Study of spatial power combining in closed waveguides

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Study of Spatial Power Combining in Closed Waveguides

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ABSTRACT

This thesis presents studies of spatial waveguide power combiners for communication systems. The previous work on spatial power combiners is reviewed thoroughly. After analyzing the pros and cons of all the existing structures, two types of spatial power combiners are proposed.

The first one is a new transitional structure from a rectangular waveguide to multiple microstrip lines for high-efficiency power combining applications. The fields of microstrip lines in a shielded rectangular waveguide are obtained by solving a set of homogeneous simultaneous equations. Mode-matching techniques are successfully implemented to analyze the N-furcated rectangular waveguide, the junction between rectangular waveguide and multiple microstrip lines, and the junction between multiple microstrip lines in one rectangular waveguide to multiple microstrip lines in multiple shielding waveguides. Calculated results are verified by the state-of-art field simulation software and excellent agreements are observed. The designed power combiner is fabricated and very good performance has been achieved.

The second one is an eight-way radial power combiner. Eight suspended substrate stripline (SSS) peripheral ports are used in the design to minimize the loss and realize a compact design. The field in a SSS is solved using a mode-matching analysis. Good experimental results are presented, which show very good return loss and insertion loss performance. Conclusions and recommendation for future works are given at the end of the thesis.

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LIST OF ACRONYMS

ABBREVIATIONS	FULL EXPRESSIONS
3D	Three-dimensional
DFT	Discrete Fourier transform
FET	Field effect transistor
GaAs	Gallium arsenide
HFSS	High Frequency Structure Simulator
IMPATT	IMPact ionization avalanche transit-time
MMIC	Monolithic microwave integrated circuit
MPIE	Mixed-potential integral-equation
PCB	Printed circuit board
SIE	Singular integral equations
SMA	SubMiniature version A
SSPA	Solid state power amplifier
SSS	Suspended substrate stripline
TDMA	Time division multiple access
TRL	Through, reflection, and line
TSA	Tapered slot antenna
TWTA	Traveling wave tube amplifier

CHAPTER 1

INTRODUCTION

1.1 Background

With the rapid growth of terrestrial and satellite communications in the 20th century, low radio frequency bands are occupied by various kinds of communication systems and resulting in heavy congestion. Improving spectrum efficiency can alleviate the current situation, but a better solution is to explore more spectrum resources in higher frequency bands, e.g., microwave and millimeter wave frequencies. The traditional way to transmit high power at these microwave frequencies is to utilize traveling wave tube amplifier (TWTAs). TWTAs, which can offer very high output power up to several kilowatts, have been the predominant choices in the early times of 20th century. Although TWTAs offer very high output power, the disadvantages of TWTAs are also very significant. TWTAs normally require very high operating voltages and a considerable warm-up time. TWTAs continually decrease output power as the tube ages and re-adjustment is required, which increases the maintenance difficulties and costs. Other disadvantages of TWTAs include their bulky size and poor linearity. With the improvement in technologies, a new kind of power device called solid state power amplifier (SSPA) have been replacing TWTAs since the early 1980's [1], [2]. SSPA devices provide many advantages compared to TWTAs. SSPAs have much more compact size and lower weight. They also have better immunity to shock and vibration. These advantages make them very suitable to be used in mobile communication applications. SSPAs can be switched on and off very fast since they need very less warm up time, which makes it possible to switch off the SSPA between burst of a TDMA signal for power saving and heat reduction. SSPAs also offer lower operating

1.1 Background

and maintenance costs, greater operating life, and better inter-modulation performance.

Although SSPA device offers so many advantages, one major problem of an SSPA device is that its output power is much smaller compared to the kW level power offered by TWTAs. To solve this problem, power combiners are widely used to combine the output power from many single SSPA devices, as shown in Figure 1.1. Figure 1.1a shows an SSPA device with chip-level power combiner and divider, while Figure 1.1b presents a 4-way waveguide power combiner with coaxial input ports. The size of an SSPA device is very small. It is therefore very difficult to squeeze a very high power amplifier into a very small chip because heat dissipation becomes a big problem. With a chip-level power combiner inside, the state-of-the-art SSPA device can now provide output power in several tens of watt in microwave and low millimeter wave frequencies. Obtaining higher output power from SSPA devices becomes the main driven force for microwave power combining to be the subjects of many investigations in recent years.

a) An SSPA device with chip-level combiner and divider.

1.2 Motivation

b) A 4-way waveguide power combiner with coaxial input ports fabricated by Universal Microwave Technology Inc.

Figure 1.1. Microwave power combiners

1.2 Motivation

Many types of power combiners have been studied in the previous literature [3], [4]. From the structure of the combiners, they can be classified into two-way power combiners and N-way power combiners. The traditional two-way power combiners include Wilkinson power combiner, 3-dB branch-line coupler, Rat-race coupler and waveguide couplers [5]. To obtain higher output power, multi-stage power combiners must be used to combine power from more SSPA devices. Tree (parallel) type and chain (series) type structures have been used to connect two-way power combiners to obtain higher combined power. With the increasing number of power devices to be combined, the loss of the two-way combiners and the transmission lines between combiners

1.3 Objectives

becomes significant. The combining efficiency is low when the number of devices is increased. An N-way power combiner can overcome these shortcomings by combining all the power devices in one step. Two major types of N-way combiners have been investigated in the recent years. They are circuit-level and space-level N-way combiners. Because of their low path loss and capability of combining a large number of power devices in one step, spatial power combiners have become very good choices for microwave and millimeter wave power combining applications. Researches are focused on the investigation of new and efficient structures, analysis method and general design procedures, combining efficiency analysis, bandwidth enhancement, heat dissipation, and graceful degradation.

1.3 Objectives

The main objective of this research is to develop low profile, high power, and high efficiency spatial power combiners for C and Ku band satellite communication applications. We will start with the review of existing structures for spatial power combiners and later present our new structures with better performance. A theoretical study will be conducted to analyze the structures and the calculated results will be verified by comparing simulation results and measurement ones.

1.4 Major Contribution of the Thesis

In this Ph. D research project on study of spatial power combining in closed waveguides, five major contributions have been made. Some of them have been published in international journals and conferences as listed in author's publications on page 121. The major contributions of the thesis are listed as follows:

1.4 Major Contribution of the Thesis

1. A thorough literature review of the previous research work on microwave power combiners has been conducted, through which we conclude that spatial combiner with closed structure is preferable in microwave power combining application. To minimize the loss, power must be combined in air. On the other hand, to integrate with planar two terminal power devices, input ports with planar PCB transmission line structure are preferred.
2. Various analytical and numerical approaches have been used to analyze the dispersion characteristics of a microstrip line in the previous literature. In this thesis, the hybrid mode analysis was employed to analyze the dispersion characteristics of shielded multiple microstrip lines for the first time. The numerical results are compared to the HFSS simulated ones and very good agreements have been achieved.
3. A new C-band eight-way power combiner structure is analyzed, fabricated and tested in this thesis. The proposed power combiner has eight microstrip lines as input ports so that it can be easily integrated with two terminal SSPA devices. The output port is a standard WR-137 rectangular waveguide with very low loss and high power handling capability. The power combiner has a tray type structure for easy assembly and this unique structure also helps to solve three major issues of the designs proposed by other researchers [60]-[66] previously. The power combiner is designed using the mode matching formulas derived in the thesis. Resistive loaded decoupling slots and microstrip decoupling networks are introduced to improve the isolation between eight microstrip line ports. Finally the design has been fabricated and very good measured results have been achieved.
4. The hybrid mode analysis has been used to analyze the suspended substrate

1.5 Organization of the Thesis

stripline successfully. The calculated results are verified by comparing with HFSS simulated results. The method is later employed to analyze the SSS ports of the eight-way radial power combiner.

5. An eight-way radial power combiner has been presented with coaxial output port and SSS peripheral ports. Mode analysis has been performed to calculate the field distribution of the input SSS ports. The combiner has a wide bandwidth from 12.1GHz to 15.7GHz, while the return loss is better than 19dB. The insertion of this combiner is measured to be better than 0.2dB. Compared to other types of radial power combiners, the proposed power combiner successfully integrates printed circuits transmission lines with a wideband 3D space combining structure, which results in high efficiency, wide bandwidth, and high power handling capability.

1.5 Organization of the Thesis

There are seven chapters in this thesis and they are organized as follows. In Chapter 1, we introduce the background of the research on microwave and millimeter wave power combiners, followed by the motivations of our research work on spatial power combiners. The objectives of our research and the organization of the dissertation are also presented in this chapter.

The literature review on microwave and millimeter power combiners is presented in Chapter 2. General power combining structures from circuit level combiners to spatial level combiners have been reviewed in this chapter. Spatial power combining technique has been reviewed as a separate section in this chapter.

1.5 Organization of the Thesis

To design a high efficiency transition from the rectangular waveguide to shielded microstrip lines for power combining application, we begin our study with the analysis of dispersion characteristics of multiple microstrip lines shielded by a rectangular enclosure in Chapter 3. Hybrid mode analysis is employed to analyze the structure. Both formulations and numerical results are presented.

Chapter 4 presents the mode-matching analysis of all the waveguide to waveguide, waveguide to microstrip lines, and microstrip lines to microstrip lines junctions. The formulations as well as extensive numerical results are presented and verified in this chapter.

Utilizing the formulas that are developed in the previous chapters, an eight-way power combiner is designed in Chapter 5. The design is then fabricated and measured. Experimental results of the fabricated power combiner with very good return loss and insertion loss are observed.

Chapter 6 presents the design of an eight-way radial power combiner. Mode analysis is performed to calculate the field distribution of the suspended substrate stripline (SSS) peripheral ports. Design details and measured results are presented. The proposed power combiner successfully integrates printed circuits transmission lines with 3D space combining structure, which results in high efficiency, wide bandwidth, and high power handling capability.

A summary of work conducted and recommendations for future work are provided in Chapter 7. A list of publications resulted from the conducted research is also given.

CHAPTER 2

LITERATURE REVIEW

2.1 Power Combiners

Power combining techniques can be used at microwave or millimeter wave frequencies to obtain higher combined output power that cannot be achieved using a single device. Russell reviewed the microwave power combining techniques in [3]. The millimeter wave power combining techniques are reviewed by Chang and Sun in [4]. There are many ways to classify the power combiners into different categories. According to the number of sources to combine in a single step, they can be classified into N-way combiners and two way combiners; according to the type of source, they can be separated into oscillator-combiners and amplifier-combiners, and according to the size and the location where the combining process occurs, they can be classified as circuit level combiners and spatial level combiners. In the rest of this section we will discuss all these types of combiners in detail.

2.1.1 Circuit Level and Spatial Level Combiners

Circuit level combiners utilize a variety of transmission line structures to combine the power from active devices. The most commonly used transmission lines include microstrip line, strip line, SSS and rectangular waveguide. Microstrip line, strip line and SSS power combiners are classified as planar printed circuit board (PCB) power combiners. The advantages of these planar combiners are their manufacturing flexibility because they can be built easily using the standard PCB process. Some traditional

2.1 Power Combiners

combiners which have been widely used in PCB applications are shown in Figure 2.1. Recent researches on planar combiners are focused on expanding the frequency bandwidth [6]-[9], dual-frequency applications [10]-[12], reducing the loss, minimizing the size and N-way combiner design [13]-[18]. Other PCB transmission line structures like substrate integrated waveguide (SIW) are also introduced in power divider/combiner applications recently [19]-[21].

Figure 2.1. Three types of PCB combiners.

The disadvantages of the PCB combiners are their high conduction and dielectric losses, especially when a large number of devices need to be combined. Rectangular waveguide couplers can be used to overcome these shortcomings because of the very low insertion loss of waveguide structures. Quadrature hybrid, magic-T and E-plane T junctions [22], [23] are widely used in high power combining applications. The disadvantages of these rectangular waveguide power combiners are their large size, manufacturing complexity, and high cost.

A spatial power combiner combines the power from devices in the space. A typical spatial power combiner radiates power in the free space by means of some antenna structures

2.1 Power Combiners

and then collects the power from the space using horn antenna. Since in most of the applications the power needs to be collected into a waveguide section for later usage, closed structures can also be used for power combining instead of open space. The combiners which directly radiate power to the waveguide structure are also called spatial power combiners [24], [25]. Closed waveguide structures have been used for power combining for many years. Many structures have been investigated during the past few decades. Some combiners use resonant type three-dimensional (3D) waveguide structures for power combining and the bandwidth is relative narrow. In this thesis, if non-resonant type 3D waveguide structures are employed for power combining, the combiner is considered as a spatial power combiner. Spatial power combiners will be analyzed in details in the next section.

2.1.2 N-way Combiners and Two-way Combiners

A two-way combiner combines power from two sources at one step. To combine power from more sources several two-way power combiners can be connected together. Two types of configurations can be used, serial (chain) and parallel (corporate) combining structures. A typical serial power combining structure can be found in [27]. As shown in Figure 2.2 a), for an N-stage power combiner, each coupler adds $1/N$ of the output power to the output. A parallel power combining structure [28] is shown in Figure 2.2 b). For two-way power combiners, the parallel combining structure is also called a corporate- or tree-type combining structure. An N-stage parallel power combining structure has $2^N - 1$ two-way combiners and can combine 2^N power devices. Both serial-type and corporate-type power combiners demonstrate wideband response. Serial type combiners can combine an arbitrary number of power sources while corporate combiners can only combine 2^N power devices. Corporate combiners can have identical combining units

2.1 Power Combiners

while in a serial structure all the couplers must have different coupling coefficients, and sometimes it is difficult to fabricate couplers with very high coupling coefficients, which are required for combining large numbers of power devices.

a) Serial power combining structure b) Parallel power combining structure
 Figure 2.2. Serial and parallel power combining structures for two-way power combiners.

The usage of two-way power combiners in combining large numbers of devices is limited by its efficiency. With the increase in the number of devices to be combined, more stages of combiners are required for both parallel- and serial-types of two-way power combiners and therefore the corresponding insertion loss becomes significant. The combiners are connected using different types of transmission lines and losses of these transmission lines can also be considerably high.

An N-way combiner is a combiner which combines the power from N sources together in one step. Although the N-way power combiners combine power in one step, they can still be classified into two types, e.g., serial combiners and parallel combiners. A serial combiner can be found in [29]. As shown in Figure 2.3, six groups of total twelve IMPATT diodes are located in the rectangular waveguide. The interspacing between the

2.1 Power Combiners

groups are $\lambda/2$. The power generated by the diodes is combined in series. Two types of parallel combiners are investigated in the pass. They are radial combiners and those combiners that are really parallel in structure. Most of the spatial combiners are of the latter type and can be further classified as tray-type and tile-type combiners. The spatial power combiners will be reviewed in detail in the next section and we will put emphasis on the radial power combiners here.

Figure 2.3. A serial rectangular waveguide combiner.

The radial power combiners can be classified into resonant-type combiners and non-resonant-type combiners. A resonant-type cylindrical power combiner was first presented to combine power from IMPATT diodes by Harp and Stover [30] in 1973. Later cylindrical resonant combiners were investigated by many researchers [31]-[38]. The resonant radial combiners demonstrate very good combining efficiencies with relative narrow frequency bandwidth. Most of the researches are focused on increasing the bandwidth of the combiners, and combining more numbers of diodes. Aston used circuit model to identify circuit parameters that increase the bandwidth of the power combiner [33]. Tokumitsu et al. [34] used a double-cavity structure to broaden the operating bandwidth of the combiner. By using this structure, an experimental 6GHz 80W GaAs FET amplifier with a combining efficiency of 85% was demonstrated. Higher order cavity resonance modes are employed to combine power from more diodes in the

2.1 Power Combiners

cylindrical cavity [35]-[38]. Mode analytical treatment of cylindrical cavity power combiners was presented by Fukui and Nogi in [39].

Compared to the resonant radial power combiner, a non-resonant radial power combiner has wider bandwidth. Many types of transmission lines have been used in the radial power combiners, which include ridged waveguide [40], double-ridged waveguide [41], parallel plate radial line [42]-[45], and microstrip line [46]. Swift and Stones [47] used techniques similar to those used for distributed element filters to design the radial wave power combiner at L-band. Chen et al. [48] hybridized the mixed-potential integral-equation (MPIE) formulation with the idea of discrete Fourier transform (DFT) to present a full-wave analysis for the microstrip line radial power combiner. A detailed design procedure of radial power combiners utilizing HFSS for optimization was presented in [49] by Fathy et al. For those structures which utilize dielectric substrate and printed transmission lines, the design is flexible but the dielectric loss is inevitable, which also limits the power handling capacity of the combiners. Another great challenge in the design of non-resonant radial power combiners is the impedance matching of the center combining port. The matching structures normally require very tight mechanical tolerance, which results in difficulties in fabrication and limits their applications at higher frequencies. Recently, de Villers et al. presented their conical transmission line power combiners in [50], [51]. Since the characteristic impedance of the parallel plate radial line varies with the radius of the plate, a conical radial transmission line is employed in the design for better impedance matching. A matched bandwidth of 74% is achieved [50] with a maximum return loss of 14.7dB from 6.5 to 14.1GHz. The maximum in-band loss is around 1.1dB. Better performance was achieved in [51] by introducing tapered line matching sections. Broadband probes were implemented to improve the overall bandwidth in [53].

2.2 Spatial Power Combining Technique

2.1.3 Oscillators and Amplifiers

The microwave or millimeter wave power sources to be combined can be either oscillators or amplifiers. The combined power from oscillators can be well used in microwave or millimeter-wave radar applications, while communication systems require for low noise and high stability power that carrying information, which can only be generated by amplifiers. The active devices to be used in the oscillators are IMPATT diodes, Gunn diodes and three-terminal grid oscillators. For amplifier combining, two power combiners/dividers must be employed: one distributes the power from the previous amplifier stage to power amplifiers and the other combines the power to the output port.

2.2 Spatial Power Combining Technique

2.2.1 Brief Introduction

Solid state power devices are often used to deliver power to guided wave structures, such as microstrip lines. A circuit level combiner combines power together in guided wave structures, while a spatial power combiner transfers the power into free space by means of some radiating structures and the power combining takes place in the free space. Compared to conventional power combiners with corporate structures, the main advantages of the spatial power combiners are their high efficiency and high power handling capacity. A detailed review on quasi-optical and spatial power combining techniques is presented by Delisio and Robert in [54].

The structures of the spatial power combiners can be categorized into two types: open space structure and closed structure. The early research on spatial power combiners were

2.2 Spatial Power Combining Technique

started with open space structures. Many open space power combiners have been investigated [55]-[60]. The first spatial power combiner was presented by Staiman *et al* [55]. A hundred transistor amplifiers were combined successfully to get a net gain of 4.75dB at 410MHz with 100 watts output. Later, the concept of spatial power combining was used in combining power from various oscillators. Mink [56] introduced a Fabry-Perot resonator to combine signals from different oscillators. Injection locking technique was introduced to control the frequency of the resonator [57]. In many applications, the power generated by the oscillators needs to be collected at an output port to be reused instead of radiating into the free space. The power generated by oscillators is radiated using antenna arrays and then lenses are used to focus the beam and horn antennas are used to collect the power.

There are two types of configurations for amplifier combiners: tile configuration and tray configuration, as shown in Figure 2.4. Tile configuration combiners use planar antenna array to radiating power to free space. One advantage of the tile configuration is its planar structure, which can be easily fabricated and very large number of active devices can be integrated using monolithic manufacturing techniques. The disadvantages of the tile configuration are that the bandwidth is limited due to the usage of resonant type antenna array and the space limitation when multistage amplifiers have to be implemented. Tray configuration combiners use wide band end-fire antennas as their radiating elements. The most commonly used antennas are tapered slot antennas and quasi-Yagi antennas. To combine power from amplifiers, two antennas arrays must be employed, one for receiving power from the previous stage and one for radiating power to free space for combining. To create better isolation between the input and output ports of the amplifiers, orthogonal polarization is used usually.

2.2 Spatial Power Combining Technique

Figure 2.4. Tile and tray configuration of power combiners.

At millimeter wave frequencies, lens and horn antennas can be used to collect the power. For lower frequencies in the microwave range, closed structures are desired because of their compact size. In many literatures, those combiners radiating power to a closed structure instead of free space are also called spatial combiners. However, resonant cavity combiners have a major difference compared to the spatial combiners. In this dissertation, we only consider those non-resonant N-way combiners with air substrate as spatial combiners.

2.2.2 Spatial Power Combiners with Closed Structure

Three major types of spatial power combiners with closed structures have been investigated in recent years. They are radial power combiners or conical power combiners, as discussed in 2.1.2, TSA or finline rectangular waveguide combiners [61]-[67], and over-sized coaxial waveguide combiners [61], [72]-[77].

2.2 Spatial Power Combining Technique

The structure of an array of tapered slotlines inserted between two rectangular waveguides was first introduced by Alexanian and York in [61]. A broadband performance can be realized because of the usage of wide band end-fired tapered slot antennas (TSAs). The structure of the combiner is illustrated in Figure 2.5, where a TSA array is placed between two rectangular waveguides. To integrate with three terminal SSPA devices, a transition from slotline to microstrip is employed. The TSA is built on 10mil thick Aluminum Nitride substrate to guarantee the low loss and good thermal performance for removing the heat generated by the SSPA devices. A 2.4Watt RF output is achieved with combining efficiency of 70% at X-band.

Figure 2.5. Topology of waveguide spatial combiner

Later, the structure was continuously improved by Cheng et al. to realize 20 Watt [63], 40 Watt [24], and finally 120 Watt X-band combiner [64]. The main efforts devoted by Cheng et al. are to improve the heat sinking, characterize and optimize passive structures, and integrate with more high-performance MMIC amplifiers. In the 20 watt design [63], the antenna cards were mounted onto a metallic test fixture for both mechanical support

2.2 Spatial Power Combining Technique

and heat removal. Thermal grease was applied between antenna cards and the test fixture to enhance the heat sinking property of the combiner. In the 40 Watt design presented in [24], a fully closed structure is employed to provide better insertion loss and better heat dissipation. The influence of non-uniform field profile was also analyzed. According to the analysis, the total effect of the input and output non-uniform field distribution only accounts for 5% power drop. An averaging 73% of combining efficiency for the four-tray combiner is estimated. The final 120 Watt X-band combiner was realized in [64] with a total of 24 MMIC amplifier devices to be combined. Six antenna cards with 4 MMIC amplifier devices on each card were put into the waveguide for combining. 126 Watt maximum output power was produced successfully. The characteristics of graceful degradation are also measured. The combiner circuit loses about 57% of original output power when half of the devices are turned off. The return loss of the passive six-tray is only 7.5dB. Oscillations were also found when the combiner was driven with weak input power. Later Jia et al. [25] established a design procedure to optimize the return loss of the array based on an extension of the Klopfenstein optimal taper design to a non-TEM waveguiding structures. The spectral-domain method was employed to compute the propagation constants in the array structure. An improved version of waveguide combiner was presented and analyzed in [65] and [66], which employed a double antipodal finline to microstrip transition structure to eliminate the bonding wire from slotline to microstrip line. The structure allowed better efficiency and repeatability and was successfully used in 24GHz, 1.6 Watt and 3.3 W power amplifier modules.

Since the electric profile of TE_{10} mode across the waveguide is not uniform, the active devices between the divider and combiner are driven differently. This will affect the combining efficiency and over linearity performance of the power amplifier. The TEM-mode waveguide concept was introduced to overcome this problem [68]. By using

2.2 Spatial Power Combining Technique

a uniplanar compact electromagnetic bandgap (UC-EBG) structure, a relatively uniform field distribution along the waveguide was obtained.

To accommodate for more active devices at high frequency, an over-sized rectangular waveguide was used in an 18 to 22GHz power combiner application [69]. Because of the usage of oversized waveguide, TE_{20} mode will propagate because of the non-ideality of the structure. Recently, a mode transformer using finline was proposed to convert TE_{10} mode to TE_{20} mode for over-sized rectangular waveguide spatial power combiner applications [70]. With this mode converter, the undesired modes are suppressed effectively.

The structure of a conical waveguide or an over-sized coaxial waveguide structure was first introduced by Quine [71] for power combining usage. The early researches are focused on combining power from two terminal devices. The structure of a TSA array in an oversized coaxial waveguide was first proposed by Alexanian and York in [61]. The structure is shown in Figure 2.6. Two main advantages of this structure compared to the rectangular waveguide combiner are claimed. First, since the excitation of a rectangular waveguide is non-uniform, the efficiency and saturation characteristics of the system are reduced when a TSA array is used in the rectangular waveguide for power combining. The oversized coaxial waveguide uses a uniform TEM wave to excite the TSA array and overcome the problem. Secondly, the TEM waveguide guarantees much wider frequency performance compared to the dispersive rectangular waveguide structure. The structure was later analyzed by Jia et al. in [72]-[74]. A 32-MIC system was built to demonstrate the wideband performance of this combining structure. Spectral-domain method was used to model the passive structure. 75% combining efficiency was achieved over 13GHz bandwidth.

