Vibro-acoustic interaction of microsystem components in data storage devices

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**VIBRO-ACOUSTIC INTERACTION OF MICRO-
SYSTEM COMPONENTS IN DATA STORAGE
DEVICES**

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SCHOOL OF MECHANICAL & AEROSPACE ENGINEERING

2006

Vibro-Acoustic Interaction of Micro-System Components in Data Storage Devices

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Abstract

Vibro-acoustic characteristics of compact data storage devices are not well understood. This study examines fundamental research issues of structure-borne noise through the sample case of 3.5-in personal computer hard disk drives. This structure-borne noise is generated from the surface vibration, including the top cover and the base plate, which is induced by the motion of the internal components transmitted through screws and rigid connections. Therefore, vibro-acoustic analysis of a HDD is divided into two analyses: vibration analysis and acoustic analysis.

Vibrations of the components inside the hard disk drive and their transmissions to the housing are first investigated. It is already known that spindle/disks assembly and voice coil motor/actuators assembly are the two major noise and vibration contributors in a hard disk drive. Noises induced by these two assemblies are called idle noise and seek noise, respectively. Therefore, two mathematical models have been developed. One is the mathematical model of the spindle/disks assembly-shaft-housing system to predict vibration transmitted to the housing due to the vibration of the spindle/disks assembly caused by noise source excitations. Gyroscopic effects and membrane stresses due to the rotation of disks are included. A few parametric studies have been presented on the effects of the disk geometry and number of disks to illustrate the model capabilities. Forced responses of the disks and the top cover under different load cases have also been explored. The other is the mathematical model of the voice coil motor/actuators assembly-shaft-housing system to model vibration transmitted to the housing due to the vibration of the voice coil motor/actuators assembly caused by corresponding noise source excitations. Governing equations are derived incorporating the elastic

deformations of the flexible components in the system represented by appropriate finite elements and corresponding shape functions. The flexible components are modelled by a simple stationary actuator to illustrate the programming implementation. The effect of the flexibility of the shaft and the housing is examined on the natural frequencies of the system. Forced responses of the top cover and the base plate are obtained as well.

Sound radiation from the housing, particularly from the top cover is then studied by simplifying the top cover as a rectangular plate in both idle and seek mode of a HDD. All the calculations are performed under harmonic forces since the distributions of the noise source excitations are unknown. For idle noise, a few parametric studies have been presented on the effects of the disk geometry and number of disks.

Necessary experiments have been carried out to verify the simulation results. Potential areas of further research are finally discussed.

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Nomenclature

Symbol	Description
a_i	inner radius of the i^{th} disk
a, b, c	length, width and height of the measurement surface
A	transform matrix between the rocking frame and the inertial frame
b_i	outer radius of the i^{th} disk
B	transform matrix between the frame $x''y''z''$ and the inertia frame
$B(m, n)$	balanced modes
$B_f(m, n)$	forward balanced modes
$B_b(m, n)$	backward balanced modes
c_0	speed of sound
C	constant in calculating the membrane stresses
C_i	origin of the i^{th} flexible component in VCM/actuators assembly
C_{mn}^i	modal participation factor corresponding to symmetric waves of the i^{th} disk expressed in frame $x''y''z''$
$\bar{C}_{mn}^i$	modal participation factor corresponding to symmetric waves of the i^{th} disk expressed in rocking frame
$\dot{C}_{mn}^i$	velocity of modal participation factor corresponding to symmetric waves of the i^{th} disk expressed in frame $x''y''z''$
$\dot{\bar{C}}_{mn}^i$	velocity of modal participation factor corresponding to symmetric waves of the i^{th} disk expressed in rocking frame
$\ddot{C}_{mn}^i$	acceleration of modal participation factor corresponding to symmetric waves of the i^{th} disk expressed in frame $x''y''z''$
$\ddot{\bar{C}}_{mn}^i$	acceleration of modal participation factor corresponding to symmetric waves of the i^{th} disk expressed in rocking frame
d	measurement distance between the reference box and the measurement surface in sound radiation measurement
d_m	cage diameter of rolling elements
dS	element of surface area
dV_j	volume element of element j of the i^{th} flexible component
D_a	diameter of rolling elements
D_e	flexural rigidity of the plate
D_e^i	flexural rigidity of the i^{th} disk
D_i	origin of the i^{th} body-fixed frame $x''y''z''$
$\mathbf{E}$	back EMF vector
f_b	rotation frequency of rolling elements
f_c	orbital revolution frequency of rolling elements
$f(F_x, F_y, M_z)$	forcing frequency components expressed as multiples of spin speed of the rotor
$\mathbf{F}$	generalized force vector
F_x	x-component of force acting on the stator of BLDC motor

F_x, F_y, F_z	components of generalized force vector $\mathbf{F}$
F_y	y- component of force acting on the stator of BLDC motor
$F_{\theta x}, F_{\theta y}, F_{\theta z}$	components of generalized force vector $\mathbf{F}$
$G(x, y)$	free field Green's function
$\mathbf{G}_d$	gyroscopic matrix due to vibration of the rotating disks
$\mathbf{G}_r$	gyroscopic matrix due to rigid body motions
$\mathbf{G}_{rd}$	gyroscopic coupling between the rigid body motions and the vibration of rotating disks
h_c	thickness of the top cover
h_i	thickness of the i^{th} disk
I	sound intensity
I_i	sound intensity normal to the i^{th} surface segment
I_x, I_y, I_z	moment of inertia of the spindle or the rigid pivot housing with respect to the origin of the body-fixed frame attached to the spindle or the rigid pivot housing
I_0	reference sound intensity of 10^{-12} W/m^2
I_1, I_2	diametral moment of inertia of the spindle/disks assembly
I_3	polar moment of inertia of the spindle/disks assembly
$\vec{I}$	sound intensity vector
$\mathbf{I}$	stator phase current vector
$\mathbf{i}'' , \mathbf{j}'' , \mathbf{k}''$	unit vectors of the body-fixed frame
$\mathbf{I}, \mathbf{J}, \mathbf{K}$	unit vectors of the inertia frame
k	acoustic wave number
k_{ax}, k_{ay}, k_{az}	stiffness of bearing A in x, y and z dimension
k_{bx}, k_{by}, k_{bz}	stiffness of bearing B in x, y and z dimension
$\mathbf{K}_a$	stiffness matrix of the actuator
$\mathbf{K}_b$	stiffness matrix of bearings
$\mathbf{K}_d$	stiffness matrix of the disks
$\mathbf{K}'_d$	stiffness matrix of the disks accounting for the effect of rotation
$\mathbf{K}_s$	modal stiffness matrix of the stationary part
$\mathbf{K}_{sb}$	stiffness matrix of the stationary part induced by the bearing deformation
$\mathbf{K}_{sr}$	stiffness coupling matrix between the stationary part and the rotating part
KE_d	kinetic energy of the elastic disks
KE_f	kinetic energy of the flexible components
KE_r	kinetic energy of the rigid pivot housing
$KE_{rotating}$	kinetic energy of the rotating part
KE_s	kinetic energy of the rigid spindle
$KE_{stationary}$	kinetic energy of the stationary part
l_1, l_2, l_3	length, width and height of the reference box
L_l	sound intensity level

L_p	sound pressure level
$\overline{L_p}$	sound pressure level averaged over the surface that surround the source
L_{pi}	sound pressure level at the i^{th} microphone position
L_w	sound power level
m	number of nodal circles
m_p	mass of the pivot housing with respect to the origin of the body-fixed frame attached to the pivot housing
m_s	mass of the spindle with respect to the origin of the frame attached to the spindle
$(m, n)_b$	backward travelling wave of (m, n) mode of the disk
$(m, n)_f$	forward travelling wave of (m, n) mode of the disk
M	total mass of the spindle and the disks/the total mass of the flexible components and the pivot housing
$M_n^i, n = 1 \cdots n_m$	components of generalized modal force $\mathbf{M}$ in vibration of VCM/actuators assembly-shaft-housing system
$M_{cmn}^i, n = 1 \cdots n$	components of generalized modal force $\mathbf{M}$ in vibration of spindle/disks assembly or in vibration of spindle/disks assembly-shaft-housing system
$M_{smn}^i, n = 1 \cdots n$	components of generalized modal force $\mathbf{M}$ in vibration of spindle/disks assembly or in vibration of spindle/disks assembly-shaft-housing system
$\mathbf{M}$	generalized modal force vector
$\mathbf{M}_a$	modal mass matrix of the flexible actuator
$\mathbf{M}_d$	inertial matrix of the disks due to the flexible vibration
$\mathbf{M}_r$	diagonal inertial matrix for the rigid-body motions of the spindle or pivot housing
$\mathbf{M}_{ra}$	inertial coupling between the rigid-body motions and the flexible vibration of the actuator
$\mathbf{M}_{rd}$	inertial coupling between the rigid body motions and the flexible vibration of disk
$\mathbf{M}_s$	modal mass matrix of the stationary part
M_z	torque acting on the stator of BLDC motor
n	number of nodal diameters
n_e	number of elements for the i^{th} flexible component in finite element analysis
n_i	running speed of the inner ring
n_m	number of mode shapes to be taken into account of the i^{th} flexible component
n_n	number of shape function of the element j of the i^{th} flexible component
n_s	number of vibration modes of the stationary part
n_y	outward normal to the disk surface at the point y
N	number of flexible disks/number of flexible components
N_k^i	k^{th} shape function of the element j of the i^{th} flexible component
$N_e(x, y)$	shape functions.

N_m	number of microphone positions
N_p	number of poles of BLDC motor
N_s	number of slots of BLDC motor
N_{se}	number of segments
N_1, N_2, N_3, N_4	shape functions
O	origin of the inertial frame
O'	O' and O are coincident initially
p	acoustic pressure induced by the vibrating disk/complex acoustic pressure amplitude at a location in space expressed in spherical co-ordinates
p^-	pressure at the bottom surface of the disk
p^+	pressure at the upper surface of the disk
p_{rms}^2	mean-square sound pressure at a point
P	sound pressure
P_0	reference pressure of 20 micropascals
PE_b	potential energy of bearings
PE_b^i	strain energy of the i^{th} disk due to pure bending
PE_b^{i*}	strain energy of the i^{th} disk due to the centrifugal stresses
PE_{fi}	strain energy of the i^{th} flexible component due to elastic deformation
$PE_{rotating}$	potential energy of the rotating part
$PE_{stationary}$	potential energy of the stationary part
q_{mn}^i	modal participation factor of the i^{th} disk expressed in frame $x''y''z''$
$\bar{q}_{mn}^i$	modal participation factor of the i^{th} disk expressed in rocking frame
q_n^i	modal participation factor of the i^{th} flexible component
$\dot{q}_n^i$	velocity of modal participation factor of the i^{th} flexible component
$\ddot{q}_n^i$	acceleration of modal participation factor of the i^{th} flexible component
$\mathbf{q}$	generalized displacement vector in vibration analysis of spindle/disks assembly/in vibration analysis of VCM/actuator assembly-shaft-housing system expressed in the body-fixed frame $x''y''z''$
$\dot{\mathbf{q}}$	generalized velocity vector in vibration analysis of spindle/disks assembly expressed in the body-fixed frame $x''y''z''$
$\ddot{\mathbf{q}}$	generalized acceleration vector in vibration analysis of spindle/disks assembly/in vibration analysis of VCM/actuator assembly-shaft-housing system expressed in the body-fixed frame $x''y''z''$
$\bar{\mathbf{q}}$	generalized displacement vector in vibration analysis of spindle/disks assembly expressed in rocking frame
$\dot{\bar{\mathbf{q}}}$	generalized velocity vector in vibration analysis of spindle/disks assembly expressed in rocking frame
$\ddot{\bar{\mathbf{q}}}$	generalized acceleration vector in vibration analysis of spindle/disks assembly expressed in rocking frame
$\hat{\mathbf{q}}$	generalized displacement vector in vibration analysis of spindle/disks assembly-shaft-housing system expressed in rocking frame

$\dot{\mathbf{q}}$	generalized velocity vector in vibration analysis of spindle/disks assembly-shaft-housing system expressed in rocking frame
$\ddot{\mathbf{q}}$	generalized acceleration vector in vibration analysis of spindle/disks assembly-shaft-housing system expressed in rocking frame
$\mathbf{Q}$	generalized force vector in Lagrange's equation
r	radial distance of an arbitrary point A measured from frame $x''y''z''$
$\mathbf{r} = (r, \theta, \phi)$	spherical co-ordinates
$\mathbf{r}_a^i$	position vector from D_i to an arbitrary point A in the i^{th} disk or from C_i to an arbitrary point A in the i^{th} flexible component
$\dot{\mathbf{r}}_a^i$	velocity vector of an arbitrary point A on the i^{th} disk with respect to D_i or velocity vector of an arbitrary point A on the i^{th} flexible component with respect to C_i
$\mathbf{r}_d$	displacement modal participation factor vector of the disks
$\dot{\mathbf{r}}_d$	velocity of displacement modal participation factor vector of the disks
$\mathbf{r}_r$	displacement vector of the rigid spindle
$\dot{\mathbf{r}}_r$	velocity vector of the rigid spindle
$\mathbf{r}_s$	displacement modal participation factor vector of the stationary part
$\dot{\mathbf{r}}_s$	velocity of displacement modal participation factor vector of the stationary part
r_{sn}	displacement modal participation factor of the stationary part
$\dot{r}_{sn}$	velocity of displacement modal participation factor of the stationary part
$\ddot{r}_{sn}$	acceleration of displacement modal participation factor of the stationary part
r_x, r_y, r_z	position coordinate of point A on the i^{th} flexible component in x, y and z direction with respect to the body-fixed frame
R	Euclidean distance between points x and y / distance from the source point to observer point
R_{mn}^i	assumed function to be determined by satisfying the inner and outer boundary conditions of the disk
$R_{sn}, n = 1 \cdots n_s$	components of the generalized modal force vector $\mathbf{S}$
R_x, R_y, R_z	rigid body translation of the rigid spindle or rigid pivot housing in x, y and z direction
$\dot{R}_x, \dot{R}_y, \dot{R}_z$	velocity of rigid body motion of the rigid spindle or the rigid pivot housing in x, y and z direction
$\ddot{R}_x, \ddot{R}_y, \ddot{R}_z$	acceleration of rigid body motion of the rigid spindle or the rigid pivot housing in x, y and z direction
$\mathbf{R}_a^i$	displacement of an arbitrary point A on the i^{th} disk/on the i^{th} flexible component
$\dot{\mathbf{R}}_a^i$	velocity of an arbitrary point A on the i^{th} disk/on the i^{th} flexible component
$\dot{R}_{ax}^i, \dot{R}_{ay}^i, \dot{R}_{az}^i$	velocity of an arbitrary point A on the i^{th} disk or on the i^{th} flexible component in x, y and z direction expressed in inertia frame
$\mathbf{R}_{je}, j = a, b$	bearing deformation due to the elastic displacements of the stationary part

$\mathbf{R}_{jr}, j = a, b$	bearing deformation due to the rigid-body motions of the pivot housing
$\mathbf{R}_{o'c_i}$	position vector of the origin of the body-fixed frame of the i^{th} flexible component C_i with respect to O'
$\dot{\mathbf{R}}_{o'c_i}$	velocity vector of the origin of the body-fixed frame of the i^{th} flexible component C_i with respect to O'
$\mathbf{R}_{o'D_i}$	position vector of the origin of the body-fixed frame of the i^{th} disk D_i with respect to O'
$\dot{\mathbf{R}}_{o'D_i}$	velocity vector of the origin of the body-fixed frame of the i^{th} disk D_i with respect to O'
$\mathbf{R}_{oo'}$	rigid body displacement vector of the entire spindle/disks assembly or VCM/actuator assembly with O and O' are coincident initially
$\dot{\mathbf{R}}_{oo'}$	velocity vector of the entire spindle/disks assembly or the entire VCM/actuators assembly
$\dot{\mathbf{R}}_s$	velocity vector of the rigid spindle
S	(measurement) surface that surrounds the source
S_i	area of the i^{th} surface segment
S_{mn}^i	modal participation factor corresponding to anti-symmetric waves of the i^{th} disk expressed in frame $x''y''z''$
$\dot{S}_{mn}^i$	velocity of modal participation factor corresponding to anti-symmetric waves of the i^{th} disk expressed in frame $x''y''z''$
$\ddot{S}_{mn}^i$	acceleration of modal participation factor corresponding to anti-symmetric waves of the i^{th} disk expressed in frame $x''y''z''$
$\bar{S}_{mn}^i$	modal participation factor corresponding to anti-symmetric waves of the i^{th} disk expressed in rocking frame
$\dot{\bar{S}}_{mn}^i$	velocity of modal participation factor corresponding to anti-symmetric waves of the i^{th} disk expressed in rocking frame
$\ddot{\bar{S}}_{mn}^i$	acceleration of modal participation factor corresponding to anti-symmetric waves of the i^{th} disk expressed in rocking frame
S_T	total surface of the disk
S_0	equal to 1 m^2
S^+	upper surface of the disk
$\mathbf{S}$	generalized modal force vector
$\mathbf{U}(m, n)$	unbalanced modes
$\mathbf{U}_f(m, n)$	forward balanced modes
$\mathbf{U}_b(m, n)$	backward balanced modes
$(\mathbf{u} \bullet \mathbf{n}_x)$	normal displacement at point x of the disk
v_e	complex velocities at nodes of element e
$v(\mathbf{x})$	complex surface normal velocity amplitude at location $\mathbf{x} = (x, y)$
V	volume of the i^{th} flexible component
V_j	volume of element j of the i^{th} flexible component

w_a^i	transverse elastic deformation of the point A on the i^{th} disk or on the i^{th} flexible component measured from the body-fixed reference frame
w_{mn}^i	mode shape of the i^{th} disk
w_{sxn}	mode shapes of the stationary part in x direction
w_{syn}	mode shapes of the stationary part in y direction
w_{szn}	mode shapes of the stationary part in z direction
$w_{sxn,j}$	mode shapes of the stationary part at the bearing support positions on the shaft in x direction
$w_{syn,j}$	mode shapes of the stationary part at the bearing support positions on the shaft in y direction
$w_{szn,j}$	mode shapes of the stationary part at the bearing support positions on the shaft in z direction
W	sound power
W_0	reference sound power of 10^{-12} watt
$\mathbf{w}_a^i$	transverse elastic deformation vector of the point A on the i^{th} disk or on the i^{th} flexible component measured from the body-fixed reference frame
$\dot{\mathbf{w}}_a^i$	velocity vector of transverse elastic deformation of the point A on the i^{th} disk or on the i^{th} flexible component measured from the body-fixed reference frame
$\mathbf{w}_s$	elastic deformation of an arbitrary point of the stationary part
$\mathbf{w}_{sn}$	n^{th} vibration mode shape of the stationary part
$\mathbf{w}_{sn,j}$	mode shape vector of the bearing support positions on the shaft
x	an arbitrary point in the fluid
xyz	intermediate frame
$x' y' z'$	rocking frame
$x'' y'' z''$	body-fixed coordinates
XYZ	inertial frame
y	an arbitrary point on the disk
z_a	distance from origin O to the bearing A
z_b	distance from origin O to the bearing B
z_i	distance from the origin of the i^{th} disk D_i to origin O or from the origin of the i^{th} flexible component C_i to origin O
α	angular position of an arbitrary point on the i^{th} disk measured from the body-fixed frame $x'' y'' z''$
α_a	absorption coefficient of the reflecting plane
α_c	contact angle of bearing
$\Delta \mathbf{R}_j$	deformation vector of the bearing A or B
δ_{mn}	Kronecker delta
θ	angular position of an arbitrary point on the i^{th} disk measured from the rocking frame
$\theta_x, \theta_y, \theta_z$	Euler angles

$\dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z$	velocity of Euler angles
$\ddot{\theta}_x, \ddot{\theta}_y, \ddot{\theta}_z$	acceleration of Euler angles
μ	pressure jump
ν_i	Poisson's ration of the i^{th} disk
ξ, η	variables in shape functions
ρ_i	density of the i^{th} disk
ρ_j	density of element j of the i^{th} flexible component in FE analysis
ρ_0	density of the air
σ_{ri}	radial membrane stresses of the i^{th} disk due to disk rotation
$\sigma_{\theta i}$	tangential membrane stresses of the i^{th} disk due to disk rotation
ω	frequency of harmonic motion
ω_c	carrier frequency of PWM control scheme
ω_{mn}^i	natural frequency of flexural mode of the i^{th} disk taking into account the membrane stresses due to the rotation
ω_n^i	natural frequency of flexural mode of the i^{th} flexible component
ω_{sn}	j^{th} natural frequencies of the stationary part
ω_{st}^i	frequency of the i^{th} stationary disk
$\boldsymbol{\omega}''$	angular velocity vector of the body-fixed coordinate $x''y''z''$
ω^2	normal surface acceleration of the disk
ϕ_n^i	n^{th} mode shape at any point A on the i^{th} stationary flexible component
φ_{nk}^i	n^{th} mode shape evaluated at nodal positions of element j of the i^{th} flexible component
Ω	angular speed of rotating disk
Ω_r	angular velocity of rotor

Chapter 1

Introduction

1.1 Background

From the mechanical point of view, hard disk drives (HDDs) have two major subsystems, which allow the HDD to function properly. One is the high-speed spindle motor/disks assembly. The other is the head actuator assembly. Due to their mechanical nature, both assemblies can generate vibrations and consequently make considerable noise. Noise and vibration in a HDD can greatly influence the quality of the products by affecting run-out characteristics and servo read/write performances, increasing seek times and track misregistration (TMR), and limiting storage capacity and drive speeds as well.

Noise and vibration issues have become critical in recent years. The demand for increased storage capacity and decreased seek times has dictated that modern disk drives should have faster spindle speeds with lower flying heights and faster actuator access times. This unavoidably causes noise and vibration levels to increase.

Today, as the HDD goes into consumer electronics systems, which may be operating in quieter environments than the typical usage in personal computers, end users desire that the HDD maintain the highest level of performances yet are quieter to fit easily into their lifestyles. For example, a personal video recorder (PVR) may be recording in the bedroom in the middle of the night. For such applications, the HDD must be inaudible, once it is mounted into the systems.

It is evident that vibro-acoustic interaction is one of the major mechanical problems that affect the performance of the HDD. Recently, increasing attention has been focused on the vibro-acoustic interaction of micro-system components in the HDD, especially analytical/numerical studies on the mechanisms of how the vibrations are transmitted and how the structure-borne noise is generated. Most of the work done in the area so far has been mainly on the vibration analysis of some of the major assemblies individually, such as voice coil motor (VCM)/actuators assembly and spindle motor/disks assembly. There is much scope to explore the vibro-acoustic interaction of the various components in the hard disk drive as a system.

Thus, this study attempts to fill part of this void by carrying out a fundamental, systematic and in-depth exploration of the noise source mechanisms and characteristics, vibration transmissions through the development of the mathematical models for the spindle motor/disks assembly-shaft-housing system and the VCM/actuators assembly-shaft-housing system, and sound radiation from the vibrating top cover.

1.2 Objectives

Noise produced by HDDs is due to the forces and motions generated by the internal components and transmitted structurally through the joints to the HDD housing, which, in turn, vibrate and radiate sound into the surrounding air. In short, this study of vibro-acoustic interaction of micro-system components in a HDD involves the study of the noise source excitations, the study of the structurally transmitted vibration in the two major assemblies (spindle/disks assembly and VCM/actuators assembly) and the housing, and the study of acoustic radiation from vibrating planar radiators (the top cover and/or the base plate). Consequently, the objectives of the dissertation are

- To understand the mechanisms of different noise sources in HDD components, their characteristics and corresponding control methods;
- To simulate the HDD in idle mode by developing a mathematical model for spindle/disks assembly-shaft-housing system with which the transfer functions that link corresponding noise sources and vibration of the housing can be determined;
- To simulate the HDD in seek mode by developing a mathematical model for VCM/actuators assembly-shaft-housing system with which the transfer functions that link corresponding noise sources and vibration of the housing can be obtained;
- To build models for calculation of sound radiation from the housing, i.e., the top cover and/or the base plate as radiators in a HDD.

1.3 Dissertation outline

This dissertation is organized as follows. Following the introduction, the vibration and acoustic analysis in HDDs are reviewed in Chapter 2, which highlights the background information and motivation of this research. Some experimental results have also been described in this chapter. Noise source mechanisms and characteristics are presented in Chapter 3. Methods for reducing noise and vibration have been reviewed as well. In Chapters 4 to 6, research methodologies in terms of analytical and/or numerical modelling are developed to investigate the vibro-acoustic behaviour in HDDs. The mathematical models for vibration analysis of the two major noise contributors are explored in Chapter 4 and Chapter 5 for spindle/disks assembly-shaft-housing system and VCM/actuators assembly-shaft-housing system, respectively. The numerical models for sound radiation calculation are presented in Chapter 6. Finally, the general conclusions are drawn and future work is recommended in Chapter 7.

Chapter 2

Review of Vibro-acoustic Analysis in HDDs

This chapter presents a literature review of vibro-acoustic analysis in HDD. With advanced technologies applied to a HDD to improve its performance, vibro-acoustic issue is becoming more critical. In this chapter, specifications in terms of general and specific drive acoustics are introduced and existing studies on vibro-acoustic analysis in HDD are reviewed. Experimental studies have also been carried out to provide a general idea of vibro-acoustic characteristics of HDDs.

2.1 Structure of a typical hard disk drive

Fig. 2.1 The structure and components of a hard disk drive
(From <http://www.seagate.com/>)

A typical HDD is a very compact, electronically controlled, rotary memory device as illustrated in Figure 2.1. From the mechanical point of view, major components in a modern HDD include:

Spindle motor assembly It includes pre-loaded ball bearings at each end of the spindle shaft and brushless direct current (BLDC) motor as shown in Figure 2.2. The BLDC motor can be either symmetric (12 pole-9 slot) or asymmetric (8 pole-9 slot) as shown in Figure 2.3.

Fig. 2.2 Mechanical structure of 3.5-in HDD spindle system (Jang *et al.*, 2002)

Fig. 2.3 Symmetric (left) and asymmetric (right) BLDC motors (Jang *et al.*, 1996)

The spindle motor assembly is responsible for rotating the hard disks. It must provide reliable, stable and consistent turning power for thousands of hours of continuous use.

Disk Every HDD contains one or more flat disks that are used to hold the data. They are composed of two main substances: a substrate material (Al-Mg alloy or glass substrate)

that forms the bulk of the disk and gives it structure and rigidity, and a magnetic media coating (Cr or Cr-alloy under-layer, Co-alloy recording medium, and carbon overcoat) which actually holds the magnetic impulses that represent the data. The disks are mounted by a centrally cutting hole and stacked onto the spindle.

Voice coil motor/actuator assembly This assembly is made up of a head gimbal assembly (HGA), actuator and voice coil motor (VCM) as shown in Figure 2.4. The HGA consists of a slider, where the read and/or write head (R/W) is attached, and a suspension. The actuator consists of an arm and a pivot ball bearing assembly. The VCM is composed of a magnetic coil and two yoke plates where permanent magnets are bonded in place.

Fig. 2.4 Annotated illustration of a typical PC VCM/actuator assembly
(From <http://www.pcguide.com/>)

The VCM is to position the head actuator arms from the current track to the destination track across the surface of the disks. This movement must be done quickly and accurately because physical motion is so slow compared to the following electronic functioning of the R/W head.

Device housing It usually consists of two parts: the base plate and the top cover, which may be sealed with a gasket and/or tape. The housing provides features for mounting, supports spindle motor/disks assembly, VCM/actuators assembly, and electronics card and keeps contamination low. In conventional HDDs, the actuator shaft is mounted on the drive base plate and supported by the cover through a screw.

2.2 Hard disk drive technology trends and challenges

From the investigations on the trends of hard disk drive technologies (Grochowski and Hoyt, 1996; Grochowski, 1998; Coughlin, 2000; Thompson and Best, 2000; Abramovitch and Franklin, 2002), it is known that HDD has increased rapidly in its storage capacity and the rate of reading and writing of the R/W heads.

Fig. 2.5 Areal density vs. year of IBM production introduction (Thompson and Best, 2000)

The increasing of the storage capacity can be realized by increasing the packing density of bits on the disk surface. Areal density is a measure of the amount of information that can be stored on the surface of the disk in megabits per square inches ($Mbits/in^2$). It is the product of track density multiplied by bit density on each track. Figure 2.5 shows the areal density evolution of IBM HDD products since IBM RAMAC was first used in PC

systems. It can be seen from Figure 2.5 that since 1991, the slope changes sharply due to the use of the magnetoresistive (MR) recording heads. In September, 2003, Seagate introduced the world's first hard disk drive to store 100 Gigabits on a single 3.5-inch disk.

Apart from the areal density growth, a decrease in the data access time is another important evolution direction for the performance of HDD. Total access time has decreased from about 20 ms in 1990 to about 9 ms in 2000 as shown in Figure 2.6. The access time is the sum of track seeking time, track settling time and rotational latency. Track seeking time is defined as the time for the head to come to the destination track and track settling time is the time for the head to decay the residual vibration caused by mechanical resonances. Rotational latency is the time required for the proper information to be found and it is mostly determined by the spindle speed which is defined as the rotation speed of the disk in terms of revolutions per minutes (RPM). The RPM of HDD has been increasing steadily. The most common HDD in current desktop PCs have a disk speed of 7200 RPM while Seagate Cheetah X15 with 15,000 RPM was announced in November, 2000.

Fig. 2.6 Performance history of IBM disk products with respect to access time (Thompson and Best, 2000)

From the mechanical and dynamics point of view, these technological trends have introduced many challenges to the development of the HDD system. The increasing improvement in areal density is projected to place extreme requirements on the precision with which the actuator assembly brings the R/W head to the data destination track and maintains the head over that track. The offset between the actual R/W head position and the data track centre is called the track misregistration (TMR). There are many factors that will contribute to TMR. The first is external shock and vibration. As today's HDD are used in portable and other light weight applications, such as laptop or notebook computers, they are easily exposed to greater vibration and shock. Besides, the run-out caused by spindle motor system and the rocking motion caused by disk stack also contribute to TMR. The run-out is the characteristic of spindle motor, which is caused by the misregistration of ball bearings inside the motor. Rocking motion of the disk stack is one of the most undesirable dynamics in a HDD. When this motion is excited, it directly increases the TMR.

The decrease of the track-settling time is mainly hampered by the residual vibrations caused by the flexibility of the head actuator assembly. The decrease of rotational latency may be achieved with an increase of spindle speed theoretically. But over-high spindle speed will cause many problems such as disk vibration and flutter induced by fluid-structure interaction and the rocking motion of the spindle motor system.

It should be noted that among all these challenges, vibration is one of the most important. Many components or subsystems in a HDD, when in operation, are subjected to vibration due to out of balance of rotation and dynamic interactions between the components. These vibrations have been found not only to have significant effect on the operating

performance of the system but also to generate noise. Noise issues have become exacerbated in recent years primarily due to the increase in spindle speed and faster actuators.

