Distributed source localization in wireless sensor networks

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Distributed Source Localization in Wireless Sensor Networks

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A thesis submitted to the Nanyang Technological University
in fulfillment of the requirements for the degree of
Doctor of Philosophy

2013

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Date

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MENG WEI

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Abstract

With the advent of low-cost, low-power wireless sensing devices, wireless sensor networks (WSNs) have found potential applications in environment monitoring, battlefield surveillance, manufacturing and logistics, healthcare, etc. WSNs have been receiving increasing attention in recent years from researchers and practitioners. Source localization using measurements from spatially distributed sensors is an important application of WSNs. Many challenges arise when sensors are networked wirelessly due to the limited resources inherent to a sensor network. The main contribution of this thesis is to provide sensor resource management strategies and develop algorithms and protocols for distributed source localization and tracking in WSNs. Several types of sensor measurements are considered in this thesis, e.g., received signal strength (RSS), range, direction-of-arrival (DOA)/angle-of-arrival (AOA), time-of-arrival (TOA), time-difference-of-arrival (TDOA), etc.

One of the main challenges in the source localization problem is sensor resource management. In Part I, we consider the sensor placement problem. The relative sensor-source geometry plays an important role in the performance of any particular localization algorithm. Cramer-Rao lower bound (CRLB) is a measure of the fundamental limitation which provides the hard bound on the accuracy of unbiased position estimates under different conditions. Firstly, we study the optimal geometry of a team of networked TDOA sensors for the source localization problem, taking into account of *a priori* uncertainty of the source location under sensor position and motion constraints. Analytic solutions to the optimal sensor geometries are derived. Secondly, we consider the optimal sensor placement problem in heterogeneous sensor

networks (HSNs), where two types of sensors are deployed for source localization. Relative optimal sensor-source configurations with the minimum number of sensors for source localization are identified under the D-optimality criterion with potential extensions to a general case. Explicit characterizations of optimal sensor-source geometry are given for hybrid range and DOA sensors, hybrid DOA and TDOA sensors as well as co-located hybrid range and DOA sensors, respectively.

Then, we proceed to investigate another important research topic in the sensor resource management: sensor pairing. An in-network decentralized sensor pairing method is proposed for TDOA data gathering for source localization. The proposed decentralized sensor pairing method is proved to be able to give the same CRLB as the centralized one at a far less communication cost.

In Part II, we focus on source localization algorithm and protocol design. Firstly, an efficient centralized algorithm is developed for a multiple source localization problem. For the energy measurements case, the problem becomes challenging because each sensor's measurement is a superimposition of energy signals emitted from multiple sources. In this work, an Expectation-Maximization (EM) algorithm for maximum likelihood estimation (MLE) is presented. The proposed algorithm can provide a good trade-off between estimation accuracy and computational complexity.

The design of fully distributed algorithms is challenging but important to source localization which can improve the scalability of sensor networks. A projection based fully distributed algorithm is proposed for the energy based source localization. It is proved that the proposed method has low complexity and can achieve global optimality at a fast speed. Then, distributed algorithms are designed for the hybrid TOA/DOA-based source localization problem. For the bounded noise case, two methods are adopted: the average consensus method and projection based consensus method. The minimum number of sensors required to meet a certain estimation error tolerance is also derived.

Symbols and Acronyms

Algebraic Operators

$[a, b]$	closed interval from a to b
A^T	transpose of matrix A
A^{-1}	inverse of matrix A
$A^\dagger$	the pseudo-inverse of matrix A
$A^{i,j}$	the element in the i -th row and j -th column of matrix A
$\mathcal{P}_A$	projection onto set A
$\text{Tr}(A)$	the trace of matrix A
$\det(A)$	the determinant of matrix A
$\lambda(A)$	an eigenvalue of A
$\lambda_M(A)$	the eigenvalue of A with the largest magnitude
I_n	$n \times n$ identity matrix
$\mathbf{1}_n$	an n -dimensional column vector with all elements of 1
$\mathbf{0}_n$	an n -dimensional column vector with all elements of 0
$\text{Re}(\cdot)$	the real component of a complex number
$\ \cdot\ $	Euclidean norm
$E(X)$	the mathematical expectation of X
$\text{Var}(X)$	the variance of X
D^{ij}	the i th-order partial derivatives with respect to the first variable and the j th-order partial derivative with respect to the second variable
$\hat{x}$	an estimate of variable x

$\nabla_{\mathbf{x}}\mathbf{z}(\mathbf{x})$	the Jacobian of the vector $\mathbf{z}(\mathbf{x})$ with respect to the parameter $\mathbf{x}$
$\triangleq$	“is defined as”

Sets and Spaces

$\mathbb{R}$	set of real numbers
$\mathbb{R}^n$	Set of n dimensional real vectors
$\mathbb{R}^{m \times n}$	set of $m \times n$ real matrices
$\mathcal{N}_i$	set of communication neighbors of sensor i

Acronyms

AOA	angle-of-arrival
CRLB	Cramer-Rao lower bound
DOA	direction-of-arrival
EKF	extended Kalman filter
EM	expectation-maximization
FIM	Fisher information matrix
HSN	heterogeneous sensor network
WSN	wireless sensor network
MLE	maximum likelihood estimate
NLS	nonlinear least-squares
PDF	probability density function
RMSE	root mean square error
RSS	received signal strength
TDOA	time-difference-of-arrival
TOA	time-of-arrival
UAV	unmanned aerial vehicle
UGV	unmanned ground vehicle

Contents

Acknowledgement	i
Abstract	iii
Symbols and Acronyms	v
List of Figures	xiii
List of Tables	xvii
1 Introduction	1
1.1 Motivations and Objective	1
1.2 Major Contributions	4
1.3 Outline of the Thesis	5
2 Literature Review	7
2.1 Measurements and Models	7
2.1.1 RSS and Signal Energy	8
2.1.2 TOA/TDOA	9
2.1.3 DOA	11

2.2	Resource Management for Source Localization	12
2.2.1	Sensor Placement	12
2.2.2	Sensor Pairing	16
2.3	Algorithms for Source Localization	18
2.3.1	Centralized Algorithms	18
2.3.2	Distributed Algorithms	21
2.4	Conclusion	23

Part I Sensor Resource Management for Source Localization 25

3	Sensor Placement with Uncertainty in Source Location	27
3.1	Problem Statement	27
3.2	Sensor Placement without Communication Constraint	29
3.2.1	Centralized Sensor Pairing	29
3.2.2	Decentralized Sensor Pairing	37
3.3	Sensor Trajectory Optimization	43
3.3.1	Trajectory Optimization Problem	44
3.3.2	Simulation Results	46
3.4	Discussions	50
3.5	Conclusion	52

4	Sensor Placement in Heterogeneous Sensor Networks	53
4.1	Problem Description	53
4.2	Optimality Metric of Sensor-Source Geometries	55
4.3	Hybrid Range and DOA Sensors	56
4.3.1	Two-sensors Case	59
4.3.2	Three-sensors Case	60
4.3.3	N Sensors Case	68
4.4	Hybrid TDOA and DOA Sensing	68
4.4.1	Three-sensors Case	71
4.4.2	Arbitrary Number of Sensors Case	73
4.5	Co-Located Heterogeneous Sensors	73
4.5.1	Two and Three Sensor Platforms	76
4.5.2	N Sensor Platforms	77
4.5.3	Comparison with DOA and Range Localization	79
4.5.4	Discussion	81
4.6	Conclusion	81
5	Decentralized Sensor Pairing	83
5.1	Problem Statement	83
5.2	TDOA Sensor Pairing in Multi-hop Networks	85
5.2.1	Network Model	85
5.2.2	Proposed TDOA Sensor Pairing Method	85
5.2.3	Performance Analysis	88
5.3	Application to Decentralized Source Localization	91
5.4	Conclusion	92

Part II Algorithm Development for Source Localization 93

6 Multiple Source Localization: Centralized Methods 95

6.1	Problem Description	96
6.2	EM Localization Technique	98
6.2.1	Expectation-Step (E-Step)	99
6.2.2	Maximization-Step (M-step)	100
6.2.3	Initialization for EM Algorithm	102
6.2.4	β -parameterized Search Method for M-step	105
6.3	Algorithm Analysis	110
6.3.1	CRLB	110
6.3.2	Computational Complexity	111
6.4	Simulation Results	112
6.4.1	Performance Comparison with Known Decay Factor	112
6.4.2	Decay Factor Estimation	116
6.4.3	Performance Analysis for the Proposed EM Algorithm	116
6.5	Conclusion	120

7 Fully Distributed Energy Based Source Localization 121

7.1	Preliminary	122
7.2	Problem Statement	122
7.3	Sequential and Parallel Projection Methods	125
7.3.1	Sequential Projection Method	126
7.3.2	Parallel Projection Method	127

7.4	Fully Distributed Projection Method	128
7.4.1	Protocol for the General Case	130
7.4.2	Convergence Behavior for the Consistent Case	131
7.4.3	Convergence Behavior for the Inconsistent Case	135
7.4.4	Estimation Accuracy Analysis	138
7.4.5	Sensor Activation Problem	139
7.5	Numerical Examples	140
7.5.1	Simulation Setup	140
7.5.2	Convergence Performance	141
7.5.3	Estimation Performance	142
7.6	Conclusion	146
8	Hybrid TOA/DOA Measurements Based Distributed Source Localization	147
8.1	Problem Statement	148
8.2	Average Consensus Method	148
8.2.1	Linear Fusion Method	150
8.2.2	Consensus Method	150
8.3	Convex Projection Method	151
8.3.1	Estimation Accuracy Analysis	152
8.4	Simulation Results	155
8.4.1	Evaluation of Convergence and Estimation Performance	156
8.4.2	Communication Cost	157
8.5	Conclusion	157

9	Conclusions and Future Work	159
9.1	Conclusions	159
9.2	Recommendation for Future Research	160
	Appendix	163
A	Proof of Lemma 7.2	163
	Author's Publications	166
	Bibliography	168

List of Figures

1.1	Wireless sensor network	1
1.2	Motivating example: human localization and tracking	2
1.3	Motivating example: disaster rescue	2
1.4	Motivating example: ground source localization using a team of UAVs	3
2.1	Relative geometry between two sensors and a single stationary source for DOA sensing, where v_{12} denotes the angle subtended at the source by two sensors	11
2.2	Two types of sensor pairing in a single-hop sensor network	16
2.3	A multi-hop sensor network	17
2.4	Types of localization algorithms	18
3.1	Optimal three-sensors geometry for centralized sensor pairing for the static source case	32
3.2	Effect of p_3	35
3.3	Optimal three-sensor geometry for centralized pairing with source location uncertainty	37
3.4	Optimal two sensor pairs geometry with known source location	40
3.5	Optimal two sensor pairs placement for decentralized sensor pairing with source location uncertainty	44

3.6	Centralized sensor pairing with only turning rate constraint	47
3.7	Centralized sensor pairing with all constraints	47
3.8	Comparison of localization performance: with and without sensor trajectory optimization for centralized sensor pairing	49
3.9	Decentralized sensor pairing with only turning rate constraint	50
3.10	Decentralized sensor pairing with all constraints	50
3.11	Comparison of localization performance: with and without sensor trajectory optimization for decentralized sensor pairing	51
4.1	Optimal sensor-source configurations: one DOA sensor and one range sensor case: (a) Two sensors are placed at the different sides of the source; (b) Two sensors are placed at the same side of the source . . .	60
4.2	Source localization by using hybrid range and DOA sensors: (a) Two DOA sensors and one range sensor; (b) Two range sensors and one DOA sensor.	61
4.3	This figure illustrates the optimal sensor-source geometries under the two range sensors and one DOA sensor case. In the figure, ϕ denotes the angle subtended at the source by the DOA sensor and the range sensor.	64
4.4	This figure illustrates the optimal sensor-source geometries under the one range sensor and two DOA sensors case. In the figure, ϕ denotes the angle subtended at the source by the DOA sensor and the range sensor.	67
4.5	Source localization by using hybrid TDOA and DOA sensors.	71
4.6	This figure illustrates the optimal sensor-source geometries under two TDOA sensors and one DOA sensor case. Note that one of the TDOA sensors will be chosen as the reference node.	74

4.7	Comparison of upper bound of the determinant of the FIM	80
5.1	Sensor pairing in a multi-hop sensor network	85
5.2	An example of finding a CDS	87
5.3	Decentralized source localization architecture.	92
6.1	Iterative search in the M-step	106
6.2	Search points of the initial iteration in the M-step	109
6.3	Sensor and source deployment	113
6.4	Sensor density versus RMS	114
6.5	Noise variance versus RMS	114
6.6	Number of the sources versus RMS	115
6.7	Source energy estimation	115
6.8	Decay factor estimation	117
6.9	Step size versus RMS	118
6.10	Sensor density versus RMS for each source	118
6.11	Noise variance versus RMS for each source	119
6.12	Location estimation with known decay factor and unknown decay factor	119
6.13	Algorithm performance analysis	120
7.1	Consistent case and inconsistent case	124
7.2	Parallel and sequential projection methods	126
7.3	Inconsistent case	135
7.4	Sensor activation	139

7.5	Sensor network configuration. In (a), dots and triangle denote the sensors and the source respectively	141
7.6	Convergence result by using protocol (7.13)	142
7.7	Convergence results by proposed diffusion protocol (7.15).	143
7.8	Convergence trajectory of the proposed protocol (7.15)	143
7.9	Comparisons in accuracy as noise variance changes	145
7.10	Comparisons in accuracy as the number of the sensors changes. . . .	145
8.1	Bounded errors	149
8.2	Discrete model	153
8.3	A connected sensor network (small circles denote the sensors and the triangle denotes the source)	155
8.4	Convergence results	156
8.5	RMSE v.s. the number of iterates	157

List of Tables

4.1	Solution list (Two range sensors and one DOA sensor)	63
4.2	Solution list with $r_{b1} = r_{b2}$ (Two DOA sensors and one range sensor)	66
6.1	The EM Algorithm	102
6.2	Sequential Dominant-Source (SDS) Localization Method	103
6.3	Pseudocode of the Search Method	110
6.4	Radius for Initial Step	113
6.5	Computational Complexity	116

Chapter 1

Introduction

1.1 Motivations and Objective

A wireless sensor network (WSN) (as shown in Figure 1.1) consists of networked small and highly densely deployed sensor nodes, which has a variety of applications. Each sensor node can take the measurements but has limited communication and computation capabilities. In particular, there are two critical problems underlying successful operations of a WSN: efficient sensor resource management and collaborative information processing [64].

Figure 1.1: Wireless sensor network

Figure 1.2: Motivating example: human localization and tracking

Figure 1.3: Motivating example: disaster rescue

Localization and tracking of one or more sources in two-dimensional (2D) or three-dimensional (3D) sensor fields via a WSN is a very challenging, multi-faceted problem [64, 67, 91, 92]. This “canonical problem” has many applications, such as localization and tracking of vehicles, surveillance, localization of pollution sources, disaster rescue and so on. Several motivating examples are shown in Figures 1.2-1.4.

As stated above, the localization problem has many applications. However, generally it is a quite difficult problem. Firstly, sensor nodes in a WSN are densely deployed and the topology of a sensor network changes very frequently [1]. How to deploy and organize these sensors in an efficient way for the source localization is a challenging problem. Another issue is that the sensor nodes in a WSN are limited in power, computational capacities, and memory. Hence, resource management for the sensor nodes is a necessary task. Several interesting research topics are worthy of studying,

Figure 1.4: Motivating example: ground source localization using a team of UAVs

e.g., sensor placement, sensor selection, sensor pairing, data aggregation, etc.

The sensors distributed in a network measure the acoustic, radio-frequency (RF), seismic or thermal signals emitted by a source and produce useful information such as RSS, DOA, TDOA, etc [22, 64]. In such cases, sensor measurements are correlated and usually corrupted by noise. In addition, the sensor measurements are highly nonlinear which makes the source localization problem difficult. The sensors need to cooperate with each other. There are many network, signal processing and even control issues that need to be addressed.

Another issue is that all the operations in a WSN are powered by batteries of limited energy, and all data communications among the sensor nodes are provided by low-power and low-data rate wireless RF links. Since the data transmissions are the most energy-consuming operation for each sensor, data compression is required before the transmissions. Efficient data compression schemes become attractive for the sensor networks. Also it is much more energy efficient to perform distributed information processing than to do central processing that requires extensive remote communications.

In this thesis, the main objective is two folds: sensor resource management and algorithm development for source localization. For the resource management, the sensor placement, sensor pairing (data aggregation), etc will be investigated. For

the algorithm development, the objective is to design algorithms for single and multiple source localization and tracking in WSNs. The performance of the proposed algorithms will be also analyzed.

1.2 Major Contributions

The following lists particular novel contributions of the thesis.

- **Sensor placement taking into account of *a priori* uncertainty of source location:** Optimal geometry of a team of networked sensors for the source localization problem is given, taking into account *a priori* uncertainty of the source location under the sensor position and motion constraints (Chapter 3, the 5th journal publication in the author's publication list).
- **Sensor placement in heterogeneous sensor networks (HSNs):** Optimal sensor placement problem in a HSN is firstly explored, where two types of sensors are deployed for source localization. Relative optimal sensor-source configurations with the minimum number of sensors for source localization are identified under the D-optimality criterion (Chapter 4, the 2nd journal publication in the author's publication list).
- **Decentralized sensor pairing:** An in-network decentralized sensor pairing method is proposed to acquire TDOA measurements in multi-hop sensor networks for source localization (Chapter 5, the 3rd journal publication in the author's publication list).
- **Multiple source localization algorithm:** An efficient Expectation Maximization (EM) algorithm is proposed for energy based multiple source localization problem (Chapter 6, the 1st journal publication in the author's publication list).

- **Fully distributed source localization algorithms:** Fully distributed algorithms are proposed for the energy based and hybrid TOA/DOA based source localization problems (Chapter 7 and Chapter 8, the 4th journal publication in the author's publication list).

1.3 Outline of the Thesis

Some related works will be reviewed in Chapter 2. The remaining chapters are organized into two parts. The first part (including Chapter 3-Chapter 5) deals with sensor resource management for source localization while the second part (including Chapter 6-Chapter 8) focuses on the algorithm development for the source localization problem.

In Chapter 3, the sensor placement problem in a homogeneous network is considered. The optimal geometry of a team of networked sensors for the source localization problem, taking into account *a priori* uncertainty of the source location under the sensor position and motion constraints, will be investigated. Analytic solutions to the optimal sensor geometries are also derived.

In Chapter 4, the sensor placement problem in a heterogeneous network will be studied. Different from homogeneous sensor networks, a heterogeneous network considered in this thesis consists of two different types of sensors. Theoretic analysis for the explicit characterizations of the optimal sensor-source geometries are given for the different combinations of hybrid sensors.

In Chapter 5, a decentralized sensor pairing strategy will be proposed for multi-hop sensor networks. The solution involves finding a connected dominating set (CDS) for the network graph. It is proved that the proposed decentralized in-network sensor pairing method can result in the same CRLB as the centralized one at a far less communication cost.

In Chapter 6, the energy based multiple source localization problem will be addressed. An efficient EM algorithm for MLE is presented. The proposed algorithm

can estimate the source parameters, such as source energy and locations, as well as the decay factor of the signal during propagation. Theoretic analysis on the algorithm convergence rate, the CRLB for the localization accuracy, and the computational complexity of the algorithm are also given.

In Chapter 7, a projection based fully distributed algorithm will be presented for the energy based single-source localization problem. The source localization problem is formulated as a convex feasibility problem (CFP). In the proposed method, sensor nodes only need to communicate with one-hop neighbors and update their estimates simultaneously based on a projection algorithm. It is shown that sensors are able to achieve consensus on a possible minimizer asymptotically. The proposed method has low complexity and can achieve global optimality fast.

Chapter 8 continues to study a fully distributed source localization algorithm using another type of sensor measurements: hybrid TOA/DOA measurements. Two methods are proposed: linear fusion method and convex projection method. The convex projection method has communication and computation advantages. The minimum number of sensors required to meet a certain estimation error tolerance is also derived.

Finally, Chapter 9 draws some concluding remarks. The contributions of thesis are recaptured and possible future research topics are highlighted.

Chapter 2

Literature Review

Source localization using measurements from spatially distributed sensors is an important application of WSNs. Considerable research has been carried out on various aspects of the source localization problem. This thesis mainly focuses on resource management and algorithm development for the source localization problem. These two topics have been attracting much attention in the literature, especially in signal processing, control, aerospace and communication societies. Different research societies focus on the different aspects of the source localization problem.

2.1 Measurements and Models

This section discusses several types of sensor measurement models used for the source localization. These models form the basis for the later chapters of this thesis. Generally, the measurement models can be classified into three categories: 1) received signal strength (RSS) related models, such as energy, range, etc. 2) Time-related measurement models, e.g., time-of-arrival (TOA) and time-difference-of-arrival (TDOA). 3) Direction-of-arrival (DOA) or angle-of-arrival (AOA). Sections 2.1.1 through 2.1.3 are used to present an introduction to measurement models, prior to the main research topics of this thesis.

2.1.1 RSS and Signal Energy

RSS is defined as the voltage measured by a sensor's received signal strength indicator (RSSI) circuit. The RSS of acoustic, RF, or other signals can be considered. Since the RSS measurements are of relatively low cost and simple for implementation, the RSS based source localization is important and also a hot research topic.

The model below is widely used in the literature [63, 62, 60, 106]. The signal received by a sensor at time instant k can be expressed as

$$z(k) = \gamma \frac{\Gamma(k)}{d(k)} + \epsilon(k) = s(k) + \epsilon(k), \quad (2.1)$$

where $z(k)$ represents the signal intensity, γ is the gain factor of the sensor, $\Gamma(k)$ denotes the intensity of the source signal measured at a location with distance of 1 meter from the source, $d(k) = \|\mathbf{x}_i - \mathbf{x}_t\|$ is the distance between the sensor and the source, $\mathbf{x}_i$ denotes the coordinates of the i -th sensor, $\mathbf{x}_t = [x_t, y_t]^T$ is the source's coordinates. $\epsilon(k)$ is modeled as a zero-mean additive white Gaussian (AWGN) noise with variance ς^2 .

Energy based model is firstly presented in [106, 62, 105], which is derived from the RSS model (2.1). In essence, the energy is the average intensity measurements over the time window $[t - T/2, t + T/2]$.

Assume that $\epsilon(k)$ and $\Gamma(k)$ are uncorrelated such that

$$E\{\Gamma(k)\epsilon(k)\} = E\{\Gamma(k)\}E\{\epsilon(k)\} = 0.$$

Then the signal energy can be represented as (setting $g = \gamma^2$, and $S(k) = E\{\Gamma^2(k)\}$)

$$E\{s^2(k)\} = \gamma^2 \frac{E\{\Gamma^2(k)\}}{d^2(k)} = g \frac{S(k)}{d^2(k)}. \quad (2.2)$$

The average energy measurement over the time window $[t - T/2, t + T/2]$ denoted

by $P(t)$ is given by

$$P(t) = \frac{1}{f_s T} \sum_{k=(t-T/2)f_s}^{(t+T/2)f_s} z^2(k) = \frac{1}{f_s T} \sum_{k=(t-T/2)f_s}^{(t+T/2)f_s} s^2(k) + \frac{1}{f_s T} \sum_{k=(t-T/2)f_s}^{(t+T/2)f_s} \epsilon^2(k). \quad (2.3)$$

In practice, the expectation is realized using the ensemble average over a time window, $T = M/f_s$, where M is the number of sample points received by the sensor during the time interval T and f_s is the sampling frequency [106].

Then, according to the result in (2.2), (2.3) can be rewritten as:

$$P(t) = g \frac{S(t)}{d^2(t)} + w(t). \quad (2.4)$$

The square of the background noise $\epsilon^2(k)$ in (2.3) will have a χ^2 distribution with a mean equal to $E\{\epsilon^2(k)\} = \zeta^2$ and a variance equal to $2\zeta^4/M$. If M is sufficiently large ($M \gg 30$), according to the central limit theorem, w can be approximated well with a normal distribution, namely, $w \sim N(\zeta^2, 2\zeta^4/M)$. In some work in the literature, w is simplified to a zero mean white Gaussian noise, i.e., $w \sim N(0, \sigma^2)$ [12, 58, 107].

Range between the sensor and the source can also be estimated from RSS measurements. Except RSS, range measurement can also be estimated from information such as TOA or TDOA. In [23], several biased and unbiased estimators are proposed to estimate the range between the source and the sensor.

2.1.2 TOA/TDOA

TOA is the measured time at which a signal (RF, acoustic, or other) first arrives at a sensor from the source [89]. The measured TOA is the time of transmission plus a propagation-induced time delay. This time delay, τ^{delay} , is equal to the range between the sensor and the source divided by the propagation velocity, v_p . As a rule of thumb, for acoustic propagation, 1 ms translates to 1 ft (0.3 meter), while

for RF, 1 ns translates to 1 ft [89]. The measurement model of TOA by the i -th sensor can be written as

$$\tau_i^{\text{delay}} = \frac{\|\mathbf{x}_i - \mathbf{x}_t\|}{v_p} + \Delta\tau_i, \quad (2.5)$$

where $\Delta\tau_i$ denotes the measurement error which is usually assumed to be Gaussian.

For the TDOA sensing, each sensor can measure the time of arrival of the signal from the source. However, the sensors can not know when the signal was transmitted from the source. The general approach is to take the time difference between two sensors, i.e., TDOA. The TDOA estimated by a sensor pair $\{i, j\}$ can be written as:

$$\hat{t}_{ij} = \frac{d_{ij}}{v} + \varepsilon_{ij}, \quad (2.6)$$

where $d_{ij} = \|\mathbf{x}_i^t - \mathbf{x}_t\| - \|\mathbf{x}_j^t - \mathbf{x}_t\|$, $\mathbf{x}_t = [x_t, y_t]^T$ is the source's coordinates, v denotes the propagation speed of signals, and $\varepsilon_{ij} \sim \mathcal{N}(0, \sigma_t^2)$ denotes the TDOA estimation error.

The cross covariance of $\hat{t}_{ij}$ and $\hat{t}_{kl}$ denoted by $\text{cov}(\hat{t}_{ij}, \hat{t}_{kl})$ is [19]

$$\text{cov}(\hat{t}_{ij}, \hat{t}_{kl}) = \begin{cases} \frac{1}{2}\sigma_t^2, & i = k \text{ and } j \neq l \text{ or } i \neq k \text{ and } j = l \\ -\frac{1}{2}\sigma_t^2, & i = l \text{ and } j \neq k \text{ or } i \neq l \text{ and } j = k \\ 0, & \text{otherwise.} \end{cases} \quad (2.7)$$

Without loss of generality, only TDOA measurements between sensor 1 and sensor i , $i = 2, \dots, N$ are considered. Thus the TDOA measurement model from N sensors can be written as

$$\hat{\mathbf{t}} = \mathbf{t}(\mathbf{x}_t) + \varepsilon = [t_{12}, t_{13}, \dots, t_{1N}]^T + [\varepsilon_{12}, \varepsilon_{13}, \dots, \varepsilon_{1N}]^T, \quad (2.8)$$

where $\varepsilon \sim \mathcal{N}(\mathbf{t}(\mathbf{x}_t), \Sigma_t)$ with $\Sigma_t = 2\sigma_t^2 \text{circulant}([1 \ \frac{1}{2} \ \dots \ \frac{1}{2}])$, where $\text{circulant}(\cdot)$ generates a square circulant matrix.

Remark 2.1. *Time-based methods (TOA and TDOA) require precise acquisition of the phases of signals arriving at different nodes. However, the features of some*

signals tend to be narrow-band, making precise phase acquisition difficult in a noisy environment.

2.1.3 DOA

Figure 2.1: Relative geometry between two sensors and a single stationary source for DOA sensing, where v_{12} denotes the angle subtended at the source by two sensors

Direction-of-arrival (DOA) based sensing method (also referred as angle-of-arrival (AOA)) provides information about the direction from sensor to source. The most common method that sensors measure DOA is to use a sensor array and employ so-called array signal processing techniques at the sensor nodes [89].

Consider the source location $\mathbf{x}_t = [x_t, y_t]^T$ and N DOA sensors located at $\mathbf{x}_j^b = [x_{bj}, y_{bj}]^T, j = 1, \dots, N$. The DOA measurements from N sensors can be stacked as follows

$$\hat{\Phi} = \Phi(\mathbf{x}_t) + \mathbf{e} = [\phi_1, \dots, \phi_N]^T + [e_1, \dots, e_N]^T, \quad (2.9)$$

where $\phi_i, i = 1, \dots, N$ denote the DOA measurements as shown in Figure 2.1. The measurement noises are assumed to be mutually independent and Gaussian distributed with zero mean and the same variance σ_b^2 , i.e., $e_i \sim \mathcal{N}(0, \sigma_b^2)$. Then the covariance of $\mathbf{e} = [e_1, \dots, e_N]^T$ is given by $\Sigma_b = \sigma_b^2 \mathbf{I}_N$.

In summary, a number of measurement techniques are available for the source localization problem. Which measurement technique to be chosen for the source localization will depend on specific applications. Typically, localization algorithms

based on TOA/TDOA and DOA/AOA can achieve better accuracy. However, the expense of equipment cost is higher [74].

2.2 Resource Management for Source Localization

This section provides a brief introduction to the sensor resource management (SRM) for source localization problems, including sensor placement, path planning, sensor pairing, etc. A related problem is sensor selection which has been considered by some other researchers [118, 57, 55, 101]. We shall not discuss the sensor selection problem in this thesis. However, the sensor placement analysis carried out in this thesis can provide some guidance to the sensor selection strategies. We note that the resource management plays an important role in the source localization problem which can improve the efficiency of the localization algorithms, optimize the bandwidth and power usage of the network, etc.

2.2.1 Sensor Placement

2.2.1.1 Sensor Placement in a Homogeneous Network

The goal of sensor placement is to place sensors to form the best configuration under given constraints so as to optimize a performance measure defined on the resulting source location estimate statistics [134]. It is well known that the relative sensor-source geometry plays an important role in the performance of any particular localization algorithm [8].

For the static source case, the sensor placement problem is investigated under the parameter estimation framework. In general, the study of the sensor-source geometry is based on the CRLB which has been explored in [133, 8, 76, 48, 30, 47, 20, 119]. CRLB is a measure of the fundamental limit which provides the hard bound on the

accuracy of unbiased estimates given measurement conditions which may be explored for better accuracy. In some works, the sensor placement analysis is based on a given source location estimate. For example, in [30], the optimal angular separation for the AOA based source localization is established. In [8], a direct and rigorous characterization of the optimal relative sensor-source geometry is given for localization with range, bearing or TOA measurements. Moreover, optimal sensor configurations with some other types of measurements, e.g., signal strength and Doppler-based measurements are discussed in [10, 11]. In [56], an algorithm named RELOCATE is proposed, which is an iterative method that places the sensors so as to minimize the position error bound at each possible source location.

Remark 2.2. *The analysis of this thesis is based on the use of CRLB. The CRLB can be achieved by a maximum likelihood estimator (MLE) for sufficiently high signal-noise-ratio (SNR); however below certain SNR the MLE departs significantly from the CRLB, displaying a threshold behavior [130]. This departure is partially attributed to the fact that a MLE with nonlinear parameter dependence is biased at low SNR while a local performance bound such as the CRLB is limited to unbiased estimates. Indeed the information theory inequality, from which the CRLB can be derived, does include some bias-related terms, which are ignored in the commonly-referred form of the CRLB (i.e., inverse of the Fisher information matrix (FIM)) due to evaluation difficulty. However, the estimation bias also makes contribution to the performance bound of the estimators. Hence, when SNR is low, a tighter bound compared to the CRLB should be derived. In [113, 99], Barankin bounds are used for source localization in an uncertain ocean environment. In [71], the author derives the Battacharyya Bound (BB) and the Abel Bound in a source localization scenario.*

Recently, there have also been some works on optimal sensor position design in the average sense, where the expected value of the FIM determinant is to be maximized [115, 54, 81]. In [54], the authors address the TDOA based sensor placement problem. The main result of the paper is based on maximizing the expected determinant of the FIM for truncated, radially-symmetric source distributions. Two

features of the optimal sensor placement have been shown in this work, e.g, the sensors should be placed as far from the expected source position as possible. Secondly, the sensors should be arranged evenly around the source. In [81], the optimal sensor placement for multiple underwater source localization is studied with acoustic range measurements. There are tradeoffs involved in the precision with which each of the sources can be localized. In this work, some techniques that borrow from Pareto optimization and estimation theory are used to address this problem.

For the movable source case, the sensor placement problem is usually investigated under the Kalman filtering or information filtering framework [134]. The trace and the determinant of the location estimation error covariance matrix are the two most commonly used matrix measures for the sensor placement in SRM. Martinez et al. [76] study the source localization and tracking problem using range sensors under an optimal sensor geometry. A sensor placement algorithm is also presented to achieve the optimal sensor configuration. However, the uncertainty of the source location is not taken into account when deriving the optimal sensor placement. In [134], sensor placements with both the Kalman filtering and information filtering are addressed. The work demonstrates how to accommodate bearing and range measurements. In [94], the posterior Cramer-Rao lower bound (PCRLB) is used to quantify and control the optimal achievable accuracy of source location estimation. The issues of imperfect sensor placement and uncertain sensor movement (e.g., sensor drift) are addressed.

In many applications, we deal with a network of moving sensor platforms such as unmanned aerial vehicles (UAVs). Due to the constraints in communications and control of UAVs, the problem becomes a constrained sensor trajectory optimization for the optimal source localization. In the literature, the problem of UAV trajectory optimization has been extensively studied, especially for the bearing-only and vision based target tracking problems [93, 95, 29, 87, 47, 37, 7, 121]. In [37], a network of UAVs is used for a radio source localization using signal strength measurements, where aircraft motion is coordinated through iterative consensus by individual receding horizon controllers on each vehicle. In [95], TDOA measurements are used

for radar localization. The optimal sensor placement rules including angle rule and coordination rule are addressed for bearing and location estimation, respectively. The results can only be applied for a three-sensors case. In [7], the authors present a cooperative vision-based estimation and tracking system. In [87], the determinant of the FIM is adopted as an objective function for DOA sensor trajectory optimization. In [28], the path planning problem is cast as a nonlinear programming problem using an approximation of the FIM and solved at successive waypoints to generate vehicle trajectories.

2.2.1.2 Sensor Placement in a Heterogeneous Network

By fusing information from different types of sensors or measurements, source localization can be achieved more easily and accurately [15, 61, 114]. In real applications, different types of sensors may spread in a sensor field, which form a heterogeneous sensor network (HSN). HSNs can support applications that may require diverse resources. Especially, HSNs with multiple sensing modalities are gaining popularity in diverse fields since they can provide flexibility and robustness [61, 72, 138, 120]. In [61], an approach for target tracking in urban environments utilizing a HSN of mote class devices equipped with acoustic sensor boards and embedded PCs equipped with web cameras is proposed. In [120], heterogeneous wireless sensor networks are used for intrusion detection in a battlefield.

The majority of the existing works consider source localization problems using homogeneous sensors. Whilst in a HSN, there may exist many different types of sensors. In [15, 9, 39, 110], the authors make efforts to study the CRLB for localization from HSNs. However, how to deploy heterogeneous sensors in the most favorable positions for source localization is not considered. The work in [135] is concerned with the geometrical aspect of sensor placement so as to optimize the tracking performance. The configurations for the optimal placement of two heterogeneous sensors in source tracking over several time steps are provided. However, the results are constrained to a two-sensors case and only the bearing and range sensors are considered.

2.2.2 Sensor Pairing

Figure 2.2: Two types of sensor pairing in a single-hop sensor network

This subsection goes to another important research topic in the SRM: sensor pairing. Sensor pairing belongs to the research area of data aggregation and data compression [53]. In this thesis, sensor pairing is mainly applied for TDOA based sensors. From the TDOA measurement model in (2.6), it can be seen that a sensor pair is needed to acquire a TDOA measurement. In a WSN, a large number of sensors will be placed in the sensing area. For better source localization, more sensor measurements are to be taken. How to construct the sensor pairs in a networked sensor system is a big issue.

Traditionally, TDOA sensor pairing is done in a centralized way which we call centralized sensor pairing as shown in Figure 2.2(a), where one sensor node is chosen as the reference node and the other nodes broadcast their raw measurements to this reference node. The centralized sensor pairing has a long history of more than 50 years and it is widely used in aerospace systems [18, 109, 19, 132]. However, due to the power and bandwidth constraints in WSNs, centralized signal processing may be difficult to be used, especially for a large scale and unreliable network. Also some sensors can not transmit their measurements to the reference sensor due to long distances between the sensors and the reference.

To overcome the communication issue, decentralized sensor pairing strategy is proposed in [52, 53]. A simple case for decentralized sensor pairing in which there is

no common reference sensor as shown in Figure 2.2(b). In [52], the sensor selection problem is studied for both the centralized and the decentralized sensor pairing methods. Then in [53], two methods are proposed where the FIM is used to determine the sensor pairing/selection. The first method optimally determines pairings as well as selections of pairs but with the constraint that no sensors are shared between pairs. The second method allows sensors to be shared between pairs.

