Nonlinear techniques for source detection and localization in shallow ocean with non-Gaussian noise

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localization in shallow ocean with non-Gaussian
noise**

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“Genius is 1 percent inspiration and 99 percent perspiration.”

Thomas A. Edison

To:

My parents Shahla and Behrouz,

and my loved ones Farzad

and

Farhad,

my lovely son who I do everything for

to make all his dreams come true.

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List of Abbreviations

1-D	One-Dimensional
2-D	Two-Dimensional
3-D	Three-Dimensional
AP	Alternating Projection
AVS	Acoustic Vector Sensor
A*Star	Agency for Science, Technology And Research
BMPT	Block Median Pyramid Transform
CDF	Cumulative Distribution Function
CRB	Cramer-Rao Bound
D20	Daubechies 20
DA-ZMNL	Data Adaptive Zero Memory Non-Linear
DOA	Direction Of Arrival
DSTA	Defense Science and Technology Agency
EM	Expectation Maximization
ENR	Energy to Noise Ratio
E-step	Expectation step
ESPRIT	Estimation of Signal Parameters by Rotational Invariance Techniques
FD	Direct Finite-Difference
FE	Finite-Element
FFP	Fast Field Program
FFT	Fast Fourier Transform
FIM	Fisher Information Matrix
FIR	Finite Impulse Response
GGN	Generalized Gaussian Noise
GG	Generalized Gaussian

List of Abbreviations

GM	Gaussian Mixture
GMM	Gaussian Mixture Model
HLA	Horizontal Linear Array
i.i.d	identically independent distributed
IMP	Imputation
LMIPT	Linearized Median Interpolation Pyramid Transform
LOD	Locally Optimal Detector
LWD	Linear Wavelet denoising
MDL	Minimum Description Length
MIPT	Median Interpolation Pyramid Transform
MFP	Matched Field Processing
MF	Matched Filter
ML	Maximum Likelihood
MMP	Matched Mode Processing
MOG	Mixture Of Gaussian
M-step	Maximization step
MUSIC	Multiple Signal Classification
NMSE	Normalized Mean Square Error
NM	Normal Mode
NWD	Nonlinear Wavelet Denoising
OD	Optimal Detector
PDF	Probability Density Function
PE	Parabolic Equation
RMSE	Root Mean Square Error
ROC	Receiver Operating Characteristic
R-MUSIC	Rayleigh MUSIC
SAGE	Space Alternating Generalized Expectation Maximization
SAGE-USL	SAGE-Underwater Source Localization
SIM	Subspace Intersection Method
SS	Scalar Sensor
SNR	Signal to Noise Ratio
SSR	Suprathreshold Stochastic Resonance
VLA	Vertical Linear Array

Abstract

Acoustic waves in water are used to detect or locate targets, measure environmental or target parameters and transmit signals. Although detection and localization of an underwater source (target) have been the primary need in military applications, they have been found many commercial applications as well. In this work, we consider the problem of detection and localization of acoustic sources in the presence of non - Gaussian noise.

Although assumption of Gaussian distribution for ambient noise provides simplicity and tractability, it is not a valid assumption when a large number of impulses appear in the background noise, such as warm water shallow oceans. Two main sources that contribute to the non - Gaussianity of ambient noise are biological noise produced by some marine organisms such as snapping shrimp, and navigation noise.

When noise is non - Gaussian, the performance of linear signal processing techniques that have been derived based on the assumption of Gaussian noise characteristics degrade. In these cases, nonlinear processing techniques are more effective. We investigate a number of nonlinear methods for detecting and localizing acoustic sources in the presence of highly non - Gaussian noise. We present a robust and easily implementable detector based on nonlinear wavelet denoising (NWD) for detection of signals. A class of nonlinear median based transforms has been chosen for denoising since the conventional denoising techniques based on linear wavelet transforms are suitable only for denoising signals in Gaussian noise. The proposed NWD detector offers the following advantages for signal detection in strongly non - Gaussian noise: (1) significantly better performance than the matched filter, (2) greater robustness than the optimal detector, (3) moderate computational complexity.

Further, we develop a computationally simple algorithm known as SAGE - USL, for 3 - dimensional (3 - D) localization of multiple acoustic sources in shallow ocean. In this algorithm, a hybrid array of sensors composed of a vertical and a horizontal linear array is used. In the SAGE - USL algorithm, after a rough estimate of different unknown parameters, a novel SAGE

- based approach is applied to update the estimates sequentially. We further analyze the computational complexity of the proposed algorithm and compare it with that of 3 - D MUSIC. Modified version of the SAGE - USL algorithm for an array of acoustic vector sensors is also presented. Simulations conducted to evaluate the performance of the algorithm illustrate: (1) Significant improvement in the source localization performance compared to 3 - D MUSIC with RMSEs that compare favorably with the corresponding Cramér - Rao Bounds (CRBs) , (2) Fast convergence rate of the proposed algorithm, (3) Significant reduction in computational complexity compared to that of 3 - D MUSIC.

We also derive an expression for CRBs of 3 - D localization of multiple sources in a range independent ocean with any symmetric noise distribution that is easy to compute and usable for different array configuration of scalar sensors and acoustic vector sensors. The CRBs derived are used for performance evaluation of SAGE - USL algorithm.

Chapter 1

Introduction

The science of underwater acoustics began in 1490, when Leonardo Da Vinci used a long tube in water in order to listen for approaching ships from afar [2]. The sinking of Titanic in 1912 and the beginning of the first World War were also events that led to a significant progress in the field of underwater acoustics. Subsequently, in 1919, the first scientific paper on underwater acoustics was published [3]. From then on, several applications of underwater acoustics were considered. After World War II, the Cold War was a driving force behind many developments that occurred especially in the area of sonar systems. As a result, significant advances were made in the field of underwater acoustics theoretically and practically. In recent years, research in fields such as underwater sensor networks, oceanographic applications, warning systems for natural disasters (e.g. seismic and tsunami monitoring), ecological applications (e.g. pollution, water quality and biological monitoring), navigation systems and offshore exploration has made considerable progress [4].

Other possible avenues for underwater applications include radio waves and optic waves. Radio waves that work effectively for radar and terrestrial communication may not be effective in underwater channels due to their strong attenuation in sea water. The attenuation of radio waves in water is related to the strong conductivity of water (especially salt water) which makes water highly dissipative. Radio wave propagation in water over long ranges is possible only at very low frequencies (30 – 300 Hz), which requires large antenna and high transmission power that are impractical.

Optical waves cannot be used for underwater long range applications because of the strong turbidity of ocean water which arises due to a large number of scattering particles (e.g. sedi-

ments and phytoplankton). In both highly absorbing and highly scattering waters, visibility in water is reduced significantly. Hence, optical waves may be considered as inefficient methodology for underwater applications.

Acoustic waves propagate in water with a speed that is four to five times higher than their propagation speed in air. Besides, they attenuate in water less than air. Therefore, they can propagate over larger distance in water compared to air. However, the level of ambient noise and undesirable echoes in water are higher than those in air.

The propagation advantages of acoustic waves in water have made them as promising and practical methodology for many underwater applications. In general, acoustic waves in water are used to detect or locate targets, measure environmental parameters (e.g. water depth, marine organism, seafloor topography), measure target parameters (such as velocity and trajectory of a moving target) and transmit signal (e.g. data acquisition, communication messages, command and control) [4].

Although detection and localization of an underwater source (target) have been the primary need in military applications, they have been found many commercial applications (such as their applications in oil and gas exploration, fisheries and geophysics) as well. Detection and localization of a target can be based on reception of either the echo of a transmitted signal from the target or the acoustic signal emitted by the target. The term *underwater channel* refers to oceans, seas, lakes, ponds or rivers that covers around 71% of the Earth's surface.

The purpose of this introductory chapter is to briefly provide some background information in this area of study (Section 1.1) and outline the motivation for this research as well as the contributions of the work in the development of the field (Sections 1.2 and 1.3 respectively). The organization of the thesis is summarized in Section 1.4.

1.1 Background

1.1.1 Sonar systems

Systems which are used for detection and localization of targets using acoustic (sound) waves are called sonars (Sound Navigation And Ranging). Two types of technologies exist for sonar systems: passive and active. Passive sonars listen to the surrounding for acoustic waves emitted by a target that one wishes to detect, locate or estimate some of its parameters. The target may

be a vessel, a submarine, an underwater animal or any other object. Active sonars transmit an acoustic signal and listen for its echoes sent back by the target. In both passive and active sonar systems, an array of hydrophones attached to a floating or submerged platform is usually used to receive the acoustic signals emitted or reflected by a target in the water.

Sonar systems have many military and commercial applications. They are used for navigation, detection, localization and speed estimation. They also can be used for mine hunting, mine counter measurement, ocean surveillance to find suspicious obstacles or lost equipments on sea bed or for measuring different physical and chemical properties of marine environments (e.g. sound speed, seafloor topography, sediment profile, water salinity, dissolved oxygen in water). Sonar systems can also be used to study the population, behavior and habitat of different marine animals such as fishes and whales or to protect them from different threats [4].

Fig. 1.1 demonstrates a general structure of a sonar system. The transmitter in this figure is shown in dashed line. An active sonar contains a transmitter to emit an acoustic signal usually with high-power and a receiver to receive its echo. The transmitted signal and the reflected signal by the target (echo) will propagate in water and on the way to the receiver will be attenuated, distorted and contaminated by ambient noise. These signals will be received at an array of sensors (transducers) whose outputs are combined properly (according to the requirements of the system). The received signal may pass through a signal conditioning unit to amplify, filter and enhance the signal to noise ratio (SNR). The role of signal conditioning is to make the signal suitable for later processes. Then, the signal is fed to the main processing part to detect targets and locate or identify them. A passive sonar only includes the receiver part and has no transmitter.

1.1.2 Sonar signal processing

Performance of a sonar system (or in general an underwater acoustic system) is affected by two factors: Constraints of the physical environment (underwater channel) and the signal processing techniques used in the system. As observed in Fig. 1.1, detection, localization and target identification are the main signal processing functions required in a sonar system.

Many studies have been conducted in recent decades for improvement of sonars' performance by developing different signal processing techniques. For active sonars, a large part of studies are carried out in parallel with the earlier or present works on active radars. However, in passive sonars development of original concepts and theories are necessary since their processing

Figure 1.1: A general structure of a sonar system (active/passive)

techniques are very specific [4]. The important and critical role of signal processing techniques in sonar systems necessitates extensive research in this field especially for passive sonars.

1.1.3 Some usual terms in underwater acoustics

The ocean as an acoustic medium is very complicated because of its different acoustic characteristics such as the effects of the sea surface, the sea bottom and internal waves. Scattering in water may occur at sea surfaces, air bubbles in water, or may have even biological sources [5]. Absorption and attenuation of acoustic waves in the ocean also affect the wave propagation significantly.

The ocean is extremely variable, both spatially and temporally. Currents, internal waves and small-scale oceanic turbulence (with the spatial scale ranging from a few centimeters to dozens of meters) affect the sound velocity in the ocean. These may also lead to spatial and temporal fluctuations in the intensity and the phase of a propagating acoustic wave. Besides, the ocean surface is rarely calm and it varies with surface wind-generated waves. This feature affects the reflection and scattering of the acoustic wave from the ocean surface. While the ocean surface only scatters the acoustic wave, the bottom of the ocean may scatter and absorb it. The effect of the ocean bottom on sound propagation is more significant at low frequency. At frequencies

above a few kHz, the effect of sea bottom on sound propagation may be disregarded. Different types of the ocean bottom sediment (e.g. muddy or rocky) may affect the reflection of sound waves in a variety of ways. For example, reflection from a very rocky bottom may be less than that from a muddy sediments [5].

Another characteristic of the ocean that affects the propagation of acoustic waves are marine organisms. Marine organisms including both plants and animals can affect the propagation of underwater acoustic waves through noise production, attenuation, scattering of signals, or even by fouling sonar transducers or presentation of false alarms in sonar systems. For example, some marine animals such as snapping shrimps, whales or various fishes produce sounds that contribute to background noise. Organisms such as large numbers of plankton and floating kelps may also lead to signal attenuation [5].

Ambient noise:

Ambient noise or background noise in the ocean is that part of noise that is independent of the source, receiver and platform characteristics. Ambient noise which is an important characteristic of the ocean depends on different parameters such as frequency, behavior of marine animals, geographical location, season, etc.

At frequencies between 0.1 – 10 Hz, ambient noise is mainly related to earthquakes, volcanic eruptions in water, or some ocean surface processes (e.g. nonlinear interaction of surface waves). However, for the frequency band of 50 – 300Hz, main source of underwater noise is related to shipping traffic at far distances (called as navigation noise) [5]. Attenuation of sound waves produced by ships in a deep ocean is small because of the creation of ‘*deep sound channel*’. Therefore, the sound waves can propagate to far distances and this leads to a continuous background noise in this frequency band. This type of noise is denoted as navigation noise. The phenomena of deep sound channel can be explained as follows. When the sea surface temperature is high enough so that it reverses the pressure effect, the sound speed as a function of temperature, pressure and salinity, reach a minimum value at a few hundred meter depth. This leads to creation of sound speed channel in which acoustic waves can propagate to far distance (thousands of kilometer). In this case, the acoustic waves are trapped in a waveguide (‘*deep sound channel*’) and they have no interaction with sea surfaces therefore, they attenuate a little. The acoustic waves approaching the sea surface are turned back toward the sea bottom and the waves approach the bottom are turned back toward the surface [5, 6].

When frequency of acoustic wave is larger than 300Hz and lower than 50kHz, ambient

noise depends directly on the ocean surface state and wind speed in the area considered. For frequencies higher than 100kHz, molecular thermal noise is dominant noise. Ocean surface state or sea state is a factor that shows the general condition and characteristic of sea and it depends on the wind speed and surface wave height. The larger sea state is associated with larger wave height and higher wind speed. [5]

Biological noise is another source of underwater noise which is generated by marine animals. Some marine animals make sounds to communicate with each other, explore their environment, locate their prey or frighten their enemies. This type of noise may be very intensive in some regions of seas and oceans such as shallow water channels [5].

Shallow water channels

Underwater channels are generally divided into deep and shallow water channels and the propagation of acoustic waves will have some differences in these two types of channels. Shallow water is defined in two ways: hypsometric and acoustic.

In the hypsometric definition, the continental shelf waters shallower than 200m are considered as shallow water. According to this definition, 7.5 percent of the total ocean area is shallow water.

From an acoustic point of view, whenever the acoustic propagation in the sea is characterized by several reflections from both the sea surface and the sea bottom, the channel is defined as shallow water [7].

Non-Gaussianity of ambient noise in shallow water

In many applications, noise is assumed to have Gaussian distribution. This assumption provides simplicity and tractability of the problem considered. But this is not a valid assumption in some applications, due to the presence of a large number of impulses in the background noise. For example, in shallow water acoustic channels, in some situations, ambient noise is highly impulsive and non-Gaussian [8, 9].

Two main sources contribute to the non-Gaussianity of ambient noise in shallow water are biological noise produced by marine organisms and navigation noise [8, 9]. The noise produced from some biological sources such as snapping shrimp is impulsive in nature and this dominates the underwater ambient noise. The shrimps are usually found in groups with large numbers and they lead to a permanent crackling background noise in warm shallow waters. Shrimps produce loud snapping sounds by very rapid closure of their snapper claw. The closure of their

snapper claw leads to the formation of cavitation bubbles. The collapsing bubbles cause a loud snapping (impulsive) sound [10]. This impulsive contribution leads to heavy tailed and highly non-Gaussian noise distributions. Consequently, assuming the Gaussian distribution for background noise in shallow water channels is not valid in some situations. Instead, the probability distribution of the noise samples in such situations should be considered as heavy-tailed non-Gaussian.

When ambient noise is non-Gaussian, the performance of linear signal processing algorithms that have been derived based on the assumption of Gaussian noise characteristics degrade [11] because of many impulses that exist in background noise. In these cases, nonlinear processing techniques are more effective for underwater applications. The motivation for working on these problems are presented in the following section.

1.2 Motivation

The oceans cover two thirds of the earth's surface and have abundant resources. They represent an important frontier for exploration and science. Detection and localization of acoustic sources in oceans is a common requirement in many military and commercial applications. Different dynamic characteristics of the ocean as an acoustic medium (shortly explained in Section 1.1.3) lead to many challenges for underwater acoustic detection and localization.

Since waters around Singapore are considered as shallow waters with non-Gaussian noise [12], we study the detection and localization problem in shallow water channels in the presence of non-Gaussian noise. This non-Gaussianity is mostly related to the noise produced by snapping shrimps [12]. The non-Gaussian nature of noise distribution in shallow water necessitates the investigation of some nonlinear approaches for detection and localization of sources.

1.3 Contributions of the thesis

The contributions of this work are divided into two main parts: detection and localization in shallow water with non-Gaussian noise. For the purpose of detection, a computationally simple and robust sub-optimal detector, denoted as Nonlinear Wavelet Denoising (NWD)-detector, for detection of weak sources in non-Gaussian noise is proposed that works significantly better than a matched filter and close to the optimal detector [13, 14]. The proposed detector contains

a nonlinear filter for denoising signal. We consider two different nonlinear filters that are based on median interpolation pyramid transform (MIPT) and block median pyramid transform (BMPT). Theoretical and experimental performance of NWD detectors, their computational complexity and their sensitivity to error in modeling noise are also analyzed in this work. These are parts of our contributions in this thesis.

For the problem of localization, a computationally simple algorithm, SAGE-USL, for 3-dimensional (3-D) localization of multiple acoustic sources in a shallow ocean with non-Gaussian noise is developed which requires only a 2-D search and a small number of 1-D searches instead of the impractical 3-D search required in 3-D MUSIC. In this algorithm, a hybrid array of sensors (composed of a vertical and a horizontal linear array) is used and after a rough estimation of different unknown parameters, a novel SAGE-based approach is applied to update the estimates [15–17]. Our main contribution in this part is adaptation of the DOA estimation algorithm presented in [18] for 3-D localization problem in a shallow ocean. This needed several modifications to reduce the complexity arising out of the higher dimensionality of the problem and the use of a hybrid array. We have also analyzed the computational complexity of the algorithm developed and compared it with that of 3-D MUSIC. Using a novel SAGE-based procedure, an algorithm for adaptive shrinkage of the search space, modification of the algorithm for acoustic vector sensor (AVS) arrays and convergence analysis of the algorithm are some of the novelties in our work over what presented in [18].

Another contribution in this thesis is derivation of a closed form expression for Cramèr-Rao Bound (CRB) for 3-D localization of multiple sources in a range independent ocean that can be used with different symmetric noise distributions. The expression derived in this work is easy to compute, and usable for different array configurations of scalar sensors [17]. Further, a modified version of SAGE-USL algorithm using an array of AVSs is also developed and the associated CRBs are computed using the the derived expression in this thesis [16].

1.4 Thesis outline

The work in this thesis is to study and discuss the problem of detection and localization of acoustic sources in presence of non-Gaussian noise specifically for shallow water applications. The first three chapters are introduction and background information in these fields. Our contributions on these areas are presented in Chapters 4-6. The detailed work in the each

chapter is summarized in the following:

Chapter 1 forms introductory material. It includes a description of the background information on the topic. Motivations for this research and our contributions are discussed in this chapter as well.

Chapter 2 presents a literature review on underwater detection and localization algorithms using scalar sensor or acoustic vector sensor arrays. Previous works done on CRBs for different localization problems are also reviewed in this chapter.

Chapter 3 introduces the characteristics of shallow water acoustic channels as the type of channel we have considered in this work. Different techniques used in different applications for modeling shallow water acoustic channels are reviewed. A suitable model (i.e. a simple normal mode-based model that incorporates most of underwater acoustic propagation features) is chosen for our work. Ambient noise model in shallow water acoustic channel is also discussed in detail in this chapter.

In **Chapter 4**, the problem of detection in non-Gaussian noise is discussed. The nonlinear approaches used in this thesis for denoising data in non-Gaussian noise are explained in detail. The computational complexity of the proposed detector, matched filter and optimal detector in non-Gaussian noise are presented as well. Different simulations considered for theoretical and experimental performance evaluation of the proposed detector are discussed in this chapter. We also evaluate the robustness of the NWD-detector and optimal detector.

Chapter 5 provides the details of our contribution in deriving CRBs for 3-D localization of multiple acoustic sources in a range-independent ocean. We also modify and present the CRBs for an array of acoustic vector sensors. Different simulated experiments are conducted for evaluation of the derived CRBs considering plane-wave and shallow water scenarios.

In **Chapter 6**, we present the details of proposed SAGE-USL algorithm for 3-D localization of multiple sources in shallow water with impulsive noise. The problem of single source and multiple source localization using the proposed SAGE-based algorithm with different array configurations are considered. The performance of the algorithm, computational complexity and convergence rate of the proposed algorithm are also considered.

In **Chapter 7**, the conclusions of this research are summarized. Some possible paths for future researches in this area are suggested as well.

Chapter 2

Literature Review

The aim of this chapter is to review the existing literature on the problem of detection and localization under non-Gaussian noise conditions. This will present an appropriate background before our work is introduced in Chapters 4-6. Detection and localization of weak signals in non-Gaussian noise are problems of interest in several applications such as sonar [12, 19–22], radar [23–29], medical equipment [30] and wireless communications [31–39]. In this work, we are interested in shallow water applications in which ambient noise is known to be leptokurtic (heavy-tailed) in many situations [8, 9]. When noise is non-Gaussian, the performance of detection and localization methods designed based on the assumption of Gaussian noise degrade significantly. This is because of many impulses that appear in background noise.

In this chapter, the problem of detection and localization are reviewed in two separate sections. Literature on the problem of detection is reviewed in Section 2.1. Source localization is discussed in Section 2.2. Since we are interested in the localization performance using both arrays of scalar sensor (SS) and acoustic vector sensor (AVS), we review the literature relevant to the problem of source localization using SS arrays in Section 2.2.1 and AVS arrays in Section 2.2.2. In Section 2.2.3, the previous works on Cramér-Rao bounds (CRBs) for source localization are examined. A summary of the chapter is presented in Section 2.3.

2.1 Source detection in non-Gaussian noise

Detection is to decide if an event of interest has occurred. When an event is detected, it is of interest to extract certain information about the event. In many signal processing systems such

as sonars and radars, we usually deal with signals that are weak, distorted by the medium, and corrupted by noise. Noise has been often assumed to have Gaussian probability distribution in conventional studies. Although this assumption makes the analysis of the problem simpler and more tractable, it is not valid in cases such as multiple access interference in communication [31, 35], sea clutter in radars [23, 25, 29], and ambient noise in shallow water [8, 9, 40, 41].

The goal of many sonar systems is to detect and/or to locate particular targets such as submarines, mines, fish or ships. Although there are some differences among these sonar systems, in most of them a matched filter is used to correlate a known signal with received (noisy) signal in order to detect the presence of the known signal associated with the particular target in the received data. Matched filter is extensively used for detection of a deterministic signal in sonar systems [42].

One approach for designing a detector is the Neyman-Pearson approach in which the probability of detection is maximized subject to a fixed probability of false alarm. This approach is used for binary hypothesis testing in many sonar and radar applications [43]. In binary hypothesis testing, two cases are considered: (1) signal presence; (2) signal absence. In the presence of Gaussian noise, the optimal detector for a deterministic signal in the Neyman-Pearson approach is a replica-correlator. Replica-correlator is a detector that correlates the received data with a replica of the deterministic signal. It consists of a multiplier that multiplies the received data and the deterministic signal sample by sample followed by a summation over all the samples. The replica-correlator can be easily implemented as a matched filter in which the multiplier and the summation are replaced by a finite impulse response (FIR) filter and a sampler. Using the Neyman-Pearson approach, the optimal detector (OD) in non-Gaussian noise can be obtained. However, the implementation of the optimal detector for non-Gaussian noise is a difficult task because it involves computation of a nonlinear function at each data sample [43]. This nonlinear function requires knowledge of the noise probability density function (PDF). Besides, the performance of OD is sensitive to errors in modeling the noise PDF [14]. Therefore, designing sub-optimal detectors which are easily implementable and robust is a matter of interest [44].

A class of suboptimal detectors in non-Gaussian noise is based on the optimal quantization of the data samples followed by the replica correlator [45, 46]. This approach stems from the fact that the optimal detector is a one-bit quantizer followed by a replica correlator when the noise is Laplacian [47]. In this class of detectors, data quantization is used to mitigate the effect of impulsive noise and improve the signal detection performance [45]. Following

this idea, different quantization techniques have been presented for signal detection in non-Gaussian noise [46, 48, 49]. Another approach that has received much attention in recent years uses the nonlinear phenomenon of stochastic resonance to implement non-linear processors for applications in which noise is non-Gaussian. In this approach, noise is used constructively to enhance the signal-to-noise ratio (SNR). Stochastic resonance is a phenomenon where a weak signal is amplified by adding white noise to the signal [50]. In this case, the frequencies of noise that are equal to those of signal will resonate together. This phenomenon boosts the signal while other frequencies of noise will not be amplified, therefore SNR increases.

An alternative for SNR enhancement in non-Gaussian noise is denoising techniques based on nonlinear wavelet transforms. The wavelet transform that has several applications [51–57] compacts a noisy signal into a small number of large coefficients. The process of wavelet denoising is to threshold the coefficients in order to discard the values that are most likely due to the additive noise. The thresholds may be either fixed (hard/soft) or adaptive [58, 59]. In the conventional wavelet denoising techniques, the signal decomposition and reconstruction involve linear transformations. The determination of the denoising thresholds is based on the assumption of Gaussian noise. However, when noise is strongly non-Gaussian, denoising based on a class of nonlinear wavelet transforms called median interpolating pyramid transform [60, 61] is found to be more effective.

In the pyramidal decomposition scheme [62, 63], a signal is divided into a decimated (approximate) part and a residual (detail) information. The residual signal at each scale is the difference between the actual signal at that scale and the interpolated signal from the adjacent coarser scale. In the median interpolating pyramid transform (MIPT) formulated by Donoho and Yu [60], the median operation is used for decimation. Starting from a set of 3^J data samples at scale J , medians at scale $J - j$; $j = 1, 2, \dots, J - 1$ are computed from non-overlapping blocks of 3^j data samples. In order to compute the detail coefficients at scale $J - j + 1$, the medians at scale $J - j + 1$ are estimated from the medians at the coarser scale $J - j$ using an interpolating polynomial. Donoho and Yu have shown that closed form expressions for the estimated medians can be obtained if a quadratic polynomial is used for interpolation [60]. A different version of MIPT, called block median pyramid transform (BMPT) has been presented by Melnik et al [61]. In BMPT, medians at scale $J - j$ are computed from non-overlapping triadic blocks of medians at scale $J - j + 1$. To compute the detail coefficients, they have proposed a simpler version of the median interpolation scheme based on a quadratic interpolating polynomial. This interpolation scheme is equivalent to the linearized version of the interpolation scheme of

Donoho and Yu [60].

2.2 Source localization in the presence of non-Gaussian noise

2.2.1 Source localization using scalar sensor arrays

Three-dimensional (3-D) localization of acoustic sources in shallow ocean is an interesting albeit a challenging problem. Conventional methods of source localization include several versions of the matched field processing (MFP) [64–66] or matched-mode processing (MMP) [67–70] methods, such as Bartlett, Capon, and MUSIC processors. In both MFP and MMP methods, a model is considered for acoustic signal propagation in the ocean which requires environmental information such as sound speed profile, density, water depth and bottom parameters [71]. The received data is compared with the predicted acoustic field obtained from the model considered over a set of possible source positions. The source location that leads to the best match between the measured and predicted acoustic field is considered as the estimated source location. MFP and MMP based methods are used to compare the measured and predicted fields. An ambiguity function is an indicator for the correlation between the measured and the predicted fields. The location corresponding to the peak of the ambiguity function is estimated as a true source location. In conventional MMP techniques, a set of linear equations are solved to estimate the modal amplitudes. MMP based techniques facilitate mode filtering for applications in which modeling errors or mismatches lead to performance degradation. However, MMP processing is restricted to a modal representation of sound fields while this limitation does not exist in MFP processing methods [67, 72].

The problem of bearing estimation of a source in the ocean has been widely studied in the literature [73–78]. Different algorithms such as beamforming [79, 80], maximum likelihood estimation [81, 82], MUSIC [83] and ESPRIT [84] have been presented for bearing estimation. The problem of estimating range and/or depth of a source has been also discussed extensively by many researchers [68–70, 72, 85–88] and 3-D localization of a source has been investigated in some papers, such as [89–91].

To localize a source in 3-D space in a shallow ocean using the simplest and most robust version of MFP, namely Bartlett processor, an ambiguity function is computed by correlating the data vector measured by a horizontal linear array (HLA) with replicas of the signal vector on a 3-D grid over the search region. An HLA is the simplest array configuration which

contains the information of all source coordinates (i.e., bearing, range and depth). The resultant ambiguity function from Bartlett estimator has a broad peak and a low peak-to-sidelobe ratio. Consequently, the Bartlett processor is susceptible to interference and has a low resolution. The Capon and MUSIC processors provide higher resolution. However, all these methods perform localization through a 3-D search which requires a high degree of computational complexity.

An HLA does not provide good estimates of range and depth since the signal vector at an HLA is not very sensitive to variation of source range and depth. An alternative approach is to use a hybrid 2-D array composed of an HLA and a vertical linear array (VLA). The VLA data may be used to perform range-depth estimation by any of the MFP methods [64–66] mentioned above. The HLA data may then be used in conjunction with the estimated range-depth of a source to estimate the bearing of that source through a 1-D search. This approach provides better range-depth estimates. The computational complexity is also reduced since the hybrid array requires one 2-D search for range-depth estimation and J_s 1-D searches for bearing estimation of J_s sources. In general, in sub-optimum approach of source localization, range and depth of a source is estimated using a vertical array and the estimated value of range and depth are used for estimating bearing with a HLA [87–89, 92–94].

In recent years, some algorithms have been developed for bearing estimation of multiple sources in the ocean without prior knowledge of their ranges and depths [78, 95–97]. These algorithms include subspace intersection method (SIM) [95, 96], Rayleigh MUSIC (R-MUSIC) [78], and modified R-MUSIC (R-MUSIC-mdf) [97]. These methods exploit the fact that the N -dimensional signal vector at an array of N sensors belongs to an M -dimensional modal subspace if $N > M$, where M is the number of normal modes supported by the underwater acoustic channel. Thus it is now possible to localize J_s sources through one 2-D search and one 1-D search if a hybrid array is employed. SIM has been developed for SS arrays and AVS arrays and provides unbiased estimate of the bearings [95, 96]. In R-MUSIC that is a method based on normal mode theory, the generalized Rayleigh ratio theory has been used to estimate bearings of sources independent from their range and depth. R-MUSIC is suitable for bearing estimation of low frequency sources in shallow ocean. When a linear array is used, bearing estimation of sources close to the broadside direction degrades [78]. To overcome the draw-back of R-MUSIC with linear arrays, Xu and Liu have presented mdf-R-MUSIC that leads to improved bearing estimation for sources close to the broadside direction [97].

It is known that the best statistical (second-order) performance in terms of parameter es-

timization error is provided by the maximum likelihood (ML) estimator which is asymptotically (as the number of data samples T tends to infinity) efficient and unbiased [98]. This means that the covariance matrix of ML estimator reaches the CRB when the number of data samples tends to infinity [76]. Tabrikian and Messer [89] have investigated the problem of ML localization of a monochromatic (single frequency) source in a range-independent ocean in the presence of Gaussian noise using linear arrays. However, ML localization of multiple narrowband sources in the ocean is a highly challenging task, especially if one takes into account the impulsive nature of underwater acoustic noise characterized by a non-Gaussian distribution with a heavy-tailed PDF [99]. ML localization involves estimation of not only the $3J_s$ coordinates of J_s sources but also a large number of nuisance parameters associated with the signals and the noise.

Fessler and Hero [100] have proposed space-alternating generalized expectation-maximization (SAGE) algorithm to decrease the complexity of ML estimation and drawbacks of classical expectation maximization (EM) algorithm. A classical EM algorithm that update all the estimates simultaneously has slow convergence rate and difficult maximization steps. However, in SAGE the estimates of unknown parameters are updated sequentially using several small hidden data sets. The convergence rate of an EM algorithm is inversely related to the Fisher information of its complete (augmented) data set. Therefore less informative complete data set leads to higher convergence rate [100,101]. SAGE approach has been used in different applications such as direction of arrival (DOA) estimation [102–106] and detection [107,108].

Kozick and Sadler [18] have applied the SAGE algorithm to the problem of estimating the directions of arrival (DOA) and signal waveforms of plane waves in non-Gaussian noise represented by a Gaussian mixture model (GMM). In their method, the unknown parameter set is partitioned into three subsets composed of (1) DOAs, (2) signal waveforms, and (3) parameters of the GM noise PDF. Initial estimates of the DOAs are obtained using a data-adaptive zero-memory nonlinear preprocessor (DA-ZMNL) and MUSIC, and these estimates are used to obtain initial estimates of the signal waveforms. A sequential search procedure is then used to update the estimates of the parameters in the three subsets, and this sequential search is repeated iteratively until convergence is reached.

When the problem of multiple source localization in shallow ocean is considered, the challenges become more significant. This is due to contribution of multiple sources to the acoustic field. In this case, sidelobes of a strong source may mask the main lobe of a weak source. One solution is to use a high resolution estimator which minimizes the sidelobes and Capon has been

found to be effective. However, it is known to be sensitive to mismatch in model parameters [72]. In [72], Mirkin and Sibul have used the deterministic ML algorithm [75] for the problem of multiple source localization in an acoustic waveguide with Gaussian noise. The localization algorithm used in [72] is limited to 2-D position finding (namely, range and depth estimation) and they have used a vertical linear array for localization. In [75], alternating projection (AP) algorithm for ML DOA estimation of multiple source has been developed. In this algorithm the complex multivariate nonlinear maximization required in ML algorithm is replaced by an iterative technique (i.e., AP) that transforms the complex multivariate maximization problem to a sequence of much simpler one-dimensional maximization problems. Problem of source localization in an uncertain ocean has been discussed in [70, 109]. In [93, 110, 111], passive localization of sources in shallow ocean has been investigated using model based algorithms in which ocean acoustic propagation model is incorporated into signal processing scheme.

2.2.2 Source localization using acoustic vector sensor arrays

An underwater acoustic wave is a propagating energy that produces a disturbance in the ambient pressure, creating a volumetric motion where the rate of this motion is particle velocity. The amplitudes of acoustic pressure and particle velocity are related. However, particle velocity is a vector that carries directional information about the propagating acoustic energy. While a hydrophone can only measure scalar acoustic pressure, an acoustic vector sensor (AVS) is capable of measuring acoustic pressure as well as velocity and direction of acoustic particle motion. The conventional hydrophone is a scalar sensor (SS) which provides only a partial characterization of the acoustic field. A more complete characterization is provided by an AVS which makes simultaneous measurements of the acoustic pressure as well as three orthogonal components of particle velocity at a point [112–114]. Each acoustic vector sensor contains two or three co-located and orthogonally oriented velocity hydrophones plus an optional pressure hydrophone [115, 116]. Utilization of the additional information provided by an AVS array can lead to a better localization performance. AVS arrays in contrast to SS arrays do not suffer from the steering vector ambiguity associated with undersampling nor the left-right ambiguity [96, 117]. The SS arrays can provide information about source location by measuring the propagation delay between sensors, however using AVS arrays this information can be obtained directly from the velocity measurements.

Acoustic vector sensors have been studied extensively from theoretical and practical per-

spective [91, 116, 116–125]. Due to the advantages that acoustic vector sensors provide, they have gained popularity for different signal processing applications such as source localization [91, 96, 121, 124, 125], tracking [126–131], inversion [132], communication [133, 134] and speech enhancement [135].

In [119], a maximum likelihood based algorithm for DOA estimation using vector sensors is developed. Problem of 2-D DOA estimation using Bartlett and Capon beamforming with acoustic vector sensors is examined in [121] and azimuth-elevation estimation using acoustic vector sensors is also discussed in [115, 136–139]. In [138], Tichavsk et al have presented an ESPRIT-based algorithm that provides closed-form direction-of-arrival estimation of azimuth and elevation of sources (up to four uncorrelated monochromatic sources) using a single vector hydrophone. Acoustic vector sensors for source localization using MUSIC based algorithms [116, 124, 137, 139, 140], ESPRIT [115, 116, 120, 141] and subspace intersection method (SIM) [96] have been studied as well. Wong and Zoltowski in [116] have developed a blind MUSIC-based source localization algorithm that is applicable to an arbitrarily spaced AVS array. The problem of wideband source localization using a distributed acoustic vector sensor array is discussed in [142–144]. Besides, Nehorai have used AVS arrays as nodes in a sensor network for localization [122]. The problem of 2-D DOA tracking has been investigated by Zhong et al. using particle filtering algorithm [128]. AVSs have been studied for underwater acoustic communication by Song et al and Abdi et al. [134, 145] and found to be effective for acoustic communication in underwater applications as well.

In [146], a near field measurement model for acoustic vector sensors is presented and a DOA estimation method based on received signal strength indication approach for near field sources is developed that requires only two passive anchor-nodes. The localization of near field wideband sources using acoustic vector sensors are also discussed in [147, 148]. The 3-D localization algorithm presented by Song and Wong in [148], has been used for estimating the azimuth, elevation and range of a near field source of unknown spectrum in presence of white/coloured noise. Besides, the problem of direction finding of far field wideband sources using acoustic vector sensors has been considered in [124, 141, 144, 149–152] as well.

2.2.3 Cramér-Rao bound

Cramér-Rao bounds (CRBs) are essential benchmarks to evaluate the performance and accuracy of any unbiased estimator and play an important role in array signal processing. CRB

determines a lower bound on the variance of estimation error and can be used for performance comparison of different estimation algorithms or estimation error evaluation of an estimator. We are interested in source localization and in this section, we provide a literature survey on CRBs for source localization.

DOA estimation as a 1-D source localization has been widely discussed in literature. Many authors have derived CRBs of DOA estimation in additive noise for performance comparison of different DOA estimation algorithms. In these, DOA estimation of deterministic and stochastic sources in presence of Gaussian noise have been considered [81, 153, 154]. A deterministic (non-random) signal in presence of a random noise is also known as ‘conditional model’ [81]. However, when signal and noise both are random, the data model is known as stochastic or ‘unconditional model’ [81]. Based on application, a suitable signal model (deterministic or random) is considered and corresponding CRB is derived. The CRBs obtained for localization of deterministic sources are called deterministic CRBs and the CRBs computed for stochastic sources are known as stochastic CRBs [81, 154].

