Modeling for active needle steering in Percutaneous surgery

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Modeling for Active Needle Steering
in Percutaneous Surgery

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Percutaneous Surgery**

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**SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING**

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**SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING**

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2008

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ABSTRACT

Precise needle insertion is very important for a number of percutaneous interventions. Yet it is very difficult to achieve in practice. Errors caused by the target movement and needle deflection have been observed for a long time. Yet to date, there are few effective physical-based needle steering systems existing for correcting the needle deflection when it occurs. In addition, many procedures are currently not amenable to needles because of obstacles, such as bone or sensitive tissues, which lie between feasible entry points and potential targets. Thus, there is a clear motivation for needle steering in order to provide accurate and dexterous targeting.

This work aims to build a needle-tissue interaction model that can be used for designing a needle steering system. A spring-beam-damper model is adopted in this work, which takes into consideration both the elastic, viscoelastic tissue reactions and the needle flexibility, as well as their interaction effects. Unconstrained modal analysis method, which is computationally efficient, is adopted to analyze the system dynamics. Considering the tissue inhomogeneity and computational efficiency, depth-varying mean parameters are proposed in this thesis to calculate the spring and damper effects. Local polynomial approximations in finite depth segments are used to approximate the unknown depth-varying mean parameters. Based on these approaches, an online parameter estimator has been designed using modified least square method with forgetting factor. Extensive experiments have been carried out in various types of phantoms to validate the needle steering model with the online parameter estimator. Results have shown that the model can track the needle tip trajectory with RMSE (Root Mean Square

ABSTRACT

Errors) less than 1mm after convergence even in the presence of large noises, which come from the reduced mode model, poor initial estimation and sensor noises etc. The convergence rate will be greatly improved if the needle gets supported.

To improve the targeting accuracy, a physical target movement reduction method, high frequency translational oscillation ($>2\text{kHz}$), has also been investigated in this research. Experimental results have shown that it can reduce the target movement up to 60%. This physical method can be used in situations where real time tracking of the target movement is not available, or computational expensive.

Acronyms

Acronym	Meaning
CT	Computed Tomography
MRI	Magnetic Resonance Imaging
MRS	Magnetic Resonance Spectroscopy
MRE	Magnetic Resonance Elastography
US	Ultrasound
TRUS	Transrectal Ultrasound
MEMS	Micro-Electro-Mechanical System
OR	Operation Room
FFD	Free-form Deformation
MSD	Mass-spring-damper
FEM	Finite Element Method
BEM	Boundary Element Method
KV	Kelvin Voigt
EDA	Exploratory Data Analysis

List of Symbols

Symbol	Description	Page
T	kinetic energy	62
V	potential energy	62
W_{nc}	work done by non-conservative forces	62
$x - y$	the fixed coordinate system	62
$\tilde{x} - \tilde{y}$	the moving coordinate system	62
v_x	the needle insertion velocity	64
M	the mass of the fixture	64
L	the length of the needle	64
ρ	the mass per unit length of the elastic beam	64
E	Young's modular	64
I	the second moment of area	64
k	the spring coefficient per unit length	64
c	the damper coefficient per unit length	64
ω	the deflection of the beam	64
$\dot{\omega}, \omega'$	the first derivative of the beam deflection with respect to time and space, respectively	64
H	the Heaviside unit step function	64
F_y	the steering force	64
$\alpha(t)$	the motion of the center of mass of the total system without perturbation	65
$\phi(\tilde{x})$	the shape function	65
$q(t)$	the time-varying amplitude of motion	65
ξ	the natural frequency of the system	66
$\bar{\varphi}(s)$	The transformed function of $\varphi(\tilde{x})$	66
$Y(\tilde{x}, t)$	the displacement of the needle point that has a distance $\tilde{x}$ from the needle base at time t	67
$\theta(s)$	the mean spring and damper coefficients	71
l	the length of the segment	72
s	the coordinate system defined for the depth-varying mean parameters	72
p	the order of the polynomial	73
s_0	the resetting point	73
A	The constant polynomial parameter vector	73
y_T	the needle tip trajectory	76
y_e	the needle base trajectory	76
$\mathcal{X}$	the system state	76
T	the sampling time	77
$\bar{\theta}$	the unknown depth-varying mean parameter vector with an additional constant '1'	77
$\mathcal{G}$	the constant polynomial parameter vector with an additional constant '1'	78
λ	forgetting factor	78
P	covariance matrix	78
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Chapter 1 Introduction and Background

1.1 Introduction

Percutaneous surgical procedures can be diagnostic or therapeutic procedures that involve inserting tubular delivery devices of materials or energy into targeted locations in the human body with the aid of intra-operative imaging devices (e.g., Computed Tomography (CT), Magnetic Resonance Imaging (MRI)/Magnetic Resonance Spectroscopy (MRS), ultrasound, and fluoroscopy). These delivery apparatuses include instruments such as needles, trocars, drills, screws, laser devices, etc. Such procedures range in complexity from superficial needle sticks, to the biopsy of deep-seated tumors. The latter involves subcutaneous insertions of long, slender surgical tools and needles into soft inhomogeneous tissue, usually with limited visual feedback from below the skin's surface. Since they are less invasive than traditional open techniques, they significantly reduce patient recovery time and discomfort. For this reason, percutaneous operations are popular. Percutaneous procedures involving flexible instrument insertions include tissue biopsy, brachytherapy, vaccinations, blood/fluid sampling, regional anaesthesia, abscess drainage, catheter insertion, cryogenic ablation, electrolytic ablation, neurosurgery, deep brain stimulation and minimally invasive surgeries, etc. [1]. Precise instrument placement is very important. Complications associated with poor placement have been studied, which can include tissue damage, misdiagnosis, poor dosimetry and tumor seeding.

Figure 1.1 Prostate biopsy: (right) prostate biopsy in fluoroscope image

Figure 1.1 illustrates an example of percutaneous needle insertion for prostate biopsy through perineum. Prostate biopsy is a procedure to extract a specimen sample from the prostate to be examined under a microscope for early detection of the cancer. Prostate biopsy is carried out by inserting a biopsy needle through the rectum, or the urethra or the perineum. Precise needle placement is essential, for a misplaced needle may lead to wrong diagnosis and miss the chance of better treatment for the patient.

Figure 1.2 Brachytherapy of the prostate (adopted from [2])

Figure 1.2 illustrates another percutaneous needle insertion example: prostate brachytherapy. Prostate Brachytherapy [3] is the transperineal implantation of radioisotopes into the patient's prostate gland that has been diagnosed to have localized prostate cancer. It carries the risk of seed misplacement, seed migration after being loaded, incomplete tumor eradication, and damage to the urethra and rectum, causing incontinence. The risks associated with the procedure must be minimized, since their effects can be significant, long-lasting and even fatal [4].

In current percutaneous intervention, a straight-line needle insertion is usually performed either manually or robotically, likely due to the lack of sufficient visual feedback and knowledge of needle steering mechanics. Needle deflection and target movement are accommodated by human dexterity in compensating errors. If a needle fails to reach its goal even after manual adjustment, due to tissue or needle movement, it must be retracted and reinserted. It is interesting to note that during clinical practice, some surgeons make use of a combination of lateral, twisting, and inserting motions of the needle under visual feedback from imaging systems, such as ultrasound, to correct the targeting errors. Surgeons accomplish this from experience, making it difficult to replicate and limiting the accuracy to that of human hand/eye coordination. Attempts to improve the targeting accuracy using crude compensation techniques have also been tried by several researchers, but only with limited success.

Precise needle placement is very difficult to achieve due to the following problems. Firstly, the targeting point will move due to the tissue deformation. The force that the needle exerts upon the organ (e.g. prostate) and the surrounding soft tissue may not only cause the deformation of the soft tissue and organ, but also produce the

translation and even rotation of the organ about the transverse and sagittal axes [5,6]. These changes in position and shape of the organ reduce the targeting accuracy, for the desired targeting locations are usually calculated in the organ's original unaltered state. Figure 1.3 clearly shows in a simulation that the needle misses the target due to the tissue deformation.

Figure 1.3 Simulated needle intercept of a small target embedded within elastic tissue (adopted from [7])

Secondly, the slender needle will deflect during insertion due to the unbalanced force caused by the needle's bevel tip (as in the biopsy needle), or the inhomogeneous human tissue. In addition, when the needle cuts through the elastic capsule of the organ, the exerting force will not only cause the organ translation, deformation, and even rotation [8], but also inversely bend the needle and deviate its course from the designed path after insertion into the organ.

Last but not the least, during the operation, the surgeon does not have a direct view of anatomic structures. Instead he/she must rely on indirect views provided by imaging modalities such as ultrasound (US), low intensity X-ray or fluoroscopy etc. It is very difficult to distinguish the needle position in US images, especially the tip position from the blurry images. Speckles, shadows and other artifacts all can degrade the visualization of the needle in the image. This is very challenging and requires the surgeon to be highly experienced. Successful targeting of an

anatomic structure is dependent on the surgeon's skill, especially when the target is small.

1.2 Project Background with the Whole Picture

1.2.1 Project Background

Currently a biopsy robot system (as shown in Figure 1.4) has been developed in CIMI Lab to assist in the prostate biopsy operation.

Figure 1.4 Photographs of biopsy urobot – close up (left) and overview of complete system (right)

This robot system adopts Taylor's Remote Center of Motion (RCM) for needle placement, which allows for orienting the needle while maintaining the location of its point at the desired skin entry point in percutaneous procedures. A straight-line needle path is assumed during the insertion [9]. Past clinical trials in robotically-assisted prostate biopsy had shown that the targeting errors are usually no less than 2mm even after some crude compensation if using the 16G (1.7mm) needle. Larger errors ($>3\text{mm}$) are expected when adopting the smaller 18G (1.2mm) needle, which is commonly used in the operation room. A mechanism that can improve the system's targeting accuracy is needed.

1.2.2 The Whole Picture

Figure 1.5 shows the whole picture of this project. In the first phase, we adopted some crude compensation method to improve the targeting accuracy, which was still unsatisfactory as abovementioned. The 2nd phase is active needle steering. That is, applying force at the needle base to correct the needle tip position. This method is straightforward, and can be easily implemented in the current robotic system by using a compact y-z table (x is insertion direction; y and z are the other two directions). To achieve this goal, a needle-tissue interaction model that can predict the needle tip position accurately and efficiently when given an input actuation force is required. The work presented in this thesis focuses on building such a model.

In the near future, a needle steering system will be designed using the proposed steering model. A compact y-z actuation table will be designed and integrated with the current robotic system for needle steering. A needle rotation system or oscillation system may also be implemented into the whole platform to reduce the target movement, if real time target tracking is unavailable or infeasible.

The last phase is to make a self-actuation smart needle, which will be discussed in Chapter 7. It will be our future direction.

Figure 1.5 The whole picture of the project

1.3 Research Objectives

The objectives of current research work are 1) to develop a needle steering model that is computationally efficient; 2) to design an effective online parameter estimator that will help achieve precise needle tip position prediction; 3) to find out a physically effective method for reducing the target movement during insertion, and investigating the influential factors.

1.4 Synopsis of Thesis Results and Approach

As abovementioned, the scope of this work mainly includes needle steering force modeling, online parameter estimator design and physical-based target movement reduction method.

Needle steering force modeling (Chapter 3)

In this research work, the actuation of needle steering will be executed at the needle base. Thus, only the resultant needle base forces during the procedure are of interests, instead of the force distribution along the needle body. The needle steering force model will give the dynamics between the lateral force at the needle base and corresponding lateral movement of the needle tip, with consideration of the needle flexibility and tissue deformation.

A spring-beam-damper model is proposed in this thesis to model the system dynamics during the steering procedure. The proposed model takes into consideration not only the elastic, viscoelastic tissue reactions and needle flexibility, but also their interaction effects. In the literature, the Mass-Spring model has been adopted by many research groups in studying the tissue

deformation [10-12]. However, the interactions between the needle and soft tissue, and the control of the needle while in collision with deformable, inhomogeneous tissues have received little attention. Some researchers studied the collision of the flexible link with the environment in the application of grinding or surface turning operation [13,14]. They modeled the impedance of the environment as a simple spring-damper system, which was assumed to be stationary and was arbitrarily placed along the trajectory such that the beam would only make contact with it at the tip. In the application of surgical needle insertion, the flexible needle interacts with the deformable tissue with changing forces along the needle body from time to time. The situation is much more complicated compared with the point contact. The traditional way to solve problems involving deformable soft tissue is to use finite element method, which is computationally expensive. In this work, a computational effective method, unconstrained modal analysis method is adopted to solve the mathematical model.

The proposed base-control steering model will be used for designing a needle steering system that can steer the needle to correct needle bending or avoid an obstacle, e.g. pubic bone. It can be easily extended to other steering methods such as the insertion point control, which will be discussed in Chapter 7.

Online parameter estimator design (Chapter 4)

A major impediment in applying the spring-beam-damper models is to accurately estimate the virtual spring and damper coefficients of the soft tissue. In the past, the coefficients were normally assumed to be constant for simplicity. The acquisition of elastic or viscoelastic properties of a tissue was usually performed

by some offline methods (e.g. rheological experiments [15], or crude estimation [16]), which may vary substantially from the true properties. In recent years, online measurement techniques via image modalities (e.g. ultrasound [17,18], CT-scanners [19,20]) have been adopted. However, they can only examine the small strain, linear portion of the response and are unable to examine the viscoelastic behavior of tissues [21]. Other methods using image modalities like MRI [22-24] and Magnetic Resonance Elastography (MRE) [25] have also been proposed, but they are unsuitable for real time application in the operation room due to the high cost and its bulky size. An effective online parameter estimation technique is still lacking.

In this thesis, depth-varying mean parameters are proposed to calculate the spring and damper coefficients with consideration of the tissue inhomogeneity and computational efficiency. Based on this approach, local polynomial approximations in finite depth segments are used to approximate the unknown depth-varying mean parameters. To estimate the parameters in the polynomials, an online parameter estimator has been designed using modified least square method with forgetting factor. Extensive experiments have been carried out in gelatin phantoms, animal tissues, inhomogeneous phantoms and multi-layer phantoms to validate the proposed needle steering model with the online parameter estimator. As will be shown in the subsequent chapters, the proposed needle steering model with the online parameter estimator can track the needle tip trajectory with fast convergence rate and satisfactory accuracy ($RMSE < 1\text{mm}$), even in the presence of large noises and disturbances, especially when adding support to the needle.

Physically-based target movement reduction method (Chapter 6)

In this thesis, a novel approach, high frequency translational oscillation ($>2\text{kHz}$) has been proposed to physically reduce the target movements. The proposed method is based on a hypothesis that for a translating needle operating in an oscillatory mode (back and forth motion), the target movement will be reduced. In the literature, high frequency oscillation has been adopted in hard metal machining (ultrasonic machining) and in bone surgery (Ultrasonic or Harmonic scalpels) for its abilities to reduce the drilling or cutting force, decrease the tissue trauma and adhesion onto cutting instrument. This research focuses on investigating the effectiveness of the high frequency translational oscillation in reducing target movement during needle insertion. Extensive experiments have been carried out in one-layer and two-layer tissue-like phantoms. Results have shown that high frequency ($>2\text{kHz}$) translational oscillation of the needle can effectively reduce the target movement. Tissue properties, oscillation frequency/amplitude and insertion speed are all influential factors to the target movement. Different types of tissue may require different factor settings in order to achieve better target movement reduction rate. In cases when multi-layer tissues of different properties are involved in the needle insertion, the tissue in which the target is embedded will play a central role in deciding the optimal factor setting.

1.5 Thesis Organization

The thesis chapters are organized as follows:

Chapter 2: This chapter provides a comprehensive survey of the real-time flexible instrument tracking methods used in assisting surgery, the needle-tissue contact studies, needle steering methods in the literature; soft tissue

biomechanics and techniques of obtaining the needed biomechanical information, which are closely related to this research work, are also reviewed. The uncertainty control methods in the literature are also briefly reviewed. Future control strategy is likely to be chosen from these methods.

Chapter 3: The spring-beam-damper model is proposed for lateral needle base steering. It gives the dynamics between the needle base lateral force and the needle tip position with consideration of the needle flexibility and tissue deformation. The detailed analysis and deduction procedures are given in this chapter.

Chapter 4: Depth-varying mean parameters for calculating the spring and damper reaction forces are proposed in this chapter. Local polynomials are used to approximate these depth-varying mean parameters. An online parameter estimator has been designed accordingly to analyze the polynomial parameters, from which the spring and damper coefficients can be obtained.

Chapter 5: This chapter focuses on the experimental validation of the needle steering model with the assistance of the online parameter estimator. The experimental setup that is used throughout this whole project and the relevant calibration work are presented as well.

Chapter 6: This chapter presents a novel approach, high frequency translational oscillation ($>2\text{kHz}$), to reduce target movement during needle insertion. The influential factors and general trends for smaller target movement in different types of tissues are also investigated and presented in this

thesis.

Chapter 7: Thesis contributions and limitations are summarized, and the scope for future work is discussed.

Chapter 2 Literature Review

Flexible instruments, like needles, endoscopes or catheters, play a central role in the field of percutaneous surgery. They provide access to even remote operating sites within the human body through natural body openings or small incisions. However, flexible instrument control is quite difficult to implement due to tissue deformation, instrument deflection and limited tactile/visual feedback during the procedure. Furthermore, limited information is available about the needle-tissue contact during insertion. Various research efforts have been put into real-time tracking of the instrument, modeling of the soft tissue deformation and the axial needle insertion force during the straight-line needle insertion. This chapter provides a review of the real-time flexible instrument tracking methods that assist in surgery, the needle-tissue contact studies, needle steering methods, soft tissue biomechanics and available techniques of obtaining the needed biomechanical information, all of which are closely related to this research work. Considering the inhomogeneous human tissue and complex needle-tissue contact procedure, the uncertainty control methods in the literature are also briefly reviewed. Future control strategy of this project is likely to benefit from these methods.

2.1 Flexible Instrument Tracking Methods in Literature

Instrument tracking plays an important role especially in percutaneous operation, in which the surgeon does not have a direct view of the operating instrument and the target area. An effective instrument tracking method should provide navigation before and during the operation and help the surgeon to better comprehend position information. The tracking methods appeared in literature can be roughly

classified into various image based segmentation techniques, and direct tracking of the instrument coupled with sensors utilizing piezoelectric, optical or electromagnetic means. The tracking errors can come from the calibration error, registration error, interpretation error, and execution error, etc. This section reviews the tracking methods grouped by the abovementioned two classes.

2.1.1 Ultrasound (CT, C-arm...) Based Image Segmentation

Among various image modalities used in the operation room, ultrasound is perhaps the most often used one because of its convenience and free of radiation. Software algorithms that segment the needle from the ultrasound image have been the subjects of recent research. A Japanese group developed a real-time 2D ultrasound-guided needle-insertion medical robot for percutaneous cholecystostomy [26], as shown in Figure 2.1. Their aim was to compensate the needle misalignment caused by unexpected organ motions occurring due to respiration, heartbeat or pain caused by needle insertion under local anesthesia. An ultrasonic image segmentation technique, known as Hough transform method, was used to detect needle tip from intraoperative ultrasound images during insertion for real time modification of the needle path.

Figure 2.1 Ultrasound-guided motion adaptive needle-insertion instrument (UMI). (a) Schematic and (b) Appearance. (adopted from [26])

Another research group in Robots Research Institute, Ontario, Canada led by Fenster [27] developed a robot aided 3D transrectal ultrasound (TRUS) guided robot, which assists in intraoperative prostate brachytherapy by positioning a template. This system allowed brachytherapy needles to be inserted into the prostate along various trajectories including oblique ones to avoid pubic arch interference. Linear least square method was adopted to automatically detect the needle from TRUS images for oblique insertion.

The performance of these two systems is highly dependent on the qualities of the ultrasound images. Speckles, shadows and other artifacts all can degrade the visualization of the needle in the images thus affecting the performance of the systems. Moreover, locational and dimensional information rely on stringent calibration procedures to get accurate results. Last but not the least, the image segmentation methods used in the abovementioned systems both presume a straight, linear needle during penetration. Needle deflection is neglected.

Some tricks have also been adopted to improve the needle detection. For example, in [28], when the needle tip position was found to be unclear under the US, the stylet would be actively oscillated within the cannula to improve the detection. The resulting oscillation of the nearby tissue can then be localized by the Doppler detection system that can be found in most modern US scanners. Real time precise needle tip tracking will be the main concern using this method.

In summary, the disadvantages of needle tracking using US segmentation are listed as follows.

- 1) Precise tracking is hard to obtain due to the blurry images. Speckle, shadows and other artifacts all can degrade the visualization of the needle

thus affecting the performance of the system.

- 2) Needle tracking is easily lost due to needle bending out of plane.
- 3) When many needles are inserted during an operation (e.g. prostate brachytherapy), the image will become extremely noisy and the needle positions will be very difficult to distinguish in the ultrasound images.
- 4) US segmentation poses a problem for real time tracking [26].

Other image modalities such as CT scanner [27-32], C-arm fluoroscopy [33], MRI/MRS [34] were also proposed for needle guidance in the literature. For example, Gabor Fichtinger from Johns Hopkins University proposed the CT-guided [32] and MRI-guided [34] robotic system for applications in abdominal, prostate and spine procedures. Some preliminary experiments have been carried out in gelatin phantoms. For the CT-guided system, the orientation accuracy was determined to be 0.6° , and the distance between the needle tip and the target was 1.04 mm. While for the MRI-guided system, the accuracy of needle placement was determined visually and it was mentioned to be below 2 mm in all trials.

Alexandru Patriciu et. al.[33] adopted image servoing method to perform positioning and tracking operations in percutaneous renal access under C-arm fluoroscopy. To facilitate the automatic detection of the needle in the image, the needle was equipped with a radio-opaque spherical ball at the tip, thus providing a well-discriminated signature. A pattern-matching algorithm running on the video acquisition board was used to rapidly locate the ball marker in the x-ray image. Their targeting accuracy was reported to be less than 0.5mm.

Although the mentioned image modalities are capable of providing better image identification and proved suitable for some conditions when the tradition US is

unsuitable for use (for example, transrectal US cannot be used when the patients have previously undergone rectal surgery), they are not the preferred modalities for common uses in operation room (OR): CT and C-arm fluoroscope have relatively high radiation; while MRI will need elaborate design to be compatible with magnetic field, and at the same time, it is costly to be adopted for real-time tracking in OR.

2.1.2 Direct Tracking of the Instrument Coupled with Sensors

Alternatively, some research groups attempted to equip the instrument with sensors and utilize some piezoelectric, optical or electromagnetic systems to detect its position in real-time during operation. For example, Clikes and Breyer [35] used a PZT ring mounted near the tip of the catheter either to receive the ultrasound pulse sent by the scanner (passive mode) or to transmit an ultrasound pulse to the scanner (active mode). So the position of the catheter tip can be known during operation. One of the problems in this approach is that PZT is rigid and the attachment of a ring near the tip increases substantially the diameter of the catheter.

K. Kirk and Brendan [36] proposed to use the piezoelectric polymer PVDF that is flexible and extremely thin to replace PZT in ultrasound-guided biopsy. The needle was modified to facilitate the mounting of film and electrical leads near its tip, as shown in Figure 2.2.

Though PVDF has a number of advantages for this application, it has a couple of drawbacks. The most important disadvantage is its low energy conversion efficiency. To make the detection of needle easier, the film should be designed to

generate as broad a beam as possible, which is presently difficult to achieve.

Figure 2.2 The modified needle (adopted from [36])

N.D.Glossop and K.Cleary [37] proposed to adopt an electromagnetic system – ‘Aurora’ magnetic tracking system to monitor and locate the needle tip coupled with a sensor coil during injection or biopsy. They validated the position accuracy by comparing with the Polaris optical tracking system, which is expected to have high tracking accuracy. The reported root-mean-square error was 0.28mm with 225 data samples. Even though in the past such electromagnetic tracking devices inside an operation room (OR) could be considered problematic because of the potential interference from this specific environment (particularly the operation table and metallic surgical tools), results from [37] and [38] show that satisfactory tracking accuracy can be obtained when the surrounding environment is properly arranged. A superb advantage of these systems is that they are not subject to the line-of-sight limitations (as reported in [39]) that currently annoyingly affect the optical tracking system, which is an alternative tracking technology used in clinical

applications.

