Fluid mechanics of flow through microchannels

Navid Kashaninejad

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FLUID MECHANICS OF FLOW THROUGH MICROCHANNELS

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2013

FLUID MECHANICS OF FLOW THROUGH MICROCHANNELS

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Abstract

The main objective of this thesis is to characterize and investigate single-phase liquid flow in microchannels, and can be divided into two parts: analytical modeling and experimental investigation. In the analytical part, velocity distribution and Darcy friction factor of liquid flow in both parallel-plate and rectangular microchannels were revised theoretically by taking into account the effects of slip boundary conditions as well as the aspect ratio of the channels. For the case of parallel-plate microchannel, generating mechanism of slip was attributed to the existence of a low-viscosity region between the liquid and the solid surface. Two different cases were considered based on the pressure gradient: a) re-circulating air region, where the net flow rate of gas phase was zero; b) reduced viscosity gas region, in which pressure gradient was assumed to be the same at both phases. It was found that the slip length in both cases was directly proportional to the ratio of liquid-to-gas viscosity as well as the air gap thickness. Unlike other studies which used empirical values of air gap, here, equations describing this parameter were derived based on different ranges of Knudsen number for the case of reduced viscosity region. For the case of rectangular microchannels, new analytical forms of velocity distribution, Darcy friction factor and Poiseuille number in laminar liquid flow in rectangular microchannels were derived using the eigenfunction expansion technique. Three different boundary conditions (BCs) were considered: no-slip BCs, constant slip BCs, and general Navier slip length BCs. Due to the symmetry of the solutions, the effects of changing the aspect ratio from 0 to ∞ were also discussed. Using finite element method (FEM), the obtained analytical results for the case of no-slip BCs were further compared with the 3D numerical simulations for the rectangular microchannels with no-slip BCs and different ranges of aspect ratio and pressure gradient.

In the experimental section, an important geometrical parameter of the hydrophobic micropillar surfaces termed as eccentricity, the center-to-center offset distance between successive columns of a square lattice of micropatterned surfaces, both micropillar and microhole arrays, was introduced. First, the combined effects of changing relative spacing, eccentricity, and viewing directions on the wetting conditions of the fabricated micropillar surfaces were experimentally investigated. The contact angle distribution was found as a periodic function of viewing angle whose period depends on the micropillar eccentricity. The results showed that anisotropy increases by increasing the micropillar normalized eccentricity, ε^* , or decreasing the pillar relative spacing, D^* . However, the effect of changing the micropillar eccentricity was much more pronounced. Micropillars with $\varepsilon^* = 1.00$ and smaller D^* showed maximum degrees of anisotropic wetting and droplet distortion corresponding to 7% and 15%, respectively. Using the measured droplet aspect ratio, corrugated shapes of the three-phase contact line of the micropillars were also reconstructed. Finally, a simple yet effective semi-analytical model, based on Fourier series curve-fitting of the experimental data, was developed to describe the equilibrium 3D shape of the droplet on anisotropic surfaces.

Subsequently, the wettability of microhole structures fabricated by replica molding of polydimethylsiloxane (PDMS) was analyzed by measuring both static and dynamic contact angles. It was found that wetting conditions can be controlled not only by changing the normalized widths but also the eccentricities. Generally, increasing the micropattern eccentricity increased the contact angle hysteresis. The effect of changing the eccentricity was more pronounced for large normalized width microhole arrays. In particular, for the case of $D^* = 4.6$ increasing ε^* from 0 to 2.6 increased the contact angle hysteresis from 18.7° to 36.5° .

Dependency of the contact angle hysteresis on microhole eccentricity was explained by the shape of the three-phase contact line on microhole configurations.

Finally, owing to the advantages of the microhole surfaces, a series of experiments was performed on the microchannels fabricated from such hydrophobic microhole arrays, and the drag reduction efficiency was evaluated. The results indicated that the impact of microhole eccentricity on drag reduction performance correlated well with the contact angle hysteresis, rather than the static contact angle. In particular, on the microchannels fabricated from microhole surfaces with the minimum contact angle hysteresis of 18.7° , the maximum percentage increase in the relative velocity, i.e., 39 % and maximum slip length of $2.49\mu m$ were observed. For the same normalized width, increasing the normalized eccentricity eventually reduces the percentage increase in relative velocity and slip length down to 16 % and $0.91\mu m$, respectively. The obtained results were in qualitative agreement with the existing theoretical and numerical models. These findings provide additional insights in design and fabrication of efficient micropatterned channels for reducing the flow resistance.

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*This thesis is lovingly dedicated to my wife, Betia, as well
as my son, Artin, whom I wish to read daddy's work
sometime in the future!*

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List of Symbols

Roman Letters

A = droplet surface area, Eq. (2.1)

$2a$ = droplet width

a^* = droplet aspect ratio

b = slip length of liquids

Bo = Bond number

C_f = Darcy friction factor

D = micropillar spacing, or microhole width

D_h = hydraulic diameter

f = Darcy friction factor

F = pressure drop parameter, Eq.(4.2)

G = Gibbs free energy

$H, 2h$ = total channel height

H_i = height parameter, Eq. (4.77)

h = droplet height

Po = Poiseuille number

Kn = Knudsen number

L = micropillar width, or microhole spacing

l_s = length scale

Q = flow rate

Re = Reynolds number

$W, 2w$ = total channel width

W_i = width parameter, Eq. (4.76)

U_x = experimental uncertainty of x

$\bar{u}$ = average velocity

u_s = slip velocity at the wall

u_b = bulk flow velocity

Greek Letters

α = aspect ratio or viewing angle

β = dimensionless slip

γ = interface/surface tension

$\dot{\gamma}$ = shear rate

Γ = channel wetted perimeter

δ = air gap thickness

$\delta_g = D/\mathcal{L}$ = shear-free (gas) fraction

ε = micropatterned eccentricity

ϵ = gas molecular slip length

ϵ_b = bias error

ϵ_s = standard error

ζ = anisotropic droplet distortion

η = height of the micropatterns

θ = contact angle

k = line tension

Λ = Percentage increase in the relative velocity

λ = degree of anisotropic wetting

μ = fluid viscosity

μ_r = ratio of liquid to air viscosities

ρ = fluid density

σ = tangential momentum coefficient

τ = shear stress

ϕ = zenith angle

ϕ_g = gas area fraction

ϕ_s = solid area fraction

Letter-Like Symbols

$\mathcal{H}$ = edge length of tetrahedral mesh

κ = mesh quality

$\mathcal{L}$ = period of micropatterned ($D + L$)

ℓ = air mean free path

$\mathcal{V}$ = mesh volume

Subscripts/superscripts

a : advancing

b : bulk flow

CB: Cassie-Baxter

eff: effective

exp: experimental

g: gas

H-P: Hagen-Poiseuille

LV: liquid-vapor phases

r : receding

s: side wall

SL: solid-liquid phases

SV: solid-vapor phases

th: theoretical

w: channel wall

+: upper

-: lower

*: dimensionless

Abbreviations

BC: boundary condition

CAH: contact angle hysteresis

DI: deionized water

MEMS: micro-electro-mechanical
systems

N-S: Navier-Stoke

ODE: ordinary differential equation

PDE: partial differential equation

PDMS: poly-dimethylsiloxane

PIV: particle image velocimetry

PMMA: poly-methylmethacrylate

SVR: surface-to-volume ratio

TCMS: trichloromethylsilane

UV: ultraviolet

Publications Arising From this Thesis

International Journal Papers

- **N. Kashaninejad**, W.K. Chan, N.T. Nguyen, “Eccentricity effect of micropatterned surface on contact angle” *Langmuir* (American Chemical Society-ACS), 2012, 28 (10), pp 4793–4799. **DOI:** 10.1021/la300416x
- **N. Kashaninejad**, N.T. Nguyen, W.K. Chan, “Three-phase contact line shape and eccentricity effect of anisotropic wetting on hydrophobic surfaces”, *Soft Matter* (Royal Society of Chemistry-RSC), 2013, 9 (2), pp. 527-535. **DOI:** 10.1039/C2SM26963E
- **N. Kashaninejad**, N.T. Nguyen, W.K. Chan, “Eccentricity effects of microhole arrays on drag reduction efficiency of microchannels with a hydrophobic wall” *Physics of Fluids* (American Institute of Physics-AIP), 2012, 24 (11), pp. 112004-18. **DOI:**10.1063/1.4767539
- **N. Kashaninejad**, W.K. Chan, and N.T. Nguyen, “Analytical modeling of slip flow in parallel plate microchannels”, *Micro and Nanosystems* (Bentham Science Publishers), manuscript accepted on 26/06/2013.

International Refereed Conference Papers

- **N. Kashaninejad**, W.K. Chan, N.T. Nguyen, “Analytical and Numerical Investigation of the Effects of Microchannel Aspect Ratio on Velocity Profile and Friction Factor”, 4th International Conference on Computational Methods, *ICCM2012*, 25-28 November 2012, Crowne Plaza, Gold Coast, Australia.

- **N. Kashaninejad**, W.K. Chan, N.T. Nguyen, “Fluid Mechanics of Flow through Rectangular Hydrophobic Microchannels”, ASME 9th International Conference on Nanochannels, Microchannels, and Minichannels, *ICNMM2011*, 19-22 June 2011, Edmonton, Alberta, CANADA.

Chapter 1 : Introduction

1.1 Background

Microchannel is a basic component in drug delivery devices, micro-electro-mechanical-systems (MEMS), lab-on-a-chip systems, and other microfluidic components. Large surface-to-volume ratio (SVR) of microchannels makes them an excellent choice for compact and efficient heat exchangers in electronic cooling devices [1-3]. In biomedical and chemical sciences, microchannels are used to deliver and analyze the micron sized biological and chemical substances [4]. Therefore, complete understanding of the flow characteristics in microchannels is essential to improve the performance of such an interdisciplinary field.

Incorporating plastic and polymer based materials such as poly-methylmethacrylate (PMMA) and poly-dimethylsiloxane (PDMS) besides silicon and glass facilitates the microchannel fabrication techniques greatly. Nevertheless, theoretical models governing small-scale domains have not been progressed as rapidly as their fabrication techniques. In other words, fundamental hypotheses, which conventional theories are based on, need to be reconsidered carefully when working in microscale domains otherwise unacceptable deviations from experimental results would occur. For instance, the continuum assumption in modeling gas flows in microchannels is not valid due to the comparable size of the system with the mean free path of the gas molecules.

1.2 Validity of Conventional Flow Theories in Microchannels

To verify the applicability of Navier-Stoke (N-S) equations in microchannels, liquid flow rate can be measured at the imposed pressure difference along the given length of the microchannels. The obtained results can then be compared to that predicted by N-S equations. Alternatively,

Poiseuille number, the product of the friction factor and Reynolds number, can be determined by gradually increasing the pressure differences and recording the corresponding values of the flow rate. For laminar flows, conventional fluid dynamics theories state that this number should be a fixed value regardless of channel roughness. However, based on the prior experimental works in the literature, there is a lack of consensus regarding this. The main sources of the discrepancies of these experimental data might be related to the following issues [5]: 1) difficulty in measuring the microchannel dimensions before and after the experiment accurately; 2) experimental difficulties in precise measurements of the flow rate and pressure drop along the microchannel length; 3) evaluating near-wall surface effects of the microchannels. The first two experimental difficulties give rise to the uncertainty of the experimental results and can be minimized by appropriate selection of the test instruments. However, the prime reason of these discrepancies is due to the dominant channel walls effects in the flow through microchannels. As the characteristic size of the channel is reduced, surface effects become more pronounced even in the laminar region. This implies that depending on channel wall conditions, drag forces on the microchannels can be greater than, equal to or even smaller than corresponding values in macrochannels. Accordingly, three different microchannel wall surfaces are distinguishable as shown in Figure 1-1.

Figure 1-1: Schematic presentation of three different microchannels wall surfaces: (a) Atomically smooth walls. (b) Naturally rough walls. (c) Microstructured rough walls.

As illustrated in Figure 1-1(b,c), roughness ratios of the solid wall might cause the near-wall flow to behave differently. . In the first scenario, liquid might penetrate into the cavities, as shown in Figure 1-1(b), and this is known as noncomposite state. Conversely, liquid may stay atop of the roughness, as shown in Figure 1-1(c), and this is known as composite state.

Larger values of the frictional losses in the microchannels can be attributed to the large values of relative roughness (roughness amplitude divided the channels height) especially in the noncomposite state. It should be noted other micro-scale effects in extremely small microchannels, typically smaller than $10\mu m$, such as electroviscous [6] and pressure-dependent viscosity effects [7] can also give rise to the frictional losses.

On the other hand, frictional drags in microchannels can be smaller than those in the macrochannels mainly as a result of slip between the liquid and solid surface. For the case of atomically smooth surfaces, two situations may lead to slip, namely very high shear rate flows and hydrophobic channels. In the first case, repulsive forces induced by the high shear rates may overcome the attractive Van der Waals forces at the solid surface and lead to intrinsic slip. For the water flows this happens when the shear rate is higher than 10^{10} sec^{-1} . In the case of hydrophobic channels, presence of tiny bubbles near the solid walls might act as a cushion. This can lead to an apparent slip of the flow at the vicinity of the solid walls. For instance, wall slip velocity up to 9% of the free stream velocity was observed for water flows in hydrophobic channels at moderate shear rates [8]. For microengineered surfaces shown in Figure 1-1(c), the effective contact areas between liquid and solid are greatly reduced and are replaced with air pockets. That means, the liquid is in contact with periodic no-slip and shear-free regions, and therefore, the frictional losses can be effectively reduced. These results imply that microchannels

surface conditions can greatly affect the liquid flow even in the laminar region, and careful characterizations of these surfaces are essential to obtain accurate results.

1.3 Objective of the Current Project

Recently, implementing the microstructured surfaces in microchannels as the passive method to reduce the frictional losses in laminar flow has attracted many attentions. Fluid flow characteristics at relatively large Reynolds number (typically, $30 \leq Re \leq 500$) in microchannels made from periodic arrays of alternating microgroove and microrib as well as aligned micropillar structures were extensively evaluated.

The main objective of this thesis is to advance the current understanding of single-phase liquid flow in microchannels. Classical theories describing the hydrodynamics of the liquid flow will be modified to account the effects of slip boundary conditions as well as the finite aspect ratio of the microchannels. An important geometrical parameter of the micropatterned surfaces termed as eccentricity is introduced. Subsequently, the combined effects of changing the micropatterns relative spacing and eccentricity on both isentropic and anisotropic wetting conditions will be evaluated. Novel techniques used to depict the corrugations of the three-phase contact line shapes of these surfaces are presented and discussed. Finally, a series of microchannels are fabricated from these micropatterned surfaces, and the volume flow rates at the different hydrostatic pressures are found experimentally for low Reynolds numbers. The effects of micropatterns eccentricity and spacing as well as the aspect ratio of the microchannels on slip lengths are discussed, and conclusive remarks are presented.

1.4 Scope of the work

In Chapter 2, a comprehensive literature review regarding wetting analysis of micropatterned surfaces as well as liquid flow characteristics through the microchannels made from such

hydrophobic surfaces is presented. In Chapter 3, Newtonian liquid flow in parallel-plate microchannels made from two different channel walls with unequal wettability and surface conditions will be evaluated. The classical formulas for velocity, flow rate, and shear stress will be revised to incorporate the effect of slip velocity near the solid walls. In addition, for the case of hydrophobic microchannels with air flows at the top and bottom, the previous works are modified by proposing the estimated value of air fraction thickness for different ranges of Knudsen number. In Chapter 4, analytical and numerical models of liquid flow in rectangular microchannels are presented. In particular, the effect of aspect ratio of rectangular microchannels is evaluated using the two-dimensional solutions in terms of dual finite series. Also, the solution is extended to the cases of a constant slip velocity at the bottom wall by superposing the eigenfunction expansion method with the separation of variables technique as well as general Navier slip length boundary conditions. In Chapter 5, the fabrication processes of the micropatterned surfaces at different values of pillars/holes eccentricity and relative spacing are described in details. Furthermore, the detailed experimental procedures to fabricate and measuring the mean velocity of microchannels made from such hydrophobic surfaces will be described. Chapter 6 is dedicated to the results and discussions of the both wettability analysis of the micropatterned surfaces and drag-reduction performance of the hydrophobic microchannels. Wettability of such surfaces is analyzed by measuring the values of static and dynamic contact angles of single droplet. Results of changing the micropatterns eccentricity and relative spacing at different viewing directions are also presented. Finally, two simple yet effective methods to depict the corrugations of the three-phase contact line for anisotropic surfaces will be also proposed. Then, the results of the pressure driven flow in hydrophobic microchannels will be presented, and the relation between the microchannel slip length and micropatterned surface

wettability, in particular eccentricity, will be thoroughly discussed. Finally, the conclusion and summary of the key findings of this study are presented in Chapter 7. Recommendations on future work are also presented in this chapter.

Chapter 2 : Literature Review*

2.1 Introduction

One of the challenging issues in the domain of micro/nanofluidics is the proper formulation of liquid-solid interaction. Interpretations of the experimental results critically depend on the precise evaluation of the liquid close to the solid wall. Furthermore, deriving the final form of fluid flow in any channels is not possible unless the interactions at the liquid-solid interface, known as boundary conditions (BCs), are well defined. Also, wetting property of a surface represented by a contact angle is an important factor affecting the near-wall liquid flows. Experiments showed that the amount of slip on hydrophobic (low energy) surfaces is much higher than that on hydrophilic ones. Therefore, it suggests that there should be a relationship between the slip length (microscopic property) and wetting conditions (macroscopic property) of a surface. This chapter covers a review of different approaches used to analyze the wettability of roughness-induced hydrophobic surfaces as well as theoretical models describing the drag-reducing ability of such hydrophobic surfaces. This chapter is divided into five sections. In the first and second sections, isotropic and anisotropic wetting conditions of micropatterned

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- N. Kashaninejad, N.T. Nguyen, W.K. Chan, “Three-phase contact line shape and eccentricity effect of anisotropic wetting on hydrophobic surfaces”, *Soft Matter*, 2013, 9 (2), pp. 527-535, DOI: 10.1039/C2SM26963E
- N. Kashaninejad, N.T. Nguyen, W.K. Chan, “Eccentricity effects of microhole arrays on drag reduction efficiency of microchannels with a hydrophobic wall”, *Physics of Fluids* (American Institute of Physics-AIP Publisher), 2012, 24 (11), pp. 112004-18, DOI: 10.1063/1.4767539

surfaces are reviewed. A brief historical review will be presented in the third part regarding the concept of slip length. In the fourth section, parameters influencing the effective slip length of superhydrophobic surfaces will be discussed in details. The last section contains the summary and outlook of this chapter.

2.2 Review of the Available Works for Quantifying Isentropic Wetting Conditions of a Microstructured Surface

Evaluating the contact angle behavior of a solid surface is of great importance for many practical applications such as self-cleaning coatings, surface tension driven flow in micro-electro-mechanical-systems (MEMS), and drag reduction of film liquid flow in micro/nanochannels. It is well-known that micro/nano arrays of pillars on a surface can greatly affect the contact angle. The wetting properties of artificially roughened surface referred to here as micropatterned surface have been evaluated extensively in the literature [9-14]. Two extreme situations may occur on these micropatterned surfaces. First, the droplet may penetrate into the gap between the pillars known as noncomposite state; and second, the droplet may stay on top of the pillars enclosing air pockets termed as composite state. The noncomposite and the composite states were evaluated by Wenzel [9] and Cassie-Baxter [15], respectively. The contact angle was determined as a function of intrinsic contact angle as well as roughness geometry with parameters such as roughness factor and wetted area of solid fraction. Figure 2-1 illustrates the side view of a typical micropatterned surface with geometrical parameters such as width (top L , bottom l), base angles (top α , bottom β), height η , and spacing D . In the same figure, ζ depicts the height of an air gap underneath the liquid drop, which is 0 for noncomposite state and h for composite state.

Park et al. [16] reported that depending on the relative values of pillar spacing and heights, observed wetting states could be considered as mixed wetting state, i.e., $0 < \zeta/\eta < 1$, . The liquid-air meniscus could either be convex or concave for hydrophobic or hydrophilic pillars, respectively. Chen et al. [17] introduced a dimensionless number as a function of pillar height and size, and experimentally showed that a large apparent contact angle is achievable with this number larger than 4 and a small solid fraction. Öner and McCarthy [18] investigated the dynamic hydrophobicity of extensive micropatterned surfaces and reported that as long as the length scale of the square micropillars, with the equal spacing and size less than $32\mu\text{m}$, composite state with large contact angle and small contact angle hysteresis could be obtained regardless of the pillar height. They also observed that changing the pillar shape would affect the dynamic contact angle which is not predictable by the Cassie-Baxter theory. Extrand [19] introduced two criteria ensuring the composite state, namely, the contact line density and the asperity height. To identify the role of the factors determining the wettability condition of the micropatterned surface, Li and Amirfazli [20, 21] performed a comprehensive theoretical work based on free energy and free energy barrier of a metastable energy state, which was the simplified version of the three-dimensional approach previously proposed by Johnson and Detree [22]. The obtained analytical formula clearly explained how geometrical parameters of a pillar affect the wetting conditions [21]. Li et al. [23] extended the previous analysis for the trapezoidal micropatterns and concluded that pillar base angle α in Figure 2-1 was also important in determining the transition from the composite state to the non-composite one.

Figure 2-1: Illustration of micropatterned surfaces with the pillar parameters introduced in the literature.

The applicability of Cassie-Baxter and Wenzel theories were supported but in some cases questioned by various researchers over the years [24-37]. From the thermodynamic point of view, the contact angle on any surface can be determined by minimization of the Gibbs free energy, G , whose change can be formulated as follows [38] :

$$dG = \gamma_{SL}dA_{SL} + \gamma_{SV}dA_{SV} + \gamma_{LV}dA_{LV} + \Delta P dV - k dl \quad (2.1)$$

where γ denotes the interfacial surface energy and subscripts S, L and V correspond to solid, liquid and vapor phases, respectively. The change in volume of the drop dV , as a result of the change in the Laplace pressure (ΔP), is negligible for incompressible substances. The line tension or energy per unit length k is associated with a length increment in the three-phase contact line dl . The effect of changing the three-phase contact line has been neglected in derivations of the both Cassie-Baxter and Wenzel formula and is the main reason of the long dispute in the literature regarding these theories. For example, Chen et al. [39] demonstrated that surfaces with the same solid-liquid area fractions can have different contact angles which proved the importance of the three-phase contact line. Similarly, Extrand [40] showed that interactions

at the contact line and not the contact area control wetting of heterogeneous surfaces. Gao and McCarthy [28] also showed a large deviation between experimentally measured contact angles and those predicted by the Cassie-Baxter equation. Anantharaju et al. [41] experimentally compared two situations differing only in the microstructure of the roughness. Their results showed that the contact angle on surfaces with microholes was independent of the void fraction due to the continuity of the contact line.

2.3 Review of the Available Works for Quantifying Anisotropic Wetting Conditions of a Microstructured Surface

In contrast to isotropic surfaces, the contact angle varies along different viewing angles on anisotropic surfaces. Directional dependency of the contact angle known as anisotropic wetting is prevalent in nature such as allowing water droplet movement along the edge of rice, pandanus (screwpine) and bamboo leaves [42-44], and rolling off (pinning) of drop along (against) the radial outward direction of the central axis of the butterfly wings [45]. In general, asymmetry of either surface chemistry or surface morphology may lead to directional dependency of the contact angle. Accordingly, two methods can be devised to artificially fabricate anisotropic surfaces, namely one-dimensional (1D) and two/three-dimensional (2D/3D) methods. In 1D approach, periodic planar regions of alternating hydrophobic and hydrophilic strips are formed using chemical coatings on macroscopically smooth surfaces. The wettability of these surfaces was analyzed experimentally, theoretically and numerically [46-52]. In 2D/3D approach, rough surfaces with asymmetrical micro/nano protrusions are fabricated as a result of highly advanced micro/nano fabrication techniques such as optical/interference lithography, embossing and imprinting, wrinkling, etc, for 2D features [53, 54] and inclined/multi-photon lithography, metal assisted chemical etching, elastocapillary self-assembly, etc [54], for 3D geometries (defined as

high aspect ratio and variable vertical direction geometry such as zigzag and helical pillars). Moreover, by fabricating gradient surfaces whose chemical and/or topographical features can be gradually changed, unique anisotropic properties such as controlling the drop motion in a desired direction, could be obtained [55-57].

Because of its potential applications, attempts to identify the roles of key parameters influencing anisotropic wetting are of great interest for the research community. Directional dependency of the contact angle of different natural surfaces, such as rice leaf structures [58], and other synthetic microstructures, including checkboard-patterned surfaces [59], wrinkled surfaces [60, 61], ratchet-like topographical structures [62], elliptical silicon arrays [63] and stooped polymer nanohairs [64], and tilted polymer nanorod arrays [26], has been evaluated. Among many asymmetric microstructures, surfaces with periodic parallel grooves at different length scales, pitches, height, etc were extensively investigated in the literature [36, 57, 65-78]. Chen et al. [66] reported a semi-analytical and an experimental study of microgroove surfaces. Higher observable contact angle in the front view of the droplet compared to the side view was attributed to the squeezing and pinning of the droplet along the direction of the grooves and stretching across that direction. Neuhaus et al. [57] investigated microstructured surfaces consisting of both groove and square pillars with three different wetting conditions, namely hydrophobic, moderately hydrophilic and very hydrophilic. They found that on moderately hydrophilic groove substrate, a water droplet pinned to the structure and became elongated. The contact angle was considerably higher (up to 37°) along the viewing direction parallel to the groove as compared to that from the perpendicular direction. In the case of square micropatterns, the droplet shape deviated from being circular especially for the case of moderate hydrophilicity, and interestingly anisotropy was observed on those topographically symmetric structures. The

contact angle was higher (around 4° for the case of hydrophobic and 9° for the case of moderately hydrophilic conditions) along the edge as compared to the diagonal viewing direction of the square pillars. For both patterns in hydrophilic state, almost the same isotropic contact angle was reported. Long et al. [75] systematically investigated the static and dynamic wetting anisotropies of eight different translationally symmetric elongated ridges by gradually increasing the length of subsequent discrete ridge to finally form a continuous microridge structure. They reported that by splitting the continuous ridges into the discrete ones, the static and dynamic contact angle anisotropies substantially decreased, but did not disappear completely.

In all these previous works, the anisotropy of the contact angle has been measured along very limited viewing directions, mostly parallel and perpendicular directions. More importantly, directional dependency of the discrete square micropillar structure has not been yet evaluated systematically in the previous works.

2.4 Solid-liquid Interactions: Slip Length

In the preceding two sections, previous works regarding the wettability analysis of surfaces were reviewed. As explained before, slip flow can occur in microchannels fabricated from surfaces with low wetting conditions, hydrophobic surfaces. Therefore, there should be a correlation between the contact angle of the surfaces and slip length of the microchannels made from such surfaces. In this section, after briefly introducing the methods used to probe the slip in hydrophobic microchannels, the concept of slip length will be further elucidated.

