Experimental study of hydrodynamic forces on a surging circular cylinder in still water and in regular waves

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**EXPERIMENTAL STUDY OF HYDRODYNAMIC
FORCES ON A SURGING CIRCULAR CYLINDER
IN STILL WATER AND IN REGULAR WAVES**

YUAN ZHIDA

School of Civil and Environmental Engineering

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Summary

For a moored semi-submersible platform, the slow-drift motions arise from resonant oscillations. The resonant slow-drift motions are bounded mainly by the hydrodynamic damping due to the weak restoring forces of the mooring system and the highly tuned low-frequency resonance of the system. Accurate prediction of hydrodynamic damping is essential for the prediction of platform motions and the design of mooring and riser systems. The hydrodynamic viscous damping arises from the flow separation around the slender cylindrical structures and contributes significantly to the total hydrodynamic damping in slow-drift oscillations. The viscous damping coefficients are determined from Morison equation by model tests.

This thesis presents an experimental study of hydrodynamic forces acting on a slowly surging vertical circular cylindrical model in still water and in regular waves, with emphasis on investigating the viscous drag force induced by both the wave-frequency motion and low-frequency slow oscillation of the cylinder. The aim is to have a well understanding of hydrodynamic forces on a cylinder under complicated flows characterized by two time scales: fast (wave motion) and slow (slow-drift) time scales.

Experiments on a circular cylinder sinusoidally oscillating in still water were firstly carried out with a wide range of frequencies and amplitudes. The drag and added mass (or inertia) and root-mean-square coefficients of in-line force were presented and discussed; results of these coefficients were also compared with those from tests of the same cylinder model fixed in regular waves to reveal the three-dimensional effects of wave motion on force coefficients. The harmonic, root-

mean-square, maximum and mean transverse force coefficients of these two types of tests were reported and compared. The motivations of conducting these tests are to provide a comprehensive data base for understanding the results of the problems involving two frequencies.

Experimental investigations of hydrodynamic forces acting on a vertical circular cylindrical model slowly surging in regular waves were then carried out; results were analyzed using both the Independent-Flow form of Morison equation and the Relative-Velocity form of Morison equation, and compared with those for the cylinder model oscillating with dual-frequency in still water, in which the high-frequency simulates the wave motion and the low-frequency represents the slow-drift motion. The mutual influences between wave-frequency motion and slow-surfing motion were discussed; special emphasis was put on the viscous drag force induced by both the wave-frequency motion and slow-surfing motion of the cylinder. Hydrodynamic coefficients based on these two force models (Independent-Flow and Relative-Velocity Morison equations) were shown against various suitable non-dimensional parameters with which these coefficients are well correlated. The applicability of these two forms of Morison equation was discussed based on extensive experimental data and their applicable ranges were given. In addition, experimental results of the transverse forces of a circular cylinder in two frequencies flow were also presented.

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List of Symbols

A	Added mass matrix
A_O	Amplitude of a cylinder sinusoidally oscillating in still water
A_{OH}	Amplitude of high-frequency component of a cylinder oscillating with dual-frequency in still water
A_{OL}	Amplitude of low-frequency component of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves
A_W	Amplitude of regular wave
B	Frequency dependent potential damping matrix
C	Geometry/position-dependent restoring stiffness matrix
C_{AO}	Added mass coefficient for a cylinder sinusoidally oscillating in still water or fixed cylinder in oscillatory flow
C_{AOH}	Added mass coefficient for the high-frequency component of a cylinder oscillating with dual-frequency in still water
C_{AOL}	Added mass coefficient for the low-frequency component of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves
C_{AW}	Added mass coefficient for a fixed cylinder in regular wave
C_D	Drag coefficient
C_{DO}	Drag coefficient for a cylinder sinusoidally oscillating in still water or fixed cylinder in oscillatory flow
C_{DOH}	Drag coefficient for the high-frequency component of a cylinder oscillating with dual-frequency in still water
C_{DOL}	Drag coefficient for the low-frequency component of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves
C_{DOW}	Drag coefficient for wave motion of a cylinder slowly surging in regular waves

C_{Dr}	Drag coefficient in Relative-Velocity Morison equation of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves
C_{DW}	Drag coefficient for a fixed cylinder in regular waves
C_{Frms}	Root-mean-square in-line force coefficient
C_{Lmax}	Maximum transverse force coefficient
C_{Lmean}	Mean transverse force coefficient
$C_L(n)$	Harmonic transverse force coefficient
C_{Lrms}	Root-mean-square transverse force coefficient
C_M	Inertia coefficient
C_{MO}	Inertia coefficient for a cylinder sinusoidally oscillating in still water or fixed cylinder in oscillatory flow
C_{MOW}	Inertia coefficient for wave motion of a cylinder slowly surging in regular waves
C_{MW}	Inertia coefficient for a fixed cylinder in regular waves
d	Water depth
D	Diameter (or typical dimension) of a model
$\mathbf{D}_1$	Linear damping matrix, including wave drift damping
$\mathbf{D}_2$	Quadratic damping matrix
f	Frequency
$\mathbf{f}$	Vector function, given by $\mathbf{x}_i \mathbf{x}_i $
f_o	Frequency of sinusoidal oscillation
f_w	Frequency of regular wave
F_L	Transverse force

F_{Lmax}	Maximum value of transverse force
F_{Lmean}	Mean value of transverse force
F_{Lrms}	Root-mean-square value of transverse force
F_{rms}	Root-mean-square value of in-line force
F_s	Sampling frequency
F_x	In-line force
g	Acceleration due to gravity
k	Wave number
KC	Keulegan-Carpenter number
KC_l	Local Keulegan-Carpenter number based on maximum flow velocity and high-frequency oscillation period (or wave period) of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves ($= (U_{mL} + U_{mH})T_{OH} / D$ or $(U_{mL} + U_{mW})T_W / D$)
KC_o	Keulegan-Carpenter number for a cylinder sinusoidally oscillating in still water ($= U_{mo}T_o / D$)
KC_{OH}	Keulegan-Carpenter number based on high-frequency components of a cylinder oscillating with dual-frequency in still water ($= U_{mH}T_{OH} / D$)
KC_{OL}	Keulegan-Carpenter number based on low-frequency components of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves ($= U_{mL}T_{OL} / D$)
KC_r	Relative Keulegan-Carpenter number of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves ($= U_rT_r / D$)
KC_W	Keulegan-Carpenter number based on wave flow components for a cylinder fixed in regular waves or slowly surging in regular waves ($= U_{mW} / T_W / D$)
L	Length of test section of the circular cylinder model
L_w	Wave length

m	Structural mass matrix
M	Total mass matrix
R^2	Goodness-of-fit
Re	Reynolds number
Re_o	Reynolds number for a cylinder sinusoidally oscillating in still water
Re_r	Reynolds number based on relative velocity of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves ($= (U_{mL} + U_{mH})D / \nu / \nu \text{ or } (U_{mL} + U_{mW})D$)
Re_w	Reynolds number for a fixed cylinder in regular waves
t	Time
T	Period
T_o	Period of sinusoidal oscillation
T_{OH}	Period of high-frequency component of a cylinder oscillating with dual-frequency in still water
T_{OL}	Period of low-frequency component of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves
T_r	Relative period between two frequencies flow
T_w	Period of regular wave
U_m	Velocity amplitude
U_{mC}	Velocity amplitude of a steady flow
U_{mH}	Velocity amplitude of high-frequency component of a cylinder oscillating with dual-frequency in still water
U_{mL}	Velocity amplitude of low-frequency component of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves
U_{mO}	Velocity amplitude of a cylinder sinusoidally oscillating in still water

U_{mr}	Relative velocity amplitude of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves ($= (U_{mL} + U_{mH})$ or $(U_{mL} + U_{mW})$)
U_{mW}	Velocity amplitude of water particle induced by wave
U_r	Reduced velocity of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves ($= U_{mL} T_{OH} / D$ or $U_{mL} T_W / D$); or Reduced velocity in the case of a cylinder oscillating in a steady flow or a fixed cylinder in wave plus current flow ($= U_{mC} T_O / D$ or $U_{mC} T_W / D$)
u_w	Horizontal water particle velocity induced by regular wave
$\dot{u}_w$	Horizontal water particle acceleration induced by wave
v_w	Vertical water particle velocity induced by regular wave
x	Displacement of cylinder oscillation
x_H	Displacement of high-frequency component of a cylinder oscillating with dual-frequency in still water
x_L	Displacement of low-frequency component of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves
$\mathbf{x}$	Position vector
$\dot{x}$	Velocity of a moving cylinder
$\dot{x}_H$	Velocity of high-frequency component of a cylinder oscillating with dual-frequency in still water
$\dot{x}_L$	Velocity of low-frequency component of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves
$\ddot{x}$	Acceleration of a moving cylinder
$\ddot{x}_H$	Acceleration of high-frequency component of a cylinder oscillating with dual-frequency in still water
$\ddot{x}_L$	Acceleration of low-frequency component of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves
β	Viscous (or frequency) parameter ($= Re / KC$)

β_o	Viscous parameter for a cylinder sinusoidally oscillating in still water ($= \text{Re}_o / KC_o$)
β_w	Viscous parameter for a fixed cylinder in regular waves ($= \text{Re}_w / KC_w$)
θ	Wave phase or phase of cylinder oscillation
ν	Kinematic viscosity coefficient of water
ω	Angular velocity
ω_o	Angular velocity of sinusoidal oscillation
ω_{OH}	Angular velocity of high-frequency component of a cylinder oscillating with dual-frequency in still water
ω_{OL}	Angular velocity of low-frequency component of a cylinder oscillating with dual-frequency in still water or slowly surging in regular waves
ω_w	Angular velocity of regular wave
ζ_a	Amplitude of regular wave
$\psi(n)$	Phase angle of transverse force of nth harmonic component
Φ	Velocity potential of regular wave
Ω	Velocity ratio of vertical and horizontal water particle velocity induced by regular wave ($= V_w / U_w$)

Chapter 1

Introduction

1.1 Background

With the increasing worldwide demand of hydrocarbons, offshore engineering is currently taking on a dominant role in energy technology. People are exploring and exploiting offshore oil & gas energy in deeper and deeper ocean regions. Offshore floating platforms such as semi-submersibles and tension leg platforms (TLPs) have received more and more attention and been used widely because of their good performance such as small motion responses and stability. Semi-submersibles and tension leg platforms often operate under severe environmental conditions and they may experience serious environmental loadings during their serving term, such as ocean waves, currents, wind as well as seismic effects. Moreover, the interactions between different environmental loads make the problem more complicated. Knowledge of environmental loads is essential in both the design and operation of offshore structures.

According to the frequency of the structure motion, four types of motions can be identified for a floating structure: wave-frequency motion, high-frequency motion, slow-drift motion and the mean-drift. The wave-frequency motion is mainly linearly excited motion by waves in the frequency range of significant wave energy. The high-frequency motion is significant for TLPs because of the high stiffness in the tension legs and the high-frequency loading can lead to the phenomenon as “springing” - the high-frequency time-harmonic oscillation of a TLP and the large resonant high-frequency transient response as “ringing”. The wave-frequency and

high-frequency motions are mainly governed by inviscid fluid effects. Slowly varying wind, ocean wave and current forces can also induce low-frequency slow-drift and mean motions. The slow-drift motions of a floating structure arise from resonant oscillations; for a moored floating structure, they may occur in surge, sway and yaw (Faltinsen, 1990).

To study the motion responses of a floating structure to ocean environmental loads, it is essential to have a good understanding of various loads and damping in the structure system. The equations of motion for a free floating or moored structure can be written, in vector form, as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{D}_1\dot{\mathbf{x}} + \mathbf{D}_2\mathbf{f}(\dot{\mathbf{x}}) + \mathbf{C}\mathbf{x} = \mathbf{q}(\mathbf{t}, \mathbf{x}, \ddot{\mathbf{x}}) \quad (1-1)$$

Where

$\mathbf{M}$ = mass matrix, $\mathbf{m} + \mathbf{A}$, including added mass

$\mathbf{m}$ = structural mass matrix

$\mathbf{A}$ = added mass matrix

$\mathbf{B}$ = frequency dependent potential damping matrix $\mathbf{B} = \mathbf{B}(\omega)$

$\mathbf{D}_1$ = linear damping matrix, including wave drift damping

$\mathbf{D}_2$ = quadratic damping matrix

$\mathbf{f}$ = vector function where each element is given by $x_i |x_i|$

$\mathbf{C}$ = geometry/position-dependent restoring stiffness matrix (including mooring stiffness)

$\mathbf{x}$ = position vector

$\mathbf{q}$ = exciting force and moment vector, including the wave exciting force (1st and 2nd), wind /current forces and any other forces, such as thrusters force, etc.

Chakrabarti (2008) pointed out that the biggest difficulty, or the most challenging task, in the computations of the system response is an accurate determination of the hydrodynamic coefficients. The added mass coefficient is usually obtained from a linear diffraction/radiation theory for a large structure; for a structure composed of small members, physical experiments are needed to determine the added mass coefficient from the Morison equation (Morison et al., 1950). The linear radiation damping term can be calculated by studying the radiation problem using a potential theory. The restoring force term arises from the structure geometry (buoyancy change) and the type of mooring system. The quadratic nonlinear damping term is evaluated from the drag force in Morison equation based on structure motion; this term can be very significant for certain types of platforms and the mode of motion (e.g. the heave of a Spar, roll of a FPSO, slow-drift for a semi-submersible), and is particularly important for a motion near resonance of the structure (Chakrabarti, 1990; Kim and Sclavounos, 2001). The higher order damping force, especially in the slow-drift oscillation of a soft-moored structure (e.g. a semi-submersible) or the high-frequency motion of a stiff vertically-moored platform (e.g. a TLP), is usually difficult to estimate from available data (Chakrabarti, 2008).

For TLPs, the “Springing” or “Ringing” is a resonant phenomenon induced by the second order “sum-frequency” loads, and the typical periods of high-frequency oscillations in heave, pitch, and roll modes are in the range of 2-4s. Submerged components of a TLP have typical dimensions of order 10 meters, and the resonant oscillation amplitudes in centimetres, indicating that the order of the Reynolds number for the induced oscillating flow is of the order 10^5 and the KC (Keulegan–Carpenter) number 10^{-2} (Chaplin, 2000; Johanning et al., 2001). Previous studies

have found that an important parameter to measure the relative importance of viscosity is the frequency number $\beta (= Re / KC = D^2 / \nu T)$, where ν is kinematic viscosity coefficient of water, D the characteristic dimension of the structure and T the oscillating period). In full scale conditions, for the submerged components of a TLP, the frequency number β is of order 10^7 . In the past, extensive studies have been carried out for the hydrodynamic loads and damping coefficients under the conditions $KC \ll 1$ and $\beta \gg 1$; these studies include theoretical work by Stokes (1851) and Wang (1968), and experimental work by Bearman et al. (1985), Sarpkaya (1986), Otter (1990), Troesch and Kim (1991), Bearman and Russell (1996), Chaplin and Subbiah (1998), Chaplin (2000) and Johanning et al. (2001). An extensive review of this topic is beyond the scope of this thesis; two excellent reviews can be found in Sarpkaya (2001 and 2010).

For a moored semi-submersible platform, a characteristic feature is the low-frequency horizontal motion in irregular waves. These slow-drift motions, which occur well outside the frequency range of typical wave spectrums, are often a predominant part of the total motion (Schellin and Kirsch, 1989). Environmental loads that excite slow-drift responses include the second order difference-frequency wave loads and the viscous drag forces from the wind and currents (Emmerhoff and Sclavounos, 1996 and Kim and Sclavounos, 2001). To optimise the mooring system design and to predict the mooring line loads, one has to calculate the motion responses of the moored platform. The first-order motion responses can be well calculated by a potential theory. Obtaining the second order (low-frequency) motion responses, however, remains a challenging task. The principal difficulties lie in obtaining quality data for low-frequency damping (Schellin and Kirsch, 1989,

Chakrabari, 2008). Due to the weak restoring forces of the mooring system and the highly tuned low-frequency resonance of the system, the resonant slow-drift motions are controlled mainly by the hydrodynamic damping, which is the most important factor influencing the amplitudes of the low-frequency motions. Therefore, accurate prediction of hydrodynamic damping is essential for the prediction of platform motions and the design of mooring and riser systems (Schellin and Kirsch, 1989; Emmerhoff and Sclavounos, 1996; Chakrabari, 2008).

For most practical slow-drift problems, wave radiation damping is relatively small and can be negligible (DNV-RP-C205, 2007; Schellin and Kirsch, 1989). Hydrodynamic damping of low-frequency motions mainly includes wave-drift damping and viscous damping. The wave-drift damping can be well treated using a potential flow theory with satisfactory accuracy. The viscous damping mainly arises from the flow separation around the structures such as columns, bracings and pontoons of semi-submersibles/TLPs, as well as drag forces on mooring lines and risers. The viscous damping is often difficult to quantify and its contribution to the total damping may differ from one structure to another. In calculating the motion responses of a platform, the viscous damping due to flow separation around the cylindrical members is represented as a sum of forces acting on columns, bracings, pontoons as well as mooring lines/risers. Due to the large amplitude of mooring line motion, the viscous damping due to the drag force acting on the mooring lines represents a significant contribution to the total surge damping in spite of their small drag area (Huse, 1986; Huse and Matsumoto, 1988; Nakamura et al., 1991; Kotera et al., 1994; Lie et al., 2007).

Therefore, obtaining these unsteady viscous forces acting on these slowly-surging members is vital for understanding slow-drift damping and predicting platform motions. The motions of a moored floating platform are typically characterized by two distinct time scales: the fast (wave) and the slow (drift) time scales. The former is of the order 10 seconds, which is the dominant period in the wave spectrum of interest. The slow (drift) time scale is characterized by the natural period of the mooring system, which is an order of magnitude larger than the wave period due to the relatively larger mass and the weak restoring forces (Emmerhoff and Scлавounos, 1996). Typical resonance periods are of the order of 1-2 minutes for conventionally moored systems (Faltinsen, 1990). Accordingly the motion curve of a moored floating structure in irregular waves consists of a slowly varying component and a component oscillating at wave frequencies. The spectrums of forces and motion curve all have two peaks: one is close to the wave frequency, and the other close to the resonance frequency (Dev, 1996).

In practice, the viscous force due to unsteady flow separation around slender bodies is usually evaluated according to Morison equation (Morison et al., 1950). It expresses the viscous force on a slender member as a nonlinear function of the relative velocity between the slender member and the ambient flow. The Morison equation was first proposed by Morison et al. (1950) to describe the in-line force acting on a vertical pile subjected to a viscous unsteady flow in a steady progressive wave; this equation has been widely used in practice to design offshore structures. For a fixed cylinder, the original Morison equation can be written in term of the water particle velocity and acceleration as

$$F_x = \rho \frac{\pi D^2}{4} C_{MW} \dot{u}_w ds + \frac{1}{2} \rho D C_{DW} |u_w| u_w ds \quad (1-2)$$

where F_x is the wave force acting on the cylinder model with its length of ds in the direction of wave propagation; D is the cylinder diameter; u_w and $\dot{u}_w$ are, respectively, the horizontal water particle velocity and acceleration normal to the cylinder axis where the wave force acts on; and ρ is the density of water. This equation consists of two components: the first is the inertia force due to flow acceleration, and the second is the drag force proportional to the square of flow velocity. The coefficients $C_{MW}(=1+C_{AW})$ and C_{DW} are, respectively, the inertia and drag force coefficients, which need to be determined experimentally. In the expression for C_{MW} , C_{AW} is the added mass coefficient and the constant “1” accounts for the hydrodynamic force due to the pressure gradient in the oscillatory flow.

On the left hand side of Equation (1-1), the inertial force associated with the added mass and the quadratic viscous damping force $D_2 f(\dot{x})$, are usually modelled by Morison equation for slender offshore members. For a slender structure moving in waves, the quadratic viscous damping force in Morison equation is usually treated by two forms (Emmerhoff and Sclavounos, 1996): The first form is the so-called “Independent-Flow” form of Morison equation, and the second form is the so-called “Relative-Velocity” form of Morison equation.

The “Independent-Flow” form of Morison equation assumes that the drag forces due to the contributions from the wave-frequency and slow-drift frequency can be

separated. Therefore, two drag coefficients are required. For this form of Morison equation, the drag force dF on a strip of a vertical cylinder can be given by

$$dF = \frac{1}{2} \rho D dz (C_{DOW} u_w |u_w| + C_{DOL} \dot{x} |\dot{x}|) \quad (1-3)$$

where u_w is the water particle velocity due to the wave motion and $\dot{x}$ is the velocity of the structure motion, and C_{DOW} and C_{DOL} are two drag coefficients for the two flows of distinct frequencies.

The second form of Morison equation, the “Relative-Velocity” form of Morison equation, expresses the drag force as a function of the relative velocity between the structure and the ambient flow, $\dot{x} + u_w$. Combing the slow- and fast- scale velocities gives the following expression for the drag force as

$$dF = \frac{1}{2} \rho D dz C_{Dr} (\dot{x} + u_w) |\dot{x} + u_w| \quad (1-4)$$

Note that only one drag coefficient C_{Dr} is required.

For a complex structure, the total drag force is obtained by summing up all the damping forces acting on columns, bracings, lower-hulls as well as mooring lines/risers.

These two forms of Morison equation are frequently used in offshore engineering. Since the typical resonance periods are of the order of 1-2 minutes for conventionally moored systems (Faltinsen, 1990), and the typical wave period is of order 10 seconds, the ratio of these two periods varies from 6-12. Intuitively, the

nonlinear interaction between these two flows of distinctive periods should be weak and a linear superposition of the hydrodynamic forces for the two flows should provide a reasonable estimation of the total hydrodynamics force (this is the assumption of the “Independent-Flow” form of Morison equation); if this can be done with sufficient accuracy, the previous results of hydrodynamic coefficients for a cylinder fixed in waves or a cylinder oscillating in still water can be used.

In the literature, the Independent-Flow Morison equation and Relative-Velocity Morison equation for a cylinder oscillating in a current, oscillatory or wave flow were discussed by many researchers (e.g. Laya et al., 1984; Chakrabarti, 1987; Demirbilek, 1987; Lipsett and Williamson, 1994; Merz et al., 2009). However, experimental verifications of either of these two forms of Morison equation are limited and quality data of the hydrodynamic coefficients for a circular cylinder slowly oscillating in waves are scarce. There is a need to carry out systematic measurements of hydrodynamic forces acting on a cylinder slowly surging in regular waves, and provide hydrodynamic coefficients in the two forms of Morison equation so that offshore engineers can use them in their designs.

A large body of knowledge about the hydrodynamic loading on cylindrical structures has been accumulated in the past decades. Comprehensive surveys on hydrodynamic loads due to the steady and unsteady oscillatory flows (or waves) around slender bodies can be found in Sarpkaya and Isaacson (1981), Chakrabarti (1987), Faltinsen (1993), Zdravkovich (1997), Sumer and Fredsoe (2006) and Sarpkaya (2010). In the next section, the current knowledge of hydrodynamic loading on cylindrical members, with an emphasis on the hydrodynamic

coefficients in these two forms of Morison equation, is briefly reviewed. The reviews of the key references are summarized in the following two groups: (i) a cylinder oscillating in steady flow or a cylinder fixed in combined wave-current fields, and (ii) a slender structure (including members of a semi-submersible and a single cylinder) oscillating in waves. In-depth reviews of the other literature on specific topics are given in the relevant chapters of this thesis.

1.2 A cylinder oscillating in a steady flow or a fixed cylinder in a combined wave-current field

Verley and Moe (1979) studied the drag and inertia coefficients in the Relative-Velocity form of Morison equation for a cylinder oscillating in a steady stream at a frequency parameter β of about 300, KC number up to 35, and reduced velocity up to 45. They concluded that both the drag and inertia coefficients were insignificantly affected by KC number when the reduced velocity U_r was larger than 20.

Koterayama (1984) conducted an extensive study of the drag and inertia coefficients for a cylinder in a combined wave-current field. Wave forces acting on a submerged circular cylinder moving with a constant velocity in regular wave and hydrodynamic forces acting on the cylinder forced to surge in a steady flow were investigated. The drag force was analyzed in terms of the relative velocity; but in his analysis, the drag force was separated into a steady component and oscillating components. The measured force eventually was expressed as a Fourier series and a drag coefficient was defined for each of the Fourier component. The single drag coefficient in the Relative-Velocity form of Morison equation was not provided.

His experimental results showed that the drag coefficients (used in the Fourier series expression of the measured drag force) depended mainly on the reduced velocity U_r when the reduced velocity U_r was small ($U_r < 20$), but also depended weakly on the KC number when the reduced velocity was large. When around $U_r > 18$, the drag coefficient for oscillating component of the drag force approaches the drag coefficient for the steady component of the drag force, which is around 1.0. The inertia coefficients C_M agreed well with $1 + C_A$ (C_A is the added mass coefficient) when $U_r < 8$, and was greater than $1 + C_A$ when $U_r > 8$. He attributed this to the experimental error (the drag force dominates and the inertia force is much smaller).

Low et al. (1989) investigated the hydrodynamic forces acting on a circular cylinder oscillating in line with a uniform flow in a wind tunnel. The drag coefficients based on the Relative-Velocity form of Morison equation were presented for the reduced velocity U_r ranging from 20 to 110 and the KC number from 2.5 to 15. Their drag coefficient varied from 0.9 to 1.25 and showed a dependence on the reduced velocity even when the reduced velocity was very large.

Shafiee-far (1997) conducted extensive experiments on hydrodynamic forces acting on a vertical cylinder oscillating in-line with a uniform flow and on the same vertical cylinder towing in waves to simulate the combined wave-current flow field. Using the Relative- Velocity form of Morison equation, he concluded that the drag coefficient was affected by the presence of the current; overall, the drag coefficients for the case of a combined wave-current flow were smaller than those for wave alone; when the ratio of current velocity to cylinder oscillating velocity (or water

particle velocity in a wave field) was high, the values of the drag coefficient approached the drag coefficient for current alone. He also analyzed the results using the Independent-Flow form of Morison equation; the presence of the current increased the drag coefficient, reduced the inertia coefficient for cylinder oscillation (or for wave motion). When at higher ratio of the current velocity to cylinder oscillating velocity (or water particle velocity in a wave field), the drag coefficient for the current approached the drag coefficient for current alone.

Chakrabarti et al. (1997) measured the total force on a vertical pile in regular waves and in combined wave-current flows. The drag coefficients were presented for the Relative-Velocity form of Morison equation. The drag coefficient for wave alone showed trends different from that for combined wave-current flow. The drag coefficients for wave alone had a mean value of about 1.4 when the KC number was in the range of 10-30. For the combined wave-current flows, the mean drag coefficient value was about 1.0 when KC number (defined in terms of the total velocity) varied from 15 to 40.

Teng and Nath (1985) and Chaplin and Subbiah (1997) conducted extensive large scale laboratory measurements to study the hydrodynamic force on a horizontal circular cylinder in waves co-existing with a current. The Relative-Velocity form of Morison equation was used to analyze the data. One important conclusion was that when the ratio of the current velocity to the wave particle velocity was large, the value of drag coefficient for wave plus current had the tendency to approach the drag coefficient for current alone.

Experiments on horizontal rectangular cylinders with different aspect ratios (breadth to depth) in combined wave-current flows were had been conducted by Vengatesan et al. (2009), and the results were analyzed using the Relative-Velocity form of Morison equation.

1.3 A slender structure oscillating in waves or oscillating with dual-frequency in still water

Cao et al. (1986), Satio and Takagi (1988), Kinoshita and Takaiwa (1988, 1989 and 1990), and Chakrabarti and Cotter (1990) showed that the waves might have significant effects on overall hydrodynamic coefficients for the low-frequency motion of a semi-submersible platform model. No systematic results were given for individual cylindrical members.

Koterayama and Nakamura (1988) investigated the hydrodynamic forces of a vertical cylinder by forcing the model oscillating in still water with dual- frequency as well as oscillating the cylinder with a low-frequency in regular waves. The drag force was analyzed using Fourier series and one drag coefficient was defined for each frequency component. They showed the added mass or inertia coefficients in high (wave)-frequency were affected by slow-drift oscillations and decrease with U_r at $U_r < 6$. In addition, the added mass coefficients for the slow oscillation from two-harmonic forced surging tests and surging tests in waves were much larger than those obtained from surging tests with a single frequency in still water. They found that the drag coefficients for high- frequency motion (or wave motion) were much different from those for low-frequency oscillation.

Kinoshita and Takaiwa (1989) showed that the ratio of the wave particle velocity to the velocity of low-frequency motion U_{mW} / U_{mL} is another important parameter influencing the low-frequency forces. Ikeda and Fujiwara (1990) studied a cylinder oscillating with two different frequencies in still water, and found that the low-frequency viscous damping was related to vortex shedding in a cycle of the high-frequency oscillation. They also experimentally confirmed that the low-frequency viscous damping depended on two parameters: the reduced velocity U_r and the ratio of the velocity of the high-frequency oscillation to the velocity of the low-frequency oscillation U_{mH} / U_{mL} . However, the following two important aspects were not discussed in their paper: (1) the hydrodynamic coefficients for the high-frequency (or wave-frequency) components, and (2) the influence of the KC number for wave motion on the coefficients for the low-frequency oscillation.

Hu and Koterayama (1993) studied a surging vertical cylinder in irregular waves and compared the results with those of Koterayama and Nakamura (1988). Again, the drag force was analyzed using Fourier series, and one drag coefficient was defined for each frequency component. They also found that the added mass coefficients for high-frequency components decreased with the reduced velocity when the reduced velocity $U_r < 6$.

Otsuka et al. (1993) investigated the viscous damping forces of a lower-hull slowly oscillating in regular waves and a circular cylinder oscillating with two different frequencies. Even though they illustrated that the low-frequency hydrodynamic coefficients significantly depended on the reduced velocity U_r and the velocity ratio of U_{mW} / U_{mL} , it was not discussed how the low-frequency coefficients vary with the

KC number for wave motion. The added mass coefficients for wave motion and the influence of the high-frequency (or wave-frequency) KC number on the low-frequency force coefficients were not discussed as well.

Downie et al. (1995) carried out experiments on a submerged pontoon-like body using a planar motion generator. The model was forced to sway, heave or in combination of both in still water as well as in the presence of a weak current. The results showed that a transverse mean current had a significant effect on the damping force.

Hu and Koterayama (1996) investigated, both experimentally and numerically, the slow-drift viscous damping forces on a horizontal circular and a horizontal rectangular cylinder in regular waves. The slow-drift damping force coefficients in the Relative-Velocity form of Morison equation were obtained using two methods: oscillating the horizontal cylinder in regular waves and oscillating the horizontal cylinder with dual-frequency in still water. For the horizontal circular cylinder, they found that the slow-drift damping force coefficients given by these two methods were similar; for the horizontal rectangular cylinder, the force coefficients obtained from forced oscillation in regular waves are much larger.

Shafiee-far (1997) conducted extensive tests on a vertical cylinder oscillating in regular waves, and analyzed the hydrodynamic forces using both the Relative-Velocity form of Morison equation and the Independent-Flow form of Morison equation. However, the experimental data in Shafiee-far (1997) is quite scattered and no definitive conclusions can be drawn on how the slow-surfing motion and the

fast wave motion mutually may affect the hydrodynamic coefficients in the Relative-Velocity and Independent-Flow forms of Morison equation.

At present, the hydrodynamic forces acting on a vertical circular cylinder slowly surging in regular waves are still not well understood. The aim of the present experimental study is (1) to better understand how the slow-surfing motion and the fast wave motion mutually affect the hydrodynamic coefficients in two forms of Morison equation, and (2) to provide a comprehensive set of hydrodynamic coefficients for Morison equation so that offshore engineers can better estimate the hydrodynamic loads and damping on offshore floating structures.

1.4 Research objectives

The objectives of this study are: (1) to carry out systematic evaluation of the two forms of Morison equation (Relative-Velocity and Independent-Flow Morison equation) using the data collected in a well-controlled laboratory environment; (2) to understand effects of the low-frequency (slow-drift) motion on the hydrodynamic coefficients for the wave-frequency component; (3) to understand effects of the wave-frequency motion on the hydrodynamic coefficients for the low-frequency (slow-drift) component; (4) to assess the applicability of the Relative-Velocity Morison equation and the Independent-Flow Morison equation. It is hoped that the finding of this study can assist offshore engineers to better model the wave loadings (especially the viscous damping forces) in their designs.

1.5 Outline of the thesis

Including this chapter, this thesis comprises seven parts. The rest of the thesis is organized as follows:

In Chapter 2, the details of experimental setup are introduced, including the water channel / wave flume, instrumented cylindrical force model, the actuator for oscillating the cylinder model, and an integrated Labview program capable of controlling/triggering/synchronizing the various signals. Methods for initial data processing and various calibrations of the cylindrical force model and factors influencing the force measurements are also presented in this chapter.

In Chapter 3, experiments on a cylinder oscillating in still water and a fixed cylinder in regular waves are reported for a wide range of frequencies and amplitudes. The characteristics of in-line forces and the corresponding hydrodynamic coefficients are reported. The motivations of conducting these tests are: 1) to validate the reliability and repeatability of the experimental setup; 2) to provide a comprehensive data set for understanding the results of the problems involving two frequencies discussed in Chapters 5 and 6.

