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MODELING AND DESIGN OF A THREE DEGREE-OF-FREEDOM PERMANENT MAGNET SPHERICAL ACTUATOR

by

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A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

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List of Symbols

A, A_t, A_i	Sectional area of different coils, $i = 1, 2, 3, 4$
a, b, c	Constants in the solution of C_{nm}
$\mathbf{B}$	Magnetic field flux density
$\mathbf{B}_i$	Magnetic field flux density of Region $i = I, II, III$ in the rotor
$\mathbf{B}_{rem}$	Remanence of PM pole
B_{Ir}^e	Experimental measurement result of magnetic flux density in radial direction
B_{Ir}^N	Magnetic flux density in radial direction after nondimensionalization and normalization of experimental result
B_{Ir}^{New}	Magnetic flux density in radial direction obtained from normalized data
$B_{ir}, B_{i\theta}, B_{i\phi}$	Components of magnetic flux density in spherical coordinates, $i = I, II, III$
$B_{X_h}, B_{Y_h}, B_{Z_h}$	Magnetic flux density based on Hall probe coordinates
C_{nm}	Coefficients in the spherical harmonic expansion M_{0r}^s
$\mathbf{D}(\theta, \phi)$	Angular term in the torque integral formula
$\mathbf{D}(\theta_i, \phi_i)$	Representation of $\mathbf{D}(\theta, \phi)$ with arguments θ_i and ϕ_i
$\mathbf{D}^e(\theta_i, \phi_i)$	Integral result of $\mathbf{D}(\theta_i, \phi_i)$
$\mathbf{D}_c^e(\theta_i, \phi_i)$	Angular terms in $\mathbf{D}^e(\theta_i, \phi_i)$ pertaining to both ζ_0 and ζ_1
$\mathbf{D}_{G_\zeta'}^e(\theta_i, \phi_i)$	Angular terms in $\mathbf{D}^e(\theta_i, \phi_i)$ pertaining to ζ_0
$\mathbf{D}_{G_\zeta''}^e(\theta_i, \phi_i)$	Angular terms in $\mathbf{D}^e(\theta_i, \phi_i)$ pertaining to ζ_1

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D_1, D_2, D_3	Diameter of different coil frames
$d_4, d_4^\top, d_4^\perp$	Particular cases of $d_n, -d_n^\top, d_n^\perp$ when $n = 4$
$d'_4, d_4^{\top'}, d_4^{\perp'}$	Nondimensionalized result of $d_4, d_4^\top, d_4^\perp$
d_a	Normal distance from one concerned point to the rotor surface
$d\mathbf{F}$	Differential force exerted on the differential segment length $d\mathbf{l}$ of winding
$d\mathbf{l}$	Differential segment length of winding
$\ d\mathbf{l}\ $	Magnitude of $d\mathbf{l}$
$d\mathbf{T}_i$	Differential torque generated by differential winding segment $d\mathbf{l}$
$d_n, -d_n^\top, d_n^\perp$	Constants in the magnetic field model
ds	Differential sectional area of $d\mathbf{l}$ computed by $ds = r dr d\zeta$
E_1, E_2	Winding efficiency of coil stacking patterns
$\mathbf{e}_{d\mathbf{l}}$	Unit vector of the differential length segment $d\mathbf{l}$
$\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi$	Unit vectors in r, θ and ϕ directions of the spherical coordinates
$\mathbf{e}_{\theta_i}, \mathbf{e}_{\phi_i}$	Unit vectors in θ_i and ϕ_i directions which specify the position of the i^{th} coil axis
F_{sx}, F_{sy}, F_{sz}	Force components based on the sensor coordinates
$f(\phi)$	Function in the torque integral formula pertaining to ϕ
$\mathbf{H}$	Magnetic field intensity
$\mathbf{H}_i$	Magnetic field intensity of Region $i = I, II, III$ in the rotor
$\mathbf{G}(\theta_i, \phi_i)$	Vector in the torque formula of i^{th} coil, $\mathbf{G}(\theta_i, \phi_i) = [g_x(\theta_i, \phi_i) \ g_y(\theta_i, \phi_i) \ g_z(\theta_i, \phi_i)]^T$
G_ζ	Combination of G'_ζ and G''_ζ , i.e. $G''_\zeta - \frac{3G'_\zeta}{4}$
G'_ζ, G''_ζ	Constants in $\mathbf{D}^e(\theta_i, \phi_i)$ pertaining to ζ_0 and ζ_1
h_1, h_2	Heights of stacked wires
$H_{It}, H_{II t}$	Tangential components of magnetic field intensity at the interface between Region I and II
H_r, H_θ, H_ϕ	Components of the magnetic field intensity in spherical coordinates

I, II, III	Number of region in the rotor space
I_w	Current passing through a single wire
$\mathbf{J}$	Current density vector
J_i	Current density in the sectional area of the i^{th} coil
k_1, k_2, k'_1, k'_2	Integers used in the expression of maximum flux density positions
L_s	Distance between the origin of the sensor coordinates and that of the intermediate transformation coordinates
L_c	Coil length, $L_c = R_1 - R_0$
l, l_1, l_2	Length of different coil frames
l_{da}, l_{bc}	Edge lengthes of the closed contour at the boundary of two neighboring medium
$\mathbf{M}_0$	Residual magnetization vector
M_0	Magnitude of $\mathbf{M}_0$
$M_{0r}, M_{0\theta}, M_{0\phi}$	Components of residual magnetization vector $\mathbf{M}_0$ in spherical coordinates
M_{0r}^s	Spherical harmonic expansion of residual magnetization vector component M_{0r}
m_c	Number of winding turns for each coil layer
N	Number of coils in the PM spherical actuator
N_{turn1}, N_{turn2}	Number of winding turns for different wire radius
n_c	Condition number
n, m	Orders of spherical harmonics and Associated Legendre functions
$P_n^m(\cos \theta)$	Associated Legendre functions with argument $\cos \theta$
$\mathbf{p}_i$	Unit vector of the i^{th} coil axis in Cartesian coordinates, $\mathbf{p}_i = [p_{i,x} \ p_{i,y} \ p_{i,z}]^T$
$\mathbf{p}_{i,0}$	Initial value of $\mathbf{p}_i$
p	Number of PM poles, $p = 1, 2, \dots, 8$
$\mathbf{Q}$	Torque matrix

$\mathbf{Q}^{RM}$	Minimum right inverse of torque matrix $\mathbf{Q}$
$\mathbf{Q}^T$	Transpose of $\mathbf{Q}$
$(\mathbf{Q}\mathbf{Q}^T)^{-1}$	Inverse of $\mathbf{Q}\mathbf{Q}^T$
R_0	Distance from the rotor center to the top surface of the coil
R_1	Distance from the rotor center to the bottom surface of the coil
R_b	Radius of Region <i>III</i> in the rotor
R_c	Constant in the torque model computed by $R_c = R_0^{-2} - R_1^{-2}$
R_r	Rotor radius
$R_i(r), \Theta_i(\theta), \Psi_i(\phi)$	Independent functions in magnetic scalar potential, $\Phi_i(r, \theta, \phi)$, i.e. $\Phi_i(r, \theta, \phi) = R_i(r)\Theta_i(\theta)\Psi_i(\phi), i = I, II, III$
r, ϕ, θ	Spherical coordinates
$\mathbf{R}_z(\phi_r), \mathbf{R}_y(\theta_r), \mathbf{R}_z(\vartheta_r)$	Rotation matrices of rotor relative to the stator
r_p, θ_p, ϕ_p	Measurement coordinates
r_w	Radius of wire
$\mathbf{T}$	Torque generated by full set of coils
$\mathbf{T}_i$	Torque generated by the i^{th} coil
T_c	Constant in the torque model
T_{mx}, T_{my}, T_{mz}	Torque components based on intermediate coordinates
T_{rx}, T_{ry}, T_{rz}	Torque components with respect to the rotor coordinates
T_{sx}, T_{sy}, T_{sz}	Torque components with respect to the sensor coordinates
t_{n_c}	Height of n_c -layer coil
t_v, t_h	Limitations of vertical height and horizontal height of coil section
u_r	Rotor speed of the induction motor by Williams <i>et al.</i>
u_s	Speed of the moving magnetic field in the induction motor by Williams <i>et al.</i>
$V_{ABCD}, V_{A'B'C'D'}$	Volume of two conical-shaped coils
V_{co}, V_{cy}	Volume of conical- and cylindrical-shaped coils with hollow cores

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X_r, Y_r, Z_r	Rotor frame coordinates
X_s, Y_s, Z_s	Stator frame coordinates
x, y, z	Cartesian coordinates
Y_n^m	Spherical harmonic functions, or the angular portion of the solution of Laplace's equation
$Y_n^{m*}(\theta, \phi)$	Complex conjugate of $Y_n^m(\theta, \phi)$
α	Longitudinal angle of PM pole
α_0	Angle that specifies the center line position of PM pole in ϕ direction
β	Latitudinal angle of PM pole
β_1	Half of β_1 , i.e. $\beta_1 = \beta/2$
ζ	Angle between the coil axis and the line from the center of spherical coordinates system to $d\mathbf{l}$
ζ_0	Inner surface angle of the coil
ζ_1	Outer surface angle of the coil
$\theta_d, \dot{\theta}_d, \ddot{\theta}_d$	Desired angular displacement, velocity and acceleration
$\theta_{i,0}, \phi_{i,0}$	Initial values of θ_i, ϕ_i
θ_i, ϕ_i	Position of i^{th} coil axis in spherical coordinates
θ_s	Tilting motion range of the rotor shaft
θ_w	Angle between the direction of rotor motion and that of moving magnetic field in the induction motor by Williams <i>et al.</i>
$\kappa_{ni}^m, \xi_{ni}^m$	Coefficients in magnetic scalar potential, $i = I, II, III$
μ_0	Permeability of the free space
μ_m	Relative recoil permeability of PM pole
μ_r	Relative permeability of the ferromagnetic material
μ_{rec}	Recoil permeability of PM pole
ν_1, ν_2	Nondimensionalized PM-pole parameters, $\nu_1 = R_r/r, \nu_2 = R_b/R_r$
Φ	Magnetic scalar potential

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$\Phi_I, \Phi_{II}, \Phi_{III}$	Magnetic scalar potential of Region <i>I</i> , <i>II</i> and <i>III</i> in the rotor
$\Delta h, \Delta S$	Thickness and area of the rectangular section at the boundary of two neighboring medium
σ	Twist angle between the intermediate coordinates and rotor coordinates
ψ	Angular parameter in the plane of one winding loop

List of Abbreviations

1/2/3D	One/two/three-dimensional
AC	Alternating current
AWG	American wire gauge
BC	Boundary condition
CCD	Charge-coupled device
DAQ	Data acquisition
DC	Direct current
DOF	Degree-of-freedom
DLIM	Double-sided linear induction motor
FE	Finite element
FEA	Finite element analysis
SLIM	Single-sided linear induction motor
MST	Maxwell stress tensor
PC	Personal computer
PM	Permanent magnet
PZT	Piezoelectric
SPM	Spherical pointing motor
SRM	Switched reluctant motor
VRSM	Variable-reluctance spherical motor

Abstract

A permanent magnet (PM) spherical actuator based on electromagnetic principle is proposed in this study. The PM spherical actuator consists of a ball-like rotor and a spherical-shell-like stator. PM poles are mounted along the rotor equator, whereas layers of air-core coils are assembled on the stator. By varying current in coils, three degree-of-freedom (3-DOF) rotational motion of the rotor can be achieved. This thesis focuses on modeling, design and experimental investigation of this proposed PM spherical actuator. Based on the PM poles pattern on the rotor, the magnetic field distribution surrounding the rotor is formulated using Laplace's equations. Magnetic scalar potential is obtained according to the magnetic properties of rotor materials. With appropriate boundary conditions, the magnetic scalar potential is solved analytically. An analytical torque model is important for real-time motion control of electric actuators. Using Lorentz force law, the differential torque exerted on a wire segment in the magnetic field of the PM-pole rotor can be expressed. Integrating this differential torque within the entire coil volume produces the torque generated by a single coil within the magnetic field. The total torque generated by a full set of coils can be formulated based on principle of superposition. The superimposed actuator torque is directly proportional to the current input, which facilitates position and velocity control of the spherical actuator. The non-singularity property of the actuator workspace is verified from the torque model. Because the actuator torque model is an analytical expression, parametric design on the spherical actuator performance, especially the torque output is possible. Based on the theoretical analysis, a research prototype

is fabricated for experimental investigation. An automatic apparatus is developed to measure the magnetic field distribution of the PM-pole rotor. The comparison between experimental and analytical results shows that the maximum difference is about 7% of the experimental result. This experimental result is also nondimensionalized and normalized as a benchmark result for other similar rotor design study. A torque measuring apparatus is designed to measure the actuator torque output correspond to different rotor orientations. The experimental measurement is compared with analytical torque result, which shows that the maximum difference is about 12% of the experimental result. Based on this thesis work, motion control algorithms of the spherical actuator can be investigated and implemented in the future.

Chapter 1

Introduction

1.1 Background and Motivation

An electric motor is a device which converts electricity into mechanical motion. Most electric motors work by electromagnetism, but motors based on other electromechanical phenomena, such as electrostatic forces and the piezoelectric effect, exist as well.

Figure 1.1: Conventional one-axis rotary motor

So far, motors with one degree-of-freedom (1-DOF) rotary or linear motion, dominate the market. The rotary motor can be categorized as direct current (DC) motors and alternating current (AC) motors. As illustrated in Fig. 1.1(a), a classical DC motor has coils laying in the magnetic field of permanent magnets (PMs). A rotary switch called a commutator reverses the direction of the electric current flowing through the coils twice in every cycle so that a torque with constant direction can be generated between the coils and PMs resided on the fixture of the motor. The AC motor in Fig. 1.1(b) uses the phase differences between the three phases of the electrical current to create a rotating electromagnetic field in the motor. The rotor often consists of a number of copper conductors embedded in the steel. Through electromagnetic induction, the rotating magnetic field induces current to flow in these conductors, which in turn generates a torque by interacting with the magnetic field and then “pulls” the rotor to turn in the direction that the field is rotating. This type of motors is known as induction motors. For operation the rotor must always run slower than the frequency of the supplied power. If the rotor coils are introduced with a separate current, one has a synchronous motor. The rotor will rotate in synchronism with the rotating magnetic field produced by the three-phase AC power.

Many industrial applications require actuators to realize more than 1-DOF motion. Since conventional motors can have only 1-DOF rotation, at least three motors connected in series or in parallel are needed to provide a multi-DOF spherical motion. However, this type of combined multi-DOF actuator system has intrinsic disadvantages:

- the friction and backlash exist in motion transmission mechanism reducing motion accuracy;
- large mass and inertia moment of the combined actuator introduce high energy consumption;
- kinematic singularities exist in the operation range of the actuator.

(a) Machining tool

(b) Omnidirectional wheels [1,2]

(c) Robotic wrist

(d) Vision system

(e) Radar [3]

(f) Joy stick [4]

Figure 1.2: Applications of spherical actuators

The desire to eliminate all these drawbacks motivated researchers to design a compact and high performance 3-DOF actuator. One promising solution is the spherical actuator. The spherical actuator is a ball-joint-like device that is capable of performing 3-DOF rotational motion in one joint. It can be employed in a variety of industrial applications where orientation must be achieved rapidly, continuously, and uniformly, for example,

- flexible drilling, turning, grinding and milling machines (Fig. 1.2(a));
- omni-directional wheels for mobile robots (Fig. 1.2(b)); a potential option of wheels used in electric cars which can reduce the environmental pollution;
- robotic wrist, elbow, and shoulder joints (Fig. 1.2(c));
- camera actuators for stereoscopic active vision systems (Fig. 1.2(d));
- actuators for object (radar) tracking (Fig. 1.2(e));
- joy sticks which are useful in system control or manipulating (Fig. 1.2(f));
- attitude control mechanism for simulation of spacecrafts.

1.2 Review of Previous Work

Currently there are three different types of driving techniques proposed for the design of spherical actuators: the mechanical means, piezoelectricity and electromagnetic forces.

A spherical actuator driven by wires mechanically has been demonstrated by Nagasawa *et al* [5]. As illustrated in Fig. 1.3, the rotor (spherical cell) is put inside the concave shell and can move like a spherical sliding bearing. Four wires are connected between the rotor and four single-axis motors so that the orientation of the rotor

Figure 1.3: A cable-driven spherical motor [5]

can be moved to a desired position through tightening and/or loosening the wires. Essentially, this spherical actuator is similar to the conventional spherical motion mechanism because four single-axis motors are utilized to drive the rotor.

Ultrasonic spherical actuators have been studied by several researchers. For instance, Amano *et al.* [6] have proposed a 3-DOF ultrasonic actuator design using bending and longitudinal vibrations of a single stator installed with three sets of piezoelectric (PZT) elements; Takemura *et al.* [7,8] have developed a multi-DOF ultrasonic motor with a compact plate stator and a spherical rotor; Aoyagi *et al.* [9] have proposed a multi-DOF ultrasonic motor by using low-cost thick-film technology; Toyama *et al.* [10–14] have developed an ultrasonic motor as a fine-orienting stage. The working principle of ultrasonic motors is based on the reverse piezoelectric effect of ceramics, i.e. an applied electric voltage can cause a PZT material to change dimensions. A typical example [10] is illustrated in Fig. 1.4(a). A PZT element layer is bonded to the back

Figure 1.4: Ultrasonic spherical motor

of the stator. It can produce expansion or contraction motions after being energized positively or negatively. Thus a traveling wave on the surface of the stator can be generated. This traveling wave can drive the rotor on top of the stator. By using three stators of the same type, a 3-DOF spherical motion of the rotor can be achieved (Fig. 1.4(b)). The ultrasonic actuator has the advantages of high resolution and low power consumption. However, it also possesses disadvantages such as small working range, complex fabrication and hysteresis. Furthermore, wear of frictional material for long term operation may cause instability of the spherical motion.

Most spherical actuators are based on the principle of electromagnetism. Williams and Laithwaite *et al.* [16, 17] have designed the first spherical induction motor. As shown in Fig. 1.5(a), the rotor is made of ferromagnetic material with a spherical barrel shaped surface. A copper mesh is inlaid on the rotor surface to allow induction current to travel in longitudinal and latitudinal directions. The stator consists of two multiphase winding blocks (Fig. 1.5(b)) that can be twisted with an angle θ_w about an axis perpendicular to the rotor's axle, generating a rotational magnetic field at velocity u_s which induces the current on the surface of the rotor. The rotor approaches the synchronous speed $u_r = u_s / \cos \theta_w$, which can be changed by varying θ_w . Including

Figure 1.5: A 2-DOF spherical induction motor by Williams *et al.* [16,17]

the rotation of the rotor and the twist of the stator blocks, this motor has two DOFs, despite the fact that the twist of the stator blocks was not accomplished by the winding currents. Davey *et al.* [18] have analyzed the magnetic field and torque of this spherical induction motor. A true 3-DOF spherical motor could not be developed by simple modification of this motor.

Vachtsevanos and Lee *et al.* [19–24] have proposed two types of 3-DOF spherical motors for robotic applications. One of them is spherical induction motor. The rotor is a ball filled with infinite permeability material and encapsulated with conducting shell. Three sets of windings are mounted on the stator and excited to generate induced current on the rotor surface, and thus to drive the rotor to achieve 3-DOF rotations. However, no prototype has been constructed for this conceptualized spherical motor, because of the complexity in mechanical design, especially inlaying three transversal windings on the inner spherical surface is practically difficult. A second realization of the spherical motor is based upon the stepper motor principle. The hemispheric stator structure is designed to support coils, iron trimmings, bearings and the position measurement devices. A pair of PM poles are mounted on the rotor. The rotor is

supported by six bearings on the stator. Energizing the stator coils sequentially, the rotor can be pulled to any desired position.

The variable reluctance spherical motor (VRSM) has been developed by Lee *et al* [26–38] (Fig. 1.6). The stator is constructed of iron, which provides a magnetic flux path for linking stator poles. The stator poles consists of evenly spaced coils wound on ferromagnetic cores. These poles are arranged on the inside of the stator surface and at the vertices of a dodecahedron. The rotor consists of iron poles embedded in a round, smooth sphere following the pattern of octahedron. The torque of VRSM depends on the current inputs as well as the magnetic reluctance of the airgaps between the rotor and the stator poles. The reluctance of the airgap is a function of the relative position between the rotor and stator, and varies as the rotor moves, hence the name of VRSM. About two-thirds of the inertia moment of the entire system is contributed by the orientation measurement system consisting of three encoders, which cannot be ignored for system control [25].

Motivated by the inadequacy of industrial rotor manipulators and a proposed coarse-

(a) Exploded view [25]

(b) Prototype [26]

Figure 1.6: Variable reluctance spherical motor by Lee *et al*.

Figure 1.7: Multi-DOF motion wrist by Hollis *et al.* [40]

fine-manipulation solution, Hollis *et al.* [39–45] have developed a multi-DOF magnetically levitated fine-motion wrist. As shown in Fig. 1.7(a), every pole of this wrist consists of two U-shaped PMs and a flat-wound rectangular coil. The two U-shaped magnets are placed in opposite to form a loop with the small airgap. Between the two U-shaped magnets is the flat-wound rectangular coil. When the coil is excited with currents, it can move in the gap under the effect of magnets. Six flat-wound coils are attached on the six surfaces of a hexagonal cylinder in an interlaced longitudinal and latitudinal pattern (Fig. 1.7(b)). Six pairs of PMs are placed on both sides of coils so that the longitudinal position coil could move in latitudinal direction and the latitudinal position coil could move in longitudinal direction. Therefore, with respect to the magnets, the cylinder has 6-DOF motion. This wrist has relatively small workspace, allowing maximum translations and rotations of approximately $\pm 5\text{mm}$ and $\pm 4^\circ$.

Foggia *et al.* [46] have designed an electromagnetic actuator rotating around three independent axes. As illustrated in Fig. 1.8, the actuator consists of a moving armature and three fixed inductors. The moving armature is composed of a hollow sphere made of solid steel and can move around its center point. The outer surface of the armature may be used to attach additional components like antennas, cameras and

Figure 1.8: 3-DOF induction motor by Foggia *et al.* [46]

light beam supports, etc. depending upon the application the user is looking after. On the inner surface of the sphere, there is a thin deposit of copper obtained through electrolytic processing. When the three inductors are adequately supplied, a moving magnetic field can be created. This magnetic field can induce current on the thin copper deposit, which in turn generates torques on the armature by interacting with the magnetic field. This electromagnetic actuator can achieve a maximum deviation of $\pm 30^\circ$. Experimental results showed that the motor prototype exhibited significant noise and had a rather long response time of 5 seconds.

Kaneko *et al.* [47] have developed a spherical DC servo motor with 3-DOF motions. The motor is constructed of a rotor, a spherical stator and a pivot bearing. As shown in Fig. 1.9(a), the rotor is a disk comprised of a yoke with four PMs attached to its periphery. The stator has three spherical windings with electric currents I_x , I_y and I_z (Fig. 1.9(b)). Three encoders are used to measure the rotor orientation. This motor can achieve a maximum spinning torque 0.30Nm and $\pm 15^\circ$ tilting motion.

The spherical pointing motor (SPM) (Fig. 1.10) designed by Bederson *et al.* [48–52]

Figure 1.9: Spherical DC servo motor by Kaneko *et al.* [47] Figure 1.10: Miniature pan-tilt actuator by Bederson *et al.* [48]

is a 2-DOF computer-controlled device which incorporates both pan and tilt motions. This actuator consists of three orthogonal windings in a PM field. The coils can be either assembled inside of the actuator, free to rotate on a gimbal in a fixed magnetic field, or mounted outside whereas the magnets are attached to the gimbal inside the coils. A charge-coupled device (CCD) camera can be mounted on the rotor. The operating principle of the SPM comes from the fact that there is always a force on a

current-carrying wire loop in a PM field in such a direction that the loop will move to make the normal to the plane of the loop align with the magnetic field. As designed for applications in active vision, this pan-tilt mechanism should be fast, small, inexpensive and have low power requirement. It is an absolute positioning device and runs in open-loop. The accuracy of the actuator is dependant on the friction of the bearings as well as rotor orientations. According to the literature [52], this actuator is capable of panning and tilting with a load of 15 grams at rotational velocity of several degrees per second with a repeatability of 0.15° at a fixed orientation.

Spherical actuators which can achieve either 2-DOF or 3-DOF motions have been developed by Wang *et al* [54–63] (Fig. 1.11). The spherical rotor is entirely made of magnetized rare-earth materials. It is housed within the spherical hollow stator on a low friction surface coating. Accommodated on the stator are several windings. On the application of current to the stator windings, the resulting torque can orientate the rotor to minimize the system potential energy. Thus, control of the rotor orientation can be achieved by varying the winding currents.

Figure 1.11: 3-DOF spherical actuator by Wang *et al*. [53]

Figure 1.12: 3-DOF spherical motor by Weck *et al.* [64]

Weck *et al.* [65] had proposed a 3-DOF spherical motor shown in Fig. 1.12. The rotor diameter is 275mm. Coils are evenly mounted on the inner surface of the hemispherical stator and NdFeB magnets are plated on the entire rotor surface. The guiding of the rotor is realized by a hydrostatic bearing to achieve high stiffness and low friction. Additionally, the oil can reduce the temperature of the coils. The torque of this spherical motor has been calculated numerically.

Chirikjian *et al.* [66–69] had designed a 3-DOF spherical step motor as shown in Fig. 1.13(a). Eighty cylindrical rare-earth PMs were placed along the inside surface of a hollow plastic sphere with a 12-inch diameter (Fig. 1.13(b)). Sixteen coils with iron cores are placed on the surface of a spherical cap and polarized to form the electromagnetic field within the stator (Fig. 1.13(c)). An adjustable magnet saddle holds the stator magnets so that the stator magnets can be repositioned and reoriented. Because the symmetry of the rotor-pole arrangement is different from that of the stator poles, the magnetic field created by energizing two or more stator coils generates a torque that can change the rotor orientation.

(a) Assembly

(b) Rotor

(c) Stator

Figure 1.13: Spherical step motor by Chirikjian *et al.* [1]

1.3 Research Objective and Thesis Scope

Generally speaking, electromagnetic actuators have advantages such as fast response, high torque and moderate voltage operation, etc. As an optional force/torque generating element of electromagnetic actuators, permanent magnet has a few virtues as follows [70].

- No electrical energy is absorbed by the field excitation system. Thus there are no excitation losses. This indicates that the efficiency of the PM actuator can be higher compared to actuators using only electromagnetic excitation.
- High torque or output power per volume can be achieved.
- Better dynamic performance can be achieved by using PM poles.
- The cost of rare-earth PM has dropped significantly in recent years for wide adoption.

For these reasons, a 3-DOF electromagnetic spherical actuator with PM poles is proposed in this work. As introduced, some other researchers have developed electromagnetic spherical actuators. However, the structures of those actuators are fixed and it is not convenient to describe the relationship between actuator structure and torque output. Unlike previous electromagnetic spherical actuators, the structure of this spherical actuator is more flexible, i.e. the structure of two major force/torque generating elements (PM and coil poles) are parameterized, which is useful for observing the relationship between structure parameters and magnetic field or torque output. This provides a way to increase the actuator torque output by selecting structure dimensions. Furthermore, in this spherical actuator, more PM and coil poles could be incorporated to increase the working range as well as the motion resolution of the actuator. The goal of this research is to investigate the design and modeling of this kind of PM spherical actuator for future commercial applications.

Figure 1.14: Schematic drawing of PM spherical actuator

Figure 1.15: 3-DOF motion of spherical actuator

As illustrated in Fig. 1.14, this proposed spherical actuator has two major parts: a ball-shaped rotor with a full circle of PM poles along the equator, and a spherical shell stator with multiple layers of circumferential air-core coils. The PMs made of rare-earth materials can produce high flux density. The use of air-core coils may simplify the control of the actuator based on linear torque model.

The basic working principle of the spherical actuator is illustrated in Fig. 1.15. By activating pairs of coils in two longitudinal directions, the rotor can tilt in two orthogonal directions as shown in Fig. 1.15(a) and (b). Energizing all coils, the rotor can spin about its own axis (Fig. 1.15(c)). Therefore, through varying the current inputs of the coils, any desirable 3-DOF spherical motion within the workspace can be achieved.

Figure 1.16 illustrates the major tasks done in this thesis [71–80]. All tasks indicated in the figure are extended from torque modeling which is extremely important for motion control of the spherical actuator. Specifically, the magnetic field modeling is the prerequisite of the torque modeling; based on the torque model, design and development of a research prototype of the spherical actuator can be carried out; experimental investigation can be conducted on the magnetic field distribution and torque variation of this research prototype to verify the theoretical models.

- **Magnetic field expression:** The purpose of magnetic field expression is to formulate the magnetic field of the PM-pole rotor analytically so that the torque modeling can be carried out. The PM rotor pole is defined based on several generic geometric parameters. The space of the rotor under study is divided into three regions according to their magnetic characteristics. Laplace's equation can be obtained for each of these three regions respectively, and thus the general solution of the magnetic scalar potential for each region can be obtained. By taking advantage of boundary conditions in between these regions, the unknowns in the general solution of magnetic scalar potential can be determined. Because the magnetic intensity is equal to the gradient of the magnetic scalar potential, the magnetic flux density of the PM-pole rotor can be obtained subsequently.