Figure 2.3: A multi-hop sensor network

In a multi-hop sensor network as shown in Figure 2.3, due to the limited communication distance, most of the nodes can not communicate with the reference node directly, and much energy is consumed in multi-hop transmissions of the raw measurements. Hence, in practice, a decentralized sensor pairing method with low requirements for both network bandwidth and power consumption is highly desirable. It is well known that the decentralized in-network signal processing and aggregation is more energy efficient than its centralized counterpart [97, 33]. In the communication area, the connected dominating set (CDS) is useful in the computation of routing for ad-hoc networks [65, 27, 125, 42, 46]. In many applications, a small CDS is used as a backbone for communications, and nodes that are not in this set communicate by passing messages through neighbors that are in the set. Intuitively, we can find certain connections between the CDS and the decentralized sensor pairing problem. For the sensor pairing in a multi-hop sensor network, if possible, a backbone can be found for the TDOA sensor pairing and data gathering.

2.3 Algorithms for Source Localization

Extensive studies have been carried out for algorithm development for source localization using different types of measurements, e.g., RSS, TOA/TDOA, range, DOA, etc. Source localization algorithms estimate the location of sources by using the knowledge of sensor locations and sensor measurements (sensor locations can be estimated through Global Positioning System (GPS) or by cooperative localization methods [123, 103, 104, 102]). In general, the localization algorithms can be broadly classified into two categories: centralized and distributed algorithms, depending on the manner of data sharing among sensors.

Figure 2.4: Types of localization algorithms

2.3.1 Centralized Algorithms

For the centralized framework as shown in Figure 2.4(a), the sensors need to transmit raw measurement data to a fusion center. The fusion center will calculate the source location estimate. For different types of sensor measurements, the localization algorithms may also be different.

Firstly, RSS based source localization has been attracting a lot of attention due to its simple implementation in which no additional hardware is required. In [89, 90], the authors consider sensor location estimation when sensors measure RSS between themselves and neighboring sensors. In this case, the sensor location estimation is similar to the source localization case. The authors derive the CRLB for RSS based algorithms and a MLE estimator is used for localization. The maximum of the MLE is found using a conjugate gradient algorithm. In [34, 51], RSS based indoor localizations are addressed. In [34], based on the compressed sensing (CS) theory, the location of the source can be recovered from a small number of noisy RSS measurements.

Compared with the RSS based method, energy (which is derived from RSS measurements) based approach is more appropriate for WSNs. This is because that the signal energy emitted by the sources usually varies slowly. As such, the signal energy time series can be sampled at a much lower rate compared with the raw acoustic time series [105, 106]. Therefore, few data will be transmitted to the fusion center via the often congested wireless communication channels. This will reduce the energy consumption for data transmissions at individual sensor node and save communication bandwidth over shared wireless channels. In [62, 63], source localization is formulated as an unconstrained least-squares problem where the source location is accurately estimated as the intersection of hyper-spheres. In [106], a MLE method is proposed where a Multi-Resolution (MR) search is needed to find the optimal solution. In [50], an accurate two stage algebraic closed-form solution is presented. The first stage computes the source location together with an auxiliary variable using a weighted least-squares method. The second stage explores the relationship between the source location and the auxiliary variable to improve the location estimation. A weighted direct least-squares formulation is presented in [79] to study the tradeoff between performance and computational complexity. A semidefinite programming approach is developed in [80], where minimax approximation and semidefinite relaxation are applied to transform traditional nonconvex problems into convex optimization problems. In [84, 59, 58], source localization using quantized

sensor signal energy readings is considered. Different quantization strategies, e.g., equally distance divided quantizer (EDQ) [59] are proposed for energy-aware source localization.

Time based source localization has been a hot topic in recent years due to its high localization precision and wide social and military applications. A TOA based source localization problem can be transformed to a range-based localization problem [5]. Compared with TOA based methods, TDOA based source localization is more popular since in some real applications, it is difficult to acquire the time delay from the source to the sensor. In the literature, a lot of methods have been proposed to solve the TDOA based source localization problem [35, 109, 136, 49, 86, 36, 75, 40, 85]. In [35], an iterative Taylor-series method is proposed for position estimation. However, a reasonable initial guess is required for convergence to the correct solution. In [109], a spherical interpolation (SI) method is presented which provides a least-squares closed-form solution. In [19], an intermediate variable is introduced so that the nonlinear equation relating TDOA estimates and source position can be transformed into a set of linear equations. A two step least-squares (LS) method is proposed to solve the problem. An explicit solution is given. In [128, 129], the authors consider the TOA measurements directly without the subtraction preprocessing for TDOA estimation, the SDP relaxation approach is proposed for source localization. An alternative way to solve the TDOA based source localization problem is to consider range-difference measurements. In [111], the authors comment on, clarify, and streamline some approximate and exact solutions to this problem. A new closed-form approximate solution is also introduced that can be viewed as an enhanced version of the unconstrained LS estimate.

DOA based source localization has also received considerable attention in the literature. DOA estimation plays an important role in the source localization [112, 17, 73, 68]. In [112], the multiple signal classification (MUSIC) is adopted for the DOA estimation from the maximum-likelihood principle. In [73], the DOA based source localization is formulated in a sparse signal representation framework. A subspace based ℓ_1 -SVD method is developed for multiple snapshots DOA estimation.

In [21, 22], a single step ML estimator is presented. For a multiple source case, an efficient alternating projection procedure based on sequential iterative search is proposed which can yield superior performance over other suboptimal techniques, including the wideband MUSIC and the two-step least-squares methods. An empirical study of DOA based source localization is developed in [2] to localize actual animals in their natural habitat.

The implementation of centralized algorithms is simple. However, due to the limited bandwidth of WSNs, this type of algorithms is not realistic. This is because that centralized algorithms have a low robustness to node failures and consume lots of energy in long-distance data transmissions.

Remark 2.3. *The problem of centralized single source localization has been well studied. It is interesting to study the multiple source localization problem. For TDOA and DOA based sensor measurements, the main difficulty in multiple source localization is data association. Although data association has been well studied in the literature (See [117, 88] and reference therein), energy based multiple source localization is quite difficult since in this case, each sensor's measurement is a superimposition of energy signals emitted from multiple sources. The existing MR search method in [106] has a high computational complexity.*

2.3.2 Distributed Algorithms

Distributed algorithms are more promising for WSNs compared with centralized algorithms since there is no fusion center required for data processing. Moreover, distributed algorithms are more energy efficient and robust to node failures. In general, distributed algorithms can be further classified into sequential algorithms and fully distributed algorithms according to the pattern of data sharing among sensors, as shown in Figures 2.4(b) and 2.4(c).

Firstly, for the RSS and the energy based source localization problems, several sequential distributed methods have been proposed. In [26], the RSS based source

localization is formulated as a multiple hypothesis testing problem, where each hypothesis corresponds to the event that the source belongs to a particular region. The asymptotic properties of the proposed regional localization problem is also provided. On the other hand, distributed algorithms design for energy based source localization is more active in the literature. In [98], Rabbat and Nowak propose a distributed implementation of the incremental gradient (IG) algorithm to solve the nonlinear least-squares problem. However, in their work, the source energy is assumed to be known. Later in [96], they propose to use a kernel averaging estimator for robust source localization. In [12], Blatt and Hero first formulate the source localization as a convex feasibility problem (CFP) and propose to use a projection-onto-convex-sets (POCS) method. This method can be implemented in a distributed manner. However, the convergence of the method is verified by simulations but no rigorous analysis is provided. An incremental optimization algorithm for maximum likelihood based source localization is proposed in [107]. In [77], an energy efficient iterative source localization scheme is proposed where the quantized data of each activated sensor is used. At each iteration, the posterior probability density function (PDF) of the source location is approximated using an importance sampling based Monte Carlo method.

In the literature, there are also distributed algorithms designed by using other types of sensor measurements. For example, in [6], two iterative schemes are proposed for a range based source localization problem. The proposed algorithms target at finding a global solution for location estimation. However, a “good” starting point is also required for both algorithms. In [137], a nonparametric iterative adaptive approach based on weighted least-squares is presented for the DOA based source localization problem. A sequential distributed odor source localization is investigated in [45]. The proposed method can be used for odor plume localization and tracing.

Remark 2.4. *There are few work on distributed algorithms for TDOA based source localization. This is mainly because that the TDOA measurement model is more complex than other types of sensing models.*

The majority of the existing distributed algorithms adopt the sequential method. Note that the sequential method requires a specified data transmitting path. How to do the path planing in such networks is a big issue. Also, the convergence rate is low when the sensor density is high and the data transmission becomes unreliable when some of the nodes fail. Hence the robustness of such networks is low and a novel type of data transmission is needed. In the literature, the recently emergent consensus technique which is a fully distributed scheme as shown in Figure 2.4(c) can be applied to the localization problem. This technique has been intensively investigated in signal processing and control area recently. Some references can be found in [100, 31, 16, 66].

2.4 Conclusion

In this chapter, a comprehensive literature review for the source localization problem was provided, including the introductions to the measurement models, the resource management strategies and the existing localization algorithms. The research works in the literature also motivate the study of this thesis.

Part I

Sensor Resource Management for Source Localization

Chapter 3

Sensor Placement with Uncertainty in Source Location

This chapter is concerned with optimal sensor placement of a team of networked sensors for the source localization problem, taking into account a priori uncertainty of the source location under sensor position and motion constraints. Analytic solutions to the optimal sensor geometries are derived. Furthermore, this work is extended to optimal sensor path planning. Specifically, in this chapter, TDOA based sensor measurements are considered, however, the main results can also be applied to the source localization problems based on other types of sensor measurements.

3.1 Problem Statement

Consider a team of N sensor nodes performing the source localization and tracking task. Each sensor pair can estimate the TDOA between them by using a generalized cross correlation method (GCC)[43, 3]. The TDOA model is reproduced here for convenience:

$$\hat{t}_{ij} = \frac{d_{ij}}{v} + e_{ij}, \quad (3.1)$$

where $d_{ij} = \|\mathbf{x}_i - \mathbf{x}_t\| - \|\mathbf{x}_j - \mathbf{x}_t\|$, $\mathbf{x}_i = [x_i, y_i]^T$ denotes the coordinates of the i -th sensor. $\mathbf{x}_t = [x_t, y_t]^T$ is the source's coordinates, v denotes the propagation speed of signal and $e_{ij} \sim \mathcal{N}(0, \sigma^2)$, $\forall 1 \leq i, j \leq N, i \neq j$, denote the TDOA estimation errors.

The cross covariance of $\hat{t}_{ij}$ and $\hat{t}_{kl}$ denoted by $\text{cov}(\hat{t}_{ij}, \hat{t}_{kl})$ is [19]

$$\text{cov}(\hat{t}_{ij}, \hat{t}_{kl}) = \begin{cases} \frac{1}{2}\sigma_t^2, & i = k \text{ and } j \neq l \text{ or } i \neq k \text{ and } j = l \\ -\frac{1}{2}\sigma_t^2, & i = l \text{ and } j \neq k \text{ or } i \neq l \text{ and } j = k \\ 0, & \text{otherwise.} \end{cases} \quad (3.2)$$

For N sensors, there are a total $N(N-1)/2$ possible sensor pairs and thus $N(N-1)/2$ TDOA measurements. Let

$$\mathcal{I} = \{\{i, j\} | 1 \leq i < j \leq N\} \quad (3.3)$$

denote the set of all sensor pairs. $\mathcal{I}'$ is a subset of $\mathcal{I}$ and contains K ($K \leq \frac{N(N-1)}{2}$) sensor pairs whose TDOA measurements are used for source localization. By introducing the $K \times 1$ vectors

$$\hat{\mathbf{t}} = \begin{bmatrix} \hat{t}_{ij} \\ \vdots \end{bmatrix}_{\{i,j\} \in \mathcal{I}'}, \mathbf{d} = \begin{bmatrix} d_{ij} \\ \vdots \end{bmatrix}_{\{i,j\} \in \mathcal{I}'}, \mathbf{e} = \begin{bmatrix} e_{ij} \\ \vdots \end{bmatrix}_{\{i,j\} \in \mathcal{I}'}, \quad (3.4)$$

the measurement model in vector form becomes

$$\hat{\mathbf{t}} = \frac{1}{v} \mathbf{d} + \mathbf{e}. \quad (3.5)$$

The problem of source localization is to estimate the source location $\mathbf{x}_t$ given $\hat{\mathbf{t}}$ and $\mathbf{x}_i, i = 1, \dots, N$. This chapter mainly focuses on how to design the optimal sensor pair geometry to improve the source localization and tracking accuracy.

3.2 Sensor Placement without Communication Constraint

In this section, theoretic analysis of optimal sensor pair geometry is given for the best source localization performance without considering communication constraints. Analytic solutions are derived for both the static and movable source cases. Except the results in Section 3.2.1.1, the results in other sections are novel contributions.

3.2.1 Centralized Sensor Pairing

Most of the existing TDOA based source localization approaches adopt a centralized sensor pairing method to extract the TDOAs in which there is one common reference node to compute TDOA measurements via the generalized cross correlation (GCC) method. See Figure 2.2(a).

3.2.1.1 Static source scenario

In the static scenario, the localization can be done by using estimation methods such as MLE, NLS methods, etc. The localization performance can be explicitly characterized by certain measures, for example, by the CRLB [133, 47]. It is well known that MLE can achieve CRLB asymptotically [8]. CRLB is related to the sensor-source geometry which gives the lowest bound for error covariance matrix among any unbiased estimators. For an unbiased estimate $\hat{\mathbf{x}}_t$ of $\mathbf{x}_t$, we find

$$\mathbb{E}[(\hat{\mathbf{x}}_t - \mathbf{x}_t)(\hat{\mathbf{x}}_t - \mathbf{x}_t)^T] \geq \mathbf{J}^{-1} = \mathbf{CRLB}, \quad (3.6)$$

where $\mathbf{J}$ is the Fisher information matrix (FIM). The (i, j) -th element of $\mathbf{J}$ is given by

$$\mathbf{J}_{i,j}(\mathbf{x}_t) = \mathbb{E}\left[\frac{\partial}{\partial \mathbf{x}_{t_i}} \ln(f(\hat{\mathbf{t}}; \mathbf{x}_t)) \frac{\partial}{\partial \mathbf{x}_{t_j}} \ln(f(\hat{\mathbf{t}}; \mathbf{x}_t))\right],$$

where $f(\hat{\mathbf{t}}; \mathbf{x}_t)$ is the probability density function (PDF) of $\hat{\mathbf{t}}$ given by

$$f(\hat{\mathbf{t}}; \mathbf{x}_t) = \frac{1}{(2\pi)^{N/2} \sqrt{\det(\Sigma)}} \exp\left[-\frac{1}{2}(\hat{\mathbf{t}} - \frac{1}{v}\mathbf{d})^T \Sigma^{-1} (\hat{\mathbf{t}} - \frac{1}{v}\mathbf{d})\right] \quad (3.7)$$

with Σ the covariance matrix of TDOA error $\mathbf{e}$.

The CRLB for TDOA based localization has been derived in [19] which is given by

$$\mathbf{J}^{-1} = v^2(\mathbf{G}\Sigma^{-1}\mathbf{G}^T)^{-1}, \quad (3.8)$$

where

$$\mathbf{G} = [g_{ij}, \dots]_{\{i,j\} \in \mathcal{I}}, \quad (3.9)$$

$$g_{ij} = g_i - g_j, \quad (3.10)$$

$$g_i = \begin{bmatrix} \frac{x_t - x_i}{\sqrt{(x_t - x_i)^2 + (y_t - y_i)^2}} \\ \frac{y_t - y_i}{\sqrt{(x_t - x_i)^2 + (y_t - y_i)^2}} \end{bmatrix} = \begin{bmatrix} \cos \alpha_i \\ \sin \alpha_i \end{bmatrix}, \quad (3.11)$$

where α_i denotes the angle of arrival measurement of the i -th sensor.

We are interested in the best sensor pair geometry for localization. In this chapter, a scalar function, i.e., the trace of the CRLB is used as the objective function to assess the optimality of the relative sensor-source geometry, i.e., A-optimality. For the centralized sensor pairing, without loss of generality, the first sensor is assigned as the reference node and $\mathcal{I}_0$ is defined as the corresponding subset of sensor pairs, i.e., $\mathcal{I}_0 = [\{21\}, \{31\}, \dots, \{N, 1\}]$, N denotes the number of the sensors. Hence, the TDOA parameter vectors become,

$$\mathbf{G} = [g_{21}, g_{31}, \dots, g_{N1}]$$

$$\Sigma = 0.5\sigma^2(\mathbf{I} + \mathbf{1}\mathbf{1}^T),$$

where $\mathbf{1} \in \mathbb{R}^N$ is the vector with all entries equal to 1.

The optimal sensor pair geometry for the centralized sensor pairing is summarized

in the following theorem.

Theorem 3.1. *Given the set of the sensor pairs $\mathcal{I}_0$, then*

$$\mathbf{Tr}(\mathbf{J}^{-1}) \geq \frac{4}{bN^2},$$

where $N \geq 3$ denotes the number of the sensors, and $b = \frac{1}{(v\sigma)^2}$. $\mathbf{Tr}(\cdot)$ denotes the trace of a matrix.

The equality holds if and only if

$$\begin{aligned} \sum_{i=1}^N \cos \alpha_i &= 0, & \sum_{i=1}^N \sin \alpha_i &= 0, \\ \sum_{i=1}^N \cos 2\alpha_i &= 0, & \sum_{i=1}^N \sin 2\alpha_i &= 0, \end{aligned} \tag{3.12}$$

where α_i denotes the angle of arrival measurement of the i -th sensor which is computed with respect to an unbiased estimate $\hat{\mathbf{x}}_t$.

Proof. A similar proof can be found in [69] but is omitted here. □

Remark 3.1. *From the results of Theorem 3.1, it can be seen that the optimal sensor pair geometry does not depend on the ranges between the source and the sensors. It only depends on the directions from the source to the sensors. This is mainly because that measurement noise e_{ij} is assumed distance-independent. Furthermore, in the optimal sensor-source geometry setting, any sensor can be chosen as the reference which results in the same lower bound of the localization performance.*

It is well known that a uniform angular array (UAA) is a special case of the above optimal sensor placement strategy, where

$$\alpha_i = \frac{2\pi}{N}(i-1) + \phi, i = 1, 2, \dots, N, \tag{3.13}$$

where $\phi \in (0, 2\pi)$ is any given constant. In Figure 3.1, an example of the three-sensors case is given, where black star and dots denote the source and sensors, respectively.

Figure 3.1: Optimal three-sensors geometry for centralized sensor pairing for the static source case

3.2.1.2 Movable source case

A movable source can be considered as a random parameter evolving under a stochastic difference equation [76]. Let $\mathbf{x}_t(k)$ denote the source location at time k which evolves according to the following state equation:

$$\mathbf{x}_t(k+1) = F_k(\mathbf{x}_t(k)) + w(k), \quad (3.14)$$

for some functions $F_k : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, where $\mathbf{x}_t(k) = [x_t(k), y_t(k)]^T$ denotes the source location at time k , $w(k) \sim (0, \Lambda_k)$ is the process noise.

Similarly as before, the TDOA measurement model is given by

$$\hat{t}_{ij}(k) = \frac{d_{ij}}{v}(k) + e_{ij}(k). \quad (3.15)$$

The extended Kalman filter (EKF) is widely employed for the source localization and tracking problem for nonlinear systems [76, 70, 93]. The EKF provides a state estimate $\hat{\mathbf{x}}_t(k+1)$ together with an estimate of the covariance of the estimation error, $\mathbf{P}_{k+1|k+1}$, which can be written as [131]:

$$\mathbf{P}_{k+1|k+1} = ((\mathbf{P}_{k+1|k})^{-1} + \mathbf{J}_{k+1})^{-1}, \quad (3.16)$$

where $\mathbf{P}_{k+1|k}$ is the one-step prediction error covariance of $\mathbf{x}_t(k+1)$ which is assumed to be positive definite and $\mathbf{J}_{k+1}$ is the FIM associated with the sensor measurements at time step $k+1$.

Here, we mainly focus on how to design a sensor-source geometry which will enable an optimal localization under certain criterion in terms of the posterior error covariance matrix $\mathbf{P}_{k+1|k}$. Note that the *a priori* source location information matrix $\mathbf{P}_{k+1|k}$ plays an important role in the geometry design, but has not been investigated in the literature.

Without loss of generality, this chapter focuses on a 2D case and assumes that

$$\mathbf{P}_{k+1|k} = \begin{bmatrix} p_1 & p_3 \\ p_3 & p_2 \end{bmatrix} \quad (3.17)$$

which is positive definite. Then, define

$$\mathbf{Q}_{k+1} = (\mathbf{P}_{k+1|k+1})^{-1}.$$

Combining (3.16) and (3.17), $\mathbf{Q}_{k+1}$ can be expressed as in (3.18), where the subscript, ' $k+1$ ' is omitted. The detail of the derivation is also omitted here.

$$\mathbf{Q} = b \begin{bmatrix} \sum_{i=1}^N \cos^2 \alpha_i - \frac{1}{N} (\sum_{i=1}^N \cos \alpha_i)^2 + \frac{p_2}{b(p_1 p_2 - p_3^2)}, \\ \sum_{i=1}^N \frac{\sin 2\alpha_i}{2} - \frac{1}{N} \sum_{i=1}^N \cos \alpha_i \sum_{i=1}^N \sin \alpha_i - \frac{p_3}{b(p_1 p_2 - p_3^2)}, \\ \sum_{i=1}^N \frac{\sin 2\alpha_i}{2} - \frac{1}{N} \sum_{i=1}^N \cos \alpha_i \sum_{i=1}^N \sin \alpha_i - \frac{p_3}{b(p_1 p_2 - p_3^2)}, \\ \sum_{i=1}^N \sin^2 \alpha_i - \frac{1}{N} (\sum_{i=1}^N \sin \alpha_i)^2 + \frac{p_1}{b(p_1 p_2 - p_3^2)} \end{bmatrix} \quad (3.18)$$

For the optimal sensor-source geometry, the following results can be obtained.

Theorem 3.2. *If $\mathcal{I}' = \mathcal{I}_0$, then we have*

$$\text{Tr}(\mathbf{P}_{k+1|k+1}) \geq \frac{4}{bN + \frac{p_1 + p_2}{p_1 p_2 - p_3^2}}, \quad (3.19)$$

where N denotes the number of sensors and $N \geq 4$.

Further, the equality in (3.19) holds if and only if

$$C1) \sum_{i=1}^N \cos \alpha_i = 0, \quad \sum_{i=1}^N \sin \alpha_i = 0;$$

$$C2) \sum_{i=1}^N \cos 2\alpha_i = \frac{p_1 - p_2}{b(p_1 p_2 - p_3^2)}, \quad \sum_{i=1}^N \sin 2\alpha_i = \frac{p_3}{2b(p_1 p_2 - p_3^2)}.$$

Proof. The proof consists of two parts. The first part is similar to Theorem 1 in [133], that is to find a lower bound for $\mathbf{Tr}(\mathbf{P}_{k+1|k+1})$ in terms of $\mathbf{Tr}(\mathbf{Q}_{k+1})$. We reproduce it here for the convenience.

Let $\lambda_i, i = 1, 2$ be the eigenvalues of the 2×2 matrix $\mathbf{Q}_{k+1}$ which is assumed to be positive definite. The eigenvalues of $\mathbf{P}_{k+1|k+1}$ are $1/\lambda_i$. It is well known that

$$\mathbf{Tr}(\mathbf{Q}_{k+1}) = \lambda_1 + \lambda_2, \quad \mathbf{Tr}(\mathbf{P}_{k+1|k+1}) = 1/\lambda_1 + 1/\lambda_2.$$

It is obvious that

$$\begin{aligned} 2 &\leq \sqrt{(\lambda_1 + \lambda_2)(1/\lambda_1 + 1/\lambda_2)} \\ &= \sqrt{\mathbf{Tr}(\mathbf{Q}_{k+1})\mathbf{Tr}(\mathbf{P}_{k+1|k+1})} \end{aligned}$$

which implies that

$$\mathbf{Tr}(\mathbf{P}_{k+1|k+1}) \geq \frac{4}{\mathbf{Tr}(\mathbf{Q}_{k+1})} \quad (3.20)$$

The equality holds if and only if the two eigenvalues of matrix $\mathbf{Q}_{k+1}$ are identical, i.e., $\lambda_1 = \lambda_2 = \lambda$. This is equivalent to

$$\mathbf{Q}_{k+1} = \lambda \mathbf{I} \quad (3.21)$$

Next, the term $\mathbf{Tr}(\mathbf{Q}_{k+1})$ is studied. From (3.18), it can be obtained that

$$\begin{aligned} \frac{1}{b} \mathbf{Tr}(\mathbf{Q}_{k+1}) &= N - \frac{1}{N} \left[\left(\sum_{i=1}^N \cos \alpha_i \right)^2 + \left(\sum_{i=1}^N \sin \alpha_i \right)^2 \right] \\ &\quad + \frac{p_1 + p_2}{b(p_1 p_2 - p_3^2)} \leq N + \frac{p_1 + p_2}{b(p_1 p_2 - p_3^2)}, \end{aligned} \quad (3.22)$$

which together with (3.20) leads to (3.19).

The equality holds if and only if

$$\sum_{i=1}^N \cos \alpha_i = 0, \quad \sum_{i=1}^N \sin \alpha_i = 0$$

i.e., the condition C1 is satisfied.

Combining (3.21) and (3.22), we can obtain

$$\begin{aligned} \sum_{i=1}^N \cos 2\alpha_i &= \frac{p_1 - p_2}{b(p_1 p_2 - p_3^2)} \\ \sum_{i=1}^N \sin 2\alpha_i &= \frac{p_3}{2b(p_1 p_2 - p_3^2)} \end{aligned} \quad (3.23)$$

i.e., the condition C2 holds. This completes the proof. $\square$

Figure 3.2: Effect of p_3

If $|\frac{p_1 - p_2}{b(p_1 p_2 - p_3^2)}| > N$ or $|\frac{p_3}{2b(p_1 p_2 - p_3^2)}| > N$, it may not be possible to get an analytic solution for the sensor-geometry problem. In this case, the following optimization can be carried out.

$$\min_{\alpha_i} \text{Tr}(\mathbf{P}_{k+1|k+1}) \quad (3.24)$$

for which, some numerical algorithms, e.g., a gradient search based method, can be used but may only give a local optimal solution.

Comparing the results in Theorem 3.1 and Theorem 3.2, it can be found that the *a priori* source location information, i.e., $\mathbf{P}_{k+1|k}$, does affect the optimal sensor

platform formations for the next time step, i.e., $k + 1$. For example, if $p_1 > p_2$, then $\sum_{i=1}^N \cos 2\alpha_i > 0$ which is different from that in Theorem 3.1. However, if $p_1 = p_2$, then $\sum_{i=1}^N \cos 2\alpha_i = 0$ which is the same as in Theorem 3.1. $p_3/\sqrt{p_1 p_2}$ denotes the error correlation coefficient between x and y directions. The value of p_3 will affect the direction of the coordinate system of sensor formation as seen in Figure 3.2. Next, a simple example is provided to illustrate the conditions in Theorem 3.2.

Example 3.1. *Consider the case with four sensors. Without loss of generality, sensor 1 is chosen as the reference. The source location estimate at time step k is denoted by $\hat{\mathbf{x}}_t(k) = [4, 4]^T$, and the estimation error covariance matrix $\hat{\mathbf{P}}_{k+1|k}$ is given by*

$$\hat{\mathbf{P}}_{k+1|k} = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix} \quad (3.25)$$

The 1-sigma uncertainty ellipse is plotted in Figure 3.3.

The objective is to minimize the mean square error (MSE), i.e., $\min \text{Tr}(\mathbf{P}_{k+1|k+1})$. Specifically, we assume that $\alpha_i, i = 1, 2, 3, 4$ are $\alpha_1 = \beta, \alpha_2 = -\beta, \alpha_3 = \pi - \beta, \alpha_4 = \pi + \beta$, where $0 \leq \beta \leq \pi/2$. Then, it can be easily checked that C1 and condition $\sum_{i=1}^4 \sin 2\alpha_i = 0$ are satisfied. Substituting the α_i into the first condition in (3.23), it can be obtained that

$$4 \cos^2 \beta = 2 + \frac{1}{2b} \left(1 - \frac{1}{10}\right).$$

Solving the above equation, it can be obtained that $\beta = \arccos \sqrt{0.5 + \frac{9}{80b}}$. Hence one solution for the optimal sensor placement is presented as follows:

$$\begin{cases} \alpha_1 = \arccos \sqrt{0.5 + \frac{9}{80b}} \\ \alpha_2 = 2\pi - \arccos \sqrt{0.5 + \frac{9}{80b}} \\ \alpha_3 = \pi - \arccos \sqrt{0.5 + \frac{9}{80b}} \\ \alpha_4 = \pi + \arccos \sqrt{0.5 + \frac{9}{80b}} \end{cases} \quad (3.26)$$

The result in (3.26) is also presented in Figure 3.3.

Note that the solutions for the above example are not unique.

Figure 3.3: Optimal three-sensor geometry for centralized pairing with source location uncertainty

3.2.2 Decentralized Sensor Pairing

Due to the power and bandwidth constraints in WSNs, the centralized sensor pairing is not realistic, especially for a large scale and unreliable network. Hence, in this chapter, the decentralized sensor pairing is also considered, which can handle the bandwidth constraint and communication issue in the centralized sensor pairing. A simple case for decentralized sensor pairing in which there is no common reference sensor is shown in Figure 2.2(b) of Chapter 2.

In the following subsections, theoretical study will be performed on the optimal sensor pair geometry for the decentralized sensor pairing strategy, for both static and movable source scenarios.

3.2.2.1 Static source case

Firstly, the decentralized sensor pairing for the static source case is considered. Similarly, the trace of CRLB is utilized as a metric when deriving the characterization of the sensor-source geometry. Define $\hat{\mathbf{x}}_t$ as an unbiased estimate of source location and $\mathcal{I}_1$ as a subset of sensor pairs, i.e., $\mathcal{I}_1 = [\{i, j\}, \{k, l\}, \dots]_{\mathcal{I}_1 \in \mathcal{I}, i \neq j \neq k \neq l}$ (without sensor sharing). Thus the corresponding TDOA parameter matrices become

$$\mathbf{G} = [g_{ij}, g_{kl}, \dots]_{\mathcal{I}_1 \in \mathcal{I}, i \neq j \neq k \neq l}$$

$$\Sigma = \sigma^2 \mathbf{I}$$

Through some algebraic manipulations, the FIM, $\mathbf{J}^d$, can be written as in (3.27) below, where superscript d means ‘decentralized’ and $b = 1/(\nu\sigma)^2$. Consequently, the trace of CRLB can be expressed as in (3.28). The detail of the derivation is omitted here.

$$\mathbf{J}^d = \frac{1}{b} \begin{bmatrix} \sum_{\{i,j\} \in \mathcal{I}_1} (\cos \alpha_i - \cos \alpha_j)^2, \\ \sum_{\{i,j\} \in \mathcal{I}_1} (\cos \alpha_i - \cos \alpha_j)(\sin \alpha_i - \sin \alpha_j), \\ \sum_{\{i,j\} \in \mathcal{I}_1} (\cos \alpha_i - \cos \alpha_j)(\sin \alpha_i - \sin \alpha_j) \\ \sum_{\{i,j\} \in \mathcal{I}_1} (\sin \alpha_i - \sin \alpha_j)^2 \end{bmatrix} \quad (3.27)$$

$$\text{Tr}((\mathbf{J}^d)^{-1}) = \frac{1}{b} \frac{\sum_{\{i,j\} \in \mathcal{I}_1} 2 - 2 \cos(\alpha_i - \alpha_j)}{\mathbf{DEN}} \quad (3.28)$$

where $\mathbf{DEN} = \sum_{\{i,j\}\{k,l\} \in \mathcal{I}_1} (\cos \alpha_i - \cos \alpha_j)(\sin \alpha_k - \sin \alpha_l)[\sin(\alpha_k - \alpha_i) - \sin(\alpha_k - \alpha_j) + \sin(\alpha_l - \alpha_j) - \sin(\alpha_l - \alpha_i)]$.

Theorem 3.3. *If $\mathcal{I}' = \mathcal{I}_1$, then we have*

$$\text{Tr}((\mathbf{J}^d)^{-1}) \geq \frac{1}{bW}, \quad (3.29)$$

where $W \geq 2$ denotes the number of sensor pairs (without sensor sharing).

The equality holds if and only if

$$C1) \alpha_i - \alpha_j = \pm\pi, \forall \{i, j\} \in \mathcal{I}_1;$$

$$C2) \sum_{i \in \{i,j\} \in \mathcal{I}} \cos^2 \alpha_i = \sum_{i \in \{i,j\} \in \mathcal{I}} \sin^2 \alpha_i = W/2, \sum_{i \in \{i,j\} \in \mathcal{I}} \sin 2\alpha_i = 0.$$

Proof. The proof is similar to Theorem 3.2, that is firstly to find a lower bound for $\text{Tr}((\mathbf{J}^d)^{-1})$ in terms of $\text{Tr}(\mathbf{J}^d)$. Similarly as before, we have

$$\text{Tr}((\mathbf{J}^d)^{-1}) \geq \frac{4}{\text{Tr}(\mathbf{J}^d)}. \quad (3.30)$$

The equality holds if and only if two eigenvalues of matrix $\mathbf{J}^d$ are identical, i.e., $\lambda_1 = \lambda_2 = \lambda$. This is equivalent to

$$\mathbf{J}^d = \lambda \mathbf{I} \quad (3.31)$$

From (3.27), it can be obtained that

$$\text{Tr}(\mathbf{J}^d) = \sum_{\{i,j\} \in \mathcal{I}_1} (2 - 2 \cos(\alpha_i - \alpha_j)) \leq 4W.$$

The equality holds if and only if

$$\cos(\alpha_i - \alpha_j) = 0, \forall \{i, j\} \in \mathcal{I}_1 \Leftrightarrow \alpha_i - \alpha_j = \pm\pi, \quad (3.32)$$

i.e., the condition C1 is satisfied.

Combining (3.31) and (3.32), C2 can be obtained

$$\sum_{i \in \{i,j\} \in \mathcal{I}_1} \cos^2 \alpha_i = \sum_{i \in \{i,j\} \in \mathcal{I}_1} \sin^2 \alpha_i = W/2, \quad (3.33)$$

$$\sum_{i \in \{i,j\} \in \mathcal{I}_1} \cos \alpha_i \sin \alpha_i = 0. \quad (3.34)$$

This completes the proof. $\square$

It is easy to check that a uniform angular array (UAA) is also a special case of the above optimal sensor placement strategy. In Figure 3.4, an example of two sensor pairs is given, where star and black dots denote the source and sensors respectively and each black line denotes the edge which connects one sensor pair.

Guideline for sensor placement and sensor selection: From Theorem 3.3, it can be seen that for the decentralized sensor pairing (without sensor sharing), the CRLB is greatly affected by two factors: (1) The angle, denoted by $v_{i,j}$, $0 \leq v_{i,j} \leq \pi$, subtended at the source by sensors i and j . A large $v_{i,j}$ is preferred. (2) The

Figure 3.4: Optimal two sensor pairs geometry with known source location

intersection angles among the sensor pairs. For the two sensor pairs case, a large intersection angle is also preferred, where intersection angle equal to $\pi/2$ is the best choice.

3.2.2.2 Movable source case

For the movable source case, the source dynamic model is the same as in Section 3.2.1.2. An EKF is also utilized for source localization and the error covariance matrix at time step $k + 1$ is denoted by $\mathbf{P}_{k+1|k+1}^{d'}$.

Through some algebraic manipulations, $\mathbf{J}_{k+1}^{d'} = (\mathbf{P}_{k+1|k+1}^{d'})^{-1}$ can be obtained as in (3.35) below, where subscript ' $k + 1$ ' is omitted and $\alpha_i, i \in \{i, j\} \in \mathcal{I}_1$ are computed based on the state estimate $\hat{\mathbf{x}}_t(k)$.

$$\mathbf{J}^{d'} = b \begin{bmatrix} \sum_{\{i,j\} \in \mathcal{I}_1} (\cos \alpha_i - \cos \alpha_j)^2 + \frac{p_2}{b(p_1 p_2 - p_3^2)}, \\ \sum_{\{i,j\} \in \mathcal{I}_1} (\cos \alpha_i - \cos \alpha_j)(\sin \alpha_i - \sin \alpha_j) - \frac{p_3}{b(p_1 p_2 - p_3^2)}, \\ \sum_{\{i,j\} \in \mathcal{I}_1} (\cos \alpha_i - \cos \alpha_j)(\sin \alpha_i - \sin \alpha_j) - \frac{p_3}{b(p_1 p_2 - p_3^2)} \\ \sum_{\{i,j\} \in \mathcal{I}_1} (\sin \alpha_i - \sin \alpha_j)^2 + \frac{p_1}{b(p_1 p_2 - p_3^2)} \end{bmatrix} \quad (3.35)$$

For the optimal sensor-source geometry, the result is summarized in the following theorem.

Theorem 3.4. *If $\mathcal{I}' = \mathcal{I}_1$, then we have*

$$\text{Tr}(\mathbf{P}_{k+1|k+1}^{d'}) \geq \frac{1}{b} \frac{4}{4W + \frac{p_1+p_2}{b(p_1p_2-p_3^2)}}, \quad (3.36)$$

where $W \geq 2$ denotes the number of pairs (without sensor sharing).