2.3 Concluding Remark

Figure 2.6. An oversized coaxial waveguide combiner using a dense finline array.

Some variations of the oversized coaxial waveguide power combiners have been investigated recently [45], [75]-[77]. A low frequency version of oversized coaxial waveguide power combiner was analyzed by Amjadi and Jafari [75], which was used in the frequency range of 520 to 1860MHz. A coaxial waveguide ten-way combiner was presented by Song and Xue in [45]. Planar probes are utilized as peripheral ports in the Ku-band ten-way combiner. With the planar probe input structure, the combiner has perfect compatibility with the standard microwave integrated circuitry. Compromising on frequency bandwidth has been made. The design was improved and applied to a Ka-band 4-way combiner [76] and U-band 32-way combiner [77] later.

2.3 Concluding Remark

Power combining techniques in microwave and millimeter wave frequencies have been reviewed. Serial or parallel connected 2-way power combiners are found to be not efficient when a large number of active devices are to be combined. Planar combiner structures using microstrip line combiners on dielectric substrates suffer from dielectric loss and conductive loss. Spatial combining technique is found to be a promising

2.4 Our Approaches

combining technique to combine a large number of active devices with high power combining efficiency. Open-space and closed structures are reviewed for spatial combining applications. At microwave frequencies, a closed structure is preferred because of its high efficiency, compact structure and low power leakage. To be compatible with two terminal SSPA devices, the design of excellent transition from the open space to multiple planar PCB transmission lines is critical for good overall combining performance.

2.4 Our Approaches

As summarized in Section 2.3, spatial combiner with closed structure is preferable in microwave power combining application. To minimize the loss, power must be combined in air. On the other hand, to integrate with planar two terminal power devices, input ports with planar PCB transmission line structure are preferred.

Bearing these basic ideas in mind, two microwave power combiners have been proposed in this thesis. The first one is an eight-way C-band power combiner with eight microstrip line input ports and one rectangular waveguide output port. A transition from two microstrip lines to one reduced-height rectangular waveguide is designed first. Four reduced-height rectangular waveguides are then combined together to one waveguide. Since E-plane combining was used in the structure, the uniformity of electric field distribution of TE_{10} mode along the waveguide is not a problem anymore. The field in the shielded microstrip lines is solved using hybrid mode analysis and the whole structure is analyzed and optimized using mode-matching method. Additional resistor loaded slots and microstrip structures are added to improve isolation between input microstrip line ports. Compared to other people's designs, this design demonstrates lower loss, wider

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band, better power linearity and better isolation between input ports.

The second design is a Ku-band eight-way radial power combiner. The combiner consists of a junction between a centered coaxial port and a large circular waveguide, a transition between a circular waveguide and eight SSS ports, and a circular waveguide section between these two transitions. Mode analysis has been employed to calculate the field distribution of the input SSS ports. The bandwidth of this combiner is measured to be 3.6GHz from 12.1GHz to 15.7GHz, with return loss better than 19dB. Compared to other types of radial power combiners, the proposed power combiner successfully integrates PCB transmission lines with a 3D space combining structure, which results in high efficiency, wide bandwidth, high power handling capability and good compatibility with the standard microwave integrated circuitry.

CHAPTER 3

DISPERSION ANALYSIS OF SHIELDED MULTIPLE MICROSTRIP LINES

Microstrip line was first introduced for microwave transmission and components in the 1950s [78]-[80]. Compared to the waveguide or coaxial structures, microstrip line has its advantages of light weight and compact size, which is particularly suitable to be used to construct compact, rugged, and inexpensive microwave components. Early studies on microstrip lines were based on TEM analysis [81]-[84]. With the increasing demands of using microstrip lines at microwave and millimeter wave frequencies, the dispersion characteristics of the microstrip lines becomes more and more important. Not only the dominant mode, but also the higher-order modes should be considered to obtain accurate results. Many authors have contributed to the dispersion analysis of a microstrip line through various analytical and numerical approaches. These approaches include the method of moments [85], spectral domain technique [86], [87], and the hybrid mode analysis [88]-[90]. Among all of the approaches, the hybrid mode analysis [88], [89] is a rigorous and accurate procedure to solve the dispersion characteristics of the microstrip lines. The electric and magnetic fields in the structure of interest are expressed in terms of finite numbers of TE and TM modes first, followed by applying the boundary conditions and matching the fields at substrate-air interface, an infinite set of homogeneous simultaneous equations can be formed. By searching the zeros of the determinant of the subsequent matrix, the propagation constant can be obtained. The numerical efficiency can be improved by transferring the equations into an auxiliary set of equations and solving it by the singular integral equations (SIE), as proposed by Mittra [90]. Dispersion

3.1 Formulation

and characteristics impedances of other variation structures like coupled microstrip lines and shielded microstrip lines were also investigated [91]-[93]

In this chapter, the dispersion characteristics of shielded multiple microstrip lines are studied using the hybrid mode analysis. The coefficients of the hybrid modes are then obtained. Numerical results are compared with those obtained by EM simulation software HFSS.

3.1 Formulation

The cross section of a shielded microstrip line with multiple strips is shown in Figure 3.1. The height and width of the shielding rectangular waveguide is h and l , respectively. Inside the shielding, Region 1 is filled with air and Region 2 is filled with dielectric material with relative permittivity of ϵ_r . The thickness of the dielectric material is d . There are totally I microstrip lines in the rectangular shielding. The distance from the i th microstrip line to the left shielding wall is s_{li} , and the width of the i th microstrip line is $t_i = s_{2i} - s_{1i}$.

Figure 3.1. Cross section of a shielded multiple microstrip line.

3.1 Formulation

The electric and magnetic fields in the shielded microstrip line can be expressed by hybrid modes, which are combinations of TE and TM modes.

Since the tangential electric field must vanish at the PEC boundaries at $x=0$, $x=l$, $y=0$ and $y=h$, it is appropriate to write the Hertzian potentials of TE modes and TM modes in Regions 1 and 2 as

$$\psi_1^{(e)} = \sum_{n=1}^{\infty} A_n^{(e)} \sinh(\alpha_n^{(1)} y) \sin(k_n x) e^{-j\beta z} \hat{a}_z, \quad (3.1)$$

$$\psi_1^{(h)} = \sum_{n=0}^{\infty} A_n^{(h)} \cosh(\alpha_n^{(1)} y) \cos(k_n x) e^{-j\beta z} \hat{a}_z, \quad (3.2)$$

$$\psi_2^{(e)} = \sum_{n=1}^{\infty} B_n^{(e)} \sinh[\alpha_n^{(2)} (h-y)] \sin(k_n x) e^{-j\beta z} \hat{a}_z, \quad (3.3)$$

$$\psi_2^{(h)} = \sum_{n=0}^{\infty} B_n^{(h)} \cosh[\alpha_n^{(2)} (h-y)] \cos(k_n x) e^{-j\beta z} \hat{a}_z, \quad (3.4)$$

where

$$\alpha_n^{(1)} = \sqrt{k_n^2 + \beta^2 - \epsilon_r k_0^2}, \quad (3.5)$$

$$\alpha_n^{(2)} = \sqrt{k_n^2 + \beta^2 - k_0^2}, \quad (3.6)$$

$$k_n = \frac{n\pi}{l}, \quad (3.7)$$

$$\text{and } n = 0 \text{ to } \infty. \quad (3.8)$$

The electric and magnetic fields can then be readily obtained by utilizing the following equations,

$$\mathbf{E} = \nabla \times \nabla \times \psi_e - j\omega\mu \nabla \times \psi_h. \quad (3.9)$$

3.1 Formulation

$$H = \nabla \times \nabla \times \psi_h + j\omega\epsilon \nabla \times \psi_e. \quad (3.10)$$

Matching the E_z components on the air-dielectric interface, where E_z field can be derived from (3.9) and (3.10), we get

$$\begin{aligned} & \sum_{n=1}^{\infty} A_n^{(e)} k_n^2 \sinh(\alpha_n^{(1)} y) \sin(k_n x) - \sum_{n=1}^{\infty} A_n^{(e)} \alpha_n^{(1)2} \sinh(\alpha_n^{(1)} y) \sin(k_n x) \\ &= \sum_{n=1}^{\infty} B_n^{(e)} k_n^2 \sinh[\alpha_n^{(2)} (h-y)] \sin(k_n x) - \sum_{n=1}^{\infty} B_n^{(e)} \alpha_n^{(2)2} \sinh[\alpha_n^{(2)} (h-y)] \sin(k_n x) \end{aligned} \quad (3.11)$$

From (3.11), $B_n^{(e)}$ can be derived as

$$B_n^{(e)} = A_n^{(e)} \frac{(\epsilon_r - \bar{\beta}^2) \sinh(\alpha_n^{(1)} d)}{(1 - \bar{\beta}^2) \sinh[\alpha_n^{(2)} (h-d)]}, \quad (3.12)$$

where

$$\bar{\beta} = \frac{\beta}{k_0}. \quad (3.13)$$

Similarly, by matching the E_x components, $B_n^{(h)}$ can be expressed in terms of $A_n^{(e)}$ and $A_n^{(h)}$ as follows.

$$B_n^{(h)} = A_n^{(e)} \frac{\beta k_n}{\omega \mu \alpha_n^{(2)}} \frac{(\epsilon_r - 1)}{(1 - \bar{\beta}^2)} \frac{\sinh(\alpha_n^{(1)} d)}{\sinh[\alpha_n^{(2)} (h-d)]} - A_n^{(h)} \frac{\alpha_n^{(1)} \sinh(\alpha_n^{(1)} d)}{\alpha_n^{(2)} \sinh[\alpha_n^{(2)} (h-d)]} \quad (3.14)$$

Since the H fields on the air-dielectric interface must be continuous and the E_x and E_z components must vanish on the microstrip lines, the following four equations must be satisfied on $y=d$.

$$E_{z1} = 0 \quad x \in s_a \quad (3.15)$$

$$H_{x1} = H_{x2} \quad x \in s_b \quad (3.16)$$

3.1 Formulation

$$E_{x1} = 0 \quad x \in s_a \quad (3.17)$$

$$H_{z1} = H_{z2} \quad x \in s_b \quad (3.18)$$

where $x \in s_a$ means $s_{l1} < x < s_{r1}$, $s_{l1} < x < s_{r1}, \dots, s_{li} < x < s_{ri}, \dots, s_{li} < x < s_{ri}$,

$x \in s_b$ means $0 \leq x \leq L$ and $x \notin s_a$.

Equations (3.12)-(3.18) lead to four sets of equations as follows,

$$\sum_{n=1}^{\infty} \bar{A}_n^{(e)} Q_{1e}(\beta) \sin(k_n x) = 0 \quad x \in s_a \quad (3.19)$$

$$\sum_{n=1}^{\infty} \bar{A}_n^{(e)} Q_{2e}(\beta) \sin(k_n x) + \sum_{n=0}^{\infty} \bar{A}_n^{(h)} Q_{2h}(\beta) \sin(k_n x) = 0 \quad x \in s_b \quad (3.20)$$

$$\sum_{n=1}^{\infty} \bar{A}_n^{(e)} Q_{3e}(\beta) \cos(k_n x) + \sum_{n=0}^{\infty} \bar{A}_n^{(h)} Q_{3h}(\beta) \cos(k_n x) = 0 \quad x \in s_a \quad (3.21)$$

$$\sum_{n=1}^{\infty} \bar{A}_n^{(e)} Q_{4e}(\beta) \cos(k_n x) + \sum_{n=0}^{\infty} \bar{A}_n^{(h)} Q_{4h}(\beta) \cos(k_n x) = 0 \quad x \in s_b. \quad (3.22)$$

where

$$\bar{A}_n^{(e)} = A_n^{(e)} \sinh(\alpha_n^{(1)} d) \quad (3.23)$$

$$\bar{A}_n^{(h)} = A_n^{(h)} \omega \mu \sinh(\alpha_n^{(1)} d) \quad (3.24)$$

$$Q_{1e}(\beta) = k_0^2 (\varepsilon_r - \bar{\beta}^2) \quad (3.25)$$

$$Q_{1h}(\beta) = 0 \quad (3.26)$$

$$Q_{2e}(\beta) = \left(\begin{aligned} &\varepsilon_r \alpha_n^{(1)} \coth(\alpha_n^{(1)} d) + \frac{\alpha_n^{(2)} (\varepsilon_r - \bar{\beta}^2)}{(1 - \bar{\beta}^2)} \coth[\alpha_n^{(2)} (h - d)] \\ &+ \frac{\bar{\beta}^2 k_n^2 (1 - \varepsilon_r)}{\alpha_n^{(2)} (1 - \bar{\beta}^2)} \coth[\alpha_n^{(2)} (h - d)] \end{aligned} \right) \quad (3.27)$$

3.1 Formulation

$$Q_{2h}(\beta) = \frac{\bar{\beta}}{k_0} k_n \left(\coth(\alpha_n^{(1)} d) + \frac{\alpha_n^{(1)}}{\alpha_n^{(2)}} \coth[\alpha_n^{(2)} (h-d)] \right) \quad (3.28)$$

$$Q_{3e}(\beta) = \beta k_n \quad (3.29)$$

$$Q_{3h}(\beta) = \alpha_n^{(1)} \quad (3.30)$$

$$Q_{4e}(\beta) = \frac{\beta k_n}{\alpha_n^{(2)}} (1 - \varepsilon_r) \coth[\alpha_n^{(2)} (h-d)] \quad (3.31)$$

$$Q_{4h}(\beta) = \left((\varepsilon_r - \bar{\beta}^2) \coth(\alpha_n^{(1)} d) - \frac{\alpha_n^{(1)} (\bar{\beta}^2 - 1)}{\alpha_n^{(2)}} \coth[\alpha_n^{(2)} (h-d)] \right) \quad (3.32)$$

For both sides of equations (3.19) and (3.20), taking inner product with $\sin(k_m x)$, adding them together, integrating over the whole intersection area, and using N truncation instead of infinite number of modes, we obtain

$$\sum_{n=1}^N \bar{A}_n^{(e)} w_1^{(e)}{}_{mn} + \sum_{n=0}^N \bar{A}_n^{(h)} w_1^{(h)}{}_{mn'} = 0 \quad m=1 \text{ to } N, \quad n' = n+1 \quad (3.33)$$

Similarly, for (3.21) and (3.22), taking inner product with $\cos(k_m x)$ and taking integration results in

$$\sum_{n=1}^N \bar{A}_n^{(e)} w_2^{(e)}{}_{mn} + \sum_{n=0}^N \bar{A}_n^{(h)} w_2^{(h)}{}_{mn'} = 0 \quad m=0 \text{ to } N, \quad n' = n+1 \quad (3.34)$$

where,

$$w_1^{(e,h)}{}_{mn'} = \begin{cases} Q_{1(e,h)}(\beta) \left(-\frac{\sin((k_n + k_m)s_2) - \sin((k_n + k_m)s_1)}{2(k_n + k_m)} + \frac{\sin((k_n - k_m)s_2) - \sin((k_n - k_m)s_1)}{2(k_n - k_m)} \right) \\ + Q_{2(e,h)}(\beta) \left(-\frac{\sin((k_n + k_m)L) - \sin((k_n + k_m)s_2) + \sin((k_n + k_m)s_1)}{2(k_n + k_m)} \right. \\ \left. + \frac{\sin((k_n - k_m)L) - \sin((k_n - k_m)s_2) + \sin((k_n - k_m)s_1)}{2(k_n - k_m)} \right) & n \neq m, \\ Q_{1(e,h)}(\beta) \left(-\frac{\sin(2k_n s_2) - \sin(2k_n s_1)}{4k_n} + \frac{s_2 - s_1}{2} \right) \\ + Q_{2(e,h)}(\beta) \left(-\frac{\sin(2k_n L) - \sin(2k_n s_2) + \sin(2k_n s_1)}{4k_n} + \frac{L - s_2 + s_1}{2} \right) & n = m \end{cases} \quad (3.35)$$

3.1 Formulation

$$w_2^{(e,h)}_{m'n'} = \begin{cases} Q_{3(e,h)}(\beta) \left(\frac{\sin((k_n + k_m)s2) - \sin((k_n + k_m)s1)}{2(k_n + k_m)} + \frac{\sin((k_n - k_m)s2) - \sin((k_n - k_m)s1)}{2(k_n - k_m)} \right) \\ + Q_{4(e,h)}(\beta) \left(\frac{\sin((k_n + k_m)L) - \sin((k_n + k_m)s2) + \sin((k_n + k_m)s1)}{2(k_n + k_m)} \right. \\ \left. + \frac{\sin((k_n - k_m)L) - \sin((k_n - k_m)s2) + \sin((k_n - k_m)s1)}{2(k_n - k_m)} \right) & n \neq m \\ \left\{ \begin{aligned} & Q_{3(e,h)}(\beta) \left(\frac{\sin(2k_n s2) - \sin(2k_n s1)}{4k_n} + \frac{s2 - s1}{2} \right) \\ & + Q_{4(e,h)}(\beta) \left(\frac{\sin(2k_n L) - \sin(2k_n s2) + \sin(2k_n s1)}{4k_n} + \frac{L - s2 + s1}{2} \right) \end{aligned} \right. & n = m \neq 0 \\ Q_{3(e,h)}(\beta)(S2 - S1) + Q_{4(e,h)}(\beta)(L - S2 + S1) & n = m = 0 \end{cases} \quad (3.36)$$

for $w_1^{(e)}_{m'n'}$, $m' = m = 1$ to N , $n' = n = 1$ to N ,

for $w_1^{(h)}_{m'n'}$, $m' = m = 1$ to N , $n = 0$ to N , and $n' = n + 1$,

for $w_2^{(e)}_{m'n'}$, $m = 0$ to N , $m' = m + 1$, and $n' = n = 1$ to N ,

and for $w_2^{(h)}_{m'n'}$, $m = 0$ to N , $m' = m + 1$, $n = 0$ to N , and $n' = n + 1$.

Equations (3.33) and (3.34) can be re-written into the following matrix form

$$\begin{bmatrix} w_1^{(e)} & w_1^{(h)} \\ w_2^{(e)} & w_2^{(h)} \end{bmatrix} \begin{bmatrix} \bar{A}^{(e)} \\ \bar{A}^{(h)} \end{bmatrix} = \mathbf{W} \begin{bmatrix} \bar{A}^{(e)} \\ \bar{A}^{(h)} \end{bmatrix} = 0, \quad (3.37)$$

where $\mathbf{W}$ is a $2N+1$ by $2N+1$ matrix. For the existence of non-zero values of $\bar{A}^{(e)}$ and $\bar{A}^{(h)}$, the determinant of matrix $\mathbf{W}$ must vanish. Sweeping β and by searching for the zeros of the determinant of $\mathbf{W}$ the correct propagation constant can finally be obtained. However, with a large matrix size, the determinant value can be extremely large or small, which will cause numerical overflow. Instead, we use singular value decomposition [94] as follows

$$\mathbf{W} = \mathbf{U} * \mathbf{S} * \mathbf{V}'. \quad (3.38)$$

By checking the minimum value of the diagonal element of $\mathbf{S}$ matrix, the propagation

3.2 Numerical Results

constant can be determined. The coefficients $\bar{A}^{(e)}$ and $\bar{A}^{(h)}$ can also be determined from the last column of $\mathbf{V}$ matrix.

3.2 Numerical Results

Figure 3.2 presents the computed propagation constants of the first five modes in a shielded microstrip line. The geometric parameters of the structure are $l=6\text{mm}$, $s_1=2.5\text{mm}$, $s_2=3.5\text{mm}$, $d=1\text{mm}$, and $h=5\text{mm}$. The relative permittivity of the substrate is $\epsilon_r=2.2$. Two hundred modes are used in the calculation. Results obtained by HFSS are also presented for verification. The solid lines in the figure are for our calculated results and the circles represent the results obtained using HFSS. It can be seen that both results are in good agreement.

Figure 3.2. Propagation constants of the first five modes vs. frequency ($l=6\text{mm}$, $s_1=2.5\text{mm}$, $s_2=3.5\text{mm}$, $d=1\text{mm}$, $h=5\text{mm}$, and $\epsilon_r=2.2$).

3.2 Numerical Results

The convergence or efficiency of our method is examined in Table 3.1. The propagation constants of the first five modes are presented with different numbers of modes used in our calculation. The numbers of modes are from ten to five hundred. The frequency is 1GHz and the structure is the same as that shown in Figure 2.1. In the table, N/A represents the situation where it cannot give proper result. HFSS results are also provided for comparison. It can be seen from the table that for the dominant mode, our method cannot provide correct results with less than 50 modes. Over 200 modes, correct result can be obtained. Unlike the dominant mode, the higher-order modes converge faster. This is because most of the higher-order modes are basically waveguide modes, which are disturbed by the centered microstrip line. They can be represented by TE or TM modes very well.

Table 3.1. Propagation constants of the first five modes vs. number of modes been used in the calculation ($L=6\text{mm}$, $s_1=2.5\text{mm}$, $s_2=3.5\text{mm}$, $d=1\text{mm}$, $h=5\text{mm}$, $\epsilon_r=2.2$ and frequency=1GHz).

	10 modes	20 modes	50 modes	100 modes	200 modes	300 modes	400 modes	500 modes	HFSS
Mode 1	N/A	N/A	N/A	28.97	28.30	28.07	27.95	27.87	27.54
Mode 2	-523.09i	-523.09i	-523.09i	-523.09i	-523.09i	-523.09i	-523.09i	-523.09i	-523.1i
Mode 3	-627.21i	-624.74i	-623.38i	-622.69i	-622.26i	-622.01i	-622i	-622i	-621.7i
Mode 4	-817.57i	-817.56i	-817.53i	-817.52i	-817.51i	-817.51i	-817.5i	-817.5i	-817.53i
Mode 5	-931.12i	-924.75i	-920.17i	-918.32i	-917.27i	-916.9i	-916.71i	-916.59i	-916.14i

The width of the microstrip line also affects the speed of convergence. The convergence speed vs. width of the microstrip line is presented in Table 3.2. It can be observed from

3.2 Numerical Results

the table that when the trace width is smaller than the substrate thickness, reducing the trace width can improve the convergence speed. When the trace width is larger than the substrate thickness, the structure with a wider trace can converge more rapidly. The results show that the speed of convergence depends on whether the dominant microstrip mode can be well represented by less numbers of TE or TM modes that are used in our calculation.

Table 3.2. Convergence speed of the dominant mode of a shielded microstrip line with different trace width ($l=6\text{mm}$, $h=5\text{mm}$, $d=1\text{mm}$, $\epsilon_r=2.2$, microstrip in the center of the waveguide, and frequency=1GHz).

t(mm)		100 modes	200 modes	300 modes	400 modes	500 modes
0.5	$\beta(\text{m}^{-1})$	28.8019	28.2874	28.1057	28.0089	27.9501
	$\Delta\beta/\beta(\%)$		1.818831	0.646488	0.345604	0.210375
1	$\beta(\text{m}^{-1})$	28.9654	28.3032	28.0727	27.9523	27.876
	$\Delta\beta/\beta(\%)$		2.339665	0.821082	0.430734	0.273712
2	$\beta(\text{m}^{-1})$	28.7762	28.3454	28.1932	28.114	28.065
	$\Delta\beta/\beta(\%)$		1.519823	0.539846	0.28171	0.174595
3	$\beta(\text{m}^{-1})$	28.723	28.3824	28.2604	28.1979	28.1591
	$\Delta\beta/\beta(\%)$		1.200039	0.431699	0.221648	0.137788

The effects of the shielding rectangular waveguide walls are shown in Figure 3.3. It can be seen from figure that when the distance between the microstrip line and the top or side waveguide wall are larger than four times the thickness of the substrate, the waveguide wall has much less effect on the propagation constant of the microstrip line. With the wall of the waveguide further approaching the microstrip line, the propagation constant drops. The side wall affects the propagation constant a lot when the distance between the side

3.2 Numerical Results

wall and the microstrip line is less than two times of the substrate thickness.

Figure 3.3. Propagation constant of the dominant mode in a shielded microstrip line vs. width and height of the shielding rectangular waveguide ($h=5\text{mm}$, $d=1\text{mm}$, $t=0.5\text{mm}$, $\epsilon_r=2.2$, microstrip in the center of the waveguide, and frequency=1GHz).

The structure with two coupled microstrip lines shielded by one rectangular waveguide is also considered. The calculated E-field patterns of the first two propagating modes are presented in Figure 3.4. The geometric parameters are based on Figure 3.1. From the E-field presented in Figure 3.4 a) it can be seen that the first propagating mode is an even mode. The second mode can be identified as an odd mode from the E-field shown in Figure 3.4 b).

