Investigations of echo signal models for medical ultrasound imaging systems

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Investigations of Echo Signal Models for Medical Ultrasound Imaging Systems

Chen Ming

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirements for the degree of
Doctor of Philosophy

2012

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Date

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Chen Ming

To Guangzhou and my parents.

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Summary

Medical ultrasound images are degraded representations of the true acoustical reflectors in the imaged anatomical tissue. In theory, the qualities of medical ultrasound images rely mainly on the collected ultrasound echo signals. If ultrasound echo signals were accurate representations of biological tissues, the resolution of medical ultrasound images would be improved correspondingly. However, ultrasound echo signals do not provide a direct description of biological tissues usually because they are regarded as the reflected echoes arising from the interactions between ultrasound system and biological tissues. In order to retrieve the true properties of tissues of interest, we need to resort to accurate models of ultrasound echo signals. A reasonable and accurate model is very instructive in understanding the essential characteristics of ultrasound imaging systems. This thesis focuses on the investigations of current models and development of novel models for ultrasound imaging systems.

A theoretical model based on classical ultrasound acoustics was developed in 1970s. This model with integration format has been verified and cited for decades. However, from both the signal and system point of view, it purely concerns ultrasound physics and is difficult to be applied in engineering field. Since 1990s, a conventional convolution model was fully derived based on the theoretical model and presented an appealing format for engineers. In this convolution model, ultrasound echo signals are modeled mathematically as a spatio-temporal convolution between the space-variant ultrasonic system impulse response or point-spread function (PSF) and the

biological tissues, with the addition of observation noise introduced in the image formation process. Furthermore, a simple discrete convolution model directly arising from continuous conventional model was proposed and widely applied in ultrasound deconvolution filtering techniques.

The first contribution of our work is the development of a new convolution model with reasonable and feasible conditions. After careful studies, we find there is a flawed approximation in the conventional convolution model which plays the key role in the original derivations such that the original convolution model is not fully theoretically sound and valid. Consequently, the deconvolution-related techniques for processing ultrasound echo signals no longer have theoretical foundation. In this thesis, a new convolution model of medical ultrasound echo process is proposed and its derivation and formulation are provided. Based on the investigations of classical acoustical equations, the new convolution model presents dominant terms under common practically feasible conditions of medical ultrasound. It provides a new theoretical foundation for the prevailing deconvolution techniques in ultrasound signal processing and new insight in exploring interactions between ultrasound pulses and body tissues in ultrasound scanning and imaging.

The second contribution of our work is the proposal of two new discrete models of ultrasound echo signals. The conventional discrete model does not provide a convincing discretization from conventional continuous convolution model. The reason behind is that time dimension is ignored deliberately during the discretization without strict proof. Meanwhile, tissue information is only conceptual 2-D sequences of signals without specific indications on how to connect the so-called tissue signals with real soft tissues' anatomy. Meanwhile, the simple discrete model is quite general and symbolic such that it is not mathematically straightforward and accurate enough. Besides, we have pointed out the problematic approximation in the derivation of conventional convolution model and proposed new convolution model

with reasonable conditions. Hence, starting from the original theoretical model and new convolution model proposed in Chapter 3, two new discrete models and their careful derivations are presented, respectively. In comparison with the conventional discrete model, they are of familiar format of standard discrete systems. Also, inside the models, there are clear indications on how the tissue property parameters interact with the input impulse signals exactly. Hence, provided the proposed discrete models, the tissue property parameters can be separated with ease from the echo signals, which are exactly what are of ultrasound imaging interest.

In summary, this thesis is a contribution to the development of novel ultrasound echo models for medical ultrasound imaging systems. Several new models and their extensive studies for medical ultrasound imaging are provided. Our work establishes a new foundation for ultrasound deconvolution techniques in modern signal processing.

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Notations and Abbreviations

The mathematical variables or symbols used within limited contexts are not listed here. They are described within their context. Symbols listed may stand for other entities or variables in different applications. Only the common associations in this thesis are listed for ready reference.

List of Symbols

Symbol	Description
$d(m, n)$	2-D tissue noise
$d(n)$	1-D tissue noise
F	FT operator
F^{-1}	IFT operator
$f(m, n)$	2-D tissue
$f(n)$	1-D tissue
$h_{pe}(\vec{r}_1, \vec{r}_5, t)$	modified pulse-echo spatial impulse response
$\vec{r}_1$	position of scatterer
$\vec{r}_5$	position of ultrasound transducer
$f(\vec{r}_1)$	inhomogeneities of the tissue
t	continuous time variable

m	horizontal discrete sample coordinate
n	vertical discrete sample coordinate
$U(n)$	Unit step function
$h(m, n)$	2-D PSF
$h(n)$	1-D PSF
$v_{pe}(t)$	ultrasonic pulse-echo wavelet
$y(m, n)$	2-D ultrasonic RF observed signal
$y(n)$	1-D ultrasonic RF observed signal
Z	ZT operator
Z^{-1}	IZT operator
$\delta(n)$	Kronecker delta (discrete Dirac impulse in n)
$*$	2-D linear convolution operator
$*$ r	spatial convolution operator
$*$ t	temporal convolution operator
∞	infinity

List of Abbreviations

1-D	one-dimensional
2-D	two-dimensional
3-D	three-dimensional
B-mode	Brightness-mode
DFT	Discrete Fourier Transform
FT	Fourier Transform
HOS	Higher-Order Statistics
IDFT	Inverse Discrete Fourier Transform
IFT	Inverse Fourier Transform
IZT	Inverse Z-Transform

LS	Least Square
MSE	Mean Square Error
NMSE	Normalized Mean Square Error
RMSE	Root Mean Square Error
pdf	probability density function
PSF	Point-Spread Function
RF	Radio Frequency
ROI	Region Of Interest
SNR	Signal-to-Noise Ratio
dB	deciBel
WSS	White Sense Stationary
ZT	Z-Transform

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Chapter 1

Introduction

Ultrasound has been an increasingly important imaging modality in medical diagnosis since late 1930s. It facilitates the non-invasive observation of human soft tissues as the 2nd widely-used tool only next to X-ray. Ultrasound is applied as a machine of detecting the body with no adverse effect being reported. Meanwhile, its real-time capability makes it suitable in monitoring the anatomical changes during surgery, chemotherapy, rehabilitation therapy and so on. Furthermore, unlike the bulky and stationary Computer Tomography (CT) and Magnetic Resonance Imaging (MRI), ultrasound is mobile for medical operators to move easily and conveniently. What's more, the cost of purchase and operating expenses of ultrasound are incomparably lower than CT and MRI.

For the reasons above, Obstetric ultrasound is a progressively indispensable modality in obstetrics and a large number of diseases since its introduction in the late 1950s. Ultrasound probes are placed in contact with maternal abdomen and particular content of the uterus are scanned, as shown in Figure 1.1. Movements such as fetal heart beat and malformations in the fetus can be assessed and accurate measurements can be made directly on the images displayed on the screen, such as gestational age, size and growth in the fetus. These measurements enable the detec-

tion of abnormalities in the fetus development and complications due to maternal diseases. Figure 1.2 shows an ultrasonic image of a 35-week fetal profile.

Figure 1.1: Ultrasound Obstetric Scanning. Image Courtesy of GE Healthcare

Medical ultrasound is developed by leaps and bounds. The first two-dimensional ultrasound scanner was named *Somascopes* shown in Figure 1.3 as appearing in the Medicine section of the Life Magazine® in 1954. This water-immersed B-mode scanner displayed the scanned kidney of the subject. In contrast, recently GE® Healthcare released a pocket-sized ultrasound scanner in Figure 1.4 and enable a more rapid diagnosis. And physicians are able to carry ultrasonic scanners along with their other paraphernalia such as stethoscopes and ophthalmoscopes.

Meanwhile, new generation of medical ultrasound house the state-of-art technologies of 3-D/4-D ultrasound imaging, as shown in Figure 1.5. Not only are 3-D images very impressive to parents, but also demonstrate the defects such as cleft lips/palate, spina bifida and polydactyl more lucidly and lead to more effective diagnosis of abnormalities. In the future, how to make full use of real-time 3-D imaging capability

Figure 1.2: An Ultrasonic Image Of A 35-Week Fetal Profile. Image Courtesy of GE Healthcare.

of ultrasound and present images or videos with higher resolution would become one of the focuses of the ultrasound research work.

Figure 1.3: Somascope: Original 2-D Ultrasound Scanner. Image Courtesy of <http://www.ob-ultrasound.net/history.html>

Figure 1.4: GE Vscan Ultrasound Scanner (Image Courtesy Of GE Healthcare).

Figure 1.5: A 3-D Ultrasound Image (Image Courtesy Of GE Healthcare).

This thesis is a part of the ongoing research interests on biomedical signal processing at the Biomedical Electronics Laboratory, in the Division of Control & Instrumentation of Nanyang Technological University, Singapore. The research activity has concentrated on ultrasound signal modeling for imaging purpose. The central to this approach is based on classical ultrasound acoustics and linear systems theory where the ultrasound echo signals and corresponding images are interpreted as a spatio-temporal convolution between the ultrasound pulse-echo wavelet or ultrasonic PSF and the biological tissue. We aim at proposing more accurate ultrasound models of echo signals and construct new theoretic basis to deconvolution techniques of ultrasound imaging systems.

1.1 Motivations

The non-invasive nature, low cost, portability, and real-time image formation make ultrasound imaging a valuable tool for medical diagnosis. It is now the second most widely used diagnostic imaging modality (next to X-ray). Ultrasound imaging has found numerous applications in today's clinics [1]– [3]. Some of these applications are the diagnosis of valvular insufficiency (flail leaflet), myocardial dyskinesis, septal defects and congenital malformations in cardiology. It has becomes widely known and commonly used in obstetrics for fetal monitoring and congenital malformations examination. Ultrasound is also generally far better than plain X-ray at distinguishing the subtle variations of soft, fluid-filled tissues structures, and can be used in any of several modes, depending on the need at hand. This makes it particularly useful in providing diagnostic images of the abdomen where identification of tumors and cysts in both liver and kidneys are performed.

Many of these diagnoses are based on relatively gross anatomical information that is available from the ultrasound image. This information is of critical value to the

clinician for diagnoses such as diffuse disease processes, the identification of tumors, quality of myocardial dynamics and so forth. Due to these reasons, the signal integrity and retention of maximum dynamic range is of key value in the image formation process.

The full potential of ultrasound imaging in diagnostic and monitoring capabilities, as well as its clinical and biological applications has not yet been realized. It still suffers from a number of limitations. The ultrasound images exhibit speckle artifacts which mask the presence of low cost lesions and hence reduce the ability of human observer to discriminate fine details in diagnostic examination. Speckle is caused by the constructive and destructive interference of backscattered echoes originating from unresolved tissue inhomogeneities which is responsible for a characteristic granularity, or texture, of the image which bears little resemblance to the actual acoustical tissue micro-structures.

An ultrasound image is a representation of the reality. It can do no more than convey information to the observer concerning structure and function as represented by the ultrasonic characteristics of the tissue. Knowing that image formation actually occurs in the RF domain, recent techniques modeled the ultrasound image formation process in the RF domain as a spatio-temporal convolution between the ultrasonic 2-D PSF and tissue response [4], [5]. Corresponding restoration techniques with deconvolution-related filtering are developed [6]–[23]. This paradigm opens a new era spring from the remarkable advances in modern signal processing techniques for the restoration of the imaged tissue structure.

Hence, given the numerous subsequent convolution-based technologies on ultrasound imaging, the accuracy of the convolution model [4], [5] are of critical importance to the development of these techniques. After careful studies of the convolution model of ultrasound echo signals, it is found that the model is subjective to some flawed approximations. Meanwhile, the popular discrete version corresponding to

the continuous convolution model is still very preliminary and may not stand for strict derivations. What's more, as the growth of modern computer-based digital signal processing technologies, an accurate discrete model of ultrasound echo signals has become more and more necessary. Current discrete model is very simple and conceptual. Hence, in this thesis, we focus on the further research work of modeling ultrasound echo signals.

1.2 Objectives

The overall goal of this study is to develop new and accurate models for ultrasound imaging systems in medical diagnosis.

In our studies, we are partly using simulated ultrasound data. This has the advantage that the test data are eliminated from effects highly susceptible by a variety of other ultrasound research topics such as phase aberrations and attenuation. This approach of investigation had also been widely adopted. Examples of which are [10], [15], [18], [9]– [32].

Existing conventional convolution model [9]- [5] provided comprehensive coverage of ultrasound modeling from theoretical ultrasound acoustics to the systematic analysis from the signal and system point of view. It also analyzed many transducers with different shapes of apertures and calculates their impulse responses. The complicated establishment of conventional convolution model is quite instructive which can perform as the smoothing transition from physics to electrical and electronic engineering in ultrasound field. With this model, researchers in ultrasound engineering fields have a new insight on how to perform signal analysis from deconvolution point of view and with solid acoustical foundation. Hence, this model is well recognized and widely applied since 1990s.

However, this conventional model incorporated a lot of physical assumptions and mathematical approximations during its derivations. After careful studies, we consider that some critical approximations are problematic which conventional convolution model cannot work without. Meanwhile existing blind deconvolution methods for ultrasound echo signals described and referred to in [6]–[23] would have no theoretical foundations. Thus, we want to develop new convolution models under improved approximations and assumptions with higher accuracy.

Conventional convolution model of ultrasound echo signals was proposed in [4], [5] and widely applied for decades. A seemingly-simple 2-D discrete convolution model is frequently mentioned in a lot of works on ultrasound deconvolution filtering techniques in [6]–[23], [79]–[80]. However, the simple discrete convolution model is only with a conceptual convolution format which has not undergone any strict derivation from the original continuous model in [4], [5]. We are interested in deriving a discrete convolution model of ultrasound echo signals with strict reasoning and providing a specific and accurate discrete model for engineering analysis.

1.3 Major Contributions of The Thesis

The first contribution of our work is the development of a new convolution model with reasonable and feasible conditions. There is a flawed approximation in the conventional convolution model which plays the key role in the original derivations. After careful study, we find that there exist some substantial mathematical problems in the original derivations of convolution model such that the original convolution model is not fully theoretically sound and valid. Consequently, the deconvolution-related techniques for processing ultrasound echo signals no longer have theoretical foundation. In this thesis, a new convolution model of medical ultrasound echo process is proposed and its derivation and formulation are provided. Based on the

investigation of classical acoustical equations, the new convolution model presents dominant terms under common practically feasible conditions of medical ultrasound. It provides a solid theoretical foundation for the prevailing deconvolution techniques in ultrasound signal processes and new insight in exploring interactions between ultrasound pulses and body tissues in ultrasound scanning and imaging.

The second contribution of our work is the proposal of discrete models of ultrasound echo processes. The conventional discrete model does not provide a convincing discretization from conventional continuous convolution model. The reason behind is that time dimension is ignored deliberately during the discretization without convincing proof. Meanwhile, tissue information is only conceptual 2-D sequences of signals without specific indications on how to connect the so-called tissue signals with real soft tissue. The simple discrete model is quite general and symbolic such that it is not mathematically straightforward and accurate enough. At the mean time, we have pointed out the problematic approximation in the derivation of conventional convolution model and proposed new convolution model with reasonable conditions. Hence, starting from the original theoretical model, a new discrete model and its careful derivation are presented.

1.4 Organization of the Thesis

The thesis is divided into five chapters in which we address the multiple connections between the essential principles of ultrasound imaging, the mathematical formulation of the ultrasonic RF convolution model, and the proposed new models. It is organized as follows.

In Chapter 2, we first review the background knowledge of ultrasound physics. Then mathematical formulations of the research problem are presented in details. Meanwhile, the basics of ultrasound discrete modeling are introduced. In addition,

Chapter 3 addresses problems in widely-applied conventional convolution model of ultrasound echo processes. A new convolution model of medical ultrasound echo processes is proposed and its derivation and formulation are provided. Based on the investigation of classical acoustical equations, the new convolution model presents dominant terms under practically feasible conditions of medical ultrasound. It thus provides a theoretical foundation for the prevailing deconvolution techniques in ultrasound signal processing and new insight in exploring interactions between ultrasound pulses and body tissues in ultrasound scanning and imaging processes.

Chapter 4 concerns the establishment of discrete models which originate from the theoretical model and new convolution model, comes with the discrete format and stands for strict mathematical reasonings. With the discrete models proposed, the ultrasound echo signals are regarded as the product of interactions between ultrasound pulse spread function (PSF) and tissue of interest. At the mean time, specific model parameters are given qualitatively in the linear model.

Finally, Chapter 5 is devoted specially to conclusion and our recommendations for further research.

Chapter 2

Background

2.1 Physics of Ultrasound Imaging Techniques

Ultrasound refers to the sound waves with frequencies greater than 20 KHz which are higher than upper limit of the human hearing range. Its sound wave is generated by vibrating piezoelectric crystals by applying an alternating current across the crystal, and the same crystals can perform as receivers of reflected sound waves. Given the high frequency, ultrasonic waves have a much shorter wavelength which allows them to respond to small changes in medium composition. The reflected echoes are collected by an electronic apparatus that measures their intensity and the position of the medium reflecting them. Hence, final electrical signals are capable to reveal details of the inner structure of the medium.

Ultrasound images are generated by ultrasound scanner and widely-used in medical diagnosis. In general, ultrasound scanner is non-invasive, of low cost, portable and capable to provide real-time image formation. Pulse-echo techniques are used to form images by sending a short ultrasonic pulse into the human body and detecting the echoes whenever sound waves encounter a material with a different density

(acoustical impedance) and part of the sound wave is reflected back to the probe. The greater the difference between acoustic impedances, the larger the echo is. The differences of density or acoustical impedance usually arise from boundaries between tissues or organs, lesions in the body, and from other small structures within tissue.

Typical diagnostic ultrasound scanners operate in the frequency ranging from 2 to 18 megahertz which is a trade-off between spatial resolution of the image and imaging depth. The higher frequencies, the shorter wavelengths, and thus the finer details can be visualized. However, ultrasonic waves are attenuated more as frequencies increase. And hence more energy is absorbed by the soft tissue, and more information reflected from deeper inside the body are lost. Given the compromise between image resolution and ultrasound penetration, ultrasound probes with different operational frequencies are applied for different body structures.

2.1.1 Interaction of Ultrasound With Tissues

Ultrasound waves interact with tissues are described as attenuation, reflection, scattering, refraction and diffraction.

Attenuation As shown in Fig.2.1, attenuation is the loss of ultrasound energy as it propagates through a medium. Parts of the energy are converted to heat, reflected, scattered, absorbed, refracted or diffracted. attenuation is increased with higher frequency of transducers, increased distance from the transducer and more heterogeneous medium to pass through. Usually, the values of attenuation is approximated as 1 dB/cm/MHz.

Reflection As shown in Fig.2.2, reflections can be strictly divided into 2 patterns - specular reflection and scattering. Specular reflection of Ultrasound waves is the process which occurs when the sound waves meet a distinct surface significantly large-

Figure 2.1: Effect of Ultrasound Attenuation.

er than the wavelength of the ultrasound. The amount of reflection at the boundary between different tissues depends on the difference of their acoustic properties. As a measure of the property, acoustic impedance varies with the mass density and compressibility of the tissue, as well as the incident angle.

Figure 2.2: Effect of Ultrasound Specular Reflection.

Refraction When an ultrasound waves encounters the boundaries and interfaces between different media at an oblique angle, some portion of energy is specular reflected and the rest is refracted and continues into the next medium. As shown

in Fig.2.3, refraction occurs when the velocities of ultrasound waves are different on each side of the tissue interface. In medical imaging, considerable refractions are caused by fat and bone which results in the image distortion.

Figure 2.3: Effect of Ultrasound Refraction.

Scattering As shown in Fig.2.4, scattering occurs when the interface is rough or the wavelength of the ultrasound wave is larger than the dimensions of the reflective structure. The reflected waves scatter in many different directions and generate weak echoes. Ultrasound scattering gives rise to much of the diagnostic information observed in medical ultrasound imaging.

Diffraction As shown in Fig.2.5, diffraction occurs when an ultrasound wave traverses an small obstacle within a distance of several wavelengths, its direction of propagation is deflected by diffraction. The diffraction becomes greater when ultrasound beam is closer to the diffracting object.

Figure 2.4: Effect of Ultrasound Scattering.

Figure 2.5: Effect of Ultrasound Diffraction.

2.1.2 General Signal Processing in Medical Ultrasound

In general, the formation of medical ultrasound images can be divided into the several steps in Figure 2.6 while each step involves many related techniques.

Figure 2.6: Ultrasound Image Formation.

It begins with designing the excitation pulses to apply them to all the elements within probe and generate a focused, steered and apodized pulsed wave. After the interaction between tissue and transmitted pulsed waves, the echoes are received by all the same elements and undergo further processing like apodization, dynamic focusing or steering delays. In beamforming, the echo signals received by

different elements are combined to form a single signal. Single line RF processing involves a wide variety of techniques like frequency demodulation, time gain compensation, signal filtering, speckle removal, envelope detection and so on. Also in this thesis, the echo signals after beamforming and before the multi-line processing are considered. Several techniques are applied in multi-line RF processing like lateral gain compensation, interpolation between RF lines and others. Scan conversion is the stage at which interpolation between envelope-detected signal lines is usually applied and also angle compounding can be used to enhance the speckle SNR. Substantial smoothing in both axial and lateral direction are made use of in post-processing while different information are combined for simultaneous display. Meanwhile, speckle reduction and boundary detection are conducted in this stage. Finally, grey-scale or color-Doppler ultrasound images are generated for display.

2.2 Modeling of Ultrasound Echo Signals

As described in Chapter 1, the advantages of medical ultrasound have made it generally recognized as an important alternative or supplement of MRI, CT and X-ray in various clinical diagnosis. And it is also the first choice of imaging modality to examine the soft tissue structure in human body safely and conveniently. Unfortunately, ultrasound images are well-known for its low quality and lots of artifacts as compared with other imaging modalities. Generally speaking, the image degradation is mainly caused by the finite bandwidth of ultrasound transducers and the beam width of ultrasound pulses.

Subsection 2.2.1 to Subsection 2.2.4 present the mathematical formulation of the physical ultrasound RF images as a convolution expression. This expression laid the groundwork for ultrasound signal modeling and using deconvolution methods as a means to improve the quality of the ultrasound images.

2.2.1 Wave Equations of Ultrasound Propagation Through A Medium

The wave equations to describe how the ultrasound waves propagates through a medium are presented in this subsection. And the formulations adopt those in [9], [36] where the scattering term for the wave equation is a function of the density and propagation velocity perturbations.

In order to facilitate the derivation, a few assumptions and approximations are made first.

Assumption 1 The instantaneous acoustic pressure and density can be written as:

$$P_{ins}(\vec{r}, t) = P + p_1(\vec{r}, t); \quad \rho_{ins}(\vec{r}, t) = \rho(\vec{r}) + \rho_1(\vec{r}, t), \quad (2.1)$$

where P is the mean pressure of the medium, ρ is the density of the undisturbed medium at continuous time t . Both the pressure variation $p_1(\vec{r}, t)$ and density change $\rho_1(\vec{r}, t)$ are small quantities of first order compared to P and ρ respectively.

