Left ventricular surface kinematics as it fills and contracts

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**LEFT VENTRICULAR SURFACE KINEMATICS
AS IT FILLS AND CONTRACTS**

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SCHOOL OF MECHANICAL AND PRODUCTION ENGINEERING

A thesis submitted to Nanyang Technological University in partial
fulfilment of the requirement for the degree of Master of Engineering

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Abstract

It is hypothesized that myocardial ischemia and other cardiac pathologies are associated with regional alterations in ventricular shape. In order to elucidate and assess the left ventricular (LV) surface shape changes during the cardiac cycle, information regarding the surface properties is particularly useful. In this report, the differential properties of the LV surface were computed via an analytic approach using local surface fitting method. The efficiency of the proposed surface fitting algorithm was validated against exact solutions for simple geometrical shapes and was found to offer great accuracy in the computation of curvatures.

A LV endocardial dataset was acquired from 4D ultrasound imaging, which provides good spatial and temporal resolution. 18 frames during one cardiac cycle were captured and used for the assessment of LV endocardial surface shape. The curvatures of the endocardial surface were presented in the form of color maps and further evaluated.

By studying the curvature maps of the LV endocardial surface during the cardiac cycle, one can easily get a good perception of the regional wall deformation in terms of changes in shape. It is possible that with more LV datasets (i.e. 30-50 cases) being tested and evaluated, one can discover specific trends in the temporal changes of surface curvatures during the cardiac cycle. This will eventually be useful for the assessment of regional cardiac function, especially for the detection of infarcted regions and unusual wall activities.

In order to track the motion of specific endocardial surface points, the point correspondences between successive frames were determined based on a thin plate bending model of the LV. This point matching algorithm was also applied to a magnetic resonance imaging (MRI) dataset to determine the LV surface motion during the isovolumic contraction phase. The results obtained showed good overall agreement with reported observations by other researchers.

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List of Symbols

E^3	three-dimensional Euclidean space
$\mathcal{R}^3$	three-dimensional vector space
$\mathbf{x}, x, y, z$	rectangular Cartesian coordinates of a point
C^m	class m
γ	curve of class ≥ 2
s	arc length
$\mathbf{t}$	tangent vector at point $\mathbf{x}$
κ	curvature of γ at $\mathbf{x}(s)$
ρ	radius of curvature of γ
I	first fundamental form of a surface
E, F, G	first fundamental coefficients of a surface
$\mathbf{N}$	normal vector of a surface at $\mathbf{x}$
$\mathbf{n}$	unit normal to a surface at $\mathbf{x}$
II	second fundamental form of a surface
L, M, N	second fundamental coefficients of a surface
κ_n	normal curvature of a surface
κ_1, κ_2	principal curvatures of a surface
K	Gaussian curvature of a surface
H	mean curvature of a surface
C	curvedness of a surface
S	shape index of a surface
ε	potential energy of a thin plate
Λ	material constant of a thin plate
ε_{be}	bending energy of a thin plate
E	energy function of a traditional snake
α, β	weighting parameters that regulate a snake's tension and rigidity respectively
E_{ext}	external energy function derived from the image

$I(x, y)$	gray-level image viewed as a function of continuous variables (x, y)
∇	gradient operator
$G_{\sigma}(x, y)$	2-D Gaussian function with standard deviation σ

List of Acronyms

DICOM	Digital Imaging and Communications in Medicine
ED	end-diastole
ES	end-systole
FE	finite element
FEM	finite element method
GVF	gradient vector flow
LV	left ventricle of a heart
MR	magnetic resonance
MRI	magnetic resonance imaging
NEMA	National Electrical Manufacturers Association
SA	sinoatrial

Chapter 1

INTRODUCTION

The heart is the beginning of life; the sun of the microcosm, even as the sun in his turn might well be designated the heart of the world

- William Harvey, 1578-1657

1.1 Background

The most fundamental function of the heart is to pump blood throughout the vasculature, thus providing oxygen and nutrients to the cells. The underlying pumping action being rhythmical is triggered by the electrical wave propagation in the cardiac cells, resulting in the mechanical contraction of the cardiac muscle tissue. This remarkable organ is so reliable that it contracts over 2 billion times during the average human life span, compensating for acute changes in blood flow demands and adapting to sustained changes in applied loads [1]. However, this intended reliability and adaptability of the cardiac function can sometimes fall short due to heart diseases which are often caused by unhealthy lifestyles, aging, congenital abnormalities etc..

Heart failure is one of the leading causes of mortality and morbidity in the world and a considerable proportion of these deaths is due to myocardial ischemia. Since 1998, ischemia and other heart diseases has been the second major cause of death in Singapore [2], and in year 2000, over 3000 patients died of ischemic heart disease and related causes, accounting for almost 25% of the total deaths in Singapore [3].

In order to gain insight into the roots and causes of heart diseases, there has been a continuing research on the mechanism and function of the heart. The thick-walled left ventricle (LV) which is responsible for pumping oxygenated blood throughout the body

at physiologically high pressures during the cardiac cycle is considered the most important chamber of the heart. Therefore, the left ventricle has been the focal point of explorations for several researchers, medical scientists and cardiologists. Numerous parameters of the LV such as wall thickness, long-axis length, ventricular volume, myocardium mass have been investigated and quantified by researchers. Also, different indices of contractility such as the ventricular wall stress and strain, wall thickening, stroke volume and ejection fraction have been used to evaluate the physical condition of the heart [4].

Despite the many efforts dedicated to the analysis of left ventricular mechanism, it remains an exigent and challenging problem with much to be discovered. It is postulated that myocardial ischemia and other cardiac pathologies are associated with regional alterations in ventricular shape. Hence, it is in the interest of the author to investigate the intricate variation of left ventricular surface curvatures, which remains an unexplained phenomenon, and may be a clinically useful parameter for assessing the healthiness of the heart.

1.2 Objectives

The primary objectives of this research were to:

1. compute the LV surface curvatures via an analytical approach, using the concepts of differential geometry,
2. elucidate the temporal variation of LV surface curvatures during the cardiac cycle,
3. assess the value and potential of LV surface curvature dynamics as a clinical tool for cardiac health evaluation,

4. determine LV surface point correspondences between successive frames based on the bending energy of a thin plate LV model.

1.3 Scope

In this report, the LV was first modeled using the 3-D data points of the endocardial surface acquired from ultrasound imaging. The surface properties were derived by means of local surface patch fitting with a second order polynomial, which adequately provides excellent estimates of curvatures of the original surface. In the process of estimating point correspondences, the displacements of selected LV surface points during the cardiac cycle were also determined. The LV endocardial surface patch was modeled as an ideal thin flexible plate, whereby the model was loosely constrained to deform in a predetermined fashion, and the bending energy of the plate was used as a gauge for matching selected surface points between temporally successive frames. In addition, the LV contours from a new magnetic resonance (MR) dataset were extracted using deformable contours, and the motion of the LV surface points during isovolumic contraction were estimated based on the thin plate LV model.

1.4 Overview of the Framework

The changes in curvatures of the left ventricle have been examined from a sequence of frames throughout the cardiac cycle. The approach consists of three key elements, which are the geometric modeling of the left ventricle, the estimation of surface normals, and the extraction of surface curvatures by basis of surface fitting. Following this chapter is a review of the methods used by researchers to extract differential properties from various surfaces. Also, a literature review on the past works done and efforts made to study the functions and mechanisms of the left ventricle will also be presented in Chapter 2. Chapter 3 will describe the basic structure and function of the heart, while Chapter 4 will

bring in some foundations on the theory of differential geometry. Chapter 5 will focus on the methodology used for the local patch fitting. In addition, Chapter 6 will explain the procedures performed to accurately retrieve the surface properties and curvatures, and validate the algorithm used for the computation of curvatures. Chapter 7 will briefly describe the process of LV surface motion tracking based on the potential energy of an ideally thin and flexible plate model. Chapter 8 will briefly cover the method used to extract the LV surface contours from the cardiac MR images. The results will be depicted in Chapter 9, and the conclusion is made in Chapter 10 together with some recommendations for future work and direction.

Chapter 2

LITERATURE REVIEW

This chapter serves to describe and appraise the various methods employed by researchers for the extraction and analysis of surface curvatures, which are essential for the readers to appreciate the algorithm used in this report. In addition, a range of related work aimed at improving the understanding of cardiac mechanism and function are reviewed.

2.1 Related Work on Computation and Application of Surface Curvatures

The local surface curvature at any point on a structure is dependent only on the neighborhood of that particular point, and is invariant under rigid body motions. The definition of surface curvature is thus made without reference to any coordinate system, making it a distinctive and viewpoint independent descriptor of local shapes. It is hence logical and practical to describe the shape of a 3-D body by means of curvature distribution of its surface. For the past two decades, researchers and scientists have developed numerous methods and algorithms for the derivation of surface curvatures, and have proven the values of these intrinsic properties in different areas of research.

It has been the choice of many authors including Ferrie and Levine [5], Sander and Zucker [6], and Besl and Jain [7], to compute surface curvatures by means of an analytic approach. The analytic approach fundamentally approximates the original surface by an analytic surface such that the differential properties for the input points can be estimated. These analytic methods used by researchers differ in their surface fitting algorithms, but can be categorized into two types, namely linear and nonlinear fitting.

Researchers such as Krsek et al. [8], Dyn et al. [9], and Hamann [10] have attempted to approximate the surface using linear approximation methods. The main advantages of using linear approximations are the robustness and low computational cost involved. Due to the simplicity of this approach, it is mostly used for surfaces which are already triangulated, i.e. meshed with triangular elements. Although the faces of these triangle elements are flat and possess no curvature, one can try to estimate the curvatures of a small unknown surface by treating a triangle mesh as a piecewise linear approximation of that particular surface [9].

In a way, only the information from the triangle mesh itself is utilized for the curvature estimation. Besides, the vertex normals can be easily obtained from the 3-D meshes and be used to approximate the surface normals. It is a common practice for authors including Dyn et al. [9], van Damme and Alboul [11], Alboul and van Damme [12] etc., to define the integral Gaussian and mean curvatures with respect to a small arbitrary area surrounding a vertex. This is done by virtually replacing each edge by a cylinder of radius r joining the adjacent triangular faces tangentially, and then blending these triangles at the vertices. However, in order to derive the curvatures at the vertex, one has to assume that the curvatures are uniformly distributed around the vertex, and can be normalized by the specified area.

There are notably some drawbacks in using such linear methods. It is nonrealistic and imprecise to first approximate a complex and irregular surface with piecewise linear functions, and further assume that the original surface can be accurately estimated by blending cylinders at each triangle mesh. There is also a question on the area chosen for the triangle mesh since different areas will bring about different curvatures at the vertex. For convenience, two definitions of this area, i.e. the barycentric area and the Voronoi area are often used. It is also an important criterion in this method to have a good triangulated surface, meaning that efforts have to be made to obtain the optimal triangulation.

Another group of authors including Sander and Zucker [6], Frobin and Hierholzer [13], Kambhamettu and Goldgof [14] etc., used a nonlinear method to compute the surface curvatures. Extraction of curvatures using this approach is mostly performed by employing a second order polynomial function to approximate the original surface. Since a surface is a two-dimensional entity in a three-dimensional space, the analytical surfaces can be represented as a function of two parameters, u and v . With this, researchers often express the fitting polynomial usually in the form, z as a function of x and y i.e. $z = f(x, y)$, which leads to a quadratic form with parameters u and v . The task is then to solve the coefficients of the approximating paraboloid, which are used to evaluate the differential parameters of the surface. In general, an over-determined system of linear equations are obtained, and the polynomial coefficients are optimized by least squares minimization of a predefined error function.

Douros and Buxton [15] proposed another analytic representation which is an implicit surface of the form $f(x, y, z) = 0$. The resulting quadric representation is of the form $x^T U x = 0$ where U is a real symmetric matrix consisting of the coefficients. Since the quadric expression is homogeneous, there is a need to introduce some form of a constraint so as to solve for the optimized coefficients of the quadric patch. The minimization problem here is to determine the coefficients of the eigenvector corresponding to the smallest eigenvalue from a 10×10 eigen-problem. This method has yielded satisfactory results. However it is rather computational expensive due to the large number of coefficients involved and the intricacy of the minimization problem.

Although it is possible to approximate the original data by different types and degrees of polynomial function, Krsek et al. [16], in their experiments found that higher ordered polynomial such as third and fourth order surfaces show little advantages over second order surfaces. Besides, higher ordered polynomials require higher computational cost and may sometimes be unstable in their approximations.

Nonlinear methods are generally more reliable as compared to linear methods in the extraction of curvatures. This is especially true when the considered surface is of

substantial irregularity and complexity, since the geometric nonlinearity of the structure can be accurately tracked when the fitting paraboloid is used to approximate a reasonably small patch of area. Furthermore, additional information such as principal directions and surface deviations can be easily acquired for a rigorous and thorough assessment of the reliability and stability of the employed technique.

The recent advances in 3-D scanning technology saw an increase in popularity in the applications of surface curvature results on reconstructed images in the field of biological and medical science. Frobin and Hierholzer [17] has performed an analysis of the human back shape in terms of curvatures and determine its possible correlations with spinal deformities. The nonlinear method was applied in this particular shape analysis, in which second order surface patches were used to approximate the surface of the human back. The author utilized the results for the classification of surface points according to the Gaussian curvature obtained. Also, the mean curvature was used for diagnosing the asymmetry of the back shape. The advantage of this curvature analysis is the reliable detection of asymmetries independent of any precautions concerning patient positioning. This method was also employed to convert the primary data points of the human back into a rasterized parallel projection for back shape measurement [13]. It was found that the results obtained were reliable except at the boundaries and in regions with high curvature, which might be due to the systematic deviations of the second order surface used. Such problems could however be resolved by using randomly distributed data such that the data points in the operator field are located irregularly and with a sufficiently constant density.

Drerup and Hierholzer [18], with the aim of determining anatomical landmarks on the body surface, used curvature results acquired from moire topography and rasterstereography to detect the location of vertebra prominens. In a test of 80 different patients, it was found that the vertebra prominens can be easily located in more than 95 % of the Gaussian curvature maps, thus validating the accuracy and usefulness of this landmark localization by shape analysis.

Besides the direct analysis of shape using curvatures, these intrinsic surface properties are also applied in a variety of biomedical problems. In many cases, the surface curvatures obtained are used for the determination of corresponding 3-D points between pairs of surfaces. Wang et al. [19] have developed an algorithm for 3-D surface correspondence using geodesics and local surface properties, and applied the results to human cerebral cortical surfaces. In their work, the determination of the surface correspondence is based on the minimization of the objection function which is comprised of the Euclidean distance measure, surface normal match measure, and the feature match measure. In the matching process, the tested surface was classified into different shapes and degrees of curvatures. Results on a pair of brain surfaces have shown satisfactory results and the classified points have been identified consistently.

It has been the interest of several researchers to tackle the problem of motion analysis using information from the surface properties. Such efforts have been made by various authors [14, 20, 21, 22] to track the motion of the left ventricular wall. In [14], Kambhamettu and Goldgof [14] proposed a method for the estimation of point correspondences on a surface undergoing conformal non-rigid motion based on the changes in its Gaussian curvature. This method is justified by the invariancy of the Gaussian curvature towards rigid body motion and the type of surface parameterization.

The algorithm involving three steps was experimented with volumetric CT data of the left ventricle of a canine's heart. The first step consists of the computation of Gaussian curvature, and coefficients of the first fundamental form and its derivatives for the tracking of conformal motion. The second step involves the formulation of hypotheses using a window around the point of consideration. Points within the window are the candidates for point correspondence, and stretching is calculated for each point and the error for each candidate correspondence is computed. The third step evaluates which of the hypotheses satisfies the underlying motion assumption by selecting the hypothesis with minimum error. There is however a problem with this algorithm, which is the possibility of having more than one hypothesis with similar minimum error. Also, the derived point correspondences will not be reliable if the minimum error is too large.

These problems however can be resolved by the acquisition of more sample points by interpolation and defining a threshold for the minimum error. The results showed that the algorithm managed to estimate contraction and expansion for most points of the left ventricle during systole and diastole respectively. Stretching between the healthy LV and the LV affected by atrial fibrillation was also performed to verify the results of the algorithm. It was observed that less stretching took place on the LV wall affected by atrial fibrillation between end-diastole and end-systole, which corresponded to the findings of radiologists. This provided an indication that conformal motion might be a reasonable approximation of the LV motion.

Amini and Duncan [20, 21] formulated an algorithm to track the left ventricular motion using a bending and stretching model. They hypothesized that the physical model of the LV can be approximated by a thin-plate with constraint to stretch in a predefined manner. In this method, the principal curvatures or the Gaussian and mean curvatures are required for the derivation of the bending energy of a thin plate. Also, the coefficients of the first fundamental form are needed for matching points according to conformal maps. The local displacement vectors of each considered point is then determined by finding the point of the deformed surface which minimizes the bending energy and deviation from the conformal stretching. This method described here provides a quick and reasonable way to track the LV wall motion, and is thus favored over tracking methods that involves only the bending energy.

Unfortunately, although the surface curvatures were used during numerous studies to track LV wall motion, there have been no further efforts made on the analysis of these intrinsic LV surface properties.

2.2 Related Work on Left Ventricular Function and Mechanism

There has been a vigorous research in the biomedical community aimed at understanding the left ventricular function and mechanism. For decades, simple indices such as the

cardiac output and ejection fraction have been used by clinicians for the measure of contractility. In addition, straightforward relationships like ventricular volume versus time, ventricular pressure versus time, and ventricular blood pressure versus volume have been employed by cardiologists as a measure of cardiac health. In recent years, it has become apparent that regional qualities and quantities of the left ventricle have significant importance in the perception of pathological conditions such as myocardial ischemia and arrhythmias. Subsequently, researchers have explored the different factors that might prove invaluable for the understanding of left ventricular mechanism.

2.2.1 Various Models of the Left Ventricle

For decades, the famous Laplace law for a sphere has been used by researchers for the determination of left ventricular wall stress. Woods [23] was one of the first to formulate a thin-walled spherical model for the evaluation of ventricular wall stress with the Laplace law. It should be noted that this formula approximates the average circumferential wall stress which is often used interchangeably with mid-wall stress [24].

Sandler and Dodge [25] recognized that the ellipsoid of revolution being a more realistic choice of left ventricular geometry as compared to the sphere, employed a different version of the Laplace law for the stress analysis. In their work, the principal radii of the ellipsoidal model was taken into account for the formulation of wall stress. The main limitation of these models was their thin-walled representation which assumed the ventricular wall thickness to be small relative to the radii of curvature. With this constraint, variations of material properties across the wall thickness were ignored and the stresses were assumed to be constant through the wall thickness. Regardless, the Laplace law is still accepted by many researchers and is being applied with reasonable assumptions, for the analysis of ventricular stress due to the simplicity and availability of this formula.

Wong and Rautaharju [26] were one of the first to come up with a thick-walled ventricular model. The wall stress was formulated with ellipsoidal shells assumed to be isotropic, linearly elastic and compressible. Despite the many assumptions and further simplification of the analysis using small-strain theory, they discovered nonlinear stress distributions through the wall thickness, an outcome that cannot be predicted by the Laplace law. It was Mirsky [27] who first applied large deformation elasticity for the evaluation of LV stress. He discovered that the stresses found at the endocardium using large strain theory were about ten times as high as the values predicted by linear small strain models.

Besides the spherical and ellipsoidal models, simple geometries like the cylinder and cone have also been used to study the LV. Authors like Reneman and Veenstra [28], Tozeren [29], and Guccione et al. [30] etc. have attempted to model the left ventricle with a cylindrical model. However, it should be noted that these cylindrical models are best suited to describe the equatorial region of the LV. Azhari et al. [31], realizing that a cylindrical model is not enough to predict the non-uniformity of the ventricular motion, constructed a conical model to better understand this non-uniformity. Chaudhry et al. [32] formulated a mathematical model based on a thick-walled truncated conical shell and incorporated large deformation elasticity. It was found that conical models leads to more realistic results than the spherical models, and can better evaluate stresses and strains from base to apex instead of only at equatorial regions in cylindrical models.

In recent years, there has been an increase in the number of left ventricular models based on the finite element method (FEM). It was very much agreed by researchers and scientist around the world, that the FEM was best suited for the stress and strain analysis of the LV. Gould et al. [33] formulated one of the first finite element models of the left ventricle. With doubts that simple geometric model were proficient in their stress prediction, an axisymmetric FE model using the longitudinal cross-section of the LV was developed. In this particular analysis, they discovered that the irregularity of the LV geometry had considerable effects on wall stresses. Thereafter, many FE models with realistic LV geometry were constructed and tested. Recently, Nash [34] formulated one

of the finest FE model of the LV based on finite deformation elasticity. This anatomically accurate model took into account the effects of muscle fibre orientation, and the difference in myocardial response during systole and diastole to realistically simulate and analyze the deformations of the heart during different phases of the heart.