Figure 2.3 The accuracy testing apparatus (adopted from [37])

Other approaches like physical modification of the needle tip, such as scoring and coatings [40] or roughen the tip [41] to improve its ultrasound reflectivity has also been proposed in some research work, but they achieved limited success in terms of recovery speed and reliability.

2.2 Needle-tissue Contact Studies in Literature

The complex needle-tissue contact procedure has raised a lot of research interests lately. Yet works in the past mainly focused on modeling the soft tissue deformation and needle axial insertion force during straight-line needle insertion. Needle force and interaction with the tissue during lateral movement have been largely neglected. This section reviews the models and methodologies presented in the literature in studying the soft, deformable tissues, and the needle axial insertion force.

2.2.1 Soft Tissue Deformation Modeling

Models that describe the behaviors of soft, deformable tissues are of interest in medical and computer science fields, with a wide range of applications such as physiological analysis, surgery planning, or interactive simulation for training purpose. Significant research efforts have been put into modeling soft tissue deformation. The choice of models is normally governed by two factors: computational efficiency and physical accuracy. This section outlines several different deformation models with these in mind.

Surface Models and Volumetric Models: As the names imply, these two types of models depict a body by the set of points that either lie on its boundary, or belong to the body, respectively. Figure 2.4 illustrates the deformation of elastic materials using these two methods.

The selection of a surface or a solid representation greatly influences achievable deformation model behavior. In terms of computation, surface models are advantageous because they have fewer vertexes than volumetric models for representing the same shapes. However, surface models tend to give physically invalid deformations. The main problems with surface models are that they do not behave like 3D objects, and lack a defined interior. The first problem is purely visual, but the second is a serious limitation to the possible applications. Some simple cuts can be modeled, such as cutting an artery or other thin structures, but general surgical incisions are impossible. Volumetric models model the complete 3D volumetric behavior of the objects. The complexity of the models rises dramatically when volumetric models are used, and real-time performance is

difficult to achieve.

Figure 2.4 The deformation of non-rigid structures using: (a) A surface representation, and (b) A solid representation (adopted from [1])

Implicit models: Implicit methods for modeling deformable objects are usually based on pure heuristic geometric techniques or use a sort of simplified physical principles to achieve the reality-like effect. Examples such as spline techniques [42], radial basis function [43], super-quadratics [44], free-form deformation (FFD) [41], potential field method [45], modal analysis [46] and discrete model [47] can be found in the literature. In essence, these models exploit parametric functions to describe complex geometry.

Implicit models are very popular in computer graphics and sometimes used in real time applications, since they are computationally efficient compared with other expensive physical approaches. But the primary drawback of this approach is that it is difficult to express the mechanics of physical bodies in terms of these abstract parameters. As a result, material deformations are often “tuned” by making manual adjustments to the model parameters. In addition, it is difficult to use this approach for volumetric model representations. Nevertheless, because relatively few parameters are required to describe complex shapes, computations such as

collision detection can be performed extremely efficiently.

ChainMail: ChainMail algorithm represents a non-rigid body as a set of linked elements that resemble a chain, as illustrated in Figure 2.5. The motion of a single mesh node is absorbed by the link interface between adjacent nodes. If the link between two nodes is stretched or compressed beyond predefined limits, then displacements are transferred to neighboring nodes. In this way, small displacements of a single node within a relaxed medium results in local deformations, while displacements within a fully stressed medium cause global motion [1].

Figure 2.5 A one dimension chainmail structure undergoing deformation (adopted from [1])

An example using ChainMail can be found in [48]. This approach is computationally efficient and accommodates simple mesh topology changes, so it is well suited for real-time deformation and cutting operations. However, for inhomogeneous materials, the first-moved-first-processed element processing order that is used in ChainMail is no longer appropriate. How realistic the deformation is and how feasible the real-time rendering of the tissue model is are no longer clear. Schill [49] used an enhanced ChainMail Algorithm to model the vitreous tumor in the eye during surgical simulation and could achieve a new

element processing order. However it was still unable to model inter-element rotation and volume conservation, an important characteristic of many tissues in the human body.

Mass-spring (MS) models: Mass-spring models consist of a set of mass points linked by springs, which have been used extensively in the literature. MS systems can be seen as a simplified model of particle interaction, since physical bodies in fact consist of discrete sub-elements, atoms and molecules. MS models for interactive deformable models were first used by Terzopoulos to simulate facial expressions [50]. They have subsequently appeared in many other applications, e.g. craniofacial surgery [51-54], human liver surgery [55] and vitrectomy [56], etc.

Figure 2.6 A mass-spring model of the muscle surface (adopted from [2])

The main advantage of MS models is their ease of implementation, for they do not require continuous parameterization. They are very simple to solve and are an order of magnitude faster than the more rigorous Finite Element models [57]. In

addition to computational efficiency, MS systems allow interactive topological modifications, e.g., cutting or suturing operation can be modeled simply by removing or adding connections between vertices.

The major drawback of MS systems is their inadequacy in exhibiting true material properties. Using a mass-spring model, there is very little real-world material parameter correlation. The only parameters to control the amount of deformation of the simulated soft tissue are the spring coefficients, which are often approximated for visual quality. MS systems are also weak in modeling complex, arbitrary shaped objects such as thin surfaces, which are resistant to bending. Furthermore, model behavior is particularly sensitive to the network topology, which can result in an under-constrained system that exhibits local minima, or an over-constrained system that tends to decrease the range of deformation. Last but not the least, the range of achievable mesh stiffness is limited by node density and the simulation sample time [1].

Several improvements to spring models have been proposed, especially with regards to their dynamical behavior. Cover et al. [58] used springs to represent the surface of the gall bladder. Surface forces were combined with internal forces to enforce shape constraints. Similarly, Kuhn et al. [59] developed a surface model of the gall bladder using springs, dampers and plastic elements where mass points were connected to parent nodes. In that system, a NURBS representation was attached to points, giving a realistic rendering of the tissue. Stone et al. [60] added a slip and a split threshold to a spring network, in order to model plasticity and fracture. Meseure et al. [61] proposed a surface representation of tissue similar to the hybrid model proposed by Terzopoulos [62]. Springs of zero length were

attached to a virtually rigid component. During deformation, both the positions of mass-points and the translation/rotation parameters of the rigid component were updated. This framework had the advantage of decoupling stiff behavior modeled by the rigid component and local deformation modeled by springs and dampers.

Some other researchers have tried to incorporate aspects of the FEM into mass-spring models, blurring the distinction between these two approaches in soft-tissue simulation. For example, Koch et al. [63] combined a finite element model for representing the skin surface, with a spring model to represent the fat tissue. The stiffness of the springs was derived from the intensity of voxels in a CT-scan image. The underlying assumption was that stiffness was proportional to tissue density and therefore to the Hounsfield units. Another research group [64,65] used a combination of pre-computed linear static FEM and a dynamic tensor-mass model, a mesh of mass points subject to forces that were derived from a nonlinear FEM formulation, instead of forces from springs. Although mass-spring models could be defined on the basis of FE meshes, springs were usually traced from each mass node to many other mass nodes, often crossing each other. This allowed efficient transfer of forces but complicated the calculation of nodal parameters.

Finite Element (FE) Method: A FE model consists of a mesh defined over a set of nodes describing the geometry of the object, and an interpolation function over the mesh approximating the real deformation field. Figure 2.7 illustrates how the continuous domain Ω is divided into a finite number of discrete elements Ω_e .

Figure 2.7 Finite element model

FE models are suitable for computing accurate and complex deformation of soft tissue. They are based on continuum mechanics and material physics; thus, it is easy to incorporate the results of biomechanical studies. Furthermore, dynamic behaviors are easily accommodated and integrated using semi-implicit or explicit methods. Unlike spring models, there is no restriction on the stiffness value of the model with respect to the time step when using semi-implicit or static schemes. Finally, visualization of finite elements is well suited for graphics hardware since it consists rendering, visible elements.

Nowadays there are many soft tissue models based on FEM [1], [66-78]. The major disadvantage of FE model is its computation inefficiency. It is extremely difficult to get real time performance on a moderately powerful workstation using FE models. In addition, its implementation is not as straightforward as that of mass-spring models.

A few significant optimization methods have been proposed in the literature. For example, Stéphane Cotin et al. [71] used a large preprocessing step to achieve real-time visual performance for surgery simulation of the human liver. Nienhuys [72]

adopted an interactive algorithm for an interactive linear FE deformation simulation to improve computational efficiency. H.W. Nienhuys and A.F. Stappen [76] used an element subdivision approach (based on edge-bisection) to ameliorate the effects of the added computational complexity when extending the 2D models to 3D neo-Hookean models. DiMaio [68,69] tried to adopt condensation technique to achieve faster interactive simulations. Similarly, Cotin [79] and Bro-Nielsen [80] adopted condensation to decrease the computation time of finite element models by at least factor of 100. However, such optimizations were not compatible with the topological change entailed by suturing or cutting where the stiffness matrix must be updated. Furthermore, the computational costs were still too high especially when applying in 3D models as indicated in their works. For example, in [76], it was mentioned that the total computational cost of a 3D (10cm*10cm*1cm) simulation was roughly a factor 120 larger than a 2D (10cm*10cm) simulation.

Boundary element method (BEM): In some cases where the motion of visible surface nodes is of more interest than that of internal nodes, BEM can be adopted. For models using BEM, the dimension of the resulting system of equations is significantly smaller than that of total volume discretization. Furthermore, the difficult volumetric mesh generation is not an issue any more. Unfortunately, the volume integrals in BEM can be completely eliminated only when the material is homogeneous and no volumetric forces are given, which are generally not the case in soft tissue modeling. For elastic registration of medical images, BEM generally is not as robust and flexible as FEM. Applications using BEM can be found in [81-83].

This section reviews different modeling methods for tissue deformation. To suit the real time needle steering application, MSD model has been selected in this research work and extended to be continuously distributed spring-damper model for calculating the tissue reaction forces during needle-tissue interaction. Instead of giving a rough estimation of the spring and damper coefficients, we have designed an online parameter estimator to real time estimate the tissue parameters. The details will be reported in following chapters.

2.2.2 Needle Insertion Models

The needle insertion force has aroused a lot of research interests in recent years. An extensive survey can be found in [84]. Needle insertion forces have been measured in tissue-like phantoms, e.g. gelatin [87,173] and PVC [1,85,86], *ex vivo* human and animal organ samples [87-93] and *in vivo* animal experiments [94-96]. Recently, *in vivo* force measurement collected from real patients during actual prostate brachytherapy procedures has been reported [97]. The influences of the needle coating [85], needle tip shape and insertion velocity [86] etc. on the insertion force have also been explored. In the majority of these works, only the resultant forces acting at the proximal ends of the needles were measured. Kataoka *et al.* [98] separated the needle tip force and friction force along the needle shaft by means of an instrumentation approach. The needle they adopted in the experiment comprised an inner needle shaft, and an outer sleeve that extended to just behind the inner needle tip. The axial tip forces were measured by the inner shaft, and the friction forces were measured by the outer sleeve. They analyzed the force variations during two insertions into a canine prostate, but did not give any quantitative model to describe this process.

Simone and Okamura [88,89] attempted to isolate stiffness force (due to the elastic properties of the organ and its capsule), friction force (due to tissue adhesion and damping) and cutting force (combination of forces required to slice through the tissue and tissue stiffness at the tip of the needle) from the driving forces measured during insertions into ex vivo bovine liver tissue. They modeled the stiffness force by a second-order polynomial of insertion depth, friction force by a modified Karnopp model, and cutting force by a constant value. This model was later tested by another researcher using the in vivo data obtained during experiments on anesthetized pigs, and compared with Maurel's models [94]. The latter model was said to better fit the experimental data. However Maurel's models only included the factor of needle insertion depth. Other factor effects such as insertion speed were not considered.

J.T.Hing [93,99] studied needle and tissue interactions during a needle insertion and withdrawal task. They used a dual C-arm fluoroscope setup and implanted radio-opaque markers to extract the internal tissue motion during the insertion and withdrawal of a needle. Both image and force data were utilized to determine important parameters such as the local effective elastic modulus during puncture and the approximate cutting force for soft tissue samples. A finite element model was then built using the data to model needle puncture of tissue.

The needle insertion studies presented in this section focused on analyzing the needle axial forces during straight-line insertion, which mainly involves the tissue deformation. The effects of the needle lateral force (normal to the needle shaft) have not been studied. The lateral force is the main source for needle steering, and will induce more complicated needle-tissue contact procedure, during which not

only the tissue deformation, but also the needle flexibility and their interactions should be taken into consideration.

2.3 Needle Steering Methods

The placement of needles for percutaneous intervention is usually performed using straight-line needle trajectories [27-30,100] likely due to the absence of sufficient visual feedback or knowledge of needle steering mechanics required to make trajectory compensation feasible. The insertion point (at the skin surface) and needle orientation are selected in order to set up a straight-line intercept with the target. If a needle fails to reach its goal, due to tissue deformation or needle deflection, it must be retracted and re-inserted. In some cases, attempts are made to steer the needle. For example, there is anecdotal evidence of needle steering using bevel deflection and direct manipulation of the needle between the template and the perineum in prostate brachytherapy. Ultrasound is used to monitor the movement of the needle. Both transverse and sagittal views are used. Surgeons accomplish this from experience, making it difficult to teach and limit the accuracy to that of human hand/eye coordination.

Several types of steerable needles have been proposed in the literature to correct the needle path. Loser [101] proposed a steerable needle with a double cannula that was constructed from two concentric pre-bent tubes. When the curvatures of the two tubes were aligned, the needle was bent. Rotating the inner tube with respect to the outer tube reduced the net bending force and straightened the needle. Steering was derived from rotation of the tubes by two motors housed in the handle.

Another steerable needle proposed in [102] made use of a telescoping combined double cannula and stylet where only the inner cannula was curved and was made from a superelastic material such as nickel titanium. Steering was derived from rotation and extension of the inner cannula.

R.Ebrahimi [103] presented a handheld, motorized device (see Figure 2.8) for steerable needle insertions. Their method employed a stylet that was longer than the cannula, so that up to 2cm of the stylet tip could be selectively exposed. The stylet had a mild curve over the exposed portion of the tip. The steering direction was selected by rotating the stylet. The exposed curve tip of the stylet could impart a lateral deflecting force on the needle as it advanced through the tissue. This caused the needle trajectory to arc in the direction of the exposed curve. One practical concern of using this handheld device is its weight and size, as indicated in the paper.

Figure 2.8 (a) Needle steering concept: stylet withdrawn and stylet exposed. (b) The steering device (adopted from [103])

Other researchers showed that needles with bevel tips bent more than symmetric-tip needles [104]. Making use of this effect, highly flexible bevel-tip slender needles made of Nitinol alloy were developed [105]. A nonholonomic model, a

generalization of a 3D bicycle model, was developed and experimentally validated using a very stiff tissue phantom. The steering was achieved by rotating the shaft to change the direction of the bevel tip, thus changing the movement direction. Recent advances in this nonholonomic model include stochastic model-based motion planning to compensate for noise bias [106], probabilistic models of dead-reckoning error in nonholonomic robots [107], a diffusion-based motion planning to search for a feasible path in full 3D space [108], and motion planning under Markov Motion Uncertainty using dynamic programming to search for a feasible route while avoiding obstacle [109].

The main drawback of this method is that the modeling method was based on an assumption that the tissue was stiff enough compared with the slender needle, which is not true in most of the needle-tissue contact. In addition, the authors assumed that the needle shaft would follow the trajectory of the tip position, which is easily violated when the needle interacts with the inhomogeneous tissue.

A Canadian researcher, Simon DiMaio did the pioneer work in proposing a needle steering simulation system [1]. He used a large-strain elastic needle model coupled with the 2D tissue models to simulate the needle and tissue phantom reaction during insertion, as well as to predict target motion and needle trajectories due to phantom deformation and needle deflection. The traditional FE model was used to solve the system kinematics. His model was later extended by other researchers to 3D models [110,111].

There are some practical considerations in adopting this method for clinical applications. Although in his work, a condensation method was adopted and showed to reduce the computational complexity of linear simulation significantly

in 2D model, it cannot be extended to 3D models for the well-known “curse of dimension”. The size of the model datasets may exceed available memory resources, requiring hierarchical representations, data caching and predictive caching, as indicated in DiMaio’s paper. Another shortcoming is that linear elastostatic homogeneous tissue model was used to model tissue deformation, which is not representative of tissues that exhibit more complex behavior, such as viscoelasticity.

In 2005 Medical Image Computing and Computer-Assisted Intervention (MICCAI) conference, Daniel Glozman and Moshe Shoham [16] presented a simplified virtual spring model for the needle insertion procedure. They assumed small tissue displacement and linear lateral force response. So the tissue forces on the needle could be modeled as lateral virtual springs distributed along the needle curve plus friction forces tangent to the needle longitudinal axis. Modeling of a flexible needle was based on the assumption of quasi-static motion and a third order polynomial was used to calculate the displacement of each element. Compromise had to be made between the computation efficiency and the model accuracy. In addition, the spring coefficient was assumed to be constant without considering the inhomogeneous tissue properties. Their method in estimating the spring coefficients was to apply force on the tissue surface and observe the corresponding tissue deformation, which was rather crude. The difficulties in accurate parameter estimation and real time computation will be the main impediments in applying his method.

2.4 Soft Tissue Biomechanics

For an accurate soft tissue modeling, it is vital to have quantitative knowledge of the biomechanical behavior of soft tissue. There exists a large bibliography on studying soft tissue biomechanics [112]. Those studies range from the determination of qualitative behavior of tissues to the recovery of quantitative parameters governing their deformation. The tissues that have been studied include skin [113], vessels [112], muscles, brain [114] and heart [115], etc. Many mathematical models of soft tissue deformations have been proposed. Although the behavior of structures and materials in engineering can be relatively accurately described by abstracted models such as those for *Hookean* elastic solids, non-viscous fluids and *Newtonian* viscous fluids, most biological materials exhibit far more complex dynamic behaviors [112]. For example, blood is better characterized as a viscoelastic material that exhibits hysteresis, relaxation and creep under dynamic loading, than as a *Newtonian* fluid. Collagen is the basic structural element for soft and hard tissues, but is present in a large variety of structural forms within different tissues and organs. Skeletal muscle is an anisotropic viscoelastic material. The membrane exhibits non-linear viscoelastic behavior [1]. Moreover, they criss-cross each other in grains, giving orderliness in the inhomogeneity. The pelvic floor muscle, for example, comprises of several symmetrically layout grains of muscle.

In summary, biological tissues may be characterized as quasi-incompressible, non-homogeneous, non-isotropic, non-linear viscoelastic materials likely to undergo large deformations [15]. Most of current works on tissue modeling and simulation focus on the elastic component of deformation, particularly linear elasticity. The

accuracy of these models generally cannot be guaranteed. A precise soft tissue model, which describes both the elastic and viscoelastic behavior with consideration of the human heterogeneity is still lacking.

2.5 Acquisition of Biomedical Information

A major impediment in building accurate soft tissue models is the lack of quantitative biomechanical information. The required information not only refers to the inner mechanical properties of the given soft tissue but also includes contact with the surrounding tissues/instrument. In terms of computation, the former corresponds to the constitutive law of motion linking the stress tensor with the strain tensor whereas the latter corresponds to the boundary conditions [15]. Mechanical properties of biological tissues have been a subject of interests in various medical applications.

In the past, the acquisition of elastic or viscoelastic properties of a tissue was usually performed by rheological experiments. Existing rheometers require the experiments to be performed *in vitro* on uniaxial samples, which may vary substantially from the true *in vivo* properties, especially with permeable tissues containing incompressible fluids (such as blood). Furthermore, rheometers do not allow characterization of the force/deformation contact between neighboring tissues. Experiments with uniaxial samples are only valid if the tissue is homogeneous and isotropic [15].

In recent years, several *in vivo* measurement techniques have been proposed to study the *in vivo* properties, which fall into two categories: non-invasive imaging-based methods and invasive methods, as discussed below.

Non-invasive imaging-based methods: Non-invasive imaging-based methods, which examine the strain field within tissue, are typically referred to as elastography. The principle is to apply some known displacement, which may include static or dynamic compression or shear displacement, to the exterior of the tissue under consideration. Since the tissue is continuous, the surface displacement produces a strain field within the tissue, which is a function of the surface displacement, the elastic properties of the tissue and other boundary conditions. The strain field is measured through the use of some non-invasive imaging techniques. In the static case, before and after scans are made, and the strain can be calculated based on the local displacements measured between them. For dynamic testing, vibration amplitude or velocity can be used instead of static strain [21].

Widely-used imaging modalities such as ultrasound [17,18], CT-scanners [19,20], MRI [22-24] and MRE [25] have been adopted in non-invasive methods. Non-invasive techniques have shown some success, but are subject to certain limitations. The majority of the non-invasive techniques can only examine the small strain, linear portion of the response. Furthermore, the static techniques cannot examine the viscoelastic behavior of tissues, while the dynamic techniques require excitation frequencies large enough so that the wavelength of sound is small enough compared with the structures under consideration [21].

Invasive methods: Invasive measurement techniques that can be directly applied to the tissues in question have also been proposed by many researchers. For testing of living tissue, the instrumentation system must not only be able to acquire the desired data, but also be suitable for use in often restricted environments like minimally invasive surgery, and must not cause trauma to the tissue. Based on

these two criteria, several indentation instruments have been developed to measure the tissue movement [21,116,117]. Surgical instruments equipped with force and position sensors [118], as well as motorized endoscopic grasper [119] and aspiration-based techniques [120,121] have also been proposed in the literature. Some researchers studied porcine esophagus using a phantom haptic interface as a robot arm, in which position was measured relative to ground using the optical encoders of the instrument, and the applied loads was recorded via a 6-axis force-torque sensor [122]. This method is more versatile than some of the other techniques, in that the phantom can move in three dimensions, whereas the devices previously described measured only one-dimensional responses.

For the invasive and non-invasive methods, it is hard to say which one is better. No single technique is capable of capturing the full range of tissue characteristics, e.g. non-linear stiffness, viscoelasticity, anisotropy and inhomogeneity. However each has its own advantages, and can be used to contribute data to and cross check the results of the others.

2.6 Uncertainty Control Methodologies in Literature

Considering future control system design, uncertainty control methodologies in literature are also reviewed here. A branch of control science can be found to deal with the problem of how to guide and regulate systems that exhibit some kind of uncertainties, so that they can achieve a certain desired behavior. To a large extent, the difficulties that arise in realizing this goal stem from disturbances that affect the system as well as uncertainties about the system behavior, which may come from poor plant knowledge, uncertain or slowly varying parameters, nonlinearities such as hysteresis or friction, errors or unknown initial conditions, reduced-order

models, additive unknown internal or external noise, environmental influence [103]. Uncertainty is the major stumbling block in most problems of design and decision. In dealing with uncertainties, several control theories have been proposed in the literature. The following is a brief review of these methods.

Stochastic optimal control: In a stochastic system, the uncertainty is modeled in a probabilistic way using random variables and stochastic processes as important tools. Dynamic programming is a classical approach in studying stochastic control problems [123]. Stochastic optimal control methods have found significant applications in areas such as aerospace control problems [124], management and finance [125], manufacturing and production [126], communication/computer networks [127], military and service industry logistics [128], etc.

Robust control: Robust control is a constant gain feedback control. Robust control and its related area of robustness analysis usually require that the true plant model lies in a (small) ball centered at a known nominal model and often assume that the controllers are either selected from certain given classes of systems or simply fixed [129]. Robust controllers are simpler to implement compared with adaptive control and no time is required to “tune” the controller to the plant variations. However, such approaches have the following drawbacks [130]: (i) the crudeness of the uncertainty description usually leads to redundant and physically impossible plants, with highly conservative designs, (ii) it is infeasible to favor scenarios with higher chance of occurrence among all the possible ones, and as a result, robust optimality is precluded, (iii) a quantitative description of the robustness of the solution is unattainable and (iv) the resulting controllers are so complex that model reduction

techniques are usually required.

Adaptive control: Adaptive control is another way to handle uncertainties not only in parameters and inputs, but also in the model of the structure. It can continuously increase the information about the instantaneous parameters of the uncertain process and to utilize them, by simultaneously calculating the optimal controller parameters. It is a nonlinear feedback technique that can perform identification and control simultaneously in the same feedback loop, and is known to be a powerful tool in dealing with systems with large uncertainties [131]. Although adaptive controllers can produce excellent control performance, it may be difficult to develop general design principles, which can be applied to various industrial processes. Moreover, there may arise problems concerning the stability of the system, the supply of sufficient amplitude and frequency richness of the process signals under identification, the differentiation between system parameter deviations and disturbance reactions, the modifications to guarantee boundness of the estimates, and large overshoots during phases of rapid adaptation (at startup and after parameter changes).

