequency [117]. Accordingly, we define

$$\Gamma_w(f) = N_w \quad (3.2)$$

and

$$\Gamma_p(f) \propto 1/f^\alpha, \quad (3.3)$$

where Γ_w and Γ_p are the power spectral densities of white and $1/f$ noise respectively, N_w is the constant power spectral density of white noise and $0 < \alpha < 2$.

Comparison of different nonuniform spline models

As depicted in Figure 3.1, a comparison between the original EEG signal and its approximations show that the nonuniform linear spline model gives the best approximation where a Cross Correlation (CC) of 99.22%, with an Root Mean Square Error ($RMSE$) of 0.0020, a Percent Root Difference (PRD) of 13.07% and a Maximum Error ($MAXERR$) of 0.0012 are achieved. An evaluation of 30 sets of 10 second EEG epochs in Table 3.1 show that the nonuniform linear and cubic spline models best fitted the EEG signals.

Figure 3.1: Comparison of different spline models of EEG. (a) Original EEG signal; (b) Linear spline model, $CC = 99.22\%$, $RMSE = 0.0020$, $PRD = 13.07\%$, $MAXERR = 0.0012$; (c) Quadratic spline model, $CC = 96.00\%$, $RMSE = 0.0046$, $PRD = 29.65\%$, $MAXERR = 0.023$; (d) Cubic spline model $CC = 99.13\%$, $RMSE = 0.0021$, $PRD = 13.65\%$, $MAXERR = 0.0011$.

Table 3.1: Comparison of different approximation models for 30 epochs of EEG signals.

Evaluation metric	Linear spline model	Quadratic spline model	Cubic spline model
Average CC (%)	97.92	90.37	98.11
Average RMSE	0.0024	0.0059	0.0020
Average PRD (%)	15.26	45.21	16.42
Average MAXERR	0.0013	0.027	0.0012

Figure 3.2: Examples of modelling different EEG signals as nonuniform linear splines with (a) $K = 37$ pieces. (b) $K = 23$ pieces. (c) $K = 53$ pieces. (d) $K = 45$ pieces.

Since the results achieved with a linear spline model are very close to that of a cubic spline model, the nonuniform linear spline model is chosen as our EEG model to minimise computation costs. Figure 3.2 illustrates the nonuniform linear spline model for different EEG signals whereby the approximations closely model the original signals with K different pieces of linear splines. As such, we conclude that with our data, different EEG signals can be modeled with $2K \in [350, 900]$ for each 10 seconds EEG segment. The nonuniform linear spline model makes up approximately 97% of the EEG signal. From our experiments, we observed that the residue signal (EEG signal - nonuniform linear splines) resembled a $1/f$ noise signal with a power spectral density $\Gamma(f) \propto 1/f^\alpha$. The range of this noise amplitude was found to be $[-0.003, 0.002]\mu V$ and α was estimated to be around 0.75 by computing the slope of a fitted line onto the log-log plot of $\Gamma(f)$ vs. f as shown in Figure 3.3(a) and 3.3(b) according to [119]. Thus, our EEG signals are modeled as nonuniform linear splines embedded in additive $1/f$ noise. In this way, the EEG signals have been cast as signals with finite rate of innovation embedded in noise and this

Figure 3.3: (a) The average power spectral density of the residue signal compared to that of $1/f$ noise; (b) The estimation of α from the log-log plot of average power spectral density vs. frequency of the residue signal.

provides a motivation to exploit the FRI framework for compressive sampling of EEG signals.

3.2 Estimation of FRI signal parameters embedded in $1/f$ noise

In this section, we consider two cases of FRI signals embedded in $1/f$ noise, namely a stream of Dirac pulses and a nonuniform linear spline. In addition, the $1/f$ noise is generated with the amplitude and α values of $[-0.003, 0.002]\mu V$ and 0.75 respectively to match closely to that observed in the earlier section. The $1/f$ noise can be generated by filtering white noise with a stable, linear time-invariant low pass filter whose frequency amplitude response is $H(f) \propto 1/\sqrt{f}$. This translates to a signal to noise ratio of approximately $20dB$. We employ the Cadzow's noise suppression technique introduced in [120] to suppress the $1/f$ noise before estimating the parameters of the recovered FRI signals. Hence, we have

$$x(t) = x_{FRI}(t) + n_p(t), \quad (3.4)$$

where $x(t)$ is the observed signal, $x_{FRI}(t)$ is the FRI signal, and $n_p(t)$ is the $1/f$ noise signal. Since

$$n_p(t) = n_w(t) * h(t) \quad (3.5)$$

the autocorrelation of the $1/f$ noise can be expressed as

$$\gamma_p(l) = \sigma_w^2 (h * h)(l), \quad (3.6)$$

where σ_w^2 is the variance of the white noise signal. Furthermore, the autocorrelation of a low pass filter impulse response is an exponentially decaying function, hence, we can view the expectation of the $1/f$ noise signal to be a scaled version of the expectation of the white noise signal. Thus, we can make use of the Cadzow noise suppression technique in

Figure 3.4: (a) The noiseless stream of Dirac pulses with $K = 5$ and the same signal embedded in $1/f$ noise; (b) The estimation of the locations and amplitudes of the Dirac pulses.

Figure 3.5: (a) The noiseless nonuniform linear splines with $K = 5$ and the same signal embedded in $1/f$ noise; (b) The estimation of the locations and amplitudes of the Dirac pulses.

the FRI reconstruction method to reduce additive $1/f$ noise. As observed in Figure 3.4 and 3.5, we are able to estimate the locations and amplitudes of the innovation parameters of the FRI signals when they are embedded in $1/f$ noise. By degrading the signals to a SNR of $-10dB$, estimation errors in the amplitude values begin to appear, although the locations of the innovation parameters are quite accurately estimated.

3.3 Sampling and reconstruction of EEG signals at rate of innovation

Figure 3.6 shows our proposed EEG signal acquisition process with finite rate of innovation. Corresponding to the representation for the EEG signals, a reconstruction method is presented in Figure 3.7. Since the EEG signals are modeled as an FRI signal with

Figure 3.6: Our proposed scheme for sampling EEG signals with finite rate of innovation.

Figure 3.7: The FRI reconstruction block diagram for EEG signals.

additive $1/f$ noise, we will employ methods developed in [79, 121] and described in Section 2.6.2 to acquire and reconstruct them. It is important that the rate of innovation of the EEG signals are known and in our case, it can be estimated from the source signals. One method of estimating the rate of innovation is shown as follows:

Step 1 Estimate the locations $\{t_k\}_{k=0}^{K-1}$ and weights $\{c_k\}_{k=0}^{K-1}$ of the peaks and troughs of the original signal.

A point in the signal is considered a maximum peak (or a minimum trough) if it has the maximal (or minimum) value, and was preceded to the left by a value lower than an empirically derived threshold value ϵ . A range of values of ϵ in $[0.001, 1]$ in steps of 0.001 has been examined and $\epsilon = 0.1$ is chosen since values less than 0.1 detect the same peaks and troughs while values greater than 0.1 result in a smoothed signal that do not comply to the original peaks and troughs. The result of the estimation is a stream of Dirac pulses.

Step 2 Integrate the stream of Dirac pulses two times to get a nonuniform linear spline approximation of the original signal.

In this way, the number of pieces of linear splines needed to represent the EEG signals are estimated and given as shown in Table 3.2. With these assumptions, we represent the EEG signals as K pieces of nonuniform linear splines embedded in $1/f$ noise and the corresponding rate of innovation will be $\rho = 2K/\tau$ with $\tau = 10$ seconds.

In order to perform the Cadzow's noise reduction, a rectangular $(L \times (L + 1), L \geq K)$ Toeplitz matrix $\mathbf{D}$ is created from the spectral values of the source signal $x(t)$ in the form

$$\mathbf{D} = \begin{pmatrix} X[0] & X[-1] & \cdots & X[-L] \\ X[1] & X[0] & \cdots & X[-L+1] \\ \vdots & \vdots & \vdots & \vdots \\ X[L-1] & X[L-2] & \cdots & X[-1] \end{pmatrix}. \quad (3.7)$$

We then perform a singular vector decomposition of the matrix $\mathbf{D}$, and enforce rank K on $\mathbf{D}$ by choosing only K most significant singular values. This is iterated until the ratio of the largest singular value of the $K^{th} + 1$ to that of the K^{th} is smaller than a preset threshold. Thus the denoised DFT coefficients can be extracted from $\mathbf{D}$ [121].

Table 3.2: The assumed values of $2K$, the respective compression ratio CR and C .

EEG epoch	No. of coefficients $2K$	Compression ratio CR (%)	C
P1_001	694	72.85	3.68
P1_002	716	72.03	3.61
P1_003	852	66.71	3.00
P1_004	640	75.00	4.00
P1_005	702	72.61	3.65
P1_006	880	65.63	2.91
P1_007	788	69.18	3.24
P1_008	784	69.34	3.26
P1_009	720	71.88	3.56
P1_010	742	70.98	3.45
P2_001	474	81.48	5.40
P2_002	582	77.27	4.40
P2_003	635	75.20	4.03
P2_004	706	72.42	3.63
P2_005	742	71.02	3.45
P2_006	490	80.86	5.23
P2_007	548	78.55	4.66
P2_008	610	76.17	4.20
P2_009	544	78.71	4.70
P2_010	888	65.31	2.89
P3_001	860	66.41	2.98
P3_002	836	67.34	3.06
P3_003	470	81.64	5.45
P3_004	682	73.36	3.75
P3_005	624	75.59	4.10
P3_006	490	80.86	5.23
P3_007	631	75.35	4.06
P3_008	755	70.51	3.39
P3_009	654	74.45	3.91
P3_010	820	67.97	3.12

Since our EEG signals are modeled as nonuniform linear splines, we perform a differentiation operation twice on the denoised signals so as to reduce them into a stream of Dirac pulses. In order to find the locations and weights of the Dirac pulses, consider a filter $A[m]$ whose z -transform has K zeros at $u_k = e^{-i\frac{2\pi t_k}{\tau}}$, that is, $A(z) = \prod_{k=0}^{K-1} (1 - u_k z^{-1})$. Since the CTFS of the twice differentiated EEG signal $x^{(2)}(t)$ is a linear combination of K complex exponentials u_k , it follows that $A[m]$ is an annihilating filter and satisfies the following condition

$$A[m] * X^{(2)}[m] = 0 \quad (3.8)$$

where

$$X^{(2)}[m] = \left(\frac{j2\pi m}{\tau}\right)^2 X[m]. \quad (3.9)$$

The coefficients of the annihilating filter are found solving Eq. (3.8) which is equivalent to the following Toeplitz linear system of equations

$$\sum_{k=0}^K A[k] X^{(2)}[m - k] = 0. \quad (3.10)$$

Thus the locations $\{t_k\}_{k=0}^{K-1}$ of the Dirac pulses are given by the roots of $A(z)$. Next, the weights $\{c_k\}_{k=0}^{K-1}$ of the Dirac pulses are given by solving the Vandermonde system of equations given by

$$X^{(2)}[m] = \frac{1}{\tau} \sum_{k=0}^{K-1} c_k e^{\frac{-j2\pi m t_k}{\tau}}, m = 0, \dots, K-1. \quad (3.11)$$

Lastly, the stream of Dirac pulses are integrated twice to obtain the reconstructed EEG signal which corresponds to the nonuniform linear spline approximation of the EEG signal.

3.4 EEG data description and performance metrics

3.4.1 EEG data description

A total of 3 sets of EEG signals comprising 72 hours were used for the study. The data is further divided into 10 second epochs for processing. All 3 patients experienced

similar seizure types at similar locations. From this dataset, 30 epochs of 10 seconds duration were selected for establishing a finite rate of innovation model of EEG signals while the rest of the data was used to evaluate our compression scheme. The EEG data were acquired using a Neurofile NT digital video EEG system with 128 channels, 256 Hz sampling rate, and a 16 bit analogue-to-digital converter. Notch or bandpass filters have not been applied. More details of the database can be found in reference [116]. In our experiments, these EEG signals are assumed to be the source signals. For each patient, there will be 360 epochs and the epochs will be referenced as Px_yyy where x represents the patient number and yyy represents the epoch number.