Assumption 2 There is no heat conduction or conversion of ultrasound to thermal energy taking place. Thus, the entropy is constant for the process such that the pressure and density satisfy the adiabatic equation:

$$P_{ins}(\vec{r}, t) = c^2 \frac{\partial \rho_{ins}(\vec{r}, t)}{\partial t}, \quad (2.2)$$

where c is the adiabatic ultrasound velocity at the given point of the medium.

Beside the two assumptions above, the pressure, density and particle velocity $\vec{v}$ must

also satisfy the *hydrodynamical equations*:

$$\rho_{ins} \frac{\partial \vec{v}}{\partial t} = -\nabla P_{ins} , \quad (2.3)$$

$$\frac{\partial \rho_{ins}}{\partial t} = -\nabla \cdot (\rho_{ins} \vec{v}) , \quad (2.4)$$

which are the *linear inviscid force equation* and the *equation of continuity*. Equation (2.2) contains total rather than partial time derivatives since the relation must be satisfied for a given particle of the medium and for a given point of space. Using (2.1) to substitute for the pressure and the density in (2.2) and using the assumption $\rho_1(\vec{r}, t) \ll \rho(\vec{r})$, we obtain:

$$\frac{1}{c^2} \frac{\partial p_1(\vec{r}, t)}{\partial t} = \frac{\partial \rho_1(\vec{r}, t)}{\partial t} + \vec{v}(\vec{r}, t) \cdot \nabla \rho(\vec{r}) . \quad (2.5)$$

In the case of a homogeneous medium, the convective term $\vec{v}(\vec{r}, t) \cdot \nabla \rho(\vec{r})$ in the last equation vanishes, and (2.2) can be written for $p_1(\vec{r}, t)$ and $\rho_1(\vec{r}, t)$ using either the total or partial time derivatives as in (2.2) and (2.5). Likewise, using the same assumption and upon substituting (2.1) into (2.3) and (2.4), we obtain:

$$\rho(\vec{r}) \frac{\partial \vec{v}(\vec{r}, t)}{\partial t} = -\nabla p_1(\vec{r}) , \quad (2.6)$$

$$\frac{\partial \rho_1(\vec{r}, t)}{\partial t} = -\nabla \cdot (\rho(\vec{r}) \vec{v}(\vec{r}, t)) . \quad (2.7)$$

Differentiating (2.7) with respect to t and inserting (2.6) gives:

$$\frac{\partial^2 \rho_1(\vec{r}, t)}{\partial t^2} = \nabla^2 p_1(\vec{r}, t) . \quad (2.8)$$

Differentiating (2.5) with respect to t yields:

$$\frac{1}{c^2} \frac{\partial^2 p_1(\vec{r}, t)}{\partial t^2} = \frac{\partial^2 \rho_1(\vec{r}, t)}{\partial t^2} + \frac{\partial \vec{v}(\vec{r}, t)}{\partial t} \cdot \nabla \rho(\vec{r}) . \quad (2.9)$$

Substituting (2.6) and (2.8) into (2.9) leads to:

$$\nabla^2 p_1(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2 p_1(\vec{r}, t)}{\partial t^2} = \frac{1}{\rho(\vec{r})} \nabla \rho(\vec{r}) \cdot \nabla p_1(\vec{r}, t). \quad (2.10)$$

We now assuming that the propagation velocity and density only vary slightly from their mean values, so that:

$$\rho(\vec{r}) = \rho_0 + \Delta\rho(\vec{r}); \quad c(\vec{r}) = c_0 + \Delta c(\vec{r}), \quad (2.11)$$

where $\rho_0 \gg \Delta\rho(\vec{r})$ and $c_0 \gg \Delta c(\vec{r})$. Then, using the approximation $\frac{1}{1+x} \approx 1 - x$ and $\rho_0 \gg \Delta\rho(\vec{r})$, (2.11) becomes:

$$\nabla^2 p_1(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2 p_1(\vec{r}, t)}{\partial t^2} = -\frac{2\Delta c(\vec{r})}{c_0^3} \frac{\partial^2 p_1(\vec{r}, t)}{\partial t^2} + \frac{1}{\rho_0} \nabla \left(\Delta\rho(\vec{r}) \cdot \nabla p_1(\vec{r}, t) \right). \quad (2.12)$$

This is the homogeneous wave equation in inhomogeneous media. The last two terms on the right side of the equation are the scattering terms that vanish for a homogeneous medium.

2.2.2 Calculation of the Scattered Field

After deriving the wave equation, the scattered field from a small inhomogeneity identified by $\vec{r}_1$ embedded in a homogeneous surrounding is calculated within the volume V . The coordinate is shown in Figure 2.7.

The scattered field is calculated at the point indicated by $\vec{r}_1$ by integrating all the spherical waves emanating from the scattering region V using the time dependent Green's function for unbounded space. The scattered field is found to be [9], [37],

Figure 2.7: Coordinate System For Calculating The Scattered Field.

[38]:

$$p_s(\vec{r}_2, t) = \int_V \int_T \left[\frac{1}{\rho_0} \nabla(\Delta \rho(\vec{r}_1)) \cdot \nabla p_1(\vec{r}_1, t_1) - \frac{2\Delta c(\vec{r}_1)}{c_0^3} \frac{\partial^2 p_1(\vec{r}_1, t_1)}{\partial t^2} \right] \times G(\vec{r}_1, t_1 | \vec{r}_2, t) dt_1 d^3 \vec{r}_1, \quad (2.13)$$

where the free space Green's function is:

$$G(\vec{r}_1, t_1 | \vec{r}_2, t) = \frac{\delta(t - t_1 - \frac{|\vec{r}_{tr} - \vec{r}_1|}{c_0})}{4\pi |\vec{r}_{tr} - \vec{r}_1|}, \quad (2.14)$$

and $|\cdot|$ denotes the magnitude. The pressure field inside the scattering region is given as:

$$p_1(\vec{r}_2, t) = p_i(\vec{r}_2, t) + p_s(\vec{r}_2, t), \quad (2.15)$$

where $p_i(\vec{r}_2, t)$ is the incident pressure field. The integral in (2.13) cannot be solved

directly. Fortunately, it is solved by applying *Born-Neumann expansion*:

$$p_s(\vec{r}_2, t) = p_{s1}(\vec{r}_2, t) + p_{s2}(\vec{r}_2, t) + p_{s3}(\vec{r}_2, t) + \dots, \quad (2.16)$$

where the scattering fields are given by:

$$p_{sn}(\vec{r}_2, t) = \begin{cases} \int_V \int_T \left[\frac{1}{\rho_0} \nabla(\Delta \rho(\vec{r}_1)) \cdot \nabla p_i(\vec{r}_1, t_1) \right. \\ \quad \left. - \frac{2\Delta c(\vec{r}_1)}{c_0^3} \frac{\partial^2 p_i(\vec{r}_1, t_1)}{\partial t^2} \right] G(\vec{r}_1, t_1 | \vec{r}_2, t) dt_1 d^3 \vec{r}_1 & , \text{if } n = 1 \\ \int_V \int_T \left[\frac{1}{\rho_0} \nabla(\Delta \rho(\vec{r}_1)) \cdot \nabla p_{s(n-1)}(\vec{r}_1, t_1) \right. \\ \quad \left. - \frac{2\Delta c(\vec{r}_1)}{c_0^3} \frac{\partial^2 p_{s(n-1)}(\vec{r}_1, t_1)}{\partial t^2} \right] G(\vec{r}_1, t_1 | \vec{r}_2, t) dt_1 d^3 \vec{r}_1 & , \text{if } n > 1 \end{cases} \quad (2.17)$$

For weak scattering throughout V , i.e., $p_{s1}(\vec{r}_2, t) \ll p_i(\vec{r}_2, t)$, the first-order Born approximation will give a satisfactory representation of the behavior of the scattered ultrasound. The scattered field is found to be [9], [37], [38]:

$$\begin{aligned} p_s(\vec{r}_2, t) &\approx p_{s1}(\vec{r}_2, t) \\ &\approx \int_V \int_T \left[\frac{1}{\rho_0} \nabla(\Delta \rho(\vec{r}_1)) \cdot \nabla p_i(\vec{r}_1, t_1) - \frac{2\Delta c(\vec{r}_1)}{c_0^3} \frac{\partial^2 p_i(\vec{r}_1, t_1)}{\partial t^2} \right] \\ &\quad \times G(\vec{r}_1, t_1 | \vec{r}_2, t) dt_1 d^3 \vec{r}_1. \end{aligned} \quad (2.18)$$

Now, we have an expression of the scattered field expressed in terms of the incident field. So, the incident field has to be calculated in order to calculate the scattered field.

2.2.3 Calculation of the Incident Field

The incident field is generated by the transducer, assuming no other sources exist in the tissue. The method presented in this section adopts the convolution method developed in [33]–[35]. The incident field is found by solving the wave equation for

the homogeneous case:

$$\nabla^2 p_i(\vec{r}, t) - \frac{1}{c_0^2} \frac{\partial^2 p_i(\vec{r}, t)}{\partial t^2} = 0. \quad (2.19)$$

The field is conveniently calculated by employing the velocity potential $\Psi(\vec{r}, t)$, and enforcing appropriate boundary conditions. From *Eular's equation* of particle velocity, $\rho(\vec{r}) \frac{\partial \vec{v}(\vec{r}, t)}{\partial t} = -\nabla p_i(\vec{r})$, and since $\nabla \times \nabla p_i(\vec{r}) = 0$ which implies $\nabla \times \vec{v}(\vec{r}, t) = 0$. Hence, the particle velocity has a scalar potential function:

$$\vec{v}(\vec{r}, t) = \nabla \Psi(\vec{r}, t). \quad (2.20)$$

Substituting (2.20) into the Euler's equation, the acoustic pressure wave can be expressed as:

$$p_i(\vec{r}, t) = \rho_0 \frac{\partial \Psi(\vec{r}, t)}{\partial t}. \quad (2.21)$$

Using the coordinate system shown in Figure 2.8 in the calculation, the velocity potential satisfies the wave equation for the homogeneous medium:

$$\Psi(\vec{r}_1, \vec{r}_3, t) = \int_S \int_T v(\vec{r}_3 + \vec{r}_4, t_3) g(\vec{r}_1, t | \vec{r}_3 + \vec{r}_4, t_3) dt_3 d^2 \vec{r}_4, \quad (2.22)$$

where the transducer is mounted in a rigid infinite planar baffle, $v(\vec{r}_3 + \vec{r}_4, t_3)$ denotes the particle velocity normal to the transducer surface. Here, S denotes the transducer surface and the Green's function for a bounded medium is:

$$g(\vec{r}_1, t | \vec{r}_3 + \vec{r}_4, t_3) = \frac{\delta(t - t_3 - \frac{|\vec{r}_1 - \vec{r}_3 + \vec{r}_4|}{c_0})}{2\pi |\vec{r}_1 - (\vec{r}_3 + \vec{r}_4)|}. \quad (2.23)$$

Assume that the surface vibration can be split into a spatial component and a temporal component:

$$\Psi(\vec{r}_1, \vec{r}_3, t) = \int_S \int_T v(t_3) a(\vec{r}_3 + \vec{r}_4) g(\vec{r}_1, t | \vec{r}_3 + \vec{r}_4, t_3) dt_3 d^2 \vec{r}_4. \quad (2.24)$$

Figure 2.8: Coordinate System For Calculating The Incident Field.

Now, if we assume that the particle velocity is uniform over the surface of the transducer, (2.24) is reduced to:

$$\begin{aligned}\Psi(\vec{r}_1, \vec{r}_3, t) &= \int_T v(t_3) \int_S a(\vec{r}_3 + \vec{r}_4) g(\vec{r}_1, t | \vec{r}_3 + \vec{r}_4, t_3) d^2 \vec{r}_4 dt_3 \\ &= v(t) *_{\vec{t}} h(\vec{r}_1, \vec{r}_3, t),\end{aligned}\quad (2.25)$$

where $*_{\vec{t}}$ denotes temporal convolution. Therefore, upon substituting (2.25) into (2.21), we arrived at:

$$p_i(\vec{r}_1, \vec{r}_3, t) = \rho_0 \frac{\partial v(t)}{\partial t} *_{\vec{t}} h(\vec{r}_1, \vec{r}_3, t). \quad (2.26)$$

Noted, the separation between the excitation and the transducer geometry and that $h(\vec{r}_1, \vec{r}_3, t)$ is spatially varying depends on the difference between $\vec{r}_1$ and $\vec{r}_3$.

2.2.4 Calculation of the Received Signal

The coordinate system for the calculation of the received signal is shown in Figure 2.9. The received signal is the scattered pressure field of the inhomogeneity integrated over the transducer surface, convolved with the electromechanical impulse response

Figure 2.9: Coordinate System For Calculating The Received Signal.

$E_m(t)$ of the transducer:

$$p_r(\vec{r}_5, t) = E_m(t) * \int_S p_s(\vec{r}_5 + \vec{r}_6, t) d^2 \vec{r}_6. \quad (2.27)$$

Substituting the scattered field derived in (2.18) and that of the incident field in (2.26) into (2.27) gives:

$$\begin{aligned} p_r(\vec{r}_5, t) &= E_m(t) * \frac{1}{2} \int_S \int_V \int_T \left[\frac{1}{\rho_0} \nabla (\Delta \rho(\vec{r}_1) \cdot \nabla p_i(\vec{r}_1, t)) \right. \\ &\quad \left. - \frac{2\Delta c(\vec{r}_1)}{c_0^3} \frac{\partial^2 p_i(\vec{r}_1, t)}{\partial t^2} \right] \cdot 2 \cdot G(\vec{r}_1, t_1 | \vec{r}_5 + \vec{r}_6, t) dt_1 d^3 \vec{r}_1 d^2 \vec{r}_6 \\ &= E_m(t) * \frac{1}{2} \int_V \left[\frac{1}{\rho_0} \nabla (\Delta \rho(\vec{r}_1) \cdot \nabla [\rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1, \vec{r}_3, t)]) \right. \\ &\quad \left. - \frac{2\Delta c(\vec{r}_1)}{c_0^3} \frac{\partial^2 [\rho_0 \frac{\partial v(t)}{\partial t} * h(\vec{r}_1, \vec{r}_3, t)]}{\partial t^2} \right] * h(\vec{r}_5, \vec{r}_1, t) d^3 \vec{r}_1. \end{aligned} \quad (2.28)$$

If the position of the transmitting and the receiving transducer are the same, i.e., $\vec{r}_3 = \vec{r}_5$, we have:

$$\begin{aligned} p_r(\vec{r}_5, t) &= \frac{\rho_0}{2} E_m(t) * \frac{\partial v(t)}{\partial t} * \int_V \left[\frac{1}{\rho_0} \nabla (\Delta \rho(\vec{r}_1) \cdot \nabla H_{pe}(\vec{r}_1, \vec{r}_5, t)) \right. \\ &\quad \left. - \frac{2\Delta c(\vec{r}_1)}{c_0^3} \frac{\partial^2 H_{pe}(\vec{r}_1, \vec{r}_5, t)}{\partial t^2} \right] d^3 \vec{r}_1. \end{aligned} \quad (2.29)$$

Here, the pulse-echo spatial impulse response is:

$$H_{pe}(\vec{r}_1, \vec{r}_5, t) = h(\vec{r}_1, \vec{r}_5, t) *_{\vec{r}_1} h(\vec{r}_5, \vec{r}_1, t). \quad (2.30)$$

Assume that $H_{pe}(\vec{r}_1, \vec{r}_5, t)$ is slowly varying so that the spatial frequency content is constant over a finite volume, we have:

$$\nabla(\Delta\rho(\vec{r}_1) \cdot \nabla H_{pe}(\vec{r}_1, \vec{r}_5, t)) = \Delta\rho(\vec{r}_1) \nabla^2 H_{pe}(\vec{r}_1, \vec{r}_5, t). \quad (2.31)$$

Since the time taken for the propagation of the pressure field between the transducer and the scatterer is given by:

$$t = \frac{|\vec{r}_1 - \vec{r}_5|}{c_0}, \quad (2.32)$$

and assuming only small deviations from the mean propagation velocity, the Laplace operator, through the use of chain rule, corresponds to the second derivative with respect to time. Thus,

$$\nabla^2 H_{pe}(\vec{r}_1, \vec{r}_5, t) = \frac{1}{c_0^2} \frac{\partial^2 H_{pe}(\vec{r}_1, \vec{r}_5, t)}{\partial t^2}. \quad (2.33)$$

Using the approximations in (2.31) and (2.33), (2.29) can be rewritten as:

$$p_r(\vec{r}_5, t) = \frac{\rho_0}{2} E_m(t) *_{\vec{r}_5} \frac{\partial v(t)}{\partial t} *_{\vec{r}_5} \int_V \left[\frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c_0} \right] \frac{1}{c_0^2} \frac{\partial^2 H_{pe}(\vec{r}_1, \vec{r}_5, t)}{\partial t^2} d^3 \vec{r}_1. \quad (2.34)$$

Symbolically, this is written as:

$$p_r(\vec{r}_5, t) = v_{pe}(t) *_{\vec{r}_5} s(\vec{r}_1) *_{\vec{r}_1} h_{pe}(\vec{r}_1, \vec{r}_5, t). \quad (2.35)$$

Here, $*_{\vec{r}_1}$ denotes spatial convolution, $v_{pe}(t)$ is the pulse-echo wavelet that accounts for the transducer excitation and the electromechanical impulse response during emission and reception of the pulse, and $h_{pe}(\vec{r}_1, \vec{r}_5, t)$ is the modified pulse-echo

spatial impulse response that relates the transducer geometry to the spatial extent of the scattered field. The term $s(\vec{r}_1) = \frac{\Delta\rho(\vec{r}_1)}{\rho_0} - \frac{2\Delta c(\vec{r}_1)}{c_0}$ accounts for the inhomogeneities in the tissue due to density and propagation velocity perturbations that give rise to the scattered signal.

2.3 Ultrasound Discrete Modeling of Echo Signals

Conventional convolution model of ultrasound echo signals was proposed in [5] [9] and widely applied for decades. As the growth of modern computer-based digital signal processing technologies, an accurate discrete model of ultrasound echo signals has become more and more necessary. A seemingly-simple 2-D discrete convolution model is frequently referred to in a lot of works on ultrasound deconvolution filtering techniques. In [19, 23, 75–81], the continuous conventional convolution model is reduced to a simple 2-D discrete convolution model

$$g(m, n) = f(m, n) * h(m, n) + v(m, n), \quad (2.36)$$

where m, n represent 2-D independent discrete spatial locations in both axial and lateral direction. $g(m, n)$ refers to ultrasound echo signals. $f(m, n)$ is the assumed tissue information in two dimensions. $h(m, n)$ is the point spread function (PSF) concerning the characteristics of ultrasound excitation systems. $v(m, n)$ is the additive noise introduced. $*$ means the convolution operation. The usual practice of processing procedure for the discrete model (2.36) is to estimate the PSF first, deconvolute the echo signals and finally generate the tissue information after deconvolution filtering.

From our point of view, this simple 2-D discrete model in (2.36) does not provide a

convincing discretization from conventional convolution model. That is because time dimension is ignored deliberately during the discretization without strict derivations. The conventional convolution model [5] [9] has three independent variables-2D spatial locations and time i.e. x, y, t . Meanwhile, more importantly, the mapping between specific tissue properties and echo signals at an arbitrary spatial location has not been established yet. In the conventional convolution model [5] [9], the tissue property information refers to the combined function of tissue mass density and ultrasound propagation velocity. However, in the discrete model (2.36), tissue information is only conceptual 2-D sequences of signals without specific indications on how to connect the so-called tissue signals with tissue anatomy. Hence, although the format of the simple 2-D discrete model in (2.36) is quite appealing, still it is quite general and symbolic such that it is not mathematically straightforward and accurate enough.

This motivates us to develop a discrete model of ultrasound echo signals. Unlike the simple model in (2.36), our development are not based on the conventional convolution model [5] [9] but the theoretical model in [69]. The reason is that the model in [69] is also the basis of the conventional model. At the mean time, we have pointed out the problematic approximation in the derivation of conventional convolution model and proposed new convolution model with reasonable conditions in Chapter 3. Hence, the reliable way to perform discretization is to derive the discrete model directly from the original theoretical model.

2.4 Machine for Experiments: Ultrasonix[®] Sonix RP

In order to verify the proposed model, the ultrasound measurements collected by Ultrasonix[®] Sonix RP are applied to generate real tissue images instead of envelop

images of ultrasound echo signals. Ultrasonix® Sonix RP, shown in Fig 2.10, is a reliable ultrasound machine and specially designed for research work in ultrasound.

Figure 2.10: Ultrasound Machine: Ultrasonix® Sonix RP

Ultrasonix® is the world leading company as an ultrasound supplier for research applications. Based on the OpenSONIX platform, the Ultrasonix system provides an open-architecture system that can be configured via software to operate in the various modes required for research. Over 150 sites are using the Sonix RP and SonixTOUCH Research systems to develop and test new ultrasound technologies. Some of these new technologies such as Elastography Imaging have already proven to be of clinical significance and have been integrated on our Sonix systems. Working in conjunction with the research groups, Ultrasonix® is leading innovation in ultrasound by providing the most flexible research platform available today. For more details, please refer to their official web site at www.Ultrasonix.com.

Biomedical Electronics Lab in Nanyang Technological University (NTU) is the first

research lab to introduce this machine into Singapore and applies it in ultrasound signal processing. Other famous universities like Columbia University and University of Cambridge also use this machine for research applications.

2.5 The Blind Deconvolution Problem

We developed the concept of the ultrasound image degradation and restoration model and its discrete approximation in Section 2.2 and Section 2.3. In this section, we explain the corresponding techniques to restore the tissue image.

In ultrasound image acquisition, the captured image is the result of the biological tissue structures being convolved with the 2-D PSF. Given the convolution expression in (2.36), if one of the functions $f(m, n)$ or $h(m, n)$ is known, restoration methods such as Wiener filtering and iterative restoration can recover the other function. The problem of separation of the two signals becomes difficult if neither of the functions is known. The problem is now termed *blind image deconvolution*.

The general blind image deconvolution problem refers to the task of separating the two convolved signals, $f(m, n)$ and $h(m, n)$, when both the signals are either unknown or partially known. This problem has been the subject of a vast literature spanning the numerous special fields of science [41]–[54]. The existing literature differs fundamentally on the notion of a solution, which in turn leads to different mathematical approaches explored to solve the problem. This recovery process is critical to many image processing applications such as astronomical imaging [45]–[47], remote sensing [41], and medical ultrasound [55]. In addition to image restoration, there are many other important applications such as seismic data analysis, blind channels equalization in communication [54], transmission monitoring, and echo cancelation in wireless telephony.