Stoica and Nehorai in [76] have derived deterministic CRB of DOA estimation in presence of uniform white noise. Ye and Degroat [155] have extended this deterministic CRB to a more general case of a deterministic source contaminated with additive unknown colored noise. In [81, 82], the stochastic CRB for DOA estimation has been derived using an indirect method in which an asymptotic covariance matrix of ML estimator is used. This derivation is extended to obtain CRB of a random source in presence of colored noise distributions [156, 157]. Gershman et al. [158] have derived a closed form expression for stochastic CRB of DOA estimation in unknown noise fields. Stochastic CRB for DOA estimation using direct method has been derived in [153, 159] for uniform and nonuniform white noise.

An approximate lower bound (ALB) for variance of direction estimation error has been derived in [160]. This ALB that has been obtained for sensor breakdown case is equal to a weighted sum of CRB when conditioned on disjoint of sensor breakdown [160]. Tam and Wong in [161] have taken into account the non-ideality of sensors gain response, phase response, or orientation of acoustic vector sensors. Further, they have derived CRB of DOA estimation (azimuth and elevation angles) considering these non-ideal situations.

In [18], an expression for deterministic CRB of DOA estimation of plane wave sources in non-Gaussian noise has been presented. Problem of range and depth estimation of underwater acoustic sources and derivation of corresponding CRBs are discussed in [162–164]. CRBs for

3-D localization of a single source in shallow ocean with Gaussian noise using a linear array of scalar sensors has been derived in [89]. In [165], an expression for the CRB of DOA and range-depth estimators of underwater acoustic sources in a shallow ocean with generalized Gaussian noise has been developed using a horizontal and a vertical linear array respectively. To best of our knowledge, a general expression for 3-D localization of multiple acoustic sources in shallow ocean that can be used for different noise distributions and different array configurations has not been considered yet.

2.3 Summary

In this chapter, we have reviewed the existing literature on the topics that we will discuss in detail in forthcoming chapters. We have reviewed the literature on the problem of source detection in non-Gaussian noise. We have focused on different algorithms for SNR enhancement and compared them by giving the advantages and drawbacks of each algorithm for detection application.

In the next step, we have focused on source localization problem and reviewed the papers in the field of underwater acoustic source localization including bearing estimation, range-depth estimation or 3-D localization (bearing-range-depth estimation). In this part, several existing algorithms in two categories, namely, matched mode processing and matched field processing have been studied and compared for localization in a shallow ocean with non-Gaussian noise. We have considered utilization of both scalar sensor and acoustic vector sensor arrays and we have reviewed the literature on both of these areas accordingly. We have also introduced acoustic vector sensors and their applications in signal processing. The mathematical models used in this work for modeling acoustic propagation and ambient noise in a shallow water channel will be presented in Chapter 3.

CRBs as potential lower bounds on the variance of different estimators have been widely used in the literature for evaluation of different estimation algorithms. Further in this chapter, the existing publications on CRBs for source localization have been reviewed. In this work, we derive CRBs for multiple source localization in a range-independent ocean with a symmetric noise distribution. The details of the CRBs derivation will be presented in Chapter 5.

Chapter 3

Acoustic field and noise modeling in shallow ocean

In a shallow-water channel, acoustic propagation of signal is affected by several reflections from sea surface and sea bottom as well as attenuation, absorption, refraction, etc [5, 166]. Designing and developing suitable signal processing algorithms for the detection and localization of acoustic sources in a shallow-water channel require a proper mathematical model for the channel. This mathematical model can be used to simulate different characteristics of the shallow-water channel and their effects on the propagation of acoustic signals. Different models have been developed for underwater acoustic channels that can be used for different applications [7, 167].

The signal traveling in a shallow-water channel is affected by characteristics of the channel and contaminated with ambient noise before it reaches the receiver. In order to simulate the received signal in the channel for further processing, we need to model ambient noise. In a deep ocean, we can model the noise as a plane-wave noise field that is a superposition of independent plane-waves in all directions [1, 168]. The plane-wave noise field is homogeneous with a power spectral density that is independent of the position of receiver sensor. However, in a shallow ocean because of the channel boundaries and inhomogeneity of noise, noise cannot be modeled as a plane-wave field. Hence, we need to consider a proper model for ambient noise in shallow ocean.

In this chapter, we describe different features of shallow water environment and their effects on the acoustic wave propagation in Section 3.1. In Section 3.2 shallow-water channel is dis-

cussed as a waveguide. In Section 3.3, we investigate main techniques for modeling acoustic field in shallow water with the objective of selecting a suitable model required for evaluation of detection and localization algorithms presented and developed in this work. We discuss Pekeris model [5] which is used for modeling acoustic field in shallow ocean in Section 3.4. The problem of ambient noise modeling and Buckingham model [1, 169] as a good model for correlation between noise samples in shallow water acoustic channel are investigated in Section 3.5. Finally, in Section 3.6 we summarize the models that are discussed.

3.1 Shallow water as a medium for propagation of acoustic waves

Many parameters affect the propagation of an acoustic wave in shallow ocean, such as the channel geometry, the wave frequency, sound-speed profile, etc. The shape of the sound-speed profile and the geo-acoustic properties of the ocean bottom affect wave propagation to a large extent. These two parameters are explained in detail in the following subsections. Other parameters that affect the propagation of the acoustic wave are roughness of the bottom, dynamic surface disturbance that depends on the wind, random inhomogeneities in the water layer, sea currents. In real situations, it is observed that the two most important factors (i.e. the sound-speed profile and the geo-acoustic properties of the bottom) are functions of geographical location, season, meteorological conditions [166]. Therefore, accurate prediction of acoustic propagation in shallow water is a complex problem.

3.1.1 Sound Speed Profile in shallow water

Sound speed in the ocean varies as a function of temperature, pressure and salinity and is determined by the following expression [170].

$$c \approx 1449 + 4.6T_w + (1.34 - 0.01T_w)(S_w - 35) + 0.016h, \quad (3.1)$$

where c is the sound speed in the water in meters per second, h is depth of the channel in meters, S_w is water salinity in parts per thousand, and T_w is water temperature in Celsius. Higher temperature and higher salinity result in higher sound speed [5]. One of the main characteristics of shallow water propagation is its sound-speed profile. Since the depth of shallow-water channel

is low, the dependence of the sound speed on depth is not significant especially in winter when temperature is low. However, during summer with increasing temperature, sound speed depends on depth to some extent [171].

3.1.2 Geo-acoustic properties of ocean bottom

The second parameter that affects the acoustic propagation in shallow water is the ocean bottom. The ocean bottom type can vary from soft clay to rigid rock. A summary of geo-acoustic properties of shallow ocean bottoms have been presented in [171]. As the rigidity of the bottom increases, the speed of sound at the bottom increases. The difference between the sound speed in the sea bottom and in water determines how the wave behaves at the interface of water and the bottom (seabed). Considering Snell's Law and the reflection properties of the ocean bottom, it is possible to determine qualitatively how acoustic waves behave and propagate in shallow water. Ocean bottoms are often modeled as fluids because they are covered with sediments (with a rigidity considerably less than that of a solid). The assumption of solid ocean bottom is applicable for the *ocean basement* or where there is no sediment overlying the basement [171].

3.2 Shallow water as an acoustic waveguide

From theoretical and practical points of view, the shallow-water channel can be considered as an acoustic waveguide, in which the limiting boundaries are sea surface and the sea bottom [166]. Acoustic propagation in the medium is described by waveguide theory that represents the acoustic field as a superposition of normal modes [171].

Reflectivity is an important factor to determine how an acoustic wave behaves at the sea bottom or sea surface-as the separating interface of two environments (water-sediment or water-air). Reflectivity is defined as the amplitude ratio of reflected and incident plane-waves at the interface of two environments [171]. We assume a plane-wave at medium 1 reaches at the interface of two media with different sound speeds (c_1, c_2) and different densities (ρ_1, ρ_2) with angle θ_1 (Fig. 3.1). The change in sound speeds leads to two phenomena: (1) specular reflection of the wave in the first environment in a direction θ_r symmetrical to the normal at the incident point; (2) refraction of the wave in the second medium with an angle θ_2 obtained from Snell's

Figure 3.1: A plane-wave at the interface of two media with different sound speeds

law [171]:

$$\frac{\cos\theta_1}{c_1} = \frac{\cos\theta_2}{c_2} \quad (3.2)$$

If $c_1 < c_2$, we have $\theta_1 > \theta_2$ which is the case for example for water-sediment interface. Critical angle θ_c is related to $\theta_2 = 0$ and can be obtained as

$$\cos\theta_c = \frac{c_1}{c_2}, \quad (3.3)$$

For angles larger than θ_c , the total reflection happens and the wave reflects only in the first medium and can not propagate in the second medium. Reflectivity at the interface of two mediums with densities ρ_1 and ρ_2 which is also known as Rayleigh reflection coefficient is computed as [171]

$$R(\theta) = \frac{\frac{\rho_2}{\rho_1} + i \frac{\sqrt{\cos^2\theta_1 - (c_1^2/c_2^2)}}{\sqrt{1 - \cos^2\theta_1}}}{\frac{\rho_2}{\rho_1} - i \frac{\sqrt{\cos^2\theta_1 - (c_1^2/c_2^2)}}{\sqrt{1 - \cos^2\theta_1}}} \equiv e^{i\epsilon} \quad (3.4)$$

where θ_1 is grazing angle in the first medium and c_1, c_2 are speed of an acoustic wave in medium 1 and medium 2 respectively. In this expression, ϵ denotes the phase change in the reflected wave with respect to the incident angle (θ_1) and is equal to

$$\begin{aligned} \epsilon &= 2 \tan^{-1} \frac{\rho_1 \sqrt{\cos^2\theta_1 - (c_1^2/c_2^2)}}{\rho_2 \sqrt{1 - \cos^2\theta_1}}, \\ \theta_r &= \epsilon + \theta_1. \end{aligned} \quad (3.5)$$

The water-air interface is usually approximated with the pressure release case, in which $\rho_2 = 0$ because of the large density difference between water and air. In this case, $R(\theta) = -1$

and phase change $\epsilon = 180^\circ$. At the critical angle of $\theta_2 = 0$, then $\cos^2\theta_1 - (c_1^2/c_2^2) = 0$, therefore $R(\theta) = 1$ and phase change in the reflected wave is $\epsilon = 0$. For a rigid interface where $\rho_2 \gg \rho_1$, the reflection coefficient is $R(\theta) = 1$ and the phase change $\epsilon = 0$ as well. This means that an acoustic wave propagating in water reflects back into the water at the sea surface (or at the rigid sea bottom) without any changes in its phase.

3.2.1 Different types of acoustic propagation in a range-independent ocean

In this section we discuss the possible propagation paths that an acoustic wave can have in a range-independent ocean. Considering the ocean as range-independent or horizontally stratified (i.e. assuming that environmental parameters such as sound-speed profile, water depth and bottom composition are invariant with range) is a good approximation to the real ocean environments [167]. Fig. 3.2 shows five types of propagation paths of an acoustic wave in a deep and a shallow ocean with corresponding sound speed profiles given in dashed line. The acoustic wave propagation path between two points is called an *acoustic ray*.

In Fig. 3.2, paths *A*, *B*, *C* and *D* are related to acoustic propagation in a deep water. Path *A* shows the propagation of an acoustic wave from a source at a depth less than 200m that undergoes multiple reflections from the sea surface. This type of propagation is known as surface duct propagation [172]. Path *B* in this figure demonstrates a near horizontal ray emitted from a source, in a deep ocean, at a depth close to the axis of *deep sound channel*. A deep sound channel is an area in water where the wave has no interaction with the boundaries and the only loss the wave suffers from, is absorption in sea water (because of its viscosity) and the usual geometrical spreading loss. Hence, this wave can travel to very far distance (in range of hundred kilometers). Attenuation due to the water viscosity reduces with decreasing frequency and waves with lower frequency can propagate over longer ranges.

A wave steeper than ray *B*, originated from a deep source may follow path *C*. Ray *C* converges toward the surface and creates an area with a high intensity level of waves called *convergence zone* [5]. The rays with angles steeper than that of ray *C* may reflect from the bottom toward the surface and from surface toward the bottom having several bounces (Path *D*). As observed in this figure, multiple reflections from bottom and surface is the significant characteristic of acoustic propagation in shallow ocean (path *E*).

Figure 3.2: Different types of acoustic propagation in a range-independent ocean (from Ref. [1])

3.3 Acoustic propagation Modeling

The goal of modeling acoustic propagation is to incorporate different features and characteristic of acoustic propagation into a theoretical and numerical formalism that can be used to predict the wave propagation in an ocean environment. Although ocean acoustics has been investigated both theoretically and experimentally, the complexity of the acoustic propagation problem especially for long range in shallow-water channel (that is the subject of interest in this work) makes it hard to predict the acoustic propagation in the ocean accurately. Models of ocean acoustic propagation are expected to be able to model different types of acoustic propagation for different bottoms and different sound-speed profiles. However in practice, models have limitations in their capability and applicability. Some can model some situations but may fail to handle other cases. Hence, models are usually selected based on the problem of interest or for a specific application. It is difficult to produce a universal model for acoustic propagation in the ocean [66, 167].

Acoustic propagation in the ocean is mathematically described by wave equation [167]. Different models presented in the literature for acoustic propagation are computer solutions of the wave equations for boundary conditions that are defined based on the ocean environment and its parameters. There are usually five types of models for acoustic propagation in the ocean [66, 167]:

- Ray Theory

- Fast Field Program (FFP)
- The Parabolic Equation (PE)
- The normal mode model (NM)
- Direct Finite-difference (FD) or finite-element (FE)

Variation of ocean environmental parameters with depth are supported by all of the above mentioned models. As explained earlier, range-independence is a good approximation to the real ocean environment. But there are some cases where the variation of the ocean parameters with range cannot be neglected such as sloping bottom or spatially variable oceans. These types of oceans are known as range-dependent. The Ray theory, PE and FD/FE models can directly be used for range-dependent environments. Although NM and FFP models are based on range-independence assumptions, they can be extended for range-dependent ocean as well [167]. In the following, a short summary of each model and their applicability for different problems is presented.

Ray theory based models that have been used for many years in underwater acoustics are derived as the asymptotic solutions (frequency $\rightarrow$ infinity) of the wave equation. Ray theory based methods provide some advantages, namely, they are quickly computable, readily understandable and simply visualizable. But diffraction effects and low frequency behavior are not usually applicable in this method. This group of models are used extensively for problems in which processing time is a critical factor or the uncertainty of environmental parameters leads to serious accuracy restrictions in other models [66, 167].

FFP approach introduced by DiNapoli and Deavenport [173] is obtained from numerical integration of the Helmholtz Green's function. Helmholtz equation is a time-independent representation form of the wave equation that results from applying the technique of separation of variables to reduce the complexity of the analysis. The Helmholtz Green's function is a type of function used to solve the inhomogeneous differential equation (Helmholtz equation) for a specific boundary condition. The integration in this method is performed using Fast Fourier transform (FFT). This is the main reason for naming this approach as FFP. Some sample applications that use FFP for modeling acoustic fields are: near-field applications, elastic boundaries (e.g. ice surface or basalt bottom) and underwater transmission loss. However, FFP is slow, computationally not efficient method and usually limited to range-independent oceans. [66, 167].

PE method is a popular wave theory approach for range-dependent oceans in low to medium frequency range. In PE technique, a parabolic wave equation that approximates the Helmholtz equation under a special condition is solved for a point source in an environment with constant density. The special condition used in the PE method makes it usable for long range applications. The PE approach includes diffraction phenomenon but does not incorporate backscattering of energy because it treats one directional wave propagation. It becomes computationally inefficient when either frequency of source or complexity of the environment increases. Another disadvantage of PE based models is related to its inherent phase errors that are higher for longer ranges [66].

In NM, a separable coordinate system (usually cylindrical) is considered. Then, Helmholtz equation is solved under the problem conditions and its solution is defined as a finite sum of depth-dependent Eigen functions or resonant modes which satisfy the local boundary conditions. The range dependency of the solutions is usually expressed in the form of Hankel functions. The NM approach provides precise and rapid computation of low frequency behavior of ocean because low frequency leads to fewer modes. NM based models are usually used in matched field processing (MFP) algorithms because they are computationally more efficient than FFP and PE models. Although the first normal mode based model presented by Pekeris in 1948 [5] as a widely cited model, considers a two layer model for the ocean, other normal mode based approaches have been also presented in the literature that model the ocean environment with an arbitrary number of layers [167, 174].

FD/FE based methods are discrete time methods proper for range-dependent oceans that incorporate reverberation and backscattering. These methods that discretized the wave equations are computationally intensive because they are required to model the temporal and spatial variation of the ocean environment in practice. Therefore, they are usually used for short range scattering application or as a part of hybrid algorithms in which the scattering part is solved using FD/FE methods. They are rarely used for general acoustic propagation applications unless for providing a benchmark [167].

In this work, we are interested in the localization of acoustic sources with low frequency (long range) in a range-independent ocean. We also use MFP algorithms for source localization that are found to be suitable for underwater acoustic applications [66]. In MFP based algorithms, the position of the source is estimated based on the correlation between the observed data and the replica data at each possible location in the search area and the location that leads

to maximum correlation will be considered as estimated source location. The replica vector is computed using the acoustic propagation model considered and should be computed several times over the search area. Therefore, it is important to use a fast algorithm for modeling the acoustic propagation and that is why normal mode based models are the best choice especially when frequency is low.

Ray theory that is based on high frequency assumption may not be a good choice for MFP algorithm and low frequency applications. FFP is a slow method and PE suffers from phase errors. However, in MFP the phase of signal is important and necessary. Therefore, FFP and PE would not be good choices. In general, Ray-theory, PE and FD/FE are considered as a model for range-dependent ocean so they are not used in this work. Normal modes have been found to be very successful in modeling and expressing shallow-water channel acoustic propagation and transmission [5]. NM based approaches can be used for different source and receiver configurations and they can be easily extended for range-dependent ocean environments [167].

3.4 Pekeris model

Pekeris model is a simple yet accurate normal mode based model that incorporates main features of acoustic propagation in a range-independent ocean. This model has been widely used in the literature for modeling acoustic propagation, especially in a shallow or range-independent ocean. In this section we present a summary of Pekeris model from reference [5]. However, interested readers looking for more details are referred to several references available for Pekeris model such as [5, 7, 167].

In Pekeris model, shallow-water channel is considered as an homogeneous water layer of constant depth with constant density and sound speed overlying an infinite fluid half-space (either of the two parts of a space divided by a plane) sediment. In this model, signal at the sea surface reflects back into water (total reflection) and the reflection at the bottom depends on the grazing angle (based on the Snell's law). Considering the fluid half-space bottom in Pekeris model implies that signal can be transmitted across the water-sediment. In other words, the model incorporates the transmission loss associated with refraction in the sediment [167].

The radiation from a source at a given frequency in the waveguide can be decomposed into an angular spectrum of plane-waves. Each of these plane-wave components undergoes multiple reflections at the ocean boundaries. The components that undergo a phase shift of

an integral multiple of 2π after each cycle (one surface and one bottom reflection) interfere constructively. The components which do not interfere constructively become progressively smaller and are negligible at far distances from the source. Each of the surviving components is called a propagating normal mode.

From a mathematical point of view, the pressure field $p(r, z)$ at range r and depth z from a point source with frequency $\omega/2\pi$ located at a specific range 0 and depth z_s in a range-independent ocean with depth h can be obtained from the Helmholtz equation by the normal mode solution [5] as

$$\begin{aligned} \nabla^2 p(r, z) + k^2 p(r, z) &\approx \delta(z - z_s) \delta(r), \quad r \gg \lambda \\ p(r, z) &= \sum_{m=1}^M B_0 \Psi_m(z_s) \Psi_m(z) \frac{e^{jk_m r - \alpha_m r}}{\sqrt{k_m r}} \end{aligned} \quad (3.6)$$

where ∇ is gradient operator, δ denotes Dirac delta function, λ is wavelength and $k = \omega/c$. In this expression k_m and α_m are horizontal wave number and attenuation coefficient of m^{th} mode, M is number of normal modes and B_0 is a complex quantity independent of r and z . In Pekeris model a good approximation for modal function $\Psi_m(z)$ are given by

$$\Psi_m(z) = \sin(m\gamma z); \quad m = 1, \dots, M \quad (3.7)$$

$$k_m = k \left(1 - \frac{m^2 \gamma^2}{2k^2} \right); \quad m = 1, \dots, M \quad (3.8)$$

$$\alpha_m = \frac{m^2 \pi^2 u}{k^2 h_e^2}; \quad m = 1, \dots, M \quad (3.9)$$

$$M = \text{int} \left(\frac{kh}{\pi} \sin \theta_c + \frac{1}{2} \right), \quad (3.10)$$

where

$$\gamma = \frac{\pi}{h_e}, \quad h_e = h + \frac{\rho_b/\rho}{k \sin \theta_c}, \quad \cos \theta_c = \frac{c}{c_b}, \quad (3.11)$$

$$u = \frac{(\rho_b/\rho) \cos^2 \theta_c}{\sin^3 \theta_c} \epsilon, \quad \epsilon_b = \frac{k \alpha_b}{40 \pi \log_{10} e}. \quad (3.12)$$

where h_e is the frequency-dependent ‘effective depth’ of the water column, ‘int’ denotes integer, θ_c is critical grazing angle of water-bottom interface, the parameter ϵ_b is the absorption coefficient of the ocean bottom in neper/m, and α_b is the corresponding attenuation in dB/λ_b and λ_b is the wave-length of acoustic waves in the water and bottom density. ρ and ρ_b are the

density of water and bottom respectively. c and c_b are sound speed in water and ocean bottom respectively. The expressions (3.7 - 3.9) have been approximated based on the low bottom attenuation, i.e.

$$\frac{2u}{kh_e} \ll 1, \quad (3.13)$$

which is a valid assumption for many realistic ocean bottom. Another assumption used is

$$\left(\frac{M\gamma}{k}\right)^2 \simeq \sin^2\theta_c \ll 1, \quad (3.14)$$

that is true in most cases of practical interest [5].

3.5 Underwater acoustic noise and Buckingham noise models

Although ambient noise in the ocean is usually stated as a superposition of independent, uncorrelated plane-waves propagating in all directions, this assumption is only valid for deep oceans [175]. Such a noise field known as a plane-wave noise field is spatially homogeneous (independent of the location in the field). In the plane-wave noise field, the second-order statistical measurements, namely, the power spectral density, cross-spectral density, the coherent function and cross correlation function do not change at different positions in the field [176].

The cross correlation function (also known as cross-spectral density function) describes the second-order statistical relationship between each pair of sensors in an array. The performance of the array in a noise field depends on the cross-spectral density function that is a function of array manifold and directional properties of the noise field [175].

A general noise field has a directional density function that determines how much noise power comes from each direction. Such a noise field can be represented as a sum of uncorrelated plane-waves in various directions.

According to Buckingham [176], to produce a plane-wave noise theoretically we may place a random distribution of independent point sources on a sphere with infinite radius. Fig. 3.3 shows such a plane-wave. We assume a receiver located at the center of this sphere. If we change the density of random noise sources on the sphere by the vertical angular direction θ , we can control the directionality of noise. This idea has been used by Faran and Hills in 1952 [177] and Jacobian in 1962 [178] to model acoustic noise fields. We define a normalized density function of noise sources in the direction (θ, ϕ) and angular frequency ω as $F(\theta, \phi, \omega)$. This implies a

Figure 3.3: Randomly distributed point sources on an infinite-radius sphere

Figure 3.4: Randomly distributed point sources on an infinite-radius sphere

directional density function that describes the density of the plane-wave sources in the direction of (θ, ϕ) in which θ is a polar angle measured from the upward direction (zenith) and ϕ is an azimuthal angle (Fig. 3.4). The normalization of this density function is performed at each frequency w so that the following integration is satisfied [175]

$$\frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi F(\theta, \phi, w) \sin\theta d\theta d\phi = 1. \quad (3.15)$$

We assume that the noise sources are impulsive sources with broad flat spectra. The noise field obtained from such impulsive sources is zero-mean, white noise with a directional density function $F(\theta, \phi)$. Further, assuming uniformly distributed noise sources in azimuth direction will lead to a single variable directional density function $F(\theta)$. In this case, $\theta = 0$ and $\theta = \pi$ are vertical directions while $\theta = \pi/2$ is horizontal direction. In this case, the normalization

required for the density function $F(\theta)$ is simplified to

$$\frac{1}{2} \int_0^\pi F(\theta) \sin \theta d\theta = 1. \quad (3.16)$$

To obtain the correlation of ambient noise at sensors in the ocean, we need to compute the cross-correlation function of noise at sensors in the array. To do this, we consider two sensors designated as sensor 1 and 2 in the plane-wave noise field with corresponding noise samples $n_1(t)$ and $n_2(t)$. The temporal cross-correlation function is defined as [175]

$$r_{12}(\tau) = \overline{n_1(t)n_2(t-\tau)}, \quad (3.17)$$

where the overbar denotes ensemble average. The cross spectral density function of the noise at sensors 1 and 2 is defined through the following Fourier transform

$$R_{12}(\omega) = \int r_{12}(\tau) e^{-i\omega\tau} d\tau, \quad (3.18)$$

where R_{12} is not normalized. The cross spectral density of the noise at sensors 1 and 2 can also be written as

$$R_{12}(\omega) = \frac{\overline{N_1(\omega)N_2^H(\omega)}}{T}, \quad (3.19)$$

where $N_1(\omega)$ and $N_2(\omega)$ denotes the Fourier transforms of time series $n_1(t)$ and $n_2(t)$, and T is the observation time. The coherence function, $\Gamma_{12}(\omega)$, is defined as the cross-spectral density function normalized to the geometric mean of power spectral densities [176]

$$\Gamma_{12}(\omega) = \frac{R_{12}(\omega)}{\sqrt{R_{11}(\omega)R_{22}(\omega)}} = \frac{R_{12}(\omega)}{R_0(\omega)}, \quad (3.20)$$

where the power spectral density at both sensors ($R_{11}(\omega)$ and $R_{22}(\omega)$) are assumed to have equal value $R_0(\omega)$. This is a valid assumption for a plane-wave noise field because a plane-wave noise field is spatially homogeneous and the power spectral density R_{jj} , has the same value everywhere in the field. In a spatially homogeneous field, the cross-spectral density function depends on the distance between sensors and their orientation (not exact position of sensors). A special case of a homogeneous field is the isotropic noise field in which the sources has been distributed uniformly in all directions [175].

Cox [175] has derived an expression for the coherence function at any two points (sensors)

in a directional noise field given as

$$\Gamma(d, \omega; \theta_d) = \frac{1}{2} \int_0^\pi J_0 \left[\left(\frac{\omega d}{c} \right) \sin \theta_d \sin \theta \right] F(\theta, \omega) \exp \left[j \left(\frac{\omega d}{c} \right) \cos \theta_d \cos \theta \right] \sin \theta \, d\theta \quad (3.21)$$

where J_0 is the Bessel function of the first kind of zero order, d is the distance between the two sensors of interest and θ_d is the angle between the line connecting the sensors and the vertical, measured from the zenith. In this expression, θ is the polar angle measured from the zenith (Fig. 3.4) and ω is the angular frequency. Without loss of generality, the origin of the spherical coordinate system is selected on the line joining the pair of sensors (sensor 1 and 2) so that the location of sensors can be defined as sensor 1 at (r, θ_d, ϕ) and sensor 2 at $(r - d, \theta_d, \phi)$. From expression 3.21, it is obvious that the coherence function is independent of azimuth angle [175].

According to Buckingham's theoretical model, ambient noise [1, 169] in shallow water is dominated by the surface generated (wind generated) noise. In this model, noise is considered to have been produced by a random distribution of a large number of independent and identical impulsive point sources situated on a horizontal plane at a small depth near the sea surface. The acoustic field associated with each of these point sources in a depth far from the channel boundaries can be expressed as sum of finite number of normal modes by ignoring the contribution of the continuous radiation spectrum of near-field sources (3.6) [5]. The continuous contribution of near field sources can be neglected when the bottom is a low-loss boundary and the point sources are distributed in far-field. The noise field is taken to be uniform in azimuth which means that no particular horizontal direction is preferred for noise propagation.

Using Buckingham model, the noise correlation matrix may be computed for vertically [1] and horizontally [169] separated points. Buckingham model for the surface generated noise in isovelocity shallow water (i.e. sound speed is the same in all parts of a given water column) gives an expression for coherence function at two arbitrary points in the ocean. This model has been shown to yield results which are virtually indistinguishable from those given by a more general computer simulated model. The results obtained from Buckingham model agree with the results from measured data taken from sites such as Eureka, North California, Jellicoe Channel, New Zealand as well [1, 85, 169]. These results show that Buckingham model can be used as an acceptable mathematical model for noise samples at two points in a shallow-water channel.

Buckingham has shown that over a large portion of the water column (a conceptual column of

water from surface to bottom sediments) far from boundaries, noise is essentially homogeneous. Under these conditions, we can represent the spatial coherence of the noise field in terms of the directional density function of plane-wave sources. The directional density function is given by [1]

$$F(\theta, \omega) = \frac{1}{M} \left[\sum_{m=-M}^M \delta \left(\cos \theta - \frac{m\pi c}{\omega h} \right) - \delta(\cos \theta) \right]. \quad (3.22)$$

Function $F(\theta, \omega)$ satisfies the normalization condition defined in (3.15). On substituting the directivity function (3.22) and the normalization condition into (3.21), the coherence function becomes [169]

$$\Gamma(d, \omega; \theta_d) = \frac{1}{M} \sum_{m=1}^M J_0 \left\{ \varpi \left[1 - \left(\frac{m}{M} \right)^2 \sin^2 \alpha_c \right]^{1/2} \sin \theta_d \right\} \cos \left[\left(\frac{m}{M} \varpi \right) \cos \theta_d \sin \alpha_c \right], \quad (3.23)$$

where $\varpi = \frac{\omega d}{c}$ and M is the total number of modes defined in (3.10) that the channel can support and α_c is the critical grazing angle at the sea bottom.

When $\theta_d = 0$, this model presents the normalized vertical density function between two vertically separated sensors as [1, 169]

$$\Gamma(d, \omega; 0) = \frac{1}{2M} \left[\frac{\sin \left((M + .5) \frac{\pi d}{h} \right)}{\sin \left(\frac{\pi d}{2h} \right)} - 1 \right]. \quad (3.24)$$

If one of the sensors is close to either the surface or bottom of the ocean, (3.24) has to be replaced by the following equation [1]

$$\Gamma(d, \omega; 0) = \frac{\frac{\sin \left((M + \frac{1}{2}) a d \right)}{\sin \left(\frac{a d}{2} \right)} - \frac{\sin \left((M + \frac{1}{2}) a (z_1 + z_2) \right)}{\sin \left(a \frac{(z_1 + z_2)}{2} \right)}}{\sqrt{2M + 1 - \frac{\sin((2M+1)az_1)}{\sin(az_1)}} \sqrt{2M + 1 - \frac{\sin((2M+1)a z_2)}{\sin(a z_2)}}}, \quad (3.25)$$

where $a = \pi/h$, and z_1 and z_2 denote the depths of sensor 1 and 2 respectively.

Considering $\theta_d = \pi/2$, expression (3.23) leads to horizontal density function [169]

$$\Gamma(d, \omega; \pi/2) = \frac{1}{M} \sum_{m=1}^M J_0 \left\{ \varpi \left[1 - \left(\frac{m}{M} \right)^2 \sin^2 \alpha_c \right]^{1/2} \right\}. \quad (3.26)$$

The cross-correlation r_{12} between the noise samples at sensors 1 and 2 can be obtained from directional density function as follows [176]. First, we obtain the cross correlation function

based on the cross spectral density function using inverse of Fourier transform as

$$r_{12}(\tau) = \frac{1}{2\pi} \int R_{12}(\omega) e^{i\omega\tau} d\omega, \quad (3.27)$$

where τ is the correlation delay time between the time series $n_1(t)$ and $n_2(t)$.

Using (3.20), we can write $R_{12}(\omega) = \Gamma_{12}(\omega)R_0(\omega)$. By substituting this in (3.27), the cross-correlation function can be defined based on directional density function as

$$r_{12}(\tau) = \frac{R_0}{2\pi} \int \Gamma_{12}(\omega) e^{i\omega\tau} d\omega, \quad (3.28)$$

where the power spectrum of noise (R_0) is taken out from the integral because the noise sources are assumed to be white and hence their power spectral density function is independent of frequency.

By replacing Γ_{12} obtained for vertical and horizontal array respectively in (3.24 - 3.25) and (3.26), the vertical and horizontal cross-correlation function can be computed accordingly.

3.6 Summary

In this chapter, we have introduced acoustic propagation and theoretical models for acoustic field and ambient noise in a shallow-water channel. These models are required for simulating observations in the channel for further processing. We have discussed different mathematical models exist for modeling acoustic propagation in the ocean including Ray Theory, Fast Field Program (FFP), Parabolic Equation (PE), normal mode model (NM) and Direct Finite-difference (FD) or finite-element (FE). Normal mode based models have been found to be suitable models for shallow water acoustic propagation and transmission especially in low frequency range for range-independent oceans. They can also be easily extended for range-dependent ocean environments. We selected Pekeris model as a simple yet accurate NM based model to simulate acoustic field in our work. The details of Pekeris model have been discussed in this chapter.

Modeling ambient noise in the ocean has been also investigated in this chapter. Buckingham model as a suitable model for surface generated ambient noise in shallow water for both vertically and horizontally separated sensors have been discussed in detail. The Buckingham model is used to simulate ambient noise in Chapter 6 of this thesis.

Chapter 4

Signal detection using nonlinear wavelet denoising

In this chapter, we consider the problem of weak signal detection in non-Gaussian noise and present a computationally simple detector that employs a different category of denoising scheme as a preprocessor to conventional detectors. This detector is based on nonlinear wavelet denoising whose performance is significantly better than matched filter and compares favorably with the optimal detector. The detector presented consists of a denoising filter for SNR enhancement using a nonlinear wavelet transform, followed by a replica correlator and a threshold detector. We consider two different filtering schemes for denoising the data. The first scheme is based on a linearized version of Median Interpolating Pyramid Transform (MIP-T) [60], designated LMIPT. The second scheme is based on Block Median Pyramid Transform (BMPT) [61]. Detectors employing these filters are designated as the LMIPT detector and the BMPT detector respectively. These detectors are collectively referred to as NWD detectors. We have derived expressions for the PDFs of the outputs of the LMIPT and BMPT filters, and used these expressions for a performance analysis of the corresponding detectors. The theoretical and experimental analysis of the NWD detectors are presented and compared with those of optimal detector and matched filter. Theoretical and simulation results are presented in this chapter to illustrate the performance of the NWD detectors and their robustness. All the analysis and derivation presented in this Chapter have a general form and can be used for different noise distributions. Since we have not access to real data, we use one of the existing non-Gaussian noise models to model shallow water noise. These mathematical models

have been widely used in literature in several applications for modeling underwater acoustic noise [12, 19, 40, 41, 91, 99, 165, 179]. We use Generalized Gaussian noise to model non-Gaussian noise as a sample model in simulations in this Chapter. The Buckingham model introduced in Chapter 3 is for modeling the correlation between noise samples at any two points in a shallow ocean. Here, we are interested in the problem of single sensor detection, so we have not used Buckingham model. This model will be used in Chapter 6 for modeling the correlation between noise samples when the problem of source localization using a linear array of sensors is discussed.

This chapter is organized as follows. The structure of the proposed detector and an outline of its performance analysis are presented in Section 4.1. A nonlinear wavelet denoising scheme, including the description of two nonlinear filters (LMIPT and BMPT), is presented in Section 4.2. Expressions for the output PDFs of the LMIPT and BMPT filters are derived in Section 4.3. These expressions are used to carry out a performance analysis of the NWD detectors. The computational complexity of the proposed detectors is analyzed in Section 4.4. Simulation results are presented in Section 4.5 and compared with the theoretical results. Summary and concluding remarks are presented in Section 4.6.

4.1 NWD detector

The schematic diagram of the proposed detector is shown in Fig. 4.1. We consider the binary hypothesis testing problem

$$\begin{aligned} H_0 : x[n] &= w[n], \\ H_1 : x[n] &= w[n] + s[n]; \quad n = 0, 1, \dots, N-1. \end{aligned} \quad (4.1)$$

where H_0 denotes the null hypothesis (signal absent) and H_1 denotes the alternative hypothesis (signal present). The noise samples $w[n]$ are assumed to be independent and identically distributed with PDF $f_w(\cdot)$, and the signal $s[n]$ is deterministic. The conventional replica correlator or matched filter (MF) detector, which is optimal in white Gaussian noise, is shown in Fig. 4.1(a). The test statistic of this detector is given by

$$T_{MF}(\mathbf{x}) = \sum_{n=0}^{N-1} x[n] s[n], \quad (4.2)$$

Figure 4.1: Block diagram of (a) matched filter, (b) nonlinear wavelet denoising (NWD) detector.

where $\mathbf{x} = [x[0], \dots, x[N-1]]^T$ denotes the data vector.