3.4.2 Performance metrics

We utilised the following metrics [43]:

- The compression ratio (CR) is defined as a ratio between the number of bits required to represent the original signal and the compressed signal. First, we define a ratio

$$C = \frac{b_{orig}}{b_{comp}} \quad (3.12)$$

where b_{orig} and b_{comp} represent the numbers of bits required for the original and compressed signals respectively.

Thus we can define a CR commonly used in literature as

$$CR(\%) = \frac{C - 1}{C} \times 100. \quad (3.13)$$

- A metric that can be used to measure distortion is percent root difference (PRD). This metric is commonly used for measuring the distortions in reconstructed biomedical signals such as Electrocardiographic (ECG) signals and EEG signals. For signals

of length N , PRD can be defined as

$$PRD(\%) = \sqrt{\frac{\sum_{n=1}^N (x_{orig}(n) - x_{recon}(n))^2}{\sum_{n=1}^N (x_{orig}(n))^2}} \times 100, \quad (3.14)$$

where $x_{orig}(n)$ and $x_{recon}(n)$ are the sampled values of the original and reconstructed signals.

- Another distortion metric is the root mean square error ($RMSE$). In data compression, we are interested in finding an optimal approximation for minimizing this metric as defined by the following formula

$$RMSE = \sqrt{\frac{\sum_{n=1}^N (x_{orig}(n) - x_{recon}(n))^2}{N}}. \quad (3.15)$$

- Since the similarity between the reconstructed and original signal is crucial from the clinical point of view, the cross correlation (CC) is used to evaluate the similarity between the original signal and its reconstruction.

$$CC = \frac{\frac{1}{N} \sum_{n=1}^N (x_{orig}(n) - \overline{x_{orig}})(x_{recon}(n) - \overline{x_{recon}})}{\sqrt{\frac{1}{N} \sum_{n=1}^N (x_{orig}(n) - \overline{x_{orig}})^2} \sqrt{\frac{1}{N} \sum_{n=1}^N (x_{recon}(n) - \overline{x_{recon}})^2}}, \quad (3.16)$$

where $\overline{x_{orig}}$ and $\overline{x_{recon}}$ are the mean values of the original and reconstructed signals respectively.

- In order to understand the local distortions between the original and the reconstructed signals, two metrics, the maximum error ($MAXERR$) and the peak amplitude related error ($PARE$) [118], will be computed. The maximum error metric is defined as

$$MAXERR = \max(x_{orig} - x_{recon}) \quad (3.17)$$

and it shows how large the error is between every sample of the original and reconstructed signals. This metric should ideally be small if both signals are similar.

- The *PARE* is defined as

$$PARE(n) = [x_{orig}(n) - x_{recon}(n)]/x_{orig}(n). \quad (3.18)$$

By plotting *PARE*, one will be able to understand the locations and magnitudes of the errors between the original and reconstructed signals.

3.5 Results and discussions

In this section, we present results for the EEG dataset described in Section 3.4.1 based on the performance metrics in Section 3.4.2. Comparisons to wavelet based compression techniques using Daubechies and Coiflets wavelet will be included in our discussions. These wavelet transforms are performed with four detailed levels and one approximation [41]. We also compared our results to those found in [45] in terms of normalised mean square error (*NMSE*), which is the ratio of mean square error of the reconstructed signals to the range of amplitudes of the signals.

We applied our method on the 3 sets of EEG signals and the results of 30 selected epochs are tabulated in Table 3.3. The *CC* is selected as the primary evaluation metric with $CC = 1$ as the best achievable value and our results are generated with the best *CC* achievable for each EEG signal. As shown in Table 3.3, consistently high *CC* ranging from 89.71 to 97.56 is achieved for our dataset. This implies that there is great similarity in the morphology between the original and reconstructed EEG signals. This result is highly desirable because such diagnostic features are extremely important and must be preserved. Our method achieves a *CR* ranging from 65.31% to 81.48%. This is due to the morphology of the EEG signals, where some signals need more linear splines to model them compared to the others. As *K* varies, the rate of innovation ρ varies accordingly and leads to an increase or adcrease in the number of spectral coefficients. Thus *CR* varies inversely as ρ . Table 3.3 also tabulates the distortions arising from our method.

Table 3.3: Performance of our method: *CC*, *RMSE*, *PRD* and *MAXERR*.

EEG	Cross correlation	Root mean square error	Percent root difference	Maximum error
	<i>CC</i> (%)	<i>RMSE</i>	<i>PRD</i> (%)	<i>MAXERR</i>
P1.001	94.05	0.0067	39.35	0.0235
P1.002	94.98	0.0082	32.37	0.0247
P1.003	91.87	0.0051	30.43	0.0212
P1.004	95.95	0.0024	34.90	0.0187
P1.005	93.52	0.0122	40.05	0.0204
P1.006	89.71	0.0066	51.37	0.0231
P1.007	92.42	0.0183	49.37	0.0213
P1.008	94.20	0.0091	35.43	0.0220
P1.009	94.20	0.0081	38.54	0.0186
P1.010	97.56	0.0039	23.05	0.0070
P2.001	95.08	0.0099	39.35	0.0102
P2.002	94.05	0.0055	39.78	0.0151
P2.003	93.98	0.0070	31.09	0.0211
P2.004	96.51	0.0069	31.52	0.0098
P2.005	95.36	0.0100	34.85	0.0178
P2.006	95.64	0.0063	30.83	0.0123
P2.007	92.28	0.0007	34.31	0.0210
P2.008	95.07	0.0008	36.31	0.0090
P2.009	97.26	0.0046	27.45	0.0012
P2.010	92.88	0.0096	43.52	0.0244
P3.001	95.25	0.0052	30.19	0.0231
P3.002	92.02	0.0031	32.11	0.0235
P3.003	90.03	0.0011	33.17	0.0209
P3.004	94.08	0.0032	33.27	0.0125
P3.005	93.44	0.0024	32.75	0.0102
P3.006	94.69	0.0011	37.07	0.0196
P3.007	92.07	0.0019	32.70	0.0180
P3.008	91.63	0.0012	28.33	0.0219
P3.009	90.20	0.0039	36.37	0.0243
P3.010	96.27	0.0025	29.95	0.0023

Figure 3.8: A typical plot of *PARE*. Errors between original and reconstructed samples are depicted as spikes in this plot.

We obtained low *RMSE* and *PRD*, implying that our method recovers signals with some distortion. Furthermore, a *MAXERR* between 0.0070 and 0.0247 is obtained, suggesting that the distortions of the reconstructed signals are very small. A typical plot of *PARE* and a histogram of the errors are shown in Figure 3.8 and 3.9. Errors between original samples of the signal and the corresponding reconstructed samples will be amplified and shown in the *PARE* plot. The histogram of errors shows a concentration of errors in the range between -0.005 to 0.005 , and some outliers in the larger error bins, thus contributing to distortions. Although the results are satisfactory, we observed that the distortions arise from the estimation of the innovation parameters. Let us make a comparison of the *CC* obtained by sampling both the original EEG epochs and their nonuniform linear spline approximation (i.e. noise free signals). If the estimation of the innovation parameters are accurate, the *CC* obtained should be the same. As shown in Figure 3.10, the *CC* obtained for the noise free signal is higher than that of the noisy signal, due to wrong estimations of locations and weights.

Figure 3.9: Histogram plot of the errors obtained for an epoch.

Figure 3.10: Comparison of the 30 epochs' CC in the event of (i) sampling the original EEG epochs; and (ii) sampling their nonuniform linear spline approximations (noise free signals).

Figure 3.11: Variation of the evaluation metrics on 343 epochs corresponding to 3430 seconds of continuous EEG from 1 patient. (a) The number of coefficient 2K vs. epochs; (b) CC vs. epochs;(c) RMSE vs. epochs;(d) CR vs. epochs.

Table 3.4: Performance of our method on the 3 EEG datasets.				
EEG	Patient 1	Patient 2	Patient 3	Average
Mean no. of coefficients	941	982	973	965
Mean CR (%)	63.24	61.53	61.98	62.3
Mean CC (%)	94.30	95.32	95.63	95.08
Mean PRD (%)	34.89	32.72	33.29	33.63
Mean RMSE	0.0080	0.0061	0.0025	0.0055
Mean MAXERR	0.0201	0.0302	0.0171	0.0225

In addition, we present results for sampling three 24-hour recording. Figure 3.11 shows how the various metrics change in a continuous EEG recording of one patient. As illustrated in Figure 3.11(a), the number of coefficients 2K varies between 200 and 1000, thus showing that for the same patient, the innovation parameters cannot be assumed to be constant. We also noted that although the value 2K varies in a large range, CC is quite consistently lying in the range of 91% to 97% as in Figure 3.11(b). Furthermore, the distortion in the EEG is kept very low, as depicted by Figure 3.11(c). Compression

Figure 3.12: Comparison of reconstruction of an EEG signal acquired by our method. (a) The original signal sampled at 256 Hz; (b) The EEG signal acquired by our method at 47.5 Hz with $2K = 474$; (c) The reconstructed EEG signal at 256 Hz with our method $CC = 97.08\%$, $RMSE = 0.0099$, $PRD = 22.43\%$, $MAXERR = 0.0016$; (d) The reconstructed EEG signal using the traditional sinc interpolation method $CC = 90.11\%$, $RMSE = 0.0147$, $PRD = 48.30\%$, $MAXERR = 0.428$.

Figure 3.13: A comparison of the performance of our method with sinc interpolation, wavelet compression using Daubechies and coiflets wavelets on an EEG epoch. (a) Variation of CC with CR . (b) Variation of PRD with CR . (c) Variation of $RMSE$ with CR . (d) Variation of ρ with CR .

ratio for this recording ranges from about 50% to 99%, which varies faithfully with $2K$. Table 3.4 tabulates the mean CR , CC , PRD , $RMSE$, $MAXERR$ and number of coefficients for each dataset. On the average, we achieved a CR of around 62.3% with low PRD , $RMSE$ and $MAXERR$ errors of around 33.63%, 0.0055 and 0.0225 respectively. In addition, the CC achieved is around 95.08%. Figure 3.12 presents an example of the strength and uniqueness of our modelling method. The EEG signal's original sampling frequency is 256 Hz (Figure 3.12(a)). Our system estimated $2K$ to be 474 and modeled the signal with 474 samples of the original signal (Figure 3.12(b)), which has a CR of 81.48%. Effectively, we are sampling the original signal at 47.5 Hz. We effectively reconstructed the 256 Hz signal as shown in Figure 3.12(c) with $CC = 97.08\%$, $RMSE = 0.0099$, $PRD = 22.43\%$, $MAXERR = 0.0016$. As a comparison, we reconstructed the signal in Figure 3.12(b) with the traditional sinc interpolation method (Figure 3.12(d)) with $CC = 90.11\%$, $RMSE = 0.0147$, $PRD = 48.30\%$, $MAXERR = 0.428$. Clearly, we are able to represent EEG signals with a low number of samples and reconstruct them with high fidelity in terms of morphology. Figure 3.13 shows the relation between CC , $RMSE$, PRD and ρ with CR respectively. The CR is inversely proportional to the rate of innovation, as shown in the earlier discussion. In order to achieve a high CC , CR has to be compromised. Similarly, as we increase CR , the error involved such as $RMSE$ and PRD will increase together, though not in a linear form. A comparison is made with traditional sinc interpolation and the performance of our method is superior since an interpolation process is unable to faithfully reconstruct signals acquired at a low sampling rate into one of a higher sampling rate. Next we compared our results with that of compressing the EEG signals using wavelet compression methods. As illustrated, our method achieves comparable results in terms of CC and PRD although the $RMSE$ achieved by wavelet methods is slightly better. Since $RMSE$ only indicates how much error is incurred in the reconstruction without reference to the original morphology of the signals, our results indicate that our reconstructed signals do not differ largely from the original

Figure 3.14: A comparison between (a) an original EEG signal and (b) the reconstructed EEG signal. $2K = 1184$, $CR = 81.48\%$ $CC = 97.52\%$, $RMSE = 0.0044$, $PRD = 24.55\%$, $MAXERR = 0.0022$.