However, not much work has been published on a systematic study of vibro-acoustics in HDDs. This dissertation attempts to provide an exploration on this issue through systematic studies on aspects of noise sources, vibration transmissions and sound radiation in a typical HDD.

2.3 Specifications in vibro-acoustic analysis of HDDs

2.3.1 Acoustic specifications

Sound pressure, sound intensity, and sound power

A variation in pressure above and below atmospheric pressure is called sound pressure, in units of pascal (Pa). This variation can cause vibration of the human eardrum. In the audible frequency range, the vibration is perceived as sound. The normal ear can perceive sound in the frequency range of roughly 15-16,000 Hz and is most sensitive at frequencies between 3000 and 6000 Hz.

Sound intensity is defined as the flow of power carried by a sound wave through an infinitesimally small area at a point in space. The units are watts per square meter (W/m^2).

In the far field of a source radiating into a free field or free field over a reflecting plane, the sound intensity may be approximated by

$$I = \frac{P_{rms}^2}{\rho_0 \cdot c_0} \quad (2.1)$$

where

I is the sound intensity in (W/m^2)

P_{rms}^2 is the mean-square sound pressure at a point in (Pa^2)

ρ_0 is the density of air in (kg/m^3)

c_0 is the speed of sound in (m/s)

Sound power is the amount of power radiated by a sound source into the surrounding air, in watts. It can be calculated from the sound intensity on a closed surface that completely surrounds the source

$$W = \int_S \vec{I} \cdot \vec{n} \cdot dS \quad (2.2)$$

where

W is the sound power in watts (W)

$\vec{I}$ is the sound intensity vector in (W/m^2)

dS is the element of surface area in square meters (m^2)

$\vec{n}$ is the unit vector normal to the surface area

S is the surface that surrounds the source in (m^2)

In practice, this integral is replaced by a summation: the surface S is divided into N_{se} segments with the i^{th} segment having an area S_i . Equation (2.2) is then approximated by the sum

$$W = \sum_{i=1}^{N_{se}} I_i \cdot S_i \quad (2.3)$$

Here, I_i is the sound intensity normal to the i^{th} surface segment and S_i is the area of the i^{th} surface segment.

Sound pressure level, sound intensity level and sound power level

By definition, sound pressure level is

$$L_p = 10 \log \left(\frac{P}{P_0} \right)^2 = 20 \log \frac{P}{P_0} \quad (2.4)$$

where

L_p is the sound pressure level in (dB)

P is the sound pressure in micropascals (μPa)

P_0 is the reference pressure of $20 \mu Pa$

The reference pressure is the pressure of the threshold of hearing.

The sound intensity level is defined as

$$L_I = 10 \log \frac{I}{I_0} \quad (2.5)$$

where

L_I is the sound intensity level in (dB)

I is the sound intensity in (W/m^2)

I_0 is the reference sound intensity of $10^{-12} W/m^2$

The sound power level is defined as

$$L_w = 10 \log \frac{W}{W_0} \quad (2.6)$$

where

L_w is the sound power level in (dB)

W is the sound power in watts (W)

W_0 is the reference sound power of $10^{-12} W$

Relation among sound power level, sound pressure level and intensity level

Combine Equation (2.5), (2.1) and (2.4), one obtains

$$L_p = L_I + 10 \log \frac{\rho_0 \cdot c_0}{400} \quad (2.7)$$

Under the condition of uniform intensity over an area S , the following relation can be obtained by substituting Equation (2.3) into (2.6)

$$L_w = 10 \log \frac{I}{10^{-12}} + 10 \log \frac{S}{S_0} = L_I + 10 \log \frac{S}{S_0} \quad (2.8)$$

Where S is the surface that surrounds the source in m^2 . S_0 is equal to $1 m^2$.

Then, an expression for the sound power level is deduced by substituting Equation (2.7) into (2.8)

$$L_w = L_p + 10 \log \frac{S}{S_0} - 10 \log \frac{\rho_0 \cdot c_0}{400} \quad (2.9)$$

In practice, the last term can usually be neglected for measurements of engineering grade.

In case of measurements over a reflecting plane as stated in international standard organization (ISO) 3744, an array of fixed microphone positions are used, the positions being distributed over the surface of a parallelepiped. The sound pressure level averaged over the measurement surface is easily determined

$$\bar{L}_p = 10 \log \left[\frac{1}{N_m} \sum_{i=1}^{N_m} 10^{0.1 L_{pi}} \right] \quad (2.10)$$

where

$\bar{L}_p$ is the sound pressure level averaged over the surface that surrounds the source

L_{pi} is the sound pressure level at the i^{th} microphone position

N_m is the number of microphone positions

2.3.2 HDD noise specifications

Idle noise and Seek noise

From the point of view of operating modes, two types of noise can be distinguished in a HDD. The spinning of the spindle motor/disks assembly (SDA), which includes the spindle motor, ball bearings and hard disks, generates steady-state periodic tones. The HDD makes this kind of noise whenever it is in operation, regardless of what it is doing. This is referred to as *idle noise*.

Seek movements of the VCM/actuators assembly, which includes VCM, actuator and HGA, generate transient and discrete noises called *seek noise*. This is the noise level when the drive is performing positioning tasks as well as the SDA since they continue to spin during seeks.

Structure-borne and Air-borne

From the point of view of transmission paths, noise in a HDD can be classified into structure-borne and air-borne noise. Structure-borne noise is generated from surface vibration, such as the HDD top cover vibration due to the motion of the internal components. In another word, vibrations of the spindle motor/disks and VCM/actuators assemblies can be directly transmitted to the drive housing, including the top cover and the base plate, through screws and rigid connections. These vibrations will in turn be amplified by resonances of the drive housing. This form of noise transmission is referred to as a *structure-borne* path.

The other is the noise generated from the air movement. The noise from the spindle/disks assembly and VCM/actuators assembly, which are contained inside the drive housing, is

also incident upon the interior housing surfaces, and transmitted through the drive housing. This is called an *air-borne* transmission path. It should be noted that reverberation effects and acoustic resonances within the drive housing could amplify the interior noise.

Table 2.1 lists the classifications of idle and seek noises in terms of excitations, transmission paths and radiators.

Noise Classifications	Internal excitations	Transmission mechanisms	Sound radiators
Idle noise	Vibration sources in spindle motor	Spindle motor/disks assembly ↓ Shaft ↓ Housing	Drive housing (top cover)
	Vibrations in bearings		
	Vibration of disks		
Seek noise	Vibration in VCM/actuators assembly	VCM/actuators assembly ↓ Shaft ↓ Housing	Drive housing (top cover)

Table 2.1 Classifications of idle and seek noise

2.4 Literature review on vibro-acoustic analysis in HDD

2.4.1 Vibration modes of spinning disks

There have been many investigations on the vibration of spinning disks. For a centrally clamped flexible disk, when the disk is stationary, a pair of (m, n) disk vibration modes, which have m nodal circles and n nodal diameters, will have repeated natural frequencies ω_{mn} (Shen, 2000). Figure 2.7 shows the calculated first four mode shapes of a stationary disk by using ANSYS.

Fig. 2.7 First four mode shapes of a stationary clamp-free disk calculated from ANSYS

When the disk rotates at a constant angular speed Ω , unlike stationary disks, modal distributions are characterized by pairs of waves propagating in opposite directions circumferentially in the disk for a ground-based observer. This phenomenon is called mode splitting or frequency splitting. The one propagating in the direction of the disk rotation is called the forward travelling wave $(m, n)_f$ and the other propagating in the opposite direction of the disk rotation called the backward travelling wave $(m, n)_b$. The corresponding frequencies are $\omega_{mn} + n\Omega$ and $\omega_{mn} - n\Omega$, respectively. Figure 2.8 presents this relationship schematically.

Fig. 2.8 Mode splitting phenomenon for a rotating disk

The rotation of the disk about its central axis induces the in-plane radial and tangential membrane stresses distributed symmetrically about the axis of rotation. These membrane stresses cause an increase of the disk natural frequencies with speed. This is called centrifugal effect. Another phenomenon due to the influence of disk rotation is the rocking motion of spindle/disks assembly. In a rocking mode, all disks have the same deformation. The inertia forces associated with the disk deformation result in an unbalanced moment about the centroid. Because the disk/spindle assembly is spinning with an angular momentum in the spindle direction, the presence of the unbalanced moment causes steady precession of the spindle (Shen, 2000). This is also called rocking modes in HDD industry. The mode shapes of rocking modes are illustrated in Figure 2.9. It should be noted that rocking mode contributes to the track misregistration directly.

Fig. 2.9 Mode shape of the two rocking modes (Wang, 1996)

2.4.2 Vibration modes of head actuator assembly

Due to the bearing stiffness in six degrees of freedom, six quasi-rigid (QR) modes of head actuator assembly have been predicated as shown in Fig. 2.10 in which solid lines represent the deformed shape and the dashed lines the undeformed edge of the structure (Xu, 2002).

Fig. 2.10 The mode shape of the QR modes (Xu, 2002)

(a) QR vertical rotation (b) QR lateral rotation (c) QR vertical translation
(d) QR lateral translation (e) QR longitudinal translation (f) QR longitudinal rotation

The mode shape of the QR vertical rotation mode at frequency 0 Hz indicates that the head actuator assembly can rotate about the shaft freely in this mode. Except QR vertical rotation mode, all other QR modes are not in pure rigid body translation or rotation motion but a combination between the rigid body motion of the entire head actuator assembly and elastic deformation of some components of the assembly. For example, it can be seen from Figure 2.10 that the QR lateral translation mode is a mode of the translation in the lateral direction combined by rotation about the longitudinal axis. The whole assembly vibrates in this mode and the suspensions twist about the longitudinal axis.

Except the out-of-plane modes the head actuator assembly also exhibits in-plane modes. One particular mode that limits the bandwidth is the butterfly mode (Liu *et al.*, 2003). It is the result of the sway mode in the arm and coil plus the translational motion of the pivot cartridge. The shape of the motion is in the form of a butterfly flapping its wings as shown in Figure 2.11.

Fig. 2.11 Butterfly mode of head actuator assembly (Liu *et al.*, 2003)

2.4.3 Review on vibro-acoustic analysis in HDD

Early studies of vibro-acoustics in a HDD are confined to experimental investigations. Noise sources, such as spindle, power supply and cooling impeller, was identified by measuring sound power level before and after design modifications on the corresponding sources (Luttrell and Dunens, 1987).

The seek noise, long distance seeks and short distance seeks, radiated by a HDD incorporating a stepper motor has been studied (Prasad *et al.*, 1987). The conclusion was that radiated noise during the seek mode was related to the actuating signal used for the seek operations of the stepper motor. The level of the radiated acoustic signal depended on the structure's response to the vibration induced by the stepper motor. However, due to the technical limitations then, the positioning system of the example HDD of the study was a stepper motor, which has been replaced by VCM in modern HDD. Furthermore, the tests were conducted by forcing the seek rate to remain constant. In practice, the seek rate varies depending on the access needs of the end user.

Fig. 2.12 Acoustic spectrum of a HDD (Woldemar and Kumano, 1990)

A common commercially available 3.5-in HDD was used to improve the general understanding of drive acoustics and to reduce the acoustic level (Woldemar and Kumano,

1990). Figure 2.12 shows the acoustic spectrum before the damping material insertion. It can be seen that there are two major contributors of sound radiation, broad band noise in the range of 2-4 kHz and a prominent discrete tone in the 8 kHz.

The authors pointed out that the first kind of acoustic radiation was due to the vibration of the spindle motor caused by either electromagnetic switching of the motor from one coil to another or imperfections in the bearings. It was also found that the printed circuit board (PCB) was a major contributor to the sound power level of the disk drive. The presence of the PCB contributed an average of 5 dB in the 4 kHz octave band and 2-3 dB in the 2 kHz band. The audible 8 kHz discrete tone was a result of a peak in the driving current of the electronics at 8.64 kHz and the amplitude of the spindle resonance in the 8.6 kHz region. Some findings have been concluded that tonal noise was amplified by motor or structural dynamics. They successfully reduced the prominent high frequency tones by inserting foam materials between the disk drive base and electronic circuit board and by adding a varnish material to the motor coils.

On the numerical simulation of acoustic radiation from a HDD, although a substantial body of literature exists concerning the general subject of acoustics (Morse and Ingard, 1968; Pierce, 1981; Beranek and Ver, 1992; Filippi *et al.*, 1999; Fahy, 2001) and sound and structural vibration (Fahy, 1985; Junger and Feit, 1986; Cremer and Heckl, 1988; Ohayon and Soize, 1998), information on the HDD acoustics is limited in open literature although the vibration part have been relatively well covered. So far, a few papers have been found on the sound radiation from a centrally clamped circular disk vibrating at its elastic modes (Lee and Singh, 1992b, 1994; Beslin and Nicolas, 1996; Cote *et al.*, 1998).

Modal radiation efficiency and modal radiation power of a rotating baffled plate was determined in a rotating frame of reference (Lee and Singh, 1992b, 1994). The rotation effect was considered through centrifugal stresses and retarded source Green's function, but the doubling of modes due to a non-rotating frame of reference was not considered. This method does not apply directly since real structures are most often unbaffled. Moreover, the calculations were done in the rotating frame of the disk, which requires the results to be further transformed to the inertia frame.

The modal radiation efficiency of an unbaffled rotating disk in both frames of reference, rotating and non-rotating, have been calculated (Beslin and Nicolas, 1996). The rotation effect was taken into account by using the concept of rotating waves instead of the retarded time potential method. The radiation efficiency was calculated at the natural frequencies of the plate, considering that all modes have the same imposed velocity. The authors have concluded that only the modes with a high number of nodal circles can radiate. They have also concluded that the effect of rotation has negligible effects on the radiation. However, equally imposed velocity was assumed, which is violated by most applications. In addition, centrifugal stresses were not considered in the study.

The acoustic response of an unbaffled annular plate due to an exciting force has been investigated (Cote *et al.*, 1998). The force was supposed to be fixed in space. All the calculations were performed in the fixed reference frame. The radiated acoustic pressure was calculated using boundary element formulation. The authors pointed out that rotation has a similar effect on the acoustic response as on the forced response. That is, a narrow-band excitation might give very different results with or without the rotation while for a wide spectral excitation there is no significant change.

A mathematical model, which linked the sound radiated by a stationary or rotating disk due to electromagnetic torque pulsations of the BLDC motor, was developed (Lee and Singh, 1996). However, this model has made a number of simplifications to make the analytical modelling tractable. The disk was assumed to be stationary and factored in the rotation effect later in sound radiation calculation. This assumption ignored the gyroscopic effect.

In summary, there is little published work devoted to a systematic study of vibro-acoustic interaction in HDDs, especially the analytical/numerical models to predict the vibration and subsequent noise. Therefore, in this dissertation, the objective is to build mathematical models for vibration analysis to simulate a HDD in both idle mode and seek mode. With the development of the transfer functions, which link the corresponding noise sources and vibration transmitted to the housing, sound radiated subsequently from the housing can be determined.

2.5 Experimental studies of vibro-acoustics in HDDs

The normal velocity measurement of the top cover and measurement of sound radiation from the HDD in terms of sound power level are carried out under the same operating conditions, respectively. Both measurements employ Seagate diagnostic commands to activate the HDD to perform random seek in which the R/W head seeks randomly among all the logical data tracks. The disks are spinning when the seek mode is activated. The commercially available Seagate U10 HDD is used as the sample with the parameters listed in Table 2.2.

Drive Model	U10 ST310212A
Disk Dimension	147×101.05×26.1mm
Formatted Capacity	10.24 GB
Heads	2
Number of Disks	1
Rotational Speed	5400 RPM
Average Seek Time	8.9 ms

Table 2.2 Parameters of the HDD under test

2.5.1 Velocity measurement of the top cover

Experiment set-up

The HDD is mounted on a bracket which is bolted onto a vibration isolator. Another PC is used to provide electrical power for the HDD under test through a power cable. A specific serial cable is connected to a serial port of the PC at the same time to run the Seagate diagnostic commands so that the HDD can be activated to operate in random seek mode. The Polytec Laser Doppler Vibrometer (LDV) is adjusted to make the laser beam perpendicular to the top cover so that the normal velocities are to be measured. Figure 2.13 shows the experiment set-up.

Fig. 2.13 Velocity measurement set-up

The vibration inside the HDD is transmitted to the top cover through the screw due to its direct connection between the head actuator shaft and the top cover. Thus, the screw position is chosen as the point of interest as shown in Figure 2.14.

Fig. 2.14 Point of interest in velocity measurements of the top cover

Velocity measurement in idle mode

FFT of the velocity response of the screw position on the top cover is plotted in Figure 2.15 when only the disk is spinning at a constant speed of 5400 RPM. There is a sharp spike at the 90 Hz. This is due to the rotational speed of the spindle of 5400 RPM, which is equivalent to 90 Hz. Other resonances are observed in the range of 2-3 kHz, 4-5 kHz, and 8-10 kHz, respectively.

Fig. 2.15 FFT of velocity response of the screw position on the top cover in idle mode

Velocity measurement in random seek

FFT of the velocity responses of the screw position on the top cover in random seek is shown in Figure 2.16. It is observed that, below 1 kHz, the most noticeable resonant peak appears at 90 Hz, which is due to the rotation of the HDD at 5400 RPM as explained previously. This is because the disk is still spinning when the HDD is in seek mode. In addition, due to the VCM/actuators assembly's movements and its direct connection with the top cover through a screw, it is understood that the FFT spectrum in random seek is different from that of the HDD in idle mode. The portion of 2-4 kHz range in the FFT spectrum shows the response due to the seeking mode.

Fig. 2.16 FFT of velocity response of the screw position in random seek

2.5.2 Sound radiation measurement

Sound radiation measurement is generally consistent with ISO standards (3744, 7779 and 11201) under essentially free-field conditions over a reflecting plane. It requires the use of an anechoic chamber. The chamber eliminates or reduces reflected noise and noise from outside sources. The reflecting plane is with absorption coefficient $\alpha < 0.06$.

A B&K PULSE system is used in the sound radiation measurement. The PULSE system consists of DSP boards, a TAXI interface, multi-channel data acquisition front-end

hardware and interface software. Multi-channel Sound Power 7748 is the specific interface for sound radiation measurements.

Spectra of sound pressure level and sound power level at the entire measurement surface can be obtained in one-third octave bands from 100 Hz to 10 kHz. Overall A-weighted sound power level is determined from band levels.

Measurement surface and microphone positions and experiment set-up

According to the ISO standards, to facilitate the location of the microphone positions on the measurement surface, a hypothetical reference box is defined as shown in Figure 2.17. It is the smallest rectangular parallelepiped that just encloses the source and terminates on the reflecting plane. The microphone positions lie on the measurement surface, a hypothetical surface of area S which envelops the source as well as the reference box and terminates on the reflecting plane.

Fig. 2.17 Measurement surface and microphone positions (paths)

Here, a rectangular parallelepiped is used for the measurement surface and to dispose microphones at strategic locations. The measurement distance, d , is the perpendicular distance between the reference box and the measurement surface. The selection of a small

measurement distance is appropriate for sources to be measured in rooms under unfavourable acoustical conditions (for example, high levels of background noise). Here, to make sure the HDD noise dominate in the measurement, we adopt

$$d = 250 \text{ mm} \quad (2.11)$$

The area S of the measurement surface according to Figure 2.12 is given by the formula

$$S = 4(ab + bc + ca) \quad (2.12)$$

where

$$a = 0.5l_1 + d = 300.525 \text{ mm} \quad (2.13)$$

$$b = 0.5l_2 + d = 323.5 \text{ mm} \quad (2.14)$$

$$c = l_3 + d = 276.1 \text{ mm} \quad (2.15)$$

l_1 , l_2 , and l_3 are the length, width and height of the reference box, and $l_1 = 101.05 \text{ mm}$, $l_3 = 26.1 \text{ mm}$, $l_2 = 147 \text{ mm}$.

In practice, a frame structure was designed and the nine microphones are mounted at the corresponding locations as shown in Figure 2.18. Table 2.3 lists the instrumentation used in the experiment.

Fig. 2.18 Experiment set-up of sound radiation measurement

<i>Name</i>	<i>Quantity</i>
4188 Prepolarized Free-Field 1/2" Microphone	9
2671 Deltatron Microphone Preamplifier	9
A00427 BNC to BNC Coaxial Cable (10 m)	9
Microphone Holder 1/2" UA1317	9
B&K Sound level Calibrator, Type 4231	1

Table 2.3 B&K microphone and its associate accessories

Background noise

Averaged background sound pressure level was measured at 23.14 dB(A) as shown in Figure 2.19.

Fig. 2.19 Spectrum of sound pressure level of background noise

Idle noise measurement

Figure 2.20 and 2.21 plot the spectra of the sound pressure level and sound power level at the entire measurement surface when only the disk is spinning at a constant speed of 5400 RPM. Overall A-weighted sound pressure level and sound power level are 33.77 dB(A) and 34.12 dB(A), respectively. This result is acceptable according to the acoustic data provided in product manual of ST310212A as listed in Table 2.4.

Fig. 2.20 Spectrum of sound pressure level of idle noise

Fig. 2.21 Spectrum of sound power level of idle noise

Sound Power level (dB)	Acoustic mode	
	Idle	Random seek
Typical value	32	36
Maximum value	35	39

Table 2.4 ST310212A drive acoustics provided in product manual

Seek noise measurement

Spectra of the sound pressure level and sound power level are shown in Figure 2.22 and 2.23, respectively, when the HDD is in random seek mode. Overall A-weighted sound pressure level and sound power level are recorded at 37.46 dB(A) and 38.10 dB(A), respectively. The results are much higher than those in HDD idle mode, by about 4 dB(A). This is reasonable since in seek mode not only the VCM/actuators assembly is in motion but also the disk is spinning at the same time and thus contributing to the resultant sound radiation. Moreover, the readings obtained fall within the range of random noise level as given in Table 2.4.

Fig. 2.22 Spectrum of sound pressure level of random seek noise

Fig. 2.23 Spectrum of sound power level of random seek noise

2.6 Summary

To summarize the literature review concerning analysis of vibro-acoustic interaction in HDD, it can be said that there is as yet little investigation in this area, especially the vibration transmission paths which link the noise source excitations and output sound radiation. It is evident that there is a need to develop a full system model for vibro-acoustic analysis in HDD.

Fig. 2.24 Three subsystems in chain of vibro-acoustic analysis

Vibro-acoustic modelling in a HDD comprises three processes: (i) The model of the noise source, which is the root cause that generates the noise phenomenon. It can be treated as the exciting force acting on the HDD and be characterized by its amplitude and frequency. (ii) The model of vibration transmission path, which can be treated as a transfer function relating the exciting forces applied at the subsystems of HDD to the generated vibration at the housing. (iii) The model of the sound radiation from the final sound radiators, the housing, in terms of sound power level. In short, the noise source models determine the forces, the vibration models calculate the corresponding velocities of the housing and finally the acoustic models predict the output sound from the housing. Figure 2.24 shows schematically the method in which these three regimes can be assembled. The dashed line means the coupling between the different regimes. For the current study, uncoupled analysis is to be performed, that is, vibration generates noise only.

Therefore, in the current study, existing literature on noise source mechanisms and characteristics are reviewed extensively in Chapter 3 to get an understanding of noise source issues in HDDs. The vibration models of spindle/disks assembly-shaft-housing system and VCM/actuators assembly-shaft-housing system are developed in Chapter 4 and 5 to simulate the HDD in idle mode and seek mode, respectively. The acoustic models are studied in Chapter 6 for radiators of the top cover.

Chapter 3

Noise Source Mechanisms and Noise Control Methods

This chapter is concerned with the noise and vibration sources in HDDs which are the exciting forces that generate noise. Noise sources identification is realized by understanding the root causes and transmission paths and by knowing the characteristics of source excitations, the amplitude and frequency. Following the noise sources identification, corresponding noise and vibration control solutions are reviewed for each source. Finally industrial approaches to create quieter HDDs are briefly introduced.

3.1 Introduction

The disk drive noise is usually a result of induced structural vibration. The sources of this vibration are found within the mechanical devices utilized in the rotation of hard disks and the positioning of the magnetic R/W heads, that is, the spindle motor/disks assembly and the VCM/actuators assembly. Figure 3.1 shows the major noise and vibration sources in HDD.

Noise control is defined as the technology of obtaining an acceptable noise level at a receiver. Reduction of noise can be accomplished both at the noise sources by reducing the amplitude of the source excitations and/or eliminating the source excitations, and by dealing with noise transmission paths, avoiding the coincidence of excitations and structural resonant frequencies. Also, various means of structural damping can be used to reduce structural vibrations that contribute to acoustic emission.

Fig. 3.1 Major noise sources in HDD

3.2 Noise source mechanisms and characteristics

3.2.1 Mechanically/Electro-magnetically induced vibration sources in spindle motor

Fig. 3.2 Mechanical structure of 3.5-in HDD spindle system (Jang *et al.*, 2002)

A typical spindle motor assembly for a 3.5-in HDD is shown in Figure 3.2. The motor mounted within the hub rotates the hub and disks. The motor is typically of the Brushless DC type, consisting of a stator attached to the spindle shaft and a permanent magnet ring attached to the hub so that the energization of the stator causes the hub, and thus the disks, to rotate. Two sets of ball bearings separate the hub from the shaft and keep rotation

smooth. Figure 3.3 is the top sectional view of a rotational symmetric (12 pole-9 slot) and an asymmetric (8 pole-9 slot) Brushless DC motor.

Fig. 3.3 Rotational symmetric (left) and asymmetric (right) Brushless DC motor (Jang *et al.*, 1996)

Spindle motor vibration sources can be categorized as: mechanically induced vibration sources, electro-magnetically induced vibration sources and bearing related vibration sources as classified in Table 3.1. Bearing related vibration sources will be discussed separately in detail in the following section.

Classification			Factors
Mechanical			Eccentricity defects, e.g. rotor eccentricity, misalignment
Electro-magnetical	Unbalanced radial forces		Unbalanced magnetic field distribution due to rotor/stator eccentricity, magnetically asymmetric motor
	Torque ripple	Cogging torque	Interaction between the magnet poles and the geometry of the stator tooth
		EMF waveform imperfection	
		Commutation	Phase current commutation
		Current ripple	PWM control

Table 3.1 Noise Sources Classification in BLDC Motor

Mechanically induced vibration sources

Almost all the mechanical vibration sources of the spindle motor can be traced to eccentricity defects such as rotor eccentricity, stator eccentricity and misalignment.

Electro-magnetically induced vibration sources

Electro-magnetically induced vibration sources include: (i) Unbalanced radial force acting on the stator or the rotor. It is the result of an uneven magnetic field distribution, which is caused by several factors. For example, the eccentricity of the rotor and stator, the unbalance of the magnetic field in magnetically asymmetric motors, and an unbalanced phase-current excitation that depends on the phase-winding patterns and connections. (ii) Cogging torque acting on the rotor. This is caused by the interaction between the magnet poles and the geometry of the stator tooth. (iii) Current torque ripple due to the pulse-width-modulation (PWM) control. (iv) Electromotive force (EMF) waveform imperfection. (v) Commutation torque ripple, created by the commutation of current from one phase to another.

There have been many investigations focused on predicting the unbalanced radial force, cogging torque and their dependence on the motor design parameters. Forcing frequency components have been investigated (Lorimer and Hartman, 1999) by considering a BLDC motor with N_s slots and N_p poles. The shaft is aligned along the z direction and the disks parallel to the x - y plane. Table 3.2 lists the common formulae for BLDC motor vibration frequencies expressed as multiples of spin speed of the rotor.

Causes		Formulae
Cogging		$f(M_z) = i \frac{N_s N_p}{GCF(N_s, N_p)}, i = 1, 2, 3 \dots$
Unbalanced Magnetic Pull		$f(F_x, F_y) = n N_p$ for n satisfying $n N_p = q N_s \pm 1, q \in \{1, 3, 5 \dots\}$
Commu- tation	Electrically symmetric 3-phase motor	$f(M_z) = i \cdot 3 \cdot N_p$
	Driven imbalance force	$f(F_x, F_y) = n N_p$ for n satisfying $n N_p = 3q \pm 1, q \in \{1, 3, 5 \dots\}$
Stator Eccentricity		$f(M_z, F_x, F_y) = i N_p, i = 1, 2, 3 \dots$
Rotor Eccentricity		$f(M_z) = i N_s$ $f(F_x, F_y) = \begin{Bmatrix} 1 \\ i N_s - 1 \\ i N_s + 1 \end{Bmatrix}, i = 1, 2, 3 \dots$

N_s : number of slots

N_p : number of poles

F_x, F_y : x and y components of force

M_z : torque

$f(F_x, F_y, M_z)$: forcing frequency components expressed as multiples of
spin speed of the rotor

$GCF(i, j)$: greatest common factor of whole numbers i and j

Table 3.2 Formulae for BLDC motor vibration sources frequencies
(Lorimer & Hartman, 1999)

Symmetric and asymmetric motors were analysed to compare various sources of vibration when rotor eccentricity exists (Kim *et al.*, 2001). The effects of rotor eccentricity on motor performance for symmetric and asymmetric motors, such as unbalanced force and cogging torque, were discussed (Hwang *et al.*, 2001). Figure 3.4 shows x and y components of the unbalanced radial forces with respect to the rotor position with the eccentricity ratio of 0.5. Figure 3.5 shows the results of cogging torque for symmetric and asymmetric motors without eccentricity, and Figure 3.6 shows the effects of eccentricity on the cogging torque.

Fig. 3.4 Unbalanced radial forces with respect to the rotor position (Hwang *et al.*, 2001)

Fig. 3.5 Cogging torque without eccentricity (Hwang *et al.*, 2001)

Fig. 3.6 Cogging torque with eccentricity (Hwang *et al.*, 2001)

Torque ripple due to PWM control was studied by using Fourier series representation models (Lee and Singh, 1992a). Table 3.3 lists the harmonics of this torque ripple without and with PWM control.

	Harmonic Group	Frequency Contents	Source
Without Pulse-Width-Modulation	(i)	Harmonics of $3N_p\Omega_r$	Dynamic interactions between E and I
	(ii)	Odd harmonics of $0.5N_sN_p\Omega_r$	Open stator slot
	(iii)	$sN_p\Omega_r + m\Omega_r$ where s is an integer and $m = q, 2q, N_pq$; $q = 1, 2, 3, \dots$	Inductance variation
With Pulse-Width-Modulation	(i)	$3qN_p\Omega_r \pm \alpha\omega_c$ where $q = 1, 2, 3, \dots$ and $\alpha = 0, 1, 2, \dots$	Dynamic interactions between E and I with side bands generated by PWM
	(ii)	Odd harmonics of $0.5N_sN_p\Omega_r$	Open stator slot
	(iii)	$sN_p\Omega_r \pm m\Omega_r \pm \alpha\omega_c$ where s is an integer, $\alpha = 0, 1, 2, \dots$ and $m = q, 2q, N_pq$; $q = 1, 2, 3, \dots$	Inductance variation with side bands generated by PWM
	(vi)	Harmonics of ω_c	Carrier frequency ω_c

E: back EMF vector (v)

I: stator phase current vector (A)

N_p : number of rotor poles

N_s : number of stator slots

ω_c : carrier frequency of PWM control scheme (rad/s)

Ω_r : angular velocity of rotor (rad/s)

Table 3.3 Harmonics of torque ripple due to PWM control (Lee & Singh, 1992a)

The relationship between motor current and back EMF was explored (Hanselman, 1994).