Robust adaptive control: The main goals of the robust adaptive control technique are to achieve guaranteed steady state and transient tracking accuracy (properties of Deterministic Robust Controls) and asymptotic tracking at the absence of disturbances without relying on discontinuous switching or infinite control gains (property of Adaptive Controls) [132]. The robust adaptive control combines the benefits of robust and adaptive control methods while avoiding their drawbacks.

Because robust adaptive control requires on-line system identification, full excitation, and additional computation, its advantages are costly.

Adaptive dual control: An adaptive dual control system can be defined as an adaptive control system with the control signal having the following properties [133]: (i) it cautiously follows the control goal and (ii) excites the plant to improve the estimation. The main idea of the adaptive dual control method lies in the design of adaptive systems that are not optimal but have at least the main dual features of the optimal adaptive dual control systems. The formal solution to the optimal adaptive dual control problem can be obtained through the use of dynamic programming. Adaptive dual controllers provide a number of advantages compared to the classical adaptive control schemes: (i) especially the accuracy of the estimation is taken into account, (ii) an optimal excitation for speeding up the estimation is provided, and (iii) the adaptation time becomes shorter and provides a smooth transitional behavior [133].

In choosing an appropriate control strategy, not only the characteristics of the abovementioned control strategies, but also the features of the model itself should be taken into consideration. Compromise may be made between the estimation accuracy and computational efficiency to achieve effective real time performance.

The following is a short review of some basic methods used to design adaptive laws.

(i) Sensitivity methods

The sensitivity method can be used to design the adaptive law to minimize a certain performance function by adjusting the estimated parameters. The adaptive

law is driven by the sensitivity function with respect to the estimated parameters multiplied by an error signal, which characterizes the mismatch between the actual and desired behavior. The main drawback of the method is that the sensitivity function cannot be generated online in most formulations of adaptive control [134].

A popular method for computing the approximate sensitivity functions is the so-called MIT rule. With this rule the unknown parameters needed for generating the sensitivity functions are replaced by their online estimates. Unfortunately, with the use of approximate sensitivity functions, it is impossible, in general, to prove global closed-loop stability and convergence of the tracking error to zero. An example of adopting the sensitivity method to design an adaptive controller for thermal treatment of prostate disease can be found in [135].

(ii) Positivity and Lyapunov design

In this approach, the differential equation of the adaptive law is chosen to satisfy certain stability conditions based on Lyapunov theory. The adaptive law developed is very similar to that based on the sensitivity method. The difference between this approach and the sensitivity method is that the sensitivity functions are replaced with ones that can be generated online. Examples using this method can be found in [136-138].

(iii) Gradient method and least squares method based on estimation error cost criteria

The main drawback of the sensitivity methods is that the minimization of the

performance cost function led to sensitivity functions that are not implementable. One way to avoid this drawback is to choose a cost function criterion that can lead to measurable sensitivity functions. A class of such cost criteria based on the estimation error can provide a measure of the discrepancy between the estimated and actual parameters. The relationship of the estimation error with the estimated parameters is chosen so that the cost function is convex, and its gradient with respect to the estimated parameters is implementable [134].

The *gradient method*, as a simple optimization technique to minimize the estimation error with respect to some adjustable parameters, has been widely used in discrete-time and, to a less extent, continuous-time adaptive systems. The use of the gradient method involves the development of an algebraic estimation error equation, which motivates the selection of an appropriate cost function that is convex over the space of the estimates for each time instant. The cost function is then minimized with respect to the estimates for each time by using the gradient method [134]. Examples using the gradient method can be found in [139,140].

Another popular method based on the estimation error cost criteria is the so-called *least squares* method. The basic idea behind the least squares method is to minimize the sum of the squares of the difference between the observed and computed data by fitting a mathematical model with a sequence of observed data. In doing so, any noise or inaccuracies in the observed data are expected to have less effect on the accuracy of the mathematical model. Several improvements have been made to deal with the problems arising when using the least square methods, e.g. least square with covariance resetting, or modified least square with forgetting factor, etc. Least square methods have been widely used in parameter estimation

both in a recursive and nonrecursive form mainly for discrete-time systems [96,141,144,171,172].

2.7 Conclusion

Real-time tracking instruments that can assist in surgery, the needle-tissue contact studies, needle steering methods, soft tissue biomechanics and techniques of obtaining the needed biomechanical information are reviewed in this chapter. These are followed by a review of control methods that involve uncertainty control, which should be considered in future control strategy design for the special application environment: the intricate human soft tissue.

Prior works in the literature were limited to soft tissue deformation modeling, needle axial insertion force analysis during straight-line needle insertion, or needle-tissue interaction study using FEM that is computational expensive. An effective needle-tissue interaction model that can be used for real time needle steering is still lacking. In the current models, the tissue is generally assumed to be homogeneous and isotropic, and the tissue reactions are calculated using rough estimation of Young's modulus and Poisson ratio (e.g. [1]), or spring coefficient (e.g. [16]), which will cause significant errors in real application. An accurate, efficient estimation of the tissue reactions during interaction is very difficult due to the tissue complexity, which is probably the main reason holding the development of an effective needle-tissue interaction model.

In this thesis, a computationally effective needle-tissue interaction model is proposed, and an online parameter estimation method is developed to estimate the parameters using online measured dataset. Following chapters present the details.

Chapter 3 Needle Steering Modeling

Flexible needle insertion has aroused a lot of research interests in recent years. Many medical procedures such as brachytherapy, biopsies, and treatment injections require inserting a long slender needle into a specific target location inside the human body to implant a radioactive seed, extract a tissue sample, or inject a drug. Precise needle placement is very important. Poor placement may cause tissue damage, misdiagnosis, poor dosimetry and tumor seeding. Yet precise needle placement is hard to achieve in practice. One of the difficulties is the needle deflection. Most of the existing needle insertion robot can only perform straight-line needle path. When the needle misses the target, in most cases, it has to be retracted and reinserted. A physically based needle steering system that can adjust the needle position in a constrained manner and in real time, is still lacking. In addition, many procedures are currently not amenable to needles because of obstacles, such as bone or sensitive tissues, which lie between feasible entry points and potential targets. Thus, there is a clear motivation for needle steering in order to provide accurate and dexterous targeting.

One simple and effective way to do the needle steering is to apply lateral force at the needle base to change the needle tip trajectory. Compared with other steering methods, e.g. specially-made steerable needles as presented in [102,103], the proposed base steering method is more straightforward, and no special needle or environment is required. Thus it is expected to have more generalized applications. The total force acted at the base can be decomposed into the insertion force and steering force, which are in x and y directions respectively. The lateral steering force is the main actuation for correcting the needle tip deflection or steering the

needle tip trajectory. This chapter proposes a spring-beam-damper model for the needle base steering, which describes the dynamics between the needle base steering force and the needle tip lateral movement with consideration of the needle flexibility and tissue deformation. The proposed steering model takes into consideration not only the elastic tissue reactions, but also the viscoelastic tissue effects while interacting with the flexible needle, which have been largely ignored in the literature. A computationally efficient method, unconstrained model analysis, is adopted in this research to solve the system dynamics. The proposed model, after being proven effective (as will be shown in the following chapters), will be used in designing a steering system that can steer the needle trajectory to correct deflection or avoid an obstacle, e.g. pubic bone.

3.1 Needle Steering Modeling

Figure 3.1 Mechanism of the spring-beam-damper system for the needle insertion procedure.

A spring-beam-damper system as shown in Figure 3.1 is considered in this study to model the system dynamics between the lateral steering forces acting at the needle base and the corresponding needle tip lateral movement during insertion in the soft

tissue [170]. According to Fung [112], most living tissues have viscoelastic behaviors, as long as small displacements are considered. Linear soft tissue's viscoelastic models, which relate the interaction forces and the displacement between two bodies, are often described by serial or parallel combinations of springs and dashpots. The classical tissue models built from an equivalent spring and an equivalent viscous element are the serial Maxwell model and the parallel Kelvin Voigt (KV) model. The latter is certainly the most used model in the viscoelastic interaction modeling literature. Some researchers have used a generalized KV model to identify the needle axial insertion force with slowly varying coefficients during straight-line insertion [96]. In this work, a continuously distributed KV model is adopted to study the tissue reaction forces during steering procedure. The initial lengths of springs are decided by the needle tip trajectory. The flexible needle is assumed to follow the Bernoulli-Euler beam model that mainly considers that needle deflection is due to bending instead of shearing [142]. At the beginning, the needle is placed next to the tissue and clamped tightly at the base. With time progressing on, the needle is inserted into the tissue. When lateral force is applied at the needle base, the needle will move laterally. Thus the spring-damper system will come into contact with the needle and exert forces on it. The forces of the springs at time instant t are determined by the needle body shape at that time and the needle tip trajectory; while the forces of the dampers are determined by the velocities of the contact points. During this procedure, not only the tissue deformation and the needle flexibility, but also their interaction should be taken into consideration.

To derive the equations of needle insertion, the following assumptions are made:

(1) For simplicity, the needle is considered to move only in the XY plane. X is the insertion direction and Y is the steering direction.

(2) There is no longitudinal compression of the beam and only lateral deflection is possible. Furthermore, the lateral deflection of the beam is small compared with the length of the beam.

(3) The rotational effect of the beam with respect to the local coordinate system is neglected.

Under the above assumptions, the system dynamic equation can be derived using Hamilton's principle as follows:

$$\int_{t_1}^{t_2} (\delta T - \delta V + \delta W_{nc}) dt = 0 \quad (3-1)$$

T : kinetic energy

V : potential energy

W_{nc} : work done by non-conservative forces

A local coordinate system is introduced by the Galilean transformation to replace the fixed coordinates (x) with a moving coordinate system ($\tilde{x}$), which is attached at the needle base and moves with it. v_x is needle insertion velocity, which is assumed to be constant for modeling simplicity. The transformation between the fixed coordinate system and moving coordinate system are shown below.

$$\begin{cases} \tilde{x} = x - v_x t \\ \tilde{y} = y \end{cases} \quad (3-2)$$

$$(3-3)$$

The system kinetic energy T includes the kinetic energy of the fixture and the needle, as shown in the first and second terms of Equation (3-4); while the potential energy V includes the potential energy of the needle caused by the needle bending (the first term), and the potential energy of the springs resulted from the

forces between the needle and the tissue (the second term), as given in Equation (3-5). The integration of the difference between the needle body positions and the needle tip trajectory gives the summation of the elongated or compressed spring length (decided by the sign of the difference) at the contact points. Because only the needle portion inside the tissue has springs exerting force on it, Equation (3-6), the Heaviside unit step function, is used to exclude the portion outside the tissue. $h(t) = L - v_x t$ is the position of the insertion point in the moving coordinates system at time instant t . Thus the potential energy of the springs can be calculated using the second term of Equation (3-5).

$$T = \frac{1}{2} M \dot{y}^2 + \int_0^L \frac{1}{2} \rho (\dot{y}(t) + \dot{\omega}(\tilde{x}, t))^2 d\tilde{x} = T(\dot{y}, \dot{\omega}) \quad (3-4)$$

$$\begin{aligned} V &= \int_0^L \frac{1}{2} EI \omega''^2 d\tilde{x} + \int_0^L H(\tilde{x} - L + v_x t) \cdot \frac{1}{2} k [y(t) + \omega(\tilde{x}, t) - y(t + \frac{\tilde{x} - L}{v_x}) - \omega(L, \frac{\tilde{x} - L}{v_x} + t)]^2 d\tilde{x} \\ &\xrightarrow{h(t) = L - v_x t} \int_0^L \frac{1}{2} EI \omega''^2 d\tilde{x} + \int_0^L H(\tilde{x} - h(t)) \cdot \frac{1}{2} k [y(t) + \omega(\tilde{x}, t) - y(\frac{\tilde{x} - h(t)}{v_x}) - \omega(L, \frac{\tilde{x} - h(t)}{v_x})]^2 d\tilde{x} \end{aligned} \quad (3-5)$$

$$H(x) = \begin{cases} 1 & \dots x \geq 0 \\ 0 & \dots x < 0 \end{cases} \quad (3-6)$$

Here, M is the mass of the fixture, which links the needle with the 3D motion platform; L is the length of the elastic beam; ρ is the mass per unit length of the elastic beam; E is the Young's modulus of the needle; I is the second moment of inertia about the z axis. k is stiffness coefficient of the spring per unit length, and c is the damper coefficient per unit length. y is the needle base position in y axis; $\dot{y}(t)$ is the corresponding velocity at time instant t . ω is the deflection of the beam along the needle body at time instance t . $\dot{\omega}$ and ω'' are the first and second derivatives of the beam deflection with respect to time and space, respectively.

The virtual work done by all the non-conservative forces (steering force F_y and

damping forces), is given by

$$\delta W_{nc} = F_y \cdot \delta y - \int_0^L H(\tilde{x} - L + v_x t) \cdot c(\dot{y} + \dot{\omega}) \cdot \delta(y + \omega) d\tilde{x} \quad (3-7)$$

The first term is the virtual work done by the steering force and the second term gives the energy consumed by the dampers along the needle body inside the tissue. Substituting equations (3-4), (3-5), (3-7) into (3-1), integrating the resulting equation by parts with consideration that the time instants t_1 and t_2 are arbitrary, and that $\delta y, \delta \omega$ are arbitrary and independent, the following dynamic equations for the spring-beam-damper system can be obtained (See Appendix G.1).

$$F_y = M\ddot{y} + \int_0^L \rho(\ddot{y} + \ddot{\omega}) d\tilde{x} + \int_0^L H(\tilde{x} - L + v_x t) [k(y + \omega - y(t_1) - \omega(L, t_1)) + c(\dot{y} + \dot{\omega})] d\tilde{x} \quad (3-8)$$

$$EI\omega'''' + \rho(\ddot{y} + \ddot{\omega}) + H(\tilde{x} - L + v_x t) [k(y + \omega - y(t_1) - \omega(L, t_1)) + c(\dot{y} + \dot{\omega})] = 0 \quad (3-9)$$

$$t_1 = t + \frac{\tilde{x} - L}{v_x} \quad (3-10)$$

Boundary conditions:

$$(1) \omega(0, t) = 0 \quad (2) \omega'(0, t) = 0 \quad (3) \omega''(L, t) = \omega'''(L, t) = 0 \quad (3-11)$$

3.2 Model Analysis

To solve the partial differential equations shown in (3-8) and (3-9), unconstrained modal analysis is adopted in this approach. By unconstrained modal analysis [142], the deflection of the beam at $\tilde{x}$, $\omega(\tilde{x}, t)$ and the position of the fixture, $y(t)$ can be represented respectively as

$$y = \alpha(t) + \beta q(t) \quad (3-12)$$

$$\omega(\tilde{x}, t) = \phi(\tilde{x}) q(t) \quad (3-13)$$

where $\alpha(t)$ describes the motion of the center of mass of the fixture without

perturbation. $\phi(\tilde{x})$ is the shape function, $q(t)$ is the time-varying amplitude of motion. β is defined to satisfy

$$\beta = -\frac{\rho}{M_t} \int_0^L \phi(\tilde{x}) d\tilde{x} \quad (3-14)$$

with $M_t = M + \rho L$.

Note that equation (3-14) guarantees that no perturbation of the motion of the fixture center of mass occurs, e.g. $\ddot{\alpha}(t) = 0$, when no force acts on the system. (See Appendix G.2)

Substituting equation (3-12) into (3-8), and taking into account equation (3-14), we get (see Appendix G.3)

$$M_t \ddot{\alpha}(t) = F_y - f(t) \quad (3-15)$$

with

$$f(t) = \int_0^L H(\tilde{x} - L + v_x t) [k(y + \omega(\tilde{x}, t) - y(t_1) - \omega(L, t_1)) + c(\dot{y} + \dot{\omega}(\tilde{x}, t))] d\tilde{x}$$

To get the normal mode solutions, where the effects of the external force are absent, one can decompose equation (3-9) into two equations as follows (see Appendix G.4)

$$\ddot{q}(t) + \xi^2 q(t) = 0 \quad (3-16)$$

$$EI \phi''''(\tilde{x}) - \xi^2 \rho \phi(\tilde{x}) = 0 \quad (3-17)$$

where ξ is the natural frequency of the system, and

$$\phi(\tilde{x}) \triangleq \beta + \phi(\tilde{x}) \quad (3-18)$$

The boundary conditions become

$$\phi(0) = \beta, \phi'(0) = 0, \phi''(L) = 0, \phi'''(L) = 0 \quad (3-19)$$

The Laplace transform of Equation (3-17), considering Equation (3-19), leads to

$$\bar{\varphi}(s) = \frac{s^3}{s^4 - \kappa^4} \beta + \frac{s}{s^4 - \kappa^4} \varphi''(0) + \frac{\varphi'''(0)}{s^4 - \kappa^4} \quad (3-20)$$

where $\bar{\varphi}(s)$ represents the transformed function of $\varphi(\tilde{x})$ and

$$\kappa^4 = \rho \xi^2 / EI \quad (3-21)$$

The inverse Laplace transform of the equation gives the general solution of Equation (3-18) as follows:

$$\varphi(\tilde{x}) = \frac{\beta}{2} (\cos \kappa \tilde{x} + \cosh \kappa \tilde{x}) - \frac{\varphi''(0)}{2\kappa^2} (\cos \kappa \tilde{x} - \cosh \kappa \tilde{x}) - \frac{\varphi'''(0)}{2\kappa^3} (\sin \kappa \tilde{x} - \sinh \kappa \tilde{x}) \quad (3-22)$$

The constants $\varphi''(0)$ and $\varphi'''(0)$ can be obtained from the last two boundary conditions of Equation (3-19) as follows:

$$\varphi''(0) = \beta \kappa^2 \frac{\sin \kappa L \sinh \kappa L}{1 + \cos \kappa L \cosh \kappa L} \quad (3-23)$$

$$\varphi'''(0) = -\beta \kappa^3 \frac{\cos \kappa L \sinh \kappa L + \sin \kappa L \cosh \kappa L}{1 + \cos \kappa L \cosh \kappa L} \quad (3-24)$$

Thus, $\varphi(\tilde{x})$ can be obtained as

$$\varphi(\tilde{x}) = \frac{\beta}{2} [\cos \kappa \tilde{x} + \cosh \kappa \tilde{x} + \frac{M\kappa}{\rho} \frac{\cos \kappa \tilde{x} - \cosh \kappa \tilde{x}}{c \tanh \kappa L + c \tan \kappa L} - \frac{M\kappa}{\rho} (\sin \kappa \tilde{x} - \sinh \kappa \tilde{x})] \quad (3-25)$$

Integrate Equation (3-9) with respect to $\tilde{x}$, and substitute Equation (3-8) into the resulting equation give

$$EI \omega'''(0, t) + M \ddot{y} = F_y \quad (3-26)$$

When $F(y, t) = 0$, substituting (3-12), (3-13), (3-16) into the above equation yields

$$\beta = \frac{EI}{M \xi^2} \phi'''(0) = \frac{EI}{M \xi^2} \varphi'''(0) = \frac{-EI}{M \xi^2} (\beta \kappa^3) \frac{\cos \kappa L \sinh \kappa L + \sin \kappa L \cosh \kappa L}{1 + \cos \kappa L \cosh \kappa L} \quad (3-27)$$

from which we can obtain the frequency equation

$$1 + \cos \kappa L \cosh \kappa L + \frac{\rho}{M \kappa} (\sin \kappa L \cosh \kappa L + \cos \kappa L \sinh \kappa L) = 0 \quad (3-28)$$

The roots of the frequency equation $\kappa_i, i=1,2,\dots$ can be obtained numerically using *Newton-Rapson method*. Substitute κ_i into Equations (3-21), (3-25) and (3-27), one can get $\xi_i, \varphi_i(\tilde{x}), \beta_i$. Assumed-mode solutions for a finite number of eigenvalues are obtained by considering the first n terms. The deflection of the elastic beam and the displacement of the fixture are expressed respectively in terms of n mode shapes using the obtained $\varphi_i(\tilde{x}), \beta_i, q_i(t)$, as given in Equations (3-29) and (3-30).

$$\omega(\tilde{x}, t) = \sum_{i=1}^n \phi_i(\tilde{x}) q_i(t) = \sum_{i=1}^n [\varphi_i(\tilde{x}) - \beta_i] q_i(t) \quad (3-29)$$

$$y(t) = \alpha(t) + \sum_{i=1}^n \beta_i q_i(t) \quad (3-30)$$

Substituting equations (3-29), (3-30) into (3-8), equation (3-31) can be obtained; substituting (3-29), (3-30), (3-17) into (3-9) and multiplying it by $\phi_i(\tilde{x})$, integrating from 0 to L , and applying the orthogonal condition ($\int_0^L \rho \phi_i(\tilde{x}) \phi_j(\tilde{x}) d\tilde{x} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$ [177]), the following equation (3-32) can be obtained.

$$M_i \ddot{\alpha}(t) + c v_x \dot{\alpha}(t) + \sum_j \dot{q}_j(t) c \left[\int_{L-v_x t}^L \phi_j(\tilde{x}) d\tilde{x} \right] + \sum_j q_j(t) k \left[\int_{L-v_x t}^L \phi_j(\tilde{x}) d\tilde{x} \right] + k v_x \alpha(t) = F_y + f_2 \quad (3-31)$$

$$\begin{aligned} & \dot{\alpha}(t) c \left[\int_{L-v_x t}^L \phi_i(\tilde{x}) d\tilde{x} \right] + \alpha(t) k \left[\int_{L-v_x t}^L \phi_i(\tilde{x}) d\tilde{x} \right] + \ddot{q}_i(t) + \sum_j \dot{q}_j(t) c \left[\int_{L-v_x t}^L \phi_i(\tilde{x}) \phi_j(\tilde{x}) d\tilde{x} \right] \\ & + \sum_j q_j(t) k \left[\int_{L-v_x t}^L \phi_i(\tilde{x}) \phi_j(\tilde{x}) d\tilde{x} \right] + \xi_i^2 q_i(t) = F_y \beta_i + f_{1i} \end{aligned} \quad (3-32)$$

with

$$\begin{aligned} f_{li} &= \int_0^t k\alpha(t_1)\varphi_i(v_x t_1 + L - v_x t)v_x dt_1 + \int_0^t k \sum_j^n \varphi_j(L)q_j(t_1)\varphi_i(v_x t_1 + L - v_x t)v_x dt_1 \\ &= \int_0^t ky_T(t_1)\varphi_i(v_x t_1 + L - v_x t)v_x dt_1 \end{aligned}$$

$$f_2 = \int_0^t k\alpha(t_1)v_x dt_1 + \int_0^t k \sum_j^n \varphi_j(L)q_j(t_1)v_x dt_1 = \int_0^t ky_T(t_1)v_x dt_1$$

here, M_i is the total mass of the fixture and needle. $y_T(t_1) = \alpha(t) + \sum_{i=1}^n \varphi_i(L)q_i$ is the needle tip position at time instant t_1 , which is time varying and derived from Equations (3-29) and (3-30). F_y is the lateral steering force, which will be acting at the needle base in the y direction. Equation (3-31) describes various forces acting on the link; while Equation (3-32) describes various forces acting on the flexible needle.

The initial conditions of the system are given as

$$\begin{aligned} 1) \omega(\tilde{x}, 0) &= 0 & 2) \alpha(0) &= 0 \end{aligned} \quad (3-33)$$

With the initial conditions, $\alpha(t)$ and $q_i(t)$ can be solved from equations (3-31) and (3-32) using Explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair, which is a one-step solver and is available in Matlab. A Matlab simulation program has been developed to simulate this model. With the applied needle base force F_y that is the input of the model, α and q_i as well as the tissue reaction forces will change with time, therefore leading to the change of the needle tip position $y_T(t_1)$ in y axis that is the output of the model.

3.3 Discussions

In this chapter, a spring-beam-damper model is proposed for the needle steering procedure. The flexible needle is assumed to follow the Bernoulli-Euler beam

model; while the tissue lateral elastic and viscoelastic reaction forces are modeled by a continuously distributed spring and damper system, by which the tissue continuum is better taken into consideration than the classical sparse spring-damper models. Unconstraint modal analysis is adopted to solve the system dynamics, which is computational efficient and suitable for real time applications.