Figure 3.15: A zoom-in view of the comparison of original and reconstructed EEGs in Figure 3.14.

signals morphologically. Furthermore, our method entails a less costly low-rate sampling device and does not waste precious computational resources collecting extra data only to discard them subsequently. Figure 3.14 illustrates a comparison between an original and a reconstructed EEG signal with the $2K = 1184$ coefficients, we reconstructed the signal and achieved $CR = 81.48\%$, $CC = 97.52\%$, $RMSE = 0.0044$, $PRD = 24.55\%$ and $MAXERR = 0.0022$. Furthermore, the $NMSE$ of the reconstructed signal is in the range of 5×10^{-9} to 7.2×10^{-7} as opposed to 0.01 achieved by the method in [45]. A zoomed in view in Figure 3.15 confirms that the morphology of the original signal is conserved in the reconstruction.

Lastly we will discuss about the computational costs of our scheme. With reference to Figure 3.7, the computational complexity can be estimated as follows:

- Compute the DFT to obtain the Fourier series coefficients $\in [-K, K]$ - $O(K \log K)$
- Denoising - $O(K^3)$
- Differentiating the denoised signal - $O(K)$
- Solving a Toeplitz system of equation of size K by K to get $A(z)$ - $O(K^2)$
- Finding the roots of $H(z)$ by factorization, to get t_k - $O(K^3)$
- Solving a Vandermonde system of equation of size K by K , to get c_k - $O(K^2)$
- Integrating the K Dirac pulses - $O(N)$

Hence effectively, the computational costs involved is $O(K^3 + N)$. For our dataset, the average time required to sample and reconstruct a 10-second epoch is 5.009 seconds on an Intel Core2 Duo 2.50GHz system with 4GB RAM. This computational time can be improved by employing fast algorithms on dedicated digital signal processors to achieve a real-time EEG signal acquisition and display.

3.6 Conclusions

We proposed a new EEG signal model and framework to acquire electroencephalographic signals at their rate of innovation based on the finite rate of innovation sampling theory. Even though a small set of data is obtained, our method retains the morphologies of the EEG signals and inherently compresses the signals. We examined our method on 3 sets of electroencephalographic signals comprising 72 hours of recording and present results based on factors commonly found in the compression literature, namely compression ratio, percent root difference and root mean square error. We also employed the cross correlation criterion to study the morphological relations between the original and reconstructed electroencephalographic signals. It is observed that the FRI model preserves the morphological information of the electroencephalographic signals and achieves high compression ratios with low reconstruction errors. It yielded promising results at a low computational cost and thus, we achieve computational savings which can be utilised in other more important signal processing stages. These results confirm the validity of our model. Furthermore, improvements to compression ratios can be achieved by considering structures that yield sparser EEG representations, such as in the time-frequency domain. More importantly, this model serves as a foundation for future exploitation of the finite rate of innovation sampling theory to analyse EEG signals, such as spike train detection, artifact removal and seizure detection. New performance measures can be established based on different parameters of the FRI framework.

Although the accuracy of the estimated K affects the entire scheme, as discussed in [121], it can be estimated from the rank of a Toeplitz matrix. However, more research is needed to determine the correct duration of EEG signals to yield optimal K values based on certain evaluation metrics such as CR , CC , PRD or $MAXERR$. Furthermore, it is observed that K changes depending on the state of the EEG, hence a scheme capable of yielding variable rates of innovation is desired and K remains a viable candidate as some form of

performance measures.

Chapter 4

Artifact removal in EEG signals

Artifacts are commonly found in EEG signals. Correct diagnoses of medical problems as abnormalities in EEG signals are often impeded because they can be masked by these artifacts. As such, reliable artifact removal methods are crucial to enable proper diagnoses of neurological conditions. In this chapter, we propose a novel EEG artifact removal method that relies on modelling the EEG and artifact signals as signals with finite rate of innovation (FRI) [79] by varying the model in Chapter 3. We focus our examination on three types of artifacts, namely electrostatic discharges, electrooculographic and electromyographic artifacts, and we show that they can be effectively removed concurrently. Firstly, we will revise and extend the theory on sampling bandlimited signals combined with nonuniform splines and Dirac pulses. Next, we will explain how different types of artifacts, namely electrostatic discharge artifact, EOG artifact and EMG artifact, can be modeled as FRI signals in the form of Dirac pulses and nonuniform splines. Subsequently, our artifact removal algorithm is introduced followed by the results achieved.

4.1 EEG signal and artifact model

In our study, three EEG artifacts, namely the electrostatic spike discharges, EOG and EMG artifacts, will be modeled as FRI signals. In particular, the electrostatic spike discharges will be modeled as a stream of Dirac pulses, the EOG artifact as a piecewise constant signal and a nonuniform spline of degree 1, and the EMG artifact as a nonuniform

spline of degree 1.

4.1.1 Electrostatic spike discharges

As described in [46], electrostatic discharges resemble that shown in Figure 4.1(a). These discharges can therefore be modeled as a stream of Dirac pulses such that

$$x_{ED}(t) = \sum_{k=1}^K c_k \delta(t - t_k), \quad (4.1)$$

where $x_{ED}(t)$ is the electrostatic discharge artifact signal, t_k represents the timings of the electrostatic discharges and c_k represents the voltage of the discharges. In this case, the rate of innovation of this artifact is $\rho = 2K/\tau$. The model of this artifact is shown in Figure 4.1(b).

Figure 4.1: Examples of artifacts in EEG signals and their FRI models. (a) An Electrostatic discharge artifact, (b) The FRI model of electrostatic discharge artifact. (c) An EOG eye movement artifact, (d) The FRI model of the EOG eye movement artifact. (e) An EOG eye blink artifact, (f) The FRI model of the EOG eye blink artifact. (g) An EMG artifact, (h) The FRI model of the EMG artifact.

4.1.2 Electrooculographic (EOG) artifacts

It has been shown in [122] that eye movements create DC shifts in EOG signals which can be represented by square voltage waveforms. Figure 4.1(c) shows an example of EOG artifacts and these can be mathematically expressed as

$$x_{EOG}^{(1)}(t) = \sum_{k=0}^{K-1} c_k \delta(t - t_k) \quad (4.2)$$

where $x_{EOG}(t)$ is a piecewise constant signal with knots at $\{t_k\}_{k=0}^{K-1}$ and the first derivative $x_{EOG}^{(1)}(t)$ is a stream of K weighted Dirac pulses defined by Eq. (4.2). In this case, the rate of innovation of this artifact is $\rho = 2K/\tau$ and this artifact is illustrated in Figure 4.1(d). During eye blinks as shown in Figure 4.1(e), the EOG signals generated can be represented as a nonuniform linear spline such that

$$x_{EOG}^{(2)}(t) = \sum_{k=0}^{K-1} c_k \delta(t - t_k) \quad (4.3)$$

where $x_{EOG}(t)$ is a nonuniform linear spline signal with knots at $\{t_k\}_{k=0}^{K-1}$ and the second derivative $x_{EOG}^{(2)}(t)$ is a stream of K weighted Dirac pulses defined by Eq. (4.3). An example of this artifact is shown in Figure 4.1(f). The rate of innovation for this artifact is $\rho = 2K/\tau$.

4.1.3 Electromyographic (EMG) artifacts

Figure 4.1(g) shows an EEG signal which is contaminated by EMG artifacts. According to [123], EMG signals can be modeled mathematically as a summation of multiple trains of muscle unit action potentials:

$$x_{EMG}(t) = \sum_{g=1}^G \sum_{k=1}^K h_g(t - t_{g,k}) + v(t), \quad (4.4)$$

where $x_{EMG}(t)$ is the EMG artifact signal, $v(t)$ is additive white noise, $h(t)$ represents a particular action potential waveform, g indexes the number of trains and k indexes the

number of action potentials. For simplicity, we look at the case where $G = 1$, that is, only 1 train of action potentials makes up the EMG signal and it is noise free. By further modelling $h(t)$ as a nonuniform linear spline, we arrive at an FRI model for the EMG artifact signal. An example of the EMG signal and its muscle unit action potential are shown in Figure 4.1(h) and 4.2 respectively. Similarly, the rate of innovation for this artifact is $\rho = 2K/\tau$.

Figure 4.2: The FRI model of 1 muscle unit action potential.

4.1.4 Our signal model

We assume that the artifact-free EEG signals are L bandlimited and the artifacts are added to the EEG signals linearly. Thus our signal model can be expressed as

$$x(t) = x_{EEG}(t) + x_A(t), \quad (4.5)$$

where $x(t)$ is the contaminated EEG signal, $x_{EEG}(t)$ is the artifact-free EEG signal and $x_A(t)$ is the artifact signal equivalent to either $x_{ED}(t)$, $x_{EOG}(t)$, $x_{EMG}(t)$ or a combination of all 3 types of artifacts. We express the artifact signal as

$$x_A(t) = \alpha_1 x_{ED}(t) + \alpha_2 x_{EOG}(t) + \alpha_3 x_{EMG}(t) \quad (4.6)$$

where α_1 , α_2 and α_3 are either 0 or 1 to indicate whether the artifact is absent or present, respectively.

According to [19], EEG signals typically range between 0.1 Hz and 100 Hz, thus we assume that L is equivalent to 100 Hz.

4.2 Artifact removal algorithm

In this section, we first revise the theory of sampling bandlimited signals that are combined with streams of Dirac pulses and nonuniform splines. Then we present our artifact removal algorithm to remove three types of artifacts.

4.2.1 Sampling bandlimited signals + a combination of stream of Dirac pulses and nonuniform splines of degrees R

Consider a τ periodic signal $x_A(t)$ which is made up of a piecewise combination of stream of Dirac pulses and nonuniform splines of degree R as described in Eq. (4.6). Table 4.1 tabulates the different rate of innovation ρ for all the possible combinations of artifact signals. Hence, following the method to sample a stream of Dirac pulses, by taking a sinc sampling kernel $h_B(t) = B\text{sinc}(Bt)$ and B is greater or equal to the rate of innovation $\rho = 2K/\tau$, and sampling $y(t) = (h_B * x_A)(t)$ at N uniform locations $t = nT; n = 0, \dots, N-1$, $M = \lfloor \frac{B\tau}{2} \rfloor$, then the samples defined by $y_n = \langle h_B(t - nT), x_A(t) \rangle, n = 0, 1, \dots, N-1$ sufficiently represent $x_A(t)$.

Suppose that $x_A(t)$ is added to a L -bandlimited signal $x_{EEG}(t)$ as described by Eq. (4.5). Here, the rate of innovation $\rho = \frac{2L+1+2K}{\tau}$, where $K = K_{ED} + K_{EOG} + K_{EMG}$. The CTFS coefficients of $x(t)$ is given by:

$$X[m] = \begin{cases} X_{EEG}[m] + X_A[m] & \text{if } m \in [-L, L] \\ X_A[m] & \text{if } m \notin [-L, L] \end{cases} \quad (4.7)$$

Table 4.1: Combination of artifacts and their rate of innovation

Artifact	Rate of innovation $\rho = 2K/\tau$
$x_A(t)$	K
$x_{ED}(t)$	K_{ED}
$x_{EOG}(t)$	K_{EOG}
$x_{EMG}(t)$	K_{EMG}
$x_{ED}(t) + x_{EOG}(t)$	$(K_{EOG} + K_{EMG})$
$x_{ED}(t) + x_{EMG}(t)$	$(K_{ED} + K_{EMG})$
$x_{EOG}(t) + x_{EMG}(t)$	$(K_{EOG} + K_{EMG})$
$x_{ED}(t) + x_{EOG}(t) + x_{EMG}(t)$	$(K_{ED} + K_{EOG} + K_{EMG})$

Since the CTFS values of the EEG signal $X_{EEG}[m]$ is zero outside the band $[-L, L]$, the CTFS values of $x(t)$ outside this band will be exactly equal to that of the artifact signal $X_A(t)$. Thus, by choosing a contiguous set of at least $2K$ CTFS values outside the band $[-L, L]$, for example, in $[L+1, L+1+2K]$, the stream of K Dirac pulses can be recovered. By integrating this stream of Dirac pulses $R+1$ times, we can recover $x_A(t)$. The CTFS coefficients of the EEG signal, $X_{EEG}[m]$ can then be obtained by subtracting $X_A[m]$ from $X[m]$ in the band $[-L, L]$.