Determining the interactions of liquid molecules adjacent to solid surface is still a challenging issue, and it is especially important in small scale domains. Experimentally these interactions, quantified by slip lengths, can be obtained by two methods: 1) indirect method, 2) direct or local method. To measure the slip length indirectly, the effective slip length is found by comparing the

value of the measured parameter with its value obtained theoretically using no-slip assumption. This measured parameter can be flow rate, viscous force, sedimentation velocity and potential difference in pressure driven flows, drainage force measurement (SFM/AFM), sedimentation and streaming potential systems respectively. In local methods, the velocity field near the solid surface is measured, and the slip velocity very close to the wall is obtained directly. There are three important techniques used for directly measuring the slip: 1) particle image velocimetry (PIV), 2) near field laser velocimetry (NFLV), and 3) fluorescence cross-correlation (FC). In the following parts after a brief historical review, the concept of slip length will be further explained through a rich source of experimental and analytical works.

2.4.1 No-slip Assumption

One of the famous assumptions which cannot be proved by the available principles is the assumption of no-slip BC, as shown in Figure 2-2(a) where u_b is the bulk flow velocity. This implies that the velocity of the molecules very close to the wall is the same as the solid wall. Based on a famous experiment, Bernoulli [79] in 1738 was among the first scientists who proposed the notion of no-slip BC. He states that velocity of the working fluid across the rigid channel decreases from a maximum at the center to zero at the solid wall. Having debated over this empirical assumption since 1810s, Stokes, commissioned by the Royal Academy of Science, finally declared the no-slip BC as the standard assumption which was closer to the experimental results. A thorough historical overview on this assumption is presented by Neto et al. [80].

2.4.2 Motionless Liquid Layer Hypothesis

Girard [81] brought up the idea of the existence of a motionless layer of liquid in the proximity of the wall which was allegedly responsible for occurrence of liquid slip. Wetting

properties of the solid surface, uniformity of its materials and wall curvature were the main factors affecting the thickness of this layer and the amount of slip [80]. This situation is shown in Figure 2-2(b).

2.4.3 Navier Slip Length

The idea of slip length was first proposed by Navier [82] in 1823. This model is among the most accepted constitutive equations. The model states that slip velocity at the wall, u_s , is directly proportional to the local shear rate, $\partial u / \partial y$. The constant of proportionality is called slip length, b , Figure 2-2(c). That is:

$$u_s \propto \frac{\partial u}{\partial y} \Rightarrow u_s = b \frac{\partial u}{\partial y} \quad (2.2)$$

Based on the values of slip length, three distinct situations are distinguishable. First, when slip length is zero, the boundary condition becomes the same as no-slip (stick). Second, for negative values of slip length, the so-called “locking” occurs [83]. In this case, bulk liquid layers and near-wall fluid layer are firmly joined together and move with the wall velocity [84]. The last situation which has many practical applications corresponds to the positive values of slip length. In that case, liquid partially slip on solid walls and the fluid resistance reduces, Figure 2-2(c).

Figure 2-2: Illustration of three solid-liquid interactions: a) No-slip BC. b) Motionless liquid layer. c) Navier slip length (redrawn with some changes from [80]).

2.5 Effective Slip Length and Drag Reduction on Microstructured Surfaces

Parameters influencing the surface wettability conditions can subsequently affect the slip length and the bulk flow resistance. In addition, pinning of the droplet to the surface heterogeneities may also affect the bulk flow friction factor. This pinning force, F_{pin} , acting like a frictional resistance, is related to the advancing, θ_a , and receding, θ_r , contact angles of the single droplet, $F_{pin} \propto (\cos \theta_r - \cos \theta_a)$, as shown in Figure 2-3. This shows the importance of evaluating the wetting conditions of the superhydrophobic surfaces in advance. As the size of the channel reduces, the flow resistance becomes higher. Therefore, techniques for minimizing the frictional drag in microchannels are extremely important for these applications. One of the passive techniques to decrease the flow frictional resistance is to pattern the microchannels with microstructured surfaces such as microgroove/rib and micropillar/hole arrays to reduce the effective solid-liquid contact area, shown in Figure 2-3. These surfaces are fabricated by incorporating roughness followed by coating with low-surface energy materials. In this situation, flow of liquid would be in Cassie-Baxter regime. In this case, intrinsic slip between molecules can be estimated via macroscopic experimental measurement. In such microchannels, the liquid is in contact with periodic no-slip and shear-free regions. Thus, the classical no-slip boundary condition (BC) is replaced by an effective slip velocity. The validity of this assumption was verified theoretically [85-87], experimentally [88-90], and numerically [91-93] in the literature.

Figure 2-3: Schematic configuration of BCs on a superhydrophobic surface

Having used conformal mapping method to solve Laplace equation with mixed boundary value problems, Philip [85] solved the flow over such surfaces with the shear-free region parallel to the applied pressure field, i.e. longitudinal configuration. The final form of the slip length can be summarized as:

$$b_{eff,||} = \frac{\mathcal{L}}{\pi} \ln(\sec(\delta_g \frac{\pi}{2})) \quad (2.3)$$

where $\mathcal{L}$ is period of shear-free regions and $\delta_g = D/\mathcal{L}$ respectively. Also for the case of transverse flow, Lauga and Stone [87] found the following expression for effective slip length:

$$b_{eff,\perp} = \frac{\mathcal{L}}{2\pi} \ln(\sec(\delta_g \frac{\pi}{2})) \quad (2.4)$$

provided that $\mathcal{L}$ approaches zero and δ_g is fixed. This relationship shows that slip length in longitudinal flow is twice the transverse flow. This indicates that the orientation of the superhydrophobic surface can affect the fluid flow resistance.

More general scaling of the slip length has been conducted by Ybert et al. [94] for micropatterned surfaces with both ribs and pillars. Their result can be expressed as follows:

$$u_s = b\dot{\gamma} \text{ (Navier slip)} \Rightarrow \dot{\gamma} = \frac{u_s}{b} \text{ \& } \tau = \mu\dot{\gamma} \Rightarrow \therefore \tau = \mu \frac{u_s}{b} \quad (2.5)$$

Assuming shear-free regions over the gaps (as a result of very small viscosity of the gases), shear stress on a micropatterned surface, with the pillar/rib width L as shown in Figure 2-3, can be scaled as $\tau = \phi_s \mu \bar{u}/L$ where ϕ_s is the solid-liquid area fraction, $\bar{u}$ is the average flow velocity, and $\bar{u}/L$ is the estimated value of shear rate for Stokes flow in this state. As the solid fraction decreases, slip velocity becomes closer to the average flow velocity. Thus, for a perfect slip model, $\phi_s \rightarrow 0$, u_s asymptotically approaches $\bar{u}$, and in this case shear stress can be scaled as:

$$\tau = \phi_s \mu \frac{\bar{u}}{L} \xrightarrow{\phi_s \rightarrow 0} \tau \sim \phi_s \mu \frac{u_s}{L} \quad (2.6)$$

Upon comparing Eq. (2.6) with Eq. (2.5), the related scale of slip length is obtained:

$$b \sim \lim_{\phi_s \rightarrow 0} \frac{L}{\phi_s} \quad (2.7)$$

For two different configurations of roughness on the micropatterned surfaces, namely micropillars as well as microribs, the general forms of the slip length become:

$$\left\{ \begin{array}{ll} b \sim \mathcal{L} & : \text{Microribs} \\ b \sim \frac{\mathcal{L}}{\sqrt{\phi_s}} & : \text{Micropillars} \end{array} \right. \quad (2.8)$$

For microribs, Eq. (2.8) shows that slip length depends more strongly on $\mathcal{L}$ and weakly depends on ϕ_s . This is in agreement with the above obtained scaling rule for slip length over micropatterned surfaces, where slip length was found as a logarithmic function of solid fraction, Eqs. (2.3) and (2.4).

Orientation effect of surface microstructures on the effective slip length was formulated by Bazant and Vinogradova [95]. They found that the general slip formula is a second rank tensor

where the effective slip length in the parallel, $b_{eff,\parallel}$, and perpendicular, $b_{eff,\perp}$, directions are eigenvalues of this general matrix, as indicated below.

$$b_{eff} = \begin{bmatrix} b_{eff,\parallel} \cos^2 \theta + b_{eff,\perp} \sin^2 \theta & (b_{eff,\parallel} - b_{eff,\perp}) \sin \theta \cos \theta \\ (b_{eff,\parallel} - b_{eff,\perp}) \sin \theta \cos \theta & b_{eff,\parallel} \sin^2 \theta + b_{eff,\perp} \cos^2 \theta \end{bmatrix} \quad (2.9)$$

This equation is capable of calculating effective slip length in any direction. Schematic view of such anisotropic surfaces is presented in Figure 2-4.

Figure 2-4: Schematic view of anisotropic texture on SH surfaces

Theoretical modeling of superhydrophobic microchannels was modified by taking into account the effect of liquid-water curvature by Sbragaglia and Prosperetti [96]. Belyaev and Vinogradova [97] considered a more realistic BC between liquid and gas interfaces. Shear-free BC is equivalent to an infinite amount of Navier slip length; however, from experimental observation, it can vary from 5 to 500 μm . This means that the shear-free assumption may overestimate the experimental results. The effective slip length of superhydrophobic microchannels with mixed no-slip and partial slip was formulated as follows [97]:

$$b_{eff,||} = \frac{\mathcal{L}}{\pi} \frac{\ln(\sec(\delta_g \frac{\pi}{2}))}{1 + \frac{\mathcal{L}}{\pi b} \ln(\sec(\delta_g \frac{\pi}{2}) + \tan(\delta_g \frac{\pi}{2}))} \quad \& \quad b_{eff,\perp} = \frac{b_{eff,||}}{2} \quad (2.10)$$

For the 2D micropillar superhydrophobic surfaces at small solid-liquid area fraction ϕ_s , Davis and Lauga [98] developed an analytical model to predict the effective slip length, $b_{eff,pillar}$.

This expression can be written as:

$$b_{eff,pillar} = \frac{\mathcal{L}}{2\pi} \left(\frac{3\pi}{8} \sqrt{\frac{\pi}{\phi_s}} - 3 \ln(1 + \sqrt{2}) \right) \quad (2.11)$$

This equation is in excellent agreement with the semi-numerical model developed by Ybert et al. [94] for micropillar configurations, Eq. (2.8).

Besides the analytical descriptions of slip length which were briefly explained here, there are many experimental works conducted on superhydrophobic microchannels. Watanabe et al. [99] used low-energy surfaces to reduce the drag of laminar flows of Newtonian fluids. A reduction of drag of 14% was observed for water flowing in a circular pipe with a diameter in the order of several millimeters. To date, no other researchers have reported such extent of drag reduction. Ou et al. [88] systematically fabricated different hydrophobic surfaces from silanized silicon whose static contact angles were in the range of 130° to 174° . The contact angle hysteresis was more than 25° . They reported a slip length of up to $25\mu m$ on these surfaces. Numerous studies were conducted to investigate the influence of different parameters of superhydrophobic surfaces on drag reduction. More details on these works were recently reviewed [84, 100, 101]. Wettability and consequently slip length are functions of the shape of microstructured surfaces. Aside from random roughness structures [132], previous works showed different values of slip length for flow in different directions (parallel or perpendicular) through microgrooves,

microribs [97], micropillars, and microholes [133]. These results suggest that slip length may also be a function of the microhole eccentricity. The effect of microhole eccentricity on static/dynamic contact angles as well as bulk frictional flow resistance has not been investigated in the literature, and is the main objective of this project. The drag reduction performance of the microchannels made of such hydrophobic microhole surfaces will be evaluated. The obtained results will be discussed in terms of relative velocity increase and the effective slip length, and will be compared with the available theoretical and numerical data in the literature.

2.6 Summary

In the literature, the effects of geometric parameters of microstructures on the slip length and drag- reduction efficiency of the hydrophobic microchannels are thoroughly investigated. Both analytical and experimental results suggest that the slip length should be independent of the microchannel height and can have the form of $b = f(\phi_s, \mathcal{L})$. These parameters also affect the wettability of the surface [102]. For instance, if the spacing of a micropillar array is large, liquid may penetrate into the cavities and the wetting condition shifts from Cassie-Baxter state into the Wenzel state leading to a decrease in the slip length. Therefore, parameters influencing the surface wettability conditions can subsequently affect the slip length and the bulk flow resistance. Also, pinning of the droplet to the surface heterogeneities may also affect the bulk flow friction factor. This pinning force was related to the contact angle hysteresis of the heterogeneous surfaces. Therefore, it seems that wetting conditions of the microstructured surfaces can greatly influence the slip flow and should be evaluated. Accordingly, in this chapter, approaches used in the literature to quantify the wettability of the micropatterned surfaces were first reviewed. Subsequently, a wide variety of analytical advances used to model the drag-

reduction ability of hydrophobic microchannels fabricated from micropatterned surfaces were also discussed.

In the next two chapters, theoretical modeling of microchannels will be presented by taking into consideration the effects of slip BCs and finite width-to-height aspect ratio of the channel. To experimentally evaluate the slip flow in microchannels, first the wettability analysis of the channel surface walls will be analyzed by introducing a new parameter termed as eccentricity. Finally, the drag-reduction capability of the hydrophobic microchannels fabricated from such micropatterned surfaces will be investigated, and the relationship between slip length of the microchannel and surface wettability will be evaluated.

Chapter 3 : Analytical Modeling of Slip flow in Parallel-Plate

Microchannels*

3.1 Slip Velocity in Parallel-Plate Microchannels

In previous works, it is always assumed that dominating BCs in liquid flows through microchannels are either no-slip or equal slip conditions. In this section, the general form of Newtonian liquid flow in two parallel-plate microchannel which consists of two different walls with unequal wettability and surface condition is considered. This situation may occur frequently in microchannel fabrication, since fabricating microchannels with a single material is not always possible. In this chapter, it is assumed that the width of the channel is much larger than its height so that effect of side walls can be neglected. That means, a high aspect ratio microchannel is investigated. Schematic view of such microchannel is illustrated in Figure 3-1.

Figure 3-1: Schematic view of the channel with slip at both walls

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As the flow enters the channel, the velocity also varies in the streamwise direction, i.e. $u = f(y, z)$, until it passes the entrance length, l_e , and the flow becomes unidirectional, independent of the streamwise direction. In this state, the velocity profile is known to be fully developed. If the channel length, l , is much larger than its hydraulic diameter, D_h , the developing region can be negligible. It is commonly accepted that if $l/D_h > 0.065 \text{ Re}$, where Re is the Reynolds number, the flow can be considered fully developed. Therefore, fully developed and unidirectional flow dominates in long, $l \uparrow$, microchannels, $D_h \downarrow$, at low Reynolds number, $\text{Re} \downarrow$.

Considering fully developed laminar flow for Newtonian liquids with constant properties at low Reynolds number, the Navier-Stokes (N-S) equation can be simplified to Stokes equation. Solving this ordinary differential equation, the velocity distribution in terms of BCs can be calculated as follows:

$$\frac{d^2 u}{dy^2} = 1/\mu \frac{dP}{dz} \Rightarrow u(y) = 1/2\mu \frac{dP}{dz} y^2 + C_1 y + C_2 \quad (3.1)$$

Corresponding Navier slip BCs at both walls are:

$$\begin{cases} u(y = +h) = -b^+ \left. \frac{du}{dy} \right|_{y=+h} \\ u(y = -h) = b^- \left. \frac{du}{dy} \right|_{y=-h} \end{cases} \quad (3.2)$$

Imposing these BCs dictates:

$$\begin{cases} 1/2\mu \frac{dP}{dz} h^2 + C_1 h + C_2 = -b^+ \left(1/\mu \frac{dP}{dz} h + C_1 \right) \\ 1/2\mu \frac{dP}{dz} h^2 - C_1 h + C_2 = b^- \left(-1/\mu \frac{dP}{dz} h + C_1 \right) \end{cases} \quad (3.3)$$

Further simplifying the first BC, leads to:

$$C_2 = -\frac{1}{2\mu} \frac{dP}{dz} h(b^+ + b^- + h) + \frac{1}{2}(b^- - b^+)C_1 \quad (3.4)$$

Also, the second BC indicates that:

$$\therefore C_1 = 1/\mu \frac{dP}{dz} h \frac{(b^- - b^+)}{(2h + b^+ + b^-)} \quad (3.5)$$

Substituting C_1 into the expression of C_2 results in:

$$\therefore C_2 = 1/2\mu \left(-\frac{dP}{dz}\right) h \left[\frac{3hb^+ + 3hb^- + 2h^2 + 4b^+ b^-}{2h + b^+ + b^-} \right] \quad (3.6)$$

Replacing these constants in Eq. (3.1) leads to:

$$\begin{aligned} u(y) = & 1/2\mu \frac{dP}{dz} y^2 + 1/\mu \frac{dP}{dz} h \frac{(b^- - b^+)}{(2h + b^+ + b^-)} y \\ & + 1/2\mu \left(-\frac{dP}{dz}\right) h \left[\frac{3hb^+ + 3hb^- + 2h^2 + 4b^+ b^-}{2h + b^+ + b^-} \right] \end{aligned} \quad (3.7)$$

Upon additional simplifications and substituting: $H = 2h$, we can rewrite the obtained Eq. (3.7)

as:

$$\begin{aligned} u(y) = & 1/\mu \left(-\frac{dP}{dz}\right) H^2 \left[-\frac{y^2}{2H^2} - \frac{(b^- - b^+)}{(H + b^+ + b^-)} \frac{y}{2H} \right. \\ & \left. + \frac{1}{8H} \frac{3Hb^+ + 3Hb^- + H^2 + 8b^+ b^-}{H + b^+ + b^-} \right] \end{aligned} \quad (3.8)$$

3.1.1 Dimensionless Form of Velocity Profile

By denoting:

$$u^* = \frac{u}{1/\mu \left(-\frac{dP}{dz}\right) H^2} \quad (3.9)$$

$$\frac{y}{H} = y^* \quad (3.10)$$

$$\frac{b}{H} = \beta \quad (3.11)$$

Eq. (3.8) can be written in dimensionless form as:

$$u^*(y^*) = -\frac{1}{2}y^{*2} - \frac{(\beta^- - \beta^+)}{(1 + \beta^+ + \beta^-)}y^* + \frac{1}{8} \left[\frac{3\beta^+ + 3\beta^- + 8\beta^+ \beta^- + 1}{\beta^+ + \beta^- + 1} \right] \quad (3.12)$$

3.1.2 Average Velocity in General Form

In the case of two-wall slip, it is also desirable to calculate the average flow velocity. In particular, the average velocity is measurable experimentally. The average velocity becomes:

$$\begin{aligned} \bar{u} &= \frac{1}{2h} \int_{-h}^h u(y) dy \\ &= \frac{1}{2h} \int_{-h}^h \frac{1}{\mu} \left(-\frac{dP}{dz} \right) h^2 \left[-\frac{y^2}{2h^2} - \frac{(b^- - b^+)}{(2h + b^+ + b^-)} \frac{y}{h} \right. \\ &\quad \left. + \frac{1}{2h} \frac{3hb^+ + 3hb^- + 2h^2 + 4b^+ b^-}{2h + b^+ + b^-} \right] dy \end{aligned} \quad (3.13)$$

After some mathematical manipulations, the average velocity can be expressed as:

$$\bar{u} = \frac{1}{2h} \left[\frac{1}{\mu} \left(-\frac{dP}{dz} \right) h^2 \left[\frac{4h^3 + 8h^2b^- + 8h^2b^+ + 12hb^+ b^-}{3h(2h + b^+ + b^-)} \right] \right] \quad (3.14)$$

The final form of the average velocity in terms of the total channel height is formulated in Eq.

(3.15):

$$\therefore \bar{u} = \frac{1}{\mu} \left(-\frac{dP}{dz} \right) \left[\frac{H^3 + 4H^2b^- + 4H^2b^+ + 12Hb^+ b^-}{12(H + b^+ + b^-)} \right] \quad (3.15)$$

For the limited case where the intrinsic slip on the upper wall is negligible, $b^+ = 0$, the above equation simplifies to:

$$\bar{u}_{slip} = \frac{1}{\mu} \left(-\frac{dP}{dz} \right) \left[\frac{H^3 + 4H^2b^-}{12(H + b^-)} \right] \quad (3.16)$$

Denoting $\beta^- = b^- / H$, for the case of existing slip on the bottom wall, slip velocity becomes:

$$\bar{u}_{slip} = 1/12\mu \left(-\frac{dP}{dz}\right)H^2 \left(\frac{1 + 4\beta^-}{1 + \beta^-}\right) \quad (3.17)$$

where the first term is the no-slip velocity profile traditionally obtained by Hagen-Poiseuille.

3.1.3 Dimensionless Average Velocity

It is more relevant to normalize the average velocity by $1/\mu (-dP/dz)H^2$ as follows:

$$\therefore \bar{u}^* = \frac{\bar{u}}{1/\mu \left(-\frac{dP}{dz}\right)H^2} = \frac{1 + 4\beta^- + 4\beta^+ + 12\beta^+ \beta^-}{12(1 + \beta^+ + \beta^-)} \quad (3.18)$$

3.2 Determination of Slip Flow Rate

Similar to the average velocity, the volumetric flow rate can also be computed as:

$$\therefore \frac{Q}{W} = 1/\mu \left(-\frac{dP}{dz}\right)H \left[\frac{H^3 + 4H^2b^- + 4H^2b^+ + 12Hb^+b^-}{12(H + b^+ + b^-)}\right] \quad (3.19)$$

Alternatively, the dimensionless flow rate in the microchannel can be written as:

$$Q^* = \frac{Q}{1/\mu \left(-\frac{dP}{dz}\right)H^3W} = \frac{1 + 4\beta^- + 4\beta^+ + 12\beta^+ \beta^-}{12(1 + \beta^+ + \beta^-)} \quad (3.20)$$

which is the same as non-dimensional average velocity.

3.3 Determination of Shear Stress and Friction Factor with Slip at the Wall

The shear stress can be calculated as [103] :

$$\tau = \mu \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right] = \mu \frac{\partial u}{\partial y} \quad (3.21)$$

By substituting this velocity distribution, shear stress becomes:

$$\therefore \tau = \mu \frac{\partial u}{\partial y} = \left(\frac{dP}{dz}\right) H^2 \left[\frac{y}{H^2} + \frac{(b^- - b^+)}{(H + b^+ + b^-)} \frac{1}{2H} \right] \quad (3.22)$$

It is common to define an average wall shear stress as follows:

$$\bar{\tau}_w = \frac{1}{\Gamma} \int_0^\Gamma \tau_w ds \quad (3.23)$$

where Γ is the perimeter of the channel and ds is the element of the arc length.

It is possible to relate the average wall shear stress to the required pressure gradient in the channel. Force balance indicates that:

$$dz \int_0^\Gamma \tau_w ds = -A dP \quad (3.24)$$

Upon replacing the average wall shear stress, i.e. $\int_0^\Gamma \tau_w ds = \bar{\tau}_w \Gamma$, the above equation simplifies to:

$$\bar{\tau}_w = \left(- \frac{dP}{dz} \right) \cdot \frac{A}{\Gamma} \quad (3.25)$$

By comparing the above equation with $\bar{\tau}_w$ of a circular cross section with diameter D_c , i.e.

$\bar{\tau}_w = (-dP/dz) \cdot D_c/4$, the equivalent diameter for non-circular channel could be defined as:

$$D_h = \frac{4A}{\Gamma} \quad (3.26)$$

In a parallel-plate microchannel, it is possible to write the hydraulic diameter in term of channel height and aspect ratio, $\alpha = W/H$ as:

$$D_h = \frac{4(H \cdot W)}{2(H + W)} = \frac{2H(W)}{H(1 + W/H)} = \frac{2\alpha H}{\alpha + 1} \quad (3.27)$$

In addition, the relationship between the wall shear stress and pressure gradient becomes:

$$\bar{\tau}_w = \left(- \frac{dP}{dz} \right) \cdot \frac{D_h}{4} \quad (3.28)$$

In the present analysis, the relationship between the pressure gradient $(-dP/dz)$ and the average velocity was obtained, Eq. (3.15). Considering Eq. (3.28) and replacing the pressure gradient from Eq. (3.15) and hydraulic diameter from Eq. (3.27), the relationship between the average wall shear stress $\bar{\tau}_w$ and average velocity $\bar{u}$ becomes as follows:

$$\therefore \bar{\tau}_w = \frac{6\mu \alpha H(H + b^+ + b^-)}{(H^3 + 4H^2b^- + 4H^2b^+ + 12H b^+ b^-)(\alpha + 1)} \bar{u} \quad (3.29)$$

Dimensionless shear stress is traditionally called the friction coefficient, C_f , in which shear stress is non-dimensioned by dynamic pressure, $1/2 \rho \bar{u}^2$, as:

$$C_f = \frac{\bar{\tau}_w}{1/2 \rho \bar{u}^2} \quad (3.30)$$

Also, it is customary to define another dimensionless friction factor as:

$$f = \frac{8\bar{\tau}_w}{\rho \bar{u}^2} \quad (3.31)$$

where f is called Darcy friction factor. It is evident that f is four times in magnitude larger than friction factor C_f .

If we substitute the obtained relationship between the average wall shear stress and the pressure gradient:

$$f = \frac{8 \left(- \frac{dP}{dz} \right) \cdot D_h / 4}{\rho \bar{u}^2} \quad (3.32)$$

which can be written as:

$$f = \frac{\left(- \frac{dP}{dz} \right) \cdot D_h}{1/2 \rho \bar{u}^2} \quad (3.33)$$

In most references, e.g. [104], Darcy friction factor appears once in the following equation:

$$\Delta P = f \frac{L}{D_h} \frac{\rho \bar{u}^2}{2} \quad (3.34)$$

Replacing $(-dP/dz) = \Delta P/L$ in Eq. (3.33), here the aforementioned equation can be thoroughly proved.

Also, if we define Reynolds number based on the hydraulic diameter, i.e. $Re = \rho \bar{u} D_h / \mu$, Eq. (3.33) becomes:

$$\therefore f = \frac{\left(-\frac{dP}{dz}\right) \cdot D_h^2}{\frac{1}{2} \mu \bar{u}} \times \frac{1}{Re} \quad (3.35)$$

Additionally, from Eq. (3.15), the relationship between the pressure drop and the average velocity in the microchannel consists of different slip conditions at the top and the bottom walls can be explicitly obtained, as:

$$\left(-\frac{dP}{dz}\right) = \frac{12\mu (H + b^+ + b^-)}{H^3 + 4H^2b^- + 4H^2b^+ + 12H b^+ b^-} \bar{u} \quad (3.36)$$

By substituting this equation into Eq. (3.22), the friction coefficient becomes:

$$\therefore f = \frac{24 (H + b^+ + b^-)}{H^3 + 4H^2b^- + 4H^2b^+ + 12H b^+ b^-} \cdot \frac{D_h^2}{Re} \quad (3.37)$$

In Eq. (3.37) by replacing the hydraulic diameter in terms of channel height and aspect ratio, i.e.,

$D_h = 2\alpha H / (\alpha + 1)$, Eq. (3.37) becomes:

$$f = \frac{24 (H + b^+ + b^-)}{H^3 + 4H^2b^- + 4H^2b^+ + 12H b^+ b^-} \cdot \frac{4\alpha^2 H^2}{(\alpha + 1)^2} \cdot \frac{1}{Re} \quad (3.38)$$

In term of dimensionless slip, i.e., $\beta = b/H$ one can get:

$$\therefore f = \frac{96}{Re} \frac{\alpha^2(1 + \beta^+ + \beta^-)}{(1 + 4\beta^- + 4\beta^+ + 12\beta^+\beta^-)(\alpha + 1)^2} \quad (3.39)$$

Equivalently, we can also define another dimensionless number by multiplying Darcy friction factor to Reynolds number, i.e., Poiseuille number, Po :

$$Po = f \cdot Re = 96 \frac{(1 + \beta^+ + \beta^-)}{(1 + 4\beta^- + 4\beta^+ + 12\beta^+\beta^-)(\alpha + 1)^2} \quad (3.40)$$

The width of the channel is much larger than its height, that is:

$$W \gg H \Rightarrow \frac{W}{H} \gg 1 \rightarrow \alpha \gg 1 \quad (3.41)$$

The friction factor calculated from Eq. (3.39) can only be used when the aspect ratio is much larger than unity. Sometimes it is completely omitted from the friction factor relation in the case of parallel-plate channels. However, it is retained here for completeness sake. In other words, the obtained formulae are valid for liquid flow through parallel-plate microchannels both for large and relatively moderate aspect ratios.