Deconvolution is necessary in order to undo the effects of the blur. The ultrasound convolution model [9] provided the theoretical foundation for the corresponding blind image deconvolution problem, where researchers seek to deconvolute the observation $y(m, n)$ to obtain a close approximation to the unknown tissue structures $f(m, n)$ without precise knowledge of the degradation $v(m, n)$. With or without the noise, $f(m, n)$ admits an infinite number of solutions. Fortunately, there is some prior knowledge on the 2-D PSF. Its axial PSF is an oscillating wavelet about the transducer center frequency and its lateral PSF is approximately Gaussian function [9]. Hence, with the ultrasound convolution model, ultrasound image degradation can be improved by means of various deconvolution techniques.

Chapter 3

New Convolution Model of Ultrasound Echo Signals

This chapter addresses problems in widely-applied conventional convolution model of ultrasound echo processes. A new convolution model of medical ultrasound echo processes is proposed and its derivation and formulation are provided. Based on the investigation of classical acoustical equations, the new convolution model presents dominant terms under practically feasible conditions of medical ultrasound. It thus provides a theoretical foundation for the prevailing deconvolution techniques in ultrasound signal processing and new insight in exploring interactions between ultrasound pulses and body tissues in ultrasound scanning and imaging processes.

3.1 Introduction

Ultrasound as an imaging modality, has been more and more important in medical diagnosis. It facilitates the non-invasive observations of human soft tissues as the 2nd widely-used tool only next to X-ray. While it is applied as a means of medical

imaging without considerably adverse effect being reported, its real-time capability makes it suitable in medical diagnoses, monitoring the anatomical changes during surgery, chemotherapy, rehabilitation therapy and so on. Unlike the bulky and stationary Computer Tomography (CT) and Magnetic Resonance Imaging (MRI), ultrasound is mobile for medical operators to move easily and conveniently. Moreover, the cost of purchase and operating expenses of ultrasound are incomparably lower than CT and MRI.

Despite its advantages, low image quality of ultrasound compared with CT and MRI poses difficulty for ultrasound operators to make accurate medical diagnosis. It is believed that one of fundamental methods to ameliorate the imaging quality lies in the full understanding of the ultrasound echo processes. In general, the ultrasound image degradation is caused by the constructive and destructive interferences of backscattered signals due to unresolved tissue inhomogeneity [66]. An accurate mathematical model in view of signal and system can provide fundamental comprehension of this complicated echo process.

Since 1980s, convolution models of ultrasound echo processes have been studied and applied. Attempts were made to enhance the resolution of scanners a posteriori with digital signal processing [68]. The ultrasound image was modeled as a convolution of the body tissue information with the point-spread function (PSF) and the inverse process was applied to retrieve the body tissue data. Hence, if being removed of the negative effect of PSF, the masked ultrasound images would be improved.

The convolution model based on linear system theory and ultrasound acoustics was first systematically formulated in [9]. With the convolution model, the influence of frequency dependent attenuation into the calculation of the response was added in [11]. A program for the simulation of ultrasound systems for arbitrary kinds of transducer geometries and excitations was described in [73]. Nonlinear effects were added in the convolution model and implemented in the simulation [74]. The pulse-

echo electromechanical characteristic in the convolution model was determined by the focal plane reflection in some cases [8]. Moreover, the noise contained in the tissue signal was investigated [81], which introduced the assumed additive tissue noise into the convolution.

The convolution model of ultrasound echo processes played a very important role in ultrasound signal processing. Based on this model, a lot of corresponding techniques have been developed to remove speckles and noises by means of deconvolution filtering and other related techniques. Abeyratne et al. [75] proposed to use higher order spectra based method for estimating the separable two-dimensional PSF. Meanwhile, Taxt [19, 76, 77] suggested using the complex cepstrum-based methods up to three dimensional analysis. Michailovich [23, 78–80] applied a wavelet-based smoothing operator to estimate the PSF and proposed to use Laplacian distribution for sparse tissue and perform deconvolution. Yeoh and Zhang [81] combined the Wiener filter and the constrained least squares (LS) algorithm to modify the classical regularization technique.

However, after careful study, we find that there exist some substantial mathematical problems in the original derivations of convolution model in [9] such that the original convolution model is not fully theoretically sound and valid. Consequently, the deconvolution-related techniques for processing ultrasound echo signals no longer have theoretical foundation.

In this chapter, a new convolution model of medical ultrasound echo processes is proposed and its derivation and formulation are provided. Based on the investigation of classical acoustical equations, the new convolution model presents dominant terms under common and practically feasible conditions of medical ultrasound. It rebuilds a theoretical foundation for the prevailing deconvolution techniques in ultrasound signal processing and provides new understanding and insight in exploring interactions between ultrasound pulses and body tissues in ultrasound scanning and

imaging.

The rest of this chapter is organized as follows. In Section 3.2, fundamental knowledge and mathematical expressions of ultrasound acoustics are introduced. Section 3.3 describes the conventional convolution model of ultrasound echo processes and analyzes its problems. Section 3.4 provides important assumptions and approximations for the rest derivations. Then in Section 3.5, a new convolution model of ultrasound echo processes is proposed and derived in details. Simulation results of the proposed convolution model are provided in Section 3.6.

3.2 Ultrasound Acoustics

The following fundamentals of ultrasound acoustics are based on two assumptions:

- 1) no heat conduction or conversion of ultrasound to thermal energy takes place;
 - 2) the first-order born approximated is satisfied [9].
- The 2-dimensional (2-D) coordinate system for representing the ultrasound echo process is shown in Figure 3.1, where the left apparatus is the ultrasound transducer and the circle on the right is the tissue of interest. As indicated in [65], following from adiabatic equation and hydrodynamic equations, the ultrasound echo process is modeled as

$$p_r(r_0, t) = E_m(t) * \int_S p_s(r_0 + r_a, t) d^2 r_a, \quad (3.1)$$

where E_m is the electrical-magnetic characteristics of medical ultrasound machine and $p_s(r_0 + r_a, t)$ is the ultrasound scattered field at the spatial location $(r_0 + r_a)$; r_0 refers to the location of transducer while r_a the location within transducer aperture S with respect to r_0 . Here r_a is a 2-D vector in transducer surface as assumed in [9]. The structure of transducer aperture S results in the 2-D integration over it in (3.1).

Let V represent the spatial volume of tissue. T is denoted as integration interval

Figure 3.1: Coordinate System For Calculating The Received Signal.

over time. ∇ refers to the gradient operation. $\tilde{\rho}(r_1)$ is the mass density perturbations at r_1 and it is calculated as the difference between the instantaneous mass density and mean density of tissue ρ_0 . $\tilde{c}(r_1)$ is the propagation velocity perturbations at r_1 and it is calculated as the difference between the instantaneous velocity and mean propagation velocity of tissue. c_0 is the average propagation velocity of ultrasound waves through the soft tissue.

Let $\nu(t)$ be the particle velocity normal to the transducer surface and the incident pressure at (r_1, t_1) is

$$p_i(r_1, t) = \rho_0 \frac{\partial \Psi(r_1, t)}{\partial t}, \quad (3.2)$$

where $\Psi(r_1, t)$ is the velocity potential and described as

$$\Psi(r_1, t) = \int_T \nu(t_1) h(r_1, r_0, t, t_1) dt_1, \quad (3.3)$$

and

$$h(r_1, r_0, t, t_1) = \frac{\delta(t - t_1 - \frac{|r_0 - r_1|}{c_0})}{2\pi|r_0 - r_1|}. \quad (3.4)$$

The incident pressure $p_i(r_1, t)$ is further derived as

$$p_i(r_1, t) = \rho_0 \nu_0 *_t \frac{\partial h(r_1, r_0, t)}{\partial t} = \rho_0 \frac{\partial \nu_0}{\partial t} *_t h(r_1, r_0, t). \quad (3.5)$$

where

$$\nu_0 = \nu(t - \frac{|r_0 - r_1|}{c_0}) 2\pi|r_0 - r_1|. \quad (3.6)$$

The ultrasound scattered field can be formulated at the point r_1 by integrating all the spherical waves from the scattering region over time:

$$p_s(r_0, t) = \int_V \int_T F_{ps}(r_1, r_0, t, t_1) h(r_1, r_0, t, t_1) dt_1 d^3 \vec{r}_1, \quad (3.7)$$

where

$$F_{ps}(r_1, r_0, t, t_1) = \frac{1}{\rho_0} \nabla(\tilde{\rho}(r_1) \cdot \nabla(p_i(r_1, t_1))) - \frac{2\tilde{c}(r_1)}{c_0^3} \frac{\partial^2 p_i(r_1, t_1)}{\partial t^2}. \quad (3.8)$$

The integration over V is usually 3-dimensional (3-D) due to the biological tissue structures. The integrated function $F_{ps}(r_1, r_0, t, t_1)$ carries tissue information of interest such as ultrasound propagation velocity perturbations $\tilde{c}(r_1)$ and gradient of tissue mass density perturbations $\nabla(\tilde{\rho}(r_1))$.

The original theoretical model in (3.1) is 4-dimensional (4-D) including three spatial

dimensions and time dimension. Without loss of generality and to simplify the problem and analysis, we will consider the case of a single element transducer and ultrasound pulses transmitted in the 1-dimensional (1-D) spatial direction. Given the fixed location of transducer r_0 , the model is reduced to 1-D and the received signals should be a single sequence over time. As a result, following developments of new convolution model in Section 3.4 and Section 3.5 are mainly based on the 1-D analysis. Analogously, the developments can also be extended to higher dimensions with similar but more complicated derivative analyses.

3.3 Conventional Convolution Model and its Problems

Based on the classical acoustical equations in Section 3.2, the output signals of the conventional convolution model in [9] are modeled as the scattered pressure field of the inhomogeneity convolved with piezo-electrical characteristics of ultrasound transducer:

$$p_r(r_0, t) = E_m(t) *_{\mathbf{r}} v_J(t) *_{\mathbf{t}} S_J(r_1) *_{\mathbf{r}} h_{pe}(r_1, r_0, t), \quad (3.9)$$

where $*_{\mathbf{r}}$ denotes the spatial convolution, $*_{\mathbf{t}}$ represents the convolution over time, $v_J(t)$ is the pulse-echo wavelet that accounts for the transducer excitation:

$$v_J(t) = \frac{\rho_0}{2c_0^2} \frac{\partial \nu^2(t)}{\partial t^3}, \quad (3.10)$$

$h_{pe}(r_1, r_0, t)$ is the modified pulse-echo spatial impulse response that relates the transducer geometry to the spatial extent of the scattered field, whereas $S_J(r_1)$ accounts for the inhomogeneities in the tissue due to density and propagation velocity

perturbations that give rise to the scattered signal and it is

$$S_J(r_1) = \frac{\tilde{\rho}(r_1)}{\rho_0} - \frac{2\tilde{c}(r_1)}{c_0}. \quad (3.11)$$

It is found that, in the conventional convolution model (3.9), some essential part of derivation is not approximated properly, so the model is not valid mathematically. A flawed approximation in the derivation of the conventional convolution model is below:

$$\nabla(\tilde{\rho}(r_1)) \cdot \nabla h_{pe}(r_1, t) \approx \tilde{\rho}(r_1) \nabla^2 h_{pe}(r_1, t), \quad (3.12)$$

where $h_{pe}(r_1, t)$ refers to the spatial impulse response. More specifically, the Laplacian operator in (3.12) is the derivative with respect to the distance, which can be approximated with the derivative with respect to time, leading to

$$\nabla h_{pe}(r_1, t) \approx \frac{1}{c_0} \frac{\partial h_{pe}(r_1, t)}{\partial t}; \quad (3.13)$$

$$\nabla^2 h_{pe}(r_1, t) \approx \frac{1}{c_0^2} \frac{\partial^2 h_{pe}(r_1, t)}{\partial t^2}. \quad (3.14)$$

Hence, with (3.13) and (3.14), (3.12) is expressed as

$$\nabla(\tilde{\rho}(r_1)) \cdot \frac{1}{c_0} \frac{\partial h_{pe}(r_1, t)}{\partial t} \approx \tilde{\rho}(r_1) \frac{1}{c_0^2} \frac{\partial^2 h_{pe}(r_1, t)}{\partial t^2}. \quad (3.15)$$

It is clear that the function in the left side of (3.15) is not mathematically equivalent to that in the right side. Hence, the important approximation in (3.12) in the conventional convolution model is not mathematically feasible and satisfied. It leads to erroneous quantification of the echo progress in ultrasound imaging as shown in later simulation studies. This motivates us to develop a more feasible convolution model of ultrasound echo processes.

We now demonstrate the flawed conventional convolution model with an example. In this example, the impulse response h_{pe} is assumed the same as that in [73] within a certain and short time range T_h determined by ultrasound machines.

$$h_{pe} = \sin(2\pi f_0 t), \quad t \in [0, T_h] \quad (3.16)$$

and its derivatives are

$$\frac{\partial h_{pe}(r_1, t)}{\partial t} = 2\pi f_0 \cos(2\pi f_0 t), \quad t \in [0, T_h], \quad (3.17)$$

$$\frac{\partial^2 h_{pe}(r_1, t)}{\partial t^2} = -4\pi^2 f_0^2 \sin(2\pi f_0 t), \quad t \in [0, T_h]. \quad (3.18)$$

The 1-D distribution of mass density is assumed to satisfy

$$\tilde{\rho} = k_\rho x, \quad (3.19)$$

where k_ρ is the constant slope and x refers to the location in axial direction.

Hence, the approximation in the conventional convolution model in (3.12) becomes

$$\frac{2\pi f_0 k_\rho}{c_0} \cos(2\pi f_0 t) \approx \frac{-4\pi^2 f_0^2 k_\rho x}{c_0^2} \sin(2\pi f_0 t). \quad (3.20)$$

If $x = c_0 t$, (3.20) is reorganized as

$$\tan(2\pi f_0 t) \approx -\frac{1}{2\pi f_0 t}, \quad t \in [0, T_h]. \quad (3.21)$$

Given the properties of tangent function and inverse proportional function as well as the short duration of ultrasound pulses, only few solutions exist for (3.21). As a result, the approximation in (3.20) cannot be satisfied in most cases.

3.4 Important and Feasible Conditions

In addition to the two assumptions on the heat exchange and Born approximation stated in Section 3.2, we introduce and incorporate the following properties and conditions for facilitating further derivation of the new convolution model.

3.4.1 Particle Velocity

The ultrasound scanning for B-mode imaging is a sequence of repetitive operations and each operation is composed of a short duration of pulsing time and a long duration of listening or measurement time. During the pulsing time, the ultrasound probe produces and transmits ultrasound pulses to body tissues controlled by the electronic circuit of the scanner. In commercial ultrasound scanners, the pulses generated are characterized by their center frequency, typically between 3–10 MHz. Thus it is practically feasible to model the particle velocity $\nu(t)$ normal to the surface of the transducer as a short duration envelop function $g(t)$ modulated by a trigonometry function of the center frequency. Without loss of generality and similar to the velocity assumption in [85], the particle velocity function can be written as

$$\nu(t) = g(t) \sin(2\pi f_0 t), \quad (3.22)$$

where $g(t)$ is the envelop function of the particle velocity and f_0 is the center frequency of the ultrasound probe. The envelop function $g(t)$ is of short time duration and its components in the frequency domain are at significantly lower frequency than the center frequency.

It is assumed that the envelop function $g(t)$ and its derivatives satisfy:

$$\|g(t)\| \gg \left\| \frac{g^{(1)}(t)}{2\pi f_0} \right\| \gg \left\| \frac{g^{(2)}(t)}{(2\pi f_0)^2} \right\| \gg \left\| \frac{g^{(3)}(t)}{(2\pi f_0)^3} \right\|, \quad (3.23)$$

where

$$g^{(n)}(t) = \frac{d^n g(t)}{dt^n}, n = 1, 2, 3 \quad (3.24)$$

This assumption has also been verified by the velocities with different shapes of envelope like sinusoidal function, Gaussian-shaped sinusoidal function, Sinc function and so on. Hence, it is believed that the assumption in (3.23) holds for the arbitrary-shaped sinusoidal velocity in (3.22).

3.4.2 Tissue Characteristics in Medical Ultrasound

As shown in Table 3.1 from National Physics Laboratory, UK and [66], average mass densities of common human soft tissues ρ_0 are between $1000(\text{kg}/\text{m}^3)$ and $1200(\text{kg}/\text{m}^3)$, while the average ultrasound propagation velocity c_0 is around 1598m/s . In most

Table 3.1: Densities and Ultrasound Velocity for Some Common Tissues in National Physical Laboratory, UK

Tissue	Density (kg/m^3)	Velocity of sound (m/s)
Liver	1050-1070	1578-1640
Kidney	1050	1560-1580
Muscle (skeletal)	1038-1056	1529-1629
Pancreas	1040-1050	1591
Skin (epidermis)	1110-1190	1729+
Spleen	1054	1567-1635
Breast	1020	1430-1570
Average	1061	1598

cases, the depth from the transducer to tissue of interest is around several centimeters. Thus, the minimum value of spatial location r_1 can be assumed as one

centimeter. Also the step size of gradient operation is defined as

$$\Delta r_1 = c_0/f_s; \quad (3.25)$$

where f_s is the sampling frequency of medical ultrasound machines and usually $f_s \in [2 \times 10^6, 20 \times 10^6](\text{Hz})$. Hence, $\Delta r_1 \in [7.9 \times 10^{-5}, 7.9 \times 10^{-4}](m)$. These specifications are going to be applied in the dominance analysis in Section 3.5.

3.4.3 Anatomical Property 1

Lemma 3.4.1 (Tissue Property 1). *For a common body soft tissue, when its average ultrasound propagation velocity is around 1580 m/s, its ultrasound perturbations of propagation velocity $\tilde{c}(r_1)$ at a spatial location r_1 within the tissue can be linearly represented by tissue perturbation of mass density $\tilde{\rho}(r_1)$ as follows*

$$\tilde{c}(r_1) \approx -a_1 \tilde{\rho}(r_1). \quad (3.26)$$

Proof. In theoretical acoustics [65], the dependence of the three acoustical parameters $c(r_1)$, the propagation velocity of ultrasound, $\rho(r_1)$, the mass density and $\kappa(r_1)$, the compressibility, at the spatial location r_1 within the tissue is given as

$$c(r_1) = \frac{1}{\sqrt{\rho(r_1)\kappa(r_1)}}. \quad (3.27)$$

It follows from [66] that, if the mass density of human soft tissue varies in a small range and typical average propagation velocity is assumed as 1580m/s, the tissue compressibility can be linearized and represented by its mass density:

$$\kappa(r_1) = \kappa_0 + \frac{d\kappa(r_1)}{d\rho} \approx 10^{-12}(1545 - 1.1\rho(r_1)) \text{ m}^2/Nt. \quad (3.28)$$

Here it is reasonable to assume that the average ultrasound propagation velocity is 1580 m/s which is quite similar to the average value 1598 m/s in Table 3.1 while it has to be fixed for the reference of (3.28).

Then we substitute $\kappa(r_1)$ in (3.27) with that in (3.28),

$$\begin{aligned} c(r_1) &= \frac{10^6}{\sqrt{\rho(r_1)(1545 - 1.1\rho(r_1))}} \\ &\triangleq G_c(\rho(r_1)). \end{aligned} \quad (3.29)$$

Meanwhile,

$$c(r_1) = c_0 + \tilde{c}(r_1), \quad (3.30)$$

$$\rho(r_1) = \rho_0 + \tilde{\rho}(r_1). \quad (3.31)$$

It is then straightforward to apply 1st-order Tailor progression to (3.29) at ρ_0 and further simplify the relationship between $c(r_1)$ and $\rho(r_1)$ as

$$\begin{aligned} c(r_1) &\approx G_c(\rho_0) + \left. \frac{dG_c(\rho)}{d\rho} \right|_{\rho=\rho_0} \tilde{\rho}(r_1) \\ &= \frac{10^6}{\sqrt{\rho_0(1545 - 1.1\rho_0)}} - \frac{10^6(772.5 - 1.1\rho_0)}{\sqrt{\rho_0(1545 - 1.1\rho_0)^3}} \tilde{\rho}(r_1). \end{aligned} \quad (3.32)$$

With (3.30) and (3.32),

$$\tilde{c}(r_1) \approx a_2 - a_1 \tilde{\rho}(r_1) - c_0, \quad (3.33)$$

where

$$a_1 = \frac{10^6(772.5 - 1.1\rho_0)}{[\rho_0(1545 - 1.1\rho_0)]^{3/2}}, \quad (3.34)$$

$$a_2 = \frac{10^6}{[\rho_0(1545 - 1.1\rho_0)]^{1/2}}. \quad (3.35)$$

It is also found that, theoretically, ultrasound propagation velocity depends on the variation of tissue mass density. In another word, if there is no change of tissue mass density, ultrasound propagation velocity should remain unchanged, i.e.

$$\tilde{c}(r_1) = 0, \text{ If } \tilde{\rho}(r_1) = 0. \quad (3.36)$$

At the mean time, for the tissue mass densities listed in Table 3.1, the difference between a_2 and c_0 is not large. For example, when $\rho_0=1000(\text{kg}/\text{m}^3)$, $a_2 \approx 1499$. Hence, $(a_2 - c_0)$ can be regarded as the error of linearization.

As a result, ultrasound perturbations of propagation velocity $\tilde{c}$ can be further modified to be (3.26). □

The ultrasound perturbation of propagation velocity $\tilde{c}(r_1)$ is not as easy to predict and estimate as that for tissue perturbation of mass density $\tilde{\rho}(r_1)$. With Lemma 3.4.1, ultrasound perturbation of propagation velocity $\tilde{c}(r_1)$ is substituted with the function of tissue perturbation of mass density $\tilde{\rho}(r_1)$. Hence, the ultrasound echo process in the further derivations can be regarded as the interaction between ultrasound excitation waves and tissue function of mass density.

