Narrowband and wideband DOA estimation with unknown number of sources

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Narrowband and Wideband DOA Estimation with Unknown Number of Sources

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गुरुर् ब्रह्म गुरुर्विष्णुः गुरुर्देवो महेश्वरः ।
गुरुर्साक्षात् परम्ब्रह्म तस्मै श्री गुरवे नमः ॥

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They are the form of absolute truth. I bow to them with gratitude.)*

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Abstract

Array processing has been an active research area for several decades. The advent of new signal processing techniques has maintained this topic afresh in the research community with new challenging problems. The estimation of direction-of-arrival (DOA) for instance has evolved from high-resolution to superresolution techniques, and of-late, is heading towards increasing the available degrees of freedom. Excited by these developments and the awaiting potential applications, we have considered the study of DOA estimation for narrowband and wideband sources in this thesis under various conditions.

Within an array processing system, the inter-dependence between the model-order estimation, DOA estimation and beamforming tasks reflects the sensitivity of one task to the outcome of the other. In view of this, beamformers have been designed in the past to incorporate robustness against look direction mismatch and array manifold errors. However, existing DOA estimation techniques are sensitive to the accuracy of estimated number of sources. In order to overcome this limitation, we propose a new narrowband DOA estimation technique which substitutes the noise subspace eigenvectors with a weight vector matrix. This allows one to obtain the spatial spectrum with unknown number of sources. Any error in model order estimation will therefore have no impact on the accuracy of DOA estimates.

Estimating the number of sources in the presence of wideband sources is a very challenging task considering the fact that existing techniques retrieve the model order either from a coherently-averaged covariance matrix or by the maximum likelihood approach. While the estimated model-order from the former method is susceptible to the initial estimates, the latter technique is computationally expensive. We therefore present a time-domain DOA estimation technique which provides distinct peaks along the source directions in its spatial spectrum without estimating the number of sources. The underlying idea relies on the array manifold approximation using Taylor series expansion across the signal bandwidth. The undesired derivative components are then suppressed by the proposed optimization problem. The effectiveness of this technique is verified with a detailed mathematical analysis and simulations.

With finite-ordered Taylor series expansion, the array manifold approximation is accurate for sources with a percentage bandwidth less than 30%. For larger source bandwidth, the estimation accuracy of the time-domain technique decreases. We therefore transform the problem to frequency domain and perform DOA estimation on a regulated signal bandwidth.

Environmental factors such as multipath, dispersion and scattering adversely affect the performance of existing DOA estimation techniques in many applications such as radio wave communication, seismic and underwater acoustic applications. Existing techniques such as matched-field processing incorporate the speed profile and introduce robustness to random perturbations in speed. However, the estimation of speed profile is itself challenging and inaccurate many times. We therefore consider redefining array manifold approximation such that robustness can be incorporated to dispersion. The optimization problem introduces derivative compensation with respect to the wavenumber which absorbs the effect of dispersion in the signal model. With this approach, one requires to only estimate the propagation speed at only a reference frequency instead of the entire source bandwidth.

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Acronyms

MVDR	Minimum Variance Distortionless Response
EVD	EigenValue Decomposition
SVD	Singular Value Decomposition
MUSIC	MUltiple Signal Classification
ESPRIT	Estimation of Signal Parameters via Rotational Invariance Technique
FIB	Frequency Invariant Beamforming
ISSM	Incoherent Signal Subspace Method
CSSM	Coherent Signal Subspace Method
TOPS	Test for Orthogonality of Projection Subspaces
BFW-MUSIC	Beamforming Framework Wideband MUSIC
STCM	STeered Covariance Matrix
GEV	Generalized EigenValue
ULA	Uniform Linear Array
SNR	Signal-to-Noise Ratio
RMSE	Root Mean Squared Error
PHAT	PHase Transform
GSCT	Generalized State Coherence Transform
KDE	Kernel Density Estimator
STCM-MVDR-MS	Multi-Stage STCM-MVDR
NTVE	Normalized Time difference Vector Error

List of Notations

Chapters 1 and 2

Δf	Fractional bandwidth of a wideband source
θ	Source direction
ϕ	Arbitrary look direction
L	Number of sensors
M	Number of sources
F	Number of DFT/FFT points
N	Number of snapshots
$\mathbf{a}(\theta)$	Steering vector in the direction θ for a narrowband signal
$\mathbf{x}(n)$	Vector of sensor array snapshot
$\mathbf{s}(n)$	Vector of sources
$\mathbf{v}(n)$	Vector of additive noise
$\mathbf{A}(\Theta)$	Array manifold matrix with steering vectors forming its columns
$\mathbf{P}_{xx}(f)$	Spectral density matrix of $\underline{\mathbf{x}}(f)$
$\mathbf{P}_{ss}(f)$	Spectral density matrix of $\underline{\mathbf{s}}(f)$
$\mathbf{P}_{vv}(f)$	Spectral density matrix of $\underline{\mathbf{v}}(f)$
$\mathbf{R}$	Array covariance matrix
$\mathbf{R}_{ss}$	Source covariance matrix
$\mathbf{R}_{vv}$	Noise covariance matrix
$\mathbf{w}$	Weight vector of a beamformer
$\mathbf{U}_s$	Matrix with signal subspace eigenvectors
$\mathbf{U}_n$	Matrix with noise subspace eigenvectors
$\mathbf{\Lambda}_s$	Diagonal matrix with signal subspace eigenvalues
$\mathbf{\Lambda}_n$	Diagonal matrix with noise subspace eigenvalues
σ_v^2	Noise power
J	Number of frequency domain snapshots
D	Temporal delays used in the time-delay line
$\underline{\mathbf{x}}(f)$	Frequency domain sensor snapshot
$\mathbf{a}(f_k, \theta)$	Steering vector in the direction θ for a wideband signal at frequency f_k
$\bar{\mathbf{x}}(n)$	Spatio-temporal sensor snapshot
$\mathbf{T}(f_k, \phi)$	Transformation matrix. Focussing matrix in the context of CSSM.
$\mathbf{R}(f_k)$	Array covariance matrix at frequency f_k
$\mathbf{R}_{\text{cssm}}$	Focussed covariance matrix employed in CSSM
$\mathbf{R}_{\text{fib}}$	Frequency invariant covariance matrix employed in FIB based DOA estimation
$\mathbf{R}(\phi)$	Steered covariance matrix used in STCM-MVDR

Chapter 3

$\mathbf{w}_\phi$	Weight vector evaluated at each look direction ϕ
$\mathbf{P}_{\mathbf{a}_\phi}$	Projection matrix of steering vector $\mathbf{a}(\phi)$
χ	Lagrangian parameter
$\chi_{\min}$	Minimum eigenvalue of a generalized eigenvalue problem
$\chi_{\mathbf{A},\min}$	Minimum eigenvalue of matrix $\mathbf{A}$
$\text{tr}\{\cdot\}$	Trace of a matrix
$\mathbf{W}$	Matrix formed with weight vectors from different directions
$\mathcal{R}\{\mathbf{A}\}$	Range space of matrix $\mathbf{A}$
$\bar{\Lambda}_s$	$\Lambda_s - \sigma_v^2 \mathbf{I}_L$.

Chapter 4

δf	Frequency deviation $f - f_0$
P	Order of derivative terms
$\mathbf{d}_p(f_0, \theta)$	p th derivative of the steering vector $\mathbf{a}(f_0, \theta)$ w.r.t. frequency
$\mathbf{D}_p(\theta)$	p th derivative of the array manifold matrix $\mathbf{A}(f_0, \Theta)$ w.r.t. frequency
$\mathbf{D}_{f_0, \phi}$	Derivative matrix with $1 : P$ order derivative vectors from look direction ϕ
$\mathbf{P}_{\mathbf{D}_\phi}$	Projection matrix of $\mathbf{D}_{f_0, \phi}$
$\bar{\mathbf{a}}(f_0, \phi)$	Spatio-temporal steering vector
$\bar{\mathbf{D}}_{f_0, \phi}$	Derivative matrix with $1 : P$ order derivative vectors of $\bar{\mathbf{a}}(f_0, \phi)$
$\mathbb{R}\{\cdot\}$	Real part of the component
$\mathbf{R}_y(\phi)$	Covariance matrix of data projected onto the derivative vector space using $\mathbf{D}_{f_0, \phi}$

Chapter 5

B	Number of frequency bins used
$\underline{\mathbf{X}}(k)$	Data matrix at a frequency f_k
$\underline{\mathbf{X}}$	Data matrix constructed over B frequency bins
$\mathbf{R}_{\text{av}}$	Average covariance matrix estimated over B frequency bins
Q	Number of subbands
$\bar{\mathbf{E}}_d$	Extended signal subspace matrix of $\mathbf{R}_{\text{av}}$

Chapter 6

ν_p	Phase velocity
ν_g	Group velocity
κ_f	frequency dependent wavenumber
$\mathbf{a}(\kappa_f, \phi)$	Steering vector from look direction ϕ

Chapter 1

Introduction

1.1 Background

Any physical process can be transduced into an electrical signal using a sensor which is then processed to extract necessary information. If multiple sensors are employed to capture a signal, additional information about the physical phenomenon can be extracted. The stream of signal processing which studies the unified treatment of signals acquired from an array of sensors is widely known as *Sensor Array Signal Processing*. The wide range of potential applications has kept array processing as an active area of research over the last three decades. The concepts, problems and solutions have been well explained in several books [1–4] and review articles [5, 6]. A few applications where sensor arrays are used are listed below [2]:

- (i) **RADAR** - RAdio Detection And Ranging is one of the well-known applications employing array signal processing. The use of antenna array over a single antenna enhances target detection rate as well as estimation of target range and speed. Thanks to array processing and adaptive filter theory, moving objects can be conveniently tracked from the received signals. MIMO-RADAR has gained recent

interest of the research community due to its pertinence in commercial and defense related applications.

- (ii) **SONAR** - SOund Navigation And Ranging extends the application of RADAR to acoustic signals underwater. With the medium uncertainties posing complicated challenges, the use of sensor array aids in improving the system performance.
- (iii) **Communications** - The seamless use of wireless communication for several applications increases the demand for bandwidth. Besides the conventional use of sensor arrays for signal enhancement and interference suppression, arrays are used to increase the channel capacity as well.
- (iv) **Seismology** - The area of reflection seismology applicable for seismic exploration uses geophone arrays to record reflections of a seismic event which can subsequently be used to construct subsurface images. Geophone arrays can also be used to detect and localize underground seismic events.
- (v) **Speech processing** - Microphone arrays are gaining popularity in order to enhance the quality of recorded speech by denoising and dereverberation. DOA estimation and blind speech/audio source separation have been another applications where the use of microphone array is advantageous.

Several other applications including the field of medical diagnosis (EEG, ECG, tomography) and radio astronomy also use sensor arrays. This explains the need for such an intensive research interest in the area of array signal processing. Several signal processing algorithms have been developed to tackle problems specific to applications. Among them, three widely observed problems are:

- **Signal detection** - The detection of all sources present in the scenario is the foremost task to be performed with the acquired signal. In RADAR and SONAR applica-

tions, the detection of targets is a very critical task which is followed by parameter estimation. Other tasks such as DOA estimation and beamforming are directly or indirectly sensitive to the accurate estimation of the number of sources. Since the order of the signal model signifies the number of sources present, this problem is also termed as model order estimation. Most common approach for signal detection is by formulating a multiple hypothesis test.

- Direction-of-Arrival (DOA) estimation - Once the sources present in a scenario are detected, additional information from the array outputs need to be extracted. The source directions are also preserved in the array outputs due to the spatial separation of array elements. This permits the signal model to be parameterized with source directions. The DOA estimates may serve as a prerequisite for beamforming.
- Beamforming - Constructive combination of signals across the sensors to recover the signal is termed as beamforming, since the signal processor digitally focusses a beam towards the direction of interest by providing appropriate weights to the sensor outputs. Advanced design of the weights include constraints for interference suppression and provide robustness to array manifold errors. Evidently, a DOA estimation module may be followed by a beamformer in an array processing system.

The sequence and inter-relation between the three tasks are shown in Figure 1.1.

1.2 Narrowband vs Wideband Signals

The system design in any application, beginning from the choice of sensors to the front-end design and the digital architecture, rely on the signal characteristics. A signal is first classified as either narrowband or wideband in nature observing its fractional bandwidth

Figure 1.1: Relation between the three major problems observed in array processing.

which is defined as

$$\Delta f = \frac{f_u - f_l}{\frac{1}{2}(f_u + f_l)}, \quad (1.1)$$

where f_l and f_u are the lowest and highest frequencies of the signal. A signal with $\Delta f \geq 0.1$ is classified as a wideband source [6], whereas the fractional bandwidth of a narrowband signal satisfies $\Delta f \ll 0.01$. We adhere to this definition across this thesis. While specific applications, such as wireless communications, further classify signals with $\Delta f > 0.2$ as ultra-wideband signals [7, 8], other applications such as speech processing consider speech sources as wideband sources although Δf is significantly larger than 0.2. In this thesis, we treat such sources as wideband sources.

Besides the fractional bandwidth, we encounter other measures of bandwidth for wideband sources such as the percentage bandwidth and the normalized bandwidth.

Percentage bandwidth of a wideband source is the fractional bandwidth expressed as a percentage, i.e., $\Delta f \times 100\%$. The normalized bandwidth of a source is defined as the bandwidth measured by normalizing all the frequencies w.r.t. the sampling frequency f_s and is evaluated as $(f_u - f_l)/f_s$.

1.3 Motivation and Objectives

As observed from Figure 1.1, the three array processing modules are very closely related. The state-of-the-art narrowband DOA estimation techniques such as multiple signal classification (MUSIC) and estimation of signal parameters via rotational invariance techniques (ESPRIT) provide good estimates with high resolution assuming accurate knowledge of the number of sources present. Existing beamforming techniques rely significantly on the source directions provided by the DOA estimation techniques. While robust beamformers are designed to handle errors in source directions and array geometry, existing DOA estimation techniques are observed to be susceptible to model-order estimation errors.

Under very low signal-to-noise (SNR) conditions, or when the number of sensor snapshots are limited, model-order estimation techniques are more likely to identify incorrect number of sources [9]. Although existing narrowband DOA estimation techniques can estimate source directions under such adverse conditions, providing inaccurate number of sources to these techniques results in incorrect DOA estimates. The model-order estimation error propagates through various stages resulting in degradation of system performance. This often translates to the need for DOA estimation techniques which are robust towards model order estimation errors. One approach is to resort to beamforming techniques which identify source directions from the steered response peaks. However, obtaining the resolution of subspace-based techniques is challenging with this approach.

Towards this, a narrowband DOA estimation technique was recently proposed in [9] which evaluates the spatial spectrum independent of the estimated model order. With the beamforming framework, the performance of this technique was shown to approach that of the MUSIC algorithm. However, the precise operation of the underlying optimization problem has not been analyzed in detail. One of the main objectives of this thesis is to provide a proper mathematical treatment to this algorithm. In this study, we also justify that the technique is indeed a “MUSIC-like” algorithm with its performance converging asymptotically to the MUSIC algorithm. This technique has limited performance at a larger computational cost relative to the MUSIC algorithm. We therefore aim to overcome these limitations with a modified approach.

A similar problem of incorrect model order estimation adversely affecting the DOA estimation task is encountered for wideband sources. While the maximum likelihood approach is computationally expensive, model-order estimation on coherently averaged covariance matrix across the signal bandwidth is susceptible to focussing errors. It is therefore essential to have a DOA estimation technique which is insensitive to the estimated model order. Noting the advantages of the MUSIC-like algorithm proposed for narrowband case earlier, it is imperative to formulate a wideband DOA estimation technique which does not rely on model-order estimation. This forms our second objective.

Radio wave propagation in the earth’s atmosphere is subject to many influences such as refraction, reflection, diffraction, fading and scattering. The change in medium due to atmospheric precipitation is known to attenuate signals beyond 10 GHz due to scattering [10]. With rain drops accounting for the maximum attenuation, it is modeled as a function of rain-rate which defines the density of rain drops. Besides this attenuation, water particles ordain as a refractive medium which introduces dispersion. Dispersion is a phenomenon in which the propagation speed changes with the signal frequency. Due to the varying density of clouds and rain, attenuation and dispersion changes with time.

It is therefore difficult to precisely model these factors and compensate the same. Imprecise knowledge of the propagation speed variations adversely affects the performance of existing wideband DOA estimation techniques. This emphasizes the need for introducing robustness against dispersion in wideband DOA estimation. Addressing this problem can aid in improving source localization performance in other applications such as seismic and underwater acoustic applications. This serves as a motivation for us to develop a wideband DOA estimation technique for dispersive medium which is robust to changes in speed with frequency.

1.4 Contributions of this Thesis

Our first contribution is dedicated to the mathematical analysis of a recently proposed narrowband DOA estimation technique [9] which is insensitive to the estimated model order. Designed with a beamforming framework, the performance of this technique is shown to approach that of the multiple signal classification (MUSIC) algorithm. We perform a detailed analysis of the optimization problem which exhibits the prime difference between this technique with the minimum variance distortionless response (MVDR) algorithm and show that it degenerates to the MUSIC algorithm asymptotically. We then rederive the bounds for the sole parameter from the solution under the desired spatial spectrum conditions. From the working principle of this algorithm, we identify a new criteria for DOA estimation which outperforms the technique proposed in [9] and approaches the performance of the MUSIC algorithm. The reduced computational cost proves the proposed technique to be more appealing over that proposed in [9].

In the second contribution we propose a wideband DOA estimation technique which is insensitive to errors in the estimated model order. For wideband sources with fractional bandwidth up to 30%, the steering vector across the bandwidth can be approximated

using Taylor series expansion w.r.t. the frequency deviation from a reference frequency. Employing Parseval's relation, we deduce an approximated time-domain signal model for wideband sources. The MUSIC-like algorithm studied in the previous contribution is then extended for wideband DOA estimation by incorporating a quadratic derivative term which reduces the problem to a narrowband problem. Besides the ability to estimate source directions with variable bandwidth, this time-domain technique requires reduced number of snapshots for DOA estimation.

The time-domain wideband DOA estimation technique relies on the validity of Taylor series approximation of array manifold across the signal bandwidth. With the order of Taylor series fixed due to design considerations, the array manifold approximation error increases with the source bandwidth which, as a consequence, causes a reduction in DOA estimation accuracy. We therefore maintain the manifold approximation in the frequency domain and construct a covariance matrix with regulated signal bandwidth within which the Taylor series approximation introduces minimum error. The resultant derivative-constrained frequency domain wideband DOA estimation technique then forms our third contribution. In order to exploit the DOA information across the entire signal bandwidth, we present a multi-band DOA estimation approach which improves the estimation accuracy.

In our fourth contribution, we consider wideband DOA estimation for an application scenario in which the signals propagate through dispersive medium. The introduced amount of dispersion is often an unknown function of rain-rate or cloud density. If we model the change in the propagation speed as an unknown nonlinear function of frequency, existing wideband DOA estimation techniques suffer from poor performance. We therefore propose a technique with derivative constraints w.r.t. the wavenumber. This technique introduces robustness to changes in the propagation speed across the signal bandwidth and can be extended to seismic and underwater acoustic applications

when the dispersion profile is a continuous function of frequency.

The organization of this thesis is as follows: Chapter 2 provides the necessary background with the fundamentals of array processing. In Chapter 3, we present detailed analysis of the MUSIC-like algorithm and propose a new narrowband DOA estimation technique. Chapter 4 presents a new time-domain wideband DOA estimation technique which does not rely on model-order estimation. In Chapter 5, we propose a frequency-domain DOA estimation algorithm employing derivative constraints. Wideband estimation in dispersive medium is then presented in Chapter 6. Chapter 7 follows with the solution provided for unambiguous DOA estimation of speech sources under reverberant environment. Chapter 8 concludes the thesis with possible future directions.

1.5 Statement of Originality and Publications Related to this Thesis

To the extent of the author's knowledge, the following contributions of this thesis are original:

- (i) The detailed mathematical analysis of the MUSIC-like algorithm and the derivation of bounds for its parameter presented in Chapter 3. This work has been submitted for publication [J4].
- (ii) The array manifold approximation using Taylor series and the time-domain wideband DOA estimation technique presented in Chapter 4. The publication related to this work is [J1].
- (iii) The derivative-constrained wideband DOA estimation discussed in Chapter 5 has been accepted for publication [J2].

- (iv) DOA estimation applicable for dispersive medium as discussed in Chapter 6 based on Taylor series expansion w.r.t. the wavenumber. The manuscript is under preparation.

Chapter 2

Fundamentals of Array Processing

This chapter provides the necessary prerequisites to array signal processing for understanding and appreciating the subsequent chapters. The chapter starts with the study of wave propagation and its sampling in the spatial domain. Defining a signal model for the acquired narrowband signals first, we study the process of beamforming. We then review a few subspace-based narrowband DOA estimation techniques and examine the problem of model-order estimation using hypothesis testing. The discussion transits to wideband source scenario under which we discuss the tasks of beamforming, DOA estimation and model-order estimation. The study of wideband DOA estimation is performed in detail as it serves as a review of existing techniques required for subsequent chapters.

2.1 Wave Propagation and Spatial Sampling

Consider a wave propagating in a homogeneous lossless medium. The wave equation in rectangular coordinates is then given by [11]

$$\nabla^2 s(t, \mathbf{r}) - \frac{1}{\nu^2} \frac{\partial^2}{\partial t^2} s(t, \mathbf{r}) = 0, \quad (2.1)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

is the Laplacian operator expressed in the Cartesian coordinates (x, y, z) , $s(t, \mathbf{r})$ is the signal at time t and $\mathbf{r}$ is the spatial position in the medium and ν is the wave propagation speed. Equation (2.1) is termed as the homogeneous wave equation since the left hand side of the equation is equal to zero indicating that there is no external force or input acting at $\mathbf{r}$.

Defining A as the amplitude and f as the frequency of a complex sinusoidal signal

$$s(t) = A \exp(j2\pi ft) \quad (2.2)$$

propagating at a spatial point $\mathbf{r}$ in the medium, the solution to the wave equation leads to the plane wave:

$$\begin{aligned} s(t, \mathbf{r}) &= s(t - \boldsymbol{\alpha} \cdot \mathbf{r}), \\ &= A \exp(j2\pi f(t - \boldsymbol{\alpha} \cdot \mathbf{r})) = A \exp(j(\omega t - \boldsymbol{\kappa} \cdot \mathbf{r})) \end{aligned} \quad (2.3)$$

where $\boldsymbol{\alpha} = (\alpha_x, \alpha_y, \alpha_z)$ is the slowness vector with $|\boldsymbol{\alpha}| = 1/\nu$, $\boldsymbol{\kappa} = (\kappa_x, \kappa_y, \kappa_z) = \kappa \mathbf{u}$ with $|\boldsymbol{\kappa}| = \kappa = \frac{2\pi f}{\nu}$ is the corresponding wavenumber vector and $\omega = 2\pi f$ is the angular frequency. The unit vector $\mathbf{u}$ signifies the direction of wave propagation. From (2.3), we note that the wave measured at an arbitrary point in a lossless medium is equal to the signal time-delayed by a factor $\boldsymbol{\alpha} \cdot \mathbf{r}$ with respect to the origin of the coordinate system. The wavenumber vector and the slowness vector are related by

$$\boldsymbol{\kappa} = \omega \boldsymbol{\alpha}. \quad (2.4)$$

Deploying an array of sensors in the propagation medium, we acquire the plane

wave at different spatial locations. The time delay at each spatial location is measured with respect to a reference point, generally chosen as the origin of the coordinate system. Figure 2.1 illustrates a uniform linear array (ULA) with L sensors positioned at

Figure 2.1: Illustration of an L -sensor ULA sampling a plane wave from direction θ .

$\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_L$ sampling a plane wave arriving from an angle θ w.r.t. the array axis. Denoting the position of the first sensor as the origin of the coordinate system and the array axis coinciding with the abscissa, the array is completely defined by the inter-element spacing d and the number of sensors L . When the array is positioned in the farfield of the complex source defined by (2.2), the sampled signal at the m th sensor is given by

$$x_m(t) = A \exp(j(\omega t - \boldsymbol{\kappa} \cdot \mathbf{r}_m)) = A \exp(j(\omega t - (m-1)\kappa d \cos \theta)). \quad (2.5)$$

In its vector form, the outputs of all the sensors can be written as

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_L(t) \end{bmatrix} = \begin{bmatrix} 1 \\ \exp(-j\kappa d \cos \theta) \\ \exp(-j2\kappa d \cos \theta) \\ \vdots \\ \exp(-j(L-1)\kappa d \cos \theta) \end{bmatrix} s(t). \quad (2.6)$$

When M sources are generating plane waves, the sensor outputs at a specific time snapshot can be written as

$$\mathbf{x}(t) = \sum_{m=1}^M \mathbf{a}(\theta_m) s_m(t), \quad (2.7)$$

where

$$\mathbf{a}(\theta_m) = [1 \ \exp(-j\kappa d \cos \theta_m) \ \dots \ \exp(-j(L-1)\kappa d \cos \theta_m)]^T \quad (2.8)$$

is the source position vector or the steering vector. Stacking all the source position vectors from directions $\Theta = [\theta_1, \dots, \theta_M]^T$ into the columns of an array manifold matrix $\mathbf{A}(\Theta) = [\mathbf{a}(\theta_1) \ \mathbf{a}(\theta_2) \ \dots \ \mathbf{a}(\theta_M)]$ and defining the source signal vector as $\mathbf{s}(t) = [s_1(t) \ s_2(t) \ \dots \ s_M(t)]^T \in \mathbb{C}^{M \times 1}$, the array snapshots can be modeled as

$$\mathbf{x}(t) = \mathbf{A}(\Theta)\mathbf{s}(t) + \mathbf{v}(t), \quad (2.9)$$

where $\mathbf{v}(t) = [v_1(t) \ v_2(t) \ \dots \ v_L(t)]^T \in \mathbb{C}^{L \times 1}$ is the vector of additive noise at each sensor. Sampling the sensor data with $t = nT_s$, where T_s is the sampling interval and n is the time index, we have

$$\mathbf{x}(n) = \mathbf{A}(\Theta)\mathbf{s}(n) + \mathbf{v}(n). \quad (2.10)$$

Since only narrowband sources are considered in obtaining the expression (2.10), it can be used as a narrowband signal model. Signal model for wideband sources will be introduced

in Section 2.3.

In the analysis of observed sensor snapshots, the sensor noise is generally modeled as a Gaussian random process while the source signals can either be deterministic or random, depending on application. For random uncorrelated sources, if we define $\mathbf{R}_{ss}$ as the $M \times M$ source covariance matrix, the second-order statistics of the sensor snapshots is obtained by the array covariance matrix as

$$\mathbf{R} = \mathbb{E}\{(\mathbf{x}(n) - \bar{\mathbf{x}})(\mathbf{x}(n) - \bar{\mathbf{x}})^H\} = \mathbf{A}(\Theta)\mathbf{R}_{ss}\mathbf{A}^H(\Theta) + \mathbf{R}_{vv}, \quad (2.11)$$

where $\bar{\mathbf{x}} = \mathbb{E}\{\mathbf{x}(n)\}$ is the mean vector of $\mathbf{x}(n)$ and $\mathbf{R}_{vv} \in \mathbb{C}^{L \times L}$ is the noise covariance matrix. For applications with non-Gaussian sources, higher-order statistics of the sensor snapshots preserve additional information which has been exploited for both DOA estimation [12] and beamforming [13].

2.2 Narrowband Array Processing

In this section, we discuss existing beamforming, DOA estimation and model order estimation techniques for narrowband sources. For such a scenario, the signal model is given by (2.10). It is important to note that the signal model in (2.10), as well as the techniques being presented henceforth, are not restricted to uniform linear array geometry.

2.2.1 Beamforming

Beamforming is the process of combining signals from the array elements such that the source from a specific direction of interest constructively add up while the signals arriving from other directions are suppressed. The most primitive beamformer is the delay-and-sum beamformer which compensates for the path delay of a source signal from the desired

direction before accumulating the compensated outputs. An equivalent phase compensation can restore the desired signal when processed in the frequency domain. If we denote the phase compensation as a vector of complex weights, the amplitude of these weights provide additional degree-of-freedom (DOF) enabling the suppression of interferers from other directions. A typical narrowband beamformer is shown in Figure 2.2. The objective of beamforming algorithms is to design the complex weight vector $\mathbf{w}$ considering the design constraints.

Figure 2.2: A typical narrowband beamformer with complex weights.

Design of the beamformer can either be data dependent or independent. Data independent design leads to the domain area of array pattern synthesis. For real-time processing, data-dependent adaptive beamformers are advantageous as they incorporate environmental factors into the design. Here, we briefly discuss minimum variance distortionless response (MVDR) beamformer [14].

As the name suggests, the output of an MVDR beamformer is expected to provide an undistorted signal from the desired direction while maintaining a minimum output power. If $\mathbf{a}(\theta_0)$ denotes the steering vector from direction θ_0 , the source signal from this direction can be maintained undistorted at the beamformer output with weight vector $\mathbf{w}$

chosen such that $\mathbf{w}^H \mathbf{a}(\theta_0) = c$, where $c > 0$ is a constant equal to the gain of the array.