2.2.2 Mechanical Properties and Constitutive Laws for the Myocardium

In order to accurately predict the stresses and strains in the left ventricle, a suitable characterization of the material properties of cardiac muscle is required. There are two major features of the mechanical properties of the myocardium, namely passive and active. The passive myocardial stress corresponds to the diastolic heart, and describes the resistance to deformation due to the dense framework of connective tissue. On the other hand, active myocardial stress is representative of the contractile properties of tissue and describes the muscle tension development responsible for segment shortening during systole.

Traditionally, the material properties used in LV models were assumed to be homogeneous and isotropic [25,26]. However, studies made by some researchers [35,36, 37, 38] have shown that the architecture and orientation of myocardial fibre have significant effects on the mechanical properties of myocardium. One of the simplest approach for deriving the passive material properties of the myocardium is the uniaxial tension test, but due to the three-dimensional constitutive behavior of the myocardium, uniaxial stress-strain data are insufficient for the purpose [39]. Triaxial testing is considered as the ideal approach, but it remains a challenging and difficult practice. It is thus the common practice of researchers [40, 41, 42] to characterize myocardial material properties by biaxial tissue testing. In these tests, thin slabs of muscle tissue are excised and the samples are usually tested for stress and strain along the fibre and cross-fibre axes. In general, the constitutive behavior of myocardium are assumed to be hyperelastic with the strain energy density function formulated from biaxial experimental data.

With the same goal in mind, researchers have come up with different constitutive laws to describe the passive myocardial behavior. Guccione and coworkers [30] formulated a transversely isotropic strain energy density function in an exponential form by considering the fibrous structure of the myocardium taken into account. Humphrey and coworkers [43, 44] developed a polynomial form of the strain energy function which also described a transversely isotropic behavior. Recently, Hunter et al. [45] proposed an alternative pole-zero formulation of the pseudo-strain energy density function. This formulation came from their observations from biaxial tests which showed that the axial stress was particularly low for small axial strains, and increased rapidly when the strain approached the limiting strain for that axis.

Active muscle law differs from the passive law in that a description of a one-dimensional relationship between fibre strain and active muscle tension is adequate [46]. In the past, models of the active tension development in cardiac muscle fibre were often based on Hill's three-component model [47], and Huxley's Sliding-filament model [48]. In Hill's model, active muscle is represented by: A contractile element, which is freely extensible at rest but when activated, can shorten and develop tension; a series elastic element which is in series with the contractile element; and a parallel elastic element which accounts for the elasticity of the muscle at rest. On the other hand, Huxley's model was formulated with the assumption that tension is generated by the interaction of myosin and actin filaments, through a side piece situated between two protrusions from the myosin filament. More recently, Hunter [49] presented a fading memory model of the active tension based on the time history of length. In this model, the active tension is influenced more by recent length changes than by the past history of length changes. Up till now, there is still a lack of reliable constitutive laws to describe the active properties of myocardium during the systolic phase.

2.2.3 Left Ventricular Twisting Motion

It is the author's belief as well as of some researchers that one of the earliest indicators of ischemic heart diseases is the regional alteration in myocardial wall motion. In general, ventricular wall motion can be characterized into longitudinal contraction, radial contraction, and rotation. In the early studies, invasive methods were used to quantify the left ventricular twisting motion. Rushmer et al. [50] implanted stainless steel wire in the heart wall of dogs and investigated the twisting motion using cineradiography. In their experimentals, a rotation about the long axis of less than 10° was noted. McDonald [51], using similar technique, reported a slight counterclockwise rotation and a slight clockwise rotation of the epicardium when viewed from apex to base during the early and late stages of systole respectively. These observations were complemented by Hansen and coworkers [52] who found similar counterclockwise rotation of the LV during systole. In addition, angles of rotation relative to the base were found smaller in the antero-apical wall as compared to the inferior and lateral walls. Also, the rotation at the mid-ventricular regions were almost half of that at the apex.

More recent imaging techniques have allowed for non-invasive study of the LV twisting motion. Lorente and coworkers [53] reported an average left ventricular rotation at the apex in 38 normal patients using 2-D echograms and cineangiography. Arts et al. [54] studied left ventricular torsion in closed-chest dogs using echocardiography and determined experimentally that the ventricular torsion was relatively constant over a wide range of preload, afterload, and contractility. Buchhalter and coworkers [55] used the magnetic resonance imaging (MRI) technique for their study. The difference in angular displacement of each epicardial and endocardial tag point at end systole from the systolic position of the corresponding tag point on the basal plane was calculated. It was reported that the mean angle of torsion in the apical slice with respect to the mean angle in the basal slice in the endocardium was around 19° counterclockwise. It was also noted that the rotation at the epicardial surface was approximately 8° less than that in the endocardial surface. Anteroseptal regions reportedly exhibited more twisting than the

posteroseptal regions for both the endocardial and epicardial surface, and the angle of twist increased with the distance relative to the base.

Recently, Kroeker et al. [56] with an optical device coupled to the LV apex, observed apex rotation with simultaneous recording of LV pressure, ECG, segment length and minor-axis diameters in 16 open-chest dogs. They reported that ischemia has a pronounced effect on the dynamics of apex rotation. Their results were complemented by Knudtson and coworkers [57], who reported the similar sensitiveness of ventricular twist to ischemia. Many other efforts made by researchers [58, 59, 60] also noted the significant relationship of the LV twisting motion with cardiac health.

Chapter 3

PHYSIOLOGY OF THE HEART

This chapter provides readers with some knowledge of the macroscopic and microscopic features of the heart. In conjunction, the basic functioning of the heart is also explained. Interested readers should refer to Humphrey [1], Martini et al. [61], Opie [62], Streeter [63], Smail and Hunter [64] etc. for more detailed explanations.

3.1 Macroscopic Structure of the Heart

The human heart is a muscular pump, approximately the size of a human fist, and is located in the mid-center of the chest cavity with approximately two thirds of its mass to the left of the body's midline. The heart consists of four chambers through which blood is pumped (refer to Figure 3.1). The upper two chambers are called the left and right atria, while the lower two are known as the left and right ventricles. The left and right ventricles are separated by a tough muscle wall known as the interventricular septum which normally functions as part of the LV. The pericardium which is a double-layered membrane, embraces the heart like a sac. The outer layer of this membrane surrounds the roots of the heart's major blood vessels, while the inner layer is attached to the heart muscle wall. There are four types of valves that facilitate the regulation of blood flow in the body. They are namely, the tricuspid valve which is located between the right atrium and right ventricle, the pulmonary valve situated between the right ventricle and the pulmonary artery, the mitral valve which is between the left atrium and left ventricle, and the aortic valve that is found between the left ventricle and the aorta.

Figure 3.1 : Structure of the heart (from [65])

The relative thin-walled atrial chambers act as low pressure blood reservoirs during the ventricular contraction phase. From Figure 3.1, it can be observed that the left ventricle have relative thick wall as compared to the other chambers, which is due to its highly pressured pumping responsibility to propel blood to distal locations of the body.

3.2 The Cardiac Cycle

The cardiac cycle can be characterized into two major phases, known as the ventricular systole and diastole. During ventricular systole, the heart contracts and ejects the blood, while in ventricular diastole, the heart relaxes and fills with blood. It is during the start of systole that sees the earliest and fastest rise in the left ventricular pressure (refer to Figure

3.2). This period of the cardiac cycle comes after the left atrial contraction and is referred to as the isovolumic contraction phase. At this stage of ventricular contraction, the mitral and aortic valves remain closed, thus preventing any change in the LV cavity volume. The second stage of systole occurs when the left ventricular pressure exceeds that of the aortic pressure, causing the aortic valve to open and consequently leading to the rapid ejection of blood from the ventricle. This stage of the systolic phase causes a rapid decrease in the LV cavity volume and a further increase in LV pressure. Following the rapid ejection phase is a longer period of systole called the reduced rejection, during which the aortic pressure decreases. The last interval of systole is known as the protodiastolic phase and it ends with the closure of the semilunar valves.

Figure 3.2: Different phases of the cardiac cycle

The diastolic phase starts with the isovolumic relaxation whereby the LV relaxes while maintaining a constant volume inside the cavity. When the ventricular pressure falls below the atrial pressure, the mitral valve opens which marks the beginning of the rapid filling stage. During this period of time, there is a rapid increase in the LV cavity volume and subsequently the slow filling phase also known as diastasis occurs. The last stage of ventricular diastole coincides with atrial systole, and there is a final surge of blood into the ventricles before the closure of the atrioventricular valves due to the higher ventricular pressures relative to those of the atria.

3.3 Microscopic Structure of the Heart

Heart muscles consist of mainly myocytes which are typically ellipsoidal cylinders with length of around 80 to 100 μm , and major-axis dimension of about 10 to 20 μm [64]. Cardiac myocytes are connected end to end and attached with several others to form a 3-dimensional network of cells. Adjacent cells are interconnected through intercalated disks. These intercalated disks contain gap junctions that provide the electrical continuity between cells. This eventually causes the electrical impulses to propagate more rapidly along the axis of the fibres, rather than across axis. Caulfield and Borg [66] observed that myocytes were connected to adjacent myocytes by numerous bundles of collagen 120 to 150 nm in diameter, which prevent the slippage of adjacent cells. It was found that radial collagen chords of similar dimension also extended between myocytes and capillaries. They also noted a meshwork of collagen bundles that surrounded groups of three or more myocytes. Figure 3.3 depicts the collagen network of the rabbit LV which shows the interconnection between the myocyte (M) and the capillary (C).

Figure 3.3: Collagen network of the rabbit LV (from [66])

Studies by Streeter and coworkers [35, 36, 37, 63, 67] have provided one of the most systematic and quantitative analyses of muscle-fiber orientations. They found that the fiber orientations are generally tangential to the endocardial and epicardial surfaces. They also discovered that the fiber paths follow a helical geometry and the fiber angle vary smoothly from a right-handed helix on the endocardium to a left-handed helix on the epicardium. With their experimental studies on a canine heart, they reported a well-ordered distribution of myocardial fibre directions varying from 60° on the endocardium to around -60° on the epicardium.

More recently, LeGrice and coworkers [68] noted that the myocytes are ordered laminae with relatively extensive cleavage planes in between muscle layers. Each layer is found to be around four cells thick. Their orientation was found to be generally normal to the ventricular surfaces, except in the subendocardial and subepicardial regions, where they become almost tangential to the surfaces. It has also suggested that one axis of the sheets is built by the myofibre orientation, while the other axis which is measured relative

to the local outer-wall normal in planes, is built by the principal axis of the heart and a vector radial to it. Figure 3.4 illustrates that the fibre orientation inside a myocardial sheet varies smoothly across the wall, and the myocytes are around four cells thick in a sheet. Endomysial collagen can be seen linking the adjacent cells within a sheet, while the adjacent sheets are bonded by perimysial collagen.

Figure 3.4: Collagen network of the rabbit LV (from [45])

3.4 Cardiac Electrical Activity

The beating of the heart is governed by the electrical impulses that come from the heart muscle. In other words, it is the electrical depolarization that causes cardiac muscle cells to generate contractile forces. In a normal heart, this electrical signal begins in the sinoatrial (SA) node, which consists of a cluster of cells located in the upper posterior portion of the right atrium (refer to Figure 3.5). The SA node is an electrically active region of the heart which exhibits automaticity and is sometimes called the natural pacemaker. When an electrical impulse is activated from the SA node, it stimulates the adjacent excitable tissue and initiates conduction, such that the electrical signal can propagate through the cardiac cells. This action potential first travels through the atria causing the atria to contract. This signal eventually travels to the atrioventricular (AV) node which is located at the lower portion of the right atrium.

The AV node, similar to the SA node, exhibits some form of automaticity, however this ability of self activation does not fire unless the SA node is nonfunctional. This AV node receives the electrical impulses and provides a short delay of time between atria and ventricles contraction so as to produce an efficient pumping action. Subsequently, the signal is passed down to the Bundle of His which is located in the upper portion of the interventricular septum. The Bundle of His connects the AV node with the bundle branches, and they are together called the AV junction. The AV node is the only electrical connection between the atria and the ventricles, which implies that the only pathway for the action potential to reach the ventricles is through the AV node. It can therefore be devastating if the AV node fails to function properly.

Figure 3.5: Electrical conduction in the heart [69]

When the Bundle of His is activated, the electrical signal will be split into the Right Bundle Branch, which directs to the right ventricle, and the Left Bundle Branch which leads to the left ventricle. The Left Bundle Branch is further divided into the Anterosuperior, Inferoposterior, and Septal fascicle fibres. These fascicles of the Left Bundle Branch and the Right Bundle Branch all terminate in the Purkinje fibres. The Purkinje fibres invade approximately one thirds into the myocardium, and serve to propagate the signal to the rest of the ventricular muscle. It is partly due to this dense network that allows for the excellent volume reduction of the ventricles during systole.

Figure 3.6: Typical Electrocardiogram (ECG)

The electrical conduction system of the heart can also be explained with the electrocardiogram (ECG) as shown in Figure 3.6. The P wave can be understood as the spread of the electrical activity over the atria after the initial depolarization of the SA node. The QRS complex represents the propagation of negative wave which is caused by the depolarization of the ventricular muscles. During this short period of time, the atrial repolarization also occurs. The T wave signifies the repolarization of the ventricular muscles, and is of a lower amplitude than the QRS complex. The T wave is also of a longer duration which indicates that the ventricular repolarization process is slower than depolarization.

Chapter 4

THEORY OF DIFFERENTIAL GEOMETRY

The theory of differential geometry is very much based on the ground-breaking work of L. Euler (1707-1783), C. Monge (1746-1818) and C. F. Gauss (1777-1855). One particular area of their work concerns the description of local curve and surface properties, which has been widely appreciated and applied in various fields of work. In this chapter, some of the basic concepts of differential geometry will be discussed. Interested readers can refer to Willmore [70], Chuiv [71], Lipschutz [72] or any other text on differential geometry for more detailed explanations.

4.1 Parametric Curves

A parametric representation serves as a convenient way of describing a curve and as a useful tool for the analysis of curve properties. In the theory of plane curves, a curve is usually described by means of a single equation or by a parametric representation. A good example will be that of a circle lying on a plane, with center (0, 0) and radius r , specified in Cartesian coordinates (x, y) by a single equation

$$x^2 + y^2 = r^2 \quad (4.1)$$

or by the parametric representation

$$x = r \cos(u), \quad y = r \sin(u), \quad 0 \leq u \leq 2\pi \quad (4.2)$$

However, in a three-dimensional Euclidean space E^3 , a single equation generally represents a surface, and is insufficient to specify a curve. Since the intersection of two surfaces generally results in a curve (refer to Figure 4.1), two equations are required to specify a regular space curve in E^3 .

Figure 4.1 : Curve defined by the intersection of two surfaces

Parametrically, a curve in three-dimensional vector space $\mathcal{R}^3$ can be specified as

$$\mathbf{x} = \mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}, \quad t \in [a, b] \subset \mathcal{R} \quad (4.3)$$

where its Cartesian coordinates x , y and z are differentiable real-valued functions of the real parameter t which falls in the interval a and b .

It is often imperative to differentiate the equation of a curve so as to determine the "smoothness" of the curve. It is therefore convenient to make some technical definitions to facilitate the classification of curves.

A real-valued function f defined on a real interval I is said to be of class m (C^m) if f can be differentiated m^{th} times and if f^m is continuous on I . Thus a curve with vector function $\mathbf{x}$ is said to be of C^m if each of the component functions (x, y, z) is C^m .

Let γ be a curve of class ≥ 2 , and assume that γ is parameterized with respect to arc length s given by

$$s = s(t) = \int_a^t |\dot{\mathbf{x}}| dt, \quad t \in [a, b] \quad (4.4)$$

such that γ can be represented by the equation $\mathbf{x} = \mathbf{x}(s)$. The tangent vector at point $\mathbf{x}(s)$ (refer to Figure 4.2) is therefore given by

$$\mathbf{t} = \mathbf{t}(s) = \dot{\mathbf{x}}(s) = \frac{d\mathbf{x}}{ds} \quad (4.5)$$

Figure 4.2: Tangent to a point on curve

The arc-rate at which the tangent changes direction as a point $\mathbf{x}(s)$ moves along the curve γ is known as the curvature of γ at $\mathbf{x}(s)$ and is denoted by

$$\kappa = \kappa(s) = |\dot{\mathbf{t}}(s)| = |\ddot{\mathbf{x}}(s)| \quad (4.6)$$

or in terms of the actual parameter t

$$\kappa = \kappa(t) = \frac{|\dot{\mathbf{x}} \times \ddot{\mathbf{x}}|}{|\dot{\mathbf{x}}|^3} \quad (4.7)$$

while the radius of curvature of γ is defined as

$$\rho = \frac{1}{\kappa} \quad (4.8)$$

4.2 Surface Geometry

A surface is a two-dimensional entity in three-dimensional space. As remarked in section 4.1, a surface in E^3 can be specified by a single equation in the three Cartesian coordinates x , y and z . Alternatively, a surface can be described by indicating each of the three coordinates x , y and z in terms of two parameters (u, v) . That is, any surface point $\mathbf{x}$ can be generally represented by a two-parametric position vector

$$\mathbf{x} = \mathbf{x}(u, v) = \begin{bmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{bmatrix} \quad (4.9)$$

the components of which are functions of the two parameters u and v . It is further assumed that these functions are continuous and differentiable as many times as required. In order to inspect the local shape of a surface in the surroundings of a point $\mathbf{x} = \mathbf{x}(u, v)$, it is necessary to consider the points $\mathbf{x} + d\mathbf{x} = \mathbf{x}(u + du, v + dv)$ in the vicinity of $\mathbf{x}$. If $\mathbf{x} = \mathbf{x}(u, v)$ is a coordinate patch on the surface of class ≥ 1 , the differential of $\mathbf{x} = \mathbf{x}(u, v)$ at (u, v) is a linear mapping

$$d\mathbf{x} = \mathbf{x}_u du + \mathbf{x}_v dv \quad (4.10)$$

of vectors (du, dv) in the uv plane onto vectors $\mathbf{x}_u du + \mathbf{x}_v dv$ parallel to the tangent plane at $\mathbf{x}(u, v)$

where $\mathbf{x}_u = \frac{d\mathbf{x}}{du}$ and $\mathbf{x}_v = \frac{d\mathbf{x}}{dv}$.

4.2.1 First Fundamental Form

Suppose γ is a curve on the surface given by the parameter t , such that $u = u(t)$ and $v = v(t)$. From equation 4.4, it can be shown that

$$\frac{ds}{dt} = \left| \frac{d\mathbf{x}}{dt} \right| = \left| \dot{\mathbf{x}} \right| \quad (4.11)$$

thus giving

$$\begin{aligned} \left(\frac{ds}{dt} \right)^2 &= \frac{d\mathbf{x}}{dt} \cdot \frac{d\mathbf{x}}{dt} = \left(\frac{d\mathbf{x}}{du} \frac{du}{dt} + \frac{d\mathbf{x}}{dv} \frac{dv}{dt} \right) \cdot \left(\frac{d\mathbf{x}}{du} \frac{du}{dt} + \frac{d\mathbf{x}}{dv} \frac{dv}{dt} \right) \\ &= \left(\frac{d\mathbf{x}}{du} \cdot \frac{d\mathbf{x}}{du} \right) \left(\frac{du}{dt} \right)^2 + 2 \left(\frac{d\mathbf{x}}{du} \cdot \frac{d\mathbf{x}}{dv} \right) \left(\frac{du}{dt} \right) \left(\frac{dv}{dt} \right) + \left(\frac{d\mathbf{x}}{dv} \cdot \frac{d\mathbf{x}}{dv} \right) \left(\frac{dv}{dt} \right)^2 \\ &= I \left(\frac{du}{dt}, \frac{dv}{dt} \right) \end{aligned} \quad (4.12)$$

By replacing the derivatives $\frac{du}{dt}$ and $\frac{dv}{dt}$ by the differentials du and dv respectively in equation 4.12, this particular quadratic form is transformed in a way that it depends only on the surface and not on a particular curve in the surface.

$$I(du, dv) = (\mathbf{x}_u \cdot \mathbf{x}_u) du^2 + 2(\mathbf{x}_u \cdot \mathbf{x}_v) du dv + (\mathbf{x}_v \cdot \mathbf{x}_v) dv^2 \quad (4.13)$$

This particular form is a homogeneous function of second degree in du and dv , and is very important in the consideration of the surface. It is known as the first fundamental form of the surface given by

$$I(du, dv) = Edu^2 + 2Fdudv + Gdv^2 \quad (4.13)$$

with

$$E = E(u, v) = \mathbf{x}_u \bullet \mathbf{x}_u \quad (4.14)$$

$$F = F(u, v) = \mathbf{x}_u \bullet \mathbf{x}_v$$

$$G = G(u, v) = \mathbf{x}_v \bullet \mathbf{x}_v$$

called the first fundamental coefficients of the surface, and are functions of u and v and vary from each point on the coordinate patch.