In Chapter 4, the transverse forces are reported for a cylinder oscillating in still water and a fixed cylinder in regular waves. The measurements of the transverse forces were conducted simultaneously with the measurements of the in-line forces reported in Chapter 3. In addition to comparing the transverse forces with the measured in-line forces, the harmonic hydrodynamic coefficients for the lift force, the maximum and root-mean-square as well as the mean hydrodynamic coefficients

for lift force are examined for different values of $\beta = Re/ KC$ number. The aim of this chapter is to have a better understanding of the hydrodynamics around a vertical circular cylinder in oscillatory and wave flows.

The Chapter 5 reports experimental results of hydrodynamic forces acting on a cylinder oscillating with dual-frequency in still water, with the high-frequency simulating the wave motion and the low-frequency the slow-drift motion. The objective of this study is to gain insights into the hydrodynamic characteristics of a cylinder slowly surging in waves. Two forms of Morison equations (the Independent-Flow form of Morison equation and the Relative-Velocity form of Morison equation) are used to analyze the in-line forces. Results of the transverse forces are also presented.

Chapter 6 investigates the hydrodynamic forces acting on a vertical circular cylinder slowly surging in regular waves, with a special emphasis on the viscous drag force induced by both the wave motion and slow oscillation of the cylinder. Experimental results are analyzed using both the Independent-Flow form of Morison equation and the Relative-Velocity form of Morison equation, and compared with those for a cylinder oscillating with dual-frequency in still water reported in Chapter 5. The applicability of these two forms of Morison equation is also discussed.

In Chapter 7, main conclusions in this thesis are summarized and some recommendations for further investigation are given.

Chapter 2

Experimental Facility and Data Acquisition Methods

2.1 Preliminary remarks

In this chapter, the experimental facilities and apparatus, together with the data acquisition and deduction methods are described. In particular, various calibrations undertaken are provided so as to establish the reliability and accuracy of our data. The flow considerations are also discussed to show the experimental conditions under which our experiments were conducted.

2.2 Description of the experimental system

2.2.1 Water channel and wave flume

Experimental works in this study were conducted in a water channel (where water is still) and a wave flume in Hydraulics Laboratory in Nanyang Technological University, Singapore. Two groups of tests were carried out in the water channel and wave flume, respectively: the water channel was used for tests of a forced oscillating cylinder in still water, and the wave flume was used for tests of a fixed or a forced surging cylinder in regular waves.

The water channel used for experiments on hydrodynamic loads acting on a forced oscillating cylinder in still water is 0.55m wide, 0.6m deep and about 36m long. In the forced oscillation tests, the model was located in the middle section of the channel so as to minimize the influence of the two ends.

The dimension of the wave flume used for measuring hydrodynamic loading on a fixed or surging cylinder in regular waves has the dimensions of 1.55m in width, 1.5 m in depth and 45m in length. The flume was constructed of concrete with three glass observation windows. A piston-type wave generator was installed at one end of the flume. A perforated wave absorbing beach was installed on the other end to dissipate wave energy and reduce wave reflection. The wave generator was equipped with an Active Wave Absorption Control System (AWACS) to absorb possible waves reflected from models and the wave absorbing beach. This Wave Synthesizer can generate both regular and irregular waves.

2.2.2 Cylindrical force model

The cylinder model used to measure hydrodynamic forces was constructed using 3-mm thick aluminium pipe. It was segmented into three parts: test section (or test ring), upper and lower dummy sections; all of which were connected by specially designed inner interlocking fixtures (see Figure 2-1 for reference). The test section, which was 50mm in length and 50mm in diameter and had a smooth outer surface, was situated at the mid-length of the cylinder. This test section was chemically treated (coating) to prevent the metal from reacting with water so that its surface was always kept smooth in the experiments. This test section was instrumented with a three-dimensional piezoelectric force transducer, which can measure the hydrodynamic forces acting on this test section in both the in-line (x) and transverse (y) directions. The upper dummy section was 300 mm long and the length of lower dummy section was adjustable. All the three sections were carefully assembled so

that the centre line was aligned and these three components formed an integrated cylindrical force model, which was attached to a stiff frame firmly.

Figure 2-1 Schematic and photograph of the experimental cylindrical model.
(Left: the design of the integrated cylindrical force model; right: a photo of the assembled force model used in the experiments)

Referring to Figure 2-1, the upper and lower dummy model are interlocked and not mechanically connected to the force sensor, so that the force transducer experiences only the local force acting on the test section. The upper and lower dummy sections are separated from the test section by two narrow slits of 1-2 mm in width. The design of the cylindrical force model requires that it should be stiff enough for the applied loads so that the natural frequency of the lowest modes would be well out of the range of interested force frequencies. Impulse response tests (the test section was hit suddenly by a rubber hammer and allowed to vibrate freely) were performed in air. Figure 2-2 shows an example of the responses of natural vibration tests. Impulse response tests showed that the test section connected to the force sensor by the interlocking rod had a natural frequency of around 24 HZ in both two directions (x and y), which is much higher than the highest wave/oscillation frequency (0.88

HZ) used in the experiments. In addition, the natural frequency of the integrated force model was found to be around 10HZ in air and 8 HZ in water, which are also far above the highest force frequency (0.88HZ) in the experiments.

Figure 2-2 Responses of natural oscillations induced by hammer tests.
(a): x-direction; (b): y-direction.

Therefore, the coupling effects between the hydrodynamic force and noise due to the model vibration can be neglected; the high-frequency noise induced by high-frequency vibrations of the model can be removed by applying a low pass digital filter on the measured force signals (see Section 2.4).

In principle, the gaps between the dummy sections and the test section should be sealed (covered) by a flexible thin rubber sheet to prevent any leakage of water into the cylinder model so that the interior of cylinder model is free of water. However,

after several attempts to prevent any leakage of water into the cylinder by using thin rubber skins or other soft waterproof materials failed, we realized that this method had several drawbacks: 1) it was not easy to quantify the small influence of the rubber skin used to seal the gap on the measured force even in the condition of dry calibration (in air); 2) the rubber skin would somewhat affect the surface smoothness. In this investigation, the narrow gaps between the dummy sections and the test section were not sealed, and the influence of trapped water on the measured forces will be discussed in Section 2.3.2.

2.2.3 Data acquisition and signal synchronization

Forced oscillations of the cylindrical force model were obtained using a Belt-driven type linear Actuator (RG-MB40, HISAKA), which has a maximum travelling stroke of 1.0 m. This actuator can be mounted across the wave flume or in the direction of wave propagation. The cylinder force model can be firmly fixed to the movable carrier for oscillating tests. Figure 2-3 is a photo of the Actuator used in this thesis.

Figure 2-3 A picture of the Actuator used in the present study.

In the present experiment, one key issue is to control the cylinder force model to move in a desired manner, and synchronize all signals such as wave propagation, two directional hydrodynamic forces, and instantaneous position of the moving

force model. To achieve the aforementioned requirements, a motion control system that consists of the components shown in Figure 2-4 has been designed and constructed.

Figure 2-4 Flow chart of the motion control system used in the experiments.

- Application software, which controls the desired target position. NI Labview 8.6 and NI-motion 7.7 installed on the main computer can offer all the functions to configure the system and develop the control applications.
- Motion Controller, which is the brain of the system, takes a target position and converts it into a voltage reference; it calculates the trajectory for each controlled move. Furthermore, the motion controller uses the trajectory to determine the information of the proper torque and send it to the motor amplifier and carries out the designed motion. In addition, a 7340-type NI motion control card inserted in the main computer was used to communicate with the motion controller.
- Amplifier or Drive, which generates the current required to drive the motor, takes the voltage supplied by the motion controller and supplies the required current to the motor. The power drive must match the motor technology and also provide the correct peak current, continuous current, and voltage to

drive the motor. A Servo Drive system, which can be capable of driving the motor, was provided by the manufacturer.

- Motor, which converts electrical energy into mechanical energy, produces the required torque to move the force mode to a desired target position.
- Mechanical Elements, including a belt, a movable stage and a shaft, actually carry out the motion in the system (see Figure 2-3).
- Feedback device, which senses a position of the motor and reports the result to the controller, thereby closing the loop to the motion controller. The encoder system provided by the manufacturer can satisfy this demand.

After the model-driving system has been established, the control of the motor is straightforward since its servo controller was custom-built by the manufacturer so as to track an input voltage corresponding to a particular absolute position of the motor shaft. In addition to motion control, timing sequences of all required operations such as triggering of data acquisition/model motion, switching off of model motion (time duration), synchronization of data acquisition/model motion and wave propagation are also of crucial importance for conducting the present precise tests. An integrated master control program used to communicate between the user and controller to generate a desired motion profile of the force model, and to carry out data acquisition was developed by the author using Labview 8.6; its main block diagram is shown below in Figure 2-5.

Figure 2-5 Main block diagram of the Labview program.
(The front panel and the sub-VI for model-driving are not shown).

This main program VI (VI-Virtual Instrument) can dynamically load and run the sub VI for model motion when the specified condition is satisfied (a time delay can be also specified); while the sub VI is loaded, the main VI can continue to run in parallel. This integrated Labview program has the capacity to acquire/synchronize all the signals and dynamically load/run the motion control program. When the cylinder force model was forced to oscillate in waves, the synchronization of the main Labview program with the wave generator was achieved by an Ethernet Interface that can trigger the two applications simultaneously. In all the measurements, the signals such as the motion of the force model, wave surface displacement, and the measured forces are all operated by their respective dedicated signal conditioning devices (i.e. the amplifiers, see Section 2.2.4 for details). The signals from all channels are logged via the NI Data Acquisition Compact (DAQ) to

the main computer for storage and further post processing. The sampling frequencies (F_s) for all signal channels are 500HZ for all the tests.

2.2.4 Wave probes and force sensors

Water surface displacements and the motion of the cylinder force model are measured and monitored by 3 sets of Ultralab sensors. Figure 2-6 shows the photos of the Ultralab sensors and the amplifier.

The sensors were calibrated by the manufacturer. However, before and after the use of these sensors, separate in-house calibrations were also carried out to confirm and obtain the correct factors of voltage/distance, and results indicated that the voltage was linearly proportional to the measured distance. It was also shown that no obvious drifts were found within the past 3 years. An example of typical calibration curve is shown in Figure 2-7 and the factors of voltage/distance for three sensors are listed in Table 2-1.

Figure 2-6 Photos of UltraLab sensors and the amplifier (ULS 40D).
(Left: UltraLab sensors; right: amplifier)

Figure 2-7 A typical calibration curve for an UltraLab sensor (Sensor-1).

Table 2-1 The factors of Volt/Distance for three Ultralab sensors

	Sensor-1	Sensor-2	Sensor-3
Volt/Distance(V/cm)	0.0895	0.092	0.091

Figure 2-8 Photos of force sensor (Kistler-9317B) and its amplifier (Kistler-4618A).
(Left: force sensor; right: amplifier-A)

In addition to the displacement signals, the in-line and transverse force signals were also recorded. The hydrodynamic forces acting on the test section were measured by using a highly-accurate and mechanically-stiff piezoelectric force transducer (Kistler-9317B, 3-axes force transducer), as shown in Figure 2-8. The sensor is constructed from quartz piezoelectric material that builds up an electric charge in response to an externally applied force. The piezoelectric element (quartz plate) yields electric charge that is proportional to the force. The charge signal is transferred via a connector to a charge amplifier, which converts the signals into a proportional analog voltage output. The piezoelectric measurement technology allows it to implement different measuring ranges without loss of accuracy and sensitivity. In this study, the available minimum measuring scale of the used charge amplifier is 0.5N/V and the output voltage ranges are either $\pm 10\text{V}$ (this is especially for the laboratory-type amplifiers); therefore, the possible minimum measuring ranges which can be obtained in the present system are $\pm 5\text{N}$ for F_x and F_y . The typical measured force ranges in this study are: 0~0.5N. Although the measured range is only 1/10 of the effective measuring range, i.e. $\pm 5\text{N}$, as mentioned above, the high sensitivity of piezoelectric force sensors, which is independent of different measuring ranges, still allows accurate measurement of small forces. Since the force transducer has been well calibrated by the manufacturer, it is ready to use after proper installation. However, for our specially designed cylindrical force model (for details, see Figure 2-1), additional calibrations are needed to account for influences such as force moment and water trapped in the cylindrical force model on the measured force acting on the test section. Detailed calibrations including both static and dynamic calibrations in air and in water are depicted in Section 2.3.2.

Both the UltraLab sensors and Kistler force transducer are operated with their respective signal conditioning devices: the amplifier (ULS 40D) for the UltraLab sensors (see Figure 2-6) and the charge amplifiers (Kistler-4618A) for the force transducer (see Figure 2-8). There are totally two charge amplifiers (A and B); the force transducer is connected to the charge amplifier-A via specially developed screened cable, and the two charge amplifiers are linked through circuit cables. The amplifier-A is placed as close as possible to the force transducer; the amplifier-B is placed near the DAQ equipment for the convenience of data acquisition. The analog voltage outputs from both the amplifiers are converted to digital signals via NI CompactDAQ (9172 USB) and fed to the main computer for storage and further processing. CompactDAQ (9172 USB) is equipped with 4 NI 9215 I/O modules (16 channels), and the main computer is equipped with NI Labview 8.6. The data acquisition/synchronization and motion driving/controlling through the integrated Labview program have been discussed in Section 2.2.3.

2.3 Calibration and error estimation

Before conducting experiments, rigorous calibrations were performed for the oscillating system (Actuator) and the specially designed instrumented cylindrical force model. The errors of individual components are estimated according to the calibrations described below.

2.3.1 Calibration of the oscillating system

Before conducting experiments, careful calibrations were performed to assess the accuracy of the Actuator within the ranges of frequencies and amplitudes used in

the experiments. The input displacements and frequencies of the oscillating cylinder force model were compared with those measured by the UltraLab sensors. The calibrations showed that the Actuator performed very well and the relative error in amplitude was accurate to 0.5%~1% throughout the test range (the relative error in amplitude is given by $|A_M - A_T| / A_T$, where A_M is the measured amplitude and A_T is the corresponding target input amplitude). In addition, the oscillation frequency was extremely accurate with its error order of about 10^{-3} during the first several cycles. However, the accumulation of frequency errors exists; for example, in our tests for the fastest oscillation frequency ($f_o = 1/1.2s = 0.833HZ$), the accumulated error in frequency can be around 0.5% at the end of 20 cycles, which means that the phase shift between the measured displacement and the input displacement is about 0.5% at the 20th oscillation cycle. To minimize the effects of the accumulated error, a “time-window” method has been used; the force coefficients were calculated within one “time-window” (a typical “time-window” consists of less than 20 successive cycles) and the coefficients obtained from different “time-windows” were averaged.

2.3.2 Calibration of the cylindrical force model

As mentioned in Section 2.2.4, the Kistler force transducer (9317B) was well calibrated by the manufacturer. However, careful and extensive calibrations were needed so as to account for influences such as force moment and the water trapped in the force model. In this section, extensive static and dynamic calibrations are described.

Extensive static calibrations were carried out in both $\pm x$ (in-line) and $\pm y$ (transverse) directions by placing the cylinder force model horizontally and hanging known weights at the centre of the test section. It was found that the measured force were linearly proportional to the applied weight in both the positive and negative x/y directions. However, due to the influence of the force moment, the measured force is somewhat different from the weight applied to the test section, and a correction factor needs to be applied to obtain the correct force acting on the test section. The linear relation between the applied weight and the measured force before force-moment correction is shown in Figure 2-9 below. It can be seen that the measured force is larger than the applied weight.

Figure 2-9 A typical static calibration curve (+x direction) before force-moment correction.

In addition to static calibrations, extensive dynamic calibrations in the air (dry calibration) were also conducted. Another standard force sensor was used to generate a time-varying force on the centre of the instrumented test section. The force given by the standard sensor was compared with that measured by the instrumented cylinder force model. The dynamic calibrations were carried out in the air at typical frequencies and amplitudes of interest. Four typical dynamic

calibration curves in both $\pm x$ and $\pm y$ directions are shown in Figure 2-10. Results indicated that the frequency responses of the force transducer (9317B) and charge amplifiers (4618A) used in this study were identical to the applied force by the standard force sensor. However, as mentioned in the static calibrations, the amplitude of the measured force should be corrected to account for the influence of the force moment. The moment-correction factor can be determined by comparing the applied and measured forces. The measured forces before and after moment-correction are shown in Figure 2-10.

Figure 2-10 Dynamic calibration curves of the instrumented cylindrical force model.

Continued on the next page.

((a): +x direction, (b): -x direction, (c): +y direction, (d): -y direction)

Figure 2-10 Continued.

Moreover, we also conducted various dynamic calibrations by oscillating the cylinder model in air at desired frequencies and amplitudes. Through the comparison between the measured inertia force time series, which was induced by the model mass due to acceleration and deceleration during oscillations and the calculated theoretical inertia force time series, which was computed by the known model mass timing the acceleration of the cylinder oscillation, the correction factors could also be determined in this way. An example of comparison between measured and theoretical inertia force time series is demonstrated in Figure 2-11.

Based on extensive static and dynamic calibrations (using two kinds of methods described above) in air, the appropriate amplification factors for $\pm x$ and $\pm y$

directions are determined and listed in Table 2-2, together with the highest normalized root-mean-square errors (NRMSE). The NRMSE was obtained by the root-mean-square error between the corrected force and applied standard force normalized by the measured force range.

Figure 2-11 An example of comparison between measured and theoretical inertia force induced by the cylinder force model oscillating in air ($A_o = 0.091m$ and $T_o = 2.0s$, $\pm x$ direction).

Table 2-2 Amplification factors and NRMS error of the cylinder force model

	+X	-X	+Y	-Y
Correction factor	0.86	0.86	0.8	0.8
NRMSE	2.0%	2.2%	2.8%	2.5%

When the cylinder force model was positioned in water, the interior space of the cylinder will be filled by water and the interlocking rod will be submerged in water. Therefore, we should also account for the effect of trapped water. Effects of the trapped water on the measured force are discussed below.

Firstly, the test section and the lower dummy section were removed from the cylindrical force model, i.e. only the interlocking rod was attached to the force sensor (refer to Figure 2-1); the interlocking rod was forced to oscillate (with the

upper dummy section) in air as well as in water with given oscillation amplitudes and frequencies. The forces measured in air and in water were then compared for the same oscillation amplitude and frequency. The highest normalized root-mean-square error (NRMSE) was found to be less than 2%, indicating that the water trapped between the dummy pipe wall and the interlocking rod has negligible influences on force measurements. An example of measured inertia force time series induced by the interlocking rod oscillating in air and oscillating submerged in water is shown in Figure 2-12 below

Figure 2-12 An example of comparison between measured inertia force induced by interlocking rod oscillating in air and in water ($A_o = 0.13m$ and $T_o = 2.0s$, $\pm x$ direction).

We can explain the conclusion that the water trapped between the dummy pipe wall and the interlocking rod has negligible influences on force measurements as follows. When the interlocking rod was forced to oscillate along with the upper dummy pipe in water, the water trapped in the space between the interlocking rod and dummy pipe wall was also forced to oscillate accordingly; as a result, there is no relative movement between the trapped water and the interlocking rod. In fact, the force required to drive the trapped water to accelerate and decelerate is provided by the dummy pipe wall, and therefore the trapped water has a negligible influence on the

force measurements. In addition, due to the gaps between the dummy sections and test section were very tiny (2mm), effects of the flows within the gaps can be negligible.

2.3.3 Overall accuracy of the experimental system

Based on extensive calibrations of the individual components in the experimental system, estimation of the overall accuracy of the experimental system would be desirable. However, due to a large number of factors involved, a value of overall accuracy of the experimental system is difficult to calculate. Even though each individual component in the experimental system described above has a nominal error bound less than 3%, their combinations to give an overall system error bound is still unknown. In addition, many other factors, such as errors in the amplifiers, blockage correction, temperature and purity of water used in the experiment, etc. would also affect the force measurements. As Gopalkrishnan (1993) pointed out, the overall system accuracy cannot be calculated from knowledge of the individual component specifications. Therefore, the overall system accuracy in this study is difficult to specify. However, comparisons of force measurements will be made with some theoretical values and published experimental data in the literature in the corresponding chapters.

2.3.4 Long-term stability of the experimental system

In this study, the experiments were conducted within a span of over 3 years, during which time the apparatus was dismantled, restored and reassembled at least ten times. No evidence showed that these operations caused any significant drifts in the

measurements. In addition, dynamic calibrations (refer to Section 2.3.2) were conducted by a standard force sensor two times (within a span of 2 years) and by using the method of oscillating the cylinder model in air three times (within a span of 3 years); results showed that long-term stability of the experimental system was good.

2.4 Initial data processing

As mentioned above, all the signals from different channels were synchronized and acquired at a frequency up to 500HZ via NI compact DAQ. The acquired data are stored in the Labview measurement (.lvm) format, which is designed so that it can be easily parsed and read when imported into a spreadsheet program, such as Microsoft Excel, or a text editor such as Notepad. In this thesis, all the data were imported into Microsoft Excel and further initial data processing of these data was accomplished by using the in-built Visual Basic Application. Extensive batch programs were coded and in-built functions in Excel were adopted so as to provide for semi-automatic data processing. Some of the important steps in the initial data processing are detailed as follows.

The piezoelectric force transducer used in the experiments essentially is a dynamic measuring instrument; intrinsically, the measured force signal exhibits a small linear drift during force measurements. Moreover, when the force amplifier is switched on (Auto-zero), a tiny “voltage jump” will occur; even though this “voltage jump” is relatively small, it needs to be corrected so as to obtain an accurate force measurement. Therefore, in order to correct these effects, careful determination of the mean zero and the drift slope for each run are needed so that

during post-processing, compensation can be performed to obtain correct raw signals (refer to Figure 3-3 and Figure 3-4 for examples). This procedure was accomplished by the Excel VBA codes developed by the author. Detailed experimental procedures and data post-processing are given in the corresponding chapters.

During the tests, a large variety of sources would cause noises which could contaminate the interested signals, such as structural vibrations, electrical noises, wave maker induced noises, as well as unknown noises from the neighboring equipments in the laboratory, etc. In order to remove the unwanted high-frequency noises from the original signals, a digital low pass filter was utilized to eliminate these effects for further data analysis. Since the phase difference between force signals and motion signals is of vital importance in the determination of hydrodynamic coefficients, in the present investigations, all the unwanted high-frequency signals were filtered out by using the zero phase shift Fast Fourier Transform (FFT) low-pass filter. The cut-off frequency was at least 4 times the interested frequency in each different runs for in-line forces and 6~8 times for the transverse components. The most advantage of the FFT filter is that no phase shift is experienced, which is of critical importance in the force coefficients determinations. Once the high-frequency, undesirable frequencies are eliminated, the inverse transform restores the force time series as filtered signals.

2.5 Flow considerations

2.5.1 End effect

For towing tests or oscillating tests, the end effect may affect the force measurement acting on the test section. In our two-dimensional oscillating tests, the gap between the bottom tip of the lower dummy pipe and the floor of the water channel was about 0.5 cm so as to minimize the 3-dimensional end effect and maintain an almost two-dimensional flow (for more detailed discussions, see Chapter 3). In addition, in wave tests, one good solution is to provide a sufficient long lower dummy cylinder, i.e. the test section is far from the cylinder model end. However, a too long lower dummy cylinder would affect the smoothness of model oscillating in waves. Thus in our tests in wave flows, the lower dummy part of the cylinder model was provided relatively long enough to reduce the end effect (at least 5 times cylinder diameter), which is believed to be long enough to minimize the end effect during oscillations (Yuan and Huang, 2010).

2.5.2 Block effect

Blockage refers to the fact that the measured forces on a cylinder model in a finite body of water may be different from the values expected in an infinite water body, due to the presence of the walls of the wave flume around the model. Therefore, empirical blockage corrections should be applied to the measured forces. These corrections are a function of the equivalent blockage ratio of the model diameter to the width of the water channel. In the present experiments, for the cylinder oscillating tests in water channel, the ratio of the model diameter to the width of the

water channel is 1/11; for wave tests in the wave flume, the ratio is 1/30. In this thesis, no corrections were made for blocking effect in both the tests of the cylinder model oscillating in the still water channel and wave tests in the wave flume.

2.5.3 Residual current effect

For the tests conducted in wave fields, in order to assess the residual current effect on the force measurements at the depth of instrumented section, we performed measurements of hydrodynamic forces on the cylinder model and water surface variation when wave maker terminated after approximately 10 minutes for the test case of maximum wave height and longest wave period. Also these measurements were repeated after longer breaks, i.e. more than 30 minutes and one hour for comparisons. Consequently, these measurements gave nearly the same results as that measured after 10 minutes. No significant residual current was found in the wave flume after wave maker stopped. It is therefore safely concluded that waiting 10 minutes between each runs can be sufficient for the water surface to become calm and the effect of the residual currents is minimal in our wave flume within the test ranges.

2.5.4 Limited wave tank length

Due to the limited length of the wave flume (45m long) and insufficient absorption of incoming waves by absorbing beach, during wave tests, the reflected waves would contaminate the wave field at the location of test model for a longer record. Therefore, the records of the useful time series for hydrodynamic force calculations are limited by the length of the wave tank. For our tests of a fixed cylinder in

regular waves with the maximum wave period ($T_w = 2.0s$), at least 15 wave cycles without contaminated by reflected waves were obtained and all selected signals for data processing were free of any reflected waves; this can be ensured by examining the time series of wave surface displacement. In addition, in tests of the cylinder oscillating in waves, at least 8 oscillation cycles were ensured for accurate data acquisition.

2.5.5 Free surface effect

When a cylinder is towed or oscillates in still water, wakes will be generated on water surface, which consequently disturb the pressure field in the immediate vicinity of the cylinder and the water surface. This pressure disturbance will result in additional force acting on the interested measuring section of the cylinder. One way to reduce the surface wake effect is to use a surface plate to damp any disturbance of the water surface generated by the cylinder movement. However, if the disturbance is slight, it can be neglected. In order to investigate the free surface effect, Bishop and Hassan (1964) have introduced a criterion that when the Froude number is much less than unity, the free surface effect can be ignored. The Froude number is defined by $Fr = U_m / \sqrt{gh_{\min}}$, where U_m is the maximum towing/oscillating velocity, g is the gravitational acceleration, and $h_{\min}$ is the minimum depth of submergence of the test section. In their experiments, the maximum Froude number $Fr_{\max}$ was 0.375, which Bishop and Hassan felt was sufficiently low. In our oscillating tests, the maximum Froude number Fr was about 0.3, so neglecting the effect of free surface is justified.

Chapter 3

In-line Force on a Cylinder Model under Oscillatory Flow and Regular Waves

3.1 Preliminary remarks

Knowledge of wave induced loads is essential in both the reliable design and successful operation of offshore structures. In offshore engineering, Morison equation is widely used in calculating wave loads on slender cylindrical structures. In using Morison equation, two empirical hydrodynamic coefficients, i.e. an inertia coefficient and a drag coefficient, are needed, which are usually determined from experiments. In this chapter, extensive tests with a single frequency, including a cylinder oscillating in still water and a fixed cylinder in regular waves are reported. The experimental setups, experimental conditions and parameter ranges, as well as the experimental procedures are depicted in details. The characteristics of in-line forces and the in-line hydrodynamic coefficients obtained experimentally are reported and discussed in this chapter. The corresponding transverse forces will be discussed in the next chapter. The present experimental results can contribute to further understanding hydrodynamics around cylindrical structures, and also provide a solid base for further studying a surging cylinder in regular waves.

3.2 Load models for in-line hydrodynamic forces

In the process of structural analysis in offshore engineering, one of the most common types of dynamic nonlinearity encountered in the exciting force comes from the quadratic drag force. The nonlinear, steady drag forces due to wind and current are well known (refer to Zdravkovich, 1997). In wavy flows, extending this

force by adding an inertia component and taking into account of reverse of force in a wave cycle, a semi-empirical formula was proposed by Morison et al. (1950), which is commonly known as Morison equation. The Morison equation was introduced to describe the in-line force on a vertical pile subjected to a viscous unsteady flow in a progressive wave, and has been widely used to design offshore structures whose members are exposed to viscous oscillatory flows. The original Morison equation can be written in term of the water particle velocity and acceleration components as

$$F_x = \rho \frac{\pi D^2}{4} C_{MW} \dot{u}_w ds + \frac{1}{2} \rho D C_{DW} |u_w| u_w ds \quad (3-1)$$

where D is the cylinder diameter, $u_w, \dot{u}_w$ are, respectively, the horizontal water particle velocity and acceleration, both normal to the cylinder axis where the wave force acts on; ρ is the density of water, and ds is the length of the cylinder. Equation (3-1) consists of two components: the first term on the right-hand side is the inertia force due to the flow acceleration, and the second term is the drag force proportional to the square of the flow velocity. $C_{MW}(=1+C_{AW})$ and C_{DW} are, respectively, the inertia and drag force coefficients, which are determined experimentally from wave tests. Here, C_{AW} is the added mass coefficient and unit “1” accounts for the hydrodynamic force due to pressure gradient which is required to build up the wave field (Chakrabarti, 1987; Garrison, 1980 and 1990). One of the pioneering works in studying these hydrodynamic coefficients was carried out by Keulegan and Carpenter (1958) on a fixed circular cylinder in standing waves; in their experiments, a systematic approach was considered; they tested horizontal

cylinders at the nodes of a standing wave in a tank so that the cylinders essentially experienced a one-dimensional flow. They successfully showed the dependence of both C_{MW} and C_{DW} on a period parameter (the so-called Keulegan-Carpenter number, KC number in short) defined as $U_{mW}T_W / D$, in which U_{mW} is the maximum horizontal water particle velocity, T_W is the wave period and D is the diameter of the horizontal cylinder. For a fixed cylindrical model exposed to oscillatory flows, extensive research effort had already been conducted by Sarpkaya (1976) to investigate the hydrodynamic coefficients C_{MO} and C_{DO} where the subscript symbol “o” on the inertia and drag coefficients is used to distinguish them from the similar coefficients C_{MW} and C_{DW} of fixed cylinder in wave field. His results show that the hydrodynamic coefficients depend not only on the Keulegan-Carpenter number KC_o ($KC_o = U_{mo}T_o / D$), but also significantly on the Reynolds number ($Re_o = U_{mo}D / \nu$). Instead of using Reynolds number, the hydrodynamic coefficients can be represented as a function of Keulegan-Carpenter number and viscous parameter $\beta_o (= Re_o / KC_o = D^2 / \nu T_o)$, which is defined as the ratio of Reynolds number to the KC number (Sarpkaya, 1976; Sarpkaya and Isaacson, 1981).

Similar to the in-line hydrodynamic loads on a cylindrical model exposed to oscillatory flows, if the model is forced to oscillate in an otherwise still water, the in-line component of the fluid force acting on the cylinder can be written as

$$F_x = -\rho \frac{\pi D^2}{4} C_{AO} \ddot{x} ds - \frac{1}{2} \rho D C_{DO} \dot{x} |\dot{x}| ds \quad (3-2)$$

where $\dot{x}$ and $\ddot{x}$ are, respectively, the velocity and acceleration of the oscillating cylinder, and C_{AO} is the added mass coefficient. It should be noted that the loading on a cylinder oscillating in still water differs from that on a fixed cylinder in oscillatory flow. For the cases where the relative motions between the cylinder and the fluid are the same, the fixed cylinder in an oscillatory flow experiences an additional Froude-Krylov force from the pressure gradient present in an accelerating flow (Chakrabarti, 1987). Garrison (1980 and 1990) theoretically showed that the oscillation of a cylinder in still water is kinematically identical to the oscillation of water past a fixed cylinder. The only difference between a cylinder oscillating in water and a fixed cylinder in oscillatory flow is that in the case of oscillating flow past a fixed cylinder, the pressure includes the additional pressure term whose gradient is just that required to accelerate the flow with an magnitude $\partial U / \partial t$. Therefore, for a cylinder oscillating in still water, only an added mass coefficient is used in Equation (3-2) since there is no longer a pressure gradient required to generate the external flow.

The Morison equation describes the in-line force perpendicular to a slender model. If the proper fluid velocity, acceleration as well as hydrodynamic coefficients can be well determined, the use of Morison equation can yield good results. However, the question under discussion is what are the appropriate values for these two coefficients. Furthermore, the importance of the surface roughness and the orientation of the cylinder, the span-wise coherence, current effects, etc., remains disputed even though a vast amount of experimental works have been published in the past decades. The accuracy of this semi-empirical formula has often been questioned, but a more acceptable theoretically-based mathematical formulation has

not yet been developed. It is easy to criticize the Morison equation, but there is nothing better from a practical point of view that can be proposed presently (Chakrabarti, 1990).

In general, there are two methods to conduct the tests in laboratories to determine hydrodynamic coefficients. In the first method, the member is forced to oscillate harmonically in still water. This method has the versatility to allow independent control over the amplitude and frequency of the forced oscillation. Equivalently, the cylinder may be held fixed and the fluid oscillates harmonically past the cylinder. It is shown that oscillating motion of a circular cylinder in still water is kinematically identical to the oscillating fluid pasting the cylinder fixed and therefore results from both types of experiments are equally applied (Garrison, 1980 and 1990). The latter technique was extensively used by Sarpkaya (1976 and 1986), Maull and Milliner (1978), Bearman et al. (1985) and Obasaju et al. (1988), etc. in a U-tube test apparatus. The method of a cylinder oscillating in still water tests was employed by Honji (1981), Williamson (1985), Justesen (1989), Garrison (1990), Tatsuno and Bearman (1990), Lam and Dai (2002), and more recent Lam et al. (2010) etc. These experimental studies were intended to reveal the flow patterns around the cylinder under oscillatory flow or examine the hydrodynamic loading on the cylinder.