Given the beamformer output $y(n) = \mathbf{w}^H \mathbf{x}(n)$, the output power can be expressed as

$$\begin{aligned} P &= \mathbb{E}\{y^2(n)\} = \mathbb{E}\{|\mathbf{w}^H \mathbf{x}(n)|^2\} \\ &= \mathbf{w}^H \mathbf{R} \mathbf{w}, \end{aligned} \quad (2.12)$$

where $\mathbf{R} = \mathbb{E}\{\mathbf{x}(n)\mathbf{x}^H(n)\}$ is the array covariance matrix of the zero-mean sensor snapshots across the array. The weight vector for an MVDR beamformer can therefore be obtained by solving the following optimization problem:

$$\begin{aligned} &\underset{\mathbf{w}}{\text{minimize}} \quad \mathbf{w}^H \mathbf{R} \mathbf{w} \\ &\text{subject to} \quad \mathbf{w}^H \mathbf{a}(\theta_0) = c. \end{aligned} \quad (2.13)$$

The Lagrangian function for this problem is given by

$$L(\mathbf{w}, \lambda) = \mathbf{w}^H \mathbf{R} \mathbf{w} + \lambda(\mathbf{w}^H \mathbf{a}(\theta_0) - c), \quad (2.14)$$

where λ is the Lagrangian parameter. With $\mathbf{R}$ assumed to be a positive definite matrix, (2.14) is a convex function. Solving for the global minimum of (2.14), we obtained the solution for $\mathbf{w}$ as

$$\mathbf{w} = \frac{\mathbf{R}^{-1} \mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0) \mathbf{R}^{-1} \mathbf{a}(\theta_0)}. \quad (2.15)$$

Substituting (2.15) into (2.12), the beamformer output power is given by

$$P_{\text{mvdr}} = \frac{1}{\mathbf{a}^H(\theta_0) \mathbf{R}^{-1} \mathbf{a}(\theta_0)}. \quad (2.16)$$

We note that the optimization problem in (2.13) consists of a single linear constraint. It is straightforward to extend this beamformer to incorporate multiple constraints to, for

instance, suppress an interference from a known direction. The beamformer is then called linear constrained minimum variance (LCMV) beamformer. From (2.8), we observe that the steering vector is a function of source direction, array geometry, signal frequency and the propagation speed. Extensive work has been done to incorporate robustness to errors in these factors into the beamformer formulation [15].

2.2.2 DOA Estimation

In the previous section, beamforming is achieved by providing weights corresponding to the desired source direction in the spatial domain. By evaluating the beamformer output power for all possible directions, we obtain the spatial spectrum which exhibits distinct peaks along the source directions. If we denote ϕ as the scanning direction which sweeps the entire azimuth range, the spatial spectrum is given by

$$P_{\text{mvdr}}(\phi) = \frac{1}{\mathbf{a}^H(\phi)\mathbf{R}^{-1}\mathbf{a}(\phi)}. \quad (2.17)$$

The expression in (2.17) evaluates the power in each direction ϕ in contrast to the beamforming context where the output power, defined in (2.16), is evaluated only along the source direction.

The MVDR algorithm and other high resolution spectral estimation techniques such as linear prediction [16] and maximum entropy [17] have been outperformed by techniques which exploit eigen structure within the array correlation matrix $\mathbf{R}$. Among them, the well-known multiple signal classification (MUSIC) algorithm [18] and estimation of signal parameters via rotational invariance technique (ESPRIT) [19] are discussed here, while the readers are referred to [2] for a description of other techniques.

2.2.2.1 MUSIC Algorithm

Since the array covariance matrix $\mathbf{R}$ is Hermitian symmetric, it can be subjected to eigen decomposition as

$$\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H, \quad (2.18)$$

where $\mathbf{U}$ and $\mathbf{\Lambda}$ are the eigenvector and eigenvalue matrices, respectively. The eigenvectors in $\mathbf{U}$ corresponding to the largest M eigenvalues form the columns of $\mathbf{U}_s$, while the remaining eigenvectors form the noise subspace matrix $\mathbf{U}_n$. We therefore have

$$\begin{aligned} \mathbf{R} &= \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix} \\ &= \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^H, \end{aligned} \quad (2.19)$$

where $\mathbf{\Lambda}_s \in \mathbb{C}^{M \times M}$ and $\mathbf{\Lambda}_n \in \mathbb{C}^{L-M \times L-M}$ are the diagonal matrix of signal and noise subspace eigenvalues. The MUSIC algorithm can be explained based on the following proposition.

Proposition 2.1 *If the sources are not coherent and the uncorrelated additive noise $\mathbf{v}(n)$ has equal variance σ_v^2 across all sensors, then the array manifold matrix $\mathbf{A}(\Theta)$ and $\mathbf{U}_s$ span the same subspace [18].*

Under the specified conditions, the array covariance matrix $\mathbf{R}$ in (2.11) simplifies to

$$\mathbf{R} = \mathbf{A}(\Theta) \mathbf{R}_{ss} \mathbf{A}^H(\Theta) + \sigma_v^2 \mathbf{I}_L. \quad (2.20)$$

It is easy to show that the range space of the signal subspace eigenvector matrix $\mathbf{U}_s$ and that of the array manifold matrix $\mathbf{A}(\Theta)$ are identical [3]. It therefore follows that the steering vectors along the source directions $\theta \in \Theta$ are orthogonal to the noise subspace,

i.e., $\mathbf{U}_n^H \mathbf{a}(\theta) = 0$. The spectral MUSIC estimator searches for the M source directions which are exquisitely orthogonal to the noise eigenvectors over the direction span in the spatial spectrum given by

$$P_{\text{music}}(\phi) = \frac{1}{|\mathbf{U}_n^H \mathbf{a}(\phi)|^2}. \quad (2.21)$$

The search over all possible directions can be circumvented by exploiting specific structures in the array geometry. For a ULA, the spectrum of MUSIC algorithm can be expressed as a polynomial, the roots of which yields the source directions. Details of this technique is presented in [2].

2.2.2.2 ESPRIT Algorithm

Estimation of signal parameters via rotational invariance technique (ESPRIT) [19] is a subspace technique which requires two subarrays for DOA estimation. Elements of the two subarrays are required to form matched pairs, i.e., the displacement vectors of the first subarray elements w.r.t. its reference sensor have to be equal to the displacement vectors of the second subarray elements w.r.t. the corresponding reference sensor. This condition requires the subarrays to be identical. For simplicity we consider a ULA of $L + 1$ sensors divided into two subarrays. The first L sensors form the first subarray and the L sensors starting from the second sensor form the second subarray.

Proposition 2.2 *Given two matched subarrays whose reference sensors, measured in wavelengths, are displaced by Δ_0 , the source directions can be estimated from the eigenvalues of a nonsingular matrix Ψ defining the linear transformation between the first subarray signal subspace $\mathbf{U}_{\text{sx}}$ and the second subarray signal subspace $\mathbf{U}_{\text{sy}}$.*

The proof for the above proposition provided in [3] shows that the matrix Ψ relates the signal subspaces of the two subarrays as

$$\mathbf{U}_{\text{sx}} \Psi = \mathbf{U}_{\text{sy}}, \quad (2.22)$$

preserves the source directions in the argument of its eigenvalues. The matrix Ψ can be estimated in several ways, of which, the total least squares approach requires one to first define a $2M \times L$ matrix $\mathbf{U} = [\mathbf{U}_{\text{sx}} \mathbf{U}_{\text{sy}}]^H$. The eigen decomposition of $\mathbf{U}\mathbf{U}^H$ yields the eigenvalue and eigenvector matrices $\Lambda_{\mathbf{U}}$ and $\mathbf{V}_{\mathbf{U}}$, respectively. The eigenvector matrix can be partitioned into a 2×2 block matrix as

$$\mathbf{V}_{\mathbf{U}} = \begin{bmatrix} \mathbf{V}_{11} & \mathbf{V}_{12} \\ \mathbf{V}_{21} & \mathbf{V}_{22} \end{bmatrix} \quad (2.23)$$

From this partitioned matrix, the estimate of Ψ is obtained as $\Psi = -\mathbf{V}_{11} \mathbf{V}_{22}^{-1}$.

Once Ψ is estimated, its eigenvalues provide the $M \times M$ diagonal matrix Φ . From (??), the source directions are retrieved as

$$\theta_m = \cos^{-1} \left(\frac{\arg(\Phi_{m,m})}{2\pi\Delta_0} \right), \quad m = 1, 2, \dots, M, \quad (2.24)$$

‘where $\Phi_{m,m}$ denotes the m th diagonal entry of Φ .

The above subspace-based techniques such as MUSIC, ESPRIT and maximum likelihood method can be generalized under a unified approach called weighted subspace fitting (WSF) as shown in [20].

2.2.3 Model-Order Estimation

The order of the narrowband signal model in (2.10) is equal to the number of sources present in the scenario. The problem of estimating the number of sources can also be posed as a detection problem in which it is required to detect all the active sources in the environment. As studied earlier, several DOA estimation techniques presume the model order to be known. While several techniques have been proposed for this task, we briefly review two popular techniques, viz., Akaike information criterion (AIC) [21] and Rissanen's minimum description length (MDL) [22] in this section.

One approach to estimate the model order is to perform sequential hypothesis tests looking for $L - q$, $q = 1, \dots, L$ smallest eigenvalues of $\mathbf{R}$ which are equal. A sufficient statistic for this task is [23],

$$L_q(q) = N(L - q) \ln \left\{ \frac{g_1(q)}{g_2(q)} \right\}, \quad (2.25)$$

where $g_1(q) = \frac{1}{L-q} \sum_{i=q+1}^L \lambda_i$ and $g_2(q) = \left\{ \prod_{i=q+1}^L \lambda_i \right\}^{\frac{1}{L-q}}$, with λ_i denoting the i th eigenvalue of $\mathbf{R}$. For $N \gg L$ and $\mathbf{A}(\Theta)$ and $\mathbf{R}_{ss}$ of rank q , $L - q$ number of smallest eigenvalues will be equal resulting in $L_q(q) = 0$.

The AIC and MDL employ the criteria of (2.25) along with a penalty function related to the available degrees of freedom. The AIC and MDL functions are given, respectively, by

$$L_{\text{AIC}}(q) = N(L - q) \ln \left\{ \frac{g_1(q)}{g_2(q)} \right\} + q(2L - q), \quad (2.26a)$$

$$L_{\text{MDL}}(q) = N(L - q) \ln \left\{ \frac{g_1(q)}{g_2(q)} \right\} + \frac{1}{2}q(2L - q) \log N. \quad (2.26b)$$

From these criteria, the model order is estimated as

$$q_{\text{AIC}} = \underset{q}{\text{argmin}} L_{\text{AIC}}(q), \quad (2.27\text{a})$$

$$q_{\text{MDL}} = \underset{q}{\text{argmin}} L_{\text{MDL}}(q). \quad (2.27\text{b})$$

The performance of these techniques are susceptible to the number of sensors and sources, signal strength, spatial separation of sources, signal correlation and the number of snapshots. A detailed comparative study of the two techniques has been presented in [2]. In the presence of coherent sources, the rank deficient $\mathbf{R}_{ss}$ adversely affects AIC and MDL. A modification to MDL has been proposed in [24] for such a scenario.

2.3 Wideband Array Processing

We now consider a wideband signal propagating in a lossless homogeneous medium. The wave equation at a spatial location $\mathbf{r}$ due to the planar wavefront is

$$s(t, \mathbf{r}) = s(t - \boldsymbol{\alpha} \cdot \mathbf{r}), \quad (2.28)$$

where $\boldsymbol{\alpha} = \frac{1}{v} \mathbf{u}$ is the slowness vector directed towards the source direction. Depending on the application, wideband sources can either have band-limited or bandpass spectrum as shown in Figure 2.3. For a wideband source with bandwidth B_s and having its spectrum within $[f_l, f_u]$, the bandpass sampling theorem furnishes an alias-free sampling frequency of $\frac{2f_u}{b} \geq f_s \leq \frac{2f_l}{b-1}$, where $b \leq \lfloor \frac{f_u}{B_s} \rfloor$. However, for a bandlimited wideband signal, the sensor outputs have to be sampled at $f_s \geq B_s$ in order to avoid the presence of aliasing components. Likewise, it is essential for the sensor array to satisfy Nyquist criterion in the spatial domain for band-limited wideband sources, $d \leq \frac{\lambda_u}{2}$, where λ_u is the wavelength

Figure 2.3: Source spectrum, (a) bandpass signal, (b) band-limited signal.

corresponding to the frequency f_u . The acquired sensor signals can then be processed using signal processing techniques.

Considering the measurement noise $\mathbf{v}(t)$, the signal acquired by an array with sensors situated at $\mathbf{r}_m, m = 1, \dots, L$, not limited to a uniform linear array, can be written in vector form as

$$\mathbf{x}(t) = \begin{bmatrix} s(t - \boldsymbol{\alpha} \cdot \mathbf{r}_1) \\ s(t - \boldsymbol{\alpha} \cdot \mathbf{r}_2) \\ \vdots \\ s(t - \boldsymbol{\alpha} \cdot \mathbf{r}_L) \end{bmatrix} + \mathbf{v}(t). \quad (2.29)$$

Employing the Fourier transform relation for a traveling wave in a homogeneous medium

$$\mathcal{F}\{s(t - \boldsymbol{\alpha} \cdot \mathbf{r})\} = \underline{s}(f)e^{-j2\pi f \boldsymbol{\alpha} \cdot \mathbf{r}}, \quad (2.30)$$

where $\underline{s}(f)$ is the Fourier transform of the signal $s(t)$. The frequency-domain represen-

tation of (2.29) is then given by the $L \times 1$ vector

$$\begin{aligned} \underline{\mathbf{x}}(f) &= \begin{bmatrix} e^{-j2\pi f \boldsymbol{\alpha} \cdot \mathbf{r}_1} \\ e^{-j2\pi f \boldsymbol{\alpha} \cdot \mathbf{r}_2} \\ \vdots \\ e^{-j2\pi f \boldsymbol{\alpha} \cdot \mathbf{r}_L} \end{bmatrix} \underline{\mathbf{s}}(f) + \underline{\mathbf{v}}(f) \\ &= \mathbf{a}(f, \theta) \underline{\mathbf{s}}(f) + \underline{\mathbf{v}}(f), \end{aligned} \quad (2.31)$$

where $\mathbf{a}(f, \theta)$ is the $L \times 1$ steering vector, $\underline{\mathbf{x}}(f)$ and $\underline{\mathbf{v}}(f)$ denote the frequency-domain representation of the sensor and noise vectors at frequency f . Extending the signal model in (2.31) for a scenario with M sources from directions $\Theta = [\theta_1, \dots, \theta_M]$, we have

$$\underline{\mathbf{x}}(f) = \mathbf{A}(f, \Theta) \underline{\mathbf{s}}(f) + \underline{\mathbf{v}}(f), \quad (2.32)$$

with $\mathbf{A}(f, \Theta) = [\mathbf{a}(f, \theta_1), \dots, \mathbf{a}(f, \theta_M)]$ and $\underline{\mathbf{s}}(f)$ denoting the array manifold matrix and the $M \times 1$ source vector, respectively. The spectral density matrix of $\underline{\mathbf{x}}(f)$, $\mathbf{P}_{xx}(f) = E\{\underline{\mathbf{x}}(f)\underline{\mathbf{x}}^H(f)\}$ is given by

$$\mathbf{P}_{xx}(f) = \mathbf{A}(f, \Theta) \mathbf{P}_{ss}(f) \mathbf{A}^H(f, \Theta) + \mathbf{P}_{vv}(f), \quad (2.33)$$

where $\mathbf{P}_{ss}(f) = E\{\underline{\mathbf{s}}(f)\underline{\mathbf{s}}^H(f)\}$ and $\mathbf{P}_{vv}(f) = E\{\underline{\mathbf{v}}(f)\underline{\mathbf{v}}^H(f)\}$ are the source and noise spectral density matrices.

In order to process the wideband data, sensor outputs are critically sampled at $f_s = 2f_u$ assuming band-limited sources. If the sensor data over a temporal window of size ΔT is subjected to discrete Fourier transform (DFT), a vector of sensor outputs $\underline{\mathbf{x}}(k, l)$ known as the frequency-domain snapshot at each discrete frequency bin index k within the signal bandwidth and time-frame index l is obtained. The signal model is then given

by

$$\underline{\mathbf{x}}(k, l) = \mathbf{A}(f_k, \Theta) \underline{\mathbf{s}}(k, l) + \underline{\mathbf{v}}(k, l). \quad (2.34)$$

The covariance matrix at bin index k can be estimated using J frequency-domain snapshots from the corresponding frequency as

$$\hat{\mathbf{R}}(f_k) = \frac{1}{J} \sum_{l=1}^J \underline{\mathbf{x}}(k, l) \underline{\mathbf{x}}^H(k, l). \quad (2.35)$$

2.3.1 Beamforming

At each frequency, we note that the wideband signal model degenerates to a narrowband model in (2.10). Therefore, a straightforward approach to achieve wideband beamforming is to perform narrowband beamforming at each frequency bin. The beamformer output in time domain can then be estimated by inverse DFT of the narrowband beamformer outputs.

Alternatively, a time-domain wideband beamformer introduces a filter at each sensor output prior to the constructive summation of the outputs as shown in Figure 2.4(a). This technique was first proposed by Frost [25]. Figure 2.4(b) shows a D -tap FIR filter $w_{i,m}$, $m = 0, \dots, D - 1$ at the output of i th sensor. The design of these filters to obtain a desired frequency and wave number response is the problem in hand. The response for the broadband beamformer is given by

$$b(\theta, f) = \sum_{l=1}^L W_l(f) e^{-j2\pi f \tau_l(\theta)}, \quad (2.36)$$

where $\tau_l(\theta)$ is the propagation delay w.r.t. the origin of the sensor array, $W_l(f)$ is the frequency response of the filter for the l th sensor. With the desired source direction known apriori, it is a common practice to pre-steer the sensor outputs such that the

source is synthetically shifted to the broadside of the array. This allows one to focus on designing a broadside beamformer with required interference suppression from all other directions.

2.3.1.1 Frost Beamforming

Let $x_l(n)$ be the output of the l th sensor, appropriately presteered to have the desired signal in the broadside of the array. At a tap delay m , we can define a spatial vector $\mathbf{x}(n-m) = [x_1(n-m) \ x_2(n-m) \ \dots \ x_L(n-m)]^T$. Vectorizing the snapshots at all the D -delays, we obtain a spatio-temporal snapshot with D previous time samples from n th sample as

$$\bar{\mathbf{x}}(n) = [\mathbf{x}^T(n) \ \mathbf{x}^T(n-1) \ \dots \ \mathbf{x}^T(n-D+1)]^T. \quad (2.37)$$

Since the array outputs are presteered, the look-direction spatial vector at a given tap-delay is a vector of ones $\mathbf{1}_L$. This results in a cumulated filter weight $h(m) = \sum_{i=1}^L w_{i,m}$, $m = 0, \dots, D-1$. The frequency response of the wideband array can be controlled by designing the filter response of $h(m)$. If $g(m)$, $m = 0, \dots, D-1$ denote the filter coefficients of the desired frequency response, we can construct a constraint for the weights of the m th tap delay across the sensors such that

$$\bar{\mathbf{w}}^H \bar{\mathbf{c}}_m = g(m), \quad (2.38)$$

where $\bar{\mathbf{w}} = [\mathbf{w}_0^T \ \dots \ \mathbf{w}_{D-1}^T]^T$ is the weight vector across the entire time-delay structure with $\mathbf{w}_m = [w_{1,m} \ w_{2,m} \ \dots \ w_{L,m}]^T$ denoting the vector of weights across the array at tap m and $\bar{\mathbf{c}}_m = [\mathbf{0}_L^T, \dots, \mathbf{1}_L^T, \dots, \mathbf{0}_L^T]^T$ is the $LD \times 1$ constraint vector with the m th block of L elements corresponding to a vector of ones and zeros elsewhere. Likewise, an $LD \times D$ constraint matrix $\mathbf{C}$ can be defined as

$$\mathbf{C} = [\bar{\mathbf{c}}_0 \ \bar{\mathbf{c}}_1 \ \dots \ \bar{\mathbf{c}}_{D-1}], \quad (2.39)$$

using which, we have D constraints given by

$$\mathbf{C}^T \bar{\mathbf{w}} = \mathbf{g}, \quad (2.40)$$

where $\mathbf{g} = [g(0) \ \dots \ g(D-1)]^T$ is the filter response of the desired frequency response.

The optimal set of filter weights $\bar{\mathbf{w}}$ can be obtained by solving the following optimization problem:

$$\begin{aligned} & \underset{\bar{\mathbf{w}}}{\text{minimize}} \quad \bar{\mathbf{w}}^H \mathbf{R}_{\bar{x}\bar{x}} \bar{\mathbf{w}} \\ & \text{subject to} \quad \mathbf{C}^T \bar{\mathbf{w}} = \mathbf{g}. \end{aligned} \quad (2.41)$$

The obtained solution assures a linear distortionless response over the signal bandwidth. As shown in [2, p. 657], this formulation can directly be extended to tackle interferences by introducing additional constraints over the $D-1$ tap delays.

For a wideband beamformer, an error in the nominal source direction can result in signal cancellation at the beamformer output. In order to introduce robustness to source position error, the unity response in the look direction can be broadened using additional constraints. While several techniques have been proposed to achieve this [3], introducing linear derivative constraints which enforce the derivative of the power pattern w.r.t. the bearing angle to zero [26] was one of the foremost techniques.

A generalized sidelobe canceller (GSC) beamformer can be extended to broadband source scenario as well. The first path ensures a fixed beam with specific frequency response while the auxiliary path blocks the desired signal by placing a null in the corresponding direction. The $L-1$ beams at the output of the blocking pre-filter are then passed through a tapped delay line filter. The filters in the auxiliary path are adaptively designed to minimize the overall beamformer output power. In the discussed beamformers, the frequency response of the filters are designed such that the distortionless response from the look direction is emphasized. On the contrary, frequency invariant beamform-

ers (FIB) are designed to have a constant frequency response over the signal bandwidth along with spatial filtering [27]. The underlying idea is to transform the sensor signals into phase mode, wherein the dependence on frequency is removed by appropriately-designed filters. The beamforming coefficients which follows the filtered signals can be estimated by employing the trapezoidal rule. The design of FIR filters following each sensor output are shown to be dilation factors of a reference filter [28]. Under the frequency invariance conditions, it is important to note that the FIR filters cannot be designed for any arbitrary array. However, it is possible to exploit inherent structures in some array geometries to design filters with frequency invariant characteristics. The design of filters with uniform concentric circular arrays for beamforming and DOA estimation has been shown in [29].

2.3.2 DOA Estimation

As described earlier, broadband array processing process spatio-temporal snapshots $\bar{\mathbf{x}}(n)$ instead of spatial snapshots used for narrowband DOA estimation. However, broadband DOA estimation using spatio-temporal snapshots has the following limitations [30]:

- (i) The signal subspace dimension of the covariance matrix $\mathbf{R} = E\{\bar{\mathbf{x}}(n)\bar{\mathbf{x}}^H(n)\}$ will be a composite subspace greater than the number of sources present in the scenario.
- (ii) With increase in the tap-delays, the dimensionality of the problem increases the computational complexity.

Due to the first limitation, the segregation of signal subspace and noise subspace is not clearly defined. Therefore, the problem is generally addressed in the frequency domain. Transforming the sensor outputs into frequency domain converts the filtering process at each sensor into multiplication of a complex weight at each frequency. Moreover, the frequency-domain snapshots follow a narrowband signal model at each frequency bin.

We now review some of the existing techniques which serve as baseline algorithms in future chapters.

2.3.2.1 Incoherent Signal Subspace Method (ISSM)

The foremost technique for wideband DOA estimation is to estimate source directions in small narrowband portions of the signal bandwidth using any of the narrowband subspace techniques and unify them incoherently to obtain a final estimate [31]. This is achieved by an arithmetic or geometric mean of the spatial spectra across the signal bandwidth as follows:

$$\begin{aligned} P_{\text{ISSM-AM}}(\phi) &= \frac{1}{\frac{1}{B} \sum_{f_k=f_1}^{f_u} \frac{1}{L-M} \sum_{i=M+1}^L |\mathbf{a}^H(f_k, \phi) \mathbf{e}_i(f_k)|^2}, \\ P_{\text{ISSM-GM}}(\phi) &= \frac{1}{\frac{1}{B} \prod_{f_k=f_1}^{f_u} \left(\frac{1}{L-M} \sum_{i=M+1}^L |\mathbf{a}^H(f_k, \phi) \mathbf{e}_i(f_k)|^2 \right)^{1/B}}, \end{aligned} \quad (2.42)$$

where $\mathbf{e}_i(f_k), i = M+1, \dots, L$ are the noise eigenvectors for bin index k and B denotes the number of narrowband frequency components across the signal bandwidth $[f_1, f_u]$. The subscripts “ISSM-AM” and “ISSM-GM” denote the arithmetic and geometric mean versions of the spatial spectrum. As highlighted in [31], $P_{\text{ISSM-AM}}(\phi)$ exhibits a peak when $\mathbf{a}(f_k, \phi)$ is “almost orthogonal” to all the noise eigenvectors across all the frequency bins. On the contrary, the geometric mean estimator is observed to exhibit a peak if the noise eigenvectors are almost orthogonal to $\mathbf{a}(f_k, \phi)$ in at least one of the narrowband components over the signal bandwidth. Therefore $P_{\text{ISSM-GM}}(\phi)$ will have a higher resolution but lower accuracy than the arithmetic mean estimator. The power variations in different narrowband segments adversely affects the ISSM spectrum. Moreover, the technique cannot resolve coherent sources which occur in the presence of multipath.

In [32], a new incoherent DOA estimation technique which does not require one to

determine the number of sources has been proposed. The arithmetic mean of the narrowband cost function across the signal bandwidth has been used to estimate wideband source directions. This technique assumes source nonstationarity in order to develop a cost function which is independent of the model order. While the solution is well suited for speech sources, the assumptions withhold its extension to stationary source scenario, in which case, the technique degenerates to ISSM [31]. Furthermore, the incoherent summation across frequencies bins, carries the limitations of ISSM to this technique in handling the coherent sources.

2.3.2.2 Coherent Signal Subspace (CSS) Approach

In this approach [33–35], the wideband data is first decomposed into non-overlapping narrowband components. The data from each frequency bin index k is then transformed to a reference frequency f_0 using pre-designed focussing matrices $\mathbf{T}(f_k, \Theta_{\text{init}})$ given by

$$\begin{aligned}\underline{\mathbf{y}}(k) &= \mathbf{T}(f_k, \Theta_{\text{init}})\underline{\mathbf{x}}(k) \\ &= \mathbf{T}(f_k, \Theta_{\text{init}})\mathbf{A}(f_k, \Theta)\underline{\mathbf{s}}(k) + \mathbf{T}(f_k, \Theta_{\text{init}})\underline{\mathbf{v}}(k),\end{aligned}\tag{2.43}$$

where Θ_{init} is the vector of approximate initial source directions. The transformed data covariance matrices, given by $\mathbf{R}_y(f_k) = E\{\underline{\mathbf{y}}(k)\underline{\mathbf{y}}^H(k)\}$, are averaged over the entire frequency band of interest $[f_l, f_u]$ to obtain

$$\begin{aligned}\mathbf{R}_{\text{cssm}} &= \sum_{f_k=f_l}^{f_u} \mathbf{R}_y(f_k) \\ &= \sum_{f_k=f_l}^{f_u} \mathbf{T}(f_k, \Theta_{\text{init}})\mathbf{R}(f_k)\mathbf{T}^H(f_k, \Theta_{\text{init}}).\end{aligned}\tag{2.44}$$

The focussing matrices are designed such that

$$\mathbf{T}(f_k, \Theta_{\text{init}})\mathbf{A}(f_k, \Theta) = \mathbf{A}(f_0, \Theta). \quad (2.45)$$

This allows (2.44) to be written as

$$\mathbf{R}_{\text{cssm}} = \mathbf{A}(f_0, \Theta) \left\{ \sum_{f_k=f_1}^{f_u} \mathbf{R}_{ss}(f_k) \right\} \mathbf{A}^H(f_0, \Theta) + \mathbf{R}_v, \quad (2.46)$$

where $\mathbf{R}_{ss}(f_k)$ is the source covariance matrix obtained at frequency f_k and $\mathbf{R}_v$ is the transformed noise covariance matrix. We note that the first term in (2.46) is similar to the covariance matrix obtained for a narrowband signal model at f_0 . Assuming additive noise across the sensors to be uncorrelated and have equal power $\sigma_v^2(f_k)$, the second term is given by

$$\mathbf{R}_v = \sum_{f_k=f_1}^{f_u} \sigma_v^2(f_k) \mathbf{T}(f_k, \Theta_{\text{init}}) \mathbf{T}^H(f_k, \Theta_{\text{init}}). \quad (2.47)$$

In order to estimate the source locations, $\mathbf{R}_v$ is first estimated using (2.47). The eigenvectors $\mathbf{e}_i, i = 1, \dots, L$ and eigenvalues $\lambda_i, i = 1, \dots, L$ of the matrix pencil $(\mathbf{R}_{\text{cssm}}, \mathbf{R}_v)$ are obtained by solving the generalized eigenvalue problem. With the number of sources assumed to be known, the narrowband MUSIC spectrum is used to estimate the source directions with noise subspace eigenvectors $\mathbf{U}_n = [\mathbf{e}_{M+1} \dots \mathbf{e}_L]$ and the steering vectors evaluated at f_0 .

From (2.45) we note that the design of focussing matrices requires approximate source directions. The design of focussing matrix has therefore gained significant interest [36–38]. While the details of focussing matrix design are not discussed here, one suggested approach [36] is to provide initial estimates from the peaks of the MVDR spatial spectrum along with the spatial points distant from the peaks by 0.125 times the beamwidth.

It is well known that CSSM performs well at low SNRs. Furthermore, the coherent

summation of focussed covariance matrices restores the rank of $\mathbf{R}_{\text{cssm}}$, thus enabling the technique to resolve coherent sources. However, a large error in the provided initial estimates reflects in the bias of the DOA estimates.

2.3.2.3 Frequency Invariant Beamforming-based DOA Estimation

This technique, proposed in [39], adopts a beamspace approach for wideband DOA estimation. The basic idea relies in forming G ($M < G < L$) beams from the sensor outputs using frequency invariant beamformers (FIB). The used FIB ensures frequency invariance, as a consequence of which, coherent summation of data across frequency is achieved. DOA parameters can then be estimated using any narrowband estimator.