The first fundamental form is positive definite, i.e. $I \geq 0$ and $I = 0$ if and only if $du = 0$ and $dv = 0$. Also, the coefficients must satisfy the conditions $E > 0$, $G > 0$ and $EG - F^2 > 0$.

4.2.2 Second Fundamental Form

Suppose $\mathbf{x} = \mathbf{x}(u, v)$ is a point on the surface of class ≥ 2 , then the partials $\mathbf{x}_u$ and $\mathbf{x}_v$ at this point span the tangent plane to the surface at $\mathbf{x}$. The normal vector $\mathbf{N}$ of the tangent plane coincides with the normal to the surface at $\mathbf{x}$, and is given by

$$\mathbf{N} = \mathbf{x}_u \times \mathbf{x}_v \quad (4.15)$$

The unit normal to the surface at $\mathbf{x}$ is obtained by normalizing the vector $\mathbf{N}$ (refer to Figure 4.3), and is given by

$$\mathbf{n} = \frac{\mathbf{x}_u \times \mathbf{x}_v}{|\mathbf{x}_u \times \mathbf{x}_v|} \quad (4.16)$$

It is obvious that $\mathbf{n}$ is a function of u and v of class C^1 with its differential defined as

$$d\mathbf{n} = \mathbf{n}_u du + \mathbf{n}_v dv \quad (4.17)$$

Figure 4.3: Unit normal to surface at a point

The second fundamental form can be defined as

$$\begin{aligned} II(du, dv) &= -d\mathbf{x} \cdot d\mathbf{n} \\ &= -(\mathbf{x}_u du + \mathbf{x}_v dv) \cdot (\mathbf{n}_u du + \mathbf{n}_v dv) \\ &= -\mathbf{x}_u \cdot \mathbf{n}_u du^2 - (\mathbf{x}_u \cdot \mathbf{n}_v + \mathbf{x}_v \cdot \mathbf{n}_u) du dv - \mathbf{x}_v \cdot \mathbf{n}_v dv^2 \end{aligned} \quad (4.18)$$

Since $\mathbf{x}_u \cdot \mathbf{n} = 0$ and $\mathbf{x}_u \cdot \mathbf{n} + \mathbf{x}_{uu} \cdot \mathbf{n} = 0$, equation 4.18 can be written as

$$\begin{aligned} II(du, dv) &= L du^2 + 2M du dv + N dv^2 \\ &= d^2 \mathbf{x} \cdot \mathbf{n} \end{aligned} \quad (4.19)$$

I is a homogeneous function of second degree in du and dv with coefficients L , M and N being continuous functions of u and v , and are given by

$$L = L(u, v) = -\mathbf{x}_u \cdot \mathbf{n}_u = \mathbf{x}_{uu} \cdot \mathbf{n}_u \quad (4.20)$$

$$M = M(u, v) = -\frac{1}{2}(\mathbf{x}_u \cdot \mathbf{n}_v + \mathbf{x}_v \cdot \mathbf{n}_u) = \mathbf{x}_{uv} \cdot \mathbf{n}$$

$$N = N(u, v) = -\mathbf{x}_v \cdot \mathbf{n}_v = \mathbf{x}_{vv} \cdot \mathbf{n}$$

4.2.3 Classification of Surface Points

Let P be a point $\mathbf{x} = \mathbf{x}(u, v)$ on the surface, with $\mathbf{n}$ being the unit normal to the surface at P as shown in Figure 4.4. Consider a point Q with $\mathbf{x} = \mathbf{x}(u + du, v + dv)$ lying in the vicinity of P , then the projection of the vector $\mathbf{PQ}$ onto the unit normal $\mathbf{n}$ at P is given as

$$d = \mathbf{PQ} \cdot \mathbf{n} \quad (4.21)$$

and $|d|$ represents the magnitude of d which measures the perpendicular distance from Q to the tangent plane at P . It can be observed from the figure that d may have different sign conventions.

Figure 4.4: Projection of vector onto the unit normal

Using Taylor's theorem,

$$\begin{aligned}
 d &= \mathbf{PQ} \cdot \mathbf{n} \\
 &= [\mathbf{x}(u + du, v + dv) - \mathbf{x}(u, v)] \cdot \mathbf{n} \\
 &= [\{\mathbf{x}(u, v) + d\mathbf{x} + \frac{1}{2} d^2 \mathbf{x} + o(du^2 + dv^2)\} - \mathbf{x}(u, v)] \cdot \mathbf{n} \\
 &= d\mathbf{x} \cdot \mathbf{n} + \frac{1}{2} d^2 \mathbf{x} \cdot \mathbf{n} + o(du^2 + dv^2)
 \end{aligned} \tag{4.22}$$

Since $d\mathbf{x}$ is tangent to the surface at P, $d\mathbf{x} \cdot \mathbf{n} = 0$. Thus from equation 4.19, the above equation can be reduced to the form

$$d = \frac{1}{2} \mathbf{II}(du, dv) + o(du^2 + dv^2) \tag{4.23}$$

Consider a function

$$\delta = \frac{1}{2} \mathbf{II}(du, dv) \tag{4.24}$$

which is known as the osculating paraboloid at P. This paraboloid has second order contact with the surface and provides a good classification of the surface at P, depending on the value of $LN - M^2$.

4.2.3.1 Elliptic point

When the value $LN - M^2 > 0$, the point considered is an elliptic point. This also implies that the osculating paraboloid δ is an elliptic paraboloid with the small cross-section (du, dv) being an ellipse. In this case, δ maintains the same sign convention across (du, dv) and the surface lies on one side of the tangent plane at P as shown in the Figure 4.5.

Figure 4.5: Elliptic point

4.2.3.2 Hyperbolic point

When the value $LN - M^2 < 0$, the point considered is a hyperbolic point. The osculating paraboloid δ in this case is a hyperbolic paraboloid, with the (du, dv) cross-sections being hyperbolae. In this case, δ can be either positive or negative, and the surface lies on both side of the tangent plane at P as shown in the Figure 4.6.

Figure 4.6: Hyperbolic point

4.2.3.3 Parabolic point

When the value $LN - M^2 = 0$ and $L^2 + M^2 + N^2 \neq 0$, the point considered is a parabolic point (refer to Figure 4.7). The osculating paraboloid δ is a parabolic cylinder, and the surface touches the tangent plane along a whole curve near P.

Figure 4.7: Parabolic point

4.2.3.4 Planar point

When $L = M = N = 0$, the osculating parabola degenerates into a plane, i.e. $\delta = 0$. In this case, the given surface has a higher order of contact with the tangent plane as compared to the previous cases.

4.2.4 Normal, Gaussian and Mean Curvatures

If a plane coincides with the unit normal $\mathbf{n}$ and unit tangent $\mathbf{t}$ to the surface at P, then this plane is orthogonal to the surface at P. The intersection of this plane and the surface is a curve γ , and the curvature of γ is the curvature of the surface at P in the direction of $\mathbf{t}$. The normal curvature of the surface at P denoted by κ_n can then be interpreted as the projection of the curvature vector $\mathbf{k}$ of γ at P onto the normal $\mathbf{n}$ at P.

$$\kappa_n = \mathbf{k} \bullet \mathbf{n} \quad (4.25)$$

Alternatively, κ_n can be specified in terms of the first and second fundamental forms,

$$\begin{aligned} \kappa_n &= \frac{II(du, dv)}{I(du, dv)} \\ &= \frac{Ldu^2 + 2Mdudv + Ndv^2}{Edu^2 + 2Fdudv + Gdv^2} \end{aligned} \quad (4.26)$$

In general, κ is a principal curvature in the direction du, dv if and only if κ, du, dv satisfy the following conditions,

$$\begin{aligned} (L - \kappa E)du + (M - \kappa F)dv &= 0 \\ (M - \kappa F)du + (N - \kappa G)dv &= 0 \\ du^2 + dv^2 &\neq 0 \end{aligned} \quad (4.27)$$

There are two extreme values κ_1 and κ_2 of $\kappa = \kappa_n$, known as the principal curvatures of the surface at P, and are obtained from the roots of

$$\det = \begin{bmatrix} L - \kappa E & M - \kappa F \\ M - \kappa F & N - \kappa G \end{bmatrix} = 0 \quad (4.28)$$

The product of the roots is known as the Gaussian curvature and is denoted by

$$K = \kappa_1 \kappa_2 = \frac{LN - M^2}{EG - F^2} \quad (4.29)$$

It should be noted that both the roots κ_1 and κ_2 change sign when the normal $\mathbf{n}$ is reversed, while K is not affected by such a reversal.

The average of the roots is called the mean curvature and is denoted by

$$H = \frac{\kappa_1 + \kappa_2}{2} = \frac{EN - 2FM + GL}{2(EG - F^2)} \quad (4.30)$$

These two properties are often used for the classification of different surfaces as shown in Figure 4.8, which depicts the relationship between the principal, Gaussian and mean curvatures for various shapes.

The Gaussian curvature K can be used in the classification of points on a surface since $EG - F^2 > 0$ at every regular point, and the sign of K follows the sign of $LN - M^2$. On the other hand, the mean curvature H can be used to differentiate convexity and concavity of a surface under consideration.

Classification of Shape	Example	Curvature			
		Gauss (K)	Mean (H)	Principal (κ_1) (κ_2)	
Planar		0	0	0	0
Parabolic		0	> 0	> 0	0
Elliptic		> 0	> 0	> 0	> 0
Hyperbolic		< 0	$<$ $= 0$ $>$	< 0	> 0

Figure 4.8: Relationship between principal, Gaussian and mean curvatures

Chapter 5

METHODOLOGY APPLIED FOR CURVATURE ESTIMATION

As described in Chapter 4, there exists a well-established theory which can be directly applied to analytical surfaces for the purpose of shape analysis. Analytical surfaces are surfaces with known equations and functions, where the surfaces geometrical properties at a particular surface point can be derived, i.e. a sphere, cylinder and ellipsoid etc. However, with irregularly shaped surfaces, the situation is different as there are no predefined equations or functions necessary for the derivation of surface properties. In this report, an analytic method to determine the curvature information by basis of surface fitting will be used.

5.1 Local Patch Fitting

In the vicinity of a point $\mathbf{x}$, the surface can be approximated by the osculating paraboloid which may be represented by a quadratic polynomial in the parameters du and dv . This polynomial can be considered as a Taylor expansion at point $\mathbf{x}$ of the surface by omitting those after the quadratic term [17, 73].

$$\mathbf{x}(u + du, v + dv) = \mathbf{x}_{00} + \mathbf{x}_u du + \mathbf{x}_v dv + (\mathbf{x}_{uu} du^2 + 2\mathbf{x}_{uv} dudv + \mathbf{x}_{vv} dv^2)/2 \quad (5.1)$$

The coefficients $\mathbf{x}_{00}$, $\mathbf{x}_u$, $\mathbf{x}_v$ etc. are the zero, first and second derivatives of $\mathbf{x}$ with respect to u and v at the surface point $\mathbf{x}_{00} = \mathbf{x}(u, v)$. With these six derivatives, the curvatures of the surface can be easily calculated. Considering the function given by $z = f(x, y)$, a second order polynomial of the form

$$z = c_1 + c_2p + c_3q + c_4p^2 + c_5pq + c_6q^2 \quad (5.2)$$

is used to fit the approximating surface patch. In this equation, $p = x - x_o$ and $q = y - y_o$ where x_o and y_o are the x and y coordinate of the center point $\mathbf{x}_o$ of the surface patch under consideration. The six coefficients of the paraboloid are then obtained by least-squares solution of an over-determined system of linear equations,

$$z_1' = c_1 + c_2p_1 + c_3q_1 + c_4p_1^2 + c_5p_1q_1 + c_6q_1^2 \quad (5.3)$$

$$z_2' = c_1 + c_2p_2 + c_3q_2 + c_4p_2^2 + c_5p_2q_2 + c_6q_2^2 \quad (5.4)$$

$$z_3' = c_1 + c_2p_3 + c_3q_3 + c_4p_3^2 + c_5p_3q_3 + c_6q_3^2 \quad (5.5)$$

$$\vdots$$

$$z_n' = c_1 + c_2p_n + c_3q_n + c_4p_n^2 + c_5p_nq_n + c_6q_n^2 \quad (5.6)$$

where n is the number of points in the neighbourhood of a selected surface point.

Alternatively, equations 5.3-5.6 can be represented in a matrix form given by

$$\begin{Bmatrix} z_1' \\ z_2' \\ z_3' \\ \vdots \\ z_n' \end{Bmatrix} = \begin{bmatrix} 1 & p_1 & q_1 & p_1^2 & p_1q_1 & q_1^2 \\ 1 & p_2 & q_2 & p_2^2 & p_2q_2 & q_2^2 \\ 1 & p_3 & q_3 & p_3^2 & p_3q_3 & q_3^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & p_n & q_n & p_n^2 & p_nq_n & q_n^2 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{Bmatrix} \quad (5.7)$$

5.2 Minimization of Error and Optimization of Coefficients

It is desired that the six coefficients c_1 to c_6 , are acquired such that the divergence of the fitted paraboloid and the data points are minimized, which is similar to minimizing the differences between z_n (measured value of z) and z_n' (calculated value of z). The minimization problem at this point is then expressed as

$$E(z) = \sum_{n=1}^N (z_n' - z_n)^2 = \sum_{n=1}^N (z(p_n, q_n) - z_n)^2 \quad (5.8)$$

where N here represents the number of points within the fitted paraboloid. In order to account for the effect due to the distance of each point with respect to the center point of the fitted paraboloid, i.e. $\mathbf{x}_0$, a weighting function in the form of a diagonal matrix is introduced

$$\mathbf{G} = \text{Diag}(G_1, G_2, G_3, \dots, G_n) \quad (5.9)$$

The components are given by

$$G_i = f \cdot (e^{-(p_i^2 + q_i^2)/d^2}), \quad i = 1, 2, 3, \dots, n \quad (5.10)$$

where f and d are arbitrary constants which can be adjusted accordingly.

The minimization problem may now be expressed as

$$E(z) = \sum_{n=1}^N G_n (z_n' - z_n)^2 = \sum_{n=1}^N G_n (z(p_n, q_n) - z_n)^2 \quad (5.11)$$

The error function E will have a minimum when

$$\frac{\partial E}{\partial c_i} = 0, \quad i = 1, 2, 3, 4, 5, 6 \quad (5.12)$$

thus yielding the following six linear equations required to compute the six coefficients.

$$\sum_{n=1}^N 2G_n(z(p_n, q_n) - z_n) = 0 \quad (5.13)$$

$$\sum_{n=1}^N 2G_n(z(p_n, q_n) - z_n)p_n = 0 \quad (5.14)$$

$$\sum_{n=1}^N 2G_n(z(p_n, q_n) - z_n)q_n = 0 \quad (5.15)$$

$$\sum_{n=1}^N 2G_n(z(p_n, q_n) - z_n)p_n^2 = 0 \quad (5.16)$$

$$\sum_{n=1}^N 2G_n(z(p_n, q_n) - z_n)p_nq_n = 0 \quad (5.17)$$

$$\sum_{n=1}^N 2G_n(z(p_n, q_n) - z_n)q_n^2 = 0 \quad (5.18)$$

The problem can then be described in the matrix form

$$\begin{aligned} \mathbf{z} + \mathbf{r} &= \mathbf{z} \\ &= \mathbf{AC} \end{aligned} \quad (5.19)$$

In this equation,

$$\mathbf{z} = (z_1, z_2, z_3, \dots, z_n)^T \quad (5.20)$$

$$\mathbf{r} = (r_1, r_2, r_3, \dots, r_n)^T \quad (5.21)$$

$$\mathbf{z}' = (z_1', z_2', z_3', \dots, z_n')^T \quad (5.22)$$

$$\mathbf{A} = \begin{bmatrix} 1 & p_1 & q_1 & p_1^2 & p_1 q_1 & q_1^2 \\ 1 & p_2 & q_2 & p_2^2 & p_2 q_2 & q_2^2 \\ 1 & p_3 & q_3 & p_3^2 & p_3 q_3 & q_3^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & p_n & q_n & p_n^2 & p_n q_n & q_n^2 \end{bmatrix} \quad (5.23)$$

$$\mathbf{C} = (c_1, c_2, c_3, c_4, c_5, c_6)^T \quad (5.24)$$

where $\mathbf{r}$ is the residual denoted by $\mathbf{z}_n' - \mathbf{z}_n$.

The polynomial coefficients in these error equations are then optimized such that the sum of weighted least squares has a minimum value

$$\mathbf{r}^T \mathbf{G} \mathbf{r} = \text{minimum} \quad (5.25)$$

With the minimum condition specified in equation 5.18, the problem is then dealt with using the normal equations

$$\mathbf{N} \mathbf{C} = \mathbf{A}^T \mathbf{G} \mathbf{z} \quad (5.26)$$

where $\mathbf{N}$ is called the normal equation matrix given by

$$\mathbf{N} = \mathbf{A}^T \mathbf{G} \mathbf{A} \quad (5.27)$$

By solving equation 5.26 by matrix manipulation, the optimized coefficients can be expressed as

$$\mathbf{C} = \mathbf{N}^{-1} \mathbf{A}^T \mathbf{G} \mathbf{z} \quad (5.28)$$

The first component of $\mathbf{C}$ is actually the z value of the center point $\mathbf{x}_0$ on fitted surface under consideration, while the other components of $\mathbf{C}$ can be interpreted as the first and second derivatives of $\mathbf{x}$ with respect to p and q at the center point $\mathbf{x}_0$.

Such surface fitting methods has been used by several authors [13, 18, 17, 73,], not just to calculate the curvatures, but also to transform a set of irregularly sampled data points into a regular grid with the help of a transformation operator. One main reason for this is to simplify the surface reconstruction process, i.e. for visualization purposes. However, problems arise when one tries to render the surface near the edges of an entity, as the transformation operator tends to discard the data points falling outside the edges of the grid. Also there are some difficulties in assigning the transformation operator to function well on a surface of revolution.

Therefore, the author proposed that the above technique be solely used for the calculation of curvatures, while the surface be reconstructed using B-Splines and area skinning. One reason is because B-Splines and area skinning have been widely and effectively used for surface reconstruction, and the resultant surface is a smoother and more aesthetical one which suits the purpose of visualization.

As described above, the paraboloid fitting is done based on the function $z = f(x, y)$, and the error minimization is done based on the surface elevation from the data's z coordinates. This implies that a surface patch can be most accurately fitted when the direction of the normal to the surface at the considered point is approximately in the z -direction. When the direction of the surface normal deviates too much from the z -direction, the accuracy of the calculated curvatures decreases. Due to this reason, such surface fitting methods is mostly applied to the shape analysis of simple surfaces such as

that of a human back [13, 17, 18], but with surfaces near the edges being disregarded. This method was also applied on a more complicated surface of a human foot [73]. Although an attempt was made to simplify the curvature calculations of the upper foot surface by dividing it into several patches, inaccuracies near the edges were still reported.

Since the endocardium is a rather complex surface of revolution, with surface normal varying greatly across the entire surface, it is necessary to solve the above-mentioned problem before the paraboloid fitting algorithm can be applied efficiently. This can be done by rotating each selected patch such that the resultant direction of the patch normal is in the z-direction before the surface fitting process is carried out.

In this way, one can ensure an accurate calculation of curvatures at each given point on the surface since the surface fitting is done with each surface normal pointing in the z-direction.

Chapter 6

ACQUISTION OF LV CURVATURES

In the previous chapter, an analytic method for computing surface curvatures by basis of surface fitting was explained. However, for a more accurate evaluation of the surface properties from the datasets, additional steps should be taken to enhance the proficiency of the surface fitting algorithm. This chapter will describe the procedures performed to efficiently and accurately retrieve the surface properties of the LV surface, and validates the accuracy of the proposed algorithm.

6.1 Initial Data Sets

The LV endocardium dataset from a healthy heart was provided by TomTec [74], using 4D ultrasound imaging system which has an essential function for the good spatial and temporal rendering of the LV. The concept of this system is based on a traditional continuously oscillating probe and an ultrasonic scanner that provides 3D spatial data together with temporal information.