The optimal detector in non-Gaussian noise has the test statistic [47]

$$T_{OD}(\mathbf{x}) = \sum_{n=0}^{N-1} \ln \frac{f_w(x[n] - s[n])}{f_w(x[n])}, \quad (4.3)$$

whose implementation imposes a heavy computational burden. The optimal detector is also sensitive to errors in modeling the noise PDF. The proposed NWD detector is an easily implementable and robust suboptimal detector. The block diagram of the proposed detector is shown in Fig. 4.1 (b). The test statistic of this detector is given by

$$T_{NWD}(\mathbf{y}) = \sum_{n=0}^{N-1} y[n] s[n], \quad (4.4)$$

where $\mathbf{y} = [y[0] \dots y[N-1]]^T$ is the output of the NWD filter. The objective of this filter is to enhance the SNR if the data vector contains the signal component. In general, the filter output $\mathbf{y}$ is non-Gaussian. But it is shown in Section 4.4 that the non-Gaussianity of $\mathbf{y}$ decreases as the number of decomposition levels in the NWD filter is increased. Secondly, in view of the central limit theorem, the test statistic $T[\mathbf{y}]$ is asymptotically ($N \rightarrow \infty$) Gaussian even if $\mathbf{y}$ is non-Gaussian. Hence, we can write the following asymptotic expressions for the probability of

false alarm P_F and the probability of detection P_D for NWD detector as follows [43].

$$\begin{aligned}
 P_F &= P(H_1; H_0) = P(T_{NWD}(\mathbf{y}) > \eta; H_0) = Q\left(\frac{\eta - E(T_{NWD}; H_0)}{\sqrt{\text{var}(T_{NWD}; H_0)}}\right), \\
 P_D &= P(H_0; H_0) = P(T_{NWD}(\mathbf{y}) > \eta; H_1) \\
 &= Q\left(\frac{\eta - E(T_{NWD}; H_1)}{\sqrt{\text{var}(T_{NWD}; H_1)}}\right) = Q\left(r Q^{-1}(P_F) - \sqrt{r} \delta^2\right),
 \end{aligned} \tag{4.5}$$

where η denotes the detector threshold, var denotes the variance,

$$\delta^2 = \frac{(E(T_{NWD}; H_1) - E(T_{NWD}; H_0))^2}{\sqrt{\text{var}(T_{NWD}; H_0) \times \text{var}(T_{NWD}; H_1)}}, \quad r = \sqrt{\frac{\text{var}(T_{NWD}; H_0)}{\text{var}(T_{NWD}; H_1)}}, \tag{4.6}$$

$Q(\cdot)$ is the complementary CDF of a standard normal random variable, and $Q^{-1}(\cdot)$ is the inverse of Q function. The mean and variance of the test statistic are given by,

$$\begin{aligned}
 E(T_{NWD}; H_i) &= \sum_{n=1}^N s[n] E(y[n]; H_i); \\
 \text{var}(T_{NWD}; H_i) &= \sum_{n=1}^N s^2[n] \text{var}(y[n]; H_i); \quad i = 0, 1.
 \end{aligned} \tag{4.7}$$

4.2 Nonlinear Wavelet Denoising (NWD) filters

In this section, three denoising techniques based on pyramid transform used in the NWD detector are described. These denoising techniques are applied to the received data as a pre-processor in NWD detectors. Detection performance and computational complexity of NWD detectors based on these denoising methods are analyzed and compared in later sections.

4.2.1 Median interpolating pyramid transform

NWD filters considered in this chapter are based on the median interpolating pyramid transform (MIPT) or its variants. Consider $N = 3^J$ samples of a time series $x[n]$

$$x[n] = x\left(\left(n + \frac{1}{2}\right)T_s\right); \quad n = 0, 1, \dots, N-1, \tag{4.8}$$

generated by uniform sampling of a continuous time-signal $x(t)$ with sampling period T_s . If the sampling is sufficiently dense, we can consider $x[n]$ to be the median of $x(t)$ over the interval $[nT_s, (n+1)T_s]$

$$x[n] = \text{med}(x(t) | [nT_s, (n+1)T_s]); \quad n = 0, 1, \dots, N-1, \quad (4.9)$$

where following [60], the median of a function on an interval I is defined as $\text{med}(x(t) | I) = \inf\{\mu : m(t \in I : x(t) \geq \mu) \geq m(t \in I : x(t) \leq \mu)\}$. Let $m_{j,k}$ denote the medians of the sequence $\{x[n], n = 0, 1, \dots, N-1\}$ over triadic blocks of size 3^{J-j} ; $0 \leq j \leq J$:

$$\begin{aligned} m_{j,k} = & \text{smed}(x[k \times 3^{J-j}], x[k \times 3^{J-j} + 1], \dots \\ & x[(k+1) \times 3^{J-j} - 1]); \quad k = 0, 1, \dots, 3^j - 1. \end{aligned} \quad (4.10)$$

where the operator ‘smed’ denotes the median of a sequence, defined as the central value of an ordered set of amplitudes. It follows that $m_{J,k} = x[k]$. Further, defining

$$\epsilon_{j,k} = m_{j,k} - \hat{m}_{j,k}, \quad (4.11)$$

where $\hat{m}_{j,k}$ is the estimate of $m_{j,k}$ obtained from the medians at the next lower scale as follows.

We define an interpolating quadratic polynomial $\pi_{j,k}(t) = a + bt + ct^2$ such that

$$\text{med}(\pi_{j,k}(t) | [r+1, r+2]) = m_{j,k+r}; \quad r = -1, 0, 1. \quad (4.12)$$

The estimates of medians at the next higher scale are given by

$$\hat{m}_{j+1,3k+l} = \text{med}\left(\pi_{j,k}(t) \left| \left[\frac{l}{3} + 1, \frac{l+1}{3} + 1\right]\right.\right); \quad l = 0, 1, 2. \quad (4.13)$$

It can be shown that [60]

$$\hat{m}_{j+1,3k+l} = m_{j,k-1} + (m_{j,k} - m_{j,k-1}) q_l(d_{j,k}); \quad l = 0, 1, 2. \quad (4.14)$$

where

$$d_{j,k} = \frac{m_{j,k+1} - m_{j,k}}{m_{j,k} - m_{j,k-1}}, \quad (4.15)$$

$$q_0(d) = \begin{cases} \frac{323-214d+35d^2}{288(1-d)} & d \in [-11, -3], \\ \frac{77+13d+8\sqrt{1-62d+d^2}}{135} & d \in [-3, -1/3], \\ \frac{2(13+8d-2\sqrt{1+16d+16d^2})}{27} & d \in [1/5, 3/7], \\ \frac{59+7d-8\sqrt{16+16d+d^2}}{27} & d \in [7/3, 5], \\ \frac{7-d}{9} & \text{otherwise,} \end{cases} \quad (4.16)$$

$$q_1(d) = \begin{cases} \frac{23+7d+2\sqrt{1-62d+d^2}}{30} & d \in \{-3, -10/7\} \cup [-7/10, -1/3], \\ \frac{274-4d+\sqrt{1-62d+d^2}}{270} & d \in [-10/7, -7/10], \\ 1 & \text{otherwise,} \end{cases} \quad (4.17)$$

$$q_2(d) = \begin{cases} \frac{122+58d+8\sqrt{1-62d+d^2}}{135} & d \in [-3, -1/3], \\ \frac{323-214d+35d^2}{288(1-d)} & d \in [-1/3, -1/11], \\ \frac{4(5-8d+2\sqrt{1+16d+16d^2})}{27} & d \in [1/5, 3/7], \\ \frac{11+d+4\sqrt{16+16d+d^2}}{27} & d \in [7/3, 5], \\ \frac{10+2d}{9} & \text{otherwise.} \end{cases} \quad (4.18)$$

MIPT with L levels of decomposition is composed of the set of approximation coefficients $\{m_{J-L,k}; k = 0, 1, \dots, 3^{J-L} - 1\}$, and the set of detail coefficients $\{\epsilon_{j,k} = m_{j,k} - \hat{m}_{j,k}, k = 0, 1, \dots, 3^j - 1; j = J - L + 1, J - L + 2, \dots, J\}$. Reconstruction of the signal is done using the equations

$$m_{j,k} = \hat{m}_{j,k} + \epsilon_{j,k}. \quad (4.19)$$

Reconstruction starts at the lowest scale, $J - L$, and proceeds to higher scales.

$$m_{J-L,k} \rightarrow \hat{m}_{J-L+1,k} \rightarrow m_{J-L+1,k} \rightarrow \hat{m}_{J-L+2,k} \dots \rightarrow m_{J,k}.$$

For denoising, the coefficients $\{\epsilon_{j,k}; k = 0, \dots, 3^j - 1; j = J - L + 1, J - L + 2, \dots, J\}$ are replaced by the thresholded coefficients $\{\tilde{\epsilon}_{j,k}, k = 0, 1, \dots, 3^j - 1; j = J - L + 1, J - L + 2, \dots, J\}$ before performing the reconstruction, as explained in greater detail in Section 4.3.4. A block diagram of the denoising filter based on MIPT is shown in Fig. 4.2.

Figure 4.2: Structure of NWD filter based on median interpolating pyramid transform (MIPT) and its linearized version (LMIPT). In this figure, IMP and MF refer to Imputation (Interpolation) and Median filter respectively

4.2.2 Linearized median interpolating pyramid transform

The linearized median-interpolating pyramid transform (LMIPT) is obtained by using the linearized versions of q_l , $l = 0, 1, 2$, defined in (4.16-4.18), i.e.

$$q_0(d) = \frac{1}{9}(7 - d), \quad q_1(d) = 1, \quad q_2(d) = \frac{2}{9}(5 + d). \quad (4.20)$$

On substituting (4.20) into (4.14) for each l and considering the definition of d in (4.15), we get the following linear relation between the medians at scale j and the imputed medians at scale $j + 1$

$$\hat{m}_{j+1,3k+l} = \sum_{r=-1}^1 \alpha_{r,l} m_{j,k+r}; \quad l = 0, 1, 2, \quad (4.21)$$

where

$$\begin{aligned} \alpha_{-1,0} &= \frac{2}{9}, \quad \alpha_{0,0} = \frac{8}{9}, \quad \alpha_{1,0} = \frac{-1}{9}, \quad \alpha_{-1,1} = 0, \quad \alpha_{0,1} = 1, \\ \alpha_{1,1} &= 0, \quad \alpha_{-1,2} = \frac{-1}{9}, \quad \alpha_{0,2} = \frac{8}{9}, \quad \alpha_{1,2} = \frac{2}{9}. \end{aligned} \quad (4.22)$$

The linearization is required for computing the PDF of the detail coefficients $\{\epsilon_{j,k}; k = 0, 1, \dots, 3^j - 1; j = J - L + 1, J - L + 2, \dots, J\}$. It is noted that linearization does not impair the perfect reconstruction property of the transform. A comparison of the functions $q_l(d)$, $l = 0, 1, 2$, for MIPT and LMIPT is shown in Figs. 4.3 - 4.5. It is seen that, for each of these functions, the difference between their values for MIPT and LMIPT is restricted to a few isolated segments, and even in these segments the differences are quite small. Hence, it is expected that the performance of the LMIPT denoising filter to be very close to that of the MIPT denoising filter. Results presented in Section 4.4 indicate that the performance of the LMIPT detector is indeed very close to that of the MIPT detector. Linearization provides the LMIPT detector with two distinct advantages over the MIPT detector: (1) the former is easier to implement, and (2) the former is amenable to a theoretical performance analysis while the latter is not.

4.2.3 Block median pyramidal transform

In MIPT, $\{m_{j,k}; k = 0, 1, \dots, 3^j - 1\}$ are medians of non-overlapping data blocks of size 3^{J-j} . In the block median pyramidal transform (BMPT), medians at scale j are medians of

Figure 4.3: Plots of $q_0(d)$ versus parameter d for median interpolating pyramid transform (MIPT) and its linearized version (LMIPT).

Figure 4.4: Plots of $q_1(d)$ versus parameter d for median interpolating pyramid transform (MIPT) and its linearized version (LMIPT).

Figure 4.5: Plots of $q_2(d)$ versus parameter d for median interpolating pyramid transform (MIPT) and its linearized version (LMIPT).

non-overlapping triadic blocks of medians at scale $j + 1$, i.e.

$$m_{j,k} = \text{smed}(m_{j+1,3k}, m_{j+1,3k+1}, m_{j+1,3k+2}); \quad k = 0, 1, \dots, 3^j - 1. \quad (4.23)$$

The imputed medians $\hat{m}_{j+1,3k+l}$ at scale $j + 1$ are obtained from the medians at the next lower scale as follows. Defining an interpolating quadratic polynomial $\pi_{j,k}(t) = a + bt + ct^2$ such that

$$\pi_{j,k}(l) = m_{j,k+l}; \quad l = -1, 0, 1. \quad (4.24)$$

The imputed medians at scale $j + 1$ are given by

$$\hat{m}_{j+1,3k+l} = \pi_{j,k}\left(\frac{l-1}{3}\right); \quad l = 0, 1, 2. \quad (4.25)$$

It can be easily shown that the imputed medians at scale $j + 1$ are linearly related to the medians at scale j . The relation between the imputed medians at at scale $j + 1$ and the medians at scale j in BMPT is defined by equation (4.21) that is identical to that for LMIPT. To show this, we start from (4.24) considering $\pi_{j,k}(t) = a + bt + ct^2$ to find the coefficients a , b , and c in terms of medians:

$$\begin{aligned} \pi_{j,k}(-1) &= m_{j,k-1} = a - b + c, \\ \pi_{j,k}(0) &= m_{j,k} = a, \\ \pi_{j,k}(1) &= m_{j,k+1} = a + b + c. \end{aligned}$$

By solving this set of equations for obtaining a , b , and c , we have:

$$\begin{aligned} a &= m_{j,k}, \\ b &= \frac{1}{2}m_{j,k+1} - \frac{1}{2}m_{j,k-1}, \\ c &= \frac{1}{2}m_{j,k+1} + \frac{1}{2}m_{j,k-1} - m_{j,k}. \end{aligned}$$

Replacing a , b , and c into (4.25) for $l = 0$, leads to

$$\begin{aligned}
 \hat{m}_{j+1,3k} &= \pi_{j,k}\left(\frac{-1}{3}\right) = a - \frac{b}{3} + \frac{c}{9}, \\
 &= m_{j,k} - \frac{1}{3}\left(\frac{1}{2}m_{j,k+1} - \frac{1}{2}m_{j,k-1}\right) \\
 &\quad + \frac{1}{9}\left(\frac{1}{2}m_{j,k+1} + \frac{1}{2}m_{j,k-1} - m_{j,k}\right), \\
 &= \frac{2}{9}m_{j,k-1} + \frac{8}{9}m_{j,k} - \frac{1}{9}m_{j,k+1}.
 \end{aligned}$$

Therefore, considering the notation used in (4.21), we can write:

$$\alpha_{-1,0} = \frac{2}{9}, \quad \alpha_{0,0} = \frac{8}{9}, \quad \alpha_{1,0} = \frac{-1}{9}.$$

It is seen that the values of the coefficients $\alpha_{-1,0}$, $\alpha_{0,0}$, and $\alpha_{1,0}$ are identical to those given in (4.22). It can be similarly shown that the values of $\alpha_{r,l}$ for $l = -1$ and $l = 1$ are also the same as in (4.22).

It is noted that BMPT coefficients are different from the LMIPT coefficients since the medians $m_{j,k}$ of these transforms are different. A block diagram of the BMPT denoising filter is shown in Fig. 4.6.

4.2.4 Denoising of Signal

Denoising is achieved by subjecting the coefficients $\epsilon_{j,k}$ to hard thresholding. The filtered signal is recovered by using the thresholded coefficients $\tilde{\epsilon}_{j,k}$ in the reconstruction procedure instead of the original coefficients $\epsilon_{j,k}$. The thresholds are level dependent, and are also dependent on the PDF of noise. It has been shown by Donoho and Yu [60] that, for zero-mean i.i.d noise with symmetric PDF, the magnitude $|\epsilon_{j,k}|$ of every coefficient is very likely to be lower than the threshold t_j defined as

$$t_j = F^{-1} \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 - \left(\frac{1}{2J3^J} \right)^{\frac{2}{3^{J-j}}}} \right), \quad (4.26)$$

where $F(\cdot)$ denotes the cumulative distribution function of noise. The coefficients, $\epsilon_{j,k}$ with absolute value lower than t_j are considered as noise and neglected. The thresholded coefficients

Figure 4.6: Structure of the nonlinear wavelet denoising (NWD) filter based on block median pyramid transform (BMPT). In this figure, IMP and MF refer to imputation (interpolation) and Median filter respectively.

are given by:

$$\tilde{\epsilon}_{j,k} = \begin{cases} \epsilon_{j,k}, & \text{if } |\epsilon_{j,k}| \geq t_j \\ 0, & \text{if } |\epsilon_{j,k}| < t_j \end{cases}; \quad j = J - L + 1, \dots, J. \quad (4.27)$$

The thresholding procedure is the same for MIPT, LMIPT and BMPT. But values of the thresholded coefficients are different for different transforms. The medians $\tilde{m}_{j,k}$ of the filtered signal are reconstructed using the equations:

$$\begin{aligned} \tilde{m}_{J-L,k} &= m_{J-L,k}, \\ \tilde{m}_{j,k} &= \tilde{\epsilon}_{j,k} + \hat{\tilde{m}}_{j,k}; \quad k = 0, 1, \dots, 3^j - 1; \quad j = J - L + 1, \dots, J. \end{aligned} \quad (4.28)$$

where $\hat{\tilde{m}}_{j,k}$ is the imputed denoised median at scale j . The imputed denoised medians $\hat{\tilde{m}}_{j+1,3k+l}$ ($l = 0, 1, 2$) at scale $j + 1$ are estimated from the denoised medians at scale j , $\tilde{m}_{j,k+r}$ ($r = -1, 0, 1$) using equations analogous to (4.14) in the case of MIPT and equations analogous to (4.21) in the case of LMIPT and BMPT. Thus, in the case of LMIPT and BMPT, we have

$$\hat{\tilde{m}}_{j+1,3k+l} = \sum_{r=-1}^1 \alpha_{r,l} \tilde{m}_{j,k+r}; \quad l = 0, 1, 2, \quad (4.29)$$

where the coefficients $\alpha_{r,l}$ are defined in (4.22). The denoised medians at scales $J - L, J - L + 1, \dots, J$ can be determined recursively using (4.28) and (4.29). The set of denoised medians $\{\tilde{m}_{J,k}; k = 0, 1, \dots, 3^J - 1\}$ at scale J constitute samples of the filtered signal $\{y_L[n]; n = 0, 1, \dots, 3^J - 1\}$ with L levels of decomposition.

4.3 PDF of denoised data vector

4.3.1 Linear Median Interpolation Pyramid Transform

PDF of $\epsilon_{j,k}$

LMIPT with L levels of decomposition is composed of the set of approximation coefficients $\{m_{J-L,k}; k = 0, 1, \dots, 3^{J-L} - 1\}$, and the set of detail coefficients $\{\epsilon_{j,k}; k = 0, 1, \dots, 3^j - 1; j =$

$J - L + 1, J - L + 2, \dots, J\}$. Combining (4.11) and (4.21), we can write

$$\epsilon_{j+1,3k+l} = z_j(k, l) + u_j(k, l) + v_j(k, l), \quad (4.30)$$

where

$$\begin{aligned} z_j(k, l) &= m_{j+1,3k+l} - \alpha_{0,l} m_{j,k}, & u_j(k, l) &= -\alpha_{-1,l} m_{j,k-1}, \\ v_j(k, l) &= -\alpha_{1,l} m_{j,k+1}; & l &= 0, 1, 2. \end{aligned} \quad (4.31)$$

Since signal $s[n]$ is deterministic and noise samples are i.i.d, the data samples $x[n]$ are i.i.d random variables under hypotheses H_0 and independent (but not identically distributed) under hypothesis H_1 defined in (4.1). It is seen from (4.30) and (4.31) that the PDFs of the transform coefficients can be determined if the PDFs of the medians $m_{j,k}$ and the joint PDFs of $m_{j,k}$ and $m_{j+1,3k+l}$ are known for all j, k, l . For MIPT / LMIPT, expressions for the cumulative distribution function (CDF) of $m_{j,k}$, denoted by $F_{med}(x; j, k)$ and the joint CDF of $m_{j,k}$ and $m_{j+1,3k+l}$, denoted by $F_{joint}(x, y; j, k, l)$ are derived in Appendices A and B. The corresponding PDF $f_{med}(x; j, k)$ and joint PDF $f_{joint}(x, y; j, k, l)$ can be obtained by differentiation.

Using (4.31), we can write the expressions for PDFs of the random variables $u_j(k, l)$, $v_j(k, l)$, and $z_j(k, l)$ denoted by $f_{u_j}(x; k, l)$, $f_{v_j}(x; k, l)$ and $f_{z_j}(x; k, l)$ respectively, as

$$f_{u_j}(x; k, l) = \frac{1}{|\alpha_{-1,l}|} f_{med}\left(\frac{-x}{\alpha_{-1,l}}; j, k\right), \quad (4.32)$$

$$f_{v_j}(x; k, l) = \frac{1}{|\alpha_{1,l}|} f_{med}\left(\frac{-x}{\alpha_{1,l}}; j, k\right), \quad (4.33)$$

$$f_{z_j}(x; k, l) = \int_{-\infty}^{+\infty} \frac{1}{|\alpha_{0,l}|} f_{joint}\left(\frac{-v}{\alpha_{0,l}}, x - v; j, k, l\right) dv. \quad (4.34)$$

It can be seen from the definitions in (4.31) that the random variables z_j , u_j and v_j are derived from non-overlapping data blocks. Since the data samples $x[n]$ are statistically independent, it follows that the random variables z_j , u_j and v_j are also mutually independent. Hence, using (4.30), we can write the PDF of the coefficients $\epsilon_{j+1,3k+l}$ as

$$f_{\epsilon_{j+1,3k+l}}(x; l) = f_{z_j}(x; k, l) * f_{u_j}(x; k, l) * f_{v_j}(x; k, l), \quad (4.35)$$

where $*$ denotes convolution.

For an L -level LMIPT, we can determine the PDFs of $m_{J-L,k}$ under hypotheses H_1 and H_0 from (A.5) and (A.7) in Appendix A, and the joint PDFs of $m_{j,k}$ and $m_{j+1,3k+l}$ under hypotheses H_1 and H_0 from (B.6), (B.8) and (B.10), (B.12) in Appendix B. The corresponding PDFs and joint PDFs for an L -level BMPT can be determined from (C.1) - (C.4) in Appendix C and (D.2) - (D.5) in Appendix D.

Recursive procedure for obtaining the PDF of $y_L[n]$

We shall now consider the procedure for obtaining the PDFs of the filtered LMIPT coefficients, culminating in the determination of the PDF of the filtered signal $y_L[n]$. From (4.27), we get the following expression for the PDF of filtered LMIPT coefficients $\tilde{\epsilon}_{j,k}$

$$\begin{aligned} f_{\tilde{\epsilon}_{j,k}}(x) &= [F_{\epsilon_{j,k}}(t_j) - F_{\epsilon_{j,k}}(-t_j)] \delta(x) + f_{\epsilon_{j,k}}(x) \\ &\quad [U(x - t_j) + U(-x - t_j)], \quad j = J - L + 1, \dots, J. \end{aligned} \quad (4.36)$$

where $\delta(x)$ denotes the Dirac delta function and $U(x)$ is the unit step function.

From (4.29) and (4.28), we get the following expressions for the PDFs of $\hat{m}_{j,k}$ and $\tilde{m}_{j,k}$:

$$\begin{aligned} f_{\hat{m}_{j+1,3k+l}}(x; l) &= \left\{ \frac{1}{|\alpha_{-1,l}|} f_{\tilde{m}_{j,k-1}}\left(\frac{x}{\alpha_{-1,l}}\right) * \frac{1}{|\alpha_{0,l}|} f_{\tilde{m}_{j,k}}\left(\frac{x}{\alpha_{0,l}}\right) \right. \\ &\quad \left. * \frac{1}{|\alpha_{1,l}|} f_{\tilde{m}_{j,k+1}}\left(\frac{x}{\alpha_{1,l}}\right) \right\}, \quad j = J - L, \dots, J - 1. \end{aligned} \quad (4.37)$$

$$f_{\tilde{m}_{j,k}}(x) = f_{\tilde{\epsilon}_{j,k}}(x) * f_{\hat{m}_{j,k}}(x); \quad j = J - L + 1, \dots, J. \quad (4.38)$$

We can determine the PDF of the filtered signal $y_L[k] = \tilde{m}_{J,k}$ by a recursive procedure starting with the initial condition $f_{\tilde{m}_{J-L,k}}(x) = f_{med}(x; J - L, k)$, to obtain

$$f_{y_L[k]}(x) = f_{\tilde{m}_{J,k}}(x); \quad k = 0, 1, \dots, 3^J - 1. \quad (4.39)$$

Equation (4.39) can be used to determine the mean and variance of $T(\mathbf{y})$ required for analyzing the asymptotic performance of the NWD detector.

4.3.2 Block Median Pyramid Transform

For BMPT, expressions for the CDF $F_{med}(x; j, k)$ and the joint CDF $F_{joint}(x, y; j, k, l)$ are derived in Appendices C and D. Procedure for determining the PDF of the detail coefficients

$\epsilon_{j,k}$ and the filtered signal $y_L[n]$ is similar to the procedure for LMIPT described in Section 4.3.1.

The PDF of the output $y_L[n]$ of the BMPT filter under hypotheses H_0 and H_1 is shown in Fig. 4.7 for different number of decomposition levels $L = 1$ to $L = 4$. The PDF of the filter input $x[n]$ is also shown for comparison. The number of samples is $N = 3^5$, the signal is a constant, the signal power-to-noise power ratio (SNR) is -26.2 dB, the signal energy-to-noise power ratio (ENR) is -2.16 dB, and the noise $w[n]$ at the filter input has the generalized Gaussian (GG) PDF

$$f_w(x) = a \exp\{-b|x|^p\}, \quad (4.40)$$

where

$$a = \frac{p \left[\Gamma(\frac{3}{p}) \right]^{1/2}}{2 \left[\Gamma(\frac{1}{p}) \right]^{3/2}}, \quad b = \left[\frac{\Gamma(\frac{3}{p})}{\Gamma(\frac{1}{p})} \right]^{p/2} \quad \text{and} \quad \Gamma(\alpha) = \int_0^\infty \exp(-t) t^{\alpha-1} dt.$$

In different experiments conducted for detection of known deterministic signals corrupted by underwater acoustic noise, different classes of PDFs have been used and compared with each other. Many of the results showed that more general results can be obtained in the case of Generalized Gaussian densities [180, 181]. Besides, since we have derived the PDF of denoised data based on input data PDF (in a general form), other noise PDFs for modeling non-Gaussian distributions can be used for evaluation and analysis of the performance of the NWD detectors.

The plots in Fig. 4.7 have been obtained for GG exponent $p = 0.55$. The PDFs of the output of the LMIPT filter have a similar appearance. The peaked and heavy-tailed PDF of $x[n]$ in Fig. 4.7 illustrates the impulsive nature of noise at the filter input. The PDF of the filter output $y_L[n]$ has a smoother peak and a lighter tail because the thresholding operation defined in (4.27) removes some of the noise impulses. As the number of decomposition levels L is increased, more noise impulses are removed by the thresholding operation and the PDF of $y_L[n]$ approaches the Gaussian PDF. To validate this statement, we have computed, for $L = 1, 2, 3, 4$, the Kullback-Leibler divergence D_L of the PDF $f_{y_L[n]}(x)$ from the Gaussian PDF $f_{g_L}(x)$ with standard deviation σ_{g_L} , where σ_{g_L} is chosen so as to minimize D_L . The Kullback-Leibler divergence, which is a measure of difference between the PDFs, is defined as [182].

$$D_L = D(f_{y_L[n]} || f_{g_L}) = \sum_{-\infty}^{\infty} \ln \frac{f_{y_L[n]}(x)}{f_{g_L}(x)} f_{y_L[n]}(x) dx. \quad (4.41)$$

In general, $\sigma_{g_L} \neq \sigma_L$, where σ_L is the standard deviation of $y_L[n]$. Values of D_L, σ_L , and σ_{g_L} are shown in Table 4.1. In Table 4.1, $L = 0$ denotes values of the parameters D, σ and

Figure 4.7: PDF of input $x[n]$ and output $y[n]$ of block median pyramid transform (BMPT) filter under hypotheses H_0 and H_1 .

σ_g before denoising (at the filter input). The reduction of D_L with increasing L implies that the denoised signal $y_L[n]$ becomes more Gaussian with increasing L . The reduction of σ_L and σ_{gL} with increasing L implies that $y_L[n]$ becomes less noisy, and consequently the detection performance is expected to improve as L is increased. The proximity of $y_L[n]$ to Gaussianity provides additional justification for the asymptotic approximation made in Section 4.2 that the test statistic $T_{NWD}(\mathbf{y})$ is Gaussian.

4.4 Computational complexity

We shall compare the complexities of different NWD detectors, the optimal detector (OD) and the replica correlator or matched filter(MF). Blocks of $N = 3^J$ data samples are considered, n digits are used for expressing each data sample, and L is the number of decomposition levels in the NWD filters. Complexity can be measured in terms of the number of elementary operations performed in the algorithm [183]. The LMIPT and BMPT detectors involve

Table 4.1: Statistics of the BMPT filter output

L	D_L	σ_L	σ_{gL}
0	0.49	1	0.416
1	0.32	0.4	0.225
2	0.26	0.4	0.225
3	0.1	0.28	0.129
4	0.02	0.24	.0745

Table 4.2: Computational complexity of the NWD detectors, Optimal detector and Matched filter (MF)

Detector	NWD-detector (LMIPT)	NWD-detector (MIPT)	NWD-detector (BMPT)	MF	Optimal-detector
Computational Complexity	$4NO(n^2)$	$19NO(n^2)$	$4NO(n^2)$	$NO(n^2)$	$3NO(n^2) +$ $3NO(n^2 \log n) +$ $2O(n^{5/2}(\log n)^2)$

the operations of median computation, linear interpolation, wavelet decomposition and reconstruction, and replica correlation, all of which require additions and multiplications only. The nonlinear interpolation in MIPT also involves a number of division and square root operations. For n -bit numbers, the complexity of an addition is $O(n)$ and the complexity of a multiplication, division, or square root operation is $O(n^2)$ [184]. The OD involves computation of the nonlinear transformation given in (4.3) for each data sample. For the GG noise PDF defined in (4.40), the nonlinear transformation (4.3) also involves the operations of division, exponentiation with an integral exponent q , exponentiation with a non-integral exponent, logarithm, and Gamma function computation, whose complexities are $O(n^2)$, $O(qn^2)$, $O(n^2 \log n)$, and $O(n^{5/2}(\log n)^2)$ respectively [185]. Table 4.2 shows the computational complexities of the NWD detectors, matched filter and optimal detector. In this table, the contribution of $O(n)$ is ignored in comparison to $O(n^2)$. It is observed that the complexities of LMIPT and BMPT detectors are similar. For $n = 32, N = 243$, and $L = 3$, the complexity ratios (OD / LMIPT), (MIPT / LMIPT) and (MF / LMIPT) are approximately 3.5, 4.75 and 0.25 respectively. When $n = 64$, the (OD / LMIPT) complexity ratio increases to 4.2, while the other ratios are not affected significantly. The effect of changes in the value of N on the complexity ratios is marginal.

Figure 4.8: ROCs of MIPT and LMIPT detectors for different values of L obtained from simulations. $N = 3^5$.

4.5 Results and discussion

In this section, asymptotic theoretical and simulation results on the performance of the NWD detectors are presented. All the simulation results presented in figures have been obtained from 20000 Monte Carlo simulation runs. The signal-to-noise power ratio (SNR) is -26.2 dB and $N = 3^5$, unless otherwise stated. Most of the results (Figs. 4.8 - 4.14) are presented for a constant signal and GG noise with $p = 0.55$. Results for other signal waveforms are presented in Fig. 4.16, and the effect of variation of noise PDF is presented in Fig. 4.15. The simulation receiver operating characteristics (ROCs) of MIPT and LMIPT detectors are compared in Fig. 4.8 for different number of decomposition levels ranging from $L = 1$ to $L = 4$.

It is seen that ROCs of detectors based on the exact MIPT and the approximate LMIPT are almost coincident for all L . This result is in conformity with the conclusion in Section 4.3 B, based on Figs. 4.3 - 4.5, that LMIPT is a very good approximation to MIPT. We shall henceforth confine our attention to LMIPT and BMPT detectors only since the MIPT detector has a higher computational complexity and also because it is not amenable to a theoretical performance analysis due to the nonlinear interpolating method used in it.

Figure 4.9: ROCs of LMIPT detector obtained from theoretical (Thr) and simulation (Exp) results for different values of L . $N = 3^5$.

Figures 4.9 and 4.10 show ROCs of the LMIPT detector and the BMPT detector respectively, obtained from theoretical and simulation results for $N = 3^5$ and $SNR = -26.2$ dB. Similar plots for $N = 3^6$ and $SNR = -26.2$ dB are shown in Figs. 4.11 and 4.12. It is seen that the asymptotic theoretical predictions of P_D are consistently higher than the corresponding values obtained from simulations. But the difference between the theoretical and simulation values reduces with increasing N , as expected. It is seen that the performance of the LMIPT detector keeps improving as the number of decomposition levels L is increased. This improvement is intuitively expected since an increase in L causes the removal of more noise impulses by the thresholding operation defined in (4.27). But the best performance of the BMPT detector occurs at $L = 3$. This apparent anomaly may be explained by noting that the thresholding operation also results in a distortion of the signal, and the signal distortion increases as L is increased. Hence, there exists an optimal value of L , above which the detection performance starts degrading. The optimal value of L depends on the structure of the filter and the signal-to-noise ratio at the filter input. A comparison of the results in Figs. 4.9 and 4.11 with those in Figs. 4.10 and 4.12 also indicates that the performance of the LMIPT detector is consistently better than that of the BMPT detector.

Figure 4.10: ROCs of BMPT detector obtained from theoretical (Thr) and simulation (Exp) results for different values of L . $N = 3^5$.

We shall now compare the performances of the nonlinear wavelet denoising (NWD) detectors with other detectors, viz. the optimal detector (OD), the conventional matched filter (MF) and a linear wavelet denoising (LWD) detector based on standard (linear) wavelet transform using the Daubechies 20 ($D20$) wavelet. The ROCs of these detectors obtained from simulations are shown in Fig. 4.13, while plots of probability of detection P_D versus SNR for a fixed false alarm probability $P_F = 0.1$ (obtained from simulations) are shown in Fig. 4.14. Three levels of decomposition ($L = 3$) have been used for denoising. It is seen that the performance of the LMIPT detector is distinctly better than that of the BMPT detector, and that both of them are markedly superior to the matched filter. The performance of the LWD detector is almost the same as that of the matched filter because linear wavelet denoising provides little improvement in SNR if the noise is strongly non-Gaussian.

It is of interest to study the performance of the NWD detectors for different noise PDFs. It is seen from (4.5) and (4.6) that the asymptotic performance of the detectors depends on the value of the parameter δ^2 known as the deflection coefficient. As explained in Section 4.1, invoking Central Limit Theorem justifies that the test statistics of the detectors are asymptotically Gaussian, hence deflection coefficient can be used for performance comparison of the detectors.

Figure 4.11: ROCs of LMIPT detector obtained from theoretical (Thr) and simulation (Exp) results for different values of L . $N = 3^6$.

Figure 4.12: ROCs of BMPT detector obtained from theoretical (Thr) and simulation (Exp) results for different values of L . $N = 3^6$.

Figure 4.13: ROCs of LM IPT detector, BMPT detector, LWT detector, matched filter (MF), and optimal detector (OD) for $N = 3^5$ obtained from simulations.

Figure 4.14: Plots of P_D versus SNR for LM IPT detector, BMPT detector, LWT detector, matched filter (MF), and optimal detector (OD) obtained from simulations for $P_F = 0.1$ and $N = 3^5$.

It is observed in Fig. 4.15 that the performance of the detectors improves if the deflection coefficient becomes larger. Figure 4.15 shows the variation of δ^2 with the parameter p of the noise PDF, for the LMIPT detector, the BMPT detector, the matched filter (MF) and the locally optimal detector (LOD). The LOD is the optimal detector for signal amplitude tending to zero and number of samples N tending to infinity. We also recall that the GG noise is Gaussian if $p = 2$, super-Gaussian (heavy-tailed) if $p < 2$, sub-Gaussian (light-tailed) if $p > 2$, and that the tail of the PDF becomes progressively heavier as p is reduced. It is seen from Fig. 4.15 that: (1) the performance of the NWD detectors and that of LOD improves as the noise becomes more non-Gaussian, (2) the LMIPT detector outperforms the BMPT detector for all values of the parameter p . The performance of MF does not depend on the noise PDF. The MF performs better than the NWD detectors only if the noise PDF is close to Gaussian.

Figure 4.15: Variation of deflection coefficient δ^2 with parameter p of GG noise for LMIPT ($L = 3$) detector, BMPT ($L = 3$) detector, matched filter (MF), and locally optimal detector (LOD).

The performance of the NWD detectors and the optimal detector depends on the signal waveform, unlike the matched filter whose performance depends only on the signal energy. ROCs of the LMIPT ($L = 3$) detector for different signal waveforms obtained from theoretical and simulation results are presented in Fig. 4.16. The signals considered are (1) constant signal:

$s[n] = A$, (2) sinusoid: $s[n] = A \sin \frac{2\pi n}{N}; n = 0, 1, \dots, N-1$; and (3) rectangular pulse: $s[n] = A$, for $n = 40, 41, \dots, N-41$ and $s[n] = 0$ for $n = 0, 1, \dots, 39, N-40, N-39, \dots, N-1$. In this figure, we have chosen $N = 3^5$ and SNR = -20 dB for all the signals. It is seen that there is a good agreement between theoretical and simulations results, and that the best performance is obtained for the constant signal.

Figure 4.16: ROCs of LMIPT detector for different signal waveforms obtained from theoretical (Thr) and simulation (Exp) results $L = 3$, $N = 3^5$.

We shall next examine the sensitivity of the detectors to error in modeling the noise PDF. Let the actual noise PDF be GG with parameter $p = 0.5$. We shall consider the performance of OD, LMIPT, and BMPT detectors designed assuming a different value of p . Figure 4.17 shows the plots of probability of detection P_D versus the assumed value of parameter p obtained from simulations, for the OD, LMIPT ($L = 3$), BMPT ($L = 3$), and MF detectors. Figures 4.18 and 4.19 contain similar plots respectively for the cases of Laplacian (GG with $p = 1$) actual noise PDF and Gaussian (GG with $p = 2$) actual noise PDF. It is seen from these figures that the LMIPT and BMPT detectors are much more robust than OD. Obviously, MF provides the most robust performance, but it is far less efficient than the LMIPT detector if the noise is strongly non-Gaussian.

Figure 4.17: Effect of PDF mismatch on performance of different detectors. Actual noise PDF is GG with $p = 0.5$. Figure shows plots of P_D versus assumed value of parameter p for $P_F = 0.1$.

Figure 4.18: Plots similar to those in Fig. 4.17. Actual noise PDF is Laplacian (GG with $p = 1$).

Figure 4.19: Plots similar to those in Fig. 4.17. Actual noise PDF is Gaussian (GG with $p = 2$).