An analytical study was developed concerning the torque ripple due to phase commutation and the results indicate that in the range where the current is effectively controlled, the relative ripple is independent of current and varies with speed (Carlson *et al.*, 1992).

3.2.2 Sound and vibration in rolling element bearings

A rolling bearing is a relatively simple machine element including raceway rings, rolling elements and a cage, as shown in Figure 3.7. However, due to the fundamental nature of

rolling bearings and limitations on machining technology, a certain degree of vibration and sound is unavoidable. The various types of vibration and sound are grouped into the four categories listed in Table 3.4 (Momono and Noda, 1999).

Fig. 3.7 A typical ball bearing (<http://www.csc.uvic.ca/>)

Among them the most basic one is race noise, which is inevitably produced from all bearings as they operate. This is due to the very small irregularities in the surface (waviness) and roughness of the raceways and rolling elements of the bearing. Race noise is characterized by a smooth, continuous sound.

Cage noise is not one sound, but can have a variety of sounds. The mechanism of each sound is different from the other. There are two kinds of cage noise: one is the noise caused by the cage colliding with rolling elements or bearing rings (CK noise); the other is a low frequency noise which is attributed to self-induced vibration of the cage caused by sliding friction between the cage guide face and the bearing component that guides the cage (CG noise).

Chapter 3

Noise source mechanisms and noise control methods

	Sound		Vibration		Features
Structural	Race noise		Free vibration of raceway ring		Continuous noise, basic unavoidable noise which all bearings generate
	Click noise		Free vibration of raceway ring, free vibration of cage		Regular noise at a certain interval, large bearings and horizontal shaft, radial load and low rpm
	Squeal noise		Free vibration of raceway ring		Intermittent or continuous, mostly large cylindrical roller bearings, radial load, grease lubrication, at particular speed
	Cage noise	"CK" noise	Free vibration of cage		Regular noise at a certain interval, all bearing types generate it
		"CG" noise	Vibration of cage		Intermittent or continuous, lubrication with particular grease
		Tapping noise	Free vibration of cage		Certain interval, but a little irregular under radial load and during initial stage
—		Rolling element passage vibration		Continuous, all bearing types under radial load	
Manufacturing	Waviness noise		Vibration due to waviness	Inner ring	Continuous noise
				Outer ring	Continuous noise
				Rolling element	Continuous with rollers, occasional with balls
Handling	Flaw noise		Vibration due to flaw	Inner ring	Regular noise at a certain interval
				Outer ring	
				Rolling element	
				Contamination noise	
Others	Seal noise		Free vibration of seal		Contact seal
	Lubricant noise		—		Irregular
	—		Runout	f_r	Continuous
				f_c	Continuous
$f_r - 2f_c$				Continuous	

Generated frequency (frequency analysis)			Source	Countermeasures
FFT of original wave		FFT after envelope (basic No.)		
Radial (angular) direction	Axial direction			
$f_{RN} \cdot f_{MI}$	$f_{AN} \cdot f_{AM}$	—	Selective resonance of waviness (rolling friction)	Improve rigidity around the bearings, appropriate radial clearance, high-viscosity lubricant, high-quality bearings
$f_{RN} \cdot f_{MI}$ Natural frequency of cage	$f_{AN} \cdot f_{AM}$	Z_c	Collision of rolling elements with inner ring or cage	Reduce radial clearance, apply preload, high-viscosity oil
($= f_{RN} \cdot f_{RN}$)	—	?	Self-induced vibration caused by sliding friction at rolling surface	Reduce radial clearance, apply preload, change the grease, replace with countermeasured bearings
Natural frequency of cage	f_c	?	Collision of cage with rolling elements or rings	Apply preload, high-viscosity lubricant, reduce mounting error
Natural frequency of cage	?	?	Self-induced vibration caused by friction at cage guide surface	Change the grease brand, replace with countermeasured cage
Natural frequency of cage	Z_c	?	Collision of cage and rolling element caused by grease resistance	Reduce radial clearance, apply preload, low-viscosity lubricant
Z_c	—	—	Displacement of inner ring due to rolling element passage	Reduce radial clearance, apply preload
$nZf_i \pm f_r$ ($nZ \pm 1$ peaks)	nZf_i (nZ peaks)	—	Inner ring raceway waviness, irregularity of shaft exterior	High-quality bearings, improve shaft accuracy
nZf_c ($nZ \pm 1$ peaks)	nZf_c (nZ peaks)	—	Outer ring raceway waviness, irregular bore of housing	High-quality bearings, improve housing bore accuracy
$2nf_b \pm f_c$ ($2n$ peaks)	$2nf_b$ ($2n$ peaks)	—	Rolling element waviness	High-quality bearings
$f_{RN} \cdot f_{MI}$	$f_{AN} \cdot f_{AM}$	Z_f	Nicks, dents, rust, flaking on inner ring raceway	Replacement and careful bearing handling
		Z_c	Nicks, dents, rust, flaking on outer ring raceway	Replacement and careful bearing handling
		$2f_b$	Nicks, dents, rust, flaking on rolling elements	Replacement and careful bearing handling
$f_{RN} \cdot f_{MI}$	$f_{AN} \cdot f_{AM}$	Irregular	Entry of dirt and debris	Washing, improve sealing
Natural frequency of seal		(f_r)	Self-induced vibration due to friction at seal contact area	Change the seal, change the grease
?	?	Irregular	Lubricant or lubricant bubbles crushed between rolling elements and raceways	Change the grease
f_r	—	—	Irregular inner ring cross-section	High-quality bearings
f_c	—	—	Ball variation in bearing, rolling elements non-equidistant	High-quality bearings
$f_r - 2f_c$	—	—	Non-linear vibration due to rigid variation by ball variation	High-quality bearings

 n : Positive integer (1, 2, 3...) Z : Number of rolling elements f_{RN} : Ring natural frequency in radial bending mode, Hz f_{MI} : Natural frequency in the mode of angular vibration in inertia of outer ring-spring system, Hz f_r : Rotation frequency of inner ring, Hz f_c : Orbital revolution frequency of rolling elements, Hz f_{AN} : Ring natural frequency in axial bending mode, Hz f_{AM} : Natural frequency in the mode of axial vibration in mass of outer ring-spring system, Hz f_r : $f_r = f_r - f_c$, Hz f_b : Rotation frequency of rolling element around its center, Hz

Table 3.4 Classification of noise and vibration in rolling bearing (Momono & Noda, 1999)

Flaw noise is caused by the rolling elements hitting the flaw on the raceway surface. The flaw can be any part of the rolling contact surface scratched, dented, or otherwise sustains a shock. Improper handling of the bearing is the main cause of such flaw noise. The vibration waveform of flawed ball bearings is shown in Figure 3.8 (Noda, 2002).

Fig. 3.8 Vibration waveform of flawed ball bearings (Noda, 2002)

Frequency of vibration and sound in bearings (Momono and Noda, 1999)

Natural frequency of raceway rings, in particular, natural frequency of the outer ring is examined systematically since the outer ring has direct contact with the housing. Basically, the modes of vibration of the outer ring are roughly divided into two categories due to the loose fit between the outer ring and the housing: (i) Rigid body modes when the outer raceway ring is considered as a rigid body, which can be divided further into angular vibration in the inertia outer ring-spring system and vibration in the mass outer ring-spring system in the axial direction of the bearing as shown in Figure 3.9 and Figure 3.10; (ii) Elastic body modes when the outer raceway ring is considered as an elastic body. One of the examples is the bending modes in the radial and axial directions as shown in Figure 3.11.

Fig. 3.9 Mode of angular vibration in inertia of outer ring-spring system (Momono & Noda, 1999)

Fig. 3.10 Mode of axial vibration in mass of outer ring-spring system (Momono & Noda, 1999)

Fig. 3.11 Bearing rings bending mode shapes (Momono & Noda, 1999)

It can be seen from Table 3.4 that rotation frequency of rolling elements, the orbital revolution frequency of the rolling elements f_c and the rolling element frequency f_b , are required in order to calculate either the frequency of vibration caused by waviness or flaw noise. They can be calculated with the following formulae (Momono and Noda, 1999):

$$f_c = \frac{1}{60} \frac{n_i}{2} \left(\frac{d_m - D_a \cos \alpha_c}{d_m} \right) \quad (3.1)$$

$$f_b = -\frac{1}{60} \frac{n_i}{2} \left(\frac{d_m}{D_a} - \frac{D_a \cos^2 \alpha_c}{d_m} \right) \quad (3.2)$$

where

n_i : Running speed of the inner ring (RPM)

d_m : cage diameter of rolling elements (mm)

D_a : Diameter of rolling elements (mm)

α_c : Contact angle ($^\circ$)

3.2.3 Sound radiation from vibrating hard disks

The subject of sound radiation from vibrating structures has been extensively investigated.

It is already known that once the vibration response has been obtained, it can be used as an input to further calculate the sound radiation. The acoustic pressure induced by the vibrating disk is generally given by the Helmholtz integral equation (Cote *et al.*, 1998)

$$p(x) = \int_{S_T} \left[p(y) \frac{\partial G(x, y)}{\partial n_y} - G(x, y) \frac{\partial p}{\partial n_y} \right] dS_y \quad (3.3)$$

where x and y represent an arbitrary point in the fluid and on the disk, respectively. n_y is the outward normal to the disk surface at the point y . S_T is the total surface of the disk.

The free field Green's function is given by

$$G(x, y) = \frac{1}{4\pi R} e^{-jkR} \quad (3.4)$$

with R the Euclidean distance between points x and y and k is the acoustic wave number.

Continuity exists in the normal velocity through the disk thickness and discontinuity exists in the acoustic pressure due to the assumption of thin disk. Thus Equation (3.3) reduces to (Cote *et al.*, 1998)

$$p(x) = \int_S \mu(y) \frac{\partial G(x, y)}{\partial n_y} dS_y \quad (3.5)$$

where the pressure jump denoted as

$$\mu = p^+ - p^- \quad (3.6)$$

with p^- and p^+ are the pressure at the bottom and upper surface of the disk, respectively.

The normal is defined from p^- to p^+ and S is the upper surface of the disk

Recall that the annular disk is in direct contact with the fluid and there is a continuity between the surface of the disk and the fluid displacement, thus Equation (3.5) becomes (Cote *et al.*, 1998)

$$\rho_0 \omega^2 (\mathbf{u} \bullet \mathbf{n}_x) = \frac{\partial}{\partial n_x} \int_S \mu(y) \frac{\partial G(x, y)}{\partial n_y} dS_y \quad (3.7)$$

where ρ_0 is the density of the fluid. ω^2 is the normal surface acceleration of the disk with $(\mathbf{u} \bullet \mathbf{n}_x)$ the normal displacement at point x of the disk. A variational approach is used to evaluate Equation (3.7) and remove the hypersingularity. Hence, the following is obtained with the condition that there is no pressure jump along the edges of a thin disk (Cote *et al.*, 1998)

$$\begin{aligned} \rho_0 \omega^2 \int_{\Sigma} \delta \mu(x) (\mathbf{u} \bullet \mathbf{n}_x) dS_x &= \iint_{\Sigma} k^2 (\mathbf{n}_x \bullet \mathbf{n}_y) G(x, y) \delta \mu(x) \mu(y) dS_y dS_x \\ &\quad - \iint_{\Sigma} [\mathbf{n}_x \wedge \nabla_x \delta \mu(x)] \bullet [\mathbf{n}_y \wedge \nabla_y \mu(y)] G(x, y) dS_y dS_x \end{aligned} \quad (3.8)$$

A boundary element method is used to solve Equation (3.8). The numerical evaluation of the singular integrals (cross influence terms) is made through a Radau-Hammer integration scheme while the integration of the singular term (self-influence terms) is

made using a semianalytical approach. Then, the solution of Equation (3.8) can be found by using the variational principle. Once the pressure jump is determined, the acoustic pressure can be calculated through Equation (3.5) (Cote *et al.*, 1998).

3.2.4 Electro-magnetically induced vibration sources in VCM/actuators assembly

The voice coil motor/actuators assembly consists of head gimbal assembly, actuator, voice coil motor, and control electronics as shown in Figure 3.12. The HGA consists of a slider, where the head is attached, and a suspension. This subsystem is connected to the actuator through the suspension. The actuator consists of an arm and a pivot ball bearing assembly. The VCM subsystem is composed of a magnetic coil and two yoke plates (top and bottom) where permanent magnets are bonded in place.

Fig. 3.12 Annotated illustration of a typical PC VCM/actuator assembly
(From <http://www.pcguide.com/>)

When the voice coil is energized by the input current from the flex circuit cable, it creates a magnetic field between the top and bottom magnetic plates as shown in Figure 3.13. The force (F) generated by the VCM serves not only as a driving force which causes the actuator arm and the HGA to rotate simultaneously, but also as a force to vibrate the

VCM/actuators assembly, thus producing noise. This kind of noise is associated with the actuator dynamics and can be reduced by damping treatment.

Fig. 3.13 Operating principles of VCM/actuator assembly (Makoto et al., 2000)

On the other hand, noise can arise in the movement of the head from one data track to another in response to the current fed into the voice coil. This motion of the head is referred to as a “seek” to a destination track. A common way of controlling a seek operation is to develop a velocity profile that provides the velocity the head should have at varying distances from the destination track (Hampshire *et al.*, 1995). A control signal, directly proportional to the difference between the profile velocity and the actual velocity of the head, is provided to the power amplifier at each of a succession of tracks terminating with the destination track. The velocity profile is shaped with respect to the number of tracks remaining in the seek to cause the head to initially accelerate toward the destination track and subsequently decelerate to the track. During acceleration and deceleration of the head, currents through the voice coil change rapidly due to large differences between the profile velocity and the actual velocity. Thus, noise is generated. Figure 3.14 shows the vibration generated in the conventional manner of developing the velocity profile. In this case, the voice coil currents only satisfy a selected current versus time relation during deceleration while the profile velocity is generated by just setting it equal to the maximum velocity during acceleration.

Fig. 3.14 Vibration generated in the conventional manner of developing the velocity profile (a) Voice coil current versus time (b) Velocity profile developed in accordance with current relation given in (a) (c) Output of an accelerometer mounted on the disk drive versus time (Hampshire et al., 1995)

3.2.5 Vibration amplified by the housing and the PCB

So far, the key noise sources mechanisms and characteristics are reviewed. However, vibrations of the spindle/disks assembly and VCM/actuators assembly can be directly transmitted to the drive housing, including the top cover and the base plate normally

made of thin aluminium or stainless steel, through screws and rigid connections. These vibrations will in turn be amplified by resonances of the drive housing.

On the other hand, the PCB is normally mounted on the bottom of the base plate and exposed to the outside. This printed circuit board, being highly flexible, is a significant source of amplification of noise and vibration mentioned above. Further efforts to reduce noise emissions from the HDD must be focused on reducing the amplification caused by vibration of the PCB.

3.3 Noise and vibration control methods

3.3.1 Noise and vibration reduction in spindle motor

Generally, the motor industry has continually improved the brushless DC motor smoothness by improvements in several aspects: higher quality bearings (steel ball & hydrodynamic) instead of the conventional ball bearings, better inertia balancing to avoid eccentricity, coil current waveform shaping, special magnetization patterns and stator tooth profiles to reduce cogging torque and symmetric pole/slot ratios to reduce magnetic radial forces.

The effect of magnet pole shaping and magnet arc length on the output torque and torque ripple was discussed and a magnet edge shaping was proposed to design a trapezoidal back EMF motor without torque ripple (Hwang and Lieu, 1995). A technique has been developed to alter the motor commutation pattern, hence, the pattern of the commutation torque ripple, to avoid exciting mechanical resonances when changing current levels in the motor windings (Brackley and Pollock, 1999). From the commutation time point of

view, the theoretical background to reduce torque ripple of BLDC motor was presented (Kang *et al.*, 2001).

Other treatments for solving the problem of spindle motor vibration sources include damping treatments on the stator and spacers. Test results show that disk vibration is significantly lowered when a constrained-layer damper is added to the spacer. Various designs of this type of damper for the disk spacer have been proposed (Landin & Hwang, 1998).

3.3.2 Sound and vibration reduction in rolling element bearings

Recent approaches, made by NSK, to reduce noise and vibration in rolling bearings were described (Furumura, 1996; Noda, 2002). (i) Reduction of race noise – The minimization of the very small surface irregularities is the most basic approach to take in reducing race noise. However, even today's excellent bearing machining and processing technologies cannot completely eliminate waviness. (ii) Reduction of flaw noise – In response, bearing design and materials are developed to provide greater resistance to shock loads. NSA Grease was developed by NSK to provide better protection against wear corrosion or friction oxidation, which is caused by shock or vibration during transportation. (iii) Reduction of cage noise – Narrowing the range of cage riding clearance is effective in reducing cage noise. Then again, excessively restricting cage movement can increase rotating torque and cause poor lubrication. For these reasons, provisions are often made to minimize cage noise, depending on the type of cage and particular bearing operating conditions, while also taking into consideration bearing performance factors.

Fluid dynamic bearing motor

Since Seagate released the world's first fluid dynamic bearing (FDB) motor in 1996, quieter drives can be achieved by using FDB motor into the spindle/disks assembly instead of ball-bearing motors. Figure 3.15 shows the structure of a fluid dynamic bearing motor. The point is that the piece parts inside the fluid dynamic bearing motors form air gaps that are filled with viscous oil. This design provides more advantages in the following aspects: (i) Shock performance: The working parts of the motor are separated by a film of oil, which acts like a shock absorber and prevents damage to the bearing surfaces. (ii) Vibration: A well-designed fluid bearing can quickly dampen any oscillation generated externally or internally. Dampening in bearings is important for a disk drive to correctly write to or retrieve the data from the disk. (iii) Acoustic performance: Fluid dynamic bearings produce very little noise because they have no metal-to-metal contact.

Fig. 3.15 Fluid dynamic bearing motor (From <http://www.byte.com/>)

3.3.3 Noise and vibration reduction in VCM/actuators assembly

There are many ways to accomplish quieter seeks. For seek induced noise mentioned in section 3.2.4, a method to reduce the vibration and noise was presented by the generation of demand velocities during acceleration (Hampshire *et al.*, 1995). At any time interval during acceleration, the demand velocity is generated by incrementing the demand velocity for the previous interval by the product of the demand velocity fraction and the difference between the current profile velocity and the previous demand velocity.

Deceleration is started when the profile velocity is less than the previous demand velocity. Thus, the vibration and noise can be reduced by selection of the demand velocity fraction to limit the voice coil current and its rate of change during acceleration. Figure 3.16 shows the vibration generated in accordance with this noise and vibration reduction method. By comparing Figure 3.14 (c) and Figure 3.14 (d), it is obviously observed that vibration generated is greatly reduced with the improvement on the current through the voice coil motor by the generation of demand velocities.

(a)

(b)

(c)

(d)

Fig. 3.16 Vibration generated in accordance with the noise and vibration reduction method (a) Demand velocity versus tracks to completion of a seek (b) Actual head velocity versus tracks to completion of a seek (c) Actuator coil current versus time (d) Output of an accelerometer mounted on the disk drive versus time (Hampshire et al., 1995)

Alternatively, another method is to increase the system damping levels of the key internal components while maintaining the drive performance (Hwang, 1998; Jiang and Miles, 1999; Blount, 2001). In the past years, a variety of damper designs have been developed to reduce the vibration and acoustic noise coming from subsystems of the VCM/actuators assembly as shown in Figure 3.17.

Fig. 3.17 Designed dampers related to head actuator assembly (Hwang, 1998)

The VCM damper is located between the cover and the magnetic plate of the VCM. The damper not only reduces the low-frequency noise but also reduces the resonance noise at high frequencies. The actuator arm damping treatment uses a constrained-layer damper. The damper is made of a metallic layer (aluminium or stainless steel) and a layer of damping polymer that is attached to the actuator arm surface. Suspension dampers have been used to reduce HGA resonance. Using damping polymers as a bonding agent

between the mounting base (through which the actuator arm is attached to the suspension element) and the actuator arm has shown an improvement of HGA performance.

3.3.4 Reduction of vibration amplified by the housing and PCB

A number of different types of material have been used as the acoustic insulator placed between the base plate and the PCB. Based on the constrained layer damping treatment, a layer of adhesive material was strategically inserted between the base and PCB to reduce resonance (Schmitz and Beach, 1993).

The cover of the HDD can be made or stamped out of a stainless steel constrained layer of viscoelastic damping material. A foam material is provided between the base and the PCB to avoid transmission of vibrations which would in turn generate noise (Woldemar and Mylabathula, 1999).

3.3.5 Low noise technologies in commercial HDDs

Table 3.5 lists some of the commercial low noise HDDs with the techniques utilized and the typical acoustic values in terms of sound power level. In the following, these technologies are introduced briefly. *Seagate Sound Barrier Technology* is a group of improvements. It includes a silent seek mode that is configured during manufacturing or enabled by a user and can be controlled through tools called Automatic Acoustic Management. With sound barrier technology, the hard disk drive can be controlled to perform in quiet mode for entertainment applications at home or to perform faster seeks for business applications where the speed of the R/W of the data is the main concern; *SeaShield* drive covers and designed-in dampers; Resonance reduction; Just-in-Time

seeks that reduce acoustics and power consumption; Applications of fluid dynamic bearing spindle motors and isolator mounting systems.

<i>Manufacturers</i>	<i>Low Noise Technology</i>	<i>HDD samples</i>	<i>Idle noise(dB)</i>	<i>Average Random Seek (dB)</i>
Seagate	Sound Barrier Technology	U Series 5 10211	29	31
Samsung	Noise Guard, Silent Seek	Spinpoint V series SV0802N	26	27
Fujitsu	Silent Drive	MPF3204AT	-	22
Quantum	Quiet Drive Technology	Fireball lct10	-	-
Maxtor	Silent Store	DiamondMax 60	-	-
IBM	Active damping	Ultrastar 36LP	-	-

Table 3.5 Low noise commercial HDD

Samsung Noise Guard Technology selected optimal combination of vibration absorbing materials to solve the problem. The cover shape was also designed to reinforce the most vibration susceptible areas. The *Silent Seek Technology* is another unique design that is to minimize the acoustic noise generated by the movements of the actuators by incorporating a performance optimized sinusoidal seek trajectory. These design improvements significantly reduces overall acoustic noise levels.

Fujitsu Silent Drive Dec. 1, 1999, Fujitsu announced the world's quietest hard disk drive with a noise level as low as 22dB(A). It utilises a combination of fluid bearing technology, specially designed acoustic damping materials and silent mode drive control firmware.

Quantum Quiet Drive Technology reduces HDD noise in terms of sound quality and sound measurement. The quality of the sound is tuned so as to eliminate sound that can be annoying. *Maxtor Silent Store* can let PC manufacturers select whether to run the HDD at maximum performance or with the lowest noise level. This provides PC manufacturers with maximum flexibility when building their systems because there will be times when they need a drive that is suitable for noise-sensitive environments. *IBM Active Damping* applies active control of the actuator to reduce impact from resonant modes and to allow increased servo bandwidth.

3.4 Summary

This chapter mainly reviews various noise source excitations in the HDD and provides source excitation characteristics wherever possible. Although it does not show any new phenomena but it is very important in analysing vibro-acoustics in HDDs since noise source identification is the first step in vibro-acoustic analysis. It helps in understanding the acoustic radiation phenomena. On the other hand, to the author's knowledge, there is no such detailed published review of noise sources in HDDs although many researchers have dealt with individual noise sources that are of specific interest. This Chapter provides readers a basic and relatively detailed review of the topic.

It is obvious that vibrations and consequently noise generated can be reduced to a very large extent if the forces produced during the HDD operation are not allowed to excite any resonances of the HDD components. Accurate determination of the exciting forces and the resonant frequencies are therefore very important. However, it is not easy because of the rather complex structures of HDD components and interactions between them.

One approach to analyse vibro-acoustic interactions is to derive computer models of the spindle/disks assembly-shaft-housing system and VCM/actuators assembly-shaft-housing system to determine the vibration frequency response functions. With these frequency response functions, it is possible to predict the vibration response of, say, the HDD housing, to forced excitations during idle and seek modes. The calculated vibration response of the housing can then be used to further predict the sound radiated from the HDD.

In reality, it is difficult to obtain accurate models of vibration sources, especially magnitude levels of the excitations. This means that an accurate prediction of the absolute vibration response cannot be achieved, even though the frequency functions are available. However, it is still useful to adopt this approach to analyse relative levels of vibration response. For instance, if certain design modifications are made to a HDD, it will be possible to evaluate the relative improvement to noise and vibration response as a result of the changes. With an understanding of the frequency characteristics of the various noise sources, the relative improvement in the noise and vibration levels during idle and seek modes can be predicted. Hence, good vibro-acoustic models, as described in the following chapters, serve as potentially useful tools for design modification studies.

Chapter 4

Dynamic Analysis of Spindle/Rotating Disks Assembly-Shaft-Housing System

This chapter presents the mathematical models of the spindle/disks-shaft-housing system for vibration analysis. A model of the rigid spindle-flexible rotating disks assembly operating in idle mode is first developed using a multi-body system Lagrangian formulation. A more complete model of spindle/disks assembly-flexible shaft-flexible housing system is then developed and the transfer functions between source excitations and induced vibration on the housing are derived. The effect of adding flexibility to the no-spinning components (i.e. housing and shaft) is examined and the model capabilities are demonstrated through parameter and load case studies.

4.1 Introduction

To further explore the sound radiation from a HDD in idle mode, it is necessary to have a thorough understanding of the vibration characteristics of the components and subsystems involved, their interaction and the vibration transmission between different components. In another word, the understanding of the dynamic behavior of the entire system of spindle, shaft, spinning disks and housing is essential. However, available open literature focuses mainly on vibration analysis of individual components and subsystems, say, a single rotating disk and spindle/disks assembly.

Vibration of a single rotating disk

The free circular plate spinning about its cylindrical axis with a constant angular velocity was first explored in 1921 (Lamb and Southwell, 1921). The problem of a circular disk, clamped at its center and free at its outer edge and rotating with a constant angular velocity was analyzed in 1922 (Southwell, 1922). Since then, there have been many analytical and experimental studies devoted to vibration analysis of rotating disks.

Early studies focused on free vibrations and critical speeds of flexible spinning disks. The increase of natural frequencies from the stiffening effect induced by the rotation was investigated (Ramaiah, 1981). Accurate representation of the disk membrane stresses caused by rotation was examined based on the membrane stress boundary condition at the clamping collar (Angelo and Mote, 1993). A finite element formulation was developed for the study of axial, torsional and flexural dynamic behavior of thin rotating disks with account taken of the gyroscopic effect and the influence of the centrifugal loadings (Genta and Tonoli, 1996). Variational formulation based on the Kirchhoff plate theory and von Karman strain theory was used to derive the governing equations for free vibration of a spinning circular disk (Chung *et al.*, 1996). A more general spinning disk model with the assumptions of the plate theories of Kirchhoff and von Karman were derived using Hamilton's principle. The traditionally ignored in-plane inertia of the spinning plate and the rotary inertia were taken into account (Baddour and Zu, 2001a, 2001b).

Research on circular disks moved toward the dynamic response and stability of stationary disks with moving load (Shen, 1993) and response and stability of spinning disks with a stationary load (Hutton *et al.*, 1987; Chen and Bogy, 1992; Liang *et al.*, 2002).

All the above work considers that the orientation of the spindle/disks assembly is fixed in space, that is, the spinning disk is centrally clamped on the shaft and constrained from translational and rotational rigid body motion. Some studies have been carried out that considered not only disk vibration but also the rigid body motion in a single disk system (Yang, 1993; Chen and Bogoy, 1993). A more general formulation based on the Kirchhoff plate theory and von Karman strain theory was used to derive the governing equations for free vibration of a flexible spinning disk with axial translation and rigid body tilting (Lee *et al.*, 1998).

Vibration of spindle-disks assembly

Traditionally, HDDs have been modeled as flexible disks mounted on a rigid spindle with rigid bearings. However, such a model could not predict the resonance of unbalanced (0, 1) mode since it did not consider the rigid-body rocking of the spindle (Shen, 1997).

The coupling mechanism between the rigid-body motion of the spindle and the flexible modes of the disks was studied extensively. A complete spindle/disks assembly model, which consists of multiple flexible disks rigidly attached to a rigid spinning spindle supported by elastic ball bearings, was developed and the concepts of balanced and unbalanced modes were proposed (Shen and Ku, 1995, 1997). Later a closed form formulation for the forced response of a damped, rotating, spindle/multiple disks system was completed (Shen, 1997). Follow-up parametric studies were performed with the focus on the rocking modes (Shen, 1998). Free and forced vibrations of a spindle/ rotating disks assembly supported by hydrodynamic bearings (HDB) were examined considering that HDB are to replace ball bearings in future hard disk drives (Jintanawan *et al.*, 1999).

More complex models including flexible spindle and disks have also been developed. Natural frequencies of a flexible shaft coupled with multiple flexible disks were investigated using a substructure technique with the assumed modes method (Lee *et al.*, 1997). Assumed modes method was used to explore the response of flexible spindle carrying multiple flexible disks (Chun and Lee, 1996; Lee and Chun, 1998). Extended operators were used to predict the vibration of flexible disk/flexible spindle systems (Parker, 1999). Later an exact, closed-form solution for the eigensolutions of an elastic disk mounted on an elastic spindle system was presented (Parker and Sathe, 1999). The dynamics of the flexible spindle/spinning disk system was also investigated by using finite element method and substructure synthesis (Lim, 2000). A more general model, considering the non-linearity of the spinning disks and the flexibility of the stationary shaft, was developed recently (Jang *et al.*, 2002).

Due to the important role of ball bearings in the spindle/disks assembly, much research has also been carried out on it (Harris and Mindel, 1973; Poon, 1978; Lim and Singh, 1990a, 1990b; Mukasa, 2002). A mathematical model for the rolling element bearings was developed and a comprehensive bearing stiffness matrix of dimension six was proposed (Lim and Singh, 1990a). The model can be used to explain how the vibration may be transmitted from the shaft to the connecting structures in rotating mechanical equipment.