3.2 Numerical Results

a) E-field of the first propagating mode

b) E-field of the second propagating mode

Figure 3.4. E-field of the first two modes in a structure of two microstrip lines shielded by a rectangular waveguide ($h=3\text{mm}$, $d=1\text{mm}$, $\epsilon_r=2.2$, $l=22.86\text{mm}$, $s_{l1}=5.5\text{mm}$, $s_{r1}=8\text{mm}$, $s_{l2}=14.86\text{mm}$, $s_{r2}=17.36\text{mm}$, and frequency= 10GHz).

3.2 Numerical Results

The propagation constants of three coupled microstrip lines in one shield are shown in Figure 3.5. Four propagating modes are found in this structure in the frequency range from 8.5 to 11.5GHz. The first propagating mode is an even mode, which is marked in squares. The second and third modes are odd modes, which are marked in asterisks and crosses. The fourth mode marked in circles is a waveguide mode.

Figure 3.5. Propagation constants of the propagating modes in a structure of three microstrip lines shielded by a rectangular waveguide ($h=3\text{mm}$, $d=1\text{mm}$, $\epsilon_r=2.2$, $a=22.86\text{mm}$, $s_{l1}=9.73\text{mm}$, $s_{r1}=10.73\text{mm}$, $s_{l2}=10.93\text{mm}$, $s_{r2}=11.93\text{mm}$, $s_{l3}=12.13\text{mm}$, $s_{r3}=13.13\text{mm}$, and frequency=10GHz).

CHAPTER 4

MODE-MATCHING ANALYSIS OF AN N-FURCATED WAVEGUIDE JUNCTION

4.1 Introduction

The mode-matching method is widely used to rigorously model various types of waveguide junctions [95]-[98]. It has been proved to be highly accurate and very efficient. In this chapter, the mode-matching method is employed to analyze an N-furcated waveguide junction. The method is later implemented to solve the problem of scattering of an N-furcated rectangular waveguide junction, a rectangular waveguide to N microstrip lines shielded in one rectangular waveguide junction, and N microstrip lines shielded in one rectangular waveguide to N microstrip lines shielded by N rectangular waveguide junctions. Numerical results for the scattering parameters of these waveguide junctions are verified by comparing our results with those obtained by the commercial software HFSS.

4.2 General Formulation

The geometry of an N-furcated waveguide junction is shown in Figure 4.1. As shown in the figure, the junction is located in the x - y plane, where $z=0$. The main waveguide is indicated as waveguide #0, which is on the left side. On the right side there are N-furcated waveguides. The area of the i th waveguide at $z=0$ is s_i , where $i=0$ to N.

4.2 General Formulation

Figure 4.1. Structure of an N-furcated waveguide.

Let the transverse electric and magnetic fields in the i th waveguides at the junction plane $z=0$ be expressed by the modal series expansion

$$\vec{E}_i^T = \sum_{k=1}^{K_i} A_{i,k}^{(e)} \vec{e}_{i,k}^T, \quad (4.1)$$

$$\vec{H}_i^T = \sum_{k=1}^{K_i} A_{i,k}^{(h)} \vec{h}_{i,k}^T, \quad (4.2)$$

where $i=0$ denotes the main waveguide on the left side, and $i=1$ to N represents the branch waveguides on the right side. $\vec{e}_{i,k}^T$ and $\vec{h}_{i,k}^T$ are the normalized E-field and H-field of the k th mode in the i th waveguide, and they satisfy the following orthogonality relation

$$\iint_{S_i} (\vec{e}_{i,k}^T \times \vec{h}_{i,k}^T) \cdot \hat{z} ds_i = 1. \quad (4.3)$$

4.2 General Formulation

Matching the transverse electric fields at $z=0$, we get

$$\sum_{k=1}^{K_0} A_{0,k}^{(e)} \vec{e}_{0,k}^T = \begin{cases} \sum_{k=1}^{K_1} A_{1,k}^{(e)} \vec{e}_{1,k}^T & s_1 \\ \sum_{k=1}^{K_2} A_{2,k}^{(e)} \vec{e}_{2,k}^T & s_2 \\ \vdots & \vdots \\ \sum_{k=1}^{K_N} A_{N,k}^{(e)} \vec{e}_{N,k}^T & s_N \\ 0 & \text{else where} \end{cases} \quad (4.4)$$

For both sides of (4.4), taking cross product with $\vec{h}_{0,j}$, integrating over the area of s_0 , and invoke the mode orthogonality, the following equation can be obtained.

$$A_{0,j}^{(e)} = \left(\iint_{s_1} \sum_{k=1}^{K_1} A_{1,k}^{(e)} (\vec{e}_{1,k}^T \times \vec{h}_{0,j}^T) \cdot \hat{z} ds_1 + \iint_{s_2} \sum_{k=1}^{K_2} A_{2,k}^{(e)} (\vec{e}_{2,k}^T \times \vec{h}_{0,j}^T) \cdot \hat{z} ds_2 + \dots + \iint_{s_N} \sum_{k=1}^{K_N} A_{N,k}^{(e)} (\vec{e}_{N,k}^T \times \vec{h}_{0,j}^T) \cdot \hat{z} ds_N \right), \quad (4.5)$$

where $j=1$ to K_0 .

Equation (4.5) can be written in matrix form as

$$\mathbf{A}_0^{(e)} = \mathbf{M} \mathbf{B}^{(e)}, \quad (4.6)$$

where

$$\mathbf{B}^{(e)} = [\mathbf{A}_1^{(e)} \quad \mathbf{A}_2^{(e)} \quad \dots \quad \mathbf{A}_N^{(e)}]^T, \quad (4.7)$$

$$\mathbf{M} = [\mathbf{M}_{01} \quad \mathbf{M}_{02} \quad \dots \quad \mathbf{M}_{0N}], \quad (4.8)$$

and the jk th element of $\mathbf{M}_{0i}$ is

$$M_{0i(j,k)} = \iint_{s_i} (\vec{e}_{i,k}^T \times \vec{h}_{0,j}^T) \cdot \hat{z} ds_i. \quad (4.9)$$

4.2 General Formulation

Similarly, matching the transverse magnetic fields at $z=0$, we get

$$\sum_{k=1}^{K_0} A_{0,k}^{(h)} \vec{h}_{0,k}^T = \begin{cases} \sum_{k=1}^{K_1} A_{1,k}^{(h)} \vec{h}_{1,k}^T & s_1 \\ \sum_{k=1}^{K_2} A_{2,k}^{(h)} \vec{h}_{2,k}^T & s_2 \\ \vdots & \vdots \\ \sum_{k=1}^{K_N} A_{N,k}^{(h)} \vec{h}_{N,k}^T & s_N \\ J_s & \text{else where} \end{cases} \quad (4.10)$$

Taking left cross product with $\vec{e}_{i,j}^T$ integrating over the area of s_i , and invoke the mode orthogonality, we obtain

$$\sum_{k=1}^{K_0} \iint_{s_i} A_{0,k}^{(h)} (\vec{e}_{i,j}^T \times \vec{h}_{0,k}^T) \cdot \hat{z} ds_i = A_{i,j}^{(h)}, \quad (4.11)$$

where $i=1$ to N and $j=1$ to K_i .

Equation (4.11) can be put in the matrix form as

$$\mathbf{B}^{(h)} = \mathbf{M}^T \mathbf{A}_0^{(h)}, \quad (4.12)$$

where $\mathbf{M}^T$ is the transpose of $\mathbf{M}$ and the jk th element of $\mathbf{M}$ is given in (4.9).

To solve the scattering matrix of the junction, let's denote the incident and reflected waves in waveguide 0 to be $\mathbf{A}_0^+$ and $\mathbf{A}_0^-$, respectively. The incident and reflected waves in waveguide 1 to N as $\mathbf{B}^+$ and $\mathbf{B}^-$. By matching the transverse electric and magnetic fields at the plane of $z=0$, we have

$$(\mathbf{A}_0^+ + \mathbf{A}_0^-) = \mathbf{M}(\mathbf{B}^+ + \mathbf{B}^-), \quad (4.13)$$

$$(\mathbf{B}^+ - \mathbf{B}^-) = \mathbf{M}^T (\mathbf{A}_0^+ - \mathbf{A}_0^-). \quad (4.14)$$

4.3 N-furcated Rectangular Waveguide Junction

Since

$$\begin{bmatrix} \mathbf{A}_0^- \\ \mathbf{B}^- \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{A}_0^+ \\ \mathbf{B}^+ \end{bmatrix}, \quad (4.15)$$

utilizing (4.6) and (4.12), the scattering parameters of the waveguide discontinuity at $z=0$ can then be derived as

$$\mathbf{S}_{22} = 2(\mathbf{I} + \mathbf{M}^T \mathbf{M})^{-1} - \mathbf{I}, \quad (4.16)$$

$$\mathbf{S}_{12} = \mathbf{M}(\mathbf{I} + \mathbf{S}_{22}), \quad (4.17)$$

$$\mathbf{S}_{21} = 2(\mathbf{I} + \mathbf{M}^T \mathbf{M})^{-1} \mathbf{M}^T, \quad (4.18)$$

$$\mathbf{S}_{11} = \mathbf{M} \mathbf{S}_{21} - \mathbf{I}, \quad (4.19)$$

Where $\mathbf{I}$ is the identity matrix.

4.3 N-furcated Rectangular Waveguide Junction

4.3.1 Formulation

The geometry of an N-furcated rectangular waveguide is shown in Figure 4.2. The left side is a standard waveguide port. The width and height of the rectangular waveguide are a and b_0 , respectively. There are $N-1$ septa inside the waveguide, which separate the right side of the waveguide into N small waveguides with reduced height. The height of the small waveguide i is b_i and the thickness of the i th septa is t_i , where $i=1$ to $N-1$.

4.3 N-furcated Rectangular Waveguide Junction

Figure 4.2. Geometry of an N-furcated rectangular waveguide.

The electromagnetic fields in a rectangular waveguide can be either TE modes or TM modes. If the j th mode in the i th rectangular waveguide is TE_{mn} mode, the normalized transverse E-field and H-field can be expressed as

$$\vec{e}_{i,j}^T = \frac{1}{\sqrt{Y_{mn}^h}} [\hat{x} k_n N_{mn} \cos(k_m x) \sin[k_n (y - y_i)] - \hat{y} k_m N_{mn} \sin(k_m x) \cos[k_n (y - y_i)]], \quad (4.20)$$

$$\vec{h}_{i,j}^T = \sqrt{Y_{mn}^h} [\hat{x} k_m N_{mn} \sin(k_m x) \cos[k_n (y - y_i)] + \hat{y} k_n N_{mn} \cos(k_m x) \sin[k_n (y - y_i)]]. \quad (4.21)$$

For TM_{mn} mode, the normalized transverse E-field and H-field are

$$\vec{e}_{i,j}^T = \frac{1}{\sqrt{Y_{mn}^e}} [\hat{x} k_m N_{mn} \cos(k_m x) \sin[k_n (y - y_i)] + \hat{y} k_n N_{mn} \sin(k_m x) \cos[k_n (y - y_i)]], \quad (4.22)$$

$$\vec{h}_{i,j}^T = \sqrt{Y_{mn}^e} [-\hat{x} k_n N_{mn} \sin(k_m x) \cos[k_n (y - y_i)] + \hat{y} k_m N_{mn} \cos(k_m x) \sin[k_n (y - y_i)]], \quad (4.23)$$

where

$$N_{mn} = \frac{\sqrt{\epsilon_m \epsilon_n}}{k_{cmn} \sqrt{ab}}, \quad \epsilon_n = \begin{cases} 1 & n = 0 \\ 2 & n > 0 \end{cases}, \quad k_{cmn}^2 = \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2,$$

4.3 N-furcated Rectangular Waveguide Junction

$$Y_{mn}^h = \frac{\gamma_{mn}}{j\omega\mu}, \quad Y_{mn}^e = \frac{j\omega\varepsilon}{\gamma_{mn}}, \quad (\gamma_{mn})^2 = k_{c_{mn}}^2 - k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2,$$

and $y_0 = 0$, $y_1 = 0$, $y_i = \sum_{k=1}^{i-1} (b_k + t_k)$ $i = 2$ to N . Substituting (4.20)-(4.23) into (4.9),

the elements of $\mathbf{M}$ matrix can be obtained. If the j th mode in waveguide 0 is TE mode and k th mode in waveguide i is TE mode, we write the jk th element of $\mathbf{M}_{0i}$ as $M_{0i}^{hh}(j, k)$. It can be written as

$$M_{0i}^{hh}(j, k) = AF + BE,$$

where

$$A = \frac{n_{0j}n_{ik}\pi^2}{b_0b_i} N_{m_{0j}n_{0j}} N_{m_{ik}n_{ik}}, \quad B = \frac{m_{0j}m_{ik}\pi^2}{a^2} N_{m_{0j}n_{0j}} N_{m_{ik}n_{ik}}, \quad C = \frac{m_{ik}n_{0j}\pi^2}{ab_0} N_{m_{0j}n_{0j}} N_{m_{ik}n_{ik}},$$

$$D = \frac{m_{0j}n_{ik}\pi^2}{ab_i} N_{m_{0j}n_{0j}} N_{m_{ik}n_{ik}}, \quad E = I1 \cdot I2, \quad F = I1 \cdot I3,$$

$$I1 = \begin{cases} 0 & m_{0j} \neq m_{ik} \\ \frac{a}{2} & m_{0j} = m_{ik} \end{cases}, \quad I2 = Ia + Ib, \quad I3 = Ib - Ia,$$

$$Ia = \begin{cases} \frac{b_0b_i}{2(n_{0j}b_i + n_{ik}b_0)\pi} \left(\sin\left(\frac{n_{0j}\pi}{b_0}(y_i + b_i) + n_i\pi\right) - \sin\frac{n_{0j}\pi y_i}{b_0} \right) & n_{0j} \neq 0 \text{ or } n_{ik} \neq 0 \\ \frac{b_i}{2} & n_{0j} = 0 \text{ and } n_{ik} = 0 \end{cases},$$

$$\text{and } Ib = \begin{cases} \frac{b_0b_i}{2(n_{0j}b_i - n_{ik}b_0)\pi} \sin\left(\frac{n_{0j}\pi}{b_0}(y_i + b_i) - n_{ij}\pi\right) - \sin\frac{n_{0j}\pi y_i}{b_0} & n_{0j}b_i - n_{ik}b_0 \neq 0 \\ \frac{b_i}{2} \cos\frac{n_{ik}\pi y_i}{b_i} & n_{0j}b_i - n_{ik}b_0 = 0 \end{cases}.$$

Similarly, $M_{0i}^{he}(j, k)$, $M_{0i}^{eh}(j, k)$, and $M_{0i}^{ee}(j, k)$ can be derived as

$$M_{0i}^{he}(j, k) = CF + DE,$$

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$$M_{0i}^{eh}(j,k) = DF + CE ,$$

$$\text{and } M_{0i}^{ee}(j,k) = BF + AE .$$

Using (4.16)-(4.19), the scattering parameters of the junction can be calculated.

4.3.2 Numerical Results

Verification of results

In order to verify the formulation described in the previous section and the written computer program, we first consider a trifurcated waveguide structure shown in Figure 4.3. The geometric parameters are $a=22.86\text{mm}$, $b=10.16\text{mm}$, $b_1=4.08\text{mm}$, $t_1=1\text{mm}$, $b_2=3.08\text{mm}$, $t_2=1\text{mm}$, and $b_3=1\text{mm}$. The relative permittivity of all the four waveguides is selected to be 1. Fifty eigenmodes are used in the calculation for each waveguide. Numerical results of the scattering parameters obtained by our mode-matching method are compared with those calculated using HFSS software in Figure 4.4. The solid lines are for the mode-matching results and the circles are HFSS results, respectively. It can be seen that they are in excellent agreement.

Figure 4.3. A trifurcated waveguide junction ($a=22.86\text{mm}$, $b_0=10.16\text{mm}$, $b_1=4.08\text{mm}$, $t_1=1\text{mm}$, $b_2=3.08\text{mm}$, $t_2=1\text{mm}$, $b_3=1\text{mm}$, and $\epsilon_{r1}=\epsilon_{r2}=\epsilon_{r3}=\epsilon_{r4}=1$).

4.3 N-furcated Rectangular Waveguide Junction

Figure 4.4. Scattering parameters of a 3-furcated rectangular waveguide junction.

The convergence behavior of the mode-matching method described is examined in Table 4.1, where the scattering parameters of a waveguide junction considered in Figure 4.3 are checked with different numbers of modes retained. It should be point out that since there is no discontinuity in the x direction, we only considered TE_{1n} and TM_{1n} modes in the calculation. The numbers of modes used in the waveguides are set to be proportional to the cross sectional area of the waveguides.

It can be seen from the table that conservative property of the total power is well maintained and numerical results converge very rapidly when the number of eigenmodes considered is greater than 50 in this structure.

4.3 N-furcated Rectangular Waveguide Junction

Table 4.1. Convergence behavior of the scattering parameters with respect to the number of modes considered in the main rectangular waveguides ($a=22.86$ mm, $b=10.16$ mm, $b_1=4.08$ mm, $t_1=1$ mm, $b_2=3.08$ mm, $t_2=1$ mm, $b_3=1$ mm, and frequency=10 GHz).

Number of eigenmodes	$ S_{11} ^2$	$ S_{21} ^2$	$ S_{31} ^2$	$ S_{41} ^2$	$ S_{11} ^2+ S_{21} ^2+ S_{31} ^2+ S_{41} ^2$
5	0.0122	0.4757	0.3834	0.1287	1
10	0.0122	0.4750	0.3848	0.1279	1
20	0.0122	0.4744	0.3838	0.1296	1
30	0.0122	0.4766	0.3832	0.1279	1
50	0.0122	0.4773	0.3829	0.1276	1
100	0.0122	0.4773	0.3828	0.1277	1
200	0.0122	0.4773	0.3827	0.1277	1

Characteristics of N-furcated waveguides

We consider a bifurcated rectangular waveguide junction first as shown in Figure 4.5. The dependency of the reflection coefficient of the main waveguide to the thickness of the septa is shown in Figure 4.6.

Figure 4.5. A bifurcated waveguide junction ($a=22.86$ mm, $b_0=10.16$ mm, $b_1=b_2$, frequency=10GHz, $\epsilon_{r1}=1$, and $\epsilon_{r2}=\epsilon_{r3}=\epsilon_r$)

4.3 N-furcated Rectangular Waveguide Junction

Figure 4.6. Reflection coefficients of the main waveguide as function of the septa thickness and dielectric constant.

It can be seen that with the increasing of the thickness of the septa, the reflection coefficient increases. For good reflection performance, the septa thickness should be less than $0.1b_0$. Otherwise, a special impedance matching is required to reduce the reflection coefficient. From the figure, we can also find that the reflection coefficient increases with the increasing of the relative permittivity of the filling material in the branch waveguides.

The transmission coefficients from the main waveguide to the branch waveguides can be adjusted by changing the height of the two branch waveguides. This can be utilized to make an uneven power combiner/divider. The transmission coefficient vs. the height of the first branch waveguide is shown in Figure 4.7, with the septa thickness and height of the main waveguide fixed. It can be seen that with the changing of height of the branch

4.3 N-furcated Rectangular Waveguide Junction

waveguide from $0.1b_0$ to $0.9b_0$, the transmission coefficient changes from -10dB to -0.5dB in magnitude, and the phase also changes from 6 to -1 degree at the same time.

Figure 4.7. Transmission coefficients from the main waveguide to the branch waveguides vs. branch waveguide height ($a=22.86\text{mm}$, $b_0=10.16\text{mm}$, frequency= 10GHz , and

$$\epsilon_{r1}=\epsilon_{r2}=\epsilon_{r3}=1).$$

Computed results of a tri-furcated waveguide filled with different dielectric materials are also verified by comparing them with HFSS results. The transmission coefficients of a tri-furcated waveguide are shown in Figure 4.8. The geometrical parameters are $a=22.86\text{mm}$, $b_0=10.16\text{mm}$, $b_1=1\text{mm}$, $b_2=4\text{mm}$, and $b_3=1.16\text{mm}$, $t_1=2\text{mm}$, and $t_2=2\text{mm}$. The main waveguide is filled with air. The three branch waveguides are filled with different dielectric materials, with $\epsilon_{r2}=1$, $\epsilon_{r3}=3$, and $\epsilon_r=5$. The isolations between the three branch waveguides are presented in Figure 4.9. In both figures, our numerical results are presented in lines and HFSS results are represented by circles.

4.3 N-furcated Rectangular Waveguide Junction

Figure 4.8. Transmission and reflection coefficients of a tri-furcated waveguide, incident from the main waveguide ($a=22.86\text{mm}$, $b_0=10.16\text{mm}$, $b_1=1\text{mm}$, $b_2=4\text{mm}$, $b_3=1.16\text{mm}$, $t_1=2\text{mm}$, $t_2=2\text{mm}$, $\epsilon_{r1}=1$, $\epsilon_{r2}=3$, and $\epsilon_{r3}=5$).

Figure 4.9. Isolation between branch waveguides of a tri-furcated waveguide ($a=22.86\text{mm}$, $b_0=10.16\text{mm}$, $b_1=1\text{mm}$, $b_2=4\text{mm}$, $b_3=1.16\text{mm}$, $t_1=2\text{mm}$, $t_2=2\text{mm}$, $\epsilon_{r1}=1$, $\epsilon_{r2}=3$, and $\epsilon_{r3}=5$).

4.4 Junction between Shielded N-Microstrip Lines and Waveguide

4.4 Junction between Shielded N-Microstrip Lines and Waveguide

4.4.1 Formulation

The structure of N microstrip lines shielded in one rectangular waveguide to a rectangular waveguide junction is shown in Figure 4.10. The left side is indicated as waveguide #0, which is a rectangular waveguide with N microstrip lines inside. The distance from the left side of the i th microstrip line to the left side wall is sl_i , and the distance from the right side of the i th microstrip line to the left waveguide wall is sr_i . The height of the shielding rectangular waveguide is b_0 and its width is a . On the right side is a rectangular waveguide filled with dielectric material, denoted as waveguide #1. The waveguide height is b_1 , which is equal to the thickness of the substrate of the microstrip lines.

Figure 4.10. Structure of N shielded microstrip lines shielded in one rectangular waveguide to a rectangular waveguide junction.

4.4 Junction between Shielded N-Microstrip Lines and Waveguide

Matching the fields in the junction area of s at $z=0$, the scattering parameters of the junction can be obtained, as detailed in Section 4.2. For the shielded microstrip lines on the left side, the normalized H-field of the j th mode $\vec{h}_{0,j}^T$ can be written as

$$\vec{h}_{0,j}^T = \sum_{n_0=0}^{N_0} [\hat{x} \bar{C}_{n_0,j}^{h_{x(1)}} \cosh(\alpha_{n_0,j}^{(1)} y) \sin(k_{n_0} x) + \hat{y} \bar{C}_{n_0,j}^{h_{y(1)}} \sinh(\alpha_{n_0,j}^{(1)} y) \cos(k_{n_0} x)] \quad (4.24)$$

If the k th mode in the rectangular waveguide is $TE_{m_1 n_1}$ mode, the transverse E-field is

$$\vec{e}_{1,k}^T = \frac{N_{m_1 n_1}}{\sqrt{Y_{m_1 n_1}^h}} (\hat{x} k_{n_1} \cos(k_{m_1} x) \sin(k_{n_1} y) - \hat{y} k_{m_1} \sin(k_{m_1} x) \cos(k_{n_1} y)). \quad (4.25)$$

The jk th element of the $\mathbf{M}$ matrix is

$$M_{j,k} = \frac{N_{m_1 n_1}}{\sqrt{Y_{m_1 n_1}^h}} \sum_{n_1=0}^{N_1} \iint_s \left(\bar{C}_{n_0,j}^{h_{x(1)}} k_{m_1} \cosh(\alpha_{n_0,j}^{(1)} y) \sin(k_{n_0} x) \sin(k_{m_1} x) \cos(k_{n_1} y) + \bar{C}_{n_0,j}^{h_{y(1)}} k_{n_1} \sinh(\alpha_{n_0,j}^{(1)} y) \cos(k_{n_0} x) \cos(k_{m_1} x) \sin(k_{n_1} y) \right) ds. \quad (4.26)$$

Similarly, if the k th mode in the rectangular waveguide is $TM_{m_1 n_1}$ mode,

$$\vec{e}_{1,k}^T = \frac{N_{m_1 n_1}}{\sqrt{Y_{m_1 n_1}^e}} (\hat{x} k_{m_1} \cos(k_{m_1} x) \sin(k_{n_1} y) + \hat{y} k_{n_1} \sin(k_{m_1} x) \cos(k_{n_1} y)). \quad (4.27)$$

$$M_{j,k} = \frac{N_{m_1 n_1}}{\sqrt{Y_{m_1 n_1}^e}} \sum_{n_1=0}^{N_1} \iint_s \left(\bar{C}_{n_0,j}^{h_{y(1)}} k_{m_1} \sinh(\alpha_{n_0,j}^{(1)} y) \cos(k_{n_0} x) \cos(k_{m_1} x) \sin(k_{n_1} y) - \bar{C}_{n_0,j}^{h_{x(1)}} k_{n_1} \cosh(\alpha_{n_0,j}^{(1)} y) \sin(k_{n_0} x) \sin(k_{m_1} x) \cos(k_{n_1} y) \right) ds. \quad (4.28)$$

Once $\mathbf{M}$ matrix is obtained, the scattering parameters can be obtained from (4.16)-(4.19).