- **Torque modeling:** Following the magnetic field modeling, torque modeling of the PM spherical actuator is worked out. According to Lorentz force law, the force

Figure 1.16: Scope of thesis

acted on a current-carrying conductor is equal to the multiplication of the current and the cross product of length segment vector and magnetic flux density. Therefore, the differential torque acted on a differential length segment of the winding with respect to the rotor center can be calculated by multiplying the moment arm with the differential force. Subsequently, integrating the differential torque within the entire volume of a single-coil results in the torque formula for a single coil. For each individual coil, similar formula can be obtained. Concatenating these formulas in a matrix form produces the torque model of the PM spherical actuator with full set of coils.

- **Prototype development:** The parameters of PM and coil poles provide an effective way for actuator design. The design objective of the actuator is to achieve high torque output of the spherical actuator. According to the derived torque model, effects of poles parameters on the torque output can be described so that appropriate values of parameters can be chosen. Additionally, in order to increase the number of winding turns, several factors such as stacking patterns of windings, wire diameters, optional coil frames are discussed. Based on theoretical analysis, dimensions of PM pole and coil are determined, and thus a research prototype of the spherical actuator has been manufactured.

- **Experimental investigation:** The developed research prototype can be used for the experimental investigation on magnetic field and torque variation of the spherical actuator. The experimental results can be utilized to verify theoretical models. In order to measure the magnetic flux distribution of PM-pole rotor, an automated apparatus that is capable of locating any point surrounding the rotor needs to be developed. Through a three-axis hall probe, the three components of the flux density at a particular point can be measured. Non-dimensionalization and normalization of the acquired data for bench marking are carried out so that the experimental data can be referred by other similar designs without mentioning specific rotor dimensions. Furthermore, the measured data of the magnetic field can be visualized and compared

with result from the theoretical model. In order to measure torque generated by stator coils, a force/torque measurement apparatus has been developed. This apparatus can obtain the 6D force/torque data for any rotor orientation within the workspace. The measured torque data is then compared with the theoretical torque model.

1.4 Thesis Organization

The remaining chapters of the thesis are arranged as follows. Chapter 2 introduces the working principle of the PM spherical actuator. According to the PM-pole arrangement, the magnetic field of the PM-pole rotor is formulated. Chapter 3 presents the torque modeling of the spherical actuator based on the magnetic field expression. By taking advantage of the magnetic field model and torque model, Chapter 4 describes the design of the spherical actuator and presents a research prototype. In Chapter 5, experimental apparatuses are developed to measure the flux density distribution of the PM-pole rotor and the torque variation of the spherical actuator. The data processing and presentation are discussed and the results are compared with theoretical models. Finally, Chapter 6 summarizes this thesis work and makes recommendations for future research on the PM spherical actuator.

Chapter 2

Magnetic Field Modeling

2.1 Introduction

Magnetic field modeling of the spherical actuator refers to the formulation of magnetic flux density distribution of the PM-pole rotor. There are mainly two purposes for the magnetic field modeling of the spherical actuator. First, torque modeling is necessary for position and velocity control of the spherical actuator. According to Lorentz force law, one prerequisite of the torque modeling is to formulate the magnetic field distribution of the PM-pole rotor analytically. Second, the magnetic field model can facilitate the comprehension of the flux density variation of the PM-pole rotor.

Various magnetic field modeling approaches have been investigated by researchers. A method based on Maxwell equation has been proposed by Prieto *et al.* [81] to obtain the magnetic component model for planar structures successfully. This method is applicable to one-dimensional (1D) magnetic field, i.e. the magnetic field vector has a constant direction. As an improvement, by using quasi-Poissonian/Laplace's equations in polar coordinates, Zhu *et al.* [82, 83] have developed an analytical technique to predict the 2D magnetic field distribution of single-axis brushless PM machines. Similar approaches have been utilized by Cho *et al.* [84] to analyze the 2D magnetic field of a planar motor, and by Polinder *et al.* [85] to analyze the 2D magnetic field

in the cylindrical airgap of a PM generator. For many practical cases, the magnetic field with 3D distribution requires different approaches for modeling. Xiong *et al.* [86] have proposed an analytical method of the magnetic field for a linear PM synchronous machine by using the concept of magnetic charge. A general analysis on the magnetic field of a spherical induction motor was introduced by Davey *et al.* [18] using magnetic vector potential. The analysis properly accounts for the diffusion of the magnetic field with changing frequency and motor speed. Wang *et al.* [57, 59] have proposed 2D/3D magnetic field modeling methods based on Laplace's equation in spherical coordinates for two spherical actuators. In these models, pole arrangements like two or four PM poles and three or four windings, are considered. The rotor is in principle regarded as a sphere entirely consisting of magnetized rare-earth materials. This approach shows less flexibility in actuator design and system analysis to a certain extent. Prieto *et al.* [87] have presented a procedure to obtain the energy in 3D magnetic structures by applying the *Double 2D* methodology. An accurate model of the magnetic component can be obtained from the magnetic field energy. This approach is based on finite-element analysis (FEA), and thus is not suitable for system control.

In this chapter, an analytical model for the magnetic field of PM-pole rotor is proposed. The PM poles are described by generic variables related to the formulation of the magnetic field. Laplace's equations are formulated for deriving the general solution of magnetic scalar potential. The analytical solution of magnetic flux density distribution of the PM-pole rotor can be obtained with suitable boundary conditions (BCs). With analytical solution of the magnetic flux density, the influence of design parameters of the rotor on the magnetic field can be determined.

2.2 Formulation of the Magnetic Field

2.2.1 Configuration of Rotor Poles

When multiple poles are used in a rotor, the PM poles are arranged along the rotor equator in an alternative polarization pattern. Putting a coil between two PMs as shown in Fig. 2.1(b), one neighboring PM will generate an attraction force with it and the other will generate a repulsion force. The attraction and repulsion will move the rotor in the same clockwise direction. Therefore, the alternative polarization pattern can achieve larger torque compared with the pattern with same polarization. In order to realize this alternative arrangement in a circle, even number of PM poles are required. Due to the intrinsically high magnetic flux produced by rare-earth magnet as indicated in Fig. 2.1(a), the rotor tends to align itself to the earth magnetic field. This phenomenon is analogous to the effect of earth magnetic field to the compass needle. Having even number of poles distributed uniformly around the equator of the

Figure 2.1: Influence of earth magnetic field on rotor poles

rotor is a method to reduce the inherent self-inducing torque generated by the earth magnetic field at the center of the rotor (Fig. 2.1(b)).

Figure 2.2(a) illustrates the shape of a single rotor pole - an approximated dihedral cone enclosed by $ABCD$ and $abcd$. The dihedral cone can be specified by four parameters: longitudinal angle α , latitudinal angle β , rotor radius R_r and rotor core radius R_b . Modeling a single pole as a dihedral cone has several benefits. First, due to the 3-DOF spherical motion of the rotor, the spherical surface of the dihedral-cone-shaped pole can avoid the interference between coils and PM poles, whilst making use of the working space of the rotor completely. Second, the volume of the rotor pole can be specified in spherical coordinates, which facilitates the formulation of the actuator torque. Third, by varying the parameters of the dihedral cone, R_b , R_r , α and β , the study of optimal magnet-pole pattern can be carried out. Figure 2.2(b) presents the PM poles in alternate magnetization directions placed around the equator of the rotor. There are air slots in between PM poles. The regions on top and bottom of the rotor can also be air or low-density materials such as aluminum. These air slots

Figure 2.2: Arrangement of rotor poles

generalize the study of poles arrangement and decreases the inertial moment of the rotor.

2.2.2 Magnetic Scalar Potential

In the formulation of the magnetic field, the rotor space under study is divided into three regions based on their magnetic characteristics. The air space outside the rotor is denoted as Region *I*. The volume enclosed by *ABCD* and *abcd*, the PM pole (filled with rare-earth magnetic material), is denoted as Region *II*. The inner core enveloped by *abcdO* filled with ferromagnetic materials such as soft iron is denoted as Region *III*. The ferromagnetic core in Region *III* can “converge” the magnetic flux created by PM poles and reduce the magnetic energy loss. It is found that filling Region *III* with magnetized rare-earth material does not have significant improvement on the strength of the magnetic flux density. Rather, the inertia moment of the rotor increases and the dynamic performance of the rotor will be affected.

Relations between **H** and **B** for three regions

Region *I* (air) and *III* (iron):

According to the material properties of Region *I* (air) and Region *III* (iron), it is readily to obtain equations relating magnetic field intensity **H** (A/m) to flux density **B** (T) for these two regions [70]:

$$\mathbf{B}_I = \mu_0 \mathbf{H}_I, \quad (2.1)$$

$$\mathbf{B}_{III} = \mu_0 \mu_r \mathbf{H}_{III}, \quad (2.2)$$

where μ_0 is permeability of the free space with a value of $4\pi \times 10^{-7}$ H/m, dimensionless quantity μ_r is the relative permeability of soft iron having typical value larger than 4000.

Figure 2.3: Typical hysteresis loop of rare-earth magnet

Region II (PM):

PM can be characterized by a hysteresis loop which relates $\mathbf{H}$ to $\mathbf{B}$. Figure 2.3(a) presents a typical full hysteresis loop or B-H curve for a rare-earth magnet. From the figure, it can be seen that the variation of $\mathbf{B}$ always lags behind that of $\mathbf{H}$. This phenomenon is called hysteresis of PMs. The second quadrant of the B-H curve, commonly referred to as the demagnetization curve, describes the behavior of magnetic characteristics in actual use. According to Fig. 2.3(b), it can be found that rare-earth magnet exhibits high coercivity and nearly linear demagnetization property. The slope of this demagnetization curve is defined as the recoil permeability μ_{rec} (H/m), or $\mu_0\mu_m$, where dimensionless quantity μ_m is called relative recoil permeability of magnet having typical value between 1.05 and 1.20. Therefore, for Region II (PM) of the rotor, the demagnetization curve can be represented by [70]

$$\mathbf{B}_{II} = \mu_0\mu_m\mathbf{H}_{II} + \mu_0\mathbf{M}_0, \quad (2.3)$$

where $\mathbf{M}_0 = \mathbf{B}_{rem}/\mu_0$ (A/m) is the residual magnetization vector and $\mathbf{B}_{rem}$ (T) is remanence of PM.

Laplace's equations for three regions

By using the relations between $\mathbf{H}$ and $\mathbf{B}$ in Eqns. (2.1)-(2.3), Laplace's equations for three regions of the rotor space can be derived as follows.

Magnetic intensity $\mathbf{H}$ vs. scalar potential Φ :

It is known that for three regions of the rotor space, following equation can be obtained [88]

$$\nabla \times \mathbf{H}_i = 0, \quad (2.4)$$

where $i = I, II$ and III . Eqn. (2.4) means that $\mathbf{H}_i$ is an irrotational field. According to Helmholtz's theorem [89], the magnetic intensity can be expressed as the gradient of a scalar potential Φ_i , i.e.,

$$\mathbf{H}_i = -\nabla \Phi_i. \quad (2.5)$$

Using the spherical coordinates (r, θ, ϕ) , the magnetic field intensity $\mathbf{H}_i$ can be expressed as

$$\begin{aligned} \mathbf{H}_i &= H_{ir}\mathbf{e}_r + H_{i\theta}\mathbf{e}_\theta + H_{i\phi}\mathbf{e}_\phi \\ &= -\left(\frac{\partial \Phi_i}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial \Phi_i}{\partial \theta}\mathbf{e}_\theta + \frac{1}{r \sin \theta}\frac{\partial \Phi_i}{\partial \phi}\mathbf{e}_\phi\right), \end{aligned}$$

where $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ are unit vectors in r , θ and ϕ directions respectively. H_{ir} , $H_{i\theta}$ and $H_{i\phi}$ are three components of the magnetic field intensity $\mathbf{H}_i$, where

$$H_{ir} = -\frac{\partial \Phi_i}{\partial r}, \quad H_{i\theta} = -\frac{1}{r}\frac{\partial \Phi_i}{\partial \theta}, \quad H_{i\phi} = -\frac{1}{r \sin \theta}\frac{\partial \Phi_i}{\partial \phi}. \quad (2.6)$$

According to Maxwell's equations [90], a source-free or solenoidal magnetic field has the following property

$$\nabla \cdot \mathbf{B}_i = 0, \quad (2.7)$$

where $i = I, II$ and III .

Laplace's equation for outside rotor (Region I):

Outside the rotor (Region I), substituting Eqn. (2.1) into Eqn. (2.5) and Eqn. (2.7), we have

$$\nabla \cdot \mathbf{B}_I = \nabla \cdot (\mu_0 \mathbf{H}_I) = \nabla \cdot [\mu_0 (-\nabla \Phi_I)] = 0.$$

Thus,

$$\nabla^2 \Phi_I = 0, \quad (2.8)$$

where ∇^2 is Laplacian operator and Φ_I is the scalar potential in Region I (air).

Laplace's equation for PM (Region II):

Similarly, from Eqns. (2.3), (2.5) and (2.7), following equation can be obtained readily

$$\nabla \cdot \mathbf{B}_{II} = \nabla \cdot [-\mu_0 \mu_m (\nabla \Phi_{II})] + \nabla \cdot (\mu_0 \mathbf{M}_0) = 0,$$

which yields

$$-\mu_m \nabla^2 \Phi_{II} + \nabla \cdot \mathbf{M}_0 = 0,$$

that is

$$\nabla^2 \Phi_{II} = \nabla \cdot \mathbf{M}_0 / \mu_m. \quad (2.9)$$

Eqn. (2.9) is in the form of Poisson's equation. Due to symmetry of the rotor pole arrangement, the divergence of the residual magnetization vector is equal to zero, i.e., $\nabla \cdot \mathbf{M}_0 = 0$. The Poisson's equation can be reduced to Laplace's equation as

$$\nabla^2 \Phi_{II} = 0. \quad (2.10)$$

Laplace's equation for iron core (Region III):

Similar to Region *I*, scalar potential Φ_{III} of the soft iron core (Region *III*) can also be obtained as

$$\nabla^2 \Phi_{III} = 0. \quad (2.11)$$

General solution of Laplace's equation

In the spherical coordinates, the Laplace's equation can be expressed as

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \Phi_i) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial \Phi_i}{\partial \theta}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial \Phi_i}{\partial \phi} = 0. \quad (2.12)$$

where $i = I, II$ and *III*, indicating the region of concern. Using separation of variables method, the scalar potential Φ_i can be represented as $\Phi_i(r, \theta, \phi) = R_i(r) \Theta_i(\theta) \Psi_i(\phi)$, where $R_i(r)$, $\Theta_i(\theta)$ and $\Psi_i(\phi)$ are functions of the spherical coordinates. Substituting $\Phi_i(r, \theta, \phi)$ into Eqn. (2.12), the general solution of the scalar potential Φ_i can be expressed as [91]

$$\Phi_i = \sum_{n=0}^{\infty} \sum_{m=-n}^n [\kappa_{ni}^m r^n + \xi_{ni}^m r^{-(n+1)}] [Y_n^m(\theta, \phi)], \quad (2.13)$$

where κ_{ni}^m and ξ_{ni}^m are constants to be determined by boundary conditions. $Y_n^m(\theta, \phi)$ is the spherical harmonic functions defined by

$$Y_n^m(\theta, \phi) = \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} [P_n^m(\cos \theta)] e^{im\phi},$$

where $P_n^m(\cos \theta)$ is associated Legendre functions, and n, m are integers with $-n \leq m \leq n$. Note that spherical harmonics are complex valued functions.

2.2.3 Spherical Harmonic Expansion of M_{0r}

Let M_0 be the magnitude of the residual magnetization vector $\mathbf{M}_0$. With reference to Fig. 2.4 that illustrates the poles placement on the equatorial plane of the rotor, the constituents M_{0r} , $M_{0\theta}$ and $M_{0\phi}$ of $\mathbf{M}_0$ in the directions $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ can be computed

Figure 2.4: Poles on the equatorial plane of the rotor in spherical coordinates

as

$$M_{0r} = (-1)^{p-1} M_0 \cos\left[\phi - \alpha_0 - \frac{2\pi}{P}(p-1)\right] \sin \theta, \quad (2.14)$$

$$M_{0\theta} = (-1)^{p-1} M_0 \cos\left[\phi - \alpha_0 - \frac{2\pi}{P}(p-1)\right] \cos \theta, \quad (2.15)$$

$$M_{0\phi} = (-1)^p M_0 \sin\left[\phi - \alpha_0 - \frac{2\pi}{P}(p-1)\right], \quad (2.16)$$

where $p = 1, 2, \dots, P$. P is the total number of PM poles. In this study, $P = 8$, and α_0 is the PM pole angle at the center line in ϕ -direction. Note that these equations are only valid within the range of

$$\frac{\pi}{4}(p-1) + \alpha_0 - \frac{\alpha}{2} < \phi < \frac{\pi}{4}(p-1) + \alpha_0 + \frac{\alpha}{2}, \quad (2.17)$$

$$\frac{\pi}{2} - \frac{\beta}{2} < \theta < \frac{\pi}{2} + \frac{\beta}{2}, \quad (2.18)$$

in the PM pole (Region II). For the rest non-magnetized regions in the rotor, the residual magnetization is equal to zero.

When applying boundary conditions to solve the unknowns in the general solution of scalar potential, only the radial component of residual magnetization vector M_{0r} will be used to express the flux density continuity between regions. Components $M_{0\theta}$ and

$M_{0\phi}$ do not exist in any boundary condition. The radial component can be expressed as an expansion of spherical harmonic functions $Y_n^m(\theta, \phi)$ as [59]:

$$M_{0r}^s(\theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^n C_{nm} Y_n^m(\theta, \phi), \quad (2.19)$$

where C_{nm} are coefficients determined from the surface integral of the following form:

$$C_{nm} = \int_0^{\pi} \int_0^{2\pi} M_{0r}(\theta, \phi) Y_n^{m*}(\theta, \phi) \sin \theta d\theta d\phi, \quad (2.20)$$

and $Y_n^{m*}(\theta, \phi)$ denotes the complex conjugate of $Y_n^m(\theta, \phi)$. Substituting Eqn. (2.14) into Eqn. (2.20) gives the coefficients

$$C_{nm} = M_0 \int_0^{2\pi} f(\phi) e^{-im\phi} d\phi \int_0^{\pi} \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} [P_n^m(\cos \theta)] \sin^2 \theta d\theta, \quad (2.21)$$

where

$$f(\phi) = (-1)^{p-1} \cos[\phi - \alpha_0 - \frac{\pi}{4}(p-1)], \quad p = 1, 2, \dots, 8. \quad (2.22)$$

It is found that $C_{nm} \neq 0$ if and only if $m = \pm 4, \pm 12, \pm 20, \dots$. Thus, the fundamental terms of the spherical harmonic functions can be taken at $n = 4$ and $m = \pm 4$. For simplicity, only these terms are used for the derivation of the magnetic field. Denote

$$a \pm bi \equiv \int_0^{2\pi} f(\phi) e^{-im\phi} d\phi \quad (m = 4 \text{ and } m = -4), \quad (2.23)$$

$$c/\sqrt{\pi} \equiv \int_0^{\pi} \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} \sin^2 \theta [P_n^m(\cos \theta)] d\theta, \quad (2.24)$$

where a , b and c are real numbers. As M_{0r} is available within the range defined by Eqns. (2.17) and (2.18), integrals in Eqns. (2.23) and (2.24) are also constrained by the same range. Out of this range, the integrals are equal to zero. It can be verified that the results of the second integral for $m = 4$ and $m = -4$ are the same. Consequently, the coefficients $C_{4,4}$ and $C_{4,-4}$ can be obtained based on Eqn. (2.21) as

$$C_{4,4} = M_0 \frac{1}{\sqrt{\pi}} (a + bi)c, \quad C_{4,-4} = M_0 \frac{1}{\sqrt{\pi}} (a - bi)c, \quad (2.25)$$

where $C_{4,-4}$ is the complex conjugate of $C_{4,4}$. Therefore, the radial component M_{0r} of residual magnetization vector can be expressed in terms of spherical harmonics as

$$M_{0r}^s(\theta, \phi) = C_{4,-4}Y_4^{-4}(\theta, \phi) + C_{4,4}Y_4^4(\theta, \phi), \quad (2.26)$$

where $Y_4^{-4} = 3/16\sqrt{35/2\pi}\sin^4\theta e^{-4i\phi}$ and $Y_4^4 = 3/16\sqrt{35/2\pi}\sin^4\theta e^{4i\phi}$.

2.2.4 Boundary Conditions

Utilizing the boundary conditions in between different regions of the rotor space as well as the spherical harmonic expansion of M_{0r} , coefficients κ_{nI}^m , κ_{nII}^m , κ_{nIII}^m , ξ_{nI}^m , ξ_{nII}^m and ξ_{nIII}^m , in the general solution of magnetic scalar potentials of Eqn. (2.13) can be obtained. Because only the flux density in Region *I* can produce actuator force/torque by interacting with the air-core coils, the coefficients κ_{nI}^m and ξ_{nI}^m are of significant importance.

Boundary condition A or far field boundary condition ($B_{Ir}|_{r \rightarrow \infty} = 0$, $B_{I\theta}|_{r \rightarrow \infty} = 0$ and $B_{I\phi}|_{r \rightarrow \infty} = 0$)

The magnetic flux density decreases when the radial distance r increases. Three components of the flux density, B_{Ir} , $B_{I\theta}$ and $B_{I\phi}$ tend to zero when $r \rightarrow \infty$. According to Eqns. (2.6) and (2.13), the boundary condition $B_{Ir}|_{r \rightarrow \infty} = 0$ can be written as

$$\begin{aligned} B_{Ir}|_{r \rightarrow \infty} &= -\mu_0 \frac{\partial \Phi_I}{\partial r} \Big|_{r \rightarrow \infty} \\ &= -\mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^n [n\kappa_{nI}^m r^{n-1} - (n+1)\xi_{nI}^m r^{-(n+2)}] [Y_n^m(\theta, \phi)] \Big|_{r \rightarrow \infty} = 0. \end{aligned}$$

Through inspection of the exponential terms, it can be concluded that $\kappa_{nI}^m = 0$. With the aid of Eqns. (2.6) and (2.13), the boundary conditions $B_{I\theta}|_{r \rightarrow \infty} = 0$ and $B_{I\phi}|_{r \rightarrow \infty} = 0$ can be expressed as follows.

$$B_{I\theta}|_{r \rightarrow \infty} = -\mu_0 \frac{1}{r} \frac{\partial \Phi_I}{\partial \theta} \Big|_{r \rightarrow \infty}$$

$$\begin{aligned}
 &= -\mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^n [\kappa_{nI}^m r^{n-1} + \xi_{nI}^m r^{-(n+2)}] \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} \frac{\partial [P_n^m(\cos \theta)]}{\partial \theta} e^{im\phi} \Big|_{r \rightarrow \infty} \\
 &= 0, \\
 B_{I\phi} \Big|_{r \rightarrow \infty} &= -\mu_0 \frac{1}{r \sin \theta} \frac{\partial \Phi_I}{\partial \phi} \Big|_{r \rightarrow \infty} \\
 &= -\mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^n [\kappa_{nI}^m r^{n-1} + \xi_{nI}^m r^{-(n+2)}] \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} \frac{P_n^m(\cos \theta)}{\sin \theta} e^{im\phi} (im) \Big|_{r \rightarrow \infty} \\
 &= 0.
 \end{aligned}$$

These two conditions lead to the same result $\kappa_{nI}^m = 0$, as that of $B_{Ir} \Big|_{r \rightarrow \infty} = 0$.

Boundary condition B ($B_{Ir} \Big|_{r=R_r} = B_{IIr} \Big|_{r=R_r}$)

On the interface of Region *I* (air) and Region *II* (PM) as shown in Fig. 2.5 (a), the components of the flux density B_{Ir} and B_{IIr} are normal to the interfacial surface of the two neighboring medium. Applying the law of conservation of the magnetic flux [92] to the cylindrical volume in Fig. 2.5 (a) and allowing $\Delta h \rightarrow 0$, the result of $B_{Ir} \Delta S = B_{IIr} \Delta S$ can be obtained, hence $B_{Ir} \Big|_{r=R_r} = B_{IIr} \Big|_{r=R_r}$, where R_r is the radius of the rotor that defines the spherical boundary between these two regions.

Projecting all terms of Eqn. (2.3) into the r -direction gives

$$B_{IIr} = \mu_0 \mu_m H_{IIr} + \mu_0 M_{0r}. \quad (2.27)$$

Figure 2.5: Continuity boundary conditions

Substituting Eqn. (2.19) into Eqn. (2.27), the radial component $B_{\mathbb{I}r}$ can be expressed as

$$B_{\mathbb{I}r} = \mu_0 \mu_m H_{\mathbb{I}r} + \mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^n C_{nm} [Y_n^m(\theta, \phi)]. \quad (2.28)$$

By taking advantage of Eqns. (2.6) and (2.13), Eqn. (2.28) can be rewritten as

$$\begin{aligned} B_{\mathbb{I}r} = & -\mu_0 \mu_m \sum_{n=0}^{\infty} \sum_{m=-n}^n [n \kappa_{n\mathbb{I}}^m r^{n-1} - (n+1) \xi_{n\mathbb{I}}^m r^{-(n+2)}] [Y_n^m(\theta, \phi)]|_{r=R_r} \\ & + \mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^n C_{nm} [Y_n^m(\theta, \phi)]. \end{aligned} \quad (2.29)$$

Therefore, the boundary condition $B_{Ir}|_{r=R_r} = B_{\mathbb{I}r}|_{r=R_r}$ can be expanded to

$$\begin{aligned} & -\mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^n [-(n+1) \xi_{nI}^m r^{-(n+2)}] [Y_n^m(\theta, \phi)]|_{r=R_r} \\ = & -\mu_0 \mu_m \sum_{n=0}^{\infty} \sum_{m=-n}^n [n \kappa_{n\mathbb{I}}^m r^{n-1} - (n+1) \xi_{n\mathbb{I}}^m r^{-(n+2)}] [Y_n^m(\theta, \phi)]|_{r=R_r} \\ & + \mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^n C_{nm} [Y_n^m(\theta, \phi)]. \end{aligned} \quad (2.30)$$

As the spherical harmonics $Y_n^m(\theta, \phi)$ are orthonormal functions [93], Eqn. (2.30) holds for each pair of n and m . The following relation can be subsequently obtained

$$(n+1) \xi_{nI}^m = -\mu_m [n \kappa_{n\mathbb{I}}^m R_r^{2n+1} - (n+1) \xi_{n\mathbb{I}}^m] + C_{nm} R_r^{n+2}. \quad (2.31)$$

Boundary condition C ($H_{I\phi}|_{r=R_r} = H_{\mathbb{I}\phi}|_{r=R_r}$ and $H_{I\theta}|_{r=R_r} = H_{\mathbb{I}\theta}|_{r=R_r}$)

According to Ampere's circuital law [92], the line integral of the magnetic intensity along any closed contour is always equal to the total real current crossing a surface limited by the contour. As shown in Fig. 2.5 (b), a narrow rectangular contour at the boundary surface is indicated by $abcd$. The lengths of l_{da} and l_{bc} tend to be close to zero. If there is no real surface currents on the interface, the result of $H_{\mathbb{I}t} l_{cd} - H_{It} l_{ab} = 0$ can be obtained, i.e., $H_{It} = H_{\mathbb{I}t}$ with $l_{cd} = l_{ab}$, where H_{It} and

H_{It} are tangent to the surface. For the spherical actuator, there are two components H_θ and H_ϕ of the magnetic field intensity which are tangent to the rotor surface between Region *I* (air) and Region *II* (PM). Therefore, $H_{It} = H_{It}$ can be rewritten as $H_{I\phi}|_{r=R_r} = H_{II\phi}|_{r=R_r}$ and $H_{I\theta}|_{r=R_r} = H_{II\theta}|_{r=R_r}$, where $r = R_r$ defines the boundary surface between Region *I* (air) and *II* (PM).

From Eqns. (2.6) and (2.13), it can be obtained that

$$H_{I\phi} = -\frac{1}{r \sin \theta} \sum_{n=0}^{\infty} \sum_{m=-n}^n \xi_{nI}^m r^{-(n+1)} \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} [P_n^m(\cos \theta)] e^{im\phi} (im),$$

$$H_{II\phi} = -\frac{1}{r \sin \theta} \sum_{n=0}^{\infty} \sum_{m=-n}^n [\kappa_{nII}^m r^n + \xi_{nII}^m r^{-(n+1)}] \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} [P_n^m(\cos \theta)] e^{im\phi} (im).$$

Substituting these two equations into the boundary condition of $H_{I\phi}|_{r=R_r} = H_{II\phi}|_{r=R_r}$ yields

$$\xi_{nI}^m R_r^{-(n+1)} = \kappa_{nII}^m R_r^n + \xi_{nII}^m R_r^{-(n+1)},$$

that is,

$$\xi_{nI}^m = \kappa_{nII}^m R_r^{2n+1} + \xi_{nII}^m. \quad (2.32)$$

Similarly, the following equations can be obtained for the θ -direction

$$H_{I\theta} = -\sum_{n=0}^{\infty} \sum_{m=-n}^n \xi_{nI}^m r^{-(n+2)} \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} \frac{\partial [P_n^m(\cos \theta)]}{\partial \theta} e^{im\phi},$$

$$H_{II\theta} = -\frac{1}{r} \sum_{n=0}^{\infty} \sum_{m=-n}^n [\kappa_{nII}^m r^n + \xi_{nII}^m r^{-(n+1)}] \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} \frac{\partial [P_n^m(\cos \theta)]}{\partial \theta} e^{im\phi}.$$

From these two equations, it can be verified that the boundary condition of $H_{I\theta}|_{r=R_r} = H_{II\theta}|_{r=R_r}$ yields the same result as Eqn. (2.32).