4.2.2 Our algorithm

The artifact removal algorithm is implemented in a discrete time domain. We note that if a Dirac pulse is differentiated P times, $P+1$ Dirac pulses will be created. In our algorithm, we consider differentiating the signal twice so that artifacts that are a combination of Dirac pulses, piecewise constants and nonuniform linear splines will be reduced to a stream of K weighted Dirac pulses. In this case,

$$K = 3K_{ED} + 2K_{EOG} + K_{EMG} \quad (4.8)$$

where K_{ED} is the number of Dirac pulses of the electrostatic discharge artifact signal, K_{EOG} is the number of pieces of constant signals in the EOG artifact signal, and K_{EMG} is the number of pieces of nonuniform linear splines of the EMG artifact signal. Thus the rate of innovation of this artifact signal is $\rho = 2K/\tau$ with K given by Eq. (4.8).

Thus we present the algorithm as:

Step 1 Sampling of $x(t)$ at the rate of innovation.

Let $B = \rho$ and consider the sampling kernel $h_B(t) = B\text{sinc}(Bt)$, then the sample values are

$$y_n = \langle h_B(t - nT), x(t) \rangle, n = 0, 1, \dots, N - 1. \quad (4.9)$$

Step 2 Find $2M + 1$, with $M = L + 2K$, contiguous spectral values of $x(t)$ from N samples

$$y_n = \sum_{m=-M}^M X[m] e^{i \frac{2\pi mn}{N}}. \quad (4.10)$$

Step 3 Determine the $2K$ spectral values of the stream of Dirac pulses from the spectral values of $x(t)$:

$$X^{(R+1)}[m] = \left(\frac{j2\pi m}{\tau}\right)^{R+1} X[m], m \in [L + 1, L + 1 + 2K], \quad (4.11)$$

where $R = 1$.

Step 4 Determine the locations of the Dirac pulses by the annihilating filter method.

The coefficients of the annihilating filter are found solving the following Toeplitz linear system of equations

$$\sum_{k=0}^K A[k] X_A^{(R+1)}[m - k] = 0, m = L + 1, \dots, L + 1 + 2K. \quad (4.12)$$

The locations $\{t_k\}_{k=0}^{K-1}$ of the Dirac pulses are given by the roots of $A(z)$.

Step 5 Determine the weights of the Diracs.

The weights $\{c_k\}_{k=0}^{K-1}$ of the Dirac pulses are given by solving the Vandermonde

system of equations given by

$$X_A^{(R+1)}[m] = \frac{1}{\tau} \sum_{k=0}^{K-1} c_k e^{\frac{-j2\pi m t_k}{\tau}}, m = L+1, \dots, L+1+K. \quad (4.13)$$

Step 6 Integrate $(R+1)$ times the stream of Dirac pulses to get the artifact signal $x_A(t)$.

Step 7 Obtain the CTFS of the artifact signal $X_A[m]$, $m \in \mathbb{Z}$.

Step 8 Subtract the spectral values of the artifact signal from the spectral values of $x(t)$ in $[-L, L]$.

Thus the original EEG signal can be recovered by taking the inverse CTFS of

$$X_{EEG}[m] = X[m] - X_A[m], m \in [-L, L]. \quad (4.14)$$

4.3 Results and discussions on semi simulated EEG data

In this section, we present the results achieved by our method and discuss our findings. A total of 100 sets of clean EEG signals comprising 23.6 seconds duration were used for the study. All EEG signals were recorded with the same 128-channel amplifier system, digitized with a sampling rate of 173.6 Hz and 12 bit A/D resolution. The data was filtered using a band pass filter with settings 0.53 to 40 Hz. More details of the database can be found in reference Andrzejak et. al. [125]. Simulated artifact signals shown in Figure 4.1(b), (d) and (f) are added randomly into the real EEG signals to create a semi simulated database of 100 contaminated EEG signals. These 100 sets of contaminated EEG signals are used to test our artifact removal algorithm. The "cleaned" EEG signals are scored using the percent root difference (*PRD*) performance metric found in Section 3.4.2 to measure distortion. Consider two EEG signals and a synthetic artifact signal shown in Figure 4.3. These signals are linearly mixed into four artifact-corrupted EEG

Figure 4.3: (a) and (b) Two EEG signals. (c) The artifact signal to be linearly mixed into signals (a) and (b).

Figure 4.4: Four sets of EEG signals linearly mixed with artifact signal.

Table 4.2: The average *PRD* (%) achieved for artifact removal.

Method	ED	EOG	EMG	Combination of all artifacts
FRI based	2.04×10^{-7}	1.04×10^{-7}	1.16×10^{-7}	2.12×10^{-7}
FastICA	89.4	90.1	94.1	92.4

Figure 4.5: Independent components (ICs) extracted by the FastICA method.

Figure 4.6: A comparison of extracted artifact using our method and the FastICA method.

signals as shown in Fig 4.4.

Using our method, the artifact signal is perfectly recovered from a single contaminated EEG signal, and thus the original EEG signals are recovered. Using an independent component analysis method, the FastICA method [124], we separated the components from these EEG signals as depicted in Figure 4.5 and Figure 4.6. By comparing the extracted artifact signals from our method and the FastICA method to the original artifact signal we clearly note that the FastICA recovers an artifact signal that closely resembles the original but is affected by permutations and scaling of the original signal. Permutations means that the order of the components estimated cannot be determined and scaling means that the variance of the independent components estimated cannot be determined. Further processing is required to remove these ambiguities. The main advantage of our method is that it can perform artifact removal on single channels. Table 4.2 tabulates the average *PRD* achieved for the removal of artifacts from the 100 contaminated EEG signals using our method and the FastICA method. The tabulation of results are divided into individual artifacts (ED, EOG, EMG) and a combination of all artifacts. The main observation for the high *PRD* obtained is due to the FastICA recovering an artifact signal that closely resembles the original artifact signal but affected by permutations and scaling of the original signal.

An example in Figure 4.7 shows an EEG signal with a seizure arising at sample 1024 and this onset has been contaminated by EMG artifacts. We show that the EMG artifacts can be removed as shown in Figure 4.8 and the average *PRD* achieved is $1.16 \times 10^{-7}\%$. An example of artifact removal achieved by our method is shown in Figure 4.9 to Figure 4.11. In this example, 3 types of artifacts shown in Figure 4.10 are introduced into the EEG signal in Figure 4.10. The removed artifacts are shown in Figure 4.10 and the "cleaned" EEG signal is depicted in Figure 4.11. We compare the extracted artifact to that extracted by FastICA in Figure 4.12 and observe that the artifacts' locations are generally correctly detected although differences in amplitudes exist.

Figure 4.7: The EEG signal with seizure onset contaminated with EMG artifacts.

Figure 4.8: The "cleaned" EEG signal with seizure onset clearly seen.

Figure 4.9: The EEG signal with combined artifacts. The locations of the artifacts are shown in the rectangle boxes.

Figure 4.10: The removed artifacts.

Figure 4.11: The cleaned EEG signal. The locations of the artifacts are shown in the rectangle boxes.

Figure 4.12: A comparison of the artifact signal found in Figure 4.9 extracted by FastICA and FRI method.

4.4 Results and discussions on real neonatal EEG data

In this section, our method is applied to 30 excerpts of 15 minutes long neonatal EEG data (description of this dataset can be found in Section 5.3.1) contaminated with various artifacts such as EOG, EMG, head movements and sucking. These artifacts occur during the normal states and do not overlap onto seizure epochs.

Figure 4.13: Extraction of EOG artifact signal (a) Original signal. (b) Detected stream of Dirac pulses. (c) Extracted EOG artifact signal as a nonuniform linear spline.

As depicted in Figure 4.13, our method detected a stream of K weighted Dirac pulses (Figure 4.13(b)) corresponding to the EOG artifact signal (Figure 4.13(a)). In this case, $K = 40$ and the artifact signal is recovered as shown in Figure 4.13(c). Figure 4.14 compares the original EOG contaminated signal with the cleaned signal. Visually, we conclude that the EOG artifacts are removed as shown in Figure 4.14(c). In Figure 4.14(d), the frequency spectra of the original and cleaned signals are compared. The low frequency spectral components of the contaminated signal is reduced while the higher frequency components are retained.

Figure 4.14: An EEG signal contaminated with EOG eyeblink artifacts. (a) Original signal. (b) Extracted artifact signal, $K = 40$. (c) Cleaned signal. (d) Power spectrum of original vs cleaned signal. Note that the low frequency components of the contaminated signal is reduced due to the removal of the EOG artifacts.

Next, we visit an example of a head movement artifact contaminated signal. Since the morphology of this artifact is similar to that of an EOG eyeblink artifact except for the duration of the artifact, we observed similar results to that of the EOG eyeblink removal, as shown in Figures 4.15 and 4.16. On the other hand, the removal of sucking artifacts, which are essentially high frequency spikes, results in the reduction of higher frequency spectral values of the signal as shown in Figures 4.17 and 4.18.

Our method failed to extract EMG artifact signals during our experiment, as depicted in Figure 4.19. Erroneous streams of weighted Dirac pulses with $2K > 1000$ are detected causing the recovered "EMG" artifacts to resemble signals of very high frequency contents. When this signal is subtracted spectrally from the original contaminated signal, an undesirable signal with high frequency contents is created. In our example, there exists two high frequency components at 40 and 80Hz. This is most likely due to the simple model adopted in Section 4.1.3. The model contains only one train of action potentials with the same morphology $h(t)$ do not match real life EMG artifacts since the EMG artifacts are

Figure 4.15: Extraction of head movement artifact signal (a) Original signal. (b) Detected stream of Dirac pulses. (c) Extracted head movement artifact signal as a nonuniform linear spline.

Figure 4.16: An EEG signal contaminated with head movement artifacts. (a) Original signal. (b) Extracted artifact signal, $K = 42$. (c) Cleaned signal. (d) Power spectrum of original vs cleaned signal. Note that the low frequency components of the contaminated signal is reduced due to the removal of the head movement artifacts.

Figure 4.17: Extraction of sucking artifact signal (a) Original signal. (b) Detected stream of Dirac pulses. (c) Extracted sucking artifact signal as a nonuniform linear spline.

Figure 4.18: An EEG signal contaminated with sucking artifacts. (a) Original signal. (b) Extracted artifact signal, $K = 10$. (c) Cleaned signal. (d) Power spectrum of original vs cleaned signal. Note that the high frequency components of the contaminated signal is reduced due to the removal of the sucking artifacts, which are essentially high frequency components.

most likely make up of multiple trains of action potentials with different $h(t)$ s.

Figure 4.19: An EEG signal contaminated with EMG artifacts. (a) Original signal. (b) Extracted artifact signal, $K = 1436$. (c) Cleaned signal. (d) Power spectrum of original vs cleaned signal. Note that our method did not remove the EMG artifact, but introduced undesirable high frequency components into the original contaminated signal.

Lastly, we show that two types of artifacts, namely head movement and sucking artifacts, are removed concurrently in Figures 4.20 and 4.21.

From our experiments, the typical values of K for EOG, head movement and sucking artifacts fall in the range of $2K \in [10, 120]$ for a 10-second EEG segment. However, that value cannot yet be determined for the EMG artifacts due the model mismatch. Hence, by choosing the value of $2K$ to be higher such as 200 will allow us to detect and extract the EOG, head movement and sucking artifacts concurrently.

4.5 Conclusions

We have introduced a novel EEG artifact removal method based on a recently developed sampling theory. Our contaminated EEG model follows a linear superposition of the artifact signals with clean EEG signals. This assumption holds as mixing of artifacts

Figure 4.20: Eight channels of neonatal EEG with head movement and sucking artifacts.