With the array geometry and source bandwidth known, G FIBs are designed to span the entire spatial range. Defining a matrix $\mathbf{C}(f_k) = [\mathbf{b}_1(f_k), \dots, \mathbf{b}_G(f_k)]$ with G beam-shaping filter responses $\mathbf{b}_j(f_k), j = 1, \dots, J$ at frequency f_k , the beam-shaped output $\underline{\mathbf{z}}(k) \in \mathbb{C}^{G \times 1}$ is given by

$$\begin{aligned} \underline{\mathbf{z}}(k, l) &= \mathbf{C}^H(f_k) \underline{\mathbf{x}}(k, l) \\ &= \mathbf{C}^H(f_k) \underline{\mathbf{s}}(k, l) + \mathbf{C}^H(f_k) \underline{\mathbf{v}}(k, l). \end{aligned} \quad (2.48)$$

The corresponding covariance matrix is

$$\mathbf{R}_{zz}(f_k) = \mathbf{A}_c(f_k, \Theta) \mathbf{R}_{ss}(f_k) \mathbf{A}_c^H(f_k, \Theta) + \mathbf{R}_v(f_k), \quad (2.49)$$

where $\mathbf{A}_c(f_k, \Theta) = \mathbf{C}^H(f_k) \mathbf{A}(f_k, \Theta)$ and $\mathbf{R}_v(f_k) = \mathbf{C}^H(f_k) E\{\underline{\mathbf{v}}(k) \underline{\mathbf{v}}^H(k)\} \mathbf{C}(f_k)$ is the correlated noise after transformation. Since $\mathbf{C}(f_k)$ is designed such that the array manifold

representation becomes frequency invariant, i.e., $\mathbf{A}_c(f_k, \Theta) = \mathbf{A}_c(\Theta), \forall f_k \in [f_1, f_u]$,

$$\begin{aligned} \mathbf{R}_{\text{fib}} &= \sum_{f_k=f_1}^{f_u} \mathbf{R}_{zz}(f_k) \\ &= \mathbf{A}_c(\Theta) \left\{ \sum_{f_k=f_1}^{f_u} \mathbf{R}_{ss}(f_k) \right\} \mathbf{A}_c^H(\Theta) + \sum_{f_k=f_1}^{f_u} \mathbf{R}_v(f_k). \end{aligned} \quad (2.50)$$

From (2.50) and (2.46), it is evident that FIB-based DOA estimation employs a similar approach to tackle coherent sources in the scenario by restoring the rank of source covariance matrix. Assuming the noise covariance matrix to be known, narrowband DOA estimation can be performed similar to that performed for CSSM post-focussing.

2.3.2.4 Steered Covariance Matrix-MVDR (STCM-MVDR) [40]

In this technique, DOA estimation is accomplished via a beamformer derived from the coherent processing of data across the signal bandwidth which is shown to provide improved stability and accuracy [40]. Towards this, the array covariance matrix $\mathbf{R}(f_k) = \text{E}\{\mathbf{x}(k, l)\mathbf{x}^H(k, l)\}$, evaluated at each frequency within the signal bandwidth $[f_1, f_u]$ are first steered to the look direction ϕ and then accumulated to form the steered covariance matrix (STCM) given by

$$\mathbf{R}(\phi) = \sum_{f_k=f_1}^{f_u} \mathbf{T}^H(f_k, \phi) \mathbf{R}(f_k) \mathbf{T}(f_k, \phi), \quad (2.51)$$

where $\mathbf{T}(f_k, \phi) = \text{diag}(\mathbf{a}(f_k, \phi))$ is the diagonal steering matrix. This coherent summation over several frequency bins enhances the signal power over the background noise prior to the estimation. The spatial response is then given by [40]

$$P(\phi) = \frac{1}{\mathbf{1}_L^T \mathbf{R}^{-1}(\phi) \mathbf{1}_L}. \quad (2.52)$$

Although STCM-MVDR is expected to exhibit good resolution, the subspace-based techniques provide superior resolution. Furthermore, performing matrix inverse in each look direction is computationally expensive. However, it is important to note that the spatial spectrum of this technique is insensitive to the errors in estimated model order.

2.3.2.5 Test for Orthogonality of Projection Subspaces (TOPS)

TOPS [41] is a relatively new wideband DOA estimation technique that has been classified as a noncoherent signal subspace method. While this technique cannot resolve coherent sources, the approach is quite different from other techniques.

At a reference frequency f_0 , the covariance matrix estimated from frequency domain snapshots is decomposed into signal and noise subspaces $\mathbf{U}_s(k_0)$ and $\mathbf{U}_n(k_0)$ assuming the number of sources to be known. The signal subspace is transformed to each of the other frequencies within the signal bandwidth $[f_l, f_u]$ using a transformation matrix $\mathbf{T}(f_k, \phi)$ which compensates for the frequency in each direction

$$\hat{\mathbf{U}}_s(f_k, \phi) = \mathbf{T}(f_k, \phi) \mathbf{U}_s(f_0), \quad k = 1, \dots, B, \quad (2.53)$$

where $\delta f_k = f_k - f_0$ and B is the number of frequency bins under consideration. A direction dependent matrix $\mathbf{D}_\phi$ is then defined as

$$\mathbf{D}_\phi = [\hat{\mathbf{U}}_s'^H(f_1, \phi) \mathbf{U}_n(f_1) \quad \hat{\mathbf{U}}_s'^H(f_2, \phi) \mathbf{U}_n(f_2) \quad \dots \quad \hat{\mathbf{U}}_s'^H(f_B, \phi) \mathbf{U}_n(f_B)], \quad (2.54)$$

where $\hat{\mathbf{U}}_s'(f_k, \phi) = [\mathbf{I}_L - (\mathbf{a}^H(f_k, \phi) \mathbf{a}(f_k, \phi))^{-1} \mathbf{a}(f_k, \phi) \mathbf{a}^H(f_k, \phi)] \hat{\mathbf{U}}_s(f_k, \phi)$. The matrix $\mathbf{D}_\phi$ has the following property for the source and non-source directions:

- (i) if $\phi \in \Theta$, $\mathbf{D}_\phi$ becomes rank deficient,
- (ii) if $\phi \notin \Theta$, $\mathbf{D}_\phi$ is a full rank matrix.

Source directions can therefore be estimated using

$$\hat{\theta}_{\text{tops}} = \arg \min_{\phi} \frac{1}{\sigma_{\min}(\phi)}, \quad (2.55)$$

where $\sigma_{\min}(\phi)$ is the smallest singular value of $\mathbf{D}_{\phi}$.

While this technique has found to outperform CSSM for mid and high SNR ranges, the technique cannot resolve sources at low SNR conditions. Furthermore, this technique is highly sensitive to any errors in model-order estimation. Since this is a noncoherent approach, the technique cannot handle coherent sources.

2.3.2.6 Beamforming-Framework Wideband MUSIC algorithm (BFW-MUSIC) [42]

This is another technique which estimates wideband source directions from time-domain snapshots. Unlike FIB-based technique, this technique uses narrowband beamforming framework for DOA estimation, i.e., spatial snapshots $\mathbf{x}(n)$ are used instead of spatio-temporal snapshots $\bar{\mathbf{x}}(n)$. For time-domain processing, as mentioned earlier, the steering vector for a given direction will span a subspace obtained over the signal bandwidth $\{\mathbf{a}(f, \phi), \forall f \in [f_l, f_u]\}$. This technique introduces robustness to the steering variations with frequency, thus enabling frequency invariance.

The narrowband MUSIC algorithm is first reformulated as a beamformer [42]

$$\begin{aligned} & \underset{\mathbf{w}_{\phi}}{\text{minimize}} \quad \|\mathbf{w}_{\phi} - \mathbf{a}(\phi)\|_2^2 \\ & \text{subject to} \quad \mathbf{U}_s^H \mathbf{w}_{\phi} = 0, \end{aligned} \quad (2.56)$$

where $\mathbf{w}_{\phi}$ is the weight vector solution. The above beamformer, which introduces deep nulls along the source directions, is steered in each look direction and $\mathbf{w}_{\phi}$ is evaluated.

Source directions can be estimated from the peaks in the direction finding function

$$\text{DF}(\phi) = 10\log_{10} \frac{1}{|\mathbf{w}_{\phi}^H \mathbf{a}(\phi)|}. \quad (2.57)$$

The formulation is directly extended to wideband case by introducing the following constraints,

$$\begin{aligned} \mathbf{w}_{\phi}^H \frac{\partial \mathbf{a}(f, \phi)}{\partial f} \Big|_{f=f_0} &= 0 \\ \vdots \\ \mathbf{w}_{\phi}^H \frac{\partial^P \mathbf{a}(f, \phi)}{\partial f^P} \Big|_{f=f_0} &= 0, \end{aligned} \quad (2.58)$$

where $\frac{\partial^P \mathbf{a}(f, \phi)}{\partial f^P}$ is the P^{th} derivative of the steering vector in the direction ϕ . These derivative constraints widen the nulls along the frequency axis, making the weight vector insensitive to wideband components. This technique is advantageous in estimating directions of sources with varying signal bandwidth with limited number of time-domain snapshots. However, the challenge appears in providing the extended signal subspace $\mathbf{U}_s$ of $\mathbf{R}$ for DOA estimation.

The above discussed techniques are observed to exploit the orthogonality between the signal and noise subspaces for DOA estimation, either directly or indirectly. Alternatively, direction parameters can also be estimated in the maximum likelihood sense [43]. However, the complete search over the parameter space in this approach is computationally very expensive. A modified maximum likelihood approach with limited search is therefore proposed in [44], which can simultaneously estimate the model order and the source directions. Although the technique reduces the search space, the estimates are searched starting from a maximally hypothesized number of sources. Therefore, the computational complexity is proportional to the maximally hypothesized number of sources and the array size. In [45], a unified approach for beamforming, DOA estimation and

model order estimation has been proposed in which a posteriori distribution parameterized with source parameters, directions and the number of sources is obtained from the likelihood function. Markov chain Monte-Carlo approach is used to estimate these parameters simultaneously. The computational complexity and the iterative nature does not make this technique attractive.

In summary, the large computational cost of maximum likelihood techniques can be substituted with subspace-based wideband DOA estimation techniques such as CSSM. However, accurate model order has to be estimated for these techniques to provide reliable estimates at low SNR conditions.

2.3.3 Model-Order Estimation

Determining the number of sources is a primitive problem, solving which propels the task of DOA estimation. As studied in Section 2.2.3, the most common criteria used for model-order estimation in the narrowband scenario is by observing the multiplicity of the smallest eigenvalue of the array covariance matrix. If q denotes the smallest eigenvalue multiplicity, $\widehat{M} = L - q$ is the estimated number of sources. Evidently, it is more challenging to estimate the number of sources in a wideband scenario by combining the information over the entire signal bandwidth. This problem has been treated with either standalone algorithms [31, 33] or as one of the outcomes from joint model order-DOA estimation schemes [43–45].

The foremost approach [31] adopts a composite likelihood ratio which is a function of M obtained by the ratio of arithmetic and geometric means of the eigenvalues obtained for array covariance matrices over the entire signal bandwidth. Due to the incoherent processing, the performance of this technique is not acceptable at low SNR values. Moreover, the presence of coherent sources cannot be identified by this technique. These two limitations are circumvented in [33] by using AIC on eigenvalues obtained by solving

the generalized eigenvalue decomposition ($\mathbf{R}_{\text{cssm}}, \mathbf{R}_v$). Besides restoring the rank in the presence of coherent sources, this technique exhibits better detection at low SNRs due to the coherent summation across all the frequency bins. The problem of simultaneous estimation of model order and source directions has been formulated in [43] as a multiple-hypothesis test with test statistics derived from the generalized likelihood ratio principle. Although the performance is acceptable at low SNRs, the computational complexity limits its applicability. As discussed before, the modification to limit its search [44] reduces the complexity marginally.

Figure 2.4: Wideband array processing in time-domain, (a) block diagram, (b) using FIR filters with pre-steering.

Chapter 3

Narrowband DOA Estimation without Order Selection

3.1 Introduction

In the previous chapter, it was observed that the subspace-based techniques for narrowband DOA estimation seek knowledge of the number of sources present for accurate estimation of source directions. As a result, the estimated source directions are susceptible to model-order estimation errors. More recently, a DOA estimator based on a beamforming-framework has been proposed in [9] which is insensitive to the estimated model order. It was remarked that the working principle of this technique follows that of the MUSIC algorithm and hence was referred to as “MUSIC-like” algorithm. However, detailed analysis of this technique which asserts the MUSIC-like operation is not provided.

In this chapter, we first review the technique proposed in [9] and perform a detailed analysis of its optimization problem. We first show that the steered response power of this technique differs from the MVDR algorithm and then demonstrate that the underlying

working principle of the technique is similar to that of the MUSIC algorithm. The bounds for the sole parameter is derived from the solution under the desired spatial spectrum conditions. From the insights obtained, we observe that the weight vector estimated in each direction resides in the noise subspace. Exploiting this fact, we construct a matrix $\mathbf{W}$ of weight vectors obtained from arbitrary directions, the columns of which, spans only the noise subspace. This matrix can substitute the use of noise subspace in subspace-based techniques, thus circumventing the order-selection problem. Furthermore, the estimation of weight vector from limited number of directions significantly reduces the computational complexity of the proposed technique (MUSIC-W algorithm) over the MUSIC-like algorithm which solves a generalized eigenvalue problem in each azimuth direction. The performance of MUSIC-like and MUSIC-W algorithms are evaluated in comparison with the MUSIC algorithm.

Within the scope of the thesis, this chapter plays a significant role. With a detailed understanding of the MUSIC-like algorithm, we realize the potential of this algorithm to be extended for wideband DOA estimation. This extension is very appealing considering the fact that model-order estimation in wideband scenario is much more challenging and computationally expensive.

3.2 MUSIC-like Narrowband DOA Estimator

3.2.1 Formulation

The algorithm proposed in [9] solves the following optimization problem in each look direction,

$$\begin{aligned} & \underset{\mathbf{w}_\phi}{\text{minimize}} && \mathbf{w}_\phi^H \mathbf{R} \mathbf{w}_\phi \\ & \text{subject to} && \mathbf{w}_\phi^H \{ \mathbf{a}(\phi) \mathbf{a}^H(\phi) + \beta \mathbf{I}_L \} \mathbf{w}_\phi = c, \end{aligned} \tag{3.1}$$

where $c, \beta > 0$ are constants. Although this optimization problem reduces to the MVDR algorithm when $\beta = 0$, the working principle significantly differs from MVDR for all $\beta > 0$. While the objective function minimizes the beamformer output power in the look direction, the quadratic equality constraint regulates the look-direction power gain $\|\mathbf{w}_\phi^H \mathbf{a}(\phi)\|_2^2$ in relation with the objective function. An in-depth discussion on this relation will be provided in the next section. The Lagrangian function can be constructed as

$$L(\mathbf{w}_\phi, \chi) = \mathbf{w}_\phi^H \mathbf{R} \mathbf{w}_\phi - \chi(\mathbf{w}_\phi^H \{\mathbf{a}(\phi) \mathbf{a}^H(\phi) + \beta \mathbf{I}_L\} \mathbf{w}_\phi - c), \quad (3.2)$$

where χ is the Lagrangian multiplier. Equating the gradient of $L(\mathbf{w}_\phi, \chi)$ w.r.t. the weight vector to zero, the solution is given by the eigenvector corresponding to the minimum eigenvalue $\chi_{\min}$ of the generalized eigenvalue (GEV) problem

$$\mathbf{R} \mathbf{w}_\phi = \chi_{\min} \{\mathbf{a}(\phi) \mathbf{a}^H(\phi) + \beta \mathbf{I}_L\} \mathbf{w}_\phi. \quad (3.3)$$

Since the weight vector is a generalized eigenvector of the matrix pencil $[\mathbf{R}, \{\mathbf{a}(\phi) \mathbf{a}^H(\phi) + \beta \mathbf{I}_L\}]$, the solution vector obtained from (3.3) is invariant to the rescaling $\mathbf{w}_\phi \rightarrow \alpha \mathbf{w}_\phi$. We therefore consider $\|\mathbf{w}_\phi\|_2 = 1$ henceforth without loss of generality. Furthermore, we note that the solution $\mathbf{w}_\phi$ in (3.3) is independent of the value of c , which is implicitly assigned to $c = \|\mathbf{w}_\phi^H \mathbf{a}(\phi)\|_2^2 + \beta \|\mathbf{w}_\phi\|_2^2$. As will be shown in the next section, the solution vector $\mathbf{w}_\phi, \forall \phi$, resides in the noise subspace with appropriate choice of β . This again is in contrast to weight vector solution for MVDR along the source directions due to the distortionless response constraint. The algorithm in (3.1) therefore employs the direction finding function given by

$$P(\phi) = 10 \log_{10} \frac{1}{|\mathbf{w}_\phi^H \mathbf{a}(\phi)|^2}, \quad (3.4)$$

unlike the MVDR algorithm which employs the steered response power spectrum for DOA estimation.

3.2.2 Detailed Analysis

In [9], it was shown that the quiescent array response of the algorithm under consideration in the absence of sources (i.e., when $\mathbf{R} = \sigma_v^2 \mathbf{I}_L$) is equivalent to that of the MUSIC algorithm. However, a detailed discussion on the relation between the two algorithms has not been presented. In order to gain insights into the optimization problem of (3.1), we first express the array covariance matrix by its signal and noise subspace components as

$$\begin{aligned} \mathbf{R} &= \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H \\ &= \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^H. \end{aligned} \quad (3.5)$$

When additive noise variance σ_v^2 is assumed to be equal across the array, we have $\mathbf{\Lambda}_n = \sigma_v^2 \mathbf{I}_L$ which simplifies (3.5) to

$$\begin{aligned} \mathbf{R} &= \mathbf{U}_s (\mathbf{\Lambda}_s - \sigma_v^2 \mathbf{I}_M) \mathbf{U}_s^H + \sigma_v^2 \mathbf{I}_L \\ &= \mathbf{U}_s \overline{\mathbf{\Lambda}}_s \mathbf{U}_s^H + \sigma_v^2 \mathbf{I}_L \end{aligned} \quad (3.6)$$

Comparing (3.6) with (2.20), we note that $\overline{\mathbf{\Lambda}}_s = \mathbf{\Lambda}_s - \sigma_v^2 \mathbf{I}_M$ preserves the signal power defined by $\mathbf{R}_{ss}$ in (2.20) since $\mathbf{A}(\Theta)$ and $\mathbf{U}_s$ span the same subspace. Substituting this expression into the solution of the optimization problem in (3.3), we have

$$\{\mathbf{U}_s \overline{\mathbf{\Lambda}}_s \mathbf{U}_s^H + \sigma_v^2 \mathbf{I}_L\} \mathbf{w}_\phi = \chi_{\min} \{\mathbf{a}(\phi) \mathbf{a}^H(\phi) + \beta \mathbf{I}_L\} \mathbf{w}_\phi \quad (3.7)$$

From (3.7), it is not straightforward to understand the influence of the objective function and the constraint on the weight vector solution. In order to simplify the analysis,

we interpret the solution for the GEV problem as the one obtained by simultaneously solving two symmetric eigenvalue problems while satisfying the unit weight vector norm constraint. The resulting problem can then be written as

$$\{\mathbf{U}_s \bar{\mathbf{\Lambda}}_s \mathbf{U}_s^H + \sigma_v^2 \mathbf{I}_L\} \mathbf{w}_\phi = \sqrt{\chi_{\min}} \mathbf{w}_\phi \quad (3.8)$$

$$\{\mathbf{a}(\phi) \mathbf{a}^H(\phi) + \beta \mathbf{I}_L\} \mathbf{w}_\phi = \frac{1}{\sqrt{\chi_{\min}}} \mathbf{w}_\phi = \bar{\chi}_{\max} \mathbf{w}_\phi \quad \text{and} \quad (3.9)$$

$$\mathbf{w}_\phi^H \mathbf{w}_\phi = 1. \quad (3.10)$$

We derive the following inferences when $\mathbf{w}_\phi$ and $\chi_{\min}$ satisfy (3.8)-(3.10):

- (i) With the weight vector norm constrained to unity by (3.10), (3.8) minimizes the term $\|\bar{\mathbf{\Lambda}}_s^{-1/2} \mathbf{U}_s^H \mathbf{w}_\phi\|_2^2$ such that the weight vector $\mathbf{w}_\phi$ resides in the noise subspace. We note that this condition is accomplished by the constraint of the MUSIC algorithm in (2.56).
- (ii) The weight vector solution maximizes the projection $\|\mathbf{w}_\phi^H \mathbf{a}(\phi)\|_2^2$ such that the L.H.S. of the constraint $\mathbf{w}^H \{\mathbf{a}(\phi) \mathbf{a}^H(\phi) + \beta \mathbf{I}_L\} \mathbf{w}_\phi$ is equal to $c = 1/\sqrt{\chi_{\min}}$. Referring back to the formulation of the MUSIC algorithm in (2.56), this maximization is accomplished by its objective function which minimizes the term $\|\mathbf{w}_\phi - \mathbf{a}(\phi)\|_2^2 = \|\mathbf{w}_\phi\|_2^2 - 2\text{Re}\{\mathbf{w}_\phi^H \mathbf{a}(\phi)\} + \|\mathbf{a}(\phi)\|_2^2$.

Although we observe from the above findings that the algorithm in (3.1) and the MUSIC algorithm employ the same principle for DOA estimation, the respective formulations are marginally different from each other. While the MUSIC algorithm employs only the signal subspace eigenvectors in its formulation, the algorithm in (3.1) employs the entire array covariance matrix which includes the components from the noise subspace as well. It is therefore appropriate to address this algorithm as the ‘‘MUSIC-like algorithm’’.

Figure 3.1: Steered response power of MVDR and the algorithm under consideration obtained from the cost function of (3.1) for $L = 5$ and $M = 2$; $\mathbf{R}$ generated using 100 snapshots.

While the problem in (3.8) is regulated by the signal subspace eigenvalues $\bar{\mathbf{\Lambda}}_s$ and the noise power σ_v^2 , the choice of β in (3.9) plays a significant role in determining the feasible region for the solution. With an appropriate choice for β in relation with the signal and noise powers, or alternatively the SNR (as discussed in Section 3.2.3), the weight vector solution can be maintained in the noise subspace irrespective of the look direction. As a consequence, the steered response power is minimized in each direction in contrast to the MVDR algorithm which provides distinct peaks along the source directions. We illustrate this difference between the two algorithms for a five-sensor ULA situated in the farfield of two sources along 70° and 90° from the array endfire in Figure 3.1. With the additive noise power $\sigma_v^2 = 0.01$ and $\text{SNR} = 5$ dB, the covariance matrix is estimated from 100 snapshots. As expected, we note that the steered response power of the algorithm under study is approximately constant in all the directions and is equal to σ_v^2 , while the MVDR

response provides peaks along the source directions. This explains the use of (3.4) for DOA estimation instead of the steered power response.

In order to further analyze the constraint of (3.1), we express it in terms of the complementary subspaces of $\mathbf{R}$. Assuming $\|\mathbf{a}(\phi)\|_2^2 = 1$ without loss of generality, the steering vector projection matrix is given by

$$\mathbf{P}_{a_\phi} = \mathbf{a}(\phi)(\mathbf{a}^H(\phi)\mathbf{a}(\phi))^{-1}\mathbf{a}^H(\phi) = \mathbf{a}(\phi)\mathbf{a}^H(\phi). \quad (3.11)$$

Noting that $\mathbf{P}_{a_\phi} = \mathbf{P}_{a_\phi}\mathbf{P}_{a_\phi}$, we can introduce the complementary subspaces of the covariance matrix using the relation $\mathbf{I}_L = \mathbf{U}_s\mathbf{U}_s^H + \mathbf{U}_n\mathbf{U}_n^H$ in the look direction gain as

$$\begin{aligned} \mathbf{w}_\phi^H \mathbf{a}(\phi) \mathbf{a}^H(\phi) \mathbf{w}_\phi &= \mathbf{w}_\phi^H \mathbf{P}_{a_\phi} \mathbf{I}_L \mathbf{P}_{a_\phi} \mathbf{w}_\phi \\ &= \mathbf{w}_\phi^H \mathbf{P}_{a_\phi} \{ \mathbf{U}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{U}_n^H \} \mathbf{P}_{a_\phi} \mathbf{w}_\phi \\ &= \|\mathbf{U}_s^H \mathbf{P}_{a_\phi} \mathbf{w}_\phi\|_2^2 + \|\mathbf{U}_n^H \mathbf{P}_{a_\phi} \mathbf{w}_\phi\|_2^2. \end{aligned} \quad (3.12)$$

The constraint and the direction finding functions which incorporate this term can be written as

$$\|\mathbf{U}_s^H \tilde{\mathbf{w}}_\phi\|_2^2 + \|\mathbf{U}_n^H \tilde{\mathbf{w}}_\phi\|_2^2 + \beta \|\mathbf{w}_\phi\|_2^2 = c \quad (3.13)$$

$$\text{DF}(\phi) = 10 \log_{10} \frac{1}{\|\mathbf{U}_s^H \tilde{\mathbf{w}}_\phi\|_2^2 + \|\mathbf{U}_n^H \tilde{\mathbf{w}}_\phi\|_2^2}, \quad (3.14)$$

where $\tilde{\mathbf{w}}_\phi = \mathbf{P}_{a_\phi} \mathbf{w}_\phi$. From the illustration in Figure 3.1, it is observed that the optimization problem successfully minimizes $\mathbf{w}_\phi^H \mathbf{R} \mathbf{w}_\phi$ for all the directions ϕ , thus enforcing the weight vector to reside in the noise subspace. It therefore follows that $\|\mathbf{U}_s^H \mathbf{P}_{a_\phi} \mathbf{w}_\phi\|_2^2 \rightarrow 0$ in each direction ϕ . Along the source directions in particular, we have the steering vector residing completely in the signal subspace while the noise subspace contains the

Figure 3.2: Spatial spectrum of MUSIC and MUSIC-like algorithms for $L = 5$ sensors with synthetic covariance matrix generated using (2.20).

weight vector solution, i.e., $\mathcal{R}\{\mathbf{w}_\phi\} \cap \mathcal{R}\{\mathbf{a}(\phi)\} = \varphi$, where φ denotes the null set. As a consequence, we have $\tilde{\mathbf{w}}_\phi = \mathbf{0}$, which induces distinct peaks in $\text{DF}(\theta)$ along the source directions similar to the MUSIC algorithm.

We illustrate this similarity between the two algorithms for the simulation setup employed for Figure 3.1 with $L = 5$ sensors and $M = 2$ sources. The covariance matrix is however generated directly from the source and noise powers using (2.20) instead of using the array snapshots. The spatial spectrum for MUSIC and MUSIC-like algorithms plotted in Figure 3.2 are observed to be almost identical. As the covariance matrix deviates from (2.20) due to finite number of snapshots, the spatial spectrum of MUSIC-like algorithm deviates from the MUSIC algorithm. This difference in the performance of the two algorithms is due to the difference in respective formulations although their asymptotic performances are similar. This phenomenon is observed for the same setup

Figure 3.3: Spatial spectrum of MUSIC and MUSIC-like algorithms with 100 snapshots for $L = 5$, $M = 2$ and SNR=5 dB.

with $L = 5$, $M = 2$ along 70° and 90° , but with 100 sensor snapshots. The sensor outputs are corrupted with additive Gaussian noise maintained at an SNR of 5 dB. Deferring the discussion on the choice of β for now, the spatial spectra obtained for MUSIC and MUSIC-like algorithms are plotted in Figure 3.3. We note that the spectrum floor for both the algorithms are still maintained at the same level. Although the source direction peaks are prominent in the spectrum for both the techniques, the magnitude is significantly reduced when compared to Figure 3.2. The difference in the spectrum of the two techniques becomes more prominent at lower SNR conditions where the use of one of the complementary subspaces in MUSIC is advantageous over MUSIC-like algorithm which uses the entire covariance matrix as will be shown Section 3.4.

3.2.3 Bounds for β

The bounds for β were derived in [9] considering the possibility of the steering vector residing in different subspaces under three different cases. We alternatively derive the bounds from the solution to the GEV in (3.3)

$$(\mathbf{a}(\phi)\mathbf{a}^H(\phi) + \beta\mathbf{I}_L)\mathbf{w}_\phi = \chi_{\min}^{-1}\mathbf{R}\mathbf{w}_\phi. \quad (3.15)$$

With $\mathbf{R}$ being positive definite, (3.15) takes the form

$$\begin{aligned} \mathbf{R}^{-1}(\mathbf{a}(\phi)\mathbf{a}^H(\phi) + \beta\mathbf{I}_L)\mathbf{w}_\phi &= \chi_{\min}^{-1}\mathbf{w}_\phi, \\ \{\mathbf{C} + \beta\mathbf{B}\}\mathbf{w}_\phi &= \chi_{\min}^{-1}\mathbf{w}_\phi, \end{aligned} \quad (3.16)$$

where $\mathbf{B} = \mathbf{R}^{-1}$ and $\mathbf{C} = \mathbf{B}\mathbf{a}(\phi)\mathbf{a}^H(\phi)$. Hence, the above results in

$$\frac{\mathbf{w}_\phi^H\{\mathbf{C} + \beta\mathbf{B}\}\mathbf{w}_\phi}{\|\mathbf{w}_\phi\|_2^2} = \chi_{\min}^{-1} \quad (3.17)$$

denoting the maximum eigenvalue of $\{\mathbf{C} + \beta\mathbf{B}\}$. For $P(\phi)$ to provide peaks along source directions, $\mathbf{w}_\phi$ has to satisfy $|\mathbf{w}_\phi^H\mathbf{a}(\phi)| = 0$. Consequently, we require $\mathbf{C}\mathbf{w}_\phi = \mathbf{B}\mathbf{a}(\phi)\mathbf{a}^H(\phi)\mathbf{w}_\phi = 0$, $\forall \phi \in \Theta$ which is satisfied when

$$\frac{\mathbf{w}_\phi^H\{\mathbf{C} + \beta\mathbf{B}\}\mathbf{w}_\phi}{\|\mathbf{w}_\phi\|_2^2} \approx \beta \frac{\mathbf{w}_\phi^H\mathbf{B}\mathbf{w}_\phi}{\|\mathbf{w}_\phi\|_2^2}. \quad (3.18)$$

This condition is assured when

$$\beta \frac{\mathbf{w}_\phi^H\mathbf{B}\mathbf{w}_\phi}{\|\mathbf{w}_\phi\|_2^2} \gg \frac{\mathbf{w}_\phi^H\mathbf{C}\mathbf{w}_\phi}{\|\mathbf{w}_\phi\|_2^2}. \quad (3.19)$$

Since $\mathbf{R}$ is a full rank matrix, we note that

$$\text{rank}(\mathbf{C}) = \text{rank}(\mathbf{R}^{-1}\mathbf{P}_{\mathbf{a}_\phi}) = \min\{\text{rank}(\mathbf{R}^{-1}), \text{rank}(\mathbf{P}_{\mathbf{a}_\phi})\}. \quad (3.20)$$

With $\text{rank}(\mathbf{P}_{\mathbf{a}_\phi}) = 1$, the rank of $\mathbf{C}$ is equal to 1. Under this condition, we have $\chi_{\mathbf{C},\max} = \text{tr}\{\mathbf{C}\}$, where we explicitly denote the maximum eigenvalue of $\mathbf{C}$ as $\chi_{\mathbf{C},\max}$. The highest lower bound for the inequality in (3.19) will therefore be

$$\beta \frac{\mathbf{w}_\phi^H \mathbf{B} \mathbf{w}_\phi}{\|\mathbf{w}_\phi\|_2^2} \gg \chi_{\mathbf{C},\max} = \text{tr}\{\mathbf{C}\}. \quad (3.21)$$

Furthermore, when (3.18) is satisfied, we have $\frac{\mathbf{w}_\phi^H \mathbf{B} \mathbf{w}_\phi}{\|\mathbf{w}_\phi\|_2^2} = \chi_{\mathbf{B},\max}$. The highest lower bound for β considering all the source directions is therefore given by

$$\beta > \max_{\phi \in \Theta} \frac{\text{tr}\{\mathbf{B}\mathbf{a}(\phi)\mathbf{a}^H(\phi)\}}{\chi_{\mathbf{B},\max}} = \max_{\phi \in \Theta} \frac{\mathbf{a}^H(\phi)\mathbf{R}^{-1}\mathbf{a}(\phi)}{\chi_{\mathbf{R}^{-1},\max}}, \quad (3.22)$$

where we have used the identity $\text{tr}\{\mathbf{EFG}\} = \text{tr}\{\mathbf{FGE}\}$ for any three matrices $\mathbf{E}, \mathbf{F}, \mathbf{G}$. It is interesting to note that the term $(\mathbf{a}^H(\phi)\mathbf{R}^{-1}\mathbf{a}(\phi))^{-1}$ signifies the signal power along the source direction. Rewriting the R.H.S. of (3.22) in terms of the minimum eigenvalue of $\mathbf{R}$ which defines the noise power, we have

$$\begin{aligned} \beta &> \max_{\phi \in \Theta} \frac{\chi_{\mathbf{R},\min}}{(\mathbf{a}^H(\phi)\mathbf{R}^{-1}\mathbf{a}(\phi))^{-1}} = \max_{\phi \in \Theta} \frac{\sigma_v^2}{(\mathbf{a}^H(\phi)\mathbf{R}^{-1}\mathbf{a}(\phi))^{-1}}, \\ \beta &> \frac{1}{\text{SNR}}. \end{aligned} \quad (3.23)$$

We therefore infer that the parameter β is guided by the reciprocal of SNR.