To summarize, there have been many studies devoted to the dynamic analysis of the spindle/disks assembly. Generally, the analytical models consist of a rigid spindle mounted with multiple disks and supported by flexible bearings. The non-spinning parts, i.e. the shaft and the housing, have not been included in these analyses. The top cover and

base plate were represented by dynamic stiffness matrices in a mathematic model for the study on sound radiation by a stationary or rotating disk caused by motor torque pulsation of the BLDC motor (Lee and Singh, 1996). However, the single disk was assumed to be stationary and the rotation effect was factored in later. In reality, the disk is spinning no matter it is in idle or seek mode, which is one of the most important characteristics of the HDD. In addition, gyroscopic effect was not considered. As the disk is rotating at high speed, the gyroscopic effect may be significant.

Therefore, a more accurate and complete mathematical model is needed. The top cover, as the main radiator, and the shaft, as the supporter of the system, are the indispensable elements in the vibration transmission and sound radiation. Moreover, the effect of flexibility of the shaft and the housing cannot be neglected. Consequently, the subject of the study in the vibration analysis of spindle/disks assembly-shaft-housing system is a rigid rotating spindle carrying multiple flexible rotating disks mounted on an assembly of the flexible shaft and the flexible housing through flexible ball bearings. The task is to determine the vibration of the housing structurally transmitted by the internal motion of spindle/disks assembly via ball bearings and flexible shaft. Vibration of the spindle/disks assembly is first studied followed by the general model for the entire spindle/disks assembly-shaft-housing system.

4.2 Vibration of spindle/rotating disks assembly

4.2.1 Frames of reference

To facilitate the formulation, a set of reference frames are defined as shown in Figure 4.1. The inertial frame XYZ has point O as the origin and has unit vectors of $\mathbf{I}$, $\mathbf{J}$ and $\mathbf{K}$. The rotation sequence of Euler angles θ_x , θ_y and θ_z are selected as (1) θ_x about X axis

followed by intermediate coordinates xyz ; (2) θ_y about y axis followed by intermediate coordinates $x'y'z'$ which is non-spinning and also called rocking frame; (3) θ_z about z' axis followed by coordinates $x''y''z''$ which is body-fixed to the spinning disk and has unit vectors of $\mathbf{i}''$, $\mathbf{j}''$ and $\mathbf{k}''$.

Fig. 4.1 Definition of frames of reference

(1) θ_x about $X \rightarrow xyz$ (2) θ_y about y axis $\rightarrow x'y'z'$ (3) θ_z about z' axis $\rightarrow x''y''z''$

The rocking coordinates $x'y'z'$ and the coordinates $x''y''z''$ body-fixed to the spinning disk are related to the inertial coordinates XYZ through the following transformation matrices,

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = [A] \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \quad (4.1)$$

$$\begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} = [B] \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \quad (4.2)$$

with

$$[A] = \begin{bmatrix} \cos \theta_y & \sin \theta_x \sin \theta_y & -\cos \theta_x \sin \theta_y \\ 0 & \cos \theta_x & \sin \theta_x \\ \sin \theta_y & -\sin \theta_x \cos \theta_y & \cos \theta_x \cos \theta_y \end{bmatrix} \quad (4.3)$$

$$[B] = \begin{bmatrix} \cos \theta_y \cos \theta_z & \sin \theta_x \sin \theta_y \cos \theta_z + \cos \theta_x \sin \theta_z & -\cos \theta_x \sin \theta_y \cos \theta_z + \sin \theta_x \sin \theta_z \\ -\cos \theta_y \sin \theta_z & -\sin \theta_x \sin \theta_y \sin \theta_z + \cos \theta_x \cos \theta_z & \cos \theta_x \sin \theta_y \sin \theta_z + \sin \theta_x \cos \theta_z \\ \sin \theta_y & -\sin \theta_x \cos \theta_y & \cos \theta_x \cos \theta_y \end{bmatrix} \quad (4.4)$$

4.2.2 Problem description

Fig. 4.2 Components of spindle/disks assembly (From www.storagereview.com)

Figure 4.2 shows the components of a typical spindle/disks assembly. Refer to Figure 3.2 in Chapter 3 for the detailed cross-sectional view of the spindle motor.

Fig. 4.3 The schematic of the analysis model of the spindle/disks assembly

Consider an analysis model consisting of a spindle and N identical flexible disks rigidly attached to the spindle as shown in Figure 4.3. The rigid spindle is simply supported by two ball bearings A and B . The bearing A is distance z_a from O and is simplified as a

spring with stiffness in three translational dimensions. The same simplification applies to the bearing B with distance z_b from O . The i^{th} disk is distance z_i from O to the origin of the body-fixed frame of the i^{th} disk D_i and has inner radius a_i and outer radius b_i .

4.2.3 Energy functions

Kinetic Energy

Kinetic energy (KE) of the rigid spindle/flexible disks assembly consists of two parts: one is from the elastic disks themselves KE_d and the other from the rigid spindle KE_s

$$KE = KE_d + KE_s \quad (4.5)$$

The motion of the material point on the i^{th} flexible disk is a combination of rigid-body motion of its body-fixed reference frame with respect to the inertia frame and the flexible deformation of the material point with respect to its body-fixed frame. Therefore, the displacement of an arbitrary point A, $\mathbf{R}_a^i$ on the i^{th} disk, as shown in Figure 4.4, can be described as

$$\mathbf{R}_a^i = \mathbf{R}_{oo'} + \mathbf{R}_{o'D_i}^i + \mathbf{r}_a^i + \mathbf{w}_a^i \quad (4.6)$$

where $\mathbf{R}_{oo'}$ is the rigid-body displacement vector of the entire assembly with the assumption that O and O' are coincident initially. $\mathbf{R}_{o'D_i}^i$ is the position vector of the origin of the body-fixed frame of the i^{th} disk D_i with respect to O' . $\mathbf{r}_a^i$ is the position vector of an arbitrary point A with respect to D_i with radial distance r and angular position α measured from the body-fixed axes $x''y''z''$. $\mathbf{w}_a^i$ is the transverse elastic deformation measured from the body-fixed reference frame $x''y''z''$. The vectors can be expressed as follows

$$\mathbf{R}_{oo'} = R_x \mathbf{I} + R_y \mathbf{J} + R_z \mathbf{K} \quad (4.7)$$

$$\mathbf{R}_{o'D_i}^i = z_i \mathbf{k}'' \quad (4.8)$$

$$\mathbf{r}_a^i = r \cos \alpha \mathbf{i}'' + r \sin \alpha \mathbf{j}'' \quad (4.9)$$

$$\mathbf{w}_a^i = w_a^i(r, \alpha, t) \mathbf{k}'' \quad (4.10)$$

Fig. 4.4 Displacement vector of an arbitrary point A on the i^{th} disk

The velocity vector of the point A $\dot{\mathbf{R}}_a^i$ on the i^{th} disk can be obtained by differentiating equation (4.6), therefore

$$\dot{\mathbf{R}}_a^i = \dot{\mathbf{R}}_{oo'} + \dot{\mathbf{R}}_{o'D_i}^i + \dot{\mathbf{r}}_a^i + \dot{\mathbf{w}}_a^i \quad (4.11)$$

where

$$\dot{\mathbf{R}}_{oo'} = \dot{R}_x \mathbf{I} + \dot{R}_y \mathbf{J} + \dot{R}_z \mathbf{K} \quad (4.12)$$

$$\dot{\mathbf{R}}_{o'D_i}^i = z_i \frac{d\mathbf{k}''}{dt} \quad (4.13)$$

$$\dot{\mathbf{r}}_a^i = r \cos \alpha \frac{d\mathbf{i}''}{dt} + r \sin \alpha \frac{d\mathbf{j}''}{dt} \quad (4.14)$$

$$\dot{\mathbf{w}}_a^i = \frac{dw_a^i}{dt} \mathbf{k}'' + w_a^i \frac{d\mathbf{k}''}{dt} \quad (4.15)$$

$$\frac{d\mathbf{i}''}{dt} = \boldsymbol{\omega}'' \times \mathbf{i}'' \quad (4.16)$$

$$\frac{d\mathbf{j}''}{dt} = \boldsymbol{\omega}'' \times \mathbf{j}'' \quad (4.17)$$

$$\frac{d\mathbf{k}''}{dt} = \boldsymbol{\omega}'' \times \mathbf{k}'' \quad (4.18)$$

where the angular velocity of the body-fixed frame $x''y''z''$, $\boldsymbol{\omega}''$, is given by

$$\boldsymbol{\omega}'' = \dot{\theta}_x \mathbf{I} + \dot{\theta}_y \mathbf{j}' + \dot{\theta}_z \mathbf{k}'' \quad (4.19)$$

Substitute the coordinates relations given in equations (4.1) – (4.4), the angular velocity ω'' , expressed in the body-fixed frame $x''y''z''$, can be derived as

$$\omega'' = (\dot{\theta}_x \cos \theta_y \cos \theta_z + \dot{\theta}_y \sin \theta_z) \mathbf{i}'' + (\dot{\theta}_y \cos \theta_z - \dot{\theta}_x \cos \theta_y \sin \theta_z) \mathbf{j}'' + (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y) \mathbf{k}'' \quad (4.20)$$

The velocity vector of the point A $\mathbf{R}_a^i$ on the i^{th} disk can be derived as follows with the substitution of equations (4.12)-(4.20) into equation (4.11),

$$\dot{\mathbf{R}}_a^i = \dot{R}_{ax}^i \mathbf{I} + \dot{R}_{ay}^i \mathbf{J} + \dot{R}_{az}^i \mathbf{K} \quad (4.21)$$

where

$$\begin{aligned} \dot{R}_{ax}^i = & \dot{R}_x + z_i \dot{\theta}_y \cos \theta_y + r \sin \alpha (\sin \theta_y \dot{\theta}_y \sin \theta_z - \cos \theta_y \cos \theta_z \dot{\theta}_z) \\ & + r \cos \alpha (-\dot{\theta}_z \sin \theta_z \cos \theta_y - \cos \theta_z \sin \theta_y \dot{\theta}_y) + w_a^i \dot{\theta}_y \cos \theta_y + \frac{dw_a^i}{dt} \sin \theta_y \end{aligned} \quad (4.22)$$

$$\begin{aligned} \dot{R}_{ay}^i = & \dot{R}_y + z_i (\dot{\theta}_y \sin \theta_x \sin \theta_y - \dot{\theta}_x \cos \theta_y \cos \theta_x) - r \sin \alpha (\sin \theta_x \sin \theta_y \cos \theta_z \dot{\theta}_z \\ & + \dot{\theta}_x \cos \theta_x \sin \theta_z \sin \theta_y + \dot{\theta}_x \sin \theta_x \cos \theta_z + \sin \theta_x \cos \theta_y \sin \theta_z \dot{\theta}_y + \cos \theta_x \sin \theta_z \dot{\theta}_z) \\ & + r \cos \alpha (-\sin \theta_x \sin \theta_y \sin \theta_z \dot{\theta}_z + \dot{\theta}_x \cos \theta_x \cos \theta_z \sin \theta_y - \dot{\theta}_x \sin \theta_x \sin \theta_z \\ & + \sin \theta_x \cos \theta_y \cos \theta_z \dot{\theta}_y + \cos \theta_x \cos \theta_z \dot{\theta}_z) \\ & + w_a^i (\dot{\theta}_y \sin \theta_x \sin \theta_y - \dot{\theta}_x \cos \theta_x \cos \theta_y) - \frac{dw_a^i}{dt} \sin \theta_x \cos \theta_y \end{aligned} \quad (4.23)$$

$$\begin{aligned} \dot{R}_{az}^i = & \dot{R}_z + z_i (-\dot{\theta}_x \cos \theta_y \sin \theta_x - \dot{\theta}_y \cos \theta_x \sin \theta_y) + r \sin \alpha (\cos \theta_x \sin \theta_y \cos \theta_z \dot{\theta}_z \\ & - \dot{\theta}_x \sin \theta_x \sin \theta_z \sin \theta_y + \dot{\theta}_x \cos \theta_x \cos \theta_z + \cos \theta_x \cos \theta_y \sin \theta_z \dot{\theta}_y - \sin \theta_x \sin \theta_z \dot{\theta}_z) \\ & + r \cos \alpha (\cos \theta_x \sin \theta_y \sin \theta_z \dot{\theta}_z + \dot{\theta}_x \sin \theta_x \cos \theta_z \sin \theta_y + \dot{\theta}_x \cos \theta_x \sin \theta_z \\ & - \cos \theta_x \cos \theta_y \cos \theta_z \dot{\theta}_y + \sin \theta_x \cos \theta_z \dot{\theta}_z) \\ & + \frac{dw_a^i}{dt} \cos \theta_x \cos \theta_y - w_a^i (\dot{\theta}_x \sin \theta_x \cos \theta_y + \dot{\theta}_y \cos \theta_x \sin \theta_y) \end{aligned} \quad (4.24)$$

Therefore, the kinetic energy KE_d of N disks is obtained as

$$\begin{aligned}
 KE_d &= \sum_{i=1}^N \frac{1}{2} \int \dot{\mathbf{R}}_a^T \cdot \dot{\mathbf{R}}_a^i dm_i \\
 &= \sum_{i=1}^N \frac{1}{2} \int \left\{ \dot{R}_x^2 + \dot{R}_y^2 + \dot{R}_z^2 + \left(\frac{dw_a^i}{dt} \right)^2 + r^2 (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)^2 \right. \\
 &\quad + r^2 \cos^2 \alpha \dot{\theta}_y^2 + r^2 \sin^2 \alpha \dot{\theta}_x^2 \cos^2 \theta_y + w_a^{i2} (\dot{\theta}_y^2 + \cos^2 \theta_y \dot{\theta}_x^2) + 2z_i w_a^i (\dot{\theta}_y^2 + \cos^2 \theta_y \dot{\theta}_x^2) \\
 &\quad - 2r \cos \alpha (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y) w_a^i \dot{\theta}_x \cos \theta_y - 2r \sin \alpha (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y) w_a^i \dot{\theta}_y \\
 &\quad \left. + 2(\dot{R}_x \sin \theta_y - \dot{R}_y \sin \theta_x \cos \theta_y + \dot{R}_z \cos \theta_x \cos \theta_y + r \sin \alpha \dot{\theta}_x \cos \theta_y - r \cos \alpha \dot{\theta}_y) \frac{dw_a^i}{dt} \right\} dm_i \\
 &\quad (4.25)
 \end{aligned}$$

Following the same procedure, the kinetic energy of the rigid spindle KE_s , with zero elastic deformation, can be derived as follows

$$\begin{aligned}
 KE_s &= \frac{1}{2} \int \dot{\mathbf{R}}_s^T \cdot \dot{\mathbf{R}}_s dm \\
 &= \frac{m_s}{2} (\dot{R}_x^2 + \dot{R}_y^2 + \dot{R}_z^2) + \frac{1}{2} [I_y \dot{\theta}_y^2 + I_x \dot{\theta}_x^2 \cos^2 \theta_y + I_z (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)^2] \\
 &\quad (4.26)
 \end{aligned}$$

where m_s and I_x, I_y, I_z are mass and moments of inertia of the spindle with respect to the origin of the body-fixed frame attached to the spindle, assuming that the spindle is symmetric about z'' .

Therefore, the total kinetic energy of the assembly can be written as

$$\begin{aligned}
 KE &= KE_d + KE_s = \frac{M}{2} (\dot{R}_x^2 + \dot{R}_y^2 + \dot{R}_z^2) + \frac{1}{2} [I_2 \dot{\theta}_y^2 + I_1 \dot{\theta}_x^2 \cos^2 \theta_y + I_3 (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)^2] \\
 &\quad + \sum_{i=1}^N \frac{1}{2} \int \left[\left(\frac{dw_a^i}{dt} \right)^2 + w_a^{i2} (\dot{\theta}_y^2 + \cos^2 \theta_y \dot{\theta}_x^2) + 2z_i w_a^i (\dot{\theta}_x^2 \cos^2 \theta_y + \dot{\theta}_y^2) \right. \\
 &\quad - 2r \cos \alpha (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y) w_a^i \dot{\theta}_x \cos \theta_y - 2r \sin \alpha (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y) w_a^i \dot{\theta}_y \\
 &\quad \left. + 2(\dot{R}_x \sin \theta_y - \dot{R}_y \sin \theta_x \cos \theta_y + \dot{R}_z \cos \theta_x \cos \theta_y + r \sin \alpha \dot{\theta}_x \cos \theta_y - r \cos \alpha \dot{\theta}_y) \frac{dw_a^i}{dt} \right] dm_i \\
 &\quad (4.27)
 \end{aligned}$$

where M is the total mass of the spindle and the disks. I_1, I_2 and I_3 are the diametral and polar moment of inertia of the spindle/disks assembly, respectively and expressed as

$$I_1 = I_x + \int r^2 \sin^2 \alpha dm_i \quad (4.28)$$

$$I_2 = I_y + \int r^2 \cos^2 \alpha dm_i \quad (4.29)$$

$$I_3 = I_z + \int r^2 dm_i \quad (4.30)$$

Potential Energy

Potential energy (PE) of the assembly comes from three parts: the strain energy from the pure bending of the disks, the strain energy due to the centrifugal stresses induced by the rotation of the disks and strain energy due to the deformation of the bearings. Parameters D_e^i , h_i and ν_i are the flexural rigidity, thickness and Poisson's ratio of the i^{th} disk, respectively. The strain energy of the i^{th} disk due to pure bending, denoted by PE_{bi} , is represented by (Wang and Yi, 2000)

$$PE_b^i = \frac{D_e^i}{2} \int_0^{2\pi} \int_{a_i}^{b_i} \left\{ \left(\frac{\partial^2 w_a^i}{\partial r^2} + \frac{1}{r} \frac{\partial w_a^i}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w_a^i}{\partial \alpha^2} \right)^2 \right. \\ \left. - 2(1-\nu_i) \left[\frac{\partial^2 w_a^i}{\partial r^2} \left(\frac{1}{r} \frac{\partial w_a^i}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w_a^i}{\partial \alpha^2} \right) - \left(\frac{1}{r} \frac{\partial^2 w_a^i}{\partial r \partial \alpha} - \frac{1}{r^2} \frac{\partial w_a^i}{\partial \alpha} \right)^2 \right] \right\} r dr d\alpha \quad (4.31)$$

Strain energy due to the centrifugal stresses, denoted by PE_{bi}^* , becomes (Wang and Yi, 2000)

$$PE_b^{i*} = \frac{h_i}{2} \int_0^{2\pi} \int_{a_i}^{b_i} \left[\sigma_{ri} \left(\frac{\partial w_a^i}{\partial r} \right)^2 + \sigma_{\alpha} \left(\frac{1}{r} \frac{\partial w_a^i}{\partial \alpha} \right)^2 \right] r dr d\alpha \quad (4.32)$$

where σ_{ri} and σ_{α} are the in-plane radial and tangential membrane stresses of the i^{th} disk due to the rotation. These membrane stresses greatly affect the disk natural frequencies. They depend strongly on the rotation speed and on the boundary condition at the inner radius, where the collars are used to space the disks.

The deformation of the bearings A and B, $\Delta \mathbf{R}_j$, $j = a, b$, are caused by the rigid-body motion of the rigid spindle as shown in Figure 4.5. Therefore,

$$\Delta \mathbf{R}_j = \mathbf{R}_{oo'} + z_j(\mathbf{k}'' - \mathbf{K}), j = a, b \quad (4.33)$$

Fig. 4.5 Deformation of bearings caused by the rigid-body motion of the rigid spindle

The corresponding potential energy PE_b is

$$\begin{aligned} PE_b &= \sum_{j=a,b} \frac{1}{2} \Delta \mathbf{R}_j^T \cdot \begin{bmatrix} k_{jx} & 0 & 0 \\ 0 & k_{jy} & 0 \\ 0 & 0 & k_{jz} \end{bmatrix} \cdot \Delta \mathbf{R}_j \\ &= \sum_{j=a,b} \left[\frac{1}{2} k_{jx} (R_x + z_j \theta_y)^2 + \frac{1}{2} k_{jy} (R_y - z_j \theta_x)^2 + \frac{1}{2} k_{jz} (R_z + z_j - z_j)^2 \right] \end{aligned} \quad (4.34)$$

Consequently, the total potential energy of the assembly is given by

$$PE = \sum_{i=1}^N (PE_b^i + PE_b^{i*}) + PE_b \quad (4.35)$$

4.2.4 Discretization

The analytical solution of the elastic transverse deformation of the stationary disk can be approximated by the following assumed mode solution

$$w_a^i(r, \alpha, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} w_{mn}^i(r, \alpha) q_{mn}^i(t) \quad (4.36)$$

where w_{mn}^i is the mode shape of the i^{th} stationary disk and expressed as

$$w_{mn}^i = \{R_{mn}^i(r) \cos(n\alpha), R_{mn}^i(r) \sin(n\alpha)\} \quad (4.37)$$

with $R_{mn}^i(r)\cos(n\alpha)$ and $R_{mn}^i(r)\sin(n\alpha)$ representing symmetric and anti-symmetric mode shapes of disks, respectively. $R_{mn}^i(r)$ is an assumed function to be determined by satisfying the inner and outer boundary conditions of the disk. And $q_{mn}^i(t)$ is the modal participation factor to be determined and can be written as

$$q_{mn}^i(t) = \{C_{mn}^i(t), S_{mn}^i(t)\}^T \quad (4.38)$$

with $C_{mn}^i(t)$ and $S_{mn}^i(t)$ corresponding to symmetric and anti-symmetric waves, respectively. m is the number of nodal circles and n is the number of nodal diameters.

Recall that

$$\int_0^{2\pi} \cos \alpha \cos n\alpha d\alpha = \int_0^{2\pi} \sin \alpha \sin n\alpha d\alpha = \begin{cases} 0 & n \neq 1 \\ \pi & n = 1 \end{cases} \quad (4.39)$$

$$\int_0^{2\pi} \sin \alpha \cos n\alpha d\alpha = \int_0^{2\pi} \cos \alpha \sin n\alpha d\alpha = 0 \quad (4.40)$$

Thus, with the assumption that the disks are spinning at a constant speed, that is, $\dot{\theta}_z = \Omega$,

the kinetic energy and potential energy are discretized as

$$\begin{aligned} KE = & \frac{M}{2}(\dot{R}_x^2 + \dot{R}_y^2 + \dot{R}_z^2) + \frac{1}{2}[I_2\dot{\theta}_y^2 + I_1\dot{\theta}_x^2 \cos^2 \theta_y + I_3(\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)^2] \\ & + \frac{\pi}{2} \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \left\{ \sum_{m=0}^{\infty} \dot{C}_{m0}^{i2} R_{m0}^{i2} + \left[\sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (\dot{C}_{mn}^{i2} + \dot{S}_{mn}^{i2}) R_{mn}^{i2} \right] \right\} r dr + (\dot{\theta}_y^2 + \cos^2 \theta_y \dot{\theta}_x^2) \\ & \sum_{i=1}^N \rho_i h_i \left\{ \frac{\pi}{2} \int_{a_i}^{b_i} \left[\sum_{m=0}^{\infty} C_{m0}^{i2} R_{m0}^{i2} + \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (C_{mn}^{i2} + S_{mn}^{i2}) R_{mn}^{i2} \right] r dr + 2\pi \omega_i \int_{a_i}^{b_i} \sum_{m=0}^{\infty} C_{m0}^i R_{m0}^i r dr \right\} \\ & + (\dot{R}_x \sin \theta_y - \dot{R}_y \sin \theta_x \cos \theta_y + \dot{R}_z \cos \theta_x \cos \theta_y) \sum_{i=1}^N 2\pi \rho_i h_i \int_{a_i}^{b_i} \left[\sum_{m=0}^{\infty} \dot{C}_{m0}^i R_{m0}^i \right] r dr \\ & - (\Omega + \dot{\theta}_x \sin \theta_y) \pi \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \left\{ \dot{\theta}_x \cos \theta_y \left[\sum_{m=0}^{\infty} C_{m1}^i R_{m1}^i \right] + \dot{\theta}_y \left[\sum_{m=0}^{\infty} S_{m1}^i R_{m1}^i \right] \right\} r^2 dr \\ & + \pi \dot{\theta}_x \cos \theta_y \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \sum_{m=0}^{\infty} \dot{S}_{m1}^i R_{m1}^i r^2 dr - \pi \dot{\theta}_y \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \left[\sum_{m=0}^{\infty} \dot{C}_{m1}^i R_{m1}^i \right] r^2 dr \end{aligned} \quad (4.41)$$

$$PE = \frac{1}{2} \pi \sum_{i=1}^N \rho_i h_i \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \int_{a_i}^{b_i} R_{mn}^{i2} r dr [(\omega_{mn}^i C_{mn}^i)^2 + (\omega_{mn}^i S_{mn}^i)^2] \\ + \sum_{j=a,b} \left[\frac{1}{2} k_{jx} (R_x^2 + 2R_x z_j \theta_y + z_j^2 \theta_y^2) + \frac{1}{2} k_{jy} (R_y^2 - 2R_y z_j \theta_x + z_j^2 \theta_x^2) + \frac{1}{2} k_{jz} R_z^2 \right] \quad (4.42)$$

where ρ_i is the density of the i^{th} disk. ω_{mn}^i is the natural frequency of flexural mode of the i^{th} disk taking into account the membrane stresses due to the rotation and can be estimated by (Ramaiah, 1981)

$$\omega_{mn}^{i2} = \omega_{st}^{i2} + C\Omega^2 \quad (4.43)$$

with ω_{st}^i corresponds to the frequency of the i^{th} stationary disk and C is a constant depending on the elastic mode, the ratio of a_i / b_i and the boundary condition.

4.2.5 Governing equations of motion

The governing equations of motion are derived as follows using Lagrange's equation

$$\frac{d}{dt} \left(\frac{\partial (KE - PE)}{\partial \dot{q}_k} \right) - \frac{\partial (KE - PE)}{\partial q_k} = Q_k \quad k = 1, 2, \dots, n \quad (4.44)$$

with a linearization assuming $\theta_x \ll 1$ and $\theta_y \ll 1$.

Therefore, equation for R_x is

$$M\ddot{R}_x + k_{ax}(R_x + z_a \theta_y) + k_{bx}(R_x + z_b \theta_y) = F_x \quad (4.45)$$

Equation for R_y is

$$M\ddot{R}_y + k_{ay}(R_y - z_a \theta_x) + k_{by}(R_y - z_b \theta_x) = F_y \quad (4.46)$$

Equation for R_z is

$$M\ddot{R}_z + 2\pi \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \sum_{m=0}^{\infty} \ddot{C}_{m0}^i R_{m0}^i r dr + (k_{az} + k_{bz})R_z = F_z \quad (4.47)$$

Equation for θ_x is

$$I_1 \ddot{\theta}_x + I_3 \Omega \dot{\theta}_y - \pi \Omega \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \dot{C}_{m1}^i + \pi \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \ddot{S}_{m1}^i + z_a k_{ay} (z_a \theta_x - R_y) + z_b k_{by} (z_b \theta_x - R_y) = F_{\theta_x} \quad (4.48)$$

Equation for θ_y is

$$I_2 \ddot{\theta}_y - I_3 \Omega \dot{\theta}_x - \pi \Omega \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \dot{S}_{m1}^i - \pi \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \ddot{C}_{m1}^i + z_a k_{ax} (R_x + z_a \theta_y) + z_b k_{bx} (R_x + z_b \theta_y) = F_{\theta_y} \quad (4.49)$$

Equation for C_{m0}^i of the i^{th} disk is

$$\pi \rho_i h_i \int_{a_i}^{b_i} \ddot{C}_{m0}^i R_{m0}^{i2} r dr + 2\pi \rho_i h_i \ddot{R}_z \int_{a_i}^{b_i} R_{m0}^i r dr + \pi \rho_i h_i \omega_{m0}^{i2} \int_{a_i}^{b_i} R_{m0}^{i2} r dr C_{m0}^i = M_{cm0}^i \quad (4.50)$$

Equation for C_{m1}^i of the i^{th} disk is

$$\pi \rho_i h_i \int_{a_i}^{b_i} \ddot{C}_{m1}^i R_{m1}^{i2} r dr + \pi \rho_i h_i \Omega \int_{a_i}^{b_i} R_{m1}^i r^2 dr \dot{\theta}_x - \pi \rho_i h_i \int_{a_i}^{b_i} R_{m1}^i r^2 dr \ddot{\theta}_y + \pi \rho_i h_i \omega_{m1}^{i2} \int_{a_i}^{b_i} R_{m1}^{i2} r dr C_{m1}^i = M_{cm1}^i \quad (4.51)$$

Equation for $C_{mn}^i, n \geq 2$ of the i^{th} disk is

$$\pi \rho_i h_i \int_{a_i}^{b_i} \ddot{C}_{mn}^i R_{mn}^{i2} r dr + \pi \rho_i h_i \omega_{mn}^{i2} \int_{a_i}^{b_i} R_{mn}^{i2} r dr C_{mn}^i = M_{cmn}^i \quad (4.52)$$

Equation for S_{m1}^i of the i^{th} disk is

$$\pi \rho_i h_i \int_{a_i}^{b_i} R_{m1}^{i2} \ddot{S}_{m1}^i r dr + \pi \rho_i h_i \int_{a_i}^{b_i} R_{m1}^i r^2 dr \ddot{\theta}_x + \pi \rho_i h_i \Omega \int_{a_i}^{b_i} R_{m1}^i r^2 dr \dot{\theta}_y + \pi \rho_i h_i \omega_{m1}^{i2} \int_{a_i}^{b_i} R_{m1}^{i2} r dr S_{m1}^i = M_{sm1}^i \quad (4.53)$$

Equation for $S_{mn}^i, n \geq 2$ of the i^{th} disk is

$$\pi \rho_i h_i \int_{a_i}^{b_i} \ddot{S}_{mn}^i R_{mn}^{i2} r dr + \pi \rho_i h_i \omega_{mn}^{i2} \int_{a_i}^{b_i} R_{mn}^{i2} r dr S_{mn}^i = M_{smn}^i \quad (4.54)$$

The equations of motion can be written in matrix form as follows

$$\begin{bmatrix} \mathbf{M}_r & \mathbf{M}_{rd} \\ \mathbf{M}_{dr} & \mathbf{M}_d \end{bmatrix} \{\ddot{\mathbf{q}}\} + \begin{bmatrix} \mathbf{G}_r & \mathbf{G}_{rd} \\ \mathbf{G}_{dr} & \mathbf{0} \end{bmatrix} \{\dot{\mathbf{q}}\} + \begin{bmatrix} \mathbf{K}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_d \end{bmatrix} \{\mathbf{q}\} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{M} \end{Bmatrix} \quad (4.55)$$

where $\mathbf{M}_r$ is the diagonal inertial matrix of the rigid-body motion of the assembly. $\mathbf{M}_d$ is the inertial matrix due to the flexible vibration of disks. $\mathbf{M}_{rd}$ and $\mathbf{M}_{dr}$ represents the inertial coupling between the rigid body motion and the flexible vibration of the disk. $\mathbf{G}_r$ is skew gyroscopic matrix and comes from the rigid body motion. $\mathbf{G}_{rd}$ and $\mathbf{G}_{dr}$ represents the gyroscopic coupling between the rigid body motion and the vibration of rotating disks. $\mathbf{K}_d$ and $\mathbf{K}_b$ are stiffness matrices of disks and bearings, respectively. $\{\mathbf{q}\}$ is the generalized coordinate vector with the definition of

$$\mathbf{q} = \{R_x \quad R_y \quad R_z \quad \theta_x \quad \theta_y \quad C_{m0}^i \quad C_{m1}^i \quad C_{mn}^i \quad S_{m1}^i \quad S_{mn}^i\}^T \quad n \geq 2 \quad (4.56)$$