Finite boundary condition D at $r = 0$ ($B_{\text{III}r}|_{r=0} \neq \infty$, $B_{\text{III}\theta}|_{r=0} \neq \infty$ and $B_{\text{III}\phi}|_{r=0} \neq \infty$)

This boundary condition comes from the fact that it is impossible to achieve an infinite value of flux density. According to Eqns. (2.6) and (2.13), the boundary condition, $B_{\text{III}r}|_{r=0} \neq \infty$, can be written as

$$B_{\text{III}r}|_{r=0} = -\mu_0\mu_r \sum_{n=0}^{\infty} \sum_{m=-n}^n [n\kappa_{n\text{III}}^m r^{n-1} - (n+1)\xi_{n\text{III}}^m r^{-(n+2)}][Y_n^m(\theta, \phi)]|_{r=0} \neq \infty.$$

This equation indicates that $\xi_{n\text{III}}^m = 0$. It can be verified that $B_{\text{III}\theta}|_{r=0} \neq \infty$ and $B_{\text{III}\phi}|_{r=0} \neq \infty$ yield the same result.

Boundary condition E ($B_{\text{II}r}|_{r=R_b} = B_{\text{III}r}|_{r=R_b}$)

This boundary condition is similar to BC-B. Following the same development, BC-E results in

$$\begin{aligned} & -\mu_0\mu_r \sum_{n=0}^{\infty} \sum_{m=-n}^n [n\kappa_{n\text{III}}^m r^{n-1}][Y_n^m(\theta, \phi)]|_{r=R_b} \\ & = -\mu_0\mu_m \sum_{n=0}^{\infty} \sum_{m=-n}^n [n\kappa_{n\text{II}}^m r^{n-1} - (n+1)\xi_{n\text{II}}^m r^{-(n+2)}][Y_n^m(\theta, \phi)]|_{r=R_b} \\ & + \mu_0 \sum_{n=0}^{\infty} \sum_{m=-n}^n C_{nm}[Y_n^m(\theta, \phi)], \end{aligned} \quad (2.33)$$

that is,

$$\mu_r n \kappa_{n\text{III}}^m R_b^{2n+1} = \mu_m [n \kappa_{n\text{II}}^m R_b^{2n+1} - (n+1) \xi_{n\text{II}}^m] - C_{nm} R_b^{n+2} \quad (2.34)$$

Boundary condition F ($H_{\text{II}\phi}|_{r=R_b} = H_{\text{III}\phi}|_{r=R_b}$ and $H_{\text{II}\theta}|_{r=R_b} = H_{\text{III}\theta}|_{r=R_b}$)

Following the same procedure of BC-C, $H_{\text{II}\phi}|_{r=R_b} = H_{\text{III}\phi}|_{r=R_b}$ can lead to

$$\begin{aligned} & -\frac{1}{r \sin \theta} \sum_{n=0}^{\infty} \sum_{m=-n}^n (\kappa_{n\text{III}}^m r^n) \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} [P_n^m(\cos \theta)] e^{im\phi}(im)|_{r=R_b} \\ & = -\frac{1}{r \sin \theta} \sum_{n=0}^{\infty} \sum_{m=-n}^n [\kappa_{n\text{II}}^m r^n + \xi_{n\text{II}}^m r^{-(n+1)}] \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} [P_n^m(\cos \theta)] e^{im\phi}(im)|_{r=R_b}, \end{aligned}$$

that is

$$\kappa_{n\text{III}}^m R_b^{2n+1} = \kappa_{n\text{II}}^m R_b^{2n+1} + \xi_{n\text{II}}^m. \quad (2.35)$$

$H_{\text{II}\theta}|_{r=R_b} = H_{\text{III}\theta}|_{r=R_b}$ also yields the same result.

Solution of coefficients ξ_{nI}^m and κ_{nI}^m

So far, the values or relationships of coefficients ξ_{nI}^m , $\xi_{n\text{II}}^m$, $\xi_{n\text{III}}^m$, κ_{nI}^m , $\kappa_{n\text{II}}^m$ and $\kappa_{n\text{III}}^m$ have been derived from boundary conditions. Specifically, BC-A and BC-D produce $\kappa_{nI}^m = 0$ and $\xi_{n\text{III}}^m = 0$ respectively; BC-B, BC-C, BC-E and BC-F lead to Eqn. (2.31), (2.32), (2.34) and (2.35) respectively. According to Lorentz force law, only the magnetic field in Region *I* (air) generates actuator torque. Therefore, solutions of ξ_{nI}^m and κ_{nI}^m are important ($\kappa_{nI}^m = 0$). By using Eqns. (2.31), (2.32), (2.34) and (2.35), coefficient ξ_{nI}^m can be calculated as follows.

First, multiplying Eqn. (2.35) by $\mu_r n$ and adding into Eqn. (2.34) yield

$$(\mu_r - \mu_m)n\kappa_{n\text{II}}^m R_b^{2n+1} + [\mu_r n + \mu_m(n+1)]\xi_{n\text{II}}^m + C_{nm}R_b^{n+2} = 0. \quad (2.36)$$

Substituting $\xi_{n\text{II}}^m$ in Eqn. (2.32) into Eqn. (2.36) gives

$$\kappa_{n\text{II}}^m = -\frac{[\mu_r n + \mu_m(n+1)]\xi_{nI}^m + C_{nm}R_b^{n+2}}{(\mu_r - \mu_m)nR_b^{2n+1} - [\mu_r n + \mu_m(n+1)]R_r^{2n+1}}. \quad (2.37)$$

From Eqns. (2.32) and (2.37), the following result can be obtained

$$\xi_{n\text{II}}^m = \xi_{nI}^m + \frac{R_r^{2n+1}\{[\mu_r n + \mu_m(n+1)]\xi_{nI}^m + C_{nm}R_b^{n+2}\}}{(\mu_r - \mu_m)nR_b^{2n+1} - [\mu_r n + \mu_m(n+1)]R_r^{2n+1}}. \quad (2.38)$$

Substituting Eqns. (2.37) and (2.38) into Eqn. (2.31) yields

$$\xi_{nI}^m = C_{nm}d_n, \quad (2.39)$$

where

$$d_n = -d_n^\top / d_n^\perp, \quad (2.40)$$

and

$$\begin{aligned} d_n^\top &= R_r^{n+2} + \frac{\mu_m(2n+1)R_b^{n+2}R_r^{2n+1}}{(\mu_r - \mu_m)nR_b^{2n+1} - [\mu_r n + \mu_m(n+1)]R_r^{2n+1}}, \\ d_n^\perp &= (\mu_m - 1)(n+1) \\ &\quad + \frac{\mu_m(2n+1)[\mu_r n + \mu_m(n+1)]R_r^{2n+1}}{(\mu_r - \mu_m)nR_b^{2n+1} - [\mu_r n + \mu_m(n+1)]R_r^{2n+1}}. \end{aligned}$$

Particularly, for $n = 4$ and $m = \pm 4$, the coefficients of ξ_{4I}^4 and ξ_{4I}^{-4} can be expressed with $C_{4,4}$ and $C_{4,-4}$ in Eqn. (2.25) as

$$\begin{aligned} \xi_{4I}^4 &= C_{4,4}d_4 = M_0 \frac{d_4}{\sqrt{\pi}}(a + bi)c, \\ \xi_{4I}^{-4} &= C_{4,-4}d_4 = M_0 \frac{d_4}{\sqrt{\pi}}(a - bi)c, \end{aligned} \quad (2.41)$$

where

$$d_4 = -d_4^\top / d_4^\perp, \quad (2.42)$$

and

$$\begin{aligned} d_4^\top &= R_r^6 + \frac{9\mu_m R_b^6 R_r^9}{4(\mu_r - \mu_m)R_b^9 - (4\mu_r + 5\mu_m)R_r^9}, \\ d_4^\perp &= 5(\mu_m - 1) + \frac{9\mu_m(4\mu_r + 5\mu_m)R_r^9}{4(\mu_r - \mu_m)R_b^9 - (4\mu_r + 5\mu_m)R_r^9}, \end{aligned}$$

2.2.5 Solutions of Scalar Potential and Flux Density

The derivation of the flux density is performed in the space outside of the rotor (Region I), because only the magnetic field in this region is used to compute the motor torque according to Lorentz force law. Substituting the coefficients of ξ_{4I}^4 and ξ_{4I}^{-4} in Eqn. (2.41) into Eqn. (2.13) and discarding the higher order harmonic terms result in

$$\begin{aligned} \Phi_I &= \xi_{4I}^{-4} r^{-5} [Y_4^{-4}(\theta, \phi)] + \xi_{4I}^4 r^{-5} [Y_4^4(\theta, \phi)] \\ &= M_0 \frac{d_4}{\sqrt{\pi}} (a - bi) cr^{-5} \left(\frac{3}{16} \sqrt{\frac{35}{2\pi}} \sin^4 \theta e^{-4i\phi} \right) + M_0 \frac{d_4}{\sqrt{\pi}} (a + bi) cr^{-5} \left(\frac{3}{16} \sqrt{\frac{35}{2\pi}} \sin^4 \theta e^{4i\phi} \right) \\ &= M_0 \frac{3cd_4}{8\pi} \sqrt{\frac{35}{2}} r^{-5} \sin^4 \theta (a \cos 4\phi - b \sin 4\phi). \end{aligned} \quad (2.43)$$

Using Eqns. (2.1) and (2.6), the flux density in Region *I* (air) can be obtained

$$B_{Ir} = -\mu_0 \frac{\partial \Phi_I}{\partial r} = \frac{15\mu_0 M_0 c d_4}{8\pi} \sqrt{\frac{35}{2}} r^{-6} \sin^4 \theta (a \cos 4\phi - b \sin 4\phi), \quad (2.44)$$

$$B_{I\theta} = -\mu_0 \frac{1}{r} \frac{\partial \Phi}{\partial \theta} = \frac{12\mu_0 M_0 c d_4}{8\pi} \sqrt{\frac{35}{2}} r^{-6} \sin^3 \theta \cos \theta (b \sin 4\phi - a \cos 4\phi), \quad (2.45)$$

$$B_{I\phi} = -\mu_0 \frac{1}{r \sin \theta} \frac{\partial \Phi}{\partial \phi} = \frac{12\mu_0 M_0 c d_4}{8\pi} \sqrt{\frac{35}{2}} r^{-6} \sin^3 \theta (a \sin 4\phi + b \cos 4\phi). \quad (2.46)$$

2.2.6 Simplification of Magnetic Field Model

It is worth mentioning that in the derivation of the magnetic field model, a generic Cartesian coordinate system is set up as indicated in Fig. 2.4 (or Fig. 2.6(a)), with angle α_0 specifying the position of PM poles. By letting $\alpha_0 = 0$ as shown in Fig. 2.6(b), the magnetic field model can be simplified. In this case, the x - and y -axes pass through the center of PM poles. Correspondingly, Eqns. (2.14) - (2.16) can be simplified as

$$M_{0r} = (-1)^{p-1} M_0 \cos[\phi - \frac{\pi}{4}(p-1)] \sin \theta, \quad (2.47)$$

$$M_{0\theta} = (-1)^{p-1} M_0 \cos[\phi - \frac{\pi}{4}(p-1)] \cos \theta, \quad (2.48)$$

$$M_{0\phi} = (-1)^p M_0 \sin[\phi - \frac{\pi}{4}(p-1)], \quad (2.49)$$

which are only valid within the range of

$$\frac{\pi}{4}(p-1) - \frac{\alpha}{2} < \phi < \frac{\pi}{4}(p-1) + \frac{\alpha}{2}, \quad (2.50)$$

$$\frac{\pi}{2} - \frac{\beta}{2} < \theta < \frac{\pi}{2} + \frac{\beta}{2}, \quad (2.51)$$

in the PM pole (Region *II*). Applying these equations into Eqn. (2.20), it can be verified that the constant b is always equal to zero. Hence, all terms related to b in Eqns. (2.44) - (2.46) vanish. Eqns. (2.44) - (2.46) become

$$B_{Ir} = \frac{15\mu_0 M_0 a c d_4}{8\pi} \sqrt{\frac{35}{2}} r^{-6} \sin^4 \theta \cos 4\phi, \quad (2.52)$$

$$B_{I\theta} = -\frac{12\mu_0 M_0 a c d_4}{8\pi} \sqrt{\frac{35}{2}} r^{-6} \sin^3 \theta \cos \theta \cos 4\phi, \quad (2.53)$$

$$B_{I\phi} = \frac{12\mu_0 M_0 a c d_4}{8\pi} \sqrt{\frac{35}{2}} r^{-6} \sin^3 \theta \sin 4\phi. \quad (2.54)$$

This simplification can facilitate the torque formulation greatly.

Figure 2.6: Coordinates definition in rotor frame

2.3 Summary

This chapter has presented the magnetic field modeling of the PM-pole rotor. According to the magnetic characteristics of the three regions of the rotor space, Laplace's equations of magnetic scalar potentials are derived. By using the boundary conditions in between neighboring regions as well as the spherical harmonic expansion of radial component of the residual magnetization vector, coefficients in the general solution of magnetic scalar potential can be determined. The magnetic field flux density of the PM rotor can be formulated analytically by taking the gradient of the scalar potential. This analytical magnetic field model is described based on the dimensional parameters of the PM poles. Hence, it can be used in the design of the spherical actuator to maximize the torque output.

Chapter 3

Torque Modeling

3.1 Introduction

As an important topic in electromagnetic actuators, the torque modeling of the actuator is to establish the relation between current input and torque output mathematically. One typical usage of the torque model is for computer simulation of actuator motions as shown in Fig. 3.1(a). Given a set of current inputs, torque output of an actuator can be calculated according to the torque model. Based on forward dynamics of an actuator, one is able to simulate how an actuator would move under the action of the torque. Another significant use of the torque model is for position and velocity servo control of actuators. According to inverse dynamics of an actuator as shown in Fig. 3.1(b), the desired actuator torque can be computed from the desired angular displacement (θ_d), angular velocity ($\dot{\theta}_d$) and angular acceleration ($\ddot{\theta}_d$) of the actuator. Then by utilizing the torque model, required currents can be calculated from the desired actuator torque. These currents are supplied to the actuator so that it can follow the desired motion. An analytical solution of the torque model in a linear fashion is preferred because it can facilitate the real-time motion control of the complex mechatronic systems.

Generally speaking, there are three types of methods that can be employed for the

(a) Application for simulation

(b) Application for motion control

Figure 3.1: Functions of the actuator torque model

calculation of force and torque of electromagnetic products, i.e., Maxwell stress tensor (MST), coenergy method¹ and Lorentz force law. The MST approach computes local stress at all points of a boundary surface of a body, then sums the local stresses to obtain the global magnetic force/torque [70]. This classic method is usually used in finite element (FE) solution of force/torque computation. For example, Hamler *et al.* [94] took advantage of MST and an interpolation-based triangular mesh to compute the torque of a one-phase brushless motor. Im *et al.* [95] computed the force of a single-sided linear induction motor (SLIM)² by applying MST. Yamasawa [96] described the calculation of the magnetic thrust of an automotive magnetic actuator by expanding the MST to cover the nonlinear magnetic region. Davey *et al.* [18] proposed a general torque computation approach of spherical induction motors by using MST. The disadvantage of MST is that the result is very sensitive to discretization density and integration contour position. In addition, the definition of the boundary surface becomes difficult in 3D problems.

¹Coenergy method is also named virtual work method.

²A linear induction motor is conceptually a rotary induction motor whose stator core has been cut and unrolled. The circular stator becomes a linear stator, defining a single-sided linear induction motor (SLIM). Likewise, if the circular stator is cut into two sections and flattened, the motor becomes a double-sided linear induction motor (DLIM).

Figure 3.2: Closed magnetic flux loop

The coenergy method [97] can be employed to obtain the torque model of electromagnetic products analytically. Its basic principle is illustrated in Fig. 3.2. Many electromagnetic products possess coils and ferromagnetic materials such as soft iron disrupted by narrow airgaps. By energizing the coils, flux lines are generated and flow through the soft iron and airgaps, forming a closed magnetic flux loop. The relative permeability of soft iron (more than 4000) is much greater than that of the airgap (approximately 1). In other words, the reluctance of airgap is much larger than that of soft iron. Therefore, the magnetic energy is mostly stored in the narrow airgaps³. In this case, the magnetic energy can be formulated easily. Coenergy is equal to the difference of electrical power input and the magnetic energy stored. By differentiating the coenergy with respect to the displacement element x , the force acted on the mover can be calculated. If the displacement is from an angular element, then a torque model can be obtained. This approach was utilized by Lee *et al.* [29, 30, 35] to derive the torque model of VRSM successfully. Sahoo [98] and Materu [99] *et al.* have also obtained an analytical torque model of switched reluctance motors (SRM) in terms of the stator phase current and rotor position by using coenergy principle. One prerequisite of the implementation of coenergy method for torque modeling is that the magnetic energy of the airgap in actuators can be formulated readily.

³A magnetic circuit is very similar to an electric circuit. The equivalent to resistance is reluctance; the equivalent to current is flux; the equivalent to voltage is mmf or magnetomotive force.

However, due to the existence of large airgap or air-core coils in some cases, the magnetic energy cannot be formulated analytically and the coenergy method becomes ineffective. Alternatively, Lorentz force law can be adopted because energy formation is no longer required. Lorentz force law is especially useful when the force/torque is generated by a current-carrying conductor laying in the magnetic field of PM. As illustrated in Fig. 3.3, Lorentz force law [92] states that the force $d\mathbf{F}$ exerted on differential length segment $d\mathbf{l}$ of current carrying conductor by external magnetic field $\mathbf{B}$ is equal to the multiplication of the current I_w and the cross product of $d\mathbf{l}$ and $\mathbf{B}$, i.e. $d\mathbf{F} = I_w d\mathbf{l} \times \mathbf{B}$. By using Lorentz force law, Sathuvalli *et al.* [100] calculated the force on an electrically conducting sphere placed in an arbitrary sinusoidally varying 1D magnetic field analytically. Another implementation of this method was found in the torque formulation of spherical actuators developed by Wang *et al* [55, 59].

Due to the existence of the air-core coils and the large airgap between the rotor and the stator in our PM spherical actuator, the magnetic energy of the PM spherical actuator is difficult to formulate analytically. Based on previous research on the three torque modeling methods, Lorentz force law is adopted for the torque formulation of

Figure 3.3: Force on a current-carrying element

this PM spherical actuator. By utilizing the magnetic field model derived in Chapter 2, an analytical solution of the torque model is able to be obtained.

3.2 Formulation of Actuator Torque

3.2.1 Torque Generating Component of Flux Density

The direction of the force generated by each component of the flux density, B_{Ir} , $B_{I\theta}$ and $B_{I\phi}$ can be determined as shown in Fig. 3.4. The differential length segment $d\mathbf{l}$ of the wire is tangential to the spherical surface at point O . Note that only B_{Ir} can produce a torque to change the rotor orientation. $B_{I\theta}$ and $B_{I\phi}$ do not produce torque on the rotor because the action lines of magnetic forces generated by $B_{I\phi}$ and $B_{I\theta}$ pass through the rotor center. As a consequence, following discussion will focus on the radial component of the flux density B_{Ir} .

3.2.2 Torque Model for a Single Coil

The torque produced by a single coil due to the existence of the PM rotor magnetic field will be first studied. An ideal air-core coil used in the spherical actuator is illus-

Figure 3.4: Force activated by three components of the flux density

trated in Fig. 3.5. The coil assumes a conical-shaped object embedded in the stator shell to facilitate the formulation of the actuator torque. Without loss of generality, the dimensions of the coil (shaded region) can be specified by four quantities: R_0 -the center distance of the inner surface of the coil; R_1 -the center distance of the outer surface of the coil; ζ_0 -the angular diameter of the air-core and ζ_1 -the angular diameter of the coil.

Figure 3.5: Section view of the conical air-core coil

Force on differential line segment of winding

Consider a differential line segment $d\mathbf{l}$ of the winding. As shown in Fig. 3.5, the differential sectional area of $d\mathbf{l}$ can be computed by

$$ds = r dr d\zeta. \quad (3.1)$$

The current passing through this section area is $Jr dr d\zeta$, where J is the current density in the section area of the coil. According to Lorentz force law, the differential force on the rotor caused by the interaction between the magnetic field of the PM-pole rotor

and current carrying conductor $d\mathbf{l}$ is

$$\begin{aligned} d\mathbf{F} &= -I d\mathbf{l} \times B_{Ir}(r, \theta, \phi) \mathbf{e}_r \\ &= -J r dr d\zeta d\mathbf{l} \times B_{Ir}(r, \theta, \phi) \mathbf{e}_r, \end{aligned} \quad (3.2)$$

where $\mathbf{e}_r$ is the unit vector in the r -direction of spherical coordinates. The negative sign indicates that the force imposed on the rotor by $d\mathbf{l}$ is the reaction force exerted by the magnetic field on $d\mathbf{l}$.

Torque on differential line segment of winding

It can be seen that the differential torque generated by $d\mathbf{l}$ is the cross product of the moment arm $r\mathbf{e}_r$ (the vector from rotor center to the differential winding segment) and force $d\mathbf{F}$, i.e.,

$$d\mathbf{T}_i = r\mathbf{e}_r \times [-J r dr d\zeta d\mathbf{l} \times B_{Ir}(r, \theta, \phi) \mathbf{e}_r]. \quad (3.3)$$

Integration on entire coil volume

Integrating the differential torque in Eqn. (3.3) within the entire volume of the coil gives the torque of a single coil under the magnetic field of PM rotor as

$$\begin{aligned} \mathbf{T}_c &= \int \int \int_V r\mathbf{e}_r \times [-J r dr d\zeta d\mathbf{l} \times B_{Ir}(r, \theta, \phi) \mathbf{e}_r] \\ &= J \int_{R_0}^{R_1} \int_{\zeta_0}^{\zeta_1} \left\{ \int_C r\mathbf{e}_r \times [B_{Ir}(r, \theta, \phi) \mathbf{e}_r \times d\mathbf{l}] \right\} r dr d\zeta. \end{aligned} \quad (3.4)$$

The symbol $\int_C$ denotes the line integral of the differential torque along a circular loop of the winding that has a “wire” section area of ds . The total volume of the coil can be completely described by the four parameters R_0 , R_1 , ζ_0 and ζ_1 . Integration of the torque value is carried out within this volume. Because the differential length $d\mathbf{l}$ is tangential to the spherical surface, $d\mathbf{l}$ is perpendicular to $\mathbf{e}_r$. Therefore, the

relationship of $\mathbf{e}_r \times (\mathbf{e}_r \times d\mathbf{l}) = -d\mathbf{l}$ can be obtained. As a result, Eqn. (3.4) is reduced to

$$\mathbf{T}_c = -J \int_{R_0}^{R_1} \int_{\zeta_0}^{\zeta_1} \left\{ \int_C r B_{Ir}(r, \theta, \phi) d\mathbf{l} \right\} r dr d\zeta. \quad (3.5)$$

Solution to torque integral

With analytical expression of B_{Ir} and Eqn. (3.5), the actuator torque generated by the i^{th} coil, denoted as $\mathbf{T}_i$, can be expressed explicitly using the i^{th} coil-axis position θ_i and ϕ_i with respect to the rotor frame, as well as the current input J_i passing through this coil. For any specific rotor orientation, the torque output can then be determined by the current input uniquely.

Assume that only one coil is mounted on the stator. The spherical coordinates (θ_i, ϕ_i) is used to represent the position of the coil in the rotor frame. One loop of winding in the i^{th} coil on the sphere is indicated in Fig. 3.6; $\mathbf{e}_{\theta i}$ and $\mathbf{e}_{\phi i}$ are unit vectors of the spherical coordinates. The magnitude and the unit vector of the differential length

Figure 3.6: One loop of wire in the i^{th} coil on the sphere

segment $d\mathbf{l}$ can be calculated respectively as

$$\|d\mathbf{l}\| = r \sin \zeta d\psi, \quad (3.6)$$

$$\mathbf{e}_{d\mathbf{l}} = \sin \psi \mathbf{e}_{\theta i} - \cos \psi \mathbf{e}_{\phi i}. \quad (3.7)$$

Combining Eqns. (3.6) and (3.7) together, we have

$$d\mathbf{l} = r \sin \zeta d\psi (\sin \psi \mathbf{e}_{\theta i} - \cos \psi \mathbf{e}_{\phi i}). \quad (3.8)$$

Substituting magnetic field component B_{Ir} of Eqn. (2.52) and $d\mathbf{l}$ of Eqn. (3.8) into torque integral formula of Eqn. (3.5) results in

$$\begin{aligned} \mathbf{T}_i = & \sqrt{\frac{35}{2}} \frac{15\mu_0 M_0 a c d_4}{8\pi} J_i \int_{R_0}^{R_1} \int_{\zeta_0}^{\zeta_1} \int_0^{2\pi} r^{-3} \sin^4 \theta \cos 4\phi \sin \zeta \\ & (\cos \psi \mathbf{e}_{\phi i} - \sin \psi \mathbf{e}_{\theta i}) d\psi dr d\zeta. \end{aligned} \quad (3.9)$$

For a particular coil, the ranges of integration are known: ζ varies from ζ_0 to ζ_1 ; r from R_0 to R_1 ; ψ from 0 to 2π . To integrate Eqn. (3.9), it is necessary to relate the terms $\sin \theta$ and $\cos 4\phi$ to ζ , ψ and r . For convenience, denote the angular portion of Eqn. (3.9) as

$$\mathbf{D}(\theta, \phi) \equiv \sin^4 \theta \cos 4\phi \sin \zeta (\cos \psi \mathbf{e}_{\phi i} - \sin \psi \mathbf{e}_{\theta i}). \quad (3.10)$$

Eqn. (3.9) can be compactly expressed as

$$\begin{aligned} \mathbf{T}_i = & \sqrt{\frac{35}{2}} \frac{15\mu_0 M_0 a c d_4}{8\pi} J_i \int_{R_0}^{R_1} \int_{\zeta_0}^{\zeta_1} \int_0^{2\pi} r^{-3} [\mathbf{D}(\theta, \phi)] d\psi dr d\zeta \\ = & \sqrt{\frac{35}{2}} \frac{15\mu_0 M_0 a c d_4 R_c}{16\pi} J_i \int_{\zeta_0}^{\zeta_1} \int_0^{2\pi} [\mathbf{D}(\theta, \phi)] d\psi d\zeta, \end{aligned} \quad (3.11)$$

where

$$R_c = R_0^{-2} - R_1^{-2}. \quad (3.12)$$

The integration of $\int_{\zeta_0}^{\zeta_1} \int_0^{2\pi} [\mathbf{D}(\theta, \phi)] d\psi d\zeta$ will be discussed below. First, rewrite $\mathbf{D}(\theta, \phi)$ as

$$\mathbf{D}(\theta, \phi) = \sin^4 \theta (\cos^4 \phi - 6 \sin^2 \phi \cos^2 \phi + \sin^4 \phi) \sin \zeta (\cos \psi \mathbf{e}_{\phi i} - \sin \psi \mathbf{e}_{\theta i}). \quad (3.13)$$

Recall the relationship of the Cartesian coordinates, x , y , z , to spherical coordinates θ and ϕ ,

$$\cos \phi = \frac{x}{r \sin \theta}, \quad \sin \phi = \frac{y}{r \sin \theta}. \quad (3.14)$$

Substituting Eqn. (3.14) into Eqn. (3.13) gives

$$\mathbf{D}(\theta, \phi) = \frac{1}{r^4} (x^4 - 6x^2y^2 + y^4) \sin \zeta (\cos \psi \mathbf{e}_{\phi_i} - \sin \psi \mathbf{e}_{\theta_i}). \quad (3.15)$$

With reference to Fig. 3.6, it can be verified that the Cartesian coordinates of the differential element on the coil, x , y and z can be represented in terms of ζ , ψ , θ_i and ϕ_i as following

$$\begin{aligned} x &= r \cos \zeta \sin \theta_i \cos \phi_i - r \sin \zeta \cos \psi \cos \theta_i \cos \phi_i + r \sin \zeta \sin \psi \sin \phi_i, \\ y &= r \cos \zeta \sin \theta_i \sin \phi_i - r \sin \zeta \cos \psi \cos \theta_i \sin \phi_i - r \sin \zeta \sin \psi \cos \phi_i, \\ z &= r \cos \zeta \cos \theta_i + r \sin \zeta \cos \psi \sin \theta_i. \end{aligned} \quad (3.16)$$

The unit vectors $\mathbf{e}_{\phi_i}$ and $\mathbf{e}_{\theta_i}$ in spherical coordinates can be represented in terms of Cartesian coordinates as

$$\mathbf{e}_{\phi_i} = -\sin \phi_i \mathbf{e}_x + \cos \phi_i \mathbf{e}_y = \begin{bmatrix} -\sin \phi_i \\ \cos \phi_i \\ 0 \end{bmatrix}, \quad (3.17)$$

$$\mathbf{e}_{\theta_i} = \cos \theta_i \cos \phi_i \mathbf{e}_x + \cos \theta_i \sin \phi_i \mathbf{e}_y - \sin \theta_i \mathbf{e}_z = \begin{bmatrix} \cos \theta_i \cos \phi_i \\ \cos \theta_i \sin \phi_i \\ -\sin \theta_i \end{bmatrix}. \quad (3.18)$$