The only computational costly parts are the space-varying integrals, $\int_{L-v_x t}^L \varphi_j(\tilde{x}) d\tilde{x}$ and $\int_{L-v_x t}^L \varphi_i(\tilde{x}) \varphi_j(\tilde{x}) d\tilde{x}$ highlighted in box in Equations (3-31) and (3-32), but they can be pre-computed once for all by choosing a small sample depth, if k and c are independent of $\tilde{x}$. Studies in literature have shown that the tissue is inhomogeneous and different types of tissues exhibit different properties. For example, [143] shows that the elastic modulus of the cancer tissue is around 3 times that of the normal tissue. This means tissue properties could be different at different places. Therefore, the k and c coefficients could be $\tilde{x}$ related. To take into consideration the tissue inhomogeneity, and at the same time, improve the computational efficiency, mean spring and damper coefficients are proposed to calculate the tissue reaction forces at each insertion depth. These mean parameters are independent of $\tilde{x}$ at each step, but they are considered to be time or depth varying due to tissue inhomogeneity. The following chapter gives more details.

Chapter 4 Depth-varying Mean Parameters and Online Parameter Estimation

A major difficulty in applying the spring-beam-damper model is the estimation of the virtual spring and damper coefficients of the tissue, which is very difficult considering the complex tissue behaviors especially when combined within the heterogeneous body. In the literature, the acquisition of the elastic or viscoelastic properties of a tissue was either borrowed from *a priori* researches in the literature, or performed using some offline methods (e.g. rheological experiments [15], or crude estimation [16]), which may differ substantially from the true properties during operation. A rough estimation of the k and c values may cause the model output to deviate from or oscillate around the real output, as observed through past simulations. An estimation method that can guide the model output to closely track the real measured output is needed.

In this chapter, depth-varying mean parameters are proposed to calculate the tissue reaction forces, with consideration of the tissue inhomogeneity and computational efficiency. Local polynomials in finite depth segments are adopted to approximate the unknown depth-varying mean parameters. Based on these approaches, an online parameter estimator has been designed using modified least square method with forgetting factor to estimate the parameters. By using this method, the tracking errors are minimized. The subsequent sections present the details.

4.1 Local Polynomial Approximation of Depth-varying Mean Parameters

4.1.1 Two Assumptions

Considering the inhomogeneous human tissue, depth-varying mean parameters are

proposed to calculate the spring and damper reaction forces, and local polynomials in finite depth segments are used to approximate the depth-varying mean parameters [162].

Assumption 1:

The spring and damper coefficients may be different at different depths of the tissue. At each insertion step, the spring and damper forces along the needle body that is inside the tissue can be calculated using mean spring and damper coefficients $\theta(s) = [\bar{c} \quad \bar{k}]^T$ (s here is the insertion depth, which is a function of time.) These mean coefficients may vary along the steps considering the tissue inhomogeneity, and the tumors that may occur along the needle trajectory.

From the definition, at time instant t , the mean spring coefficient is calculated as

$$\int_{L-v_x t}^L k(\tilde{x}, t) * \Delta y(\tilde{x}, t) d\tilde{x} = \bar{k}(t) \int_{L-v_x t}^L \Delta y(\tilde{x}, t) d\tilde{x} \quad (4-1)$$

where $\Delta y(\tilde{x}, t)$ is the spring elongated or compressed length at place $\tilde{x}$ along the needle body. The mean damper coefficient $\bar{c}(t)$ can be calculated using similar method. It is easy to prove that $\bar{k}(t)$ and $\bar{c}(t)$ are continuously distributed.

This assumption takes into considerations the tissue inhomogeneity, and at the same time, reduces the computational intensity by using mean values to calculate the spring and damper forces along the needle body at each insertion step. In addition, the adoption of the mean values guarantees that $\theta(s)$ is continuously distributed regardless of the abrupt changes of tissue properties (e.g. pathological changes of the tissue, or multi-layer insertion), since the effect of abrupt changes will be evened or smoothed in calculating the mean value along the whole needle body inside tissue.

Assumption 2:

The depth-varying mean parameters $\theta(s)$ are assumed to be $(p+1)^{th}$ order differentiable within finite segments. Thus it can be represented by a series of local polynomial approximations in these segments.

This can be achieved using Taylor series expansion. Recall that the functions $\theta(s)$ can be expanded around certain points s_0 , as shown below.

$$\theta(s) = \theta(s_0) + (s - s_0) \frac{d\theta^{(1)}(s_0)}{ds} + \dots + \frac{(s - s_0)^p}{p!} \frac{d\theta^{(p)}(s_0)}{ds} + \int_{s_0}^s \frac{(s - \xi)^{p+1}}{(p+1)!} \frac{d\theta^{(p+1)}(\xi)}{ds} d\xi \quad (4-2)$$

Here, $\theta(s)$ is approximated by the first $p+1$ terms. The last term represents the error due to the approximation.

4.1.2 Local Polynomials for the Depth-Varying Mean Parameters

From the above assumptions, we can divide the whole insertion length into several segments and adopt piecewise continuous p -order differentiable functions θ_{ij} , $i=1,2, j=1,2,\dots,n$ to represent the depth-varying mean parameter $\theta_i(s)$ $i=1,2$ in each segment, as illustrated in Figure 4.1. Here, the index i refers to the i^{th} parameter (spring or damper coefficient); while the index j refers to the j^{th} segment and n is the number of segments. s coordinate system is adopted for convenient representation of the parameter polynomials. The origin of the s coordinate system is set at the insertion point, as shown in Figure 4.1. The transformation between s domain and $\tilde{x}$ domain is given by

$$\begin{cases} \tilde{x} = s + L - \dot{s}t \\ \dot{s} = v_x \end{cases} \quad (4-3)$$

under the assumption that the needle is inserted at constant velocity.

Figure 4.1 Local polynomial approximations of the parameter functions

Thus the polynomial approximation of θ_{ij} is represented as

$$\begin{aligned}\theta_{ij}(s) &= a_{ij0}(s_{0,j}) + a_{ij1}(s_{0,j})(s - s_{0,j}) + \cdots + a_{ijp}(s_{0,j})(s - s_{0,j})^p \\ &= \sum_{m=0}^p a_{ijm}(s_{0,j})(s - s_{0,j})^m, \quad s \in [s_{0,j}, s_{0,j} + l] \quad (4-4) \\ &= \eta_{ij}^T(s, s_{0,j}) A_{ij}(s_{0,j})\end{aligned}$$

where l is the length of the segment and p is the order of the polynomial. $s_{0,j}$ refers to the resetting depth (joints of each segments) at which the j^{th} segment of the local polynomial approximation for parameter θ_i begins. $s_{0,j}$ is given by the sequence $s_0 = \{s_{0,j}\}, j = 1, \dots, n$ and $s_{0,(j+1)} - s_{0,j} = l$.

$a_{ijm}(s_{0,j}) = (1/k!) \theta_{ij}^{(m)}(s_{0,j}), k = 0, \dots, p$, $\theta_{ij}^{(m)}(s_{0,j})$ is the m^{th} depth derivative evaluated at $s = s_{0,j}$. $A_{ij}(s_{0,j}) = [a_{ij0}(s_{0,j}), a_{ij1}(s_{0,j}), \dots, a_{ijp}(s_{0,j})]^T$ is the unknown constant vector and $\eta_{ij}(s, s_{0,j}) = [1, (s - s_{0,j}), \dots, (s - s_{0,j})^p]^T$ is a column vector. Notice that $A_{ij}(s_{0,j})$ is constant only within each segment $[s_{0,j}, s_{0,j+1})$ and in general differs from one segment to another for the inhomogeneous tissue.

The truncation error, $\delta = \int_{s_0}^s \frac{(s - \xi)^{p+1}}{(p+1)!} \theta_{ij}^{(p+1)}(\xi) d\xi$ resulted from equation (4-4) is

negligible if the length of segment is sufficiently small. It is easy to prove that

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when the $(p+1)^{th}$ derivative of $\theta_{ij}(s)$ is bounded, that is, $\sup_t \|\theta_{ij}^{(p+1)}(s)\| \leq C$, δ can be bounded by

$$|\delta| \leq \frac{C(s - s_{0,j})^{p+1}}{(p+1)!} \quad (4-5)$$

Therefore, it is possible to use (4-4) to approximate $\theta_{ij}(s)$ more precisely by choosing either a higher order polynomial (i.e. large p), or a smaller segment l , or both. If one partitions the whole insertion length into segments with the length of each segment equal to l , then the depth-varying function $\theta_i(s)$ can be approximated by a number of polynomials $\theta_{ij}(s)$ located in each segment with constant coefficients a_{ijk} , as shown in Equation (4-4).

4.2 Some Useful Properties of the Local Polynomial

Approximation

4.2.1 Relationship of the Parameter Vectors at Two Adjacent Resetting

Depths

To get the relationship of the parameter vectors at two adjacent resetting depths, with reference to the analytical method shown in [145], we approximate $\theta_{ij}(s)$ using another point in the j^{th} segment, that is, $s_r \neq s_{0,j}$, $s_r - s_{0,j} \leq l$ by

$$\begin{aligned} \theta_{ij}(s) &= a_{ij0}(s_r) + a_{ij1}(s_r)(s - s_r) + \cdots + a_{ijp}(s_r)(s - s_r)^p \\ &= \sum_{m=0}^p a_{ijm}(s_r)(s - s_r)^m \end{aligned} \quad (4-6)$$

where $a_{ijm}(s_r) = (1/m!)\theta_{ij}^{(m)}(s_r)$, $m = 0, 1, \dots, p$.

From Equations (4-4) and (4-6), we can obtain the n^{th} derivative

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$$\theta_{ij}^n(s) = \sum_{m=n}^p a_{ijm}(s_{o,j})(s - s_{o,j})^{m-n} \frac{m!}{(m-n)!} \quad (4-7)$$

$$\theta_{ij}^n(s) = \sum_{m=n}^p a_{ijm}(s_r)(s - s_r)^{m-n} \frac{m!}{(m-n)!} \quad (4-8)$$

Evaluating (4-7) at $s = s_r$, we get

$$\theta_{ij}^n(s_r) = \sum_{m=n}^p a_{ijm}(s_{o,j})(s_r - s_{o,j})^{m-n} \frac{m!}{(m-n)!} \quad (4-9)$$

Similarly, evaluating (4-8) at $s = s_r$, we obtain

$$\theta_{ij}^n(s_r) = \sum_{m=n}^p a_{ijm}(s_r)(s_r - s_r)^{m-n} \frac{m!}{(m-n)!} = n! a_{ijn}(s_r) \quad (4-10)$$

Comparing (4-9) and (4-10) results in

$$a_{ijn}(s_r) = \sum_{m=n}^p a_{ijm}(s_{o,j})(s_r - s_{o,j})^{m-n} \frac{m!}{n!(m-n)!} \quad (4-11)$$

Therefore, the relationship between $a_{ijm}(s_r)$ and $a_{ijm}(s_{o,j})$ is given by

$$\begin{bmatrix} a_{ij0}(s_r) \\ a_{ij1}(s_r) \\ \vdots \\ a_{ijp}(s_r) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & s_r - s_{o,j} & \cdots & (s_r - s_{o,j})^p \\ 0 & 1 & \cdots & p(s_r - s_{o,j})^{p-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}}_{\psi(s_r, s_{o,j})} \begin{bmatrix} a_{ij0}(s_{o,j}) \\ a_{ij1}(s_{o,j}) \\ \vdots \\ a_{ijp}(s_{o,j}) \end{bmatrix} \quad (4-12)$$

that is,

$$A_{ij}(s_r) = \psi(s_r, s_{o,j}) A_{ij}(s_{o,j}) \quad (4-13)$$

Set $s_r = s_{0,j+1}$, we obtain

$$A_{ij}(s_{0,j+1}) = \psi(s_{0,j+1}, s_{0,j}) A_{ij}(s_{0,j}) \quad (4-14)$$

By using (4-14), we can get the relationship of the parameter vectors at two adjacent resetting depths using the following equation.

$$A(s_{0,j+1}) = \begin{bmatrix} \psi(s_{0,j+1}, s_{0,j}) & \\ & \psi(s_{0,j+1}, s_{0,j}) \end{bmatrix} A(s_{0,j})^1 \quad (4-15)$$

¹ Zeros are neglected in diagonal matrices.

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Notice that $A(s_{0,j})$ is constant in the j^{th} segment, that is, $A(s) = A(s_{0,j})$ for all $s \in [s_{0,j}, s_{0,j+1})$.

4.2.2 Recovery of the Original Parameter Vector

Equation (4-4) can be used to get the parameter vector $\theta_i(s)$. The order of the polynomials p for different $\theta_{ij}(s)$ can be chosen based on some *a priori* knowledge; for convenience, p is chosen to be the same for the two depth-varying parameters. Therefore, the original parameter vector $\theta_i(s)$ is related to the approximated polynomial coefficient vector $A_i(s_0)$ by

$$\theta(s) = \begin{bmatrix} \eta^T(s, s_0) \\ \eta^T(s, s_0) \end{bmatrix} A(s_0)^{-1} \quad (4-16)$$

Here, $s_0 = \{s_{0,j}\}$, $j=1,2,\dots,n$ is the j^{th} segment of the local polynomials. Equation (4-16) is used to simplify the parameter estimation problem by using a constant vector $A_i(s_0)$ to replace the depth/time varying vector $\theta_i(s)$.

4.3. Online Parameter Estimator Design

4.3.1 Discretization of the Needle Steering Force Model

The discretized needle steering model is considered here. The procedure is shown below.

The needle steering force model can be reorganized as

$$M \ddot{\chi} + c\Pi \dot{\chi} + (k\Pi + \Pi_1)\chi = u - kH \quad (4-17)$$

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where $\chi = \begin{pmatrix} \alpha \\ q \end{pmatrix}$, $M = \begin{pmatrix} M_t & 0 \\ 0 & 1 \end{pmatrix}$, $u = \begin{pmatrix} 1 \\ \beta \end{pmatrix} F_y$, $\Pi_1 = \begin{pmatrix} 0 & 0 \\ 0 & \xi^2 \end{pmatrix}$

$$\Pi = \begin{pmatrix} v_x t & \sum \int_{L-v_x t}^L \varphi(\tilde{x}) d\tilde{x} \\ \int_{L-v_x t}^L \varphi(\tilde{x}) d\tilde{x} & \sum \int_{L-v_x t}^L \varphi^2(\tilde{x}) d\tilde{x} \end{pmatrix}, \quad H = \begin{pmatrix} -\int_0^t y_T(t_1) v_x dt_1 \\ -\int_0^t \varphi_1(v_x t_1 + L - v_x t) y_T(t_1) v_x dt_1 \end{pmatrix}$$

The measured output

$$\vec{Y}(t) = \begin{pmatrix} y_T \\ y_e \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & \varphi(L) \\ 1 & \varphi(0) \end{pmatrix}}_{\mathbf{B}} \chi = \mathbf{B} \chi \quad (4-18)$$

Here, y_T refers to the needle tip trajectory; y_e is the needle base trajectory that is measured to facilitate the computation of the system state χ .

From (4-18), we can obtain

$$\chi = \mathbf{B}^{-1} \vec{Y} \quad (4-19)$$

Substitute (4-19) into (4-17), we get

$$\mathbf{M} \mathbf{B}^{-1} \ddot{\vec{Y}} + c \mathbf{\Pi} \mathbf{B}^{-1} \dot{\vec{Y}} + (k \mathbf{\Pi} + \mathbf{\Pi}_1) \mathbf{B}^{-1} \vec{Y} = u - kH \quad (4-20)$$

Discretize (4-20) by

$$\begin{cases} \vec{Y}_k(k) \triangleq \vec{Y}(kT) \\ \dot{\vec{Y}}_k = \frac{1}{T} (\vec{Y}_{k+1} - \vec{Y}_k) \\ \ddot{\vec{Y}}_k = \frac{1}{T^2} (\vec{Y}_{k+2} - 2\vec{Y}_{k+1} + \vec{Y}_k) \end{cases} \quad (4-21)$$

with $t = kT$ (T : sampling time).

After some algebraic manipulation, we can get

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$$\vec{Y}_k = \Phi_{k-1}^T \bar{\theta} \quad (4-22)$$

where

$$\Phi_{k-1}^T = \begin{bmatrix} -T\mathbf{B}\mathbf{M}^{-1}\mathbf{\Pi}_{k-2}\mathbf{B}^{-1}(\vec{Y}_{k-1}-\vec{Y}_{k-2}) & -T^2\mathbf{B}\mathbf{M}^{-1}(\mathbf{\Pi}_{k-2}\mathbf{B}^{-1}\vec{Y}_{k-2}-H_{k-2}) & 2\vec{Y}_{k-1}-\vec{Y}_{k-2}+T^2\mathbf{B}\mathbf{M}^{-1}u_{k-2}-\mathbf{B}\mathbf{M}^{-1}\mathbf{\Pi}_k\mathbf{B}^{-1}\vec{Y}_{k-2} \end{bmatrix},$$

$\bar{\theta} = [\bar{c} \quad \bar{k} \quad 1]^T \in R^m$ is the unknown depth-varying mean parameter vector with an additional constant ‘1’, which is achieved after reorganizing the model.

To facilitate the computation of the dynamic equation, the sequence $s_0 = \{s_{0,j}\}$,

$j=1,2,\dots,n$ can be converted to the time domain using

$$t_0 = \left\{ t_{0,j} = \frac{s_{0,j}}{v_x} \right\}, \quad j=1,2,\dots,n \quad (4-23)$$

In discretized form: $k_{0,j} = \{t_{0,j} / T\}$, $j=1,2,\dots,n$ (4-24)

Substituting Equation (4-16) into (4-22), we can get the discretized model represented by the constant polynomial parameters in each segment.

$$\begin{aligned} \vec{Y}_k &= \Phi_{k-1}^T \bar{\theta} = \underbrace{\Phi_{k-1}^T \begin{bmatrix} \eta^T(k, k_{0,j}) \\ \eta^T(k, k_{0,j}) \\ 1 \end{bmatrix}}_{\mathbf{X}_{k-1}^T} \underbrace{\begin{bmatrix} A_1 \\ A_2 \\ 1 \end{bmatrix}}_{\mathbf{g}} \\ &= \mathbf{X}_{k-1}^T \mathbf{g} \end{aligned} \quad (4-25)$$

where A_1, A_2 are components of A .

4.3.2 Design of the Online Parameter Estimator

Estimation of the system parameters, given noisy input/output data, is a major field in control and signal processing. Many different estimation methods have been proposed in recent years [146-156]. Considering the noisy measurements and complex needle-tissue contact procedure, modified least square method with

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forgetting factor is adopted here to estimate the above constant polynomial parameter vector $\mathcal{G}$ in each segment. The problem here is to determine the parameter vector $\hat{\mathcal{G}}$ in such a way that the outputs computed from the system model agree as closely as possible with the system output measurements in the sense of least squares, where $\hat{\mathcal{G}}$ is the estimate of $\mathcal{G}$. The parameter vector $\hat{\mathcal{G}}$ should be chosen to minimize the least-square minimization criterion

$$\begin{aligned} V(\hat{\theta}, n) &= \frac{1}{2} \sum_{k=1}^n \lambda (\bar{Y}_k - \hat{Y}_k)^2 \\ &= \frac{1}{2} \sum_{k=1}^n \lambda (\bar{Y}_k - \mathbf{X}_{k-1}^T \hat{\mathcal{G}})^2 \end{aligned} \quad (4-26)$$

that is, $\hat{\mathcal{G}} = \arg \min_{\mathcal{G}} V(\mathcal{G})$. λ is the forgetting factor.

Thus, the modified least-square estimation with covariance resetting and forgetting factor takes the form [157]:

$$\begin{aligned} \mathcal{G}_k &= \mathcal{G}_{k-1} + K_k (\bar{Y}_k - \mathbf{X}_{k-1}^T \hat{\mathcal{G}}_{k-1}) \\ K_k &= P_{k-1} \mathbf{X}_{k-1} [\lambda I + \mathbf{X}_{k-1}^T P_{k-1} \mathbf{X}_{k-1}]^{-1} \\ P_k &= [P_{k-1} - K_k \mathbf{X}_{k-1}^T P_{k-1}] / \lambda \end{aligned} \quad (4-27)$$

where P_k and K_k are the computed covariance matrix and gain respectively at step k .

1) Selection of the forgetting factor

Forgetting factor λ is added into Equation (4-26) to attach smaller weight to older data and more weight to recent data. As a rule of thumb, λ can be decided by [153]

$$N = \frac{2}{1 - \lambda} \quad (4-28)$$

where N is the number of steps.

The adoption of the forgetting factor takes into consideration the stochastic disturbances due to the inhomogeneous human tissue and the noisy measurements,

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as well as the inaccuracy caused by the simplified model. The assumed constant polynomial parameters are actually allowed to vary in certain degree, which can give better tracking of the system output.

2) Covariance resetting

The covariance matrix P is reset at the beginning of each segment or within each segment if P becomes less than some preset value P_1 to prevent the windup phenomenon².

$$P_k = P_0, \quad \text{if } k = \{k_{o,j}\} \mid P_{k-1} < P_1 \quad (4-29)$$

3) Initial estimates resetting

At the beginning of the insertion, the initial estimation should be set based on *a priori* knowledge. During the insertion, when the needle penetrates across the segments, the initial estimation of each segment should also be set using the following Equation (4-30) to guarantee continuity of the parameters at the resetting points.

$$\hat{\mathcal{G}}_{s_{0,j+1}} = \begin{bmatrix} \psi(s_{0,j+1}, s_{0,j}) & & \\ & \psi(s_{0,j+1}, s_{0,j}) & \\ & & 1 \end{bmatrix} \hat{\mathcal{G}}_{s_{0,j}} \quad (4-30)$$

which can be transformed to the time domain

$$\hat{\mathcal{G}}_{k_{0,j+1}} = \begin{bmatrix} \psi(k_{0,j+1}, k_{0,j}) & & \\ & \psi(k_{0,j+1}, k_{0,j}) & \\ & & 1 \end{bmatrix} \hat{\mathcal{G}}_{k_{0,j}} \quad (4-31)$$

The following proof shows that the coefficients will be continuous at two adjacent segments when using Equation (4-30).

² Windup phenomenon [153]: The estimates converge to the true values very quickly. After a while, they deviate from the true values and the deviation keeps increasing. When the deviation becomes large enough, after some sharp oscillations, the estimates converge to the true values again and this cycle continues. This type of estimator windup happens when the regression vector is poorly excited.

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Proof:

Just before resetting for the $(j+1)^{th}$ segment, $s \in [s_{0,j}, s_{0,j+1})$, from Equation (4-25), we obtain

$$\hat{\theta}(s_{0,j+1}^-) = \begin{bmatrix} \eta^T(s_{0,j+1}^-, s_{0,j}) \\ \eta^T(s_{0,j+1}^-, s_{0,j}) \\ 1 \end{bmatrix} \hat{\mathcal{G}}_{s_{0,j}} \quad (4-32)$$

At the resetting point of $(i+1)^{th}$ segment, $s \in [s_{0,j+1}, s_{0,j+2})$

$$\hat{\theta}(s_{0,j+1}) = \begin{bmatrix} \eta^T(s_{0,j+1}, s_{0,j+1}) \\ \eta^T(s_{0,j+1}, s_{0,j+1}) \\ 1 \end{bmatrix} \hat{\mathcal{G}}_{s_{0,j+1}} \quad (4-33)$$

Substitute Equation (4-30) into (4-33), we get

$$\hat{\theta}(s_{0,j+1}) = \begin{bmatrix} \eta^T(s_{0,j+1}, s_{0,j+1}) \\ \eta^T(s_{0,j+1}, s_{0,j+1}) \\ 1 \end{bmatrix} \begin{bmatrix} \psi(s_{0,j+1}, s_{0,j}) \\ \psi(s_{0,j+1}, s_{0,j}) \\ 1 \end{bmatrix} \hat{\mathcal{G}}(s_{0,j}) \quad (4-34)$$

From the definition, $\eta(s_{0,j+1}, s_{0,j+1}) := [1, 0, \dots, 0]^T = e_1$

Thus,

$$\begin{aligned} \hat{\theta}(s_{0,j+1}) &= \begin{bmatrix} e_1^T \\ e_1^T \\ 1 \end{bmatrix} \begin{bmatrix} \psi(s_{0,j+1}, s_{0,j}) \\ \psi(s_{0,j+1}, s_{0,j}) \\ 1 \end{bmatrix} \hat{\mathcal{G}}(s_{0,j}) \\ &= \begin{bmatrix} \eta^T(s_{0,j+1}, s_{0,j}) \\ \eta^T(s_{0,j+1}, s_{0,j}) \\ 1 \end{bmatrix} \hat{\mathcal{G}}(s_{0,j}) \end{aligned} \quad (4-35)$$

Because

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$$\begin{bmatrix} \eta^T(s_{0,j+1}^-, s_{0,j}) \\ \eta^T(s_{0,j+1}^-, s_{0,j}) \\ 1 \end{bmatrix} = \begin{bmatrix} \eta^T(s_{0,j+1}, s_{0,j}) \\ \eta^T(s_{0,j+1}, s_{0,j}) \\ 1 \end{bmatrix} \quad (4-36)$$

From (4-32) and (4-35), we can easily obtain

$$\hat{\theta}(s_{0,j+1}^-) = \hat{\theta}(s_{0,j+1}) \quad (4-37)$$

which means that the coefficients will be continuous at two adjacent segments when using (4-30).