3.4.4 Anatomical Property 2

In this section, another lemma is introduced to facilitate the development of the new convolution model. Here, two functions concerning tissue properties and ultrasound machines are named as A_ρ and A_c , where A_ρ is related to tissue mass densities and

A_c to ultrasound propagation velocities through the soft tissue, i.e.

$$A_p(r_1, t_1) = \nabla(\tilde{\rho}(r_1)) \frac{2\pi f_0^2}{c_0 r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right), \quad (3.37)$$

and

$$A_c(r_1, t_1) = \tilde{c}(r_1) \frac{8\rho_0}{c_0^3} \frac{\pi^2 f_0^3}{r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right). \quad (3.38)$$

Lemma 3.4.2 (Tissue Property 2). *Given the medical specifications in Section 3.4.2 and Lemma 3.4.1, the tissue properties of most human soft organs listed in Table 3.1 satisfy in two different cases:*

$$\begin{cases} A_c(r_1, t_1) \gg A_p(r_1, t_1), & \text{if } \tilde{\rho}(r_1) \text{ is large;} \\ A_c(r_1, t_1) \text{ and } A_p(r_1, t_1) \text{ are small,} & \text{if } \tilde{\rho}(r_1) \text{ is small.} \end{cases} \quad (3.39)$$

Proof. The gradient of tissue variation of mass density is usually defined as

$$\nabla(\tilde{\rho}(r_1)) = \frac{\Delta(\tilde{\rho}(r_1))}{\Delta r_1}, \quad (3.40)$$

where Δ represents the differential operation, i.e.,

$$\Delta(\tilde{\rho}(r_1)) = \tilde{\rho}(r_1 + \Delta r_1) - \tilde{\rho}(r_1). \quad (3.41)$$

If the sampling frequency f_s is substituted by ultrasound center frequency f_0 and over-sampling factor N as

$$f_s = N f_0, \quad (3.42)$$

with (3.50), (3.51) and (3.25), A_ρ is rewritten as

$$\begin{aligned}
 A_\rho(r_1, t_1) &= \frac{2\pi f_0^2}{c_0 r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \frac{\Delta \tilde{\rho}(r_1)}{\Delta r_1} \\
 &= \frac{2\pi f_0^2 f_s}{c_0 r_1 c_0} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \Delta \tilde{\rho}(r_1) \\
 &= \frac{2\pi N f_0^3}{c_0^2 r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \Delta \tilde{\rho}(r_1) \\
 &\triangleq R_\rho(r_1, t_1) (\tilde{\rho}(r_1 + \Delta r_1) - \tilde{\rho}(r_1)), \tag{3.43}
 \end{aligned}$$

where

$$R_\rho(r_1, t_1) \triangleq \frac{2\pi N f_0^3}{c_0^2 r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right). \tag{3.44}$$

On the other hand, $A_c(r_1, t_1)$ is further developed as

$$\begin{aligned}
 A_c(r_1, t_1) &= \tilde{c}(r_1) \frac{8\rho_0 \pi^2 f_0^3}{c_0^3 r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \\
 &= \frac{2N\pi f_0^3}{c_0^2 r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \left[\frac{\frac{8}{N}\pi |a_1| \rho_0}{c_0} \tilde{\rho}(r_1) \right] \\
 &= R_\rho(r_1, t_1) R_c \tilde{\rho}(r_1), \tag{3.45}
 \end{aligned}$$

where

$$R_c = \frac{\frac{8}{N}\pi |a_1| \rho_0}{c_0}. \tag{3.46}$$

With the average mass density ρ_0 1061 kg/m³ and average ultrasound propagation velocity c_0 1580 m/s, the constant $|a_1|$ is calculated by (3.34) and $|a_1| \in [1.2332, 2.6954]$. Thus, if the over-sampling factor $N < 4$, R_c becomes

$$1.6860 \leq R_c \leq 3.6264. \tag{3.47}$$

Most soft organs listed in Table 3.1 are of the smooth anatomical structures. In

addition, compared with r_1 , the absolute step size Δr_1 is very small as indicated in (3.25). Here there are two possible cases for $\tilde{\rho}(r_1)$.

Case 1 If $\tilde{\rho}(r_1)$ is not small while Δr_1 is very small, the difference between $\tilde{\rho}(r_1 + \Delta r_1)$ and $\tilde{\rho}(r_1)$ is very small and negligible, i.e.,

$$\tilde{\rho}(r_1) \gg \tilde{\rho}(r_1 + \Delta r_1) - \tilde{\rho}(r_1). \quad (3.48)$$

Meanwhile, as $R_c > 1$ shown in (3.63),

$$\begin{aligned} A_c(r_1, t_1) &= R_c R_\rho(r_1, t_1) \tilde{\rho}(r_1) \\ &> R_\rho(r_1, t_1) \tilde{\rho}(r_1) \\ &\gg R_\rho(r_1, t_1) (\tilde{\rho}(r_1 + \Delta r_1) - \tilde{\rho}(r_1)) \\ &= A_\rho(r_1, t_1). \end{aligned} \quad (3.49)$$

Case 2 If $\tilde{\rho}(r_1)$ is very small, $[\tilde{\rho}(r_1 + \Delta r_1) - \tilde{\rho}(r_1)]$ are also very small correspondingly. Since R_c and R_ρ have finite values, $A_c(r_1, t_1)$ and $A_\rho(r_1, t_1)$ are both very small.

As a result, (3.39) is proved.

□

The simulation of Lemma 3.4.2 will be shown in Section 3.6.

Remark We also found that Lemma 3.4.2 can also be extended to the two-dimensional case. The proof is given as below.

Proof. The gradient of tissue variation of mass density is usually defined as

$$\nabla(\tilde{\rho}(r_1)) = \left(\frac{\Delta(\tilde{\rho}(r_1^x))}{\Delta r_1^x}, \frac{\Delta(\tilde{\rho}(r_1^y))}{\Delta r_1^y} \right), \quad (3.50)$$

where Δ represents the differential operation, i.e.,

$$\Delta(\tilde{\rho}(r_1^x)) = \tilde{\rho}(r_1^x + \Delta r_1^x) - \tilde{\rho}(r_1^x), \quad (3.51)$$

$$\Delta(\tilde{\rho}(r_1^y)) = \tilde{\rho}(r_1^y + \Delta r_1^y) - \tilde{\rho}(r_1^y), \quad (3.52)$$

and

$$r_1^2 = (r_1^x)^2 + (r_1^y)^2, \quad (3.53)$$

$$(\Delta r_1)^2 = (\Delta r_1^x)^2 + (\Delta r_1^y)^2. \quad (3.54)$$

If the sampling frequency f_s is substituted by ultrasound center frequency f_0 and over-sampling factor N as

$$f_s = N f_0, \quad (3.55)$$

with (3.50), (3.51) and (3.25), A_ρ is rewritten as

$$\begin{aligned} \|A_\rho(r_1, t_1)\| &= \frac{2\pi f_0^2}{c_0 r_1} \left| g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \right| (|\nabla(\tilde{\rho}(r_1))|) \\ &= \frac{2\pi f_0^2}{c_0 r_1} \left| g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \right| \\ &\quad \sqrt{\left(\frac{\Delta(\tilde{\rho}(r_1^x))}{\Delta r_1^x}\right)^2 + \left(\frac{\Delta(\tilde{\rho}(r_1^y))}{\Delta r_1^y}\right)^2}. \end{aligned} \quad (3.56)$$

Since

$$(\Delta r_1^x)^2 < (\Delta r_1)^2, \quad (3.57)$$

$$(\Delta r_1^y)^2 < (\Delta r_1)^2, \quad (3.58)$$

$$\begin{aligned} \|A_\rho(r_1, t_1)\| &< \frac{2\pi f_0^2}{c_0 r_1} \left| g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \right| \\ &\quad |\Delta(\tilde{\rho}(r_1))| \sqrt{\left(\frac{1}{\Delta r_1^x}\right)^2 + \frac{1}{\Delta r_1^y}} \\ &= \frac{2\pi f_0^2}{c_0 r_1} \left| g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \right| \\ &\quad |\Delta(\tilde{\rho}(r_1))| \frac{|\Delta r_1|}{|\Delta r_1^x \Delta r_1^y|} \\ &= \frac{2\pi f_0^2}{c_0 r_1} \frac{f_s}{c_0} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \Delta \tilde{\rho}(r_1) \\ &= \frac{2\pi f_0^2}{c_0 r_1} \frac{f_s}{c_0} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \Delta \tilde{\rho}(r_1) \\ &= \frac{2\pi N f_0^3}{c_0^2 r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \Delta \tilde{\rho}(r_1) \\ &\triangleq R_\rho(r_1, t_1) (\tilde{\rho}(r_1 + \Delta r_1) - \tilde{\rho}(r_1)), \end{aligned} \quad (3.59)$$

where

$$R_\rho(r_1, t_1) \triangleq \frac{2\pi N f_0^3}{c_0^2 r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right). \quad (3.60)$$

On the other hand, $A_c(r_1, t_1)$ is further developed as

$$\begin{aligned} A_c(r_1, t_1) &= \tilde{c}(r_1) \frac{8\rho_0}{c_0^3} \frac{\pi^2 f_0^3}{r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \\ &= \frac{2N\pi f_0^3}{c_0^2 r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \left[\frac{\frac{8}{N}\pi|a_1|\rho_0}{c_0} \tilde{\rho}(r_1) \right] \\ &= R_\rho(r_1, t_1) R_c \tilde{\rho}(r_1), \end{aligned} \quad (3.61)$$

where

$$R_c = \frac{\frac{8}{N}\pi|a_1|\rho_0}{c_0}. \quad (3.62)$$

With the average mass density ρ_0 1061 kg/m³ and average ultrasound propagation velocity c_0 1580 m/s, the constant $|a_1|$ is calculated by (3.34) and $|a_1| \in [1.2332, 2.6954]$. Thus, if the over-sampling factor $N < 4$, R_c becomes

$$1.6860 \leq R_c \leq 3.6264. \quad (3.63)$$

Most soft organs listed in Table 3.1 are of the smooth anatomical structures. In addition, compared with r_1 , the absolute step size Δr_1 is very small as indicated in (3.25). Here there are two possible cases for $\tilde{\rho}(r_1)$.

Case 1 If $\tilde{\rho}(r_1)$ is not small while Δr_1 is very small, the difference between $\tilde{\rho}(r_1 + \Delta r_1)$ and $\tilde{\rho}(r_1)$ is very small and negligible, i.e.,

$$\tilde{\rho}(r_1) \gg \tilde{\rho}(r_1 + \Delta r_1) - \tilde{\rho}(r_1). \quad (3.64)$$

Meanwhile, as $R_c > 1$ shown in (3.63),

$$\begin{aligned} A_c(r_1, t_1) &= R_c R_\rho(r_1, t_1) \tilde{\rho}(r_1) \\ &> R_\rho(r_1, t_1) \tilde{\rho}(r_1) \\ &\gg R_\rho(r_1, t_1) (\tilde{\rho}(r_1 + \Delta r_1) - \tilde{\rho}(r_1)) \\ &= A_\rho(r_1, t_1). \end{aligned} \quad (3.65)$$

Case 2 If $\tilde{\rho}(r_1)$ is very small, $[\tilde{\rho}(r_1 + \Delta r_1) - \tilde{\rho}(r_1)]$ are also very small correspondingly. Since R_c and R_p have finite values, $A_c(r_1, t_1)$ and $A_p(r_1, t_1)$ are both very small.

As a result, (3.39) is proved.

□

Hence, our model proposed by Section 3.5 is also application for 2-D cases.

3.5 New Convolution Model of Ultrasound Echo Signals

In this section, a new convolution model of ultrasound echo processes is derived from the theoretical model in [69], which is also the basis of the conventional ultrasound convolution model in [9]. This derivation begins with the description of the theoretical model, and then continues to develop the convolution model by focusing on the integrated function of original model. Finally, a new convolution model of ultrasound echo signals is presented.

Theorem 3.5.1. *For typical echo process of ultrasound, under medical specifications in Section 3.4.2, with Lemma 3.4.1, Lemma 3.4.2 and approximations of particle velocity in (3.23), its one-dimensional echo signals can be modeled as:*

$$p_r(r_0, t) \approx E_m(t) \underset{t}{*} v_{cc}(t) \underset{t}{*} f_{cc}(r_1) \underset{r}{*} h_{pe}(r_1, r_0, t) \quad (3.66)$$

where v_{cc} is the pulse-echo wavelet of the excitation, f_{cc} accounts for the inhomogeneities in the tissue due to propagation velocity perturbations, h_{pe} is the modified

pulse-echo spatial impulse response that relates the transducer geometry to the spatial extent of the scattered field:

$$v_{cc}(t) = \frac{\rho_0}{c_0^2} \frac{\partial^3 \nu(t)}{\partial t^3}, \quad (3.67)$$

$$f_{cc}(r_1) = -\frac{\tilde{c}(r_1)}{c_0}, \quad (3.68)$$

$$h_{pe}(r_1, r_0, t_1) = h(r_1, r_0, t_1) *_{\tilde{t}} h(r_0, r_1, t_1). \quad (3.69)$$

Proof. Based on the formulas (3.1)-(3.3), integrations are carried out. If it is assumed that $r_0 = 0$ and $r_1 > 0$, the integrated function F_{ps} in (3.8) is specifically written as:

$$F_{ps}(r_1, t_1) = F_{ps}^{\rho}(r_1, t_1) + F_{ps}^c(r_1, t_1). \quad (3.70)$$

It has two terms concerning information of tissue mass density and propagation velocity respectively, where

$$\begin{aligned} F_{\rho}(r_1, t_1) = & \nabla(\bar{\rho}(r_1)) \left\{ \frac{2\pi f_0^2}{c_0 r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \sin\left(2\pi f_0 \frac{c_0 t_1 - r_1}{c_0}\right) \right. \\ & - \frac{2f_0}{c_0 r_1} g^{(1)}\left(\frac{c_0 t_1 - r_1}{c_0}\right) \cos\left(2\pi f_0 \frac{c_0 t_1 - r_1}{c_0}\right) \\ & \left. - \frac{1}{2\pi c_0 r_1} g^{(2)}\left(\frac{c_0 t_1 - r_1}{c_0}\right) \sin\left(2\pi f_0 \frac{c_0 t_1 - r_1}{c_0}\right) \right\}, \end{aligned} \quad (3.71)$$

$$\begin{aligned} F_c(r_1, t_1) = & -2 \frac{\tilde{c}(r_1) \rho_0}{c_0^3} \left\{ \left[\frac{1}{2\pi r_1} g^{(3)}\left(\frac{c_0 t_1 - r_1}{c_0}\right) \right. \right. \\ & - \frac{6\pi f_0^2}{r_1} g^{(1)}\left(\frac{c_0 t_1 - r_1}{c_0}\right) \left. \right] \sin\left(2\pi f_0 \frac{c_0 t_1 - r_1}{c_0}\right) \\ & + \left[\frac{3f_0}{r_1} g^{(2)}\left(\frac{c_0 t_1 - r_1}{c_0}\right) - \frac{4\pi^2 f_0^3}{r_1} g\left(\frac{c_0 t_1 - r_1}{c_0}\right) \right] \\ & \left. \cos\left(2\pi f_0 \frac{c_0 t_1 - r_1}{c_0}\right) \right\}. \end{aligned} \quad (3.72)$$

With the assumptions and approximations in Section 3.4, the integrated function in (3.70) is rewritten as

$$F_{ps}(r_1, t_1) \approx F_\rho^D(r_1, t_1) + F_c^D(r_1, t_1), \quad (3.73)$$

where

$$F_\rho^D(r_1, t_1) = A_\rho(r_1, t_1) \sin(2\pi f_0 \frac{c_0 t_1 - r_1}{c_0}), \quad (3.74)$$

$$F_c^D(r_1, t_1) = A_c(r_1, t_1) \cos(2\pi f_0 (\frac{c_0 t_1 - r_1}{c_0})), \quad (3.75)$$

As shown by Lemma 2, when the magnitude $A_c(r_1, t_1)$ of the dominant term F_c^D is much greater than the magnitude $A_\rho(r_1, t_1)$ of the other term $F_\rho^D(r_1, t_1)$, the integrated function of the ultrasound theoretical model in (3.73) is mainly determined by the perturbation of propagation velocity $\tilde{c}(r_1)$.

Hence, the integrated function of ultrasound echo processes is simplified and approximated as

$$F_{ps}(r_1, t_1) \approx 8 \frac{\tilde{c}(r_1) \rho_0}{c_0^3} \frac{\pi^2 f_0^3}{r_1} g(\frac{c_0 t_1 - r_1}{c_0}) \cos(2\pi f_0 (\frac{c_0 t_1 - r_1}{c_0})). \quad (3.76)$$

Meanwhile, when $A_c(r_1, t_1)$ and $A_\rho(r_1, t_1)$ are both very small, we may choose to neglect $A_\rho(r_1, t_1)$ and (3.76) is also valid.

Accordingly, the ultrasound theoretical model in (3.7) becomes

$$p_s(r_0 + r_a, t) = \int_V \int_T \left[-\frac{2\tilde{c}(r_1)}{c_0^3} \frac{\partial^2 p_i(r_1, t_1)}{\partial t^2} \right] \frac{\delta(t - t_1 - \frac{|r_0 + r_a - r_1|}{c_0})}{2\pi|r_0 + r_a - r_1|} dt_1 d^3\vec{r}_1. \quad (3.77)$$

With the theoretical model (3.1)-(3.3) and all the derivations before, ultrasound echo signals are mainly determined by ultrasound perturbation of propagation velocity and the effects from the gradient of tissue perturbation of mass density are negligible. Hence, ultrasound echo signals can be approximated as

$$p_r(r_0, t) = E_m(t) *_t \int_S p_s(r_0 + r_a, t) d^2r_a \quad (3.78)$$

where

$$p_s(r_0 + r_a, t) = \int_V \int_T \left[-\frac{2\tilde{c}(r_1)}{c_0^3} \frac{\partial^2 p_i(r_1, t_1)}{\partial t^2} \right] h(r_1, r_0, t, t_1) dt_1 d^3r_1 \quad (3.79)$$

Following from the derivations in [9], it is straightforward to establish the new convolution model of ultrasound echo processes in (3.78). With (3.78), (3.79) and (3.5), ultrasound echo signals are further developed as

$$\begin{aligned}
p_r(r_0, t) &= E_m(t) *_t \frac{1}{2} \int_V \left[-\frac{2\tilde{c}(r_1)}{c_0^3} \frac{\partial^2}{\partial t^2} \right. \\
&\quad \left. \left(\rho_0 \frac{\partial \nu(t)}{\partial t} *_t h(r_1, r_0, t, t_1) \right) *_t h(r_1, r_0, t, t_1) \right] d^3 r_1 \\
&= E_m(t) *_t \frac{\rho_0}{2c_0^2} \frac{\partial \nu(t)}{\partial t} \\
&\quad *_t \int_V \left(-\frac{2\tilde{c}(r_1)}{c_0} \frac{\partial^2 h_{pe}(r_1, r_0, t, t_1)}{\partial t^2} \right) d^3 r_1 \\
&= E_m(t) *_t \frac{\rho_0}{c_0^2} \frac{\partial^3 \nu(t)}{\partial t^3} \\
&\quad *_t \int_V \left(-\frac{\tilde{c}(r_1)}{c_0} h_{pe}(r_1, r_0, t, t_1) \right) d^3 r_1 \\
&= E_m(t) *_t \frac{\rho_0}{c_0^2} \frac{\partial^3 \nu(t)}{\partial t^3} \\
&\quad *_t \left[-\frac{\tilde{c}(r_1)}{c_0} \right] *_r h_{pe}(r_1, r_0, t, t_1)
\end{aligned} \tag{3.80}$$

Hence (3.66)-(3.69) are proved. $\square$

3.6 Simulation Results

In this section, the new convolution model is simulated and compared with theoretical model while the conventional convolution model with flawed approximation is also simulated.

Before the simulation, several definitions are provided first. Similar to the assumption of particle velocity in [85], the particle velocity $\nu(t)$ involved can be assumed as $\sin(2\pi f_0 t)$ with Gaussian-shaped envelope, i.e.

$$\nu(t) = e^{-\frac{1}{2} \left(\frac{2.5(t - \frac{T_D}{2})}{T_D} \right)^2} \sin(2\pi f_0 t), \tag{3.81}$$

where T_D is the duration of the particle velocity for each ultrasound pulse.

Meanwhile, the spatial impulse response h_{pe} is defined the same as the one in the widely-applied ultrasound simulation software *Field II* [73], and the homogeneous spatial impulse response is assumed as

$$h_{pe} = \text{Han}(t) \sin(2\pi f_0 t), \quad (3.82)$$

where $\text{Han}(t)$ refers to the Hanning window function while f_0 the center frequency of ultrasound transducer. And rest specifications in the simulated are listed in Table 3.2. In addition, because of the same electrical-magnetic characteristic E_m involved, the simulations excluded the convolution effects resulting from it.

Table 3.2: Specifications for Simulation

f_0	Ultrasound Center Frequency	$3.5 \times 10^6(\text{Hz})$
f_s	Sampling Frequency	$20 \times 10^6(\text{Hz})$
c_0	Average Ultrasound Propagation Velocity	$1580(\text{m/s})$
ρ_0	Average Mass Density	$1061(\text{kg/m}^3)$
Δt	Step-size over time	$5 \times 10^{-8}(\text{sec})$
Δr	Step-size in axial direction	$7.7 \times 10^{-5}(\text{m})$

Remark on simulation We also would like to conduct experiments to verify the new convolution model. Actually, we are in need of phantoms which should have known distributions of mass density or ultrasound propagation velocity, while the phantoms should include at least one part which has no abrupt changes on their distributions. Hence, we searched among current commercial phantoms and failed to find such one. We also tried to make phantoms by agar power which should have ideal distributions of density or velocity as we expected and found the self-made phantoms have relatively irregular distributions without any indications of possible distributions. As a result, we only provided simulations to justify the validity of the new model.

Table 3.3: Root Mean Square Error of Conventional Model and Proposed Model

Conventional Convolution Model	38.2665%
Proposed Convolution Model	$2.8496 \times 10^{-6}\%$

3.6.1 Simulation of Sinusoidal Phantom

As shown in Figure 3.2.a, the mass density of tissue of interest varies like sinusoidal functions added with random changes. In order to highlight the echoes from middle tissue which represent the important texture information, only the part of echoes from the tissue in the rectangular are shown. Ultrasound echo signals simulated by classical theoretical model, conventional convolution model and proposed convolution model, are shown in (b), (c) and (d) of Figure 3.2, respectively. It is shown that the echo values simulated by conventional convolution model are much larger than those of classical theoretical model and proposed convolution model. Meanwhile, proposed convolution model and theoretical model are all very sensitive to the changes of perturbations of mass density while conventional convolution model fails to detect them. This is because that conventional model erroneously overestimates the effects of mass density while underestimates perturbations of the ultrasound propagation velocity.

The errors of the two convolution model are shown in Figure 3.3. It indicates that the error of conventional convolution model is remarkably larger than that of proposed new convolution model. The results corroborate well with our theoretical analysis.

The root mean square error (RMSE) of the two models are listed in Table 3.3. The new convolution model improves the accuracy enormously.

The validity of Lemma 3.4.2 is essential to that of the new convolution model (3.66)-(3.69). In Figure 3.4, the comparison of two magnitudes in Lemma 3.4.2 is simulated

Figure 3.2: Ultrasound Echoes Simulated For Sinusoidal Phantom By Three Models. (a) Tissue Perturbations Of Mass Density; (b) Echoes By Theoretical Model; (c) Echoes By Conventional Convolution Model; (d) Echoes By Proposed New Convolution Model.

Figure 3.3: Error Of Echoes Simulated For Step Phantom: Error Of Conventional Model (Dashed Line); Error Of Proposed New Model (Solid Line)

for sinusoidal phantom. For this simulated phantom, the average absolute value of A_p is $1.6420\text{e}+018$ whereas that of A_c is $1.5238\text{e}+020$. It is verified that the magnitude concerning ultrasound propagation velocity is much greater than that concerning tissue mass density.

Figure 3.4: A_p and A_c Simulated For Sinusoidal Phantom: A_p (Dashed line); A_c (Solid line)

3.6.2 Simulation of Random Phantom

As shown in Figure 3.5.a, the mass density of tissue of interest is randomly distributed. In order to highlight the echoes from middle tissue which represent the important texture information, only the part of echoes from the tissue are shown.