By substituting Eqns. (3.16), (3.17) and (3.18) into Eqn. (3.15), $\mathbf{D}(\theta, \phi)$ can be expressed as a function of ζ and ψ . Because the integration range of ψ is from 0 to 2π , many terms in $\int_{\zeta_0}^{\zeta_1} \int_0^{2\pi} [\mathbf{D}(\theta, \phi)] d\psi d\zeta$ vanish. Let

$$\mathbf{D}^e(\theta_i, \phi_i) \equiv \int_{\zeta_0}^{\zeta_1} \int_0^{2\pi} [\mathbf{D}(\theta, \phi)] d\psi d\zeta, \quad (3.19)$$

we have

$$\mathbf{D}^e(\theta_i, \phi_i) = \pi \left\{ \mathbf{e}_{\phi_i} \left[G'_\zeta (-3 \sin \theta_i \cos^4 \phi_i \cos^3 \theta_i - 18 \sin \theta_i \cos^2 \phi_i \cos \theta_i \sin^2 \phi_i \right. \right.$$

$$\begin{aligned}
& -3 \sin \theta_i \sin^4 \phi_i \cos^3 \theta_i + 18 \sin \theta_i \cos^2 \phi_i \cos^3 \theta_i \sin^2 \phi_i + 3 \sin \theta_i \cos^4 \phi_i \\
& \cos \theta_i + 3 \sin^4 \phi_i \sin \theta_i \cos \theta_i) + G''_{\zeta} (-4 \sin^3 \theta_i \cos^4 \phi_i \cos \theta_i - 4 \sin^3 \theta_i \\
& \sin^4 \phi_i \cos \theta_i + 24 \sin^3 \theta_i \cos^2 \phi_i \sin^2 \phi_i \cos \theta_i) - \mathbf{e}_{\theta_i} [G'_{\zeta} (12 \sin \theta_i \cos^3 \phi_i \\
& \cos^2 \theta_i \sin \phi_i + 12 \sin \theta_i \cos \phi_i \sin^3 \phi_i - 12 \sin \theta_i \sin^3 \phi_i \cos^2 \theta_i \cos \phi_i - 12 \sin \theta_i \\
& \sin \phi_i \cos^3 \phi_i) + G''_{\zeta} (16 \sin^3 \theta_i \cos^3 \phi_i \sin \phi_i - 16 \sin^3 \theta_i \sin^3 \phi_i \cos \phi_i)] \}, (3.20)
\end{aligned}$$

Eqn. (3.20) can be simplified as following

$$\begin{aligned}
\mathbf{D}^e(\theta_i, \phi_i) = & (G''_{\zeta} - \frac{3G'_{\zeta}}{4}) \pi \left\{ \mathbf{e}_{\phi_i} (-4 \sin^3 \theta_i \cos^4 \phi_i \cos \theta_i - 4 \sin^3 \theta_i \sin^4 \phi_i \cos \theta_i \right. \\
& + 24 \sin^3 \theta_i \cos^2 \phi_i \sin^2 \phi_i \cos \theta_i) - \mathbf{e}_{\theta_i} (16 \sin^3 \theta_i \cos^3 \phi_i \sin \phi_i - 16 \sin^3 \theta_i \\
& \left. \sin^3 \phi_i \cos \phi_i) \right\}. \quad (3.21)
\end{aligned}$$

where

$$G'_{\zeta} = 1/5 \sin^5 \zeta_1 - 1/5 \sin^5 \zeta_0, \quad (3.22)$$

$$\begin{aligned}
G''_{\zeta} = & 1/5 \cos^4 \zeta_0 \sin \zeta_0 - 1/15 \cos^2 \zeta_0 \sin \zeta_0 - 2/15 \sin \zeta_0 - 1/5 \cos^4 \zeta_1 \sin \zeta_1 \\
& + 1/15 \cos^2 \zeta_1 \sin \zeta_1 + 2/15 \sin \zeta_1. \quad (3.23)
\end{aligned}$$

Eqn. (3.21) can be rewritten in a compact form as

$$\mathbf{D}^e(\theta_i, \phi_i) = \pi G_{\zeta} [\mathbf{D}_c^e(\theta_i, \phi_i)]. \quad (3.24)$$

where

$$G_{\zeta} = G''_{\zeta} - \frac{3G'_{\zeta}}{4}, \quad (3.25)$$

which is completely determined by the geometry of coils, and

$$\begin{aligned}
\mathbf{D}_c^e(\theta_i, \phi_i) = & \mathbf{e}_{\phi_i} (-4 \sin^3 \theta_i \cos^4 \phi_i \cos \theta_i - 4 \sin^3 \theta_i \sin^4 \phi_i \cos \theta_i + 24 \sin^3 \theta_i \cos^2 \phi_i \sin^2 \phi_i \\
& \cos \theta_i) - \mathbf{e}_{\theta_i} (16 \sin^3 \theta_i \cos^3 \phi_i \sin \phi_i - 16 \sin^3 \theta_i \sin^3 \phi_i \cos \phi_i), \quad (3.26)
\end{aligned}$$

which is specified by the coil arrangement patterns.

Consequently, the integration of Eqn. (3.11) can be obtained as

$$\mathbf{T}_i = \frac{15}{16} \sqrt{\frac{35}{2}} \mu_0 M_0 a c d_4 R_c G_\zeta [\mathbf{D}_c^e(\theta_i, \phi_i)] J_i. \quad (3.27)$$

Note that $\mathbf{D}_c^e(\theta_i, \phi_i)$ is a 3×1 Cartesian vector. Parameters θ_i and ϕ_i represent the position of the i^{th} coil's axis relative to the rotor frame. Without loss of generality, let

$$\mathbf{G}(\theta_i, \phi_i) = [g_x(\theta_i, \phi_i) \ g_y(\theta_i, \phi_i) \ g_z(\theta_i, \phi_i)]^T \equiv \mathbf{D}_c^e(\theta_i, \phi_i). \quad (3.28)$$

Eqn. (3.27) can be written in the matrix form

$$\mathbf{T}_i = T_c \mathbf{G}(\theta_i, \phi_i) J_i \quad (3.29)$$

$$\text{or} \quad \begin{bmatrix} T_{xi} \\ T_{yi} \\ T_{zi} \end{bmatrix} = T_c \begin{bmatrix} g_x(\theta_i, \phi_i) \\ g_y(\theta_i, \phi_i) \\ g_z(\theta_i, \phi_i) \end{bmatrix} J_i, \quad (3.30)$$

where

$$T_c = \frac{15}{16} \sqrt{\frac{35}{2}} \mu_0 M_0 a c d_4 R_c G_\zeta. \quad (3.31)$$

3.2.3 Torque Model for Complete Set of Coils

For the i^{th} coil, the torque model of the actuator can be expressed as Eqn. (3.29). Hence, there will be N torque equations like Eqn. (3.30) with N coils mounted on the stator. The actuator torque is thus a vector sum of all the individual torque equations. Concatenating all N torque equations, we can obtain the torque model of the actuator in the rotor frame as

$$\mathbf{T} = T_c \begin{bmatrix} g_x(\theta_1, \phi_1) & g_x(\theta_2, \phi_2) & \cdots & g_x(\theta_N, \phi_N) \\ g_y(\theta_1, \phi_1) & g_y(\theta_2, \phi_2) & \cdots & g_y(\theta_N, \phi_N) \\ g_z(\theta_1, \phi_1) & g_z(\theta_2, \phi_2) & \cdots & g_z(\theta_N, \phi_N) \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_N \end{bmatrix}. \quad (3.32)$$

or

$$\mathbf{T} = T_c \mathbf{QJ}, \quad (3.33)$$

Figure 3.7: Two layers of twelve coils distributed around the PM-pole rotor

where

$$\mathbf{Q} = \begin{bmatrix} g_x(\theta_1, \phi_1) g_x(\theta_2, \phi_2) \cdots g_x(\theta_N, \phi_N) \\ g_y(\theta_1, \phi_1) g_y(\theta_2, \phi_2) \cdots g_y(\theta_N, \phi_N) \\ g_z(\theta_1, \phi_1) g_z(\theta_2, \phi_2) \cdots g_z(\theta_N, \phi_N) \end{bmatrix} \text{ is the torque matrix; } \theta_i \text{ and } \phi_i \text{ present the position of the } i^{th} \text{ coil in the rotor frame;} \quad (3.34)$$

$$\mathbf{J} = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_N \end{bmatrix} \text{ is the current density vector of electric currents passing through Coil 1, Coil 2, } \cdots, \text{ and Coil } N. \quad (3.35)$$

The torque model of Eqn. (3.33) can be generically applicable to the study of spherical actuators with any number of coils. In this research, two layers of twelve coils are employed on the actuator as illustrated in Fig 3.7. Pairs of coils at positions 1, 2, 3 and 4 are for tilting motion of the rotor in two different directions. Through the attraction and repulsion force generated by rest of the coils (Fig. 3.7(b)), a spinning motion about the rotor axis can be achieved. The following discussion is based on this coil layout.

3.2.4 Orientation Dependence of Torque Model

Further inspection of Eqn. (3.32) shows that the positions of the coils (θ_i, ϕ_i) are defined based on the rotor frame. Because coils are fixed on the stator, when the rotor moves and change its orientation, (θ_i, ϕ_i) will change its value and thus the matrix $\mathbf{Q}$ and the torque value. This section shows how to obtain the torque matrix from the rotor orientation.

Representation of the rotor orientation

The workspace of the PM spherical actuator is shown in Fig. 3.8(a). A series of PM poles are mounted along the equator of the rotor, whereas the air-core coils are symmetrically mounted on the stator with respect to the stator equatorial plane, with two layers separated by an angular distance of θ_s . It can spin about its own z-axis, Z_r , in 360° without constraint. The rotor can also incline about its equatorial plane to an extreme position that the axes of a PM pole and the coil are aligned. Thus the z-axis of the rotor can move within a conical workspace with a conical angle of θ_s as shown in Fig. 3.8(a).

The orientation of the rotor frame with respect to the stator frame can be expressed by using Euler angles as follows. Let the rotor frame be (X_r, Y_r, Z_r) and the stator frame be (X_s, Y_s, Z_s) . In order to arrive at an arbitrary final orientation within the workspace, three rotations of the rotor have to take place in sequence (Fig. 3.8(b)). Suppose that the stator frame is coincident with the rotor frame initially, the Euler ZYZ angle expression of the rotor orientation can be written as the matrix multiplication of the three body rotation matrices $\mathbf{R}_z(\phi_r)$, $\mathbf{R}_y(\theta_r)$ and $\mathbf{R}_z(\delta_r)$ as [101]

$$\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} = \mathbf{R} \begin{bmatrix} X_r \\ Y_r \\ Z_r \end{bmatrix}, \quad (3.36)$$

Figure 3.8: Representation of the rotor orientation

where

$$\mathbf{R} = \mathbf{R}_z(\phi_r) \mathbf{R}_y(\theta_r) \mathbf{R}_z(\delta_r), \quad (3.37)$$

$$\mathbf{R}_z(\phi_r) = \begin{bmatrix} \cos \phi_r & -\sin \phi_r & 0 \\ \sin \phi_r & \cos \phi_r & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (3.38)$$

$$\mathbf{R}_y(\theta_r) = \begin{bmatrix} \cos \theta_r & 0 & \sin \theta_r \\ 0 & 1 & 0 \\ -\sin \theta_r & 0 & \cos \theta_r \end{bmatrix}, \quad (3.39)$$

$$\mathbf{R}_z(\delta_r) = \begin{bmatrix} \cos \delta_r & -\sin \delta_r & 0 \\ \sin \delta_r & \cos \delta_r & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (3.40)$$

Computation of torque matrix $\mathbf{Q}$ from rotor orientation

By utilizing Euler angle representation of the rotor orientation, the computing process of torque matrix $\mathbf{Q}$ from the rotor orientation is carried out as follows (Fig. 3.9).

Step one: *Computation of initial values of θ_i and ϕ_i*

Figure 3.9: Calculation of torque matrix from rotor orientation

Because the positions of all coil axes with respect to the rotor frame at the initial rotor orientation are known, it is easy to obtain the initial values of (θ_i, ϕ_i) of the i^{th} coil, i.e.,

$$\theta_{i,0} = \pi/2 - \theta_s/2, \quad \phi_{i,0} = \pi(i-1)/6, \quad (3.41)$$

for coils $(i = 1, 2, \dots, 12)$ at the upper layer, and

$$\theta_{i,0} = \pi/2 + \theta_s/2, \quad \phi_{i,0} = \pi(i-13)/6, \quad (3.42)$$

for coils $(i = 13, 14, \dots, 24)$ at the lower layer. The subscript “0” represents the initial values of θ_i, ϕ_i .

Step two: *Calculate the initial position of a unit vector on the coil axis*

Take a unit vector starting from the rotor center along the axis of the i^{th} coil. The Cartesian coordinates of the end point of this vector in the rotor frame can be calculated as

$$\mathbf{p}_{i,0} = \begin{bmatrix} \sin \theta_{i,0} \cos \phi_{i,0} \\ \sin \theta_{i,0} \sin \phi_{i,0} \\ \cos \theta_{i,0} \end{bmatrix}. \quad (3.43)$$

Step three: *Compute the final position of the end point of the unit vector after rotor rotations*

The final coordinates $\mathbf{p}_i$ of the point $\mathbf{p}_{i,0}$ in the rotor frame can be obtained via

$$\mathbf{p}_i = \begin{bmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{bmatrix} = [\mathbf{R}_z(\phi_r) \mathbf{R}_y(\theta_r) \mathbf{R}_z(\delta_r)]^T \mathbf{p}_{i,0},$$

where p_{ix}, p_{iy} and p_{iz} are components of $\mathbf{p}_i$.

Step four: *Computation of (θ_i, ϕ_i) or their trigonometric functions*

In order to obtain the torque matrix $\mathbf{Q}$, values of θ_i, ϕ_i or their trigonometric functions can be calculated from the final coordinates of $\mathbf{p}_i$.

(i) A simple way to obtain $\mathbf{Q}$ is to calculate θ_i and ϕ_i then substitute them into the torque matrix formula. This approach is easy to understand and can be employed to analyze or observe the torque variation corresponding to θ_i and ϕ_i . From the final position of the unit vector $\mathbf{p}_i$ in the rotor frame, the values of θ_i and ϕ_i can be obtained in terms of p_{ix} , p_{iy} and p_{iz} as follows.

$$\theta_i = \cos^{-1}\left(\frac{p_{iz}}{\sqrt{p_{ix}^2 + p_{iy}^2 + p_{iz}^2}}\right) = \cos^{-1}(p_{iz}), \quad (3.44)$$

$$\phi_i = \begin{cases} \cos^{-1}\left(\frac{p_{ix}}{\sqrt{p_{ix}^2 + p_{iy}^2}}\right), & p_{iy} \geq 0, \\ -\cos^{-1}\left(\frac{p_{ix}}{\sqrt{p_{ix}^2 + p_{iy}^2}}\right), & p_{iy} < 0. \end{cases} \quad (3.45)$$

Note that θ_i takes values within $(0, \pi)$. Due to the different signs of cosine function in $(0, \pi/2]$ and in $[\pi/2, \pi)$, the value of θ_i can be uniquely determined through Eqn. (3.44). In contrast, for the implementation of Eqn. (3.45), two situations for $p_{iy} \geq 0$ and $p_{iy} < 0$ are considered. It can be found that when $p_{ix}^2 + p_{iy}^2 = 0$, Eqn. (3.45) is no longer available. This condition implies that the axis of this coil is co-linear with the shaft of the rotor. However, in practice, this situation will never happen due to the constraints of the mechanical design in the actual spherical actuator.

(ii) The previous method facilitate the analysis and observation of actuator torque variation with respect to θ_i and ϕ_i . But it takes long computation time due to the trigonometric functions in the torque matrix, which may not be feasible for real-time motion control. Therefore, an alternative approach is used to calculate the torque matrix $\mathbf{Q}$ by computing trigonometric functions of θ_i and ϕ_i from $\mathbf{p}_i$.

Inspection of Eqn. (3.26) shows that $\mathbf{D}_c^e(\theta_i, \phi_i)$ or torque matrix $\mathbf{Q}$ is composed of $\sin \theta_i$, $\cos \theta_i$, $\sin \phi_i$ and $\cos \phi_i$. Solving functions of $\sin \theta_i$, $\cos \theta_i$, $\sin \phi_i$ and $\cos \phi_i$ from the final position $\mathbf{p}_i$ of the axis point directly instead of from θ_i and ϕ_i could improve the computing efficiency considerably because computation of trigonometric functions

takes much time. Specifically, trigonometric functions of θ_i and ϕ_i are calculated as

$$\begin{aligned}\cos \phi_i &= \frac{p_{ix}}{\sqrt{p_{ix}^2 + p_{iy}^2}}, \\ \sin \phi_i &= \frac{p_{iy}}{\sqrt{p_{ix}^2 + p_{iy}^2}}, \\ \cos \theta_i &= \frac{p_{iz}}{\sqrt{p_{ix}^2 + p_{iy}^2 + p_{iz}^2}} = p_{iz}, \\ \sin \theta_i &= \sqrt{p_{ix}^2 + p_{iy}^2}.\end{aligned}\tag{3.46}$$

Step five: *Complete solution of torque matrix*

Repeat the above computation for the all coils. A complete set of θ_i and ϕ_i or their trigonometric functions can be calculated. Substituting all these values into Eqn. (3.32), the torque matrix $\mathbf{Q}$ can be obtained finally.

3.3 Solution of Inverse Electromagnetics

With matrix $\mathbf{Q}$, the inverse electromagnetic solution of the actuator can be solved. The purpose of the inverse electromagnetic solution is to determine the required current input from the desired torque output based on torque model. The discussion of inverse electromagnetics includes two parts. One is on the existence of inverse electromagnetics solution within the workspace, i.e., $\mathbf{Q}$ is a full rank matrix, the other is to obtain the minimum right-inverse solution of electromagnetics because $\mathbf{Q}$ is not a square matrix.

3.3.1 Nonsingularity of the Workspace

Singularities here are defined as orientations of the rotor where no torque can be generated with respect to the rotor center even though large currents are supplied.

These singularities have to be strictly avoided in the design because they may cause severe malfunctions.

In mathematical sense, the non-singular orientation of the spherical actuator are the orientations that there exists at least one set of coil currents $[J_1, J_2, \dots, J_N]^T$ to produce the desired actuator torque $\mathbf{T}$. This implies that the matrix $\mathbf{Q}$ must be a full rank matrix for any rotor orientation within the workspace. A simple way to verify the nonsingularity of the spherical actuator is to compute the rank of torque matrix $\mathbf{Q}$ for every attainable rotor orientation within the workspace. If the rank is equal to 3, i.e., full rank, the spherical actuator is singularity-free. Otherwise, singularities exist within the workspace. However, this method does not reveal how close the system is to singularity points. Therefore, a method based on the condition number of $\mathbf{Q}$ is proposed to satisfy this requirement.

The condition number for square matrices is computed from the characteristic roots or eigenvalues⁴ of the matrix. It is defined as the ratio of largest eigenvalue and the smallest eigenvalue (in magnitude), i.e.,

$$\text{cond}(\mathbf{Q}) = \left| \frac{\text{largest eigenvalue}}{\text{smallest eigenvalue}} \right|.$$

It can be generalized to non-square matrices with singular values⁵

$$\text{cond}(\mathbf{Q}) = \left| \frac{\text{largest singular value}}{\text{smallest singular value}} \right|.$$

Ideally, when $\text{cond}(\mathbf{Q}) = 1$, the system being evaluated is in good condition. However, when $\text{cond}(\mathbf{Q}) \gg 1$, the system is ill-conditioned or nearly singular.

Figure 3.10: Computation of condition number of torque matrix Q within the workspace of the spherical actuator

Computation of condition number

The computation of the condition numbers of torque matrix $\mathbf{Q}$ is illustrated in Fig. 3.10. Torque matrix $\mathbf{Q}$ can be calculated corresponding to a rotor orientation within the workspace, and thus the condition number. If the condition number of $\mathbf{Q}$ is much great than 1, then it is assumed a singular matrix, i.e., the corresponding rotor orientation is a singular point. Repeat this computation by varying the angles ϕ_r , θ_r and δ_r within their ranges. Based on all condition numbers obtained, the singularity property of the PM spherical actuator workspace can be evaluated.

Visualization of computational result of condition numbers

The variation of the condition numbers can be visualized. By fixing ϕ_r at certain values such as 0° , 15° , 30° , etc, the condition numbers corresponding to different values of θ_r and δ_r can be obtained. Thus, a 3D plot of the condition number, denoted as n_c , with respect to the variation of θ_r and δ_r can be presented visually as a 2D surface. By choosing different ϕ_r , a set of 2D surfaces can be obtained, some of which are shown in Fig. 3.11. Due to the symmetric arrangement of the PM poles about the shaft, only the ranges of $\phi_r = 0^\circ \sim 45^\circ$ and $\delta_r = 0^\circ \sim 45^\circ$ are considered. Referring to Fig. 3.11, we can find that the condition numbers n_c do not vary too much with respect to δ_r and ϕ_r because of the evenly distributed PM poles along the rotor equator.

Through computing the condition number of $\mathbf{Q}$ within the workspace of the spherical actuator, it can be found that the minimum value of the condition number is 4.877, whereas the maximum value is 5.145, which is in the same order magnitude of 1. Therefore the workspace of this PM spherical actuator is completely singularity-free.

⁴Let $\mathbf{Q}$ be a square matrix. Then λ is called an eigenvalue of $\mathbf{Q}$ if there exists a nonzero vector $\mathbf{v}$ such that $\mathbf{Q}\mathbf{v} = \lambda\mathbf{v}$ [102].

⁵For any matrix $\mathbf{Q}$, the square roots of the nonzero eigenvalues of $\mathbf{Q}^*\mathbf{Q}$ are called singular values of $\mathbf{Q}$, where $\mathbf{Q}^*$ denotes the adjoint or conjugate transpose of $\mathbf{Q}$ [103].

Figure 3.11: Condition number of torque matrix Q

3.3.2 Minimum Right-inverse Solution of Electromagnetics

As there is no singularity within the workspace of the rotor, there are infinite solutions to Eqn. (3.32). This fact offers an opportunity to optimize the solution of inverse electromagnetics. In order to improve the actuator efficiency, the optimization of the inverse electromagnetics solution in this thesis aims at minimizing the electric power consumption under current input. This can be formulated as follows.

$$\text{Minimize} \quad \frac{1}{2} \mathbf{J}^T \mathbf{W} \mathbf{J}, \quad (3.47)$$

$$\text{Subject to} \quad \mathbf{T} = T_c \mathbf{Q} \mathbf{J}, \quad (3.48)$$

where $\mathbf{J}$ is current input vector, $\mathbf{W}$ is a positive definite matrix in the form of

$$\mathbf{W} = \begin{bmatrix} w_1 & 0 & \cdots & 0 \\ 0 & w_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & w_N \end{bmatrix},$$

$w_1, w_2, \dots, w_N$ are positive weightings determined by coils' resistance. As an effective technique to solve this problem, the *minimum right-inverse solution* [104] is employed to obtain the solution of current input

$$\mathbf{J} = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_N \end{bmatrix} = \mathbf{Q}^{RM} \mathbf{T} / T_c, \quad (3.49)$$

where $\mathbf{Q}^{RM}$ is the minimum right inverse of $\mathbf{Q}$ computed by

$$\mathbf{Q}^{RM} = \mathbf{W}^{-1} \mathbf{Q}^T (\mathbf{Q} \mathbf{W}^{-1} \mathbf{Q}^T)^{-1}, \quad (3.50)$$

and $\mathbf{Q}^T$ is the transpose of $\mathbf{Q}$, $\mathbf{W}^{-1}$ is the inverse of $\mathbf{W}$. The similar minimum right-inverse solution approach was applied to VRSM by Zhou [36] to obtain the control vector of currents.

3.4 Summary

By utilizing the magnetic field model derived in Chapter 2, this chapter has presented the formulation of the actuator torque caused by the interaction between current carrying coils and the magnetic field of the PM-pole rotor according to Lorentz force law. This torque model relates the torque output of the spherical actuator to the current inputs of coils as well as the rotor orientation, which indicates that the torque output of the PM spherical actuator is orientation dependant. Nonsingularity is one of the important advantages of this PM spherical actuator. Based on the torque model,

existence of inverse electromagnetic solution or nonsingularity workspace of the PM spherical actuator is verified through the condition number of the torque matrix. In addition, the minimum right-inverse electromagnetics solution is proposed to calculate the required current inputs for desired torque output. This solution can minimize the electric power consumption of the spherical actuator.

In VRSM developed by Lee *et al.* [36], a non-linear torque model has been derived, i.e., the torque output is a quadratic function current inputs. The non-linearity makes the real-time motion control of VRSM very complex. In the study of the proposed PM spherical actuator, the torque output is directly proportional to the current input, which facilitates the position and velocity control of the actuator. This analytical torque model can be used as a tool for the spherical actuator design to maximize the actuator torque output.

Chapter 4

Prototype Development

Prototype development is an essential step to ensure the functionalities of the proposed 3-DOF PM spherical actuator. The research prototype can be used for conducting experimental works on magnetic field distribution as well as actuator torque output to verify the derived analytical models. This chapter discusses the prototype development of the PM spherical actuator with detail considerations in the design of the rotor poles and stator poles. In nature, both rotor and stator poles are crucial components for generating the actuator torque. The prototype design here is aimed at an actuator with the following features.

- A 3-DOF rotational motion can be achieved for the rotor.
- The actuator torque output is maximized by choosing appropriate values of structural parameters of rotor and stator poles.
- The friction torque exerted on the rotor is reduced as it may affect the working efficiency of the actuator and make the actuator motion control complex.
- This actuator prototype facilitates the experimental measurement on magnetic field and torque output.
- The eddy current on the stator created by the rotor is avoided because it may retard the rotor motion.

4.1 Introduction

Here, an overview of the developed prototype of the PM spherical actuator is presented and the equations that will be used for actuator design are listed. The component design will be described in detail subsequently.

4.1.1 Prototype of PM Spherical Actuator

A research prototype of the 3-DOF PM spherical actuator has been developed. As shown in Fig. 4.1 (exploded view) and Fig. 4.2 (assembly view), this prototype consists of a ball-like rotor housed in a shell-like stator. The rotor is connected to the stator through a spherical bearing. Eight PM poles are evenly arranged along the rotor equator. The stator has twenty-four coils which are evenly grouped into two layers and symmetrically placed with respect to its equator. The specification of the prototype is listed in Table 4.1. The maximum tilting range can be increased up to about 45° by incorporating more layers of coils. This research prototype can be employed for the experimental investigation on magnetic field and torque variation to verify the theoretical models of magnetic field and torque output.

Table 4.1: Specifications of 3-DOF PM spherical actuator prototype

Inner / outer stator radius	95/112.5 (mm)
Rotor radius	46.5 (mm)
Rotor core radius	23 (mm)
PM pole material	NdFeB 37N25EH
PM pole angular parameters	$\alpha=40^\circ$, $\beta = 70^\circ$
Number of rotor poles (PM)	8
Number of stator poles (coil)	24/2 layers
Number of coil turns	900
Coil Wire type	AWG24
Maximum tilting angle	$\pm 11^\circ$
Maximum torque	4 (Nm)

Figure 4.1: Exploded view of spherical actuator prototype

Figure 4.2: Fabricated 3-DOF PM spherical actuator

4.1.2 Equations for Actuator Design

In Chapter 3, an analytical torque model of the spherical actuator has been formulated in terms of the critical parameters of the PM poles and air-core coils. Based on the torque model, appropriate values of structural parameters can be selected so that high actuator torque can be achieved. To facilitate the discussion, the equations which will be used for actuator design are listed in Table 4.2. From the torque model of Eqn. (3.33), it can be seen that the torque matrix $\mathbf{Q}$ is completely determined by positions of coil axis (θ_i, ϕ_i) which in turn is determined by rotor orientations with respect to the stator frame. The constant T_c is defined by structural parameters of PM poles (longitudinal angle α , latitudinal angle β , rotor core radius R_b and rotor radius R_r in Fig. 2.2) and those of coils (the center distance of the inner surface of the coil R_0 , the center distance of the outer surface of the coil R_1 , the angular diameter of the air-core ζ_0 and the angular diameter of the coil ζ_1 in Fig. 3.5). Each of constants a , c ,

Table 4.2: Equations used for prototype design

Torque model	$\mathbf{T} = T_c \mathbf{QJ}$	Eqn. (3.33)
Torque matrix	$\mathbf{Q} = \begin{bmatrix} g_x(\theta_1, \phi_1) g_x(\theta_2, \phi_2) \cdots g_x(\theta_N, \phi_N) \\ g_y(\theta_1, \phi_1) g_y(\theta_2, \phi_2) \cdots g_y(\theta_N, \phi_N) \\ g_z(\theta_1, \phi_1) g_z(\theta_2, \phi_2) \cdots g_z(\theta_N, \phi_N) \end{bmatrix}$	Eqn. (3.34)
Constant T_c	$T_c = \frac{15}{16} \sqrt{\frac{35}{2}} \mu_0 M_0 a c d_4 R_c G_\zeta$	Eqn. (3.31)
Constants a, b, c	$a \pm bi \equiv \int_0^{2\pi} f(\phi) e^{-im\phi} d\phi, (m = \pm 4)$	Eqn. (2.23)
	$c/\sqrt{\pi} \equiv \int_0^\pi \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} \sin^2 \theta [P_n^m(\cos \theta)] d\theta$ $(n = 4, m = \pm 4)$	Eqn. (2.24)
	$f(\phi) = (-1)^{p-1} \cos[\phi - \frac{\pi}{4}(p-1)], p = 1, 2, \dots, 8.$	Eqn. (2.22)
	$\frac{\pi}{4}(p-1) - \frac{\alpha}{2} < \phi < \frac{\pi}{4}(p-1) + \frac{\alpha}{2}$	Eqn. (2.17)
	$\frac{\pi}{2} - \frac{\beta}{2} < \theta < \frac{\pi}{2} + \frac{\beta}{2}$	Eqn. (2.18)
Constant d_4	$d_4 = -d_4^\top / d_4^\perp$	Eqn. (2.42)
	$d_4^\top = R_r^6 + \frac{9\mu_m R_b^6 R_r^9}{4(\mu_r - \mu_m) R_b^9 - (4\mu_r + 5\mu_m) R_r^9}$	
	$d_4^\perp = 5(\mu_m - 1) + \frac{9\mu_m(4\mu_r + 5\mu_m) R_r^9}{4(\mu_r - \mu_m) R_b^9 - (4\mu_r + 5\mu_m) R_r^9}$	
Constant R_c	$R_c = R_0^{-2} - R_1^{-2}$	Eqn. (3.12)
Constant G_ζ	$G_\zeta = G_\zeta'' - \frac{3G_\zeta'}{4}$	Eqn. (3.25)
	$G_\zeta' = \frac{1}{5} \sin^5 \zeta_1 - \frac{1}{5} \sin^5 \zeta_0$	
	$G_\zeta'' = \frac{1}{5} \cos^4 \zeta_0 \sin \zeta_0 - \frac{1}{15} \cos^2 \zeta_0 \sin \zeta_0 - \frac{2}{15} \sin \zeta_0 - \frac{1}{5} \cos^4 \zeta_1 \sin \zeta_1 + \frac{1}{15} \cos^2 \zeta_1 \sin \zeta_1 + \frac{2}{15} \sin \zeta_1$	
Magnetic field	$B_{Ir} = \frac{15\mu_0 M_0 a c d_4}{8\pi} \sqrt{\frac{35}{2}} r^{-6} \sin^4 \theta \cos 4\phi$	Eqn. (2.52)

d_4 , R_c and G_ζ in T_c can be calculated from pole parameters. Therefore, the effect of pole parameters on the torque output of the actuator can be evaluated through these constants.