Figure 4.21: Eight cleaned channels of neonatal EEG.

and EEG processes are linear [47]. In fact, externally imposed electrical artifacts are also mixed linearly at the scalp electrodes [47]. In this chapter, our artifacts model typical biological artifact patterns. It is important to understand that there are other artifacts for which further study is required. As such, work will continue to include more patterns of artifacts. By modelling EEG signals and artifacts as signals with finite rate of innovation, we demonstrated that in the case of simulated experiments, EEG artifacts can be removed, resulting in EEG signals with very low percent root difference. By employing our method to recover FRI signals, we illustrated that these artifacts can be removed from contaminated EEG signals and the EEG signal recovery is achieved with very low *PRD*. We also demonstrated that combinations of such artifacts can be effectively removed. The application of this method to real life neonatal EEG data showed that some types of artifacts such as EOG, head movements and sucking can be removed effectively and concurrently on single channels. However, due to the oversimplified EMG model, the real EMG artifacts cannot be effectively removed. Further work can be carried out to determine a suitable EMG artifact model.

Chapter 5

Detection of neonatal EEG seizures

Central nervous system dysfunctions in neonates are often presented as seizures. Currently, the primary means of detection and identification of seizures is through manual interpretation of the neonatal electroencephalogram (EEG) by experts. It is crucial to allow quick relief to the factors causing seizures, thus automatic seizure detection methods are necessary to significantly improve medical intervention methods. In this chapter, a new time-frequency seizure detection method based on the local-energy variance measure of the Stockwell transform is proposed. The algorithm's performance is evaluated on multichannel neonatal EEG seizure signals and compared to some commonly cited neonatal seizure detection methods. It is shown that the local-energy variance measure of the Stockwell transform is a simple and yet effective feature to detect seizures with both high specificity and sensitivity.

5.1 The performance of the Stockwell transform

In this section, we first show a comparison between Stockwell transform, Choi-Williams and B distribution. Different types of synthetic and real neonatal EEG signals, in particular normal background and two types of seizure signals were evaluated. Then, we derive the variance of the local energy of the Stockwell spectra, and show how it distinguishes seizure signals from normal signals. This measure will be the main feature used in our seizure detection method.

5.1.1 Synthetic data

In this section, we compare the three time-frequency distributions (TFDs) over synthetic data. The time-frequency (TF) representations of all data after applying the TFDs with the parameters listed in Table 5.1 will be presented next.

Table 5.1: Parameter values of the TFDs

TFD	Parameter Value	Window Length
CW	$\varsigma = 11$	128
BD	$\beta=0.01$	128
ST	Nil	Nil

Two Parallel Linear FM signals

Consider a multicomponent signal consisting of two parallel linear FM components. The signal length is 1024, the sampling frequency is 1 Hz, the first component frequency range lies between 0.1 Hz to 0.3 Hz and the second component frequency range lies between 0.13 Hz to 0.33 Hz. At time instant $n = 20$, slices of the TFDs are taken and compared. In Figure 5.1, the normalised amplitudes of the distributions are displayed. All three distributions can resolve the two components, with BD and CW resolving the components at 0.091 Hz and 0.132 Hz respectively, and ST resolving the components at 0.109 Hz and 0.130 Hz respectively. Hence the frequency resolution performance of the three distributions in the case of closely spaced frequency components is very similar. Moreover, BD has the best resolution as it completely separates both components.

Spike signals

As neonatal seizures often contain sharp spike waves, a train of spikes is simulated and analysed using the three TFDs to study how the TFDs treat spikes signals. Figure 5.2(a) shows the spike signal of length 1000, sampled at 1 Hz, and the spikes are triangles with

Figure 5.1: Slices taken at the same time instant $n = 20$ of the CW, BD and ST for a multicomponent signal composed of two closely spaced parallel FM components.

amplitude 1 and width 3 samples. As seen in Figure 5.2(b), ST and CW are able to show these high frequency spikes clearly, whereas BD is unable to resolve them. However, BD is able to resolve the spikes components if the data window length is set to very small. This, however, will lead to unwanted distortions. As such, ST or CW is the TFD that best characterises spikes.

Low frequency sinusoid with spikes signals

In order to simulate a signal that contains both low frequency components and short high frequency components, a sinusoid with centre frequency at 0.02 Hz is combined with the train of spikes from Figure 5.2(a). Figure 5.3(a) shows the low frequency with spikes signal, Figure 5.3(b) shows a slice taken from the TFD distributions at $n = 30$ and Figure 5.4 shows the TFDs for the signal. As seen in Figure 5.3(b), all three TFDs have similar narrow lobe around the signal instantaneous frequency (IF) (0.02Hz). Hence, the TFDs are able to distinguish low frequency components very well. Consider Figure 5.4, where the TFDs for the signal are shown. Very clearly, ST is the only TFD that can concurrently

(a)

(b)

Figure 5.2: (a) Sharp spike time series; (b) CW, BD and ST for the sharp spike time series.

(a)

(b)

Figure 5.3: (a) Sinusoidal with centre frequency 0.02 Hz with sharp spike time series; (b) Slices of CW, BD and ST at $n = 30$ for time series in (a)

Figure 5.4: Comparison of distributions for sinusoid with sharp spike using CW, BD and ST.

show low frequency components and high frequency spike components clearly around the components' IF. BD can either show the low frequency component at window length 128, or the spikes components at window length 8, but not at the same time. CW shows the low frequency well, but the energy distributions of the high frequency components are not apparent. Hence, we conclude that ST is best suited for this type of signal.

Simulated neonatal seizure with spikes signals

In order to simulate a seizure signal that contains both low frequency components with sharp spike waves as commonly seen in neonatal seizures [11, 17], a seizure consisting of 2 frequency components and 16 spikes of duration between 20 msec to 70 msec is simulated. Figure 5.5 shows the signal and Figures 5.6 to 5.8 show the TFDs of the simulated signal. As illustrated in Figure 5.6, clearly the ST can concurrently show low frequency components around the components' IF and high frequency spike components

Figure 5.5: Simulated neonatal seizure with spikes time series.

Figure 5.6: Time-frequency representation of simulated neonatal seizure with the ST.

Figure 5.7: Time-frequency representation of simulated neonatal seizure with the BD.

Figure 5.8: Time-frequency representation of simulated neonatal seizure with the CW.

in their correct positions as high energy ridge-like localised patterns. The BD can only detect the low frequency components and the CW shows the low frequency well, but the energy distributions of the high frequency components are not apparent. Hence, we conclude that the ST is well suited for these types of signals.

5.1.2 Neonatal EEG data

Next, we apply the three TFDs to real neonatal signals as depicted in Figure 5.9. Three types of neonatal EEG signals, namely a normal EEG, a sharp spike seizure and a slow wave seizure are considered in our study.

Sharp spike neonatal seizure signals

Figure 5.10 shows the strengths of the ST as it clearly captures low frequency activities and high frequency activities during a spike seizure. Each ridge across the frequency range represents a spike in the time domain and coincides with spikes in the time domain signal. Low frequency components existing over the entire time span of the signal are also captured and displayed. Both the BD and CW, however, show only the low frequency activities. This can lead to a false representation of sharp spike seizures as slow wave seizures. Figure 5.11 shows a slice of the ST at $n = 200$ for the spike seizures and also illustrates that the ST is able to separate the components of the signal well, even when the two low frequency components are relatively close together.

Slow wave neonatal seizure signals

In this section, the ST for a slow wave seizure is analysed. Figure 5.12 shows a close up TF representation for all three TFDs in the frequency range of 0 to 0.2 Hz¹. The variation of the low frequency component of the slow wave seizure over time is shown clearly by all the TFDs. Figure 5.13 shows a slice of the ST at $n = 200$ for the slow wave seizure.

¹No components exist above 0.2 Hz in this analysis.

Figure 5.9: Different types of neonatal EEG signals. (a) normal: (b) sharp spike seizure: (c) slow wave seizure [11].

Figure 5.10: Representation of sharp spikes seizure EEG using (a) CW; (b) BD; (c) ST.

Figure 5.11: Slices of the CW, BD and ST taken at the same time instant $n = 200$ of sharp spike seizure.

As shown in Figure 5.13, the ST accurately estimates the IF of the signal in terms of the narrow lobe around the signal IF. Since we have determined that the ST can capture the energy concentrations of the neonatal EEG signals accurately, we will proceed to utilise this observation and derive a dominant feature, namely the variance of the local energy of the Stockwell spectra, to discriminate seizures from normal background signals.

5.1.3 Variance of the local energy of the Stockwell spectra

The moments of a time-frequency distribution provide important information about the signal. In particular, the first and second order moments, in time and in frequency, describe the average position and spreads in time and in frequency of the signal. At first inspection, one would consider computing the time moments of the Stockwell transform

Figure 5.12: Representation of slow wave seizure EEG using (a) CW; (b) BD; (c) ST.

Figure 5.13: Slices of the CW, BD and ST taken at the same time instant $n = 200$ of slow wave seizure.

at time t [126]. However, from the Parseval's energy conservation law of a signal $x(t)$,

$$\begin{aligned} E &= \int_{-\infty}^{\infty} |x(t)|^2 dt \\ &= \int_{-\infty}^{\infty} |X(f)|^2 df \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\text{TFR}_x(t, f)|^2 dt df, \end{aligned} \quad (5.1)$$

where $\text{TFR}_x(t, f)$ is the time-frequency representation of signal $x(t)$.

If we substitute Eq. (2.12) into Eq. (5.1) we arrive at

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left| \int_{-\infty}^{\infty} S(t, f) dt \right|^2 df \neq \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\text{TFR}_x(t, f)|^2 dt df \quad (5.2)$$

which clearly does not satisfy the Parseval's law. Hence, the concept of moments cannot be applied directly. As such, estimations of the first and second order time moments for the Stockwell transform are used instead.

Consider the 1-D function of the variable f and fixed parameter t_1 of the Stockwell transform, $S(t_1, f)$ as the local spectrum. The local spectrum for a particular slice at time t_1 is

$$S(t_1, f) = \frac{|f|}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(\tau) e^{-\frac{(\tau-t_1)^2 f^2}{2}} e^{-j2\pi f \tau} d\tau. \quad (5.3)$$

We first define the local-energy of the local spectrum for a particular time slice in the Stockwell spectra as

$$E_L(t_1, f) = |S(t_1, f)|^2. \quad (5.4)$$

Next, we define the mean of $E_L(t_1)$ of each time slice as

$$\mu_{E_L}(t_1) = \frac{\int_0^{f_{\max}} E_L(t_1, f) df}{f_{\max}} \quad (5.5)$$

where $f_{\max}$ is the maximum frequency present in the signal. The $\mu_{E_L}(t_1)$ gives a representation of local-energy variation at time slice t_1 . As neonatal EEG signals are highly nonstationary, the $\mu_{E_L}(t_1)$ of an abnormal or seizure EEG signal is expected to show

Figure 5.14: Comparison of variances of local-energies of the Stockwell spectra for different types of neonatal EEG signals.

greater variations with time. Lastly, we define the variance of the local-energy as

$$\sigma_{E_L}^2(t_1) = \frac{\int_0^{f_{\max}} [E_L(t_1, f) - \mu_{E_L}(t_1)]^2 df}{f_{\max}}. \quad (5.6)$$

In the case where there are multicomponents in the neonatal EEG signal, such as during a seizure, the variance of the local-energy will become larger. As illustrated in Figure 5.14 during a normal EEG state, the local-energy's variance is very low, close to zero. However, during a slow wave seizure, the variance progressively increases in amplitude. Finally, in a sharp spike seizure, very high amplitude variances can be seen.

5.1.4 Modifications to the Stockwell transform

The Stockwell transform of a signal $x(t)$ is defined as

$$S(t, f) = \frac{|f|}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(\nu) e^{-\frac{(\nu-t)^2 f^2}{2}} e^{-j2\pi f \nu} d\nu = \int_{-\infty}^{\infty} x(\nu) w(\nu - t, f) e^{-j2\pi f \nu} d\nu \quad (5.7)$$

where t and f represent the time and frequency variables. The analysis window $w(t, f) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2}}$ is Gaussian, hence $\int_{-\infty}^{\infty} w(t, f) dt = 1$ and the localizing time window becomes

$1/|f|$ in the ST, i.e., the window width is scaled according to the inverse of the temporal frequency. In order to improve the energy concentration of the ST to achieve better visual representations and variances of the local-energies of the Stockwell spectra, some modifications to the analysis window are introduced. An additional parameter α is added to the analysis window as follows:

$$\sigma(f) = \frac{\alpha}{f}, \quad (5.8)$$

where σ is the standard deviation of the Gaussian analysis window. In this case, the modified ST becomes:

$$S(t, f, \alpha) = \frac{|f|}{\sqrt{2\pi\alpha}} \int_{-\infty}^{\infty} x(\nu) e^{-\frac{(\nu-t)^2 f^2}{2\alpha^2}} e^{-j2\pi f\nu} d\nu \quad (5.9)$$

and the analysis window

$$w(t, f, \alpha) = \frac{|f|}{\sqrt{2\pi\alpha}} e^{-\frac{t^2 f^2}{2\alpha^2}}. \quad (5.10)$$

The time resolution that determines the event onset and offset time and frequency smearing is controlled by the factor α . If α is too small the frequency resolution degrades at higher frequency. If α is too high the time resolution degrades at lower frequencies. Thus, α is varied within the frequency range of our interest, and can be represented as:

$$\alpha(f) = c(f)f, \quad (5.11)$$

where $c(f)$ varies between different frequency ranges from low to high frequencies.