So far, the general expressions describing the velocity, flow rate as well as friction factor in parallel-plate microchannels were derived in terms of the yet-unknown general slip lengths at both top and bottom channel walls. In the following part, we try to relate this unknown Navier slip length to the known geometrical parameters of the channel by considering the realistic models causing the slip velocity at the micron scale.

3.4 Apparent Slip in Hydrophobic Microchannel

As it was explained before, one possible reason for the violation of no-slip BC could be the existence of a low-viscosity region very close to the wall that fluid slip on it, as it is called the

apparent slip. Fundamental difference between apparent and true slip, usually explained by Navier slip length, is shown in Figure 3-2.

Figure 3-2: Two possible mechanism of slip over low-energy surfaces [105].

In this section, the apparent slip can be justified due to the existence of a uniform layer of air gap and/or low-viscosity region between bulk liquid flow and solid (non-permeable) surfaces of microchannels whose widths and lengths are much larger than their height (depth). This model is schematically shown in Figure 3-3.

Figure 3-3: two-phase flow in parallel-plate hydrophobic microchannel. Air gap or low viscosity region can explain possible non-sticky behavior of the fluid near the wall.

3.4.1 Trapped air model

In this case, it is assumed that air re-circulates in the vicinity of the wall and the relationship between air region pressure drop and liquid flow can be found from the fact that net flow rate of air flow is zero. The problem is further simplified by assuming: (1) No gravity and no mixing; (2) Air-liquid interface is a straight line. In this case, the BC between gas and wall is assumed to be no-slip. Additionally since both liquid and gas are assumed to be continuum; Eq. (3.1) can be applied separately for each phase subject to following BCs:

$$BC's: \begin{cases} y = 0: \begin{cases} u_{air} = u_{liquid} \\ \tau_{nt air} = \tau_{nt liquid} \Rightarrow \mu_{liquid} \frac{\partial u_{liquid}}{\partial y} = \mu_{air} \frac{\partial u_{air}}{\partial y} \end{cases} \\ y = -\delta : u_{air} = 0 \\ y = h : u_{liquid} = 0 \end{cases} \quad (3.42)$$

At the interface ($y = 0$), the velocity and shear stress are assumed to be continuous.

The liquid velocity profile can be calculated as:

$$u_{liquid} = 1/\mu_{liquid} \left(-\frac{dP}{dz} \right)_{liquid} h^2 \left(-1/2 \left(\frac{y}{h} \right)^2 + \frac{y}{h} + 1/4 \mu_r \xi \right) \quad (3.43)$$

where $\mu_r = \mu_{liquid}/\mu_{air}$ and $\xi = \delta/h$.

Following Navier hypothesis, the effective slip length was calculated as follows:

$$b_{eff} = 1/4 \mu_r \delta \quad (3.44)$$

This equation indicates that the effective slip length is related to the viscosity ratio between the liquid and the gas, and the height ratio of the air gap and the liquid flow.

3.4.2 Near Wall Reduced-Viscosity Model

This approach was first adopted by Tretheway and Meinart [106] who considered the existence of a uniform gas layer near hydrophobic surfaces at both continuum and rarefied gas flow conditions. The main difference between this model and the previous one lies in the pressure drop condition. In this model, known as reduced viscosity model, it is assumed that near wall flows with the same pressure drop as that of the liquid flow. Similar to the velocity profile of the previous case of liquid, the effective slip length can be calculated by the following equations:

$$u_{liquid} = \frac{h^2}{2\mu_{liquid}} \left(-\frac{dP}{dz} \right)_{liquid} \left(-\frac{y^2}{h^2} + \frac{2y}{h} + 2\xi\mu_r + \mu_r\xi^2 \right) \quad (3.45)$$

$$b_{eff} = \mu_r \left(\frac{\delta^2}{2h} + \delta \right) \quad (3.46)$$

Both Eq. (3.44) and Eq. (3.46) depend directly on the thickness of air/low-viscosity δ . Previous results have established the slip lengths without any estimation of the air gap thickness. This limitation will be addressed below by providing some expressions for the air-gap thickness.

3.4.3 Calculation of Air-gap Thickness for Flow at Continuum Region

($Kn \leq 10^{-3}$)

Under the same assumptions of the previous section, due to the small air viscosity, we can assume that at the interface, the shear stress is zero, hence the BCs become:

$$BC's: \begin{cases} y = -\delta : & u_{air} = 0 \\ y = 0 : & \frac{\partial u_{liquid}}{\partial y} = \frac{\partial u_{air}}{\partial y} = 0 \\ y = 2h : & u_{liquid} = 0 \end{cases} \quad (3.47)$$

By imposing these 4 boundary conditions, the 4 unknowns of air and fluid velocity profiles can be calculated. Air velocity profile becomes:

$$u_{air}(y) = 1/2\mu_{air} \left(-\frac{dP}{dz} \right)_{air} \delta^2 \left[1 - \frac{y^2}{\delta^2} \right] \quad (3.48)$$

For the liquid section, similarly we can find:

$$u_{liquid}(y) = 2/\mu_{liquid} \left(-\frac{dP}{dz} \right)_{liquid} h^2 \left[1 - \frac{y^2}{4h^2} \right] \quad (3.49)$$

Now, if we assume no velocity jump at the interface, it requires that:

$$u_{liquid}(y = 0) = u_{air}(y = 0) \quad (3.50)$$

Substituting Eq. (3.49) and Eq. (3.48) into Eq. (3.50) results:

$$\therefore 2/\mu_{liquid} \left(\frac{dP}{dz} \right)_{liquid} h^2 = -1/2\mu_{air} \delta^2 \left(\frac{dP}{dz} \right)_{air} \quad (3.51)$$

For equal pressure gradient at both phases (reduced-viscosity approach), the above equation simplifies to:

$$4/\mu_r h^2 = \delta^2 \quad (3.52)$$

Hence, the thickness of the reduced viscosity zone must be:

$$\delta = \sqrt{\frac{4h^2}{\mu_r}} \quad (3.53)$$

It is noted that the above equation is valid for the Knudsen number less than 0.001.

$$Kn = \frac{\ell}{\delta} \Rightarrow \delta = \frac{\ell}{Kn} \quad (3.54)$$

At sea level, the mean free path of air, ℓ , is in the order of $0.1\mu m$

$$\delta_{min} = \frac{0.1\mu m}{0.001} = 100\mu m \quad (3.55)$$

Hence the minimum allowable height of microchannel in this case is H_{min} , ($H = 2h$):

$$H_{min} = \sqrt{\frac{\mu_{liquid}}{\mu_{air}}} \delta \cong 700\mu m \quad (3.56)$$

3.4.4 Calculation of Air-gap Thickness for Flow at Slip Region ($10^{-3} \leq Kn \leq 10^{-1}$)

In this case, slip exists between air molecules and the solid surface. To quantify the amount of slip of gases, Maxwell first used the general Navier slip model as follows:

$$@ \text{ wall: } u_{air} - u_{solid} = \epsilon \frac{\partial u_{air}}{\partial y} \xrightarrow{\text{stationary wall}} u_{air} = \epsilon \frac{\partial u_{air}}{\partial y} \quad (3.57)$$

We will discuss the amount of slip length for air and solid surface (ϵ) but it is presently assumed to be a known parameter. Then the BCs become:

$$BC's: \begin{cases} y = -\delta & : & u_{air} = \epsilon \frac{\partial u_{air}}{\partial y} \\ y = 0 & : & \frac{\partial u_{liquid}}{\partial y} = \frac{\partial u_{air}}{\partial y} = 0 \\ y = 2h & : & u_{liquid} = 0 \end{cases} \quad (3.58)$$

By imposing these 4 BCs, the corresponding velocity profile for air and liquid can be found:

$$u_{air}(y) = \frac{1}{2\mu_{air}} \left(\frac{dP}{dz} \right)_{air} y^2 + C_5 \quad (3.59)$$

$$\therefore C_5 = -\frac{1}{2\mu_{air}} \left(\frac{dP}{dz} \right)_{air} \delta^2 \left[1 + 2 \frac{\epsilon}{\delta} \right] \quad (3.60)$$

Finally, air velocity profile is obtained:

$$u_{air}(y) = -1/2\mu_{air} \left(\frac{dP}{dz} \right)_{air} \delta^2 \left[1 + 2\frac{\epsilon}{\delta} - \frac{y^2}{\delta^2} \right] \quad (3.61)$$

For the liquid velocity, the equation remains the same as before:

$$u_{liquid}(y) = -2/\mu_{liquid} \left(\frac{dP}{dz} \right)_{liquid} h^2 \left[1 - \frac{y^2}{4h^2} \right] \quad (3.62)$$

Again, if we assume no velocity jump between air and liquid at the interface, it requires that:

$$-1/2\mu_{air} \left(\frac{dP}{dz} \right)_{air} \delta^2 \left[1 + 2\frac{\epsilon}{\delta} \right] = -2/\mu_{liquid} \left(\frac{dP}{dz} \right)_{liquid} h^2 \quad (3.63)$$

For equal pressure gradient at both phases (reduced-viscosity approach), the two pressure drops should be the same, so Eq. (3.63) simplifies to:

$$\delta^2 + 2\epsilon\delta - \frac{4}{\mu_r}h^2 = 0 \quad (3.64)$$

Eq. (3.64) is a quadratic equation for δ , therefore, the air-gap height can be computed as:

$$\delta = -\epsilon \pm \sqrt{\epsilon^2 + \frac{4}{\mu_r}h^2} \quad (3.65)$$

Because the air-gap cannot be negative, the negative sign before the square root is not acceptable:

$$\delta = -\epsilon + \sqrt{\epsilon^2 + \frac{4}{\mu_r}h^2} \quad (3.66)$$

If we assume no-slip at the wall, as in the previous case, the same formula describing the air-gap height can be obtained, i.e Eq. (3.53):

$$\delta_{\epsilon \rightarrow 0} = 0 + \sqrt{0 + \frac{4}{\mu_r}h^2} = \sqrt{\frac{4h^2}{\mu_r}} \quad (3.67)$$

The maximum allowable Knudsen number in this case should not exceed 0.1, it requires that:

$$\delta_{min} = \frac{\ell}{(Kn)_{max}} \frac{0.1\mu m}{0.1} = 1\mu m \quad (3.68)$$

From Eq. (3.66), the height of the channel ($2h$) can be found in terms of gap-height:

$$2h = \sqrt{\mu_r \left[1 + 2 \frac{\epsilon}{\delta} \right]} \delta \quad (3.69)$$

To find the minimum height of the channel, we need to find the typical value of ϵ . Accordingly, the Maxwell equation can be used:

$$\epsilon = \left(\frac{2 - \sigma}{\sigma} \right) \left(\frac{2}{3} \ell \right) \quad (3.70)$$

where σ is the tangential momentum accommodation coefficient. This parameter is a function of the solid material, gas type and surface roughness. The experiments suggest that σ should range from 0.5 to 1. The maximum value $\sigma = 1$ is typical for most surfaces, while the minimum is for very smooth surface with roughness height in the order of nanometers.

Therefore, σ can be approximated as unity, so the slip length of air and solid surface becomes:

$$\epsilon_{air} = \left(\frac{2}{3} \ell \right) \xrightarrow{\ell=0.1\mu m} \epsilon_{air} = 0.0667\mu m \quad (3.71)$$

Finally, the minimum height of the channels becomes:

$$(2h)_{min} = \sqrt{\mu_r \left[1 + 2 \frac{\epsilon}{\delta} \right]} \delta \cong 7 \times \sqrt{1 + \frac{2 \times 0.0667\mu m}{1\mu m}} \times 1\mu m \approx 7.523\mu m \quad (3.72)$$

Therefore, the range of the channel height for the model to be valid is:

$$7.5\mu m < H < 700\mu m \quad (3.73)$$

which corresponds to an air-gap in the range of:

$$1\mu m < \delta < 100\mu m \quad (3.74)$$

According to the obtained results, not only the estimated values of the air-gap thickness were determined, but the related range of the channel height was also estimated. It was found that when the channel height is larger than $700\mu m$, air flow is in continuum regime and no-slip BC can be used. For the case where the channels height is smaller than $700\mu m$ and larger than $7.5\mu m$, slip BC should be used to model the air flow in the channel. Finally, for the channel with the height smaller than $7.5\mu m$, the N-S equation cannot be used to model the air flow and molecular-based approaches should be used. The key findings of this section can be summarized in the Table 3-1.

Table 3-1: summary of obtained analytical results of hydrophobic microchannels with reduced viscosity gas region assumption

	Continuum Region	Slip Region
Effective slip length	$b_{eff} = \varepsilon \left(\frac{\delta^2}{2h} + \delta \right)$	$b_{eff} = \varepsilon \left(\frac{\delta^2}{2h} + \delta \right)$
Air gap thickness	$\delta = \sqrt{\frac{4h^2}{\mu_r}}$	$\delta = -\varepsilon + \sqrt{\varepsilon^2 + \frac{4h^2}{\mu_r}}$
Range of air gap	$\delta \geq 100\mu m$	$1\mu m < \delta < 100\mu m$
Channel size	$H \geq 700\mu m$	$7.5\mu m < H < 700\mu m$

3.5 Summary

This chapter presented analytical modeling of liquid flow in parallel-plate microchannels, and can be divided in two parts. In the first part, classical relationships describing velocity, flow rate, pressure gradient, and shear stress were extended to the more general cases where there exist two different values of yet-unknown slip length at the top and bottom walls of the channel. In the second part, the emphasis was on the quantification of the slip length. Generating mechanism of slip was attributed to the existence of a low-viscosity region between the liquid and the solid surface. Two different cases were considered based on the pressure gradient: a) re-circulating air

region, where the net flow rate of gas phase was zero; b) reduced viscosity gas region, in which pressure gradient was assumed to be the same at both phases. It was found that the slip length in both cases is directly proportional to the ratio of liquid-to-gas viscosity as well as the air gap thickness. Therefore, it is necessary to estimate the air gap thickness to quantify the value of slip length. Unlike other studies which used empirical values of air gap, here, equations describing this parameter were derived based on the range of Knudsen number for the case of reduced viscosity region.

In this chapter, the width of the channel was assumed to be much larger than its height so that the effect of side walls on velocity profile was negligible. In the next chapter, liquid flow will be evaluated in a rectangular microchannel by taking into account the effect of aspect ratio of the channel.

Chapter 4 : Analytical and Numerical Investigation of Flow in Rectangular Microchannels*

4.1 Introduction

For fully developed incompressible flow in microchannels at low Reynolds number, partial differential equation (PDE) of the momentum equation simplifies to the classic Poisson equation [103]. Although classical 1D analysis of Hagen-Poiseuille for cylindrical pipe is usually adapted to solve noncircular cross-section microchannels by considering the equivalent hydraulic diameter [107], analytical modeling for rectangular cross-section microchannels needs 2D analysis [103, 108, 109]. Because of the complexity of this approach, most previous research works merely focused on high aspect ratio microchannel, which is in fact the flow between parallel plates. One of the drawbacks of such analysis is that the effect of the side walls is ignored. From an experimental point of view, difference may occur due to the materials used as the side walls which affect the wettability conditions and consequently affect the accuracy of the measured velocity and friction factor.

Two dimensional solution of Poisson equation is similar to the theory of elasticity. Therefore, the 2D velocity profile was obtained by an analogy with the torsion problem in elasticity using

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- N. Kashaninejad, W.K. Chan, N.T. Nguyen, “Fluid Mechanics of Flow through Rectangular Hydrophobic Microchannels”, ASME Conference Proceedings 2011(44632), pp 647-655.

Lagrange stress function [110] and presented by Knudsen and Katz [111], Berker [112]. These models were well summarized by Wang [113] for different channel cross sections. In the obtained series solutions for the rectangular channels, the variables of the lateral coordinates were expressed by two different functions, i.e., one variable with hyperbolic function and the other one with trigonometric function. Spiga and Morino [109] used finite Fourier transform to find the velocity profile of the finite aspect ratio channels and verified their results by comparing the obtained friction factors with those presented in the literature. Additionally, approximate solutions for the channels with side walls effect also exist. Knowing the maximum velocity, Purday [114] proposed an approximate expression to calculate the average velocity of the channels with the aspect ratio greater than 2. Using finite difference method for channels with the aspect ratio greater than 3, Natarajan and Lakshmanan [115] presented another approximate solution for the velocity field. Savino and Siegel [116] also suggested an approximate series solution for the channels with the aspect ratio from 1 to ∞ . The other exact series solutions of the Poisson equation were also proposed [117, 118]. Even though the experimental results verified the accuracy of these models, the convergences of these series solutions were slow and had many computational complexities. Moreover, the lateral coordinates in all, except the solution presented by Spiga and Morino [109], of these classical solutions of the Poisson equation describing the velocity profile of the finite aspect ratio channels were in the asymmetric form with different numerical exponents. Furthermore, to relate the frictional losses to the average velocity in the channels, classical Darcy-Weisbach equation is used. Subsequently, in the literature (see for example a recent review by Dey et al. [119]) friction factor and Poiseuille number are multiplied by Hartnett–Kostic correction factor to take into account the effect of the microchannels aspect ratio.

In the present chapter, new analytical forms of velocity distribution, Darcy friction factor and Poiseuille number in laminar liquid flow in rectangular microchannels were derived using the eigenfunction expansion technique [120]. The effects of changing the aspect ratio from 0 to ∞ are discussed. Using finite element method (FEM), the obtained analytical for the case of no-slip BCs are further compared with the numerical simulations of COMSOL Multiphysics for microchannels with different values of aspect ratio and pressure gradient.

4.2 Rectangular Microchannel with No-Slip Boundary Conditions

Velocity distributions of a creeping flow in a rectangular microchannel are derived. This type of flow is dominant at small length scale, low velocity or for very viscous fluid. By considering a hydrophilic channel walls, the boundary conditions are the homogeneous Dirichlet boundary conditions. Yet, the governing equation is still non homogeneous and the classical method of separation of variables cannot be used.

In order to solve this type of problem, the method of eigenfunction expansion can be used corresponding to the homogeneous boundary conditions and non homogeneous linear governing PDE equation [120]. The schematic of the problem is depicted in Figure 4-1. It is common that the origin of coordinates describing the velocity distribution is chosen at the center of the channel (Figure 4-1(a)) but it is more convenient to transform it to the bottom corner of the channel, (Figure 4-1(c)), the new coordinates X and Y become:

$$X = x + w \qquad Y = y + h$$

Figure 4-1: Schematic of the model: a) rectangular cross-section of the channel with the flow velocity at z direction (normal to the page) and original coordinate located at the middle of the channel. b) Related boundary conditions (no-slip) at the walls and Stokes governing equation. c) Transformation of the coordinate to X and Y .

The momentum equation in the stream-wise direction becomes:

$$\frac{\partial^2 u}{\partial X^2} + \frac{\partial^2 u}{\partial Y^2} = 1/\mu_w \frac{\partial P}{\partial z} \quad (4.1)$$

where u and $\partial P/\partial z$ is the velocity and pressure gradient in the streamwise direction (i.e., z) respectively. Since $\partial P/\partial z$ and fluid viscosity, μ_w , are constant across the channel cross section, we denote them with a single variable say, F , that is,

$$F = 1/\mu_w \frac{\partial P}{\partial z} \Rightarrow \frac{\partial^2 u}{\partial X^2} + \frac{\partial^2 u}{\partial Y^2} = F \quad (4.2)$$

Subject to the following boundary conditions:

$$BC's \begin{cases} u(0, Y) = 0 & ; & u(2w, Y) = 0 \\ u(X, 0) = 0 & ; & u(X, 2h) = 0 \end{cases} \quad (4.3)$$

If we can find the function (say, ψ) by which the Laplacian operator can be simplified to the operation, i.e., $\nabla^2(\psi) = \lambda \cdot \psi$, then we can find the solution to the problem. In mathematical terminology, such function is called eigenfunction with the corresponding λ as the eigenvalue. In

such problem, such eigenfunction should also satisfy the boundary conditions. 2D form of the solution can be written as follows:

$$u(X, Y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{mn} \sin\left(\frac{m\pi}{2w} X\right) \sin\left(\frac{n\pi}{2h} Y\right) \quad (4.4)$$

where A_{mn} is a constant needed to be determined. By substituting this form of the solution into the governing Eq. (4.2), it results:

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} -A_{mn} \left\{ \left(\frac{m\pi}{2w}\right)^2 + \left(\frac{n\pi}{2h}\right)^2 \right\} \sin\left(\frac{m\pi}{2w} X\right) \sin\left(\frac{n\pi}{2h} Y\right) = F \quad (4.5)$$

Considering the above equation as the double Fourier sinusoidal series expansion of F , we can find the as-yet unknown A_{mn} :

$$A_{mn} = \frac{-4}{(2w)(2h) \left\{ \left(\frac{m\pi}{2w}\right)^2 + \left(\frac{n\pi}{2h}\right)^2 \right\}} \int_0^{2h} \int_0^{2w} F \sin\left(\frac{m\pi}{2w} X\right) \sin\left(\frac{n\pi}{2h} Y\right) dX dY \quad (4.6)$$

Since F is not function of neither X nor Y , it comes out of the integration and due to independency of each integrand, they can be treated separately as a single integration:

$$\int_0^{2w} \sin\left(\frac{m\pi}{2w} X\right) dX = \left(\frac{2w}{m\pi}\right) [1 - \cos m\pi] \quad (4.7)$$

$$\int_0^{2h} \sin\left(\frac{n\pi}{2h} Y\right) dY = \left(\frac{2h}{n\pi}\right) [1 - \cos n\pi] \quad (4.8)$$

Therefore, the A_{mn} becomes:

$$A_{mn} = \frac{-4F}{\left\{ \left(\frac{m\pi}{2w}\right)^2 + \left(\frac{n\pi}{2h}\right)^2 \right\}} \left(\frac{1}{m\pi}\right) \left(\frac{1}{n\pi}\right) [1 - \cos m\pi] [1 - \cos n\pi] \quad (4.9)$$

By introducing the aspect ratio as:

$$\alpha = \frac{2w}{2h} \quad (4.10)$$

The half width of the channel can be expressed as:

$$w = \alpha h \quad (4.11)$$

Hence,

$$A_{mn} = \frac{-16 F \alpha^2 h^2}{\{(m\pi)^2 + (\alpha n\pi)^2\}} \left(\frac{1}{m\pi}\right) \left(\frac{1}{n\pi}\right) [1 - \cos m\pi][1 - \cos n\pi] \quad (4.12)$$

By replacing Eq. (4.12) into Eq. (4.4) we obtain:

$$u(X, Y) = \frac{-16 F \alpha^2 h^2}{\pi^4} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi][1 - \cos n\pi]}{m n (m^2 + \alpha^2 n^2)} \sin\left(\frac{m\pi}{2\alpha h} X\right) \sin\left(\frac{n\pi}{2h} Y\right) \quad (4.13)$$

By replacing the (X, Y) with the original (x, y) the final form of the velocity profile becomes:

$$u(x, y) = \frac{-16 F \alpha^2 h^2}{\pi^4} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi][1 - \cos n\pi]}{m n (m^2 + \alpha^2 n^2)} \sin\left(\frac{m\pi}{2\alpha h} (x + w)\right) \sin\left(\frac{n\pi}{2h} (y + h)\right) \quad (4.14)$$

By introducing the dimensionless velocity u^* [103] :

$$u^*(x, y) = \frac{u(x, y)}{\frac{\left(-\frac{\partial P}{\partial z}\right) h^2}{\mu_w}} = \frac{u(x, y)}{-F h^2} \quad (4.15)$$

Eq. (4.15) will have the final form as follows:

$$u^*(x, y) = \frac{16 \alpha^2}{\pi^4} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi][1 - \cos n\pi]}{m n (m^2 + \alpha^2 n^2)} \sin\left(\frac{m\pi}{2\alpha h} (x + w)\right) \sin\left(\frac{n\pi}{2h} (y + h)\right) \quad (4.16)$$

Figure 4-2 illustrates the variations of the normalized velocity distribution with respect to the aspect ratio, i.e. Eq. (4.16). The results show that by increasing the aspect ratio, velocity profile becomes more parabolic and the effects of side walls on velocity distributions become negligible. That means, the classical Hagen-Poiseuille equation is the limiting case of the obtained 2D analytical formula when aspect ratio approaches infinity.

Figure 4-2: Variations of the normalized velocity (Eq. (4.16)) at different values of aspect ratio: (a) $\alpha = 1$. (b) $\alpha = 3$. (c) $\alpha = 5$. (d) $\alpha = 10$. (e) $\alpha = 20$. (f) $\alpha = 30$.

Also the flow rate can be calculated as:

$$Q = \int_0^{2h} \int_0^{2w} u(X, Y) dX dY \quad (4.17)$$

By substituting Eq. (4.13) into Eq. (4.17), we will have:

$$Q = \int_0^{2h} \int_0^{2w} \frac{-16 F \alpha^2 h^2}{\pi^4} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sin\left(\frac{m\pi}{2\alpha h} X\right) \sin\left(\frac{n\pi}{2h} Y\right) dX dY \quad (4.18)$$

By considering Eqs. (4.7) and (4.8), then Eq. (4.18) becomes:

$$Q = \frac{-16 F \alpha^2 h^2}{\pi^4} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi][1 - \cos n\pi]}{m n (m^2 + \alpha^2 n^2)} \left(\frac{2w}{m\pi}\right) \left(\frac{2h}{n\pi}\right) [1 - \cos n\pi][1 - \cos m\pi] \quad (4.19)$$

Finally, by further simplifying, the flow rate can be expressed by Eq. (4.20):

$$Q = \frac{64 \left(-\frac{\partial P}{\partial z}\right) \alpha^3 h^4}{\mu_w \pi^6} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)} \quad (4.20)$$

The solution for the two infinitely large parallel-plate channels separated at a small distance should be similar to a rectangular channel when its width is much larger than its height. The flow rate for one dimensional channel with hydrophilic wall can be obtained as:

$$Q_{\alpha \rightarrow \infty} = -\frac{4F\alpha h^4}{3} \quad (4.21)$$

By normalizing Eq. (4.20) with Eq. (4.21), we obtain:

$$\frac{Q}{Q_{\alpha \rightarrow \infty}} = \frac{48\alpha^2}{\pi^6} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)} \quad (4.22)$$

It can be shown that Eq. (4.31) approaches unity as the value of aspect ratio increases confirming the accuracy of the obtained 2D flow rate for rectangular microchannel.