Ultrasound echo signals simulated by classical theoretical model, conventional convolution model and proposed convolution model, are shown in (b), (c) and (d) of Figure 3.5, respectively. It is shown that the echo values simulated by conventional convolution model are much higher than those of classical theoretical model and proposed convolution model.

Figure 3.5: Ultrasound Echoes Simulated For Random Phantom By Three Models. (a) Tissue Perturbations Of Mass Density; (b) Echoes By Theoretical Model; (c) Echoes By Conventional Convolution Model; (d) Echoes By Proposed New Convolution Model.

The errors of the two convolution model are shown in Figure 3.6. It indicates that the error of conventional convolution model is remarkably larger than that of proposed new convolution model. The results corroborate well with our theoretical analysis.

Figure 3.6: Error Of Echoes Simulated For Random Phantom: Error of Conventional Model (Dashed line); Error of Proposed New Model (Solid line)

The root mean square error (RMSE) of the two models are listed in Table 3.4. It is shown that the new convolution model improves the accuracy enormously.

Table 3.4: Root Mean Square Error of Conventional Model and Proposed Model

Conventional Convolution Model	1281.4%
Proposed Convolution Model	$6.7103 \times 10^{-5}\%$

3.7 Summary

This chapter presented a new convolution model of ultrasound echo signals. It is based on careful studies of the classical ultrasound acoustics and the existing convolution model. The problematic approximation in the conventional convolution model is identified which motivated the proposal of our new convolution model. The new model takes the practical specifications of medical ultrasound into account and evaluates the echo effects arising from different anatomical properties. It is shown that, under practical circumstances and in tissues of small inhomogeneities, medical ultrasound echo signals are dominated by perturbations of ultrasound propagation velocity. With the new model, the convolution format of ultrasound echo processes remains valid and prevailing deconvolution techniques for ultrasound signal processing and imaging have gained a new theoretical foundation.

Chapter 4

Discrete Modeling of Ultrasound Echo Signals

4.1 Introduction

Conventional convolution model of ultrasound echo processes was proposed in [9] and widely applied for decades. As the growth of modern computer-based digital signal processing technologies, an accurate discrete model of ultrasound echo processes has become more and more necessary. A seemingly-simple 2-D discrete convolution model is frequently mentioned in a lot of works on ultrasound deconvolution filtering techniques. In [19, 23, 75–81], the continuous conventional convolution model is transformed to a simple 2-D discrete convolution model

$$g(m, n) = f(m, n) * h(m, n) + v(m, n) \quad (4.1)$$

where m, n represent 2-D independent discrete spatial locations in both axial and lateral direction, respectively. $g(m, n)$ refers to ultrasound echo signals. $f(m, n)$ is the assumed 2-D tissue information. $h(m, n)$ is the point spread function (PS-

F) concerning the characteristics of ultrasound excitation systems. $v(m, n)$ is the additive noise introduced. $*$ means the convolution operator. The usual practice of processing procedure for the discrete model (4.1) is to estimate the PSF first, deconvolute the echo signals and finally generate the tissue information after deconvolution filtering.

From our point of view, this simple 2-D discrete model in (4.1) does not provide a convincing discretization from conventional convolution model. That is because time dimension is ignored during the discretization process without convincing proof. The conventional convolution model [5] [9] are the function of three independent variables - 2D spatial locations and time i.e. x, y, t . More importantly, no mapping between specific tissue properties and echo signals at an arbitrary spatial location has been established yet. In the conventional model [5] [9], the tissue property information refers to the combined function of tissue mass density and ultrasound propagation velocity. However, in the discrete model (4.1), tissue information is only conceptual 2-D sequences of signals without specific indications on how to connect the so-called tissue signals with real soft tissue. Hence, although the format of the simple 2-D discrete model in (4.1) is quite appealing, still it is quite general and symbolic such that it is not mathematically straightforward and accurate enough.

This motivates us to develop new discrete models of ultrasound echo processes. Unlike the simple model in (4.1), our developments are not based on the conventional convolution model [5] [9] but the theoretical model in [69]. The reason is that the model in [69] is also the basis of the conventional model [5] [9]. At the mean time, we have pointed out the problematic approximation in the derivation of conventional convolution model and proposed new convolution model with reasonable conditions in Chapter 3. Hence, the reliable way to perform discretization is to derive the discrete model directly from the original theoretical model.

This chapter considers the establishment of such discrete models which originate

from the theoretical model, comes with the discrete format and stands for strict mathematical proof. As shown in the discrete models proposed, the ultrasound echo processes are regarded as the product of interactions between ultrasound pulse spread function (PSF) and tissue of interest. At the mean time, specific model parameters are given qualitatively in the linear model. With such discrete models, the recovery of tissue information signals will be made much easier. In order to verify the proposed models, the ultrasound measurements collected by Ultrasonix® Sonix RP are applied to generate real tissue images instead of envelop images of ultrasound echo signals. Ultrasonix® Sonix RP, is a reliable ultrasound machine and specially designed for research work in ultrasound.

The chapter is organized as follows. At the beginning, we develop the continuous ultrasound theoretical model further in 2-D framework to make it appropriate for the discretization later. Then based on the theoretical model, the corresponding discrete model is derived and verified by simulations and experiments. What's more, another discrete model based on the new convolution model in Chapter 3 is also developed and verified.

4.2 Continuous Model of Ultrasound Echo Signals

This section is the preparation part for discretization. It may be ignored if the basic knowledge are enough to facilitate the further derivations for discretization.

4.2.1 Calculation of the scattered field

The wave equation in [65] and is

$$\nabla^2 p_1 - \frac{1}{c_0^2} \frac{\partial^2 p_1}{\partial t^2} = -\frac{2\Delta c}{c_0^3} \frac{\partial^2 p_1}{\partial t^2} + \frac{1}{\rho_0} \nabla(\tilde{\rho}) \cdot \nabla p_1, \quad (4.2)$$

where p_1 is the pressure variation resulting from the ultrasound wave w.r.t. $\vec{r}_1$ and t , $\tilde{c}$ and $\tilde{\rho}$ are slight differences from the mean propagation velocity and density in the undisturbed medium, respectively, ∇ is the gradient operator, and Δc is in (2.11).

The two terms on the right of Equation (4.2) are the scattering terms which disappear in a homogeneous medium. Now we compute the scattered field from a small inhomogeneity located in a homogeneous surrounding. The inhomogeneity area is represented by $\vec{r}_1$ and enclosed in the volume V . The pressure of the scattered field is obtained by means of integrating all the spherical waves from the scattering region V and using the time dependent Green's function for unbounded space.

Hence, the scattered field is:

$$P_s(\vec{r}_{tr}, t) = \int_{S_{ap}} \int_V \int_T \left[\frac{1}{\rho_0} \nabla(\tilde{\rho}(\vec{r}_1)) \cdot \nabla p_i(\vec{r}_1, t_1) - \frac{2\tilde{c}(\vec{r}_1)}{c_0^3} \frac{\partial^2 p_i(\vec{r}_1, t_1)}{\partial t^2} \right]. \quad (4.3)$$

$$G(\vec{r}_1, t_1 | \vec{r}_{tr} + \vec{r}_{ap}, t) dt_1 d^3 \vec{r}_1,$$

where $G(\cdot)$ is the boundary space Green's function:

$$G(\vec{r}_1, t_1 | \vec{r}_{tr} + \vec{r}_{ap}, t) = \frac{\delta(t - t_1 - \frac{|\vec{r}_{tr} + \vec{r}_{ap} - \vec{r}_1|}{c_0})}{2\pi |\vec{r}_{tr} + \vec{r}_{ap} - \vec{r}_1|}, \quad (4.4)$$

$d^3 \vec{r}_1$ means integrating w.r.t. $\vec{r}_1$ over the volume V , p_i can be referred in Section.2.2.3, $\vec{r}_{tr}$ is the location of the transducer, $\vec{r}_{ap}$ is the location of the aperture of transducer.

4.2.2 Pressure Variation $p_1(x, y, z, t)$

The calculation is to obtain the incident pressure of the ultrasound waves on the tissue propagating from ultrasound transducer. Here, it is assumed that there is no other sources existing in the tissue. By employing the velocity potential $\Psi(\vec{r}, t)$, the field is conveniently calculated, which satisfies the following wave equation for the homogeneous medium:

$$\nabla^2 \Psi(\vec{r}_1, t) - \frac{1}{c_0^2} \frac{\partial^2 \Psi(\vec{r}_1, t)}{\partial t^2} = 0, \quad (4.5)$$

where ∇ is the gradient operator, c_0 is the mean propagation velocity of ultrasound waves in the medium, $\vec{r}_1$ is the spatial location vector referring to one point in the medium, $\vec{r}_{tr}$ is the location of the transducer, t is the travelling time of the waves from the transducer to the point with its location $\vec{r}_1$ and can be calculated by

$$t = \frac{|\vec{r}_1 - \vec{r}_{tr}|}{c_0}, \quad (4.6)$$

The pressure is calculated from:

$$p_i(\vec{r}, t) = \rho_0 \frac{\partial \Psi(\vec{r}, t)}{\partial t}. \quad (4.7)$$

The solution to the homogeneous wave equation is [Jensen,1999]:

$$\Psi(\vec{r}_{tr} + \vec{r}_{ap}, t) = \int_{S_{ap}} \int_T \nu(\vec{r}_{tr} + \vec{r}_{ap}, t_1) G(\vec{r}_1, t | \vec{r}_{tr} + \vec{r}_{ap}, t_1) dt_1 d^2 \vec{r}_{ap}, \quad (4.8)$$

where S_{ap} denotes the transducer surface, T is the reception time interval and $t_1 = |\vec{r}_1 - \vec{r}_{tr}|/c_0$, $\nu(\vec{r}_{tr} + \vec{r}_{ap}, t)$ identifies the particle velocity normal to the transducer surface with $\vec{r}_{tr}$ referring to the position of the transducer and $\vec{r}_{ap}$ the location of a point on the transducer surface relative to $\vec{r}_{tr}$, $G(\vec{r}_1, t | \vec{r}_{tr} + \vec{r}_{ap}, t_1)$ is the Green's

function for a bounded medium and is

$$G(\vec{r}_1, t | \vec{r}_{tr} + \vec{r}_{ap}, t_1) = \frac{\delta(t - t_1 - \frac{|\vec{r}_1 - \vec{r}_{tr} - \vec{r}_{ap}|}{c_0})}{2\pi|\vec{r}_1 - \vec{r}_{tr} - \vec{r}_{ap}|}, \quad (4.9)$$

with $|\vec{r}_1 - \vec{r}_{tr} - \vec{r}_{ap}|$ being the distance from S_{ap} to the point where the field is calculated and c_0 the mean propagation velocity. It is assumed that radiation is into an isotropic, homogeneous, non-dissipative medium.

4.2.3 2-D Continuous Ultrasound Signal Model

For single-element transducer, signals propagate in both x direction and y direction are modeled as below.

4.2.3.1 Velocity potential

Velocity potential is

$$\Psi(x, y, t) = \int_T \nu(t_1) \frac{\delta\left(t - t_1 - \frac{\sqrt{(x-x_0)^2 + (y-y_0)^2}}{c_0}\right)}{2\pi\sqrt{(x-x_0)^2 + (y-y_0)^2}} dt_1, \quad (4.10)$$

where (x_0, y_0) is the location of transducer, and (x, y) is the spacial location of interest.

If assuming

$$t_{[(x,y)(x_0,y_0)]} = \frac{\sqrt{(x-x_0)^2 + (y-y_0)^2}}{c_0}, \quad (4.11)$$

then (4.10) becomes

$$\Psi(x, y, t) = \frac{\nu(t - t_{[(x,y)(x_0,y_0)]})}{2\pi\sqrt{(x-x_0)^2 + (y-y_0)^2}}, \quad (4.12)$$

4.2.3.2 Incident pressure

$$\begin{aligned}
p_i(x, y, t) &= \rho_0 \frac{\partial \Psi(x, y, t)}{\partial t} \\
&= \frac{\rho_0}{2\pi \sqrt{(x - x_0)^2 + (y - y_0)^2}} \frac{\partial \nu(t - t_{[(x, y)(x_0, y_0)]})}{\partial t} \quad (4.13)
\end{aligned}$$

Then its gradient is

$$\begin{aligned}
\nabla p_i(x, y, t) &= \frac{\partial p_i(x, y, t)}{\partial x} + \frac{\partial p_i(x, y, t)}{\partial y} \\
&= \frac{-\rho_0(x - x_0)}{2\pi \left(\sqrt{(x - x_0)^2 + (y - y_0)^2} \right)^3} \frac{\partial \nu(t - t_{[(x, y)(x_0, y_0)]})}{\partial t} \\
&\quad + \frac{\rho_0}{2\pi \left(\sqrt{(x - x_0)^2 + (y - y_0)^2} \right)^2} \frac{\partial^2 \nu(t - t_{[(x, y)(x_0, y_0)]})}{\partial t^2} \frac{\partial(t - t_{[(x, y)(x_0, y_0)]})}{\partial x} \\
&\quad + \frac{-\rho_0(y - y_0)}{2\pi \left(\sqrt{(x - x_0)^2 + (y - y_0)^2} \right)^3} \frac{\partial \nu(t - t_{[(x, y)(x_0, y_0)]})}{\partial t} \\
&\quad + \frac{\rho_0}{2\pi \left(\sqrt{(x - x_0)^2 + (y - y_0)^2} \right)^2} \frac{\partial^2 \nu(t - t_{[(x, y)(x_0, y_0)]})}{\partial t^2} \frac{\partial(t - t_{[(x, y)(x_0, y_0)]})}{\partial y} \\
&= \frac{-\rho_0[(x - x_0) + (y - y_0)]}{2\pi \left(\sqrt{(x - x_0)^2 + (y - y_0)^2} \right)^3} \frac{\partial \nu(t - t_{[(x, y)(x_0, y_0)]})}{\partial t} \\
&\quad - \frac{\rho_0[(x - x_0) + (y - y_0)]}{2\pi c_0 [(x - x_0)^2 + (y - y_0)^2]} \frac{\partial^2 \nu(t - t_{[(x, y)(x_0, y_0)]})}{\partial t^2} \\
&= \frac{-\rho_0[(x - x_0) + (y - y_0)]}{2\pi [(x - x_0)^2 + (y - y_0)^2]} \\
&\quad \cdot \left(\frac{1}{\left(\sqrt{(x - x_0)^2 + (y - y_0)^2} \right)} \frac{\partial}{\partial t} + \frac{1}{c_0} \frac{\partial^2}{\partial t^2} \right) \nu(t - t_{[(x, y)(x_0, y_0)]}). \quad (4.14)
\end{aligned}$$

And the second-order partial differentiation becomes

$$\begin{aligned}
 \frac{\partial^2 p_i(x, y, t)}{\partial t^2} &= \frac{\partial^2}{\partial t^2} \left(\frac{\rho_0}{2\pi \left(\sqrt{(x - x_0)^2 + (y - y_0)^2} \right)} \frac{\partial \nu(t - t_{[(x, y)(x_0, y_0)]})}{\partial t} \right) \\
 &= \frac{\rho_0}{2\pi \left(\sqrt{(x - x_0)^2 + (y - y_0)^2} \right)} \frac{\partial^3 \nu(t - t_{[(x, y)(x_0, y_0)]})}{\partial t^3} \quad (4.15)
 \end{aligned}$$

4.2.3.3 Scattered pressure

$$\begin{aligned}
p_s(x, y, t) &= \int_{X_1} \int_{Y_1} \int_T \left[\frac{1}{\rho_0} \nabla(\tilde{\rho}(x_1, y_1)) \cdot \nabla p_i(x_1, y_1, t_1) - \frac{2\tilde{c}(x_1, y_1)}{c_0^3} \frac{\partial^2 p_i(x_1, y_1, t_1)}{\partial t_1^2} \right] \\
&\quad G(x, y, t|x_1, y_1, t_1) dt_1 dx_1 dy_1 \\
&= \int_{X_1} \int_{Y_1} \int_T \left[\frac{-\nabla(\tilde{\rho}(x_1, y_1))[(x_1 - x_0) + (y_1 - y_0)]}{2\pi[(x_1 - x_0)^2 + (y_1 - y_0)^2]} \right. \\
&\quad \cdot \left(\frac{1}{\left(\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}\right)} \frac{\partial}{\partial t} + \frac{1}{c_0} \frac{\partial^2}{\partial t^2} \right) \\
&\quad \left. - \frac{2\rho_0\tilde{c}(x_1, y_1)}{2\pi c_0^3 \left(\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}\right)} \frac{\partial^3}{\partial t^3} \right] \\
&\quad \cdot \nu(t_1 - t_{[(x_1, y_1)(x_0, y_0)]}) \cdot G(x, y, t|x_1, y_1, t_1) dt_1 dx_1 dy_1 \\
&= \int_{X_1} \int_{Y_1} \int_T \frac{-1}{2\pi\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}} \\
&\quad \cdot \left[\frac{\nabla(\tilde{\rho}(x_1, y_1))[(x_1 - x_0) + (y_1 - y_0)]}{(x_1 - x_0)^2 + (y_1 - y_0)^2} \frac{\partial}{\partial t_1} \right. \\
&\quad + \frac{\nabla(\tilde{\rho}(x_1, y_1))[(x_1 - x_0) + (y_1 - y_0)]}{c_0\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}} \frac{\partial^2}{\partial t_1^2} \\
&\quad \left. + \frac{2\rho_0\tilde{c}(x_1, y_1)}{c_0^3} \frac{\partial^3}{\partial t_1^3} \right] \\
&\quad \cdot \nu(t_1 - t_{[(x_1, y_1)(x_0, y_0)]}) \cdot G(x, y, t|x_1, y_1, t_1) dt_1 dx_1 dy_1 \\
&= \int_{X_1} \int_{Y_1} \int_T \frac{-1}{2\pi\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}} \\
&\quad \cdot \left[\frac{\nabla(\tilde{\rho}(x_1, y_1))[(x_1 - x_0) + (y_1 - y_0)]}{(x_1 - x_0)^2 + (y_1 - y_0)^2} \frac{\partial}{\partial t_1} \right. \\
&\quad + \frac{\nabla(\tilde{\rho}(x_1, y_1))[(x_1 - x_0) + (y_1 - y_0)]}{c_0\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}} \frac{\partial^2}{\partial t_1^2} \\
&\quad \left. + \frac{2\rho_0\tilde{c}(x_1, y_1)}{c_0^3} \frac{\partial^3}{\partial t_1^3} \right] \\
&\quad \cdot \nu(t_1 - t_{[(x_1, y_1)(x_0, y_0)]}) \cdot \frac{\delta(t - t_1 - t_{[(x_1, y_1)(x_0, y_0)]})}{2\pi\sqrt{(x - x_1)^2 + (y - y_1)^2}} dt_1 dx_1 dy_1
\end{aligned} \tag{4.16}$$

$$\begin{aligned}
&= \int_{X_1} \int_{Y_1} \frac{-1}{4\pi^2 \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \sqrt{(x - x_1)^2 + (y - y_1)^2}} \\
&\quad \left[\frac{\nabla(\tilde{\rho}(x_1, y_1))[(x_1 - x_0) + (y_1 - y_0)]}{(x_1 - x_0)^2 + (y_1 - y_0)^2} \frac{\partial}{\partial t_1} \right. \\
&\quad \left. + \frac{\nabla(\tilde{\rho}(x_1, y_1))[(x_1 - x_0) + (y_1 - y_0)]}{c_0 \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}} \frac{\partial^2}{\partial t_1^2} + \frac{2\rho_0 \tilde{c}(x_1, y_1)}{c_0^3} \frac{\partial^3}{\partial t_1^3} \right] \\
&\quad \nu(t_1 - t_{[(x_1, y_1)(x_0, y_0)]}) \Big|_{t_1=t-t_{[(x, y)(x_1, y_1)]}} dx_1 dy_1,
\end{aligned} \tag{4.17}$$

When $(x, y) = (x_0, y_0)$, (4.17) becomes

$$\begin{aligned}
p_s(x_0, y_0, t) &= \int_{X_1} \int_{Y_1} \frac{-1}{4\pi^2 [(x_1 - x_0)^2 + (y_1 - y_0)^2]} \\
&\quad \left[\frac{\nabla(\tilde{\rho}(x_1, y_1))[(x_1 - x_0) + (y_1 - y_0)]}{(x_1 - x_0)^2 + (y_1 - y_0)^2} \frac{\partial}{\partial t_1} \right. \\
&\quad \left. + \frac{\nabla(\tilde{\rho}(x_1, y_1))[(x_1 - x_0) + (y_1 - y_0)]}{c_0 \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}} \frac{\partial^2}{\partial t_1^2} + \frac{2\rho_0 \tilde{c}(x_1, y_1)}{c_0^3} \frac{\partial^3}{\partial t_1^3} \right] \\
&\quad \nu(t_1 - t_{[(x_1, y_1)(x_0, y_0)]}) \Big|_{t_1=t-t_{[(x_1, y_1)(x_0, y_0)]}} dx_1 dy_1
\end{aligned} \tag{4.18}$$

4.3 Continuous Model for Discretization

The continuous model of ultrasound echo processes for further discretization has been derived in Section 4.2.3. From the previous development, the model indicates that, in practice, the received echo signals are products of interactions between pulse excitation and tissue of imaging interest. In more details, for an ultrasound receiver located at a fixed position, its echo signals are regarded as a time trace, which is composed of a 2-D integration over the whole cross-section plain where the ultrasound waves propagate. The integrand consists of several terms of multi-order

derivatives of input particle velocity and tissue characteristic parameters.

The specific mathematical form of the continuous model is

$$\begin{aligned}
 p_s(x_0, y_0, t) = & \int_{X_1} \int_{Y_1} \frac{-1}{4\pi^2 [(x_1 - x_0)^2 + (y_1 - y_0)^2]} \\
 & \left[\frac{\nabla(\tilde{\rho}(x_1, y_1))[(x_1 - x_0) + (y_1 - y_0)]}{(x_1 - x_0)^2 + (y_1 - y_0)^2} \frac{\partial}{\partial t_1} \right. \\
 & \left. + \frac{\nabla(\tilde{\rho}(x_1, y_1))[(x_1 - x_0) + (y_1 - y_0)]}{c_0 \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}} \frac{\partial^2}{\partial t_1^2} + \frac{2\rho_0 \tilde{c}(x_1, y_1)}{c_0^3} \frac{\partial^3}{\partial t_1^3} \right] \\
 & \nu(t_1 - t_{10}^{(2)}) \Big|_{t_1=t-t_{10}^{(2)}} dx_1 dy_1,
 \end{aligned} \tag{4.19}$$

where $p_s(x, y, t)$ is ultrasound echo signals received by ultrasound receiver located at (x, y) at the time of t . (x_0, y_0) is the coordinate of the location of ultrasound receiver. X_1, Y_1 are the integrating ranges in both x axis and y axis, respectively, in the ultrasound propagation plain. (x_1, y_1) is the coordinate of an arbitrary location inside the tissue of imaging interest. $\tilde{\rho}(x_1, y_1)$ is the mass density variation function of the tissue, which is one of the main tissue characteristics to image. $\tilde{c}(x_1, y_1)$ is the velocity variation function of wave propagation, which is also one of the main tissue characteristics to image. $\nu(t)$ is the particle velocity on the surface of ultrasound receiver; $t_{10}^{(2)}$ is the time for ultrasound waves to propagate from (x_1, y_1) to (x_0, y_0) , which is defined as:

$$t_{10}^{(2)} = \frac{\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}}{c_0}; \tag{4.20}$$

ρ_0, c_0 are the average mass density of the tissue and average propagation velocity of ultrasound waves, respectively.