The results of these modifications are shown in Figures 5.15 to 5.17.

Clearly by comparing Figures 5.15 and 5.16, the transition from seizure onset to cessation, and from low frequency to high frequency activities can be visually captured. The energy concentration is more distinct in the modified ST as compared to that of the standard ST. As illustrated in Figure 5.17 variances of the local-energies of the Stockwell spectra remains similar and we can still clearly distinguish the seizure types and duration as low frequency activities between 28 and 55 seconds and high frequency activities between 52 and 80 seconds.

Figure 5.15: (a) A neonatal seizure from the NUH test signal; (b) The standard ST spectrum.

Figure 5.16: (a) The same neonatal seizure from the NUH test signal from the example in Figure 5.15; (b) The modified ST spectrum. Note the contrast between low frequency and high frequency components.

Figure 5.17: The variance of the local-energies of standard ST ((a) and (b)) and modified ST ((c) and (d)). (a) variance of the low frequency coefficients (LFC) of standard ST; (b) variance of the high frequency coefficients (HFC) of standard ST. (c) variance of the low frequency coefficients (LFC) of modified ST; (d) variance of the high frequency coefficients (HFC) of modified ST.

5.2 Neonatal seizure detection system

Our proposed seizure detection system is illustrated in Figure 5.18 and is applied to every EEG channel. The main idea behind our proposed seizure detection method is to exploit the markedly differing local-energy variances of the Stockwell spectra during a normal EEG state and a seizure EEG state. A step by step detailed description follows:

Step 1 Stockwell transform

Firstly, the epoched EEG signal is transformed from the time domain into the time-frequency domain via the Stockwell transform. We recall that most neonatal EEG seizures have discharges in the range of 0.4 to 15 Hz and most of the power lies between 0.4 and 7.5 Hz and sometimes, as high as 75 Hz [63]. To suit these characteristics, we separate the Stockwell spectra into low frequency coefficients (LFC), $\leq 15\text{Hz}$, and high frequency coefficients (HFC), $> 15\text{Hz}$, for the next processing

Figure 5.18: Our proposed 4-step seizure detection method where LFC and HFC refer to low frequency and high frequency coefficients of the Stockwell transform respectively.

steps. Consider the neonatal test signal shown in Figure 5.19(a) and its Stockwell spectra (the output of this stage) is illustrated in Figure 5.19(b). As shown in Figure 5.19(b), a low frequency component can be observed from 28 seconds to 55 seconds. High frequency components are also present from 52 seconds to 80 seconds. At this point, we can conclude that there is a slow wave seizure in this test signal although it is uncertain if spike seizures are present.

Step 2 Variance estimator

Next, the variances of the local-energies for the Stockwell LFC and the HFC of each EEG epoch are computed based on the definitions in Section 5.1.3. It is noted that during normal EEG states, the variances are close to 0. During ictal states, the variances exhibit large amplitudes over the entire ictal state (usually larger than 10 seconds) before returning near-zero amplitudes. Usually, the variance of the local-energy for the LFC is sufficient to identify seizures since most of the power of neonatal seizures reside in this frequency interval. The variance of the local-energy for the HFC serves to identify high frequency contents in seizures, usually those of sharp spikes type. The computed variances of the Stockwell spectra from Step 1 can be seen in Figure 5.20. As shown in Figure 5.20(a), the variance of the LFC

Figure 5.19: (a) A neonatal seizure from the NUH test signal; (b) its corresponding Stockwell transform. Note the definitions of LFC and HFC.

Figure 5.20: The variances of the local-energy of the Stockwell spectra of the test signal in Figure 5.19 where S_o indicates the detected seizure onset and S_c indicates the detected seizure cessation locations respectively. (a) variance of the low frequency coefficients (LFC); (b) variance of the high frequency coefficients (HFC).

Figure 5.21: The histogram of the variance of the local-energy of the Stockwell spectra. (a) a normal neonatal EEG; (b) a slow wave neonatal seizure; (c) a sharp spikes neonatal seizure.

increases at S_o and diminishes after 55 seconds. The variance of the HFC, as shown in Figure 5.20(b) starts increasing at 52 seconds and drops to 0 at S_c . This confirms that both low frequency slow waves and high frequency spikes are present in the test signal.

Step 3 Thresholding

Two types of thresholding are performed, namely the variance and duration thresholding.

Variance thresholding is done based on the histogram of the variances. As shown in Figure 5.21, distribution of the variances for different neonatal EEG signals are concentrated in different bins of the histogram. In Figure 5.21(a), the variance corresponding to the normal EEG state is concentrated in the bins of up to 250 μV^2 . However, in Figure 5.21(b) and 5.21(c), we observe that the variances for slow wave and sharp spike seizure states are concentrated in much higher values than that of

Figure 5.21(a). From experimental results, it is observed that setting the variance threshold at the variance of the bin containing the largest number of elements yields favourable results. Thus only the variances above this threshold are accepted. Variances selected in this step will be assigned a state value of 1, and those that are not will be assigned a state value of 0.

In some cases, normal neonatal EEG also contain short durations of higher-than-zero amplitudes or at other times, transients activities [127]. These activities appear in the variances as short-duration high-amplitude readings normally lasting at most 4 seconds. Hence, to improve the variance feature, we need to select the seizure activities and eliminate those unwanted activities. This can be achieved using a simple duration thresholding technique. The conditions of selecting the correct segments are that if high amplitude variances with durations less than 4 seconds are close to each other (i.e. less than 10 seconds apart) then they will be combined, otherwise, isolated 4-second activities will be discarded. Figure 5.20 depicts a typical thresholding process on the test signal of Figure 5.19(a). As illustrated in Figure 5.19(b), the variance threshold varies according to the values of the variance, thus providing an adaptive threshold related to the variance. The onset of the seizures can be obtained at this juncture as the transition locations of the state values from 0 to 1 while the cessation of the seizures will be the transition locations of the state values from 1 to 0.

Step 4 Classification

After thresholding is completed, the resultant outputs comprise of 2 sets of information, namely whether a seizure has occurred, and whether the seizure is a slow wave or a sharp spike seizure. A simple way to classify these outputs can be achieved by employing a lookup table as shown in Table 5.2.

Table 5.2: Lookup table for seizure classification.

Value	EEG state
00	No seizure
01	Transients
10	Slow wave seizure
11	Sharp spikes seizure

5.3 Neonatal EEG data description and performance metrics

In this section, a description of two sets of the neonatal EEG is presented. We also discuss the different performance metrics used to evaluate the effectiveness of our neonatal seizure detection algorithm.

5.3.1 Neonatal EEG data description

Singapore neonatal EEG dataset

EEG data acquisition was performed on 5 newborns and were collected from the Paediatric EEG Laboratory at the National University Hospital (NUH) and from the Department of Paediatric Medicine at the KK Women’s and Children’s Hospital (KKH) in Singapore. A Nihon Kodan EEG 9100K machine was used to record 16 channels of EEG data based on the ipsilateral montage of electrode placement for neonates as shown in Figure 2.1. These recordings were then sampled at 200Hz. A 50 Hz notch filter and a 15 Hz low-pass filter were applied on the signals. This dataset is artifact-free as the artifacts epochs had been manually removed and unavailable for evaluation. The dataset does not contain sleep segments. The seizure activities on the recordings were visually labelled by neurologists from the respective hospitals. One set of EEG signal from NUH (test signal), which was not low-pass filtered, was used to validate our seizure detection method and four sets of EEG signals from KKH were used for verification. A total of 534 10-second epochs of EEG

data comprising 258 nonseizure epochs and 276 seizure epochs were used. In our study, an electrographic seizure is defined as a repetitive EEG discharge of at least 10 seconds duration with evolution in the morphology, frequency and amplitude. Furthermore, an epoch containing $\geq 50\%$ of seizures is classified as a seizure epoch. The electrographic seizure characteristics for each patient in the dataset are tabulated in Table 5.3: GA is the gestational age of the neonate in weeks; Number of seizure refers to the total number of seizures for each neonate; Seizure location indicates the spatial location of the seizures; Frequency and Amplitude refers to the frequency and amplitude of the seizures as indicated by the neurologist. Two examples of the neonatal EEG collected in Singapore are shown in Figure 5.22 and 5.23.

Table 5.3: Electrographic seizure characteristics for each patient in the Singapore dataset.

Patient	GA (weeks)	Sex	Number of seizures	Seizure location	Frequency (Hz)	Amplitude (μ V)
1	42	F	6	Generalised	3 – 12	20 – 30
2	42	M	2	C4	3 – 7	40 – 100
3	40	M	9	Generalised	2 – 5	60 – 250
4	37	F	4	C4, T4, Cz	3 – 12	100 – 250

Cork neonatal EEG dataset

Another set of neonatal EEG data was obtained from University College Cork, Cork, Ireland. A Viasys NicOne video EEG machine was used to record multichannel EEG at 256Hz using the 10 – 20 system of electrode placement modified for neonates. In this study, 8 bipolar EEG channels are used (F4-C4, F3-C3, T4-C4, C4-CZ, CZ-C3, C3-T3, T4-O2, T3-O2). This dataset contained 75.83 hours of EEG. A total of 228 seizure events were annotated by 2 experienced neonatal electroencephalographers, further information on the dataset can be found in Table 5.4. The dataset does not contain sleep segments and was used regardless of length and quality, and artifacts are present and unannotated.

An example of the Cork dataset is shown in Figure 5.24.

Figure 5.22: A 16-channel neonatal EEG tracing from NUH. Normal background activities are observed.

Figure 5.23: A 16-channel neonatal EEG tracing from KKH. Electrographic seizures are observed in all channels on the entire tracing.

Table 5.4: Electrographic seizure characteristics for each patient in the University College Cork dataset.

Patient	length (hr)	No. of seizure events	Mean seizure duration (mins)
5	10.06	25	5.44
6	24.04	156	2
7	5.69	37	1.17
8	12	4	1.09
9	24.04	6	1.07

Figure 5.24: A 16-channel neonatal EEG tracing from University College Cork. Seizure onset at around 25 seconds to the end of the tracing.

5.3.2 Performance metrics

We utilised the following metrics [128, 129]:

- Sensitivity (SENS) is defined as the percentage of seizure epochs (as labelled by an expert in neonatal EEG) correctly identified as seizure epochs by the system;
- Specificity (SPEC) is defined as the percentage of labelled non-seizure epochs correctly classified as non-seizure by the system;
- Positive predictive value (PPV) is defined as the percentage of total positive detected seizure epochs that matched seizure epochs;
- Negative predictive value (NPV) is defined as the percentage of total detected non-seizure epochs that matched non-seizure epochs;
- False detection rate (FDR) is defined as the percentage of nonseizure epochs incorrectly identified as seizure epochs and is equal to $(100 - \text{specificity})\%$;
- Receiver Operating Characteristics (ROC) curve is a plot of class sensitivity against specificity as a threshold parameter is varied. The area under the ROC curve can be computed using the trapezoidal numerical integration method. It is an effective way of comparing the performance of a feature, or a group of features. A random discrimination will give an area of 0.5 under the curve while perfect discrimination between classes will give unity area under the ROC curve.