4.3.3 Properties of the Online Parameter Estimator

With reference to [145], we can derive the following properties for the proposed online parameter estimator.

Set $\varepsilon \triangleq Y(t) - X^T(t)\hat{A}$, $n_s^2 \triangleq X^T(t)X(t)$, we have

- i) $\varepsilon \in n_s, \hat{A}, \hat{A} \in L_\infty$, L_∞ is the Hilbert space of infinite-integrable signals.
- ii) $\varepsilon \in n_s, \hat{A} \in L_2$, L_2 is the Hilbert space of square-integrable signals.
- iii) If $n_s, X \in L_\infty$ and X satisfies the following persistence of excitation (PE) condition:

$$\alpha_1 l_0 I \geq \int_s^{s+l_0} X(x)X^T(x)dx \geq \alpha_0 l_0 I, \forall s \geq 0 \text{ and } l_0 < l$$

for some $0 < \alpha_0 < \alpha_1$, then $\hat{A}(s_0)$ converges exponentially to $A(s_0)$,

and the estimation error, $\tilde{A}(s_0)$ converges exponentially to zero.

- iv) The estimate of $\theta(s)$, $\hat{\theta}(s)$ is continuous and bounded.
- v) If X satisfies the PE condition given in (iii), the estimation error of

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$\hat{\theta}(s)$, $\tilde{\theta}(s)$ exponentially converges to zero within each depth segment.

Proof:

For i), ii) and iii):

The property i), ii) and iii) are guaranteed by the extended least square method itself, as shown in [157]. In iii), the additional constraint in the PE condition, $l_0 < l$, is necessary for the coefficients of the local polynomial approximation to converge to their true values within any depth segment.

For iv):

From Equation (4-16), the estimate of $\theta(s)$, $\hat{\theta}(s)$ is given by

$$\hat{\theta}(s) = \underbrace{\begin{bmatrix} \eta^T(s, s_0) \\ \eta^T(s, s_0) \end{bmatrix}}_{\mathbf{\Omega}(s, s_0)} \hat{A}(s_0) = \mathbf{\Omega}(s, s_0) \hat{A}(s_0) \quad (4-38)$$

Thus

$$\begin{aligned} \|\hat{\theta}(s)\| &\leq \|\mathbf{\Omega}(s, s_0)\|_2 \|\hat{A}(s_0)\| \leq \sqrt{\lambda_{\max}(\mathbf{\Omega}^T(s, s_0)\mathbf{\Omega}(s, s_0))} \|\hat{A}(s_0)\| \\ &\leq \gamma(s) \|\hat{A}(s_0)\| \end{aligned} \quad (4-39)$$

where

$$\lambda_{\max}(\mathbf{\Omega}^T(s, s_0)\mathbf{\Omega}(s, s_0)) = 1 + (s - s_0)^2 + \dots + (s - s_0)^{2p} \text{ and } \gamma(s) = \sqrt{1 + s^2 + \dots + s^{2p}}.$$

From i), $\hat{A}(s)$ is bounded. Thus $\hat{\theta}(s)$ is bounded.

Take the derivative of (4-38), we get

$$\dot{\hat{\theta}}(s) = \dot{\mathbf{\Omega}}(s, s_0) \hat{A}(s_0) + \mathbf{\Omega}(s, s_0) \dot{\hat{A}}(s_0) \quad (4-40)$$

Because $\dot{\mathbf{\Omega}}(s, s_0)$ and $\mathbf{\Omega}(s, s_0)$ are bounded within each depth segment, and from

i), $\hat{A}(s_0)$ and $\dot{\hat{A}}(s_0)$ are bounded, $\dot{\hat{\theta}}(s)$ is bounded within each depth segment. Hence,

$\hat{\theta}(s)$ is continuous within each depth segment. Recall that the continuity of $\hat{\theta}(s)$ at

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each resetting point can be guaranteed by (4-30). Therefore, it follows that $\hat{\theta}(s)$ is uniformly continuous.

For v):

Subtracting Equation (4-16) from (4-38), we get

$$\tilde{\theta}(s) = \mathbf{\Omega}(s, s_0) \tilde{A}(s_0) \quad (4-41)$$

Therefore, the estimation error $\tilde{\theta}(s)$ is bounded by

$$\|\tilde{\theta}(s)\| \leq \gamma(s) \|\tilde{A}(s_0)\| \quad (4-42)$$

From iii), we can get that $\tilde{A}(s_0)$ exponentially converges to zero, which implies that $\tilde{\theta}(s)$ exponentially converges to zero within each depth segment.

4.4 Discussion

In this chapter, depth-varying mean parameters are proposed to estimate the spring and damper effects. The adoption of ‘mean’ parameters can not only reduce the computation intensity, but also cope with the effects of the abrupt changes of the tissue properties, e.g. pathological changes of the tissue, or multi-layer insertion, and guarantee that the parameters are continuously distributed. One disadvantage of using the mean parameters is that the variation of the tissue properties cannot be clearly represented by the mean parameters, since the changes along the course will be evened into a mean value.

Directly estimation of the time or step varying parameters is complicated. Local polynomials in finite depth segments are adopted here to approximate the depth-varying mean parameters. The accuracy of the approximation depends on the order of the polynomials and the width of the segments, which can be chosen. The

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polynomial coefficients vary from one segment to the other, but within a segment they are constant. Thus, each depth-varying mean parameter can be approximated independently in each segment by a set of constant coefficients. With this approach, the estimation of time or step varying parameters can be simplified by estimating the constant coefficients instead.

Based on this approach, an online parameter estimator has been designed using modified least square method with forgetting factor. Forgetting factor is added in the equations to attach smaller weight to older data and more weight to recent data. The assumed constant polynomial parameters are actually allowed to vary in certain degree, which can give better tracking of the system output. The method that divides the whole insertion depth into several segments and adopts local polynomials to approximate the depth-varying mean parameter acts like the macro-adjustment for accurate approximations; while the use of forgetting factor in the estimation equations acts like the micro-adjustment of the approximations to adapt to the noisy procedure, so that better output estimation can be achieved.

The following chapter shows the experiment validation of the needle steering model with the assistance of the online parameter estimator. The experiment setup that is used throughout this project is described as well.

Chapter 5 Experimental Validation

This chapter presents the experimental validation of the needle steering model with the assistance of the online parameter estimator. The experiment setup that is used throughout the whole project and the relevant calibration work are presented as well.

5.1 Materials and Methods

5.1.1 Experimental Setup

The whole experimental setup and the main components are shown in Figure 5.1.

Figure 5.1 Experimental setup: (a) whole view (b) 5DOF needle probe

A 3DOF motion platform is used to drive the needle along three axes. DMC-1842 controller (from Galil Motion Control, California, USA) is installed in the computer to control the platform motion by sending commands through the software interface. A 6DOF force/torque (F/T) sensor (Nano17-SI-12-0.12 from ATI Industrial Automation, Inc., Apex, North Carolina, USA) is mounted at the needle base to measure the needle base force. The sensor has force and torque resolutions of 12.5mN and 0.0625Nmm force/torque resolution, respectively.

The needle used here is a customized 5DOF MagTrax Needle Probe (from Traxtal Technology, Canada). It is 130mm in length, has a trihedral tip, and a 19G stylette with 18G cannula. It has two sensors located at the stylet's distal tip and base respectively. The movement of the two points can be observed in real time via an electromagnetic system called Aurora® (from Northern Digital Inc. Canada). It has a Field Generator that can generate a known electromagnetic field with a characterized measurement volume of 500mm*500mm*500mm (as shown in Figure 5.2). This cubic volume is projected outwards from the Field Generator's labeled face. The measurement volume is offset by approximately 50mm from the Field Generator. When the tools with the sensor coils are exposed to this field, signals are produced inside the sensor coil. The Aurora System measures these signals to calculate the exact position and orientation of the sensor coils. This system can track the Aurora tool's movement regardless of the tool's bending and does not subject to the line-of-sight limitations that affect the optical tracking systems.

Figure 5.2 The measurement volume

Since it is an electromagnetic system, it is susceptible to surrounding metal interference. Any object that may produce magnetic field disturbances should be kept away from the Field Generator as far as possible. Also the cable of the Field Generator should be placed at a distance of 3 cm or more from the tool cables and it should not be coiled around itself or other objects, including the Field Generator. Otherwise, misleading or inaccurate transformation can occur. When the Generator is put at a high position from where the measurement volume is projected into the test field, the system is found capable of achieving better accuracy.

A Visual C++ program has been developed to track the needle positions in real time during the experiment. The GUI is shown in Figure 5.3.

Figure 5.3 GUI

5.1.2 Phantom Preparation and Calibration

The gelatin phantom molded from gelatin and water has been used a lot in this research to represent soft tissue, as shown in Figure 5.4. By varying the relative composition of gelatin powder, water and the settling time, one can produce soft elastic phantoms with elasticities (Young's Modulus) ranging from below 10kPa to well over 50kPa, a range that covers many of the body's soft elastic tissues [112]. The calibration of phantom elasticity was carried out using the "Instron" tensile machine as shown in Figure 5.5.

Figure 5.4 Pure gelatin phantom

Figure 5.5 'Instron' compression testing

Figure 5.6 Phantom testing for composition ratio 1:6, settling time 16hrs

Figure 5.6 is one of the stress-strain curves obtained from the experiment. The stress-strain curve is almost linear with strains up to 25%. The relationship is non-linear for strains greater than 25%, due to a combination of material and geometric non-linearities (e.g., local strains are not uniform throughout the sample) and the phantom was observed to begin to crack for strain greater than 25%. The initial slope (Young's Modulus) is of interest. It is approximately 6.4kPa for this particular composition. The relationship between some phantom compositions and corresponding elasticities is shown in Table A-1 (refer to Appendix A).

5.1.3 Platform Transformation and Calibration

There are two types of coordinates system in the current platform: one is the Aurora coordinate system (A), whose origin (x_a, y_a, z_a) is approximately at the center of the Generator's front panel with 50mm offset away and the directions of the three axes relative to the whole platform are shown in Figure 5.7. The needle

positions are measured in the Aurora coordinate system, which is set by the manufacturer and is hard to tell its exact position in the platform space.

The other is the 3D motion stage coordinate system (G), which is regarded as the global coordinate system. The needle's movement is controlled in this coordinate system. For easy conversion, the origin (x_b, y_b, z_b) is set at the needle distal tip, which is the needle sensor coil's position. The XYZ axes are parallel to the motion stage's three axes as shown in Figure 5.7.

Transformation from Aurora coordinate system to the global coordinate system was carried out for easy conduction of the experiment. The following section shows the procedure.

Figure 5.7 Two coordinate systems

a) Transformation Procedure

The needle was driven to several positions along the axes by sending counts (of encoder) to the motion controller. Its position data were recorded when the needle was at rest. Its corresponding coordinates in 3D motion table were calculated via

multiplying the aggregation of counts ΣC sent from the start position by the position/count ratio α . For example, in X axis, 14000 counts correspond to 10mm, so $\alpha_x=10/14000$ (mm/count); while for Y and Z axes, $\alpha_y=\alpha_z=10/5000$ (mm/count). So if the needle was moved by 28000 counts from the start point in X-axis, the resulted x coordinate would be $x=28000*10/14000=20$ mm.

In order to get the transformation matrix H, which translates the data from Aurora coordinates system (A) to global coordinates system (G), points should be chosen at the origin, X and Y axes respectively of the coordinates system A, and calculated using its coordinates in G. Because the exact positions and orientations of Aurora system are difficult to locate, so are the required A, B, C points. A more convenient way is to first find the transformation matrix H_A from system G to coordinates system A, then reverse it to find H.

Figure 5.8 Coordinates system transformation

b) Validation & Compensation

To validate the accuracy of the transformation matrix, the needle was driven to several positions and the needle tip positions were recorded. These data were first

transformed into the global coordinates system using the transformation matrix and then compared with the expected positions, which were calculated by converting the sent counts. The mean and maximum errors are shown in the Table 5-1.

To compensate the errors, which mainly came from the motion inaccuracy of 3D motion table, the noisy measurements of the Aurora system and the inaccuracy of the transformation matrix H , needle position data were sampled continuously while the needle was driven at a low velocity of 1mm/s at different offsets along the X, Y axes. Currently only a X-Y plane at certain Z position was calibrated, since the needle movement would be mainly controlled in a plane for simplicity. These original data were transformed into 3D coordinates system using the transformation matrix, and compared with the expected values. The errors (in 3D coordinates system) vs. positions (in Aurora coordinates system) are shown in the following Figure 5.9.

Figure 5.9 Error shapes

- (a) Error shape in X axis vs. positions in Y, Z axes
- (b) Error shape in Y axis vs. positions in X, Z axes
- (c) Error shape in Z axis vs. positions in X, Y axes

Note: Errors are in 3D coordinates (transformed), while positions are in Aurora coordinates system.

Table 5-1 Errors before compensation

	Mean (mm)	Max (mm)
X	1.1734	2.5189
Y	0.8922	1.6114
Z	0.1397	1.7307

The patterns shown in the Figure 5.9 indicate that the errors can be modeled by quadratic equations for x and y axes, and a linear equation for z axis. Quadratic Regression method was used to model the errors. The error model would be used to calculate the expected error after transformation, and thus compensate it.

The error model was obtained as

$$\begin{cases}
 X_{error} = 16.7913 + [3.1060 & 0.0763 & 0.0358] \begin{bmatrix} x \\ y \\ z \end{bmatrix} + [x & y & z] \begin{bmatrix} 0.0090 & 0.0100 & 0.0074 \\ 0.0100 & 0 & -0.0003 \\ 0.0074 & -0.0003 & -0.0001 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\
 Y_{error} = -3.3050 + [0.7717 & -0.0683 & -0.0311] \begin{bmatrix} x \\ y \\ z \end{bmatrix} + [x & y & z] \begin{bmatrix} -0.0370 & -0.0136 & 0.0004 \\ -0.0136 & 0.1570 & -0.0004 \\ 0.0004 & -0.0004 & 0.0002 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\
 Z_{error} = 3.8981 + [1.0000 & 0.0050 & -0.0045] \begin{bmatrix} x \\ y \\ z \end{bmatrix}
 \end{cases} \quad (5-8)$$

Additional experiment runs were carried out to validate the accuracy after compensation. The errors after compensation are shown in Figure 5.10 and Table 5-2.

Table 5-2 Errors after compensation

	X	Y	Z
Mean (mm)	0.0933	0.0987	3.1812e-013
Max (mm)	0.7755	0.5475	4.7740e-014

Figure 5.10 Error shapes after compensation

- (a) Errors in X axis
- (b) Errors in Y axis
- (c) Errors in Z axis

5.2 Experimental Validation Procedures

An ‘active’ way of validating the model by steering the needle tip to a defined position is infeasible now, since it will require a steering strategy, which is currently not available and out of this research scope. Instead a passive validation method is adopted to validate the model as shown below.

Figure 5.11 shows the experimental validation procedures of the needle steering model. The needle was driven into the prepared phantom by the 3DOF platform following some pre-determined trajectories with various insertion speeds. The needle tip/base positions and corresponding needle base force data were collected during the procedure. These collected datasets were first passed through a filter designed using 50 moving average to remove the measurement noises and smooth

the data. After that, the datasets went through the online parameter estimator to estimate the parameters $(\bar{k}_k, \bar{c}_k)$. At the same time, the model was simulated using the online estimated parameters and the collected dataset $(y_k, y_{k-1}, \dots)$ to predict the output, the needle tip position (y_{k+1}) . As shown in the Figure, the output was regulated using the collected output data $(y_k, y_{k-1}, \dots)$ instead of the simulated ones $(\hat{y}_k, \hat{y}_{k-1}, \dots)$ during the simulation. This regulation method can prevent the simulator from accumulating estimation errors, which will gradually lead to the divergence of the estimation. The simulated outputs, the needle tip position data, were compared with the measured position data during the experiments and the estimation errors went into the parameter estimator to adjust the estimations.

Figure 5.11 Experimental validation procedures

5.3 Experimental Results & Discussions

5.3.1 Experiments in Gelatin Phantom

i) Experimental results

Phantoms made of different gelatin/water ratios (1:3 and 1:4) were first adopted to simulate the soft tissue. Such phantoms are easy to obtain and their properties are easy to control and replicate. The needle was driven into the phantom for 8cm in x direction, and 2cm in y direction. The insertion speed was set to be 8mm/s, 4mm/s and 2mm/s respectively. The lateral speed was chosen accordingly in order to keep the movements in x and y to start and stop simultaneously. The results shown in the subsequent figures are one example obtained from phantom 1:3.

5th order polynomials were chosen to represent the spring and damper coefficients. The initial coefficients were set to be $[3 \times 10^5 \times \text{ones}(6,1); 2 \times 10^6 \times \text{ones}(6,1); 1]^3$. The initial covariance matrix was set to be $[10^{16} \times \text{eye}(13,12) \text{ zeros}(13,1)]^3$. The forgetting factor was selected to be 0.99. For comparison purpose, one segment was chosen first. The simulated output vs. the measured output is shown in Figure 5.12. The measurement errors and the reconstructed spring and damper coefficients using the estimated polynomial parameters are shown in Figures 5.13, 5.14 and 5.15 respectively. x and y in the Figures refer to insertion depth and lateral movement respectively. k and c are spring and damper coefficients per meter length.

To improve the tracking accuracy, two segments were then chosen. The same initial settings were used as in the one segment estimation. The simulated tip trajectories of the two segments are shown in Figures 5.16 and 5.17, respectively. The simulation errors of one segment and two segments are compared and shown

³ ones(m,n) is an m-by-n matrix of ones. eye(m,n) is an m-by-n matrix with 1's on the diagonal and 0's elsewhere. zeros(m,n) is an m-by-n matrix of zeros.

in Figure 5.18. For one segment, RMSE after convergence is 0.432mm; while for two segments RMSE after convergence is 0.218mm. We can see that the overall accuracy has been improved when using two segments.

Figure 5.12 Simulated vs. measured tip trajectories (gelatin phantom)

Figure 5.13 Simulation errors (gelatin phantom)

Figure 5.14 Estimated depth-varying mean spring coefficients (gelatin phantom)

Figure 5.15 Estimated depth-varying mean damper coefficients (gelatin phantom)

Figure 5.16 Simulated vs. measured tip trajectory in the 1st segment (gelatin phantom) Figure 5.17 Simulated vs. measured tip trajectory in the 2nd segment (gelatin phantom)

Figure 5.18 Error comparisons (gelatin phantom)

ii) Discussions

In this set of gelatin experiments, 5th order polynomials were adopted for the spring and damper coefficients. Orders lower than five have shown larger estimation errors; while orders larger than five can give better accuracy, but no significant improvement. Dividing the whole insertion depth into more segments will improve the overall tracking accuracy, but no much improvement on the convergence rate, as can be detected in Figure 5.18. The initial covariance matrix with a magnitude larger than 10^{16} showed better convergence rate, but no much improvement could be achieved especially when the magnitude is larger than 10^{18} ; while anything less than 10^{16} would give poor convergence rate and the overall accuracy suffered.

The design of the online parameter estimator guarantees that the estimated coefficients will lead to minimum errors between the measured and estimated needle positions; this means that errors due to the reduced order plant model,

needle bending out of plane, platform calibration errors and measurement noises during the procedure all will be incorporated into the parameter estimates: $\bar{k}$ and $\bar{c}$, which reflect the mean elastic and viscoelastic tissue properties along the needle insertion length. In addition, the gelatin phantom was not ideally homogeneous as expected, with bubbles and unevenly dissolved gelatin lumps inside. The area near the boundary was stiffer than the inner area due to loss of water after lying in air for near one day. Furthermore, with more and more forces exerting on the phantom by the needle, the phantom started to crack at some delicate places, which made the change of phantom properties that also introduced ‘noise’ into the estimation procedure. All these abovementioned factors resulted in the estimated mean spring and damper coefficients to be non-uniform in depth.

The large tip tracking errors at the beginning were caused by the poor initial estimation and the large sensor noises due to the sudden oscillation of the sensors when the needle was accelerated to penetrate into the phantom. The mean spring coefficient of the phantom can be roughly calculated using the method introduced in [178] and was selected as the initial estimate in the simulation. As shown in Figure 5.14, the online estimated spring coefficient was greatly biased at the beginning mainly due to the noisy sensor readings; but it finally went back to be around the calculated value with insignificant variation. The initial estimate of the damper coefficient was set randomly due to the unavailability of the prediction method and lack of prior work in the literature as a reference. The adjustment of the initial estimation has been found to be capable of decreasing the magnitude of the initial estimation errors; but cannot give much improvement on the convergence rate, which is crucial for improving the overall tracking accuracy.

Another simulation was run to test the model performance when using the constant mean estimates ($\bar{k} : 3 \times 10^5$, $\bar{c} : 8 \times 10^4$). The results are shown in the following figures. The large errors at the beginning are caused by the noisy sensor outputs and the underestimated spring coefficient (As mentioned, the area near the boundary was relatively stiffer than inner area due to lost of water.). The errors during the insertion trajectory are caused by the low mode model, needle bending out of plane, platform calibration errors and measurement noises as mentioned. The possible reason for the coincide point near 0.03m in Figure 5.19 may be that all the process noises are coincidently compensated with each other at this point. The estimation errors are plotted in Figure 5.20.

Figure 5.19 simulation using mean estimates

Figure 5.20 Simulation errors

5.3.2 Experiments in Animal Meat Phantom

i) Experimental results

The inhomogeneous animal meat (pork) has also been used in experiment to test the robustness of the proposed model simulator with the online parameter estimator. The same procedure was carried out as mentioned in the previous section. 6th order polynomials were chosen for the spring and damper coefficients. An especially poor initial estimation was set to start the simulation: $[-1 \times 10^6 \times \text{ones}(7,1); -2 \times 10^6 \times \text{ones}(7,1); 1]$, which would give negative coefficient estimations at the beginning. The initial covariance matrix was set to be $[10^{16} \times \text{eye}(13,12) \text{ zeros}(13,1)]$. Considering the noisy process and measurements, the forgetting factor was set to be 0.96. Figures 5.21 to 5.24 show the simulated output, simulation errors and the estimated spring\damper coefficients, respectively. We can see that even with poorly excited initial estimation and noisy data collection,

the estimation can still achieve satisfactory accuracy. The RMSE after convergence was 0.692mm.

Figure 5.21 Simulated vs. measured tip trajectories (animal meat)

Figure 5.22 Simulation errors (animal meat)

Figure 5.23 Estimated depth-varying mean spring coefficients (animal meat)

Figure 5.24 Estimated depth-varying mean damper coefficients (animal meat)

ii) Discussions

In this set of experiment, inhomogeneous animal tissue (pork) was adopted and poor initial estimation was set to test the robustness of the estimation method. Higher order, 6th order polynomials were selected for the coefficients. The forgetting factor was chosen to be 0.96 for the noisy process and measurements. The results clearly showed that the convergence rate was similar to that in the phantom experiment, and was not affected by the poorly excited initial estimation and noisy process/measurement errors. A method that can improve the convergence rate must be investigated.

5.3.3 Experiments in Inhomogeneous Phantom

i) Straight-line needle insertion

The biological tissue exhibits complex behaviors, particularly when combined within heterogeneous bodies. Neither the gelatin phantom nor the animal meat can authentically represent the real tissue behaviors during operation. Since in-vivo experiments are currently not available and making a real tissue – like phantom will be difficult and beyond the scope of this study, we made an especially inhomogeneous phantom with different stiffness PVC lumps, undissolved gelatin lumps and bubbles inside the gelatin solution to represent the glands, tumors, vessels and nerves, etc. (as shown in Figure 5.25), which may exist in the tissue. As long as the model can work properly in this inhomogeneous phantom under large disturbances, we can expect it to work well in the *in vivo* condition.