For directions $\phi \notin \Theta$, it is equally important to ensure $\mathbf{C}\mathbf{w}_\phi \neq 0$, failing which false peaks will appear in the spatial spectrum. In view of this, β cannot be arbitrarily high

and has to be constrained such that

$$\begin{aligned} \beta \frac{\mathbf{w}_\phi^H \mathbf{B} \mathbf{w}_\phi}{\|\mathbf{w}_\phi\|_2^2} &< \frac{\mathbf{w}_\phi^H \mathbf{C} \mathbf{w}_\phi}{\|\mathbf{w}_\phi\|_2^2}, \text{ or} \\ \beta \chi_{\mathbf{B}, \max} &< \chi_{\mathbf{C}, \max}. \end{aligned} \quad (3.24)$$

Since (3.24) has to be maintained for all $\phi \notin \Theta$, the lowest upper bound for β is obtained as

$$\beta < \min_{\phi \notin \Theta} \frac{\mathbf{a}_\phi^H \mathbf{R}^{-1} \mathbf{a}_\phi}{\chi_{\mathbf{B}, \max}}. \quad (3.25)$$

Therefore, the bound for β is given by

$$\begin{aligned} \max_{\phi \in \Theta} \frac{\mathbf{a}^H(\phi) \mathbf{R}^{-1} \mathbf{a}(\phi)}{\chi_{\mathbf{R}^{-1}, \max}} &< \beta < \min_{\phi \notin \Theta} \frac{\mathbf{a}^H(\phi) \mathbf{R}^{-1} \mathbf{a}(\phi)}{\chi_{\mathbf{R}^{-1}, \max}} \\ \beta_{\min} &< \beta < \beta_{\max}. \end{aligned} \quad (3.26)$$

Emphasizing on the peaks along the source directions, a good choice for β can be obtained w.r.t. the minimum value of $\mathbf{a}^H(\phi) \mathbf{R}^{-1} \mathbf{a}(\phi)$ as

$$\beta = \eta \min_{\phi} \frac{\mathbf{a}^H(\phi) \mathbf{R}^{-1} \mathbf{a}(\phi)}{\chi_{\mathbf{R}^{-1}, \max}}, \quad (3.27)$$

where $\eta > 1$ ensures reliable DOA estimates when β is contained within the upper bound.

3.3 Proposed DOA Estimation Technique (MUSIC-W)

In the previous section, we studied that the working principle of the MUSIC-like algorithm is similar to that of MUSIC algorithm asymptotically. Even with finite number of snapshots, it was noted that the spatial spectrum of the MUSIC-like algorithm at

moderate SNR conditions approached that of the MUSIC algorithm when β is chosen within the derived bounds. Under adverse conditions however, the covariance matrix deviates significantly from the ideal conditions assumed in the earlier analysis which may adversely affect the estimation accuracy. Although this algorithm is insensitive to the errors in model-order estimation, it is computationally intensive to evaluate the generalized eigenvector in each look direction, compared to the evaluation of a single SVD in the case of MUSIC algorithm. In this section, we propose a new technique developed over the MUSIC-like algorithm which provides improved performance over the latter and has reduced the computational complexity.

Interpreting the GEV problem solved by the MUSIC-like algorithm in each look direction as two eigenvalue problems ((3.8) and (3.9)) being simultaneously solved, it is noted that the generalized eigenvector and the corresponding eigenvalue have to satisfy both the problems. Since we have $\mathbf{a}(\phi) \in \mathcal{R}\{\mathbf{U}_s\} \cup \mathcal{R}\{\mathbf{U}_n\}$ along the non-source directions, the solution weight vector yields $|\mathbf{w}_\phi^H \mathbf{a}(\phi)|^2 \neq 0$. Along the source directions however, we have the steering vector completely residing in the signal subspace. While the objective function ensures the weight vector solution to reside in the noise subspace, it is contradicting to maximize the look-direction power gain $\|\mathbf{w}_\phi^H \mathbf{a}(\phi)\|_2^2$. Due to this contrast between the two symmetric eigenvalue problems along the source directions, the generalized eigenvalue $\chi_{\min}$ corresponding to the solution vector is expected to be large compared to the value of $\chi_{\min}$ in all other directions. In order to verify this phenomenon, we study the generalized eigenvalue corresponding to the solution vector for the previous simulation setup with $L = 5$ sensors and two sources along 70° and 90° . With 100 snapshots and SNR= 5 dB, $\chi_{\min}$ is plotted against the scan direction ϕ in Figure 3.4. We note an increase in $\chi_{\min}$ along the source direction over a nominal value in all directions. This increase in eigenvalue indicates the difficulty for the weight vector solution along the source direction to achieve concurrent minimization of $|\bar{\mathbf{\Lambda}}_s^{-1/2} \mathbf{U}_s^H \mathbf{w}_\phi|^2$

Figure 3.4: Generalized eigenvalue $\chi_{\min}$ corresponding to the solution weight vector plotted against the scan direction for $L = 5, M = 2$ with 100 snapshots and SNR=5 dB.

and the maximization of $|\mathbf{w}_{\phi}^H \mathbf{a}(\theta)|^2$. It therefore follows that the choice of β is crucial in providing a solution which results in $|\mathbf{w}_{\phi}^H \mathbf{a}(\phi)|^2 = 0$ required for showing distinct peaks in $\text{DF}(\phi)$ along the source directions. Under adverse conditions, the estimation accuracy is susceptible to the choice of β and the number of snapshots used.

It was earlier observed that the weight vector solution resides in the noise subspace for both source and non-source directions. While each vector $\mathbf{w}_{\phi}$ is orthogonal to the signal subspace, the vector may be contained in a subset of noise eigenvectors of $\mathbf{R}$ unlike the minimum norm solution. We therefore construct a weight vector matrix given by

$$\mathbf{W} = [\mathbf{w}_{\phi_1} \ \mathbf{w}_{\phi_2} \ \dots \ \mathbf{w}_{\phi_K}], \quad (3.28)$$

where the solutions from K different directions are used to construct the matrix. Al-

though the columns of this weight vector matrix are not orthogonal, with $K > L - M$, the range space of $\mathbf{W}$ is expected to be the same as the noise subspace of $\mathbf{R}$. We can therefore replace the use of noise subspace in existing subspace-based DOA estimation techniques such as MUSIC and minimum norm algorithms. In order to emulate the MUSIC algorithm, the source directions can then be estimated as the peaks of

$$P_{\text{music-w}}(\phi) = 10 \log_{10} \frac{1}{\mathbf{a}^H(\phi) \mathbf{W} \mathbf{W}^H \mathbf{a}(\phi)}, \quad (3.29)$$

where the subscript “music-w” is used to denote the proposed algorithm which uses weight matrix $\mathbf{W}$ to approximate MUSIC algorithm. The performance of this technique will rely on the number of weight vectors K used to construct $\mathbf{W}$. This is because, sufficient number of weight vectors are required to ensure that the weight vector matrix $\mathbf{W}$ and the signal subspace eigenvector matrix $\mathbf{U}_n$ have the same range space, failing which will give rise to false peaks in the spatial spectrum. We therefore require at least $K \geq L - M$ vectors to construct $\mathbf{W}$. Since the weight vector evaluated in each direction over the entire azimuth range reside in the noise subspace, constructing $\mathbf{W}$ with $K \geq L$ will still provide similar performance, thus avoiding the need for the model order.

For the earlier setup with $L = 8$ and $M = 2$, we now study the spatial spectrum of the proposed MUSIC-W algorithm in comparison with the MUSIC and MUSIC-like algorithms. We employ $K = 8$ weight vectors from equidistant directions over an azimuth span of 10° to 170° to construct $\mathbf{W}$. At an SNR of 5 dB, the spatial spectra for the three techniques obtained for $N = 100$ snapshots is plotted in Figure 3.5. We observe that the proposed technique provides prominent peaks along the source directions similar to the MUSIC and MUSIC-like algorithms. However, the spectrum floor for the proposed technique differs from the other techniques which have similar spatial spectra over the entire azimuth range. The spectrum floor obtained for the MUSIC-W is susceptible to

Figure 3.5: Spatial spectrum of MUSIC-W algorithm in comparison with that of MUSIC and MUSIC-like algorithms at SNR=5 dB.

the directions from which the weight vectors are used to construct the matrix $\mathbf{W}$.

The above phenomenon is illustrated with the previous simulation setup for two cases with the weight vectors chosen from an azimuth range of 10° to 90° for the first case and 90° to 170° for the second. The spatial spectra are plotted in Figure 3.6(a) and (b), respectively. For the first case, we note that the spectrum floor is suppressed only in the corresponding region. While the absence of significant false peaks in the spectrum ensures that the range space $\mathcal{R}\{\mathbf{W}\}$ is equal to the noise subspace, the use of weight vectors from a specific region in the azimuth range emphasizes the suppression of noise floor corresponding to the same azimuth range. Since $\mathbf{W}$ cannot control the spectrum floor in the complementary region, we observe an increase in the corresponding segment in both the subplots of Figure 3.6.

Figure 3.6: Spatial spectrum of MUSIC-W with $\mathbf{W}$ constructed with weight vectors obtained at SNR=5 dB from the azimuth range (a) 10° to 90° , (b) 90° to 170° . $K = L$ weight vectors are used in $\mathbf{W}$.

3.3.1 Computational Complexity

For the total number of search directions N_{tot} , implementation of MUSIC-like algorithm is optimized in [9] by employing two algorithms to solve the GEV. Along few coarse directions N_{coarse} , an iterative algorithm which requires $O(L^2)$ to evaluate GEV [46] is used. In the remaining directions N_{fine} , a fast minor component analysis (MCA) algorithm [47] has been used to reduce the total computational cost to $O(N_{\text{coarse}}L^2 + 2N_{\text{fine}}N_{\text{iter}}L + N_{\text{tot}}L)$, where N_{iter} is the number of iterations required for the MCA algorithm to converge. In comparison, the proposed technique requires one to solve GEV in only K directions which can be solved employing the approach in [46]. The resultant complexity of this technique is therefore $O(KL^2 + N_{\text{tot}}KL)$. With K comparable with N_{coarse} , the use of MCA over N_{fine} directions is completely circumvented, thus reducing

the computations significantly. With the evaluation of GEV reduced to only K directions, the computational cost involved in the proposed algorithm is comparable with the MUSIC algorithm which requires $O(ML^2 + L(L - M)N_{\text{tot}} + (L - M)N_{\text{tot}})$.

3.4 Simulation Results

In this section, we evaluate the performance of the analyzed MUSIC-like algorithm and the proposed MUSIC-W algorithm in comparison with the MUSIC algorithm. It was observed in Section 3.2.3 that β is implicitly related to the SNR. We therefore expect the performance of MUSIC-like algorithm to be influenced by the choice of β , and hence η . We now study the performance of the MUSIC-like estimator over various values of η from the root-mean-squared error (RMSE) of the estimates, evaluated over 100 trials. Across all the simulations, we consider the scenario which consists of an eight-sensor ULA and three sources at 50° , 65° and 110° unless specified. We fix the number of snapshots to 100 and study the effect of η , defined after (3.27), at SNR = -5 , 0 and 5 dB. Figure 3.7 shows the RMSE of estimates averaged across the three sources and plotted against η for the MUSIC-like algorithm. At a moderate SNR of 5 dB, we note that any choice of η which ensures β within the bounds has marginal effect on the estimation accuracy. However, for SNRs of 0 dB and -5 dB, a large RMSE is observed when $\eta < 1.5$ suggesting that the parameter introduces significant error in the accuracy when chosen close to the derived lower bound. The estimation error gradually increases for $\eta > 1.5$ as β approaches the upper bound.

For the proposed MUSIC-W algorithm, the weight vectors used for $\mathbf{W}$ are evaluated with $\eta = 1.5$ based on this study. Besides this factor, the performance of the MUSIC-W algorithm is also influenced by the number of weight vectors used to construct $\mathbf{W}$ and the azimuth region over which these vectors are evaluated. While the influence of azimuth

Figure 3.7: RMSE of source estimates using the MUSIC-like algorithm plotted against η at various SNR values.

region on the spatial spectrum of MUSIC-W algorithm was observed in Figures 3.5 and 3.6, we now study the influence of the number of weight vectors in $\mathbf{W}$ on the performance of the technique. For the earlier simulation setup used to generate Figure 3.7 with $L = 8$, the estimation accuracy of the MUSIC-W technique is evaluated as a function of K . The RMSE of the three source estimates are averaged and plotted against K at SNR values of -5 dB, 0 dB and 5 dB in Figure 3.8. When the SNR is 5 dB, we note that the choice of $K > L$ is not of high importance. However, for SNRs of 0 and -5 dB, the estimation error is observed to decrease for $K > L$. This indicates that limiting the number of weight vectors marginally deteriorates the estimation performance at low SNR conditions since the weight vectors may not be completely orthogonal to the signal subspace. Therefore, employing $\mathbf{W}$ with $K \geq L$ is suggested for MUSIC-W algorithm to provide accurate estimates.

Figure 3.8: RMSE of source estimates obtained with the MUSIC-W algorithm plotted against K at various SNR values.

Since the weight vector solutions for both MUSIC-like and MUSIC-W algorithms are obtained by solving the GEV problem, the spatial spectrum cannot be expressed as a closed-form expression in terms of the covariance matrix and steering vector unlike the MVDR or MUSIC algorithms. We therefore study the performance of these algorithms with simulations observing the estimation accuracy and the probability of resolving closely-situated sources. We first study the estimation accuracy by observing the root mean squared error (RMSE) plotted against the SNR before analyzing the probability of resolution. We conducted a Monte Carlo simulation with 100 trials. For a simulation setup with an eight-sensor ULA and three sources at 50° , 70° and 80° , the DOA estimates for MUSIC-like algorithm are obtained with $\eta = 1.5$. We have used $K = 10$ vectors to construct $\mathbf{W}$ for the proposed MUSIC-W algorithm. The RMSE of source estimates are obtained with 100 snapshots for various SNR values and is compared

with the MVDR and MUSIC algorithms in Figure 3.9.

Figure 3.9: RMSE of source estimates plotted against SNR for the MUSIC-like, MUSIC-W, MVDR and MUSIC algorithms.

With the clear distinction of the signal and noise subspaces known, we note that the MUSIC algorithm has the best estimation performance and sets the bench mark for other techniques. Due to the poor resolution capabilities of the MVDR algorithm,

we note that the RMSE of the obtained DOA estimates for second and third sources is significantly larger than all the other techniques. We observe that the source direction estimates obtained by the MUSIC-like algorithm exhibit large RMSE at low SNR conditions. Although the RMSE decreases with an increase in the SNR, the non-diagonal noise covariance matrix with limited number of snapshots withhold its performance from approaching that of the MUSIC algorithm. It is interesting to note that the MUSIC-W algorithm shows better estimation accuracy over MUSIC-like algorithm when the SNR is increased. This advantage in the performance is due to the prime difference between the two algorithms. Along the source directions, it was earlier studied that the generalized eigenvalue of the MUSIC-like optimization problem is significantly large compared to all other directions, thus indicating the difficulty in simultaneously satisfying both the eigenvalue problems. This condition will therefore introduce marginal error in the DOA estimates of the MUSIC-like algorithm. Since the matrix $\mathbf{W}$ employed in the MUSIC-W algorithm is constructed with weight vectors obtained from arbitrary directions, the range space of this matrix is confined to the noise subspace. The estimation accuracy of MUSIC-W DOA estimates is therefore observed to approach that of MUSIC as the SNR is increased. At lower SNR conditions, the solution vectors in $\mathbf{W}$ may not completely reside in the noise subspace thus introducing marginal errors in the estimates.

We now study the capability of these techniques to resolve two closely-situated sources. The simulation setup involves $L = 8$ sensor ULA and two sources situated at 70° and 76° from the array axis. The two sources are considered to be resolved when

$$\begin{aligned} |\hat{\theta}_1 - \theta_1| &\leq \frac{\theta_2 - \theta_1}{2} \quad \text{and} \\ |\hat{\theta}_2 - \theta_2| &\leq \frac{\theta_2 - \theta_1}{2}, \end{aligned} \tag{3.30}$$

where $\hat{\theta}_1$ and $\hat{\theta}_2$ are the DOA estimates corresponding to the source directions θ_1 and

θ_2 respectively, obtained from the estimator. The probability of resolution is evaluated as the ratio of the number of times the algorithm resolves over the total number of Monte Carlo trials. With 100 sensor snapshots synthesized, the probability of resolution, evaluated over 150 trials, is plotted against SNR in Figure 3.10.

Figure 3.10: The probability of resolving two closely situated sources with 100 snapshots.

We note that the MUSIC-like algorithm and the proposed MUSIC-W algorithm can resolve the two source directions under adverse conditions where the MUSIC algorithm supported with AIC for model order estimation fails to resolve the sources. Furthermore, it is observed that MVDR requires 8 – 10 dB higher SNR to resolve the two sources. This simulation asserts the advantage of the proposed technique and the MUSIC-like algorithm over the MUSIC algorithm in conjunction with existing model order estimation techniques under adverse conditions.

We next study the effect of the number of snapshots used on the estimation accuracy of these algorithms for the scenario with $L = 8$ sensor ULA and three sources situated at 50° , 70° and 80° . Setting the SNR at 5 dB, the RMSE for each source direction is plotted in Figure 3.11 over increasing number of snapshots. We note that the DOA estimation accuracy of the MUSIC-like and MUSIC-W algorithms improves with an increase in the number of snapshots. However, we observe that the performance of the MUSIC-like algorithm does not approach that of the MUSIC algorithm. This is due to the non-diagonal noise covariance matrix obtained even when the number of snapshots is significantly large. In contrast, the performance of the MUSIC-W algorithm approaches that of the MUSIC algorithm when the number of snapshots is large. Due to the limited capability of the MVDR algorithm to resolve the closely-situated sources, we note that the MVDR estimates exhibit large RMSE error when the SNR is 5 dB.

When the SNR is increased to 15 dB, we note that the performance of the MVDR and the MUSIC-W algorithms approach that of the MUSIC algorithm while the performance of the MUSIC-like algorithm exhibits larger RMSE even when the number of snapshots is significantly large. This difference is again due to the small deviation of the noise covariance matrix from being a diagonal matrix. We also note that the RMSE of MUSIC-W decreases with increase in the number of snapshots and outperforms the MVDR algorithm even at 15 dB SNR.

3.5 Conclusion

In this chapter, we have presented a mathematical analysis of the MUSIC-like algorithm starting from its optimization problem and have shown that the working principle of this technique is similar to that of the MUSIC algorithm when the effect of finite snapshots is not considered for the estimation of the array covariance matrix. The bounds for the

parameter β have been rederived to ensure reliable estimates. From the insights obtained, we then proposed a new technique developed over the working principle of the MUSIC-like algorithm. The performance of the proposed MUSIC-W algorithm is observed in simulations to outperform the MUSIC-like algorithm with lesser computational cost.

With the insights obtained from this chapter, we identify the possibility of extending the MUSIC-like formulation for wideband DOA estimation. Considering the advantage of circumventing the model order estimation, we dedicate the next two chapters to develop wideband DOA estimation techniques based on this technique. The bounds for β derived in Section 3.2.3 can directly be extended to wideband source scenarios.

Figure 3.11: The RMSE of source estimates plotted against snapshots for the MUSIC-like, MUSIC-W, MUSIC and MVDR algorithms.

Chapter 4

Time-Domain Wideband DOA Estimation

4.1 Introduction

The problem of estimating wideband source directions from sensor snapshots is an integral part of many applications considering the fact that most of the real-world signals are wideband in nature. As studied in Section 2.3, the problem definition and the signal model for wideband DOA estimation are an extension of their narrowband counterpart. Since elegant solutions are available in the literature for narrowband DOA estimation, wideband DOA estimation is in general accomplished by employing narrowband algorithms either directly [31] or indirectly [33]. The appealing high resolution achieved by subspace-based narrowband techniques requires precise knowledge of the model order. The inaccuracy and computation complexity of model order estimation in wideband scenario requires an alternative approach to tackle the problem.

From the wideband data, parameter estimation can either be accomplished by incoherent processing of estimates obtained at each frequency [31] or by coherent processing

of data, followed by DOA estimation [33] as studied in Section 2.2.2. The latter approach generally outperforms the former under adverse SNR conditions due to an increase in signal power obtained by summing the coherent covariance matrices across the signal bandwidth. In coherent signal subspace method (CSSM), the focussing matrices transform data from different frequencies to a reference frequency prior to the joint processing. However, this technique requires initial approximate estimates of source directions for the design of focussing matrices. Under adverse conditions, large errors in the initial estimates introduce bias to the estimates. The frequency-invariant DOA estimation proposed in [39] is equivalent to coherent processing of the wideband data. Nevertheless, it is difficult to design filters with frequency-invariant property for arbitrary array geometries. The number of wideband sources present in the scenario has been estimated in [33] employing either the AIC or MDL criterion on the coherently averaged covariance matrix. However, if the initial estimates deviate from the true source directions, estimation of the model order and hence the DOA estimates will be adversely affected.

In this chapter, we provide a new approach to coherently process the wideband data across the signal bandwidth. This is accomplished by approximating the array manifold at a frequency within the signal bandwidth w.r.t. a reference frequency using Taylor series expansion. This approximation, detailed in Section 4.2, permits us to have a time-domain signal model for wideband DOA estimation. The expression for the corresponding data covariance matrix, consists of a coherent component at the reference frequency along with additional derivative components. We then propose to extend the optimization problem of the MUSIC-like algorithm for wideband DOA estimation in Section 4.3. This enables one to obtain DOA estimates without estimating the model order. We also perform a detailed analysis of the optimization problem which enables one to understand the working principle of the proposed technique. Computer simulations and corresponding discussions are presented in Section 4.5.

4.2 Array Manifold Approximation for Wideband Signal Model

The wideband signals acquired by the sensor array can be modeled in frequency domain as shown in (2.32). From the Fourier transform relation, the continuous-time sensor signals can be expressed using (2.32) as

$$\begin{aligned}\mathbf{x}(t) &= \int_{-\infty}^{\infty} \underline{\mathbf{x}}(f) e^{j2\pi ft} df \\ &= \int_{-\infty}^{\infty} \mathbf{A}(f, \Theta) \underline{\mathbf{s}}(f) e^{j2\pi ft} df + \mathbf{v}(t),\end{aligned}\tag{4.1}$$

where $\mathbf{v}(t) = \int_{-\infty}^{\infty} \underline{\mathbf{v}}(f) e^{j2\pi ft} df$ is the additive noise vector, uncorrelated with the sources. It is important to note that the vectors $\mathbf{a}(f, \theta)$ in $\mathbf{A}(f, \Theta)$ corresponding to each source direction are continuous functions of frequency. The steering vector at a frequency f in the proximity of a reference frequency f_0 can hence be approximated using Taylor series expansion up to the P th order as

$$\mathbf{a}(f, \theta) \approx \mathbf{a}(f_0, \theta) + \sum_{p=1}^P \frac{(\delta f)^p}{p!} \frac{\partial^p}{\partial f^p} \mathbf{a}(f, \theta) \Big|_{f=f_0},\tag{4.2}$$

where $\frac{\partial^p}{\partial f^p} \mathbf{a}(f, \theta)$ is the p th-order derivative of the steering vector w.r.t. frequency and $\delta f = f - f_0$ is the frequency deviation from f_0 . Correspondingly, the array manifold matrix can be written as

$$\mathbf{A}(f, \Theta) \approx \mathbf{A}(f_0, \Theta) + \sum_{p=1}^P \frac{(\delta f)^p}{p!} \frac{\partial^p}{\partial f^p} \mathbf{A}(f, \Theta) \Big|_{f=f_0},\tag{4.3}$$

where the matrix $\frac{\partial^p}{\partial f^p} \mathbf{A}(f, \Theta) = [\frac{\partial^p}{\partial f^p} \mathbf{a}(f, \theta_1) \dots \frac{\partial^p}{\partial f^p} \mathbf{a}(f, \theta_M)]$ is the p th-order derivative array manifold. If the steering vector at each frequency over the entire signal bandwidth

is approximated with the P th-order Taylor series, we can substitute (4.3) into (4.1) giving

$$\mathbf{x}(t) \approx \int_{-\infty}^{\infty} \left\{ \mathbf{A}(f_0, \Theta) + \sum_{p=1}^P \frac{(\delta f)^p}{p!} \frac{\partial^p}{\partial f^p} \mathbf{A}(f, \Theta) \Big|_{f=f_0} \right\} \underline{\mathbf{s}}(f) e^{j2\pi f t} df + \mathbf{v}(t). \quad (4.4)$$

The discrete-time sensor snapshots can be obtained by replacing Fourier transform in (4.4) with an F –point discrete Fourier transform as

$$\mathbf{x}(n) \approx \frac{1}{F} \sum_{k=0}^{F-1} \left\{ \mathbf{A}(f_0, \Theta) + \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} \frac{\partial^p}{\partial f^p} \mathbf{A}(f, \Theta) \Big|_{f=f_0} \right\} \underline{\mathbf{s}}(f_k) e^{j2\pi k n / F} + \mathbf{v}(n), \quad (4.5)$$

where f_k denotes the frequency corresponding to bin index k . The array covariance matrix can then be expressed as

$$\begin{aligned} \mathbf{R} &= \mathbb{E}\{\mathbf{x}(n)\mathbf{x}^H(n)\} \\ &\approx \mathbb{E}\left\{ \left[\frac{1}{F} \sum_{k=0}^{F-1} \left(\mathbf{A}(f_0, \Theta) + \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} \frac{\partial^p}{\partial f^p} \mathbf{A}(f, \Theta) \Big|_{f=f_0} \right) \underline{\mathbf{s}}(f_k) e^{j\frac{2\pi k n}{F}} + \mathbf{v}(n) \right] \right. \\ &\quad \left. \left[\frac{1}{F} \sum_{k=0}^{F-1} \left(\mathbf{A}(f_0, \Theta) + \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} \frac{\partial^p}{\partial f^p} \mathbf{A}(f, \Theta) \Big|_{f=f_0} \right) \underline{\mathbf{s}}(f_k) e^{j\frac{2\pi k n}{F}} + \mathbf{v}(n) \right]^H \right\}. \end{aligned} \quad (4.6)$$

Assuming that the source spectrum is uncorrelated across frequency bins, i.e., $E\{\underline{\mathbf{s}}(f_1)\underline{\mathbf{s}}^H(f_2)\} = \mathbf{0}, \forall f_1 \neq f_2$, (4.6) simplifies to

$$\begin{aligned}
 \mathbf{R} &\approx \frac{1}{F^2} \sum_{k=0}^{F-1} \sum_{p,q=0}^P \frac{(\delta f_k)^p}{p!} \frac{\partial^p}{\partial f^p} \mathbf{A}(f, \Theta) \Big|_{f=f_0} \mathbf{R}_{ss}(f_k) \frac{(\delta f_k)^q}{q!} \frac{\partial^q}{\partial f^q} \mathbf{A}^H(f, \Theta) \Big|_{f=f_0} + \sigma_v^2 \mathbf{I}_L \\
 &\approx \frac{1}{F^2} \mathbf{A}(f_0, \Theta) \left\{ \sum_{k=0}^{F-1} \mathbf{R}_{ss}(f_k) \right\} \mathbf{A}^H(f_0, \Theta) \\
 &\quad + \frac{2}{F^2} \text{Re} \left\{ \sum_{k=0}^{F-1} \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} \mathbf{A}(f_0, \Theta) \mathbf{R}_{ss}(f_k) \frac{\partial^p}{\partial f^p} \mathbf{A}(f, \Theta) \Big|_{f=f_0}^H \right\} \\
 &\quad + \frac{1}{F^2} \sum_{k=0}^{F-1} \sum_{p,q=1}^P \frac{(\delta f_k)^p}{p!} \frac{\partial^p}{\partial f^p} \mathbf{A}(f, \Theta) \Big|_{f=f_0} \mathbf{R}_{ss}(f_k) \frac{(\delta f_k)^q}{q!} \frac{\partial^q}{\partial f^q} \mathbf{A}(f, \Theta) \Big|_{f=f_0}^H + \sigma_v^2 \mathbf{I}_L,
 \end{aligned} \tag{4.7}$$

where $\mathbf{R}_{ss}(f_k)$ is the source covariance matrix at frequency f_k , σ_v^2 is the noise variance assumed to be equal across the array and $\text{Re}\{\cdot\}$ denotes the real part of the expression within. We note that the data from the entire signal bandwidth is coherently averaged in the first term. Besides improving the SNR, this term restores the rank of the averaged source covariance matrix when the sources are highly correlated as shown in [33]. The second and third terms contain derivatives arising due to the array manifold approximation. These additional terms may increase the eigenvalue spread of $\mathbf{R}$, making it difficult to estimate the model order employing conventional techniques.