4.4 Junction between Shielded N-Microstrip Lines and Waveguide

4.4.2 Numerical Results

A junction between a rectangular waveguide and two shielded microstrip lines is considered. We use a reduced height X-band rectangular waveguide in the calculation. The geometrical parameters are $a=22.86\text{mm}$, $b_0=3\text{mm}$, $b_l=0.81\text{mm}$, $sl_1=5.5\text{mm}$, $sr_1=8\text{mm}$, $sl_2=14.86\text{mm}$, and $sr_2=17.36\text{mm}$. The dielectric constant of the substrate is $\epsilon_{r1}=2.2$ and $\epsilon_{r2}=1$. The calculated scattering parameters are compared with HFSS results in Figure 4.11. The lines are calculated results and the circles are HFSS results. It can be seen in the figure that they are in very good agreement. With two microstrip lines shielded in a reduced height X-band waveguide, there are at least three propagating modes in the waveguide. It can be seen in the figure that the transmission coefficient is around 0.9 and some of the power is reflected as the third propagating mode. To utilize this structure as an efficient power combiner/divider, the propagating of high order modes in the shielded microstrip lines must be stopped.

Figure 4.11. Scattering parameters of two shielded microstrip lines to a rectangular waveguide junction.

4.5 Junction between N-Microstrip Lines in One Shield and N Shields

As we discussed in Section 4.4, besides the microstrip modes, there are higher-order modes propagating in the N microstrip lines inside one rectangular waveguide structure. To prevent the higher-order modes from propagating, we introduce metallic walls between microstrip lines to shield the N microstrip lines into N rectangular waveguides.

4.5.1 Formulation

The junction between N microstrip lines in one shield and in N shields is shown in Figure 4.12. On the left side is one rectangular waveguide enclosing N microstrip lines, which is denoted as waveguide #0.

Figure 4.12. Structure of N shielded microstrip lines in one rectangular shielding to a rectangular waveguide junction.

4.5 Junction between N-Microstrip Lines in One Shield and N Shields

For the shielded microstrip lines on the left side, the normalized H-field of the j th mode

$\vec{h}_{0,j}^T$ can be written as

$$\vec{h}_{0,j}^T = \begin{cases} \sum_{n_0=0}^{N_0} \left(\hat{x} \bar{C}_{n_0,j}^{h_{x(1)}} \cosh(\alpha_{n_0,j}^{(1)} y) \sin(k_{n_0} x) \right) \\ + \hat{y} \bar{C}_{n_0,j}^{h_{y(1)}} \sinh(\alpha_{n_0,j}^{(1)} y) \cos(k_{n_0} x) \end{cases} \\ \sum_{n_0=0}^{N_0} \left(\hat{x} \bar{C}_{n_0,j}^{h_{x(2)}} \cosh(\alpha_{n_0,j}^{(2)} (h-y)) \sin(k_{n_0} x) \right) \\ + \hat{y} \bar{C}_{n_0,j}^{h_{y(2)}} \sinh(\alpha_{n_0,j}^{(2)} (h-y)) \cos(k_{n_0} x) \end{cases} \quad (4.29)$$

For the N shielded microstrip lines on the right side, the normalized E-field of the k th

mode $\vec{e}_{i,k}^T$ in the i th waveguide can be written as

$$\vec{e}_{i,k}^T = \begin{cases} \sum_{n_i=0}^{N_i} \left(\hat{x} \bar{C}_{n_i,k}^{e_{x(1)}} \sinh(\alpha_{n_i,k}^{(1)} y) \cos(k_{n_i} (x - xl_i)) \right) \\ + \hat{y} \bar{C}_{n_i,k}^{e_{y(1)}} \cosh(\alpha_{n_i,k}^{(1)} y) \sin(k_{n_i} (x - xl_i)) \end{cases} \\ \sum_{n_i=0}^{N_i} \left(\hat{x} \bar{C}_{n_i,k}^{e_{x(2)}} \sinh(\alpha_{n_i,k}^{(2)} (h-y)) \cos(k_{n_i} (x - xl_i)) \right) \\ + \hat{y} \bar{C}_{n_i,k}^{e_{y(2)}} \cosh(\alpha_{n_i,k}^{(2)} (h-y)) \sin(k_{n_i} (x - xl_i)) \end{cases} \quad (4.30)$$

The jk th element of $\mathbf{M}_{0i}$ is

$$M_{j,k} = \sum_{n_0=0}^{N_0} \sum_{n_i=0}^{N_i} \begin{pmatrix} -\bar{C}_{n_i,k}^{e_{x(1)}} \bar{C}_{n_0,j}^{h_{y(1)}} SS(0, d, i\alpha_{n_i,k}^{(1)}, i\alpha_{n_0,j}^{(1)}, 0, 0) CC(xl_i, xl_i + a_i, k_{n_i}, k_{n_0}, xl_i, 0) \\ -\bar{C}_{n_i,k}^{e_{y(1)}} \bar{C}_{n_0,j}^{h_{x(1)}} CC(0, d, i\alpha_{n_i,k}^{(1)}, i\alpha_{n_0,j}^{(1)}, 0, 0) SS(xl_i, xl_i + a_i, k_{n_i}, k_{n_0}, xl_i, 0) \\ -\bar{C}_{n_i,k}^{e_{x(2)}} \bar{C}_{n_0,j}^{h_{y(2)}} SS(d, b_0, i\alpha_{n_i,k}^{(2)}, i\alpha_{n_0,j}^{(2)}, h, h) CC(xl_i, xl_i + a_i, k_{n_i}, k_{n_0}, xl_i, 0) \\ -\bar{C}_{n_i,k}^{e_{y(2)}} \bar{C}_{n_0,j}^{h_{x(2)}} CC(d, b_0, i\alpha_{n_i,k}^{(2)}, i\alpha_{n_0,j}^{(2)}, h, h) SS(xl_i, xl_i + a_i, k_{n_i}, k_{n_0}, xl_i, 0) \end{pmatrix}, \quad (4.31)$$

where

$$SS(x_1, x_2, a_1, a_2, b_1, b_2) = \int_{x_1}^{x_2} \sin(a_1(x - b_1)) \sin(a_2(x - b_2)) dx, \quad (4.32)$$

$$CC(x_1, x_2, a_1, a_2, b_1, b_2) = \int_{x_1}^{x_2} \cos(a_1(x - b_1)) \cos(a_2(x - b_2)) dx. \quad (4.33)$$

4.5 Junction between N-Microstrip Lines in One Shield and N Shields

4.5.2 Numerical Results

Numerical results for the scattering parameters of the junction are presented in Figure 4.6. Our calculated results are denoted lines, while the HFSS results are represented in circles. In the figure, $|S(m:n, o:p)|$ represents the transmission/reflection coefficient from the p th mode of port o to the n th mode of port m .

Figure 4.13. Scattering parameters of the junction between two shielded microstrip lines inside one shielding waveguide and two separate shielding waveguides.

CHAPTER 5 DESIGN OF AN EIGHT-WAY POWER COMBINER

5.1 Introduction

As discussed in Chapter 2, spatial power combiners with closed waveguide structures are desired because of their high combining efficiency and compact structure. One of the very good candidates among these combiners is the TSA waveguide combiner proposed by Alexanian and York in [61] and was investigated thoroughly in the subsequent publications [61]-[67]. In a later version of this design [64], with six antenna cards and a total number of 24 MMIC amplifier devices, a waveguide power combiner was realized with maximum output power of 126W in X-band. However, there are still some major issues to be addressed in this structure, which can be summarized as follows.

- 1) To integrate with three-terminal SSPA devices, a slotline-to-microstrip transition is required, and the electric field in the waveguide section must be rotated by 90 degrees so that it can be converted from the rectangular waveguide mode to the microstrip mode. Such transition will unavoidably introduce extra loss.
- 2) The input return loss of the whole structure cannot be accurately calculated because of the complexity of the structure. Even if the impedance of the slotline itself can be analyzed, the mutual coupling effect between TSA cards is very difficult to be characterized accurately.
- 3) The distribution of the TE_{10} mode in the rectangular waveguide is not

5.2 Description of the Structure

uniform, which will affect the combining efficiency. The center cards receive more power than the outer cards. Therefore, the solid state amplifiers on the center cards will reach saturation earlier than the whole amplifier groups reach the output saturation power, which will greatly affect the linearity of the whole power amplifier.

Since most of the three-terminal SSPA devices utilize 50Ω microstrip lines as their input-output interface, to design an efficient rectangular waveguide to multiple microstrip lines transition becomes extremely important in the design of rectangular waveguide power combiners. In this chapter, a new rectangular waveguide to multiple microstrip lines transition is proposed to solve the above mentioned three problems. Using the mode matching formulas derived in the previous chapters, an eight-way C-band power combiner is designed. To improve the isolation between eight microstrip line ports, resistive loaded decoupling slots and microstrip decoupling networks are introduced. The slots and the microstrip network are optimized using HFSS to obtain better isolations between microstrip line ports. Finally, the designed power combiner is fabricated and experimental measurements are carried out. Measured results are compared with those from HFSS, and very good insertion loss and return loss results are achieved.

5.2 Description of the Structure

The proposed eight-way power combiner/divider structure is described in Figure 5.1. The combining port is a standard WR137 rectangular waveguide port, which is denoted as port 1. The width and height of the rectangular waveguide are 34.85mm and 15.85mm, respectively. Port 2 to Port 9 are eight microstrip line ports. As shown in Figure 5.1 a), two microstrip line ports are on one PCB and there are totally four PCBs that contain

5.2 Description of the Structure

eight microstrip line ports altogether. As shown in the side cross section view of the structure in Figure 5.1 b), the rectangular waveguide port is in the left-hand side, and in the right-hand side there are the eight microstrip line ports. There are totally nine junctions in this structure, denoted as J1 to J9, as illustrated in the figure. J1 is a 4-furcated rectangular waveguide junction. The rectangular waveguide is divided into four reduced-height rectangular waveguide by J1. J2 to J5 are waveguide steps for matching purpose. J6 is the interface between air and PCB substrate. J7 is a rectangular waveguide to 2-microstrip line junction. The two microstrip lines are shielded by one rectangular waveguide. To stop the higher-order modes from propagating, a metallic wall is introduced after junction 8 between the two microstrip lines. Junction 9 is a microstrip line step transition which is introduced for matching purpose. Together with the via holes on the printed circuit board between two microstrip lines, two separate rectangular waveguides are formed for shielding. Finally, on the right-hand side there are eight microstrip lines shielded by eight individual rectangular waveguides.

a) Eight microstrip line ports.

5.2 Description of the Structure

b) Side cross section view

Figure 5.1. Structure of the eight way power combiner/divider.

To achieve good isolation between the reduced-height rectangular waveguides, three resistive-loaded coupling slots have been introduced on the three septa [99]. A microstrip decoupling network is introduced to further increase the isolation between two adjacent microstrip lines on the same substrate, as shown in Figure 5.2. The microstrip decoupling network consists of three branch lines between the two microstrip lines, which are loaded with three thin film resistors. Since the resistors are loaded at the center of the branch lines, they will not affect the propagating of even modes and only absorb the odd modes and therefore create isolation between the two microstrip ports.

5.2 Description of the Structure

Figure 5.2. Microstrip network for isolation improvement.

The physical parts of the combiner are designed for easy fabrication and assembly. The whole structure consists of nine parts. There are one bottom metallic block, one top metallic block, three metallic cards and four PCBs in between, as shown in Figure 5.3. The first PCB is attached to the bottom block and the rest of the PCBs are attached to the three metallic cards. Covering the bottom block with three cards and the top metallic block, a rectangular waveguide to eight shielded microstrip lines power combiner/divider is constructed.

The detailed drawing of a card with a PCB attached is shown in Figure 5.4. There are two microstrip lines on the same PCB. Between the two microstrip lines is a ground plane. With enough via holes connected to the ground, the card and the ground plane form a conductive wall to separate the two microstrip lines into two shielding waveguides.

5.2 Description of the Structure

Figure 5.3. Structure of a eight-way microstrip line to waveguide power combiner.

Figure 5.4. A card with a PCB attached.

5.3 Design Procedure

To design the eight-way multiple microstrip lines to rectangular waveguide power combiner, a rectangular waveguide to four-reduced height waveguides junction is designed initially. A four-furcated waveguide is used to construct this junction. Four stages of E-planes steps are employed to reduce the waveguide height for further integrating it with microstrip lines later. The height of the four reduced-height rectangular waveguide is selected to be the same as the PCB thickness.

The second step is to design the transition from reduced-height waveguide to two microstrip lines. There are four junctions in this structure. The first one is a junction from an air-filled waveguide to a dielectric-filled waveguide junction; the second is a junction from dielectric-filled waveguide to two microstrip lines; the third one is to add a metallic wall between the two microstrip lines; the forth junction is a microstrip step junction for impedance matching purpose.

The two designed transitions are cascaded together to form an eight-way power combiner. The whole structure is calculated using the formulations developed in Chapter 3 and Chapter 4. To improve the isolation between the microstrip lines, resistive-loaded slots are added to the waveguide transition designed in the first step. The size and location of the slot are optimized using the commercial software HFSS. A microstrip network between two adjacent microstrip lines is added to further improve the isolation between adjacent ports. The network is also simulated and optimized using HFSS.

The final step is to take manufacturing details into account. Necessary fillets are added and some small modifications are made for practical implementation reasons. With all the

5.4 Calculated and Measured Results

changes presented, the whole structure is simulated and fine-tuned in HFSS.

5.4 Calculated and Measured Results

5.4.1 Calculated Results without Isolation Improvement Mechanism

Calculated scattering parameters of a 4-furcated rectangular waveguide are presented in Figure 5.6. The main waveguide is a WR137 C-band rectangular waveguide. The width and height of the waveguide are 34.85 mm and 15.85 mm, respectively. Other geometric parameters are $t=0.5$ mm and $b_I=3.575$ mm as shown in Figure 5.5. It can be seen Figure 5.6 that the power are divided into four ports quite evenly. Very small differences of 0.05dB in power dividing ratio between port 2 and port 3 (or port 4 and port 5) are observed, which is due to the asymmetry of port 2 and port 3 (or port 4 and port 5). Good return loss of better than 25 dB from the main waveguide port is observed. The isolations between all ports are found to be better than 9dB. However, the return losses of port 2 to port 5 are not satisfactory, which are only around 2.4dB. For a lossless network without passive loading for isolation improvement, this is not unexpected.

Figure 5.5. A 4-furcated rectangular waveguide ($b_0=15.85$ mm, $b_I=3.575$ mm, and $t=0.5$ mm).

5.4 Calculated and Measured Results

Figure 5.6. Calculated transmission coefficients of a 4-furcated waveguide.

Figure 5.7. Calculated reflection coefficients and isolations of a 4-furcated waveguide.

5.4 Calculated and Measured Results

For further integrating the reduced-height waveguide with microstrip line structure later, a waveguide section with four matching steps is introduced to reduce the height of the waveguide from 3.575 mm to the height of the substrate material. The substrate used in this design is RT5880 with a thickness of 0.508mm and dielectric constant of 2.2. Considering the copper thickness of 0.04 mm on both sides of the PCB, the height of the waveguide b_5 is selected to be 0.588mm. After optimization, the geometric parameters of the structure are $b_1=3.575$ mm, $b_2=2.6$ mm, $b_3=1.4$ mm, $b_4=0.75$ mm, $b_5=0.588$ mm, $l_2=13.81$ mm, $l_3=13.94$ mm, and $l_4=13.95$ mm.

Figure 5.8. Structure of a waveguide section with four matching steps.

Calculated scattering parameters of this waveguide section are shown in Figure 5.9. The return loss is better than 25dB and the insertion loss is lower than 0.02dB over the entire frequency range of 5.8GHz to 8.2GHz.

5.4 Calculated and Measured Results

Figure 5.9. Calculated scattering parameters of a waveguide section with four matching steps.

The detailed structure of the reduced-height waveguide to two microstrip lines transition is shown in Figure 5.10. The height of the shield b_6 is 3.075 mm and the wall between the two microstrip lines is 2 mm. The width of the 50 Ω microstrip lines for port 2 and port 3 is 1.56 mm. l_6 is selected to be 1 mm to maintain a good contact between the microstrip lines and the top wall of the waveguide. Other geometric parameters are optimized to achieve the best $|S_{11}|$ results. After optimization, the rest of the geometric parameters are determined as $l_7=1.309$ mm, $l_8= 5.344$ mm, $w_{ml}=2.856$ mm, and $w_d=5.4$ mm.

5.4 Calculated and Measured Results

Figure 5.10. The transition between a reduced-height waveguide and two microstrip lines.

Calculated scattering parameters of this transition are shown in Figure 5.11. It can be seen from the figure that the return loss of the combining port is better than 22dB and the incident power is distributed evenly to port 2 and port 3. Since there is no mechanism for improving isolation between ports in this structure, the return loss of ports 2 and 3 is only 6dB and the isolation between ports 2 and 3 is around 5.7dB.

5.4 Calculated and Measured Results

Figure 5.11. Calculated scattering parameters for the transition between a reduced-height waveguide and two microstrip lines.

Calculated scattering parameters for the whole structure are shown in Figure 5.12. The reflection coefficient at port 1 is better than -21 dB. The transmission coefficient from port 1 to port 2 is 9 dB, which is very constant over the entire frequency range of interest. However, the maximum reflection coefficient of the microstrip line ports are only -1.3 dB and the isolation between port 2 and port 3 is only 1.3 dB also. In an ideal case for power combining, the input magnitudes and phases from all the ports should be same. Therefore, large reflection coefficients at port 2 to port 9 and couplings between ports will not cause actual power loss. In a practical case, to have better isolation between ports will give much stable output when the input is not ideal.

5.4 Calculated and Measured Results

Figure 5.12. Calculated scattering parameters of the eight-way power combiner.

5.4.2 Simulation Results with Improved Isolation

Three slots are added in the center of the septa in the 4-furcated rectangular waveguide to improve the isolation performance. The 3-D HFSS model of the 4-furcated waveguide with slots is shown in Figure 5.13. The slots are loaded with 100Ω thin film resistors at the center. The width of the slot is selected to be 1mm. The distance from the center of the slot to the junction d_s is 9.5 mm and the slot length l_s is 30 mm, and they are optimized in HFSS for best isolation results.

5.4 Calculated and Measured Results

Figure 5.13. Scattering parameters of junction 5.

A microstrip line decoupling network is introduced to improve the isolation between adjacent ports on the same PCB. Three branch lines with three 100Ω resistors loaded in the center connect the two microstrip lines together. With a distance of 5.4 mm between the two microstrip lines, considering the gap of 0.4 mm for the resistors, the length of all the stubs l_m are set to be 2.5 mm. The width of the stubs and the distance between stubs are optimized in HFSS to obtain the best isolation between ports. After optimization, the geometric parameters are determined as follows: $l_{m1}=2.70$ mm, $l_{m2}=7.61$ mm, $l_{m3}=7.87$ mm, $w_1=0.21$ mm, $w_2=0.78$ mm, and $w_3=0.22$ mm.

Figure 5.14. Microstrip line network to improve the isolation between ports.

5.4 Calculated and Measured Results

After having optimized the slots and microstrip network, via holes are added below the center wall in the HFSS model to connect the center wall to the ground. Fillets are also added to simulate the actual fabricated parts. Copper thickness of the microstrip lines is also considered and some other small changes have been made for fabrication and assembling consideration. The final structure is then simulated in HFSS. The simulated scattering parameters of the entire structure are presented in Figure 5.15. The simulated $|S_{11}|$ is better than -20dB over the whole frequency range. Due to the symmetry of the structure, the transmission coefficients results can be separated into two groups, where $|S_{21}|$, $|S_{31}|$, $|S_{81}|$ and $|S_{91}|$ are identical and $|S_{41}|$, $|S_{51}|$, $|S_{61}|$ and $|S_{71}|$ are the same. The power unevenness between the two groups is simulated to be around 0.05dB . The reflection coefficients of port 2 to port 9 are better than -10dB over the entire frequency range. The simulated isolations between ports are also better than 13dB .

Figure 5.15 Simulated scattering parameters of the power combiner.

5.4 Calculated and Measured Results

5.4.3 Measured Results

The photos of the fabricated eight-way power combiner are shown in Figure 5.16. The top and bottom views of a metallic card with PCB attached are presented in Figure 5.16 a). Figure 5.16 b) are the assembled eight-way power combiner and a metal plate with two SMA connectors attached. The metallic plate with SMAs attached is used to measure the transmission coefficients from the waveguide port to the microstrip lines and the isolations between the microstrip ports.

Figure 5.16 a) A metallic card with PCB attached.

5.4 Calculated and Measured Results

Figure 5.16 b) A fully assembled combiner and a metallic plate with SMA connectors for testing

Figure 5.16. Photos of the fabricated eight-way waveguide power combiner.

The measured $|S_{11}|$ results are compared with the simulation ones in Figure 5.17. Due to the space constrain, it is difficult to tape out all eight microstrip lines and load them with perfect 50Ω terminators. The results are obtained by terminating all microstrip lines with

5.4 Calculated and Measured Results

two 100 Ω thin film resistors internally. It is seen from Figure 5.17 that measured $|S_{11}|$ results are better than -22dB over the frequency range of 5.8GHz to 8.2GHz, which is slightly better than the simulated results of -20dB .

Figure 5.17. Measured and simulated $|S_{11}|$ of the eight-way power combiner.

Measured transmission coefficients of the eight-way power combiner are shown in Figure 5.18. Since the distances between the eight microstrip line ports are very close, it is very difficult to tape out all the eight ports simultaneously. During the measurement of transmission coefficients, only two of the microstrip lines are connected to the SMA connectors at one time and other ports are loaded with two 100 Ω thin film resistors internally. Since the input and output ports of the combiner are of different forms, the calibration of the measurement system can only be done up to the coaxial ports. A coaxial

5.4 Calculated and Measured Results

to rectangular waveguide transition and an SMA microstrip line to coaxial transition have to be added for the measurement purpose. The obtained transmission coefficients include the loss of the two transitions. The measured transmission coefficients vary from -9.3dB to -10.1dB over the frequency range of 5.8GHz to 8.2GHz . The measured maximum power unevenness between different ports over frequency is less than 0.8dB .

Figure 5.18. Measured transmission coefficients of the eight-way power combiner.

The measured mutual couplings between adjacent ports on the same PCB are presented in Figure 5.19 together with the simulated results. It can be seen that measured and simulated results are in great agreement. The isolations between adjacent ports on the same PCB are measured to be better than 12dB over the frequency range of 5.8GHz to 8.2GHz . Due to the symmetry of the combiner structure, $|S_{23}|$, $|S_{89}|$ are identical and $|S_{45}|$,

5.4 Calculated and Measured Results

$|S_{67}|$ are identical.

Figure 5.19. Simulated and measured mutual coupling results between microstrip lines on the same PCB layer of the eight-way power combiner.

The measured mutual couplings between ports on different PCB layers are presented in Figure 5.20. The simulated results are also presented in the same figure for comparison. The maximum mutual couplings are found to be between ports on adjacent PCB layers, which are measured to be around -18dB. The measured maximum mutual couplings are about -21dB between microstrip line ports on the first and fourth layers, and -27dB between microstrip lines on the first and third layers.

Due to the structural difficulties, most of the N-way combiners don't have the mechanism to absorb the uneven power between input ports [61]-[77]. For a lossless N-way combiner,

5.4 Calculated and Measured Results

if only one of the input ports is excited and other ports are loaded ideally, $1/N$ of the power will be transmitted from the input port to the combining port, and the rest $(N-1)/N$ of the power will contribute to either return loss of this input port or mutual coupling between input ports. This will result in very high return loss of the input ports and high mutual coupling between input ports. In the previous publications, the mutual coupling between adjacent input ports are typically in the range of 6 to 9 dB and the return loss of the input ports are normally left unmentioned and they can be extremely high. In this design, better isolation between input ports and better input return loss are realized by introducing the resistive loaded slots and the microstrip decoupling network.

Figure 5.20. Mutual couplings between microstrip lines on different PCB layers of the eight-way power combiner.

5.4 Calculated and Measured Results

The measured reflection coefficients of the eight microstrip line ports are shown in Figure 5.21 and Figure 5.22. Due to the symmetry of the structure, the reflection coefficients of the microstrip line ports can be separated into two groups. $|S_{22}|$, $|S_{33}|$, $|S_{88}|$ and $|S_{99}|$ are identical and they are presented in Figure 5.21. $|S_{44}|$, $|S_{55}|$, $|S_{66}|$ and $|S_{77}|$ are identical and they are shown in Figure 5.22. The measured reflection coefficients are better than -9dB for all microstrip line ports. The results agree well with our simulated results. Unlike the return loss of port 1, the reflection coefficients of these microstrip line ports will not affect the insertion loss of the combiner directly. Due to the existence of coupling between ports, large reflection coefficients at these ports are only indicating high coupling between ports.