4.6 Summary and Conclusions

In this chapter, we have presented a robust and easily implementable detector based on nonlinear wavelet denoising (NWD) for detection of signals in non-Gaussian noise. In the proposed detector, the data vector is passed through a denoising filter based on a nonlinear wavelet transform to enhance the signal-to-noise ratio, and the filtered data vector is correlated with the signal vector to obtain the test statistic. A class of nonlinear median based transforms has been chosen for denoising since the conventional denoising techniques based on linear wavelet transforms are suitable only for denoising signals in Gaussian noise. Three denoising filters have been considered. These filters are based on median interpolating pyramid transform (MIPT), linearized median interpolating pyramid transform (LMIPT) and block-median pyramid transform (BMPT). LMIPT and BMPT employ a linear interpolation scheme to estimate medians at scale $j+1$ from the medians at the coarser scale j . LMIPT is a linearization of the nonlinear interpolation scheme of MIPT. A theoretical analysis of the LMIPT and BMPT detectors has been performed by deriving the relation between the PDF of the filter output and the noise PDF under the assumption of i.i.d noise samples. The PDF of the filtered data vector has been used to find the mean and variance of the test statistic under both hypotheses and thus

determine the asymptotic (N tending to infinity) performance of the detectors. A theoretical analysis of the MIPT detector is difficult since MIPT employs a nonlinear interpolation scheme.

The performance of the NWD detectors has been investigated using the generalized Gaussian PDF as a model for non-Gaussian noise. Receiver operating characteristics of the LMIPT and BMPT detectors have been obtained from theoretical and simulation results, and they have been shown to agree with one another. Only simulation results have been presented for the MIPT detector since it is not amenable to a theoretical analysis. The results show that (1) the performance of the MIPT detector is marginally better than that of the LMIPT detector, but the computational complexity of the former is approximately 5 times higher than that of the latter, (2) the performance of the LMIPT detector is consistently better than that of the BMPT detector, (3) the performance of the LMIPT and BMPT detectors is significantly better than that of the matched filter in strongly non-Gaussian noise, and (4) the performance of all NWD detectors improves as the non-Gaussianity of noise increases. The computational complexity of the LMIPT and BMPT detectors is lower than that of the optimal detector by a factor of approximately 4, and higher than that of the matched filter by a factor of approximately 4.

An analysis of sensitivity of the NWD detectors to noise PDF modeling error has also been carried out. It has been shown that the LMIPT and BMPT detectors have the same degree of robustness, and that both of them are far more robust than the optimal detector.

In summary, the LMIPT detector offers the following advantages for signal detection in strongly non-Gaussian noise: (1) significantly better performance than the matched filter, (2) greater robustness than the optimal detector, (3) moderate computational complexity. The study in this chapter is confined to the problem of detection of a known signal. The NWD approach may also be employed for the detection of an unknown signal in non-Gaussian noise. Such an extension would require replacement of the replica correlator defined in (4) by an estimator-correlator which involves estimation of the unknown signal vector.

Chapter 5

Cramér-Rao bound for 3-D source localization

Detection of a source in non-Gaussian noise was discussed in Chapter 4. In this chapter and the next chapter, we consider the problem of source localization in non-Gaussian noise. This chapter discusses the subject of lower bound on source localization and Chapter 6 introduces a computationally simple algorithm for localization of multiple sources in non-Gaussian noise. Three-dimensional (3-D) source localization of a deterministic source in shallow water in the presence of ambient noise is an important research problem in many underwater signal processing applications. Obtaining a lower bound on the variance of the source position estimates is very useful as a measure of estimation performance. If the estimator attains the bound for all the possible source positions (in ideal case), we can claim that the estimator is a minimum variance estimator. However, in practice, it provides a benchmark to compare the performance of different unbiased estimators. The Cramér-Rao bound (CRB) commonly used in signal processing is a well-known and easy to compute lower bound on the variance of unbiased estimators [186].

To be able to evaluate the performance of 3-D localization algorithm proposed in this thesis and its accuracy with the corresponding lower bound on each coordinate, we require to derive an expression for the CRB of 3-D source localization in shallow ocean with non-Gaussian noise. This is the main motivation for us to derive an expression for the CRB of 3-D localization that can be used for different array configurations and different noise distributions. Since the sources we consider are deterministic, we derive expressions for the deterministic CRBs in a channel with symmetric noise distributions considering the approach used by Kozick and Sadler in [18].

The CRBs derived in this chapter can be used to evaluate the performance of different source localization algorithms when noise has symmetric PDF including Gaussian and non-Gaussian noise. The CRB expressions derived will be used in several simulations in this thesis to evaluate the performance of the localization algorithm that we have developed.

The details of the CRB derivation are presented in Section 5.1 of this chapter. We obtain the expressions required for computation of the CRBs in shallow ocean (Pekeris model) with non-Gaussian noise using an array of scalar sensors in Section 5.2. The expressions for CRBs using an array of acoustic vector sensors are also computed and presented in Section 5.3. Simulation results are presented and discussed in Section 5.4. Finally, Section 5.5 ends this chapter with a summary of the results and concluding remarks.

5.1 CRB derivation

Consider an array of N sensors receiving narrowband signals from J_s far-field point sources in a range independent shallow ocean. The array output at snapshot t is denoted by the $N \times 1$ vector $\mathbf{y}(t)$ as

$$\mathbf{y}(t) = \mathbf{P}(\mathbf{x})\mathbf{s}(t) + \mathbf{w}(t); \quad t = 1, \dots, T, \quad (5.1)$$

where $\mathbf{w}(t) = [w_1(t), \dots, w_N(t)]^T$ is the additive noise vector, and superscript T denotes matrix transpose. The vector $\mathbf{s}(t)$ is defined as $\mathbf{s}(t) = [s_1(t), \dots, s_{J_s}(t)]^T$, where $s_j(t)$ is the complex amplitude of the j^{th} source signal at t . We assume a ‘conditional model’ [81] for the signals, wherein the signal envelopes $\{s_j(t); t = 1, \dots, T\}$, for $j = 1, \dots, J_s$, are modeled as unknown deterministic signals. Based on the definition in [81], in ‘conditional model’ signals are non-random (deterministic) and therefore, the same in all realizations. However, in an ‘unconditional model’, signals are random. The vector $\mathbf{x} = [\mathbf{x}_1, \dots, \mathbf{x}_{J_s}]$ is a row vector of the source positions where $\mathbf{x}_j = [r_j, z_j, \theta_j]$ denotes the j^{th} source position in terms of range r_j , depth z_j , and bearing θ_j . It is assumed that any change in the source position over the set of T snapshots is negligible. The matrix

$$\mathbf{P}(\mathbf{x}) = [\mathbf{p}(\mathbf{x}_1), \dots, \mathbf{p}(\mathbf{x}_{J_s})], \quad (5.2)$$

is the steering matrix of the array and $\mathbf{p}(\mathbf{x}_j)$ is a steering vector denoting the response of the array to a source of unit strength located at the position $\mathbf{x}_j$.

The real and imaginary part of a complex parameter w is shown with $\bar{w}$ and $\overline{\bar{w}}$ respectively.

Besides, the superscripts T and H denote the transpose and conjugate transpose operations respectively. In (5.1), the complex noise samples $\{w_n(t), n = 1, 2, \dots, N; t = 1, \dots, T\}$ at the array are assumed to be independent and identically distributed (i.i.d) in time and space with bivariate PDF of $f_W(\bar{w}, \bar{\bar{w}})$ that is symmetric, as

$$f_W(\pm \bar{w}, \pm \bar{\bar{w}}) = f_W(\bar{w}, \bar{\bar{w}}). \quad (5.3)$$

The symmetric assumption of noise samples defined above implies that the real and imaginary component of noise samples have zero mean and are uncorrelated. To make the steps of CRB derivation clear and easy to follow, we express the data vector $\mathbf{y}(t)$ given in (5.1) as:

$$\mathbf{y}(t) = \mathbf{g}(\Omega(t), t) + \mathbf{w}(t); \quad t = 1, \dots, T, \quad (5.4)$$

where $\mathbf{g}(\Omega(t), t) = [g_1(\Omega(t), t), \dots, g_N(\Omega(t), t)]^T$ is an $N \times 1$ vector and $\Omega(t)$ is the vector of unknown parameters related to the sources defined as

$$\Omega(t) = [\mathbf{x}_j, \bar{s}_j(t), \bar{\bar{s}}_j(t) \quad j = 1, \dots, J_s]; \quad t = 1, \dots, T, \quad (5.5)$$

in which $\bar{s}_j(t)$ and $\bar{\bar{s}}_j(t)$ are real and imaginary parts of signal $s_j(t)$ at time t respectively.

It is assumed that i.i.d noise samples $w_n(t)$ satisfy the conditions required for the existence of CRB. CRB matrix for the vector Ω is defined as the inverse of the Fisher information matrix (FIM) denoted as $\mathbf{J}_\Omega$. This implies that the variance of any unbiased estimator for i^{th} element of Ω (denoted by $\hat{\Omega}_i$) is bounded by the corresponding CRB as

$$\text{var}(\hat{\Omega}_i) \geq [\mathbf{J}_\Omega^{-1}]_{ii}, \quad (5.6)$$

The number of elements in $\Omega(t)$ is $3J_s$, therefore $i = 1, \dots, 3J_s$. The $(i, j)^{th}$ element of the Fisher information matrix $\mathbf{J}_\Omega$, a $3J_s \times 3J_s$ matrix, can be defined as [186]

$$[\mathbf{J}_\Omega]_{i,j} = E\left[\left(\frac{\partial \ln f_Y(\mathbf{Y})}{\partial \Omega_i}\right)\left(\frac{\partial \ln f_Y(\mathbf{Y})}{\partial \Omega_j}\right)\right], \quad (5.7)$$

where $\mathbf{Y} = [\mathbf{y}(1), \dots, \mathbf{y}(T)]$ is the set of array observations and $\mathbf{y}(t) = [y_1(t), \dots, y_N(t)]$ is the array output at t .

PDF of $y_n(t)$ is $f_w(\bar{w}_n(t), \bar{\bar{w}}_n(t))$. Since the noise samples $w_n(t)$ are i.i.d and the source signals are deterministic, the set of array observations are independent. Therefore PDF of the

data vector $\mathbf{Y}$ is defined as the multiplication of data samples' PDF:

$$f_Y(\mathbf{Y}) = \prod_{t=1}^T \prod_{n=1}^N f_w(\bar{w}_n(t), \bar{\bar{w}}_n(t)). \quad (5.8)$$

By taking the natural logarithm of (5.8) and substituting it into (5.7), the following expression is obtained

$$[\mathbf{J}_\Omega]_{i,j} = E\left[\left(\sum_{t=1}^T \sum_{n=1}^N \frac{1}{f_W(\bar{w}_n, \bar{\bar{w}}_n)} \frac{\partial f_W(\bar{w}_n, \bar{\bar{w}}_n)}{\partial \Omega_i}\right) \left(\sum_{t=1}^T \sum_{n=1}^N \frac{1}{f_W(\bar{w}_n, \bar{\bar{w}}_n)} \frac{\partial f_W(\bar{w}_n, \bar{\bar{w}}_n)}{\partial \Omega_j}\right)\right]. \quad (5.9)$$

Using the chain rule:

$$\frac{\partial f_W(\bar{w}, \bar{\bar{w}})}{\partial \Omega_i} = \frac{\partial f_W(\bar{w}, \bar{\bar{w}})}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \Omega_i} + \frac{\partial f_W(\bar{w}, \bar{\bar{w}})}{\partial \bar{\bar{w}}} \frac{\partial \bar{\bar{w}}}{\partial \Omega_i}, \quad (5.10)$$

and considering $\bar{w}_n(t) = \bar{y}_n(t) - \bar{g}_n(\Omega, t)$, and $\bar{\bar{w}}_n(t) = \bar{\bar{y}}_n(t) - \bar{\bar{g}}_n(\Omega, t)$ the expression in (5.9) can be written as:

$$[\mathbf{J}_\Omega]_{i,j} = I_{cr}(\Lambda) \sum_{t=1}^T \sum_{n=1}^N \frac{\partial \bar{g}_n(\Omega, t)}{\partial \Omega_i} \frac{\partial \bar{g}_n(\Omega, t)}{\partial \Omega_j} + I_{ci}(\Lambda) \sum_{t=1}^T \sum_{n=1}^N \frac{\partial \bar{\bar{g}}_n(\Omega, t)}{\partial \Omega_i} \frac{\partial \bar{\bar{g}}_n(\Omega, t)}{\partial \Omega_j}, \quad (5.11)$$

where

$$\begin{aligned} I_{cr}(\Lambda) &= E\left\{\left(\frac{1}{f_W(\bar{w}, \bar{\bar{w}})} \frac{\partial f_W(\bar{w}, \bar{\bar{w}})}{\partial \bar{w}}\right)^2\right\}, \\ I_{ci}(\Lambda) &= E\left\{\left(\frac{1}{f_W(\bar{w}, \bar{\bar{w}})} \frac{\partial f_W(\bar{w}, \bar{\bar{w}})}{\partial \bar{\bar{w}}}\right)^2\right\}, \end{aligned} \quad (5.12)$$

and Λ denotes noise PDF parameters. Since the noise PDF is symmetric, considering (5.3) we can write: $\frac{\partial f_W(\bar{w}, \bar{\bar{w}})}{\partial \bar{w}} = \frac{\partial f_W(\bar{w}, \bar{\bar{w}})}{\partial \bar{\bar{w}}}$, therefore $I_{cr}(\Lambda) = I_{ci}(\Lambda)$.

In Appendix E of this thesis, we have shown that

$$I_c(\Lambda) \triangleq I_{cr}(\Lambda) = I_{ci}(\Lambda) = \pi \int_0^\infty \frac{[f'(\rho)]^2}{f(\rho)} \rho \, d\rho, \quad \rho = \sqrt{\bar{w}^2 + \bar{\bar{w}}^2}. \quad (5.13)$$

Therefore (5.11) can be written as

$$[\mathbf{J}_\Omega]_{i,j} = I_c(\Lambda) \sum_{t=1}^T \operatorname{Re}\left[\left(\frac{\partial \mathbf{g}(\Omega, t)}{\partial \Omega_i}\right)^H \left(\frac{\partial \mathbf{g}(\Omega, t)}{\partial \Omega_j}\right)\right]. \quad (5.14)$$

where Re denotes real part. The FIM matrix is:

$$\mathbf{J}_\Omega = I_c(\Lambda) \sum_{t=1}^T Re[(\frac{\partial \mathbf{g}(\Omega, t)}{\partial \Omega})^H (\frac{\partial \mathbf{g}(\Omega, t)}{\partial \Omega})]. \quad (5.15)$$

To obtain the CRB, inverse of $\mathbf{J}_\Omega$ can be computed using the results presented in [76]. Using the computed inverse of $\mathbf{J}_\Omega$, the CRB for source coordinate estimators can be obtained from the following expression

$$CRB(\mathbf{x}) = \frac{1}{I_c(\Lambda)} \left\{ \sum_{t=1}^T Re[\mathbf{S}(t)^H \mathbf{Q}(\mathbf{x})^H (\mathbf{I} - \mathbf{P}(\mathbf{x})(\mathbf{P}(\mathbf{x})^H \mathbf{P}(\mathbf{x}))^{-1} \mathbf{P}(\mathbf{x})^H) \mathbf{Q}(\mathbf{x}) \mathbf{S}(t)] \right\}^{-1}, \quad (5.16)$$

where $\mathbf{I}$ is an $N \times N$ identity matrix and

$$\mathbf{Q}(\mathbf{x}) = [\mathbf{q}(\mathbf{x}_1), \dots, \mathbf{q}(\mathbf{x}_{J_s})], \quad \mathbf{q}(\mathbf{x}_j) = \left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_j}. \quad (5.17)$$

For 1-D source localization in which, $\mathbf{x}_j$ is a scalar (i.e., bearing or range or depth), $\mathbf{S}(t) = \text{diag}(s_1(t) \dots s_{J_s}(t))$ and $\mathbf{Q}(\mathbf{x}) = [\mathbf{q}(x_1), \dots, \mathbf{q}(x_{J_s})]$ where $\mathbf{q}(x_j) = \left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial x} \right|_{x=x_j}$.

In 3-D localization problem where $\mathbf{x}_j = [r_j, z_j, \theta_j]$ is a vector, $\mathbf{S}(t) = \text{diag}(\mathbf{s}_1(t), \dots, \mathbf{s}_{J_s}(t)) \otimes \mathbf{I}_3(t)$ is a $3N \times 3N$ diagonal matrix and $\otimes$ denotes the Kronecker product. $\mathbf{Q}(\mathbf{x})$ is an $N \times 3J_s$ matrix in which

$$\mathbf{q}(\mathbf{x}_j) = \left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_j} = \left[\frac{\partial \mathbf{p}(\mathbf{x})}{\partial r} \quad \frac{\partial \mathbf{p}(\mathbf{x})}{\partial z} \quad \frac{\partial \mathbf{p}(\mathbf{x})}{\partial \theta} \right] \bigg|_{\mathbf{x}=\mathbf{x}_j}. \quad (5.18)$$

For the problem of DOA estimation of plane waves in Gaussian noise, (5.16) reduces to the CRB expression given in [76] and the proposed CRB in (5.16) for DOA estimation of plane waves in mixture of Gaussian noise is similar to the expression presented in [18]. The only assumption we have made for deriving the CRB is symmetric noise distribution, therefore, the CRB expression in (5.16) can be used for different symmetric noise distributions and different array configurations. In the following sections, we use the expression (5.16) for deriving the CRB for 3-D localization in shallow ocean with non-Gaussian noise using an array of scalar sensors and acoustic vector sensors.

5.2 CRB for 3-D source localization in shallow ocean with non-Gaussian noise using an array of scalar sensors

In this section, we consider the problem of multiple source localization in shallow ocean and we present the CRB expressions for source range, depth and bearing estimates using an array of scalar sensors (SS). Because of poor performance of SS horizontal arrays in range and depth [92], we consider a hybrid array composed of a horizontal and a vertical array of scalar sensors. The derivation presented in the section can be easily used for a single horizontal array as well by a slight modification.

We consider a hybrid array composed of a uniform HLA and a coplanar VLA. The HLA and VLA have the following parameters:

- N_H : Number of sensors in HLA
- d_H : Inter-sensor spacing in HLA
- z_H : Depth of sensors in HLA
- $N_v = N - N_H$: Number of sensors in VLA
- d_V : sensors located at depths
- $z_{V_n} = z_{V_1} + (n - 1)d_V$, $n = 1, \dots, N_V$: Depth of sensors in VLA

The array output vector $\mathbf{y}(t)$ can be expressed as

$$\mathbf{y}(t) = [\mathbf{y}_H(t)^T, \mathbf{y}_V(t)^T]^T; \quad t = 1, \dots, T, \quad (5.19)$$

$$\mathbf{y}_H(t) = \mathbf{P}_H(\mathbf{x})\mathbf{s}(t) + \mathbf{w}_H(t), \quad \mathbf{y}_V(t) = \mathbf{P}_V(\mathbf{x})\mathbf{s}(t) + \mathbf{w}_V(t), \quad (5.20)$$

where $\mathbf{y}_H(t) = [y_{H,1}(t), \dots, y_{H,N_H}(t)]^T$ and $\mathbf{y}_V(t) = [y_{V,1}(t), \dots, y_{V,N_V}(t)]^T$ are the received data vectors at the HLA and VLA respectively, $\mathbf{w}_H(t) = [w_{H,1}(t), \dots, w_{H,N_H}(t)]^T$ and $\mathbf{w}_V(t) = [w_{V,1}(t), \dots, w_{V,N_V}(t)]^T$ are the corresponding additive noise vectors ($y_{H,n}(t)$ and $y_{V,n}(t)$ are the horizontal and vertical array output at sensor n ; $w_{H,n}(t)$ and $w_{V,n}(t)$ are the additive noise at sensor n of the HLA and VLA respectively). The vector $\mathbf{s}(t)$ is the source signal vector as defined in the previous section. Other parameters in this section are similar to those in previous

section unless otherwise stated.

$$\mathbf{P}_H(\mathbf{x}) = [\mathbf{p}_H(\mathbf{x}_1), \dots, \mathbf{p}_H(\mathbf{x}_{J_s})], \quad \mathbf{P}_V(\mathbf{x}) = [\mathbf{p}_V(\mathbf{x}_1), \dots, \mathbf{p}_V(\mathbf{x}_{J_s})], \quad (5.21)$$

are the HLA and VLA steering matrices, and $\mathbf{p}_H(\mathbf{x}_j)$ and $\mathbf{p}_V(\mathbf{x}_j)$ are the steering vectors denoting the response of the arrays to a source of unit strength located at the position $\mathbf{x}_j$. $\mathbf{x}_j = [r_j, z_j, \theta_j]$ denotes the j^{th} source position in terms of range r_j , depth z_j , and bearing θ_j . The range r_j is measured with reference to the VLA and the bearing θ_j is measured with reference to the endfire direction of the HLA. It is assumed that any change in the source position over the set of T snapshots is negligible.

Under the far-field approximation in a range-independent shallow ocean, the steering vectors associated with the HLA can be expressed as [5]

$$\mathbf{p}_H(\mathbf{x}_j) = \mathbf{A}(\theta_j)\mathbf{b}(r_j, z_j); \quad j = 1, \dots, J_s, \quad (5.22)$$

where $\mathbf{b}(r_j, z_j) = [b_{1j} \dots b_{Mj}]^T$ is the mode amplitude vector whose elements are

$$b_{mj} = \Psi_m(z_H)\Psi_m(z_j)\frac{e^{(ik_m - \alpha_m)r_j}}{\sqrt{k_m r_j}}; \quad m = 1, \dots, M, \quad (5.23)$$

and $\mathbf{A}(\theta_j)$ is an $N_H \times M$ matrix expressed as

$$\mathbf{A}(\theta_j) = [\mathbf{a}(k_1 \cos \theta_j), \dots, \mathbf{a}(k_M \cos \theta_j)], \quad (5.24)$$

$$\mathbf{a}(k_m \cos \theta_j) = \left[1, e^{ik_m d_H \cos \theta_j}, \dots, e^{i(N_H - 1)k_m d_H \cos \theta_j} \right]^T; \quad m = 1, \dots, M. \quad (5.25)$$

The signal vectors of the VLA are independent of the bearing because of the cylindrical symmetry of the range independent ocean, and hence we have $\mathbf{p}_V(\mathbf{x}_j) = \mathbf{p}_V(\mathbf{u}_j)$ where $\mathbf{u}_j = (r_j, z_j)$. The elements of $\mathbf{p}_V(\mathbf{u}_j)$ are given by [5]

$$p_{V_n}(\mathbf{x}_j) = \sum_{m=1}^M \Psi_m(z_{V_n})\Psi_m(z_j)\frac{e^{(ik_m - \alpha_m)r_j}}{\sqrt{k_m r_j}}; \quad n = 1, \dots, N_V. \quad (5.26)$$

In (5.23) - (5.26), $\Psi_m(z)$ is the eigenfunction of the m^{th} normal mode of the channel at depth z and k_m, α_m denote respectively the associated wavenumber and attenuation coefficient, and M is the number of normal modes.

The complex noise samples at the array are i.i.d in time and space with the following zero-mean L -component Gaussian mixture (GM) PDF:

$$f_W(\bar{w}_n, \overline{\bar{w}}_n) = \sum_{l=1}^L \frac{\lambda_l}{2\pi\sigma_l^2} \exp\left(-\frac{\bar{w}^2 + \overline{\bar{w}}^2}{2\sigma_l^2}\right). \quad (5.27)$$

This is a circularly symmetric bivariate PDF for the complex-valued random variable $W = \bar{W} + i\overline{\bar{W}}$ where $\bar{W}$ and $\overline{\bar{W}}$ are the real and imaginary parts of W respectively. The GM has the ability of modeling a variety of different probability distributions including those representing heavy-tailed/impulsive noise. A relatively small value of L is usually adequate to obtain a good model in most applications. The parameter λ_l is the weight of the l^{th} component in the Gaussian mixture, i.e., λ_l is the probability that W is chosen from the l^{th} term in the Gaussian mixture PDF with $\sum_{l=1}^L \lambda_l = 1$. The quantity $2\sigma_l^2$ is the variance of the l^{th} component in the mixture, and the variance of W is given by

$$\text{var}(W) = \sigma^2 = 2 \sum_{l=1}^L \lambda_l \sigma_l^2. \quad (5.28)$$

Parameters (λ_l, σ_l^2) , $l = 1, \dots, L$ of the GMM considered for modeling the noise PDF in the ocean are unknown and should be estimated. The details of estimators used for estimation of these parameters are presented in the next chapter. For the Gaussian mixture noise PDF defined in (5.27), $I_c(\Lambda)$ in (5.13) can be computed numerically from the following expression [18]

$$\begin{aligned} & I_c(\lambda_1, \dots, \lambda_L, \sigma_1^2, \dots, \sigma_L^2) \\ &= \frac{1}{2} \int_0^\infty \frac{\sum_{l=1}^L \sum_{m=1}^L \frac{\lambda_l \lambda_m}{\sigma_l^4 \sigma_m^4} \exp\left[-\frac{\rho^2}{2} \left(\frac{1}{\rho_l^2} + \frac{1}{\rho_m^2}\right)\right]}{\sum_{r=1}^L \frac{\lambda_r}{\sigma_r^2} \exp\left[-\frac{\rho^2}{2\sigma_r^2}\right]} d\rho. \end{aligned} \quad (5.29)$$

For obtaining the CRB expressions in this problem, we start from the general expression derived in (5.16). According to this expression, we need to compute the partial derivatives of the steering vectors, $\mathbf{q}(x_j) = \left. \frac{\partial \mathbf{p}(x)}{\partial x} \right|_{\mathbf{x}=\mathbf{x}_j}$ in order to obtain $Q(\mathbf{x}_j)$. For the hybrid array considered, partial derivatives of the steering vector, $\mathbf{q}(\mathbf{x}_j)$ can be obtained as follows.

$$\begin{aligned}
 \left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial r} \right|_{\mathbf{x}=\mathbf{x}_j} &= \left[\frac{\partial \mathbf{p}_H^T(\mathbf{x})}{\partial r}, \frac{\partial \mathbf{p}_V^T(\mathbf{x})}{\partial r} \right]^T \bigg|_{\mathbf{x}=\mathbf{x}_j}, \\
 \left. \frac{\partial \mathbf{p}_H(\mathbf{x})}{\partial r} \right|_{\mathbf{x}=\mathbf{x}_j} &= \mathbf{A}(\theta_j) \left. \frac{\partial \mathbf{b}(r, z)}{\partial r} \right|_{\substack{r=r_j \\ z=z_j}} = \mathbf{A}(\theta_j) \frac{\partial [b_{1j}, b_{2j}, \dots, b_{Mj}]}{\partial r_j}, \\
 &= \psi_m(z_H) \psi_m(z_j) \frac{e^{ik_m r_j - \alpha_m r_j}}{\sqrt{k_m r_j}} (ik_m - \alpha_m - \frac{1}{2r_j}), \\
 &= b_{mj} (ik_m - \alpha_m - \frac{1}{2r_j}), \\
 \left. \frac{\partial \mathbf{p}_V(\mathbf{x})}{\partial r} \right|_{\mathbf{x}=\mathbf{x}_j} &= \left. \frac{\partial [p_{V1}, p_{V2}, \dots, p_{VN_V}]}{\partial r} \right|_{\mathbf{x}=\mathbf{x}_j} \\
 \left. \frac{\partial p_{Vn}}{\partial r} \right|_{\mathbf{x}=\mathbf{x}_j} &= \sum_{m=1}^M \psi_m(z_n) \psi_m(z_j) \frac{e^{ik_m r_j - \alpha_m r_j}}{\sqrt{k_m r_j}} (ik_m - \alpha_m - \frac{1}{2r_j}). \tag{5.30}
 \end{aligned}$$

$$\begin{aligned}
 \left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial z} \right|_{\mathbf{x}=\mathbf{x}_j} &= \left[\frac{\partial \mathbf{p}_H^T(\mathbf{x})}{\partial z}, \frac{\partial \mathbf{p}_V^T(\mathbf{x})}{\partial z} \right]^T \bigg|_{\mathbf{x}=\mathbf{x}_j}, \\
 \left. \frac{\partial \mathbf{p}_H(\mathbf{x})}{\partial z} \right|_{\mathbf{x}=\mathbf{x}_j} &= \mathbf{A}(\theta_j) \left. \frac{\partial \mathbf{b}(r, z)}{\partial z} \right|_{\substack{r=r_j \\ z=z_j}} = \mathbf{A}(\theta_j) \frac{\partial [b_{1j}, b_{2j}, \dots, b_{Mj}]}{\partial z_j}, \\
 \frac{\partial b_{mj}}{\partial z_j} &= \psi_m(z_H) \frac{e^{ik_m r_j - \alpha_m r_j}}{\sqrt{k_m r_j}} \left. \frac{\partial \psi_m(z)}{\partial z} \right|_{z=z_j}, \\
 \left. \frac{\partial \mathbf{p}_V(\mathbf{x})}{\partial z} \right|_{\mathbf{x}=\mathbf{x}_j} &= \left. \frac{\partial [p_{V1}, p_{V2}, \dots, p_{VN_V}]}{\partial z} \right|_{\substack{r=r_j \\ z=z_j}}, \\
 \left. \frac{\partial p_{Vn}}{\partial z} \right|_{\substack{r=r_j \\ z=z_j}} &= \sum_{m=1}^M \psi_m(z_n) \frac{e^{ik_m r_j - \alpha_m r_j}}{\sqrt{k_m r_j}} \left. \frac{\partial \psi_m(z)}{\partial z} \right|_{z=z_j}. \tag{5.31}
 \end{aligned}$$

$$\left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial \theta} \right|_{\mathbf{x}=\mathbf{x}_j} = \left[\frac{\partial \mathbf{p}_H^T(\mathbf{x})}{\partial \theta}, \frac{\partial \mathbf{p}_V^T(\mathbf{x})}{\partial \theta} \right]^T \bigg|_{\mathbf{x}=\mathbf{x}_j} = \left[\frac{\partial \mathbf{A}(\theta)}{\partial \theta} \bigg|_{\theta=\theta_j} \mathbf{b}(r_j, z_j), \quad \mathbf{0} \right]. \tag{5.32}$$

To compute the CRB for 3-D localization in shallow ocean with an array of scalar sensors, $\mathbf{Q}(\mathbf{x})$ in the expression (5.16) is replaced by what has been computed in (5.30) - (5.32). In

the CRB expression given in (5.16), signal vector $\mathbf{s}(t)$ is equal to $[s_1(t), \dots, s_{J_s}(t)]^T$ and $\mathbf{P}(\mathbf{x})$ is obtained from (5.21). As explained earlier, $I_c(\Lambda)$ in the CRB expression is computed from (5.13) for an arbitrary symmetric noise PDF and from (5.29) for the Gaussian mixture noise PDF defined in (5.27).

5.3 CRB for 3-D source localization in shallow ocean with non-Gaussian noise using an array of acoustic vector sensors (AVS)

In previous section, the CRBs for 3-D localization of J_s sources in shallow ocean using an array of scalar sensors have been derived. In this section, we consider the problem of 3-D source localization in shallow ocean using AVS arrays and we derive CRB expressions for range, depth and bearing estimates of J_s source.

Consider a linear AVS array of N sensors receiving a narrowband signal of center-frequency $(\omega/2\pi)$ from J_s far-field acoustic point sources at $\mathbf{x}_j = [r_j, z_j, \theta_j]; j = 1, \dots, J_s$ in a range independent shallow ocean with water column depth h , sound speed c and density ρ . We consider two alternative array geometries, namely, a horizontal linear array (HLA) at depth z_H , and a vertical linear array (VLA) with sensors at depths $z_{V_n} = z_{V_1} + (n-1)d_V$, $n = 1, \dots, N$. The inter-sensor spacing is d in both cases. Each acoustic vector sensor measures acoustic pressure and three orthogonal components of particle velocity. We shall discard the vertical component of particle velocity since its contribution to the localization performance is insignificant. We shall also scale the particle velocity measurements by factor $\sqrt{2}\rho c$ to render the measurements dimensionally uniform. The scaling factor $\sqrt{2}$ is included to make the variances of acoustic pressure and particle velocity components of ambient noise to be equal to each another. The $3N$ -dimensional data vectors are therefore defined as

$$\mathbf{y}(t) = [y_1(t), \dots, y_{3N}(t)]^T = \mathbf{P}(\mathbf{x}) \mathbf{s}(t) + \mathbf{w}(t), \quad t = 1, \dots, T, \quad (5.33)$$

where the vector $\mathbf{s}(t)$ is defined as $\mathbf{s}(t) = [s_1(t), \dots, s_{J_s}(t)]^T$ and $s_j(t)$ is the slowly varying

complex amplitude of the j^{th} source signal at time t .

$$\begin{aligned}\mathbf{P}(\mathbf{x}) &= [\mathbf{p}(\mathbf{x}_1), \dots, \mathbf{p}(\mathbf{x}_{J_s})]; \\ \mathbf{p}(\mathbf{x}_j) &= \left[p_1(t), \sqrt{2}\rho c v_{x1}, \sqrt{2}\rho c v_{y1}, \dots, p_N(t), \sqrt{2}\rho c v_{xN}, \sqrt{2}\rho c v_{yN}(t) \right]^T \Big|_{\mathbf{x}=\mathbf{x}_j}, \\ &= [p_1(t), v_{x1}, v_{y1}, \dots, p_N(t), v_{xN}, v_{yN}(t)]^T \Big|_{\mathbf{x}=\mathbf{x}_j},\end{aligned}\quad (5.34)$$

where p_n, v_{xn}, v_{yn} denote respectively the complex amplitudes of acoustic pressure and orthogonal horizontal components of particle velocity measured at the n^{th} sensor for a unit-amplitude source at position x_j . In this expression, v_{xn} and v_{yn} denote scaled particle velocities, $\mathbf{w}(t) = [\mathbf{w}_1^T(t), \dots, \mathbf{w}_{3N}^T(t)]^T$ is the array noise vector, and $\mathbf{w}_n(t) = [w_{3n-2}(t), w_{3n-1}(t), w_{3n}(t)]^T$ is the noise vector at the n^{th} sensor. Elements of $\mathbf{p}(\mathbf{x}_j)$, obtained from the normal mode theory of sound propagation, are given by [96]

$$p_n \Big|_{\mathbf{x}=\mathbf{x}_j} = \sum_{m=1}^M p_{mn} \phi_{mn}(\theta) \Big|_{\mathbf{x}=\mathbf{x}_j}, \quad (5.35)$$

$$v_{xn} \Big|_{\mathbf{x}=\mathbf{x}_j} = \sqrt{2}\rho c v_{xn} = \sqrt{2}\cos\theta \sum_{m=1}^M \frac{k_m}{k} p_{mn} \phi_{mn}(\theta) \Big|_{\mathbf{x}=\mathbf{x}_j}, \quad (5.36a)$$

$$v_{yn} \Big|_{\mathbf{x}=\mathbf{x}_j} = \sqrt{2}\rho c v_{yn} = \sqrt{2}\sin\theta \sum_{m=1}^M \frac{k_m}{k} p_{mn} \phi_{mn}(\theta) \Big|_{\mathbf{x}=\mathbf{x}_j}, \quad (5.36b)$$

$$p_{mn} \Big|_{\mathbf{x}=\mathbf{x}_j} = \Psi_m(z_H) \Psi_m(z) \frac{e^{(ik_m - \alpha_m)r}}{\sqrt{k_m r}} \Big|_{\mathbf{x}=\mathbf{x}_j}; \quad m = 1, \dots, M, \quad (5.37)$$

where r, z, θ are range, depth and bearing of the source, $\Psi_m(z)$ and k_m are the eigenfunction and wavenumber of the m^{th} normal mode, M is the total number of modes, $k = w/c$, z_n is the depth of the n^{th} sensor respectively, and $\phi_{mn}(\theta) \Big|_{\mathbf{x}=\mathbf{x}_j}$ is a phase term corresponding to the source j^{th} defined as

$$\phi_{mn}(\theta) \Big|_{\mathbf{x}=\mathbf{x}_j} = e^{i(n-1)k_m d \cos\theta_j} \text{ for HLA, } \phi_{mn}(\theta) \Big|_{\mathbf{x}=\mathbf{x}_j} = 1 \text{ for VLA.} \quad (5.38)$$

To compute CRB for range, depth and bearing of the source, we consider the general expression derived in (5.16). For obtaining $\mathbf{Q}(\mathbf{x})$ in (5.16), derivative of the steering vector

corresponding to each source with respect to range, depth and bearing should be obtained.