For a given array geometry, if the p th derivative of the steering vector $\mathbf{a}(f, \theta)$ w.r.t. frequency evaluated at the reference frequency f_0 is denoted as

$$\mathbf{d}_p(f_0, \theta) = \frac{\partial^p \mathbf{a}(f, \theta)}{\partial f^p} \Big|_{f=f_0}, \tag{4.8}$$

a matrix with p th-order derivatives from all the source directions can be formed as

$$\begin{aligned}\mathbf{D}_p(\Theta) &= \left. \frac{\partial^p}{\partial f^p} \mathbf{A}(f, \Theta) \right|_{f=f_0} \\ &= \begin{bmatrix} \mathbf{d}_p(f_0, \theta_1) & \mathbf{d}_p(f_0, \theta_2) & \dots & \mathbf{d}_p(f_0, \theta_M) \end{bmatrix}.\end{aligned}\tag{4.9}$$

The covariance matrix in (4.7) can then be written as

$$\begin{aligned}\mathbf{R} \approx & \frac{1}{F^2} \left[\mathbf{A}(f_0, \Theta) \left\{ \sum_{k=0}^{F-1} \mathbf{R}_{ss}(f_k) \right\} \mathbf{A}^H(f_0, \Theta) + 2 \sum_{k=0}^{F-1} \text{Re} \left\{ \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} \mathbf{A}(f_0, \Theta) \mathbf{R}_{ss}(f_k) \mathbf{D}_p^H(\Theta) \right\} \right. \\ & \left. + \sum_{p,q=1}^P \mathbf{D}_p(\Theta) \left\{ \sum_{k=0}^{F-1} \frac{(\delta f_k)^{p+q}}{p!q!} \mathbf{R}_{ss}(f_k) \right\} \mathbf{D}_q^H(\Theta) \right] + \sigma_v^2 \mathbf{I}_L.\end{aligned}\tag{4.10}$$

Note that this approximation is valid over a small signal bandwidth across which δf is small. When the signal bandwidth is large, finite order Taylor series approximation may introduce error resulting in estimation inaccuracy.

4.3 Proposed Technique

It is interesting to note from (4.10) that the expression for the wideband data covariance matrix deviates from its narrowband counterpart only due to the presence of the second term, suppression of which, reduces the problem to a coherently averaged narrowband DOA estimation problem. Referring to the BFW-MUSIC algorithm discussed in Section 2.3.2.6, it is interesting to note that the linear derivative constraints are precisely aimed at annulling the effect of these terms. Although BFW-MUSIC is efficient in suppressing these terms, the optimization problem is susceptible to the choice of the signal subspace dimension. This is because, it may not be optimal to choose the dimension of signal subspace for $\mathbf{R}$ expressed in (4.10) to be equal to the model order. We therefore consider the structure of a MUSIC-like formulation for wideband DOA estimation, which

does not explicitly require one to define the signal subspace dimension.

4.3.1 Formulation

We first define a matrix $\mathbf{D}_{f_0, \phi}$ with derivatives of steering vectors up to the P th-order evaluated at f_0 from a given scan direction ϕ as

$$\mathbf{D}_{f_0, \phi} = \begin{bmatrix} \mathbf{d}_1(f_0, \phi) & \mathbf{d}_2(f_0, \phi) & \dots & \mathbf{d}_P(f_0, \phi) \end{bmatrix}. \quad (4.11)$$

With projection matrix of $\mathbf{D}_{f_0, \phi}$ constructed as

$$\mathbf{P}_{\mathbf{D}_\phi} = \mathbf{D}_{f_0, \phi} (\mathbf{D}_{f_0, \phi}^H \mathbf{D}_{f_0, \phi})^{-1} \mathbf{D}_{f_0, \phi}^H,$$

the sensor snapshots $\mathbf{x}(n)$ are then projected onto the derivative vector space, $\mathbf{y}_\phi(n) = \mathbf{P}_{\mathbf{D}_\phi} \mathbf{x}(n)$, which can loosely be termed as derivative-transformed data snapshots. Since the columns of $\mathbf{P}_{\mathbf{D}_\phi}$ span a P -dimensional subspace, the rank of the covariance matrix

$$\begin{aligned} \mathbf{R}_y(\phi) &= \mathbb{E}\{\mathbf{y}_\phi(n) \mathbf{y}_\phi^H(n)\} \\ &= \mathbf{P}_{\mathbf{D}_\phi} \mathbf{R} \mathbf{P}_{\mathbf{D}_\phi}^H \end{aligned} \quad (4.12)$$

is restricted to P with its eigenvectors spanning only the derivative vector space.

From the analysis of narrowband MUSIC-like algorithm in Section 3.2.2, it was noted that minimizing the beamformer output power in the look direction enforces the weight vector solution to reside in the noise subspace. Besides this orthogonality to signal subspace, the weight vector is now required to be robust against frequency variation. Therefore, we propose a new formulation for wideband DOA estimation by incorporating (4.12)

as

$$\begin{aligned}
 & \underset{\mathbf{w}_\phi}{\text{minimize}} \quad \mathbf{w}_\phi^H \mathbf{R} \mathbf{w}_\phi + \zeta \mathbf{w}_\phi^H \mathbf{R}_y(\phi) \mathbf{w}_\phi \\
 & \text{subject to} \quad \mathbf{w}_\phi^H \{ \mathbf{a}(f_0, \phi) \mathbf{a}^H(f_0, \phi) + \beta \mathbf{I}_L \} \mathbf{w}_\phi = c,
 \end{aligned} \tag{4.13}$$

where $\zeta, \beta, c > 0$ are constants which will be discussed subsequently. Based on the eigenvalue spread of $\mathbf{R}$, the first term in the objective function $\mathbf{w}_\phi^H \mathbf{R} \mathbf{w}_\phi$ minimizes the projection of $\mathbf{w}_\phi$ onto the composite signal subspace. The inclusion of the second term provides additional emphasis on the derivative vector space, thus ensuring the weight vector to be invariant to frequency variations. The solution to this problem is obtained by setting the gradient of the Lagrangian function

$$L(\mathbf{w}_\phi, \chi) = \mathbf{w}_\phi^H (\mathbf{R} + \zeta \mathbf{R}_y(\phi)) \mathbf{w}_\phi + \chi (\mathbf{w}_\phi^H \mathbf{a}(f_0, \phi) \mathbf{a}^H(f_0, \phi) \mathbf{w}_\phi + \beta \|\mathbf{w}_\phi\|_2^2 - c), \tag{4.14}$$

where χ is the Lagrangian parameter, with respect to the conjugate of $\mathbf{w}_\phi$ to zero. The eigenvector corresponding to the minimum eigenvalue $\chi_{\min}$ of the generalized eigenvalue (GEV) problem,

$$(\mathbf{R} + \zeta \mathbf{R}_y(\phi)) \mathbf{w}_\phi = \chi_{\min} (\mathbf{a}(f_0, \phi) \mathbf{a}^H(f_0, \phi) + \beta \mathbf{I}_L) \mathbf{w}_\phi \tag{4.15}$$

serves as the optimal weight vector in each scan direction. Since the solution weight vector is independent of c , the value of c need not be determined. However, for any $\beta > 0$, the matrix $\mathbf{a}(f_0, \phi) \mathbf{a}^H(f_0, \phi) + \beta \mathbf{I}_L$ is positive definite which assures $c > 0$ from the constraint. Solving for $\mathbf{w}_\phi$ in each direction, source directions are identified from the direction finding function

$$P(\phi) = \frac{1}{|\mathbf{w}_\phi^H \mathbf{a}(f_0, \phi)|^2}. \tag{4.16}$$

4.3.2 Discussion

We note that the proposed optimization problem incorporates $\mathbf{R}_y(\phi)$ into the cost function of the narrowband MUSIC-like algorithm. It is essential to analyze the impact of this term on the weight vector solution, and hence the DOA estimates. Towards that we consider a single-source scenario with the source positioned in direction θ . The array covariance matrix in (4.10) simplifies to

$$\begin{aligned} \mathbf{R} \approx \frac{1}{F^2} \sum_{k=0}^{F-1} \sigma_s^2(f_k) & \left[\mathbf{a}(f_0, \theta) \mathbf{a}^H(f_0, \theta) + 2\text{Re} \left\{ \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} \mathbf{a}(f_0, \theta) \mathbf{d}_p^H(f_0, \theta) \right\} \right. \\ & \left. + \sum_{p,q=1}^P \frac{(\delta f_k)^{p+q}}{p!q!} \mathbf{d}_p(f_0, \theta) \mathbf{d}_q^H(f_0, \theta) \right] + \sigma_v^2 \mathbf{I}_L, \end{aligned} \quad (4.17)$$

where $\sigma_s^2(f_k)$ is the signal power at frequency f_k . When $\mathbf{w}_\phi^H \mathbf{R} \mathbf{w}_\phi$ is minimized in the MUSIC-like algorithm, the components corresponding to the narrowband steering vector in (4.17) are suppressed efficiently, while the suppression of the derivative terms is not ensured. This is because, the power of derivative terms in (4.17) is relatively lower than the signal power, requiring external assistance to suppress these terms.

When $\mathbf{R}_y(\phi)$ is included into the objective function in (4.13), the expression for $\mathbf{R} + \zeta \mathbf{R}_y(\phi)$ along $\phi \neq \theta$ is given by

$$\begin{aligned} \mathbf{R} + \zeta \mathbf{R}_y(\phi) = \frac{1}{F^2} \sum_{k=0}^{F-1} \sigma_s^2(f_k) & \left[\mathbf{a}(f_0, \theta) \mathbf{a}^H(f_0, \theta) + \zeta \mathbf{P}_{\mathbf{D}_\phi} \mathbf{a}(f_0, \theta) \mathbf{a}^H(f_0, \theta) \mathbf{P}_{\mathbf{D}_\phi} \right. \\ & + 2\text{Re} \left\{ \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} \left(\mathbf{a}(f_0, \theta) \mathbf{d}_p^H(f_0, \theta) + \zeta \mathbf{P}_{\mathbf{D}_\phi} \mathbf{a}(f_0, \theta) \mathbf{d}_p^H(f_0, \theta) \right) \right\} \\ & + \sum_{p,q=1}^P \frac{(\delta f_k)^{p+q}}{p!q!} \left(\mathbf{d}_p(f_0, \theta) \mathbf{d}_q^H(f_0, \theta) + \zeta \mathbf{P}_{\mathbf{D}_\phi} \mathbf{d}_p(f_0, \theta) \mathbf{d}_q^H(f_0, \theta) \mathbf{P}_{\mathbf{D}_\phi} \right) \Big] \\ & + \sigma_n^2 (\mathbf{I}_L + \zeta \mathbf{P}_{\mathbf{D}_\phi}). \end{aligned} \quad (4.18)$$

For $\phi \neq \theta$, we note that the range space of $\mathbf{R}_y(\phi)$ is not the same as that of the derivative terms present in $\mathbf{R}$. Although the inclusion of $\mathbf{R}_y(\phi)$ does not contribute to the spectrum in non-source directions, the performance of the proposed technique is not adversely affected since the solution in these directions still satisfies $\mathbf{w}_\phi^H \mathbf{a}(f_0, \phi) \neq 0, \forall \phi \neq \theta$. Moreover, the weight vectors obtained in these arbitrary directions may not span the noise subspace. We therefore cannot employ these weight vectors for the construction of the weight matrix $\mathbf{W}$ to realize the MUSIC-W algorithm for wideband DOA estimation.

On the contrary, the range space of $\mathbf{R}_y(\theta)$ is equal to that of $\mathbf{D}_{f_0, \theta}$ along the source directions. As a consequence, the inclusion of $\mathbf{R}_y(\theta)$ provides additional emphasis on the derivative vector space, thus enforcing the cost function to minimize the corresponding components. In order to analyze the expression for $\mathbf{R} + \zeta \mathbf{R}_y(\theta)$, we consider the following property of projection matrices:

Given a projection matrix $\mathbf{P} \in \mathbb{C}^{L \times L}$ and a vector $\mathbf{u} \in \mathbb{C}^L$, if $\mathbf{u} \in \mathcal{R}(\mathbf{P})$, where $\mathcal{R}(\cdot)$ denotes the range space of the underlying matrix, then $\mathbf{P}\mathbf{u} = \mathbf{u}$ [48, pp.108]. We therefore have $\mathbf{P}_{\mathbf{D}_\theta} \mathbf{d}_p(f_0, \theta) = \mathbf{d}_p(f_0, \theta)$ along the source directions, thus simplifying (4.18) to

$$\begin{aligned} \mathbf{R} + \zeta \mathbf{R}_y(\theta) = & \frac{1}{F^2} \sum_{k=0}^{F-1} \sigma_s^2(f_k) \left[\mathbf{a}(f_0, \theta) \mathbf{a}^H(f_0, \theta) + \zeta \mathbf{P}_{\mathbf{D}_\theta} \mathbf{a}(f_0, \theta) \mathbf{a}^H(f_0, \theta) \mathbf{P}_{\mathbf{D}_\theta} \right. \\ & + 2\text{Re} \left\{ \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} \left(\mathbf{a}(f_0, \theta) \mathbf{d}_p^H(f_0, \theta) + \zeta \mathbf{P}_{\mathbf{D}_\theta} \mathbf{a}(f_0, \theta) \mathbf{d}_p^H(f_0, \theta) \right) \right\} \\ & \left. + (1 + \zeta) \sum_{p,q=1}^P \frac{(\delta f_k)^{p+q}}{p!q!} \mathbf{d}_p(f_0, \theta) \mathbf{d}_q^H(f_0, \theta) \right] + \sigma_v^2 (\mathbf{I}_L + \zeta \mathbf{P}_{\mathbf{D}_\theta}). \end{aligned} \quad (4.19)$$

Since $\mathcal{R}(\mathbf{a}(f_0, \theta)) \notin \mathcal{R}(\mathbf{P}_{\mathbf{D}_\theta})$ for most array geometries, the contribution of $\mathbf{R}_y(\theta)$ to the derivative subspace is prominent over the contribution to the steering vector. It therefore follows that minimizing the objective function in (4.13) suppresses the derivative terms, thus reducing it to a narrowband DOA estimation problem. Although the matrix $\mathbf{R}_y(\theta)$

provides data dependent weight to the derivative components, we note that it does not vanish when only narrowband sources are present unless the derivative vector matrix $\mathbf{D}_{f_0, \theta}$ is orthogonal to the steering vector. We therefore introduce the parameter ζ which can regulate the contribution of this term as a function of signal bandwidth. Letting

$$\zeta = \frac{f_u - f_l}{f_c} = \Delta f, \quad (4.20)$$

we note that ζ scales the contribution from $\mathbf{R}_y(\theta)$ in accordance with the source bandwidth. For a specific case with narrowband sources, we have $\zeta = 0$, reducing the proposed formulation to the narrowband MUSIC-like algorithm. Although the assigned value for ζ is intuitively sensible for reliable DOA estimates, a detailed study on the influence of this parameter calls for attention. In Section 4.5, we study the sensitivity of DOA estimation performance with the choice of ζ for various SNR values.

Following the same procedure as that for the narrowband MUSIC-like algorithm provided in Section 3.2.3, the bounds for β can be derived with $\zeta \geq 0$ as

$$\min_{\phi} \frac{\mathbf{a}^H(f_0, \phi)(\mathbf{R} + \zeta \mathbf{R}_y(\phi))^{-1} \mathbf{a}(f_0, \phi)}{\chi(\mathbf{R} + \zeta \mathbf{R}_y(\phi))^{-1}, \max} < \beta < \max_{\phi} \frac{\mathbf{a}^H(f_0, \phi)(\mathbf{R} + \zeta \mathbf{R}_y(\phi))^{-1} \mathbf{a}(f_0, \phi)}{\chi(\mathbf{R} + \zeta \mathbf{R}_y(\phi))^{-1}, \max}. \quad (4.21)$$

While the lower bound ensures distinct peaks along the source directions, the upper bound prevents the false peaks from occurring in the spatial spectrum. The value of β is therefore chosen with reference to the lower bound as

$$\beta = \eta \min_{\phi} \frac{\mathbf{a}^H(f_0, \phi)(\mathbf{R} + \zeta \mathbf{R}_y(\phi))^{-1} \mathbf{a}(f_0, \phi)}{\chi(\mathbf{R} + \zeta \mathbf{R}_y(\phi))^{-1}, \max}, \quad (4.22)$$

where $\eta > 1$ provides unambiguous spatial spectrum such that β is within the bounds of (4.21). From (4.22), we observe that the value of β is chosen based on the reciprocal of MVDR response for $\mathbf{R} + \zeta \mathbf{R}_y(\phi)$. We therefore expect the nulls of this function to occur

approximately at the source directions. In order to verify this, we consider a scenario with ten-sensor double-ring array in the presence of three wideband sources with a fractional bandwidth of 33% along 30° , 36° and 60° . The sensor snapshots are synthesized with an SNR of 5 dB. The function on the R.H.S. of (4.22) is plotted for $\zeta = 0$ and 1 in Figure 4.1. We observe that the MVDR response identifies approximate source directions although it fails to resolve all the sources directions present in the scenario. While the response for the two values of ζ are distinct, it is interesting to note that the amplitude of the function is almost equal along the source directions. This implies that the null-depth of this function is not significantly affected by the contribution of $\mathbf{R}_y(\phi)$ suggesting the choice of β based on the latter case with $\zeta = 0$, i.e.,

$$\beta = \eta \min_{\phi} \frac{\mathbf{a}^H(f_0, \phi) \mathbf{R}^{-1} \mathbf{a}(f_0, \phi)}{\chi_{\mathbf{R}^{-1}, \max}}. \quad (4.23)$$

In order to estimate the source directions using the proposed technique for the above illustration, we assign $\eta = 2.5$ ensuring that β is within the bounds defined by (4.21). Following the expression in (4.20), we have $\zeta = 0.33$. With 500 time-domain snapshots, the spatial spectrum obtained for the proposed technique is plotted in Figure 4.2. The enlarged window shows that the technique accurately estimates all the three source directions without requiring the model order with as few as 500 time-domain snapshots.

Note that the optimization problem in (4.13) is similar to that proposed in [49] in the context of adaptive beamforming to introduce robustness against jammer motion. While the technique in [49] imposes derivative constraints w.r.t. the wavenumber to achieve invariance in the spatial spectrum around a given source direction θ , the proposed technique imposes derivative constraints w.r.t. frequency so that $\mathbf{w}_\phi$ is invariant to the steering vector variations around the center frequency f_0 .

Figure 4.1: Plot of $\frac{\mathbf{a}^H(f_0, \phi)(\mathbf{R} + \zeta \mathbf{R}_y(\phi))^{-1} \mathbf{a}(f_0, \phi)}{\chi_{(\mathbf{R} + \zeta \mathbf{R}_y(\phi))^{-1}, \max}}$ and $\frac{\mathbf{a}^H(f_0, \phi) \mathbf{R}^{-1} \mathbf{a}(f_0, \phi)}{\chi_{\mathbf{R}^{-1}, \max}}$ in the presence of three sources.

4.3.3 Computational Complexity

Evaluating the precise computational complexity for the proposed technique involves the computation of the steering vector derivatives, matrix multiplication and solving a GEV problem for each direction. With steering vector derivatives independent of data, the run-time computational load can be reduced significantly with a look-up table of values evaluated apriori. Therefore, solving GEV in each direction is the only run-time computationally intensive task. An iterative algorithm to realize generalized eigen decomposition with a computational complexity of $O(L^2)$ has been proposed in [46]. Including the computation cost for matrix inversion and maximum eigenvalue of $\mathbf{R}^{-1}$ required for evaluating β (which together consumes $O(L^3) + O(L^2)$), the total computational cost of the proposed technique is $O(L^3) + (N_{\text{tot}} + 1)O(L^2)$, where N_{tot} is the number of search directions.

Figure 4.2: Illustrative spatial spectrum of the proposed technique.

Besides ignoring the computational cost involved in the estimation of the signal subspace dimension, the BFW-MUSIC algorithm requires the inversion of a $D \times D$ matrix ($D = \dim(\mathbf{E}_s) + P$) in each direction resulting in a total cost of $N_{\text{tot}}O(D^3)$. Although $D < L$, the third-degree complexity indicates an increase in complexity with increasing signal subspace dimension. DOA estimation using the TOPS algorithm requires an eigen decomposition of the $L \times L$ autocorrelation matrix at each frequency bin, followed by an eigen decomposition of a smaller $P \times P$ matrix in each direction. This results in a total computational complexity of $N_{\text{bins}}O(L^2) + N_{\text{tot}}O(P^2)$, where N_{bins} is the number of frequency bins used. This is again a conservative measure without considering the computations required for estimating the model order. While a single SVD is required to be solved by the CSSM technique for DOA estimation, the process of finding the focussing matrices requires an $L \times L$ matrix SVD in each frequency bin of consideration. Although this technique is computationally less intensive, it additionally requires an ini-

tial estimate of the source directions obtained from another technique. Considering the complexity of the model-order estimation techniques and the prerequisites required for the CSSM algorithm, the proposed technique has comparable computational cost with existing subspace-based techniques.

4.3.4 The $\{f, \phi\}$ Ambiguity

The steering vector in the context of narrowband DOA estimation is a function of only the source direction since all the sources are assumed to be operating at the same frequency. For wideband source scenario, however, the steering vector is a function of frequency and direction. A combined processing of data across frequencies gives rise to an ambiguity over the $\{f, \phi\}$ domain for some array configurations as reported in [30]. In order to understand this ambiguity, consider a ULA where, the steering vector is given by

$$\begin{aligned} \mathbf{a}(f, \phi) &= [1 \ e^{j2\pi f \tau_\phi} \ \dots \ e^{j2\pi f (L-1) \tau_\phi}]^T \\ &= [1 \ e^{j2\pi f \frac{d}{c} \cos \phi} \ \dots \ e^{j(L-1)2\pi f \frac{d}{c} \cos \phi}]^T. \end{aligned} \tag{4.24}$$

From the argument of each element in the steering vector, we note that a few combinations of frequency-direction pairs satisfy

$$(f + \delta f) \tau_\phi = f(\tau_{\phi + \delta \phi}) \tag{4.25}$$

over the azimuth range. Along these directions, the steering vectors $\mathbf{a}(f + \delta f, \phi)$ and $\mathbf{a}(f, \phi + \delta \phi)$ are indistinguishable, resulting in an ambiguity. We therefore observe false peaks in some non-source directions for the proposed technique which exploits monotonicity of steering vector across frequency. Although the use of 2-D array configurations such as double-ring array can resolve this ambiguity, the technique cannot provide reliable estimates with primitive 1-D arrays. In order to resolve this problem we construct

spatio-temporal observation vectors using D delays at the sensor outputs as suggested in [30]. The number of taps D which introduces a temporal delay of T_D is chosen such that the product $[\omega \times T_D] \bmod 2\pi$ is unambiguous over all signal frequencies ω within the observation bandwidth. The number of delays D can however be relaxed with the choice of appropriate array configuration which ensures unambiguous $[\omega.T_D] \bmod 2\pi$ product [30].

For an L -sensor array, the time-domain snapshots $\mathbf{x}(n)$ are used to construct spatio-temporal snapshots with D -tap delays as

$$\bar{\mathbf{x}}(n) = [\mathbf{x}^T(n) \ \mathbf{x}^T(n-1) \ \dots \ \mathbf{x}^T(n-D)]^T. \quad (4.26)$$

While existing subspace-based techniques can be employed for DOA estimation with these snapshots, the broadband signal subspace dimension has to be determined based on the observed source time-bandwidth product [30]. This product, and hence the signal subspace dimension, varies with the scenario rather than the actual number of sources present in the scenario. Alternatively, with the array manifold approximation discussed in Section 4.2, we employ the MUSIC-like formulation to resolve the source directions without estimating the model order. The array covariance matrix corresponding to the spatio-temporal snapshots $\bar{\mathbf{x}}(n)$ is given by

$$\begin{aligned} \bar{\mathbf{R}} &= \mathbb{E}\{\bar{\mathbf{x}}(n)\bar{\mathbf{x}}^H(n)\} \\ &\approx \frac{1}{F^2} \left[\bar{\mathbf{A}}(f_0, \Theta) \left\{ \sum_{k=0}^{F-1} \mathbf{R}_{ss}(f_k) \right\} \bar{\mathbf{A}}^H(f_0, \Theta) + 2 \sum_{k=0}^{F-1} \text{Re} \left\{ \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} \bar{\mathbf{A}}(f_0, \Theta) \mathbf{R}_{ss}(f_k) \bar{\mathbf{D}}_p^H(\Theta) \right\} \right. \\ &\quad \left. + \sum_{k=0}^{F-1} \sum_{p,q=1}^P \frac{(\delta f_k)^{p+q}}{p!q!} \bar{\mathbf{D}}_p(\Theta) \mathbf{R}_{ss}(f_k) \bar{\mathbf{D}}_q^H(\Theta) \right] + \sigma_v^2 \mathbf{I}_L, \end{aligned} \quad (4.27)$$

Figure 4.3: Performance of the proposed technique under the $\{f, \phi\}$ ambiguity at an SNR of 5 dB (a) without time delay taps, (b) with $D = 3$ time delay taps.

where the matrix $\bar{\mathbf{A}}(f_0, \Theta)$ consists of DL -dimensional steering vectors constructed as

$$\bar{\mathbf{a}}(f, \theta) = [1, e^{j2\pi f T_s}, \dots, e^{j2\pi f (D-1)T_s}]^T \otimes \mathbf{a}(f, \theta), \quad (4.28)$$

with T_s denoting the time-delay between the temporal samples and $\otimes$ denotes the Kronecker product. The columns of $\bar{\mathbf{D}}(f_0, \Theta)$ are obtained by the P derivatives of $\bar{\mathbf{a}}(f, \theta)$.

Employing the proposed technique on the spatio-temporal snapshots with the steering vector given in (4.28), the spatial spectrum is expected to provide distinct peaks along the source directions for any array configuration. In order to demonstrate this, we consider a ULA with five sensors and a source scenario with two wideband sources from 50° and 70° having a normalized bandwidth of 0.1 and centered at $f_c = 0.3$. The spatio-temporal snapshots $\bar{\mathbf{x}}(n)$ are synthesized at an SNR of 5 dB with a tap delay line of $D = 3$. In Figure 4.3(a), the spatial spectrum of the proposed technique without any time-delay taps is plotted. We note that the technique fails to estimate accurate source

directions employing only the spatial snapshots. Furthermore, an ambiguous false peak is observed in a non-source direction which satisfies the condition in (4.25). Figure 4.3(b) shows the spatial spectrum when spatio-temporal structure is employed for DOA estimation. It is observed that the technique resolves the ambiguity in identifying the true source directions. Besides resolving the ambiguity, the incorporation of tap-delay line increases the degrees-of-freedom, thus improving the DOA estimation accuracy at the cost of increased computation complexity.

4.4 Derivation of the Cramer-Rao Lower Bound (CRLB)

The Cramer-Rao bound provides a lower bound on the estimation variance of a parameter θ . If we have a vector of parameters Θ , the covariance matrix of estimation errors for any unbiased estimate is bound by

$$\mathbf{C}(\Theta) \geq \mathbf{C}_{\text{CR}}(\Theta) \triangleq \mathbf{J}^{-1}, \quad (4.29)$$

where $\mathbf{C}_{\text{CR}}(\Theta)$ denotes the CRLB for the covariance matrix which is equal to the inverse of the Fisher's information matrix (FIM) $\mathbf{J}$.

For an L -element array, if each snapshot $\mathbf{x}$ follows a zero-mean complex Gaussian random vector parameterized by Θ , the probability density function is given by

$$p_{\mathbf{x}|\Theta}(\mathbf{x}) = \frac{1}{\det[\pi \mathbf{C}_{\mathbf{x}}(\Theta)]} \exp \{ -\mathbf{x}^H \mathbf{C}_{\mathbf{x}}^{-1}(\Theta) \mathbf{x} \}, \quad (4.30)$$

where $\mathbf{C}_{\mathbf{x}}(\Theta)$ is the covariance matrix of $\mathbf{x}$. If we assume that the successive N snapshots

are statistically independent, the log-likelihood function is given by

$$\mathbf{L}_{\mathbf{x}}(\Theta) = \sum_{i=1}^N \ln p_{\mathbf{x}_i|\Theta}(\mathbf{x}_i). \quad (4.31)$$

The elements of the FIM are then evaluated as

$$\mathbf{J}_{ij} \triangleq \mathbb{E} \left[\frac{\partial \mathbf{L}_{\mathbf{x}}(\Theta)}{\partial \theta_i} \cdot \frac{\partial \mathbf{L}_{\mathbf{x}}(\Theta)}{\partial \theta_j} \right] = \mathbb{E} \left[\frac{\partial^2 \mathbf{L}_{\mathbf{x}}(\Theta)}{\partial \theta_i \partial \theta_j} \right], \quad (4.32)$$

Exercising ergodicity, the covariance matrix of sensor snapshots $\mathbf{R}$ can be estimated using the temporal snapshots. After simplifying (4.32), the FIM for Θ is given by

$$\mathbf{J}_{ij} = N \text{Tr} \left(\frac{\partial \mathbf{R}}{\partial \theta_i} \mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial \theta_j} \mathbf{R}^{-1} \right), \quad \forall, i, j = 1, \dots, M. \quad (4.33)$$

For detailed derivation of (4.33), please refer to [2, Chap. 8].