Furthermore, the average velocity can be calculated as:

$$Q = \bar{u} \times A \Rightarrow \bar{u} = \frac{Q}{4 \alpha h^2} \quad (4.23)$$

Substituting Eq. (4.20) in Eq. (4.23) results:

$$\bar{u} = \frac{16 \left(-\frac{\partial P}{\partial z}\right) \alpha^2 h^2}{\mu_w \pi^6} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)} \quad (4.24)$$

Moreover, Hagen-Poiseuille average velocity was formulated as:

$$\bar{u}_{H-P} = \frac{h^2}{3\mu_w} \left(-\frac{\partial P}{\partial z} \right) \quad (4.25)$$

Hence, average velocity can be expressed as:

$$\bar{u} = \frac{h^2 \left(-\frac{\partial P}{\partial z} \right)}{3\mu_w} \frac{48\alpha^2}{\pi^6} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)} \quad (4.26)$$

which results in the following final form:

$$\bar{u} = \bar{u}_{H-P} \phi(\alpha) \quad (4.27)$$

where:

$$\phi(\alpha) = \frac{48\alpha^2}{\pi^6} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)} \quad (4.28)$$

Furthermore, the Darcy friction factor f can be written as:

$$f = \frac{2 \left(-\frac{\partial P}{\partial z} \right) \cdot D_h}{\rho \bar{u}^2} \quad (4.29)$$

Multiplying both sides by Reynolds number, $Re = \rho \bar{u} D_h / \mu_w$, results:

$$f = \frac{\left(-\frac{\partial P}{\partial z} \right) \cdot D_h^2}{\frac{1}{2} \mu_w \bar{u}} \times \frac{1}{Re} \quad (4.30)$$

where hydraulic diameter D_h is:

$$D_h = \frac{4(2h \cdot 2w)}{2(2h + 2w)} = \frac{4h(w)}{h(1 + w/h)} = \frac{4\alpha h}{\alpha + 1} \quad (4.31)$$

Substituting Eq. **Error! Reference source not found.** and Eq. (4.31) into Eq. (4.30) results:

$$f = \frac{\left(-\frac{\partial P}{\partial z}\right) \cdot \left(\frac{4\alpha h}{\alpha+1}\right)^2}{\frac{1}{2}\mu_w \left(\frac{16 \left(-\frac{\partial P}{\partial z}\right) \alpha^2 h^2}{\mu_w \pi^6} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)} \right)} \frac{1}{Re} \quad (4.32)$$

Further simplifying the above equation results:

$$f = \frac{2 \pi^6}{\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)} (\alpha + 1)^2} \frac{1}{Re} \quad (4.33)$$

Equivalently, we can also define another dimensionless number by multiplying Darcy friction factor to Reynolds number which is usually called the Poiseuille number, Po :

$$Po = \frac{2 \pi^6}{\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)} (\alpha + 1)^2} \quad (4.34)$$

Similarly, Poiseuille number can be written as:

$$Po = 96 \eta(\alpha) \quad (4.35)$$

where:

$$\eta(\alpha) = \frac{\pi^6}{48 (\alpha + 1)^2 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)}} \quad (4.36)$$

To further illustrate the effect of aspect ratio on frictional losses, Poiseuille number is plotted as a function of aspect ratio in Figure 4-3. Generally, Poiseuille number increases with increasing the aspect ratio. For very large aspect ratio, the value of Poiseuille number approaches a value of 96 which corresponds to the value for parallel-plate channels. Generally, it can be concluded for the laminar flow at the same Reynolds number, square cross-section channels ($\alpha = 1$) generates

the lowest frictional losses and as the channel cross-section deviates from being square shape, frictional losses increases (up to 74%).

Figure 4-3: Poiseuille number Po vs. aspect ratio α .

4.3 Rectangular Microchannel with Constant Slip at the Wall

In this case, slip is assumed to exist only at the bottom plate (i.e. non symmetry case) while the three remaining walls are hydrophilic so that no-slip BC can be applied. The domain of this type of the problem is shown in the Figure 4-4.

Figure 4-4: Rectangular microchannel with constant slip velocity at wall

Since the governing equation is still linear, we can take advantage of superposition principle and decompose the domain into two separate ones and solve each domain independently. Then by summing up each solution, the final solution of the desired domain would be obtained. The procedure is shown in Figure 4-5:

$$\begin{array}{c}
 \begin{array}{|c|} \hline \mathbf{v} = \mathbf{0} \\ \hline \mathbf{v} = \mathbf{0} \quad \boxed{\nabla^2 \mathbf{v} = \nabla P / \mu} \quad \mathbf{v} = \mathbf{0} \\ \hline \mathbf{v} = \mathbf{v}_s \\ \hline \end{array} \\
 (a)
 \end{array}
 =
 \begin{array}{c}
 \begin{array}{|c|} \hline \mathbf{v} = \mathbf{0} \\ \hline \mathbf{v} = \mathbf{0} \quad \boxed{\nabla^2 \mathbf{v} = \nabla P / \mu} \quad \mathbf{v} = \mathbf{0} \\ \hline \mathbf{v} = \mathbf{0} \\ \hline \end{array} \\
 (b)
 \end{array}
 +
 \begin{array}{c}
 \begin{array}{|c|} \hline \mathbf{v} = \mathbf{0} \\ \hline \mathbf{v} = \mathbf{0} \quad \boxed{\nabla^2 \mathbf{v} = 0} \quad \mathbf{v} = \mathbf{0} \\ \hline \mathbf{v} = \mathbf{v}_s \\ \hline \end{array} \\
 (c)
 \end{array}$$

Figure 4-5: Required procedure for obtaining velocity distribution in a microchannel with constant velocity at bottom wall

The first part (Figure 4-5(b)) has already been solved in the previous section by the so-called eigenfunction expansion method. The second part is Laplace equation in rectangular domain with non homogeneous Dirichlet BCs. Therefore, for this type of the problem, the solution has one additional term as a result of the existence of slip at the wall.

To solve the Laplace equation, we can use the separation of variables because it is a linear homogeneous partial differential equation.

$$u(X, Y) = f(X) \cdot g(Y) \xrightarrow{\nabla^2 v = 0} f''g + g''f = 0 \Rightarrow \frac{f''}{f} = -\frac{g''}{g} = \text{Constant} \quad (4.37)$$

To avoid trivial solution according to the boundary conditions, the constant must be negative (say, $-\lambda^2$) and the two obtained ordinary differential equations become:

$$f'' + \lambda^2 f = 0 \Rightarrow f(X) = a_1 \cos \lambda X + a_2 \sin \lambda X \quad (4.38)$$

Imposing $f(0) = 0 \Rightarrow a_1 = 0$ and $f(2w) = 0 \Rightarrow \sin 2\lambda w = \sin k\pi \Rightarrow \lambda = \frac{k\pi}{2w}$.

The other ordinary differential equation (ODE) becomes:

$$g'' - \lambda^2 g = 0 \Rightarrow g(Y) = a_3 \cosh \lambda(2h - Y) + a_4 \sinh \lambda(2h - Y) \quad (4.39)$$

Imposing $g(2h) = 0 \Rightarrow a_3 = 0$.

Finally the velocity profile can be written as:

$$u(X, Y) = \sum_{k=1}^{\infty} A_k \sin \frac{k\pi}{2w} X \sinh \frac{k\pi}{2w} (2h - Y) \quad (4.40)$$

To find the only unknown A_k , we impose the last remaining slip boundary condition at the bottom wall:

$$u(X, 0) = \mathbf{v}_s \Rightarrow \sum_{k=1}^{\infty} A_k \sinh \frac{k\pi}{2w} (2h) \sin \frac{k\pi}{2w} X = \mathbf{v}_s \quad (4.41)$$

Therefore, the coefficient $A_k \sinh \frac{k\pi}{2w} (2h)$ can be the Fourier sine coefficient of $\mathbf{v}_s$ on the interval $0 < X < 2w$:

$$A_k = \frac{1}{\sinh \frac{k\pi}{2w} (2h)} \int_0^{2w} \mathbf{v}_s \sin \frac{k\pi}{2w} X \, dX \quad (4.42)$$

Since $\mathbf{v}_s$ is constant, the above equation becomes:

$$A_k = \frac{\mathbf{v}_s}{w \sinh \left[k\pi \frac{h}{w} \right]} \frac{2w}{k\pi} [1 - \cos k\pi] = \frac{2\mathbf{v}_s}{\pi} \frac{[1 - \cos k\pi]}{k \sinh \left[k\pi \frac{h}{w} \right]} \quad (4.43)$$

The final form of the velocity distribution becomes:

$$u(X, Y) = \frac{2\mathbf{v}_s}{\pi} \sum_{k=1}^{\infty} \frac{[1 - \cos k\pi]}{k \cdot \sinh \left[k\pi \frac{h}{w} \right]} \sin \frac{k\pi}{2w} X \sinh \frac{k\pi}{2w} (2h - Y) \quad (4.44)$$

By replacing $X = x + w$ and $Y = y + h$, the velocity distribution in (x, y) coordinates becomes:

$$u(x, y) = \frac{2\mathbf{v}_s}{\pi} \sum_{k=1}^{\infty} \frac{[1 - \cos k\pi]}{k \cdot \sinh \left[k\pi \frac{h}{w} \right]} \sin \frac{k\pi}{2w} (x + w) \sinh \frac{k\pi}{2w} (h - y) \quad (4.45)$$

The flow rate in this case can be expressed as:

$$Q = \int_0^{2h} \int_0^{2w} \frac{2\mathbf{v}_s}{\pi} \sum_{k=1}^{\infty} \frac{[1 - \cos k\pi]}{k \cdot \sinh \left[k\pi \frac{h}{w} \right]} \sin \frac{k\pi}{2w} X \sinh \frac{k\pi}{2w} (2h - Y) dX dY \quad (4.46)$$

After integration and further simplification, flow rate becomes:

$$Q = \frac{4\mathbf{v}_s w}{\pi^2} \sum_{k=1}^{\infty} \frac{[1 - \cos k\pi]^2}{k^2} \left[\frac{\cosh \left[\frac{k\pi h}{w} \right] - 1}{\sinh \left[\frac{k\pi h}{w} \right]} \right] \quad (4.47)$$

The velocity profile for the entire domain can be found by summing Eq. (4.14) and Eq. (4.45):

$$\begin{aligned} u_{total}(x, y) = & \frac{16 \left(-\frac{dP}{dz} \right) \alpha^2 h^2}{\mu \pi^4} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi][1 - \cos n\pi]}{mn(m^2 + \alpha^2 n^2)} \sin \left[\frac{m\pi}{2\alpha h} (x \right. \\ & \left. + \alpha h) \right] \sin \left[\frac{n\pi}{2h} (y + h) \right] \\ & + \frac{2\mathbf{v}_s}{\pi} \sum_{k=1}^{\infty} \frac{[1 - \cos k\pi]}{k \cdot \sinh \left[\frac{k\pi}{\alpha} \right]} \sin \left[\frac{k\pi}{2\alpha h} (x + \alpha h) \right] \sinh \left[\frac{k\pi}{2\alpha h} (h - y) \right] \end{aligned} \quad (4.48)$$

Also, total flow rate, $Q_{total} = Q_{No-slip} + Q_{slip}$, becomes:

$$\begin{aligned} Q_{total} = & \frac{64 h w^3}{\mu \pi^6} \left(-\frac{dP}{dz} \right) \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)} \\ & + \frac{4\mathbf{v}_s w}{\pi^2} \sum_{k=1}^{\infty} \frac{[1 - \cos k\pi]^2}{k^2} \left[\frac{\cosh \left[\frac{k\pi h}{w} \right] - 1}{\sinh \left[\frac{k\pi h}{w} \right]} \right] \end{aligned} \quad (4.49)$$

Rearranging, it yields the relationship for slip velocity at the wall:

$$\mathbf{v}_s = \frac{\pi^2}{4w} \frac{(Q_{total} - \frac{64 h w^3}{\mu \pi^6} (-\frac{dP}{dz}) \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)})}{\sum_{k=1}^{\infty} \frac{[1 - \cos k\pi]^2}{k^2} \left[\frac{\cosh \left[\frac{k\pi h}{w} \right] - 1}{\sinh \left[\frac{k\pi h}{w} \right]} \right]} \quad (4.50)$$

Or in terms of aspect ratio, it is:

$$\mathbf{v}_s = \frac{\pi^2}{4\alpha h} \frac{(Q_{total} - \frac{64 \alpha^3 h^4}{\mu \pi^6} (-\frac{dP}{dz}) \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1 - \cos m\pi]^2 [1 - \cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)})}{\sum_{k=1}^{\infty} \frac{[1 - \cos k\pi]^2}{k^2} \left[\frac{\cosh \left[\frac{k\pi}{\alpha} \right] - 1}{\sinh \left[\frac{k\pi}{\alpha} \right]} \right]} \quad (4.51)$$

4.4 General Navier Slip Boundary Conditions

In this section, the general Navier slip BCs are assumed at the solid-liquid interfaces:

$$BC's \begin{cases} u(0, Y) = b_s^- \frac{\partial u}{\partial X} \Big|_{X=0} & ; & u(2w, Y) = -b_s^+ \frac{\partial u}{\partial X} \Big|_{X=2w} \\ u(X, 0) = b_w^- \frac{\partial u}{\partial Y} \Big|_{Y=0} & ; & u(X, 2h) = -b_w^+ \frac{\partial u}{\partial Y} \Big|_{Y=2h} \end{cases} \quad (4.52)$$

The domain of this type of the problem is shown in the Figure 4-6.

Figure 4-6: Rectangular microchannel with general Navier slip length BCs at the walls

The slip length b can be obtained from different analytical models as discussed in Chapter 2.

Table 4-1 summarized the most commonly accepted models of slip length.

Table 4-1: Analytical formulas of effective slip length of different superhydrophobic surfaces

Slip Length Formula	Applicable Geometry	Reference
$b = \frac{\mathcal{L}}{\pi} \ln \left(\sec \left(\phi_g \frac{\pi}{2} \right) \right)$	Periodic 1D strips of no-slip and no-shear BCs parallel to the flow direction	[85, 86]
$b = \frac{\mathcal{L}}{2\pi} \ln \left(\sec \left(\phi_g \frac{\pi}{2} \right) \right)$	Periodic 1D strips of no-slip and no-shear BCs perpendicular to the flow direction	[87]
$b = \frac{\mathcal{L}}{2\pi} \left(\frac{3\pi}{8} \sqrt{\frac{\pi}{1 - \phi_g}} - 3 \ln(1 + \sqrt{2}) \right)$	Square arrays of 2D circular micropillars ($\phi_g \rightarrow 1$)	[98]

Here, both PDE and BCs are non-homogeneous. Specifically, BCs are the so-called Robin BC which is the combination of the function and its derivatives. Therefore, we try the eigenfunction as the combination of sine and cosine terms, that is:

$$\psi(X, Y) = F(X)G(Y) = [C_1 \sin(\mu X) + C_2 \cos(\mu X)] [C_3 \sin(\nu Y) + C_4 \cos(\nu Y)] \quad (4.53)$$

Hence the velocity profile in this general case can be written as:

$$u(X, Y) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} A_{ij} [C_1 \sin(\mu_i X) + C_2 \cos(\mu_i X)] [C_3 \sin(\nu_j Y) + C_4 \cos(\nu_j Y)] \quad (4.54)$$

First derivative of this velocity distribution with respect to X becomes:

$$\frac{\partial u}{\partial X} = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} A_{ij} [\mu_i C_1 \cos(\mu_i X) - \mu_i C_2 \sin(\mu_i X)] [C_3 \sin(v_j Y) + C_4 \cos(v_j Y)] \quad (4.55)$$

Similarly, the derivative with respect to Y reads as:

$$\frac{\partial u}{\partial Y} = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} A_{ij} [v_j C_3 \cos(v_j Y) - v_j C_4 \sin(v_j Y)] [C_1 \sin(\mu_i X) + C_2 \cos(\mu_i X)] \quad (4.56)$$

By imposing the first BC, $u(0, Y) = b_s^- \frac{\partial u}{\partial X} \Big|_{X=0}$, we have:

$$\begin{aligned} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} A_{ij} [0 + C_2] [C_3 \sin(v_j Y) + C_4 \cos(v_j Y)] \\ = b_s^- \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} A_{ij} [\mu_i C_1 - 0] [C_3 \sin(v_j Y) + C_4 \cos(v_j Y)] \end{aligned} \quad (4.57)$$

Simplifying Eq. (4.57), results:

$$\therefore C_2 = b_s^- \mu_i C_1 \quad (4.58)$$

By imposing the second BC, $u(2w, Y) = -b_s^+ \frac{\partial u}{\partial X} \Big|_{X=2w}$, we can get:

$$\begin{aligned} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} A_{ij} [C_1 \sin(2\mu_i w) + C_2 \cos(2\mu_i w)] [C_3 \sin(v_j Y) + C_4 \cos(v_j Y)] \\ = -b_s^+ \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} A_{ij} [\mu_i C_1 \cos(2\mu_i w) \\ - \mu_i C_2 \sin(2\mu_i w)] [C_3 \sin(v_j Y) + C_4 \cos(v_j Y)] \end{aligned} \quad (4.59)$$

Further simplifying results:

$$\tan(2w\mu_i) = \frac{(C_2 + b_s^+ \mu_i C_1)}{(b_s^+ C_2 - C_1)} \quad (4.60)$$

Replacing Eq. (4.58) into the above equation results:

$$\therefore \tan(2w\mu_i) = \frac{\mu_i(b_s^- + b_s^+)}{(b_s^+ b_s^- \mu_i^2 - 1)} \quad (4.61)$$

Similarly, by imposing the other two BCs one can obtain the following expressions:

$$\therefore C_4 = b_w^- v_j C_3 \quad (4.62)$$

And:

$$\therefore \tan(2h v_j) = \frac{v_j(b_w^- + b_w^+)}{(b_w^+ b_w^- v_j^2 - 1)} \quad (4.63)$$

Eqs. (4.61) and (4.63) should be solved numerically to obtain the values of μ_i and v_j . For a special case where the slip lengths are zero, similar results as described in Section 4.1 can be obtained. For instance, let us assume $b_s^- = b_s^+ = b_w^+ = 0$, then Eq. (4.61) becomes:

$$\tan(2w\mu_i) = 0 \Rightarrow 2w\mu_i = i\pi \Rightarrow \mu_i = \frac{i\pi}{2w} \quad (4.64)$$

Also, if the bottom wall of the microchannel is fabricated from Periodic 1D strips of no-slip and no-shear BCs parallel to the flow direction, from Table 4-1, the effective slip at the bottom wall reads as:

$$b_w^- = \frac{\mathcal{L}}{\pi} \ln \left(\sec \left(\phi_g \frac{\pi}{2} \right) \right) \quad (4.65)$$

In that case, Eq. (4.63) becomes:

$$\tan(2hv_j) = \frac{v_j(b_w^- + b_w^+)}{(b_w^+ b_w^- v_j^2 - 1)} = -b_w^- v_j \quad (4.66)$$

To guess the approximate solutions, the graph of the functions on the both sides of Eq. (4.66) should be plotted, Figure 4-7:

Figure 4-7: Finding the roots of Eq. (4.66) graphically ($2h = 27\mu\text{m}$).

Using the numerical method for the channel with $127\mu\text{m}$ height, the first six solutions can be approximated as: $v_1 = 0.0241$, $v_2 = 0.0482$, $v_3 = 0.0723$, $v_4 = 0.0965$, $v_5 = 0.1206$, $v_6 = 0.1448$. It can be seen that unlike Fourier series in the no-slip solutions, the arguments of sine and cosine in the general solution with Navier slip boundary conditions are not periodic.

Replacing Eq. (4.58) and Eq. (4.62) into Eq. (3.56) and denoting $B_{ij} = A_{ij}C_1C_3$, then:

$$u(X, Y) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} B_{ij} [\sin(\mu_i X) + b_s^- \mu_i \cos(\mu_i X)] [\sin(v_j Y) + b_w^- v_j \cos(v_j Y)] \quad (4.67)$$

After differentiating twice from the above equation with respect to X , it reads:

$$\begin{aligned} \frac{\partial^2 u}{\partial X^2} = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} B_{ij} [-\mu_i^2 \sin(\mu_i X) - b_s^- \mu_i^3 \cos(\mu_i X)] [\sin(v_j Y) \\ + b_w^- v_j \cos(v_j Y)] \end{aligned} \quad (4.68)$$

And differentiating Eq. (4.67) twice with respect to Y results:

$$\begin{aligned} \frac{\partial^2 u}{\partial Y^2} = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} B_{ij} [-v_j^2 \sin(v_j Y) - b_w^- v_j^3 \cos(v_j Y)] [\sin(\mu_i X) \\ + b_s^- \mu_i \cos(\mu_i X)] \end{aligned} \quad (4.69)$$

Substituting Eq. (4.68) and Eq. (4.69) into Eq. (4.2) reads as:

$$\begin{aligned} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} -(\mu_i^2 + v_j^2) B_{ij} [\sin(\mu_i X) + b_s^- \mu_i \cos(\mu_i X)] [\sin(v_j Y) \\ + b_w^- v_j \cos(v_j Y)] = F \end{aligned} \quad (4.70)$$

Multiplying both sides of the above equation to $[\sin(\mu_{i'} X) + b_s^- \mu_{i'} \cos(\mu_{i'} X)] [\sin(v_{j'} Y) + b_w^- v_{j'} \cos(v_{j'} Y)]$ and integrating results:

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} -B_{ij} (\mu_i^2 + v_j^2) I_{ij i' j'} = F I_{i' j'} \quad (4.71)$$

where

$$\begin{aligned}
I_{iji'j'} = \int_0^{2h} \int_0^{2w} [\sin(\mu_i X) + b_s^- \mu_i \cos(\mu_i X)] [\sin(v_j Y) \\
+ b_w^- v_j \cos(v_j Y)] [\sin(\mu_{i'} X) + b_s^- \mu_{i'} \cos(\mu_{i'} X)] [\sin(v_{j'} Y) \\
+ b_w^- v_{j'} \cos(v_{j'} Y)] dX dY
\end{aligned} \quad (4.72)$$

And:

$$\begin{aligned}
I_{i'j'} = \int_0^{2h} \int_0^{2w} [\sin(\mu_{i'} X) + b_s^- \mu_{i'} \cos(\mu_{i'} X)] [\sin(v_{j'} Y) \\
+ b_w^- v_{j'} \cos(v_{j'} Y)] dX dY
\end{aligned} \quad (4.73)$$

Using the orthogonal property:

$$\begin{aligned}
\int_0^{2h} \int_0^{2w} [\sin(\mu_i X) + b_s^- \mu_i \cos(\mu_i X)] [\sin(v_j Y) + b_w^- v_j \cos(v_j Y)] [\sin(\mu_{i'} X) \\
+ b_s^- \mu_{i'} \cos(\mu_{i'} X)] [\sin(v_{j'} Y) + b_w^- v_{j'} \cos(v_{j'} Y)] dX dY \\
= 0 \text{ if } (i \neq i' \text{ \& } j \neq j')
\end{aligned} \quad (4.74)$$

$I_{iji'j'}$ can be calculated as:

$$\begin{aligned}
\int_0^{2h} \int_0^{2w} [\sin(\mu_i X) + b_s^- \mu_i \cos(\mu_i X)]^2 [\sin(v_j Y) + b_w^- v_j \cos(v_j Y)]^2 dX dY \\
= W_i H_j
\end{aligned} \quad (4.75)$$

where W_i and H_j can be found from Eq. (3.73) and Eq. (3.74), respectively:

$$W_i = \left(w(1 + \mu_i^2 b_s^{-2}) + b_s^- \sin^2(2\mu_i w) + \frac{1}{4\mu_i} \sin(4\mu_i w)(\mu_i^2 b_s^{-2} - 1) \right) \quad (4.76)$$

$$H_j = \left(h(1 + v_j^2 b_w^{-2}) + b_w^- \sin^2(2v_j h) + \frac{1}{4v_j} \sin(4v_j h)(v_j^2 b_w^{-2} - 1) \right) \quad (4.77)$$

Also, $I_{i'j'}$ becomes:

$$\begin{aligned} I_{i'j'} &= \int_0^{2h} \int_0^{2w} [\sin(\mu_i X) + b_s^- \mu_i \cos(\mu_i X)] [\sin(v_j Y) + b_w^- v_j \cos(v_j Y)] dX dY \\ &= \beta_i^s \beta_j^w \end{aligned} \quad (4.78)$$

where:

$$\beta_i^s = \frac{1}{\mu_i} [1 - \cos(2\mu_i w)] + b_s^- \sin(2\mu_i w) \quad (4.79)$$

And:

$$\beta_j^w = \frac{1}{v_j} [1 - \cos(2v_j h)] + b_w^- \sin(2v_j h) \quad (4.80)$$

Finally Eq. (4.71) can be simplified to:

$$\therefore B_{ij} = \frac{-F}{W_i H_j (\mu_i^2 + v_j^2)} \beta_i^s \beta_j^w \quad (4.81)$$

Eventually by replacing Eq. (4.81) into Eq. (4.67), velocity profile can be obtained:

$$\begin{aligned} u(X, Y) &= -F \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{\beta_i^s \beta_j^w}{W_i H_j (\mu_i^2 + v_j^2)} [\sin(\mu_i X) + b_s^- \mu_i \cos(\mu_i X)] [\sin(v_j Y) \\ &\quad + b_w^- v_j \cos(v_j Y)] \end{aligned} \quad (4.82)$$

4.4.1 General Velocity Profile

By replacing the (X, Y) with the original (x, y) the final form of the velocity profile becomes:

$$u(x, y) = 1/\mu_w \left(-\frac{\partial P}{\partial z} \right) \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{\beta_i^s \beta_j^w}{(\mu_i^2 + \nu_j^2) W_i H_j} [\sin(\mu_i (x + w)) + b_s^- \mu_i \cos(\mu_i (x + w))] [\sin(\nu_j (y + h)) + b_w^- \nu_j \cos(\nu_j (y + h))] \quad (4.83)$$

4.4.2 General Flow Rate

By substituting (4.82) into Eq. (4.17), we will have:

$$Q = \int_0^{2h} \int_0^{2w} -F \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{\beta_i^s \beta_j^w}{(\mu_i^2 + \nu_j^2) W_i H_j} [\sin(\mu_i X) + b_s^- \mu_i \cos(\mu_i X)] [\sin(\nu_j Y) + b_w^- \nu_j \cos(\nu_j Y)] dX dY \quad (4.84)$$

After replacing Eq. (4.78) and some mathematical simplifications, the final form of the flow rate assuming general Navier BCs becomes:

$$Q = \left(-\frac{\partial P}{\partial z} \right) \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{\beta_i^{s^2} \beta_j^{w^2}}{(\mu_i^2 + \nu_j^2) W_i H_j} \quad (4.85)$$

4.4.3 General Average Velocity Distribution

The average velocity can be calculated from the flow rate, Eq. (4.85):

$$\bar{u} = \frac{\left(-\frac{\partial P}{\partial z}\right)}{4wh\mu_w} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{\beta_i^{s^2} \beta_j^{w^2}}{(\mu_i^2 + \nu_j^2)W_iH_j} \quad (4.86)$$

4.4.4 General Friction Factor

Substituting Eq. (4.86) into Eq. (4.30) results:

$$f = \frac{\left(-\frac{\partial P}{\partial z}\right) \cdot \left(\frac{4hw}{h+w}\right)^2}{\frac{1}{2}\mu_w \left(\frac{\left(-\frac{\partial P}{\partial z}\right)}{4wh\mu_w} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{\beta_i^{s^2} \beta_j^{w^2}}{(\mu_i^2 + \nu_j^2)W_iH_j} \right)} \frac{1}{Re} \quad (4.87)$$

Upon further simplifications, general form of Darcy friction factor can be obtained from Eq. (4.88):

$$f = \frac{128 h^3 w^3}{(h+w) \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{\beta_i^{s^2} \beta_j^{w^2}}{(\mu_i^2 + \nu_j^2)W_iH_j}} \frac{1}{Re} \quad (4.88)$$

4.4.5 General Poiseuille number

General form of Poiseuille number Po becomes:

$$Po = \frac{128 \alpha^3 h^5}{(1+\alpha) \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{\beta_i^{s^2} \beta_j^{w^2}}{(\mu_i^2 + \nu_j^2)W_iH_j}} \quad (4.89)$$

4.5 Numerical Modeling of Rectangular Microchannels with No-Slip BCs

To numerically simulate the flow in the rectangular microchannel and validate the results of the theoretical modeling with no-slip BCs, COMSOL Multiphysics 3.5a was used which implements the finite element method (FEM) approach to discretize and solve the governing

equations. To this aim, first 3D models of the microchannels with the geometrical sizes shown in Table 4-2 were sketched:

Table 4-2: Geometrical sizes of the microchannels used for numerical simulation

$2h \text{ } (\mu m)$	$2w \text{ } (\mu m)$	$L \text{ } (mm)$	$\alpha = w/h$	$D_h \text{ } (\mu m)$
20, 10	60, 20	1	3, 2	30, 13

Subsequently, DI water with density of $\rho = 1000 \frac{kg}{m^3}$, dynamic viscosity of $\mu_w = 0.001 \text{ Pa.s}$, and zero body forces were defined. Inlet pressure was varied from 500Pa to 4000Pa while atmospheric pressure was defined as the outlet BC. Other domains of the microchannel were considered as the solid walls with no-slip BCs. Three dimensional form of Navier-Stokes equation was used to simulate the results.