The ultrasonic imager is usually triggered by the R-wave during the ECG and runs freely with fixed frame rate. Therefore the image frames acquired by such ultrasound system are generally equi-temporal with 33 ms or 40 ms spacing.

In this research, a set of 18 image frames which are equally spaced with European PAL timing (40 ms), were acquired during a complete cardiac cycle. This set of LV data was received in the form of a stack of cleaned contour segments, with each frame consisting of over 700 3D data points, as displayed in Figure 6.1.

Figure 6.1: 3D data points of LV endocardial surface

6.2 Reconstruction and Meshing of LV Endocardial Surface

In order to portray the variation of the left ventricle's geometry and proficiently analyze the endocardial surface properties across the temporal frames, the construction of an anatomically accurate model of the LV endocardial surface is desirable. Since the geometrical data generated from ultrasound imaging is in the form of contour stacks as shown in Figure 6.1, the reconstruction can be done by first curve fitting the contours to form the boundary of the endocardium, followed by the surface rendering. The author decided on the use of B-splines for the contour curve fitting, and surface skinning for the rendering of the endocardial surface, both of which have been widely used in various fields including medical science, biomedical engineering, manufacturing and animation industries etc..

B-splines are specially constructed piecewise polynomials [75, 76], which obey a certain set of continuity conditions. They provide local or regional control of the curve shape as opposed to global control by employing a special set of blending functions that provide

local influence. They also have the capability to insert additional control points without increasing the degree of the curve. Furthermore, B-spline curves have the ability to interpolate or approximate a set of given data points accurately and efficiently. Since the shape of the left ventricle is rather complex, piecewise polynomials such as B-splines are more suited for interpolation as compared to pure polynomials. Due to these advantages and more, B-splines are employed by several authors [77, 78, 79, 80] to facilitate the reconstruction of various models with complex geometrical representations. Figure 6.2 shows the B-splines fitted contours of the LV endocardium for the first frame, whose data points are displayed in Figure 6.1.

Figure 6.2: B-splines fitting of LV endocardial contours

Surface skinning, also known as lofting, is a process of passing a surface through a set of cross-sectional curves. In other words, skinning is a process of fitting an interpolatory surface through a set of contours. This technique has considerable generality in that the required cross-sectional curves can be arbitrary, which means that curves can be of

different degrees, rational or non-rational, and can be defined over arbitrary knot vectors. This implies that the skinning process possesses great flexibility which makes it possible to pass a surface through complex shapes. Thus, the surface skinning technique has been used by various researchers [77,80, 81] to render irregular surface models.

Figures 6.3 and 6.4 depict the reconstructed endocardial surfaces for the entire cardiac cycle. It appears, from the sequential set of reconstructed endocardial surface-frames during systole (Figure 6.3), that the LV is twisting as per the left-hand thumb rule. In other words, if the left hand thumb is placed pointing upwards the long axis, the fingers indicate the direction of rotation. It is also observed from the reconstructed endocardial surface-frames during diastole (Figure 6.4), that the LV appears to be un-twisting (as per the right-hand thumb rule).

Figure 6.3: Reconstructed LV endocardial surfaces during systole

Figure 6.4: Reconstructed LV endocardial surfaces during diastole

As there are six unknown coefficients (c_1, c_2, c_3, c_4, c_5 and c_6) in equation 5.2, at least six points are required in the vicinity of the centre point of each surface patch to solve these unknowns. However, six points are barely enough to depict a surface accurately, especially one with such complexity like the endocardial surface.

Figure 6.5: Approximation of a surface with different number of points

As portrayed in Figure 6.5, the estimated surface matches the original surface more fully as the number of points increases. Therefore, it is recommended that at least 20 neighborhood points in each surface patch be involved to accurately compute the curvature. Since the initial datasets are sparse and are arranged in such a way that points are in close proximity in the circumferential direction but relative far apart in the long

axis direction, an adequate and unbiased selection of sufficient points for curvature estimation would imply a large area involved. As the tangential directions as well as curvatures could vary considerably along the endocardial surface, especially during late systole, the 2nd order polynomial might not be sufficient to describe the relatively large area selected. To avoid the above-mentioned problem, it is thus useful to obtain more data points by interpolation.

With the formerly reconstructed surface using B-Splines and surface skinning, a new set of denser and scattered data points can be easily designated by meshing the surface with triangular elements. The triangulated endocardial surfaces over the entire cycle are depicted in Figures 6.6 and 6.7. In future, this set of triangulated endocardial surfaces can be employed to determine the surface-strain distribution as follows: On frame 1, some twisting motion will be imposed to the apical nodes, and its modality will be determined such that the resulting deformed shape of frame 1 matches frame 2. Subsequently, displacements will be applied to the next set of nodes such that the deformed frame 2 matches frame 3. This upward progression of the deformation wave will be continued for the rest of the frames, and at each time the incremental strains on the endocardial surface will be computed.

Figure 6.6: Triangulation of LV endocardial surfaces during systole

Figure 6.7: Triangulation of LV endocardial surfaces during diastole

6.3 Algorithm for Computation of LV Surface Curvatures

With a new set of point clouds created for the entire cardiac cycle, one can now retrieve the surface curvatures accurately. For convenience, an operator in the form of a search window was brought in to determine the neighborhood points required for the necessary calculations. This operator serves to capture all the neighboring points falling within a predefined distance from the centre point as depicted in Figure 6.8.

Figure 6.8: Selection of neighborhood points

For the purpose of obtaining a smooth distribution of curvatures, the operator was assigned to move to each and every data point, making each reference point the new origin to release the search window for neighboring points selection. These selected points within the search window thus form the region utilized for local surface fitting. As mentioned in Section 5.2, the accuracy of the fitted surface decreases as the direction of the selected region's normal deviates from the z-direction, which calls for a need to diminish this discrepancy. This can easily be resolved by rotating the selected region such that the direction of the normal is approximately aligned in the z-direction.

Making use of the connectivity information from the triangulated LV surface, one can readily estimate the unit normal to each region required for the construction of the rotation matrices. Suppose a particular reference point is also the vertex v^{tx} of a small

mesh M formed by N number of triangles, and the edges are defined as $\mathbf{e}_i = \mathbf{v}_{i+1}^{\text{tx}} - \mathbf{v}_i^{\text{tx}}$ where $\mathbf{v}_i^{\text{tx}}$ are ordered neighbor vertices of $\mathbf{v}^{\text{tx}}$ (refer to Figure 6.9), then the unit normal of a triangular face between edges $\mathbf{e}_i$ and $\mathbf{e}_{i+1}$ can be obtained from

$$\mathbf{n}_t = \frac{\mathbf{e}_i \times \mathbf{e}_{i+1}}{|\mathbf{e}_i \times \mathbf{e}_{i+1}|} \quad (6.1)$$

Figure 6.9: Unit normal of triangulated mesh M

Using this approach, the unit normal $\mathbf{n}_s$ of the selected region can then be found by taking the average of the unit normals of all triangular faces forming mesh M , with the assumption that the search window is sufficiently small and there is no drastic change in surface normal directions within the selected region.

$$\mathbf{n}_s = \frac{\sum_{j=1}^N \mathbf{n}_{t_j}}{N} \quad (6.2)$$

Subsequently, the selected region goes through a series of rotations which first rotates its normal into the z-y plane, and then rotating the normal in this plane to coincide with the

z-axis. This set of rigid body motions does not affect the surface properties of the selected patch and is achieved by post-multiplying the row vectors of the selected coordinates by matrices $\mathbf{R}_1$ and $\mathbf{R}_2$, which are defined as,

$$\mathbf{R}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{N}{\sqrt{M^2 + N^2}} & \frac{M}{\sqrt{M^2 + N^2}} \\ 0 & \frac{-M}{\sqrt{M^2 + N^2}} & \frac{N}{\sqrt{M^2 + N^2}} \end{bmatrix} \quad (6.3)$$

$$\mathbf{R}_2 = \begin{bmatrix} \sqrt{M^2 + N^2} & 0 & L \\ 0 & 1 & 0 \\ -L & 0 & \sqrt{M^2 + N^2} \end{bmatrix} \quad (6.4)$$

where L, M and N are the direction cosines of the estimated patch normal.

With the alignment of the patch normal in the z-direction, the local surface fitting can now take place effectively, and the surface properties calculated accurately. Besides calculating the mean curvature H and the Gaussian curvature K, two other curvature metrics are also computed. They are namely, the curvedness C and the shape index S, which are both presented by Koenderink and Doorn [82].

The curvedness C measures the degree of curvature, and is defined as

$$C = \sqrt{\frac{\kappa_1^2 + \kappa_2^2}{2}} \quad (6.5)$$

It is a positive value that describes the magnitude of the curvedness at a point, thus it is a measure of how gently or highly curved a point is.

On the other hand, the shape index S classifies surfaces into nine types and is defined as

$$S = \frac{2}{\pi} \arctan \left[\left(\frac{\kappa_1 + \kappa_2}{\kappa_1 - \kappa_2} \right) \right], \quad (6.6)$$

where κ_1 and κ_2 are the maximum and minimum principal curvatures respectively. This shape index function is scale invariant, i.e. two spherical patches with different radii will have the same shape index values. Also, convexities and concavities fall on the opposite sides of the shape index scale, separated by saddle-like shapes. Furthermore, two shapes from which the shape index differs only in sign convention represent complementary pairs that will fit to each other like stamp and mold if they are of identical scale.

As in the case of the mean curvature (H), the definition of convexity and concavity of a particular surface is based on which side the observer views the surface from. For example, if a sphere is viewed from the interior, it will appear concave to the observer. On the other hand, if the sphere is viewed from the outside, it will appear convex to the observer. Therefore the usage of sign convention in H can be rather subjective, depending on which side of the surface the curvature values are derived from.

This condition is also true for the classification of surface types using the shape index. If one defines negative H values as concavities, the computed S will be negative for spherical cup, trough, rut and saddle rut, and positive for saddle ridge, ridge, dome and spherical cap (refer to Table 6.1). On the other hand, if one defines positive H values as concavities, the classification of shapes will be reversed as illustrated by Table 6.2 (i.e. spherical cup, trough, rut and saddle rut represented by positive S values, and saddle ridge, ridge, dome and spherical cap represented by negative S values).

Surface Type	Shape Index Range
Spherical Cup	$[-1, -7/8)$
Trough	$[-7/8, -5/8)$
Rut	$[-5/8, -3/8)$
Saddle Rut	$[-3/8, -1/8)$
Saddle	$[-1/8, +1/8)$
Saddle Ridge	$[+1/8, +3/8)$
Ridge	$[+3/8, +5/8)$
Dome	$[+5/8, +7/8)$
Spherical Cap	$[+7/8, +1]$

Table 6.1 :Classification of surfaces using shape index (–ve values taken as concavities)

Surface Type	Shape Index Range
Spherical Cap	$[-1, -7/8)$
Dome	$[-7/8, -5/8)$
Ridge	$[-5/8, -3/8)$
Saddle Ridge	$[-3/8, -1/8)$
Saddle	$[-1/8, +1/8)$
Saddle Rut	$[+1/8, +3/8)$
Rut	$[+3/8, +5/8)$
Trough	$[+5/8, +7/8)$
Spherical Cup	$[+7/8, +1]$

Table 6.2: Classification of surfaces using shape index (+ve values taken as concavities)

6.4 Validation for Computation of Curvatures

In order to probe the implementation of the proposed algorithm and evaluate the accuracy of the curvature calculations by the surface fitting method, two simple shapes, namely a sphere and a cylinder with known curvatures are tested. These two shapes are first tested with the surface fitting method without any normal estimation, and then tested again with the normal estimation and re-orientation of the estimated surface normal towards the z-axis.

6.4.1 Curvatures of Cylinder

A cylinder is a simple surface of revolution which can be defined parametrically as

$$\mathbf{x}(u, v) = \begin{bmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{bmatrix} = \begin{bmatrix} r \cos(u) \\ r \sin(u) \\ v \end{bmatrix}, \quad (6.7)$$

where r denotes the radius of the cylinder.

The cylinder has zero Gaussian curvature, i.e. $K = 0$ and a constant mean curvature given by

$$H = \frac{\kappa_1 + \kappa_2}{2} = -\frac{1}{2r} \quad (6.8)$$

The negative sign here represents concavity when the cylindrical surface is observed from the interior. In this particular test for the accuracy of curvatures computation, the radius of the cylinder was chosen to be 20 mm giving a constant mean curvature value of $-2.5 \times 10^{-2} \text{ mm}^{-1}$. Figure 6.10 depicts the hollow cylinder under consideration, while Figure 6.11 and Figure 6.12 illustrate the plots of curvatures along the cylinder's

circumferential direction, with surface fitting performed without prior estimation and re-alignment of surface normals.

Figure 6.10: Cylinder of radius 20 mm

Graph of Gaussian curvature along circumferential direction
(without normal estimation and re-alignment)

Figure 6.11: Comparison of computed cylinder's Gaussian curvature with exact value (without re-alignment of surface normal to z-axis)

Figure 6.12: Comparison of computed cylinder's mean curvature with exact value (without re-alignment of surface normal to z-axis)

It is observed from Figure 6.11 that the computed Gaussian curvatures enjoy the best accuracies at regions around 90° and 270° . Although the surface normals at regions around 270° are pointing in the reverse direction of the z-axis (90°), the accuracy is hardly affected since the Gaussian curvature as a product of κ_1 and κ_2 , is not affected by the change in signs in both κ_1 and κ_2 . However, the accuracy of the Gaussian curvature deteriorates as the normals of the considered regions moves away from the z-axis, and diverges at regions with normals perpendicular to the z-axis (i.e. at 0° , 180° and 360°).

As shown in Figure 6.12, the results for the mean curvatures are the most accurate around the 90° mark, where the surface normals directions coincide with the z-axis direction. This accuracy however depreciates gradually as the region under consideration moves away from the 90° spot, and suddenly diverges as it approaches the 0° and 180° spots. On the other half of the graph, similar divergence are experienced near regions with normals perpendicular to the z-axis (i.e. 180° to 360°). It is also found that although the magnitude of the computed mean curvatures around the 270° point are close to the

theoretical value, the sign convention is reversed. This is due to the surface normals at these regions pointing in reversed direction from the z-axis. Such reversal in sign convention is undesirable as the cylinder is completely concave when viewed from the interior, which implies a negative mean curvature throughout.

The above-mentioned discrepancies and problems can be solved if the surface fitting are done with prior knowledge of surface normals and re-alignment of these estimated normals to coincide with the z-axis direction. The following two figures depict the plots of curvatures along the cylinder's circumference, this time with surface fitting done with prior normal estimation and re-orientation.

Figure 6.13: Comparison of computed cylinder's Gaussian curvature with exact value (with re-alignment of surface normal to z-axis)

Figure 6.14: Comparison of computed cylinder's mean curvature with exact value (with re-alignment of surface normal to z-axis)

It can be observed from Figures 6.13 and 6.14 that the proficiency and efficiency of the proposed algorithm has been justified, providing results closed to the theoretical values as shown in Table 6.3.

	Cylinder (tested with normal estimation and re-alignment)	
	<i>Gaussian curvature (mm^{-2})</i>	<i>Mean curvature (mm^{-1})</i>
<i>Theoretical value</i>	0	$- 2.500 \times 10^{-2}$
<i>Average Computed value</i>	$- 2.960 \times 10^{-7}$	$- 2.518 \times 10^{-2}$

Table 6.3: Comparison of theoretical and computed cylinder curvatures

6.4.2 Curvatures of Sphere

A sphere of radius r can be described by its parametric equation,

$$\mathbf{x}(u, v) = \begin{bmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{bmatrix} = \begin{bmatrix} r \cos(u) \cos(v) \\ r \cos(v) \sin(u) \\ r \sin(v) \end{bmatrix}$$

A sphere has constant Gaussian and mean curvatures which are easily obtained from equations 6.9 and 6.10 respectively.

$$K = \kappa_1 \kappa_2 = \frac{1}{r^2} \quad (6.10)$$

$$H = \frac{\kappa_1 + \kappa_2}{2} = -\frac{1}{r} \quad (6.11)$$

Similar to the case of a cylinder, the negative sign in H represents concavity when viewed from the sphere's interior. The radius of the sphere being tested was chosen to be 20 mm resulting in a constant Gaussian curvature of $2.5 \times 10^{-3} \text{ mm}^{-2}$ and constant mean curvature of $5 \times 10^{-2} \text{ mm}^{-1}$. Figure 6.15 depicts the tested sphere, while Figure 6.16 and Figure 6.17 illustrate the plots of curvatures along a chosen circumferential path, with surface fitting performed without estimation and re-alignment of surface normals.

Figure 6.15: Sphere of radius 20 mm

Graph of Gaussian curvature along circumferential direction
(without normal estimation and re-alignment)

Figure 6.16: Comparison of sphere's computed Gaussian curvature with exact value (without re-alignment of surface normal to z-axis)

Figure 6.17: Comparison of sphere's computed mean curvature with exact value (without re-alignment of surface normal to z-axis)

From Figure 6.16, it can be seen that the computed values for Gaussian curvature differ considerably except at regions in close proximity with the 90° and 270° spots. Similar divergence problems are also found at regions with normals perpendicular to the z-axis (i.e. 0° , 180° and 360°). The results for the computed mean curvatures exhibit similar divergence problems and inaccuracies at regions away from the 90° mark as illustrated in Figure 6.17. Similar to that of the tested cylinder, the sign of the computed mean curvatures at the lower half of the sphere are opposite from those at the upper half, which is unfavorable.

Similarly, these discrepancies can be easily solved when surface fitting are done with former knowledge and re-alignment of surface normals, and the achieved accuracy can be observed from Figures 6.18 and 6.19, and Table 6.4.

Figure 6.18: Comparison of sphere's computed Gaussian curvature with exact value (with re-alignment of surface normal to z-axis)

Figure 6.19: Comparison of sphere's computed mean curvature with exact value (with re-alignment of surface normal to z-axis)

	Sphere <i>(tested with normal estimation and re-alignment)</i>	
	<i>Gaussian curvature (mm^{-2})</i>	<i>Mean curvature (mm^{-1})</i>
<i>Theoretical value</i>	2.5×10^{-3}	$- 5 \times 10^{-2}$
<i>Average Computed value</i>	2.533×10^{-3}	$- 5.049 \times 10^{-2}$

Table 6.4: Comparison of theoretical and computed sphere curvatures

Chapter 7

DETERMINATION OF LV SURFACE MOTION

The LV ultra-structure is comprised of spiral myocardial fibers (refer to Figure 7.1). As these fibers contract, the LV surface is twisted and pushed upwards. In this process the region *abcd* is deformed and strained. The surface triangulation scheme (Figure 6.6) will enable determination of the surface strains. Now, if a segment (such as *abcd*) is infarcted, it will not contract; in other words it will deform passively due to the active deformation of its neighbouring segments. Its strain profile is expected to be quite different from that of an active normal segment. It is this differential strain profile that the author expects to uncover, in his quest for characterization of “LV surface kinematography”.

Figure 7.1: Deformation of LV endocardial segment

It is the author's hypothesis that the LV surface properties may be a valuable tool in assessing cardiac health. In order to study the change in surface curvature at a particular endocardial point with respect to time, the knowledge of endocardial points positioning and movement during the cardiac cycle is essential. Therefore the author at this stage, is developing the basis of characterizing left ventricular surface, with a view to delineate surface motion. It is expected that tracking of endocardial surface kinematics over a large patient population will yield hitherto unearthed information concerning how LV

excitation-contraction wave affects the endocardial surface and depicts some surface motion irregularities over ischemic and infarcted zones.

Unlike magnetic resonance (MR) tagging, a non-invasive imaging technique that offers the tracking of a sparse set of LV myocardial points, the LV dataset acquired from 4D ultrasound imaging does not provide points correspondences of the surface between successive frames. Thus, the problem of LV surface motion tracking was embarked upon with the formulation of an algorithm which is based on the assumption that deformation of LV surface patches can be modeled with that of thin plates.

7.1 Thin Plate Bending Model of the LV

It has been the practice of some researchers [83, 84] to track the motion of the left ventricle via a minimum distance based approach. The myocardial surface exhibits a composite movement during the cardiac cycle consisting of rigid body translation and rotation, and non-rigid deformations. Due to such complex LV motion, myocardial points tracking approaches based solely on the minimum distance criterion may often not be enough to produce consistently good matches.

In this part of the report, the prediction of endocardial surface points motion was performed based on the potential energy of an ideally thin flexible plate. This approach has been employed by several authors [20, 21, 85, 86] to track left ventricular motion and has apparently yielded somewhat satisfactory results.