Figure 5.21. Reflection coefficient at the microstrip line ports of the eight-way power combiner.

5.4 Calculated and Measured Results

Figure 5.22. Reflection coefficients at the microstrip line ports of the eight-way power combiner.

In order to characterize the insertion loss of the combiner accurately, the combiner is measured in a back-to-back configuration. Figure 5.23 shows the measurement setup of the back-to-back power combiner. TRL calibration of the whole measurement system is carried out on the rectangular waveguide ports. By doing this, the measurement errors caused by the extra insertion losses of the coaxial to waveguide transition and microstrip line to SMA transitions in the previous measurement of a single combiner are fully removed.

5.4 Calculated and Measured Results

Figure 5.23. A back-to-back eight-way power combiner under measurement.

The measured transmission coefficients of the back-to-back configuration are presented in Figure 5.24. The measured transmission coefficients vary in the range of -0.62dB to -0.78dB over the frequency range of 5.8GHz to 8.2GHz . Therefore, for a single eight-way power combiner, the insertion loss is better than 0.39dB over the frequency range of 5.8GHz to 8.2GHz .

The measured reflection coefficients of the eight-way combiner in a back-to-back configuration are compared with the simulated ones in Figure 5.25. The measured results are in solid line and the simulated ones are in dot line. It can be seen from the figure that the measured $|S_{11}|$ of the back-to-back power combiner is better than -14.5dB . Very good agreements are observed between measured and simulated results.

5.4 Calculated and Measured Results

Figure 5.24. Measured transmission coefficient of the power combiner in a back to back configuration.

Figure 5.25 Reflection coefficient of the eight-way power combiner in a back to back configuration.

5.5 Discussions

An eight-way microstrip line to rectangular waveguide power combiner has been presented in this chapter. Utilizing the formulas developed in Chapter 3 and Chapter 4, an initial design of C-band eight-way power combiner is carried out. The combiner is designed to cover the entire band from 5.8GHz to 8.2GHz for C-band satellite communication applications. The design consists of a 4-furcated rectangular waveguide, rectangular waveguide matching steps, and four reduced height waveguide to microstrip line transitions. Three slots and microstrip line networks are added in the structure to improve the isolation between ports. The slots and the microstrip line networks are optimized using HFSS. Experimental measurement has been carried out. The unevenness of the transmission coefficients among the eight microstrip line ports is measured to be around 0.8dB over the whole frequency range. The return loss is measured to be better than 22dB at the combining port. Measured isolations are better than 12dB for the microstrip lines on the same layer and 17dB for the microstrip lines on different boards. The return losses of all microstrip line ports are measured to be better than 9.5dB. The insertion loss is accurately characterized by measuring the combiner in a back-to-back configuration. Better than 0.39dB loss for a single combiner over the frequency range of 5.8 to 8.2GHz is achieved. It has been found that all measured results agree with simulated ones very well.

CHAPTER 6

DESIGN OF AN EIGHT-WAY KU-BAND RADIAL POWER COMBINER

6.1 Introduction

With the wide use of solid state power amplifiers (SSPAs) in modern microwave communication systems, combining many single SSPA devices to obtain high output power has been a research topic of extensive attention in recent years. Among all the existing power combiners [3], the radial combiner is one of the most promising structures, which demonstrate very good amplitude and phase balance due to its axially symmetric structure. A resonant-type cylindrical power combiner was first presented to combine power from IMPATT diodes by Harp and Stover [30] in 1973. Conventional cylindrical power combiners have very high combining efficiency with relatively narrow bandwidth. Compared to resonant radial combiners, non-resonant radial power combiners have a wider bandwidth. Various radial power combiners have been proposed by many researchers [40]-[51].

In order to take advantage of the output microstrip ports of most three-terminal SSPA devices, printed transmission line peripheral ports are preferred. Otherwise, a transitional structure must be included, which will introduce additional loss and phase imbalance. To achieve high power combining efficiency and high power handling capability, the power combining process should take place in air, instead of in substrate materials. Based on these two considerations, a novel 8-way radial combiner with microstrip peripheral ports was briefly introduced in our previous work [52]. In this chapter, an improved design is

6.2 Description of the Structure

presented by replacing eight microstrip line ports with SSS ports to further reduce the loss. Experimental results show that the bandwidth of this combiner is very wide due to its non-resonant nature. All ports are DC isolated and no output DC blocking capacitors are required at the output of SSPA devices.

6.2 Description of the Structure

The proposed 8-way radial power combiner is shown in Figure 6.1. The structure consists of a cylindrical cavity, a subminiature version A (SMA) coaxial output port and eight SSS input ports. As shown in Figure 6.1, a printed circuit board is inserted into the air-filled cylindrical cavity. Along the cavity wall, there are eight open windows in the radial direction, which allow eight SSSs to extend to the eight peripheral ports outside the cavity. The output coaxial port is located at the center of the cavity bottom and its inner conductor is extended into the cavity serving as a coupling probe.

Figure 6.1 a) 3-D view

6.3 Mode Analysis of a Suspended Substrate Stripline

Figure 6.1. Geometry of the 8-way radial power combiner.

6.3 Mode Analysis of a Suspended Substrate Stripline

The cross sectional view of a SSS is shown in Figure 6.2. The whole structure is shielded by a rectangular waveguide as shown in the figure. Region I_a , I_c , Region II and Region III_b are filled with air. Region I_b and Region III_a are filled with substrate material with dielectric constant of ϵ_r . The suspended stripline is located between Region II and Region III . The width and height of the rectangular waveguide are $2a$ and b , respectively. The thickness of the substrate is t , and the distance between the substrate to the top and bottom wall of the rectangular waveguide are d and s , respectively. Since the stripline is located at the center of the waveguide, a PMC is put at the center of the rectangular waveguide to simplify the structure.

6.3 Mode Analysis of a Suspended Substrate Stripline

Figure 6.2. Cross sectional view of a shielded SSS.

6.3.1 Field Expressions in Region I

In region I, the field can be expressed by combination of LSE modes and LSM modes. Since the tangential electric field must vanish at the PEC boundaries of $y=0$, $y=b$, and $x=0$, it is appropriate to write the Hertzian potential of LSE modes and LSM modes as

$$\Pi_n^{lh} = \begin{cases} A_n^{lh} \frac{\sin(k_{yn}^{lah}(b-y))}{\sin(k_{yn}^{lah}d)} \frac{\cos(k_{xn}^{lh}x)}{\cos(k_{xn}^{lh}x_0)} e^{-j\beta z} \hat{a}_y & s+t \leq y \leq b \\ \left(B_n^{lh} \frac{\sin(k_{yn}^{lbh}(y-s))}{\sin(k_{yn}^{lbh}t)} + C_n^{lh} \frac{\sin(k_{yn}^{lbh}(s+t-y))}{\sin(k_{yn}^{lbh}t)} \right) \frac{\cos(k_{xn}^{lh}x)}{\cos(k_{xn}^{lh}x_0)} e^{-j\beta z} \hat{a}_y & s \leq y \leq s+t \\ D_n^{lh} \frac{\sin(k_{yn}^{lch}y)}{\sin(k_{yn}^{lch}s)} \frac{\cos(k_{xn}^{lh}x)}{\cos(k_{xn}^{lh}x_0)} e^{-j\beta z} \hat{a}_y & 0 \leq y \leq s \end{cases} \quad (6.1)$$

$$\Pi_n^{le} = \begin{cases} A_n^{le} \frac{\cos(k_{yn}^{lae}(b-y))}{\cos(k_{yn}^{lae}d)} \frac{\sin(k_{xn}^{le}x)}{\sin(k_{xn}^{le}x_0)} e^{-j\beta z} \hat{a}_y & s+t \leq y \leq b \\ \left(B_n^{le} \frac{\cos(k_{yn}^{lbe}(y-s))}{\cos(k_{yn}^{lbe}t)} + C_n^{le} \frac{\cos(k_{yn}^{lbe}(s+t-y))}{\cos(k_{yn}^{lbe}t)} \right) \frac{\sin(k_{xn}^{le}x)}{\sin(k_{xn}^{le}x_0)} e^{-j\beta z} \hat{a}_y & s \leq y \leq s+t \\ D_n^{le} \frac{\cos(k_{yn}^{lce}y)}{\cos(k_{yn}^{lce}s)} \frac{\sin(k_{xn}^{le}x)}{\sin(k_{xn}^{le}x_0)} e^{-j\beta z} \hat{a}_y & 0 \leq y \leq s \end{cases} \quad (6.2)$$

6.3 Mode Analysis of a Suspended Substrate Stripline

where $k_{xn}^{lp} = \sqrt{\varepsilon_r k_0^2 - (k_{yn}^{lp})^2 - \beta^2}$, $k_{yn}^{lp} = \sqrt{(k_{yn}^{lap})^2 + (\varepsilon_r - 1)k_0^2}$, $k_{yn}^{lap} = k_{yn}^{lcp}$, and $p = h, e$.

For LSE modes, utilizing the following equations

$$\vec{E} = \nabla \times \nabla \times \Pi^e - j\omega\mu\nabla \times \Pi^h \quad (6.3)$$

$$\vec{H} = \nabla \times \nabla \times \Pi^h + j\omega\varepsilon(y)\nabla \times \Pi^e, \quad (6.4)$$

E_z and H_x can be obtained as

$$E_z^{lh} = \begin{cases} A_n^{lh} j\omega\mu k_{xn}^{lh} \frac{\sin(k_{yn}^{lah}(b-y))}{\sin(k_{yn}^{lah}d)} \frac{\sin(k_{xn}^{lh}x)}{\cos(k_{xn}^{lh}x_0)} e^{-j\beta z} & s+t \leq y \leq b \\ j\omega\mu k_{xn}^{lh} \left(B_n^{lh} \frac{\sin(k_{yn}^{lbh}(y-s))}{\sin(k_{yn}^{lbh}t)} + C_n^{lh} \frac{\sin(k_{yn}^{lbh}(s+t-y))}{\sin(k_{yn}^{lbh}t)} \right) \frac{\sin(k_{xn}^{lh}x)}{\cos(k_{xn}^{lh}x_0)} e^{-j\beta z} & s \leq y \leq s+t \\ D_n^{lh} j\omega\mu k_{xn}^{lh} \frac{\sin(k_{yn}^{lch}y)}{\sin(k_{yn}^{lch}s)} \frac{\sin(k_{xn}^{lh}x)}{\cos(k_{xn}^{lh}x_0)} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.5)$$

$$H_x^{lh} = \begin{cases} A_n^{lh} k_{xn}^{lh} k_{yn}^{lah} \frac{\cos(k_{yn}^{lah}(b-y))}{\sin(k_{yn}^{lah}d)} \frac{\sin(k_{xn}^{lh}x)}{\cos(k_{xn}^{lh}x_0)} e^{-j\beta z} & s+t \leq y \leq b \\ k_{xn}^{lh} k_{yn}^{lbh} \left(-B_n^{lh} \frac{\cos(k_{yn}^{lbh}(y-s))}{\sin(k_{yn}^{lbh}t)} + C_n^{lh} \frac{\cos(k_{yn}^{lbh}(s+t-y))}{\sin(k_{yn}^{lbh}t)} \right) \frac{\sin(k_{xn}^{lh}x)}{\cos(k_{xn}^{lh}x_0)} e^{-j\beta z} & s \leq y \leq s+t \\ -D_n^{lh} k_{xn}^{lh} k_{yn}^{lch} \frac{\cos(k_{yn}^{lch}y)}{\sin(k_{yn}^{lch}s)} \frac{\sin(k_{xn}^{lh}x)}{\cos(k_{xn}^{lh}x_0)} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.6)$$

Matching E_z at $y=s$ and $y=s+t$ we have

$$D_n^{lh} = C_n^{lh} \quad (6.7)$$

$$A_n^{lh} = B_n^{lh} \quad (6.8)$$

Match H_x at $y=s$ and $y=s+t$ we obtain

$$-B_n^{lh} k_{yn}^{lbh} \frac{1}{\sin(k_{yn}^{lbh}t)} + C_n^{lh} k_{yn}^{lbh} \cot(k_{yn}^{lbh}t) = -D_n^{lh} k_{yn}^{lch} \cot(k_{yn}^{lch}s) \quad (6.9)$$

$$-B_n^{lh} k_{yn}^{lbh} \cot(k_{yn}^{lbh}t) + C_n^{lh} k_{yn}^{lbh} \frac{1}{\sin(k_{yn}^{lbh}t)} = A_n^{lh} k_{yn}^{lah} \cot(k_{yn}^{lah}d) \quad (6.10)$$

6.3 Mode Analysis of a Suspended Substrate Stripline

Solving (6.7) to (6.9), we get

$$C_n^{lh} = A_n^{lh} \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbh}} \sin(k_{yn}^{lbh} t) \cot(k_{yn}^{lah} d) + \cos(k_{yn}^{lbh} t) \right) \quad (6.11)$$

$$B_n^{lh} = A_n^{lh} \quad (6.12)$$

$$D_n^{lh} = A_n^{lh} \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbh}} \sin(k_{yn}^{lbh} t) \cot(k_{yn}^{lah} d) + \cos(k_{yn}^{lbh} t) \right) \quad (6.13)$$

Substitute (6.11), (6.12) and (6.13) into (6.10), for a non-trivial A_n^{lh} to exist, we obtain

$$\begin{aligned} & -\frac{k_{yn}^{lbh}}{\sin(k_{yn}^{lbh} t)} + k_{yn}^{lah} \cot(k_{yn}^{lah} d) \cos(k_{yn}^{lbh} t) + \frac{k_{yn}^{lah} k_{yn}^{lch}}{k_{yn}^{lbh}} \cot(k_{yn}^{lah} d) \sin(k_{yn}^{lbh} t) \cot(k_{yn}^{lch} s) \\ & + k_{yn}^{lbh} \cos(k_{yn}^{lbh} t) \cot(k_{yn}^{lbh} t) + k_{yn}^{lch} \cos(k_{yn}^{lbh} t) \cot(k_{yn}^{lch} s) = 0 \end{aligned} \quad (6.14)$$

Utilizing $k_{yn}^{lbh^2} = (k_{yn}^{lah})^2 + (\varepsilon_r - 1)k_0^2$, $k_{yn}^{lah} = k_{yn}^{lch}$ and equation (6.14), infinite numbers of k_{yn}^{lah} and k_{yn}^{lbh} can be calculated.

Once all the coefficients are determined, the transverse field components normal to x-axis can be written as

$$E_{yn}^{lh} = 0 \quad (6.15)$$

$$E_{zn}^{lh} = \begin{cases} A_n^{lh} j\omega\mu k_{xn}^{lh} \frac{\sin(k_{yn}^{lah} (b-y))}{\sin(k_{yn}^{lah} d)} \frac{\sin(k_{xn}^{lh} x)}{\cos(k_{xn}^{lh} x_0)} e^{-j\beta z} & s+t \leq y \leq b \\ A_n^{lh} j\omega\mu k_{xn}^{lh} \left[\frac{\sin(k_{yn}^{lbh} (y-s))}{\sin(k_{yn}^{lbh} t)} + \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbh}} \cot(k_{yn}^{lah} d) + \cot(k_{yn}^{lbh} t) \right) \sin(k_{yn}^{lbh} (s+t-y)) \right] \frac{\sin(k_{xn}^{lh} x)}{\cos(k_{xn}^{lh} x_0)} e^{-j\beta z} & s \leq y \leq s+t \\ A_n^{lh} j\omega\mu k_{xn}^{lh} \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbh}} \sin(k_{yn}^{lbh} t) \cot(k_{yn}^{lah} d) + \cos(k_{yn}^{lbh} t) \right) \frac{\sin(k_{yn}^{lch} y)}{\sin(k_{yn}^{lch} s)} \frac{\sin(k_{xn}^{lh} x)}{\cos(k_{xn}^{lh} x_0)} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.16)$$

6.3 Mode Analysis of a Suspended Substrate Stripline

$$H_{yn}^{lh} = \begin{cases} A_n^{lh} (k_0^2 - k_{yn}^{lah^2}) \frac{\sin(k_{yn}^{lah} (b-y))}{\sin(k_{yn}^{lah} d)} \frac{\cos(k_{xn}^{lh} x)}{\cos(k_{xn}^{lh} x_0)} e^{-j\beta z} & s+t \leq y \leq b \\ A_n^{lh} (k_0^2 - k_{yn}^{lah^2}) \left(\frac{\sin(k_{yn}^{lbh} (y-s))}{\sin(k_{yn}^{lbh} t)} + \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbh}} \cot(k_{yn}^{lah} d) + \cot(k_{yn}^{lbh} t) \right) \sin(k_{yn}^{lbh} (s+t-y)) \right) \frac{\cos(k_{xn}^{lh} x)}{\cos(k_{xn}^{lh} x_0)} e^{-j\beta z} & s \leq y \leq s+t \\ A_n^{lh} \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbh}} \sin(k_{yn}^{lbh} t) \cot(k_{yn}^{lah} d) + \cos(k_{yn}^{lbh} t) \right) (k_0^2 - k_{yn}^{lah^2}) \frac{\sin(k_{yn}^{lch} y)}{\sin(k_{yn}^{lch} s)} \frac{\cos(k_{xn}^{lh} x)}{\cos(k_{xn}^{lh} x_0)} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.17)$$

$$H_{zn}^{lh} = \begin{cases} A_n^{lh} j\beta k_{yn}^{lah} \frac{\cos(k_{yn}^{lah} (b-y))}{\sin(k_{yn}^{lah} d)} \frac{\cos(k_{xn}^{lh} x)}{\cos(k_{xn}^{lh} x_0)} e^{-j\beta z} & s+t \leq y \leq b \\ A_n^{lh} j\beta \left(-k_{yn}^{lbh} \frac{\cos(k_{yn}^{lbh} (y-s))}{\sin(k_{yn}^{lbh} t)} + \left(k_{yn}^{lah} \cot(k_{yn}^{lah} d) + k_{yn}^{lbh} \cot(k_{yn}^{lbh} t) \right) \cos(k_{yn}^{lbh} (s+t-y)) \right) \frac{\cos(k_{xn}^{lh} x)}{\cos(k_{xn}^{lh} x_0)} e^{-j\beta z} & s \leq y \leq s+t \\ -A_n^{lh} j\beta k_{yn}^{lch} \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbh}} \sin(k_{yn}^{lbh} t) \cot(k_{yn}^{lah} d) + \cos(k_{yn}^{lbh} t) \right) \frac{\cos(k_{yn}^{lch} y)}{\sin(k_{yn}^{lch} s)} \frac{\cos(k_{xn}^{lh} x)}{\cos(k_{xn}^{lh} x_0)} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.18)$$

For LSM modes, using (6.2), (6.3) and (6.4), H_z and E_x can be obtained as

$$E_{xn}^{le} = \frac{\partial^2 \phi^{le}}{\partial x \partial y} = \begin{cases} A_n^{le} k_{xn}^{le} k_{yn}^{lae} \frac{\sin(k_{yn}^{lae} (b-y))}{\cos(k_{yn}^{lae} d)} \frac{\cos(k_{xn}^{le} x)}{\sin(k_{xn}^{le} x_0)} e^{-j\beta z} & s+t \leq y \leq b \\ \left(-B_n^{le} k_{yn}^{lbe} \frac{\sin(k_{yn}^{lbe} (y-s))}{\cos(k_{yn}^{lbe} t)} + C_n^{le} k_{yn}^{lbe} \frac{\sin(k_{yn}^{lbe} (s+t-y))}{\cos(k_{yn}^{lbe} t)} \right) k_{xn}^{le} \frac{\cos(k_{xn}^{le} x)}{\sin(k_{xn}^{le} x_0)} e^{-j\beta z} & s \leq y \leq s+t \\ -D_n^{le} k_{xn}^{le} k_{yn}^{lce} \frac{\sin(k_{yn}^{lce} y)}{\cos(k_{yn}^{lce} s)} \frac{\cos(k_{xn}^{le} x)}{\sin(k_{xn}^{le} x_0)} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.19)$$

6.3 Mode Analysis of a Suspended Substrate Stripline

$$H_{zn}^{le} = j\omega\epsilon(y)\frac{\partial\phi^{le}}{\partial x} = \begin{cases} A_n^{le} j\omega\epsilon_0 k_{xn}^{le} \frac{\cos(k_{yn}^{lae}(b-y))}{\cos(k_{yn}^{lae}d)} \frac{\cos(k_{xn}^{le}x)}{\sin(k_{xn}^{le}x_0)} e^{-j\beta z} & s+t \leq y \leq b \\ j\omega\epsilon_0 \epsilon_r k_{xn}^{le} \left(\begin{aligned} & B_n^{le} \frac{\cos(k_{yn}^{lbe}(y-s))}{\cos(k_{yn}^{lbe}t)} \\ & + C_n^{le} \frac{\cos(k_{yn}^{lbe}(s+t-y))}{\cos(k_{yn}^{lbe}t)} \end{aligned} \right) \frac{\cos(k_{xn}^{le}x)}{\sin(k_{xn}^{le}x_0)} e^{-j\beta z} & s \leq y \leq s+t \\ D_n^{le} j\omega\epsilon_0 k_{xn}^{le} \frac{\cos(k_{yn}^{lce}y)}{\cos(k_{yn}^{lce}s)} \frac{\cos(k_{xn}^{le}x)}{\sin(k_{xn}^{le}x_0)} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.20)$$

Similarly, by matching H_z and E_x at $y=t$, we obtain

$$A_n^{le} k_{yn}^{lae} \tan(k_{yn}^{lae}d) = -B_n^{le} k_{yn}^{lbe} \tan(k_{yn}^{lbe}t) \quad (6.21)$$

$$C_n^{le} k_{yn}^{lbe} \tan(k_{yn}^{lbe}(t)) = -D_n^{le} k_{yn}^{lce} \tan(k_{yn}^{lce}s) \quad (6.22)$$

$$A_n^{le} = \left(B_n^{le} + C_n^{le} \frac{1}{\cos(k_{yn}^{lbe}t)} \right) \epsilon_r \quad (6.23)$$

$$\left(B_n^{le} \frac{1}{\cos(k_{yn}^{lbe}t)} + C_n^{le} \right) \epsilon_r = D_n^{le} \quad (6.24)$$

Solving (6.21), (6.22) and (6.23), the coefficients B_n^{le} , C_n^{le} and D_n^{le} can be obtained as

$$B_n^{le} = -A_n^{le} \frac{k_{yn}^{lae}}{k_{yn}^{lbe}} \tan(k_{yn}^{lae}d) \cot(k_{yn}^{lbe}t) \quad (6.25)$$

$$C_n^{le} = A_n^{le} \cos(k_{yn}^{lbe}t) \left(\frac{1}{\epsilon_r} + \frac{k_{yn}^{lae}}{k_{yn}^{lbe}} \tan(k_{yn}^{lae}d) \cot(k_{yn}^{lbe}t) \right) \quad (6.26)$$

$$D_n^{le} = -A_n^{le} \left(\frac{1}{\epsilon_r} \frac{k_{yn}^{lbe}}{k_{yn}^{lce}} \sin(k_{yn}^{lbe}t) \cot(k_{yn}^{lce}s) + \frac{k_{yn}^{lae}}{k_{yn}^{lce}} \cos(k_{yn}^{lbe}t) \tan(k_{yn}^{lae}d) \cot(k_{yn}^{lce}s) \right) \quad (6.27)$$

Substituting (6.25), (6.26) and (6.27) into (6.24), for a non-trivial A_n^{le} to exist, we have

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$$\begin{aligned}
& -\varepsilon_r \frac{k_{yn}^{Iae}}{k_{yn}^{Ibe}} \tan(k_{yn}^{Iae} d) \frac{1}{\sin(k_{yn}^{Ibe} t)} + \cos(k_{yn}^{Ibe} t) + \varepsilon_r \frac{k_{yn}^{Iae}}{k_{yn}^{Ibe}} \tan(k_{yn}^{Iae} d) \cot(k_{yn}^{Ibe} t) \cos(k_{yn}^{Ibe} t) \\
& + \frac{k_{yn}^{Ibe}}{\varepsilon_r k_{yn}^{Ice}} \sin(k_{yn}^{Ibe} t) \cot(k_{yn}^{Ice} s) + \tan(k_{yn}^{Iae} d) \cos(k_{yn}^{Ibe} t) \cot(k_{yn}^{Ice} s) = 0
\end{aligned} \tag{6.28}$$

Solving (6.28), $k_{yn}^{Ibe^2} = (k_{yn}^{Iae})^2 + (\varepsilon_r - 1)k_0^2$ and $k_{yn}^{Iae} = k_{yn}^{Ice}$ we can get infinite numbers of k_{yn}^{Iae} and k_{yn}^{Ibe} .