For the time-domain wideband signal model with array manifold approximation given in (4.4), the covariance matrix $\mathbf{R}$ is given by (4.10). The partial derivative of $\mathbf{R}$ w.r.t. each parameter is given by

$$\begin{aligned} \frac{\partial \mathbf{R}}{\partial \theta_i} = & \frac{1}{F^2} \sum_{k=0}^{F-1} \left[\mathbf{A}(f_0, \Theta) \mathbf{R}_{ss}(f_k) \dot{\mathbf{A}}^H(f_0, \Theta) + \dot{\mathbf{A}}(f_0, \Theta) \mathbf{R}_{ss}(f_k) \mathbf{A}^H(f_0, \Theta) \right. \\ & + 2\text{Re} \left\{ \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} (\mathbf{A}(f_0, \Theta) \mathbf{R}_{ss}(f_k) \dot{\mathbf{D}}_p^H(f_0, \Theta) + \dot{\mathbf{A}}(f_0, \Theta) \mathbf{R}_{ss}(f_k) \mathbf{D}_p^H(f_0, \Theta)) \right\} \\ & \left. + \sum_{p,q=1}^P \frac{(\delta f_k)^{p+q}}{p!q!} \left(\mathbf{D}_p(f_0, \Theta) \mathbf{R}_{ss}(f_k) \dot{\mathbf{D}}_q^H(f_0, \Theta) + \dot{\mathbf{D}}_p(f_0, \Theta) \mathbf{R}_{ss}(f_k) \mathbf{D}_q^H(f_0, \Theta) \right) \right], \end{aligned} \quad (4.34)$$

where $\dot{\mathbf{A}}(f_0, \Theta) = \frac{\partial}{\partial \theta_i} \mathbf{A}(f_0, \Theta)$ and $\dot{\mathbf{D}}_p(f_0, \Theta) = \frac{\partial^{p+1}}{\partial \theta_i \partial f^p} \mathbf{A}(f_0, \Theta)$. The noise variance at each

frequency bin is estimated as $\hat{\sigma}_v^2(f_k)$ as

$$\hat{\sigma}_v^2(f_k) = \frac{1}{L-M} \text{trace} \left\{ \mathbf{I}_L - \mathbf{A}(f_k, \Theta) (\mathbf{A}^H(f_k, \Theta) \mathbf{A}(f_k, \Theta))^{-1} \mathbf{A}^H(f_k, \Theta) \right\}. \quad (4.35)$$

The source covariance matrix can then be estimated as

$$\hat{\mathbf{R}}_{ss}(f_k) = (\mathbf{A}^H(f_k, \Theta) \mathbf{A}(f_k, \Theta))^{-1} \mathbf{A}^H(f_k, \Theta) (\hat{\mathbf{R}}(f_k) - \hat{\sigma}_v^2(f_k) \mathbf{I}_L) \mathbf{A}(f_k, \Theta) (\mathbf{A}^H(f_k, \Theta) \mathbf{A}(f_k, \Theta))^{-1}, \quad (4.36)$$

where $\hat{\mathbf{R}}(f_k)$ is the array covariance matrix estimated from the sensor snapshots at frequency f_k . With the source covariance matrix estimated, the FIM, and hence the CRB can be estimated by substituting (4.34) into (4.33).

4.5 Simulation Results and Discussions

In this section, the performance of the proposed technique is evaluated using Monte-Carlo simulations. For all the simulations, we employ a double-ring array of ten elements ($L = 10$) shown in Figure 4.4 with the array phase center coinciding with the origin of the coordinate system. Since this array geometry does not exhibit the $\{f, \phi\}$ ambiguity unlike the ULA, we use $D = 0$ tap delays. The source azimuths are measured from the horizontal x-axis. Despite the radius of the two rings being greater than $\lambda_{\min}/2$, the farfield planar wavefront from any source direction is sampled by a set of sensors complying with the $\lambda_{\min}/2$ phase delay, thus avoiding any source direction ambiguity due to spatial aliasing. The wideband sources are generated as a sum of sinusoids with random phase and magnitude within the bandwidth,

$$s(t) = \sum_{i=1}^{B_{\text{bw}}} a_i \exp\{j(2\pi f_i t + c_i(t))\}, \quad (4.37)$$

Figure 4.4: Ten-element double ring circular array.

where the amplitude a_i is an instance of Rayleigh distribution, $c_i(t)$ is uniformly distributed in $[-\pi, \pi]$ and B_{bw} is the number of sinusoids that constitute the wideband source. The sources are generated with a normalized bandwidth of 0.1 centered at a normalized frequency of 0.3 accounting for a percentage bandwidth of 33% unless specified explicitly for specific experiments. The frequency-domain signal model given by (6.12) is used to first obtain the frequency-domain sensor snapshots with $F = 256$ -point DFT. The time-domain sensor signals are then obtained by the inverse discrete Fourier transform of these snapshots. For baseline algorithms such as CSSM and TOPS which operate on the frequency-domain snapshots, the source signal expressed in (4.37) will then consists of 26 sinusoidal components. Additive complex Gaussian noise is used to generate sensor data snapshots at required SNR in accordance with the signal model defined in (2.32).

For the proposed technique, as well as the BFW-MUSIC algorithm, the derivative constraints play a significant role in handling the wideband sources. While the use

Figure 4.5: Spatial spectra with different derivative orders P at an SNR of 5 dB for (a) BFW-MUSIC, (b) proposed technique.

of higher-order derivative constraints is expected to completely suppress the derivative terms in (4.10), the optimization problem of BFW-MUSIC and the proposed technique restricts the use of large number of derivative constraints due to the limited degrees of freedom available. This is observed with a simulation consisting of three wideband sources at 50° , 70° and 80° . With 500 time-domain snapshots at an SNR of 5 dB, the spatial spectra for both the techniques are plotted for different derivative orders P in Figure 4.5. In Figure 4.5(a), we note that the BFW-MUSIC algorithm provides reliable DOA estimates by incorporating derivative constraints. However, an increase in

Figure 4.6: RMSE of the DOA estimates plotted for various SNR values against (a) η , (b) ζ .

derivative order is observed to adversely affect the spectrum with false peaks, especially when $P = 4$. This is because of the strict linear constraints of BFW-MUSIC which severely deter the available degrees of freedom giving rise to false peaks for larger P . On the contrary, as shown in Figure 4.5(b), the soft derivative constraints introduced into the objective function of the proposed technique does not exhibit false peaks. Nevertheless, increasing P beyond 2 has less effect on the performance of the proposed algorithm for sources with percentage bandwidth (fractional bandwidth) up to 50%.

We now study the influence of the two parameters η and ζ on the performance of the proposed technique. Extending the procedure adopted in the context of narrowband DOA estimation, it is straight forward to derive the bounds for β provided in (4.21). However, considering the computational advantage, β is chosen using (4.23). The performance of the technique within this bound is observed for the earlier simulation setup at various SNR values by plotting the RMSE of DOA estimates averaged over the three sources against η in Figure 4.6(a). As η is increased from 1 to a value of 2, we note that the RMSE of the estimates decreases under all SNR conditions. When the SNR is greater than 0 dB,

further increase in η beyond 2 is observed to have less significance on the estimation accuracy. However, as η guides β towards its upper bound, the spatial spectrum will be affected with false peaks due to the relaxation of the constraint explained while deriving the bounds for β in Section 3.2.3. When $\text{SNR} \leq 0$, the RMSE gradually increases with increasing η . We therefore employ $\eta = 2$ for all the simulations.

With this choice for η , we observe the effect of ζ on the DOA estimates for the same simulation setup. Figure 4.6(b) shows the RMSE of DOA estimates plotted against ζ for various SNR values. We note that the RMSE for DOA estimates is large for $\zeta < 0.5$. This is due to the insufficient suppression of derivative components in $\mathbf{R}$. For $\zeta \geq 0.5$, which is equal to the fractional bandwidth BW/f_c the estimation accuracy is not significantly affected.

Frequency-domain techniques such as CSSM and TOPS require the estimation of covariance matrix at each frequency bin within the signal bandwidth. For reliable DOA estimation, sufficient frequency-domain snapshots are essential for the covariance matrix estimation in each frequency bin. If J denotes the number of frequency-domain snapshots used for DOA estimation, the corresponding temporal snapshots $N = JF$ required is significantly large. Since the proposed technique operates on time-domain snapshots, the covariance matrix $\mathbf{R}$ encompasses information from the entire signal bandwidth. This simultaneous processing across the signal bandwidth reduces the number of snapshots required for DOA estimation. We first evaluate the proposed technique in comparison with the BFW-MUSIC algorithm employing reduced number of temporal snapshots, followed by which, the technique is compared with other baseline algorithms using larger snapshots.

Three wideband sources with a percentage bandwidth of 33% are situated at 40° , 50° and 70° . Employing 1500 time-domain snapshots for DOA estimation, the RMSE of direction estimates are plotted against SNR in Figure 4.7. The performance of the

proposed technique with $P = 1$ and $P = 2$ are shown with the BFW-MUSIC algorithm being used as a baseline algorithm. Since the signal subspace dimension of $\mathbf{R}$ includes additional terms in (4.10), the BFW-MUSIC algorithm is implemented with $\dim\{\mathbf{E}_s\} = M + 2$. Linear derivative constraints of up to $P = 2$ are used to obtain the spatial spectrum of BFW-MUSIC. The RMSE of DOA estimates is plotted against SNR for the two techniques in Figure 4.7 along with the square root of the Cramer-Rao lower bound (sqrtCRB).

With prior knowledge of the approximate signal subspace dimension and the strict linear constraints, BFW-MUSIC is observed to perform better when the SNR is low. It is important to note that an inappropriate signal subspace dimension adversely affects the performance of this algorithm which is not shown in the figure. On the contrary, the estimation accuracy of the proposed technique without using prior information of the signal subspace dimension, is observed to have marginally lower performance at lower SNR conditions. However, as the SNR is increased we note that the proposed technique outperforms the BFW-MUSIC algorithm. This is because, at higher SNR values, the noise subspace is clearly segregated from the composite subspace in the covariance matrix, which enables one to direct the weight vector to reside in the noise subspace. In such scenario, the data dependent suppression of derivative components will be more effective than the linear constraints of the BFW-MUSIC algorithm. The improvement in the performance for $P = 2$ over $P = 1$ reflects the importance of including $\mathbf{R}_y(\phi)$ and its order. Knowledge of the source covariance matrix used in evaluating the CRLB explains the difference between the performance of the two techniques and the CRLB.

We now investigate the estimation performance as a function of the number of snapshots that is used to construct $\mathbf{R}$ and $\mathbf{R}_y(\theta)$. Fixing the SNR at 5 dB for the same source-sensor configuration as the previous simulation, source directions are estimated using the proposed technique and the BFW-MUSIC algorithm. In Figure 4.8, we note the

estimation accuracy of both the techniques increases with the number of snapshots. The RMSE of DOA estimates obtained by the proposed technique approaches that of BFW-MUSIC and outperforms the latter when the number of snapshots is increased further. While the provided signal subspace dimension and the strict linear derivative constraints aid to have relatively better performance for BFW-MUSIC with limited number of snapshots, the improvement in the estimation of $\mathbf{R}$ with larger number of snapshots has less impact on its solution.

The capability of the proposed technique to resolve two closely situated sources is now studied for two wideband sources with a percentage bandwidth of 50%. The probability of resolution is studied with increase in SNR for 5° , 7° and 9° of angular separation between the two sources in Figure 4.9. The number of snapshots used is 1000. When the sources are closely-situated, $\mathbf{A}(f_0, \Theta)$ and $\mathbf{D}_p(\Theta)$ are highly ill-conditioned. In the presence of large additive noise σ_v^2 and the approximation errors, the two sources are indistinguishable by the proposed technique. Therefore, we note that larger SNR is required to resolve the two sources when they are separated by 5° . When the angular separation between the sources is increased to 7° and 9° , the technique can resolve the source directions at much smaller SNR.

In order to compare the performance of the proposed technique with frequency domain techniques such as CSSM and TOPS, we study the RMSE of the source DOA estimates. The number of frequency bins used for DOA estimation with TOPS is equal to the source bandwidth. The frequency-domain snapshots are obtained with a 256-point FFT performed on the sensor snapshots. While $J = 25$ frequency-domain snapshots are used for the TOPS algorithm, equivalent number of time-domain snapshots 256×25 is used for the proposed technique. Three sources with 50% bandwidth are positioned at 12° , 40° and 48° . Figure 4.10 shows the RMSE of the source estimates for all the techniques averaged over 200 trials along with the CRLB (sqrtCRB). Among all the

compared techniques, CSSM is observed to have the best performance at low SNR conditions. However, the solution is highly susceptible to the initial estimates provided to the technique. Moreover, the performance saturates with the increase in SNR as the array manifold fitting error cannot be eliminated with an increase in SNR. The TOPS algorithm is observed to have the worst performance at lower SNR conditions and the plots shown in Figure 4.10 are consistent with the observation made in [41]. The performance of TOPS is comparable with other techniques only at mid and high SNR ranges. At lower SNR conditions, the BFW-MUSIC algorithm shows better performance over the proposed technique with the support of the approximate signal subspace dimension and the linear derivative constraints. Nevertheless, its performance does not improve significantly with the SNR. In contrast, the estimation performance of the proposed technique improves with SNR compared to other techniques. In view of the insensitivity to the incorrect model order estimated at lower SNR conditions, the performance of the proposed technique is acceptable.

Figure 4.11 shows the ability of all the techniques under comparison to resolve the three source directions in the earlier source scenario. Each source is considered to be resolved when the algorithm provides a significant peak within the spatial region $[\hat{\theta}_i - \Delta\theta, \hat{\theta}_i + \Delta\theta]$, where $\Delta\theta = \min\{\theta_i - \theta_j\}/2, \forall i, j, i \neq j \in M$. The CSSM and BFW-MUSIC algorithms are observed to have good resolution capability with the advantage of partial information of the model order. Since the distinction between the noise and signal subspace eigenvalues is not significant at low SNR conditions, the proposed technique is observed to have marginally lower resolution capability than the BFW-MUSIC algorithm. As expected, the TOPS algorithm estimate source directions only when the SNR is significantly high.

4.6 Conclusion

In this chapter, a new approach to coherently process the wideband data across the signal bandwidth using the Taylor series expansion of the array manifold w.r.t. frequency has been proposed. The covariance matrix obtained from the time-domain signal model has been identified to have a coherently averaged narrowband component which preserves the DOA information along with the additional derivative components. Exploiting this observation, a new DOA estimation technique has been proposed which suppresses the unnecessary derivative terms before estimating the source directions using the MUSIC-like structure. The effective suppression of these terms by the proposed technique has been analyzed in detail. Being insensitive to the estimated model order, the performance of this technique has been shown to be comparable with existing techniques through simulations.

Figure 4.7: RMSE of DOA estimates against SNR with 1500 time-domain snapshots.

Figure 4.8: RMSE of DOA estimates as a function of the number of snapshots in comparison with the BFW-MUSIC algorithm. The snapshots are obtained at 5 dB SNR.

Figure 4.9: Probability of resolving two sources with angular separations $\Delta\theta = 5^\circ$, $\Delta\theta = 7^\circ$ and $\Delta\theta = 9^\circ$.

Figure 4.10: RMSE of DOA estimates against SNR in comparison with the CSSM, TOPS and BFW-MUSIC algorithms when the sources are situated at 12° , 40° and 48° .

Figure 4.11: The probability of resolving all the three sources by CSSM, TOPS, BFW-MUSIC and the proposed technique.

Chapter 5

Derivative-Constrained Frequency-Domain Wideband DOA Estimation

5.1 Introduction

In Chapter 4, a time-domain DOA estimation technique was presented which estimates wideband source directions with relatively reduced number of time-domain snapshots. The use of MUSIC-like formulation provides distinct peaks in the spatial spectrum of the technique without estimating the number of sources. For a wideband source scenario with center frequency f_c , if the maximum frequency deviation is defined as $\delta f_{\max} = f_u - f_c$ such that the bandwidth is equal to $f_u - f_l = 2\delta f_{\max}$, the underlying idea is to express the steering vector at a frequency f within the signal bandwidth as a Taylor series expansion with respect to the reference frequency f_0 as

$$\hat{\mathbf{a}}(f, \theta, P) \approx \mathbf{a}(f_0, \theta) + \sum_{p=1}^P \frac{(\delta f)^p}{p!} \frac{\partial^p}{\partial f^p} \mathbf{a}(f, \theta) \Big|_{f=f_0}. \quad (5.1)$$

However, the consequence of truncating the Taylor series expansion (TSE) up to an order of P introduces approximation error as the frequency deviation δf increases. In order to observe the effect of P th-order approximation and the frequency deviation, we evaluate the approximation error $e = \|\mathbf{a}(f, \theta) - \hat{\mathbf{a}}(f, \theta, P)\|_2$ in an arbitrary direction for the double-ring array shown in Figure 4.4. A frequency deviation of $\delta f = 0.04$ from a normalized frequency of $f_0 = 0.3$ corresponding to a percentage bandwidth of $2\delta f/f_0 \times 100 \approx 27\%$ is used for the illustration. The approximation error is plotted against TSE order in Figure 5.1(a). We note that the error is considerably low for $P \geq 2$ indicating that a P -ordered TSE is a good approximation. In the associated simulation, we study the error in approximation as the steering vector deviates from f_0 . The error e is plotted against δf for various orders of the Taylor series expansion in Figure 5.1(b). For $P \geq 2$, we note that the approximation error is acceptable up to $\delta f = 0.04$, or equivalently 27% bandwidth, beyond which the error increases significantly. It can hence be inferred that the error in approximation increases drastically when the fractional source bandwidth is larger than 0.27 or 0.33. In such cases, the performance of the time-domain DOA estimation (TD-DOA) technique proposed in Chapter 4 deteriorates considerably. In order to observe this phenomenon, we study the estimation accuracy of this technique with the source bandwidth with a simulation consisting of three wideband sources at 12° , 50° and 60° using the double-ring array with $L = 10$ sensors. The RMSE of the three DOA estimates are averaged and plotted against the fractional bandwidth of the sources. With 1500 snapshots and an SNR of 5 dB, the RMSE of the sources are plotted against the fractional bandwidth Δf in Figure 5.2.

Although the estimation errors for different sources vary due to their spatial positions, we observe that the RMSE increases with an increase in the signal bandwidth. This reflects the limitation of time-domain processing for a source scenario with large bandwidth due to inaccurate approximation. In order to reduce this approximation error,

Figure 5.1: Error in steering vector approximation using TSE against (a) derivative order for $\delta f = 0.04$, (b) δf for various values of P .

it is essential to regulate the signal bandwidth under consideration while constructing the covariance matrix. Towards this, we translate the problem to the frequency domain and redefine the signal model with array manifold approximation.

In the literature we have several frequency-domain techniques which have shown good DOA estimation performance. While ISSM, and the more recent TOPS algorithm yield accurate DOA estimates in uncorrelated source scenario, the CSSM, reviewed in Section 2.3.2.2, can estimate DOA in the presence of completely correlated sources as well. The coherent averaging of the array covariance matrix across the signal bandwidth restores the effective rank of the source covariance matrix enabling the DOA estimation of correlated sources. However, the performance of all these techniques are sensitive to the estimated number of sources.

Figure 5.2: RMSE of DOA estimates plotted against fractional bandwidth.

Since the coherently averaged covariance matrix $\mathbf{R}_{\text{cssm}}$ exhibits narrowband structure with transformed noise covariance matrix $\mathbf{R}_v$, statistical techniques such as MDL and AIC have been suggested in [33] for model order estimation even in wideband source scenario. The generalized eigenvalues of the matrix pencil $\{\mathbf{R}_{\text{cssm}}, \mathbf{R}_v\}$ are subject to either MDL or AIC for model order estimation. Although these techniques provide reliable estimates under moderate SNR conditions, the performance is unreliable when the number of frequency-domain snapshots is low either due to the inaccurate estimation of the correlated noise covariance matrix or due to adverse source scenarios.

We verify this inaccuracy with a simulation example where we study the model order estimated by MDL and AIC employing the generalized eigenvalues of $\{\mathbf{R}_{\text{cssm}}, \mathbf{R}_v\}$ for two source scenarios. In the first scenario, two wideband sources are situated at 40° and 48° . For the second scenario, the sources are situated at 40° and 58° . The two sources are uncorrelated with each other and have a percentage bandwidth of 27%. The matrix $\mathbf{R}_{\text{cssm}}$ is estimated using (2.44) with 10 frequency-domain snapshots. In order to observe

Figure 5.3: Performance of MDL and AIC techniques against SNR when $J = 10$ frequency-domain snapshots are used.

the performance of existing model order estimation techniques, we study the ratio of the true number of sources M to the estimated model order $\widehat{M}$. The ratio approaches unity when the algorithm estimates correct model order. The ratio $M/\widehat{M}$ is averaged over 100 trials and plotted against SNR for the two source scenarios in Figure 5.3. For the first scenario where the sources are closely situated, we note that the ratio for both the algorithms decreases below 1 with the increase in SNR. This indicates that the two algorithms over estimate the number of sources present in the scenario with the increase in the SNR. When the two sources are widely-spaced, both the algorithms estimate M accurately over specific SNR values and over estimate the number of sources when the SNR is increased further due to the inaccurate estimation of the transformed noise covariance matrix. The spatial spectrum of CSSM would therefore exhibit more number of peaks than the true model order. Under adverse conditions, it is therefore a challenging problem to accurately estimate the model order estimation in the context of wideband

sources.

While the time-domain wideband DOA estimation technique proposed in Chapter 4 provides accurate DOA estimation with unknown model order, it was observed that the estimation accuracy of the algorithm deteriorates with an increase in the signal bandwidth. In this chapter, we therefore translate the problem to the frequency domain where the source bandwidth being processed can be regulated such that the array manifold Taylor series approximation error is acceptable. We first redefine the frequency-domain signal model with array manifold approximation in Section 5.2. Since the BFW-MUSIC algorithm also relies on this approximation, we first present its extension to the frequency domain in Section 5.3.1. However, this algorithm still depends on the estimated dimension of the signal subspace of the array covariance matrix. We then present the extension of the proposed formulation for wideband DOA estimation in Chapter 4 over a regulated signal bandwidth in Section 5.3.2. Although these algorithms provide accurate DOA estimates, they do not use the DOA information present in the unused portion of source bandwidth. We therefore present a multi-band DOA estimation approach by incoherently combining the DOA spectra obtained over various subbands. The three techniques discussed in Section 5.3 are evaluated in Section 5.4 using simulated experiments. The conclusion and the performance advantage of the presented techniques are summarized in Section 5.5.

5.2 Signal Model with Array Manifold Approximation

In Section 4.2, sensor snapshots in the time domain were modeled by introducing array manifold approximation into the signal model. The inverse-DFT relation includes fre-

quency components corresponding to the entire source bandwidth. However, as observed in the previous section, inclusion of components which are farther from the reference frequency increases the Taylor series approximation error, yielding inaccurate results. In order to regulate the signal bandwidth under consideration, we estimate wideband source directions with an averaged covariance matrix obtained across a smaller frequency band.

In the frequency domain, the sensor snapshots at a given frequency are modeled as shown in (2.32). Expressing the array manifold matrix $\mathbf{A}(f, \Theta)$ in terms of the reference frequency f_0 with Taylor series approximation, the signal model at frequency f_k is given by

$$\underline{\mathbf{x}}(k, l) \approx \left(\mathbf{A}(f_0, \Theta) + \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} \frac{\partial^p \mathbf{A}(f, \Theta)}{\partial f^p} \bigg|_{f=f_0} \right) \underline{\mathbf{s}}(k, l) + \underline{\mathbf{v}}(k, l). \quad (5.2)$$

With this approximation, the spatial covariance matrix of the zero-mean snapshots $\underline{\mathbf{x}}(k, l)$ can be expressed as

$$\begin{aligned} \mathbf{R}(f_k) &= \mathbb{E}\{\underline{\mathbf{x}}(k, l)\underline{\mathbf{x}}^H(k, l)\} \\ &\approx \mathbf{A}(f_0, \Theta)\mathbf{R}_{ss}(f_k)\mathbf{A}^H(f_0, \Theta) + \sigma_v^2(f_k)\mathbf{I}_L \\ &\quad + \sum_{p=1}^L \frac{(\delta f_k)^p}{p!} \operatorname{Re} \left[\mathbf{A}(f_0, \Theta)\mathbf{R}_{ss}(f_k) \frac{\partial^p}{\partial f^p} \mathbf{A}^H(f, \Theta) \bigg|_{f=f_0} \right] \\ &\quad + \sum_{p,q=1}^P \frac{(\delta f_k)^{p+q}}{p!q!} \frac{\partial^p}{\partial f^p} \mathbf{A}(f, \Theta) \bigg|_{f=f_0} \mathbf{R}_{ss}(f_k) \frac{\partial^q}{\partial f^q} \mathbf{A}^H(f, \Theta) \bigg|_{f=f_0}, \end{aligned} \quad (5.3)$$

where $\sigma_v^2(f_k)$ is the variance of additive uncorrelated sensor noise at frequency f_k . We first define a fractional bandwidth Δf_B corresponding to $B + 1$ frequency bins, with B even, in the proximity of f_0 , over which a P th-order Taylor series approximates the array manifold up to an acceptable error. An average covariance matrix across this bandwidth can then be defined as

$$\mathbf{R}_{\text{av}} = \frac{1}{B+1} \sum_{f_k=f_0-B/2}^{f_0+B/2} \mathbf{R}(f_k). \quad (5.4)$$

Defining the data matrix at each frequency f_k as

$$\underline{\mathbf{X}}(k) = [\underline{\mathbf{x}}(k, 1), \dots, \underline{\mathbf{x}}(k, J)], \quad (5.5)$$

where J is the number of frequency domain snapshots, the data matrices across the $B+1$ frequency bins are then concatenated to form

$$\underline{\mathbf{X}} = [\underline{\mathbf{X}}(-B/2), \dots, \underline{\mathbf{X}}(B/2)] \in \mathbb{C}^{L \times J(B+1)}. \quad (5.6)$$

The covariance matrix $\mathbf{R}_{\text{av}}$ can then be estimated as

$$\mathbf{R}_{\text{av}} = \frac{1}{J(B+1)} \underline{\mathbf{X}} \underline{\mathbf{X}}^H. \quad (5.7)$$

If the source spectrum is uncorrelated across frequency, i.e., $\mathbb{E}\{\mathbf{s}(k_1, l) \mathbf{s}^H(k_2, l)\} = \mathbf{0}_M, \forall f_1 \neq f_2$ and the additive sensor noise is uncorrelated with sources, we have

$$\begin{aligned} \mathbf{R}_{\text{av}} \approx & \frac{1}{B+1} \left[\mathbf{A}(f_0, \Theta) \left\{ \sum_{k=k_0-B/2}^{k_0+B/2} \mathbf{R}_{ss}(f_k) \right\} \mathbf{A}^H(f_0, \Theta) \right. \\ & + 2 \sum_{k=k_0-B/2}^{k_0+B/2} \text{Re} \left\{ \sum_{p=1}^P \frac{(\delta f_k)^p}{p!} \mathbf{A}(f_0, \Theta) \mathbf{R}_{ss}(f_k) \mathbf{D}_p^H(\Theta) \right\} \\ & \left. + \sum_{k=k_0-B/2}^{k_0+B/2} \sum_{p,q=1}^P \frac{(\delta f_k)^{p+q}}{p!q!} \mathbf{D}_p(\Theta) \mathbf{R}_{ss}(f_k) \mathbf{D}_q^H(\Theta) + \sum_{k=k_0-B/2}^{k_0+B/2} \sigma_v^2(f_k) \mathbf{I}_L \right], \end{aligned} \quad (5.8)$$

where $\mathbf{D}_p(\Theta)$ is the p th-order derivative matrix defined in (4.9). The coherent summation of $\mathbf{R}_{ss}(f_k)$ across the source bandwidth in the first term restores the rank of the effective source covariance matrix when the sources are completely correlated. It is therefore possible to resolve coherent source directions when this covariance matrix is used for DOA estimation. We note that the average covariance matrix $\mathbf{R}_{\text{av}}$ has a similar expression as

that of the time-domain covariance matrix $\mathbf{R}$ in (4.10) with a coherent narrowband component and components with derivative terms. However, the signal bandwidth beyond Δf_B will have less impact on this matrix, and hence on the DOA estimation. For an F -point DFT, the maximum frequency deviation from f_0 is $\delta f_{\max} = \frac{B+1}{2F}$. With $\delta f_{\max}$ known, the order of TSE necessary for $\mathbf{A}(f, \Theta)$ to constrain the approximation error can be chosen based on Figure 5.1(b).

5.3 Proposed DOA Estimation

As discussed in Section 1.2, a signal is classified as narrowband when the fractional bandwidth $\Delta f \ll 0.1$, in which case, the narrowband MUSIC-like algorithm or MUSIC-W algorithm can be employed to estimate source directions. However, for $\Delta f \geq 0.1$, the sources have to be considered as wideband in nature. When $0.1 < \Delta f < 0.3$, or equivalently $0.015 < \delta f < 0.045$, it was observed in Figure 5.1 that the approximation error is not significant when the derivative order is restricted to $P = 2$. The TD-DOA estimation technique proposed in Chapter 4 is well suited for such scenarios. For a source scenario with $\Delta f > 0.3$, the estimation error increases significantly due to the array manifold approximation of order $P = 2$. We therefore need to restrict the processing bandwidth to $\Delta f \leq 0.3$ for the specified derivative order. This can be accomplished with the frequency-averaged covariance matrix $\mathbf{R}_{\text{av}}$ given by (5.8). Since the BFW-MUSIC algorithm also relies on the concept of array manifold approximation, we first present its extension to the frequency domain. While the derivative constraints can successfully suppress the derivative components, the formulation requires the knowledge of the signal subspace dimension of $\mathbf{R}_{\text{av}}$. In order to circumvent the model order or the signal subspace dimension, we then present the extension of the MUSIC-like formulation for wideband DOA estimation in frequency domain.