To mesh the model, tetrahedral meshes with six edge lengths, $\mathcal{H}$, were used. Firstly, the mesh quality, $\kappa = 72\sqrt{3}\mathcal{V}/\sqrt{\sum_{i=1}^6 \mathcal{H}_i^2}$, where $\mathcal{V}$ is the element volume, should be larger than 0.1 to not affect the solution quality. Secondly, the numerical results should be both grid independent and close to the theoretical values. Finally, due to the nonlinearity of the governing equations, iterations are needed for the convergence of the solutions. To this aim, the relative error and the pre-factor should be properly defined so that the product of relative residual and pre-factor becomes less than the desired tolerance, convergence criteria. Accordingly, the mesh size was refined to extra fine meshes with the minimum mesh quality of 0.1985. Total element numbers of 17191 tetrahedral grids were generated after testing the grid independency. Using BiCGStab as a linear system solver and appropriate pre-factor of 50, it was found that all the numerical results converged with the relative error of 10^{-16} after 32 number of iterations.

The grid shapes as well as velocity distributions on z-plane are shown in Figure 4-8.

Figure 4-8: Numerical modeling of the microchannels: (a) Extra fine tetrahedral meshes structures. (b) Numerical velocity distribution (m/s) on z-plane for pressure gradient of 500 Pa.

Comparison between the derived analytical velocity distributions, i.e., Eq. (4.14), and the obtained numerical results across the channel height and width (along the vertical and horizontal directions of the microchannel, respectively) are illustrated in Figure 4-9 for two different values of channel aspect ratio at the same pressure gradient (500 Pa).

Figure 4-9: Velocity distributions at different channel positions and aspect ratios ($P=500$ Pa) as compared with the obtained analytical formula and numerical results, 1D refers to Eq. (4.25). (a), (b) Velocity profiles across the channel height. (c), (d) Velocity profiles across the channel width.

The analytical results are in excellent agreement with the numerical simulations at different locations of the channel cross-section, except on the planes very near to the channel walls. As expected, maximum velocity occurs at the mid planes ($x = y = 0$). Also, it is shown that classical equation of Hagen-Poiseuille, i.e. Eq. (4.25), can only predict the maximum velocity distributions on the mid plane, i.e. $x = 0$. Further, this equation overestimates the numerical values of 2D velocity.

Figure 4-9(a) also indicates that on the plane at $2\mu m$ near to the side walls of the channel, i.e. $x = 28\mu m$, the numerical results are notably less than the analytical ones. This can be related to the numerical errors of the simulated software as a result of low resolutions of the grids on the plane very close to the channel walls. As indicated earlier, total number of the elements used in this simulation for the current channel geometry was 17191. It was observed that these element numbers were the maximum number of the grids a normal computer can generate. That is, using the finer mesh or increasing the number of grids resulted in incomplete numerical simulation. Considering these element numbers and the total volume of the channel, each grid had an approximate size of $10\mu m$. Therefore, on the planes very close to the channel wall, e.g. $2\mu m$, the numerical results describing the velocity distribution are not accurate. Yet, the obtained analytical formula can precisely predict the velocity profile even on the near wall planes.

4.6 Conclusions

In this chapter, classical Hagen-Poiseuille and Darcy-Weisbach equations were modified by introducing the effect of aspect ratio of the channel assuming three different BCs: no-slip BCs, constant slip velocity BCs, and general Navier slip length BCs. Compared to the previous 2D exact solutions, the present series solutions were symmetrical (x and y lateral coordinates are interchangeable) and converged very fast (owing to the 3rd and 4th powers of the computational indexes, i.e. m and n , in the denominator of the formulae, c.f., Eq. (4.14) or Eq. (4.28)). In addition, the obtained exact solutions are valid for any range of the channel aspect ratio. It was also shown that 1D classical equations were the limiting cases of the obtained 2D analytical formulae for velocity distributions and friction factor when the channel aspect ratio approached infinity. The results also indicated that for the laminar flow at the same Reynolds number, square cross-section channels ($\alpha = 1$) generated the lowest frictional losses. As the channel cross-

section deviated from square shape, Poiseuille number increased significantly from 55 (square section) to 96 (parallel-plate channel). Moreover, the obtained analytical formula was compared with the numerical simulations of the 3D pressure driven laminar flow in rectangular microchannels with hydraulic diameters of $13\mu m$ and $30\mu m$, and excellent agreements were found, except for on the plane very close to the microchannel walls. On these near-wall planes, for instance on the plane at $2\mu m$ near to the side walls of the microchannel, due to the limited mesh size of the numerical simulation, the numerical velocity profile was not accurate. However, the obtained analytical formula describing the velocity distribution predicted the trend perfectly well. Although the obtained results are applicable to any rectangular ducts, they are more useful in interpreting the pressure driven flows in microchannels, in which careful comparison is crucial in interpreting the results.

Having obtained the analytical modeling of microchannels by considering the width-to-height aspect ratio as well as slip BCs in Chapters 3 and 4, in the next two chapters experimental procedures of fabrication and drag reduction analysis of hydrophobic microchannels will be presented by introducing a new geometrical parameter of micropatterned surfaces.

Chapter 5 : Fabrication, Characterization and Experimental Procedures *

5.1 Introduction

Controlling the wetting behavior of a droplet on a solid surface is of critical importance owing to its wide range of applications including surface coating, surface tension driven flow and liquid transport in emerging areas such as micro/nanofluidics, biomedical devices, biomimetics, and digital microfluidics. Wettability is quantified by the contact angle which macroscopically manifests the intermolecular interactions at the three-phase contact line.

Apart from those geometrical parameters predicted by classical wetting theories and those addressed in the previous works, other factors influencing the topology of the three-phase contact line should also be taken into account in evaluating wetting behavior. One of these factors is the orientation of the micropatterns, in particular the center-to-center offset distance between successive pillars in a column. This distance is termed as eccentricity ϵ . This parameter is represented in Figure 5-1.

* This chapter was partially published in the following journal papers:

- N. Kashaninejad, W.K. Chan, N.T. Nguyen, “Eccentricity effect of micropatterned surface on contact angle” *Langmuir* (American Chemical Society-ACS), 2012, 28 (10), pp 4793–4799. DOI: 10.1021/la300416x
- N. Kashaninejad, N.T. Nguyen, W.K. Chan, “Three-phase contact line shape and eccentricity effect of anisotropic wetting on hydrophobic surfaces”, *Soft Matter*, 2013, 9 (2), pp. 527-535, DOI: 10.1039/C2SM26963E
- N. Kashaninejad, N.T. Nguyen, W.K. Chan, “Eccentricity effects of microhole arrays on drag reduction efficiency of microchannels with a hydrophobic wall”, *Physics of Fluids* (American Institute of Physics-AIP Publisher), 2012, 24 (11), pp. 112004-18, DOI: 10.1063/1.4767539

In the literature, the eccentricity effects of the micropatterned surface on contact angle values have not been investigated before. Experiments will be conducted on square micropatterns which are fabricated on silicon wafers and will be transferred to PDMS. These micropatterned surfaces have different ranges of eccentricity and spacing/width. The detailed fabrication techniques of microchannels made from such micropatterned surfaces as well as experimental procedure to measure the velocity as a function of static pressure will be also described in this chapter.

5.2 Fabrication of Micropatterned Surfaces

5.2.1 Microhole Arrays

Following a standard photolithography with AZ4620 positive photoresist and etching processes, micropillars were first etched on silicon wafer. The details of the fabrication technique are as follows:

The pillar geometries were designed and patterned on a glass mask. Since positive photoresist (AZ4620) was used during photolithography, the pillars on the mask appear opaque, so that the patterns of the pillars would remain on the wafer. Silicon wafers were cleaned with piranha following RCA (Radio Corporation of America) procedure to achieve maximum process reliability. After the cleaning process, the wafer was dried by blowing with nitrogen gas and then baked at 100°C for 1 minute on a hot plate. Pre-bake was required to improve the surface adhesion of the silicon wafer before the coating process. The wafer was baked at 100°C for 90 seconds. Two-step coating of the photoresist AZ4620 was applied to achieve uniform resist on the silicon wafer. The first step slowly spreads the resist on the whole wafer surface. The second step distributes the resist uniformly. The coated wafer was then soft baked at 100°C for 90 sec on a hotplate to remove excess solvent and to improve the adhesion of photo resist on the wafer.

Next, the wafer was exposed to UV light for about 170 sec at an intensity of 8.3 mW/cm^2 . As the photoresist used was a positive type, exposed region of the resist was removed, leaving the unexposed photoresist on the wafer. The developer was diluted with deionized (DI) water with a volume ratio of 1:4 to gain more control over the development process of the photoresist. The wafer was examined under a microscope to ensure that there were no residues of photoresist on the patterns of the wafer as it would affect the etching process. If there were still some residues, the wafer was immersed again in the developer until the resist was completely removed. The silicon wafer with the patterns was subsequently rinsed in DI water and slowly dried by nitrogen gas. Before etching the actual wafer, the etch rate was determined by measuring the depth of the etched wafer and the etching time. With an etch rate of about $3 \mu\text{m}$ per minute, the time required for etching $8 \mu\text{m}$ depth is 160 sec. A representative image of such fabricated micropillars along with the related parameters is shown in Figure 5-1.

Figure 5-1: Representative optical microscopic image of the fabricated pillars: (a) Micro patterns with the same D^* but different eccentricity were fabricated on a single silicon wafer. The y-axis shows the viewing direction for the contact angle measurement. (b) The varying pillar geometrical parameters under investigation.

Subsequently, micromolding technique was used to transfer the pillar geometries and form the microhole features on PDMS. Prepolymer (Sylgard 184, Dow Corning) was mixed with its curing agent in a ratio of 10:1 (w/w) and degassed in a vacuum pump for 1 hour. The cross-linked PDMS was carefully poured on the silicon master mold and baked in an oven for 2 hours at 65°C. After peeling off the PDMS, the microholes were formed. To facilitate the peeling off process, the surface of the silicon wafer was treated with trichloromethylsilane (TCMS, Sigma Aldrich) as the releasing agent. The SEM images of these microhole morphologies are shown in Figure 5-2.

Figure 5-2: Characteristics of the fabricated microholes in PDMS: (a) Geometric parameters of the microholes reported in the literature. (b), (c) SEM images of the fabricated microholes and the definition of the eccentricity ϵ . (d) Confocal microscopic image of the microhole to measure the depth η .

The geometrical parameters of the microhole arrays, width D , spacing L , and eccentricity ε , are illustrated in Figure 5-2(a,b). To minimize the amount of experimental data, all these parameters are normalized with respect to the spacing L . Also, normalized eccentricity $\varepsilon^* = \varepsilon/L$ is varied systematically at two ranges of normalized microhole width $D^* = D/L$ of 2.4 and 4.6, respectively. The detailed geometric values of the fabricated microhole surfaces are listed in Table 5-1.

Table 5-1. Geometrical parameters of the fabricated PDMS square microholes ($\eta = 10\mu\text{m}$)

L (μm)	D (μm)	ε (μm)	D^*	ε^*
1.5	7	0, 2, 4, 6	4.6	0, 1.3, 2.6, 4
2.5	6	0, 2, 4, 6	2.4	0, 0.8, 1.6, 2.4

5.2.2 PDMS Micropillar Arrays

PDMS micropillars were formed using three-step double casting replication procedure. First, master mold micropillars as well as microhole arrays were first fabricated on silicon wafer and PDMS, respectively, as discussed in the previous section. Micropillars were formed on another PDMS substrate using double casting technique. To avoid irreversible bonding between two PDMS substrates, the surface of the PDMS microholes template was treated with trichloromethylsilane (TCMS, Sigma Aldrich) as the releasing agent [61] with the following procedure. First, oxygen plasma was used to render the PDMS microholes hydrophilic before the deposition of TCMS. Then, the PDMS slabs were placed in a desiccator while 5mL of TCMS was dripped in a Petri dish. The pressure of the chamber was reduced by pumping it for 90 minutes. Afterwards, the valve of the chamber connecting the pump was closed, and the pump was switched off. The total deposition process was conducted under a fume hood and the chamber was left overnight for the deposition to take place. After pouring another liquid PDMS

on the TCMS-coated microhole substrate and solidifying it in an oven for 30 minutes at 85°C, PDMS micropillars were successfully formed. The SEM images of these micropillar morphologies are shown in Figure 5-3(a,b).

Figure 5-3: SEM images of the fabricated pillars: (a) Top view of the micropillars with coordinates and geometrical parameters used in this study. (b) Isometric view of the micropillars, $\eta = 10\mu\text{m}$.

The detailed geometrical parameters of the pillars are listed in Table 5-2.

Table 5-2: Geometrical parameters of the fabricated PDMS square micropillars ($\eta = 10\mu\text{m}$).

$L (\mu\text{m})$	$D (\mu\text{m})$	$\varepsilon (\mu\text{m})$	D^*	ε^*
6	2	0, 2, 4, 6	0.33	0, 0.33, 0.66, 1.00
5.3	2.7	0, 2, 4, 6	0.50	0, 0.37, 0.75, 1.13

5.3 Fabrication of Hydrophobic Microchannels

The fabricated microhole surfaces, as described in the previous section, were implemented as the bottom wall of microchannels to investigate the pressure drop performance under the varying microhole normalized eccentricities and widths. Microhole surfaces withstand more static pressure when the wetting condition is the composite state. Therefore, liquid cannot penetrate into the air-pockets formed under the continuous flow in the channel, and the drag-reduction of

hydrophobic channels, if any, would be more pronounced. In contrast, using micropillar surfaces as the channel wall of the hydrophobic channels, the percentage increase of flow velocity may become as small as the uncertainty of the experiments.

Serpentine microchannels were fabricated via soft lithography followed by replica molding as summarized in Figure 5-4(a-d). First, the channels were designed with AutoCAD software. For calibration purpose, precise ruler marks at intervals of $250\mu m$ were indicated next to the measurement channel. The serpentine microchannel was designed with 11 straight channels and 18 bends forming a total length $L_{tot} = 41.25mm$ and a width of $w = 220\mu m$, Figure 5-4(a). As the next step, SU-8 photoresist was first spin coated on an ultrasonically cleaned silicon wafer. Finally, the UV exposure time determined the thickness of the SU-8.

To highlight the drag reducing effect, the height of the microchannel or the SU-8 thickness was selected to be $20\mu m$. After pillar baking the wafer for 10 minutes at $85^{\circ}C$, the SU-8 thickness was measured using a stylus profilometer (Dektak Brand) to be $17.95 \pm 0.05\mu m$, Figure 5-4(b). Following the same procedure for PDMS replica molding, the channel features were transferred into PDMS, Figure 5-4(c). Both PDMS substrates were thoroughly soaked in DI water and cleaned with ultrasound cleaner for 30 minutes. Then, they were preheated in an oven for another 15 minutes at $80^{\circ}C$ and finally bonded together with the help of a 30 sec exposure time to oxygen plasma. To enhance the bonding quality, the device was heated in the oven for 2 hours at $80^{\circ}C$. However, because of the ragged surface of the microholes, oxygen plasma bonding only sealed the smooth part of the PDMS channel. To bond the microhole channel irreversibly, a thin intermediate layer of dilute PDMS was used as an adhesive [134]. First, $2mL$ PDMS mixture was dissolved in tert-Butanol with 85% (w/w) ratio. The solution was then spin-coated on a $2.4cm \times 5cm$ clean glass substrate in two steps: first at $550 rpm$ for 6 sec to evenly

spread the PDMS-tertButanol solution, then the speed was increased to 1200 *rpm* and maintained for 60 sec. Subsequently, the PDMS slab with microchannels was gently placed in contact with the PDMS coated substrate. After pressing the channel slab on the PDMS slab with microhole patterns and baking for 2 hours at 65°C, an irreversible bond was formed and the microchannels walls were effectively sealed, Figure 5-4(d).

Figure 5-4: Microchannel fabrication process: (a) Channel features designed on plastic mask for photolithography. (b) Master mold of SU-8 on a silicon wafer. (c) Microchannel features after PDMS replica molding (top) along with the microhole arrays (bottom). (d) Sealed microchannel after inlet/outlet punching and bonding the top and bottom walls.

5.4 Experimental Procedure for Wettability Analysis of Micropillar Arrays

The previous section showed that the wetting conditions of the micropatterned surfaces can be altered by changing the eccentricity of micropatterns. Eccentricity can also be regarded as the degree of deviation of the micropatterned surface from the symmetry position. Therefore, by changing this parameter, the surface is no longer isotropic and becomes directional dependence. This directional dependency, known as anisotropic wetting, is not only a peculiar phenomenon but also common in some natural surfaces such as rice leaves [42], which have the ability to directionally control the movement of water droplets [126]. Anisotropic wetting of the 1D strips of hydrophobic and hydrophilic regions as well as 2D microgroove have already been evaluated in the literature [36, 127] and the effect of changing the viewing direction along and across the groove on the contact angle was reported and it was found that Cassie-Baxter equation was only valid when the contact line moved along the strip [36]. As Cassie-Baxter emphasized in their original work [15], anisotropic wetting is completely different along the perpendicular direction.

It is worth investigating and comparing the directional dependency of the contact angle as a function of eccentricity at different micropillar relative spacing as it is an important parameter influencing the wettability of a surface.

In this section, the combined effect of changing the micropillar eccentricity and relative spacing on contact angle anisotropy of the discrete micropillar structures will be evaluated. For each value of the micropillar eccentricity and relative spacing, the contact angle is measured by gradually rotating the sample along different viewing angles α ranging from 0° to 180° . The results indicate the degree of anisotropic wetting on each sample, and further comparisons can be made by changing both pillar eccentricity and relative spacing.

5.4.1 Droplet Volume and Deposition Technique

The degree of anisotropic wetting could be affected by the size of the droplet [73]. We needed to compromise between a large and a small volume of the droplet. A large droplet volume is more stable but more prone to gravitational sagging. On the other hand, a small droplet volume is dominated by capillary pressure but evaporates quickly and may have a transient regime. Here, a water droplet of $2\mu\text{L}$ was selected for the experiment. Having automatically pumped $2\mu\text{L}$ volume of DI water from the delivery syringe, the droplet was detached at the minimum distance between the tip of the needle and the surface. The droplet was placed gently on the sample without compressing it to keep the spherical shape and to minimize all external disturbances. After a minute, the image of the drop was captured and further analyzed using the built-in image analyzer of the tensiometer.

5.4.2 Methodology and Statistical Error

To evaluate the contact angle at each viewing direction systematically, each PMDS substrate consisting of four different micropatterned surfaces was attached to a rotary stage (Edmund Optics Inc.) along the y axis shown in the Figure 5-3(a). The corresponding angle on the rotary stage was fixed to 90° as the reference point. With the employed rotary stage, the sample can be rotated 360° with $\pm 5^\circ$ fine rotations.

Each measurement was repeated at least 6 times at different locations of the sample (at each specific D^* , ε^* and α) to ensure the repeatability of the data and the average value of the contact angle was found at 95% confidence interval. Corresponding statistical errors of the static contact angles were less than $\pm 4^\circ$. Further error propagation and uncertainty analyses of the experimentally obtained parameters are presented in the Appendix.

Figure 5-5 depicts a typical measurement of contact angle anisotropy of a single droplet with counterclockwise (CCW) 90° rotation of the sample ($D^* = 0.33$ and $\varepsilon^* = 1$). This figure clearly shows the anisotropy of the fabricated surface, as the contact angle differs greatly at different viewing directions. Along the x-axis ($\alpha = 0^\circ$) the contact angle is more than 124° and as the sample rotates CCW, it starts to decrease gradually, and the minimum occurs along the orthogonal direction with a contact angle of less than 109° .

Figure 5-5: Typical images of the contact angle anisotropy of the micropillar surfaces ($D^* = 0.33$ and $\varepsilon^* = 1.00$) of a single droplet by rotating the sample from ($0^\circ - 90^\circ$) in 20° step increment.

5.5 Experimental Procedure for Wettability Analysis of Microhole Arrays

To quantify the wettability of the fabricated microhole surfaces, an optical tensiometer (Theta Attension, BiolinScientific, Finland) was used. Since changing the eccentricity of the microhole arrays may change the anisotropy of the surface, all the contact angles were measured along a

single viewing direction, y . To ensure the spherical shape of the droplet, the maximum volume of the droplet was selected to be $15 \mu L$ corresponding to the Bond number, Bo , of 0.31, $Bo = \rho g a^2 / \gamma$ where a is the droplet radius and γ is the surface tension. To measure the static contact angle, a small volume of de-ionized (DI) water ($V = 5 \mu L$) was placed carefully on the surface of each sample. When the droplet has stabilized, its image was captured and analyzed using the tensiometer software. The static contact angles of different microhole arrays are shown in Figure 5-6.

Figure 5-6: Static contact angle variations with different geometric parameters of the microholes. (a) Contact angle on smooth PDMS. (b) Contact angle on microhole PDMS with $D^* = 2.4$, $\epsilon^* = 2.4$ (general effect of microhole arrays). (c) Contact angle on microhole PDMS with $D^* = 2.4$, $\epsilon^* = 0$ (effect of decreasing ϵ^*). (d) Contact angle on microhole PDMS with $D^* = 4.6$, $\epsilon^* = 0$ (effect of increasing D^*).

A bare, smooth PDMS substrate has an advancing contact angle of 107° , Figure 5-6(a). By patterning the holes on the surface, the contact angle increases, Figure 5-6(b). In the case depicted in Figure 5-6(c), the effect of eccentricity can be seen by comparing it with Figure 5-6(b). In both cases, microhole relative width is 2.4 while eccentricity changes from the maximum to the minimum value. The results show that the contact angle of micropatterned surfaces with the same microhole geometry increases from 123° to 132° just by decreasing the eccentricity from 6 to 0 μm . Figure 5-6(d) represents the sample with the microholes of the same height and eccentricity as those depicted in Figure 5-6(c) but with a larger relative width. In this case, the apparent contact angle increases significantly. This observation is consistent with the Cassie-Baxter theory, which predicts that surfaces with lower solid-liquid area fraction have higher contact angle values.

5.6 Experimental Procedure for Pressure-Velocity Measurement in the Hydrophobic Microchannels

To investigate the effect of geometric parameters of the microholes, average velocity of the DI water flow was recorded as a function of the hydrostatic pressure. The actual as well as schematic experimental setups are shown in Figure 5-7(a,b). To generate the small, yet stable flow rate, a DI water reservoir was mounted on a precise height gauge (with 0.01 – mm accuracy). Different values of hydrostatic pressure were obtained by altering the effective gap (denoted as H_{static} in Figure 5-7) from 5cm to 15cm. Moreover, to measure the relative gap between the DI water reservoir and the channel inlet more precisely, an adjustable ruler and a bubble leveler were used. A thermocouple was used to measure the temperature change on the DI water during the experiment. It was observed that the temperature difference between the channel inlet and outlet was less than 0.2°C . Hence, the properties of the DI water such as

dynamic viscosity and density were assumed to be constant at an average temperature of $\bar{T} = 21.9\text{ }^{\circ}\text{C}$. As the working liquid passed through the straight measurement channel with the ruler marks, a CCD camera was used to monitor the flow in a known time $T = n_t/f_r$ where n_t is the total number of images (usually 143) and f_r is the frame rate. Image processing for evaluating the flow displacement $\Delta y \pm 50\mu\text{m}$ was conducted using Image-Pro Plus software, Figure 5-8. The average velocity was subsequently calculated. The channels are coded as ABC-D-E where AB shows the channel materials, C indicates whether the bottom wall of the channel is smooth (S) or patterned with microholes (H), and D and E correspond to the microhole normalized width and eccentricity, respectively. For instance, PPH-D2.4-E1.6 means PDMS/PDMS (PP) microchannel fabricated from microhole bottom wall with $D^* = 2.4$ and $\varepsilon^* = 1.6$. Accordingly, nine different microchannels were tested in this experiment under the identical conditions.

Figure 5-7: (a) Actual experimental test rig. (b) Schematic illustration of the set up.

Figure 5-8: Method for velocity measurement

5.6.1 Experimental Uncertainties of the Data

In this work, the total experimental uncertainty, U_{x_i} , of the independent variables x_i , was calculated from the following relationship [135]:

$$U_{x_i} = 2 \sqrt{\left(\frac{\epsilon_b}{2}\right)^2 + \left(\frac{\epsilon_s}{\sqrt{N}}\right)^2} \quad (5.1)$$

where ϵ_b is the bias error due to the limited accuracy of the measuring devices, ϵ_s is the standard deviation as a result of random error during the experiment, and N is the number of the samples. Here, the bias error of the pressure measurement is negligible because the maximum relative error of the hydrostatic height is $\pm 0.02\%$ corresponding to a minimum height of $H_{static} = 50 \text{ mm}$. Also, the bias error of the velocity measurement is only related to the error in measuring the displacement Δy ($\pm 0.05 \text{ mm}$) since the time is accurately determined by the number of images and the frame rate of the CCD camera. Thus, to decrease the relative importance of this error, the maximum allowable value of Δy (more than 2 mm but less than 6 mm) was recorded by adjusting the frame rate of the CCD camera. Accordingly, the maximum relative error of the measured velocity was found to be $\pm 2\%$.

Moreover, the experimental error U_y of the dependent variable y , calculated from the directly measured quantities, i.e. $y = f(x_i)$, was found based on the error propagation formula given by:

$$U_y = \sqrt{\sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} U_{x_i}\right)^2} \quad (5.2)$$

Throughout the present chapter, the uncertainty for each variable is represented by the corresponding error bar in the related figures and detailed calculations are provided in Appendix B and Appendix C.

5.7 Summary

In this chapter detailed experimental procedures of fabricating micropatterned surfaces as well as hydrophobic microchannels were presented. Using DRIE technique, first micropillars were formed on silicon wafer. Then, the pillars were transformed on PDMS using the standard micro-molding technique. To fabricate the micropillars on PDMS, double replica-casting technique was used. Following the standard photo-lithography as well as soft-lithography techniques, hydrophobic microchannels were fabricated. The microchannels fabricated from the microhole arrays were bonded irreversibly using a thin intermediate layer of dilute PDMS as an adhesive.