The first five coordinates correspond to the rigid-body motion of the spindle and the rest the displacement modal participation factors of the disks. The same goes to the generalized force vector, that is, the first five coordinates correspond to the forces acting on the spindle $\{\mathbf{F}\}$ and the rest the forces on the disks $\{\mathbf{M}\}$ and defined as follows,

$$\mathbf{F} = \{F_x \quad F_y \quad F_z \quad F_{\theta_x} \quad F_{\theta_y}\}^T \quad (4.57)$$

$$\mathbf{M} = \{M_{cm0}^i \quad M_{cm1}^i \quad M_{cmn}^i \quad M_{sm1}^i \quad M_{smn}^i\}^T \quad n \geq 2 \quad (4.58)$$

The details of the coefficient matrices are described as follows for the case of one disk

$$\mathbf{M}_r = \begin{bmatrix} M & 0 & 0 & 0 & 0 \\ 0 & M & 0 & 0 & 0 \\ 0 & 0 & M & 0 & 0 \\ 0 & 0 & 0 & I_1 & 0 \\ 0 & 0 & 0 & 0 & I_2 \end{bmatrix} \quad (4.59)$$

$$\mathbf{M}_{rd} = \pi \rho_1 h_1 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \int_{a_i}^{b_i} \sum_{m=0}^{\infty} R_{m0}^1 r dr & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \int_{a_i}^{b_i} \sum_{m=0}^{\infty} R_{m1}^1 r^2 dr \\ 0 & 0 & - \int_{a_i}^{b_i} \sum_{m=0}^{\infty} R_{m1}^1 r^2 dr & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.60)$$

$$\mathbf{M}_{dr} = \pi \rho_1 h_1 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \int_{a_i}^{b_i} R_{m0}^1 r dr & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \int_{a_i}^{b_i} R_{m1}^1 r^2 dr \\ 0 & 0 & - \int_{a_i}^{b_i} R_{m1}^1 r^2 dr & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.61)$$

$$\mathbf{G}_r = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_3 \Omega \\ 0 & 0 & 0 & -I_3 \Omega & 0 \end{bmatrix} \quad (4.62)$$

$$\mathbf{G}_{rd} = \pi \rho_1 h_1 \Omega \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & - \int_{a_i}^{b_i} \sum_{m=0}^{\infty} R_{m1}^1 r^2 dr & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & - \int_{a_i}^{b_i} \sum_{m=0}^{\infty} R_{m1}^1 r^2 dr \end{bmatrix} \quad (4.63)$$

$$\mathbf{G}_{dr} = \pi \rho_1 h_1 \Omega \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \int_{a_i}^{b_i} R_{m1}^1 r^2 dr & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \int_{a_i}^{b_i} R_{m1}^1 r^2 dr \end{bmatrix} \quad (4.64)$$

$$\mathbf{M}_d = \pi \rho_1 h_1 \left[\text{diag} \left[\int_{a_i}^{b_i} (R_{mn}^1)^2 r dr \right] \right]_{7 \times 7} \quad (4.65)$$

$$\mathbf{K}_d = \pi \rho_1 h_1 \left[\text{diag} \left[(\omega_{mn}^1)^2 \int_{a_i}^{b_i} (R_{mn}^1)^2 r dr \right] \right]_{7 \times 7} \quad (4.66)$$

$$\mathbf{K}_b = \begin{bmatrix} k_{ax} + k_{bx} & 0 & 0 & 0 & k_{ax} z_a + k_{bx} z_b \\ 0 & k_{ay} + k_{by} & 0 & -k_{ay} z_a - k_{by} z_b & 0 \\ 0 & 0 & k_{az} + k_{bz} & 0 & 0 \\ 0 & -z_a k_{ay} - z_b k_{by} & 0 & z_a k_{ay} z_a + z_b k_{by} z_b & 0 \\ z_a k_{ax} + z_b k_{bx} & 0 & 0 & 0 & z_a k_{ax} z_a + z_b k_{bx} z_b \end{bmatrix} \quad (4.67)$$

with the generalized displacement vector taking the following form

$$\mathbf{q} = \{R_x \ R_y \ R_z \ \theta_x \ \theta_y \ C_{m2}^1 \ C_{m3}^1 \ C_{m1}^1 \ S_{m2}^1 \ C_{m0}^1 \ S_{m3}^1 \ S_{m1}^1\}^T \quad (4.68)$$

4.2.6 Governing equations expressed in the rocking frame

To better understand the mode splitting phenomenon in vibration of the rotating disks, there is a need to express the elastic deformation of the disk in the intermediate rocking frame $x'y'z'$ which is a non-spinning frame in the undeformed plane of the disk, rather than in the frame $x''y''z''$ which is body-fixed to the spinning disk.

Fig. 4.6 Position vector of point A determined in different reference frames
 (1) θ_x about $X \rightarrow xyz$ (2) θ_y about y axis $\rightarrow x'y'z'$ (3) θ_z about z' axis $\rightarrow x''y''z''$

The location of an arbitrary point A on the i^{th} disk as illustrated in Figure 4.6 can also be determined in the coordinates $x'y'z'$ with the same radial distance r as in frame $x''y''z''$ but different angular position θ measured from the frame $x'y'z'$. The angular relation between frame $x''y''z''$ and frame $x'y'z'$ can be described as follows

$$\theta = \alpha + \Omega t \quad (4.69)$$

Therefore, the assumed mode solution of the elastic transverse deformation of the disk expressed in the non-spinning frame $x'y'z'$ can be derived as follows by substituting equations (4.37), (4.38) and (4.69) into (4.36)

$$\begin{aligned} w_a^i(r, \alpha, t) &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} w_{mn}^i(r, \alpha) q_{mn}^i(t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} [R_{mn}^i(r) \cos(n\alpha) C_{mn}^i(t) + R_{mn}^i(r) \sin(n\alpha) S_{mn}^i(t)] \\ &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} [R_{mn}^i(r) \cos(n\theta - n\Omega t) C_{mn}^i(t) + R_{mn}^i(r) \sin(n\theta - n\Omega t) S_{mn}^i(t)] \\ &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \{ R_{mn}^i(r) \cos n\theta [\cos n\Omega t C_{mn}^i(t) - \sin n\Omega t S_{mn}^i(t)] \\ &\quad + R_{mn}^i(r) \sin n\theta [\cos n\Omega t S_{mn}^i(t) + \sin n\Omega t C_{mn}^i(t)] \} \\ &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \{ R_{mn}^i(r) \cos n\theta \bar{C}_{mn}^i(t) + R_{mn}^i(r) \sin n\theta \bar{S}_{mn}^i(t) \} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} w_{mn}^i(r, \theta) \bar{q}_{mn}^i(t) = w_a^i(r, \theta, t) \end{aligned} \quad (4.70)$$

The relation between the displacement modal participation factors of the disks expressed in the non-spinning frame $\bar{q}_{mn}^i = \{\bar{C}_{mn}^i, \bar{S}_{mn}^i\}^T$ and those expressed in the frame attached to the spinning disk $q_{mn}^i = \{C_{mn}^i, S_{mn}^i\}^T$ can be obtained as follows

$$\begin{Bmatrix} \bar{C}_{mn}^i \\ \bar{S}_{mn}^i \end{Bmatrix} = \begin{bmatrix} \cos n\Omega t & -\sin n\Omega t \\ \sin n\Omega t & \cos n\Omega t \end{bmatrix} \begin{Bmatrix} C_{mn}^i \\ S_{mn}^i \end{Bmatrix} \quad (4.71)$$

Substituting equation (4.71) into equation (4.55), the governing equations of motion in the rocking frame becomes

$$\begin{bmatrix} \mathbf{M}_r & \mathbf{M}_{rd} \\ \mathbf{M}_{dr} & \mathbf{M}_d \end{bmatrix} \{\ddot{\bar{\mathbf{q}}}\} + \begin{bmatrix} \mathbf{G}_r & \mathbf{G}'_{rd} \\ \mathbf{G}'_{dr} & \mathbf{G}_d \end{bmatrix} \{\dot{\bar{\mathbf{q}}}\} + \begin{bmatrix} \mathbf{K}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{K}'_d \end{bmatrix} \{\bar{\mathbf{q}}\} = \begin{Bmatrix} \bar{\mathbf{F}} \\ \bar{\mathbf{M}} \end{Bmatrix} \quad (4.72)$$

with the generalized displacement vector taking the following form

$$\bar{\mathbf{q}} = \{R_x \ R_y \ R_z \ \theta_x \ \theta_y \ \bar{C}_{m2}^{-1} \ \bar{C}_{m3}^{-1} \ \bar{C}_{m1}^{-1} \ \bar{S}_{m2}^{-1} \ \bar{C}_{m0}^{-1} \ \bar{S}_{m3}^{-1} \ \bar{S}_{m1}^{-1}\}^T \quad (4.73)$$

By comparing equation (4.72) with equation (4.55), one may see that when the elastic deformation of the disks is expressed in rocking frame $x'y'z'$, a skew gyroscopic matrix $\mathbf{G}_d$ due to vibration of the disks appears. The stiffness matrix of the disk $\mathbf{K}_d$, the gyroscopic coupling matrices $\mathbf{G}_{rd}$ and $\mathbf{G}_{dr}$ have changed to $\mathbf{K}'_d$, $\mathbf{G}'_{rd}$ and $\mathbf{G}'_{dr}$ to account for the effect due to disk rotation in this case. The details of these matrices are described as follows for the case of one disk with the modal coordinates of the disk taking the same form as in equation (4.73)

$$\mathbf{G}_d = \pi\rho_1 h_1 \int_{a_i}^{b_i} (R_{mn}^1)^2 r dr \times \begin{bmatrix} \mathbf{0} & \text{diag}(2n\Omega) \\ \text{diag}(-2n\Omega) & \mathbf{0} \end{bmatrix}_{7 \times 7} \quad (4.74)$$

$$\mathbf{K}'_d = \pi\rho_1 h_1 \left[\text{diag}\{[(\omega_{mn}^1)^2 - n^2\Omega^2]\} \int_{a_i}^{b_i} (R_{mn}^1)^2 r dr \right]_{7 \times 7} \quad (4.75)$$

$$\mathbf{G}'_{rd} = \pi \rho_1 h_1 \Omega \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 \int_{a_i}^{b_i} \sum_{m=0}^{\infty} R_{m1}^1 r^2 dr & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 \int_{a_i}^{b_i} \sum_{m=0}^{\infty} R_{m1}^1 r^2 dr \end{bmatrix} \quad (4.76)$$

$$\mathbf{G}'_{dr} = \pi \rho_1 h_1 \Omega \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 \int_{a_i}^{b_i} R_{m1}^1 r^2 dr & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \int_{a_i}^{b_i} R_{m1}^1 r^2 dr \end{bmatrix} \quad (4.77)$$

4.2.7 Free vibration analysis

A Quantum Fireball HDD with two disks and three heads is used as a sample in this section. The parameters of the disks are listed in Table 4.1 with natural frequencies of the disks calculated using finite element analysis software ANSYS and listed in Table 4.2.

The calculated natural frequencies of the spindle/disks assembly are then plotted in Figure 4.7 together with the experimental results obtained by measuring transmissibility functions for the rotation speed up to 7200 RPM. It may be seen that the calculated results agree well with the experimental ones. In the graph, $B(m, n)$ and $U(m, n)$ represent the balanced and unbalanced modes which have m nodal circles and n nodal diameters. Subscript b and f denote the backward and forward waves.

<i>Parameters</i>	<i>Disk</i>
Outer Diameter (<i>mm</i>)	95
Inner Diameter (<i>mm</i>)	31
Thickness (<i>mm</i>)	0.8
Density (<i>kg/m³</i>)	2800
Young's Modulus (<i>GPa</i>)	69
Poisson's Ratio	0.3

Table 4.1 Parameters of the Quantum HDD disk

<i>Mode</i>	<i>Numerical results (Hz)</i>	<i>Experiment results (Hz)</i>
(0, 0)	608.49	602.25
(0, 1)	604.22	592.5
(0, 2)	720.79	706.25

Table 4.2 Calculated and experimentally derived natural frequencies

Fig. 4.7 Comparison of natural frequencies between numerical and experimental results (—: numerical results •: experimental results)

Fig. 4.8 Balanced and unbalanced modes of the spindle/disks assembly with multiple flexible disks (a) Balanced (0, 0) modes and the rigid spindle in no motion
(b) Unbalanced (0, 0) modes and the rigid spindle in axial motion
(c) Balanced (0, 1) modes and the rigid spindle in no motion
(d) Unbalanced (0, 1) modes and the rigid spindle in rotation (Park, 2003)

From equations (4.45)-(4.54) derived, one may see that axial translation of the spindle and the axisymmetric modes of the disk are coupled and the rigid-body rotation of the

spindle and the disk vibration modes with one nodal diameter are also coupled. The coupled vibration of the spindle/disks assembly splits into balanced modes and unbalanced modes when all the disks are identical (Shen and Ku, 1997). In a balanced mode, two adjacent disks vibrate entirely out of phase, whereas the other disks undergo no deformation. The inertia forces of the two deformed disks are completely balanced and the orientation of the spindle remains fixed in space (Shen, 2000). In an unbalanced mode, all the disks vibrate with the same phase and the inertia force from the left half of the disk is entirely equal and opposite to that from the right half of the disk caused by the disk deformation. These two inertia forces create an unbalanced moment. Because the spindle/disks assembly is spinning with an angular momentum, the presence of the unbalanced moment causes a steady precession of the spindle (Shen, 2000). Figure 4.8 schematically shows some of the balanced and unbalanced modes.

It is illustrated in Figure 4.7 that balanced and unbalanced (0, 1) modes are observed. Take a closer view of the frequency range from 580-650 Hz as shown in Figure 4.9, it is clear that balanced and unbalanced (0, 0) modes appear. It should also be noted that natural frequencies of unbalanced modes are lower than those of the corresponding balanced ones.

Fig. 4.9 Closer view of balanced and unbalanced (0, 0) modes

It can also be observed from equations (4.45) – (4.54) that the rigid-body motion of the spindle does not affect the disk vibration modes with more than one nodal diameter. Therefore, only balanced (0, 2) modes appear in Figure 4.7. It can also be shown from Figure 4.7 that mode splitting phenomenon has occurred for all modes with one or more nodal diameters, no matter the modes are balanced or unbalanced. This is true observing from the non-spinning rocking frame.

Experiment validation by using an impact hammer

To validate the numerical model developed, an experiment by using an impact hammer was first performed. The experiment set-up is shown in Figure 4.10. The HDD was fixed on the vibration isolation table after the top cover was removed. A small disturbance was applied to the top disk through an impact hammer. The velocity of the top disk was measured by using a Laser Doppler Vibrometer (LDV) and its frequency spectrum was obtained through dynamic signal analyzer HP35670A. A HC5802 Motor Driver together with a power supply was used to rotate the disks at different rotational speeds. Experimental results are plotted in Figure 4.11.

It may be seen from Figure 4.11 that harmonics of rotational speed are clearly shown while the disk vibration modes are not easily identified. The reason may be that there is not sufficient energy to excite all the disk modes by using an impact hammer. Moreover, it is also not easy to manually control the action point and force quality of the impact hammer.

(a)

(b)

(c)

Fig. 4.10 Experiment set-up by using an impact hammer (a) Measurement set-up (b) HP dynamic signal analyzer (c) For rotating disks at different rotational speed

Fig. 4.11 Waterfall plot obtained by using an impact hammer

Experiment validation by using a shaker

An alternative approach was used to investigate the vibration modes. Transmissibility function measurements were performed. The experiment set-up is shown in Figure 4.12. The HDD with the top cover removed was fixed on the shaker through a head expander. A sweeping sinusoidal signal was produced by a Nicolet dynamic signal generator and analyzer and applied to the shaker to excite the base of the HDD. The velocity of the top disk and the velocity of the base were measured using a Polytec LDV and B&K accelerometer, respectively. The transmissibility ratio of disk velocity to base velocity was obtained. A HC5802 Motor Driver together with a power supply was used to spin the disks at different speeds. Figure 4.13 gives the transmissibility functions of rotational speeds at 3600 and 7200 RPM.

(a)

(b)

(c)

Fig. 4.12 The experiment set-up of vibration of the spindle/disks assembly
 (a) For measurement and post-processing (b) Closer view of the shaker with head expander (c) For rotating disks at different rotational speed

Fig. 4.13 Transmissibility functions of disk rotating at 3600 and 7200 RPM

Transmissibility method measurement indicates the relative vibration levels between the disk and the base. The disk vibration modes are much easily excited because of the sweeping sinusoidal signal as excitation compared with an impact hammer as excitation.

4.2.8 Non-constant rotation speed case

Till now, the analysis assumes that the disk is spinning with a constant angular speed Ω . However, due to the current ripple induced by the PWM control and the commutation torque ripple created by the commutation of current from one phase to the other, the driving torque generated is not a perfect one. Therefore, in practice, the rotation speed of the disk is not a constant. In this case, $\dot{\theta}_z \neq \Omega$. Following the same procedure as in section 4.2.3, 4.2.4, 4.2.5 and 4.2.6, the kinetic energy and potential energy of the spindle/disks assembly, with the elastic deformation of the disk expressed in frame $x'y'z'$, can be discretized as

$$\begin{aligned}
 KE = & \frac{M}{2} (\dot{R}_x^2 + \dot{R}_y^2 + \dot{R}_z^2) + \frac{1}{2} [I_2 \dot{\theta}_y^2 + I_1 \dot{\theta}_x^2 \cos^2 \theta_y + I_3 (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)^2] \\
 & + \frac{\pi}{2} \sum_{i=1}^N \rho_i h_i \left\{ \int_{a_i}^{b_i} \sum_{m=0}^{\infty} \bar{C}_{m0}^{i2} R_{m0}^{i2} r dr + \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (\bar{C}_{mn}^{i2} + \bar{S}_{mn}^{i2}) R_{mn}^{i2} r dr + n^2 \dot{\theta}_z^2 \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (\bar{C}_{mn}^{i2} + \bar{S}_{mn}^{i2}) R_{mn}^{i2} r dr \right. \\
 & + 2n \dot{\theta}_z \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (\bar{C}_{mn}^i \bar{S}_{mn}^i - \bar{S}_{mn}^i \bar{C}_{mn}^i) R_{mn}^{i2} r dr \left. \right\} + (\dot{\theta}_y^2 + \cos^2 \theta_y \dot{\theta}_x^2) \\
 & \sum_{i=1}^N \rho_i h_i \frac{\pi}{2} \left\{ \int_{a_i}^{b_i} \left[\sum_{m=0}^{\infty} \bar{C}_{m0}^{i2} R_{m0}^{i2} + \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (\bar{C}_{mn}^{i2} + \bar{S}_{mn}^{i2}) R_{mn}^{i2} \right] r dr + 2\pi \dot{\theta}_z \int_{a_i}^{b_i} \sum_{m=0}^{\infty} \bar{C}_{m0}^i R_{m0}^i r dr \right\} \\
 & + (\dot{R}_x \sin \theta_y - \dot{R}_y \sin \theta_x \cos \theta_y + \dot{R}_z \cos \theta_x \cos \theta_y) \sum_{i=1}^N 2\pi \rho_i h_i \int_{a_i}^{b_i} \left[\sum_{m=0}^{\infty} \bar{C}_{m0}^i R_{m0}^i \right] r dr \\
 & - (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y) \pi \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \left\{ \dot{\theta}_x \cos \theta_y \left[\sum_{m=0}^{\infty} \bar{C}_{m1}^i R_{m1}^i \right] + \dot{\theta}_y \left[\sum_{m=0}^{\infty} \bar{S}_{m1}^i R_{m1}^i \right] \right\} r^2 dr \\
 & + \pi \dot{\theta}_x \cos \theta_y \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \sum_{m=0}^{\infty} (\bar{S}_{m1}^i - \dot{\theta}_z \bar{C}_{m1}^i) R_{m1}^i r^2 dr - \pi \dot{\theta}_y \sum_{i=1}^N \rho_i h_i \int_{a_i}^{b_i} \left[\sum_{m=0}^{\infty} (\bar{C}_{m1}^i + \dot{\theta}_z \bar{S}_{m1}^i) R_{m1}^i \right] r^2 dr
 \end{aligned} \tag{4.78}$$

$$\begin{aligned}
 PE = & \frac{1}{2} \pi \sum_{i=1}^N \rho_i h_i \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \int_a^b R_{mn}^{i2} r dr [(\omega_{mn}^i C_{mn}^i)^2 + (\omega_{mn}^i S_{mn}^i)^2] \\
 & + \sum_{j=a,b} \left[\frac{1}{2} k_{jx} (R_x^2 + 2R_x z_j \theta_y + z_j^2 \theta_y^2) + \frac{1}{2} k_{jy} (R_y^2 - 2R_y z_j \theta_x + z_j^2 \theta_x^2) + \frac{1}{2} k_{jz} R_z^2 \right]
 \end{aligned} \tag{4.79}$$

Therefore, the governing equations of motion are derived as follows using Lagrange's equations

Equation for R_x is

$$M\ddot{R}_x + k_{ax}(R_x + z_a \theta_y) + k_{bx}(R_x + z_b \theta_y) = \bar{F}_x \tag{4.80}$$

Equation for R_y is

$$M\ddot{R}_y + k_{ay}(R_y - z_a \theta_x) + k_{by}(R_y - z_b \theta_x) = \bar{F}_y \tag{4.81}$$

Equation for R_z is

$$M\ddot{R}_z + 2\pi \sum_{i=1}^N \rho_i h_i \int_a^b \sum_{m=0}^{\infty} \ddot{\bar{C}}_{m0}^i R_{m0}^i r dr + (k_{az} + k_{bz}) R_z = \bar{F}_z \quad (4.82)$$

Equation for θ_x is

$$\begin{aligned} I_1 \ddot{\theta}_x + I_3 \dot{\theta}_y \dot{\theta}_z + I_3 \theta_y \ddot{\theta}_z - 2\pi \dot{\theta}_z \sum_{i=1}^N \rho_i h_i \int_a^b \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \dot{\bar{C}}_{m1}^i - 2\pi \ddot{\theta}_z \sum_{i=1}^N \rho_i h_i \int_a^b \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \bar{C}_{m1}^i \\ + \pi \sum_{i=1}^N \rho_i h_i \int_a^b \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \ddot{\bar{S}}_{m1}^i + z_a k_{ay} (z_a \theta_x - R_y) + z_b k_{by} (z_b \theta_x - R_y) = \bar{F}_{\theta_x} \end{aligned} \quad (4.83)$$

Equation for θ_y is

$$\begin{aligned} I_1 \ddot{\theta}_y - I_3 \dot{\theta}_x \dot{\theta}_z - 2\pi \dot{\theta}_z \sum_{i=1}^N \rho_i h_i \int_a^b \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \dot{\bar{S}}_{m1}^i - 2\pi \ddot{\theta}_z \sum_{i=1}^N \rho_i h_i \int_a^b \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \bar{S}_{m1}^i \\ - \pi \sum_{i=1}^N \rho_i h_i \int_a^b \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \ddot{\bar{C}}_{m1}^i + z_a k_{ax} (R_x + z_a \theta_y) + z_b k_{bx} (R_x + z_b \theta_y) = \bar{F}_{\theta_y} \end{aligned} \quad (4.84)$$

Equation for θ_z is

$$\begin{aligned} I_3 \ddot{\theta}_z + I_3 \ddot{\theta}_x \theta_y + I_3 \dot{\theta}_x \dot{\theta}_y - 2\pi \ddot{\theta}_y \sum_{i=1}^N \rho_i h_i \int_a^b \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \bar{S}_{m1}^i - 2\pi \dot{\theta}_y \sum_{i=1}^N \rho_i h_i \int_a^b \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \dot{\bar{S}}_{m1}^i \\ - 2\pi \ddot{\theta}_x \sum_{i=1}^N \rho_i h_i \int_a^b \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \bar{C}_{m1}^i - 2\pi \dot{\theta}_x \sum_{i=1}^N \rho_i h_i \int_a^b \sum_{m=0}^{\infty} R_{m1}^i r^2 dr \dot{\bar{C}}_{m1}^i \\ + \pi \sum_{i=1}^N \rho_i h_i \int_a^b n^2 \ddot{\theta}_z \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (\bar{C}_{mn}^{i2} + \bar{S}_{mn}^{i2}) R_{mn}^{i2} + \pi \sum_{i=1}^N \rho_i h_i \int_a^b n^2 \dot{\theta}_z \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (2\bar{C}_{mn}^i \dot{\bar{C}}_{mn}^i + 2\bar{S}_{mn}^i \dot{\bar{S}}_{mn}^i) R_{mn}^2 \\ + n \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (\ddot{\bar{C}}_{mn}^i \bar{S}_{mn}^i R_{mn}^{i2} - \ddot{\bar{S}}_{mn}^i \bar{C}_{mn}^i R_{mn}^{i2}) r dr = \bar{F}_{\theta_z} \end{aligned} \quad (4.85)$$

Equation for $\bar{C}_{m0}^i$ of the i^{th} disk is

$$\pi \rho_i h_i \int_a^b \ddot{\bar{C}}_{m0}^i R_{m0}^{i2} r dr + 2\pi \rho_i h_i \ddot{R}_z \int_a^b R_{m0}^i r dr + \pi \rho_i h_i \omega_{m0}^{i2} \int_a^b R_{m0}^{i2} r dr \bar{C}_{m0}^i = \bar{M}_{cm0}^i \quad (4.86)$$

Equation for $\bar{C}_{m1}^i$ of the i^{th} disk is

$$\begin{aligned} \pi \rho_i h_i \int_a^b \{ \ddot{\bar{C}}_{m1}^i R_{m1}^{i2} - \dot{\theta}_z^2 \bar{C}_{m1}^i R_{m1}^{i2} + \ddot{\theta}_z \bar{S}_{m1}^i R_{m1}^{i2} + 2 \dot{\theta}_z \dot{\bar{S}}_{m1}^i R_{m1}^{i2} \} r dr \\ + 2 \pi \rho_i h_i \dot{\theta}_z \int_a^b R_{m1}^i r^2 dr \dot{\theta}_x - \pi \rho_i h_i \int_a^b R_{m1}^i r^2 dr \ddot{\theta}_y + \pi \rho_i h_i \omega_{m1}^{i2} \int_a^b R_{m1}^{i2} r dr \bar{C}_{m1}^i = \bar{M}_{cm1}^i \end{aligned} \quad (4.87)$$

Equation for $\bar{C}_{mn}^i, n \geq 2$ of the i^{th} disk is

$$\pi \rho_i h_i \int_a^b \{ \ddot{\bar{C}}_{mn}^i R_{mn}^{i2} - n^2 \dot{\theta}_z^2 \bar{C}_{mn}^i R_{mn}^{i2} + n \ddot{\theta}_z \bar{S}_{mn}^i R_{mn}^{i2} + 2n \dot{\theta}_z \dot{\bar{S}}_{mn}^i R_{mn}^{i2} \} r dr + \pi \rho_i h_i \omega_{mn}^{i2} \int_a^b R_{mn}^{i2} r dr \bar{C}_{mn}^i = \bar{M}_{cmn}^i \quad (4.88)$$

Equation for $\bar{S}_{m1}^i$ of the i^{th} disk is

$$\begin{aligned} \pi \rho_i h_i \int_a^b \{ R_{m1}^{i2} \ddot{\bar{S}}_{m1}^i - \dot{\theta}_z^2 R_{m1}^{i2} \bar{S}_{m1}^i - 2 \dot{\theta}_z R_{m1}^{i2} \dot{\bar{C}}_{m1}^i - \ddot{\theta}_z R_{m1}^{i2} \bar{C}_{m1}^i \} r dr \\ + \pi \rho_i h_i \int_a^b R_{m1}^i r^2 dr \ddot{\theta}_x + 2 \pi \rho_i h_i \dot{\theta}_z \int_a^b R_{m1}^i r^2 dr \dot{\theta}_y + \pi \rho_i h_i \omega_{m1}^{i2} \int_a^b R_{m1}^{i2} r dr \bar{S}_{m1}^i = \bar{M}_{sm1}^i \end{aligned} \quad (4.89)$$

Equation for $\bar{S}_{mn}^i, n \geq 2$ of the i^{th} disk is

$$\pi \rho_i h_i \int_a^b \{ \ddot{\bar{S}}_{mn}^i R_{mn}^{i2} - n^2 \dot{\theta}_z^2 \bar{S}_{mn}^i R_{mn}^{i2} - 2n \dot{\theta}_z \dot{\bar{C}}_{mn}^i R_{mn}^{i2} - n \ddot{\theta}_z \bar{C}_{mn}^i R_{mn}^{i2} \} r dr + \pi \rho_i h_i \omega_{mn}^{i2} \int_a^b R_{mn}^{i2} r dr \bar{S}_{mn}^i = \bar{M}_{smn}^i \quad (4.90)$$

It should be noted that the equations for the case of non-constant rotation speed of the disk can be reduced to those for the constant rotation speed case by substituting $\dot{\theta}_z = \Omega$.

4.3 Dynamic analysis of spindle/rotating disks assembly-shaft-housing system

The analysis is now extended to a spindle/disks assembly connected to the flexible stator, the top cover and the base plate. For the convenience of formulation, this system is divided into three main parts: rotating part, ball bearings, and stationary part, as shown in Figure 4.14. The rotating part is the spindle/disks assembly studied in the previous sections. The stationary part is an assembly of the flexible shaft (stator) and the flexible housing (top cover and base plate). The flexible shaft is attached at both ends to the top cover and the base plate.

Fig. 4.14 The schematic of the spindle/disks assembly-shaft-housing system

4.3.1 Formulation

Formulation of the rotating part

The rotating part consists of a rigid spindle carrying N flexible disks. Referring to the derivations in the previous sections, the kinetic energy of the rotating part, which comes from the elastic vibration of rotating disks expressed in frame $x'y'z'$ and rigid body motion of the spindle, can be derived as follows from equation (4.72)

$$\begin{aligned}
 KE_{rotating} = & \frac{1}{2} \{ \dot{\mathbf{r}}_r^T \mathbf{M}_r \dot{\mathbf{r}}_r + \dot{\mathbf{r}}_r^T \mathbf{G}_r \mathbf{r}_r \} + \sum_N \frac{1}{2} \{ \dot{\mathbf{r}}_r^T \mathbf{M}_{rd} \dot{\mathbf{r}}_d + \dot{\mathbf{r}}_r^T \mathbf{G}'_{rd} \mathbf{r}_d \} \\
 & + \sum_N \frac{1}{2} \{ \dot{\mathbf{r}}_d^T \mathbf{M}_d \dot{\mathbf{r}}_d + \dot{\mathbf{r}}_d^T \mathbf{M}_{dr} \dot{\mathbf{r}}_r + \dot{\mathbf{r}}_d^T \mathbf{G}'_{dr} \mathbf{r}_r + \dot{\mathbf{r}}_d^T \mathbf{G}_d \mathbf{r}_d \}
 \end{aligned} \tag{4.91}$$

where $\mathbf{M}_r$ and $\mathbf{M}_d$ are the mass matrices of the rotating part and the disks, respectively.