For LSM modes, the transverse field components normal to x-axis can be solved as

$$E_{yn}^{Ie} = \begin{cases} A_n^{Ie} \left(k_0^2 - k_{yn}^{Iae^2} \right) \frac{\cos(k_{yn}^{Iae} (b-y))}{\cos(k_{yn}^{Iae} d)} \frac{\sin(k_{xn}^{Ie} x)}{\sin(k_{xn}^{Ie} x_0)} e^{-j\beta z} & s+t \leq y \leq b \\ \left(-\frac{k_{yn}^{Iae}}{k_{yn}^{Ibe}} \tan(k_{yn}^{Iae} d) \frac{\cos(k_{yn}^{Ibe} (y-s))}{\sin(k_{yn}^{Ibe} t)} \right. \\ \left. + \left(\frac{1}{\varepsilon_r} + \frac{k_{yn}^{Iae}}{k_{yn}^{Ibe}} \tan(k_{yn}^{Iae} d) \cot(k_{yn}^{Ibe} t) \right) \cos(k_{yn}^{Ibe} (s+t-y)) \right) \left(k_0^2 - k_{yn}^{Iae^2} \right) \frac{\sin(k_{xn}^{Ie} x)}{\sin(k_{xn}^{Ie} x_0)} e^{-j\beta z} & s \leq y \leq s+t \\ -A_n^{Ie} \left(\frac{1}{\varepsilon_r} \frac{k_{yn}^{Ibe}}{k_{yn}^{Iae}} \sin(k_{yn}^{Ibe} t) + \cos(k_{yn}^{Ibe} t) \tan(k_{yn}^{Iae} d) \right) \left(k_0^2 - k_{yn}^{Iae^2} \right) \frac{\cos(k_{yn}^{Iae} y)}{\sin(k_{yn}^{Iae} s)} \frac{\sin(k_{xn}^{Ie} x)}{\sin(k_{xn}^{Ie} x_0)} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \tag{6.29}$$

$$E_{zn}^{Ie} = \begin{cases} -A_n^{Ie} j\beta k_{yn}^{Iae} \frac{\sin(k_{yn}^{Iae} (b-y))}{\cos(k_{yn}^{Iae} d)} \frac{\sin(k_{xn}^{Ie} x)}{\sin(k_{xn}^{Ie} x_0)} e^{-j\beta z} & s+t \leq y \leq b \\ \left(k_{yn}^{Iae} \tan(k_{yn}^{Iae} d) \frac{\sin(k_{yn}^{Ibe} (y-s))}{\sin(k_{yn}^{Ibe} t)} \right. \\ \left. + \left(\frac{k_{yn}^{Ibe}}{\varepsilon_r} + k_{yn}^{Iae} \tan(k_{yn}^{Iae} d) \cot(k_{yn}^{Ibe} t) \right) \sin(k_{yn}^{Ibe} (s+t-y)) \right) \frac{\sin(k_{xn}^{Ie} x)}{\sin(k_{xn}^{Ie} x_0)} e^{-j\beta z} & s \leq y \leq s+t \\ -A_n^{Ie} j\beta \left(\frac{1}{\varepsilon_r} k_{yn}^{Ibe} \sin(k_{yn}^{Ibe} t) + k_{yn}^{Iae} \cos(k_{yn}^{Ibe} t) \tan(k_{yn}^{Iae} d) \right) \frac{\sin(k_{yn}^{Iae} y)}{\sin(k_{yn}^{Iae} s)} \frac{\sin(k_{xn}^{Ie} x)}{\sin(k_{xn}^{Ie} x_0)} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \tag{6.30}$$

$$H_{yn}^{Ie} = 0 \tag{6.31}$$

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$$H_{zn}^{le} = \begin{cases} A_n^{le} j\omega\epsilon_0 k_{xn}^{le} \frac{\cos(k_{yn}^{lae}(b-y)) \cos(k_{xn}^{le}x)}{\cos(k_{yn}^{lae}d) \sin(k_{xn}^{le}x_0)} e^{-j\beta z} & s+t \leq y \leq b \\ A_n^{le} j\omega\epsilon_0 \epsilon_r k_{xn}^{le} \left[-\frac{k_{yn}^{lae}}{k_{yn}^{lbe}} \tan(k_{yn}^{lae}d) \frac{\cos(k_{yn}^{lbe}(y-s))}{\sin(k_{yn}^{lbe}t)} + \left(\frac{1}{\epsilon_r} + \frac{k_{yn}^{lae}}{k_{yn}^{lbe}} \tan(k_{yn}^{lae}d) \cot(k_{yn}^{lbe}t) \right) \cos(k_{yn}^{lbe}(s+t-y)) \right] \frac{\cos(k_{xn}^{le}x)}{\sin(k_{xn}^{le}x_0)} e^{-j\beta z} & s \leq y \leq s+t \\ -A_n^{le} j\omega\epsilon_0 k_{xn}^{le} \left(\frac{k_{yn}^{lbe}}{\epsilon_r k_{yn}^{lae}} \sin(k_{yn}^{lbe}t) + \cos(k_{yn}^{lbe}t) \tan(k_{yn}^{lae}d) \right) \frac{\cos(k_{yn}^{lce}y) \cos(k_{xn}^{le}x)}{\sin(k_{yn}^{lae}s) \sin(k_{xn}^{le}x_0)} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.32)$$

6.3.2 Field Expressions in Region II

In region II, the Hertzian potential for LSE and LSM modes can be written as

$$\Pi_n^{lh} = A_n^{lh} \sin(k_{yn}^{lh}(b-y)) \frac{\sin(k_{xn}^{lh}(a-x))}{\sin(k_{xn}^{lh}(a-x_0))} e^{-j\beta z} \hat{a}_y \quad \text{for LSE modes} \quad (6.33)$$

$$\Pi_n^{le} = A_n^{le} \cos(k_{yn}^{le}(b-y)) \frac{\cos(k_{xn}^{le}(a-x))}{\cos(k_{xn}^{le}(a-x_0))} e^{-j\beta z} \hat{a}_y \quad \text{for LSM modes} \quad (6.34)$$

Utilizing (6.3) and (6.4), the field components for LSE modes can be derived as

$$E_{yn}^{lh} = 0 \quad (6.35)$$

$$E_{zn}^{lh} = A_n^{lh} j\omega\mu k_{xn}^{lh} \sin(k_{yn}^{lh}(b-y)) \frac{\cos(k_{xn}^{lh}(a-x))}{\sin(k_{xn}^{lh}(a-x_0))} e^{-j\beta z} \quad (6.36)$$

$$H_{yn}^{lh} = A_n^{lh} (k_0^2 - k_{yn}^{lh2}) \sin(k_{yn}^{lh}(b-y)) \frac{\sin(k_{xn}^{lh}(a-x))}{\sin(k_{xn}^{lh}(a-x_0))} e^{-j\beta z} \quad (6.37)$$

$$H_{zn}^{lh} = A_n^{lh} j\beta k_{yn}^{lh} \cos(k_{yn}^{lh}(b-y)) \frac{\sin(k_{xn}^{lh}(a-x))}{\sin(k_{xn}^{lh}(a-x_0))} e^{-j\beta z} \quad (6.38)$$

The field components for LSM modes are

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$$E_{yn}^{IIe} = A_n^{IIe} (k_0^2 - k_{yn}^{IIe2}) \cos(k_{yn}^{IIe} (b - y)) \frac{\cos(k_{xn}^{IIe} (a - x))}{\cos(k_{xn}^{IIe} (a - x_0))} e^{-j\beta z} \quad (6.39)$$

$$E_{zn}^{IIe} = A_n^{IIe} k_{yn}^{IIe} (-j\beta) \sin(k_{yn}^{IIe} (b - y)) \frac{\cos(k_{xn}^{IIe} (a - x))}{\cos(k_{xn}^{IIe} (a - x_0))} e^{-j\beta z} \quad (6.40)$$

$$H_{yn}^{IIe} = 0 \quad (6.41)$$

$$H_{zn}^{IIe} = A_n^{IIe} j\omega\epsilon_0 k_{xn}^{IIe} \cos(k_{yn}^{IIe} (b - y)) \frac{\sin(k_{xn}^{IIe} (a - x))}{\cos(k_{xn}^{IIe} (a - x_0))} e^{-j\beta z} \quad (6.42)$$

where $k_{yn}^{IIe} = \frac{(n-1)\pi}{b}$, $k_{yn}^{IIIh} = \frac{n\pi}{b}$ and $k_{xn}^{IIp} = \sqrt{k_0^2 - k_{yn}^{IIp2} - \beta^2}$.

6.3.3 Field Expressions in Region III

The Hertzian potential in Region III can be written as

$$\Pi_n^{IIIh} = \begin{cases} A_n^{IIIh} \frac{\sin(k_{yn}^{IIIh} (s + t - y))}{\sin(k_{yn}^{IIIh} t)} \frac{\sin(k_{xn}^{IIIh} (a - x))}{\sin(k_{xn}^{IIIh} (a - x_0))} e^{-j\beta z} \hat{a}_y & s \leq y \leq s + t \\ B_n^{IIIh} \frac{\sin(k_{yn}^{IIIh} y)}{\sin(k_{yn}^{IIIh} s)} \frac{\sin(k_{xn}^{IIIh} (a - x))}{\sin(k_{xn}^{IIIh} (a - x_0))} e^{-j\beta z} \hat{a}_y & 0 \leq y \leq s \end{cases} \quad \text{LSE modes} \quad (6.43)$$

$$\Pi_n^{IIIe} = \begin{cases} A_n^{IIIe} \frac{\cos(k_{yn}^{IIIe} (s + t - y))}{\cos(k_{yn}^{IIIe} t)} \frac{\cos(k_{xn}^{IIIe} (a - x))}{\cos(k_{xn}^{IIIe} (a - x_0))} e^{-j\beta z} \hat{a}_y & s \leq y \leq s + t \\ B_n^{IIIe} \frac{\cos(k_{yn}^{IIIe} y)}{\cos(k_{yn}^{IIIe} s)} \frac{\cos(k_{xn}^{IIIe} (a - x))}{\cos(k_{xn}^{IIIe} (a - x_0))} e^{-j\beta z} \hat{a}_y & 0 \leq y \leq s \end{cases} \quad \text{LSM modes} \quad (6.44)$$

where $k_{xn}^{IIIp} = \sqrt{\epsilon_r k_0^2 - (k_{yn}^{IIIp})^2 - \beta^2}$, $p = h, e$

Utilizing (6.3) and (6.4), for LSE modes E_z and H_x can be derived as

$$E_{zn}^{IIIh} = -j\omega\mu \frac{\partial \phi_n^{IIIh}}{\partial x} = \begin{cases} A_n^{IIIh} j\omega\mu k_{xn}^{IIIh} \frac{\sin(k_{yn}^{IIIh} (s + t - y))}{\sin(k_{yn}^{IIIh} t)} \frac{\cos(k_{xn}^{IIIh} (a - x))}{\sin(k_{xn}^{IIIh} (a - x_0))} e^{-j\beta z} & s \leq y \leq s + t \\ B_n^{IIIh} j\omega\mu k_{xn}^{IIIh} \frac{\sin(k_{yn}^{IIIh} y)}{\sin(k_{yn}^{IIIh} s)} \frac{\cos(k_{xn}^{IIIh} (a - x))}{\sin(k_{xn}^{IIIh} (a - x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.45)$$

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$$H_{xn}^{IIIh} = \frac{\partial^2 \phi_n^{IIIh}}{\partial x \partial y} = \begin{cases} A_n^{IIIh} k_{xn}^{IIIh} k_{yn}^{IIIh} \frac{\cos(k_{yn}^{IIIah}(s+t-y))}{\sin(k_{yn}^{IIIah}t)} \frac{\cos(k_{xn}^{IIIh}(a-x))}{\sin(k_{xn}^{IIIh}(a-x_0))} e^{-j\beta z} & s \leq y \leq s+t \\ -B_n^{IIIh} k_{xn}^{IIIh} k_{yn}^{IIIh} \frac{\cos(k_{yn}^{IIIbh}y)}{\sin(k_{yn}^{IIIbh}s)} \frac{\cos(k_{xn}^{IIIh}(a-x))}{\sin(k_{xn}^{IIIh}(a-x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.46)$$

Match E_z and H_x at $y=t$, we get

$$A_n^{IIIh} = B_n^{IIIh} \quad (6.47)$$

$$A_n^{IIIh} k_{yn}^{IIIah} \cot(k_{yn}^{IIIah}t) = -B_n^{IIIh} k_{yn}^{IIIbh} \cot(k_{yn}^{IIIbh}s) \quad (6.48)$$

Solving (6.47), (6.48) and $(k_{yn}^{IIIah})^2 = (k_{yn}^{IIIbh})^2 + (\epsilon_r - 1)k_0^2$, we can get infinite numbers of

k_{yn}^{IIIah} and k_{yn}^{IIIbh} .

The field in region III for LSE mode can be written as

$$E_{yn}^{IIIh} = 0 \quad (6.49)$$

$$E_{zn}^{IIIh} = \begin{cases} A_n^{IIIh} j\omega\mu k_{xn}^{IIIh} \frac{\sin(k_{yn}^{IIIah}(s+t-y))}{\sin(k_{yn}^{IIIah}t)} \frac{\cos(k_{xn}^{IIIh}(a-x))}{\sin(k_{xn}^{IIIh}(a-x_0))} e^{-j\beta z} & s \leq y \leq s+t \\ A_n^{IIIh} j\omega\mu k_{xn}^{IIIh} \frac{\sin(k_{yn}^{IIIbh}y)}{\sin(k_{yn}^{IIIbh}s)} \frac{\cos(k_{xn}^{IIIh}(a-x))}{\sin(k_{xn}^{IIIh}(a-x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.50)$$

$$H_{yn}^{IIIh} = \begin{cases} A_n^{IIIh} (k_0^2 - k_{yn}^{IIIbh^2}) \frac{\sin(k_{yn}^{IIIah}(s+t-y))}{\sin(k_{yn}^{IIIah}t)} \frac{\sin(k_{xn}^{IIIh}(a-x))}{\sin(k_{xn}^{IIIh}(a-x_0))} e^{-j\beta z} & s \leq y \leq s+t \\ A_n^{IIIh} (k_0^2 - k_{yn}^{IIIbh^2}) \frac{\sin(k_{yn}^{IIIbh}y)}{\sin(k_{yn}^{IIIbh}s)} \frac{\sin(k_{xn}^{IIIh}(a-x))}{\sin(k_{xn}^{IIIh}(a-x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.51)$$

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$$H_{zn}^{IIIh} = \begin{cases} A_n^{IIIh} j\beta k_{yn}^{IIIah} \frac{\cos(k_{yn}^{IIIah}(s+t-y))}{\sin(k_{yn}^{IIIah}t)} \frac{\sin(k_{xn}^{IIIh}(a-x))}{\sin(k_{xn}^{IIIh}(a-x_0))} e^{-j\beta z} & s \leq y \leq s+t \\ -A_n^{IIIh} j\beta k_{yn}^{IIIbh} \frac{\cos(k_{yn}^{IIIbh}y)}{\sin(k_{yn}^{IIIbh}s)} \frac{\sin(k_{xn}^{IIIh}(a-x))}{\sin(k_{xn}^{IIIh}(a-x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.52)$$

where $k_{xn}^{IIIh} = \sqrt{\epsilon_r k_0^2 - (k_{yn}^{IIIah})^2 - \beta^2}$.

For LSM modes,

$$E_{xn}^{IIIe} = \begin{cases} A_n^{IIIe} k_{xn}^{IIIe} k_{yn}^{IIIae} \frac{\sin(k_{yn}^{IIIae}(s+t-y))}{\cos(k_{yn}^{IIIae}t)} \frac{\sin(k_{xn}^{IIIe}(a-x))}{\cos(k_{xn}^{IIIe}(a-x_0))} e^{-j\beta z} & s \leq y \leq s+t \\ -B_n^{IIIe} k_{xn}^{IIIe} k_{yn}^{IIIbe} \frac{\sin(k_{yn}^{IIIbe}y)}{\cos(k_{yn}^{IIIbe}s)} \frac{\sin(k_{xn}^{IIIe}(a-x))}{\cos(k_{xn}^{IIIe}(a-x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.53)$$

$$E_{yn}^{IIIe} = \begin{cases} A_n^{IIIe} (k_0^2 - k_{yn}^{IIIbe^2}) \frac{\cos(k_{yn}^{IIIae}(s+t-y))}{\cos(k_{yn}^{IIIae}t)} \frac{\cos(k_{xn}^{IIIe}(a-x))}{\cos(k_{xn}^{IIIe}(a-x_0))} e^{-j\beta z} & s \leq y \leq s+t \\ B_n^{IIIe} (k_0^2 - k_{yn}^{IIIbe^2}) \frac{\cos(k_{yn}^{IIIbe}y)}{\cos(k_{yn}^{IIIbe}s)} \frac{\cos(k_{xn}^{IIIe}(a-x))}{\cos(k_{xn}^{IIIe}(a-x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.54)$$

$$E_{zn}^{IIIe} = \begin{cases} -A_n^{IIIe} j\beta k_{yn}^{IIIae} \frac{\sin(k_{yn}^{IIIae}(s+t-y))}{\cos(k_{yn}^{IIIae}t)} \frac{\cos(k_{xn}^{IIIe}(a-x))}{\cos(k_{xn}^{IIIe}(a-x_0))} e^{-j\beta z} & s \leq y \leq s+t \\ B_n^{IIIe} j\beta k_{yn}^{IIIbe} \frac{\sin(k_{yn}^{IIIbe}y)}{\cos(k_{yn}^{IIIbe}s)} \frac{\cos(k_{xn}^{IIIe}(a-x))}{\cos(k_{xn}^{IIIe}(a-x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.55)$$

$$H_{zn}^{IIIe} = \begin{cases} A_n^{IIIe} j\omega\epsilon_0\epsilon_r k_{xn}^{IIIe} \frac{\cos(k_{yn}^{IIIae}(s+t-y))}{\cos(k_{yn}^{IIIae}t)} \frac{\sin(k_{xn}^{IIIe}(a-x))}{\cos(k_{xn}^{IIIe}(a-x_0))} e^{-j\beta z} & s \leq y \leq s+t \\ B_n^{IIIe} j\omega\epsilon_0 k_{xn}^{IIIe} \frac{\cos(k_{yn}^{IIIbe}y)}{\cos(k_{yn}^{IIIbe}s)} \frac{\sin(k_{xn}^{IIIe}(a-x))}{\cos(k_{xn}^{IIIe}(a-x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.56)$$

Similarly, by matching H_z and E_x at $y=t$ we get

$$A_n^{IIIe} \epsilon_r = B_n^{IIIe} \quad (6.57)$$

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$$A_n^{IIIe} k_{yn}^{IIIae} \tan(k_{yn}^{IIIae} t) = -B_n^{IIIe} k_{yn}^{IIIbe} \tan(k_{yn}^{IIIbe} s) \quad (6.58)$$

$$k_{yn}^{IIIae} \tan(k_{yn}^{IIIae} t) = -\varepsilon_r k_{yn}^{IIIbe} \tan(k_{yn}^{IIIbe} s) \quad (6.59)$$

Solve $\begin{cases} k_{yn}^{IIIae} \tan(k_{yn}^{IIIae} t) = -\varepsilon_r k_{yn}^{IIIbe} \tan(k_{yn}^{IIIbe} s) \\ (k_{yn}^{IIIae})^2 = (k_{yn}^{IIIbe})^2 + (\varepsilon_r - 1)k_0^2 \end{cases}$ we can compute infinite numbers of k_{yn}^{IIIae}

and k_{yn}^{IIIbe} .

Using $B_n^{IIIe} = \varepsilon_r A_n^{IIIe}$, the field for LSM mode can be written as

$$E_{yn}^{IIIe} = \begin{cases} A_n^{IIIe} (k_0^2 - k_{yn}^{IIIbe2}) \frac{\cos(k_{yn}^{IIIae} (s+t-y))}{\cos(k_{yn}^{IIIae} t)} \frac{\cos(k_{xn}^{IIIe} (a-x))}{\cos(k_{xn}^{IIIe} (a-x_0))} e^{-j\beta z} & s \leq y \leq s+t \\ A_n^{IIIe} \varepsilon_r (k_0^2 - k_{yn}^{IIIbe2}) \frac{\cos(k_{yn}^{IIIbe} y)}{\cos(k_{yn}^{IIIbe} s)} \frac{\cos(k_{xn}^{IIIe} (a-x))}{\cos(k_{xn}^{IIIe} (a-x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.60)$$

$$E_{zn}^{IIIe} = \begin{cases} -A_n^{IIIe} j\beta k_{yn}^{IIIae} \frac{\sin(k_{yn}^{IIIae} (s+t-y))}{\cos(k_{yn}^{IIIae} t)} \frac{\cos(k_{xn}^{IIIe} (a-x))}{\cos(k_{xn}^{IIIe} (a-x_0))} e^{-j\beta z} & s \leq y \leq s+t \\ A_n^{IIIe} \varepsilon_r j\beta k_{yn}^{IIIbe} \frac{\sin(k_{yn}^{IIIbe} y)}{\cos(k_{yn}^{IIIbe} s)} \frac{\cos(k_{xn}^{IIIe} (a-x))}{\cos(k_{xn}^{IIIe} (a-x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.61)$$

$$H_{yn}^{IIIe} = 0 \quad (6.62)$$

$$H_{zn}^{IIIe} = \begin{cases} A_n^{IIIe} j\omega\varepsilon_0 \varepsilon_r k_{xn}^{IIIe} \frac{\cos(k_{yn}^{IIIae} (s+t-y))}{\cos(k_{yn}^{IIIae} t)} \frac{\sin(k_{xn}^{IIIe} (a-x))}{\cos(k_{xn}^{IIIe} (a-x_0))} e^{-j\beta z} & s \leq y \leq s+t \\ A_n^{IIIe} j\omega\varepsilon_0 \varepsilon_r k_{xn}^{IIIe} \frac{\cos(k_{yn}^{IIIbe} y)}{\cos(k_{yn}^{IIIbe} s)} \frac{\sin(k_{xn}^{IIIe} (a-x))}{\cos(k_{xn}^{IIIe} (a-x_0))} e^{-j\beta z} & 0 \leq y \leq s \end{cases} \quad (6.63)$$

where $k_{xn}^{IIIe} = \sqrt{\varepsilon_r k_0^2 - (k_{yn}^{IIIae})^2 - \beta^2}$.