In the second method, the cylinder is held fixed in waves; in this case, the water particle velocity varies in both the horizontal and vertical directions. The waves, although regular, are not necessarily sinusoidal due to some nonlinear effects. The frequency of the regular waves is specified as a constant value while the real

amplitude and shape of the waves are obtained from the measurement of the free surface profile at the location of the cylinder axis. The undisturbed water particle velocity in the wave field is measured by placing a velocity meter beside the cylinder and in line with the center of the cylinder. In the absence of the measured velocity, a wave theory needs to be used, such as Airy, second-order Stokes wave theory, or stream function wave theory (Dean, 1965), to calculate the wave kinematics at the cylinder center for a given surface profile.

Extensive experiments using both oscillatory flows (or oscillating cylinders) and waves have been performed to investigate the hydrodynamic coefficients both in regular and irregular waves. Fundamental differences exist in the nature of the flow between a planar motion and wave motion. For example, a planar motion is strictly one dimensional while the path of water particle in waves is orbital. The development of a boundary layer adjacent to the surface of the cylinder and the pattern of flow separation (including the wake formation) are expected to be different in these two different test methods; the pressure distribution around the cylinder will also be different, resulting in different hydrodynamic forces. In addition, in a wavy flow, the span-wise correlation along the cylinder is not well defined and the coherence of the wake field is disturbed by the orbital motion (Ramberg and Niedzwecki, 1982; Chakrabarti, 1990). Comprehensive reviews of cylindrical models in oscillatory flows and wave fields can be found in Sarpkaya and Isaacson (1981), Chakrabarti (1987 and 1990), Sumer and Fredsoe (2006), Sarpkaya (2010), etc. In the present study, both of the two methods will be used to experimentally examine the hydrodynamic coefficients C_M and C_D . Forced oscillation tests were conducted with various periods and amplitudes, and the in-line

hydrodynamic force is described by Equation (3-2). In the second method, the cylindrical model, which was equipped with a force sensor, was firmly fixed in regular waves with different periods and amplitudes; the in-line wave force acting on the test section is evaluated by Equation (3-1). Based on the experimental measurements, the results are compared with those published in the literature, and the differences between the hydrodynamic loads are discussed. The main purpose of these tests is to validate our cylinder model used to study the hydrodynamics forces in the subsequent chapters and to provide a data base for more complicated problems of a surging cylinder in regular waves.

3.3 Experimental setups

3.3.1 Experimental setup for a cylinder oscillating in still water

Figure 3-1 A schematic diagram of the experimental setup of the forced oscillation test in still water.

(Not drawn to scale)

In the present experiments, the oscillation tests were performed in a small water channel, which was 0.55 m in width and 0.6 m in depth and 36 m in length. The Actuator carriage can be forced to oscillate at any desired frequencies and amplitudes (subjected to the motor capacity) by an in-house Labview program developed by the author (for details, see Chapter 2). The detailed descriptions and calibrations of the instrumented cylindrical force model have been depicted in Chapter 2.

During the experiments, the phase shift between a measured force and the motion profile is of critical importance in determining the hydrodynamic coefficients. Therefore, the motion of the test model attached to the Actuator carriage needs to be synchronized with the force signals. The measurement of the instantaneous displacement of the model was achieved by an UltraLab sensor, which was well calibrated before tests (refer to Section 2.2.4 in Chapter 2). This UltraLab sensor was well aligned with the model in the oscillation direction. Figure 3-1 illustrates the schematic diagram of the experimental setup of the forced oscillation test in still water. The integrated Labview program can trigger the start of the carriage motion and synchronize the data acquisition from all different signal channels, including signals from the force transducer in the in-line (x) and transverse (y) directions as well as the displacement sensor. In the present tests, the water depth was maintained at 0.4 m, and the distance between the still water surface and the upper edge of the test section which was 0.05m long, was 0.25 m so as to minimize the influence of any tiny disturbance at the water surface when oscillating the cylinder in water. The gap between the bottom tip of the lower dummy pipe and the floor of the channel

was about 0.5 cm to minimize possible three-dimensional end effects (refer to Obasaju et al., 1988; Nakamura et al., 1992 and Yuan and Huang, 2010).

3.3.2 Experimental setup for a fixed cylinder in regular waves

To test a fixed cylinder in regular waves, the cylinder model was vertically fixed in the wave flume about 10 m away from the wave generator paddle, which ensured the incident regular waves were well developed. In addition, a sufficient long (5 times of the model diameter) lower dummy part of the cylinder model (below the instrumented test section) was used to reduce possible end effects (see Figure 3-2). The influence of the length of the lower dummy section on wave force measurement was discussed by Yuan and Huang (2010). In their experiments, the flow field was considered to be quasi-two dimensional at the test section with the lower dummy section 0.2 m in length (being 4 times of the model diameter) and the end effects were found to be negligible. It is believed the lower dummy section with its length being 5 times of the model diameter is long enough to minimize possible end effects.

The water surface displacement was measured by two UltraLab sensors. One sensor was placed upstream of the model (about 2.0 m from the model) to monitor and measure the undisturbed incident waves. Another one was positioned 0.5 m away from one side of the cylinder, and the line through the centres of the model and the sensor was parallel to the wave crest. Note that special care was taken to align the centre of the side UltraLab sensor (the Sensor-1 shown in Figure 3-1) with the model centre line since the phase shift between the force signals and wave surface is of critical importance in determining hydrodynamic coefficients.

Figure 3-2 A schematic diagram of the experimental setup for a fixed cylinder in regular waves.
(Not drawn to scale)

3.4 Ranges of experimental parameters

Extensive tests were performed for oscillating the cylinder in still water: a combination of nine periods (T_o) and several amplitudes (A_o), subject to the capacity of the motor power of the Actuator system. The tests were carried out for a constant water depth of 0.4m, as described in Section 3.3 throughout the entire range of the tested periods. For tests of a fixed cylinder model in regular waves, three wave periods T_w and a range of wave amplitudes A_w were tested. In the present experiments, the selection of the wave amplitudes was subjected to both the capacity of the wave generator and the cylindrical model: since the force sensor should be kept “dry” at all the times, an extremely large incident wave would overtop and damage the force sensor. Two water depths were tested: 0.75 m and 0.95 m; the distances between the centre of the instrumented test section and the

still water surface were 0.14 m and 0.17 m, respectively, in these two water depths. The ranges of the parameters are listed in Table 3-1 for a cylinder oscillating in still water and Table 3-2 for a fixed cylinder in regular waves.

Table 3-1 Experimental parameter ranges for a cylinder oscillating in still water

Period T_o (s)	Amplitude A_o (m)	$KC_o =$ $2\pi A_o / D$	$Re_o = U_{mo} D / \nu$	$\beta_o = Re_o / KC_o$ $= D^2 / (\nu T_o)$
1.2	0.039~0.065	4.9~8.2	10210~17017	2083
1.6	0.039~0.091	4.9~11.4	7658~17868	1563
2.0	0.039~0.117	4.9~14.7	6126~18378	1250
2.4	0.0455~0.117	5.7~14.7	5956~15315	1042
3.0	0.0455~0.156	5.7~19.6	4765~16336	833
4.0	0.039~0.195	4.9~ 24.5	3063~15315	625
5.0	0.0455~0.234	5.7~29.4	2859~14703	500
6.0	0.0455~0.234	5.7~29.4	2382~12252	417
8.0	0.065~0.338	8.2~42.5	2552~13272	313

Table 3-2 Experimental parameter ranges for a fixed cylinder in regular waves

Period T_w (s)	Amplitude A_w (m)	$KC_w =$ $U_{mw} T_w / D$	$Re_w = U_{mw} D / \nu$	$\beta_w = Re_w / KC_w$
1.2	0.02~0.08	1.7~7.2	3427~15012	2083
1.6	0.02~0.09	2.2~10.5	3433~16412	1563
2.0	0.02~0.1	2.7~13.7	3406~17166	1250

3.5 Experimental procedures

3.5.1 Experimental procedure for a cylinder oscillating in still water

The experimental procedure is as follows,

1. Switch on the amplifiers of the force transducer and the Ultralab sensor.
2. Auto-zero the amplifier of the force transducer before data logging.

3. Start the data logging; 3 seconds will be lasted before starting to oscillate the cylinder.
4. Oscillate the cylinder at a specified period T_o and amplitude A_o for each run.
5. Stop automatically the oscillation after a specified number of oscillation cycles.
6. Continue data logging for additional at least 20 seconds.

For both the in-line and transverse force signals, as depicted in Section 2.4 of Chapter 2, the slope of the steady drift should be removed to recover the real signal. The recorded data within the first 3 seconds before starting the oscillation and the data recorded within 20 seconds after stopping the oscillations were used to determine the slope of the steady drift, which was used to recover the real force signal (e.g. see Figure3-3(b) and (d)). In addition, for each test run, when the force amplifier was switched on (Auto-zero), a tiny “voltage jump” would always occur. Even though this “voltage jump” was usually small compared to the real force signals, this “voltage jump” should also be compensated to ensure acquire accurate force signals. This was particularly important when the force magnitude was small, especially for the measured transverse forces where a mean component always exists in most of test conditions (for details, see Chapter 4). After the raw voltage signals were acquired, the factor of “voltage (Volt)/force (N)”, as well as the calibrated “correction factor” (for details, see Chapter 2) were applied to obtain the measured forces.

As for motion signals, the motion signal was used as a reference signal in determining the model velocity and acceleration, and subsequently in the

calculation of the hydrodynamic force coefficients. The motion signals were acquired by using an Ultralab sensor via the UltraLab amplifier. The mean value within the first 3 seconds (during that time, the model was stationary) was subtracted from the time series of the motion profile, and the absolute motion profile was calculated in terms of the well calibrated “voltage (Volt)/distance (m)” factor (for details, see Chapter 2). Samples of the measured and the compensated oscillation profile, in-line and transverse raw force signals, as well as the filtered signals for a cylinder oscillating in still water are shown in Figure 3-3.

Figure 3-3 Samples of the measured and corrected oscillation profile, the in-line and transverse forces as well as the filtered forces for a cylinder oscillating in still water.

$$(T_o = 1.6s, A_o = 0.091m \text{ and } KC_o = 11.4)$$

3.5.2 Experimental procedure for a fixed cylinder in regular waves

The experimental procedure for testing a fixed cylinder in regular waves is straightforward since there is no need to trigger the motion of the model. The key steps are listed below:

1. Switch on the Amplifiers of the force transducer and the Ultralab sensor.
2. Auto-zero the amplifier of the force transducer before data logging.
3. Start the data logging and wave generator simultaneously.
4. Record the force signals and the wave surface signals of the regular wave for a number of specified wave cycles.
5. Continue data logging for additional 30 seconds.

Similar to the tests of a cylinder oscillating in still water, for each run in wave tests, the steady drift and “voltage jump” were also corrected and compensated. The calm water surface was used as the reference level for wave surface displacement calculation. Therefore, a sufficient long time record of the undisturbed water surface was recorded during data acquisition; in the present experiments, at least 5~10 seconds were allowed to record the undisturbed water surface before the wave passed the cylindrical model. Samples of the measured and compensated water surface profile, the in-line and transverse raw force signals as well as the filtered signals for a fixed cylinder in regular waves are shown in Figure 3-4.

Figure 3-4 Samples of the measured and corrected water surface profile, the in-line and transverse forces, as well as the filtered forces for a fixed cylinder in regular wave tests.
 $(T_w = 1.6s, A_w = 0.07m \text{ and } KC_w = 8.0)$

3.6 Data analysis and the determination of hydrodynamic coefficients

This section discusses the data analysis and the methods used to calculate the hydrodynamic coefficients for a cylinder oscillating in still water and a fixed cylinder in regular waves.

3.6.1 A cylinder oscillating in still water

Since the cylinder model was forced to oscillate at given amplitude and frequency, the measured motion profile of the model was compared with the theoretical displacement of the cylinder; the experimental results showed that the agreement was quite well for all runs (the frequency and amplitude error analysis was given in Chapter 2). When both the in-line and transverse force became steady (e.g. refer to Figure 3-3(e)), the data can be used for further data processing. Since the displacement x was specified as a simple harmonic motion, the calculations of the cylinder velocity $\dot{x}$ and the acceleration $\ddot{x}$ were straightforward. In addition, the inertia force associated with the model mass during oscillation was subtracted from the total measured force so as to obtain the net hydrodynamic force acting on the model.

3.6.2 A fixed cylinder in regular waves

Since the wave absorber at another end of the wave flume cannot absorb the incident waves perfectly, the reflected waves would affect and contaminate the force measurements after running the wave generator for a long time. When analyzing the data, the portion of the wave force signals that are free of the contamination by the reflected wave should be selected for further processing; this can be insured by examining the measured surface displacement (e.g. see Figure 3-4(a)). For the present experiments (see Table 3-2), according to Le Mehaute (1976), the linear (Airy) or 2nd order Stokes wave theory should be appropriate to calculate the water particle kinematics. In this study, the water particle kinematics was calculated using an appropriate wave theory.

3.6.3 Calculation of hydrodynamic coefficients

For a cylinder oscillating in still water, the hydrodynamic coefficients can be obtained by the Fourier averaging technology (Keulegan and Karpenter, 1958; Sarpkaya and Isaacson, 1981). In this case, it is assumed that the force can be represented as a Fourier series containing both the drag and inertia terms. In an oscillatory flow, where the water particle velocity is assumed to be $u(t) = U_m \cos \theta$, with $\theta = 2\pi t / T = \omega t$, the hydrodynamic coefficients are considered time-invariant within one cycle and the Fourier averages of C_M and C_D have been given by Keulegan and Karpenter (1958) as follows:

$$\begin{aligned} C_M &= -(2U_m T / \pi^3 D) \int_0^{2\pi} F_m \sin \theta / (\rho U_m^2 DL) d\theta \\ C_D &= 0.75 \int_0^{2\pi} F_m \cos \theta / (\rho U_m^2 DL) d\theta \end{aligned} \quad (3-3)$$

where F_M is the measured force signal.

In the present analysis, the experimental data were mainly analyzed using the least square method, and the hydrodynamic coefficients were also considered as time-invariant over one cycle. In the least square method, the hydrodynamic coefficients are found by minimizing the global error between the measured force and the prediction by Morison equation. The best fit between the data and the prediction by Morison equation occurs when the error, defined by the sum of the squares of the differences between each data point value calculated from Morison equation and its corresponding measured value, is a minimum (Dunn, 2005). A thorough derivation of the expressions for the hydrodynamic coefficients using the least square method

is given in Appendix-B. This least square method is much general and can be applied in cases where the water particle velocity is not necessarily sinusoidal. A Matlab program was developed by the author to calculate the hydrodynamic coefficients by the least square method for both a cylinder oscillating in still water and a fixed cylinder in regular waves.

Sarpkaya (1976) and Sarpkaya and Isaacson (1981) showed that these two methods (Fourier averaging and least square) could yield the exactly same C_M value, but the C_D values could differ slightly. In this thesis, the experimental results also showed that both methods yielded the same added mass coefficient; but the drag coefficients obtained by the Fourier averaging method were slightly larger than those by the least square method. Because the least square method is much general and can be used for complicated flow fields, such as highly nonlinear waves, moving cylinder in waves, etc, only force coefficients obtained by the least square method are presented and discussed in this thesis.

3.7 Experimental results and discussion

3.7.1 Added mass and drag coefficients for a cylinder oscillating in still water

Sarpkaya (1976) and Sarpkaya and Isaacson (1981) have shown in sinusoidal flows, the hydrodynamic coefficients depend on both KC number ($KC_o = U_{mo}T_o / D$) and Reynolds number ($Re_o = U_{mo}D / \nu$); alternatively, the hydrodynamic coefficients can be represented as a function of the KC number and a viscous parameter, which is defined as the ratio of Reynolds and KC number, $\beta_o (= Re_o / KC_o = D^2 / \nu T_o)$. For a given model size and fluid, β is strictly a function of flow frequency (in

small-scale laboratory tests, the β parameter typically takes a value in the range of $10^2 \sim 10^3$). Our tests on a cylinder oscillating in still water were conducted at 9 frequencies, i.e. $T_o = 1.2s, 1.6s, 2.0s, 2.4s, 3.0s, 4.0s, 5.0s, 6.0s$ and $8.0s$. The hydrodynamic coefficients (averaged at least 40 cycles) obtained for 9 β_o values, namely 2083, 1563, 1250, 1042, 833, 625, 500, 417 and 313 are shown in Figures 3-5~3-13 below, together with comparisons with available data published in the literature.

Figure 3-5 In-line hydrodynamic coefficients C_{AO} and C_{DO} for a cylinder oscillating in still water with $T_o = 1.2s$.

Figure 3-6 In-line hydrodynamic coefficients C_{AO} and C_{DO} for a cylinder oscillating in still water with $T_O = 1.6s$.

Figure 3-7 In-line hydrodynamic coefficients C_{AO} and C_{DO} for a cylinder oscillating in still water with $T_O = 2.0s$. Continued on the next page.

Figure 3-7 Continued.

Figure 3-8 In-line hydrodynamic coefficients C_{A0} and C_{D0} for a cylinder oscillating in still water with $T_O = 2.4s$.

Figure 3-9 In-line hydrodynamic coefficients C_{AO} and C_{DO} for a cylinder oscillating in still water with $T_O = 3.0s$.

Figure 3-10 In-line hydrodynamic coefficients C_{AO} and C_{DO} for a cylinder oscillating in still water with $T_O = 4.0s$. Continued on the next page.

Figure 3-10 Continued.

Figure 3-11 In-line hydrodynamic coefficients C_{A0} and C_{D0} for a cylinder oscillating in still water with $T_O = 5.0s$.

Figure 3-12 In-line hydrodynamic coefficients C_{AO} and C_{DO} for a cylinder oscillating in still water with $T_o = 6.0s$.

Figure 3-13 In-line hydrodynamic coefficients C_{AO} and C_{DO} for a cylinder oscillating in still water with $T_o = 8.0s$. Continued on the next page.

Figure 3-13 Continued.

Since Sarpkaya (1976) and Sarpkaya and Isaacson (1981), the in-line hydrodynamic coefficients are mainly dependent on KC number and β number; the in-line hydrodynamic coefficients in this study are also presented against KC_0 number for different β_0 numbers. Some of the inertia coefficients ($C_{AO} + 1$) and drag coefficients C_{D0} have been compared with other experimental results (Sarpkaya, 1986; Obasaju et al., 1988), and results are generally in reasonable agreement with their data at approximate β numbers; However, some small discrepancies exist between the present results with their results. For example, at $\beta_0 = 1042$ (refer to Figure 3-8), our drag coefficients C_{D0} are slightly lower than the results of Sarpkaya (1986) within the test range; also in the range of $10 < KC_0 < 12$, our added mass coefficients C_{AO} are somewhat higher than his results. In addition, at $\beta_0 = 500$ (refer to Figure 3-11), in the range of $KC_0 > 12$, our drag coefficients drop more rapidly and the added mass coefficients are a little higher than the data in Obasaju et al. (1988). In order to resolve these discrepancies, a large number of tests were repeated in these parameter ranges within about 6 months. The

repeatability of our data is well and the relative uncertainty (standard deviation divided by mean value) is less than 5%. The results presented in Figures 3-5~3-13 were averaged over many different runs conducted at different times.

It should be kept in mind that all the referred data and our data were calculated by the least square method, where the calculated hydrodynamic coefficients are very sensitive to the phase shift between the force and the water particle velocity (i.e. the model oscillation velocity $\dot{x}$) signals. In the present experiments, both force and model oscillation signals were strictly synchronized and more attentions were paid to ensure the same starting points of the selected two signals when performing the least square analysis. In our calculations, careful comparisons were conducted between the input motion profiles with the measured motion profiles to ensure the correctness in the calculations of the model velocity and acceleration in the least square analysis. In order to avoid the accumulation of the tiny error in frequency (for details, see Section 2.3.1 of Chapter 2), a “time-window” procedure was adopted; the least square fitting was performed within a time window containing 5~10 cycles.

The experimental systematic errors in this study have been discussed in details in Chapter 2 (see Section 2.3.1). However, random errors differ from each other due to random, unpredictable variations in the measurement process. Random errors can be reduced by repeating those measurements. In this thesis, each presented data point was obtained by repeating the experimental measurements at least three times, and the random errors were then analyzed by statistic methods. The random errors are quantified by the standard deviation normalized by the mean, and for all the

presented data points, the standard deviation normalized by the mean are less than 5%. Based on the laboratory observations during experiments and discussion of other researchers (Garrison, 1990), the following sources may have contributed to the small difference between our experimental data and the data found in the literature:

1) Difference in the experimental parameters used for comparison.

The hydrodynamic coefficients are very sensitive to the β_o value; however, our β_o values are not identical to those used for comparison.

2) Difference in the experimental settings.

Obviously, the experimental settings may have some influence on the results. The present experiments were conducted for a cylinder oscillating in still water and other experiments were conducted in U-tubes. The water channel used in the present study is over 30 m long and thus can be considered to be unbounded in the direction of oscillation; however, the U-tube is a bounded facility, as stated by Garrison (1990):

The U-tube tunnel walls may limit the influx of momentum into the wake so that the wake-velocity profile is hindered, and therefore the wake is more pronounced than it would be in unbounded flow. The return flow in the second half-cycle of the motion generates an unnaturally strong wake effect and associated drag force resulting in values of the drag coefficient that are apparently large.

3) Other factors.

Furthermore, the differences in water temperature and purity of water, facility dimensions (block ratio) may also have little influence on the results.

Generally, the added mass and drag coefficients in the present study agree favorably with other published data, even though there are some small discrepancies between them. These discrepancies may be due to the error sources listed above.

The in-line hydrodynamic coefficients for different periods T_o are presented in Figure 3-14 for a comprehensive comparison. The KC_o number of our data ranges from around 5 to 42; the highest experimental KC_o numbers for different oscillation periods are different due to the limited power capacity of the model-driving system.

From Figure 3-14, it can be seen that, in the range of $5 \leq KC_o \leq 8$, the added mass coefficients C_{AO} are weakly dependent on β_o at a given KC_o number, showing that higher β_o number (Re_o number) yields a slightly higher C_{AO} value; however, the drag coefficients C_{DO} are remarkably influenced by β_o number (Re_o number): at a given KC_o number, the lower β_o number (Re_o number), the higher drag coefficient C_{DO} is. This indicates that at a given KC_o number, for tests with lower β_o numbers (i.e. Re_o numbers), the drag force is more predominant than the added mass (or inertia) force. This finding is in agreement with Garrison (1990) and Sarpkaya (1976). However, due to the different ranges of Reynolds numbers, it is difficult to have a direct comparison.

For all β_o values, the added mass coefficients C_{AO} generally decrease with increasing KC_o number for $8 \leq KC_o \leq 16$; this may be attributed to the strong

vortex shedding. The reduction in C_{AO} is so obvious and large that around $10 \leq KC_o \leq 15$, the added mass coefficients C_{AO} may take on even negative values for some β_o values. This is commonly referred as “inertia crisis” (Sarpkaya, 1976; Chakrabarti, 1990; Sumer and Fredsoe, 2006); accordingly, the drag coefficient C_{DO} assumes a largest value (“drag crisis”) in this regime. For a given β_o value, the drag coefficient C_D rises rapidly to a maximum; accordingly added mass coefficient C_{AO} drops to a minimum. Furthermore, from Figure 3-14(b), it can be found that the largest C_D is also obviously dependent on β_o numbers; the higher β_o number, the lower largest C_{DO} is. For example, the largest drag coefficient C_{DO} can reach a value of about 2.2 for $\beta_o = 313$, but the largest C_{DO} value is only about 1.8 for $\beta_o = 833$.

For $KC_o > 16$, the drag coefficients C_{DO} start to drop only slightly with increasing KC_o number, and the added mass coefficients also do not vary a lot. For most test cases, the added mass coefficients C_{AO} are positive, i.e. the “inertia crisis” is less profound when $KC_o > 16$.

For $KC_o \geq 25$, the flow can be considered as quasi-steady flow and the drag force dominates over the inertia force (Laya et al., 1984). In fact, according to Obasaju et al. (1988), the regime of $22 \leq KC_o \leq 30$ is the third-vortex regime and the regime of $KC_o \geq 30$ is called the quasi-steady regime. Lam et al. (2010) showed that when $KC_o > 24$, vortices are shed behind the moving cylinder, similar to the case of a towed cylinder in still water (for details, refer to Section 4.4 in Chapter 4). In this

situation, with increasing KC_o number, the drag coefficients C_{Do} tend to the values for the steady flow (Laya et al., 1984). Extensive studies have been conducted about the flow and force characteristics of steady flow past circular cylinders, Comprehensive reviews have been given by Zdravkovich (1997) and Sumer and Fredsoe (2006). For steady flow past a smooth circular cylinder, throughout the subcritical Reynolds number range (about $300 < Re < 3 \times 10^5$), the mean drag coefficient $\bar{C}_D$ assumes a practically constant value, namely, $\bar{C}_D = 1.2$. Our obtained drag coefficients C_{Do} , at larger $KC_o (\geq 25)$ number, especially for tests of $\beta_o = 313$, are in well agreement with the widely recognized drag coefficient value of $\bar{C}_D = 1.2$; this also validates the reliability of our experimental data (the $Re_o = U_{mo}D/\nu$ numbers in the present tests well lie in the subcritical Reynolds range).

For the added mass coefficients presented in this chapter, negative added mass (“inertia crisis”) were found in some test cases. A qualitative explanation is proved below based on the experimental observations.

When the cylinder reaches the extreme excursion in each oscillation cycle, its velocity is zero but its acceleration reaches its maximum. When the cylinder experiences acceleration in still water, it also experiences an additional added mass force which is proportional to the relative acceleration between the cylinder and the water, inducing the “added mass”. However, in some runs, due to the strong vortex shedding behind the cylinder model, the shed vortices will affect the movement of the adjacent water particle around the cylinder, especially when the cylinder reaches

its extreme excursion. In this situation, when the cylinder reaches its extreme excursion and starts to accelerate, the strong vortices shed from the cylinder makes the adjacent water particles around the cylinder move in the direction opposite to the direction of acceleration, as shown in Figure 3-15, causing a viscous drag force which cancels out part of the added mass force, and thus the “inertia crisis” occurs. This is in particular obvious in the range of about $10 \leq KC_o \leq 15$. For even higher KC_o numbers (i.e. larger oscillation amplitudes A_o), the shed vortices are mostly decayed and much weak at the extreme excursion of the cylinder oscillation; therefore, have less influence on the movement of the water particles around the cylinder, and no negative added mass is caused.

Figure 3-14 In-line hydrodynamic coefficients versus KC_o number for all β_o values.

Continued on the next page.

Figure 3-14 Continued.

Figure 3-15 PIV water particle velocity vectors around the cylinder model at extreme excursion, from Lam and Dai (2002) with $KC_o = 12$ and $\beta_o = 200$.

(Left figure: the cylinder velocity is zero and would be accelerated to the neutral position on the right side, right figure: the cylinder velocity is zero and would be accelerated to the neutral position on the left side)

3.7.2 Root-Mean-Square(RMS) coefficients for a cylinder oscillating in still water

An alternative way to present the hydrodynamic coefficients is to calculate the root-mean-square value of the measured in-line force. Accordingly, the root-mean-square hydrodynamic coefficients can be obtained by

$$C_{F_{rms}} = \frac{F_{rms}}{0.5\rho DU_{mO}^2 L} \quad (3-4)$$

As shown in Equation (3-5), the root-mean-square value of the measured in-line force in Morison equation may be shown to be a function of C_M and C_D , as well as KC number (e.g. see Bearman et al., 1985 and SarpKaya, 1986). A detailed derivation of Equation (3-5) is given in the Appendix -A.

$$C_{F_{rms}} = \left(\frac{3}{8} C_D^2 + \frac{\pi^4 C_M^2}{2 KC^2} \right)^{\frac{1}{2}} \quad (3-5)$$

The values of $C_{F_{rms}}$ calculated using the measured in-line force for all β_o values are presented in Figure 3-16, together with the values of $C_{F_{rms}}$ calculated using Equation (3-5) with the inertia coefficients $C_{MO} = (C_{AO} + 1)$ and C_{DO} values in Figure 3-14. It should be pointed out that when calculating F_{rms} using the measured in-line force, the force component induced by pressure gradient, which is absent in the forced-oscillation tests in still water, was artificially included for the convenience of comparisons. Furthermore, the inviscid theoretical values of $C_{MO}=2$ and $C_{DO}=0$ (Sarpkaya, 1986; Dean and Dalrymple, 1991; Sumer and Fredsoe, 2006)

were used to calculate the root-mean-square coefficient $C_{Frms}(=\sqrt{2\pi^2 / KC_o})$ for comparisons.

From Figure 3-16, it can be seen, for all of our tests, that the values of C_{Frms} obtained using the two methods are in general good agreement, suggesting that the present coefficients $C_{MO}=(C_{AO}+1)$ and C_{DO} were accurately obtained from the corresponding measured in-line force time series. When $KC_o \geq 5$, the measured $C_{MO}=(C_{AO}+1)$ is slightly smaller than 2, which is the value for inviscid potential flows; when C_{Frms} is calculated using Equation (3-5), even though the small drag coefficient C_{DO} would also contribute to the value of C_{Frms} , the value of C_{Frms} is still slightly smaller than the value for inviscid potential flows. In the inertia-drag dominated regime (around $5 \leq KC_o \leq 25$) and the drag-dominated regime (around $KC_o \geq 25$), the C_{Frms} gradually overtakes the corresponding theoretical value for inviscid flows, indicating that the drag force becomes more and more predominant. In the inertia-drag dominated regime, the values of C_{Frms} for smaller values of β_o were slightly larger than those for larger values of β_o . A comprehensive comparison between the values of C_{Frms} for all values of β_o is shown in Figure 3-17. Moreover, in the drag-dominated regime, the value of C_{Frms} would approach a constant around 0.75 for the present tests (e.g. see Figure 3-16 (g), (h) and (i) and Figure 3-17).

Figure 3-16 In-line root-mean-square coefficients C_{Frms} versus KC_O number for all β_O values.
Continued on the next page.

Figure 3-16 Continued.

Figure 3-16 Continued.

Figure 3-17 Comparison of the C_{Frms} coefficients for all β_o values.

3.7.3 Inertia and drag coefficients for a fixed cylinder model in regular waves

For a vertical fixed cylinder in wavy flows, the flow velocity is not always perpendicular to the cylinder axis. The water particle path is orbital, which can be characterized by the ratio of the vertical to horizontal velocity amplitudes $\Omega = V_{mW} / U_{mW}$, with V_{mW} and U_{mW} being the vertical and horizontal velocity amplitude, respectively. As stated by Ramberg and Niedzwecki (1982) and Chakrabarti (1990), the instantaneous velocity and acceleration in a wave field will have an axial variation, and the wave flow is not always uniform along the cylinder, which can also induce significant three-dimensional effects on the hydrodynamic force. The water particle velocity and acceleration in a wavy flow are not explicitly known, and they are usually computed by a wave theory with a given wave

measurement. Since different wave theories might produce slightly different hydrodynamic coefficients for the same data, Chakrabarti (1980) suggested that the user should be consistent in the selection of wave theory. In the present calculations, as mentioned in Section 3.6, 2nd Stokes wave theory is appropriate to compute water particle velocities and accelerations.

Due to the limited capacity of the wave maker, the maximum values of KC_w in the wave tests are relatively small compared to that in the tests on a cylinder oscillating in still water: around 7 for $T_w = 1.2s$, about 12 for $T_w = 1.6s$, and around 14 for $T_w = 2.0s$. For the convenience of a better comparison, the experimental data from the tests on a cylinder oscillating in still water at the same β number are also included in Figures 3-18 to 3-20. For all of the tests for a fixed cylinder in regular waves (three periods $T_w = 1.2s, 1.6s$ and $2.0s$), the inertia force dominates the drag force when $KC_w \leq 5$, and the measured inertia coefficients C_{MW} vary from 1.97 to 2.02, very close to the value for potential flow ($C_M = 2$). (Dean and Dalrymple, 1991; Sumer and Fredsoe, 2006). The favourable agreement of the measured C_{MW} with the theoretical value can also serve as a way to validate the accuracy of the present measurements. For $KC_w \leq 5$, the drag force is much smaller compared to the inertia force, and it is therefore difficult to obtain drag coefficients C_{DW} accurately, especially for large values of β_w . In the present tests, the critical phase shift between the force signal and the wave surface was also accurately measured to determine hydrodynamic coefficients accurately; this was achieved by the well alignment between the wave probe and the cylinder centre, as well as the high

sampling frequency (up to 500HZ). Therefore, the values of drag coefficients C_{DW} , even though they are small, are also acceptable and reliable.

Figure 3-18 Comparison of the in-line hydrodynamic inertia and drag coefficients between a cylinder oscillating in still water ($T_o = 1.2s$) and a fixed cylinder in regular waves ($T_w = 1.2s$).

Figure 3-19 Comparison of the in-line hydrodynamic inertia and drag coefficients between a cylinder oscillating in still water ($T_o = 1.6s$) and a fixed cylinder in regular waves ($T_w = 1.6s$).