The relation between pole parameters and the torque output is illustrated in Fig. 4.3 specifically. The longitudinal angle α of the PM pole (Fig. 2.2) is related to constant a through Eqns. (2.23) and (2.17), whereas latitudinal angle β is related to constant c through Eqns. (2.24) and (2.18). Constant d_4 is specified by the rotor radius R_r and rotor core radius R_b through Eqn. (2.42). According to the torque model of Eqns. (3.33), (3.31) and the magnetic field model of Eqn. (2.52), large a , c and d_4 can increase the torque output of the actuator as well as the magnetic flux density of PM rotor, which is consistent with the fact that high magnetic flux density can produce high torque generally. By inspecting the variation of a , c and d_4 with respect to pole parameters, values of PM-pole parameters can be determined to achieve high torque output capacity as well as high magnetic flux density of the actuator.

Similarly, R_c is defined by coil parameters R_0 and R_1 through Eqn. (3.12) and G_ζ is defined by angular parameters ζ_0 and ζ_1 through Eqn. (3.25). Eqn. (3.31) indicates that high values of R_c and G_ζ are preferred by high torque output. Therefore, coil dimensions can be designed to achieve high torque output capacity through observing the variation of R_c and G_ζ with respect to coil parameters.

According to the above discussion, it can be found that the constants a , c , d_4 , R_c and G_ζ are affected by different pole parameters separately. Therefore, appropriate pole parameters can be selected to achieve maximum value for each constant, thus to maximize the torque output.

Figure 4.3: Relation between pole (PM & coil) parameters and torque output

4.2 Rotor Pole Design

The task of the rotor pole design is to select appropriate rotor pole dimensions α , β , R_r and R_b to maximize the actuator torque output capability.

4.2.1 Longitudinal Angle α versus a

Constant a can be calculated from α through Eqn. (2.23) and the integral range in Eqn. (2.17). Because eight PM poles are installed along the equator of the rotor, the maximum value of α is 45° . By varying the value of α between 0 to 45 degrees, the relation between α and a is shown in Fig. 4.4. According to Fig. 4.4, a increases with the increase of α . Furthermore, according to Eqns. (3.31) and (3.32), large value of a is helpful for high torque output. Therefore, in order to achieve high torque output of the spherical actuator, the optimum value of α is supposed to be 45° . From physical perspective, large α represents large volume of magnetized material, which can create large flux density thus large torque output generally.

Figure 4.4: Pole parameter α vs. constant a

4.2.2 Latitudinal Angle β versus c

Similarly, according to Eqns. (3.32) and (3.31), large c also contributes to high torque output. Constant c is calculated from Eqn. (2.24) and the integral range in Eqn. (2.18) specified by the angle β ($0 \sim 180^\circ$) of the PM pole. Let $\beta_1 = \beta/2$, where β_1 varies from 0 to 90° . The relation between β_1 and the constant c is shown in Fig. 4.5 and it shows that after $\beta_1 = 60^\circ$, c reaches the largest value. Hence, high torque output can be achieved within the range of $60^\circ \leq \beta_1 \leq 90^\circ$. For $\beta_1 > 60^\circ$, the inertia moment of the rotor increases, which affects the dynamic performance of the spherical actuator. Thus, the optimum value of β_1 should be 60° . The curve in Fig. 4.5 has the following physical meaning. The increase of β_1 indicates increase of magnet volume, which leads to high flux density and thus high torque output. However, when β_1 increases to a certain degree, the volume increase of magnet is much less. Furthermore, as illustrated in Fig. 4.6, when β is large ($\beta'' > \beta'$), the distance between neighboring PM poles becomes smaller ($L'' < L'$), thus the reducing effect of flux density between these two opposite magnetized poles becomes evident.

Figure 4.5: Pole parameter β_1 vs. c

Figure 4.6: Distance between PM poles

Therefore, the effect of β on the increase of actuator torque output is negligible when it is large.

4.2.3 Rotor Radius R_r versus d_4

According to Eqns. (3.31) and (2.52), both torque output and magnetic flux density are proportional to the constant d_4 . In order to simplify the study of R_r , let $R_b = 0$. Figure 4.7 illustrates the relationship between R_r and d_4 explicitly. Approximately, d_4 increases exponentially with respect to R_r . This is due to the fact that the increase of magnet volume with respect to R_r is fast when R_r is large, which yields fast increase of magnetic field and torque output. Therefore, large rotor is preferred to create a high flux density as well as high torque output. Due to mechanical constraint in the spherical actuator, the rotor size is chosen as 46.5mm for prototype development.

4.2.4 Rotor Core Radius R_b versus d_4

Similarly, from the torque model, large d_4 is preferred in order to achieve high torque. With $R_r = 46.5\text{mm}$, the relation between R_b and d_4 is shown in Fig. 4.8. It can be seen that small R_b produces large d_4 , and thus high torque output as well as high

Figure 4.7: R_r vs. d_4

flux density. However, when R_b is reduced to a certain size, i.e., 23mm, there is no increase of constant d_4 . In other words, when $R_b < 23$, the decrease of R_b does not contribute much to increase magnetic flux density and torque output. This justifies the assumption of $R_b = 0$ when evaluating the effect of R_r in Section 4.2.3. The relation in Fig. 4.8 can be explained as follows. It is known that the smaller is R_b , the larger the size of PM pole is. Thus the magnetic field and torque output increase. However, when R_b is less than a certain degree, the volume increase of magnetized material is much less and the contribution to the magnetic flux density and torque output is negligible. As a result, the optimum R_b is equal to 23mm. This design creates a hollow space at the center of the rotor core for assembly of a spherical bearing. The hollow core also reduces the inertia moment of the PM-pole rotor.

4.2.5 Relative Permeability μ_r versus d_4

Constant d_4 in Eqn. (2.42) can also be used to observe the effect of relative permeability μ_r of ferromagnetic materials on the torque output of the spherical actuator.

Figure 4.8: Determination of R_b

According to Eqn. (2.42), the relation between μ_r and d_4 can be seen in Fig. 4.9. When μ_r is less than 50 in Fig. 4.9(a), its effect on d_4 is evident. However, when μ_r is greater than 400 (Fig. 4.9(b)), d_4 does not change with respect to μ_r . In other words, when the permeability of Region III is very large, the impact of material selection of the rotor core in Region III on the torque output of the PM spherical actuator becomes

(a) $0 < \mu_r < 50$ (b) $0 < \mu_r < 2000$ Figure 4.9: Effect of μ_r on d_4

invariable.

4.2.6 Result of PM Pole Design

The final dimensions of the PM pole using rare-earth material NdFeB 37N25EH have been determined based on the above discussion, i.e., $\alpha = 45^\circ$, $\beta = 120^\circ$, $R_r = 46.5\text{mm}$ and $R_b = 23\text{mm}$. However, for the actual prototype, $\alpha = 40^\circ$ is used to replace $\alpha = 45^\circ$ and $\beta = 70^\circ$ is used to replace $\beta = 120^\circ$ to facilitate the fabrication of PM poles. It can be seen from Fig. 4.4 and 4.5, this replacement does not change the torque output much. Instead, the size reduction of PM poles provides a space for a fixture design to hold PM poles. Furthermore, the mass/inertia moment of the rotor is reduced considerably (about 50%).

4.3 Coil Pole Design

Besides the PM poles, the other critical element generating torque in the PM spherical actuator is the air-core coil. Similar to the PM pole design, the effect of coil parameters on the torque output of the actuator can be described so that appropriate values of coil parameters can be chosen to achieve high torque output capacity of the spherical actuator. In addition to the geometric parameters of coils, large number of winding turns also benefits high torque output. Several factors of fabricating coils that can increase the winding turns are discussed here, such as the packing patterns of coils, diameter of wires and optional coil-frame structures.

4.3.1 Geometric Parameters of Coil

As shown in Fig. 3.5, four parameters are used to specify the coil shape, i.e., R_0 , R_1 , ζ_0 and ζ_1 . R_0 and R_1 can be replaced by the length of coil denoted as $L_c = R_1 - R_0$. In the

Figure 4.10: Effect of L_c on R_c

following discussion, the values of three parameters ζ_0 , ζ_1 and L_c will be determined.

Determination of coil length

Constant R_c is defined with R_0 and R_1 as indicated in Eqn. (3.12). Let $R_0 \approx R_r(46.5\text{mm})$, i.e., the coil tip is very close to the rotor surface, because the flux density is high near the rotor surface. By using L_c , Eqn. (3.12) becomes

$$R_c = R_0^{-2} - R_1^{-2} = R_r^{-2} - (R_r + L_c)^{-2}. \quad (4.1)$$

The relation between L_c and R_c is illustrated in Fig. 4.10(a). It can be seen that increasing L_c , R_c will increase, and thus torque output increases. Large L_c implies that more wires are enclosed in the winding volume, for high actuator torque output. The rate of increment in R_c versus L_c is illustrated in Fig. 4.10(b). It can be found that when the length of coil exceeds 230mm, $\Delta R_c / \Delta L_c$ is close to zero. R_c does not increase further when $L_c > 230\text{mm}$ as the magnetic flux density becomes very small at the distance far away from the rotor surface. The increased wire winding in this region, therefore, has negligible effect to the torque generation. Hence, to achieve high torque output, the optimum value of L_c can be kept at 230mm theoretically. Nevertheless,

according to $\Delta R_c/\Delta L_c$ -curve in Fig. 4.10(b), the increment rate $\Delta R_c/\Delta L_c$ for $L_c = 30\text{mm}$ is only one quarter of the maximum $\Delta R_c/\Delta L_c$, i.e., the torque contribution from coil turns beyond $L_c = 30\text{mm}$ is not evident. Excessive winding increases the electrical power consumption. Therefore, $L_c = 30\text{mm}$ can be chosen for the actual coil design of the PM spherical actuator.

Determination of coil angles

As indicated in Eqn. (3.25), G_ζ is defined with two coil angles ζ_0 and ζ_1 . Therefore, by describing the relation between G_ζ and these angular parameters, appropriate ζ_0 and ζ_1 can be determined to obtain high torque.

Determination of ζ_1 :

To facilitate the discussion, let $\zeta_0 = 0$, hence G_ζ is a function of angle ζ_1 . According

Figure 4.11: Coil parameter ζ_1 vs. G_ζ ($\zeta_0 = 0$)

to Eqn. (3.25), the relation between ζ_1 and G_ζ is plotted in Fig. 4.11. It can be found that the effect of ζ_1 on G_ζ is prominent. With reference to Fig. 3.5, when ζ_1 is large, the same increment of ζ_1 results in a large increment of wire length, and thus the torque increases fast. The maximum value of ζ_1 is approximately 15° because there are twelve coils for each layer of stator poles. To achieve high torque output, $\zeta_1 = 15^\circ$ is the optimum value.

Determination of ζ_0 :

The core angle ζ_0 in the coil also affects the torque output capacity of the spherical actuator. By fixing ζ_1 at 15° , the relation between G_ζ and ζ_0 is plotted in Fig. 4.12 according to Eqn. (3.25). When ζ_0 increases, the value of G_ζ decreases, thus the torque output decreases. However, when ζ_0 is less than 2° , G_ζ is approximately a constant of 1.39×10^{-3} . Usually the smaller ζ_0 , the longer the winding length and

Figure 4.12: Coil parameter ζ_0 vs. G_ζ ($\zeta_1 = 15^\circ$)

Figure 4.13: Different coil geometry

the larger the torque output. But when ζ_0 is smaller than a certain value, variation of ζ_0 does not affect the wire length on the coil evidently and the variation of torque is ignorable. This justifies the condition of $\zeta_0 = 0$ when we discuss the optimum value of ζ_1 . The value of $\zeta_0 = 2^\circ$ offers two additional advantages. First, it reduces the electrical power consumption whereas the torque output is not affected. Second, it provides the space for the design of coil frame on which the wire is wound. Therefore, $\zeta_0 = 2^\circ$ can be used as the optimum value for the coil design of the PM spherical actuator.

Optimized and actual coil structures

Thus far, an optimized coil geometry has been obtained as illustrated in Fig. 4.13(a). However, in the actual prototype, off-the-shelf cylindrical coils (Fig. 4.13(b)) are utilized to facilitate the fabrication. It has inner radius (R_{ci}) of 2mm, outer radius (R_{co}) of 10mm, length (L_c) of 30mm and 900 turns of winding. It can be approximated with a conical coil with $\zeta_0 = 2^\circ$, $\zeta_1 = 9.5^\circ$ and $L_c = 30$ mm in terms of the winding volume. It can be seen from Fig. 4.11 that the value of G_ζ (1.39×10^{-3}) at $\zeta_1 = 15^\circ$ is approximately four times the value (3.5×10^{-4}) at $\zeta_1 = 9.5^\circ$.

Volume comparison between two coil geometries:

In previous study, it is known that the torque model is formulated by integrating the differential torque within the entire coil volume. In order to show the cylindrical coil can be approximated with a conical coil for torque computation, the volumes of these two types of coils will be compared.

1. Volume of conical coil

The conical coil presented in Fig. 4.13(b) can be computed with the difference between the volume enclosed by $ABCD$ (V_{ABCD}) and that enclosed by $A'B'C'D'$ ($V_{A'B'C'D'}$). V_{ABCD} and $V_{A'B'C'D'}$ can be calculated by following equations [105],

$$\begin{aligned} V_{ABCD} &= \frac{2}{3}\pi R_1^3(1 - \cos \zeta_1) - \frac{2}{3}\pi R_0^3(1 - \cos \zeta_1) \\ &= \frac{2}{3}\pi(R_1^3 - R_0^3) + \frac{2}{3}\pi(R_0^3 - R_1^3)\cos \zeta_1, \\ V_{A'B'C'D'} &= \frac{2}{3}\pi R_1^3(1 - \cos \zeta_0) - \frac{2}{3}\pi R_0^3(1 - \cos \zeta_0) \\ &= \frac{2}{3}\pi(R_1^3 - R_0^3) + \frac{2}{3}\pi(R_0^3 - R_1^3)\cos \zeta_0. \end{aligned}$$

Therefore, the volume of the conical coil is

$$\begin{aligned} V_{co} &= V_{ABCD} - V_{A'B'C'D'} \\ &= \frac{2}{3}\pi(R_0^3 - R_1^3)(\cos \zeta_1 - \cos \zeta_0). \end{aligned} \quad (4.2)$$

2. Volume of cylindrical coil

The volume of the cylindrical coil is

$$V_{cy} = \pi R_{co}^2 L_c - \pi R_{ci}^2 L_c = \pi(R_{co}^2 - R_{ci}^2)L_c. \quad (4.3)$$

Substituting parameter values into Eqns. (4.2) and (4.3), it can be found that the ratio between V_{co} and V_{cy} is 1.03. The volumes of two coil geometries are approximately the same. As a result, replacement of the cylindrical coil with the conical coil does not have significant effect on the torque computation.

4.3.2 Increase Number of Winding Turns

Current density of the coil is defined as the electric current passing through unit sectional area of the coil, i.e., [88]

$$J = \frac{\Delta I}{\Delta A} = \frac{\Delta N I_w}{\Delta A}, \quad (4.4)$$

where I_w is the current passing through a single wire, $\Delta I = \Delta N I_w$ is the total current passing through a cross section ΔA of coil. Therefore, increasing the number of winding turns within per unit sectional area of coil improves the current density of the coil and thus the torque output capacity of the spherical actuator. Several approaches are discussed in order to increase the number of winding turns such as, packing patterns of wires, reduction of wire diameters, geometry of coil frames, etc.

Comparison of packing patterns

The packing pattern of winding affects the number of wire turns placed within a given area. Two possible stacking patterns are studied here. Figure 4.14(a) illustrates the regular packing pattern in which the wires are stacked on row upon another, and Fig. 4.14(b) is the staggering pattern in which each wire conductor is placed above and between the previous rows wires. The density of the patterns can be computed and compared for their effectiveness. Here, the stacking density is defined as the ratio of the occupied cross sectional area of the wires and the area of the designated square space (Fig. 4.14).

For the regular pattern, stacking density can be computed as following

$$E_1 = \frac{4\pi r_w^2}{16r_w^2} = 78.5\%, \quad (4.5)$$

where r_w is the radius of the wire.

As for the staggering packing pattern, the total area of wire sections within the square

Figure 4.14: Wire packing patterns

in Fig. 4.14(b) can be expressed as

$$A_t = 4\pi r_w^2 + 2[r_w^2 \arccos(\frac{h_1}{r_w}) - \sqrt{r_w^2 - h_1^2}h_1] + 2[r_w^2 \arccos(\frac{h_2}{r_w}) - \sqrt{r_w^2 - h_2^2}h_2] \quad (4.6)$$

Hence, the stacking density is

$$\begin{aligned} E_2 &= \frac{A_t}{16r^2} \\ &= \{4\pi r^2 + 2[r^2 \arccos(\frac{h_1}{r}) - \sqrt{r^2 - h_1^2}h_1] + 2[r^2 \arccos(\frac{h_2}{r}) - \sqrt{r^2 - h_2^2}h_2]\} / 16r^2 \\ &= 87.2\%. \end{aligned} \quad (4.7)$$

Comparing E_1 and E_2 shows that the staggering packing pattern offers 10% more stacking density than the regular packing pattern.

* Square wire

The above discussion is based on wires with round section. An alternative option is to use square section wire as shown in Fig. 4.15. With the square wire, the packing density will be much larger than the round wire. This square wire also aids in heat dissipation. However, the winding of square wires is more complex than that of round

Figure 4.15: Coil wound with square wire

wires due to the fragileness and cost of the square wire. Therefore, in this prototype development, the round wire is employed for coils.

Properties of wires

Wire material:

Presently, most available wires are made either of copper or of aluminum. Specialty magnet wire can be made from silver, gold or platinum whose prices are very high. Aluminum has the advantage of light weight. According to Table 4.3, its density (2.7 g/cm^3) is less than $1/3$ of copper's (8.9 g/cm^3). However, comparing with aluminum wires, copper wires also possess significant advantages.

Table 4.3: Properties of copper and aluminum wires [106]

Properties	Copper	Aluminum
Density (g/cm^3)	8.9	2.7
Cross section for same conductivity (cm^2)	100	160
Tensile strength (lb/in^2)	55,000	25,000
Corrosion resistant	good	not good
Compatibility with connectors	good	not good

- The conductivity of copper wires is larger than that of aluminum wires. To

offer the same conductance, copper wires can have smaller cross-sections than the equivalent aluminum wires. This means windings wound with copper wires will likely have smaller volume compared with an equivalent aluminum windings. This also indicates that copper building wire requires less insulation and smaller conduits than aluminum.

- The strength of copper (55,000 lb/in²) is much higher than aluminum conductors (25,000 lb/in²). It can resist stretching, neck-down, creep, nicks and breaks.
- Copper is corrosion resistant. As a noble metal, copper gives a system high reliability and longevity. The pure copper used in building wire is not subject to galvanic corrosion when connected to other, less noble metals and alloys. Copper wiring will also resist corrosion from moisture, humidity, industrial pollution and other atmospheric influences - to insure safe, trouble-free performance for the life of a system.
- One of the inherent properties of aluminum is that in the presence of oxygen, it quickly forms a very tight oxide skin. A connection to aluminum wire must be properly plated and crimped (or soldered with a special solder) to be gas-tight. This prevents the formation of aluminum oxide film that could raise the resistance of the joint and cause a problem. In contrast, copper, with its compatibility with connectors and other devices, safeguards against these conditions occurring.

Based on the comparison between copper and aluminum wires, copper wire is employed for the windings of this spherical actuator.

Reduction of wire diameter:

Diameter of wires affects the number of winding turns which in turn is closely related to the torque output of the spherical actuator. Generally, small diameter of wires

Figure 4.16: Number of winding turns vs. wire diameter

allows more turns of winding to occupy the same area. Analysis is carried out to estimate the increased turns of coils due to diameter reduction of wires. It can be seen from Fig. 4.16 that the height of a single row of coil t_1 is equal to the diameter of the wire $2r_w$. The height of an n_c -layer coil can be computed as:

$$t_{n_c} = 2r_w \left[1 + \frac{\sqrt{3}}{2}(n_c - 1) \right]. \quad (4.8)$$

Assuming that the section area of winding is constraint by the height t_v and the width t_h as shown in Fig. 4.16. The height of the coil stack must be equal or less than the height t_v , i.e.

$$2r_w \left[1 + \frac{\sqrt{3}}{2}(n_c - 1) \right] \leq t_v. \quad (4.9)$$

The number of layers can be obtained as:

$$n_c = \text{Int} \left[1 + \frac{2}{\sqrt{3}} \left(\frac{t_v}{2r_w} - 1 \right) \right], \quad (4.10)$$

where Int represents the integer of the result value. Similarly, the number of turns m_c for each layer can be computed as

$$m_c = \text{Int} \left(\frac{t_h}{2r_w} \right). \quad (4.11)$$

Hence, the number of turns that occupies the sectional area in Fig. 4.16 is

$$N_{turn} = Int[1 + \frac{2}{\sqrt{3}}(\frac{t_v}{2r_w} - 1)]Int(\frac{t_h}{2r_w}). \quad (4.12)$$

Suppose that there are two type of wires with radius of r_{w1} and r_{w2} . Correspondingly, the winding numbers of coils using these two type of wires are denoted as N_{turn1} and N_{turn2} . The ratio of N_{turn1} and N_{turn2} can be calculated as

$$\frac{N_{turn1}}{N_{turn2}} = \frac{Int[1 + \frac{2}{\sqrt{3}}(\frac{t_v}{2r_{w1}} - 1)]Int(\frac{t_h}{2r_{w1}})}{Int[1 + \frac{2}{\sqrt{3}}(\frac{t_v}{2r_{w2}} - 1)]Int(\frac{t_h}{2r_{w2}})}. \quad (4.13)$$

Normally, $t_v/2r_{w1}, t_v/2r_{w2} \gg 1$. In this case, Eqn. (4.13) is reduced to

$$\frac{N_{turn1}}{N_{turn2}} \simeq (\frac{r_{w2}}{r_{w1}})^2. \quad (4.14)$$

Given two examples of wires, i.e., American Wire Gauge (AWG) 21 with $r_{w2} = 0.38\text{mm}$ (bare wire $\phi 0.70\text{mm}$, with isolation $\phi 0.76\text{mm}$) and AWG24 with $r_{w1} = 0.225\text{mm}$ (bare wire $\phi 0.40\text{mm}$, with isolation $\phi 0.45\text{mm}$), it can be known from Eqn. (4.14) that the ratio of winding turns N_{turn1}/N_{turn2} is nearly equal to 3. Hence, the number of winding turns with AWG24 can increase about three folds than with the coil using AWG21 wire.

Operating temperature of wires:

Besides wire diameters, another important factor that affects the choice of the wire type is the operating temperature of insulation layer of wires. The electric wire used for winding includes the bare wire and the insulation layer. For each kind of insulation layer, there is a maximal working temperature, i.e., heat resistance grade. The highest heat resistance grade (Grade H) of wire, can withstand 220°C . While the coil with thin wire is preferred for generating high torque, it also produces more heat due to the increased wire turns. Experiments are carried out on the working temperature of AWG21 and AWG24 wires. By using AWG21 wire, it is found that the coil can work continuously with an input current of 3A. However, the torque produced is small relatively because of the less number of turns. Using AWG24 wire will have more

turns and produce more heat. By using a simple cooling system, AWG24 wire can work stably in the spherical actuator.

Winding geometry

In formulating the torque model, the conical coil is used. However, this type of coils is difficult to fabricate massively in practice. Therefore, several other types of winding geometries like straight coil, stepped coil and trapezoid coil, are introduced as alternative options. Large sectional area of coils allows more wire turns, which helps in achieving large torque output. Sectional areas of these type of coil frames are shown in Fig. 4.17. According to Fig. 4.17(a), the sectional winding area of the straight coil is computed as

$$A_1 = lD_1 - ld. \quad (4.15)$$

Similarly, the sectional areas of the stepped coil and the trapezoid coil are

$$A_2 = l_1D_1 + (l - l_1)D_3 - l_2(D_2 - d) - ld, \quad (4.16)$$

$$A_3 = \frac{l(D_1 + D_2)}{2} - ld. \quad (4.17)$$

In order to show how much the stepped and trapezoid coils can increase the winding turns, let

$$D_2 = 2D_1, \quad (4.18)$$

$$D_3 = \frac{3}{2}D_1, \quad (4.19)$$

$$l = 2l_1, \quad (4.20)$$

Using Eqns. (4.18)-(4.20) and ignoring the terms of ld and $l_2(D_2 - d)$, Eqns. (4.15)-(4.17) become

$$A_1 = 2D_1l_1, \quad (4.21)$$

$$A_2 = \frac{5}{2}D_1l_1, \quad (4.22)$$

$$A_3 = 3D_1l_1, \quad (4.23)$$

(a) Straight coil

(b) Stepped coil

(c) Trapezoid coil

Figure 4.17: Section view of different coil frames and coil geometry

It can be seen that, compared to the straight coil, the stepped coil can increase the winding area by 25%, while the trapezoid coil can increase it by 50%, which is preferable for large torque generation. However, special winding approaches are necessary to be considered for the trapezoid coil to prevent from loosing wires. By contrast, the stepped coil and straight coil are relatively easy to be wound. Selection of the coil frame type could depend on specific requirements.

4.3.3 Material of Coil Frame

The material of the coil frame used is important as it might generate eddy current under the action of the PM-pole rotor. If the size of the coil frame, especially the inner core angle ζ_0 , is large, nonmetal material such as Delrin or laminated metal material is favorable for the coil frame because there is no or little eddy current produced. However, if angle ζ_0 is very small, the eddy current produced in the coil frame can be neglected. In this case, metal material such as aluminium can be chosen as the coil frame material.

4.4 Stator

The primary function of the spherical-shell-like stator is to house air-core coils. As shown in Fig. 4.18, twenty-four circumferential holes are drilled through the stator shell and arranged in two layers symmetrically about the stator equator. Coils will be mounted through these holes and pointed to the center of the stator. The threads in the holes allow coil frames to be screwed in and out for adjusting the radial position of coils. The assembling slot is to facilitate the operation of tools such as pliers.

In the assembly of the spherical actuator, it is required that the rotor is concentric with the spherical-shell-like stator. Two cylindrical holes separated 90° apart are drilled at the equator plane of the stator as shown in Fig. 4.19. Likewise, two holes

Figure 4.18: Stator

that are separated 90° apart and pointed to the rotor center are drilled at the equator plane of the rotor. Two centering pins can be inserted through the holes on the stator shell to the holes on the rotor. In this way, the concentricity of the rotor and the stator can be maintained. The stator can also be used for hosting optical/Hall-effect sensors to detect the rotor orientation for motion control of the spherical actuator.

The stator is made of aluminum due to its low cost and ease of fabrication. The inner radius of the stator (95mm) is much larger than that of the rotor (46.5mm) due to the following facts.