The endocardial surface patches are modeled as thin flexible plates which are constrained to deform in a predetermined manner. The potential energy of an ideally thin flexible flat plate of elastic material is a gauge of the strain energy of the deformation and can be defined as [86, 87]:

$$\varepsilon = \Lambda \frac{\kappa_1^2 + \kappa_2^2}{2} \quad (7.1)$$

where Λ is a material constant of the thin plate while κ_1 and κ_2 are the maximum and minimum principal curvatures of the plate respectively. It should be apparent that this particular measure is a function of the principal curvatures and is invariant to 3D rotation and translation [20]. The potential energy of the thin plate can also be expressed in the following form:

$$\varepsilon = \Lambda(2H^2 - K) \quad (7.2)$$

with H and K being the mean and Gaussian curvature of the particular plate. From this particular expression, it can be noted that the variation of local surface features of the LV can be brought into consideration and accounted for in the process of point tracking by means of potential energy of thin plate.

The energy required to bend a curved plate to a newly deformed state has been defined in [85] as:

$$\varepsilon_{be} = \Lambda \frac{\kappa_1^2 + \kappa_2^2}{2} - \Lambda \frac{\bar{\kappa}_1^2 + \bar{\kappa}_2^2}{2} \quad (7.3)$$

where the principal curvatures of the surface at time t_o are specified with no bars, while the principal curvatures at $t_o + \Delta t$ are specified with bars.

However, it is noticed that this form of bending energy tends to miss out the detection of large deformation when the principal curvatures change their signs but not their magnitudes. In such cases, the bending energy attained spell the value of zero, which is an indication of no deformation despite the large deformation involved.

The author therefore proposed that the bending energy of the LV thin plate model be defined in the following form which has also been suggested in [86]:

$$\varepsilon_{be} = \Lambda \frac{(\kappa_1 - \bar{\kappa}_1)^2 + (\kappa_2 - \bar{\kappa}_2)^2}{2} \quad (7.4)$$

where κ_1 and κ_2 represent principal curvatures of the initial undeformed surface patch, while the same quantities of the deformed surface patch are specified as $\bar{\kappa}_1$ and $\bar{\kappa}_2$. In this way, the sign convention of the principal curvatures will be accounted for in the computation of bending energy.

7.2 Algorithm for Endocardial Surface Points Matching

It is the aim of the proposed LV surface motion tracking algorithm to determine the point correspondences between successive temporal image frames throughout the entire cardiac cycle. The bending energy formulated in equation 7.4 will be used as the principal matching criterion to obtain the unique point correspondences. The deformation of the endocardial surface patches are assumed to correspond to the deformation of the thin flexible plate, and can be fully characterized by the bending energy of the plate.

The primary essence of this point tracking criterion is the postulation that left ventricular surface patches deform as little as possible during the small time interval between pairs of successive image frames. In other words, it is assumed that minimum bending energy is required to deform the LV surface patches between small time gaps. In this research, 18 temporal frames are acquired for a complete cardiac cycle, which should be a fine enough temporal sampling for the current purpose.

Since the LV rigid motions, such as (global and local) translation and rotation, do not affect the local shape properties of the endocardial surface (implying that κ_1 and κ_2 remain constant), point correspondences without any deformation can be determined by

matching surface patches with zero bending energy. On the other hand, non-rigid deformation of a LV surface patch has been assumed to deform in a manner similar to that of the thin flexible plate and can thus be determined by the required bending energy. As such, it is possible to use the bending energy measure to recover the rigid body motion and non-rigid deformation of the LV surface.

In the process of point correspondence recovery, it is hypothesized that the surface patch surrounding the point of interest at time t_i deforms only locally and as little as possible within a small time interval Δt . For any point on the LV surface at time t_i (i.e. frame i), a search boundary (SB) is defined as a small region surrounding the nearest point on the corresponding surface at time t_{i+1} (i.e. frame $i + 1$).

Figure 7.2: Construction of search window

As depicted in Figure 7.2, the search window is centered at the closest surface point at time t_{i+1} to the surface point at t_i , and will include neighboring points within a small distance from the selected point. This set of surface points at t_{i+1} within the search

boundary is denoted as $\mathbf{x}_{SB,i}$, where $i = 1, 2, 3, \dots, n$. The bending energy measured between the surface patch surrounding point $\mathbf{x}$ at t_i and the surface patches surrounding points $\mathbf{x}_{SB,i}$ are then computed as:

$$\varepsilon_{be}(\mathbf{x}, \mathbf{x}_{SB,i}) = \Lambda \frac{(\kappa_1(\mathbf{x}) - \kappa_1(\mathbf{x}_{SB,i}))^2 + (\kappa_2(\mathbf{x}) - \kappa_2(\mathbf{x}_{SB,i}))^2}{2} \quad (7.4)$$

The surface point $\bar{\mathbf{x}}$ within the search boundary (SB) at t_{i+1} that incurred the minimum bending energy is then chosen as the point corresponding to the initial surface point $\mathbf{x}$ at t_i . It should be noted that the material constant of the thin plate Λ is assumed to be constant throughout the surface patch in consideration at t_i and the corresponding surface patches within the search boundary at t_{i+1} , and will remain constant within the small time interval.

This point tracking system can thus be considered as a minimization problem which is expressed as follows:

$$\begin{aligned} \bar{\mathbf{x}} &= \arg \min_{\mathbf{x}_{SB,i}} \{\varepsilon_{be}(\mathbf{x}, \mathbf{x}_{SB,i})\} \\ &= \arg \min_{\mathbf{x}_{SB,i}} \left\{ \Lambda \frac{(\kappa_1(\mathbf{x}) - \kappa_1(\mathbf{x}_{SB,i}))^2 + (\kappa_2(\mathbf{x}) - \kappa_2(\mathbf{x}_{SB,i}))^2}{2} \right\} \end{aligned} \quad (7.5)$$

where $i = 1, 2, 3, \dots, n$.

As the LV surface points are dense, multiple matching or crossover of local corresponding points can occur. In order to alleviate this problem, an operator was introduced to points with multiple matches. This operator functions in a way that when multiple matches are detected, the matching point that is nearest to the initial surface point $\mathbf{x}$ at time t_i is selected, while the rest of the matches are dropped. This operation thus corresponds to the initial assumption that the LV surface patches deform as slightly and locally as possible. From this point onwards, the path motion vectors of the points correspondences are then computed as:

$$\begin{aligned} \mathbf{dx} &= \bar{\mathbf{x}} - \mathbf{x} \\ &= (\bar{x} - x)\mathbf{i} + (\bar{y} - y)\mathbf{j} + (\bar{z} - z)\mathbf{k} \end{aligned} \quad (7.6)$$

With the functioning of this operator, the surface points of interest at time t_i are in a way sub-sampled and are reduced to a sparser set. This sparse set of points matches achieved will be sufficient for current purposes. However it can be advantageous that all surface points at t_i are matched so as to obtain a more complete motion field of the LV surface. This is especially useful towards the author's future work in analyzing and relating LV strains and stresses with LV surface curvatures. Based on the assumption that the surface motion of the LV varies smoothly in direction and magnitude over space (which is a reasonable assumption for most materials of biological origin), a device can be brought in to estimate the motion vectors of the unselected points.

For example, this device can function in a way such that for any unselected points $\mathbf{x}_{us,j}$, a small search region is defined to find the previously matched points $\mathbf{x}_i$, and a weighting factor W_i is assigned to these selected matched points $\mathbf{x}_i$ according to their distance away from $\mathbf{x}_{us,j}$. It should be noted that the matched points $\mathbf{x}_i$ which are near to $\mathbf{x}_{us,i}$ are given higher weight factors while those points further away from $\mathbf{x}_{us,i}$ are given lower weight factors. In this way, the points are constrained to be more affected by near points than by faraway points. The motion vectors for these unselected points can then be estimated by:

$$\mathbf{dx}_{us,j} = \frac{\sum_i^n W_i \mathbf{dx}_i}{n} \quad (7.7)$$

where $i = 1, 2, 3, \dots, n$ and n is the number of matched points $\mathbf{x}_i$ within the search region.

Chapter 8

LV SURFACE CONTOURS EXTRACTION (FROM MRI)

In addition to the ultrasound dataset, a different set of LV data (from a healthy heart) during the isovolumic contraction phase from MR imaging was received. This particular dataset was received in the form of DICOM (Digital Imaging and Communications in Medicine) standard images, which is a standard created by the National Electrical Manufacturers Association (NEMA) to aid the distribution and viewing of medical images. As medical images are often corrupted with noise and sampling artifacts, difficulties often occur when classical segmentation techniques such as edge detection and thresholding are being applied. Therefore, active snakes, which have been widely used in medical image segmentation with promising results, are used for the segmentation process of the MR images in this research.

In this particular chapter, the process of extracting the LV surface contours based on gradient vector flow (GVF) snakes introduced by Xu and Prince [88, 89] will be briefly described. The segmented contours are then used as inputs for further surface rendering and shape-based points tracking to recover the motion of LV surface points during isovolumic contraction.

8.1 Traditional Snakes

Snakes also known as active or deformable contours, were first introduced by Kass et al. [90]. A traditional snake can be defined parametrically as a curve $\mathbf{x}(s) = [x(s), y(s)]$, $s \in [0, 1]$, that deforms through the spatial domain of an image so as to minimize the energy function

$$E = \int_0^1 \frac{1}{2} (\alpha |\mathbf{x}'(s)|^2 + \beta |\mathbf{x}''(s)|^2) + E_{ext}(\mathbf{x}(s)) ds \quad (8.1)$$

where α and β are weighting parameters that regulate the snake's tension and rigidity respectively, and $\mathbf{x}'(s) = \frac{\partial \mathbf{x}}{\partial s}$ and $\mathbf{x}''(s) = \frac{\partial^2 \mathbf{x}}{\partial s^2}$ denote the first and second derivatives of $\mathbf{x}(s)$ with respect to s . E_{ext} is the external energy function derived from the image such that it takes on smaller values at object boundaries and features of interest. This energy function is often set equal to the negative value of the magnitude of the image gradient, computed from the image itself or from the image convolved by a Gaussian kernel. Therefore with a gray-level image $I(x,y)$ viewed as a function of continuous variables (x, y) , typical external potential functions intended to lead a deformable contour toward step edges are given as [88, 89]

$$E_{ext}^1(x, y) = -|\nabla I(x, y)|^2 \quad (8.2)$$

$$E_{ext}^2(x, y) = -|\nabla(G_\sigma(x, y) * I(x, y))|^2 \quad (8.3)$$

where ∇ is the gradient operator, $G_\sigma(x, y)$ is the 2-D Gaussian function with standard deviation σ , and $*$ is the 2-D image convolution operator. If the desired image features are black lines on a white background, then adequate potential energies are [89, 91]

$$E_{ext}^3(x, y) = I(x, y) \quad (8.4)$$

$$E_{ext}^4(x, y) = G_\sigma(x, y) * I(x, y) \quad (8.5)$$

On the other hand, if white lines are to be extracted from a black background, the potential energies can be defined as

$$E_{ext}^5(x, y) = -[I(x, y)] \quad (8.6)$$

$$E_{ext}^6(x, y) = -[G_{\sigma}(x, y) * I(x, y)] \quad (8.7)$$

From variational calculus [87], it is known that if a curve $\mathbf{x}(s)$ minimizes the energy function E and is sufficiently regular, then it must satisfy the Euler equation [88, 89]

$$\alpha \mathbf{x}''(s) - \beta \mathbf{x}''''(s) - \nabla E_{ext} = 0 \quad (8.8)$$

This can be treated as a force balance equation [88,89]

$$\mathbf{F}_{int} + \mathbf{F}_{ext} = 0 \quad (8.9)$$

where $\mathbf{F}_{int} = \alpha \mathbf{x}''(s) - \beta \mathbf{x}''''(s)$ denotes the internal force which discourages stretching and bending, and $\mathbf{F}_{ext} = -\nabla E_{ext}$ denotes the external potential force which attracts the contour towards the desired object boundaries. To find a solution to equation 8.8, the active contour is made dynamic by treating $\mathbf{x}$ as a function of s and time t (i.e. $\mathbf{x}(s, t)$).

The partial derivative of $\mathbf{x}$ with respect to time t is then equated to the left hand side of equation 8.8 as follows

$$\mathbf{x}_t(s, t) = \alpha \mathbf{x}''(s, t) - \beta \mathbf{x}''''(s, t) - \nabla E_{ext} \quad (8.10)$$

With the stabilization of the solution $\mathbf{x}(s, t)$, the term $\mathbf{x}_t(s, t)$ vanishes and a solution of equation 8.8 is achieved. Equation 8.10 can be solved numerically by discretizing the equation and solving the discrete system iteratively.

8.2 Gradient Vector Flow Snake

There are some general problems with the formulations and behavior of traditional snakes. One problem is associated with the initialization and poor convergence to boundary concavities. Another problem is the limited capture range where the magnitude of the external forces fade away rapidly from the object boundary. To address these problems, Xu and Prince [88, 89, 92] defined a new external force gradient vector flow (GVF), which is computed as a diffusion of the gradient vectors of a gray-level or binary edge map derived from the image.

The edge map $f(x, y)$ derived from the image $I(x, y)$ is defined by setting the property to be larger at desired features of interest (i.e. image edges). For example, the edge map can be expressed as

$$f(x, y) = -E_{ext}(x, y) \quad (8.11)$$

The gradient of the edge map ∇f consists of vectors pointing toward the edges, and are normal to the edges at the boundaries. These vectors typically have large magnitudes only in the immediate neighbourhood of the edges. Also, ∇f is negligible in homogeneous regions where $I(x, y)$ is constant. The basic approach is to keep the advantageous characteristics of the gradients near the edges, and diffuse the gradient map further away from the edges into the homogeneous regions.

The gradient vector flow field is defined as the vector field $\mathbf{v}(x, y) = [u(x, y), v(x, y)]$ that minimizes the energy function

$$\mathcal{E} = \iint \mu(u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |\mathbf{v} - \nabla f|^2 dx dy \quad (8.12)$$

where μ is a regularization parameter governing the tradeoff between the first and second term in the integrand. μ is typically decided according to the amount of noise present in

the studied image. In practice, the value of μ is set higher when there is more noise, and the value is set lower when there is less noise.

The variational formulation is rather standard, making the result smooth when there is no data. In particular, when $|\nabla f|$ is small, the energy $\mathcal{E}$ is dominated by the sum of squares of the partial derivatives of the vector field, resulting in a smooth field. In cases where $|\nabla f|$ is large, the second term dominates this energy, and is minimized by setting $\mathbf{v} = \nabla f$. In this way, $\mathbf{v}$ is kept nearly equal to the gradient of the edge map $f(x, y)$ when it is large, but the field is forced to vary slowly in homogeneous region, which is a desired effect.

Using the principles of variational calculus [87], the GVF field can be obtained by solving the following Euler equations

$$\mu \nabla^2 u - (u - f_x)(f_x^2 + f_y^2) = 0 \quad (8.13)$$

$$\mu \nabla^2 v - (v - f_y)(f_x^2 + f_y^2) = 0 \quad (8.14)$$

where ∇^2 denotes the Laplacian operator. From the above Euler equations, it can be observed that in a homogeneous region, the second term in both equations is zero since ∇f is zero. Therefore in regions like this, both u and v are determined by Laplace's equation, and the resulting GVF field is interpolated from the region's boundary. In doing so, the GVF field is able to yield vectors that point into the boundary concavities.

The solutions to equations 8.13 and 8.14 can be found by treating u and v as functions of time and solving

$$u_t(x, y, t) = \mu \nabla^2 u(x, y, t) - (u(x, y, t) - f_x(x, y))(f_x(x, y)^2 + f_y(x, y)^2) \quad (8.15)$$

$$v_t(x, y, t) = \mu \nabla^2 v(x, y, t) - (v(x, y, t) - f_y(x, y))(f_x(x, y)^2 + f_y(x, y)^2) \quad (8.16)$$

Equations 8.15 and 8.16 are also known as generalized diffusion equations, and the steady state solution of these two equations is the desired solution for equations 8.13 and 8.14. Readers who are interested in this area can refer to [93] where a finite difference implementation for solving the steady state solution of equations 8.15 and 8.16 is described.

Following the computation of $\mathbf{v}(x, y)$, the potential force term $(-\nabla E_{\text{ext}})$ in equation 8.10 is replaced by $\mathbf{v}(x, y)$, yielding the following equation

$$\mathbf{x}_t(s, t) = \alpha \mathbf{x}''(s, t) - \beta \mathbf{x}''''(s, t) + \mathbf{v} \quad (8.17)$$

This parametric curve solving the above dynamic equation is known as GVF snake, and is solved numerically by discretization and iteration.

8.3 Segmentation and Rendering of the LV surfaces

In particular, 2 time frames of the LV during the isovolumic contraction phase were acquired from MR imaging. The left ventricular dataset received is consisted of both short-axis and long-axis slices of the left ventricle. After organizing the images, the LV surface contours from each image slice are obtained by using gradient vector flow (GVF) snakes as described in section 8.2.

Figure 8.1 depicts a single short-axis MR image and a long-axis MR image of the left ventricle.

(a) Short-axis image slice of the LV

(b) Long-axis image slice of the LV

Figure 8.1: Magnetic resonance images of the left ventricle

To begin, some Gaussian filtering was applied to the images so as to reduce noise. The edge maps are then derived before computing the gradient vector flow field for the images. Figure 8.2 depicts a selected image of a short-axis section of the LV and the computed gradient vector flow field (normalized and scaled-up for visualization purposes).

(a) Short-axis image slice of the LV

(b) GVF field (sub-sampled)

Figure 8.2: GVF field derived from a short-axis image slice of the LV

An example of the application of the GVF snake in extracting the endocardial contour from the left ventricular short-axis section is illustrated in Figure 8.3.

Figure 8.3: Evolution of a GVF snake on the image of a LV short-axis section

As shown in Figure 8.3 (a), the snake was initialized as a small circle within the endocardial boundary. From Figure 8.3 (b) - (f), one can observe the evolution and deformation of the snake during the iteration. It is apparent that the snake deformed in a way so as to conform and create an accurate representation of the endocardial contour shape.

Following the segmentation process, the contour stacks were arranged and positioned accordingly to their respective image orientation and position so as to form an accurate outline of the LV geometry. Subsequently, the LV model is reconstructed using similar surface rendering and triangulation schemes described in Chapter 6. The LV contour stacks, rendered surfaces and triangulation of the LV surfaces for the first time frame during isovolumic contraction are shown in Figures 8.4, 8.5 and 8.6 respectively.

Figure 8.4: LV contour lines for the first time frame during isovolumic contraction

Figure 8.5: LV rendered surfaces for the first time frame during isovolumic contraction

Figure 8.6: Triangulation of LV surfaces for the first time frame during isovolumic contraction

Figure 8.7: Rendered LV surfaces during isovolumic contraction

Figure 8.7 depicts the rendered surfaces of the LV model during the isovolumic contraction phase. The epicardial surface is made translucent so as to allow a better view of both the endocardial and epicardial surfaces.

With the reconstructed LV model, the algorithms used for the determination of surface curvatures and points correspondences for the ultrasound endocardial dataset in Chapters 6 and 7, can now be applied to estimate the motion of the LV surface points during isovolumic contraction.

Chapter 9

RESULTS AND DISCUSSION

In this particular chapter, the results obtained from the computation of various curvature metrics for the ultrasound dataset will first be presented. The outcomes of these LV surface curvatures will be depicted in the form of color plots and evaluated. In addition, the LV surface motion predicted from Chapter 7 will also be briefly discussed and used for depicting the change in surface curvatures at sample points. As there seems to be no reference of past work done on the detailed analysis of LV surface curvatures and variation, it remains an uncharted and challenging problem. Therefore, the color plots of all the curvature metrics (mean curvature H , Gaussian curvature K , curvedness C and shape index S) defined in Chapter 6 will be presented.

As there are 18 frames (from the ultrasound dataset) acquired during the cycle, the time instances for the LV frames will be designated as $t_0 + i$, where $i = 0, 1, 2, \dots, 17$ for subsequent analysis. Frame 1, which is captured during the early ejection phase will be denoted as t_0 , and $t_0 + 10$ will represent frame 11 which is captured at the end of systole (ES). The diastolic phase will thus consist of $t_0 + 10$ (at ES or start of diastole) to $t_0 + 17$ (frame 18). It should however be noted here that frame 18 is the image captured around late diastole instead of end diastole (ED) as the imager missed out a number of frames from this point onwards, thus excluding some instances during end-diastole (ED) and isovolumic contraction. In order to concentrate on the prime features, instead of presenting a large number of images acquired, only curvature plots at selected time instances will be discussed here.