The equality holds if and only if

$$C1) \alpha_i - \alpha_j = \pm\pi, \forall \{i, j\} \in \mathcal{I}_1$$

$$C2) \sum_{i \in \{i, j\} \in \mathcal{I}} \cos 2\alpha_i = \frac{p_1-p_2}{4b(p_1p_2-p_3^2)} \text{ and } \sum_{i \in \{i, j\} \in \mathcal{I}} \sin 2\alpha_i = \frac{p_3}{2b(p_1p_2-p_3^2)}$$

Proof. The proof is similar to that of Theorem 3.3. Similarly as before,

$$\text{Tr}(\mathbf{P}_{k+1|k+1}^{d'}) \geq \frac{4}{\text{Tr}(\mathbf{J}_{k+1}^{d'})} \quad (3.37)$$

The equality holds if and only if two eigenvalues of matrix $\mathbf{J}_{k+1}^{d'}$ are identical, i.e., $\lambda'_1 = \lambda'_2 = \lambda'$. This is equivalent to

$$\mathbf{J}_{k+1}^d = \lambda' \mathbf{I} \quad (3.38)$$

On the other hand, from (3.35), we have

$$\begin{aligned} \text{Tr}(\mathbf{J}_{k+1}^{d'}) &= b \left(\sum_{\{i, j\} \in \mathcal{I}_1} (2 - 2 \cos(\alpha_i - \alpha_j)) + \frac{p_1 + p_2}{b(p_1p_2 - p_3^2)} \right) \\ &\leq 4bW + \frac{p_1 + p_2}{(p_1p_2 - p_3^2)}. \end{aligned} \quad (3.39)$$

The equality holds if and only if the condition C1 is satisfied:

$$\cos(\alpha_i - \alpha_j) = 0, \forall \{i, j\} \in \mathcal{I}_1 \Leftrightarrow \alpha_i - \alpha_j = \pm\pi. \quad (3.40)$$

Combining the results in (3.38) and (3.40), the following two conditions need to be satisfied

$$\sum_{i \in \{i, j\} \in \mathcal{I}_1} 4 \cos^2 \alpha_i + \frac{p_2}{b(p_1p_2 - p_3^2)}$$

$$= \sum_{i \in \{i,j\} \in \mathcal{I}_1} 4 \sin^2 \alpha_i + \frac{p_1}{b(p_1 p_2 - p_3^2)} = 2W + \frac{p_1 + p_2}{2b(p_1 p_2 - p_3^2)} \quad (3.41)$$

$$\sum_{i \in \{i,j\} \in \mathcal{I}_1} \sin 2\alpha_i = \frac{p_3}{2b(p_1 p_2 - p_3^2)} \quad (3.42)$$

Through some simple derivations, (3.41) can be written as $\sum_{i \in \{i,j\} \in \mathcal{I}_1} \cos^2 \alpha_i = W/2 + \frac{p_1 - p_2}{8b(p_1 p_2 - p_3^2)}$, $\sum_{i \in \{i,j\} \in \mathcal{I}_1} \sin^2 \alpha_i = W/2 + \frac{p_2 - p_1}{8b(p_1 p_2 - p_3^2)}$, which is equivalent to $\sum_{i \in \{i,j\} \in \mathcal{I}} \cos 2\alpha_i = \frac{p_1 - p_2}{4b(p_1 p_2 - p_3^2)}$. This completes the proof. $\square$

Note that when $|\frac{p_1 - p_2}{8b(p_1 p_2 - p_3^2)}| > W/2$ or $|\frac{p_3}{2b(p_1 p_2 - p_3^2)}| > W$ the optimality condition in C1 and C2 can not be satisfied. In this case, a numerical optimization can be carried out to find an optimal sensor-source geometry, which is similar with that in the centralized case.

Example 3.2. Here, a simple example is given to illustrate the conditions in Theorem 3.4. More specifically, two pairs of sensors $\{i, j\}, \{k, l\}$ will be placed for source localization. The source location estimate at time step k , $\hat{\mathbf{x}}_t(k) = [4, 4]^T$, and $\hat{\mathbf{P}}_{k+1|k}$ is given by

$$\hat{\mathbf{P}}_{k+1|k} = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix} \quad (3.43)$$

The 1-sigma error covariance ellipse is plotted in Figure 3.5.

The objective is to minimize the mean square error, i.e., $\min \text{Tr}(\mathbf{P}_{k+1|k+1}^{d'})$. Without loss of generality, we assume that $\alpha_i \leq \alpha_j$, and $\alpha_k \leq \alpha_l$. At first, for the condition stated in (3.42), we can get that

$$\sin 2\alpha_i = -\sin 2\alpha_k$$

Hence, it is easy to obtain that $\alpha_k = \pi/2 + \alpha_i$ or $\alpha_k = \pi - \alpha_i$. When we substitute these two intermediate results into (3.41), we can easily find that the result $\alpha_k =$

$\pi/2 + \alpha_i$ is invalid. Continuing the computation, we can obtain that

$$2 \cos^2 \alpha_i = 1 + \frac{1}{8b} \left(1 - \frac{1}{10}\right).$$

Hence, $\alpha_i = \arccos \sqrt{0.5 + \frac{9}{160b}}$ or $\alpha_i = \pi - \arccos \sqrt{0.5 + \frac{9}{160b}}$. Then the angle of arrivals for all the sensors are summarized as follows:

$$\begin{cases} \alpha_i = \arccos \sqrt{0.5 + \frac{9}{160b}} \\ \alpha_j = \pi + \arccos \sqrt{0.5 + \frac{9}{160b}} \\ \alpha_k = \pi - \arccos \sqrt{0.5 + \frac{9}{160b}} \\ \alpha_l = 2\pi - \arccos \sqrt{0.5 + \frac{9}{160b}} \end{cases} \quad (3.44)$$

The result in (3.44) is also plotted in Figure 3.5. From the result and the figure, it can be seen that it is better to place the sensors with the direction close to the x -axes. This is because the error variance of the state estimate for $\hat{x}_t$ is larger than that for $\hat{y}_t$.

Guideline for sensor placement and sensor selection: Assume that the two eigenvalues of $\hat{\mathbf{P}}_{k+1|k}$ are denoted by λ_1 and λ_2 associated with their corresponding eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. Intuitively, more sensor pairs should be placed close to the direction of the eigenvector with a larger eigenvalue.

Remark 3.2. Comparing Figure 3.5 with Figure 3.3, it can be seen that sensors in centralized pairing are placed closer to the direction of the eigenvector with larger eigenvalue. This is because that the centralized pairing can access more information compared with the decentralized pairing method by using the same number of sensors. However, in real applications, due to communication and computation constraints, the centralized pairing method may not be applicable.

3.3 Sensor Trajectory Optimization

Until now, the optimal sensor pair geometry has been investigated without considering sensor motion and communication constraints. Although the results for

Figure 3.5: Optimal two sensor pairs placement for decentralized sensor pairing with source location uncertainty

sensor placement in the past sections can provide an intuition about the expected trajectory shape, in real applications, generally, sensors are not around the sources. In many cases, they are movable and may start from the same places when they take over the source localization and tracking task. Due to the motion constraint of the sensor platforms, such as speed and turning rate constraints of unmanned aerial vehicles (UAVs) and unmanned ground vehicles (UGVs), the optimal geometry obtained above may not be achieved directly. Hence path planning for UAVs/UGVs is needed in order to achieve the best localization performance under motion, communication and/or environment constraints..

3.3.1 Trajectory Optimization Problem

The sensor platform, e.g., unmanned ground vehicle (UGV), is assumed to have a constant velocity and a maximal turn rate constraint. The source of interest is stationary.

The discrete-time sensor kinematic equations of motion are given by

$$x_i(k+1) = x_i(k) + vT \cos u_i(k), \quad x_i(0) = x_{i_0}, \quad (3.45)$$

$$y_i(k+1) = y_i(k) + vT \sin u_i(k), \quad y_i(0) = y_{i_0}, \quad (3.46)$$

where $[x_{i_0}; y_{i_0}]$ is the initial sensor position. $u_i(k)$ denotes the control input (heading angle) for the i -th sensor at time step k . v and T denote the velocity and sampling interval of the sensor platforms respectively.

Since the source is assumed to be stationary, hence the parameters in source motion model (3.14) are given by $F_k = [1, 0; 0, 1]$ and $\Lambda_k = [0, 0; 0, 0]$. At each step, after a sensor takes a measurement, an EKF is used to update the state estimation, then the sink node will run the sensor geometry optimization for the next step. In this work, the online TDOA sensor pair path planning problem is cast as a constrained nonlinear optimization problem. The objective is to minimize the localization error under the sensor motion and communication constraints, etc. The problem is formulated as follows:

$$\min_{u(k+1)} \mathcal{F}(u(k+1)) = \mathbf{Tr}(\mathbf{J}_{k+1}^{-1}) \quad (3.47)$$

subject to:

$$\|u_i(k+1) - u_i(k)\| \leq u_{max} \quad (3.48)$$

$$h(\mathbf{x}_i(k+1), \hat{\mathbf{x}}_t(k)) \geq r_i \quad (3.49)$$

$$h(\mathbf{x}_i(k+1), \hat{\mathbf{x}}_t(k)) \leq r'_i \quad (3.50)$$

$$h(\mathbf{x}_i(k+1), \mathbf{x}_j(k+1)) \geq a_{ij} \quad (3.51)$$

$$h(\mathbf{x}_i(k+1), \mathbf{x}_j(k+1)) \leq b_{ij} \quad (3.52)$$

where $u(k) = [u_1(k), \dots, u_N(k)]$. The condition (3.48) presents the sensor platform input constraint, e.g., UAV turning rate constraint. The inequality (3.49) is concerned with the sensor to source distance which must be larger than a given threshold where $r_i > 0$. The condition (3.50) is about the baseline expansion constraint where r'_i is a given constant and $r'_i > r_i$. This is because that in certain situations, minimizing the objective function may result in favoring baseline expansion which is not desired. The condition (3.51) addresses the collision avoidance between the sensors where a_{ij} is a constant. Actually, we can also consider the collision avoidance

between sensor and potential obstacles. The inequality (3.52) shows the network connectivity conditions, e.g., sensor communication distance constraints. In this chapter, the channel noise and packet loss issues are not considered, but it is assumed that sensors can communicate with each other if the distance between them is not larger than a given constant value.

From the problem formulation (Equations (3.47-3.52)), it can be seen that it is a nonlinear optimization problem with both linear and nonlinear constraints. Direct gradient method can not be used for this kind of problems. Generally, the sequential quadratic programming (SQP) algorithm can be utilized. For a numerical solution for this kind of SQP problems, the Lagrangian based method is developed in [14].

3.3.2 Simulation Results

Some simulation results are provided for the two types of sensor pairing methods. Four UAVs will be used to localize a stationary source. The true source location is $\mathbf{x}_t = [0; 0]$ with initial guess $\hat{\mathbf{x}}_t(0) = [-50; -20]$, $P_0 = [50, 0; 0, 50]$; The initial positions of the UAVs are $[-100; -50]$, $[-98; -50]$, $[-96; -50]$, $[-94; -50]$, respectively. The initial heading for UAVs are all equal to $\frac{\pi}{2}$ (North). The other parameters are $u_{max} = 15^\circ$, $r_i = 10$ m $\forall i$, $r'_i = 200$ m $\forall i$, $a_{ij} = 1$ m, $b_{ij} = 20$ m, $v = 30$ /s, $T = 1$, $\sigma = 1$.

3.3.2.1 Sensor trajectory optimization with centralized pairing

As seen from the initial parameters defined above, the starting points of the four UAVs are close to each other. Each UAV is supposed to take 20 measurements only.

Firstly, the sensor pair geometry optimization with only UAV turning rate constraint is investigated. Hence the objective function is as in (3.47) with (3.48) as the only constraint. The simulation results are presented in Figure 3.6. The red curve in Figure 3.6(a) is the trajectory of the reference node. The red triangle and dot denote the true source location and the initial guess of the source location. The

Figure 3.6: Centralized sensor pairing with only turning rate constraint

Figure 3.7: Centralized sensor pairing with all constraints

small rectangle denotes the location estimate at each time step. From the figure, it can be seen that two sensors are trying to move away from the reference node to obtain a large angle subtended at the source. This coincides with the analysis in Section 3.2. It is interesting to note that during the first several steps, the UAVs have not obtained much information around the x axes due to the motion constraints. After the 5th time step, the localization errors in the x and y axes drop sharply when the UAVs encompass the source.

However, there are several problems for the above scenario. The first one is the baseline expansion problem. From Figure 3.6(a), it can be found that minimizing the localization error may result in favoring baseline expansion, which is not desired. Another issue is about the sensor communication problem. In real applications,

sensors need to communicate with the reference node to do the TDOA estimations. However, as shown in Figure 3.6(a), two sensors (green and black curves) move far away from the reference node (red curve). In this case, the communications between them may not be guaranteed. Also in some cases, collision between sensors may occur.

Accordingly, all constraints are included in the optimization problem. The simulation results are shown in Figure 3.7. Different from the results in Figure 3.6(a), the reference sensor has to keep close to the other three sensors to ensure the quality of communications. On the contrary, the sensors tend to keep away from each other to provide a large angle subtended at the source. In addition, there is no baseline expansion when UAVs move towards the source. After 20 measurements, localization error almost converges to zero.

In addition, for comparison purposes, source localization without the trajectory optimization is also considered. In this case, UAVs is assumed to follow a random path with the turning rate constraint. Figure 3.8 shows the comparison result of localization performance: with and without sensor trajectory optimization for centralized sensor pairing. Clearly it can be seen that by optimizing the UAV trajectory, localization error covariance will converge to 0 fast.

3.3.2.2 Sensor trajectory optimization with decentralized pairing

Similar to the centralized pairing case, firstly, the sensor pair geometry optimization with only UAV turning rate constraint is studied. The simulation results are presented in Figure 3.9. The red and black curves in Figure 3.9(a) represent two separate sensor pairs. From the figure, it can be seen that two pairs tend to move towards opposite directions, i.e., southwest versus northeast. This is reasonable because the sensors need to surround the source to get more information about the source location. In addition, within each pair, two sensors are trying to move far away from each other to obtain a large angle subtended at the source. This also coincides with the analysis in Section 3.2.

Figure 3.8: Comparison of localization performance: with and without sensor trajectory optimization for centralized sensor pairing

Similarly, during the first several steps, the UAVs have not obtained much information around the x axes due to the motion constraints. After 5 time steps, the localization errors in the x and y axes drop sharply when the UAVs encompass the source. Besides, there are baseline expansion and communication issues for the above scenario.

Then the full constraints are considered in the optimization problem. The simulation results are shown in Figure 3.10. Compared with the results in Figure 3.10(a) and 3.10(b), due to the communication constraint, two sensors in a pair have to keep close to each other to ensure the quality of communications. On the contrary, the sensors tend to keep distance from each other to have a large angle subtended at the source. Furthermore, the sensors are trying to surround the source as much as possible while still maintaining as close to orthogonality between two pairs as possible. In addition, there is no baseline expansion when UAVs move towards the source. From the figures, it can be seen that after around 20 measurements, localization error almost converges to zero.

In addition, similar to the centralized sensor pairing case, the comparison results

Figure 3.9: Decentralized sensor pairing with only turning rate constraint

Figure 3.10: Decentralized sensor pairing with all constraints

can be obtained for the source localization with and without trajectory optimization. As seen from Figure 3.11, the localization performance using sensor trajectory optimization is better than the random path case.

3.4 Discussions

The results in Section 3.2, the theoretical analysis on the sensor pair geometry, can provide intuitions about the expected trajectory shape. In addition, it provides some insights about the expected optimization results in Section 3.3. Generally, the search based methods are used for path planning. However, it is well known

Figure 3.11: Comparison of localization performance: with and without sensor trajectory optimization for decentralized sensor pairing

that it can only converge to local extrema. As the number of the sensors, N , increases, the objective function has a growing number of local extrema causing severe sensitivities to initialization and often leading to suboptimal solutions if not initialized properly. Hence, the initialization phase plays an important role in this kind of method. The analytic results in Section 3.2 can be helpful for a proper choice of initial measurement configuration. For example, in our case, the initial input heading angles or initial positions of the UAVs can be designed by using the theoretical results in Section 3.2.

Furthermore, a multi-step trajectory optimization problem can also be considered, however, the computational resources required for a multi-step optimization is several orders of magnitude larger than that of the single-step optimization and the resulting algorithm can not be implemented in real-time using onboard UAV computers. Hence, in this chapter, only the single-step case is considered even though the multi-step trajectory optimization often provides better localization performance.

3.5 Conclusion

In this chapter, the characterizations of the optimal centralized and decentralized sensor pair geometries for static and movable source cases, have been provided. The CRLB and posterior error covariance matrix were utilized to assess the optimality of the relative sensor-source geometry respectively for two different cases. In addition, the study has been extended to the sensor pair trajectory optimization problem. The SQP method was utilized to solve this constrained nonlinear optimization problem. Simulations results showed that the proposed method can handle the sensor pair path planning under sensor motion and communication constraints, etc. Besides, it has a good localization performance.

Chapter 4

Sensor Placement in Heterogeneous Sensor Networks

In the last chapter, the optimal geometry of a team of networked sensors has been studied in a homogeneous network, where a single type of sensors is used for source localization. This chapter is concerned with the sensor placement problem in heterogeneous sensor networks (HSNs). In a WSN, many types of sensors can be used for source localization, e.g., range sensors, DOA sensors and TDOA sensors, etc. In this chapter, we consider a HSN consisting of two types of sensors. Relative optimal sensor-source configurations with the minimum number of sensors for source localization are identified under the D-optimality criterion with potential extensions to a general case. Explicit characterizations of optimal sensor-source geometries are given for hybrid range and DOA sensors, hybrid DOA and TDOA sensors as well as co-located hybrid range and DOA sensors, respectively. The results of this chapter can be applied to the sensor path planning problem for optimal source localization.

4.1 Problem Description

The problem of interest is to study the optimal sensor-source geometries for source localization in a heterogeneous sensor network (HSN). The source is assumed stationary with its location $\mathbf{x}_t = [x_t, y_t]^T$. There are several types of sensors that can

be placed in the sensor field: range sensors, DOA sensors and TDOA sensors. The numbers of these three types of sensors are N_1 , N_2 and N_3 respectively. The locations of the sensors are denoted by $\mathbf{x}_i^r = [x_{ri}, y_{ri}]^T, i = 1, \dots, N_1$ (range sensors), $\mathbf{x}_j^b = [x_{bj}, y_{bj}]^T, j = 1, \dots, N_2$ (DOA sensors), $\mathbf{x}_k^t = [x_{tk}, y_{tk}]^T, k = 1, \dots, N_3$ (TOA sensors), respectively.

The measurement models for the above three sensing techniques have been discussed in Chapter 2. For simplicity, here a simplified range measurement model is given in the following equation.

$$\hat{\mathbf{r}} = \mathbf{r}(\mathbf{x}_t) + \epsilon = [r_1, \dots, r_{N_1}]^T + [\epsilon_1, \dots, \epsilon_{N_1}]^T, \quad (4.1)$$

where $r_i = \|\mathbf{x}_t - \mathbf{x}_i^r\|$ denotes the true range between the i th range sensor and the source. The measurement noises $\epsilon_i, \forall i \in \{1, \dots, N_1\}$ are assumed to be mutually independent and Gaussian distributed with zero mean and the same variance σ_r^2 , i.e., $\epsilon_i \sim \mathcal{N}(0, \sigma_r^2)$. Hence $\hat{\mathbf{r}} \sim \mathcal{N}(\mathbf{r}(\mathbf{x}_t), \mathbf{\Sigma}_r)$, where $\mathbf{\Sigma}_r = \sigma_r^2 \mathbf{I}_{N_1}$ and $\mathbf{I}_{N_1}$ is the N_1 -dimensional identity matrix.

In this chapter, the following problems will be considered.

Problem 4.1. *In a HSN, there exist N_1 range sensors and N_2 DOA sensors performing the source localization task. Then the problem is to find an optimal sensor configuration for the best source localization performance.*

Problem 4.2. *In a HSN, there exist N_2 DOA sensors and N_3 time of arrival (TOA) based sensors performing the source localization task. Then the problem is to find an optimal sensor configuration for the best source localization performance.*

Problem 4.3. *In a HSN, there exist N sensor platforms. Each sensor platform can obtain both DOA and range measurements. Then the problem is to find an optimal sensor-source configuration for the best source localization performance.*

In Section 4.2, the introduction to optimality metrics used for sensor-source geometry analysis will be addressed. Some analysis results will be presented in Sections

4.3-4.5. In the following sections, the readers can see that some expressions of the CRLB or the FIM for single type of sensor measurements, may have already appeared in the existing research works. However, the novel contribution of this chapter is to study the optimal sensor placement for hybrid sensor measurements based source localization.

4.2 Optimality Metric of Sensor-Source Geometries

The main objective of this chapter is to study the optimal sensor placement in a HSN, where several different types of sensors are used for source localization. Commonly used criteria for the optimal sensor-source geometries are based on the FIM or CRLB. For a general measurement vector $\hat{\mathbf{z}} = \mathbf{z}(\mathbf{x}) + \mathbf{w}$, where $\mathbf{x}$ is an unknown parameter vector which is to be estimated from the measurement $\hat{\mathbf{z}} \in \mathbb{R}^N$ and $\mathbf{w} \in \mathbb{R}^N$ is a vector of Gaussian noises with zero mean and a covariance matrix Σ . Then the likelihood function of $\mathbf{x}$ given the measurement vector $\hat{\mathbf{z}}$ is given by

$$f_{\hat{\mathbf{z}}}(\hat{\mathbf{z}}; \mathbf{x}) = \frac{1}{(2\pi)^{N/2} |\Sigma|^{1/2}} \exp \left(-\frac{1}{2} (\hat{\mathbf{z}} - \mathbf{z}(\mathbf{x}))^T \Sigma^{-1} (\hat{\mathbf{z}} - \mathbf{z}(\mathbf{x})) \right), \quad (4.2)$$

where $|\Sigma|$ is the determinant of Σ .

For an unbiased estimate $\hat{\mathbf{x}}$ of $\mathbf{x}$, the CRLB gives the lowest bound for error covariance matrix among any unbiased estimators [116], i.e.,

$$\mathbb{E}[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T] \geq \mathbf{J}(\mathbf{x})^{-1} \triangleq \mathbf{CRLB}(\mathbf{x}), \quad (4.3)$$

where $\mathbf{J}(\mathbf{x})$ is called the FIM. The (i, j) -th element of $\mathbf{J}(\mathbf{x})$ is given by

$$(\mathbf{J}(\mathbf{x}))_{i,j} = \mathbb{E} \left[\frac{\partial}{\partial x_i} \ln(f_{\hat{\mathbf{z}}}(\hat{\mathbf{z}}; \mathbf{x})) \frac{\partial}{\partial x_j} \ln(f_{\hat{\mathbf{z}}}(\hat{\mathbf{z}}; \mathbf{x})) \right], \quad (4.4)$$

where x_i, x_j are the i -th and j -th entries of $\mathbf{x}$ respectively. Under the assumption of Gaussian measurement noise and when the noise covariance is independent of the parameter, the FIM is simply given by

$$\mathbf{J}(\mathbf{x}) = \nabla_{\mathbf{x}} \mathbf{z}(\mathbf{x})^T \Sigma^{-1} \nabla_{\mathbf{x}} \mathbf{z}(\mathbf{x}) \quad (4.5)$$

where $\nabla_{\mathbf{x}} \mathbf{z}(\mathbf{x})$ is the Jacobian of the measurement vector $\mathbf{z}(\mathbf{x})$ with respect to the parameter $\mathbf{x}$.

The FIM, i.e., $\mathbf{J}(\mathbf{x})$, quantifies the amount of information about the unknown parameter $\mathbf{x}$ that measurement vector $\hat{\mathbf{z}}$ carries. However, for optimization, we need a scalar measure of the corresponding FIM or CRLB. Note that $\mathbf{J}(\mathbf{x})$ is symmetric and semi-positive definite, therefore its eigenvalues are real and nonnegative. The eigenvalues of the $\mathbf{J}(\mathbf{x})$ are related to the uncertainty ellipsoid of the source location estimation. Commonly used optimality measures include the D-optimality criterion [115] given by

$$\text{D-optimality: } f(\mathbf{J}(\mathbf{x})) = \max_{\mathbf{s}} (\log \det(\mathbf{J}(\mathbf{x}))) \quad (4.6)$$

where $\mathbf{s}$ denotes the vector of sensor locations.

There are many other measures that can be used, e.g., A-optimality criterion $f(\mathbf{J}(\mathbf{x})) = \min_{\mathbf{s}} (\text{Tr}(\mathbf{J}(\mathbf{x})^{-1}))$ and E-optimality criterion $f(\mathbf{J}(\mathbf{x})) = \max_i (\text{eig}(\mathbf{J}(\mathbf{x})^{-1}))$ [38]. In this work, the D-optimality criterion is used to analyze the optimal sensor-source geometries for source localization. D-optimality has a long history in experimental design [115]. A D-optimality design for the system parameters will minimize the volume of the uncertainty ellipsoid generated by $\mathbf{J}(\mathbf{x})^{-1}$.

4.3 Hybrid Range and DOA Sensors

In this section, the hybrid range and DOA sensors case is studied. Firstly, considering a general case: N_1 range sensors and N_2 DOA sensors need to be placed in

the sensor field. Given the measurement vectors $\hat{\mathbf{r}}$ and $\hat{\Phi}$, combine them together, i.e., $\mathbf{z}_1 = [\hat{\mathbf{r}}, \hat{\Phi}]$. Then the FIM i.e., $\mathbf{J}_{br}(\mathbf{x}_t)$, for the hybrid range and DOA sensors based localization is given by

$$\begin{aligned} \mathbf{J}_{br}(\mathbf{x}_t) &= \nabla_{\mathbf{x}_t} \mathbf{r}(\mathbf{x}_t)^T \Sigma_r^{-1} \nabla_{\mathbf{x}_t} \mathbf{r}(\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \Phi(\mathbf{x}_t)^T \Sigma_b^{-1} \nabla_{\mathbf{x}_t} \Phi(\mathbf{x}_t) \\ &= \frac{1}{\sigma_r^2} \sum_{i=1}^{N_1} \begin{bmatrix} \sin^2(\phi_{ri}) & \sin \phi_{ri} \cos \phi_{ri} \\ \sin \phi_{ri} \cos \phi_{ri} & \cos^2(\phi_{ri}) \end{bmatrix} \\ &\quad + \frac{1}{\sigma_b^2} \sum_{j=1}^{N_2} \frac{1}{r_{bj}^2} \begin{bmatrix} \cos^2(\phi_{bj}) & -\sin \phi_{bj} \cos \phi_{bj} \\ -\sin \phi_{bj} \cos \phi_{bj} & \sin^2(\phi_{bj}) \end{bmatrix}, \end{aligned} \quad (4.7)$$

where $\phi_{ri}, \phi_{bj}, \forall i \in \{1, \dots, N_1\}, \forall j \in \{1, \dots, N_2\}$ denote the angular positions of range sensors and DOA sensors, respectively. $r_{bj}, \forall j \in \{1, \dots, N_2\}$ denotes the range between the j -th DOA sensor and the source. Equation (4.7) is a summation of the FIM for range measurements and the FIM for DOA measurements. Each term of the summation also appears in [8] by Bishop et. al. equations (13) and (28), respectively.

Theorem 4.1. *Consider the source localization problem in the hybrid range and DOA sensors case, i.e., Problem 4.1. Then the determinant of the FIM, i.e., $\det(\mathbf{J}_{br}(\mathbf{x}_t))$ can be expressed by*

$$\det(\mathbf{J}_{br}(\mathbf{x}_t)) = A + B + C,$$

where

$$\begin{aligned} A &= \frac{1}{\sigma_r^4} \sum_{\substack{j > i \\ \in \mathcal{Q}_r}} \sin^2(\phi_{rj} - \phi_{ri}), \\ B &= \frac{1}{\sigma_r^2} \frac{1}{\sigma_b^2} \sum_{k=1}^{N_1} \sum_{l=1}^{N_2} \frac{1}{r_{bl}^2} \cos^2(\phi_{rk} - \phi_{bl}), \\ C &= \frac{1}{\sigma_b^4} \sum_{\substack{n > m \\ \in \mathcal{Q}_b}} \frac{\sin^2(\phi_{bn} - \phi_{bm})}{r_{bm}^2 r_{bn}^2}, \end{aligned}$$

where $\varrho_r = \{\{i, j\}\}$ is defined as the set of all combinations of i and j with $i, j \in \{1, \dots, N_1\}$ and $j > i$; $\varrho_b = \{\{m, n\}\}$ is defined as the set of all combinations of m and n with $m, n \in \{1, \dots, N_2\}$ and $n > m$.

Proof. Firstly, according to (4.7), $\det(\mathbf{J}_{br}(\mathbf{x}_t))$ can be expressed as follows:

$$\begin{aligned} \det(\mathbf{J}_{br}(\mathbf{x}_t)) &= \det \begin{bmatrix} a_1 + b_1 & a_3 + b_3 \\ a_3 + b_3 & a_2 + b_2 \end{bmatrix} \\ &= (a_1 a_2 - a_3^2) + (a_1 b_2 + b_1 a_2 - 2a_3 b_3) + (b_1 b_2 - b_3^2), \end{aligned} \quad (4.8)$$

where $a_1 = \frac{1}{\sigma_r^2} \sum_{i=1}^{N_1} \sin^2(\phi_{ri})$, $a_2 = \frac{1}{\sigma_r^2} \sum_{i=1}^{N_1} \cos^2(\phi_{ri})$, $a_3 = \frac{1}{\sigma_r^2} \sum_{i=1}^{N_1} \sin(\phi_{ri}) \cos(\phi_{ri})$,
 $b_1 = \frac{1}{\sigma_b^2} \sum_{j=1}^{N_2} \frac{1}{r_{bj}^2} \cos^2(\phi_{bj})$, $b_2 = \frac{1}{\sigma_b^2} \sum_{j=1}^{N_2} \frac{1}{r_{bj}^2} \sin^2(\phi_{bj})$, $b_3 = -\frac{1}{\sigma_b^2} \sum_{j=1}^{N_2} \frac{1}{r_{bj}^2} \sin(\phi_{bj}) \cos(\phi_{bj})$.

Denoting $A = (a_1 a_2 - a_3^2)$, $B = (a_1 b_2 + b_1 a_2 - 2a_3 b_3)$ and $C = (b_1 b_2 - b_3^2)$, then (4.8) can be further simplified as

$$\det(\mathbf{J}_{br}(\mathbf{x}_t)) = A + B + C. \quad (4.9)$$

For the calculation of A and C , we refer to [8]. Here, we derive the expression of B .

From (4.8) and the notation of B , we have

$$\begin{aligned} B &= (a_1 b_2 + b_1 a_2 - 2a_3 b_3) \\ &= \frac{1}{\sigma_r^2} \sum_{k=1}^{N_1} \sin^2(\phi_{rk}) \frac{1}{\sigma_b^2} \sum_{l=1}^{N_2} \frac{1}{r_{bl}^2} \sin^2(\phi_{bl}) + \frac{1}{\sigma_b^2} \sum_{l=1}^{N_2} \frac{1}{r_{bl}^2} \cos^2(\phi_{bl}) \frac{1}{\sigma_r^2} \sum_{k=1}^{N_1} \cos^2(\phi_{rk}) \\ &\quad + 2 \frac{1}{\sigma_r^2} \sum_{k=1}^{N_1} \sin(\phi_{rk}) \cos(\phi_{rk}) \frac{1}{\sigma_b^2} \sum_{l=1}^{N_2} \frac{1}{r_{bl}^2} \sin(\phi_{bl}) \cos(\phi_{bl}) \\ &= \frac{1}{\sigma_r^2} \frac{1}{\sigma_b^2} \sum_{k=1}^{N_1} \sum_{l=1}^{N_2} \frac{1}{r_{bl}^2} \sin^2(\phi_{rk}) \sin^2(\phi_{bl}) + \frac{1}{\sigma_r^2} \frac{1}{\sigma_b^2} \sum_{k=1}^{N_1} \sum_{l=1}^{N_2} \frac{1}{r_{bl}^2} \cos^2(\phi_{rk}) \cos^2(\phi_{bl}) \\ &\quad + \frac{1}{\sigma_r^2} \frac{1}{\sigma_b^2} \sum_{k=1}^{N_1} \sum_{l=1}^{N_2} 2 \frac{1}{r_{bl}^2} \sin(\phi_{rk}) \cos(\phi_{rk}) \sin(\phi_{bl}) \cos(\phi_{bl}) \\ &= \frac{1}{\sigma_r^2} \frac{1}{\sigma_b^2} \sum_{k=1}^{N_1} \sum_{l=1}^{N_2} \frac{1}{r_{bl}^2} (\cos(\phi_{rk}) \cos(\phi_{bl}) + \sin(\phi_{rk}) \sin(\phi_{bl}))^2 \end{aligned}$$

$$= \frac{1}{\sigma_r^2} \frac{1}{\sigma_b^2} \sum_{k=1}^{N_1} \sum_{l=1}^{N_2} \frac{1}{r_{bl}^2} \cos^2(\phi_{rk} - \phi_{bl}).$$

This completes the proof. $\square$

4.3.1 Two-sensors Case

Based on the expression of determinant of the FIM, the optimal sensor-source geometry can be derived according to the D-optimality measure defined in Section 4.2.

Firstly the simplest case is considered: two-sensors case, i.e., one DOA sensor and one range sensor, and the range sensor is of an arbitrary sensing range.

Proposition 4.1. *Consider the source localization problem in the hybrid range and DOA sensors case, i.e., Problem 4.1. For the case of one DOA sensor and one range sensor, given the minimum sensing range between the DOA sensor and the source $r_{\min}$, an optimal sensor-source geometry should satisfy: (1) The angle subtended at the source by the two sensors, i.e., v_{12} should be equal to 0 or π ; (2) the range between the DOA sensor and the source should be $r_{\min}$.*

Proof. Based on the results of Theorem 4.1, for the case of one DOA sensor and one range sensor,

$$\det(\mathbf{J}_{br}(\mathbf{x}_t)) = \frac{1}{\sigma_r^2} \frac{1}{\sigma_b^2} \frac{1}{r_b^2} \cos^2(\phi_1 - \phi_2), \quad (4.10)$$

where ϕ_1 and ϕ_2 denote the angular positions of the DOA sensor and the range sensor, respectively. r_b denotes the range between the DOA sensor and the source.

From (4.10), it can be seen that $\det(\mathbf{J}_{br}(\mathbf{x}_t))$ attains its maximum when $\cos^2(\phi_1 - \phi_2) = 1$ and $r_b = r_{\min}$. $\square$

In Figure 4.1, an example is given to illustrate the optimal sensor-source configurations for the case of one DOA sensor and one range sensor. From the figure we

can see that in the optimal settings, the DOA sensor and the range sensor should be arranged to be collinear with the source. Furthermore, the DOA sensor should be placed as close as possible to the source.

Figure 4.1: Optimal sensor-source configurations: one DOA sensor and one range sensor case: (a) Two sensors are placed at the different sides of the source; (b) Two sensors are placed at the same side of the source

4.3.2 Three-sensors Case

The three-sensors case is particularly important in practice. This is because that in a heterogeneous network with three sensors containing both DOA and range sensors, the source location can be uniquely determined in general¹. In Fig. 4.2, we plot a circle with the range sensor as its center and the distance from the range sensor to the source as its radius. It can be easily known that if the bearing sensor is inside the circle, the source location can be uniquely determined by using only two sensors, i.e., one DOA sensor and one range sensor. However, as seen from Figure 4.2, if the DOA sensor is outside the circle, i.e., DOA sensor can only be placed at a location where the distance between the DOA sensor and the source is greater than the maximum sensing range of the range sensor, we need at least three sensors to determine the source location uniquely, i.e., two DOA sensors and one range sensor as in Figure 4.2 (a) or two range sensors and one DOA sensor as in Figure 4.2 (b). In the following part of this subsection, we consider these two scenarios separately.

¹Note that in a homogeneous network, the source location may be uniquely determined by two DOA sensors.

Figure 4.2: Source localization by using hybrid range and DOA sensors: (a) Two DOA sensors and one range sensor; (b) Two range sensors and one DOA sensor.

4.3.2.1 Two Range Sensors and One DOA Sensor

In this case, the determinant of the FIM can be written as

$$\begin{aligned} \det(\mathbf{J}_{br}(\mathbf{x}_t)) = & \frac{1}{\sigma_r^2 \sigma_b^2 r_b^2} \cos^2(\phi_b - \phi_{r1}) + \frac{1}{\sigma_r^2 \sigma_b^2 r_b^2} \cos^2(\phi_b - \phi_{r2}) \\ & + \frac{1}{\sigma_r^4} \sin^2(\phi_{r2} - \phi_{r1}), \end{aligned} \quad (4.11)$$

where r_b denotes the distance between the DOA sensor and the source. ϕ_b , ϕ_{r1} , ϕ_{r2} denote the angular positions of the DOA sensor and two range sensors, respectively.