- The eddy current induced on the aluminum stator due to PM poles on the rotor may affect the rotor motion, large space between the rotor and the stator can reduce it significantly.
- Large space between the rotor and the stator facilitates the flux density measurement inside the actuator and heat dissipation.

For industrial products, the stator can be made from nonmetal material such as

Figure 4.19: Positions of centering pins (section view on the equator plane)

Delrin or laminated metal materials so that stator size can be reduced about 50%. Laminated ferromagnetic material not only reduces the eddy current significantly, but also improves the efficiency of the spherical actuator by “converging” the magnetic flux and reducing the magnetic energy loss.

4.5 Spherical Bearing

In order to secure the rotor position in the spherical actuator, a supporting mechanism is necessary. In the spherical actuator developed by Wang *et al.* [54,59], the spherical rotor is housed within the spherical stator directly. Although a low friction coating was made on the stator inner surface, the sliding friction between the rotor and the stator may cause unfavorable effect on the rotor motion. In some spherical motors [67,107], transfer bearings or analog are used to support the rotor. A typical transfer bearing is shown in Fig. 4.20(a). This bearing consists of a rolling ball on the top, which can create a rolling motion with respect to the contact surface. The screw

Figure 4.20: Support spherical rotor with transfer bearings

on the other end can fix the bearing on the stator for adjusting its position. As indicated in Fig. 4.20(b), to support a spherical rotor stably, at least three transfer bearings are necessary. The rolling motion of these transfer bearings produces rolling frictions lower than the sliding frictions. Nevertheless, the torque of friction from the transfer bearing relative to the rotor center is large due to the rotor radius. The non-negligible resultant frictional torque generated by three transfer bearings may suppress the system efficiency. The effect of frictional torque on the rotor of the VRSM has been analyzed by Zhou [36]. To effectively reduce the friction, an air-bearing system has been proposed by Lee *et al* [108,109]. Because the rotor floats on air, the friction can be played down significantly. However, the requirement of the air source complicates the application of this bearing system.

In our spherical actuator, a spherical bearing (Fig. 4.21(a)) [111] is employed to support the rotor housed in the stator. This spherical bearing can achieve a smooth 3-DOF rotational motion. The tilting motion range of this bearing is up to $\pm 35^\circ$. It can be assembled at the rotor center as shown in Fig. 4.21(b). Because the friction produced by this spherical bearing is very small and the moment arm of the frictional force is less than 6mm, the frictional torque produced by the spherical bearing is negligible. Compared with the air bearing, the implementation of this spherical bearing

(a) Spherical bearing [111]

(b) Spherical bearing at the rotor center

Figure 4.21: Implementation of spherical bearing on the rotor

is also easy.

4.6 Summary

In order to achieve high torque output of the PM spherical actuator, the design of two major force/torque generating elements, PM-pole rotor and coil has been the main concerns in this chapter. Due to the parametrization of PM poles and coils, the relationship between these parameters and torque output can be described. According to these relationship, suitable values of the parameters can be determined to achieve high torque output of the actuator. In addition to the analysis of geometric dimensions of PM and coil poles, several winding techniques are proposed to increase the number of coil turns and the torque output. A spherical bearing is employed to reduce the frictional torque exerted on the rotor. Based on the theoretical analysis, a research prototype is fabricated. This prototype will be used for experimental measurement on magnetic field and torque output to verify the theoretical models.

Chapter 5

Experimental Investigation

In this chapter, the experimental investigation is conducted on the prototype of the PM spherical actuator to verify the analytical magnetic field model and torque model proposed in Chapter 2 and Chapter 3. The experimental result also facilitate the visualization of the magnetic field distribution and the torque variation of the actuator. As shown in Fig. 5.1, the experimental investigation of the PM spherical actuator consists of two major tasks: measurement of magnetic field distribution and torque variation.

This chapter begins with description of magnetic field measurement procedure of the PM-pole rotor, and the corresponding data processing and analysis. The force/torque

Figure 5.1: Experimental investigation on PM spherical actuator

measurement is carried out on the PM spherical actuator and compared with the theoretical torque model. The experimental results confirm the proposed magnetic field model and torque model for further control study of the actuator.

5.1 Measurement of PM Rotor Magnetic Field

Experimental measurement on the magnetic flux density can be done readily in some special cases. For example, the magnetic flux distribution of rectangular cross-section electromagnets proposed by Mintchev *et al.* [113, 114] is relatively simple. As illustrated in Fig. 5.2(a), a magnetic field is excited by a current carrying coil. The magnetic flux lines form a closed loop through ferromagnetic material such as soft iron disrupted by a narrow airgap. Because of the small size of the airgap, the fringing flux can be ignored and the flux lines can be regarded as evenly distributed within the airgap volume. In other words, the magnetic flux in the airgap forms a 1D field. In this case, the flux density data can be taken easily at points inside the airgap by using the single-axis Hall probe that has a tip as thin as 0.5mm (Fig. 5.2(b)). The rigorous requirement on the airgap in the magnetic flux loop limits the application of this measurement method. Furlani [115] simplified the flux density measurement of a PM motor by sampling the data in a characteristic point within the workspace. However, the data at this point can only reflect the variation of the magnetic field

Figure 5.2: Flux density measurement in a narrow airgap

roughly. It is no longer applicable for precise measurement. As an improvement, Enokizono *et al.* [116] have proposed a method to measure the flux density on a 2D plane of a single-axis motor accurately. Due to the axial symmetric structure of this motor, the magnetic field on one plane perpendicular to the axis suffices to represent the entire magnetic field. However, this method is not suitable for the 3D flux density measurement of magnetic elements with complex structures. In recent years, 3-DOF electromagnetic spherical actuators have been developed by researchers to overcome the drawbacks of conventional spherical motion mechanisms realized by several single-axis actuators. FEM approach has been used to simulate the variation of the flux density [58,60]. However, so far complete experimental measurement on the three components of the flux density distribution in 3D space of these motors has not been reported in the literature. The challenge for experimental study of the magnetic field of the spherical actuator lies in the complexity of measurement for all three components of the flux density varying in the 3D space around the rotor.

In our study, an apparatus for the 3D flux density measurement of the PM rotor is developed. By using this apparatus, the complete experimental measurement of the magnetic field distribution of the PM-pole rotor is carried out in 3D space. The measured data can be nondimensionalized and normalized to eliminate the effect of the specific rotor dimensions. The result can be referred by other similar rotor designs by multiplying several pre-defined constants. Furthermore, these data can be visualized and compared with the analytical result from the magnetic field model derived previously.

5.1.1 Flux Density Measurement Apparatus

The flux measurement apparatus developed for this project can move a magnetic field measuring probe (Hall probe) to any 3D point of concern and take the flux density data at the point. It includes a three-axis Hall probe, a Gauss meter, a three-axis

translational stage, a personal computer (PC) and a single-axis servo motor.

Three-axis Hall probe

The flux density is a 3×1 vector. A complete measurement of the magnetic field needs to acquire all three components of the flux density at any measuring point surrounding the rotor. A three-axis Hall probe (Lake Shore MMZ-2502-UH [117]) as shown in Fig. 5.3 is employed for this task with specifications listed in Table 5.1. According to the theoretical analysis, the maximum flux density of the PM poles on the rotor surface is about $\pm 6\text{kG}$ which is within the measuring range of the probe, $\pm 30\text{kG}$. With a resolution of 0.1G and the operating temperature range from 10°C to 40°C , this Hall probe meets the measurement requirement in most cases.

Table 5.1: Specifications of a three-axis Hall probe [117]

Model	LakeShore MMZ-2502-UH (three-axis)
Type	HSE-1 (High sensitivity probe)
Stem material	Aluminum
Measuring range	$\pm 30\text{kG}$
Measuring resolution	0.1G
Operating temperature	10°C to 40°C

Figure 5.3: Three-axis Hall probe (Lake Shore MMZ-2502-UH) [112]

Setup of measurement apparatus

Figure 5.4 shows the setup of the flux density measurement apparatus with specifications listed in Table 5.2. The Hall probe is mounted on a high precision three-axis translational motion stage (NSK XY-HRS 040-H201 [118]) with a motion resolution of 0.01mm, so that it can pinpoint to any location near the rotor. This probe is connected to a Gauss meter that can display the measured flux density in three components. The Gauss meter is linked to a PC through a data acquisition (DAQ) card. The measured flux density then can be automatically stored on the PC. The rotor is mounted on a motorized fixture that can spin along the rotor axis in 360° . Thus, the Hall probe can measure the magnetic flux density in the longitudinal direction of the rotor by simply rotating the rotor using the fixture without re-orienting the Hall probe.

Figure 5.4: Apparatus setup

Table 5.2: Specifications of the magnetic field measurement apparatus

Gauss meter	Model	LakeShore 460 [117]
	Number of Channel	3 Channels
	Update rate	18 readings/s
Platform	Size	340x440mm
Three-axis translational stage	Model	NSK XY-HRS 040-H201 [118]
	Resolution	0.01mm
	Maximum velocity	1200mm/s
Motorized fixture	Model	PowerCube PDU 070 [119]
	Resolution	6 Arcsec
	Maximum angular velocity	238°/s

Measurement coordinates

To facilitate measurement of the magnetic field of the PM-pole rotor, the measurement coordinates r_p , ϕ_p and θ_p are introduced as illustrated in Fig. 5.5. The origin of the measurement coordinate system (r_p, θ_p, ϕ_p) coincides with that of the rotor coordinate system (r, θ, ϕ) . The latitudinal angle θ starts from the z -axis whereas θ_p starts from

Figure 5.5: Definition of measurement coordinate system for Hall probe

the equatorial plane of the rotor; the longitudinal angle ϕ coincides with ϕ_p ; the radial coordinates r_p and r are identical.

Measurement process

The Hall probe moves along a pre-determined path illustrated in Fig. 5.6 and takes measurement of flux density at sampling points along the path. As the rotor structure is symmetric about the equatorial plane, the measurement is only carried out for the upper hemisphere. The measuring path starts from a point along the center axis of a PM pole and very close to the rotor surface. The measuring path is kept along a vertical plane called a *measuring plane*. The probe moves along an arc upwards on the plane while keeping at a constant normal distance d_a to the rotor surface. The neighboring sampling points keep a constant angle of $\Delta\theta_p$ with respect to the rotor center as shown in Fig. 5.6. After the probe completes an arc of $\theta_p = 25^\circ$, the probe offsets a distance of Δr_p radially and then carries out measurement along the subsequent arc path. This measuring process is repeated with increasing radial distance r_p until the flux density is significantly small. For our measurement,

Figure 5.6: Magnetic flux density measurement of the PM-pole rotor

Figure 5.7: Magnetic flux density measurement process

the maximum motion in the r_p -direction is 30mm. This radial distance is beyond the possible position of the stator coil. Because of the constant angular sampling pattern, the measuring points are sparsely located when the radius r_p increases. It is coincident with the fact that the gradient of the flux density decreases with increasing r_p . Therefore, this sampling method requires less sampling points than that of the equal distance sampling method.

After completing the measurement task in one measuring plane, the PC sends commands to the controller of the rotor fixture to turn the rotor with an angle of $\Delta\phi_p$, while the probe keeps stationary. The Hall probe is then positioned in a new measuring plane with respect to the rotor and the same data acquisition procedure is repeated. The entire process of positioning the probe, taking measurement and turning the rotor can be fully automated as shown in Fig. 5.7.

By using this apparatus, the flux density around the rotor has been measured several times. It was found that the maximum difference between measurements is about 50G or 1% of the maximum flux density. This repeatability could be acceptable.

Calibration of spinning center of rotor

In the magnetic field measurement apparatus, the rotor of the PM spherical actuator is mounted on a single-axis servo motor (PowerCube PR070 [120]). This single-axis servo motor rotates the rotor in the ϕ_p -direction. Thus, alignment of the axes of the servo motor and the rotor is important for accurate magnetic field measurement. In order to ensure the alignment of the two axes, a calibration of the eccentricity of the rotor axis is carried out by using the apparatus shown in Fig. 5.8.

The servo motor along with the rotor of the spherical actuator is first secured on a stage. This single-axis servo motor can attain a resolution as high as 6 arcsec. A displacement sensor probe (Probe 5313180 [119]) is put in contact with the rotor

Figure 5.8: Calibration of alignment

surface gently. When the rotor rotates by the single-axis servo motor, the probe tip detects the variation of the rotor surface and transmits the variation information to the Millitron (Mahr Millitron-1240 [121]). The Millitron has a resolution of $10\mu\text{m}$ and a measurement range of $\pm 300\mu\text{m}$. With distance variation data, the rotor position on the servo motor is adjusted to minimize the distance variation or the misalignment of the servo motor axis and the rotor axis. In this experiment, the distance variation is calibrated below $50\mu\text{m}$ to minimize its effect on the magnetic field measurement.

5.1.2 Flux Density Data Processing

Nondimensionalization and normalization techniques are used to process the measured magnetic flux density from the Hall probe. In this way, the experimental data can be used as a reference for similar spherical actuator designs without referring to specific rotor dimensions.

Transformation of coordinates

When the tip of the Hall probe reaches a measuring point as shown in Fig. 5.9, three components of the flux density, B_{X_h} , B_{Y_h} and B_{Z_h} are obtained based on the coordinates of the Hall probe, X_h , Y_h and Z_h . The Hall-probe is always kept in horizontal direction. As the magnetic field model of the PM rotor is expressed in the spherical coordinates (r, θ, ϕ) on the rotor (The Hall probe does not align with r .), it is necessary to convert these data from the sensor coordinates to the spherical coordinates on the rotor. According to Fig. 5.9, the following relation can be derived:

$$\begin{aligned} B_{Ir} &= -B_{Y_h} \cos \theta + B_{Z_h} \sin \theta, \\ B_{I\theta} &= B_{Y_h} \sin \theta + B_{Z_h} \cos \theta, \\ B_{I\phi} &= B_{X_h}. \end{aligned} \tag{5.1}$$

Table 5.3 presents some sampling data converted from sensor coordinates to spherical coordinates. Because $B_{I\theta}$ and $B_{I\phi}$ do not contribute to the actuator torque in spherical configuration, and only the radial component B_{Ir} will do, the following discussion is focused on B_{Ir} only.

Nondimensionalization and normalization of experimental result

Nondimensionalization is the partial or full removal of units from a mathematical equation by a suitable substitution of variables. It is generally used in sociological, economic or mechanical areas to evaluate the weight of different inputs, such as the oil gap in the hydrodynamic lubrication system [122–124]. By using the technique of nondimensionalization, the behavior of the system can be analyzed regardless of the units used to measure the variables. One notable example is the Reynolds number frequently used in fluid mechanics that is the ratio of inertial force and the viscous force. In addition, the number of variables in the flux density function can be reduced.

Figure 5.9: Transformation from Hall-probe coordinates to spherical coordinates

After nondimensionalization of experimental data of the magnetic flux density measurement, the flux density becomes a function of dimensionless arguments. Normalization of the nondimensionalized data is then followed. Normalized data set can be used as a benchmark result to be used by other spherical actuators with similar rotor design.

Nondimensionalization of the radial parameters in magnetic field model:

The radial parameter r in the magnetic field model can be scaled by rotor radius R_r .

Thus Eqn. (2.52) and the related constant d_4 can be reorganized as

$$B_{Ir} = \frac{15\mu_0 M_0 a c d'_4}{8\pi} \sqrt{\frac{35}{2}} \left(\frac{R_r}{r}\right)^6 \sin^4 \theta \cos 4\phi, \quad (5.2)$$

where

$$\begin{aligned} d'_4 &= -d_4^\top / d_4^\perp, \\ d_4^{\top'} &= d_4^\top / R_r^6 \\ &= 1 + \frac{9\mu_m R_b^6 R_r^3}{4(\mu_r - \mu_m)(R_b^9 - (4\mu_r + 5\mu_m)R_r^9)} \end{aligned} \quad (5.3)$$

Table 5.3: Magnetic flux density data conversion from sensor coordinates to spherical coordinates ($\phi_p = 0$, $d_a = 0.5\text{mm}$)

θ_p ($^\circ$)	B_{X_h} (kG)	B_{Y_h} (kG)	B_{Z_h} (kG)	B_r (kG)	B_θ (kG)	B_ϕ (kG)
0	0.7638	0.5037	-6.2135	-6.2135	0.5037	0.7638
1	1.0086	0.4979	-6.1921	-6.1998	0.3898	1.0086
2	1.2179	0.4800	-6.1845	-6.1975	0.2639	1.2179
3	1.3691	0.4703	-6.1746	-6.1908	0.1465	1.3691
4	1.5860	0.4427	-6.1028	-6.1188	0.0159	1.5860
5	1.7680	0.4382	-6.0478	-6.0630	-0.0906	1.7680
6	1.9618	0.3867	-5.9676	-5.9753	-0.2392	1.9618
7	2.1247	0.3879	-5.8748	-5.8783	-0.3309	2.1247
8	2.2865	0.3683	-5.8010	-5.7958	-0.4426	2.2865
9	2.4550	0.3485	-5.6722	-5.6569	-0.5431	2.4550
10	2.6562	0.3164	-5.5846	-5.5547	-0.6582	2.6562
11	2.8175	0.2670	-5.4552	-5.4059	-0.7788	2.8175
12	2.9740	0.2550	-5.2905	-5.2279	-0.8505	2.9740
13	3.1255	0.2470	-5.1099	-5.0345	-0.9088	3.1255
14	3.2383	0.2226	-4.9629	-4.8693	-0.9846	3.2383
15	3.4186	0.1903	-4.7742	-4.6608	-1.0518	3.4186
16	3.6008	0.1677	-4.5640	-4.4334	-1.0968	3.6008
17	3.7007	0.1658	-4.3939	-4.2504	-1.1261	3.7007
18	3.8790	0.1472	-4.1720	-4.0133	-1.1492	3.8790
19	4.0733	0.1103	-3.9549	-3.7753	-1.1833	4.0733
20	4.2697	0.1033	-3.7040	-3.5160	-1.1698	4.2697
21	4.4738	0.0193	-3.4245	-3.2040	-1.2092	4.4738
22	4.7021	0.0135	-3.1232	-2.9008	-1.1575	4.7021
23	4.9161	0.0099	-2.7956	-2.5772	-1.0832	4.9161
24	5.2382	0.0034	-2.3262	-2.1265	-0.9430	5.2382
25	5.6152	-0.0090	-1.7727	-1.6028	-0.7573	5.6152

$$= 1 + \frac{9\mu_m}{4(\mu_r - \mu_m)\left(\frac{R_b}{R_r}\right)^3 - (4\mu_r + 5\mu_m)\left(\frac{R_r}{R_b}\right)^6}, \quad (5.4)$$

$$\begin{aligned}
d_4^{\perp'} &= d_4^{\perp} \\
&= 5(\mu_m - 1) + \frac{9\mu_m(4\mu_r + 5\mu_m)R_r^9}{4(\mu_r - \mu_m)R_b^9 - (4\mu_r + 5\mu_m)R_r^9} \\
&= 5(\mu_m - 1) + \frac{9\mu_m(4\mu_r + 5\mu_m)}{4(\mu_r - \mu_m)\left(\frac{R_b}{R_r}\right)^9 - (4\mu_r + 5\mu_m)}. \quad (5.5)
\end{aligned}$$

Let $\nu_1 = R_r/r$, $\nu_2 = R_b/R_r$, Eqns. (5.2)-(5.5) can be rewritten as

$$B_{Ir} = \frac{15\mu_0 M_0 a c d_4'}{8\pi} \sqrt{\frac{35}{2}} \nu_1^6 \sin^4 \theta \cos 4\phi, \quad (5.6)$$

where

$$d'_4 = -d_4^{\top'}/d_4^{\perp'}, \quad (5.7)$$

$$d_4^{\top'} = 1 + \frac{9\mu_m}{4(\mu_r - \mu_m)\nu_2^3 - (4\mu_r + 5\mu_m)(\frac{1}{\nu_2})^6}, \quad (5.8)$$

$$d_4^{\perp'} = 5(\mu_m - 1) + \frac{9\mu_m(4\mu_r + 5\mu_m)}{4(\mu_r - \mu_m)\nu_2^9 - (4\mu_r + 5\mu_m)}. \quad (5.9)$$

Both ν_1 and ν_2 are dimensionless coefficients varying between 0 and 1. Therefore, computing the flux density based on the dimensional parameters R_b , R_r and r becomes computing the flux density from dimensionless parameters ν_1 and ν_2 .

Normalization of measured flux density:

The normalization technique is applied to the measured magnetic field. For convenience, denote radial component of the measured flux density under rotor coordinates with coordinate transformation from the probe coordinates, Eqn. (5.1) as B_{Ir}^e . The process of normalization is as follows.

Step One: The relationship between ν_2 and d'_4 is plotted in Fig. 5.10 based on Eqns. (5.7)-(5.9). When $0 < \nu_2 < 0.5$, the value of d'_4 does not change much, but always kept at 0.1087. It means that the magnetized material at the core of the rotor does not help much to improve the flux density of the rotor. Between $0.5 < \nu_2 < 0.7$, d'_4 decreases slightly. After $\nu_2 = 0.7$, d'_4 decreases drastically, which indicates that when R_b is close to R_r or when the thickness of the PM pole is very small, the flux density of the rotor drops quickly. Recall the design parameters of the PM-pole rotor. It is known that for this spherical actuator, $\nu_2 = 23/46.5 < 0.5$, and d'_4 maintains the maximum value. Hence, the maximum value of d'_4 is chosen as the normalization constant for the measured data. In other words, the experimental result on the rotor can be normalized by $B_{Ir}^e/0.1087$ or $9.2B_{Ir}^e$.

Step Two: Inspection of Eqn. (5.6) shows that the term $\mu_0 M_0 a c$ is related to the parameters of this particular rotor design. In order to produce useful benchmark data,

Figure 5.10: Dimensionless variables ν_2 vs. d_4'

the measured result needs to be normalized further by $B_{Ir}^e/(\mu_0 M_0 ac)$. Note that this is also a nondimensionalization process because all variables are dimensionless.

Eventually, the dimensionless flux density, denoted as B_{Ir}^N , will be related to the measured flux density B_{Ir}^e by

$$B_{Ir}^N = \frac{9.2 B_{Ir}^e}{\mu_0 M_0 ac}. \quad (5.10)$$

The dimensionless B_{Ir}^N is a function of dimensionless variables ν_1 , ϕ and θ .

Computation of magnetic flux density from B_{Ir}^N

After the normalized data set B_{Ir}^N is established, it is necessary to consider how to obtain the magnetic flux density of another PM-pole rotor with similar structure from this data set.

To distinguish from the proposed spherical actuator design, a generic spherical rotor

can be specified with parameters denoted as M_0^g , a^g , c^g and d_4^g , where a^g can be obtained from α^g with Eqn. (2.23), c^g from β^g with Eqn. (2.24) and d_4^g from R_b^g and R_r^g with Eqn. (5.3). The magnetic flux density of the new spherical rotor can be computed based on the normalized measurement data as

$$B_{Ir}^{New} = \mu_0 M_0^g a^g c^g d_4^g B_{Ir}^N, \quad (5.11)$$

where B_{Ir}^{New} is the data for the new rotor.

5.1.3 Visualization and Analysis of Experimental Result

The flux density B_{Ir} is a function of the rotor coordinates (r, θ, ϕ) or the measurement coordinates (r_p, θ_p, ϕ_p) with suitable coordinate transformation. Visualizing the distribution of B_{Ir} in the coordinates is difficult because it becomes a 3D manifold in a 4D space. Hence, by fixing the value of one of the coordinates, B_{Ir} can be plotted or visualized as a 2D manifold in a 3D space. Logically, by fixing r_p , the flux density B_{Ir} can be plotted against θ_p and ϕ_p as a 2D surface.

Magnetic flux density B_{Ir} variation with respect to θ_p and ϕ_p

Let d_a be the normal distance from the measuring point to the rotor surface. Besides the angular parameters θ_p and ϕ_p , magnetic flux density B_{Ir} is a function of r_p (Eqn. (2.52)) which in turn can be replaced by the normal distance from measuring point to the rotor surface d_a as

$$r_p = d_a + R_r. \quad (5.12)$$

Comparisons between the measured result and the analytical model of B_{Ir} are summarized in Fig. 5.11 and 5.12. The experimental and analytical values of B_{Ir} at $d_a = 0.5\text{mm}$, $d_a = 2.5$, $d_a = 4.5$ and $d_a = 6.5\text{mm}$ are presented in Fig. 5.11(a), 5.11(b), 5.12(a) and 5.12(b). The values of B_{Ir} decrease when d_a becomes large as

(a) 3D view of experimental vs. analytical results ($d_a = 0.5\text{mm}$)

(b) 3D view of experimental vs. analytical results ($d_a = 2.5\text{mm}$)

(c) Difference of experimental and analytical results ($d_a = 0.5\text{mm}$)

(d) Difference of experimental and analytical results ($d_a = 2.5\text{mm}$)

(e) 2D view of experimental vs. analytical results ($d_a = 0.5\text{mm}$, $\theta_p = 0$)

(f) 2D view of experimental vs. analytical results ($d_a = 2.5\text{mm}$, $\theta_p = 0$)

Figure 5.11: Experimental and analytical values of magnetic flux density B_{Ir} ($d_a = 0.5\text{mm}$ and 2.5mm)

(a) 3D view of experimental vs. analytical results ($d_a = 4.5\text{mm}$)

(b) 3D view of experimental vs. analytical results ($d_a = 6.5\text{mm}$)

(c) Difference of experimental and analytical results ($d_a = 4.5\text{mm}$)

(d) Difference of experimental and analytical results ($d_a = 6.5\text{mm}$)

(e) 2D view of experimental vs. analytical results ($d_a = 4.5\text{mm}$, $\theta_p = 0$)

(f) 2D view of experimental vs. analytical results ($d_a = 6.5\text{mm}$, $\theta_p = 0$)

Figure 5.12: Experimental and analytical values of magnetic flux density B_{Ir} ($d_a = 4.5\text{mm}$ and 6.5mm)

the magnetic flux density reduced its strength away from the rotor. Along the equator of the rotor in the ϕ_p -direction, eight positive/negative ridges can be observed due to the eight alternately magnetized PM poles on the rotor equator. This variation can be clearly seen in 2D presentation of the flux density data. Figures 5.11(e), 5.11(f), 5.12(e) and 5.12(f) show the variation of B_{Ir} along the rotor equator at $\theta_p = 0$.

Differences between the experimental data and analytical model can be observed in Figure 5.11(c), 5.11(d), 5.12(c) and 5.12(d) for flux density at $d_a = 0.5\text{mm}$, $d_a = 2.5\text{mm}$, $d_a = 4.5\text{mm}$ and $d_a = 6.5\text{mm}$ respectively. This difference of flux density modeling is mainly caused by the omission of high order harmonics terms in developing the scalar potential function of the magnetic field. The maximum difference between the experimental data and analytical result is about 400G or 7% of the actual measured data, which indicates that the accuracy of the proposed analytical magnetic field model is acceptable. With the inclusion of high order harmonics, the magnetic field model can be more accurate.

Magnetic flux density B_{Ir} with respect to d_a

By fixing the angular parameters (θ_p, ϕ_p) , the relationship between magnetic flux density B_{Ir} and d_a can be illustrated. Figure 5.13(a) and 5.13(b) present the change of B_{Ir} along the radial direction at $\theta_p = 0^\circ$, $\phi_p = 45^\circ$ and $\theta_p = 10^\circ$, $\phi_p = 35^\circ$. It can be seen that the analytical model also fits the experimental result well along the radial direction.

5.2 Measurement of Actuator Torque Output

To verify the analytical torque model, a series of experiments are conducted on the torque measurement of the spherical actuator.

Figure 5.13: Magnetic flux density B_{Ir} in radial direction

Torque measurements for single-axis actuators is relatively simple because the fixed rotor-axis position facilitates the design of measuring structure. Stark *et al.* [125] have conducted the torque measurement on an outer-rotor micromotor by attaching a beam to the rotor on one side, and a fixed pad at the other side. The torque value can be obtained from the deflection of the beam. Lin *et al.* [126] have designed a structure to measure the torque of a large gap magnetic suspension system. A pendulum is mounted on the rotor. From the angular position measurement of pendulum motion, the magnetic torque can be obtained. Beihoff [127] has made a survey of torque transduction methodologies applicable to single-axis actuators. In recent years, the emergence of multi-DOF spherical actuators brings forward requirement of different force/torque measurement techniques because the torque of spherical actuators is dependant on rotor orientations. Amano *et al.* [6] have conducted the measurement of the horizontal direction torque on a 3-DOF ultrasonic actuator by adding pre-loading weight on the rotor. Mitchell *et al.* [62] have measured the reluctance torque of a 2-DOF spherical PM actuator by using an experimental test rig in which the rotor is fixed on the base and the stator angle can be adjusted. Roth [35] and Zhou [36] have conducted the torque measurement of VRSM by using a customized apparatus

which can change the rotor shaft orientation.

A force/torque measurement apparatus is developed here to measure the six components of force/torque generated by the 3-DOF PM spherical actuator at any rotor orientation within the workspace. The actuator torque produced by a single stator coil within the magnetic field of the PM rotor is investigated first. The measured result is then compared with the analytical torque model of Eqn. (3.30) for the single coil. The linear relationship between torque output and current input in the torque model can be verified. Subsequently, experiments on measuring the actuator torque simultaneously produced by two stator coils are conducted. The measurement result should verify the principle of superposition on the actuator torque produced by multiple coils described by Eqn. (3.32).