$\mathbf{M}_{rd}$ and $\mathbf{M}_{dr}$ are the mass coupling matrices between the spindle and the disks.

$\mathbf{G}_r$ and $\mathbf{G}_d$ are the gyroscopic matrices of the rotating part and the disks, respectively.

$\mathbf{G}'_{rd}$ and $\mathbf{G}'_{dr}$ are the gyroscopic coupling matrices between the spindle and the disks. $\mathbf{r}_r$

is defined as the displacement vector of the spindle and has the form of

$$\mathbf{r}_r = \{R_x \ R_y \ R_z \ \theta_x \ \theta_y\}^T \quad (4.92)$$

and $\mathbf{r}_d$ is the displacement modal participation factors vector of the disk taking the

following form

$$\mathbf{r}_d = \{\bar{C}_{m2}^1 \ \bar{C}_{m3}^1 \ \bar{C}_{m1}^1 \ \bar{S}_{m2}^1 \ \bar{C}_{m0}^1 \ \bar{S}_{m3}^1 \ \bar{S}_{m1}^1\}^T \quad (4.93)$$

The potential energy of the rotating part includes only the flexible disks and is given by

$$PE_{rotating} = \sum_N \frac{1}{2} \mathbf{r}_d^T \mathbf{K}'_d \mathbf{r}_d \quad (4.94)$$

where $\mathbf{K}'_d$ is the stiffness matrix of the disk. The values of these matrices can be found from equations (4.59) - (4.65), (4.74) - (4.77) in the previous sections for the one disk case.

Formulation of the stationary part

The stationary part is modeled as an elastic assembly including the flexible top cover, flexible base plate and flexible shaft as shown in Figure 4.15.

Fig. 4.15 The model of the stationary part

Let $\mathbf{w}_s$ be the elastic deformation of an arbitrary point of the stationary part. This can be approximated in terms of n_s vibration modes of the stationary part as

$$\mathbf{w}_s = \sum_{n=1}^{n_s} \mathbf{w}_{sn} r_{sn} \quad (4.95)$$

where $\mathbf{w}_{sn}$ is the n^{th} vibration mode shape of the stationary part and takes the following form

$$\mathbf{w}_{sn} = w_{sxt} \mathbf{I} + w_{syn} \mathbf{J} + w_{szr} \mathbf{K} \quad (4.96)$$

and r_{sn} is the displacement modal participation factor.

In addition, the mode shapes satisfy the orthonormality condition

$$\int \mathbf{w}_{sm} \cdot \mathbf{w}_{sn} dm = \delta_{mn} \quad (4.97)$$

where δ_{mn} is the Kronecker delta.

Thus, the kinetic energy and potential energy of the stationary part are given by

$$KE_{stationary} = \frac{1}{2} \dot{\mathbf{r}}_s^T \mathbf{M}_s \dot{\mathbf{r}}_s \quad (4.98)$$

$$PE_{stationary} = \frac{1}{2} \mathbf{r}_s^T \mathbf{K}_s \mathbf{r}_s \quad (4.99)$$

where $\mathbf{M}_s$ is the modal mass matrix of the stationary part and $\mathbf{K}_s$ is the modal stiffness matrix of the stationary part. With the above normalization,

$$\mathbf{M}_s = \text{DIAG}\{1 \quad 1 \quad \cdots \quad 1\}_{n_s \times n_s} \quad (4.100)$$

$$\mathbf{K}_s = \text{DIAG}\{\omega_{s1}^2 \quad \omega_{s2}^2 \quad \cdots \quad \omega_{s,n_s}^2\}_{n_s \times n_s} \quad (4.101)$$

with $\omega_{sn}, n=1, n_s$ are the natural frequencies of the stationary part. DIAG means diagonal matrix and subscript n_s denotes the dimension of the matrix. The displacement modal participation factors vector of the stationary part takes the following form

$$\mathbf{r}_s = \{r_{s1} \quad r_{s2} \quad r_{s3} \quad \cdots \quad r_{sn_s}\}^T \quad (4.102)$$

Formulation of the ball bearing part

In this case, the shaft is not fixed and has elastic deformation due to the flexible stationary part. Thus, the deformations of the bearings A and B, $\Delta \mathbf{R}_j, j = a, b$ are the difference between the rigid-body motions due to the rigid spindle and the elastic displacement due to the stationary part, thus, referring to Figure 4.5,

$$\Delta \mathbf{R}_j = \mathbf{R}_{oo'} + z_j(\mathbf{k}' - \mathbf{K}) - \sum_{n=1}^{n_s} \mathbf{w}_{sn,j} r_{sn}, j = a, b \quad (4.103)$$

where $\mathbf{w}_{sn,j}, j = a, b$ represents the modal response of the shaft at the bearing support positions.

Therefore, the potential energy PE_b is

$$PE_b = \sum_{j=a,b} \frac{1}{2} \Delta \mathbf{R}_j^T \cdot \begin{bmatrix} k_{jx} & 0 & 0 \\ 0 & k_{jy} & 0 \\ 0 & 0 & k_{jz} \end{bmatrix} \cdot \Delta \mathbf{R}_j = \sum_{j=a,b} \left[\frac{1}{2} k_{jx} (R_x + z_j \theta_y - \sum_{n=1}^{n_s} w_{sxn,j} r_{sn})^2 \right. \\ \left. + \frac{1}{2} k_{jy} (R_y - z_j \theta_x - \sum_{n=1}^{n_s} w_{syn,j} r_{sn})^2 + \frac{1}{2} k_{jz} (R_z - \sum_{n=1}^{n_s} w_{szn,j} r_{sn})^2 \right] \quad (4.104)$$

where $w_{sxn,j}, w_{syn,j}, w_{szn,j}, j = a, b$ are the mode shapes of the stationary part at the bearing support positions on the shaft in x, y and z directions, respectively.

4.3.2 Governing equations of motion

The total kinetic and potential energy of the spindle/disks assembly-shaft-housing system can be written as

$$KE = KE_{rotating} + KE_{stationary} \\ = \frac{1}{2} \{ \dot{\mathbf{r}}_r^T \mathbf{M}_r \dot{\mathbf{r}}_r + \dot{\mathbf{r}}_r^T \mathbf{G}_r \mathbf{r}_r \} + \sum_N \frac{1}{2} \{ \dot{\mathbf{r}}_r^T \mathbf{M}_{rd} \dot{\mathbf{r}}_d + \dot{\mathbf{r}}_r^T \mathbf{G}'_{rd} \mathbf{r}_d \} \\ + \sum_N \frac{1}{2} \{ \dot{\mathbf{r}}_d^T \mathbf{M}_d \dot{\mathbf{r}}_d + \dot{\mathbf{r}}_d^T \mathbf{M}_{dr} \dot{\mathbf{r}}_r + \dot{\mathbf{r}}_d^T \mathbf{G}'_{dr} \mathbf{r}_r + \dot{\mathbf{r}}_d^T \mathbf{G}_d \mathbf{r}_d \} + \frac{1}{2} \dot{\mathbf{r}}_s^T \mathbf{M}_s \dot{\mathbf{r}}_s \quad (4.105)$$

$$\begin{aligned}
 PE &= PE_{rotating} + PE_{stationary} + PE_b \\
 &= \sum_N \frac{1}{2} \mathbf{r}_d^T \mathbf{K}'_d \mathbf{r}_d + \frac{1}{2} \mathbf{r}_s^T \mathbf{K}_s \mathbf{r}_s + \sum_{j=a,b} \left[\frac{1}{2} k_{jx} (R_x + z_j \theta_y - \sum_{n=1}^{n_s} w_{sxn,j} r_{sn})^2 \right. \\
 &\quad \left. + \frac{1}{2} k_{jy} (R_y - z_j \theta_x - \sum_{n=1}^{n_s} w_{syn,j} r_{sn})^2 + \frac{1}{2} k_{jz} (R_z - \sum_{n=1}^{n_s} w_{szn,j} r_{sn})^2 \right]
 \end{aligned} \quad (4.106)$$

Therefore, governing equations of motion are derived as follows by applying the Lagrange's equation,

$$\begin{bmatrix} \mathbf{M}_s & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_r & \mathbf{M}_{rd} \\ \mathbf{0} & \mathbf{M}_{dr} & \mathbf{M}_d \end{bmatrix} \left\{ \ddot{\hat{\mathbf{q}}} \right\} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_r & \mathbf{G}'_{rd} \\ \mathbf{0} & \mathbf{G}'_{dr} & \mathbf{G}_d \end{bmatrix} \left\{ \dot{\hat{\mathbf{q}}} \right\} + \begin{bmatrix} \mathbf{K}_s + \mathbf{K}_{sb} & \mathbf{K}_{sr} & \mathbf{0} \\ \mathbf{K}_{sr}^T & \mathbf{K}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}'_d \end{bmatrix} \left\{ \hat{\mathbf{q}} \right\} = \begin{Bmatrix} \hat{\mathbf{S}} \\ \hat{\mathbf{F}} \\ \hat{\mathbf{M}} \end{Bmatrix} \quad (4.107)$$

with

$$\mathbf{K}_{sb} = \begin{bmatrix} \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si1,j} w_{si1,j} & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si1,j} w_{si2,j} & \cdots & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si1,j} w_{sin_s,j} \\ \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si2,j} w_{si1,j} & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si2,j} w_{si2,j} & \cdots & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si2,j} w_{sin_s,j} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{sin_s,j} w_{si1,j} & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{sin_s,j} w_{si2,j} & \cdots & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{sin_s,j} w_{sin_s,j} \end{bmatrix}_{n_s \times n_s} \quad (4.108)$$

$$\mathbf{K}_{sr} = - \begin{bmatrix} \sum_{j=a,b} k_{jx} w_{sx1,j} & \sum_{j=a,b} k_{jy} w_{sy1,j} & \sum_{j=a,b} k_{jz} w_{sz1,j} & - \sum_{j=a,b} k_{jy} z_j w_{sy1,j} & \sum_{j=a,b} k_{jx} z_j w_{sx1,j} \\ \sum_{j=a,b} k_{jx} w_{sx2,j} & \sum_{j=a,b} k_{jy} w_{sy2,j} & \sum_{j=a,b} k_{jz} w_{sz2,j} & - \sum_{j=a,b} k_{jy} z_j w_{sy2,j} & \sum_{j=a,b} k_{jx} z_j w_{sx2,j} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sum_{j=a,b} k_{jx} w_{sxn_s,j} & \sum_{j=a,b} k_{jy} w_{syn_s,j} & \sum_{j=a,b} k_{jz} w_{szn_s,j} & - \sum_{j=a,b} k_{jy} z_j w_{syn_s,j} & \sum_{j=a,b} k_{jx} z_j w_{sxn_s,j} \end{bmatrix}_{n_s \times 5} \quad (4.109)$$

where $\mathbf{K}_{sr}$ is the stiffness coupling matrix between the stationary part and the rotating spindle. $\mathbf{K}_{sb}$ is the stiffness matrix of the stationary part induced by the bearing deformation. $\hat{\mathbf{S}}$, $\hat{\mathbf{F}}$ and $\hat{\mathbf{M}}$ are the generalized force vectors. $\{\hat{\mathbf{q}}\}$ is the generalized displacement vector, which consists of the displacement modal participation factors of the

stationary part, rigid-body motion of the spindle and the displacement modal participation factors of the disk and takes the following form

$$\{\hat{\mathbf{q}}\} = \{r_{s1} \ r_{s2} \ \dots \ r_{sn_s} \ R_x \ R_y \ R_z \ \theta_x \ \theta_y \ \bar{C}_{m2}^{-1} \ \bar{C}_{m3}^{-1} \ \bar{C}_{m1}^{-1} \ \bar{S}_{m2}^{-1} \ \bar{C}_{m0}^{-1} \ \bar{S}_{m3}^{-1} \ \bar{S}_{m1}^{-1}\}^T \quad (4.110)$$

It should be noted from equation (4.107) that the displacement modal participation factors of the stationary part can be solved provided the corresponding noise source excitations $(\hat{\mathbf{S}} \ \hat{\mathbf{F}} \ \hat{\mathbf{M}})$ are known. That is to say, the mobility transfer function which links the noise source excitations to the vibration transmitted to the housing has been built. This is an important stage for the further calculation of sound radiation from the vibrating top cover and base plate. Mobility transfer functions of the first six modes of the stationary part for a point harmonic force applied at position of $r = b_1, \theta = 0$ on the top disk are shown in Figure 4.16.

(a)

(b)

Fig. 4.16 Mobility transfer functions of the stationary part modes

(a) r_{s1} mode (b) r_{s2} mode (c) r_{s3} mode (d) r_{s4} mode (e) r_{s5} mode (f) r_{s6} mode

4.3.3 Free vibration analysis

Natural frequencies of the entire spindle/disks assembly-shaft-housing system are calculated with the modal analysis of the stationary part performed in finite element software ANSYS. To examine the effect of the flexibility of the stationary part, the comparisons of natural frequencies of vibration modes without and with the stationary part installed are given in Table 4.3, when the disks rotate at 5400 RPM. From Table 4.3 one may see that the flexibility of the stationary part does not affect disk vibration modes

with two or more nodal diameters. It also does not affect balanced (0, 0) and balanced (0, 1) modes. The flexibility of the stationary part only affects natural frequencies of unbalanced modes. From Table 4.3, it is shown that natural frequencies of unbalanced modes have been reduced with the consideration of the flexibility of the stationary part. For example, unbalanced forward (0, 1) mode has shifted from 508.732 Hz to 508.728 Hz, unbalanced backward (0, 1) mode from 330.113 Hz to 330.109 Hz and unbalanced (0, 0) mode from 611.92 Hz to 608.67 Hz.

<i>Vibration modes</i>	<i>Flexibility of stationary part excluded (Hz)</i>	<i>Flexibility of stationary part included (Hz)</i>
$U_b(0,1)$	330.113	330.109
$U_f(0,1)$	508.732	508.728
$B_b(0,1)$	522.63	522.63
$B_f(0,1)$	702. 63	702. 63
$U(0,0)$	611.92	608.67
$B(0,0)$	613.31	613.31
$B_b(0,2)$	555.86	555.86
$B_f(0,2)$	915.58	915.58

Table 4.3 Effect of considering flexibility of the stationary part on natural frequencies

When the flexibility of the stationary part is present, equation 4.107 supports the above observed phenomena, together with the details of the coefficient matrices expressed in equations (4.59) – (4.67), (4.74) - (4.77), (4.108) and (4.109). The flexibility of the stationary part does not affect disk modes with two or more nodal diameters since there are no parameters associated with the stationary part and bearings are contained in governing equations of disk modes with two or more nodal diameters. The flexibility of

the stationary part affects unbalanced modes because the stiffness coupling exists between the stationary part and the rigid spindle through bearings and modes of the stationary part. Therefore, unbalanced vibration modes are possibly the key modes in vibration transmission and subsequent noise radiation since they are coupled with the stationary part.

On the other hand, the comparisons of natural frequencies of the stationary part before and after assembly into the system are examined and listed in Table 4.4 when the disks rotate at 5400 RPM. It can be observed that the natural frequencies of the stationary part are slightly changed after added into the system.

<i>Mode</i>	<i>Before (Hz)</i>	<i>After (Hz)</i>
1	973.4	973.40
2	1874.8	1873.99
3	1969.4	1969.86
4	2008.7	2012.0
5	2330.7	2331.24
6	2863.6	2863.4

Table 4.4 Changes of natural frequencies of the stationary part before and after added into the system

4.3.4 Frequency response analysis

The top disk is subject to a point harmonic force at position of $r = b_1, \theta = 0$. First the response of the force action point is depicted in Figure 4.17 when the disk is stationary and rotating at a speed of 5400 RPM. It may be observed that except for the B(0, 0) and U(0, 0) modes, the number of resonant frequencies of both balanced and unbalanced

modes are doubled due to the rotation effect. The reason is that the excitation force is fixed in space and a mode is the superposition of a forward traveling wave and a backward traveling wave in a non-spinning frame. For example, the stationary unbalanced (0, 1) mode initially at 409.31 Hz has split into a forward mode at 508.81 Hz and a backward mode at 330.13 Hz. The difference 178.68 Hz between $U_f(0, 1)$ and $U_b(0, 1)$ is not equal to $2n\Omega = 2*1*90=180$ Hz as in the case where only the rotating disk is considered. This is because of the existence of the coupling between the disk and the rigid body motion of the spindle. The same applies to the stationary balanced (0, 2) mode which occurs at 915.92 Hz for $B_f(0, 2)$ and 555.76 Hz for $B_b(0, 2)$ with the stationary resonance at 721.12 Hz.

Fig. 4.17 Frequency response of the force action point of the top disk under stationary and rotating conditions with the stationary part

Moreover, the resonant frequencies have increased due to the centrifugal stresses induced by rotation. This is apparent from mode $B(0, 0)$ and $U(0, 0)$ since they do not present mode splitting effect. $B(0, 0)$ has increased from 608.31 to 613.21 Hz and $U(0, 0)$ from 606.91 Hz to 611.81 Hz.

The displacement of the centre point of the top cover is then predicted in Figure 4.18 for both stationary disk and disk rotating at 5400 RPM. Except the natural modes of the

stationary part, unbalanced (0, 0) and (0, 1) modes are also excited in the stationary disk case and unbalanced (0, 0) and unbalanced forward and backward (0, 1) modes in the rotating disk case. For the displacement of the centre point of the base plate as shown in Figure 4.19, the resonant frequencies appear at unbalanced (0, 0) and unbalanced (0, 1) modes for the stationary disk and unbalanced (0, 0) and unbalanced backward (0, 1) mode for the rotating disk. Moreover, the amplitude of the deformation of the base plate is much smaller since the base plate is much stiffer and thicker.

Fig. 4.18 Displacement of the centre of the top cover under point force on the top disk

Fig. 4.19 Displacement of the centre of the base plate under point force on the top disk

4.3.5 Parametric studies

The vibration of the spindle/disks assembly-shaft-housing system is affected by various parameters of the overall model. For the sake of illustration, a few parametric studies are

presented on the effect of i^{th} disk geometry, such as thickness, outer radius and number of disks. Limited parametric studies are conducted by varying the disk thickness from $0.75h_i$ to $1.25h_i$ and the outer radius from $0.875b_i$ to $1.125b_i$ where h_i and b_i are the current values of the i^{th} disk thickness and outer radius, respectively. The effect of disk number is examined through 1-disk, 2-disk and 4-disk configurations. A point force at the position of $r = b_1, \theta = 0$ on the top disk is applied and the disk is rotating at 5400 RPM.

The effect of disk thickness is first examined. For the displacement of the force action point, it is illustrated in Figure 4.20 (a) that the resonant frequencies of disk vibration modes have changed accordingly. For the displacement of the center point of the top cover and base plate as shown in Figure 4.20 (b) and Figure 4.20 (c), the change of disk thickness does not affect the resonant frequencies of the stationary part. However, the unbalanced (0, 0) mode, unbalanced backward (0, 1) and unbalanced forward (0, 1) mode will change with the increase of the disk thickness.

(a)

(b)

(c)

Fig. 4.20 Effect of disk thickness under point force on the top disk
 (a) Displacement of force action point of the disk (b) Displacement of the centre of the top cover (c) Displacement of the centre of the base plate

The effects of disk outer radius are shown in Figure 4.21. The resonant frequencies of disk modes increase with the decrease of the outer radius of the disk as illustrated in Figure 4.21 (a). From Figure 4.21 (b), it is observed that the change of disk outer radius does not affect the resonant frequencies of the stationary part. However, the unbalanced (0, 0) mode, unbalanced backward (0, 1) mode and unbalanced forward (0, 1) mode will increase with the decrease of the disk outer radius. Similar phenomenon appears for the effect of outer radius on the displacement of the base plate as shown in Figure 4.21 (c).

(a)

(b)

(c)

Fig. 4.21 Effect of disk outer radius under point force on the top disk
(a) Displacement of the force action point of the disk (b) Displacement of the centre of the top cover (c) Displacement of the centre of the base plate

The effect of disk number is compared in Figure 4.22. It is obvious in Figure 4.22 (b) and (c) that as disk number decreases, unbalanced (0, 1) modes have higher frequencies.

Peaks A1, A2 and A3 denote unbalanced backward (0, 1) modes and peaks B1, B2 and B3 unbalanced forward (0, 1) modes. This is because smaller inertia is induced in 1-disk and 2-disk configurations than 4-disk configuration.

(a)

(b)

(c)

Fig. 4.22 Effect of number of disks under point force on the top disk
(a) Displacement of the force action point of the disk (b) Displacement of the centre of the top cover (c) Displacement of the centre of the base plate

4.3.6 Load Cases studies

Two load cases are compared for the disk rotating at 5400 RPM. One is a point force acting at the position of $r = b_1, \theta = 0$ on the top disk. The other is the force applied on the stationary part and the rotating part in X direction, which is to simulate the unbalanced radial force, the electro-magnetically induced vibration source in the spindle motor. For the displacement of the force action point of the disk, it can be seen from Figure 4.23 (a) that unlike under the first load case in which each natural mode of the disk is excited with a forward and backward mode, only unbalanced forward and backward (0, 1) modes are excited in the second load case. While for the displacement of the center point of the top cover and base plate as shown in Figure 4.23 (b) and Figure 4.23 (c), unbalanced (0, 0) mode can not be excited in the second load case as in the first load case.

(a)

(b)

(c)

Fig. 4.23 Effect of load cases

(a) Displacement of disk (b) Displacement of the centre of the top cover

(c) Displacement of the centre of the base plate

4.4 Summary

Governing equations of spindle/disks assembly are derived in terms of rigid-body translation, rigid-body rotation of the spindle and the displacement modal participation factors of the disks. The coupling between the axial translation of the spindle and the axisymmetric modes of the disk, and between the rigid-body rotation of the spindle and the disk vibration modes with one nodal diameter elastic modes results in balanced and unbalanced modes of the disks. Balanced and unbalanced modes, except those with zero diameter, will exhibit mode splitting phenomenon consisting of a forward and a backward traveling wave. In addition, the natural frequencies of the unbalanced modes are lower than those of balanced modes. The calculation of natural frequencies is validated by experiment.

The analysis has been extended to a spindle/disks assembly connected to the flexible shaft and the flexible housing. The formulation of a spindle/disks assembly-shaft-housing system is essential for the analysis of vibration transmission, in particular, the vibration transmission to the housing which subsequently becomes a radiator for noise. The

governing equations of the entire spindle/disks assembly-shaft-housing system are derived in terms of displacement modal participation factors of the stationary part, rigid-body translation and rotation of the spindle, as well as displacement modal participation factors of each disk by using a Lagrangian formulation. The transfer function which links the corresponding noise source excitations and the induced vibration on the housing by the spindle/disks assembly is also obtained. From the equations derived, it can be concluded that the unbalanced modes of the disks are coupled with the vibration modes of the stationary part and play an important role in vibration transmission to the stationary part via the ball bearings. The comparisons have been performed in terms of natural frequencies of the disk and natural frequencies of the stationary part as well, before and after the stationary part is installed into the system. The results show that natural frequencies of unbalanced modes have been reduced with the consideration of the flexibility of the stationary part. Displacement of the disk, top cover and base plate under corresponding harmonic forces are predicted. Some effects of disk geometry and number of disks on the forced responses have been explored. Forced responses under different load cases have also been explored.

Chapter 5

Dynamic Analysis of VCM/actuators Assembly-Shaft-Housing System

This chapter presents a mathematical model for the dynamic analysis of the head actuator assembly in seek mode. Governing equations of motion of the VCM/actuators assembly-shaft-housing system is derived by using a Lagrangian formulation. The transfer function which links the noise source excitations to the induced vibration on the housing is derived. In addition, finite element analysis is used to determine the elastic deformation of the flexible components.

5.1 Introduction

As mentioned in Chapter 4, an accurate determination of the vibration characteristics and the dynamic behavior of the HDD and its subsystems needs to be done prior to analyzing the sound radiation from a HDD. When the HDD is in seek operation mode, the components and subsystems involved are the VCM/actuators assembly, ball bearings, shaft and housing.

However, most of the available literature is devoted to the electro-mechanical components and subsystems individually. Several investigations have studied characteristics of the vibration of the head suspension assemblies (Yoneoka *et al.*, 1989; Frees and Miu, 1990; Henze *et al.*, 1990; Miu and Karam, 1991). Experimental results of the tracking dynamics of suspension were presented (Chious and Miu, 1992). The

dynamic properties of a certain type of suspension assembly were examined numerically and experimentally (Zeng and Bogy, 1998a, 1998b).

Although the analysis of the individual components is important in understanding the dynamic behavior of the system, it is also important to analyze the VCM/actuators assembly as a whole under actual operating conditions. Modal analysis of the actuator assembly was performed by using finite element method (Ryota et al., 2002). A detailed FE model of a hard disk drive was established using finite element method and the characteristics of quasi-rigid body modes were studied (Xu *et al.*, 2000; Xu, 2002; Xu and Guo, 2003). It is found that the existence of six quasi-rigid body modes is due to the flexibility of the ball bearings.

In addition, much attention has been paid to certain modes of the VCM/actuators assembly. A theoretical model of the actuator rocking mode was proposed and verified by experiments to show its significant impact on servo stability (Liu *et al.*, 2002). The butterfly mode was investigated in terms of FEM and experiment analysis (Liu and Lim, 2003). The effect of the pivot assembly on the dynamic behavior of the VCM/actuators assembly was also explored (Gao *et al.*, 2000; Xu *et al.*, 2001). There is also research on the pivot bearing itself (Liu and Liu, 1999).

It should be noted that in these models, the housing has been excluded, or simulated by just considering the boundary conditions of the shaft. The shaft has been assumed to be rigid and fixed, and the bearing has been represented as a linear spring. These assumptions are not appropriate for vibor-acoustic analysis of a HDD. When the structure-borne sound radiation from the housing (due to the vibration transmitted by the

VCM/actuators assembly) is the main concern, these assumptions have some limitations. Firstly, in conventional HDDs, the actuator shaft is mounted on the drive base plate and connected to the top cover through a screw. The cover, as the sound radiator, is an indispensable component in vibro-acoustic analysis. Consequently, a thorough understanding of the dynamic behavior of the VCM/actuators assembly with the flexible housing is essential in order to analyze sound radiation from the HDD, and to determine the dominant frequencies contributing to the seek noise level. Secondly, the assumption of a fixed and rigid shaft cannot explain how the vibration of the VCM/actuators assembly may be transmitted from the shaft to the housing. The flexibility of the shaft must be considered. Thirdly, the spring stiffness assumption of bearings can only simulate the in-plane translational type motions between the components. Such assumptions may be adequate for the free and forced vibration analysis of the VCM/actuators assembly, but not sufficient for the vibro-acoustic analysis in HDDs since in vibro-acoustic analysis of HDDs out-of-plane motions of the components are often dominant. The new model should incorporate bearings with stiffness for all six degrees of freedom of motion. This is essential in modeling the vibration transmission between the VCM/actuators assembly and the shaft, hence, the housing. It can be seen that the existing models are inadequate for analyzing the sound radiation of HDDs in which the bearings and the shaft act as vibration transmission elements and the housing as a radiator. Consequently, a finer model is needed.

In addition, for the above-mentioned numerical investigations, the finite element (FE) method has been used. However, finite element method itself is not the best approach for the present study. It may be time consuming not only when building the model but also when performing analyses at the system level, since the system involves many

components and the structures of some of them are relatively complicated. It is also difficult to perform design parameter studies because it requires the re-building of the model of the entire system when certain design variables are changed.

Therefore, the aim of this chapter is to provide a novel flexible multi-body dynamic model for the vibration analysis of the entire VCM/actuators assembly-shaft-housing system by using Lagrangian equations. With the model, the vibration transmitted to the housing can be determined provided the corresponding noise source excitations have been characterized in advance.

5.2 Dynamic analysis of VCM/actuators assembly-shaft-housing system

5.2.1 Frames of reference

A set of reference frames is defined and illustrated in Figure 5.1. The inertial frame XYZ is with point O being the origin and has unit vectors of $\mathbf{I}$, $\mathbf{J}$ and $\mathbf{K}$. The elastic deformation of the i^{th} flexible component in the system is defined in the body-fixed frame $x''y''z''$. It has C_i as the origin and unit vectors $\mathbf{i}''$, $\mathbf{j}''$ and $\mathbf{k}''$. The rotation sequence of Euler angles θ_x , θ_y and θ_z are selected as (1) θ_x about X axis followed by intermediate coordinates xyz ; (2) θ_y about y axis followed by intermediate coordinates $x'y'z'$; (3) θ_z about z' axis followed by the body-fixed coordinates $x''y''z''$. The body-fixed frame $x''y''z''$ is related to the inertial reference frame XYZ through the following transformation matrix,

$$\begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} = [B] \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \quad (5.1)$$

with

$$[B] = \begin{bmatrix} \cos \theta_y \cos \theta_z & \sin \theta_x \sin \theta_y \cos \theta_z + \cos \theta_x \sin \theta_z & -\cos \theta_x \sin \theta_y \cos \theta_z + \sin \theta_x \sin \theta_z \\ -\cos \theta_y \sin \theta_z & -\sin \theta_x \sin \theta_y \sin \theta_z + \cos \theta_x \cos \theta_z & \cos \theta_x \sin \theta_y \sin \theta_z + \sin \theta_x \cos \theta_z \\ \sin \theta_y & -\sin \theta_x \cos \theta_y & \cos \theta_x \cos \theta_y \end{bmatrix} \quad (5.2)$$

Fig. 5.1 Definition of frame of references

(1) θ_x about $X \rightarrow xyz$ (2) θ_y about y axis $\rightarrow x'y'z'$ (3) θ_z about z' axis $\rightarrow x''y''z''$

5.2.2 Problem description

VCM/actuators assembly-shaft-housing system, as schematically shown in Figure 5.2, comprises three main parts: rotating part, ball bearings and stationary part.

Fig. 5.2 Schematic of VCM/actuators assembly-shaft-housing system

The rotating part is made up of the pivot housing and the subsystem of the coil and N assemblies of arm and suspension as shown in Figure 5.3. The pivot housing shown in Figure 5.4 is assumed to be rigid, while the flexible components, as shown in Figure 5.5, the coil and N assemblies of arm and suspension, are rigidly attached to the pivot housing.