The two variants discussed henceforth can be considered as alternate formulations of BFW-MUSIC and the proposed wideband TD-DOA technique suitable for source with fractional bandwidth greater than 0.3. Although these algorithms provide accurate DOA estimates employing snapshots from a regulated processing bandwidth, they do not exploit the DOA information present in the signal bandwidth unused for the construction of the averaged covariance matrix $\mathbf{R}_{\text{av}}$. We therefore present a multi-band DOA estimation approach by incoherently combining the spatial spectra obtained over various subbands.

5.3.1 Frequency-Averaged BFW-MUSIC Formulation (Formulation 1)

The time-domain covariance matrix employed in the BFW-MUSIC algorithm is known to have a composite signal subspace whose dimension is larger than the model order. With the array manifold approximation introduced into the signal model, it is evident from (5.8) that the extended signal subspace is due to the derivative terms. For a large signal bandwidth, the order of TSE should be large enough to achieve an accurate approximation. Alternatively, if we employ the averaged covariance matrix $\mathbf{R}_{\text{av}}$, the constrained processing bandwidth permits us to construct the signal subspace $\bar{\mathbf{U}}_s$ of $\mathbf{R}_{\text{av}}$ with $Q, M < Q < 2M$ dominant eigenvectors. Therefore, the DOA estimation problem

can be formulated as

$$\begin{aligned}
 & \underset{\mathbf{w}_\phi}{\text{minimize}} \quad \|\mathbf{w}_\phi - \mathbf{a}(f_0, \phi)\|_2^2 \\
 & \text{subject to} \quad \bar{\mathbf{U}}_s^H \mathbf{w}_\phi = 0, \\
 & \quad \quad \quad \left. \frac{\partial \mathbf{a}(f, \phi)}{\partial f} \right|_{f=f_0}^H \mathbf{w}_\phi = 0, \\
 & \quad \quad \quad \vdots \\
 & \quad \quad \quad \left. \frac{\partial^P \mathbf{a}(f, \phi)}{\partial f^P} \right|_{f=f_0}^H \mathbf{w}_\phi = 0.
 \end{aligned} \tag{5.9}$$

The linear derivative constraints in the above formulation suppress the additional terms of (5.8). Furthermore, the use of covariance matrix $\mathbf{R}_{\text{av}}$ is advantageous in restoring the rank of the source covariance matrix in the presence of coherent sources which is achieved by coherently averaging across the signal bandwidth. For improved spatial spectrum, the signal subspace matrix $\bar{\mathbf{U}}_s$ used to obtain the spectrum is replaced with a weighted matrix given by [42]

$$\bar{\mathbf{E}}_d = \left[\bar{\mathbf{U}}_s \left(\frac{\Lambda}{\chi_M} \right)^q, \left(\frac{\chi_1}{\chi_M} \right)^q \left. \frac{\partial \mathbf{a}(f, \phi)}{\partial f} \right|_{f=f_0}^H, \dots \right], \tag{5.10}$$

where $\Lambda = \text{diag}(\chi_1, \dots, \chi_M)$ are the eigenvalues of $\mathbf{R}_{\text{av}}$ and $q > 1$ is a constant. The corresponding Lagrangian function is given by

$$L(\mathbf{w}_\theta, \boldsymbol{\lambda}) = \|\mathbf{w}_\phi - \mathbf{a}(f_0, \phi)\|_2^2 + \boldsymbol{\lambda}^H (\bar{\mathbf{E}}_d^H \mathbf{w}_\phi) + \boldsymbol{\lambda}^T (\bar{\mathbf{E}}_d^T \mathbf{w}_\phi^*), \tag{5.11}$$

where $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers. The solution to this problem is given by

$$\mathbf{w}_\phi = \left\{ \mathbf{I}_L - \bar{\mathbf{E}}_d (\bar{\mathbf{E}}_d^H \bar{\mathbf{E}}_d)^{-1} \bar{\mathbf{E}}_d^H \right\} \mathbf{a}(f_0, \phi). \tag{5.12}$$

The distinct peaks in the direction finding function (2.57) correspond to the source directions.

5.3.2 Frequency-Averaged Wideband MUSIC-like Algorithm (Formulation 2)

When the model order is not available, it was noted in Chapter 3 that the framework of narrowband MUSIC-like algorithm provides better resolution over the MVDR algorithm. In order to extend this algorithm to resolve wideband source directions, we first project the concatenated data matrix $\underline{\mathbf{X}}$ defined in (5.6), which constitutes the observation vectors from the bandwidth being processed, onto the derivative vector space in each look direction

$$\underline{\mathbf{Y}} = \mathbf{P}_{\mathbf{D}_\phi} \underline{\mathbf{X}}. \quad (5.13)$$

The corresponding covariance matrix is given by

$$\mathbf{R}_y(\phi) = \mathbf{P}_{\mathbf{D}_\phi} \mathbf{R}_{\text{av}} \mathbf{P}_{\mathbf{D}_\phi}^H. \quad (5.14)$$

Introducing this term into the optimization problem, we have

$$\begin{aligned} \underset{\mathbf{w}_\phi}{\text{minimize}} \quad & \mathbf{w}_\phi^H \{ \mathbf{R}_{\text{av}} + \zeta \mathbf{R}_y(\phi) \} \mathbf{w}_\phi \\ \text{subject to} \quad & \mathbf{w}_\phi^H \{ \mathbf{a}(f_0, \phi) \mathbf{a}^H(f_0, \phi) + \beta \mathbf{I}_L \} \mathbf{w}_\phi = c, \end{aligned} \quad (5.15)$$

where β, ζ, c are constants. The optimal weight vector is given by the eigenvector corresponding to the minimum eigenvalue ($\chi_{\min}$) of the GEV,

$$(\mathbf{R}_{\text{av}} + \zeta \mathbf{R}_y(\phi)) \mathbf{w}_\phi = \chi_{\min} (\mathbf{a}(f_0, \phi) \mathbf{a}^H(f_0, \phi) + \beta \mathbf{I}_L) \mathbf{w}_\phi. \quad (5.16)$$

Although the above optimization problem is very similar to that of the earlier proposed time-domain DOA estimation technique, the use of frequency-domain snapshots corresponding to a reduced processing bandwidth is expected to improve the performance of

the technique over its time-domain counterpart. In particular, when the source bandwidth is large, a P -ordered Taylor series approximation introduces error into the solution as shown in Figure 5.2 if the processing bandwidth is not constrained.

The choice parameters ζ and β follows the same procedure provided for the time-domain DOA estimation technique. With the processing bandwidth limited to $B + 1$ frequency bins, the parameter ζ is set equal to the corresponding fractional bandwidth $\Delta f_B = \frac{B+1}{f_0}$. Likewise, the bounds for β provided in Section 4.3.2 can be extended for the frequency-domain approach with the time-domain covariance matrix $\mathbf{R}$ in (4.21) replaced with $\mathbf{R}_{\text{av}}$. Considering the computational advantage, we choose the value of β with reference to the lower bound setting $\zeta = 0$ as

$$\beta = \eta \min_{\phi} \frac{\mathbf{a}^H(f_0, \phi) \mathbf{R}_{\text{av}}^{-1} \mathbf{a}(f_0, \phi)}{\chi_{\mathbf{R}_{\text{av}}^{-1}, \max}}, \quad (5.17)$$

where $\chi_{\mathbf{R}_{\text{av}}^{-1}, \max}$ is the maximum eigenvalue of $\mathbf{R}_{\text{av}}^{-1}$ and $\eta > 1$ provides unambiguous spatial spectrum when chosen such that β is within the bounds.

5.3.3 Multiband DOA Estimation (Formulation 3)

In applications where the source bandwidths are not equal, estimating DOA over a small frequency bandwidth which is common to all sources is ideally suited. However, in other applications where the source bandwidths are equal and known apriori, DOA estimation over the entire frequency range is advantageous to reduce the effect of SNR fluctuations across frequency bins.

For the proposed technique, the order of derivative terms introduced in the optimization problem regulates the band of frequencies over which the additional terms in (5.8) can be completely suppressed. Therefore, with the increase in processing bandwidth, using limited derivative terms may introduce approximation errors into the solution. Since

the maximum number of derivative terms that can be introduced is bounded by the dimensionality of the problem, we present a multi-band DOA estimation henceforth.

For a scenario with large source bandwidth, the entire bandwidth is divided into multiple subbands with respective center frequencies identified to be $f_{0,1}, f_{0,2}, \dots, f_{0,Q}$, where Q is the number of subbands. The optimization problem in either (5.9) or (5.15) is solved in each subband with steering vector and the derivative vectors obtained at $\{f_{0,q}, q = 1, \dots, Q\}$ corresponding to the subband. A unified direction finding function can then be obtained by incoherently combining the individual spatial spectra as

$$P_{\text{mb}}(\phi) = 10 \log_{10} \frac{1}{\sum_{q=1}^Q |\mathbf{w}_{\phi,q}^H \mathbf{a}(f_{0,q}, \phi)|^2}, \quad (5.18)$$

where $\mathbf{w}_{\phi,q}$ is the weight vector obtained for each subband q either using Formulation 1 or 2. While the coherent processing within each subband resolves the directions of coherent sources, incoherent processing over several bands can enhance the estimation accuracy as will be observed in the following section.

In Section 4.3.4, it was observed that the time-domain technique exhibits an $\{f, \phi\}$ ambiguity for some array geometries. This phenomenon exists for the frequency-domain approach as well. However, the use of spatio-temporal snapshots as suggested in [30] resolves this ambiguity with very few time-delay taps.

5.4 Simulation Results

5.4.1 Simulation Setup and Illustrations

The three formulations proposed for DOA estimation in frequency domain are evaluated under various conditions in this section. All simulations are performed for the double-ring array shown in Figure 4.4. With a normalized bandwidth of 0.2 centered at $f_c = 0.3$,

the sources occupy a percentage bandwidth of 66.6%. The frequency-domain snapshots are obtained with non-overlapping 256-point DFT of the sensor snapshots. Two sources are referred to as completely correlated when one source is a delayed version of the other source and are generated as suggested in [33]. The array snapshots in each frequency bin is obtained using the conventional frequency-domain signal model given in (6.12). The wideband sources are generated as a sum of sinusoids with random phase and magnitude within the signal bandwidth as defined in (4.37).

From Figure 5.1(b), it was inferred that the array manifold approximation with a derivative order of $P = 2$ introduces error which is acceptable up to a percentage signal bandwidth of 30%. Although increasing the derivative order reduces the approximation error, the two proposed formulations exhibit less performance advantage due to the derivative order as will be studied soon. Deferring the discussion on the signal dimension for the first formulation for now, we use $\dim\{E_s\} = M + 2$, where M is the number of sources present. For multiband DOA estimation, the entire bandwidth is divided into two subband with fractional bandwidths of 0.33 each. While either of the first two formulations can be used for estimation in each subband, we present the performance employing only the second formulation. In line with our the earlier discussion, we assign $\zeta = \Delta f_B$ and $\eta = 2.5$ for all the simulations. The performance of the proposed algorithms is compared with the CSSM. The focussing matrices are obtained using Rotational signal-subspace (RSS) method [36]. As suggested in [36], the directions with maxima observed in the MVDR spectrum are provided as initial estimates along with directions separated by 0.25 times the beamwidth on either sides of these estimates. It was earlier observed in Figure 5.3 that the use of AIC and MDL on the generalized eigenvalues of the matrix pencil $\{\mathbf{R}_{\text{cssm}}, \mathbf{R}_v\}$ exhibits over estimation of the model order. We therefore assume the model order to be known apriori for the CSSM algorithm.

For the source directions to be identified from the spatial spectrum, the corresponding

Figure 5.4: Spatial spectrum obtained for the proposed formulations.

peaks have to be distinct from the spectrum floor. In order to observe this, we consider a scenario with three wideband sources from 220° , 250° and 258° with a percentage bandwidth of 67%. For an SNR of 5 dB and $J = 20$ frequency-domain snapshots, the spatial spectrum obtained by the three formulations are plotted in Figure. 5.4. We note that all the three techniques provide precise DOA estimates from the spatial spectrum. Without knowledge of the model order, the second and third formulations are observed to provide peaks only along the source directions while the spatial spectrum in all other directions is almost flat.

The appropriate choice of signal subspace dimension of $\mathbf{R}_{av}$ is essential for the first formulation which realizes the BFW-MUSIC algorithm in frequency domain. To observe the effect of this factor on the performance, we plot the spatial spectrum of Formulation 1 when sources are situated at 12° , 40° and 48° and $\dim\{\bar{E}_s\}$ is varied from M to $M + 3$. From the spatial spectra plotted in Figure 5.4.1, we observe that the technique cannot

Figure 5.5: Spatial spectrum for various signal subspace dimensions of $\mathbf{R}_{av}$.

resolve the three sources when $\dim\{\bar{E}_s\}$ is equal to M or $M + 1$, whereas the sources are accurately resolved for the rest of the values. When $\dim\{\bar{E}_s\} = M + 3$, we observe false peaks along some non-source directions due to insufficient degrees-of-freedom for the weight vector solution. For the case where $L = 10$ and $P = 2$, if $\dim\{\bar{E}_s\} = M + 3 = 6$, the degrees of freedom available for the weight vector solution is only 2 which is insufficient to withhold false peaks. One potential way to estimate the signal subspace dimension of the averaged covariance matrix $\mathbf{R}_{av}$ is to employ either AIC or MDL with a modified penalty function. While further investigation of this approach is a part of our future work, we employ $\dim\{\bar{E}_s\} = M + 2$ for all our simulations which has been observed to provide consistent results for various combinations of L , M and P .

We now consider the case when sources are completely correlated. In [33], it was shown that the frequency averaging of coherent signal subspaces obtained from various frequency bins can restore the rank of source correlation matrix. Likewise, the rank of the source correlation matrix is restored with frequency averaging performed in (5.8).

Therefore, the proposed formulations are expected to resolve coherent sources present in the scenario. For illustration, we consider three sources situated at 12° , 40° and 48° and DOA is estimated with the double-ring array. The coherent source at 48° is obtained as a delayed version of the second source by $40T_s$, where T_s is the sampling period. At an SNR of 5 dB, the spatial spectrum obtained for the proposed formulations using $J = 20$ snapshots is shown in Figure 5.6. We note that the three formulations are able

Figure 5.6: Spatial spectrum obtained for the two proposed formulations. Sources located at 40° and 48° are completely correlated.

to resolve all the source directions. More specifically, accurate DOA estimation using Formulation 2 and Formulation 3 are appealing considering the fact that model order estimation in coherent scenario is very challenging.

5.4.2 Performance Evaluation

Since the proposed methods incorporate capability to handle wideband sources into the formulations of existing narrowband DOA estimation techniques, it is essential to study the performance of the Formulations 1 and 2 with the processing bandwidth used to evaluate $\mathbf{R}_{av}$. For a scenario with three sources situated at 12° , 40° and 48° , the DOA is estimated from $J = 20$ frequency-domain snapshots acquired by the double-ring array. With the spectrum of all the wideband sources centered at a normalized frequency of $f_c = 0.3$, their fractional bandwidths is increased from 0.01 to 0.67. We first study the probability of resolving the three sources against the fractional bandwidth of the sources in Figure 5.10(a) for the narrowband MUSIC algorithm with model order estimated using MDL and Formulations 1 and 2.

It is evident that the MUSIC algorithm completely fails to resolve the sources as the processing fractional bandwidth increases to 0.1. However, the proposed formulations can resolve the sources for larger source bandwidths with a probability of 1. When the fractional bandwidth is increased, it is important to note that the signal subspace of $\mathbf{R}_{av}$ is also increased. Since the first formulation employs a fixed signal subspace dimension over the entire simulation, we note that the techniques fails to resolve the source directions after a specific signal bandwidth. The second formulation however continues to resolve the source directions even when the source bandwidth is increased beyond with larger bandwidths.

In the accompanied simulation, we study the RMSE of the DOA estimates over 100 Monte-Carlo trials as a function of the processing bandwidth for the same source-sensor setup. The RMSE of the three DOA estimates are averaged and plotted against the fractional bandwidth in Figure 5.10(b). Since the narrowband MUSIC algorithm fails to resolve the three sources, the corresponding RMSE plot is not shown. We note that the

Figure 5.7: DOA estimation performance against fractional bandwidth being processed: (a) probability of resolving the closely-situated sources, (b) RMSE of estimates averaged over the three sources.

RMSE from Formulations 1 and 2 starts to increase with the fractional bandwidth when Δf is approximately greater than 0.25.

We now evaluate the performance of the DOA estimates obtained from the three formulations in comparison with existing techniques. We limit our comparison only to the TD-DOA technique proposed in the previous chapter and CSSM since the TOPS algorithm and ISSM are known to have limited performance at low SNR conditions as shown in Section 4.5. With $J = 10$ frequency domain snapshots and a fractional bandwidth of $\Delta f = 0.25$, the RMSE of DOA estimates evaluated over 100 independent trials is plotted against SNR in Figure 5.8.

We note that the RMSE of the estimates decrease with increasing SNR for all the techniques except for TD-DOA proposed in Chapter 4. The approximation of array manifold over such a large bandwidth with Taylor series of order 2 introduces a large error in the estimated DOA. Therefore, the estimation performance does not improve significantly when the SNR is increased. The CSSM exhibits the best performance for the first and third sources, while the RMSE is relatively large for the second source till an SNR of 10 dB. This is because of its dependence on the initial estimates. The inability to resolve the three sources by MVDR [36] provides inaccurate initial estimates for the focussing matrix design, resulting in an estimation error. As the SNR increases beyond 10 dB, the MVDR algorithm succeeds in providing the three initial directions estimates, which improves the DOA estimation. This observation reflects the sensitivity of CSSM to initial estimates. Among the three proposed formulations, we observe that Formulation 1 performs better than the other two formulations at low SNR values. As the SNR is increased, the performance of Formulation 2 approaches that of Formulation 1 even without the knowledge of model order. This observation can be accounted for the improved segregation between signal and noise subspaces at high SNR values and the automated choice of β . As expected, the processing of multiple subbands yields improved results over the entire SNR range over Formulation 2 which uses only one subband.

Figure 5.9 shows the probability of resolving all the three sources against SNR for the previous simulation. Among the proposed formulations, the first formulation has better resolution over the other two formulations. It is interesting to note that the second formulation has the same resolution capability as that of the TD-DOA technique. This is due to the MUSIC-like framework being employed for DOA estimation in contrast to the subspace-based approach which provides better resolution for Formulation 1. The corresponding multiband scheme is observed to yield marginal improvement in resolving the sources. Although CSSM exhibits better resolution over the proposed techniques,

this performance is susceptible to the initial estimates and the model order.

In Figure 5.8, it was noted that the RMSE for Formulations 1 and 2 almost saturate when the SNR is increased beyond 10 dB for the last two sources. This phenomenon surfaces when the SNR is high, for which case the effect of sensor noise is lesser than the residual error introduced due to the incomplete suppression of all the derivative terms in $\mathbf{R}_{av}$ by the derivative constraints. In order to study this phenomenon, the derivative order was fixed to $P = 2$ and we study the performance of Formulation 2 with SNR for various processing bandwidths of the sources. The array geometry and source directions are maintained the same as the previous simulation and the RMSE of the second source direction estimate obtained by Formulation 2 is observed in Figure 5.10. We note that the RMSE for various processing bandwidths of the sources decreases as the SNR increases upto a specific SNR and saturates without decreasing further. This saturation point can be identified as the situation when the additive noise variance is lower than the residual error due to incomplete suppression of the derivative terms. Furthermore, the SNR at which the RMSE curve saturates is observed to decrease with increasing processing bandwidth. For a fixed order of derivative compensation, this observation indicates that the residual error increases with the processing bandwidth.

We next study the effect of the number of frequency-domain snapshots on the DOA estimates for the same simulation setup at 5 dB SNR in Figure 5.11. Among the three sources, we note that the estimate for the first source shows lower RMSE due to the large spatial separation from other sources. The first formulation is observed to have the least RMSE when compared to all other techniques. CSSM is seen to be less sensitive to the variations in the number of snapshots, while the RMSE of the proposed Formulation 2 decreases as the number of snapshots increases. The multiband DOA estimation developed over Formulation 2 is observed to have better performance over it, indicating the need to exploit DOA estimation across the entire bandwidth.

5.5 Discussions and Concluding Remarks

In this chapter, we first observed that the AIC and MDL techniques over-estimate the number of sources present in the scenario when used with the coherently averaged covariance matrix $\mathbf{R}_{\text{cssm}}$. This result would adversely affect the performance of CSSM if the model order is obtained incorrectly. From the simulations studies, it is observed that all the three formulations can estimate wideband source directions. The first formulation, built over the BFW-MUSIC, requires to estimate the signal subspace dimension which is generally greater than the model order. With the signal subspace dimension approximately known, this formulation can suppress the derivative components present in the covariance matrix effectively, thus yielding better estimation accuracy over the other two formulations. However, the problem of model order estimation is still unresolved with the outcome of this formulation.

In order to estimate wideband source directions independent of the model order estimation problem, we present the second formulation employing the MUSIC-like formulation. This is a frequency domain realization of the proposed algorithm in Chapter 4. This algorithm maintains the accuracy of estimates even for sources with large bandwidth by regulating the processing bandwidth. The third formulation extends the second formulation over multiple subbands in order to exploit the DOA information preserved in other source subbands. We therefore achieve a means to estimate directions of wideband sources whose percentage bandwidth can be from 10 – 67%.

Figure 5.8: The RMSE of DOA estimates for CSSM, TD-DOA and the proposed frequency-domain formulations plotted against SNR.

Figure 5.9: The probability of resolving all the three sources plotted against SNR for various formulations.

Figure 5.10: The plot of RMSE for the second source direction estimate using Formulation 2 for various processing bandwidths.

Figure 5.11: The RMSE of DOA estimates plotted against the number of frequency-domain snapshots used.

Chapter 6

Wideband DOA Estimation in Dispersive Medium

6.1 Introduction

The DOA estimation techniques discussed in previous have been developed considering homogeneous nondispersive medium. In practical scenarios however, environmental factors such as multipath, scattering and dispersion [50] may have to be incorporated into the signal model. Conventional techniques therefore cannot be directly employed to localize sources in such conditions. Among these factors, dispersion is a common phenomenon due to which different frequency components within a wave propagate at different speeds. In a homogeneous medium, dispersion is observed when the relative permittivity and/or permeability of the medium are/is a function of frequency. Whereas in a heterogeneous medium, effects such as scattering due to suspended particles affect the speed of propagation. Often, dispersion is studied with the “dispersion curve” obtained by plotting the wavenumber or speed against angular frequency.

For wireless applications, the access bandwidth within the conventional frequency

spectrum is very limited; the spectrum with wavelength in millimeter range corresponding to frequencies beyond 30 GHz is available for usage. At these frequencies, however, the atmospheric transfer function exhibits significant frequency-dependent attenuation and phase delay as pointed out in [51]. The absorption, dispersion and scattering of signals due to atmospheric elements as well as the rain drops, can be cumulatively incorporated within the atmospheric transfer function [50]. For ultra-wideband pulse propagation, the expressions for time-delay and the pulse width variations are shown in [50] to be functions of first and second-order derivatives of attenuation and phase delay factors w.r.t. frequency. This dependence over frequency translates to dispersive effect. Besides these factors which exist in clear sky conditions, variations in time-delay due to rain are susceptible to parameters such as rain drop size and rain-rate. It is therefore challenging to precisely estimate and compensate for dispersive effect prior to DOA estimation. A robust estimator is hence essential to obtain reliable estimates in such situations.

In other applications such as seismic and underwater acoustics, the medium is heterogeneous in nature due to the presence of suspended particles. As a consequence, the wave encounters absorption, refraction, reflection and dispersion while propagating through the medium. Furthermore, the wave propagation speed is a continuous function of depth in these applications and has to be compensated based on location specific sound speed profiles. Under such conditions, signal parameters are estimated by modeling these uncertainties as either multiplicative noise [52] or as wavefront amplitude distortion [53]. An alternative approach is to employ matched-field processing [54] framework to estimate the source location by evaluating the received power from each possible location. This framework permits the incorporation of wave solution and robustness to random speed perturbations into the constraint equations of MVDR algorithm as shown in [55]. The maximum likelihood estimator in [55] is however developed for the narrow-band signal model. This principle can easily be extended to wideband sources with the

STCM-MVDR algorithm [40] reviewed in Section 2.3.2.4. Across the signal bandwidth, the steered covariance matrix in a given azimuth ϕ is constructed as

$$\mathbf{R}(\phi) = \sum_{f_k=f_1}^{f_u} \mathbf{T}^H(f_k, \phi) \mathbf{R}(f_k) \mathbf{T}(f_k, \phi), \quad (6.1)$$

where $\mathbf{T}(f_k, \phi) = \text{diag}\{\mathbf{a}(f_k, \phi)\}$. If we define a diagonal matrix $\mathbf{U}(f_k)$ which incorporates a frequency-dependent speed perturbation, the STCM can be modified as

$$\mathbf{R}_1(\phi) = \sum_{f_k=f_1}^{f_u} \mathbf{U}(f_k) \mathbf{T}^H(f_k, \phi) \mathbf{R}(f_k) \mathbf{T}(f_k, \phi) \mathbf{U}^H(f_k). \quad (6.2)$$

The source directions can then be estimated from the peaks of the steered response as

$$P(\phi) = \frac{1}{\mathbf{1}_L^T \mathbf{R}_1^{-1}(\phi) \mathbf{1}_L}. \quad (6.3)$$

We use this technique as the baseline algorithm in our study.

Our objective in this chapter is to estimate wideband source directions in a dispersive medium. Due to inaccurate knowledge of the dispersion curve, we assume that the propagation speed is an unknown continuous function of frequency. With this assumption, we can express the steering vector at a reference frequency as a Taylor series expansion w.r.t. the wavenumber within the signal bandwidth. An averaged covariance matrix can then be estimated across the signal bandwidth which consists of derivative terms w.r.t. wavenumber. We then employ the structure of MUSIC-like algorithm for DOA estimation by introducing robustness to dispersion. With an average propagation speed known only at the reference frequency, the proposed technique estimates DOA without requiring the dispersion curve. The chapter is structured as follows: In Section 6.2, we first derive the signal model for homogeneous dispersive medium. The array manifold

approximation is incorporated into the signal model in Section 6.3. We then propose a DOA estimation technique in Section 6.4 which introduces robustness to variations in speed over frequency. The performance of the proposed technique is evaluated with simulations in Section 6.5 and Section 6.6 concludes this chapter.

6.2 Signal Model

A wideband signal, if represented as a sum of sinusoids, can be written as a composite modulated signal. For instance, a signal with two sinusoids at angular frequencies $\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$ is given by

$$\begin{aligned} s(t) &= \sin(\omega_1 t) + \sin(\omega_2 t) \\ &= 2 \sin(\omega t) \cos(\delta\omega t), \end{aligned} \tag{6.4}$$

where $\omega = (\omega_1 + \omega_2)/2$ and $\delta\omega = (\omega_1 - \omega_2)/2$. In a homogeneous medium, this travelling wave at a spatial point $\mathbf{r}$ can be expressed as

$$s(t - \boldsymbol{\alpha} \cdot \mathbf{r}) = 2 \cos(\delta\omega t - \delta\boldsymbol{\kappa} \cdot \mathbf{r}) \sin(\omega t - \boldsymbol{\kappa} \cdot \mathbf{r}), \tag{6.5}$$

where $\boldsymbol{\alpha}$ is the slowness vector and $\delta\boldsymbol{\kappa} = \delta\omega\boldsymbol{\alpha}$ is the differential wavenumber vector for the two frequencies. We note that the signal consists of a carrier signal component $\sin(\omega t - \boldsymbol{\kappa} \cdot \mathbf{r})$ and a modulating signal component $\cos(\delta\omega t - \delta\boldsymbol{\kappa} \cdot \mathbf{r})$. The velocities estimated from the respective components are termed as the phase and group velocities. Phase velocity is the rate at which the carrier phase propagates in the wave propagation direction. In its scalar form, we have

$$\nu_p = \frac{\omega}{\kappa}, \tag{6.6}$$

where $\kappa = |\boldsymbol{\kappa}|$ is the magnitude of the wavenumber vector. The group velocity is the propagation speed of the modulating signal or the envelop. In its scalar form

$$\nu_g = \frac{\delta\omega}{\delta\kappa} = \frac{\omega_2 - \omega_1}{\kappa_2 - \kappa_1}, \quad (6.7)$$

where κ_1 and κ_2 are the wavenumbers at frequencies ω_1 and ω_2 , respectively. For a nondispersive medium, the phase velocity is a constant over frequency. From (6.6), we hence have

$$\omega_1 = \nu_p \kappa_1, \quad (6.8)$$

$$\omega_2 = \nu_p \kappa_2.$$

Substituting (6.8) into (6.7), it is easy to note that the group velocity is equal to the phase velocity. However, since the permittivity and/or permeability of a dispersive medium varies with frequency, this equality is not maintained. It is also possible to show that these velocities, and the corresponding slowness vectors ($\boldsymbol{\alpha}$) are related by simple geometric relations [56].