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6.3.4 Dispersion Analysis

Matching the E field at $z=0$ and $x=x_0$, we get

$$E_t^I = \begin{cases} E_t^{II} & t \leq y \leq b \\ E_t^{III} & 0 \leq y \leq t \end{cases} \quad (6.64)$$

$$\sum_{n=1}^N (A_n^{Ih} \bar{e}_m^{Ih} + A_n^{Ie} \bar{e}_m^{Ie}) = \begin{cases} \sum_{n=0}^N (A_n^{IIh} \bar{e}_m^{IIh} + A_n^{IIe} \bar{e}_m^{IIe}) & t \leq y \leq b \\ \sum_{n=0}^N (A_n^{IIIh} \bar{e}_m^{IIIh} + A_n^{IIIe} \bar{e}_m^{IIIe}) & 0 \leq y \leq t \end{cases} \quad (6.65)$$

Taking cross product with $\bar{h}_j^I$ and invoking the following orthogonality relation

$$\int_0^b \bar{e}_n^{Ip} \times \bar{h}_{ij}^{Ip} dy = 0, \quad p = h, e \quad \text{we obtain}$$

$$\mathbf{U}^I \mathbf{A}^I = \mathbf{P}^{II} \mathbf{A}^{II} + \mathbf{P}^{III} \mathbf{A}^{III} \quad (6.66)$$

$$\text{where } \mathbf{A}^I = \begin{bmatrix} \mathbf{A}^{Ih} \\ \mathbf{A}^{Ie} \end{bmatrix}, \quad \mathbf{A}^{II} = \begin{bmatrix} \mathbf{A}^{IIh} \\ \mathbf{A}^{IIe} \end{bmatrix}, \quad \mathbf{A}^{III} = \begin{bmatrix} \mathbf{A}^{IIIh} \\ \mathbf{A}^{IIIe} \end{bmatrix}, \quad \mathbf{U}^I = \begin{bmatrix} \mathbf{U}_{hh}^I & \mathbf{U}_{eh}^I \\ \mathbf{0} & \mathbf{U}_{ee}^I \end{bmatrix}, \quad \mathbf{P}^{II} = \begin{bmatrix} \mathbf{P}_{hh}^{II} & \mathbf{P}_{eh}^{II} \\ \mathbf{0} & \mathbf{P}_{ee}^{II} \end{bmatrix},$$

$$\mathbf{P}^{III} = \begin{bmatrix} \mathbf{P}_{hh}^{III} & \mathbf{P}_{eh}^{III} \\ \mathbf{0} & \mathbf{P}_{ee}^{III} \end{bmatrix}.$$

The elements of the matrix are

$$\begin{aligned} U_{hh}^I(j, j) &= \int_0^b \bar{e}_{ij}^{Ih} \times \bar{h}_{ij}^{Ih} dy \\ &= -C_{zj}^{e-} C_{yj}^{h-} \int_0^s \sin^2(k_{yj}^{Iah} y) dy - C_{zj}^{e-} C_{yj}^{h-} \int_s^{s+t} \sin^2(k_{yj}^{Ibh} (y-s)) dy \\ &\quad - (C_{zj}^{e-} C_{yj}^{h-} \int_s^{s+t} \sin^2(k_{yj}^{Ibh} (y-s)) dy + C_{zj}^{e-} C_{yj}^{h-} \int_s^{s+t} \sin^2(k_{yj}^{Ibh} (s+t-y)) dy) \\ &\quad - C_{zj}^{e-} C_{yj}^{h-} \int_s^{s+t} \sin^2(k_{yj}^{Ibh} (s+t-y)) dy - C_{zj}^{e-} C_{yj}^{h-} \int_{s+t}^b \sin^2(k_{yj}^{Iah} (b-y)) dy \end{aligned}$$

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$$\begin{aligned}
U_{ee}^I(j, j) &= \int_0^b \bar{e}_{ij}^{Ie} \times \bar{h}_{ij}^{Ie} dy \\
&= C_{yj}^{e-Ice} C_{zj}^{h-Ice} \int_0^s \cos^2(k_{yj}^{Iae} y) dy + C_{yj}^{e-Ibe1} C_{zj}^{h-Ibe1} \int_s^{s+t} \cos^2(k_{yj}^{Ibe}(y-s)) dy \\
&\quad + (C_{yj}^{e-Ibe1} C_{zj}^{h-Ibe2} + C_{yj}^{e-Ibe2} C_{zj}^{h-Ibe1}) \int_s^{s+t} \cos(k_{yj}^{Ibe}(y-s)) \cos(k_{yj}^{Ibe}(s+t-y)) dy \\
&\quad + C_{yj}^{e-Ibe2} C_{zj}^{h-Ibe2} \int_s^{s+t} \cos^2(k_{yj}^{Ibe}(s+t-y)) dy + C_{yj}^{e-Iae} C_{zj}^{h-Iae} \int_{s+t}^b \cos^2(k_{yj}^{Iae}(b-y)) dy \\
\\
U_{eh}^I(i, j) &= \int_0^b \bar{e}_{ij}^{Ie} \times \bar{h}_{ii}^{Ih} dy \\
&= C_{yj}^{e-Ice} C_{zi}^{h-Ich} \int_0^s \cos(k_{yj}^{Iae} y) \cos(k_{yi}^{Iah} y) dy + C_{yj}^{e-Ibe1} C_{zi}^{h-Ibh1} \int_s^{s+t} \cos(k_{yj}^{Ibe}(y-s)) \cos(k_{yi}^{Ibh}(y-s)) dy \\
&\quad + C_{yj}^{e-Ibe1} C_{zi}^{h-Ibh2} \int_s^{s+t} \cos(k_{yj}^{Ibe}(y-s)) \cos(k_{yi}^{Ibh}(s+t-y)) dy + C_{yj}^{e-Ibe2} C_{zi}^{h-Ibh1} \int_s^{s+t} \cos(k_{yj}^{Ibe}(s+t-y)) \cos(k_{yi}^{Ibh}(y-s)) dy \\
&\quad + C_{yj}^{e-Ibe2} C_{zi}^{h-Ibh2} \int_s^{s+t} \cos(k_{yj}^{Ibe}(s+t-y)) \cos(k_{yi}^{Ibh}(s+t-y)) dy + C_{yj}^{e-Iae} C_{zi}^{h-Iah} \int_{s+t}^b \cos(k_{yj}^{Iae}(b-y)) \cos(k_{yi}^{Iah}(b-y)) dy \\
&\quad - C_{zj}^{e-Ice} C_{yi}^{h-Ich} \int_0^s \sin(k_{yj}^{Iae} y) \sin(k_{yi}^{Iah} y) dy - C_{zj}^{e-Ibe1} C_{yi}^{h-Ibh1} \int_s^{s+t} \sin(k_{yj}^{Ibe}(y-s)) \sin(k_{yi}^{Ibh}(y-s)) dy \\
&\quad - C_{zj}^{e-Ibe1} C_{yi}^{h-Ibh2} \int_s^{s+t} \sin(k_{yj}^{Ibe}(y-s)) \sin(k_{yi}^{Ibh}(s+t-y)) dy - C_{zj}^{e-Ibe2} C_{yi}^{h-Ibh1} \int_s^{s+t} \sin(k_{yj}^{Ibe}(s+t-y)) \sin(k_{yi}^{Ibh}(y-s)) dy \\
&\quad - C_{zj}^{e-Ibe2} C_{yi}^{h-Ibh2} \int_s^{s+t} \sin(k_{yj}^{Ibe}(s+t-y)) \sin(k_{yi}^{Ibh}(s+t-y)) dy - C_{zj}^{e-Iae} C_{yi}^{h-Iah} \int_{s+t}^b \sin(k_{yj}^{Iae}(b-y)) \sin(k_{yi}^{Iah}(b-y)) dy \\
\\
P_{hh}^{II}(i, j) &= \int_{s+t}^b \bar{e}_{ij}^{IIh} \times \bar{h}_{ii}^{IIh} dy = -C_{zj}^{e-IIh} C_{yi}^{h-Iah} \int_{s+t}^b \sin(k_{yj}^{IIh}(b-y)) \sin(k_{yi}^{Iah}(b-y)) dy, \\
\\
P_{eh}^{II}(i, j) &= \int_{s+t}^b \bar{e}_{ij}^{IIe} \times \bar{h}_{ii}^{IIh} dy \\
&= C_{yj}^{e-IIe} C_{zi}^{h-Iah} \int_{s+t}^b \cos(k_{yj}^{IIe}(b-y)) \cos(k_{yi}^{Iah}(b-y)) dy - C_{zj}^{e-IIe} C_{yi}^{h-Iah} \int_{s+t}^b \sin(k_{yj}^{IIe}(b-y)) \sin(k_{yi}^{Iah}(b-y)) dy, \\
\\
P_{he}^{II}(i, j) &= 0, \\
\\
P_{ee}^{II}(i, j) &= \int_{s+t}^b \bar{e}_{ij}^{IIe} \times \bar{h}_{ii}^{Ie} dy = C_{yj}^{e-IIe} C_{zi}^{h-Iae} \int_{s+t}^b \cos(k_{yj}^{IIe}(b-y)) \cos(k_{yi}^{Iae}(b-y)) dy, \\
\\
P_{hh}^{III}(i, j) &= \int_0^{s+t} \bar{e}_{ij}^{IIIh} \times \bar{h}_{ii}^{IIIh} dy \\
&= -C_{zj}^{e-IIIbh} C_{yi}^{h-Ich} \int_0^s \sin(k_{yj}^{IIIbh} y) \sin(k_{yi}^{Iah} y) dy - C_{zj}^{e-IIIah} C_{yi}^{h-Ibh1} \int_s^{s+t} \sin(k_{yj}^{IIIah}(s+t-y)) \sin(k_{yi}^{Ibh}(y-s)) dy \\
&\quad - C_{zj}^{e-IIIah} C_{yi}^{h-Ibh2} \int_s^{s+t} \sin(k_{yj}^{IIIah}(s+t-y)) \sin(k_{yi}^{Ibh}(s+t-y)) dy \\
\\
P_{eh}^{III}(i, j) &= \int_0^{s+t} \bar{e}_{ij}^{IIIe} \times \bar{h}_{ii}^{IIIh} dy \\
&= C_{yj}^{e-IIIbe} C_{zi}^{h-Ich} \int_0^s \cos(k_{yj}^{IIIbe} y) \cos(k_{yi}^{Iah} y) dy + C_{yj}^{e-IIIae} C_{zi}^{h-Ibh1} \int_s^{s+t} \cos(k_{yj}^{IIIae}(s+t-y)) \cos(k_{yi}^{Ibh}(y-s)) dy \\
&\quad + C_{yj}^{e-IIIae} C_{zi}^{h-Ibh2} \int_s^{s+t} \cos(k_{yj}^{IIIae}(s+t-y)) \cos(k_{yi}^{Ibh}(s+t-y)) dy \\
&\quad - C_{zj}^{e-IIIbe} C_{yi}^{h-Ich} \int_0^s \sin(k_{yj}^{IIIbe} y) \sin(k_{yi}^{Iah} y) dy - C_{zj}^{e-IIIae} C_{yi}^{h-Ibh1} \int_s^{s+t} \sin(k_{yj}^{IIIae}(s+t-y)) \sin(k_{yi}^{Ibh}(y-s)) dy \\
&\quad - C_{zj}^{e-IIIae} C_{yi}^{h-Ibh2} \int_s^{s+t} \sin(k_{yj}^{IIIae}(s+t-y)) \sin(k_{yi}^{Ibh}(s+t-y)) dy
\end{aligned}$$

6.3 Mode Analysis of a Suspended Substrate Stripline

$P_{he}^{III}(i, j) = 0$, and

$$\begin{aligned}
 P_{ee}^{III}(i, j) &= \int_0^{s+t} \bar{e}_{ij}^{IIIe} \times \bar{h}_{ii}^{Ie} dy \\
 &= C_{yj}^{e-IIIbe} C_{zi}^{h-Ice} \int_0^s \cos(k_{yj}^{IIIbe} y) \cos(k_{yi}^{Iae} y) dy \\
 &\quad + C_{yj}^{e-IIIae} C_{zi}^{h-Ibe1} \int_s^{s+t} \cos(k_{yj}^{IIIae} (s+t-y)) \cos(k_{yi}^{Ibe} (y-s)) dy \\
 &\quad + C_{yj}^{e-IIIae} C_{zi}^{h-Ibe2} \int_s^{s+t} \cos(k_{yj}^{IIIae} (s+t-y)) \cos(k_{yi}^{Ibe} (s+t-y)) dy
 \end{aligned}$$

The field components in Region *I*, *II* and *III* at $z=0$ and $x=x_0$ are provided in Appendix A.

All the coefficients we used in the above equations can be found in Appendix A.

Matching the H field at $z=0$, $x=x_0$ we arrive at the following equations

$$H_t^I = \begin{cases} H_t^{II} & t \leq y \leq b \\ H_t^{III} & 0 \leq y \leq t \end{cases} \quad (6.67)$$

$$\sum_{n=1}^N (A_n^{Ih} \bar{H}_{tn}^{Ih} + A_n^{Ie} \bar{H}_{tn}^{Ie}) = \begin{cases} \sum_{n=0}^N (A_n^{IIh} \bar{H}_{tn}^{IIh} + A_n^{IIe} \bar{H}_{tn}^{IIe}) & t \leq y \leq b \\ \sum_{n=0}^N (A_n^{IIIh} \bar{H}_{tn}^{IIIh} + A_n^{IIIe} \bar{H}_{tn}^{IIIe}) & 0 \leq y \leq t \end{cases} \quad (6.68)$$

Taking cross product with $\bar{E}_i^{II}$ and $\bar{E}_i^{III}$ and invoking the following orthogonality relation

$$\int_t^b \bar{e}_{ii}^{IIP} \times \bar{h}_{ij}^{IIP} dy = 0 \quad \text{and} \quad \int_0^t \bar{e}_{ii}^{IIIP} \times \bar{h}_{ij}^{IIIP} dy = 0 \quad \text{where } p=e, h, \text{ we obtain}$$

$$\mathbf{Q}^{II} \mathbf{A}^I = \mathbf{U}^{II} \mathbf{A}^{II} \quad (6.69)$$

$$\mathbf{Q}^{III} \mathbf{A}^I = \mathbf{U}^{III} \mathbf{A}^{III} \quad (6.70)$$

where

$$\mathbf{U}^{II} = \begin{bmatrix} \mathbf{U}_{hh}^{II} & \mathbf{0} \\ \mathbf{U}_{he}^{II} & \mathbf{U}_{ee}^{II} \end{bmatrix}, \quad \mathbf{U}^{III} = \begin{bmatrix} \mathbf{U}_{hh}^{III} & \mathbf{0} \\ \mathbf{U}_{he}^{III} & \mathbf{U}_{ee}^{III} \end{bmatrix}, \quad \mathbf{Q}^{II} = \begin{bmatrix} \mathbf{Q}_{hh}^{II} & \mathbf{0} \\ \mathbf{Q}_{he}^{II} & \mathbf{Q}_{ee}^{II} \end{bmatrix}, \quad \mathbf{Q}^{III} = \begin{bmatrix} \mathbf{Q}_{hh}^{III} & \mathbf{0} \\ \mathbf{Q}_{he}^{III} & \mathbf{Q}_{ee}^{III} \end{bmatrix}.$$

The elements of the matrix are

6.3 Mode Analysis of a Suspended Substrate Stripline

$$U_{hh}^{\text{II}}(j, j) = \int_{s+t}^b \bar{e}_{ij}^{\text{II}h} \times \bar{h}_{ij}^{\text{II}h} dy = - \int_{s+t}^b e_{zj}^{\text{II}h} h_{yj}^{\text{II}h} dy,$$

$$= -C_{zj}^{e-} C_{yj}^{h-} \int_{s+t}^b \sin^2(k_{yj}^{\text{II}h} (b-y)) dy$$

$$U_{he}^{\text{II}}(i, j) = \int_{s+t}^b \bar{e}_i^{\text{II}e} \times \bar{h}_j^{\text{II}h} dy = \int_{s+t}^b e_{yi}^{\text{II}e} h_{zj}^{\text{II}h} dy - \int_{s+t}^b e_{zi}^{\text{II}e} h_{yj}^{\text{II}h} dy,$$

$$= C_{yi}^{e-} C_{zj}^{h-} \int_{s+t}^b \cos(k_{yi}^{\text{II}e} (b-y)) \cos(k_{zj}^{\text{II}h} (b-y)) dy - C_{zi}^{e-} C_{yj}^{h-} \int_{s+t}^b \sin(k_{yi}^{\text{II}e} (b-y)) \sin(k_{yj}^{\text{II}h} (b-y)) dy$$

$$U_{ee}^{\text{II}}(j, j) = \int_{s+t}^b \bar{e}_j^{\text{II}e} \times \bar{h}_j^{\text{II}e} dy = \int_{s+t}^b e_{yj}^{\text{II}e} h_{zj}^{\text{II}e} dy,$$

$$= C_{yj}^{e-} C_{zj}^{h-} \int_{s+t}^b \cos^2(k_{yj}^{\text{II}e} (b-y)) dy$$

$$U_{hh}^{\text{III}}(j, j) = \int_0^{s+t} \bar{e}_{ij}^{\text{III}h} \times \bar{h}_{ij}^{\text{III}h} dy = - \int_0^{s+t} e_{zj}^{\text{III}h} h_{yj}^{\text{III}h} dy$$

$$= -C_{zj}^{e-} C_{yj}^{h-} \int_0^s \sin^2(k_{yj}^{\text{III}h} y) dy - C_{zj}^{e-} C_{yj}^{h-} \int_s^{s+t} \sin^2(k_{yj}^{\text{III}h} (s+t-y)) dy$$

$$U_{he}^{\text{III}}(i, j) = \int_0^{s+t} \bar{e}_i^{\text{III}e} \times \bar{h}_j^{\text{III}h} dy = \int_0^{s+t} (e_{yi}^{\text{III}e} h_{zj}^{\text{III}h} - e_{zi}^{\text{III}e} h_{yj}^{\text{III}h}) dy$$

$$= C_{yi}^{e-} C_{zj}^{h-} \int_0^s \cos(k_{yi}^{\text{III}e} y) \cos(k_{zj}^{\text{III}h} y) dy + C_{yi}^{e-} C_{zj}^{h-} \int_s^{s+t} \cos(k_{yi}^{\text{III}e} (s+t-y)) \cos(k_{zj}^{\text{III}h} (s+t-y)) dy$$

$$- C_{zi}^{e-} C_{yj}^{h-} \int_0^s \sin(k_{yi}^{\text{III}e} y) \sin(k_{yj}^{\text{III}h} y) dy - C_{zi}^{e-} C_{yj}^{h-} \int_s^{s+t} \sin(k_{yi}^{\text{III}e} (s+t-y)) \sin(k_{yj}^{\text{III}h} (s+t-y)) dy$$

$$U_{ee}^{\text{III}}(j, j) = \int_0^{s+t} \bar{e}_j^{\text{III}e} \times \bar{h}_j^{\text{III}e} dy = \int_0^{s+t} e_{yj}^{\text{III}e} h_{zj}^{\text{III}e} dy$$

$$= C_{yj}^{e-} C_{zj}^{h-} \int_0^s \cos^2(k_{yj}^{\text{III}e} y) dy + C_{yj}^{e-} C_{zj}^{h-} \int_s^{s+t} \cos^2(k_{yj}^{\text{III}e} (s+t-y)) dy$$

$$Q^{\text{II}} = P^{\text{II}T},$$

$$\text{and } Q^{\text{III}} = P^{\text{III}T}.$$

All the coefficients can be found in Appendix A. Solving (6.51), (6.54) and (6.55) we get

$$\mathbf{XA}^{\text{I}} = \mathbf{0} \quad (6.71)$$

$$\mathbf{X} = \mathbf{U}^{\text{I}} - \mathbf{P}^{\text{II}} (\mathbf{U}^{\text{II}})^{-1} (\mathbf{P}^{\text{II}})^{\text{T}} - \mathbf{P}^{\text{III}} (\mathbf{U}^{\text{III}})^{-1} (\mathbf{P}^{\text{III}})^{\text{T}} \quad (6.72)$$

For the existence of non-zero values of $\mathbf{A}^{\text{I}}$, the determinant of matrix $\mathbf{X}$ must vanish.

Sweeping β and by searching the zeros of the determinant of $\mathbf{X}$ the propagation constant

6.3 Mode Analysis of a Suspended Substrate Stripline

can be obtained.

6.3.5 Numerical Results

The calculated propagation constant of an SSS line is presented and compared to HFSS simulated results in Figure 6.3. The geometric parameters are $w_I=1.04$ mm, $h=1$ mm, $h_I=0.5$ mm, $t=0.203$ mm and $w=4$ mm. The relative permittivity of the substrate is $\epsilon_r=3.55$. It can be seen in the figure that our calculated results agree with the HFSS simulated results very well.

Figure 6.3. Propagation constant vs. frequency of a SSS line with relative permittivity of 3.55 and geometric parameters of $w_I=1.04$ mm, $h=1$ mm, $h_I=0.5$ mm, $t=0.203$ mm, and $w=4$ mm.

6.3 Mode Analysis of a Suspended Substrate Stripline

The characteristic impedances of a SSS line at 14GHz with different geometric parameters are shown in Figure 6.4. Same RO4003 material with thickness $t=0.203$ mm and dielectric constant of 3.55 is used. The width of the metallic shield is fixed as $w=4$ mm and h is set to be $h_1/2$. It can be seen from the figure that the characteristic impedance drops with the increasing in strip line width or reducing in the height of the shield. The characteristic impedance of the SSS can be well controlled by changing the width of the strip line and the height of the shield.

Figure 6.4. Characteristic impedance of an SSS line with relative permittivity of 3.55 and geometric parameters of $t=0.203$ mm, $h=h_1/2$, and $w=4$ mm.

6.4 Combiner/Divider Design

The 8-way power combiner can be divided into three parts: a junction between a centered coaxial line (SMA connector) and a large circular waveguide, a transition between a circular waveguide and eight SSS ports, and a circular waveguide section between these two transitions. Based on the proposed axially symmetric structure, the propagating mode excited in the circular waveguide is TM_{01} mode and the next higher-order mode that can be strongly coupled to the input and output ports is TM_{02} mode. The diameter of the circular waveguide d_l is selected to ensure that TM_{01} mode can propagate and TM_{02} mode is cut-off in our frequency range of interest. Therefore, we have

$$\frac{P_{02}}{\pi f_{\max} \sqrt{\mu_0 \epsilon_0}} > d_l > \frac{P_{01}}{\pi f_{\min} \sqrt{\mu_0 \epsilon_0}} \quad (6.73)$$

where $p_{01}=2.405$, $p_{02}=5.520$, $f_{\min}=12$ GHz, $f_{\max}=16$ GHz, ϵ_0 and μ_0 are the permittivity and permeability in free space, respectively. Equation (6.73) gives the range of d_l between 19.12 mm and 29.26 mm. On the other hand, considering the physical layout constrain of the cavity that houses eight packaged high power GaAs FETs, the required diameter of the circular waveguide is chosen to be $d_l=23$ mm.

Once the diameter of the cylindrical cavity is determined, a transition from the circular waveguide to eight 50Ω SSS ports is designed using HFSS. The substrate used in our design is RO4003 with thickness of 0.203 mm, relative permittivity of 3.55, and loss tangent of 0.0027 at 10GHz. The 50Ω SSS port is calculated using our mode analysis method. The cross section of the designed SSS port is shown in Figure 6.5, where $w_s=4$ mm, $w_{sl}=1.04$ mm, $h_s=1$ mm, $h_{sl}=0.5$ mm and $h_{s2}=0.297$ mm. The surface of the SSS is plated with silver to minimize loss.

6.4 Combiner/Divider Design

Figure 6.5. Cross section of the designed 50 Ω SSS line.

The extension of SSS into the cavity serves as a radiating probe. Taking advantage of the symmetry of the structure, we can set virtual perfect magnetic conductor (PMC) boundaries along the symmetric planes. The structure can then be simplified as one probe radiating into one sector of the circular waveguide with two PMC side walls. The HFSS model of the simplified transition is shown in Figure 6.6. Port 1 is the input SSS port and port 2 is one-eighth of the output cylindrical port.

Figure 6.6. Simplified HFSS model of a SSS to cylindrical waveguide TM_{01} mode transition.

6.4 Combiner/Divider Design

The initial length of the SSS extension l_I is set to $\lambda_{gs}/4$, where λ_{gs} is the guided wavelength of the SSS. The distance h_{Ia} is set to $\lambda_{gw}/4$ initially to provide an open circuit at the plane where the SSS probe is located, where λ_{gw} is the guided wavelength of the TM_{01} mode in the circular waveguide. However, the distance h_{Ia} can vary greatly from the initial value of $\lambda_{gw}/4$ to provide a shunt susceptance to the radiating probe. The simulated scattering parameters of the junction with different l_I and h_{Ia} are shown in Figure 6.7 and Figure 6.8. It can be seen from the figure that by varying distance h_{Ia} and length l_I , the resonant frequency, bandwidth, as well as the input impedance at the SSS port can be adjusted.

Figure 6.7. Scattering parameters of a SSS to cylindrical waveguide TM_{01} mode junction with different SSS probe length l_I ($d_I=23\text{mm}$, $h_{Ia}=6\text{mm}$).

6.4 Combiner/Divider Design

Figure 6.8. Scattering parameters of a SSS to cylindrical waveguide TM_{01} mode junction with different cylindrical waveguide height h_{1a} ($l_I=4.5\text{mm}$, $h_{1a}=6\text{mm}$)..

The next step is to design the transition from SMA port to a circular waveguide. By adjusting the length of the feeding probe h_I around the initial value of $\lambda_{gw}/4$, the resonant frequency can be controlled.

These two designed transitions are then connected together by a circular waveguide section. Two resonances are introduced by the two transitions. The overall height of the circular waveguide is selected to be around $\lambda_{gw}/2$ to create a third resonance by exciting the TM_{011} mode in the cavity. By changing h_{1b} and other geometrical parameters, good transmission and reflection coefficients can be achieved. All geometric parameters are fine-tuned using HFSS and the final geometric parameters of the combiner shown in Figure 6.1 are obtained as $l_I=4.4\text{ mm}$, $d_I=23.05\text{ mm}$, $h_{1a}=3.5\text{ mm}$, $h_{1b}=11.5\text{ mm}$, and $h_I=6.27\text{ mm}$.

6.4 Combiner/Divider Design

Like most power combiners employing dielectric substrates, the losses in our power combiner include Ohmic and dielectric losses. Compared to a planar power combiner, the air-filled cylindrical cavity obviously offers lower dielectric loss. Also, for a planar combiner, with the microwave power mainly confined in a small volume of the dielectric substrate, the current flow over the conductor traces will be much higher compared to this cylindrical cavity with relatively large space, which results in a smaller Ohmic loss. The maximum average power handling capability is also improved accordingly.