Figure 3-20 Comparison of the in-line hydrodynamic inertia and drag coefficients between a cylinder oscillating in still water ($T_o = 2.0s$) and a fixed cylinder in regular waves ($T_w = 2.0s$).

Continued on the next page.

Figure 3-20 Continued.

The comparison between hydrodynamic coefficients obtained from the tests for a fixed cylinder in waves and the tests for a cylinder oscillating in still water showed that the hydrodynamic coefficients for wavy flows were different from those for oscillatory flows: for the same values of KC number and β , the drag coefficient C_D is slightly smaller and the inertia coefficient C_M is slightly larger for orbital flows. This finding appears to be in favorable agreement with that reported by Stansby et al. (1983) who compared the drag and inertia coefficients from wave flume tests (the ellipticity of the orbital motion $\Omega = V_m / U_m$ varies from 0.3~0.9, the Reynolds number range of $0.4 \sim 1.9 \times 10^4$ and KC number range of around 3~25) with the results of Sarpkaya (1976) for oscillatory flows with $\beta_o = 784$. Generally, in the range of around $8 \leq KC \leq 20$, their drag coefficients for wave flume tests are smaller than those found in the tests for oscillatory flows, but the inertia coefficients for wave flume tests are larger than those found in the tests for oscillatory flows. For the present experimental results with $T_w(T_o) = 1.6$ and $2.0s$ (see Figures 3-19 and 3-20), when the KC number is greater than about 7~8, the difference between

the hydrodynamic coefficients obtained using these two flows becomes significant, especially for inertia coefficients, i.e. the “inertia crisis” in two-dimensional oscillatory flows is much pronounced than that in wavy flows.

Figure 3-21 Comparison of the inertia and drag coefficients for a cylinder oscillating in still water and a fixed cylinder in regular wave with other published data.

Figure 3-21 compares the present results with those of Charkrabarti (1990) for a fixed cylinder in waves under $\beta_W = 1085$ and those Sarpkaya (1976) for a cylinder oscillating in still water under $\beta_O = 1107$. It is obvious that, the “inertia crisis” is much less pronounced for wave flume tests; the difference between the two sets of the results may be largely attributed to the three-dimensional flow effects in wave

flume tests. To facilitate further comparisons, our results with the value of β close to theirs are also included in Figure 3-21; our data also support the conclusion that the orbital motion of water particle in wave field could increase the inertia coefficient C_{MW} and decrease the drag coefficients C_{DW} .

“It is usually recognized that the span-wise coherence of vortices shed behind the cylinder is less evident in waves than in oscillatory flow. This randomness of the shedding of vortex along the length of the cylinder has the effect of increasing the value of the inertia coefficient, C_M (for KC larger than 10 and $Re < 10^5$) There is a corresponding decrease in the value of the drag coefficient, C_D (Charkrabarti, 1990, page 114).

3.7.4 Root-Mean-Square coefficients for a fixed cylinder in regular waves

Now we examine the root mean square in-line force coefficients $C_{F_{rms}}$. It can be seen from Figure 3-22 that when $KC_w < 4$, especially for the cases of $T_w = 1.2$ and $1.6s$, the values of $C_{F_{rms}}$ agree well with the value for inviscid flows, indicating that the present experimental system has small system errors.

Figure 3-22 In-line-root-mean square coefficients C_{Frms} for a fixed cylinder in regular waves.
($T_w = 1.2s, 1.6s$ and $2.0s$).

3.8 Goodness-of-Fit of Morison equation

In this section, the applicability of both forms of Morison equation, Equation (3-1) and (3-2), is assessed. Both Morison equation, the statistic R-square (R^2), as defined in Equation (3-6), is used to assess the goodness-of-fit of these two equations to the measured data; this parameter measures how successful an equation can fit the measured data. R-square is the square of the correlation between the measured values and the predicted values, as defined by

$$R^2 = 1 - \frac{\sum (F_{Mi} - F_{Pi})^2}{\sum (F_{Mi} - \bar{F}_M)^2} \approx 1 - \frac{\sum (F_{Mi} - F_{Pi})^2}{\sum (F_{Mi})^2} \quad (3-6)$$

where F_{Mi} is a measured data point, F_{Pi} is the corresponding value predicted using the equation, and $\bar{F}_M$ is the mean of all measured values of F_{Mi} . In the present experiments, $\bar{F}_M$ is almost zero. The value of R-square can take on any value between 0 and 1, with a value closer to 1 indicating that a greater proportion of variance is accounted for by Morison equation.

Figure 3-23 shows the values of R^2 values for tests of a cylinder oscillating in still water, together with the values of their mean and standard deviation. The data quality is a little poor (a relatively small value of R^2) for $T_o = 1.2s$, which is probably due to the higher noise in the measured force caused by the fast oscillation of the cylinder; the mean value is about 0.89, and its standard (STD) about 0.04. For other periods, all the values of R^2 lie in the range of 0.91~0.95, suggesting that Morison equation (3-2) can model the hydrodynamic force with a certain degree of accuracy.

Figure 3-24 gives the values of R^2 for tests of a fixed cylinder in regular waves. It can be seen that a slightly larger values of R^2 appear in wave flume tests, which may be due to the relatively lower signal noise (refer to Figure 3-4(b)) in the force measurements.

The measured and fitted hydrodynamic forces for selected KC numbers and different periods for both a cylinder oscillating in still water and a fixed cylinder in regular waves are shown in Figures 3-25~3-35 for reference. A detailed assessment of the Morison equation of Equations (3-1) and (3-2) is beyond of the scope of this chapter; a critical assessment of the Morison equation and a form of modified Morison equation with more terms for a cylinder in oscillatory flow or regular waves can be found in Sarpkaya and Isaacson (1981) and Sarpkaya (2010).

Figure 3-23 R^2 error for tests of a cylinder oscillating in still water. Continued on the next page.

(a): $\beta_o = 2083$, (b): $\beta_o = 1563$, (c): $\beta_o = 1250$, (d): $\beta_o = 1042$

(e): $\beta_o = 833$, (f): $\beta_o = 625$, (g): $\beta_o = 500$, (h): $\beta_o = 417$

Figure 3-23 Continued.

Figure 3-24 R^2 error for tests of a fixed cylinder in regular waves.

(a): $\beta_w = 2083$, (b): $\beta_w = 1563$, (c): $\beta_w = 1250$

Figure 3-25 Measured and fitted in-line force for a cylinder oscillating in still water with $T_o = 1.2s$.

Solid lines: measured, dot lines: fitted

(a) Cylinder position (not plotted to scale)

(b): $KC_o = 4.9$, (c): $KC_o = 5.7$, (d): $KC_o = 6.5$, (e): $KC_o = 7.4$, (f): $KC_o = 8.2$

Figure 3-26 Measured and fitted in-line force for a cylinder oscillating in still water with $T_o = 1.6s$.

Solid lines: measured, dot lines: fitted

(a) Cylinder motion position (not plotted to scale)

(b): $KC_o = 4.9$, (c): $KC_o = 6.5$, (d): $KC_o = 8.2$, (e): $KC_o = 9.8$, (f): $KC_o = 11.4$

Figure 3-27 Measured and fitted in-line force for a cylinder oscillating in still water with $T_o = 2.0s$.

Solid lines: measured, dot lines: fitted

(a) Cylinder motion position (not plotted to scale)

(b): $Ke = 5.7$, (c): $Ke = 7.4$, (d): $Ke = 9.8$, (e): $Ke = 12.3$, (f): $Ke = 13.9$

Figure 3-28 Measured and fitted in-line force for a cylinder oscillating in still water with $T_o = 2.4s$.

Solid lines: measured, dot lines: fitted

(a) Cylinder motion position (not plotted to scale)

(b): $KC_o = 5.7$, (c): $KC_o = 8.2$, (d): $KC_o = 10.6$, (e): $KC_o = 12.3$, (f): $KC_o = 13.9$

Figure 3-29 Measured and fitted in-line force for a cylinder oscillating in still water with $T_o = 3.0s$.

Solid lines: measured, dot lines: fitted

(a) Cylinder motion position (not plotted to scale)

(b): $KC_o = 6.5$, (c): $KC_o = 9.8$, (d): $KC_o = 13.1$, (e): $KC_o = 16.3$, (f): $KC_o = 19.6$

Figure 3-30 Measured and fitted in-line force for a cylinder oscillating in still water with $T_o = 4.0s$.

Solid lines: measured, dot lines: fitted

(a) Cylinder motion position (not plotted to scale)

(b): $KC_o = 6.5$, (c): $KC_o = 9.8$, (d): $KC_o = 13.1$, (e): $KC_o = 16.3$, (f): $KC_o = 19.6$

Figure 3-31 Measured and fitted in-line force for a cylinder oscillating in still water with $T_o = 5.0s$.

Dark solid lines: measured, red dot lines: fitted

(a) Cylinder motion position (not plotted to scale)

(b): $KC_o = 5.7$, (c): $KC_o = 8.2$, (d): $KC_o = 13.1$, (e): $KC_o = 18$, (f): $KC_o = 22.9$

Figure 3-32 Measured and fitted in-line force for a cylinder oscillating in still water with $T_o = 6.0s$.

Solid lines: measured, dot lines: fitted

(a) Cylinder motion position (not plotted to scale)

(b): $KC_o = 6.5$, (c): $KC_o = 11.4$, (d): $KC_o = 16.3$, (e): $KC_o = 21.2$, (f): $KC_o = 29.4$

Figure 3-33 Measured and fitted in-line force for a cylinder fixed in regular waves with $T_w = 1.2\text{ s}$.

Dark solid lines: measured, red dot lines: fitted

(a) Wave surface (not plotted to scale)

(b): $KC_w = 1.6$, (c): $KC_w = 2.5$, (d): $KC_w = 3.4$, (e): $KC_w = 4.9$, (f): $KC_w = 5.7$

Figure 3-34 Measured and fitted in-line force for a cylinder fixed in regular waves with $T_w = 1.6s$.

Solid lines: measured, dot lines: fitted

(a) Wave surface (not plotted to scale)

(b): $KC_w = 2.3$, (c): $KC_w = 4.8$, (d): $KC_w = 7.2$, (e): $KC_w = 9.1$, (f): $KC_w = 11.5$

Figure 3-35 Measured and fitted in-line force for a cylinder fixed in regular waves with $T_w = 2.0s$.

Solid lines: measured, dot lines: fitted

(a) Wave surface (not plotted to scale)

(b): $KC_w = 2.7$, (c): $KC_w = 5.5$, (d): $KC_w = 8.5$, (e): $KC_w = 11.6$, (f): $KC_w = 12.9$

3.9 Summary

This chapter has reported the results of in-line forces for a cylinder oscillating in still water and a fixed cylinder in regular waves. The motivations of conducting these tests are: 1) to validate the reliability / repeatability of our experimental setups described in Chapter 2, and to build up the confidence in our experimental setup for more complicated problems involving two frequencies; 2) to provide a comprehensive data base for understanding the results of the problems involving two frequencies. Based on the results reported in this chapter, the following conclusions can be drawn:

- 1) The measured hydrodynamic coefficients generally agree well with the available data published in the literature, indicating that our equipment and experimental setup can perform properly with satisfactory accuracy.
- 2) For a cylinder oscillating in still water, the drag coefficients are relatively sensitive to the frequency number β_o , in particular in the range of $5 \leq KC_o \leq 15$. However, the added mass coefficient is not as sensitive to β_o as the drag coefficient. For those tests with $\beta_o = 1250$ and 1042 and $10 \leq KC_o \leq 15$, the “inertia crisis” is more pronounced.
- 3) For a fixed cylinder in regular waves, three-dimensional effects could reduce the drag force, but increase inertia force.
- 4) For both a cylinder oscillating in still water and a fixed cylinder in waves, the root-mean-square coefficients C_{Fms} that are obtained using the measured inertia and drag coefficients agree well with those directly computed using the measured force.

Chapter 4

Transverse Force on a Cylinder Model under Oscillatory Flow and Regular Waves

4.1 Preliminary remarks

The alternating nature of vortex shedding and the induced lift forces are of significant importance in predicting structural responses, such as vortex-induced vibration and the associated fatigue. In this chapter, the experimental results of the transverse force acting on a cylinder oscillating in still water and a fixed cylinder in regular waves are presented. The measurements on transverse forces were conducted simultaneously with the measurements on the in-line forces reported in Chapter 3. In addition to comparing the transverse forces with the measured in-line forces, the harmonic hydrodynamic coefficients for lift force, the maximum and root-mean-square (RMS) as well as the mean hydrodynamic coefficients for lift force are examined for different $\beta = Re/ KC$ numbers. The aim of this chapter is to have a better understanding of the hydrodynamics around a vertical circular cylinder in oscillatory and wave flows.

4.2 Load model for transverse force

When a cylinder is exposed to oscillatory flows or waves, the total force will have an in-line component (in-line force) and a transverse component (transverse force or lift force). The transverse force is induced by the cyclic vortex shedding which gives an asymmetrical pressure gradient across the wake. The lift force is usually expressed as an empirical model with multiple frequencies, see the expression of the second term in Equation (4-1) (Isaacson and Maull, 1976; Chakrabarti et al.,

1976 and 1983; Cotter and Chakrabarti, 1984; Williamson, 1985; Obasaju et al., 1988; Justesen, 1989 and 1991, etc).

The lift force on a cylinder oscillating in still water or a fixed cylinder in regular waves, in general, has multiple frequencies. For certain KC number and Reynolds number, the vortex shedding may take place mainly on one side of the cylinder and the cylinder may also have a steady component in the lift force in addition to the fluctuating component (see, Williamson, 1985; Obasaju et al., 1988; Tatsuno and Bearman, 1990; Lam and Dai, 2002 and Lam et al., 2010). Therefore, a mean steady component should be included in the model describing the transverse force, and it seems appropriate to express the lift force using Equation (4-1),

$$F_L(t) = F_{Lmean} + \frac{1}{2} \rho D L U_m^2 \sum_{n=1}^N C_L(n) \cos[2\pi n f t + \psi(n)] \quad (4-1)$$

in which D , L are the diameter and length of the cylinder, respectively; U_m is the maximum horizontal velocity component of the flow; $C_L(n)$ is the lift coefficient for the n -th harmonic force component and is a function of both the KC number and Reynolds number; f is the fundamental frequency of oscillatory flow or wave flow; N is the number of harmonic components used in the analysis; and $\psi(n)$ is the phase angle associated with the n -th harmonic force component. Coefficients up to the 6th harmonic component are presented in this study.

4.3 Determination of lift coefficients

Lift coefficients for the harmonic components of the lift force $C_L(n)$ in Equation (4-1), can be determined from the measured transverse force by a Fourier analysis

(Chakrabarti et al., 1976 and 1983; Cotter and Chakrabarti, 1984), and the procedure is given below.

It can be found by performing the Fast Fourier Transform (FFT) on the lift force that the energy of the measured lift force is mainly concentrated at frequencies that are multiples of the fundamental frequency, indicating that the lift force is approximately periodic. After removing the mean component, a periodic function $f(t)$ can be represented by an infinite sum of sine and/or cosine functions, as shown in Equation (4-2).

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)] \quad (4-2)$$

where $n = 1, 2, 3, \dots$; a_0, a_n and b_n are called Fourier coefficients; $\omega_0 = 2\pi / T$ is the fundamental angular frequency of the periodic function $f(t)$ with period T ; and $n\omega_0$ is known as the frequency of the n -th harmonic component of $f(t)$. The Fourier coefficients a_0, a_n and b_n can be calculated as follows:

$$\left. \begin{aligned} a_0 &= \frac{1}{T} \int_{t_0}^{t_0+T} f(t) dt \\ a_n &= \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos(n\omega_0 t) dt \\ b_n &= \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \sin(n\omega_0 t) dt \end{aligned} \right\} \quad (4-3)$$

The amplitude of the n -th harmonic component is calculated by $A_L(n) = \sqrt{a_n^2 + b_n^2}$ and the corresponding lift coefficient for this harmonic component is defined by $C_L(n) = A_L(n) / (0.5\rho D U_m^2 L)$.

When calculating the Fourier coefficients, no window functions are needed for periodic force signals, but the length of the measured lift force should be chosen as multiples of the fundamental period to avoid any amplitude ambiguity (Dunn, 2005). The total number of the data points for FFT is obtained by the length of the selected force record multiplied by F_s (F_s is the sampling frequency, here $F_s=500$). Normally, about a length of 24~48 seconds was selected from the measured force for analysis. Accordingly, the frequency resolution is 0.021~0.042 HZ.

In addition to the lift coefficients described above, other lift force coefficients reflecting other aspects of transverse force characteristics will also be investigated; these coefficients are defined below.

Maximum lift force coefficient ($C_{L_{\max}}$) The maximum magnitude of lift force $F_{L_{\max}}$ is directly associated with the strength of shed vortices and it is assessed by the coefficient which is defined as the maximum of the measured lift force normalized by $0.5\rho DU_m^2 L$,

$$C_{L_{\max}} = \frac{F_{L_{\max}}}{0.5\rho DU_m^2 L} \quad (4-4)$$

Root-Mean-Square (RMS) lift coefficient ($C_{L_{rms}}$) Since the lift force is solely due to the asymmetrical vortex patterns and motions, and the RMS value $F_{L_{rms}}$ of the measured transverse force is closely associated with the vortex shedding and the history of fluid motion; therefore, it is also worth to investigate the root-mean-square lift force coefficient, defined by

$$C_{Lrms} = \frac{F_{Lrms}}{0.5\rho DU_m^2 L} \quad (4-5)$$

Mean lift coefficient (C_{Lmean}) As for a given $\beta = Re/ KC$ number, within a certain range of KC numbers, the vortex activity mainly takes place on one side of the cylinder; consequently the cylinder may experience a mean transverse force in addition to the fluctuating transverse force, and the magnitude can be comparable to that of the fluctuating component (Williamson, 1985 ; Obasaju et al., 1988; Tatsuno and Bearman, 1990; Lam and Dai, 2002; and Lam et al., 2010). The mean transverse force F_{Lmean} can be obtained by integrating the transverse force to obtain the value of a_0 in Equation (4-3). As a result, the mean transverse force coefficient can be defined by,

$$C_{Lmean} = \frac{F_{Lmean}}{0.5\rho DU_m^2 L} \quad (4-6)$$

4.4 A brief review of key findings in the literature related to lift force

Investigations in the past decades (e.g. see Maull and Miller, 1978; Bearman et al., 1979; Bearman et al., 1981; Williamson, 1985; Obasaju et al., 1988; Justesen, 1989; Tatsuno and Bearman, 1990; Lam and Dai, 2002 and Lam et al., 2010, etc) have revealed that certain repeatable vortex shedding patterns can occur for certain flow amplitudes. Each of these repeatable patterns reflects the shedding of a particular number of vortices per half cycle; as the flow reverses in another half cycle, the shed vortices in the first half cycle are carried back over the cylinder, and one or more pairs with vortices of opposite sign can be formed. The pairs of vortices usually are carried away from the body through convection at a large angle to the

direction of main flow. For the convenience of understating and discussion of transverse force induced by vortex shedding, some key findings about vortex shedding patterns of a circular cylinder oscillating in still water (or a fixed cylinder in oscillatory flows) are reviewed below.

Based on the results obtained using a U –tube, Bearman et al. (1981) and Obasaju et al. (1988) observed nearly repeatable vortex patterns. Following the results of Bearman et al. (1981) and Obasaju et al. (1988) with $\beta_o = 416$, for KC_o numbers ranging from 3 to 55, the following regimes can be defined:

- 1) $KC_o \leq 4$: symmetric regime. In this regime, no major vortices shed from the cylinder during each half cycle although pairs of small symmetric vortices are formed each time when the flow reverses.
- 2) $4 \leq KC_o \leq 8$: asymmetrical regime. In this regime, the visualization made at $KC_o \approx 7$ shows that the vortex pattern tends to be concentrated on one side of the cylinder. A vortex shed from the cylinder in the previous half cycle is carried back during the flow reversal phase. This vortex is present in each half cycle and remains close to the cylinder and seems to be dissipated as it is carried back during the flow reversal phase.
- 3) $8 \leq KC_o \leq 15$: transverse regime (sideways street). In this regime, the visualization made at $KC_o \approx 10$ shows that the vortex activity is also one-sided and two vortices with opposite signs are shed from the cylinder in one complete cycle. The vortices form a pair and can be carried through convection to large distances from the cylinder. Vortex shedding takes place at about 45° to the main flow.

- 4) $15 \leq KC_o \leq 22$: diagonal regime. In this regime, Obasaju et al. (1988) show that at $KC_o \approx 18$, a pair of vortices is shed in each half cycle and the vortex pair in the one half-cycle is shed diagonally opposite to the pair in the previous half-cycle. Also, vortex pairs are shed at about 45° to the main flow.
- 5) $22 \leq KC_o \leq 30$: three vortex regime. In this regime, three full vortices are shed each half cycle.
- 6) $KC_o \geq 30$: the quasi-steady regime.

Williamson (1985) suggested approximate boundaries between these regimes based on flow visualization made for a cylinder oscillating in still water. The value of β_o number was estimated to be 1020 by Justesen (1991) and 255 by Lam and Dai (2002). Williamson (1985) suggested the following flow regimes:

- 1) $0 < KC_o < 7$: pairing of attached vortices. In this range, for $KC_o \leq 4$, a symmetrical pair of small “attached” vortices form in the wake of the cylinder in each half cycle. When the flow reverses, the attached vortices split up and pair with new vortices in the new half cycle; the vortices are converted to distances from the cylinder, and no lift force is generated. Between $KC_o = 4$ and 7 , the attached vortices become unequal in strength and the vortex pairs do not form simultaneously upon flow reversal, generating a lift force with low magnitude at the oscillation frequency f_o .
- 2) $7 < KC_o < 15$: transverse street/single pair. In this regime, the vortex patterns are most repeatable and one large vortex is shed during each half cycle. In the range of $7 < KC_o < 13$, each successive main vortex shed from the cylinder per

half cycle is convected to distances from one side of the cylinder roughly perpendicular to the direction of oscillation, forming a street or “jet” of vortices (therefore, named as sideways street). Between $13 < KC_o < 15$, the main vortex pattern comprises a series of paired vortices being convected to distances from the cylinder at roughly 45° to the direction of oscillation. In both the transverse street and single pair regimes, the vortices tend to convect to one side predominantly. Williamson (1985) also pointed out the repeatable vortices will induce lift force, and the fundamental frequency of the lift force is twice the oscillation frequency f_o .

- 3) $15 < KC_o < 24$: double pairs. In this regime, the wake pattern comprises two large vortices in each half cycle and the formation of two pairs in each cycle. A series of vortex pairs convect to distances from the cylinder in opposite directions and from opposite quadrants. In this case, the frequency of the lift force is three times the oscillation frequency f_o .
- 4) $24 < KC_o < 40$: three pairs. In this regime, three pairs of vortices shed in each half cycle and the frequency of the lift force is four times the oscillation frequency f_o .
- 5) $32 < KC_o < 40$: four pairs. In this range, four pairs of vortices are formed in each half cycle and the frequency of the lift force is five times the oscillation frequency f_o .

As a rough rule, the fundamental frequency of the lift is $(m+1)$ times the oscillation frequency f_o , where m is the number of vortices shed from the cylinder in each half cycle.

Based on flow visualizations of a circular cylinder oscillating in still water, Tatsuno and Bearman (1990) proposed up to eight different vortex patterns within the parameter ranges of KC_o (1.6 ~ 15) and β_o (5 ~ 160), and showed that for a given KC number, the flow pattern was weakly dependent on β_o number. The instability and three-dimensional structure of the flow along the cylinder axis were discussed in details (see also Honji, 1981).

Lam and Dai (2002), using a phase-averaged particle image velocimetry (PIV) studied the symmetrical pair of vortices attached to the rear of the cylinder for $KC_o = 4.25$ with $\beta_o = 1290$. They also found that for $KC_o = 6.28$ with $\beta_o = 940$, symmetric vortex pairs were observed to form on the leeward side of the cylinder for about half of the observation time and asymmetric vortex pairs were observed for the other half of the time. It was also observed that even when an asymmetric vortex pair was formed, the vortices did not appear to be convected in an oblique direction. This observation is not consistent with that of Tatsuno and Bearman (1990) for the smaller values of β_o number ; their visualization showed that in this regime, asymmetric vortex pairs were found to be shed from the cylinder and complex vortex - vortex and vortex - cylinder interactions gave rise to a V-shaped pattern of vortices. The inability of the vortex pair to be detached from the cylinder at a lower KC number was attributed by Lam and Dai (2002) to the short stroke of the cylinder oscillation. For $KC_o = 12$ and $\beta_o = 200$, they characterized the flow regime by a periodic vortex street at a lateral direction to the line of cylinder oscillation. The flow pattern exhibited a high degree of periodicity and regularity over each cycle, and the asymmetric and stronger vortex was shed as a result of the

longer cylinder stroke. Their observation is consistent with those of Williamson (1985), Obasaju et al. (1988) and Tatsuno and Bearman (1990).

Recently, Lam et al. (2010), using a time-resolved particle image velocimetry (PIV), also reported vortex shedding patterns for a circular cylinder oscillating in still water for $8 \leq KC_o \leq 36$ and $Re_o = 2400$. Three vortex patterns were identified:

- 1) $8 \leq KC_o < 16$: transverse vortex street or single pair regime.
- 2) $16 \leq KC_o \leq 24$: two pairs or diagonal shedding regime.
- 3) $24 < KC_o$: three or more vortices in every half cycle. The vortices lie mainly in the direction of the cylinder oscillation.

A schematic figure illustrating the three vortex patterns is shown below (Lam et al., 2010).

Figure 4-1 A schematic of vortex patterns, from Lam et al. (2010).

(a) Mode I-a lateral vortex street; (b): Mode I- an inclined vortex street (single pair regime);

(c) Mode II- two pairs regime; (d) three or more vortices.

Bold arrows-- secondary flow stream, dotted arrows--ambient water flow, dot-dashed lines-- typical length of cylinder stroke.

Some transverse forces were also measured and related to the associated shed vortex patterns by Williamson (1985) and Obasaju et al. (1988). Transverse force acting on a circular cylinder in oscillatory flow or regular waves were also reported by Isaacson and Maull (1976), Chakrabarti et al. (1976), Sarpkaya (1976), Chaplin (1988), Skomedal et al. (1989), Justesen (1989), etc. However, systematic measurements of transverse force acting on a circular cylinder in oscillatory flows or regular waves are absent and quality data are limited in the literature. In the subsequent sections, extensive measurements and analysis of transverse force will be reported in order to better understand the characteristics of the transverse force acting on a circular cylinder in oscillatory flows or regular waves. Whenever possible, the results will be compared with those reported in the literature under similar test conditions.

4.5 Results and discussion of the transverse force

In this section, results of the measured transverse forces are presented, and various lift coefficients are calculated and discussed for 8 oscillation periods ($T_o = 1.2s, 1.6s, 2.0s, 2.4s, 3.0s, 4.0s, 5.0s \text{ and } 6.0s$), and 3 wave periods ($T_w = 1.2s, 1.6s \text{ and } 2.0s$).

4.5.1 Transverse force for a cylinder oscillating in still water

In the present experiment, the measured transverse forces are quite stable except for certain KC numbers where the measured lift forces showed certain degree of intermittent characteristics caused by the out-of-phase vortex shedding along the cylinder length (Maull and Miller, 1978; Bearman et al., 1981; Williamson, 1985).

Figures 4-2~4-9 show samples of the measured transverse forces; it can be observed that at some small values of KC_o number, the transverse force mainly took on negative values, indicating that the vortex activity mainly concentrated on one side of the cylinder at these values of KC_o number. The vortex activity that concentrates on one side would generate a considerable mean transverse force. By comparing the transverse forces for a certain value of KC number, such as $KC_o = 9.8$, with different oscillation periods, the transverse forces showed so different characteristics, showing a strong dependency on β_o number. In both the asymmetrical and transverse street /single pair regimes (where $KC_o < 15$, according to Williamson (1985), etc), the phenomena that vortex shedding concentrates on one side was more obvious for smaller values of β_o . Therefore, the vortex shedding and the associated transverse force are obviously dependent on both KC_o and β_o numbers, rather than only on KC_o number. As a result, the classification of vortex shedding patterns should be given in terms of both KC_o and β_o numbers.

To the author's knowledge, the random, intermittent characteristic of the lift force was firstly reported by Maull and Miller (1978). Bearman et al. (1981) and Williamson (1985) suggested this intermittent characteristic in the measured lift force was caused by the out-of-phase vortex shedding along the cylinder length. In the present experiment, the intermittent characteristic of the lift force was mainly found for KC_o numbers ranging from 6.5 to 8 and larger values of β_o number; for tests with smaller values of β_o number, this intermittent characteristic was less

profound. Some examples of the measured transverse forces with intermittent characteristics are shown in Figures 4-10 through 4-12.

Figure 4-2 Transverse forces measured at different KC_o numbers for $T_o = 1.2s$.
The topmost graph shows the displacement of the oscillating cylinder (not drawn to scale)

Figure 4-3 Transverse forces measured at different KC_o numbers for $T_o = 1.6$ s .

Figure 4-4 Transverse forces measured at different KC_o numbers for $T_o = 2.0s$.

Figure 4-5 Transverse forces measured at different KC_o numbers for $T_o = 2.4$ s .

Figure 4-6 Transverse forces measured at different KC_o numbers for $T_o = 3.0s$.

Figure 4-7 Transverse forces measured at different KC_o numbers for $T_o = 4.0s$.

Figure 4-8 Transverse forces measured at different Ke numbers for $T_o = 5.0s$.

Figure 4-9 Transverse forces measured at different KC_o numbers for $T_o = 6.0s$.

Figure 4-10 An example of lift force time series with intermittent characteristics for $T_o = 1.2s$.

Figure 4-11 Examples of lift force time series with intermittent characteristics for $T_o = 1.6s$.

Figure 4-12 Examples of the measured lift forces showing intermittent characteristics for $T_o = 2.0s$.

The total force, which comprises both the in-line and transverse components are demonstrated in Figures 4-13 through 4-20 for various KC_o for a circular cylinder oscillating in still water. When compared to the in-line component, the lift

(transverse) force magnitude can be considerable to or even larger than that of the corresponding in-line component. In some cases, the transverse component is nearly 2 times the magnitude of the in-line component, see, for example, Figure 4-14(d) for $KC_o = 9.0$ and $T_o = 1.6s$. Therefore, in the design of offshore structures, the transverse force should be a design factor as important as the in-line force, especially when studying the responses of offshore structures subjected to ocean waves.

Figure 4-13 Force-vector trajectories showing the time histories of in-line and transverse forces for $T_o = 1.2s$.

(a): $KC_o = 5.7$, (b): $KC_o = 6.5$, (c): $KC_o = 7.4$, (d): $KC_o = 8.2$

Figure 4-14 Force-vector trajectories showing the time histories of in-line and transverse forces for $T_o = 1.6s$.

(a): $KC_o = 6.5$, (b): $KC_o = 7.4$, (c): $KC_o = 8.2$, (d): $KC_o = 9.0$, (e): $KC_o = 9.8$, (f): $KC_o = 11.4$

Figure 4-15 Force-vector trajectories showing the time histories of in-line and transverse forces for $T_o = 2.0s$.

(a): $KC_o = 8.2$, (b): $KC_o = 9.8$, (c): $KC_o = 11.4$, (d): $KC_o = 12.3$, (e): $KC_o = 13.1$, (f): $KC_o = 13.9$

Figure 4-16 Force-vector trajectories showing the time histories of in-line and transverse forces for $T_o = 2.4s$.

(a): $KC_o=7.4$, (b): $KC_o=8.2$, (c): $KC_o=9.8$, (d): $KC_o=11.4$, (e): $KC_o=13.1$, (f): $KC_o=14.7$

Figure 4-17 Force-vector trajectories showing the time histories of in-line and transverse forces for $T_o = 3.0s$.

(a): $Ke = 9.8$, (b): $Ke = 11.4$, (c): $Ke = 13.1$, (d): $Ke = 14.7$, (e): $Ke = 16.3$, (f): $Ke = 19.6$

Figure 4-18 Force-vector trajectories showing the time histories of in-line and transverse forces for $T_o = 4.0s$.

(a): $KC_o = 9.8$, (b): $KC_o = 11.4$, (c): $KC_o = 13.1$, (d): $KC_o = 14.7$, (e): $KC_o = 16.3$, (f): $KC_o = 19.6$

Figure 4-19 Force-vector trajectories showing the time histories of in-line and transverse forces for $T_o = 5.0s$.

(a): $KC_o = 9.8$, (b): $KC_o = 13.1$, (c): $KC_o = 14.7$, (d): $KC_o = 16.3$, (e): $KC_o = 19.6$, (f): $KC_o = 21.2$

Figure 4-20 Force-vector trajectories showing the time histories of in-line and transverse forces for $T_o = 6.0s$.

(a): $KC_o = 9.8$, (b): $KC_o = 13.1$, (c): $KC_o = 14.7$, (d): $KC_o = 16.3$, (e): $KC_o = 19.6$, (f): $KC_o = 21.2$

4.5.2 Harmonic lift coefficients for a cylinder oscillating in still water

As discussed by Williamson (1985), the fundamental frequency of transverse force is closely associated with the number of shed vortices each half cycle of oscillation. In this section, the amplitude spectrums of the measured transverse force (after removing the steady component) and the lift coefficient of each harmonic component are discussed for a cylinder oscillating in still water. For latter discussion, the amplitude spectrums and lift coefficients are presented in Figures 4-21 through 36.