Fig. 5.3 Definition of the rotating part

Fig. 5.4 Rigid pivot of the rotating part

Fig. 5.5 Flexible components of the rotating part

The stationary part shown in Figure 5.6 is the assembly of the flexible shaft and the flexible housing, which is the same as that in spindle/disks assembly-shaft-housing system except that the shaft position is different.

Fig. 5.6 Definition of stationary part

Referring to Figure 5.2, the rigid pivot housing is supported by two ball bearings *A* and *B*. The two bearings are simplified as two springs with stiffness in three translational dimensions. The bearing *A* is distance z_a from the origin *O* while the bearing *B* is distance z_b from *O*.

5.2.3 Formulation of the rotating part

Kinetic Energy

Kinetic energy of the rotating part consists of the energy from the rigid pivot housing KE_r , and that from the flexible components, that is, the coil and the *N* assemblies of arm and suspension KE_f as shown in Figure 5.5. It can be written as

$$KE = KE_r + KE_f \quad (5.3)$$

The motion of a material point on the i^{th} flexible component is defined as the rigid-body motion of its body-fixed reference plus the motion of the material point on the body with respect to its reference. Therefore, the displacement of an arbitrary point *A* on the i^{th} flexible component $\mathbf{R}_a^i$, as shown in Figure 5.7, can be described as

$$\mathbf{R}_a^i = \mathbf{R}_{oo'} + \mathbf{R}_{o'c_i} + \mathbf{r}_a^i + \mathbf{w}_a^i \quad (5.4)$$

where $\mathbf{R}_{oo'}$ is the rigid-body displacement vector of the entire assembly with the assumption that O and O' are coincident initially. $\mathbf{R}_{o'c_i}$ is the position vector of the origin of the body-fixed frame of the i^{th} flexible component, C_i , with respect to O' . $\mathbf{r}_a^i$ is the position vector of an arbitrary point A with respect to C_i . $\mathbf{w}_a^i$ is the transverse elastic deformation measured from the body-fixed reference frame $x''y''z''$. They can be described as follows

$$\mathbf{R}_{oo'} = R_x \mathbf{I} + R_y \mathbf{J} + R_z \mathbf{K} \quad (5.5)$$

$$\mathbf{R}_{o'c_i} = z_i \mathbf{k}'' \quad (5.6)$$

$$\mathbf{r}_a^i = r_x \mathbf{i}'' + r_y \mathbf{j}'' + r_z \mathbf{k}'' \quad (5.7)$$

$$\mathbf{w}_a^i = w_a^i(r_x, r_y, r_z, t) \mathbf{k}'' \quad (5.8)$$

Fig. 5.7 Displacement vector of an arbitrary point A on the i^{th} flexible component

Therefore, velocity vector of the point A , $\dot{\mathbf{R}}_a^i$, on the i^{th} flexible component can be obtained by differentiating Equation (5.4)

$$\dot{\mathbf{R}}_a^i = \dot{\mathbf{R}}_{oo'} + \dot{\mathbf{R}}_{o'c_i} + \dot{\mathbf{r}}_a^i + \dot{\mathbf{w}}_a^i \quad (5.9)$$

where

$$\dot{\mathbf{R}}_{oo'} = \dot{R}_x \mathbf{I} + \dot{R}_y \mathbf{J} + \dot{R}_z \mathbf{K} \quad (5.10)$$

$$\dot{\mathbf{R}}_{o'c_i} = z_i \frac{d\mathbf{k}''}{dt} \quad (5.11)$$

$$\dot{\mathbf{r}}_a^i = r_x \frac{d\mathbf{i}''}{dt} + r_y \frac{d\mathbf{j}''}{dt} + r_z \frac{d\mathbf{k}''}{dt} \quad (5.12)$$

$$\dot{\mathbf{w}}_a^i = \frac{dw_a^i}{dt} \mathbf{k}'' + w_a^i \frac{d\mathbf{k}''}{dt} \quad (5.13)$$

$$\frac{d\mathbf{i}''}{dt} = \boldsymbol{\omega}'' \times \mathbf{i}'' \quad (5.14)$$

$$\frac{d\mathbf{j}''}{dt} = \boldsymbol{\omega}'' \times \mathbf{j}'' \quad (5.15)$$

$$\frac{d\mathbf{k}''}{dt} = \boldsymbol{\omega}'' \times \mathbf{k}'' \quad (5.16)$$

with the angular velocity of the body-fixed frame $x''y''z''$, $\boldsymbol{\omega}''$, is given by

$$\boldsymbol{\omega}'' = (\dot{\theta}_x \cos \theta_y \cos \theta_z + \dot{\theta}_y \sin \theta_z) \mathbf{i}'' + (\dot{\theta}_y \cos \theta_z - \dot{\theta}_x \cos \theta_y \sin \theta_z) \mathbf{j}'' + (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y) \mathbf{k}'' \quad (5.17)$$

Thus, the velocity vector of the point A, $\dot{\mathbf{R}}_a^i$, on the i^{th} flexible component can be derived by substituting Equations (5.10)-(5.17) into Equation (5.9)

$$\dot{\mathbf{R}}_a^i = \dot{R}_{ax}^i \mathbf{I} + \dot{R}_{ay}^i \mathbf{J} + \dot{R}_{az}^i \mathbf{K} \quad (5.18)$$

where

$$\begin{aligned} \dot{R}_{ax}^i = & \dot{R}_x + [z_i \dot{\theta}_y + r_z \dot{\theta}_y - r_y (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)] \cos \theta_y + (r_y \dot{\theta}_x \cos \theta_y - r_x \dot{\theta}_y) \sin \theta_y \\ & + w_a^i \dot{\theta}_y \cos \theta_y + \frac{dw_a^i}{dt} \sin \theta_y \end{aligned} \quad (5.19)$$

$$\begin{aligned} \dot{R}_{ay}^i = & \dot{R}_y + [z_i \dot{\theta}_y + r_z \dot{\theta}_y - r_y (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)] \sin \theta_x \sin \theta_y \\ & + [r_x (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y) - r_z \dot{\theta}_x \cos \theta_y - z_i \dot{\theta}_x \cos \theta_y] \cos \theta_x + (r_x \dot{\theta}_y - r_y \dot{\theta}_x \cos \theta_y) \sin \theta_x \cos \theta_y \\ & + w_a^i \dot{\theta}_y \sin \theta_x \sin \theta_y - w_a^i \dot{\theta}_x \cos \theta_x \cos \theta_y - \frac{dw_a^i}{dt} \sin \theta_x \cos \theta_y \end{aligned} \quad (5.20)$$

$$\begin{aligned} \dot{R}_{az}^i = & \dot{R}_z - [z_i \dot{\theta}_y - r_y (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y) + r_z \dot{\theta}_y] \cos \theta_x \sin \theta_y \\ & + [r_x (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y) - r_z \dot{\theta}_x \cos \theta_y - z_i \dot{\theta}_x \cos \theta_y] \sin \theta_x + (r_y \dot{\theta}_x \cos \theta_y - r_x \dot{\theta}_y) \cos \theta_x \cos \theta_y \\ & - w_a^i \dot{\theta}_y \cos \theta_x \sin \theta_y - w_a^i \dot{\theta}_x \sin \theta_x \cos \theta_y + \frac{dw_a^i}{dt} \cos \theta_x \cos \theta_y \end{aligned} \quad (5.21)$$

Therefore, the kinetic energy KE_f of the $(N + 1)$ flexible components is expressed as

$$\begin{aligned}
 KE_f = & \frac{1}{2} \sum_{i=1}^{N+1} \int \dot{\mathbf{R}}_a^{iT} \cdot \dot{\mathbf{R}}_a^i dm = \frac{1}{2} \sum_{i=1}^{N+1} \int \{ \dot{R}_x^2 + \dot{R}_y^2 + \dot{R}_z^2 + \left(\frac{dw_a^i}{dt} \right)^2 \\
 & + (r_x^2 + r_z^2) \dot{\theta}_y^2 + (r_z^2 + r_y^2) \dot{\theta}_x^2 \cos^2 \theta_y + (r_y^2 + r_x^2) (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)^2 \\
 & + w_a^{i2} \dot{\theta}_y^2 + \cos^2 \theta_y w_a^{i2} \dot{\theta}_x^2 - 2r_x \dot{\theta}_z w_a^i \dot{\theta}_x \cos \theta_y - 2r_y \dot{\theta}_z w_a^i \dot{\theta}_y \\
 & + 2(\dot{R}_x \sin \theta_y - \dot{R}_y \sin \theta_x \cos \theta_y + \dot{R}_z \cos \theta_x \cos \theta_y + r_y \dot{\theta}_x \cos \theta_y - r_x \dot{\theta}_y) \frac{dw_a^i}{dt} \\
 & + 2(r_z \dot{\theta}_y - r_y \dot{\theta}_z) \cos \theta_y \dot{R}_x + 2(r_x \dot{\theta}_z - r_z \dot{\theta}_x \cos \theta_y) \cos \theta_x \dot{R}_y \\
 & + 2[r_x \dot{\theta}_z \sin \theta_x + (r_y \dot{\theta}_x \cos \theta_y - r_x \dot{\theta}_y) \cos \theta_x \cos \theta_y + r_y \dot{\theta}_z \cos \theta_x \sin \theta_y] \dot{R}_z \\
 & - 2r_x r_y \dot{\theta}_x \dot{\theta}_y \cos \theta_y - 2r_z r_y \dot{\theta}_z \dot{\theta}_y - 2r_x r_z \dot{\theta}_z \dot{\theta}_x \cos \theta_y \} dm
 \end{aligned} \quad (5.22)$$

Following the same procedure, the kinetic energy of the rigid pivot housing KE_r , with zero elastic deformation, can be derived as follows

$$KE_r = \frac{m_p}{2} (\dot{R}_x^2 + \dot{R}_y^2 + \dot{R}_z^2) + \frac{1}{2} [I_y \dot{\theta}_y^2 + I_x \dot{\theta}_x^2 \cos^2 \theta_y + I_z (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)^2] \quad (5.23)$$

where m_p and I_x, I_y, I_z are the mass and moment of inertia of the pivot housing with respect to the origin of the body-fixed frame attached to the pivot housing, assuming that the pivot housing is symmetric about z'' .

Therefore, the total kinetic energy of the assembly is written by

$$\begin{aligned}
 KE = KE_r + KE_f = & \frac{M}{2} (\dot{R}_x^2 + \dot{R}_y^2 + \dot{R}_z^2) + \frac{1}{2} [I_y \dot{\theta}_y^2 + I_x \dot{\theta}_x^2 \cos^2 \theta_y + I_z (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)^2] \\
 & + \frac{1}{2} \sum_{i=1}^{N+1} \int \{ \left(\frac{dw_a^i}{dt} \right)^2 + (r_x^2 + r_z^2) \dot{\theta}_y^2 + (r_z^2 + r_y^2) \dot{\theta}_x^2 \cos^2 \theta_y + (r_y^2 + r_x^2) (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)^2 \\
 & + w_a^{i2} \dot{\theta}_y^2 + \cos^2 \theta_y w_a^{i2} \dot{\theta}_x^2 - 2r_x \dot{\theta}_z w_a^i \dot{\theta}_x \cos \theta_y - 2r_y \dot{\theta}_z w_a^i \dot{\theta}_y \\
 & + 2(\dot{R}_x \sin \theta_y - \dot{R}_y \sin \theta_x \cos \theta_y + \dot{R}_z \cos \theta_x \cos \theta_y + r_y \dot{\theta}_x \cos \theta_y - r_x \dot{\theta}_y) \frac{dw_a^i}{dt} \\
 & + 2(r_z \dot{\theta}_y - r_y \dot{\theta}_z) \cos \theta_y \dot{R}_x + 2(r_x \dot{\theta}_z - r_z \dot{\theta}_x \cos \theta_y) \cos \theta_x \dot{R}_y \\
 & + 2[r_x \dot{\theta}_z \sin \theta_x + (r_y \dot{\theta}_x \cos \theta_y - r_x \dot{\theta}_y) \cos \theta_x \cos \theta_y + r_y \dot{\theta}_z \cos \theta_x \sin \theta_y] \dot{R}_z \\
 & - 2r_x r_y \dot{\theta}_x \dot{\theta}_y \cos \theta_y - 2r_z r_y \dot{\theta}_z \dot{\theta}_y - 2r_x r_z \dot{\theta}_z \dot{\theta}_x \cos \theta_y \} dm
 \end{aligned} \quad (5.24)$$

where M is the total mass of the flexible components and the pivot housing.

Potential Energy

Potential energy of the system comes from the strain energy induced by the elastic deformation of the i^{th} flexible component PE_{fi} . Consequently, the total potential energy of the assembly is given by

$$PE = \sum_{i=1}^{N+1} PE_{fi} \quad (5.25)$$

Discretization

The kinetic energy of the rotating part in Equation (5.24) needs to be discretized, in particular, the elastic deformation of the flexible components measured from the body-fixed frame. Since the geometries of the flexible components are relatively complicated, finite element analysis is employed here.

Assume that the selected arbitrary point A on the i^{th} flexible component is located in volume element dV_j of element j of the i^{th} flexible component as shown in Figure 5.8.

Fig. 5.8 Description of point A in finite element analysis

Therefore, the kinetic energy of flexible components can be rewritten as follows

$$KE_f = \sum_{i=1}^{N+1} \frac{1}{2} \int \dot{\mathbf{R}}_a^{iT} \cdot \dot{\mathbf{R}}_a^i dm = \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \frac{1}{2} \int_{V_j} \rho_j \dot{\mathbf{R}}_a^{iT} \cdot \dot{\mathbf{R}}_a^i dV_j \quad (5.26)$$

where n_e is the number of elements for the i^{th} flexible component in the finite element model and ρ_j is the density of element j .

From modal finite element analysis using, for example, ANSYS, it is easy to obtain vibration modes of a flexible component. Therefore, the n^{th} mode shape at any point A on the i^{th} flexible component can be expressed as

$$\phi_n^i(r_x, r_y, r_z) = \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i \quad (5.27)$$

where ϕ_{nk}^i is the n^{th} mode shape evaluated at nodal positions of element j , N_k^i is the k^{th} shape function of the element and n_n is the number of shape function of the element j .

Therefore, the solution of the elastic transverse deformation of the arbitrary point A on the i^{th} flexible component can be approximated by the following assumed solution

$$w_a^i(r_x, r_y, r_z) = \sum_{n=1}^{n_m} \phi_n^i(r_x, r_y, r_z) q_n^i(t) = \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i q_n^i(t) \quad (5.28)$$

where $\phi_n^i(r_x, r_y, r_z)$ is the mode shape of the i^{th} flexible component and n_m is the number of mode shapes to be taken into account. $q_n^i(t)$ is the corresponding modal participation factor to be determined.

Substitution of Equation (5.28) into Equation (5.24) results in the discretized form of the kinetic energy

$$\begin{aligned}
 KE = KE_r + KE_f = & \frac{M}{2}(\dot{R}_x^2 + \dot{R}_y^2 + \dot{R}_z^2) + \frac{1}{2}[I_y \dot{\theta}_y^2 + I_x \dot{\theta}_x^2 \cos^2 \theta_y + I_z (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)^2] \\
 & + \frac{1}{2} \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j \{ (\sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i \dot{q}_n^i(t))^2 + (r_x^2 + r_z^2) \dot{\theta}_y^2 + (r_z^2 + r_y^2) \dot{\theta}_x^2 \cos^2 \theta_y \\
 & + (r_y^2 + r_x^2) (\dot{\theta}_z + \dot{\theta}_x \sin \theta_y)^2 + \dot{\theta}_y^2 \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i q_n^i(t))^2 + \cos^2 \theta_y \dot{\theta}_x^2 \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i q_n^i(t))^2 \\
 & - 2r_x \dot{\theta}_z \dot{\theta}_x \cos \theta_y \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i q_n^i(t) - 2r_y \dot{\theta}_z \dot{\theta}_y \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i q_n^i(t) \\
 & + 2(\dot{R}_x \sin \theta_y - \dot{R}_y \sin \theta_x \cos \theta_y + \dot{R}_z \cos \theta_x \cos \theta_y + r_y \dot{\theta}_x \cos \theta_y - r_x \dot{\theta}_y) \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i \dot{q}_n^i(t) \\
 & + 2(r_z \dot{\theta}_y - r_y \dot{\theta}_z) \cos \theta_y \dot{R}_x + 2(r_x \dot{\theta}_z - r_z \dot{\theta}_x \cos \theta_y) \cos \theta_x \dot{R}_y \\
 & + 2[r_x \dot{\theta}_z \sin \theta_x + (r_y \dot{\theta}_x \cos \theta_y - r_x \dot{\theta}_y) \cos \theta_x \cos \theta_y + r_y \dot{\theta}_z \cos \theta_x \sin \theta_y] \dot{R}_z \\
 & - 2r_x r_y \dot{\theta}_x \dot{\theta}_y \cos \theta_y - 2r_z r_y \dot{\theta}_z \dot{\theta}_y - 2r_x r_z \dot{\theta}_z \dot{\theta}_x \cos \theta_y \} dV_j
 \end{aligned}
 \tag{5.29}$$

The potential energy can be discretized as follows

$$PE = \frac{1}{2} \sum_{i=1}^{N+1} \sum_{n=1}^{n_m} (\omega_n^i q_n^i)^2
 \tag{5.30}$$

where ω_n^i is the natural frequency of the n^{th} flexural mode of the i^{th} flexible component.

5.2.4 Formulation of the stationary part

The stationary part is modeled as an elastic assembly including the flexible top cover, flexible base plate and flexible shaft as shown in Figure 5.6.

Let $\mathbf{w}_s$ be the elastic deformation of an arbitrary point on the stationary part and it can be approximated in terms of n_s vibration modes of the stationary part as

$$\mathbf{w}_s = \sum_{n=1}^{n_s} \mathbf{w}_{sn} r_{sn}
 \tag{5.31}$$

where $\mathbf{w}_{sn}$ is the n^{th} vibration mode shape of the stationary part and takes the following form

$$\mathbf{w}_{sn} = w_{sxt} \mathbf{I} + w_{syn} \mathbf{J} + w_{syt} \mathbf{K} \quad (5.32)$$

and r_{sn} is the displacement modal participation factor.

In addition, the mode shapes satisfy the orthonormality condition

$$\int \mathbf{w}_{sm} \cdot \mathbf{w}_{sn} dm = \delta_{mn} \quad (5.33)$$

where δ_{mn} is the Kronecker delta.

Thus, the kinetic energy and potential energy of the stationary part are given by

$$KE_{stationary} = \frac{1}{2} \sum_{n=1}^{n_s} (\dot{r}_{sn})^2 \quad (5.34)$$

$$PE_{stationary} = \frac{1}{2} \sum_{n=1}^{n_s} (\omega_{sn} r_{sn})^2 \quad (5.35)$$

with $\omega_{sn}, n = 1, n_s$ are the corresponding natural frequencies of the stationary part.

5.2.5 Formulation of the ball bearing part

The deformations of the bearings A and B, $\Delta \mathbf{R}_j, j = a, b$, are the differences between the rigid-body motions due to the rigid pivot housing $\mathbf{R}_{jr}, j = a, b$, and the elastic displacements due to the stationary part $\mathbf{R}_{je}, j = a, b$

$$\Delta \mathbf{R}_j = \mathbf{R}_{jr} - \mathbf{R}_{je}, j = a, b \quad (5.36)$$

where the rigid-body motions due to the rigid pivot housing as illustrated in Figure 5.9 is given by

$$\mathbf{R}_{jr} = \mathbf{R}_{oo'} + z_j (\mathbf{k}'' - \mathbf{K}), j = a, b \quad (5.37)$$

with bearing A moves to A' and bearing B to B' . The elastic displacements due to the stationary part is determined by

$$\mathbf{R}_{je} = \sum_{n=1}^{n_s} \mathbf{w}_{sn,j} r_{sn}, j = a, b \quad (5.38)$$

with $\mathbf{w}_{sn,j}, j = a, b$ represents the mode shapes of the bearing support positions on the shaft of the stationary part.

Fig. 5.9 Deformation of bearings due to the rigid-body motions of the rigid pivot housing

Therefore, the potential energy PE_b is

$$\begin{aligned} PE_b &= \sum_{j=a,b} \frac{1}{2} \Delta \mathbf{R}_j^T \begin{bmatrix} k_{jx} & 0 & 0 \\ 0 & k_{jy} & 0 \\ 0 & 0 & k_{jz} \end{bmatrix} \Delta \mathbf{R}_j \\ &= \sum_{j=a,b} \left[\frac{1}{2} k_{jx} (R_x + z_j \theta_y - \sum_{n=1}^{n_s} w_{sxn,j} r_{sn})^2 \right. \\ &\quad \left. + \frac{1}{2} k_{jy} (R_y - z_j \theta_x - \sum_{n=1}^{n_s} w_{syn,j} r_{sn})^2 + \frac{1}{2} k_{jz} (R_z - \sum_{n=1}^{n_s} w_{szn,j} r_{sn})^2 \right] \end{aligned} \quad (5.39)$$

where $w_{sxn,j}, w_{syn,j}, w_{szn,j}, j = a, b$ are the mode shapes of the stationary part at the bearing support positions on the shaft of the stationary part in x, y and z directions, respectively.

5.2.6 Governing equations of motion

The governing equations of motion are derived as follows using Lagrange's equation after a linearization assuming $\theta_x \ll 1$ and $\theta_y \ll 1$

$$\frac{d}{dt} \left(\frac{\partial(KE - PE)}{\partial \dot{q}_k} \right) - \frac{\partial(KE - PE)}{\partial q_k} = Q_k \quad k = 1, 2, \dots, n \quad (5.40)$$

with the generalized coordinates defined as

$$\mathbf{q} = \{R_x \ R_y \ R_z \ \theta_x \ \theta_y \ \theta_z \ q_n^i \ r_{sn}\}^T \quad (5.41)$$

and the generalized force defined as

$$\mathbf{Q} = \{F_x \ F_y \ F_z \ F_{\theta x} \ F_{\theta y} \ F_{\theta z} \ M_n^i \ R_{sn}\}^T \quad (5.42)$$

The generalized coordinates consist of six rigid body motions, the modal coordinates of the flexible actuator q_n and the modal coordinates of the stationary part r_{sn} . Similarly, the generalized force vector consists of forces acting on the spindle, actuators and the stationary part.

Thus, equation for R_x is

$$\begin{aligned} M\ddot{R}_x + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j (\ddot{\theta}_y r_z - r_y \ddot{\theta}_z) dV_j \\ + k_{ax} (R_x + z_a \theta_y - \sum_{n=1}^{n_s} w_{syn,a} r_{sn}) + k_{bx} (R_x + z_b \theta_y - \sum_{n=1}^{n_s} w_{syn,b} r_{sn}) = F_x \end{aligned} \quad (5.43)$$

Equation for R_y is

$$\begin{aligned} M\ddot{R}_y + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j (r_x \ddot{\theta}_z - r_z \ddot{\theta}_x) dV_j \\ + k_{ay} (R_y - z_a \theta_x - \sum_{n=1}^{n_s} w_{syn,a} r_{sn}) + k_{by} (R_y - z_b \theta_x - \sum_{n=1}^{n_s} w_{syn,b} r_{sn}) = F_y \end{aligned} \quad (5.44)$$

Equation for R_z is

$$\begin{aligned} M\ddot{R}_z + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j (-r_x \ddot{\theta}_y + r_x \ddot{\theta}_z \theta_x + r_x \dot{\theta}_z \dot{\theta}_x + r_y \ddot{\theta}_x + r_y \ddot{\theta}_z \theta_y + r_y \dot{\theta}_z \dot{\theta}_y + \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \varphi_{nk}^i \ddot{q}_n^i) dV_j \\ + (k_{az} + k_{bz} - k_{az} \sum_{n=1}^{n_s} w_{syn,a} r_{sn} - k_{bz} \sum_{n=1}^{n_s} w_{syn,b} r_{sn}) R_z = F_z \end{aligned} \quad (5.45)$$

Equation for θ_x is

$$\begin{aligned} \{I_x + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j (r_z^2 + r_y^2) dV_j\} \ddot{\theta}_x - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_z dV_j \ddot{R}_y + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y dV_j \ddot{R}_z \\ - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x r_y dV_j \ddot{\theta}_y - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x r_z dV_j \ddot{\theta}_z + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \varphi_{nk}^i dV_j \ddot{q}_n^i \\ + \{I_z + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j (r_x^2 + r_y^2) dV_j\} \dot{\theta}_y \dot{\theta}_z + \{I_z + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j (r_x^2 + r_y^2) dV_j\} \theta_y \ddot{\theta}_z \\ - \sum_{n=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \varphi_{nk}^i dV_j \dot{q}_n^i \ddot{\theta}_z - \sum_{n=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \varphi_{nk}^i dV_j \dot{q}_n^i \dot{\theta}_z \\ - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x dV_j \dot{R}_z \dot{\theta}_z + z_a k_{ay} (z_a \theta_x - R_y + \sum_{n=1}^{n_s} w_{syn,a} r_{sn}) + z_b k_{by} (z_b \theta_x - R_y + \sum_{n=1}^{n_s} w_{syn,b} r_{sn}) = F_{\theta_x} \end{aligned} \quad (5.46)$$

Equation for θ_y is

$$\begin{aligned} \{I_y + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j (r_z^2 + r_x^2) dV_j\} \ddot{\theta}_y + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_z dV_j \ddot{R}_x - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x dV_j \ddot{R}_z \\ - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x r_y dV_j \ddot{\theta}_x - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y r_z dV_j \ddot{\theta}_z - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \varphi_{nk}^i dV_j \ddot{q}_n^i \\ + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \varphi_{nk}^i dV_j \dot{q}_n^i \ddot{\theta}_z - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \varphi_{nk}^i dV_j \dot{q}_n^i \dot{\theta}_z \\ - \{I_z + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j (r_y^2 + r_x^2) dV_j\} \dot{\theta}_x \dot{\theta}_z - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y dV_j \dot{\theta}_z \dot{R}_z \\ + z_a k_{ax} (R_x + z_a \theta_y - \sum_{n=1}^{n_s} w_{syn,a} r_{sn}) + z_b k_{bx} (R_x + z_b \theta_y - \sum_{n=1}^{n_s} w_{syn,b} r_{sn}) = F_{\theta_y} \end{aligned} \quad (5.47)$$

Equation for θ_z is

$$\begin{aligned}
& \{I_z + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j (r_y^2 + r_x^2) dV_j\} \ddot{\theta}_z - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_y dV_j \ddot{R}_x + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_x dV_j \ddot{R}_y - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_x r_z dV_j \ddot{\theta}_x \\
& - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_y r_z dV_j \ddot{\theta}_y + \{I_z + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j (r_y^2 + r_x^2) dV_j\} \ddot{\theta}_x \theta_y + \{I_z + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j (r_y^2 + r_x^2) dV_j\} \ddot{\theta}_y \theta_x \\
& - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_y \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i dV_j \dot{q}_n^i \ddot{\theta}_y + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_x \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i dV_j \dot{q}_n^i \ddot{\theta}_x \\
& + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_x \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i dV_j \dot{q}_n^i \dot{\theta}_x - \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_y \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i dV_j \dot{q}_n^i \dot{\theta}_y \\
& + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_x dV_j \dot{\theta}_x \dot{R}_z + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_x dV_j \theta_x \ddot{R}_z + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_y dV_j \dot{\theta}_y \dot{R}_z + \sum_{i=1}^{N+1} \sum_{j=1}^{n_e} \int \rho_j r_y dV_j \ddot{R}_z \theta_y = F_{\theta_z}
\end{aligned} \tag{5.48}$$

Equation for q_n^i of the i^{th} flexible component is

$$\begin{aligned}
& \sum_{j=1}^{n_e} \int \rho_j \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i dV_j \ddot{q}_n^i(t) + \sum_{j=1}^{n_e} \int \rho_j \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i dV_j (r_y \ddot{\theta}_x - r_x \ddot{\theta}_y + \ddot{R}_z) \\
& + \sum_{j=1}^{n_e} \int \rho_j r_y \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i dV_j \dot{\theta}_y \dot{\theta}_z + \sum_{j=1}^{n_e} \int \rho_j r_x \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k^i \phi_{nk}^i dV_j \dot{\theta}_x \dot{\theta}_z + \omega_n^2 q_n^i = M_n^i
\end{aligned} \tag{5.49}$$

Equation for r_{sn} of the stationary part is

$$\begin{aligned}
& \ddot{r}_{sn} + \omega_{sn}^2 r_{sn} + w_{sxn,a} k_{ax} \sum_{n=1}^{n_s} w_{sxn,a} r_{sn} + w_{sxn,b} k_{bx} \sum_{n=1}^{n_s} w_{sxn,b} r_{sn} + w_{syn,a} k_{ay} \sum_{n=1}^{n_s} w_{syn,a} r_{sn} \\
& + w_{syn,b} k_{by} \sum_{n=1}^{n_s} w_{syn,b} r_{sn} + w_{sxn,a} k_{az} \sum_{n=1}^{n_s} w_{sxn,a} r_{sn} + w_{sxn,b} k_{bz} \sum_{n=1}^{n_s} w_{sxn,b} r_{sn} \\
& - (w_{sxn,a} k_{ax} + w_{sxn,b} k_{bx}) R_x - (w_{syn,a} k_{ay} + w_{syn,b} k_{by}) R_y - (w_{sxn,a} k_{az} + w_{sxn,b} k_{bz}) R_z \\
& + (k_{ay} z_a w_{syn,a} + k_{by} z_b w_{syn,b}) \theta_x - (k_{ax} z_a w_{sxn,a} + k_{bx} z_b w_{sxn,b}) \theta_y = R_{sn}
\end{aligned} \tag{5.50}$$

It should be noted that although it is not quite easy to provide a matrix form of the governing equations at this stage, the link between the noise source excitations and the vibration on the housing has been established.

5.3 Programming implementation for a simple actuator

To illustrate the programming implementation, the flexible components of the VCM/actuators assembly-shaft-housing system is represented by a simple actuator as shown in Figure 5.10.