5.4 Results and discussions

The seizure detection system considered in this study is epoch-based. As a result, classification accuracy is an epoch-based measure. We will validate our performance with the performance metrics found in Section 5.3.2 on the datasets described in Section 5.3.1. The

performance of our seizure detection system is dependent on the sensitivity of the thresholds obtained during **Step 3** in Section 5.2. On the one hand, adjusting the thresholds in a way to achieve a high SENS would increase the FDR. On the other hand, a seizure detection system with low SENS may miss seizures. Therefore, a trade-off between SENS and SPEC must be considered to determine the thresholds. In our case, the thresholds were automatically adjusted to simultaneously achieve maximum SENS and SPEC.

5.4.1 Results and discussions on the Singapore dataset

Figure 5.25 shows an example of the onset of a clinical seizure in the NUH test signal. As clearly shown, it comprises of sharp spikes with slow waves. According to the variance measures in Figure 5.25(b) and (c), our system reports a slow wave seizure from 20 to 145 seconds, with a sharp spike seizure superimposed from 85 to 130 seconds. This detected duration coincides with the annotations given by the neurologist from NUH. The results

Figure 5.25: Closeup view of the onset and cessation of a clinical seizure with slow waves and sharp spikes. (a) EEG signal; (b) the variance computed from the LFC; (c) the variance computed from the HFC; (d) the Stockwell spectra of the signal in (a). Notice that the variance of the LFC reflects the slow wave activities (from 20 to 145 seconds) while the HFC reflects the spike activities (from 85 to 130 seconds) of the seizure signal.

Figure 5.26: Neonatal EEG signal from channel 1 of Patient 1 and its corresponding variance. (a) Neonatal EEG seizure signal of Patient 1 (Seizure duration: 0 to 108 seconds); (b) Stockwell spectra of a seizure section of Patient 1; (c) variance of the local-energy of the Stockwell spectra of signal in (a).

that follow are obtained from evaluating our seizure detection system on the KKH seizure signals. As these signals were low pass filtered at 15 Hz, we present the results obtained using only the variance of the LFC.

Consider a 120-second section of EEG signal from Patient 1. Figure 5.26 shows the EEG signal, its Stockwell spectra and its variance. Clearly, the Stockwell spectra in Figure 5.26(b) shows two frequency components progressing from the start of the signal to 108 seconds later. This observation points to a slow wave seizure and we also note that the variance measure demonstrates fluctuations from the start to 108 seconds of the signal, and falls close to 0 promptly. Indeed, this seizure is correctly detected in Figure 5.27 which further shows the comparison of all detected seizures vs annotated seizures for Patient 1. In this case, the value of 1 indicates a seizure is present, and the value of 0 indicates that a seizure is not present. Note that the second seizure detected by our algorithm corresponds to an artifact that is periodic in nature, and our system failed to rule this out.

Figure 5.27: The entire annotated seizure locations vs detected seizure locations of Patient 1.

Figure 5.28: An example of multichannel seizure detection vs the annotated seizures of Patient 4 (right hemisphere).

Figure 5.29: An example of multichannel seizure detection vs the annotated seizures of Patient 4 (left hemisphere).

Next, we extend the detection to all channels of the EEG data. Figure 5.28 and 5.29 compare the detected seizures from affected locations on the scalp of Patient 4 to the seizure annotations. According to the annotations, seizures occurred in the C4, T4 and Cz regions. However, our system detects 4 seizure occurrences in all the channels in the right hemisphere and 2 seizure occurrences in the beginning of all the left hemisphere channels except T3. Figure 5.28 also depicts the progression of the seizures from the focal points of C4, T4 into other regions that are close by. The onset of the detected seizures differ slightly from that of the annotations due to the fact that our algorithm detects the onset of progressive frequency components corresponding to the start of the evolution of repetitive discharges. Interictal epileptiform events are also detected especially in the period between the third and final seizure.

The Receiver Operator Characteristics (ROC) of our system are investigated by varying the thresholds of our system such that the SENS and SPEC of our system vary from 0 to 1 (or from 0% to 100%). The ROC curves for each patient and their respective areas under the curves are depicted in Figure 5.30. As illustrated, the areas achieved are

Figure 5.30: The Receiver Operating Characteristics curves for the KKH dataset. The respective areas under the curves are listed in the legend.

Table 5.5: Comparison of the algorithm performance on the KKH dataset				
Assessment	Poh et al.	Gotman et al.	Liu et al.	Hassanpour et al.
Detected seizures	21	15	21	8
Missed seizures	0	6	0	13
True positives	250	189	260	96
True negatives	238	201	60	62
False positives	20	57	198	200
False negatives	26	87	16	176
Average SENS(%)	90.6	68.4	94.2	35
Average SPEC(%)	92.2	77.9	23.2	23.7
Average FDR(%)	7.76	22.1	76.8	76.3
PPV(%)	92.6	76.8	56.8	32.4
NPV(%)	90.2	69.8	78.9	25.4

between 0.7 and 0.9, which prove that our variance measure is a good discriminator of seizure and non-seizure. Table 5.5 tabulates the comparison of our system to that of Liu, Gotman and Hassanpour. A total of 534 epochs (10 seconds each) comprising 276 seizure epochs and 258 non-seizure epochs have been tested. We were able to achieve an average SENS of 90.6% and an average SPEC of 92.2%. This translates to an average FDR of 7.76%. These results indicated that 90.6% of the patients with seizures have a positive test result, while 92.2% of patients who do not have seizures test negative. Since the SPEC is high and the patients have a positive test result, we can safely decide that the patients suffer from seizures. Furthermore, with the PPV of 92.6% and the NPV of 90.2%, one can conclude that out of all positive seizure detections, 92.6% are true positives and out of all negative seizure detections, 90.2% are truly non-seizures. The results obtained for Liu and Gotman are comparable to those obtained in [65]. Hence we are convinced that with the KKH dataset, our seizure detection method performs better than the previously cited methods.

5.4.2 Results and discussions on the Cork dataset

Our seizure detection method is also tested using the neonatal EEG dataset obtained from University College Cork, Cork, Ireland. The details of the dataset are described in Section 5.3.1. The results of our method are tabulated in Table 5.6 together with those obtained using the methods by Liu, Gotman and Hassanpour.

As seen in Table 5.6, an average SENS and SPEC of 87.65% and 70.42% is achieved respectively. Due to the presence of many artifacts throughout the dataset, the performance of our detection algorithm is lower than that obtained with the earlier dataset. As the artifacts have not been annotated, they are not removed and the dataset is used regardless of their quality. However, this also reflects the true situation that we would expect to encounter during bedside monitoring. As such, these results are satisfactory. In

Table 5.6: Comparison of the algorithm performance on the University College Cork dataset

Assessment	Poh et al.	Gotman et al.	Liu et al.	Hassanpour et al.
Detected seizures	226	135	210	97
Missed seizures	2	93	18	131
True positives	5144	3067	4758	2231
True negatives	15092	15823	4021	3927
False positives	6340	6840	17399	16281
False negatives	725	1571	1123	4862
Average SENS(%)	87.65	66.1	80.9	31.5
Average SPEC(%)	70.42	69.8	18.8	19.4
Average FDR(%)	29.58	30.2	81.2	80.6
PPV(%)	44.79	31.00	21.47	12.1
NPV(%)	95.42	90.97	78.17	44.7

Table 5.7, the FP values are typically higher than expected and these translate to higher number of false seizure epochs detected. As discussed earlier, this is due to the artifacts which are of durations longer than 10 seconds and which cause the variance measure to behave like a seizure detection. However, most of the seizures present in the dataset are detected with onsets within an average of 3.61 seconds deviation from the annotations and durations within an average of 4.64 seconds deviation from the annotations.

Table 5.7: Detection results for each patient in the University College Cork dataset

Patient	TP	TN	FP	FN	SENS %	SPEC %	Onset variability (secs)	Duration variability (secs)
5	713	2138	689	80	89.91	75.6	4.04	3.21
6	4215	2842	1031	568	88.12	73.38	2.32	3.65
7	158	1141	683	67	70.22	62.55	7.10	10.21
8	19	2709	1589	4	82.61	63.03	2.76	4.01
9	39	6262	2348	6	86.7	72.73	1.87	2.12

Lastly, the ROC for this dataset is plotted and illustrated in Fig 5.31 where the ROC curves for each patient and their respective areas under the curves are depicted. As illustrated, the areas achieved are between 0.64 and 0.81. As artifacts are present in this dataset, especially in Patient 7, the discriminating power of the variance measure becomes less contrasting. As for the rest of the dataset, the ROC results show that the variance

Figure 5.31: The Receiver Operating Characteristics curves for the University College Cork dataset. The respective areas under the curves are listed in the legend.

measure can discriminate seizures from non-seizures well.

5.4.3 Results and discussions on datasets modelled as FRI signals

In this section, we present the results of seizure detection performed on FRI modelled signals. The neonatal EEG dataset is modelled and reconstructed according to the methods discussed in Chapter 3. Table 5.8 depicts the average number of coefficients required for each set of data with respect to seizure and non-seizure epochs. Their corresponding average CC are also shown.

Figure 5.32 depicts the FRI model the original signal shown in Figure 5.25. If we compare the results from these two signals, we note that there are visually no difference in the Stockwell spectra. Minor differences in the variances of the local-energies exist but are not sufficient to alter the seizure detection decisions since the thresholds are determined based on the actual values of the variances. The same type of results are evident when

Table 5.8: The average values of $2K$ and CC for FRI modelled seizure and non-seizure epochs.

Patient	Ave. no. of coef. per seizure epoch	Ave. $CC(\%)$	Ave. no. of coef. per non-seizure epoch	Ave. $CC(\%)$
1	802	95.85	670	96.12
2	714	96.03	602	96.25
3	850	97.71	720	9.733
4	642	96.00	540	97.15
5	706	97.61	624	95.23
6	866	96.63	732	95.21
7	784	96.18	686	97.45
8	782	97.34	674	96.92
9	750	96.88	680	95.35

Figure 5.32: Closeup view of the onset and cessation of an FRI modelled clinical seizure first depicted in Figure 5.25. (a) FRI modelled EEG signal; (b) the variance computed from the LFC; (c) the variance computed from the HFC; (d) the Stockwell spectra of the signal in (a). Notice that the variance of the LFC reflects the slow wave activities (from 20 to 145 seconds) while the HFC reflects the spike activities (from 85 to 130 seconds) of the seizure signal.

Figure 5.33: An FRI modelled version of the signal shown in Figure 5.26. (a) FRI modelled signal); (b) Stockwell spectra of the signal in (a); (c) variance of the local-energy of the Stockwell spectra of signal in (a).

we compare Figure 5.33 with Figure 5.26. By examining the power spectra of the original neonatal EEG signal and that of the FRI modelled signal, we see, in Figure 5.34, that the two power spectra match closely and the energy of the signals are concentrated in the low frequency components. These show that the FRI modelled neonatal EEG signals match closely to their original counterparts in both morphologies and in frequency contents.

The performance of the seizure detection algorithm on the FRI modelled datasets is shown in Table 5.9. Clearly, we note that our seizure detection algorithm performs very similarly on both original and modelled datasets. The minor differences in the variances of the local-energies of the Stockwell spectra did not contribute to wrong or false detections on FRI modelled datasets. Hence we can conclude that our seizure detection method is robust and our FRI EEG model preserves the morphologies and frequency contents of the original signals.

Figure 5.34: A comparison of the power spectrum of the original signal and the FRI modelled signal.

Table 5.9: The performance of our seizure detection algorithm on FRI modelled datasets.

Assessment	KKH dataset	Cork dataset
Detected seizures	21	226
Missed seizures	0	2
True positives	250	5144
True negatives	238	15092
False positives	20	6340
False negatives	26	725
Average SENS(%)	90.6	87.65
Average SPEC(%)	92.2	70.42
Average FDR(%)	7.76	29.58
PPV(%)	92.6	44.79
NPV(%)	90.2	95.42

5.4.4 Results and discussions on artifact-removed datasets

Based on the methods and results discussed in Section 4.4, we present the seizure detection results of our algorithm on artifact-removed dataset. The dataset used in this section is the Cork dataset since the KKH dataset is artifact-free. The dataset is processed in 10-seconds epochs by first going through the artifact removal process. Subsequently, the cleaned dataset will go through the seizure detection process. The EMG and other artifact epochs remain unchanged in the dataset. Consider a segment of EOG contaminated

Figure 5.35: Comparison of the variance of the local-energies of the Stockwell spectra. (a) EOG contaminated and cleaned EEG. (b) Variance measures of the contaminated and cleaned EEG.