Derivative of the pressure component $p_n(t)$ with respect to θ is:

$$\begin{aligned}
 \frac{\partial p_n(t)}{\partial \theta} &= \sum_{m=1}^M p_{mn}(t) \frac{\partial \phi_{mn}(\theta)}{\partial \theta}, \\
 &= \sum_{m=1}^M p_{mn}(t) (-i(n-1)k_m d \sin \theta) e^{i(n-1)d k_m \cos \theta} \\
 &= \begin{cases} -i(n-1) \sum_{m=1}^M p_{mn}(t) \phi_{mn}(\theta) k_m d \sin \theta. & \text{HLA} \\ 0. & \text{VLA} \end{cases}
 \end{aligned} \tag{5.39}$$

Derivative of the pressure component with respect to r (for both HLA and VLA) is:

$$\begin{aligned}
 \frac{\partial p_n(t)}{\partial r} &= \sum_{m=1}^M \frac{\partial p_{mn}(t)}{\partial r} \phi_{mn}(\theta), \\
 &= \sum_{m=1}^M \frac{2\sqrt{2\pi}}{h} \Psi_m(z_n) \Psi_m(z) \phi_{mn}(\theta) \\
 &\quad \times \left(\frac{\sqrt{k_m r} (ik_m - \alpha_m) e^{ik_m r - \alpha_m r} - \frac{1}{2} k_m^{1/2} r^{-1/2} e^{ik_m r - \alpha_m r}}{k_m r} \right), \\
 &= \sum_{m=1}^M p_{mn}(t) \phi_{mn}(\theta) (ik_m - \alpha_m - \frac{1}{2r}).
 \end{aligned} \tag{5.40}$$

Derivative of the pressure component with respect to z (for both HLA and VLA) is:

$$\begin{aligned}
 \frac{\partial p_n(t)}{\partial z} &= \sum_{m=1}^M \frac{\partial p_{mn}(t)}{\partial z} \phi_{mn}(\theta), \\
 &= \sum_{m=1}^M \frac{2\sqrt{2\pi}}{h} \Psi_m(z_n) \frac{\partial \Psi_m(z)}{\partial z} \phi_{mn}(\theta).
 \end{aligned} \tag{5.41}$$

Considering the velocity components (i.e., $v_{xn}(t), v_{yn}(t)$) of an AVS given in (5.36), their derivatives with respect to θ are computed as follows:

$$\frac{\partial v_{xn}(t)}{\partial \theta} = -\sin\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} p_{mn}(t) \phi_{mn}(\theta) + \cos\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} p_{mn}(t) \frac{\partial \phi_{mn}(\theta)}{\partial \theta}. \quad (5.42)$$

$$\frac{\partial v_{yn}(t)}{\partial \theta} = \cos\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} p_{mn}(t) \phi_{mn}(\theta) + \sin\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} p_{mn}(t) \frac{\partial \phi_{mn}(\theta)}{\partial \theta}. \quad (5.43)$$

Considering equations in (5.42) - (5.43) and (5.36a), derivatives of velocity components of an HLA of AVS are simplified to

$$\frac{\partial v_{xn}(t)}{\partial \theta} = -v_{yn}(t) + \cos\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} p_{mn}(t) \phi_{mn}(\theta) (-i(n-1) k_m d\sin\theta), \quad (5.44)$$

$$\frac{\partial v_{yn}(t)}{\partial \theta} = v_{xn}(t) + \sin\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} p_{mn}(t) \phi_{mn}(\theta) (-i(n-1) k_m d\sin\theta). \quad (5.45)$$

and these derivatives for an VLA of AVS are simplified as

$$\frac{\partial v_{xn}(t)}{\partial \theta} = -v_{yn}(t), \quad (5.46)$$

$$\frac{\partial v_{yn}(t)}{\partial \theta} = v_{xn}(t). \quad (5.47)$$

Derivative of velocity components (i.e., $v_{xn}(t), v_{yn}(t)$) of an AVS with respect to r is:

$$\frac{\partial v_{xn}(t)}{\partial r} = \cos\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} \frac{\partial p_{mn}(t)}{\partial r} \phi_{mn}(\theta), \quad (5.48)$$

$$\frac{\partial v_{yn}(t)}{\partial r} = \sin\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} \frac{\partial p_{mn}(t)}{\partial r} \phi_{mn}(\theta). \quad (5.49)$$

Considering equations in (5.48) - (5.49) and (5.23), derivatives of velocity components of an

AVS array (both HLA and VLA) with respect to r are simplified as

$$\begin{aligned}
 \frac{\partial v_{xn}(t)}{\partial r} &= \cos\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} \frac{\partial p_{mn}(t)}{\partial r} \phi_{mn}(\theta), \\
 &= \cos\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} \frac{2\sqrt{2\pi}}{h} \Psi_m(z_n) \Psi_m(z) \phi_{mn}(\theta) \\
 &\quad \times \left(\frac{\sqrt{k_m r}(ik_m - \alpha_m)e^{ik_m r - \alpha_m r} - \frac{1}{2}k_m^{1/2} r^{-1/2}e^{ik_m r - \alpha_m r}}{k_m r} \right), \\
 &= \cos\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} p_{mn}(t) \phi_{mn}(\theta) (ik_m - \alpha_m - \frac{1}{2r}).
 \end{aligned} \tag{5.50}$$

$$\begin{aligned}
 \frac{\partial v_{yn}(t)}{\partial r} &= \sin\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} \frac{\partial p_{mn}(t)}{\partial r} \phi_{mn}(\theta), \\
 &= \sin\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} \frac{2\sqrt{2\pi}}{h} \Psi_m(z_n) \Psi_m(z) \phi_{mn}(\theta) \\
 &\quad \times \left(\frac{\sqrt{(k_m r)}(ik_m - \alpha_m)e^{ik_m r - \alpha_m r} - \frac{1}{2}k_m^{1/2} r^{-1/2}e^{ik_m r - \alpha_m r}}{k_m r} \right), \\
 &= \sin\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} p_{mn}(t) \phi_{mn}(\theta) (ik_m - \alpha_m - \frac{1}{2r}).
 \end{aligned} \tag{5.51}$$

Finally derivatives of velocity components with respect to z are

$$\begin{aligned}
 \frac{\partial v_{xn}(t)}{\partial z} &= \cos\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} \frac{\partial p_{mn}(t)}{\partial z} \phi_{mn}(\theta), \\
 &= \cos\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} \frac{2\sqrt{2\pi}}{h} \Psi_m(z_n) \frac{\partial \Psi_m(z)}{\partial z} \phi_{mn}(\theta).
 \end{aligned} \tag{5.52}$$

$$\begin{aligned}
 \frac{\partial v_{yn}(t)}{\partial r} &= \sin\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} \frac{\partial p_{mn}(t)}{\partial r} \phi_{mn}(\theta), \\
 &= \sin\theta \sum_{m=1}^M \frac{\sqrt{2}k_m}{k} \frac{2\sqrt{2\pi}}{h} \Psi_m(z_n) \frac{\partial \Psi_m(z)}{\partial z} \phi_{mn}(\theta).
 \end{aligned} \tag{5.53}$$

Now for each array configuration, it is enough to put the derivative of different AVS components (computed above) in an order identical to their order in the steering vector. Therefore, for the

steering vector $\mathbf{p}(\mathbf{x}) = [p_1 \ v_{x1} \ v_{y1} \dots p_N \ v_{xN} \ v_{yN}]^T$, the $3N \times 3$ matrix $\mathbf{Q}(\mathbf{x})$ defined in (5.16) is identical to

$$\begin{aligned}
 \mathbf{Q}(\mathbf{x}) &= [\mathbf{q}(\mathbf{x}_1), \dots, \mathbf{q}(\mathbf{x}_J)], \\
 \mathbf{q}(\mathbf{x}_j) &= \left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_j}, \\
 &= \left[\left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial \theta} \right|_{\mathbf{x}=\mathbf{x}_j} \left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial r} \right|_{\mathbf{x}=\mathbf{x}_j} \left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial z} \right|_{\mathbf{x}=\mathbf{x}_j} \right], \\
 \left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial \theta} \right|_{\mathbf{x}=\mathbf{x}_j} &= \left[\frac{\partial p_1}{\partial \theta} \ \frac{\partial v_{x1}}{\partial \theta} \ \frac{\partial v_{y1}}{\partial \theta} \dots \frac{\partial p_N}{\partial \theta} \ \frac{\partial v_{xN}}{\partial \theta} \ \frac{\partial v_{yN}}{\partial \theta} \right]^T \bigg|_{\mathbf{x}=\mathbf{x}_j}, \\
 \left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial r} \right|_{\mathbf{x}=\mathbf{x}_j} &= \left[\frac{\partial p_1}{\partial r} \ \frac{\partial v_{x1}}{\partial r} \ \frac{\partial v_{y1}}{\partial r} \dots \frac{\partial p_N}{\partial r} \ \frac{\partial v_{xN}}{\partial r} \ \frac{\partial v_{yN}}{\partial r} \right]^T \bigg|_{\mathbf{x}=\mathbf{x}_j}, \\
 \left. \frac{\partial \mathbf{p}(\mathbf{x})}{\partial z} \right|_{\mathbf{x}=\mathbf{x}_j} &= \left[\frac{\partial p_1}{\partial z} \ \frac{\partial v_{x1}}{\partial z} \ \frac{\partial v_{y1}}{\partial z} \dots \frac{\partial p_N}{\partial z} \ \frac{\partial v_{xN}}{\partial z} \ \frac{\partial v_{yN}}{\partial z} \right]^T \bigg|_{\mathbf{x}=\mathbf{x}_j}. \tag{5.54}
 \end{aligned}$$

By replacing $\mathbf{Q}(\mathbf{x})$ computed in (5.54) and $\mathbf{P}(\mathbf{x})$ from (5.34)-(5.38) in (5.16), the CRB for 3-D localization in shallow ocean with an array of acoustic vector sensors is obtained.

5.4 Simulation results and discussion

Results of simulations conducted for study and evaluation of the expressions derived in Sections 5.1- 5.3 for CRB are presented and discussed in this section. As discussed in Chapters 1 - 2 of this thesis, we are interested in non-Gaussian dominated ambient noise environment in a shallow ocean. In the simulations considered in this section, non-Gaussian noise is generated with a GM model with two components as defined in (5.27). We consider three sets of simulations to study the problem of source localization in the shallow ocean using the derived CRBs. The goal of each set is as follows:

- Set 1: To study the effect of GMM parameters in (5.27) on non-Gaussianity of noise
- Set 2: To study the CRBs of source localization in shallow ocean using the expressions derived in (5.16)
- Set 3: To study the dependence of the CRBs as a bench mark for 3-D source localization performance on the source position
- **Variation of GMM parameters (Set 1)**

In the first set, we investigate the effect of parameters of GMM given in (5.27) on non-Gaussianity of the noise. We use a GMM with parameters $\sigma_1^2 = 1$ and $\sigma_2^2 = 1000$ and $\lambda_2 = 0.01$ to generate noise samples. The impulsiveness of the GMM noise is varied by changing its parameters σ_1^2 , σ_2^2 , λ_2 and $\lambda_1 = 1 - \lambda_2$.

As an example, we fix all the GMM parameters except λ_2 and $\lambda_1 = 1 - \lambda_2$. We consider a frame of noise samples for two values of $\lambda_2 = 0.01, 0.11$. Fig. 5.1 (a) demonstrates time series of the noise samples for value of $\lambda_2 = 0.01$ and Fig. 5.1 (b) is for $\lambda_2 = 0.11$. It is observed that by increasing λ_2 with other parameters fixed, the number of impulses appearing on background noise has increased significantly.

Figure 5.1: Impulsive noise generated from a GMM with two components with variances $\sigma_1^2 = 1$ and $\sigma_2^2 = 1000$ and (Top) $\lambda_1 = 0.99$ and $\lambda_2 = 0.01$ (bottom) $\lambda_1 = 0.89$ and $\lambda_2 = 0.11$

We also assess the effect of changing the GMM model parameters on the PDF of the noise samples. To see how the PDF of the non-Gaussian noise samples is affected by variations in the GMM parameters, we consider two cases. In the first case, we fix $\lambda_2 = 0.3$ with σ_2^2/σ_1^2 varying. Fig. 5.2 (a) shows the results of this simulation. The effect of varying λ_2 on the noise PDF with fixed $\sigma_2^2/\sigma_1^2 = 1000$ is illustrated in Fig. 5.2 (b). It is observed that by increasing λ_2 or σ_2^2/σ_1^2 the noise PDF becomes more heavy-tailed and more non-Gaussian.

• **CRBs of source localization (Set 2)**

In the second set of simulations, we use the CRB expression derived to study the source localization performance in non-Gaussian noise. As explained in Section 5.1, the CRB expression in (5.16) can be used for obtaining the CRB for DOA estimation of plane wave sources as well. For a plane wave source, 3-D localization problem is converted to a 1-D localization problem in which $x_j = \theta_j$. The steering vector for a plane wave source can be defined as

$$\mathbf{p}(x_j) = \mathbf{p}(\theta_j) = [p_1(\theta_j) \ p_2(\theta_j) \ \dots \ p_N(\theta_j)]; \ j = 1, \dots, J_s, \ p_n(\theta_j) = \exp^{-i(\frac{2\pi f}{c}(x_n \cos \theta_j + y_n \sin \theta_j))}, \quad (5.55)$$

where x_n and y_n are the coordinates of n^{th} sensor position, c is the propagation speed, f is source frequency and θ_j is the DOA of j^{th} source with respect to x-axis.

By replacing the steering vector and other parameters of a plane wave source in (5.16), CRB for DOA estimation is computed. In this simulation, we use an HLA consisting of 14 scalar sensors with inter-element spacing $7.5m$ (half-wavelength) at the depth of $50m$ in a shallow ocean. The depth of the shallow ocean is $100m$ and a source is at range $r = 5000m$, depth $z = 30m$ and bearing $\theta = 30^\circ$. A plane wave source with direction of arrival DOA) of 30° is also considered. We consider both Gaussian and non-Gaussian noise distributions and we compute the CRBs for both cases accordingly. We obtain the CRB for DOA estimation of the plane-wave source as well as the source in shallow ocean using (5.16) and compare their results. Here SNR is defined as the ratio of total signal power to total noise power at the array. To generate non-Gaussian noise, a GMM with two components defined in (5.27) is used. We consider three simulations in this set with the following objectives:

- to study the CRBs variation versus SNR
- to study the effect of λ_2 variation on the CRBs
- to study the effect of σ_2^2/σ_1^2 variation on the CRBs

(a)

(b)

 Figure 5.2: The effect of σ_2^2 / σ_1^2 and λ_2 on PDF of a GM noise with two components

We obtain the CRBs for range, depth and bearing of the source in the shallow ocean. The effect of variation of λ_1 on the CRBs is assessed for the source in the shallow ocean. Fig. 5.3 shows the CRBs for different SNRs and different λ_1 . In this simulation, the total noise power ($2\lambda_1 \sigma_1^2 + 2\lambda_2 \sigma_2^2$) is fixed and the signal power is varied to have different SNRs. It is observed that the CRBs for all source coordinates increase when SNR decreases for each value of λ_1 . Besides, the figure also shows that for a specific value of SNR, the CRBs are higher for larger value of λ_1 and the highest CRB is related to $\lambda_1 = 1$ which is identical to Gaussian noise.

In another simulation, total noise power, SNR and the ratio σ_2^2/σ_1^2 are kept fixed and λ_2 is varied. In this simulation, $\sigma_2^2/\sigma_1^2 = 100$ and SNR is fixed on $-20dB$ and Fig. 5.4 illustrates CRBs for the source localization. It is observed that CRB for the plane-wave DOA estimation is lower than the CRB for bearing estimation of the source in shallow ocean. It is also seen that CRB for non-Gaussian noise is always lower than that for Gaussian noise which is the case for both the plane-wave source and the source in shallow ocean. This can be explained as follows. For a fixed SNR and a fixed signal power, the total noise power will also be fixed. We call the first Gaussian component in the GMM with smaller variance (σ_1^2) as background noise. The second component with larger variance (σ_2^2) appears in the background noise as impulses. When impulses are added to the background noise (i.e., $\lambda_2 > 0$), a part of noise power will be assigned to the impulses ($2\lambda_2 \sigma_2^2$) and the power of background noise ($2\lambda_1 \sigma_1^2$) decreases compared to the Gaussian noise case in which $\lambda_2 = 0$. This leads to the lower CRB for the source localization in non-Gaussian noise compared to Gaussian noise. This trend is similar for all source coordinates. The reason for poor performance of 3D-MUSIC can be attributed to inaccurate estimation of correlation matrix because of low SNR. This leads to poor estimation of noise subspace matrix (obtained from eigen decomposition of the correlation matrix) and this causes poor parameter estimates.

By increasing λ_2 from 0 to 0.5 with a fixed SNR, the share of impulsive component in the total noise power increases and the power of background noise decreases. In this case, impulses affect minority of the samples ($\leq 50\%$) in each frame, while background noise exists in majority of the samples in a frame. Therefore, by increasing λ_2 from 0 to 0.5, power of background noise is the dominant parameter in the performance of localization. By decreasing the power of this dominant component, CRB decreases as well. When the impulsive component becomes the dominant component in the GMM (i.e., when $\lambda_2 > 0.5$), the impulses affects the performance of localization more than the background noise. Hence by increasing λ_2 from 0.5 to 1, CRB increases until it reaches the maximum value (that is related to a Gaussian noise). We observed

 Figure 5.3: CRB vs. SNR for source localization in shallow ocean with different values of λ_1

in Fig.5.4 that the CRBs for Gaussian noise does not change over λ_2 , because in this simulation, noise power and SNR is kept fixed. A similar trend is observed for all source coordinates in Fig. 5.4 (a-c).

In the third simulation of this set, the source power, SNR and λ_2 are fixed (SNR=-20dB, $\lambda_2 = 0.1$) and σ_2^2/σ_1^2 is varied. We evaluate the effect of σ_1^2/σ_2^2 variation on the CRBs. The results are illustrated in Fig .5.5. It is observed that the largest CRB for all source coordinates is associated to Gaussian noise. The CRBs in non-Gaussian noise decreases when σ_2^2/σ_1^2 increases. The decrease in CRB is more significant when $\sigma_2^2/\sigma_1^2 > 10$. This can be related to this fact that when σ_2^2/σ_1^2 increases, impulses can be distinguished and removed from the background noise more easily. To explain this observation, we notice that when SNR is fixed and power of impulses increases, power of background noise (i.e. the Gaussian component with smaller variance and larger weight) decreases to keep the SNR fixed. This leads to lower CRBs. Theoretically, this decrease in CRBs is related to the value of $1/I_c(\Lambda)$ in (5.16). For any non-Gaussian noise distribution, $1/I_c(\Lambda)$ defined in (5.13) is smaller than that of a Gaussian noise (i.e., $(1/I_c(\Lambda)) \leq \sigma^2$). This observation coincides with what Kozick and Sadler observed in DOA estimation of plane-wave sources [18]. A similar trend is observed for CRBs of all source coordinates in Fig .5.5.

In Fig.5.5 (c), the comparison between CRB of bearing estimation of the source in shallow ocean and the CRB of DOA estimation of a plane-wave source shows that the CRB for bearing estimation in shallow ocean is significantly larger than that for plane-wave DOA estimation. This shows that source localization is more challenging in shallow ocean.

• CRBs and source location (Set 3)

It is also of interest to compare the achievable 3-D localization accuracy of different array configurations (viz. HLA, VLA, and hybrid array) using the corresponding CRBs, and to study the dependence of the localization errors on the source coordinates. For this study, we consider a hybrid array and an HLA of 30 scalar sensors each and an VLA of 10 scalar sensors in a shallow ocean with depth 100m. Inter-sensor spacing is 7.5 m for all arrays. Values of other parameters are: source depth $z_1 = 40$ m, range $r_1 = 5000$ m, bearing $\theta_1 = 30^\circ$, $\sigma_2^2/\sigma_1^2 = 1000$, $\lambda_2 = 0.1$, and $SNR = -5$ dB.

Figure 5.6 shows the plots of CRB versus range. Similar plots of CRB versus source depth and CRB versus source bearing are shown in Figs. 5.7 and 5.8 respectively. These results

 Figure 5.4: CRB vs. λ_2 for 3-D source localization in shallow ocean with fix SNR

(a)

(b)

(c)

 Figure 5.5: CRB vs. σ_2^2 / σ_1^2 for 3-D source localization in shallow ocean with fix SNR

(a)

(b)

(c)

Figure 5.6: Dependence of CRBs on source Range. (a) Range error (b) Depth error (c) Bearing error.

Figure 5.7: Dependence of CRBs on source Depth. (a) Range error (b) Depth error (c) Bearing error.

(a)

(b)

(c)

Figure 5.8: Dependence of CRBs on source bearing. (a) Range error (b) Depth error (c) Bearing error.

are CRBs that do not depend on noise. Therefore, there is no random parameter in these simulations that need to get average over a large number of runs. We have used the CRB expressions derived in this chapter, to compute the CRB of range, depth and bearing in Figs. 5.6 - 5.8.

We draw the following conclusions from Figs. 5.6 - 5.8. Variations in range, depth, or bearing of the source have very little effect on the 3-D localization performance of the hybrid array. The range-depth estimation performance of the 10-sensor VLA is almost as good as that of the 30-sensor hybrid array. Improvement in range-depth estimation performance due to the addition of the 20-sensor HLA to the 10-sensor VLA is negligible. But the VLA cannot estimate bearing whereas the hybrid array can. The HLA has poor range-depth estimation performance for sources near the broadside direction ($\theta = 90^\circ$) and poor bearing estimation performance for sources near the endfire direction ($\theta = 0^\circ$ or 180°). In general, the CRBs for the HLA are larger respectively than the corresponding CRBs for the hybrid array. It is also seen that changes in source range and depth cause fluctuations in the CRBs for the HLA, unlike the CRBs for the hybrid array.

5.5 Summary

In this chapter we have derived a closed form expression for the CRB of 3-D source localization in the presence of symmetric noise distributions which can be used for different array configurations of scalar sensors and acoustic vector sensors. Furthermore, using this expression, we have computed the CRBs for range, depth and bearing estimation of sources in shallow ocean with non-Gaussian noise using an array of scalar sensors. Besides, we have derived the CRB expressions for the multiple source localization problem when an array of acoustic vector sensors is used. Different simulations conducted using the derived CRBs to study the problem of 3-D localization in shallow ocean with both Gaussian and non-Gaussian noise. The CRBs derived in this chapter are used extensively as a bench mark for the performance evaluation of our proposed 3-D localization algorithm in the next chapter as well.

Chapter 6

Localization in shallow ocean with non-Gaussian noise

In Chapter 4, we have studied the problem of source detection in non-Gaussian noise and we proposed a simple and robust nonlinear detector for detection of sources. We derived an expression for Cramér-Rao bound on 3-D localization of multiple sources in presence of non-Gaussian noise in Chapter 5 that can be used for performance evaluation of different localization algorithms. In this chapter, we consider the problem of 3-D localization of multiple sources in shallow ocean with non-Gaussian noise. The exact location of a source in the ocean is usually stated in terms of its range, depth and bearing. Many localization algorithms presented in the literature to estimate the source bearing [73–78, 95, 96], or range and/or depth of sources [68–70, 72, 85–88] and some papers consider 3-D localization of sources [89–91].

The main issue in 3-D localization of multiple sources is the computational complexity of the algorithm. Conventional methods of 3-D localization include several versions of the matched field processing (MFP) techniques such as Bartlett, Capon, and MUSIC processors require a 3-D search over range-depth-bearing space which need a high degree of computational complexity. We develop a computationally simple method based on maximum likelihood (ML) algorithm that uses space-alternating generalized expectation-maximization (SAGE) approach for source localization (SAGE-USL) in a range-independent ocean. The proposed algorithm (SAGE-USL) is an adaptation of the Kozick and Sadler algorithm [18] for 3-D localization of multiple acoustic sources in a range-independent ocean with non-Gaussian noise. This adaption has included several modifications explained in this Chapter to reduce the complexity arising

out of the higher dimensionality of the problem and the use of a hybrid array. The derived CRBs in Chapter 5 are used for evaluating the localization performance of the proposed algorithm in different simulations conducted.

This chapter is organized as follows. In Section 6.1, the proposed 3-D localization method with an array of scalar sensors is introduced. The data model for the scalar sensor array is presented in Section 6.2. Details of the proposed 3-D localization algorithm are presented in Section 6.3. The CRB expression for range, depth and bearing estimation in this problem are presented in Section 6.4. The computational complexity of the proposed algorithm is analyzed in Section 6.5. Section 6.6 presents the modified version of the SAGE-USL algorithm for an array of acoustic vector sensors. Simulation results and conclusions are presented in Sections 6.7 and 6.8 respectively. The chapter ends with a summary in Section 6.9.

6.1 SAGE-USL algorithm with an array of scalar sensors

We observed in Chapter 5 that a horizontal array has poor range-depth estimation for sources near the broadside direction ($\theta = 90^\circ$) and poor bearing estimation for sources near the endfire direction ($\theta = 0^\circ$ or 180°). Besides, a vertical array cannot estimate bearing because it is not sensitive to the changes in source bearing. Results of simulation in Chapter 5 showed that a hybrid array is a better choice for 3-D localization that can estimate all the source coordinates in all bearing directions.

The SAGE-USL algorithm uses data from a hybrid array consisting of an HLA and a VLA. The signals are modeled as complex sinusoids (narrowband signal) and the noise is modeled as a complex zero-mean Gaussian mixture. The unknown parameters are divided into two groups: (1) the desired parameters, consisting of the number of sources and their coordinates, and (2) the nuisance parameters, consisting of the signal envelopes and the parameters required to model the Gaussian mixture noise PDF. The SAGE-USL algorithm involves initial estimation of all parameters using known methods followed by an iterative procedure based on a modified version of the SAGE algorithm for updating the estimates sequentially. The noise parameters are initialized using a procedure similar to that of Kozick and Sadler [18]. The number of sources is estimated using the minimum description length (MDL) criterion [187]. Initial estimates of source ranges and depths are obtained from the VLA data using 2-D MUSIC algorithm and initial bearing estimates are obtained from HLA data using the R-MUSIC-mdf algorithm [97].

A simple search-maximize-discard algorithm described in Section 6.3.1, is used to determine the pairings of the range-depth and bearing estimates. The full hybrid array data and the estimated source locations are used to obtain initial estimates of the source envelopes. A modified version of the SAGE algorithm is then used to update the initial estimates. The proposed algorithm is an extension of our earlier work [15] on localization of a single source in shallow ocean with non-Gaussian noise using a hybrid array.

We use CRB expressions for 3-D localization of underwater acoustic sources in non-Gaussian noise derived in Chapter 5 and compare the root mean square localization errors of the proposed SAGE-USL algorithm and the classical 3-D MUSIC algorithm with the CRB. Simulation results show that the performance of the proposed algorithm is significantly better than that of 3-D MUSIC.

6.2 Data model for an array of scalar sensors

Consider an array of N sensors receiving narrowband signals from J_s far-field point sources in a range independent shallow ocean. The array output at time t is denoted by the $N \times 1$ vector $\mathbf{y}(t)$. Let the array be composed of a uniform N_H -sensor HLA with inter-sensor spacing d_H located at depth z_H , and a coplanar VLA of $N_V = N - N_H$ sensors located at depths $z_{V_n} = z_{V_1} + (n - 1)d_V$, $n = 1, \dots, N_V$. Figure 6.1 shows the geometry of the hybrid array. We consider T snapshots of $\mathbf{y}(t)$, which can be expressed as

$$\mathbf{y}(t) = [\mathbf{y}_H(t)^T \ \mathbf{y}_V(t)^T]^T; \ t = 1, \dots, T, \quad (6.1)$$

$$\mathbf{y}_H(t) = \mathbf{P}_H(\mathbf{x})\mathbf{s}(t) + \mathbf{w}_H(t), \quad \mathbf{y}_V(t) = \mathbf{P}_V(\mathbf{x})\mathbf{s}(t) + \mathbf{w}_V(t), \quad (6.2)$$

where $\mathbf{y}_H(t) = [y_{H,1}(t) \dots y_{H,N_H}(t)]^T$ and $\mathbf{y}_V(t) = [y_{V,1}(t) \dots y_{V,N_V}(t)]^T$ are the received data vectors at the HLA and VLA respectively, $\mathbf{w}_H(t) = [w_{H,1}(t) \dots w_{H,N_H}(t)]^T$ and $\mathbf{w}_V(t) = [w_{V,1}(t) \dots w_{V,N_V}(t)]^T$ are the corresponding additive noise vectors, and superscript T denotes matrix transpose. The vector $\mathbf{s}(t)$ is defined as $\mathbf{s}(t) = [s_1(t) \dots s_{J_s}(t)]^T$, where $s_j(t)$ is the slowly varying complex amplitude of the j^{th} source signal at time t . We assume a ‘conditional model’ [81] for signals where the signal envelopes $\{s_j(t); \ t = 1, \dots, T\}$, for $j = 1, \dots, J_s$, are modeled as unknown deterministic functions. The vector $\mathbf{x} = [\mathbf{x}_1 \dots \mathbf{x}_{J_s}]$ is a row vector of the source positions where $\mathbf{x}_j = [r_j, z_j, \theta_j]$ denotes the j^{th} source position in terms of range r_j , depth z_j , and bearing θ_j . The range r_j is measured with reference to the VLA and the bearing

θ_j is measured with reference to the endfire direction of the HLA. It is assumed that any change in the source position over the set of T snapshots is negligible. The matrices

$$\mathbf{P}_H(\mathbf{x}) = [\mathbf{p}_H(\mathbf{x}_1) \dots \mathbf{p}_H(\mathbf{x}_{J_s})], \quad \mathbf{P}_V(\mathbf{x}) = [\mathbf{p}_V(\mathbf{x}_1) \dots \mathbf{p}_V(\mathbf{x}_{J_s})] \quad (6.3)$$

are the steering matrices of the HLA and the VLA, and $\mathbf{p}_H(\mathbf{x}_j)$ and $\mathbf{p}_V(\mathbf{x}_j)$ are the steering vectors denoting the response of these arrays to a source of unit strength located at the position $\mathbf{x}_j$.

Figure 6.1: Geometry of the Hybrid array and sources

Under the far-field approximation, the signal vectors associated with the HLA can be expressed as [5]

$$\mathbf{p}_H(\mathbf{x}_j) = \mathbf{A}(\theta_j) \mathbf{b}(r_j, z_j); \quad j = 1, \dots, J_s, \quad (6.4)$$

where $\mathbf{b}(r_j, z_j) = [b_{1j} \dots b_{Mj}]^T$ is the mode amplitude vector whose elements are

$$b_{mj} = \Psi(z_H) \Psi(z_j) \frac{e^{(ik_m - \alpha_m)r_j}}{\sqrt{k_m} r_j}; \quad m = 1, \dots, M, \quad (6.5)$$

and $\mathbf{A}(\theta_j)$ is an $N_H \times M$ matrix expressed as

$$\mathbf{A}(\theta_j) = [\mathbf{a}(k_1 \cos \theta_j) \dots \mathbf{a}(k_M \cos \theta_j)], \quad (6.6)$$

$$\mathbf{a}(k_m \cos \theta_j) = \left[1 \ e^{ik_m d_H \cos \theta_j} \ \dots \ e^{i(N_H-1)k_m d_H \cos \theta_j} \right]^T; \ m = 1, \dots, M. \quad (6.7)$$

In (6.5) - (6.7), $\Psi_m(z)$ is the eigenfunction of the m^{th} normal mode of the channel, k_m and α_m denote respectively the associated wavenumber and attenuation coefficient and M is the number of normal modes. The signal vectors of the VLA are independent of bearings because of the cylindrical symmetry of the range-independent ocean, and hence we have $\mathbf{p}_V(\mathbf{x}_j) = \mathbf{p}_V(\mathbf{u}_j)$ where $\mathbf{u}_j = (r_j, z_j)$. The elements of $\mathbf{p}_V(\mathbf{u}_j)$ are given by [5]

$$p_{V_n}(\mathbf{x}_j) = \sum_{m=1}^M \Psi_m(z_{V_n}) \Psi_m(z_j) \frac{e^{(ik_m - \alpha_m)r_j}}{\sqrt{k_m r_j}}; \ n = 1, \dots, N_V. \quad (6.8)$$

The complex noise samples at each sensor are assumed to be i.i.d in time. Using Buckingham's model [1] for ambient noise in a shallow water channel, the variance of noise at depth z can be expressed as

$$v(z) = \sigma^2 \beta^2(z), \quad \beta^2(z) = \sum_{m=1}^M \Psi_m^2(z), \quad (6.9)$$

where σ^2 is a scaling factor. The PDF of each noise sample at depth z is modeled as the following zero-mean L -component Gaussian mixture (GM):

$$f_W(\bar{w}_n, \bar{\bar{w}}_n; z) = \sum_{l=1}^L \frac{\lambda_l}{2\pi\beta^2(z)\sigma_l^2} \exp\left(-\frac{\bar{w}^2 + \bar{\bar{w}}^2}{2\beta^2(z)\sigma_l^2}\right). \quad (6.10)$$

This is a circularly symmetric bivariate PDF for the complex-valued random variable $W = \bar{W} + i\bar{\bar{W}}$ where the cumulative distribution function $F_W(w_n = \bar{w}_n + i\bar{\bar{w}}_n; z) = P(W \leq w_n)$ and $f_W(w_n; z) = \frac{dF_W(w_n; z)}{dw_n}$. The GM has the ability of modeling a variety of different probability distributions including those representing heavy-tailed/impulsive noise. A relatively small value of L is usually adequate to obtain a good model in most applications. The parameter $\beta^2(z)$ is defined in (6.9). The parameter λ_l is the weight of the l^{th} component in the Gaussian mixture, i.e. λ_l is the probability that W is chosen from the l^{th} term in the Gaussian mixture PDF, with $\sum_{l=1}^L \lambda_l = 1$. The quantity $2\beta^2(z)\sigma_l^2$ is the variance of the l^{th} component in the mixture, and

the variance of W is given by

$$\text{var}(W) = v(z) = \sigma^2 \beta^2(z), \quad \sigma^2 = 2 \sum_{l=1}^L \lambda_l \sigma_l^2, \quad \beta^2(z) = \sum_{m=1}^M \Psi_m^2(z). \quad (6.11)$$

Parameters (λ_l, σ_l^2) , $l = 1, \dots, L$ of the GMM considered for modeling the noise PDF in the ocean are unknown and should be estimated. Noise is assumed to be spatially uncorrelated in the horizontal direction for spatial separation greater than or equal to $\lambda/2$ (half-wavelength) [169]. Hence, assuming that $d_H \geq \lambda/2$ and using (6.11), covariance matrix of the HLA noise vector can be written as

$$\mathbf{C}_H \triangleq E[\mathbf{w}_H(t) \mathbf{w}_H(t)^H] = \sigma^2 \mathbf{C}_{0H}, \quad \mathbf{C}_{0H} = \beta^2(z_H) \mathbf{I}_{N_H}, \quad (6.12)$$

where the scaling factor σ^2 is defined in (6.11), $\mathbf{I}_{N_H}$ is the $N_H \times N_H$ identity matrix, and superscript H denotes the conjugate transpose of a matrix. Noise is spatially correlated in the vertical direction, and the covariance matrix of the VLA noise vector is given by [1]

$$\begin{aligned} \mathbf{C}_V &\triangleq E[\mathbf{w}_V(t) \mathbf{w}_V(t)^H] = \sigma^2 \mathbf{C}_{0V}, \\ \mathbf{C}_{0V}(i, j) &= \sum_{m=1}^M \Psi_m(z_i) \Psi_m(z_j), \quad i, j = 1, \dots, N_V. \end{aligned} \quad (6.13)$$

Since the HLA noise vector $\mathbf{w}_H(t)$ and the VLA noise vector $\mathbf{w}_V(t)$ are uncorrelated, the covariance matrix of the hybrid array noise vector $\mathbf{w}(t) = [\mathbf{w}_H(t)^T \mathbf{w}_V(t)^T]^T$ is expressed as

$$\mathbf{C} \triangleq E[\mathbf{w}(t) \mathbf{w}(t)^H] = \sigma^2 \mathbf{C}_0, \quad \mathbf{C}_0 = \begin{bmatrix} \mathbf{C}_{0H} & \mathbf{O}_{N_H N_V} \\ \mathbf{O}_{N_V N_H} & \mathbf{C}_{0V} \end{bmatrix}. \quad (6.14)$$

where $\mathbf{O}_{N_H N_V}$ and $\mathbf{O}_{N_V N_H}$ are null matrices of dimension $N_H \times N_V$ and $N_V \times N_H$ respectively. The SAGE algorithm presented in the next section is based on the assumption that the elements of the array noise vector are i.i.d random variables. To satisfy this requirement, whitening transformations are applied to the data vectors $\mathbf{y}_H(t)$ and $\mathbf{y}_V(t)$ to obtain the pre-whitened

vectors

$$\begin{aligned}\tilde{\mathbf{y}}_H(t) &= \mathbf{U}_H^T \mathbf{y}_H(t) = \tilde{\mathbf{P}}_H(\mathbf{x}) \mathbf{s}(t) + \tilde{\mathbf{w}}_H(t), \\ \tilde{\mathbf{P}}_H(\mathbf{x}) &= \mathbf{U}_H^T \mathbf{P}_H(\mathbf{x}), \quad \tilde{\mathbf{w}}_H(\mathbf{x}) = \mathbf{U}_H^T \mathbf{w}_H(\mathbf{x})\end{aligned}\tag{6.15}$$

$$\begin{aligned}\tilde{\mathbf{y}}_V(t) &= \mathbf{U}_V^T \mathbf{y}_V(t) = \tilde{\mathbf{P}}_V(\mathbf{x}) \mathbf{s}(t) + \tilde{\mathbf{w}}_V(t), \\ \tilde{\mathbf{P}}_V(\mathbf{x}) &= \mathbf{U}_V^T \mathbf{P}_V(\mathbf{x}), \quad \tilde{\mathbf{w}}_V(\mathbf{x}) = \mathbf{U}_V^T \mathbf{w}_V(\mathbf{x})\end{aligned}\tag{6.16}$$

$$\mathbf{U}_H = \beta^{-1}(z_H) \mathbf{I}_{N_H}, \quad \mathbf{U}_V = \text{diag} \left(\lambda_1^{-1/2} \dots \lambda_{N_V}^{-1/2} \right) [\mathbf{v}_1 \dots \mathbf{v}_{N_V}],\tag{6.17}$$

where $\mathbf{v}_1, \dots, \mathbf{v}_{N_V}$ are the eigenvectors of $\mathbf{C}_{0V}$ and $\lambda_1, \dots, \lambda_{N_V}$ are the corresponding eigenvalues. The columns of the pre-whitened steering matrices $\tilde{\mathbf{P}}_H(\mathbf{x})$ and $\tilde{\mathbf{P}}_V(\mathbf{x})$; $j = 1, \dots, J_s$ are denoted by $\tilde{\mathbf{p}}_H(\mathbf{x}_j)$ and $\tilde{\mathbf{p}}_V(\mathbf{x}_j)$; $j = 1, \dots, J_s$. For the hybrid array, the pre-whitened data vector $\tilde{\mathbf{y}}(t) = [\tilde{y}_1(t) \dots \tilde{y}_N(t)]^T$, the associated steering matrix $\tilde{\mathbf{P}}(\mathbf{x})$ and steering vectors $\tilde{\mathbf{p}}(\mathbf{x}_j)$, and the whitened noise vector $\tilde{\mathbf{w}}(t) = [\tilde{w}_1(t) \dots \tilde{w}_N(t)]^T$ is expressed as

$$\begin{aligned}\tilde{\mathbf{y}}(t) &= [\tilde{\mathbf{y}}_H(t)^T \quad \tilde{\mathbf{y}}_V(t)^T]^T = \tilde{\mathbf{P}}(\mathbf{x}) \mathbf{s}(t) + \tilde{\mathbf{w}}(t), \\ \tilde{\mathbf{P}}(\mathbf{x}) &= [\tilde{\mathbf{P}}_H(t)^T \quad \tilde{\mathbf{P}}_V(t)^T]^T; \quad \tilde{\mathbf{p}}(\mathbf{x}_j) = [\tilde{\mathbf{p}}_H(\mathbf{x}_j)^T \quad \tilde{\mathbf{p}}_V(\mathbf{x}_j)^T]^T, \\ \tilde{\mathbf{w}}(t) &= [\tilde{\mathbf{w}}_H(t)^T \quad \tilde{\mathbf{w}}_V(t)^T]^T.\end{aligned}\tag{6.18}$$

The whitened noise samples $\{\tilde{w}_n(t); n = 1, \dots, N; t = 1, \dots, T\}$ are i.i.d with the Gaussian mixture PDF

$$f_{\tilde{W}}(\bar{w}_n, \bar{\bar{w}}_n) = \sum_{l=1}^L \frac{\lambda_l}{2\pi\sigma_l^2} \exp \left[-\frac{\bar{w}^2 + \bar{\bar{w}}^2}{2\sigma_l^2} \right].\tag{6.19}$$

6.3 Proposed multiple source localization algorithm

The set of unknown parameters in this problem is as follows

$$\Phi = \{\mathbf{x}, s_j(t), \lambda_l, \sigma_l^2, t = 1, \dots, T; j = 1, \dots, J_s; l = 1, \dots, L\}.\tag{6.20}$$

The first step in this algorithm is an initialization procedure that produces rough estimate of unknown parameters and it is followed by a SAGE-based approach for updating the initial estimates iteratively. These two steps are explained in the following subsections.