A straight-line needle insertion was performed first. The needle was driven into the inhomogeneous phantom for 8cm at 8mm/s, 4mm/s and 2mm/s respectively. In the

homogeneous phantom and the animal tissue phantom, we have observed that the symmetric tip needle hardly deflect during the straight-line needle insertion. While in the inhomogeneous phantom, the needle deflected due to the unbalanced force acting on the needle body, as shown in the Figure 5.26.

Figure 5.25 Inhomogeneous phantom

Figure 5.26 Needle tip trajectory in inhomogeneous phantom

The collected datasets were very noisy as shown in the above figures. To improve the estimation accuracy, four segments with 2cm per depth were selected. 6th order polynomials were chosen for the spring and damper coefficients. The initial coefficients were set to be $[3 \times 10^5 \times \text{ones}(6,1); 2 \times 10^6 \times \text{ones}(6,1); 1]$. The initial covariance matrix was set to be $[10^{18} \times \text{eye}(13,12) \text{ zeros}(13,1)]$. The forgetting factor was selected to be 0.95. The results are shown in Figures 5.27 and 5.28. The estimated spring and damper coefficients are shown in Figures 5.29 and 5.30. We can see that the model predicts the tip trajectory with satisfactory accuracy. But the estimation of the parameters is not good, for large fluctuations (even negative values) appear during the procedure. In the presence of large disturbances and noises, the proposed estimation method sacrifices the estimation of the parameters to guarantee the accuracy of the model prediction by incorporating the noises and errors into the parameter estimation, thus causing the unrealistic estimations. The RMSE after convergence is 0.176mm.

Figure 5.27 Simulated vs. measured tip trajectory (inhomogeneous phantom, straight-line insertion)

Figure 5.28 Simulation errors (inhomogeneous phantom, straight-line insertion)

Figure 5.29 Estimated depth-varying mean spring coefficients (inhomogeneous phantom, straight-line insertion)

Figure 5.30 Estimated depth-varying mean damper coefficient (inhomogeneous phantom, straight-line insertion)

ii) Needle steering with support

In the previous experiments, the performance of the model at the beginning was very poor, which has been diagnosed mainly due to the poor initial estimation and the noisy sensor output caused by the sudden oscillation of the long slender needle. An accurate initial estimation is hard to achieve especially in applications involving complex biological tissue. But the oscillation of the needle can be suppressed by adding support to the needle. Therefore, experiment was designed and carried out to test the model performance when the needle got supported during steering procedure.

In this experiment, the needle steering was actuated after the needle had been inserted into the phantom for some distance. So when the needle steering was actuated, part of the needle body had already been inside the phantom, which acted

like the support to the needle body. This procedure is more like what is performed in the real surgery: Normally straight-line needle insertion is performed at start. The needle steering will only be actuated when the needle is found deviating from its planed trajectory, when part of the needle has already been inside the tissue. The same initial settings were chosen as in part i). Figures 5.31 and 5.32 show the estimation results. Even though at the beginning the error was still large due to the initial estimation, but it converged very fast and the accuracy was greatly improved. The estimated spring and damper coefficients are shown in Figures 5.33 and 5.34. As can be depicted from the figures, the estimation of the coefficients was sacrificed again to guarantee the output accuracy. The RMSE after convergence is 0.121mm.

Figure 5.31 Simulated vs. measured tip trajectory (inhomogeneous phantom, with support)

Figure 5.32 Simulation errors (inhomogeneous phantom, with support)

Figure 5.33 Estimated depth-varying mean spring coefficients (inhomogeneous phantom, with support)

Figure 5.34 Estimated depth-varying mean damper coefficient (inhomogeneous phantom, with support)

iii) Discussions

In this set of experiment, experiments were carried out to further test the robustness of the model performance in an especially inhomogeneous phantom, which would give large process noises. Different trajectories were chosen for the needle insertion. The needle was inserted following a straight line without applying the steering force. The measured forces and tip/base positions were used for the estimation. It was found that in this inhomogeneous phantom, the tip would deflect due to the unbalanced forces acting on the needle body. The output estimation has shown satisfactory results. But the initial estimation was poor and no significant improvement can be made to the convergence rate by changing the initial settings.

The low convergence rate was suspected to be caused by the sudden oscillation of

the slender needle at start, which would give erratic sensors outputs. Thus another set of experiments was designed and carried out to test the model behavior when the needle got supported. The phantom itself was made use of to support the needle, which can decrease the needle oscillation, thus giving better sensor outputs. This simple but effective method has proved to greatly improve the convergence rate and the whole estimation accuracy.

5.3.4 Experiments in Two-layer Phantoms

i) Needle steering without support

In the actual surgery, the needle may need to penetrate through several layers before reaching the target position. So in this experiment, a two-layer phantom was made to simulate this kind of terrain. The first layer was made of inhomogeneous gelatin phantom as shown in the previous section; while the second layer was made by the animal meat (pork). The thickness of the two layers was both around 4cm.

Figure 5.35 Two-layer phantom. First layer: inhomogeneous phantom with thickness around 4cm.
Second layer: animal meat with thickness around 4cm.

Two segments have been chosen corresponding to the two layers. The initial settings were chosen as the same as in section 5.3.3. The simulated tip trajectories vs. the measured ones in these two segments and corresponding errors are shown in Figures 5.36-5.39. The estimated spring and damper coefficients are shown in Figures 5.40 and 5.41. As shown in the figures, the estimations of the coefficients are greatly distorted near the joint of the two layers due to the noisy sensor output, needle bending out of plane, etc. But the output estimation is satisfactory. The RMSE of the tracking errors after convergence is 0.464mm.

Figure 5.36 Simulated vs. measured tip trajectory in the 1st segment (two-layer phantom, without support)

Figure 5.37 Simulation errors in the 1st segment (two-layer phantom, without support) Figure 5.38 Simulated vs. measured tip trajectory in the 2nd segment (two-layer phantom, without support)

Figure 5.39 Simulation errors in the 2nd segment (two-layer phantom, without support)

Figure 5.40 Estimated depth-varying mean spring coefficients (two-layer phantom, without support)

Figure 5.41 Estimated depth-varying mean damper coefficient (two-layer phantom, without support)

ii) Needle steering with support

Needle steering with tissue support has also been tested in the two-layer phantom. The same initial settings were chosen as in previous section. The results are shown in Figures 5.42 to 5.45. As shown in the figures, the convergence rate and estimation accuracy have been greatly improved when the needle got supported. The RMSE of the tracking errors after convergence is 0.102mm.

Figure 5.42 Simulated vs. measured tip trajectory in the 2nd segment (two-layer phantom, with support)

Figure 5.43 Simulation errors in the 2nd segment (two-layer phantom, with support)

Figure 5.44 Estimated depth-varying mean spring coefficients (two-layer phantom, with support)

Figure 5.45 Estimated depth-varying mean damper coefficient (two-layer phantom, with support)

iii) Discussions

Needle steering with/without support was carried out in the two-layer phantom to simulate the situation encountered in the real surgery. The estimated parameters have shown to be continuously distributed along the whole trajectory regardless the disturbances and the different layers, after adopting the ‘mean’ concept. It has been further proved in the two-layer phantom that the convergence rate and overall estimation accuracy can be greatly improved when the needle gets supported.

5.4. Conclusions

In this chapter, the experimental setup that is used throughout this project and its corresponding calibration procedures are discussed first. Experimental validation of the needle steering model based on the online measurements in various types of phantoms is then presented. Results have shown that the model can track the needle tip trajectory with RMSE less than 1mm after convergence even in the presence of large disturbances and noises. The estimation errors at the beginning are large due to the poor initial estimation and the noisy sensor output caused by the sudden oscillation of the long slender needle. To solve this problem, experiments with the ‘supported’ needle were carried out by making use of the tissue as the support. Results have shown that this simple but effective method can greatly improve the convergence rate and overall accuracy.

Experiments in this chapter have shown that the proposed polynomial approximation and online micro-adjustment of the coefficients can give the model more flexibility and compatibility to the uncertainties and variations of the tissue properties in real clinical or surgical applications. In addition, some unmodeled tissue behaviors, e.g. tissue translation or even rotation (e.g. when the needle

punctures the elastic capsule of the prostate) can be better accommodated by adjusting the parameters.

The parameter estimation may be greatly biased as shown in the previous experiments to guarantee the tip tracking accuracy, which is our final objective. One way to improve the parameter estimation accuracy is to reduce the noises. For example, use a high-mode model to improve the model accuracy, and adopt a more effective noise filter to remove the sensor noises. To include more modes in the model will need more sensors to observe the displacements at certain points along the needle body, for the system equations to be solvable. Currently, only two sensors are inside the needle. Thus a very low mode model that only includes the first primary mode is used, which causes great noises in the estimation procedure.

The proposed model and online parameter estimation method can be used to design an adaptive needle steering system using adaptive control algorithm. This steering system will be actuated when the needle is found to deflect from the planned trajectory; or if one wants to steer the needle path to avoid some obstacles (e.g. public bone), and reach some areas that are currently unreachable under straight line needle insertion. The accuracy of the steering system will depend not only on the needle tip prediction accuracy, which has been proved to be effective using the proposed model and estimation method, but also on other factors, e.g. the accuracy of the imaging system. This will be left for future investigation.

In the next chapter, a novel approach, high frequency translational oscillation, is proposed to deal with the target movement that is also a very important factor affecting accurate needle placement.

Chapter 6 Target Movement Reduction Methodology

6.1 Introduction

Another problems affecting needling accuracy is the target movement caused by tissue deformation. Target movement cannot be prevented due to the soft tissue nature. It is also difficult to track it via image modalities like ultrasound probe or C-arm/CT scanner, for the former's noisy images and the latter's harmful radiation to the patient and the surgeon. Furthermore, real time tracking will be another impediment when adopting image modalities. A more practical way is to reduce the target movement as much as possible via some feasible physical methods.

This chapter proposes a novel approach, high frequency ($>2\text{kHz}$) translational oscillation of the needle to reduce target movement. The proposed method is based on a hypothesis that for a translating needle operating in an oscillatory mode (back and forth motion), target movement can be reduced. One possible reason for this hypothesis is that an oscillating needle has a tendency to allow the test subject to return elastically to its original shape between pushes of the needle; thus reducing its deformation. This kind of translational oscillation has been studied in a low frequency band (10Hz –30Hz) [158]. Experiments results showed that oscillating the needle could reduce the peak force up to 60%, and the surface deformation of a soft tissue (chicken breast) around 14%. Yet the low frequency oscillation has been observed to cause the long, slender needle to vibrate randomly and also interfere with the needle tip position tracking during the process. In the literature, high frequency oscillation has been adopted in hard metal machining (ultrasonic machining) [159] and in bone surgery (Ultrasonic scalpels) [160] for its abilities to reduce the drilling or cutting force and decrease the tissue trauma and adhesion. So

in this research, the high frequency band oscillation (from 2kHz to 20kHz) that has negligible displacement is selected for investigation [167].

This chapter focuses on proving that high frequency translational oscillation can effectively reduce the target movement. The influential factors are investigated as well. Another possible target movement reduction method, rotational drilling of the needle, is also shortly discussed in this chapter.

6.2 Material and Method

6.2.1 Equipments for Translational Oscillation

The equipments for generating and measuring the translational oscillation are shown in Figure 6.1.

Figure 6.1 Equipments for oscillation

An oscillation system named ‘Minishaker’ (from Gearing & Watson Electronics Limited, UK) is used to generate the needle’s translational oscillation. The

adjustable frequency range of the ‘Minishaker’ is from 2Hz to 20kHz; while the maximum amplitude is decided according to the chosen frequency, under the constraint that the maximum force the power amplifier can generate is 17.8N. The oscillation frequency and amplitude can be adjusted by tuning the knobs on the power amplifier. The accelerometer is fixed on one side of the fixture near the needle base to measure the needle’s oscillating frequency and amplitude, and then display them through sine curves on the oscilloscope.

Two 5DOF Aurora tools are adopted in this experiment as shown in Figure 6.2. One is the 5DOF MagTrax Needle Probe. The other is a light-weighted catheter that will be inserted into the phantom as the target during the experiments, as shown in Figure 6.3. It also has one sensor coil near the distal end. Thus its movement can be observed through ‘Aurora’ system.

Figure 6.2 5DOF Aurora tools

Figure 6.3 Catheter in the phantom as the target

6.2.2 Experimental Design Method – 2^k Factorial Design

Factorial designs are widely used in experiments involving several factors where it is necessary to study the joint effect of these factors on a response. There are several special cases of the general factorial design that are important because they are widely used in research work, and also because they form the basis of other designs of considerable practical value.

One of these special cases is that of k factors, each at only two levels. These levels can always be represented by using plus and minus signs, or '1' and '-1'. These levels may be quantitative, such as two values of temperature, pressure, or time; or they may be qualitative, such as two machines, two operators, the 'high' and 'low' levels of a factor, or perhaps the presence and absence of a factor. A complete replicate of such a design requires 2^k observations, and is called a 2^k factorial design [161]. The 2^k design provides the smallest number of treatment combinations with which k factors can be studied in a complete factorial arrangement; thus it is particularly efficient in investigating the factor effects to the

response. The 2^k design has been adopted in this research to study the oscillation effect and investigate the influential factors.

In this set of experiments, the effect of translational oscillation is of primary interests. Factors like the phantom properties, needle insertion speed, oscillation frequency and amplitude will influence target movement. Table A-2 (refer to Appendix A) shows the relating factors with their two levels. The low and high levels for the oscillation frequency are chosen to be 2kPa to 20kPa respectively, to avoid needle random shaking. The two levels of oscillation amplitude are set accordingly. The two levels of the translation velocity are chosen to be 0.4mm/s and 10mm/s respectively, which is the range of clinical needle insertion velocity [1]. Gelatin phantoms with composition ratio 1:2 and 1:6, and fixed configuration time 20hrs are chosen to represent the two levels of the tissue elastic properties in both the one-layer phantom experiment and the two-layer phantom experiment. The phantom factor is treated as the block factor. Each block holds 32 experimental runs, 16 for oscillation and 16 for without oscillation. The detailed experimental plan is shown in Table A-3 (refer to Appendix A). Randomization and replication are applied to exclude noises from the operator faults or equipments etc.

Figure 6.4 Experimental procedure

A systematic procedure for experimentally observing and characterizing translational oscillation effects on target movement is described in Figure 6.4. The needle is driven into the prepared phantom following the designed experimental plan. The target movement during the procedure is tracked by the ‘Aurora’ system. Experimental measurements are used to analyze the oscillation effect by comparing the target movement with/without translational oscillation, and investigate the influential factors.

6.2.3 Experimental Analysis Method – Exploratory Data Analysis (EDA)

Approach

Target movements during the needle penetration process are calculated using

$$\Delta P = \sqrt{(\overline{X}_e - \overline{X}_s)^2 + (\overline{Y}_e - \overline{Y}_s)^2 + (\overline{Z}_e - \overline{Z}_s)^2} \quad (6-1)$$

ΔP represents the target moving distance. $\overline{X}_s$, $\overline{Y}_s$ and $\overline{Z}_s$ are the original positions of the target, while $\overline{X}_e$, $\overline{Y}_e$ and $\overline{Z}_e$ are the final positions of the target when the needle stops insertion. Then target movement with/ without oscillation can be compared to investigate the oscillation effect.

A graphical method, exploratory data analysis (EDA) approach, is adopted for its straightforward graphical illustration. Figure 6.5 shows one example of using EDA approach to analyze oscillation effect from the DEX (Design of Experiments) Mean plot.

In the DEX Mean plot, the horizontal axis is the ‘oscillation’ factor with the two levels (‘-1’ refers to needle movement ‘without oscillation’ and ‘1’ refers to needle movement with oscillation); while the vertical axis corresponds to the mean target movement distance. As shown in the graph, the target movement with oscillation is

less than without oscillation, which means that oscillation can reduce target movement.

Figure 6.5 DEX mean plot for oscillation effect analysis

Figure 6.6 DEX mean plot for factor effect analysis

Figure 6.6 gives another example of using EDA approach to investigate the factor effects on target movement. As can be detected from the plot, the two levels of the

speed factor produce the largest difference in target movement. So it is the most important factor for target movement reduction. The other two factors, frequency and amplitude, are less important: frequency is the second influential factor and amplitude has the least effects on target movement reduction. In order to achieve smaller target movement, the factor settings, lower frequency (-1), larger amplitude (1) and lower speed (-1) should be considered.

6.3 Experimental Results

6.3.1 Experiment in One-layer Pure Gelatin Phantoms

One-layer gelatin phantoms were first chosen for experiment. Totally 224 experiments were carried out following the design shown in Table A-2 and A-3 in Appendix A to study the oscillation effect. The results are shown in Figure 6.7.

From these two plots, one can clearly see that target movement with oscillation is less than that without oscillation in both types of phantoms; that means, translational oscillation can reduce target movement. The target movement reduction rate is also analyzed statistically and shown in Table 6-1.

(a)

Figure 6.7 DEX mean plot a) in softer phantom b) in stiffer phantom

Table 6-1 Target movement reduction rate

	Softer Phantom (128 runs)			Stiffer Phantom (96 runs)			Overall (224 runs)		
	With Osci. (mm)	Without Osci. (mm)	Red. (%)	With Osci. (mm)	Without Osci. (mm)	Red. (%)	With Osci. (mm)	Without Osci. (mm)	Red. (%)
RMS	1.05	1.33	21	1.06	1.68	37	1.06	1.49	29
Max	0.53	1.19	55	0.58	1.17	50	0.53	1.19	55

6.3.2 Experiments in Two-layer Phantoms

Experiments were also carried out in two-layer phantoms to simulate the multiple layers that the needle would penetrate through during prostate operation. The two layers were composed by the gelatin phantom and chicken breast. Before the experiment, gelatin mixtures of different elasticities (two levels are chosen as in the above section) were made to mimic surrounding soft tissue. Then the chicken breast, which was used to simulate the prostate organ, was put into the solution immediately after it cooled down. The mixture was then left to solidify for around 20hrs.

Figure 6.8 Gelatin phantom with chicken breast

Experiment was carried out using the same plan sheet as shown in Table A-1 and A-2. Figure 6.9 shows the results. From the plots, we can draw the same conclusion, that is, translational oscillation can reduce target movement. The reduction rate was also analyzed using statistical method as shown in Table 6-2.

(b)

Figure 6.9 DEX mean plot with chicken breast hosted in a) stiffer phantom b) softer phantom

Table 6-2 Reduction rate (two-layer phantom)

	Organ with Softer Phantom (80 runs)			Organ with Stiffer Phantom (64 runs)			Overall (144 runs)		
	With Osci. (mm)	Without Osci. (mm)	Red. (%)	With Osci. (mm)	Without Osci. (mm)	Red. (%)	With Osci. (mm)	Without Osci. (mm)	Red. (%)
RMS	1.47	1.98	25	1.4	2.5	44	1.48	2.16	31
Max	1.2	2.9	59	1.4	2.80	50	1.2	2.9	59

6.3.3 Investigation of Factors Effects

Figure 6.10 DEX mean plot (factor effect) a) in softer phantom b) in stiffer phantom

Experiments in gelatin phantoms and two-layer phantoms have proved that translational oscillation can effectively reduce target movement up to near 60%. The reduction rate is related to many factors such as tissue properties, insertion speed, oscillation frequency and amplitude. As can be detected from Figure 6.10, in softer phantoms, the three influential factors seem to have similar importance on

the target movement. Lower frequency, higher speed and smaller amplitude can lead to smaller target movement. While in stiffer phantoms, insertion speed becomes the most influential factor. Lower frequency, lower speed and larger amplitude will produce smaller target movement. The change of optimal settings in different types of phantom suggests that interactions between the decision factors exist. This can be detected from Figure 6.11, the interaction effect matrix plot.

Figure 6.11 DEX interaction effect main plot (x1:oscillation frequency, x2:oscillation amplitude, x3: speed, x4: phantom property)

The interaction effects matrix plot is an upper right-triangular matrix of mean plots consisting of k main effects plots on the diagonal and $k*(k-1)/2$ 2-factor interaction effects plots on the off-diagonal. All the mean plots, for both main effects and 2-factor interactions, have a common scale to facilitate comparisons. Each mean plot has

1. Vertical Axis: The mean response for a given setting (- or +) of a given

- factor or a given 2-factor interaction.
2. Horizontal Axis: The 2 settings (- and +) within each factor, or within each 2-factor interaction.
 3. Legend: The least squares estimate of the factor or 2-factor interaction effect (scaled). These effect estimates are large in magnitude for important factors and near zero in magnitude for unimportant factors.

From this plot, one can see that the interaction between speed and phantom property ($x_3 \times x_4$), amplitude and speed ($x_2 \times x_3$) are important. This can also be clearly illustrated by the following statistical analysis as shown in Table 6-3.

Table 6-3 ANOVA table

ANOVA: Tests of factor-Interaction Effects

Dependent Variable: TargetMovement

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Model	87.860 ^a	11	7.987	22.571	.000
Frequency * Amplitude	.001	1	.001	.003	.956
Frequency * Velocity	.059	1	.059	.167	.684
Frequency * Phantom	.026	1	.026	.073	.788
Amplitude * Velocity	2.448	1	2.448	6.917	.010
Amplitude * Phantom	.196	1	.196	.555	.459
Velocity * Phantom	2.666	1	2.666	7.535	.007
Error	28.664	81	.354		
Total	116.524	92			

a. R Squared = .754 (Adjusted R Squared = .721)

In cases where multi-layer tissues of different properties are involved in the needle insertion, the tissue in which the target is embedded will play a central role in deciding the factor setting. This conclusion can be drawn from Figure 6.12. As can be detected from the plot, the factor settings for smaller target movement in the two-layer phantoms are higher frequency, higher speed and smaller amplitude regardless what the surrounding tissue is, even though the relative importance of

each influential factor to the reduction rate is different due to the outside layer interaction.

Figure 6.12 DEX mean plot (factor effect): chicken breast hosted in a) in stiffer phantom b) in softer phantom

6.3.4 Discussion

Experimental results have shown that high frequency oscillation can reduce the target movement. The reduction rate is related to many factors such as tissue

properties, insertion speed, oscillation frequency and amplitude. An appropriate setting of the factors can result in a reduction rate up to 60%. As shown in previous section, different tissues may require different optimal settings to achieve the minimum target movement. Thus at current stage, only the general trends for the more effective parameters are studied, instead of the exact optimal settings. In future, *in-vivo* or at least *ex-vivo* experiments should be carried out to determine the most effective factor setting for a specific operation, e.g. prostate surgery.

6.4 Investigation of Rotational Drilling Effects

The rotation of the needle during insertion is expected to have the potential of reducing the target movement, for the turning effect of the needle can reduce the forward thrust caused by the tip of the needle and thus, reduce the displacement of the target. Dr. T.K.Podder etc. [86] has studied the rotational oscillation and rotational drilling effects. Their conclusion is low frequency rotational oscillations (less than 5Hz) cannot reduce target movement. The higher frequency oscillation was supposed to be more effective, but they did not do any experimental validation due to hardware limitations. Full rotational drilling effects had also been tested at a rotational speed 60rpm, 300rpm and 600rpm respectively. It was concluded that for each insertion speed, there might exist a threshold of the rotational velocity, larger than which rotating the needle would reduce target movement.

A final year project student Kiong Chee Hung [165] in our lab carried out some preliminary experiments to investigate the rotational drilling effects using the setup as shown in Figure 6.13. The rotary motor shown in the figure was mounted on the 3D motion platform replacing the Minishaker to produce the rotary effect. The rotational speed of the motor was selected to be 3000rpm and 1600rpm

respectively. A tachometer was used to detect the rotational speed of the needle mount in relation to the voltage supply from the power supply. The two levels of the needle insertion speed were set to be 10mm/s and 0.4mm/s, respectively. Experiment was carried out in gelatin phantoms of 1:5 to test the rotational drilling effect. Results show that it can reduce the target movement from 67% to 84%.

One issue involved in adopting the rotational drilling on our current biopsy robot is the wire-wrapping problem. That is, when rotating the needle, the wire of the force sensor or of the tracking needle will wrap around the rotating shaft. This phenomenon has been observed in both of the abovementioned works. Kiong Chee Hung avoided this problem by adopting the normal brachytherapy needle instead of the trackable needle; while Dr. T.K. Podder mentioned that they could not take the force/torque and target movement data (when needle was rotated) simultaneously. Instead the experiments were done identically with same composition and time elapse of the phantom materials.