EEG as shown in Figure 5.35. The variance of the two EOG artifacts present as high amplitude values lasting about 2 seconds and 2 seconds apart and would not have been rejected by our seizure detection algorithm (see Section 5.2). However, after the EOG artifact removal, the high amplitude variances do not exist. Another example shows an EEG segment contaminated with head movement and sucking artifacts as depicted in Figure 5.36. Clearly, the variance of the contaminated EEG captures the sucking

Figure 5.36: Comparison of the variance of the local-energies of the Stockwell spectra. (a) Head movement plus sucking artifact contaminated and cleaned EEG. (b) Variance measures of the contaminated and cleaned EEG. The spikes in the variance of the contaminated EEG corresponds to sucking artifacts.

artifacts as high amplitude sharp spikes with moderately high amplitude variance from 5 to 30 seconds. This will have been accepted as a seizure segment and cause a false positive. With the removal of these artifacts, the variance then corresponds to a normal EEG segment.

Table 5.10 tabulates the comparison of the performance of different seizure detection algorithms on the Cork dataset after artifact removal. An overall improvement for all the algorithms is observed in terms of better SPEC, FDR, PPV and NPV. This improvement is due to the reduction of false positive detections and the corresponding increase in true negative detections when artifacts that confuse the detection are removed.

Table 5.10: Comparison of the algorithm performance on the University College Cork dataset after artifact removal

Assessment	Poh et al.	Gotman et al.	Liu et al.	Hassanpour et al.
Detected seizures	226	135	210	97
Missed seizures	2	93	18	131
True positives	5144	3067	4758	2231
True negatives	16322	17023	5998	5147
False positives	5110	5640	16202	15061
False negatives	725	1571	1123	4862
Average SENS(%)	87.65	66.1	80.9	31.5
Average SPEC(%)	76.16	75.11	27.02	25.47
Average FDR(%)	23.84	24.89	72.98	74.53
PPV(%)	50.17	35.22	22.7	12.9
NPV(%)	95.75	91.55	84.23	51.42

5.5 Conclusions

We presented a simple neonatal EEG seizure detection method based on the variance measure of the local-energy of the Stockwell transform. It was shown that the characteristics of the variance of the local-energy of the Stockwell spectra can be a dominant feature in seizure detection. By incorporating this feature into our seizure detection algorithm, our results indicate that we can detect slow wave and sharp spike seizures with a SENS of 90.6%, a SPEC of 92.2% and a FDR of 7.76%. Furthermore, our system achieved a PPV of 92.6% and a NPV of 90.2%. In the event when the dataset is corrupted by artifacts, we also showed that our system achieved a SENS of 87.65%, a SPEC of 70.42% and a FDR of 29.58%. Our seizure detection algorithm performed very similarly on both original and the FRI modelled datasets. The minor differences in the variances of the local-energies of the Stockwell spectra did not contribute to wrong or false detections on FRI modelled datasets. It was also demonstrated that the SPEC, FDR, PPV and NPV values of our seizure detection method improved upon applying our artifact removal scheme onto the Cork dataset. Hence we can conclude that our seizure detection method is robust and our FRI EEG model preserved the morphologies and frequency contents of the original signals.

Chapter 6

Conclusions and future research directions

6.1 Conclusions

This thesis is focused on a neonatal seizure monitoring and representation system. This system is presented in three modules, namely an EEG modelling and acquisition module, an artifact removal module and lastly, a seizure detection module. By representing neonatal EEG signals as signals with finite rate of innovation (FRI), we developed the first two modules. Through investigating a variety of time-frequency signal processing techniques, we developed an automatic neonatal seizure detection module based on the Stockwell transform. Our modules are tested using various datasets and compared to existing methods in literature. It is found that the performance of our modules are generally better than those in existing literature.

The first objective of this thesis is to propose a new FRI-based EEG model to model EEG signals. To this end, we investigated the morphologies of EEG signals and concluded that they can be closely approximated with nonuniform linear splines with additive $1/f$ noise. Thus, a neonatal EEG acquisition that compresses the signals at source is developed based on the finite rate of innovation framework. The important contribution of this module is the introduction of a new framework to acquire EEG signals at their rate of innovation. Although promising results in terms of good compression ratio and low distortions are

achieved, an improved signal model that further improves the compression ratio is possible.

The second objective of this thesis is to develop new FRI-based EEG signal processing methods such as artifact removal to clean the neonatal EEG signals, thus easing the diagnosis of brain abnormalities. In this thesis, a method to sample and perfectly reconstruct bandlimited signals with additive FRI signals is revised from the existing FRI theory to address artifact removal in neonatal EEG signals. With the assumptions that the artifact-free EEG signals are L -bandlimited FRI signals and three types of artifact signals (electrostatic discharges, electrooculographic and electromyographic) are non-bandlimited FRI signals, we are able to extract the artifacts from the EEG signals. In the thesis, semi-simulated contaminated EEG signals are processed and artifact signals are accurately extracted as compared to FastICA method. Real, contaminated neonatal EEG signals are also processed and favourable results are achieved.

The final objective of this thesis is to monitor and detect electrographic neonatal seizures. A comparison of the Stockwell transform with existing time-frequency distributions reveal that the Stockwell transform provides a good visual representation of neonatal seizure signals which contrasts that of normal background signals. It is also shown that the characteristics of the variance of the local-energy of the Stockwell spectra captures the visual representation and can be a dominant feature in seizure detection. Based on empirically derived seizure durations and thresholds, a seizure detection algorithm is developed. Based on a single time-frequency feature, our method achieved results comparable to existing methods that utilised many features. Our results indicate that slow wave and sharp spike seizures are detected with a sensitivity of 90.6%, a specificity of 92.2% and a false detection rate of 7.76%. Furthermore, our system achieved a positive predictive value of 92.6% and a negative predictive value of 90.2%. In the event when the dataset is corrupted by artifacts, we also showed that our system achieved a sensitivity of 87.65%, a specificity of 70.42% and a false detection rate of 29.58%. Lastly, validations to our

EEG signal model and artifact removal algorithm are made by applying our seizure detection algorithm to datasets that have been modelled as FRI signals and artifact-removed. Good results are achieved, comparable to that obtained from the original datasets. In accordance to the results achieved, our methods outperform existing methods and the objectives of this thesis have been met.

6.2 Future research directions

A number of future research areas have been identified. Specifically, the following research areas could be addressed:

- **Neonatal EEG database**

Neonatal EEG databases are not easily available in the research literature. In order for neonatal EEG research to reach new heights, a concerted effort to collect, collate and share neonatal EEG datasets from various sources is necessary. In this way, a common database is available to all researchers and a benchmark to quantify new EEG signal processing methods can be derived.

- **FRI seizure detection method**

In this thesis, the EEG model is based on nonuniform linear splines and the rate of innovation of the EEG dataset varies between epochs. In view of this, sparser EEG models such as time-frequency FRI models can be examined and new measures or feature sets based on the FRI sampling theory may be derived. With this feature set, new FRI seizure detection and artifact removal methods may be developed.

- **Stockwell feature set**

In our work, only one feature is extracted from the Stockwell transform. Investigations into how the phase of the Stockwell spectra behaves may lead to a new feature suitable for seizure detection. Features that characterise how the signals transit

from non seizure epochs to seizure epochs may also be derived from the Stockwell spectra.

Appendix A:

Implementation of other neonatal seizure detection techniques

In order to compare the performance of our method with other neonatal seizure detection methods, three methods, namely the Gotman, Liu and Hassanpour methods, are implemented. Here, the specific parameters used are given.

Liu algorithm

The Liu algorithm segments the EEG signals into 30 seconds epoch. The autocorrelation is then performed on 5 windows of the 30 second epoch. For nonperiodic signals the peaks and troughs in the autocorrelation are irregularly spaced. The time interval between the peaks of the autocorrelation become constant in periodic signals. As the newborn EEG seizure is characterised by repetitive rhythmic patterns, this algorithm assumes that the seizure EEG should have peaks in the autocorrelation that are spaced evenly. This opposes the case of nonseizure EEG, which should have irregular spacing.

To classify the EEG data, the moment center of the peaks of the autocorrelation are determined and ratios between centers are calculated. The closer the ratios are to whole numbers, the higher the score that window gets. Either of the three criteria must be met for a channel to be seizure positive:

- two consecutive window scores within a channel are ≥ 2 , with the sum of the two

Table 6.1: Boundary values for the features of Gotman algorithm.

	Dominant frequency (Hz)	Width of dominant peak	Power ratio
First combination	0.5 – 1.5	≤ 0.6	3 – 4
Second combination	1.5 – 10	≤ 0.6	2 – 4
Third combination	1.5 – 10	≤ 1	4 – 80

window scores ≥ 10 ;

- three consecutive window scores within a channel are ≥ 2 , with the sum of the two window scores ≥ 14 ;
- a score from a single window is ≥ 12 .

In the event that two or more channels in the epoch are seizure positive, the epoch is considered a seizure epoch. Finally, the 30 second epoch is divided into 310 second epochs so that comparisons can be made to the other algorithms.

Gotman algorithm

The Gotman method takes a 10.24 second sliding window with a 75% overlap. The power spectrum of the EEG window is obtained through FFT. The power spectrum of a nonseizure newborn EEG is assumed to exhibit a peak frequency of between 0.5 – 0.8Hz with an exponential decay. On the other hand, the rhythmic characteristic of the newborn EEG seizure is represented as significant large peaks around the seizure frequency in the power spectrum. The spectrum of the window under investigation is compared to the spectra of two occurring 60 seconds prior. These are referred to as background windows. The features of the Gotman algorithm include dominant frequency, width of dominant frequency peak, and power ratio between dominant spectral band of the current window and background window. Table 6.1 shows the boundary values used to distinguish between a seizure and a nonseizure window. A seizure detection is made if the current window meets any of the criteria listed in the table.

Hassanpour algorithm

The Hassanpour algorithm is composed of three stages namely spike detection, training

and seizure detection. The spike detection stage involves segmenting data into non-overlapping epochs of 10 seconds. The epoch is then transformed into the time-frequency domain using CWD. Two frequency slices at approximately 60 and 65 Hz are taken from the TFD and fed into the SNEO to emphasise the spike event. Two separate thresholds, which are empirically chosen, are used for each slice. The threshold values are typically in the range of 1000 – 3000 μV for the 60 Hz slice and 10000 – 20000 μV for the 65 Hz slice. The centers of the local maximums which are above the thresholds are selected as possible spike locations. If the spike locations are observed in both frequency slices, a positive spike detection is made. Hassanpour suggests that distributions of the intervals between spikes differ between seizure and nonseizure EEG. Therefore a histogram of successive spike intervals (HSSI) is used as a feature for classification. Six seizure classes were constructed using the k -nearest neighbour algorithm.

In the seizure detection stage, the 10 second epochs are transformed into the time-frequency domain. The time-frequency spectrum is enhanced using the SVD-based enhancement method and the HSSI is obtained. The HSSI is then compared to the classes and if the output is less than 0.1 (meaning this epoch is close to any of the six classes), a seizure detection is made for that epoch.

Author's publications

The following is a list of publications that has been written by the author during the course of this project:

- K. Poh, P. Marziliano, "Compression of neonatal EEG seizure signals with finite rate of innovation", *Proc. IEEE International Conference on Acoustic, Speech and Signal Processing (ICASSP)*, Las Vegas, Nevada, USA, March 2008.
- K. Poh, P. Marziliano, "Analysis of Neonatal EEG Signals using Stockwell Transform", *29th Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, Lyon, France, August 2007.
- K. Poh, P. Marziliano, "Compressive sampling of EEG signals with finite rate of innovation", *EURASIP Journal on Advances in Signal Processing*, vol. 2010, Article ID 183105, February 2010.
- K. Poh, P. Marziliano, "Detection of Neonatal EEG Seizure Signals with Stockwell Transform", to be submitted.
- K. Poh, P. Marziliano, "Removal of artifacts in EEG signals using the finite rate of innovation framework", to be submitted.

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