First we examine the results for $T_o = 1.2s$ ($\beta_o = 2083$). When $4.9 \leq KC_o \leq 7.5$, the flow pattern mainly lies in asymmetrical regimes according to Williamson (1985). It can be seen from Figures 4-21 and 4-22 that the predominant frequency is the frequency of oscillation f_o . When $KC_o > 7.5$, the 2nd harmonic begins to dominates; this is because when $KC_o > 7$, the flow is in the “transverse street/single pair” regime (Williamson, 1985), and the fundamental frequency of the lift force is now two times the frequency of oscillation f_o .

Next, we examine the results for $T_o = 1.6s$ ($\beta_o = 1563$). When KC_o number ranges from 4.9 to 11.5, the flow pattern changes from asymmetrical regime to transverse street /single pair regime, according to Williamson (1985). As shown in Figures 4-23 and 4-24, the predominant frequency is the 2nd harmonic frequency, and the 2nd harmonic lift coefficients increase with increasing KC_o number and local maximum values of the 2nd harmonic lift coefficients were found around $KC_o = 9 \sim 11$. The 4th harmonic force is more significant than the 2nd and 3rd harmonic

forces. When $KC_o < 6.5$, the lift coefficients of the 1st harmonic are a little larger than those of the 4th harmonic, and the lift coefficient of the 1st harmonic decreases with increasing KC_o number. However, the magnitude of the 4th harmonic lift force becomes more considerable with increasing KC_o number and its lift coefficient increase with increasing KC_o number. It should be pointed out that the 4th harmonic component should be caused by the vortex-vortex interactions, but not induced by three-pair vortices; because the shedding of three-pair vortices in one half cycle only occurs at large KC numbers, i.e. $24 < KC_o < 32$. Furthermore, Figure 4-23 also shows that the fundamental frequency of the lift force is two times the frequency of oscillation f_o for 4.9~11.5.

Now, we examine even longer oscillation periods $T_o = 2.0s (\beta_o = 1250)$, and $T_o = 2.4s (\beta_o = 1042)$. For $T_o = 2.0s$, KC_o numbers ranging from 4.9~13.9; for $T_o = 2.4s$, KC_o numbers varying from 4.9~14.7. For these two periods, the flow pattern changes from asymmetrical regime to transverse street/single pair regime according to Williamson (1985) and Obasaju et al. (1988). As shown in Figures 4-25 through 4-28, the 2nd harmonic lift force dominates. The 2nd harmonic lift coefficients reach their peak values around $KC_o = 8\sim 9$. The 1st harmonic lift force coefficients are a little higher when compared to values of the 4th harmonic for $KC_o < 7$; however, the magnitude of the 4th harmonic lift force becomes considerable when $KC_o > 7$ and the lift coefficient of the 4th harmonic increases with increasing KC_o number. For these two groups of tests with $T_o = 2.0s$ and $2.4s$,

the fundamental frequency of lift force is two times the frequency of oscillation f_o within the tested KC number range.

From Figures 4-4 and 4-5, Figures 4-25 and 4-27 for $T_o = 2.0s$ and $2.4s$, and the lift coefficients up to 6 harmonics for $T_o = 1.2s, 1.6s, 2.0s$ and $2.4s$, it may be inferred that the transition from asymmetrical regime to the transverse street /single pair regime might not be abrupt at the critical value of $KC_o = 7$, (as suggested by Williamson (1985)), but could be weakly dependent on β_o number. The fundamental frequency of lift force becomes two times the frequency of oscillation f_o when the vortex shedding pattern falls into the regime of transverse street /single pair regime; reducing β_o number reduces the KC_o number at which the fundamental frequency of lift force becomes two times the frequency of oscillation f_o .

For further longer periods of oscillation $T_o = 3.0s, 4.0s, 5.0s$ and $6.0s$ (i.e. relatively smaller β_o values), the general trend of the coefficients of each harmonic is similar to that for a shorter T_o . However, the lift coefficients of the 2nd harmonic for $T_o = 3.0s$ and $4.0s$ (see Figures 4-30 and 4-32) reach their maximums around $KC_o = 7 \sim 7.5$, which is slightly smaller than $KC_o = 8 \sim 9$ for the maximum lift coefficients of the 2nd harmonic when $T_o = 2.0s$ and $2.4s$. It can be seen from Figure 4-24 that the maximum lift coefficient of the 2nd harmonic for $T_o = 1.6s$ appears at $KC_o = 9 \sim 11$. Therefore, it may be reasonably concluded that the longer the oscillation period T_o , the smaller KC_o number is at which the maximum lift

coefficient of the 2nd harmonic coefficient occurs. This conclusion can also be verified by the lift coefficients for the oscillation periods of $T_o = 5.0s$ and $6.0s$ (see Figures 4-34 and 4-36). According to Williamson (1985) and Obasaju et al. (1988), in the present tests, some flow patterns for $T_o = 3.0s, 4.0s, 5.0s$ and $6.0s$ lie in the double pairs regime ($15 < KC_o < 24$). From the amplitude spectrums (Figures 4-31, 4-33 and 4-35), it can be observed that the 3rd harmonic frequency becomes more and more pronounced with increasing KC_o number; we found that the fundamental frequency of lift force was now three times the frequency of oscillation f_o when $KC_o > 16$. The spectrums of lift force (Figures 4-31, 4-33 and 4-35) also suggest that the transition from the transverse street/single pair regime to the double pairs regime should not occur sharply at a critical KC_o number, but be weakly dependent on β_o number. The smaller the β_o number, the smaller the KC_o number probably is at which the fundamental frequency of lift force becomes three times the frequency of oscillation f_o . At large KC_o numbers (e.g. see Figure 4-33 (h), Figure 4-35 (g) and (h)), the magnitude of the 3rd harmonic component can be comparable to those of the 2nd and 4th harmonic components.

Figure 4-21 Amplitude spectrums measured for different KC_O number with $T_o = 1.2s$.

Figure 4-22 Lift coefficients up to 6 harmonics versus KC_O number for $T_o = 1.2s$.

Figure 4-23 Amplitude spectrums measured for different KC_o number with $T_o = 1.6s$.

Figure 4-24 Lift coefficients up to 6 harmonics versus KC_o number for $T_o = 1.6s$.

Figure 4-25 Amplitude spectrums measured for different KC_o number with $T_o = 2.0s$.

Figure 4-26 Lift coefficients up to 6 harmonics versus KC_o number for $T_o = 2.0s$.

Figure 4-27 Amplitude spectrums measured for different KC_o number with $T_o = 2.4s$.

Figure 4-28 Lift coefficients up to 6 harmonics versus KC_o number for $T_o = 2.4s$.

Figure 4-29 Amplitude spectrums measured for different KC_o number with $T_o = 3.0s$.

Figure 4-30 Lift coefficients up to 6 harmonics versus KC_o number for $T_o = 3.0s$.

Figure 4-31 Amplitude spectrums measured for different KC_o number with $T_o = 4.0s$.

Figure 4-32 Lift coefficients up to 6 harmonics versus KC_o number for $T_o = 4.0s$.

Figure 4-33 Amplitude spectrums measured for different KC_O number with $T_O = 5.0s$.

Figure 4-34 Lift coefficients up to 6 harmonics versus KC_O number for $T_O = 5.0s$.

Figure 4-35 Amplitude spectrums measured for different KC_O number with $T_o = 6.0s$.

Figure 4-36 Lift coefficients up to 6 harmonics versus KC_O number for $T_o = 6.0s$.

4.5.3 Root-mean-square (RMS) and maximum lift coefficients for a cylinder oscillating in still water

As described in Section 4.3, the lift force is solely due to the asymmetrical vortex patterns, and the root-mean-square (RMS) value F_{Lrms} of the measured force is closely related to the vortex shedding and the motion history; the maximum magnitude of lift force F_{Lmax} is directly associated with the strength of the shed vortices. Therefore, it is much worth to investigate the RMS and maximum lift coefficients, C_{Lrms} and C_{Lmax} . Figure 4-37 shows that the maximums may be observed in the RMS coefficient for $\beta_o \geq 833$ (i.e. $T_o \leq 3s$). In addition, the KC_o numbers at which the maximum values of C_{Lrms} and C_{Lmax} occur are obviously dependent on the oscillation period T_o (i.e. β_o number). Similar to the conclusion drawn in Section 4.5.2 for the KC_o number where the maximum values of the 2nd harmonic lift coefficients occur, reducing β_o number reduces the KC_o number at which the RMS coefficients take on maximum values. For smaller β_o numbers (such as $\beta_o = 625, 500$ and 417), no obvious peak values are found for the RMS coefficients. For a given KC_o number, reducing β_o number increased the C_{Lrms} value, suggesting that the C_{Lrms} values should be strongly dependent on the β_o number. Experimental results also showed the C_{Lrms} dependence on β_o number was consistent throughout the KC_o numbers for all tests. Figure 4-38 shows that the maximum lift coefficients (C_{Lmax}) have very similar trends to those of the RMS lift coefficients (C_{Lrms}) for all β_o numbers.

Figure 4-37 Comparisons of root-mean-square lift coefficients versus KC_o number for all oscillation periods.

Figure 4-38 Comparisons of maximum lift coefficients versus KC_o number for all oscillation periods.

In published data sources, even though systematic data about lift coefficients are few (this is also one major reason why we performed the present experimental study), some data can be found about the RMS coefficients of lift force for a cylinder oscillating in still water (or a fixed cylinder in oscillatory flow). Figure 4-39 shows a comparison of the present RMS coefficients C_{Lrms} for $T_O = 1.6s$ ($Re_O : 7.7 \times 10^3 \sim 1.8 \times 10^4$) and $T_O = 1.2s$ ($Re_O : 1 \times 10^4 \sim 1.7 \times 10^4$) with the data from Chaplin (1988). In the experiments of Chaplin (1988), the hydrodynamic loads on a smooth circular cylinder oscillating in still water at high Reynolds numbers ($Re_O : 2 \times 10^4 \sim 2.8 \times 10^5$) were measured using pressure transducers. It can be seen that the C_{Lrms} coefficients are dependent on Reynolds number: small values of C_{Lrms} occur at high Reynolds numbers.

The lift coefficients C_{Lrms} for $T_O = 4.0s$ and $5.0s$ are compared in Figure 4-40 with some published data with the β_O values close to those in the present study. The lift coefficients of the present study are slightly higher than those of Obasaju et al. (1988) and Skomedal et al. (1989). In the present study, the lift force were measured at a 5 cm test ring in length with the length-to diameter ratio 1.0, while in the experiments of Obasaju et al. (1988) and Skomedal et al. (1989), the force were obtained from the measurements over a much longer length of the cylinder. As pointed out by Chaplin (1988) and Justesen (1989), a relatively short measuring length gave a higher correlation in the vortex shedding process along the cylinder length, but the correlation was relatively weak for a long length. This could be the possible causes of these discrepancies found in Figure 4-40. For $4 \leq KC_O \leq 8$, the discrepancy between the present results and those of Obasaju et al. (1988) and

Skomedal et al. (1989) is remarkable; this may be attributable to the fact that in the present measurements in this range, the measured transverse forces are still quite stable and repeatable over continuous multiple cycles, resulting in larger RMS values of lift force (refer to Figures 4-7 and 4-8). The smaller values of the lift coefficients C_{Lrms} in Obasaju et al. (1988) and Skomedal et al. (1989) may be due to the irregular vortex patterns and the associated unstable transverse forces, as they stated.

Figure 4-39 Comparison of root-mean-square lift coefficients versus KC_o number between tests of $T_o = 1.6s$ and $T_o = 1.2s$ with data from Chaplin (1988).

Figure 4-40 Comparison of root-mean-square lift coefficients versus KC_o number between tests of $T_o = 5.0s$ with data from Obasaju et al. (1988) and Skomedal et al. (1989).

4.5.4 Mean lift coefficients for a cylinder oscillating in still water

As suggested by Williamson (1985), Obasaju et al. (1988), Lam and Dai (2002) and Lam et al. (2010), the vortex shedding activity in the asymmetrical and transverse street/single pair regime mainly takes place on one side of the cylinder, leading to a considerable time-invariant steady transverse force. The measured transverse forces (refer to Section 4.5.1) show that, in both the asymmetrical and transverse street /single pair regimes (where $KC_o < 15$, according to Williamson, 1985), vortex shedding and the associated transverse force are obviously dependent on both KC_o and β_o numbers. Figure 4-41 shows that the mean lift coefficient C_{Lmean} is not only dependent on KC_o number, but also significantly affected by the period of oscillation T_o (i.e. β_o number). The present results indicate that for a given KC_o number, vortex shedding is more likely to take place on one side of the cylinder when the β_o number is small. For all β_o numbers, increasing KC_o number generally reduces the mean lift coefficients C_{Lmean} .

Although Williamson (1985) and Obasaju et al. (1988) mentioned that the cylinder would experience a steady mean transverse force, few data are available for comparison with the present data. It is believed that the various transverse coefficients reported here can provide an extensive data set for understanding the transverse force acting on a circular cylinder in oscillatory flow.

Figure 4-41 Comparisons of mean lift coefficients versus KC_o number for all oscillation periods.

4.5.5 Results for a fixed cylinder in regular waves with $T_w=1.2s$, $T_w=1.6s$ and $T_w=2.0s$

This section reports the transverse forces for a fixed cylinder model in regular waves. The results will also be compared with those for a cylinder oscillating in still water at the same β number.

From Figures 4-44, 4-45, 4-51, 4-52, 4-58 and 4-59, it can be seen when $KC_w < 5$, the fundamental frequency of lift force is the wave frequency f_w , which is true for all three tested wave frequencies; when $5 < KC_w < 13$ ($KC_w = 13$ is the maximum number that can be generated in the present facility), the fundamental frequency of lift force is two times the wave frequency f_w . The present results

agree with the following conclusions drawn by Isaacson and Maull (1976) and Chakrabarti et al. (1976): one vortex is shed per half cycle in the range of $6 \leq KC_w \leq 15$ and thus the flow is asymmetrical and the fundamental frequency of lift force is two times the wave frequency.

Figures 4-45, 4-52 and 4-59 show the comparisons of the lift coefficients (up to the 4th order) for two types of tests: a fixed cylinder in regular waves and the same cylinder oscillating in still water; results for three test periods are included in the figures. Within the range of the KC numbers that are common for both types of tests, the 1st harmonic lift coefficients agree well for these two types of tests. The 2nd harmonic lift coefficients for a fixed cylinder in regular waves ($T_w = 1.2s$) are slightly larger than those for the same cylinder oscillating in still water ($T_o = 1.2s$); the maximum and RMS lift coefficients for $T_w = 1.2s$ also showed this trend, as shown in Figures 4-46 and 4-47. The 2nd harmonic lift coefficients for a fixed cylinder in regular waves ($T_w = 1.6s$ and $2.0s$) are generally smaller than those for the same cylinder oscillating in still water (see Figures 4-52 and 4-59). As shown in Figures 4-53, 4-54, 4-60 and 4-61, the maximum and RMS coefficients show the similar trends as that of the 2nd harmonic lift coefficients; this may be due to the fact that two-dimensional flows, compared to three-dimensional flows, may induce stronger vortex shedding and consequently larger lift forces.

As shown in Figures 4-42, 4-49 and 4-56, mean transverse forces can be observed, indicating that asymmetric shedding of vortex occurred. The corresponding mean lift force coefficients are shown in Figures 4-48, 4-55 and 4-62. Referring to

Figures 4-50(f), 4-57(e) and (f), the maximum transverse force can be of the same magnitude as that of the in-line component.

Figure 4-63 shows that the 1st and 2nd harmonic coefficients of the present study agree favourably with those of Chakrabarti et al. (1976): the 2nd harmonic lift forces starts to be predominant when $KC_w > 5$, and reaches the peak values in the range of about $KC_w = 8 \sim 10$.

Figure 4-42 Transverse forces measured at different KC_w numbers for $T_w = 1.2s$.

Figure 4-43 The in-line and transverse forces measured at different numbers for $T_w = 1.2s$.

(a): $KC_w = 1.6$, (b): $KC_w = 2.5$, (c): $KC_w = 3.4$, (d): $KC_w = 4.2$, (e): $KC_w = 5.0$, (f): $KC_w = 6.0$

Figure 4-44 Figure 4-44 Amplitude spectrums measured at different KC_W numbers for $T_W = 1.2s$.

Figure 4-45 Comparison between harmonic coefficients versus KC_O (KC_W) number for $T_O = T_W = 1.2s$.

(Solid symbols: a fixed cylinder in regular waves; hollow symbols: a cylinder oscillating in still water)

Figure 4-46 Maximum lift coefficients versus KC_O (KC_W) number for $T_O = T_W = 1.2s$.

Figure 4-47 Root-mean-square lift coefficients versus KC_O (KC_W) number for $T_O = T_W = 1.2s$.

Figure 4-48 Mean lift coefficients versus KC_O (KC_W) number for $T_O = T_W = 1.2s$.

Figure 4-49 Transverse forces measured at different KC_w numbers for $T_w = 1.6s$.

Figure 4-50 The in-line and transverse forces measured at different numbers for $T_w = 1.6s$.

(a): $KC_w = 2.2$, (b): $KC_w = 3.3$, (c): $KC_w = 4.4$, (d): $KC_w = 5.6$, (e): $KC_w = 6.7$, (f): $KC_w = 8.0$

Figure 4-51 Amplitude spectra measured at different KC_w numbers for $T_w = 1.6s$.

Figure 4-52 Comparison between harmonic coefficients versus KC_O (KC_w) number for

$$T_O = T_w = 1.6s.$$

(Solid symbols: a fixed cylinder in regular waves; hollow symbols: a cylinder oscillating in still water)

Figure 4-53 Maximum lift coefficients versus KC_O (KC_W) number for $T_O = T_W = 1.6s$.

Figure 4-54 Root-Mean-Square lift coefficients versus KC_O (KC_W) number for $T_O = T_W = 1.6s$.

Figure 4-55 Mean lift coefficients versus KC_O (KC_W) number for $T_O = T_W = 1.6s$.

Figure 4-56 Transverse forces measured at different KC_w numbers for $T_w = 2.0s$.

Figure 4-57 The in-line and transverse forces measured at different numbers for $T_w = 2.0s$.

(a): $KC_w = 4.2$, (b): $KC_w = 5.5$, (c): $KC_w = 6.9$, (d): $KC_w = 8.3$, (e): $KC_w = 11.2$, (f): $KC_w = 12.7$

Figure 4-58 Amplitude spectrums measured at different KC_w numbers for $T_w = 2.0s$.

Figure 4-59 Comparison between harmonic coefficients versus KC_o (KC_w) number for

$$T_o = T_w = 2.0s .$$

(Solid symbols: a fixed cylinder in regular waves; hollow symbols: a cylinder oscillating in still water)

Figure 4-60 Maximum lift coefficients versus KC_O (KC_W) number for $T_O = T_W = 2.0s$.

Figure 4-61 Root-mean-square lift coefficients versus KC_O (KC_W) number for $T_O = T_W = 2.0s$.

Figure 4-62 Mean lift coefficients versus KC_O (KC_W) number for $T_O = T_W = 2.0s$.

Figure 4-63 Comparison of 1st and 2nd harmonic lift coefficients with data from Chakrabarti et al. (1976).

4.6 Summary

Transverse forces for two types of tests (a cylinder oscillating in still water and a fixed cylinder in regular waves) have been reported. The following conclusions can be drawn:

- 1) At some KC_o numbers, the magnitude of lift force component can be larger than the corresponding in-line component: for example, at $\beta_o = 1563$, the magnitude of the lift forces at some numbers can be twice as large as that of the corresponding in-line forces.
- 2) For $6 \leq KC_o \leq 8$, the flow pattern changed from the asymmetrical regime to the transverse street/single pair regime. For $\beta_o = 2083, 1563$ and 1250 , the vortex shedding could be unstable and intermittent characteristics on the hydrodynamic force could be observed.

- 3) Mean lift force were observed due to the asymmetrical vortex shedding from the cylinder. This is more noticeable for smaller values of β_o number. It was found that the mean lift coefficients decreased with increasing KC_o number.
- 4) It was found from the amplitude spectrums of lift force that the transition from one vortex shedding regime to another was affected by both KC_o numbers and β_o number.
- 5) Various lift coefficients (harmonic lift coefficients, maximum lift coefficient, root-mean-square lift coefficient and mean lift coefficient) were influenced by the frequency number β_o . In general, reducing β_o number reduced the KC_o number at which the peak values of both C_{Lrms} and C_{Lmax} occur; for a given β_o number, the 2nd harmonic coefficients approximately follow the same trend as those of C_{Lrms} and C_{Lmax} . For very small β_o numbers ($\beta_o = 625, 500$ and 417), no obvious peaks of C_{Lrms} and C_{Lmax} were found.
- 6) Lift forces for a fixed cylinder in regular waves showed similar characteristics as those for the same cylinder oscillating in still water. However, the lift forces for a fixed cylinder in regular waves were slightly smaller than those for the same cylinder oscillating in still water.

Chapter 5

Hydrodynamic Forces on a Cylinder Oscillating with Dual-frequency in Still Water

5.1 Preliminary remarks

In this chapter, experimental results of hydrodynamic forces acting on a cylinder oscillating with dual-frequency in still water are reported, with the high-frequency simulating wave-frequency and the low-frequency the slow-drift motion. The objective of this study is to gain insights into the hydrodynamic characteristics of a slowly surging cylinder in waves in an easily-to-control environment. Two forms of Morison equations (Relative-Velocity Morison equation and Independent-Flow Morison equation) will be used to analyze the in-line forces. Results of the transverse forces are also presented.

5.2 Modeling the hydrodynamic force on a cylinder oscillating with dual-frequency in still water

Morison equation has been used in Chapter 3 for a fixed cylinder in waves or a cylinder sinusoidally oscillating in still water. For problems involving two frequencies (for example, a surging cylinder in regular waves), two forms of Morison equation have been proposed: the Relative-Velocity Morison equation and the Independent-Flow Morison equation (e.g. Laya et al., 1984; Chakrabarti, 1987).

5.2.1 Relative-Velocity Morison equation

For a cylinder surging in ocean waves, the Relative-Velocity Morison equation expresses the drag force as a function of the relative velocity $u_w - \dot{x}$ to give

$$F_x = \frac{1}{2} \rho D C_{Dr} (u_w - \dot{x}) |u_w - \dot{x}| ds + \rho \frac{\pi D^2}{4} C_{MOW} \dot{u}_w ds - C_{AOL} \rho \frac{\pi}{4} D^2 \ddot{x} ds \quad (5-1)$$

where the subscript “W” and “L” refer to the wave-frequency and low (surge)-frequency, respectively. In the Relative-Velocity Morison equation, the drag force is assumed to be proportional to the square of the relative velocity and a drag coefficient C_{Dr} is introduced; the inertia force contains one term proportional to the water particle acceleration associated with high-frequency component and one term proportional to the water particle acceleration associated with low-frequency component. Note that C_{MOW} is not necessarily equal to $C_{AOL} + 1$, since they are related to two different flows. As stated by Laya et al. (1984), the Relative-Velocity Morison equation is rather heuristic even though it may seem to be an obvious extension of the original Morison equation.

The objective of this chapter is to use a cylinder oscillating with dual-frequency in still water to simulate a surging cylinder in regular waves. Accordingly, for a cylinder oscillating with dual-frequency in still water, the Relative-Velocity Morison equation can be written as

$$F_x = -\frac{1}{2} \rho D C_{Dr} (\dot{x}_L + \dot{x}_H) |\dot{x}_L + \dot{x}_H| ds - C_{AOH} \rho \frac{\pi}{4} D^2 \ddot{x}_H ds - \rho \frac{\pi D^2}{4} C_{AOL} \ddot{x}_L ds \quad (5-2)$$

where the subscript “H” refers to the high-frequency and “L” the low-frequency. The inertia force term related to wave motion in Equation (5-1) is replaced by the added mass force related to the high-frequency component of the cylinder acceleration.

5.2.2 Independent-Flow Morison equation

Based on Moe and Verley (1980)'s experimental work for a steady current past an oscillating cylinder, Laya et al. (1984) argued that for the case of a rapidly vibrating cylinder in a relatively slowly oscillating external flow (or a cylinder oscillating with a low-frequency in a high-frequency flow, e.g. a slowly- drift moored structure in waves), any vortex initiation is virtually eliminated by the cylinder due to its high rate of vibrations. Under this condition, the water particles near the cylinder's field of influence follow a wildly disorganized and unsteady flow pattern, and thus the suitability of the Relative-Velocity Morison equation is highly suspected. Therefore, Laya et al. (1984) extended the Independent-Flow Morison equation for a cylinder oscillating in a steady current to a cylinder oscillating in waves:

$$F_x = \frac{1}{2} \rho D C_{DOW} u_w |u_w| ds + \rho \frac{\pi D^2}{4} C_{MOW} \dot{u}_w ds - \frac{1}{2} \rho C_{DOL} D |\dot{x}| \dot{x} ds - C_{AOL} \rho \frac{\pi}{4} D^2 \ddot{x} ds \quad (5-3)$$

Take the slowly surging cylinder in a wave field for example; the Independent-Flow Morison equation assumes that the resultant hydrodynamic force acting on the surging cylinder is the superposition of the forces due to two independent flow fields, i.e. the interaction between the flows with different frequencies can be ignored. When using a cylinder oscillating with dual-frequency in still water to simulate a surging cylinder in regular waves, the Independent-Flow Morison equation can be expressed as

$$F_x = -\frac{1}{2} \rho D C_{DOL} \dot{x}_L |\dot{x}_L| ds - C_{AOL} \rho \frac{\pi D^2}{4} \ddot{x}_L ds - \frac{1}{2} \rho D C_{DOH} \dot{x}_H |\dot{x}_H| ds - C_{AOH} \rho \frac{\pi}{4} D^2 \ddot{x}_H ds \quad (5-4)$$

where the total drag force is the sum of the drag forces associated with the two oscillation frequencies. Two drag coefficients are introduced for each frequency: the drag coefficient C_{DOL} for the low-frequency component of the dual-frequency oscillation and C_{DOH} for the high-frequency component of the dual-frequency oscillation.

5.3 Experimental setup and procedure

The experimental setup and procedure are exactly the same as those described in Chapters 3 and 4, except that in dual-frequency oscillations, the motion profile is a combination of a high-frequency and a low-frequency oscillation with different amplitudes.

5.4 Experimental program

5.4.1 Test conditions

Three sets of tests were carried out. The first set of tests examined the hydrodynamic force acting on a cylinder oscillating with a single high frequency. For the first set of tests, two high frequencies were selected: $T_o = 1.2s$ and $1.6s$, and for each frequency, various amplitudes were tested, as detailed in Table 5-1. The second set of tests examined the hydrodynamic force acting on a cylinder oscillating with a single low frequency. Three low frequencies were selected: $T_o = 4.0s, 6.0s$ and $8.0s$, for each frequency, various amplitudes were tested, as detailed in Table 5-2. The third set of tests was designed to understand how the

interaction between the low- and high- frequencies affects the hydrodynamic force, and the test conditions are listed in Table 5-3.

Table 5-1 Test conditions for a cylinder oscillating with a single high-frequency in still water

Period T_o (s)	Amplitude A_o (m)	$KC_o = 2\pi A_o D$	$Re_o = U_{mo} D / \nu$	$\beta_o = Re_o / KC_o$ $= D^2 / (\nu T_o)$
1.2	0.0325, 0.039	4.1, 4.9	8508, 10210	2083
1.6	0.039, 0.0455	4.9, 5.7	7658, 8934	1563

Table 5-2 Test conditions for a cylinder oscillating with a single low-frequency in still water

Period T_o (s)	Amplitude A_o (m)	$KC_o = 2\pi A_o D$	$Re_o = U_{mo} D / \nu$	$\beta_o = Re_o / KC_o$ $= D^2 / (\nu T_o)$
4.0	0.039~0.104	4.9~13.1	3063~8168	625
6.0	0.039~0.143	4.9~18	2042~7487	417
8.0	0.039~0.182	4.9~22.9	1531~7147	313

Table 5-3 Test conditions for a cylinder oscillating with dual-frequency in still water

	$T_{OL}=4.0s$	$T_{OL}=6.0s$	$T_{OL}=8.0s$
$T_{OH}=1.2s$	$A_{OH} : 0.0325, 0.039m$ $A_{OL} : 0.039 \sim 0.091m$	$A_{OH} : 0.0325, 0.039m$ $A_{OL} : 0.039 \sim 0.13m$	$A_{OH} : 0.0325, 0.039m$ $A_{OL} : 0.039 \sim 0.156m$
$T_{OH}=1.6s$	$A_{OH} : 0.039, 0.0455m$ $A_{OL} : 0.039 \sim 0.104m$	$A_{OH} : 0.039, 0.0455m$ $A_{OL} : 0.039 \sim 0.13m$	$A_{OH} : 0.039, 0.0455m$ $A_{OL} : 0.039 \sim 0.182m$

5.4.2 Displacement of a cylinder oscillating with dual-frequency

The displacement of a cylinder oscillating with dual-frequency can be expressed by

$$x(t) = x_H(t) + x_L(t) = A_{OH} \sin(\omega_{OH}t) + A_{OL} \sin(\omega_{OL}t) \quad (5-5)$$

where the first term is the high-frequency component with A_{OH} being the high-frequency amplitude and ω_{OH} the high-frequency angular frequency; the second term is the low-frequency component, with A_{OL} being the low-frequency amplitude and ω_{OL} the low-frequency angular frequency. Figure 5-1 shows the samples of the displacement and velocity of the cylinder oscillating with dual-frequency.

As shown in Figure 5-1(b), when the amplitude of the low frequency component is small, the velocity of high-frequency dominates the total velocity. When the amplitude of the low-frequency component is much larger than that of the high-frequency component, the velocity of the low-frequency component dominates the total velocity.

Figure 5-1 Samples of the displacement (a) and velocity (b) of the cylinder oscillating with dual-frequency. Continued on the next page.
 $(T_{OH} = 1.6s, A_{OH} = 0.039m, T_{OL} = 8.0s)$.

Figure 5-1 Continued.

For later discussion, the excursion between two turning peak points is defined as “one swing”, as shown in Figure 5-2: if the velocity is positive, it is called a “forward swing”; and if the velocity is negative, then the swing is a “backward swing. In each swing, the cylinder experiences either a local maximum or a minimum velocity, because in each “swing” the cylinder either accelerates or decelerates. When the low-frequency and high-frequency velocities are in the same direction, the excursion is increased; while when the low-frequency and high-frequency velocities are in the opposite direction, the excursion is reduced due to the cancellation effect. In each “swing”, the duration is half the period of the high-frequency component, i.e. $T_{OH} / 2$. The difference in the magnitude or velocity between the adjacent forward and backward “swing” differentiates the dual-frequency motion from the single-frequency motion and may have a significant influence on both the low- and high-frequency hydrodynamic forces.

Figure 5-2 Definitions of forward swing and backward swing.

$$(T_{OH} = 1.2s, A_{OH} = 0.039m, T_{OL} = 8.0s, A_{OL} = 0.117m)$$

5.5 Methods of hydrodynamic coefficients determination

As stated in Section 3.6.3 of Chapter 3, the least square method is more general in calculating hydrodynamic coefficients (Dunn, 2005). A detailed derivation of least square analysis for calculating the hydrodynamic coefficients is given in Appendix-B. Matlab codes were developed for various forms of Morison equation to calculate the hydrodynamic coefficients.

For a cylinder oscillating with dual-frequency in still water, the measured in-line force time series $F_M(t)$ can be expanded into a Fourier series in two fundamental frequencies (ω_{OL} and ω_{OH}). Our results showed that the added mass coefficients for the two frequencies were the same in the two forms of Morison equation (5-2) and (5-4), because the added mass forces are treated in the same ways in these two

equations. The present results showed that Fourier analysis (see Appendix-C) and least square method gave almost the same added mass coefficients.

5.6 Results and discussion

In this section, the hydrodynamic coefficients obtained by using the two forms of the Morison equation (Independent-Flow equation and Relative-Velocity Morison equation) are reported. All the hydrodynamic coefficients were obtained using the least square method.

5.6.1 Hydrodynamic coefficients for Independent-Flow Morison equation

It is intuitively reasonable to present the hydrodynamic coefficients for both the high- and low- frequency components as functions of a KC number defined by $KC_{OL} = U_{mL} T_{OL} / D$, where U_{mL} is the velocity magnitude of slow oscillation, T_{OL} is period of slow oscillation and D is diameter of the model.

5.6.1.1 Hydrodynamic coefficients for the low-frequency component

The hydrodynamic coefficients C_{DOL} and C_{AOL} for the low-frequency component are shown in Figures 5-3 through 5-8, where the added mass and drag coefficients for a cylinder oscillating with a single frequency (with its period $T_o = T_{OL}$) in still water are also included for comparison. At the same $KC_{OL}(KC_o)$ number, the coefficients C_{DOL} and C_{AOL} are much larger than their corresponding values for a cylinder oscillating with a single frequency in still water, and this is especially obvious for smaller values of $KC_{OL}(= U_{mL} T_{OL} / D)$ number. Therefore, the coefficients C_{DOL} and

C_{AOL} for the low-frequency component could be strongly affected by the high-frequency component at smaller values of KC_{OL} number, but would have the tendency to approach the values for the cylinder oscillating in a single frequency in still water when the KC number is large enough (i.e. the flow is dominated by the large amplitude, low-frequency component). Due to the capacity limitation of the actuator, in the present experiments, few data can be obtained at large KC_{OL} numbers for the cylinder oscillating with dual-frequency in still water.