Due to its axially symmetric structure, there is no amplitude and phase imbalance among the eight peripheral ports in the ideal case. *However*, mechanical and fabrication tolerance must be considered in reality. The size of the stripline and its location can be accurately ensured by modern lithographic technology for PCB fabrication and its tolerance can easily be less than 0.01 mm. Fabrication of metallic cavity can also meet similar tolerance by using modern NC machine. The assembly tolerance is probably the major concern in this power combiner. To assess the effect of assembly tolerance, we define amplitude imbalance ΔMag and phase imbalance ΔAng as the maximum difference between the amplitude and phase of transmission coefficients when the PCB axis has an offset Δa from the waveguide center and those of an ideal centered case, as described in equation (6.74) and (6.75), where $i=2$ to 9.

$$\Delta Mag = \max(|S_{i1}|) - \min(|S_{i1}|) \quad (6.74)$$

$$\Delta Ang = \max(\angle(S_{i1})) - \min(\angle(S_{i1})) \quad (6.75)$$

Table 6.1 presents the simulated amplitude imbalance ΔMag and phase imbalance ΔAng with different PCB offsets at 14 GHz. It can be seen that with 0.1 mm to 0.2 mm

6.5 Experimental Measurements

PCB offset, the amplitude imbalance is around 0.67 dB and phase imbalance about 1.78 degrees. Although the combining efficiency is not affected significantly by introducing such a small PCB offset, the assembly tolerance should be minimized to ensure the linearity of the whole SSPA system.

Table 6.1. Amplitude and phase imbalance due to PCB offset.

Δa	0	0.1	0.2	0.3	0.4	0.5
ΔMag (dB)	0.07	0.67	1.25	1.96	2.7	3.32
ΔAng ($^{\circ}$)	0.64	1.5	2.8	3.95	5.32	7.2

6.5 Experimental Measurements

The photos of our fabricated radial SSS 8-way power combiner are shown in Figure 6.9, where the left side is the top part of the cavity. The substrate is in the middle and the bottom part of the cavity as well as the output SMA port is shown in the right side.

Figure 6.9. Photos of the fabricated 8-way radial power combiner.

6.5 Experimental Measurements

To take advantage of the universal substrate test fixture from Inter-Continental Microwave for testing, a transition from microstrip line to SSS is introduced. Also, the eight input ports are bent towards two directions since the test fixture can only test microstrip line ports from these two directions.

The measured reflection coefficients from the combining port of the 8-way power combiner are shown in Fig. 4. During the measurement, ports 2 to 9 are terminated by matched loads, which are implemented by partially filling the SSS with microwave absorbers. Simulated results by HFSS are also presented in the same figure for comparison. It is seen that the measured results agree very well with the simulated ones. Within the frequency range from 12.1 GHz to 15.7 GHz, the measured return loss is better than 19 dB.

Figure 6.10. Reflection coefficient of the 8-way power combiner.

6.5 Experimental Measurements

Measured and simulated isolation results between peripheral ports are presented in Figure 6.11. Our measured isolation results range from -7 to -20 dB in the frequency range of 12 to 16 GHz, which also agree well with simulated ones. To improve the isolation, eight slots may be cut on the cylindrical cavity wall between peripheral ports. When the slots are loaded by absorbers, the isolation between peripheral ports can be improved.

Figure 6.11. Reflection coefficient of the 8-way power combiner.

The transmission coefficients from eight peripheral ports to the combining port are measured and presented in Figure 6.12. The loss of the test fixture with two back-to-back transitions from microstrip line to SSS is measured to have an average value of 1.4dB. Half of this loss is excluded from the final transmission results. The loss of the SSS is

6.5 Experimental Measurements

measured to be 0.047dB/cm. An average value of 0.13dB loss for the outside four ports with longer SSS lines and 0.08dB for the remaining four ports are also excluded from the results to account for the losses of the SSS lines. It can be seen from Figure 6.12 that the measured maximum amplitude imbalance among the eight peripheral ports is around 1dB. The measured average transmission coefficient within the frequency range from 12GHz to 16GHz is -9.2dB , which suggests a loss of 0.17dB compared to the transmission coefficient of 9.03dB for a lossless 8-way combiner.

Figure 6.12. Transmission coefficients of the 8-way power combiner.

6.6 Conclusion

6.6 Conclusion

In this chapter, a Ku-band 8-way radial combiner has been presented with coaxial output port and SSS peripheral ports. Mode analysis has been performed to calculate the field distribution of the input SSS ports. The bandwidth of this combiner is measured to be 3.6GHz from 12.1GHz to 15.7GHz, with return loss better than 19dB. Compared to other types of radial power combiners, the proposed power combiner successfully integrates printed circuits transmission lines with a 3D space combining structure, which results in high efficiency, wide bandwidth, and high power handling capability.

CHAPTER 7

CONCLUSION AND FUTURE WORK

7.1 Summary of Work Conducted

In this dissertation, a thorough study of spatial power combining in a closed waveguide structure has been carried out. The microwave power combining techniques have been reviewed first. Various radial power combiners and the TSA array in waveguide structures have been discussed in detail. These discussions lead to two new power combining structures which are proposed in this dissertation.

To prepare for our new power combiner design, the dispersion characteristics and field expressions of shielded multiple microstrip lines are obtained by solving a set of homogeneous simultaneous equations. Mode-matching techniques are implemented to solve all the junctions we needed for constructing a multiple microstrip lines to rectangular waveguide transition for high efficient power combining application. All the numerical results are compared with those obtained by HFSS for verification.

A C-band eight-way microstrip lines to rectangular waveguide combiner is constructed using the above mentioned numerical techniques. Both numerical and experimental results have been carried out. Very good measured results have been achieved. The measured insertion loss is better than 0.39dB and return loss of the combining port is better than 22dB over the whole frequency range of 5.8GHz to 8.2GHz.

The second design is a Ku-band eight-way radial power combiner. The dispersion

7.2 Recommendations for Future Work

characteristics of an SSS have been analyzed. Measured results show return loss of better than 19dB and insertion loss better than 0.2dB over the frequency range of 12.1GHz to 15.7GHz.

7.2 Recommendations for Future Work

A number of recommendations for future works are given as follows.

First of all, instead of using TE and TM modes to represent the modes in the shielded microstrip lines and solving it by matching the fields on the air-substrate interface as shown in Figure 7.1 a), LSM and LSE modes can be used and the waveguide can be separated into several regions vertically, as shown in Figure 7.1 b). By matching the electromagnetic fields between these regions the propagation constants can be obtained. There are several advantages by employing this method.

- a. The thickness of the microstrip line can be taken into consideration easily, which will lead to more accurate results.
- b. The microstrip modes can be better represented by LSM and LSE modes than by TE and TM modes. The convergence can be improved greatly, which improves the modeling efficiency.

Figure 7.1 Illustration of different region divisions for better results.

7.2 Recommendations for Future Work

Secondly, an over-heighted rectangular waveguide can be employed to construct microstrip to waveguide power combiner with more input ports. Since the electromagnetic field distribution of TE_{10} mode is constant along the y-axis, by utilizing an over-heighted rectangular waveguide, more metallic cards and PCBs can be easily added to combine power from more microstrip ports.

Finally, for the Ku-band eight-way radial combiner, further research should focus on improving of isolation between ports. Eight slots may be cut on the cylindrical cavity wall between peripheral ports. With the slots properly loaded by absorbers, the isolation between peripheral ports can be improved.

APPENDIX A

FIELD EXPRESSIONS OF A SUSPENDED SUBSTRATE STRIPLINE

Field Expressions in Region I

For LSE mode, at $z=0$, $x=x_0$, the transverse field components normal to x -axis can be written as

$$E_{yn}^{lh} \Big|_{x=x_0, z=0} = 0$$

$$e_{zn}^{lh} \Big|_{x=x_0, z=0} = \begin{cases} C_{zn}^{e-lah} \sin(k_{yn}^{lah} (b-y)) & s+t \leq y \leq b \\ C_{zn}^{e-lbh1} \sin(k_{yn}^{lbh} (y-s)) + C_{zn}^{e-lbh2} \sin(k_{yn}^{lbh} (s+t-y)) & s \leq y \leq s+t \\ C_{zn}^{e-lch} \sin(k_{yn}^{lah} y) & 0 \leq y \leq s \end{cases}$$

$$h_{yn}^{lh} \Big|_{x=x_0, z=0} = \begin{cases} C_{yn}^{h-lah} \sin(k_{yn}^{lah} (b-y)) & s+t \leq y \leq b \\ C_{yn}^{h-lbh1} \sin(k_{yn}^{lbh} (y-s)) + C_{yn}^{h-lbh2} \sin(k_{yn}^{lbh} (s+t-y)) & s \leq y \leq s+t \\ C_{yn}^{h-lch} \sin(k_{yn}^{lah} y) & 0 \leq y \leq s \end{cases}$$

$$h_{zn}^{lh} \Big|_{x=x_0, z=0} = \begin{cases} C_{zn}^{h-lah} \cos(k_{yn}^{lah} (b-y)) & s+t \leq y \leq b \\ C_{zn}^{h-lbh1} \cos(k_{yn}^{lbh} (y-s)) + C_{zn}^{h-lbh2} \cos(k_{yn}^{lbh} (s+t-y)) & s \leq y \leq s+t \\ C_{zn}^{h-lch} \cos(k_{yn}^{lah} y) & 0 \leq y \leq s \end{cases}$$

where

$$C_{zn}^{e-lah} = \frac{j\omega\mu k_{xn}^{lh} \tan(k_{xn}^{lh} x_0)}{\sin(k_{yn}^{lah} d)}$$

$$C_{zn}^{e-lbh1} = \frac{j\omega\mu k_{xn}^{lh} \tan(k_{xn}^{lh} x_0)}{\sin(k_{yn}^{lbh} t)}$$

$$C_{zn}^{e-lbh2} = j\omega\mu k_{xn}^{lh} \tan(k_{xn}^{lh} x_0) \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbh}} \cot(k_{yn}^{lah} d) + \cot(k_{yn}^{lbh} t) \right)$$

$$C_{zn}^{e-lch} = j\omega\mu k_{xn}^{lh} \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbh}} \sin(k_{yn}^{lbh} t) \cot(k_{yn}^{lah} d) + \cos(k_{yn}^{lbh} t) \right) \frac{\tan(k_{xn}^{lh} x_0)}{\sin(k_{yn}^{lah} s)}$$

APPENDIX A

Field Expressions of a Suspended Substrate Stripline

$$C_{yn}^{h_lah} = \frac{(k_0^2 - k_{yn}^{lah^2})}{\sin(k_{yn}^{lah} d)},$$

$$C_{yn}^{h_lbt} = \frac{(k_0^2 - k_{yn}^{lbt^2})}{\sin(k_{yn}^{lbt} t)},$$

$$C_{yn}^{h_lbt2} = (k_0^2 - k_{yn}^{lbt^2}) \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbt}} \cot(k_{yn}^{lah} d) + \cot(k_{yn}^{lbt} t) \right),$$

$$C_{yn}^{h_lch} = \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbt}} \sin(k_{yn}^{lbt} t) \cot(k_{yn}^{lah} d) + \cos(k_{yn}^{lbt} t) \right) \frac{(k_0^2 - k_{yn}^{lah^2})}{\sin(k_{yn}^{lah} s)}$$

$$C_{zn}^{h_lah} = \frac{j\beta k_{yn}^{lah}}{\sin(k_{yn}^{lah} d)},$$

$$C_{zn}^{h_lbt} = -\frac{j\beta k_{yn}^{lbt}}{\sin(k_{yn}^{lbt} t)},$$

$$C_{zn}^{h_lbt2} = j\beta (k_{yn}^{lah} \cot(k_{yn}^{lah} d) + k_{yn}^{lbt} \cot(k_{yn}^{lbt} t)),$$

$$C_{zn}^{h_lch} = -\frac{j\beta k_{yn}^{lah}}{\sin(k_{yn}^{lah} s)} \left(\frac{k_{yn}^{lah}}{k_{yn}^{lbt}} \sin(k_{yn}^{lbt} t) \cot(k_{yn}^{lah} d) + \cos(k_{yn}^{lbt} t) \right)$$

For LSM mode, at $z=0$, $x=x_0$, the transverse field components normal to x -axis can be written as

$$e_{yn}^{le} \Big|_{x=x_0, z=0} = \begin{cases} C_{yn}^{e_lae} \cos(k_{yn}^{lae} (b-y)) & s+t \leq y \leq b \\ C_{yn}^{e_lbt} \cos(k_{yn}^{lbt} (y-s)) + C_{yn}^{e_lbt2} \cos(k_{yn}^{lbt} (s+t-y)) & s \leq y \leq s+t \\ C_{yn}^{e_lce} \cos(k_{yn}^{lce} y) & 0 \leq y \leq s \end{cases}$$

$$e_{zn}^{le} \Big|_{x=x_0, z=0} = \begin{cases} C_{zn}^{e_lae} \sin(k_{yn}^{lae} (b-y)) & s+t \leq y \leq b \\ C_{zn}^{e_lbt} \sin(k_{yn}^{lbt} (y-s)) + C_{zn}^{e_lbt2} \sin(k_{yn}^{lbt} (s+t-y)) & s \leq y \leq s+t \\ C_{zn}^{e_lce} \sin(k_{yn}^{lce} y) & 0 \leq y \leq s \end{cases}$$

$$h_{zn}^{le} = \begin{cases} C_{zn}^{h_lae} \cos(k_{yn}^{lae} (b-y)) & s+t \leq y \leq b \\ C_{zn}^{h_lbt} \cos(k_{yn}^{lbt} (y-s)) + C_{zn}^{h_lbt2} \cos(k_{yn}^{lbt} (s+t-y)) & s \leq y \leq s+t \\ C_{zn}^{h_lce} \cos(k_{yn}^{lce} y) & 0 \leq y \leq s \end{cases}$$

where

APPENDIX A

Field Expressions of a Suspended Substrate Stripline

$$C_{yn}^{e_l ae} = \frac{(k_0^2 - k_{yn}^{l ae^2})}{\cos(k_{yn}^{l ae} d)},$$

$$C_{yn}^{e_l be1} = -\frac{k_{yn}^{l ae}}{k_{yn}^{l be}} \frac{\tan(k_{yn}^{l ae} d)}{\sin(k_{yn}^{l be} t)} (k_0^2 - k_{yn}^{l ae^2}),$$

$$C_{yn}^{e_l be2} = \left(\frac{1}{\epsilon_r} + \frac{k_{yn}^{l ae}}{k_{yn}^{l be}} \tan(k_{yn}^{l ae} d) \cot(k_{yn}^{l be} t) \right) (k_0^2 - k_{yn}^{l ae^2}),$$

$$C_{yn}^{e_l ce} = -\left(\frac{1}{\epsilon_r} \frac{k_{yn}^{l be}}{k_{yn}^{l ae}} \sin(k_{yn}^{l be} t) + \cos(k_{yn}^{l be} t) \tan(k_{yn}^{l ae} d) \right) \frac{(k_0^2 - k_{yn}^{l ae^2})}{\sin(k_{yn}^{l ae} s)}$$

$$C_{zn}^{e_l ae} = -\frac{j\beta k_{yn}^{l ae}}{\cos(k_{yn}^{l ae} d)},$$

$$C_{zn}^{e_l be1} = -j\beta k_{yn}^{l ae} \frac{\tan(k_{yn}^{l ae} d)}{\sin(k_{yn}^{l be} t)},$$

$$C_{zn}^{e_l be2} = -j\beta \left(\frac{k_{yn}^{l be}}{\epsilon_r} + k_{yn}^{l ae} \tan(k_{yn}^{l ae} d) \cot(k_{yn}^{l be} t) \right),$$

$$C_{zn}^{e_l ce} = \frac{-j\beta}{\sin(k_{yn}^{l ae} s)} \left(\frac{1}{\epsilon_r} k_{yn}^{l be} \sin(k_{yn}^{l be} t) + k_{yn}^{l ae} \cos(k_{yn}^{l be} t) \tan(k_{yn}^{l ae} d) \right)$$

$$C_{zn}^{h_l ae} = j\omega\epsilon_0 k_{xn}^{l e} \frac{\cot(k_{xn}^{l e} x_0)}{\cos(k_{yn}^{l ae} d)},$$

$$C_{zn}^{h_l be1} = -j\omega\epsilon_0 \epsilon_r k_{xn}^{l e} \cot(k_{xn}^{l e} x_0) \frac{k_{yn}^{l ae}}{k_{yn}^{l be}} \frac{\tan(k_{yn}^{l ae} d)}{\sin(k_{yn}^{l be} t)},$$

$$C_{zn}^{h_l be2} = j\omega\epsilon_0 \epsilon_r k_{xn}^{l e} \cot(k_{xn}^{l e} x_0) \left(\frac{1}{\epsilon_r} + \frac{k_{yn}^{l ae}}{k_{yn}^{l be}} \tan(k_{yn}^{l ae} d) \cot(k_{yn}^{l be} t) \right),$$

$$C_{zn}^{h_l ce} = -j\omega\epsilon_0 k_{xn}^{l e} \left(\frac{k_{yn}^{l be}}{\epsilon_r k_{yn}^{l ae}} \sin(k_{yn}^{l be} t) + \cos(k_{yn}^{l be} t) \tan(k_{yn}^{l ae} d) \right) \frac{\cot(k_{xn}^{l e} x_0)}{\sin(k_{yn}^{l ae} s)}.$$

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Field Expressions in Region II

For LSE modes, at $z=0$, $x=x_0$, the transverse field components normal to x -axis can be written as

$$e_{zn}^{llh} \Big|_{x=x_0, z=0} = C_{zn}^{e-llh} \sin(k_{yn}^{llh}(b-y))$$

$$h_{yn}^{llh} \Big|_{x=x_0, z=0} = C_{yn}^{h-llh} \sin(k_{yn}^{llh}(b-y))$$

$$h_{zn}^{llh} \Big|_{x=x_0, z=0} = C_{zn}^{h-llh} \cos(k_{yn}^{llh}(b-y))$$

where $C_{zn}^{e-llh} = j\omega\mu k_{xn}^{llh} \cot(k_{xn}^{llh}(a-x_0))$, $C_{yn}^{h-llh} = (k_0^2 - k_{yn}^{llh^2})$ and $C_{zn}^{h-llh} = j\beta k_{yn}^{llh}$.

For LSM modes, at $z=0$, $x=x_0$, the transverse field components normal to x -axis can be written as

$$e_{yn}^{lle} \Big|_{x=x_0, z=0} = C_{yn}^{e-lle} \cos(k_{yn}^{lle}(b-y))$$

$$e_{zn}^{lle} \Big|_{x=x_0, z=0} = C_{zn}^{e-lle} \sin(k_{yn}^{lle}(b-y))$$

$$h_{zn}^{lle} \Big|_{x=x_0, z=0} = C_{zn}^{h-lle} \cos(k_{yn}^{lle}(b-y))$$

where $C_{yn}^{e-lle} = (k_0^2 - k_{yn}^{lle^2})$, $C_{zn}^{e-lle} = k_{yn}^{lle}(-j\beta)$ and $C_{zn}^{h-lle} = j\omega\epsilon_0 k_{xn}^{lle} \tan(k_{xn}^{lle}(a-x_0))$.

Field Expressions in Region III

For LSE mode, at $z=0$, $x=x_0$, the transverse field components normal to x -axis can be written as

$$e_{zn}^{lllh} \Big|_{x=x_0, z=0} = \begin{cases} C_{zn}^{e-lllh} \sin(k_{yn}^{lllh}(s+t-y)) & s \leq y \leq s+t \\ C_{zn}^{e-llbh} \sin(k_{yn}^{llbh}y) & 0 \leq y \leq s \end{cases}$$

$$h_{yn}^{lllh} \Big|_{x=x_0, z=0} = \begin{cases} C_{yn}^{h-lllh} \sin(k_{yn}^{lllh}(s+t-y)) & s \leq y \leq s+t \\ C_{yn}^{h-llbh} \sin(k_{yn}^{llbh}y) & 0 \leq y \leq s \end{cases}$$

APPENDIX A

Field Expressions of a Suspended Substrate Stripline

$$h_{zn}^{IIIh} \Big|_{x=x_0, z=0} = \begin{cases} C_{zn}^{h_IIIah} \cos(k_{yn}^{IIIah} (s+t-y)) & s \leq y \leq s+t \\ C_{zn}^{h_IIIbh} \cos(k_{yn}^{IIIbh} y) & 0 \leq y \leq s \end{cases}$$

where

$$C_{zn}^{e_IIIah} = j\omega\mu k_{xn}^{IIIh} \frac{\cot(k_{xn}^{IIIh} (a-x_0))}{\sin(k_{yn}^{IIIah} t)},$$

$$C_{zn}^{e_IIIbh} = j\omega\mu k_{xn}^{IIIh} \frac{\cot(k_{xn}^{IIIh} (a-x_0))}{\sin(k_{yn}^{IIIbh} s)}$$

$$C_{yn}^{h_IIIah} = \frac{(k_0^2 - k_{yn}^{IIIbh^2})}{\sin(k_{yn}^{IIIah} t)},$$

$$C_{yn}^{h_IIIbh} = \frac{(k_0^2 - k_{yn}^{IIIbh^2})}{\sin(k_{yn}^{IIIbh} s)}$$

$$C_{zn}^{h_IIIah} = \frac{j\beta k_{yn}^{IIIah}}{\sin(k_{yn}^{IIIah} t)},$$

$$C_{zn}^{h_IIIbh} = \frac{-j\beta k_{yn}^{IIIbh}}{\sin(k_{yn}^{IIIbh} s)}$$

For LSM mode, at $z=0$, $x=x_0$, the transverse field components normal to x -axis can be written as

$$e_{yn}^{IIIe} \Big|_{x=x_0, z=0} = \begin{cases} C_{yn}^{e_IIIae} \cos(k_{yn}^{IIIae} (s+t-y)) & s \leq y \leq s+t \\ C_{yn}^{e_IIIbe} \cos(k_{yn}^{IIIbe} y) & 0 \leq y \leq s \end{cases}$$

$$e_{zn}^{IIIe} \Big|_{x=x_0, z=0} = \begin{cases} C_{zn}^{e_IIIae} \sin(k_{yn}^{IIIae} (s+t-y)) & s \leq y \leq s+t \\ C_{zn}^{e_IIIbe} \sin(k_{yn}^{IIIbe} y) & 0 \leq y \leq s \end{cases}$$

$$h_{zn}^{IIIe} \Big|_{x=x_0, z=0} = \begin{cases} C_{zn}^{h_IIIae} \cos(k_{yn}^{IIIae} (s+t-y)) & s \leq y \leq s+t \\ C_{zn}^{h_IIIbe} \cos(k_{yn}^{IIIbe} y) & 0 \leq y \leq s \end{cases}$$

Where

$$C_{yn}^{e_IIIae} = \frac{(k_0^2 - k_{yn}^{IIIbe^2})}{\cos(k_{yn}^{IIIae} t)},$$

$$C_{yn}^{e_IIIbe} = \frac{\epsilon_r (k_0^2 - k_{yn}^{IIIbe^2})}{\cos(k_{yn}^{IIIbe} s)}$$

APPENDIX A

Field Expressions of a Suspended Substrate Stripline

$$C_{zn}^{e_IIIae} = \frac{-j\beta k_{yn}^{IIIae}}{\cos(k_{yn}^{IIIae} t)},$$

$$C_{zn}^{e_IIIbe} = \frac{\varepsilon_r j\beta k_{yn}^{IIIbe}}{\cos(k_{yn}^{IIIbe} s)}$$

$$C_{zn}^{h_IIIae} = j\omega\varepsilon_0\varepsilon_r k_{xn}^{IIIe} \frac{\tan(k_{xn}^{IIIe} (a - x_0))}{\cos(k_{yn}^{IIIae} t)},$$

$$C_{zn}^{h_IIIbe} = j\omega\varepsilon_0\varepsilon_r k_{xn}^{IIIe} \frac{\tan(k_{xn}^{IIIe} (a - x_0))}{\cos(k_{yn}^{IIIbe} s)}$$

PUBLICATIONS

Related Publications

- [1] X. Shan, Z. Shen, P. Kumaresh, and R. M. Jayasuriya, "A rectangular waveguide to multiple microstrip lines transition for power combining application," in *Asia-Pacific Microw. Conf.*, pp. 996-999, Dec. 2009.
- [2] X. Shan, Z. Shen, P. Kumaresh, and R. M. Jayasuriya, "A novel 8-way radial power combiner," in *Asia-Pacific Microw. Conf.*, pp. 2625-2628, Dec. 2009.
- [3] X. Shan and Z. Shen, "A suspended-Substrate Ku-band symmetric radial power combiner," *IEEE Microw. Wireless Compon. Lett.*, pp. 652-654, Dec. 2011.
- [4] X. Shan and Z. Shen, "A microstrip to rectangular waveguide eight-way power combiner," Accepted by *IEEE Trans. Microw. Theory Tech.*, 2013.

Other Journal Publications

- [1] Z. Jiang, Z. Shen, and X. Shan, "Mode-matching analysis of waveguide T-junction loaded with an H-plane dielectric slab," *Journal of Electromagnetic Waves and Applications*, vol.16, pp.1613-1614, Nov. 2002.
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