The slowness vector is related to the phase velocity as

$$\boldsymbol{\alpha} = |\boldsymbol{\alpha}| \mathbf{u} = \frac{1}{\nu_p} \mathbf{u}. \quad (6.9)$$

where $\mathbf{u}$ denotes a unit vector in the direction of wave propagation. Since the propagation speed varies with frequency for a dispersive medium, we note that the slowness vector is a function of frequency, i.e., $\boldsymbol{\alpha}(f)$. When a propagating signal is acquired by an array of L sensors at spatial locations $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_L$, the sensor outputs in frequency domain can

be obtained using the Fourier transform relation in (2.30) as

$$\begin{aligned}\underline{\mathbf{x}}(f) &= \begin{bmatrix} e^{-j2\pi f \boldsymbol{\alpha}(f) \cdot \mathbf{r}_1} \\ e^{-j2\pi f \boldsymbol{\alpha}(f) \cdot \mathbf{r}_2} \\ \vdots \\ e^{-j2\pi f \boldsymbol{\alpha}(f) \cdot \mathbf{r}_L} \end{bmatrix} \underline{\mathbf{s}}(f) + \underline{\mathbf{v}}(f) \\ &= \mathbf{a}(f, \nu(f), \theta) \underline{\mathbf{s}}(f) + \underline{\mathbf{v}}(f),\end{aligned}\tag{6.10}$$

where $\nu(f) = 1/|\boldsymbol{\alpha}(f)|$ is the frequency-dependent propagation speed. Note that this model holds true even when the source signal has more than two sinusoids. The source model can be expressed as

$$s(t) = \sum_{i=1}^{B_{\text{bw}}} a_i \exp\{j(2\pi f_i t + c_i)\},\tag{6.11}$$

where the amplitude a_i is an instance of Rayleigh distribution, c_i is uniformly distributed in $[-\pi, \pi]$ and B_{bw} is the number of sinusoids that constitute the wideband source. Since the phase velocity and group velocity are related by geometric relations in a dispersive medium [56], an explicit presence of group velocity in the signal model need not be considered. Extending the signal model in (6.10) for a scenario with M sources from directions $\Theta = [\theta_1, \dots, \theta_M]$, we have

$$\underline{\mathbf{x}}(f) = \mathbf{A}(f, \nu(f), \Theta) \underline{\mathbf{s}}(f) + \underline{\mathbf{v}}(f),\tag{6.12}$$

with $\mathbf{A}(f, \nu(f), \Theta) = [\mathbf{a}(f, \nu(f), \theta_1), \dots, \mathbf{a}(f, \nu(f), \theta_M)]$ is the array manifold matrix. The problem in hand is to estimate the source directions Θ from the sensor snapshots.

6.3 Array Manifold Approximation

Since the slowness vector and the wavenumber vector are related by

$$\boldsymbol{\kappa}_f = \omega \boldsymbol{\alpha}(f) = \frac{2\pi f}{\nu(f)} \mathbf{u} = \kappa_f \mathbf{u}, \quad (6.13)$$

the wavenumber $\kappa_f = 2\pi f/\nu(f)$ absorbs the dispersion factor which affects the steering vector. We can therefore rewrite the steering vector as a function of wavenumber instead of frequency as

$$\mathbf{a}(\kappa_f, \phi) = [e^{-j\kappa_f \mathbf{u} \cdot \mathbf{r}_1} \ e^{-j\kappa_f \mathbf{u} \cdot \mathbf{r}_2} \ \dots \ e^{-j\kappa_f \mathbf{u} \cdot \mathbf{r}_L}]^T. \quad (6.14)$$

If we define a reference frequency f_0 at which the propagation speed is estimated $\nu(f_0)$, the wavenumber at a frequency bin in the proximity of f_0 can be approximated with Taylor series of order P w.r.t. κ_f as

$$\mathbf{a}(\kappa_f, \phi) \approx \mathbf{a}(\kappa_0, \phi) + \sum_{p=1}^P \frac{(\delta\kappa_f)^p}{p!} \frac{\partial \mathbf{a}(\kappa_f, \phi)}{\partial \kappa_f} \Big|_{\kappa_f=\kappa_0}, \quad (6.15)$$

where $\kappa_0 = 2\pi f_0/\nu(f_0)$ is the wavenumber at frequency f_0 and $\delta\kappa_f = \kappa_f - \kappa_0$. We can express the array manifold matrix $\mathbf{A}(\kappa_f, \Theta)$ in terms of κ_0 with Taylor series approximation using which, the signal model at frequency f_k is given by

$$\underline{\mathbf{x}}(k, l) \approx \left(\mathbf{A}(\kappa_0, \Theta) + \sum_{p=1}^P \frac{(\delta\kappa_f)^p}{p!} \frac{\partial^p \mathbf{A}(\kappa_f, \Theta)}{\partial \kappa_f^p} \Big|_{\kappa_f=\kappa_0} \right) \underline{\mathbf{s}}(k, l) + \underline{\mathbf{v}}(k, l). \quad (6.16)$$

With this approximation, the spatial covariance matrix for the zero-mean snapshots $\underline{\mathbf{x}}(k, l)$ at f_k is given by

$$\begin{aligned} \mathbf{R}(f_k) &\approx \mathbf{A}(\kappa_0, \Theta) \mathbf{R}_{ss}(f_k) \mathbf{A}^H(\kappa_0, \Theta) + \sigma_v^2(f_k) \mathbf{I}_L \\ &+ \sum_{p=1}^L \frac{(\delta \kappa_f)^p}{p!} \text{Re} \left[\mathbf{A}(\kappa_0, \Theta) \mathbf{R}_{ss}(f_k) \frac{\partial^p}{\partial \kappa_f^p} \mathbf{A}^H(\kappa_f, \Theta) \Big|_{\kappa_f = \kappa_0} \right] \\ &+ \sum_{p,q=1}^P \frac{\delta \kappa_f^{p+q}}{p!q!} \frac{\partial^p}{\partial \kappa_f^p} \mathbf{A}(\kappa_f, \Theta) \Big|_{\kappa_f = \kappa_0} \mathbf{R}_{ss}(f_k) \frac{\partial^q}{\partial \kappa_f^q} \mathbf{A}^H(\kappa_f, \Theta) \Big|_{\kappa_f = \kappa_0}, \end{aligned} \quad (6.17)$$

where $\sigma_v^2(f_k)$ is the variance of the additive uncorrelated sensor noise at frequency f_k . We note that the derivatives of $\mathbf{A}(\kappa_f, \Theta)$ w.r.t. the wavenumber distinguishes (6.17) from the expression for $\mathbf{R}(f_k)$ in (5.3). Therefore, following the previous approach adopted for frequency-domain wideband DOA estimation, we first define a fractional bandwidth $\Delta f_B < \Delta f$ corresponding to $B + 1$ frequency bins in the proximity of f_0 , over which a P th order Taylor series approximates the array manifold up to an acceptable error. An average covariance matrix across this bandwidth can then be defined as

$$\begin{aligned} \mathbf{R}_{\text{av}} &= \frac{1}{B+1} \sum_{k=-B/2}^{B/2} \mathbf{R}(f_k) \\ &\approx \frac{1}{B+1} \left[\mathbf{A}(\kappa_0, \Theta) \left\{ \sum_{k=k_0-B/2}^{k_0+B/2} \mathbf{R}_{ss}(f_k) \right\} \mathbf{A}^H(\kappa_0, \Theta) \right. \\ &+ 2 \sum_{k=k_0-B/2}^{k_0+B/2} \text{Re} \left\{ \sum_{p=1}^P \frac{(\delta \kappa_f)^p}{p!} \mathbf{A}(\kappa_0, \Theta) \mathbf{R}_{ss}(f_k) \frac{\partial^p}{\partial \kappa_f^p} \mathbf{A}^H(\kappa_f, \Theta) \Big|_{\kappa_f = \kappa_0} \right\} \\ &+ \left. \sum_{k=k_0-B/2}^{k_0+B/2} \sum_{p,q=1}^P \frac{(\delta \kappa_f)^{p+q}}{p!q!} \frac{\partial^p}{\partial \kappa_f^p} \mathbf{A}(\kappa_f, \Theta) \Big|_{\kappa_f = \kappa_0} \mathbf{R}_{ss}(f_k) \mathbf{A}^H(\kappa_f, \Theta) \Big|_{\kappa_f = \kappa_0} + \sum_{k=k_0-B/2}^{k_0+B/2} \sigma_v^2(f_k) \mathbf{I}_L \right]. \end{aligned} \quad (6.18)$$

This matrix consists of the familiar coherently averaged component and the derivative terms observed in (5.8). Suppressing the derivative terms will therefore reduce the prob-

lem to narrowband DOA estimation.

6.4 Proposed Technique

When the model order is not available, employing the framework of narrowband MUSIC-like algorithm provides better resolution over the MVDR algorithm. We first define a matrix $\mathbf{D}_{\kappa_0, \phi}$ with derivative vectors up to P th order from each look direction ϕ as

$$\mathbf{D}_{\kappa_0, \phi} = \begin{bmatrix} \mathbf{d}_1(\kappa_0, \phi) & \mathbf{d}_2(\kappa_0, \phi) & \dots & \mathbf{d}_P(\kappa_0, \phi) \end{bmatrix}, \quad (6.19)$$

where $\mathbf{d}_p(\kappa_0, \phi) = \left. \frac{\partial^p \mathbf{a}(\kappa_f, \phi)}{\partial \kappa_f^p} \right|_{\kappa_f = \kappa_0}$. The projection matrix of $\mathbf{D}_{\kappa_0, \phi}$ is given by

$$\mathbf{P}_{\mathbf{D}_\phi} = \mathbf{D}_{\kappa_0, \phi} (\mathbf{D}_{\kappa_0, \phi}^H \mathbf{D}_{\kappa_0, \phi})^{-1} \mathbf{D}_{\kappa_0, \phi}^H.$$

The data matrices at each frequency $\underline{\mathbf{X}}(k)$, are concatenated over the processing bandwidth ($B + 1$ bins) to obtain

$$\underline{\mathbf{X}} = \begin{bmatrix} \underline{\mathbf{X}}(-B/2), \dots, \underline{\mathbf{X}}(B/2) \end{bmatrix} \in \mathbb{C}^{L \times J(B+1)}. \quad (6.20)$$

This matrix is projected onto the derivative vector space in each look direction as $\underline{\mathbf{Y}} = \mathbf{P}_{\mathbf{D}_\phi} \underline{\mathbf{X}}$. The corresponding covariance matrix is given by

$$\mathbf{R}_y(\phi) = \mathbf{P}_{\mathbf{D}_\phi} \mathbf{R}_{\text{av}} \mathbf{P}_{\mathbf{D}_\phi}^H. \quad (6.21)$$

Introducing this term into the optimization problem, we have

$$\begin{aligned}
 & \underset{\mathbf{w}_\phi}{\text{minimize}} && \mathbf{w}_\phi^H \{ \mathbf{R}_{\text{av}} + \zeta \mathbf{R}_y(\phi) \} \mathbf{w}_\phi \\
 & \text{subject to} && \mathbf{w}_\phi^H \{ \mathbf{a}(\kappa_0, \phi) \mathbf{a}^H(\kappa_0, \phi) + \beta \mathbf{I}_L \} \mathbf{w}_\phi = c,
 \end{aligned} \tag{6.22}$$

where β, ζ, c are constants. The optimal weight vector is given by the eigenvector corresponding to the minimum eigenvalue $\chi_{\min}$ of the GEV,

$$(\mathbf{R}_{\text{av}} + \zeta \mathbf{R}_y(\phi)) \mathbf{w}_\phi = \chi_{\min} (\mathbf{a}(\kappa_0, \phi) \mathbf{a}^H(\kappa_0, \phi) + \beta \mathbf{I}_L) \mathbf{w}_\phi. \tag{6.23}$$

With the processing bandwidth limited to $B + 1$ frequency bins, the parameter ζ can be assigned a value equal to the fractional bandwidth $\Delta f_B = \frac{B+1}{f_0}$ based on the discussion presented in Section 4.3.2. Likewise, the bounds derived for β in the TD-DOA algorithm can be extended to this context by replacing the time-domain covariance matrix $\mathbf{R}$ with $\mathbf{R}_{\text{av}}$. The value of β is chosen with reference to the lower bound as

$$\beta = \eta \min_{\phi} \frac{\mathbf{a}^H(\kappa_0, \phi) (\mathbf{R} + \zeta \mathbf{R}_y(\phi))^{-1} \mathbf{a}(\kappa_0, \phi)}{\chi(\mathbf{R} + \zeta \mathbf{R}_y(\phi))^{-1}, \max}, \tag{6.24}$$

where $\eta > 1$ provides unambiguous spatial spectrum when it is chosen such that β is within the bounds. We observe that the proposed technique for DOA estimation under dispersive medium is an extension of the technique discussed in Chapter 5 and only differs in using the derivative terms w.r.t. the wavenumber instead of frequency. Since the dispersion was preserved in the derivative terms of (6.18), their suppression using the proposed formulation enhances its robustness to dispersion.

6.5 Simulation Results and Discussions

In this section, we evaluate the performance of the proposed technique under a simulated dispersive environment. In this section all the frequencies are normalized such that the sampling frequency is equal to unity. The propagation speed is synthesized from a simple nonlinear function of frequency f given by

$$\nu(f) = \nu(f_c) \left(1 + \Delta\nu_{\max} \tanh(\gamma(f - f_c)) \right), \quad (6.25)$$

where $\Delta\nu_{\max} = \frac{\nu(f_{\max}) - \nu(f_{\min})}{\nu(f_c)}$ is the fractional speed variation across the signal bandwidth which is centered at the normalized frequency f_c with speed $\nu(f_c)$ and $\gamma > 0$ regulates the nonlinearity of the function. When $\gamma = 0$, the propagation speed remains constant across the signal bandwidth. Noting that the dispersion relation for freely propagating ocean surface gravity waves obeys a hyperbolic tangent function [57], we have introduced $\Delta\nu_{\max}$ and $\gamma > 0$ in order to regulate the degree of nonlinearity in (6.25). Moreover, the theoretical Love and Rayleigh dispersion curves for a seismic two-layer model [58] can be synthesized with (6.25). The dispersion observed in atmosphere can similarly be modeled using the same function. The frequency-domain signal model given by (6.12) is used for the simulations in this section and the source signals are generated using (6.11). The frequency dependent array manifold matrix $\mathbf{A}(f, \nu(f), \Theta)$ is obtained using the wave velocity function given by (6.25).

The proposed technique is compared with the STCM-MVDR algorithm, which follows a matched-field processing structure. If the dispersion curve is available, it can be incorporated in the steered vectors used while estimating the steered covariance matrix. For the proposed technique, we provide only the propagation speed $\nu(f_c)$ at f_c , while for the STCM-MVDR we provide the dispersion curve given by (6.25). A derivative order of $P = 2$ is used to construct $\mathbf{D}_{\kappa_0, \phi}$. The propagation speed at frequency $f_c = 0.3$

considering wireless applications is chosen to be $\nu(f_c) = 3 \times 10^8$ m/s. We use $J = 20$ frequency-domain snapshots for all the simulations.

We first study the spatial spectrum of the proposed technique for a scenario with a 4×4 planar array with an inter-element spacing of $d = \lambda_{\min}/2$ is used for this simulation. Two wideband sources with a fractional bandwidth of $\Delta f = 0.33$ and centered at $f_c = 0.3$ are situated at azimuth angles 40° and 60° in the same plane as the array. The elevation angle is therefore equal to zero. The propagation speed varies according to the dispersion function given by (6.25) with $\nu(f_c) = 3 \times 10^8$ m/s and a percentage speed deviation of 5% ($\Delta\nu_{\max} = 0.05$). Two instances with of dispersion curve with $\gamma = -60$ and $\gamma = 60$ shown in Fig. 6.1(a) are used for the simulation. While the dispersion curve for $\gamma = -60$ is a monotonically decreasing function, the speed increases with frequency for $\gamma = 60$. The frequency-domain snapshots are obtained by introducing dispersion according to (6.12) at an SNR of 5 dB. The spatial spectra obtained by the proposed technique for the two dispersion functions with 20 frequency-domain snapshots are plotted in Fig. 6.1(b). We note that the spatial spectrum for the proposed technique exhibits distinct peaks along the two source directions. With only the center frequency provided, the algorithm is observed to be robust for changes in speed over a percentage speed deviation of 5% within the source bandwidth.

It is important to note that the proposed technique provides accurate estimates without the use of dispersion curve. This indicates that the proposed technique exhibits robustness for various degrees of dispersion nonlinearity. In order to verify this, we investigate the performance of the proposed technique with the RMSE of DOA estimates over various degrees of nonlinearity obtained by regulating the parameters γ and $\Delta\nu_f$ in (6.25). For the simulation setup used before, we plot the RMSE of the estimates averaged over the two sources against $\Delta\nu_f$ and γ in Fig. 6.2(a) and (b), respectively. We plot the performance of the STCM-MVDR algorithm under the same simulation setup. We note

Figure 6.1: (a) Dispersion function with the propagation speed plotted against frequency for $\gamma = -60$ and $\gamma = 60$. (b) Spatial spectrum obtained by the proposed technique for the two values of γ with SNR set to 5 dB.

that the proposed technique has improved estimation accuracy over the STCM-MVDR algorithm for all values of $\Delta\nu_f$ and γ . From Figure 6.2(a) and (b), it is important to note that the estimation accuracy is almost constant over the entire range of $\Delta\nu_f$ and γ , which implies that the proposed technique is robust to different nonlinear functions. This is due to the incorporation of $\mathbf{R}_y(\phi)$ which provides robustness to any monotonic change in speed from ν_c at the center frequency.

We next study the variation in performance of the proposed technique with SNR in comparison with the STCM-MVDR algorithm for the same simulation setup. The simulation is conducted for $\gamma = -50$ and $\gamma = 50$ maintaining a $\Delta\nu_f = 0.05$. The RMSE of individual source estimates are plotted in Figure 6.3. The proposed technique is observed to provide improved estimation accuracy over the STCM-MVDR algorithm at lower SNR values. For both the values of γ , we note that the accuracy is very similar as expected.

In all the above simulations, the dispersion function employed is idealistic with speed

Figure 6.2: Estimation performance of the proposed technique plotted against the non-linearity function parameters, (a) γ , (b) $\Delta \nu_{\max}$.

monotonically increasing or decreasing as frequency increases. We now consider a practical situation with propagation speed perturbed with random fluctuations across the signal bandwidth as shown in Figure 6.4(a). This scenario is encountered in seismic applications where phase velocity is sensitive over different layers of the soil [58]. For the same source directions and array geometry used for the previous simulation, the dispersion function shown in Figure 6.4(a) is obtained as

$$\nu_f = \nu_c \left(1 + \Delta \nu_{\max} \tanh(\gamma(f - f_c)) \right) + \delta \nu_c, \quad (6.26)$$

where $\delta \nu_c$ is a random speed fluctuation derived from Gaussian distribution.

Allowing the speed at the center frequency to be $\nu(f_c) = 1000$ m/s, a percentage speed dispersion of 10% ($\Delta \nu_{\max} = 0.1$) is introduced for this simulation. With only the speed at center frequency known, the spatial spectrum of the proposed technique obtained at an SNR of 0 dB is plotted in Figure 6.4(b) in comparison with the performance of the STCM-

Figure 6.3: (a) Estimation performance of the proposed technique plotted against SNR for, (a) Source 1, (b) Source 2.

MVDR algorithm. We note that the spatial spectrum of the proposed technique shows accurate peaks along the source directions in contrast to STCM-MVDR which has poorer resolution. This robustness to speed perturbations can be explained by first realizing that narrowband MUSIC-like formulation introduces deep nulls along the source directions. The introduced derivative compensation widens the null width along the wavenumber axis depending on the derivative order included.

Figure 6.4: (a) Dispersion function with the propagation speed obtained using (6.26) plotted against frequency. (b) Spatial spectrum obtained by the proposed technique and the STCM-MVDR algorithm at an SNR of 0 dB.

6.6 Conclusion

In this chapter, we first observed that the incorporation of dispersion into the wideband signal model affects only the array manifold matrix. The array manifold approximation which has been adopted in previous chapters can therefore be extended to dispersive environment with steering vectors approximated with Taylor series expansion w.r.t. the wavenumber instead of frequency. The effect of dispersion on wideband DOA estimation was then countered by incorporating a data-dependent correlation matrix into the MUSIC-like formulation. With the propagation speed known only at a reference frequency, the algorithm was observed to resolve source directions accurately. The proposed technique has been observed to be robust to different degrees of nonlinearity verified with synthesized dispersive curve.

Chapter 7

Conclusion and Future Directions

7.1 Conclusion

The central theme of this thesis has been the DOA estimation of narrowband and wide-band sources. Having known the need to estimate the number of sources from the review of existing techniques, the working principle of the narrowband MUSIC-like algorithm [9], which does not require the model order information, has been investigated. The performance of this technique is observed to approach that of MUSIC algorithm when the SNR is increased. More interestingly, the weight vector solution obtained in each scan direction has been identified to reside in the noise subspace. This observation is exploited to construct a weight vector matrix, the columns of which spans the noise subspace of the array covariance matrix. With this matrix, the MUSIC-W algorithm has been presented in Chapter 3 by replacing the noise eigenvector matrix in the MUSIC spectrum with the weight vector matrix. From our investigations, it has been inferred that the MUSIC-W algorithm out-performs the MUSIC-like algorithm and approaches the performance of the MUSIC algorithm. The computational advantage and performance improvement makes this algorithm appealing over MUSIC-like algorithm when the number of sources

is unknown.

With the performance of MUSIC-like and MUSIC-W algorithms comparable with the MUSIC algorithm for narrowband sources, wideband DOA estimation has been considered with unknown number of sources in Chapter 4. Noting that the steering vector is a function of frequency, an array manifold approximation has been exercised using Taylor series expansion w.r.t. frequency. The expression for time-domain array covariance matrix is then shown to have additional derivative terms along with a coherently averaged narrowband component. Introducing derivative compensation to the beamforming-framework of MUSIC-like algorithm, the technique has been extended for wideband source scenario. From the presented analysis however, it is inferred that this compensation cannot be incorporated into the MUSIC-W algorithm. Nevertheless, the proposed TD-DOA technique with modified MUSIC-like formulation has been observed to provide accurate source directions by incorporating robustness against frequency variations. Unlike the BFW-MUSIC algorithm which imposes linear derivative constraints, this technique is less sensitive to the choice of derivative order. The performance of this technique has been shown to be comparable with CSSM and outperforms BFW-MUSIC algorithm and TOPS algorithms when the number of snapshots is large. With model order assumed to be unknown, the probability of resolution is marginally decreased for this technique.

When the source bandwidth is large, the performance of TD-DOA technique has been observed to decrease in Figure 5.2 due to the array manifold approximation error. In order to address this problem, the BFW-MUSIC and MUSIC-like wideband DOA estimation techniques have been realized in the frequency domain by constructing an average covariance matrix over a regulated source bandwidth. Investigations reveal that the proposed techniques have estimation performance comparable with CSSM and other existing techniques. With the number of sources unknown, the frequency-domain MUSIC-like

formulation has been observed to provide reliable estimates with marginally lower probability of resolution. The multi-band extension of this technique to extract the DOA information from the entire signal bandwidth has been shown to improve the estimation performance. Furthermore, the capability of these techniques to resolve closely-situated coherent sources with unknown number of sources has been examined, in which case, other techniques fail when incorrect number of sources is estimated.

When the medium is dispersive in nature, DOA estimation accuracy is adversely affected by the speed variations. In Chapter 6, the array manifold approximation has been redefined such that the formulation incorporates robustness to dispersion. With derivative compensation incorporated w.r.t. the wavenumber in the MUSIC-like formulation, source directions are estimated in dispersive medium with high accuracy. The robustness of the proposed technique to variations in propagation speed has been shown in our investigations. It is important to emphasize that this technique estimates source directions with the propagation speed known only at the reference frequency.

Besides the topics discussed in the thesis, we also pursued with the study of DOA estimation of speech sources under aliasing conditions. For speech sources which are bandlimited from 400 Hz to 4 kHz, the output of a microphone array has to be sampled abiding the spatial Nyquist criterion which requires the sensor spacing $d \leq 0.5\lambda_{\min} = 0.04$ m, where $\lambda_{\min}$ is the wavelength corresponding to the maximum signal frequency. With number of microphones often constrained by the system design, the aperture length, and hence the resolution of DOA estimation is limited by this condition. While the extension of microphone spacing beyond Nyquist criterion improves the resolution, it is complemented with the introduction of spatial aliasing.

As shown in [59], the effect of aliasing on wideband DOA estimation exhibits itself with raised spatial spectrum floor unlike in the narrowband scenario which provides distinct aliasing peaks. This phenomenon adversely affects the identifiability of source

directions from the spatial spectrum in a reverberant scenario, more so when the number of sources is not known accurately. Therefore, it is required to suppress the spectrum floor while maintaining the source direction peaks following a detailed analysis of aliasing effect on wideband speech sources.

In order to improve the identifiability of source directions, we have proposed a multi-stage scheme in [J5] based on a careful analysis, where we note that the effect of aliasing on microphone outputs varies over different bands within the source bandwidth. Based on this observation, we first decompose the microphone signals into different subbands depending on the degree of aliasing that affects each subband. The alias-free spatial spectrum obtained by performing DOA estimation on the first subband is employed in the second stage to inhibit any DOA ambiguity introduced due to aliasing in the spatial spectrum of the second subband. The resultant spectrum is then employed in the succeeding stages to suppress the effect of aliasing while maintaining the high resolution speech DOA estimation obtained by the latter stage. This multi-stage scheme has been verified with a beamforming-based technique (STCM-MVDR) and a time-difference of arrival-based technique (approximate kernel density estimator (KDE) [60], 2012Nesta). It is observed that the proposed scheme enhances the identifiability of the source directions in the spatial spectrum while maintaining the accuracy of the baseline algorithm employed. This discussion was not included as a part of the thesis due to its mismatch with the rest of the chapters in the thesis.

7.2 Future Directions

In the due course of this research, a few potential extensions were identified. Since the involving work steps beyond the outline of this thesis, we enlist them here inviting interest from researchers.

- One of our interesting findings is the MUSIC-W algorithm developed for narrowband DOA estimation. Constructing the weight vector matrix from arbitrary directions was observed to substitute the need to estimate the noise subspace, as well as the model order for DOA estimation. However, this idea could not be directly extended to localize wideband sources for the reason discussed in Section 4.3.2. It may require us to reformulate the optimization problem with alternative means to incorporate wideband capabilities. Noting the improved performance of the narrowband MUSIC-W algorithm with reduced computational cost compared to the MUSIC-like algorithm, extending this technique to wideband DOA estimation is an interesting topic for further research.
- DOA estimation of narrowband sources when the number of sources is greater than the number of sensors is a challenging task. Model order estimation is equally difficult to accomplish in such scenario. Existing techniques increase the degrees-of-freedom by exploiting statistical characteristics of sources such as non-Gaussianity [61] or quasi-stationarity [62]. MUSIC-like and MUSIC-W algorithms may suit well for such cases where the performance of existing subspace-based are sensitive to the chosen noise subspace. A thorough study of the performance of MUSIC-like algorithm and detailed analysis in underdetermined source scenario deserves more research resources.
- In order to handle more number of wideband sources as well as larger bandwidth, increasing the degrees of freedom is very essential. Once the extension of MUSIC-like algorithm with increased degrees-of-freedom is validated, we can extend the proposed TD-DOA and the frequency-domain techniques for wideband DOA estimation with higher accuracy due to the additional degrees-of-freedom.
- Besides dispersion, many factors affect the performance of DOA estimation in prac-

tical SONAR and seismic applications. A thorough investigation is essential to verify the applicability of the proposed technique in such conditions.

- In the design of sensor arrays, it is difficult to ensure all the array elements to have identical gain and phase characteristics. Direct application of DOA estimation techniques is observed to provide inaccurate results in such scenarios. One therefore has to calibrate the array prior to the DOA estimation. It is interesting for us to study the sensitivity of the proposed algorithms to the problem of gain and phase errors. As a part of future work, it is essential to study the performance of these techniques in conjunction with gain/phase calibration techniques.
- In the problem definition for speech source localization, the signal model is generally simplified by segregating the cumulative effect of reverberation as a diffused component. While the suppression of this term is not explicitly discussed, employing the PHAT compensation for beamforming-based techniques introduces robustness to mild reverberation. In order to counter the effect of reverberation more effectively, approximate KDE estimates time-difference of arrival from the filter coefficients obtained of the convolutive BSS. However, it is important to note that the output of a convolutive BSS algorithm is a filtered version of the sources which implies that the filter coefficients of convolutive BSS are not completely exempted of the reverberation effect. It therefore follows that the performance of DOA estimation for speech sources can be enhanced by integrating it with multichannel blind deconvolution techniques. We look forward to work in this direction for speech source localization problem.

Author's Publications

Journal papers

- [J1] V. V. Reddy, B. P. Ng, Z. Ying and Andy W. H. Khong, "DOA estimation of wide-band sources without estimating the number of sources", *Signal Process.*, vol. 92, no. 4, pp. 1032-1043, Apr. 2012.
- [J2] V. V. Reddy, B. P. Ng, and Andy W. H. Khong, "Derivative-Constrained Frequency-Domain Wideband DOA Estimation", accepted in *Multidim. Systems and Signal Process.*
- [J3] Wen Fuxi, B. P. Ng and V. V. Reddy, "Extending the concept of IIR filtering to array processing using approximate spatial IIR structure", *Multidim. Systems and Signal Process.*, issn: 0923-6082, pp. 1-23, 2011.
- [J4] V. V. Reddy, B. P. Ng, and Andy W. H. Khong, "Insights into MUSIC-like Algorithm", accepted for publication in *IEEE Trans. Signal Process.*
- [J5] V. V. Reddy, Andy W. H. Khong, and B. P. Ng, "Unambiguous Speech DOA Estimation under Spatial Aliasing Conditions", to be submitted to *IEEE Trans. Audio, Speech and Lang. Process.*

Conference papers

- [C1] V. V. Reddy, V. Divya, Andy W. H. Khong and B. P. Ng, "Footstep detection and denoising using a single triaxial geophone," *Circuits and Systems (APCCAS), IEEE Asia Pacific Conf.*, pp.1171-1174, Dec. 2010.
- [C2] V. V. Reddy, F. Sattar, B. P. Ng and P. F. Driessen, "Two-stage underdetermined speech source separation using frequency normalization," *Communications, Computers and Signal Process. (PacRim), IEEE Pacific Rim Conf.*, pp.648-653, Aug. 2011.
- [C3] V. Divya, V. V. Reddy, Andy W. H. Khong and B. P. Ng, "Polarization-cum-energy metric for footstep detection using vector-sensor," *Technologies for Homeland Security*

(*HST*), *IEEE Intl. Conf.*, pp.196-201, Nov. 2011.

[C4] V. Divya, V. V. Reddy and Andy W. H. Khong, "A study of the ambiguity problem in footstep bearing estimation using tri-axial geophone," *Information, Communications and Signal Process. (ICICS)*, 8th Intl. Conf. , pp.1-5, Dec. 2011.

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