Figure 5-3 The added mass (a) and drag (b) coefficients for the low-frequency component in the Independent-Flow Morison equation ($T_{OH} = 1.2s, KC_{OH} = 4.1$ and $4.9, T_{OL} = 4.0s$). The results for $T_O = 4.0s$ are for the cylinder oscillating with a single frequency in still water.

Figure 5-4 The added mass (a) and drag (b) coefficients associated with the low-frequency component in the Independent-Flow Morison equation ($T_{OH} = 1.6s, KC_{OH} = 4.9$ and $5.7, T_{OL} = 4.0s$). The results for $T_O = 4.0s$ are for the cylinder oscillating with a single frequency in still water.

Figure 5-5 The added mass (a) and drag (b) coefficients associated with the low-frequency component in the Independent-Flow Morison equation ($T_{OH} = 1.2s, KC_{OH} = 4.1$ and $4.9, T_{OL} = 6.0s$). The results for $T_O = 6.0s$ are for the cylinder oscillating with a single frequency in still water.

Figure 5-5 Continued.

Figure 5-6 The added mass (a) and drag (b) coefficients associated with the low-frequency component in the Independent-Flow Morison equation ($T_{OH} = 1.6s, KC_{OH} = 4.9$ and $5.7, T_{OL} = 6.0s$).

The results for $T_O = 6.0s$ are for the cylinder oscillating with a single frequency in still water.

Figure 5-7 The added mass (a) and drag (b) coefficients associated with the low-frequency component in the Independent-Flow Morison equation ($T_{OH} = 1.2s, KC_{OH} = 4.1$ and $4.9, T_{OL} = 8.0s$).

The results for $T_O = 8.0s$ are for the cylinder oscillating with a single frequency in still water.

Figure 5-8 The added mass (a) and drag (b) coefficients associated with the low-frequency component in the Independent-Flow Morison equation ($T_{OH} = 1.6s, KC_{OH} = 4.9$ and $5.7, T_{OL} = 8.0s$).

The results for $T_O = 8.0s$ are for the cylinder oscillating with a single frequency in still water.

Continued on the next page.

Figure 5-8 Continued.

Figures 5-3 through 5-8 suggest that the flows of different frequencies are not independent in the present experiments. The added mass and drag coefficients obtained for a cylinder oscillating with a single frequency in still water cannot be used in the Independent-Flow Morison equation for the same cylinder oscillating with dual-frequency in still water. In the investigation of the low-frequency damping forces for oscillating cylinders and plates, Ikeda and Fujiwara (1990) also pointed out the difference in the drag forces between the two-half cycles defined by the high-frequency period yields the damping force for a low-frequency oscillation. Moreover, from the present results of added mass coefficients, it is believed that the large values of the added mass coefficient for the low-frequency component in the Independent-Flow Morison equation can also be attributable to the difference in the two successive forward and backward wings.

As shown in Figures 5-3 through 5-8, the drag (C_{DOL}) and added mass (C_{AOL}) coefficients for low-frequency component of the same oscillation amplitude (A_{OL}) increased with increasing amplitude of the high-frequency amplitude (A_{OH}) in the

Independent-Flow Morison equation, which means the drag and added mass coefficients for low-frequency components are dependent on the high-frequency amplitude (A_{OH}). Therefore, it is noteworthy to further investigate the influence of the amplitude of the high-frequency component (A_{OH}) on the hydrodynamic coefficients for the low-frequency component.

By comparing Figures 5-7 and 5-8 between force coefficients of different KC_{OH} numbers for $T_{OH} = 1.2s$ and $T_{OH} = 1.6s$, respectively, it can be found that KC_{OL} is not a suitable parameter to describe the drag and added mass coefficients for the low-frequency component in Independent-Flow Morison equation, because these coefficients do not consistently vary with different KC_{OH} numbers.

In order to account for the influence of the period of the high-frequency component (T_{OH}), the hydrodynamic coefficients for the low-frequency component will be analyzed using a new dimensionless parameter called as the “reduced velocity”. The reduced velocity has been used by many researchers in the past (e.g. Verley and Moe, 1979 and 1980; Koterayama, 1984; Saprkaya and Storm, 1985; Teng and Nath, 1985; Low et al., 1989; Shafiee-far, 1997) in the investigation of a cylinder oscillating in a current flow. For the case a cylinder oscillating in a current, the reduce velocity physically represents the ratio of the travelling length of a water particle due to the current during one cycle of the oscillation of the cylinder to the cylinder diameter (or the ratio of the oscillation period to the wake development time). Koterayama and Nakamura (1988) and Hu and Koterayama(1996) used the reduced velocity to investigate a vertical or horizontal cylinder oscillating in waves.

Figure 5-9 The added mass coefficients for the low-frequency component in the Independent-Flow Morison equation versus reduced velocity U_r for various KC_{OH} numbers.
($T_{OH} = 1.2$ and $1.6s, T_{OL} = 4.0, 6.0$ and $8.0s$)

In the present study, the reduced velocity is defined as $U_r = U_{mL} T_{OH} / D$. For the problems studied here, the reduced velocity represents the ratio of the travelling length of a water particle associated with the low-frequency motion during one cycle of the high-frequency oscillation of the cylinder to the cylinder diameter. The hydrodynamic coefficients for the low-frequency component (C_{AOL} and C_{DOL}) are shown in Figures 5-9 and 5-10 as functions of the reduced velocity $U_r = U_{mL} T_{OH} / D$ for various KC_{OH} numbers. Figure 5-9 shows that the added mass coefficient for the low-frequency component (C_{AOL}) decreases with increasing U_r for a given KC_{OH} numbers. Similarly, Figure 5-10 also shows that the drag coefficient for the low-frequency component (C_{DOL}) decreases with increasing reduced velocity U_r for

a given KC_{OH} number. In general, at a given reduced velocity U_r number, reducing KC_{OH} number reduce the added mass and drag coefficients for low-frequency component.

Figure 5-10 The drag coefficients for the low-frequency component in the Independent-Flow Morison equation versus reduced velocity U_r for various KC_{OH} numbers.
($T_{OH} = 1.2 \text{ and } 1.6s, T_{OL} = 4.0, 6.0 \text{ and } 8.0s$)

5.6.1.2 Hydrodynamic coefficients for the high-frequency component

The added mass coefficients C_{AOH} and drag coefficients C_{DOH} for the high-frequency component are also firstly plotted against the KC number defined by $KC_{OL} = U_{mL} T_{OL} / D$ in Figures 5-11~5-16. For a given amplitude of the high-frequency component A_{OH} , the added mass coefficient C_{AOH} decreases with increasing KC_{OL} , suggesting that the amplitude of the low-frequency component A_{OL}

has a remarkable influence on the added mass coefficients for the high-frequency component C_{AOH} . For the drag coefficients C_{DOH} , increasing amplitude of the low-frequency component A_{OL} significantly increases the drag coefficients for the high-frequency component C_{DOH} . The present results showed that both the added mass coefficient C_{AOH} and the drag coefficient C_{DOH} in the Independent-Flow Morison equation for a cylinder oscillating with dual-frequency in still water are very different from the corresponding values for a cylinder oscillating with a single frequency in still water.

Figure 5-11 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_{OL} number ($T_{OH} = 1.2s, KC_{OH} = 4.1$ and 4.9 , $T_{OL} = 4.0s$).

Figure 5-12 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation KC_{OL} number ($T_{OH} = 1.6s, KC_{OH} = 4.9$ and $5.7, T_{OL} = 4.0s$).

Figure 5-13 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_{OL} number ($T_{OH} = 1.2s, KC_{OH} = 4.1$ and $4.9, T_{OL} = 6.0s$). Continued on the next page.

Figure 5-13 Continued.

Figure 5-14 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_{OL} number ($T_{OH} = 1.6s, KC_{OH} = 4.9$ and 5.7 , $T_{OL} = 6.0s$).

Figure 5-15 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_{OL} number ($T_{OH} = 1.2s, KC_{OH} = 4.1$ and 4.9 , $T_{OL} = 8.0s$).

Figure 5-16 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_{OL} number ($T_{OH} = 1.6s, KC_{OH} = 4.9$ and 5.7 , $T_{OL} = 8.0s$). Continued on the next page.

Figure 5-16 Continued.

Figures 5-11(a) through 5-16(a) show the effects of the amplitude of the high-frequency component (A_{OH}) on the hydrodynamic coefficients C_{AOH} at given T_{OH} , T_{OL} and A_{OL} . The present results showed that A_{OH} had a little influence on the added mass coefficient C_{AOH} ; at a given value of $KC_{OL}(=U_{mL}T_{OL}/D)$ number, reducing A_{OH} slightly increased the C_{AOH} value.

From Figures 5-11(b) through 5-16(b), it can also be found that C_{DOH} is distinctly dependent on A_{OH} (or KC_{OH} number). Given a combination of T_{OH} and T_{OL} , when A_{OL} is small, the flow is dominated by the high-frequency component: C_{DOH} increases with increasing A_{OH} (or higher KC_{OH}). However, with increasing A_{OL} , the flow becomes more dominated by the low-frequency component, causing a strong drag force during each oscillation cycle of high-frequency. As shown in Figure 5-14 (b), 5-15(b) and 5-16 (b), C_{DOH} increases with decreasing A_{OH} (KC_{OH} number).

For the hydrodynamic coefficients for the high-frequency component in the Independent-Flow Morison equation, the reduced velocity U_r is not a suitable parameter to present C_{DOH} and C_{AOH} , because the physical meaning is not easily understandable when these values are plotted against the reduced velocity U_r .

An attempt was made in the present study to find a new parameter to present the results of C_{AOH} and C_{DOH} . A new local KC number is defined as $KC_l = (U_{mL} + U_{mH})T_{OH} / D$, where the maximum velocity of a water particle is the summation of the low- and high-frequency velocity components and the time scale is the period of the high-frequency component T_{OH} . Here, the subscript “l” signifies that this new KC number is defined in terms of the “local” flow characteristics. Figures 5-17 through 5-22 present the coefficients C_{AOH} and C_{DOH} as functions of KC_l number. It can be seen that both C_{AOH} and C_{DOH} are correlated well with KC_l .

Figure 5-23 and 5-24 present force coefficients C_{AOH} and C_{DOH} as functions of KC_l number for various KC_{OH} numbers. Both C_{AOH} and C_{DOH} are correlated well with KC_l number for all the KC_{OH} numbers examined in the present study. For a given value of KC_{OH} number, C_{AOH} decreases with increasing KC_l ; for a given value of KC_l , C_{AOH} increases with increasing KC_{OH} number. For a given value of KC_{OH} number, C_{DOH} increases with increasing KC_l ; for a given value of KC_l , roughly, C_{DOH} decreases with increasing KC_{OH} number after $KC_l > 7$.

Figure 5-17 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_I number ($T_{OH} = 1.2s, KC_{OH} = 4.1$ and 4.9 , $T_{OL} = 4.0s$)

Figure 5-18 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_I number ($T_{OH} = 1.6s, KC_{OH} = 4.9$ and 5.7 , $T_{OL} = 4.0s$). Continued on the next page.

Figure 5-18 Continued.

Figure 5-19 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_I number ($T_{OH} = 1.2s, KC_{OH} = 4.1$ and 4.9 , $T_{OL} = 6.0s$).

Figure 5-20 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_I number ($T_{OH} = 1.6s$, $KC_{OH} = 4.9$ and 5.7 , $T_{OL} = 6.0s$).

Figure 5-21 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_I number ($T_{OH} = 1.2s$, $KC_{OH} = 4.1$ and 4.9 , $T_{OL} = 8.0s$). Continued on the next page.

Figure 5-21 Continued.

Figure 5-22 The added mass (a) and drag (b) coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_I number ($T_{OH} = 1.6s$, $KC_{OH} = 4.9$ and 5.7 , $T_{OL} = 8.0s$).

Figure 5-23 The added-mass coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_I for various KC_{OH} numbers.
 $(T_{OH} = 1.2 \text{ and } 1.6 \text{ s}, T_{OL} = 4.0, 6.0 \text{ and } 8.0 \text{ s})$

Figure 5-24 The drag coefficients for the high-frequency component in the Independent-Flow Morison equation versus KC_I for various KC_{OH} numbers.
 $(T_{OH} = 1.2 \text{ and } 1.6 \text{ s}, T_{OL} = 4.0, 6.0 \text{ and } 8.0 \text{ s})$

5.6.1.3 Comments on the hydrodynamic coefficients for the Independent-Flow Morison equation

When a cylinder oscillates with dual-frequency in still water, the flows generated by the two frequency components are not independent. Therefore, when one uses the Independent-Flow Morison to describe the in-line hydrodynamic force acting on a cylinder oscillating with dual-frequency in still water, it is not appropriate to use the added mass and drag coefficients obtained from the experiments on a cylinder oscillating with a single frequency in still water.

5.6.2 Hydrodynamic coefficients for the Relative-Velocity Morison equation

For a given measured force acting on a cylinder oscillating with dual-frequency in still water, the added mass coefficients in both the Independent-Flow Morison equation and the Relative-Velocity Morison equation are exactly the same. This section analyzes the drag coefficient C_{Dr} used in the Relative-Velocity Morison equation.

The drag coefficients C_{Dr} in Relative-Velocity Morison equation were also calculated using the least square method. When the drag coefficients C_{Dr} are plotted against the KC number ($KC_{OL} = U_{mL} T_{OL} / D$), for a given oscillation with dual-frequency, for example, $T_{OH} = 1.2s$ and $T_{OL} = 4.0s$, the drag coefficient C_{Dr} increased generally with increasing KC_{OL} . As shown in Figures 5-25 (a) through 5-30(a), for a given KC_{OL} number, C_{Dr} increases slightly with increasing A_{OH} (or KC_{OH} number). As a result, the drag coefficient C_{Dr} is dependent on both the KC_{OL}

number and KC_{OH} number. Therefore, it is needed to find suitable parameters to present the drag coefficients C_{Dr} . After lots of trial and error, a relative KC number, defined as $KC_r = U_{mr} T_r / D$, is found suitable to present the drag coefficients C_{Dr} ; in this KC number, $U_{mr}(=U_{mL} + U_{mH})$ is the maximum relative velocity and T_r is the relative period between two sinusoidal motions (in this study, T_r is roughly taken as T_{OL}). Figures 5-25(b) ~5-30(b) show the drag coefficients C_{Dr} as functions of the KC_r number; the drag coefficients C_{Dr} correlate well with KC_r . Figure 5-31 (a), Figure 5-31 (c) and Figure 5-31 (c) show the drag coefficients obtained at $T_{OH} = 1.2s$ and $1.6s$ for $T_{OL} = 4s, 6s$, and $8s$. The drag coefficients C_{Dr} for $T_{OH} = 1.6s$ are in general slightly larger than those for $T_{OH} = 1.2s$. The reason should be due to the fact that within the test range with period $T_{OL}(= 4.0s, 6.0s$ or $8.0s)$, generally the high-frequency oscillation dominates the flow; therefore, the drag coefficients are influenced by the high-frequency oscillation period T_{OH} , showing that the drag coefficients for $T_{OH} = 1.6s$ are slightly higher than those for $T_{OH} = 1.2s$. When slow oscillation amplitude A_{OL} becomes larger, the flow will be dominated by the low-frequency component, and the drag coefficient C_{Dr} would approach that for the same cylinder oscillating with a single frequency T_{OL} in still water. More experimental results and discussions will be presented in the next chapter in which larger amplitude slow oscillations of a slowly surging cylinder in regular wave tests are reported.

Figure 5-25 The drag coefficients in the Relative-Velocity Morison equation versus KC_{OL} (a) and KC_r (b) for $T_{OH} = 1.2s$ and $T_{OL} = 4.0s$.

Figure 5-26 The drag coefficients in the Relative-Velocity Morison equation versus KC_{OL} (a) and KC_r (b) for $T_{OH} = 1.6s$ and $T_{OL} = 4.0s$. Continued on the next page.

Figure 5-26 Continued.

Figure 5-27 The drag coefficients in the Relative-Velocity Morison equation versus KC_{OL} (a) and KC_r (b) for $T_{OH} = 1.2s$ and $T_{OL} = 6.0s$.

Figure 5-28 The drag coefficient in the Relative-Velocity Morison equation versus KC_{OL} (a) and KC_r (b) for $T_{OH} = 1.6s$ and $T_{OL} = 6.0s$.

Figure 5-29 The drag coefficient in the Relative-Velocity Morison equation versus KC_{OL} (a) and KC_r (b) for $T_{OH} = 1.2s$ and $T_{OL} = 8.0s$. Continued on the next page.

Figure 5-29 Continued.

Figure 5-30 The drag coefficient in the Relative-Velocity Morison equation versus KC_{OL} (a) and KC_r (b) for $T_{OH} = 1.6s$ and $T_{OL} = 8.0s$.

Figure 5-31 Comparisons showing the effects of high-frequency flow on the drag coefficient in the Relative-Velocity Morison equation.

(a): ($T_{OH} = 1.2s$ and $1.6s$, $T_{OL} = 4.0s$), (b): ($T_{OH} = 1.2s$ and $1.6s$, $T_{OL} = 6.0s$),

(c): ($T_{OH} = 1.2s$ and $1.6s$, $T_{OL} = 8.0s$)

5.7 Remarks on the Independent-Flow Morison equation and the Relative-Velocity Morison equation

In the Relative-Velocity Morison equation, only one drag coefficient is needed, while in the Independent-Flow Morison equation, two drag coefficients are required. The two drag coefficients in the Independent-Flow Morison equation show complicated dependence on dimensionless parameters such as KC_{OH} number, the reduced velocity U_r , or the local KC_l number. Even though it is convenient in engineering practice for engineers to use the Relative-velocity Morison equation with the drag coefficients given as a function of KC_r number, it is difficult to make recommendation on which force model is more appropriate for a given complicated problem without assessing the applicability of them. Due to the limited test range in the present tests, the applicability of the Independent-Flow versus Relative-Velocity Morison equation would not be investigated in this chapter. Detailed explorations about the applicability of these two force models will be given in Section 6.7 in the next chapter based on more extensive range of data from tests of a cylinder slowly surging in regular waves. In general, the R^2 values calculated from both two force models (see the definition of R^2 in Section 3.8 of Chapter 3) fall into the range of 0.75~0.95 for tests with $T_{OH} = 1.2s$ (see Figures 5-32 (a), (c) and (e)), and the range of 0.8~0.95 for tests with $T_{OH} = 1.6s$ (see Figures 5-32 (b), (d) and (f)), suggesting that both these two force models can reflect the in-line force of a cylinder oscillating with dual-frequency in still water with certain accuracy. Selected measured force and the corresponding fitted force time series with both two force models are shown in Figures 5-33 through 5-38 for reference.

Figure 5-32 R-square error for a cylinder oscillating with dual-frequency in still water.
(The R-square values are calculated from both the Independent-Flow and Relative-Velocity Morison equations)

((a): $T_{OH} = 1.2s$ and $T_{OL} = 4.0s$, (b): $T_{OH} = 1.6s$ and $T_{OL} = 4.0s$, (c): $T_{OH} = 1.2s$ and $T_{OL} = 6.0s$)
((d): $T_{OH} = 1.6s$ and $T_{OL} = 6.0s$, (e): $T_{OH} = 1.2s$ and $T_{OL} = 8.0s$, (f): $T_{OH} = 1.6s$ and $T_{OL} = 8.0s$)

Figure 5-33 Examples of measured and fitted forces for a cylinder oscillating with dual-frequency in still water for $T_{OH} = 1.2s$ and $T_{OL} = 4.0s$.

((a): $KC_{OL} = 4.9$, (b): $KC_{OL} = 6.5$, (c): $KC_{OL} = 8.2$, (d): $KC_{OL} = 9.8$, (e): $KC_{OL} = 11.4$)
 (Solid line: measured force time series; dot line: the Relative-Velocity Morison equation;
 dash-dot line: the Independent-Flow Morison equation)

Figure 5-34 Examples of measured and fitted forces for a cylinder oscillating with dual-frequency in still water for $T_{OH} = 1.6s$ and $T_{OL} = 4.0s$
 ((a): $KC_{OL} = 6.5$, (b): $KC_{OL} = 8.2$, (c): $KC_{OL} = 9.8$, (d): $KC_{OL} = 11.4$, (e): $KC_{OL} = 13.1$)
 (Solid line: measured force time series; dot line: the Relative-Velocity Morison equation; dash-dot line: the Independent-Flow Morison equation)

Figure 5-35 Examples of measured and fitted forces for a cylinder oscillating with dual-frequency in still water for $T_{OH} = 1.2s$ and $T_{OL} = 6.0s$
 ((a): $KC_{OL} = 4.9$, (b): $KC_{OL} = 8.2$, (c): $KC_{OL} = 11.4$, (d): $KC_{OL} = 14.7$, (e): $KC_{OL} = 18$)
 (Solid line: measured force time series; dot line: the Relative-Velocity Morison equation; dash-dot line: the Independent-Flow Morison equation)

Figure 5-36 Examples of measured and fitted forces for a cylinder oscillating with dual-frequency in still water for $T_{OH} = 1.6s$ and $T_{OL} = 6.0s$
 ((a): $KC_{OL} = 4.9$, (b): $KC_{OL} = 8.2$, (c): $KC_{OL} = 11.4$, (d): $KC_{OL} = 14.7$, (e): $KC_{OL} = 16.3$)
 (Solid line: measured force time series; dot line: the Relative-Velocity Morison equation; dash-dot line: the Independent- Flow Morison equation)

Figure 5-37 Examples of measured and fitted forces for a cylinder oscillating with dual-frequency in still water for $T_{OH} = 1.2s$ and $T_{OL} = 8.0s$
 ((a): $KC_{OL} = 4.9$, (b): $KC_{OL} = 8.2$, (c): $KC_{OL} = 11.4$, (d): $KC_{OL} = 14.7$, (e): $KC_{OL} = 19.6$)
 (Solid line: measured force time series; dot line: the Relative-Velocity Morison equation; dash-dot line: the Independent-Flow Morison equation)

Figure 5-38 Examples of measured and fitted forces for a cylinder oscillating with dual-frequency in still water for $T_{OH} = 1.6s$ and $T_{OL} = 8.0s$

((a): $KC_{OL} = 4.9$, (b): $KC_{OL} = 9.8$, (c): $KC_{OL} = 14.7$, (d): $KC_{OL} = 19.6$, (e): $KC_{OL} = 22.9$)
 (Solid line: measured force time series; dot line: the Relative-Velocity Morison equation;
 dash-dot line: the Independent- Flow Morison equation)

5.8 Transverse force of Dual-frequency oscillation tests

In this section, results of the transverse forces for a cylinder oscillating with dual-frequency in still water are presented.

5.8.1 Definitions of various transverse force coefficients

Maximum lift force coefficient ($C_{L\max}$) The maximum lift force coefficient is defined as

$$C_{L\max} = \frac{F_{L\max}}{0.5\rho DU_{rms}^2 L} \quad (5-6)$$

where $F_{L\max}$ is the maximum lift force.

Root-Mean-Square lift coefficient (C_{Lrms}) The root-mean-square lift force coefficient is defined as

$$C_{Lrms} = \frac{F_{Lrms}}{0.5\rho DU_{rms}^2 L} \quad (5-7)$$

where F_{Lrms} is the root mean-square-value of the measured lift force.

Mean lift force coefficient (C_{Lmean}) The mean transverse force coefficient is defined as

$$C_{Lmean} = \frac{F_{Lmean}}{0.5\rho DU_{rms}^2 L} \quad (5-8)$$

Unlike a cylinder oscillating with a single frequency in still water, the transverse lift force is normalized by the value of the mean-root-square velocity U_{rms}^2 rather than

the squared maximum velocity U_m^2 . Here, using U_{rms}^2 is considered to be more reasonable in that the root-mean-square lift force is more closely associated with the instantaneous velocity time series rather than the maximum velocity U_m . In single-frequency oscillation tests, the velocity magnitude U_m equals to $\sqrt{2}U_{rms}$, therefore, using any velocity value (U_m or U_{rms}) to normalize the root-mean-square lift force is acceptable.

5.8.2 Characteristics of transverse force coefficients of a cylinder oscillating with dual-frequency in still water

Selected transverse forces are shown in Figures 5-39 and 5-40. Similar to the transverse force in single frequency tests reported in Chapter 4, mean lift force can also be observed, suggesting that vortex shedding took place mainly on one side of the cylinder, especially when the period of the low-frequency was very long, such as $T_{OL} = 8.0s$. Figures 5-41 and 5-42 show that the mean transverse coefficients C_{Lmean} for $T_{OL} = 8.0s$ are obviously higher than those for $T_{OL} = 4.0s$ and $6.0s$.

The amplitude spectrums of transverse force are shown in Figures 5-43 and 5-44 for the transverse forces shown in Figures 5-39 and 5-40. These spectrums were obtained by a FFT analysis as described in Chapter 4, it can be found that the sum-frequency $f_{OH} + f_{OL}$ and the difference-frequency $f_{OH} - f_{OL}$ are also generated in addition to the low-frequency f_{OL} and high-frequency f_{OH} . These sum- and difference-frequency might be caused by vortex-vortex interactions during dual-frequency oscillations. The amplitudes at these sum- and difference-frequencies are quite considerable, signifying that a large portion of the energy in the transverse

force is distributed around these frequencies. In some situations, the amplitudes at the sum-frequency $f_{OH} + f_{OL}$ and the difference-frequency $f_{OH} - f_{OL}$ are even higher than that at f_{OH} (refer to Figure 5-44). Therefore, in the design of offshore structures, special attention should be given to these sum-frequency and difference-frequency components, particularly in the investigation of vortex induced transverse vibration problems.

Figures 5-45 and 5-46 show the root-mean-square coefficients C_{Lrms} . It seems that at the same KC_{OL} number, the root-mean-square coefficients C_{Lrms} for $T_{OL} = 8.0s$ are slightly higher than those for $T_{OL} = 4.0s$ and $6.0s$. The experimental results of the maximum transverse force coefficients C_{Lmax} are also shown in Figures 5-47 and 5-48 for reference. These maximum transverse coefficients are quite scattered, and no definitive conclusion could be drawn for them.

Figure 5-39 Transverse forces measured at different KC_{OL} numbers for $T_{OH} = 1.2s$, $A_{OH} = 0.039m$ and $T_{OL} = 8.0s$.

Figure 5-40 Transverse forces measured at different KC_{OL} numbers for $T_{OH} = 1.6s$, $A_{OH} = 0.0455m$ and $T_{OL} = 8.0s$.

Figure 5-41 Mean transverse force coefficients versus KC_{OL} number for $T_{OH} = 1.2s$.
(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

Figure 5-42 Mean transverse force coefficients versus KC_{OL} number for $T_{OH} = 1.6s$.
(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

Figure 5-43 Amplitude spectra of the transverse force for $T_{OH} = 1.2s$, $A_{OH} = 0.039m$ and $T_{OL} = 8.0s$.

Figure 5-44 Amplitude spectra of the transverse force for $T_{OH} = 1.6s$, $A_{OH} = 0.0455m$ and $T_{OL} = 8.0s$.

Figure 5-45 Root-mean-square transverse force coefficients versus KC_{OL} number for $T_{OH} = 1.2s$.
(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

Figure 5-46 Root-mean-square transverse force coefficients versus KC_{OL} number for $T_{OH} = 1.6s$.
(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

Figure 5-47 Maximum transverse force coefficients versus KC_{OL} number for $T_{OH} = 1.2s$.
(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

Figure 5-48 Maximum transverse force coefficients versus KC_{OL} number for $T_{OH} = 1.6s$.
(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

5.9 Summary

From the present experiments on a cylinder oscillating with dual-frequency in still water, the following conclusions can be drawn:

- 1) Due to the interaction between the two frequencies, the hydrodynamic coefficients in the Morison equation for the same cylinder oscillating with a single frequency in still water cannot be used in the Independent-Flow Morison equation.
- 2) The added mass coefficients evaluated by both the Independent-Flow Morison equation and the Relative-Velocity Morison equation are the same.
- 3) In the Independent-Flow Morison equation, the hydrodynamic coefficients C_{DOL} and C_{AOL} for the low-frequency component correlate well with the reduced velocity U_r ; while the hydrodynamic coefficients C_{DOH} and C_{AOH} for the high-frequency component correlate well with the local KC_l number.
- 4) In the Relative-Velocity Morison equation, the drag coefficients C_{Dr} are well correlated with the KC_r number; when the high-frequency motion dominates the flow, the drag coefficients C_{Dr} are influenced by high-frequency motion.
- 5) The characteristics of vortex shedding mainly occurred on one side of the cylinder when oscillating a cylinder with two frequencies in still water. A lot of energy in the transverse force was distributed at the sum-frequency and difference- frequency.

Chapter 6

Hydrodynamic Forces on a Slowly Surging Cylinder in Regular Waves

6.1 Preliminary remarks

This chapter investigates the hydrodynamic forces acting on a slowly surging vertical circular cylinder in regular waves, with a special emphasis on the viscous drag force induced by both the wave motion and slow oscillation of the cylinder. Experimental results are analyzed using both the Independent-Flow Morison equation and Relative-Velocity Morison equation, and compared with those for a cylinder oscillating with dual-frequency in still water.

6.2 Two forms of Morison equation for a slowly surging cylinder in regular waves

For the convenience of later discussion, Equations (6-1) and (6-2) are, respectively, the Relative-Velocity Morison equation and the Independent-Flow Morison equation, which are re-written using the notations for regular waves and cylinder oscillation. The Relative-Velocity Morison Equation relates the drag force to the relative velocity $u_w - \dot{x}$ through a drag coefficient C_{Dr} :

$$F_x = \frac{1}{2} \rho D C_{Dr} (u_w - \dot{x}) |u_w - \dot{x}| ds + \rho \frac{\pi D^2}{4} C_{MOW} \dot{u}_w ds - C_{AOL} \rho \frac{\pi}{4} D^2 \ddot{x} ds \quad (6-1)$$

where the subscript “W” or “L” is to represent the high-frequency wave motion and the low-frequency oscillation, respectively. The inertia force contains one term related to the wave motion and another term related to the cylinder acceleration. It

is stressed here that C_{MOW} is not necessarily equal to $C_{AOL}+1$, since they are related to two different flows.

In the Independent-Flow Morison equation, the hydrodynamic forces induced by the wave motion and the oscillation of the cylinder are treated separately, and therefore two drag coefficients and two added mass (or inertia) coefficients are required in Equation (6-2),

$$F_x = \frac{1}{2} \rho D C_{DOW} u_w |u_w| ds + \rho \frac{\pi D^2}{4} C_{MOW} \dot{u}_w ds - \frac{1}{2} \rho C_{DOL} D |\dot{x}| \dot{x} ds - C_{AOL} \rho \frac{\pi}{4} D^2 \ddot{x} ds \quad (6-2)$$

where C_{DOW} and C_{MOW} are the drag and inertia coefficients related to the fast wave motion, and C_{DOL} and C_{AOL} are the drag and added mass coefficients related to the slow oscillation of the cylinder.

6.3 Experimental setup

In the present study, all the experiments were carried out in a wave flume of 45 m long, 1.5 m wide. The water depth was fixed at 0.95 m. The same model-driving apparatus and instrumented cylindrical model as described in the preceding chapters were used. In order to determine the force coefficients in Equations (6-1) and (6-2), synchronization among the hydrodynamic force acting on the cylinder, the instantaneous displacement of the cylinder and the water surface displacement is required. Both the motion of the cylinder and the water surface displacement were measured using two non-resistance Ultralab sensors: one was positioned in line with the centre of the cylinder at its side about 0.5m away (the cylinder was stationary

before each test run) so that the line through the centres of the cylinder and the sensor was parallel to the wave crest. Another Ultralab sensor was aligned with the cylinder so that the line through the centres of the cylinder and the sensor is perpendicular to the wave crest, as shown in Figure 6-1. A view of the experimental setup is shown in Figure 6-2.

Figure 6-1 A schematic of the experimental setup for a slowly surging cylinder in regular waves.
(Not drawn to scale)

The present experiment involves measurements of various signals, such as the wave forces, the water surface and the displacement of the oscillating cylinder. A coordinate system therefore is needed. In this study, the origin “*o*” of the fixed coordinate system is the point where the stationary cylinder pierces through the still water surface. The *x* axis points in the direction of wave propagation, and the *z* axis points vertically upwardly.

Figure 6-2 A view of the experimental setup in the wave flume.

The 2nd order Stokes theory is used to calculate the water particle kinematics. In terms of velocity potential, the incident waves can be expressed by

$$\Phi = \frac{\omega \zeta_a}{k} \frac{\cosh k(z+d)}{\sinh kd} \sin \theta + \frac{3}{8} \omega \zeta_a^2 \frac{\cosh 2k(z+d)}{\sinh^4 kd} \sin 2\theta \quad (6-3)$$

where $\theta = kx - \omega_w t$ is the wave phase, ζ_a wave amplitude, $k = 2\pi / L_w$ the wave number, $\omega_w = 2\pi / T_w$ the angular frequency, L_w the wave length, T_w the wave period and d the water depth. The linear dispersion relation is $kd \tanh kd = \frac{\omega^2 d}{g}$.