Theorem 4.2. *Consider the source localization problem in the hybrid range and DOA sensors case, i.e., Problem 4.1. For the case of one DOA sensor and two range sensors, the results are given as follows:*

(1) If $\frac{\alpha_1}{2\alpha_2} \leq 1$ or equivalently $r_b \geq \frac{\sqrt{2}\sigma_r}{2\sigma_b}$, where $\alpha_1 = \frac{1}{\sigma_r^2 \sigma_b^2 r_b^2}$ and $\alpha_2 = \frac{1}{\sigma_r^4}$, then the determinant of the FIM, i.e., $\det(\mathbf{J}_{br}(\mathbf{x}_t))$ is **upper-bounded** by $\frac{(\alpha_1 + 2\alpha_2)^2}{4\alpha_2}$. This upper-bound is achieved if and only if the angular positions of the sensors satisfy

$$\begin{aligned} \phi_{r1} - \phi_b &= \frac{1}{2} \arccos \frac{\alpha_1}{2\alpha_2}, \quad \phi_{r2} - \phi_b = -\frac{1}{2} \arccos \frac{\alpha_1}{2\alpha_2}, \\ \text{or } \phi_{r1} - \phi_b &= \frac{1}{2} \arccos \frac{\alpha_1}{2\alpha_2}, \quad \phi_{r2} - \phi_b = \pi - \frac{1}{2} \arccos \frac{\alpha_1}{2\alpha_2}. \end{aligned} \quad (4.12)$$

(2) If $\frac{\alpha_1}{2\alpha_2} > 1$ or equivalently $r_b < \frac{\sqrt{2}\sigma_r}{2\sigma_b}$, then $\det(\mathbf{J}_{br}(\mathbf{x}_t))$ is **upper-bounded** by $2\alpha_1$. This upper-bound is achieved if and only if the angular positions of the sensors

satisfy

$$\sin(\phi_{r2} - \phi_{r1}) = 0, \quad \cos(\phi_{r1} - \phi_b) = \pm 1, \quad \cos(\phi_{r2} - \phi_b) = \pm 1. \quad (4.13)$$

Proof. Without loss of generality, firstly, it is assumed that $\phi_b = 0$. Then (4.11) can be simplified as

$$\begin{aligned} \det(\mathbf{J}_{br}(\mathbf{x}_t)) &= \frac{1}{\sigma_r^2 \sigma_b^2 r_b^2} \cos^2(\phi_{r1}) + \frac{1}{\sigma_r^2 \sigma_b^2 r_b^2} \cos^2(\phi_{r2}) \\ &\quad + \frac{1}{\sigma_r^4} \sin^2(\phi_{r2} - \phi_{r1}). \end{aligned} \quad (4.14)$$

Assume r_b is fixed. Taking the partial derivative of (4.14) with respect to the variables ϕ_{r1} and ϕ_{r2} respectively, we can get

$$\begin{aligned} \frac{\partial \det(\mathbf{J}(\mathbf{x}_t))}{\partial \phi_{r1}} &= -\frac{1}{\sigma_r^4} \sin(2(\phi_{r2} - \phi_{r1})) - \frac{1}{\sigma_r^2 \sigma_b^2 r_b^2} \sin(2(\phi_{r1})) = 0, \\ \frac{\partial \det(\mathbf{J}(\mathbf{x}_t))}{\partial \phi_{r2}} &= \frac{1}{\sigma_r^4} \sin(2(\phi_{r2} - \phi_{r1})) - \frac{1}{\sigma_r^2 \sigma_b^2 r_b^2} \sin(2(\phi_{r2})) = 0. \end{aligned} \quad (4.15)$$

Then, by solving the above two equations, we stack all the possible solutions in Table 4.1. Note that exchanges of the values of ϕ_{r1} and ϕ_{r2} will result in the same solutions and the results in the last two lines of Table 4.1 hold if and only if $\frac{\alpha_1}{2\alpha_2} = \frac{\sigma_r^2}{2\sigma_b^2 r_b^2} \leq 1$.

From Table 4.1, it can be seen that there are 3 possible local optima under the D-optimality measure, i.e., $\max_s(\det(\mathbf{J}_{br}(\mathbf{x}_t)))$. Thus we need to identify which one is the global optimal solution.

From the notations of α_1 and α_2 , we have $\alpha_1 > 0$ and $\alpha_2 > 0$, hence

$$\frac{(\alpha_1 + 2\alpha_2)^2}{4\alpha_2} = \alpha_1 + \alpha_2 + \frac{\alpha_1^2}{4\alpha_2} > \alpha_1 + \alpha_2.$$

Table 4.1: Solution list (Two range sensors and one DOA sensor)

ϕ_b	ϕ_{r1}	ϕ_{r2}	$\det(\mathbf{J}_{br}(\mathbf{x}_t))$
0	0	0	$2\alpha_1$
0	π	π	$2\alpha_1$
0	0	π	$2\alpha_1$
0	$\frac{\pi}{2}$	0	$\alpha_1 + \alpha_2$
0	π	$\frac{\pi}{2}$	$\alpha_1 + \alpha_2$
0	$\frac{3\pi}{2}$	π	$\alpha_1 + \alpha_2$
0	$-\frac{1}{2} \arccos \frac{\alpha_1}{2\alpha_2}$	$\frac{1}{2} \arccos \frac{\alpha_1}{2\alpha_2}$	$\frac{(\alpha_1 + 2\alpha_2)^2}{4\alpha_2}$
0	$\pi - \frac{1}{2} \arccos \frac{\alpha_1}{2\alpha_2}$	$\frac{1}{2} \arccos \frac{\alpha_1}{2\alpha_2}$	$\frac{(\alpha_1 + 2\alpha_2)^2}{4\alpha_2}$

Then we compare $(\alpha_1 + \alpha_2 + \frac{\alpha_1^2}{4\alpha_2})$ and $2\alpha_1$. Taking the difference between them, we have

$$\frac{(\alpha_1 + 2\alpha_2)^2}{4\alpha_2} - 2\alpha_1 = \frac{(\alpha_1 - 2\alpha_2)^2}{4\alpha_2} \geq 0,$$

where the equality holds if and only if $\frac{\alpha_1}{2\alpha_2} = 1$. Hence we can conclude that if $\frac{\alpha_1}{2\alpha_2} \leq 1$ which is the condition that the solutions in the last two lines of Table 4.1 exist, then $\frac{(\alpha_1 + 2\alpha_2)^2}{4\alpha_2}$ is an upper bound of $\det(\mathbf{J}_{br}(\mathbf{x}_t))$. If $\frac{\alpha_1}{2\alpha_2} > 1$, then $\det(\mathbf{J}_{br}(\mathbf{x}_t))$ is upper-bounded by $2\alpha_1$. The angular positions of the sensors should be as in the first six lines in Table 4.1. $\square$

Corollary 4.1. *In the optimal sensor-source geometries, the limit value of the angle subtended at the source by the DOA sensor and the range sensor, i.e., ϕ , is 45 degrees.*

Proof. This result can be easily obtained from (4.12). $\square$

Example 4.1. *Assuming that $\sigma_r = 2$ and $\sigma_b = 1$, then the **critical scenario** for the two possible optimal sensor-source topologies, i.e., cases (1) and (2) in Theorem 4.2, is $r_b = \frac{\sqrt{2}\sigma_r}{2\sigma_b} = \sqrt{2}$. When $r_b \geq \sqrt{2}$, according to the result of the case (1) in Theorem 4.2, the optimal sensor angular configurations with different r_b are shown in Figure 4.3. From the figure, we can see that when the DOA sensor is far away from the source, then the angles subtended at the source by the DOA sensor and the range sensor, i.e., ϕ in Figure 4.3 is nearly equal to 45 degrees. However, when the*

DOA sensor moves closer to the source, ϕ becomes smaller and smaller. Finally, when $r_b < \sqrt{2}$, $\phi = 0$.

Figure 4.3: This figure illustrates the optimal sensor-source geometries under the two range sensors and one DOA sensor case. In the figure, ϕ denotes the angle subtended at the source by the DOA sensor and the range sensor.

4.3.2.2 Two DOA Sensors and One Range Sensor

For the case of two DOA sensors and one range sensor, it is more complicated to analyze the optimal sensor-source geometry than the case of two range sensors and one DOA sensor. Firstly, similarly as before, we can get the determinant of the FIM as follows

$$\begin{aligned} \det(\mathbf{J}_{br}(\mathbf{x}_t)) = & \frac{1}{\sigma_r^2 \sigma_b^2 r_{b1}^2} \cos^2(\phi_{b1} - \phi_r) + \frac{1}{\sigma_r^2 \sigma_b^2 r_{b2}^2} \cos^2(\phi_{b2} - \phi_r) \\ & + \frac{1}{\sigma_b^4 r_{b1}^2 r_{b2}^2} \sin^2(\phi_{b2} - \phi_{b1}), \end{aligned} \quad (4.16)$$

where r_{b1} and r_{b2} denote the distances between the two DOA sensors and the source respectively. ϕ_r , ϕ_{b1} , ϕ_{b2} denote the angular positions of the range sensor and two DOA sensors, respectively.

Without loss of generality, we assume $\phi_r = 0$. Firstly, we assume that r_{b1} and r_{b2} are fixed. Denote $\beta_1 = \frac{1}{\sigma_r^2 \sigma_b^2 r_{b1}^2}$, $\beta_2 = \frac{1}{\sigma_r^2 \sigma_b^2 r_{b2}^2}$ and $\beta_3 = \frac{1}{\sigma_b^4 r_{b1}^2 r_{b2}^2}$. Then (4.16) can be simplified as

$$\det(\mathbf{J}_{br}(\mathbf{x}_t)) = \beta_1 \cos^2(\phi_{b1}) + \beta_2 \cos^2(\phi_{b2}) + \beta_3 \sin^2(\phi_{b2} - \phi_{b1}). \quad (4.17)$$

Then, by taking the partial derivative with respect to ϕ_{b1} , and setting the derivative to be zero, we can get

$$\frac{\partial \det(\mathbf{J}(\mathbf{x}_t))}{\partial \phi_{b1}} = -\beta_1 \sin(2\phi_{b1}) - \beta_3 \sin(2(\phi_{b2} - \phi_{b1})) = 0. \quad (4.18)$$

Similarly, by taking the partial derivative with respect to ϕ_{b2} and setting the derivative to be zero, we have

$$\frac{\partial \det(\mathbf{J}(\mathbf{x}_t))}{\partial \phi_{b2}} = -\beta_2 \sin(2\phi_{b2}) + \beta_3 \sin(2(\phi_{b2} - \phi_{b1})) = 0. \quad (4.19)$$

Case 1: $r_{b1} = r_{b2} = r_b$.

From the definitions of β_1 and β_2 , we have $\beta_1 = \beta_2$. In this case, we can see that equations (4.18) and (4.19) are similar to (4.15), and hence their solutions are similar too. Thus we list all the possible results in Table 4.2. For the optimal sensor-source configurations, the results are summarized in Theorem 4.3.

Theorem 4.3. *Consider the source localization problem under the case of one range sensor and two DOA sensors, we have the following results:*

(1) If $\frac{\beta_1}{2\beta_3} \leq 1$ or equivalently $r_b \leq \frac{\sqrt{2}\sigma_r}{\sigma_b}$, then the determinant of the FIM, i.e., $\det(\mathbf{J}_{br}(\mathbf{x}_t))$ is **upper-bounded** by $\frac{(\beta_1 + 2\beta_3)^2}{4\beta_3}$. This upper-bound is achieved if and only if the angular positions of the sensors satisfy

$$\phi_{b1} - \phi_r = \frac{1}{2} \arccos \frac{\beta_1}{2\beta_3}, \quad \phi_{b2} - \phi_r = -\frac{1}{2} \arccos \frac{\beta_1}{2\beta_3},$$

Table 4.2: Solution list with $r_{b1} = r_{b2}$ (Two DOA sensors and one range sensor)

ϕ_r	ϕ_{b1}	ϕ_{b2}	$\det(\mathbf{J}_{br}(\mathbf{x}_t))$
0	0	0	$2\beta_1$
0	π	π	$2\beta_1$
0	0	π	$2\beta_1$
0	$\frac{\pi}{2}$	0	$\beta_1 + \beta_3$
0	π	$\frac{\pi}{2}$	$\beta_1 + \beta_3$
0	$\frac{3\pi}{2}$	π	$\beta_1 + \beta_3$
0	$-\frac{1}{2} \arccos \frac{\beta_1}{2\beta_3}$	$\frac{1}{2} \arccos \frac{\beta_1}{2\beta_3}$	$\frac{(\beta_1+2\beta_3)^2}{4\beta_3}$
0	$\pi - \frac{1}{2} \arccos \frac{\beta_1}{2\beta_3}$	$\frac{1}{2} \arccos \frac{\beta_1}{2\beta_3}$	$\frac{(\beta_1+2\beta_3)^2}{4\beta_3}$

$$\text{or } \phi_{b1} - \phi_r = \frac{1}{2} \arccos \frac{\beta_1}{2\beta_3}, \quad \phi_{b2} - \phi_r = \pi - \frac{1}{2} \arccos \frac{\beta_1}{2\beta_3}. \quad (4.20)$$

(2) If $\frac{\beta_1}{2\beta_3} > 1$ or equivalently $r_b > \frac{\sqrt{2}\sigma_r}{\sigma_b}$, then $\det(\mathbf{J}_{br}(\mathbf{x}_t))$ is **upper-bounded** by $2\beta_1$. This upper-bound is achieved if and only if the angular positions of the sensors satisfy

$$\sin(\phi_{b2} - \phi_{b1}) = 0, \quad \cos(\phi_{b1} - \phi_r) = \pm 1, \quad \cos(\phi_{b2} - \phi_r) = \pm 1. \quad (4.21)$$

Proof. The proof is similar to Theorem 4.2. □

Example 4.2. Assume that $\sigma_r = 2$ and $\sigma_b = 1$. Then the **critical scenario** for the two possible optimal sensor-source topologies is $r_b = \frac{\sqrt{2}\sigma_r}{\sigma_b} = 2\sqrt{2}$. When $r_b \leq 2\sqrt{2}$, the optimal sensor angular configurations with different r_b are shown in Figure 4.4. From the figure, we can see that when the two DOA sensors move closer to the source, the angles subtended at the source by the DOA sensor and the range sensor, i.e., ϕ in Figure 4.4 become larger which vary from 0 to 45° .

Remark 4.1. From the results in Example 4.1 and Example 4.2, we can see that the optimal sensor-source configurations (as stated in Theorem 4.2 and Theorem 4.3) for these two different three-sensors cases are contrary to each other. For instance, in

Figure 4.4: This figure illustrates the optimal sensor-source geometries under the one range sensor and two DOA sensors case. In the figure, ϕ denotes the angle subtended at the source by the DOA sensor and the range sensor.

Example 4.1, when the range sensor moves closer to the source, the angle subtended at the source by the DOA sensor and the range sensor, i.e., ϕ , becomes smaller and smaller. However, it is opposite in Example 4.2.

Remark 4.2. From Figures 4.3 and 4.4, we can see that in the optimal sensor-source geometry settings for the three-sensors case, sensors are placed at one-side of the source. This is promising since the results for the single type of sensors require that the sensors be placed around the source [8, 76] which may be impossible in real implementations. However, for a larger number of sensors in a HSN, the sensors may also encircle the source in the optimal settings as in a homogeneous network.

Case 2: $r_{b1} \neq r_{b2}$.

In this case, it is more difficult to find the global optimal sensor-source geometries. Combing (4.18) and (4.19), we can get

$$\beta_1 \sin(2\phi_{b1}) + \beta_2 \sin(2\phi_{b2}) = 0 \quad (4.22)$$

Hence, ϕ_{b1} can be represented by

$$\phi_{b1} = -\frac{1}{2} \arcsin \left(\frac{\beta_2 \sin(2\phi_{b2})}{\beta_1} \right) \quad (4.23)$$

Then substituting (4.23) back into (4.19), we can get the solution of ϕ_{b2} . If r_{b1} and r_{b2} are given, then we can get the explicit solutions of ϕ_{b1} and ϕ_{b2} .

4.3.3 N Sensors Case

In the three-sensors case, the source location can be uniquely determined. A small number of sensors will require less communication and computation cost which is important in a resource limited WSN.

If we have a large number of range and DOA sensors, it is quite complicated to analyze the optimal sensor-source configurations. There is no explicit solution for this case. What we can do is to apply the results in Theorem 4.1 for the expression of the determinant of the FIM, then solve an optimization problem. One approach is to utilize the gradient search method with initial guesses for the unknown variables. But there is no guarantee for obtaining a global optimal solution.

4.4 Hybrid TDOA and DOA Sensing

In this section, we study the hybrid DOA and TDOA sensing case. Firstly, we consider a general case: N_2 DOA sensors and N_3 TDOA sensors. Given the measurement vectors $\hat{\Phi}$ and $\hat{\mathbf{t}}$, we combine them together, i.e., $\mathbf{z}_2 = [\hat{\Phi}, \hat{\mathbf{t}}]$. The overall FIM i.e., $\mathbf{J}_{bt}(\mathbf{x}_t)$, for hybrid DOA and TDOA localization is given by

$$\begin{aligned} \mathbf{J}_{bt}(\mathbf{x}_t) &= \nabla_{\mathbf{x}_t} \Phi(\mathbf{x}_t)^T \Sigma_b^{-1} \nabla_{\mathbf{x}_t} \Phi(\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \mathbf{t}(\mathbf{x}_t)^T \Sigma_t^{-1} \nabla_{\mathbf{x}_t} \mathbf{t}(\mathbf{x}_t) \\ &= \frac{1}{\sigma_b^2} \begin{bmatrix} b'_1 & b'_3 \\ b'_3 & b'_2 \end{bmatrix} + \frac{1}{(v\sigma_t)^2} \begin{bmatrix} c_1 & c_3 \\ c_3 & c_2 \end{bmatrix} \end{aligned} \quad (4.24)$$

where $b'_1 = \sum_{i=1}^{N_2} \frac{\cos^2(\phi_{bi})}{r_{bi}^2}$, $b'_2 = \sum_{i=1}^{N_2} \frac{\sin^2(\phi_{bi})}{r_{bi}^2}$, $b'_3 = -\sum_{i=1}^{N_2} \frac{\sin(\phi_{bi}) \cos(\phi_{bi})}{r_{bi}^2}$, $c_1 = \sum_{j=1}^{N_3} \cos^2(\phi_{tj}) - \frac{1}{N_3} \left(\sum_{j=1}^{N_3} \cos(\phi_{tj}) \right)^2$, $c_2 = \sum_{j=1}^{N_3} \sin^2(\phi_{tj}) - \frac{1}{N_3} \left(\sum_{j=1}^{N_3} \sin(\phi_{tj}) \right)^2$, $c_3 = \sum_{j=1}^{N_3} (\sin(\phi_{tj}) \cos(\phi_{tj})) - \frac{1}{N_3} \sum_{j=1}^{N_3} \sin(\phi_{tj}) \sum_{j=1}^{N_3} \cos(\phi_{tj})$.

The first term in (4.24) appears in [8] whereas a similar expression for the second term can be found in [69].

Theorem 4.4. *Consider the source localization problem in the hybrid DOA and TDOA sensing, i.e., Problem 4.2. Then, $\det(\mathbf{J}_{bt}(\mathbf{x}_t))$ can be expressed by*

$$\det(\mathbf{J}_{bt}(\mathbf{x}_t)) = D + E + F, \quad (4.25)$$

where

$$\begin{aligned} D &= \frac{1}{\sigma_b^4} \sum_{i>j}^{N_2} \frac{\sin^2(\phi_{bi} - \phi_{bj})}{r_{bi}^2 r_{bj}^2}, \\ E &= \frac{1}{(v\sigma_t)^4} \left[\left(1 - \frac{2}{N_3}\right) \sum_{i>j}^{N_3} \sin^2(\phi_{ti} - \phi_{tj}) \right. \\ &\quad \left. + \frac{2}{N_3} \sum_{l=1}^{N_3} \sum_{i>j}^{N_3} \sin(\phi_{tl} - \phi_{tj}) \sin(\phi_{ti} - \phi_{tl}) \right], \\ F &= \frac{1}{(v\sigma_t\sigma_b)^2} \left[\sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} (\sin(\phi_{bi} + \phi_{tj}))^2 \right. \\ &\quad \left. - \frac{1}{N_3} \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \sum_{k=1}^{N_3} \frac{1}{r_{bi}^2} \sin(\phi_{bi} + \phi_{tj}) \sin(\phi_{bi} + \phi_{tk}) \right]. \end{aligned}$$

In the above, $\phi_{bi}, \phi_{tj}, \forall i \in \{1, \dots, N_2\}, \forall j \in \{1, \dots, N_3\}$ denote the angular positions of the DOA sensors and the TDOA sensors, respectively. $r_{bi}, \forall i \in \{1, \dots, N_2\}$ denotes the range between the i -th DOA sensor and the source.

Proof. Firstly, it can be easily checked that

$$\begin{aligned} \det(\mathbf{J}_{bt}(\mathbf{x}_t)) &= \frac{1}{\sigma_b^4} (b'_1 b'_2 - b_3'^2) + \frac{1}{(v\sigma_t)^4} (c_1 c_2 - c_3^2) \\ &\quad + \frac{1}{(v\sigma_t\sigma_b)^2} (b'_1 c_2 + b'_2 c_1 - 2b'_3 c_3) \end{aligned} \quad (4.26)$$

By denoting $D = \frac{1}{\sigma_b^4} (b'_1 b'_2 - b_3'^2)$, $E = \frac{1}{(v\sigma_t)^4} (c_1 c_2 - c_3^2)$, $F = \frac{1}{(v\sigma_t\sigma_b)^2} (b'_1 c_2 + b'_2 c_1 - 2b'_3 c_3)$, then (4.26) can be written as $\det(\mathbf{J}_{bt}(\mathbf{x}_t)) = D + E + F$. Next, we will calculate these three terms separately.

For the expression of D , it is the same with C in Theorem 4.1. The derivation for the expression of E is omitted here. A similar proof can be found in [69].

Next we focus on deriving the expression of F . From the definition of F , we have

$$\begin{aligned}
b'_1 c_2 + b'_2 c_1 &= \sum_{i=1}^{N_2} \frac{\cos^2(\phi_{bi})}{r_{bi}^2} \times \left(\sum_{j=1}^{N_3} \sin^2(\phi_{tj}) - \frac{1}{N_3} \left(\sum_{j=1}^{N_3} \sin \phi_{tj} \right)^2 \right) \\
&\quad + \sum_{i=1}^{N_2} \frac{\sin^2(\phi_{bi})}{r_{bi}^2} \left(\sum_{j=1}^{N_3} \cos^2(\phi_{tj}) - \frac{1}{N_3} \left(\sum_{j=1}^{N_3} \cos \phi_{tj} \right)^2 \right) \\
&= \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} \cos^2(\phi_{bi}) \sin^2(\phi_{tj}) + \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} \sin^2(\phi_{bi}) \cos^2(\phi_{tj}) \\
&\quad - \frac{1}{N_3} \sum_{i=1}^{N_2} \frac{1}{r_{bi}^2} \cos^2(\phi_{bi}) \left(\sum_{j=1}^{N_3} \sin(\phi_{tj}) \right)^2 \\
&\quad - \frac{1}{N_3} \sum_{i=1}^{N_2} \frac{1}{r_{bi}^2} \sin^2(\phi_{bi}) \left(\sum_{j=1}^{N_3} \cos(\phi_{tj}) \right)^2, \tag{4.27}
\end{aligned}$$

$$\begin{aligned}
2b'_3 c_3 &= -2 \sum_{i=1}^{N_2} \frac{\sin(\phi_{bi}) \cos(\phi_{bi})}{r_{bi}^2} \times \left(\sum_{j=1}^{N_3} (\sin(\phi_{tj}) \cos(\phi_{tj})) \right. \\
&\quad \left. - \frac{1}{N_3} \sum_{j=1}^{N_3} \sin(\phi_{tj}) \sum_{j=1}^{N_3} \cos(\phi_{tj}) \right) \\
&= -2 \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} \sin(\phi_{bi}) \cos(\phi_{bi}) \sin(\phi_{tj}) \cos(\phi_{tj}) \\
&\quad + \frac{2}{N_3} \sum_{i=1}^{N_2} \frac{1}{r_{bi}^2} \sin(\phi_{bi}) \cos(\phi_{bi}) \sum_{j=1}^{N_3} \sin(\phi_{tj}) \sum_{j=1}^{N_3} \cos(\phi_{tj}) \tag{4.28}
\end{aligned}$$

From (4.27) and (4.28), we can obtain that

$$\begin{aligned}
&b'_1 c_2 + b'_2 c_1 - 2b'_3 c_3 \\
&= \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} [\cos^2(\phi_{bi}) \sin^2(\phi_{tj}) + \sin^2(\phi_{bi}) \cos^2(\phi_{tj}) \\
&\quad + 2 \sin(\phi_{bi}) \cos(\phi_{bi}) \sin(\phi_{tj}) \cos(\phi_{tj})] - \frac{1}{N_3} \left(\sum_{j=1}^{N_3} \sin(\phi_{tj}) \right)^2 \times
\end{aligned}$$

$$\begin{aligned}
& \left[\sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} (\cos^2 \phi_{bi} \sin(\phi_{tj}) + \sin(\phi_{bi}) \cos(\phi_{bi}) \cos(\phi_{tj})) \right] \\
& - \frac{1}{N_3} \left(\sum_{j=1}^{N_3} \cos(\phi_{tj}) \right) \left[\sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} (\sin^2(\phi_{bi}) \cos(\phi_{tj}) \right. \\
& \quad \left. + \sin(\phi_{bi}) \cos(\phi_{bi}) \sin(\phi_{tj})) \right] \\
& = \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} (\cos(\phi_{bi}) \sin(\phi_{tj}) + \sin(\phi_{bi}) \cos(\phi_{tj}))^2 \\
& \quad - \frac{1}{N_3} \left(\sum_{j=1}^{N_3} \sin(\phi_{tj}) \right) \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} \cos(\phi_{bi}) \sin(\phi_{bi} + \phi_{tj}) \\
& \quad - \frac{1}{N_3} \left(\sum_{j=1}^{N_3} \cos(\phi_{tj}) \right) \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} \sin(\phi_{bi}) \sin(\phi_{bi} + \phi_{tj}) \\
& = \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} (\sin(\phi_{bi} + \phi_{tj}))^2 - \frac{1}{N_3} \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \sum_{k=1}^{N_3} \frac{1}{r_{bi}^2} (\sin(\phi_{tk}) \\
& \quad \cos(\phi_{bi}) \sin(\phi_{bi} + \phi_{tj}) + \cos(\phi_{tk}) \sin(\phi_{bi}) \sin(\phi_{bi} + \phi_{tj})) \\
& = \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \frac{1}{r_{bi}^2} (\sin(\phi_{bi} + \phi_{tj}))^2 - \frac{1}{N_3} \sum_{i=1}^{N_2} \sum_{j=1}^{N_3} \sum_{k=1}^{N_3} \frac{1}{r_{bi}^2} \times \\
& \quad (\sin(\phi_{bi} + \phi_{tj}) \sin(\phi_{bi} + \phi_{tk})). \tag{4.29}
\end{aligned}$$

This completes the proof. $\square$

4.4.1 Three-sensors Case

Figure 4.5: Source localization by using hybrid TDOA and DOA sensors.

Similarly as before, the three-sensors case is the main focus of this section. This is because that in this case, the source location can be uniquely determined with a minimum number of sensors. As shown in Figure 4.5, we consider the case that in the sensing field, there are two TDOA sensors and one DOA sensor to be deployed optimally for the source localization application. Note that one of the TDOA sensors will be chosen as the reference node for TDOA estimation. Firstly, similarly as before, we can get the determinant of the FIM as follows

$$\begin{aligned}
\det(\mathbf{J}_{bt}(\mathbf{x}_t)) &= \frac{1}{(v\sigma_t\sigma_b)^2} \left(\frac{1}{r_b^2} [\sin^2(\phi_b + \phi_{t1}) + \sin^2(\phi_b + \phi_{t2})] \right. \\
&\quad \left. - \frac{1}{2} \frac{1}{r_b^2} [\sin(\phi_b + \phi_{t1}) + \sin(\phi_b + \phi_{t2})]^2 \right) \\
&= \frac{1}{2(v\sigma_t\sigma_b)^2} \frac{1}{r_b^2} (\sin(\phi_b + \phi_{t1}) - \sin(\phi_b + \phi_{t2}))^2.
\end{aligned} \tag{4.30}$$

In order to maximize $\det(\mathbf{J}_{bt}(\mathbf{x}_t))$, we should let $\sin(\phi_b + \phi_{t1}) - \sin(\phi_b + \phi_{t2}) = \pm 2$ and r_b be the smallest possible. Without loss of generality, we let $\sin(\phi_b + \phi_{t1}) = \pm 1$ and $\sin(\phi_b + \phi_{t2}) = \mp 1$. Since $0 \leq \phi_b, \phi_{t1}, \phi_{t2} \leq 2\pi$, hence the solution for the angular positions of the sensors must satisfy one of the following equation groups:

$$\begin{aligned}
&\phi_b + \phi_{t1} = \frac{\pi}{2}, \quad \phi_b + \phi_{t2} = \frac{3\pi}{2}, \\
&\text{or} \\
&\phi_b + \phi_{t1} = \frac{\pi}{2}, \quad \phi_b + \phi_{t2} = \frac{7\pi}{2}, \\
&\text{or} \\
&\phi_b + \phi_{t1} = \frac{5\pi}{2}, \quad \phi_b + \phi_{t2} = \frac{3\pi}{2}, \\
&\text{or} \\
&\phi_b + \phi_{t1} = \frac{5\pi}{2}, \quad \phi_b + \phi_{t2} = \frac{7\pi}{2}.
\end{aligned} \tag{4.31}$$

From (4.30), we can easily get that the determinant of the FIM for this three-sensors case is **upper-bounded** by $\frac{2}{(v\sigma_t\sigma_b r_{\min})^2}$, where $r_{\min}$ denotes the minimum possible distance between the DOA sensor and the source. For example, if $\phi_b = 0$, the

solution for (4.31) is $\{\phi_{t1}, \phi_{t2}\} = \{\frac{\pi}{2}, \frac{3\pi}{2}\}$ or $\{\frac{3\pi}{2}, \frac{\pi}{2}\}$.

Remark 4.3. *For the three-sensors case, from (4.30), we can see that the optimal sensor-source configurations depend on the angular positions of the sensors, i.e., $\phi_b, \phi_{t1}, \phi_{t2}$, as well as the range between the DOA sensor and the source, i.e., r_b . Furthermore, in the optimal setting, r_b is independent of the other three variables. However, the optimal angular positions for the DOA sensor and the two TDOA sensors are mutually correlated with each other.*

Example 4.3. *Here, we give an example to show the optimal sensor-source configurations for the three-sensors case. Firstly, we let $r_b = r_{\min}$. Then the optimal sensor configurations for the three scenarios with $\phi_b = 0$, $\phi_b = \frac{\pi}{6}$ and $\phi_b = \frac{\pi}{4}$ are presented in Figure 4.6. From the figure, we can see that when the angular position of the DOA sensor changes, the shapes of the optimal sensor-source geometries also change. This is mainly due to that the optimal angular positions for the three sensors are mutually correlated with each other as shown in (4.31).*

4.4.2 Arbitrary Number of Sensors Case

If we have an arbitrary number of TDOA and DOA sensors, it is also quite complicated to analyze the optimal sensor-source configurations. In this case, it may not be possible to get an analytic solution for the sensor-source geometry problem. Some numerical algorithms, e.g., gradient search based method, can be used but may only give rise to a local optimal solution.

4.5 Co-Located Heterogeneous Sensors

In the previous sections, we have assumed that each sensor can only obtain one type of measurements. However, in some applications, sensor platforms containing multiple sensors may be deployed, i.e., each sensor platform can have more than one

Figure 4.6: This figure illustrates the optimal sensor-source geometries under two TDOA sensors and one DOA sensor case. Note that one of the TDOA sensors will be chosen as the reference node.

type of measurements [25, 41]. Hence, in this section, we will investigate the case that each platform has two types of co-located heterogeneous measurements. More specifically, as an example, we assume that these two heterogeneous measurements are hybrid range and DOA measurements. Similarly, co-located hybrid TDOA and DOA sensors case can also be investigated.

Given N sensor platforms with each of them taking both range and DOA measurements, the measurement vector can be represented by $\mathbf{z}_3 = [\hat{\mathbf{r}}', \hat{\Phi}']$. Without abuse of notations, we assume that the measurement noises are both zero mean Gaussian distributed with variances σ_r and σ_b for the range and DOA measurements respectively. The FIM for this case, denoted by $\mathbf{J}_{br}(\mathbf{x}_t)$, is given by [110]

$$\mathbf{J}_{br}(\mathbf{x}_t) = \nabla_{\mathbf{x}_t} \mathbf{r}'(\mathbf{x}_t)^T \Sigma_r^{-1} \nabla_{\mathbf{x}_t} \mathbf{r}'(\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \Phi'(\mathbf{x}_t)^T \Sigma_b^{-1} \nabla_{\mathbf{x}_t} \Phi'(\mathbf{x}_t)$$

$$= \sum_{i=1}^N \begin{bmatrix} \frac{1}{\sigma_r^2} \sin^2(\phi_i) + \frac{1}{\sigma_b^2 r_i^2} \cos^2(\phi_i) & \frac{1}{2} \left(\frac{1}{\sigma_r^2} - \frac{1}{\sigma_b^2 r_i^2} \right) \sin(2\phi_i) \\ \frac{1}{2} \left(\frac{1}{\sigma_r^2} - \frac{1}{\sigma_b^2 r_i^2} \right) \sin(2\phi_i) & \frac{1}{\sigma_r^2} \cos^2(\phi_i) + \frac{1}{\sigma_b^2 r_i^2} \sin^2(\phi_i) \end{bmatrix} \quad (4.32)$$

where $\phi_i, \forall i \in \{1, \dots, N\}$ denotes the angular position of the i -th sensor platform. $r_i, \forall i \in \{1, \dots, N\}$ denotes the range between the i -th sensor platform and the source.

Theorem 4.5. *Consider the source localization problem using the co-located heterogeneous sensors, i.e., Problem 4.3. The determinant of the FIM, i.e., $\det(\mathbf{J}_{\bar{b}r}(\mathbf{x}_t))$, can be expressed by*

$$\det(\mathbf{J}_{\bar{b}r}(\mathbf{x}_t)) = \left(\sum_{i=1}^N \left(\varsigma_i + \frac{1}{2} \zeta_i \right) \right)^2 - \frac{1}{4} \left(\sum_{i=1}^N \zeta_i \cos(2\phi_i) \right)^2 - \frac{1}{4} \left(\sum_{i=1}^N \zeta_i \sin(2\phi_i) \right)^2 \quad (4.33)$$

where $\varsigma_i = \frac{1}{\sigma_b^2 r_i^2}$, $\zeta_i = \frac{1}{\sigma_r^2} - \frac{1}{\sigma_b^2 r_i^2}$.

Proof. The proof of (4.33) is straightforward. Firstly, we write (4.32) as

$$\mathbf{J}_{\bar{b}r}(\mathbf{x}_t) = \sum_{i=1}^N \begin{bmatrix} \varsigma_i + \zeta_i \sin^2(\phi_i) & \frac{1}{2} \zeta_i \sin(2\phi_i) \\ \frac{1}{2} \zeta_i \sin(2\phi_i) & \varsigma_i + \zeta_i \cos^2(\phi_i) \end{bmatrix} \quad (4.34)$$

Then $\det(\mathbf{J}_{\bar{b}r}(\mathbf{x}_t))$ can be derived as follows:

$$\begin{aligned} \det(\mathbf{J}_{\bar{b}r}(\mathbf{x}_t)) &= \left(\sum_{i=1}^N \varsigma_i + \sum_{i=1}^N \zeta_i \sin^2(\phi_i) \right) \left(\sum_{i=1}^N \varsigma_i + \sum_{i=1}^N \zeta_i \cos^2(\phi_i) \right) \\ &\quad - \frac{1}{4} \left(\sum_{i=1}^N \zeta_i \sin(2\phi_i) \right)^2 \\ &= \left(\sum_{i=1}^N \varsigma_i \right)^2 + \left(\sum_{i=1}^N \varsigma_i \sum_{i=1}^N \zeta_i \right) + \left(\sum_{i=1}^N \zeta_i \sin^2(\phi_i) \right) \\ &\quad \left(\sum_{i=1}^N \zeta_i \cos^2(\phi_i) \right) - \frac{1}{4} \left(\sum_{i=1}^N \zeta_i \sin(2\phi_i) \right)^2 \end{aligned}$$

$$\begin{aligned}
&= \left(\sum_{i=1}^N \varsigma_i \right)^2 + \left(\sum_{i=1}^N \varsigma_i \sum_{i=1}^N \zeta_i \right) + \left(\sum_{i=1}^N \zeta_i \frac{1 - \cos(2\phi_i)}{2} \right) \\
&\quad \left(\sum_{i=1}^N \zeta_i \frac{1 + \cos(2\phi_i)}{2} \right) - \frac{1}{4} \left(\sum_{i=1}^N \zeta_i \sin(2\phi_i) \right)^2 \\
&= \left(\sum_{i=1}^N \varsigma_i \right)^2 + \left(\sum_{i=1}^N \varsigma_i \sum_{i=1}^N \zeta_i \right) + \frac{1}{4} \left(\sum_{i=1}^N \zeta_i \right)^2 \\
&\quad - \frac{1}{4} \left(\sum_{i=1}^N \zeta_i \cos(2\phi_i) \right)^2 - \frac{1}{4} \left(\sum_{i=1}^N \zeta_i \sin(2\phi_i) \right)^2 \\
&= \left(\sum_{i=1}^N \left(\varsigma_i + \frac{1}{2} \zeta_i \right) \right)^2 - \frac{1}{4} \left(\sum_{i=1}^N \zeta_i \cos(2\phi_i) \right)^2 \\
&\quad - \frac{1}{4} \left(\sum_{i=1}^N \zeta_i \sin(2\phi_i) \right)^2. \tag{4.35}
\end{aligned}$$

The proof is completed. $\square$

After deriving the determinant of the FIM, we can get the following results concerning the optimal sensor-source configurations for source localization using the co-located heterogeneous measurements.