The results obtained for the MRI dataset will be presented separately and briefly discussed at the end of the chapter. This is because the two datasets came from different sources (i.e. different subjects) and represent different phases of the heart cycle. In addition, these two datasets were acquired from two different imaging techniques.

9.1 Endocardial Surface Mean Curvature

As mentioned in Chapter 4, the mean curvature H can be used to differentiate convexity and concavity of a surface under consideration. As the left ventricular surface is known to vary greatly in shape during the cardiac cycle, the mean curvature measure can be a potentially useful tool in diagnosing the regional surface dynamics of the left ventricle.

9.1.1 Mean Curvature Plots of the Endocardial Surface

In this section, the color plots for the mean curvature of the endocardial surface will be presented in different views. The long-axis views of the endocardial surface mean curvatures will first be presented, followed by closed-up views of selected regions. The nomenclature used in defining the endocardial regions in this chapter is based on the standard myocardial segmentation and nomenclature used for tomographic imaging of the heart [94]. Readers unfamiliar with the standardized LV segmentation and nomenclature may refer to Appendix A for a more detailed explanation.

Figure 9.1 to 9.4 depict the temporal variation of endocardial surface mean curvatures in four different views during the ejection phase. The author has used the same range of values in the color plots for the four different long axis views (lateral, inferior, septal and anterior) for each time frame, so as to allow readers to compare the differences in LV regional surface curvatures at each instant.

It should be noted that the negative sign here represents concavity when the LV surface is viewed from the cavity interior, or convexity when viewed from the exterior. Thus, positive values here represent convexity when viewed from the LV interior, or concavity when viewed from the LV exterior. For convenience and consistency, the author will employ the convention when the LV is viewed from the exterior in the rest of this chapter.

Figure 9.1: Long axis (lateral) view of LV surface mean curvature (mm⁻¹) during systole

Figure 9.2: Long axis (inferior) view of LV surface mean curvature (mm⁻¹) during systole

Figure 9.3: Long axis (septal) view of LV surface mean curvature (mm⁻¹) during systole

Figure 9.4: Long axis (anterior) view of LV surface mean curvature (mm^{-1}) during systole

From the long axis view of the LV shown in Figure 9.1, it can be seen that at the start of ejection (t_0), a long sector of the lateral LV (bounded by dotted lines) is concaved inwards with a maximum value of around 0.06 mm^{-1} at the mid-cavity area. As time progresses to $t_0 + 4$, a large portion of this surface segment becomes slightly convex, leaving a small portion of this surface segment (at mid-cavity) slightly concaved. However, the area of concavity starts to spread to the rest of the long sector, with a slight increase in magnitude at $t_0 + 6$. As the LV contracts further from $t_0 + 6$ to $t_0 + 10$ (ES), the whole stretch of this section becomes highly concaved with a maximum value of around 0.25 mm^{-1} at the mid-cavity and apical segments.

Referring to Figure 9.1 (a), the left portion of the LV surface adjoining the encircled region at t_0 is slightly convex with values ranging from around -0.04 mm^{-1} to -0.09 mm^{-1} .

The extent of this convexity reduces slightly at time $t_0 + 2$, but starts to increase during mid-ejection and reaches maximum convexity (-0.37 mm^{-1}) at end-systole. This highly convex region at $t_0 + 10$ can be easily detected as a protrusion of the endocardial surface on the left of Figure 9.1 (f).

The inferior view of the LV from Figure 9.2 shows that the sector (enclosed by dotted lines) is slightly convex (shaded in green with value around -0.04 mm^{-1}) near the periphery of the enclosed zone, and gradually decreases in convexity as it moves inwards becoming almost flat (indicated by a lighter shade of green). Small regions of minor concavity represented by yellow shade can also be found. As time reaches $t_0 + 2$, this slight concavity starts to spread throughout a large portion of the enclosed sector. However, at time $t_0 + 4$ and $t_0 + 8$, this sector appears mostly flat until ES when gradual convexity comes into sight for most of the defined section. It is also noted that the apical region in the left vicinity of the bounded section at t_0 starts with a considerable convexity and remains substantially convex during ejection. Apparently, this LV view shows modest variation of mean curvature as compared to the lateral view.

Figure 9.3 depicts the long axis septal view of the LV during ejection phase. As illustrated in the enclosed boundary in Figure 9.3 (a), the upper half of the LV surface is

rather uneven and has a high variation of mean curvatures even at the start of ejection, with prominent convex regions lying next to convex ones. The magnitude of curvature starts to grow gradually as time increases, and at mid-ejection the septal LV surface becomes substantially irregular with prominent grooves and humps covering the surface. Subsequently at the end of ejection, the mid anteroseptal and apical septal segments can be found extruding with a convexity value as high as 0.23 mm^{-1} in magnitude. As the ventricular septum is bounded by the right ventricle (RV) wall, the large variation in curvature in this view might be caused by the synchronized contraction of the RV wall and RV cavity pressure acting on the septum.

As depicted in Figure 9.4 (a) and (b), the LV surface in the anterior view is rather smooth and slightly convex during early ejection. As the LV goes into mid-ejection phase, the anterior LV surface starts to crease, with indentations found at the basal and mid-cavity areas. At end systole, a large portion of the anterior LV surface area becomes slightly concaved (represented by yellow shade in Figure 9.4 (f)).

It is also observed from Figure 9.1 to 9.4 that the lower apical region is extremely convex during late systole, forming almost a pointed tip at the apical end (apex). The convexity at this apical end is found to increase greatly during ejection, from a magnitude of 0.137 mm^{-1} at t_0 to a magnitude of about 0.55 mm^{-1} at ES. This large deformation at the lower end of the LV cavity is probably one of the main contributors in propelling the blood upwards and towards the aorta at relatively high pressures. It is however the author's opinion that the mean curvature values computed at the pointed tip of the apical region and its close vicinity should be treated with care. This is because the apex region is very small and constricted, which makes it difficult to capture the precise shape with present imaging techniques due to noise, let alone an accurate surface interpolation and representation at this small locality.

The long axis views of the LV surface mean curvatures during filling phase will be presented in the following four figures. Similarly to that of systole, six frames will be presented here and they are timed at $t_0 + 11$, $t_0 + 12$, $t_0 + 13$, $t_0 + 15$, $t_0 + 16$ and $t_0 + 17$.

Figure 9.5: Long axis (lateral) view of LV surface mean curvature (mm⁻¹) during diastole

Figure 9.6: Long axis (inferior) view of LV surface mean curvature (mm⁻¹) during diastole

Figure 9.7: Long axis (septal) view of LV surface mean curvature (mm⁻¹) during diastole

Figure 9.8: Long axis (anterior) view of LV surface mean curvature (mm⁻¹) during diastole

It can be observed from Figure 9.5 to 9.8 that the temporal variation of the LV surface mean curvature during diastole generally acts in the reverse fashion from that during systole. This phenomenon of reverting from a more irregular surface to a smoother one can be explained from the untwisting action of the LV during diastole.

From these curvature plots during the diastolic phase, one can observe that the LV shape at time $t_0 + 11$ or early diastole (represented by Figure 9.5 (a), 9.6 (a), 9.7 (a) and 9.8 (a)) generally corresponds to the LV shape at time $t_0 + 10$ or ES except for some disparities in magnitude of the mean curvature. This insignificant change in LV shape from $t_0 + 10$ (ES) to $t_0 + 11$ (isovolumic relaxation) corresponds to the state of the LV cavity volume whereby the volume remains constant or almost unchanged during isovolumic relaxation.

An interesting situation arises at the basal anteroseptal region as the LV functions from ES to early diastole ($t_0 + 11$ and $t_0 + 12$), which can be noted from the encircled region in Figure 9.7 (a) and (b), and Figure 9.8 (a) and (b). This portion of the LV is only slightly irregular with a small bulging area (convex value of -0.16 mm^{-1}) and an insignificant indentation (concave value of 0.073 mm^{-1}) below the bulge at ES as depicted in Figure 9.4 (f).

As time reaches $t_0 + 11$ (refer to Figure 9.8 (a)), the corresponding region becomes relatively uneven with the convexity of the bulge (shaded in blue) reaching a value of about -0.34 mm^{-1} , resulting in a rather pointed region. Concurrently, the indentation grows deeper and reaches a concavity value of 0.2 mm^{-1} (represented by shade of red in Figure 9.8 (a)) at time $t_0 + 11$. The curvature at this locality is found to increase to a maximum at $t_0 + 12$ with a convexity value of almost -0.4 mm^{-1} and concavity value of 0.24 mm^{-1} . This abrupt and drastic change in the local shape at the basal anteroseptal area is most probably due to the premature untwisting of the LV, when most of the myocardium tries to spring back or undeform, while some regions of the myocardium remains stiff (due to activation during systole). This will eventually cause the corresponding adjacent regions to push against each other resulting in the acute surface irregularity.

Also illustrated in the above figures on diastolic LV mean curvature, the LV surface generally gets smoother during filling and at time $t_0 + 17$ which is during late diastole (Figure 9.5, 9.6, 9.7 and 9.8 (f)), the LV shape is quite similar to that of the LV at time t_0 (Figure 9.1, 9.2, 9.3 and 9.4 (a)). The LV shape at ED should correspond better to that of the LV at time t_0 .

There is however some contradicting increase in the mean curvatures obtained at the apex from time $t_0 + 13$ to $t_0 + 17$. As explained earlier, these unexpected results are most likely due to the imprecise image resolution and surface interpolation at this small locality.

To allow readers to get a better perception on the LV surface curvature changes and regional deformation, magnified views of the mean curvature plots at selected regions of the LV during the cardiac cycle will also be presented.

As depicted in Figure 9.9 to 9.16, the distribution and temporal variation of mean curvatures at each region can be more clearly assessed. From these plots of LV regional views, one can truly appreciate the value of mean curvature plots in the evaluation of LV regional profile and deformation during the cardiac cycle.

Figure 9.9: Regional (basal inferoseptal) view of surface mean curvature (mm^{-1}) during systole

Figure 9.10: Regional (basal inferoseptal) view of surface mean curvature (mm^{-1}) during diastole

Figure 9.11: Regional (basal inferolateral) view of surface mean curvature (mm^{-1}) during systole

Figure 9.12: Regional (basal inferolateral) view of surface mean curvature (mm⁻¹) during diastole

Figure 9.13: Regional (mid anterolateral) view of surface mean curvature (mm^{-1}) during systole

Figure 9.14: Regional (mid anterolateral) view of surface mean curvature (mm⁻¹) during diastole

Figure 9.15: Regional (mid anteroseptal) view of surface mean curvature (mm^{-1}) during systole

Figure 9.16: Regional (mid anteroseptal) view of surface mean curvature (mm^{-1}) during diastole

9.1.2 ‘Hills and Valleys’ Plots of the Endocardial surface

In this particular section, the LV endocardial surface will be rolled into a flat sheet, and the corresponding mean curvatures will be mapped onto the plane as ‘hills and valleys’. Suppose the LV surface is being sliced along the division line between the inferior and lateral sections, and forces are applied along both edges of the sliced segments (from base to apex), the surface can be stretched into a flat sheet as shown in Figure 9.17.

Figure 9.17: LV surface mapped onto a flat sheet

Figure 9.18 to 9.21 depict the mean curvatures of the LV as ‘hills and valleys’ plots at time t_0 , $t_0 + 3$, $t_0 + 7$ and $t_0 + 10$ during systole. Similar to Section 9.1.1, positive values imply concavities when viewed from the LV exterior and negative values imply convexities when viewed from the interior. Therefore in the following figures, convexities (-ve H) are represented by ‘hills’ and concavities (+ve H) are represented by ‘valleys’. Readers should take note of the mean curvature values shown to the right of the color bar in each figure so as to better understand the degree of curvatures at different regions.

Figure 9.18: LV surface mean curvature ('hills and valleys') at time t_0

Figure 9.19: LV surface mean curvature ('hills and valleys') at time $t_0 + 3$

Figure 9.20: LV surface mean curvature ('hills and valleys') at time $t_0 + 7$

Figure 9.21: LV surface mean curvature ('hills and valleys') at time $t_0 + 10$

From the above figures, one can easily investigate the distribution of surface curvatures and effortlessly differentiate concave regions from convex regions with just a glimpse. In addition, the temporal change in the height and depth of the ‘hills and valleys’ can be easily studied using these plots. The author believes that if more LV datasets are put under test and their surface curvatures evaluated, it is very likely to discover certain trends in the distribution and temporal variation of curvatures. Therefore, these ‘hills and valleys’ plots of LV curvatures may in future be very useful tools for determining sites of myocardial activation during systole, and for detecting sites of myocardial infarction.

9.2 Endocardial Surface Gaussian Curvature

As discussed in Chapter 4, the Gaussian curvature (K) classifies the shape of a particular surface under planar, parabolic, elliptic or hyperbolic. When $K = 0$, the considered surface is either planar or parabolic depending on the value of the mean curvature. When $K > 0$, the surface is elliptic, and when $K < 0$, the surface is hyperbolic.

9.2.1 Gaussian Curvature Plots of the Endocardial Surface

The magnitudes of the Gaussian curvature computed for the LV surface are in general much smaller than those of the mean curvature. Therefore, a large portion of the color bar is used to depict the high curvatures found at the apex, leaving the Gaussian curvatures on the rest of the LV represented by only one or two shades of color. This is especially true during late systole when the apex tip becomes substantially pointed, creating the visual effect of a single colored representation of the LV Gaussian curvature at most of the LV surface area. As such, the variation and distribution of Gaussian curvatures can hardly be represented thoroughly in a full long axis view of the LV (Appendix B). This problem can however be solved if one increases the ranges of color shades used for the curvature plots, or truncate the apex to allow a better distribution.

It is doubtful that the investigation of Gaussian curvature and its variation alone can infer any useful information for clinical purposes. However, the Gaussian curvature is particularly useful in nonrigid motion analysis due to its invariancy towards rigid motion and the type of surface parameterization [14]. Besides, the LV Gaussian curvatures if viewed in close conjunction with the corresponding mean curvatures can be used to describe and characterize the LV shape more precisely.

As such, the variation of Gaussian curvatures at a particular region (mid anterolateral) of the endocardial surface will be shown in order for readers to get a more detailed perception on the local LV shape and changes during the cardiac cycle. As shown in Figure 9.22 and 9.23, the hyperbolic areas (represented in dark blue shades) can be easily distinguished from the elliptic ones by the sign convention of the Gaussian curvature. It is also observed that the mid anterolateral region remains elliptic over a large portion of the surface area during the cardiac cycle.

In addition, the magnitudes of the extreme K values at this particular region (i.e. -ve for hyperbolic areas, +ve for elliptic areas) are found to increase during systole and decrease during diastole. In fact, the magnitudes of the maximum (+ve) and minimum (-ve) K values at the end of ejection ($t_0 + 10$) are found to be more than 10 times larger than those found during the start of ejection (t_0). At the start of diastole, the maximum and minimum values of K are found to be 0.071 and -0.0339 respectively, and at time $t_0 + 17$ these values change drastically to 0.00442 and -0.0038.

With the reasonable assumption that the LV surface patch deforms slightly with minimum change in its shape and Gaussian curvature at small time interval, one can appropriately track the movement of the surface patch by means of shape correspondences. For regions with relatively high elliptic and hyperbolic values such as those found in the encircled regions of Figure 9.22 (e) and (f), and Figure 9.23 (a), (b), (e) and (f), one can readily detect the movement of these particular regions between successive frames with just a glimpse of the Gaussian curvature plots.

Figure 9.22: Regional (mid anterolateral) view of surface Gaussian curvature (mm^{-2}) during systole

Figure 9.23: Regional (mid anterolateral) view of surface Gaussian curvature (mm^{-2}) during diastole

9.3 Endocardial Surface Curvedness

As mentioned in Chapter 6, the curvedness metric (C) is a measure of how gently or highly a curved point is. Unlike the mean curvature (H), it disregards the sign convention used for differentiating convexities and concavities. This metric C is thus a useful tool in describing the magnitude of the curvedness at a point on the surface.

9.3.1 Curvedness Plots of the Endocardial Surface

Similar to Section 9.1.1, the whole endocardial surface curvedness will first be depicted in four different views (lateral, interior, septal and anterior), followed by selected segmental plots. By comparing the curvedness plots of the LV with the mean curvature plots in Section 9.1.1, it can be observed that the relative curvedness of the surface corresponds to the extent of mean curvatures obtained at the surface. In other words, the degree of convexities and concavities of the endocardial surface patches are reflected through the curvedness (C) magnitudes of the corresponding surface patches. This therefore justifies the feasibility of the curvedness metric.

Figures 9.24 to 9.27 each illustrates the curvedness of the LV surface through systole from different long axis views. From these figures, one can easily observe that the maximum curvedness values achieved near the end of ejection (i.e. at $t_0 + 8$ and $t_0 + 10$) are considerably higher than those attained during early ejection (t_0 and $t_0 + 2$) and mid-ejection ($t_0 + 4$ and $t_0 + 6$). As illustrated in the figures below, the apical cap remains considerably curved (which is predictable), and achieved a C value as high as 0.6 mm^{-1} which is much higher than the value obtained at t_0 .

Figure 9.24: Long axis (lateral) view of LV surface curvedness (mm^{-1}) during systole

Figure 9.25: Long axis (inferior) view of LV surface curvature (mm⁻¹) during systole

Figure 9.26: Long axis (septal) view of LV surface curvedness (mm⁻¹) during systole

Figure 9.27: Long axis (anterior) view of LV surface curvedness (mm⁻¹) during systole

From the curvedness plots, there is no indication that every region of the LV surface should increase in curvedness during the ejection period. This is logical as the myocardium does not activate all at once, in fact the cardiac muscle cells at different regions of the LV generate contractile forces through electrical depolarization which propagates and spreads through the LV during systole. Suppose at time t , a small section of the myocardium attached to the endocardial surface at R2 starts to contract and becomes relatively stiffer than R1 and R3 (refer to Figure 9.28), with relative movement towards R2. Eventually, R1 and R2 become crumpled up at $t + 1$ while R3 experiences some stretching and becomes flatter. Thus, when a section of the LV surface increases in its curvedness, its neighboring surface may sometimes become flatter instead.

Figure 9.28: Stretching of the endocardial surface

From Figure 9.24 to 9.27, one can easily see that at sections with high curvedness, the adjacent surface regions appear to be relatively flatter. Such situations can be easily detected from Figure 9.24 (a)-(d), 9.25 (a) etc.. It can be noted from Figure 9.26 that the maximum curvedness attained at time t_0 by the LV is found at the lower basal anteroseptal region, while the maximum curvedness at time $t_0 + 2$ and $t_0 + 4$, the maximum curvedness is achieved at the upper end of the basal inferoseptal region. Subsequently, the maximum curvedness is found at the apical end for the rest of the ejection period.

Figure 9.29: Long axis (lateral) view of LV surface curvedness (mm⁻¹) during diastole

Figure 9.30: Long axis (inferior) view of LV surface curvedness (mm⁻¹) during diastole

Figure 9.31: Long axis (septal) view of LV surface curvedness (mm⁻¹) during diastole

Figure 9.32: Long axis (anterior) view of LV surface curvedness (mm⁻¹) during diastole

Figures 9.29-9.32 depicts the curvedness computed for the LV surface during diastole. From these figures, one can observe that the variation of the LV surface curvedness during diastole is generally going through a reverse process from that of systole, which is similar to that of the LV surface mean curvatures. Although some contradicting results were found at the apex (most probably due to poor image resolution and surface interpolation at this small locality), the general change in curvedness for the rest of the LV has been reversed.

The magnified curvedness plots of the basal inferoseptal, basal inferolateral, mid anterolateral and mid antero-septal regions during the cardiac cycle are shown in Figure 9.33 to Figure 9.40. From these figures, the regional distribution and variation in LV surface curvedness can be assessed more thoroughly.

Figure 9.33: Regional (basal inferoseptal) view of surface curvedness (mm^{-1}) during systole

Figure 9.34: Regional (basal inferoseptal) view of surface curvedness (mm^{-1}) during diastole

Figure 9.35: Regional (basal inferoseptal) view of surface curvedness (mm^{-1}) during systole

Figure 9.36: Regional (basal inferolateral) view of surface curvedness (mm^{-1}) during diastole

Figure 9.37: Regional (mid anterolateral) view of surface curvedness (mm^{-1}) during systole

Figure 9.38: Regional (mid anterolateral) view of surface curvedness (mm⁻¹) during diastole

Figure 9.39: Regional (mid anteroseptal) view of surface curvedness (mm⁻¹) during systole

Figure 9.40: Regional (mid anteroseptal) view of surface curvedness (mm⁻¹) during diastole

9.3.2 Average Curvedness of the Endocardial Surface

It is noticed from Figure 9.33 to Figure 9.40 that the distribution and temporal variation of curvedness at each region of the LV surface can be quite different from each other. However it is believed that the overall curvedness of the LV surface should increase during systole and decrease during diastole. As a test, the average curvedness are computed at 4 segmental contours (i.e. taking the average of the sum of C values from the number of data points on each contour,) located at near base (B1), mid-cavity (B2), apical (B3) and near apex (B4) and put under investigation.