Furthermore, experimental methods used in this project to analyze the wetting conditions of micropillar and microhole surfaces were described. Owing to the anisotropic of micropillar surfaces, the detailed method used to measure the static contact angles on these surfaces at different viewing directions was introduced. For microhole surfaces both techniques used to measure the static and dynamic contact angles were described. Finally, the experimental apparatus used to measure the velocity as a function of hydrostatic pressure in the microchannels was described. The general procedure for determination of the uncertainty of the experimental data was also introduced in this chapter.

In the next chapter, the results of the both wetting analysis of the micropatterned surfaces (both micropillar and microhole surfaces) and slip flow in hydrophobic microchannels will be

presented. In addition, these results will be discussed in terms of different parameters including the eccentricity of the micropatterned surfaces, contact angle hysteresis and slip length.

Chapter 6 : Results and Discussions*

6.1 Micropillar Surface

6.1.1 Raw Experimental Data

The average values of the contact angle on the micropillar surfaces of different D^* and ε^* as a function of viewing angles are shown in Figure 6-1. In addition, the experimental values of the contact angle are also compared with the theoretical equation proposed by Cassie and Baxter, θ_{CB} , [128]:

$$\cos \theta_{CB} = \phi_s \cos \theta_{smooth} + (1 - \phi_s) \cos \pi \quad (6.1)$$

where θ_{smooth} and π are contact angles on the smooth solid and gas portions, respectively, and ϕ_s is the solid area fraction of the micropillars. The contact angle on smooth PDMS was measured as $\theta_{smooth} = 109^\circ$. The experimental contact angles on samples with zero eccentricities are in good agreement with Cassie-Baxter equation. However, the results indicate that this equation can only predict the maximum achievable contact angle irrespective of the micropillar eccentricities.

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- N. Kashaninejad, W.K. Chan, N.T. Nguyen, “Eccentricity effect of micropatterned surface on contact angle” *Langmuir* (American Chemical Society-ACS), 2012, 28 (10), pp 4793–4799. DOI: 10.1021/la300416x
- N. Kashaninejad, N.T. Nguyen, W.K. Chan, “Three-phase contact line shape and eccentricity effect of anisotropic wetting on hydrophobic surfaces”, *Soft Matter*, 2013, 9 (2), pp. 527-535, DOI: 10.1039/C2SM26963E
- N. Kashaninejad, N.T. Nguyen, W.K. Chan, “Eccentricity effects of microhole arrays on drag reduction efficiency of microchannels with a hydrophobic wall”, *Physics of Fluids* (American Institute of Physics-AIP Publisher), 2012, 24 (11), pp. 112004-18, DOI: 10.1063/1.4767539

Generally, the results show a periodicity trend for contact angle when the viewing angle gradually varies up to 180° . The period and amplitude (corresponding to the degree of contact angle anisotropy) of the samples are found to be a function of both micropillars normalized eccentricity and relative spacing. Most importantly, the amplitude is larger for the case of lower pillar spacing, where the degree of hydrophobicity is lower. This implies that contact angle anisotropy becomes more important as the hydrophobicity decreases. Specifically, the results indicate that even on micropillars with zero eccentricity, the contact angle is not isotropic. In this case, contact angles measured along the sides of the pillars ($\alpha = 0^\circ, 90^\circ$) are the same but decrease as viewing angle approaches the diagonal direction ($\alpha = 45^\circ, 135^\circ$).

This is in qualitative agreement with the results reported by Neuhaus et al. [57], and can be related to the different geometrical features. Thus, the droplet width at the three-phase contact line along the side is not equal to the one obtained along the diagonal direction. In that case, the droplet contact line deviates from the perfect circular shape.

By increasing the pillar normalized eccentricities to nonzero values, the period increases to 180° . That means, for the pillars with nonzero eccentricities, the maximum contact angle occurs along the x-axis shown in Figure 5-3. As the sample rotates, the contact angle decreases down to its minimum value along the perpendicular direction, i.e. y-axis in Figure 5-3. Additionally, the results indicate that along a fixed viewing angle, contact angle decreases by increasing the eccentricity, which is in good agreement with the previous results.

Figure 6-1: Experimental contact angle θ versus viewing angle α at different values of normalized eccentricity ϵ^* and relative pillar spacing D^* . (a) $D^* = 0.33$. (b) $D^* = 0.50$.

6.1.2 Degree of Anisotropic Wetting

To further quantify the contact angle anisotropy, the degree of anisotropic wetting, λ , can be defined as:

$$\lambda = \frac{\theta(\alpha)_{max} - \theta(\alpha)_{min}}{\theta(\alpha)_{max} + \theta(\alpha)_{min}} \times 100\% \quad (6.2)$$

where $\theta(\alpha)_{max}$ and $\theta(\alpha)_{min}$ are maximum and minimum contact angles on the same micropillar surface obtained at different viewing directions. According to the experimental data, degree of anisotropic wetting λ can be found as a function of normalized eccentricity ε^* of the micropillars, Figure 6-2.

Figure 6-2: Percentage of anisotropic wetting λ versus normalized eccentricity ε^* at different values of relative pillar spacing D^* .

The results suggest that the degree of anisotropic wetting of a surface can be tailored by changing both micropillar normalized eccentricity and relative spacing. As shown in Figure 6-2,

for the case of lower pillar relative spacing, increasing the pillar eccentricity also increases the degree of anisotropic wetting. This can be attributed to the increasing of the anisotropic features of micropillars by increasing the eccentricity.

Moreover, Figure 6-2 indicates that the degree of contact angle anisotropy decreases by increasing the relative pillar spacing. For instance, on the surface with $D^* = 0.33$ and $\varepsilon^* = 1$, the degree of anisotropic wetting is as high as 7% while it falls below 5% by increasing the relative spacing to 0.50. It was observed that as the relative pillar spacing increases, the degree of hydrophobicity also increases. Therefore, the degree of contact angle anisotropy is inversely proportional to the degree of hydrophobicity. This result suggests that for anisotropic but highly hydrophobic micropatterned surfaces, directional dependency of contact angle is less pronounced. This finding is also in agreement with the isotropic contact angle on the microstructures of lotus leaf [129].

6.1.3 Degree of Anisotropic Droplet Distortion

In the present work, at each viewing angle, the average drop width at the three-phase contact line, $2a$, and the drop height h were measured from the recorded images of the droplet with an error of $\pm 0.0005 \text{ mm}$ after proper calibration. Accordingly, the droplet aspect ratio, $a^* = a/h$, was determined. Subsequently, similar to the degree of wetting anisotropy, we can define the degree of the anisotropic droplet distortion ζ as follows:

$$\zeta = \frac{a^*(\alpha)_{\max} - a^*(\alpha)_{\min}}{a^*(\alpha)_{\max} + a^*(\alpha)_{\min}} \times 100\% \quad (6.3)$$

where $a^*(\alpha)_{\max}$ and $a^*(\alpha)_{\min}$ are the maximum and minimum droplet aspect ratios obtained at different viewing angles α on the same micropillar surface.

The corresponding values of the degree of anisotropic droplet distortion ζ are plotted as a function of the normalized eccentricity ε^* in Figure 6-3. The trend of increasing anisotropic drop distortion ζ is favored by decreasing the pillar relative spacing and increasing the pillar normalized eccentricity. This trend is consistent with the contact angle anisotropy, which confirms the accuracy of the measurements. The maximum anisotropic droplet distortion ζ occurs for the case of $D^* = 0.33$ and $\varepsilon^* = 1$ where $\zeta = 15\%$ leads to $\lambda = 7\%$. For the case of $D^* = 0.50$ and $\varepsilon^* = 0$, anisotropic droplet distortion ζ is less than 5% and this leads to an anisotropic wetting λ of 1.5%. This fact explains the reason why top-view macroscopic images of the drop placed on the micropillar surfaces with zero eccentricities represent non-distorted, circular drop shape.

Figure 6-3: Percentage of the degree of anisotropic droplet distortion ζ versus normalized eccentricity ε^* at different values of relative pillar spacing D^* .

Furthermore, it should be noted that micropillars with $D^* = 0.33$ and $\varepsilon^* = 0.33, 1.00$ as well as $D^* = 0.50$ and $\varepsilon^* = 0.37, 1.13$ are mirror images of each other. Thus, the morphologies of the pillars are identical. Consistently, the results in Figure 6-2 and Figure 6-3 confirm that the wetting anisotropy is similar on such surfaces, and droplets are elongated along the same mirror plane, i.e. y-axis in Figure 5-3.

6.1.4 Three-phase Contact Line Shape

6.1.4.1 Two-Dimensional Approach

Owing to the existence of the roughness, the actual three-phase contact line of a droplet on rough surfaces is not smooth but rather corrugated which would give rise to the effect of contact line tension, an extra term in Gibbs free energy, as explained in previous sections. This micro/nanoscale corrugation may be very small, thus, indistinguishable from smooth line macroscopically. For isotropic wetting/roughness, the shape of the contact line is assumed to be perfectly circular [66]. However, unequal droplet widths in contact with an isotropically rough surface are key parameters in directional dependency of the contact angle. If the degree of anisotropy is high, such as surfaces with hydrophilic and hydrophobic strips as well as groove structures, the elongation of the drop along a specific direction is macroscopically observable. Chen et al. [66] suggested an elliptical geometry as the equivalent noncircular smooth contact line shape for the case of parallel groove structures as supported by their experimental data and numerical simulation.

Figure 6-4: Droplet aspect ratio a^* , as a function of viewing angle α ($^\circ$) on polar coordinate. Spline interpolation between the experimental data represents the three-phase contact line of the droplet (a) $D^* = 0.33$. (b) $D^* = 0.50$ (line legend is the same as that of Figure 6-1).

In Figure 6-4, the values of droplet aspect ratio a^* are plotted as a function of viewing angle α on a polar coordinate. To depict the contact line shape of the droplet, spline interpolation is used to fit these experimental data. It should be noted that the values of aspect ratio for $\alpha = 180^\circ - 360^\circ$ are found based on the symmetrical conditions of the microstructures, i.e. the obtained curves from $\alpha = 0^\circ - 180^\circ$ are mirrored with respect to the horizontal line.

Figure 6-4 clearly illustrates the corrugation of the three-phase contact line of the microstructured surfaces as a function of the pillar eccentricity as well as the relative spacing. The result also illustrates that the variations of the contact line shape are more pronounced by changing the micropillar eccentricities. That means wetting anisotropy is highly affected by the micropillar eccentricity, rather than relative spacing, as confirmed by the experimental contact angles shown in Figure 6-1.

6.1.4.2 Three-Dimensional Approach

To obtain the equilibrium drop shape on any surface, the Gibbs free energy of the surface should be minimized subject to some constraints such as constant drop volume condition forming the basis of numerical tools such as the Surface Evolver software [130]. However, for microstructured surfaces where the size of the droplet is much larger than the characteristic length of the patterns, it was shown that finding the equilibrium drop shape is extremely difficult due to the computational complexity of the problem [66]. Another alternative approach to find the equilibrium drop shape is to solve nonlinear partial differential equation of Young-Laplace equation developed for the sessile drop. It can be proved mathematically that if the Bond number is less than unity, and the characteristic length of the drop is less than the capillary length δ_c of the liquid, the shape of the drop becomes spherical cap which is the solution of Young-Laplace

equation by neglecting the hydrostatic pressure. Such axisymmetric drop placed on a hydrophobic surface is illustrated in Figure 6-5.

Figure 6-5: Axisymmetric drop shape on a hydrophobic surface

For an anisotropic surface at each viewing angle α the value of the contact angle θ is different. Thus, contact angle can be expressed as a function of α , $\theta = f(\alpha)$.

Using the current experimental data in Figure 6-1, $f(\alpha)$ can be approximated as a Fourier series (with periods $T = \pi$ for nonzero eccentricities and $T = \pi/2$ for zero eccentricities) as follows.

$$\theta = f(\alpha) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{2n\pi}{T} \alpha \approx a_0 + a_1 \cos \frac{2\pi}{T} \alpha + a_2 \cos \frac{4\pi}{T} \alpha \quad (6.4)$$

where cosine Fourier series up to the first two sums are used for curve-fitting of the measured data. The values of a_0 , a_1 , and a_2 are listed in Table 6-1 for each micropillar surfaces.

Table 6-1: Coefficients of the Fourier series, Eq. (6.4), for the micropillars used in this study.

	$D^* = 0.33$				$D^* = 0.50$			
	$\varepsilon^* = 0$	$\varepsilon^* = 0.33$	$\varepsilon^* = 0.66$	$\varepsilon^* = 1.00$	$\varepsilon^* = 0$	$\varepsilon^* = 0.37$	$\varepsilon^* = 0.75$	$\varepsilon^* = 1.13$
a_0	2.168	2.017	2.098	2.001	2.33	2.128	2.22	2.144
a_1	0.04303	0.1069	0.07173	0.1183	0.02459	0.1026	0.07195	0.09158
a_2	-0.0007	0.01263	0.001257	0.00059	0.00164	-0.0097	-0.0092	0.00925

Using the geometrical relations as well as the fact that the height of the drop h is constant at any viewing angle α , the 2D form of the drop becomes:

$$r(\phi) = \frac{h}{(1 - \cos \theta)} \sin \phi \quad (6.5)$$

$$z(\phi) = h - \frac{h}{(1 - \cos \theta)} (1 - \cos \phi) \quad (6.6)$$

where ϕ is the zenith angle, as shown in Figure 6-5. For an anisotropic surface at different viewing angles α , the values of the contact angle θ are different. This continuous function was obtained using Eq. (6.4). Therefore, the final 3D form of the equilibrium droplet shape on anisotropic surfaces becomes:

$$x^* = \frac{\sin \phi}{1 - \cos(f(\alpha))} \cos \alpha \quad (6.7)$$

$$y^* = \frac{\sin \phi}{1 - \cos(f(\alpha))} \sin \alpha \quad (6.8)$$

$$z^* = 1 - \frac{1 - \cos \phi}{1 - \cos(f(\alpha))} \quad (6.9)$$

where $0 \leq \phi \leq \theta$, $0 \leq \alpha \leq 2\pi$, and $x^* = \frac{x}{h}$; $y^* = \frac{y}{h}$; $z^* = \frac{z}{h}$.

Finally, by varying α from 0 to 360°, the 3D shape of the drop can be plotted. These shapes at different values of pillar relative spacing and eccentricity are presented in Figure 6-6(a-h). The results shown in Figure 6-6 confirmed the previous discussions regarding the degrees of

anisotropic wetting and droplet distortion by changing micropillars eccentricity and relative spacing. As illustrated in Figure 6-6, increasing the relative pillar spacing D^* as well as decreasing the pillar normalized eccentricity ε^* make the droplet shape closer to the symmetrical configuration. If the geometric parameters of the micropillars are tailored such that the anisotropic wetting increases, it can be concluded that the energy barriers of the surface also increases as compared to the isotropic features and wettability increases.

This is in good agreement with the thermodynamic model presented by Zhao et al. [69] derived by calculating the wetting surface energy of the system as contact line moves along two orthogonal directions for parallel groove structures.

For the case of discrete micropillars, the obtained results can be explained by taking into consideration the corrugation of the three-phase contact line which is the main reason of the directional dependency of the contact angle. Distorted nature of the three-phase contact line, as illustrated in Figure 6-4, as well as corrugated drop shapes found from the simulated results (Figure 6-6) by increasing anisotropic wetting, increases the value of the line tension term of Gibbs free energy, i.e. the energy required to change the length of the three-phase contact line.

As explained in previous sections, the importance of the line tension becomes more pronounced when the contact line is corrugated. The corrugation gives rise to the energy barrier of the system, and hence decreases the degree of hydrophobicity. For the case of zero eccentricity, i.e., Figure 6-6(a,e) the energy barrier is larger on the diagonal direction of the pillars, and as a result, the contact line would be distorted as shown in Figure 6-4. Distortion of the contact line along the diagonal direction causes unequal drop widths and leads to contact angle anisotropy.

Figure 6-6: Three-quarter of equilibrium drop shape with $V = 2 \mu L$. From top to bottom, the effects of increasing the pillars eccentricity are shown, i.e. (a-d) and (e-h), while from left to right, the effects of increasing the pillars relative spacing are illustrated, e.g. (a,e) or (d,h).

On the surfaces with nonzero eccentricities, line tension, equivalent to the energy barrier and contact line distortion, increases as the along $\alpha = 90^\circ$ (y-axis in Figure 5-3(a)). Likewise, as the sample rotates toward the features along x-axis (orthogonal direction) energy barrier decreases, and contact line corrugation becomes smaller and closer to the circular shape.

6.2 Microhole Surfaces

6.2.1 Static and Dynamic Contact Angles

Since the contact line shape is continuous on microhole geometry, the hysteresis of the contact angle is not negligible. The static contact angle cannot quantify the wettability of the surface because its value lies between those of the advancing and receding contact angles. To measure the dynamic contact angle, the so-called imbedded needle technique was used as recommended by the supplier for superhydrophobic surfaces. To measure the advancing contact angle, the droplet volume was increased very slowly at a rate of $0.05\mu\text{L}/\text{s}$ from its initial volume of $2\mu\text{L}$ to the maximum volume of $10\mu\text{L}$. The images were recorded with 0.6 frames per second. Subsequently, the volume was increased to $15\mu\text{L}$ and then decreased back to its initial volume of $10\mu\text{L}$. Maximum achievable contact angle during this process was recorded as the advancing contact angle. The reverse process was performed to measure the receding contact angle. The pixel error of the tensiometer is reported to be around 0.1 degrees. All measurements were conducted at least six times at different locations on the sample, and the results are shown in Figure 6-7(a,b). The reported contact angles are the average values with the error bars indicating the standard deviations of the data set. The contact angle hysteresis (CAH) $\Delta\theta$ in Figure 6-7(a,b) is the difference between the advancing, θ_a , and receding, θ_r , contact angles.

Figure 6-7: Experimental results of static, advancing and receding contact angles as a function of microhole normalized eccentricity. (a) $D^* = 2.4$. (b) $D^* = 4.6$.

The static Cassie-Baxter contact angle, θ_{CB} , on heterogeneous surface was theoretically obtained using the expression below:

$$\cos \theta_{CB} = (1 - \phi_g) \cos \theta_{smooth} + \phi_g \cos \pi \quad (6.10)$$

where θ_{smooth} and π are contact angles on smooth solid and gas portions, respectively. Considering the present microhole arrays with $\phi_g = D^2/(D + L)^2$ and the measured smooth contact angle on PDMS, $\theta_{smooth} = 107^\circ$, the static contact angles on such microhole morphology are calculated and compared with the experimentally obtained static and dynamic contact angles in Figure 6-7(a,b).

Firstly, the results indicate that Cassie-Baxter contact angle can predict reasonably well the average static and advancing contact angles on microhole arrays independent of the microhole eccentricity. However, the receding contact angle and subsequently the CAH $\Delta\theta$ cannot be predicted by Cassie-Baxter theory. Secondly, the results suggest that static contact angle increases by increasing the normalized microhole width D^* while changing the microhole eccentricity does not affect the contact angle significantly. However, contact angle hysteresis can be altered by changing the microhole normalized eccentricity ε^* . In particular, for the case of $D^* = 4.6$ increasing ε^* to 2.6 increases the CAH from 18.7° to 36.5° , and consequently affects the resistant force and mobility of a sessile droplet. In addition, the static and dynamic contact angles of $D^* = 2.4$ and $\varepsilon^* = 0.8, 2.4$ as well as $D^* = 4.6$ and $\varepsilon^* = 1.3, 4.0$ are very similar, which may be attributed to the periodicity of the microhole arrays. Dependency of the contact angle hysteresis on microhole eccentricity can be explained by the shape of the three-phase contact line on microhole configurations. For small values of D^* , the CAH has a large value of more than 35° implying that the contact line is continuous and pinned to the edge of the solid parts. In this case, changing the microhole eccentricity does not significantly affect the shape of

contact line and thus $\Delta\theta$ is almost independent of the microhole eccentricity. However, by increasing D^* , the shear-free portion increases the mobility of the droplet with a contact angle hysteresis as low as 18.7° . The decrease in contact angle hysteresis implies that the three-phase contact line shape is rather discontinuous. In this case, changing the microhole eccentricity has a higher impact. This finding emphasizes the important role of contact line topology on wettability condition, which is not included in Cassie-Baxter equation.

6.2.2 Percentage Increase of Contact Angle Hysteresis

For better understanding of the trend of increasing CAH as a function of microhole eccentricity, the following non-dimensional variable is defined:

$$\Delta\theta^* = \frac{\Delta\theta(\varepsilon) - \Delta\theta(0)}{\Delta\theta(0)} \times 100 \% \quad (6.11)$$

where $\Delta\theta(\varepsilon)$ is the CAH at different values of eccentricity, $\Delta\theta(0)$ is the CAH at zero eccentricity, corresponding to minimum CAH.

For the investigated samples, $\Delta\theta^*$ is plotted against the normalized eccentricity for both cases of low and high microhole width, Figure 6-8(a,b). Figure 6-8(a) reveals that at a smaller microhole width, percentage increase of CAH is less than 10% in all ranges of microhole normalized eccentricity. However, CAH can increase up to 95% for higher microhole width for the case of $\varepsilon^* = 1.6$. For both cases of higher and lower microhole width, maximum obtainable hydrophobicity corresponds to the microhole patterns with zero eccentricity. As the microhole relative width increases, the effect of increasing the eccentricity becomes much more pronounced.

Figure 6-8: Percentage increase of CAH $\Delta\theta^*$ as a function of normalized eccentricity. (a) $D^* = 2.4$.
(b) $D^* = 4.6$.

Dependency of contact angles as well as CAH on microhole eccentricity cannot be explained by the original form of Cassie-Baxter equation since the effect of three-phase contact line was neglected as mentioned in the introduction. Drelich and Miller [122] derived the modified version of this equation by taking into account the effect of contact line tension k for a surface consisting of n heterogeneous structures, as follows:

$$\cos \theta_C^M = \sum_{i=1}^n \phi_{si} \cos \theta_i - \frac{1}{\gamma_{LV}} \sum_{i=1}^n \phi_{si} k_i \frac{1}{\mathcal{R}_i} \quad (6.12)$$

In Eq. (6.12), θ_C^M is the modified Cassie contact angle, ϕ_s is the wetted area of the solid fraction, γ_{LV} is interfacial tension of liquid and vapor, and $\mathcal{R}$ is the radius of curvature of the three-phase contact line. The first term on the right hand side is the same originally derived by Cassie [123]. The second term is the additional term due to the existence of the contact line tension. According to the above equation, the effect of this additional term becomes important in two situations. They are namely, when the contact line is pinned to the microholes where the role of line tension k is significant and when the radius of curvature $\mathcal{R}$ is small. Eq. (6.12) indicates that for the radii of deformation in the order of several microns, the effect of contact line should be taken into account [124]. Also, different contact line corrugations may lead to different values of contact angle despite the constant Cassie term [125]. This hypothesis agrees with the observation made in this experiment. Changing the microhole eccentricity while keeping the solid area fraction constant may lead to a change in the contact line radius of curvature and consequently a change in the apparent contact angle.

Furthermore, we found that the influence of contact angle as well as CAH variations with eccentricity is more pronounced for microholes with a higher relative width D^* . It was shown that decreasing microhole width makes the wetting state to be closer to the composite state. The

three-phase contact line de-pins from the microhole because of the small contact line tension k . Consequently CAH is smaller and the surface becomes more hydrophobic. These experimental results confirm the increase of CAH by increasing the microhole relative width. This also implies that the line tension is smaller for the case of lower microhole width. Smaller values of line tension correspond to lower microhole width, and the effect of deformation of the three-phase contact line ($d\mathcal{L}$ and $1/\mathcal{R}_i$ in Eq. (6.12)) on the contact angle as well as CAH becomes insignificant. The reverse is true for the case of higher relative microhole width. This is in good agreement with the current experimental results.

6.3 Hydrophobic Microchannels

Wetting condition of the microhole arrays differs from those of micropillar morphology reported in the literature. On the one hand, the three-phase contact line shape on the microhole configuration is real and continuous [41] which may increase the possibility of droplet pinning [18]. On the other hand, subject to the bulk flow, the confined air inside the microhole features resists the invading water, and thus the Cassie-Baxter state becomes stable [131]. Therefore, it is of great interest to investigate the drag-reducing performance of such configurations.

6.3.1 Data Analysis and Pressure Loss Calculation

Since the maximum Reynolds number Re in this work was less than 0.035, the effect of inertia was negligible, and Stokes flow was dominant. Considering the microchannel with a finite width ($2w$) to height ($2h$) aspect ratio, $\alpha = w/h$, velocity distribution u can be found by solving the Stokes equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = \frac{\nabla P}{\mu_w} \quad (6.13)$$

where x and z are the cross-section coordinates of the microchannel, ∇P is the pressure gradient in the streamwise direction y , and μ_w is the dynamic viscosity of the fluid. Considering the no-slip BCs on the channel walls and using eigenfunction expansion technique, in Chapter 4, Eq. (4.27), it was shown that the average velocity of the flow by considering the side wall effects, $\bar{u}_{2D}$, becomes:

$$\bar{u}_{2D} = \bar{u}_{H-P} \phi(\alpha) \quad (6.14)$$

where $\bar{u}_{H-P} = -\frac{h^2 \nabla P}{12\mu_w}$ is the classical Hagen-Poiseuille (H-P) average velocity with no slip BCs,

and $\phi(\alpha) = \frac{48\alpha^2}{\pi^6} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1-\cos m\pi]^2 [1-\cos n\pi]^2}{m^2 n^2 (m^2 + \alpha^2 n^2)}$ is the additional term resulting from the finite aspect ratio.

Figure 6-9 shows the comparison of experimental and analytical values of average velocity as a function of hydrostatic pressure for smooth microchannels.

Figure 6-9: Comparison of experimental and analytical values of average velocity as a function of hydrostatic pressure for smooth microchannel.

As the results suggest, the obtained analytical formula can better predict the experimental data because the aspect ratio of the channel is small. In this case, considering the effect of side walls makes the analysis more realistic. The pressure drop was calculated based on the hydrostatic height H_{static} , i.e., $\Delta P = P_{in} - P_{out} = \rho g H_{static}$. However, there might be some head losses along the channel which can reduce H_{static} :

$$H_{loss} = H_{l,in} + H_{l,out} + H_{l,dev} + H_{l,tube} + H_{l,minor} + H_{l,ev} \quad (6.15)$$

where H_{loss} is the total head loss, $H_{l,in}$, $H_{l,out}$ are entrance and exit head losses respectively, $H_{l,dev}$ is the head loss of developing region, $H_{l,tube}$ is the head loss of the connecting tubes, $H_{l,minor}$ is the minor loss due to the meandering nature of the channel as well as the fittings, and $H_{l,ev}$ is the head loss as a result of electroviscous effect. Since we intended to compare the performance of the microhole fabricated microchannels, rather than computing these losses, the experimental data of the smooth PDMS channel serve as reference to calibrate the effective height of the channel to indirectly find these losses. As shown in Figure 6-9, effective channel height of $h = 16.4\mu m$ can fit the experimental data reasonably well. Therefore, the total loss can reduce the height of the channel by $1.5\mu m$.

6.3.2 Raw Experimental Data of Pressure-Velocity Measurement

The experimental results of the average velocity versus hydrostatic pressure of the channels with smooth and microhole patterns are presented in Figure 6-10(a,b). The results suggest that the average velocity at the same hydrostatic pressure is higher in microhole channels as compared to the smooth one. Furthermore, the geometric parameters of the microhole arrays can also affect the values of the velocity and pressure drop.