Figure 6.13 Motor and needle mounting for rotational drilling experiment

Since in our proposed steering system, online force and position measurements will be needed for real time needle steering, so we cannot remove the force sensor

and trackable needle from the setup to avoid wire-wrapping phenomenon. Thus using high frequency translational oscillation to reduce target movement is an appropriate solution at this stage.

6.5 Discussion

This chapter investigates and proves the hypothesis that high frequency translational oscillation can effectively reduce target movement. High frequency oscillation is chosen due to its indiscernible oscillation displacement. Thus it is expected to have neglected interference with the insertion movement, compared with the low frequency oscillation. The reduction rate has found to be influenced by many factors such as tissue properties, insertion speed, oscillation frequency and amplitude. To achieve smaller target movement, different types of tissue will require different factor settings due to the factor interaction. In cases when multi-layer tissues of different properties are involved in the needle insertion, the tissue in which the target is embedded will play a central role in deciding the optimal factor setting.

Current research work only focuses on proving that high frequency translational oscillation can effectively reduce target movement, and investigating the influential factors. In future, *in vivo* or at least *ex vivo* experiments should be carried out to find the optimal factor setting for a specific operation. When integrating such oscillation system into the needle steering system, factors such as technical simplicity, size and weight of the oscillator etc. should all be considered. Compromise may be made between the maximum target movement reduction and technical simplicity.

The rotational drilling has also been shortly discussed in this chapter. Even though it may be a promising target reduction method, the wire wrapping cannot be avoided in current biopsy robot. Thus it will cause a big trouble in adopting it into the steering system. In future, we will carry out more experiments to study rotational drilling effects. At the same time, we may consider to modify the hardware design to avoid the wire-wrapping problem, if it is proven to be highly effective.

Chapter 7 Conclusions and Future Works

7.1 Conclusions

Needle steering has aroused a lot of research interests in recent years. It has the potential to improve targeting accuracy and guide the needle to some areas that are currently not amenable to needles. In this work, a base steering method is proposed to steer the needle during insertion. To achieve this goal, a needle-tissue interaction model that can precisely and efficiently predict the needle tip trajectory with given input forces is crucial for the future control system design. In this thesis, a spring-beam-damper model is proposed to describe the system dynamics between the lateral steering forces acting at the needle base and the corresponding needle tip lateral movement during insertion into the soft tissue. The proposed spring-beam-damper model takes into consideration not only the elastic/viscoelastic tissue reactions, but also needle flexibility and their interaction effects.

For the spring-beam-damper model, the setting of the spring and damper parameters is found to be important for the model convergence. A rough estimation can cause the overshooting or oscillation of the needle tip position, as observed from past simulations. Considering tissue inhomogeneity and computational efficiency, depth-varying mean parameters are first proposed to calculate the tissue reaction forces. Then an online parameter estimation method is designed using modified least square method with forgetting factor. This estimation method guarantees that the estimated model output can closely track the real measured one. Extensive experiments have been carried out in gelatin phantoms, animal tissues, inhomogeneous phantoms and multi-layer phantoms to validate the needle steering model with the online parameter estimator. Results

have shown that the model can track the needle tip trajectory with RSME less than 1mm after convergence even in the presence of large disturbances and noises, but the estimation of the parameters cannot be guaranteed especially when large process errors (caused by the unmodeled system dynamics, calibration errors, or needle bending out of plane etc.) or sensor noises exist. The estimation errors at the beginning are large due to the poor initial estimation and the noisy sensor output caused by the sudden oscillation of the long slender needle. This can be greatly improved by adding support to the needle. In this work, the tissue itself is made use of to support the needle. Experimental results have shown that this simple but effective method can greatly improve the convergence rate and overall estimation accuracy.

Compared with other proposed steering methods in the literature, e.g. rotating a highly flexible bevel-tip slender needles to change its course [104], or using a specially-made steering needle with curved tip for steering [103], the proposed base steering method does not have any special requirements on the needle or environment. It is more straightforward and expected to have more generalized applications. In addition, it is computationally efficient compared with those steering models built on FEM (e.g. [1]). Thus it is suitable for real time application. The only concern with this method is that the force acting at the base may not be strong enough to steer the needle especially when it is deep inside the tissue. As an alternative, insertion point control can be considered. The current method can be easily extended to insertion point control, as will shown in section 7.2.3.

To reduce the target movement during needle insertion, a novel approach, high frequency translational oscillation ($>2\text{kHz}$) is investigated. This method can be

used in situations where real-time tracking of the target movement is not available or infeasible. Experimental results have shown that high frequency translational oscillation can effectively reduce target movement. The reduction rate is related to many factors such as tissue properties, insertion speed, oscillation frequency and amplitude. The importance of the influential factors to the target movement reduction is also discussed for different types of tissue-like phantoms. It has been shown that different types of tissue will require different factor settings to achieve smaller target movement. In cases when multi-layer tissues of different properties are involved in the needle insertion, the tissue in which the target is embedded will play a central role in deciding the factor setting. The investigation of the optimal factors for a specific operation will be our future task.

The specific contributions of this thesis are summarized as follows.

Needle steering force model: A spring-beam-damper model that describes the dynamics between the needle base lateral force and the needle tip position has been proposed for the lateral needle steering. The proposed steering model takes into consideration not only the elastic tissue reactions, but also the viscoelastic tissue effects that have been largely ignored in the literature. Instead of using the traditional Finite Element Method (FEM) that has been adopted by many other researchers in solving problems involving the deformable tissue [1,66-78], a computationally efficient method, unconstrained model analysis, is adopted to solve the system dynamics. The only computational expensive parts in the model are the space-varying integrals, $\int_{L-v_x t}^L \varphi_j(\tilde{x}) d\tilde{x}$ and $\int_{L-v_x t}^L \varphi_i(\tilde{x}) \varphi_j(\tilde{x}) d\tilde{x}$, which can be pre-computed once and for all with the usage of depth varying mean parameters in

this work. The simulation can be achieved in just a few seconds after directly adopting the pre-computed integrals.

Control or vibration analysis of a flexible beam under static or dynamic moving loads has been studied by many researchers (e.g. [174-176]). In their applications, the loads are normally known or with certain degree of bounded uncertainty, and the contacts between the loads and beams are point-contacts with known distribution. The model proposed in this thesis is intended for applications that involve time-changing continuously distributed loads, whose magnitudes are dependent on the real time needle deflection. Both the needle flexibility and the tissue deformation are decisive in the model behaviors, and at the same time, interacting with each other.

Online parameter estimation: An effective online parameter estimation method has been proposed in this thesis to estimate the spring and damper coefficients in the model. The proposed method can be applied to other applications that have time/space varying parameters, involve large disturbances and noises in the process, or have no effective physical measurement methods available for online estimation.

- Considering the tissue inhomogeneity, depth-varying mean parameters are proposed to calculate the spring and damper effects. The adoption of ‘mean’ parameters can not only reduce the computational intensity, but also cope with the effects of the abrupt changes of the tissue properties, e.g. pathological changes of the tissue, or multi-layer insertion, and guarantee that the parameters are continuously distributed.

- Local polynomials in finite depth segments are used to approximate the unknown depth-varying mean parameters. The accuracy of the approximation depends on the order of the polynomials and the width of the segments, which can be adjusted. The polynomial coefficients vary from one segment to the other, but within a segment they are constant. Thus, each depth-varying parameter can be approximated independently in each segment by a set of constant coefficients. This local polynomial method is more data-centric. More accurate and realistic estimations can be obtained based on this method.
- An online parameter estimator has been designed using modified least square method with forgetting factor to estimate the constant polynomial coefficients. The depth-varying mean parameters can then be reconstructed based on these estimates. Forgetting factor is added in the equations to attach smaller weight to older data and more weight to recent data. The assumed constant polynomial parameters are actually allowed to vary in certain degree to adapt to the noisy measurements, which can give better tracking of the system output. The method that divides the whole insertion depth into several segments and adopts local polynomials to approximate the depth-varying mean parameter acts like the macro-adjustment for accurate approximations; while the use of forgetting factor in the estimation equations acts like the micro-adjustment of the approximations to adapt to the noisy measurements, so that better output estimation can be achieved. The inhomogeneous tissue properties, the disturbances and noises involved in the whole process are fully taken into consideration.

Target movement reduction method: A novel approach, high frequency translational oscillation (>2kHz), is proposed to reduce the target movement during needle insertion. Prior works in the literature focus on either predicting the target movement by FE modeling that is computational expensive, or adopting imaging modalities (e.g. CT, Ultrasound, etc) to track the target movement, whose performances are highly dependent on the quality of the images. In addition, image segmentation will pose another problem for real time tracking. The target reduction method proposed in this thesis is simple and effective. Off-the-shelf products are already available to be integrated in the robotic system to generate the high frequency translational oscillation.

7.2 Future works

Some aspects of future works are addressed as follows.

7.2.1 3D Model

The current steering force model is developed in 2D space (x-y) for simplicity, which can easily be extended to 3D by superposition under the assumption that the coupling effects (nonlinear interactions) between the lateral movements can be neglected. The needle lateral deflection in the 3D space can be decomposed into y and z directions, and the tissue reaction forces can be represented by springs and dampers in the specific axis. Thus the 3D model can be obtained as follows.

$$\begin{aligned} & \dot{\alpha}(t) \int_{L-v_x t}^L (c_y + c_z) \varphi_i(\tilde{x}) d\tilde{x} + \alpha(t) \int_{L-v_x t}^L (k_y + k_z) \varphi_i(\tilde{x}) d\tilde{x} + \ddot{q}_i(t) + \sum_j \dot{q}_j(t) \int_{L-v_x t}^L (c_y + c_z) \varphi_i(\tilde{x}) \varphi_j(\tilde{x}) d\tilde{x} \\ & + \sum_j q_j(t) \int_{L-v_x t}^L (k_y + k_z) \varphi_i(\tilde{x}) \varphi_j(\tilde{x}) d\tilde{x} + \xi_i^2 q_i(t) = (F_y + F_z) \beta_i + f_{li} \end{aligned} \quad (7-1)$$

$$\begin{aligned}
 & M_t \ddot{\alpha}(t) + (c_y + c_z) v_x \dot{\alpha}(t) + \sum_j \dot{q}_j(t) \int_{L-v_x t}^L (c_y + c_z) \varphi_j(\tilde{x}) d\tilde{x} + \sum_j q_j(t) \int_{L-v_x t}^L (k_y + k_z) \varphi_j(\tilde{x}) d\tilde{x} + (k_y + k_z) v_x \alpha(t) t \\
 & = F_y + F_z + f_2
 \end{aligned} \tag{7-2}$$

with

$$\begin{aligned}
 f_{li} &= \int_0^t (k_y + k_z) \alpha(t_1) \varphi_i(v_x t_1 + L - v_x t) v_x dt_1 + \int_0^t (k_y + k_z) \sum_j \varphi_j(L) q_j(t_1) \varphi_i(v_x t_1 + L - v_x t) v_x dt_1 \\
 &= \int_0^t (k_y + k_z) Y(t_1) \varphi_i(v_x t_1 + L - v_x t) v_x dt_1 \\
 f_2 &= \int_0^t (k_y + k_z) \alpha(t_1) v_x dt_1 + \int_0^t (k_y + k_z) \sum_j \varphi_j(L) q_j(t_1) v_x dt_1 = \int_0^t (k_y + k_z) Y(t_1) v_x dt_1
 \end{aligned}$$

Note the current model is built under an assumption that the needle inserts at constant velocity to facilitate and simplify the modeling procedure. Under this assumption, the forces in the insertion direction will be equivalent. Thus they are not included in this model. In a situation where a variable velocity will apply, insertion force model must be considered.

7.2.2 Adaptive Needle Steering System

Considering the unknown, space/time varying spring and damper coefficients involved in the system model, adaptive control can be adopted for future controller design. An adaptive controller is formed by combining an online parameter estimator, which provides estimates of unknown parameters at each time instant, with a control law that is motivated from the known parameter case.

There are two schemes for the adjustment as discussed below.

Direct Scheme: The plant model is parameterized in terms of the controller parameters that are estimated directly without intermediate calculations involving plant parameter estimates. This approach is referred to as implicit adaptive control.

In this scheme, the control law must be chosen first. Based on the control law, the

controller parameters are re-parameterized in the linear regression form and thus estimated online. It is simple as no intermediate calculation of controller parameters is required, but inflexible to choose control law.

Indirect Scheme: The plant parameters are estimated online and used to calculate the controller parameters. This approach has also been referred to as explicit adaptive control.

The advantage of using the direct structure is potentially more efficient in the implementation as the design block can be rather computationally demanding. However not all control algorithms can be parameterized for use in the direct structure. In comparison, the indirect structure is more flexible in implementing different controller algorithms and can be considered as a potential choice for the needle steering controller design.

The indirect adaptive control system can be illustrated using the following block diagram. The proposed online parameter estimator can be adopted in designing the indirect adaptive control system.

Figure 7.1 Diagram of indirect adaptive control system

The needle steering system is originally designed to be used in a fully automatic

system, which may arouse a lot issues under current practice guidelines. In a situation where a fully automatic system is not permitted, a semi-automatic system can be considered, which means, that the surgeon will control the insertion operation with the guide of the robot, while the steering system control the lateral movement that is not too aggressive (normally a few mm) compared with insertion movement. In this case, the interactions between the surgeon and the steering system should also be taken into consideration. The dynamic model should be extended to use variable velocity and include insertion forces as mentioned in previous section. In addition, if high frequency translational oscillation will be adopted in the system, the interactions between the oscillation and steering forces should be investigated too, even though high frequency oscillation is expected to have neglectable effects.

7.2.3 Insertion Point Control

The proposed needle steering strategy is the needle base steering, that is, applying force at the needle base to steer the needle tip position during insertion. Another possible strategy is the insertion point steering. This idea comes from an observation that in prostate brachytherapy, the surgeon corrects the needle bending using a hook to directly manipulate the needle between the template and the perineum. Figure 7.2 illustrates the idea of insertion point steering. The insertion force F_x still acts at the base to control needle insertion, while the steering force F_y changes to the insertion point. The proposed steering model in this thesis can be easily extended to the insertion point steering with just a little modification as shown below.

Figure 7.2 Insertion point control

In the original base steering model, the steering force F_y is fixed at the origin in the $\tilde{x}-\tilde{y}$ coordinate system. When the actuation point changes to the insertion point, which is a moving point with respect to the $\tilde{x}-\tilde{y}$ coordinate system, F_y should be replaced by $\int_0^L \delta(\tilde{x}-L+v_x t) \cdot F_y d\tilde{x}$. Thus the insertion point steering model can be obtained as

$$\begin{aligned} & \dot{\alpha}(t) \int_{L-v_x t}^L c \varphi_i(\tilde{x}) d\tilde{x} + \alpha(t) \int_{L-v_x t}^L k \varphi_i(\tilde{x}) d\tilde{x} + \ddot{q}_i(t) + \sum_j \dot{q}_j(t) \int_{L-v_x t}^L c \varphi_i(\tilde{x}) \varphi_j(\tilde{x}) d\tilde{x} \\ & + \sum_j q_j(t) \int_{L-v_x t}^L k \varphi_i(\tilde{x}) \varphi_j(\tilde{x}) d\tilde{x} + \omega_i^2 q_i(t) = \int_0^L \delta(\tilde{x}-L+v_x t) \cdot F_y \cdot \beta_i d\tilde{x} + f_{li} \end{aligned} \quad (7-3)$$

$$\begin{aligned} & M_i \ddot{\alpha}(t) + c v_x \dot{\alpha}(t) + \sum_j \dot{q}_j(t) \int_{L-v_x t}^L c \varphi_j(\tilde{x}) d\tilde{x} + \sum_j q_j(t) \int_{L-v_x t}^L k \varphi_j(\tilde{x}) d\tilde{x} + k v_x \alpha(t) t \\ & = \int_0^L \delta(\tilde{x}-L+v_x t) \cdot F_y d\tilde{x} + f_2 \end{aligned} \quad (7-4)$$

with

$$\begin{aligned} f_{li} &= \int_0^t k \alpha(t_1) \varphi_i(v_x t_1 + L - v_x t) v_x dt_1 + \int_0^t k \sum_j \varphi_j(L) q_j(t_1) \varphi_i(v_x t_1 + L - v_x t) v_x dt_1 \\ &= \int_0^t k Y(t_1) \varphi_i(v_x t_1 + L - v_x t) v_x dt_1 \\ f_2 &= \int_0^t k \alpha(t_1) v_x dt_1 + \int_0^t k \sum_j \varphi_j(L) q_j(t_1) v_x dt_1 = \int_0^t k Y(t_1) v_x dt_1 \end{aligned}$$

7.2.4 Smart Needle Steering

Steering the needle tip trajectory during insertion is the ‘core’ of this research. There were some initial thoughts of using electro-pneumatics or electro-hydraulics power to drive the needle by dividing the needle into several chambers and

controlling the internal pressure, or using electro-magnetic field to guide the needle motion through the change of magnetic field intensity. It may be technically stringent and financially expensive to implement these systems.

Another possible method is to use MEMS (Micro-Electro-Mechanical System) structure to develop ‘smart needles’. Smart needle steering is an important future direction for the needle steering. Smart structure has been a hot topic in recent years, which is man-made engineered system that can mimic biological systems’ ability to react to stimuli. They are composed of materials that can determine their present state, decide what is the most desirable state, and carry out an appropriate response in a controlled manner. A smart needle can be developed using this concept by coupling the needle with sensors to monitor the needle displacements, and actuators to respond to an undesirable structural response (e.g. needle deflection). A control system can be designed accordingly to handle the transfer of information and real time computation.

To make such a smart needle is very challenging. The needle should not deviate much from the traditional needles, so that it is easier for the users to accept it. Considering electronic power will be connected with the needle for actuation and sensing purposes, the needle should be well shielded in order to protect the patient from being injured by the electricity. The material of the shield should also be biocompatible. That is, it should not poison the patient, as well as leave stains after the needle is withdrawn. The following are some preliminary thoughts on the smart needle.

A variety of different materials can be utilized as sensor/actuator elements, e.g. piezoelectric materials, shape memory alloys, fiber optics, electrostrictive

materials, magnetostrictive materials. Depending on the specific material used, the sensor and actuator elements can be controlled through electrical, magnetic, thermal, or optical energy. Among these materials, piezoelectric materials have several advantages, such as lightweight, simple integration into the structure and commercial availability. A deciding factor is that, in piezoelectric materials, electrical charge is produced due to mechanical strain and mechanical strain is produced due to an applied electric field. Such unique mechanical and electrical properties of the piezoelectric materials make them well suited for use as sensors and actuators. The two common types of piezoelectric materials are lead zirconate titanate (PZT) ceramics and polyvinylidene fluoride (PVDF) polymers. Piezoceramics and piezopolymers are usually produced in thin sheets with films of metal deposited on the opposite surfaces to form electrodes. Piezoceramics are brittle and stiff, while piezopolymers are tough and flexible. Piezopolymers can be used for the sensors because of their small stiffness, which adds minimum local stiffening to the needle; while piezoceramics can be used for the actuator as they have greater elastic modulus for effective mechanical coupling to the needle.

There are two configurations for incorporating the piezoelectric elements with the original standard brachytherapy/biopsy needle: surface mounting or sandwich structure. Needle steering can be designed based on these two structures.

Surface-mounted piezoelectric actuators are known to induce longitudinal strains (extend or contract) when the applied electric field is parallel (same or opposite) to the poling direction of the piezoelectric element. This is known as extension actuation mechanism. Figure 7.3 illustrates the mechanism.

The other structure is a sandwich structure, in which shear actuators will be used.

Shear actuators in the sandwich structure are normally longitudinally poled. When subjected to transverse electric field, they will experience shear deformations through the so-called e_{15} piezoelectric constant. This leads to the shear actuation mechanism. Figure 7.4 shows the mechanism.

Figure 7.3 Extension actuation mechanism: (a) Sensor/actuators mounted on the inner surface of the needle cannula. (b) Bended needle cannula due to extension actuation (not to scale)

Figure 7.4 Shear actuation mechanism: (a) Undeformed sandwich structured needle stylet (b) Bended needle stylet due to shear actuation (not to scale)

Surface mounted sensors/actuators are relatively easier to be placed on the needle, but likely to be damaged when contacting with surroundings or by the largest bending stress since the sensors/actuators are located at the farthest locations from the neutral plane. In contrast, the sensors/actuators in the sandwich structure sustain the lowest bending stress level within the structure, since the piezoelectric core is located near the neutral plane of the beam, which has been proven to minimize debonding problems. In addition, from the weight and size consideration,

sandwich structure normally is more efficient than the surface-mounted actuation structure.

The choice of a suitable structure depends on not only the characteristics of the structures themselves, but also the applied applications. For example, for needles used in biopsy operation, sandwich structure can be adopted to build the needle stylet; while for needles used in brachytherapy operation, surface mounting structure should be used, in which the piezoelectric elements will be mounted on the inner surface of the needle cannula. This is due to the fact that the needle stylet will be withdrawn during the brachytherapy procedure in order to load the seeds. To avoid needle rebounding and keep the needle position traceable after the core is withdrawn, as well as protect the actuators/sensors during interaction with the tissue, the inner face of the needle cannula should be chosen to place the piezoelectric elements.

In the figures above, only one sensor and one actuator are shown on the needle. The actual smart needle can have several sensors/actuators along the needle body to make it more effective. Depending on the number of sensors/actuators on the smart needle, the needle steering system can be single input single output (SISO) or multiple inputs multiple outputs (MIMO). Some preliminary simulation study has been carried out to investigate the most effective configuration for the actuators that will require smallest voltage in order to achieve certain needle displacement. More details can be found in [166]. Work in this topic is ongoing.

Author's Publications

Journal articles:

1. K. G. Yan, T. Podder, D. Xiao, Y. Yu, T.-I Liu, C.W.S Cheng, W. S. Ng (2006), *An Improved Needle Steering Model with Online Parameter Estimator*, International Journal of Computer Assisted Radiology and Surgery, ISSN1861-6410, vol.1, pp 205-212.
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1. K. G. Yan, T.-I Liu, K. V. Ling, Y.Yu, O.D. Walter., W. S. Ng (2004), *Preliminary Work of A Smart Needling Project*, 3rd International Conference on Experimental Mechanics, SPIE, Singapore.
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3. K. G. Yan, W. S. Ng, K. V. Ling , Y. Yu, T. Podder (2005), *High Frequency Translational Oscillation & Rotational Drilling of the Needle in Reducing Target Movement*, IEEE International Symposium. on. Computational Intelligence in Robotics and Automation, pp163-168. Espoo, Finland.
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Appendix A Test Results of Needling

Table A-1 Phantom elasticity testing results (refer to pp.83)

Number of testing	Configuration ratio	Configuration time (hr)	Young's Modulus E (kPa)	Mean of E
1	1:2	16	26.3	
2	1:2	16	27.9	
3	1:2	16	26.8	
				27
4	1:2	24	49.7	
5	1:2	24	51.1	
6	1:2	24	44.3	
				48.4
7	1:3	16	21.2	
8	1:3	16	19.8	
9	1:3	16	21.4	
				20.8
10	1:3	24	28.8	
11	1:3	24	29.6	
12	1:3	24	35.6	
				30.1
13	1:6	16	6.1	
14	1:6	16	6.8	
15	1:6	16	6.4	
				6.4
16	1:6	24	11.8	
17	1:6	24	10.4	
18	1:6	24	9.6	
				10.6

Table A-2 Factorial design for oscillation effect testing (refer to pp. 119)

Important factors		Levels		
		1		-1
Oscillation (X ₁)	Frequency (X ₁₀)	Yes	High (20khz)/ Low (2khz)	No
	Amplitude (X ₁₁)		Large/ Small	
Velocity (X ₂)		High (10mm/s)		Low (0.4mm/s)
Phantom (X ₃)		Softer (1:6)		Stiffer (1:2)
Response (Y)		Target movement		

Appendix A

Test Results of Needling

Table A-3 Randomized experiment design with two replications (refer to pp. 122)

Run Order	Block (X_3)	X_1		X_2
		X_{10}	X_{11}	
1	1	1	1	-1
2	1	-1	-1	-1
3	1	1	-1	-1
4	1	-1	1	1
5	1	1	1	1
6	1	-1	1	1
7	1	-1	-1	-1
8	1	1	1	-1
9	1	1	-1	1
10	1	1	-1	1
11	1	-1	-1	1
12	1	-1	1	1
13	1	-1	-1	1
14	1	-1	1	1
15	1	-1	1	1
16	1	1	1	1
17	1	-1	-1	-1
18	1	-1	-1	-1
19	1	-1	-1	-1
20	1	-1	1	-1
21	1	-1	1	-1
22	1	-1	1	1
23	1	-1	-1	-1
24	1	1	-1	-1
25	1	-1	1	1
26	1	-1	-1	-1
27	1	-1	1	1
28	1	-1	-1	-1
29	1	-1	-1	-1
30	1	-1	-1	-1
31	1	-1	-1	1
32	1	-1	-1	1

Appendix B Aurora System Technical Specifications

The contents of this section are extracted from Aurora Technical Specifications Manual, which can be found in NDI's website: www.ndigital.com.