Figure 9.41: Average curvedness at selected contours during systole

Figure 9.42: Average curvedness at selected contours during diastole

Figure 9.41 and 9.42 depict the average curvedness computed at the selected contours (B1-B4) during systole and diastole respectively. It can be observed that the average curvedness computed at the contours generally increases during systole and decreases during diastole.

The average curvedness of the whole endocardial surface during the cardiac cycle is also computed, and the values are normalized against the maximum value and plotted against

time as shown in Figure 9.43. It can be observed that the average curvedness computed for the LV during systole increases with time and reaches a maximum value at ES ($t_0 + 10$). As predicted, the average curvedness of the endocardial surface decreases over time during diastole.

Figure 9.43: Average curvedness (normalized) of the endocardial surface

9.4 Endocardial Surface Shape Index

It was mentioned in Chapter 6 that the numerical relationship between the two principal curvatures can be used in a more detailed classification of surface types where the shape index function (S) mapped all surface patches, except for plane patches ($\kappa_1 = \kappa_2 = 0$) onto $S \in [-1, +1]$.

9.4.1 Shape Index Plots of the Endocardial Surface

In this particular section, the shape index defined over the endocardial surface in the lateral and septal view during systole will be shown. In viewing these plots, readers should follow the convention described in Table 6.2 for the ranges of values used in the surface classification.

Figure 9.44 depicts the shape index identified over the endocardial surface in the lateral view during systole, while Figure 9.45 illustrates the LV surface shape index in the septal view. As shown in the figures, the shape index can easily be used to identify highly curved regions from flatter ones (i.e. most curved when classified as a spherical cup or cap, while flattest when classified as a saddle). In other words, this shape index function is an appropriate tool for the detection of extreme curvatures.

It should be noted that spherical cups and troughs are actually concave ellipsoids, while ruts are concave cylinders. On the other hand, spherical caps and domes are convex ellipsoids, and ridges are convex cylinders. Therefore, it is obvious that the shape index can also be used to differentiate concave regions from convex ones just like the mean curvature (H).

Figure 9.44: Long axis (lateral) view of LV surface shape index during systole

Figure 9.45: Long axis (septal) view of LV surface shape index during systole

From Figures 9.44 and 9.45, one can see that the LV surface has been nicely classified into various shapes represented by different shades of color. Thus it is evident that with such shape index plots, one can easily define landmarks on the complex LV surface especially at regions with prominent feature for future assessments.

As the shape index function has a fair sensitivity to changes in surface and is invariant to geometric transformation, one can probably make use of the shape index plots to identify unusual changes in the LV surface shape from images (i.e. ultrasound, magnetic resonance (MR), X-Ray computed tomography (CT) etc.) taken from the same patient at different periods of time. For example, if the LV shape index plots derived from MR images taken from the same person have been observed to be similar and consistent for a number of years, but in the near future found to differ greatly from the previous plots, one can almost confirm that the LV has undergone some changes in its functionality and efficiency. To be specific, suppose some prominent landmarks have been consistently spotted at certain regions on the surface of a healthy heart at end systole (i.e. from MR images taken at different times), and in the next MR session if one or few areas at these landmarks are infarcted, one can probably perceive these infarctions when the landmarks are found to diminish greatly or are nowhere to be found on the shape index plot at end systole. It should be highlighted that the imaging technique, surface parameterization and algorithm etc. used should be consistent in carrying out such investigations.

From the shape index maps, it is observed that regions with prominent features (such as cup, trough, rut, ridge, dome, or cap) run through several temporal frames or even the entire sequence. This consistency of shape features over time can thus provide the basis for visually matching surface regions based upon shape cues. This approach is justified by the scale invariancy of the shape index and the reasonable assumption that LV surface patches deform locally and minimally in a shape-preserving manner during short time intervals.

As shown in Figure 9.46, one can say that the regions denoted as 1 and 2 moves approximately to regions 1' and 2' and subsequently to 1'' and 2'' respectively during

short time intervals. However for a more precise point matching estimation, one should utilize the derived differential properties to resolve the ambiguity inherent to the point correspondence estimation problem using a shape-based approach.

(a) prominent features classified by shape index around mid cavity section

(b) prominent features classified by shape index around basal section

Figure 9.46: Visual estimation of regional movements

9.5 Endocardial Surface Points Matches and Curvature Strain

The algorithm described in Chapter 7 has provided a basis for the estimation of LV surface motion field. In this section, the results attained from the surface points matching process will be briefly discussed and used for depicting the change in curvatures at sampled surface points.

9.5.1 Endocardial Surface Motion

The complete trajectories of 2 sample points over the cardiac cycle predicted from the algorithm are superimposed onto the endocardial surface at end systole as shown in Figures 9.47 and 9.48. Evidently, these endocardial points experiences out of plane motion which is why 3D images are so important for the investigation of LV motion. It can be observed that the trajectory of each point forms almost a complete loop, implying that the point moves back near the region where it started.

Figure 9.47: Trajectory of single endocardial point at apical region during the cardiac cycle

Figure 9.48: Trajectory of single endocardial point at basal region during the cardiac cycle

The trajectories of sample points during ejection are superimposed onto the endocardial surface at ES and shown in Figure 9.49. Clearly, the points move inwards towards the cavity long-axis during ejection showing the extent of radial contraction.

Figure 9.49: Trajectories of sample endocardial points during ejection

Figure 9.50 (a) depicts the dense field displacement vector map at the middle slice of the LV surface from time $t_0 + 4$ to $t_0 + 5$ during systole, while Figure 9.50 (b) shows the displacement field at the middle slice from time $t_0 + 14$ to $t_0 + 15$. The non-homogeneous nature of the endocardial motion can be easily observed from the figures as different regions of the slice display different motion characteristics in direction and magnitude. The displacement maps show reasonable deformation at the slice such as overall contraction during systole (Figure 9.50 (a)) and dilation during diastole (Figure 9.50 (b)).

Figure 9.50: Dense displacement vector field at the middle slice of the LV surface (vectors are projected in 2D and scaled-up for visualization purposes)

The average short-axis rotation with respect to the start of ejection was computed at different segments of the endocardial surface and depicted in Figure 9.51. The results were compared to the average twist angles found in Nash [34] and Arts [54]. For comparison purposes, a trend-line was fitted for results from Nash [34] and the slope was extended to cross the intercept for those of Arts et al. [54]. Anticlockwise rotations were defined as positive when the LV is viewed from the apex. The maximum rotation

computed was around 10° and was found around the apical region, while the basal region was found to attain minimal twist during ejection. Some over-estimation of twist angles were found when compared with the finite element model from Nash [34], however the trend is quite similar. Arts et al. [54] measured the left ventricular twist during ejection in canines using 2D ultrasonic echocardiography, and found that the basal regions rotated clockwise during ejection (when viewed from apex). This clockwise rotation however was representative of the mean LV wall twist, not just the endocardial surface alone. Although reasonable results are obtained from the points matching algorithm, they cannot be guaranteed as there is no further comparison with gold standard points tracking system such as MR tagging. More work will be involved in the determination of LV twist using MR tagging in the near future.

Figure 9.51: Short-axis rotation with respect to start of ejection state

9.5.2 LV Surface Curvature Strain

In this section, a new index defined as curvature strain (ϵ_c) is used for quantifying the change in curvatures at sample endocardial points. As implied by the term 'strain', this new index is defined as the change in curvature at a point divided by the original

curvature. As the mean curvature (H) can have both negative and positive values, it is obviously not suitable for the derivation of ϵ_c unless only the absolute values are considered. On the other hand, the curvedness metric (C) is a positive value which is used to gauge the degree of curvature at a point despite of its normal direction, and is thus a good choice for measuring ϵ_c .

This curvature strain is used to investigate the change in curvatures at LV endocardial points during small time intervals and is represented by the following equation:

$$\epsilon_{c,i} = \frac{C_i - C_{i-1}}{C_{i-1}} \quad (9.1)$$

where $\epsilon_{c,i}$ is the instantaneous LV curvature strain, while C_i and C_{i-1} denote the curvedness measured from the same surface point at time $t = i$ and $t = i - 1$ respectively.

The instantaneous curvature strain computed at 3 endocardial points from different LV segments during systole are plotted against time and shown in Figure 9.52 to 9.54.

Figure 9.52: Curvature strain of single point from basal region

Figure 9.53: Curvature strain of single point from mid cavity region

Figure 9.54: Curvature strain of single point from apical region

From Figure 9.52, a considerable change in curvedness at the basal region is detected when time approaches mid-ejection from $t_0 + 3$ to $t_0 + 5$, and for the rest of ejection, this change remains relatively small.

It is observed from Figure 9.53 that the instantaneous curvature strain computed at the point around mid cavity level fluctuates substantially throughout the ejection phase, and

are relatively large as compared to those at the basal point. This inevitably implies a considerable amount of wall deformation at this locality during ejection.

From Figure 9.54, it is evident that the curvature strain computed at the point selected from the apical region is very small at most time during ejection as compared to the other two points. Therefore it is apparent that the surface patch at this point experiences relatively small changes in its curvedness during the ejection phase.

It should be clear that the curvature strain index introduced in this particular section can be used as a tool to assess local LV surface activity in terms of shape changes.

9.6 LV Surface Kinematics during Isovolumic Contraction (from MRI)

In this section, the results obtained from a different set of data acquired from cardiac MR imaging during the isovolumic contraction phase will be presented and briefly discussed. Although the main interest of this particular section is to elucidate the motion of the LV surfaces during isovolumic contraction, the mean curvature and curvedness maps computed prior to the point-based tracking will also be shown for the reference of interested readers. Since only two time instances were acquired during the isovolumic contraction phase, the first frame will be considered as the original undeformed state while the second will be treated as the deformed configuration in the following analyses.

9.6.1 Curvature Plots of the Endocardial and Epicardial Surfaces during Isovolumic Contraction

Although the same surface parameterization and algorithm used in Chapter 6 are applied to this dataset, the imaging technique or modality used for this set of data (MR imaging) is different from the first set of data (Ultrasound imaging). In addition, the author received no prior information on the image processing schemes applied to the first dataset acquired from ultrasound imaging. Therefore, readers should note that the curvature maps computed for this particular set of data (MR imaging) should not be directly associated with those acquired for the first dataset (Ultrasound imaging).

Figures 9.55 and 9.56 depict the mean curvature maps of the endocardial surface and epicardial surface respectively for both the original and deformed configurations in the anterior view. The curvedness maps of the endocardial and epicardial surface are illustrated in Figures 9.57 and 9.58 respectively.

From the curvature maps of the endocardial surface depicted in Figures 9.55 and 9.57, it can be seen that the computed values are generally higher than those acquired for the first

few instances of the ultrasound dataset acquired during the ejection phase. As mentioned in Chapter 6, the ultrasound dataset was provided in the form of cleaned contour points without information on the segmentation and interpolation methods used. Therefore, one possible reason for the lower curvature values acquired for the ultrasound dataset might be due to the extensive smoothing used on this dataset for the purpose of visualization in commercial softwares. This is another reason why readers should not correlate the curvature values computed for these two datasets which are acquired by different imaging modalities. More research will be done on the designing of a set of rules and procedures for accurate and consistent computation of LV surface curvatures in the near future.

Endocardial Surface

Figure 9.55: LV endocardial surface mean curvature (mm^{-1}) during isovolumic contraction

Epicardial Surface

Figure 9.56: LV epicardial surface mean curvature (mm^{-1}) during isovolumic contraction

From Figures 9.55 and 9.56, it is apparent that the epicardial surface has a smoother shape than that of the endocardial surface, resulting in lower mean curvature values as compared to the computed values for the endocardial surface. The two extreme values computed for the endocardial surface in the original configuration are -0.3596 mm^{-1} and 0.19823 mm^{-1} , while those acquired for the endocardial surface in the deformed state are -0.3367 mm^{-1} and 0.2967 mm^{-1} . On the other hand, the two extreme mean curvature values obtained for the epicardial surface in the original and deformed configurations are -0.1778 mm^{-1} and 0.0884 mm^{-1} , and -0.2021 mm^{-1} and 0.17364 mm^{-1} respectively, which are relatively low. By observing and tracking the landmarks which are more concaved or convexed, one can easily perceive that both the LV surfaces experience a twisting action in the anti-clockwise direction (when viewed from the apex) during isovolumic contraction.

Endocardial Surface

Figure 9.57: LV endocardial surface curvedness (mm^{-1}) during isovolumic contraction

Epicardial Surface

Figure 9.58: LV epicardial surface curvedness (mm^{-1}) during isovolumic contraction

From Figures 9.57 and 9.58, it is apparent that the apical region of the LV surfaces is generally more curved than the rest of the LV. It can be observed that both the endocardial and epicardial surfaces experience some form of deformation which caused the surfaces to become more curved during the isovolumic contraction phase. The highest curvedness values acquired for both the endocardial and epicardial surfaces are found to be 0.47311 mm^{-1} and 0.2675 mm^{-1} respectively. From these curvedness plots, one can also identify an anti-clockwise twisting action of the LV surfaces when viewed from the apex. This twisting motion is especially notable at regions with relatively high curvedness value.

9.6.2 Displacement of Endocardial and Epicardial Surfaces during Isovolumic Contraction

The displacement vectors computed for the LV endocardial and epicardial surfaces during isovolumic contraction for the MRI dataset are depicted in Figures 9.59 and 9.60 respectively. The surface points are sub-sampled for the purpose of clarity when viewing the displacement vectors.

Figure 9.59: Dense displacement vector field of the LV endocardial surface (mm)

Figure 9.60: Dense displacement vector field of the LV epicardial surface (mm)

From Figure 9.59, it can be observed that most of the endocardial surface points experienced a twisting motion in the anti-clockwise direction when viewed from the apex, and a general downward motion (i.e. towards the apex) during the isovolumic contraction phase. Similar twisting motion of the epicardial surface points can also be detected from the vector plots shown in Figure 9.60. The downward movement of the epicardial surface points can also be observed from the anterior view of the vector plot (Figure 9.60 (a)), however this general downward motion is not observed with the epicardial surface in the inferior view (Figure 9.60 (b)). Instead, much of the epicardial points (from the inferior view) seemed to be experiencing displacement within their short axis planes (i.e. little movement in the long axis direction), while some of the points at the lower half of the LV appeared to be moving upwards (i.e. towards the base) along with twist.

In order for readers to have a better perception of the LV motion at different locations, the LV is divided into nine different sections along the long-axis as shown in Figure 9.61. The displacement vectors at each level are then shown as 2D projections onto the surface contours.

Figure 9.61: Different short-axis (SA) planes of the LV

Figure 9.62 depicts the displacement vector field obtained for the LV surface contours at different short-axis (SA) planes during the isovolumic contraction phase. Although the LV surface deformation is observed to be non-homogeneous in nature, it is evident that the surface contours at each plane are experiencing a twisting motion in the anti-clockwise direction (when viewed from the apex) during isovolumic contraction. One can therefore deduce that the LV is being twisted in the anti-clockwise direction prior to the start of ejection, when viewed from the apex.

It can also be observed that the magnitude of displacement of the LV surface points is generally higher at the apical segment and at regions near to it (i.e. SA planes 6, 7, 8 and 9). This is possibly due to the relatively earlier depolarization or activation of the apical segment, causing larger displacements due to myo-fiber shortening to be detected at this region.

Figure 9.62: Dense displacement vector field of surface contours at different SA planes (mm)

Figure 9.62: Dense displacement vector field of surface contours at different SA planes (mm)

To illustrate the magnitude of the different components of the displacement vectors, a coordinate system in terms of r , θ and z (cylindrical coordinate system) is defined as shown in Figure 9.63. The origin is set at the base level, with the z axis defined along the long axis of the LV.

Figure 9.63: Coordinate system defined for the LV

The total displacement of the LV surfaces from SA plane 1 to apex during isovolumic contraction are depicted in Figure 9.64 in the form of color plots. In addition, the displacement in the radial (r), circumferential (θ) and longitudinal (z) directions are shown in Figure 9.65, 9.66 and 9.67 respectively. Readers should note that the unit used in the color plots of the circumferential displacement is in millimeters instead of degrees so as to allow comparison to be made for the magnitude of displacements in each direction (i.e. r , θ and z). The average short axis rotation in degrees at different segments of the LV will be shown later in this section.

Figure 9.64: Total displacement at LV surfaces (mm)

As illustrated in Figure 9.64, the maximum displacement values for both the endocardial and epicardial surfaces are 5.838 mm and 5.005 mm respectively, and are found to take place at the apical anterior segment. In general, the LV deformation at the anterior sector is observed to be relatively higher than that at the inferior sector.

(a) LV endocardial surface

(b) LV epicardial surface

Figure 9.65: Radial (r) displacement at LV surfaces (mm)

From Figure 9.65, it is apparent that the LV surfaces experienced both radial contraction ($-ve\ r$) and expansion ($+ve\ r$) during the isovolumic contraction phase. This radial deformation is reasonable since during isovolumic contraction, the LV blood volume remains constant, and while radial contraction occurs at certain regions, some form of

expansion in the radial or longitudinal direction due to fluid pressure can be expected at some other LV regions so as to keep the blood volume constant. This radial deformation in both directions might also be due to the non-homogeneous activation of the myocardium.

(a) LV endocardial surface

(b) LV epicardial surface

Figure 9.66: Circumferential (θ) displacement at LV surfaces (mm)

From Figure 9.66, it is evident that the LV is generally being twisted in the anticlockwise direction (+ve θ) when viewed from apex. The maximum circumferential displacement at the endocardial surface during isovolumic contraction is found to be 5.073 mm which is relatively larger than that (4.389 mm) recorded at the epicardial surface.

Figure 9.67: Longitudinal (z) displacement at LV surfaces (mm)

As illustrated in Figure 9.67, the LV experienced minimal longitudinal displacement as compared to the displacement in the radial and circumferential directions. The magnitude of the largest z-direction displacement at the LV surfaces during isovolumic contraction is found to be 2.534 mm, which is much smaller than those recorded for radial contraction (4.577 mm) and circumferential rotation (5.073 mm).

From the results obtained, it was found that a significant share of the total displacement at the LV surfaces is contributed by the displacement in the circumferential direction. It is thus possible that the high increase in LV blood pressure during isovolumic contraction is largely due to the twisting action of the LV.

The average short-axis rotation of the endocardial and epicardial surfaces during the isovolumic contraction phase was computed at different segments of the LV surfaces from near base to near apex and is depicted in Figure 9.68. From this graph, it is apparent that the LV is twisted in the anti-clockwise direction (when viewed from the apex) during isovolumic contraction. It can be also be observed that the apical segment of the LV generally experience a greater amount of rotation as compared to the rest of the LV during the isovolumic contraction phase.

Figure 9.68: Short-axis rotation during isovolumic contraction

Chapter 10

CONCLUSION AND FUTURE WORK

10.1 Conclusion

The results obtained from the computation of LV surface properties based on an analytic approach have been presented in the form of surface curvature plots in Chapter 9. The accuracy of the proposed algorithm was also verified with the exact solutions of simple geometrical shapes in Chapter 6. It is evident that the surface curvature plots have enabled us to view and study the LV surface shape in a more detailed manner.

By putting together the LV surface curvature maps derived from different instances, one can easily and effectively delineate the deformation of the LV surface in terms of shape changes during the cardiac cycle. The distribution and changes in endocardial surface curvatures were found to vary to such great extent and non-homogeneously that it is hardly possible for one to accurately observe and quantify these changes with a naked-eye.

The mean curvature metric has been particularly useful in distinguishing concave regions from convex regions on the LV surface. It was observed that the variation in the endocardial surface mean curvatures during diastole generally acts in the opposite manner from that during systole, similar to the untwisting action of the LV during diastole. Shape changes were found to be minimal during isovolumic relaxation. The ‘hills and valleys’ plots of the endocardial surface provide an effective means to view the distribution and variation of mean curvatures, and could be helpful in detecting unusual wall deformation.