The main concern in ultrasound imaging is how to extract the tissue characteristic parameters, such as $\tilde{\rho}(x, y)$ and $\tilde{c}(x, y)$, from the echo signals $p_s(x_0, y_0, t)$ accurately. Then the continuous model in (4.19) will be under more manipulations in the later

part of this chapter.

In a word, ultrasound signal processing is applied in order to recover the tissue property parameters from the noisy received ultrasound echo signals as much and accurately as possible. Hence, the tissue properties can be imaged.

With the approximation on the relationship between tissue mass density and ultrasound propagation velocity in Lemma 3.4.1, Section 3.4.3, ultrasound received echo signals becomes

$$\begin{aligned}
 p_s(x_0, y_0, t) = & \int_{X_1} \int_{Y_1} \frac{-1}{4\pi^2 [(x_1 - x_0)^2 + (y_1 - y_0)^2]} \\
 & \left[\frac{\nabla(\tilde{\rho}(x_1, y_1)) [(x_1 - x_0) + (y_1 - y_0)]}{(x_1 - x_0)^2 + (y_1 - y_0)^2} \frac{\partial}{\partial t_1} \right. \\
 & + \frac{\nabla(\tilde{\rho}(x_1, y_1)) [(x_1 - x_0) + (y_1 - y_0)]}{c_0 \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}} \frac{\partial^2}{\partial t_1^2} \\
 & \left. - \frac{2\rho_0 a_1 \tilde{\rho}(x, y)}{c_0^3} \frac{\partial^3}{\partial t_1^3} \right] \nu(t_1 - t_{10}^{(2)}) \Big|_{t_1=t-t_{10}^{(2)}} dx_1 dy_1. \quad (4.21)
 \end{aligned}$$

It is shown in (4.21) that, the echo signal $p_s(x, y, t)$ in (4.19) is manipulated to be the interactions simply between tissue mass density variation $\tilde{\rho}(x, y)$ and input pulse signals $\nu(t)$. Hence, the complicated model (4.19) is reduced to be a problem of parameter estimation, given the measured echo signals and input pulse signals.

In the following sections, the simplified model (4.21) will be under discretization and other more manipulations. New discrete models of ultrasound echo processes with straightforward representations and clear indication for further processing will be generated.

4.4 Discretization Description

In this section, the model (4.21) developed in Sec 4.3 will be discretized to develop a mathematical connection between conventional ultrasound echo signal model [9].

The followings are the assumptions on discretization and the discrete versions of the variables (x, y, t) are

$$x(n_x) = x_0 + T_x \cdot n_x \quad (4.22)$$

$$y(n_y) = y_0 + T_y \cdot n_y \quad (4.23)$$

$$t(n_t) = t_0 + T_t \cdot n_t \quad (4.24)$$

where T_x, T_y and T_t are step sizes of discretization in x dimension, y dimension, and time dimension, respectively.

To be simple, we can assume $x_0=y_0=t_0=0$, hence

$$x \xrightarrow{\text{Discretization}} T_x \cdot n_x \quad (4.25)$$

$$y \xrightarrow{\text{Discretization}} T_y \cdot n_y \quad (4.26)$$

$$t \xrightarrow{\text{Discretization}} T_t \cdot n_t \quad (4.27)$$

For an arbitrary function $f_c(x, y, t)$, it can be discretized into a discrete function $f_d(n_x, n_y, n_t)$, i.e.

$$\begin{aligned} f_c(x, y, t) &\xrightarrow{\text{Discretization}} f_c(T_x n_x, T_y n_y, T_t n_t) \\ &\triangleq f_d(n_x, n_y, n_t) \end{aligned} \quad (4.28)$$

Here, the functions in continuous convolution model (4.21) subjective to discretization operation are listed in the Table 4.1.

Table 4.1: Functions Under Discretization

Continuous	Discrete	Representation
$\tilde{\rho}(x, y)$	$\tilde{\rho}_d(n_x, n_y)$	Perturbation of Tissue Mass Density
$\tilde{c}(x, y)$	$\tilde{c}_d(n_x, n_y)$	Perturbation of Ultrasound Propagation Velocity
$\nu(t)$	$\nu_d(n_t)$	Particle Velocity On The Surface Of Ultrasound Transducer
$p_s(x, y, t)$	$p_{sd}(n_x, n_y, n_t)$	Scattered Pressure Received By Ultrasound Transducer
$t_{10}^{(2)}$	$k_{10}^{(2)}$	Time Interval For Ultrasound Waves To Propagate from (x_1, y_1) to (x_0, y_0)

For $f_c(x, y, t)$, its partial derivatives are discretized to be:

$$\begin{aligned}
\frac{\partial f_c(x, y, t)}{\partial t} &\xrightarrow{\text{Discretization}} \frac{z_t - 1}{T_t} f_d(n_x, n_y, n_t) = \frac{f_d(n_x, n_y, n_t + 1) - f_d(n_x, n_y, n_t)}{T_t} \quad (4.29) \\
\frac{\partial^2 f_c(x, y, t)}{\partial t^2} &\xrightarrow{\text{Discretization}} \frac{(z_t - 1)^2}{T_t^2} f_d(n_x, n_y, n_t) \\
&= \frac{f_d(n_x, n_y, n_t + 2) - 2f_d(n_x, n_y, n_t + 1) + f_d(n_x, n_y, n_t)}{T_t^2} \quad (4.30) \\
\frac{\partial^3 f_c(x, y, t)}{\partial t^3} &\xrightarrow{\text{Discretization}} \frac{(z_t - 1)^3}{T_t^3} f_d(n_x, n_y, n_t) \\
&= \frac{1}{T_t^3} [f_d(n_x, n_y, n_t + 3) - 3f_d(n_x, n_y, n_t + 2) \\
&\quad + 3f_d(n_x, n_y, n_t + 1) - f_d(n_x, n_y, n_t)], \quad (4.31)
\end{aligned}$$

and the gradient is discretized as

$$\begin{aligned}
\nabla f_c(x, y, t) &\xrightarrow{\text{Discretization}} \frac{z_x - 1}{T_x} f_d(n_x, n_y, n_t) + \frac{z_y - 1}{T_y} f_d(n_x, n_y, n_t) \\
&= \frac{f_d(n_x + 1, n_y, n_t) - f_d(n_x, n_y, n_t)}{T_x} \\
&\quad + \frac{f_d(n_x, n_y + 1, n_t) - f_d(n_x, n_y, n_t)}{T_y}, \quad (4.32)
\end{aligned}$$

where z_x, z_y, z_t are the differential operators in x dimension, y dimension and t dimension, respectively.

4.5 Ultrasound Discrete Model Based on Ultrasound Theoretical Model

4.5.1 2-D Ultrasound Discrete Model

With the discretization operations in Section 4.4, scattered pressure $p_s(x_0, y_0, t)$ in (4.21) is discretized to be $p_{s_d}(n_{x0}, n_{y0}, n_t)$, which is written as:

$$\begin{aligned}
 p_{s_d}(n_{x0}, n_{y0}, n_t) = & \sum_{n_{x1}} \sum_{n_{y1}} \frac{-1}{4\pi^2 [T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2]} \\
 & \left[\left\{ \left(\frac{z_x - 1}{T_x} \right) [\tilde{\rho}_d(n_{x1}, n_{y1})] + \left(\frac{z_y - 1}{T_y} \right) [\tilde{\rho}_d(n_{x1}, n_{y1})] \right\} \right. \\
 & \quad \cdot \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})] \left(\frac{z_t - 1}{T_t} \right)}{[T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2]} \\
 & \quad + \left\{ \left(\frac{z_x - 1}{T_x} \right) [\tilde{\rho}_d(n_{x1}, n_{y1})] + \left(\frac{z_y - 1}{T_y} \right) [\tilde{\rho}_d(n_{x1}, n_{y1})] \right\} \\
 & \quad \cdot \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})] \left(\frac{z_t - 1}{T_t} \right)^2}{c_0 \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\
 & \quad \left. - \frac{2\rho_0 a_1 \tilde{\rho}_d(n_{x1}, n_{y1})}{c_0^3} \left(\frac{z_t - 1}{T_t} \right)^3 \right] \\
 & \cdot \nu_d(n_t - 2k_{10}^{(2)})
 \end{aligned} \tag{4.33}$$

where $k_{10}^{(2)}$ is the discretized value of $\frac{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}}{c_0}$.

After reorganization, (4.33) becomes

$$\begin{aligned}
p_{s_d}(n_{x0}, n_{y0}, n_t) = & \sum_{n_{x1}} \sum_{n_{y1}} \frac{-1}{4\pi^2 T_x^2 [T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2]} \\
& \left\{ \left[\left(\frac{z_x - 1}{T_x} \right) [\tilde{\rho}_d(n_{x1}, n_{y1})] + \left(\frac{z_y - 1}{T_y} \right) [\tilde{\rho}_d(n_{x1}, n_{y1})] \right] \right. \\
& \cdot \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\
& \cdot \left(\frac{-1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} + \frac{1}{T_t^2 c_0} \right) \\
& \left. + \frac{2\rho_0 a_1 \tilde{\rho}_d(n_{x1}, n_{y1})}{T_t^3 c_0^3} \right] \\
& + \left[\left(\frac{z_x - 1}{T_x} \right) [\tilde{\rho}_d(n_{x1}, n_{y1})] + \left(\frac{z_y - 1}{T_y} \right) [\tilde{\rho}_d(n_{x1}, n_{y1})] \right] \\
& \cdot \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\
& \cdot \left(\frac{1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} - \frac{2}{T_t^2 c_0} \right) \\
& \left. - \frac{6\rho_0 a_1 \tilde{\rho}_d(n_{x1}, n_{y1})}{T_t^3 c_0^3} \right] z_t \\
& + \left[\left(\frac{z_x - 1}{T_x} \right) [\tilde{\rho}_d(n_{x1}, n_{y1})] + \left(\frac{z_y - 1}{T_y} \right) [\tilde{\rho}_d(n_{x1}, n_{y1})] \right] \\
& \cdot \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{c_0 T_t^2 \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\
& \left. + \frac{6\rho_0 a_1 \tilde{\rho}_d(n_{x1}, n_{y1})}{T_t^3 c_0^3} \right] z_t^2 \\
& - \left[\frac{2\rho_0 a_1 \tilde{\rho}_d(n_{x1}, n_{y1})}{T_t^3 c_0^3} \right] z_t^3 \Big\} \cdot \nu_d(n_t - 2k_{10}^{(2)}).
\end{aligned} \tag{4.34}$$

Then we reorganize (4.34) as the format of standard description of differential sys-

tems:

$$\begin{aligned}
 p_{s_d}(n_{x0}, n_{y0}, n_t) \triangleq & \sum_{n_{x1}} \sum_{n_{y1}} \frac{-1}{4\pi^2 T_x^2 [T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2]} \\
 & \left\{ \left[\left(\sum_{i=1}^2 \sum_{j=1}^2 A_{ij}^{xy} \tilde{\rho}_d(n_{x1} + i - 1, n_{y1} + j - 1) \right) + A_{00}^{xy} \right] \nu_d(n_t - 2k_{10}^{(2)}) \right. \\
 & + \left[\left(\sum_{k=1}^2 \sum_{l=1}^2 B_{kl}^{xy} \tilde{\rho}_d(n_{x1} + k - 1, n_{y1} + l - 1) \right) + B_{00}^{xy} \right] \nu_d(n_t - 2k_{10}^{(2)} + 1) \\
 & + \left[\left(\sum_{p=1}^2 \sum_{q=1}^2 C_{pq}^{xy} \tilde{\rho}_d(n_{x1} + p - 1, n_{y1} + q - 1) \right) + C_{00}^{xy} \right] \nu_d(n_t - 2k_{10}^{(2)} + 2) \\
 & \left. + \left[\left(\sum_{r=1}^2 \sum_{s=1}^2 D_{rs}^{xy} \tilde{\rho}_d(n_{x1} + r - 1, n_{y1} + s - 1) \right) + D_{00}^{xy} \right] \nu_d(n_t - 2k_{10}^{(2)} + 3) \right\}
 \end{aligned} \tag{4.35}$$

where

$$\left\{ \begin{aligned}
 & A_{22}^{xy} = 0; \\
 & A_{21}^{xy} = \frac{1}{T_x} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\
 & \quad \left(\frac{-1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} + \frac{1}{T_t^2 c_0} \right); \\
 & A_{12}^{xy} = \frac{1}{T_y} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\
 & \quad \left(\frac{-1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} + \frac{1}{T_t^2 c_0} \right); \\
 & A_{11}^{xy} = \left(\frac{-1}{T_x} + \frac{-1}{T_y} \right) \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\
 & \quad \left(\frac{-1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} + \frac{1}{T_t^2 c_0} \right) + \frac{2\rho_0 a_1}{T_t^3 c_0^3}; \\
 & A_{00}^{xy} = 0.
 \end{aligned} \right. \tag{4.36}$$

$$\left\{ \begin{array}{l} B_{22}^{xy} = 0; \\ B_{21}^{xy} = \frac{1}{T_x} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \cdot \\ \quad \left(\frac{1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} - \frac{2}{T_t^2 c_0} \right); \\ B_{12}^{xy} = \frac{1}{T_y} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \cdot \\ \quad \left(\frac{1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} - \frac{2}{T_t^2 c_0} \right); \\ B_{11}^{xy} = \left(\frac{-1}{T_x} + \frac{-1}{T_y} \right) \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\ \quad \cdot \left(\frac{1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} - \frac{2}{T_t^2 c_0} \right) - \frac{6\rho_0 a_1}{T_t^3 c_0^3}; \\ B_{00}^{xy} = 0. \end{array} \right. \quad (4.37)$$

$$\left\{ \begin{array}{l} C_{22}^{xy} = 0; \\ C_{21}^{xy} = \frac{1}{T_x T_t^2 c_0} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}}; \\ C_{12}^{xy} = \frac{1}{T_y T_t^2 c_0} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}}; \\ C_{11}^{xy} = \left(\frac{-1}{T_x} + \frac{-1}{T_y} \right) \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{T_t^2 c_0 \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\ \quad + \frac{6\rho_0 a_1}{T_t^3 c_0^3}; \\ C_{00}^{xy} = 0 \end{array} \right. \quad (4.38)$$

$$\begin{cases} D_{22}^{xy} = 0; \\ D_{21}^{xy} = 0; \\ D_{12}^{xy} = 0; \\ D_{11}^{xy} = -\frac{2\rho_0 a_1}{T_t^3 c_0^3}; \\ D_{00}^{xy} = 0 \end{cases} \quad (4.39)$$

If reorganizing (4.34) with respect to tissue density variation, then it becomes to be

$$\begin{aligned} p_{s_d}(n_{x0}, n_{y0}, n_t) = & \sum_{n_{x1}} \sum_{n_{y1}} \frac{-1}{4\pi^2 T_x^2 [T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2]} \\ & \left\{ \left[\sum_{i=1}^4 E_i^{xy} \nu(n_t - 2k_{10}^{(2)} + i - 1) \right] \tilde{\rho}_d(n_{x1} + 1, n_{y1}) \right. \\ & + \left[\sum_{j=1}^4 F_i^{xy} \nu(n_t - 2k_{10}^{(2)} + j - 1) \right] \tilde{\rho}_d(n_{x1}, n_{y1} + 1) \\ & \left. + \left[\sum_{m=1}^4 G_i^{xy} \nu(n_t - 2k_{10}^{(2)} + m - 1) \right] \tilde{\rho}_d(n_{x1}, n_{y1}) \right\} \quad (4.40) \end{aligned}$$

where

$$\begin{cases} E_1^{xy} = \frac{1}{T_x} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\ \quad \left(\frac{-1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} + \frac{1}{T_t^2 c_0} \right); \\ E_2^{xy} = \frac{1}{T_x} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\ \quad \left(\frac{1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} - \frac{2}{T_t^2 c_0} \right); \\ E_3^{xy} = \frac{1}{T_x T_t^2 c_0} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}}; \\ E_4^{xy} = 0. \end{cases} \quad (4.41)$$

$$\left\{ \begin{array}{l} F_1^{xy} = \frac{1}{T_y} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \cdot \\ \quad \left(\frac{-1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} + \frac{1}{T_t^2 c_0} \right); \\ F_2^{xy} = \frac{1}{T_y} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \cdot \\ \quad \left(\frac{1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} - \frac{2}{T_t^2 c_0} \right); \\ F_3^{xy} = \frac{1}{T_y T_t^2 c_0} \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \\ F_4^{xy} = 0 \end{array} \right. \quad (4.42)$$

$$\left\{ \begin{array}{l} G_1^{xy} = \left(\frac{-1}{T_x} + \frac{-1}{T_y} \right) \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \cdot \\ \quad \left(\frac{-1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} + \frac{1}{T_t^2 c_0} \right) + \frac{2\rho_0 a_1}{T_t^3 c_0^3}; \\ G_2^{xy} = \left(\frac{-1}{T_x} + \frac{-1}{T_y} \right) \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} \cdot \\ \quad \left(\frac{1}{T_t \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} - \frac{2}{T_t^2 c_0} \right) - \frac{6\rho_0 a_1}{T_t^3 c_0^3}; \\ G_3^{xy} = \left(\frac{-1}{T_x} + \frac{-1}{T_y} \right) \frac{[T_x(n_{x1} - n_{x0}) + T_y(n_{y1} - n_{y0})]}{T_t^2 c_0 \sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}} + \frac{6\rho_0 a_1}{T_t^3 c_0^3}; \\ G_4^{xy} = -\frac{2\rho_0 a_1}{T_t^3 c_0^3}. \end{array} \right. \quad (4.43)$$

4.5.2 Reduced 1-D Ultrasound Discrete Model

Usually, ultrasound signals are processed line by line and one-dimensional model analysis is good enough to produce reasonable estimation.

In one dimension, the arbitrary discrete n_{y1} becomes fixed and can be assumed to be equal to n_{y0} , i.e.

$$n_{y1} - n_{y0} = 0. \quad (4.44)$$

As a result, the differential operation in discrete y axis will produce none. For variation function of tissue mass density $\tilde{\rho}(n_x, n_y)$,

$$\frac{z_y - 1}{T_y} \tilde{\rho}(n_x, n_y) = 0. \quad (4.45)$$

Hence, (4.33) is simplified to be

$$p_{s_d}(n_{x0}, n_t) = \sum_{n_{x1}} \frac{-1}{4\pi^2 T_x^2 (n_{x1} - n_{x0})^2} \left[\frac{(z_x - 1) \tilde{\rho}_d(n_{x1})}{T_x^2 (n_{x1} - n_{x0})} \left(\frac{z_t - 1}{T_t} \right) \right. \\ \left. + \frac{(z_x - 1) \tilde{\rho}_d(n_{x1})}{T_x c_0} \frac{(z_t^2 - 2z_t + 1)}{T_t^2} \right. \\ \left. - \frac{2\rho_0 a_1 \tilde{\rho}_d(n_{x1})}{c_0^3} \frac{(z_t^3 - 3z_t^2 + 3z_t - 1)}{T_t^3} \right] \cdot \nu_d(n_t - 2k_{10}^{(1)}). \quad (4.46)$$

Then, after reorganization with respect to input particle velocity at different times,

$$\begin{aligned}
 p_{sd}(n_{x0}, n_t) = & \sum_{n_{x1}} \frac{-1}{4\pi^2 T_x^2 (n_{x1} - n_{x0})^2} \left[\left(\frac{-(z_x - 1)\tilde{\rho}_d(n_{x1})}{T_x^2 T_t (n_{x1} - n_{x0})} + \frac{(z_x - 1)\tilde{\rho}_d(n_{x1})}{T_x T_t^2 c_0} \right. \right. \\
 & + \frac{2\rho_0 a_1 \tilde{\rho}_d(n_{x1})}{T_t^3 c_0^3} \\
 & + \left(\frac{(z_x - 1)\tilde{\rho}_d(n_{x1})}{T_x^2 T_t (n_{x1} - n_{x0})} - \frac{2(z_x - 1)\tilde{\rho}_d(n_{x1})}{T_x T_t^2 c_0} \right. \\
 & \left. \left. - \frac{6\rho_0 a_1 \tilde{\rho}_d(n_{x1})}{T_t^3 c_0^3} \right) z_t \right. \\
 & + \left(\frac{(z_x - 1)\tilde{\rho}_d(n_{x1})}{T_x T_t^2 c_0} + \frac{6\rho_0 (a_1 \tilde{\rho}_d(n_{x1}) - a_1 + c_0)}{T_t^3 c_0^3} \right) z_t^2 \\
 & \left. - \frac{2\rho_0 a_1 \tilde{\rho}_d(n_{x1})}{T_t^3 c_0^3} z_t^3 \right] \nu_d(n_t - 2k_{10}^{(1)}). \quad (4.47)
 \end{aligned}$$

We can also define (4.47) to be the format of standard discrete systems:

$$\begin{aligned}
 p_{sd}(n_{x0}, n_t) \triangleq & \sum_{n_{x1}} \frac{-1}{4\pi^2 T_x^2 (n_{x1} - n_{x0})^2} \left\{ \left[\left(\sum_{r=1}^2 A_r^{(x)} \tilde{\rho}_d(n_{x1} + r - 1) \right) + A_0^{(x)} \right] \nu_d(n_t - 2k_{10}^{(1)}) \right. \\
 & + \left[\left(\sum_{q=1}^2 B_q^{(x)} \tilde{\rho}_d(n_{x1} + q - 1) \right) + B_0^{(x)} \right] \nu_d(n_t - 2k_{10}^{(1)} + 1) \\
 & + \left[\left(\sum_{p=1}^2 C_p^{(x)} \tilde{\rho}_d(n_{x1} + p - 1) \right) + C_0^{(x)} \right] \nu_d(n_t - 2k_{10}^{(1)} + 2) \\
 & \left. + \left[\left(\sum_{m=1}^1 D_m^{(x)} \tilde{\rho}_d(n_{x1} + m - 1) \right) + D_0^{(x)} \right] \nu_d(n_t - 2k_{10}^{(1)} + 3) \right\}, \quad (4.48)
 \end{aligned}$$

where

$$\begin{cases} A_2^{(x)} = \frac{-1}{T_x^2 T_t (n_{x1} - n_{x0})} + \frac{1}{T_x T_t^2 c_0}; \\ A_1^{(x)} = \frac{1}{T_x^2 T_t (n_{x1} - n_{x0})} - \frac{1}{T_x T_t^2 c_0} + \frac{2\rho_0 a_1}{T_t^3 c_0^3}; \\ A_0^{(x)} = 0; \end{cases} \quad (4.49)$$

$$\begin{cases} B_2^{(x)} = \frac{1}{T_x^2 T_t (n_{x1} - n_{x0})} - \frac{2}{T_x T_t^2 c_0}; \\ B_1^{(x)} = \frac{-1}{T_x^2 T_t (n_{x1} - n_{x0})} + \frac{2}{T_x T_t^2 c_0} - \frac{6\rho_0 a_1}{T_t^3 c_0^3}; \\ B_0^{(x)} = 0; \end{cases} \quad (4.50)$$

$$\begin{cases} C_2^{(x)} = \frac{1}{T_x T_t^2 c_0}; \\ C_1^{(x)} = \frac{6\rho_0 a_1}{T_t^3 c_0^3} - \frac{1}{T_x T_t^2 c_0}; \\ C_0^{(x)} = 0; \end{cases} \quad (4.51)$$

$$\begin{cases} D_1^{(x)} = \frac{-2\rho_0 a_1}{T_t^3 c_0^3}; \\ D_0^{(x)} = 0. \end{cases} \quad (4.52)$$

Based on (4.48), it is summed up that, echo on a specific discrete time index n_t and a certain depth n_{x0} is regarded as the summary of all the discrete echoes from all the locations inside the tissue of interest; For any discrete location n_{x1} , its echo is the summary of manipulations of input impulse $\nu(n_t)$ of up to the fourth order and the variation function of tissue mass density distribution $\tilde{\rho}(n_x)$ of up to the first order.