Fig. 5.10 FE model of a simple actuator

In addition, the actuator is assumed to be stationary. Thus, the governing equations of motion reduce to

Equation for R_x :

$$M\ddot{R}_x + k_{ax}(R_x + z_a\theta_y - \sum_{n=1}^{n_s} w_{sxn,a}r_{sn}) + k_{bx}(R_x + z_b\theta_y - \sum_{n=1}^{n_s} w_{sxn,b}r_{sn}) = F_x \quad (5.51)$$

Equation for R_y :

$$M\ddot{R}_y + k_{ay}(R_y - z_a\theta_x - \sum_{n=1}^{n_s} w_{syn,a}r_{sn}) + k_{by}(R_y - z_b\theta_x - \sum_{n=1}^{n_s} w_{syn,b}r_{sn}) = F_y \quad (5.52)$$

Equation for R_z :

$$\begin{aligned} M\ddot{R}_z + \sum_{j=1}^{n_s} \int_{V_j} \rho_j (-r_x \ddot{\theta}_y + r_y \ddot{\theta}_x + \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k \phi_{nk} \ddot{q}_n) dV_j \\ + (k_{az} + k_{bz} - k_{az} \sum_{n=1}^{n_s} w_{sxn,a}r_{sn} - k_{bz} \sum_{n=1}^{n_s} w_{sxn,b}r_{sn}) R_z = F_z \end{aligned} \quad (5.53)$$

Equation for θ_x :

$$\begin{aligned} & \{I_x + \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y^2 dV_j\} \ddot{\theta}_x + \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y dV_j \ddot{R}_z - \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x r_y dV_j \ddot{\theta}_y + \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k \varphi_{nk} dV_j \ddot{q}_n \\ & + z_a k_{ay} (z_a \theta_x - R_y + \sum_{n=1}^{n_s} w_{syn,a} r_{sn}) + z_b k_{by} (z_b \theta_x - R_y + \sum_{n=1}^{n_s} w_{syn,b} r_{sn}) = F_{\theta_x} \end{aligned} \quad (5.54)$$

Equation for θ_y :

$$\begin{aligned} & \{I_y + \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x^2 dV_j\} \ddot{\theta}_y - \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x dV_j \ddot{R}_z \\ & - \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x r_y dV_j \ddot{\theta}_x - \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k \varphi_{nk} dV_j \ddot{q}_n \\ & + z_a k_{ax} (R_x + z_a \theta_y - \sum_{n=1}^{n_s} w_{sxn,a} r_{sn}) + z_b k_{bx} (R_x + z_b \theta_y - \sum_{n=1}^{n_s} w_{sxn,b} r_{sn}) = F_{\theta_y} \end{aligned} \quad (5.55)$$

Equation for q_n^1 of the flexible actuator:

$$\begin{aligned} & \sum_{j=1}^{n_e} \int_{V_j} \rho_j \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k \varphi_{nk} \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k \varphi_{nk} dV_j \ddot{q}_n^1 \\ & + \sum_{j=1}^{n_e} \int_{V_j} \rho_j \sum_{n=1}^{n_m} \sum_{k=1}^{n_n} N_k \varphi_{nk} dV_j (r_y \ddot{\theta}_x - r_x \ddot{\theta}_y + \ddot{R}_z) + \omega_n^2 q_n^1 = M_n^1 \end{aligned} \quad (5.56)$$

Equation for r_{sn} of the stationary part:

$$\begin{aligned} & \ddot{r}_{sn} + \omega_{sn}^2 r_{sn} + w_{sxn,a} k_{ax} \sum_{n=1}^{n_s} w_{sxn,a} r_{sn} + w_{sxn,b} k_{bx} \sum_{n=1}^{n_s} w_{sxn,b} r_{sn} + w_{syn,a} k_{ay} \sum_{n=1}^{n_s} w_{syn,a} r_{sn} \\ & + w_{syn,b} k_{by} \sum_{n=1}^{n_s} w_{syn,b} r_{sn} + w_{sxn,a} k_{az} \sum_{n=1}^{n_s} w_{sxn,a} r_{sn} + w_{sxn,b} k_{bz} \sum_{n=1}^{n_s} w_{sxn,b} r_{sn} \\ & - (w_{sxn,a} k_{ax} + w_{sxn,b} k_{bx}) R_x - (w_{syn,a} k_{ay} + w_{syn,b} k_{by}) R_y - (w_{sxn,a} k_{az} + w_{sxn,b} k_{bz}) R_z \\ & + (k_{ay} z_a w_{syn,a} + k_{by} z_b w_{syn,b}) \theta_x - (k_{ax} z_a w_{sxn,a} + k_{bx} z_b w_{sxn,b}) \theta_y = R_{sn} \end{aligned} \quad (5.57)$$

Therefore, the matrix form can be written as follows

$$\begin{bmatrix} \mathbf{M}_s & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_r & \mathbf{M}_{ra} \\ \mathbf{0} & \mathbf{M}_{ra}^T & \mathbf{M}_a \end{bmatrix} \{\ddot{\mathbf{q}}\} + \begin{bmatrix} \mathbf{K}_s + \mathbf{K}_{sb} & \mathbf{K}_{sr} & \mathbf{0} \\ \mathbf{K}_{sr}^T & \mathbf{K}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_a \end{bmatrix} \{\mathbf{q}\} = \begin{Bmatrix} \mathbf{S} \\ \mathbf{F} \\ \mathbf{M} \end{Bmatrix} \quad (5.58)$$

where $\mathbf{M}_s$ and $\mathbf{K}_s$ are the modal mass matrix and the modal stiffness matrix of the stationary part, respectively. $\mathbf{K}_{sr}$ and $\mathbf{K}_{sb}$ are the stiffness coupling matrices between the stationary part and the rotating part induced by the bearings. $\mathbf{M}_r$ is the inertial matrix for the rigid-body motions of the pivot housing. $\mathbf{M}_a$ is the modal mass matrix of the flexible actuator. $\mathbf{M}_{ra}$ represents the inertial coupling between the rigid-body motions and the flexible vibration of the actuator. $\mathbf{K}_a$ and $\mathbf{K}_b$ are stiffness matrices of the actuator and bearings, respectively. $\mathbf{S}$, $\mathbf{F}$ and $\mathbf{M}$ are generalized force vector. $\{\mathbf{q}\}$ is the generalized coordinate vector, which consists of the modal coordinates r_{sn} of the stationary part, rigid body translation, rigid body rotation and the modal coordinates q_n of the flexible actuator.

The details of the coefficient matrices are described as follows

$$\mathbf{M}_s = \text{DIAG}\{1 \quad 1 \quad \cdots \quad 1\}_{n_s \times n_s} \quad (5.59)$$

$$\mathbf{K}_s = \text{DIAG}\{\omega_{s1}^2 \quad \omega_{s2}^2 \quad \cdots \quad \omega_{s,n_s}^2\}_{n_s \times n_s} \quad (5.60)$$

$$\mathbf{K}_{sr} = - \begin{bmatrix} \sum_{j=a,b} k_{jx} w_{sx1,j} & \sum_{j=a,b} k_{jy} w_{sy1,j} & \sum_{j=a,b} k_{jz} w_{sz1,j} & - \sum_{j=a,b} k_{jy} z_j w_{sy1,j} & \sum_{j=a,b} k_{jx} z_j w_{sx1,j} & 0 \\ \sum_{j=a,b} k_{jx} w_{sx2,j} & \sum_{j=a,b} k_{jy} w_{sy2,j} & \sum_{j=a,b} k_{jz} w_{sz2,j} & - \sum_{j=a,b} k_{jy} z_j w_{sy2,j} & \sum_{j=a,b} k_{jx} z_j w_{sx2,j} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sum_{j=a,b} k_{jx} w_{sxn_s,j} & \sum_{j=a,b} k_{jy} w_{syn_s,j} & \sum_{j=a,b} k_{jz} w_{szn_s,j} & - \sum_{j=a,b} k_{jy} z_j w_{syn_s,j} & \sum_{j=a,b} k_{jx} z_j w_{sxn_s,j} & 0 \end{bmatrix}_{n_s \times 5} \quad (5.61)$$

$$\mathbf{K}_{sb} = \begin{bmatrix} \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si1} w_{si1} & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si1} w_{si2} & \cdots & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si1} w_{sin_s} \\ \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si2} w_{si1} & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si2} w_{si2} & \cdots & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{si2} w_{sin_s} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{sin_s} w_{si1} & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{sin_s} w_{si2} & \cdots & \sum_{j=a,b} \sum_{i=x,y,z} k_{ji} w_{sin_s} w_{sin_s} \end{bmatrix}_{n_s \times n_s} \quad (5.62)$$

$$\mathbf{M}_r = \begin{bmatrix} M & 0 & 0 & 0 & 0 \\ 0 & M & 0 & 0 & 0 \\ 0 & 0 & M & \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y dV_j & -\sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x dV_j \\ 0 & 0 & \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y dV_j & I_x + \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y^2 dV_j & -\sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x r_y dV_j \\ 0 & 0 & -\sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x dV_j & -\sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x r_y dV_j & I_y + \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x^2 dV_j \end{bmatrix} \quad (5.63)$$

$$\mathbf{M}_{ra} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \sum_{j=1}^{n_e} \int_{V_j} \rho_j \sum_{k=1}^{n_n} N_k \phi_{1k} dV_j & \cdots & \sum_{j=1}^{n_e} \int_{V_j} \rho_j \sum_{k=1}^{n_n} N_k \phi_{n_m k} dV_j \\ \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y \sum_{k=1}^{n_n} N_k \phi_{1k} dV_j & \cdots & \sum_{j=1}^{n_e} \int_{V_j} \rho_j r_y \sum_{k=1}^{n_n} N_k \phi_{n_m k} dV_j \\ -\sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x \sum_{k=1}^{n_n} N_k \phi_{1k} dV_j & \cdots & -\sum_{j=1}^{n_e} \int_{V_j} \rho_j r_x \sum_{k=1}^{n_n} N_k \phi_{n_m k} dV_j \end{bmatrix}_{5 \times n_m} \quad (5.64)$$

$$\mathbf{M}_a = \text{DIAG}\{1 \quad 1 \quad \cdots \quad 1\}_{n_m \times n_m} \quad (5.65)$$

$$\mathbf{K}_a = \text{DIAG}\{\omega_1^2 \quad \omega_2^2 \quad \cdots \quad \omega_{n_m}^2\}_{n_m \times n_m} \quad (5.66)$$

$$\mathbf{K}_b = \begin{bmatrix} k_{ax} + k_{bx} & 0 & 0 & 0 & k_{ax} z_a + k_{bx} z_b \\ 0 & k_{ay} + k_{by} & 0 & -k_{ay} z_a - k_{by} z_b & 0 \\ 0 & 0 & k_{az} + k_{bz} & 0 & 0 \\ 0 & -z_a k_{ay} - z_b k_{by} & 0 & z_a k_{ay} z_a + z_b k_{by} z_b & 0 \\ z_a k_{ax} + z_b k_{bx} & 0 & 0 & 0 & z_a k_{ax} z_a + z_b k_{bx} z_b \end{bmatrix} \quad (5.67)$$

with the generalized displacement vector taking the following form

$$\mathbf{q} = \{r_{s1} \ r_{s2} \ \cdots \ r_{sn_s} \ R_x \ R_y \ R_z \ \theta_x \ \theta_y \ q_1^1 \ q_2^1 \ \cdots \ q_{n_m}^1\}^T \quad (5.68)$$

and the generalized force vector taking the form of

$$\mathbf{S} = \{R_{s1} \ R_{s2} \ \cdots \ R_{sn_s}\}^T \quad (5.69)$$

$$\mathbf{F} = \{F_x \ F_y \ F_z \ F_{\theta x} \ F_{\theta y}\}^T \quad (5.70)$$

$$\mathbf{M} = \{M_1^1 \ M_2^1 \ \cdots \ M_{n_m}^1\}^T \quad (5.71)$$

Shape functions for the four-node isoparametric element, which means that displacements and geometry of the element are described by using the same shape functions, are used and given below

$$N_1(\xi, \eta) = \frac{1}{4}(1 - \xi)(1 - \eta) \quad (5.72)$$

$$N_2(\xi, \eta) = \frac{1}{4}(1 + \xi)(1 - \eta) \quad (5.73)$$

$$N_3(\xi, \eta) = \frac{1}{4}(1 + \xi)(1 + \eta) \quad (5.74)$$

$$N_4(\xi, \eta) = \frac{1}{4}(1 - \xi)(1 + \eta) \quad (5.75)$$

It can be observed that displacement modal participation factors of the stationary part can be solved through equation (5.58) provided the noise source excitations have been predetermined. In another word, the transfer function between the noise source excitations and the vibration transmitted to the housing has been established. Mobility transfer functions of the first six modes of the stationary part for harmonic force acting in pair on the stationary part and the rotating part in the x direction are shown in Figure 4.16.

(a)

(b)

Fig. 5.11 Mobility transfer functions of the stationary part modes

(a) r_{s1} mode (b) r_{s2} mode (c) r_{s3} mode (d) r_{s4} mode (e) r_{s5} mode (f) r_{s6} mode

5.3.1 Free vibration analysis

Finite element analyses of the flexible actuator and the stationary part are performed first. The 4-node shell elements (SHELL63) are used for the model of the simple flexible actuator and the 4-node solid elements (SOLID72) for the stationary part. The material properties used in the models are listed in Table 5.1. The calculated first seven natural frequencies of the flexible actuator and the first six natural frequencies of the stationary part are listed in Table 5.2 and Table 5.3, respectively. Figure 5.12 and Figure 5.13 show

the corresponding mode shapes of the flexible actuator and the stationary part, respectively.

<i>Materials</i>	<i>Young's Modulus (GPa)</i>	<i>Density (kg/m³)</i>	<i>Poisson's Ratio</i>
Aluminum	73	2700	0.3
Steel	200	7800	0.3

Table 5.1 Material properties used in the FE models

<i>Modes</i>	<i>Frequency (Hz)</i>
1	129.15
2	616.69
3	964.43
4	1602.2
5	2284.5
6	3085.3
7	3830.3

Table 5.2 Calculated natural frequencies of the flexible actuator

<i>Mode</i>	<i>Natural frequency (Hz)</i>
1	803.18
2	1444.9
3	1749
4	2022.5
5	2543.2
6	2785.3

Table 5.3 Calculated natural frequencies of the stationary part

Fig. 5.12 Mode shapes of the flexible actuator

Fig. 5.13 Mode shapes of the stationary part

First, free vibration of the VCM/actuators assembly, including actuator, pivot housing, coil and bearings, is analyzed. Natural frequencies of the system are predicted with the elastic deformation of the flexible actuator approximated by its first seven natural modes. The system frequencies can be classified into frequencies related to the vibration of the simple flexible actuator and those of the rigid-body motion of the assembly as shown in Table 5.4.

<i>Frequency description</i>	<i>Frequencies (Hz)</i>
Related to the rigid-motion of the assembly	551.97, 552.91, 954.54, 954.56, 1126.88
Related to the flexible actuator	129.1429, 616.93, 964.4657, 1602.63, 2284.51, 3085.57, 3830.31

Table 5.4 Natural frequencies of the VCM/actuators assembly

Free vibration of the VCM/actuator assembly-shaft-housing system is then performed. Natural frequencies are predicted with the elastic deformation of the flexible actuator and the stationary part approximated by first seven and first six natural modes, respectively.

To examine the effect of the flexibility of the stationary part on natural frequencies, natural frequencies of VCM/actuators assembly-shaft-housing system and those of VCM/actuators assembly are compared in Table 5.5. One may see from Table 5.5 that natural frequencies of VCM/actuators assembly-shaft-housing system have reduced due to the flexibility of the stationary part. Natural frequencies of the stationary part before and after assembly into the system are listed in Table 5.6. One can see that the natural frequencies of the stationary part have changed after it is added to the system due to the coupling between the stationary part and the rotating part as derived in Equation (5.58) - Equation (5.67).

<i>Mode</i>	<i>VCM/actuators assembly(Hz)</i>	<i>VCM/actuators assembly-shaft- housing system(Hz)</i>
1	129.1429	129.1428
2	616.93	616.92
3	964.4657	964.46
4	1602.63	1602.62
5	2284.51	2284.5068
6	3085.57	3085.57
7	3830.31	3830.31

Table 5.5 Effect of adding flexibility of the stationary part

<i>Mode</i>	<i>Stationary part (Hz)</i>	<i>VCM/actuators assembly-shaft-housing system(Hz)</i>
1	803.18	800.8
2	1444.9	1453.17
3	1749.	1749.46
4	2022.5	2025.03
5	2543.2	2551.82
6	2785.3	2786.31

Table 5.6 Changes of natural frequencies of the stationary part

5.3.2 Frequency response analysis

Harmonic forced responses of the top cover and the base plate are performed with frequencies up to 4000 Hz. The forces are applied to the stationary part and the rotating part in pair in the x direction. Figure 5.14 and Figure 5.15 show the displacements of the centre point of the top cover and the base plate, respectively. Resonant frequencies of the stationary part are observed at peaks of c and C, f and F, g and G, h and H, i and I, j and J in both graphs, which are attributed to the 1st, 2nd, 3rd, 4th, 5th and 6th mode, respectively. Peaks a and A, e and E are related to the rigid body motion of the pivot housing while peaks at b and B, d and D are due to the second and third mode of the flexible actuator, respectively. This shows that the motion of the VCM/actuators assembly can be transmitted to the top cover and the base plate via the flexible shaft and ball bearings.

Fig. 5.14 Displacement of the centre of the top cover

Fig. 5.15 Displacement of the centre of the base plate

5.4 Vibration measurement of a Seagate HDD

This experiment was performed for future validation of the mathematical model developed above when the simple actuator is substituted by the actual flexible components of VCM/actuators assembly. A Seagate HDD with three arms and two R/W heads is under test as shown in Figure 5.16. The measurement points include the top suspension tip, arm tip and the VCM tip.

Fig. 5.16 VCM/actuators assembly under test and points of interest

The velocity frequency response functions of the points of interest in the lateral direction without and with the top cover were measured by using Polytec PSV-300 system and dynamic signal analyzer HP35670A DSA. The experiment set-up is shown in Figure 5.17.

Fig. 5.17 FRF measurement set-up

A sinusoidal sweeping signal is produced by HP analyzer and applied to the VCM to excite the VCM/actuators assembly. LDV is employed to measure the motion of the selected points of interest. The excitation signal and LDV output are then fed to the four channels HP analyzer where a frequency response function between the velocity and the excitation signal is extracted for the range of frequencies from 100 Hz to 10 KHz.

Figure 5.18 and Figure 5.19 show the velocity frequency response functions in the vertical direction of all the points without and with the top cover, respectively.

Fig. 5.18 The measured velocity FRFs of the VCM/actuators assembly in the vertical direction

Fig. 5.19 The measured velocity FRFs of the VCM/actuators assembly-shaft-housing system in the vertical direction

5.5 Summary

The mathematical model for dynamic analysis of the entire VCM/actuators assembly-shaft-housing system has been developed in terms of displacement modal participation factors of the stationary part, the rigid-body motion of the pivot housing and displacement modal participation factors of the flexible components. With the derivation of the

governing equations, the link between the noise source excitations and the vibration transmitted to the housing due to the vibration of VCM/actuators assembly has been established. The mode shapes and natural frequencies of the flexible components are obtained by modal finite element analyses. A simple actuator is used to illustrate the programming implementation. Natural frequencies of the actuator and natural frequencies of the stationary part, before and after the stationary part is installed into the system, have been compared. The displacements of the top cover and the base plate are predicted under harmonic force as well. Finally, velocity frequency response functions of a Seagate HDD's VCM/actuators assembly without and with the top cover have been measured in the vertical direction for future validation of the developed mathematical model.

Chapter 6

Sound Radiation Analysis From The Housing

This chapter presents the numerical models for the acoustic part of vibro-acoustic analysis of HDDs. Sound radiation in terms of sound power level from the stationary part due to harmonic forcing is investigated in both idle and seek mode of a HDD. For idle noise, a few parametric studies have been presented on the effects of the disk geometry and number of disks.

6.1 Introduction

Fig. 6.1 Three subsystems in chain of vibro-acoustic analysis

In principle, all vibro-acoustic problems are coupled because there is always an action from the vibrating structure on the fluid and a reaction from the acoustic pressure distribution in the fluid on the structure as shown in Figure 6.1. However, for the current study, the coupling effect is not considered. This uncoupled analysis is also known as a pure acoustic analysis in which vibration generates noise only.

In the current study, the radiated sound power is used as the vibro-acoustic indicator. A numerical approach was presented for calculating the sound pressure and acoustic power radiated by vibrating structures using finite element method (Li and Shen, 2000). This

method can be readily applicable to any structural configurations at any frequencies of excitation under any boundary conditions. The following sound radiation calculation employs this approach.

Consider the top cover as a rectangular thin plate, fixed on four edges and set in an infinite rigid baffle as shown in Figure 6.2. The total acoustic power radiated from the plate can be obtained by integrating the far field acoustic intensity over a hemisphere of radius r (Xie *et. al.*, 2004),

$$W = \int_0^{2\pi} \int_0^{\pi} \frac{|p(\mathbf{r})|^2}{2\rho_0 c_0} r^2 \sin \theta d\theta d\phi \quad (6.1)$$

Where $p(\mathbf{r})$ is the complex acoustic pressure amplitude at a location in space expressed in spherical co-ordinates $\mathbf{r} = (r, \theta, \phi)$ at frequency ω of harmonic motion. ρ_0 and c_0 are the density of air and the speed of sound in air, respectively.

Fig. 6.2 Three subsystems in chain of vibro-acoustic analysis (Xie *et. al.*, 2004)

The sound pressure in (6.1) can be written in terms of the plate surface velocity using Rayleigh integral (Xie *et. al.*, 2004),

$$p(\mathbf{r}) = \frac{j\omega\rho_0}{2\pi} \int_S v(\mathbf{x}) \frac{e^{-jkR}}{R} ds \quad (6.2)$$

Where the integral is evaluated over the plate surface S with $v(\mathbf{x})$ being the complex surface normal velocity amplitude at location $\mathbf{x} = (x, y)$. R is the distance from the source point to observer point with $R = |\mathbf{r} - \mathbf{x}|$.

By applying the finite element method, the complex velocity of an arbitrary point on element e of the top cover can be expressed as follows through interpolation with shape functions (Li and Shen, 2000)

$$v(x, y) = v_e^T N_e^T(x, y) \quad (6.3)$$

Where v_e denotes complex velocities at nodes of element e , $N_e(x, y)$ are shape functions.

Substituting equation (6.3) into (6.2), the complex sound pressure can be rewritten as

$$p(r, \theta, \phi) = \sum_e \int_e \frac{j\omega\rho_0}{2\pi} v_e^T N_e^T(x, y) \frac{\exp(-jkR)}{R} dx dy \quad (6.4)$$

Recall

$$|p|^2 = p\bar{p} \quad (6.5)$$

Therefore, the total acoustic power in equation (6.1) is rewritten as

$$W = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{p\bar{p}}{2\rho_0 c_0} r^2 \sin \theta d\theta d\phi \quad (6.6)$$

Equation (6.4) is solved numerically by a commonly used Gauss numerical integration scheme and equation (6.6) an extended Simpson numerical integration scheme for the dual integration on the θ and ϕ variables on a surface of constant radius r .

6.2 Idle noise

6.2.1 Load Cases Studies

The vibration response calculated in Chapter 4 will now be used as an input to obtain sound radiation. Due to the complex of noise sources in a HDD, three load cases are considered when the disks are rotating at 5400 RPM. One is a point force acting at the position of edge $r = b_1, \theta = 0$ of the top disk. The others are the force applying in pair between the stationary part and the rotating part in Y direction and Z direction, respectively.

Figure 6.3 shows the radiated sound power under three forces. From the noise spectrum, it may be observed that unbalanced mode $(0, 0)$ and natural modes of the stationary part are found for forces on the disk and in Z direction. For the force acting in Y direction, only natural modes of the stationary part are excited. Unbalanced $(0, 1)$ modes, which are coupled with the stationary part in vibration analysis, cannot be seen in the sound spectrum due to its mode characteristics. All balanced modes also do not show in the noise spectrum.

Fig. 6.3 Radiated sound power from the top cover under different load cases

6.2.2 Parametric Studies

Noise radiated from the top cover is examined through a few parametric studies. The parameters studied include disk thickness, disk outer radius, and number of disks. Limited studies are conducted by varying the disk thickness from $0.75h_i$ to $1.25h_i$ and the outer radius from $0.875b_i$ to $1.125b_i$ where h_i and b_i are the current values of the i^{th} disk thickness and outer radius, respectively. The effect of disk number is examined through 1-disk, 2-disk and 4-disk configurations. A point force at the position of $r = b_1$, $\theta = 0$ on the top disk is applied and the disk is rotating at 5400 RPM.

The effect of disk thickness is shown in Figure 6.4. The acoustic power results indicate that only unbalanced (0, 0) modes and natural modes of the stationary part have been excited. Also with the increase of the disk thickness, unbalanced (0, 0) modes have higher frequencies.

Fig. 6.4 Radiated sound power from the top cover with different disk thickness

The effect of change of disk outer radius on the radiated acoustic power is compared in Figure 6.5. Similar with the effect of change of disk thickness, only unbalanced (0, 0) modes and natural modes of the stationary part have been excited in the noise spectrum.

But with the increase of the disk outer radius, unbalanced (0, 0) modes have lower frequencies.

Fig. 6.5 Radiated sound power from the top cover with different outer radius

The effect of disk number is compared in Figure 6.6. The results show that unbalanced (0, 0) modes and natural modes of the stationary part are the modes which will be excited in the noise spectrum.

Fig. 6.6 Radiated sound power from the top cover with different number of disks

6.3 Seek noise

From vibration models developed in Chapter 5, normal displacement, hence, normal velocity, of an arbitrary point on the top cover can be obtained and then used as input to calculate the radiated sound power. Due to the complex of noise sources in a HDD, three

load cases applying in pair between the stationary part and the rotating part in X , Y and Z direction are considered. Figure 6.7 shows the noise spectrum in terms of sound power.

Fig. 6.7 Radiated sound power from the top cover under different load cases

It can be seen from Figure 6.7 that peaks along lines b, e, f, g, h, i are due to natural modes of the stationary part; Peaks along lines a and d are associated with rigid body motion of the pivot housing; Peaks along line c are related to the third mode of the flexible actuator. The results indicate that seek noise level is determined by the forcing functions, the coupling between the rotating part and the stationary part and the amplification of resonances of the stationary part.

6.4 Summary

Sound radiation from the stationary part, particularly from the top cover, is calculated by using finite element method. For idle noise, the results indicate that unbalanced mode (0, 0) and natural modes of the stationary part are the key modes for sound radiation. Unbalanced (0, 1) modes, which are coupled with the stationary part in vibration analysis, cannot be seen in the sound spectrum. For seek noise, noise level is determined by the forcing functions, the coupling between the rotating part and the stationary part and the amplification of resonances of the stationary part.

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

Vibration and noise problems arising from vibro-acoustic interaction of micro-system components in HDD is unavoidable due to the inherent mechanical properties and operating processes in the system. These affect not only the performance of HDDs but may potentially limit its scope of application. This dissertation has presented novel fundamental methodology for studying the structure-borne noise generated from the vibration of the HDD housing due to the motion of the internal components which can be directly transmitted to the drive housing.

Based on the source-vibration transmission paths-sound radiation thread, this study has systematically explored the overall strategy for analysing vibro-acoustic interaction of the components and subsystems in a HDD in idle and seek modes.

(i) Experimental studies

Velocity measurements as well as sound radiation measurements have been performed. In addition, the acoustic measurement is consistent with the velocity measurement.

(ii) Vibration models

Two vibration models have been built to simulate the vibration transmission in the subsystems in a HDD for idle noise and seek noise, respectively. In particular, the

vibration transmitted to the housing, which is treated as an intermediate stage for sound radiation calculation, are explored with the development of a set of transfer functions between noise source excitations and induced vibration on the housing.

Vibration analysis of spindle/disks assembly-shaft-housing system

This mathematical model can be applied to analyze the dynamics of the housing due to the vibration of the spindle/disks assembly transmitted through the shaft via ball bearings. The governing equations of motion are derived in terms of displacement modal participation factors of the non-spinning part, rigid-body motion of the spindle and displacement modal participation factors of the disks, by using Lagrange's equation. The flexibility of the stationary part results in the stiffness coupling between the stationary part and the rotating part. In particular, the unbalanced vibration modes of the disks with zero and one nodal diameter, which are coupled with the rigid-body motion of the spindle, affect the vibration of the stationary part and vice versa due to bearing stiffness. Hence, these unbalanced modes of the disks are the key modes in the vibration transmission of the HDD in idle mode. In addition, the effects of component geometries and different load cases on the forced responses of the top cover and the disk are investigated.

Vibration analysis of the VCM/actuators assembly-shaft-housing system

This mathematical model has been developed to predict the vibration of the housing in HDD seek mode. The generalized displacement vector consists of the displacement modal participation factors of the stationary part, the rigid-body motion of the pivot housing and the displacement modal participation factors of the flexible components. Due to the complex nature of the geometries, finite element method is employed to determine the mode shapes and natural frequencies of the flexible components. These are then

incorporated into the equations of motion for the whole assembly. The effect of the flexibility of the stationary part on the natural frequencies is examined and forced responses of the top cover and base plate are obtained for a simple actuator.

(iii) Acoustic models

Sound radiation from the housing in idle and seek mode of a HDD are calculated with the vibration response obtained from the developed vibration models as the input. For idle noise, unbalanced mode (0, 0) and natural modes of the stationary part are the key modes for sound radiation. Unbalanced (0, 1) modes, although coupled with the stationary part in vibration analysis, do not show in the sound spectrum. For seek noise, noise level is determined by the forcing functions, the coupling between the rotating part and the stationary part and the amplification of resonances of the stationary part.

These numerical vibration and acoustic models provide some insight into vibro-acoustic interaction in a HDD, in particular, how vibration is transmitted from one part to another, how different noise sources will affect the vibration generated and how design modifications, for example, the physical geometry of a component, will affect the noise.

With these capabilities, the models developed in the current studies may enable the optimisation of vibration and consequent noise generated to be carried out at the design stage or to help to modify an existing model. It is becoming necessary to be able to predict accurately the noise produced from the design data. In addition, controlling acoustics at the design stage is easier, more effective and less costly.

7.2 Recommendations

Vibro-acoustic issues are becoming increasingly critical as higher density, high rotation speed and faster access time are required for future HDDs. Although the results obtained so far have been encouraging, much work remains to be done on enhancing the models to provide an accurate prediction of vibration transmitted to the housing, and the noise radiated.

(i) Vibration models

With further refinement of the models, it is hoped that an accurate prediction of vibration transmission to the housing, hence, the noise radiated, can be realized for both idle and seek modes.

Vibration analysis of spindle/disks assembly-shaft-housing system

The refinement to the model of the stationary part, especially to the base plate, is necessary. How to simulate the boundary conditions of the top cover is another factor that will affect the sound radiated greatly. It is also challenging to consider the case of non-constant rotation speed of the disks even though the governing equations have been derived in this study.

Vibration analysis of the VCM/actuators assembly-shaft-housing system

The above-mentioned refinements to the stationary part also apply to the model of the VCM/actuators assembly-shaft-housing system. In addition, more realistic finite element models of the head actuator will be used to derive the component mode shape and natural frequencies.

(ii) Noise sources excitations

In the current studies, the structural and acoustic models are subjected to harmonic forces applied to the subsystems and the sound power transfer functions were derived. For future studies, much effort is needed on modelling the noise source excitations, especially on determining their characteristics in terms of amplitude and frequency.

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