5.2.1 Experiment on Torque Generated by a Single Coil

Apparatus of torque measurement

An apparatus is designed for the measurement of the torque generated between the PM-pole rotor and a single coil as shown in Fig. 5.14. The specifications of this torque measurement apparatus are listed in Table 5.4. A DC power supply (Topward 3303D [128]) provides current to the coil. A six-axis force/torque sensor (Fig. 5.15) that can measure three torque components and three force components is mounted on the shaft of the rotor. This sensor has a measuring range of 0.5Nm for torque component and 36N for force component. The rotor shaft is connected to a guide shaft. This guide shaft can slide along the slot of the arc guide so that the rotor can tilt. The tilting angle is indicated by the dials on the arc guide. The guide shaft can rotate about its own axis along with the rotor. The amount of rotation angle is indicated by the longitudinal rotation angle scale. The tilting and rotating motions determine the position of the coil axis (θ_p, ϕ_p) in the rotor frame. The relation

Figure 5.14: An apparatus for force/torque measurement of single coil

(a) Six-axis force/torque sensor

(b) DAQ interface

Figure 5.15: Six-axis force/torque sensor and DAQ interface (ATI Nano43 [129])

Table 5.4: Specifications of the torque measurement apparatus

DC power supply	Model	Topward 3303D
	Range	0~30V, 0~3A
Torque sensor	Model	ATI Nano 43
	Channel	6-axis
	Force range	$\pm 36\text{N}$
	Torque range	$\pm 0.5\text{Nm}$
Measuring structure (Guide shaft)	Tilting range	$\pm 40^\circ$
	Rotating range	0~360°

between (θ_p, ϕ_p) and (θ, ϕ) is indicated in Fig. 5.5, i.e., $\theta_p = \pi/2 - \theta$, $\phi_p = \phi$. By using this apparatus, the measurement of actuator torque with respect to (θ_p, ϕ_p) can be obtained. In this experiment, the measuring range of ϕ_p is from 0° to 360° , and θ_p is from -20° to 20° , which completely covers the rotor orientations in the workspace.

Coordinate transformation from sensor to motor torque

The raw data acquired from the six-axis force/torque sensor is based on the sensor coordinates as shown in Fig. 5.16; (x_s, y_s, z_s) represents the sensor coordinate system whereas (x_r, y_r, z_r) represents the rotor coordinate system. The measured data obtained from the sensor can be translated into the actual motor torque by

$$\begin{bmatrix} T_{rx} \\ T_{ry} \\ T_{rz} \end{bmatrix} = \begin{bmatrix} -\sin \delta & \cos \delta & 0 \\ \cos \delta & \sin \delta & 0 \\ 0 & 0 & -1 \end{bmatrix} \left\{ \begin{bmatrix} T_{sx} \\ T_{sy} \\ T_{sz} \end{bmatrix} + \begin{bmatrix} F_{sy} \\ -F_{sx} \\ 0 \end{bmatrix} L_s \right\}, \quad (5.13)$$

where F_{sx} , F_{sy} , F_{sz} and T_{sx} , T_{sy} , T_{sz} are force and torque components based on the sensor coordinates, T_{rx} , T_{ry} and T_{rz} are torque components based on the rotor coordinates, L_s is the distance from the origin of the sensor coordinates to the rotor center. According to the assembly of this apparatus, there is a twist angle δ between the sensor coordinate system and the rotor coordinate system.

Figure 5.16: Coordinate transformation of torque measurement

Analysis of experimental result

Visualization of experimental result:

The actuator torque is a (θ_p, ϕ_p) -dependant 3×1 vector consisting of three components T_{rx} , T_{ry} and T_{rz} . Visualization of the actuator torque is done on each of the torque components, i.e., $(\theta_p, \phi_p, T_{rx})$, $(\theta_p, \phi_p, T_{ry})$ and $(\theta_p, \phi_p, T_{rz})$. In conducting the torque measurement, the distance from the coil to the rotor surface is kept very close to zero. 3A current is supplied into the coil.

Figures 5.17, 5.18 and 5.19 present experimental measurement and analytical data of three torque components, T_{rx} , T_{ry} and T_{rz} respectively. The eight ridges appear in Fig. 5.19(a) are caused by the eight alternately arranged PM poles along the rotor equator. The variation of T_{rx} and T_{ry} in the ϕ_p -direction (Fig. 5.17(a) and Fig. 5.18(a)) are not in phase with the PM poles arrangement because the rotor coordinate is used for

(a) T_{rx} torque variation in 3D space

(b) Difference of analytical and experimental torque data

(c) T_{rx} torque variation at $\theta_p = 18^\circ$

(d) T_{rx} torque variation at $\phi_p = 21^\circ$

Figure 5.17: Actuator torque in x -direction T_{rx}

(a) T_{ry} torque variation in 3D space

(b) Difference of analytical and experimental torque data

(c) T_{ry} torque variation at $\theta_p = 18^\circ$

(d) T_{ry} torque variation at $\phi_p = 21^\circ$

Figure 5.18: Actuator torque in y -direction T_{ry}

(a) T_{rz} torque variation in 3D space

(b) Difference of analytical and experimental torque data

(c) T_{rz} torque variation at $\theta_p = 0^\circ$

(d) T_{rz} torque variation at $\phi_p = 21^\circ$

Figure 5.19: Actuator torque in z -direction T_{rz}

Figure 5.20: Rotor motion generated by torque components

the presentation. It can be verified that after transforming the torque data into stator coordinates, eight ridges can be observed for all three torque components (Appendix C). The maximum difference between the experimental and analytical results is about 12%. The difference is mainly due to the omission of high order terms in formulating magnetic flux density, approximation of coil shape, fabrication error, etc.

Figure 5.17(d), 5.18(d) and 5.19(d) illustrates the torque variation at $\phi_p = 21^\circ$. It can be found that T_{rx} and T_{ry} change the direction when θ_p varies, whereas T_{rz} is always positive. In order to facilitate the comprehension of the torque variation shown in Fig. 5.17-5.19, Fig. 5.20 presents the torque values corresponding to three positions of coil axis at $(\theta_p, \phi_p) = (0^\circ, 21^\circ)$, $(18^\circ, 21^\circ)$ and $(-18^\circ, 21^\circ)$ respectively. In Fig. 5.20(a), the coil axis is on the rotor equator plane and between two neighboring PM poles. The interaction between the coil and the pole produces an attraction force (PM 1) and a repulsion force (PM 2). Due to the symmetry of PM poles, directions of both forces are on the rotor equator plane. Thus, a torque T_{rz} is generated about the z -axis of the rotor to create a spinning motion, whereas the torque components in the x - and

y -direction T_{rx} and T_{ry} are negligible. Therefore, one can see clearly that at $\theta_p = 0$ in Fig. 5.17(a) and 5.18(a), the torque in x - and y -directions are very small. In Fig. 5.20(b) and 5.20(c), the attraction and repulsion force can pull the rotor to spin in the z -axis. Because the resultant force is not on the rotor equator plane, T_{rx} and T_{ry} can also be generated to tilt the rotor in different directions.

Torque variation with respect to airgap:

The torque output of the spherical actuator is also a function of the gap between coils and rotor surface. Based on the torque model of a single coil Eqns. (3.30) and (3.31), the actuator torque is related to the coil parameters R_0 and R_1 , which, in turn, can be expressed by the gap d_a between the rotor surface and the coil tip as

$$R_0 = R_r + d_a, \quad (5.14)$$

$$R_1 = R_0 + L_c = R_r + d_a + L_c, \quad (5.15)$$

where L_c is the coil length. By fixing the values of θ_p and ϕ_p and varying d_a , the relation between torque output and d_a can be obtained. In the torque measurement apparatus, the airgap d_a can be adjusted by screwing the coil in or out the fixture. Figure 5.21 shows the torque variation with respect to d_a at $(\theta_p, \phi_p) = (15^\circ, 25^\circ)$. It can be seen that the torque decreases with increasing d_a . Therefore, to achieve high torque output, smaller value of d_a is preferred.

Linearity of torque model:

According to Eqn. (3.27) and (3.32), the torque output of this PM spherical actuator is always directly proportional to the current input in a linear fashion. This linearity of the torque model can facilitate real-time motion control of the actuator. Experiments are carried out here to verify this linearity relationship. In the experiment, the coil is mounted on a fixed position with respect to the rotor frame, $\phi_p = 21^\circ$, $\theta_p = -9^\circ$. By varying the current input, the torque output are measured. Figure 5.22

(a) Torque z (T_{rz}) (b) Torque x (T_{rx}) (c) Torque y (T_{ry})Figure 5.21: Torque variation in the radial direction d_a ($\theta_p = 15^\circ$, $\phi_p = 25^\circ$)

(a) Torque x T_{rx} ($k = 10.2$, $k' = 9.1$)

(b) Torque y T_{ry} ($k = 5.2$, $k' = 5.6$)

(c) Torque z T_{rz} ($k = 72.0$, $k' = 74.3$)

Figure 5.22: Linearity of actuator torque and coil currents ($\phi_p = 21^\circ$, $\theta_p = -9^\circ$, k and k' : slopes of analytical and experimental results respectively)

shows both experimental data and analytical calculation results. It can be found that three components of the torque are all proportional to the current input. The experimental result is interpolated with straight line of slope k' , whereas the analytical linear relationship is characterized by the straight line of slope k . It can be shown that both analytical and experimental lines are close to each other. The difference is caused by approximation of coil shape, measurement error etc.

5.2.2 Experiment on Torque Generated by Multiple Coils

According to Eqn. (3.32), the total torque output of the spherical actuators is the superposition of torques generated by individual coils. To verify the viability of superposition principle, torque measurement is conducted on two arbitrarily chosen coils.

Torque measurement on multiple coils

Figure 5.23 shows that the same apparatus for torque measurement is used for multiple coils. The coils are mounted on the stator of the spherical actuator prototype instead of a special fixture.

Two coils, Coil 1 and Coil 2 indicated in Fig. 5.23 are used for the experiment. The initial position of these coils in the rotor frame are $(\theta_p, \phi_p)=(11^\circ, 0)$ and $(\theta_p, \phi_p)=(11^\circ, 30^\circ)$ respectively. The supplied current is 2.5A. Sliding the guide shaft along the arc guide will change orientation of the rotor. For any specified rotor orientation, three sets of data are captured: torque obtained by energizing Coil 1 alone, Coil 2 alone, and Coils 1 and 2 simultaneously. Then we compare the summation of the separate torque from the two coils and torque from the two simultaneously activated coils. The same measuring process is carried out for another pair of coils at Coil 3 ($\theta_p=-11^\circ, \phi_p=120^\circ$) and Coil 4 ($\theta_p=11^\circ, \phi_p=150^\circ$).

Figure 5.23: Measurement apparatus of torque generated by multiple coils

Experimental result

The experimental result of the actuator torque is shown in Fig. 5.24 and 5.25. In each of these figures, there are five curves representing experimental torque generated by Coil 1 alone, by Coil 2 alone, sum of these two sets of data, experimental torque by Coil 1 and 2, as well as analytical torque produced by Coil 1 and 2. The sum of two sets of data and the experimental torque by Coil 1 and 2 fit closely to each other. The maximum differences between analytical and experimental results (Coil 1 and 2) of three torque components T_{rx} , T_{ry} and T_{rz} are about 18%, 12% and 9% of the experimental result respectively. For Coil 3 and 4, the maximum differences are about 12%, 15% and 16% respectively. Additionally, the torque generated by these four coils are also measured. The seven curves in each figure of Fig. 5.26 represents the experimental torque generated by Coil 1 alone, Coil 2 alone, Coil 3 alone, Coil 4 alone, sum of these four sets of data, experimental torque by Coil 1-4 together and analytical torque produced by Coil 1-4. The trend of analytical result matches with

(a) T_{rx} (b) T_{ry} (c) T_{rz}

Figure 5.24: Superposition of actuator torque for multiple coils (Coil 1 and Coil 2)

(a) T_{rx} (b) T_{ry} (c) T_{rz}

Figure 5.25: Superposition of actuator torque output (Coil 3 and Coil 4)

(a) T_{rx} (b) T_{ry} (c) T_{rz}

Figure 5.26: Superposition of actuator torque output (Coils 1-4)

the experimental result closely. The maximum differences between analytical and experimental results of three torque components are about 14%, 13% and 11% of the experimental result respectively. The different between analytical and experimental results could be caused by omission of higher order terms of magnetic scalar potential, approximation of coil shape, measurement error etc.

5.3 Summary

By taking advantage of the developed spherical actuator with dihedral-shaped PM rotor poles, experimental investigation has been carried out on the magnetic field distribution of the PM rotor and actuator torque variation in this chapter. An automated apparatus is developed to measure the magnetic field distribution in 3D space surrounding the PM-pole rotor. The experimental result verifies the analytical magnetic field expression of the PM rotor. The maximum difference between analytical and experimental result is about 7%. Furthermore, nondimensionalization approach is used to process the measured magnetic field as benchmark result to be referred by other similar rotor designs. An apparatus is designed to measure the actuator torque output within the workspace. The measurement result is compared with analytical model, which shows that the maximum difference is about 12% of the experimental result. The difference can be produced by the omission of higher-order terms of magnetic scalar potential, approximation of coil shape, measurement error etc. One of the advantages of the PM spherical actuator is the linear torque model, i.e., the torque output is always directly proportional to the current input. Compare with nonlinear model, the computation of linear torque model is more effective for real-time control algorithm. In this chapter, experimental torque measurement with respect to current input verifies the linearity of the torque model. In addition, according to the actuator torque model for full set of coils, the total actuator torque is equal to the sum of torques generated by individual coils. Torque measurement on multiple coils mounted

on the stator verifies the superposition principle of the actuator torque output.

Chapter 6

Conclusions

6.1 Accomplishments and Contributions

This thesis has presented study on a permanent magnet spherical actuator with the following tasks: (1) formulation of the magnetic field produced by the PM-pole rotor; (2) torque modeling of the spherical actuator; (3) design and prototype of the spherical actuator; (4) experimental investigation on the magnetic field distribution and torque variation. All these works are fundamental and critical to the position and velocity servo control of the PM spherical actuator. Specifically, the magnetic field modeling is the prerequisite of the actuator torque, design and prototype of the actuator for experimental investigation of the magnetic field distribution and actuator torque variation.

Based on the PM pole arrangement, the magnetic field surrounding the rotor is formulated using Laplace's equation and Poisson's equation with the general solution of the magnetic scalar potentials. By utilizing appropriate boundary conditions and the spherical harmonic expansion of the radial component of the residual magnetization vector, the coefficients in the general solution of the scalar potential can be determined. Because magnetic intensity is the gradient of magnetic scalar potential, the magnetic flux density distribution in the air space surrounding the PM-pole rotor is

obtained.

With knowledge of the magnetic field expression, the torque modeling of the actuator which relates current inputs to the torque output of the spherical actuator can be carried out. According to Lorentz force law, the differential torque caused by a differential wire segment can be formulated. The torque created between a single coil and the PM-pole rotor can be obtained by integrating the differential torque within the coil volume. Using the principle of superposition, the torque for full set of coils can be finally obtained, which is linearly proportional to the current inputs. This linearity facilitates the real-time control of the spherical actuator.

Inverse electromagnetic solution of the torque model, i.e., solving the required current inputs for desired torque output, has been discussed. It is verified that there is no singularity in the torque model within the workspace of the spherical actuator. Because the torque model is a multiple input (coil currents) multiple output (3D torque) system, there are infinite possible inverse electromagnetic solutions. A minimum right-inverse solution has been proposed to obtain the current inputs for the actuator based on minimization of power consumption.

The tunable parameters of the PM pole and coil in the torque model provide a tool for prototype design of the PM spherical actuator aiming at maximizing torque output. According to the proposed torque model, the effect of poles parameters on the torque output can be described. Thus, appropriate values for different parameters can be chosen to achieve maximum torque output. Based on the theoretical analysis, a research prototype of the spherical actuator has been developed. It can be used for the experimental investigation on magnetic field distribution and torque variation of the spherical actuator.

Experimental investigation of the spherical actuator includes two major parts: magnetic field distribution of the PM-pole rotor and torque variation of the spherical

actuator. An automated apparatus has been developed to measure the flux density of the rotor in 3D space. Because the flux density is a 3×1 vector, a three-axis hall probe has been employed for the measurement. The acquired data of the magnetic field has been nondimensionalized and normalized as a benchmark result. It can be referred by similar rotor designs without regard to the specific dimensions of PM poles. The experimental result of magnetic flux density is visualized in a 3D fashion and compared with the analytical magnetic field model, which shows that the maximum different is about 7% and the theoretical model is feasible for torque modeling.

The second apparatus has been designed for the torque measurement of the spherical actuator. This apparatus can measure the torque output generated by the spherical actuator at any rotor orientation within the workspace. A six-axis force/torque sensor has been employed for the torque measurement. The torque measurement is conducted based on a single coil configuration and a two-coil configuration. The first experiment is used to verify the torque model of a single coil as well as the linear relation between torque output and current input. The second experiment is used to verify the superposition principle of the torque model for multiple coils, i.e. the torque generated by multiple coils is superposition of torques generated by every individual coil. The experiment shows that the difference between analytical model and experimental result is about 12%, which indicates that the proposed torque model of the spherical actuator is feasible.

In order to reduce the fabrication cost, a second prototype has been developed as described in Appendix B, in which cylindrical PMs are utilized as the rotor poles. Due to the wide availability of this type of PMs, the cost of the spherical actuator can be reduced significantly. The torque output expression of this prototype has been obtained by modifying the proposed torque model according to the torque measurement of the actuator with cylindrical PM poles.

The above work is based on the spherical actuator with single-layer PM poles. In the study of this spherical actuator, it is found that the tilting torque is relatively low compared with the spinning torque. In order to increase the tilting torque, a spherical actuator with two layers of PM poles has been proposed in Appendix C. Using the same modeling method for the single-layer configuration, the magnetic field model and torque model of double-layer configuration are obtained. The analytical result of torque output is compared with that of single-layer configuration. It shows that the spinning torque of the actuator with single-layer PM poles is larger than that of the actuator with double-layer PM poles by 41%, whereas the tilting torque of the actuator with double-layer PM poles is larger than that of the actuator with single-layer PM poles by 88%. Hence, selection of the pole pattern depends on the task requirements. The modeling approach developed here is generic to spherical actuators with multi-layer PM poles of different patterns.

This research has established a fundamental basis for the study of 3-DOF PM spherical actuators. The major contributions of this investigation are acknowledged as follows.

- A design concept of the spherical actuator using multiple PM poles and multiple layers of stator air-core coils has been proposed. One advantage of this configuration is that different numbers and patterns of stator and rotor poles are allowed to be incorporated to improve the working range as well as the motion resolution of the spherical actuator. This configuration also offers a linear relation between torque output and current inputs. This linearity can facilitate the real-time motion control of the PM spherical actuator.
- Formulation of the magnetic field of PM-pole rotor is a prerequisite of actuator torque modeling. The challenge of formulating the magnetic field of the spherical actuator is that the magnetic flux density varies at every point of the 3D space surrounding the PM-pole rotor. By using the Laplace's equation of the magnetic scalar

potential, the flux density of the PM-pole rotor has been formulated. The parameters of PM poles in the model can be adjusted to observe the magnetic field variation.

- The torque output of single-axis actuators has a constant direction, which makes the torque formulation much easier. However, for 3-DOF spherical motion actuators, the torque output is an orientation dependant 3D vector. Based on Lorentz force law as well as the magnetic field expression, the torque model of the spherical actuator has been formulated analytically, which could facilitate the real-time position and velocity control of the spherical actuator. Another feature of this analytical torque model is that the two major force/torque generating elements, PM poles and coils, are parameterized. This provides an opportunity to study the effect of each parameter on the torque output of the spherical actuator. As a result, the dimensions of PM pole and coil can be determined to achieve maximum torque output.
- Experimental measurement of magnetic field can be used to verify the theoretical model as well as observe the magnetic flux density distribution. So far, complete measurement on the 3D magnetic field of spherical actuator has not been reported in the literature. Only FEM approach or simple measurement is used to verify the theoretical models. In this research, an automated apparatus has been developed to measure the magnetic flux density distribution of the PM-pole rotor in 3D space. A three-axis Hall probe is employed to measure the three components of flux density of the PM-pole rotor. This probe can be mounted on the apparatus and pinpoint to any point surrounding the rotor then acquire the data of flux density at that point. The acquired data has been processed with nondimensionalization and normalization so that it could be used to compare with analytical magnetic field model as well as referred by designs with other rotor configuration.

6.2 Recommendation for Future Research

To continue this research effort, the following aspects are recommended.

- **Development and implementation of a servo control algorithm**

Development of the control system is critically important for practical uses of the spherical actuators. The schematic block diagram of motion control of the spherical actuator is illustrated in Fig. 6.1. The angular acceleration, angular velocity and angular displacement of the spherical actuator are provided as the desired actuator motion. Based on the inverse dynamics of the spherical actuator, the desired torque can be computed. Inverse electromagnetic solution is carried out to compute the required current inputs corresponding to the desired torque output. Finally, these

Figure 6.1: Block diagram of motion control of the spherical actuator

(a) Electric circuit

(b) Interface

Figure 6.2: Twelve-channel current controller

current inputs are applied to the spherical actuator so that the actuator can follow the desired motion. Presently, a twelve-channel current controller as shown in Fig. 6.2 has been developed for energizing stator coils. This current controller can manage twelve-channel current inputs at the same time. By using one or two pieces of this current controller, the current inputs to the coils on the PM spherical actuator can be varied. In the future, a control algorithm will be developed and implemented on the system by using the current controller to achieve real-time feedback control of the actuator. The rotor dynamics and external payload will be considered for actuator control.

• Development of a spherical displacement sensing system

As indicated in Fig. 6.1, to realize the closed-loop high precision position control of the spherical actuator, a high performance sensing system is necessary. This sensing system can detect the rotor orientation and then feedback it to the system controller. Some orientation sensing systems have been proposed by researchers. For example, three single-axis encoders are used in the rotor orientation measurement of VRSM [38, 130]. This type of orientation measurement system requires a mechanism that consists of two arc-shaped guides and a sliding block. This bulky structure contributes to two-thirds of the total system inertia, which complicates the control implementation. To avoid this additional inertial moment, the non-contact type sensor is preferred, i.e., there is no contact between the measurement system and the rotor. A real-time vision based absolute orientation sensor has been proposed by Garner *et al* [131, 132]. In order for the implementation of this sensor, the rotor surface is required to be grided and the resolution of the sensor depends on the density of grids. Lee *et al.* [133] have proposed a microscopic-surface-based optical sensor. The grid pattern is no longer required for the rotor surface. However, this type of sensors is very sensitive to the gap as well as the relative motion speed between the rotor surface and the sensor tip. Its availability needs to be validated further.

Because the magnetic field of the PM-pole rotor has been formulated, a possible orientation measurement sensor could be the Hall-effect sensor. Several Hall-effect sensors can be placed at different positions relative to the rotor. Based on the magnetic field formulated, the rotor orientation could be calculated from the measured value of flux density by these Hall-effect sensors. The implementation of this sensor needs to be considered further and the performance of this orientation detection approach is necessary to be evaluated in the future.

- **Using laminated soft-iron as the stator material**

Currently, large size is chosen for the aluminum stator to avoid the eddy current on the stator generated by the PM poles and to facilitate the experimental measurement of flux density. In order to reduce the actuator size, nonmetal materials such as Delrin could be used to replace the aluminum for commercialized products as eddy current would not be created for nonmetal materials. However, these materials normally cannot withstand the high temperature produced by the coils. In this situation, laminated soft-iron may be a good option as it can bear the high temperature resulted from the electrical power consumption. Furthermore, the eddy current generated by the PM poles is ignorable. More importantly, due to the high permeability of the soft-iron, the loss of magnetic energy could be reduced and torque produced by the spherical actuator could be improved greatly.

Appendix A

Higher-order terms in Scalar Potential

Theoretically, a complete expression of the magnetic scalar potential of the PM rotor ($\Phi_I = \sum_{n=0}^{\infty} \sum_{m=-n}^n [\kappa_{nI}^m r^n + \xi_{nI}^m r^{-(n+1)}] [Y_n^m(\theta, \phi)]$) can be used to derive a complete magnetic field model, and thus a torque model including all terms. These magnetic field model as well as torque model can provide a more accurate comprehension of the PM spherical actuator for researchers.

Because the fundamental term has the most contribution to the magnetic scalar potential, at the same time, there are infinite terms in the expression of magnetic scalar potential and the torque integration for higher-order terms is very complex, we chose the fundamental term for magnetic field and torque computation. Here, we select the next higher-order term to analyze its effect on the magnetic field and torque output of the PM spherical actuator as follows.

In Chapter 2, we know that $C_{nm} \neq 0$ (C_{nm} is related to the coefficients of the magnetic scalar potential) if and only if $m = \pm 4, \pm 12, \pm 20, \dots$. Thus, the fundamental terms of the spherical harmonics functions can be taken at $n = 4$ and $m = \pm 4$. Furthermore, it is found that when $n = 5$ and $m = \pm 4$, C_{nm} is also equal to zero. Therefore, we take the term when $n = 6$ and $m = \pm 4$ as the next higher-order term.

Following the similar deriving process mentioned in Chapter 2, the magnetic scalar potential contributed by this higher-order term can be obtained as

$$\begin{aligned}
\Phi_I' &= \xi_{6I}^{-4} r^{-7} [Y_6^{-4}(\theta, \phi)] + \xi_{6I}^4 r^{-7} [Y_6^4(\theta, \phi)] \\
&= M_0 \frac{d_6}{\sqrt{\pi}} (a' - b' i) c' r^{-7} \left[\sqrt{\frac{13}{2\pi}} \frac{1}{10!} \frac{945}{2} \sin^4 \theta (11 \cos^2 \theta - 1) e^{-4i\phi} \right] + \\
&\quad M_0 \frac{d_6}{\sqrt{\pi}} (a' + b' i) c' r^{-7} \left[\sqrt{\frac{13}{2\pi}} \frac{1}{10!} \frac{945}{2} \sin^4 \theta (11 \cos^2 \theta - 1) e^{4i\phi} \right] \\
&= \frac{945}{\pi} \sqrt{\frac{13}{2 \cdot 10!}} M_0 d_6 c' r^{-7} \sin^4 \theta (11 \cos^2 \theta - 1) (a' \cos 4\phi - b' \sin 4\phi), \quad (\text{A.1})
\end{aligned}$$

where

$$d_6 = - \frac{R_r^8 + \frac{13\mu_m R_b^8 R_r^{13}}{6(\mu_r - \mu_m) R_b^{13} - [6\mu_r + 7\mu_m] R_r^{13}}}{7(\mu_m - 1) + \frac{13\mu_m [6\mu_r + 7\mu_m] R_r^{13}}{6(\mu_r - \mu_m) R_b^{13} - [6\mu_r + 7\mu_m] R_r^{13}}},$$

and a' , b' , c' can be obtained from Eqns. (2.23) and (2.24). The total scalar potential with the fundamental term and the next higher-order term is

$$\begin{aligned}
\Phi_I &= \frac{M_0}{\pi} \sin^4 \theta \left[\sqrt{\frac{35}{2}} \frac{3cd_4}{8} r^{-5} (a \cos 4\phi - b \sin 4\phi) + \right. \\
&\quad \left. 945 \sqrt{\frac{13}{2 \cdot 10!}} c' d_6 r^{-7} (11 \cos^2 \theta - 1) (a' \cos 4\phi - b' \sin 4\phi) \right]. \quad (\text{A.2})
\end{aligned}$$

Correspondingly, the magnetic flux density B_{Ir} can be obtained as

$$\begin{aligned}
B_{Ir} &= -\mu_0 \frac{\partial \Phi_I}{\partial r} = \frac{\mu_0 M_0}{\pi} \sin^4 \theta \left[\sqrt{\frac{35}{2}} \frac{15cd_4}{8} r^{-6} (a \cos 4\phi - b \sin 4\phi) + \right. \\
&\quad \left. 6615 \sqrt{\frac{13}{2 \cdot 10!}} c' d_6 r^{-8} (11 \cos^2 \theta - 1) (a' \cos 4\phi - b' \sin 4\phi) \right]. \quad (\text{A.3})
\end{aligned}$$

Then, by using the expression of the magnetic flux density, the actuator torque can be obtained. According to Chapter 3, the formulation used to integrate the torque generated by a single coil is

$$\mathbf{T}_i = -J_i \int_{R_0}^{R_1} \int_{\zeta_0}^{\zeta_1} \left\{ \int_C r B_{Ir}(r, \theta, \phi) dl \right\} r dr d\zeta. \quad (\text{A.4})$$

As shown in Section 2.2.6, by choosing appropriate coordinates, b and b' in the magnetic flux density can vanish. Substituting the magnetic flux density B_{Ir} into Eqn.