4.5.1 Two and Three Sensor Platforms

Here we give the optimal sensor placement results for two and three sensor platforms cases.

Proposition 4.2. *Consider the source localization using co-located heterogeneous measurements with two sensor platforms. ϕ_1, ϕ_2 denote the angular positions of the sensor platforms. Given arbitrary sensor-source ranges r_1 and r_2 , then the optimal sensor angular separation is $v_{12} = \frac{\pi}{2}$.*

Proof. According to Theorem 4.5, for the two sensor platforms case, we have

$$\det(\mathbf{J}_{br}(\mathbf{x}_t)) = \left(\sum_{i=1}^2 \left(\varsigma_i + \frac{1}{2} \zeta_i \right) \right)^2$$

$$-\frac{1}{4}(\zeta_1^2 + \zeta_2^2 + 2\zeta_1\zeta_2 \cos 2(\phi_1 - \phi_2)) \quad (4.36)$$

Then using the D-optimality measure, we obtain the following optimization problem

$$\arg \min_{\phi_1, \phi_2} \cos 2(\phi_1 - \phi_2) \quad (4.37)$$

which implies $v_{12} = \frac{\pi}{2}$. □

From the result of Proposition 4.2, we can see that the optimal sensor angular positions do not depend on the sensor-source ranges, i.e., r_1 and r_2 . However, when $N > 2$, the problem becomes more complicated and we will show that the optimal sensor angular separations are related to the sensor-source ranges.

For the three sensor platforms case, we can still compute the optimal sensor angular positions given the sensor-source ranges. We can adopt similar results of Theorem 7 in [8]. The optimal angular separation v_{12} , v_{13} and v_{23} can be obtained by solving the following equations:

$$\begin{aligned} v_{12} &= \arccos \left(\frac{\zeta_2^4 \zeta_1^4 - \zeta_3^4 \zeta_1^4 - \zeta_3^4 \zeta_2^4}{2\zeta_3^4 \zeta_2^2 \zeta_1^2} \right), \\ v_{13} &= \arccos \left(\frac{\zeta_3^4 \zeta_1^4 - \zeta_3^4 \zeta_2^4 - \zeta_2^4 \zeta_1^4}{2\zeta_3^2 \zeta_2^4 \zeta_1^2} \right), \\ v_{23} &= 2\pi - v_{12} - v_{13}. \end{aligned} \quad (4.38)$$

A similar case has been investigated in [8]. For $N > 3$, it is difficult to obtain explicit solutions for the optimal sensor configurations. But for some special cases, we can get the solutions which will be introduced in the following subsection.

4.5.2 N Sensor Platforms

Here, we give the main results for the general case.

Theorem 4.6. *Consider the source localization using co-located heterogeneous measurements, where $\phi_i, \forall i \in \{1, \dots, N\}$ denotes the angular position of the i -th sensor platform. Let r_i be given for all $i \in \{1, \dots, N\}$. The determinant of the FIM, i.e., $\det(\mathbf{J}_{\bar{b}r}(\mathbf{x}_t))$, is upper-bounded by $\frac{1}{4} \left(\sum_{i=1}^N \left(\frac{1}{\sigma_b^2 r_i^2} + \frac{1}{\sigma_r^2} \right) \right)^2$. The upper-bound is achieved if and only if the following conditions*

$$\begin{aligned} \sum_{i=1}^N \left(\frac{1}{\sigma_r^2} - \frac{1}{\sigma_b^2 r_i^2} \right) \sin(2\phi_i) &= 0 \\ \sum_{i=1}^N \left(\frac{1}{\sigma_r^2} - \frac{1}{\sigma_b^2 r_i^2} \right) \cos(2\phi_i) &= 0 \end{aligned} \quad (4.39)$$

are satisfied by some $\phi_i, \forall i \in \{1, \dots, N\}$.

Proof. The proof is straightforward from the result in (4.33). $\square$

Corollary 4.2. *Consider the source localization using co-located heterogeneous measurements, where $\phi_i, \forall i \in \{1, \dots, N\}$ denotes the angular position of the i -th sensor platform. Given $r_i = r_j, \forall i \in \{1, \dots, N\}$. Then the upper-bound of $\det(\mathbf{J}_{\bar{b}r}(\mathbf{x}_t))$ is achieved if and only if the following conditions hold.*

$$\sum_{i=1}^N \sin(2\phi_i) = 0, \sum_{i=1}^N \cos(2\phi_i) = 0. \quad (4.40)$$

From Theorem 4.6, we can see that the optimal sensor angular positions are correlated with the ranges from the sensors to the source. However, when $r_i = r_j, \forall i \in \{1, \dots, N\}$, the problem becomes simpler. It can be easily verified that if $v_{ij} = v_{ji} = \frac{2\pi}{N}, \forall i, j \in 1, \dots, N$, then the conditions stated in (4.40) can be satisfied.

From Corollary 4.2, we can see that for the special case in which sensors are placed on a circle with the source as its center, the sensors should be symmetrically placed on the circle in an optimal sensor-source geometry setting.

4.5.3 Comparison with DOA and Range Localization

In this subsection, we compare the optimal conditions of maximizing the determinant of the Fisher information matrices of three methods: DOA, range and the co-located hybrid DOA and range based localization.

In [8], the authors give analytic results concerning the optimal sensor-source configuration for the DOA and range based source localization.

Theorem 4.7. [8] *Consider the range localization problem, with $\phi_i, \forall i$ denoting the angular position of the i -th sensor. Given an arbitrary number of range sensors N , the Fisher information determinant for range localization is upper-bounded by $\frac{N^2}{4\sigma_r^4}$. This upper-bound is achieved if and only if the angular positions of the sensors simultaneously satisfy*

$$\sum_{i=1}^N \sin(2\phi_i) = 0, \sum_{i=1}^N \cos(2\phi_i) = 0. \quad (4.41)$$

Theorem 4.8. [8] *Consider the DOA localization problem, with $\phi_i, \forall i \in \{1, \dots, N\}$ denoting the angular position of the i -th sensor. Given an arbitrary number of DOA sensors N . Let r_i be given for all $i \in \{1, \dots, N\}$. The determinant of the FIM is upper-bounded by $\frac{1}{4\sigma_b^4} \left(\sum_{i=1}^N \frac{1}{r_i^2} \right)^2$. The upper-bound is achieved if and only if the following conditions*

$$\sum_{i=1}^N \frac{\sin(2\phi_i)}{r_i^2} = 0, \sum_{i=1}^N \frac{\cos(2\phi_i)}{r_i^2} = 0 \quad (4.42)$$

are satisfied by some $\phi_i, \forall i \in \{1, \dots, N\}$.

Compared Theorem 4.6 with Theorem 4.7 and Theorem 4.8, we can find that the upper-bound of the determinant of the FIM of the co-located hybrid DOA and range based method, is the largest, i.e., $\frac{1}{4} \left(\sum_{i=1}^N \left(\frac{1}{\sigma_b^2 r_i^2} + \frac{1}{\sigma_r^2} \right) \right)^2 > \frac{N^2}{4\sigma_r^4}$ and $\frac{1}{4} \left(\sum_{i=1}^N \left(\frac{1}{\sigma_b^2 r_i^2} + \frac{1}{\sigma_r^2} \right) \right)^2 > \frac{1}{4\sigma_b^4} \left(\sum_{i=1}^N \frac{1}{r_i^2} \right)^2$, which means that the heterogeneous method can provide more accurate position estimation. This is because that with the same

number of sensor platforms, the heterogeneous method can get more information than the other two methods.

Example 4.4. Here, we provide an example illustrating the comparison of upper-bound of the determinant of the FIM using different methods. Assume that $\sigma_r = 2$, $\sigma_b = 1$ and $r_i = 2 \forall i$. The numerical results are shown in Figure 4.7. From the figure, we can see that with the increase in the number of sensors, the amount of Fisher information obtained from the hybrid sensing is higher than both DOA and range based methods. In addition, in this example, the upper-bound of the determinant of the FIM using range based method is slightly higher than the DOA based method.

Figure 4.7: Comparison of upper bound of the determinant of the FIM

For the conditions on achieving the upper-bounds of the determinant of the FIMs, from Theorem 4.6, Theorem 4.7 and Theorem 4.8, we can see that the conditions for the range based localization are simplest since they are not related to the ranges from the sensors to the source, i.e., $r_i, \forall i \in 1, \dots, N$. But if we assume that $r_i = r_j, \forall i \in \{1, \dots, N\}$, then the conditions of maximizing the determinant of the FIM for these three cases will be the same.

4.5.4 Discussion

The following points are worth noting:

- From the results of this section, we can see that when two types of measurements are co-located in the same sensor platforms, the resultant optimal sensor-source geometries are similar with the existing results for the single type of sensors. But the co-located heterogeneous case can provide more information of the source location. This is intuitively understood since co-located heterogeneous sensor platform have two types of measurements.
- In the optimal sensor configuration settings for the co-located sensors case (also for the single type of sensors), the sensor platforms are expected to form a circular formation around the source. This is different from the results when heterogeneous sensors are placed in separate platforms in Sections 4.3 and 4.4, where sensors are all placed at one side of the source in the optimal settings.

4.6 Conclusion

In this chapter, the optimal sensor placement problem in a HSN was firstly explored. General expressions for the determinant of the FIM of the three cases, i.e., hybrid range and DOA sensors, hybrid DOA and TDOA sensors as well as co-located hybrid DOA and range sensors, were derived, from which the explicit solutions of the optimal sensor-source geometries were derived.

As we know, different from WSN with a single type of sensors, it is difficult to obtain an explicit solution to the optimal sensor configuration problem for an arbitrary number of sensors case in a HSN, especially when N is large. Therefore, we focused on cases where the number of heterogeneous sensors is small but sufficient for a source localization task. It is worth pointing out that the results of the present work can be applied to the path planning problem for unmanned aerial/ground vehicles for optimal source localization.

In real source localization applications, source location is exactly the parameter to be estimated. However, the analysis presented in this chapter depends on the known source location. Hence, it is suggested that the sensor placement should be done iteratively based on the estimated source location. In addition, some other constraints should be added in the optimization, such as communication constraints, sensor dynamic constraints, etc.

Chapter 5

Decentralized Sensor Pairing

In Chapter 3 and Chapter 4, the sensor placement problem was studied for homogeneous and heterogeneous sensor networks, respectively. This chapter is concerned with another important issue in sensor resource management: TDOA sensor pairing. In Chapter 3, TDOA sensor pairing was already mentioned where the main focus is on sensor placement strategies. This chapter mainly deals with the sensor pairing protocol in a multi-hop sensor network.

As introduced in Chapter 2 and Chapter 3, most of the existing works adopt a centralized sensor pairing strategy, where one sensor node is chosen as the common reference. However, due to the bandwidth and power constraints of multi-hop WSNs, it is well known that this kind of centralized methods is energy consuming due to the need of single and multi-hop transmissions of raw measurement data. In this chapter, we propose a decentralized in-network sensor pairing method to acquire TDOA measurements for source localization. It is proved that the proposed decentralized in-network sensor pairing method can result in the same CRLB as the centralized one at a far less communication cost.

5.1 Problem Statement

A multi-hop WSN is considered. Given a team of N nodes performing a source localization task. Each sensor pair can estimate the TDOA between them by using

the generalized cross correlation method (GCC)[3]. A TDOA measurement by a sensor pair $\{i, j\}$ is reproduced here for convenience:

$$\hat{t}_{ij} = \frac{d_{ij}}{v} + e_{ij}, \quad (5.1)$$

where $d_{ij} = \|\mathbf{x}_i - \mathbf{x}_t\| - \|\mathbf{x}_j - \mathbf{x}_t\|$, $\mathbf{x}_i = [x_i, y_i]^T$ denotes the coordinates of sensor i , $\mathbf{x}_t = [x_t, y_t]^T$ is the source's coordinates, v is the propagation speed of the source signal. $e_{ij} \sim N(0, \sigma^2)$ is the TDOA estimation error where for simplicity we assume the variances of all TDOA measurements are equal.

For N sensors, there is a total of $N(N-1)/2$ possible TDOA sensor pairs. Let

$$\mathcal{I} = \{\{i, j\} | 1 \leq i < j \leq N\} \quad (5.2)$$

denote the set of all sensor pairs. $\mathcal{I}'$ is a subset of $\mathcal{I}$ and contains K ($K \leq \frac{N(N-1)}{2}$) sensor pairs whose TDOAs are used in source localization. By introducing the $K \times 1$ vectors

$$\hat{\mathbf{t}} = \begin{bmatrix} \hat{t}_{ij} \\ \vdots \end{bmatrix}_{\{i,j\} \in \mathcal{I}'}, \mathbf{d} = \begin{bmatrix} d_{ij} \\ \vdots \end{bmatrix}_{\{i,j\} \in \mathcal{I}'}, \mathbf{e} = \begin{bmatrix} e_{ij} \\ \vdots \end{bmatrix}_{\{i,j\} \in \mathcal{I}'}, \quad (5.3)$$

the measurement model in matrix form becomes

$$\hat{\mathbf{t}} = \frac{1}{v} \mathbf{d} + \mathbf{e}. \quad (5.4)$$

The problem of source localization is to estimate the source location $\mathbf{x}_t$ given the measurement vector $\hat{\mathbf{t}}$ and the sensor location $\mathbf{x}_i, i = 1, \dots, N$.

Most of the existing research is focused on algorithm development for source localization. Differently, in this chapter, the research focuses on how to form TDOA sensor pairs or how to collect TDOA measurements in a bandwidth and power limited multi-hop sensor network for source localization.

5.2 TDOA Sensor Pairing in Multi-hop Networks

5.2.1 Network Model

Let us represent a multi-hop WSN as a graph defined by $\mathcal{G} := (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the node set, $\mathcal{N} := \{1, \dots, N\}$ and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the edge set. If node k and node l can communicate with each other directly, we define the link by $(k, l) \in \mathcal{E}$.

Several definitions from graph theory are used in this chapter.

Definition 5.1. *Maximum Degree is the maximum count of edges emanating from a single node.*

Definition 5.2. *Dominating Set S is defined as a subset of $\mathcal{N}$ such that each node in $\mathcal{N} - S$ is adjacent to at least one node in S .*

Definition 5.3. *Connected Dominating Set (CDS): C is a dominating set of $\mathcal{G}$ which induces a connected subgraph of $\mathcal{G}$.*

Figure 5.1: Sensor pairing in a multi-hop sensor network

5.2.2 Proposed TDOA Sensor Pairing Method

In the literature, most of the works adopt a centralized sensor pairing strategy, as shown in Figure 5.1(a), where one sensor node (black node) is chosen as the reference node. In a centralized sensor pairing method, all the TDOAs can be estimated without redundancy. However, it may not work well in a multi-hop sensor network. This

is mainly because that raw measurements are involved in transmissions resulting in high communication overhead and high power consumption.

To reduce the requirements for both network bandwidth and power consumption, we propose an in-network sensor pairing method to collect TDOA measurements while guarantee the quality of source localization, as shown in Figure 5.1(b). The solution involves finding a CDS for the network graph [13]. The nodes in the CDS work as a network backbone to collect the TDOA measurements of the whole network. The proposed in-network sensor pairing method consists of the following two steps.

5.2.2.1 Finding a CDS

Finding a CDS has been well studied in the literature [42, 13]. In this thesis, a sequential greedy algorithm presented in [13] is adopted to find a CDS which is presented in Algorithm 1. For other algorithms, we refer readers to [42] and the references therein. Note that the CDS for a graph may not be unique.

Algorithm 1 Sequential Greedy Algorithm

- 1: Coloring all nodes white.
 - 2: Selecting a node that causes the maximum reduction of the number of white nodes. Once a node is selected, it is marked black and its white neighbors are marked blue.
 - 3: The algorithm iteratively scans the blue nodes and their white neighbors, and selects a blue node or a pair of nodes (a blue node and one of its white neighbors), whichever has the maximal number of white neighbors. The selected node or the selected pair of nodes are marked black, with their white neighbors marked blue.
 - 4: Algorithm terminates when there is no white node left.
 - 5: All the black nodes form a connected dominating set (CDS).
-

An example of finding a CDS is presented in Figure 5.2. From the above algorithm, the nodes in a CDS will be marked black and their neighbors are blue. We assume that there are p sensors in the dominating set, then $\frac{N}{1+\Delta} \leq p \leq \frac{N}{2}$, where Δ denotes the maximum degree of the graph [46].

In real applications, a CDS of a sensor network can be determined offline if the topology of the network remains fixed during the online sensing tasks.

Figure 5.2: An example of finding a CDS

Remark 5.1. *It is important to keep the cardinality of the dominating set small. In this case, we can find a CDS with minimum cardinality, which is called minimum CDS (MCDS). Finding the MCDS in a connected network was proved to be NP-complete. Some heuristic algorithms can be found in [27] and the references therein.*

5.2.2.2 Decentralized TDOA sensor pairing

After the CDS is determined, one node in the CDS will be selected as the **Team Leader**. The team leader is responsible for collecting the TDOA information from the other CDS nodes and computing a source location estimate. The TDOA sensor pairing can be achieved by using the following decentralized in-network TDOA sensor pairing method as presented in Algorithm 2.

Remark 5.2. *As stated in the Algorithm 2, the blue nodes will do the sensor pairing with the nearest black node in the CDS.*

Remark 5.3. *The proposed sensor pairing method may give rise to redundant TDOA information which will be eliminated by the **Team Leader** when calculating the source location.*

In the next subsection, performance analysis for the proposed in-network sensor pairing method is given in terms of estimation accuracy and communication cost.

Algorithm 2 Sensor Pairing in Multi-hop Sensor Networks

- 1: Finding a connected dominating set using Algorithm 1.
 - 2: The nodes in the dominating set (black nodes) broadcast their **raw measurement data** to their corresponding one-hop neighbors (including the blue and/or black nodes), respectively.
 - 3: The blue/black nodes estimate the TDOAs accordingly using their **local** processor. For two or more black nodes which are neighbours, only one of them needs to compute TDOAs.
 - 4: The blue nodes transmit their estimated TDOAs (which are **scalars**) back to their neighboring black nodes.
 - 5: The black nodes forward their collected/estimated TDOAs to the **Team Leader**.
 - 6: The **Team Leader** estimates the source location using the received TDOAs.
-

5.2.3 Performance Analysis

In this work, we use the CRLB as a metric to evaluate the estimation accuracy. CRLB is a lower bound for the error covariance matrix of any unbiased estimator.

The CRLB for TDOA based localization has been derived in [19]

$$\mathbf{J}^{-1} = v^2(\mathbf{G}\Sigma^{-1}\mathbf{G}^T)^{-1}, \quad (5.5)$$

where

$$\begin{aligned} \mathbf{G} &= [\mathbf{g}_{ij}, \dots]_{\{i,j\} \in \mathcal{I}}, \\ \mathbf{g}_{ij} &= \mathbf{g}_i - \mathbf{g}_j, \\ \mathbf{g}_i &= \begin{bmatrix} \frac{x_t - x_i}{\sqrt{(x_t - x_i)^2 + (y_t - y_i)^2}} \\ \frac{y_t - y_i}{\sqrt{(x_t - x_i)^2 + (y_t - y_i)^2}} \end{bmatrix}, \end{aligned}$$

and Σ denotes the covariance matrix of TDOA measurement noises.

Theorem 5.1. *If the network is connected, then the proposed decentralized in-network sensor pairing results in the same CRLB with the centralized one.*

Proof. Without loss of generality, we assume that the sensors with IDs $1, 2, \dots$ are in a chosen CDS. TDOA measurements for centralized pairing and the proposed

decentralized pairing methods can be represented by

$$\hat{\mathbf{t}}_{ce} \triangleq \begin{bmatrix} \hat{t}_{21} \\ \hat{t}_{31} \\ \vdots \\ \hat{t}_{N1} \end{bmatrix} = \mathbf{T}_1 \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{bmatrix}, \hat{\mathbf{t}}_{de} \triangleq \begin{bmatrix} \hat{t}_{i1} \\ \vdots \\ \hat{t}_{j2} \\ \vdots \end{bmatrix} = \mathbf{T}_2 \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{bmatrix} \quad (5.6)$$

respectively, where $\hat{\mathbf{t}}_{ce}, \hat{\mathbf{t}}_{de} \in \mathbb{R}^{(N-1) \times 1}$ and $t_i, \forall i$ denotes the time of arrival of the i -th sensor which is actually not known. $\mathbf{T}_1$ and $\mathbf{T}_2$ are transformation matrices and both of them are of dimension $(N-1) \times N$. $\mathbf{T}_1$ and $\mathbf{T}_2$ only have one ‘1’ and one ‘-1’ in each row. $\mathbf{T}_1$ can be represented by

$$\mathbf{T}_1 = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -1 & 0 & \cdots & 0 & 1 \end{bmatrix} \quad (5.7)$$

$\mathbf{T}_2$ is a little more complicated. Here we give an example to show how $\mathbf{T}_2$ looks like. We assume nodes 1 and 2 are the only two nodes in a CDS and they are connected. For simplicity, we further assume that node 3 is connected to 2 and the other nodes are all connected to node 1. Then $\mathbf{T}_2$ can be written as

$$\mathbf{T}_2 = \begin{bmatrix} -1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 & \ddots & \vdots \\ -1 & 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ -1 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \quad (5.8)$$

From (5.8), we can see that through a simple elementary transformation, i.e., the first row is added to the second row, then $\mathbf{T}_2$ can be transformed to $\mathbf{T}_1$.

Since the network is connected, it can be easily verified by induction method that for a general $\mathbf{T}_2$, it can also be transformed to $\mathbf{T}_1$ through an elementary transformation, i.e.,

$$\mathbf{T}_2 = \mathbf{U}_{cd}\mathbf{T}_1. \quad (5.9)$$

where $\mathbf{U}_{cd}$ is a $(N-1) \times (N-1)$ elementary transformation matrix.

The CRLBs for the centralized pairing and the proposed decentralized in-network pairing methods are denoted by $\mathbf{CRLB}_{ce} = v^2(\mathbf{G}_{ce}\Sigma_{ce}^{-1}\mathbf{G}_{ce}^T)^{-1}$ and $\mathbf{CRLB}_{de} = v^2(\mathbf{G}_{de}\Sigma_{de}^{-1}\mathbf{G}_{de}^T)^{-1}$, respectively.

According to (5.6) and (5.9), it is easy to obtain that

$$\mathbf{G}_{de} = \mathbf{G}_{ce}\mathbf{U}_{cd}^T.$$

For the covariance matrix, we have

$$\begin{aligned} \Sigma_{de}/v^2 &= E[(\mathbf{t}_{de} - E(\mathbf{t}_{de}))(\mathbf{t}_{de} - E(\mathbf{t}_{de}))^T] \\ &= \mathbf{U}_{cd}E[(\mathbf{t}_{ce} - E(\mathbf{t}_{ce}))(\mathbf{t}_{ce} - E(\mathbf{t}_{ce}))^T]\mathbf{U}_{cd}^T \\ &= \mathbf{U}_{cd}\Sigma_{ce}/v^2\mathbf{U}_{cd}^T \end{aligned} \quad (5.10)$$

Hence $\Sigma_{de} = \mathbf{U}_{cd}^T\Sigma_{ce}\mathbf{U}_{cd}$. Then, we can obtain that

$$\begin{aligned} \mathbf{CRLB}_{de} &= v^2(\mathbf{G}_{de}\Sigma_{de}^{-1}\mathbf{G}_{de}^T)^{-1} \\ &= v^2((\mathbf{G}_{ce}\mathbf{U}_{cd}^T)(\mathbf{U}_{cd}\Sigma_{ce}\mathbf{U}_{cd}^T)^{-1}(\mathbf{G}_{ce}\mathbf{U}_{cd}^T)^T)^{-1} \\ &= v^2(\mathbf{G}_{ce}(\mathbf{U}_{cd}^T(\mathbf{U}_{cd}^T)^{-1})\Sigma_{ce}^{-1}((\mathbf{U}_{cd})^{-1}\mathbf{U}_{cd})\mathbf{G}_{ce}^T)^{-1} \\ &= v^2(\mathbf{G}_{ce}\Sigma_{ce}^{-1}\mathbf{G}_{ce}^T)^{-1} \\ &= \mathbf{CRLB}_{ce}. \end{aligned} \quad (5.11)$$

This completes the proof. $\square$

Remark 5.4. *From the above theorem, it can be seen that if combining all the TDOA measurements collected by the nodes (redundant TDOAs are ignored) in a chosen CDS, then the corresponding CRLB will be the same as the centralized one. One important issue for the above result is that the network should be connected, i.e., there is at least a spanning tree in the network. If not, (5.9) does not hold and then not all the TDOA information can be collected by the black nodes in the CDS.*

Remark 5.5. *In-network aggregation in sensor networks will reduce communication cost and power consumption [33]. In the proposed method, only one-hop communications are required to accomplish the sensor pairing task which is a better choice compared with the centralized sensor pairing where long-distance transmissions are involved. Hence the proposed decentralized sensor pairing method requires a less communication cost. Some remarks on the communication cost of the CDS can be found in [65].*

5.3 Application to Decentralized Source Localization

One important application of the proposed sensor pairing method is the decentralized source localization. In the literature, most of the algorithms developed are centralized which are not realistic in general. In the proposed method, each black node collects several TDOA measurements (for source localization in a 2D space, at least two TDOAs are required) and is capable of estimating the source location by using the existing methods. Then an overall source location estimate can be obtained by fusing all the estimates from the black nodes in a centralized or a sequential manner.

In Figure 5.3, an example of the decentralized source localization is given by using the structure of the proposed in-network sensor pairing method, where the unknown parameters of the source are estimated sequentially by the black nodes in the CDS.

Many kinds of estimators can be applied, such as extended Kalman filter (EKF), nonlinear least-squares (NLS) method, particle filter (PF), etc.

Figure 5.3: Decentralized source localization architecture.

5.4 Conclusion

In this chapter, an in-network sensor pairing method has been proposed to collect TDOA measurements. Compared to the centralized pairing method, it has lower requirements for both network bandwidth and energy consumptions in data transmissions. It has been proved that the proposed in-network sensor pairing can result in the same CRLB as the centralized one at a far less communication cost. Finally, it is necessary to point out that the proposed sensor pairing method can be applied to the decentralized source localization problem.

Part II

Algorithm Development for Source Localization

Chapter 6

Multiple Source Localization: Centralized Methods

In Part I, we focused on the the research of sensor resource management which can improve the efficiency of localization algorithms, optimize the bandwidth and power usage of the network, etc. From this chapter, we start to study the algorithm development for the source localization problem.

This chapter will explore how to localize multiple sources using energy measurements in WSNs. As discussed in Chapter 2, energy based multiple source localization is quite difficult since in this case, each sensor's measurement is a superimposition of energy signals emitted from multiple sources. Existing algorithms for this problem such as Multi-Resolution (MR) search and exhaustive search methods are of either high computational complexity or low estimation accuracy. In this chapter, an efficient EM algorithm for MLE is presented. The basic idea of the algorithm is to decompose each sensor's energy measurement into components, each of which corresponds to an individual source, then estimate the source parameters such as source energy and location, as well as the decay factor of signal during propagation. An efficient Sequential Dominant-Source (SDS) initialization scheme and an incremental parameterized search refinement scheme are introduced to speed up the algorithm and improve the estimation accuracy.

6.1 Problem Description

Assume that a total of N sensors are deployed and K static sources are present in the sensor field, where K is the known number of sources¹. A fusion center is used to collect the measurement data of the sensors and to run the source localization algorithm. The locations of sensors, denoted by $\mathbf{x}_i = [x_i, y_i]^T, i = 1, \dots, N$, are known to the fusion center. The locations of sources, denoted by $\mathbf{x}_{sk} = [x_{sk}, y_{sk}]^T, k = 1, \dots, K$, are unknown. The signal energy received by sensor i at time t is reproduced here for convenience:

$$P_i(t) = P_{si}(t) + w_i(t) = g_i \sum_{k=1}^K \frac{S_k(t)}{d_{ik}^\alpha(t)} + w_i(t), \quad (6.1)$$

where $d_{ik} = \|\mathbf{x}_i - \mathbf{x}_{sk}\|$ is the Euclidean distance between the i th sensor and the k th source; α denotes the unknown decay factor with typical value in the range from 2 to 4; $S_k(t)$ is the signal energy at one meter away from the k -th source; g_i is the gain factor of the i -th sensor; $w_i(t)$ is a Gaussian measurement noise with mean μ and variance σ^2 , i.e., $w_i(t) \sim N(\mu, \sigma^2)$.

In the following, the time t is omitted for brevity. Introduce the following notations:

$$\begin{aligned} \mathbf{Y} &= [P_1 - \mu, P_2 - \mu, \dots, P_N - \mu]^T, \\ \mathbf{H} &= \text{diag} \underbrace{\{1/\sigma, \dots, 1/\sigma\}}_N, \\ \mathbf{D} &= \begin{bmatrix} \frac{g_1}{d_{11}^\alpha} & \frac{g_1}{d_{12}^\alpha} & \cdots & \frac{g_1}{d_{1K}^\alpha} \\ \frac{g_2}{d_{21}^\alpha} & \frac{g_2}{d_{22}^\alpha} & \cdots & \frac{g_2}{d_{2K}^\alpha} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{g_N}{d_{N1}^\alpha} & \frac{g_N}{d_{N2}^\alpha} & \cdots & \frac{g_N}{d_{NK}^\alpha} \end{bmatrix}, \\ \mathbf{S} &= [S_1, S_2, \dots, S_K]^T, \\ \boldsymbol{\varepsilon} &= [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N]^T, \end{aligned}$$

¹Estimating the number of sources is out of the scope of this work. The readers can refer to [106] and [82] for more discussions on how to solve this problem, e.g., using the MDL (minimum description length) method.

where $\varepsilon_i \triangleq (w_i - \mu)/\sigma \sim N(0, 1)$, $i = 1, 2, \dots, N$, are independent Gaussian random variables.

Using these matrices, (6.1) can be rewritten as

$$\mathbf{H}\mathbf{Y} = \mathbf{H}\mathbf{D}\mathbf{S} + \varepsilon. \quad (6.2)$$

In addition, denote

$$\mathbf{R} = \mathbf{D}\mathbf{S} = \left[\sum_{k=1}^K R_{1k}, \sum_{k=1}^K R_{2k}, \dots, \sum_{k=1}^K R_{Nk} \right]^T \in \mathcal{R}^{N \times 1}, \quad (6.3)$$

where $R_{ik} = g_i \frac{S_k}{d_{ik}^\alpha}$, $i = 1, \dots, N$.

Then the joint probability density function of $\mathbf{Y}$ can be expressed as

$$f(\mathbf{Y}|\Theta) = (2\pi)^{-N/2} \exp\left(-\frac{1}{2}(\mathbf{H}\mathbf{Y} - \mathbf{H}\mathbf{R})^T(\mathbf{H}\mathbf{Y} - \mathbf{H}\mathbf{R})\right), \quad (6.4)$$

where

$$\Theta = [\mathbf{x}_{s1}^T, \mathbf{x}_{s2}^T, \dots, \mathbf{x}_{sK}^T; S_1, S_2, \dots, S_K; \alpha]^T.$$

The negative log-likelihood function is proportional to the quadratic form

$$\ell(\mathbf{Y}|\Theta) = (\mathbf{H}\mathbf{Y} - \mathbf{H}\mathbf{R})^T(\mathbf{H}\mathbf{Y} - \mathbf{H}\mathbf{R}), \quad (6.5)$$

which can be expressed as

$$\ell(\mathbf{Y}|\Theta) = (\mathbf{Y} - \mathbf{R})^T \Sigma^{-1} (\mathbf{Y} - \mathbf{R}), \quad (6.6)$$

where $\Sigma^{-1} = \mathbf{H}^T \mathbf{H}$ and $\Sigma = \underbrace{\text{diag}\{\sigma^2, \dots, \sigma^2\}}_N$.

6.2 EM Localization Technique

Denote by $P_{ik}, i = 1, 2, \dots, N, k = 1, 2, \dots, K$, the signal energy received by the i -th sensor from the k -th source. Further, $\mathbf{Y}'_k, k = 1, \dots, K$ represents the vector of signal energy received by N sensors from the k -th source. The relationship among $\mathbf{Y}, \mathbf{Y}'_k$ and P_{ik} can be given by the following two equations:

$$\mathbf{Y}' = [\mathbf{Y}'_1, \mathbf{Y}'_2, \dots, \mathbf{Y}'_K]^T \in \mathcal{R}^{KN \times 1}, \quad (6.7)$$

$$\mathbf{Y} = \mathbf{A}\mathbf{Y}' \in \mathcal{R}^{N \times 1}, \quad (6.8)$$

where $\mathbf{Y}'_k = [P_{k1}, P_{k2}, \dots, P_{kN}] \in \mathcal{R}^{1 \times N}, k = 1, \dots, K$ and $\mathbf{A}$ is a $N \times KN$ matrix which has K ($N \times N$) identity matrices.

$$\mathbf{A} = \underbrace{[I_{N \times N}, I_{N \times N}, \dots, I_{N \times N}]}_K. \quad (6.9)$$

Similar to the measurement matrix $\mathbf{Y}', \mathbf{R}'$ can be expressed as another form:

$$\mathbf{R}' = [\mathbf{R}'_1, \mathbf{R}'_2, \dots, \mathbf{R}'_K]^T \in \mathcal{R}^{KN \times 1} \quad (6.10)$$

where $\mathbf{R}'_k = [R_{k1}, R_{k2}, \dots, R_{kN}], k = 1, \dots, K$, and

$$\mathbf{R} = \mathbf{A}\mathbf{R}' \in \mathcal{R}^{N \times 1}. \quad (6.11)$$

In this chapter, an efficient EM algorithm is proposed to estimate source locations, source energy and the decay factor. In the algorithm, $\mathbf{Y}'_k, k = 1, \dots, K$ are considered as hidden variables.

6.2.1 Expectation-Step (E-Step)

In this chapter, for any $\mathbf{u}$ which can be any scalar, vector or matrix, the notation $\hat{\mathbf{u}}$ is used to denote its estimated value. From (6.3), it can be deduced that

$$\mathbf{R}'_k = S_k \mathbf{d}_k, \quad (6.12)$$

where $\mathbf{d}_k = [g_1/d_{1k}^\alpha, g_2/d_{2k}^\alpha, \dots, g_N/d_{Nk}^\alpha] \in \mathcal{R}^{1 \times N}$.

In the E-Step of the EM algorithm, the expectation value of hidden information $\mathbf{Y}'_k, k = 1, \dots, K$, can be obtained by

$$\hat{\mathbf{Y}}'_k(\Theta_k) = S_k \hat{\mathbf{d}}_k + 1/K(\mathbf{Y} - \hat{\mathbf{D}}\hat{\mathbf{S}})^T, \quad (6.13)$$

where the second term of the right side of the equation is obtained by assuming that every element of $\mathbf{Y}'_k(\Theta_k)$, i.e., $P_{ik}, i = 1, \dots, N$, has an approximate noise variance σ^2/K .

Proposition 6.1. *Let α be given, then the unknown parameter of each source $\Theta_k = [\mathbf{r}_{sk}, S_k]$ can be estimated independently by*

$$\hat{\Theta}_k = \arg \min_{\Theta_k} \ell(\hat{\mathbf{Y}}'_k | \Theta_k) = \arg \min_{\Theta_k} (\hat{\mathbf{Y}}_k - \mathbf{R}_k)^T (\hat{\mathbf{Y}}_k - \mathbf{R}_k) \quad (6.14)$$

where $\mathbf{Y}'_k, k = 1, \dots, K$, is the hidden information for the EM algorithm as defined in (6.7).

Proof: Similar to (6.6), the negative log-likelihood function of $\mathbf{Y}'$ can be written as

$$\begin{aligned} \ell(\mathbf{Y}' | \Theta) &= (\mathbf{Y}' - \mathbf{R}')^T \Sigma'^{-1} (\mathbf{Y}' - \mathbf{R}') \\ &= \frac{K}{\sigma^2} \sum_{k=1}^K (\mathbf{Y}'_k - \mathbf{R}'_k)(\mathbf{Y}'_k - \mathbf{R}'_k)^T, \end{aligned} \quad (6.15)$$

where $\mathbf{\Sigma}' = (\sigma^2/K)I_{NK} \in \mathcal{R}^{NK \times NK}$. This is because every element in $\mathbf{Y}'$, *i.e.*, $P_{ik}, i = 1, \dots, N, k = 1, \dots, K$, is assumed to have an approximate noise variance σ^2/K .