Figure 6-10: Average velocity as a function of hydrostatic pressure for microchannels at different values of microhole eccentricity. (a) $D^* = 2.4$. (b) $D^* = 4.6$. For comparison, the experimental data of a smooth channel with the same channel height ($h = 17.9 \mu\text{m}$) is also presented.

6.3.3 Percentage Increase of Velocity in the Microhole Channels

To further clarify the results, percentage increase in velocity of the microhole channels at the given pressure is defined as:

$$\Lambda = \frac{\bar{u}_{microhole} - \bar{u}_{smooth}}{\bar{u}_{smooth}} \times 100\% \quad (6.16)$$

where $\bar{u}_{microhole}$ and $\bar{u}_{smooth}$ are the average velocities of the microchannel with microholes and smooth bottom walls, respectively. Figure 6-11(a,b) shows the variations of this parameter as a function of hydrostatic pressure at different values of normalized width and eccentricity. The uncertainty calculation of Λ is presented in Appendix A, and its corresponding values are shown by error bars in Figure 6-11(a,b).

The results demonstrate that patterning the bottom channel wall with microhole arrays increases the average velocity at the same hydrostatic pressure as compared to the smooth channel wall. Figure 6-11(a) depicts the percentage increase for channels with $D^* = 2.4$. The average velocity can increase up to 17% and is almost independent of the values of the eccentricities. However, Figure 6-11(b) reveals that drag reduction performance of the microchannels may be strongly affected by changing the eccentricity as well. As in the case of $D^* = 4.6$ and $\varepsilon^* = 2.6$, the lowest increase in the average velocity was observed.

Figure 6-11: Percentage increase in the relative velocity λ vs. hydrostatic pressure for the microchannels at different values of microhole eccentricity. (a) $D^* = 2.4$. (b) $D^* = 4.6$.

To justify the observed trend of the increase in velocity, the drag reduction performance of the microhole microchannels is plotted as a function of microhole eccentricity, Figure 6-12.

Figure 6-12: Average Λ as a function of the microhole normalized eccentricity ϵ^* for different D^* .

Figure 6-12 illustrates that the average percentage increase of the velocity of the microhole channels can be controlled by changing both width and eccentricity of the microhole. In particular, for $D^* = 4.6$ and $\epsilon^* = 0$ the average velocity increases up to 39%. While at the same D^* , increasing the normalized eccentricity to 2.6 lowers the average Λ down to 16%. At lower values of D^* , microchannels drag-reducing ability seems to be independent of the microhole eccentricity. In this case, the average values of Λ are around 15%. Generally, it can be concluded that at the same microhole eccentricity, channels with larger normalized width have higher

values of Λ . Importantly, at the same normalized width, increasing the eccentricity decreases the average values of Λ . Although this behavior differs from the trend of the static contact angle observed on these microhole surfaces, it correlates well with the trend of the contact angle hysteresis. Contact angle hysteresis directly relates to the critical line force caused the droplet to roll on a solid surface [101]. In other words, as the contact angle hysteresis decreases, the droplet moves faster on the surface. Hence, the drag reducing efficiency of the bulk flow should be proportional to the contact angle hysteresis, which is in good agreement with the present results. Therefore, we can conclude that the static contact angle alone is unable to predict the drag reduction ability of microchannels patterned with microholes. Advancing and receding contact angles should also be quantified to further understand the trend of drag reduction.

6.3.4 Experimental Slip Length

Here, we assume that intrinsic slip on the smooth PDMS top wall is negligible, $b^+ = 0$. Moreover, by denoting $\beta = b^- / h$ as the effective normalized slip length of the bottom wall of the channel, the average slip velocity $\bar{u}_{slip}$ can be written as: (Refer to Eq. (3.17) in Chapter 3 for detailed derivation)

$$\bar{u}_{slip} = \bar{u}_{H-P} \left(\frac{1 + 4\beta}{1 + \beta} \right) \quad (6.17)$$

The normalized experimental effective slip length, β_{exp} , then becomes:

$$\beta_{exp} = \frac{\bar{u}_{slip}/\bar{u}_{H-P} - 1}{4 - \bar{u}_{slip}/\bar{u}_{H-P}} \quad (6.18)$$

In this work, β_{exp} was obtained by considering $\bar{u}_{slip} = \bar{u}_{microhole}$ and $\bar{u}_{H-P} = \bar{u}_{smooth}$.

6.3.5 Theoretical Slip Length

For superhydrophobic surface containing 1D periodic BCs of shear-free and no-slip patterns, the following analytical formula is derived [85, 87]:

$$\beta_{th,1D,||[\perp]} = \frac{1}{[2]\pi h} \mathcal{L} \ln(\sec(\delta_g \frac{\pi}{2})) \quad (6.19)$$

where $\beta_{th,1D,||[\perp]}$ is the normalized (with respect to channel height h) slip length of the 1D strip patterns parallel [perpendicular] to flow direction, and $\delta_g = D/\mathcal{L}$ is the shear-free fraction. If the gas area fraction is large, 1D periodic patterns can be considered as the limiting case for microhole arrays. Ybert et al. [94] proposed a scaling law for the effective slip length of the 2D micropillar arrays in terms of the geometry-dependent coefficients. These coefficients were numerically determined by Cheng et al. [133]. Similarly, it is hypothesized that the normalized theoretical slip length of the 2D microhole, $\beta_{th,2D}$, can be scaled as:

$$\beta_{th,2D} \sim \frac{\mathcal{L}}{h} \ln(\sec(\phi_g \frac{\pi}{2})) \rightarrow \beta_{th,2D} = \mathcal{A} \mathcal{L} \ln(\sec(\phi_g \frac{\pi}{2})) + \mathcal{B} \quad (6.20)$$

where $\mathcal{A}$, $\mathcal{B}$ are the two normalized shape factors which depend on the geometry of the microholes. In Figure 6-13, the experimental values of slip length β_{exp} from Eq. (6.18), are compared graphically to the theoretical counterparts of $\beta_{th,1D,||[\perp]}$ and $\beta_{th,2D}$ predicted from Eq. (6.19) and Eq. (6.20), respectively. The corresponding values of the shape factor $\mathcal{A}$, $\mathcal{B}$ are listed in Table 6-2.

Figure 6-13: Theoretical and experimental values of slip length as a function of Reynolds number for the microchannels at different values of microhole eccentricity: (a) $D^* = 2.4$. (b) $D^* = 4.6$.

Table 6-2: Normalized shape factors corresponding to the normalized slip lengths in different configurations.

	1-D Patterns Parallel (Perpendicular)	PPH- D2.4(4.6)- E0(0)	PPH- D2.4(4.6)- E0.8(1.3)	PPH- D2.4(4.6)- E1.6(2.6)	PPH- D2.4(4.6)- E2.4(4.0)
$\mathcal{A}$	0.0194 (0.0097)	0.028299	0.016037	0.00051239	0.0091044
$\mathcal{B}$	0 (0)	-0.0222	0.010847	0.052388	0.032306

Generally, the obtained results in Figure 6-13 suggest that the theoretical slip length corresponding to the 1D strip patterns perpendicular to the flow direction [Eq. (6.19)] with $\frac{1}{2}$ factor) can be used well to predict the experimental results. Yet, the 1D parallel model overestimates the values of the experimental slip length. This observation is consistent with the reported results of Ybert et al. [94] who stated that the flow over the microholes with low surface area fraction, i.e. $\phi_g \rightarrow 1$, should be similar to the 1D grooves perpendicular to the flow direction. In particular, for the case of microholes with $D^* = 4.6$, Eq. (6.20) with the shape factors listed in Table II, can be used to predict the experimental values of slip length accurately, even as a function of microhole eccentricity.

Furthermore, Figure 6-13(a) indicates that the values of slip length for microchannels with $D^* = 2.4$ are independent of the eccentricity in the investigated creeping regime. In this case, the slip length cannot exceed more than $b = 0.99\mu m$. However, results of Figure 6-13(b) state that slip length can be reduced by changing the microhole eccentricity. In particular, slip length decreases from $b = 2.49\mu m$ for $\varepsilon^* = 0$ to $b = 0.91\mu m$ for $\varepsilon^* = 2.6$. This finding is in good agreement with the numerical simulations conducted by Koopae et al. [136]. They showed that friction factor of aligned micropillars, corresponding to $\varepsilon^* = 0$ in this study, is smaller than the staggered micropillars, corresponding to $\varepsilon^* \neq 0$ in this study. Here, we observed that further

increasing the eccentricity led to increasing the slip length as a result of periodicity of the patterns.

6.4 Summary

In this chapter the results of wettability analysis of micropatterned surfaces as well as slip flow in such hydrophobic microchannels were presented and discussed.

Anisotropic wetting analysis was performed on micropillar surfaces. Contact angles were measured by gradually rotating the sample at different viewing angles α . The contact angle anisotropy was found to be periodic function of viewing angle whose period was $\pi/2$ and π for zero and nonzero values of eccentricity, respectively. To quantify the directional dependency of the contact angle on such geometries both degrees of anisotropic wetting λ and droplet distortion ζ were defined. It was observed that both λ and ζ increased by increasing the pillar eccentricity as well as decreasing the pillar relative spacing. However, the effect of changing the pillar eccentricity was much more profound. On the surface with $D^* = 0.50$ and $\varepsilon^* = 0$, minimum anisotropy was observed, i.e. $\lambda_{min} = 1.5\%$ and $\zeta_{min} = 4.8\%$. By decreasing the relative pillar spacing to 0.33 and increasing the normalized eccentricity to 1.00, maximum anisotropy was observed, i.e. $\lambda_{max} = 7\%$ and $\zeta_{max} = 15\%$. Also, on the patterns which were mirror images of each other, the contact angle anisotropy was the same. Additionally, by measuring the droplet aspect ratio a^* at each viewing angle, the three-phase contact line shape of the droplet at each D^* and ε^* was found. The three-phase contact line shapes on discrete anisotropic micropillars were highly corrugated at the microscopic scale. This fact explains why macroscopic images of the drop placed on the micropillar surfaces with zero eccentricities represent non-distorted, circular drop shape. Furthermore, a semi-analytical model based on the low Bond number, was

developed to describe the equilibrium 3D shape of the droplet. The model was strictly based on the relationship between droplet contact angle θ and viewing angle α . This relationship was approximated by curve fitting of the experimental data using Fourier series. Based on the model, equilibrium 3D shapes of the droplet at different D^* and ε^* were found, and good qualitative agreement with the experimental results was obtained. Generally, it was found that both wetting anisotropy of the micropillars and the degree of the hydrophobicity of the surface have the same trend and interrelated. That means, by increasing micropillar normalized eccentricity, the energy barriers (fluctuations in the free energy) of the system increase giving rise to the three-phase contact line distortion. The corrugation of the contact line is the main source of anisotropy. The results also showed that energy barrier of the micropillar structures with nonzero eccentricities increases gradually in the orthogonal direction with a period of 180° whereas for micropillars with zero eccentricities has a period of 90° . Finally, these results suggest that combined effects of pillar relative spacing, normalized eccentricity, and viewing angle should be taken into consideration when evaluating the contact angle anisotropy and wetting conditions of the micropillar hydrophobic surfaces.

Owing to the advantages of microhole surfaces in sustaining the Cassie-Baxter wetting conditions under higher static pressures, they were better candidates for drag-reducing experiment compared to the micropillar surfaces. Accordingly, on these surfaces not only static contact angle but also advancing and receding contact angles were measured. It was found that wetting conditions on these surfaces can be controlled not only by changing the normalized widths but also the eccentricities. Most importantly, increasing the microhole eccentricity enlarged the contact angle hysteresis. This relationship was more pronounced for large

normalized width microhole arrays. In particular, for the case of $D^* = 4.6$, increasing ε^* from 0 to 2.6 increased the contact angle hysteresis from 18.7° to 36.5° .

Finally, the effects of microhole eccentricity on drag reduction of Stokes flow through hydrophobic microchannels were investigated. A series of experiments was performed for the pressure driven flow through these hydrophobic microhole channels, and drag reduction performance was evaluated. For microchannels with $D^* = 4.6$ and $\varepsilon^* = 0$, the average velocity increases up to 39% as compared to a smooth PDMS microchannel, with a corresponding slip length of $2.49\mu m$. For the same D^* , increasing the normalized eccentricity to 2.6 decreases the average percentage increase of the velocity Λ and the slip length to 16% and $0.91\mu m$, respectively. For a lower value of D^* , microchannels drag-reducing ability was independent from the microhole eccentricity. In this case, the average values of Λ were around 15%, and the corresponding slip length is less than $1\mu m$. These observations were consistent qualitatively with the analytical as well as numerical results reported previously in the literature.

In the next chapter the general conclusions of the whole thesis will be summarized and recommendations for future work will be presented.

Chapter 7 : Conclusions and Future Work

7.1 Introduction

In this study, the effect of two important parameters have been thoroughly investigated for pressure driven liquid flows in microchannel in laminar regime by analytical modeling, followed by experimental measurements. These parameters were wettability conditions of microchannel surfaces and aspect ratio of rectangular microchannels.

7.2 Analytical and Numerical Models

Firstly, for slip liquid flow in parallel-plate microchannels, classical relationships describing velocity, flow rate, pressure gradient, and shear stress were extended to the more general cases where there existed two yet-unknown slip lengths at the top and bottom walls of the channel. Subsequently, the values of the previously unknown slip length were quantified by assuming the existence of a low-viscosity region very close to the liquid and the solid surface. Two different cases were considered based on the pressure gradient: a) re-circulating air region, where the net flow rate of gas phase was zero; b) reduced viscosity gas region, in which pressure gradient was assumed to be the same at both phases. It was found that the slip length in both cases was directly proportional to the ratio of liquid-to-gas viscosity as well as the air gap thickness. Unlike other studies which used empirical values of air gap thickness, the equations describing this parameter were derived based on different ranges of Knudsen number for the case of reduced viscosity region.

Secondly, new analytical forms of velocity distribution, Darcy friction factor and Poiseuille number in laminar liquid flow in rectangular microchannels were derived using the eigenfunction

expansion technique. Three different boundary conditions (BCs) were considered: no-slip BCs, constant slip BCs, and general Navier slip length BCs. Due to the symmetry of the solutions, the effects of changing the aspect ratio from 1 to ∞ were also discussed. Using finite element method (FEM), the obtained analytical results for the case of no-slip BCs were further compared with the 3D numerical simulations for the rectangular microchannels with different range of aspect ratio and pressure gradient, and excellent agreements were found except on the near-wall planes. Although using the normal computer with limited accuracy of the mesh sizes resulted in overestimating the velocity profile in the microchannels, the obtained analytical formula predicted the trend of velocity distribution perfectly well.

7.3 Experimental Methods

Another important parameter of the micropatterned surfaces, termed as eccentricity, was introduced. Subsequently, the eccentricity effects of the micropatterned surface on wettability of hydrophobic micropatterns were investigated. Experiments were conducted on square micropatterns fabricated from both microhole and micropillar arrays at different ranges of eccentricity ε^* and spacing/width D^* . In addition, due to the better performance of microhole arrays, hydrophobic microchannels were fabricated from such micropatterned surfaces, and the hydrodynamics of the slip flow was evaluated in terms of the slip length.

For wettability analysis of micropillar surfaces, contact angles were measured by gradually rotating the sample at different viewing angles α . The contact angle anisotropy was found to be periodic function of viewing angle whose period was $\pi/2$ and π for zero and nonzero values of eccentricity, respectively. The directional dependency of the contact angle on such geometries are functions of the degree of anisotropic wetting λ and droplet distortion ζ . It was observed that both λ and ζ increase by increasing the pillar eccentricity as well as decreasing the pillar relative

spacing. However, the effect of changing the pillar eccentricity was much more profound. On the surface with $D^* = 0.50$ and $\varepsilon^* = 0$, minimum anisotropy was observed, i.e. $\lambda_{min} = 1.5\%$ and $\zeta_{min} = 4.8\%$. By decreasing the relative pillar spacing to 0.33 and increasing the normalized eccentricity to 1.00, maximum anisotropy was observed, i.e. $\lambda_{max} = 7\%$ and $\zeta_{max} = 15\%$. Also, on the patterns which were mirror images of each other, the contact angle anisotropy was the same. The three-phase contact line shapes on discrete anisotropic micropillars are highly corrugated at the microscopic scale. This fact explains why macroscopic images of the drop placed on the micropillar surfaces with zero eccentricities represent non-distorted, circular drop shape. A semi-analytical model based on the low Bond number, was developed to describe the equilibrium 3D shape of the droplet. The model is strictly based on the relationship between droplet contact angle θ and viewing angle α . This relationship was approximated by curve fitting of the experimental data using Fourier series. Based on the model, equilibrium 3D shapes of the droplet at different D^* and ε^* were found, and good qualitative agreement with the experimental results was obtained. Generally, it was found that both wetting anisotropy of the micropillars and the degree of the hydrophobicity of the surface have the same trend and interrelated. That means, by increasing micropillar normalized eccentricity, the energy barriers (fluctuations in the free energy) of the system increase giving rise to the three-phase contact line distortion. The corrugation of the contact line is the main source of anisotropy. The results also showed that energy barrier of the micropillar structures with nonzero eccentricities increases gradually in the orthogonal direction with a period of 180° whereas for micropillars with zero eccentricities has a period of 90° . The results of these experiments suggest that combined effects of pillar relative spacing, normalized eccentricity, and viewing angle should be taken into consideration when evaluating the contact angle anisotropy and wetting conditions of the micropillar hydrophobic

surfaces. Subsequently, two simple yet effective approaches to quantify the corrugations shape of three-phase contact line structures were also introduced and discussed.

On microhole surfaces, in addition to measuring the static contact, both advancing and receding contact angles were measured, and the trend of contact angle hysteresis (CAH) in terms of the eccentricity of the microhole surfaces was found. The results revealed that at a smaller microhole width, percentage increase of CAH was less than 10% in all ranges of microhole normalized eccentricity. However, CAH can increase up to 95% for higher microhole width for the case of $\epsilon^* = 1.6$. For both cases of higher and lower microhole width, maximum obtainable hydrophobicity corresponded to the microhole patterns with zero eccentricity. As the microhole relative width increased, the effect of increasing the eccentricity became much more pronounced.

Finally, the effects of microhole eccentricity on the slip lengths of Stokes flow in microchannels with the bottom wall made of microhole arrays were experimentally investigated. The results showed that the impact of microhole eccentricity on drag reduction performance correlated well with the contact angle hysteresis rather than with the static contact angle. In microchannels made from microhole arrays with large normalized width and zero eccentricity, the maximum percentage increase in the relative velocity was 39 % corresponding to a slip length of 2.49 μm . For the same normalized width, increasing the normalized eccentricity to 2.6 eventually reduces the percentage increase in relative velocity and slip length down to 16 % and 0.91 μm , respectively. The obtained results were in qualitative agreement with the existing theoretical and numerical models. In conclusion, the shape of three-phase contact line plays a crucial role for both single droplet wettability and bulk flow drag reduction performance, which can be controlled by changing both normalized width and eccentricity of the microholes.

7.4 Outlook and Future Work

Apart from the analytical modifications of the classical hydrodynamic theories presented in this work, we showed for the first time that a lattice of hydrophobic pillars whereby each successive column was offset by a certain distance, termed as eccentricity, results in a different superhydrophobic contact angle. It was found that surfaces with large eccentricity have higher values of contact angle hysteresis. This implies that droplet may pin to the microstructures by increasing their eccentricities giving rise to the importance of line tension previously neglected by Cassie-Baxter theories. In addition, it was found that surfaces with larger eccentricities have larger contact angle anisotropy, consistent with the trend of contact angle hysteresis. This shows the importance of the three-phase contact line. By fabricating the microchannel from these surfaces, the pillar eccentricity also affected the bulk flow frictional resistance. It is believed that these results can be used as guidelines for the theoreticians and experimentalists to further modify the existing wetting theories in terms of the contact line. Recently, Zhang et al. [137] proposed a 3D thermodynamic model to relate the macroscopic properties of wetting such as contact angle, contact angle hysteresis, changes in free energy, free energy barrier, and adhesion work to the microscopic features of micropatterned surfaces. Similar wetting model can be developed to take into account the effect of micropatterned eccentricity. We believe that such model should incorporate the line tension effect resulting from the tortuosity of three-phase contact line. These theoretical models can be extended to the slip length in microchannels. Here, empirical formula, i.e. Eq. (6.20), was used to relate the slip length of microhole surfaces to the eccentricity of the patterns. Nevertheless, it is expected that more experimental and theoretical works be conducted in near future to explicitly predict the effects of changing the microhole eccentricity on the slip length.

From numerical point of view, the obtained analytical formulas of velocity distributions of the microchannels with both constant slip velocity, Section 4.3, and Navier slip boundary conditions, Section 4.4, can be numerically verified. In addition, the eccentricity effect of microhole arrays on drag-reduction in hydrophobic channels can be further evaluated. Koopaee et al. [136] used only aligned and staggered micropatterned surfaces to compare the pressure drop performance. Aligned micropillars are similar to micropatterned surfaces with zero eccentricity, while staggered micropillars corresponds to micropatterned surfaces with nonzero eccentricity used in this study. Using finite element/volume approach on an appropriate unit cell, the drag reduction capability of microchannels can be systematically quantified as a function of different values of surface eccentricity. In that case, higher ranges of pressure gradient can be implemented, and the influence of micropatterned eccentricity at Reynolds number higher than unity can also be investigated numerically.

In addition, more experiments can also be done by taking into consideration the effect of micropatterned eccentricity. For instance, the effects of eccentricity on the coffee ring phenomenon and the mechanism of droplet evaporation as well as the droplet impact can be experimentally investigated.

In summary, the findings of the present work provide additional insights in design and fabrication of efficient micropatterned channels for reducing the flow resistance and leave open questions for theoreticians and experimentalists to further investigate in this field.

Appendix

A) Error Propagations of the Degrees of Anisotropic Wetting and Drop Distortion

Generally, if z is a function of independent variables x and y , $z = f(x, y)$, the uncertainty of z , ΔU_z , becomes:

$$\Delta U_z = \sqrt{\left(\frac{\partial f}{\partial x} \Delta U_x\right)^2 + \left(\frac{\partial f}{\partial y} \Delta U_y\right)^2} \quad (\text{A1})$$

where ΔU_x and ΔU_y are uncertainties of x and y , respectively. For $z = (x - y)/(x + y)$, it becomes:

$$\frac{\partial f}{\partial x} = \frac{2y}{(x + y)^2} \quad ; \quad \frac{\partial f}{\partial y} = \frac{-2x}{(x + y)^2} \quad (\text{A2})$$

Substituting Eq. (A2) in Eq. (A1) results:

$$\Delta u_z = \frac{2}{(x + y)^2} \sqrt{y^2 \Delta U_x^2 + x^2 \Delta U_y^2} \quad (\text{A3})$$

This formula is used to calculate the uncertainties of λ by replacing $x \rightarrow \theta(\alpha)_{max}$ and $y \rightarrow \theta(\alpha)_{min}$ and are shown by error bars in Figure 6-2.

Similarly, the error propagation of ζ can be calculated by replacing $x \rightarrow a^*(\alpha)_{max}$ and $y \rightarrow a^*(\alpha)_{min}$ in Eq. (A3). To find the uncertainty of maximum/minimum droplet aspect ratio, $\Delta U_{a^*max/min}$, one can write:

$$\Delta U_{a^*max/min} = \sqrt{\left(\frac{\Delta U_a}{h}\right)^2 + \left(\frac{-a}{h^2} \Delta U_h\right)^2} = \frac{\Delta U_a}{h} \sqrt{1 + a^2} \quad (A4)$$

where the uncertainties in a and h are calculated from:

$$\Delta U_h = \Delta U_a = 2 \sqrt{e_b^2 + \left(\frac{e_s}{\sqrt{N}}\right)^2} \quad (A5)$$

In Eq. (A5), e_b is the bias error as a result of the limited accuracy of the measuring the a and h , $e_b = 0.0005 \text{ mm}$, and e_s is the systematic error due to deviation of the measured volume from the desired volume ($2\mu\text{L}$), and N is the number of the samples.

The uncertainties of ζ were found and represented by error bars in Figure 6-3.

B) Uncertainty of Percentage Increase of Velocity

Considering Eq. (6.16), and denoting $\bar{u}_{microhole} = A$, $\bar{u}_{smooth} = B$ and the percentage increase of velocity $\Lambda = z$, then Eq. (6.16) can be rewritten as $z = (A - B)/B$. Therefore:

$$\frac{\partial z}{\partial A} = \frac{1}{B} \quad ; \quad \frac{\partial z}{\partial B} = -\frac{A}{B^2} \quad (B1)$$

Substituting Eq. (B1) in Eq. (A1) results:

$$\Delta z = \frac{1}{B} \sqrt{\Delta A^2 + \left(\frac{A}{B}\right)^2 \Delta B^2} \quad (B2)$$

By replacing A, B, z , the uncertainty of percentage increase of velocity, $\Delta\Lambda$, becomes:

$$\Delta\Lambda = \frac{1}{\bar{u}_{smooth}} \sqrt{\Delta\bar{u}_{microhole}^2 + \left(\frac{\bar{u}_{microhole}}{\bar{u}_{smooth}}\right)^2 \Delta\bar{u}_{smooth}^2} \quad (B3)$$

The percentages of these uncertainties are shown as error bars in Figure 6-11.

C) Uncertainty of Experimental Slip Length

Eq. (6.18), can be written as $\beta_{exp} = (\bar{u}_{slip} - \bar{u}_{H-P}) / (4\bar{u}_{H-P} - \bar{u}_{slip})$. Thus by denoting $\bar{u}_{slip} = A$, $\bar{u}_{H-P} = B$ and $\beta_{exp} = z$, then this equation becomes:

$$Z = \frac{A - B}{4B - A} \quad (C1)$$

Therefore:

$$\frac{\partial Z}{\partial A} = \frac{3B \Delta A}{(4B - A)^2} \quad ; \quad \frac{\partial Z}{\partial B} = \frac{-3A \Delta B}{(4B - A)^2} \quad (C2)$$

Substituting Eq. (C2) in Eq. (A1) results:

$$\Delta Z = \frac{3}{(4B - A)^2} \sqrt{B^2 \Delta A^2 + A^2 \Delta B^2} \quad (C3)$$

Finally, by replacing A, B, z , the uncertainty of experimental slip length, $\Delta\beta_{exp}$, becomes:

$$\therefore \Delta\beta_{exp} = \frac{3}{(4\bar{u}_{H-P} - \bar{u}_{slip})^2} \sqrt{\bar{u}_{H-P}^2 \Delta\bar{u}_{slip}^2 + \bar{u}_{slip}^2 \Delta\bar{u}_{H-P}^2} \quad (C4)$$

The percentages of these uncertainties are shown by the error bars in Figure 6-13.