(A.3) gives

$$\begin{aligned} \mathbf{T}_i = & \frac{\mu_0 M_0}{\pi} J_i \left[\sqrt{\frac{35}{2}} \frac{15acd_4}{8} \int_{R_0}^{R_1} \int_{\zeta_0}^{\zeta_1} \int_C r^{-3} \sin^4 \theta \cos 4\phi \sin \zeta (\cos \psi \mathbf{e}_{\phi i} - \sin \psi \mathbf{e}_{\theta i}) \right. \\ & d\psi dr d\zeta + 6615 \sqrt{\frac{13}{2 \cdot 10!}} a' c' d_6 \int_{R_0}^{R_1} \int_{\zeta_0}^{\zeta_1} \int_C r^{-5} \sin^4 \theta (11 \cos^2 \theta - 1) \cos 4\phi \sin \zeta \\ & \left. (\cos \psi \mathbf{e}_{\phi i} - \sin \psi \mathbf{e}_{\theta i}) d\psi dr d\zeta \right]. \end{aligned} \quad (\text{A.5})$$

The integration of $\int_{\zeta_0}^{\zeta_1} \int_C \sin^4 \theta \cos 4\phi \sin \zeta (\cos \psi \mathbf{e}_{\phi i} - \sin \psi \mathbf{e}_{\theta i}) d\psi d\zeta$ is obtained as

$$\begin{aligned} \mathbf{D}^e(\theta_i, \phi_i) = & (G''_{\zeta} - \frac{3}{4} G'_{\zeta}) \pi [\mathbf{e}_{\phi i} (-4 \sin^3 \theta_i \cos^4 \phi_i \cos \theta_i - 4 \sin^3 \theta_i \sin^4 \phi_i \cos \theta_i + 24 \\ & \sin^3 \theta_i \cos^2 \phi_i \sin^2 \phi_i \cos \theta_i) - \mathbf{e}_{\theta i} (16 \sin^3 \theta_i \cos^3 \phi_i \sin \phi_i - 16 \sin^3 \theta_i \sin^3 \phi_i \cos \phi_i)], \end{aligned} \quad (\text{A.6})$$

where G'_{ζ} , G''_{ζ} are defined in Eqns. (3.22) and (3.23).

Denote the angular portion in the second integral term of Eqn. (A.5) as

$$\begin{aligned} \mathbf{D}_2(\theta, \phi) & \equiv \sin^4 \theta (11 \cos^2 \theta - 1) \cos 4\phi \sin \zeta (\cos \psi \mathbf{e}_{\phi i} - \sin \psi \mathbf{e}_{\theta i}) \\ & = \sin^4 \theta (10 - 11 \sin^2 \theta) (\cos^4 \phi - 6 \sin^2 \phi \cos^2 \phi + \sin^4 \phi) \sin \zeta (\cos \psi \mathbf{e}_{\phi i} - \sin \psi \mathbf{e}_{\theta i}). \end{aligned} \quad (\text{A.7})$$

According to the relationship of the Cartesian coordinates, x , y , z to spherical coordinates, θ and ϕ ,

$$\cos \phi = \frac{x}{r \sin \theta}, \quad \sin \phi = \frac{y}{r \sin \theta}, \quad \sin^2 \theta = \frac{x^2 + y^2}{r^2}, \quad (\text{A.8})$$

$\mathbf{D}_2(\theta, \phi)$ can be rewritten as

$$\mathbf{D}_2(\theta, \phi) = \frac{1}{r^4} (10 - 11 \frac{x^2 + y^2}{r^2}) (x^4 - 6x^2 y^2 + y^4) \sin \zeta (\cos \psi \mathbf{e}_{\phi i} - \sin \psi \mathbf{e}_{\theta i}). \quad (\text{A.9})$$

In order to facilitate the computation, $\mathbf{D}_2(\theta, \phi)$ is split into two terms

$$\mathbf{D}'_2(\theta, \phi) = \frac{10}{r^4} (x^4 - 6x^2 y^2 + y^4) \sin \zeta (\cos \psi \mathbf{e}_{\phi i} - \sin \psi \mathbf{e}_{\theta i}), \quad (\text{A.10})$$

$$\mathbf{D}''_2(\theta, \phi) = -\frac{11}{r^6} (x^2 + y^2) (x^4 - 6x^2 y^2 + y^4) \sin \zeta (\cos \psi \mathbf{e}_{\phi i} - \sin \psi \mathbf{e}_{\theta i}). \quad (\text{A.11})$$

In Chapter 3, we know that

$$x = r \cos \zeta \sin \theta_i \cos \phi_i - r \sin \zeta \cos \psi \cos \theta_i \cos \phi_i + r \sin \zeta \sin \psi \sin \phi_i,$$

$$\begin{aligned}
y &= r \cos \zeta \sin \theta_i \sin \phi_i - r \sin \zeta \cos \psi \cos \theta_i \sin \phi_i - r \sin \zeta \sin \psi \cos \phi_i, \\
z &= r \cos \zeta \cos \theta_i + r \sin \zeta \cos \psi \sin \theta_i.
\end{aligned}$$

By using these equations, $\mathbf{D}'_2(\theta, \phi)$ and $\mathbf{D}''_2(\theta, \phi)$ can be expressed as a function of ζ , ψ , ϕ_i and θ_i , in which the former two parameters are integrable, whereas the latter two are dependant on the orientation of the coil in the rotor frame. Because the range of ψ is from 0 to 2π , many terms in the integral of $\mathbf{D}'_2(\theta, \phi)$, $\mathbf{D}''_2(\theta, \phi)$ vanish. Finally, the integral results of $\mathbf{D}'_2(\theta, \phi)$, $\mathbf{D}''_2(\theta, \phi)$, denoted as $\mathbf{D}'_2{}^e(\theta_i, \phi_i)$, $\mathbf{D}''_2{}^e(\theta_i, \phi_i)$ are obtained as follows.

$$\begin{aligned}
\mathbf{D}'_2{}^e(\theta_i, \phi_i) &= 10(G''_\zeta - \frac{3G'_\zeta}{4})\pi \{ [-\sin \phi_i \cos \phi_i \ 0]^T (-4 \sin^3 \theta_i \cos^4 \phi_i \cos \theta_i - 4 \sin^3 \theta_i \\
&\sin^4 \phi_i \cos \theta_i + 24 \sin^3 \theta_i \cos^2 \phi_i \sin^2 \phi_i \cos \theta_i) - [\cos \theta_i \cos \phi_i \quad \cos \theta_i \sin \phi_i \quad -\sin \theta_i]^T \\
&(16 \sin^3 \theta_i \cos^3 \phi_i \sin \phi_i - 16 \sin^3 \theta_i \sin^3 \phi_i \cos \phi_i) \}, \quad (\text{A.12})
\end{aligned}$$

$$\begin{aligned}
\mathbf{D}''_2{}^e(\theta_i, \phi_i) &= -11 \left\{ \left[\frac{1}{4} \sin^3 \theta_i \cos \theta_i \pi [G'_{\zeta 2} (15 \cos^2 \theta_i + 5) + G''_{\zeta 2} (10 - 90 \cos^2 \theta_i) + \right. \right. \\
&G'''_{\zeta 2} (99 \cos^2 \theta_i - 39)] (-8 \cos^2 \phi_i + 1 + 8 \cos^4 \phi_i) \mathbf{e}_{\phi_i} + [2 \sin \phi_i \sin^3 \theta_i \cos \phi_i \pi (5 G'_{\zeta 2} \\
&(\cos^2 \theta_i + 1) - G''_{\zeta 2} (30 \cos^2 \theta_i + 10) + G'''_{\zeta 2} (33 \cos^2 \theta_i - 3)) (-1 + 2 \cos^2 \phi_i)] \mathbf{e}_{\theta_i} \left. \right\}, \quad (\text{A.13})
\end{aligned}$$

where

$$\begin{aligned}
G'_{\zeta 2} &= \frac{1}{3}(\sin^3 \zeta_1 - \sin^3 \zeta_0), \\
G''_{\zeta 2} &= \frac{1}{5} \sin \zeta_0 \cos^4 \zeta_0 - \frac{1}{15} \sin \zeta_0 \cos^2 \zeta_0 - \frac{2}{15} \sin \zeta_0 - \frac{1}{5} \sin \zeta_1 \cos^4 \zeta_1 + \\
&\frac{1}{15} \sin \zeta_1 \cos^2 \zeta_1 + \frac{1}{15} \sin \zeta_1, \\
G'''_{\zeta 2} &= \frac{1}{7} \sin \zeta_0 \cos^6 \zeta_0 - \frac{1}{35} \sin \zeta_0 \cos^4 \zeta_0 - \frac{4}{105} \sin \zeta_0 \cos^2 \zeta_0 - \frac{8}{105} \sin \zeta_0 - \\
&\frac{1}{7} \sin \zeta_1 \cos^6 \zeta_1 + \frac{1}{35} \sin \zeta_1 \cos^4 \zeta_1 + \frac{4}{105} \sin \zeta_1 \cos^2 \zeta_1 + \frac{8}{105} \sin \zeta_1.
\end{aligned}$$

Therefore, the torque output by considering the fundamental term and the next higher-order term is

$$\begin{aligned}
\mathbf{T}_i &= \frac{\mu_0 M_0}{\pi} J_i \left\{ \sqrt{\frac{35}{2}} \frac{15acd_4}{16} [(R_0^{-2} - R_1^{-2}) \mathbf{D}^e(\theta_i, \phi_i)] + \sqrt{\frac{13}{2 \cdot 10!}} \frac{6615a'c'd_6}{4} \right. \\
&\left. \{ (R_0^{-4} - R_1^{-4}) [\mathbf{D}'_2{}^e(\theta_i, \phi_i) + \mathbf{D}''_2{}^e(\theta_i, \phi_i)] \} \right\}. \quad (\text{A.14})
\end{aligned}$$

It is found that the maximum difference between measured result and analytical model of the magnetic field is 4% of the experimental result, and the maximum difference between the measured result and analytical model of the torque output is 9% of the experimental result. Compared with the difference created by the fundamental term only (7% for magnetic field model, 12% for torque model), using more higher-order terms in the magnetic scalar potential may aid in reducing the model error of magnetic field and torque output. However, the effect is not significant and the computation of torque is much more complex for higher-order terms. Thus, currently, we didn't use this higher-order term in our magnetic field model and torque model derivation. Similar approach was also adopted by other researchers in this field [59].

Appendix B

Low-cost PM Rotor

B.1 Rotor Poles with Cylindrical PMs

In previous study, dihedral-shaped PMs are utilized as the rotor poles. This type of PM pole benefits the torque modeling as well as actuator design. However, it is relatively complicated to fabricate and the cost is comparatively high. Rotor poles with stacked cylindrical PMs are proposed as an alternative. Because the cylindrical PMs are widely available on the market, the cost of PM poles can be reduced by 80%. Several cylindrical PMs of different sizes can be stacked together to form a rotor pole

Figure B.1: Utilization of cylindrical PMs for rotor poles

as shown in Fig. B.1(a).

B.2 Torque Model through experimentation

Because the torque model developed in Chapter 3 is based on dihedral-shaped PM poles, it cannot be directly used on the cylindrical PM pole rotor design. Due to similar PM poles arrangement on the rotor surface, however, the magnetic field distribution surrounding the rotor and the corresponding actuator torque could be similar. Thus, it is possible to formulate the torque expression of the actuator with the low-cost rotor design through experimentation. In other words, by adjusting the terms in the torque model according to the experimental torque measurement, the torque model for this low-cost rotor could be obtained.

Two possible approaches can be used for modeling the actuator torque. First is to obtain the magnetic field model from experiment measurement on the actual magnetic field of the rotor and to adjust the terms in the magnetic field model to match the experimental result. The actuator torque can be obtained by using Lorentz force law. This approach is not favorable because of accumulated errors from the measurement of the magnetic field flux density and the computation of the torque integration (Eqn. (3.5)). The alternative approach is to use experimental measurement on torque variation to formulate the torque model directly.

Experimental measurement of the torque occurred between the low-cost rotor and a single coil is carried out on the torque measuring apparatus. Through comparing the analytical torque model with the experimental result, Eqns. (3.30) and (3.32) can be modified as the torque model of the spherical actuator with cylindrical pole rotor. As indicated in Eqn. (B.1), there are three modifications. First, according to experimental result, the frequency of torque variation in the θ_p -direction increases 2.1 times compare with the previous torque model. Hence, $\theta'_p = 2.1\theta_p$. The rotor

coordinates and the measurement coordinates have the relationship: $\theta = \pi/2 - \theta_p$. Therefore, $\theta' = \pi/2 - 2.1\theta_p$. For the i^{th} coil, it is $\theta'_i = \pi/2 - 2.1\theta_{pi}$. Second, ratios among three torque components can be observed from experimental result. It is found that torque components in x - and y -directions need to be multiplied approximately by 2.4. Third, the magnitude of the actuator torque can also be modified according to experimental result. As shown in Fig. B.1(b), the cylindrical PM pole is enclosed by a dihedral-shaped PM pole. As introduced in Chapter 4, by varying dimensions of the PM pole, the magnitude of the theoretical torque output can be adjusted. Therefore, dimensions of PM pole can be chosen to make the magnitude of theoretical result coincident with that of experimental result. In this prototype construction, the PM-pole dimensions are determined as $\alpha = 30^\circ$, $\beta = 30^\circ$, $R_b = 15\text{mm}$ and $R_r = 46.5\text{mm}$. The actuator torque for single coil is thus

$$\begin{aligned} \mathbf{T}_i = \begin{bmatrix} T_{xi} \\ T_{yi} \\ T_{zi} \end{bmatrix} &= T_c \begin{bmatrix} 2.4g_x(\pi/2 - 2.1\theta_{pi}, \phi_i) \\ 2.4g_y(\pi/2 - 2.1\theta_{pi}, \phi_i) \\ g_z(\pi/2 - 2.1\theta_{pi}, \phi_i) \end{bmatrix} J_i \\ &= T_c \mathbf{G}(\pi/2 - 2.1\theta_{pi}, \phi_i) J_i, \end{aligned} \quad (\text{B.1})$$

where

$$T_c = \sqrt{\frac{35}{2}} \frac{15\mu_0 M_0 a c d_4}{16\pi} (R_0^{-2} - R_1^{-2}).$$

The torque model for a full set of coils is thus replaced by

$$\mathbf{T} = T_c \begin{bmatrix} 2.4g_x(\pi/2 - 2.1\theta_{p1}, \phi_1) & 2.4g_x(\pi/2 - 2.1\theta_{p2}, \phi_2) & \cdots & 2.4g_x(\pi/2 - 2.1\theta_{pN}, \phi_N) \\ 2.4g_y(\pi/2 - 2.1\theta_{p1}, \phi_1) & 2.4g_y(\pi/2 - 2.1\theta_{p2}, \phi_2) & \cdots & 2.4g_y(\pi/2 - 2.1\theta_{pN}, \phi_N) \\ g_z(\pi/2 - 2.1\theta_{p1}, \phi_1) & g_z(\pi/2 - 2.1\theta_{p2}, \phi_2) & \cdots & g_z(\pi/2 - 2.1\theta_{pN}, \phi_N) \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_N \end{bmatrix}. \quad (\text{B.2})$$

These equations are valid within the range of $\theta_{pi} \in [-\pi/6, \pi/6]$ and $\phi_i \in [0, 2\pi]$. The normal distance d_a from the rotor surface to the top of the coil is approximately equal to zero. Magnets with $\mathbf{B}_{rem} = 1.29\text{T}$ (NdFeB 26 N-42H) are utilized. The comparison between the analytical torque model and experimental result of three torque components are presented in Fig. B.2, B.3 and B.4. The maximum difference between the theoretical model and experimental result is about 15%.

(a) T_{rx} torque in x -direction (b) Difference of analytical and experimental results (T_{rx}) (c) T_{rx} variation at $\theta_p = 15^\circ$ Figure B.2: Actuator torque T_{rx} in x -direction of low-cost rotor

(a) T_{ry} torque in y -direction

(b) Difference of analytical and experimental results (T_{ry})

(c) T_{ry} variation at $\theta_p = 15^\circ$

Figure B.3: Actuator torque T_{ry} in y -direction of low-cost rotor

(a) T_{rz} torque in z -direction (b) Difference of analytical and experimental results (T_{rz}) (c) T_{rz} variation at $\theta_p = 0^\circ$ Figure B.4: Actuator torque T_{rz} in z -direction of low-cost rotor

Appendix C

Double-layer PM-Pole Spherical Actuator

In this appendix, a rotor configuration with double-layer PM poles is presented. Corresponding to this configuration, the magnetic field model as well as the torque model are obtained. The formulation of these models has been mentioned in Chapters 2 and 3. According to the analytical models, the magnetic field and torque output for both single-layer and double-layer configurations are compared, which helps selection of appropriate PM pole patterns.

C.1 Arrangement of PM Poles

As shown in Fig. C.1, two layers of PM poles in alternate magnetization directions are placed along the rotor equator. The neighboring PM poles residing in different layers are magnetized in opposite directions. Imagine that a coil is placed between these two PM-pole layers. One PM pole at the bottom layer creates an attraction force by interacting with the coil, whereas the other PM pole at the top layer can generate a repulsion force with the coil. Both forces cause a tilting torque in the same direction. Therefore, this pole arrangement can achieve larger tilting torque compared with the one using same magnetization direction for both layers of PM poles.

Figure C.1: Arrangement of double-layer PM rotor poles

Similar to the PM-pole geometry of single-layer configuration (Fig. 2.2(a)), the PM-pole of double-layer configuration is also specified by four parameters as indicated in Fig. C.1(b), i.e., longitudinal angle α , latitudinal angle β , rotor radius R_r and rotor core radius R_b .

C.2 Magnetic Field Model

Following the same magnetic field modeling process for the single-layer PM pole configuration, the magnetic field model for double-layer PM pole configuration can be obtained as

$$\begin{bmatrix} B_{Ir} \\ B_{I\theta} \\ B_{I\phi} \end{bmatrix} = \frac{945\mu_0 C_{5,4} d_1}{d_2} \sqrt{\frac{11}{9!\pi}} r^{-7} \sin^3 \theta \begin{bmatrix} 6 \sin \theta \cos \theta \cos 4\phi \\ (\sin^2 \theta - 4 \cos^2 \theta) \cos 4\phi \\ 4 \cos \theta \sin 4\phi \end{bmatrix}, \quad (\text{C.1})$$

where $C_{5,4}$ is determined from the surface integral of the following form:

$$C_{n,m}|_{n=5,m=4} = \int_0^\pi \int_0^{2\pi} M_{0r}(\theta, \phi) [Y_n^{m*}(\theta, \phi)] \sin \theta d\theta d\phi. \quad (\text{C.2})$$

$M_{0r}(\theta, \phi)$ in Eqn. (C.2) is the radial component of residual magnetization vector $\mathbf{M}_0$ which can be specified in spherical coordinates as follows.

- For the top PM-pole layer, the residual magnetization vector $\mathbf{M}_0$ can be represented in spherical coordinates as

$$\mathbf{M}_0 = \begin{bmatrix} M_{0r} \\ M_{0\theta} \\ M_{0\phi} \end{bmatrix} = (-1)^{p-1} \frac{\sqrt{2}}{2} |\mathbf{M}_0| \begin{bmatrix} \cos(\phi - \alpha_p) \sin \theta + \cos \theta \\ \cos(\phi - \alpha_p) \cos \theta - \sin \theta \\ -\sin(\phi - \alpha_p) \end{bmatrix}, \quad (\text{C.3})$$

which is valid within the range of

$$-\frac{\alpha}{2} < \phi - \frac{\pi}{4}(p-1) < \frac{\alpha}{2}, \quad \frac{\pi}{4} - \frac{\beta}{2} < \theta < \frac{\pi}{4} + \frac{\beta}{2}, \quad p = 1, 2, \dots, 8.$$

- For the bottom PM-pole layer, $\mathbf{M}_0$ can be represented as

$$\mathbf{M}_0 = \begin{bmatrix} M_{0r} \\ M_{0\theta} \\ M_{0\phi} \end{bmatrix} = (-1)^{p-1} \frac{\sqrt{2}}{2} |\mathbf{M}_0| \begin{bmatrix} \cos(\phi - \alpha_p) \sin \theta - \cos \theta \\ \cos(\phi - \alpha_p) \cos \theta + \sin \theta \\ -\sin(\phi - \alpha_p) \end{bmatrix}, \quad (\text{C.4})$$

which is valid within the range of

$$-\frac{\alpha}{2} < \phi - \frac{\pi}{4}(p-1) < \frac{\alpha}{2}, \quad \frac{3\pi}{4} - \frac{\beta}{2} < \theta < \frac{3\pi}{4} + \frac{\beta}{2}, \quad p = 1, 2, \dots, 8.$$

C.3 Torque Model

The torque generated by a single coil is obtained as following

$$\mathbf{T}_i = \begin{bmatrix} T_{xi} & T_{yi} & T_{zi} \end{bmatrix}^T = T_c \mathbf{G}(\theta_i, \phi_i) J_i, \quad (\text{C.5})$$

where

$$\begin{aligned} \mathbf{G}(\theta_i, \phi_i) &= \begin{bmatrix} g_x(\theta_i, \phi_i) & g_y(\theta_i, \phi_i) & g_z(\theta_i, \phi_i) \end{bmatrix}^T \\ &= \mathbf{e}_{\phi_i} (-4 \sin^3 \theta_i \sin^4 \phi_i \cos^2 \theta_i - 4 \sin^3 \theta_i \cos^4 \phi_i \cos^2 \theta_i - 6 \sin^5 \theta_i \cos^2 \phi_i \sin^2 \phi_i \\ &\quad + \sin^5 \theta_i \sin^4 \phi_i + \sin^5 \theta_i \cos^4 \phi_i + 24 \sin^3 \theta_i \cos^2 \phi_i \cos^2 \theta_i \sin^2 \phi_i) - \mathbf{e}_{\theta_i} (16 \sin^3 \theta_i \\ &\quad \cos^3 \phi_i \cos \theta_i \sin \phi_i - 16 \sin^3 \theta_i \cos \phi_i \sin^3 \phi_i \cos \theta_i), \\ T_c &= \frac{630 \mu_0 C_{5,4} d_1}{d_2} \sqrt{\frac{11\pi}{8!}} (R_0^{-3} - R_1^{-3}) G_\zeta, \\ G_\zeta &= \frac{1}{8} G'_\zeta - \frac{3}{2} G''_\zeta + G'''_\zeta, \\ G'_\zeta &= \int_{\zeta_0}^{\zeta_1} \sin^6 \zeta d\zeta, \end{aligned}$$

$$G''_{\zeta} = \int_{\zeta_0}^{\zeta_1} \cos^2 \zeta \sin^4 \zeta d\zeta,$$

$$G'''_{\zeta} = \int_{\zeta_0}^{\zeta_1} \cos^4 \zeta \sin^2 \zeta d\zeta.$$

Similarly, θ_i and ϕ_i are used to define the position of the i^{th} coil's axis in spherical coordinates. Corresponding to N coils assembled in the stator, N torque equations like Eqn. (C.5) can be obtained. Concatenating them in a matrix form yields the torque model of the spherical actuator with complete set of coils

$$\mathbf{T} = T_c \begin{bmatrix} g_x(\theta_1, \phi_1) g_x(\theta_2, \phi_2) \cdots g_x(\theta_N, \phi_N) \\ g_y(\theta_1, \phi_1) g_y(\theta_2, \phi_2) \cdots g_y(\theta_N, \phi_N) \\ g_z(\theta_1, \phi_1) g_z(\theta_2, \phi_2) \cdots g_z(\theta_N, \phi_N) \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_N \end{bmatrix}, \quad (\text{C.6})$$

where $[J_1 \ J_2 \ \cdots \ J_N]^T$ is a vector of the current densities passing through Coil 1, Coil 2, $\cdots$, Coil N .

C.4 Comparison of Single and Double-layer Configurations

For convenience, let Configuration *I* be the rotor with single-layer PM poles, whereas Configuration *II* be the rotor with double-layer PM poles. The magnetic field and torque output of Configuration *I* and *II* are compared here.

C.4.1 Comparison of Magnetic Field

The radial component of flux density B_{Ir} (Region *I*) and two angular coordinates θ , ϕ can form a 3D coordinate system. The variation of B_{Ir} appears as a surface in this coordinate system visually. For ease of comparison, the same PM-pole parameters are used for both configurations: $\alpha = 40^\circ$, $\beta = 40^\circ$, $R_r = 46.5\text{mm}$, $R_b = 30\text{mm}$, $r = 48\text{mm}$ and $\mathbf{B}_{rem} = 1.0\text{T}$. The magnetic field distributions are plotted in Fig. C.2. It can be found that the distribution of peak values are coincident with the PM-pole

arrangement. Specifically, for Configuration *I*, eight positive/negative peaks appears at $\theta = 90^\circ$, which is caused by alternatively arranged PM poles along the rotor equator. As for Configuration *II*, two rows of peak values appear in the ϕ -direction. This is also coincident with the PM pole arrangement on the rotor surface. Computation of

(a) Magnetic field for single-layer of PM poles (Configuration *I*)

(b) Magnetic field for double-layer of PM poles (Configuration *II*)

Figure C.2: Magnetic field distribution of PM-pole rotors of single and double layer ($r = 48\text{mm}$)

magnetic field indicates that the maximum value of B_{Ir} for Configuration *I* in Fig. C.2(a) (0.40T) is larger than the maximum value for Configuration *II* in Fig. C.2(b) (0.34T). This is because the magnetized PM-pole volume for Configuration *I* is larger than that of Configuration *II*. However, for Configuration *II*, two positive/negative peaks appear in the θ -direction, which may help in increasing tilting torque of the actuator.

C.4.2 Comparison of Torque Output

As indicated in Eqn. (C.6), the total torque generated by multiple coils are the superposition of torques generated by individual coils. Therefore, analyzing the torque generated by single coil is essential. In a spherical actuator, the torque output is an orientation dependant 3×1 vector, $\mathbf{T} = [T_x \ T_y \ T_z]^T$. To facilitate torque comparison, coil parameters for both PM-pole configurations are given as $R_0 = 47\text{mm}$, $R_1 = 75\text{mm}$, $\zeta_0 = 2^\circ$ and $\zeta_1 = 9^\circ$. The number of winding turn is 800 turns, and 3A current is supplied into the coil. It can be found in Fig. C.3 that the maximum value of T_z of Configuration *I* (120.24Nmm) is larger than that of Configuration *II* (84.70Nmm), i.e., Configuration *I* is able to generate a high spinning torque about the rotor shaft. However, the maximum value of T_x or T_y of Configuration *II* (75.16Nmm) is higher than that of Configuration *I* (39.65Nmm), i.e. Configuration *II* is able to generate a high tilting torque.

Torque variation in stator coordinates:

Further inspection of Fig. C.3 shows that T_z for both configurations has eight positive/negative peaks in the ϕ -direction, which is coincident with the PM poles arrangement along the rotor equator. However, it seems that for T_x and T_y , the distribution of peak values is not coincident with the poles arrangement. This is because the torque computation is based on rotor coordinates. In order to observe the torque

(a) Torque x (single)

(b) Torque x (double)

(c) Torque y (single)

(d) Torque y (double)

(e) Torque z (single)

(f) Torque z (double)

Figure C.3: Torque variation in rotor coordinates (single & double layers)

Figure C.4: Position of Coil in stator coordinates

variation in stator coordinates, a stator frame is defined in Fig. C.4. As indicated in Fig. C.4, the position of the coil is fixed with respect to the stator frame on the plane of $X_s O Y_s$. The torque obtained in rotor coordinates can be transferred into stator coordinates (denoted as T_{sx} , T_{sy} and T_{sz}) and plotted in Fig. C.5. For both PM-pole configurations, eight peaks can be observed for all torque components in the ϕ -direction in Fig. C.5, which coincides with the eight PM pole arrangement along the rotor equator.

(a) Torque x (single)

(b) Torque x (double)

(c) Torque y (single)

(d) Torque y (double)

(e) Torque z (single)

(f) Torque z (double)

Figure C.5: Torque variation in stator coordinates (single & double layers)

Appendix D

Design drawings of spherical actuator prototype

This appendix includes the major design drawings of the PM spherical actuator prototype: stator, coil frame, stator base, rotor frame, iron core, PM pole, rotor shaft, magnet pad, bearing support, counterweight. Assembly drawings are used to illustrate the relation among parts.

Figure D.1: Permanent magnet spherical actuator

Figure D.2: Rotor assembly with dihedral-shaped PM poles

Figure D.4: Stator (b)

Figure D.5: Stator (c)

Figure D.6: Coil frame

Figure D.7: Stator base

Figure D.8: Rotor frame for dihedral-shaped PM poles

Figure D.9: Iron core

Figure D.10: Permanent magnet pole

Figure D.11: Rotor shaft

Figure D.12: Magnet pad

Figure D.13: Bearing support

Figure D.14: Counterweight

Figure D.15: Rotor frame for cylindrical-shaped PM poles

Appendix E

Drawings of torque measurement apparatus

This appendix presents the drawings of the torque measurement apparatus. This apparatus can be used to measure the torque generated by single coil or multiple coils interacting with the PM-pole rotor. A six-axis force/torque sensor is employed for the measurement. Following part drawings are included: arc guide, arc guide support, base of arc guide, slider, base of coil fixture, rotor base, fixture of coil frame, torque sensor connector, torque sensor shaft, rotation scale, indicator beam, stator constraint. Assembly drawings are used to illustrate the relation among parts.

Figure E.1: Apparatus of torque measurement on single coil

Figure E.2: Apparatus of torque measurement on multiple coils

Figure E.3: Arc guide

Figure E.4: Arc guide support

Figure E.5: Base of arc guide

Figure E.6: Slider on the arc guide

Figure E.7: Base of coil fixture

Figure E.8: Rotor base in torque measurement apparatus

Figure E.9: Coil frame fixture

Figure E.10: Force/torque sensor connector

Figure E.11: Force/torque sensor shaft

Figure E.12: Rotation scale

Figure E.13: Indicator beam of spinning angle

Figure E.14: Stator constraint

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