As shown in (6.15), it is easy to see that the negative log-likelihood $\ell(\mathbf{Y}|\Theta)$ is proportional to a sum over K terms. For each individual log-likelihood, it only depends on the k -th source's unknown parameters. Hence, it can be obtained that

$$\hat{\Theta}_k = \mathbf{arg} \min_{\Theta_k} (\hat{\mathbf{Y}}'_k - \mathbf{R}'_k)(\hat{\mathbf{Y}}'_k - \mathbf{R}'_k)^T. \quad (6.16)$$

□

6.2.2 Maximization-Step (M-step)

Substituting (6.12) into (6.16), then it can be obtained that

$$\hat{\Theta}_k = \mathbf{arg} \min_{\Theta_k} (\hat{\mathbf{Y}}'_k - S_k \mathbf{d}_k)(\hat{\mathbf{Y}}'_k - S_k \mathbf{d}_k)^T \quad (6.17)$$

In order to compute (6.17), we first eliminate unknown parameter S_k by setting $S_k = \mathbf{Y}'_k \mathbf{d}_k^\dagger$, where $\mathbf{d}_k^\dagger = \mathbf{d}_k^T (\mathbf{d}_k \mathbf{d}_k^T)^{-1}$ is the pseudo-inverse of the matrix $\mathbf{d}_k \in \mathcal{R}^{1 \times N}$.

Then the negative log-likelihood function can be modified as

$$\ell(\hat{\mathbf{Y}}'_k | \mathbf{x}_{sk}) = \mathbf{Y}'_k (\mathbf{I} - \mathbf{P}_d) (\mathbf{I} - \mathbf{P}_d)^T \mathbf{Y}'_k{}^T \quad (6.18)$$

$$= \mathbf{Y}'_k \mathbf{Y}'_k{}^T - \mathbf{Y}'_k \mathbf{P}_d \mathbf{Y}'_k{}^T, \quad (6.19)$$

where $\mathbf{P}_d = \mathbf{d}_k^T (\mathbf{d}_k \mathbf{d}_k^T)^{-1} \mathbf{d}_k = \mathbf{d}_k^T \mathbf{d}_k / \|\mathbf{d}_k\|^2$ is a projection matrix. Since $\mathbf{Y}'_k \mathbf{Y}'_k{}^T$ is known, minimizing $\ell(\hat{\mathbf{Y}}'_k | \mathbf{x}_{sk})$ is equivalent to maximizing

$$\ell'(\hat{\mathbf{Y}}'_k | \mathbf{x}_{sk}) = \mathbf{Y}'_k \mathbf{P}_d \mathbf{Y}'_k{}^T = \mathbf{Y}'_k \frac{\mathbf{d}_k^T}{\|\mathbf{d}_k\|} \frac{\mathbf{d}_k}{\|\mathbf{d}_k\|} \mathbf{Y}'_k{}^T \quad (6.20)$$

$$= \|\mathbf{Y}'_k \mathbf{d}_k^T / \|\mathbf{d}_k\|\|^2, \quad (6.21)$$

where $\mathbf{Y}'_k$ is defined in (6.7).

So the ML estimates of $\mathbf{x}_{sk}$ and S_k are given by

$$\hat{\mathbf{x}}_{sk} = \mathbf{arg} \max_{\mathbf{x}_{sk}} \|\mathbf{Y}'_k \mathbf{d}_k^T / \|\mathbf{d}_k\|\|^2, \quad (6.22)$$

$$\hat{S}_k = \frac{\mathbf{Y}'_k \mathbf{d}_k^T}{\|\mathbf{d}_k\|^2}. \quad (6.23)$$

After getting $\hat{\mathbf{x}}_{sk}$ and $\hat{S}_k$, another unknown parameter, decay factor α can be estimated. Note that the least-squares solution for α is given by

$$\begin{aligned} \hat{\alpha} &= \mathbf{arg} \min_{\alpha} \sum_{i=1}^N (P_i - g_i \sum_{k=1}^K \frac{S_k}{d_{ik}^{\alpha}} - \mu)^2 \\ &= \mathbf{arg} \min_{\alpha} f(\alpha). \end{aligned} \quad (6.24)$$

To solve (6.24), a normalized incremental gradient (NIG) method is given by

$$\hat{\alpha}_{m+1} = \hat{\alpha}_m - a \nabla_m f(\hat{\alpha}_m) / \|\nabla_m f(\hat{\alpha}_m)\|, m = 1, 2, \dots, M, \quad (6.25)$$

where M is the maximum number of iterations which is assumed predetermined. $\hat{\alpha}_m$ denotes an estimate for α at the m th iteration, $\nabla_m f(\hat{\alpha}_m)$ denotes the gradient of $f(\hat{\alpha}_m)$, and a is the step size which is a small positive scalar. Usually the initial value of α is chosen randomly from 2 to 4.

Using the above results for estimation of unknown parameters, the detailed pseudo-code description of the EM algorithm is shown in Table 6.1.

The algorithm starts with the initialization based on a sequential dominant-source (SDS) scheme described in the next subsection. Each iteration consists of an E-step, a M-step and a decay factor parameter update step. In the E-Step, the received energy of sensors is decomposed to get the hidden data $\mathbf{Y}'_k$ which represents the signal energy received by sensors from a source. In the M-Step, a search method will be used to get the optimal source location estimate. In this chapter, an incremental parameterized search refinement scheme, i.e., β -parameterized search method is used. After getting the source location estimates, source energy can be computed by

(6.23). Then using the estimated $\hat{\mathbf{x}}_{sk}$ and S_k , decay factor can be obtained according to (6.25) by using the normalized incremental gradient method.

In summary, using the proposed EM algorithm, the aggregative energy signal is decomposed into individual components and then the corresponding location parameters are estimated separately for each source.

Table 6.1: The EM Algorithm

Input: $P_i, i = 1, 2, \dots, N$
Output: Θ

Initialization: $\mathbf{x}_{sk}, \alpha, i = 0$.
 $\mathbf{S} = \mathbf{D}^\dagger \mathbf{Y}$, where $\mathbf{D}^\dagger$ is the *pseudo-inverse* of the matrix $\mathbf{D}$.
while $\|\Theta^i - \Theta^{i-1}\| > \text{threshold}$ (we set $\text{threshold} = 10^{-3}$ in our simulation) **do**
 $i = i + 1$;
 for $k = 1$ to K ,
 E-step
 $Q(\Theta, \Theta^i) = E[\log f(\hat{\mathbf{Y}}'_k | \Theta) | \mathbf{Y}, \Theta^i]$,
 where $\hat{\mathbf{Y}}'_k(\hat{\Theta}_k^i) = S_k \hat{\mathbf{d}}_k + 1/K(\mathbf{Y} - \hat{\mathbf{D}}\hat{\mathbf{S}})^T$, with $\hat{\mathbf{d}}_k, \hat{S}_k, \hat{\mathbf{D}}, \hat{S}$ being all
 estimated using $\hat{\mathbf{x}}_{sk}$ obtained from the previous iteration.
 M-step
 $\hat{\mathbf{x}}_{sk} = \arg \max_{\mathbf{x}_{sk}} \|\mathbf{Y}'_k \mathbf{d}_k^T / \|\mathbf{d}_k\|\|$,
 $\hat{S}_k = \frac{\mathbf{Y}'_k \mathbf{d}_k^T}{\|\mathbf{d}_k\|^2}$,
 $\hat{\alpha} = \arg \min_{\alpha} \sum_{i=1}^N (P_i - g_i \sum_{k=1}^K \frac{S_k}{d_{ik}^\alpha} - \mu)^2$.
 Then **update**
 $\Theta^{i+1} = [\mathbf{x}_{s1}^T, \mathbf{x}_{s2}^T, \dots, \mathbf{x}_{sK}^T; \hat{S}_1, \hat{S}_2, \dots, \hat{S}_K; \hat{\alpha}]$.

6.2.3 Initialization for EM Algorithm

6.2.3.1 SDS Initialization

The EM algorithm could be highly sensitive to the initialization and may trap into a local optima. As such, several improved strategies have been proposed. A natural solution is to start from multiple random initializations and choose the final estimate with the highest likelihood [78]. This method increases the probability of converging to the global optima but requires extra computation effort. Another

method is the k -means clustering algorithm [44]. Similar to the EM algorithm, the k -means clustering algorithm depends on the initial condition and also may trap into local optima easily.

In this chapter, a sequential dominant source (SDS) initialization scheme is proposed to get the initial location estimates for sources. It will sequentially select the sensors which receive locally maximum received energy as the closest nodes to the sources and initiate the locations of the sources to their corresponding closest nodes.

First, denote y'_i as the energy received by the i th sensor from $K' \leq K$ sources whose closest nodes have been selected.

$$P'_i = g_i \sum_{k=1}^{K'} \frac{\hat{S}_k}{d_{ik}^\alpha}, i = 1, \dots, N.$$

Then the next closest nodes can be selected using the following algorithm presented in Table 6.2. Note that the selected closest nodes will not participate the node selection for the next dominant source. This ensures that d_{ik} in the above equation will never be equal to zero.

Table 6.2: Sequential Dominant-Source (SDS) Localization Method

Input: $P_i, i = 1, 2, \dots, N$

Output: $\hat{\mathbf{x}}_s$

For $K' = 0$ To $K - 1$

$\hat{\mathbf{x}}_{s(K'+1)} = \mathbf{arg} \max_{\mathbf{x}_s} (P_i - P'_i)$
 $= \mathbf{arg} \max_{\mathbf{x}_s} (P_i - g_i \sum_{k=1}^{K'} \frac{\hat{S}_k}{d_{ik}^\alpha}), i = 1, 2, \dots, N$

End

Remark 6.1. *The spacing of sensors is critical to obtaining initial source location estimates by using the above method. For example, we assume that there are two sensors labeled by i and j respectively, receiving higher energy than other sensors with $y(i) > y(j)$, and they are both close to the same source labeled by C . If sensor i has been selected as the closest sensor of C , then sensor j may be selected as the*

closest sensor of a remaining source. To avoid such a situation, in the following we propose an effective strategy to make a refinement for the SDS initialization method based on sensor spacing analysis.

6.2.3.2 Sensor Spacing Analysis

Consider a region of interest with an area A , N sensors are uniformly placed in the region. The density of sensors is

$$\lambda = N/A.$$

Then the number of sensors in the subregion with area s , has the poisson distribution of parameter λs . Let τ denote the distance between source and its closest sensor. Similar to [32], we can know that τ has the cumulative distribution function

$$F(\tau) = 1 - e^{-\pi\lambda\tau^2}. \quad (6.26)$$

Further, the expected distance of τ is given by

$$\bar{\tau} = \int_0^{+\infty} dF'(\tau)d(\tau) = \frac{1}{2\sqrt{\lambda}}, \quad (6.27)$$

where $F'(\tau)$ denotes the derivative of $F(\tau)$.

6.2.3.3 Complement for SDS Method

To avoid the case mentioned in Remark 6.1, after getting the closest sensor for one of the sources, the neighbors (which will be defined in the following part) of the sensor should be eliminated for the next sensor selection.

According to the above analysis on the expected distance between the source and its closest sensor, a neighboring sensor should satisfy

$$\|\hat{\mathbf{x}}_{\mathbf{s}} - \mathbf{x}_i\|_2 \leq \bar{\tau}$$

where $\hat{\mathbf{x}}_{\mathbf{s}}, \mathbf{x}_i$, denote the coordinates of the selected closest sensor for each source and its neighboring sensors respectively. This complementary strategy for sequential dominant-source method is very effective that will be validated in our simulations.

Remark 6.2. *To some extent, using the above initialization method the initial location estimates will be near to the true locations for each source especially when the sources are well-separated. But if several sources are very close to each other, it is difficult to get good initial location estimates for them simultaneously. In this case, however, the localization accuracy will be refined and improved in the search part for the M-step of the EM algorithm that will be presented in the following subsection.*

6.2.4 β -parameterized Search Method for M-step

In the M-step of the EM algorithm, in order to obtain the maximum likelihood estimation for source locations according to (6.22), a β -parameterized search method will be designed. It has a lower complexity and better localization accuracy than the existing MR search method [106].

First of all, according to the convergence property of the EM algorithm, a dart-board search region is proposed as shown in Figure 6.1. The basic idea is to adjust the search regions iteratively, where at the initial steps, large regions are discovered for global search, and gradually refined small regions are selected to improve the accuracy of the solution.

6.2.4.1 Radius of Search

Intuitively, the radius of the search region should depend on the sensor density which is presented in (6.27). Usually $\mathbf{U}_0 \geq \bar{\tau}$ is set as the radius for the first step of search method to get better performance. For a sensing region with area A and N sensors, $\bar{\tau} = 1/2\sqrt{N/A}$. So we should set $\mathbf{U}_0$ slightly larger than $\bar{\tau}$.

To design the step size of search, first it is necessary to analyze the convergence rate of the EM algorithm.

Figure 6.1: Iterative search in the M-step

Theorem 6.1. (*Convergence property of the EM algorithm*) For sufficiently large i , there exists a constant $0 \leq \beta < 1$ such that

$$\|\Theta^i - \Theta^*\| \leq \beta \|\Theta^{i-1} - \Theta^*\|.$$

where Θ^* is the limiting point.

Proof. The update of the EM algorithm can be written as:

$$\Theta^{i+1} = \mathbf{arg} \max_{\Theta} \sum_{k=1}^K E[\text{Log}f(\mathbf{Y}'_k|\Theta)|\mathbf{Y}, \Theta^i]. \quad (6.28)$$

According to the algorithm in Table 6.1, for the k th source,

$$\Theta_k^{i+1} = \mathbf{arg} \max_{\Theta_k} E[\text{Log}f(\mathbf{Y}'_k|\Theta_k)|\mathbf{Y}, \Theta_k^i], \quad (6.29)$$

where $\Theta_k = [\mathbf{x}_{sk}, S_k]$.

Then, it can be obtained that

$$E[\text{Log}f(\mathbf{Y}'_k|\Theta_k)|\mathbf{Y}, \Theta_k^i] = \text{Log}f(\mathbf{Y}|\Theta_k) + E[\text{Log}f(\mathbf{Y}'_k|\mathbf{Y}, \Theta_k)|\mathbf{Y}, \Theta_k^i]$$

Define

$$L(\Theta_k) = \text{Log}f(\mathbf{Y}|\Theta_k),$$

$$H(\Theta_k|\Theta_k^i) = E[\text{Log}f(\mathbf{Y}'_k|\mathbf{Y}, \Theta_k)|\mathbf{Y}, \Theta_k^i].$$

Then $Q(\Theta_k|\Theta_k^i) = L(\Theta_k) + H(\Theta_k|\Theta_k^i)$, and (6.29) can be expressed as:

$$\Theta_k^{i+1} = \mathbf{arg} \max_{\Theta_k} Q(\Theta_k|\Theta_k^i).$$

Because Θ_k is assumed to converge to a point Θ_k^* , the gradient $D^{10}Q(\Theta_k^{i+1}, \Theta_k^i)$ can be expanded by the Taylor series at point (Θ^*, Θ^*) as

$$\begin{aligned} D^{10}Q(\Theta_k^{i+1}, \Theta_k^i) &\approx D^{10}Q(\Theta_k^*, \Theta_k^*) + D^{20}Q(\Theta_k^*, \Theta_k^*)(\Theta_k^{i+1} - \Theta_k^*) \\ &\quad + D^{11}Q(\Theta_k^*, \Theta_k^*)(\Theta_k^i - \Theta_k^*) = 0, \end{aligned} \quad (6.30)$$

where D^{ij} denotes the i -th-order partial derivatives with respect to the first variable and the j -th-order partial derivative with respect to the second variable. This kind of Taylor expansion is similar to $f(\Theta_1, \Theta_2) \approx f(\Theta^*, \Theta^*) + (\Theta_1 - \Theta^*)f_{\Theta_1}(\Theta^*, \Theta^*) + (\Theta_2 - \Theta^*)f_{\Theta_2}(\Theta^*, \Theta^*)$. Note that $D^{10}Q(\Theta_k^*, \Theta_k^*) = 0$ since Θ_k^* is a fixed point for the k -th source. Rearranging the equation (6.30), it can be obtained that

$$\Theta_k^{i+1} - \Theta_k^* = [-D^{20}Q(\Theta_k^*, \Theta_k^*)]^{-1} D^{11}Q(\Theta_k^*, \Theta_k^*)(\Theta_k^i - \Theta_k^*).$$

Taking the 2-norm on each side, it can be obtained that

$$\begin{aligned} \|\Theta_k^{i+1} - \Theta_k^*\| &= \|[-D^{20}Q(\Theta_k^*, \Theta_k^*)]^{-1} D^{11}Q(\Theta_k^*, \Theta_k^*)(\Theta_k^i - \Theta_k^*)\| \\ &\leq \|[-D^{20}Q(\Theta_k^*, \Theta_k^*)]^{-1} D^{11}Q(\Theta_k^*, \Theta_k^*)\| \times \|(\Theta_k^i - \Theta_k^*)\| \\ &= \|[D^2L(\Theta_k^*) + D^{20}H(\Theta_k^*|\Theta_k^*)]^{-1} D^{20}H(\Theta_k^*|\Theta_k^*)\| \|(\Theta_k^i - \Theta_k^*)\|, \end{aligned}$$

where $D^{11}Q(\Theta_k^*, \Theta_k^*) = -D^{20}H(\Theta_k^*|\Theta_k^*)$ is positive definite, and $D^2L(\Theta_k^*)$ is negative

definite. Let $V = D^2L(\Theta_k^*) + D^{20}H(\Theta_k^*|\Theta_k^*)^{-1}D^{20}H(\Theta_k^*|\Theta_k^*)$, then it can be easily obtained that

$$\beta_k = \|V\| < \lambda_M(V) < 1,$$

where $\lambda_M(P)$ denotes the largest eigenvalue of P .

$$\|\Theta_k^{i+1} - \Theta_k^*\| \leq \beta_k \|\Theta_k^i - \Theta_k^*\|.$$

So for $\Theta = [\Theta_1, \dots, \Theta_K]^T$, it can be known that

$$\|\Theta^i - \Theta^*\| \leq \beta \|\Theta^{i-1} - \Theta^*\|,$$

where $\beta = \max(\beta_k), 0 \leq \beta < 1, k = 1, \dots, K$. □

From Theorem 6.1, it can be seen that convergence rate of the EM algorithm depends on the largest eigenvalue of V which however is difficult to obtain. An alternative is to fix β as a rational value.

Also according to Theorem 6.1, for the convergence property of location estimates of sources, we have the following result

$$\|\mathbf{x}_{sk}^{i+1} - \mathbf{x}_{sk}^*\|_2 \leq \beta \|\mathbf{x}_{sk}^i - \mathbf{x}_{sk}^*\|_2 \quad (6.31)$$

where $\|\cdot\|_2$ denotes the 2-norm; $\mathbf{x}_{sk}^*$ denotes the limiting point for iteration. $\|\mathbf{x}_{sk}^i - \mathbf{x}_{sk}^*\|_2$ can be considered as the radius for the i th iteration in our search method. Then the following rule for the radius of each iteration is proposed

$$\mathbf{U}_l = \beta \mathbf{U}_{l-1}, l = 1, 2, \dots, L \quad (6.32)$$

where $\mathbf{U}_l$ denotes the radius of search of the l -th iteration.

Theorem 6.2. *Let the sequence $\{\Theta^i\}_{i=0}^\infty$ be generated by $\Theta^{i+1} \in M(\Theta^i)$, where M is a point-to-set map on Θ . Let solution set $\Omega \subset \Theta$ be given, then all the limiting points of $\{\Theta^i\}$ are in the solution set Ω .*

Proof. See [124] (page 4). $\square$

In the proposed search method, the solution set is restricted into the circle with radius $\mathbf{U}_0$ around the initial search point. Then according to Theorem 6.2, the limiting points (stationary points) will be in this region. Because $\ell(\mathbf{x}_{sk}^0) \leq \ell(\mathbf{x}_{sk}^1) \leq \dots \leq \ell(\mathbf{x}_{sk}^L)$ hold (a property of the EM algorithm), where $\ell(\mathbf{x}_{sk}^L)$ is a value of the log-likelihood for the L -th iteration, so iterative estimates will gradually approach local optimal solutions.

6.2.4.2 Procedure of the Search Method

Based on the analysis of the convergence rate of the EM algorithm, the β -parameterized search method for the M-step can be designed using (6.32). From Figure 6.1, it can be seen that the radius for each iteration decreases as in (6.32).

Figure 6.2: Search points of the initial iteration in the M-step

For the initialization, the first step, the initial estimation of the source location is set as the center of the first search region and $\mathbf{U}_0$ can be obtained from the last subsection as the radius of the circle. In the second step, this region is divided into q rings with radius difference $\mathbf{U}_0/q$. On each circle, p points will be selected for comparison to obtain the local best location estimate for each source. As shown in Figure 6.2, where $q = 3, p = 8$ and the area within the solid circle denotes the search region for the first iteration, dashed circles denote the divided search region and the small black dots denote search grids. The pseudocode of the search method is shown in Table 6.3.

Table 6.3: Pseudocode of the Search Method

Input: $\mathbf{U}_0, p, q$ **Output:** $\hat{\mathbf{x}}_s$ For $l = 1$ To L $\mathbf{U}_l = \beta \mathbf{U}_{l-1}$, $interval = \mathbf{U}_l / q$; For $i = 1$ To q For $j = 1$ To p

$$\mathbf{x}_{sk}^l = \mathbf{x}_{sk}^{l-1} + interval \cdot i \cdot \begin{bmatrix} \cos(2\pi \cdot j/p) \\ \sin(2\pi \cdot j/p) \end{bmatrix},$$

$$\hat{\mathbf{x}}_{sk}^l = \arg \max_{\mathbf{x}_{sk}} \|\mathbf{d}_k^T \mathbf{Y}_k / \|\mathbf{d}_k\|\|^2.$$

End

End

End

6.3 Algorithm Analysis

6.3.1 CRLB

From (6.11), it is known that unknown parameter Θ_k of each source can be estimated independently, i.e.,

$$\hat{\Theta}_k = \arg \min_{\Theta_k} L(\hat{\mathbf{Y}}'_k | \Theta_k), \quad (6.33)$$

$$L(\hat{\mathbf{Y}}'_k | \Theta_k) = (\hat{\mathbf{Y}}'_k - \mathbf{R}_k)^T (\hat{\mathbf{Y}}'_k - \mathbf{R}_k) \quad (6.34)$$

$$= (\hat{\mathbf{Y}}'_k - S_k \mathbf{d}_k)(\hat{\mathbf{Y}}'_k - S_k \mathbf{d}_k)^T. \quad (6.35)$$

Here, it is assumed that α is known and we focus on the CRLB for each source's location estimation, that is $\hat{\mathbf{x}}_{sk} = [\hat{x}_{sk}, \hat{y}_{sk}]^T$ which involves $\mathbf{d}_k$ because $d_{ik} = \|\mathbf{x}_i - \mathbf{x}_{sk}\|$. Define $\Theta_k = \hat{\mathbf{x}}_{sk}$, $\Theta_{k1} = x_{sk}$, $\Theta_{k2} = y_{sk}$, then the FIM can be expressed as:

$$I(\Theta_{k(\mathbf{x}_s)}) = \begin{bmatrix} I_{1,1} & I_{1,2} \\ I_{2,1} & I_{2,2} \end{bmatrix}. \quad (6.36)$$

For each element of information matrix,

$$I_{mn}(\Theta_{k(\mathbf{r}_s)}) = E\left[\frac{\partial L(\hat{\mathbf{Y}}'_k | \Theta_k)}{\partial \Theta_{km}} \left(\frac{\partial L(\hat{\mathbf{Y}}'_k | \Theta_k)}{\partial \Theta_{kn}}\right)^T\right], \quad (6.37)$$

where $m = 1, 2; n = 1, 2$,

$$I_{1,1} = \frac{4\alpha^2 S_k^2}{\sigma^2} \sum_{i=1}^N \frac{g_i^2(x_{sk} - x_i)^2}{d_{ik}^{2(\alpha+2)}}, \quad (6.38)$$

$$I_{1,2} = I_{2,1} = \frac{4\alpha^2 S_k^2}{\sigma^2} \sum_{i=1}^N \frac{g_i^2(x_{sk} - x_i)(y_{sk} - y_i)}{d_{ik}^{2(\alpha+2)}}, \quad (6.39)$$

$$I_{2,2} = \frac{4\alpha^2 S_k^2}{\sigma^2} \sum_{i=1}^N \frac{g_i^2(y_{sk} - y_i)^2}{d_{ik}^{2(\alpha+2)}}. \quad (6.40)$$

The CRLB is computed by taking the inverse of the information matrix, $Var(\Theta_{k(\mathbf{x}s)}) \geq I^{-1}(\Theta_{k(\mathbf{x}s)})$.

Remark 6.3. *Note that the CRLB analysis for location estimation depends on α . In the above CRLB analysis, it is assumed that the decay factor α is given in order to give a comparison in the simulation study in Section 6.4 between the proposed algorithm and existing algorithms where decay factor α is assumed to be known. Actually, if decay factor α is unknown, it can be estimated using the NIG (normalized incremental gradient) method as presented in (6.25).*

6.3.2 Computational Complexity

In a 2D sensor field, let there be K sources. For the exhaustive search method ([106]), it is assumed there are γ grid points to be searched in each dimension. Then the total number of search points will be equal to γ^{2K} . By using the MR search method, assuming that φ points in each dimension per iteration will be examined, that is $\gamma = \varphi^m$ with m being the number of iterations. So the total search points will be $m\varphi^{2K}$ rather than $\gamma^{2K} = \varphi^{2mK}$. However for our EM algorithm, only $p \times q \times K \times \text{iteration_number}$ points need to be examined. Obviously, it is a polynomial function rather than an exponential function in the exhaustive search and MR search methods.

6.4 Simulation Results

6.4.1 Performance Comparison with Known Decay Factor

To our knowledge, the majority of the existing algorithms for source localization assume that the decay factor α is known. In order to give a fair comparison between the proposed algorithm and those existing algorithms, here it is assumed that the decay factor $\alpha = 2$. The performance of the proposed EM algorithm, MR search method [106], exhaustive search method, as well as the CRLB are compared through a series of Monte Carlo simulations. The size of the sensor field is 100 m×100 m. Sensor nodes are placed randomly in each run. The locations of the static sources are assumed to have a uniform distribution within the sensor field. 100 different sensor topologies with 1000 trials per topology are used to average the noise and the effect of sensor locations. The model in (6.1) is used to generate the signal energy readings for each sensor. The gain factors for all sensors are equal to 1. The CRLB and RMSE are computed as follows:

$$\text{CRLB} = \sqrt{\sum_{k=1}^K \sum_{i=1}^{N_t} \frac{[I^{-1}]_{11} + [I^{-1}]_{22}}{N_t K}},$$

$$\text{RMS} = \sqrt{\sum_{k=1}^K \sum_{i=1}^{N_t} \frac{\|\hat{\mathbf{x}}_{ski} - \mathbf{x}_{ski}\|^2}{N_t K}},$$

where $N_t = 100$ and K denotes the number of sources.

Figure 6.3 depicts a scenario with 30 sensors deployed in the sensor field. Three sources are located at $[-20, -20]$, $[20, 20]$ and $[0, 0]$ respectively. According to the analysis in Section 6.3, for the proposed EM algorithm, in the simulations, the radius of search region is set according to Table 6.4.

The three level grid resolutions for the MR search method are used, *i.e.*, 4×4 , 2×2 and 1×1 , which means that $[(100 \times 100)/(4 \times 4)]^K = (25 \times 25)^K$, $[(4 \times 4)/(2 \times 2)]^K = (2 \times 2)^K$ and $[(2 \times 2)/(1 \times 1)]^K = (2 \times 2)^K$ grid points are needed to be searched

Figure 6.3: Sensor and source deployment

Table 6.4: Radius for Initial Step

Sensor density	Radius of search($\mathbf{U}_0$)
10	20
30	15
50	10

in three levels respectively. For the exhaustive search method, the grid resolution is set as 1×1 . For the proposed EM algorithm, $p = 8, q = 5, \beta = 0.7$. Because the likelihood function is nonconvex, the optimization algorithm may reach the local optima if the initial estimates are far away from the true values.

The localization performance of each method with different sensor densities is illustrated in Figure 6.4. The source number is assumed to be $K = 3$ with source energy $S_1 = S_2 = S_3 = 500$. The results show that the exhaustive search method and the proposed EM algorithm outperform the MR search method. With an increasing number of sensors, the performance of the proposed EM algorithm and the exhaustive search method approach the CRLB. This is because with higher sensor density, the better initial estimates can be obtained for the EM algorithm.

The localization performance of each method with various sensor noise variances and number of sources are examined and illustrated in Figure 6.5 and Figure 6.6

Figure 6.4: Sensor density versus RMS

Figure 6.5: Noise variance versus RMS

respectively. The number of sensors is $N = 30$. From these two figures, it can also be found that the exhaustive search method and the proposed EM algorithm outperform the MR search method. With the increase of SNR, sensors can obtain better signals from sources and more accurate localization performance can be achieved. If the number of the sources is larger, the localization performance will be worse. Figure 6.7 illustrates a performance comparison on source energy estimation. It is assumed that source number is $K = 3$ with source energy $S_1 = S_2 = S_3 = 500$. Sensor deployments and sources are shown in Figure 6.3. Obviously, the exhaustive search method and the proposed EM algorithm have lower estimation error than the MR search method. When sensor density is higher, the proposed EM algorithm can achieve better energy estimation which is close to the true value.

Finally the complexity of the three methods is compared. Setting the minimum

Figure 6.6: Number of the sources versus RMS

Figure 6.7: Source energy estimation

grid search resolution as 1×1 . Specifically, assuming the number of sources is $K = 3$ and sensor density $N = 30$. Obviously, the exhaustive search method has the largest computational burden. For the MR search method, two kinds of three-level grid resolutions are considered, $4 \times 4, 2 \times 2, 1 \times 1$ and $25 \times 25, 5 \times 5, 1 \times 1$. Because the area of the region of interest is $100 \text{ m} \times 100 \text{ m}$, the MR search method for the two kinds of grid resolutions require $(25 \times 25)^K + (2 \times 2)^K + (2 \times 2)^K$ and $(4 \times 4)^K + (5 \times 5)^K + (5 \times 5)^K$ trials respectively which can be obtained by computing the ratio of the total area and the grid area. For example, for the choice of grid resolution $4 \times 4 \rightarrow 2 \times 2 \rightarrow 1 \times 1$, the number of grids in each search level can be computed by $[(100 \times 100)/(4 \times 4)]^K = (25 \times 25)^K$, $[(4 \times 4)/(2 \times 2)]^K = (2 \times 2)^K$ and $[(2 \times 2)/(1 \times 1)]^K = (2 \times 2)^K$, respectively. Actually, by using the

Table 6.5: Computational Complexity

method	trials
exhaustive search	$(100 \times 100)^K$
MR($4 \times 4, 2 \times 2, 1 \times 1$)	$(25 \times 25)^K + (2 \times 2)^K + (2 \times 2)^K$
MR($25 \times 25, 5 \times 5, 1 \times 1$)	$(4 \times 4)^K + (5 \times 5)^K + (5 \times 5)^K$
proposed EM algorithm	$p \times q \times K \times [\text{iteration_number}(< 20)]$

latter choice of grid resolution, the MR search method may more easily trap into local optima. For the proposed EM algorithm, the expected number of trials is $p \times q \times K \times \text{iteration_number}$, where $p = 8, q = 5, \text{iteration_number} \leq 20$. Clearly, the proposed EM algorithm has the lowest computational complexity. Comparison of computation complexity is presented in Table 6.5.

Remark 6.4. *For the MR search method, to obtain higher estimation accuracy, the more number of levels for grid resolution and more grid points are needed in search.*

6.4.2 Decay Factor Estimation

Using the proposed EM algorithm, each source's unknown parameters and the decay factor can be estimated simultaneously. In the algorithm, the unknown parameter α is initialized to a random value between 2 and 4.

In the simulations, the true value of α is assumed equal to 2.3. The step size is set to be $a = 0.0005$. Figure 6.8 illustrates the decay factor estimation by using the proposed EM algorithm. Through the analysis of the mean and standard deviation of the estimation error, it can be seen that the proposed algorithm has a good performance for decay factor estimation.

6.4.3 Performance Analysis for the Proposed EM Algorithm

Further simulations are also performed to examine the performance of the proposed EM algorithm. Here, we assume that $K = 3$.

Figure 6.8: Decay factor estimation

First the performance of the proposed EM algorithm is investigated with the changing variable β which was set as a constant in the previous simulations. Sensor deployment and sources are shown in Figure 6.3. Energy intensity for each source is set as $S = 500$. Noise variance is $\sigma^2 = 1$. The RMS error is presented in Figure 6.9. The results show that when $0.6 \leq \beta \leq 0.7$, the algorithm has the better performance. This is because that with a large β , the convergence rate is lower, the estimation of the EM algorithm will hover around the initial estimations. On the other hand, with a smaller β , the algorithm will converge very fast to local optima rather than the global optima.

Different source energy may also affect the localization accuracy. In the above simulations, it is assumed that sources have equal source energy. In the experiments, one of the sources is assumed to have a different source energy from the other two sources. Also sensor and source deployments are shown in Figure 6.3. Specifically, one source is located at $(0, 0)$ with source energy $S_1 = 600$, and the other two sources have the source energy $S_2 = S_3 = 500$. The rest parameters are the same as in the previous experiments. The localization performance for each source is illustrated in Figure 6.10 and Figure 6.11. In Figure 6.10 noise variance is set as $\sigma^2 = 1$, and in Figure 6.11, the number of sensors is set as $N = 50$. It can be seen that localization error for the source with larger energy is lower than the other two sources.

Using the proposed method, the decay factor can also be estimated, but with

some estimation errors as shown in Figure 6.8. Figure 6.12 shows a localization performance comparison when the decay factor is known and unknown where $S_1 = S_2 = S_3 = 500, \sigma^2 = 1$. When the decay factor is unknown, the estimation accuracy is lower as seen from Figure 6.12.

Figure 6.9: Step size versus RMS

Figure 6.10: Sensor density versus RMS for each source

The distances between the sources can also affect the robustness of the localization algorithm. The results are shown in Figure 6.13 where diamonds and circles denote sensors and sources respectively; crosses and triangles denote initial estimated coordinates and estimation results of each iteration for sources respectively. Figure 6.13 (a), (b), and (c) depict the scenarios with three sources located at $[20, 20]$, $[0, 0]$, $[-20, -20]$ and the number of sensors being 10, 30, 50 respectively. Similarly, in Figure 6.13 (d), (e), and (f), the sources' locations are $[0, 0]$, $[5, 5]$, $[20, 20]$ and in

Figure 6.11: Noise variance versus RMS for each source

Figure 6.12: Location estimation with known decay factor and unknown decay factor

Figure 6.13 (g), (h), and (i), the sources' locations are $[-45, -45]$, $[0, 0]$, and $[45, 45]$ respectively. From (a)-(f), we can find that if two sources are positioned closely, then when sensor density is low as showed in (d), the algorithm has a worse performance than the result showed in (a). With an increasing number of sensors, our algorithm can successfully distinguish and estimate the closely located sources as showed in (e) and (f). From (a)-(c), (g)-(i), we can see that when some of the sources are close to the boundary of the region of interest, the estimation accuracy will be degraded as there are not enough sensors surrounding these sources. In conclusion, as seen from Figure 6.13, better performance can be achieved with the increase of sensor density.

Figure 6.13: Algorithm performance analysis

6.5 Conclusion

In this chapter, an efficient EM algorithm for energy based multiple source localization was proposed in WSNs. It can estimate the source locations individually and can efficiently avoid the local optima through effective SDS initialization and β -parameterized search methods. Simulations demonstrated that the proposed algorithm has a better estimation accuracy and lower computation complexity.

Chapter 7

Fully Distributed Energy Based Source Localization

In Chapter 6, an efficient EM algorithm was proposed for the energy based multiple source localization. In fact, the proposed EM algorithm can be applied to both single and multiple source cases. However, it is a centralized method. It is well known that the main drawbacks of the centralized methods are that they require the transmission of measurements from each node in the network to a central point for processing and have high computational complexity. The robustness to the network failures is low. Due to the limited bandwidth and power in WSNs, distributed algorithms are desirable. In this chapter and Chapter 8, fully distributed algorithms will be designed for the single source localization problem.

In this chapter, the source localization problem using energy measurements is studied. It is well known that a solution to this problem is generally associated with a non-convex optimization which may have multiple local optima and saddle points. Most of the existing algorithms can not achieve a global optimum. In this chapter, the source localization problem is formulated as a convex feasibility problem (CFP) and a fully distributed method is proposed for it. In the proposed method, sensor nodes only need to communicate with one-hop neighbors and update their estimates simultaneously based on a projection algorithm. Finally they are able to achieve

consensus on a possible minimizer asymptotically. The proposed method has low complexity and can achieve global optimality fast. Theoretical analysis and simulation results show that the proposed approach has good estimation performance, regardless of whether the CFP is consistent or not.

7.1 Preliminary

As mentioned above, in this chapter, the source localization problem is formulated as a CFP. The mathematical formulation of CFP is as follows.

Suppose that $C_1, \dots, C_N$ are closed convex subsets in a Hilbert space with intersection C :

$$C = C_1 \cap \dots \cap C_N$$

Convex feasibility problem (CFP): Find some point x in C .