6.3.1 Parameter initialization

Initial estimates of noise parameters are obtained using the expressions proposed by Kozick and Sadler [18]. Pre-whitened data $\tilde{\mathbf{y}}(t)$ from the full hybrid array (both HLA and VLA) is used to obtain the following estimates

$$\begin{aligned}\lambda'_1 &= 0.8, \quad \lambda'_2 = \dots = \lambda'_L = \frac{0.2}{L-1}, \\ \sigma_1'^2 &= 0.5(\bar{\sigma}/0.7)^2, \\ \sigma_L'^2 &= 0.5\max\{|\tilde{y}_n(t)|^2; n=1, \dots, N; t=1, \dots, T\}, \\ \sigma_l'^2 &= (\sigma_L'^2/\sigma_1'^2)^{1/(L-1)}\sigma_{l-1}'^2 \quad \text{for } l=2, \dots, L-1, \\ \bar{\sigma} &= \text{median}\{|\tilde{y}_n(t)|; n=1, \dots, N; t=1, \dots, T\}.\end{aligned}\tag{6.21}$$

The initial estimates of $\sigma_l'^2$ are based on the assumption that the signal is weak compared to noise. The number of sources J_s is estimated using the minimum description length (MDL) criterion [187]. This approach involves estimation of the array data correlation matrix from a finite number of snapshots. When the noise is heavy-tailed, the estimate of the correlation matrix is prone to large errors due to the occurrence of large noise impulses. The correlation matrix estimation can be rendered robust by preprocessing the data so as to suppress the effect of large impulses. The data-adaptive zero-memory nonlinear preprocessor (DA-ZMNL) used in [18] is therefore employed to process each sample of transformed data $\tilde{y}_n(t)$ to obtain

$$\begin{aligned}z_n(t) &= \begin{cases} \tilde{y}_n(t), & |\tilde{y}_n(t)| \leq \tau; \\ \frac{\tilde{y}_n(t)}{|\tilde{y}_n(t)|} \tau \exp\left(-\frac{(|\tilde{y}_n(t)|-\tau)^2}{2\sigma_1'^2}\right), & |\tilde{y}_n(t)| > \tau \end{cases}; \\ \tau &= 3\sqrt{2}\sigma_1' + \bar{\sigma}.\end{aligned}\tag{6.22}$$

The DA-ZMNL in (6.22) incorporates an initial linear part and an exponentially decaying tail. Any received sample below the threshold τ is assumed to contain no impulsive component and is passed unchanged. Samples above the threshold are assumed to be corrupted by a large noise impulse and are suppressed (bounded on Gaussian tails). Let $\mathbf{z}(t)$, $\mathbf{z}_H(t)$ and $\mathbf{z}_V(t)$ denote the vectors obtained by preprocessing the pre-whitened data vectors $\tilde{\mathbf{y}}(t)$, $\tilde{\mathbf{y}}_H(t)$ and $\tilde{\mathbf{y}}_V(t)$ provided, respectively, from the full hybrid array, the HLA, and the VLA respectively. The

correlation matrices of these data vectors are estimated as

$$\begin{aligned}\hat{\mathbf{R}} &= \frac{1}{T} \sum_{t=1}^T \mathbf{z}(t) \mathbf{z}^H(t), \quad \hat{\mathbf{R}}_H = \frac{1}{T} \sum_{t=1}^T \mathbf{z}_H(t) \mathbf{z}_H^H(t), \\ \hat{\mathbf{R}}_V &= \frac{1}{T} \sum_{t=1}^T \mathbf{z}_V(t) \mathbf{z}_V^H(t).\end{aligned}\tag{6.23}$$

Let $\eta_1, \dots, \eta_N$ denote the eigenvalues of $\hat{\mathbf{R}}$ in ascending order. The estimate of the number of sources using MDL algorithm [18, 187] is given by

$$\hat{J}_s = \arg \max_j \left\{ -\log \left[\frac{\prod_{i=1}^{N-j} \eta_i^{1/(N-j)}}{\frac{1}{N-j} \sum_{i=1}^{N-j} \eta_i} \right]^{(N-j)T} + \frac{1}{2} j(2N-j) \log(T) \right\}.\tag{6.24}$$

Initial estimates of the source positions are obtained using a subspace based approach. Initial range-depth estimates of all sources are obtained by applying the 2-D MUSIC algorithm to the VLA data. The 2-D MUSIC spectrum is defined as

$$B_{\text{MUSIC}}(\mathbf{u}) = \frac{1}{\tilde{\mathbf{p}}_V(\mathbf{u})^H \mathbf{E}_{VN} \mathbf{E}_{VN}^H \tilde{\mathbf{p}}_V(\mathbf{u})},\tag{6.25}$$

where $\mathbf{E}_{VN}$ is the noise subspace matrix obtained from eigen decomposition of the estimated correlation matrix $\hat{\mathbf{R}}_V$. The J_s largest peaks of $B_{\text{MUSIC}}(\mathbf{u})$ provide the initial range-depth estimates $\hat{\mathbf{u}}_j = (\hat{r}_j, \hat{z}_j); j = 1, \dots, J_s$. These estimates do not require prior knowledge of the source bearings due to the cylindrical symmetry of the range independent ocean.

Initial bearing estimates are obtained from the HLA data using the R-MUSIC-mdf algorithm developed recently by Xu and Liu [97]. R-MUSIC-mdf belongs to the class of bearing estimation methods that use a 1-D search to estimate bearings without prior knowledge of source ranges and depths. Other methods in this class include SIM [95] and R-MUSIC [78]. These methods exploit the normal mode structure of the acoustic field generated by the sources, and the fact that the N_H -dimensional signal vector due to each source belongs to an M -dimensional subspace if $N_H > M$. We have chosen the R-MUSIC-mdf algorithm because it is computationally simpler than SIM and provides a uniformly better performance than R-MUSIC. The spectrum of R-MUSIC-mdf is given by

$$B_{RM}(\theta) = \text{sum}\{\mathbf{T}^{-1}(\theta)\},\tag{6.26}$$

$$\mathbf{T}(\theta) = \mathbf{A}^H(\theta) \mathbf{E}_{HN} \mathbf{E}_{HN}^H \mathbf{A}(\theta) + l_r \mathbf{I}_M,\tag{6.27}$$

where E_{HN} is the noise subspace matrix obtained from eigen decomposition of estimated correlation matrix $\hat{\mathbf{R}}_H$ of the HLA data vector, $\mathbf{A}(\theta)$ is the modal steering matrix defined in (6.6), l_r is a regularization parameter, and $\mathbf{I}_M$ is the $M \times M$ identity matrix and $\text{sum}(\cdot)$ denotes the summation of all elements of a matrix. The value of l_r is chosen experimentally so as to minimize the mean square estimation error. The J_s largest peaks of the spectrum defined in (6.26) provide the initial bearing estimates $\hat{\theta}_j; j = 1, \dots, J_s$.

We have thus obtained J_s initial range-depth estimates $\hat{\mathbf{u}}_i, i = 1, \dots, J_s$ and J_s initial bearing estimates $\hat{\theta}_j; j = 1, \dots, J_s$, using one 2-D search and one 1-D search. There exist $J_s!$ ways of associating the J_s range-depth estimates with the J_s bearing estimates. The ambiguity is resolved by computing the 3-D Bartlett spectrum from the ZMNL-processed hybrid array data at the J_s^2 points $(\hat{\mathbf{u}}_i, \hat{\theta}_j); i, j = 1, \dots, J_s$ and employing a simple search-maximize-discard procedure to determine the correct association. At each of these points, we compute the 3-D Bartlett spectrum of the DA-ZMNL processed data

$$B_{\text{Bartlett}}(\mathbf{u}, \theta) = \tilde{\mathbf{p}}^H(\mathbf{u}, \theta) \hat{\mathbf{R}} \tilde{\mathbf{p}}(\mathbf{u}, \theta), \quad (6.28)$$

where $\tilde{\mathbf{p}}(\mathbf{u}, \theta)$ is the replica of pre-whitened signal vector due to a unit source at $(\mathbf{u}, \theta)$. The location of the largest peak of Bartlett spectrum (6.28) is assigned to the first source (e.g. $(\hat{\mathbf{u}}_1, \hat{\theta}_1)$). We discard all the points containing either $\hat{\mathbf{u}}_1$ or $\hat{\theta}_1$ as one of the coordinates from the original set of J_s^2 points. Now only $(J_s - 1)^2$ points remain. By finding the location of the largest peak of (6.28) among the remaining points, the location of the second source is obtained and its coordinates are removed from the original set. This procedure is continued until the association for J_s sources is complete.

Initial estimate of the signal vector envelope $\{\mathbf{s}(t); t = 1, \dots, T\}$ is obtained as the best linear unbiased estimate (BLUE) [186], using the ZMNL-processed hybrid array data $\mathbf{z}(t)$ and the initial source position estimates $\hat{\mathbf{x}}$. This estimate is given by the equation

$$\mathbf{s}'(t) = \left\{ \tilde{\mathbf{P}}(\hat{\mathbf{x}})^H \tilde{\mathbf{P}}(\hat{\mathbf{x}}) \right\}^{-1} \tilde{\mathbf{P}}(\hat{\mathbf{x}})^H \mathbf{z}(t); \quad t = 1, \dots, T. \quad (6.29)$$

6.3.2 Iterative updating of initial estimates

The initial estimates are updated iteratively using the SAGE algorithm. While the basic approach is the same as that adopted by Kozick and Sadler [18] to solve the one-dimensional

problem of DOA estimation, several modifications explained in this Section have been introduced to reduce the complexity arising out of the higher dimensionality of the problem and the use of a hybrid array. Details of the algorithm are given below.

The set of unknown parameters Φ defined in (6.20) is partitioned into five subsets as follows

$$\begin{aligned}\Phi_1 &= \{\theta_1, \dots, \theta_{J_s}\}, \quad \Phi_2 = \{r_1, \dots, r_{J_s}\}, \quad \Phi_3 = \{z_1, \dots, z_{J_s}\}, \\ \Phi_4 &= \{s_j(t); t = 1, \dots, T; j = 1, \dots, J_s\}, \\ \Phi_5 &= \{\lambda_1, \dots, \lambda_L, \sigma_1^2, \dots, \sigma_L^2\}.\end{aligned}\tag{6.30}$$

so that $\Phi = \Phi_1 \cup \Phi_2 \cup \Phi_3 \cup \Phi_4 \cup \Phi_5$. A sequential updating procedure is used, wherein estimates of parameters in one subset are updated at a time while the values of all other parameters are held fixed. The current estimate of an unknown parameter ϕ is denoted by $\hat{\phi}$.

In the EM algorithm, the goal is to find the maximum likelihood estimation of unknown parameters. The EM algorithm is usually used in cases where the likelihood function cannot be solved directly (e.g. finding the derivatives of the likelihood function with respect to all unknown parameters is very complicated). EM algorithm is usually used with mixture models. A mixture model can be described more simply by assuming that each observed data sample has a corresponding unobserved (hidden) variables that determine each data sample belongs to which component in the mixture model. Therefore, the model can be formulated more simply by assuming the existence of additional unobserved data samples [188]. Hence, in the EM algorithm (and also the SAGE algorithm), the observed data is regarded as an incomplete data set. This incomplete data set D is augmented with some hidden data to generate a complete data set D_C . The hidden data is so chosen that maximization of the likelihood function of the complete data D_C is easier than maximization of the likelihood function of the incomplete data D . The EM algorithm involves an iterative implementation of the following two steps:

1. E-step: Find the expectation of log-likelihood function of the complete data set D_C conditioned on the incomplete (observable) data D and the current estimate $\hat{\Phi}$ of the parameter set Φ . Let this expectation be denoted by $Q(\Phi|\hat{\Phi})$.
2. M-step: Maximize $Q(\Phi|\hat{\Phi})$ with respect to Φ . In the present problem, the maximization is done sequentially with respect to each subset Φ_i of Φ , after assigning the current estimated values to the other subsets of Φ .

In the present problem, the incomplete data set D is

$$D = \{\tilde{\mathbf{y}}(t); t = 1, \dots, T\} = \{\tilde{y}_n(t); n = 1, \dots, N; t = 1, \dots, T\}. \quad (6.31)$$

When noise is modeled as a Gaussian mixture, each noise sample $w_n(t)$ can be regarded as being randomly selected from a set of Gaussian random variables $\{N(0, \sigma_l^2); l = 1, \dots, L\}$, the probability of selecting the l^{th} member of the set being λ_l . It is therefore convenient to augment the data D by a hidden data set $\{l_n(t); n = 1, \dots, N; t = 1, \dots, T\}$ whose members are i.i.d random variables with the conditional probability distribution

$$p(l_n(t) = l | \Phi) = \lambda_l; l = 1, \dots, L. \quad (6.32)$$

In multiple source localization problems, it is also convenient to consider the following unobservable data vectors

$$\tilde{\mathbf{y}}^{(j)}(t) = [\tilde{y}_1^{(j)}(t) \dots \tilde{y}_N^{(j)}(t)]^T; j = 1, \dots, J_s. \quad (6.33)$$

In (6.33), $\tilde{y}_n^{(j)}(t)$ is the unobservable data at sensor n due to source j , satisfying the constraint

$$\sum_{j=1}^{J_s} \tilde{y}_n^{(j)}(t) = \tilde{y}_n(t); n = 1, \dots, N; t = 1, \dots, T, \quad (6.34)$$

where $\tilde{y}_n(t)$ is the pre-whitened observation at sensor n at time t . Therefore, following Kozick and Sadler method [18], we introduce two sets of complete data D_{C1} and D_{C2} , defined as

$$D_{C1} = \{\tilde{y}_n(t), l_n(t); n = 1, \dots, N; t = 1, \dots, T\}, \quad (6.35)$$

$$D_{C2} = \{\tilde{y}_n^{(j)}(t), l_n(t); j = 1, \dots, J_s; n = 1, \dots, N; t = 1, \dots, T\}, \quad (6.36)$$

where D_{C1} is the less informative complete data set and D_{C2} is the more informative complete data set. Inclusion of the pre-whitened individual source observation at sensor n , $\tilde{y}_n^{(j)}(t)$ in D_{C2} provides more information compared to D_{C1} in which the pre-whitened observation at the sensor related to J_s sources $\tilde{y}_n(t)$ is used. The idea of decomposing the observed data into their individual signals, when we deal with superimposed signals has been discussed by Feder and Weinstein in [189].

The constraint (6.34) is satisfied by defining

$$\tilde{\mathbf{y}}^{(j)}(t) = \tilde{\mathbf{p}}(\mathbf{x}_j)s_j(t) + \tilde{\mathbf{w}}^{(j)}(t); \quad j = 1, \dots, J_s, \quad (6.37)$$

where $\tilde{\mathbf{p}}(\mathbf{x}_j) = [\tilde{p}_1(\mathbf{x}_j) \dots \tilde{p}_N(\mathbf{x}_j)]^T$ is the pre-whitened array signal vector defined in (6.18) due to a unit source at $\mathbf{x}_j$ and $\tilde{\mathbf{w}}^{(j)}(t) = [\tilde{w}_1^{(j)}(t) \dots \tilde{w}_N^{(j)}(t)]^T$, where $\{\tilde{w}_n^{(j)}(t); j = 1, \dots, J_s; n = 1, \dots, N; t = 1, \dots, T\}$ are i.i.d random variables satisfying the equation

$$\sum_{j=1}^{J_s} \tilde{w}_n^{(j)}(t) = \tilde{w}_n(t); \quad n = 1, \dots, N; \quad t = 1, \dots, T, \quad (6.38)$$

and $\tilde{w}_n(t)$ is the whitened noise at sensor n at time t . We also note that $\tilde{w}_n(t)$ and $\tilde{w}_n^{(j)}(t)$ are Gaussian when conditioned on $l_n(t)$, and therefore we have

$$\tilde{w}_n(t)|l_n(t) \sim N(0, \sigma_{l_n(t)}^2); \quad n = 1, \dots, N; \quad t = 1, \dots, T \quad (6.39)$$

$$\tilde{w}_n^{(j)}(t)|l_n(t) \sim N(0, \frac{1}{J_s} \sigma_{l_n(t)}^2); \quad j = 1, \dots, J_s; \quad n = 1, \dots, N; \quad t = 1, \dots, T \quad (6.40)$$

We also define the J_s -dimensional vectors

$$\check{\mathbf{y}}_n(t) = [\tilde{y}_n^{(1)}(t) \dots \tilde{y}_n^{(J_s)}(t)]^T; \quad n = 1, \dots, N; \quad t = 1, \dots, T. \quad (6.41)$$

Since $\tilde{w}_n^{(j)}(t)$ are independent over j , n and t , we can write the following expressions for the conditional PDFs of the complete data

$$f(\check{\mathbf{y}}_n(t), l_n(t)|\Phi) = \frac{\sum_{l=1}^L \lambda_l \delta(l_n(t) - l)}{2\pi \sigma_{l_n(t)}^2} \exp \left[-\frac{1}{2\sigma_{l_n(t)}^2} \left| \check{\mathbf{y}}_n(t) - \sum_{j=1}^{J_s} \tilde{p}_n(\mathbf{x}_j) s_j(t) \right|^2 \right], \quad (6.42)$$

$$f(\check{\mathbf{y}}_n(t), l_n(t)|\Phi) = \frac{\sum_{l=1}^L \lambda_l \delta(l_n(t) - l)}{(2\pi \sigma_{l_n(t)}^2 / J_s)^{J_s}} \exp \left[-\frac{1}{2\sigma_{l_n(t)}^2 / J_s} \sum_{j=1}^{J_s} \left| \tilde{y}_n^{(j)}(t) - \tilde{p}_n(\mathbf{x}_j) s_j(t) \right|^2 \right], \quad (6.43)$$

where $\delta(\cdot)$ denotes the Dirac delta function.

The log-likelihood functions of the complete data sets D_{C1} and D_{C2} , conditioned on Φ , can be

written as

$$L(D_{C1}|\Phi) = \sum_{t=1}^T \sum_{n=1}^N \log f(\tilde{y}_n(t), l_n(t)|\Phi) \quad (6.44)$$

$$L(D_{C2}|\Phi) = \sum_{t=1}^T \sum_{n=1}^N \log f(\check{y}_n(t), l_n(t)|\Phi). \quad (6.45)$$

E-step

The likelihood functions in (6.44) and (6.45) are not known since $l_n(t)$ and $\tilde{\mathbf{y}}_n(t)$ are not observable. The E-step of the SAGE algorithm consists in finding the expectations of $L(D_{Ci}|\Phi)$, $i = 1, 2$ conditioned on the available pre-whitened observations $\{\tilde{y}_n(t); n = 1, \dots, N; t = 1, \dots, T\}$ and the current estimates $\hat{\Phi}$. The conditional expectation of $L(D_{Ci}|\Phi)$ can be written as

$$\begin{aligned} Q_1(\Phi|\hat{\Phi}) &\triangleq E[L(D_{C1}|\Phi)|\{\tilde{y}_n(t); n = 1, \dots, N; t = 1, \dots, T\}, \hat{\Phi}] \\ &= B_1 + \sum_{t=1}^T \sum_{n=1}^N \sum_{l=1}^L [\log \lambda_l - \log \sigma_l^2 - \frac{1}{2\sigma_l^2} |\tilde{y}_n(t) - \sum_{j=1}^{J_s} \tilde{p}_n(\mathbf{x}_j) \dot{s}_j(t)|^2] \dot{g}_{l,n,t}(\tilde{y}_n(t)), \end{aligned} \quad (6.46)$$

where $\dot{g}_{l,n,t} = P(l_n(t)|\tilde{y}_n(t), \hat{\Phi})$ is the conditional probability of $l_n(t)$ defined in (6.47), and B_1 is independent of the parameters Φ . Recalling that

$$\begin{aligned} \tilde{y}_n(t)|\{l_n(t) = l, \hat{\Phi}\} &\sim N\left(\sum_{j=1}^{J_s} \tilde{p}_n(\mathbf{x}_j) \dot{s}_j(t), \sigma_l^2\right), \\ P(l_n(t) = l|\hat{\Phi}) &= \dot{\lambda}_l, \end{aligned}$$

and employing Bayes' rule, we get

$$\begin{aligned} \dot{g}_{l,n,t}(\tilde{y}_n(t)) &\triangleq P(l_n(t)|\tilde{y}_n(t), \hat{\Phi}) \\ &= \frac{\frac{\dot{\lambda}_l}{2\pi\sigma_l^2} \exp\left(\frac{-1}{2\sigma_l^2} \left|\tilde{y}_n(t) - \sum_{j=1}^{J_s} \tilde{p}_n(\mathbf{x}_j) \dot{s}_j(t)\right|^2\right)}{\sum_{q=1}^L \frac{\dot{\lambda}_q}{2\pi\sigma_q^2} \exp\left(\frac{-1}{2\sigma_q^2} \left|\tilde{y}_n(t) - \sum_{j=1}^{J_s} \tilde{p}_n(\mathbf{x}_j) \dot{s}_j(t)\right|^2\right)}; \\ &l = 1, \dots, L; n = 1, \dots, N; t = 1, \dots, T. \end{aligned} \quad (6.47)$$

The conditional expectation of $L(D_{C2}|\Phi)$ requires the conditional estimate of $\check{\mathbf{y}}_n(t)$. The estimate of $\tilde{y}_n^{(j)}(t)$ conditioned on $\{\tilde{y}_n(t); n = 1, \dots, N; t = 1, \dots, T\}$ and the current estimates of $\mathbf{x}_j$ and $s_j(t)$ can be written as

$$\check{\mathbf{y}}_n(t) = [\check{y}_n^{(1)}(t) \dots \check{y}_n^{(J_s)}(t)]^T; n = 1, \dots, N; t = 1, \dots, T. \quad (6.48)$$

$$\check{y}_n^{(j)}(t) = \tilde{p}_n(\mathbf{x}_j) \acute{s}_j(t) + \frac{1}{J_s} [\tilde{y}_n(t) - \sum_{j=1}^{J_s} \tilde{p}_n(\mathbf{x}_j) \acute{s}_j(t)]; j = 1, \dots, J_s; n = 1, \dots, N. \quad (6.49)$$

On the right hand side of (6.49), the first term is the conditional estimate of the signal from the j^{th} source at sensor n and time t , and the other terms are the conditional estimates of the whitened noise $\tilde{w}_n^{(j)}(t)$. On replacing $\check{\mathbf{y}}_n(t)$ by $\check{\mathbf{y}}_n(t)$ in (6.45), the conditional expectation of $L(D_{C2}|\Phi)$ can be written as

$$\begin{aligned} Q_2(\Phi|\acute{\Phi}) &\triangleq E[L(D_{C2}|\Phi)|\{\tilde{y}_n(t); n = 1, \dots, N; t = 1, \dots, T\}, \acute{\Phi}] \\ &= B_2 + \sum_{t=1}^T \sum_{n=1}^N \sum_{l=1}^L [\log \lambda_l - J_s \log \sigma_l^2 - \frac{J_s}{2\sigma_l^2} \sum_{j=1}^{J_s} |\tilde{y}_n^{(j)'}(t) - \tilde{p}_n(\mathbf{x}_j) \acute{s}_j(t)|^2] \acute{g}_{l,n,t}(\tilde{y}_n(t)), \end{aligned} \quad (6.50)$$

where B_2 is independent of Φ .

M-step

The M-step of the SAGE algorithm consists of maximizing either $Q_1(\Phi|\acute{\Phi})$ or $Q_2(\Phi|\acute{\Phi})$ with respect to each parameter in the set Φ holding other parameters fixed at their current values. In each cycle of iteration, the updating is done in the following sequence until all the parameters are updated. At the start of each iteration, the conditional probabilities $g_{l,n,t}(\tilde{y}_n(t))$ defined in (6.47) are updated using the current estimates of Φ . The estimate of each parameter in the set $\Phi_1 \cup \Phi_2 \cup \Phi_3 \cup \Phi_4 \cup \Phi_5$ is updated using the more informative complete data set D_{C2} by maximizing $Q_2(\Phi|\acute{\Phi})$ with respect to that parameter after assigning the current estimates to the other parameters. The condition $\frac{\partial Q_2(\Phi|\acute{\Phi})}{\partial s_j(t)} = 0$ yields the conditional estimate

$$\acute{s}_j(t|\mathbf{x}_j) = \frac{\tilde{\mathbf{p}}(\mathbf{x}_j)^H \mathbf{H}_t'(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{y}}^{(j)'}(t)}{\tilde{\mathbf{p}}(\mathbf{x}_j)^H \mathbf{H}_t'(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{p}}(\mathbf{x}_j)}, \quad (6.51)$$

where

$$\mathbf{H}'_t(\tilde{\mathbf{y}}(t)) = \text{diag} \left(\sum_{l=1}^L \frac{1}{\sigma_l^2} \dot{g}_{l,1,t}(\tilde{y}_1(t)), \dots, \sum_{l=1}^L \frac{1}{\sigma_l^2} \dot{g}_{l,N,t}(\tilde{y}_N(t)) \right); \quad t = 1, \dots, T. \quad (6.52)$$

On replacing $\dot{s}_j(t)$ by $\dot{s}_j(t|\hat{\mathbf{x}}_j)$ in (6.50) and maximizing $Q_2(\Phi|\hat{\Phi})$ with respect to $\hat{\mathbf{x}}_j$, we get the following updated estimate of $\mathbf{x}_j$

$$\hat{\mathbf{x}}_j = \underset{\mathbf{x}_j}{\text{argmax}} \sum_{t=1}^T \left\{ \left[\tilde{\mathbf{p}}(\mathbf{x}_j)^H \mathbf{H}'_t(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{p}}(\mathbf{x}_j) \right]^{-1} \left| \tilde{\mathbf{p}}(\mathbf{x}_j)^H \mathbf{H}'_t(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{y}}^{(j)'}(t) \right|^2 \right\};$$

$$j = 1, \dots, J_s. \quad (6.53)$$

Determination of $\mathbf{x}_j$ in (6.53) requires a computationally expensive 3-D search. An approximate evaluation of $\hat{\mathbf{x}}_j$ is therefore obtained by replacing the 3-D search by three sequential 1-D searches. The range is updated first using the equation

$$\hat{r}_j = \underset{r}{\text{argmax}} \sum_{t=1}^T \left\{ \left[\tilde{\mathbf{p}}(\mathbf{x}_j(r))^H \mathbf{H}'_t(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{p}}(\mathbf{x}_j(r)) \right]^{-1} \left| \tilde{\mathbf{p}}(\mathbf{x}_j(r))^H \mathbf{H}'_t(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{y}}^{(j)'}(t) \right|^2 \right\};$$

$$j = 1, \dots, J_s. \quad (6.54)$$

where $\mathbf{x}_j(r) = (r, \hat{z}_j, \hat{\theta}_j)$, and $\hat{\theta}_j$, $\hat{z}_j$ are the current estimates of θ_j and z_j respectively. The depth is updated next using the equation

$$\hat{z}_j = \underset{z}{\text{argmax}} \sum_{t=1}^T \left\{ \left[\tilde{\mathbf{p}}(\mathbf{x}_j(z))^H \mathbf{H}'_t(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{p}}(\mathbf{x}_j(z)) \right]^{-1} \left| \tilde{\mathbf{p}}(\mathbf{x}_j(z))^H \mathbf{H}'_t(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{y}}^{(j)'}(t) \right|^2 \right\};$$

$$j = 1, \dots, J_s. \quad (6.55)$$

where $\mathbf{x}_j(z) = (\hat{r}_j, z, \hat{\theta}_j)$. The range and depth are estimated using the full hybrid array data $\tilde{\mathbf{y}}(t)$. The bearing is estimated next in a similar fashion using the full hybrid array data to obtain

$$\hat{\theta}_j = \underset{\theta}{\text{argmax}} \sum_{t=1}^T \left\{ \left[\tilde{\mathbf{p}}(\mathbf{x}_j(\theta))^H \mathbf{H}'_t(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{p}}(\mathbf{x}_j(\theta)) \right]^{-1} \left| \tilde{\mathbf{p}}(\mathbf{x}_j(\theta))^H \mathbf{H}'_t(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{y}}^{(j)'}(t) \right|^2 \right\}; \quad j = 1, \dots, J_s. \quad (6.56)$$

where $\mathbf{x}_j(\theta) = (\hat{r}_j, \hat{z}_j, \theta)$. Alternatively, bearing updates may also be optimized using only the HLA data $\tilde{\mathbf{y}}_H(t)$ instead of the full hybrid array data $\tilde{\mathbf{y}}(t)$, since the VLA are insensitive to

change in bearing. Results based on these alternative strategies are presented in Section 6.7.1. The updated estimates of signal envelopes in (6.51) and the source coordinates in (6.54) - (6.56) are used to update the conditional probabilities $g'_{l,n,t}(\tilde{y}_n(t))$ in (6.47) and the matrix functions $\mathbf{H}'_t(\tilde{y}(t))$ in (6.52). The estimate of the signal vector envelope $\{\mathbf{s}'(t) = [s'_1(t) \dots s'_{J_s}(t)]^T; t = 1, \dots, T\}$ is then updated from the less informative complete data set D_{C1} by applying the condition $\frac{\partial Q_1(\Phi|\hat{\Phi})}{\partial \mathbf{s}(t)} = \mathbf{0}$, to obtain

$$\hat{\mathbf{s}}(t) = \left[\tilde{\mathbf{P}}(\hat{\mathbf{x}})^H \mathbf{H}'_t(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{P}}(\hat{\mathbf{x}}) \right]^{-1} \left[\tilde{\mathbf{P}}(\hat{\mathbf{x}})^H \mathbf{H}'_t(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{y}}(t) \right], \quad t = 1, \dots, T. \quad (6.57)$$

The updates in (6.57) are improved versions of updates in (6.51). Next, the conditional probabilities $\hat{g}_{l,n,t}(\tilde{y}_n(t))$ and the matrix functions $\mathbf{H}'_t(\tilde{y}(t))$ are updated once again using the updated values of $\hat{s}_j(t)$ obtained from (6.57).

Finally, the noise parameters $\{\lambda_l, \sigma_l^2; l = 1, \dots, L\}$ are updated from the less informative data set D_{C1} by maximizing $Q_1(\Phi|\hat{\Phi})$ with respect to Φ_5 after assigning $\Phi_i = \hat{\Phi}_i$ for $i = 1, \dots, 4$. The conditions $\left\{ \frac{\partial Q_1(\Phi|\hat{\Phi})}{\partial \lambda_l} = 0, \frac{\partial Q_1(\Phi|\hat{\Phi})}{\partial \sigma_l^2} = 0, l = 1, \dots, L \right\}$, subject to the constraint $\sum_{l=1}^L \lambda_l = 1$, yield the estimates

$$\hat{\lambda}_l = \frac{1}{NT} \sum_{n=1}^N \sum_{t=1}^T g'_{l,n,t}(\tilde{y}_n(t)), \quad l = 1, \dots, L. \quad (6.58)$$

$$\hat{\sigma}_l^2 = \frac{1}{2NT\hat{\lambda}_l} \sum_{t=1}^T [\tilde{\mathbf{y}}(t) - \tilde{\mathbf{p}}(\hat{\mathbf{x}}') \mathbf{s}'(t)]^H \mathbf{G}'_{l,t}(\tilde{\mathbf{y}}(t)) [\tilde{\mathbf{y}}(t) - \tilde{\mathbf{p}}(\hat{\mathbf{x}}') \mathbf{s}'(t)], \quad l = 1, \dots, L. \quad (6.59)$$

where

$$\mathbf{G}'_{l,t}(\tilde{\mathbf{y}}(t)) = \text{diag} \{g'_{l,1,t}(\tilde{y}_1(t)), \dots, g'_{l,N,t}(\tilde{y}_N(t))\}, \quad l = 1, \dots, L, \quad (6.60)$$

Estimates of the signal envelopes and noise parameters are updated by using the closed form expressions in (6.51) and (6.57) - (6.59). But updating the estimates of source-coordinates involves maximization of the objective functions in (6.54) - (6.56) through a search procedure. To reduce the computational complexity of the search, we use the algorithm for adaptive shrinkage of the search space proposed by Chung and Bohme [104]. Let $[a_j^{[0]}, b_j^{[0]}]$ denote the search interval for parameter ϕ_j for the first iteration, and let $\phi_j^{[i]}$ be the estimate of ϕ_j after the i^{th} iteration. Define

$$\Delta_j^{[i]} = |\phi_j^{[i]} - \phi_j^{[i-1]}|. \quad (6.61)$$

The end points of the search interval $[a_j^{[i]}, b_j^{[i]}]$ for the $(i + 1)$ st iteration are given by

$$\begin{aligned}
 a_j^{[i]} &= a_j^{[0]}, \quad b_j^{[i]} = b_j^{[0]}, \quad \text{for } i = 1, 2 \\
 a_j^{[i]} &= \begin{cases} a_j^{[0]}, & \Delta_j^{[i]} > \Delta_j^{[i-1]} \\ \phi_j^{[i]} - c\Delta_j^{[i]}, & \text{otherwise} \end{cases}; \\
 b_j^{[i]} &= \begin{cases} b_j^{[0]}, & \Delta_j^{[i]} > \Delta_j^{[i-1]} \\ \phi_j^{[i]} + c\Delta_j^{[i]}, & \text{otherwise} \end{cases}; \\
 &\text{for } i > 2,
 \end{aligned} \tag{6.62}$$

where c is a constant greater than 1. The rate of shrinkage of the search space is measured by the ratio

$$\rho_j^{[i]} = \frac{\Delta_j^{[i+1]}}{\Delta_j^{[i]}}. \tag{6.63}$$

Since $c > 1$, the ratio $\rho_j^{[i]}$ is larger than 1 for small i . But the control loop ensures that $\rho_j^{[i]}$ begins to converge and tends to zero. Convergence is said to have occurred if the difference between the current and the previous location estimates is below a prescribed threshold $[\gamma_r, \gamma_z, \gamma_\theta]$ for two successive iterations. A summary of the SAGE-USL algorithm is given in Fig. 6.2.

6.3.3 Convergence of the proposed SAGE-USL algorithm

Some remarks on the convergence of the proposed SAGE-USL algorithm are given here. In a multi-parameter estimation problem such as the present one, generalized EM algorithms (e.g. SAGE) are preferred to the pure EM algorithm because the former involve updating of parameters in smaller groups rather than updating all the parameters simultaneously [190]. In general, the complete data set required for estimating each group of parameters is smaller than the complete data set required for estimating the entire set of unknown parameters. The likelihood function in a generalized EM algorithm is non-decreasing for all estimates of each unknown parameter [101], no matter whether they are obtained from separate complete data sets or from one single complete data set. In the SAGE-USL algorithm, we have used two complete data sets, viz. the less informative data set D_{C1} and the more informative data set D_{C2} . The convergence of the SAGE algorithm with different complete data sets

Initialization

1. Apply the whitening transformations $\mathbf{U}_H$ and $\mathbf{U}_V$ to the HLA and VLA data to obtain the pre-whitened data vectors $\tilde{\mathbf{y}}_H(t)$ and $\tilde{\mathbf{y}}_V(t)$.
2. Obtain initial noise parameter estimates $\{(\lambda'_l, \sigma'^2_l); l = 1, \dots, L\}$ from (6.21).
3. Apply DA-ZMNL processing (6.22) to the pre-whitened observations $\tilde{\mathbf{y}}_n(t)$, and use (6.23) to estimate the correlation matrices $\hat{\mathbf{R}}, \hat{\mathbf{R}}_H$ and $\hat{\mathbf{R}}_V$ corresponding to the hybrid array, HLA, and VLA data vectors.
4. Estimate the number of sources J from (6.24).
5. Obtain initial range-depth estimates $\{(\hat{r}'_j, \hat{z}'_j); j = 1, \dots, J\}$ from (6.25).
6. Obtain initial bearing estimates $\{\theta'_j; j = 1, \dots, J\}$ from (6.26) and (6.27).
7. Establish the pairing between range-depth estimates $(\hat{r}'_i, \hat{z}'_i)$ and bearing estimates θ'_j using (6.28) and the search-maximize-discard method described in Section 6.4.1.
8. Obtain initial signal waveform estimates $\hat{s}'_j(t)$ from (6.29).

Iterative updates

1. Compute the conditional probability distribution, $g'_{l,n,t}(\tilde{\mathbf{y}}_n(t))$ from (6.47).
2. Obtain the conditional estimates of the hidden data vectors from (6.48) and (6.49).
3. Determine the search spaces for source coordinate estimates adaptively for each iteration using the procedure defined in (6.62).
4. Update range estimates $\hat{r}'_j$ from (6.54).
5. Update depth estimates $\hat{z}'_j$ from (6.55).
6. Update bearing estimates θ'_j from (6.56).
7. Update the conditional probability distribution $g'_{l,n,t}(\tilde{\mathbf{y}}_n(t))$ from (6.47) and the matrix function $\mathbf{H}'_t(\tilde{\mathbf{y}}(t))$ from (6.52).
8. Update the joint estimate $\mathbf{s}'(t)$ of all signal envelopes from (6.57).
9. Update $g'_{l,n,t}(\tilde{\mathbf{y}}_n(t))$ from (6.47).
10. Update noise parameter estimates $(\lambda'_l, \sigma'^2_l)$ from (6.58) and (6.59).