According to the 2nd order Stokes wave theory, the water surface elevation is given by

$$\zeta(x, t) = \zeta_a \cos \theta + \frac{1}{4} k \zeta_a^2 \frac{\cosh kd}{\sinh^3 kd} [2 + \cosh 2kd] \cos 2\theta \quad (6-4)$$

and the horizontal components of the orbital velocity and acceleration are given by

$$\left. \begin{aligned} u(x, z, t) &= \omega \zeta_a \frac{\cosh k(z+d)}{\sinh kd} \cos \theta + 0.75(k \zeta_a)^2 c \frac{\cosh 2k(z+d)}{\sinh^4 kd} \cos 2\theta \\ \dot{u}(x, z, t) &= \omega^2 \zeta_a \frac{\cosh k(z+d)}{\sinh kd} \sin \theta + \frac{3}{2} (\omega \zeta_a)^2 k \frac{\cosh 2k(z+d)}{\sinh^4 kd} \sin 2\theta \end{aligned} \right\} \quad (6-5)$$

where the wave celerity is $c = L_w / T_w = \omega_w / k$.

Since the water surface displacement is measured at, where $x=0$, the water surface elevation at time t can be described by Equation (6-4) and the horizontal components of the orbital velocity and acceleration by Equation (6-5).

Figure 6-3 Examples of the measured water surface displacement, the displacement of the cylinder, and the hydrodynamic forces.

$$(T_w = 1.2s, A_w = 0.06m(KC_w = 4.8); T_{OL} = 6.0s, A_{OL} = 0.143m(KC_{OL} = 18))$$

During the tests, when the cylinder is forced to oscillate in the direction of the wave propagation, the instantaneous displacement of the oscillating cylinder can be monitored by an Ultralab sensor (see Section 6.3.1). In the rest of this chapter, we take $t = 0$ when a wave crest is passing $x = 0$. The oscillation of the cylinder is synchronized with the wave motion such that the cylinder is passing $x = 0$ at $t = 0$. Figure 6-3 shows examples of the measured water surface displacement, the displacement of the oscillating cylinder, the in-line and transverse forces; all these signals were synchronized.

6.4 Test programs and procedure

6.4.1 Test programs

In the present experiments, the distance between the centre of test section and the still water surface was kept at 0.17m. The test programs for a slowly surging cylinder in regular waves are summarized in Table 6-1: tests were performed with a combination of two periods for waves ($T_w = 1.2\text{s}$ and $T_w = 1.6\text{s}$) and three oscillation periods for the oscillating cylinder ($T_{OL} = 4.0\text{s}$, 6.0s and 8.0s). For each wave period, either one or three wave amplitudes A_w were examined, depending on the oscillation period T_{OL} . The amplitude of the slow oscillation A_{OL} varied within a wide range in a 0.026m increment. For tests with $T_{OL} = 4.0\text{s}$ and $T_{OL} = 8.0\text{s}$, only one wave amplitude A_w was investigated at each wave period. The effects of wave amplitude A_w (or $KC_w = U_{mw}T_w / D$ number) on hydrodynamic coefficients were examined for two wave periods: $T_w = 1.2\text{s}$ and $T_w = 1.6\text{s}$ and a fixed oscillation period $T_{OL} = 6.0\text{s}$.

Table 6-1 Test programs for a slowly surging cylinder in regular waves

	$T_{OL}=4.0s$	$T_{OL}=6.0s$	$T_{OL}=8.0s$
$T_w=1.2s$	$A_w: 0.04m$ $A_{OL}: 0.065\sim 0.195m$	$A_w: 0.03, 0.04, 0.06m$ $A_{OL}: 0.065\sim 0.273m$	$A_w=0.04m$ $A_{OL}: 0.065\sim 0.325m$
$T_w=1.6s$	$A_w: 0.05m$ $A_{OL}: 0.065\sim 0.195m$	$A_w: 0.03, 0.05, 0.07m$ $A_{OL}: 0.065\sim 0.273m$	$A_w=0.05m$ $A_{OL}: 0.065\sim 0.325m$

The test programs were designed in such an attempt as to assess the combined influence by the waves and the slow oscillation of the cylinder on hydrodynamic coefficients. The results reported here can be directly compared with the results in Chapter 3 for a cylinder oscillating with a single frequency in still water or fixed in regular waves, and the results in Chapter 5 for a cylinder oscillating with dual-frequency in still water.

6.4.2 Test procedure

The test procedure for oscillating a cylinder in regular waves was similar to that for a fixed cylinder in regular waves described in Chapter 3, except that the movement of cylinder now needs to be triggered at a specified instant during the tests. In the present experiment, in order to ensure the repeatability, an Ethernet Interface was used to trigger the wave maker and the motion of the cylinder with a specified time delay so that the cylinder model was triggered when a wave crest passes the stationary cylindrical model; this time delay needs to be determined in advance for each run. Each test condition was repeated three times, and good repeatability has been achieved with the present apparatus.

6.5 Determination of hydrodynamic coefficients

All the hydrodynamic coefficients were obtained using a least square method. In calculating the hydrodynamic coefficients, the water particle kinematics is computed in light of the wave phase $\theta = kx - \omega_w t$ at the instantaneous position of oscillating cylinder; the inertia force induced by the mass of the cylinder needs to be subtracted from the measured force.

6.6 Hydrodynamic coefficients for in-line force

Again, for a given measured hydrodynamic force, the inertia coefficient C_{MOW} and the added mass coefficient C_{AOL} take the almost same values in the Independent-Flow Morison equation and the Relative-Velocity Morison equation. Therefore, in the following discussions, the results of the inertia coefficients C_{MOW} and the added mass coefficients C_{AOL} are applicable for both Equation (6-1) and Equation (6-2).

6.6.1 Hydrodynamic coefficients in the Independent-Flow Morison equation

6.6.1.1 Hydrodynamic coefficients for the surging component

Figure 6-4 and Figure 6-5 show the coefficients C_{AOL} and C_{DOL} for two slow oscillation periods $T_{OL} = 4.0$ and $8.0s$, respectively. For small values of KC_{OL} number, the values of C_{AOL} and C_{DOL} are obviously influenced by the wave motion: at the same KC_{OL} (or KC_o) number, both the added mass coefficient C_{AOL} and the drag coefficient C_{DOL} are larger than their corresponding values for the same cylinder oscillating with a single frequency in still water. With increasing KC_{OL}

number, the influence becomes weak: the hydrodynamic coefficients for the low-frequency oscillation have the tendency to approach to the values for the same cylinder oscillating with a single frequency in still water. For a large KC_{OL} number, the flow will be dominated by the slow oscillation rather than waves, and the influence of regular waves on the hydrodynamic forces for the low-frequency oscillation will become minimal; this is especially true when KC_W is small. Figures 6-4 and 6-5 show that the hydrodynamic coefficients for the low-frequency oscillation are also remarkably affected by the KC_W number.

Figure 6-4 The added mass (a) and drag coefficients (b) for the surging motion in the Independent-Flow Morison equation. Force coefficients for a cylinder oscillating with $T_O = 4.0s$ in still water are included for comparisons ($T_W = 1.2s$ and $1.6s$, $KC_W = 3.3$ and 5.5 , $T_{OL} = 4.0s$).

Figure 6-5 The added-mass (a) and drag coefficients (b) for the surging motion in the Independent-Flow Morison equation. Force coefficients for a cylinder oscillating with $T_O = 8.0s$ in still water are included for comparisons ($T_W = 1.2s$ and $1.6s, KC_W = 3.3$ and $5.5, T_{OL} = 8.0s$).

Figures 6-6 and 6-7 show the influence of KC_W number on the hydrodynamic coefficients for the low-frequency oscillation at $T_{OL} = 6.0s$; for all three KC_W numbers, both the added mass and drag coefficients for the low-frequency oscillation decrease with increasing KC_{OL} number. At very small KC_W numbers, such as $KC_W = 2.5$ for $T_W = 1.2s$ and $KC_W = 3.5$ for $T_W = 1.6s$, the drag coefficients C_{DOL} for the low-frequency oscillation are very close to the values for a cylinder oscillating with a single frequency in still water. However, when the KC_W number

is large, effects of waves on the added mass and drag coefficients cannot be ignored:

both C_{DOL} and C_{AOL} increase with increasing KC_W . The conclusions drawn above

are supported by both two groups of results for $T_W = 1.2$ and 1.6 s with $T_{OL} = 6.0$ s .

Figure 6-6 The added-mass (a) and drag coefficients (b) for the surging motion in the Independent-Flow Morison equation. Force coefficients for a cylinder oscillating with $T_O = 6.0$ s in still water are included for comparisons ($T_W = 1.2$ s, $KC_W = 2.5, 3.3$ and 4.8 , $T_{OL} = 6.0$ s).

Figure 6-7 The added-mass (a) and drag coefficients (b) for the surging motion in the Independent-Flow Morison equation. Force coefficients for a cylinder oscillating with $T_o = 6.0s$ in still water are included for comparisons ($T_w = 1.6s$, $KC_w = 3.5, 5.5$ and 7.9 , $T_{OL} = 6.0s$).

When the added mass or drag coefficients for the low-frequency oscillation from both tests of $T_w = 1.2$ and $1.6s$ are plotted in a same figure against the KC_{OL} number, it is found that the KC_{OL} number is not a suitable parameter to present these two force coefficients because of the dependence on the KC_w number. Therefore, in order to account for the influence of the wave period T_w , both C_{AOL} and C_{DOL} are shown in Figure 6-8 as function of the reduced velocity $U_r (= U_{mL} T_w / D)$. Also, the fitted mean curve of low-frequency coefficients from dual-frequency oscillation

tests in still water reported in Chapter 5 are shown for comparisons. Now, the drag coefficient C_{DOL} is well correlated with U_r for all values of the KC_W (and KC_{OH}) number. Note that, the drag coefficients C_{DOL} obtained in the two-dimensional oscillatory flow and in the three-dimensional wave field are different: three-dimensionality seemed having reduced the coefficient C_{DOL} (see Figure 6-8(b)). Note also that scatter in the data for C_{AOL} is still obvious, due possibly to the fact that for the low-frequency oscillation, the added mass force is much smaller compared with the drag force. But it still can be observed that the added mass coefficient C_{AOL} generally increases with increasing KC_W (and KC_{OH}) number.

Figure 6-8 The added mass (a) and drag coefficients (b) for the surging motion in the Independent-Flow Morison equation versus the reduced velocity $U_r = U_{mL} T_W / D$ ($T_w = 1.2s$ and $1.6s$, $T_{OL} = 6.0s$).

Figure 6-8 shows that the coefficient C_{DOL} is well correlated with the reduced velocity U_r for a given KC_W number. In order to examine the dependence of C_{DOL} and C_{AOL} on slow oscillation period T_{OL} , the results for $T_{OL} = 4.0, 6.0$ and $8.0s$ are compared in Figure 6-9 for two groups of tests with $KC_W = 3.3$ and 5.5 ; results show that the coefficient C_{DOL} has no clear dependence on slow oscillation period T_{OL} , but the low-frequency added mass coefficients C_{AOL} are still scattered.

Figure 6-9 The added-mass (a) and drag coefficients (b) for the surging motion in the Independent-Flow Morison equation versus $U_r = U_{mL} T_W / D$ for two values of KC_W number.
($KC_W = 3.3$ and 5.5 , $T_{OL} = 4.0s, 6.0s$ and $8.0s$)

Therefore, it can be safely concluded that in Independent-Flow Morison equation, the low-frequency drag coefficients C_{DOL} can be well presented against the reduced velocity U_r for a given value of the KC_w number. Moreover, no obvious dependence on slow oscillation period T_{OL} is found based on the present test results. Due to the relatively smaller value of low-frequency added mass force, the low-frequency added mass coefficients C_{AOL} are a little scattered.

6.6.1.2 Hydrodynamic coefficients for the wave component

The hydrodynamic coefficients for the wave component in the Independent-Flow Morison equation are shown in Figures 6-10 and 6-11 as functions of the KC_{OL} number. The results indicate that the amplitude of the surging motion A_{OL} may have a profound influence on both the drag coefficient C_{DOW} and the inertia coefficient C_{MOW} . For a given combination of wave period T_w and slowly surging period T_{OL} , the flow is dominated by wave motion when A_{OL} is small; in this case, the measured drag coefficient C_{DOW} generally increased with increasing wave amplitude A_w (or KC_w number). However, the flow is dominated by the surging motion when the surging amplitude A_{OL} is large; in this case, the viscous separation is mainly attributable to the large amplitude surging motions, causing strong drag force during each wave cycle, and the drag coefficient C_{DOW} increased with decreasing wave amplitude A_w (or KC_w) (see Figure 6-10(b) and Figure 6-11(b)).

As for the inertia coefficients C_{MOW} shown in Figure 6-10 (a) and Figure 6-11 (a)), C_{MOW} firstly decreased with increasing surging amplitude A_{OL} (KC_{OL}), reached a

minimum of C_{MOW} at a certain value of the KC_{OL} number, then rose slightly with increasing KC_{OL} number. It seems that C_{MOW} was not as sensitive to the wave amplitudes A_w (or KC_w number) as the drag coefficient C_{DOW} was. The conclusions drawn above are in general agreement with those for the added mass coefficients C_{AOH} for a cylinder oscillating with dual-frequency in still water (see Section 5.6.1.2).

Figure 6-10 The inertia (a) and drag (b) coefficients for the wave motion versus KC_{OL} number.

($T_w = 1.2s$, $KC_w = 2.5, 3.3$ and 4.8 , $T_{OL} = 6.0s$).

Figure 6-11 The inertia (a) and drag (b) coefficients for the wave motion versus KC_{OL} number ($T_W = 1.6s$, $KC_W = 3.5, 5.5$ and 7.9 , $T_{OL} = 6.0s$).

Since both C_{MOW} and C_{DOW} are closely associated with the local water particle motion in each wave cycle, a local KC number KC_l defined by $KC_l = (U_{mL} + U_{mW})T_W / D$ based on maximum flow velocity in the wave -oscillation flow field and wave period T_W should be considered as an appropriate parameter to present these two hydrodynamic coefficients. Figure 6-12 (b) shows that the drag coefficient C_{DOW} is correlated well with the KC_l number for each given value of the KC_W number: generally, C_{DOW} increased with decreasing KC_W for a given value of

KC_I number. Figure 6-12 (a) shows that the inertia coefficient C_{MOW} is also well correlated with the local KC_I number; however, C_{MOW} is not strongly affected by KC_W (or the wave amplitude A_W). Compared with the results for a cylinder oscillating with dual-frequency in still water for the same KC_I number and similar KC_W and KC_{OH} numbers, the three-dimensionality of the wave field had reduced the drag coefficient C_{DOW} , but increased the inertia coefficient C_{MOW} , similar to the conclusions in chapter 3.

Figure 6-12 The inertia (a) and drag (b) coefficients for the wave motion versus KC_I number

$$(T_W = 1.2s \text{ and } 1.6s, T_{OL} = 6.0s).$$

To show the dependence of inertia coefficients C_{MOW} and drag coefficients C_{DOW} on the surging period T_{OL} , the results for $KC_W = 3.3$ and 5.5 and $T_{OL} = 4.0, 6.0$ and $8.0s$ are also compared in Figure 6-13. Results showed that both the inertia coefficients C_{MOW} and drag coefficients C_{DOW} had no obvious dependence on the surging period T_{OL} .

Figure 6-13 The inertia (a) and drag (b) coefficients for the wave motion versus KC_I number ($KC_W = 3.3$ and 5.5 , $T_{OL} = 4.0, 6.0$ and $8.0s$).

6.6.1.3 Comments on the hydrodynamic coefficients in the Independent-Flow Morison equation

From the results for the hydrodynamic coefficients of C_{DOL} , C_{AOL} , C_{DOW} and C_{MOW} in the Independent-Flow Morison equation, it can be safely concluded that when one adopts the Independent-Flow Morison equation to calculate the in-line hydrodynamic force acting on a surging cylinder in regular waves, the nonlinear interactions between the moving cylinder and the wave field cannot be neglected. Since two drag coefficients C_{DOL} and C_{DOW} are needed in the Independent-Flow Morison equation, the values of these two coefficients need to be obtained from specially designed experiments; the present experimental results showed that these two drag coefficients are highly dependent on the KC_w number, the reduced velocity U_r and the KC_l numbers. Using the drag coefficients for a fixed cylinder in waves or a cylinder oscillating with a single frequency in still water, would significantly under estimate the drag (damping) force acting on a surging cylinder in waves.

6.6.2 Hydrodynamic coefficients in the Relative-Velocity Morison equation

6.6.2.1 Drag coefficients

For the drag coefficients C_{Dr} in the Relative-Velocity Morison equation, the relative KC number defined by $KC_r = U_{mr} T_r / D$ is found to be an appropriate parameter to present the data; in the definition of KC_r , the velocity U_{mr} is the maximum relative velocity ($U_{mr} = U_{mL} + U_{mW}$) and T_r the relative period between wave motion and surging motion. In the present tests, the relative oscillation period

T_r is roughly taken as the surging period T_{OL} . Figures 6-14~6-16 show the drag coefficients C_{Dr} as functions of KC_r ; it can be seen that C_{Dr} correlated well with the KC_r number. In the following three ranges: (1) $16 \leq KC_r \leq 26$ for $T_{OL} = 4.0s$, (2) $20 \leq KC_r \leq 35$ for $T_{OL} = 6.0s$, and (3) $30 \leq KC_r \leq 40$ for $T_{OL} = 8.0s$, C_{Dr} increased with increasing KC_r number. In these three ranges, the flow velocity is dominated by the wave motion; as a result, the drag coefficient C_{Dr} was strongly influenced by the period T_w and amplitude A_w of the wave motion. In other test ranges examined in the present study, the flow is dominated by the flow induced by the surging motion, and the drag coefficient C_{Dr} was well correlated with KC_r . For reference, Tables 6-3 through 6-8 list the values of U_{mL} / U_{mW} , which is the velocity ratio between the maximum velocity of the surging motion and maximum velocity of the wave motion.

6.6.2.2 Comparisons with other data reported in the literature

The present results of the drag coefficient C_{Dr} are compared with those reported in the literature, which are listed in Table 6-2. It is stressed here that high quality data of the drag coefficients for a cylinder surging in waves are scarce and most of the available data are for a cylinder oscillating in a steady flow, a fixed cylinder in regular waves, or a fixed cylinder in a combined wave-current flow. The values of the drag coefficient listed in Table 6-2 are the values at large reduced velocities (where the drag coefficients are insignificantly affected by the wave motion). The drag coefficient C_{Dr} in the present study is around 0.9~1.1 for large values of KC_r number, which is close to those listed in Table 6-2. Note that for the mean drag

coefficient for a cylinder in a steady current is 1.2 when $300 < Re < 3 \times 10^5$ (see Zdravkovich, 1997; Sumer and Fredsoe, 2006). The present drag coefficient C_{Dr} value of 0.9~1.1 is a little smaller than 1.2.

Table 6-2 Drag coefficients reported in the literature

Reference	Methods	KC_{OH} (or KC_W)	U_r	Re	C_D
Verley and Moe (1979)	oscillating in current	1~35	1~45	$3.0 \times 10^2 \sim 2.4 \times 10^4$	1.0
Koterayama (1984)	oscillating in current	4~100	0~60	$4.0 \times 10^3 \sim 5.0 \times 10^4$	1.0
Koterayama (1984)	wave and current	6~24	0~60	$3.0 \times 10^3 \sim 4.0 \times 10^4$	1.0
Low et al. (1989)	oscillating in current	2.5~15	20~100	$2.7 \times 10^4 \sim 5.4 \times 10^4$	0.9~1.25
Shafiee-far (1997)	oscillating in current	3.1~28.3	1.7~33.3	$1.8 \times 10^4 \sim 1.0 \times 10^5$	0.8~1.0
Shafiee-far (1997)	wave and current	3.1~9.2	3.3~19	$2.0 \times 10^4 \sim 6.0 \times 10^4$	0.8~1.2
Chakrabarti (1997)	wave and current	15~30	~	$1.2 \times 10^4 \sim 4.0 \times 10^4$	1.0
Shafiee-far (1997)	Surging in wave	2.3~9.3	1~36	$1.4 \times 10^4 \sim 6.0 \times 10^4$	0.8~1.0

Compared with the drag coefficients C_{Dr} for a cylinder oscillating with dual-frequency in still water, the drag coefficients C_{Dr} were reduced by the three-dimensionality in the wave field in the small KC_r ranges where wave motion dominates the flow.

Figure 6-14 The drag coefficients in the Relative-Velocity Morison equation for a cylinder oscillating with dual-frequency in still water and a surging cylinder in regular waves.

($T_{OH} = T_W = 1.2s$ and $1.6s$, $T_{OL} = 4.0s$).

Figure 6-15 The drag coefficients in the Relative-Velocity Morison equation for a cylinder oscillating with dual-frequency in still water and a surging cylinder in regular waves.

($T_{OH} = T_W = 1.2s$ and $1.6s$, $T_{OL} = 6.0s$).

Figure 6-16 The drag coefficients in the Relative-Velocity Morison equation for a cylinder oscillating with dual-frequency in still water and a surging cylinder in regular waves.

($T_{OH} = T_W = 1.2s$ and $1.6s$, $T_{OL} = 8.0s$).

Table 6-3 Values of velocity ratio U_{mL}/U_{mW} and the drag coefficients C_{Dr} ($T_w = 1.2s$ and $T_{OL} = 4.0s$).

$A_w = 0.04m, KC_w = 3.3$			
KC_r	Re_r	U_{mL}/U_{mW}	C_{Dr}
18.9	11832.6	0.74658	0.65198
21.9	13715.7	1.04521	0.80492
25.6	16024.2	1.34384	0.95443
28.9	18064.6	1.64248	0.98922
32.2	20103.6	1.94111	0.98878
35.4	22151.9	2.23974	0.99335

Table 6-4 Values of velocity ratio U_{mL}/U_{mW} and the drag coefficients C_{Dr} ($T_w = 1.6s$ and $T_{OL} = 4.0s$).

$A_w = 0.05m, KC_w = 5.5$			
KC_r	Re_r	U_{mL}/U_{mW}	C_{Dr}
21.3	13325.8	0.60539	0.89048
24.6	15399.2	0.84755	1.00956
27.9	17436.0	1.08971	0.99532
29.7	18589.9	1.33187	0.99893
34.1	21297.8	1.57402	0.98537
38.0	23747.0	1.81618	0.99166

Table 6-5 Values of velocity ratio U_{mL}/U_{mW} and the drag coefficients C_{Dr} ($T_W = 1.2s$ and $T_{OL} = 6.0s$).

$A_W = 0.03, 0.04 \text{ and } 0.06m, KC_W = 2.5, 3.3 \text{ and } 4.8$			
KC_r	Re_r	U_{mL}/U_{mW}	C_{Dr}
20.7	8610.5	0.653572	0.414903
22.9	9556.3	0.915001	0.702392
24.9	10375.7	0.486121	0.703397
27.1	11284.8	1.17643	0.795006
28.2	11738.2	0.68057	0.752355
30.4	12681.2	1.437859	0.864122
31.3	13046.0	0.875018	0.811701
32.0	13350.4	0.336839	0.752733
33.3	13889.7	1.069467	0.890813
33.7	14051.1	1.699288	0.980401
35.0	14598.8	0.471575	0.785765
35.6	14812.7	1.263915	0.904339
37.0	15410.3	1.960717	0.987044
39.0	16229.9	0.606311	0.90725
40.1	16712.4	2.222146	1.016998
40.4	16853.3	0.741046	0.906458
41.1	17122.2	1.458364	0.948803
43.5	18133.1	2.483575	1.026978
44.4	18480.7	1.652813	1.012189
45.5	18950.4	0.875782	0.970341
46.8	19493.8	2.745004	0.971965
47.6	19841.6	1.847261	1.036915
48.8	20313.9	1.010518	0.986105
49.0	20405.5	2.04171	0.983291
52.0	21675.0	1.145253	1.002909
55.3	23030.0	1.279989	1.005925
58.6	24397.0	1.414725	0.991379

Table 6-6 Values of velocity ratio U_{mL} / U_{mW} and the drag coefficients C_{Dr} ($T_w = 1.6s$ and $T_{OL} = 6.0s$).

$A_w = 0.03, 0.05 \text{ and } 0.07m, KC_w = 3.5, 5.5 \text{ and } 7.9$			
KC_r	Re_r	U_{mL} / U_{mW}	C_{Dr}
21.2	8823.5	0.627258	0.728366
24.4	10186.9	0.878161	0.886944
27.7	11541.3	1.129064	1.025722
28.8	11998.3	0.395711	0.731718
31.0	12912.0	1.379967	1.096162
32.1	13365.4	0.553995	0.815947
34.2	14261.7	1.63087	1.085704
35.3	14726.8	0.712279	0.922168
37.5	15612.7	1.881773	1.006746
37.7	15726.0	0.276174	1.061061
38.5	16058.4	0.870564	0.974009
40.7	16974.3	2.132676	1.005802
41.0	17075.3	0.386643	1.04113
41.9	17447.9	1.028848	0.968972
44.0	18345.5	2.383579	1.032201
44.1	18382.1	0.497113	1.041994
45.0	18761.1	1.187132	0.981437
47.3	19705.7	2.634482	0.989528
47.5	19801.4	0.607583	1.047351
48.4	20171.8	1.345417	0.961245
50.8	21170.9	0.718052	1.064149
51.7	21527.9	1.503701	0.980714
54.1	22533.4	0.828522	1.04678
54.9	22892.8	1.661985	0.976062
57.3	23894.5	0.938991	1.048596
60.6	25250.7	1.049461	1.023952

Table 6-7 Values of velocity ratio U_{mL}/U_{mW} and the drag coefficients C_{Dr} ($T_w = 1.2s$ and $T_{OL} = 8.0s$).

$A_w = 0.04m, KC_w = 3.3$			
KC_r	Re_r	U_{mL}/U_{mW}	C_{Dr}
30.0	9380.2	0.373289	0.24054
33.3	10400.5	0.522605	0.3982
36.6	11429.5	0.671921	0.58691
39.8	12448.8	0.821237	0.74398
43.1	13465.6	0.970553	0.80703
46.2	14445.9	1.119868	0.84146
49.6	15501.2	1.269184	0.90333
52.8	16514.7	1.4185	0.91156
56.1	17516.2	1.567816	0.93077
59.4	18568.3	1.717131	0.97494
62.6	19574.9	1.866447	0.96746

Table 6-8 Values of velocity ratio U_{mL}/U_{mW} and the drag coefficients C_{Dr} ($T_w = 1.6s$ and $T_{OL} = 8.0s$).

$A_w = 0.05m, KC_w = 5.5$			
KC_r	Re_r	U_{mL}/U_{mW}	C_{Dr}
35.7	11152.0	0.296783	0.69182
39.0	12174.1	0.415496	0.72179
42.2	13192.4	0.53421	0.78098
45.5	14215.9	0.652923	0.83491
48.8	15236.4	0.771636	0.88015
51.4	16067.2	0.890349	0.89266
55.3	17276.9	1.009062	0.90902
56.9	17783.1	1.127776	0.90198
59.6	18619.6	1.246489	0.89885
65.0	20325.9	1.365202	0.92532
65.8	20572.2	1.483915	0.92001

6.6.3 Root-mean-square force coefficients

Again, the root-mean-square in-line force coefficient is defined as

$$C_{F_{rms}} = \frac{F_{rms}}{0.5\rho D U_{rms}^2 L} \quad (6-6)$$

where F_{rms} is the root-mean-square value of the measured in-line force. The root-mean-square in-line force is normalized by the root-mean-square of the combined velocity U_{rms}^2 ; the combined velocity is the resultant velocity between two components: the velocity of the undisturbed wave field and the velocity of the surging cylinder. Figures 6-17 through 6-22 show that the root-mean-square drag coefficients ($C_{F_{rms}}$) as functions of the KC_{OL} number for a cylinder oscillating with dual-frequency in still water as well as a surging cylinder in regular waves. The influence of the high-frequency waves of large amplitudes on $C_{F_{rms}}$ is that when the surging motion is weak, in this case, a smaller KC_{OH} or KC_{OW} number gives large $C_{F_{rms}}$; with the increase of the surging amplitude (A_{OL}), the values of $C_{F_{rms}}$ for larger KC_{OH} or KC_{OW} are larger than those for smaller KC_{OH} or KC_{OW} . This conclusion is supported by all of the tests. Note that for the convenience of comparison, the force due to pressure gradient (Froude-Krylov force) in high-frequency oscillation is included which lacks in a cylinder oscillating with dual-frequency in still water.

Figure 6-17 The root-mean-square coefficients C_{Frms} for a cylinder oscillating with dual-frequency in still water and a surging cylinder in regular waves ($T_{OH} = T_W = 1.2s$ and $T_{OL} = 4.0s$).

Figure 6-18 The root-mean-square coefficients C_{Frms} for a cylinder oscillating with dual-frequency in still water and a surging cylinder in regular waves ($T_{OH} = T_W = 1.6s$ and $T_{OL} = 4.0s$).

Figure 6-19 The root-mean-square coefficients C_{Frms} for a cylinder oscillating with dual-frequency in still water and a surging cylinder in regular waves ($T_{OH} = T_W = 1.2s$ and $T_{OL} = 6.0s$).

Figure 6-20 The root-mean-square coefficients C_{Frms} for a cylinder oscillating with dual-frequency in still water and a surging cylinder in regular waves ($T_{OH} = T_W = 1.6s$ and $T_{OL} = 6.0s$).

Figure 6-21 The root-mean-square coefficients C_{Frms} for a cylinder oscillating with dual-frequency in still water and a surging cylinder in regular waves ($T_{OH} = T_W = 1.2s$ and $T_{OL} = 8.0s$).

Figure 6-22 The root-mean-square coefficients C_{Frms} for a cylinder oscillating with dual-frequency in still water and a surging cylinder in regular waves ($T_{OH} = T_W = 1.6s$ and $T_{OL} = 8.0s$).

6.7 Comments on the Independent- Flow Morison equation versus the Relative-Velocity Morison equation

In Section 6.6, the hydrodynamic coefficients for two forms of Morison equation: the Independent-Flow Morison equation and the Relative-Velocity Morison equation were discussed. In this section, the applicability of these two forms of Morison equation will be investigated for a slowly surging in regular waves. For convenience, the statistic R-square (R^2) as defined in Section 3.8 in Chapter 3 is copied in Equation (6-7) and used to assess the goodness-of-fit the Morison equation to the present data.

$$R^2 = 1 - \frac{\sum (F_{Mi} - F_{Pi})^2}{\sum (F_{Mi} - \bar{F}_M)^2} \approx 1 - \frac{\sum (F_{Mi} - F_{Pi})^2}{\sum (F_{Mi})^2} \quad (6-7)$$

where F_{Mi} is a measured force, F_{Pi} is the corresponding prediction using Morison equation, and the $\bar{F}_M$ is the mean of all the measured forces during a period of time. .

In the present experiments, $\bar{F}_M$ is almost zero.

Figure 6-23 shows the R^2 values against the reduced velocity U_r for the two forms of Morison equation. In general, both the Independent-Flow Morison equation and the Relative-Velocity Morison equation can fit the measured in-line force well, with the R^2 values in the range of 0.9~1.0 for all tests. Figure 6-23 also reveals that the value of R^2 is not only closely related to the reduced velocity U_r , but also the KC_w number. When the reduced velocity U_r is less than 7, the Independent-Flow Morison equation can give better prediction for $KC_w = 2.8 \sim 8.0$. When $U_r > 7$, the Relative-Velocity Morison equation performs slightly better for large values of KC_w number

(e.g. see Figure 6-23 (b), (f), (g) and (h)). However, the difference in the two periods (wave and surging) may also affect the accuracy of the predicted force using Morison equation. To show this, let us compare Figure 6-23 (b) with 6-23(j), the Relative-Velocity Morison equation performs slightly better in the case of Figure 6-23 (b), but the Independent-Flow Morison equation gives more accurate prediction in the case of Figure 6-23 (j). Based on the present results, the difference between the R^2 values (i.e. R^2 of Independent-Flow Morison equation minus R^2 of Relative-Velocity Morison equation) for the two forms of Morison equation is shown in Figure 6-24. In terms of Figure 6-23 and Figure 6-24, within the limited experimental range in this thesis, roughly, it can be seen that when the reduced velocity $U_r > 7$ and $KC_w > 5$, the Relative-Velocity Morison equation predicts the in-line hydrodynamic force slightly better, otherwise, the Independent-Flow Morison equation is preferred.

For reference, Figures 6-25 through 6-30 show examples of the measured and the fitted in-line forces for a surging cylinder in regular waves. These examples serve the purpose of demonstrating the performance of the two forms of Morison equation.

Figure 6-23 Goodness-of- fit (R-square) for a cylinder surging in waves.

(Solid star ★: the Independent-Flow Morison equation; hollow star ☆: the Relative-Velocity Morison equation)

(a)

(b)

Figure 6-24 Distribution of the difference of the R-square values the two forms of Morison equation over the parameter space of U_r and KC_w

((a):3D scatter plot of the R^2 difference, (b): 3D bar plot of R^2 difference)

Figure 6-25 Examples of the measured and the fitted forces for $T_w = 1.2s$ and $T_{OL} = 4.0s$.