We call the CFP consistent if $C \neq \emptyset$, and otherwise call it inconsistent. The projection based method is well studied and can be used for solving the CFP. Most of the existing projection methods are centralized and implemented in a sequential or parallel manner. In [12], Blatt and Hero propose to use the sequential projection method for source localization but no rigorous analysis is provided. There are few works on distributed projection method for CFP in the literature. In [83], the authors propose a constrained consensus method for convex optimization problems, however it is restricted to a consistent case and can not be used for source localization which is usually inconsistent.

7.2 Problem Statement

The problem of interest is to determine the location of an active source in a sensor network. Let the source be located at an unknown coordinate pair $\mathbf{x}_t = [x_t, y_t]^T$

and emit a signal with power level S . We assume that there are N sensor nodes performing sensing based on energy detection. At the i -th sensor with its known coordinate $\mathbf{x}_i = [x_i, y_i]^T, i = 1, 2, \dots, N$, the received energy can be written as [106]

$$P_i = g_i \frac{S}{d_{is}^\alpha} + \varepsilon_i, \quad (7.1)$$

where $d_{is} = \|\mathbf{x}_t - \mathbf{x}_i\|$ is the Euclidean distance between the i -th sensor and the source, g_i is the gain factor of the i -th sensor. α is the power-loss factor (in this work, α is assumed equal to 2) and ε_i is the receiver noise at the i -th sensor with an AWGN channel, i.e., $\varepsilon_i \sim N(0, \sigma_i^2), i = 1, 2, \dots, N$. Note that energy in (7.1) is derived from signal strength measurements, which has been introduced in Chapter 2.

In this chapter, the source energy S is assumed known. The only parameter required to estimate is the source's location vector $\mathbf{x}_t = [x_t, y_t]^T$.

The maximum likelihood estimator (MLE) is found by solving the nonlinear least-squares problem when the noise is Gaussian

$$\mathbf{x}_t^{MLE*} = \arg \min \sum_{i=1}^N \left[P_i - g_i \frac{S}{d_{is}^2} \right]^2 = \arg \min \sum_{i=1}^N f_i(\mathbf{x}_t), \quad (7.2)$$

where $f_i(\mathbf{x}_t) = \left[P_i - g_i \frac{S}{d_{is}^2} \right]^2$. Clearly, $f_i(\mathbf{x}_t)$ attains its minimum 0 on the circle

$$\mathcal{C}_i = \{\mathbf{x}_t \in \mathbb{R}^2 : \|\mathbf{x}_t - \mathbf{x}_i\| = \sqrt{g_i S / P_i}\}. \quad (7.3)$$

However, due to the observation noise, the source may not appear on the circles as defined in (7.3), but is located in a ring area, i.e., $\sqrt{\frac{g_i S}{P_i + \zeta \sigma_i}} \leq \|\mathbf{x}_t - \mathbf{x}_i\| \leq \sqrt{\frac{g_i S}{P_i - \zeta \sigma_i}}$ if a $\zeta - \sigma$ area of the noise distribution is adopted, where ζ is a constant (Generally, $\zeta = 1$ to 3). This will lead the source localization problem to a non-convex optimization problem which is generally difficult to handle. To overcome the difficulty, a relaxation method needs to be used. Define a convex set which is a

circular area

$$\mathcal{D}_i = \{\mathbf{x}_t \in \mathbb{R}^2 : \|\mathbf{x}_t - \mathbf{x}_i\| \leq \sqrt{\frac{g_i S}{P_i - \zeta \sigma_i}}\}, \quad (7.4)$$

where only the outer ring is considered. In this chapter, ζ is set to 1. Intuitively, the source may or may not reside in $\mathcal{D}_i$. If the source is inside $\mathcal{D}_i, \forall i$, then it is easy to see that the source localization problem can be solved by letting the estimator be a point in the intersection of the sets $\mathcal{D}_i, i = 1, 2, \dots, N$. That is,

$$\hat{\mathbf{x}}_t \in \mathcal{D} = \bigcap_{i=1}^N \mathcal{D}_i \subset \mathbb{R}^2. \quad (7.5)$$

However, due to the observation noise, the feasibility problems may turn out to be inconsistent, i.e., the intersection $\mathcal{D}$ may be empty. An illustration for the consistent and inconsistent cases is given in Figure 7.1.

Figure 7.1: Consistent case and inconsistent case

Since the convex feasibility problem may turn out to be inconsistent, the localization problem can be reformulated as finding a point $\mathbf{x}_t^*$ which minimizes the sum of the squares of the distances to the convex set $\mathcal{D}_i$ s.

$$\mathbf{x}_t^* = \arg \min_{\mathbf{x}_t \in \mathbb{R}^2} \sum_{i=1}^N \|\mathbf{x}_t - \mathcal{P}_{\mathcal{D}_i}(\mathbf{x}_t)\|^2, \quad (7.6)$$

where for a closed convex set $S \subseteq \mathbb{R}^2$ and vector $x \in \mathbb{R}^2$, $\mathcal{P}_S(x)$ is the orthogonal

projection of x onto S . That is,

$$\mathcal{P}_S(x) = \arg \min_{y \in \mathbb{R}^2} \|x - y\|, y \in S. \quad (7.7)$$

For the source localization problem, the projection operator in (7.7) has a closed-form expression given as:

$$\mathcal{P}_{\mathcal{D}_i}(\mathbf{a}) = \begin{cases} \mathbf{a}, & \|\mathbf{a} - \mathbf{x}_i\| \leq \sqrt{\frac{g_i S}{P_i - \zeta \sigma_i}}; \\ \mathbf{x}_i + \sqrt{\frac{g_i S}{P_i - \zeta \sigma_i}} \frac{\mathbf{a} - \mathbf{x}_i}{\|\mathbf{a} - \mathbf{x}_i\|}, & \text{otherwise.} \end{cases} \quad (7.8)$$

Let G denote the set of the least-squares solutions of the convex feasibility problem in (7.6), then we have the following result.

Proposition 7.1. *The set G is nonempty, closed, convex and bounded.*

Proof: Since $\mathcal{D}_i, i = 1, 2, \dots, N$ are convex and bounded, then according to Proposition 7 in [24], G is nonempty, closed, convex, and bounded. $\square$

It can be easily checked that if the problem is consistent, i.e., $\mathcal{D} = \bigcap_{i=1}^N \mathcal{D}_i \neq \emptyset$, then $\sum_{i=1}^N \|\hat{\mathbf{x}}_t - \mathcal{P}_{\mathcal{D}_i}(\hat{\mathbf{x}}_t)\| = 0$, where $\hat{\mathbf{x}}_t \in G$ and $G = \bigcap_{i=1}^N \mathcal{D}_i$.

Until now, the source localization problem has been formulated as a convex feasibility problem (CFP).

7.3 Sequential and Parallel Projection Methods

In this section, firstly, a sequential projection method proposed by Blatt and Hero [12] is reviewed. Then a parallel projection method proposed in [24] which is a centralized algorithm, is used to solve the source localization problem.

Figure 7.2: Parallel and sequential projection methods

7.3.1 Sequential Projection Method

The sequential projection method, also known as POCS (projection onto convex sets), is a cyclic algorithm. From Figure 7.2, it can be seen that data is processed across sensors in sequence. The update rule of the sequential projection method is given as follows

$$\mathbf{x}_t(k+1) = \mathbf{x}_t(k) + \lambda(k) \left[\mathcal{P}_{\mathcal{D}_{\tau(k)}}(\mathbf{x}_t(k)) - \mathbf{x}_t(k) \right] \quad (7.9)$$

where $\{\lambda(k)\}$ is a sequence of relaxation parameters satisfying that for all k , $\epsilon_1 \leq \lambda(k) \leq 2 - \epsilon_2$ for some $\epsilon_1, \epsilon_2 > 0$, $\tau(k) = k \bmod N$. The pseudocode of the sequential projection method is given by Algorithm 3.

Algorithm 3 Sequential Projection Method

- 1) Initialization: Set $\mathbf{x}_t(0)$ to be an arbitrary value.
- 2) Iteration step: $k \geq 0$

$$\mathbf{x}_t(k+1) = \mathbf{x}_t(k) + \lambda(k) \left[\mathcal{P}_{\mathcal{D}_{\tau(k)}}(\mathbf{x}_t(k)) - \mathbf{x}_t(k) \right].$$

- 3) Stopping rule: $\|\mathbf{x}_t(k+1) - \mathbf{x}_t(k)\| \leq \epsilon$. where ϵ is a predefined threshold.
-

Theorem 7.1. [4] If $\mathcal{D} = \bigcap_{i=1}^N \mathcal{D}_i \neq \emptyset$, any sequence $\mathbf{x}_t(k), k \geq 0$ converges to a point in $\mathcal{D}$. □

From Theorem 7.1, it can be seen that the POCS algorithm has good convergence performance in the consistent case of CFP. However the convergence behavior of POCS in the inconsistent case is generally unsatisfactory. A new relaxation sequence $\{\lambda(k)_{(k \geq 0)}\}$ needs to be used which is given by

$$\sum_{k=0}^{+\infty} \lambda(k) = +\infty, \lambda(k+1) \leq \lambda(k), \lim_{k \rightarrow +\infty} \lambda(k) = 0. \quad (7.10)$$

By using the relaxation sequence stated above, the POCS will converge to a point in G , which has been verified by simulations in [12] but no theoretical analysis is provided.

Another issue of the sequential distributed algorithm is that a specified data transmitting path is required. How to do path planing in a large sensor network is challenging. Also, the convergence rate is low when the sensor density is high and data transmission becomes unreliable when some of the nodes fail. Hence the robustness of such networks is low.

7.3.2 Parallel Projection Method

The parallel projection method is a centralized algorithm, where each sensor transmits its measurement to a fusion center. The parallel projection method involves finding a point $\mathbf{x}_t^*$ which minimizes the weighted sum of the squares of the distances to the convex set $\mathcal{D}_i$ s.

$$\mathbf{x}_t^* = \arg \min_{\mathbf{x}_t \in \mathbb{R}^2} \sum_{i=1}^N w_i \|\mathbf{x}_t - \mathcal{P}_{\mathcal{D}_i}(\mathbf{x}_t)\|^2, \quad (7.11)$$

where $(w_i)_{1 \leq i \leq N}$ are strictly positive weights such that $\sum_{i=1}^N w_i = 1$.

Assume that G' is the set of the weighted least-squares solutions of the convex feasibility problem in (7.11). The algorithm for the parallel projection method is

given as follows

$$\mathbf{x}_t(k+1) = \mathbf{x}_t(k) + \lambda(k) \left(\sum_{i=1}^N w_i (\mathcal{P}_{\mathcal{D}_i}(\mathbf{x}_t(k)) - \mathbf{x}_t(k)) \right), \quad (7.12)$$

where $\lambda(k)$ is a relaxation sequence.

The pseudocode of the parallel projection method is given by Algorithm 4 and its convergence analysis is addressed by Theorem 7.2.

Algorithm 4 Parallel Projection Method

- 1) Initialization: Set $\mathbf{x}_t(0)$ to be an arbitrary value.
- 2) Iteration step: $k \geq 0$

$$\mathbf{x}_t(k+1) = \mathbf{x}_t(k) + \lambda(k) \left(\sum_{i=1}^N w_i \mathcal{P}_{\mathcal{D}_i}(\mathbf{x}_t(k)) - \mathbf{x}_t(k) \right).$$

- 3) Stopping rule: $\|\mathbf{x}_t(k+1) - \mathbf{x}_t(k)\| \leq \epsilon$, where ϵ is a predefined threshold.
-

Theorem 7.2. [24] *Suppose that $\lambda(k) \in [\epsilon, 2 - \epsilon]$, where $0 < \epsilon < 1$. Then any sequence $\mathbf{x}_t(k)_{k \geq 0}$ generated by (7.12) converges to a point in G' .* $\square$

The advantage of the parallel projection method is that it has good convergence performance especially for the inconsistent case of CFP. Also we can design a weight sequence $(w_i)_{1 \leq i \leq N}$ according to the reliability of each sensor's measurements, i.e., a higher weight can be assigned to the sensor with a higher received energy. The main drawback of this centralized method is that it requires the transmission of measurements from each node to a fusion center for processing. The probability of congestion around the sink node is high and also some long-distance transmissions are required.

7.4 Fully Distributed Projection Method

Now, we will introduce the main results of this chapter. To avoid the path planning problem in the sequential distributed method and improve the robustness of the

network, a fully distributed method is useful, in which no fusion center is required and sensor nodes only need to communicate with their closest neighbors, therefore reduces the probability of congestion around the sink node and increases the robustness of the network against node failures or unpredictable switching to sleeping mode. Note that data transmission path planning is not needed for a distributed method.

Before introducing the proposed method, a brief introduction is given to a diffusion network. The network can be represented as an undirected graph defined by $\mathcal{G} := (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the node set $\mathcal{N} := 1, \dots, N$ and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the edge set. If node k and node l can communicate with each other directly, undirected link is defined by $(k, l) \in \mathcal{E}$.

If the CFP of the source localization problem is consistent, a fully distributed protocol that can be used for source localization can be found in [83]. It works as follows: sensor i at time step $k+1$ generates its estimate according to the following protocol:

$$\mathbf{x}_t^i(k+1) = \mathcal{P}_{\mathcal{D}_i} \left(\sum_{j=1}^N w_j^i(k) \mathbf{x}_t^j(k) \right), \quad (7.13)$$

where $w_j^i(k), i = 1, \dots, N, j = 1, \dots, N$ denote the weights at time step k ; $\mathbf{x}_t^i(0), i = 1, \dots, N$ are arbitrary.

Assumption 7.1. (*Network connectivity*) *The network is connected, i.e., there exists a direct or indirect path between any two nodes.*

Assumption 7.2. (*Weighting rule*) *$W(k)$ whose i -th row is the vector $w_i(k) = [w_1^i(k), \dots, w_N^i(k)]$, is an $N \times N$ doubly stochastic weighting matrix with the following properties:*

$$\mathbf{1}^T W(k) = \mathbf{1}^T, W(k) \mathbf{1} = \mathbf{1}. \quad (7.14)$$

Theorem 7.3. [83] (*Consensus*) *Let the intersection set $\mathcal{D} = \bigcap_{i=1}^N \mathcal{D}_i$ be nonempty and Assumptions 7.1 and 7.2 hold. Then for some $\mathbf{x}_t^* \in \mathcal{D}$, $\lim_{k \rightarrow \infty} \|\mathbf{x}_t^i(k) - \mathbf{x}_t^*\| = 0, \forall i = 1, \dots, N$. $\square$*

From Theorem 7.3, it can be seen that in the consistent case, the protocol (7.13) has a good convergence property. The estimates of all sensors will converge to the same optimal solution $\mathbf{x}_t^*$ which is in the intersection of convex sets determined by each sensor. However, its convergence behavior in the inconsistent case is generally unsatisfactory and an improved protocol for both the consistent and inconsistent cases is demanded.

7.4.1 Protocol for the General Case

In the above, the intersection of the convex sets is assumed nonempty, i.e., $\mathcal{D} \neq \emptyset$. However, this assumption does not generally hold in the presence of observation noises which may lead to an empty intersection $\mathcal{D}$. To deal with the inconsistent case, it can be done by enforcing that $\mathcal{D} \neq \emptyset$. For example, by expanding $\mathcal{D}_i$ (tuning ζ) to increase the probability in which the intersection of the convex sets determined by sensors is not empty. However, since it is not known how much expansion should be done, the estimation error will increase even though the algorithm does converge. Instead, a new protocol is proposed in this chapter, which will be demonstrated to work well for both the consistent and inconsistent cases:

$$\mathbf{x}_t^i(k+1) = \sum_{j=1}^N w_j^i(k) \mathbf{x}_t^j(k) + \beta(k) \left[\mathcal{P}_{\mathcal{D}_i} \left(\sum_{j=1}^N w_j^i(k) \mathbf{x}_t^j(k) \right) - \sum_{j=1}^N w_j^i(k) \mathbf{x}_t^j(k) \right], \quad (7.15)$$

where $\beta(k)$ is the relaxation sequence, $w_j^i(k)$ is the same as in (7.13). It can be seen that if $\beta(k)_{k \geq 0} = 1$, then the protocol (7.15) is the same as (7.13). The relaxation sequence $\beta(k)$ plays an important role in the convergence of the method. Next the case when $\beta(k) \neq 1$ is studied. The details will be given below.

7.4.2 Convergence Behavior for the Consistent Case

First, the consistent case is considered. Before proving the convergence of the algorithm, let

$$v^i(k) = \sum_{j=1}^N w_j^i(k) \mathbf{x}_t^j(k). \quad (7.16)$$

Then, some lemmas are introduced which will be used for the convergence analysis.

Lemma 7.1. *[83] Let $\mathcal{D}$ be a nonempty closed convex set in $\mathbb{R}^2$. Then for any $x \in \mathbb{R}^2$, we have*

$$\|\mathcal{P}_{\mathcal{D}}[x] - y\|^2 \leq \|x - y\|^2 - \|\mathcal{P}_{\mathcal{D}}[x] - x\|^2$$

$\forall y \in \mathcal{D}$.

We also make the following assumptions.

Assumption 7.3. $\sum_{k=1}^{\infty} \beta(k)(1 - \beta(k)) = \infty$.

Note that Assumption 7.3 is not restrictive. Clearly, if the relaxation sequence is a constant, i.e., $\beta(k) = \beta \in (0, 1)$, then Assumption 7.3 holds.

Lemma 7.2. *Let the Assumptions 7.1-7.3 hold, then we have*

- 1) $\|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\|^2 \leq \|v^i(k) - \mathbf{x}_t^*\|^2 - \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - v^i(k)\|^2, \forall i,$
- 2) $\sum_{i=1}^N \|v^i(k) - \mathbf{x}_t^*\|^2 \leq \sum_{i=1}^N \|\mathbf{x}_t^i(k) - \mathbf{x}_t^*\|^2,$
- 3) $\lim_{k \rightarrow \infty} \mathcal{P}_{\mathcal{D}_i}(v^i(k)) = v^i(k), \forall i = 1, \dots, N.$

Proof. See Appendix A. □

From protocol (7.15), the relation between $\mathbf{x}_t^i(k+1)$ and $\mathbf{x}_t^1(s), \dots, \mathbf{x}_t^N(s)$ at time $0 \leq s \leq k$ is given by

$$\begin{aligned} \mathbf{x}_t^i(k+1) = & \sum_{j=1}^N \Phi_j^i(k, 0) \mathbf{x}_t^j(0) + \sum_{m=1}^k \beta(m-1) \\ & \left(\sum_{j=1}^N \Phi_j^i(k, m) (\mathcal{P}_{\mathcal{D}_j}(v^j(m-1)) - v^j(m-1)) \right) \\ & + \beta(k) (\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - v^i(k)). \end{aligned} \quad (7.17)$$

where $\Phi(k, m) = W(k)W(k-1) \dots W(m+1)W(m)$ is a transition matrix. $\Phi_j^i(k, m)$ denotes the (i, j) -th entry of $\Phi(k, m)$.

Lemma 7.3. [126] *Let Assumptions 7.1 and 7.2 hold, then $\lim_{k \rightarrow \infty} \Phi(k, m) = \frac{1}{N} \mathbf{1} \mathbf{1}^T$, where $\mathbf{1} \in \mathbb{R}^N$ is the vector with all entries equal to 1.*

From Lemma 7.3, it can be seen that the (i, j) -th entry of $\Phi(k, m)$ converges to $\frac{1}{N}$ as $k \rightarrow \infty$. In order to study the convergence behavior of $\mathbf{x}_t^i(k)$, define an auxiliary sequence $\{y(k)\}$, where

$$\begin{aligned} y(k) = & \frac{1}{N} \sum_{i=1}^N \mathbf{x}_t^i(k) = \frac{1}{N} \sum_{i=1}^N \mathbf{x}_t^i(0) \\ & + \frac{1}{N} \sum_{m=1}^k \beta(m-1) \left(\sum_{i=1}^N (\mathcal{P}_{\mathcal{D}_i}[v^i(m-1)] - v^i(m-1)) \right). \end{aligned} \quad (7.18)$$

Proposition 7.2. *Let Assumptions 7.1-7.3 hold, $\lim_{k \rightarrow \infty} \|\mathbf{x}_t^i(k) - y(k)\| = 0, \forall i$.*

Proof. From (7.17) and (7.18), it can be obtained that

$$\begin{aligned} & \lim_{k \rightarrow \infty} \|\mathbf{x}_t^i(k) - y(k)\| \\ = & \lim_{k \rightarrow \infty} \left\| \sum_{m=1}^k \beta(m-1) \sum_{j=1}^N \left(\Phi_j^i(k, m) - \frac{1}{N} \right) \left(\sum_{j=1}^N (\mathcal{P}_{\mathcal{D}_j}[v^j(m-1)] - v^j(m-1)) \right) \right\| \end{aligned}$$

Since $\mathcal{P}_{\mathcal{D}_i}[v^i(0)] - v^i(0)$ is bounded, and $\lim_{k \rightarrow \infty} (\mathcal{P}_{\mathcal{D}_i}[v^i(k)] - v^i(k)) = 0$, this implies that sequences $\mathcal{P}_{\mathcal{D}_i}[v^i(k)] - v^i(k), \forall i, k$ and therefore $\sum_{i=1}^N \mathcal{P}_{\mathcal{D}_i}[v^i(k)] - v^i(k)$ are

bounded where the bound is defined as δ . So for all i

$$\begin{aligned}
& \lim_{k \rightarrow \infty} \|\mathbf{x}_t^i(k) - y(k)\| \\
& \leq \lim_{k \rightarrow \infty} \delta \left\| \sum_{m=1}^k \beta(m-1) \sum_{j=1}^N \left(\Phi_j^i(k, m) - \frac{1}{N} \right) \right\| \\
& \leq \lim_{k \rightarrow \infty} \delta \sum_{m=1}^k \beta(m-1) \left\| \sum_{j=1}^N \left(\Phi_j^i(k, m) - \frac{1}{N} \right) \right\|
\end{aligned}$$

From Lemma 7.3, it can be known that $\Phi_j^i(k, m)$ converges to $\frac{1}{N}$ as $k \rightarrow \infty$, and therefore $\lim_{k \rightarrow \infty} (\Phi_j^i(k, m) - \frac{1}{N}) = 0$. Hence, it can be obtained that

$$\begin{aligned}
& \lim_{k \rightarrow \infty} \|\mathbf{x}_t^i(k) - y(k)\| \\
& \leq \lim_{k \rightarrow \infty} \delta \sum_{m=1}^k \beta(m-1) \times 0 \\
& = 0.
\end{aligned}$$

□

Proposition 7.3. *The sequence $\{y(k)\}$ is bounded.*

Proof. From Equation (A.5), it can be obtained that

$$\begin{aligned}
\|\mathbf{x}_t^i(k+1) - \mathbf{x}_t^*\| &= \|(1 - \beta(k))(v^i(k) - \mathbf{x}_t^*) + \beta(k)(\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*)\| \\
&\leq (1 - \beta(k))\|v^i(k) - \mathbf{x}_t^*\| + \beta(k)\|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\| \quad (7.19)
\end{aligned}$$

Thus

$$\begin{aligned}
\sum_{i=1}^N \|\mathbf{x}_t^i(k+1) - \mathbf{x}_t^*\| &\leq (1 - \beta(k)) \sum_{i=1}^N \|v^i(k) - \mathbf{x}_t^*\| \\
&\quad + \beta(k) \sum_{i=1}^N \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\| \\
&\leq \sum_{i=1}^N \|v^i(k) - \mathbf{x}_t^*\| \leq \sum_{i=1}^N \|\mathbf{x}_t^i(k) - \mathbf{x}_t^*\|
\end{aligned}$$

$$\leq \sum_{i=1}^N \|\mathbf{x}_t^i(0) - \mathbf{x}_t^*\| < \infty \quad (7.20)$$

This implies that the sequence $\|\mathbf{x}_t^i(k) - \mathbf{x}_t^*\|$ is bounded, and therefore $\mathbf{x}_t^i(k)$ is bounded.

Then according to triangle inequality, it can be obtained that

$$\|y(k)\| \leq \|y(k) - \mathbf{x}_t^i(k)\| + \|\mathbf{x}_t^i(k)\|$$

According to Proposition 7.2, it follows that sequence $y(k)$ is bounded. $\square$

Theorem 7.4. (*Consensus*) Let the intersection set $\mathcal{D} = \bigcap_{i=1}^N \mathcal{D}_i$ be nonempty and Assumptions 7.1-7.3 hold, then $\lim_{k \rightarrow \infty} \mathbf{x}_t^i(k) = \mathbf{x}_t^*, \forall i = 1, \dots, N$, where $\mathbf{x}_t^* \in \mathcal{D}$.

Proof. From Propositions 7.2 and 7.3, it can be seen that the estimates of all the sensors will converge to $y(k)$, which implies that the sensors will reach a consensus. Then $\lim_{k \rightarrow \infty} v^i(k) = \sum_{j=1}^N w_j^i(k) \mathbf{x}_t^j(k) = \mathbf{x}_t^j(k), \forall i, j = 1, \dots, N$. Also according to Lemma 7.3, we have $\lim_{k \rightarrow \infty} \mathcal{P}_{\mathcal{D}_i}(v^i(k)) = v^i(k), \forall i = 1, \dots, N$. Then we have $\lim_{k \rightarrow \infty} \mathcal{P}_{\mathcal{D}_i}(\mathbf{x}_t^j(k)) = \mathbf{x}_t^j(k), \forall i, j = 1, \dots, N$ which implies that $\lim_{k \rightarrow \infty} (\mathbf{x}_t^i(k) - \mathbf{x}_t^*) = 0$, for all $i = 1, \dots, N$, where $\mathbf{x}_t^* \in \mathcal{D}$. $\square$

From Theorem 7.4, it can be seen that the estimate of every sensor will converge to the same optimal solution $\mathbf{x}_t^*$ which is in the intersection of convex sets determined by each sensor.

Remark 7.1. The analysis of the convergence rate of the algorithm is complicated since nonlinear projection operators are involved. Nonetheless, it is noted that the convergence rate mainly depends on the network connectivity, the properties of the weighting matrix W and the initial distance, i.e., $\|\sum_{i=1}^N \{\mathcal{P}_{\mathcal{D}_i}[v^i(0)] - v^i(0)\}\|$.

Figure 7.3: Inconsistent case

7.4.3 Convergence Behavior for the Inconsistent Case

In Section 7.4.2, the convergence analysis of the proposed protocol (7.15) is provided for the consistent case. In this subsection, the conditions of convergence for the inconsistent case will be derived. First, the following assumption is made.

Assumption 7.4. $\sum_{k=1}^{\infty} \beta(k)(1 - \beta(k)) = \infty$, $\beta(k+1) \leq \beta(k)$, $\lim_{k \rightarrow +\infty} \beta(k) = 0$.

Let Assumptions 7.1-7.2 and 7.4 hold, and suppose $\mathbf{x}_t^* \in G$, where G is the set of least-squares solutions of (7.6). Then defining the new convex sets by $\mathcal{D}'_i, i = 1, \dots, N$, dashed circles as shown in Figure 7.3. Furthermore, define the sequence $\{e^i(k)\}$ as the projection error as follows:

$$e^i(k) = \mathcal{P}_{\mathcal{D}_i}[v^i(k)] - \mathcal{P}_{\mathcal{D}'_i}[v^i(k)] \quad (7.21)$$

Then the protocol (7.15) can be cast as

$$\mathbf{x}_t^i(k+1) = v^i(k) + \beta(k) [(\mathcal{P}_{\mathcal{D}'_i}[v^i(k)] + e^i(k)) - v^i(k)]. \quad (7.22)$$

Theorem 7.5. $\lim_{k \rightarrow \infty} \mathcal{P}_{\mathcal{D}'_i}(v^i(k)) = v^i(k), \forall i$, if $\sum_{k \geq 0} \sum_{i=0}^N \beta(k) \|e^i(k)\| < +\infty$ and Assumptions 7.1-7.2 and 7.4 hold.

Proof. According to the protocol (7.15) and the definition of $e^i(k)$ in (7.21), it can be obtained that

$$\begin{aligned} \mathbf{x}_t^i(k+1) - \mathbf{x}_t^* &= (1 - \beta(k))(v^i(k) - \mathbf{x}_t^*) \\ &\quad + \beta(k)(\mathcal{P}_{\mathcal{D}'_i}(v^i(k)) - \mathbf{x}_t^*) + \beta(k)e^i(k) \end{aligned} \quad (7.23)$$

Then, it is easy to get

$$\begin{aligned} \sum_{i=1}^N \|\mathbf{x}_t^i(k+1) - \mathbf{x}_t^*\| &\leq (1 - \beta(k)) \sum_{i=1}^N \|v^i(k) - \mathbf{x}_t^*\| \\ &\quad + \beta(k) \sum_{i=1}^N \|\mathcal{P}_{\mathcal{D}'_i}(v^i(k)) - \mathbf{x}_t^*\| + \beta(k) \sum_{i=1}^N \|e^i(k)\|. \end{aligned} \quad (7.24)$$

Since $\beta(k)$ in Assumption 7.4 is a special case of Assumption 7.3, Lemma 7.2 can still be applied. Hence in view of Lemma 7.2, from (7.24) it can be further obtained that

$$\sum_{i=1}^N \|\mathbf{x}_t^i(k+1) - \mathbf{x}_t^*\| \leq \sum_{i=1}^N \|\mathbf{x}_t^i(k) - \mathbf{x}_t^*\| + \beta(k) \sum_{i=1}^N \|e^i(k)\| \quad (7.25)$$

Since $\sum_{k \geq 0} \beta(k) \sum_{i=1}^N \|e^i(k)\| < \infty$, $\sum_{i=1}^N \|\mathbf{x}_t^i(k+1) - \mathbf{x}_t^*\|$ converges as $k \rightarrow \infty$.

Next defining $\eta^i(k) = (1 - \beta(k))v^i(k) + \beta(k)\mathcal{P}_{\mathcal{D}'_i}(v^i(k))$, then from (7.23), it can also be obtained that

$$\begin{aligned} \|\mathbf{x}_t^i(k+1) - \mathbf{x}_t^*\|^2 &= \\ &\quad \|\eta^i(k) - \mathbf{x}_t^*\|^2 + 2\beta(k)\langle \eta^i(k) | e^i(k) \rangle + \beta^2(k)\|e^i(k)\|^2 \\ &\leq \|\eta^i(k) - \mathbf{x}_t^*\|^2 + 2(\|\eta^i(k) - \mathbf{x}_t^*\| + \beta(k)\|e^i(k)\|)\beta(k)\|e^i(k)\| \\ &\leq \|\mathbf{x}_t^i(k) - \mathbf{x}_t^*\|^2 - \beta(k)(1 - \beta(k))\|\mathcal{P}_{\mathcal{D}'_i}(v^i(k)) - v^i(k)\|^2 \\ &\quad + 2(\|\eta^i(k) - \mathbf{x}_t^*\| + \beta(k)\|e^i(k)\|)\beta(k)\|e^i(k)\| \end{aligned} \quad (7.26)$$

Hence,

$$\begin{aligned}
\sum_{i=1}^N \|\mathbf{x}_t^i(k+1) - \mathbf{x}_t^*\|^2 &\leq \sum_{i=1}^N \|\mathbf{x}_t^i(k) - \mathbf{x}_t^*\|^2 \\
&\quad - \beta(k)(1 - \beta(k)) \sum_{i=1}^N \|\mathcal{P}_{\mathcal{D}'_i}(v^i(k)) - v^i(k)\|^2 \\
&\quad + B \sum_{i=1}^N \beta(k) \|e^i(k)\|
\end{aligned} \tag{7.27}$$

where $B = \sup_{k \geq 0} (\|\eta^i(k) - \mathbf{x}_t^*\| + \beta(k) \|e^i(k)\|) < +\infty$.

From (7.27), as $\sum_{k \geq 0} \beta(k)(1 - \beta(k)) = +\infty$, it is easy to obtain that

$$\begin{aligned}
&\sum_{k \geq 0} \beta(k)(1 - \beta(k)) \sum_{i=1}^N \|\mathcal{P}_{\mathcal{D}'_i}(v^i(k)) - v^i(k)\|^2 \\
&\leq \sum_{i=1}^N \|\mathbf{x}_t^i(0) - \mathbf{x}_t^*\|^2 + B \sum_{k \geq 0} \left(\beta(k) \sum_{i=1}^N \|e^i(k)\| \right) < +\infty \\
&\Rightarrow \lim_{k \rightarrow \infty} \sum_{i=1}^N \|\mathcal{P}_{\mathcal{D}'_i}(v^i(k)) - v^i(k)\|^2 = 0 \\
&\Rightarrow \lim_{k \rightarrow \infty} \mathcal{P}_{\mathcal{D}'_i}(v^i(k)) = v^i(k)
\end{aligned} \tag{7.28}$$

□

From Theorem 7.5, it has been proved that if $\sum_{k \geq 0} \sum_{i=0}^N \beta(k) \|e^i(k)\| < +\infty$ is satisfied, then the estimate of the sensors will converge to some point $\mathbf{x}_t^* \in G$. While this condition may not be easy to verify in general, it does provide a theoretical basis to justify the convergence of the algorithm for the inconsistent case.

Since the convergence of the algorithm depends on the projection error $e^i(k)$. Here, some explanations are given on it. When the weight estimate (each sensor and its neighbors) $v^i(k)$ falls in both the convex sets $\mathcal{D}_i$ and $\mathcal{D}'_i$, $\mathcal{P}_{\mathcal{D}_i}[v^i(k)] = \mathcal{P}_{\mathcal{D}'_i}[v^i(k)]$ which implies $\|e^i(k)\| = 0$. Otherwise, $\|e^i(k)\|$ is the absolute difference between two radiuses, i.e., $\|e^i(k)\| = \|r_{\mathcal{D}_i} - r_{\mathcal{D}'_i}\|$, where $r_{\mathcal{D}_i}$, and $r_{\mathcal{D}'_i}$ denote the radius of the convex sets $\mathcal{D}_i$ and $\mathcal{D}'_i$ respectively. Intuitively, this difference is related to the

measurement noises. Since in the definition of convex sets $\mathcal{D}_i$ in Section 7.2, $\zeta - \sigma$ area of noise distribution is adopted. Hence if $\zeta = 1$, the probability of the source reside in each $\mathcal{D}_i$ is 68.2%. Therefore, the projection error $\|e^i(k)\|, \forall i$, is either equal to 0 or a small constant since both $\mathcal{D}_i$ and $\mathcal{D}'_i$ do not change with the iterations.

On the other hand, the value of relaxation sequence $\beta(k)$ can be adjusted to avoid the potential instability for the convergence condition. Here, two strategies are provided for designing the relaxation sequence. First, the potential instability for the convergence condition can be avoided by using a sufficiently small relaxation sequence $\beta(k)$. For example, if the variance of measurement noises is small, the relaxation sequence can be set as $\beta(k) = 1/k$. Otherwise, it can be set as $\beta(k) = a \times (1/k)$, where a is a small constant value which is related to the variance of measurement noises. Alternately, a similar strategy can be adopted for choosing $\beta(k)$ in [12]. In the first phase, the relaxation sequences are set to 1 if $k < N$, where N is the number of the sensors. In phase 2, the relaxation parameters are decreased at a rate of $1/k$.

Remark 7.2. *From the analysis for both the consistent and inconsistent cases, it can be seen that the proposed distributed projection based algorithm can be applied to the source localization problem without a priori knowledge whether or not the problem is consistent by using a decreasing relaxation sequence $\beta(k)$ as defined in Assumption 7.4.*

7.4.4 Estimation Accuracy Analysis

Noiseless case: It can be easily seen that if the sensor observation is noiseless, then the algorithm can converge to the true source location as long as there are at least three non-collinear sensors in the sensing field.

Noisy case: For a noisy case, the proposed estimator $\min_{\mathbf{x}_t \in S} \sum_{i=1}^N \|\mathbf{x}_t - \mathcal{P}_{\mathcal{D}_i}(\mathbf{x}_t)\|$ is optimal in the least-squares (LS) sense. Obviously, if the intersection of convex sets determined by each sensor is nonempty, i.e., $\mathcal{D} \neq \emptyset$, then $\min_{\mathbf{x}_t \in S} \sum_{i=1}^N \|\mathbf{x}_t -$

$\mathcal{P}_{\mathcal{D}_i}(\mathbf{x}_t) \parallel = 0$. Also according to the fact that “The intersection of closed sets is closed”, it can be seen that because the convex sets (disks) determined by sensors are closed sets, the intersection of these sets is closed. Thus, when the number of sensors increases to infinity, the variance of the estimates will decrease to zero, i.e., $\mathbf{E}(\mathbf{x}_t^* - \bar{\mathbf{x}}_t^*)^2 = 0$, where $\bar{\mathbf{x}}_t^*$ denotes the mean value of $\mathbf{x}_t^*$.

If the intersection of convex sets determined by each sensor is empty, i.e., $\mathcal{D} = \emptyset$, besides the number of sensors, there are two other factors affecting the estimation accuracy. One is the geometrical configuration of the sensor field and the other is the sensor observation noise.

Remark 7.3. *In this chapter, we have assumed that the communication channels between sensors are perfect without transmission noises and a node can transmit its data perfectly and reliably without packet loss to its neighbors. However, in many situations this is not realistic and will be considered in future work.*

7.4.5 Sensor Activation Problem

Figure 7.4: Sensor activation

For the source localization problem, there is no need to activate all the sensors to localize one source. When a source appears in the region of interest, one sensor with its identity number A which is awake, will perform sensing task and detect the existence of a source. If the source is detected, the sensor A will send a message to its 1-hop neighbors to let them start the sensing task. By continuing the process,

A's 1-hop neighbors will activate their own 1-hop neighbors. In all, sensor A's 2-hop neighbors will be activated to carry out the source localization task.