Figure 6.2: Summary of the SAGE-USL algorithm for multiple source localization

(including complete data sets similar to D_{C1} and D_{C2}) has been analyzed and demonstrated in [100,188,189]. It has been shown in [190] that use of a less informative complete data set leads to faster convergence. In view of the foregoing remarks, the SAGE-USL algorithm is expected to have good convergence properties. A detailed evaluation of convergence of the SAGE-USL algorithm is presented in Section 6.7.

6.4 Cramér-Rao Bound

In this section, we obtain an expression for CRB for multiple source localization problem considered in the previous section. The pre-whitened data vector $\tilde{\mathbf{y}}(t)$ can be expressed as

$$\tilde{\mathbf{y}}(t) = \sum_{j=1}^{J_s} \tilde{\mathbf{p}}(\mathbf{x}_j) \mathbf{s}_j(t) + \tilde{\mathbf{w}}(t) = \tilde{\mathbf{P}}(\mathbf{x}) \mathbf{s}(t) + \tilde{\mathbf{w}}(t), \quad t = 1, \dots, T. \quad (6.64)$$

Using this pre-whitened data, the CRB expression is modified as

$$CRB(\mathbf{x}) = \frac{1}{I_c(\Lambda)} \left\{ \sum_{t=1}^T \text{Re}[\mathbf{S}(t)^H \tilde{\mathbf{Q}}(\mathbf{x})^H (\mathbf{I} - \tilde{\mathbf{P}}(\mathbf{x})(\tilde{\mathbf{P}}(\mathbf{x})^H \tilde{\mathbf{P}}(\mathbf{x}))^{-1} \tilde{\mathbf{P}}(\mathbf{x})^H) \tilde{\mathbf{Q}}(\mathbf{x}) \mathbf{S}(t)] \right\}^{-1}, \quad (6.65)$$

where

$$\tilde{\mathbf{Q}}(\mathbf{x}) = \begin{bmatrix} \frac{\partial \tilde{\mathbf{p}}(\mathbf{x}_1)}{\partial \mathbf{r}_1} \frac{\partial \tilde{\mathbf{p}}(\mathbf{x}_1)}{\partial \mathbf{z}_1} \frac{\partial \tilde{\mathbf{p}}(\mathbf{x}_1)}{\partial \theta_1} \cdots \frac{\partial \tilde{\mathbf{p}}(\mathbf{x}_{J_s})}{\partial \mathbf{r}_{J_s}} \frac{\partial \tilde{\mathbf{p}}(\mathbf{x}_{J_s})}{\partial \mathbf{z}_{J_s}} \frac{\partial \tilde{\mathbf{p}}(\mathbf{x}_{J_s})}{\partial \theta_{J_s}} \end{bmatrix}, \quad (6.66)$$

and

$$\mathbf{S}(t) = \text{diag}(\mathbf{s}_1(t), \dots, \mathbf{s}_{J_s}(t)) \otimes \mathbf{I}_3(t) \quad (6.67)$$

is a $3N \times 3N$ diagonal matrix. For a hybrid array with pre-whitened data, we have $\tilde{\mathbf{P}}(\mathbf{x}) = [\tilde{\mathbf{p}}(\mathbf{x}_1) \dots \tilde{\mathbf{p}}(\mathbf{x}_{J_s})]$, $\tilde{\mathbf{p}}(\mathbf{x}_j) = [\mathbf{p}_H(\mathbf{x}_j)^T \mathbf{U}_H \quad \mathbf{p}_V(\mathbf{x}_j)^T \mathbf{U}_V]^T$ where $\mathbf{U}_H$ and $\mathbf{U}_V$ are respectively the whitening matrices for the horizontal and vertical segments of the hybrid array. The CRB can be computed using the expressions for $\mathbf{p}_H(\mathbf{x}_j)$ and $\mathbf{p}_V(\mathbf{x}_j)$ defined in (6.4) - (6.8). For the Gaussian mixture noise PDF defined in (6.19), $I_c(\Lambda)$ in (6.65) can be computed numerically from the expression (5-29) in Chapter 5.

Table 6.1: Computational complexity of SAGE-USL algorithm

Computation of correlation matrix estimate $\hat{\mathbf{R}}$	$O(TN_H^2) + O(TN_V^2)$
Eigen decomposition of $\hat{\mathbf{R}}$	$O(N_H^3) + O(N_V^3)$
R-MUSIC-mdf for initialization of bearing	$O(\{N_H^2 M + M^2 N_H + N_H^2 (N_H - J_s)\} N_\theta)$
2-D MUSIC for range and depth initialization	$O(N_V^3 N_r N_z)$
SAGE algorithm to update bearing estimate	$J_s \times O(N_{itr}(N^2 + N)TN_\theta)^*$
SAGE algorithm to update range estimate	$J_s \times O(N_{itr}(N^2 + N)TN_r)^*$
SAGE algorithm to update depth estimate	$J_s \times O(N_{itr}(N^2 + N)TN_z)^*$

Table 6.2: Computational complexity of 3-D MUSIC

Computation of correlation matrix estimate $\hat{\mathbf{R}}$	$O(TN^2)$
Eigen decomposition of $\hat{\mathbf{R}}$	$O(N^3)$
3-D MUSIC for estimation of bearing, range and depth	$O(N^3 N_\theta N_r N_z)$

6.5 Computational complexity

We compare the complexities of the SAGE-USL algorithm and the 3-D MUSIC algorithm using the O-notation [183]. The SAGE-USL algorithm includes an initialization stage to obtain initial estimates of all unknown parameters followed by a SAGE based algorithm to update the initial estimates. To compute the complexity of SAGE-USL algorithm, we consider the following computations which are relatively more time consuming steps in the algorithm:

- Estimation of correlation matrix and its eigen decomposition
- R-MUSIC-mdf for initial estimate of source bearings
- 2-D MUSIC for initial estimate of source ranges and depths
- SAGE algorithm for updating bearing, range and depth estimates

An alternative to the proposed SAGE-USL algorithm is 3-D MUSIC. The computational complexities of the SAGE-USL and 3-D MUSIC algorithms for 3-D localization of J_s sources are presented in Table 6.1 and Table 6.2 respectively. ¹

The parameters used in table. 6.1 and table. 6.2 are defined as follows:

¹* The complexity considered in Table 6.1 is based on the original search space defined in the problem using a standard SAGE algorithm. However, for the adaptive search algorithm used in this chapter, the values of N_θ , N_r and N_z diminish rapidly after the first 3 iterations since the search space for each coordinate is determined adaptively in the neighborhood of the previous estimate. This procedure leads to a drastic reduction in computational complexity.

- N : Number of total sensors, $N = N_H + N_V$
- N_H : Number of sensors in HLA
- N_V : Number of sensors in VLA
- T : Number of snapshots (samples) in a frame of data
- M : Number of normal modes in shallow ocean
- N_θ : Number of positions searched in bearing space
- N_r : Number of positions searched in range space
- N_z : Number of positions searched in depth space
- N_{itr} : Number of iterations of SAGE algorithm in SAGE-USL

We observe in Section 6.7 that the number of iterations required for convergence of the algorithm is not large (< 10). A comparison of Tables 6.1 and 6.2 shows that the complexity of SAGE-USL algorithm is significantly less than that of 3-D MUSIC. Recalling that $N = N_H + N_V$, we observe that entries in rows 1 and 2 of Table. 6.2 are larger than the corresponding entries in Table. 6.1. The entry in row 3 of Table. 6.2 is much larger than the sum of the entries in rows 3-7 of Table. 6.1. To show this, we assume $N_\theta = 300$, $N_z = 100$, $N_r = 200$, $T = 100$, $N_V = 10$, $N_H = 20$, $N_{itr} = 6$. On substituting these values in Tables 6.1 and 6.2, the complexity of SAGE-USL and 3-D MUSIC will be $O(10^8)$ and $O(10^{11})$ respectively. We observe that SAGE-USL has 1000 times less complex than 3-D MUSIC for the search space considered in this example. The computational advantage of SAGE-USL becomes even more significant if a larger search volume is considered.

6.6 SAGE-USL algorithm with an array of acoustic vector sensors

The conventional hydrophone is a scalar sensor (SS) that provides only a partial characterization of the acoustic field. A complete characterization is provided by an acoustic vector sensor (AVS) which makes simultaneous measurements of the acoustic pressure and three orthogonal components of particle velocity at a point [112]. Utilization of the additional information provided by an AVS array would lead to a better localization performance. In this section, we

present an extended version of the SAGE-USL algorithm [15] for 3-D localization of an acoustic source using a small AVS array. We consider two AVS array configurations, namely, HLA and VLA, and compare their performance with that of an SS HLA and an SS hybrid array.

6.6.1 Data model for an array of acoustic vector sensors

Consider a linear AVS array of N sensors receiving a narrowband signal of center-frequency $(\omega/2\pi)$ from a far-field acoustic point source at $\mathbf{x} = [r, z, \theta]$ in a range independent shallow ocean with water column depth h , sound speed c and density ρ . We consider two alternative array geometries, namely, a horizontal linear array (HLA) at depth z_H , and a vertical linear array (VLA) with sensors at depths $z_1, \dots, z_N$. The inter-sensor spacing is d in both cases. Each sensor measures acoustic pressure and three orthogonal components of particle velocity. We discard the vertical component of particle velocity since its contribution to enhancement of the localization performance is insignificant. We also scale the particle velocity measurements by factor $\sqrt{2}\rho c$ to render the measurements dimensionally uniform. The scaling factor $\sqrt{2}$ is included to make the variances of acoustic pressure and particle velocity components of ambient noise to be equal to one another. The $3N$ -dimensional data vectors are therefore defined as

$$\mathbf{y}(t) = [y_1(t) \dots y_{3N}(t)]^T = \mathbf{p}(\mathbf{x}) s(t) + \mathbf{w}(t), \quad t = 1, \dots, T, \quad (6.68)$$

where $s(t)$ is the slowly varying complex amplitude of the transmitted signal at time t ,

$$\mathbf{p}(\mathbf{x}) = [p_1(t) \quad \sqrt{2}\rho c v_{x1} \quad \sqrt{2}\rho c v_{y1} \dots p_N(t) \quad \sqrt{2}\rho c v_{xN} \quad \sqrt{2}\rho c v_{yN}(t)]^T, \quad (6.69)$$

p_n, v_{xn}, v_{yn} denote the complex amplitudes of acoustic pressure and orthogonal horizontal components of particle velocity respectively at the n^{th} sensor for a unit-amplitude source. In this expression $\mathbf{w}(t) = [\mathbf{w}_1^T(t) \dots \mathbf{w}_N^T(t)]^T$ is the array noise vector, and $\mathbf{w}_n(t) = [w_{3n-2}(t) \ w_{3n-1}(t) \ w_{3n}(t)]^T$ is the noise vector at the n^{th} sensor. Elements of $\mathbf{p}(\mathbf{x})$, obtained from the normal mode theory of sound propagation are given by [96]

$$p_n = \sum_{m=1}^M p_{mn} \Omega_{mn}(\theta), \quad (6.70)$$

$$\sqrt{2}\rho cv_{xn} = \sqrt{2}\cos\theta \sum_{m=1}^M \frac{k_m}{k} p_{mn} \Omega_{mn}(\theta), \quad (6.71a)$$

$$\sqrt{2}\rho cv_{yn} = \sqrt{2}\sin\theta \sum_{m=1}^M \frac{k_m}{k} p_{mn} \Omega_{mn}(\theta), \quad (6.71b)$$

$$p_{mn} = \frac{2\sqrt{2\pi}}{h} \Psi_m(z_n) \Psi_m(z) \frac{e^{jk_m r - \alpha_m r}}{\sqrt{k_m r}}. \quad (6.72)$$

where $\Psi_m(z)$, k , k_m and α_m have been defined in Section 6.2. z_n is the depth of the n^{th} sensor, and $\Omega_{mn}(\theta)$ is a phase term defined as

$$\Omega_{mn}(\theta) = e^{j(n-1)k_m d \cos\theta} \text{ for HLA, } \Omega_{mn}(\theta) = 1 \text{ for VLA.} \quad (6.73)$$

The elements of the noise vectors $\{\mathbf{w}_n(t); t = 1, \dots, T\}$ for a given sensor n are i.i.d. We model them as complex circular random variables $w_{3n-k}(t) = w_{3n-k}^{(r)}(t) + jw_{3n-k}^{(i)}(t); k = 0, 1, 2$, with the following L -component Gaussian mixture PDF:

$$f_{W_{3n-k}}(\bar{w}_{3n-k}, \bar{\bar{w}}_{3n-k}) = \sum_{l=1}^L \frac{\lambda_l}{2\pi\beta_n^2\sigma_l^2} \exp\left(-\frac{|w_{3n-k}|^2}{2\beta_n^2\sigma_l^2}\right); \quad n = 1, \dots, N; k = 0, 1, 2. \quad (6.74)$$

In (6.74), $\bar{w}_{3n-k}$ and $\bar{\bar{w}}_{3n-k}$ denote the real and imaginary parts of w_{3n-k} respectively, and λ_l is the probability that W_n is chosen from the l^{th} term in the Gaussian mixture PDF with $\sum_{l=1}^L \lambda_l = 1$, the parameter σ_l^2 is proportional to the variance of the l^{th} component, and

$$\beta_n^2 = \sum_{m=1}^M \Psi_m^2(z_n), \quad (6.75)$$

is a variance scaling factor that depends on the depth of the n^{th} sensor [96]. It is also assumed that the vectors $\{w_n(t); n = 1, \dots, N\}$, for a given snapshot t , are mutually statistically independent. Hence the covariance matrix of the array noise vector is the following diagonal matrix

$$\mathbf{C} = \sigma^2 \mathbf{C}_0; \mathbf{C}_0 = \text{diag}(\beta_1^2, \beta_2^2, \dots, \beta_N^2) \otimes \mathbf{I}_3, \quad (6.76)$$

where $\sigma^2 = 2 \sum_{l=1}^L \lambda_l \sigma_l^2$, and $\mathbf{I}_3$ is the 3×3 identity matrix and $\otimes$ denotes the Kronecker product. In the case of the HLA, all the sensors are located at depth z_H , and hence (6.76) reduces to $\mathbf{C}_{HLA} = \sigma^2 \beta_H^2 \mathbf{I}_{3N}$, where β_H^2 is obtained on replacing z_n by z_H in (6.75).

The SAGE algorithm presented in the next section is based on the assumption that the elements of the array noise vector $\mathbf{w}(t)$ are i.i.d random variables. This condition is satisfied if the data vectors $\mathbf{y}(t)$ are pre-multiplied by the inverse of the noise covariance matrix to obtain the transformed data vectors

$$\tilde{\mathbf{y}}(t) = \tilde{\mathbf{p}}(\mathbf{x})s(t) + \tilde{\mathbf{w}}(t); t = 1, \dots, T, \quad (6.77)$$

where

$$\tilde{\mathbf{y}}(t) = \mathbf{C}_0^{-\frac{1}{2}} \mathbf{y}(t), \quad \tilde{\mathbf{p}}(\mathbf{x}) = \mathbf{C}_0^{-\frac{1}{2}} \mathbf{p}(x), \quad \tilde{\mathbf{w}}(t) = \mathbf{C}_0^{-\frac{1}{2}} \mathbf{w}(t). \quad (6.78)$$

Elements of $\tilde{\mathbf{w}}(t)$ have the Gaussian mixture PDF as

$$f_{\tilde{\mathbf{w}}_n}(\bar{w}_n, \bar{\bar{w}}_n) = \sum_{l=1}^L \frac{\lambda_l}{2\pi\sigma_l^2} \exp\left(-\frac{|w_n|^2}{2\sigma_l^2}\right); \quad n = 1, \dots, 3N. \quad (6.79)$$

6.6.2 Modified SAGE-USL Algorithm for acoustic vector sensor array

A block diagram of the proposed algorithm is shown in Fig. 6.3. The set of unknown parameters in this problem is as follows

$$\Phi = \{\theta, r, z; s(t), t = 1, \dots, T; \lambda_1, \dots, \lambda_L, \sigma_1^2, \dots, \sigma_L^2\}. \quad (6.80)$$

As explained before, the first step in this algorithm is initialization procedure that produces rough estimates of unknown parameters and it is followed by the SAGE based approach for updating the initial estimates iteratively. These two steps of modified SAGE-USL algorithm are explained in the following subsections.

6.6.3 Parameter initialization

Initialization of GMM model considered for modeling ambient noise PDF with AVS array is similar to the method used for scalar sensor array with minor changes. In the modified algorithm expressions given in (6.21) are used to estimate parameters $(\lambda_l, \sigma_l^2), l = 1, \dots, L$ of the GMM using $\tilde{y}_n(t)$ that is n^{th} element of the transformed data vector $\tilde{\mathbf{y}}(t)$ defined in (6.77). When noise is heavy-tailed, large noise impulses leads to errors in estimating correlation matrix. The transformed data vector $\tilde{\mathbf{y}}(t)$ is processed with the DA-ZMNL defined in (6.22) to attenuate the effect of strong impulses and the processed data vector $\mathbf{z}(t)$ is used to estimate data correlation

Figure 6.3: Block diagram of SAGE-USL algorithm for AVS array

matrix. The data correlation matrix is estimated as

$$\hat{\mathbf{R}}_{zz} = \frac{1}{T} \sum_{t=1}^T \mathbf{z}(t) \mathbf{z}^H(t). \quad (6.81)$$

The initialization method used in the SAGE-USL algorithm requires only a 1-D search for bearing estimation and a 2-D search for range and depth estimation. Bearing estimation by SIM does not require prior knowledge of source range and depth [96]. A necessary condition for using SIM is $3N \geq M + J_s$ and the sufficient condition is $3N \geq M(J_s + 1)$ where J_s is the number of sources. In SIM, the estimated data correlation matrix $\hat{\mathbf{R}}_{zz}$ is decomposed to obtain estimates of signal eigen vectors, $\hat{\mathbf{u}}_1, \dots, \hat{\mathbf{u}}_{J_s}$ and noise eigen vectors, $\hat{\mathbf{u}}_{J_s+1} \dots \hat{\mathbf{u}}_N$. The signal eigen vectors are used to form the following matrix:

$$\mathbf{U}(\theta) = [\mathbf{a}(k_1, \theta) \dots \mathbf{a}(k_M, \theta) \hat{\mathbf{u}}_1 \dots \hat{\mathbf{u}}_{J_s}], \quad (6.82)$$

where

$$\mathbf{a}(k_m, \theta) = [\mathbf{a}_1^T(k_m, \theta) \dots \mathbf{a}_N^T(k_m, \theta)]^T, \quad m = 1, \dots, M$$

$$\mathbf{a}_n^T(k_m, \theta) = \begin{cases} \mathbf{g}_m(\theta) e^{j(n-1)dk_m \cos \theta}, & \text{for HLA} \\ \mathbf{g}_m(\theta) \Psi_m(z_n), & \text{for VLA} \end{cases}$$

$$\mathbf{g}_m(\theta) = [1 \quad \frac{\sqrt{2}k_m \cos \theta}{k} \quad \frac{\sqrt{2}k_m \sin \theta}{k}]. \quad (6.83)$$

The QR-factorization of $\mathbf{U}(\theta)$ leads to $\mathbf{U}(\theta) = \mathbf{Q}(\theta)\mathbf{R}(\theta)$. Finally source bearings are estimated by obtaining the location of J_s largest peaks of the following spectrum

$$B_{SIM}(\theta) = [\min_{M+1 \leq j \leq M+J_s} |r_{jj}(\theta)|]^{-1}, \quad (6.84)$$

where $r_{jj}(\theta)$ is j^{th} diagonal element of $\mathbf{R}(\theta)$. We consider a single source and hence we have $J_s = 1$. Let the bearing estimate be denoted by $\hat{\theta}$. The range and depth of the source are estimated by finding the peak of the 2D-MUSIC spectrum defined as

$$B_{MUSIC}(r, z) = \left(\tilde{\mathbf{p}}(r, z, \hat{\theta})^H \mathbf{E}_N \mathbf{E}_N^H \tilde{\mathbf{p}}(r, z, \hat{\theta}) \right)^{-1}, \quad (6.85)$$

where $\mathbf{E}_N$ is the noise subspace matrix ($\mathbf{E}_N = [\hat{\mathbf{u}}_2 \dots \hat{\mathbf{u}}_N]$). As observed in (6.85) for initializing range and depth of the source its estimated bearing $\hat{\theta}$ is required. Therefore, for initialization of source coordinates in multiple source localization with AVS array, the pairing procedure explained in Section 6.3 is not required.

Finally, the initial estimate of the signal waveform is obtained from the preprocessed data $\mathbf{z}(t)$ using the following ML estimator

$$\hat{s}(t) = \left(\tilde{\mathbf{p}}(\hat{\mathbf{x}})^H \tilde{\mathbf{p}}(\hat{\mathbf{x}}) \right)^{-1} \tilde{\mathbf{p}}(\hat{\mathbf{x}})^H \mathbf{z}(t), \quad t = 1, \dots, T. \quad (6.86)$$

where $\hat{\mathbf{x}}$ is the initial estimate of the source position.

6.6.4 SAGE Algorithm to update the initial estimates

The initial estimates are updated iteratively using the SAGE approach explained in Section 6.3. The set of unknown parameters defined in (6.80) is divided into five subsets:

$$\begin{aligned} \Phi_1 &= \{\theta\}, \quad \Phi_2 = \{r\}, \quad \Phi_3 = \{z\}, \quad \Phi_4 = \{s(t), t = 1, \dots, T\}, \\ \Phi_5 &= \{\lambda_1, \dots, \lambda_L, \sigma_1^2, \dots, \sigma_L^2\}. \end{aligned} \quad (6.87)$$

so that $\Phi = \Phi_1 \cup \Phi_2 \cup \Phi_3 \cup \Phi_4 \cup \Phi_5$. The subsets are updated using the sequential procedure, wherein a subset is updated while other subsets are kept fixed. Expressions for the update

functions of the parameters are given below. The update function for source bearing is

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}} \sum_{t=1}^T \frac{\left| \tilde{\mathbf{p}}(\hat{\mathbf{x}}(\theta))^H \mathbf{H}_t'(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{y}}(t) \right|^2}{\tilde{\mathbf{p}}(\hat{\mathbf{x}}(\theta))^H \mathbf{H}_t'(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{p}}(\hat{\mathbf{x}}(\theta))}, \quad (6.88)$$

where $\hat{\mathbf{x}}(\theta) = (r', z', \theta)$, and r', z' are current estimates of source range and depth respectively. Now define $\hat{\mathbf{x}}(r) = (r, z', \theta')$, and $\hat{\mathbf{x}}(z) = (r', z, \theta')$. Range/depth update is obtained on replacing $\hat{\mathbf{x}}(\theta)$ by $\hat{\mathbf{x}}(r)/\hat{\mathbf{x}}(z)$ in (6.88) and maximizing with respect to r/z . Thus each iteration for updating the source coordinates requires three 1-D searches instead of a 3-D search.

The update function for the signal waveforms is similar to the expression used for scalar sensors in (6.57) but here the AVS data vector and associated steering vector are used.

$$\hat{\mathbf{s}}(t) = (\tilde{\mathbf{p}}(\mathbf{x}')^H \mathbf{H}_t'(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{p}}(\mathbf{x}'))^{-1} \tilde{\mathbf{p}}(\mathbf{x}')^H \mathbf{H}_t'(\tilde{\mathbf{y}}(t)) \tilde{\mathbf{y}}(t), \quad t = 1, \dots, T. \quad (6.89)$$

Similarly the functions derived for updating the noise parameters in Section 6.3 are used to update noise parameters with AVS data vector

$$\lambda_l = \frac{1}{\lambda T} \sum_{n=1}^{3N} \sum_{t=1}^T g'_{l,n,t} \tilde{y}_n(t), \quad l = 1, \dots, L. \quad (6.90)$$

$$\sigma_l^2 = \frac{1}{6NT\lambda_l} \sum_{t=1}^T [\tilde{\mathbf{y}}(t) - \tilde{\mathbf{p}}(\mathbf{x}') \hat{\mathbf{s}}(t)]^H \mathbf{G}'_{l,t}(\tilde{\mathbf{y}}(t)) [\tilde{\mathbf{y}}(t) - \tilde{\mathbf{p}}(\mathbf{x}') \hat{\mathbf{s}}(t)]. \quad (6.91)$$

The expression for $\mathbf{H}_t'$ used in (6.88-6.91) is

$$\mathbf{H}_t'(\tilde{\mathbf{y}}(t)) = \sum_{l=1}^L \frac{1}{\sigma_l^2} \mathbf{G}'_{l,t}(\tilde{\mathbf{y}}(t)), \quad t = 1, \dots, T, \quad (6.92)$$

$$\mathbf{G}'_{l,t}(\tilde{\mathbf{y}}(t)) = \operatorname{diag} \{ g'_{l,1,t}(\tilde{y}_1(t)) \dots, g'_{l,N,t}(\tilde{y}_{3N}(t)) \}, \quad l = 1, \dots, L, \quad t = 1, \dots, T; \quad (6.93)$$

$$\begin{aligned} \hat{g}_{l,n,t} &= \frac{\frac{\lambda_l}{2\pi\sigma_l^2} \exp\left(\frac{-1}{2\sigma_l^2} |\tilde{y}_n(t) - \tilde{p}_n(\hat{\mathbf{x}}) \hat{s}(t)|^2\right)}{\sum_{q=1}^L \frac{\lambda_q}{2\pi\sigma_q^2} \exp\left(\frac{-1}{2\sigma_q^2} |\tilde{y}_n(t) - \tilde{p}_n(\hat{\mathbf{x}}) \hat{s}(t)|^2\right)}, \quad l = 1, \dots, L; \\ &t = 1, \dots, T; \quad n = 1, \dots, 3N. \end{aligned} \quad (6.94)$$

One iteration of the algorithm includes an update of all the source and noise parameter estimates. In each iteration after updating each estimate, $\mathbf{H}_t'$ is computed based on the current estimates before updating the next parameter.

6.7 Simulation results

6.7.1 Source localization with hybrid array of scalar sensors

In this section, simulation results for evaluation of the proposed algorithm in Section 6.2 are presented and discussed. We consider a T-shaped hybrid array composed of an HLA with $N_H = 20$ and a VLA with $N_V = 10$ sensors. Inter-sensor distance is $d = 7.5\text{m}$ in both HLA and VLA. Ocean depth is $h = 100\text{m}$, sound speed in water is 1500m/s , sound speed in the bottom is 1700 m/s , bottom attenuation is 0.5dB/wavelength , and density ratio is 1.5 . The HLA is placed at a depth of 50m in the ocean and the top-most sensor of the VLAs is positioned at a depth $z_1 = 5\text{m}$. Non-Gaussian noise is generated using a GMM with two components. All the results are based on 150-200 Monte Carlo simulations.

The PDF of the 2-component GM noise can be written as

$$f_{cgm}(w) = \frac{(1 - \lambda_2)}{2\pi\sigma_1^2} \exp\left[-\frac{\bar{w}^2 + \overline{\bar{w}}^2}{2\sigma_1^2}\right] + \frac{\lambda_2}{2\pi\sigma_2^2} \exp\left[-\frac{\bar{w}^2 + \overline{\bar{w}}^2}{2\sigma_2^2}\right]. \quad (6.95)$$

Equation (6.95) implies that the noise is $CN(0, 2\sigma_1^2)$ with probability $1 - \lambda_2$ and $CN(0, 2\sigma_2^2)$ with probability λ_2 . The noise variance is

$$\sigma^2 = 2[(1 - \lambda_2)\sigma_1^2 + \lambda_2\sigma_2^2]. \quad (6.96)$$

The degree of non-Gaussianity of a PDF is typically measured by its kurtosis. The kurtosis for a zero-mean complex circular random variable is defined as [191]

$$\kappa = \frac{\text{E}[|w|^4]}{\{\text{E}[|w|^2]\}^2} - 2. \quad (6.97)$$

It can be readily shown that the kurtosis for the PDF defined in (6.95) is given by

$$\kappa = \frac{3}{2} \left[\frac{\lambda_2((\sigma_2^4/\sigma_1^4) - 1) + 1}{(\lambda_2((\sigma_2^2/\sigma_1^2) - 1) + 1)^2} - 1 \right], \quad (6.98)$$

where κ is non-negative for all values of λ_2 and σ_2^2/σ_1^2 . We can assume without any loss of generality that $\sigma_2^2/\sigma_1^2 \geq 1$. For $\sigma_2^2/\sigma_1^2 = 1$, noise is Gaussian and $\kappa = 0$. For a given λ_2 and for $\sigma_2^2/\sigma_1^2 \geq 1$, κ increases monotonically as σ_2^2/σ_1^2 is increased. For a given $\sigma_2^2/\sigma_1^2 > 1$, κ attains the maximum value of $\frac{3((\sigma_2^2/\sigma_1^2)-1)^2}{8\sigma_2^2/\sigma_1^2}$ at $\lambda_2 = \frac{1}{((\sigma_2^2/\sigma_1^2)+1)}$. Thus kurtosis becomes arbitrarily

large by increasing σ_2^2/σ_1^2 and choosing $\lambda_2 \simeq 1/(\sigma_2^2/\sigma_1^2)$. Figure 6.4 (a) shows plots of κ versus σ_2^2/σ_1^2 for two different values of λ_2 , and Fig. 6.4 (b) shows plots of κ versus λ_2 for two different values of σ_2^2/σ_1^2 . We are interested in considering heavy-tailed noise with a large kurtosis, which corresponds to large values of σ_2^2/σ_1^2 , typically $\sigma_2^2/\sigma_1^2 \geq 100$ and small values of λ_2 , typically $\lambda_2 \leq 0.2$. Results of simulations to assess the performance of the SAGE-USL algorithm under different conditions are presented below.

Figure 6.4: Variation of kurtosis of 2-component GM noise with parameters σ_2^2/σ_1^2 and λ_2 and fixed noise power. (a) fixed λ_2 and varying σ_2^2/σ_1^2 , (b) fixed σ_2^2/σ_1^2 and varying λ_2 .

In the first simulation, we evaluate the performance of the proposed SAGE-USL algorithm by comparing the root mean square error (RMSE) of range, depth and bearing estimates with those of 3-D MUSIC and CRBs. 3-D MUSIC algorithm is applied to the pre-whitened data after applying DA-ZMNL preprocessor to ensure a fair comparison with the SAGE-USL algorithm. In

this simulation, we have considered two point sources of the same frequency $f = 100\text{Hz}$ located at $r_1 = 4000$, $z_1 = 40$, $\theta_1 = 30^\circ$ and $r_2 = 5000\text{m}$, $z_2 = 60\text{m}$, $\theta_2 = 40^\circ$. Bearings are measured with respect to the endfire direction of the HLA. Parameters of the GMM noise are $\sigma_1^2 = 1$, $\sigma_2^2 = 1000$ and $\lambda_2 = 0.1$. The results are shown in Fig. 6.5. RMSEs of the initial range-depth estimate by 2D-MUSIC, (6.25) using the VLA, and the initial bearing estimate by R-MUSIC-mdf, (6.26-6.27) using the HLA, are also plotted in this figure to demonstrate the improvement in performance due to the iterative updates performed by the SAGE-USL algorithm. It is seen that the SAGE-USL algorithm provides a significant reduction in RMSE not only over 3-D MUSIC but also over the initial estimates provided by 2-D MUSIC and R-MUSIC-mdf. CRBs of source coordinates are also presented in Fig. 6.5 for comparison. It is observed that the range, depth and bearing RMSEs are close to the corresponding CRBs. Poor performance of 3-D MUSIC in this Figure may be related to this fact that the noise subspace matrix obtained from eigen decomposition of estimated covariance matrix is not accurate enough to lead to good estimate of source locations. This problem is more serious in multiple source and very low SNR that is the case in the experiments conducted for Fig. 6.5.

In the second simulation (Figs. 6.6 - 6.7), we study the effect of variation of the GM noise parameters on the performance of the SAGE-USL algorithm. As indicated by Fig. 6.4, non-Gaussianity is varied either by varying the frequency λ_2 of the large-variance component or by varying the ratio of variances σ_2^2/σ_1^2 . Figure 6.6 shows the effect of varying σ_2^2/σ_1^2 for two fixed values of λ_2 , namely, $\lambda_2 = 0.05$ and $\lambda_2 = 0.2$. Values of noise variance $\sigma^2 = 250$, $SNR = -5\text{dB}$, and the source location are also held fixed. The three panels in this figure show plots of RMSEs of range, depth and bearing estimates versus σ_2^2/σ_1^2 . Plots of the corresponding CRBs are also shown in this figure. It is seen that the CRBs decrease with increasing σ_2^2/σ_1^2 , and the same trend is followed by the RMSEs. This trend can be explained by observing from Fig. 6.4 (a) that kurtosis (which is a measure of non-Gaussianity) increases as σ_2^2/σ_1^2 is increased, and the increase in non-Gaussianity leads to a reduction in CRB.

Figure 6.7 shows the effect of varying λ_2 for two fixed values of σ_2^2/σ_1^2 , namely, $\sigma_2^2/\sigma_1^2 = 100$ and $\sigma_2^2/\sigma_1^2 = 1000$. It is seen that there is a sharp initial decrease in the values of RMSEs and CRBs as λ_2 is increased. This initial trend is predicted from Fig. 6.4 (b) which shows an initial increase in kurtosis at small values of λ_2 . CRBs continue to decrease when λ_2 is increased further even though Fig. 6.4 (b) shows a reduction of kurtosis at larger values of λ_2 . Therefore, we infer that kurtosis is not always a proper measure of non-Gaussianity for predicting the behavior of CRB or the estimator performance. It is also observed from Figs. 6.6 and 6.7 that

(a)

(b)

(c)

Figure 6.5: Comparison of RMSEs of SAGE-USL algorithm with those of 3-D MUSIC, initial estimates and CRB for different SNRs. (a) Range RMSE (b) Depth RMSE (c) Bearing RMSE.

Figure 6.6: Comparison of RMSEs of SAGE-USL algorithm with CRB for different values of σ_2^2/σ_1^2 and a fixed $\lambda_2 = 0.05, 0.2$, (a) Range RMSE (b) Depth RMSE (c) Bearing RMSE.

the values of RMSEs are very close to the values of the corresponding CRBs over almost the entire range of the noise parameter values.

in the final simulation, we study the convergence rate of SAGE-USL algorithm by computing the average number of iterations required for convergence. Convergence is said to have occurred if the difference between the updated estimates and the previous estimates is below a prescribed threshold (1m for range estimates, 0.1m for depth estimates and 0.005° for bearing estimates) for two successive iterations for all the source coordinates. The average number of iterations required for convergence under different conditions is shown in Fig. 6.8. Figure 6.8 (a) shows the variation of average number of iterations i_{av} with SNR for localization of two sources at two different values of σ_2^2 . Values of all other parameters are the same as in the first simulation. It is observed that $i_{av} \simeq 6$, and it increases a little at very low SNR ($SNR \leq -15\text{dB}$). Figures 6.8 (b) and 6.8 (c) show the variation of i_{av} with the noise parameters σ_2^2/σ_1^2 and λ_2 respectively, for the problem of single-source localization. All the experimental conditions in these plots are identical to those in the second simulation. It is seen that $i_{av} \simeq 2$ for small values of λ_2 . The value of i_{av} increases steadily for larger values of λ_2 , probably because of the steadily decreasing effectiveness of the ZMNL preprocessor.

6.7.2 Source localization using AVS arrays and it comparison with scalar sensor arrays

In this section, the results of simulations with AVS arrays are presented. In these set of simulations, the ocean depth is $h = 100\text{m}$ and the sound speed in water and sediment are $c = 1500\text{m/s}$ and $c_b = 1700\text{m/s}$ respectively. The non-Gaussian noise is generated using a GMM with two components with $\sigma_1^2 = 1$, $\sigma_2^2 = 1000$, $\lambda_1 = 0.9$, $\lambda_2 = 0.1$. We compare the performance of an AVS HLA and an AVS VLA with one another and also with that of an SS HLA and an SS hybrid array consisting of an HLA and a VLA. The HLAs are placed at a depth $z_a = 50\text{m}$ and the topmost sensor of the VLAs is positioned at depth $z_1 = 5\text{m}$. The HLA and the VLA include $N = 13$ sensors each, and the SS hybrid array has 13 sensors in each arm. The inter-sensor distance is $d = 7.5\text{m}$ (half wavelength) for all the arrays. A source of center frequency $f = 100\text{Hz}$ is located at range $r = 5000\text{m}$, depth $z = 30\text{m}$, and bearing $\theta = 30^\circ$ with respect to the endfire direction of the HLA. The source waveform $\{s(t); t = 1, \dots, T\}$ is a realization of an uncorrelated zero-mean Gaussian random process with variance σ_s^2 . All the results have been obtained for $T = 150$ from 100 Monte Carlo simulations.

Figure 6.7: Comparison of RMSEs of SAGE-USL algorithm with CRB for different values of λ_2 and a fixed noise power and $\sigma_2^2/\sigma_1^2 = 100, 1000$, (a) Range RMSE (b) Depth RMSE (c) Bearing RMSE.

Figure 6.8: Average number of iterations required for convergence of the SAGE-USL algorithm: (a) two sources, varying Signal to Noise Ratio (b) single source, varying σ_2^2/σ_1^2 , fixed λ_2 , and (c) single source, varying λ_2 , fixed σ_2^2/σ_1^2 .

Table 6.3: Values of η for different array configurations.

Array Configuration	Value of η
SS HLA	-127.4812
SS Hybrid	-129.3
AVS HLA	-133.2283
AVS VLA	-134.2257

We present the performance of the SAGE-USL algorithm for different array configurations in terms of root mean square error (RMSE) of the source coordinate estimates. We also compare the RMSE of estimates obtained from each array with the corresponding CRBs. We define SNR for each array as the ratio of total signal power to total noise power at the array. Thus, from the data model described in Section 6.6.1, we have

$$(SNR)_{AVS} = 10 \log_{10} \frac{\sigma_s^2 \sum_{n=1}^N [|p_n|^2 + 2\rho^2 c^2 (|v_{xn}^2| + |v_{yn}^2|)]}{3\sigma^2 \sum_{n=1}^N \beta_n^2}. \quad (6.99)$$

$$(SNR)_{SS} = 10 \log_{10} \frac{\sigma_s^2 \sum_{n=1}^N |p_n|^2}{\sigma^2 \sum_{n=1}^N \beta_n^2}. \quad (6.100)$$

We can rewrite (6.99) and (6.100) as

$$SNR = \sigma_{SN} + \eta, \quad (6.101)$$

where $\sigma_{SN} = 10 \log_{10}(\sigma_s^2/\sigma^2)$ is a measure of signal-to-noise ratio that is independent of the array configuration, and η is an array-dependent quantity whose value for different array configurations is tabulated in Table 6.3. It is clear that the performance of different array configurations should be compared for a fixed value of σ_{SN} rather than a fixed value of SNR.