The Gaussian curvature and shape index derived for the endocardial surface are useful tools for tracking LV surface motion using a shape-based approach. This is especially true for the LV shape index, which effectively classifies the LV into different shapes. Due to its scale invariance property, it is particularly useful for defining landmarks on the LV surface.

The maximum curvedness attained near end ejection was found to be considerably higher than those acquired during early ejection. The same reverse trend in curvature changes can be observed from the LV curvedness plots. In addition, the average curvedness computed for the endocardial surface was found to increase during systole and decrease during diastole.

Satisfactory results were obtained from the LV endocardial surface motion tracking process. The average short-axis rotation computed for the endocardial surface was found to be analogous to the results obtained by previous researchers, thus proving the efficiency and accuracy of the point matching algorithm used. The instantaneous curvature strain derived from the computed curvedness values at endocardial points can be used to assess and quantify LV wall property at small time intervals in terms of changes in the degree of curvature.

The point matching algorithm was also applied to a MRI dataset to track the LV surface deformation during isovolumic contraction. In the process, a relatively significant amount of circumferential displacement was detected at the LV surfaces during the isovolumic contraction phase. In general, a twisting motion of the LV in the anti-clockwise direction (when viewed from the apex) was observed during this short period of time, and this twisting action is most prominent in the apical region.

From this research, it is believed that with a more detailed and refined analysis on a range of LV image datasets (i.e. 30-50 cases) under different physiological conditions, the investigation of LV surface curvatures would prove to be clinically useful especially for the detection of myocardial infarction and unusual wall activity. For example, one may

discover that LV regions with comparatively small or negligible curvature changes are likely to be infarcted. In addition, one may also discover specific trends or patterns in the temporal variation of LV surface curvatures which can be used as a guide to identify unusual wall deformations.

10.2 Future Work

As the heart functions with such intricacy, scientists and researchers should approach their investigations from every direction so as to improve the understanding of its remarkable mechanism. In this chapter, the author briefly describes some of the future work that may be done in order to gain greater insight in the complex mechanism of the left ventricle.

10.2.1 Left Ventricular Surface Curvatures

In this report, an additional step was taken to rotate each surface patch in interest to the z -axis for local surface fitting and computation of curvatures. This eventually slows down the computation process to retrieve surface properties.

One way of solving this problem is to define a local coordinate system at each point $\mathbf{x}$ on the LV surface such that the origin of each coordinate system lies on the point $\mathbf{x}$ being studied at each time. At each point $\mathbf{x}$ on the surface, the z -axis of its local coordinate system will be directed in the outward normal direction of the surface at that particular point. From this point onwards, the first and second derivatives of $\mathbf{x}$, and the first and second fundamental forms can be calculated easily and accurately for the computation of surface curvatures.

In this report, the LV geometrical datasets are obtained from ultrasound and magnetic resonance imaging which do not provide any point correspondence. Although efforts

have been made to track the LV surface motion using a thin plate bending model, the resultant points matches obtained may not be truthful. In the near future, the LV geometrical dataset will be acquired from magnetic resonance (MR) tagging which allows an accurate tracking of a set of myocardial points during the cardiac cycle. In this way, the change in curvatures at this set of tagged points can be studied more precisely.

In order to achieve a better understanding of the LV surfaces changes, more LV datasets (from both healthy and diseased hearts) will be tested and evaluated in the near future. By comparing the surface curvatures of healthy and diseased hearts, one may discover the usefulness of the LV surface curvatures in detecting unusual wall activity and determining areas of cardiac infarction. As the endocardial and epicardial surfaces have different fiber orientations and are subjected to different boundary conditions, their motions and deformations can be quite different. Therefore, both the endocardial and epicardial surface curvatures should be assessed in order to better understand the regional deformation of the LV.

10.2.2 Left Ventricular Pressure Development during Isovolumic Contraction

The left ventricular pressure is known to rise substantially from about 15 mmHg to around 80 mmHg during the short period of isovolumic contraction. During this small fraction of the cardiac cycle, the LV cavity volume remains constant and yet experiences such a great rise in pressure. It is postulated that this great pressure development during isovolumic contraction is largely due to the twisting action of the LV.

As a start, the LV will be modeled as a thick-walled cylinder to avoid the computational complexity, and the required simulation will be done using finite element method. The LV model will be fixed at the base to represent the relative stiffness of the papillary muscles at this region. The cylindrical model will be closed at both ends and the cavity will be completely filled with fluid as the valves are closed during this short period of

time. In addition, the LV wall will be approximately mapped with myocardial fiber orientation as shown in Figure 11.1 so as to include the effects of the fiber structure.

Figure 11.1: LV cylindrical model mapped with myofiber orientation

At each time step, contractile forces along the myofibers direction will be applied on each layer of elements, starting from the bottom-most layer to the top-most layer of the LV model. During each load step, the layers which are under activation or have already been activated previously will be modeled with a greater modulus of elasticity while those which are at rest will have a relatively smaller modulus of elasticity. This is done so as to simulate the stiffness of the myocardium during systole.

In this way, the twisting action of the model will commence from the apex and work upwards towards the base, pressurizing the cavity and providing the fluid with an initial rush towards the base which is essential for left ventricular ejection. A range of suitable contractile forces will be applied in an iterative manner until the cavity pressure is generated from 15 mmHg to 80 mmHg, so as to determine the appropriate set of loading and mechanical conditions for this pressure development. The rotation achieved and myocardial wall stress/strain generated at each layer of elements will also be determined

and further assessed in order to validate the accuracy and usefulness of the model. Simulation will also be performed using a thick-walled prolate spheroidal model which is definitely a more appropriate representation of the LV's geometry.

Subsequently, a reconstructed model of the left ventricle (acquired from MR tagging) with reasonable myofibers orientation will be tested. Contractile forces will be imposed along these myofibers in an iterative manner such that the deformed shape of the original frame (i.e. start of isovolumic contraction) matches the following frame (i.e. end of isovolumic contraction), and the cavity pressure is generated from 15 mmHg to 80 mmHg. In this way, one can realistically simulate the LV pressure development during isovolumic contraction and gain great insight in the LV mechanism.

10.2.3 Left Ventricular Stress/Strain and Mechanical Properties

There has been a substantial amount of research done to determine the stress and strain in the left ventricle. In order to model the behavior the whole heart, a suitable description of mechanical properties of the cardiac tissue is necessary. Although there are a number of reliable material laws to describe the nonlinear material properties of the passive heart, there is still a lack of well-established constitutive laws to describe the active properties of the heart during systole. The author therefore suggests that more research in determining the active properties of the myocardium should be carried out. One of the most promising methods for acquiring mathematical solutions to the above-mentioned problem is the finite element method.

It is known that biological tissues including the cardiac tissues can in general undergo large deformation in normal physiologic activity, and thus cannot be appropriately dealt with using the infinitesimal theory of linear strain and stress. As such, the theory of finite deformation elasticity should be employed for the numerical analysis of the LV model.

In order to test the implementation of the finite element method on large deformation, a cylindrical model subjected to some straightforward loading conditions such as uniform extension and torsion is considered (Refer to Appendix C for a more detailed description). The stress distributions obtained from the FE analysis are compared with the closed form solutions for the extension and torsion of a circular cylindrical tube, and the results are found to be in good agreement with the exact solutions.

The LV geometrical data will be obtained from MR tagging so as to provide a basis for wall motion analysis. In tradition, researchers have applied uniform pressure on the LV cavity wall at each time instance to simulate the boundary condition on the endocardial wall. However, this form of load application should only be applied during diastole when the pressure is considered the active component in myocardium stress generation as blood rushes in to fill and pressurized the chamber. In contrary, it is the contractile properties of the active myocardium that induces the rise in cavity pressure during systole. As such, the cavity should be filled with fluid, and the cavity pressure at each instant will be defined by the amount of wall motion and deformation of the LV.

As the MR tagged images only provide a sparse set of tracked points during the cycle, these sampled points are insufficient to depict the precise local wall deformation between successive frames. As such, the surface curvatures of the LV at each frame will be computed, and the dense motion field of the rest of the myocardium will be predicted such that wall deformation at the regional level can be accurately determined by means of shape matching between successive frames. At this point, one should ask him/herself this question:

What should be the mechanical properties of the myocardium such that the LV cavity pressure of the model increases and matches the clinically monitored pressure at each time instant, when the LV model is subjected to realistic boundary conditions and wall motion and deformation tracked by MR tagging and shape-correspondences?

Figure 11.2: Finite element analysis of an anatomically accurate left ventricular model

Figure 11.2 depicts the boundary and loading conditions of the LV model during systole. The LV will be attached to the aorta whereby clinically measured aortic pressure will be applied to the valves. The wall motion of the LV between successive frames will be applied as displacements at each time step, and iteration will be performed to find the matching set of material parameters that will eventually cause an increase in the LV cavity pressure by the amount that is clinically measured. With the matching set of material parameters defined over systole, the material properties of the active myocardium can be determined. In addition, the variation in surface curvatures will be compared with the LV wall stress generated.

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Appendix A

LEFT VENTRICULAR SEGMENTATION AND NOMENCLATURE

The LV segments are named and localized with reference to both the long axes of the ventricle and the 360° circumferential locations on the short-axis views according to [94]. The left ventricle is divided into equal thirds perpendicular to the long axis of the heart. Using basal, mid-cavity, and apical as part of the name defines the location along the long axis of the ventricle from the apex to base (refer to Figure A. 1).

Figure A. 1 : LV long axis division

With respect to the circumferential location, the basal and mid-cavity slices are divided into 6 equal segments, while the apical slices are divided into 4 equal segments. The attachment of the right ventricular wall to the left ventricle is used to identify and separate the septum from the left ventricular anterior and inferior free walls. Figure A.2

shows the 17 myocardial segments and the nomenclature when viewed from the apex on a circumferential polar plot.

Figure A.2: LV circumferential polar plot for myocardial segmentation and nomenclature

Appendix B

LEFT VENTRICULAR GAUSSIAN CURVATURES

Figure B.1: Long axis (anterior) view of LV surface Gaussian curvature (mm⁻²) during systole

Appendix C

VALIDATION OF FEM ON THE APPLICATION OF FINITE DEFORMATION ELASTICITY

In order to investigate the implementation of the finite element method and the application of Ansys on the theory of finite elasticity, a simple model subjected to some straightforward load conditions is considered here. In this section, stress distributions from the FE analysis will be compared with the closed form solutions for the extension and torsion of a circular cylindrical tube. For mathematical simplicity, it is assumed that the cylinder is homogeneous, isotropic and incompressible, while the material is assumed to be of Mooney-Rivlin nature with a strain energy function of $W = c_1(I_1 - 3) + c_2(I_2 - 3)$, where c_1 and c_2 are the material constants, and I_1 and I_2 are the principal invariants of the left and right Cauchy-Green strain tensors. Another important assumption for the validity of the closed form solutions is to treat the cylinder as an infinitely long cylinder.

C.1 Exact Solution for Extension and Torsion of a Circular Cylinder

Consider a homogeneous hollow circular cylinder of incompressible material with initial length, internal and external radii denoted by L , R_i and R_o respectively subjected to the following deformation:

1. a uniform extension parallel to the axis of the cylinder, where the ratio of the deformed to undeformed lengths is denoted by the axial stretch ratio, λ ;
2. a uniform torsion in which planes perpendicular to the axis of the cylinder are rotated in their own plane through an angle proportional to the distance from one end, with the constant of proportionality denoted by ψ .

Since there is no volume change due to the incompressibility condition, there is a change of radii associated with the extension such that the internal and external radii are $\lambda^{-1/2} R_i$ and $\lambda^{-1/2} R_o$ respectively, and the length is λL .

The above deformation can be described by

$$\begin{aligned} x_1 &= \lambda^{-1/2} R \cos(\lambda \psi X_3 + \theta) = \lambda^{-1/2} \{X_1 \cos(\lambda \psi X_3) - X_2 \sin(\lambda \psi X_3)\} \\ x_2 &= \lambda^{-1/2} R \sin(\lambda \psi X_3 + \theta) = \lambda^{-1/2} \{X_1 \sin(\lambda \psi X_3) + X_2 \cos(\lambda \psi X_3)\} \\ x_3 &= \lambda X_3 \end{aligned} \quad (C.1)$$

where $X_1 = R \cos \theta$, $X_2 = R \sin \theta$ and $X_3 = X_3$ are the undeformed x, y and z coordinates respectively and R is the corresponding radius at each point.

The tensors $\mathbf{B}$ and $\mathbf{B}^{-1}$ in terms of Cartesian coordinates take the forms,

$$\mathbf{B} = \begin{pmatrix} \lambda^{-1} + (\lambda \psi x_2)^2 & -\lambda^2 \psi^2 x_1 x_2 & -\lambda^2 \psi x_2 \\ -\lambda^2 \psi^2 x_1 x_2 & \lambda^{-1} + (\lambda \psi x_1)^2 & \lambda^2 \psi x_1 \\ -\lambda^2 \psi x_2 & \lambda^2 \psi x_1 & \lambda^2 \end{pmatrix} \quad (C.2)$$

$$\mathbf{B}^{-1} = \begin{pmatrix} \lambda & 0 & \lambda \psi x_2 \\ 0 & \lambda & -\lambda \psi x_1 \\ \lambda \psi x_2 & -\lambda \psi x_1 & \lambda^{-2} + \lambda \psi^2 (x_1^2 + x_2^2) \end{pmatrix} \quad (C.3)$$

and the stress components are given by

$$\begin{aligned} \sigma_{11} &= -p + 2\{\lambda^{-1} + (\lambda \psi x_2)^2\}W_1 - 2\lambda W_2 \\ \sigma_{22} &= -p + 2\{\lambda^{-1} + (\lambda \psi x_1)^2\}W_1 - 2\lambda W_2 \\ \sigma_{33} &= -p + 2\lambda^2 W_1 - 2(\lambda^{-2} + \psi^2 R^2)W_2 \\ \sigma_{12} &= -2\lambda^2 \psi^2 x_1 x_2 W_1 \\ \sigma_{23} &= \psi\{2\lambda(\lambda W_1 + W_2)\}x_1 \\ \sigma_{33} &= -\psi\{2\lambda(\lambda W_1 + W_2)\}x_2 \end{aligned} \quad (C.4)$$

with p being the hydrostatic pressure derived from

$$\frac{dp}{dr} = 2 \frac{d}{dr} (\lambda^{-1} W_1 - \lambda W_2) - 2(\lambda\psi)^2 r W_1 \quad (C.5)$$

In order to prevent additional change of the inner radius due to torsion, the application of a uniform internal pressure P_i on the inner surface is necessary, and is denoted by,

$$P_i = 2\lambda\psi^2 \int_{R_i}^{R_0} (RW_1) dR \quad (C.6)$$

Equations (C.1) to (C.4) quoted directly from Atkin and Fox [95], are in terms of Cartesian coordinates which are in this case, quite cumbersome to deal with. In order to simplify the calculations, and check the implementation of the FEM for curvilinear coordinates, it is desired that the problem be dealt with using cylindrical polar coordinates.

Thus, the deformation shown above can now be described by a material particle initially at (R, θ, Z) being mapped to the final configuration of (r, Φ, z) such that:

$$r = \lambda^{-1/2} R, \quad \Phi = \theta + \lambda\psi Z, \quad z = \lambda Z$$

The deformation gradient tensor $\mathbf{F}$, followed by the left Cauchy-Green tensor $\mathbf{B}$ and its inverse $\mathbf{B}^{-1}$ in cylindrical coordinates are then computed via:

$$\mathbf{F} = \begin{pmatrix} \frac{\partial r}{\partial R} & \frac{1}{R} \frac{\partial r}{\partial \theta} & \frac{\partial r}{\partial Z} \\ r \frac{\partial \Phi}{\partial R} & r \frac{\partial \Phi}{\partial \theta} & r \frac{\partial \Phi}{\partial Z} \\ \frac{\partial z}{\partial R} & \frac{1}{R} \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial Z} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{\lambda}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{\lambda}} & r\lambda\psi \\ 0 & 0 & \lambda \end{pmatrix} \quad (C.7)$$

$$\mathbf{B} = \mathbf{F}\mathbf{F}^T = \begin{pmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} + (r\lambda\psi)^2 & r\lambda^2\psi \\ 0 & r\lambda^2\psi & \lambda^2 \end{pmatrix} \quad (C.8)$$

$$\mathbf{B}^{-1} = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & -r\lambda\psi \\ 0 & -r\lambda\psi & \frac{1}{\lambda^2} + (r\psi)^2 \lambda \end{pmatrix} \quad (\text{C.9})$$

The nonzero physical Cauchy stress components and the corresponding hydrostatic pressures are now given by

$$\begin{aligned} \sigma_{RR} &= -p + 2\lambda^{-1}W_1 - 2\lambda W_2 \\ \sigma_{\theta\theta} &= -p + 2\{\lambda^{-1} + \lambda(R\psi)^2\}W_1 - 2\lambda W_2 \\ \sigma_{ZZ} &= -p + 2\lambda^2 W_1 - 2\{\lambda^{-2} + (R\psi)^2\}W_2 \\ \sigma_{\theta Z} &= 2R\lambda^{1/2}\psi(\lambda W_1 + W_2) \\ p(R) &= 2\lambda^{-1}W_1 - 2\lambda W_2 + 2\lambda\psi^2 \int_R^{R_0} (RW_1)dR \end{aligned} \quad (\text{C.10})$$

C.2 Finite Element Analysis Versus Exact Solutions

From equation (C.10), it can be understood that the stress components and hydrostatic pressures are independent of the circumferential (θ) and axial (Z) coordinate, and varies only according to the undeformed radial position (R). Therefore comparison of the results from the FEM and exact solutions can be performed by plotting the stress distributions along the radial direction only.

To check for consistency of the software, the FE model will first be tested on simple extension, followed by uniform torsion and finally a combination of extension and torsion. The parameters which are kept constant during these 3 different load conditions are presented in Table C.1.

Parameter	Value used
External radius, R_o (mm)	35
Internal radius, R_i (mm)	25
Length, L (mm)	80
Material constant 1, c_1 (Pa)	2000
Material constant 2, c_2 (Pa)	6000

Table C. 1: Parameters for validation model.

C.2.1 Load condition 1 (Extension only)

In this particular load case, the cylinder is uniformly extended in the axial direction with a stretch ratio of $\lambda = 1.2$. The deformation is depicted in Figure C.1 where the internal and external radii are decreased to $r_i = 1.2^{-1/2}R_i$ and $r_o = 1.2^{-1/2}R_o$ respectively.

Figure C.1: Deformation of cylinder under extension

In this case, the only nonzero stress component is σ_{zz} , and the stress distribution along the undeformed radial coordinate is shown in Figure C.2. With such a simple deformation,

the model which was meshed with just 32 quadratic elements obtained the exact solution for σ_{zz} .

Figure C.2: FEM versus exact solution (Extension only)

C.2.2 Load condition 2 (Torsion only)

A uniform torsion is applied to the cylinder such that one end of the cylinder is being rotated by an angle of 30° , which gives a value of $\psi = \pi(30^\circ)/(180^\circ \times L)$ rad/cm. In addition, a uniform internal pressure of $P_i = 51.40$ Pa is applied to the inner wall to prevent a change of radii. The deformation of this load case is depicted in Figure C.3.

Figure C.3: Deformation of cylinder under torsion

The stress distribution along R of the nonzero Cauchy stress components are depicted in Figure C.4. Due to the more complicated deformation, more elements are required for the FE stress distributions to converge to the closed form solutions as shown in the plots below.

(a) σ_{RR}

Figure C.4: FEM versus exact solution (Torsion only)

Graph of $\sigma_{\theta\theta}$ along wall thickness

(b) $\sigma_{\theta\theta}$

Graph of σ_{zz} along wall thickness

(c) σ_{zz}

Graph of $\sigma_{\theta z}$ along wall thickness

(d) $\sigma_{\theta z}$

Figure C.4: FEM versus exact solution (Torsion only)

C.2.3 Load condition 3 (Extension and torsion)

In this load case, the cylinder is being uniformly extended and rotated with values of λ and ψ equal to those of load case 1 and 2. In addition to these, a uniform internal pressure of $P_i = 61.69\text{Pa}$ is also applied to the inner surface. The deformation of the cylinder is shown in the figure below.

Figure C.5: Deformation of cylinder under extension and torsion

The stress distribution along R of the nonzero Cauchy stress components are illustrated in Figure C.6. Similarly to the above analysis, the FEM obtained the expected stress distributions and the results converged towards the closed form solutions as the number of elements increased.

(a) σ_{RR}

(b) $\sigma_{\theta\theta}$

(c) σ_{ZZ}

Figure C.6: FEM versus exact solution (Extension and torsion)

Figure C.6: FEM versus exact solution (Extension and torsion)