An illustrative example of this sensor activation process is presented in Figure 7.4. As plotted in the Figure 7.4(b) and Figure 7.4(c), 1-hop and 2-hop neighboring sensors of the duty sensor as in Figure 7.4(a) are activated respectively.

According to our experiences, the convergence speed of the proposed algorithm is fast (Elapsed CPU time: 0.1 second with 20 nodes active). Also because only a small number of sensors will be activated for the source localization task, therefore the communication overhead may not be a big problem. However, packet collisions may happen even though the density of sensors is not high.

7.5 Numerical Examples

7.5.1 Simulation Setup

This section presents simulation experiments for a sensor network. The algorithms are run by the MATLAB software using a HP L1908w desktop computer. 30 sensor nodes are randomly placed in a 10 m $\times$ 10 m field. The maximal communication radius of each sensor is set to 5 m. At each sensor, a measurement of the source energy is generated according to (7.1). The gain factors $g_i, i = 1, \dots, N$, for all sensors are equal to 1. The source is located at $\mathbf{x}_t = [0.5, 0.5]^T$ and emits a signal with S set to 50 dB and the background noise level is set at $0 \leq \sigma_i \leq 2, i = 1, \dots, N$ for all sensors in the sensor field. The actual receiver SNR at different sensors depends on the sensor to source distance. For example, if the variance of noise is 1, then for a sensor that is 5 m away from the source, its receiver SNR is $10 \times \log_{10}(50/5^2) = 3$ dB.

The weighting rule for the proposed diffusion based method adopts Metropolis weights [126] which is a time-invariant matrix and its (i, j) -th element is described

Figure 7.5: Sensor network configuration. In (a), dots and triangle denote the sensors and the source respectively

as:

$$w_j^i = \begin{cases} 1/(\max\{|\mathcal{N}_i|, |\mathcal{N}_j|\} + 1), \{i, j\} \in \mathcal{E}, \\ 1 - \sum_{\{i, k\} \in \mathcal{E}} (w_k^i), j = i, \\ 0, \text{otherwise}, \end{cases} \quad (7.29)$$

where $|\mathcal{N}_i|$ represents the number of the neighbors of the i -th sensor.

7.5.2 Convergence Performance

In this subsection, the convergence property of the proposed algorithm is evaluated compared with the protocol as in (7.13). The variance of background noise is set to 1 dB. The number of the active sensors is 10. The communication range of each sensor is set as 5 m to guarantee network connectivity (Assumption 7.1). The step-size sequence used for the proposed projection method is as in [12], that is

$$\beta(k) = \begin{cases} 1, & k \leq N; \\ 1/(k - N), & \text{otherwise.} \end{cases} \quad (7.30)$$

Due to the measurement noises, in this simulation, in most of the cases, the CFP is inconsistent. Figure 7.5(a) shows a source localization scenario. The corresponding

Figure 7.6: Convergence result by using protocol (7.13)

convex sets determined by sensors is plotted in Figure 7.5(b), from which it can be seen that the intersection is empty, i.e., $\mathcal{D} = \emptyset$.

Figure 7.6 and Figure 7.7 present the estimation results by using protocol (7.13) and the proposed protocol, respectively, where each curve denotes the estimation results by one sensor. In the figures, “Distance” in the y -axes denotes the Euclidean distance between the estimated and true source location, i.e., $\|\hat{\mathbf{x}}_t^i - \mathbf{x}_t\|, \forall i$. From the figures, it can be seen that the proposed method converges after around 20 steps even though the CFP is inconsistent. However, in most of the cases, the protocol (7.13) will diverge or oscillate at some point as seen in Figure 7.6.

In addition, the convergence trajectory by using the proposed protocol (7.15) is provided in Figure 7.8. Each route represents the convergence trajectory of a sensor over time. The red circles denote the estimation results of sensors after 20 time steps. From the figure, it can be seen that the red circles nearly overlap with each other, demonstrating the convergence of consensus.

7.5.3 Estimation Performance

As a benchmark, the estimation performance of the proposed projection methods is also compared against weighted least-squares (LS) method in [50] and MLE in

Figure 7.7: Convergence results by proposed diffusion protocol (7.15).

Figure 7.8: Convergence trajectory of the proposed protocol (7.15)

[106]. The MLE is found by performing a grid search over the field area. In the implementation, the grid search resolution is set to $1 \text{ m} \times 1 \text{ m}$. Grid search and weighted least-squares methods are both operated in a centralized approach. Here we mainly compare the localization accuracy among three methods. Two localization accuracy metrics, mean of absolute error (MAE) and root mean square error (RMSE), are utilized for performance evaluation of the methods. The performance of the estimators is evaluated through 1000 trials.

$$\begin{aligned} \text{MAE} &= \frac{1}{N_t} \sum_{i=1}^{N_t} \|\hat{\mathbf{x}}_{t_i} - \mathbf{x}_t\|, \\ \text{RMSE} &= \sqrt{\sum_{i=1}^{N_t} \frac{\|\hat{\mathbf{x}}_{t_i} - \mathbf{x}_t\|^2}{N_t}}, \end{aligned} \quad (7.31)$$

where N_t denotes the number of trials, $\hat{\mathbf{x}}_{t_i}$ denotes the estimate at i -th trial, $i = 1, \dots, N_t$.

In Figure 7.9, the MAE and RMSE of the three methods are presented as a function of variance of background noise, where 20 sensors are active. As shown in Figure 7.9, the weighted least square method has the worst performance and is more sensitive to the change of the noise. Also it can be seen that when the noise level is low, for instance, $\sigma^2 < 1$, the proposed method is comparable or better than the grid search method.

Similarly in Figure 7.10, the MAE and RMSE of the three methods are presented versus the number of sensors, where the variance of background noise is set to 1.5. Still, the grid search method for MLE has the best performance. Also, it can be seen that the performance of the projection based method is better than the weighted least-squares method, which is mainly because the least-squares method is more sensitive to the density of the sensors.

Figure 7.9: Comparisons in accuracy as noise variance changes

Figure 7.10: Comparisons in accuracy as the number of the sensors changes.

7.6 Conclusion

In this chapter, a fully distributed projection method was proposed for source localization with energy measurements. The solution has global convergence properties and acceptable estimation accuracy. The convergence of the proposed protocol was analyzed. Theoretic analysis and simulation results showed that the proposed method can be applied to the distributed source localization problem without *a priori* knowledge of whether or not the problem is consistent.

Chapter 8

Hybrid TOA/DOA Measurements Based Distributed Source Localization

This chapter continues to study the distributed source localization problem with a different type of sensor measurements: hybrid TOA/DOA measurements. That is to say, each sensor provides both TOA and DOA measurements. In this chapter, we assume that sensor noise is bounded. The distributed source localization is posed as a consensus problem for which the average consensus in the literature can be applied. The disadvantage of this average consensus method is that it can only be used for a linear model and needs to wait for the final convergence to achieve an acceptable accuracy. However, generally its convergence rate is low. Alternatively, a convex projection consensus method, which is similar to that in Chapter 7, is proposed in this chapter where the estimate of each sensor converges to a common point lying in the intersection of the convex regions that are determined based on local sensor measurements. The minimum number of sensors required to meet a certain estimation error tolerance is also derived. Numerical results are presented, which demonstrate the convergence properties, and RMSEs under both the average consensus and the convex projection methods are compared.

8.1 Problem Statement

The problem of interest is to determine the location of an active source in a sensor network. Let the source be located at an unknown location given by the coordinate pair $\mathbf{x}_t = [x_t, y_t]^T$. It is assumed that there are N sensor nodes with known coordinates, $\mathbf{x}_i = [x_i, y_i]^T, i = 1, 2, \dots, N$, performing location estimation.

The measurement models of TOA and DOA are the same with that in Chapter 2. The only difference is that here bounded noise is considered. In this chapter, hybrid TOA/DOA measurements are used for the source localization problem. In a sensor network, each sensor can be equipped with an antenna array. Both TOA and DOA information of the source can be extracted from the same noise-contaminated signal received at each antenna array. The estimation accuracy will surely be improved by using a hybrid TOA/DOA than using only either TOA or DOA measurement [122]. The estimated TOA and DOA by sensor i from the same noise-contaminated signal received at each antenna array are $\hat{\tau}_i = \tau_i + \Delta\tau_i$ and $\hat{\phi}_i = \phi_i + e_i$, respectively, where $\Delta\tau_i$ and e_i are measurement noises. Usually, the TOA and DOA measurement noises are assumed to be distributed according to $\Delta\tau_i \sim N(0, \sigma_{\tau_i}^2)$ and $\Delta\phi_i \sim N(0, \sigma_{\phi_i}^2)$. The variances¹ are given by $\sigma_{\tau_i}^2 = \sigma_{\tau}^2 d_i^\alpha$, where d_i denotes the distance between the source and the i -th sensor, and $\sigma_{\phi_i}^2 = \sigma_b^2 \|\phi_i - \frac{\pi}{2}\|^\beta$. $\{\alpha, \beta\} \geq 0$ are path-loss exponents and σ_{τ}^2 is the variance when $d_i = 1$ and σ_b^2 is the variance at $\phi_i = \frac{\pi}{2}, \forall i$. However, in practice, the measurement errors $\Delta\phi_i$ and $e_i, i = 1, \dots, N$ may be bounded, i.e., $|e_i| \leq p_i, |\Delta\tau_i| \leq q_i$, where $p_i \geq 0, q_i \geq 0$ are state dependent. Assume that the bounded errors follow the truncated Gaussian distribution, i.e., they are in a 3σ region, which means $p_i = 3\sigma_{\phi_i}, q_i = 3\sigma_{\tau_i}$.

8.2 Average Consensus Method

Under the assumption of bounded errors, for each sensor, the source location estimate is restrained in a shaded region as shown in Figure 8.1. It is easy to see that

¹In this chapter, the DOA measurement noise assumption is more practical and is different from that in Chapter 4.

Figure 8.1: Bounded errors

this shaded region is not convex. However this region can be bounded by a convex region such as the rectangular region “A'B'C'D'” or the circular region with center O_i . In this chapter, we utilize the circular region as the “Bounded Box” since it leads to lower computational complexity in determining the projection operator (which will be introduced later) and a lower communication burden.

Thus, each sensor determines a convex set $\mathcal{C}_i, i = 1, \dots, N$, given by the circular region shown in Figure 8.1, where the source surely lies in. To represent the sets $\mathcal{C}_i, i = 1, \dots, N$, we only need to know the coordinates of the center and the radius of the circles. Taking the sensor i for example, given $(\hat{\tau}_i, \hat{\phi}_i)$, the coordinates of the center O_i as shown in Figure 8.1 are the same with the source location estimate by sensor i , $\hat{\mathbf{x}}_t^{(i)} := (\hat{x}_t^{(i)}, \hat{y}_t^{(i)})$ can be written as [39]

$$(\hat{x}_t^{(i)}, \hat{y}_t^{(i)}) : \begin{cases} \hat{x}_t^{(i)} = x_i + c\hat{\tau}_i \cos \hat{\phi}_i \\ \hat{y}_t^{(i)} = y_i + c\hat{\tau}_i \sin \hat{\phi}_i \end{cases} \quad (8.1)$$

where c is the speed of light. The radius R_i can be calculated using the cosine law:

$$R_i = \sqrt{[c(\hat{\tau}_i + q_i)]^2 + [c\hat{\tau}_i]^2 - 2[c\hat{\tau}_i][c(\hat{\tau}_i + q_i)] \cos p_i} \quad (8.2)$$

8.2.1 Linear Fusion Method

A linear fusion rule can be applied to make a global decision by linearly combining the individual estimate $\{\hat{\mathbf{x}}_t^{(i)}\}_{i=1}^N$ as follows:

$$\hat{\mathbf{x}}_t = \alpha_1 \hat{\mathbf{x}}_t^{(1)} + \alpha_2 \hat{\mathbf{x}}_t^{(2)} + \dots + \alpha_N \hat{\mathbf{x}}_t^{(N)}, \quad (8.3)$$

where $\{\alpha_i\}_{i=1}^N$ are the linear weights with $\alpha_i = \frac{\mathcal{A}^{-1}}{\sum_{j=1}^N \mathcal{A}^{-1}}$, and $\mathcal{A}$ being the area of the circular region, i.e., πR_i^2 . Then the estimator of $\mathbf{x}_t$ can be obtained:

$$\hat{\mathbf{x}}_t = \frac{\sum_{i=1}^N \mathbf{b}_i}{\sum_{i=1}^N a_i} = \frac{(\sum_{i=1}^N \mathbf{b}_i)/N}{(\sum_{i=1}^N a_i)/N} = \frac{\varrho_b}{\varrho_a}, \quad (8.4)$$

where $a_i = (\pi R_i^2)^{-1}$, $\mathbf{b}_i = (\pi R_i^2)^{-1} \hat{\mathbf{x}}_t^{(i)}$, $\hat{\mathbf{x}}_t^{(i)}$ is the same with that in (8.1).

From (8.4), we can observe that $\hat{\mathbf{x}}$ can be written as the ratio of two averages. The problem of computing averages in a distributed manner has been well studied in the literature and can be solved using an average consensus method [127].

8.2.2 Consensus Method

The average consensus method² involves information exchange between neighboring nodes of the underlying connectivity graph corresponding to the sensor network. The average consensus protocol can be written as:

$$\hat{\varrho}_a(k+1) = W(k) \hat{\varrho}_a(k), \hat{\varrho}_b(k+1) = W(k) \hat{\varrho}_b(k) \quad (8.5)$$

where $\hat{\varrho}_a(k) \in \mathbb{R}^{N \times 1}$, $\hat{\varrho}_b(k) \in \mathbb{R}^{N \times 2}$ denote the estimates of ϱ_a and ϱ_b respectively, and $W(k)$ is an $N \times N$ doubly stochastic weighting matrix with the following properties:

$$\mathbf{1}^T W(k) = \mathbf{1}^T, W(k) \mathbf{1} = \mathbf{1}, \quad (8.6)$$

²The average consensus method also can be used to the Gaussian noise case, however, in this work, the bounded noise case is considered.

where $\mathbf{1}$ is a vector of ones.

Assumption 8.1. (*Network connectivity*) *The network is connected, i.e., there exists a direct or indirect path between any two nodes in the network.*

It is well known that, under Assumption 8.1, $\lim_{k \rightarrow \infty} \hat{\varrho}_a(t) = \frac{1}{N} \mathbf{1} \mathbf{1}^T \hat{\varrho}_a(0)$, and $\lim_{k \rightarrow \infty} \hat{\varrho}_b(k) = \frac{1}{N} \mathbf{1} \mathbf{1}^T \hat{\varrho}_b(0)$, where $\hat{\varrho}_a(0) = [a_1, a_2, \dots, a_N]^T$, $\hat{\varrho}_b(0) = [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_N]^T$ are the initial state vectors of the sensors [127]. Hence, the estimates of all the sensors will converge to the weighted linear fusion, i.e., $\hat{\mathbf{x}}_t$ as shown in (8.4).

The disadvantage of this average consensus method is that it can only be used for a linear model and needs to wait for the final convergence to achieve an acceptable accuracy since it has to compute the ratio of two averages. However, generally, its convergence rate is low. Next, an alternative method is proposed which has a faster convergence and a lower communication cost.

8.3 Convex Projection Method

In this section, an alternative method: convex projection method is proposed for the source localization in the bounded noise case. With multiple sensors, the idea is to solve the source localization problem by seeking an estimator that is a point in the intersection of the sets $\mathcal{C}_i, i = 1, \dots, N$, that is $\hat{\mathbf{x}}_t \in \mathcal{C} = \bigcap_{i=1}^N \mathcal{C}_i$. After knowing the convex sets $\mathcal{C}_i, i = 1, \dots, N$, the problem is how to obtain some common point lying in the intersection of the sets in a distributed way. Towards this end, we propose the following distributed convex projection method, based on the method of projections introduced in [83]. For the closed convex set $\mathcal{C}_i \subseteq \mathbb{R}^2$ and $x \in \mathbb{R}^2$, denote $\mathcal{P}_{\mathcal{C}_i}(x)$ as the orthogonal projection of x onto $\mathcal{C}_i$. That is,

$$\mathcal{P}_{\mathcal{C}_i}(x) = \arg \min_{y \in \mathbb{R}^2} \|x - y\|, \quad y \in \mathcal{C}_i. \quad (8.7)$$

Now, for a point x inside the circular area $\mathcal{C}_i$, $\mathcal{P}_{\mathcal{C}_i}(x)$ is equal to x . Otherwise, its projection onto $\mathcal{C}_i$ is the point on the circle given by $\mathbf{0}_i + R_i \frac{x - \mathbf{0}_i}{\|x - \mathbf{0}_i\|}$. We thus have

the following closed-form expression for (8.7):

$$\mathcal{P}_{\mathcal{C}_i}(x) = \begin{cases} x, \|x - \mathbf{0}_i\| \leq R_i, \\ \mathbf{0}_i + R_i \frac{x - \mathbf{0}_i}{\|x - \mathbf{0}_i\|}, \text{ otherwise.} \end{cases} \quad (8.8)$$

Then, the protocol for the proposed distributed convex projection method can be written as

$$\hat{\mathbf{x}}_t^{(i)}(k+1) = \mathcal{P}_{\mathcal{C}_i} \left(\sum_{j=1}^N w_{ij}(k) \hat{\mathbf{x}}_t^{(j)}(k) \right), \quad (8.9)$$

where $w_{ij}(k)$ is the (i, j) -th element of $W(k)$ and the initial values $\hat{\mathbf{x}}_t^i(0)$ can be chosen randomly.

As in the average consensus approach, a connected network is required as stated in Assumption 8.1 and the weight assumption (8.6) is also needed. Then the convergence result can be stated as follows [83].

Theorem 8.1. (*Consensus*) *Let the intersection set $\mathcal{C} = \bigcap_{i=1}^N \mathcal{C}_i$ be nonempty and Assumption 8.1 and weight rule (8.6) hold. Then for some $\mathbf{x}_t^* \in \mathcal{C}$, $\lim_{k \rightarrow \infty} \|\hat{\mathbf{x}}_t^{(i)}(k) - \mathbf{x}_t^*\| = 0, \forall i = 1, \dots, N$.*

It can be easily checked that if the measurement error bounds $\{p_i\}_{i=1}^N, \{q_i\}_{i=1}^N$ are small, then $\mathcal{C} = \bigcap_{i=1}^N \mathcal{C}_i$ has a small area, which means the algorithm can converge to the true source location. Alternatively, if the number of sensors is large enough, then the estimation error will be low. To be more specific, an estimation accuracy analysis will be provided in the next subsection.

8.3.1 Estimation Accuracy Analysis

The discrete model approach of [108] is adopted to analyze estimation accuracy. The bounded circular region determined by each sensor is approximated as a square region $\mathcal{C}_i$ with $\mathbf{0}_i, i = 1, \dots, N$ as its center. Further, since the measurement errors

Figure 8.2: Discrete model

$\{p_i, q_i\}_{i=1}^N$ are bounded, the entire region containing the estimated regions of all the sensors is also bounded. For simplicity, it can be approximated as a square region G with the source as its center. A discrete model is utilized, where G is partitioned into n^2 congruent squares with unit area, as shown in Figure 8.2. Assuming $\{C_i\}_{i=1}^N$ have sides of the same length $2r$ and that $\{O_i = (O_{xi}, O_{yi})\}_{i=1}^N$ may appear at each discrete point in the region G' , $[1+r, n-r] \times [1+r, n-r]$ with the same probability, i.e., $\frac{1}{(n-2r)^2}$. Based on the above discrete model and assumptions, the intersection area C of $\{C_i\}_{i=1}^N$ is given as

$$C = [O_{x_+} - r, O_{x_-} + r] \times [O_{y_+} - r, O_{y_-} + r], \quad (8.10)$$

where $O_{x_+/-} = \max / \min(O_{x1}, \dots, O_{xN})$ and similarly for O'_{yi} s. Hence the area of C is

$$A_{\mathbf{u}} = (O_{x_-} - O_{x_+} + 2r)(O_{y_-} - O_{y_+} + 2r).$$

Before going to the estimation accuracy analysis, the following lemma from basic probability theory is required.

Lemma 8.1. *If a random variable X has mass function f and $g: \mathbb{R} \rightarrow \mathbb{R}$, then*

$$E(g(X)) = \sum_x g(x)f(x),$$

whenever this sum is absolutely convergent.

Then, the estimation accuracy analysis result is given by the following proposition.

Proposition 8.1. *Let $\mathcal{O}_i, i = 1, \dots, N$, be randomly distributed in a square area G' .*

Then, the expectation of $A_{\hat{\mathbf{x}}_t}$ is

$$E(A_{\hat{\mathbf{x}}_t}) = \left(1 + \frac{2(n-2r)}{N+1}\right)^2,$$

and $\lim_{N \rightarrow \infty} E(A_{\hat{\mathbf{x}}_t}) = 1$.

Proof. From Lemma 8.1, it can be obtained that

$$\begin{aligned} E(A_{\hat{\mathbf{x}}_t}) &= \frac{1}{(n-2r)^{2N}} \sum_{(\mathbf{o}_1, \dots, \mathbf{o}_N) \in G'} (O_{x_-} - O_{x_+} + 2r) \\ &\quad \times (O_{y_-} - O_{y_+} + 2r). \end{aligned}$$

Then according to Lemma 6.2 in [108], it can be obtained that

$$\begin{aligned} E(A_{\hat{\mathbf{x}}_t}) &= \frac{1}{(n-2r)^{2N}} \left[(n-2r)^N + 2 \sum_{k=1+r}^{n-r} k^N \right]^2 \\ &\leq \frac{1}{(n-2r)^{2N}} \left[(n-2r)^N + 2 \frac{1}{N+1} (n-2r)^{N+1} \right]^2 \\ &= \left(1 + \frac{2(n-2r)}{N+1} \right)^2. \end{aligned} \tag{8.11}$$

Finally, it is easy to see that $\lim_{N \rightarrow \infty} E(A_{\hat{\mathbf{x}}_t}) = 1$. □

Corollary 8.1. *The minimum number of sensors necessary to achieve that $\|E(A_{\hat{\mathbf{x}}_t}) - 1\| < \epsilon$ satisfies*

$$N > 2(n-2r) \frac{1 + \sqrt{1 + \epsilon}}{\epsilon}$$

where ϵ is a prespecified error tolerance.

Proof. From (8.11), it can be obtained that

$$E(A_{\hat{\mathbf{x}}_t}) - 1 = \left(\frac{2}{N+1}\right)^2(n-2r)^2 + \frac{4}{N+1}(n-2r).$$

Hence, $E(A_{\hat{\mathbf{x}}_t}) - 1 < \epsilon$ if

$$N > 2(n-2r)\frac{1+\sqrt{1+\epsilon}}{\epsilon}.$$

□

Thus, the expected area of the region in which the estimate lies in tends to unity as the number of sensors grows. From Proposition 8.1, a relationship between the number of sensors and the estimation accuracy can be obtained. It can be seen that if the number of sensors is large, the estimation error would be low.

8.4 Simulation Results

Figure 8.3: A connected sensor network (small circles denote the sensors and the triangle denotes the source)

Consider a sensor network with 15 sensor nodes randomly placed in a $10 \text{ m} \times 10 \text{ m}$ field. The path-loss exponents α and β are both set to be 1. The source is

located at $\mathbf{x}_t = [0, 0]$ as shown in Figure 8.3. In the simulations, for convenience, the TOA measurements $\{\tau_i\}_{i=1}^N$ are transformed into distance $\{d_i\}_{i=1}^N$ between the sensor and source. The background noise variances are set as $\sigma_b^2 = 0.5, \sigma_d^2 = 0.5$. A time-invariant weight matrix W is utilized with elements given by the Metropolis weights [127], i.e. the (i, j) -th element is given by

$$w_{ij} = \begin{cases} 1/(\max\{|\mathcal{N}_i|, |\mathcal{N}_j|\} + 1), & \text{if } \{i, j\} \in \mathcal{E}, \\ 1 - \sum_{\{i, k\} \in \mathcal{E}} (w_{ik}), & \text{if } j = i, \\ 0, & \text{otherwise,} \end{cases} \quad (8.12)$$

where $|\mathcal{N}_i|$ represents the number of neighbors of the i -th sensor.

Figure 8.4: Convergence results

8.4.1 Evaluation of Convergence and Estimation Performance

The convergence property of the average consensus and the distributed convex projection methods for source localization is evaluated in Figure 8.4. In the figure, “Distance” in the y-axes denotes Euclidean distance estimation error, i.e., $\|\hat{\mathbf{x}}_t^i - \mathbf{x}_t\|, \forall i$. From the figure, we can see that both methods have good convergence performance and acceptable estimation accuracy.

Then the estimation accuracy comparison between the two methods for the bounded noise case is performed. From Figure 8.5, it can be seen that projection method has a better performance.

Figure 8.5: RMSE v.s. the number of iterates

8.4.2 Communication Cost

From Figure 8.4, it can be seen that the convergence speed of convex projection method is a little faster than the average consensus method. Also at each iteration step, the communication cost for the average consensus method (3 scalars are transmitted) is larger than the convex projection method (2 scalars are transmitted). Note that the scalars for two methods are in the same order of magnitude. Therefore, the communication burden for two methods is roughly 3 : 2.

8.5 Conclusion

In this chapter, two distributed algorithms for source localization using hybrid TOA/DOA measurements were studied for bounded noise cases. The convex projection method has a better performance than the average consensus method in terms of both the communication and computation requirements. Simulations showed the promising performance of the proposed methods.

Chapter 9

Conclusions and Future Work

9.1 Conclusions

This thesis has discussed some issues of resource management and algorithm development for source localization problems. For the resource management in source localization, several research topics such as sensor placement, sensor selection and sensor pairing, etc, were studied. For the algorithm development in source localization, a centralized algorithm for energy based multi-source localization and fully distributed algorithms for single-source localization were presented, respectively. The contributions of the thesis are summarized in the following.

- In Chapter 3, several optimal sensor placement strategies have been proposed for a homogeneous sensor network. Analytical solutions for the optimal sensor configurations were derived based on CRLB and posterior error covariance matrix. By using the proposed sensor placement strategies, source localization and tracking performance can be improved.
- In Chapter 4, the optimal sensor-source geometries for source localization in heterogeneous sensor networks have been derived. For hybrid range and DOA sensors, hybrid TDOA and DOA sensors as well as co-located hybrid range and DOA sensors, the optimal sensor configurations for the best source localization performance, have been provided.

- In Chapter 5, a decentralized in-network sensor pairing method has been proposed for the TDOA data gathering. The proposed decentralized sensor pairing method was shown to provide the same CRLB as the centralized one.
- In Chapter 6, an efficient EM algorithm has been presented for the energy based multiple source localization problem. The proposed algorithm can estimate the multiple source locations simultaneously. It was also shown that the proposed EM algorithm can provide a good tradeoff between the localization accuracy and the computational complexity.
- In Chapter 7 and Chapter 8, distributed algorithms have been developed for the single source localization problem with the energy and the hybrid TOA/DOA measurements, respectively. A projection convex optimization method was proposed which provides a fully distributed solution for the source localization problem. Simulation results showed that the proposed methods have global convergence properties and acceptable estimation accuracy.

9.2 Recommendation for Future Research

1. In Chapter 4, the optimal sensor-source geometry has been derived in HSNs. The results are only applied for a small number of sensors case. It is expected that we can develop some highly efficient heuristics for sensor placement in HSNs with a large number of sensors. It is also worthy working on optimal design for sensor positions in the average sense, i.e., maximizing the expected value of the FIM determinant.
2. We have developed an in-network TDOA sensor pairing method in Chapter 5. The topology of the network graph is assumed fixed during the source localization and tracking task. An interesting research topic is that the sensor network topology should be optimized to pair the sensors so as to improve the localization performance and reduce the communication cost.

3. In Chapter 6, the energy based multiple source localization problem has been addressed. In real applications, multiple source tracking is also an interesting problem. Sensor allocation to multiple sources would also deserve to be studied. The problem can be cast as a Pareto optimization and distributed algorithms can be developed to assign the tracking tasks efficiently.
4. In Chapters 7 and 8, distributed algorithms have been designed for the source localization problem. For the proposed localization protocol, packet collisions may happen even though the density of sensors is not high. Collision avoidance is still a common problem for fully distributed algorithms. Some scheduling protocols are to be designed to handle this problem. Another research direction is that it is expected to develop a distributed source localization algorithm which can achieve the CRLB.
5. In Part II, several algorithms have been proposed for the source localization problem. In real applications, the communication cost and quality of service (QoS) should also be taken into account when designing the localization algorithms. Thus, both the QoS and quality of information (QoI) should be incorporated in the algorithm development and analysis.

Appendix A

Proof of Lemma 7.2

According to Lemma 7.1, it is straight forward to obtain the result of part 1).

Proof of Part 2): By the definition of $v^i(k)$ and weight rule assumption, we have for every sensor i and any $\mathbf{x}_t^* \in \mathcal{D}$,

$$v^i(k) - \mathbf{x}_t^* = \sum_{j=1}^N w_j^i(k)(\mathbf{x}_t^j(k) - \mathbf{x}_t^*). \quad (\text{A.1})$$

Thus, we have

$$\|v^i(k) - \mathbf{x}_t^*\|^2 = \left\| \sum_{j=1}^N w_j^i(k)(\mathbf{x}_t^j(k) - \mathbf{x}_t^*) \right\|^2 \leq \sum_{j=1}^N w_j^i(k) \|\mathbf{x}_t^j(k) - \mathbf{x}_t^*\|^2.$$

By summing the preceding relations over $i = 1, 2, \dots, N$, we obtain

$$\begin{aligned} \sum_{i=1}^N \|v^i(k) - \mathbf{x}_t^*\|^2 &\leq \sum_{i=1}^N \sum_{j=1}^N w_j^i(k) \|\mathbf{x}_t^j(k) - \mathbf{x}_t^*\|^2 \\ &= \sum_{j=1}^N \left(\sum_{i=1}^N w_j^i(k) \right) \|\mathbf{x}_t^j(k) - \mathbf{x}_t^*\|^2 = \sum_{j=1}^N \|\mathbf{x}_t^j(k) - \mathbf{x}_t^*\|^2. \end{aligned}$$

Then we have

$$\sum_{i=1}^N \|v^i(k) - \mathbf{x}_t^*\|^2 \leq \sum_{i=1}^N \|\mathbf{x}_t^i(k) - \mathbf{x}_t^*\|^2. \quad (\text{A.2})$$

Similarly, from relation (A.1), we can obtain

$$\sum_{i=1}^N \|v^i(k) - \mathbf{x}_t^*\| = \left\| \sum_{i=1}^N \sum_{j=1}^N w_j^i(k) (\mathbf{x}_t^j(k) - \mathbf{x}_t^*) \right\| \leq \sum_{i=1}^N \|\mathbf{x}_t^i(k) - \mathbf{x}_t^*\|. \quad (\text{A.3})$$

Proof of Part 3):

Combining the results of part 1), 2), we have

$$\sum_{i=1}^N \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\|^2 \leq \sum_{i=1}^N \|v^i(k) - \mathbf{x}_t^*\|^2 \leq \sum_{i=1}^N \|\mathbf{x}_t^i(k) - \mathbf{x}_t^*\|^2. \quad (\text{A.4})$$

From protocol (7.15), we can obtain

$$\mathbf{x}_t^i(k+1) = (1 - \beta(k))v^i(k) + \beta(k)\mathcal{P}_{\mathcal{D}_i}(v^i(k)) \quad (\text{A.5})$$

Then we can deduce that

$$\begin{aligned} \|\mathbf{x}_t^i(k+1) - \mathbf{x}_t^*\|^2 &= \|(1 - \beta(k))(v^i(k) - \mathbf{x}_t^*) - \beta(k)(\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*)\| \\ &= (1 - \beta(k))^2 \|v^i(k) - \mathbf{x}_t^*\|^2 + \beta(k)^2 \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\|^2 \\ &\quad + 2\beta(k)(1 - \beta(k)) \|v^i(k) - \mathbf{x}_t^*\| \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\| \\ &= (1 - \beta(k)) \|v^i(k) - \mathbf{x}_t^*\|^2 + \beta(k) \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\|^2 + (\beta(k)^2 - \beta(k)) (\|v^i(k) \\ &\quad - \mathbf{x}_t^*\|^2 + \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\|^2) + 2\beta(k)(1 - \beta(k)) \|v^i(k) - \mathbf{x}_t^*\| \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\| \\ &= (1 - \beta(k)) \|v^i(k) - \mathbf{x}_t^*\|^2 + \beta(k) \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\|^2 - \beta(k)(1 - \beta(k)) \\ &\quad (\|v^i(k) - \mathbf{x}_t^*\|^2 + \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\|^2 - 2\|v^i(k) - \mathbf{x}_t^*\| \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\|) \\ &= (1 - \beta(k)) \|v^i(k) - \mathbf{x}_t^*\|^2 + \beta(k) \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\|^2 \\ &\quad - \beta(k)(1 - \beta(k)) \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - v^i(k)\|^2. \end{aligned} \quad (\text{A.6})$$

Hence, by using the results of (A.4) and (A.6), we have

$$\sum_{i=1}^N \|\mathbf{x}_t^i(k+1) - \mathbf{x}_t^*\|^2 = (1 - \beta(k)) \sum_{i=1}^N \|v^i(k) - \mathbf{x}_t^*\|^2$$

$$\begin{aligned}
& + \beta(k) \sum_{i=1}^N \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - \mathbf{x}_t^*\|^2 \\
& - \beta(k)(1 - \beta(k)) \sum_{i=1}^N \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - v^i(k)\|^2.
\end{aligned} \tag{A.7}$$

From (A.7) and in view of (A.2) of Part 1), we can further obtain that

$$\begin{aligned}
\sum_{i=1}^N \|\mathbf{x}_t^i(k+1) - \mathbf{x}_t^*\|^2 & \leq \sum_{i=1}^N \|\mathbf{x}_t^i(k) - \mathbf{x}_t^*\|^2 \\
& - \beta(k)(1 - \beta(k)) \sum_{i=1}^N \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - v^i(k)\|^2
\end{aligned} \tag{A.8}$$

Then, we have that

$$\begin{aligned}
& \sum_{k \geq 0} \beta(k)(1 - \beta(k)) \sum_{i=1}^N \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - v^i(k)\|^2 \\
& \leq \sum_{i=1}^N \|\mathbf{x}_t^i(0) - \mathbf{x}_t^*\|^2 < \infty
\end{aligned} \tag{A.9}$$

Since $\sum_{k \geq 0} \beta(k)(1 - \beta(k)) = \infty$, hence, $\lim_{k \rightarrow \infty} \sum_{i=1}^N \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - v^i(k)\|^2 = 0$. So $\lim_{k \rightarrow \infty} \|\mathcal{P}_{\mathcal{D}_i}(v^i(k)) - v^i(k)\| = 0 \Rightarrow \lim_{k \rightarrow \infty} \mathcal{P}_{\mathcal{D}_i}(v^i(k)) = v^i(k)$.

Author's Publications

Journal papers:

1. W. Meng, W. Xiao and L. Xie, "An efficient EM algorithm for energy based multi-source localization in wireless sensor networks," *IEEE Transactions on Instrumentation and Measurement*, vol. 60, no. 3, pp. 1017-1027, 2011.
2. W. Meng, L. Xie and W. Xiao, "Optimality analysis of sensor-source geometries in heterogeneous sensor networks," *IEEE Transactions on Wireless Communications*, vol. 12, no. 4, pp. 1958-1967, 2013.
3. W. Meng, L. Xie and W. Xiao, "Decentralized TDOA sensor pairing in multi-hop sensor networks," *IEEE Signal Processing Letters*, vol. 20, no. 2, pp. 181-184, 2013.
4. W. Meng, W. Xiao and L. Xie, "A projection based fully distributed approach for source localization in wireless sensor networks," *Ad Hoc and Sensor Wireless Networks*, vol. 18, pp. 131-158, 2013.
5. W. Meng, L. Xie and W. Xiao, "Optimal sensor pairing for source localization and tracking," *IEEE Transactions on Aerospace and Electronic Systems*, 2013, in revision.

Conference papers:

1. W. Meng, L. Xie, and W. Xiao, "Sensor placement in heterogeneous sensor networks," in *ICARCV 2012*, Guang Zhou, China, 2012.
2. W. Meng, L. Xie, and W. Xiao, "Optimal sensor pairing for source localization and tracking," in *Fusion 2012*, Singapore, 2012, pp. 1897-1902.
3. W. Meng, L. Xie, and W. Xiao, "TDOA sensor pairing in multi-hop sensor networks," in *IPSN 2012*, Beijing, China, 2012, pp. 91-92.
4. W. Meng, L. Xie, and W. Xiao, "Optimal sensor pairing for TDOA source localization," in *ICICS 2011*, Singapore, 2011, pp. 1-5.
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7. W. Meng, W. Xiao, L. Xie, A. Pandharipande, "A projection based distributed source localization in wireless sensor networks," in *INFOCOM Workshops*, Shanghai, China, 2011, pp. 543-546.

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