If reorganized (4.46) with respect to mass density property at different locations, (4.46) becomes to be

$$\begin{aligned} p_s(n_{x0}, n_t) = & \sum_{n_{x1}} \frac{-1}{4\pi^2 T_x^2 (n_{x1} - n_{x0})^2} \left\{ \left[\frac{1}{T_x T_t^2 c_0} z_t^2 + \left(\frac{1}{T_x^2 T_t (n_{x1} - n_{x0})} - \frac{2}{T_x T_t^2 c_0} \right) z_t \right. \right. \\ & + \left. \left(\frac{2}{T_x T_t^2 c_0} - \frac{1}{T_x^2 T_t (n_{x1} - n_{x0})} \right) \right] \nu_d(n_t - 2k_{10}^{(1)}) \tilde{\rho}_d(n_{x1} + 1) + \left[-\frac{2\rho_0 a_1}{T_t^3 c_0^3} z_t^3 \right. \\ & + \left. \left(\frac{6\rho_0 a_1}{T_t^3 c_0^3} - \frac{1}{T_x T_t^2 c_0} \right) z_t^2 + \left(\frac{-1}{T_x^2 T_t (n_{x1} - n_{x0})} - \frac{6\rho_0 a_1}{T_t^3 c_0^3} + \frac{2}{T_x T_t^2 c_0} \right) z_t \right. \\ & + \left. \left. \left(\frac{1}{T_x^2 T_t (n_{x1} - n_{x0})} + \frac{2\rho_0 a_1}{T_t^3 c_0^3} - \frac{1}{T_x T_t^2 c_0} \right) \right] \nu_d(n_t - 2k_{10}^{(1)}) \tilde{\rho}_d(n_{x1}) \right\}. \quad (4.53) \end{aligned}$$

Then, (4.53) is further simplified to be defined as:

$$p_s(n_{x0}, n_t) \triangleq \sum_{n_{x1}} \frac{-1}{4\pi^2 T_x^2 (n_{x1} - n_{x0})^2} \left\{ \left[\sum_{j=0}^2 E_j^{(x)} \nu_d(n_t - 2k_{10}^{(1)} + j) \right] \tilde{\rho}_d(n_{x1} + 1) + \left[\sum_{k=0}^3 F_k^{(x)} \nu_d(n_t - 2k_{10}^{(1)} + k) \right] \tilde{\rho}_d(n_{x1}) \right\}, \quad (4.54)$$

where $E_j^{(x)}, j = 0, 1, 2$ are

$$\begin{cases} E_2^{(x)} = \frac{1}{T_x T_t^2 c_0}; \\ E_1^{(x)} = \frac{1}{T_x^2 T_t (n_{x1} - n_{x0})} - \frac{2}{T_x T_t^2 c_0}; \\ E_0^{(x)} = \frac{1}{T_x T_t^2 c_0} - \frac{1}{T_x^2 T_t (n_{x1} - n_{x0})}; \end{cases} \quad (4.55)$$

$F_k^{(x)}, k = 0, 1, 2, 3$ are

$$\begin{cases} F_3^{(x)} = -\frac{2\rho_0 a_1}{T_t^3 c_0^3}; \\ F_2^{(x)} = \frac{6\rho_0 a_1}{T_t^3 c_0^3} - \frac{1}{T_x T_t^2 c_0}; \\ F_1^{(x)} = \frac{-1}{T_x^2 T_t (n_{x1} - n_{x0})} - \frac{6\rho_0 a_1}{T_t^3 c_0^3} + \frac{2}{T_x T_t^2 c_0}; \\ F_0^{(x)} = \frac{1}{T_x^2 T_t (n_{x1} - n_{x0})} + \frac{2\rho_0 a_1}{T_t^3 c_0^3} - \frac{1}{T_x T_t^2 c_0}; \end{cases} \quad (4.56)$$

If we regard the differential mass density $\tilde{\rho}_d(n_x)$ as inputs to the discrete model, the parameters in (4.54) can be simplified and thus, received pressure in (4.54) is

$$p_s(n_{x0}, n_t) = \sum_{n_{x1}} \left[E_v(n_{x1}, n_t) \tilde{\rho}_d(n_{x1} + 1) + F_v(n_{x1}, n_t) \tilde{\rho}_d(n_{x1}) \right], \quad (4.57)$$

where

$$E_v(n_{x1}, n_t) = \frac{-1}{4\pi^2 T_x^2 (n_{x1} - n_{x0})^2} \sum_{j=0}^2 E_j^{(x)} \nu_d(n_t - 2k_{10}^{(1)} + j); \quad (4.58)$$

$$F_v(n_{x1}, n_t) = \frac{-1}{4\pi^2 T_x^2 (n_{x1} - n_{x0})^2} \sum_{k=0}^3 F_k^{(x)} \nu_d(n_t - 2k_{10}^{(1)} + k). \quad (4.59)$$

Then, in order to describe the relationship between the received pressure p_s in (4.57), mass density $\tilde{\rho}_d$ are reorganized to be

$$\begin{pmatrix} p_s(n_{x0}, 1) \\ p_s(n_{x0}, 2) \\ \vdots \\ \vdots \\ p_s(n_{x0}, N) \end{pmatrix} = \begin{pmatrix} a_{1,1} & \dots & a_{1,i} & a_{1,i+1} & \dots \\ a_{2,1} & \dots & a_{2,i} & a_{2,i+1} & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{N,1} & \dots & a_{N,i} & a_{N,i+1} & \dots \end{pmatrix} \begin{pmatrix} \tilde{\rho}_d(1) \\ \vdots \\ \tilde{\rho}_d(i) \\ \tilde{\rho}_d(i+1) \\ \vdots \end{pmatrix} \quad (4.60)$$

where

$$\begin{aligned} a_{1,1} &= F_v(n_{x1}, n_t) \mid_{n_{x1}=1; n_t=1}; \\ &= F_v(1, 1) \end{aligned} \quad (4.61)$$

...

$$\begin{aligned} a_{1,i} &= [F_v(n_{x1}, n_t) + E_v(n_{x1} - 1, n_t)] \mid_{n_{x1}=i; n_t=1} \\ &= F_v(i, 1) + E_v(i - 1, 1); \end{aligned} \quad (4.62)$$

$$\begin{aligned} a_{1,i+1} &= [F_v(n_{x1}, n_t) + E_v(n_{x1} - 1, n_t)] \mid_{n_{x1}=i+1; n_t=1} \\ &= F_v(i + 1, 1) + E_v(i, 1); \end{aligned} \quad (4.63)$$

...

$$\begin{aligned} a_{2,1} &= [F_v(n_{x1}, n_t)] \mid_{n_{x1}=1; n_t=2} \\ &= F_v(1, 2); \end{aligned} \quad (4.64)$$

...

$$\begin{aligned} a_{2,i} &= [F_v(n_{x1}, n_t) + E_v(n_{x1} - 1, n_t)] \mid_{n_{x1}=i; n_t=2} \\ &= F_v(i, 2) + E_v(i - 1, 2); \end{aligned} \quad (4.65)$$

$$\begin{aligned} a_{2,i+1} &= [F_v(n_{x1}, n_t) + E_v(n_{x1} - 1, n_t)] \mid_{n_{x1}=i+1; n_t=2} \\ &= F_v(i + 1, 2) + E_v(i, 2); \end{aligned} \quad (4.66)$$

...

$$\begin{aligned} a_{N,1} &= [F_v(n_{x1}, n_t)] \mid_{n_{x1}=1; n_t=N} \\ &= F_v(1, N); \end{aligned} \quad (4.67)$$

...

$$\begin{aligned} a_{N,i} &= [F_v(n_{x1}, n_t) + E_v(n_{x1} - 1, n_t)] \mid_{n_{x1}=i; n_t=N} \\ &= F_v(i, N) + E_v(i - 1, N); \end{aligned} \quad (4.68)$$

$$\begin{aligned} a_{N,i+1} &= [F_v(n_{x1}, n_t) + E_v(n_{x1} - 1, n_t)] \mid_{n_{x1}=i+1; n_t=N} \\ &= F_v(i + 1, N) + E_v(i, N); \end{aligned} \quad (4.69)$$

...

In order to simplify the relationship between received echoes and tissue variation of mass density, received ultrasound echoes in (4.60) can be rewritten to be of the format of matrix equation:

$$P_S \triangleq \Omega \tilde{\rho}, \quad (4.71)$$

where

$$P_S = \begin{pmatrix} p_s(n_{x0}, 1) \\ p_s(n_{x0}, 2) \\ \vdots \\ \vdots \\ p_s(n_{x0}, N) \end{pmatrix}_{N_t \times 1} \quad (4.72)$$

$$\tilde{\rho} = \begin{pmatrix} \tilde{\rho}_d(1) \\ \vdots \\ \tilde{\rho}_d(i) \\ \tilde{\rho}_d(i+1) \\ \vdots \end{pmatrix}_{1 \times N_x} \quad (4.73)$$

$$\Omega = \begin{pmatrix} a_{1,1} & \dots & a_{1,i} & a_{1,i+1} & \dots \\ a_{2,1} & \dots & a_{2,i} & a_{2,i+1} & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{N,1} & \dots & a_{N,i} & a_{N,i+1} & \dots \end{pmatrix}_{N_t \times N_x} \quad (4.74)$$

where N_t is the maximum discrete time index; N_x is the maximum discrete depth index.

In the discrete model (4.71), ultrasound received echo signals are simply the functions of tissue variation of mass density and some parameters concerning input particle velocity. With such a model and priori information of input particle velocity, it is feasible to estimate the tissue variation of mass density and image the density distribution instead of amplitudes of echo signals.

Hence, new ultrasound images of tissue density distribution represent the tissue properties in a much more deterministic manner.

The following part concerns how to solve the tissue variation of mass density, given ultrasound echo signals, input particle velocity and proposed ultrasound discrete model. With (4.71), the estimated tissue variation of mass density is

$$\tilde{\rho} = \text{inverse}(\Omega) \bullet P_S, \quad (4.75)$$

where $\text{inverse}(\cdot)$ is to inverse the matrix.

Due to the length of input particle velocity, usually $N_t > N_x$. As a result, Ω is not a square matrix and its inversion should be a pseudo inversion instead of common inversion, i.e.

$$\Omega^\dagger = (\Omega^T \Omega)^{-1} \bullet \Omega^T. \quad (4.76)$$

Hence, finally (4.75) becomes

$$\tilde{\rho} = \Omega^\dagger \bullet P_S, \quad (4.77)$$

With (4.77), tissue variation of mass density can be estimated from the ultrasound

echo signals in theory.

4.5.3 Simulation Results for 1-D Ultrasound Discrete Model Based On Ultrasound Theoretical Model

Ultrasound discrete model of echo signal (4.71) has been derived mathematically and the model verification should be followed. Before the model verification applied to real ultrasound data, mathematical simulation should be performed to ensure that the discrete model (4.71) is able to work well under some typical circumstances.

In this section, two typical simulation scenarios are selected carefully to test the validity of the discrete model (4.71). The first scenario is that particle velocity is input to a tissue phantom model whose mass density increases linearly w.r.t. depth, i.e. its variation of mass density w.r.t. depth is none-zero constant; the second scenario is the same as the first except that its variation of mass density is with sine distribution.

4.5.3.1 Introduction

In the simulation process shown in Figure 4.1, ultrasound echo signals are generated line by line by the discrete model (4.71) based on input particle velocity and tissue phantom model; In the estimation process shown in Figure 4.2, tissue variation of mass density is estimated line by line and imaged by (4.77); finally the estimated tissue variation of mass density is compared with the original tissue density variation.

Simulation specifications are listed in Table 4.2, where T_x refers to step size in discrete x dimension; T_t refers to step size in discrete time dimension; c_0 refers to average ultrasound propagation velocity; ρ_0 is the average mass density of tissue of interest, which may change according to tissue properties.

Figure 4.1: Simulation Flowchart

Figure 4.2: Estimation Flowchart

Table 4.2: General Specifications in 1-D Ultrasound Echo Signal Model

Number	Name	Value
1	T_x	1.54e-004
2	T_t	1.000e-007
3	c_0	1540m/s
4	ρ_0	800-1260kg/m ³

This process has been shown in Figure 4.1, and the simulated ultrasound signals are generated by (4.54).

In the following two simulations, two different distributions of tissue mass density $\tilde{\rho}_d(n_{x1})$ will be input to discrete model (4.54) respectively. Thus, simulated ultrasound echo signals are generated and subject to further processing.

4.5.3.2 Simulation Scenario 1

In this simulation, it is assumed that the tissue of interest is a spherical soft tissue and hence, its cross section in 2-D imaging is a circle. And as show in Figure 4.3, its distribution of mass density increases linearly with respect to tissue depth, i.e. the deeper part of tissue have larger mass density. And its tissue variation of mass density is shown in Figure 4.4.

Model specifications in the simulation are listed in Table 4.3 while model parameters in Table 4.4. Average mass density all over the tissue is assumed to 1260kg/m³ in the simulation because 1260kg/m³ is the average tissue mass density of human muscle.

Figure 4.3: Original Linear Mass Density Image 70*240 Pixels

Figure 4.4: Original Tissue Variation of Linear Mass Density Image 70*240 Pixels

Table 4.3: Model Specification in Simulation Scenario 1

Number	Name	Value
1	T_x	1.54e-004
2	T_t	1.000e-007
3	c_0	1540m/s
4	ρ_0	1260kg/m ³

As shown in Table 4.4, the discrete model is specifically described as:

$$\begin{aligned}
 p_s(n_{x0}, n_t) = & \sum_{n_{x1}} \frac{-10^{22}}{(n_{x1} - n_{x0})^2} \left\{ \left[3.1499z_t^2 + \left(\frac{3.1499}{n_{x1} - n_{x0}} - 6.2997 \right) z_t \right. \right. \\
 & + \left. \left(\frac{-3.1499}{n_{x1} - n_{x0}} + 3.1499 \right) \right] \nu_d(n_t - 2k_{10}^{(1)}) \tilde{\rho}_d(n_{x1} + 1) \\
 & + \left[35.2647z_t^3 - 108.9423z_t^2 + \left(\frac{-3.1499}{n_{x1} - n_{x0}} + 112.0930 \right) z_t \right. \\
 & + \left. \left. \left(\frac{3.1499}{n_{x1} - n_{x0}} - 32.1140 \right) \right] \nu_d(n_t - 2k_{10}^{(1)}) \tilde{\rho}_d(n_{x1}) \right\}. \quad (4.78)
 \end{aligned}$$

Table 4.4: Model Parameters In Simulation Scenario 1

i	E_i^x	F_i^x
0	$\left(\frac{-2.7133}{n_{x1} - n_{x0}} + 2.7133 \right) \times 10^{15}$	$\left(\frac{2.7133}{n_{x1} - n_{x0}} - 27.663 \right) \times 10^{15}$
1	$\left(\frac{2.7133}{n_{x1} - n_{x0}} - 5.4266 \right) \times 10^{15}$	$\left(\frac{-2.7133}{n_{x1} - n_{x0}} + 96.557 \right) \times 10^{15}$
2	2.7133×10^{15}	-9.3843×10^{16}
3	-	3.0377×10^{16}

With the soft phantom model in Figure 4.3 and Figure 4.4, ultrasound echo signals are simulated and shown in Figure 4.5.

Figure 4.5: Simulated Linear Ultrasound Echoes

Then, based on the inversed model (4.77), the tissue variation of mass density can be estimated from the simulated ultrasound signals in Figure 4.5. As shown in Figure 4.6, the estimated tissue variation of mass density is quite similar to the original one shown in Figure 4.4. And then after cumulation, the estimated tissue mass density shown in Figure 4.7, also corroborate well with the original tissue mass density shown in Figure 4.3.

4.5.3.3 Simulation Scenario 2

In this simulation, it is assumed that the tissue of interest is a spherical soft tissue and hence, its cross section in 2-D imaging is a circle. And as show in Figure 4.8, its distribution of mass density is like sine waves. And its tissue variation of mass density is shown in Figure 4.9.

Figure 4.6: Estimated Tissue Variation of Linear Mass Density

Model specifications in the simulation are listed in Table 4.3 while model parameters in Table 4.4. Average mass density all over the tissue is assumed to $800\text{kg}/\text{m}^3$ in the simulation because $800\text{kg}/\text{m}^3$ is the average tissue mass density of tissue-mimic phantom.

Table 4.5: Model Specification in Simulation Scenario 2

Number	Name	Value
1	T_x	$1.54\text{e-}004$
2	T_t	$1.000\text{e-}007$
3	c_0	$1540\text{m}/\text{s}$
4	ρ_0	$800\text{kg}/\text{m}^3$

Figure 4.7: Estimated Linear Mass Density

As shown in Table 4.6, the discrete model is specifically described as:

$$\begin{aligned}
 p_s(n_{x0}, n_t) = & \sum_{n_{x1}} \frac{-10^{21}}{(n_{x1} - n_{x0})^2} \left\{ \left[31.499z_t^2 + \left(\frac{31.499}{n_{x1} - n_{x0}} - 62.997 \right) z_t \right. \right. \\
 & + \left. \left(\frac{-31.499}{n_{x1} - n_{x0}} + 31.499 \right) \right] \nu_d(n_t - 2k_{10}^{(1)}) \tilde{\rho}_d(n_{x1} + 1) \\
 & + \left[9.0664z_t^3 - 58.6974z_t^2 + \left(\frac{-31.499}{n_{x1} - n_{x0}} + 90.1961 \right) z_t \right. \\
 & + \left. \left. \left(\frac{31.499}{n_{x1} - n_{x0}} + 22.4321 \right) \right] \nu_d(n_t - 2k_{10}^{(1)}) \tilde{\rho}_d(n_{x1}) \right\}. \quad (4.79)
 \end{aligned}$$

With the soft phantom model in Figure 4.8 and Figure 4.9, ultrasound echo signals are simulated and shown in Figure 4.10.

Then, based on the inversed model (4.77), the tissue variation of mass density can be estimated from the simulated ultrasound signals in Figure 4.10. As shown in

Figure 4.8: Original Sine Mass Density

Table 4.6: Model Parameters In Simulation Scenario 2

i	E_i^x	F_i^x
0	$\left(\frac{-2.7133}{n_{x1} - n_{x0}} + 2.7133\right) \times 10^{15}$	$\left(\frac{2.7133}{n_{x1} - n_{x0}} + 1.9323\right) \times 10^{15}$
1	$\left(\frac{2.7133}{n_{x1} - n_{x0}} - 5.4266\right) \times 10^{15}$	$\left(\frac{-2.7133}{n_{x1} - n_{x0}} + 7.7695\right) \times 10^{15}$
2	2.7133×10^{15}	-5.0562×10^{15}
3	-	7.8098×10^{14}

Figure 4.11, the estimated tissue variation of mass density is quite similar to the original one shown in Figure 4.9. And then after cumulation, the estimated tissue mass density shown in Figure 4.12, also corroborate well with the original tissue

Figure 4.9: Original Tissue Variation of Sine Mass Density

mass density shown in Figure 4.8.

4.5.4 Experiment Results

Ultrasound discrete model of echo signal (4.71) has been derived and verified by simulation. In this section, model verification is applied to real ultrasound data and evaluate its practical performance.

In the experiment, tissue-mimic phantom used is shown in Figure 4.13. The phantom is "Precision Small Parts Grey Scale Phantoms(GAMMEX 404GS LE AND 404 LE)" made by *GAMMEX*[®], WI, USA. This phantom provides advanced technology for measuring image quality of small parts and intra-cavity ultrasound scanning systems.

In this section, two different targets in the tissue-mimic phantom are selected care-

Figure 4.10: Simulated Sine Ultrasound Echoes

fully to test the validity of the discrete model (4.71). In practice, only part of the tissue-mimic phantom is used due to the fact that the discrete model (4.71) is only valid when the average mass density of tissue of interest is constant or doesn't vary much.

As shown in Figure 4.14, the first experiment focuses on the pin targets of the tissue-mimic phantom. And the second experiment focuses on the cyst target in Figure 4.15.

4.5.4.1 Experiment on Pin Target

In this experiment, pin targets of the phantom in Figure 4.13, are selected to estimate their corresponding images of tissue density variation.

Experiment specifications are similar to those in simulations.

Figure 4.11: Estimated Tissue Variation of Sine Mass Density

Table 4.7: Model Specification in Experiment on Pin Target

Number	Name	Value
1	T_x	1.54e-004
2	T_t	1.000e-007
3	c_0	1540m/s
4	ρ_0	900kg/m ³

Table 4.8 presents the model parameters in the experiment on pin targets.