($KC_w = 3.3$; (a): $U_r = 2.5$, (b): $U_r = 3.4$, (c): $U_r = 4.4$, (d): $U_r = 5.4$, (e): $U_r = 7.4$)

(Solid line: measured force time series; dot line: fitted by the Relative-Velocity Morison equation; dash-dot line: fitted by the Independent-Flow Morison equation)

Figure 6-26 Examples of the measured and the fitted forces for $T_w = 1.6s$ and $T_{OL} = 4.0s$.

($KC_w = 5.5$; (a): $U_r = 3.3$, (b): $U_r = 4.6$, (c): $U_r = 5.9$, (d): $U_r = 8.5$, (e): $U_r = 9.8$)

(Solid line: measured force time series; dot line: fitted by the Relative-Velocity Morison equation; dash-dot line: fitted by the Independent-Flow Morison equation)

Figure 6-27 Examples of the measured and the fitted forces for $T_w = 1.2s$ and $T_{OL} = 6.0s$.

($KC_w = 3.3$; (a): $U_r = 1.6$, (b): $U_r = 2.9$, (c): $U_r = 4.2$, (d): $U_r = 5.6$, (e): $U_r = 6.9$)

(Solid line: measured force time series; dot line: fitted by the Relative-Velocity Morison equation; dash-dot line: fitted by the Independent-Flow Morison equation)

Figure 6-28 Examples of the measured and the fitted forces for $T_w = 1.6s$ and $T_{OL} = 6.0s$.

($KC_w = 7.9$; (a): $U_r = 2.2$, (b): $U_r = 3.9$, (c): $U_r = 5.7$, (d): $U_r = 7.4$, (e): $U_r = 8.3$)

(Solid line: measured force time series; dot line: fitted by the Relative-Velocity Morison equation; dash-dot line: fitted by the Independent-Flow Morison equation)

Figure 6-29 Examples of the measured and the fitted forces for $T_w = 1.2s$ and $T_{OL} = 8.0s$.

($KC_w = 3.3$; (a): $U_r = 1.2$, (b): $U_r = 2.2$, (c): $U_r = 3.7$, (d): $U_r = 5.1$, (e): $U_r = 6.1$)

(Solid line: measured force time series; dot line: fitted by the Relative-Velocity Morison equation; dash-dot line: fitted by the Independent-Flow Morison equation)

Figure 6-30 Examples of the measured and the fitted forces for $T_w = 1.6s$ and $T_{OL} = 8.0s$.

($KC_w = 5.5$; (a): $U_r = 1.6$, (b): $U_r = 2.9$, (c): $U_r = 4.9$, (d): $U_r = 6.9$, (e): $U_r = 8.2$)

(Solid line: measured force time series; dot line: fitted by the Relative-Velocity Morison equation; dash-dot line: fitted by the Independent-Flow Morison equation)

6.8 Transverse forces acting on a surging cylinder in regular waves

Transverse coefficients for a slowly surging cylinder in regular wave were examined in this section.

The mean transverse coefficients are shown in Figure 6-31 and Figure 6-32 for $T_w = 1.2s$ and $1.6s$ respectively. Generally, the mean transverse coefficients C_{Lmean} for longer surging periods ($T_{OL} = 6.0s$ and $8.0s$) are obviously larger than those for shorter surging period of $T_{OL} = 4.0s$.

Figures 6-33 and 6-34 show the root-mean-root-square transverse coefficients. Compared to results for a cylinder oscillating with dual-frequency in still water (refer to Figures 5-45 and 5-46), the three-dimensionality associated with the wave motion reduced the mean-root-square transverse coefficients.

The maximum transverse coefficients C_{Lmax} are shown in Figures 6-35 and 6-36 for $T_w = 1.2s$ and $1.6s$, respectively. In general, the maximum transverse coefficients are comparable to those for the cylinder oscillating with dual-frequency in still water (see Figures 5-47 and 5-48).

For reference, some examples of transverse forces for the slowly surging cylinder in regular waves are given in Figure 6-37, and the corresponding amplitude spectrums are given in Figure 6-38. Again peaks at the sum-frequency and difference-frequency can also be found; which is consistent with results for the cylinder oscillating with dual-frequency in still water.

Figure 6-31 Mean transverse force coefficients versus KC_{OL} number for $T_w = 1.2s$.
(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

Figure 6-32 Mean transverse force coefficients versus KC_{OL} number for $T_w = 1.6s$.
(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

Figure 6-33 Root-mean-square transverse force coefficients versus KC_{OL} number for $T_w = 1.2s$

(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

Figure 6-34 Root-mean-square transverse force coefficients versus KC_{OL} number for $T_w = 1.6s$

(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

Figure 6-35 Maximum transverse force coefficients versus KC_{OL} number for $T_W = 1.2s$

(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

Figure 6-36 Maximum transverse force coefficients versus KC_{OL} number for $T_W = 1.6s$

(Square: $T_{OL} = 4.0s$, circle: $T_{OL} = 6.0s$, star: $T_{OL} = 8.0s$)

Figure 6-37 Sample transverse forces at different KC_{OL} numbers for $T_w = 1.2s$ and $A_w = 0.06m$ and $T_{OL} = 6.0s$

Figure 6-38 Sample transverse force amplitude spectrums at different KC_{OL} numbers for $T_w = 1.2s$ and $A_w = 0.06m$ and $T_{OL} = 6.0s$

6.9 Summary

A slowly surging cylinder in regular waves was studied in this chapter and the following main conclusions are drawn from the present experimental results:

- 1) For the Independent-Flow Morison equation, the hydrodynamic coefficients C_{AOL} and C_{DOL} would approach to the corresponding values for a cylinder oscillating with a single frequency in still water as long as the flow generated by the surging motion dominates the total flow.
- 2) The hydrodynamic coefficients C_{AOL} and C_{DOL} for the surging component in the Independent-Flow Morison equation are well correlated with the reduced velocity U_r for each KC_w number.
- 3) Compared with a cylinder oscillating with dual-frequency in still water, three-dimensionality in wave field could reduce the drag force.
- 4) The hydrodynamic coefficients for the wave (high-frequency) component in the Independent-Flow Morison equation were remarkably influenced by the large amplitude of the slowly surging motion when the flow is dominated by the slowly surging motion: the slowly surging motion increased the drag coefficients C_{DOW} , but slightly reduced the inertia coefficients C_{MOW} .
- 5) The hydrodynamic coefficients for the wave (high-frequency) component in the Independent-Flow Morison equation are well correlated with the KC_l number for a given value of KC_w . Compared with the cylinder oscillating with dual-

frequency in still water, the 3-dimensional feature of the wave flow reduced the drag coefficients C_{DOW} , but increased the inertial coefficients C_{MOW} .

- 6) The drag coefficient C_{Dr} in the Relative-Velocity Morison equation is generally correlated well with the KC_r number. When the flow is dominated by the wave motion, the drag coefficient C_{Dr} is influenced strongly by the wave motion; when the flow is dominated by the slowly surging motion, the drag coefficient C_{Dr} fall within the range of 0.9 and 1.1.
- 7) Compared to a cylinder oscillating with dual-frequency in still water, the behaviour of the transverse force acting on a surging cylinder in regular waves is similar, except that the three-dimensional feature of the wave motion slightly reduced the values of the root-mean-square transverse coefficients.

Chapter 7

Conclusions and Recommendations

7.1 Summary of main conclusions of this study

This thesis is an attempt toward improving the understanding of the hydrodynamic forces acting on a smooth vertical circular cylinder slowly surging in still water and in regular waves, with an emphasis on the hydrodynamic coefficients in two forms of Morison equation. The experiments were designed to fill the gap that exists in using Morison equation to estimate the hydrodynamic forces acting on a cylinder surging slowly with dual-frequency in still water or in regular waves. In addition to providing hydrodynamic coefficients, the applicability of the Relative-Velocity form of Morison equation and the Independent-Flow form Morison equation were evaluated. In the subsequent sections, main conclusions are summarized below for Chapter 3 through Chapter 6.

7.1.1 Hydrodynamic force of a circular cylinder under oscillatory flow or regular waves

In Chapters 3 and 4, the hydrodynamic forces (in-line and transverse) acting on a vertical circular cylinder oscillating sinusoidally in still water or fixed in regular waves were measured and analyzed. Even though the in-line forces had been studied extensively in the past decades, we believe the results in this thesis can contribute to further understanding hydrodynamics around cylindrical structures; the work on transverse forces is original since few such data were reported in the literature. Main conclusions drawn from these two chapters were summarized below:

- **Influence of frequency number β on the hydrodynamic coefficients**

The present experimental results showed that the frequency number β had significant influences on both the in-line and transverse force coefficients. For the in-line component, the drag coefficients were much sensitive to the β number; but the added mass (or inertia) force coefficients were relatively not sensitive to the β number. For the transverse component, the magnitudes of mean, maximum, RMS and harmonic coefficients should be dependent on both KC number and β number, rather than on KC number alone.

- **Three-dimensional effects in waves on the hydrodynamic coefficients**

Comparisons between a cylinder oscillating sinusoidally in still water and the same cylinder fixed in regular waves showed that three-dimensional effects in waves could reduce the drag force and increased inertia force; the various transverse force coefficients for a fixed cylinder in regular waves were slightly smaller than those for the same cylinder oscillating in still water in the same flow conditions.

- **Drag coefficients for flows in the drag-dominated regime**

In the drag-dominated regime (about $KC > 25$), the present drag coefficients for a smooth circular cylinder in oscillatory flows tended to the mean drag coefficient value of 1.2 for a smooth circular cylinder in steady flows within the subcritical range ($300 < Re < 3 \times 10^5$).

- **Joint distribution of the in-line and transverse components**

The present results showed that the transverse force could be comparable to or even larger than the corresponding in-line force for a circular cylinder oscillating sinusoidally in still water. The transverse forces at some KC numbers could be twice as large as the corresponding in-line forces. For a vertical circular cylinder fixed in regular waves, the maximum transverse force could be of the same magnitude as that of the in-line force.

7.1.2 Hydrodynamic force on a circular cylinder oscillating with dual-frequency in still water or in-line oscillating sinusoidally in regular waves

Chapters 5 and 6 presented the experimental results of hydrodynamic force (and related coefficients) for a circular cylinder oscillating with dual-frequency in still water or in-line oscillating sinusoidally (surging) in regular waves. Systematic experimental investigations of hydrodynamic forces acting on a circular cylinder slowly surging in waves are rare. The present study, with all of the experimental data and findings presented in Chapters 5 and 6, filled the gaps that exist in the literature in the following areas:

- **Appropriateness of the Independent-Flow form of Morison equation**

The wave-frequency motion and the low-frequency motion would mutually affect each other significantly. When one adopts the Independent-Flow Morison equation to calculate the in-line hydrodynamic force acting on a surging cylinder in regular waves, the nonlinear interactions between the moving cylinder and the wave field should not be neglected. The force coefficients need to be obtained from specially designed experiments. Using the drag coefficients obtained from tests involving a

single frequency (wave only or oscillating in still water only) would significantly under-estimate the damping force.

- **Hydrodynamic coefficients in Independent-Flow form of Morison equation**

The measured hydrodynamic coefficients for the low-frequency flow were well correlated with the reduced velocity U_r and these coefficients consistently varied with the KC_w number. At large values of the reduced velocity and small values of KC_w , the hydrodynamic coefficients for the low-frequency flow could approach to the values for a cylinder oscillating sinusoidally in still water. The measured hydrodynamic coefficients for the wave-frequency flow were well correlated with the local KC_l number and these coefficients consistently varied with the KC_w number. Especially, the drag coefficient for wave-frequency flow increased with reducing KC_w .

- **The drag coefficients in the Relative-Velocity form of Morison equation**

In Relative-Velocity form of Morison equation, the present results showed that the drag coefficients C_{Dr} were generally correlated well with KC_r number. In the wave-dominated flow regime, the drag coefficient C_{Dr} was found to be noticeably influenced by the wave motion. In the slow oscillation-dominated flow regime, the drag coefficient C_{Dr} was found to fall in between 0.9 and 1.1. The present drag coefficient C_{Dr} is consistent with the drag coefficient reported in the literature for a circular cylinder oscillating in a steady flow (or a fixed vertical circular in wave-current flow) at large values of reduced velocity.

- **The Independent-Flow form of Morison equation and the Relative-Velocity form of Morison equation: applicability**

By examining the goodness-of-fit using the R^2 values for the Independent-Flow form of Morison equation and the Relative-Velocity form of Morison equation, roughly, within the test range in this thesis, it was found that when the reduced velocity $U_r > 7$ and $KC_w > 5$, the Relative-Velocity form of Morison equation was preferable, otherwise, the Independent-Flow form of Morison equation.

- **Characteristics of transverse forces**

Due to the nonlinear interactions between the vortices shed by the flow with two time scales, a significant portion of the transverse force was found to be distributed at the sum- and difference- frequencies of the high (or wave)-frequency and low-frequency.

7.2 Recommendations for future work

The present experimental apparatus, including instrumented cylindrical force model and model-driving system, as well as the integrated Laview Main program for data acquisition and synchronization, could give us highly stable, repeatable and reliable data. Further improvement can focus on extending the capability of the present experimental apparatus for exploration of new promising ideas focusing on viscous damping force on offshore elements in complex flow fields.

7.2.1 Higher Reynolds numbers

The highest Reynolds number in this thesis is less than 30,000. The Reynolds numbers are well in the range of subcritical regime, and not sufficient for most of practical applications. In offshore engineering, the Reynolds numbers for flow around cylinder members are around $10^5 \sim 10^6$ or even higher (i.e. in the critical or supercritical regime). However, it is not clear at the moment how to assess the scale/ (or Reynolds number) effects.

In the present investigation of a circular cylinder surging slowly in regular waves, the highest Reynolds number was restrained by the model diameter, the capacities of the wave maker and the actuator. In the future, it is recommended to increase the Reynolds number up to about 10^5 by increasing the model diameter from 5 cm to 15 cm (wave flume width should be at least 150 cm wide) and using a new actuator to provide an oscillation speed up to 1m/s.

7.2.2 Flow visualization

For a better understating of the generation mechanism of low-frequency force, flow visualization using Particle Image Velocimetry (PIV) is recommended to provide information on the details of vortex shedding process under different flow conditions so as to associate the hydrodynamic forces with vortex dynamics.

7.2.3 Hydrodynamic forces on an elastically mounted cylinder in waves

This phenomenon of “lock-in” between the frequency of vortex shedding and the frequency of the vibrating cylinder in a current flow has received much attention in the past. A similar phenomenon may also occur under certain conditions for an

elastically mounted cylinder in waves. The understanding of the fluid-structure interactions for a flexibly mounted cylinder in waves is still in its infancy due to the complexity of the driving forces and the response processes. In waves, the transverse force caused by vortex shedding may also activate the resonant response if the vortex shedding frequency approaches the natural frequencies of these structures. Research of hydrodynamic forces and responses of an elastically mounted vertical cylinder in waves is scarce, and extensive experimental investigation in this research area is therefore recommended.

7.2.4 Combined surging and heaving motion for pontoons

In Chapter 5 and Chapter 6, we reported the hydrodynamic loading on a vertical circular cylinder in flows with two frequencies. In offshore practice, in addition to viscous damping force due to flow separation around vertical cylinder members, viscous damping forces acting on the lower-hulls such as pontoons of semi-submersible or TLPs are equally important in predicting response of moored floating structures. For floating structures, the surging and heaving motions are often coupled. Few data are available for the force coefficients of pontoons under combined surging and heaving motions in waves. Experimental study of hydrodynamic forces acting on a pontoon undergoing a combined surging and heaving motion in waves is a promising research topic.

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Appendix-A

Derivation of Equation (3-5)

For a cylinder in an oscillatory flow, Morison equation can be written as

$$F_x = \rho \frac{\pi D^2}{4} C_M \dot{u} ds + \frac{1}{2} \rho D C_D |u| u ds \quad (\text{A-1})$$

After assuming $u = U_m \cos \omega t$, $\dot{u} = -\omega U_m \sin(\omega t)$, substituting them into equation (A-1), and integrating F_x^2 within one period, the following equation is obtained

$$\begin{aligned} \int_0^T F_x^2 dt &= \int_0^T \left(\rho \frac{\pi D^2}{4} C_M \dot{u} ds + \frac{1}{2} \rho D C_D |u| u ds \right)^2 dt \\ &= \int_0^T \left(-\rho \frac{\pi D^2}{4} C_M \omega U_m \sin \omega t ds + \frac{1}{2} \rho D C_D |U_m \cos \omega t| U_m \cos \omega t ds \right)^2 dt \end{aligned} \quad (\text{A-2})$$

The Root-mean-square coefficient of the in-line force $C_{F_{rms}}$, is defined as

$$C_{F_{rms}} = \frac{1}{0.5 \rho U_m^2 D ds} \left(\frac{1}{T} \int_0^T F_x^2 dt \right)^{\frac{1}{2}} \quad (\text{A-3})$$

After carrying out the integration and using $KC = \frac{U_m T}{D} = \frac{2\pi U_m}{D\omega}$, it gives

$$C_{F_{rms}} = \frac{1}{0.5 \rho U_m^2 D ds} \left(\frac{\pi^4 C_M^2 \rho^2 U_m^4 D^2 ds^2}{8 KC^2} + \frac{3 C_D^2 \rho^2 U_m^4 D^2 ds^2}{32} \right)^{\frac{1}{2}} = \left(\frac{\pi^4 C_M^2}{2 KC^2} + \frac{3 C_D^2}{8} \right)^{\frac{1}{2}} \quad (\text{A-4})$$

Appendix-B

Least Square Analysis in Calculating Force Coefficients

The hydrodynamic coefficients in force models can be evaluated by a best fit to the measured force time series in the least square sense (Least Square Technique). In this method, hydrodynamic coefficients are obtained by minimizing the following error function (below, we take the Independent-Flow Morison model for a cylinder oscillating with dual-frequency in still water for example),

$$E^2 = \sum_{i=1}^N (F_{Mi}(t) - C_{DOL}F_{DLi}(t) - C_{AOL}F_{ILi}(t) - C_{DOH}F_{DHi}(t) - C_{AOL}F_{IHl}(t))^2 \quad (B-1)$$

where $F_M(t)$ is the measured hydrodynamic force time series and N is the total number of data points at an equal interval, Δt , within the whole computational range. Therefore, it is inherently assumed that the hydrodynamics coefficients are constant over one cycle. In addition, in Equation (B-1), $F_{DL}(t) = -(1/2)\rho D\dot{x}_L|\dot{x}_L|ds$ is the low-frequency drag force acting on the force ring unit; $F_{IL}(t) = -(1/4)\rho\pi D^2\ddot{x}_Lds$ is the low-frequency added mass force acting on the force ring unit; $F_{DH}(t) = -(1/2)\rho D\dot{x}_H|\dot{x}_H|ds$ is the high-frequency drag force acting on the force ring unit; $F_{IH}(t) = -(1/4)\rho\pi D^2\ddot{x}_Hds$ is the high-frequency added mass force acting on the force ring unit.

The error function E^2 is then to be minimized, that is, the value of the square of the calculated and measured force difference is to be the least of all possible values. This minimum is found by setting the total derivative of the error function E^2 equal to zero. This leads to

$$\left. \begin{aligned} \frac{\partial E^2}{\partial C_{DOL}} &= 0 \\ \frac{\partial E^2}{\partial C_{AOL}} &= 0 \\ \frac{\partial E^2}{\partial C_{AOL}} &= 0 \\ \frac{\partial E^2}{\partial C_{AOH}} &= 0 \end{aligned} \right\} \quad (B-2)$$

The resulting equations are

$$\left. \begin{aligned} &C_{DOL} \sum_{i=1}^N F_{DLi}^2(t) + C_{AOL} \sum_{i=1}^N F_{ILi}(t)F_{DLi}(t) + C_{DOH} \sum_{i=1}^N F_{DHi}(t)F_{DLi}(t) + C_{AOL} \sum_{i=1}^N F_{IHi}(t)F_{DLi}(t) \\ &= \sum_{i=1}^N F_{Mi}(t)F_{DLi}(t) \\ &C_{DOL} \sum_{i=1}^N F_{DLi}(t)F_{ILi}(t) + C_{AOL} \sum_{i=1}^N F_{ILi}^2(t) + C_{DOH} \sum_{i=1}^N F_{DHi}(t)F_{ILi}(t) + C_{AOL} \sum_{i=1}^N F_{IHi}(t)F_{ILi}(t) \\ &= \sum_{i=1}^N F_{Mi}(t)F_{ILi}(t) \\ &C_{DOL} \sum_{i=1}^N F_{DLi}(t)F_{DHi}(t) + C_{AOL} \sum_{i=1}^N F_{ILi}(t)F_{DHi}(t) + C_{DOH} \sum_{i=1}^N F_{DHi}^2(t) + C_{AOL} \sum_{i=1}^N F_{IHi}(t)F_{DHi}(t) \\ &= \sum_{i=1}^N F_{Mi}(t)F_{DHi}(t) \\ &C_{DOL} \sum_{i=1}^N F_{DLi}(t)F_{IHi}(t) + C_{AOL} \sum_{i=1}^N F_{ILi}(t)F_{IHi}(t) + C_{DOH} \sum_{i=1}^N F_{DHi}(t)F_{IHi}(t) + C_{AOL} \sum_{i=1}^N F_{IHi}^2(t) \\ &= \sum_{i=1}^N F_{Mi}(t)F_{IHi}(t) \end{aligned} \right\} \quad (B-3)$$

In expanded matrix rotation, this set of Equation (B-3) becomes

$$\begin{bmatrix}
\sum_{i=1}^N F_{DLi}^2(t) & \sum_{i=1}^N F_{ILi}(t)F_{DLi}(t) & \sum_{i=1}^N F_{DHi}(t)F_{DLi}(t) & \sum_{i=1}^N F_{IHi}(t)F_{DLi}(t) \\
\sum_{i=1}^N F_{DLi}(t)F_{ILi}(t) & \sum_{i=1}^N F_{ILi}^2(t) & \sum_{i=1}^N F_{DHi}(t)F_{ILi}(t) & \sum_{i=1}^N F_{IHi}(t)F_{ILi}(t) \\
\sum_{i=1}^N F_{DLi}(t)F_{DHi}(t) & \sum_{i=1}^N F_{ILi}(t)F_{DHi}(t) & \sum_{i=1}^N F_{DHi}^2(t) & \sum_{i=1}^N F_{IHi}(t)F_{DHi}(t) \\
\sum_{i=1}^N F_{DLi}(t)F_{IHi}(t) & \sum_{i=1}^N F_{ILi}(t)F_{IHi}(t) & \sum_{i=1}^N F_{DHi}(t)F_{IHi}(t) & \sum_{i=1}^N F_{IHi}^2(t)
\end{bmatrix}
\begin{bmatrix}
C_{DOL} \\
C_{AOL} \\
C_{DOH} \\
C_{AOH}
\end{bmatrix}$$

$$= \begin{bmatrix}
\sum_{i=1}^N F_{Mi}(t)F_{DLi}(t) \\
\sum_{i=1}^N F_{Mi}(t)F_{ILi}(t) \\
\sum_{i=1}^N F_{Mi}(t)F_{DHi}(t) \\
\sum_{i=1}^N F_{Mi}(t)F_{IHi}(t)
\end{bmatrix} \quad (B-4)$$

In matrix notation, this becomes

$$[F_{D-I}][C] = [F_{M-D-I}] \quad (B-5)$$

$[F_{D-I}]$, $[C]$ and $[F_{M-D-I}]$ represent the matrices shown in the expanded form (B-4).

The solution to Equation (B-5) for the coefficients is

$$[C] = [F_{D-I}]^{-1} [F_{M-D-I}] \quad (B-6)$$

This procedure can be implemented in Matlab. Similarly, the hydrodynamic coefficients of the Relative-Velocity Morison equation (three terms) and two terms Morison equation described in Chapter 3 can be obtained in the same way. Various Matlab program were developed for the coefficients calculations in different force models in this thesis.

Appendix-C

Fourier series Analysis in Calculating Force Coefficients

We also take the Independent-Flow Morison equation for a cylinder oscillating with dual-frequency in still water for example, in which the in-line hydrodynamic force is written as

$$\begin{aligned} F_x &= F_{xL} + F_{xH} \\ &= -\frac{1}{2}\rho DC_{DOL}\dot{x}_L|\dot{x}_L|ds - C_{AOL}\rho\frac{\pi D^2}{4}\ddot{x}_Lds - \frac{1}{2}\rho DC_{DOH}\dot{x}_H|\dot{x}_H|ds - C_{AOH}\rho\frac{\pi}{4}D^2\ddot{x}_Hds \end{aligned} \quad (C-1)$$

For each low- and high-frequency motion component, the motion profile is specified as

$$\left. \begin{aligned} x_L(t) &= A_{OL} \sin(\omega_{OL}t) \\ x_H(t) &= A_{OH} \sin(\omega_{OH}t) \end{aligned} \right\} \quad (C-2)$$

The respective motion velocity and acceleration is then expressed as

$$\left. \begin{aligned} \dot{x}_L(t) &= A_{OL}\omega_{OL} \cos(\omega_{OL}t) \\ \ddot{x}_L(t) &= -A_{OL}\omega_{OL}^2 \sin(\omega_{OL}t) \\ \dot{x}_H(t) &= A_{OH}\omega_{OH} \cos(\omega_{OH}t) \\ \ddot{x}_H(t) &= -A_{OH}\omega_{OH}^2 \sin(\omega_{OH}t) \end{aligned} \right\} \quad (C-3)$$

Substitute the velocity and acceleration in expressions (C-3) into the Equation (C-1), the Independent-Flow Morison Equation is expressed as

$$\begin{aligned}
F_x &= F_{xL} + F_{xH} \\
&= -C_{DOL} \frac{1}{2} \rho D ds A_{OL}^2 \omega_{OL}^2 \cos(\omega_{OL} t) |\cos(\omega_{OL} t)| + C_{AOL} \rho \frac{\pi D^2}{4} ds A_{OL} \omega_{OL}^2 \sin(\omega_{OL} t) \\
&\quad - C_{DOH} \frac{1}{2} \rho D ds A_{OH}^2 \omega_{OH}^2 \cos(\omega_{OH} t) |\cos(\omega_{OH} t)| + C_{AOH} \rho \frac{\pi D^2}{4} ds A_{OH} \omega_{OH}^2 \sin(\omega_{OH} t)
\end{aligned} \tag{C-4}$$

Both the low- and high-frequency drag forces are nonlinear due to the square of the velocity; the term $\cos(\omega t) |\cos(\omega t)|$ can be developed into a series

$$\begin{aligned}
\cos(\omega t) |\cos(\omega t)| &= \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 8 \cos(2n+1)\omega t}{\pi(2n+1)[4 - (2n+1)^2]} \\
&= \frac{8}{3\pi} \cos(\omega t) + \frac{8}{15\pi} \cos(3\omega t) - \frac{8}{105\pi} \cos(5\omega t) + \dots
\end{aligned} \tag{C-5}$$

Then, the drag force of low-frequency and high-frequency can be expressed as

$$\begin{aligned}
F_{DL} &= -C_{DOL} \frac{1}{2} \rho D ds A_{OL}^2 \omega_{OL}^2 \cos(\omega_{OL} t) |\cos(\omega_{OL} t)| \\
&= -C_{DOL} \frac{1}{2} \rho D ds A_{OL}^2 \omega_{OL}^2 \left(\frac{8}{3\pi} \cos(\omega_{OL} t) + \frac{8}{15\pi} \cos(3\omega_{OL} t) - \frac{8}{105\pi} \cos(5\omega_{OL} t) + \dots \right)
\end{aligned} \tag{C-6}$$

$$\begin{aligned}
F_{DH} &= -C_{DOH} \frac{1}{2} \rho D ds A_{OH}^2 \omega_{OH}^2 \cos(\omega_{OH} t) |\cos(\omega_{OH} t)| \\
&= -C_{DOH} \frac{1}{2} \rho D ds A_{OH}^2 \omega_{OH}^2 \left(\frac{8}{3\pi} \cos(\omega_{OH} t) + \frac{8}{15\pi} \cos(3\omega_{OH} t) - \frac{8}{105\pi} \cos(5\omega_{OH} t) + \dots \right)
\end{aligned} \tag{C-7}$$

And the added mass forces are written as

$$F_{IL} = C_{AOL} \rho \frac{\pi D^2}{4} ds A_{OL} \omega_{OL}^2 \sin(\omega_{OL} t) \tag{C-8}$$

$$F_{IH} = C_{AOH} \rho \frac{\pi D^2}{4} ds A_{OH} \omega_{OH}^2 \sin(\omega_{OH} t) \tag{C-9}$$

Through FFT analysis, the measured in-line force time series $F_M(t)$ is found to be a periodic signal with two fundamental frequencies ω_{OL} and ω_{OH} and can be represented in a Fourier series in

$$F_M(t) = a_{0L} + a_{1L} \cos(\omega_{OL}t) + b_{1L} \sin(\omega_{OL}t) + a_{2L} \cos(2\omega_{OL}t) + b_{2L} \sin(2\omega_{OL}t) + \dots + a_{0H} + a_{1H} \cos(\omega_{OH}t) + b_{1H} \sin(\omega_{OH}t) + a_{2H} \cos(2\omega_{OH}t) + b_{2H} \sin(2\omega_{OH}t) + \dots \quad (C-10)$$

The expressions for the Fourier coefficients are

$$\left. \begin{aligned} a_0 &= \frac{1}{T} \int_{t_0}^{t_0+T} F_M(t) dt \\ a_n &= \frac{2}{T} \int_{t_0}^{t_0+T} F_M(t) \cos(n\omega t) dt \\ b_n &= \frac{2}{T} \int_{t_0}^{t_0+T} F_M(t) \sin(n\omega t) dt \end{aligned} \right\} \quad (C-11)$$

From numerical analysis, we found the coefficients of sine terms, except for the first term, are much smaller (in fact, only the added mass force contributes to the first sine term, no other sources contribute to higher order sine terms, the coefficients for sine terms higher than first harmonic obtained from Fourier analysis are due to some experimental errors). Besides, the coefficients for even cosine terms are also very smaller, compared to the odd terms with the same order.

In Morison equation, the drag force is nonlinear due to the square of velocity. In the analysis of the dynamic response of an offshore structure, a linearized Morison equation is often desirable to reduce computational time and therefore the linearized drag coefficient is needed. According to Fourier analysis, we can find the first order drag force dominates the drag force and the magnitude of the first order is

much greater than that of higher order terms; therefore, in Equations (C-6) and (C-7), the higher order drag force terms are then ignored, and the linearized drag coefficients for low- and high-frequency components are obtained. Therefore, by comparing Equations (C-6)~(C-9) to the Fourier expansion of $F_M(t)$ in Equation (C-10), the expressions of added mass and linearized drag coefficients from Fourier expansion are given below

$$\left. \begin{aligned} C_{AOL} &= \frac{b_{1L}}{\rho \frac{\pi D^2}{4} ds A_{OL} \omega_{OL}^2} \\ C_{DOL} &= -\frac{a_{1L}}{\rho D ds A_{OL}^2 \omega_{OL}^2 \frac{4}{3\pi}} \\ C_{AOH} &= \frac{b_{1H}}{\rho \frac{\pi D^2}{4} ds A_{OH} \omega_{OH}^2} \\ C_{DOH} &= -\frac{a_{1H}}{\rho D ds A_{OH}^2 \omega_{OH}^2 \frac{4}{3\pi}} \end{aligned} \right\} \quad (C-12)$$

where in Equation (C-12), the C_{DOL} and C_{DOH} are linearized drag coefficients for low-and high- frequency components in Independent-Flow Morison equation.

Similarly, in Relative-Velocity Morison equation of a cylinder oscillating with dual-frequency in still water, the drag force associated with relative velocity can be linearized as (not limited to this technique of linearization)

$$F_D = -\frac{1}{2} \rho D C_{Dr} (\dot{x}_L + \dot{x}_H) |\dot{x}_L + \dot{x}_H| ds \approx -\frac{1}{2} \rho D C_{Drl} \frac{8}{3\pi} (\dot{x}_L + \dot{x}_H) (U_{mL} + U_{mH}) ds \quad (C-13)$$

where C_{Drl} is the linearized drag coefficient, U_{mL} and U_{mH} are the low- and high-frequency oscillation velocity amplitude. However, this linearized drag force is still proportional to the total velocity which contains velocity components from two frequencies; therefore, it is not so straightforward to derive the drag coefficient in Relative-Velocity Morison equation by Fourier series method as in Independent-Flow form. But for the inertia associated coefficients, such as C_{AOL} and C_{AOH} , for a given measured force time series from a combined fast–slow oscillation test, the expressions in Formula (C-12) are applicable to both the Relative-Velocity Morison equation (5-2) and Independent-Flow Morison equation (5-4) since in these two force models, the added mass forces are represented in the same ways.

From the discussions above, we can see the Fourier series method can be applied to derive the acceleration related added mass coefficients and linearized drag coefficients for both the low- and high-frequency components in the Independent-Flow Morison equation; but it is not so straightforward to obtain the linearized drag coefficient in the Relative -Velocity Morison equation. In addition, from Fourier analysis, we can find for a given measured force time series from a combined fast–slow oscillation test, the added mass coefficients (either for low- or high-frequency component) are the same in the two force models (5-2) and (5-4), because the added mass forces are treated in the same ways, i.e. the added mass forces are only related to their respective first sine terms of low- and high-frequency in a Fourier expansion (e.g. see Equation C-10).