Figure 6.9 shows the variation of root mean square estimation error (RMSE) with σ_{SN} for different array configurations. RMSEs for estimation of bearing, range and depth are shown in three separate panels. Plots of CRB versus σ_{SN} are also shown in the same figure. For each array, we have considered a range of σ_{SN} that corresponds to a variation of SNR from -20dB to 10dB. It is seen that the AVS VLA has the best range-depth estimation performance. Bearing estimation performance of AVS VLA is better than that of AVS HLA and SS HLA for all σ_{SN} , and better than that of SS hybrid array also except at low values of σ_{SN} . But it must be remembered that the SS hybrid array has twice as many sensors as the AVS VLA. It is seen that, at -20dB SNR, the 13-sensor AVS VLA provides estimates with RMSEs of 0.05 radian for bearing, 3 m for range and 0.6 m for depth. We therefore conclude that the AVS

VLA is the best among the alternative array configurations considered in this chapter. The worst performance is provided by the SS HLA.

In the conventional subspace-based method of 3-D localization, bearing estimation is done by SIM (6.84) and range-depth estimation is done by 2D-MUSIC (6.85). The RMSEs for this conventional method are also shown in Fig. 6.9 for both AVS HLA and AVS VLA. It is seen that the SAGE-USL algorithm provides a significant reduction in RMSE over the subspace-based method. A comparison of RMSEs for different arrays with the corresponding CRBs is instructive. It is seen from Fig. 6.9 that the CRB for bearing estimation by the AVS HLA is much lower than that for other arrays. But the corresponding RMSE is larger than that for the AVS VLA and the SS hybrid array. The much smaller difference between bearing CRB and bearing RMSE for the AVS VLA and the SS hybrid array is due to the fact that these arrays provide much better estimates of range and depth at every stage. Better range-depth estimates (illustrated in Fig. 6.9) lead to better bearing estimates also since the two are inter-related. For AVS HLA initial estimate of the source coordinates is not very accurate. It might be because SNR is low and noise is highly non-Gaussian. Therefore the noise subspace matrix required for MUSIC algorithm can not be estimated accurately. This leads to inaccurate estimation of source coordinates (even bearing). In the update procedure, the estimation of range and depth using HLA is also poor. Therefore, the estimates can not improved significantly. However, in the case of AVS VLA, the good estimate of range and depth of the source (especially during update procedure using the presented SAGE approach) improves all the estimates significantly.

In another simulation, we have evaluated the convergence rate of SAGE-USL algorithm with different array configurations. Figure 6.10 shows plots of the average number of iterations required for convergence versus σ_{SN} for different array configurations obtained from 100 Monte Carlo simulations. Convergence is said to have occurred if the difference between the updated estimates and the previous estimates is below a prescribed threshold for two successive iterations for all the coordinates. It is observed that convergence for AVS VLA is much faster than that for other array configurations. In general, the average number of iterations required for convergence of the algorithm is small (< 10) for all the arrays. This result demonstrates the low complexity of the SAGE-USL algorithm compared to 3-D MUSIC and the standard ML algorithm.

Figure 6.9: Performance of SAGE-USL algorithm in bearing, range and depth estimation for different arrays: (a) Bearing RMSE/CRB (b) Range RMSE/CRB and (c) Depth RMSE/CRB for a source at ($r = 5000m, z = 30m, \theta = 30^\circ$).

Figure 6.10: Average number of iterations required for convergence of the SAGE-USL algorithm with different array configurations

6.8 Summary

In this chapter, we have presented a computationally simple algorithm called SAGE-USL algorithm for 3-D localization of multiple acoustic sources in a shallow ocean with heavy-tailed non-Gaussian noise, using a hybrid array of scalar sensors and its modified version for an AVS array. The hybrid array composed of an HLA and a VLA is used to estimate source coordinates, signal waveform and noise parameters. In the proposed algorithm, the unknown parameter set is divided into five subsets. An initial estimate of all unknown parameters using a conventional MUSIC-based approach is followed by sequential iterative updates of all the estimates. The SAGE-USL algorithm is ML based and hence its level of performance is expected to be close to that of the conventional ML algorithm. However, the SAGE-USL algorithm has a significantly reduced computational complexity compared to that of 3-D MUSIC and conventional ML algorithms. The MUSIC algorithm requires a 3-D search, while the conventional ML algorithm requires a $3J_s$ -D search for 3-D localization of J_s sources. The SAGE-USL algorithm requires one 2-D search and one 1-D search for initialization, and $3nJ_s$ 1-D searches for n update iterations. The number of iterations required for convergence is quite small (< 10).

The results of simulations with the hybrid array of scalar sensors show that (1) SAGE-USL provides a significantly better performance than 3-D MUSIC for multiple source localization, (2) the performance of SAGE-USL improves when noise PDF become more heavy-tailed (a similar trend has been observed for the corresponding CRBs), and (3) the SAGE-USL algorithm

converges very fast and the RMSEs of the final estimates are very close to the corresponding CRBs.

In the modified version of SAGE-USL algorithm, two AVS array configurations have been considered, namely, HLA and VLA, and the performance of the modified SAGE-USL algorithm with these arrays has been compared with that of (1) SAGE-USL algorithm with SS HLA or SS hybrid array, and (2) the existing method involving a combination of the SIM and MUSIC algorithms with AVS HLA or AVS VLA. We have shown that (1) the proposed SAGE-USL algorithm provides a significantly better performance than the combination of the SIM and MUSIC algorithms; (2) for the SAGE-USL algorithm, the AVS VLA outperforms all the other arrays considered in this chapter; and (3) for the SAGE-USL algorithm with AVS VLA, the RMSEs of the estimates are very close to the corresponding CRBs.

A study of the convergence rate of the SAGE-USL algorithm with different array configurations shows that the convergence rate for AVS VLA is significantly higher than that for other array configurations. Good position estimates (with RMSEs of range, depth and bearing close to the corresponding CRBs) are achieved by a 13-sensor AVS VLA at low SNR with one 2-D search and a small number of 1-D searches. SAGE-USL algorithm has been presented for 3-D localization of ‘conditional’ sources. However, it can be applied to ‘unconditional’ signals (random sources) by a few modifications. The derived CRBs also should be modified for random signals accordingly.

Chapter 7

Conclusions and future works

This chapter summarizes contributions and conclusions of our work. Further, we describe possible future directions for this work briefly.

7.1 Contributions

Background information on the topics related to source detection and localization in shallow ocean was explained in Chapter 1. This chapter provided information about sonar signal processing, shallow water channels, ambient noise in the ocean, our motivations and main contributions in this work. Chapter 2 reviewed the existing literature on topics associated with this work. This review incorporated two main topics: (1) detection in non-Gaussian noise; (2) localization in shallow ocean with non-Gaussian noise using SS arrays and AVS arrays. The existing work on CRBs for source localization was also reviewed. Chapter 3 introduced different methods of modeling acoustic field in a shallow ocean and provided a comparison between these methods. The suitable method for our application was explained and the related mathematical expressions were presented. Further in this chapter, ambient noise modeling in a shallow ocean was discussed and a suitable model was introduced. In general, the first three chapters provided background knowledge for other chapters. The details of our contributions presented in Chapter 4 to Chapter 6. Our contributions can be summarized as follows.

In Chapter 4, we presented a robust and easily implementable detector based on nonlinear wavelet denoising (NWD) for detection of signals in non-Gaussian noise. A class of nonlinear median-based wavelet transforms was chosen for denoising, since the conventional denoising

techniques based on linear wavelet transforms are suitable only for denoising signals in Gaussian noise. Three denoising filters (namely, LMIPT, BMPT and MIPT) based on median interpolating pyramid transform were considered. A theoretical analysis of the proposed detectors was performed by deriving the relation between the PDF of the filter output and the noise PDF under the assumption of i.i.d noise samples. Further, we analyzed the computational complexity of the NWD detectors and compared it with that of the optimal detector. The performance of the proposed NWD detectors was investigated using the generalized Gaussian PDF as a model for non-Gaussian noise. Receiver operating characteristics of the detectors were obtained theoretically and through different simulations, and the theoretical and simulation results were found to be agreed with one another. An analysis of sensitivity of the NWD detectors to the error in noise modeling was also carried out. The results showed that NWD detectors are significantly more robust than optimal detector.

In Chapter 5, we derived a closed form expression for the Cramèr-Rao Bound for 3-D localization of multiple acoustic sources in the presence of any symmetric noise distribution. The expression derived in this work is easy to compute and usable for different array configuration of scalar sensors and acoustic vector sensors. Furthermore, using this expression, we computed the CRBs for range, depth and bearing estimation of sources in a shallow ocean with non-Gaussian noise using different SS and AVS arrays. Using the derived CRBs, the effect of variation of non-Gaussianity of noise distribution on the performance of source localization was studied as well. We also examined the dependence of the localization errors on the source coordinates for different array configurations using the derived CRBs.

In Chapter 6, we presented a computationally simple algorithm called SAGE-USL algorithm for 3-D localization of multiple acoustic sources in a shallow ocean with heavy-tailed non-Gaussian noise, using a hybrid array of scalar sensors and its modified version for an AVS array. The hybrid array of scalar sensors, composed of an HLA and a VLA, was used to estimate source coordinates, signal waveforms and noise parameters. Computational complexity of the proposed algorithm was analyzed and compared with that of 3-D MUSIC. Since the proposed algorithm was an iterative algorithm, the convergence rate of the proposed algorithm was investigated through several simulations. A comparison between the localization performance of the SAGE-USL algorithm with different array configurations of scalar sensors and acoustic vector sensors were also performed.

7.2 Conclusions

The results of our work on detection in non-Gaussian noise showed that the LMIPT detector offers the following advantages for signal detection in strongly non-Gaussian noise: (1) significantly better performance than the matched filter, (2) greater robustness than the optimal detector, (3) moderate computational complexity. Hence the LMIPT detector can be considered as a suitable suboptimal detector for signals in non-Gaussian noise. Although the study in this work is confined to the problem of detection of a known signal, the NWD approach can also be employed for the detection of an unknown signal in non-Gaussian noise.

The results of different simulations using the derived CRBs in Chapter 5 of this thesis showed that variations in range, depth, or bearing of a source have very little effect on 3-D localization performance of a hybrid array. But this is not the case for a horizontal array. Besides, the results also showed that a horizontal array has poor range-depth estimation performance for sources near the broadside direction and poor bearing estimation performance for sources near the endfire direction. Hence, a hybrid array is a better choice compared to a horizontal array for 3-D localization of sources. The results also showed that the CRB expression derived for source localization can be used for different array configurations and different symmetric noise distributions.

The analysis of the presented localization (SAGE-USL) algorithm in Chapter 6 demonstrated fast convergence-rate of the proposed algorithm with a required number of iterations less than ten even for multiple source case. The computational complexity analysis of SAGE-USL algorithm showed that the presented algorithm has a computational complexity significantly lower than that of 3-D MUSIC and conventional ML algorithm. Different Monte-Carlo simulations conducted for performance evaluation of SAGE-USL algorithm demonstrated significant improvement in the source localization performance of SAGE-USL algorithm compared to that of 3-D MUSIC.

A comparison between performance of SAGE-USL algorithm with different array configurations of scalar sensors and acoustic vector sensors showed the significantly better performance of a vertical AVS array and a hybrid SS array compared to other array configurations considered in this work. According to different simulations conducted, we conclude that for problem of single source localization, SAGE-USL algorithm with a vertical AVS array is a proper algorithm that converges very fast and localizes the source accurately. For multiple source localization problem, the initialization method used for AVS array requires J_s 2-D searches over the range-depth

space for estimating the range and depth of J_s sources (associated with J_s bearing estimates obtained from a 1-D search over the bearing space). However, using SAGE-USL algorithm with a hybrid SS array, only a 2-D search over the range-depth space and a few 1-D searches over each coordinate space are needed to estimate all the coordinates of J_s sources. Therefore, the SAGE-USL algorithm with a hybrid array of scalar sensors is a better choice for multiple source localization as long as lower computational complexity is necessary.

7.3 Future work

In this section, we suggest some future directions for further research in the field of source detection and localization in a shallow water channel with non-Gaussian noise.

7.3.1 Evaluating detection performance of the NWD detectors in other non-Gaussian noise distributions

Although, we have considered Generalized Gaussian noise in our simulations with the proposed NWD detectors, the expressions that we have derived for PDF of the denoised data and theoretical analysis of the detectors' performance are general expressions and can be used for other noise distributions. This opens up a research direction for future in which other non-Gaussian noise distributions can be considered and the performance of the proposed detectors can be evaluated both experimentally and theoretically in other non-Gaussian noise PDFs. Any required modifications in the proposed detectors may also be applied accordingly.

7.3.2 Detection of unknown signal in non-Gaussian noise

We have considered the problem of known signal detection in non-Gaussian noise in this dissertation. Since the performance of matched filter degrades in presence of non-Gaussian noise, we have proposed NWD detectors whose performance in non-Gaussian noise is significantly better than that of matched filter. One direction for further research would be investigating detection of an unknown signal in non-Gaussian noise. Although problem of detection of unknown signal in Gaussian noise has been discussed in the literature [43], it is worth to assess detection of unknown signals in non-Gaussian noise as well.

7.3.3 Decreasing the computational complexity of SAGE-USL algorithm

In the current version of SAGE-USL algorithm that we have proposed for 3-D source localization in shallow water with non-Gaussian noise, a 2-D search over range-depth space is required to initialize range and depth of sources. Replacing the 2-D MUSIC algorithm used in the current version of the algorithm for range-depth initialization by other estimation methods that have lower complexity (e.g. require 1-D searches or no search over the spaces) can decrease the computational complexity of SAGE-USL algorithm. This modification and complexity reduction would be more significant for the SAGE-USL algorithm with AVS array in multiple source localization problem in which J_s 2-D searches are required for initialization of range and depth of J_s sources.

7.3.4 3-D source localization and tracking in shallow ocean considering the uncertainty of medium parameters

MFP based algorithms that are obtained based on known parameters of the shallow water channel are sensitive to errors in channel parameters. To remove sensitivity of the algorithm to channel parameters, one solution is using Extended Kalman filter, Unscented Kalman filter or particle filtering (commonly known as sequential Bayesian filtering) that facilitate signal processing in non-stationary dynamic systems with nonlinear equations and non-Gaussian distributions [110, 192, 193].

In this work, we have not examined the sensitivity of SAGE-USL algorithm to uncertainty in medium parameters. Hence, it would be interesting to assess the sensitivity of SAGE-USL algorithm to uncertainty in the underwater channel parameters. Further, different modifications or developments can be applied to make the algorithm more robust to the channel parameters. In general, investigating the use of sequential Bayesian filtering for more robust 3-D source localization and tracking in shallow oceans would be an interesting yet challenging research direction for future [129].

Appendix A

CDF of $m_{j,k}$ in MIPT/ LMIPT

By definition, we have

$$m_{j,k} = \text{med}(B_{jk}); \quad j = 0, 1, \dots, J-1; \quad k = 0, 1, \dots, 3^j - 1, \quad (\text{A.1})$$

where B_{jk} is the k^{th} data block when the data samples $\{x[n]\}$ are partitioned into 3^j blocks

$$B_{jk} = \{x[n]; \quad n = k.3^{J-j}, k.3^{J-j} + 1 \dots, (k+1).3^{J-j} - 1\}. \quad (\text{A.2})$$

The number of elements in B_{jk} is given by

$$2a_j + 1 = 3^{J-j}. \quad (\text{A.3})$$

Since the noise samples $w[n]$ are i.i.d, it follows that the elements of B_{jk} are i.i.d under hypothesis H_0 , and they are statistically independent but not identically distributed under hypothesis H_1 .

Let the cumulative distribution function (CDF) of $w[n]$ and CDF of $m_{j,k}$ under hypothesis H_i ; $i = 0, 1$ be denoted respectively by $F_w(x)$ and $F_{\text{med}}(x; j, k; H_i)$. We have

$$P(m_{j,k} \leq x) = \sum_{i=a_j+1}^{2a_j+1} P(\text{any } i \text{ elements of } B_{jk} \text{ are } \leq x \text{ and the rest are } > x). \quad (\text{A.4})$$

Hence, the CDF of $m_{j,k}$ under hypothesis H_1 is given by

$$\begin{aligned}
 F_{med}(x; j, k; H_1) &= P(m_{j,k} \leq x; H_1) \\
 &= \sum_{i=a_j+1}^{2a_j+1} \sum_{p=1}^{\binom{2a_j+1}{i}} \prod_{n \in I_{jk,ip}^B} F_w(x - s[n]) \prod_{m \in I_{jk,i'p}^B} (1 - F_w(x - s[m]))
 \end{aligned} \tag{A.5}$$

where

$$\begin{aligned}
 B_{jk,ip} &= p^{th} \text{ subset of } B_{jk}, \text{ each subset containing } i \text{ elements;} \\
 i &= 1, \dots, 2a_j + 1; \quad p = 1, \dots, \binom{2a_j + 1}{i},
 \end{aligned} \tag{A.6a}$$

$$\begin{aligned}
 B_{jk,i'p} &= B_{jk} - B_{jk,ip} = \text{subset containing } i' = 2a_j + 1 - i \\
 &\text{elements of } B_{jk} \text{ not belonging to } B_{jk,ip}.
 \end{aligned} \tag{A.6b}$$

and $I_{jk,ip}^B$ and $I_{jk,i'p}^B$ are sets of indices corresponding to the data samples in the set $B_{jk,ip}$ and $B_{jk,i'p}$ respectively.

The expression for $F_{med}(x; j, k; H_0)$ can be obtained from (A.5) by choosing $s[n] = 0$ for all n . Thus we get

$$F_{med}(x; j, k; H_0) = \sum_{i=a_j+1}^{2a_j+1} \frac{(2a_j+1)!}{i! (2a_j+1-i)!} F_w^i(x) (1 - F_w(x))^{2a_j+1-i}. \tag{A.7}$$

The CDFs are independent of the index k under hypothesis H_0 .

Appendix B

Joint CDF of $m_{j,k}$ and $m_{j+1,3k+l}$ in MIPT/ LMIPT

Define, for $l = 0, 1, 2$

$$C_{jkl} = B_{j+1,3k+l} = \{x[n]; n = (3k+l).3^{J-j-1}, \\ (3k+l).3^{J-j-1} + 1, \dots, (3k+l+1).3^{J-j-1} - 1\}. \quad (\text{B.1})$$

C_{jk0} , C_{jk1} , and C_{jk2} partition B_{jk} into 3 subsets of $2b_j + 1$ elements each, where

$$2b_j + 1 = \frac{1}{3}(2a_j + 1) = 3^{J-j-1}. \quad (\text{B.2})$$

The joint CDF of $m_{j,k}$ and $m_{j+1,3k+l}$ under hypothesis H_i , $i = 0, 1$, can be written as

$$F_{joint}(x, y; j, k; H_i) = P(A_{jkl}), \quad (\text{B.3})$$

where $P(A_{jkl})$ is the probability of the event A_{jkl} defined as

$$A_{jkl} = \{m_{j,k} \leq x, m_{j+1,3k+l} \leq y\} = \{\text{at least } 3b_j + 2 \text{ elements} \\ \text{of } B_{jk} \text{ are in } (-\infty, x]\} \text{ and } \{\text{at least } b_j + 1 \text{ elements of } C_{jkl} \\ \text{are in } (-\infty, y]\}. \quad (\text{B.4})$$

Case 1: $x \geq y$

In this case, we can write

$$\begin{aligned}
 A_{jkl} = & \bigcup_{i=b_j+1}^{2b_j+1} \bigcup_{t=0}^{2b_j+1-i} \bigcup_{m=0}^{b_j+i+t} \{C_{jkl} \text{ has } i \text{ elements in } (-\infty, y], \\
 & t \text{ elements in } (y, x], \text{ and } (2b_j + 1 - i - t) \text{ elements in} \\
 & (x, \infty)\} \text{ and } \{B_{jk} - C_{jkl} \text{ has } (3b_j + 2 - i - t + m) \text{ elements in} \\
 & (-\infty, x] \text{ and } (b_j + i + t - m) \text{ elements in } (x, \infty)\}.
 \end{aligned} \tag{B.5}$$

Under hypothesis H_1 , it follows from (B.3) and (B.5) that

$$\begin{aligned}
 F_{joint}(x, y; j, k, l; H_1) = & \sum_{i=b_j+1}^{2b_j+1} \sum_{t=0}^{2b_j+1-i} \sum_{m=0}^{b_j+i+t} \sum_{p=1}^{\binom{2b_j+1}{i}} \sum_{q=1}^{\binom{2b_j+1-i}{t}} \sum_{r=1}^{\binom{4b_j+2}{3b_j+2-i-t+m}} \\
 & \{ \prod_{n \in I_{jkl,ip}^C} F_w(y - s[n]) \prod_{n \in I_{jkl,i'p,tq}^C} (F_w(x - s[n]) - F_w(y - s[n])) \\
 & \prod_{n \in I_{jkl,g(i+t,m)r}^D} F_w(x - s[n]) \prod_{\substack{n \in \{I_{jkl,i'p,t'q}^C \\ \cup I_{jkl,g'(i+t)m'r}^D\}}} (1 - F_w(x - s[n])) \}, \\
 & \text{for } x \geq y,
 \end{aligned} \tag{B.6}$$

where

$C_{jkl,ip} = p^{th}$ subset of C_{jkl} , each subset containing i elements; $i = 1, \dots, 2b_j + 1$; $p = 1, \dots, \binom{2b_j+1}{i}$.

$C_{jkl,i'p} = C_{jkl} - C_{jkl,ip} =$ Subset containing $i' = 2b_j + 1 - i$ elements of C_{jkl} not belonging to $C_{jkl,ip}$.

$C_{jkl,i'p,tq} = q^{th}$ subset of $C_{jkl,i'p}$, each subset containing t elements; $t = 1, \dots, 2b_j + 1 - i$; $q = 1, \dots, \binom{2b_j+1}{i}$.

$C_{jkl,i'p,t'q} = C_{jkl,i'p} - C_{jkl,i'p,tq} =$ Subset containing $t' = 2b_j + 1 - i - t$ elements of $C_{jkl,i'p}$ not belonging to $C_{jkl,i'p,tq}$.

$D_{jkl} = B_{jk} - C_{jkl} =$ Subset containing $4b_j + 2$ elements of B_{jk} not belonging to C_{jkl} .

$D_{jkl,g(u,m)r} = r^{th}$ subset of D_{jkl} , each subset containing $g(u, m) = 3b_j + 2 - u + m$ elements; $u = 0, 1, \dots, 2b_j + 1; m = 0, 1, \dots, b_j + u; r = \binom{4b_j+2}{3b_j+2-u+m}$.

$D_{jkl,g'(u,m)r} = D_{jkl} - D_{jkl,g(u,m)r}$ = Subset containing $g'(u, m) = b_j + u - m$ elements of D_{jkl} not belonging to $D_{jkl,g(u,m)r}$.

(B.7)

and $I_{jkl,ip}^C, I_{jkl,i'p,tq}^C, I_{jkl,i'p,t'q}^C, I_{jkl,g(i+t,m)r}^D$ and $I_{jkl,g'(i+t,m)r}^D$ are sets of indices corresponding, respectively, to the data samples in the sets $C_{jkl,ip}, C_{jkl,i'p,tq}, C_{jkl,i'p,t'q}, D_{jkl,g(i+t,m)r}$ and $D_{jkl,g'(i+t,m)r}$.

On choosing $s[n] = 0$ for all n in (B.6), we get the following expression for the joint CDF under hypothesis H_0 ,

$$\begin{aligned}
 F_{joint}(x, y; j, k; H_0) = & \\
 & \sum_{m=0}^{b_j+i+t} \sum_{t=0}^{2b_j+1-i} \sum_{i=b_j+1}^{2b_j+1} \left\{ \frac{(2b_j+1)!}{i! t! (2b_j+1-i-t)!} \right. \\
 & \frac{(4b_j+2)!}{(3b_j+2-i-t+m)!(b_j+i+t-m)!} F_w^i(y) (F_w(x) - F_w(y))^t \\
 & \left. F_w^{3b_j+2-i-t+m}(x) (1 - F_w(x))^{1+3b_j-m} \right\}; \quad \text{for } x \geq y.
 \end{aligned}
 \tag{B.8}$$

Case 2: $x \leq y$

In this case, we can write

$$\begin{aligned}
 A_{jkl} = & \bigcup_{i=b_j+1}^{2b_j+1} \bigcup_{t=0}^i \bigcup_{m=0}^{b_j+t} \{C_{jkl} \text{ has } t \text{ elements in } (-\infty, x], \\
 & i - t \text{ elements in } (x, y], \text{ and } (2b_j + 1 - i) \text{ elements in} \\
 & (y, \infty)\} \text{ and } \{B_{jk} - C_{jkl} \text{ has } (3b_j + 2 - t + m) \text{ elements in} \\
 & (-\infty, x] \text{ and } (b_j + t - m) \text{ elements in } (x, \infty)\}.
 \end{aligned}
 \tag{B.9}$$

From (B.3) and (B.9), it follows that

$$\begin{aligned}
 F_{joint}(x, y; j, k, l; H_0) = & \sum_{i=b_j+1}^{2b_j+1} \sum_{t=0}^i \sum_{m=0}^{b_j+t} \sum_{p=1}^{\binom{2b_j+1}{i}} \sum_{q=1}^{\binom{i}{t}} \sum_{r=1}^{\binom{4b_j+2}{3b_j+2-t+m}} \\
 & \prod_{\substack{n \in \{I_{jkl,ip,tq}^C \\ \cup I_{jkl,g(t,m)r}^D\}}} F_w(x - s[n]) \prod_{n \in I_{jkl,ip,t'q}^C} (F_w(y - s[n]) - F_w(x - s[n])) \\
 & \prod_{n \in I_{jkl,i'p}^C} (1 - F_w(y - s[n])) \prod_{n \in I_{jkl,g'(t,m)r}^D} (1 - F_w(x - s[n])) \\
 & \text{for } x \leq y,
 \end{aligned} \tag{B.10}$$

where

$$\begin{aligned}
 C_{jkl,ip,tq} &= q^{th} \text{ subset of } C_{jkl,ip}, \text{ each subset containing } t \text{ elements; } t = 1, \dots, i; \ q = 1, 2, \dots, \binom{i}{t}. \\
 C_{jkl,ip,t'q} &= C_{jkl,ip} - C_{jkl,ip,tq} = \text{Subset containing } t' = i - t \text{ elements of } C_{jkl,ip} \text{ not belonging to } \\
 & C_{jkl,ip,tq}.
 \end{aligned} \tag{B.11}$$

and $I_{jkl,ip,tq}^C$ and $I_{jkl,ip,t'q}^C$ are sets of indices corresponding to the data samples in the sets $C_{jkl,ip,tq}$ and $C_{jkl,ip,t'q}$ respectively.

On choosing $s[n] = 0$ for all n in (B.10), we get the following expression for the joint CDF under hypothesis H_0 ,

$$\begin{aligned}
 F_{joint}(x, y; j, k, l; H_0) = & \sum_{i=b_j+1}^{2b_j+1} \sum_{t=0}^i \sum_{m=0}^{b_j+t} \left\{ \frac{(2b_j+1)!}{(i-t)! t! (2b_j+1-i)!} \right. \\
 & \frac{(4b_j+2)!}{(3b_j+2-t+m)!(b_j+t-m)!} F_w^{3b_j+2+m}(x) (1 - F_w(x))^{b_j+t-m} \\
 & \left. (F_w(y) - F_w(x))^{(i-t)} (1 - F_w(y))^{2b_j+1-i} \right\}; \quad \text{for } x \leq y.
 \end{aligned} \tag{B.12}$$

It may be noted from (B.8) and (B.12) that the joint CDFs are independent of indices k and l under hypothesis H_0 .

Appendix C

CDF of $m_{j,k}$ in BMPT

In BMPT, $m_{J-1,k}$ is the median of triplet $\{x[3k+l]; l = 0, 1, 2\}$ and $m_{j,k}$ is the median of the triplet $\{m_{j+1,3k+l}; l = 0, 1, 2\}$ for $0 \leq j \leq J-2$. We can therefore readily obtain the following expressions for the CDFs of the medians

$$F_{med}(x; J-1, k; H_1) = \sum_{r=0}^2 F_w(x - s[3k+r]) F_w(x - s[3k+(r+1) \bmod 3]) (1 - F_w(x - s[3k+(r+2) \bmod 3])) + \prod_{r=0}^2 F_w(x - s[3k+r]), \quad (C.1)$$

$$F_{med}(x; j, k; H_1) = \sum_{r=0}^2 F_{med}(x; j+1, 3k+r; H_1) F_{med}(x; j+1, 3k+(r+1) \bmod 3; H_1) (1 - F_{med}(x; j+1, 3k+(r+2) \bmod 3; H_1)) + \prod_{r=0}^2 F_{med}(x; j+1, 3k+r; H_1). \quad \text{for } 0 \leq j \leq J-2. \quad (C.2)$$

where 'mod' denotes the *module* operation. The corresponding expressions under hypothesis H_0 are

$$F_{med}(x; J-1, k; H_0) \equiv F_{J-1}(x) = F_w^2(x) (3 - 2F_w(x)); \quad (C.3)$$

$$F_{med} \equiv F_j(x) = F_{j+1}^2(x) (3 - 2F_{j+1}(x)); \quad \text{for } 0 \leq j \leq J-2 \quad (C.4)$$

The CDFs of $m_{j,k}$ for $j \leq J - 2$ can be determined recursively from (C.2) for hypothesis H_1 , and from (C.4) for hypothesis H_0 .

Appendix D

Joint CDF of $m_{j,k}$ and $m_{j+1,3k+l}$ in BMPT

In accordance with (B.4) in Appendix B, the event A_{jkl} is defined, for $l = 0, 1, 2$, as

$$\begin{aligned} A_{jkl} = \{m_{j,k} \leq x, m_{j+1,3k} \leq y\} = \{\text{at least 2 elements} \\ \text{of the set } \{m_{j+1,3k}; r = 0, 1, 2\} \text{ are in } (-\infty, x]\} \text{ and} \\ \{m_{j+1,3k} \leq y\}. \end{aligned} \quad (\text{D.1})$$

Recalling that $m_{j,k}$ and $m_{j,k'}$ are statistically independent for $k \neq k'$ and that $m_{J,k} = x[k]$, we get

$$\begin{aligned} F_{joint}(x, y; j, k, l; H_1) = P(A_{jkl}; H_1) = \\ \{F_{med}(x; j+1, 3k + (l+1) \bmod 3; H_1) + F_{med}(x; j+1, 3k + \\ (l+2) \bmod 3; H_1) - 2F_{med}(x; j+1, 3k + (l+1) \bmod 3; H_1) \\ F_{med}(x; j+1, 3k + (l+2) \bmod 3; H_1)\} F_{med}(\min(x, y); \\ j+1, 3k+l; H_1) + F_{med}(x; j+1, 3k + (l+1) \bmod 3; H_1) \\ F_{med}(x; j+1, 3k + (l+2) \bmod 3; H_1) F_{med}(y; j+1, 3k+l; H_1); \\ \text{for } 0 \leq j \leq J-2. \end{aligned} \quad (\text{D.2})$$

$$\begin{aligned}
 F_{joint}(x, y; J-1, k, l; H_1) = & \{F_w(x - s[3k + (l+1) \bmod 3]) + \\
 & F_w(x - s[3k + (l+2) \bmod 3]) - 2F_w(x - s[3k + (l+1) \bmod 3]) \\
 & F_w(x - s[3k + (l+2) \bmod 3])\} F_w(\min(x, y) - s[3k + l]) + \\
 & F_w(x - s[3k + (l+1) \bmod 3]) F_w(x - s[3k + (l+2) \bmod 3]) \\
 & F_w(y - s[3k + l]),
 \end{aligned} \tag{D.3}$$

where ‘ $\min(x, y)$ ’ denotes the smaller of the two arguments.

On substituting $s[n] = 0$ for all n in (D.2) and (D.3), we get the following expressions for the joint CDF under hypothesis H_0

$$\begin{aligned}
 F_{joint}(x, y; j, k, l; H_0) = & 2F_{j+1}(x)(1 - F_{j+1}(x))F_{j+1}(\min(x, y)) \\
 & + F_{j+1}^2(x)F_{j+1}(y), \quad \text{for } 0 \leq j \leq J-2,
 \end{aligned} \tag{D.4}$$

$$\begin{aligned}
 F_{joint}(x, y; J-1, k, l; H_0) = & 2F_w(x)(1 - F_w(x))F_w(\min(x, y)) \\
 & + F_w^2(x)F_w(y),
 \end{aligned} \tag{D.5}$$

where $F_j(x) \equiv F_{med}(x; j, k; H_0)$, as defined in (C.4).

The joint CDFs of $m_{j,k}$ and $m_{j+1,3k+l}$ for $0 \leq j \leq J-2$ can be determined recursively from (D.3) for hypothesis H_1 , and from (D.5) for hypothesis H_0 .

Appendix E

Computation of $I_c(\Lambda)$

$$\begin{aligned} I_c(\Lambda) &= I_{cr}(\Lambda) = E\left\{\left(\frac{1}{f_w(\bar{w}, \bar{\bar{w}})} \frac{\partial f_w(\bar{w}, \bar{\bar{w}})}{\partial \bar{w}}\right)^2\right\}, \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left\{\left(\frac{1}{f_w(\bar{w}, \bar{\bar{w}})} \frac{\partial f_w(\bar{w}, \bar{\bar{w}})}{\partial \bar{w}}\right)^2\right\} f_w(\bar{w}, \bar{\bar{w}}) d\bar{w} d\bar{\bar{w}} \end{aligned} \quad (\text{E.1})$$

By changing the variables $\bar{w}$ and $\bar{\bar{w}}$ into $\rho = \sqrt{\bar{w}^2 + \bar{\bar{w}}^2}$ and $\theta = \arctan(\bar{\bar{w}}/\bar{w})$, we can write

$$\begin{aligned} \frac{\partial \rho}{\partial \bar{w}} &= \frac{\partial \sqrt{\bar{w}^2 + \bar{\bar{w}}^2}}{\partial \bar{w}} = \frac{\bar{w}}{\rho} \\ \frac{\partial \rho}{\partial \bar{\bar{w}}} &= \frac{\partial \sqrt{\bar{w}^2 + \bar{\bar{w}}^2}}{\partial \bar{\bar{w}}} = \frac{\bar{\bar{w}}}{\rho} \\ \bar{w} &= \rho \cos \theta \\ \bar{\bar{w}} &= \rho \sin \theta \end{aligned} \quad (\text{E.2})$$

By replacing the new variables given in (E.2) in (E.1), $I_c(\Lambda)$ can be expressed as

$$\begin{aligned}
 I_c(\Lambda) &= \int_0^{2\pi} \int_0^{+\infty} \left(\frac{1}{f(\rho)} \frac{\partial f(\rho)}{\partial \rho} \frac{\partial \rho}{\partial \bar{w}} \right)^2 f(\rho) \frac{\rho \partial \rho}{\rho \cos \theta} \partial \rho \cos \theta \partial \theta \\
 &= \int_0^{2\pi} \int_0^{+\infty} \left(\frac{1}{f(\rho)} \frac{\partial f(\rho)}{\partial \rho} \cos \theta \right)^2 f(\rho) \frac{\partial \rho}{\cos \theta} \rho \partial \rho \cos \theta \partial \theta \\
 &= \int_0^{2\pi} \left\{ \int_0^{+\infty} \frac{[\dot{f}(\rho)]^2}{f(\rho)} \rho \partial \rho \right\} \cos^2 \theta \partial \theta \\
 &= \left\{ \int_0^{+\infty} \frac{[\dot{f}(\rho)]^2}{f(\rho)} \rho \partial \rho \right\} \int_0^{2\pi} \cos^2 \theta \partial \theta \\
 &= \left\{ \int_0^{+\infty} \frac{[\dot{f}(\rho)]^2}{f(\rho)} \rho \partial \rho \right\} \int_0^{2\pi} \frac{(1 + \cos 2\theta)}{2} \partial \theta \\
 &= \left\{ \int_0^{+\infty} \frac{[\dot{f}(\rho)]^2}{f(\rho)} \rho \partial \rho \right\} \pi \\
 &= \pi \int_0^{+\infty} \frac{[\dot{f}(\rho)]^2}{f(\rho)} \rho \partial \rho,
 \end{aligned} \tag{E.3}$$

where $\dot{f}(\rho) = \frac{\partial f(\rho)}{\partial \rho}$.

Author's Publications

Journal Papers

1. Z. Madadi, G. V. Anand and A. B. Premkumar, "Signal detection in generalized Gaussian noise by nonlinear wavelet denoising", IEEE Transaction on Circuits and systems I, in press, 2013.
2. Z. Madadi, G. V. Anand and A. B. Premkumar, "A SAGE-based algorithm for 3-D localization of multiple sources in shallow ocean with non-Gaussian noise", submitted to Digital Signal Processing, 2013.

Conference papers

1. Z. Madadi, G. V. Anand and A. B. Premkumar, "3-D source Localization in shallow ocean with Non-Gaussian noise using a linear array of acoustic vector sensors", in Proceedings of ISSPA 2012, Montreal, Canada, 2012.
2. Xionghu Zhong, Z. Madadi and A. B. Premkumar, "Performance study of vertical acoustic vector sensor array based 3-D position tracking in a shallow ocean environment", in Proceedings of ISSPA 2012, Montreal, Canada, 2012.
3. Z. Madadi, G. V. Anand and A. B. Premkumar, "A new method for 3-D source localization using a hybrid array in shallow ocean with non-Gaussian noise", in Proceedings of OCEANS 2011, Santander, Spain, 2011.
4. Z. Madadi, G. V. Anand, A. B. Premkumar, C. T. Lau, "Detection of weak signals in non-Gaussian noise using nonlinear wavelet denoising", in Proceedings of OCEANS 2010, Sydney, Australia, 2010.

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