References

- [1] N. T. Nguyen and S. T. Wereley, *Fundamentals and applications of microfluidics*, Artech House Publishers, 2002.
- [2] P. Tabeling, *Introduction to microfluidics*, Oxford University Press, USA, 2005.
- [3] C. Y. Zhao and T. J. Lu, *Analysis of microchannel heat sinks for electronics cooling*. International Journal of Heat and Mass Transfer, 45 (2002), pp. 4857-4869.
- [4] G. L. Morini, *Single-phase convective heat transfer in microchannels: a review of experimental results*. International Journal of Thermal Sciences, 43 (2004), pp. 631-651.
- [5] J. Judy, D. Maynes and B. W. Webb, *Characterization of frictional pressure drop for liquid flows through microchannels*. International Journal of Heat and Mass Transfer, 45 (2002), pp. 3477-3489.
- [6] D. Li, *Electro-viscous effects on pressure-driven liquid flow in microchannels*. Colloids and Surfaces A: Physicochemical and Engineering Aspects, 195 (2001), pp. 35-57.
- [7] Z. Silber-Li, H. Cui, Y. Tan and P. Tabeling, *Flow characteristics of liquid with pressure-dependent viscosities in microtubes*. Acta Mechanica Sinica, 22 (2006), pp. 17-21.
- [8] D. C. Tretheway and C. D. Meinhart, *Apparent fluid slip at hydrophobic microchannel walls*. Physics of Fluids, 14 (2002), pp. L9-L12.
- [9] R. N. Wenzel, *Resistance of solid surfaces to wetting by water*. Ind. Eng. Chem, 28 (1936), pp. 988-994.
- [10] A. B. D. Cassie, *Contact angles*. Discussions of the Faraday Society, 3 (1948), pp. 11-16.
- [11] H. Dettre Robert and E. Johnson Rulon, in *Contact Angle, Wettability, and Adhesion*, AMERICAN CHEMICAL SOCIETY, 1964, vol. 43, ch. 8, pp. 136-144.

- [12] B. He, N. A. Patankar and J. Lee, *Multiple Equilibrium Droplet Shapes and Design Criterion for Rough Hydrophobic Surfaces*. Langmuir, 19 (2003), pp. 4999-5003.
- [13] H. Nakae, M. Yoshida and M. Yokota, *Effects of roughness pitch of surfaces on their wettability*. Journal of materials science, 40 (2005), pp. 2287-2293.
- [14] N. A. Patankar, *On the modeling of hydrophobic contact angles on rough surfaces*. Langmuir, 19 (2003), pp. 1249-1253.
- [15] A. B. D. Cassie and S. Baxter, *Wettability of porous surfaces*. Transactions of the Faraday Society, 40 (1944), pp. 546-551.
- [16] C. I. Park, H. E. Jeong, S. H. Lee, H. S. Cho and K. Y. Suh, *Wetting transition and optimal design for microstructured surfaces with hydrophobic and hydrophilic materials*. Journal of colloid and interface science, 336 (2009), pp. 298-303.
- [17] L. Chen and Y. K. Lee, *New dimensionless number for superhydrophobicity study of micron/submicron patterned surfaces*. IEEE 23rd International Conference on Micro Electro Mechanical Systems (MEMS), (2010), pp. 1051-1054.
- [18] D. Öner and T. J. McCarthy, *Ultrasuperhydrophobic surfaces. Effects of topography length scales on wettability*. Langmuir, 16 (2000), pp. 7777-7782.
- [19] C. Extrand, *Criteria for ultrasuperhydrophobic surfaces*. Langmuir, 20 (2004), pp. 5013-5018.
- [20] W. Li and A. Amirfazli, *A thermodynamic approach for determining the contact angle hysteresis for superhydrophobic surfaces*. Journal of colloid and interface science, 292 (2005), pp. 195-201.
- [21] W. Li and A. Amirfazli, *Microtextured superhydrophobic surfaces: A thermodynamic analysis*. Advances in Colloid and Interface Science, 132 (2007), pp. 51-68.

- [22] R. Johnson and R. H. Dettre, *Contact angle hysteresis*. Contact angle, wettability, and adhesion. Advances in Chemistry Series, 43 (1964), pp. 112-135.
- [23] W. Li, X. Cui and G. Fang, *Optimal Geometrical Design for Superhydrophobic Surfaces: Effects of a Trapezoid Microtexture*. Langmuir, 26 (2010), pp. 3194-3202.
- [24] P. S. Swain and R. Lipowsky, *Contact angles on heterogeneous surfaces: a new look at Cassie's and Wenzel's laws*. Langmuir, 14 (1998), pp. 6772-6780.
- [25] A. Marmur, *Wetting on hydrophobic rough surfaces: to be heterogeneous or not to be?* Langmuir, 19 (2003), pp. 8343-8348.
- [26] J. Jopp, H. Gröll and R. Yerushalmi-Rozen, *Wetting behavior of water droplets on hydrophobic microtextures of comparable size*. Langmuir, 20 (2004), pp. 10015-10019.
- [27] M. Callies, Y. Chen, F. Marty, A. Pépin and D. Quéré, *Microfabricated textured surfaces for super-hydrophobicity investigations*. Microelectronic Engineering, 78 (2005), pp. 100-105.
- [28] L. Gao and T. J. McCarthy, *How Wenzel and Cassie were wrong*. Langmuir, 23 (2007), pp. 3762-3765.
- [29] G. McHale, *Cassie and Wenzel: were they really so wrong?* Langmuir, 23 (2007), pp. 8200-8205.
- [30] M. Nosonovsky, *On the range of applicability of the Wenzel and Cassie equations*. Langmuir, 23 (2007), pp. 9919-9920.
- [31] M. V. Panchagnula and S. Vedantam, *Comment on how Wenzel and Cassie were wrong by Gao and McCarthy*. Langmuir, 23 (2007), pp. 13242; discussion 13243.
- [32] L. Gao and T. J. McCarthy, *Reply to "comment on how Wenzel and Cassie were wrong by Gao and McCarthy"*. Langmuir, 23 (2007), pp. 13243-13243.

- [33] E. Bormashenko, *Why does the Cassie-Baxter equation apply?* Colloids and Surfaces A: Physicochemical and Engineering Aspects, 324 (2008), pp. 47-50.
- [34] A. Marmur and E. Bittoun, *When Wenzel and Cassie are right: reconciling local and global considerations.* Langmuir, 25 (2009), pp. 1277-1281.
- [35] L. Gao and T. J. McCarthy, *An attempt to correct the faulty intuition perpetuated by the Wenzel and Cassie "laws".* Langmuir, 25 (2009), pp. 7249-7255.
- [36] W. Choi, A. Tuteja, J. M. Mabry, R. E. Cohen and G. H. McKinley, *A modified Cassie-Baxter relationship to explain contact angle hysteresis and anisotropy on non-wetting textured surfaces.* Journal of Colloid and Interface Science, 339 (2009), pp. 208-216.
- [37] Y. Kwon, S. Choi, N. Anantharaju, J. Lee, M. V. Panchagnula and N. A. Patankar, *Is the Cassie–Baxter Formula Relevant?* Langmuir, 26 (2010), pp. 17528-17531.
- [38] R. Tadmor, *Line energy and the relation between advancing, receding, and young contact angles.* Langmuir, 20 (2004), pp. 7659-7664.
- [39] W. Chen, A. Fadeev, M. Hsieh, D. Oner, J. Youngblood and T. McCarthy, *Ultrahydrophobic and ultralyophobic surfaces: some comments and examples.* Langmuir, 15 (1999), pp. 3395-3399.
- [40] C. Extrand, *Contact angles and hysteresis on surfaces with chemically heterogeneous islands.* Langmuir, 19 (2003), pp. 3793-3796.
- [41] N. Anantharaju, M. V. Panchagnula, S. Vedantam, S. Neti and S. Tatic-Lucic, *Effect of Three-Phase Contact Line Topology on Dynamic Contact Angles on Heterogeneous Surfaces.* Langmuir, 23 (2007), pp. 11673-11676.

- [42] L. Feng, S. Li, Y. Li, H. Li, L. Zhang, J. Zhai, Y. Song, B. Liu, L. Jiang and D. Zhu, *Super-Hydrophobic Surfaces: From Natural to Artificial*. Advanced Materials, 14 (2002), pp. 1857-1860.
- [43] T. Sun, L. Feng, X. Gao and L. Jiang, *Bioinspired Surfaces with Special Wettability*. Accounts of Chemical Research, 38 (2005), pp. 644-652.
- [44] H. Wu, R. Zhang, Y. Sun, D. Lin, Z. Sun, W. Pan and P. Downs, *Biomimetic nanofiber patterns with controlled wettability*. Soft Matter, 4 (2008), pp. 2429-2433.
- [45] Y. Zheng, X. Gao and L. Jiang, *Directional adhesion of superhydrophobic butterfly wings*. Soft Matter, 3 (2007), pp. 178-182.
- [46] A. Neumann and R. Good, *Thermodynamics of contact angles. I. Heterogeneous solid surfaces*. Journal of Colloid and Interface Science, 38 (1972), pp. 341-358.
- [47] H. Gau, S. Herminghaus, P. Lenz and R. Lipowsky, *Liquid Morphologies on Structured Surfaces: From Microchannels to Microchips*. Science, 283 (1999), pp. 46-49.
- [48] S. Brandon, N. Haimovich, E. Yeger and A. Marmur, *Partial wetting of chemically patterned surfaces: The effect of drop size*. Journal of Colloid and Interface Science, 263 (2003), pp. 237-243.
- [49] M. Morita, T. Koga, H. Otsuka and A. Takahara, *Macroscopic-wetting anisotropy on the line-patterned surface of fluoroalkylsilane monolayers*. Langmuir, 21 (2005), pp. 911-918.
- [50] O. Bliznyuk, E. Vereshchagina, E. S. Kooij and B. Poelsema, *Scaling of anisotropic droplet shapes on chemically stripe-patterned surfaces*. Physical Review E, 79 (2009), pp. 041601-041606.

- [51] H. P. Jansen, O. Bliznyuk, E. S. Kooij, B. Poelsema and H. J. W. Zandvliet, *Simulating Anisotropic Droplet Shapes on Chemically Striped Patterned Surfaces*. Langmuir, 28 (2011), pp. 499-505.
- [52] E. S. Kooij, H. P. Jansen, O. Bliznyuk, B. Poelsema and H. J. W. Zandvliet, *Directional wetting on chemically patterned substrates*. Colloids and Surfaces A: Physicochemical and Engineering Aspects, 413 (2012), pp. 328-333.
- [53] D. Xia, L. M. Johnson and G. P. López, *Anisotropic Wetting Surfaces with One-Dimensional and Directional Structures: Fabrication Approaches, Wetting Properties and Potential Applications*. Advanced Materials, 24 (2012), pp. 1287-1302.
- [54] S. Tawfick, M. De Volder, D. Copic, S. J. Park, C. R. Oliver, E. S. Polsen, M. J. Roberts and A. J. Hart, *Engineering of Micro- and Nanostructured Surfaces with Anisotropic Geometries and Properties*. Advanced Materials, 24 (2012), pp. 1628-1674.
- [55] Y. Ito, M. Heydari, A. Hashimoto, T. Konno, A. Hirasawa, S. Hori, K. Kurita and A. Nakajima, *The Movement of a Water Droplet on a Gradient Surface Prepared by Photodegradation*. Langmuir, 23 (2006), pp. 1845-1850.
- [56] O. Bliznyuk, H. P. Jansen, E. S. Kooij, H. J. W. Zandvliet and B. Poelsema, *Smart Design of Stripe-Patterned Gradient Surfaces to Control Droplet Motion*. Langmuir, 27 (2011), pp. 11238-11245.
- [57] S. Neuhaus, N. D. Spencer and C. Padeste, *Anisotropic Wetting of Microstructured Surfaces as a Function of Surface Chemistry*. ACS Applied Materials & Interfaces, 4 (2011), pp. 123-130.

- [58] D. Zhu, X. Li, G. Zhang, X. Zhang, X. Zhang, T. Wang and B. Yang, *Mimicking the Rice Leaf—From Ordered Binary Structures to Anisotropic Wettability*. *Langmuir*, 26 (2010), pp. 14276-14283.
- [59] X. Zhang, Y. Cai and Y. Mi, *Anisotropic Wetting on Checkerboard-Patterned Surfaces*. *Langmuir*, 27 (2011), pp. 9630-9637.
- [60] J. Y. Chung, J. P. Youngblood and C. M. Stafford, *Anisotropic wetting on tunable micro-wrinkled surfaces*. *Soft Matter*, 3 (2007), pp. 1163-1169.
- [61] C. Bukowsky, J. M. Torres and B. D. Vogt, *Slip-stick wetting and large contact angle hysteresis on wrinkled surfaces*. *Journal of Colloid and Interface Science*, 354 (2011), pp. 825-831.
- [62] A. Buguin, L. Talini and P. Silberzan, *Ratchet-like topological structures for the control of microdrops*. *Applied Physics A: Materials Science & Processing*, 75 (2002), pp. 207-212.
- [63] T. Wang, X. Li, J. Zhang, X. Wang, X. Zhang, X. Zhang, D. Zhu, Y. Hao, Z. Ren and B. Yang, *Elliptical Silicon Arrays with Anisotropic Optical and Wetting Properties*. *Langmuir*, 26 (2010), pp. 13715-13721.
- [64] T.-i. Kim and K. Y. Suh, *Unidirectional wetting and spreading on stooped polymer nanohairs*. *Soft Matter*, 5 (2009), pp. 4131-4135.
- [65] H. Schonhorn, *Anisotropic wetting of liquids on finely grooved surfaces*. *The Journal of Adhesion*, 23 (1987), pp. 147-161.
- [66] Y. Chen, B. He, J. Lee and N. A. Patankar, *Anisotropy in the wetting of rough surfaces*. *Journal of Colloid and Interface Science*, 281 (2005), pp. 458-464.

- [67] A. D. Sommers and A. M. Jacobi, *Creating micro-scale surface topology to achieve anisotropic wettability on an aluminum surface*. Journal of Micromechanics and Microengineering, 16 (2006), pp. 1571-1578.
- [68] F. Zhang and H. Y. Low, *Anisotropic wettability on imprinted hierarchical structures*. Langmuir, 23 (2007), pp. 7793-7798.
- [69] Y. Zhao, Q. Lu, M. Li and X. Li, *Anisotropic Wetting Characteristics on Submicrometer-Scale Periodic Grooved Surface*. Langmuir, 23 (2007), pp. 6212-6217.
- [70] A. Sommers and A. Jacobi, *Wetting phenomena on micro-grooved aluminum surfaces and modeling of the critical droplet size*. Journal of Colloid and Interface Science, 328 (2008), pp. 402-411.
- [71] H. Kusumaatmaja, R. J. Vrancken, C. W. M. Bastiaansen and J. M. Yeomans, *Anisotropic Drop Morphologies on Corrugated Surfaces*. Langmuir, 24 (2008), pp. 7299-7308.
- [72] W. Li, G. Fang, Y. Li and G. Qiao, *Anisotropic Wetting Behavior Arising from Superhydrophobic Surfaces: Parallel Grooved Structure*. The Journal of Physical Chemistry B, 112 (2008), pp. 7234-7243.
- [73] J. Yang, F. R. A. J. Rose, N. Gadegaard and M. R. Alexander, *Effect of sessile drop volume on the wetting anisotropy observed on grooved surfaces*. Langmuir, 25 (2009), pp. 2567-2571.
- [74] L. Liu, A. M. Jacobi and D. Chvedov, *A surface embossing technique to create micro-grooves on an aluminum fin stock for drainage enhancement*. Journal of Micromechanics and Microengineering, 19 (2009), pp. 035026-035034.

- [75] C. J. Long, J. F. Schumacher and A. B. Brennan, *Potential for Tunable Static and Dynamic Contact Angle Anisotropy on Gradient Microscale Patterned Topographies*. Langmuir, 25 (2009), pp. 12982-12989.
- [76] X. Yong and L. T. Zhang, *Nanoscale wetting on groove-patterned surfaces*. Langmuir, 25 (2009), pp. 5045-5053.
- [77] D. Xia, X. He, Y. B. Jiang, G. P. Lopez and S. Brueck, *Tailoring Anisotropic Wetting Properties on Submicrometer-Scale Periodic Grooved Surfaces*. Langmuir, 26 (2010), pp. 2700-2706.
- [78] D. Wu, Q. D. Chen, J. Yao, Y. C. Guan, J. N. Wang, L. G. Niu, H. H. Fang and H. B. Sun, *A simple strategy to realize biomimetic surfaces with controlled anisotropic wetting*. Applied Physics Letters, 96 (2010), pp. 053704-053706.
- [79] D. Bernoulli, *1738 Hydrodynamica*. Argentorati (Strassburg), Germany: Johann Reinhold Dulsecker.
- [80] C. Neto, D. Evans, R. , E. Bonaccorso, H.-J. Butt and S. J. V. Craig, *Boundary slip in Newtonian liquids: a review of experimental studies*. Reports on Progress in Physics, 68 (2005), pp. 2859-2897.
- [81] P. S. Girard, *Mémoires de la Classe des Sciences Mathématiques et Physiques de l'Institut de France*. 14 (1815), pp. 329.
- [82] C. L. M. H. Navier, *Mémoire sur les lois du mouvement des fluids*. Mem. Acad. Sci. Inst. Fr, 6 (1823), pp. 389-416.
- [83] P. A. Thompson and M. O. Robbins, *Shear flow near solids: Epitaxial order and flow boundary conditions*. Physical Review A, 41 (1990), pp. 6830-6837.

- [84] R. S. Voronov, D. V. Papavassiliou and L. L. Lee, *Review of Fluid Slip over Superhydrophobic Surfaces and Its Dependence on the Contact Angle*. Industrial & Engineering Chemistry Research, 47 (2008), pp. 2455-2477.
- [85] J. R. Philip, *Flows satisfying mixed no-slip and no-shear conditions*. Zeitschrift für Angewandte Mathematik und Physik (ZAMP), 23 (1972), pp. 353-372.
- [86] J. R. Philip, *Integral properties of flows satisfying mixed no-slip and no-shear conditions*. Zeitschrift für Angewandte Mathematik und Physik (ZAMP), 23 (1972), pp. 960-968.
- [87] E. Lauga and H. A. Stone, *Effective slip in pressure-driven Stokes flow*. Journal of Fluid Mechanics, 489 (2003), pp. 55-77.
- [88] J. Ou, B. Perot and J. P. Rothstein, *Laminar drag reduction in microchannels using ultrahydrophobic surfaces*. Physics of Fluids, 16 (2004), pp. 4635-4643.
- [89] P. Joseph, C. Cottin-Bizonne, J. M. Benoit, C. Ybert, C. Journet, P. Tabeling and L. Bocquet, *Slippage of water past superhydrophobic carbon nanotube forests in microchannels*. Physical Review Letters, 97 (2006), pp. 156104.
- [90] R. Truesdell, A. Mammoli, P. Vorobieff, F. van Swol and C. J. Brinker, *Drag Reduction on a Patterned Superhydrophobic Surface*. Physical Review Letters, 97 (2006), pp. 044504.
- [91] N. V. Priezjev, A. A. Darhuber and S. M. Troian, *Slip behavior in liquid films on surfaces of patterned wettability: Comparison between continuum and molecular dynamics simulations*. Physical Review E, 71 (2005), pp. 041608.
- [92] B. Woolford, D. Maynes and B. Webb, *Liquid flow through microchannels with grooved walls under wetting and superhydrophobic conditions*. Microfluidics and Nanofluidics, 7 (2009), pp. 121-135.

- [93] C. Teo and B. Khoo, *Analysis of Stokes flow in microchannels with superhydrophobic surfaces containing a periodic array of micro-grooves*. Microfluidics and Nanofluidics, 7 (2009), pp. 353-382.
- [94] C. Ybert, C. Barentin, C. Cottin-Bizonne, P. Joseph and L. Bocquet, *Achieving large slip with superhydrophobic surfaces: Scaling laws for generic geometries*. Physics of Fluids, 19 (2007), pp. 123601-123610.
- [95] M. Z. Bazant and O. I. Vinogradova, *Tensorial hydrodynamic slip*. Journal of Fluid Mechanics, 613 (2008), pp. 125-134.
- [96] M. Sbragaglia and A. Prosperetti, *A note on the effective slip properties for microchannel flows with ultrahydrophobic surfaces*. Physics of Fluids, 19 (2007), pp. 043603.
- [97] A. V. Belyaev and O. I. Vinogradova, *Effective slip in pressure-driven flow past superhydrophobic stripes*. Journal of Fluid Mechanics, 652 (2010), pp. 489-499.
- [98] A. M. J. Davis and E. Lauga, *Hydrodynamic friction of fakir-like superhydrophobic surfaces*. Journal of Fluid Mechanics, 661 (2010), pp. 402-411.
- [99] K. Watanabe, Y. Udagawa and H. Udagawa, *Drag reduction of Newtonian fluid in a circular pipe with a highly water-repellent wall*. Journal of Fluid Mechanics, 381 (1999), pp. 225-238.
- [100] E. Lauga, M. P. Brenner and H. A. Stone, *Microfluidics: the no-slip boundary condition*. Handbook of Experimental Fluid Dynamics (Chapter 19), J Foss, C Tropea, AL Yarin (Eds.), Springer (2007), pp. 1219-1240.
- [101] J. P. Rothstein, *Slip on superhydrophobic surfaces*. Annual Review of Fluid Mechanics, 42 (2010), pp. 89-109.

- [102] O. I. Vinogradova and A. V. Belyaev, *Wetting, roughness and flow boundary conditions*. Journal of Physics: Condensed Matter, 23 (2011), pp. 184104-184118.
- [103] F. M. White, *Viscous fluid flow*, McGraw-Hill Science/Engineering/Math, 1991.
- [104] K. V. Sharp, Ph.D. 3017205, University of Illinois at Urbana-Champaign, 2001.
- [105] S. Granick, Y. Zhu and H. Lee, *Slippery questions about complex fluids flowing past solids*. Nature Materials, 2 (2003), pp. 221-227.
- [106] D. C. Tretheway and C. D. Meinhart, *A generating mechanism for apparent fluid slip in hydrophobic microchannels*. Physics of Fluids, 16 (2004), pp. 1509.
- [107] S. P. Sutera and R. Skalak, *The history of Poiseuille's law*. Annual Review of Fluid Mechanics, 25 (1993), pp. 1-20.
- [108] R. K. Shah and A. L. London, *Laminar flow forced convection in ducts: a source book for compact heat exchanger analytical data*, Academic press New York, 1978.
- [109] M. Spiga and G. L. Morino, *A symmetric solution for velocity profile in laminar flow through rectangular ducts*. International Communications in Heat and Mass Transfer, 21 (1994), pp. 469-475.
- [110] S. P. Timoshenko and J. N. Goodier, McGraw-Hill, New York, 1970.
- [111] J. G. Knudsen and D. L. Katz, *Fluid dynamics and heat transfer*, McGraw-Hill New York, 1958.
- [112] R. Berker, Springer, Berlin, 1963.
- [113] C. Wang, *Exact solutions of the steady-state Navier-Stokes equations*. Annual Review of Fluid Mechanics, 23 (1991), pp. 159-177.
- [114] H. Purday, *An introduction to the mechanics of viscous flow*. Dover, New York, (1949).

- [115] N. Natarajan and S. Lakshmanan, *Laminar Flow in rectangular ducts: prediction of velocity profiles and friction factor*. Indian J. Technol, 10 (1972), pp. 435-438.
- [116] J. M. Savino and R. Siegel, *Laminar forced convection in rectangular channels with unequal heat addition on adjacent sides*. International Journal of Heat and Mass Transfer, 7 (1964), pp. 733-741.
- [117] D. Holmes and J. Vermeulen, *Velocity profiles in ducts with rectangular cross sections*. Chemical Engineering Science, 23 (1968), pp. 717-722.
- [118] E. M. Sparrow, C. Hixon and G. Shavit, *Experiments on laminar flow development in rectangular ducts*. Journal of Basic Engineering, 89 (1967), pp. 116-123.
- [119] R. Dey, T. Das and S. Chakraborty, *Frictional and heat transfer characteristics of single-phase microchannel liquid flows*. Heat Transfer Engineering, 33 (2012), pp. 425-446.
- [120] N. Asmar, *Partial differential equations and boundary value problems with fourier series*. Prentice Hall, second edition (2005).
- [121] X. Zheng and Z. Silber-Li, *Measurement of velocity profiles in a rectangular microchannel with aspect ratio = 0.35*. Experiments in Fluids, 44 (2008), pp. 951-959.
- [122] J. Drelich and J. D. Miller, *Modification of the Cassie equation*. Langmuir, 9 (1993), pp. 619-621.
- [123] A. B. D. Cassie, *Contact angles*. Discussions of the Faraday Society, (1948), pp. 11-16.
- [124] J. Drelich and J. D. Miller, *The line/pseudo-line tension in three-phase systems*. Particulate science and technology, 10 (1992), pp. 1-20.
- [125] F. J. M. Ruiz-Cabello, H. Kusumaatmaja, M. A. Rodriguez-Valverde, J. Yeomans and M. A. Cabrerizo-Vilchez, *Modeling the Corrugation of the Three-Phase Contact Line Perpendicular to a Chemically Striped Substrate*. Langmuir, 25 (2009), pp. 8357-8361.

- [126] D. Wu, J.-N. Wang, S.-Z. Wu, Q.-D. Chen, S. Zhao, H. Zhang, H.-B. Sun and L. Jiang, *Three-Level Biomimetic Rice-Leaf Surfaces with Controllable Anisotropic Sliding*. Advanced Functional Materials, 21 (2011), pp. 2927-2932.
- [127] D. Xia, X. He, Y.-B. Jiang, G. P. Lopez and S. R. J. Brueck, *Tailoring Anisotropic Wetting Properties on Submicrometer-Scale Periodic Grooved Surfaces*. Langmuir, 26 (2010), pp. 2700-2706.
- [128] A. Cassie and S. Baxter, *Wettability of porous surfaces*. Trans. Faraday Soc., 40 (1944), pp. 546-551.
- [129] D. Quéré, *Wetting and roughness*. Annu. Rev. Mater. Res., 38 (2008), pp. 71-99.
- [130] K. A. Brakke, *The Surface Evolver*. Experimental Mathematics, 1 (1992), pp. 141-165.
- [131] M. C. Salvadori, M. Cattani, M. R. S. Oliveira, F. S. Teixeira and I. G. Brown, *Design and fabrication of superhydrophobic surfaces formed of microcavities*. Applied Physics Letters, 96 (2010), pp. 074101-074103.
- [132] M. A. Samaha, H. V. Tafreshi and M. Gad-el-Hak, *Modeling drag reduction and meniscus stability of superhydrophobic surfaces comprised of random roughness*. Physics of Fluids, 23 (2011), pp. 012001-012008.
- [133] Y. P. Cheng, C. J. Teo and B. C. Khoo, *Microchannel flows with superhydrophobic surfaces: Effects of Reynolds number and pattern width to channel height ratio*. Physics of Fluids, 21 (2009), pp. 122004-122012.
- [134] C. Liu, *Rapid fabrication of microfluidic chip with three-dimensional structures using natural lotus leaf template*. Microfluidics and Nanofluidics, 9 (2010), pp. 923-931.
- [135] M. E. Steinke and S. G. Kandlikar, *Single-phase liquid friction factors in microchannels*. International Journal of Thermal Sciences, 45 (2006), pp. 1073-1083.

- [136] M. K. Koopaee, M. Jahanmiri and H. M. Zadeh, *Numerical investigation of drag reduction in microchannels with superhydrophobic walls consist of aligned and staggered micropillars*. International Conference on Fluid Dynamics and Thermodynamics Technologies (FDTT 2012), 33 (2012), pp. 104-109.
- [137] H. Zhang, W. Li, H. Liu and D. Cui, *Thermodynamic analysis on superhydrophobicity based on the design of a pillar model*. Soft Matter, 8 (2012), pp. 10360-10369.