Hence, the discrete model is specifically described as:

Figure 4.12: Estimated Sine Mass Density

Table 4.8: Model Parameters In Experiment 1

i	E_i^x	F_i^x
0	$\left(\frac{-4.2166}{n_{x1} - n_{x0}} + 4.2166\right) \times 10^{14}$	$\left(\frac{4.2166}{n_{x1} - n_{x0}} + 1.1032\right) \times 10^{14}$
1	$\left(\frac{4.2166}{n_{x1} - n_{x0}} - 8.4331\right) \times 10^{14}$	$\left(\frac{-4.2166}{n_{x1} - n_{x0}} + 17.773\right) \times 10^{14}$
2	4.2166×10^{14}	-13.557×10^{14}
3	-	3.1133×10^{14}

$$\begin{aligned} p_s(n_{x0}, n_t) = & \sum_{n_{x1}} \frac{-10^{21}}{(n_{x1} - n_{x0})^2} \left\{ \left[4.8951 z_t^2 + \left(\frac{4.8951}{n_{x1} - n_{x0}} - 9.7900 \right) z_t \right. \right. \\ & \left. \left. + \left(\frac{-4.8951}{n_{x1} - n_{x0}} + 4.8951 \right) \right] \nu_d \left(n_t - 2k_{10}^{(1)} \right) \tilde{\rho}_d(n_{x1} + 1) \right. \\ & \left. + \left[3.6142 z_t^3 - 15.7383 z_t^2 + \left(\frac{-4.8951}{n_{x1} - n_{x0}} + 20.6327 \right) z_t \right. \right. \\ & \left. \left. + 4.8951 \right] \right\} \end{aligned}$$

Figure 4.13: Bmode Image Of Tissue-Mimic Phantom

Figure 4.14: B-mode Image Of Pin Target

Figure 4.15: B-mode Image Of Cyst Target

Then, tissue mass density estimation is performed and the estimated image of tissue variation of mass density is shown in Figure 4.16. From the comparison images in Figure 4.16, the speckles in original Bmode image have been removed and the contrast between tissue (white part) and media (black part) is improved a lot. In the mean time, the information of pin targets is kept well.

4.5.4.2 Experiment on Cyst Target

In this experiment, a cyst target of the phantom in Figure 4.13, is selected to estimate its corresponding image of tissue density variation.

Experiment specifications are similar to those in simulations, as shown in Table 4.9.

Table 4.9: Model Specification in Experiment on Cyst Target

Number	Name	Value
1	T_x	1.54e-004
2	T_t	1.000e-007
3	c_0	1540m/s
4	ρ_0	850kg/m ³

Table 4.10 presents the model parameters in the experiment on cyst target.

Hence, the discrete model is specifically described as:

Figure 4.16: Original Bmode Image (Left) and Estimated Tissue Image of Mass Density Variation (Right)

Table 4.10: Model Parameters In Experiment 2

i	E_i^x	F_i^x
0	$\left(\frac{-4.2166}{n_{x1} - n_{x0}} + 4.2166\right) \times 10^{14}$	$\left(\frac{4.2166}{n_{x1} - n_{x0}} + 2.1339\right) \times 10^{14}$
1	$\left(\frac{4.2166}{n_{x1} - n_{x0}} - 8.4331\right) \times 10^{14}$	$\left(\frac{-4.2166}{n_{x1} - n_{x0}} + 14.681\right) \times 10^{14}$
2	4.2166×10^{14}	-10.464×10^{14}
3	-	2.0826×10^{14}

$$\begin{aligned}
p_s(n_{x0}, n_t) = \sum_{n_{x1}} \frac{-10^{21}}{(n_{x1} - n_{x0})^2} & \left\{ \left[4.8951 z_t^2 + \left(\frac{4.8951}{n_{x1} - n_{x0}} - 9.7900 \right) z_t \right. \right. \\
& + \left. \left(\frac{-4.8951}{n_{x1} - n_{x0}} + 4.8951 \right) \right] \nu_d(n_t - 2k_{10}^{(1)}) \tilde{\rho}_d(n_{x1} + 1) \\
& + \left[2.4177 z_t^3 - 12.1477 z_t^2 + \left(\frac{-4.8951}{n_{x1} - n_{x0}} + 17.0432 \right) z_t \right. \\
& + \left. \left. \left(\frac{4.8951}{n_{x1} - n_{x0}} + 2.4772 \right) \right] \nu_d(n_t - 2k_{10}^{(1)}) \tilde{\rho}_d(n_{x1}) \right\}. \quad (4.81)
\end{aligned}$$

Then, tissue mass density estimation is performed and the estimated image of tissue variation of mass density is shown in Figure 4.17. From the comparison images in Figure 4.17, the speckles in original Bmode image have been removed to some extent and the contrast between tissue (white part) and media (black part) is improved a lot.

Figure 4.17: Original B-mode Image (Upper) and Estimated Tissue Image of Mass Density Variation (Lower)

4.6 Ultrasound Discrete Model Based on New Ultrasound Convolution Model

In Chapter 3, a new convolution model was proposed and provided new understanding to ultrasound echo process. During the course of its development, it was found there exists a dominant term in the received scattered pressure. Hence, in this section, the discrete version of the new model is developed.

4.6.1 New 2-D Ultrasound Discrete Model

With the discretization operations in Section 4.4, scattered pressure $p_s(x_0, y_0, t)$ in (3.77) is discretized to be $p_{s_d}(n_{x0}, n_{y0}, n_t)$, which is written as:

$$p_{s_d}(n_{x0}, n_{y0}, n_t) = \sum_{n_{x1}} \sum_{n_{y1}} \frac{1}{4\pi^2 \left[T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2 \right]} \frac{2\rho_0 a_1 \tilde{\rho}_d(n_{x1}, n_{y1})}{c_0^3} \left(\frac{z_t - 1}{T_t} \right)^3 \cdot \nu_d(n_t - 2k_{10}^{(2)}) \quad (4.82)$$

where $k_{10}^{(2)}$ is the discretized value of $\frac{\sqrt{T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2}}{c_0}$.

Then we reorganize (4.82) as the format of standard description of differential systems:

$$p_{s_d}(n_{x0}, n_{y0}, n_t) \triangleq \sum_{n_{x1}} \sum_{n_{y1}} \frac{1}{4\pi^2 T_x^2 \left[T_x^2(n_{x1} - n_{x0})^2 + T_y^2(n_{y1} - n_{y0})^2 \right]} \left\{ \frac{2\rho_0 a_1}{T_t^3 c_0^3} \tilde{\rho}_d(n_{x1}, n_{y1}) \right\} \nu_d(n_t - 2k_{10}^{(2)} + 3) \quad (4.83)$$

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If reorganizing (4.83) with respect to tissue density variation, then it becomes to be

$$p_{s_d}(n_{x0}, n_{y0}, n_t) = \sum_{n_{x1}} \sum_{n_{y1}} \frac{1}{4\pi^2 T_x^2 [T_x^2 (n_{x1} - n_{x0})^2 + T_y^2 (n_{y1} - n_{y0})^2]} \left[\frac{2\rho_0 a_1}{T_t^3 c_0^3} \nu(n_t - 2k_{10}^{(2)} + 3) \right] \tilde{\rho}_d(n_{x1}, n_{y1}) \quad (4.84)$$

4.6.2 New 1-D Ultrasound Discrete Model

Usually, ultrasound signals are processed line by line and one-dimensional model analysis is good enough to produce reasonable estimation.

In one dimension, the arbitrary discrete n_{y1} becomes fixed and can be assumed to be equal to n_{y0} , i.e.

$$n_{y1} - n_{y0} = 0. \quad (4.85)$$

As a result, the differential operation in discrete y axis will produce none. For variation function of tissue mass density $\tilde{\rho}(n_x, n_y)$,

$$\frac{z_y - 1}{T_y} \tilde{\rho}(n_x, n_y) = 0. \quad (4.86)$$

Hence, (4.84) is simplified to be

$$p_{s_d}(n_{x0}, n_t) = \sum_{n_{x1}} \frac{1}{4\pi^2 T_x^2 (n_{x1} - n_{x0})^2} \left[\frac{2\rho_0 a_1 \tilde{\rho}_d(n_{x1})}{T_t^3 c_0^3} z_t^3 \right] \nu_d(n_t - 2k_{10}^{(1)}). \quad (4.87)$$

Based on (4.87), it is summed up that, echo on a specific discrete time index n_t and a certain depth n_{x0} is regarded as the summary of all the discrete echoes from all the locations inside the tissue of interest. For any discrete location n_{x1} , its echo is the summary of manipulations of input impulse $\nu(n_t)$ of the third order and the

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variation function of tissue mass density distribution $\tilde{\rho}(n_x)$.

If reorganizing (4.87) with respect to mass density property at different locations, it becomes to be

$$\begin{aligned} p_s(n_{x0}, n_t) &= \sum_{n_{x1}} \frac{1}{4\pi^2 T_x^2 (n_{x1} - n_{x0})^2} \left\{ \frac{2\rho_0 a_1}{T_t^3 c_0^3} z_t^3 \nu_d(n_t - 2k_{10}^{(1)}) \tilde{\rho}_d(n_{x1}) \right\} \\ &\triangleq \sum_{n_{x1}} h(i, j) \tilde{\rho}_d(n_{x1}). \end{aligned} \quad (4.88)$$

where

$$h(i, j) = \frac{1}{4\pi^2 T_x^2 (n_{x1} - n_{x0})^2} \frac{2\rho_0 a_1}{T_t^3 c_0^3} \nu_d(n_t - 2k_{10}^{(1)} + 3) \Big|_{n_t=i, n_{x1}=j} \quad (4.89)$$

Then, in order to describe the relationship between the received pressure p_s in (4.88), mass density $\tilde{\rho}_d$ are reorganized to be

$$\begin{pmatrix} p_s(n_{x0}, 1) \\ p_s(n_{x0}, 2) \\ \vdots \\ \vdots \\ \vdots \\ p_s(n_{x0}, N) \end{pmatrix} = \begin{pmatrix} h_{1,1} & 0 & \dots & 0 & \dots & \dots \\ h_{2,1} & h_{2,2} & 0 & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & h_{i,i-1} & h_{i,i} & 0 & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & h_{N,N-1} & h_{N,N} & \dots \end{pmatrix} \begin{pmatrix} \tilde{\rho}_d(1) \\ \vdots \\ \tilde{\rho}_d(i) \\ \tilde{\rho}_d(i+1) \\ \vdots \\ \vdots \end{pmatrix}. \quad (4.90)$$

In order to simplify the relationship between received echoes and tissue variation of mass density, received ultrasound echoes in (4.90) can be rewritten to be of the format of matrix equation:

$$P_S \triangleq \Phi \tilde{\rho}, \quad (4.91)$$

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where

$$P_S = \begin{pmatrix} p_s(n_{x0}, 1) \\ p_s(n_{x0}, 2) \\ \vdots \\ \vdots \\ p_s(n_{x0}, N) \end{pmatrix}_{1 \times N_t} \quad (4.92)$$

$$\tilde{\rho} = \begin{pmatrix} \tilde{\rho}_d(1) \\ \vdots \\ \tilde{\rho}_d(i) \\ \tilde{\rho}_d(i+1) \\ \vdots \end{pmatrix}_{1 \times N_x} \quad (4.93)$$

$$\Phi = \begin{pmatrix} h_{1,1} & 0 & \dots & 0 & \dots \\ h_{2,1} & h_{2,2} & 0 & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & h_{i,i-1} & h_{i,i} & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & h_{N,N-1} & h_{N,N} \end{pmatrix}_{N_t \times N_x} \quad (4.94)$$

where N_t is the maximum discrete time index; N_x is the maximum discrete depth index.

In the discrete model (4.91), ultrasound received echo signals are simply the functions of tissue variation of mass density and some parameters concerning input particle velocity. With such a model and priori information of input particle ve-

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locity, it is feasible to estimate the tissue variation of mass density and image the density distribution instead of amplitudes of echo signals.

Hence, new ultrasound images of tissue density distribution represent the tissue properties in a much more deterministic manner.

The following part concerns how to solve the tissue variation of mass density, given ultrasound echo signals, input particle velocity and proposed ultrasound discrete model. With (4.91), the estimated tissue variation of mass density is

$$\tilde{\rho} = \text{inverse}(\Phi) \bullet P_S, \quad (4.95)$$

where $\text{inverse}(\cdot)$ is to inverse the matrix.

Due to the length of input particle velocity, usually $N_t > N_x$. As a result, Φ is not a square matrix and its inversion should be a pseudo inversion instead of common inversion, i.e.

$$\Phi^\dagger = (\Phi^T \Phi)^{-1} \bullet \Phi^T. \quad (4.96)$$

Hence, finally (4.95) becomes

$$\tilde{\rho} = \Phi^\dagger \bullet P_S, \quad (4.97)$$

With (4.97), tissue variation of mass density can be estimated from the ultrasound echo signals in theory.

4.6.3 Simulation Results for 1-D Ultrasound Discrete Model Based on New Convolution Model

The ultrasound discrete model of echo signal (4.91) based on new convolution model has been derived mathematically and the model verification will follow. Before the model verification applied to real ultrasound data, mathematical simulations are performed to ensure that the discrete model (4.91) is able to improve ultrasound image quality.

In this section, two typical simulation scenarios are designed carefully to test the validity of the discrete model (4.91). The first scenario is that particle velocity is input to a tissue phantom model whose mass density increases linearly w.r.t. depth, i.e. its variation of mass density w.r.t. depth is none-zero constant; the second scenario is the same as the first except that its variation of mass density is with sine distribution.

4.6.3.1 Introduction

As shown by Figure 4.1, ultrasound echo signals are generated line by line by the discrete model (4.91) based on input particle velocity and tissue phantom model; In the estimation process shown in Figure 4.2, tissue variation of mass density is estimated line by line and imaged by (4.97); finally the estimated tissue variation of mass density is compared with the original tissue density variation.

Simulation specifications are listed in Table 4.11, where T_x refers to step size in discrete x dimension; T_t refers to step size in discrete time dimension; c_0 refers to average ultrasound propagation velocity; ρ_0 is the average mass density of tissue of interest which is assumed as shown by Table 3.1; It may change according to tissue properties.

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Table 4.11: General Specifications in New 1-D Ultrasound Echo Signal Model

Number	Name	Value
1	T_x	1.54e-004 m
2	T_t	1.000e-007 s
3	c_0	1580 m/s
4	ρ_0	1061 kg/m^3

4.6.3.2 Simulation Scenario 1

In this simulation, it is assumed that the tissue of interest is a spherical soft tissue and hence, its cross section in 2-D imaging is a circle. And as show in Figure 4.3, its distribution of mass density increases linearly with respect to tissue depth, i.e. the deeper part of tissue have larger mass density. And its tissue variation of mass density is shown in Figure 4.4.

The discrete model is specifically described as:

$$p_s(n_{x0}, n_t) = \sum_{n_{x1}} \frac{-10^{16}}{(n_{x1} - n_{x0})^2} \left\{ 2.6125 z_t^3 \nu_d \left(n_t - 2k_{10}^{(1)} \right) \tilde{\rho}_d(n_{x1}) \right\}. \quad (4.98)$$

With the soft phantom model in Figure 4.3 and Figure 4.4, ultrasound echo signals are simulated and shown in Figure 4.5.

Then, based on the inverse model (4.97), the tissue variation of mass density can be estimated from the simulated ultrasound signals. As shown in Figure 4.18, the estimated tissue variation of mass density is quite similar to the original one shown in Figure 4.4. And then after cumulation, the estimated tissue mass density shown in Figure 4.19, also corroborate well with the original tissue mass density shown in Figure 4.3.

Figure 4.18: Estimated Tissue Variation of Linear Mass Density

4.6.3.3 Simulation Scenario 2

In this simulation, it is assumed that the tissue of interest is a spherical soft tissue and hence, its cross section in 2-D imaging is a circle. And as show in Figure 4.8, its distribution of mass density is with sinusoidal distribution. And its tissue variation of mass density is shown in Figure 4.9.

Model specifications in the simulation are listed in Table 4.11.

The discrete model is specifically described as:

$$p_s(n_{x0}, n_t) = \sum_{n_{x1}} \frac{-10^{16}}{(n_{x1} - n_{x0})^2} \left\{ 3.0377 z_t^3 \nu_d \left(n_t - 2k_{10}^{(1)} \right) \tilde{\rho}_d(n_{x1}) \right\}. \quad (4.99)$$

With the soft phantom model in Figure 4.8 and Figure 4.9, ultrasound echo signals are simulated. Then, based on the inverse model (4.97) and specific model (4.99),

Figure 4.19: Estimated Linear Mass Density

the tissue variation of mass density can be estimated from the simulated ultrasound signals. As shown in Figure 4.20, the estimated tissue variation of mass density is quite similar to the original one shown in Figure 4.9. And then after cumulation, the estimated tissue mass density shown in Figure 4.21, also corroborate well with the original tissue mass density shown in Figure 4.8.

4.6.4 Experiment Results

Ultrasound discrete model of echo signal (4.91) has been derived mathematically and verified by simulation. In this section, model verification is applied to real ultrasound data and evaluate its performance practically.

In the experiment, tissue-mimic phantom used is shown in Figure 4.13. The phantom is "Precision Small Parts Grey Scale Phantoms(GAMMEX 404GS LE AND 404

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Figure 4.20: Estimated Tissue Variation of Sine Mass Density For New Discrete Model

Figure 4.21: Estimated Sine Mass Density For New Discrete Model

LE)” made by *GAMMEX*[®], WI, USA. This phantom provides advanced technology for measuring image quality of small parts and intra-cavity ultrasound scanning systems.

In this section, two different targets in the tissue-mimic phantom are selected carefully to test the validity of the discrete model (4.91). In practice, only part of the tissue-mimic phantom is used due to the fact that the discrete model (4.91) is only valid when the average mass density of tissue of interest is consistent or doesn’t vary very much.

As shown in Figure 4.14, the first experiment focuses on the pin targets of the tissue-mimic phantom. And the second experiment focuses on the cyst target in Figure 4.15.

4.6.4.1 Experiment on Pin Target

In this experiment, pin targets of the phantom in Figure 4.13, are selected to estimate their corresponding images of tissue density variation. Experiment specifications are similar to those in simulations.

Then, tissue mass density estimation is performed and the estimated image of tissue variation of mass density is shown in Figure 4.22. From the comparison images in Figure 4.22, the speckles in original B-mode image have been removed and the contrast between tissue (white part) and media (black part) is improved a lot. In the mean time, the information of pin targets is kept well.

4.6.4.2 Experiment on Cyst Target

In this experiment, a cyst target of the phantom in Figure 4.13, is selected to estimate its corresponding image of tissue density variation. Experiment specifications are similar to those in simulations.

With similar procedure as that in simulation part, tissue mass density estimation is performed and the estimated image of tissue variation of mass density is shown in Figure 4.24. From the images compared, the speckles in original B-mode image have been removed and the contrast between tissue (white part) and media (black part) is improved a lot. Besides, some texture information are shown in finer details.

4.7 Summary

In this chapter, we developed two discrete ultrasound models based on theoretical model and new ultrasound model developed in Chapter 3, respectively. Simulations and experiments are performed to verify the validity of the proposed models. In

Figure 4.22: Original B-mode Image by New Discrete Model (Left) and Estimated Tissue Image of Mass Density Variation (Right)

Figure 4.23: Original Tissue B-mode Image

Figure 4.24: Estimated Tissue Image of Mass Density Variation by New Discrete Model

comparison with the conventional model, they are of familiar format of standard discrete systems. Meanwhile, inside the models, there are clear indications on how the tissue property parameters interact with the input impulse signals exactly. Hence, with the proposed discrete models, the tissue property parameters can be separated from the echo signals.

Chapter 5

Conclusions and Recommendations

5.1 Conclusions

In this thesis, we explored both continuous and discrete modeling work on ultrasound echo processes. The details of a new convolution model are proposed in Chapter 3 and two discrete models are developed in Chapter 4.

The thesis starts with a proposal of a new ultrasound convolution model in Chapter 3. It is based on careful studies of the works presented by researchers working in the field of ultrasound convolution modeling and corresponding post-processing techniques. The establishment of the new convolution model pointed out the flawed approximation in the conventional model which has not been mentioned in the existing results in the area of ultrasound convolution modeling. If the conventional model doesn't hold, the deconvolution-related techniques for processing ultrasound echo signals no longer have theoretical foundation.

Based on the investigation of classical acoustical equations, the new convolution

model presents dominant terms under common practically feasible conditions of medical ultrasound. It provides a solid theoretical foundation for the prevailing deconvolution techniques in ultrasound signal processes and new insight in exploring interactions between ultrasound pulses and body tissues in ultrasound scanning and imaging.

The new convolution model takes the practical specifications of medical ultrasound into account and evaluates the echo effects arising from different anatomical properties. It is shown that, under practical circumstances and in tissues of small inhomogeneities, medical ultrasound echo signals are dominated by perturbations of ultrasound propagation velocity. With the new model, the convolution format of ultrasound echo processes remains valid and prevailing deconvolution techniques for ultrasound signal processing and imaging have gained a new theoretical foundation.

In Chapter 4, we have proposed two discrete ultrasound models based on different continuous models. They are derived in Section 4.5 and Section 4.6 respectively. In comparison with the traditional conventional model, they have shown the formats of standard discrete systems. In the models, there are clear indications on how the tissue property parameters interact with the input impulse signals exactly. Hence, provided the discrete models, the tissue property parameters can be separated with ease from the echo signals, which are exactly what are of ultrasound imaging interest.

The results of these discrete models on both the real and simulated ultrasound RF data can show superior improvement as compared to the existing models and raw ultrasound images. Meanwhile, we can see that the embedded object in the simulated tissue phantoms and the tissue structures of the real ultrasound image are now more easily observed.

In summary, this thesis is a contribution to the development of novel ultrasound modeling for medical ultrasound imaging. Our proposed new ultrasound convolution model in Chapter 3 is in the center of the development. Under this model, this

thesis provides an extensive study of the corresponding discrete models and presents them in a unified framework to give a new paradigm of the ultrasound echo process modeling.

5.2 Recommendations for Further Research

Nowadays, 3-D or even 4-D ultrasound imaging is quite popular among young pre-parents, given the fancy and vivid visual effects produced by 3-D and 4-D ultrasound processing techniques. However, current new ultrasound convolution model in Chapter 3 is mainly based on 1-D because ultrasound signal processing is usually performed line by line. Some tentative works extended to 2-D have also been done simultaneously in Chapter 3, but still more detailed derivations and simulation or experiments verification are needed to facilitate the accurate extension to 2-D and 3-D. Hence, a complete and unified framework of ultrasound convolution modeling could be established.

Meanwhile, from the results in Chapter 3, ultrasound echo signals are mainly determined by the ultrasound velocity variation during wave propagation through soft tissues. It indicates a very interesting topic in ultrasound physics. The effects caused by velocity perturbations are much stronger than those by tissue mass density variation. It is quite different from our common knowledge on ultrasound imaging that ultrasound echo signals are essentially generated by the difference of anatomical properties. Hence, researchers in ultrasound physics may be interested in exploring the essential reasons.

Besides, during the development and verification of ultrasound discrete models, it was found that the accuracy of final density estimation depends very much on that of estimated input impulse responses. That is because less accurate impulse response will lead to wrong quantification of the parameter matrix Ω and hence the inverse

procedure for density estimation are not correct, too. Hence, the robustness of the discrete models should be analyzed more to generate more robust models which can perform well even under some ill conditions.

Author's Publications

- [1] Ming Chen and Cishen Zhang, "A new convolution model of ultrasound imaging systems," Submitted to *IEEE Transactions on Biomedical Imaging*, under review.
- [2] Ming Chen and Cishen Zhang, "On Convolution Model of Ultrasound Echo Process," accepted by *IFAC 2011*.
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