B.1 Features

- ◆ No line-of sight requirements provide greater measurement
- The Aurora System's unique electromagnetic technology accurately tracks tools even when they are not within line-of-sight of the system.
- ◆ Ultra-small sensor coils provide greater flexibility
- The ultra-small dimensions of the Aurora sensor coils enable you to develop new and more versatile tools and applications.
- ◆ Seamless system integration with new or existing systems
- NDI's Application Programmers' Interface (API) and supporting documentation make it easy to adapt the Aurora System to the needs.
- Harmonized with NDI's Polaris optical measurement system, the API provides effective integration with the most popular optical system on the market today.
- A small footprint and standard interface make the Aurora System convenient and adaptable.
- ◆ Automatic tool identification
- The SROM device located in a tool's connector stores information that describes the nature of the tool. The System Control Unit reads this SROM device whenever you plug the tool into the system, automatically recognizing which tool it is.
- ◆ User friendly
- The Field Generator is pre-calibrated for immediate, "out of the box" operation.

Appendix B

Aurora System Technical Specifications

- The Aurora System is equipped with easy-to-use documentation designed to guide you through the simple set-up process.
- The Aurora System has a compact, lightweight, and robust design.
 - ◆ Accurate, reliable measurement and tracking
- Automated tool detection helps maintain system accuracy during use.

B.2 Technical Specifications

SENSOR COIL	
Dimensions (L x Dia.)	8.0 mm x 0.8 mm
Weight	0.7 g
FIELD GENERATOR	
Dimensions (L x W x H)	220 mm x 220 mm x 170 mm
Weight	2.2 kg
SYSTEM CONTROL UNIT	
Dimensions (L x W x H)	295 mm x 235 mm x 88 mm
Weight	3.4 kg
SENSOR INTERFACE UNIT	
Dimensions (L x W x H)	50 mm x 90 mm x 32 mm
Weight	0.25 kg
POWER SUPPLY	
100 – 240 V~ (universal input) 50/60 Hz, 0.5 A	
DATA COMMUNICATION INTERFACE	
Interface	RS-232
Maximum data rate	115 kBaud
TOOLS	
Tool types	single sensor coil tools; dual sensor coil tools; sensorless tools
Maximum number of sensor coils per tool	2
Tool change	Automatic
Tool options	3 LED/switch lines per tool
PERFORMANCE	
Maximum number of sensor coils	8 (four-port system)
Maximum measurement rate	5 or less sensor coils = 45 Hz 6 or more sensor coils = 22 Hz

B.3 Accuracy (Without Interference)

ACCURACY – 5DOF TOOLS*			ACCURACY – 6DOF TOOLS*		
Note: Typical results based on more than 300 random positions and orientations distributed throughout the sub-volume.			Note: Typical results obtained using a calibration artifact consisting of more than 40 predetermined random positions and orientations distributed over a 15 cm diameter hemisphere.		
Position			Position		
Radial distance from Field Generator (mm)	RMS	95% confidence level	Artifact distance from Field Generator	RMS	95% confidence level
100–200	0.9 mm	1.7 mm	Position 1 (250 mm)	0.9 mm	2.0 mm
200–300	0.7 mm	1.3 mm	Position 2 (450 mm)	1.6 mm	3.0 mm
300–400	0.8 mm	1.4 mm	Orientation		
400–500	1.3 mm	2.1 mm	Position 1 (250 mm)	0.8°	1.5°
Orientation			Position 2 (450 mm)	1.1°	1.7°
Entire volume	0.3°	0.6°			

Appendix C 3D Motion Platform

C.1 Main components

Figure C.1 Main components of 3D Motion Platform

The 3D motion platform includes the following components.

1. DMC-1842 motion controller, 1 100-pin cable, and 1 interconnect module (ICM-2900).
2. 1 servo motor to control the insertion movement in x direction and 2 stepper motors to control the lateral movements in y and z directions.
3. x rail(travel range: 340mm); y rail(travel range: 100mm);z rail(travel range: 100mm);
4. Power amplifier.
5. PC (Personal Computer-PCI bus).
6. Communications software: DMCWIN32.

C.2 Installation steps

Installation of a complete, operational DMC-1842 system consists of 9 steps.

1. ***Determine overall motor configuration:*** For the servo motor operation, no hardware or software configuration is required, for the DMC-1842 has been setup by the factory providing an analog command signal of +/- 10V. For stepper motor configuration, the controller requires a jumper for each stepper motor and the command, MT, must be given.
2. ***Install jumpers on the DMC-1842:*** For each axis that will be used for stepper motor operation, the corresponding stepper mode (SM) jumper must be connected.
3. ***Install the communications software.***
4. ***Install the DMC-1842 in the PC:*** The DMC-1842 is installed directly into the PCI expansion bus.
5. ***Establish communications with the Galil communication software.***
6. ***Make connections to amplifier and encoder:*** refer to Figure C_1 for the system connection diagram.
7. ***Connect the standard servomotor and step motors.***
8. ***Tune the servo system.***

C.3 Motion Program

The programming for motion control can refer to DMC 18X2 Command reference.

The following are some of the most often used commands.

moveforward.dmc

SH Servo X,Y,Z motors

Appendix C

3D Motion Platform

SPX 14000	Specify speed in the X axes(10mm/s)
PRX 140000	Set motion in x axis at 100mm
SRX 14000	Set velocity in x axis at 10mm/s
BGX	Begin insertion
EN	End program

movebackward.dmc

SPX 14000	Set speed in the X axes(10mm/s)
PRX -140000	Set relative motion in x axis at -100mm
SRX 14000	Set velocity in X axis at 10mm/s
BGX	Begin insertion
EN	End program

moveXY.dmc

SP 14000,5000,5000	Set speeds in the X/Y/Z axes at 10mm/s
PRX 140000,25000,0	Set relative motion in X/Y/Z axis at
100mm,50mm,0mm	
BG	Begin motion
EN	End program

Linesegment.dmc

DP 0,0,0	Begin Program, define position to 0 on all axes
PR 140000,20000,0	Specify X/Y/Z Positions
SP 14000,2000,5000	Specify X/Y/Z Velocities

BGX	Start Motion in X axis
AD 70000	Wait until X position is 50mm
BGY	Start Motion in Y axis
EN	End program

Figure C.2 System connection for a standard DC servo Motor and encoder (adopted from DMC-18X2 Manual Ref. 1.0f)

Appendix D ATI Force Sensor System

Figure D.1 Main components of force sensor system (from ATI manual)

The main system components are listed as below.

1. **Transducer:** Nano17 is selected for its small size and adequate resolution. Table D-1 shows its specifications.
2. **Transducer Cable:** The high-flex life transducer cable is electrically shielded to protect transmission from the transducer Power Supply or Interface Power Supply boxes.
3. **Power Supply and Interface Supply Boxes:** It supplies power to the transducer and electronics as well as conditioning the transducer signals to be used with a data acquisition system. The transducer's 12-pin cable plugs into this box.
4. **Power Supply Cable:** The robust power supply cable connects the Power Supply box or Interface Power Supply box to the data acquisition system.

Appendix D

NTI Force Sensor System

This cable usually has a connector on the data acquisition end, but is also available unterminated.

5. **Data Acquisition System:** The data acquisition system converts the transducer signals from analog voltages into data

Table D-1 Nano 17 transducer specifications (from ATI manual)

	English (US)			Metric (SI)		
Calibration	US-3-1	US-6-2	US-12-4	SI-12-0.12	SI-25-0.25	SI-50-0.5
Rated Sensing Ranges						
Fx, Fy	±3 lb	±6 lb	±12 lb	±12.0 N	±25 N	±50 N
Fz	±4.25 lb	±8.5 lb	±17 lb	±17 N	±35 N	±70 N
Tx, Ty, Tz	±1 in-lb	±2 in-lb	±4 in-lb	±120 Nmm	±250 Nmm	±500 Nmm
Resolution*						
Fx, Fy	1/5120 lb	1/2560 lb	1/1280 lb	1/1280 N	1/320 N	1/160 N
Fz	1/5120 lb	1/2560 lb	1/1280 lb	1/1280 N	1/640 N	1/320 N
Tx, Ty	1/32000 in-lb	1/16000 in-lb	1/8000 in-lb	1/256 Nmm	1/128 Nmm	1/64 Nmm
Tz	1/32000 in-lb	1/16000 in-lb	1/8000 in-lb	1/256 Nmm	1/128 Nmm	1/64 Nmm
Physical Properties			US		Metric	
Stiffness (Calculated)						
X-axis and Y-axis force (Kx, Ky)			53×10 ³ lb/in		9.3×10 ⁶ N/m	
Z-axis force (Kz)			69×10 ³ lb/in		12×10 ⁶ N/m	
X-axis and Y-axis torque (Ktx, Kty)			2.2×10 ³ in-lb/rad		250 Nm/rad	
Z-axis torque (Ktz)			3.5×10 ³ in-lb/rad		390 Nm/rad	
Resonant Frequency (Measured)						
Fx, Fy, Tz			7.2 kHz			
Fz, Tx, Ty			7.2 kHz			
Maximum Single-axis Load						
Fx, Fy			±79 lb		±350 N	
Fz			±170 lb		±750 N	
Tx, Ty			±21 in-lb		±2.4 Nm	
Tz			±28 in-lb		±3.1 Nm	
Weight						
Transducer with standard aluminum plates			0.021 lb		9.4 g	
Transducer with stainless steel plates			0.073 lb.		33 g	
Material						
Transducer			Hardened Stainless Steel			
Mounting and tool adaptors			Aircraft Aluminum			

Appendix E Phantom Making Procedures

The inhomogeneous phantom as shown in Figure 5.23 was made following the procedures shown below.

1. Prepare hot water; pour 600ml hot water into the measuring jug.
2. Measure 400g gelatin powder using the electric beam balance and slowly pure the gelatin powder into the water, while stirring it using a spoon. An observation from past experiments has shown that gelatin-to-water ratio 1:2 is near the saturation point. After adding around 300g gelatin powder into the water, the gelatin power become to form some undissolved lumps.
3. Add other pre-prepared lumps of different properties (e.g. PVC lumps) into the solution to create the inhomogeneous phantom. Use the spoon to spread the undissolved gelatin lumps and PVC lumps inside the whole solution.
4. The solution is left to coagulate around 20hrs.

To make the two-layer phantom as shown in Figure 5.33, the above steps 1- 4 were performed first. A rectangular slot was then cut at a place 4cm away from the insertion point. After it, the prepared animal meat – pork was put inside the slot as the second layer.

Appendix F Insertion Force Study

This appendix presents some preliminary analysis of the resultant needle base insertion forces that are in x direction during both straight-line insertion and steering procedure. The effects of the insertion velocities on the insertion forces are investigated as well.

F.1 Material and Method

Experiments were carried out first to obtain the data for analysis. Porcine liver was chosen for experiment. It was hosted by the gelatin phantom in the container to prevent it from sliding during insertion, as shown in Figure F.1. Five insertion speeds, 2mm/s, 4mm/s, 6mm/s, 8mm/s and 10mm/s were adopted for experiments. Insertion length was set to be 6cm for the constraint length of the liver. The needle steering distance was set to be 2cm, larger than which may cause the permanent damage of the needle. The steering speed was set accordingly to guarantee the movements in x and y directions to start and stop simultaneously to examine the steering effects in the whole insertion length. Totally 3 sets of experiments that included both straight-line insertion and steering-insertion were carried out using 3 porcine livers. The needle insertion force data together with the needle position data were collected during the procedure.

Figure F.1 Needle insertion study in animal organ (porcine liver)

F.2 Insertion Force Study

Figure F.2 Needle base insertion force profile at speed 4mm/s

Figure F.2 shows one typical force profile during straight-line insertion and steering procedure. There are mainly two phases during the insertion procedure. The first phase starts from the beginning and stops at the puncture points; while the second phase starts from the puncture point and ends when the needle stops insertion. The puncture points indicate the needle base insertion forces and corresponding displacements when puncturing into the tissue. They were obtained by examining the force profile together with observations during experiment.

During the first phase, the needle compressed the tissue without puncturing through the membrane. The resultant needle base insertion force was mainly the tissue stiffness force, which linearly increased with the needle insertion until puncturing into the tissue. During the straight-line insertion, the tissue reaction force mainly acted at the needle tip in axial direction before punctuation. The needle base force equaled to the needle tip force when the needle moved at constant velocity, and the needle insertion depth equaled to the tissue deformation length at the contact point. While during the steering procedure, the needle bent. Part of the needle body had contact with the tissue, thus receiving tissue reaction forces at the contact surface. The resultant needle base insertion force was the summation of the tissue reaction forces. Thus the needle needed larger force in order to puncture into the tissue.

After the needle punctured into the tissue, which was indicated by the sudden dropdown of the forces in the figure, the needle moved inside the tissue. The forces acting at the needle body included the cutting force at the tip and the stick-slip friction force along the needle axis during the straight-line insertion. For the steering procedure, an additional stiffness forces acted perpendicularly to the

tangential lines along the needle body that was inside the tissue. Thus the needle would need larger insertion force during steering procedure than during straight-line insertion in order to keep the needle move at the same velocity. The small spikes occurred during this phase were mainly caused by internal cutting of the inhomogeneous tissue and the stick-slip friction forces.

F.3 Effects of Velocity on Insertion Forces

Figures F.3 and F.4 show the whole view of the insertion forces with zoom-in view of the puncture points at various speeds during straight-line insertion and steering procedure. The puncture points are marked in Figure F.4 for different velocities. As can be seen from the figures, the insertion forces increase linearly until reaching the puncture force limit. Higher insertion velocity will need larger insertion force, but its effect is not as significant as the insertion depth. Thus the spring effect is more influential than the damper effect in this phase. A general trend that can be detected from the figures is, the higher the insertion speed is, the faster the insertion force increases, and the less the tissue will deform before the needle punctures into the tissue. Table F-1 lists the tissue surface deformation lengths in the first phase with different insertion velocities in the three sets of experiment. In most of the cases, increasing the speed can reduce the surface deformation and make the puncture easier. But this can be affected by other factors such as the tissue inhomogeneity, the thickness of the membrane etc. That's why the tissue deforms more at 6mm/s than at 4mm/s and 2mm/s in experiment II (without steering).

Figure F.3 Needle base insertion force profile during straight-line insertion

a) whole view b) Zoom-in view of the puncture points

Figure F.4 Needle base insertion force profile during steering procedure a) whole view b) Zoom-in view of the puncture points

Table F-1 Puncture positions in 3 sets of experiment

Puncture Positions in 3 sets of experiments (m)		2mm/s	4mm/s	6mm/s	8mm/s	10mm/s
1	Without steering	0.042177	0.037921	0.03584	0.032953	0.033147
	With steering	0.046926	0.044492	0.040409	0.036396	0.035956
2	Without steering	0.036176	0.030158	0.041087	0.023881	0.018766
	With steering	0.037641	0.037779	0.034024	0.033931	0.025424
3	Without steering	0.045087	0.043713	0.037025	0.028665	0.020783
	With steering	0.046575	0.034915	0.038889	0.02624	0.023589

F.4 Discussion

In this appendix, the needle insertion forces during straight-line insertion and steering procedure have been studied. The effects of the velocity on the insertion forces are investigated as well. It has been found that higher insertion velocity can reduce the tissue surface deformation and make the puncture easier. Even though a sophisticated insertion force model is not really necessary to keep the needle insertion at constant velocity, for a commercial DC motor with controller can complete the task perfectly. Better understanding of this insertion procedure and its influential factors can help us design an effective steering system in future.

Appendix G Equation Derivation

G.1 Derivation of Equations (3-8) and (3-9)

Substituting $\delta T, \delta V, \delta W_{nc}$ into (3-1), one obtains

$$\int_{t_1}^{t_2} \left\{ \frac{\partial T}{\partial \dot{y}} \delta \dot{y} + \frac{\partial T}{\partial \dot{\omega}} \delta \dot{\omega} - \left(\frac{\partial V}{\partial \omega''} \delta \omega'' + \frac{\partial V}{\partial y} \delta y + \frac{\partial V}{\partial \omega} \delta \omega \right) + F_y \cdot \delta y - \int_0^L H(\tilde{x} - L + v_x t) \cdot c(\dot{y} + \dot{\omega}) \cdot \delta(y + \omega) d\tilde{x} \right\} dt = 0 \quad (G.1-1)$$

First, grouping terms in (G.1-1) with respect to the independent variations $\delta \dot{y}$ and δy , and using calculus of variations, the following equation can be obtained.

$$\int_{t_1}^{t_2} \left(-\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}} \right) - \frac{\partial V}{\partial y} + F_y - \int_0^L H(\tilde{x} - L + v_x t) \cdot c(\dot{y} + \dot{\omega}) \cdot d\tilde{x} \right) \delta y dt = 0 \quad (G.1-2)$$

Owing to the arbitrary nature of δy , one can get

$$-\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}} \right) - \frac{\partial V}{\partial y} + F_y - \int_0^L H(\tilde{x} - L + v_x t) \cdot c(\dot{y} + \dot{\omega}) \cdot d\tilde{x} = 0 \quad (G.1-3)$$

From (3-4) and (3-5), one can get

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}} \right) = M\ddot{y} + \int_0^L \rho(\ddot{y} + \ddot{\omega}) d\tilde{x} \quad (G.1-4)$$

$$\frac{\partial V}{\partial y} = \int_0^L H(\tilde{x} - L + v_x t) \cdot k[y + \omega - y(t_1) - \omega(L, t_1)] d\tilde{x} \quad (G.1-5)$$

Substituting (G.1-4) and (G.1-5) into (G.1-3), Equation (3-8) can be derived.

Following similar steps to grouping terms in (G.1-1) with respect to

$\partial \omega, \partial \dot{\omega}$ and $\partial \omega''$.

$$\int_{t_1}^{t_2} \left\{ \frac{\partial T}{\partial \dot{\omega}} \delta \dot{\omega} - \left(\frac{\partial V}{\partial \omega''} \delta \omega'' + \frac{\partial V}{\partial \omega} \delta \omega \right) - \int_0^L H(\tilde{x} - L + v_x t) \cdot c(\dot{y} + \dot{\omega}) \cdot \delta \omega d\tilde{x} \right\} dt = 0 \quad (G.1-6)$$

From (3-4) and (3-5), one can get

$$\int_{t_1}^{t_2} \frac{\partial T}{\partial \dot{\omega}} \delta \dot{\omega} dt = - \int_{t_1}^{t_2} \int_0^L \rho (\ddot{y} + \ddot{\omega}) \delta \omega d\tilde{x} dt \quad (G.1-7)$$

$$\int_{t_1}^{t_2} \frac{\partial V}{\partial \omega''} \delta \omega'' dt = \int_{t_1}^{t_2} \left(\delta \omega' \cdot EI \omega'' \Big|_0^L - \delta \omega \cdot \frac{\partial (EI \omega'')}{\partial \tilde{x}} \Big|_0^L + \int_0^L \delta \omega \cdot \frac{\partial^2 (EI \omega'')}{\partial \tilde{x}^2} d\tilde{x} \right) dt \quad (G.1-8)$$

$$\int_{t_1}^{t_2} \frac{\partial V}{\partial \omega} \delta \omega dt = \int_{t_1}^{t_2} \int_0^L H(\tilde{x} - L + v_x t) \cdot k(y + \omega - y(t_1) - \omega(L, t_1)) \cdot \delta \omega d\tilde{x} dt \quad (G.1-9)$$

Substituting (G.1-7), (G.1-8) and (G.1-9) into (G.1-6), the following equations can be derived

$$\int_{t_1}^{t_2} \int_0^L (\rho (\ddot{y} + \ddot{\omega}) + EI \omega'''' + H(\tilde{x} - L + v_x t) \cdot k(y + \omega - y(t_1) - \omega(L, t_1)) + H(\tilde{x} - L + v_x t) \cdot c(\dot{y} + \dot{\omega})) \delta \omega d\tilde{x} dt = 0 \quad (G.1-10)$$

$$\delta \omega' \cdot (EI \omega'') \Big|_0^L = 0 \quad (G.1-11)$$

$$\delta \omega \cdot (EI \omega''') \Big|_0^L = 0 \quad (G.1-12)$$

From (G.1-10), equation (3-9) can be easily obtained.

From (G.1-11) and (G.1-12), boundary condition (3-11) can be obtained.

G.2 Derivation of Equation (3-14)

Considering unforced system, Equation (3-8) reduces to

$$M \ddot{y} + \int_0^L \rho (\ddot{y} + \ddot{\omega}) d\tilde{x} = 0 \quad (G.2-1)$$

Substituting (3-12), (3-13) into (G.2-1), one obtains

$$M (\ddot{\alpha}(t) + \beta \ddot{q}(t)) + \int_0^L \rho (\ddot{\alpha}(t) + \beta \ddot{q}(t) + \phi(\tilde{x}) \ddot{q}(t)) d\tilde{x} = 0 \quad (G.2-2)$$

To guarantee that no perturbation of the motion of the fixture center of mass occurs, $\ddot{\alpha}(t)$ should be 0. Thus one can get

$$M \beta \ddot{q}(t) + \rho \beta \ddot{q}(t) L + \rho \int_0^L \phi(\tilde{x}) \ddot{q}(t) d\tilde{x} = 0 \quad (G.2-3)$$

Considering $\ddot{q}(t)$ is time-varying, the following equation must be hold:

$$M\beta + \rho\beta L + \rho \int_0^L \phi(\tilde{x})d\tilde{x} = 0 \quad (\text{G.2-4})$$

From (G.2-4), Equation (3-14) can be easily derived.

G.3 Derivation of Equation (3-15)

Substitute equation (3-12) into (3-8), one can obtain

$$\begin{aligned} F_y - f(t) &= M\ddot{\alpha} + M\beta\ddot{q} + \int_0^L \rho(\ddot{\alpha} + \beta\ddot{q} + \phi(\tilde{x})\ddot{q})d\tilde{x} \\ &= M(\ddot{\alpha} + \beta\ddot{q}) + \rho L(\ddot{\alpha} + \beta\ddot{q}) + \int_0^L \rho\phi(\tilde{x})\ddot{q}d\tilde{x} \\ &= M_t(\ddot{\alpha} + \beta\ddot{q}) + \int_0^L \rho\phi(\tilde{x})\ddot{q}d\tilde{x} \end{aligned} \quad (\text{G.3-1})$$

Substitute equation (3-14) into (G.3.1), equation (3-15) can be derived.

G.4 Derivation of Equations (3-16) and (3-17)

Considering the unforced system, equation (3-9) will be reduced to

$$EI\omega'''' + \rho(\ddot{y} + \ddot{w}) = 0 \quad (\text{G.4-1})$$

Substitute (3-12) and (3-13) into (G.4.1), one can get

$$EI\phi^{(4)}(\tilde{x})q(t) + \rho(\ddot{\alpha}(t) + \beta\ddot{q}(t) + \phi(\tilde{x})\ddot{q}(t)) = 0 \quad (\text{G.4-2})$$

Considering (3-18) and $\ddot{\alpha}(t) = 0$, one can get

$$EI\phi^{(4)}(\tilde{x})q(t) + \rho\phi(\tilde{x})\ddot{q}(t) = 0 \quad (\text{G.4-3})$$

By using the separation of variable, one can get

$$\frac{EI\phi^{(4)}(\tilde{x})}{-\rho\phi(\tilde{x})} = \frac{\ddot{q}(t)}{q(t)} = -\xi^2 \quad (\text{G.4-4})$$

From (G.4.4), (3-16) and (3-17) can be derived.